# Growth, Evolution and Scaling in Transport Networks

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## *Intellectual Property and Publication Statements*

The candidate confirms that the work submitted is his/her own, except where work which has formed part of jointly-authored publications has been included. The contribution of the candidate and the other authors to this work has been explicitly indicated below. The candidate confirms that appropriate credit has been given within the thesis where reference has been made to the work of others.

A jointly authored paper is related to this thesis:

Huang J; Connors RD; Maher M (2014) Modelling Network Growth with Scaling Laws in a Linear Monocentric City, *Transportation Research Record: Journal of the Transportation Research Board*, pp.134-143.

Huang J; Levinson ML. Circuity in Urban Transit Networks, *Journal of Transport Geography* (Under review).

The first paper contains major materials from Chapter 3 and major materials in Chapter 5 come from the second paper. The major contributions to these two papers regarding to major ideas and all writing parts were made by the author of this thesis. Other co-authors provided constructive comments and suggestions for revision and helped on polishing the paper respectively.

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# Abstract

Under urbanisation, transport infrastructures may be improved when urban population grows. Meanwhile, land use patterns may vary and this urban dynamics may drive variations in mode choice of commuters and spatial features of transport networks. Empirical studies have observed scaling laws between the amount of transport infrastructures and city sizes. This thesis is aiming to provide a modelling framework for the analytical investigation of network growth and present some empirical observations of the variation in spatial features of transport networks.

First, a simple linear monocentric city model is formulated and the global performance of transport systems is derived. Two cases according to strategies of urban intensification and sprawl have been studied to examine the consequence of the scaling-law growth in transport infrastructures.

Second, this thesis proposes a modelling framework. The framework includes two congestible modes, the scaling-law growth of transport infrastructures and housing allocation of residents so that phenomena under urban dynamics could be modelled. The experiments show that the proposed modelling framework could investigate the trade-off of investment on the highway and public transport system.

Third, empirical observations of spatial features in transport networks are reported in this thesis. The thesis measures circuity of transport networks, because this indicator could examine how aggregate transport networks are and the efficiency of network structures. Then research methods that can deal with several data sources are developed. The empirical observation shows that there is an exponential decay between the circuity and travel time in public transport networks. Meanwhile, this thesis also presents that the average circuity in road networks is less than that in public transport networks for the same sample of trips, which to some extent show the difference of spatial features between road and public transport networks. Additionally, correlations between circuity, accessibility and mode share are analysed.

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| Notations | |
|---|---|
| $b_i$ | Betweenness centrality for node $i$ |
| $\delta_{st}$ | The number of the shortest paths going from node $s$ to node $t$ |
| $\delta_{st}^i$ | The number of the shortest paths going from $s$ to $t$ via node $i$ |
| $b_e$ | Betweenness centrality for an edge $e$ |
| $\delta_{st}^e$ | The number of the shortest paths going from $s$ to $t$ via edge $e$ |
| $C_{ij}$ | The circuity between OD pair $i$ and $j$ |
| $D_{ij}^n$ | The network distance between $s$ and $t$ a |
| $D_{ij}^e$ | The Euclidean distance between $s$ and $t$ |
| $C_i$ | Circuity for a single node $i$ |
| $Q$ | The total number of nodes in the network |
| $x$ | The distance from that location $x$ to city centre |
| $\bar{x}$ | The location of equilibrium point along the corridor |
| $l$ | The location of city boundary |
| $N$ | Total demand or total population in a city |
| $n(x)$ | The demand density at location $x$ |
| $n_C(x)$ | The demand density at location $x$ choosing car |
| $n_T(x)$ | The demand density at location $x$ choosing train |
| $p_C(x)$ | The proportion of demand density choosing car at location $x$ |
| $p_T(x)$ | The proportion of demand density choosing train at location $x$ |
| $v_C(x)$ | The cumulative traffic volume by car at location $x$ |
| $v_T(x)$ | The cumulative traffic volume by train at location $x$ |
| $G_T(x)$ | Generalised travel time by train from location $x$ |
| $G_C(x)$ | Generalised travel time by car from location $x$ |
| $t_T$ | Travel time per unit distance by train |
| $f_T$ | Travel time per unit distance by train |
| $f_C$ | Free flow travel time by car per unit distance |
| $C$ | Road capacity per unit distance |
| $A$ | A constant in BPR function |
| $B$ | A constant in BPR function |
| $\theta$ | A constant denotes $\sqrt[B]{(f_T/f_C-1)}/A$ |
| $TTT$ | Total Travel Time |
| $MTT$ | Mean Travel Time |
| $ATS$ | Average Travel Speed |

| | |
|---|---|
| $\rho_0$ | A constant in scaling-law function |
| $\beta$ | Scaling exponent in scaling-law function |
| $N_T$ | The number of commuters using train |
| $C_T$ | Capacity constraint on the train |
| $C_{max}$ | Capacity constraint on the highway |
| $N_0$ | The initial demand level |
| $l_0$ | The initial city boundary |
| $k$ | A constant denotes $C\theta/N_0$ |
| $r$ | Demand ratio, i.e. $N/N_0$ |
| $d_0$ | Demand density at city centre |
| $\mu$ | A constant to describe how fast the density decays as $x$ |
| $M$ | A constant denotes $\sqrt[B]{(f_T/f_C - 1)/A} \cdot (\mu C/d_0) + e^{-\mu l}$ |
| $TLC$ | Total-Length-Capacity |
| $z$ | The amount of daily income on the consumption of |
| $g$ | House space |
| $U(z, \mathrm{g})$ | Utility under a certain $z$ and the house space $\mathrm{g}$ |
| $\alpha$ | Parameter in the utility function |
| $\lambda$ | Parameter in the utility function |
| $\gamma$ | Parameter in the utility function |
| $Y$ | the daily income per capita |
| $R(x)$ | Rent price per unit area at location $x$ |
| $T_m(x)$ | Monetary travel cost from location $x$ to city centre |
| $\Psi(x, u)$ | Bid rent per unit area at location $x$ under the utility level $u$ |
| $L(x)$ | Urban area on residential blocks at location $x$ |
| $\pi$ | The ratio of urban area on residential blocks |
| $V$ | The nominal width of city |
| $F$ | A positive constant denoting the monetary cost per unit distance |
| $u^*$ | Equilibrium utility |
| $H$ | A constant in the derivation |
| $R_0$ | Rent price at city centre |
| $S_T$ | Average operated speed of train |
| $w_T$ | average waiting time by train |
| $\rho_1$ | A constant in the function of waiting time by train |
| $h$ | Train headway |
| $K$ | A constant in the function of discomfort cost by train |

| | |
|---|---|
| $\rho_2$ | A constant in the function of discomfort cost by train |
| $\tau$ | Value of time |
| $T_t(x)$ | Travel time from location $x$ to city centre (after mode choices) |
| $T_l$ | Leisure time per day |
| $T_D$ | Hours per day |
| $T_{fw}$ | The fixed working hours per day |
| $T_{ew}$ | The extra working hours per day |
| $W$ | Wage of an extra working hour |
| $H'$ | A constant in the derivation |
| $C_{ave}$ | The average circuity of studied trips |
| $P$ | The total number of studied trips |
| $D_{ij}^{Transit}$ | Network distance by transit |
| $D_{ij}^{Auto}$ | Network distance by Auto |
| $C_{Transit}$ | The average transit circuity of studied trips |
| $C_{Auto}$ | The average road circuity of studied trips |
| σ | The standard deviation |
| $C_t$ | The average circuity of studied trips in travel time interval $t$ |
| $\beta_1$ | The parameter to be estimated in regression |
| $\beta_2$ | The parameter to be estimated in regression |
| $a_w$ | Weighted accessibility |
| $a_t$ | Accessibility value within time threshold $t$ |
| $a_{Auto}$ | Weighted accessibility by auto |
| $a_{Transit}$ | Weighted accessibility by transit |
| $M_{Auto}$ | Mode share by auto |
| $\alpha_0$ | A parameter to be estimated in regression |
| $\alpha_1$ | A parameter to be estimated in regression |
| $\alpha_1$ | A parameter to be estimated in regression |
| $\hat{a}_{Transit}$ | Weighted accessibility by transit from a regression model |

# Chapter 1
# Introduction

Chapter 1 contains an introduction to the thesis. The chapter begins by introducing the theme discussed within the thesis (Section 1.1).The research questions of the thesis are identified in Section 1.2, followed by the research objectives (Section 1.3). The chapter then concludes with the organisation of the thesis (Section 1.4).

## 1.1 Background

As cities develop, urban populations are increasing and urban areas are expanding. It is reported that 54 per cent of the world's population was residing in urban areas in 2014 and this proportion is predicted to increase to 66 percent by 2050 (United Nations, 2014). There are various advantages for urban agglomeration, such as the emergence of clusters of firms and the aggregation of labour. The benefits that firms obtain by locating together are that they attract more customers and decrease costs on the delivery of goods etc. Workers prefer living in cities in order to save commuting time and find residential locations that are accessible to firms and locations of social activities. As people move into cities, many public services and facilities (such as hospitals, schools) exhibit economies of scale. However, diseconomies of agglomeration arise simultaneously, such as high land rents, environmental pollution, the crowding of residential space and congestion in transport systems (Richardson, 1995; Downs, 1992). With the growth of the urban population, power-law relationships (i.e. scaling laws) were observed when studies (Bettencourt et al., 2007; Kühnert et al., 2006) investigated how urban characteristics (such as GDP, wage, the number of petrol stations) scaled with population sizes. Hence, urban characteristics with economies/diseconomies of scale increase with population sizes disproportionally, which to some extent underpins the growth and evolution of cities.

Under the evolution of cities, a rise in population means that more people may travel on transport systems. Taking driving (one major mode) as an example, the population growth may increase car ownership and the total number of trips taken. It is reported that by 2040 road traffic is forecast to be 46% higher than in 2010 (Report of Road Transport Forecasts by Department for Transport, 2013). This evidence implies an increase in congestion (measured as travel time) of about 114%. Indeed, the increasing number of travellers may affect the travel time of a journey on the road network or crowding on public transport. The congestion arising under increasing demand is worth discussing, because not only transport networks grow and evolve but also urban populations increase. Moreover, the performance of transport systems may be determined by how efficiently people can commute (e.g. Mean Travel Time per trip). Therefore, under population growth, the efficiency of transport systems gains a greater significance.

The congestion under increasing travel demand not only causes time delays but also environmental pressure (Anas and Lindsey, 2011), such as greenhouse emissions from private and public vehicles, other air pollutants and noise. To reduce the negative externalities of transport systems, cities need sustainable transport systems as they develop. The term sustainable transport is used to describe modes of transport and systems of transport planning, which are consistent with wider concerns of sustainability. There are many definitions of sustainable transport. One definition has been given by the European Union Council of Ministers of Transport (Dobranskyte-Niskota et al., 2007) and it stated that sustainable transport systems in cities should ensure the supply for increasing demand and co-operation amongst modes, so sustainable transport means at least guaranteeing the efficiency of systems and reducing car emissions.

In addition, sustainability of cities means both social and environmental sustainability. For transport systems, cities need sustainable mobility and land use policies (Button and Nijkamp, 1997). Corresponding to the sustainable urban planning strategies, proper land use patterns can reduce total distance travelled. In the models of urban economics (Alonso, 1964), people may consider their travel cost when they make decisions on residential locations. Then the population density at locations influences the land use patterns. Hence, the development of urban transport system could decrease total travel cost effectively, which also contributes to the sustainable mobility.

As mentioned before, with economies/diseconomies of scale urban characteristics, growing cities could bring a lot of benefits (e.g. economy) whilst generating various costs. Two costs arising from transport are congestion and environmental pollution. Moreover, there may be a conflict between network economy of transport system and the emergent urban congestion according to urban agglomeration. To be successful in the future, cities need to reduce urban congestion and their environmental pressures and ensure sustainability mobility. Hence, for transport systems, the significance of efficiency and sustainability increases with urban growth.

## 1.2 Motivations

In daily commuting, people can choose driving, transit systems, cycling or walking. Of these modes, driving is the most common form, for instance, 57.5 percent of trips to work are commuting by driving in England and Wales (Gower, 2013). As there are many people that choose to drive, road congestion may be severe. Note that the number of people that choose to drive may increase as the total number of people increases, even if the driving proportion declines. The growth of private vehicle use in urban areas has prompted the expansion or enhancement of road networks in many cities, e.g. the ring system of highways in Beijing.

Meanwhile, with the ideas of transit-oriented development[1], compact city[2] or urban intensification[3], many cities have improved their public transport

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1 A transit-oriented development is a mixed-use residential and commercial area designed to maximize access to public transport and often incorporates features to encourage transit ridership.

2 Compact city is an urban planning and urban design concept, which promotes relatively high residential density with mixed land uses. It is based on an efficient public transport system and has an urban layout which – according to its advocates – encourages walking and cycling, low energy consumption and reduced pollution.

3 Term ‘urban intensification’ is a similar concept to compact city.

systems to deal with increasing travel demand in order to facilitate future sustainability, such as the city of Melbourne's spatial master plan and regional master plan for Île-de-France (Matsumoto, 2010). Clercq and de Vries (2000) explained the relationship between compact city plans and public transport. On the one side, public transport should be a suitable mode to facilitate the plan of a compact city because a compact city is designed to reduce space for car parking. On the other side, compact cities may improve the patronage of public transport by intensifying land use near public-transport stops.

As mentioned above, road networks and public transport systems have been improved in many cities. A question arising from this is how to invest in highways and/or public transport systems for a certain population size? This question can be investigated by spatial analysis of network growth, optimisation in Network Design Problem with some objectives (e.g. minimise congestion) and constraints (e.g. budget), analysing the economies of network effects etc. Indeed, plenty of studies investigated how to enhance the street capacity or build a new link on road networks (e.g. Drezner and Wesolowsky, 2003, Lo and Szeto, 2004 and Szeto and Lo, 2005). Also, different models were developed in order to give suggestions on the expansion of rail line in a compact city (e.g. Li et al., 2012b; Li et al., 2012c and Ma and Lo, 2012). As suggested in (Farahani et al., 2013), urban transport systems integrate public and private transport modes, so it is necessary to include both of them in the investigation of strategies of improvement and investment in transport systems.

The amount of physical transport infrastructure may decrease *per capita* under the growth of population since Bettencourt et al. (2007) reported that the total area of road surface was scaling with population in a sub-linear regime. Moreover, another empirical study (Louf and Barthélemy, 2014) observed that congestion and car emissions were super-linearly scaling with the population size. The super-linearly scaling relationship indicates that congestion and car emissions could increase per capita as population goes up. Many empirical studies (such as Bettencourt et al., 2007; Kühnert et al., 2006; Lämmer et al. 2006) to capture scaling laws in urban systems looked at data from various cities (i.e. different city sizes) at different time period.

These studies believed that the city size mattered in the growth and evolution of urban systems. Furthermore, they suggested that the growth and evolution of transport networks are independent of various network structures. One assumption summarised from these studies is that one synthetic city or transport network may grow and evolve following scaling laws. As studies (e.g. Lämmer et al. 2006; Louf and Barthélemy, 2014) reported that characteristics of transport systems were individually governed by a single scaling exponent, the trajectories of urban characteristics versus city sizes could be forecast. Those trajectories can be regarded as the evolutionary trajectories of network performance (or other features) when a city grows from a small size to a large one. Hence, scaling phenomena could help us to investigate the performance of transport systems (e.g. Mean Travel Time) under network growth and evolution.

First, studies of scaling laws have given some general principles that are independent of network structures, instead of models in discrete networks (with details of network structures or micro-simulation model) a highly aggregate model is the most suitable way to model the scaling-law phenomena and to investigate some global features in transport systems in this thesis. Second, many empirical observations of scaling laws only studied road networks, the influence of public transport should be included in the investigation of scaling phenomena in the performance of road networks (Louf and Barthélemy, 2014). Third, these empirical studies need more theoretical foundation, such as the scaling of network performance when network growth follows different strategies (e.g. compact cities versus plans of urban sprawl). To sum up, the first research question in this thesis can be proposed as follows:

**RQ 1: How can we perform strategic analysis to investigate the aggregate performance of a congestible[4] bi-modal transport system (driving and public transport) in different urban configurations?**

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[4] A congestible mode indicates that the travel time by that mode is dependent of the traffic volume. Namely, the congestible mode is different from a congestion-free mode by which the travel time is independent of the traffic volume.

When this thesis answers RQ 1, a simple model can be developed for the investigation of transport network growth and some aggregate phenomena. Two directions to develop the study for RQ 1 can be broadly proposed: (i) extending the theoretical model with urban dynamics and (ii) examining various planning strategies of investment on transport networks.

Urban spatial structures and land use patterns have been shaped by the influence of transport (McDonald and Osuji, 1995; Anas et al., 1998). Urban dynamics refer to some changes in land use patterns that could arise from population growth. For example, between 2001 and 2011, urban areas where the population density had changed over 10 per cent (Office for National Statistics, 2012) are around 38.67 per cent in the United Kingdom. The changes of population density may re-shape land use patterns and the distribution of population in cities. As land use patterns vary and transport systems improve, where and how people commute may change. A report (Department for Transport, 2014) studied the variation in mode choice in England over time. It reported that the mode choice of driving was decreasing while that by some public transport modes (e.g. bus in London and surface rail) was increasing. This correlation between transport and land use has been studied in many recent studies (e.g. Chang and Mackett, 2006; Bravo et al., 2010; Ma and Lo, 2012), but the network features stay constant in those studies so that the co-evolution of land use and transport influence under urban growth has not been examined well. Hence, modelling the co-evolution according to urban dynamics can extend the model from RQ 1.

Similar to the Network Design Problem with the budget constraint, there should be a trade-off of investment in two modes (highways and public transport systems) for a certain population size in the decision making for transport planners. Moreover, the consequence of improving highways or public transport systems may be different under various population sizes. For instance, improving the road capacity for a small population may drive car emissions to increase, but for a large population, the improvement of road capacity could make car emissions decline (more details will be given in Chapter 5). Hence, investigating network performance of a bi-modal transport system under various strategies of investment for different population sizes is valuable to help transport planners in making decisions. Then the other direction to extend the analysis of aggregate phenomena in RQ1 is to investigate the trade-off of investment in two modes for various population sizes.

In such cases, a research question can be summarized as:

**RQ 2: How can we extend the model from RQ1 to investigate the co-evolution of land use and transport infrastructure, and hence compare different investment strategies?**

After introducing RQ 2, this thesis then introduces motivations for RQ 3 and they are briefly introduced in the following paragraphs.

Firstly, as mentioned in Section 1.1, economies of urban agglomeration exist in cities and drive cities to grow and develop. In urban economics, economies of urban agglomeration have been investigated in the configuration of a monocentric city (Alonso, 1964) for decades. In the standard monocentric city model, land rents tend to increase and travel costs decrease as the Euclidean distance from the city centre while travel costs may decline. In fact, the distance via the transport networks affects the travel cost, namely, travel costs increase when network distance increases if all other factors (including Euclidean distance) stay constant (Levinson and El-Geneidy, 2009). Hence, the economies of scale in cities will depend not simply on the Euclidean distance but on the network distance (by road or public transport). The ratio of network to Euclidean distance is defined as circuity (Barthélemy, 2011) and this will influence the effective aggregate spatial dimension of the city.

With this in mind, the trade-off between investments in the two travel modes to be analysed in answering RQ2 may have different investment efficiencies in the monocentric city model. For example, if the network distance along the highway is shorter than that along the railway, the investment efficiency of the highway may be higher when other factors (e.g. the cost to improve highway/railway per unit distance) are equivalent. Investigating the comparative circuity of road versus public transport networks may therefore be valuable.

Finally, there are some differences in spatial features between road and public transport networks. One main reason is that public transport networks were designed to ensure the accessibility with a large coverage and circuitous lines (Black, 1995; Murray et al., 1998) while people choose to drive on road networks as they may find more direct routes. Indeed, the design of public transport networks is not independent of their use. For example, how long a journey by transit systems takes (compared to alternatives) and how easily destinations are accessed by each mode explains much of the share of public transit use. One hypothesis proposed is that the circuity by transit is higher than that by auto for the same sample of trips. Moreover, the preference of mode choice has been included in the choice of residential locations, based on studies of self-selection in residential allocation (such as Krizek, 2003; Mokhtarian and Cao, 2008; Cao et al., 2009). For example, people who prefer commuting by transit may choose residences where they can easily access transit stations that are served by routes connecting relatively directly with desired destinations. Following the discussion in these studies, this thesis posits that people who commute by transit select residential locations with less circuitous transit routes than those who do not. In this thesis, another hypothesis is that the circuity of actual transit trips on public transport networks is lower than actual auto trips (some studies on circuity are introduced in Section 2.2.2). Investigating this hypothesis can help us understand the self-selection in mode choice by commuters and the need of travellers for direct routes. Hence, circuity is an additional aspect in the investigation of mode choice and it is necessary to be included in the study of mode choice, even in the aggregate scenario of the monocentric city choice model.

Then the third research question can be proposed:

**RQ 3: How does the spatial efficiency of urban public transport networks compare with urban road networks?**

After the thesis introduced motivations with three research questions, the theme of this thesis is outlined. Based on the earlier statement, a flow chart for this is drawn in Figure 1.1. As the agglomeration and growth of population drives cities to grow, expand and evolve, the distribution of population may vary and then urban dynamics should emerge. Under urban growth, variations in urban characteristics occur and some phenomena were observed, for instance, the scaling-law relationship between the population size and the amount of transport infrastructures. Urban dynamics and the growth of transport infrastructures could transform spatial features in transport networks. Because the residential locations of people may change under the agglomeration of population and transport systems may be improved under urban dynamics, mode choices of commuters could alter. As shown in Figure 1.1, these interdependences outline the theme of this thesis. To investigate the growth, evolution and scaling in transport networks, three research questions have been proposed when Section 1.2 discussed some phenomena under urban growth.

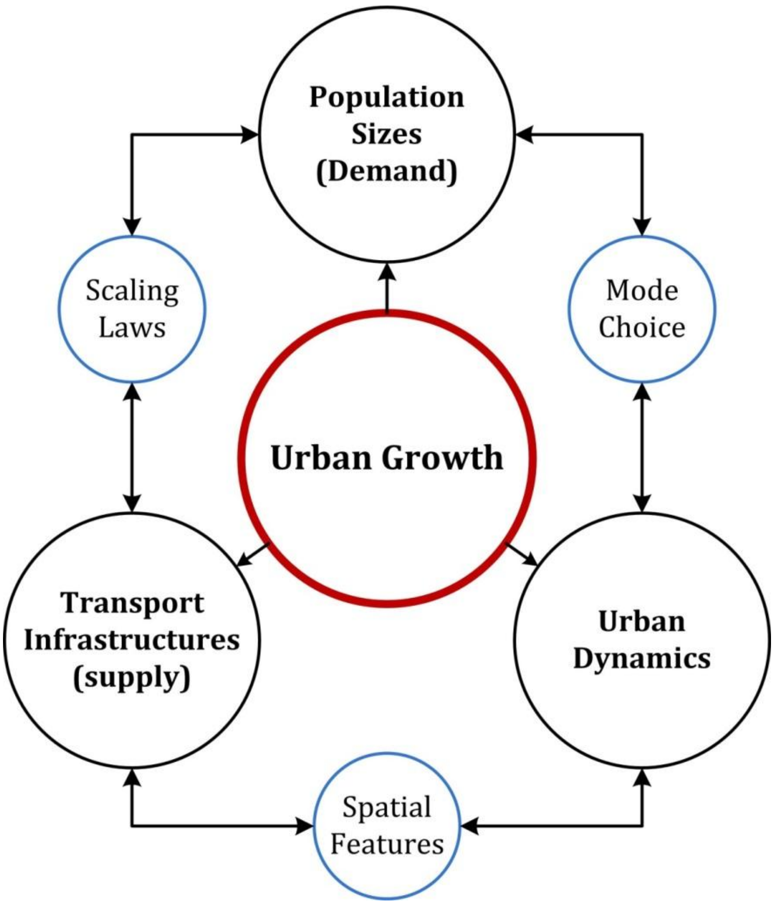


**Figure 1. 1  Flow chart of this thesis.**

## 1.3 Research Objectives (RO)

The research questions in Section 1.2 can be answered by achieving the following research objectives. They are briefly listed in this section.

As there were various scaling phenomena that have been revealed in empirical observations (Section 1.2), many researchers concluded that the growth following scaling laws was independent of network structures such as Lämmer et al. (2006). Continuum models can be adopted in the investigation of scaling-law growth because it is a network-free modelling approach. Another two reasons for using continuum models are: (i) the continuum modelling approach is useful to investigate global features of transport systems at a macroscopic level (Ho and Wong, 2006) so that this work can investigate the evolutionary trajectories of network performance; (ii) the economies of urban agglomeration can be studied in the monocentric city model that is a continuum model. Hence, the objective to answer RQ1 can be summarised as:

**RO 1: Model the interdependence between urban growth following scaling laws, and the aggregate performance of transport infrastructure provision.**

As stated previously in RQ2, according to urban dynamics, this thesis then considers the residential location and mode choice to extend the model obtained from RO 1. With scaling-law growth, a modelling framework can be designed so that variations in land use patterns and strategic plans for the improvement in transport systems can be investigated. Then the second objective is written as follows:

**RO 2: Extend the model from RO1 with the co-evolution of residential location and mode choice and propose a modelling framework for urban growth.**

Following RQ 1 and RQ 2, this thesis needs to develop some experiments. Three aims for the experiments are (i) to ensure that the proposed model or modelling framework works properly, (ii) to investigate the evolutionary trajectories of network performance under the scaling-law growth, (iii) to understand possible consequences of some strategic plans for the improvement of transport networks in order to facilitate future urban sustainability. Among these aims, the first two aims are the preparation for the third, so the third objective of this thesis is summarised as:

**RO 3: Design experiments that can study different planning strategies and the trade-off of investment in two modes (highways and public transport) for urban sustainability.**

By achieving the objectives above, this thesis theoretically investigates possible phenomena when road and public transport networks grow. This thesis then attempts to perform an empirical study to answer RQ3. According to the statement before RQ3 (see pages 7 and 8), the spatial features in public transport networks have not been reported much and circuity can be a candidate indicator to investigate the spatial feature of public transport networks in this thesis.

An objective of this thesis is written as:

**RO 4: Examine circuity in public transport networks**

According to the statement in Section 1.2, circuity may help to identify the difference in road and public transport networks. Their differences could affect how commuters make decisions on mode choice so that the mode share in cities may be related to the circuity of road and public transport networks (Levinson, 2012). Moreover, some other spatial features should have some correlations with the circuity of networks. Then another objective is to study the correlation of circuity with other indicators. The fifth research objective can be summarised as:

**RO 5: Investigate the topological difference in road and public transport networks and study the correlation between circuity and other relevant indicators.**

Following the research questions in Section 1.2, this section listed five research objectives of this thesis. More details of these questions and objectives are explained in the literature review (Chapter 2).

## 1.4 Layout of the thesis

Following Sections 1.1, 1.2 and 1.3, Section 1.4 introduces the organisation of this thesis and its layout is shown in Figure 1.2.

Chapter 1 includes a brief introduction and the theme of this thesis is broadly outlined.

Chapter 3 adopts the contents of a continuum model and includes the scaling-law growth of population size and transport infrastructure. Then a continuum model is developed to theoretically investigate aggregate phenomena under the growth of transport networks. Moreover, as stated in Research Objective 3, experiments are designed and the results from them are shown.

Chapter 4 extends the model developed in Chapter 3 considering the urban dynamics, because population densities change and land use patterns vary consequently (see Section 1.2). In the model, residential allocation of residents and their mode choices are included. A solution procedure is designed for the extended model. A modelling framework is then proposed with simultaneous improvements in road and public transport networks. At the end of Chapter 4, how the modelling framework can be used is briefly shown by several experiments (RO 3).

Motivated by RQ 3, the thesis performs an empirical study of network analysis with real data in Chapter 5. This chapter provides research methods for specific data sources to investigate circuity in public transport networks (RO 4). Through the analysis of circuity, Chapter 5 not only shows differences of network structures in road and public transport systems but also studies the implication of circuity on mode share in real cities (RO 5).

Chapter 6 includes conclusions from this thesis and provides possible directions for future work.

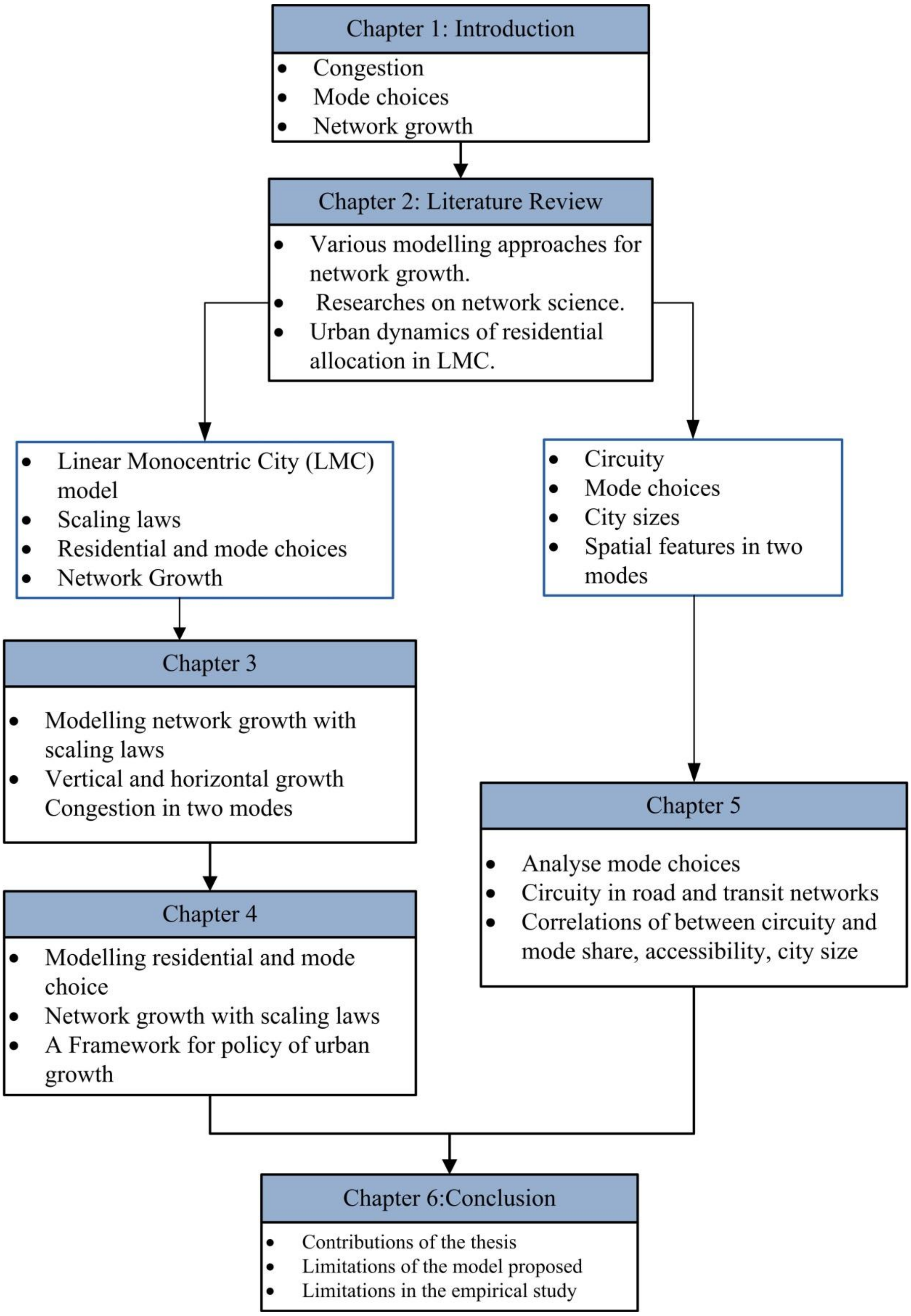


**Figure 1. 2 Layout of the thesis.**

# Chapter 2
# Literature Review

There have been many studies on modelling and analysing the growth of transport networks. Xie and Levinson (2009a) have reviewed these studies following five streams: network growth in transport geography; transport planning considering network growth; statistical analyses of network growth; economies of network growth; and network science. Instead of a comprehensive review, Chapter 2 provides a more specific literature review for this thesis. As introduced in Section 1.4, the work of this thesis can be summarised in two parts: (i) a model to theoretically study aggregate phenomena and the trade-off of investment under the growth of transport networks (RO 1, 2 and 3), (ii) an emprical investigation of circuity in road and public transport networks (RO 4 and 5).

For the first part, Section 2.1 reviews several theoretical models that have been used to study issues of transport planning under the growth of transport systems. Meanwhile, models in network science are reviewed in Section 2.2.1 since they were used in the investigation of network growth. With the review of these models, their advances and limitations will be stated.

According to the second part, definitions of indicators including circuity have been included in Section 2.2.2 as these studies belong to the field of network science. Moreover, the relevant empirical observations are reviewed in Sections 2.2.3 and 2.2.4.

Different from models reviewed in Sections 2.1 and 2.2, this thesis is going to propose a continuum approach in the study of growth and evolution in transport networks. Hence, Section 2.3 provides a literature review of continuum models in transport studies and finalises one model in Section 2.3.3. Furthermore, as stated in Section 1.2 and RQ 2, urban dynamics will be considered when this thesis develops the model for network growth (Chapter 4). Hence, Section 2.4 offers an introduction of the basic theory in residential allocation and reviews studies on urban dynamics and transport influence.

Following the literature review (in Sections 2.1, 2.2, 2.3 and 2.4), Section 2.5 summarises research gaps from previous studies for this thesis.

## 2.1 Models of Transport Planning in Discrete Networks

The aim of Section 2.1 is to review some models of transport planning in discrete networks because the models included may be relevant to some issues of transport network growth. Section 2.1.1 firstly reviews the Four-Step Model that is a traditional model used in transport planning and it could help transport planners make decisions on the expansion or improvement of transport systems. Then Section 2.1.2 reviews a model that is extended from the Four-Step Model, namely the System of Network Growth (SONG) model. Moreover, a principle in one step of the Four-Step Model (i.e. traffic assignment) is introduced in Section 2.1.3. With the advances in traffic assignment, the Network Design Problem has been studied well and some research that is relevant to the study of network growth is to be reviewed in Section 2.1.4.

### 2.1.1 Four-Step Model

In classic transport planning, the Four-Step Model was applied to many transport studies from the 1950s, such as the Detroit Metropolitan Area Traffic Study (1955). The Four-Step Model treated transport planning as a sequential process, namely trip generation, trip distribution, mode choice, and route assignment. The framework of the Four-Step Model is summarised in Figure 2.1.

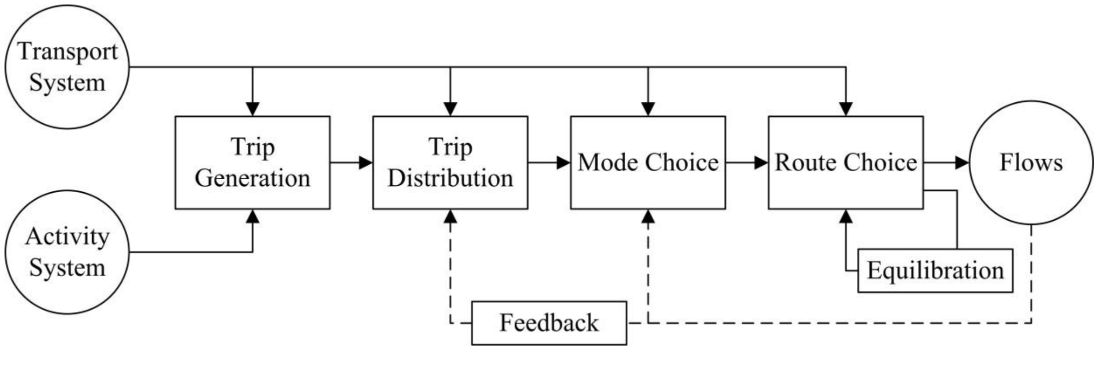


**Figure 2. 1 Flow chart of the Four-Step Model (Source: McNally, 2008).**

In the Four-Step model, there is an urban region that is divided into zones and transport networks that provide mode choices across the region. A transport network consists of a set of nodes and links between nodes. Each link has a link performance function[5] representing the relationship between travel cost (or travel time) and volume of traffic. First, trip generation determines the frequency of origins/destinations of trips in each zone by several trip purposes. Next, trip distribution matches origins with destinations. Then mode choice computes the proportion of trips between each origin and destination that uses a particular transport mode. Finally, trips are allocated to a route by a particular mode. In the step of route assignment, equilibration (Figure 2.1) indicates that a process to ensure that no forces drive commuters to switch their routes, namely at equilibrium (Sheffi, 1985). If the transport network is not at equilibrium, the flow patterns are forced toward equilibrium by a route-switching mechanism.

### 2.1.2 System of Network Growth (SONG) Model

The model introduced in Section 2.1.1 provides the traditional procedure for urban transport planning to estimate the amount of demand that would use a specific travel mode. The forecast from the Four-Step Model could help transport planners make decisions on how to invest on transport infrastructures. Furthermore, the Four-Step Model can be the basis of the investigation of some emergent phenomena under the growth of transport networks. As an example in the studies on network growth, one model that is an extension of the Four-Step Model is reviewed in Section 2.1.2.

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[5] For each link there is a function stating the relationship between resistance and volume of traffic. For example, the Bureau of Public Roads (BPR) (1964) developed a link congestion (or volume-delay, or link performance) function. BPR function is introduced in Chapter 3 and 4 in detail.

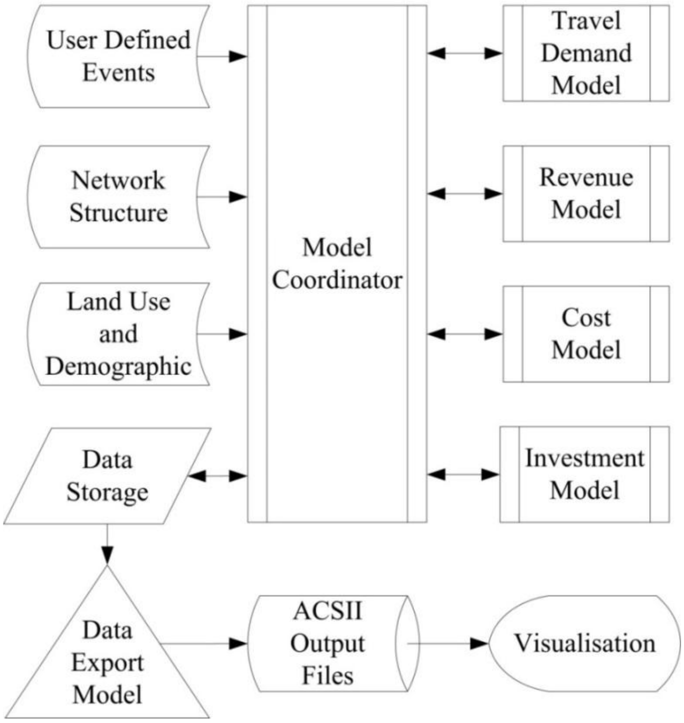


**Figure 2. 2 Framework of the SONG Model (Source: Levinson and Yerra, 2006).**

Motivated by the Four-Step Model, Zhang and Levinson (2004), Levinson and Yerra (2006) and Yerra and Levinson (2005) developed a model (i.e. SONG model) to treat the self-organisation and growth of road networks with fixed or random land use patterns. The flowchart of the SONG model is shown in Figure 2.2. The travel demand model in their studies used population and employment data following trip generation, trip distribution, and traffic assignment. Since the SONG model investigated networks with a single mode under growth, the step of mode choices has been excluded. Meanwhile, the revenue model determines the price the traffic should pay for using the road depending on speed, flow and length of the link. The cost model calculates the cost required to maintain link speeds depending on traffic flow. The revenue is equal to the excess of maintenance costs and all net revenue is consumed as an investment into the links so that these links are upgraded. On the other hand, if the amount of revenue is less than the cost, those links are downgraded. Hence, the SONG model showed how increased traffic volume may encourage transport planners to expand transport infrastructures and the varation in transport infrastructures may change the land use pattern, which could alter residential locations and travel behaviour. The SONG model could be used in urban transport planning and give suggestions on land use (Zhang et al., 2009; Xie and Levinson, 2011). Moreover, the SONG model has investigated the variation in some spatial features under network growth and the temporal change of topological attributes in networks (Xie and Levinson, 2009b).

In summary, Sections 2.1.1 and 2.1.2 provide a short review of two models in transport planning for discrete networks because these models could be used in the research of network growth. Similar to the SONG model, models that are extended from the Four-Step Model may capture some phenomena under network growth in detail, such as the consequence of an upgraded link. It is worth noting that in these models, structures of transport networks are pre-determined and specific, like the grid network in the SONG model. As stated in Section 1.2, urban dynamics need to be included so that this thesis can investigate RQ 2. Within the structure specified, urban dynamics under urban growth may be ignored. In addition, when a study is aiming to observe phenomena when urban area expands or some spatial variations occur in network structures, these models may not work well.

### 2.1.3 User Equilibrium in Transport Networks

Following the statement of equilibration in route choice, Section 2.1.3 introduces a particular route-switching mechanism that can drive the system at equilibrium.

In the process of route choice for all travellers, one sensible assumption is that every traveller tries to save their travel costs (or travel time) as much as possible by switching route choices. User Equilibrium arises as every traveller tries to minimise their travel cost. The statement of User Equilibrium is given by Wardrop's first principle (Wardrop, 1952):

*the journey times in all routes actually used are equal and less than those which would be experienced by a single vehicle on any unused route; each user non-cooperatively seeks to minimise his cost of transportation*.

In other words, User Equilibrium is a state where no user may lower his travel costs through unilateral actions. The mathematical formulation of traffic assignment at user equilibrium will be explained in Chapters 3 and 4 for the specific problems in this thesis.

Note that there may be other rules in the process of route choice (i.e. route assignment or traffic assignment). Here, this thesis only reviews the one which will be adopted in the following chapters.

### 2.1.4 Network Design Problem in Discrete Networks

As stated in Section 1.1, transport networks need to deal with growing travel demand under population growth, so transport planners may expand system capacities with the limited availability of resources. In such cases, the Network Design Problem (NDP) has emerged to be one task in transport planning. Hence, Section 2.1.4 reviews some studies of NDP that are relevant to the subject of this thesis. Moreover, issues of route assignment (Section 2.1.3) have been widely investigated since the 1950s, including mathematical programming methods (e.g. Beckmann et al., 1956) and algorithms (e.g. Dafermos and Sparrow, 1969), which are good sources for NDP.

NDP typically involves determining a set of optimal solutions for decision variables (e.g. link locations) by optimising different system performance (e.g. minimising the total travel time) based on traffic assignment. Some studies in the subject of NDP are related to issues of network growth, such as the objective is to minimise the total travel cost when expanding the road capacity of existing network or adding a new link on the network. Reviews of the NDP have been provided by Fernandez and Friesz (1983), Boyce (1984), Magnanti and Wong (1984), Friesz (1985), Migdalas (1995) and Yang and H. Bell (1998). A recent review by Farahani et al. (2013) has discussed the NDP in both road and transit networks, as well as mutil-modal transport systems. This review has provided a bigger picture of the NDP when both road and public transport networks need to be improved or expanded under urban growth.

For the growth of transport networks, two types of the NDP have been summarised by (Yang and H. Bell, 1998): (1) the determination of an optimal location for a new road added to the existing transport network; (2) the optimal capacity enhancements for an existing road (such as Abdulaal and LeBlanc, 1979; Davis, 1994; Meng et al., 2001). In an empirical study (Bettencourt et al., 2007), the relationship between road surface and population size has been observed, which may be related to the second type if the road capacity can be a proxy for road surface. Moreover, the optimal capacity enhancements for an existing road can be formulated in a bi-level optimisation problem. For example, Chiou (2005) formulated bi-level programming and solved the problem in a congested network. The objective of the upper level problem was to minimise the system performance (i.e. total travel time) and investment costs of link capacity expansions. At the lower level problem, the route choices of travellers were determined by Wardrop's first principle (Section 2.1.3). Indeed, increasing travel demand requires the improvement of the existing transport infrastructures, while resources and budgets available for expanding/building transport systems to enhance the system efficiency are limited.

Section 2.1 has firstly introduced a conventional transport planning model, the Four-Step Model. As one extension from the Four Step Model, the SONG model has been reviewed as it may investigate some emergent phenomena under network growth. Following the introduction of User Equilibrium, Section 2.1 reviews some studies of NDP that are relevant to some issues in network growth. Models reviewed in Section 2.1 are developed of the investigation for discrete networks. Within some specific network structures, those models have been used in the small-scale growth of transport systems. In discrete networks, details can be examined well, such as travel time on each link and various objectives for the NDP can be included. But there may be high computational cost when we try to observe the system performance or make the decision for all links when the whole network is growing. In order to answer RQ 1, this thesis is aiming to observe the trajectories of system performance in a long term so that this thesis can perform strategic analysis. In such cases, models of transport planning in discrete networks are not very proper.

## 2.2 Network Science in Network Growth and Evolution

Recently, the number of studies on network science has been increasing. Among those studies, some models from network science investigated emergent phenomena as networks grow and a short review for some models is included in Section 2.2.1. Furthermore, studies on network science not only provided some models of investigating issues of network growth but also some indicators to capture the spatial features of transport networks. Then Section 2.2.2 introduces two indicators that have been mentioned or adopted in this thesis (Chapter 5). Moreover, there are many empirical observations in the studies of network science. Those observations with real data help us understand some principles of network growth and the spatial features with various city sizes. One of principles introduced in Sections 2.2.3 and 2.2.4 is the scaling-law relationship between urban characteristics and population size as Chapters 3 and 4 of this thesis will investigate phenomena under the scaling-law growth of transport networks.

### 2.2.1 Models of Network Science in Network Growth

Approaches from network science have been used in the study of phenomena under the growth of general networks. The Barabási-Albert Model (Barabási and Albert, 1999) explained why uneven distributions arose under network growth, such as the formulation of hubs where nodes connected more links. Moreover, the Barabási-Albert model can be a stochastic process of network growth (Barabási and Frangos, 2014); then it became a standard model to generate ‘complex networks’. Note that the term ‘complex network’ is used for networks where various structures arise and display different topological features with some simple rules of network generation. Barabási et al. (2000) summarised that these features were neither entirely regular nor purely random and they emerged as networks grew.

Transport networks have been recently studied by using approaches or models in network science (e.g. Andersson et al., 2006), because the complex network approach can be used to understand the growth and evolution of transport networks. For transport networks, one principle in network growth is to minimise the global cost. Motivated by this idea, Barthélemy and Flammini(2008) proposed a complex network approach based on a local optimisation process, namely every new road is built to connect a new location to the existing road network in the most efficient way. Furthermore, Rui et al. (2013) suggested a local optimisation process can understand the topological evolution of road network structures. In their papers, the local optimisation only considers geographical features to obtain the most efficient topology, namely, the efficiency for travel demand has been neglected. Barthélemy and Flammini (2009) extended models for the local optimisation process and analysed how road networks evolved and interacted with the evolution of population density. The urban evolution observed from their work was that locations with high accessibility may have high population density resulting in densely populated city centres appearing, which is in consistency with the formulation of a linear monocentric city model in urban economics (see Section 2.3.3). But their work isolated two important factors of rent price and travel costs and their work did not show how the rent price and travel cost varied with various population densities.

In the meantime, Louf and Barthelemy (2014) showed that polycentricity (i.e. a self-organised feature) emerged with a complex network approach and real data of traffic congestion. Their study reported that a super-linear relation between car emissions and city sizes, so they suggested that the city size is one significant factor for urban sustainability. Their finding was also supported by some emprical studies (e.g. Chan et al., 2011 and Louf et al., 2014). According to these studies, it was suggested that the analytical investigation of the relationship between aggregate performance of transport networks and cities sizes was needed.

To sum up, the complex network approach reported how the evolution of road networks may be driven by a simple mechanism so that those studies in network science suggested that some principles in the growth of networks should be independent of various networks structures. Although those studies show that approaches of complex networks are suitable in the investigation of large-scale networks, there are two main limitations of approaches in complex networks: (i) those studies examine the evolution of road network structures and exclude public transport so that they did not consider that mode choices of commuters may alter under network evolution; (ii) only several studies have considered the traffic features, such as congestion, which is significant in models of transport planning. Additionally, there are some emergent small parts in complex networks that are rarely seen in transport networks, such as a node connecting eight links.

### 2.2.2 Indicators to Measure Spatial Features

Following Section 2.2.1, this section includes some indicators in network science. Among those indicators, Section 2.2.2.1 introduces the definition of circuity which will be used in an empirical observation in transport networks (Chapter 5) to answer RQ 3 (Section 1.2). Meanwhile, some studies on circuity in transport networks are reviewed. Because the indicator circuity could help study how aggregate a network is; there is another indicator that may capture a similar topological feature of networks in network science, namely betweenness centrality. Some studies of betweenness centrality are reviewed in Section 2.2.4. Here, Section 2.2.2.2 gives the definition and implication of betweenness centrality as a preparation for the following literature review.

Network structures had an impact on the performance of transport systems and their spatial features affected land use and urban form (Marshall, 2004). There is a long history on measuring the structure of transport networks from the 1960s, such as Garrison (1960), Mohring (1961), Garrison and Marble (1962), Smeed (1968), (Kansky, 1969) as well as Haggett and Chorley (1969). Later, studies explored how traffic flows and travel patterns are affected by various geometric network structures (Newell 1980; Vaughan 1987) when travel demand models were developed. Indeed, many studies provided quantifiable indicators about how to abstract the properties of network structures. Those spatial measurements could also help us understand the performance of transport systems and investigate the evolution of networks.

Recently, research shifted its focus from simple topologic and geometric properties to large-scale statistical properties of complex networks according to the basis of studies by Albert et al. (1999), Barabási and Bonabeau (2003) and Newman (2003). Xie and Levinson (2007) reviewed some measurements from network science and how they may be used in the investigation of network structures. Moreover, Barthélemy (2011) reviewed various indicators to explore geometric characteristics in complex networks.

#### 2.2.2.1 Circuity

Kansky (1969) proposed up to 14 indicators for topological characteristics of transport networks. Some indicators have been used in empirical studies to understand the structure and performance of transport systems, such as Levinson (2012). Circuity is one of those indicators and it is defined as the ratio of network distance over Euclidean distance between a OD pair (Barthélemy, 2011). It is given by:

$$C_{ij} = \frac{D_{ij}^{n}}{D_{ij}^{e}}, \tag{2.1}$$

where $C_{ij}$ denotes the circuity between a single OD pair between origin $i$ and destination $j$, $D_{ij}^{n}$ is the network distance between origin $i$ and destination $j$ and $D_{ij}^{e}$ is the Euclidean distance between origin $i$ and destination $j$. Theoretically, this ratio is always larger than 1. The closer circuity is to one, the more efficient commuting between origin $i$ and destination $j$ is.

From Equation 2.2, circuity $C_i$ for a single node $i$ can be derived as

$$C_i = \frac{1}{Q-1}\sum_j C_{ij}, \tag{2.2}$$

where $Q$ is the total number of nodes in the network, $j$ is the total number of nodes connecting node $i$. Circuity of a node $C_i$ measures how accessible the node is in the network. Indeed, the smaller it is, the easier it is to reach the node $i$. In other words, circuity of a node is measuring 'accessibility' of a node. There should be some correlation between circuity and accessibility in networks, which has not been investigated well in previous studies of circuity.

Furthermore, measuring the average circuity of trips has helped in the estimation of travel distance (Ballou et al., 2002) and understanding the topological features of road networks (Levinson and El-Geneidy, 2009). More recently, Giacomin and Levinson (2015) examined the average circuity of trips with the evolution of road networks. With the dataset of census trips and randomly generated trips, the research shows that the circuity of networks has generally increased over time. Taking this research as an example, indicators from network science could not only capture some topological and spatial features of transport systems but also help us understand how these features vary under network evolution. There are many studies (reviewed in Chapter 5) looking at the circuity of road networks while that of public transport systems has been rarely examined. Hence, this thesis will give a literature review in detail and investigate the circuity of trips in public transport networks in Chapter 5.

#### 2.2.2.2 Betweenness centrality

As stated before RQ 3 (Section 1.2), the indicator circuity could help us examine how aggregate a network is. In network science, there are several topological indices that could capture similar topological features. In Section 2.2.2.2, this thesis reviews one of them (i.e. betweenness centrality) and its definition is given as this indicator has been investigated in public transport networks for the preparation of following literature review (Section 2.2.4).

Betweenness centrality was defined by (Freeman, 1977) and this indicator characterises the importance of a node or an edge in the network. For a node $i$, the betweenness centrality is defined as:

$$b_i = \sum_{s \neq t} \frac{\delta_{st}^i}{\delta_{st}} \tag{2.3}$$

where $\delta_{st}$ is the total number of shortest paths from node $s$ to node $t$ and $\delta_{st}^i$ is the number of those paths via node $i$. Note that there may be not only one shortest path particularly in a complex network and the shortest path here denotes the geographical shortest path excluding the influence of congestion etc. With the definition of betweenness centrality of a node, if the number of the shortest paths is large, it indicates that the node is important in the networks.

Similarly, the betweenness centrality for an edge $e$ is defined as:

$$b_e = \sum_{s \neq t} \frac{\delta_{st}^{e}}{\delta_{st}} \tag{2.4}$$

where $\delta_{st}$ is the number of the shortest paths going from node $s$ to node $t$ and $\delta_{st}^{e}$ is the number of the shortest paths going from $s$ to $t$ via edge $e$. A high value of betweenness centrality for an edge indicates that the node or edge is a critical element in the network. If a network has many edges and nodes with high values of betweenness centrality (i.e. a large value of average betweenness centrality for a node/edge), the network may be aggregate.

### 2.2.3 The Origin of Scaling Laws

As stated in Section 1.1, the consequence of scaling-law growth in transport network growth will be investigated in this thesis. Many studies have looked into scaling laws in urban systems with data sources and approaches from network science. Before reviewing the studies of scaling laws, this thesis introduces the origin of scaling laws in Section 2.2.3, which is also a preparation of the literature review in Section 2.2.4.

Scaling laws were observed between the size of an organism (e.g. a biological entity, a network) and its characteristics (West et al., 1997). As a small change of organism size can lead to a disproportionate increase/decrease in different characteristics of that organism, the relationship between two quantities (one of them is the organism size) is given by (West et al., 1997)

$$Y = Y_0 M^{\beta}, \tag{2.5}$$

where $Y$ is a variable describing a certain characteristic in the biological organism, $M$ is body mass and $\beta$ is the scaling-law exponent. Note that $Y_0$ is the normalisation constant. With this expression, a scaling law is defined where two variables in a system (e.g. a city, a mammal) change simultaneously following a power law. Extending the study on the size of network and the growth speed, Banavar et al. (1999) discussed the scaling law in efficient transportation (very general) networks, such as river systems, power networks and blood-supply systems. Banavar et al. (1999) summarised the relationship of the volume of the blood-supply network $V_{net}$, the volume of the organism $V_{org}$, and the density of nutrient-supply sites (vessels or cells) $\rho$ in a biological organism, that is,

$$V_{net} \propto \rho V_{org}^{(D-1)/D}, \tag{2.6}$$

where $D$ represents the number of dimensions for the organism, in their work $D = 3$. Equation 2.6 indicated that the volume of the blood-supply network $V_{net}$ scaled with the volume of the biological organism and the density of nutrient-supply sites (vessels or cells) $\rho$. Because road networks may use a similar process to deliver the traffic flow as a cardiovascular system, an analogy between an urban road network and a cardiovascular system was interpreted by (Samaniego and Moses, 2008) and a relationship for a plane road network was summarised as

$$A_{road} \propto N A_{city}^{1/2}, \tag{2.7}$$

where $A_{road}$ is the length of the road network, $A_{city}$ is the spatial coverage of the city and $N$ is the number of population. Equation 2.7 denoted that the length of the road network scaled with the total number of population and the coverage of the city. Moreover, the statistical results by (Samaniego and Moses, 2008) suggested that urban road networks were different from biological vascular networks. One feature of road networks is that they are much less centralised. As there should be some similarities but many differences between the biological vascular and various urban systems, it is necessary to study the scaling-law phenomena in urban systems (including road and public transport networks) and some research is reviewed in Section 2.2.4.

### 2.2.4 Scaling Laws in Urban Systems

So far Section 2.2 has reviewed some models that could help us investigate network growth theoretically in network science, but the empirical observations with some indicators from network science have not been reviewed. As mentioned in Section 2.2.2, measuring some indicators over time or with various city sizes could help us understand the evolution of networks. Section 2.2.4 then reviews relevant empirical studies.

Since the empirical scaling laws were observed, the nonlinear agglomeration of urban characteristics needs to be examined. For instance, Bettencourt et al. (2010) proposed indicators, namely the Scale-Adjusted Metropolitan Indicators to examine the scaling phenomena of cities. The investigation by those indicators suggested that the general scaling properties of cities should be derived from a set of local principles about population, geometry and economy. Their work provided an approach to measure the social-economic metrics of cities and concluded that urban dynamics were governed by city size. Moreover, socio-economic and spatial characteristics of urban systems grow with population following scaling laws (Kühnert et al., 2006; Bettencourt et al., 2007). These studies provided insights into whether various characteristics increase or decrease per capita. Such understanding should underpin any examination of the sustainability and future prosperity of cities.

For road networks, Lämmer et al. (2006) specifically presented empirical results on geographical features of road networks which obeyed invariant power laws. Chan et al. (2011) reported that the small-scale geometry of all examined road networks was very similar. Since the distributions of geometric quantities of road segments and cellular structures shared the similarity in these two studies, they suggested that the global similarity may exist in road networks of cities. Levinson (2012) examined the variations of network features with city sizes in 50 metropolitan areas.

Meanwhile, scaling of spatial features in public transport systems has also been investigated. (Derrible and Kennedy, 2010a) found how the public transport networks were connected and reported the regional accessibility of 33 metro systems. Then a study (Derrible, 2012) reported how aggregate the metro systems are with the investigation of betweeness centrality and it suggested that the average betweenness centrality[6] of nodes grows linearly with the network size (i.e. the power-law exponent is 1). (Louf et al., 2014)

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[6] Section 2.2.2 includes the definition of betweenness centrality.

suggested that not only spatial characteristics but also socio-economic features were independent of the structures of subway systems worldwide with some scaling-law phenomena reported.

In summary, scaling-law phenomena have been investigated according to economic, spatial and other characteristics in cities and their sub-systems, e.g. road networks and transit systems. These observations help understanding of the feature of transport networks with city sizes. But congestion or delay due to population growth is not easy to explicitly understand from the empirical studies. So how urban congestion scales under population and network growth needs more study. Moreover, instead of investigating some geographic features of road and public transport networks respectively, it is valuable to study/compare them together.

## 2.3 Continuum Approaches in Transport Studies

As Sections 2.1.1, 2.1.2, 2.1.4 and 2.2.1 mentioned before, there are various types of models that have been used in the study of network growth. This thesis has summarised their advantages and limitations at the end of each section. Meanwhile, Section 2.2.4 introduced that scaling-law phenomena were observed in transport networks and one important property is that scaling-law phenomena are independent of network structures. According to this property, this thesis proposes another method, namely the continuum modelling approach, to investigate the growth of transport networks. To this end, Section 2.3 briefly introduces the history of the continuum modelling approach from the basic two-dimensional model (Section 2.3.1) and the ring-radial model (Section 2.3.2). How the Linear Monocentric City (LMC) model, as one commonly used continuum model that is developed from previous models has been included in Section 2.3.3. Additionally, Section 2.3.3 reviews the applications of the LMC model in transport studies.

### 2.3.1 The Original Continuum Model: Two-dimensional Model

Beckmann (1952) first introduced the idea of the continuum modelling approach in transport studies. In his paper, the commodity production (i.e.

‘origin’ in this thesis) and export (i.e. destinations) are continuously distributed in a geographical terrain, in such case, a two-dimensional model was formulated. The most cost-effective flow pattern was found for a given commodity production and export. The paper by (Beckmann, 1952) suggested that the cost-effective travel pattern problem be formulated as a constrained minimisation problem, which provided a framework for the traffic equilibrium problem in a continuum model. The continuum approach in a two dimensional terrain does not require the details of road networks. As the continuum modelling approach reduces the problem size compared to the discrete modelling approach, it is useful to represent some phenomena or the investigating of global features. For example, Smeed (1965) estimated the average distance travelled, the distance travelled per unit area, the maximum number of persons that can enter or leave the central area of the town within a given period.

With the continuum approach, a dense network is approximated by a continuum, in which the users are free to choose their routes in a two-dimensional space. The fundamental assumption is that the difference between adjacent areas within a network is relatively small compared with variation over the entire network. Hence, the characteristics of a network, such as the flow intensity, demand, and travel cost, can be represented by smooth mathematical functions. Furthermore, for a continnum model in two dimensions, some studies solved the traffic assignment problem with the consideration of mutiple user classes, such as (Wong, 1998), (Ho et al., 2006) and (Ho et al., 2007).

Motivated by Beckmann (1952), Zitron (1974) and Puu (1977) have formulated a slightly different model with the continuum approach. In their studies, a circular network configuration was used with a city centre where all destinations are located and the traffic flow is continuously distributed in the circular city. (Zitron, 1974) was aimed at the determination of optimal-cost routes of traffic flow. (Puu, 1977) explained how paths of traffic flows to enter the city centre were related to how the road capacity is distributed in the city. Within this formulation of a monocentric city model, there are many studies focusing on traffic assignment, such as the flow conservation equations (Dafermos, 1980), user-equilibrium traffic assignment (Sasaki et al., 1990) and sustainable cordon toll pricing schemes (Li et al., 2013b). Additionally, an example of a two-dimensional monocentric city model is shown in Figure 2.3.

The theoretical development of the continuum modelling approach allows us to represent a city or transport systems more easily. A similar model integrated a discrete network and continuum models to better represent transport systems of a city (Yang et al., 1994; Wong et al., 2003). Meanwhile, the continuum modelling approach helps to investigate some transport planning problems at an initial stage, for example, facility allocation (Wong and Sun, 2001), congestion pricing (Ho et al., 2005; Li et al., 2013b), housing allocation and transport influence (Ho and Wong, 2007).

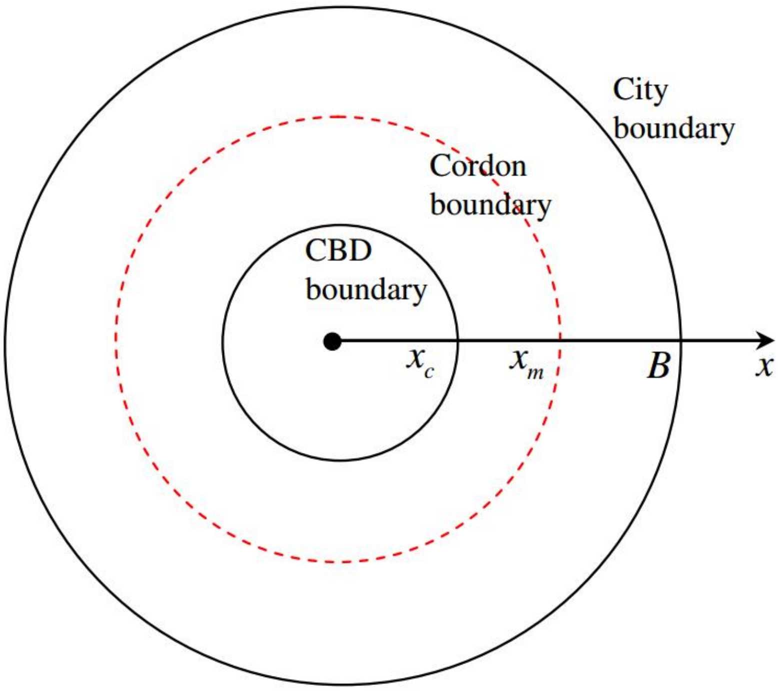


**Figure 2. 3 Two-dimensional monocentric city model (Source: Li et al., 2013b).**

### 2.3.2 Ring-radial Model

Following the two-dimensional continuum model, Lam and Newell (1967) proposed a more specific network configuration, namely a circular network with radial streets. The circular configuration can represent the ring-road system in cities and it was formulated for ease of computation compared to the general two-dimensional model. Based on the Lam-Newell formulation, studies by Blumenfeld and Weiss (1970a, 1970b) considered two rings in a radial network to investigate route choice. Similarly, D'Este (1987) solved the trip assignment problem in a circular city with radial major roads to minimise the individual travel time. Furthermore, a formulation of this minimisation problem has been proposed by (Wong, 1994) and his work has proved the existence of a unique minimum. Li et al. (2013a) have adopted the ring-radial configuration in the analysis of optimal radial density. The proposed ring-radial system (Figure 2.4) can reduce the complexity and the amount of computation required of the two dimensional monocentric city model. It is worth noting that a ring-radial system can be regarded as the orginal two dimensional model if the radials in the ring-radial system are extremely dense. Considering this extreme situation, the ring-radial system can be regarded as a simplified model of the two dimensional monocentric city model.

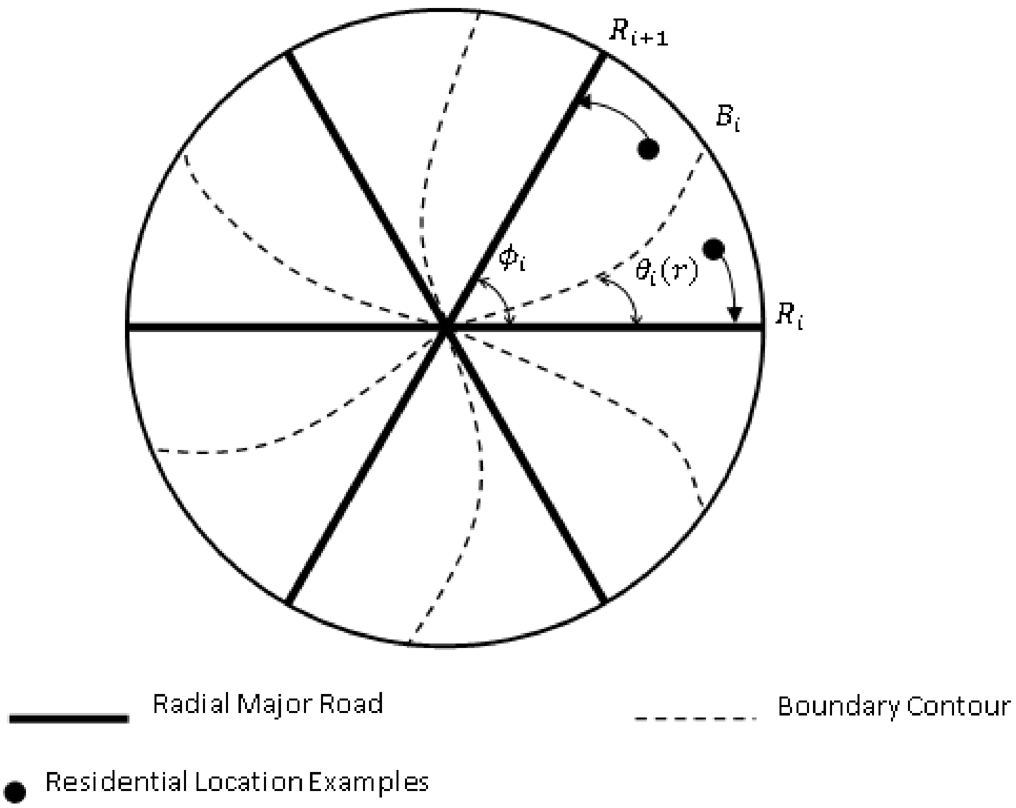


**Figure 2. 4  Ring-radial Model (Source: Wong, 1994).**

### 2.3.3 Linear Monocentric City (LMC) Model

If all radials in a ring-radial system (reviewed in Section 2.3.2) attract the same amount of demand or are evenly located, the system performance can be estimated from the result of one radial. In such cases, one radial could represent the global phenomena in a circular city, which formulated the idea that a linear monocentric city can be considered as a representation of a circular city. Indeed, the configuration of the LMC has been widedly used in transport studies as shown in Table 2.1. The LMC model can represent the morning commute into the CBD along a corridor. Commuters make the trip from their own location to the city centre.

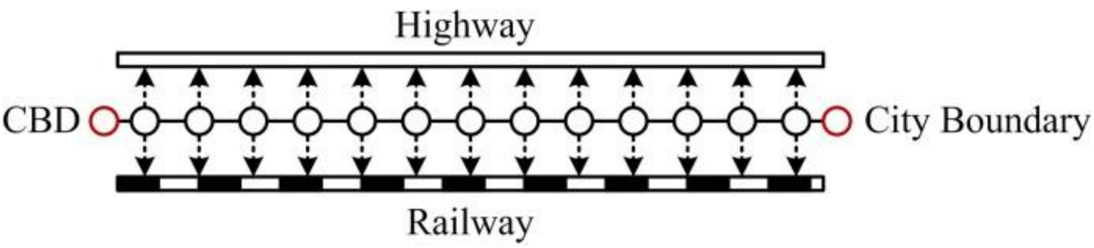


**Figure 2. 5 Linear Monocentric City Model.**

Taguchi and Iri (1982) suggested that (1) the LMC model is easier to build from the practical standpoint of data gathering; (2) its solution helps us to intuitively understand the global characteristics of road networks; (3) the amount of computation does not depend on the size of road networks. Following their work, the LMC model with a single mode has been widely used in bus/rail line design (Wirasinghe and Seneviratne, 1986; Liu et al., 1996; Furth and Rahbee, 2000; Tian et al. 2007; Chen et al., 2014) and congestion pricing (e.g. Mun et al, 2003; Liu et al., 2014).

Moreover, Jehiel (1993) has explained how people make choices among several congetible modes in the LMC. In Figure 2.5, a simple example of the LMC model with two modes is shown and it is formulated for this thesis. Commuters will choose the cheaper mode based only on travel time from their origin to the common destination at the CBD. Commuters do not change mode along their journey. In such cases, Jehiel (1993) has derived mode choices at user equilibrium.  His work provides the theoretical basis for studies of the LMC model.

**Table 2. 1 Linear Monocentric City model in transport studies.**

| Authors | Congestible modes | Modes | Themes |
|---|---|---|---|
| Taguchi and Iri (1982) | Single | Single | theoretical advance |
| Wirasinghe, Seneviratne (1986) | Single | Single | rail line design |
| Jehiel (1993) | Several | Several | theoretical advance |
| Liu et al. (1996) | Single | Single | rail line design |
| Furth and Rahbee (2000) | Single | Single | bus line design |
| Mun et al. (2003) | Single | Single | congestion pricing |
| Wang et al. (2004) | Single | Two | park-and-ride facilities |
| Tian et al. (2007) | Single | Single | congested transit system |
| Liu et al. (2009) | Two | Two | park-and-ride facilities |
| Li et al. (2012a) | Two | Two | congestion pricing |
| Li et al. (2012b) | Single | Two | rail line design |
| Liu et al. (2014) | Single | Single | congestion pricing |
| Yushimito et al. (2012) | Single | Two | park-and-ride facilities |
| Chen et al. (2014) | no | Single | transit line design |
| Du and Wang (2014) | Two | Two | park-and-ride facilities |

As shown in Table 2.1, the LMC model has been applied in various themes of transport studies. Table 2.1 summaries the formulation of transport modes in previous studies. For the design of a public transport line, studies often formulated the LMC model with a single mode (e.g. Wirasinghe and Seneviratne, 1986; Lit et al., 1996; Furth and Rahbee, 2000; Tian et al., 2007; Chen et al., 2014), as these studies focus the performance of single mode, for instance, profit maximisation of a railway line. For the investigation of locations of park-and-ride facilities, studies then considered two transport modes in the LMC model (Wang et al., 2004; Liu et al., 2009; Yushimito et al., 2012; Du and Wang, 2012). In the study of congestion pricing, the formulation of transport may be different for various purposes and transport modes are usually congestible, because the emergent congestion is one crucial factor. Therefore, formulations of transport modes in the LMC model are designed differently for various topics. In this thesis, two transport modes will be considered since the development of road and public transport network is the theme of the thesis (Section 1.2). Meanwhile, these two modes are congestible as the arising congestion under urban growth will be discussed. In addition, traffic assignment in the LMC model with two congestible transport modes will provide some theoretical advance as only few studies considered two congestible transport modes.

In the end of Section 2.3, the thesis summaries reasons of using the LMC model as follow:

First, if the discrete modelling approach is used to investigate the evolution of cities, historical data of urban features for setting up changes in the network is necessary. But the collection of data is generally time consuming and labour intensive and some resources may not be available, particularly for historical data. When the historic data of a transport system is insufficient, the LMC model (a continuum modelling approach) can be a candidate since the LMC model needs a small amount of data to set up the network.

Second, the LMC model can be used for the initial phase of planning and modelling for a regional study, in which the general trend and patterns of population distribution and travel choice of users are easy to investigate, and their changes in response to policy changes in the transport system at the macroscopic level can be observed.

Third, this thesis is more concerned about how the performance of transport systems scales under urban growth, the possible strategic plan of improvement in transport systems and the expansion of the transport system under different population sizes of a city etc. In such cases, it may be premature and ineffective to form a detailed transport network for analysis. Additionally, the results that are obtained from the discrete analysis are so detailed that they may obscure the proper interpretation of the system responses.

Finally, the LMC model can be used for the investigation of mode choices and congestion under network growth since mode choices and urban dynamics under the urban growth have not been examined well in the previous research.

Hence, this thesis is suggesting that the LMC model is valuable for the macroscopic modelling of a very dense transport system under urban growth. The LMC model can facilitate the interpretation of features during network growth, and help to observe the macroscopic phenomena of transport systems in response to major changes in a city.

## 2.4 Residential Allocation and Transport Influence

Section 2.4 prepares the basic material for modelling urban dynamics in this thesis. Land Use and Transport Interaction models have been briefly introduced in section 2.4.1. Studies on the interpretation of how people make decisions on their residential locations will be briefly introduced in Section 2.4.2. Section 2.4.3 includes a short review of residential location choice and transport influence under economic growth.

### 2.4.1 Land Use and Transport Interaction Models

Urban spatial structures and land use patterns have been shaped by improvements in transport systems (Anas et al., 1998). Some empirical evidence of travel mode choice and land use were reported by (Zhang, 2004) and McDonald and Osuji (1995) studied how land use patterns varied only under the improvement of transport systems. Moreover, the relationship between the mode choices and housing allocation of residents can be regarded as the interaction between land-use and transport (Chang, 2006).

In order to investigate urban dynamics in cities, this thesis is going to examine how transport influences land use patterns. Following the model by (Carroll, 1955), the integration of residential locations and the use of transport have been studied in many Land Use and Transport Interaction (LUTI) models for transport planning (Wilson, 1998). In recent studies, one stream of LUTI models is based on urban economics. For example, UrbanSim (Waddell 2002) is a microeconomic model of location choice of households and firms. Economic models like UrbanSim themselves have not developed transport models but relied on existing transport model or only considered monetary travel cost. Another stream is to consider system dynamics in LUTI model. For instance, MARS model (Emberger et al. 2006) uses causal loop diagrams from systems dynamics to obtain the optimal development strategy for transport systems (Pfaffenbichler et al., 2010; Shepherd, 2014). It worth noting that models similar to MARS include some subsystems and many traffic analysis zones that need plenty of data sources to set up the model.

### 2.4.2 Bid Rent Theory in Residential Allocation

After Alonso (1964) established the foundations of a mathematical model for urban land use, studies of urban economics investigated urban land use problems using the ‘monocentric city’ as a prototype. Books by Muth (1969), Mills (1972), Anas (1982) and Fujita (1989) established the basis of urban land use, bid rent theory. Note that the amount of money people are willing to pay is called ‘bid rent’. The term ‘bid’ describes the behaviour of decision-makers. Individuals are assumed to bid for a preferred location. These pioneers studied the bid rent process in the configuration of the LMC model. Similar to Section 2.3.3, the city formulation is given as follows:

(1) The city is monocentric; that is, it has a single prespecified centre of fixed size called CBD. All job opportunities are located in the CBD.

(2) There is a dense transport system. It is free of congestion. Furthermore, the only travel is that of workers commuting between residences and work places. Travel within the CBD is ignored.

(3) The land is a featureless plain. All land parcels are identical and ready for residential use.

In this context, one spatial characteristic in housing allocation that matters to individuals is the distance from the CBD. In the economic analysis of consumer behaviour, it is assumed that an individual will maximize his utility subject to a budget constraint when the individual seeks a residence in the city. If a utility function is defined as $U(z, \mathrm{g})$ where $z$ represents the amount of consumption of goods or services and $\mathrm{g}$ represents the area of house space. The individual earns a fixed income $Y$, which is spent on rent, travel costs and the consumption of goods or services. If the household is located at distance $x$ from the CBD, the budget constraint is given by

$$z + \mathrm{g}R(x) = Y - T(x) \tag{2.8}$$

where $R(x)$ is the rent per unit of land at $x$ and the travel cost from location $x$ is represented by $T(x)$.

In the LMC model, accessibility to the city centre worsens as the distance increases even if there is no congestion. People prefer the high accessibility of being close to the city centre and to live in a big house. However, people's income is limited and the house space at each location is finite. In such cases, people need to trade off the accessibility of being close to the city centre and move to the suburbs of the city when they bid for their houses. The result of bidding for houses is that the rent price per unit space may be different, namely locations with high accessibility should have a high rental price and those with low accessibility should have a low rental price. It is reasonable that people living at the edge of a city may be able to buy more land for the same amount of money. With the bid rent process, a spatial equilibrium arises when nobody in LMC would like to move (Fujita, 1989). At the spatial equilibrium, everybody enjoys the same utility. In this thesis, the process of bid rent in the LMC model will be introduced in detail in Chapter 4.

### 2.4.3 Urban Dynamics in the Linear Monocentric City

The spatial equilbrium that has been introduced in Section 2.4.1 may affect land use patterns in cities. Moreover, it is affected by congestion at each location of the transport systems, because the travel cost will affect individuals' choice of residence (as the budget constraint in Equation 2.8). Since this thesis considered the LMC model as a candidate model, Section 2.4.2 focuses on a brief review of the relationship between land use and transport influence in the LMC. This relationship was firstly investigated by (Solow and Vickrey, 1971) and (Solow, 1972). The optimal density of households along the corridor in the LMC was discussed by Vaughan (1985) and Huang (1997). Moreover, this relationship has been included in the study of congestion pricing (Verhoef, 2005) and the development between property and railway line (Li et al., 2013c).

In the literature (e.g. Muth, 1969; Mills, 1972; Anas, 1982; Fujita, 1989), two assumptions of city configuration have been proposed: (1) in the open-city model, households are assumed to be able to move costlessly across the city boundary; hence, the utility of residents equals that of the rest of the economy, which is exogenously fixed, while the population of the city is determined endogenously; (2) in the closed-city model, the population of the city is exogenous. The closed-city model under the absentee landownership (CCA model) can be used to represent the residential location choice. In a CCA model, all landlords own and rent out properties but do not live within the city. Then the rent revenue of landlords and the increase in their income due to rent may be excluded for simplification.

Wheaton (1974) considered an open and closed city in a comparative static analysis of urban spatial structure and the paper showed the difference in rent revenue in both types of cities. (Wheaton, 1982) studied the residential location choices when landlords maximised their rent and then the research extended the investigation to population and economy growth. Wheaton (1982) focused on the investigation in an open city, whereas (Pines and Sadka, 1986) extended the study in a fully closed city and examined the effects of the exogenous variables on per capita income and aggregate travel demand. In a closed city, Wheaton (1998) also investigated variations of urban forms with transport influence when landlords maximised their rent revenue in three cases: (1) without congestion; (2) with congestion and exogenous transport capacity; (3) with congestion and optimal transport capacity. One limitation of these works is that the mode choice is excluded or the parameter in BPR function is 1, and thus two congestible travel modes have rarely been included. In the meantime, urban economic features have been examined well in previous studies, such as Anas and Moses (1979), DeSalvo and Huq (1996) and DeSalvo and Huq (2005). In summary, there has been a long history of investigating urban economic features when variations of land use patterns arise in a monocentric city.

For the housing allocation in a monocentric city, (Arnott et al., 1999) provided a general model that could be used for the static/dynamic situation of residential allocation. (Braid, 2001) presented a theoretical model of housing allocation under spatial growth. (Lin et al., 2004) examined the spatial equilbrium under a certain urban land policy where the housing investment and pricing have been increased. (Zhang, 2007) dealt with a dynamic interaction between land use, the housing market, and conditions of transport networks under economic growth. Following the discussion in Capozza(1976), a recent study (Anas, 2012) presented a general equilibrium framework of optimal housing allocation dealing with the pricing, finance and supply of urban transport systems. The study investigated the revenue from congestion tolls under population growth and the increasing investment in public transit and roads. Note that the public transport was assumed to be congestion free and the congestion on roads arises under a large population.

In the previous studies, residential allocation and transport influence have been examined under various types of growth, e.g. economic, spatial. Here, the thesis concentrates on how transport influences land use patterns or spatial features instead of urban economic features. It is also necessary to investigate the variation in mode choice and congestion in transport systems under the simultaneous improvement of road capacity and public transport.

## 2.5 Summary of Chapter 2

In Chapter 2, this thesis reviewed various previous studies that enlighten the work from different aspects, as there are some research gaps that need to be investigated further. The following statements will summarise the research gaps from the previous studies.

In the classic modelling approaches (Section 2.1), although they can help us make decisions on how to improve the capacity of an individual link or build a new link etc, they could not represent the phenomena of urban dynamics well. Because they simplified the process of transport planning in several steps and ignored the emergent variations of transport networks. If we use these models to investigate the evolution of transport networks, it means we need to set up the network and demographic data for every demand size. In such cases, the amount of computation would be quite large if these models investigated aggregate performance in the whole transport network.

Models in network science (Section 2.2.1) may be feasible in a large network and examine global features well and they have been used in the study of network growth. However, traffic features were not included in models of network science. In particular, the travel cost along the link a complex network is only dependent on the physical distance but independent on the traffic volume. Moreover, models in network science usually examine the global features of road and public transport networks respectively. Hence, they have not represented a city with integrated transport systems well. In this thesis, the aim is to investigate the aggregate performance of an integrated transport system under urban growth, but the formulation of models in network science is not very suitable here.

As mentioned in Section 2.3, continuum modelling approaches could represent a city (including a dense transport network) and the global features (e.g. total travel time, mean travel time per trip) could be easily obtained. With the review of the continuum modelling approach, the thesis finds that the LMC model is a feasible method to investigate emergent phenomena under network growth. With the literature review in Section 2.3.3, this thesis suggested that two modes (public transport and highway) within the LMC model be appropriate in the investigation of network growth.

Since Section 1.1 explained the correlation between urban dynamics and urban growth, Section 2.4 has reviewed studies of land use for the LMC model following one theory of residential allocation. In many studies of land use and transport interaction, the housing allocation in a linear monocentric city with two congestible travel modes has not attracted much attention so that the study in this case will provide theoretical advance. Furthermore, the co-evolution of residential location and mode choice under the growth of transport networks has not been examined in a continuum model. Investigating this co-evolution, we can understand urban dynamics and the influence of development in transport systems further.

When this chapter ascertains research gaps in modelling approaches, some research gaps in empirical observations of spatial features in transport networks have been explained in Sections 2.2.2 and 2.2.4. In brief, circuity in public transport networks has not been reported and the correlation between circuity and accessibility need to be examined. Then the comparison of spatial efficiency in road and public transport networks has attracted little attention, which could empirically explain the reason of mode choice (Page 8).

In summary, this chapter provides a literature review for this thesis so that research gaps in theoretical modelling approaches and empirical studies have been stated.  With these research gaps, the following three chapters in this thesis will investigate the research questions (Section 1.2).

# Chapter 3
# Modelling Urban Growth in a Linear Monocentric City

Since Chapter 1 proposed RQ 1and Chapter 2 summarised several research gaps in Sections 2.3.3 and 2.5, the LMC model has been a candidate model to investigate the aggregate phenomena in transport systems. So Chapter 3 starts to adopt and develop the LMC model in modelling urban growth.

Following RQ 1, Section 3.1 explains the research question further and introduces scaling laws and previous studies of the Linear Monocentric City (LMC) in detail. Section 3.2 includes a model formulation of the LMC for the analysis in Chapter 3 so that Section 3.3 derives the global performance (i.e. Total Travel Time and Mean Travel Time) at User Equilibrium. The data needed in this chapter is sourced in Section 3.4.

With the work in Sections 3.2, 3.3 and 3.4, Section 3.5 investigates the network performance under network growth. In the end, Section 3.6 summarises findings in Chapter 3.

## 3.1 Introduction

As introduced in Section 1.1, increasing urban population is concomitant with rapid urbanisation, the physical growth of urban areas and social and economic development. By defining the city boundary as the perimeter of a populous region (Glaeser, 1998), urban population has been used as an important indicator to measure city size. In recent years, empirical studies observed that characteristics of urban systems grew with population following scaling laws (Kühnert et al., 2006 and Bettencourt et al., 2007). These studies provided insights into how various urban characteristics changed as a city grew, and highlight whether characteristics increased or decreased per capita. Such understanding should underpin any examination of the sustainability and future prosperity of cities. In particular, Lämmer et al. (2006) presented empirical results on geographical features of road networks which obey invariant scaling laws. The growth of transport networks obviously contributes to reducing urban congestion, while increasing population exacerbates it. The question arises: how does transport network performance (urban congestion) evolve under different regimes of simultaneous growth in the transport network and urban population?

Based on the literature review in Chapter 2 (Sections 2.2.2 and 2.2.3), scaling laws often arose when describing the relationship between the size of an organism (e.g. a biological entity, a network) and its characteristics. A scaling law occurred when two variables in a system (e.g. a city, a mammal) changed simultaneously following a power law. A general formula of scaling law is $Y = Y_0 N^{\beta}$ (Bettencourt et al., 2007), in which for transport systems $Y$ could represent a network characteristic (e.g. road surface area, total road length) and $N$ would be the city population or traffic demand. A scaling law resides in one of three growth regimes: (i) $0 < \beta < 1$, sub-linear; (ii) $\beta = 1$, linear; (iii) $\beta > 1$, super-linear. According to empirical observations across different cities (Bettencourt et al., 2007), the road surface area scales with population following a sub-linear power law ($\beta = 0.83$). The sub-linear regime means that when the population doubles, the area of road surface increases but does not double; with $\beta = 0.83$, the per capita share of road surface therefore decreases. The empirical pervasiveness of such scaling laws in urban networks suggested that some physical characteristics should be independent of network topology and population distribution. They may therefore be relevant to studying both collections of different cities and when applied to an individual city as it grows over time.

As stated in Section 2.3.3, the continuum modelling approach has been employed to investigate global features of transport systems at a macroscopic level in several previous studies (Taguchi and Iri, 1982; Jehiel, 1993; Ho and Wong, 2006). These emergent features in a continuum model are independent of network structures, so are scaling-laws phenomena. One continuum model, namely the LMC model, has been used on road pricing, e.g. (Verhoef, 2005), park-and-ride facilities (Wang et al., 2004; Liu et al., 2009) and regulatory regimes for bi-modal transport systems (Li et al., 2012a). In the LMC model, the detailed network configuration is summarised by simple spatial factors such as the distance between the city boundary and

Central Business District (CBD), and the road capacity along the corridor. This representation fits well with the global aggregated characteristics that appear in empirically identified urban scaling laws, whose resolution does not extend to details of the network (spatially) nor within day dynamics (temporally). Furthermore, the LMC model is relatively tractable and simple scenarios can be examined analytically.

Hence, within the LMC model, Chapter 3 considers the morning commute into the CBD along a corridor that extends from the city boundary to the CBD. Two modes are available to all travellers: train (public transport mode) and private car. This chapter assumes that commuters choose the cheaper mode based only on travel time from their origin to the common destination at the CBD. Meanwhile, travellers do not change mode along their journey. In this basic scenario, Jehiel (1993) explained how “watershed” solutions arose where travellers’ mode choice gave a simple partition about some point $\bar{x}$ i.e. at every location closer to the CBD than $\bar{x}$, everyone takes the train, and from further out than $\bar{x}$ everyone chooses car. The watershed $\bar{x}$ is also called the “equilibrium point” since this is where the travel cost to the CBD is equal on both modes. In this context, Wang et al. (2004) derived the watershed solution for the uniform demand distribution by assuming that (i) the travel time by train at CBD is less than by car at CBD; (ii) the travel time by car from the city boundary to CBD under free flow condition is less than by train. With these conditions, travel time curves intersect at the equilibrium point $\bar{x}$ (Figure 3.1 (a)). Figure 3.1 (a) only shows a watershed case with one congestible and one congestion-free mode under the deterministic mode choice and it will be discussed in sections 3.2, 3.3 and 3.5. This watershed case may not arise when the intersection of two travel cost profiles are not unique (e.g. Figure 3.1 (b)). In Figure 3.1 (c) and (d), two congestible cases are considered and the difference is that two travel cost profiles in Figure 3.1 (c) has only one intersection. The watershed case has not been observed in cases of Figure 3.1 (c) and (d). Hence, with the deterministic mode choice, the watershed case is still dependent on the assumption of travel cost profiles.

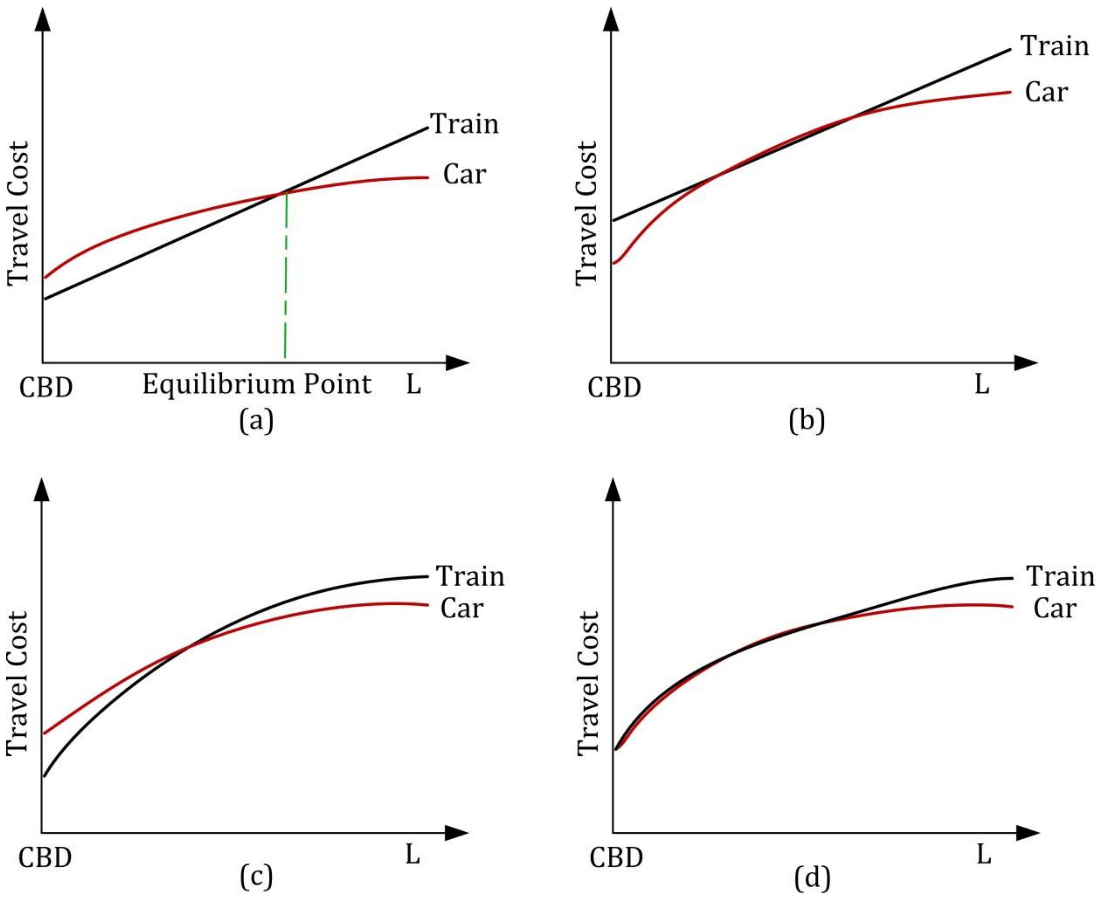


**Figure 3. 1 (a) a watershed case with one congestion-free train line and one congestible highway; three cases without the watershed point : (b) one congestion-free train line and one congestible highway, (c) and (d) considered two congestible modes. The red line shows travel time by car (if all commuters take cars). (Wang et al. 2004, Liu et al. 2009)**

## 3.2 The Configuration of a Linear Monocentric City

In the LMC, a corridor of length $l > 0$ connects the city boundary and the CBD. Total demand in the city is $N > 0$. In Chapter 3, exogenous demand distributions are considered as previous studies (e.g. Wang et al, 2004). Here, it is assumed that all commuters are continuously distributed along the corridor with demand density $n(x)$ where $x$ is the distance from CBD; $x \in [0, l]$. Hence, total demand is given by

$$N = \int_0^l n(x)dx, \tag{3.1}$$

where $n(x)$ could represent various demand distributions, such as a uniform demand distribution $n(x) = N/l$. $N/l$ indicates the constant demand density per unit distance along the corridor.

Even though there may be various traffic modes for residents to commute in a city, this thesis summarises the available transport into two modes: rail (public transport mode) and highway for private cars. All commuters can access the highway or railway everywhere along the corridor. At an arbitrary location $x$ on the corridor, the density of commuters choosing car or train is $n_C(x)$ and $n_T(x)$, respectively, so that

$$N = \int_0^l [n_T(x) + n_C(x)]dx. \tag{3.2}$$

According to mode split, for each location, the proportion of choosing car and train is $p_C(x)$ and $p_T(x)$, respectively. It holds that $p_T(x) + p_C(x) = 1$. For the mode choice density at $x$, the number of commuters choosing one mode is given by

$$n_T(x) = p_T(x)n(x), \tag{3.3}$$

$$n_C(x) = p_C(x)n(x). \tag{3.4}$$

The cumulative traffic volume by train and car is determined by $p_T$ and $p_C$ along the corridor from the city boundary to location $x$. The volumes of passengers on train and car (i.e. $v_T$ and $v_C$) are given by:

$$v_T(x) = \int_x^l n_T(w)dw \ = \int_x^l p_T(w)n(w)dw\ , \tag{3.5}$$

$$v_C(x) = \int_x^l n_C(w)dw \ = \int_x^l p_C(w)n(w)dw\ . \tag{3.6}$$

### 3.2.1 Generalised Travel Time by Train

In the beginning investigation, Section 3.2 assumes that the railway line is congestion free following many previous studies (see Table 2.1), because the public transport mode is less sensitive to congestion compared to highways. Hence the travel time by train is independent of the traffic volumes, $v_T$ and $v_C$. Travel time by train from $x$ to CBD is given by

$$G_T(x) = \int_0^x t_T(w)\, dw \tag{3.7}$$

where $t_T(w) \ = f_T$ and $f_T$ is the constant travel time per unit distance by train. Moreover, the function $t_T(w)$ can be changed for a congestible train line.

### 3.2.2 Generalised Travel Time by Car

In Section 3.2, this work assumes that travel time on the highway is affected by congestion (see Table 2.1). Then traffic volume $v_C$ can vary continuously along the corridor. Moreover, Section 3.2.2 assumes that road capacity $C$ is constant along the corridor. Using a BPR function, travel time per unit distance is given by

$$t(v_C(w)) = f_C\left[1 + A\left(\frac{v_C(w)}{C}\right)^B\right] \tag{3.8}$$

where $\mathrm{A}$ and $B$ are parameters, $f_C$ is free-flow travel time per unit distance. A high value of parameters means that an increase of traffic volume $v_C$ drives the delay of commuting longer. Additionally, the increasing value of $B$ may make congestion more severe as it is a power in BPR function.

The generalised travel cost by car from $x$ to CBD is

$$G_C(x) = \int_0^x t_C\big(v_C(w)\big)dw. \tag{3.9}$$

### 3.2.3 Model Assumptions

Following Sections 3.2.1 and 3.2.2, some assumptions are needed for the arising user equilibrium in this two-mode monocentric city model (Jehiel, 1993).

**Assumption 3.1** $G_T(0) \leq G_C(0)$.

With the generalised travel time above, $G_T(0) = G_C(0) = 0$ at $x = 0$ where indicates there is no travel time if people are in the city centre.

**Assumption 3.2** *The free flow travel cost by car is less than the free flow travel cost by train.*

Then this work requires $f_T > f_C$ indicating the free-flow travel time per unit distance by train is longer than that by car. In such a case, commuters choose driving before the highway is congested; then commuters choose to take public transport when the highway is in the congestion. Note that if this work was to set $f_T \leq f_C$ , commuters would not drive car, because commuters may save more travel time and the train line is congestion free.

## 3.3 User Equilibrium and Network Performance

### 3.3.1 User Equilibrium in the Linear Monocentric City

As Figure 3.1 shows that a watershed solution will be found (Section 3.1), this chapter directly seeks the location of equilibrium $\bar{x}$ where everyone chooses to use car for locations $x \in [\bar{x}, l]$ and no one chooses car for locations $x \in [0, \bar{x}]$. At "inner" locations the car traffic flow is constant, no additional car flow arises and the traffic volume arises from locations outside of $\bar{x}$. In such a case, the total number of commuters using car and train at location $x$ is given by

$$v_C(x) = \begin{cases} \int_{\bar{x}}^{l} n(x)dx & , x \in [0, \bar{x}] \\ \int_{x}^{l} n(x)dx & , x \in [\bar{x}, l] \end{cases}. \tag{3.10}$$

$$v_T(x) = \begin{cases} \int_{x}^{l} n(x)dx - \int_{\bar{x}}^{l} n(x)dx & , x \in [0, \bar{x}] \\ 0 & , x \in [\bar{x}, l] \end{cases}. \tag{3.11}$$

As the watershed case of the volume by Equation 3.10 and 3.11, the travel cost by car at user equilibrium $G_C$ has two regimes, as follows:

$$G_C(x) = \begin{cases} \int_{0}^{x} t\big(v_C(\bar{x})\big)dw & , x \in [0, \bar{x}] \\ \int_{0}^{\bar{x}} t\big(v_C(\bar{x})\big)dw + \int_{\bar{x}}^{x} t\big(v_C(w)\big)dw & , x \in [\bar{x}, l] \end{cases} \tag{3.12}$$

where the second term for $x \in [\bar{x}, l]$ is zero when $x = \bar{x}$. The equilibrium solution is found from $G_C(\bar{x}) = G_T(\bar{x})$. For a uniform demand distribution (i.e. $(x) = Q/l$ ), the location of equilibrium can be written in closed form:

$$\bar{x} = l - \frac{lC}{N} \sqrt[B]{\frac{1}{A}\left(\frac{f_T}{f_C} - 1\right)} = l\left(1 - \frac{C\theta}{N}\right), \tag{3.13}$$

where for convenience the constant $\theta = \sqrt[B]{\frac{1}{A}\left(\frac{f_T}{f_C} - 1\right)}$ is defined. By Assumption 3.2, $\theta > 0$. The equilibrium point cannot exceed the city boundary since $C\theta/N$ is always strictly positive. For small values of total demand, namely $N\epsilon(0, C\theta)$, $C\theta/N > 1$ implies that $\bar{x} \leq 0$. The road never becomes congested and all commuters drive by car to the CBD. For $N\epsilon[C\theta, +\infty)$ or $0 < C\theta/N \leq 1$ the equilibrium solution $\bar{x}\epsilon(0, l)$. Note that the location of equilibrium itself is not our main focus, but it aids our analysis of the network performance in the following sections.

### 3.3.2 Total-Travel-Time(TTT) under User Equilibrium

Total Travel Time is useful to measure the network performance. For the LMC model with a simple watershed solution, the TTT can be disaggregated into three parts: (a) $TTT_1$ for commuters between $\bar{x}$ and CBD who take train; (2) $TTT_2$ is the cost for car commuters between $\bar{x}$ and the CBD, dependent on the traffic volume $v_C(\bar{x})$; (3) $TTT_3$ for car commuters between the city boundary and $\bar{x}$, resulting from the accumulating traffic volume $v_C(x)$.

For the uniform demand distribution, the travel time by train to CBD is given by Equation 3.7 and $n_T(x) = N/l$, where $p_T = 1$. Hence,

$$TTT_1 = \int_0^{\bar{x}} G_T(x) \cdot n(x) dx, \tag{3.14}$$

$$TTT_1 = \begin{cases} 0, & \bar{x} \le 0 \\ \frac{f_T Q \bar{x}^2}{2l}, & 0 \le \bar{x} \le l \end{cases}. \tag{3.15}$$

$TTT_2$ is given by

$$TTT_2 = v_C(\bar{x}) \cdot G_C(\bar{x}). \tag{3.16}$$

since $G_C(\bar{x}) = G_T(\bar{x})$, $TTT_2$ is :

$$TTT_2 = \begin{cases} 0, & \bar{x} \le 0 \\ f_T \bar{x} \cdot N\left(1 - \frac{\bar{x}}{l}\right), & 0 \le \bar{x} \le l \end{cases}. \tag{3.17}$$

Finally

$$TTT_3 = \int_{\bar{x}}^{l} n(x) \int_{\bar{x}}^{x} t_C\big(v_C(w)\big) dw dx. \tag{3.18}$$

For the uniform demand distribution, $TTT_3$ is given by

$$TTT_3 = \begin{cases} \frac{N f_C l}{2} + \frac{l f_C A N^{B+1}}{C^B (B+2)}, & \bar{x} \le 0 \\ \frac{N f_C l}{2}\left(1 - \frac{\bar{x}}{l}\right) + \frac{l f_C A N^{B+1}\left(1 - \frac{\bar{x}}{l}\right)^{B+2}}{C^B (B+2)}, & 0 \le \bar{x} \le l \end{cases}. \tag{3.19}$$

To sum up, $TTT = TTT_1 + TTT_2 + TTT_3$. With low demand, $0 < N \leq C\theta$ then $\bar{x} \leq 0$ and $TTT_3 = \frac{Nf_C l}{2} + \frac{lf_C AN^{B+1}}{C^B(B+2)}$ with $TTT_1 = TTT_2 = 0$. In the limit, with everyone driving at near free flow conditions, $TTT = TTT_3 \rightarrow \frac{f_C Nl}{2}$.

### 3.3.3 Mean-Travel-Time (MTT) under User Equilibrium

This thesis considers network performance under urban growth. Whether or not the transport network performance improves (however it is measured) it is likely that population growth causes TTT to increase. Moreover, if the city expands horizontally then TTT also tends to increase due to the increasing commuting distance. For these reasons, Mean Travel Time may be a useful indicator of network performance. MTT is obtained from:

$$MTT = \frac{TTT}{N}. \tag{3.20}$$

From Equations 3.15, 3.17, 3.19 and 3.20, MTT can be written for the uniform distribution as follow:

$$MTT \begin{cases} \frac{f_C l}{2} + \frac{lf_C AN^B}{C^B(B+2)}, & \bar{x} \leq 0 \\ \frac{f_T l}{2} - \frac{Bl(f_T - f_C)}{2(B+2)}\left(\frac{C\theta}{N}\right)^2, & 0 \leq \bar{x} \leq l \end{cases} \tag{3.21}$$

Note that for $N \gg C$ (i.e. $C/N \rightarrow 0)$, the road is highly congested and the dominant proportion of commuters take the train, in which case, MTT approaches $f_T l/2$. Whereas, as noted above, for very low demand $TTT \rightarrow f_C Nl/2$ and hence $MTT \rightarrow f_C l/2$.

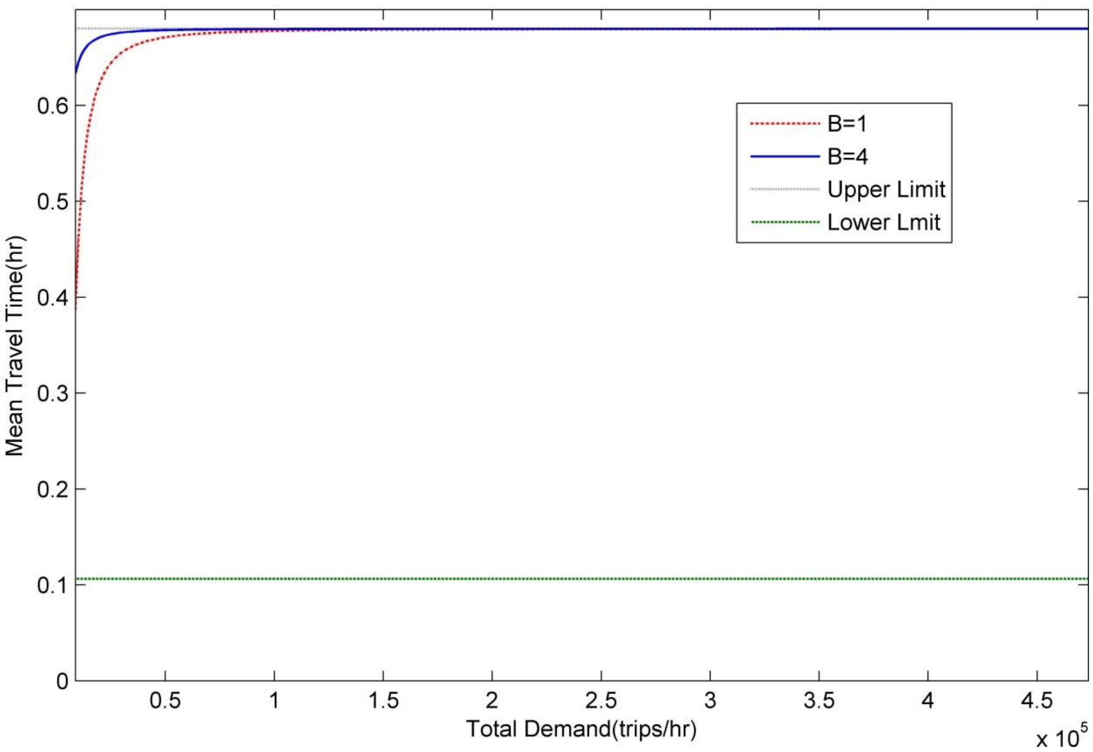


**Figure 3. 2 Mean-Travel-Time in Base Scenario (only the total demand grows).**

To begin investigating the network performance via MTT, this section consider growth in total (uniformly distributed) demand. The MTT changes according to

$$\frac{dMTT}{dN} = \frac{BlC^2\theta^2}{N^3(B+2)}(f_T - f_C), \qquad 0 \le \bar{x} \le l \tag{3.22}$$

Since $B > 0$ and $f_T > f_C$ , $\frac{dMTT}{dN}$ is positive and the second derivative is negative. Therefore, as shown in Figure 3.2, MTT is increasing and concave-down with respect to $N$. Figure 3.2 also shows the limits of the MTT, the lower limit (i.e. $\frac{f_C l}{2}$) arises from the free flow travel time on the highway and the upper limit (i.e. $\frac{f_T l}{2}$) when all commuters use the train. As cities develop, such upper and lower limits give us comparators to evaluate the evolution trajectory of a city. Not surprisingly, if the city is populous and the roads are congested especially toward the CBD, people choose public transport. On the contrary, if the city is sparsely populated, every resident can drive at near free flow conditions.

## 3.4 Data Inputs in Chapter 3

For the scenarios in this chapter, data are sourced for the synthetic city based on various empirical studies. A LMC is considered with the corridor between the city boundary and CBD of length $l = 17km$ and urban area $22.5km^2$(Tong and Wong, 1997). This chapter assumes that the population density of the city increases and take the lower limit from (Levinson, 2012) to be $776pp/km^2$ and the upper limit (Tong and Wong, 1997) for a high density city to be $42{,}089\ pp/km^2$. The morning commute is taken to be a two-hour peak period with the average number of trips per resident = 1 trip in the studied period; therefore, the total demand ranges from $8730\ trips/hr$ to $473501\ trips/hr$.

Other parameters are given by $A = 1$ , $f_T = 0.08\ hr/km$ (average travel speed of using train to commute is $12.5km/hr$), $f_C = 0.0125\ hr/km$ which is given by the free flow travel speed $80km/hr$. The exponent $B$ represents how fast the road becomes congested, the following sections investigate its effect with $B = 1, 4$ respectively.

Initially in Case I (Section 3.5.2.1), the low density city begins with small road capacity ( $C = 500\ veh/hr$) which increases following the scaling law. Just as population density cannot increase indefinitely, cities have physical spatial constraints limiting the capacity of roads. As road capacity increases, it is necessary to consider the physical space used by the transport network and account for the trade-off with other land use (e.g. residential, business). To include this in our model, this chapter sets a nominal upper limit to the road capacity ($C_{max} = 10000\ veh/hr$). Under the urban growth modeled here, the road capacity will not be increased beyond this limit, even if demand continues to increase.

Case II (Section 3.5.2.2) examines the effect on network performance of increasing demand while extending the physical city size. In Case II, the road capacity is fixed ($C = 2000\ veh/hr$).

## 3.5 Investigation of Network Growth

### 3.5.1 Preliminary Case with Demand Growth

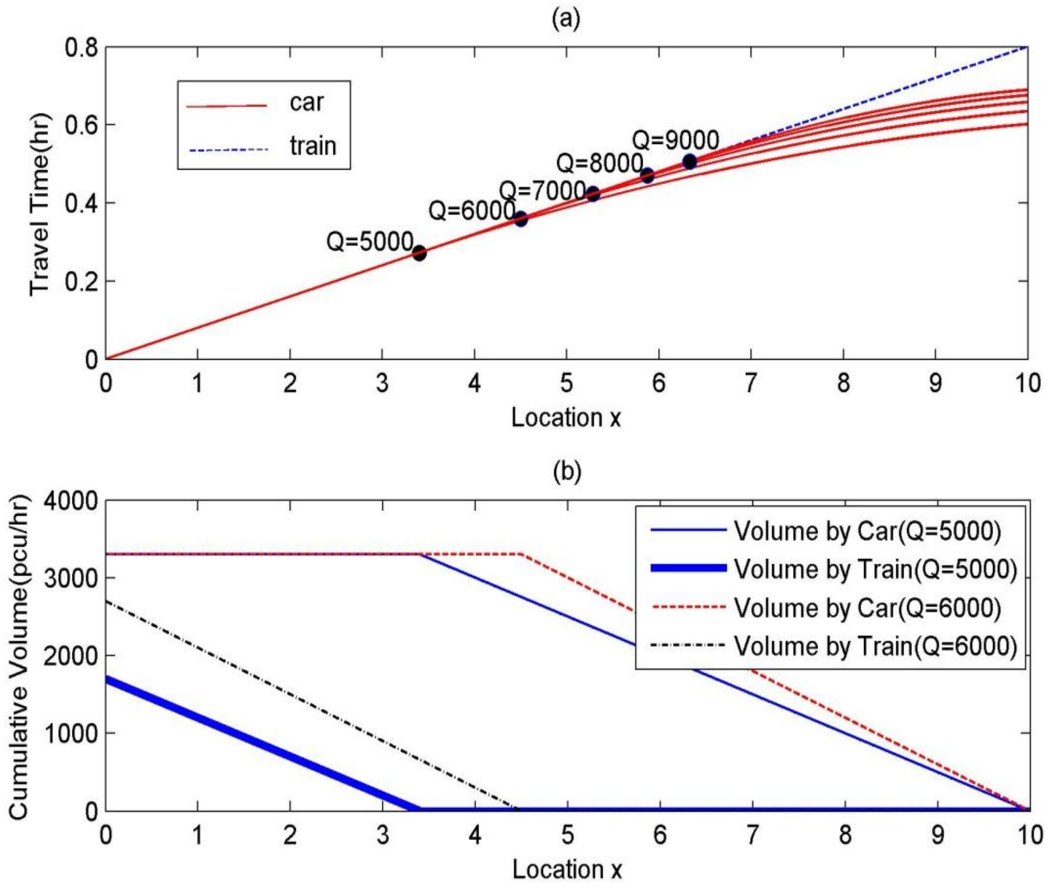


**Figure 3. 3 Variation of Equilibrium Point and Cumulative Volume in Basic Scenario (B=1). (a)Travel Time by car and by train with various demand levels (black dots are Equilibrium points and B=4); (b) Constant traffic volume by car with the increased total demand.**

According to Equation 3.13, if both $L$ and $\mathrm{C}$ are held constant, as demand increases the term $lC\theta/N$ decreases and hence $\bar{x}$ moves toward the city boundary. Figure 3.3(a) shows how the equilibrium point varies as the total demand growth. Figure 3.3(b) provides an example of demand split by train and car when total demand increases from 5000 to 6000.

For the purposes of investigating urban growth scaling laws in the context of this model, this chapter considers population (city size) growth to coincide with the increasing traffic demand. Under the conditions above the simple watershed solutions can be found. The resulting network performance depends on network characteristics (e.g. total demand, road capacity).

### 3.5.2 Exerting the Influence of Scaling Laws

Since empirical observations have been reported of scaling laws linking road surface area and city size (population) (Bettencourt et al., 2007), a measure describing the land area used by roads is necessary to apply such scaling laws in our model. Road capacity is easily included in the analysis and can be thought of a proxy for road width; roads of small width having lower capacity than those with large width (multiple lanes). The measure of Total-Length-Capacity ($TLC$) is therefore proposed to represent the area of road surface in the network, $TLC = \sum_{i=1}^{k} l_i C_i \, (km \cdot veh/hr)$, where $l_i$ and $C_i$ are the length and capacity of link $i$, $\mathrm{k}$ is the number of roads in the given area and the unit of $TLC$ is defined as kilometre times vehicles per hour. If $\mathrm{TLC}$ grows with demand according to a super-linear scaling law ( $\beta > 1$ ), this corresponds to road surface area expanding more rapidly than demand, so that even as demand increases the $\mathrm{TLC}$ per capita is increasing. This would require high investment in the road infrastructure. Whereas if the road infrastructure expands more slowly under demand growth, in the sub-linear regime as is reported empirically ($\beta < 1$), the $\mathrm{TLC}$ per capita decreases.

In Chapter 3, growth in population and road infrastructure are connected by the following scaling law:

$$lC = \rho_0 N^{\beta} \tag{3.23}$$

where $\rho_0$ is a constant, $l$ indicates the location of city boundary, $C$ is the road capacity and $\beta$ is the scaling exponent. The constant $\rho_0 = \frac{lC_0}{N_0^{\beta}}$ is determined by the initial link capacity $C_0$ and initial traffic demand $N_0$ and as such identifies the initial state of network. To discuss the urban sustainability, the road surface per capita $\mathrm{y}$ is given by,

$$y = \frac{lC}{N} = \rho_0 N^{\beta - 1}. \tag{3.24}$$

The change in road surface per capita is

$$\frac{\Delta y}{y} = (\beta - 1)\frac{\Delta N}{N}. \tag{3.25}$$

The sub-linear ($\beta < 1$) and super-linear ($\beta > 1$) scaling regimes therefore correspond to the road surface decreasing/increasing per capita (Bettencourt et al., 2007).

In this chapter, two cases of growth are considered: vertical and horizontal. In Case I (vertical growth) the population density increases. The network has fixed corridor length while demand and road capacity increase together, according to a scaling law. In Case I the consequences of assuming infinite capacity on the train are also examined. In Case II (horizontal growth) the city physically expands, so that the corridor length grows as demand increases while the road capacity remains constant. In Case II a non-uniform distribution of demand is considered.

### 3.5.2.1 Vertical Growth Scenarios (Case I)

#### 3.5.2.1.1 Case I with a congestion-free train line

In Case I, the corridor length is fixed while the population density (hence total demand) and road capacity grow together. The railway line has fixed per-km cost and is effectively uncongested. The road capacity increases until it reaches the imposed upper limit. This occurs when the demand increases to 322490trips/hr ($\beta = 0.83$), 174600trips/hr ($\beta = 1$) or 112980trips/hr ($\beta = 1.17$).

With $Q_0$ and $C_0$, the constant $\rho_0$ (from Equation 3.23) can be calculated. The MTT is then:

$$MTT_V = \frac{f_T l}{2} - \frac{B{\rho_0}^2 N^{2\beta-2}\theta^2(f_T - f_C)}{2l(B+2)} \tag{3.26}$$

where $MTT_V$ denotes MTT with the vertical growth.

Taking the derivative analysis, it holds

$$\frac{dMTT_V}{dN} = (1-\beta)\frac{B{\rho_0}^2 N^{2\beta-3}\theta^2(f_T - f_C)}{l(B+2)}, \tag{3.27}$$

$$\frac{d^2MTT_V}{dN^2} = (1-\beta)(2\beta-3)\frac{B{\rho_0}^2 N^{2\beta-3}\theta^2(f_T - f_C)}{l(B+2)}. \tag{3.28}$$

Based on Equations 3.27 and 3.28, the nature of $MTT_V(N)$ is independent of the road congestion function (BPR exponent $B$). But the scaling law exponent $\beta$ can help to understand to what extend the investment on the road infrastructure can reduce/affect the network performance with the population growth. It is no doubt that more investment can decrease congestion when $\beta > 1$ and the congestion can increase for $0 < \beta < 1$. Even though $\frac{dMTT_V}{dN}$ is always decreasing under the super-linear regimes, urban congestion will be eased efficiently for $1 < \beta < 1.5$. These results apply only when the road capacity is below its finite upper limit.

Figure 3.4(a) (b) shows MTT curves for different values of $\beta$ and $B$. For road capacity below the upper limit, the shapes of the MTT curves are governed by the scaling exponent as Equations 3.27 and 3.28 explain. Once the road capacity reaches $C_{max}$, the road infrastructure remains constant and hence the MTT curves then follow trajectories such as those seen in Figure 3.3, increasing toward the MTT upper limit.

Another indicator is the Average Travel Speed (ATS). This will be of more interest in later examples but is introduced here for completeness (Figure 3.4 (c) and (d)).

$$ATS = \frac{MTD}{MTT} \qquad (3.29)$$

where MTD denotes Mean Travel Distance. For the uniform demand distribution, MTD is $L/2$. So $ATS_V$ is written as

$$ATS_V = \frac{1}{f_T - N^{2\beta-2} . \frac{B{\rho_0}^2\theta^2(f_T - f_C)}{l(B+2)}}. \qquad (3.30)$$

With Equation 3.30, the derivative is found with $\frac{dATS_V}{dN} > 0$ when $\beta > 1$ corresponding to increasing traffic speed in the case of rapid, super-linear growth in road capacity. When $\beta = 1$, $\frac{dATS_V}{dN} = 0$, and for the sub-linear case ($0 < \beta < 1$) the slower growth in road capacity versus traffic demand gives $\frac{dATS_V}{dN} < 0$.

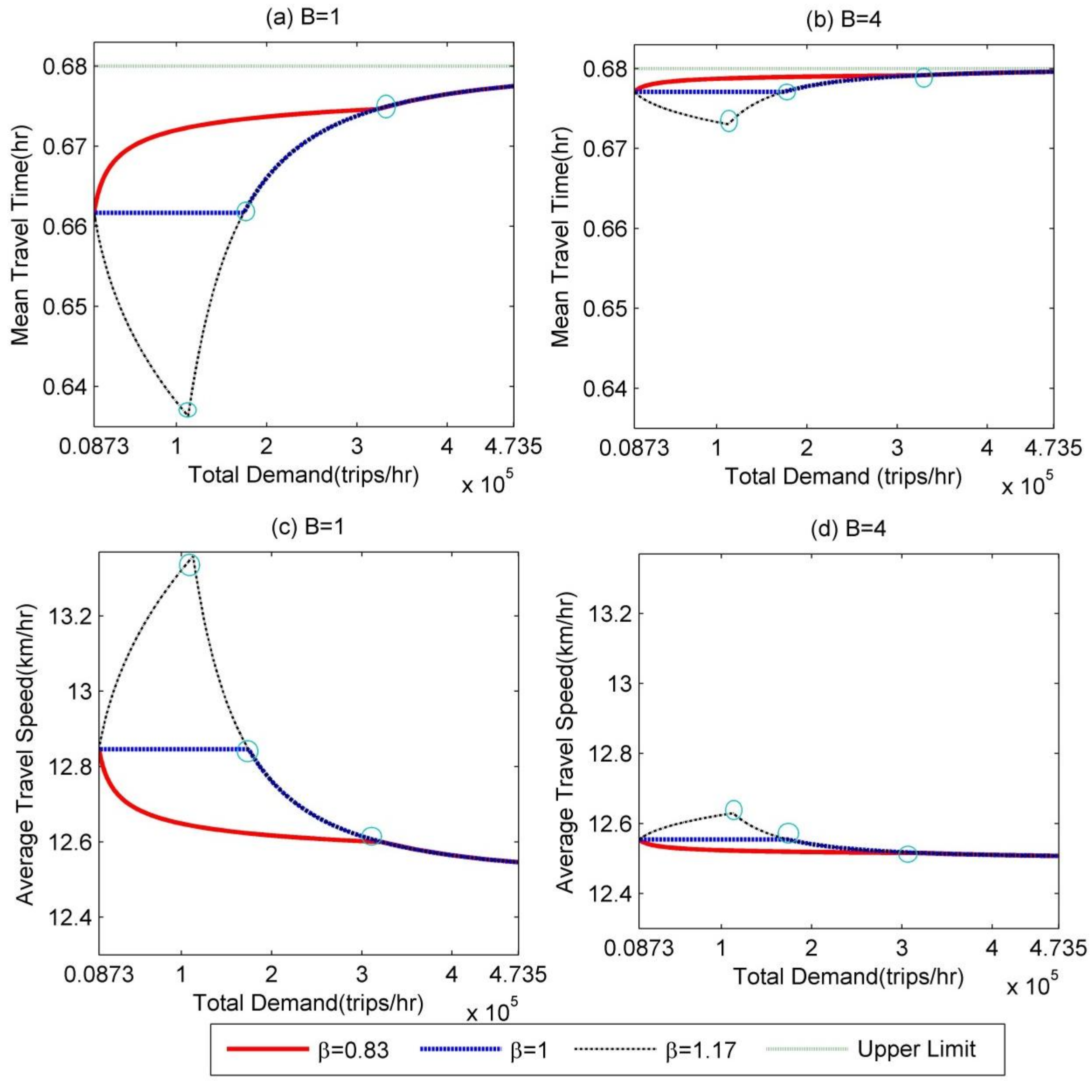


**Figure 3. 4 Network Performance under Scaling-law Growth with the Road Capacity Constraint and Congestion-free Train. Circles denote where road capacity reaches its upper limit.**

#### 3.5.2.1.2 Case I with a congestible train line

Thus far the railway has been congestion free, and able to absorb all 'excess' traffic once the road is sufficiently congested so that per-km travel time by car is the same as that by rail. Inherently this suggests that the rail capacity is sufficient to accommodate whatever portion of the total demand is assigned to it. As noted above, this effectively establishes an upper limit on the network MTT.

It is unrealistic to imagine that the rail mode has unbounded capacity, and instead this section investigates the rail mode expands as required to accommodate the assigned demand. This would allow us to examine the nature, cost and plausibility, of the rail capacity expansion to allow the scaling law relationship to hold between total demand and road capacity.

The number of people using rail is given by

$$N_T = \int_0^l n(x)dx - \int_{\bar{x}}^l n(x)dx \tag{3.31}$$

which is equal to the total demand minus the number of people using car between $\bar{x}$ and $l$.

For the uniform distribution,

$$N_T = N - C\theta. \tag{3.32}$$

Under scaling law growth,

$$N_T = N - \frac{\rho_0 N^\beta}{l}\theta, \tag{3.33}$$

where the second term represents the number of people using car between $\bar{x}$ and $l$. As the road capacity is increasing by the scaling law, how fast the train needs to accommodate the increasing demand is dependent on the derivative of Equation 3.33, $\frac{dN_T}{dN} = 1 - \beta\frac{\rho_0 N^{\beta-1}}{l}\theta$. Once the road capacity reaches the capacity constraint, it can be known from the derivative of Equation 32, $\frac{dN_T}{dN} = 1$.

Figure 3.5 shows how the number of commuters increases on the train under various scaling-law regimes with/without the road capacity constraint. Compare to Figure 3.5 (a), the change of slopes is founded in Figure 3.5 (b) where the capacity reaches $C_{max}$, although the difference can be hardly seen for $B = 4$. Meanwhile, following the derivation in Equation 3.33, the number of trip by car varies as Figure 3.6 shown. The number of trips by car shows the opposite tendency, compared to $N_T$. With a higher scaling exponent, the road capacity has been increased more under the demand growth so that the number of trips by car is higher (the black dashed line).

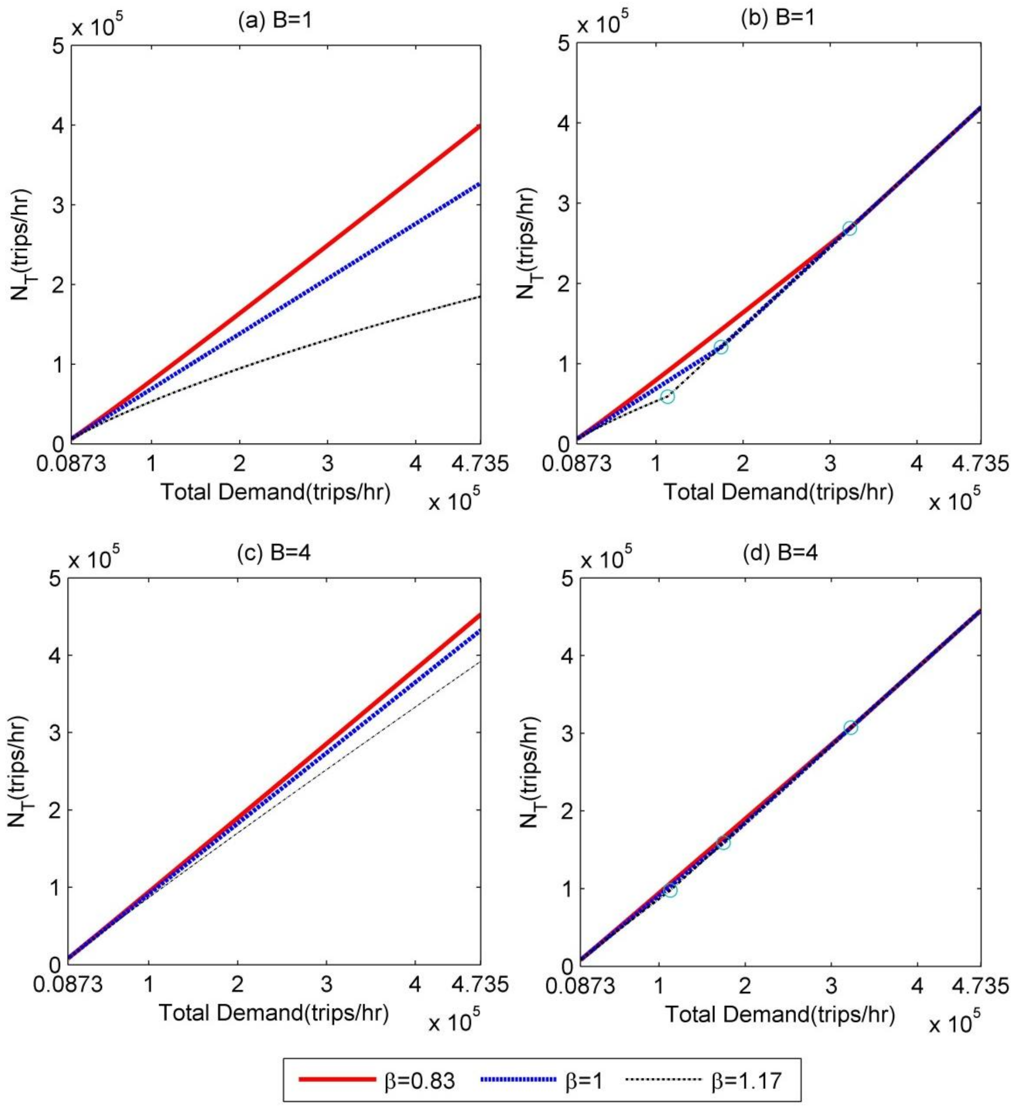


**Figure 3. 5 Required rail capacity under scaling law growth. (a) and (c) without road capacity constraint; (b) and (d) with road capacity constraint. Circles denote where road capacity reaches its upper limit.**

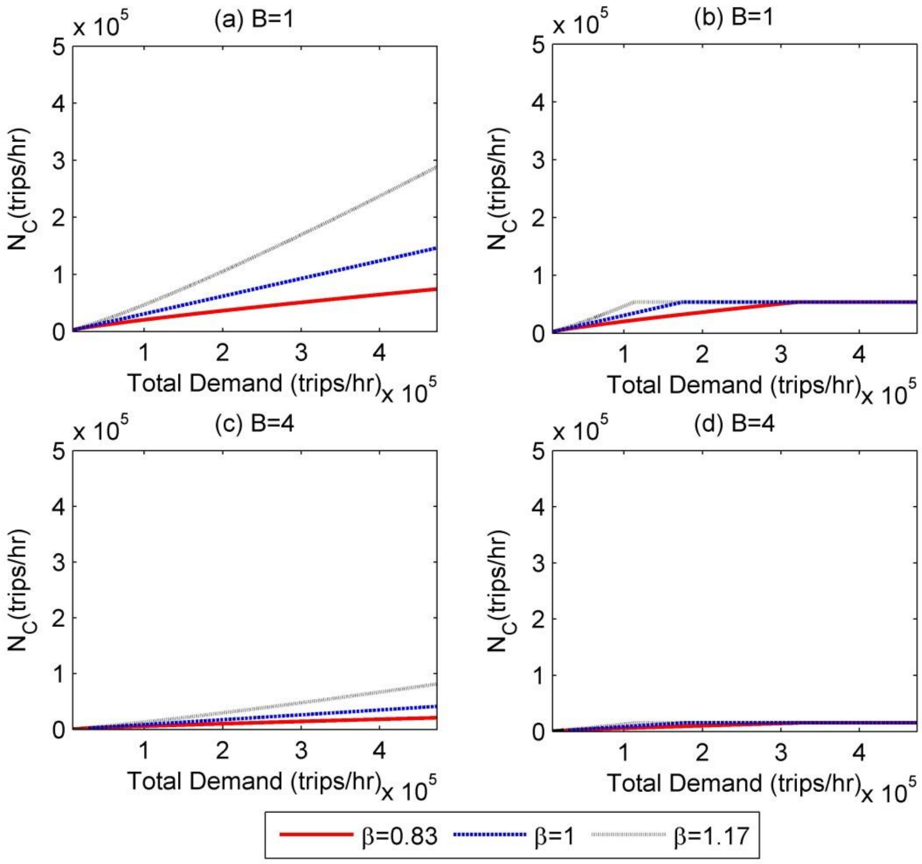


**Figure 3. 6 $N_C$ under scaling law growth. (a) and (c) without road capacity constraint; (b) and (d) with road capacity constraint.**

As mentioned after Equation 3.8, for the same traffic volume $N_C$, the travel cost is not only dependent on the road capacity $C$ but also the power $B$ in BPR function. The increasing value of power $B$ in BPR function may make congestion severe as a high value of power $B$ means that an increase of traffic volume $N_C$ drives the delay of commuting longer. In Figures 3.6 (b) and (d), the maximal road capacity is assumed to be 10,000 trips/hour regarding to the spatial limit but it does not mean that the number of trips on the road will not exceed 10,000 tips. In the mode choice, the reason that people choose train is the travel cost by car is not lower than that by train. For example, for $\mathrm{B} = 1$, the travel cost by car is not lower than that by train when the cumulative traffic volume by car is 54000 trips; for $\mathrm{B} = 4$, the travel cost by car is not lower than that by train when the cumulative traffic volume by car is 15244 trips. Though road capacity is limited to 10000 trips/hr, the power in BPR function influenced the travel cost by car. Hence, the maximal number of car trips is 54000 trips in Figure 3.6 (b) and 15244 trips in Figure 3.6 (d).

To extend the current model, this section considers the case where the train line will be congested once the number of people increases around some amount $C_T$, with travel cost per unit distance $t_T(w)$ given by:

$$t_T(v_T) = \frac{f_T}{(1-v_T/C_T)^2}, 0 \le v_T < C_T. \quad (3.34)$$

At flows well below capacity, the per-km train travel cost increases only slightly with $v_T$, loosely representing in-vehicle congestion. As $v_T$ approaches the capacity limit $C_T$, the cost of train travel increases very rapidly. Other rail cost functions could be used, but the important phenomenon this chapter wants to represent is that the travel cost by train is triggered to go up fast only under sufficiently high levels of demand so that the mean travel time of systems is increasing.

When the total travel demand is sufficiently small, the resulting mode choice is similar to the watershed cases described earlier. However, if the total demand is large (relative to road plus rail capacity), the model will have three regimes: (1) similar to the basic scenario, commuters near the city boundary will choose to drive by car according to Assumption 3.2 and give rise to highway congestion; (2) when the per-km cost of congested car travel rises to equal that of the uncongested railway, commuters begin to split between road and rail. At high levels of demand density, the road becomes congested very quickly and hence the train is in use very close to the city boundary; (3) when the travel cost by train tends to be infinite as the cumulative traffic flow on the train exceeds $C_T$ at a location, people have to drive to city centre from that location.

As before in Case I, this section considers vertical growth so the corridor length is fixed. As demand grows the capacity of the road network is set to increase following a scaling law till the capacity constraint $C_{max}$. The train line is now congestible with $C_T = 20000 trips/hr$. One difference between the road capacity and train capacity $C_T$ is that the maximal trips by train cannot exceed the constraint as Equation 3.34 defines, but the number of trips by car may exceed $C_{max}$ (see discussion on Page 61). As Figure 3.7

shows, the results are markedly different from the congestion-free train line (Figure 3.4). In Figure 3.7 (a) (c), MTT curves are increasing and the ATS curves are decreasing before the capacity increases to $C_{max}$. Once the road capacity reaches $C_{max}$, the network performance curves for different scaling exponents converge onto the same line, for a given $B$. Figures 3.7 (b) and (d) include a reasonable range ($< 3\mathrm{hr}$) for commuting time for a better comparison with Figures 3.7 (a) and (c).

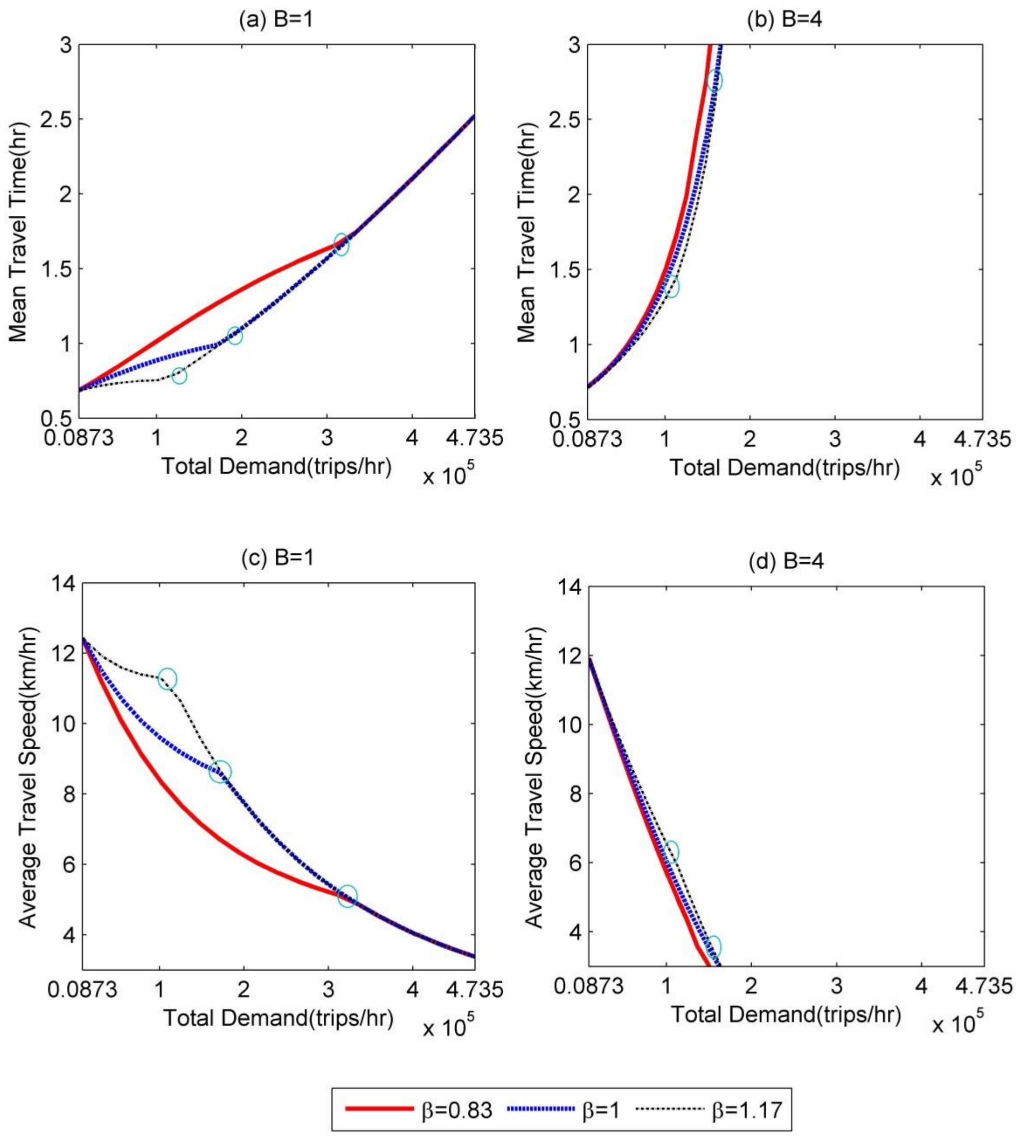


**Figure 3. 7 Network performance under scaling law growth with the road capacity constraint and a congestible train. Circles denote if road capacity reaches its upper limit.**

Note that, in Figure 3.4 the congestion-free train establishes a fixed limit on the cost of travel in this simple city network. Whereas in Figure 3.7 it is seen that the city can evolve along a wide variety of performance trajectories, with MTT increasing unboundedly, depending on the relative growth in transport infrastructure as the city population increases.

#### 3.5.2.2 Horizontal Growth Scenarios (Case II)

##### 3.5.2.2.1 Case II with the uniform demand distribution

In this scenario the city grows horizontally, albeit in a rather idealised way where the corridor extends with constant capacity per unit distance. Meanwhile, the railway line of the city is always available as an alternative and hence the railway also extends to the same length as the highway. This assumption is to simplify the scenario for a simple investigation. As demand increases, the city will extend horizontally with the TLC, and hence the length of corridor, determined by the scaling law. The road length can be calculated by Equation 3.23 and MTT by Equation 3.21.

The results of Case II are shown in Figure 3.8. In this scenario, the per-km road capacity is fixed while total demand and road length increase from some initial point $N_0$ and $l_0$, so $l = r^{\beta} l_0$. Since the capacity is held constant the constant $k = \frac{C}{Q_0}\theta$ is defined, so that $0 < k < 1$. If write the demand ratio is defined as $r = \frac{N}{N_0}$, and $r \geq 1$. Using $l_0$, the constant $k$ and $r$, Equation 3.21 is rearranged as follows:

$$MTT_H = \frac{f_T l_0 r^{\beta}}{2} - \frac{B l_0 r^{\beta-2} k^2 (f_T - f_C)}{2(B+2)}, 0 \leq \bar{x} \leq l \tag{3.35}$$

$$\frac{dMTT_H}{dr} = \frac{\beta f_T l_0 r^{\beta-1}}{2} - \frac{B(\beta-2) l_0 r^{\beta-3} k^2 (f_T - f_C)}{2(B+2)} \tag{3.36}$$

where H is for Horizontal. For $\beta > 2$, the derivative in Equation 36 is positive, though this would require enormous investment to increase the transport infrastructure so rapidly with population growth. Otherwise, the second term in Equation 36 is negative and $\frac{dMTT_H}{dr} > 0$ so that the MTT is always increasing for $0 < \beta < 2$. In this case the city limit extends and the (average) travel distance of commuters increases. In this case, the performance measure MTT cannot distinguish between changes to the average distance travelled and changes to congestion. Therefore ATS is perhaps a more useful measure of network performance. In Case II, ATS is given by:

$$ATS_H = \frac{1}{f_T - \frac{Bk^2(f_T - f_C)}{(B+2)r^2}}, \; 0 \le \bar{x} \le l. \quad (3.37)$$

According to Equation 3.37, $ATS_H$ is a function of the total demand $Q$, but surprisingly is independent of both corridor length $L$ and scaling exponent $\beta$. Under different scaling law exponents, at the same level of demand it can be therefore concluded: (1) small $\beta$ giving rise to a compact city with short corridor length; (2) large $\beta$ giving a rapidly expanding city with long corridor length. The demand density is changing like $q \sim \frac{C}{\rho_0 N^{\beta-1}}$ and the corridor length is extending as $l \sim N^{\beta}$. Here it is found that the growth in demand, the growth in corridor length and the change at equilibrium point, hence the mode split all balance so as to cancel out the scaling law exponent. As before, the results shown in Figure 3.8 include different scaling exponents, but the effect is cancelled out. It is only the power in the road link cost function that distinguishes between different networks. Figure 3.8 also includes the lower limit of ATS, which occurs when $N \to +\infty$, and as all travelers take the train so that $ATS_H \to \frac{1}{f_T}$.

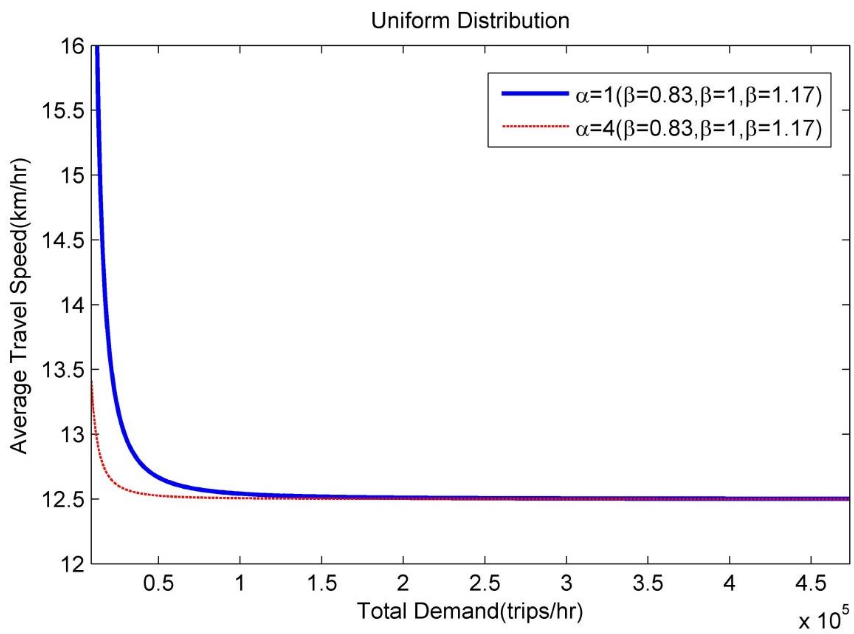


**Figure 3. 8 Network performance in Case II with the uniform demand distribution.**

#### 3.5.2.2.2 Case II with a non-uniform demand distribution

To extend our model, a non-uniformly demand distribution can be considered. As has often been assumed in previous urban studies (Li, Lam, Wang, 2012a), a population density following a negative exponential distribution between CBD and the city boundary is considered,

$$n(x) = d_0 e^{-\mu x} \tag{3.38}$$

where $d_0$ is the population density at the CBD and $\mu$ is a constant to describe how fast the density decays as $x$ increase. When $\mu = 0$, the demand is uniformly distributed as Chapter 3 has already discussed above.

The Total Demand is given by

$$N = \frac{d_0}{\mu}(1 - e^{-\mu l}). \tag{3.39}$$

As Chapter 3 already discussed for the uniform demand distribution, travel time for driving and by train in two regimes at user equilibrium can be calculated as Equation 3.13. The equilibrium solution is found from $G_C(\bar{x}) = G_T(\bar{x})$ and in this case:

$$\bar{x} = -\frac{\ln M}{\mu} \tag{3.40}$$

where $M = \sqrt[B]{\frac{1}{A}\left(\frac{f_T}{f_C} - 1\right) \cdot \frac{\mu C}{d_0} + e^{-\mu l}}$. $M$ is positive. $M \epsilon [0,1]$ corresponds to the equilibrium point being between the CBD and the city boundary. When $M \epsilon (1, +\infty)$, $\bar{x} \leq 0$, in which case every resident uses car. $M$ will decrease when $d_0$ or/and $l$ increase, whereas $f_T$ and $C$ are proportional to the value of $M$.

When $\mu \to 0$, the demand density is a uniform distribution and $N = l d_0$, the equilibrium solution using the equation above is given by:

$$\lim_{\mu \to 0} \bar{x}(\mu) = l - \frac{lC}{N}\theta \tag{3.41}$$

in agreement with previous calculations for the uniform distribution.

Similarly, when $\mu \to +\infty$, it holds

$$\lim_{\mu \to +\infty} \bar{x}(\mu) = 0 \tag{3.42}$$

which is the theoretical case when all residents live in CBD.

Here, this section still considers that the city grows horizontally, extending its city boundary but with a non-uniform demand distribution. Our synthetic city starts with 17460trips demand and city corridor (17km); the demand density at the CBD remains unchanged (5238trips). The city corridor extends as demand increases under different scaling laws. For simplicity, the capacity per unit distance is constant at $C = 2000pcu/hr$. For instance, as Figure 3.9 shows, from the black-circle line to red-triangle line, the total demand increases while maintaining constant demand density at the CBD. Here, this section tries to model the city size increasing with demand growth. As people move into the city, the urban space expands and the decay of demand density with distance from the CBD slows, hence the constant $\mu$ increases.

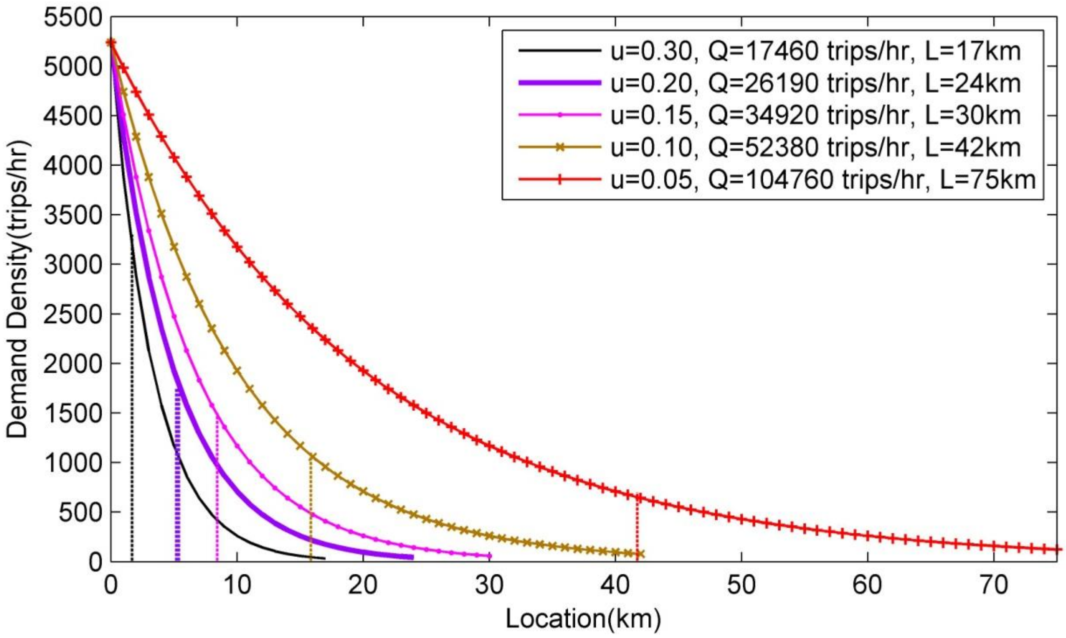


**Figure 3. 9 Demand growth with non-uniform demand distribution ($\boldsymbol{\beta = 0.83}$). Dashed lines indicate the equilibrium point of modal split ($\boldsymbol{A = 1, B = 1}$).**

The scaling law is introduced as previous case, $lC = \rho_0 Q^{\beta}$. Via Equation 3.39, the value of $Q$ and $l$ can be calculated, but it does not readily offer a closed form solution. A more practical way is: for increasing $\mu$, the total demand is estimated from $N = \frac{d_0}{\mu}$ where $l \to +\infty$. On this basis the corridor length using the scaling law can be calculated. This approximation is reasonable, because the demand density is very small when the corridor length is long as Figure 3.9 shows.

As a non-uniform demand distribution has been introduced, the calculation of MTT and ATS becomes complicated. A conventional approach in a continuum model that is the discrete approximation, e.g. (Liu et al., 2009) provides a tractable way to get the network performance. Through this method, the modal split for various demand levels and system performance (ATS) with various values of scaling exponent $\beta$ are in Figures 3.9 and 3.10. Similarly, people at locations close to city centre take the train and others drive to the city centre. Because the mode choice is dependent on the travel cost by train and car at each location, the cumulative traffic volume by train or car at each location affects the arising congestion and then mode choices. For cases where $A = 1$ and $B = 1$, when the cumulative traffic volume by car reaches 11500 trips/hr, people start to take public transport as the travel cost by train is lower. The proportion of demand choosing driving decreases from 66% to 11%, while the proportion of urban space where people prefer driving increases from 10% to 56%. Note that the train line is congestion free, so the increasing demand is attracted by the public transport. Meanwhile, for cases where $A = 1$ and $B = 4$, people prefer public transport when the cumulative traffic volume by car reaches 3500 trips/hr, as the road is sensitive to congestion (i.e. $B = 4$). In such cases, the percentage of demand choosing driving decreases from 20% to 5%, while the proportion of urban space where people prefer driving increases from 5% to 30%.

In the experiment above of Case II, it can be concluded that the nature of ATS is independent of $\beta$ and $l$. Here, different behaviours of ATS with different scaling regimes under the exponential demand distribution have been observed (Figure 3.10). In this case the ATS depends on the scaling-law exponent in an intuitive way: if $0 < \beta < 1$, ATS is decreasing; if $\beta > 1$, ATS is increasing. It is worth noting that there are no capacity constraints on the train in Figure 3.10. The cases expanded the length of the road and kept the same road capacity. In these cases, the watershed case arises at the location close to city centre, where the congestion occurs under a high cumulative traffic flow with an exponential demand distribution. However, for the uniform demand distribution, the congestion occurs at the location close to the city boundary, which reduces the system efficiency more. Hence, ATS in Figure 3.10 is higher than cases in Figure 3.7. Furthermore, the comparison between these two cases highlights how the demand distribution may affect the system performance in the same urban planning strategy.

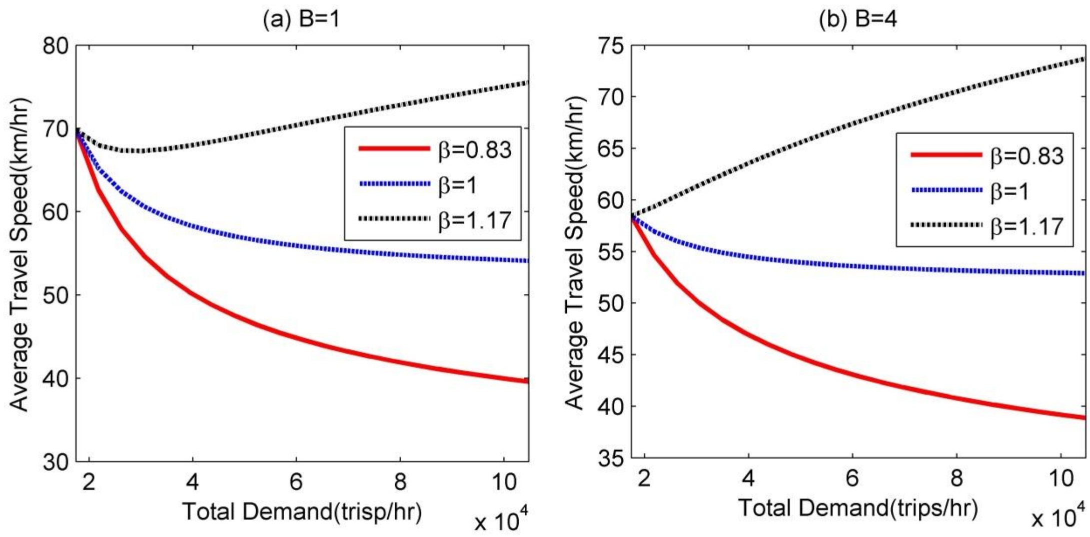


**Figure 3. 10 Network performance in Case II with the non-uniform demand distribution.**

So far, Case I models the urban growth with fixed geographical size. If road capacity is below its finite upper limit, the scaling law exponent regarding to population density and road capacity governs the network performance. The network performance is dependent on $\beta$ if the capacity is increasing following the scaling law. Once the road capacity reaches its upper limit, population continues to grow but the road infrastructure no longer expands and hence MMT increases toward an upper limit determined by the rail mode. The consequences of assuming a congestion-free train line have been examined. Through the comparison of Figure 3.4 and 3.8, Chapter 3 can observe the variation in evolutionary trajectories before and after the congestion on the train mode is included. The interpretation of the assumption is insightful for the future research.

Meanwhile, Case II considers the case where new demand appears at locations further from the CBD, extending the physical size of the city. With a uniform demand distribution, the interaction of the growth in demand, corridor length and the change in average network performance can be derived analytically and it is found that changes to the ATS are independent of both corridor length and the scaling-law exponent. The network performance evolves differently when a negative exponential demand distribution is considered. Case II highlights the significance of scrutinizing different demand distribution in this LMC model with scaling-law growth.

Since the work in Chapter 3 has considered a bi-modal transport system, the variations of mode choice or modal split is one focus here. But the mode choice arises in a particular range of total demand for different demand distribution. So the demand levels are different from cases of vertical growth. It indicates that the model proposed here has limitations of the demand levels and distributions. Moreover, it suggests that transport planners should not only consider the demand levels but also the demand distributions when we consider a bi-modal transport system, as demand distributions influence the cumulative traffic volume (i.e. congestion) significantly. Furthermore, it is worth noting that the work in Case II is not suggesting a negative exponential demand distribution is better than a uniform demand distribution, though it display better system performance, because the demand distribution in real cities is difficult to be pre-determined and it is usually dependent on the residential allocations of commuters. Here, cases of different demand distributions from previous studies (e.g. Wang et al., 2004 and Li et al., 2012 etc.) show how the demand distribution may affect the system performance. In future work, the assumption of demand distribution may be relaxed when we consider the housing allocation.

In summary, the cases examined here, which are based on reported empirical studies, show some possible evolutionary trajectories for cities under various scaling-law regimes, with different road congestion functions, under horizontal and vertical growth and with different demand distributions. Scaling-law regimes with various scaling exponents could drive the different investment/expansion strategies. These results suggest that the investment/expansion regimes and network features need to be integrally considered in the investigation of network evolution.

## 3.6 Summary of Chapter 3

This chapter uses a continuum approach, the LMC model, as the basis to interrogate the plausibility of empirical scaling laws that purport to describe the evolution of urban transport networks under population/demand growth. Though the model and the cases in Section 3.5.2 are simple, they present that various scaling exponents (or different scaling regimes) could drive the network performance trajectories to vary. It is noted that the average network performance can be independent of scaling exponents, when the growth in both the urban area and the transport systems and the impact of increasing demand balance each other, as illustrated by the constant user equilibrium solution stay constant in Section 3.5.2.2.1. Moreover, one suggestion on modelling the growth of transport infrastructures from Section 3.5.2.1 is the necessity of including two congestible modes.

There are three possible directions to extend the model in Chapter 3 and they are listed below.

(i) Variations of network performance may be dominated by exogenously applied demand distribution. The model should include the housing allocation so that the demand distribution may vary endogenously alongside the evolution of the transport networks.

(ii) As introduced in Section 1.1, the sustainable transport system should not only ensure the efficiency but also reduce environmental pressure. However, cases in Chapter 3 only consider the mean travel time or average travel speed. An indicator of environmental pressure should be included.

(iii) In Chapter 3, improvement of the public transport system has not been considered. As suggested in Sections 2.1.4 and 2.2.4, the simultaneous improvements of road and public transport system are valuable in modelling the growth of urban transport infrastructures and should be included.

# Chapter 4

# Co-evolution of Residential Location and Mode Choice in a Linear Monocentric City

As concluded in Chapter 2 (Section 2.4.2), the variation in mode choice under urban growth has rarely been examined in the LMC model. Moreover, Chapter 3 developed an approach with the LMC model (a continuum modelling approach) to investigate the aggregate phenomena under urban growth. To extend the model from Chapter 3, Chapter 4 attempts to model the co-evolution of residential location and mode choice in the LMC.

As stated in Section 1.2, variations of land use patterns should be emergent under urban growth. Section 4.1 introduces several empirical observations about variations in mode choice and changes in population density in urban areas. Meanwhile, Section 4.1 also reviews some studies on the correlation of land use and transport in detail. In the end of Section 4.1, a framework is designed for urban growth, particularly including the co-evolution of residential location and mode choice.

For the model formulation in Section 4.4, Section 4.2 introduces the bid rent theory in detail and a classic model of spatial equilibrium in the LMC model is included in Section 4.3.

Section 4.5 represents how the mechanism of housing allocation works in the model and then performs the analyses of urban growth. Meanwhile Section 4.5 shows the possible experiments that can be investigated in the modelling framework.

Finally, Section 4.6 gives a summary of Chapter 4.

## 4.1 Introduction

There is a report (Department for Transport, 2014) studying the variation in mode choice in England over years. It reports that the trip rate by private transport modes is decreasing while that by some public transport modes (e.g. bus in London and surface rail) is increasing overall. The trip rate denotes the average number of trips per person per day. In the report, around 80.5 % of trips occur in urban regions. Moreover, the population increases over years as Section 1.1 introduced. It can be concluded that the mode share by driving declines and that by public transport increases under population growth. One reason behind may be that congestion becomes severe in cities. A similar tendency occurs in many cities worldwide although this thesis only includes variations in mode share in England.

Public transport systems should alleviate urban congestion under population growth. Indeed, variations in trip rate by modes shown in the report (Department for Transport, 2014) show that the public transport takes a significant role in reducing urban congestion. Instead of exploring the reason of the variations in trip rate, Chapter 4 is going to model these phenomena so that the modelling framework is useful to investigate the performance of transport systems under population growth. Moreover, the population density at locations or areas varies when cities evolve. As the total number of population in areas increases, the population density of locations then change in different rates, for example, the population density of many areas in UK has changed from 2001 to 2011 (Office for National Statistics, 2012). Once again, similar changes occur in many urban areas. Here, this thesis only shows an example in the United Kingdom.

Since the population density drives land use patterns and residential location changes, urban spatial structures and land use patterns have been shaped by improvements in transport systems (McDonald and Osuji, 1995; Anas et al., 1998). There are many recent studies examining the correlation between transport and land-use in discrete networks (e.g. Chang and Mackett, 2006; Bravo et al., 2010; Ma and Lo, 2012). However, these specific networks stay constant in the experiments so that the consequence of network evolution within cities is not considered. A continuum model, namely the LMC model, is usually adopted to investigate the macroscopic phenomena in cities and also has been applied to analyse residential allocation and mode choice. In the LMC, residential allocation is typically modelled as an individual utility maximisation problem, with equilibrium being reached when no one can achieve higher utility by unilaterally moving to a different location. Hence, at equilibrium all residents have the same maximal utility value (i.e. equilibrium utility). Many researchers have considered the residential allocation problem following a classic model (Fujita, 1989) such as investigating the correlation between congestion tolls and residential allocation (Anas and Xu, 1999) and the calibration of land use patterns under congestion (McDonald, 2009). In particular, (Li et al., 2012c) considered railway line design under property development with a single congestion-free railway line. In their paper, therefore, the system costs are independent of flow and the equilibrium utility is insensitive to population increase and urban growth. Jehiel (1993) has derived user equilibrium among modes so that mode choice within a bi-modal transport system in the LMC model can be investigated for various problems, such as locations of park-and-ride facilities (e.g. Wang et al., 2004) and road pricing (e.g. Anas, 2012). In this context mode choice between two congestible modes has not received much attention. Hence, this chapter will jointly consider mode choices within two congestible modes and residential allocation in the LMC model. Land use patterns and mode choice at each location will be determined under utility equilibrium. In this case, the equilibrium utility is dependent on the demand level so the model allows us to investigate the consequences of urban growth.

For a given population, the residential land use pattern and travel mode choices can be derived in the LMC. Similar to Chapter 3, a scaling law is employed that drives transport infrastructures to increase with population size (Lämmer et al., 2006; Kühnert et al., 2006 and Bettencourt et al., 2007). The underlying notion of scaling laws is that urban growth is accompanied by an increase in many quantifiable urban characteristics (e.g. GDP, road surface). It is noteworthy that the per capita quantity of such characteristics may increase or decrease even in the case of an expanding city. Motivated by empirical studies, this chapter will analyse the emergent phenomena of characteristics in transport systems theoretically, particularly for residential location and mode choice. Note that the growth of economic attributes is excluded from Chapter 4, because there are various pieces of research investigating this problem in the linear monocentric city model (See Sections 2.3.3 and 2.4.2).

In Chapter 4, a modelling framework is designed as shown in Figure 4.1. More details of the modelling framework are introduced in the following sections. The proposed framework is helpful to understand the co-evolution of residential location and mode choice as cities grow. Moreover, the framework can help investigate the performance of transport systems in the LMC. Then the framework can help to answer questions such as when to enhance transport systems and how much to invest in them according to their performance. To sum up, this chapter investigates the co-evolution of transport and land use in order to help policy decision making regarding transport infrastructure improvement under urban growth, with the end goal of achieving urban sustainability.

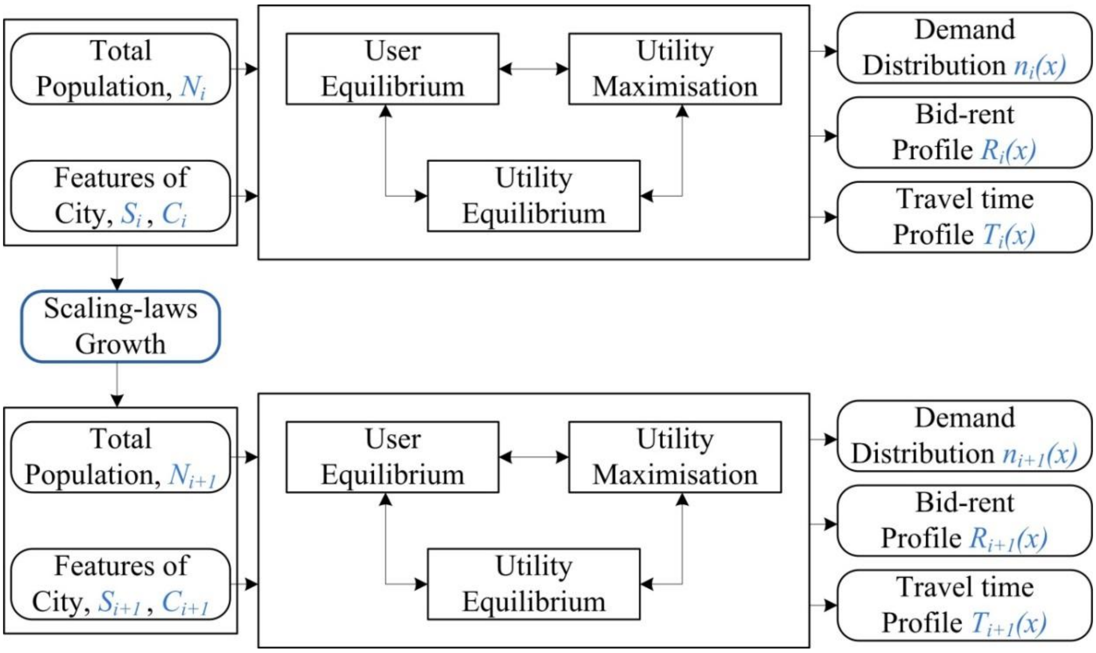


**Figure 4. 1 Modelling Framework of Network Evolution.**

## 4.2 Bid-rent Process and Utility Maximisation for Individuals

In Section 4.2, this thesis introduces how people choose some location $x$ to occupy individually in a linear monocentric city as they trade off their income to seek the maximal utility with an income constraint.

In the LMC model, for each individual, one typical decision is where to live among all available locations. If an individual chooses a certain location, the travel cost from the location to the city centre is then fixed. One trade-off of residents' income is between the total rent and consumption of goods. In such case, individual utility $U$ can be defined as:

$$U(z, g) = \alpha \ln z + \lambda \ln \mathrm{g} \tag{4.1}$$

where $z > 0$, $\mathrm{g} > 0$. This simple utility function implies that residents only weigh two factors (the daily consumption of goods/services $\mathrm{z}$ and the space of property $\mathrm{g}$) in the decision. Parameters $\alpha$ and $\lambda$ are the proportion of income assigned on $z$ and $\mathrm{g}$ respectively. Note that $\alpha, \lambda \in (0,1)$. This thesis defines that $\alpha + \lambda = 1$ here. The value of $\alpha$ and $\lambda$ indicates how much a resident weights the corresponding factor. With the definition of utility in Equation 4.1, the utility is a continuous function and increasing for $z > 0$ and $\mathrm{g} > 0$.

As in typical economic analyses, individuals would maximize their utility subject to an income constraint, that is

$$\max_{\mathrm{g},z} U(z, \mathrm{g}), \text{ subject to } Y = z + R(x)\mathrm{g} + T_m(x), \forall x \in (0, l) \tag{4.2}$$

where $T_m(x)$ denotes the monetary travel cost from location $x$, $R(x)$ denotes the rent at location $x$, $l$ represents the city size, $Y$ indicates the daily income per capita and $x$ is the distance from location $x$ to city centre. The second term $R(x)\mathrm{g}$ on the right hand of the constraint represents the individual's total rent. With the income constraint (Equation 4.2), an individual chooses some location $x$ to occupy and trades off the income to seek the maximal utility.

**Assumption 4.1** Travel cost function is continuous and monotonically increasing with the distance from the city centre. In Equation 4.2, it holds $\frac{\partial T_m}{\partial x} > 0$ here.

Instead of a given market rent curve $R(x)$, Section 4.2 introduces the concept of a bid-rent function representing a household's ability to pay the rent at location $x$ under a fixed utility level $\mathrm{u}$ (Fujita, 1989). As Figure 4.2 shows, the bid rent is the tangent of the utility curve when the budget line and utility curve only has one intersection.

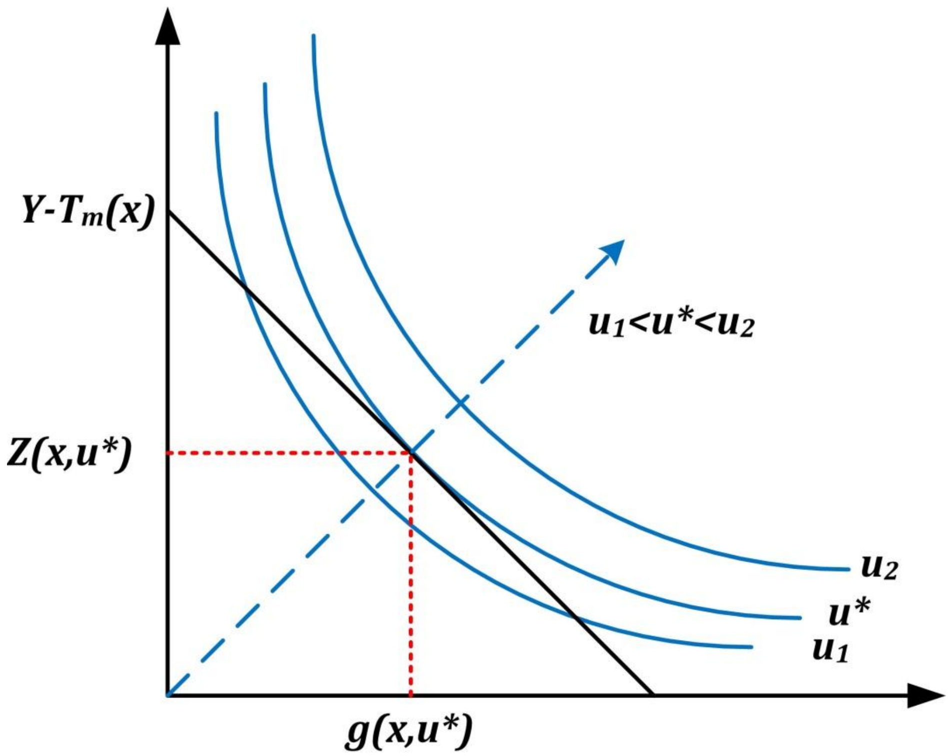


**Figure 4. 2 Definition of Bid-rent Price. The blue arrow shows the increasing direction of utility. (Source: Alonso, 1964)**

**Definition 4.1** The bid rent $\Psi(x, u)$ is the maximum rent per unit area that the household can pay at location $x$ when enjoying a fixed utility level $u$.

Given the definition, the mathematical expression of a bid rent profile is

$$\Psi(x, u) = \max_{z,\mathrm{g}} \left\{ \frac{Y - T_m(x) - z}{\mathrm{g}} \middle| U(z, \mathrm{g}) = u \right\}. \tag{4.3}$$

Solving this maximisation problem, a profile of house space along the corridor for a certain utility level, i.e. $\mathrm{g}(x, u)$, can be obtained. As Equation 4.3 explains, the bid rent $\Psi$ depends on the location $x$ and utility $u$, as does the consumption of goods/services $z$.

Since Section 4.2 has defined the utility function, with $U(z, \mathrm{g}) = u$, a function $Z(x, u)$ can be obtained. Equation 4.1 can be rearranged as

$$Z(x, u) = \mathrm{g}(x)^{-\lambda/\alpha} e^{u/\alpha}. \tag{4.4}$$

Substituting $z$ with Equation 4.4, the bid rent function becomes an unconstrained maximisation problem. The first-order condition of the objective in Equation 4.3 is given by

$$\frac{d}{d\mathrm{g}}\left(\frac{Y-T_m(x)-Z(x,u)}{\mathrm{g}}\right)=0. \tag{4.5}$$

According to Equation 4.5, the following equation can be obtained:

$$-\frac{\partial \mathrm{Z(x,u)}}{\partial \mathrm{g}}=\Psi(x,u). \tag{4.6}$$

For the given relationship between the house space and utility level in Equation 4.4, it holds that

$$\frac{Y-T_m(x)-Z(x,u)}{\mathrm{g}}=\frac{\lambda}{\alpha}Z(x,u)\frac{1}{\mathrm{g}}. \tag{4.7}$$

Equation 4.7 can be rearranged as follows:

$$\alpha\big(Y-T_m(x)\big)=\mathrm{g}(x)^{-\lambda/\alpha}e^{u/\alpha}. \tag{4.8}$$

Then the house space that is affordable for an individual at location $x$ for a fixed utility level $\mathrm{u}$ can be expressed as

$$\mathrm{g}(x,u)=\alpha^{-\alpha/\lambda}\big(Y-T_m(x)\big)^{-\alpha/\lambda}e^{u/\lambda}. \tag{4.9}$$

Then the consumption of goods/services $Z(x,u)$ is given as $Z(x,u)=\alpha(Y-T_m(x))$, which means that the weight residents give to the consumption of goods/services in the utility function determines how much income they would spend on $Z(x,u)$. In the case when $\alpha=0$, residents are not willing to spend income on $z$.

Following Equation 4.9, for a given utility level, the bid rent function can be rewritten as

$$\Psi(x,u)=\lambda(1-\lambda)^{\alpha/\lambda}\big(Y-T_m(x)\big)^{1/\lambda}e^{-u/\lambda}, \tag{4.10}$$

which represents the bid rent for an individual when he chooses the location $x$ and enjoys the utility $u$.

**Remark 4.1** Bid rent function $\Psi(x,u)$ is continuous and decreasing in both $x$ and $u$. The house space profile $\mathrm{g}(x,u)$ under the bid rent process is continuous and increasing in both $x$ and $u$.

**Remark 4.2** As $\lambda \to 1$, it holds $\lim_{\lambda\to 1} \mathrm{g}(x,u) = e^u$, $\lim_{\lambda\to 1} z = 0$ and $\lim_{\lambda\to 1} \Psi(x,u) = \frac{Y-T_m(x)}{e^u}$.

**Remark 4.3** When $\lambda \to 0$, $\lim_{\lambda\to 0} \mathrm{g}(x,u) = e^u$, $\lim\limits_{\lambda\to 0}\Psi(x,u) = 0$ and $\frac{e^u}{Y-T_m(x)} = 1$.

Remark 4.2 indicates that residents tend to spend all spare income bidding for a house when $\lambda \to 1$. Remark 4.3 indicates that residents tend not to spend their income bidding houses if $\lambda \to 0$. Note that the derivations of Remark 4.1, 4.2 and 4.3 are included in Appendix A (See A.1, A.2 and A.3).

Recalling the individual utility maximisation problem (Equation 4.2), residents choose to locate at $x$ for a given rent profile $R(x)$ which follows the rule:

$$R(x) = \Psi(x,u), \text{ if } n(x) > 0, \tag{4.11}$$

where $n(x)$ represents the demand density at location $x$. This rule indicates that $R(x) = \Psi(x,u)$ in the utility maximisation and implies that bid rent is the maximal affordable rent at a certain location for each individual.

To sum up, Section 4.2 introduces the bid-rent process where an individual finds a certain location and obtains the maximal utility (Fujita, 1989). Although the derivation of how to identify the optimal location for an individual under a given market rent curve is not included, the bid rent function has been derived for a simple utility function. Bid rent is a significant concept and it will be used when Chapter 4 extents the house allocation for all residents in a linear monocentric city.

## 4.3 Land Use Patterns under Utility Equilibrium (CCA model)

All residential decisions by individuals will be affected by the residential decisions made by all other individuals in the city. This chapter assumes that the available residential space at each location is always distributed evenly among residents who choose that location. For simplification, this chapter ignores people getting various property spaces due to their allocating orders. The house space varies with the population density, in such case, the relationship is given by $g(x) = V\pi/n(x)dx$, where $V$ is the width of the city and $\pi$ is the proportion of urban space used for residential purposes. The influx of residents at an arbitrary location decreases the utility in terms of house space, while a less populous location can offer a higher utility of house space. Because every resident is seeking the maximal utility (see Section 4.2), residents will move among locations till nobody can increase their utility through shifting the house allocation. Hence, the concept of household's utility equilibrium arises. Under the equilibrium land use state, the bid rent of all individuals follows the same bid rent function under the equilibrium utility (i.e. $\Psi(x, u^*)$ ). Note that the land market under utility equilibrium is not implying that the resulting land use is optimal for landlords. This thesis only discusses how households find the location and their utility equilibrium. Chapter 4 adopts a CCA model (Fujita, 1989) in urban economics and this section introduces it. The CCA model formulates on the following context:

(1) City configuration: There is a monocentric city with a single city centre where all job opportunities are located. It is assumed that the city is a linear rectangle of land.

$$L(x) = V\pi, \forall x \in (0, l). \tag{4.12}$$

Then the total residential area is given as $\int_0^l L(x)\, dx$. The population $N$ is exogenously determined so that

$$\int_0^l n(x)\, dx = N, \tag{4.13}$$

where $n(x)$ denotes the household density at location $x$ and it will be derived from the CCA model.

(2) Residents: All households are assumed to be homogeneous with the same utility function and identical values for all parameters. The only travel for residents is commuting between their residential location and the city centre. The individual daily income $Y$ is given exogenously.

(3) Transport system: A congestion free transport system with a single mode is considered and only the monetary cost is considered in CCA model (Fujita, 1989).

According to Assumption 4.1 and statement (3) above, the travel cost function is considered as

$$T_m(x) = Fx, \tag{4.14}$$

where $F$ is a positive constant denoting the monetary cost per unit distance.

As each individual would seek the maximal utility at a certain location through the trade-off between income on rent and the consumption of goods/services, residents must achieve the same maximum utility level independent of locations so that nobody would move to some location with higher utility. This common maximal utility is the equilibrium utility $u^*$. The demand distribution at equilibrium is given as

$$n(x) = L(x)/\mathrm{g}(x, u^*), \quad \forall x \in (0, l). \tag{4.15}$$

According to Remark 4.1, the bid rent function is decreasing with respect to location $x$, a condition which was proposed so that the city boundary is determined (Fujita, 1989). When the bid rent decreases to zero, the boundary is determined as

$$\Psi(l, u^*) = 0, \tag{4.16}$$

where $l$ indicates the distance between city centre and city boundary. With the derivation of bid rent profile in Equation. 4.10 and the travel cost given by Equation. 4.14, we can obtain the equilibrium utility in the CCA model,

$$\lambda(\alpha^{\alpha/\lambda})\big(Y - T_m(l)\big)^{1/\lambda} e^{-u^*/\lambda} = 0 \tag{4.17}$$

where we can obtain $Y = T_m(l) = Fl$. With the simple monetary travel cost function, the city size $l$ is determined by the average income level in CCA model.

The population constraint (Equation. 4.13) is written as:

$$\int_0^l \frac{L(x)}{\mathrm{g}(x,u^*)} dx = N. \tag{4.18}$$

According A.4 (Appendix A), the derived equilibrium utility and bid-rent profile are given as

$$u^* = \lambda \ln \frac{V\pi(1-\lambda)^{(1-\lambda)/\lambda} H}{N}, \tag{4.19}$$

$$R(x) = \frac{N\lambda Y^{1/\lambda}}{VH}\left(1 - \frac{T_m(x)}{Y}\right)^{1/\lambda} \tag{4.20}$$

where $H = \int_0^l \left(Y - T_m(x)\right)^{\alpha/\lambda} dx$ and $H$ is a positive constant which is independent of $\mathrm{g}(x)$ and $n(x)$. In the basic CCA model, it is easy to obtain $H$ when we only consider a single mode and a congestion-free transport system. Let $R_0$ denote the rent in the city centre and $\frac{N\lambda Y^{1/\lambda}}{VrH} = R_0$. The rent decays from $R_0$ for $x\epsilon(0, l)$. Note that the rent in the city center $R_0$ is increasing with the total population and decreasing with the daily income, city width, corridor length and average travel cost per unit distance.

**Remark 4.4** $\frac{\partial u^*}{\partial N} < 0$ indicates that equilibrium utility is decreasing with respect to population $N$.

**Remark 4.5** For a given travel cost function $T_m(x)$, a known total population $N$, the predetermined daily income per capita $Y$, the city length $l$ and a constant $\lambda$, the demand distribution under the equilibrium utility $u^*$ is given by $n(x, u^*) = \frac{N}{H}(Y - T_m(x))^{(1-\lambda)/\lambda}$ where $H$ is a positive constant and $H = \int_0^l \left(Y - T_m(x)\right)^{(1-\lambda)/\lambda} dx$.

The CCA model has explicitly formulated how to find the utility equilibrium in the closed-city under absentee landlord ownership. However, the transport cost here only depends on distance and it is a monetary cost, so the transport influence has not been fully discussed. Note that the bid rent function is the key to finding the equilibrium utility here.

## 4.4 Model Formulation

With the context of the CCA model in Section 4.3, Section 4.4 formulates a linear monocentric city model with two congestible modes and the housing allocation problem of all residents will be solved. The city configuration is the same as in Section 4.3. However, the transport systems are congestible and the details of two transport modes are introduced in Section 4.4.1 and 4.4.2.

For the railway, congestion is represented by the crowding discomfort cost. For the highway, the commuting time is dependent on the traffic flow.

### 4.4.1 Bi-modal Transport System

#### 4.4.1.1 Travel time by train

The constant travel time $f_T$ by train per unit distance is determined by the travel speed of trains and it is defined as

$$f_T = \frac{1}{s_T}, \tag{4.21}$$

where $S_T$ is the average operating speed of trains ($s_T = 12.5km/hr$).

This section assumes that stations are evenly spaced and dense and commuters can easily access the train line, the access time of train trips is ignored here. With this assumption, the average waiting time for a train for each trip is calculated as:

$$w_T = \rho_1 h, \tag{4.22}$$

where $h$ is the headway of the train system in hours (which is constant) and $\rho_1 = 0.5$ implying the uniform random arrival distribution of passengers at each location (Liu et al., 2009). It is worth noting that increasing train capacity will reduce average waiting time in realism, which is the significant difference between road and public transport networks. Hence, increasing train capacity can be represented by reducing waiting time (i.e. shortening the headway) when we attempted to represent the development in public transport. Similarly, increasing train speed (Equation 4.21) means reducing average in-vehicle travel time by public transport, which could be equivalent to the development of train line.

A discomfort cost function $t_T(v_T)$ is used to represent the impact of congestion when the number of passengers on a train is $v_T$. This requires $\frac{dt_T(v_T)}{dv_T} > 0$ and $t_T(0) = 0$. In this chapter $t_T$ is chosen to be

$$t_T(v_T) = -\rho_2 \ln(1 - v_T/K) \tag{4.23}$$

where constants are given by $K = 20000$ and $\rho_2 = 0.5$ (Liu et al., 2009), which additionally imposes a finite strict capacity constraint. Note that the discomfort of train travel is measured in units of hr/km and the discomfort cost is calculated at each location and hence will vary along the route for each commuter.

Considering the components above, the generalised travel cost by train from a location $x$ to the CBD is written as:

$$G_T(x) = \tau \cdot \left[w_T + f_T x + \int_0^x t_T(v_T(w))dw\right] + T_m x, \tag{4.24}$$

where $\tau$ represents the value of time for residents and is measured in £/hr and $T_m$ denotes the monetary travel cost per unit distance. Since $\frac{dG_T}{dx} = f_T + t_T(v_T(x)) > 0$, the generalised travel cost by train is strictly increasing with respect to the distance from location $x$ to the city centre. This chapter will use $\tau = £30/hr$. It assumes that the monetary travel cost by train is fixed per unit distance ($T_m = £2/km$).

#### 4.4.1.2 Travel time by car

This chapter assumes that the road capacity $C$ is constant along the corridor. Using a standard BPR function, the travel time per unit distance is given by

$$t_C(v_C) = f_C\left[1 + A\left(\frac{v_C}{C}\right)^B\right], \tag{4.25}$$

where A and $B$ are parameters ($A = 0.15$, $B = 4$), $v_C$ is the traffic volume on the highway at each location and $f_C$ is the free-flow travel time per unit distance. The generalised travel cost by car from $x$ to the CBD is

$$G_C(x) = \tau \cdot \left[\int_0^x t_C\big(v_C(w)\big)dw + a_C\right] + T_m x \tag{4.26}$$

where $a_C$ is the access time by car. The derivative of the generalised travel cost is given by $\frac{dG_C}{dx} = t_C(v_C(x)) > 0$, so $G_C(x)$ is strictly increasing with respect to the distance from the centre for $v_C > 0$.

The focus of this chapter is investigating congestion (i.e. the travel time) under urban growth. For simplicity, the monetary travel cost by car is the same as that by train ($T_m = £2/km$).

#### 4.4.1.3 User Equilibrium on mode choices

In the LMC model, all commuters are continuously distributed along the corridor with demand density $n(x)$ where $x$ is the distance from the CBD. It assumes that commuters can access the highway or railway everywhere along the corridor. At an arbitrary location $x$ on the corridor, the number of commuters choosing car or train is $n_C(x)$ or $n_T(x)$. Recall the population constraint (Equation. 4.13) so that

$$N = \int_0^l [n_T(x) + n_C(x)]dx. \tag{4.27}$$

Similar to previous studies (e.g. Liu et al., 2009), the standard method for solving the problem in the LMC model is by discretisation. Suppose the corridor of the monocentric city is discretised into segments, at user equilibrium for each small segment the modal split is determined by:

$$n_T(x) \geq 0 \Rightarrow G_T(x) \leq G_C(x), \tag{4.28}$$

$$n_C(x) \geq 0 \Rightarrow G_C(x) \leq G_T(x). \tag{4.29}$$

The travel time curve $T_t(x)$ can be obtained in the continuum model:

$$T_t(x) = \begin{cases} w_T + f_T x + \int_0^x t_T(v_T(w))dw\,, & if\ G_T(x) \leq G_C(x) \\ \int_0^x t_C\big(v_C(w)\big)dw + a_C, & if\ G_C(x) \leq G_T(x) \end{cases}. \tag{4.30}$$

Hence, it still holds that $\frac{dT_t}{dx} > 0$. The travel cost $T_t(x)$ is measured as hours per trip.

### 4.4.2 Utility Function and Constraints of Residents

Based on Section 4.1, the utility is defined to measure the satisfaction of residents in the LMC. This section is going to extend the utility function (Equation 4.1), because the emergent congestion should affect individuals' utility. It is assumed that all households are homogeneous as aforementioned. Moreover, they have an identical utility function as follow

$$U(z, \mathrm{g}, T_l) = \alpha \ln z + \lambda \ln \mathrm{g} + \gamma \ln T_l \tag{4.31}$$

where $\alpha, \lambda, \gamma \geq 0$ and $\alpha + \lambda + \gamma = 1$ indicating the relative importance of each utility component. In Equation 4.31, $T_l$ represents leisure time. With this component, it implies that people should have less leisure time when they spend longer commuting under congestion. Then a residents' time constraint can be written:

$$T_t(x) + T_l = T_D - T_{fw} - T_{ew} \tag{4.32}$$

where $T_{fw}$ denotes fixed working hours ($T_{fw} = 8hr$), $T_{ew}$ denotes extra working hours, $T_t$ represents the commuting time from location $x$ to the city centre and $T_D$ represents the hours in one day ($T_D = 24hr$). Extra working hours represent the opportunity to earn additional income. Suppose $W$ is the average wage per extra working hour ($W = £15$), then the income constraint for each individual is rewritten as

$$Y + WT_{ew} = z + R\text{g} + T_m(x) \tag{4.33}$$

where $Y$ is the fixed salary per day, $R$ is the rent per unit area per day at location $x$, and $T_m(x)$ is the monetary travel cost from location $x$ to the city centre. Within the time constraint, a trade-off between leisure time and extra working hours arises so that the utility of consumption and leisure time has a contradictory relationship. Suppose $Y = £100/day$.

With income and time constraints, residents explicitly trade off both time and income. It is worth noting that in previous studies, the nature of these trade-offs were implicit in the model and hence difficult to discern. For instance, in Li et al. (2012c), an increase in commuting time directly resulted in a decline in income without sufficient explanation for the mechanism causing this.

### 4.4.3 Utility Maximisation and Utility Equilibrium in the Extended Model

Similar to Section 4.2, each resident seeks to maximise their utility though the trade-off of income between rent and consumption of other goods and services, and the trade-off of time between leisure activities and earning via fixed and extra working hours. The mathematical expression of the individual utility maximisation problem with these constraints is given as:

$$\max_{\text{g},z,T_l} U(z, \text{g}, T_l) = \alpha \ln z + \lambda \ln \text{g}(x) + \gamma \ln T_l(x), \tag{4.34.a}$$

$$\text{subject to } I(x) = z + R\text{g}(x) + WT_l(x). \tag{4.34.b}$$

**Assumption 4.2** $W \geq 0$ and $\gamma \geq 0$.

As shown in Equation. 4.34, the trade-off between working hours and leisure time and between house rent and consumption have been considered for each resident. For a neat expression, $I(x) = Y + WT_D - WT_{fw} - WT_t(x) - T_m(x)$ here and $I > 0$. Following the concept of a bid rent price, the bid rent $\Psi(x, u)$ is defined as

$$\Psi(x, u) = \max_{z,\mathrm{g},T_l} \left\{ \frac{I(x) - z - WT_l(x)}{\mathrm{g}} \middle| U(z, \mathrm{g}, T_l) = u \right\}. \tag{4.35}$$

Substituting $z$, Equation 4.35 can be written as an unconstrained optimisation problem so that the first order conditions are given by $\frac{\partial \Psi}{\partial \mathrm{g}} = 0$ and $\frac{\partial \Psi}{\partial T_l} = 0$. We then have the following relationships (see Appendix A.6):

$$Z(x) = \frac{\alpha}{\alpha+\lambda}(Y + WT_{ew}(x) - T_m(x)), \tag{4.36}$$

$$T_l(x) = (\gamma/W)I(x), \tag{4.37}$$

$$\mathrm{g}(x, u) = (\gamma/W)^{-\gamma/\lambda}\alpha^{-\alpha/\lambda}I(x)^{(\lambda-1)/\lambda}e^{u/\lambda}, \tag{4.38}$$

$$\Psi(x, u) = \lambda(\gamma/W)^{\gamma/\lambda}\alpha^{\alpha/\lambda}I(x)^{1/\lambda}e^{-u/\lambda}. \tag{4.39}$$

Equation 4.37 illustrates that the proportion of net income spent on consumer goods is dependent on the parameters $\alpha$ and $\lambda$. For a resident enjoying utility $u$ at location $x$, the house space is given by Equation 4.38 and the bid rent is derived as Equation 4.39.

Following Remark 4.5 (Section 4.3), the demand distribution at equilibrium is given as (see Appendix A.7)

$$n(x, u^*) = \frac{N}{V\pi I(x)^{(\lambda-1)/\lambda}H'}. \tag{4.40}$$

The demand distribution is dependent on $\lambda$, monetary travel cost $T_m(x)$ and the commuting time $T_t(x)$, hence the influence of congestion has been considered in housing allocation. The utility at the spatial equilibrium is given as

$$u^* = \lambda \ln V\pi + \gamma \ln \gamma - \gamma \ln W + \alpha \ln \alpha + \lambda \ln H' - \lambda \ln N, \tag{4.41}$$

where $H'$ is a constant ($H' = \int_0^l I(x)^{(1-\lambda)/\lambda}\, dx$).

**Proposition 4.1** The derivative of $u^*$ is derived as $\frac{\partial u^*}{\partial N} = -\frac{\lambda}{N} < 0$ and $\frac{\partial^2 u^*}{\partial N^2} = \frac{\lambda}{N^2} > 0$.

With Proposition 4.1, the derived equilibrium utility decreases as the total demand increases in the model extended. As the equilibrium utility depends on the total population, so do the residential location and mode choice at equilibrium. This shows the feasibility of the model in investigating urban growth. Meanwhile, Proposition 4.1 is consistent with Remark 4.4, which shows that the extended model works properly. Furthermore, the bid rent function at equilibrium is given by

$$\Psi(x, u^*) = \frac{N\lambda}{H'V\pi} I(x)^{1/\lambda}. \tag{4.42}$$

With this expression, the bid rent increases with the growth of total demand. The value of $\lambda$ drives the bid rent up. The rent profile is derived as follows

$$R(x) = R_0{}'(1 - \frac{T_m(x) + WT_t(x)}{Y + WT_D})^{1/\lambda} \tag{4.43}$$

where $\frac{N\lambda}{H'V}(Y + WT_D)^{1/\lambda} = R_0{}'$ and $H'$ is a constant ($H' = \int_0^l I(x)^{(1-\lambda)/\lambda}\, dx$). When $W = 0$, $\gamma = 0$ and $Y > 0$, the equation recover to Equation 4.20.

**Proposition 4.2** As $\lambda \rightarrow 1$, $H' = l$, $\lim_{\lambda\rightarrow 1} n(x, u^*) = \frac{N}{l}$, $\lim_{\lambda\rightarrow 1} g(x, u^*) = \frac{lV\pi}{N}$ and $\lim_{\lambda\rightarrow 1} \Psi(x, u^*) = \frac{NI(x)}{lV\pi}$.

**Proposition 4.3** As $\lambda \rightarrow 0$, according to population constraint, $\lim_{\lambda\rightarrow 1} n(x_0) = N$ and $\lim_{\lambda\rightarrow 1} g(x_0) = V\pi/N$ at the city centre $x_0$.

Proposition 4.2 indicates that the derived demand distribution is uniform when $\lambda \rightarrow 1$. Because people only weight the house space in the utility function in the case of $\lambda \rightarrow 1$, the equilibrium is reached when every resident has the same house. Proposition 4.3 indicates that all people tend to locate at the city centre due to a very low value of $\lambda$. To sum up, the demand distribution at equilibrium varies from a uniform distribution to the distribution where almost all residents live in the city centre as $\lambda$ decreases from 1 to 0.

So far, the mechanism of housing allocation has been formulated, namely residents individually maximise their utility through trade-offs in time and income at a particular location and they seek the equilibrium utility through changing the location.

### 4.4.4 Solution Procedure for the Extended Model

The equilibrium problem defined in Equation. 4.34 can be solved via the following procedure:

**Step 0: Initialisation.** Choose an initial demand distribution $n^0(x)$ where a uniform demand distribution is chosen. Then the space of a property profile $g^0(x)$ is obtained. Determine the initial network features: link length $l$, road capacity $C$ and other parameters as given above. Set the iteration counter $k = 1$.

**Step 1: Mode Choice Equilibrium.** Solve the travel mode equilibrium for iteration $k$.

Determine the travel time function ${T_t}^k(x)$ to the city centre for residents and obtain the monetary travel cost at all locations ${T_m}^k(x)$. Adopt the MSA algorithm (Sheffi, 1985) for each location.

**Step 2: Individual Time Split.**

Step 2.1 Determine the consumption $Z^k(x)$ at each location using Equation 4.36.

Step 2.2 Generate the trade-off of time between work and leisure for each location, namely, ${T_l}^k(x) = {p_l}^k(x) \cdot (T_D - T_{fw} - T_t)$ and ${T_{ew}}^k(x) = (1 - {p_l}^k(x)) \cdot (T_D - T_{fw} - T_t)$, where ${p_l}^k(x)$ denotes the proportion of leisure time and ${p_l}^k(x) \in (0.1)$.

Step 2.3 Obtain the utility profile $U^k(x)$ and time split $T_l(x)$ and $T_{ew}(x)$ that drives the utility at each location.

Step 2.4 Obtain the bid rent profile $\Psi^k(x) = \left[Y + {T_{ew}}^k(x)W - Z^k(x)\right]/g^k(x)$.

**Step 3: Utility Equilibrium.** Solve the residential choice process adopting the MSA algorithm.

Step 3.1 Direction finding. Perform the All-or-Nothing algorithm based on the current utility profile. This yields an auxiliary demand distribution $y^k(x)$.

Step 3.2 Move. Find the new demand distribution by setting

$n^{k+1}(x) = n^k(x) + (1/k)((y^k(x) - n^k(x))$.

Step 3.3 Update. Update the demand distribution as $n^{k+1}(x)$.

**Step 4: Convergence criterions.** If the gap between the highest and lowest utility $U^k_{max} - U^k_{min} < \varepsilon$, the procedure stops. Otherwise, set $k = k + 1$ and go to Step 1.

It is worth noting that the house space is determined by the demand distribution here so the amount of income on consuming goods is firstly determined and the remaining net income is used on bid rent in Step 2. Step 2 is designed with the derivation of Equation 4.36, which is similar to the solution procedure proposed by (Li et al. 2012) about how to determine the residential density along the corridor. Moreover, Ma and Lo (2012) have formulated an unconstrained optimisation problem with the combined equilibrium bid-rent and nested multinomial choice framework. As the conditions of using MSA to solve the optimisation problem are (i) the objective function is twice continuously differentiable; (ii) its gradient vanishes only once in the feasible region, namely it can be an unconstrained problem. Their work indicates the combined utility and mode choice equilibrium can be solved by MSA algorithm. Here, the thesis procedure a heuristic solution procedure that provides a method to solve this combined equilibrium problem. The result from the solution is a demand distribution when everybody enjoys the equal utility after the mode choice. In future, more work to proof the convergence of the solution procedure proposed is necessary.

## 4.5 Analysis for Network Evolution

Using the procedure in Section 4.4.4, this section investigates the housing allocation under the variation in $\alpha$, $\gamma$ and $\lambda$ in the utility function (Equation. 4.31) in Section 4.5.1. Section 4.5.2 provides a preliminary policy analysis under urban growth. Section 4.5.3 investigates the phenomena in the proposed modelling framework.

### 4.5.1 Housing Allocation Mechanism

Figures 4.3, 4.4 and 4.5 show that the trade-off of time and income works simultaneously and how the mechanism for housing allocation works as Section 4.4.3 states with the solution procedure in Section 4.4.4. Figures 4.3 and 4.4 display the splits of utility, daily time (24 hours), expenses and income of an individual.

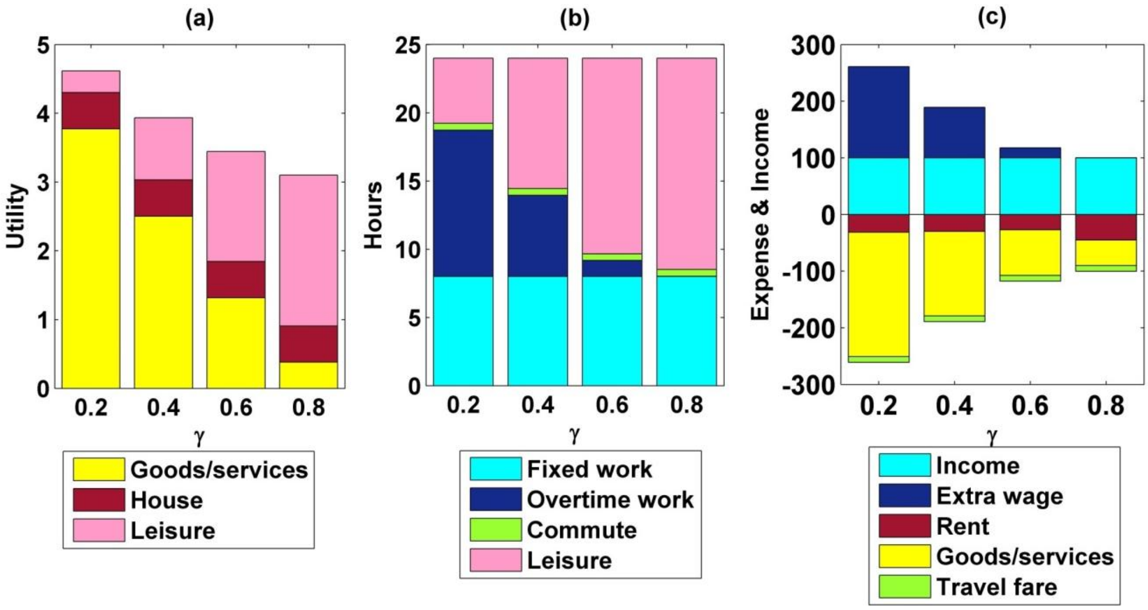


**Figure 4. 3 Parameter γ increases and α decreases, λ and total demand N are constant. (All income is shown as positive and expense are negative at the bottom.)**

In Figure 4.3, the case where the parameter $\gamma$ increases and $\alpha$ decreases simultaneously is investigated, other parameters (e.g. total population) and features of the monocentric city stay constant. The variation in parameters ($\alpha$, $\lambda$, $\gamma$) in the utility function will change a resident's preference for house space, leisure time and other consumption. For example, if $\alpha > \gamma$, an increasing unit of consumption $z$ is more valuable than gaining one unit of leisure time in utility maximisation. For the case in Figure 4.3, residents prefer to work less since they are not interesting in a large amount of consumption, so the component $\gamma \ln T_l$ (i.e. utility from leisure time) increases while $\alpha \ln z$ (i.e. utility from the goods and service) declines. Consequently, the extra working hours of residents are shorter and extra income $WT_{ew}$ goes down on Figure 4.3 (c). Similarly, consumption $z$ declines due to the decreasing duration of extra working. Note that travel cost $T_m$ is always the same because this chapter assumes the monetary travel cost per trip per unit distance by car or train is the same. The monetary travel cost is only included here for a general model formulation.

As stated in Section 4.3, the mechanism in housing allocation includes individual utility maximisation and overall utility equilibration. In Figure 4.3 (c), when $\gamma$ increases to 0.8, residents tend to have a lot of leisure time, but due to the time constraint they can only spend time left over on leisure. In such cases, their individual utility at each location has not been maximised due to the time constraint. Meanwhile, as $\alpha$ decreases, the consumer goods that residents are willing to pay for are still decreasing so the remaining income spent on rent increases. That's why the total rent per day for $\gamma = 0.8$ is higher than the other three cases.

In Figure 4.4, the scenario where parameter $\lambda$ increases and $\alpha$ decreases with $\gamma$ kept constant has been examined. As Propositions 4.2 and 4.3 state, the demand distribution tends to be uniform as the growth of $\lambda$ on Figure 4.5 (a) so does house space (Figure 4.5 (c)). In this case, residents value house space more in the utility estimation, so the term $\lambda \ln g$ (i.e. the utility from house space) increases and $\alpha \ln z$ (i.e. the utility from the consumption of goods or services) declines in Figure 4.4 (a). Also, the rent per $m^2$ per day and total rent per day per person are increasing when $\lambda$ increases from $0.2$ to $0.6$. As $\alpha$ decreases, residents tend to spend less time on extra working hours. At first, the decreasing income does not affect the bid rent since residents have sufficient income from both fixed and extra working hours. But the income is not enough to afford consumption and the bid rent when $\lambda$

increases to 0.8. The proportion of total rent declines when $\lambda = 0.8$ in Figure 4.4 (c).

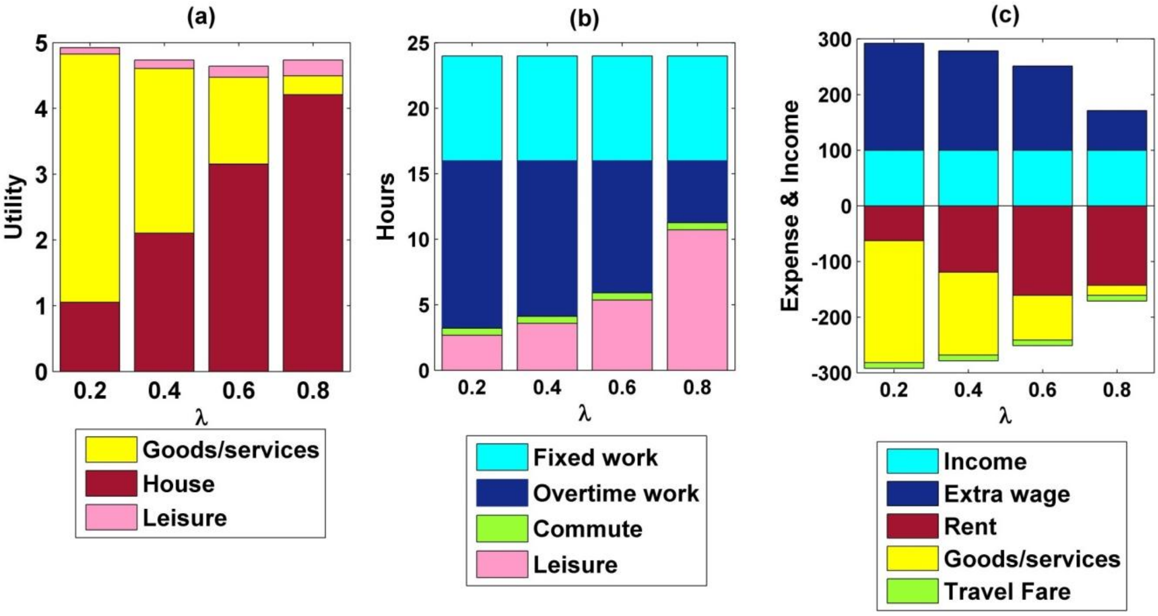


**Figure 4. 4 Parameter λ increases and α decreases, γ and total demand N are constant.**

Consequently, as shown in Figure 4.5 (b), the rent deceases for $\lambda = 0.8$ compared to $\lambda = 0.6$. It is worth noting that the rent profile is flat for $\lambda = 0.8$, which indicates that people may spend all income on rents. As stated in Proposition 4.2, the demand distribution tends to be uniform as $\lambda$ increases (Figure 4.5 (a)) and then an opposite tendency in the profile of house space.

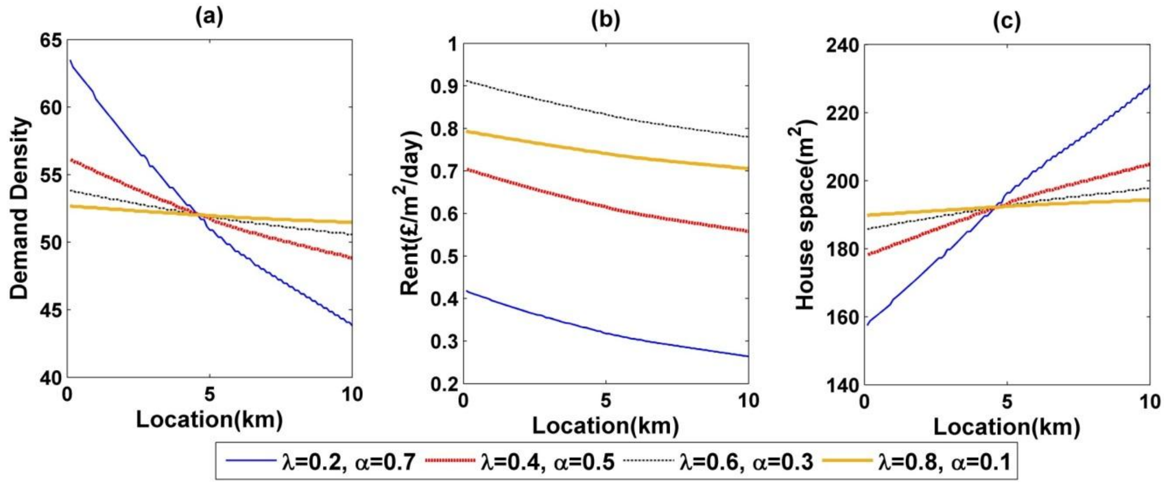


**Figure 4. 5 Parameter λ increases and α decreases, γ and total demand N are constant.**

### 4.5.2 Analyses for Urban Growth

The aim of this chapter is to investigate the performance when the highway and the train line are improved simultaneously, namely the experiments with the modelling framework in Figure 4.1. Before those experiments, this section firstly explores the performance when only one of modes (the road capacity on the highway or the train speed) has been improved. Note that the total population remains constant in each experiment in Section 4.5.2.

To measure the efficiency and sustainability in a bi-modal transport system under urban growth, this chapter observes the average travel time per trip and CO emissions per trip. As discussed in Chapter 3, the average travel time per trip in a continuum model can be calculated (Section 3.3.3). Section 4.5.2.1 explains how to calculate the average car emissions per trip in a continuum model.

#### 4.5.2.1 Indicator: Car emissions per trip

There is a study (Yin and Lawphongpanich, 2006) proposing a method of measuring car emissions in a discrete network. It is given by

$$e_a(v_a) = 0.2038 \cdot t(v_a) \cdot e^{0.7962 \cdot (l_a / t(v_a))}. \tag{4.44}$$

Equation 4.44 calculates the CO emissions of one vehicle on link a, $l_a$ is the link length in kilometers, $t(v_a)$ is the travel time on the link and $t(v_a)$ is given by the standard BPR function. The CO emissions are measured as grams per hour (i.e. g/hr). Since the average speed $s_C$ is $l_a/t(v_a)$, in the continuum model, this chapter can measure CO emissions for $dx$ and the CO emissions of one vehicle on each minor segment is calculated by

$$e(x) = \frac{0.2038}{s_C(x)} \cdot e^{0.7962 \cdot s_C(x)} dx. \tag{4.45}$$

Total CO emissions of all vehicles is given by

$$E = \int_0^{x_B} \frac{0.2038}{s_C(x)} \cdot e^{0.7962 \cdot s_C(x)} \cdot v_C(x) dx. \tag{4.46}$$

The average CO emissions per trip is given by

$$E_{ave} = \frac{E}{N}. \tag{4.47}$$

#### 4.5.2.2 Results

In Figure 4.6 (a) and 4.7 (a), the average travel time and the average car emissions of the bi-modal transport system when the road capacity is increased are shown. In Figure 4.6 (a), there is no surprise that the average travel speed declines with the increase in road capacity, in such case the number of commuters who are choosing driving increases.

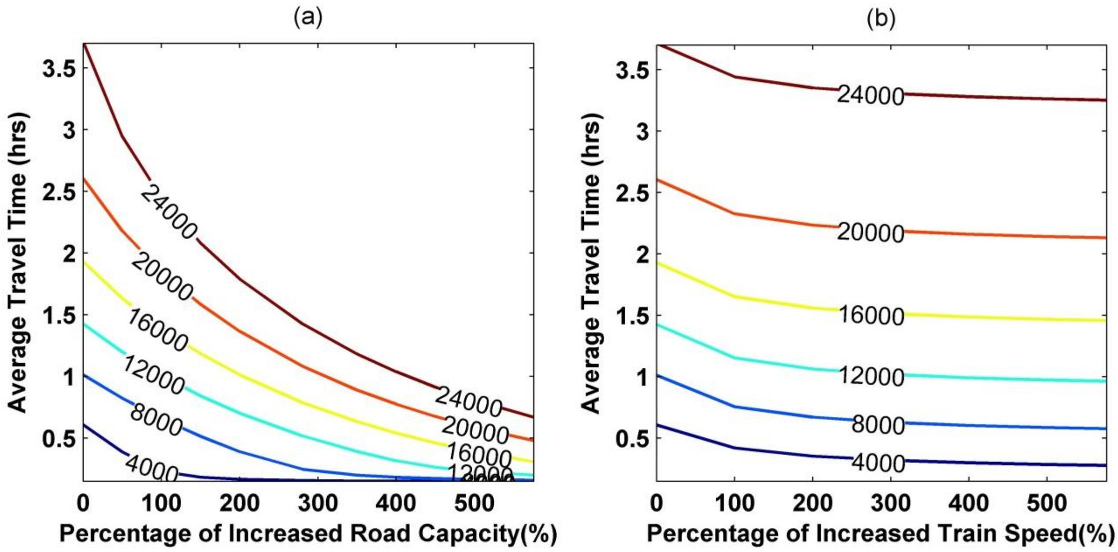


**Figure 4. 6 Average Travel Time (a) Only road capacity increases; (b) Only train speed increases. Different demand levels (i.e. the total number of population N) are labelled on each line.**

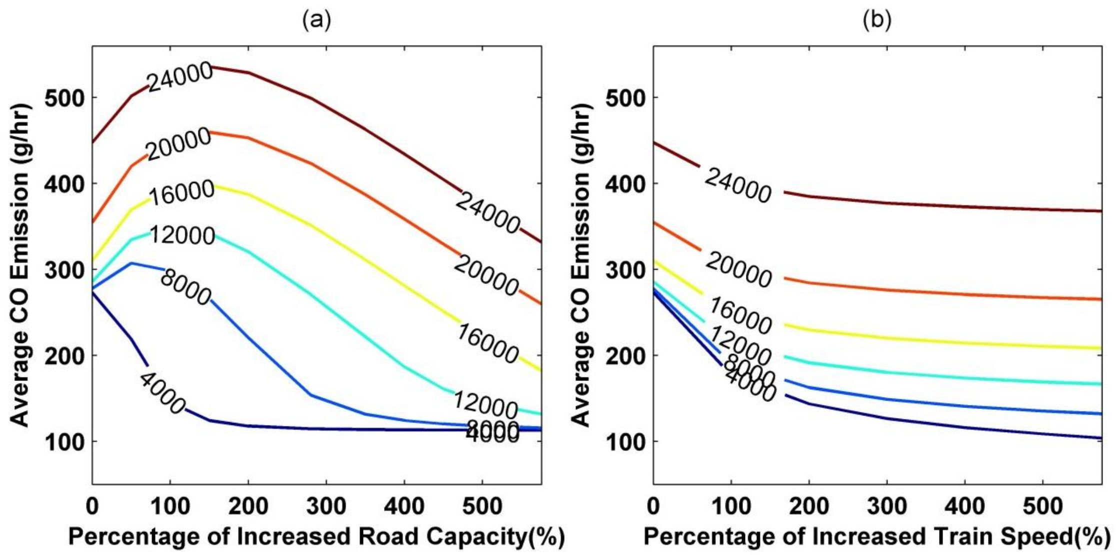


**Figure 4. 7 Average CO Emission per Trip (a) Only road capacity increases; (b) Only train speed increases. Different demand levels are labelled on each line.**

There are two reasons for the total amount of car emissions decreasing: (i) the total travel time of cars is shortened; (ii) the total distance driven is decreasing. With a constant total population, increasing road capacity can increase the average travel speed and therefore decrease the total travel time. When the road capacity increased attracts more drivers, the total distance driven increases even if there is no congestion on the link. Furthermore, both the total distance driven and the total travel time increases when the increasing travel demand causes congestion on the link. Additionally, if the increasing rate of road capacity is much faster than the increasing travel demand, car emissions could reduce. In Figure 4.7 (a), the average car emission per trip increases and then decreases as the road capacity is increased except for a low demand level (i.e. $N = 4000$). The reason behind this phenomenon is that the increasing distance driven is dominant in the performance of the bi-modal transport system, so the average car emissions increase at first. Then the decreasing total travel time of car exceeds the impact of the increasing distance driven on car emissions, consequently the average car emissions go down. Under the low demand level, the decreasing total travel time of car is dominant from the beginning.

Figures 4.6 (b) and 4.7 (b) show the performance of the bi-modal transport system with increasing train speed. With the improvement of the transport system, the average travel time slowly decreases compared to Figure 4.5 (a). In Figure 4.7 (b), the average car emission decreases as the train speed increases, in such case, the total travel time of cars is shortened and the total distance driven is less.

Figure 4.6 and 4.7 help us understand the phenomena when the highway or train line infrastructure is enhanced respectively. In the proposed model, the improvement of highways can sufficiently decrease the average travel time but it may sometimes have some negative impact on the environment. However, the enhancement of train lines can always reduce the environmental pressure though it has less positive impact on improving the efficiency of transport system.

### 4.5.3 Results from the Modelling Framework

As shown the proposed modelling framework in Section 4.4, this chapter adopts scaling laws to describe the growth of urban quantities and total demand (population) like Chapter 3. The general formula of scaling is given as:

$$\xi = \rho_0 N^{\beta}, \tag{4.43}$$

where $\rho_0$ denotes the constant which is determined by the initial quantity and demand, namely $\rho_0 = \frac{Q_0}{N_0^{\beta}}$. Here, $\xi$ represents the amount of an urban quantity. $\beta$ is the scaling exponent and determines how the urban quantity grows. Based on the literature (Bettencourt et al., 2007), the economic factor usually scales in the super linear regime, which means $\beta > 1$ and the urban infrastructure scales under the sub-linear regime (i.e. $\beta < 1$). Additionally, the linear regime is when $\beta = 1$. It is worth noting that the urban quantity stays constant as total demand grows for $\beta = 0$.

In the proposed LMC model, fixed income, wage, city width, residential ratio, road capacity, operating speed of train can be included. Because the main focus of this thesis is to investigate the growth of transport systems, the road capacity and the train speed will be increased following some certain scaling laws, which can be regarded as the improvement of a bi-modal transport system. Suppose $\beta_C$ denotes the scaling exponent on the road network (it is known $\beta_C = 0.83$) corresponding to road capacity,

$$C = \rho_0 N^{\beta_C}. \tag{4.44}$$

It is worth noting that the corridor length is constant in the experiments in Chapter 4, so the corridor length is excluded from Equation 4.44.

For the improvement of rail line, $\beta_T$ is for railway systems and it drives train operating speed. Suppose $\beta_T = 0.83$ as the urban infrastructure. It is given by

$$s_T = \rho_0 N^{\beta_T}. \tag{4.45}$$

Note that the initial parameters for all cases are given by $Y_0 = 20$, $W_0 = 5$, $V_0 = 0.5$, $r_0 = 0.2$, $N_0 = 2600$, $\alpha = 0.25$, $\lambda = 0.5$ and $\gamma = 0.25$. Case 0 is designed to observe the phenomenon stated in Proposition 4.1. The road capacity stays constant and $C_0 = 2000veh/hr$ in Case 0 so that the congestion is not severe under urban growth. As shown in Figure 4.8, the equilibrium utility is decreasing as total population increases. Note that all other parameters (including the road capacity) are kept constant in this case. It also corroborates observation in a classic model as explained in Proposition 4.1.

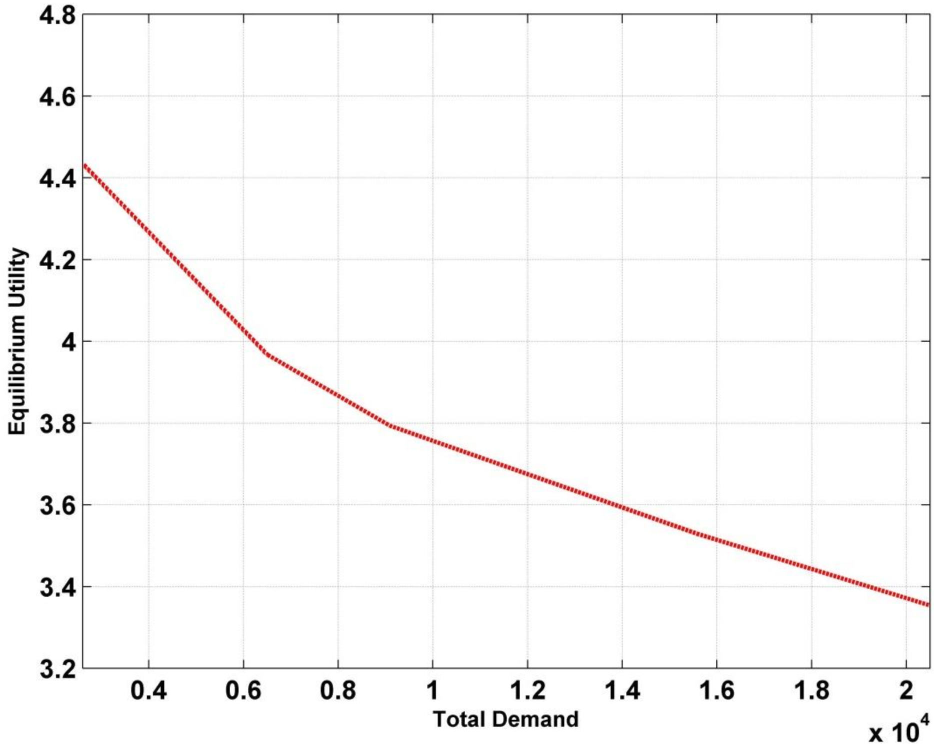


**Figure 4. 8 Equilibrium utility with total demand increasing in Case 0.**

Table 4.1 shows the scaling exponents in Figure 4.8. Case 1 is designed to investigate the improvement on the road network under demand growth. Compared to Case 0, the only initial setting that is different is road capacity, $C_0 = 500veh/hr$ here since the road capacity is increasing under urban growth. Case 2 is designed to observe the situation where the railway is improving as total demand grows. Case 3 is to analyse when the quantities in both highways and train lines are increasing simultaneously as suggested by the scaling laws (Equation 4.44 and 4.45). The utility is a measure in the housing allocation mechanism, but is not used to assess the system performance. Hence, Figure 4.9 has not shown the utility in various cases.

**Table 4. 1  Scaling exponents in cases.**

| | $\beta_C$ | $\beta_T$ | $C_0$ |
|---|---|---|---|
| Case 0 | 0 | 0 | 2000 |
| Case 1 | 0.83 | 0 | 500 |
| Case 2 | 0 | 0.83 | 500 |
| Case 3 | 0.83 | 0.83 | 500 |
| Case 4 | 0.7 | 0.96 | 500 |
| Case 5 | 0.75 | 0.91 | 500 |
| Case 6 | 0.8 | 0.86 | 500 |
| Case 7 | 0.85 | 0.81 | 500 |
| Case 8 | 0.90 | 0.76 | 500 |
| Case 9 | 0.95 | 0.71 | 500 |

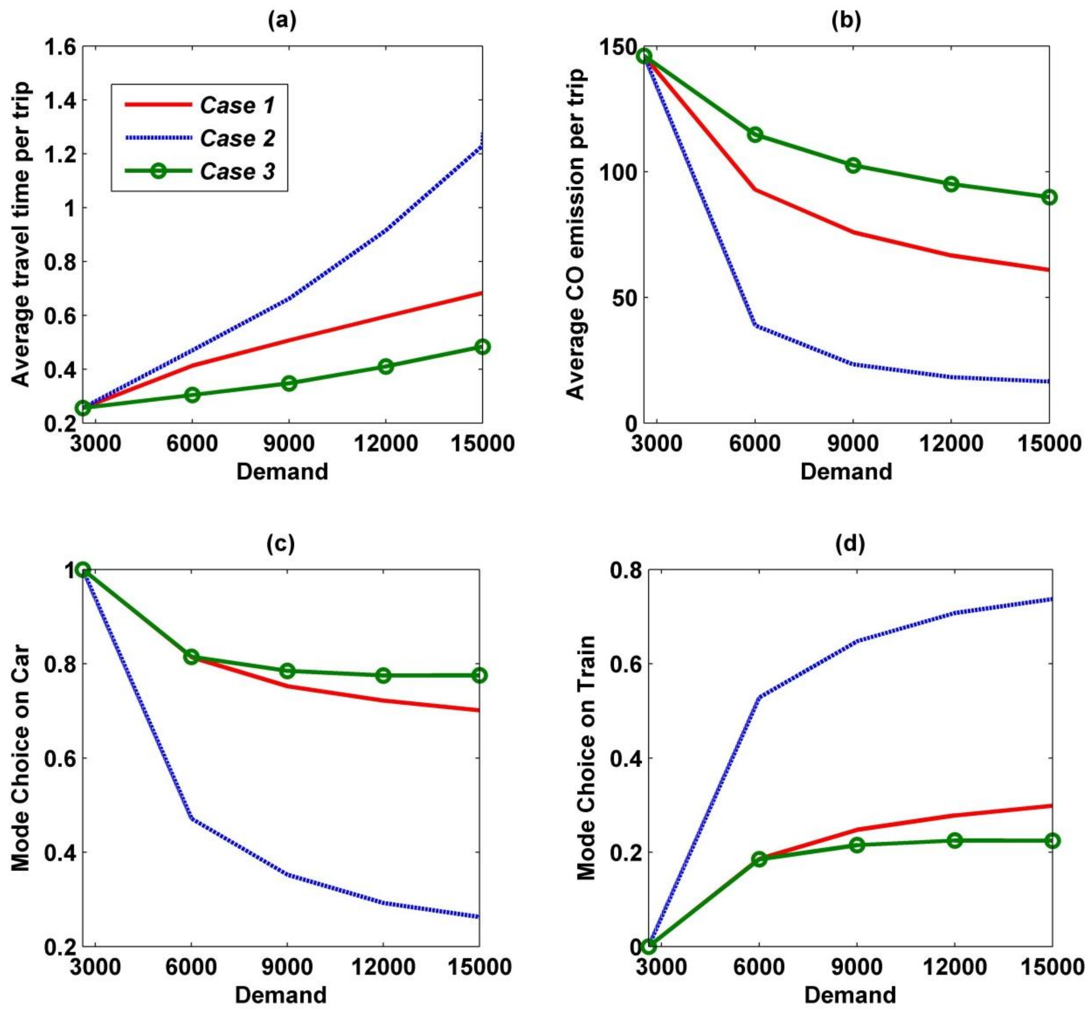


**Figure 4. 9  Results in Case 1, 2, and 3.**

In Figure 4.9 (a) and (b), average travel time and CO emissions per trip have been shown for all cases. Since this model assumes that the operating train speed is much slower than the free-flow speed on the highway, the average travel time in Case 2 where only operating train speed has been improved is higher than the other two cases. Not surprisingly, although Case 1 is more efficient than Case 2, Case 2 displays better performance on sustainability. Case 1 shows the worst performance on CO emissions per trip. This simple observation implies the significance of how railway systems can cooperate with road networks and it highlights that the cooperation of two modes can guarantee both efficiency and sustainability.

Because the operating average train speed is designed to be lower than the free flow travel speed on the highway, the train line can alleviate the arising congestion on the highway under urban growth in the proposed modelling framework. One important reason for this model formulation is that the public transport system usually takes the responsibility of reducing congestion (see Section 1.1 and 1.2). Moreover, this framework can better represent the variation in mode choices as shown in Figure 4.9. With this advance in modelling the growth of cities or networks, mode choices is included and investigated.

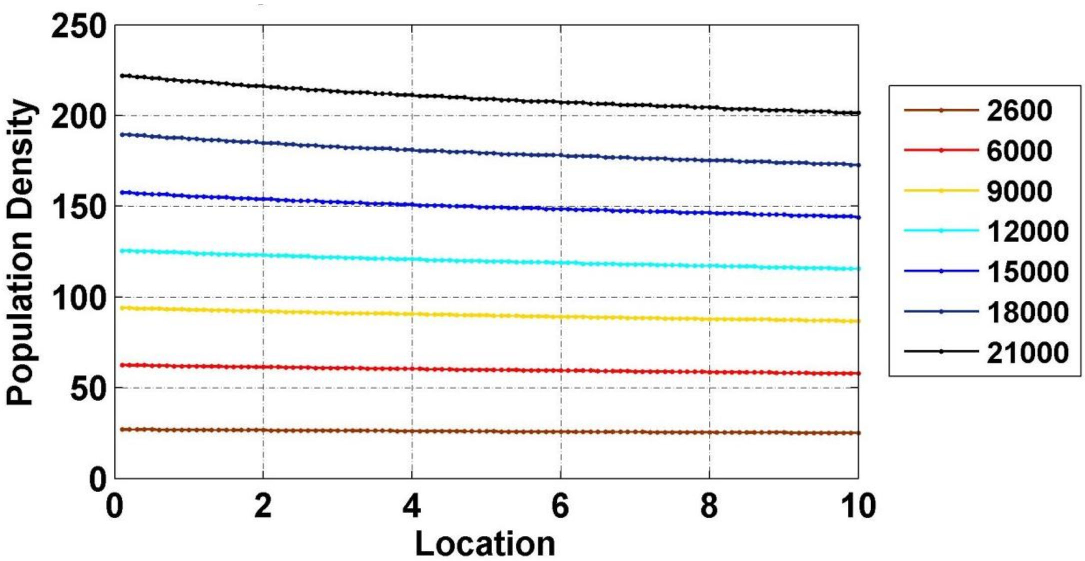


**Figure 4. 10 Evolution of population distribution in Case 3. (The legend shows the numbers of total population.)**

Figure 4.10 shows another derived phenomenon from the modelling framework, namely the variation in population distribution in the LMC. With a small population, the derived population distribution is uniform. It is obvious that the transport system becomes congested under the growth in population. People prefer the residential locations close to the city centre under the emergent congestion so that they can save the commuting time. Hence, the population distribution becomes non-uniform as the total population increases.

After the experiments in Case 1, 2 and 3, an arising question is how the various combinations of scaling exponent $\beta_C$ and $\beta_T$ affect the performance of the bi-modal transport system. Table 4.1 summarises another six possible combinations of scaling exponents (Case 4, 5, 6, 7, 8 and 9) and the results are shown in Figure 4.11. Note that they are all sub-linear scaling exponents according to the principle for infrastructures in cities. A high value of the scaling exponent $\beta_C$ drives a road capacity increase so that the road capacity per capita decreases less than the case with a lower $\beta_C$. As expected, the rate of increasing average travel time decreases with the increase of scaling exponent $\beta_C$, whereas the rate of average car emissions show the opposite tendency. These phenomena corroborate the observations in Figures 4.6 and 4.9. Moreover, Figure 4.11 shows what the possible mode shares under various combinations of scaling exponents are. It is worth noting that different combinations of scaling exponents can represent different strategic plans on the improvement of the bi-modal transport system. For example, a high value of $\beta_C$ indicates that a city improves the highway networks. Different cities may have different value of $\beta_C$. So the proposed framework can give some suggestions on the transport planning for urban growth.

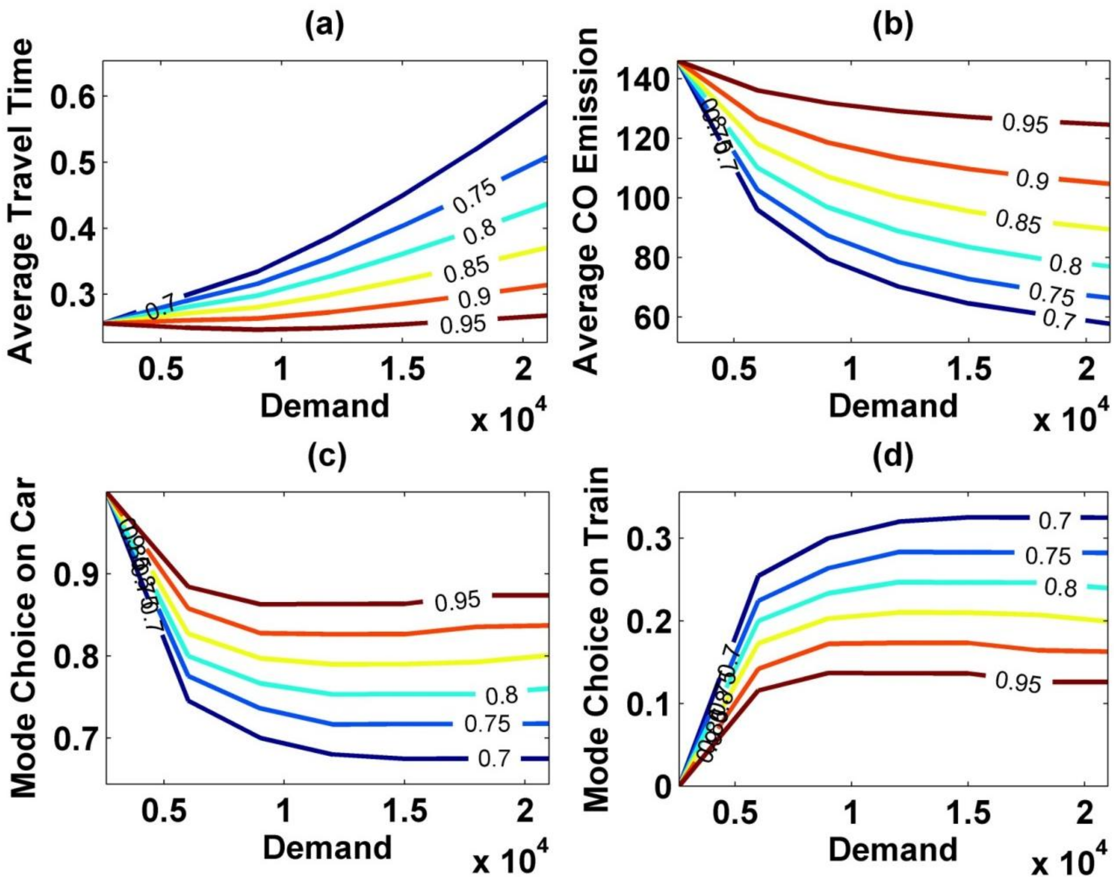


**Figure 4. 11 Results from Case 4, 5, 6, 7, 8 and 9. The values of scaling exponent are labelled on the lines.**

## 4.6 Summary of Chapter 4

This chapter has formulated a more general LMC model that includes separate time and income constraints. With two congestible modes, the utility is estimated by consumption, house space and leisure time. Hence, congestion in transport systems can affect all components in the utility function. The model extended helps to improve the investigation between transport congestion/growth and network performance under urban growth. Following the work in Chapter 3, the model includes the process of housing allocation instead of an exogenously demand distribution. Meanwhile, in the policy analysis, average travel time and CO emissions per trip are scrutinised according to the definition of sustainable transport systems in Section 1.1.

An integrated modelling framework is proposed for network evolution, in which the land use equilibrium and mode choices (at user equilibrium) have been jointly solved for each demand level. The analysis investigates various combinations of $\beta_C$ and $\beta_T$ for understanding the consequences of investment on road and public transport networks, which may give suggestions on transport planning for urban growth and can be the practical application of the modelling framework proposed.

Additionally, the modelling framework has modelled variations of mode choice in road and public transport networks. Indeed, the network efficiency on these two modes has influence on mode choice of commuters. After the analytical work in Chapter 4, an investigation of spatial efficiency in transport networks of real cities needs to be performed.

# Chapter 5
# Spatial Efficiency in Transport Networks

Public transport networks (including bus, metro systems etc.) have been built, improved or expanded in many cities, aligning network investment policies of transit-oriented development to maximize access to transit systems and encourage transit ridership (Cervero, 2004). In transport planning, public transport networks are often designed to ensure a large spatial coverage with circuitous lines (Black, 1995; Murray et al., 1998) so that they could increase ridership. However, the high indirectness of transit lines that trades off coverage for frequency (Walker, 2012) may discourage ridership, because people could commute by other modes to reduce trip circuity (and thus travel time). A question arises: to what extent the circuity of trips by transit would be accepted by travellers? Furthermore, increasing transit ridership implies reducing auto mode share, presently the most widely used commuting mode in every US metropolitan area (American Community Survey, 2012). One reason that more people commute by auto instead of transit is that they may have more circuitous transit routes on public transport networks. This thesis posits that the public transport network circuity of actual transit trips is lower than the transit circuity of travellers who chose to use auto.

The correlations between auto mode share and accessibility in road and public transport networks have been explained by Levinson (2012) and Owen and Levinson (2015). For instance, an increase in transit accessibility may reduce travel time by transit, which should reduce auto mode share. Correlations of circuity, accessibility and auto mode share are valuable to investigate.

In brief, this chapter investigates circuity in public transport networks with a comparison of that in road networks and examines the correlations among circuity, accessibility and auto mode share. Section 5.1 provides a literature review for the research in Chapter 5 followed by the introduction in Section 5.2 of measures that will be used. Section 5.3 sources data and Section 5.4 provides the research methods and sampling strategies. Results and investigation of circuity, accessibility and mode share in 36 cities are shown in Section 5.5. In the end of Chapter 5, Section 5.6 gives a summary.

## 5.1 Introduction

### 5.1.1 Measuring Circuity in Transport Networks

Following the review in Section 2.2.2.1, circuity is defined as the ratio of the shortest network distance over Euclidean distance between one origin-destination pair (Levinson and El-Geneidy, 2009; Barthélemy, 2011). The theoretical minimal value of circuity is 1 when the shortest network distance equals the Euclidean distance. The average circuity of all trips can assess the global performance of network (Vragović et al., 2005). The lower the value of circuity, the more efficient is the network. In road networks, the average circuity was estimated around 1.2 (Newell, 1980). At transit station catchments, the average circuity was found to be between 1.21 and 1.23 (O'Sullivan and Morrall, 1996). The average circuity may be used in estimating the travel distance (Ballou et al., 2002) as it presents the efficiency of road networks.

More recently, Levinson and El-Geneidy (2009) investigated the circuity in road networks of home-to-work trips (using Census LEHD data) in twenty metropolitan areas and the average circuity was lower for real trips than similar random trips. With a dataset of randomly generated trips for networks from 1990, 2000, and 2010, Giacomin and Levinson (2015) found the circuity of networks has generally increased over time, namely road networks have become less efficient in fifty metropolitan areas. These two studies revealed that the circuity of random OD pairs may be higher than that of real trips. In other words, real travellers select OD pairs which are less circuitous than random OD pairs, perhaps because such trips are more efficient, or appear more efficient. Real trips are posited to select for lower cognitive burden, that is they require fewer turns (or thought about navigation) on the part of travellers. They also showed the value of investigating the circuity by random trips since in some areas (or historically) because we may lack real trip records by modes. Though many studies have studied the average circuity of trips on road networks, the average circuity in public transport networks has not been reported. A study by Lee (2012) explained how circuity could examine the topology of public transport networks in a synthetic network. Hence, this chapter is going to investigate the average circuity of real and random trips in public transport networks of metropolitan areas.

As mentioned in Section 2.2.2.1, circuity of a node $C_i$ measures how accessible the node is in the network. The smaller $C_i$ is, the easier it is to reach the node $i$. Measuring the circuity of a node may find the most accessible node in a transport network and it has been used in Crucitti et al. (2006) and Porta et al. (2009). With this simple derivation, there should be some correlations between circuity and accessibility in transport networks. Furthermore, in terms of configurations of public transport networks, networks with a large number of lines may provide direct routes for trips while those with fewer lines may have circuitous routes for travellers (Lee, 1998). These two types of network configuration can ensure the accessibility of transit systems and they make the correlations between accessibility and circuity not obvious. This chapter attempts to survey their correlations.

### 5.1.2 Public Transport Network Performance

In assessing the performance of public transport networks, following the study of Lam and Schuler (1982), researchers have explored the complexity and robustness from the topology aspect, found network centrality to be an emergent property (Derrible, 2012; Derrible and Kennedy, 2009, 2010b). Furthermore, Mishra et al. (2012) measured the connectivity of public transport networks in multiple levels through investigating nodes, lines, transfer centres and regions. Roth et al. (2012) studied the topological evolution of public transport networks. Studies on topological characteristics can help us understand the performance of transit systems (Black, 2003), however these studies do not consider traveller behaviour or response to topological changes.

When the comparison of the circuity between real transit and auto trips may help to explain individual mode choices as the proposed hypothesis, one further question is whether auto and transit circuity could explain the aggregate mode share. In terms of mode share, Levinson (2012) reported that an increase in accessibility reduced the commute time and that made the auto mode share decrease and Owen and Levinson (2015) showed that accessibility explained mode share at the Census tract level. Based on the discussion after Equation 2.2 (see Section 2.2.2), one assumption can be proposed that transit circuity is positively correlated with transit accessibility. If so, auto mode share should be negatively correlated with transit circuity. These correlations between circuity and accessibility need to be studied. In brief, this chapter will investigate the correlations of circuity, accessibility and auto mode share in multiple metropolitan areas.

## 5.2 Definitions in Chapter 5

### 5.2.1 Average Circuity of Trips

This chapter measures the average circuity by mode of trips to investigate the performance of transport networks. Following Equation 5.1, the average circuity of trips $C_{ave}$ is given by

$$C_{ave} = \frac{\sum C_{ij}}{P}, \tag{5.1}$$

where $p$ indicates the total number of studied trips. To investigate the mode choice of commuters, the average transit circuity of trips ($C_{Transit}$) and the average auto circuity of trips ($C_{Auto}$) will be estimated respectively for each area.

### 5.2.2 Weighted Accessibility

For the analysis of correlations between circuity and accessibility, this section gives the definition of accessibility that will be used. Here, accessibility has been defined as the number of job locations that can be reached within a time threshold since reports (Levinson, 2013, Owen and Levinson, 2014) provided the number of job locations in a certain time threshold by driving and transit respectively. As the number of reachable job locations rises with travel time thresholds, the closer job locations are more attractive and weighted higher by commuters in the application of travel choice. So jobs reachable within the shortest time threshold should be weighted most heavily, and jobs are given decreasing weights as travel time increases. Then this chapter considers the weighted accessibility that can be defined as follow

$$a_w = \sum_t (a_t - a_{t-10}) \times e^{\theta t},\ t = 10, 20, 30, \dots, 60\ minutes, \tag{5.2}$$

where $a_w$ denotes weighted accessibility for the whole area and $a_t$ and $a_{t-10}$ represent the accessibility value within each time threshold. Based on Levinson and Kumar (1994), the decay coefficient used is $\theta = -0.08$. With Equation 5.2, the sum of weighted accessibility in all time periods $a_w$ can be obtained for each area. Then each area has weighted accessibility by auto (i.e. $a_{Auto}$) and transit (i.e. $a_{Transit}$).

## 5.3 Data

In the Minneapolis - St. Paul region, trips are sourced from the 2010 Travel Behaviour Inventory (collected between 2010 and 2012) (TBI) that records where, when and how commuters travel. Here, the analysis includes all home-to-work trips (302 transit trips and over 8000 auto trips). Note that the definition of a transit trip includes a primary mode (such as bus, light rail, or commuter rail) with walking as a mode of access/egress. This analysis thus excludes auto-access transit trips (e.g. trips using a park-and-ride facility). Auto trips are defined as trips with the single mode of driving, and exclude carpools and transit access trips. Chained trips are also excluded from the analysis.

General Transit Feed Specification (GTFS) provides a common format for public transport schedules and associates geographic information. For all areas studied, schedules of public transport systems were sourced from publicly available GTFS data. For example, GTFS data give the transit schedule in November 2011 in the Minneapolis - St. Paul, Minnesota to be compatible with the TBI. For other cities, GTFS data from January 2014 were used, consistent with the accessibility data described below.

Meanwhile, the data of road networks in areas studied are provided by Open Street Map (OSM). OSM is a collaborative project to create a free editable map worldwide. For instance, the OSM road network map of Minneapolis - St. Paul includes 154,571 road segments (links). It is worth noting that OSM data are continuously updated, and so represents the state of the network at the time of download (Fall 2014). Since the road network changes slowly, differences between the dates of OSM and GTFS networks will not significantly affect results.

Additionally, for the investigation of circuity, accessibility and auto mode share, this chapter obtains the accessibility data from ’Access Across America’ reports (Levinson, 2013, Owen and Levinson, 2014) and mode share data from American Community Survey (2012). With data sources mentioned, this analyses transit circuity in the Minneapolis - St. Paul region in Sections 5.5.1 and 5.5.2. Moreover, Chapter 5 uses OSM, GTFS, accessibility and mode share data of 35 additional regions for the analysis in Sections 5.5.2 and 5.5.3.

## 5.4 Research Methods and Sampling

### 5.4.1 Research Methods

Measuring circuity among trips obtained from empirical data and those randomly generated in transport networks can better help us better investigate network efficiency, because we could present how the network serves demand overall (Giacomin and Levinson, 2015, Levinson and El-Geneidy, 2009). Moreover, the investigation of random trips may be helpful when the data of home-to-work trips are limited in some areas or historically. With the hypothesis proposed, this section further tests the implication about travel responses to network structure in the form of circuity. In order to compare the transit and auto circuity for transit trips, auto trips, and trips randomly generated, research methods are developed below for the analysis in the Minneapolis - St. Paul region.

First, date of transit trips (such as the location of origins and destinations, departure time) are sourced from TBI records so that the spatial coverage of real transit trips is determined. Within the coverage determined by real transit trips in Minneapolis - St. Paul region, this chapter randomly selected 300 real auto work trips from the TBI data. In the generation of random trips, 300 origins and 300 destinations were randomly generated in the spatial coverage of real transit trips. Then each origin matched one destination. The trip length of random trips on road networks was constrained to have the same distribution as real transit trips. Because the majority of transit trips in the Minneapolis - St. Paul region are within 10-70 minutes travel time and the Euclidean distance between origin and destination is within 21 km, auto trips and those randomly generated keep these ranges consistent. Additionally, the period of departure time for random trips is same as that for real transit trips. So a further 300 trips and their information were randomly generated in the same spatial coverage. Since this chapter measures circuity in transit and road networks, the Euclidean distance is used as the criteria.

Second, this chapter finds the shortest travel time route by transit for three types of trips (transit users, auto users, random OD pairs) in OpenTripPlanner (OTP) software with GTFS and OSM data. In this step, this research sets the allowed waiting time at origin as zero in OTP in such cases the trip travel time is estimated from departure time in TBI data or the predetermined time for random trips. As the shortest travel time route by transit includes the walking time to access/egress and travel time along transit lines, the network distance in the estimation of circuity consists of the walking routes on road networks and the routes on public transport networks.

Finally, importing origins and destinations of all trips using the Geographic Information System (GIS) software (ArcGIS.10.2), the shortest road network distance of trips was calculated with the Network Analyst tool. In such cases, points join the road network via the most adjacent link. With the impedance of trip length, one-way restrictions and allowed U-turns at junctions, Dijkstra’s algorithm (Dijkstra, 1959) was used to find the shortest paths. The connectivity policy was set as Any Vertex when the Network Dataset generated for each metropolitan area. The Minneapolis - St. Paul regional network was projected onto North American Datum (NAD) coordinate system in UTM zone 15N.

Additionally, for the analysis of circuity, accessibility, mode share in transport networks, this chapter calculates the weighted accessibility and sources the auto mode share for each metropolitan area as well as the Minneapolis – St. Paul region. Note that the report by Owen and Levinson (2014) has observed transit accessibility in Core Based Statistical Areas (CBSA) in the United States, but the geometries of areas studied in this chapter are based on the rectangle map generated by OSM. Maps from OSM mainly record areas with high accessibility, other areas with low accessibility and the fringes or edges of metropolitan areas have been excluded. Since this chapter used the sum of weighted accessibility and relatively sparse OD pairs (compared to report of Owen and Levinson, 2014), the relatively few trips in the fringes or areas with low accessibility are ignored. This will not importantly affect the analysis in Section 5.5.3.

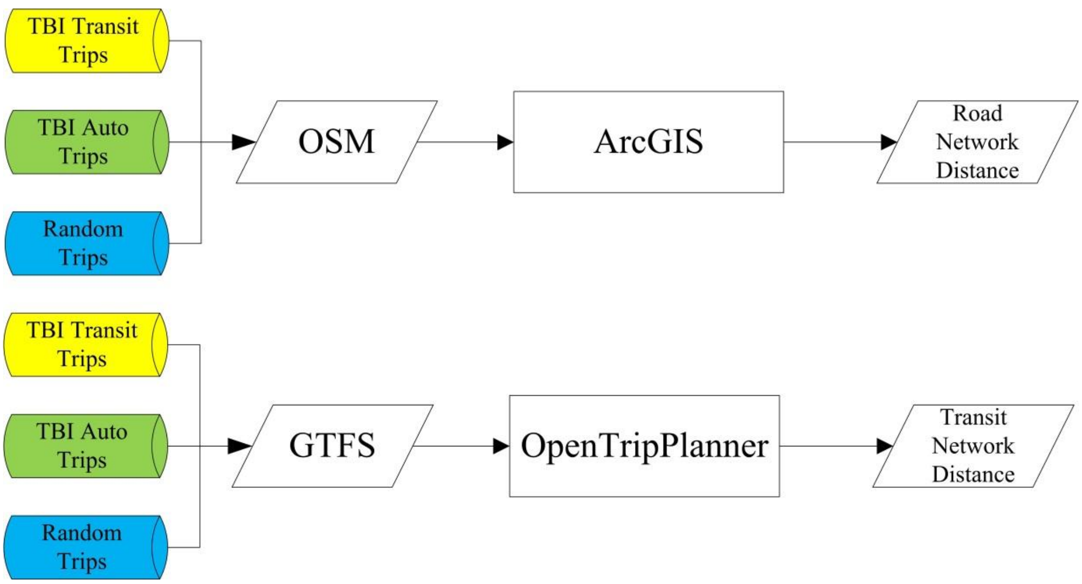


**Figure 5. 1 Flowchart of Research Methods in Chapter 5.**

In summary, the shortest road network distance is calculated with ArcGIS and OSM; the shortest travel time distance by transit is estimated by OpenTripPlanner. With the Euclidean distance from haversine formula, road and transit circuity are calculated respectively. The research methods are summarised in Figure 5.1.

### 5.4.2 Sampling

First, the outcome from OTP suggests that some transit trip have large circuity values, which may be caused by errors in the official GTFS schedules. The proportion of trips with large transit circuity (over 8) is around 3 percent. The low rate implies that those trips are affected by bugs in schedules, so they are excluded in the sample. Second, some trips cannot be estimated or have high auto circuity due to the accuracy of OSM and TBI data and these trips are also excluded from the sample. Third, if the value of circuity (by auto or transit) is lower than 1, the corresponding trips are excluded.

## 5.5 Results

### 5.5.1 Real Transit, Real Auto, and Random Trips

With the research methods proposed, real transit trips, real auto trips and those randomly generated can be analysed in the Minneapolis - St. Paul region. For a trip between origin $i$ and destination $j$, the distance of the shortest travel time route by transit ($D_{ij}^{Transit}$), the distance of the shortest route on road networks ($D_{ij}^{Auto}$), and the Euclidean distance ($D_{ij}^{e}$) between the origin and destination can be calculated. The average distances for three trip types are shown in Table 5.1. The average distance of transit routes for real transit trips is much shorter than that for real auto trips and random OD pairs. Compared to this large difference, the average distances of routes by auto are close, so are Euclidean distances. Note that the Euclidean distance is the criteria here. As stated before, this chapter examines the average circuity of transit, auto and random trips in the Minneapolis - St. Paul region and the results are shown in Table 5.1. Between each origin and destination, the shortest travel time route is found on the public transport network with OpenTripPlanner and the shortest route is found on the road network with GIS software. Hence, for each trip type, the average transit circuity $C_{Transit}$ and the average auto circuity $C_{Auto}$ can be estimated. Since the average distances of routes by auto are similar, average auto circuities in Table 5.1 only have minor differences with small standard deviations. This provides a fair comparison basis for transit circuity. However, the transit circuity of trips has a larger spread and the coefficients of variation are 50.59%, 65.23% and 55.06% for transit trips, auto trips, and random OD pairs respectively. Since transit stops are dense in some parts of the region (e.g. the city center and transit hubs), transit circuity in such regions will be low or even close to the auto circuity. But for parts of the region with few transit stops, transit circuity will typically be very high.

**Table 5. 1 Comparison of the average distance of shortest travel time routes by transit (km), the average distance of shortest routes by auto(km), the Euclidean distance(km), transit and auto circuity and stand deviations in the Minneapolis - St. Paul Region.**

| Trip Type | $D_{ij}^{Transit}$ | | $D_{ij}^{Auto}$ | | $D_{ij}^{e}$ | | $C_{Transit}$ | | $C_{Auto}$ | |
|---|---|---|---|---|---|---|---|---|---|---|
| | **mean** | $\sigma$ | **mean** | $\sigma$ | **mean** | $\sigma$ | **mean** | $\sigma$ | **mean** | $\sigma$ |
| Real transit trips | 14.72 | 6.90 | 8.79 | 4.74 | 7.7 | 4.31 | 2.19 | 1.02 | 1.15 | 0.09 |
| Real auto trips | 22.69 | 11.36 | 11.36 | 9.53 | 8.12 | 3.91 | 3.28 | 2.14 | 1.18 | 0.09 |
| Random trips | 24.40 | 9.716 | 9.71 | 10.90 | 9.53 | 4.18 | 2.98 | 1.64 | 1.16 | 0.07 |

In Figure 5.2, the map of public transport networks in the Minneapolis - St. Paul region (source: Metropolitan Council and The Lawrence Group, 2015) is shown. Meanwhile, all origins of real transit trips are shown with the values of transit circuity. As the public transport network mainly covers the region of Hennepin, Ramsey and Anoka Counties, most transit trips occur in those counties. Though there are no records of real transit trips in Carver and Scott counties in the Travel Behaviour Inventory, the coverage of random trips covers seven counties so that the average transit circuity of random trips can be computed in every county. In Table 5.2, it can be observed that auto mode share should be positively correlated with the transit circuity of random trips.

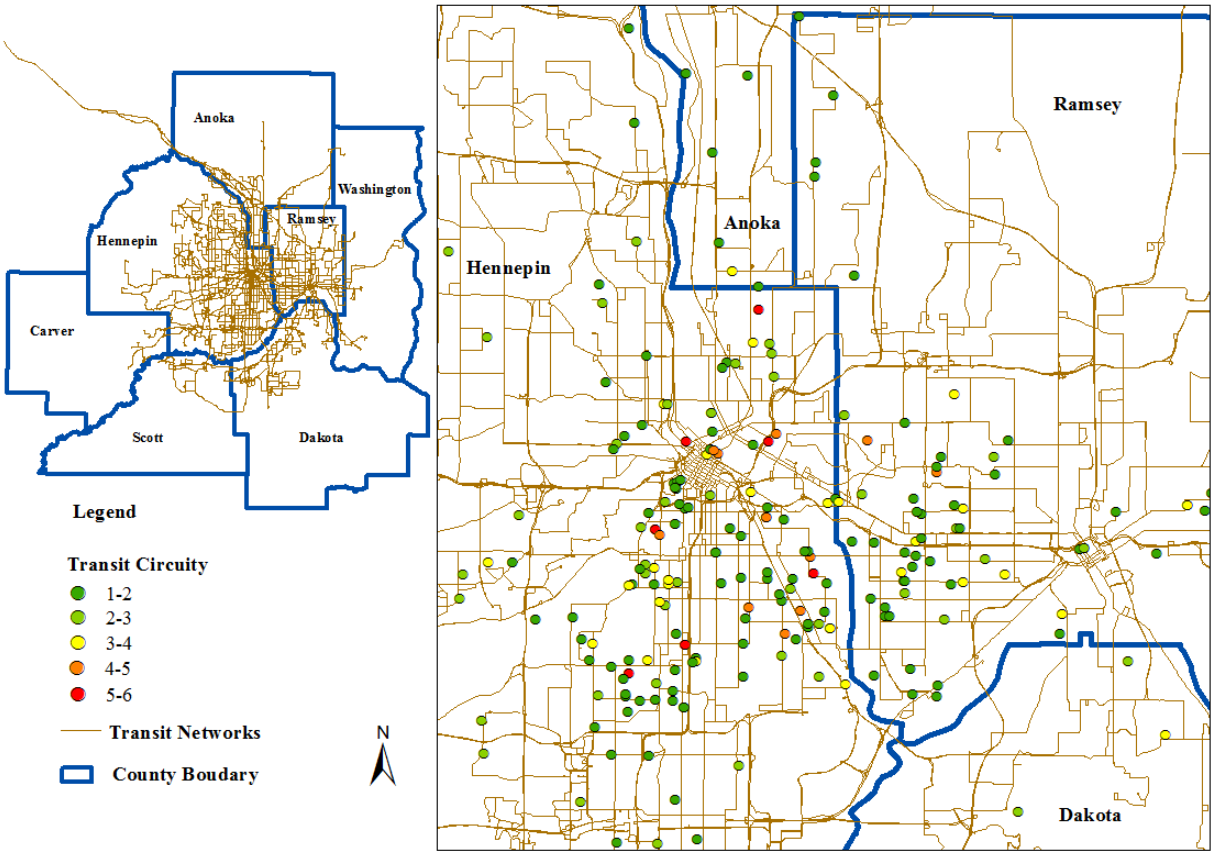


**Figure 5. 2 Transit circuity of each real transit trip in the Minneapolis - St. Paul Region**

**Table 5. 2 Auto mode share and average circuity of trips in seven counties, the Minneapolis - St. Paul Region. (– indicates that there are no real transit trips.)**

| County name | Auto mode share | Real transit trip | | Random trips | |
|---|---|---|---|---|---|
| | | $C_{Transit}$ | $C_{Auto}$ | $C_{Transit}$ | $C_{Auto}$ |
| Hennepin | 0.8255 | 2.2885 | 1.1550 | 2.2838 | 1.1569 |
| Ramsey | 0.8455 | 2.0174 | 1.1627 | 2.9642 | 1.1457 |
| Anoka | 0.8600 | 1.8697 | 1.1009 | 2.7550 | 1.1527 |
| Carver | 0.8895 | – | – | 2.9085 | 1.2457 |
| Scott | 0.8929 | – | – | 3.1570 | 1.1220 |
| Dakota | 0.8983 | 2.2020 | 1.1296 | 2.9450 | 1.1503 |
| Washington | 0.9163 | 1.4364 | 1.0938 | 3.3032 | 1.1281 |

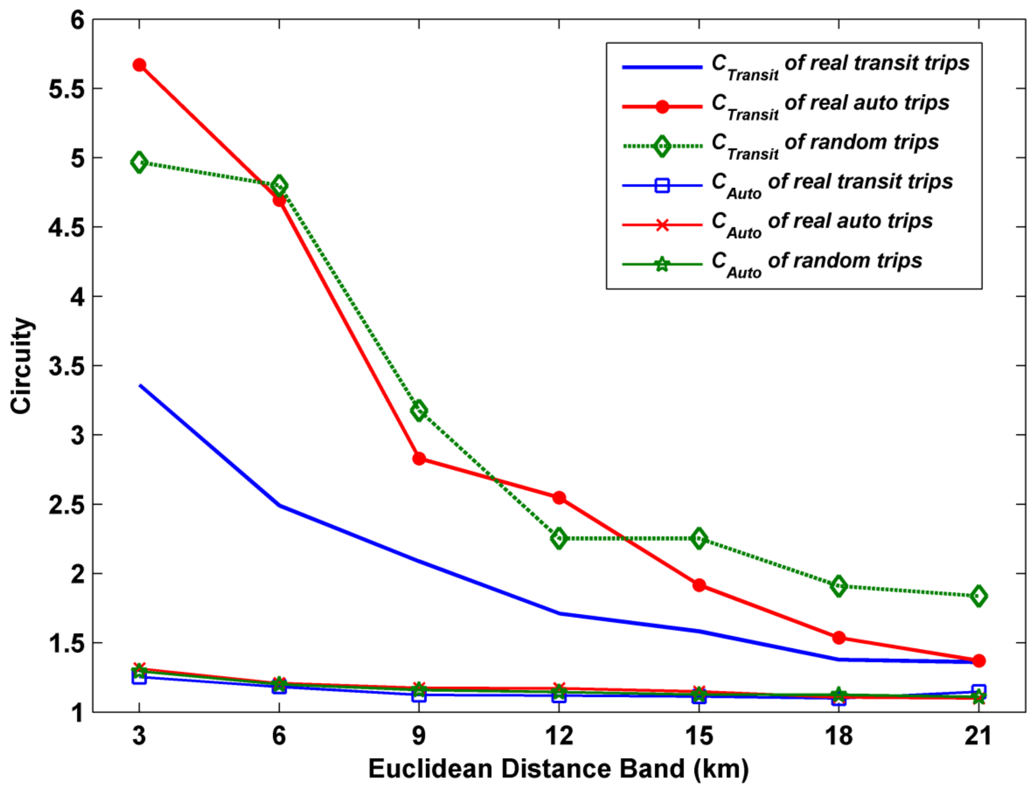


**Figure 5. 3 Circuity in each Euclidean distance interval in the Minneapolis - St. Paul Region.**

Figure 5.3 displays the decay of transit circuity over the distance interval which is consistent with the conclusion that the auto circuity declines with the increase of trip length (Giacomin and Levinson, 2015, Levinson and El-Geneidy, 2009). So it supports the claim that samples are selected properly. It is observed that trips of short distances have much higher transit circuity. One reason can be that commuters in practice take some off-road routes to get on/off transit systems, which are not reflected in the network (so the transit circuity may be somewhat overstated). Along one transit trip, the most circuitous route are usually the “first mile” of access or/and “last mile” of egress, and local distribution. These elements arise in every transit trip. But for long transit trips, the proportion of circuitous route decreases with overall trip length. Moreover, auto circuity serves as the lower limit of transit circuity for all on-road or road-adjacent transit facilities. Moreover, Figure 5.3 corroborates that the transit circuity of random OD pairs and real auto trips is higher than the transit circuity of real transit trips. The significant difference of transit circuity between transit and auto trips explains in large part the choice of modes. People served by very circuitous transit routes are much less likely to use transit than people served by relatively more direct transit routes.

Overall, average transit circuity of trips is higher than auto circuity since the public transport network is designed to provide a large area of coverage so that commuters have a shorter time to access the network. The trade-off with greater spatial coverage is longer in-vehicle travel times and lower frequencies, so transit trips generally have higher travel times than trips on the road network. (The exception is when public transport networks are grade-separated and running on more direct off-road facilities). This observation about circuity explains why the public trsnport network has lower overall accessibility, corroborating Levinson (1998).

### 5.5.2 Travel Time and Transit Circuity

Travel time is the most important factor affecting travel behavior. Hence, this chapter focuses on the correlation between transit circuity and travel time. As shown in Figure 5.4, consistent with previous findings about auto circuity, transit circuity declines as the travel time increases. Similar to Giacomin and Levinson (2015), a regression analysis is performed using Equation 5.3:

$$C_t = \beta_1 t^{\beta_2}, \tag{5.3}$$

where $t$ represents each travel time interval, $C_t$ represents the transit circuity at that interval, $\beta_1$ and $\beta_2$ are parameters to be estimated. Table 5.3 shows the regression results from the Matlab nonlinear regression tool. As shown in Figure 5.4, the transit circuity of real transit trips may decay from 3.5. The combination of $\beta_1$ and $\beta_2$ controls from where and how fast transit circuity declines as travel time increases. The situation where $\beta_1 > 0$ and $\beta_2 < 0$ indicates that the transit circuity exponentially decays as the travel time increases. The exponential decay between transit circuity and travel can be found in the cases for real transit trips and randomly generated trips. The goodness of fit ($R^2$) by transit and random trips is close to one. Table 5.3

and the regression in Figure 5.4 show that the correlation between transit circuity and travel time by random trips is similar to real transit trips. Hence, the generation of random OD pairs can provide information when data about the spatial location of actual trips in multiple areas are limited. Moreover, more evidence in more areas could help us better collaborate findings in Minneapolis – St. Paul region. To that end, the chapter replicates the investigation of transit circuity by random OD pairs for another 35 metropolitan areas in the United States. In areas studied, road networks are projected onto state planar coordinates NAD 1983 with their suitable UTM zones.

**Table 5. 3 Regression results of Transit Circuity in the Minneapolis - St. Paul Region.**

| Trip name | $\beta_1$ | $\beta_2$ | $R^2$ |
|---|---|---|---|
| Real transit trips | 14.3593 | -0.5557 | 0.9971 |
| Random trips | 29.8804 | -0.6300 | 0.9874 |

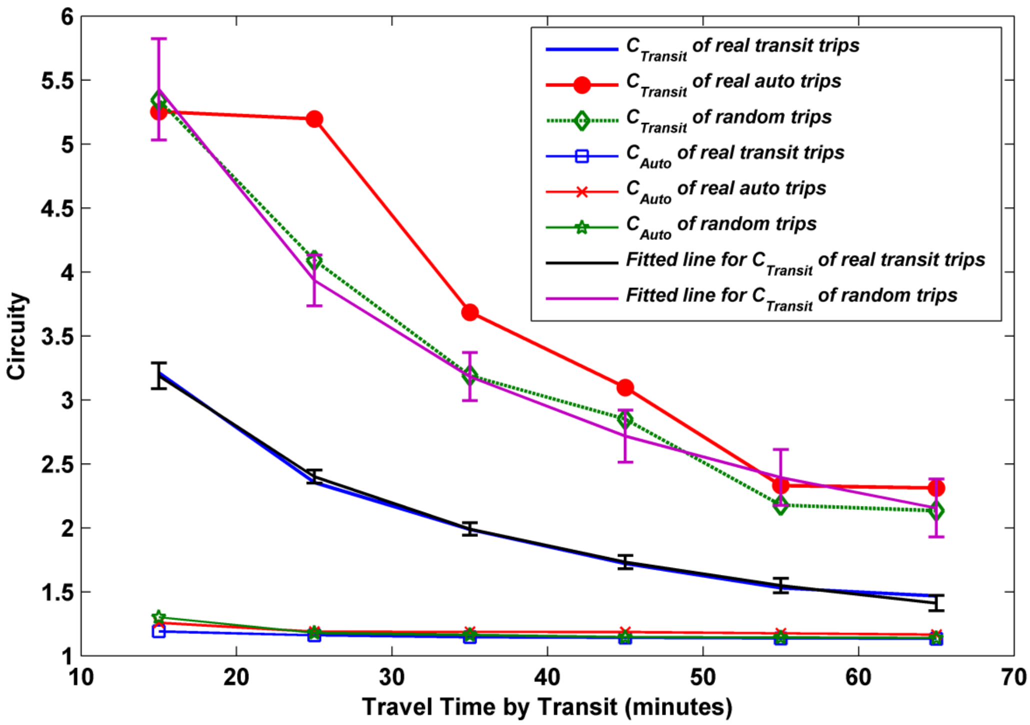


**Figure 5. 4 Circuity in each travel time interval in the Minneapolis - St. Paul Region.**

In order to generate OD pairs with a proper spatial coverage as home-to-work trips, this research first generated 300 destinations and 300 origins within the city centre. The coordinate of city centre point for each metropolitan area is obtained from (Date and time). With this nominal city centre, the region of city centre is defined as a circle with 5 km radius around its centre point. Another 300 destinations and 300 origins are randomly generated across the whole Open Street Map rectangular region for that metropolitan area. Each origin is randomly matched to one destination giving 600 trips for each area. Consequently, an origin and destination pair may represent trips with both ends in the city centre, neither end in the city centre, or one end in the centre and one not. It is worth noting that a monocentric configuration has been assumed in the generation of random trips. Although cities may not be absolutely monocentric, the polycentric city can be regarded as a combination of several monocentric models.

In these 35 areas studied, trips up to 90 minutes travel duration are included in the analysis and the travel time distribution is kept constant across areas studied. The time distribution is similar to that in the Minneapolis – St. Paul region where the number of trips in the time interval firstly increases and then decays as travel time increases. In such cases, the number of trips has a peak between 40 and 60 minutes in these additional areas. Note that the range of travel time by random trips in the Minneapolis - St. Paul region is within 90 minutes in Section 5.5.2.

To test whether 600 trips are sufficient to ascertain circuity, a more detailed analysis is conducted for the Chicago region. Figure 5.5 shows that the transit circuity slightly varies in a small range from 600 up to 2000 random trips. The average circuity of real transit trips from the household survey (2007) in Chicago (source: Metropolitan Travel Survey Archive) is 2.3842 and that of random trips is 2.3275, which show the number of random trips is substantial once again. Hence, this chapter surveys the average transit circuity of random trips due to the limited data of household surveys.

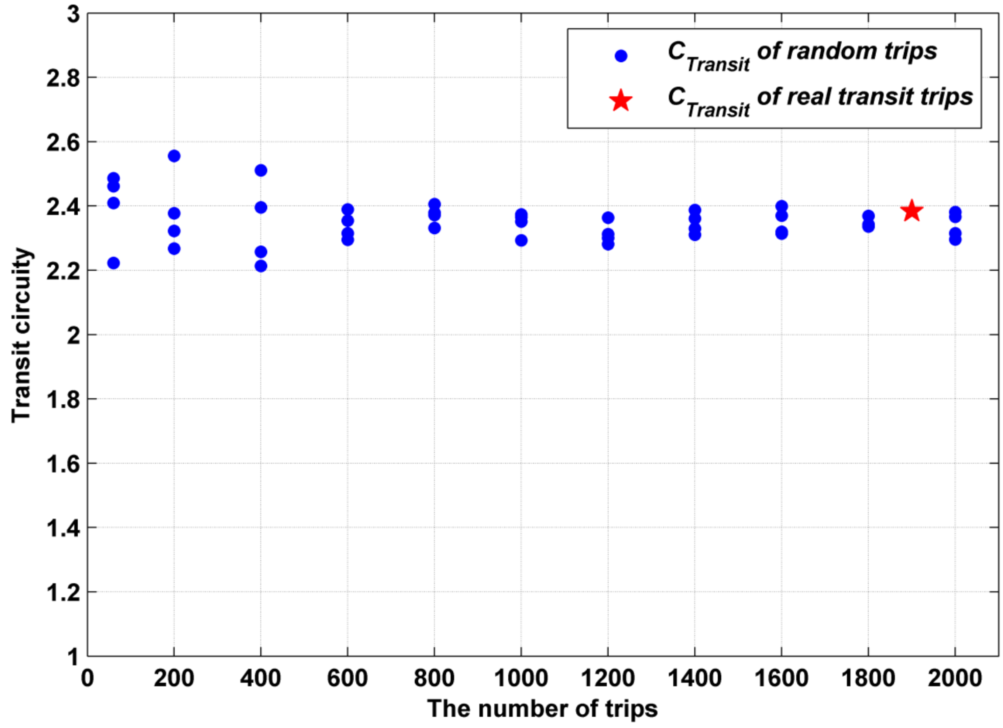


**Figure 5. 5 Sensitivity analysis of trip number and transit circuity in Chicago.**

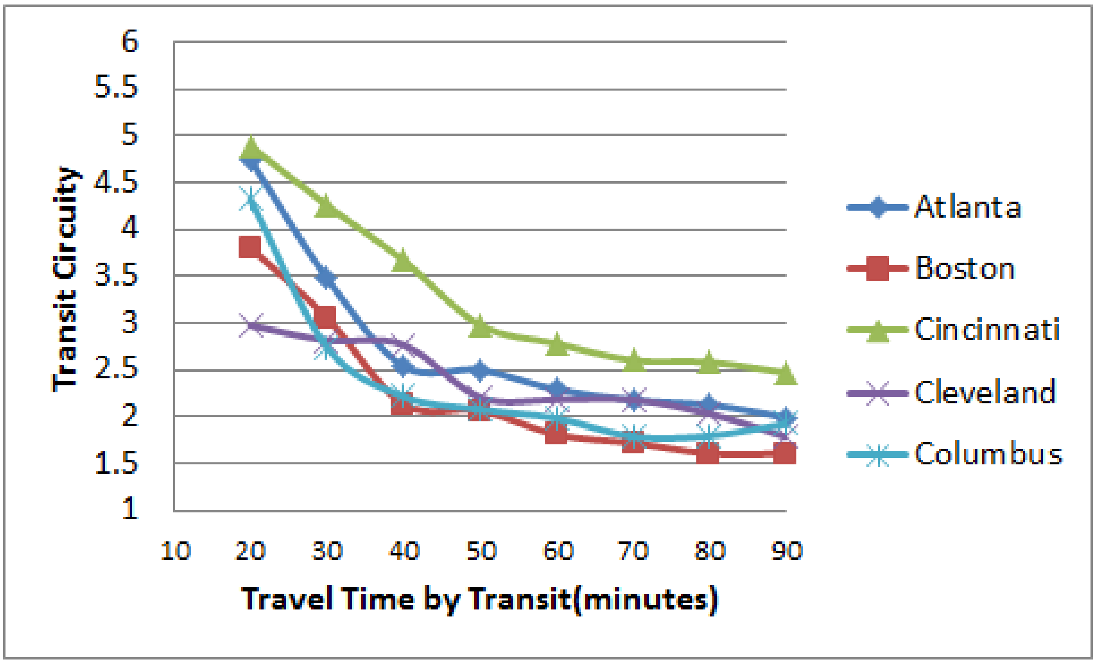


**Figure 5. 6 Decay of Transit Circuity in metropolitan areas.**

Results from 35 metropolitan areas are shown in Figures 5.6 and 5.7. In Figure 5.6, the decay between transit circuity and travel time can be observed in five example areas. This section then used the same regression method as Equation 5.3 and the result of the regression is shown in Table 5.4. The exponential decay ($\beta_1 > 0$ and $\beta_2 < 0$) has been observed across metropolitan areas with high goodness of fit. The auto circuity $C_{Auto}$ is in a proper range, which supports the use of the random trip generation method. In Figure 5.7, auto mode share of these areas are ranked and the average transit circuity of random trips and the ratio of $C_{Auto}$ to $C_{Transit}$ are listed. The average transit circuity is higher than auto circuity for the same sample of random trips (i.e. $C_{Auto} / C_{Transit} < 1$) and the transit circuity is variable compared to auto circuity (see the standard deviation in Table 5.4). One conjecture from Figure 5.7 is that the ratio of $C_{Auto}$ to $C_{Transit}$ may increase as auto mode share decreases. This should be caused by two variations: (i) the average auto circuity increases and/or (ii) the average transit circuity decreases with auto mode share decreases (non-auto mode share increases). However, the correlation between auto mode share and circuity is not obvious and it needs more discussion shown in Section 5.5.3.

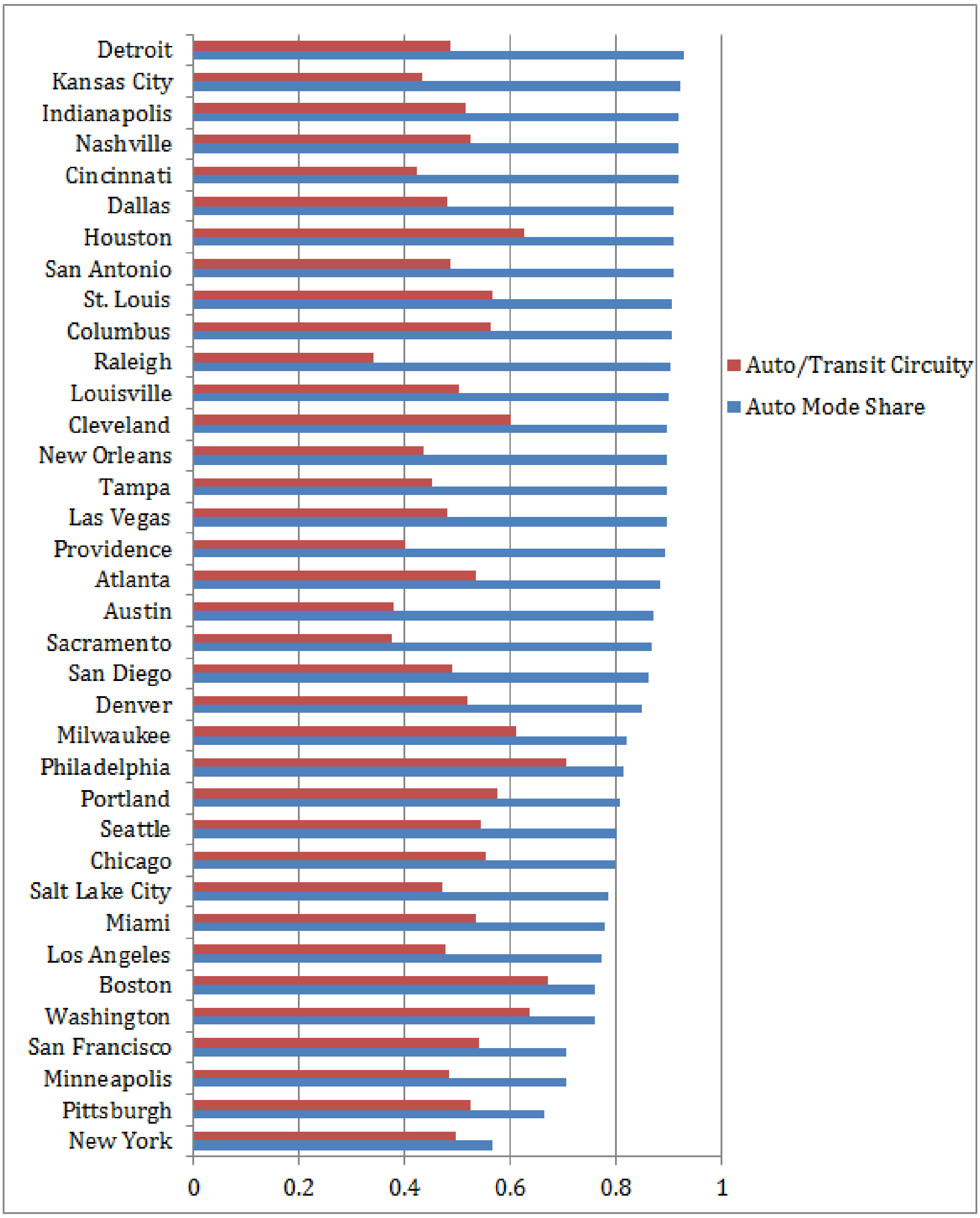


**Figure 5. 7 Transit and auto circuity in 36 metropolitan areas.**

**Table 5. 4 Auto circuity and regression results of transit circuity as a function of time for random OD pairs.**

| Metropolitan area | $C_{Auto}$ | $\sigma_{Auto}$ | $C_{Transit}$ | $\sigma_{Transit}$ | $\beta_1$ | $\beta_2$ | $R^2$ |
|---|---|---|---|---|---|---|---|
| Atlanta | 1.24 | 0.18 | 2.33 | 0.96 | 19.36 | -0.53 | 0.96 |
| Austin | 1.27 | 0.27 | 3.34 | 1.48 | 49.86 | -0.66 | 0.96 |
| Boston | 1.24 | 0.22 | 1.86 | 0.60 | 16.79 | -0.55 | 0.97 |
| Chicago | 1.29 | 0.24 | 2.33 | 0.98 | 17.61 | -0.58 | 0.99 |
| Cincinnati | 1.29 | 0.28 | 3.05 | 1.30 | 15.76 | -0.42 | 0.97 |
| Cleveland | 1.28 | 0.44 | 2.14 | 0.93 | 6.45 | -0.27 | 0.88 |
| Columbus | 1.26 | 0.20 | 2.23 | 1.05 | 18.14 | -0.55 | 0.91 |
| Dallas | 1.19 | 0.20 | 2.48 | 1.27 | 38.77 | -0.68 | 0.99 |
| Denver | 1.22 | 0.12 | 2.36 | 1.31 | 27.83 | -0.66 | 0.96 |
| Detroit | 1.23 | 0.21 | 2.54 | 1.33 | 19.14 | -0.56 | 0.98 |
| Houston | 1.20 | 0.16 | 1.91 | 0.50 | 5.06 | -0.24 | 0.96 |
| Indianapolis | 1.19 | 0.17 | 2.31 | 0.96 | 8.94 | -0.34 | 0.93 |
| Kansas City | 1.24 | 0.15 | 2.87 | 1.48 | 17.16 | -0.44 | 0.93 |
| Las Vegas | 1.23 | 0.15 | 2.57 | 1.34 | 32.90 | -0.67 | 0.98 |
| Los Angeles | 1.22 | 0.16 | 2.55 | 1.44 | 26.36 | -0.64 | 1.00 |
| Louisville | 1.33 | 0.31 | 2.65 | 1.00 | 15.31 | -0.45 | 0.94 |
| Miami | 1.25 | 0.16 | 2.35 | 1.17 | 44.20 | -0.72 | 0.95 |
| Milwaukee | 1.17 | 0.12 | 1.92 | 0.51 | 5.87 | -0.29 | 0.96 |
| Minneapolis | 1.23 | 0.03 | 2.55 | 1.09 | 29.88 | -0.63 | 0.99 |
| Nashville | 1.30 | 0.32 | 2.48 | 0.99 | 14.04 | -0.44 | 0.95 |
| New Orleans | 1.44 | 0.44 | 3.32 | 2.00 | 20.02 | -0.46 | 0.97 |
| New York | 1.47 | 0.37 | 2.97 | 1.19 | 12.84 | -0.38 | 0.99 |
| Philadelphia | 1.23 | 0.23 | 1.74 | 0.56 | 15.59 | -0.55 | 0.89 |
| Pittsburgh | 1.30 | 0.15 | 2.47 | 1.25 | 41.94 | -0.68 | 0.90 |
| Portland | 1.24 | 0.16 | 2.16 | 0.86 | 23.18 | -0.59 | 1.00 |
| Providence | 1.43 | 0.30 | 3.56 | 1.40 | 29.74 | -0.55 | 0.97 |
| Raleigh | 1.33 | 0.36 | 3.91 | 1.39 | 21.68 | -0.43 | 0.97 |
| Sacramento | 1.24 | 0.12 | 3.31 | 1.91 | 33.90 | -0.57 | 0.90 |
| Salt Lake City | 1.32 | 0.29 | 2.80 | 1.41 | 15.22 | -0.45 | 0.95 |
| San Antonio | 1.27 | 0.36 | 2.60 | 1.10 | 24.15 | -0.59 | 0.98 |
| San Diego | 1.29 | 0.20 | 2.65 | 1.54 | 34.37 | -0.63 | 0.93 |
| San Francisco | 1.23 | 0.14 | 2.27 | 0.99 | 21.30 | -0.59 | 0.94 |
| Seattle | 1.29 | 0.29 | 2.38 | 1.07 | 12.63 | -0.44 | 0.86 |
| St. Louis | 1.29 | 0.26 | 2.28 | 0.95 | 32.69 | -0.60 | 0.97 |
| Tampa | 1.34 | 0.37 | 2.97 | 1.26 | 17.23 | -0.45 | 1.00 |
| Washington | 1.38 | 0.19 | 2.17 | 1.15 | 33.85 | -0.72 | 0.92 |

### 5.5.3 Circuity, Accessibility, and Mode Share

#### 5.5.3.1 Correlation of circuity, accessibility and mode share

The variation between transit and auto circuity may explain mode share in metropolitan areas (Figure 5.7). Because either transit or road networks become efficient with lower circuity; they may attract more commuters. Meanwhile, the urban mode share may attract more users with the higher accessibility of transit or road network (Levinson, 2012). Considering these possible relationships, this chapter examines both circuity and accessibility on both transit and road networks in order to better explain mode share.

**Table 5. 5 Correlation Matrix of Circuity, Accessibility and Mode Share in 36 metropolitan areas.**

| | $\boldsymbol{M_{Auto}}$ | $\boldsymbol{C_{Auto}}$ | $\boldsymbol{A_{Auto}}$ | $\boldsymbol{C_{Transit}}$ | $\boldsymbol{a_{Transit}}$ |
|---|---|---|---|---|---|
| $\boldsymbol{M_{Auto}}$ | 1 | | | | |
| $\boldsymbol{C_{Auto}}$ | -0.2243 | 1 | | | |
| $\boldsymbol{a_{Auto}}$ | -0.5865 | -0.1007 | 1 | | |
| $\boldsymbol{C_{Transit}}$ | 0.1761 | 0.5378 | -0.2896 | 1 | |
| $\boldsymbol{a_{Transit}}$ | -0.7460 | 0.3768 | 0.6645 | -0.0698 | 1 |

For the analysis of auto mode share $M_{Auto}$, this section has four prospective explanatory variables, auto circuity $C_{Auto}$, auto accessibility $a_{Auto}$, transit circuity $C_{Transit}$ and transit accessibility $a_{Transit}$. In order to investigate correlations among these explanatory variables, this chapter first looks at the correlation matrix of them. In the metropolitan areas studied, auto mode share ranges from 56.55 to 92.65 percent. As shown in Table 5.5, auto circuity is negatively correlated with auto mode share; because high circuity on road networks could reduce the network efficiency and then auto mode share decreases. Transit accessibility is also negatively correlated with auto mode share since the high accessibility of public transport networks can attract more commuters and then less people commute by driving. Perhaps unexpectedly, auto accessibility is negatively correlated with auto mode share. This corroborates previous findings in Levinson (2012) and is posited to be due to the high correlation between auto and transit accessibility with the value 0.6645. Large cities have both high auto and high transit accessibility, and tend to have relatively low auto mode shares. Additionally, auto and transit circuity are significantly correlated as auto and transit accessibility, which may indicate the possible correlation between the circuity of a node and accessibility.

### 5.5.3.2 Circuity and accessibility

Compared to circuity, transit and auto accessibility are significantly correlated with auto mode share (see Table 5.5). A question remained: how does circuity of transport networks influence auto mode share? Based on the discussion below Equation 2.2 (see Section 2.2.2) and the statements of correlation matrix, there may be some correlation between circuity and accessibility in transport networks. Hence, this Section attempts to explain transit accessibility with auto and transit circuity via the following regression model:

$$a_{Transit} = \tau_1 C_{Auto} + \tau_2 C_{Transit} + \tau_3, \tag{5.4}$$

where $\tau_1$ and $\tau_2$ are the coefficients to be estimated and $\tau_3$ is the normalisation constant.

**Table 5. 6 Results in OLS regression for transit accessibility.**

| $\boldsymbol{a_{Transit}}$ | **Coef.** | **Std. Err** | $\boldsymbol{t}$ | $\boldsymbol{P} > \|t\|$ |
|---|---|---|---|---|
| $\boldsymbol{C_{Auto}}$ | 62875.82 | 19330.43 | 3.25 | 0.003 |
| $\boldsymbol{C_{Transit}}$ | -5912.791 | 2764.528 | -2.14 | 0.040 |
| **Constant** | -60002.24 | 21729.12 | -2.76 | 0.009 |
| | | | **Observations** | 36 |
| | | | $\boldsymbol{R^2}$ | 0.2465 |
| | | | $\boldsymbol{Prob} > F$ | 0.0094 |

Both auto and transit circuity are statistically significant factors affecting transit accessibility, because $P$ value are all less than 0.05 and $t$ value has a high absolute value (see Table 5.6). When $t$ is negative, it means the independent variable is negatively correlated with the dependent variable, namely the corresponding coefficient is negative. When $t$ is positive, the independent variable is positively correlated with the dependent variable, namely the corresponding coefficient is positive. Therefore, in Table 5.6, a one-unit increase of transit circuity decreases metropolitan average transit accessibility (by 5912 weighted jobs) because increases in circuity decrease the efficiency of transit systems for users. Perhaps less obviously a one-unit increase of auto circuity increases transit accessibility. When, after controlling for transit circuity, road networks are less efficient, more people may use transit, improving transit service (a positive feedback effect of transit service and use dubbed the “Mohring Effect” (Mohring, 1972) in the field), and thus improving transit accessibility. The accessibility of public transport networks can be estimated from the model in Table 5.6 and $\hat{a}_{Transit}$ denotes the transit accessibility estimated from $C_{Auto}$ and $C_{Transit}$. With this model, this chapter concludes that auto and transit circuity represents the efficiency of road and public transport networks and influence transit accessibility which significantly affect auto mode share.

#### 5.5.3.3 Regression Analysis for Mode Share

This section tests how accessibility by auto and transit affects auto mode share in a bi-variate model. Since this chapter has censored data of mode share, a logit function is given by

$$logit(p) = log\,(\frac{M_{Auto}}{1-M_{Auto}}), \tag{5.4}$$

where $M_{Auto}$ denotes auto mode share in a metropolitan area. The estimated transit accessibility ( $\hat{a}_{Transit}$ ) from the preceding section and auto accessibility (from Levinson, 2013) can be used in the regression for auto mode share and a regression model is used

$$logit(p) = \alpha_0 + \alpha_1 \alpha_{Auto} + \alpha_2 \hat{a}_{Transit}, \tag{5.5}$$

where $\alpha_0$ is the normalisation constant and $\alpha_1$ and $\alpha_2$ are coefficients to be estimated. Similar to the discussion of Table 5.6, the auto and transit accessibility are significantly correlated with auto mode share with small $P$ value and high absolute values of $t$. Furthermore, with negative coefficients and $t$ values, auto and transit accessibility are negatively correlated with auto mode share. As Table 5.7 shows, metropolitan areas with higher transit accessibility have lower auto mode share (higher non-auto mode share). The transit accessibility estimated $\hat{a}_{Transit}$ from transit and auto circuity shows statistical significance in the regression model. Similarly, those areas with higher auto accessibility also have lower auto mode share, as accessibility is correlated with density, crowding, and congestion.

**Table 5. 7 Results in OLS regression for auto mode share.**

| $\boldsymbol{logit(p)}$ | **Coef.** | **Std. Err** | $\boldsymbol{t}$ | $\boldsymbol{P > \lvert t \rvert}$ |
|---|---|---|---|---|
| $\boldsymbol{a_{Auto}}$ | $-4.75 \times 10^{-6}$ | $1.19 \times 10^{-6}$ | -4.00 | 0.000 |
| $\boldsymbol{\hat{a}_{Transit}}$ | $-4.92 \times 10^{-5}$ | $2.09 \times 10^{-5}$ | -2.35 | 0.025 |
| $\boldsymbol{\alpha_0}$ | 2.6482 | 0.1937 | 13.67 | 0.000 |
| | | | **Observations** | 36 |
| | | | $\boldsymbol{R^2}$ | 0.4191 |
| | | | $\boldsymbol{Prob} > F$ | 0.0001 |

## 5.6 Summary of Chapter 5

This chapter adopts circuity, the ratio of network to Euclidean distance, to investigate attributes of public transport networks. For Minneapolis - St. Paul region, transit circuity is compared to that of road networks among real transit trips, real auto trips and random OD pairs. Transit circuity for transit users is lower than transit circuity for auto users and for random trips, which helps explain mode choice. For thirty-six metro areas in the United States, trips randomly generated with systematic methods are examined. The results show that average transit circuity is higher than that in road networks. The higher value of transit circuity shows that public transport networks have been designed to ensure a large spatial coverage, giving circuitous routes for commuters. In practice, public transport networks in the United States are always more circuitous on average than road networks. Moreover, the exponential decay between transit circuity and travel time shows the possible range of transit circuity in public transport networks (from 1.5 to 6). It also shows that the increases in travel time could reduce the transit circuity as the distance of transit routes is a large proportion of total travel distance, which implies that the ‘first mile’ to access or the ‘last mile’ to egress should be the reason of high transit circuity.

Meanwhile, this chapter shows that circuity helps explain transit accessibility to jobs and that transit accessibility can help explain much of the variation in mode share across metropolitan areas. This analysis suggests that transit circuity may affect accessibility of public transport networks and then commuting mode share. This finding empirically shows the correlation between regional accessibility and circuity.

Overall, this chapter displays the variation of transit circuity in 36 different metropolitan areas, and finds these differences are important in explaining travel behaviour, demonstrating the value of measuring circuity in public transport networks.

# Chapter 6
# Conclusions

Chapter 6 reviews how this thesis answer the research questions proposed in Section 1.1 when achieving research objectives listed in Section 1.3. To this end, Section 6.1 reviews the work conducted in this thesis and Section 6.2 provides conclusions from the work of this thesis and possible directions for future work.

## 6.1 Research Summary

In this thesis, there are two parts of work: (i) investigating the emergent phenomena under the growth of transport networks with modelling approaches; (ii) empirical observation of spatial features in transport networks. Table 6.1 recapitulates the research questions that were introduced in Chapter 1 and how research objectives are correlated with each question has been shown. Moreover, the table also lists the relevant sections of literature review for each research questions as well as chapters.

In the search of an appropriate modelling approach, this thesis formulates a linear monocentric city model with two travel modes (Section 3.2). To investigate the aggregate phenomena in transport systems, the scaling-laws growth between transport infrastructures and population sizes are considered in the model and the vertical and horizontal growth have been designed according to planning strategies of urban intensification and sprawl (Section 3.5). With the analysis of Chapter 3, this thesis answered RQ 1.

Furthermore, in order to model urban dynamics in a simple way (RQ 2), Chapter 4 includes the bid-rent process in housing allocation so that the model developed in Chapter 3 has been extended. As stated in motivation of RQ 2, there may be a trade-off between investment on road and public transport networks. Hence, Sections 4.5.2 and 4.5.3 examine network performance under various plans of improving transport systems.

**Table 6. 1 Research Summary**

| **Research Question** | **Associated Research Objectives** | **Associated Literature Review** | **Supporting Chapter** |
|---|---|---|---|
| RQ 1: How can we perform strategic analysis to investigate the aggregate performance of a congestible bi-modal transport system (driving and public transport) in different urban configurations? | RO 1: Model the interdependence between urban growth following scaling laws, and the aggregate performance of transport infrastructure provision.<br>RO 3a: Design experiments that can study different planning strategies. | Sections 2.1, 2.2.1, 2.2.3, 2.2.4 and 2.3 | Chapter 3 |
| RQ 2: How can we include the co-evolution of land use and transport influence in the model from RQ 1 and investigate the trade-off of investment between two transport modes? | RO 2: Extend the model from RO1 with the co-evolution of residential location and mode choice and propose a modelling framework for urban growth.<br>RO 3b: Design experiments that can study the trade-off of investment in two modes (highways and public transport) for urban sustainability. | Sections 2.2.3, 2.2.4, 2.3 and 2.4 | Chapter 4 |
| RQ 3: How can we investigate circuity in road and public transport networks and make a comparison? | RO 4: Examine circuity in public transport networks.<br>RO 5: Investigate the topological difference in road and public transport networks and study the correlation between circuity and other relevant indicators. | Sections 2.2.2 and 2.2.4 | Chapter 5 |

As the thesis studies RQ 1 and RQ 2, the first part in modelling has been finished. Then the thesis starts the empirical study in transport networks to answer RQ 3.

With several data sources (TBI, OSM, GTFS etc.) and OpenTripPlanner and GIS software, the thesis surveys the circuity of public transport systems with travel time in 36 cities. Because the thesis attempts to find out how people make decisions on travel modes is affected by how aggregate transport networks are, a comparison of circuity between road and public transport networks has been performed. As Section 2.2.2 explained the potential correlation between the indicator 'accessibility' and circuity based on their definitions, the thesis explores their correlation with a regression analysis. Meanwhile, regression analyses have been shown between the mode share of cities and circuity in order to better understand the relationship between mode choice and spatial features in transport networks.

## 6.2 Discussions

### 6.2.1 Contributions of Thesis

As the beginning of conclusions, Section 6.2.1 will summarise the contribution of this thesis when this thesis managed to investigate the research gaps from Chapter 2.

In the literature review, this thesis pointed out that the continuum approach has advantages in the investigation of network growth, compared to the discrete models in transport planning and models in network science. Those models usually study development in road and public transport respectively and the interaction between two transport modes may be ignored, such as commuters may prefer different modes in different congestion levels. So there is a need to investigate the simultaneous growth of two transports modes and the modelling framework has following contributions:

(i) Two congestible transport modes have been formulated and the mode choice between them has been investigated;

(ii) Simultaneous development of road and public transport networks has been modelled with the scaling-law growth.

Mode choice in a linear monocentric city with two congestible transport modes did not attract much attention in the previous studies (Section 2.3.3), hence the model in this thesis provides some methodological advance in model formulation and solving the mode choice in the linear monocentric city. The investigation of simultaneous development of road and public transport networks may help transport planners more on the trade-off of investment on two transport modes and do not isolated one of them in the transport planning.

The other research gap mentioned in the literature review is that the interaction between transport and land use is necessary to include in the study of transport network growth. Because the spatial features of transport networks and mode choice of commuters varied and affected by this interaction. Therefore, the work has these contributions:

(iii) The thesis included urban dynamics in the investigation of transport network growth;

(iv) The thesis included various trade-offs of time and income by residents in the bid rent process.

To investigate the urban dynamics under network growth, this thesis considered the bid rent process of residents. To better represent how transport influence land use patterns, the model included the time and income trade-offs respectively, instead of considering the generalised travel cost. The analysis in Section 4.5.1 displays that the model proposed provides more flexibility to examine the interaction between transport and land use. Then the variation of land use pattern could be analytically examined (Figures 4.5 and 4.10).

In addition, considering three pillars of sustainability of transport systems in Chapter 1, the thesis has contributed on the study of system efficiency, car emissions and social aspect in the experiments. First, travel time is one important indicator in the decision of commuters' mode choice. Though various studies have adopted the continuum approaches in transport planning (Section 2.3), the different meanings of Total Travel Time, Mean Travel Time and Average Travel Speed have not been clarified and they may have different implication in the study of transport network growth.

Second, Chapter 4 derived and observed car emission per trip in the continuum model. Third, the variations of land use pattern with residential allocation and mode choice of commuter have been studied. In brief, one contribution is:

(v) This thesis explained why we need to measure different features (e.g. Mean Travel Time, Average Travel Speed, car emissions or land use patterns) in different situations to understand efficiency and sustainability of transport systems.

With the modelling framework, the growth of transport infrastructures and the variation of land use patterns are considered under urban growth. When models include the scaling-law growth in road networks, different values of scaling exponent may drive the amount of transport infrastructure per capita varies. In many situations, different scaling exponents could drive the performance trajectories scale diversely (Sections 5.5.2.1 and 3.5.2.2.2). But the average network performance could be independent of the scaling exponent if the impact on congestion by population growth counteracts the influence of the expansion in urban areas or/and the improvement of transport infrastructures (Section 3.5.2.2.1).

For transport planners, different scaling exponents of road and public transport networks can indicate different strategic plans on the highway and public transport systems. According to this indication, the trade-off between investment on highway and public transport system has been examined in Sections 4.5.2 and 4.5.3. This thesis has not given an optimal combination of scaling exponents for highway and public transport system, because cities may need various urban planning strategies at different developing stage as well as transport systems. As studied in Chapter 4, experiments in Section 4.5.3 show the spectrum of system performance with various scaling-law growth and the performance predicted helps transport planners make decisions on the investment of transport infrastructures. It is the possible application of the modelling framework for transport planning.

In the review of empirical observations in scaling laws of urban growth, the thesis found that empirical studies usually isolated the observation in road or public transport networks (Section 2.24). To study this research gap and examine the difference between road and public transport networks, the thesis examine the network efficiency according to circuity of trips in 36 cities in the United States. Meanwhile, there may be correlation between circuity and regional accessibility (see Section 2.2.2), but this correlation has not been examined in the empirical studies. Hence, the work in Chapter 5 has analysed this correlation and the conclusion from the analysis is that the investigation of circuity can help us understand the regional accessibility (i.e. they are significantly correlated). In detail, the comparison of circuity in road and public transport network for different real trips presents the self-selection in mode choice and residential location, which has not been shown in previous studies.

To sum up, this thesis developed a modelling framework for the investigation of growth and evolution in transport networks. Meanwhile, the thesis observed spatial features with various data sources in 36 cities, which could help us understand the possible variation in spatial networks as city sizes expand.

### 6.2.2 Limitations and Future Work

This section is going to discuss the limitations of models proposed and empirical observations shown in this thesis.

The work in Chapters 3 and 4 is based on deterministic user equilibrium assignment, which is one limitation in the models proposed. Deterministic user equilibrium assumes that all individuals have complete information and consistently make the correct decisions (Sheffi, 1985), which rarely occur in reality. Furthermore, particular assumptions of transport modes have been considered so that the mode choice or watershed case arises. However, due to these assumptions, the model only works well for a certain demand levels (see Chapter 3). For instance, due to the road capacity constraint and high demand levels, only people who live close to the city boundary will use car (e.g. 2% of city space) while all other people will take public transport. The dominant proportion of taking public transport is not realistic and it may be caused by the deterministic user equilibrium. Hence, these assumptions of transport modes can be relaxed in future and the model may consider stochastic user equilibrium.

Though the thesis included the waiting time and in-vehicle travel time by public transport, other aspects (e.g. train station density etc.) that can affect the travel time by public transport have not been included. In the meantime, the thesis assumes a low average speed of train (12.5km/hr) in order to investigate the mode choice. This assumption can be relaxed when we consider that the waiting time varies with different train headway and train capacity. In future, when studies investigate the development of public transport network, the study can consider more features and the correlations among feature of public transport rather than only considering the train speed.

Moreover, this thesis assumed that every residents/commuters are homogenous, which limited the investigation. As the thesis proposed a general model including various trade-off and several components in the utility function, there are different ways to extend the work. For example, multiple classes of residents with the different levels of income can be considered or various weights of house space (or leisure time) may be investigated.

In the analytical work and empirical study, this thesis assumes all cities are monocentric for simplification. In fact, the monocentric formulation is not the only morphology in cities. Hence, more work for various city formulations could be conducted and examined to relax this assumption.

In the empirical study, this thesis selected circuity as the indicator to measure how aggregate public transport networks of real cities are. The empirical observation of circuity proofs that the routes provided by public transport are more circuitous than those on road networks. This finding provides useful indicators of spatial efficiency in road and public transport networks, which can be used to extend the modelling framework of Chapter 4 in future.

Finally, the thesis not only discusses the possible correlation between circuity and accessibility based on their definition but also reflects the implication of circuity with the empirical report on accessibility. The thesis shows that the investigation of circuity could help in the analysis of regional accessibility and then in the explanation of mode share in cities with regression analysis. More work in detail can be conducted to present these correlations (particularly for circuity and regional accessibility) in regions and traffic analysis zones.

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# Appendix A

## A.1 Derivation of Remark 4.1

**Remark 4.1** Bid rent function $\Psi(x,u)$ is continuous and decreasing in both $x$ and $u$. The house space profile $g(x,u)$ under the bid rent process is continuous and increasing in both $x$ and $u$.

Because the derivative of bid rent function can be expressed as $\frac{\partial \Psi}{\partial x} = \frac{\partial \Psi}{\partial T_m} \cdot \frac{\partial T_m}{\partial x}$, where $\frac{\partial \Psi}{\partial T_m} = -(1-\lambda)^{\alpha/\lambda} e^{-u/\lambda} \big(Y - T_m(x)\big)^{\frac{1-\lambda}{\lambda}} < 0$.

According to Assumption 4.1, $\frac{\partial T_m}{\partial x} > 0$, so $\frac{\partial \Psi}{\partial x} < 0$.

$\frac{\partial \Psi}{\partial u} = -(1-\lambda)^{\alpha/\lambda} \big(Y - T_m(x)\big)^{\frac{1}{\lambda}} e^{-\frac{u}{\lambda}} < 0$.

Similarly, $\frac{\partial \mathrm{g}}{\partial x} = \frac{\partial \mathrm{g}}{\partial T_m} \cdot \frac{\partial T_m}{\partial x}$,

where $\frac{\partial \mathrm{g}}{\partial T_m} = (\alpha/\lambda) \alpha^{-\alpha/\lambda} \big(Y - T_m(x)\big)^{-\alpha/\lambda - 1} e^{\frac{u}{\lambda}} > 0$.

Hence, $\frac{\partial \mathrm{g}}{\partial x} > 0$.

$\frac{\partial \mathrm{g}}{\partial u} = \left(\frac{1}{\lambda}\right) \alpha^{-\alpha/\lambda} \big(Y - T_m(x)\big)^{-\alpha/\lambda} e^{\frac{u}{\lambda}} > 0$.

## A.2 Derivation of Remark 4.2

**Remark 4.2** $\lambda \to 1$, it holds $\lim_{\lambda \to 1} \mathrm{g}(x,u) = e^u$, $\lim_{\lambda \to 1} z = 0$ and $\lim_{\lambda \to 1} \Psi(x,u) = \frac{Y - T_m(x)}{e^u}$.

Because $\alpha + \lambda = 1$ where $\lambda \epsilon (0,1)$. The space house and bid rent under a certain utility level are given in the equations above,

$\mathrm{g}(x,u) = (1-\lambda)^{(\lambda-1)/\lambda} \big(Y - T_m(x)\big)^{(\lambda-1)/\lambda} e^{u/\lambda}$,

$\Psi(x,u) = \lambda (1-\lambda)^{(1-\lambda)/\lambda} \big(Y - T_m(x)\big)^{1/\lambda} e^{-u/\lambda}$.

If $\lambda \to 1$, the following derivation can be written as,

$\lim_{\lambda \to 1} (1-\lambda)^{\frac{\lambda-1}{\lambda}} = \lim_{\lambda \to 1} e^{\left(\frac{\lambda-1}{\lambda}\right)\log(1-\lambda)} = e^{\lim_{\lambda \to 1} \frac{\log(1-\lambda)}{\lambda/(\lambda-1)}} = e^{\lim_{\lambda \to 1} \frac{-1/(1-\lambda)}{-1/(\lambda-1)^2}} = 1$ ;

$\lim_{\lambda \to 1} (1-\lambda)^{\frac{1-\lambda}{\lambda}} = \lim_{\lambda \to 1} e^{\left(\frac{1-\lambda}{\lambda}\right)\log(1-\lambda)} = e^{\lim_{\lambda \to 1} \frac{\log(1-\lambda)}{\lambda/(1-\lambda)}} = e^{\lim_{\lambda \to 1} \frac{-1/(1-\lambda)}{1/(1-\lambda)^2}} = 1$.

So the house space and bid rent profile are given as

$\lim_{\lambda \to 1} \mathrm{g}(x,u) = e^u$,

$\lim_{\lambda \to 1} \Psi(x,u) = \frac{Y - T_m(x)}{e^u}$.

The house space at location $x$ under the utility $u$ is equal to the left income divided by the corresponding bid rent, namely,

$$g(x,u) = \frac{Y - T_m(x) - z}{\Psi(x,u)}.$$

So $\frac{Y - T_m(x) - z}{\Psi(x,u)} = e^u$ when $\lambda \to 1$.

Substituting $\Psi(x,u)$, $\frac{Y - T_m(x) - z}{\frac{Y - T_m(x)}{e^u}} = e^u$. Hence, z is zero when $\lambda \to 1$. In such case, residents tend to spend all spared income for bidding a house.

## A.3 Derivation of Remark 4.3

**Remark 4.3** When $\lambda \to 0$, $\lim_{\lambda \to 0} g(x,u) = e^u$, $\lim_{\lambda \to 0} \Psi(x,u) = 0$ and $e^u = Y - T_m(x)$.

When $\lambda \to 0$, the space house profile can be written as

$$g(x,u) = (1-\lambda)(Y - T_m(x))\left(\frac{1}{1-\lambda}\right)^{\frac{1}{\lambda}}\left(\frac{e^u}{(Y - T_m(x))}\right)^{\frac{1}{\lambda}},$$

where the limitation of term $\left(\frac{1}{1-\lambda}\right)^{\frac{1}{\lambda}}$ is given as

$$\lim_{\lambda \to 0}\left(\frac{1}{1-\lambda}\right)^{\frac{1}{\lambda}} = e^{\lim_{\lambda \to 0}\left[\frac{\log(1/(1-\lambda))}{\lambda}\right]} = e.$$

Let $\rho = \frac{e^u}{Y - T_m(x)}$, then $\rho > 0$.

If $0 < \rho < 1$, $\lim_{\lambda \to 0} \rho^{\frac{1}{\lambda}} = 0$, so $\lim_{\lambda \to 0} g(x,u) = 0$.

If $\rho = 1$, $\lim_{\lambda \to 0} \rho^{\frac{1}{\lambda}} = 1$, so $\lim_{\lambda \to 0} g(x,u) = \left(Y - T_m(x)\right)e = e^u$.

If $\rho > 1$, $\lim_{\lambda \to 0} \rho^{\frac{1}{\lambda}} = \infty$, then $\lim_{\lambda \to 0} g(x,u) = \infty$.

First, no matter how people spare their income on the consumption of goods, transport fare and rent, people get some house space at their locations. So $\lim_{\lambda \to 0} g(x,u) = 0$ is not valid in CCA model. Second, the linear monocentric city has a space constraint which will be defined in Section 4.2 in this thesis. So $\lim_{\lambda \to 0} g(x,u) = \infty$ is not valid here. To sum up, when $\lambda \to 0$ and $\frac{e^u}{Y - T_m(x)} = 1$, $\lim_{\lambda \to 0} g(x,u) = e^u$.

When $\lambda \to 0$, the bid rent profile can be written as

$\Psi(x,u) = \lambda(1-\lambda)^{\frac{1-\lambda}{\lambda}}\left(\frac{1}{\rho}\right)^{\frac{1}{\lambda}} = \lambda^{\frac{\lambda-1}{\lambda}}(1-\lambda)^{\frac{1-\lambda}{\lambda}}\left(\frac{\lambda}{\rho}\right)^{\frac{1}{\lambda}}$,

where $\rho = 1$ as discussed above. The following limits are derived as

$\lim_{\lambda\to 0}\lambda^{\frac{\lambda-1}{\lambda}} = e^{\lim\limits_{\lambda\to 0}\left[\left(\frac{\lambda-1}{\lambda}\right)\log\lambda\right]} = 0,$

$\lim_{\lambda\to 0}(1-\lambda)^{\frac{1-\lambda}{\lambda}} = \lim_{\lambda\to 0} e^{\left(\frac{1-\lambda}{\lambda}\right)\log(1-\lambda)} = e^{\lim_{\lambda\to 0}\frac{\log(1-\lambda)}{\lambda/(1-\lambda)}} = e^{\lim_{\lambda\to 0}\frac{-1/(1-\lambda)}{1/(\lambda-1)^2}} = e^{-1}.$

In such case, it holds

$\lim_{\lambda\to 0}\Psi(x,u) = 0.$

Therefore, in the case where $\lambda \to 0$, $\lim_{\lambda\to 0} g(x,u) = e^{u}$ and $\lim_{\lambda\to 0}\Psi(x,u) = 0$ when $eu=Y-Tm(x)$.

## A.4 Derivation of Rent Equation 4.20

Based on Equation 4.9 in Section 4.1, we have

$g(x,u^*) = (1-\lambda)^{(\lambda-1)/\lambda}\big(Y-T_m(x)\big)^{(\lambda-1)/\lambda}e^{u^*/\lambda}$,

where $1-\lambda = \alpha$. And the city configuration is given by $L(x) = V\pi$, $\forall x \in (0,l)$.

The population constraint $\int_0^l \frac{L(x)}{g(x,u^*)}dx = N$ can be written as

$\int_0^l V\pi(1-\lambda)^{(1-\lambda)/\lambda}\big(Y-T_m(x)\big)^{(1-\lambda)/\lambda}e^{-u^*/\lambda}\,dx = N.$

Then the relation is obtained from population constraint as follow

$e^{-u^*/\lambda} = \frac{N}{V\pi(1-\lambda)^{(1-\lambda)/\lambda}\int_0^l\big(Y-T_m(x)\big)^{(1-\lambda)/\lambda}dx}$,

where the integral $\int_0^l\big(Y-T_m(x)\big)^{(1-\lambda)/\lambda}dx$ is a positive constant and let $H = \int_0^l\big(Y-T_m(x)\big)^{(1-\lambda)/\lambda}dx$, namely,

$e^{-u^*/\lambda} = \frac{N}{V\pi(1-\lambda)^{(1-\lambda)/\lambda}H}.$

The equilibrium utility is derived as

$$u^* = \lambda\, ln \frac{V\pi(1-\lambda)^{(1-\lambda)/\lambda}H}{N}.$$

Because $\Psi(x,u^*) = \lambda(1-\lambda)^{(1-\lambda)/\lambda}\big(Y - T_m(x)\big)^{1/\lambda} e^{-u^*/\lambda}$ and the rule in Equations 4.11 and 4.12, the rent profile is written as follow,

$$R(x) = \Psi(x,u^*) = \lambda(1-\lambda)^{(1-\lambda)/\lambda}\big(Y - T_m(x)\big)^{1/\lambda} e^{-u^*/\lambda}.$$

Substituting $e^{-u^*/\lambda}$, the rent profile is given by

$$R(x) = \frac{N\lambda Y^{1/\lambda}}{V\pi H}\left(1 - \frac{T_m(x)}{Y}\right)^{1/\lambda}.$$

## A.5 Derivation of Remark 4.5

**Remark 4.5** For a given travel cost function $T_m(x)$, a known total number of population $N$, the predetermined daily income per capita $Y$, the city length $l$ and a constant $\lambda$, the demand distribution under the equilibrium utility $u^*$ is given by $n(x,u^*) = \frac{N}{H}(Y - T_m(x))^{(1-\lambda)/\lambda}$ where $H$ is a positive constant and $H = \int_0^l \big(Y - T_m(x)\big)^{(1-\lambda)/\lambda} dx$.

**Proof of Remark 4.5.** The population constraint is defined as Equation 4.19, so the demand distribution under $u^*$ is written as

$$n(x,u^*) = L(x)/g(x,u^*).$$

It is known that $g(x,u^*) = (1-\lambda)^{(\lambda-1)/\lambda}(Y - T_m(x))^{(\lambda-1)/\lambda} e^{u^*/\lambda}$ in Section 4.1. Meanwhile, $L(x) = V\pi$ and $e^{u^*/\lambda} = \frac{V\pi H(1-\lambda)^{(1-\lambda)/\lambda}}{N}$. Substituting $e^{u^*/\lambda}$, the demand distribution is obtained

$$n(x,u^*) = \frac{N}{H}(Y - T_m(x))^{(1-\lambda)/\lambda}.$$

It can be known that the demand density $n(x)$ at an arbitrary location $x$ is constant when the travel cost from $x$ to city centre is a known constant $T_m$. So the demand distribution $n(x,u^*)$ is unique. This completes the proof of Remark 4.5.

## A.6 Derivation of Equations 4.37-39

With the definition of bid rent function in Equation 4.35, this can be written as an unconstraint problem with $Z(\mathrm{g}, T_l, u) = \mathrm{g}^{-\lambda/\alpha} {T_l}^{-\gamma/\alpha} e^{u/\alpha}$ from the utility definition. The first order optimal conditions are written as (a) $\frac{\partial \Psi}{\partial \mathrm{g}} = 0$; (b) $\frac{\partial \Psi}{\partial T_l} = 0$.

With Condition (a), the following correlation is obtained from

$$\frac{\partial}{\partial \mathrm{g}}\left(\frac{I(x) - z(\mathrm{g}, u, T_l) - WT_l(x)}{\mathrm{g}}\right) = \frac{\left(-\frac{dz}{d\mathrm{g}}\right)\mathrm{g} - (I(x) - z(\mathrm{g}, u, T_l) - WT_l(x))}{\mathrm{g}^2} = 0.$$

The following relations can be derived from Condition (a):

$$-\frac{\partial Z(\mathrm{g}, T_l, u)}{\partial \mathrm{g}} = \Psi,$$

$$\Psi = (\lambda/\alpha)\mathrm{g}^{-1} Z,$$

$$I(x) - WT_l(x) = (\lambda + \alpha)\alpha^{-1} Z.$$

Then two equations are given by

$$Z(\mathrm{g}, T_l, u) = \frac{\alpha}{\alpha + \lambda}(Y + WT_{ew}(x) - T_m(x)),$$

$$\mathrm{g}(x, u) = \left(\frac{\lambda + \alpha}{\alpha}\right)^{\alpha/\lambda} \left(I(x) - WT_l(x)\right)^{-\alpha/\lambda} T_l(x)^{-\gamma/\lambda} e^{u/\lambda}.$$

Substituting these two equations in the bid rent function,

$$\Psi(x, u) = \lambda \alpha^{\alpha/\lambda} (\alpha + \lambda)^{(\lambda - \alpha)/\lambda} \left(Y - T_m(x) + WT_w(x)\right)^{(1-\gamma)/\lambda} T_l(x)^{\gamma/\lambda} e^{-u/\lambda},$$

where $W = 0$, $\gamma = 0$ and then $I(x) = Y - T_m(x)$, $\alpha = 1 - \lambda$, the equations above is consistent with the CCA model (Fujita, 1989) in Section 4.2. Note that Condition (b) only exists when $W > 0$ and $\gamma > 0$. With Condition (b), it holds

$$\frac{\partial}{\partial T_l}\left(\frac{I(x) - z(\mathrm{g}, u, T_l) - WT_l(x)}{\mathrm{g}}\right) = -\frac{\partial Z(\mathrm{g}, T_l, u)}{\partial T_l} \cdot \frac{1}{\mathrm{g}} - \frac{W}{\mathrm{g}} = 0,$$

$$\frac{\partial Z}{\partial T_l} = -W.$$

Meanwhile, $(\lambda/\alpha)\mathrm{g}^{-1} z = \Psi$, which is derived from Condition (a). With $z = (\alpha/\gamma)WT_l$, $\Psi = \frac{W\lambda T_l}{\gamma \mathrm{g}}$.

Moreover, $\Psi = \frac{I(x)-z-WT_l}{g} = (\lambda/\alpha)g^{-1}z$, the following relation can be obtained

$I(x) - WT_l = \frac{(\alpha+\lambda)z}{\alpha}$.

Since $WT_l = \frac{z\gamma}{\alpha}$, $Z(g, T_l, u) = \alpha I(x)$, the following equations can be written

$T_l(x) = (\gamma/W)I(x)$,

$g(x,u) = (\gamma/W)^{-\gamma/\lambda}\alpha^{-\alpha/\lambda}I(x)^{(\lambda-1)/\lambda}e^{u/\lambda}$,

$\Psi = \frac{I(x)-z-WT_l}{g} = \lambda(\gamma/W)^{\gamma/\lambda}\alpha^{\alpha/\lambda}I(x)^{1/\lambda}e^{-u/\lambda}$.

This neat expression here is all described by $I(x) = Y + WT_D - WT_{fw} - WTtx - Tmx$ with Condition (b).

## A.7 Derivation of Equations 4.40-4.43

The population constraint is $\int_0^l \frac{L(x)}{g(x,u^*)} dx = N$ and the house space profile is known. So

$g(x,u^*) = (\gamma/W)^{-\gamma/\lambda}\alpha^{-\alpha/\lambda}I(x)^{(\lambda-1)/\lambda}e^{u^*/\lambda}$,

$e^{-u^*/\lambda}V\pi(\gamma/W)^{\gamma/\lambda}\alpha^{\alpha/\lambda}\int_0^l I(x)^{(1-\lambda)/\lambda}\,dx = N$,

where the integral $\int_0^l I(x)^{(1-\lambda)/\lambda}\,dx$ is a positive constant and let $H' = \int_0^l I(x)^{(1-\lambda)/\lambda}\,dx$.

It holds that $e^{-u^*/\lambda} = \frac{N}{H'V\pi(\gamma/W)^{\gamma/\lambda}\alpha^{\alpha/\lambda}}$ and $u^* = \lambda \ln \frac{H'V\pi(\gamma/W)^{\gamma/\lambda}\alpha^{\alpha/\lambda}}{N}$.

Note that the equilibrium utility can be written as

$u^* = \lambda \ln V\pi + \gamma \ln \gamma - \gamma \ln W + \alpha \ln \alpha + \lambda \ln H' - \lambda \ln N$.

Moreover, $g(x,u^*) = \frac{I(x)^{(\lambda-1)/\lambda}H'V\pi}{N}$ and the demand distribution under equilibrium is given as $n(x,u^*) = \frac{N}{I(x)^{(\lambda-1)/\lambda}H'}$.

Substituting $e^{-u^*/\lambda}$, the bid rent function is

$\Psi(x,u^*) = \frac{N\lambda}{H'V\pi}I(x)^{1/\lambda}$.

Following Equation 4.11 and 4.12, the rent profile is derived as

$R(x) = \Psi(x, u^*) = \frac{N\lambda}{H'V}\pi(Y + WT_D)^{1/\lambda}(1 - \frac{T_m(x)+WT_t(x)}{Y+WT_D})^{1/\lambda}$.

This bid rent profile is equivalent to the Equation 4.20 above when $W = \gamma = 0$.

For simplification, suppose $\frac{N\lambda}{H'V\pi}(Y + WT_D)^{1/\lambda} = R_0{}'$,

$R(x) = R_0{}'(1 - \frac{T_m(x)+WT_t(x)}{Y+WT_D})^{1/\lambda}$.

## A.8 Derivations of Propositions 4.2 and 4.3

**Proposition 4.2** $\lambda \to 1$, $H' = l$, $\lim_{\lambda\to 1} n(x, u^*) = \frac{N}{l}$, $\lim_{\lambda\to 1} g(x, u^*) = \frac{lV\pi}{N}$ and $\lim_{\lambda\to 1} \Psi(x, u^*) = \frac{NI(x)}{lV\pi}$.

**Proposition 4.3** $\lambda \to 0$, according to population constraint, $\lim_{\lambda\to 1} n(x_0) = N$ and $\lim_{\lambda\to 1} g(x_0) = V\pi/N$ at the city centre $x_0$.

**Derivation of Proposition 4.2**

The demand distribution under equilibrium utility is given by

$n(x, u^*) = \frac{N}{I(x)^{(\lambda-1)/\lambda}H'}$.

The house space profile under equilibrium utility is given by

$g(x, u^*) = \frac{L(x)}{n(x,u^*)} = \frac{I(x)^{(\lambda-1)/\lambda}H'V\pi}{N}$.

The bid rent profile is given by

$\Psi(x, u^*) = \frac{N\lambda}{H'V\pi}I(x)^{1/\lambda}$.

The population constraint is known, namely

$\int_0^l n(x, u^*)\,dx = N$.

Because $H' = \int_0^l I(x)^{(1-\lambda)/\lambda}\,dx$, $\lim_{\lambda\to 1} I(x)^{(1-\lambda)/\lambda} = 1$ , $\forall x \in (0, l)$.

Note that $I(x) \in (0, Y)$, $\forall x \in (0, l)$.

$\lim_{\lambda\to 1} H' = l$ .

So $\lim_{\lambda\to 1} n(x, u^*) = \frac{N}{l}$, $\lim_{\lambda\to 1} g(x, u^*) = \frac{lV\pi}{N}$ and $\lim_{\lambda\to 1} \Psi(x, u^*) = \frac{NI(x)}{lV\pi}$.

**Derivation of Proposition 4.3**

When $\lambda \to 0$, $\alpha + \gamma \to 1$.

Hence, the following correlations can be summarised:

(1) When $\lambda \to 0$ and $1 < I(x) < Y$,

$\lim_{\lambda \to 0} I(x)^{(1-\lambda)/\lambda} = \infty$.

$\lim_{\lambda \to 0} n(x, u^*) = \infty$, $\lim_{\lambda \to 0} g(x, u^*) = 0$.

(2) When $\lambda \to 0$ and $I(x) = 1$,

$\lim_{\lambda \to 0} I(x)^{(1-\lambda)/\lambda} = 1$, $\lim_{\lambda \to 0} I(x)^{1/\lambda} = 1$. $\lim_{\lambda \to 1} H' = l$ .

$\lim_{\lambda \to 0} n(x, u^*) = \frac{N}{l}$, $\lim_{\lambda \to 0} g(x, u^*) = \frac{lV\pi}{N}$, and $\lim_{\lambda \to 0} \Psi(x, u^*) = 0$.

(3) When $\lambda \to 0$ and $0 < I(x) < 1$,

$\lim_{\lambda \to 0} I(x)^{(1-\lambda)/\lambda} = 0$, $\lim_{\lambda \to 0} I(x)^{1/\lambda} = 0$.

$\lim_{\lambda \to 0} n(x, u^*) = 0$, $\lim_{\lambda \to 0} g(x, u^*) = \infty$, and $\lim_{\lambda \to 0} \Psi(x, u^*) = 0$.

As shown earlier $\frac{dT_t}{dx} > 0$ and $\frac{dT_m}{dx} > 0$, so $\frac{dI(x)}{dx} < 0$. In such case, as the distance between city centre and locations increases, $I(x)$ decays. According to the population constraint and the case in (1), it holds $n(x_0) = N$ where $x_0$ is the location of city centre. Moreover, $g(x_0) = V\pi/N$.

# Appendix B

## B.1 Data in Chapter 6

| Metropolitan | $M_{Auto}$ | $M_{Transit}$ | $a_{Auto}$ | $a_{Transit}$ |
|---|---|---|---|---|
| Atlanta | 0.8849 | 0.0291 | 111294. | 1772.9 |
| Austin | 0.8701 | 0.0231 | 83929. | 1382.5 |
| Boston | 0.7602 | 0.1219 | 182718 | 10087. |
| Chicago | 0.7969 | 0.1108 | 208950 | 10684. |
| Cincinnati | 0.9185 | 0.0177 | 98699. | 1280.4 |
| Cleveland | 0.8969 | 0.0317 | 103314 | 2082.5 |
| Columbus | 0.9054 | 0.0161 | 102715 | 2042.0 |
| Dallas | 0.9102 | 0.0151 | 179480 | 2636.6 |
| Denver | 0.8476 | 0.0441 | 113500 | 5123.2 |
| Detroit | 0.9265 | 0.0156 | 129669 | 1592.6 |
| Houston | 0.9077 | 0.0341 | 163201 | 3682.2 |
| Indianapolis | 0.9198 | 0.0118 | 89808. | 1468.5 |
| Kansas City | 0.9217 | 0.0109 | 116613 | 1466.2 |
| Las Vegas | 0.8952 | 0.0373 | 98691. | 2496.5 |
| Los Angeles | 0.7726 | 0.1090 | 381088 | 11220. |
| Louisville | 0.8979 | 0.0317 | 71946. | 1502.9 |
| Miami | 0.7787 | 0.1222 | 150293 | 3665.7 |
| Milwaukee | 0.8206 | 0.0858 | 102581 | 4341.9 |
| Minneapolis | 0.7041 | 0.2958 | 181535 | 4210.9 |
| Nashville | 0.9197 | 0.12111 | 83709. | 1094.9 |
| New Orleans | 0.8959 | 0.0266 | 60481. | 1630.3 |
| New York | 0.5654 | 0.3104 | 279605 | 44615. |
| Philadelphia | 0.8126 | 0.0937 | 145955 | 7281.9 |
| Pittsburgh | 0.6646 | 0.1723 | 98342. | 2432.2 |
| Portland | 0.8056 | 0.0600 | 96586. | 4516.3 |
| Providence | 0.8920 | 0.0289 | 75474. | 1605.3 |
| Raleigh | 0.9013 | 0.0096 | 58787. | 1311.5 |
| Sacramento | 0.8663 | 0.0225 | 84147. | 2614.5 |
| Salt Lake City | 0.7843 | 0.0812 | 78759. | 3607.2 |
| San Antonio | 0.9075 | 0.0229 | 92691. | 2481.2 |
| San Diego | 0.8605 | 0.0278 | 134670 | 3236.6 |
| San Francisco | 0.7054 | 0.1559 | 232263 | 13368. |
| Seattle | 0.8013 | 0.0846 | 109845 | 6000.7 |
| St. Louis | 0.9056 | 0.0249 | 96940. | 2021.1 |
| Tampa | 0.8954 | 0.0122 | 96378. | 1599.8 |
| Washington | 0.7599 | 0.1410 | 180893 | 10893. |

# Appendix C

## C.1 List of conferences during PhD

Huang J, Connors RD and Maher M. Scaling Laws in Urban Transport Networks, 45th Universities' Transport Study Group Conference, Oxford, 2nd-4th January 2013.

Huang J, Connors RD and Maher M. Urban Congestion under Scaling Laws, 46th Universities' Transport Study Group Conference, Newcastle, 6th-8th January 2014.

Huang J, Connors RD and Maher M. Modeling network growth with scaling laws in a linear monocentric city, Transportation Research Board 93rd Annual Meeting, Washington D.C., 12th-16th January 2014.

Huang J. Modelling co-evolution of residential location and mode choice in a linear monocentric city, 47th Universities' Transport Study Group Conference, London, 5th-7th January 2015.