General Equilibrium Models of Monetary Economics

By

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Contents

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Abstract.

The thesis contains a literature survey plus five original chapters which address different topics in the area of money in general equilibrium macro models that consist of contemporaneously infinitely lived agents who operate in an economy with an imperfect financial structure; namely missing markets. Particular issues addressed are the necessary conditions for the valuation of intrinsically worthless fiat money, issues pertaining to the optimum quantity of money, money's intrinsic nature, and the occurrence and persistence of macroeconomic fluctuations. The original chapters are as follows:

Chapter two exposits a model where money is held as a precaution against idiosyncratic productivity shocks. I derive necessary conditions for money to have value and find that despite the presence of an alternative to money as a value store, no monetary equilibrium with circulating money exists with a positive rate of interest on money.

In chapter three I investigate a conjecture that if we possessed information sufficient to allow the correlation of taxes with the states of nature so as to avoid the kind of problems outlined in chapter 2 then we could not have the imperfections that generated a monetary economy in the first place. The result I find refutes this conjecture.

Chapter four addresses the issue of the optimum quantity of money in a framework where other assets are available to agents. I show that an economy with fiat money may be Pareto dominated by one without it.

Chapter five addresses the issue of the superneutrality of money. I show that the problem that led to the non-existence of monetary equilibrium in the Bewley(1983) model can cause money to be superneutral in another scenario.

Chapter six addresses the issue of macroeconomic fluctuations in models where the distribution of money holdings is important. I show two models that each demonstrate such issues. The first idea I exposit is that we can develop an economy where market participation is endogenous and gain a richer picture of endogenous fluctuations than the exogenous participation overlapping generations models provide. The second idea demonstrated is that in an economy where productive investment opportunities are intrinsically risky, the incompleteness of markets means that poor productivity shocks can yield persistence of low output levels, and reduce the frequency of investment.

Chapter 1

Literature Review

1. Literature Review.

1.1. Setting the Scene.

I begin by setting out some of the existing results in the field and summarising the contributions of the original chapters.

This thesis contains a survey of, and original contributions to the field of modelling fiat money in a general equilibrium setting. The models can be placed under the headings of the microfoundations of both macro and monetary economics. All models are sequence economies in the sense of Radner(1972) and Hahn (1971a) with trading at all dates of the model, Walrasian prices and rational expectations. Trading at each date is motivated by the lack of market completeness in the sense of Arrow-Debreu. Despite the topic of money being a traditionally macroeconomic issue, the frame of reference is of course one that historically has been 'micro' based so that its point of departure is the Arrow Debreu model and many of the contributors to the field are seen as micro rather than macro economists; issues of existence, uniqueness and efficiency of equilibrium loom large on the research agenda. Of course such micro/macro distinctions are increasingly blurred and the models in the field provide insights for macroeconomics, even though some of them might be termed to be concerned with 'philosophising' about money.

The issue of how an economy deals with the absence of perfect financial markets in the presence of individual risk is however clearly a matter of macroeconomic interest. It is sensible to relate inefficiencies due to liquidity constraints in these 'micro' models with those pertaining to unemployment for instance. Fundamentally they are both simply low levels of economic exchange.

All the original models in chapters two to six are non-representative agent models where the distribution of money among agents plays an important role. Indeed, as the work of Scheinkman and Weiss (1986), Levine (1991) and the models of my final chapter show, the distribution of liquidity can play an important role in explaining aggregate fluctuations. Due to their non representative agent nature we might hence also put the models alongside the overlapping generations framework, for reasons that will be explained later.

Despite the common theme we can identify each chapter with different issues in the field. Not all of these topics are covered in the survey so far and some of the chapters commence with some survey material which will aid understanding of the particular model presented. Below I outline the issues addressed in each chapter and the answers found.

Chapter two exposits a model where money is held as precaution against idiosyncratic productivity shocks. I derive necessary conditions for money to have value and find a role of the rate of time preference different to that found by Bewley(1980) and Levine(1989). I then investigate the issue of implementing the optimum quantity of money proposal of Friedman(1969) under a regime of lump sum taxation, where the taxes are independent of the state of nature. I find a result that is at odds with the intuition provided by the Bewley (1983) model. Despite the presence of an alternative to money as a value store, no monetary equilibrium with circulating money exists with a positive rate of interest on money.

Chapter three moves away from the restriction that taxes have to be independent of the level of income of each agent. I investigate a conjecture made by Hellwig(1982) and Woodford(1990) that if we possessed information sufficient to allow the correlation of taxes with the states of nature so as avoid the kind of problems outlined in chapter two then we could not have the imperfections that generated a monetary economy in the first place. The result I find refutes this conjecture. A scheme of taxing individuals in a way that varies with the state of nature and then uses the proceeds to change money's value is shown to be less informationally demanding than a scheme of taxation and direct redistribution.

The reason I put forward for this is based on an essential property of fiat money; its homogeneity, anonymity and common usage. This chapter's reflections shed new light on the optimum quantity of money proposal when it is examined in a framework that is specific about the information structure.

Chapter four addresses the issue of the optimum quantity of money in a framework where other assets are available to agents, that in a sense are better suited for the insurance purpose that money is valued for in the model. For a particular class of preferences I address a question posed by Levine(1985) to show that an economy with fiat money may be Pareto dominated by one without it. I show that the result derives from money's intrinsic uselessness as an insurance asset, since its payoff is the same in all states of nature.

Chapter five addresses the issue of the superneutrality of money. I show that the problem that led to the non-existence of monetary equilibrium in the Bewley(1983) model can cause money to be superneutral in another scenario; where agents have finite horizons.

Chapter six addresses the issue of macroeconomic fluctuations in models where the distribution of money holdings is important. I develop two models to show two ideas. The first one uses a non convexity in the form of a minimum size of investment and indivisiblity of projects across agents together with an endogenously determined level of liquidity to show that the model can display multiple steady states with high or low levels of activity and also may cycle between the two states in a deterministic fashion. The endogenously varying market participation is the cause of these results, and hence paints a picture of endogenous aggregate fluctuations different to that which invokes exogenous participation restrictions. The second model examines how the endogenous distribution and redistribution of liquidity can lead to the persistence of shocks. If the stock of liquidity is not held by the more 'risk loving' agents in the economy then a low level of economic activity will result through a lack of investment in the productive technology which is intrinsically risky. Incomplete markets means no mechanism is available for the speedy transfer of liquidity to the less risk averse agents when a bad shock to productivity leaves them at a level of wealth below that required to undertake the investment.

I complete the thesis by assessing the contributions and suggesting avenues for further research that it opens up.

I have identified questions which the chapters are going to address. The survey material that follows will set the scene for these. The initial survey chapter covers the topics of the valuation of fiat money, particularly relevant for chapter two, implications of incomplete markets, multiple budget constraints, the optimum quantity of money, of relevance for all chapters, and the record keeping role of money, of particular relevance for chapter three. Chapter five will address the issue of the superneutrality of money and chapter six looks at endogenous competitive equilibrium fluctuations, and the survey material most relevant for those chapters is contained within them.

1.2. Valuing Fiat Money: Some Issues

As stated the focus of the thesis is on models of contemporaneously infinitely lived agents where money plays a role in the model that allows it to have positive value. For the sake of providing background for the model of chapter 2 and to aid comparison with other classes of monetary models, a general survey of how the challenge of giving money value has been met will be of much interest.

The problem with finding a role for money is that it is an intrinsically worthless piece of paper. Holding it yields no direct utility (except in some reduced form models) and ultimately optimising agents must look to spend it. There are hence two pre-requisites for money to have value. There must be agents in the economy at all points in time who might be prepared to hold money. Secondly, its yield, pecuniary or otherwise must be such that it is sufficiently attractive to these agents. Even if a theorist were to insist that money's key role is as a medium of exchange, it must serve as a store of value as a necessary condition to it being demanded by agents.

The first modern seminal paper in the field was by Hahn (1965). The 'Hahn' problem he noted was that every model that has a monetary equilibrium also has a non monetary equilibrium. This result was subsequently seen to be true in any model where money is truly fiat in nature (intrinsically worthless paper given value only by decree, at best). In such economies money is only demanded for its value relative to other goods. If its value is zero its demand is zero. It will hence make no contribution to individual wealth either, so a non monetary equilibrium will result, if such an equilibrium exists. One further implication is that no monetary equilibrium can exist in a finite horizon economy if money is truly fiat in nature. All money will be spent in the last period of the agent's life so zero money demand forces a zero price of money in the last period if competitive equilibrium is to obtain. Money hence cannot serve as a store of value and will not be demanded in the penultimate period. The argument holds recursively for all preceding periods of the model and so its price must be zero in all periods of the model. Put another way, as long as utility functions are strictly monotonic in at least one real good then the value of purchases will exceed the value of sales of goods unless the value of money is zero. If there are I agents, and vectors $\underline{c}_i$ and

$\underline{\omega}_i$ denote consumption and endowments respectively of consumer i over all goods and time periods, and $\underline{p}$ denotes the vector of money prices then this implies;

$$\sum_{i=1}^{I} \underline{p}'\underline{c}_i > \sum_{i=1}^{I} \underline{p}'\underline{\omega}_i \tag{1}$$

Since;

$$\overline{M} + \sum_{i=1}^{I} \underline{p}'\underline{\omega}_i > \sum_{i=1}^{I} \underline{p}'\underline{\omega}_i \tag{2}$$

Where $\overline{M}$ is the total nominal money stock.

This holds unless agents violate their transversality conditions, or all money prices of the goods are infinite. This kind of problem in fact also has to be addressed in infinite horizon economies, that of a potential mismatch between the value of expenditures and the value of income. We shall see that if the problem is to be overcome then we must either make sure that the economy is never ending in the sense that there are always new agents arriving or in the case that agents live contemporaneously we need to give agents a motive to hold the money throughout their lives instead of spending it. We will firstly deal with the issue of a finite horizon economy and how the terminal date problem can be solved.

Since the finite horizon economy is such an intrinsically unforgiving environment for fiat money, it is ironic that some of the approaches taken to answer this remove the Hahn problem completely. The paradox arises because the solutions are exogenously imposed and in some sense remove the true fiat property from money. A method used by Starr (1974) for example is to induce the return of money at the end of the final period to an outside authority by imposing a real valued tax demand for which money may be used as payment, though this is not compulsory. A final period positive price of money is hence a possibility. The effect on the budget constraints (2) of the tax when looked at in the above context is obvious; the tax reduces the value of income; i.e. the left hand side of the inequality. If the tax bill is to be paid in fiat money by a compulsory arrangement and the tax demand is expressed in real terms then a unique positive final period value of money is guaranteed. Arbitrage will guarantee a positive price in all preceding periods if utility is monotonic in at least one good in the final period. As subsequent models will show the value of money will be higher than the base level implied by the tax if it is given an extra role to perform. If the total tax demand is denominated in nominal terms equal to $\overline{M}$ (the total money supply) then a monetary equilibrium will be possible, but not guaranteed as the final period price level will be indeterminate (see Starr(1974) and Geanakoplos and Mas-Collel(1989)).

The technique used by Bewley (1980, 1983) is to put terminal money balances into the utility function. If nominal balances are put in the utility function then a positive price level is guaranteed in all periods. If the balances in the utility function are real then the positive price of money is possible but not assured.

Alternative methods are employed by Kultti (1995), Faust (1989) and Duffie (1990). Kultti assumes the existence of some agents who are indifferent about consuming their goods or not in the last period. The lack of value store problem does not apply here since effectively there is no value to store. In Faust's model, he invokes the assumption of continuous trading and notes that by the continuum property, at any instant before the final date there is always another future date of trading and hence a date at which money might have value. So Faust introduces the effects of an infinite horizon model 'via the back door'.

The method of Duffie (1990) is ingenious and creates a scenario where indeed the value of purchases exceeds the values of sales. He arrives at this by noting that in an economy with intermediated costly transacting, buying and selling prices diverge. Duffie sets up such an economy with two transacting possibilities, one is a non monetary transactions technology which is operated privately, the other is a monetary transactions technology operated by the government. It is assumed that the latter dominates the former. The agents are endowed with fiat money at the start of the model. The government administers the transactions intermediation service and collects the money in payment from buyers. Such a scenario holds for the final period also, and hence the terminal date problem is negated. Duffie notes that his model achieves this result without giving backing to money, though of course we should note that the transactions agency is essentially modelled as an external body.

An approach popular in the 1970's belonged to the temporary equilibrium literature. The models here established equilibrium for a current period only with expectations of the price level in the next period dependent upon the current price level. Grandmont (1983) shows that the key to solving the problem of existence of monetary equilibrium in these models lies in making the point price expectations relatively inelastic to current price changes. If the current nominal price level increases, the expected future price level is prevented from rising too much, and the expected rate of return on money is sufficient to guarantee that a monetary equilibrium occurs.

I now turn to models where the economy described has an infinite horizon and money is truly fiat in nature. Some of these are populated by a finite number of contemporaneous infinitely lived agents, and are the focus of the thesis, and some by an infinite sequence of agents with finite lives. The latter of course are known as overlapping generations models with finitely lived agents. It has been shown however that the crucial element of such models is the arrival of new agents rather than the death of existing ones, as overlapping generations of infinitely lived agent models have shown, see for example Weil (1985, 1991)

Let us begin by considering an economy which is essentially an infinite horizon version of the Arrow Debreu model; markets are complete and there is no population growth. Assuming that money is dominated as a store of value with probability one because of the complete set of markets, then money will all be spent on goods and assets by all agents in the first period, so there is no money demand. As a formal expression we can again turn to inequality (2) shown above. If the individuals are not to violate their own transversality conditions, then the present value of their consumption must equal the present value of their income, and this must hold in the aggregate. To convert inequality (2) to an equality, a necessary condition for equilibrium, we have two possibilities. One is as in the finite horizon economy, by setting the money value of all goods equal to infinity whilst preserving their finite value in real terms. The second method is not applicable in this context of complete markets but we mention it here for future reference. This method again implies making both sides of the expression infinite. Here the money value of each good is finite but we allow the infinite sequence of goods to be priced in money terms in a way which allows their present value in real terms to be unbounded. In the complete markets setting with no population growth this contradicts conditions necessary for existence of equilibrium. How the argument works with financial imperfections or the entry of new agents will be exposited in chapter 6, but I will proceed with an alternative style of exposition of the solution to the valuing fiat money problem in this chapter, covering both overlapping generations models and debt constrained models.

These results underline the notion that since fiat money is simply a piece of paper is not part of the net wealth of the economy. If the group of agents that constitute the economy treat it as net wealth and try to spend it instead of holding it, its value will disappear.

What if we seek an equilibrium with money earning a return equal to that on the safe stores of value provided by the complete set of markets?. Assuming the economy is stationary that gross rate of return would be equal to $\frac{1}{\beta}$, one plus the pure rate of time preference (see the subsequent coverage of the Townsend model for instance). The transversality condition of the typical consumer is;

$$\lim_{t \longrightarrow \infty} \beta^t . MU(c_t) . W_t = 0 \tag{3}$$

Where W_t denotes real wealth at time t, and $MU(c_t)$ denotes the marginal utility of consumption at time t (See e.g. Lucas and Stokey (1989)). Since the endowment of the economy's consumption goods is finite in all periods, and the utility function is assumed to be monotonically increasing then $MU(c_t)$ is strictly positive in all periods. Since the gross return on real money balances is equal to $\frac{1}{\beta}$ by assumption and some agent must hold the large and growing real stock of money in the economy then this agent must violate their transversality condition, so that such a situation cannot be an equilibrium. If we stick to the assumption that agents are infinitely lived and there is no population growth then we must have some device to ensure that individuals will hold money instead of spending it. One of the first approaches to the problem was that of Patinkin (1965) and Sidrauski(1967) of putting real balances into the utility function; variations with utility as a function of beginning of period balances and end of period balances have both been used. Examples will be covered in chapter 5. Hahn (1965, 1971,1973) criticised this approach on the grounds that it made money 'inessential' to the model. By this he means that if money is removed from the model then the allocation of real resources is unaffected. According to Hahn the use of money should be closely tied to the way in which economic activity is determined. Hahn's criticism was influential and has to a large extent set the tone for the original contributions that will follow. In all the models in chapters 2 -6 inclusive, omitting money changes the allocation of resources.

In those models we will see that the inefficiency of the non monetary equilibrium is crucial, since it leaves unexploited gains from trade, and that money can have value since it 'travels round' the system rather than being passed on to newly arriving agents. Such a scenario has been depicted by Bewley(1980, 1983) and Levine(1989) for the endowment case but a gap exists for the case of productive capital as the generator of income in such models.

The other main approaches expounded in the literature have been the cash in advance models of Lucas(1980) Lucas and Stokey (1987), the debt constrained infinitely lived models of Bewley(1980) and Townsend (1980), the medium of exchange models of Romer (1986) and Kiyotaki and Wright (1989) plus the aforementioned overlapping generations models of Samuelson (1958), Gale (1973) etc. A review of some of these models and their conclusions follows. Particular attention will be paid to aspects of the models that show the effects of multiple budget constraints, money as a record keeping device, money in a Pareto improving role, and the issue of the optimum quantity of money.

1.3. Economies with Financial Imperfections.

These various approaches invoke primitive assumptions about time or spatial separation of agents or the costs of transacting in the economy. Missing markets are the extreme form of costly transacting which is the focus of this thesis. They can all be contrasted with the trading structure of the Arrow Debreu model which has the features of;

{1} A single market at the start of time, on which all present and future commodities can be traded against each other.

{2} Costless transacting.

{3} Agents meeting in the same place.

In fact,{2} and {3} are major requirements for {1}.

For instance if we relax {3} we arrive at either the communication cost or 'trading process' models in which agents are separated spatially and so their meeting is restricted (though there is no reference to calendar time in many such models); or we can have finitely lived agents separated through time, restricting their meeting in that dimension; yielding the structure of overlapping generations models.

Specifying a transactions technology or imposing incomplete markets is a less primitive approach, and leads to markets re-opening over time (contrast this with the inessential sequence economies discussed by Hahn (1973), where trading at dates other than the initial one does not change the set of equilibria). Money is suggested in the trading process models because of its informational properties, providing an important linkage across agents, tackling moral hazard related problems that hamper the working of inside assets; see for instance the work by Ostroy and Starr(1974) and Levine (1991).

The key factor in all approaches of valuing money that seek to be 'essential' in some sense is that there must be some divergence in marginal rates of substitution at the non monetary equilibrium to allow potential gains from trade so that money can be valued in a trading role; money may be passed around the system in equilibrium, or passed on to new agents as in the overlapping generations case.

General discussion of monetary models and the task facing monetary economists is contained in Karaken and Wallace(1980). In their introduction to the book the authors voice their concern over the implications of macroeconomic models of money. Such concerns form the motivation for much of the thesis. They forward two main criticisms. Firstly, cash in advance models and money in the utility function models tend to "assume the conclusions "of important monetary questions: in particular; why fiat money is valued. Secondly, the models tend to be inconsistent. That is, they specify forms of demand for money which implicitly appeal to some underlying model. However, it is common for the implicit underlying model to be inconsistent with the model into which the demand for money is then embedded (recall Hahn's comments on Patinkin's inessential money). For instance, the standard portfolio specification of asset choice under uncertainty is sometimes embedded in macro models which are used to determine a non stochastic equilibrium (such a challenge will be met in chapter 2). This standard example also has implications for the question of explaining the valuation of intrinsically useless fiat money. The intrinsic uselessness of any asset says that it is wanted only for the consumption stream it supports upon its liquidation. Since welfare analysis of alternative monetary and fiscal policies is about their effects on consumption allocations, to assume the demand function for money in advance is to remove any hope of performing an adequate welfare analysis of monetary policy. The welfare theme is the focus of much of the thesis. To analyze the concepts of different kinds of assets a primitive approach based on intrinsic uselessness is also required. For instance to put all forms of money and financial assets in the utility function would lead to a very general specifications,with little restriction on results. Chapter four will show implications of addressing such an issue. Similar criticisms apply to the Clower constraint models. For instance, Lucas appeals to the obvious conclusion that exchange is really more difficult than the Arrow Debreu model implies, but to impose the Clower constraint is 'starting too far along,' according to Karaken and Wallace.

I commence this part of the survey with a look at models that are very primitive in a 'fundamental' sense and look to show in a very detailed way how fiat money is essential for efficiency in trade. The key relevance of these models for the thesis is in showing the role of money as a decentralised record keeping device, which can costlessly keep track of the value of individual's total value of purchases and sales, and its abilities to overcome inefficiencies created by extra budget constraints in addition to the one that the value of expenditure and income should match over an agent's lifetime. The lack of inside assets is the cause of these. Exactly how well money can work to overcome these problems is a major theme of the thesis; the efficiency of a monetary equilibrium.

1.3.1. Money as a De-centralised Record Keeping Device, and the Enforcement of Budget Constraints.

The model described below examines the process of decentralized trading and the record keeping function of money in very general terms, and examine where trading in pairs (hence, subject to obvious physical, informational and incentive compatibility limitations) leads to in terms of allocations for society as a whole. Though quite old, the paper still stand as true 'originals' that give a clear exposition of the issues.

Although my thesis does not concern itself with pairwise trading the issues of how extra budget constraints can cause inefficiencies and how money might overcome these by acting as a record of purchases are of great relevance for the rest of the thesis. its record keeping role as exposited by Ostroy will in particular provide a useful backdrop to the idea I present in chapter 3.

Starr (1972) and Ostroy (1973) consider the difficulty of decentralized trading when it is carried out in pairwise fashion and how money can ease problems, by acting as a record keeping device. These two articles, (and the one of Ostroy and Starr (1974)) have the structure of a Walrasian economy where equilibrium prices have already been determined, and traders know their own excess demands, but the problem is how can they be cleared without the intervention of the auctioneer. The chief difficulties of this coordination problem are information and motivation (incentive compatibility); knowing what trades to make, and ensuring that agents do not break their budget constraints. This last point is the most interesting one for us, since pairwise trading is not the focus of the thesis. Only Ostroy is considered in detail, as the methods of the two are very similar.

Ostroy's model consists of a sequence of simultaneous bilateral trades, such that each individual meets only one other in a given unit interval of time. The criterion for the efficiency of the exchange process is the number of periods it takes to go from state (W), where the Walrasian equilibrium prices are set, and agents hold their endowments, to state (A) where individual excess demands are zero.

The fewer periods taken, the more efficient is the process.

Ostroy focuses on 3 properties of trading sequences;

(i) Technical feasibility - i.e. based on physical restrictions that trades are bilateral and not multilateral.

(ii) Informational feasibility - requires (i) and the restriction that the trading sequence is not dependent on information available to other pairs.

(iii)'Equilibrium' or 'behavioral' feasibility - that the sequence is incentive compatible - i.e. individuals have no incentive to depart from it (this is the part of most interest to us).

This is later manifested in the need for 'BB' - bilateral balance with purchase and sale values by an individual equal in any exchange.

'BB' becomes the focus of the paper, since Ostroy shows that trades satisfying (i) and (ii) and minimizing the number of periods for (W$\rightarrow$A) do not satisfy BB. Hence, with BB the process takes longer. If the concern is to make sure that individuals balance their budgets over all trades taken together, BB is of course one way to do this, but money's use is another, which hence allows the saving of time, as it allows the unbalance of trades in terms of real goods.

The economy consists of n individuals, m goods, and for a typical agent i, $\underline{w}_i$ is his endowment vector and $\underline{a}_i$ his utility maximizing choice of consumption, where $\underline{p}'\underline{a}_i = \underline{p}'\underline{w}_i$,and for each commodity c;

$$\sum_i a_{ic} = \sum_i w_{ic} \tag{4}$$

i.e. market equilibrium.

Ostroy derives the minimum number of periods for which the allocation is technically feasible (i.e. such that the sum of a pairs 'goods holdings' before and after a pair's bilateral trade is the same). This number is K = log n where K is the minimum number of periods for n agents (for n$\neq 2^k$, a slight modification is required).

Ostroy's next proposition is that if the BB requirement is imposed on trades then the 'A' allocation is not technically feasible for the above indirect exchange model (i.e. defined as one that uses log n periods) this is really just the same as saying that imposing BB will mean the allocation 'A' takes longer to achieve.

In terms of direct meetings, i.e. for everyone to meet each other once, meetings are required which take a minimum number of (n-1) periods. This is termed a direct exchange economy.

Ostroy then turns his attention to the informational feasibility of trades - i.e. how trades are actually made between individuals, when they have limited information, (i.e. as opposed to the presence of an 'all-knowing' broker) for both direct and indirect exchange economies. The implications of a pair (or a broker coordination-coordinating them) not knowing the excess demand of other agents is contained in Ostroy's propositions 5-7.

Proposition 5 is that the competitive equilibrium allocation (CE) is not informationally feasible in the indirect exchange model. Intuitively this is quite obvious, as agents will not know the 'needs' of future trading partners (direct or indirect) and hence do not know what they need to give/receive now. (This is even without imposing BB.)

Proposition 6 is that if trades must satisfy BB the CE allocation is not informationally feasible in the direct exchange model.

The basic reason for this is that since we assume there isn't a good present in sufficient supply to act as a balancing item, what is accepted in payment has to be what one is able to sell in the future. This is a strong informational requirement.

Ostroy then notes that the use of money instead of IOU's betrays that the parties lack information about where and when the debt will be settled.

Proposition 7 is that when BB is not imposed, the CE allocation is informationally feasible in the direct exchange model. This can be proved by the use of a rule developed in Starr (1972), that of excess demand diminishing trades (E.D.D. trades). These are such that agents never engage in trades which change the sign of their excess demands. That this should cause the allocations to converge towards the CE allocation is obvious.

Intuitively, this rule only requires the 'bilateral' knowledge of excess demands.(Of course, it is impossible to use E.D.D. trades in the context of an indirect exchange model.)

Ostroy, in looking towards the behavioral feasibility of exchange sequences notes

(i) E.D.D. trades are unlikely to satisfy BB. To do this would in fact require a double coincidence of wants.

(ii) E.D.D. trades will not form a utility increasing sequence. Obviously, the total effect of the sequence is to increase utility. But (by the same lack of double coincidence as above) it will not be increasing at each step.

Ostroy then looks at the question of whether a sequence of trades, technically and informationally feasible, which reaches 'A' is in fact an equilibrium i.e. is it compatible with individual incentives. He tackles this by noting that the trade sequence (organized by a broker) is to be based only upon the information 'revealed' by a trader. Here we have the incentive to default - individuals can overstate their position and hope to get extra goods, breaking their budget balance position.

Of course, if BB over all trades together (BUB) is imposed somehow (without the requirement of BB on each individual trade) then obviously individuals cannot default, and hence we arrive at Proposition 8; If the CE allocation is informationally feasible, and if BUB is imposed, the CE allocation is an equilibrium.

However, due to the informational restrictions, we know that the only way this can be satisfied is if B-B is imposed.

Hence we have proposition 9 which is that the CE allocation (A) is not an equilibrium for the direct exchange model. (From proposition 6 and the need to have BB for equilibrium).

Ostroy then proposes an alternative to keep BUB satisfied. This is by the use of 'money' in the form of credits given by the purchaser to the seller and monitored by a central monetary authority. This treatment of money as a balancing item of course ignores any notion of liquidity constraints; the occurrence of these through an insufficient quantity of money in the economy is of course a major theme of this thesis.

Hence we arrive at proposition ten which is that in the monetary version of the direct exchange model, the CE allocation is an equilibrium.

In summary, Ostroy draws attention to the fact that the role of money he wishes to concentrate on is its record keeping property (hence his cheque book approach) to avoid the usual approach of simply saying money is distinguished from other commodities by its durability, portability, etc. However he acknowledges that to equally carry out the record keeping, it would obviously be prudent to select a medium with those desirable quantities, rather than a higher cost central book-keeping method, or a cumbersome commodity money. The advantages over a book keeping system were also mentioned by Levine (1991) and will be touched on in again in chapter 3.

The set up of Ostroy et al. points the way towards the introduction of money as a method of overcoming restrictions yielded by extra budget constraints. In later parts of the thesis we shall see that such restrictions equate to the absence of inside assets, which create a non monetary equilibrium that is non Pareto optimal and allows money to be valued.

1.3.2. Overlapping Generations Models.

An important class of models that points out the relation of inefficiency of a non monetary equilibrium and the existence of a monetary equilibrium, (a topic which will be approached in chapter 2) are Overlapping Generations Models which began with the contributions of Allais (1947) and Samuelson (1958). Deeper analysis of this class of models and their relation to models with infinitely lived agents is reserved until chapter 6. Here we content ourselves by expostulating some of the basic results of the simpler kind of overlapping generations models that conveys the basic notions. The role of money in improving welfare is an important point that these models bring out and one that will be addressed and countered in chapter four.

To these ends we now review an important paper by Cass, Okuno, and Zilcha(1979) that exposits Samuelson's basic model,generalizes it and examines its two central conclusions,which are

(1) Existence Proposition

There is a monetary equilibrium if and only if there is no barter equilibrium which is Pareto optimal.

(2) Optimally Proposition

If there is a monetary equilibrium then there is a monetary equilibrium which is Pareto optimal.

Cass, Okuno and Zilcha (COZ) find that quite plausible alterations in the taste and endowment patterns (notably heterogeneous households, non convexities and non stationarities) of Samuelson's basic model change the above conclusions dramatically. We shall not go into these variations in this survey. Instead we shall simply cover some basic results and intuition in the standard Samuelson model. Further detailed consideration of this model will be presented in the final chapter, particularly as to why the equilibria may be indeterminate.

The terminology of a 'barter economy' is borrowed from Samuelson and Cass, Okuno and Zilcha for consistency and simply refers to a non monetary economy. The story traced out below looks at when the absence of assets that allow the transfer of goods across generations will allow money to have positive value in equilibrium. The absence of the assets is usually motivated by noting that one cannot strike a debt contract with someone who will not be around the following period. Alternative structures that allow for a social security system, a sequentially complete asset market or a complete asset market that allows agents to meet and trade before they are even born are however sometimes used by theorists in the field of overlapping generations.

I now review the familiar O-G framework in its simplest form. It consists of one type of consumers in every generation, (hence in a representative agent style; just 1 consumer), each generation living for just 2 periods, with a new generation born each period. The model is started at time zero with the initial old generation (born at time -1 and having no-one to trade with when young)given one unit of money between them. The initial old consumer will obviously consume his second period endowment and supply his money inelastically. For other generations the problem is for the generation born in period t; in which consumer h maximizes;

$$u^h\left(\underline{c}^h\right) \tag{5}$$

Subject to;

$$p_t c_t^h + p_{t+1} c_{t+1}^h \leq p_t y_t^h + p_{t+1} y_{t+1}^h \tag{6}$$

Hence equivalently;

$$p_t\left(y_t^h - c_t^h\right) \geq M_t^h \tag{7}$$

$$p_{t+1} c_{t+1} \leq M_t^h + p_{t+1} y_{t+1}^h \tag{8}$$

A competitive equilibrium in this model is a set of prices, and optimal consumption plans that satisfy the constraints for a feasible allocation:

$$\sum_{h \in G_{t-1} \cup G_t} c_t^h \leq \sum_{h \in G_{t-1} \cup G_t} y_t^h \tag{9}$$

Where G_t denotes the generation born at time t. The generation subscripts are dropped in situations where no confusion is created.

A useful diagrammatic tool in this analysis is the reflected generational offer curve shown overleaf. Looking at the curve for Generation t, we define it as a plot of z_{t+1} against z_t where z_t is the excess supply of the (single) consumption good in the first period of life, and z_{t+1} is the excess demand in the second period of life, by the same set of agents (i.e. within the same generation). Since the COZ paper considers only scenarios of zero population growth, the use of the reflected generational offer curve to characterize the equilibrium is very simple, as we shall see.

Since we require that the excess goods demand of old = excess goods supply of young, then competitive equilibrium requires that ${}_{G_{t-1}}z_t = {}_{G_t}z_t$. As will be elucidated later, the situation in figure one depicts a case where stationary monetary equilibrium exists. Diagrammatically, we impose the equilibrium condition in the spot market by selecting the point on the vertical axis to represent;

$${}_{G_{t-1}}z_t \tag{10}$$

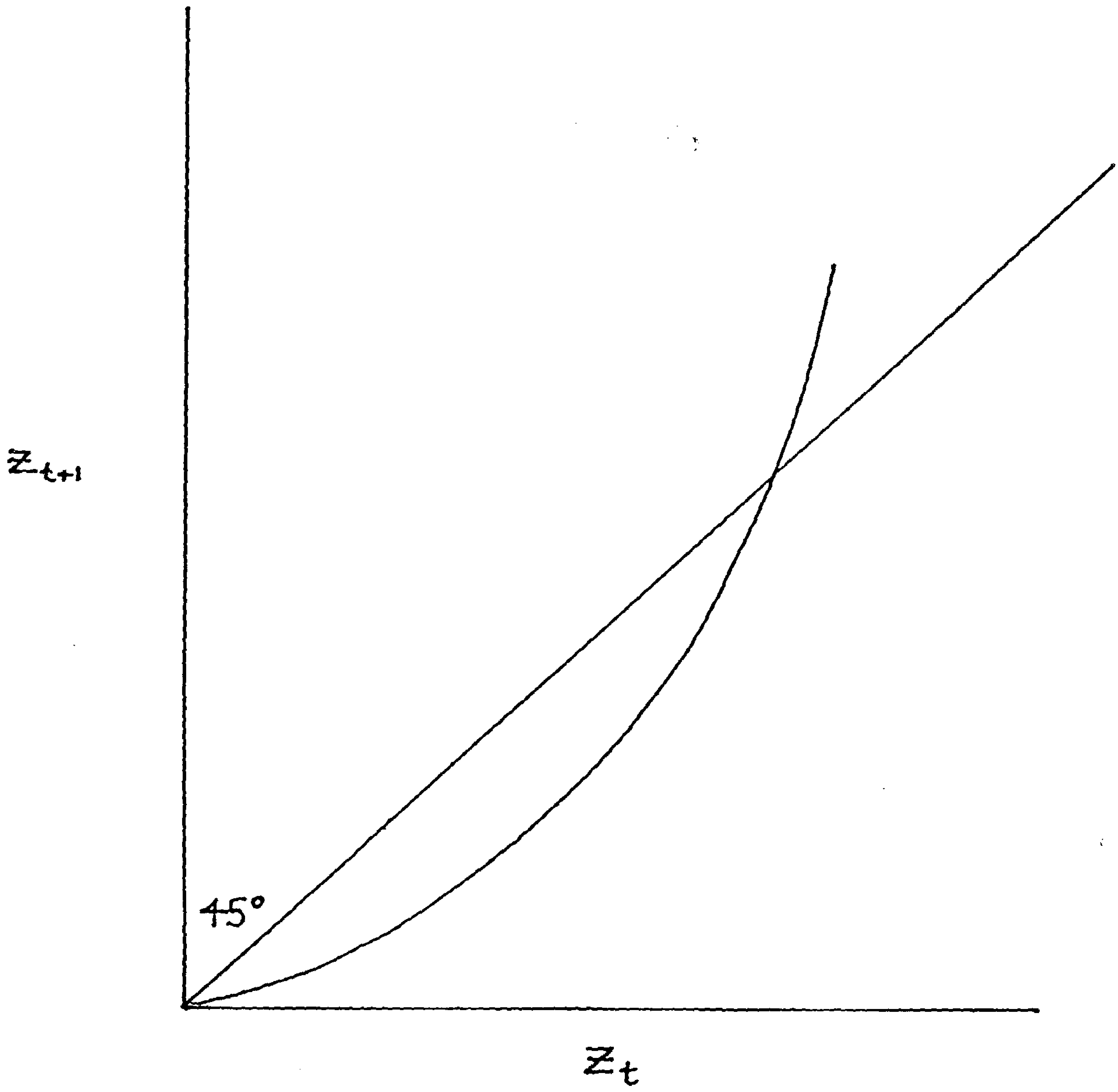

FIGURE 1.

and then tracing across to the 45 degree line to find the excess supply of the following generation that will yield equilibrium, namely;

$$_{G_t}z_t \tag{11}$$

Other points to note about the offer curve in general are that at the origin (the endowment point), its slope will be equal to that of the relevant indifference curve. Secondly, as we change the price ratio to sketch out the reflected offer curve, we have that the slope of the line from the origin to the relevant point on the offer curve is equal to the price ratio. This is obvious when one remembers that z_t and z_{t+1} are derived from the agent's constrained optimization (see equation 6).

COZ then go forward to examine the central story of Samuelson's original model using these techniques. In this case, with strictly quasi-concave utility, the offer curve will cross the 45 degree line just once. Hence a unique barter equilibrium will exist ($_{G_{t-1}}z_t = {}_{G_t}z_t = 0$ which implies autarchy, with each generation consuming its endowment). Note that will occur for a range of price ratios below a critical value and although above we define p_t, p_{t+1} in money terms, the price of money is in fact zero (p_t,p_{t+1} infinite) in an autarchic equilibrium. A key result is that there will exist a unique stationary monetary equilibrium only if the offer curve, g intersects the 45 degree line in the positive quadrant. This will only occur if the slope of the offer curve at the origin (endowment point) is less than one. Define this slope as r,where

$$r = \frac{\frac{\partial U(y_1,y_2)}{\partial c_1}}{\frac{\partial U(y_1,y_2)}{\partial c_2}} < 1 \tag{12}$$

If this quantity is greater than unity no competitive monetary equilibrium will exist. The essence of the key condition necessary for the existence of monetary equilibrium is that we need to find a rate of return on money as provided by the path of prices so that people will be willing to hold it over one period and that this rate of return is compatible with the rate of arrival of new resources into the economy. The key point on this was highlighted by Tirole (1985) in noting that the value of money is essentially a bubble since it is intrinsically worthless, and that its value grows at the rate of return, which hence must be supportable by the real resources of the economy. The higher is the rate of population growth the higher is the feasible rate of return.

In addition to the unique monetary equilibrium (with r < 1) we can sketch multiple non-stationary monetary equilibria. An example is shown in figure 2.

Note: as we follow the path down the offer curve, we effectively trace out

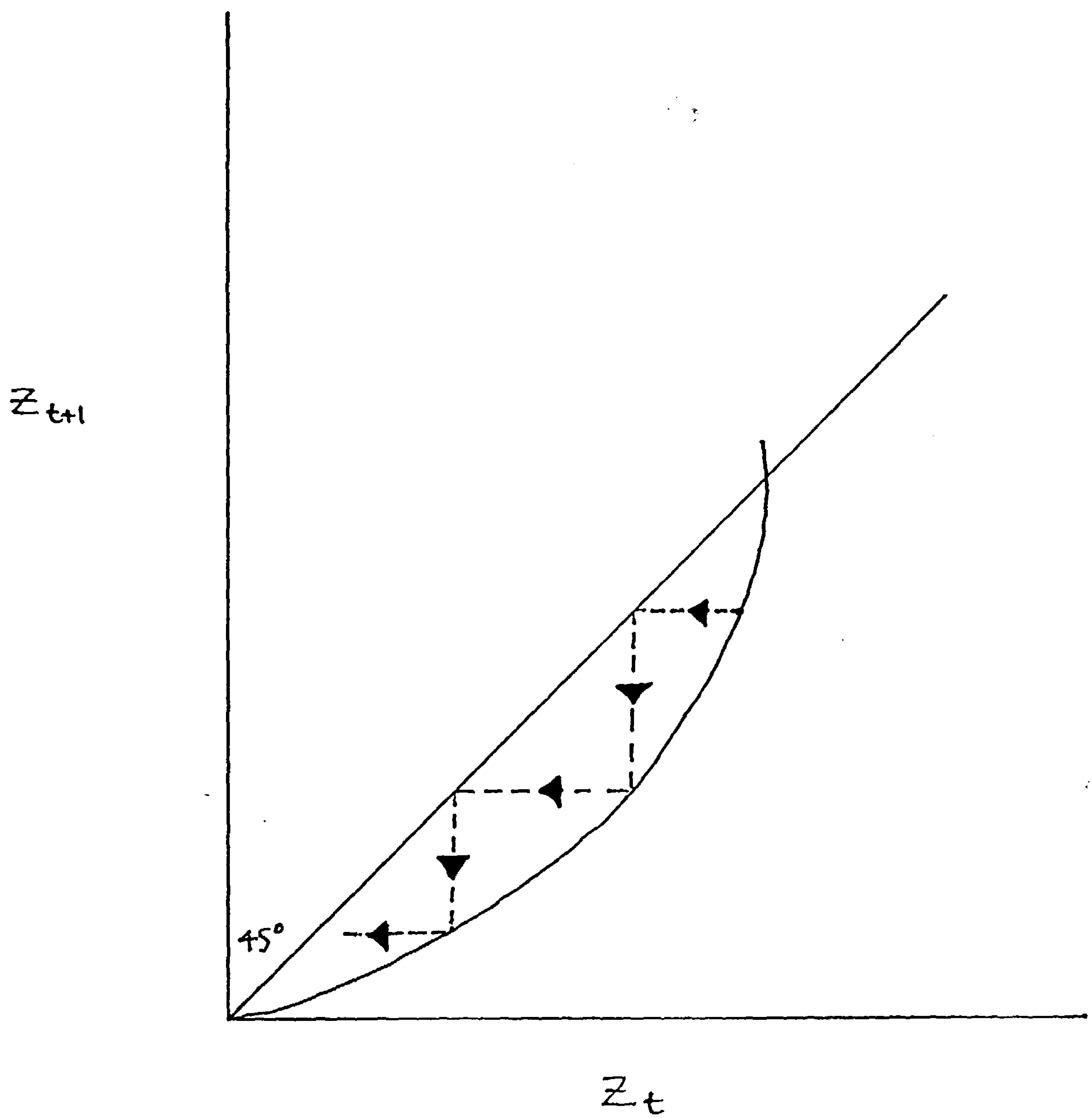

FIGURE 2.

situations in which the excess supply of the young equals the excess demand of the old with the price ratio falling each time, as does the equilibrium level of trade in each round, with the non-stationary equilibria converging asymptotically to autarchy. The occurrence of such equilibria will be analysed in chapter 6.

We now look at the welfare implications of these cases. In the case where we have just a single possible equilibrium - barter, with the marginal utility ratio (r)$\geq$1, then this barter equilibrium is Pareto optimal. The basis for this statement is that under such arrangements of endowments and preferences (giving r> 1) then we might suspect that a passing of goods from old to young is required to improve welfare. However, it is common to assume in consumption-loan models that the economy has a definite starting date. Hence the existence of the initial generation who is 'old' in the first period of the model. (This feature is discussed by Starrett ('72) and Shell ('72)). It means that to try and make Pareto improvements by the passing of goods from the old to young will unambiguously make the initial generation worse off. In effect, as Shell notes, this is really symptomatic of the fact that we pass some of the good forward into infinity. In contrast, the passing of goods from young to old accomplishes the opposite - i.e. we have some of the good brought forward from infinity. If we allowed a doubly infinite period, the old to young transfers would be Pareto improving (if r >1).

In the case where the marginal utility ratio is $r < 1$ at the endowment point, then the barter equilibrium is not Pareto efficient (nor are the non-stationary monetary equilibria). The role of the efficiency or otherwise of the non monetary equilibrium in determining the existence of a monetary equilibrium is intuitively simple. The process of looking to see if we can reallocate goods across generations by bringing goods back from infinity is analogous to seeing if we can find a rate of return on money so that agents are willing to transfer goods from young to old age, and that rate of return is compatible with equilibrium, in the same vein as the Tirole argument.

In summary these result of course demonstrates the propositions of Samuelson's model, concerning the efficiency/otherwise of the barter equilibrium and the existence of the monetary equilibrium and its efficiency. In particular the role of money in a Pareto improving role is demonstrated in an 'essential monetary model'. Indeed the stationary monetary equilibrium is Pareto optimal, though we shall see that non-interventionist monetary equilibrium in overlapping generations models and non-interventionist monetary equilibria in models with contemporaneously and infinitely lived agents have different welfare properties.

We shall see that in other respects there are some similarities between these two classes of models however, for instance on the issue of the number of equilibria in the model. However the latter class of models, consisting of infinitely and contemporaneously lived agents has the advantage of modelling money as something other than a purely intergenerational phenomenon and highlights the role of market imperfections in generating a role for money. It is to this class of models that I now turn.

1.3.3. Models of Money with Contemporaneously and Infinitely Lived Agents.

This section of the thesis considers models where agents who live contemporaneous and infinite lives operate under an imperfect non-monetary financial system so that the possibility of a monetary equilibrium is created. The models covered set the scene for the original contributions examined later and particularly introduce the notion of inefficient monetary equilibria and the optimum quantity of money. Other models of this class will however be covered later in the thesis, in order to facilitate understanding of particular points of interest.

The model covered here in detail is the most basic of this 'class' and was proposed by Townsend (1980). This is mathematically equivalent to a special case of the model presented by Bewley (1980, 1983) and the issues addressed are again trading, optimality, the valuation of fiat money and the optimum quantity of money. Studying this model will enable us to see the relation between a model in which agents are contemporaneously and infinitely lived when the financial structure is 'perfect', and secondly the same set of agents when money is the only asset and thirdly the overlapping generations models of fiat money. The relation of an inefficient non monetary equilibrium and the existence of a monetary equilibrium is highlighted again. Another important issue addressed is the efficiency or otherwise of the monetary equilibrium; i.e. the ability of money to overcome the incompleteness of markets. This in turn leads on to consideration of the optimum quantity of money.

The model is one in which agents wish to smooth their consumption in the face of uneven income streams. The optimal feasible allocations, the competitive allocation under loans and the allocation with money when we have an interventionist and non-interventionist monetary authority are addressed. The exclusion of inside debt can be imposed exogenously as in the Bewley model or endogenously as in the Townsend version.

The physical set-up of the model is of N agents each living for T periods (A horizon later taken to be unbounded) whose only heterogeneity is their endowment streams of the single perishable good. There are two types of agents (equal in number) such that for 'Type A' the endowment pattern is;

y^A = 1 when t even (0, 2,...)

y^A = 0 when t odd (1, 3,...)

and y^B = 1 - y^A.

Restricting attention to those allocations treating all agents within a given type identically, the social planner's problem is to maximise;

$$\lambda \sum_{t=0}^{T} \beta^t u\left(c_t^A\right) + (1-\lambda) \sum_{t=0}^{T} \beta^t u\left(c_t^B\right) \tag{22}$$

Subject to;

$$c_t^A + c_t^B \leq 1, \forall t \tag{23}$$

Yielding first order conditions

$$\frac{u'\left(c_{t+1}^A\right)}{u'\left(c_t^A\right)} = \frac{u'\left(c_{t+1}^B\right)}{u'\left(c_t^B\right)}, \forall t \tag{24}$$

Optimal allocations are then characterised by;

$$c_t^A = \overline{c}_A, \forall t \tag{25}$$

$$c_t^B = \overline{c}_B, \forall t \tag{26}$$

(Where $\overline{c}_A = 1-\overline{c}_B$, the split between the two being dependent on the planner's weights).

We now examine competitive equilibrium with loans.

The problem for an agent is to maximise;

$$\sum_{t=0}^{T} \beta^t u\left(c_t\right) \tag{27}$$

Subject to;

$$c_t + l_t \leq y_t + l_{t-1}\left(1 + r_{t-1}\right) \tag{28}$$

$$c_0 + l_0 \leq y_0 \tag{29}$$

Where l_t denotes the outstanding stock of assets at the end of period t (they begin the model with no debt).1+r_{t-1} denotes the gross real interest rate on period t-1 assets. Clearly $l_t < 0$ indicates borrowing.

For the last period we impose the condition that l_T must be zero. This naturally imposes the solvency condition on agents and the flow form of the budget constraint can be easily transformed into its lifetime or 'present value' form:

$$PV\left(consumption\right) = PV\left(income\right)$$

The first order conditions yield, for any type of agent;

$$\frac{u'(c_{t+1})}{u'(c_t)} = \frac{1}{\beta(1+r_t)} \tag{31}$$

Hence, in equilibrium, imposing either goods or loan market conditions for equilibrium;

$$\frac{u'\left(c_{t+1}^A\right)}{u'\left(c_t^A\right)} = \frac{u'\left(1-c_{t+1}^A\right)}{u'\left(1-c_t^A\right)} = \frac{1}{\beta(1+r_t)} \tag{32}$$

Clearly;

$$(1+r_t) = \frac{1}{\beta} \tag{33}$$

yields an equilibrium.

It is obvious that the equilibrium will be Pareto optimal. The equations above can represent either the special case of Bewley's model or Townsend's Turnpike model. Townsend's model specifies the environment in which agents can meet each other and then derives the asset structure endogenously. The set-up is one in which there is an infinite number of West heading agents and an infinite number of East heading agents travelling along a Turnpike of infinite length consisting of a countably infinite number of trading posts. Each set of agents shifts along by one post on the Turnpike each time period, and each time trades with the agent on the opposite side of the Turnpike. The crucial point about the structure is that no pair of agents will meet together again. Hence private loan markets are infeasible. Trade can only be facilitated by money. Again, the need for trade is created by the pattern of endowments over time, which is such that an East-heading agent is endowed with one unit of the consumption good at an odd-numbered trading post and zero units at an even numbered trading post. The opposite pattern prevails for the West-heading agent. Hence the opportunity to transfer the consumption good across agents is created. As before, we term agents endowed at even time points (including zero) as type A and the others B, who have M_0 units of the currency at time zero. Type A agents have no currency endowment initially.

Each agent maximizes;

$$\sum_{t=0}^{\infty} \beta^t u\left(c_t\right) \tag{34}$$

Subject to;

$$p_t c_t + M_{t+1} \leq p_t y_t + M_t \tag{35}$$

And;

$$M_{t+1} \geq 0 \tag{36}$$

Forming the optimization problem into a Lagrangian, with λ_t as the lagrange multiplier on the constraint at time t, then the F.O.C. are;

$$\begin{aligned} \beta^t(u'(c_t) - \lambda_t p_t) &< 0, \text{ if } \mathrm{c}_t = 0 \\ &= 0, \text{ if } \mathrm{c}_t > 0 \end{aligned} \tag{37}$$

$$\begin{aligned} -\beta^t\lambda_t + \beta^{t+1}\lambda_{t+1} &< 0, \text{ if } \mathrm{M}_t = 0 \\ &= 0, \text{ if } \mathrm{M}_t > 0 \end{aligned} \tag{38}$$

The transversality condition is;

$$\lim_{t\to\infty} \beta^t\lambda_t M_t = 0 \tag{39}$$

Since we require, for Pareto optimality, that;

$$\frac{u'\left(c^A_{t+1}\right)}{u'\left(c^A_t\right)} = \frac{u'\left(c^B_{t+1}\right)}{u'\left(c^t_B\right)} \tag{40}$$

For a steady consumption stream, we desire;

$$\frac{p_t}{p_{t+1}} = \frac{1}{\beta} \tag{41}$$

As was noted in the discussion of the Bernhardt paper, having this rate of deflation is incompatible with having a constant money stock, and we again choose an equilibrium with a constant price level.

We shall here, as Townsend and Bewley do, restrict attention to the deterministic steady state of the model. Non-stationary equilibria in this model as well as stochastic equilibria may occur as in the overlapping generations models of fiat money. As I have noted before, this topic is further covered in the final chapter of the thesis. In the meantime we should note that the presence of the liquidity constraint truncates the planning horizon of the consumers. The first order conditions below, since the liquidity constraint binds can hence be seen as equivalent to those of a sequence of 2 period lived generations, although the welfare implications of such models are different.

Since we have established above that the liquidity constraint must bite (confirmed by the argument below also), the total money stock will be passed across the Turnpike each period, in alternate directions.

Returning to the first order conditions we guess naturally that agents endowed with the good in period t will carry money balances over, and be liquidity constrained in the following period. In the unconstrained period;

$$\frac{u'(c_{t+1})}{u'(c_t)} = \frac{\lambda_{t+1}p_{t+1}}{\lambda_t p_t} = \frac{\lambda_{t+1}}{\lambda_t} \tag{42}$$

(Since the price level is constant)

For these unconstrained agents it must be that;

$$\frac{\lambda_{t+1}}{\lambda_t} = \frac{1}{\beta} \tag{43}$$

If we denote consumption of the agents who are endowed with a unit of the good in a given period as c* and consumption when not endowed as c** then;

$$\frac{u'(c^{**})}{u'(c^{*})} = \frac{1}{\beta} \tag{44}$$

With c** $+ c^* = 1$.

In the following period for this same agent;

$$\frac{u'(c^{*})}{u'(c^{**})} = \beta \tag{45}$$

So, with $\beta < 1$, we have that the marginal rate of substitution fluctuates either side of the unit value required for Pareto efficiency. If we treated agents symmetrically, then of course $c^* = c^{**}=0.5$ is the desired smoothing outcome.

The intuition behind this is that the friction of time preference (impatience) causes agents to carry over too little money to smooth consumption in the following period i.e. the liquidity constraint bites as a result of last period's optimization; or equivalently viewed in a general equilibrium context, that the side of the market that is supplying goods supplies too little, since the return on savings is too low.

Looking back to the first order conditions, we see that (assuming $c_t > 0$);

$$u'(c_t) = \lambda_t p_t \tag{46}$$

i.e. λ_t is the extra utility of an increment of money devoted to consumption in the current period t.

Also we can see that if $M_{t+1} > 0$ (i.e. when agents are selling and the liquidity constraint does not bite) then;

$$\lambda_t = \lambda_{t+1}\beta \tag{47}$$

Agents are able to optimize in their choice of money balances at that point.

However, when money holdings are run down to zero (i.e. when spending money) then;

$$\frac{\lambda_{t+2}}{\lambda_{t+1}} < \frac{1}{\beta} \tag{48}$$

i.e. - agents would in fact like to devote more of their resources to consumption in (t+1). However the non negative constraint on money holdings stops them. i.e. the liquidity constraint bites. We now turn to the question of interventions that need to be used to correct the inefficiency, a major theme for the rest of the thesis.

The interventions that are performed are to engineer a deflation of the price level over time to parallel the Friedman (1969) proposal of bringing private and social opportunity sets into line; the optimum quantity of money proposal. In an economy such as this one a social planner has the opportunity to redistribute goods within a period across agents who are endowed and not endowed respectively. The opportunity that faces agents however is an intertemporal one and their calculations in redistributing goods across periods when endowed and not endowed involves taking into account their rate of time preference and the rate of return on the means of storage. Impatience thus provides a friction. To see this we can note that the fluctuations in consumption as described above disappear if we have $\beta = 1$. The task of the monetary authority, according to Friedman, is to pay interest on money equal to the rate of time preference to correct the market failure of money and create opportunities identical to those that the debt mechanism provides. An alternative method is to deflate the money supply over time to make the price level fall at the same rate, which is the method we follow below.

In computing the equilibrium by anticipating the constant rate of deflation we seek, we expect that agents will carry positive amounts of money when they are sellers and spend all their cash when they are buyers (this will be seen to be still valid even though they are strictly speaking unconstrained when buyers in an efficient equilibrium). Given that we assume a velocity of circulation of one then a quantity type equation would be expected;

$$p_t = kM_t \tag{49}$$

Proportional money taxes are levied such that;

$$\text{Nominal } Tax = T_t = M_t - M_{t+1} = (1 - \beta)\, M_t \tag{50}$$

The taxes must be lump sum, and note also that a contrast with the second welfare theorem is implied, with continuous redistribution required, rather than simply one distribution at the initial date.

An interesting footnote to these efficiency issues is provided by Townsend. If we look back to our stationary overlapping generations models, in a steady state we had a constant money stock and a constant price level and agents optimised according to;

$$u'(c_t) = \beta u'(c_{t+1}) \tag{51}$$

(this is with a fixed money stock). This gives us a Pareto optimal solution. However, in a model such as the Turnpike, a steady money stock and price level giving;

$$u'(c_t) = \beta u'(c_{t+1}) \tag{52}$$

is sub Pareto optimal.

This puzzle is explained by Townsend by noting that the Turnpike model pairs agents of the same age. The overlapping generations models, in contrast, pair agents of different ages. Hence, when we pass goods between young and old (assuming, for symmetries' sake, that we do it for all generations) we effectively carry out a redistribution of goods across time periods for all agents. Hence the equality expressed above indicates a situation that is Pareto optimal in an overlapping generations model.

The analogy to production is pertinent here as we can trade off the amounts of consumption for young and old in an overlapping generations model. This is to be contrasted with the re-allocation carried out in the Turnpike model, where we cannot change the total consumption of a certain age group, and the optimum re-distribution requires that;

$$u'(c_t) = u'(c_{t+1}) \tag{53}$$

Of course, in the Turnpike model, if we could achieve equality(52) for both agents (whilst in all periods using all the economy's endowment) - that allocation would be Pareto optimal, but could only be achieved if we had total resources available in period (t) greater than those in period (t+1) enough to allow the above equality for both.

The Townsend model and its equivalent, a special case of the Bewley model both have deterministic endowments. Bewley's (1980, 1983) model is the more general case and has endowments generated by a stochastic process, and will be covered in detail in the next chapter. Other variants of this framework are developed by Levine (1991), Kehoe, Levine and Woodford (1992) and Scheinkman and Weiss (1986), whose work is extended by Dutta and Polemarchakis (1990). These will be mentioned again in the final chapter. Other liquidity distribution models are developed by Fuerst (1990) and Christiano and Eichenbaum (1992). These latter two look at liquidity distribution of a more temporary nature; a representative household is split into sectors and then re-united at the end of a period. The former set of models mentioned in this section allow an evolving distribution of liquidity through time.

In the Scheinkman and Weiss model there are 2 agents whose positions as buyer and seller of the economy's single consumption good are prone to reverse as in the Townsend model. Here however, the change in positions is random and determined by a Poisson process, which determines the allocation of the opportunity to work and sell the output. In the case of even an inefficient equilibrium of the Townsend model, where the total level of liquidity is too low, what liquidity is there is always 'conveniently' held by the buyer. In Scheinkman and Weiss the random changes in position of the 2 agents means that in cases where the level of liquidity is too low then a long period of time with the buyers and sellers in the same position causes problems. Scheinkman and Weiss and Dutta and Polemarchakis (1990) show that this factor can yield persistence in output after a single shock.

The mechanism is that previous shocks cause the agent who is now a buyer to have low real money balances, and the seller/worker has high balances that yield a low incentive to work. Contrary to the Friedman notion of the benefits of anticipated deflation, Scheinkman and Weiss focus on the benefits of a once and for all unexpected injection of money into the economy in a 'helicopter' drop fashion. This is symmetrically distributed to all agents and has the effects of redistributing liquidity towards the 'poor' buyer who holds low real money balances. The crucial extra factor of liquidity distribution as opposed to simply its quantity is thus highlighted. This issue is then taken up by Levine (1991) and Kehoe, Levine and Woodford (1992) who look at the effects of anticipated money injections. Levine looks at the extreme case of linear utility so that corner solutions in consumption occur and so only the distribution of liquidity is important, not its quantity (due to bang-bang behavior) so that in that case inflation is always the best policy. KLW examine the balance between the 2 policies using concave utility functions. The optimal policy balances the redistribution of liquidity towards previously unlucky agents who are hence low money holders now with the Friedman type gains from increasing the quantity of real money balances. As in Scheinkman and Weiss, aggregate fluctuations occur in the KLW model since there are only two types of agents in the model; and the authors note that the class of models is evocative of the Leijonhufvudd (1973) concept of a classical corridor. The economy has sufficient buffer stocks of liquidity to deal with small shocks but not large ones, where the size of the shock here corresponds to the duration of the time that buyers and sellers stay in the same configuration without reversing.

Mehrling (1995)shows that even in the case of a model with a continuum of random variables that yields a deterministic steady state, the distribution of liquidity can play a role when its quantity is finite.

The models that follow in the original chapters have the common factor of being non representative agent models where money changes hands between agents through time, hence contributing to the field of study of Bewley, Levine, Townsend, et al. We might hence even also put the models alongside the overlapping generations framework, due to some similarities that have been highlighted; chapter 6 will bring out similarities also.

I have exposited models in the survey which cover a number of issues. Not all of these are uniquely related to models of infinitely and contemporaneously lived agents but the models examined provided useful insights and contrasts. We saw how an inefficient non-monetary equilibrium created the opportunity for a monetary equilibrium, in the overlapping generations framework and the Townsend model, and more suggestively in other models (e.g. the work of Ostroy). The record keeping role of money in enforcing budget constraints was also noted, most particularly by Ostroy. Money's Pareto improving role was also noted. The question of whether money's Pareto improving role extended to make the monetary equilibrium Pareto efficient was then addressed, introducing the notion of the optimum quantity of money. All these issues are addressed in the original chapters that follow but many of the issues that specifically pertain to models of non-representative agents have not been emphasised in this initial 'general' survey, and will be covered in later chapters as necessary.

1.4. Issues to be Addressed

Despite the common theme we can identify each chapter with different issues in the field. As I have said, not all of these topics have been covered in the survey so far and some of the chapters commence with some survey material which will aid understanding of the particular model presented. Below I repeat an outline of the issues addressed in each chapter.

Chapter 2 addresses the issue of the valuation of fiat money in a scenario of uninsured idiosyncratic production risk for a continuum of contemporaneously and infinitely lived agents, since such a gap in the literature currently exists. I then examine the issue of the existence of equilibrium under the regime of lump sum taxation that is traditionally associated with the implementation of the optimum quantity of money proposal, since this is an area in which Bewley found a problem. Such an issue is of interest in the field of non representative liquidity distribution models, since it is closely tied up with the way in which money is modelled as changing hands between individuals, and so far no model has examined the issue with productive capital instead of endowments. The chapter commences with a non technical exposition of the important parts of Bewley's 1980 and 1983 papers since the originals are somewhat opaque in their presentation.

Chapter 3 takes an alternative approach to the issue of the tax system required to implement the optimum quantity of money proposal. Instead of restricting attention to lump sum taxation as other authors do, I set up a model with the same basic mathematical structure as the model of chapter 2 but this time specifying an information structure that restricts the type of state contingent taxes and redistribution schemes that are available to the government to reallocate resources. In the context of this model I then address a conjecture of Hellwig (1982) and Woodford (1990) that if taxes could be levied on individuals in a way that avoided the 'Bewley difficulty' then the information structure would be such as to make money inessential in the sense of Hahn (1965, 1973); that is, that money's value would vanish since it is replaced by a complete set of markets. Their conjecture was made in the context of a pure exchange model but my analysis will be seen to carry over easily.

As part of the analysis I am able to shed light on the way in which the implementation of the optimum quantity of money proposal operates, by noting fiat money's essential properties of anonymity and common usage. Paying out the proceeds of taxation by raising the value of real balances is seen to be superior to an insurance scheme.

Chapter 4 examines a basic notion about money on which the optimum quantity of money literature is based; that more money is a good thing, as we have seen in its Pareto improving role. More particularly I will question if it is possible to have an equilibrium in which money has too much value in a model of infinitely and contemporaneously lived agents. I answer the question by appealing to a fundamental shortcoming of money as an asset. The roles of assets in changing the distribution of liquidity across the agents in the economy is essential to the story and demands the non representative agent framework used.

Chapter 5 presents a model which addresses the issue of the superneutrality or otherwise of fiat money. I investigate the implications of the underlying cause of Bewley's difficulty for the effects of monetary policy in a case where that problem does not lead to non existence of monetary equilibrium under deflationary policy;

namely an economy with a finite horizon.

Chapter 6 addresses a set of issues different to those of 'pure' monetary theory by examining the effects of the distribution of liquidity in the economy on economic fluctuations, as opposed to looking purely at deterministic steady states and their welfare characteristics as in previous chapters. I present two models, one where a non convexity in investment opportunities leads to multiple equilibria and also the possibility of an endogenously generated investment cycle, and a second model where the distribution of liquidity over time is endogenised and interacts with the non-convexity of investment opportunities to cause persistence of low output levels; the distribution of liquidity between agents of differing risk attitudes is important here.

Before I commence chapter 2 it will be useful to stop and think about what the important functions of money are that we might wish to capture, the best ways to model them and what various models represent in terms of monetary factors.

As mentioned earlier, the point of departure of the thesis was seen to be the Arrow-Debreu model rather than 'practical' monetary theory, and the deletion of markets might leave us wondering what role money is supposed to play in the models of the type of Townsend and Bewley which form the basis of the thesis. The obvious answer would be to say that they show the role of fiat money as a financial asset in guarding agents against the risk of low income. A critic of the model might say that since money is not dominated in rate of return with probability one in any of the models I present (with the exception of the first model of chapter 6) then its properties as a medium of exchange are not represented; namely that adding another alternative store of value would cause money to disappear from the model. We can answer such a criticism by splitting it into two parts, so to speak. We might firstly ask if a model such as Townsend's captures some of the essential aspects of money as a medium of exchange as well as being a value store model, even if it is in a less than literal sense. The key factor in the models such as Townsend and the ones that will be encountered below is that money changes hands in equilibrium, and we can note the defence by Wallace (1980) of overlapping generations models that this captures an essential property of the exchange process. Indeed we note that money in such a context does overcome a lack of double coincidence of wants, even if it is only temporal. The key difference between the exchange property and a value store that allows self sufficiency in the Robinson Crusoe sense is then depicted. The 'acceptability' of money by agents that is crucial to its value and which is often quoted in the medium of exchange literature is also captured.

However, addressing the second part of our suggested split of the issue means asking if the model is a good literal representation of the role of money, particularly in regard to its rate of return dominance. We need to explain why there are no other safe assets in the model. In models that do have money dominated in rate of return, for instance the model adapted from Fuerst (1992) in chapter 6, the rate of return dominance comes from money's liquidity property. A way to defend models such as Townsend and Bewley is thus to say that the holding period of money is short and that the modeler has simply chosen to abstract from consideration of 'lower frequency' activity. However, an alternative way is to question the concept of rate of return dominance by other assets. If we looked behind inside assets that dominate money we might ask what backs them and suggest that money is doing this job. If people did not feel that their balances at a financial intermediary were not convertible into cash, then their view of these assets as safe might not be so obvious. If we search for a real store of value that dominates money then we might even be driven to look at something such as land. The land versus money debate is of course evocative of the overlapping generations literature, and on this note we can counter the competition from land by noting that its transactions costs are too large for anything but a very long term store of value role. We might view this second line of defence as a 'Keynesian' one that pushed the role of money as the only available safe asset in the portfolio. This is a particularly strong argument if we invoke the notion of sticky prices over the business cycle.

The arguments above are disparate but I tend to side with Wallace's view that distinctions between the differing roles of money as depicted in economic models are sometimes overplayed. Wallace notes that if the crucial characteristics of a medium of exchange function are in the way that it is actually exchanged between agents then the role is adequately covered by an overlapping generations model, and therefore certainly also for models infinitely lived heterogeneous agents.

I will also mention briefly different possible interpretations of models with regard to what the money of the models in all parts of the thesis can be taken to represent, with relevance to what insights the models can yield. A literal interpretation of course would be of fiat money literally passing round a system, which is what the models puport to show. Alternatively, a more metaphorical approach might view the fiat money as playing a role in backing inside assets, of either short or long maturity, but influencing economic activity in all cases.

In conclusion of this short discussion, I would say a very strict interpretation of the models is not necessary for them to have a meaning. This goes for models both surveyed and original. **Whatever individual opinions are about the role given to money in the various models, we can say that the field does indeed yield interesting macroeconomic insights into economies** with insufficient liquidity; the models are more than purely monetary.

Chapter 2

Money and Risky Production

2. Money and Risky Production.

This chapter picks up on aspects of two topics covered in the literature survey; the existence of an equilibrium with intrinsically useless fiat money and the optimum quantity of money with idiosyncratic income or output risk. The chapter splits into two parts. The first part contains a detailed survey of the work of Bewley and Levine which was briefly touched on in the literature survey. The second part exposits an original model that approaches the issues from a different angle by including risky capital instead of risky endowments, hence filling a gap in the monetary equilibrium existence literature, since no non representative agent models with risky production exist. Money's common usage among agents is important here, since it yields a constant price level in the face of idiosyncratic shocks. In the conditions for existence I find a role for the rate of time preference different to the role it plays in the work of Bewley and Levine. I then examine the issue of existence of equilibrium under the regime of taxation and interest payments on money. I find a problem that echoes a difficulty Bewley found but has a fundamentally different cause which is endemic to the nature of the model. I show that no monetary equilibrium exists with a strictly positive rate of interest on money if we restrict attention to the case where all agents carry strictly positive precautionary balances at all points in time. The result arises in spite of the presence of an alternative value store to money such that no individual ever has to hold an amount in money equal to the per capita money stock to meet his tax bill with probability one, so that the cause of the difficulty in Bewley's model is eliminated. Furthermore I manage to draw on the insights gained in the model to suggest a problem that might occur in Bewley's model even if Bewley's original difficulty does not occur.

An important point to clear up at this stage is the difference between two different ways of implementing the optimum quantity of money proposal. One method, as outlined in the Townsend model covered in the literature survey, is to deflate the money stock so that the price level will fall at the same rate. Another is to pay interest on money and keep the nominal money stock constant by taxing individuals and using the proceeds to withdraw money from circulation to subsequently pay the interest on money. The latter method is used in this chapter, to achieve continuity of presentation with Bewley's paper. In many senses the difference between the two schemes is unimportant. As Woodford(1990) points out the only difference arises when attention changes away from steady states, and we allow per capita real money balances to change over time. If the money stock is deflated then a glance at the transversality condition covered in the literature survey reveals that a wider range of equilibria may be possible. Chapter three however uncovers another difference.

2.1. Bewley's Contributions.

Bewley(1980) develops a model of individual endowment risk to address the issue raised by Friedman's Optimum Quantity of Money, and like Friedman considers an economy where agents live infinite and contemporaneous lives. Fiat money is the only asset and is used to offset short run income and taste fluctuations.

Bewley defines a monetary equilibrium to be an infinite sequence of random spot market equilibria where all prices in terms of money are bounded away from zero and infinity. Inflationary and deflationary equilibria are hence ruled out by construction. Bewley then investigates the existence and optimality of monetary equilibrium under various conditions. He shows (incorrectly) that if the state dependent utility functions and preferences are such that consumers might need to arrange self insurance by holding money then a monetary equilibrium can exist if the interest rate on money is less than the rate of time preference of all consumers. In such a case the low interest rate on money leads consumers to economise on money balances and the resulting equilibrium is not Pareto optimal. Full insurance would require an interest rate equal to the rate of time preference (see Schectman and Escudero (1977)). However in such a case no monetary equilibrium exists as long as the stochastic process that generates endowments and preferences is sufficiently random. Bewley is referring to cases where the optimum quantity of money is infinite and full insurance would mean a zero price level for all goods and unbounded consumption sets. Monetary equilibrium only exists with r $=\theta$ in these types of models in the case of periodic endowments, where r denotes the rate interest on money and θ is the pure per period rate of time preference. The specific case we can consider is that of Townsend (1980) which I reviewed in the survey chapter.

Bewley cements the concept of an optimum quantity of money in his model by considering the notion of stationary equilibrium where the only constraint on consumers is their long run average income (insurance is available) so that consumers can allocate their income according to what Bewley calls the permanent income hypothesis, where the marginal utility of money or nominal expenditure is constant across time and states, even in the face of price fluctuations. The almost equivalent situation that Bewley seeks in the monetary economy is a situation of incomplete markets, no borrowing but large real money balances so that the marginal utility of money is almost constant, and cash rarely constrains consumers. He conjectures that it might be possible to make the equilibrium allocation of such a monetary economy arbitrarily close to a Pareto optimal allocation by paying interest on money, but he is unable to prove this. As he subsequently showed in his 1983 paper, there is a problem with this statement beyond that of burden of proof.

Bewley makes the point that the optimum quantity of money is infinite since agents live forever and have risk in all periods of their life. However, efficiency is not gained in an economy of consumers with finite lives since the incentives for precautionary saving are too low (see Schectman and Escudero (1977)) to generate the high real balances required, even though the optimum quantity is finite.

Bewley's economy is populated by I consumers and contains L commodities plus fiat money. Consumer i's endowment for period n is governed by a stationary Markov process with no transient states. The total fiat money stock is normalised to 1 and interest payments on money are financed by lump sum taxes which are used to keep the total money stock constant. Hence;

$$\sum_{i=1}^{I} \tau_i = r \tag{1}$$

Period utility functions, like endowments are generically state dependent, but we will suppress the notation (s_0..........s_t) used by Bewley to denote the history of states and hope that state dependence is clear. At time t consumer i maximises;

$$E_t \sum_{n=t}^{\infty} \delta_i^{n-t} U_{in}(\underline{x}_{in}) \tag{2}$$

(Where $\underline{x}_{in}$ denotes his vector of consumption of the L goods in time period n.)

Subject to M_{i0} given, and;

$$M_{in} = (1+r)M_{i,n-1} + \underline{p}_n^{'}(\underline{\omega}_{in} - \underline{x}_{in}) - \tau_i \tag{3}$$

$$M_{in} \geq 0, \forall n \tag{4}$$

So that borrowing is prohibited.

M_{in} denotes the nominal money balances held at the end of period n by consumer i, $\underline{\omega}_{in}$ is his endowment vector over the L goods in time period n.

The first order conditions that arise from this problem and in particular the marginal utilities of money will be discussed when we come to Bewley's 1983 paper.

A monetary equilibrium is defined to be a vector ($\underline{p}$,($\underline{x}_i$)) consisting of a price system and an allocation such that $(\underline{x}_i)$is a feasible allocation, $\underline{x}_i$ is optimal given (2) , (3) and (4) above and the price system $\underline{p}$ must be such that all its components are bounded away from zero and infinity in all time periods and states.

Bewley states and proves five theorems of which three are of particular interest to us. Firstly he shows, incorrectly, that a monetary equilibrium will exist if $\delta_i < (1+r)^{-1}$ for all i, and the δ_i are all sufficiently large. We will here adopt the chronological approach of describing Bewley's 1980 'proof' and giving his 1983 correction later, since the ideas and method of his 1980 paper are instructive. Bewley's technique essentially contains three steps. He firstly shows the existence of monetary equilibrium in a finite horizon economy. Secondly he shows money prices of the goods are bounded both above and away from zero by a-priori values. Finally he shows that this holds in the infinite horizon case.

Bewley constructs a general finite horizon economy lasting N periods that has the same basic structure as the infinite horizon model described above. In order for money to be valued in the face of a finite horizon, Bewley modifies an N period horizon version of the utility function in (2) by giving end of period N balances one unit of utility per unit of money. Existence for this economy is then established by a standard fixed point argument. Monotonicity of utility in all L+1 goods is sufficient to guarantee strict positivity of their prices, including money.

In periods prior to the last one, monotonicity again guarantees strict positivity of prices of the real goods. Money yields no direct utility in these periods, but arbitrage yields a strictly positive value. Existence of a monetary equilibrium for the finite horizon is hence established.

However, since Bewley's final target is a strictly positive value of money in an infinite horizon economy, the final period utility approach will ultimately not suffice to give money value since such a device is by definition no longer present in that case. Bewley solves the problem by invoking three important assumptions. These assumptions ensure that the consumers have income fluctuating relative to their consumption needs sufficient to generate a demand for self insurance. This he does in the process of establishing that the money prices of goods in an N period equilibrium are bounded above by $\underline{p}^*$ and below by $\underline{p}_*$. The crucial assumptions for his proof of boundedness above are assumptions 4, 8 and 9 of his paper. Intuitively, assumption 4 says that in each state the economy's total endowment of each good is bounded below by a positive quantity. Assumption 8 says that each consumer has a positive probability of having an endowment of every good less than a sufficiently small positive constant, which is below the average of the relevant bound defined in assumption 4.The notion of some degree of individual uncertainty exceeding aggregate uncertainty is hence established and so the opportunity for money mediated Pareto improving trades is created. Assumption 9 gives us that in the states of bad luck defined above, marginal utility of expenditure on each of the goods is bounded below and will exceed the marginal utility of expenditure when consumption is at the level of the lower bound on the per capita average endowment of the respective goods.

Bewley uses these facts to establish that $\underline{p}_*$ and $\underline{p}^*$ exist for the finite horizon monetary economy. His existence proof for the infinite horizon economy then consists of showing that these bounds are valid for the infinite horizon economy.

Bewley's second theorem is that with an interest rate strictly less than the rate of time preference, no equilibrium is Pareto optimal as long as all consumers consume something in all states and time periods, since agents economise on money balances. I will shortly provide a sketch of the proof of this for an example given in his 1983 paper.

Bewley's third theorem is that no monetary equilibrium exists in an economy where all agents have an identical rate of time preference, and the rate of return on money is equal to this rate. The exception to this is the Townsend model. (This result of Bewley's is of course rendered redundant by his 1983 paper's result.) The intuition behind this result is (as Schectman and Escudero have shown) that the willingness to accumulate real balances increases without bound as 1+r approaches $\frac{1}{\delta}$ from below. This implies a zero price in equilibrium for all real goods and hence unbounded consumption sets if the limiting situation is reached. A counter-example to this result is provided by Mehrling (1995) whose choice of a piecewise linear utility function with a kink point at the average endowment means that 1+r = δ yields a finite money demand.

A complement to the work of Bewley is provided by Levine (1989). As in the work of Bewley, the rate of time preference plays a crucial role in the conditions for existence of monetary equilibrium; that the consumers must be sufficiently interested in the future to want to hold money. Levine's contribution is to make the restrictions on endowments and preferences more elegant. Levine's condition is simply that there be a unique Pareto inefficient barter equilibrium. Hence 'a little' diversity in marginal rates of substitution at the barter allocation instead of 'sufficient' diversity will suffice. Levine considers real rates of return on money in his model that are greater than or equal to zero, hence avoiding the problems discussed below.

Simultaneously, Bewley (1983) and Hellwig (1982) discovered a difficulty with Bewley's first theorem one of his 1980 paper. Bewley was forced to amend this to say that a monetary equilibrium will exist only if the rate of interest on money is sufficiently small. The essence of Bewley's result can be seen if we consider the case where each agent has a positive probability of income equal to zero for an arbitrarily large number of periods, hence his minimum non interest income is zero. If for instance all consumers have initial nominal money balances of $\overline{M}$ and the same tax liabilities, then each agent faces a tax bill of r$\overline{M}$ per period. Due to our assumption on endowment incomes the balances of $\overline{M}$ are the only certain resource that the consumer has to meet his taxes, which must be paid with probability one. The consumer thus has to hold balances of $\overline{M}$ each and every period and use the interest payment to meet his tax bill. It is easy to calculate that if he ever runs down his balances below $\overline{M}$ then there is a positive probability that he will fail to meet his tax bill. The problem that arises in this case will be that no monetary equilibrium exists with a strictly positive rate of interest. There will be a demand for precautionary balances which will be insatiable, since the money is already used up totally by agents holding it to pay taxes. These balances cannot serve simultaneously as precautionary balances since precautionary balances must be available for liquidation in the face of a bad income shock. The fact that the balances cannot serve two purposes at once is symptomatic of the model's non representative agent form. Ex post heterogeneity means that money circulates in equilibrium. This is an essential part of my original model also, which follows shortly. If the minimum endowment income is bounded away from zero, Bewley shows that equilibria may exist for positive rates of interest on money, provided that these rates are sufficiently low. A relatively low interest rate means a price level that is relatively high. The higher the price level is the higher is the nominal value of the lower bound on income. As long as this quantity is greater than or equal to tax bill in each period, no autonomous tax demand is created, and the Bewley problem is avoided. However, an upper bound on the feasible interest rate will still exist. As Taub's 1988 paper shows, even with the case of deterministic endowments an upper limit exists.

Bewley presents a simple special case of his model where all agents have an identical rate of time preference. He uses this to illustrate the result that no monetary equilibrium exists with a rate of interest too close to the rate of time preference. The economy described in the example consists of two agents indexed by 1 and 2. Each agent has an identical rate of time preference equal to 0.1 per period and identical period utility function $\ln(x_{it} + 1)$. There is a single non storable consumption good. The random variable a_t is identically and independently distributed across time and takes on values a or b with equal probability. In state a, consumer 1 has an endowment of $\frac{1}{4}$ and consumer 2 has an endowment of $1\frac{3}{4}$. In state b the positions reverse. The stock of money is normalised to unity and taxes are equally distributed across agents and hence equal to $\frac{r}{2}$. Bewley's notation emphasises how the equilibrium values of variables in any period depend on the whole history of a_t values realised, but I suppress this notion wherever possible.

M_{it} denotes nominal money balances held by consumer i at the end of period t

x_{it} is the consumption by consumer i in period t

Consumer i will maximise at time j;

$$E_{ij} \sum_{t=j}^{\infty} \left(\frac{1}{1.1}\right)^{t-j} \ln(x_{it}+1) \tag{5}$$

Subject to;

$$M_{it} = (1+r)M_{it-1} - \frac{r}{2} + p_t\omega_{it} - p_t x_{it} \tag{6}$$

First order conditions give us;

$$\lambda_{it} \geq \frac{du_i\,(x_{it})}{dx} = \frac{1}{p_{it}\,(x_{it}+1)} \tag{7}$$

This holds with equality if $x_{it} > 0$.

$$\lambda_{it} \geq (1+r)\,(1.1)^{-1} E_{it}(\lambda_{it+1}) \tag{8}$$

This holds with equality if $M_{it} > \underline{M}$.

$\underline{M}$ is the minimum level of money balances a consumer can hold and still ensure solvency, and is key to Bewley's argument.

To show that no monetary equilibrium of the model is Pareto optimal Bewley firstly shows that no monetary equilibrium is Pareto optimal with $r < 0.1$. Bewley's proof is by contradiction and proceeds as follows. He firstly shows that if the interest rate is strictly less than the rate of time preference then at some point the liquidity constraint must bind (i.e. given the relatively low rate of return on money the agent will optimally plan to run out of money balances at some point in equilibrium). Formally proving this shows by contradiction that if not then we have a first order condition for each agent (choose 1 w.l.g.);

$$\lambda_{it} = (1 + r)\,(1.1)^{-1}\, E_t \lambda_{it+1} \tag{9}$$

An interest rate less than 0.1 then implies an expected marginal utility of money that will grow without bound through time. At least one consumer must also have a consumption level each period greater than or equal to one (let this be consumer 1 w.l.g.). We must also note that Bewley's definition of monetary equilibrium precludes the price level going to zero asymptotically. From the definition of λ_{it} in (7) we can then see that these two facts preclude a monetary equilibrium where the expected marginal utility of money grows without bound.

Having established that the constraint must bind in equilibrium at least at some point in time, Bewley considers a scenario where the ensuing period's endowment realisation is a bad one, namely $\omega_1 = \frac{1}{4}$. If the Pareto optimal allocation described is to be continued then consumer 1's nominal net outflow for that period is at least $\frac{3}{4}p_t$.If money balances were equal to $\underline{M}$ going into the period, an outflow of $\frac{3}{4}p_t$ will push M_{t+1} below $\underline{M}$, which is impossible in equilibrium. This is so by definition of $\underline{M}$, since entering a period with this amount of money, and a bad endowment realisation occurring,only consumption equal to zero will suffice to ensure solvency.

Given this, to show that no equilibrium of the monetary economy is Pareto optimal, Bewley simply has to use his second result which is that there exists an a-priori upper bound (determined by the parameters of the model and not by any particular equilibrium or state) on the rate of interest that will yield a monetary equilibrium. This also yields the result that no monetary equilibrium can be made arbitrarily close to a Pareto optimal allocation.

Bewley first proves that no monetary equilibrium exists with r >0.1. Recursive application of (8) with r>0.1 yields the result that expected marginal utility of money falls throughout time such that;

$$\lim_{t\to\infty} E\lambda_{it} = 0 \tag{10}$$

However, Bewley's definition of monetary equilibrium in his model precludes the possibility that the price of money will go asymptotically to zero (p^* is an a-priori upper bound as in the 1980 paper) and since the real endowment of the economy is bounded, such a situation is infeasible.

Bewley's next task is to prove the existence of an a-priori bound $\underline{r}$, $0<\underline{r}<0.1$ such that no monetary equilibrium exists if $0.1\geq r \geq \underline{r}$.

The thrust of Bewley's proof is to note that if the interest rate is close to the rate of time preference, the depletion of an individual's money holdings necessary to finance the consumption stream of an agent facing such an interest rate will cause the individual's holdings to fall below those necessary to ensure solvency. In other words, we have the excess money demand problem alluded to earlier; the money needed to finance a consumption stream close to full insurance (in a sense clarified later) in the face of a run of bad luck is more than the given money stock can provide.

Bewley begins by defining a bound on the marginal utility of money: $\underline{\lambda}$ is defined as the infimum of all possible values of λ for all consumers in all possible time periods and states in equilibrium. This of course corresponds to a supremum on the set of all possible consumption values, given the bounds put on p. Since $x_{1t}+x_{2t} = 2$ for all t, then the supremum of consumptions is at least one, then since U(x)=log(x+1) Bewley obtains an upper bound on the price level in terms of $\underline{\lambda}$. This is his equation 4.6;

$$p_t \leq (2\underline{\lambda})^{-1} \tag{11}$$

Bewley then establishes $\underline{M}$, the minimum money balances that an agent can hold if he is to stay solvent. To do this he simply uses the budget constraint (6) with $x = 0$ and $\omega = \frac{1}{4}$ to obtain a transition equation for the agent's money balances in the face of a continued run of bad luck. Since he has shown that the price level is bounded above as shown, for all t, then at most $(8\underline{\lambda})^{-1}$can be earned by endowment sale. Unless the consumer holds at least $\frac{1}{2} - (8\underline{\lambda})^{-1}$ he will hold negative balances with positive probability.(Similar manipulation with the transition equation will later yield the non-existence result). Bewley's next step is to examine values of variables in the economy in a situation where the value of λ is sufficiently close to $\underline{\lambda}$, specifically he selects a state and time period for consumer 1 (w.l.g) such that $\lambda_{11} < \underline{\lambda}(1+\epsilon)$, where $\epsilon > 0$ and has a meaning that will become clear later. Given this he demonstrates that if r is sufficiently high then λ_{it} will stay close to$\underline{\lambda}$ for a number of periods sufficient to yield money balances that fall below $\underline{M}$, contradicting the conditions for existence. More specifically, he shows that there exists an ϵ and an r such that $\lambda_{it} \leq (1+\delta)\underline{\lambda}$ if $1 \leq t \leq 41$. Note that in this case δ does not denote the time preference factor. The integer 41 is a partly arbitrary choice by Bewley which is a length of time sufficient to prove the result in his model. Putting together (Bewley 4.2) and (Bewley 4.9) yields;

$$\underline{\lambda}(1+\varepsilon) \geq \lambda_{11} \geq \left(\frac{1+r}{1.1}\right)^{t-1} (\left(\frac{1}{2}\right)^{t-1} \lambda_{1t} + (1 - \left(\frac{1}{2}\right)^{t-1})\underline{\lambda}) \qquad (12)$$

The right hand part of the above expression comes from the fact that all the other possible values of λ_{1t} other than the 'general' one chosen are $\geq \underline{\lambda}$ by definition. Clearly upon rearranging the above equation (4.10 in Bewley), if ϵ is sufficiently small, r sufficiently high and t sufficiently small then we can establish that $\lambda_{it} \leq (1+\delta)\underline{\lambda}$, for any δ >0. Bewley then uses this result to derive a lower bound on the level of nominal expenditure as a function of $\underline{\lambda}$.The bound is given by Bewley's equation (4.11);

$$p_t x_{1t} > 3(8\underline{\lambda})^{-1} \text{ for t} = 1,......41 \qquad (13)$$

All Bewley's 'preliminary' calculations for the proof of his result are therefore complete and he uses the consumer's intertemporal budget constraint combined with these calculations to do so. In deriving the result Bewley assumes $\underline{r} > \frac{1}{20}$; such a simplification is permissible for the style of his proof using a numerical example.

If we assume that consumer 2, according to his no bankruptcy condition, has money balance at least equal to $\underline{M} = \frac{1}{2} - (8r\underline{\lambda})^{-1}$, then we have that (normalising the period date to one);

$$M_{11} \leq \frac{1}{2} + (8r\underline{\lambda})^{-1} \tag{14}$$

Using this inequality and substituting the upper bound on pω and the lower bound on px (the crucial inequality that arises from the sufficiently high interest rate) into the budget constraint or transition equation (6) allows Bewley by to demonstrate by induction the effects of a run of bad luck of 40 periods low income on the money balances of consumer 1. By continued recursion on the transition equation;

$$M_{1,41} \leq \frac{1}{2} + (8r\underline{\lambda})^{-1} - 40(8\underline{\lambda})^{-1} \tag{15}$$

$$< \frac{1}{2} - (8r\underline{\lambda})^{-1} \tag{16}$$

As Bewley states, the second inequality follows if $r \geq \underline{r} > \frac{1}{20}$. If we follow Bewley's proof through then we can see the contribution of a high r is two-fold; in creating a large tax demand, and in providing a lower bound in the proof for px_{1t} .

2.2. Valuing Fiat Money with Idiosyncratic Production Risk.

I now exposit an original model which establishes two major propositions.

From the literature survey on the valuation of money in general equilibrium models we have identified a gap exists in the current body of research in this area and also in the examination of the optimum quantity of money under conditions of idiosyncratic risky production. This second half of the chapter fills a gap in the general equilibrium monetary models literature by showing that money can be valued by contemporaneously and infinitely lived agents as insurance against idiosyncratic production uncertainty, hence putting Tobin's (1958) liquidity preference demand for money into a general equilibrium setting. The importance of money as a safe portfolio asset is well known in 'Keynesian' circles, and was expounded upon at the end of the literature survey. Common usage of money among agents is important here in yielding a constant price level with idiosyncratic shocks and the safety of money. In the conclusion to the thesis I make conjectures about the possibility of constructing a model where money remains a safe asset in the face of aggregate fluctuations.

For comparison we may view the paper as endogenising output in the class of pure exchange models of Bewley(1980, 1983) and Levine (1989) that we have discussed in this chapter already (hence making the field more general equilibrium based) and contrasting with representative agent money and capital models such as DenHaan (1990) and Danthine, Donaldson and Smith (1987), where there is aggregate uncertainty but putting real balances in the utility function is the source of money's value. We might also wish to contrast the result here with the proof of Kitigawa (1994) for the overlapping generations model with risky capital. It is interesting to note that Kitigawa's model has idiosyncratic production uncertainty, but all the mathematics of his model would be unchanged with aggregate uncertainty, since the realisation of the output process only affects ex-post utility, not the actions of the old.

In the absence of inside assets I show that a non interventionist (zero interest rate) monetary equilibrium can exist if agents are sufficiently patient. Though similar to the conclusions derived by Bewley and Levine, the role of the rate of time preference is more complicated in my model. The rate of time preference, for the simple log case examined, is the sole determinant of the rate of saving. A finite rate of time preference will suffice to give positive savings of some sort, which will include some money if the return on money is high enough, given the variance of the production process. However since capital accumulation and hence output and inflation are endogenous in my economy, the rate of time preference has an extra role to play by endogenously determining inflation, given a constant nominal money stock. I also show that no non - interventionist monetary equilibrium can be Pareto optimal. This lends some support to the claim of Hahn (1973) that the conditions necessary for the existence of valued fiat money will be such as to make the monetary equilibrium inefficient. The next chapter tries to counter this claim however.

In essence money is valued in the model as it is passed about amongst existing agents, travelling between those experiencing good shocks and bad shocks, much as in the Townsend and Bewley models. Again the lack of aggregate uncertainty is crucial.

An interesting result comes about when we look at the issue of the optimum quantity of money as addressed by Bewley. I show that if we attempt to pay a positive rate of interest on money, no equilibrium with circulating fiat money will exist under the necessary lump sum taxation regime. This mimics a problem discovered by Bewley (1983). However, the cause of the problem uncovered in the model below is different to that of Bewley's, since the holding of money for payment of taxes is here a matter of choice rather than necessity to meet his solvency constraint with probability one. No strictly solvent consumer is ever implicitly forced to hold money balances equal to the per capita money supply to pay his tax bill, yet the agents in this model still choose to do so. The difficulty is shown to be endemic to the type of money demand that arises in the model. I prove that the result is essentially different to Bewley's by the following argument. One would conjecture that if an alternative store of value existed such that agents had no need to hold money equal to the per capita money supply to ensure solvency with probability one then equilibria with a positive rate of interest on money could exist. I show that indeed no such Bewley style tax demand is present and yet if we concentrate attention upon equilibria in which all agents hold strictly positive precautionary balances at all times then no such monetary equilibrium can exist with a strictly positive rate of interest on money. I then make some conjectures about cases in which the problem I have highlighted may be mitigated to allow monetary equilibria with circulating fiat money and a positive rate of return on money to exist.

2.2.1. The Model

The economy is populated by a continuum of infinitely lived consumer/producer agents, indexed;

$$i \in [0, 1]$$

who engage in the production of the economy's single homogenous consumption/production good. Each agent has an identical linear stochastic production function to which capital is the only input. Production takes place according to the geometric Brownian motion process;

$$y_{it} = k_{it} r dt + k_{it} \sigma dz_{it} \tag{1}$$

Where y_{it} is the flow of output per unit time at time t, k_{it} is the capital stock held by individual i at time t, r is the average marginal product of capital per unit time which is constant across individuals and time. I assume;

$$-\infty < r < \infty \tag{1a}$$

σ^2 is the variance of the marginal product of capital per unit time. I assume;

$$0 < \sigma < \infty \tag{1b}$$

dz_{it} is a Wiener process. These shocks to productivity are i.i.d. across agents. We can invoke a strong law of large numbers argument to yield a deterministic price level.

Agents derive utility from consumption as follows;

$$V_{it} = E_{it} \int_0^{\infty} e^{-\rho s} \ln(c_{it+s}) ds \tag{2}$$

Where c_{it} is the rate of consumption per unit time at time t by agent i,and ρ , the rate of time preference, is the same for all agents. The unique ex-ante first best Pareto optimal allocation that assigns equal welfare weights to all agents is one of zero variance in the consumption stream of each individual, which depletes the capital stock at the instantaneous proportionate rate of ρ per unit time period. This result can simply be derived by maximising (2) subject to (1) with σ set equal to zero (see chapter 3 for a full derivation). The setting of σ equal to zero arises from the lack of per capita aggregate uncertainty facing an omnipotent social planner. See Judd (1985) and Feldman and Gilles (1985) for discussion of the technicalities of this.

In the absence of any financial assets, inside or outside, the barter equilibrium will be autarchic. Each individual will consume from his own capital stock and the continuum of individuals allows no opportunity for reducing the variance of consumption streams below their physical fundamental level.

Each agent will maximise (2) subject to (1). The proof of the validity of the consumption policy below can be inferred from the analysis that follows for the monetary economy, but standard results yield;

$$c_{it} = \rho k_{it} \tag{2a}$$

$$dk_{it} = k_{it}\left(\left(r-\rho\right)dt + \sigma dz_{it}\right) \tag{2b}$$

Averaging (2*b*) across all agents yields;

$$d\overline{k}_t = \overline{k}_t\left(r-\rho\right)dt \tag{2c}$$

To consider the question of existence of monetary equilibrium each agent is dowed with an equal amount, $\overline{M}$ of fiat money at time zero.(A bar above a ter will denote a per-capita average). I assume the existence of a spot market each date t$\in$ [0,∞) on which money can be traded against the real good whose ney price at time t is p_t. Agent i's real wealth at time t is;

$$w_{it} = m_{it} + k_{it} \tag{2d}$$

denotes $\frac{M_{it}}{p_t}$, real money holdings and k_{it} capital holdings at time t, w_{it} evolves er time according to the stochastic differential equation;

$$dw_{it} = k_{it}(rdt + \sigma dz_{it}) + m_{it}(x - \pi)dt - c_{it}dt - \frac{T}{p_t}dt \tag{3}$$

x is the rate of interest on money per unit time, financed by a lump sum tax the rate of T units of money per unit time interval. This rate is constant across ne and agents. π is $\frac{\frac{dp_t}{dt}}{p_t}$ the rate of inflation at time t. Attention is restricted to nstant π, without prejudicing the analysis.

There are the additional constraint on agents of;

$$M_{it} \geq 0 \tag{3a}$$

$$w_{it} \geq 0 \tag{3b}$$

Maximising(2) subject to(3), ($3a$) and (3b) is a standard problem of the form studied by Merton (1971). The optimal policies are;

$$c_{it} = \rho \left[w_{it} - \frac{T}{p_t x} \right] \tag{4}$$

$$m_{it} = (1 - \alpha)[w_{it} - \frac{T}{p_t x}] + \frac{T}{p_t x} \tag{5}$$

Where α is the proportion of the net present value of wealth held in capital and is given by;

$$\alpha = \frac{r + \pi - x}{\sigma^2} \tag{6}$$

I will now show the derivation of Merton's solutions since the detail does not appear in the literature. Although solving the problem with the constraints 3a and 3b explicitly considered I deliberately omit them and then subsequently show this procedure is valid, for purposes which will become apparent later. I shall set the problem up in a general form to begin with and then specialise to the logarithmic utility function.

We begin by defining V* the maximum value function (dropping the i subscript without loss of clarity) as;

$$V^*\left(w_{t_0}, t_0\right) = \max_{c_s, \alpha_s} E\left(t_0\right)\left[\int_{t_0}^{t} e^{-\rho s} u\left(c_s\right) ds + V^*\left(w_t, t\right)\right] \tag{6a}$$

Where c_s and α_s are the rate of consumption and portfolio share of capital at time s.

Application of the Dynkin differential operator yields the Bellman-Dreyfus fundamental equation of optimality;

$$0 = \max_{c_t, \alpha_t}\left(e^{-\rho t} u\left(c_t\right) + \frac{\partial V_t^*}{\partial t} + \frac{\partial V_t^*}{\partial w_t}\left(\left[\alpha_t\left(r + \pi - x\right) - \pi + x\right] w_t - T - c_t\right) + \frac{1}{2}\frac{\partial^2 V_t^*}{\partial w_t^2}\sigma^2 \alpha_t^2 w_t^2\right) \tag{6b}$$

Since the consumers have an infinite time horizon, and the underlying stochastic process is stationary we can remove unnecessary time subscripts and also anticipate stationary policies (the validity will be seen later). We define;

$$J(w_t) = e^{\rho t} V^*\left(w_t\right) \tag{6c}$$

If we evaluate $\frac{\partial V_t^*}{\partial t}$ according to this new definition, and then multiply through by

$e^{\rho t}$ we have (now letting dashes denote partial derivatives);

$$0 = \max_{c_t,\alpha} \left(u\left(c_t\right) - \rho J\left(w_t\right) + J'\left(w_t\right)\left[\left(\alpha\left[r + \pi - x\right] - \pi + x\right) w_t - c_t - T\right] + \frac{1}{2} J''\left(w_t\right) \sigma^2 \alpha^2 w_t^2 \right) \tag{6d}$$

We now solve for the optimum policies in 2 stages, in a manner that will be seen not to prejudice the final outcome. A portion of wealth is set aside into money, equal to the present value of the tax bill, discounted by the rate of return on money. The rest of the wealth is divided between money and capital, and then consumption proceeds on the basis of the net wealth stock with the tax 'already taken care of'. Define;

$$w_t' = w_t - \frac{T}{p_t x} \tag{6e}$$

The optimisation problem then becomes;

$$0 = \max_{c_t,\alpha} \left(u\left(c_t\right) - \rho J\left(w_t'\right) + J'\left(w_t'\right)\left[\left(\alpha\left(r + \pi - x\right) - \pi + x\right) w_t' - c_t\right] + \frac{1}{2} J''\left(w_t'\right) \sigma^2 \alpha^2 \left(w_t'\right)^2 \right) \tag{6f}$$

First order conditions in c_t and α are then;

$$0 = u'\left(c_t\right) - J'\left(w_t'\right) \tag{6g}$$

$$0 = (r + \pi - x) w_t' J'(w_t') + J''(w_t') \alpha (w_t')^2 \sigma^2 \tag{6h}$$

Adding the transversality condition then completes the sufficient conditions for optimality;

$$\lim_{t \to \infty} E\left(e^{-\rho t} J(w_t')\right) = 0 \tag{6i}$$

This general case above can then be specialised to our case of log utility and solved for optimal consumption and portfolio policies if we make a guess for J(w_t) of;

$$A + \frac{\ln(w_t')}{B} \tag{6j}$$

Substituting into the first order conditions above, we can obtain solutions for c_t in terms of w_t and B, and α in terms of r,π, x and σ^2 (independent of B and A). Substituting these into the Bellman-Dreyfus Equation yields values for A and B which confirm that the value function is correct, and gives us the final form of the policies.

We should note at this point that under the optimum policies, the constraint $w_t \geq 0$ does not bind. The constraint $M_t \geq 0$ will be shown not to bite under the conditions we derive below, for the type of equilibrium we define.

How do we know that the non negativity constraint on wealth will not bite?

Shimko(1993, p10) and Merton(1990, p.546) show that a Geometric Brownian Motion process θ, say, hits the zero barrier with probability zero. By looking at the optimal policies derived above we can see that since the consumer sets aside $\frac{T}{p_t x}$ of his wealth and then consumes at a rate given by a constant (defined in (4)) proportion of the remainder (w′), then if we look at (3) we can see that his net wealth w' will follow a geometric Brownian motion process. Hence w′ is strictly greater than zero with probability one forever. An important part of this which we will use later is that no upper bound on α has been used in this argument; it may be above unity if desired.

As in Bewley's model the government uses the proceeds of taxation to pay interest on money, in such a way to leave the total money stock unchanged;

$$T = x\bar{M} \tag{7}$$

2.2.2. Monetary Equilibrium.

A monetary equilibrium is a set of goods prices; p_t, t$\in [0, \infty)$ such that;

(A) The price of money is strictly positive at all points in time;

$$\frac{1}{p_t} > 0, \forall t \in [0, \infty)$$

(B) agents maximise(2) subject to (3),(3a) and (3b), given the set of spot market prices defined above

(C) (Social feasibility) Each of the spot markets clears such that;

$$\int_0^\infty \frac{M_{it}}{p_t}\phi_{it} = \frac{\bar{M}}{p_t}, \quad \forall t \in [0, \infty) \tag{8}$$

Where ϕ_{it} is the density function for agent i's money holdings at time t conditional on time zero information (since the strong law of large numbers allows us to say that with probability one the mean of the distribution of real money balances across individuals will be equal to the mean of the individual distribution conditional on time zero information). A non-interventionist monetary equilibrium is one that satisfies the above conditions plus the additional requirement that the rate of interest on money is zero.

Proposition (1). A non - interventionist monetary equilibrium exists if;

$$\sqrt{\rho} < \sigma \tag{8a}$$

Proof.

For the typical agent i I propose a path of money holdings, capital holdings and consumption and show it satisfies the conditions for a monetary equilibrium defined above;

(a)Individual capital holdings are described by the stochastic differential equation;

$$dk_{it} = k_{it}(r - \sigma\sqrt{\rho})dt + k_{it}\sqrt{\rho}\,dz_{it} \tag{9}$$

The aggregate capital stock is described almost surely by the differential equation

$$d\bar{k}_t = \bar{k}_t(r - \sigma\sqrt{\rho})dt \tag{10}$$

(note the absence of aggregate uncertainty).

(b) Consumption as a function of the individual's capital holdings is given by;

$$c_{it} = \sigma\sqrt{\rho}\,k_{it} \tag{11}$$

(c) Money holdings as a function of the individual's capital holdings are given by;

$$m_{it} = (\frac{\sigma}{\sqrt{\rho}} - 1)k_{it} \tag{12}$$

(d) I propose a path of prices given by;

$$\pi = \sigma\sqrt{\rho} - r, \;\; p_0 = \frac{\overline{M}}{\overline{k}_0(\frac{\sigma}{\sqrt{\rho}} - 1)} \tag{13}$$

It is simple to show that (a)-(d) describes an equilibrium as defined, and the proof is as follows;

With T and x set equal to zero, (4) gives:

$$c_{it} = \rho \left[k_{it} + m_{it}\right] \tag{13a}$$

(5) gives:

$$m_{it} = \left(1 - \frac{r + \pi}{\sigma^2}\right)(k_{it} + m_{it}) \tag{13b}$$

From (13) substituting $\pi = \sigma\sqrt{\rho} - r$ into (12) and rearranging yields

$$k_{it} + m_{it} = \frac{\sigma k_{it}}{\sqrt{\rho}} \tag{13c}$$

substituting this into $(13a)$ yields

$$c_{it} = \sigma\sqrt{\rho}k_{it} \tag{13d}$$

hence (11) is confirmed. Rearranging (13c) also confirms (12).

Since we have;

$$k_{it} = \alpha w_{it} \tag{13e}$$

then we also have

$$dk_{it} = \alpha dw_{it} \tag{13f}$$

Substitution of (3) into (13f) and further simplification using (13d) and (11) confirms (9) as the equation describing the path of individual capital holdings. Given this, averaging over all agents yields (10) as the equation for the path of the per capita capital stock.

To confirm social feasibility we firstly show that given a level of the per capita capital stock $\bar{k}_t$ at time t, the money market clears, then that the proposed path of prices is compatible with money market equilibrium at all points in time given the path of the per capita capital stock determined in (10) by equilibrium consumption.

Step 1

Given $\bar{k}_t$ and the proposed inflation rate average money demand is (from 12);

$$\left(\frac{\sigma}{\sqrt{\rho}} - 1\right) \bar{k}_t \tag{13g}$$

We have proposed a price level formula of;

$$p_t = \frac{\bar{M}}{\bar{k}_t}\left(\frac{1}{\left(\frac{\sigma}{\sqrt{\rho}}-1\right)}\right) \tag{13h}$$

and hence per capita real money supply is;

$$\left(\frac{\sigma}{\sqrt{\rho}}-1\right)\bar{k}_t \tag{13i}$$

Hence given $\bar{k}_t$ money market equilibrium holds at time t.

Step 2

Examining (13*i*), since the per capita nominal money stock is constant, we require that if the price path in (13) is to be compatible with equilibrium, then it must be that;

$$\frac{\dot{\bar{k}}}{\bar{k}} = r - \sigma\sqrt{\rho} \tag{13j}$$

Turning to equation (10) we can see that this is the case for the proposed equilibrium.

Proposition 2: No non interventionist monetary equilibrium can be Pareto optimal.

Proof.

We have noted that a necessary condition for ex-ante Pareto optimality is that the individual consumption stream must have zero variance. From equations (9) and (3) we can see that such an allocation is possible either if k_t=0 for all agents which is clearly not an equilibrium given our parameter restrictions (a finite rate of time preference), or if $p_t = 0$, for all t. In this latter case, an agent's real wealth will be insensitive to realisations of his production process. This is just another way to say that the optimum quantity of money is infinite. As Brock (1975) and Bewley found, such an equilibrium will be infeasible, since the budget set will be unbounded. An obvious corollary is that non - interventionist monetary equilibria are Pareto inefficient.

■

Comparison of the barter equilibrium and the non-interventionist monetary equilibrium reveals that the path of individual and average consumption and capital stocks respectively differ across regimes. Comparison of (2*a*) and (11) shows that the rate of consumption is higher in the monetary economy than in the barter economy. Comparison of (2*b*) and (9) shows that as a result individual capital is eroded on average more quickly in the monetary economy and the same result holds for the aggregate capital stock as comparison of (2*c*) and (10) will show. The reason for the differing rates of consumption across the regimes is the role of money as a buffer stock in reducing the variance of the consumption stream (compare (2*b*) and (9)) and changing the precautionary incentive for saving.

2.2.3. A New Type of Equilibrium

An equilibrium with circulating fiat money is an equilibrium as defined in section 2.2.2 with the additional qualification that the parameter α, as defined in (6) is strictly less than one.

The economic interpretation of this is that agents hold money to guard against fluctuations caused by output shocks and not just to pay taxes. This money held for precautionary purposes changes hands between agents in equilibrium. The equilibrium described above in the proof of proposition (1) is of this kind. However, with a tax level of zero in that case, the 'circulating' caveat was unnecessary

Proposition (3.1)

If $\sigma < \sqrt{\rho}$ no non-interventionist monetary equilibrium exists.

Proof. This is a direct corollary of proposition one.

■

Proposition (3.2)

If $\sigma < \sqrt{\rho}$ a monetary equilibrium exists at a critical positive rate of interest but at no other interest rate. The allocation in the equilibrium is the same as the barter equilibrium. The monetary equilibrium is not a circulating monetary equilibrium.

Proof. We must locate a unique value of x (as a function of ρ and σ) so that the conditions for a monetary equilibrium, previously given in (A), (B) and (C) are satisfied. The method is to apply the conditions for equilibrium to find the value of x that will be consistent with equilibrium.

From (8), the social feasibility condition for money, (5), the equation for money demand and (7) the government budget constraint, we must have that;

$$(1-\alpha)\int k_{it}\phi_{it} + \frac{\overline{M}}{p_t} = \frac{\overline{M}}{p_t} \tag{13k}$$

Note that this holds since a corollary of proposition (3.1) is that a positive rate of interest on money must be paid for monetary equilibrium to exist.

Given ($13k$) a requirement is then that $\alpha = 1$. Adapting a per capita aggregate version of ($13b$) to the case of a positive rate of interest on money yields;

$$\frac{\overline{M}}{p_t} = \left(1 - \frac{r+\pi-x}{\sigma^2}\right)\overline{k}_t \tag{13l}$$

Totally differentiating with respect to time and dividing through by ($13l$)yields;

$$r + \pi = \rho \tag{13m}$$

The right hand side of ($13m$) is derived from a per capita average version of (4).

Substitution of ($13m$) into (6) yields an expression for the equilibrium α;

$$\alpha = \frac{\rho - x}{\sigma^2} \tag{13n}$$

The only value of x consistent with monetary equilibrium is then;

$$x = \rho - \sigma^2 \tag{13o}$$

That money does not circulate in this equilibrium can be seen from (5). Individuals money demand will be the same independent of their wealth level when $\alpha = 1$. To see that consequently the allocation and consumption is the same as in the autarchic equilibrium, we can note that essentially individuals consume only from their capital stock and hold money to pay taxes. That the rate of consumption expressed as a function of the individual's capital stock will be the same as in the autarchic equilibrium can be seen from (4) if ($2a$) and (7) are substituted in.

■

Proposition (3.3)

If $\sigma > \sqrt{\rho}$ no monetary equilibrium with x >0 exists .

Proof. It will be sufficient to note that from (5), the requirement of $\alpha <$ 1, along with the government budget constraint means that money demand will exceed supply and no such equilibrium will exist.

Substituting (7) into (5) and averaging (5) over all consumers, per capita money demand at time t is;

$$(1-\alpha)[\bar{k}_t + \frac{\bar{M}}{p_t} - \frac{\bar{M}}{p_t}] + \frac{\bar{M}}{p_t} \tag{14}$$

Since $\bar{k}_t > 0$ is an implicit requirement of a monetary equilibrium (recall that the price of money must be positive) we have that per capita money demand exceeds per capita money supply almost surely;

$$(1-\alpha)\bar{k} + \frac{\bar{M}}{p_t} > \frac{\bar{M}}{p_t} \tag{15}$$

■

Recall that Bewley (1983) showed that in a model of infinitely lived agents with endowment risk such that the probability of an arbitrarily long run of zero income is positive, no equilibrium with a strictly positive interest rate on money exists. If for instance the taxes are levied symmetrically across agents then each agent must hold per capita balances equal to the per capita money stock if he is to stay solvent with probability one. A desire to hold precautionary balances in addition to this means that money demand will exceed money supply.

The problem of non - existence sketched in our model above echoes this. However, the tax component of money demand, namely $\frac{T}{xP_t} = \frac{\bar{M}}{p_t}$ appears in the solution to the consumer problem without the need to invoke the constraint of solvency as we noted. This motivated the method of derivation that omitted the solvency constraint. The source of the non - existence problem in my model is endemic to its nature and how money is valued in the model. The positive component of each agent's wealth (initial productive capital and money holdings) is totally capitalised. In planning to meet the tax liability he faces, (and as long as $\alpha \leq 1$) a solvent agent optimally chooses to set aside a portion of his capitalised wealth in a safe asset equal to the present value of the tax liability. Referring back to equation (3) we see that such a policy is motivated by a desire to 'repair the damage' to the deterministic drift component of his wealth process that the tax causes. In the model above money is the only safe asset and hence this is where the burden of meeting the tax payment falls, as motivated by individual's preferences. In a Sidrauski type model (see e.g. Cohen (1985)) wealth is capitalised but money is dominated in rate of return with probability one, and hence such a tax demand does not occur. In a pure exchange model such as Bewley's in cases where a positive interest equilibrium does exist, namely, where minimum income is strictly positive the non capitalisation of wealth stops the problem highlighted in this paper occurring.

We can remind ourselves that the source of the result in this model is different to that of Bewley's if we consider a perfectly general one period utility maximisation problem, separated out from the consumption decision (as is valid in the model above, since utility is time separable). Assume a consumer has wealth W, a differentiable utility function U and faces a tax t after returns are realised, and can invest in either a safe asset with return r or a risky asset (no further restrictions). Under such general terms, it can be easily shown that the optimum policy can be solved in 2 stages. The consumer first sets aside;

$$\frac{t}{1+r} \tag{16}$$

in the safe asset and then solves the problem of allocating;

$$W' = W - \frac{t}{1+r} \tag{17}$$

between the safe and risky assets. This is the intuition that underlies the portfolio rule of Merton and that of Hakannson (1971) for the discrete time case, and hence for my result above. I have clearly restricted the scope of analysis so far in the sense of saying that solutions to the second stage of the problem with none of the safe asset are uninteresting. The motivation for that statement is a desire to look at equilibria where money performs a Pareto improving role. In the model I derived where all agents are identical we can envisage a case where the parameters are such that no non-interventionist monetary equilibrium exists. If a sufficiently small rate of interest is paid on money, then still no monetary equilibrium will exist. However, if the rate of interest on money reaches some critical point, where we obtain a parameter α=1 then a monetary equilibrium exists where money is held only to pay taxes. Beyond that value of α, of course no monetary equilibrium will exist.

We can further cement the notion that the problem derived in the model above is different to the Bewley problem by showing the Bewley problem is not in fact present in the model. If we return to the discussion of why individual's net wealth is strictly greater than zero at all points in time, we noted that there was no need to have $\alpha \leq 1$ to ensure solvency, so an agent can hold balances such that;

$$M_t < \overline{M}, \text{ for all t}$$

and yet stay solvent with probability one. The intuition is that if the agent suffers a run of bad luck then the negative shocks to wealth will become progressively smaller, which can be seen from the very essence of the geometric stochastic process that governs wealth.

2.2.4. Discussion and Conclusions.

What would happen if we extended the model by adding some kind of heterogeneity to the set of agents? I think it is fair to say that we could potentially cover the major factors of interest by increasing the 'ex-post heterogeneity' of agents in the form of non constant relative risk aversion, so that agents with differing shock histories and hence wealth levels would have demands for money as a proportion of wealth that were now a function of wealth. From our analysis so far it should be fairly clear that we cannot have a monetary equilibrium with interest on money in which all agents hold a positive quantity of precautionary money balances at all times, whatever utility function generated the results. However the kind of economy we have just tried to imagine might be such as to allow a monetary equilibrium in which money circulates in the economy so that at any point in time some agents are holding a positive precautionary component and hence also a positive tax component whilst others allow their money balances to fall below the tax bill level. Such cases would be somewhat analogous to choices in our model where $\alpha > 1$ is the portfolio choice. Hence in the kind of model I have only conjectured, that of non constant relative risk aversion utility functions, we can hence conjecture that a monetary equilibrium with taxation and interest might then exist with money truly circulating, though clearly an upper bound would exist on the rate of interest, as money became more attractive. It would certainly be an interesting exercise to try to obtain such results for a capital economy, since they would re-enforce the point that capital can serve as an alternative value store to money and in some circumstances allow an equilibrium with interest on money.

Quite reasonably I suggested that the result I obtained for the non existence of a circulating fiat money equilibrium with taxation where all agents hold positive balances at all times would be quite general. I also showed as part of the existence proof in the model that without taxation, a monetary equilibrium would exist where all agents do hold positive balances at all times. I feel we can make some interesting comparisons with the Bewley model on these points. Firstly, if we look at the equation in Bewley's 1983 paper which is reproduced as equation (9) in this chapter we see that no monetary equilibrium can exist where the liquidity constraint never binds, if the rate of interest is strictly less than the rate of time preference; average marginal utility of money would increase without bound through time which is ruled out in Bewley's model. Liquidity constraints not binding is of course a necessary condition for holding strictly positive balances at all times. The difference between the results for existence of such equilibria between my work and Bewley's is the fact that all wealth is capitalised in my model,(i.e. rising marginal utility through time with a with low r is hence possible). However, when we come to the respective results on non existence under taxation, we can make a conjecture about imposing a result of my model onto the framework of a non-capitalised wealth model such as Bewley's. Imagine a situation where the minimum income in each period was strictly positive, at $\underline{y}$ say. Would it be true to say that a monetary equilibrium can then exist with interest and taxation as long as the Bewley problem does not come into play? I suggest that if we hypothesised a utility function of the form ln(c-x) where $0<x<\underline{y}$, then we can envisage an economy where non-interventionist monetary equilibria may exist (i.e. Bewley's corner solutions are permissible), but attempts to tax and pay interest don't lead to the Bewley difficulty if the tax is small enough but they do prevent existence if the period tax demand is $\geq \underline{x}$. Foley and Hellwig (1975) showed that if consumers have an infinite marginal utility of income at the minimum possible income then they will arrange their assets such that the liquidity constraint never binds. In this case that means holding money at least equal to the per capita money stock as soon as the tax demand hits $\underline{x}$. We can hence suggest a kind of "preference endogenised" Bewley problem. This is clearly similar to the kind of result I have suggested above in the model of this chapter, though we should note whilst infinite marginal utility at minimum wealth is sufficient to generate the result of my capital model it is in principle not necessary. All we require is that the agents choose a strictly positive proportion of wealth in money in their portfolios at all points in time.

Chapter 3

The Optimum Quantity of Money with Imperfect Information

3. The Optimum Quantity of Money with Imperfect Information.

In the work contained in this chapter I shed new light on the optimum quantity of money proposal viewed in an environment where information availability is considered. I show that a scheme of using the proceeds of taxation to retire nominal money balances and raise the value of money is less informationally demanding than a scheme of paying out the proceeds of taxation in the form of insurance and also less demanding than paying interest directly on money balances . I base this on the notion that verification of a transfer can only be carried out costlessly by the receiver of the transfer, and raising the value of money avoids such problems since fiat money has essential properties of common usage, anonymity and homogeneity; an informational advantage is hence demonstrated. I apply this to refute a conjecture about the optimum quantity of money made by Hellwig(1982) and Woodford (1990).

3.1. Introduction.

Hellwig (1982) and Woodford (1990) conjecture that in a pure exchange model of the type of Bewley (1980, 1983) if taxes could be correlated with endowment realisations so as to avoid the Bewley difficulty then money would be inessential in the sense of Hahn (1973). In other words no economy with idiosyncratic income risk could be made arbitrarily close to Pareto optimality if money is to be an essential part of the economy. Or expressed in a further different way, the authors claim that the information required to achieve the perfect correlation of the taxes with the endowment stream would be such as to allow the replacement of money by a complete set of markets (recall the issue of the inefficiency of the non monetary equilibrium and the existence of a monetary equilibrium) . Below I address this notion in the context of the model I developed in chapter 2. I hence test to see if such a notion is valid in a stochastic production economy rather than the pure exchange models of Bewley et al, but the analysis will be seen to be equally valid for an endowment model. I find that I am able to construct an environment where the information required to apply the deflationary policy in a style that avoids the problem I uncovered in chapter 2 is not sufficient to yield full insurance that would drive money's value to zero.

In the environment I describe information is 'manipulable' by the agents, but taxes are used which in effect lead to sufficient revelation of information (although some redistribution schemes will not). The taxes the government uses, though non lump sum in nature, do not cause any distortion when the effects of the high real balances generated is taken into account, hence avoiding problems pointed out by Phelps(1973) who noted that the gains from a Friedman style policy may have to be weighed against distortions created if non lump sum taxes are used to finance the deflation or interest payment on money.

I shall later conjecture about the operation of the optimum quantity of money proposal in a framework when the information about an individual's income realisation is public knowledge and not manipulable, so that taxes may be lump sum in nature and state dependent. I conjecture that if we separate out the issue of the observation of the exogenous state of nature from verification of whether a payment has been received then the result will also be a refutation of the Hellwig-Woodford conjecture. This also has implications for whether it is easier to implement the Friedman proposal by paying interest on money or by using the proceeds of taxation to deflate the price level over time.

I find some interesting differences between a scheme of taxation and a scheme of insurance when record keeping is costly, that favours fiat money and changes in its value as a way of paying out insurance, due to its essentially anonymous nature and common usage. This will show the benefits of raising the value of fiat money as a method of making insurance payouts from an angle not yet covered in the literature. The method of analysis used in the argument is informal but adequate to tell the required story. The original conjecture of Woodford and Hellwig related to the endowment stream models of Bewley et al, but I have taken the liberty of superimposing their conjecture onto my model of chapter 2. I feel this is justified especially since the essence of the results presented below is not connected to any difference between pure exchange and production models; the comments made about information and detectability can easily carry over to the endowment model and the issue of the attainability of an allocation arbitrarily close to a Pareto optimal one is essentially contained in the discussion of Bewley's 1980 paper, since that paper ignores the tax problem.

I will suggest an information structure such that the consumption purchases of agents are observable and hence the government can apply a consumption tax sufficient to raise the resources to achieve an allocation sufficiently close to Pareto optimality. I also investigate the possibility that an incentive compatible redistribution scheme could carry out such a job. I find there would be problems with such a scheme and hence we can stake a case for money's positive value consistent with an allocation arbitrarily close to Pareto optimality, since no incentive compatible redistribution scheme exists. As I have said the argument used to back up such a suggestion contains no formal analysis of the incentives involved simply because of the limited scope of the thesis, but the argument given is sufficient to rule out a scheme of perfect insurance that would eliminate money's value. What would be interesting for future research is an analysis of such an economy where the information structure allowed some mixture of insurance, inside money, outside money and non lump sum tax policies.

3.2. The Economy.

The basic mathematical structure of the model is the same as that exposited in chapter 2; namely a continuum of ex-ante identical consumer/ producers engaging in the production and consumption of the economy's single homogenous consumption/investment good, according to the geometric Brownian motion process of equation (1) of chapter 2. This time we will expand the set of possible economies by considering general isoelastic utility, not just the logarithmic form. This appears here for the sake of generality but does not add anything essential to the results. The essential changes I make concern the specification of an information structure and an analysis of its implications.

The structure of the economy that I assume can be described by the following story. If the economy is such that assets other than capital are available then any portfolio adjustment between assets that is desired can be carried out on the spot market open at that date. If agents wish to consume however, they must obtain the good in a consumable form which is different to its basic capital form. They can only obtain the good in a consumable form from the government who acts as a kind of giant producer/shopkeeper. The government takes in the good from the agents in its capital form and processes it into its consumable form using a 1:1 production function. In an economy with assets other than capital, where agents can purchase consumption with assets other than 'raw' capital, the feasibility of the production process is ensured by the ability of the government to trade on the spot market for money and capital.

The information structure I assume is that the government as shopkeeper is able to observe the consumption of agents but not establish their identity in any effective sense, so that their intrinsic need as determined by the state of nature and any transfers they have received in the past remain hidden; i.e. the true state of nature as it pertains to the individual remains hidden. In other words there is a lack of centralised record keeping in the form of a name together with detail of the amount of their transaction. Each time an agent visits the 'shop' he is effectively a stranger.

The key factor that I have assumed is costly effective observation and storage of information in common with Levine (1991). Levine's model is made up of two types of agents who experience random taste shocks such that in any one period one has a high consumption need and one a low consumption need. These taste shocks are private information and any redistribution scheme must cope with this fact. Levine shows that if the observation and storage of information in a simple pure exchange economy is prohibitively costly, then the use of money is motivated as a decentralised record keeping device (in the spirit of Ostroy (1973)). The same result occurs in my model. This can be interpreted as an (extremely) imperfect financial system. Taub(1994) examines the workings of money and credit as replacements for an absent insurance mechanism. He derives the equivalence of money and credit as record keeping devices and enforcers of budget constraints, which enable some insurance to take place in the sense that individuals who hold finite credit or cash balances will optimally allocate them to times of greatest need. He makes no distinction between centralised and decentralised record keeping, in contrast to Levine and myself and consequently derives equivalence of credit and currency mechanisms.

In deciding upon his rate of consumption the agent chooses two things, his rate of consumption purchases per trip to the shop(to obtain the consumable form of the good with his wealth) and secondly his rate of trips to the shops. In a discrete time model his choice variables would be the number of trips to the shops in any given period and his consumption purchases in any given trip. The key distinction between trips to the shops and what they do at each trip will soon become clear.

We shall now examine possible redistribution schemes and see if the information structure we have suggested will permit them. First of all imagine that the government tries to implement an insurance scheme where it looks at the instantaneous consumption rate of each agent as it comes to the shop(which is all it can detect) and adjusts it via a transfer so that it is at the per capita average consumption level (which the government hopes will also be at an optimal level). Since a consumption rate at the government's naive target level is guaranteed for each agent, is it the case that the agents have no incentive to cheat under such a scheme? The answer is no. Under this scheme simple underconsumption now at any given rate of trips to the shops in a given time period (to gain a positive transfer above that which the agent's production outcome fairly merits) means that any benefits now will simply be dissipated later since consumption of $\bar{c}_t$ the per capita average at that time point is always guaranteed, i.e. the excess will be taken away from him when he attempts to spend his gains. However the agent can gain unfair insurance payouts by increasing his rate of trips to the shops and making a low (pre-adjustment) consumption claim each time. In fact the optimal policy will be a rate of visits as fast as he can manage with zero unadjusted consumption each time, to maximise his insurance payouts. The key factor is that the identity of agents and their payouts cannot be effectively recorded by the government, each new shopping trip may as well be by a different person. This describes problems with a system that aimed for full insurance, and is clearly sufficient to eliminate the first best redistribution scheme that would drive money's value to zero. A second best system would have to address the difficulty of the frequent trips factor. Compensating individuals for a proportion of their loss (i.e. a low coinsurance rate) would clearly provide no solution. Consumers would simply collect a smaller payout each time but the same incentive as before; a high frequency of trips with zero consumption each time would still be the optimal policy. The government may well have to resort to a system where only consumers consuming above a certain threshold level are compensated. The second best nature of the arrangement is hence illustrated and so we have cleared the way for money to be valued in equilibrium, since some variance in the consumption stream would have to be retained. The analysis remains partly suggestive however since I proceed under the assumption that no insurance at all is provided, and the government instead relies upon a monetary system of insurance. A full analysis of partial insurance together with money and taxation would of course be interesting.

Since the optimum quantity of money schemes involve taxing and redistributing the proceeds, one might ask what would happen if the government tried such a scheme with this economy just containing the single physical good; i.e. a non monetary economy, in which all redistribution is done in terms of the physical good. The problem with such a scheme is in achieving a socially efficient distribution of the proceeds of the tax essential to the idea of insurance. The problem arises since identification of genuinely needy individuals is impossible. Those who turned up more often would get more shareouts and the payout would be driven to zero. In fact such a scheme is identical to the insurance model; attempting to take from the lucky and give to the unlucky, yet of course failing to do so in this situation.

The problem can basically said to be occur on the paying out side rather than the tax side. Consumers cannot escape the tax or the component of the insurance scheme that is equivalent to the tax, unless they suboptimally lower their consumption forever. Frequent trips to the shops with lower unadjusted consumption each time however means that consumers can claim more and more of the payout part of the insurance. What is needed is either identifying record keeping, which we assume to be too costly, or a way of distributing the tax revenue to the needy. Such a way is present in the presence of fiat money. When the government deflates the money supply with its tax revenues it raises the value of money in equilibrium by making its return larger and raising money demand. Since fiat money is essentially anonymous then the distribution of the gains will be towards those holding the money, whose distribution is determined by total consumption of each agent over time as opposed to the number of shopping trips. The essence of fiat money in such a decentralised record keeping role is exactly the same as in the work of Levine (1991), and Ostroy and Starr (1974). **However, the 'dynamic' process of taxing individuals and changing the money supply as a method of paying out the proceeds as insurance is an angle which is new to the literature, and is the main contribution of this chapter.** The contrast between the record keeping and paying out aspects can be emphasised if we look at a system of paying out the proceeds from taxation directly to the agents in the form of fiat money. They can spend this money only once as Levine (1991) points out but can claim extra payments from the government in this environment of imperfect information. We can hence also say that such a scenario favours the implementation of the Friedman proposal by reducing the rate of inflation rather than paying interest on the money.

Must the money be fiat in nature? We might imagine another institution such as a bank which can identify the individuals and debit an inside money account for them when they purchase and also the value of their balances would be raised by the negative seignorage. We can simply assume this is impossible however, in the spirit of keeping with the assumptions of my model and also with Levine's (1991) comments on record keeping, for instance.

Two essential points need to be summarised from this discussion then. The first concerns a difference between tax and insurance which is an essential one if record keeping is costly. The difference arises from the fact that receipt of a payment under these circumstances can only be verified by the receiver. Since the government is benevolent this does not cause a problem with taxation. (Of course given the way that this model is set up it is not clear that the government even has the ability to cheat in the collection of consumption tax revenues). But with state contingent payouts to agents, the self motivated agents can effectively claim to have never received payment; problems hence result. This is the essence of the contribution of this chapter although the way the model I propose functions leads to a partial merging with the notion of the concealment of the original state of nature.

The second major point concerns essential properties of fiat money; those of anonymity and common usage. Appreciation in its value can potentially benefit all agents, as a method of paying out insurance, as we shall see.

One final point we should note is that it is possible that Hellwig and Woodford implicitly assumed scenarios in which information was passively observed by the government and then used to apply state dependent lump sum taxation. The kind of scenario I have sketched here is with information potentially manipulable by agents, and the situation within the monetary economy reduces to consideration of whether the taxes used to 'track' the information were distortionary or not. However the kind of situation that Hellwig and Woodford seemed to imply deserves consideration. To make the refutation of the conjecture carry over to this case, we need to assume that the observation of an individual's income realisation is separate from the observation of whether they have actually received an insurance payout. Problems with the second stage would make the argument carry over. We might assume that the central authority has a finite quantity of resources with which to observe economic activity, and this will not stretch to the procurement of both pieces of information.

Having given an argument that is sufficient to ensure we will have a Pareto inefficient non-monetary equilibrium, we now move to examine the operation of a monetary economy where no usage of imperfect insurance is made.

3.3. Derivation of the Agent's Consumption and Portfolio Policies with a Consumption Tax.

The technology describing the agent's production opportunities is as in equation (1) of chapter 2. His objective function is now generalised to the general case of isoelastic utility;

$$\frac{c^{\gamma}}{\gamma} \text{ where } \gamma < 1 \tag{1}$$

Now that a consumption tax is to be applied to agents we must define a kind of 'net' consumption which will enter the utility function, rather than the gross consumption level which will enter the budget constraint. Net consumption, denoted by c* is;

$$c_{it}^{*} = (1 - t)\, c_{it} \tag{2}$$

where t is the constant rate of taxation per unit of gross consumption, which is denoted by c_{it}.

c_{it} and c_{it}^{*} are both measured in units of the good per unit time. (Hopefully the time subscript and the tax rate utilising the same letter will not cause confusion).

The objective function can be written as;

$$V_{i0} = E_{i0} \int_{0}^{\infty} e^{-\rho t} \frac{(c_{it}^{*})^{\gamma}}{\gamma} dt \tag{3}$$

Inflation is denoted by π (restricting attention to the steady state);

$$\pi = \frac{\frac{dp}{dt}}{p}, \text{ and is constant over time} \tag{4}$$

Real wealth at time t is given by;

$$w_{it} = k_{it} + m_{it} \tag{5}$$

α will denote the proportion of wealth held in capital.

The stochastic differential equation that describes the evolution of real wealth for the typical agent is;

$$dw_{it} = \alpha\left(rdt + \sigma dz_{it}\right)w_{it} - (1-\alpha)\pi w_{it}dt - c_{it}dt \tag{6}$$

We shall henceforth simplify the analysis by assuming a mean rate of return on capital equal to zero, (r =0).

In keeping with what I said earlier about the difference between paying out interest on money directly and allowing the price level to fall over time I will follow the later method for its 'informational efficiency', i.e. the proceeds of taxation will be used to retire nominal money from circulation over time.

It is simple to show that since the consumption tax takes away a constant fraction of gross consumption each period it is optimal for the agent to consume at a gross rate of consumption at the same rate as in the absence of a tax. The simplest way to see this is to substitute for net consumption in the objective function and factor out the tax component; it is simply a linear transformation. Hence note that (3) can be written as

$$V_{i0} = (1-t)^{\gamma} E_{i0} \int_0^{\infty} e^{-\rho t} \frac{(c_{it})^{\gamma}}{\gamma} dt \tag{6a}$$

The resultant gross consumption policy is;

$$c_{it} = w_{it} \left[\frac{\rho}{1-\gamma} - \gamma \left[\frac{\pi^2}{2\sigma^2 (1-\gamma)^2} - \frac{\pi}{1-\gamma} \right] \right] \tag{7}$$

and;

$$\alpha = \frac{\pi}{\sigma^2 (1-\gamma)} \tag{8}$$

See Merton (1969) p.253 eqn. 42 for proof.

3.4. Calculating Equilibrium.

Calculating equilibrium follows the same basic principles as in the proof of existence in the model of chapter 2. The first step is to state an equation for equilibrium in the money market at time t given the capital stock and the per capita nominal money supply. We then differentiate the equilibrium condition with respect to time to obtain an expression that will yield equilibrium at all points in time. We can then use expressions for tax revenue and optimal consumption to gain an expression for the endogenous variables of the system in terms of monetary policy.

Application of money market equilibrium at time t (using the fact that α is common to all agents) yields;

$$\frac{\overline{M_t}}{p_t} = \frac{(1-\alpha)}{\alpha}\overline{k_t} \tag{9}$$

Where $\overline{M_t}$ is the per capita money supply at time t and $\overline{k_t}$ is the per capita capital stock at time t

Differentiating this totally with respect to time, dividing each side by the relevant side of equation (9) and noting that inflation,π, is constant yields;

$$\frac{-\frac{dp_t}{dt}}{p_t}+\frac{\frac{d\overline{M}_t}{dt}}{\overline{M}_t}=\frac{\frac{d\bar{k}_t}{dt}}{\bar{k}_t} \tag{10}$$

Changing notation we hence have;

$$\mu-\pi=\frac{\frac{d\bar{k}_t}{dt}}{\bar{k}_t} \tag{11}$$

Where;

$$\mu=\frac{\frac{d\overline{M}_t}{dt}}{\overline{M}_t}$$

We must now derive an expression for the right hand side of this equation; the proportional rate of change of the per capita capital stock at time t. A little thought should reveal that $\frac{d\bar{k}_t}{dt}$ is equal to the difference between per capita output and per capita net consumption. We have of course simplified this by setting the mean return to capital, net of depreciation equal to zero.

Before we simplify the above expression I define some consumption fractions or propensities. The propensity for net consumption from total real wealth, (which is constant over time) is given by;

$$c_n \equiv \frac{c_{it}^*}{w_{it}} = \frac{c_{it}(1-t)}{w_{it}} \tag{12}$$

The propensity for net consumption from real capital is;

$$c_k \equiv \frac{c_{it}^*}{k_{it}} = \frac{c_n}{\alpha} \tag{13}$$

This is the propensity that we shall pay most attention to in the analysis since it relates the real physical consumption that takes place to the per capita capital stock, given that all agents have the same consumption propensities. The key notational point to remember is that c subscripted with w,n and k denote propensities for gross consumption from total real wealth, net consumption from real wealth and net consumption from capital, respectively. The expression c_{it} denotes total consumption per unit time interval by i without any further divisor. We can hence note that by averaging over all agents at time t; (consumption propensities are stationary)

$$\frac{d\overline{k}_t}{dt} = -c_k \overline{k}_t \tag{14}$$

The equation that relates the rate of inflation to the rate of money growth and the rate of consumption is then;

$$\mu - \pi = -c_k \tag{15}$$

Our task is now to use this equation and the expressions for individual consumption propensities to derive a reduced form expression for c_k in terms of the nominal rate of money growth. To do this we require one last piece of information. This is the government budget constraint that shows the rate of taxation required to bring about a given rate of contraction of the money supply. This relationship will depend upon the agent's consumption behavior, since we are considering the revenue from consumption taxation.

Since the government pursues a policy of using the proceeds of taxation to retire money balances we have that;

$$\frac{d\overline{M}_t}{dt} = -p_t(\text{real value of the tax revenue}) \tag{16}$$

$$= -p_t t(\text{gross per capita consumption per unit time}) \tag{17}$$

$$= -p_t t\overline{c}_t \tag{18}$$

Defining μ we have;

$$\mu = \frac{\frac{d\overline{M}}{dt}}{\overline{M}} \tag{19}$$

From (18) and (19) we have;

$$\mu = \frac{-p_t t \overline{c}_t}{\overline{M}_t} \tag{19a}$$

From a per capita counterpart of (8) we can substitute for per capita real balances as a function of per capita wealth to obtain;

$$\mu = \frac{-t\overline{c}_t}{(1-\alpha)\,\overline{w}_t} \tag{19b}$$

We define;

$$c_w = \frac{c_{it}}{w_{it}} = \frac{\overline{c}_t}{\overline{w}_t} \tag{19c}$$

(Recall that consumption propensities are the same for all agents)

Which gives us;

$$t = \frac{-(1-\alpha)\,\mu}{c_w} \tag{20}$$

The interpretation of this equation is simple. For a given rate of monetary contraction ($\mu < 0$) a high level of 1-α means a high demand for money and a low price level. A high real tax will therefore be needed to obtain the rate of change of the nominal money supply desired. The higher is c_w, the lower will be the tax on consumption needed to bring about a given μ. This is obviously because higher consumption means a higher tax base and a lower rate of tax is required.

Combining (20) with (15), (8), (7), (12) and (13) we have;

$$(c_k + \mu)^2 \left(1 + \frac{\gamma}{2(1-\gamma)}\right) - \gamma\sigma^2 (\mu + c_k) - \rho\sigma^2 - \mu\sigma^2 (1-\gamma) = 0 \tag{21}$$

The proof is as follows;

We have from (12) and (13) that;

$$c_k = \frac{(1-t)}{\alpha} c_w \tag{21a}$$

From (20) we can substitute for $(1-t)$ into this to obtain;

$$c_k = \frac{1}{\alpha} (c_w + (1-\alpha)\mu) \tag{21b}$$

Substituting for c_w from (7), and for α from (8) yields;

$$c_k = \frac{\sigma^2 (1-\gamma)}{\pi} \left(\left(1 - \frac{\pi}{\sigma^2 (1-\gamma)}\right)\mu + \frac{\rho}{1-\gamma} - \frac{\gamma\pi^2}{2\sigma^2 (1-\gamma)} + \frac{\gamma\pi}{1-\gamma}\right) \tag{21c}$$

Substitution from (15) for π and algebraic simplification will yield (21).

Jumping ahead to equation 44, we will see that if we have $\gamma \leq 0$ then we have a unique strictly positive solution for π if (22) below holds.

$$\mu + \frac{\rho}{1-\gamma} > 0 \tag{22}$$

If $\gamma > 0$ then condition (22) is sufficient for a unique strictly positive solution for π. However condition (22a) yields 2 possible solutions for π which are strictly positive.

$$\mu + \frac{\rho}{1-\gamma} < 0 \tag{22a}$$

Despite the multiplicity in that case we shall later note that the existence of one of the solutions is enough to yield the answer we seek.

3.5. Welfare and Optimal Monetary Policy under a Consumption Tax Regime

Our datum will be a Pareto optimal allocation in which each of the continuum of ex-ante identical agents is treated symmetrically. Hence such an allocation will correspond to the maximisation of the lifetime expected utility of one of these agents subject to the per- capita economy wide feasibility constraint, that yields the absence of aggregate uncertainty.

We now restate the value function and develop it in a way that will allow us to study the roles of intertemporal allocation and risk bearing in welfare more clearly than does a naive expression. It is possible to derive the value function by a guess and verify procedure as used in the derivation of the agent's optimal policies, but the method used below will allow us to see the source of the different components of the formula. Expected lifetime utility conditional on time zero information, remembering that all individuals are ex-ante identical; E(V_{i0}) is;

$$E_{i0} \int_0^\infty e^{-\rho t} \frac{(c^*)_{it}^\gamma}{\gamma} \text{ dt} \tag{23}$$

or by definition of c_w;

$$E_{i0}\left(Vi_0\right) = \frac{c_w^\gamma \left(1-t\right)^\gamma}{\gamma} \int_0^\infty e^{-\rho t} E_{i0}\left(w_{it}\right)^\gamma .dt \tag{24}$$

Equation (6) tells us that wealth follows a stochastic differential equation. For ease of manipulation we can relabel its components as;

$$\alpha r - (1-\alpha)\pi - c_w \equiv a \tag{25}$$

And;

$$\alpha\sigma \equiv b \tag{25a}$$

We then have that;

$$dw_{it} = w_{it}\left(adt + bdz_{it}\right) \tag{26}$$

From such an equation it is well known (e.g. Dixit (1992) p8) that $\ln(w_{it})$ will be normally distributed with mean;

$$\ln w_0 + \left(a - \frac{1}{2}b^2\right)t \tag{27}$$

And variance $b^2 t$, when conditioned on time zero information. If we let $y_{it} = \ln w_{it}$ then we have;

$$E_{i0}\left(w_{it}\right)^{\gamma} = E_{i0}\left(e^{\gamma y_{it}}\right) \tag{28}$$

Since y_t is normally distributed as described above, we can use the formula for the moment generating function of a normal variable to obtain;

$$E_0\left(e^{\gamma y_{it}}\right) = e^{\left(\ln w_0 + \left(a - \frac{1}{2}b^2\right)t\right)\gamma + \frac{1}{2}\left(b^2 t\right)\gamma^2} \tag{29}$$

$$= e^{\gamma \ln w_0 + at\gamma + \frac{1}{2}b^2 t\gamma(\gamma - 1)} \tag{30}$$

So that the value function becomes;

$$E\left(V_{i0}\right)=\frac{\left(c_{w}\right)^{\gamma}\left(1-t\right)^{\gamma}}{\gamma}\int_{0}^{\infty}e^{-\rho t}.e^{\gamma\ln w_{0}+at\gamma+\frac{1}{2}b^{2}t\gamma(\gamma-1)}.dt \tag{31}$$

$$=\frac{\left(c_{w}w_{0}\right)^{\gamma}\left(1-t\right)^{\gamma}}{\gamma}\int_{0}^{\infty}e^{-\left(\rho-a\gamma-\frac{1}{2}b^{2}\gamma(\gamma-1)\right)t}.dt \tag{32}$$

$$=\frac{\left(c_{w}w_{0}\right)^{\gamma}\left(1-t\right)^{\gamma}}{\gamma}.\frac{1}{\rho-\gamma\left(a+\frac{1}{2}b^{2}\left(\gamma-1\right)\right)} \tag{33}$$

Substituting back for a and b (noting that r=0) we have;

$$\frac{\left(c_{w}w_{0}\right)^{\gamma}\left(1-t\right)^{\gamma}}{\gamma}\frac{1}{\rho-\gamma\left(-\left(1-\alpha\right)\pi-c_{w}+\frac{1}{2}\alpha^{2}\sigma^{2}\left(\gamma-1\right)\right)} \tag{34}$$

This formulation uses c_w, the agent's propensity to consume from total real wealth, composed of an individual's capital and real money holdings. We can re-express this to better reflect the equilibrium of the model in terms of real capital alone. In the numerator we can use;

$$\left(1-t\right)c_{w}w_{0}\equiv c_{k}k_{0} \tag{35}$$

In the denominator, we can use an expression previously derived;

$$c_k = \frac{1}{\alpha}\left(c_w + \mu\left(1 - \alpha\right)\right) \tag{36}$$

to re-express (37) as (38);

$$-\left(1 - \alpha\right)\pi - c_w \tag{37}$$

$$\left(1 - \alpha\right)\left(\mu - \pi\right) - \alpha c_k \tag{38}$$

Substituting for $\mu - \pi$ from (15) gives us;

$$-\left(1 - \alpha\right)c_k - \alpha c_k = -c_k \tag{39}$$

This expresses $\mathrm{E}\left(\frac{dw}{w}\right)$ in terms of the path of the capital stock as it is eroded by consumption (remember that r = 0). The substitutions made account for the erosion of the value of money caused by the inflation that results from consumption depleting the capital stock. This reflects the notion again that money is not net wealth. Making the substitutions described then yields;

$$\frac{(k_0)^\gamma}{\gamma}\frac{(c_k)^\gamma}{\rho-\gamma\left(-c_k+\frac{1}{2}(\gamma-1)\alpha^2\sigma^2\right)} \tag{40}$$

If we carefully follow through the derivation of this equation we can intuitively see the role of the two endogenous parameters in the value function. The $\alpha^2\sigma^2$ term relates to the variance of the consumption stream which is increasing in the proportion of capital in the portfolio. The level of c_k determines the rate of decumulation of the capital stock. The term c_k in the numerator reflects the effect on instantaneous utility of the propensity to consume from the current stock of capital. The -c_k term in the denominator reflects how consumption depletes the capital stock over time. Hence it is obvious that a rise in c_k now, for a given level of the capital stock, will raise utility for the moment but will mean the economy has less capital to consume from in the future. The optimum level of c_k to be derived below reflects the trade off between these two considerations.

3.5.1. Social Planning Optimum.

We now carry out the maximisation of (40) subject to the constraints $c_k \geq 0$, and $0 \leq \alpha \leq 1$. We assume that the social planner can manipulate these two variables independently of each other. This is the scenario that would face a social planner if he could freely re-distribute the consumption/capital good and control the rate of consumption, restricted only by the initial capital stock and physical production process which is deterministic in the per capita aggregate. Notice that we carry out this operation ignoring the information constraints we talked of in the introduction that gave a justification for money's value. We are hence looking at an artificial first best situation for the social planner, simply to provide a benchmark for the monetary case. We shall see that the monetary authority's problem is different to the social planner's since α and c_k are interrelated as the previous analysis has shown.

In looking at (40), manipulation of α is not a portfolio problem for the social planner, but is merely a method of controlling the variance of the consumption stream. Differentiating (40) partially with respect to α yields;

$$\frac{\partial V_0}{\partial \alpha} = \frac{(k_0)^\gamma}{\gamma} \frac{(c_k)^\gamma}{\left(\rho - \gamma\left(-c_k + \frac{1}{2}(\gamma-1)\,\alpha^2\sigma^2\right)\right)^2}\gamma(\gamma-1)2\alpha\sigma \tag{41}$$

Not surprisingly this is negative for all values of $c_k > 0$. Risk aversion $(\gamma < 1)$ determines that the variance should be driven to zero.

Setting $\alpha = 0$, and differentiating V_0 partially with respect to c_k and setting the resultant expression equal to zero yields;

$$\frac{\partial V_0}{\partial c_k} = \frac{\partial}{\partial c_k}\left(\frac{1}{\gamma}\frac{(c_k k_0)^\gamma}{(\rho + \gamma c_k)}\right) = 0 \tag{42}$$

This yields;

$$c_k = \frac{\rho}{1-\gamma} \tag{43}$$

3.5.2. The Monetary Authority's Problem.

This is to maximise (40) subject to the restrictions on possible values of α and c_k imposed by (21) which relates c_k to μ, (15) which relates μ to π and c_k and (8) which relates α to π, (these of course come from the restrictions that monetary equilibrium impose). There is of course one further constraint that we must add which is that $\alpha > 0$, which ensures that the existence problem with a zero nominal interest rate is not encountered. This obviously means that no monetary equilibrium can be made Pareto optimal, but we can make allocations arbitrarily close to Pareto optimality as we shall see. The first question is; under a consumption tax regime can we make π and hence α arbitrarily close to zero ? Referring back to equation (40), we can substitute $c_k = \pi - \mu$ to get;

$$\pi = \frac{\gamma\sigma^2 \pm \sigma\sqrt{\gamma^2\sigma^2 + 2\left(\frac{2-\gamma}{1-\gamma}\right)\rho + 2\mu(2-\gamma)}}{\frac{(2-\gamma)}{1-\gamma}} \tag{44}$$

By inspection we can note the following; for $\gamma \leq 0$, if (22) holds,(45) holds by taking the positive solution of the above equation;

$$\lim_{\mu \to \frac{-\rho}{1-\gamma}} (\pi) = 0 \tag{45}$$

Hence it is clear that an allocation that displays arbitrarily small variance can be achieved for those parameter values. If $\gamma > 0$ and if (22a) holds then selecting the negative solution above yields (45). An interesting difference between the 2 sets of parameter values is that if $\gamma \leq 0$ then the limiting value of μ is approached from above. In the case of γ >0 the limiting value of μ is approached from below.

We must now address the question of efficient intertemporal allocation. We need to know what making π arbitrarily close to zero means for consumption. Referring back to (40) we know that;

$$c_k = \frac{\gamma\sigma^2 + \sigma\sqrt{\gamma^2\sigma^2 + 2\left(\frac{2-\gamma}{1-\gamma}\right)\rho + 2\mu(2-\gamma)}}{\frac{(2-\gamma)}{1-\gamma}} - \mu \tag{46}$$

We can see by inspection that if we take the appropriate roots for the γ values as defined above then;

$$\lim_{\mu \longrightarrow \frac{-\rho}{1-\gamma}} (c_k) = \frac{\rho}{1-\gamma} \tag{47}$$

We have confirmed that the desired allocation can hence be approached asymptotically, for all values of γ.

We now need to gain some intuition as to why we can obtain such a result. In particular we might ask how it is that real wealth becomes arbitrarily large but the c_k value approaches $\frac{\rho}{1-\gamma}$.

The reason for this is that the wealth effect of the tax required to bring about the deflation exactly cancels out the positive wealth effect of the large real money balances. This is of course just a restatement of the notion that money is not net wealth with infinitely lived agents. Money is passed around the system between agents in this model and hence we can amend the statement that money is not net wealth to be money is not net wealth 'on average'. Furthermore, the tax usable in this model given the information structure itself creates no distortion in the life-cycle allocation of resources; as the relation of the optimal net consumption policy in relation to the optimal gross consumption policy shows (equation (2)).

These notions can be clarified if we note the formulation of c_k as follows;

$$c_k = \frac{(1-t)\,c_w}{\alpha} \tag{48}$$

Formal application of L'Hopital's rule to find the limit of this expression is not necessary since we already have the answer in (47), but we can note that;

$$\lim_{\pi \to 0} (c_w) = \frac{\rho}{1-\gamma} \tag{49}$$

$$\lim_{\pi \to 0} \left(\frac{1}{\alpha}\right) = \infty \tag{50}$$

$$\lim_{\pi \to 0} (1-t) = 0 \tag{51}$$

As a final point in this section we can note some equations which further describe the allocation of real resources over time. The stochastic differential equation that describes the path of an individual's capital holdings in the monetary economy, noting that the mean return on capital is zero, is as follows;

$$dk_{it} = -c_k k_{it} dt + \alpha\sigma k_{it} dz_{it} \tag{52}$$

The stochastic integral (see Merton (1990)) that yields k_{iT} is then;

$$k_{iT} = k_0 - \int_0^T c_k k_{it}.dt + \int_0^T \alpha\sigma k_{it} dz_{it} \tag{53}$$

consumption at time t is then;

$$c_T = c_k \left[k_o - \int_0^T c_k k_{it}.dt + \int_0^T \alpha\sigma k_{it} dz_{it} \right] \tag{54}$$

The right hand integral reflects the cumulative effects of the shocks to the production process over time. Clearly if α is equal to zero, the effects will disappear.

3.6. Conclusions.

What are the contributions of this chapter ? We have shown that with the suggested information structure that taxes can be arranged in a fashion that enables the deflationary policy to achieve an allocation arbitrarily close to a Pareto optimal one, without being so as to yield an economy in which money is inessential. An obvious criticism might be that we have considered only one type of possible information set up. A response to this is to say that we have at least broken an implied 1:1 relation between the information necessary for a Friedman deflation and the information required for complete markets. What about the particular information assumption I have used to break the link ? To enable the use of non-distortionary taxes on consumption we must obviously have consumption purchases observable. There is surely nothing particularly objectionable about this, and the lack of ability to effectively record the identity of an agent and the details of his transaction are used in models such as Levine (1991), Townsend (1980) etc. to justify the usage of money as a decentralised record keeping device. In the model I exposited the key factor was the inescapability of taxation but possible abuse of insurance by private agents. The important point was that only the recipient could verify receipt of a transfer. Added to the asymmetry between the private interest of agents and the social welfare maximising goal of the government, this means an asymmetry between the operation of the two schemes; the government has no incentive to impose 'extra' income tax.

The essential property of money that solved the problem was seen to be its anonymity and common usage, as raising its value did not cause any problems of claiming payments for a second time. This shed new light on the way in which the optimum quantity of money proposal might work in such a 'primitive' situation. The role of money as an information efficient record keeping device is well known, but the advantages of raising its value through allowing the price level to fall over time as opposed to paying out insurance to those who appear to be the needy and paying out interest on money as highlighted in this chapter are new to the literature.

In the model just considered, we had a situation where information was manipulable, but the taxes used were non distortionary and so caused no problems despite their non-lump sum nature required by the manipulability of information. The inability of the authorities to verify an insurance payout was based essentially on the anonymity of agents; their economic activity in consumption terms could be observed but the frequency of their visits to the shops could not, so that effectively bad luck could be falsified. We also assumed that the government was benevolent and would not abuse its ability to cheat if it had one; alternatively, we could describe its opportunities merely by the publicly known rate of taxation.

Can we carry over such an idea to a scenario where information on the exogenous state of nature is not manipulable and is known fully to all agents. I shall sketch an argument that suggests we can do so.

I firstly add a dimension to the story that separates out the tax/monetary authority from the 'law enforcement' authority. I will assume that endowment realisations are public knowledge to all but whether a tax/transfer has taken place is private knowledge between an individual and the taxation/monetary authority. I may also make the initial state of nature private knowledge to them also, but this does not affect the argument. I assume that the receiver or intended receiver of a transfer can costlessly verify to the law enforcers whether a transfer has taken place. I assume that they may also lie however (i.e. say that they have not received a transfer), and if the body/agent who actually made the transfer wishes to verify this to the law enforcers then it must pay a verification cost. If we model the costs as once and for all and 'lump sum' in the sense of investment in some kind of costly verification technology say, then the system of taxing individuals and using the proceeds to raise the value of money means that only private agents need to invest in verification technology, not the government, showing the information cost advantage of the monetary economy. Alternatively we might tell a story where the legal authorities always demand verification of a transfer upon appeal by a supposed non-receiver, but the agents know that the government has no incentive to cheat and so does not bother with investment in verification technology. The key point being made however is the informational advantage of paying out the proceeds of taxation via the change in money's value over paying out insurance directly which would entail costly verification, and is a separate issue to that of the observability of the original state.

The analysis above is of course speculative and requires formal analysis before we could make any final definite judgement about the Hellwig/Woodford conjecture. We have however at least broken the certainty of a solid link between the necessary information for taxes to avoid the Bewley/chapter 2 difficulties and the information necessary for full insurance. This rested upon an application of a new insight into the optimum quantity of money proposal as an advantageous way of distributing insurance payments. As Ostroy(1973), Levine (1991) and Taub (1994) show the role of money as a record keeping device in enforcing budget constraints is not new to the literature. It leaves a record of past expenditures by its 'absence'. However, this can be separated from the channel of providing insurance by raising money's value instead of directly handing out money as insurance payouts. The latter method would allow individuals to spend the transfer and claim another transfer, unless costly registering is carried out. An advantage of allowing the price level to fall over time instead of paying out interest on money is also indicated.

Chapter 4

An Economy with too much Money

4. An Economy with too much Money.

4.1. Introduction

In the literature on the optimum quantity of money, the unanimous view is of fiat money in a welfare improving role. Market incompleteness leaves potential gains to trade which are facilitated by the addition of money. The only qualifications to 'more money' come from potential problems with administering the tax system required to implement the policy as highlighted by Bewley(1983), Levine (1991), chapter two of the thesis and Phelps(1973). Otherwise money is a 'good thing' and following the essence of this means reducing the opportunity cost of holding money to as low a level as possible. It would also seem that maximising per capita real balances is a good thing to do. It would be an interesting question to pose if we asked "is it possible that there is an upper limit to the amount of real balances that we might wish to create?" A more taxing and hence even more interesting question would be "can we construct an equilibrium in which money has too much value?". I shall suggest that such an event is relatively easy to envisage in an overlapping generations model but harder to do in a model with contemporaneously and infinitely lived agents, which is the context of this thesis. Even under an "extreme" monetary policy, the way in which the Walrasian auctioneer prices money implies a limit to the amount of money that can be created, as I will subsequently demonstrate. Moreover it is hard to see too much money being created. My response to this challenge is to create "extra" inefficiency in the barter economy beyond that needed to create conditions for money's existence. Further, the type of model I present builds on an essential property of fiat money as a financial asset. Purely in terms of a method of insurance that redistributes wealth across states, it is useless. its payoff is the same in all states, and its role in providing self insurance analysed in the work of Bewley, Levine and my chapter 2 is as a buffer stock, to be traded against goods on the sequence of spot markets, rather than directly redeemed.

Firstly, to demonstrate the essence of my idea, I construct a model based on the model of Taub(1988) that incorporates linear utility and taste shocks. An inside insurance asset is incorporated into the model in a way that in some sense makes it carry 'too low' a rate of return. If we follow a second best version of Friedman's rule of the sort proposed by Taub, namely making the rate of return on money as close to the rate of time preference as the Bewley difficulty will allow, we investigate under what conditions such a rule is a bad one to follow. Due to the extreme structure of preferences in the model, namely linear utility, the choice agents make is to hold either insurance alone or money alone, not both. (For the sake of simplicity, we ignore the case of indifference based on equally attractive returns on the assets). I will show that if agents are sufficiently patient then the equilibrium with fiat money is Pareto dominated by the one without it. In this example, I am hence able to address a conjecture of Levine (1985), which I shall describe later. I then show a model with strictly concave utility where the warning against too much money is less extreme, but we still obtain the result that pushing the return on money too close to the rate of time preference in the sort of second best situation I described, is a sub optimal policy. I will use the intuition behind Friedman's original result where more money is a good thing to modify the proposition when assets intrinsically better suited to insurance are present.

As we have seen, amongst the literature examining the optimum quantity of money issue have been papers where money is the only asset and is used for self insurance in the face of income and taste fluctuations. Amongst these papers are Bewley (1980,1983) and Taub(1988). In such models they point out that a necessary condition for a monetary equilibrium to be Pareto efficient is that the gross rate of return on money, 1+r, is equal to $\frac{1}{\beta}$ where β is equal to $\frac{1}{1+\alpha}$, α being the pure rate of time preference common to all consumers. As we have seen elsewhere in the thesis, in the first of these papers Bewley pointed out that no monetary equilibrium would exist if 1+r is equal to $\frac{1}{\beta}$ if the endowments/preferences were sufficiently random, and hence no monetary equilibrium can be Pareto optimal. He conjectured that if 1+r was made arbitrarily close to $\frac{1}{\beta}$ then an equilibrium arbitrarily close to a Pareto optimal one would exist. In my analysis I shall refer to such an arrangement as first best for the sake of simplicity though of course such a term is,rigorously speaking, inaccurate.

Bewley's 1983 paper then pointed out that such an arrangement is infeasible under certain conditions (which hold in my model also), since the value of the taxes necessary to finance such a deflation would exceed the nominal value of resources available. A definite limit exists on the rate of return on money that can be achieved in equilibrium and the allocation will be clearly second best. Such a second best environment becomes the background of my model. Following Lucas and Taub I will consider an economy with a continuum of agents suffering idiosyncratic taste shocks. These agents are infinitely lived and can use either money or an insurance asset to help them tailor their consumption stream to their taste shocks as desired. The insurance asset is a security that pays off one unit of the economy's single non storable consumption good in the following period contingent on the realisation of a taste shock in the highest proportion p of possible shocks. I shall initially conduct the analysis for p exogenously given and then consider an information structure that may justify it.

The first task will be to examine portfolio choice and consumption behavior when agents are faced with a choice of these two assets. Due to the linearity of the state dependent instantaneous utility function, agents will exhibit 'bang-bang' behavior. Because of this our analysis of the optimum quantity of money in this model will be reduced to a discussion of whether welfare is higher in an equilibrium with money than without it. However the basic underlying question of in what circumstances more money is a good thing remains the central issue. The question of comparing two such equilibria was raised by Levine(1985). Levine presented a model in which money would be valued if the gains to trade using money are sufficiently great. On page 2 of his paper he goes on to say 'unless money is the only asset and the only alternative to a monetary equilibrium is autarchy, a subtle second best issue, which we do not explore is whether an inefficient monetary equilibrium will Pareto dominate an equilibrium in which money has no value.'The model I present in this paper, allows us to address this question. Of course, if we were to compare two equilibria achieved using exogenously imposed asset structures(by physically prohibiting the use of each of the assets in turn) the analysis would be relatively uninteresting. What makes the model presented here interesting is the endogenous asset selection. We shall see that if the monetary authority pays a return on the currency that is sufficiently high then individual agents will choose to hold money instead of the insurance asset. However if the agents are sufficiently patient and the set of realisations in which the insurance asset pays off sufficiently small, then the monetary equilibrium will yield a lower level of ex-ante per capita welfare than the equilibrium in which money is not valued. In such circumstances a monetary equilibrium would be third best as opposed to the second best insurance asset equilibrium.

The parameter restrictions discussed above on p are needed to ensure that insurance is a superior asset to money in the second best scenario of this model. The need for these restrictions is not surprising since we have made no other restrictions as yet on the nature of the insurance asset. One would not in general expect money to be always and everywhere inferior to any other single arbitrary asset.

The reason why we are able to obtain results converse to those of a stylised second best Friedman style deflation/interest rule (see e. g. Taub (1988) p. 581) can be seen by reflecting on the intuition of Friedman's original paper. On page 15 of his paper he explains that the welfare loss in his model is due to too low a rate of return on money presenting opportunities to agents that do not match the opportunities faced by society. The target for a welfare improving policy is then obviously to bring these two opportunity sets closer together. However in my model with the insurance alternative present the target of bringing individual and social opportunities closer together will be hampered if the financial instrument that the price taking utility maximising agents choose to hold is not the best for this task. The essence of the task in this economy is to make sure that the current aggregate endowment of the consumption good is consumed only by the most hungry agents. In terms of actual asset payoffs, the insurance asset I have described is clearly superior to money in this aspect since it redistributes income towards hungry agents. Purely in terms of asset payoffs money is clearly useless in this respect since it pays the same return in all states of the world(note the economy here lacks aggregate uncertainty). Money's usefulness comes in its use as a buffer stock. In equilibrium hungry agents can liquidate their money balances and buy the consumption good because of the willingness of the non-hungry agents to save in money for such future contingencies. Such saving behavior is influenced by β and r, the rate of interest on money. As we shall see these values will determine the maximum level of welfare in a monetary economy, which will always be below the first best level since we assume that agents have a strictly positive rate of time preference.

When agents choose the insurance asset, per capita utility will again be below the level yielded by the symmetric Pareto optimal allocation, where only the hungriest agents consume. As we shall see the welfare obtainable with this asset will depend upon β and p. The desirability of a low p in diverting resources more accurately towards only the most hungry agents is obvious. The role of β in the determination of welfare with this asset is basically the same as in the monetary regime. It so happens that a very low β reduces the efficacy of the insurance mechanism more severely than it does the monetary mechanism. Hence such values would confirm the second best Friedman rule. However we find that if β lies outside this region (i.e. if agents are sufficiently patient), then the "intended" second best counterpart of the Friedman rule will fail to be the actual second best policy.

4.2. Structure of the Economy.

Endowments and preferences are identical to a specialised version of Taub's (1988) model, where the distribution function of multiplicative shocks to individual preferences is uniform. The economy is populated by a continuum of infinitely lived agents with instantaneous linear utility of consumption, which is buffeted by stochastic multiplicative shocks which are i.i.d. across agents, hence there is no aggregate uncertainty. Agents discount instantaneous utility with a pure rate of time preference which is strictly positive. If we define;

$$\beta = \frac{1}{1+\alpha}$$

then the typical agent solves the problem in period t;

$$\max E_t \sum_{s=0}^{\infty} \beta^s \theta_{t+s} c_{t+s} \tag{1}$$

where θ_{t+s} is the random multiplicative utility shock in period t+s. The random variable θ_{t+s} has a uniform distribution on $[0,\overline{\theta}\ ]$. Note that this is independent of t and s. The budget constraint faced by individuals will depend upon the financial asset they choose to hold, and will be examined in the next section. It is also assumed that each agent will have an endowment stream of the economy's single consumption good equal to y each period. Note that unlike the Taub model there is no cash in advance constraint here.

4.3. Agent's Consumption Policy if Money is the Chosen Asset.

I shall begin by examining the consumption policy of the typical agent taking the choice of asset as given and then go on to examine the asset choice problem. I commence by assuming that the agent has chosen to hold money.

I assume the economy is in a steady state, so that the price of the consumption good is stationary through time. I assume that the monetary authority conducts its monetary policy by paying interest on money balances. The interest payments are financed totally by taxing all agents in a symmetrical lump sum fashion and using the proceeds to retire money from circulation. (This means that the nominal money stock in the economy will be constant also). These taxes will be fixed across time also. If we denote the gross rate of return on money, 1+r, as ρ, and y-t, the real post tax endowment as $\overline{y}$ (which is stationary given the policy variable ρ), then the individual maximises (1) subject to;

$$c_t + m_{t+1} \leq \overline{y} + \rho m_t \tag{2}$$

Where m_{t+1} denotes real money balances held at the end of period t.

It is intuitively obvious that since agents have linear utility they will choose to either consume their current wealth which is the right hand side of (1) or save it all in money balances (for the sake of simplicity I will assume that when agents are indifferent they will choose to consume). Determination of the optimum consumption policy then reduces to finding the critical level of the consumption shock at which they are indifferent. I will call this value θ^*, which will be stationary in the steady state of the model. This will occur when the marginal utility of present consumption equals the discounted expected marginal utility of saving(conditional on the optimum value θ^*)

This value is then determined by equation (3);

$\theta^*= \beta\rho$ [(prob. consume) x (expected m.u. if do consume)]

+ $\beta^2\rho^2$[(prob. did not consume last period) x (prob cons.this period) x (expected. m.u. if do consume)]+.............

Given a level of θ^*, we can make some substitutions into the above equation (3) using the following;

$$prob.consume = x^* = 1 - \frac{\theta^*}{\overline{\theta}} \tag{3a}$$

(remember that θ is uniformly distributed)

The value x* is simply the probability that $\theta > \theta^*$

Expected m.u. if do consume in a period is given by;

$$\frac{\theta^* + \overline{\theta}}{2} \tag{3b}$$

(i.e. conditional on consumption taking place).

Substituting ($3b$) into (3) above, and using the formula for the infinite sum of a geometric progression, we have an equation for θ^*;

$$\theta^* = (\beta \rho x^*) \left(\frac{\theta^* + \overline{\theta}}{2} \right) \left(\frac{1}{1 - \beta\rho(1 - x^*)} \right) \tag{4}$$

We shall see that $\beta\rho$ <1 holds and so convergence will occur.

If we then substitute for x* from ($3a$) into (4) and use implicit differentiation we can obtain the result that;

$$\frac{d\theta^*}{d\rho} > 0$$

i.e. that a higher return money encourages saving. Also we have that;

$$\lim_{\rho \to \frac{1}{\beta}} \theta^* = \overline{\theta}$$

4.4. Consumption Policy if Insurance is the Chosen Asset.

We will assume that the probability of an agent receiving an insurance payout is less than the probability that he will consume (in equilibrium), which greatly aids simplicity. When we calculate the equilibrium we will derive parameter restrictions such that this assumption will be valid. We shall see that such an assumption is necessary if the second best form of the Friedman style rule is to be 'locally' inferior to the insurance asset. However, such conditions will be encompassed by the conditions necessary to achieve a global inferiority of money as we shall see later.

We assume a form of financial asset that yields a payout only to the top proportion p of most hungry individuals(p is given exogenously). The asset can be bought at date t and one unit yields a return of R units of the consumption good at date t+1, if the event of extreme hunger (with probability p) occurs. I assume that the payout on the asset does not discriminate between different levels of hunger within that group p. We can hence note if p was set equal to 1 the return structure would be the same as fiat money. (Note that short sales are prohibited). If we define θ_p as the minimum hunger level at which the asset will pay out then the probability of receiving an insurance payout is;

$$p = 1 - \frac{\theta_p}{\overline{\theta}} \tag{4a}$$

Hence we can now calculate the level of θ at which the agent is indifferent between saving all his income and consuming it all. We denote this by θ_c and calculate it from the following F. O. C (5);

$\theta_c \quad = \quad \beta$ [prob. insurance pays out] x [Gross return on insurance given that it pays out]

x [expected m.u. given that payment occurs]

Remember that we assume the probability of consuming is at least as great as the probability of receiving the insurance payout, hence the sole term on the right hand side of the above equation, in contrast to the money equation (3).

If we denote the gross return to insurance conditional on payout as R, then (5) becomes;

$$\theta_c = \frac{\beta p R \left[\overline{\theta} + \theta_p\right]}{2} \tag{6}$$

Substituting for θ_p,we have that

$$\theta_c = \frac{\beta p R \bar{\theta}(2-p)}{2} \tag{6a}$$

By definition, the probability x_c of consuming when holding insurance is equal to;

$$x_c = 1 - \frac{\theta_c}{\bar{\theta}} \tag{6b}$$

4.5. Optimal Portfolio Behavior.

Thanks to the analysis of the above section our task in analysing the portfolio choice of agents is simple. Equations 4 and 5 above express the first order conditions for consumption behavior under the holding of the respective assets. However the right hand side of each equation is the discounted expected marginal utility of saving in each respective asset. Hence when the consumption policy is calculated optimally, as given by equations 4 and 6, the respective marginal utilities of saving in the assets are given by θ^* for money and θ_c for insurance. Hence if we have equilibrium values of R and ρ and exogenously given values for β and p, we can easily calculate which asset all identical agents will prefer. If θ^* is the greater then money will be chosen. If θ_c is the greater then insurance will be chosen. For simplicity we will assume that if the values are equal then money will be chosen.

4.6. Calculation of Monetary Equilibrium.

We firstly calculate the unique steady state equilibrium.

Since the underlying structure of the model is very similar to that of Taub (1988), our calculation of monetary equilibrium is all but identical to his, the only difference being that my monetary economy omits the cash in advance constraint and concentrates entirely on the precautionary motive for holding money, as in the models of Bewley(1980, 1983). Due to the similarity of this section to Taub's analysis, the derivation of results will be kept relatively brief.

The stationary state will be characterised by a stationary distribution of real money balances across individuals. its distribution function is denoted by ψ. In common with Taub I ignore the path to the steady state by assuming that the economy begins in this steady state with the individuals' positions in this distribution allocated randomly. The welfare criterion we shall use later is that of expected lifetime utility evaluated at time zero before the outcome of the random allocation is known. This allows us to consider improvements in the mean level of welfare which will also be Pareto improving.

We saw earlier that the agents' threshold level of consumption was increasing in ρ, the gross nominal (and real, in the steady state) rate of return on money. The question to be addressed is exactly the same as that addressed by Taub, which is to calculate the maximum feasible rate of interest that can be paid on money in a monetary equilibrium, which is the essence of the problem exposited by Bewley(1983). The problem in the economies of those two papers (and the one presented here)is that the infiniteness of the agents' horizon means that the optimum quantity of precautionary balances is infinite. As the monetary authority attempts to increase the level of money balances, the price level must necessarily fall and nominal income, other than the interest payments on money, falls to zero. This creates an autonomous demand for money, which agents hold purely to finance the tax. No equilibrium can exist since the demand for money for both tax and precautionary purposes will exceed the supply. As we have already documented at length and as Taub (1988) describes, the limit of the feasible return on money occurs when the agent's post tax non interest income becomes zero. If the auctioneer raises the return on money beyond the level that yields $\overline{y}$ =0, he will raise the precautionary demand for money by agents. To satisfy this, the price level must fall, but this raises the demand for autonomous tax-financing balances, hence no equilibrium can exist beyond that point. Note also that this problem occurs even when income is deterministic, as the results that follow show.

(Taub produces a Pareto optimal equilibrium when the support of taste shock is discrete. Hence the continuity of support here is crucial to us.)

The task in finding the maximum feasible return on money then reduces to finding the return that yields a level of $\overline{y}$ equal to zero. We now go forward to deduce this through calculating the monetary equilibrium.

In the stationary distribution of agents according to their real money balances the following past histories(including the current period) of agents are possible.

(i) The agent is hungry this period i.e. $\theta > \theta^*$. End of period balances are hence zero. The probability of this event is x*. (See equation 3a)

(ii) he was hungry last period but not this period. His end of period balances will then be $\overline{y}$. The probability of this event is x*(1-x*)

(iii) The agent was hungry two periods ago, but not in this period or the preceding period. His end of period money balances will then be $\overline{y}\rho + \overline{y}$ with probability x*(1-x*)^2

.... and so on.

The support of the stationary distribution of money balances ψ, is therefore;

$$\overline{y}\left\{\frac{1-\rho^k}{1-\rho}\right\}_{k=0}^{\infty} \tag{6c}$$

with a probability function;

$$d\psi\left[\overline{y}\left[\frac{1-\rho^k}{1-\rho}\right]\right] = (1-x^*)^k x^* \tag{6d}$$

Per capita real balances can then be calculated as;

$$\overline{m} = \sum_{k=0}^{\infty}\left[\frac{1-\rho^k}{1-\rho}\right]\overline{y}F^k(1-F) \tag{7}$$

where we have substituted F= 1-x*, and 1-F = x*. Equation 7 then can be evaluated as;

$$\overline{m} = \frac{F\overline{y}}{1-\rho F} \tag{8}$$

Note that (8) effectively gives us per capita end of period real money balances.

Our next task is to use this mean level of real money balances (given $\overline{y}$ and x*) to calculate the monetary equilibrium. Since only the hungry spend their cash balances (plus current period net income) we have that goods market equilibrium in per capita terms is given by

$$(1-F)\left[\frac{\rho F\overline{y}}{1-\rho F}+\overline{y}\right] = y \tag{8a}$$

$$\text{Hence } \overline{y} = \frac{(1-\rho F)\, y}{1-F} \tag{8b}$$

Note that $\overline{y} = 0$ when ρF = 1. Since 1-F is the probability of consuming, and is equal to x*, we can show that ρF=1 is satisfied when;

$$\rho = \rho_{\max} = \left[\frac{2-\beta}{\beta}\right]^{\frac{1}{2}} \tag{8c}$$

We obtain this from equation 4 by substituting out θ^* from equation 3a, yielding an equation in ρ and x*, substituting for x* with 1-F then setting ρF =1.

As long as agents are impatient so that β is strictly less than one, then $\rho_{\max}$ will be greater than one, indicating that a positive return on money is possible, but will be strictly less than $\frac{1}{\beta}$, the level that would yield per capita maximum utility and a Pareto optimal outcome. (i. e. the first best Friedman rule).

4.7. Calculation of Equilibrium if Insurance is the Chosen Asset.

The unique stationary equilibrium of the model under the use of the insurance asset is simple to calculate given the assumption of p< x_c where x_c is the probability that an agent consumes. This greatly simplifies the analysis since no agent will carry wealth in the form of the insurance asset over more than one period(note also how that simplified the calculation of θ_c above. (see equations (5) and (6))

Equilibrium in the goods market is then determined by equating per capita endowment y to per capita consumption. By definition a proportion x_c of the population will consume in any given period. This proportion can be divided into 3 groups, (where proportions are defined relative to the whole population of agents): a proportion (x_c -p) who are hungry enough to consume but not to receive an insurance payout, a proportion(1- x_c)p who are hungry enough to be eligible for an insurance payout and who did buy insurance in the previous period (i. e. they were in the saving group last period), and a group x_c p who are hungry enough to qualify for an insurance payout, but were also hungry last period and did not therefore buy insurance. If R is the gross return on the insurance asset then goods market equilibrium is given by;

$$y = (x_c - p)y + p(1 - x_c)(y + yR) + x_c py \tag{9a}$$

hence we find that x_c drops out of the equation to yield:

$$pR = 1 \tag{9b}$$

If we use equation (6b) to substitute out θ_c in (6a) and then substitute pR=1 into (6a) we find;

$$x_c = 1 - \frac{\beta\,(2-p)}{2} \tag{10}$$

Hence if our analysis with p< x_c is to be valid, we desire that:

$$1 - \frac{\beta(2-p)}{2} > p \tag{11}$$

Because of the intertemporal nature of the insurance asset we need a 'start up' condition to be specified. I assume that the economy begins in the stationary state, hence analogously to the assumption for the monetary economy. We endow a fraction (1-x_c) of the top proportion p of agents according to hunger with assets equal to yR in addition to their initial endowment y. In equilibrium this matches purchases of the insurance asset by those who wish to save for the second period.

We now ask under what circumstances will money be valued. From our calculation of agent's portfolio and consumption policies we know that money is valued when the threshold hunger level for consumption under the monetary asset is greater than or equal to the threshold hunger level for the insurance asset. Since the question here is whether there exists a rate of return on money that is sufficient to induce agents to hold it instead of insurance it will suffice to do this comparison using the level of $\rho_{\max}$. We hence seek to locate parameter values such that $\theta^*_{\max} > \theta_c$ holds. Using equations 4, 6 and 8c this desired inequality becomes;

$$\overline{\theta}\left[\frac{\beta}{2-\beta}\right]^{\frac{1}{2}} \succeq \frac{\overline{\theta}\beta\,(2-p)}{2} \tag{11}$$

Or equivalently that;

$$\frac{2}{2-p} \succeq \left[\beta\,(2-\beta)\right]^{\frac{1}{2}} \tag{12}$$

This inequality will be satisfied as long as p>0 or β <1. Hence we can say that in the model agents can always be induced to hold money by pushing its return to its highest feasible level. This is an important lemma for the welfare analysis of a later section. An interesting footnote to this section is that money will not be valued in an equilibrium if the monetary authority does not pay a strictly positive rate of interest on balances. This can be seen if we substitute ρ=1 into equation 4 and consider inequality (11) for this value of ρ. We find that as long as p < 1 holds, then government intervention is needed for money to be valued.

4.8. Welfare Evaluation and the Optimum Monetary Policy.

We have established that the monetary authority can create a return on money balances such that money will be valued in equilibrium. We now ask whether such a policy is optimal in this second best environment. I answer this question in two parts. Firstly I will show that under my maintained assumption that the probability of consuming is strictly greater than the probability of receiving an asset payout, a particular local change in monetary policy will yield an unambiguous fall in ex ante per capita welfare (as evaluated at date zero). This local change is at the point of the attainment of the rate of return on money such that it becomes valued instead of insurance. I then derive conditions such that the presence of money is a bad thing for all feasible rates of return. In performing the first part of this analysis, we know the switch in the holding of the assets occurs when the respective values of the critical θ levels are equal (remember that for simplicity we assumed that when indifferent the agents would choose money). If we begin by taking an arbitrary value for this and call it θ_e, then we can easily derive an expression for V_m, the expected per capita lifetime utility of the economy under the monetary regime. We can do this by noting that the distribution of money balances and hence purchasing power is uncorrelated with the level of the taste shock suffered by the individuals within the group who choose to consume. Since per capita consumption in equilibrium is equal to y, we have that the ex-ante per capita expected utility in a monetary economy is;

$$V_m = \frac{y\left(\overline{\theta} + \theta_e\right)}{2} \cdot \left(\frac{1}{1-\beta}\right) \tag{13}$$

To gain insight on how to calculate the per capita welfare yielded by the insurance asset we can recall our analysis of the three groups of consumers who consume in the insurance asset regime (refer back to our equation for the goods market equilibrium, ($9a$)) If we assign the appropriate expected marginal utilities (with expectations evaluated conditionally on the event of being in the particular group), we obtain (assuming e>p holds)

$$V_I = \frac{p}{1-\beta}\left((1-e)\,yR + y\right)\left(\frac{\overline{\theta}+\theta_p}{2}\right) + \frac{1}{1-\beta}\left((e-p)\,y\right)\left(\frac{\theta_p+\theta_e}{2}\right) \tag{14}$$

Where e is the probability of consuming, for our arbitrarily chosen θ_e value. V_m and V_I respectively can then be re-expressed as;

$$V_m = y\overline{\theta}\frac{(2-e)}{2\,(1-\beta)} \tag{15}$$

$$V_I = \frac{y\overline{\theta}}{2\,(1-\beta)}\left[\left((1-e)+p\right)(2-p) + (e-p)\,(2-p-e)\right] \tag{16}$$

Hence to obtain the partial result that the monetary equilibrium 'at the point of its introduction' brings a fall in welfare we desire to show that:

$$\left[(1-e)+p\right](2-p) + (e-p)\,(2-p-e) > (2-e) \tag{17}$$

which reduces to:

$$e > p \tag{18}$$

This is precisely the condition required for the validity of our insurance asset calculations. It should come as no surprise that this condition is also sufficient to yield the result that introducing money at its lowest rate of return compatible with the existence of monetary equilibrium will bring an unambiguous fall in the welfare measure we have used. Recall that the task in raising such a welfare measure is to divert consumption towards the most hungry agents. The efficacy of a particular asset in achieving this goal depends on two factors; its success in yielding a high willingness to save (by the less hungry agents) and how it redistributes income towards the more hungry agents. In the analysis performed above, we have effectively 'fixed' the first of these factors to be the same for both assets. In both asset regimes the top proportion e of agents in terms of hunger consume. However, regarding the second factor, if $p<e$ holds then insurance has an obvious advantage over money. Money yields the same return to an agent in all states (remember there is no aggregate uncertainty) and provides no redistribution within the group of agents who consume. Insurance on the other hand can perform this task and so in these circumstances has a distinct advantage. The 'victory'

of insurance in this limited case shows our argument that the essence behind Friedman's theory when carried over to a second best situation (as Taub p. 581 does with money as the only asset) may mean that money is a bad thing and not a good thing. His argument that individual and social opportunities need to be brought closer together can be used in this example to overturn the Taub style second best form of the Friedman rule.

The next task is to see how well the 'inversion' of Friedman's rule described above survives a more realistic but still second best situation, where we are less 'artificially' harsh to money than in the above case. We do this by looking at the welfare achievable by the policy of setting ρ equal to $\rho_{\max}$. The final welfare outcome might be viewed as the outcome of competition between the advantage that (e-p) gives to insurance in the above case and the welfare gain within the monetary regime that accrues from allowing ρ to rise to $\rho_{\max}$. The two key parameters are p and β. If we look at equation (8c) we can see that $\beta\rho_{\max}$, the discounted gross real return on money is increasing in β, so lowering the value of β by considering more impatient agents will retard the workings of money. However, a low β also damages the workings of the insurance asset, so the trade off is complicated. The reason for the influence of β is of course that both money and insurance have to be held intertemporally and a low value of βwill mean a distortion of 'individual' opportunities away from those faced by society. This is the argument for the usual Friedman style rule (as in Taub 1988) in trying to influence consumption/saving behavior by correcting this failure.

From this situation we can derive a result to show that our basic intuition on the merits of two assets is basically correct. We compare the utility obtained in respective asset regimes when we let p, the probability of receiving an insurance payout go to zero (this of course is an artificial limiting value). In this case the condition for insurance to yield a per capita utility level higher than money is that;

$$\lim_{p \to 0} (V_I) > V_{\rho_{\max}} \tag{19}$$

Equation (14) gives an expression for V_I as a function of p and an arbitrarily chosen value of x_c , the probability of consuming ($x_c = e$ in that case). Equation (10) is an expression for x_c in terms of p so upon substitution of (10) into (14) and taking the limit of the resulting expression we have the left hand side of (20). Equation (13) is an expression for V_m as a function of θ_e the level of marginal utility at which agents will consume. Equation (8c) gives the maximum return on money that is feasible in equilibrium and substituting this into (4) and using ($3a$) to eliminate x_c gives the maximum feasible value for θ^* which can then be substituted into (13) to give the right hand side of (20).

$$\left(1+2\beta-\beta^2\right)\frac{\overline{\theta}y}{2\left(1-\beta\right)} > \left[1+\left(\frac{2-\beta}{\beta}\right)^{\frac{-1}{2}}\right]\frac{\overline{\theta}y}{2\left(1-\beta\right)} \tag{20}$$

Hence we need to locate values of β such that;

$$2\beta-\beta^2 > \left[\frac{2-\beta}{\beta}\right]^{\frac{-1}{2}} \tag{21}$$

Relabelling the left hand side of this expression as A_I and the right hand side as A_m we hence require;

$$A_I > A_m \tag{22}$$

A_I is a monotonically non decreasing function over the interval [0, 1]. It has a point of inflection at $\beta = \frac{1}{2}$ and is concave to the left of this point and convex to the right of it.

A_m and A_I are equal at three points in the interval [0, 1] including the end-points zero and one. The two curves intersect internally at β^* which is approximately equal to 0.16. To the left of this point A_m lies above A_I but the positions reverse to the right. We can therefore say that on the interval (0, 1) to the left of β^* money yields a higher level of per capita utility than insurance, whilst the converse holds for $\beta > \beta^*$. Note that as β tends to 1 both assets' utility yields will approach the feasible per capita maximum, but in our model we retain the assumption of $\beta < 1$.

We noted in equation (11) that certain parameter restrictions must be used to make the calculations with the insurance asset valid. It can be shown easily that rearrangement of (11) shows that $\beta < 1$ is a sufficient condition for (11) to be valid when we let p become arbitrarily small.

We can conclude that if $\beta > \beta^*$ holds there exists a strictly positive value of p, the size of the insurance payout region, such that a stylised second best Friedman policy (see Taub 1988 p 581) will yield a level of per capita utility lower than any monetary policy that causes money not to be valued.

4.9. Conclusions.

I have highlighted sufficient conditions in which a particular economy with individual risk but no aggregate uncertainty calls for a second best policy that ensures money will not be valued, rather than using a second best Friedman style policy. The sufficient conditions we have located reflect the intuition on the workings of the 2 assets we have described. We have shown that if $\beta < 1$ then a monetary equilibrium can exist in our model, with agents choosing money instead of insurance to save in. Yet if p is sufficiently small and β large enough then this choice of the monetary asset will yield a sub second best social outcome. The need for p to be small is that the requirement that the insurance asset should be sufficiently superior to money (and sufficiently superior to make our style of proof work) in achieving the social welfare improving task of redistributing wealth to the more hungry agents. The need for the restriction on β is from the fact that this affects the consumption/saving decision of agents no matter what asset they hold. The lower region of possible values does not yield our desired result because such values restrict the efficacy of insurance more than money.

As we have seen qualifications to Friedman's result have not been uncommon in the literature. For instance, we have the results discussed above of Bewley (1983) and Taub (1988) that consider the restrictions on the feasible tax level and return on money (see chapter 2 also). Another example is the issue raised by Phelps (1973) that in the presence of distorting taxation a government might wish to finance some of its expenditure by the inflation tax (implying a return on money below that implied by the Friedman rule), rather than use just the distortionary taxes. The counter example presented in this paper however is a more fundamental qualification to Friedman's result (or its second best counterpart, more accurately). This is because it builds directly on a notion that Friedman raised himself, that of bringing individual and social opportunities closer together. In our model, under the circumstances highlighted this is something that insurance is clearly better at. Yet the return on money, if high enough will be such that agents are induced to hold the "inferior" asset, which is money, through their individually rational optimising behavior.

We have two issues to be settled that arise from this model. The first concerns the issue of the form of the utility function. Although the basic argument of the idea is about budget constraints and matching private opportunities to social opportunities, we are in a second best situation where the poor insurance asset performance of money must be balanced against its function as a buffer stock. The following example I sketch is an illustration that the essence of my argument is retained with concave utility, that an equilibrium with too much money is possible but that the extreme conclusion of no money being optimal is not so robust.

Consider an economy with two types of agents A and B. In odd periods, the type A agents have an endowment of one unit of the single good, and type B agents have an endowment of a with probability p and an endowment of b with probability 1-p. I assume 1>b>a and that a and b are sufficiently close so that in the equilibrium the agents will always be liquidity constrained in these poor states. This greatly simplifies calculations, since a complex state dependent evolution of real balance holdings does not now occur. All the money stock is held at the end of the period by agents who had the endowment of one unit of the good in that period. I will restrict attention to steady states again, and assume the same kind of insurance asset set up, so that with the insurance asset paying off only in the 'worst' state for agents, we have that pR=1 yields the equilibrium return on the insurance asset. Given the restriction on parameters that will yield liquidity constraints in the poor states, then in equilibrium the relevant first order conditions will be as given below, where MU_1,MU_a and MU_b denote marginal utility in states with endowments 1, a and b respectively.

The first order condition for insurance is;

$$MU_1 = \beta pR \,\mathrm{MU}_a \tag{1}$$

pR =1 in equilibrium hence gives;

$$MU_1 = \beta \mathrm{MU}_a \tag{2}$$

In Money;

$$MU_1 = \beta I \left(pMU_a + (1-p)MU_b\right) \tag{3}$$

Therefore;

$$\frac{MU_1}{MU_b} = \frac{\beta\,(1-p)}{(1-pI)} \tag{4}$$

And;

$$\frac{MU_a}{MU_b} = \frac{(1-p)}{(1-pI)} \tag{5}$$

What do these equations reveal ? The crucial ones are (4) and (5) above. Equation (5) implies that to optimally balance the allocation of resources across states a and b, a gross return on money of one would be required. This would be a return that balanced correctly the choices of agents between insurance and money. its rationale in terms of equating individual and social opportunities can be seen in noting that the utility return to both assets is impeded by the factor β in both assets, hence the absence of β in equation (5). Agents can transfer funds out of state b and into state a at a rate of $\frac{R-I}{I}$ by buying more insurance and less money. The social marginal rate of transformation between those two states (treating all agents in an ex-ante identical fashion) is given by the ratio of the numbers of agents in the two groups. This ratio is $\frac{1-p}{p}$. Since R = $\frac{1}{p}$ holds, equating private and social opportunity costs means that I=1 must hold. Note that on the grounds of allocation of wealth across assets the result implies that the return on money must equal the expected rate of return on the insurance asset. This sounds like a kind of "modified" optimum quantity of money rule. **This is the first model to investigate the interaction of money and state contingent assets in this fashion and this result is hence a new contribution to the optimum quantity of money literature.**

Returning to the set of equations, (4) implies that if I=1, then the return on money is too low, given the assumption β<1, and hence needs to be raised to make the buffer stock mechanism work better. The clash between these two goals is hence clear. The optimum monetary policy in this case clearly lies in the open interval $(1, \mathrm{I}_{\max})$ (where $\mathrm{I}_{\max}$ is the maximum return permitted by the Bewley difficulty. Friedman could clearly lay claim to both ends of that interval. The structure of the model is such that the optimum quantity of money is infinite, but that demand for money with $\mathrm{I} = \frac{1}{\beta}$ will yield a finite money demand, and also that rate of return is feasible. To show the essence of the result I chose to ignore any consideration of the Bewley difficulty that may arise, and assume that I=$\frac{1}{\beta}$ is a feasible rate of return. I then examine under what conditions $\mathrm{I} = \frac{1}{\beta}$ is suboptimal. This is investigated below.

The method used is to use the equations above (one to five) to determine the equilibrium allocation as a function of the rate of return on money and to apply these results to derive conditions under which the derivative of the function that describes ex-ante per capita welfare with respect to the rate of return on money is negative.

Firstly we assume that the period utility function takes a logarithmic form, then assigning equal welfare weights to each consumer in the economy means the maximisation of the welfare function (in the stationary allocation);

$$V = \ln c_1 + p \ln c_a + (1-p) \ln c_b \tag{6}$$

Using equations (4) and (5) to substitute out c_1 and c_a yields;

$$V = \ln\left(\frac{1-pI}{\beta(1-p)}\right) + p\ln\left(\frac{1-pI}{1-p}\right) + 2\ln c_b \tag{7}$$

Goods market equilibrium dictates;

$$c_1 + pc_a + (1-p)\,c_b = 1 + pa + (1-p)\,b \equiv x \tag{8}$$

This relabelling simply aids notation. Use again of equations (4) and (5) to eliminate c_1 and c_a from the left hand side of this expression yields;

$$c_b \left(\frac{(1-pI) + \beta(1-pI)p + \beta(1-p)^2}{\beta(1-p)} \right) = x \tag{9}$$

Hence;

$$c_b = \frac{x\beta(1-p)}{(1-pI) + \beta(1-pI)p + \beta(1-p)^2} \tag{10}$$

Maximisation of (7) then reduces to maximisation of;

$$(1+p)\ln(1-pI) - 2\ln\left((1-pI)(1+\beta p) + \beta(1-p)^2\right) \tag{11}$$

Differentiation with respect to I, setting I equal to $\frac{1}{\beta}$ and rearranging yields conditions that must hold if we are to have the second best Friedman rule suboptimal;

$$\frac{1+p}{1-\frac{p}{\beta}} > \frac{2(1+\beta p)}{\left(1-\frac{p}{\beta}\right)(1+\beta p) + \beta(1-p)^2} \tag{12}$$

Inspection will reveal that if β is sufficiently small (and less than p) then this inequality will hold.

The final issue is to what extent to such "too much money " results in other models. Answering that question will help to distinguish my result. Reference to such a notion is certainly hard to find in the literature, and if present at all it seems that such a possibility arises in the overlapping generations literature. If we imagine a simple Samuelson type overlapping generations with money the only asset, and a double continuum of agents to avoid clashes between definitions of symmetric Pareto optimal allocations and maximisation of welfare of the typical generation, then any deflation of the money supply will yield a level of welfare below that obtained in the non-interventionist monetary equilibrium. In such models, the marginal rate of transformation across generations is unity and the gross rate of return on money should equal this. With too high a rate of return on money in such a model, too much trade across generations is the problem. A natural question to ask is can we produce such a result in a model with contemporaneously and infinitely lived agents? In a model such as Townsend(1980), a gross rate of return on money $I>\frac{1}{\beta}$ would produce an inefficient situation of too much trade. However in this kind of model no equilibrium with $I > \frac{1}{\beta}$ is possible. If we trace out the first order conditions of agents in such an economy then we can see that such an equilibrium could not occur since it would require endowments to increase over time in line with the desired path of consumption (see the coverage of this model in the literature survey). The finite lives in the overlapping generations models avoids such a problem; the pattern of consumption over time and across generations will be 'sawtoothed' rather than continually growing as in the Townsend model.

On the face of it then, such a difficulty of non-existence might occur in the model I have presented. To overcome such a difficulty in the model I have introduced a margin 'within' the model at which mis-allocation by money is possible without violating conditions for money's positive value in equilibrium, so taking an approach different to that of 'too much trade'. The necessary conditions were two fold. Firstly an asset that performed a role that money could not do within a second best environment had to be present, and secondly a way of pricing that asset that made it 'inefficiently' unattractive to agents. Might we envisage other similar results using the same approach ? A type of model we might look to for such a case is the model of Lucas and Stokey (1987), since we have an economy where cash buys some goods and credit others. If we kept the classification of goods rigid and made sure that credit had an imperfection making it unattractive then such a result of too much money could clearly be envisaged, leading to mis-allocation between cash and credit goods. However if the choice between the two media of exchange was endogenous then we would have a different conclusion. All exchange would become monetised and the possibility of too much money disappears. We would have to make sure there is essentially something in the model that money cannot do, which the Lucas/Stokey assumption imposes exogenously and artificially.

Another type of model we may consider is the Sidrauski/Brock model of money in the utility function. Too much money in this model means a return on money strictly greater than that on capital, and inefficiency would be implied by the lack of capital held to produce the output of the economy. The problem with such a situation however is of course that no such equilibrium can exist; money cannot have any real value at all if there is no output. A solution might be found however if we look at the stochastic form of the Sidrauski model, as examined by Danthine, Donaldson and Smith(1987) and DenHaan (1990). Due to the aggregate random return on capital the return on money is also random. Also note that money enters into the utility function, and so we might envisage an equilibrium in which the rate of return on money is pushed above the optimal level recommended by DenHaan and yet some capital is still held by the agents so that production is allowed. The yielding of an 'incorrect' level of consumption is the only possible fault in such a case, and since wealth effects are absent in such a representative agent framework (see chapter 5) a utility function separable in consumption and money would have to be assumed, and at this level of analysis the conclusions are unclear.

In summary of this discussion about possible alternative situations of 'too much money' we can make some comments about how likely such a scenario is in 'various' macro models. The possibility in the overlapping generations framework is clear. In models of contemporaneously and infinitely lived agents the possibilities are less clear when we move outside of the case I have highlighted. A model such as the Townsend set up where money is the only asset is certainly not a situation where 'too much money' is possible. Models with artificial restrictions on the functioning of different assets (particularly inside assets) are clearly undesirable, as my comments on the Lucas/Stokey model notes. We are hence forced to look at models with a real asset that has a function distinct from money; such as the money and growth models, although I noted that the conclusions are unclear. Alternatively we can turn again to inside assets, but in a situation where there clearly is 'something that money cannot do' , such as in the insurance example of this chapter. The poor insurance performance of money pointed out in this chapter is a case that seems to be 'essential' to money. In support of this chapter's result we can also again recall the comments of Karaken and Wallace (1980) who object to the way that some assets are simply placed into the utility function or their functions exogenously specified (e.g. the Lucas and Stokey set up). The approach of this chapter has been to derive results for the equilibrium of the model by showing the assets 'in operation', in regard to the consumption allocation they support. The model fits the tone of the whole thesis since the redistribution role of the assets is what is important here and a non-representative agent framework is hence essential.

Chapter 5

Superneutrality with Risky Income

5. Superneutrality with Risky Income.

The literature discussed in this chapter and the model that follows deal with the issue of the superneutrality or otherwise of money in the type of dynamic optimising monetary models that form the basis of the study carried out in this thesis.

I apply the configuration of endowments that Bewley uses to yield a problem with his existence proof to show that another potential difficulty with the optimum quantity of money can arise in models with endowment risk and money. This is that in a finite horizon model with a positive probability of zero income in each period, an attempt to improve economic welfare by implementing the optimum quantity of money proposal by deflation or paying interest on money will leave the set of equilibria of the model unchanged, if money is neutral, where neutrality is appropriately defined. We can hence say that in such circumstances neutrality is sufficient for superneutrality as long as attention is restricted to negative rates of money growth; a kind of one-sided superneutrality result. I shall furthermore suggest that if we have a situation where the probability of a reversal in the sense of Levine (1991) is high, in that a continued run of bad luck has a sufficiently small probability then in principle the optimum policy would be one of the Friedman kind, but because of the result outlined above, doing nothing yields just as high a level of welfare.

Though I exposit the idea as an application of Bewley's result, I also suggest that a potentially even stronger result holds in a finite horizon version of my chapter 2 model with risky capital, such that the superneutrality result is fully two sided; the optimum policy of 'repairing' the consumption stream will hold for inflationary monetary policy also.

When the superneutrality literature is reviewed below it will be seen that generally the results hinge on whether the real factors of the equilibrium are immune to changes in the rate of inflation and the level of real money balances. The implication of the Bewley/ chapter 2 factor applied to this question yields an essentially different cause of superneutrality. Here we find that the rate of inflation is independent of the rate of growth of the money supply. This is a conclusion that is new to dynamic general equilibrium monetary models. The reason for this factor arising is that the timing of the withdrawals from the economy is unimportant. This has echoes of a Ricardian equivalence type result, but such a conclusion is new to the money superneutrality literature due to the role that fiat money takes in the models of Bewley and myself. In both types of model we have money valued as a financial asset since it is not dominated in rate of return with probability equal to one. Contrast this for instance with models of money in the utility function and cash in advance models that have capital accumulation. There money is dominated in rate of return with probability one. When the tax is levied on individuals across the time periods in the model, then at least part of the saving to meet the tax liability(we assume all wealth is capitalised) will be in the form of the dominating asset, capital. The demand for money at time zero will then change less than one for one with the present value of the total tax liability, discounted by the return on money. The demand for money will then not rise to cancel out the fall in the money supply as it does in the case shown below. A similar argument holds in all the subsequent time periods also, so that a gradual fall in the money supply over time yields a resulting fall in the price level over time also. As might be anticipated from the discussion in my chapter 2, faced with a tax liability arriving at various points in time, the agent whose wealth is fully capitalised must set aside enough wealth from his present wealth to meet the tax liability, as they do in the Sidrauski/Brock productive capital model. However in the models of Bewley and myself the wealth set aside goes into money; this yields non existence in the infinitely lived agent consumer case or superneutrality in the finite horizon case. If the Bewley model when the endowment stream has a sufficiently large lower bound that the tax bill can be met with probability one, we can say that wealth is in a sense almost surely non capitalised so that the tax demand for money does not arise. In contrast where there is a positive probability of zero income forever then we can say that there is a positive probability that money plus income in the current period is the limit of their lifetime wealth, so we might call it capitalised in this probabilistic sense.

The only other model in which money is not dominated in rate of return with probability one is in overlapping generations models. A consumer who has an endowment stream (1,0) will set aside a nominal amount equal to the nominal amount of the tax bill in the first period of life in money but the ineffectiveness of the timing of the taxes does not occur; the arrival of new generations is important here, as the rest of this survey will show.

5.1. Review of the Literature.

We firstly need to clarify some definitions/parameters of study. The issue of neutrality of money firstly needs to be defined. This relates to the effects of a once and for all change in the nominal stock of money. In this study we shall restrict the definition of monetary policy to injections/withdrawals of fiat money by means of lump sum transfers. This might seem at odds with a traditional definition of monetary policy that separates monetary and fiscal policy, but it is as standard in the field of study that this thesis covers. Examining the usual form of the budget constraint;

$$G - T + Br = \Delta M + \Delta B \tag{1}$$

Monetary policy in some circles is described as a change in the composition of the right hand side of the equation above. The transfers that operate in the models examined in this thesis have a T to accompany the change in M; i.e. helicopter drops for instance.

Having clarified this we must firstly address the issue of when money is neutral. If prices are Walrasian then clearly the only effect of any change in the money supply can be through distributional effects. If the lump sum transfers are levied across agents in proportion to their existing money holdings then money will be neutral. Note that the efficacy of money in the models of Levine (1991) and Scheinkman and Weiss (1986) depends on the uneven distribution of money balances before the injection.

Superneutrality questions whether change in the rate of growth of the money supply will have real effects. We must of course demand a model in which real effects might potentially occur. A pure exchange version of the Sidrauski/Brock model with a representative agent having an endowment stream of y per period cannot exhibit non superneutrality in the conventional sense. In this model the agent maximises;

$$\sum_{t=0}^{\infty} \beta^t U_t\left(c_t, m_t\right) \tag{2}$$

subject to;

$$c_t + m_{t+1} = y + m_t, \text{ m}_0 \text{ given} \tag{3}$$

There is essentially no margin for reallocation or change in output to be affected in the equilibrium of this model, only the level of real money balances will change, as $c_t = y_t$ is the goods market equilibrium condition, with y_t exogenous for all t. Money is deemed to be superneutral in such a case, although Marty (1994) questions the definition since a change in the level of real money balances m has real effects; on the utility of agents. Marty's argument however is against the general tradition of the literature which tends to back the view of Hahn (1965, 1973) that money should be essential and have real effects on the allocation of resources if we are to say it is grounded in a good theoretical model.

If we were to make the utility function into one of the form U(c,m) = min (c,m) where m here denotes the start of period money balances then, we will have that money must be superneutral, even by Marty's definition. This is of course equivalent to the cash in advance constraint formulation of money's role.

To study the issue properly we need a margin of allocation to be affectable. For instance in the above model with a cash in advance constraint we might look to endogenise labour supply, and hence output. In mathematical terms such a model is equivalent to the cash/credit good set up Lucas and Stokey, with leisure as the credit good and consumption as the cash good. Since the key first order condition of the model is;

$$MU\left(leisure\right) = \beta \frac{P_0}{P_1} MU\left(consumption\right) \tag{4}$$

then the time lag in payments can create a way in which monetary policy can affect the marginal rate of transformation and hence output. Clearly with a more general utility function $\mathrm{U}(c, l, m)$ (that is not cash in advance) we must have $\mathrm{U}_{clm} \neq 0$ for superneutrality to fail.

A perennial topic in the field is the effects of money growth on capital accumulation. Following a debate started by Tobin in the nineteen sixties in non optimising models, the first model to address the issue in an optimising framework was given by Sidrauski (1967). This representative agent model had no leisure in the utility function, with an inelastic labour supply. Sidrauski found that independent of the value of U_{cm}, money was superneutral in the steady state, i.e. the steady state capital stock was independent of the rate of monetary growth. Fischer (1979) examined the issue of superneutrality on the transition path to the steady state. He found that money is superneutral if $U_{cm} = 0$, but superneutrality is violated if separability does not hold. Danthine, Donaldson and Smith (D.D.S)(1987) covered the issue for the case of uncertainty in the production process and found that unless $U_{cm} = 0$ holds money is non superneutral. D.D.S give the explanation of such a result by saying that since the economy is continually buffeted by stochastic shocks then it is always away from the steady state.

As Fischer's comments in his paper reveal, these results were not fully understood at the time. The non superneutrality results in the overlapping generations models of Drazen (1981) and Weiss (1980) deepened the puzzle. The issues were clarified by Cohen (1985) and Weil (1991), and we describe the sum of their explanations below. The basic problem of the consumer is to maximise;

$$\sum_{t=0}^{\infty} \beta^t U(c_t, m_t) \tag{5}$$

subject to;

$$k_{t+1} + m_{t+1} = m_t + k_t' - c_t \tag{6}$$

and the production function;

$$k_t' = f(k_t) \tag{7}$$

First order conditions are;

In money;

$$U'(c_t) = \beta \frac{p_t}{p_{t+1}} \left(U'(c_t) + U'(m_{t+1}) \right) \tag{8}$$

In capital;

$$U'(c_t) = \beta f'(k_{t+1}) U'(c_{t+1}) \tag{9}$$

If we look at the equation that governs the transition of the economy's capital stock;

$$k_{t+1} = f(k_t) - c_t \tag{10}$$

then clearly the only factor that can affect capital accumulation is the path of consumption over time. We can characterise the path of consumption simply and heuristically in terms of its slope and its level. Firstly, we look at the 'level' of the consumption path. The issue that clarifies this was addressed by Weil (1991) and questions whether money is net wealth or not. Consider the representative agent framework of Sidrauski, Fischer, et al., where there is no population growth. The question of whether money is net wealth and hence the wealth status of monetary injections/withdrawals can be answered in an intuitive fashion. Fiat money is of course simply an intrinsically worthless piece of paper and only has value if it is not spent. Hence the representative agent must hold it perpetually if it is to have a positive price, so that it will drop out of the consumer budget constraint when we look simply at wealth used for pure consumption purposes in the physical sense, which of course is all that affects the path of capital accumulation. The essence of money as a worthless piece of paper and the possibility of its positive value are of course addressed elsewhere in this thesis. We have also suggested elsewhere that things are different in an overlapping generations model (it should be noted that the arrival of new agents is the crucial factor rather than the dying of existing agents). The difference is that the pieces of paper can be passed on to newly arrived generations, as I document in chapters one and six. In addressing the question of for instance inflationary monetary policy, a similar argument can show that the injections are not net wealth either. Obviously inflation is the result of the injections and the real monetary wealth that 'nets out' in the process can be given a formal representation in terms of budget constraints as follows. The technique is to define total consumption in the individual consumer's budget constraint as the sum of 'physical ' consumption and the foregone return through holding money instead of capital, since money is dominated in rate of return. We will follow Weil at this point and switch to the continuous time formulation of the model, which allows easier manipulation of budget constraints for the representative agent. Without loss of generality we restrict attention to the steady state of the model, and denote the steady state net marginal product of capital as r and the rate of inflation as π.

$$PV\,(wealth) = PV(total\ consumption) \tag{11}$$

$$k_0 + \frac{M_0}{P_0} = \int_0^\infty e^{-rt}\left(c_t + (r+\pi)\, m_t\right) dt \tag{12}$$

If the argument above is to be correct then we need to prove;

$$\frac{M_0}{P_0} = \int_0^\infty e^{-rt}\,(r+\pi)\, m_t.dt \tag{13}$$

Since in the steady state of the model with zero money growth inflation is equal to zero;

$$\frac{M_0}{P_0} = \int_0^{\infty} e^{-\,\mathrm{rt}} r m_t . dt \tag{14}$$

Hence;

$$\frac{M_0}{P_0} = r \frac{M_0}{P_0} \int_0^{\infty} e^{-\mathrm{rt}} dt \tag{15}$$

This simply implies that;

$$\frac{M_0}{P_0} = \frac{M_0}{P_0} \tag{16}$$

And hence the non net wealth proposition is proved.

In the overlapping generations models things are different. Transfers of money spread through time cause inflation that lower real money balances now and redistribute wealth towards the later born. The opposite redistribution occurs with a contraction.

Having categorised the difference between representative agent models and overlapping generations models regarding wealth effects, we must now address the question of the slope of the consumption path. There is no key difference between the two types of models on this question. The key factors here are whether the utility function is separable in money and consumption and whether the economy is in a steady state or not; i.e. whether the rate of inflation is constant or not (though of course superneutrality will fail in the overlapping generations model anyway!). Examine the first order condition in capital (9); if the money stock does not affect U_c then the slope of the consumption path is independent of inflation. If $U_{cm} \neq 0$ then a time varying nominal interest rate, yields a changing level of money demand and real money balances over time. The link from equation (9) to the slope of the consumption path is hence broken. This accounts for the difference between the steady state and non steady state results.

Interesting results are also yielded by the cash in advance approach of Stockman (1981). He shows that a cash in advance constraint on consumption yields superneutrality in the steady state and even the level of real money balances in the steady state is independent of the rate of growth of the money stock. Asako (1983) and Abel (1985) show that this superneutrality even holds on the approach to the steady state. Stockman and Abel show that things are different however when investment expenditures are also subjected to a cash in advance constraint; superneutrality fails and the intuition can be easily thought out by looking at the first order condition in capital again.

The surveys by Orphanides and Solow (1990) and Wang and Yip (1993) show that violations of superneutrality can also occur even in the steady state of the representative agent model if we include money in the production function or allow labour to be supplied elastically. Again, the first order condition in capital is the key. In the steady state the rate of time preference will pin down the marginal product of capital, independently of the rate of inflation. However, for instance with money in the production function, the link from the marginal product of capital to the capital stock is broken.

5.2. The Model.

I shall now demonstrate the result formally in a Bewley type model, showing that a fall in the money stock over time is equivalent to a fall in the money stock at the initial date. The case that I choose to exposit below is equivalent to the Bewley model with a positive probability of zero income in all periods. However the result can also be extended in two ways. We have seen from the work of Bewley and those who followed him (Mehrling(1995) for instance) that in the case of a strictly positive lower bound on income then there exists a strictly positive upper bound on the rate of interest that can be payed on money and equivalently a maximum possible level of real balances that can be used for insurance purposes. A similar concept applies here. Unlike the infinite horizon model, interest paid beyond the 'upper bound' (beyond which some money is demanded for tax purposes) still allows equilibria to exist, but the welfare levels will be no higher than those that result from paying interest at the 'critical' rate. Hence below the critical rate of return money is non superneutral, above it is superneutral.

When agents set aside money equal to the present value of the total tax liability (discounted by the return on money) this is in effect equivalent to reducing the money supply at date zero by the total amount of the tax. This is so because the money is held out of circulation, only to pay the taxes. A staggered reduction in the money supply is then identical to a once and for all change. If the once and for all change is neutral then money will also be superneutral and so Bewley's difficulty with the optimum quantity of money yields another difficulty. We can easily conjecture that my version of the difficulty has a wider application also; to the superneutrality question.

The economy lasts for N periods. It is populated by a large number of agents who are born at date zero and die after the N^{th} period. Income in each period is random and has a strictly positive probability of an outcome of zero in each period of the agent's life. To overcome the terminal date problem, we artificially attach one unit of utility to each unit of money held at the end of the last period. This is not of course strictly necessary to give money a value under taxation but it provides a better 'backdrop' to the superneutrality issue. We will simply state the fundamentals of an economy with a given nominal money stock distributed equally among agents at date zero and show that the fundamentals of the economy are the same as this under deflation financed by equally distributed lump sum taxation (as opposed to paying interest on balances; the differences have no particular importance, in contrast to the environment of chapter 3).

(1) Non - interventionist monetary economy

Agent i maximises at time t;

$$E_{it} \sum_{s=0}^{N-t} \beta^s U_t \left(c_{it+s}\right) + M_{iN} \tag{17}$$

Subject to;

$$M_{t+1} = M_t - p_t c_{it} + p_t y_{it} \tag{18}$$

$$M_{t+1} \geq 0 \tag{19}$$

$$M_1 = \overline{M} \tag{20}$$

(2) Monetary economy with taxes

The government levies lump sum taxes symmetrically across agents;

$$T_{it} = \overline{M}_{t-1}.x \tag{21}$$

Where;

$$\overline{M}_t = (1-x)\,\overline{M}_{t-1} \tag{22}$$

The problem for each agent is now to maximise the utility function above subject to;

$$M_{t+1} = M_t - p_t c_{it} + p_t y_{it} - T_{it} \tag{23}$$

At any date t there are N-t periods left and by assumption there is a positive probability that the agent's income will be zero in each period. To meet his tax liability with probability one he must hence hold money equal to his total future tax liability. If the per capita money supply is $\overline{M_t}$ at time t then he must hold balances at least equal to;

$$\overline{M}_T = x\overline{M_t} + x\overline{M}_t(1-x) + x\overline{M}_t(1-x)^2 + \ldots.. \tag{24}$$

Each consumer solves his problem in two stages. Firstly he sets aside money balances as described above. Then he manages his remaining wealth as though the tax liability did not exist. If we look at the monetary economy at date zero, it can be represented in terms of the same set of agents, with real endowments and preferences as before. This time the only change is that each individual's real balances are split between the 'tax account' and the remaining balances are held for trading. We can hence say that the monetary economy with taxes is equivalent to the economy without taxes but an initial level of nominal money balances in per capita terms equal to $\overline{M} - \overline{M}_T$. Under the assumption that the monetary policy has been applied symmetrically to match the symmetric distribution of real balances, all real variables of the economy (interestingly also including the level of real money balances in circulation) are independent of the size of the money stock or its rate of change. A one sided superneutrality is hence shown.

5.3. Conclusion

I have shown that in a finite horizon economy adapted from Bewley (1980, 1983) even when we have monetary equilibrium existing with a deflationary monetary policy then problems with Friedman's proposal still occurs, namely that money can be superneutral.

I would conjecture that if I was to develop a version of my model of chapter 2 with a finite horizon then a full two sided superneutrality result would occur, since the capitalisation of wealth is absolute, and not just with a positive probability as in the Bewley case above.

It is also interesting to note that the effects of a positive probability of zero income for the rest of life is not considered in the Ricardian equivalence literature, and such an application of the Bewley/chapter 2 difficulty would be of benefit.

Chapter 6

Financial Imperfections, Money, Endogenous Cycles and Persistence

6. Financial Imperfections, Money, Endogenous Cycles and Persistence.

I present two ideas in this final original chapter. I firstly show that a model with financial imperfections can yield multiple equilibria based on endogenous market participation, including the possibility of cycling between high and low states of activity. The driving force is the minimum real size of investment required to enter into an investment project, and the restriction brought about through imperfect financial intermediation that the investment projects are indivisible; i.e. that no more than one individual may invest in any given project. Low economic activity yields low liquidity levels and an inability to invest in productive projects. This is what yields the low economic activity. The endogenous participation yields the result that in states of low economic activity, the signals provided by prices to keep the competitive economy at full employment do not get through. I find that I am able to generate endogenous cycles without the need to violate the condition of gross substitutability between consumption at different dates as Grandmont (1985) was forced to do.

The second model exhibits an essentially dynamic movement of liquidity round the economy. The distribution of the constant real stock of liquidity between agents of differing risk attitudes influences the investment level of the economy, since investment is essentially risky. The redistribution of liquidity round the economy and hence the rate of investment is influenced by financial structure and in the monetary economy potential investors must wait to accumulate liquidity. If a low shock has occured then the monetary economy has to stay at a low level of activity for a further period.

The key factor common to both models is the indivisibility of investment projects both in terms of the minimum size of investment and the indivisibility across individuals, making the participation in certain projects endogenous to the state of the economy. The effects of market participation are key to both models presented in this chapter and the review of the literature set out below outlines some of the implications of incomplete market participation.

6.1. Review of the Literature.

The focus of the thesis so far has been on economies where incomplete markets led to multiple budget constraints and inefficient steady states, and we looked at money's ability or otherwise to overcome these difficulties. The topic of study would not be complete however without looking at the role of liquidity constraints and the distribution of liquidity in the cause and propagation of aggregate fluctuations. I will forward two results which provide some contrast with the existing literature in those two areas. I will firstly survey the literature that pertains to the indeterminacy of equilibria in models with restricted market participation. In overlapping generations models the restriction is exogenously specified. In the models of Bewley and Townsend already covered the liquidity constraint is exogenous and its biting occurs in each period for at least one agent and for each agent in at least one period, in the presence of impatience and a constant money stock. In the original models I will present, the financial imperfection is that the intermediary sector cannot facilitate the divisibility of projects and dependent upon the state of the aggregate economy, the constraint on the level of investment may or may not bite. This endogeneity contrasts importantly with the overlapping generations models and the Bewley class of models. We can also put the model alongside the emerging class of endogenous market participation such as Allen and Gale(1994) and Chatterjee and Corbae (1992) but these do not deal with aggregate fluctuations in economic activity.

The second model I present has fundamentally exogenous shocks, but the way the economy redistributes liquidity has important implications for the frequency of investment and the persistence of shocks.

We have already encountered models that link the distribution of liquidity to aggregate fluctuations in the literature survey. We also briefly documented the possibility of multiplicity of equilibria in an overlapping generations model. I now turn to a more detailed consideration of indeterminacy in overlapping generations models and hence how they relate to models with contemporaneously lived agents, which are the focus of the thesis. The issue of the valuation of money will be touched on again also.

A vast and still growing strand of literature concerns itself with the issue of multiplicity of equilibria in dynamic general equilibrium models. This issue is not always everywhere an essentially monetary phenomenon, indeed an overlapping generations model with complete markets and no money might still posses a continuum of distinct equilibria. Conversely not all dynamic monetary models display indeterminacy. However, the common framework typically used to examine the two areas and the extent of common occurrence of the two phenomena make some discussion of such a topic in a thesis such as this absolutely essential. As I have said the literature in this area is vast but a number of useful surveys have appeared on the topic. Among those are Woodford(1988), Farmer (1993), Kehoe (1992), Boldrin and Woodford (1990) and Azariadis (1993), and the discussion of the background to the original models presented in this chapter draws on these.

We need to clarify a point of definition before we proceed. Indeterminacy refers to the existence of an uncountably infinite number of distinct competitive equilibria (Geanakoplos and Polemarchakis (1991)). This describes a lack of local uniqueness of equilibria.

Apart from the purely theoretical interest aroused by the issue of indeterminacy in these models, the area has attracted much attention from macroeconomists since it allows a demonstration that the Keynesian notion of animal spirits in investment can yield instability in the level of employment and output, yet be fully consistent with the presence of Walrasian prices, market clearing and rational expectations. The accomplishment of this has been termed the macroeconomics of self fulfilling prophecies. As I have noted the indeterminacy of equilibria does not always and everywhere rest upon the existence of valued fiat money. However I will shortly describe an environment in which indeterminacy does indeed rest upon the presence of outside money, and the discussion of such a case allows us to shed further light on an issue discussed in the thesis already, that of the valuation of fiat money. Indeed if we stick to a framework in which there is a single consumption good, then the presence of valued fiat money is a necessary condition for the source of indeterminacy of equilibrium we consider in this chapter, since a necessary condition is the presence of two or more goods. The discussion centres around a comparison made by Kehoe (1992) between economies with a finite number of infinitely lived agents and one with an infinite number of finitely lived agents.

Before looking at the difference between economies with a finite number of infinitely lived agents and an infinite number of finitely lived agents, we must examine the 'challenges' we have to face in creating an economy with fiat money and indeterminacy of equilibrium, some of which we saw in the survey chapter. If we have an economy with a definite starting date then we saw that in trying to derive a model with a positive value for fiat money we must face the challenge of a potential mis-match between the present value of expenditure and resources in the economy. In the case where money enters the utility function the solution is clearly achieved by having money 'held' throughout the infinite horizon of the economy, so that in terms of the budget constraints, money is held as a kind of consumption in itself (recall this discussion from chapter 5). In an overlapping generations model we have a situation where money is essentially held briefly then to be spent, and no such argument can be invoked. Recalling the budget constraints appearing in equations one and two in the survey that aggregates the market valuation of the economy's wealth, we here adapt them to where we have one consumption good so that ω and c are scalars and p_t denotes the money price of the good in period t. We have, by aggregating over all consumers;

$$\sum_{t=o}^{\infty} p_t \omega_t + \overline{M} = \sum_{t=0}^{\infty} p_t c_t \qquad (1)$$

We saw that positivity of the value of money would violate the feasibility constraints in such a situation of 'spending' unless the present value of real wealth is infinite. This can be seen if we divide (1) through by p_0. We shall later note that a non decreasing price level through time is sufficient to achieve this. With a finite number of agents this implies that the wealth of at least one consumer is unbounded and clearly no equilibrium will exist. It is clear that if such a problem is to be avoided we must either look for an economy where the unbounded real wealth is divided among an infinite number of agents, each who have finite wealth, or to stop any agent with infinite real wealth from using his wealth in a manner that would yield unbounded consumption. The formal representation of the former is the overlapping generations framework covered in the survey chapter for the Cass, Okuno and Zilcha paper, and the latter is the infinitely lived agent debt constrained or incomplete market type model which is the focus of this thesis. An alternative way to view the solution provided by the overlapping generations framework is that the infinite resources in the economy necessary for money to have value in the economy where it is continually spent are 'arriving from infinity' via the arrival of newly born agents with their endowments. It should be noted in passing that this is of course not a sufficient condition in itself for money to have positive value, as should be apparent from work in the rest of the thesis. In particular, we must have that the return on money is sufficiently attractive to agents and that the economy can support such a rate of return as an equilibrium. This of course alludes to the relation between the inefficiency of the barter equilibrium and the existence of a monetary equilibrium. In other words the rate at which the bubble which is money must grow for agents to value it must not exceed the rate at which new resources arrive in the economy to match this value.

The difference between economies consisting of a finite number of infinitely lived agents without liquidity constraints and an economy with constraints (or an overlapping generations framework) is usually expressed as a difference in market participation. The phrase has a clear meaning when we look at the effect of the liquidity constraints, but it should be noted that for instance we can introduce some infinitely lived agents who are not debt constrained into an overlapping generations economy and retain the basic results of the overlapping generations models as long as these agents only control a small amount of the economies resources compared to the total, or more precisely that their endowment has finite value in equilibrium, so that unbounded consumption is prevented. In the interpretation of an overlapping generations type model as a model of infinitely lived agents with debt constraints then it will be the case that in the monetary economy the value of some agent's wealth will be infinite, but the presence of liquidity constraints truncates the planning horizon of the consumer, and the way that prices value future wealth is now 'irrelevant'. Despite the focus of the thesis, exposition is aided by from here only comparing the models of overlapping generations with those of infinitely lived unconstrained agents, hopefully all equivalent cases should be obvious.

Two other important conclusions follow from the double infinity of goods and agents. They concern the Pareto optimality or otherwise of equilibrium and the number of equilibria in the model. Of course as we have noted the inefficiency of the non monetary equilibrium is very important. Firstly, an overlapping generations monetary economy with an infinite valued endowment stream resulting from strictly positive inflation would not be Pareto optimal. If we have a positive rate of inflation yielding an infinite present value of resources then the equilibrium will not be Pareto efficient. We can increase the welfare of all agents by 'bringing resources back from infinity'. We hence raise the consumption of the initial generation and improve the intertemporal allocation of the subsequent generations. (The closer the return on money balances is to zero the closer individual and social opportunities are brought together). Amongst the continuum of equilibria that typically exist, that which has a constant price level will be Pareto efficient (crucially we assume zero population growth), although the endowment stream has infinite value. This can be explained as a 'borderline case '; pushing the rate of return higher by seeking a return on savings that is strictly positive would yield a Pareto optimal allocation (though no such case will exist with money in these simple cases since the value of money will grow too fast relative to the resources provided by a constant population whose total endowment value given by discounting with respect to the return on money will yield a finite number). A lower return yields a non optimal allocation. We can specialise the results of Balasko and Shell (1980) which give us a necessary and sufficient condition for Pareto efficiency of equilibrium to the one good case;

$$\sum_{t=0}^{\infty} \frac{1}{p_t} = \infty \tag{2}$$

The other interesting, crucial and influential result concerns the number of equilibria in the model. The infinite horizon model without liquidity constraints as Kehoe (1992) shows can be proven to have under quite general conditions a finite number of locally unique equilibria. Kehoe uses the approach of Negeshi/Bewley that utilises the second welfare theorem. As we might guess in the overlapping generations model with money such an application will not be possible. In fact (as we showed graphically in the survey) that such a simple model as we consider with one good plus money might possess a continuum of inflationary equilibria that converge asymptotically to autarchy.

Since in this simple model the reason for this indeterminacy is the presence of money, we can explain the presence of these two factors in one step. The key phrase is that there is a lack of market clearing at infinity. If the reader refers to the coverage of the question of money as net wealth in general equilibrium models (chapter 5) then the following discussion will come as no surprise.

Following the argument of Kehoe, we can envisage an artificial finite horizon economy of T periods in length where there is a final young generation as well as the initial old generation. If money is to have value then the final young generation must accept it and relinquish goods. However we know from looking at finite horizon models of truly fiat money that such a situation is infeasible, since the final generation will be unwilling to do this. A situation therefore where we envisage money being spent by existing generations and hence ultimately generation T-1 which is matched by the delivery of goods from outside the first T-1 generations is characterised by non market clearing. Alternatively we can note that if we aggregate the budget constraints of the first T-1 generations then we have our original perplex of the total value of expenditure exceeding the total value of the endowment, hence Walras law will not hold. The extra degree of freedom in the determination of the 'equilibrium' in the T period model comes from our ability to vary the nominal price level that determines the net transfer of goods from the final young generation to the initial old generation. The obvious question is then; can we have a monetary economy where this indeterminacy remains without ever 'explicitly' forcing a final generation to literally behave sub optimally? The answer of course lies in the infinity of agents. Their never ending arrival means that we can retain the degree of freedom that varying the initial level of the money stock gives us without ever observing sub optimal actions by agents; the lack of market clearing is at infinity. Put another way, the budget constraint will be seen to hold asymptotically as we increase the horizon of the economy without bound. We can show, as in the analysis of the Cass, Okuno and Zilcha paper of the literature survey that we can start the economy off at any of a continuum of real money balances and show the existence for each of an inflationary equilibrium converging asymptotically to autarchy.

We can show the same result of a dimension of indeterminacy by counting equations and unknowns as we move forward in time. We can view such a process in the context of a model of infinitely lived agents with borrowing constraints by noting that the extra constraints add restrictions that make some equations unable to contribute to the determination of equilibrium, though they are not essentially redundant. Some express this by saying that multiple versions of Walras law are present. Staying with the one good plus money framework, in the initial period, application of Walras law for the money and good market for that period yields one equation in two unknowns,p_0 and p_1, the respective money prices of the consumption good in the first two periods. As we move forward and apply equilibrium conditions in the subsequent markets, in adding one equation we 'lose' an unknown but gain another. The net effect is to yield after n applications of this procedure, n equations in n+1 unknowns. The indeterminacy problem can be clearly seen.

The discussion above centres totally around the indeterminacy of equilibria by showing why we can have an economy with a continuum of deterministic perfect foresight equilibria all converging asymptotically to autarchy. A further question is whether such economies can exhibit perfect foresight equilibria which cycle from period to period or fluctuate in a stochastic fashion. The answers to these questions and indeed even the issue of when indeterminacy will occur still have no definitive general answers, hence the continually growing field of research output. We can nevertheless note some of the more central findings in the field. In general a sufficient condition (see Azariadis (1993) p 86-87 for example) for a dynamical system to exhibit a cycle of period 2 is that the map that gives the transition from x_t to x_{t+1}, (call it f) should have two fixed points at zero and $\overline{x} > 0$, and a point $a > \overline{x}$ such that $a > f(a)$ and $a > f^2(a)$ (simply the difference equation applied twice) then $f'(\overline{x}) < -1$ is a sufficient condition for a cycle of period 2 to exist. This effectively means that the offer curve is strongly backward bending near the steady state, $\overline{x}$. The need for the existence of two possible x_{t+1} values for a given x_t in the neighbourhood of the steady state for the existence of a 2 cycle is intuitively obvious, and can be seen if we try to "use" the offer curve.

We can use the intuition behind the notion of a cycle to examine the idea of a sunspot. A sunspot variable is a random variable whose realisation may potentially affect the real equilibrium allocations of the economy without having any effect on its fundamentals (endowments, preferences etc.). The sunspot variable effectively selects the expectation of what will happen in the following period. It is thus known as extrinsic uncertainty. In a standard real business cycle model with stochastic shocks to fundamentals (e.g. King and Plosser (1984) the uncertainty is termed intrinsic uncertainty. In the context of a simple one good plus money 2 period lives overlapping generations model a stationary sunspot equilibrium consists of prices p_a, p_b in states a and b respectively and transition probabilities π_{aa} and π_{bb} in the interval (0,1). For the sunspot to matter (i.e. to affect real allocations) we must have that $p_a \neq p_b$. Building on these definitions we can clarify the notion of an endogenous cycle. If we note that a cycle is simply a degenerate stationary sunspot equilibrium with a transition matrix $\pi_{aa}= \pi_{bb} = 0$, then an endogenous 2 period cycle is a fluctuation in real variables that arises without the need for any fluctuation (stochastic or otherwise) in the fundamentals of the economy, although there may be intrinsic uncertainty present as well.

When will a sunspot exist in the class of overlapping generations models that we have discussed? There are two basic factors to consider. Firstly for some sunspots an essential difference between the nature of a cycle and a sunspot is brought out. Indeterminacy and the possibility of periodic orbits in the real variables determined in equilibrium does not necessitate the incompleteness of markets; the double infinity of agents and goods as in the overlapping generations model is what is important. Even the inability of agents to meet together and trade at the start of time is not important. However if we are to have the presence of sunspots that matter, then they must be uninsurable. The original example of Cass and Shell (1983) emphasises not just the completeness of insurance markets but the question of whether all agents can participate in the insurance markets; similar to the notion of participation in the overlapping generations models. A possible reason why some agents can't participate in an insurance market that is relevant to them is that they are not born in the period when the insurance asset trading takes place. Hence the ability of agents to meet together at the start of time may be important. As Woodford (1988) shows, the basic notion is that sunspot realisations cannot matter if the first welfare theorem holds. If we suggest 2 different allocations that correspond respectively to 2 realisations of the sunspot variable, the property of strict convexity of preferences coupled with convexity of the set of feasible allocations means that the sunspot allocation cannot be ex- ante Pareto optimal. Hence for sunspots to matter we must depart from the Arrow-Debreu structure in some way that causes the first welfare theorem to fail. An overlapping generations structure is an obvious and popular one to go for. The indeterminacy it creates is useful in 'constructing' sunspot equilibria. Farmer describes the exercise as randomising over a set of multiple equilibria based on indeterminacies of the kind we have described, non-convexities or externalities.

Woodford (1988) presents two examples of economies consisting of a finite number of infinitely lived agents under imperfect financial intermediation that can mimic the kind of dynamic behavior associated with overlapping generations models. This is in the spirit of the type of original model that I present in the second half of this chapter. An advantage of these types of models over overlapping generation models is that the periods implied for the cycle will be less than those of the lifetime of the consumer that the overlapping generations models imply.

The first type of model presented by Woodford is the cash in advance economy of Wilson (1979) and Lucas and Stokey (1984). It is a representative agent model but one that has a representation of a financial structure imposed upon it in the form of a cash in advance constraint on the purchase of the consumption good. The worker has direct access to a production technology y = n, where n is labour supply. The single consumption good that is produced is perishable. The proceeds from the sale of the good cannot however be spent until the following period; the renowned payments lag that motivates a holding of money. The consumer problem is hence;

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left(u\left(c_t\right) - v\left(n_t\right)\right) \tag{3}$$

subject to;

$$\begin{aligned} p_t c_t &\leq M_t \\ M_{t+1} &= M_t - p_t c_t + p_t n_t \end{aligned}$$

The first order conditions are then;

$$v'(n_t) = \beta E_t \left(u'(c_{t+1}) \right) \frac{p_t}{p_{t+1}} \tag{4}$$

and;

$$u'(c_t) \geq v'(n_t) \tag{5}$$

Woodford notes that the first order conditions of the model and also its equilibrium conditions are the same as would obtain in a standard overlapping generations model. By linearising around the monetary deterministic steady state of the model, Woodford derives the result that sunspot equilibria exist in the neighbourhood of this steady state if there is a sharply backward bending labour supply curve. This implies strong income effects of a change in the ratio $\frac{p_t}{p_{t+1}}$ that are felt to be empirically implausible in general. Such a case would also violate the 'gross substitutability condition'.

A solution to this problem found in the literature is to add the possibility of capital accumulation to the model. Reichlin (1986) was the first author to do so. Woodford shows how this can be done in a model of contemporaneously and infinitely lived agents by ensuring that the capital stock is owned only by a set of agents who are separate to those who supply labour. Another solution is displayed in the original model that follows this survey.

Some of the models surveyed in Farmer (1993) consider alternatives to overlapping generations and infinite lives debt constrained models as ways to generate multiple equilibria, cycles and sunspots. The focus of part of Farmer's work is in looking at models of infinitely lived consumers that display non - convexities in the aggregate technology; externalities in production is one possible case. Another framework is akin to the Diamond style search economy (1984) where there are transacting externalities created by the holding of money as a stimulus to the trade of others (see Benhabib and Farmer (1991)). Farmer uses this case to exposit the main ideas behind an important strand of his research, the response of real variables to changes in monetary policy, a recent tradition of models started by Farmer and Woodford (1984) and Geanakoplos and Polemarchakis (1985). He shows that in the face of a change in the nominal money supply, when the monetary steady state of the model is stable, that restricting attention to the rational expectations equilibria of the economy does not necessarily imply the policy ineffectiveness of the new classical tradition as would be the case with a unique dynamically unstable steady state. The local stability of the monetary steady state means that the economy could follow a path with a level of real money balances which gradually converges to the steady state level over time.

Before presenting the original contributions of this chapter, two other types of models that may display non-uniqueness of equilibria should be mentioned. Matsuyama (1991) addresses the issue of multiple equilibria in the Sidrauski-Brock money in the utility function model, for the case of endowments rather than capital (see Brock 1974). Azariadis (1993) summarises the model and the essential results of Matsuyama. The model is that of a pure exchange economy with a single representative agent who lives forever and maximises the discounted value of utility in consumption and end of period real money balances;

$$V = \sum_{t=0}^{\infty} \beta^t w\left(c_t, m_t\right) \tag{6}$$

Azariadis notes that the crucial factor in determining the number of equilibria of the model, where y is the perpetual per-capita endowment stream of the economy's consumption good, is the value of $\mathrm{mw}_c\left(y, m\right)$ (in goods market equilibrium). If for instance the period utility function is separable in consumption and money balances then mw_c is increasing in m, then the steady state is unstable and this is the unique equilibrium of the model. If however the function mw_c is locally decreasing in m then the map of the difference equation m_{t+1} as a function of m_t is non monotone and may admit the possibility of cycles, since the monetary steady state may then be asymptotically stable.

Another type of model exhibiting indeterminacy of equilibria deserves mention here. The 'source' of price indeterminacy specified in the model below is different from that of the overlapping generations models. The way in which it affects real allocations in some ways resembles the mechanism in the sunspot overlapping generations literature, since money is a nominal asset in both models whose variation in value across states of nature mean a variation in the way in which the asset pays off in real terms across the states. In the overlapping generations model the indeterminacy of the price level comes from the lack of market clearing at infinity already described. In the work of Cass (1985) and Geanakoplos and Mas-Collel (1989), the source of indeterminacy is Walrasian indeterminacy. The context of such work is to explain how a factor that is innocuous in the context of real assets or complete markets, has definite real effects when the assets are financial and the asset market complete. The notion of Walrasian indeterminacy describes the result that the 'absolute' price level is indeterminate in a set of spot markets (say). By the absolute price level we mean prices expressed in some abstract non-traded unit of account; 'moon dust' say. Imagine an economy with 2 dates, no uncertainty at the first date, but two or more states of nature at the second date. If we express absolute spot market prices at the second date in moon dust and the asset market is complete then by definition there are enough degrees of freedom in the pricing of the nominal assets so that varying the absolute price level in across states will not affect the real allocation. When there is market incompleteness (i.e. a payoff matrix with rank less than the number of states) then the requisite degrees of freedom are not present in the asset structure. If the payoff in the assets is real (i.e. paying off in a numeraire commodity whose price is determinate) then the nominal indeterminacy is not important and has therefore no real effect. If however the assets are nominal and pay off in the unit of account whose value is indeterminate then the indeterminacy becomes real also, since the payoff of the asset is changed in real terms across states. For instance let us say that we are in an economy with uncertainty at the second date where the only asset is fiat money, (which is used as the numeraire) whose nominal return in all states is the same, and the value of money is indeterminate: then real indeterminacy would result. If the money price of goods was determinate then no real indeterminacy would occur. Geanakoplos and Mas-Collel exposit a situation where the assets are 'monetary' and the money price of goods is indeterminate in the states at the second date due to an assumption about the way in which money is given value. The case selected is that of a nominal tax demand at the end of the second period which as we have discussed earlier in the thesis leaves the price level indeterminate (a real tax would remove the indeterminacy).

6.2. Endogenous Market Participation and Endogenous Economic Fluctuations.

Imperfect financial intermediation can manifest itself in many forms and one of these is the indivisibility of investment projects across individuals. Potential financiers may have difficulty in finding partners to join in a project and this particularly becomes a problem when the minimum scale of investment is such that a single investor cannot finance any investment project on his own. If the level of liquidity that determines his participation or otherwise in the market is determined endogenously then the possibility of multiple equilibria may be created.

Following this conjecture and the survey above I present an original result that demonstrates that a model with endogenous market participation can display endogenous fluctuations, in contrast to the exogenous participation restriction models previously described. The model I present hence complements the type of literature cited above on endogenous fluctuations and also a new and growing literature on endogenous market participation, for instance see Allen and Gale (1994). The result I derive shows that violations of gross substitutability in preferences are not necessary to generate endogenous cycles and also has market participation playing a role subtly different to that in the overlapping generations models. The essence of the mechanism is the same in both the model below and the overlapping generations models in that the non participation means that signals provided by some prices do not get through. However, in the overlapping generations framework the result is the opening up of a dimension of indeterminacy, whereas the model below exhibits switching to a different yet locally unique level of economic activity, if financial market participation does not take place.

The models described in the short literature survey above showed the implications of exogenously imposed liquidity constraints for the impediment of the signals transmitted by prices that otherwise prevail in a complete markets set up. Results analogous to the overlapping generations literature were then shown to be possible. It would be interesting to know if a constraint whose binding was endogenous and dependant on the state of the economy could produce economic fluctuations. I show that such a result is possible by 'telling' the following story of activity in an economy. Imperfect financial intermediation manifested as a lack of divisibility of investment projects with a minimum size can leave 'poor' households unable to participate in the market for funds which will finance these projects; with only one person able to supply finance for any one project. If these projects are what ultimately generate the income of households then we have a scenario where participation is endogenous and is the source of multiple equilibria.

I present a model of an economy which has a large number of firms and households so that we can suggest that a relatively low level of liquidity in the aggregate would be no barrier to participation in the investment in firms if the projects were divisible between individuals. Given a high interest rate funds would be channelled through the 'intermediary sector' and would flow through to investment projects. The only possible competitive equilibrium with $\frac{1}{p_t} > 0$ would be the 'full employment' level. To provide intuition on this, we can try to imagine a situation where real money balances are too low to finance all projects but enough to finance at least one project. Say that all funds are invested in just one project. Given the assumption of constant returns to labour, agents could work as much as they wished at the going wage rate and the level of output produced and consumed would be such as to raise the level of real money balances and make investment in all projects possible, so that the hypothesised low level of activity could not be an equilibrium with divisibility of projects across agents. The high level of real money balances generated means that in equilibrium there is no rationing of funds and all projects are funded to the same amount, by symmetry of the firms. If there was some upper limit to the scale of one firm then it might be interesting to see if low level equilibria with divisibility of projects were possible, whilst retaining the minimum participation requirement.

To reiterate, the mechanism in operation is that in a slump a low level of real money balances means insufficient liquidity for one agent to finance an investment project, which requires a minimum level of investment, x. We can interpret the financial market therefore as a market for units of funding of minimum size x. There is hence no participation by savers in the bond market. They are hence unable to take advantage of the high real interest rate which arises to clear the bond market. Since the firms have to borrow to employ labour this reduces labour demand and hence output. Since money is used as a medium of exchange in this model economy with velocity equal to one, this generates the low level of real money balances that caused the illiquidity problem in the first place. The source of multiple steady states in the model (high and low activity levels are both possible) and of cycles between high and low activity levels is hence clear.

The low level of real money balances is characteristic of the form of the model in slump states, with the price level being determined by the ensuing level of real activity (see the Townsend, Bewley and overlapping generations models also). As an analogous situation we might imagine a formulation of the model with purely inside money where people are constrained by an inability to borrow an amount greater than their next pay cheque, a constraint which like the case above may or may not bite, dependent upon the endogenously determined state of the economy. The absence of such a constraint would allow full market access in all states and the interest rate would always guide the economy to a unique full employment steady state.

6.2.1. The Economy

I begin by reviewing the basic mathematical model, based on that of Fuerst (1992) and Christiano and Eichenbaum (1992). This structure is selected simply because it demonstrates the kind of configuration of households, firms and the intermediary sector within which the story I have sketched can be demonstrated. Indeed this is the kind of approach taken by Christiano and Eichenbaum in following Fuerst's work. The economy is populated by a large number of identical and infinitely lived households. The motivation of Fuerst and Christiano and Eichenbaum in their models was to examine the short run effects of the distribution of liquidity in the economy whilst avoiding the complexity of a distribution of wealth evolving over time. They employed a trick first introduced by Lucas who splits the household into sectors to exposit the mechanism of trade in the economy and then reunites them at the end of the period. The story that is told and used below is that the household separates out into 3 separate entities. The 3 sectors of the economy are workers/savers, a financial intermediary sector (essentially imperfect in this model) and a firm sector. At the start of each period after splitting up to trade we assume that all the stock of fiat money in the economy is held by the worker/saver sector. The worker/saver has two options for his money in the current period. He can hold it to finance consumption expenditure in the current period or save with the intermediary sector. I part company with Fuerst and Christiano and Eichenbaum by assuming that there is a minimum size of investment required if a project is undertaken and that investment in any one firm can only be carried out by one agent at most. Hence the projects are indivisible, so I am assuming that one of the essential roles of the financial intermediation sector works only imperfectly. I shall retain the terminology of 'saving with the intermediary sector' to describe the channelling of funds from households to firms even though the intermediary sector only manages to carry out a one to one matching of 'savers' and firms in my model.

If the saving in the inside asset marketed by the intermediary sector does take place then they will earn a gross nominal interest rate of R_t payable in period t+1. The funds acquired by the financial intermediary in that period are then loaned out by the profit maximising intermediary (As long as $R_t \geq 1$ holds) to firms who use the money to employ the labour of the workers/savers (they must borrow to do this). The money earned as wages can then be spent on the output of the firms which is produced from labour. (Note there is no cash in advance constraint on consumption purchases, only a within period cash constraint). The firms (price taking profit maximisers) then use the receipts to repay the loans to the bank. The intermediary then repays the debt to the worker/saver with interest, at the end of the period, for use in the following period. I assume that the production technology is fixed and has labour as the only input. As part of the story we assume some hidden heterogeneity so that the household's firm does not employ its own labour or consume its own good so that the decentralisation imposed makes sense; again it is a story in the spirit of Lucas(1980) (or see the Blanchard/Kiyotaki type models (Blanchard and Fischer (1989)).

Another change to the set up of Fuerst et al is in addition to allow the firm sector to have an endowment $\overline{y}$ of the economy's good each period which they can sell on the goods market. The motivation for that action can be seen by remembering that the firm's objective function is end of period nominal profits, noting that the good is not storable. When the period is over the profit will be returned straight to the worker/saver (or effectively to the household as a whole) for the start of the next period. Shares in firms are not tradeable. Note that the cash in advance constraint is on labour used in the investment projects and not consumption, since the wages earned in the current period can be spent on the consumption good.

6.2.2. Individual Optimisation Problems.

All households are distinct yet identical, so no index is necessary to distinguish different decisions, asset holdings etc. All variables below are in per capita terms.

The worker/saver maximises;

$$\sum_{t=0}^{\infty} \beta^t \left[\ln c_t + \ln (1 - l_t)\right] \tag{7}$$

(where c_t denotes consumption at time t and l_t is labour supply).$0< \beta < 1$ is assumed.

Subject to;

$$M_{t+1} = p_t \overline{y} + S_t R_t \tag{8}$$

This says that total money held at the start of period t+1 will be the sum of saving with the intermediary and interest plus the profits of firms. S_t is the level of saving at the intermediary in period t. $p_t\overline{y}$ is the nominal profits of the firm at the end of period t, where $\overline{y}$ is the exogenous endowment of the good that the firm owns, noting that there are constant returns to labour. R_t is the gross nominal return on savings in period t, the yield being paid at the end of the period. Initial money balances equal to the per capita money supply, $\overline{M}$ are given to each household at the start of the model.

This form of the budget constraint (8) (which 'short circuits the maximisation problem) can be derived from the following information:

$$M_t = S_t + E_t \tag{8a}$$

Where M_t is the stock of money held at the start of period t and E_t is the money allocated to augment wage income in the purchase of the consumption good in period t. I have assumed in advance that all this money held after the saving decision (given by the amount S_t put into the intermediary) is spent on goods during the period and that all wages earned are spent during the period, so that saving in 'bonds' (i.e. at the intermediary) and the profits of the non-market-tradeable firms are the worker/savers only form of saving for the next period. This is later justified firstly by the return on saving at the intermediary being greater than that on money for the case where such saving does occur and secondly that when there is no participation in that debt market then the return on money is too low to encourage saving in it. Hence M_{t+1}denotes the amount of money held at the start of period t+1 after the return on saving at the intermediary and profits have been payed out, rather than saving directly in fiat money.

The cash constraint on consumption is:

$$p_t c_t \leq E_t + w_t l_t \tag{8b}$$

w_t denotes the nominal wage in period t.

Total funds held at the start of period t+1 are then the sum of profit and saving plus return at the intermediary plus any unspent 'consumption' balances:

$$M_{t+1} = p_t \overline{y} + S_t R_t + E_t - p_t c_t + w_t l_t \tag{8c}$$

Following the argument I sketched out (to be validated later), the value of unspent consumption balances will be zero and hence (8) will obtain.

In addition we have the minimum participation constraint;

$$\frac{S_t}{P_t} \geq x, \text{ unless } S_t \text{ is zero.} \tag{9}$$

We might describe the financial market as a market for 'bundles' of liquidity of size at least equal to x.

The income constraint from the labour market is;

$$\text{Nominal labour income} = \mathrm{w}_t l_t \tag{10}$$

First order conditions are then in labour supply;

$$\frac{1}{1-l_t} = \frac{w_t}{P_t}\frac{1}{c_t} \tag{10a}$$

And in saving at the intermediary;

$$\frac{1}{c_t} = \beta R_t \frac{p_t}{p_{t+1}}\frac{1}{c_{t+1}}, \text{ if } \mathrm{S}_t > 0 \tag{10b}$$

The financial intermediary takes deposits from the workers/savers and as profit maximisers will clearly loan them all if $\mathrm{R}_t \geq 1$. (We assume that indifference causes no problems if $\mathrm{R}_t = 1$).

The firm will maximise the nominal value of profit in period t;

$$\pi = p_t Y_t - w_t R_t l_t + p_t \overline{y} \tag{11}$$

Subject to;

$$Y_t = l_t \tag{12}$$

Where Y_t denotes output.

I shall now describe some of the equilibria of the model;

6.2.3. Full Employment Steady State.

We obtain the full employment steady state in the model where the stock of liquidity held by households is sufficiently large to enable investment in a productive project via the intermediary.

The nominal interest rate can be obtained from the worker/saver's first order condition in saving at the intermediary, (10b) since by definition the level of consumption here is the same from period to period. The upper bars denote steady state boom values $\left(\overline{y}\text{ is exogenous, as is }\overline{M}\right)$;

$$\frac{1}{\overline{pc}} = \beta\overline{R}\frac{1}{\overline{pc}} \tag{13}$$

Therefore we have;

$$\overline{R} = \frac{1}{\beta} \tag{14}$$

The real wage is determined by (14) and the first order condition for the firm hiring of labour, which comes from the maximisation of (11) subject to (12);

$$\frac{\overline{R}\overline{w}}{\overline{p}} = 1 \qquad (15)$$

With (14) , (15) yields;

$$\frac{\overline{w}}{\overline{p}} = \beta \qquad (16)$$

(14) yields the result that the nominal interest rate will be strictly positive since the rate of time preference is strictly positive. Money is hence dominated in rate of return.

Goods market equilibrium is given by;

$$\overline{c} = \overline{y} + \overline{Y} \qquad (16a)$$

From the production function we have;

$$\overline{l} = \overline{Y} \qquad (17)$$

Substituting (16a) and (17) into the first order condition for labour supply then determines output;

$$\frac{1}{(1-\overline{Y})w} = \frac{1}{\overline{p}(\overline{Y}+\overline{y})} \tag{18}$$

With (16) this implies

$$\overline{Y} = \frac{\beta - \overline{y}}{1+\beta} \tag{19}$$

(note that at even at 'full employment' impatience is an impediment to economic activity)

The price level is then determined by the quantity equation;

$$\frac{\overline{M}}{\overline{p}} = \overline{y} + \overline{Y} \tag{20}$$

Where $\overline{M}$ denotes the per capita money supply, all held equally among the households at the start of the model.

Clearly $\overline{y}+\overline{Y}$ >x is necessary for the existence of such a steady state, given (9), the minimum level of real wealth holdings for participation in the debt market, which comes from the minimum size of investment and the indivisibility of projects across individuals. We assume this holds, so that we have participation in this state.

Is the quantity equation valid? If the velocity of circulation is unity in all periods then nominal income is the same in all periods and we then have this first order condition in money;

$$\frac{1}{c_t} > \beta \frac{p_t}{p_{t+1}} \frac{1}{c_{t+1}}, \forall t \tag{21}$$

since we have $\beta < 1$ and the quantity equation dictates;

$$\overline{M} = p_t c_t = p_{t+1} c_{t+1} \tag{21a}$$

This is hence self supporting and will later be seen to hold for both steady state booms and slumps and the cycle that follows these (indeed it should be apparent from the formulae at this stage). In other words we have the result that agents do not wish to save by holding fiat money. The flow of fiat money around the economy is that E_t the level of 'consumption' balances is all spent, as is established above and the amount that passes to the intermediary is all loaned out to the firm, as follows from the positive nominal interest rate from (14). This money is all used to pay wages. In other words;

$$w_t l_t = S_t \tag{21b}$$

These are then all spent on consumption by the workers. Combining this information with equation (8a) then establishes the unit velocity quantity equation.

6.2.4. Steady State Slump.

In the scenario that follows we have that a low level of real balances generates the result of no participation in the 'bond' market and hence no money channels through to investment in productive projects. The indivisibility of the investment projects as a result of imperfect financial intermediation is crucial here; only one person can invest in any one project (via the intermediary), and so low individual liquidity is the impediment to full employment. The high interest rates that result to choke off the demand for funds have no effect on the supply of funds. The endogenously determined non-participation in the bond market means that the signals that would otherwise guide the economy to full employment do not get through.

Lower bars will denote steady state slump values, and again all variables are in per capita terms.

Output, given that no savings flow through to firms to finance investment projects, is given by the level of exogenous output $\overline{y}$ (which is state independent). The equation $\frac{\overline{M}}{\underline{p}} = \overline{y}$ yields the price level if the velocity of circulation is equal to one. A glance at equation (21) and the accompanying argument will reveal that this holds. Clearly a necessary condition for the existence of such a steady state slump is then $\overline{y} < x$, so that the non-participation occurs. Can we also find a full set of prices such that the above is an equilibrium? For zero labour demand, we must have that;

$$\frac{\underline{R}\,\underline{w}}{\underline{p}} \geq 1 \tag{22}$$

must hold.

On the other hand for zero labour supply, with equilibrium consumption equal to $\overline{y}$, (10a) dictates that we must have;

$$\frac{\underline{w}}{\underline{p}} \leq \overline{y} \tag{23}$$

Clearly if we choose $\frac{W}{P}$ to satisfy this condition for zero labour supply then we can make R arbitrarily high to choke off labour demand, since no matter how high is the interest rate, participation by the savers is effectively prohibited. The indeterminacy of the nominal and real interest rate and the real wage rate is a result to be noted.

6.2.5. Business Cycle.

The scenario that I described above for the economy in a steady state slump in which the interest rate cannot transmit the 'required' signals provides intuition as to why the economy may fluctuate between states of high and low activity in successive periods. The spot markets that are held in a slump period are effectively isolated from those in the ensuing boom period due to the lack of voluntary holding of any intertemporal value store.

As a result of this 'isolation', in the slump part of the business cycle, allocations and prices will be the same as in the section above. (R or $\frac{W}{P}$ may again be indeterminate). We should further remind ourselves that the unit velocity of circulation of money that we hypothesised is 'self supporting' and does indeed hold since we have;

$$\frac{1}{p_t c_t} > \beta \frac{1}{p_{t+1} c_{t+1}} \tag{24}$$

if we have $\beta < 1$ then the inequality will clearly be satisfied, for both booms and slumps; the same argument applies as previously.

The prices and interest rates in the boom part of the cycle are the same as in the boom steady state equilibrium, as is verified by the intertemporal saving first order condition;

$$\frac{1}{\overline{p}\,\overline{c}} = \beta R \frac{1}{\underline{p}\underline{c}} \tag{25}$$

This implies that $\beta R = 1$ holds since velocity of circulation is equal to one.

In conclusion, as well as demonstrating the existence of two possible steady states of the economy I have also shown the possibility of fluctuating activity from period to period as determined endogenously by the level of real money balances. The economy switched between participation and non-participation in investment into firms, via participation and non-participation in the 'bond' market. An interesting contrast with the surveyed literature is that in the model I have exposited the non participation in the 'bond' market causes activity to switch to a different level of activity which is itself locally determinate. This compares with the situation in overlapping generations models for instance where the non-participation opens new dimensions of indeterminacy.

An important result is also that I have shown the existence of an endogenous two period cycle without the need to violate gross substitutability between labour and consumption. This is contrary to the assumptions of Grandmont (1985) and although Authors such as Reichlin (1986) have found solutions, the method used above is a new application to the problem.

6.3. Distribution of Liquidity and Investment Behavior.

6.3.1. Introduction.

The focus of the thesis prior to this chapter has been on ex-ante identical agent economies which evolved over time to exhibit a non-degenerate distribution of liquidity across agents. The model above had a 'temporary' distribution effect as the representative household separated into different units to trade. The model I develop below has some ex-ante heterogeneity. I appeal to the intuitive notion that when investment projects are intrinsically risky and no insurance is available then only the less risk averse agents in the economy may wish to invest. It then becomes an important issue as to whether these agents have a stock of liquidity sufficient to yield the highest possible level of investment that is feasible in the economy, and which financial structure can provide that. The distribution of liquidity in the economy is hence important, and I show that it can affect the rate of investment and lead to the persistence of low levels of output over time, when we invoke the assumption of the indivisibility of investment projects and a minimum size of investment. The key factor of the model here is again endogenous participation in investment, as determined here by the risk attitudes of the agents in the model. I assume again that the projects are indivisible across agents. The shocks to productivity are assumed to be aggregate in nature.

We can contrast the result I obtain below with the work of Scheinkman and Weiss (1986) who obtain a result for the persistence of shocks by noting that those with a low income shock have to run down their assets now and so cannot spend later, and those with a high income shock can accumulate wealth, which means that future work incentive is reduced. I develop a model where the heterogeneity is exogenous and liquidity flows round the economy in a non-stochastic fashion (simply to aid tractability), but the persistence of a low shock to output can be explained in terms of the need for investors to accumulate wealth to reach the minimum level of wealth required to finance a project. In an economy with a debt market I shall show that the acquisition of liquidity can be 'immediate' and so low output levels need not persist. In a pure monetary economy with debt constraints however the wealth needs to be accumulated over time. In Scheinkman and Weiss a process of accumulation is associated with undesirable results whereas the opposite is true in my model.

6.3.2. The Model.

The changes in the level of liquidity in the model exposited in the last section are endogenous and unconnected to any variation in the fundamentals. The fluctuations sketched out are deterministic, but one might imagine the construction of a sunspot equilibrium, where the economy switches between high and low states of activity according to a stationary 2 state Markov process. The model exposited below exhibits random aggregate fluctuations which have a different source to the class of fluctuations examined above. They are of the real business cycle type, with random productivity the underlying cause. The role of liquidity that I examine is in explaining the persistence of a low shock to productivity on output. I develop a scenario where investment projects are fundamentally risky and only some agents in the economy will possibly wish to invest in these projects. The lack of any opportunity for investors to borrow following a poor productivity shock interacts with the indivisibility of investment projects and a minimum investment size to yield a situation where they must spend time accumulating liquidity to be able to invest again. In some aspects this is in a similar vein to the effect examined by Scheinkman and Weiss (1986). What I present below is different in that the model shows the effect of liquidity distribution on the ability to invest in productive but risky projects, again utilising the indivisibility of investment projects. The effect of what I show is that any initial period of low output is followed by another period of output below the 'maximum' output possible with probability one. A period of high or low output will in turn follow this dependent upon the realisation of the productivity process. The economy can be viewed as an endowment economy augmented with a risky production technology. This requires a minimum investment size which is one that would make investment in all projects in each period feasible given the total level of resources in the economy. However I model consumer preferences relative to the production technology such that only one set of consumers will wish to invest in the technology. Models such as mine with an infinite horizon and a potentially infinite number of shocks might lead to a distribution of money that is difficult to handle. Authors such as Taub (1988) and Lucas and Stokey(1989) whose methods I have followed in chapter 4 employ utility functions that yield corner solutions, as does Levine (1991), and this is the method I employ here. The degenerate utility functions employed here yield a simple price path, and aids the logistics of the model. I make agents on one side of the economy behave in a manner that implies a liking for the risky asset. I do this by assuming that they only value consumption at some particular level, and that their rate of time preference is arbitrarily high; they are hence impatient to reach this target. I assume that level is sufficiently high in relation to the per period income of the consumer B type, so that he prefers to gamble in the risky investment technology which will yield him the possibility of a return quicker than through waiting for accumulation through the safe asset.

The financial set up we analyse first is a pure monetary economy in which money flows from one side of the economy to the other in alternate periods. This alternating flow is essentially driven by periodically fluctuating tastes on the part of the risk averse agents. Such a situation is created for the sake of simplicity in the flow of money around the economy and does not prejudice the analysis of the persistence of shocks, but is essentially tied up with the lags in liquidity accumulation inherent in the credit constrained economy.

To trace out the story that the model will tell, we begin by imagining a period following a poor production outcome from which the type B agents carry over no wealth as a result (this will be justified later). The endowment of the good the consumer of type B has is insufficient to finance an investment project (remember that no more than one agent can finance any given investment project) and is also below the minimum level of consumption at which he derives utility; so his only available policy (other than disposal) will be to sell his endowment y on the market for money to save until the next period. Given this saving, in the following period he then has enough wealth to meet the minimum size of investment required, 2y (I will assume that this is also the maximum investment per project; i.e. complete indivisibility in that sense). The level of wealth however is still not enough to meet his minimum consumption requirement, 3y and so he will choose to invest in the risky production technology. He makes this choice since there is a positive probability of reaching the threshold level of consumption in this period since the investment pays off this period if it is successful. Coupled with impatience this makes the risky asset more attractive than saving in money. This he does by spending his money on the competitive spot market to obtain the real resources from consumer A to invest. If the project is successful (with probability p) then the investment of 2y will yield a return of 4y this period and 3y next period. The shocks to productivity are aggregate shocks; they are perfectly correlated across all individual projects. I assume that the consumers of type B are satiated at a level of consumption 3y and so consume the proceeds in this period. Given my assumption about the rate of time preference he will obviously consume no less, and any potential difficulties in the calculation of B's optimal policy is avoided by the arbitrarily large rate of time preference; he will always favor consumption over saving as long if he has liquidity enough to take him to his minimum consumption level, but of course he will consume no more than 3y. I assume that only one round of investment and one round of trading is permitted in any one period. This is a simplifying assumption to add a time dimension to activity and make the model 'move forward'. In the following period the second part of the output of 3y is then added to the endowment of y for each B agent. The optimum strategy is then to consume 3y this period (since he is satiated at this level of consumption) and to carry over the value of y in money to the next period to carry out investment, and so on.

The desirability of the risky asset for this investor might be termed to be the 'liquidity' that it provides in the sense of providing the possibility of reaching his required target of a minimum consumption level more quickly. However, a poor shock will yield no output at all and the ensuing period must be spent by the investor accumulating enough wealth to invest during the period that follows that. We may view that period as one during which the bankrupt investor must earn to accumulate enough wealth to invest again in the risky asset. I shall show that in a pure credit economy this problem does not occur, and indeed that investment takes place in every period of the model in that case, even when seemingly unnecessary.

6.3.3. The monetary economy.

The economy is populated by a large number of agents who are divided equally into 2 types, A and B. They differ only in respect of their utility functions, which are, for type A;

$$U_A = E_0 \sum_{t=0}^{\infty} \beta_A^t u_{At} \tag{1}$$

I assume $\beta_A > 0$ but that β_A may be assumed to be as low as needed to prove required results that will follow.

$$\begin{aligned} Where\ u_{At} &= H \ln (x + c_{At}) \text{ if t is even and } c_{At} \leq 3y \\ u_{At} &= H \ln (x + 3y) \text{ if t is even and } c_{At} \geq 3y \end{aligned}$$

$$\begin{aligned} u_{At} &= L \ln (x + c_{At}) \text{ if t is odd and } c_{At} \leq 3y \\ u_{At} &= L \ln (x + 3y) \text{ if t is odd and } c_{At} \geq 3y \end{aligned}$$

I also assume;

$$H > L$$

I will later invoke the assumption that this dominance by H can be assumed arbitrarily large to prove required results.

For type B;

$$U_B = E_0 \sum_{t=0}^{\infty} \beta_B^t u_{Bt} \tag{2}$$

$$Where\ \mathrm{u}_{Bt} = 3y \text{ if } \mathrm{c}_{Bt} \geq 3y$$

$$u_{Bt} = 0 \text{ otherwise}$$

Each agent (regardless of type) has an endowment stream of y per period of the economy's homogenous consumption/investment good. In addition each agent of type A has $2\overline{M}$ in nominal money balances at the start of period zero.

All agents have access to a random production technology whose gross multiplicative return for a minimum investment of 2y is 2 in the period of investment and $\frac{3}{2}$ in the following period with probability p (hence yielding a sequence of 4y and 3y in total). Failure of the project occurs with probability (1-p) and yields a gross multiplicative return of $\frac{1}{2}$ in the period of investment and zero in the following period. I will assume that in a period where investment has taken place then no trade after the production outcome has taken place is permitted. Reinvestment within a period is also forbidden. In the following period however any output that accrues from the investment in that initial period may be traded or reinvested. In other words, only one round of trading and one round of investment may take place in any period.

There is a sequence of spot markets, one at each date on which money can be traded against the homogenous consumption/investment good. There are no other markets or assets in existence in this monetary economy. I shall denote the per capita nominal money stock as $\overline{\text{M}}$. The money price of the good at time t is p_t. Social feasibility at each date hence requires per capita consumption less than or equal to y and per capita money demand less than or equal to $\overline{M}$.

We shall determine the equilibrium by firstly suggesting a path of output, trade, consumption and prices, and then demonstrating that it satisfies individual optimisation conditions subject to these prices, and of course social feasibility.

The monetary equilibrium I propose is one with a constant nominal price of goods in which the whole stock of fiat money changes hands each period across the two types of agents, starting the model off at time zero (note this is an 'even period') with the A type consumers holding $2\overline{M}$, and type B consumers not carrying out any investment. This will later be seen to be equivalent to the period following a poor productivity shock). I propose that type A consumers will consume 2y each in this period upon spending their money, and zero in the next period (one) as they sell their endowment for money. We are hence proposing a price level of $\frac{2\overline{M}}{y}$. In period zero the accumulation of wealth by type B's is taking place in readiness for investment in the following period. When this period arrives (period one in this scenario) the type B agents will spend their accumulated balances on the market for goods to obtain the extra y they need to make a real investment which then occurs. If the investment fails then an output of y accrues which by assumption cannot be retraded (i.e. the agents cannot sell it for money to gain liquidity for future use since investment and trading have each taken place once in the period already). Re-investment is ruled out by assumption but of course there is now not enough to meet the minimum investment level anyway, neither is the level y enough to give agent B utility. The output is then simply and costlessly disposed of by B. The economy then returns to the initial situation in the following period (two), as I suggested, as it will be a post 'low outcome ' period during which the type B agents must accumulate liquidity. The per capita level of output of $\frac{y}{2}$ in for that initial investment period will then hence be followed by a period of no investment (an accumulation period), with per capita output equal to y, the exogenous endowment which is below that which a successful investment would yield. This is the essence of the persistence in the monetary economy, and will be seen even more clearly when we contrast this with a credit economy later.

If the initial investments in period one are successful however then a return of 4y per project is yielded. The type B agents will consume 3y and then dispose the rest, since they are satiated at that level and the arguments sketched above on uselessness will follow also. The 'extra' output will come into play when we analyse the non monetary economy.

In the period following this output of 3y per project accrues to the type B's and is added to their endowment of y. Given their utility function and arbitrarily high degree of impatience they will consume the amount 3y. The remaining quantity y of the good they will not consume this period as it yields them no utility. Investment using this quantity y is not prohibited by the assumptions of the model but is not sufficient to finance investment in this period due to the fixed size of each undertaking. We shall see that the credit economy on the other hand will allow investment again in this period. The only option in a monetary economy for the type B consumers is to sell the amount y on the spot market for fiat money to gain balances of $2\overline{M}$. As a result of the successful investment in the preceding period, output was hence high in the period we have just described. 'Positive' persistence is hence forced on the model exogenously. It is 'negative' persistence that we seek to explain in talking about financial structure. We should also note however that in the credit economy which follows we can have investment in every period of the model, regardless of the shock history.

The period (period 3) two periods after the investment was first undertaken then begins with the type B agents each holding $2\overline{M}$ in nominal money balances and the type A agents holding zero balances. The trade that occurs then is that the type B agents spend their balances and the type A agents sell their endowment. Investment by the type B agents then can take place at the level of 2y. Whether the investment is successful or not determines which of the paths described above will be subsequently followed by the economy. Note that investment will occur in alternate periods in this monetary economy, which correspond to periods in which consumers of type A consume zero.

In summary of this position, we can describe the equilibrium sketched above as follows. It is a sequence of consumptions for agents of type A of 2y, 0, 2y, 0,........., starting with an even period. The sequence of activity for Agent B types begins with them holding zero money balances Corresponding to the first period of 2y consumption for the type A agents, type B will have a consumption sequence starting with zero (an accumulation period) followed by a period of investment, and consumption of 3y if the investment is successful, with probability p. Such an occurrence would mean consumption of 3y in the subsequent period also. If the investment fails, with probability 1-p , consumption is zero in the period of investment and zero in the subsequent period as accumulation of money takes place. Investment is zero in this period following failure; we have hence returned to the initial situation we described and the process starts again. This period of no investment that follows means the persistence of output below the maximum level possible. This is the key result; the rate of investment that is facilitated by the financial structure of the economy. If we use final per capita output potentially available for consumption as our measure of economic activity (including output that might be disposed) then we have a stochastic sequence as follows. Beginning in a period of investment we have with probability p per capita output of 2y in this period and output of $\frac{5}{2}$ y in the following period. (Remember that only the type B agents will invest). This latter figure includes the exogenous endowments of the two agents (y in per capita terms, with no investment taking place in that period) . Returning to the initial period of investment, per capita output will be $\frac{y}{2}$ with probability 1-p and y in the subsequent period (i.e with no investment taking place whilst accumulation occurs) . Whatever the outcome of the initial investment, two periods later the process will 'start again'. The persistence I have described can be seen from the period of low output following the failed investment period with probability one. Investment will occur in this monetary economy in alternate periods with probability one. In the intervening periods agents of type B are accumulating money to buy goods for investment in the next period. This is not a problem when the investment has been successful as output is high for two periods running, but when investment has failed output will remain below the highest possible level with probability one in the ensuing period. The problem is the liquidity constraint on agents of type B that arises because of the lack of a market to channel the endowment of agent type A to the investor 'class'. The contrast will be seen most sharply when we later examine an economy that has such a set up, where we may have investment occurring in every single period of the model, regardless of the economy's history. Serial correlation for high shocks will remain but low production outcomes need not hamper future investment.

Having sketched the suggested equilibrium for the monetary economy, I now need to show that the policies of the two types of agents described are indeed optimal, including their decisions on participation in risky investment.

I must firstly show that for the agents of type A a consumption sequence of 2y, 0, 2y, 0,...... is optimal (beginning in an even period), and that they achieve this only by holding money and choosing not to participate in the risky investment. If they do choose to hold only money then given a constant price level of $\frac{2\bar{M}}{y}$ the consumption stream described can be supported by selling all their endowment in the odd period and spending all the proceeds in the following even period. If we look back to equation (1) that gives us the utility function of type A agents then we can see that given a constant price level then the conditions below will suffice to ensure that such a consumption sequence is optimal.

$$H \ln (x + 2y) \geq \beta_A L \ln (x) \tag{3}$$

$$L \ln x \leq \beta_A H \ln (x + 2y) \tag{4}$$

Clearly (4) is sufficient for both, and we can ensure that this holds by making the value of H sufficiently high relative to the value of L. This will suffice even if β_A is made arbitrarily low. In other words, whatever the value of $\beta_A > 0$, there will exist a value of $\frac{H}{L}$ such that (4) holds.

What parameter restrictions will suffice to yield non participation in risky investment by the type A's ? The minimum size of participation in a project is 2y and so like the type B's they must wait to accumulate wealth if they are to invest. Given that H>L is assumed to hold (along with discounting) then if we can show that the policy of accumulation in an 'L' period and investment in an 'H' period is dominated then so is a policy of accumulation in an 'H' period and investment in an 'L' period. Furthermore, following this we can rule out a policy of waiting to accumulate precautionary balances over a longer period and then investing only part of them in the risky asset by making β_A arbitrarily small. We begin the analysis by assuming that they have chosen the former. We start by assuming that they arrive in the 'H' period with 2y as their disposable wealth (note that (4) still holds). Their margin of decision is between consumption of 2y with probability one now, and investment with a probability p of consumption 3y in both this period and the next, or with probability 1-p consumption of y this period (they have no lower threshold for deriving utility) and zero next period, when accumulation occurs after the investment failure (i.e. we return to the original situation). Remember that type A's as for type B's are satiated at a level of consumption 3y. If successful the extra output above 3y is discarded in the first period, as by assumption it cannot be stored, traded or invested again, and the second period excess is sold on the market for money in the process of accumulation for reinvestment next period, as in the scenario I described for the B agents. Following this the sequence will repeat and so the analysis can be reduced to what appears below.

If investment in the risky technology is to be less attractive then we must have;

$$
\begin{aligned}
H \ln (x+2y) + \beta_A L \ln (x) \quad > & \qquad\qquad\qquad\qquad\qquad\qquad\qquad (5) \\
& H \left(p \ln (x+3y) + (1-p) \ln (x+y) \right) \\
& + \beta_A L \left(p \ln (x+3y) + (1-p) \ln x \right)
\end{aligned}
$$

Given risk aversion on the part of the type A agents then clearly a sufficiently low L relative to H will suffice to make this inequality hold if we have;

$$p \leq \frac{1}{2} \tag{6}$$

We now have to show that the actions of the type B agents that I described in the equilibrium are optimal under such conditions. In the period following a poor production outcome the type B agent has no choice other than accumulation in money. His wealth of y is below both his investment and consumption thresholds. In the period following that, he has wealth of 2y and faces the choice between saving again in money to obtain a level of consumption of 3y the following period with probability one and gambling in the investment project which has a positive probability of success this period, allowing consumption of 3y now if that is the case and 3y next period also. Utilising this information then given the form of the type B's utility function in (2) a sufficient condition for 'investment now' to be more attractive is;

$$p\,(3y + \beta_B 3y) > \beta_B 3y \tag{7}$$

Rearrangement will yield that for any strictly positive value of p, if β_B is sufficiently low then the 'invest now' strategy is optimal.

I have hence verified that the optimising actions of the two types of agents are compatible with the monetary equilibrium I described.

The scenario analysed above is such that only one set of agents will ever wish to invest in the productive technology available. The result was the persistence of a 'low' level of output with probability one for the next period, due to the distribution of liquidity in the economy. A level of wealth of 2y that will make investment feasible but is held by an agent of type A will not cause investment to take place. The financial structure I impose on the model in the analysis below is such as to remove that tendency, by allowing investment in every period.

6.3.4. A Credit Economy.

The financial structure I assume here is such that there exists equal access to the risky technology as previously, but the riskless debt asset I introduce has restricted usage. I allow Type B agents to lend and borrow in the asset, but type A's are constrained only to lend. This aids the tractability of the model by avoiding an embarrassment of riches in the wish of agents to participate in the risky asset. Endowments and preferences are exactly the same as in the monetary economy, except that the endowment of fiat money is no longer present. Preferences and technology are exactly the same as in the monetary economy.

The inside asset is allows the lender to lend and receive repayment for consumption purposes within the same period with probability one. Hence type A agents may loan to type B agents and receive repayment within the period so that consumption in that period by type A's may take place. Given the lack of ability to borrow in the safe asset that I impose on type A agents and the absence of the intertemporal value store (money) in this economy the only options open to these agents are either autarchy or investment in the 'within period' safe asset (they can never accumulate the level of wealth required to invest). Clearly if the gross return on the safe asset is greater than or equal to one then the type A agents will be willing to invest in it. If not they will choose autarchy.

We shall again assume that the projects are not divisible among the investors.

We again begin the analysis in a period in which investment is required (post failure or two periods after a success). We will call this period zero. All agents of type B have an endowment of y each and if the interest rate is favourable then they will borrow to invest in the risky asset at the threshold level of 2y. Since they are required to repay the debt within the period with probability one then the minimum return of y on the investment means that a gross interest rate of one or less will induce the desire to borrow, as such a scenario means that if the investment was successful then this leaves 3y for consumption. As we noted a gross interest rate of one will be sufficient to induce lending and so that the equilibrium gross interest rate for that period will be one. Investment by type B agents will take place and if the project fails the return per project of y will be repaid to each agent A. If the project succeeds then the total return per project is 4y, so that the type B agents can consume 3y and repay the loan of y to the type A agents. The period following a success sees the type B agents with their endowment of y plus the second round of returns on investment, 3y. They will consume 3y and borrow y from the type A agents at a gross interest rate of one and reinvest during this period. Recall that one investment per period may be made. A repayment of y will then be made to the type A agents no matter what the state. If the investment fails then the following period the economy will return to the initial state we began with. If the investment fails then the payoff of 3y above that needed to repay the type A agents will be costessly disposed of as the type B agents are satiated and have no opportunity to save the output. This was the case with the original excess y from the previous period; i.e. a lack of options. The investment is carried out in this post success period since there is a zero opportunity cost to its usage and since β_B is strictly positive then expected discounted utility gains can be made through the possibility of the successful payoff next period. If this period one investment is successful then in period two the type B agents will each have 4y to handle. We are hence back in the post success scenario we just described. If the investment in period one failed then we simply begin again in the period zero scenario. We summarise the equilibrium as follows. In all periods the 'within period' interest rate is unity and each time the type A agent chooses to 'invest' in this asset. In a period after a failed investment, then in contrast to the monetary economy investment will take place again. Per capita output will then be 2y with probability p and $\frac{y}{2}$ and with probability 1-p. In the period following a success investment will occur again and may result in per capita output as high as $\frac{7y}{2}$ if successful.

I have shown in this credit economy that investment will take place in every period of the model, even in those where it is apparently unecessary, namely those periods following an initial success where agents of type B are satiated for that period. This speeding up of the rate of activity or the frequency of investment (which effectively doubles) is a result of the financial structure I have assumed. This is of course the credit instrument which allows the transfer of resources 'immediately' from the risk averse side of the economy to the risk loving investors. This contrasts with the lags inherent in the pure monetary economy where all agents are liquidity constrained. The key point of interest however is that low outcomes of production process do not mean low output with probability one in the following period. The persistence of low output states disappears.

6.3.5. Conclusions

This chapter looked at the implications of financial imperfections for aggregate fluctuations that were endogenously generated or persistence that was explained endogenously. The key contributions flowed from the indivisibility of investment projects among the potential investors and the minimum size of investment required. Endogenous market participation was the key which was determined by the state of the economy or by the time taken to accumulate the necessary liquidity.

In the first model I showed that a minimum size of investment coupled with an endogenously determined level of liquidity could yield market participation that was endogenous to the model and lead to multiple equilibria. The possibility of endogenous cycles without the need to violate gross substitutability was also noted.

The second model again used minimum investment size and indivisibility of projects across investors, but this time to demonstrate the possibility of persistence in exogenous aggregate productivity shocks. In general we can say that non-convexities and incomplete markets were shown to be crucial and results suggest that more research along these lines may be of benefit. For instance we might look at the implications for endogenously generated fluctuations of irreversible investment and persistence. In particular we might view the irreversibility as coming from the thickness or thinness of the second hand capital goods market. This can influence willingness to participate in investment in the first place, with the kinds of implications that we have traced out already.

Chapter 7

Conclusions

7. Conclusions.

In this thesis I have presented a number of original models which all consist of contemporaneous and infinitely lived agents who face an imperfect financial system, which creates the opportunity for fiat money to be valued and so adds to the strand of literature begun by Bewley (1980). The essential nature of these models is that individuals are essentially heterogeneous in the equilibrium of the model; buffeted by stochastic shocks or affected by endogenous fluctuations in the model; also they may be in different sectors of the economy, or carry differing attitudes to risk. Such a style of modelling is surely important if we are to gain the full benefits of a general equilibrium modelling strategy to answer monetary and macroeconomic questions. In these models money changes hands amongst agents in equilibrium, affecting the real allocation of resources and allowing the claim that the models are 'essentially' monetary in the sense of Hahn (1965, 1973).

In all models presented, the distribution of currency played a central role. Chapters 2 to 5 exposited models where money was valued as insurance against idiosyncratic production, income or taste risk. In chapters 2 and 5 the fact that money had to change hands between agents to perform its role led to interesting conclusions about the implementation of the optimum quantity of money proposal of Friedman (1969).

As I suggested in the introduction to the thesis, the concern of those first four chapters was anchored in a field that essentially grew out of microeconomic general equilibrium theory and pure monetary theory; i.e. that part of monetary theory which does not deal with macroeconomic fluctuations. Also, as Lucas (1992) points out, the study of the means by how an economy deals with individual risk can shed light on 'current' unequal income distributions; inequality may be the result of an uneven distribution of past income or taste shocks not corrected for by the market. The area of pure monetary theory that the models addressed principally was of course the optimum quantity of money, and I found some interesting results which contribute to that question by applying a non-representative agent framework. However, the quantitative importance of these models has not been addressed. The key results in the literature that address this issue, namely the welfare costs of inflation under imperfect insurance, are by Kehoe, Levine and Woodford (1992) and Imrohoroglu (1992), who present contrasting results. Imrohoroglu finds that an inflation rate of ten per-cent yields a welfare loss equivalent to one per-cent of GNP. Kehoe, Levine and Woodford show in their model that an increase of inflation of about one percent yields only a 0.004 percent GNP equivalent change in welfare, though they concede that a model with a more complex role for money (i.e. with other assets) may yield different results. Kehoe et al correctly ascribe the difference to the fact that their model is one with aggregate uncertainty and the possibility of a maldistribution of liquidity that Scheinkman and Weiss(1986) and Levine(1991) also consider. It is this kind of problem that inflationary policy can aid; redistributing the liquidity in the economy rather than raising its level by the Friedman style policy. Imrohoroglu has a model in which there is no aggregate uncertainty and high transition probabilities between states so that such a distribution issue is less important. What I feel is interesting about these two results is that they show that the welfare costs of inflation may not be a trivial issue per se. What drives the small welfare cost in the Kehoe et al model is that there are both positive and negative effects of inflation on welfare, <u>not</u> that the absolute values of these two factors are necessarily low. This also implies that more sophisticated models of money might yield larger figures for the effects of inflation than Kehoe et al.

The results can hence be billed as being of theoretical interest for those fields of pure monetary theory and distribution but also yielding some 'macroeconomic' insights; the functioning of a general equilibrium model with money and an imperfect financial structure is clearly of interest to that discipline. Abstract though many of the models seem to be, there is no reason why the low level of exchange that results from incomplete markets and an insufficient stock of liquidity cannot be considered as an explanation of the level of macroeconomic activity. Again I can say that the models are more than purely monetary. We can indeed follow chronologically the appearance of models in the literature and the themes they tackle; from Bewley (1980) through to Scheinkman and Weiss (1986) and Fuerst (1992) who apply the study of the quantity and distribution of liquidity to aggregate fluctuations. We might also hence trace such a thread through the models presented in the thesis. The final theoretical chapter contains two models, the first one of which displays the possibility of the level of liquidity fluctuating in equilibrium in an economy which also displays an indeterminate steady state, due to the endogenously determined possibility of incomplete market participation. As the literature reviewed in that chapter emphasises, the kind of financial imperfections that are investigated in the thesis have profound implications for the set of equilibria that the model possesses and for the role of expectations in endogenously generating aggregate fluctuations. The second model shows how the distribution of liquidity in a real business cycle type model with ex-ante heterogeneous agents can have implications for the persistence of real shocks. A model such as that presented by Farmer(1993) that is essentially a non representative agent model in its form shows the link between the two strands of literature and a promising direction for the future. Farmer exposits the idea that the opportunity cost to society of holding money is higher than that which the nominal interest rate implies. This is due to the externalities in trade of holding money of the kind first suggested by Diamond(1984), who then builds on this to show the possibility of multiple equilibria.

I shall now briefly review the contributions of each chapter and discuss possible extensions.

Chapter 2 filled a gap in the valuation of money in general equilibrium literature by showing that we could obtain a positive value for intrinsically worthless fiat money when it was held as a precaution against poor idiosyncratic production outcomes. The role of the rate of time preference in determining when it would be valued was seen to be essentially different to the role it took in the models of Bewley (1980,1983), since the rate of capital accumulation and inflation were endogenous. The role given to money in the model is that of a portfolio asset as described by Tobin(1958), and we can justify the model's importance in a literal sense by saying that it depicts an essentially Keynesian financial asset motive for holding fiat money. I have talked about the role given to money in the models at the end of the literature survey but we can again recall the defence of Wallace

(1980) of overlapping generations models that the fundamentally non representative agent form of the model with money changing hands is a better representation of the medium of exchange representation in representative agent models (cash in advance or money in the utility function models for instance) . We might in the future fruitfully examine a portfolio model with a financial system in which money is ascribed the role of the only acceptable means of payment in the economy as the ultimate backing for inside financial assets so that these assets served as the medium of exchange but fiat money played a role in the background. We could also then examine money's interaction with real aggregate fluctuations. It would be particularly interesting to see if a model could be constructed where the price level stayed constant despite the aggregate fluctuations, reinforcing its safe asset role (i.e. surety of real value as well as of nominal value; as opposed to assets with a possibility of default). This might be fruitfully approached within the context of a model of imperfect competition, so that the price level was truly endogenous.

I then looked at the implementation of Friedman's optimum quantity of money proposal under the conventional restriction that the taxes should be lump sum in nature and independent of the agent's income. I showed that no equilibrium with circulating fiat money could exist if the rate of interest payed on money was strictly positive. This was shown to hold in spite of the presence of an alternative store of value; namely capital, so that agents could hold money strictly less than the present value of the tax bill at all points in time and still stay solvent with probability one, so that the Bewley difficulty was proved to be absent from that case and I proved that the result I found has an essentially different cause to the Bewley problem. I then applied the intuition gained from the model to conjecture another difficulty that could arise in the Bewley type pure exchange model; that of a kind of 'preference' motivated Bewley difficulty, based on the period utility functions having infinite marginal utility at the point of minimum income.

Chapter 3 used the same mathematical framework for the model that I developed in chapter 2, but specified an information structure which allowed taxes to vary with individual actions in an attempt to prevent the problem described in chapter 2 occurring. Due to the similarity of the nature of that problem to Bewley's in that it results from the nature of the taxation, I addressed the study to a conjecture of Hellwig (1982) and Woodford (1990) that if taxes could be levied on individuals in a way that avoided the 'Bewley difficulty' in an endowment model then the information structure would have to be such as to render money inessential to the model; i.e. its value would go to zero. I examine this conjecture in the context of my risky capital model and find that it is not necessarily true. The basis of the result is that the verification of a transfer is only truly costless to the receiver. Tax payment can be verified costlessly by the government but insurance payouts can only be verified by the consumers who receive them. It is clearly in their interest to cheat and claim non receipt whereas the government is assumed to be essentially benevolent to society as a whole. The difficulty can be overcome using an alternative form of payout scheme. Instead of directly redistributing the proceeds to individuals who may not be truly needy, the government can just carry out the optimum quantity of money proposal since the tax difficulties are avoided. We assume an information structure in the model that means the form of taxation used is in fact non-distortionary, so there are no problems of the kind first noticed by Phelps (1973). Nevertheless I have identified an essential asymmetry that can be used to refute the Hellwig/Woodford conjecture. I also suggested that in a model where the state of nature (as governed by the exogenous stochastic process) was directly observable, then the same result could be derived (i.e. refutation of the Hellwig/Woodford conjecture) by assuming an asymmetry between observation and recording of the initial state and the registering of the reallocation of goods legally with a third party.

Of course, the optimum quantity of money issue is not new, but its interpretation as overcoming an information problem certainly is. The way it overcomes the information problem builds on some of fiat money's essential properties of anonymity, common usage and ability to store information implicitly about an agent's previous expenditures. Carrying out a policy to change its value will benefit all money holders; namely all those in the economy, by giving them improved means to insure against poor production outcomes. The result also implied an advantage of deflating the price level over paying interest directly on money balances; verification of a transfer is again the key factor.

The notion of money being an aid to information is established, but its possibilities as a restrictor of the flow of information is less well commented on. The problems created by saving in an anonymous asset instead of buying contingent claims for goods in the future and its implications for macroeconomics are well known, but I feel that more consideration of money's information properties may be worthwhile; perhaps the anonymity of money may be a two edged sword. For instance we might conjecture that in some circumstances the level of trade supportable in the long run by an enduring trade relationship formed in the absence of money would exceed that supportable by the continued usage of the immediately beneficial fiat money.

Chapter 4 looks at another essential property of fiat money, that its nominal return is the same in all states of nature, and this is used to construct an equilibrium in which money is relatively 'not a good thing'. This led us to an answer to the conjecture of Levine (1985) as to whether an equilibrium with fiat money could be Pareto dominated by one without it. The key question was how we could construct an economy with infinitely lived agents where the rate of return on money was higher than that matching individual and social opportunities, and yet was compatible with the existence of monetary equilibrium,(I gave a few examples which showed the nature of the challenge). The crucial factor in achieving this result was to add extra assets to the model, which is of course an important future direction for the field of study to take. The result I found was achieved by showing that money was 'wrongly' accepted by agents instead of an insurance asset which was overpriced. I suggested that this was connected very essentially to money's imperfections as an insurance asset as opposed to a buffer stock.

Chapter 5 looked at the issue of the superneutrality of fiat money and found a superneutrality result that is new to the monetary growth literature in its form. Usually superneutrality results show that real activity is independent of the rate of inflation. This result, which builds on Bewley's difficulty shows that the rate of inflation is independent of the rate of monetary growth for negative rates of money growth; hence a new difficulty with the optimum quantity of money proposal is uncovered, for this finite horizon economy. The source of the result can be said to be that the wealth of the agents is non-capitalised with a probability less than one and since the tax bill must be met with probability one, then this produces a Ricardian type 'timing independence' result. Two obvious extensions of this for the future are hence to apply the 'positive probability of zero income forever' factor to a model that examines Ricardian equivalence explicitly (which is yet to be done), and secondly to produce a finite horizon version of my chapter 2 model so that the problem I uncover there can be used to produce a superneutrality result, but one that holds for both positive and negative rates of money growth.

Chapter 6 turns to issues different to those of 'pure' monetary theory by examining the effects of the level and distribution of liquidity in the economy on macroeconomic fluctuations. Two models were presented which had non-convexity in investment opportunities in the form of a minimum size of real investment required and indivisibility across agents. In the first model I showed how, when coupled with a shortage of liquidity held by investors; the non convexity led to the possibility of high and low steady states of activity and also cycles. Participation was endogenous in contrast to the overlapping generations models of endogenous fluctuations and violation of the gross substitutes assumption was not required to produce periodic solutions. In the second model I showed how the distribution of liquidity can determine the rate of investment and cause the persistence of low output. Again, non convexity of the set of investment choices is crucial in interacting with the imperfections of the financial system. The effect of incomplete financial markets was in restricting the flow of liquidity to the agents whose preferences led them to undertake investment whenever possible. Impatience by them to reach a certain threshold level of consumption was seen to be the key factor. This way of using general equilibrium models with financial imperfections to explain macroeconomic fluctuations and persistence is one on which I feel fruitful research time may be spent in the future. As I have documented, the role of financial imperfections in allowing expectations to influence aggregate economic activity is already a large and growing literature. However it is surprising in some ways that models of the type of Scheinkman and Weiss that use the distribution of liquidity have received much less attention than for instance menu costs of changing prices in the search for explanations of the persistence of shocks. One possible area of synthesis has been suggested by Laidler (1988): He notes that since the costs of individual agents not changing prices is a crucial factor then the level and distribution of precautionary balances present in the economy is can help to determine agent's price setting actions.

As a direction for future research I would suggest that an issue that needs clarifying is exactly in what circumstances restricted market participation can have the kinds of effects that we have described, particularly in allowing expectations to influence the real equilibrium. The models in the literature so far are only really suggestive of what happens when no set of consumers of positive measure are allowed to participate in all markets. With many markets, goods and agents it would be interesting to know what configurations of ability to participate in which markets led to which sets of possible allocations.

8. References.

M.Allais(1947). Economie et interet. Paris: Imprimerie Nationale.

F.Allen and D.Gale(1994). Limited Market Participation and Volatility of Asset Prices, *American Economic Review*, 84, 933-955.

K.Asako(1983). The Utility Function and the Superneutrality of Money on the Transition Path, *Econometrica*, 51, 5, 1593-1596.

C.Azariadis(1993). Intertemporal Macroeconomics, Blackwell.

Y.Balasko, D.Cass and K.Shell(1980). Existence of Competitive Equilibrium in a General Overlapping Generations Model, *Journal of Economic Theory*,23, 307-322.

D.Bernhardt(1989). Money and Loans, *The Review of Economic Studies*, 56, 89-100.

T. Bewley(1980). The Optimum Quantity of Money, in Models of Monetary Economies, eds. J.H.Karaken and N.Wallace, Federal Reserve Bank of Minneapolis.

T.Bewley(1983). A Difficulty with the Optimum Quantity of Money, *Econometrica*, 51, 1485-1504.

W.A.Brock(1974). Money and Growth: the Case of Long Run Perfect Foresight, *International Economic Review*, 15, 750-777.

J.Benhabib and R.Farmer(1991) Aggregate Effects of Monetary Externalities. University of Cambridge Discussion paper.

M.Boldrin and M.Woodford(1990). Equilibrium Models Displaying Endogenous Fluctuations and Chaos, *Journal of Monetary Economics*, 25, 189-222.

D.Cass(1985). On the "number" of equilibrium allocations with incomplete financial markets. CARESS working paper, University of Pennsylvania.

D.Cass and K.Shell(1983). Do Sunspots Matter, *Journal of Political Economy*, 91, 193-227.

D.Cass, M.Okuno and I.Zilcha(1979). The Role of Money in Supporting the Pareto Optimality of Competitive Equilibrium in Consumption Loan Type Models, *Journal of Economic Theory*, 20, 41-80

S.Chatterjee and D.Corbae(1992). Endogenous Market Participation and the General Equilibrium Value of Money, *Journal of Political Economy*, 100, 3, 615-646.

L.Christiano and M.Eichenbaum(1992). Liquidity Effects and the Monetary Transmissions Mechanism, *American Economic Review*, 82, 346-353.

D.Cohen(1985). Inflation, Wealth and Interest Rates in an Intertemporal Optimising Model, *Journal of Monetary Economics*, 16, 1, 73-85.

J.P.Danthine, J.B.Donaldson and L.Smith(1987). On the Superneutrality of Money in a Stochastic Dynamic Macroeconomic Model, *Journal of Monetary Economics*, 20, 3, 475-499.

W.J.DenHaan(1990). The Optimal Inflation Path in a Sidrauski Type Model with Uncertainty, *Journal of Monetary Economics*, 25, 389-409.

P.Diamond(1984). Money in Search Equilibrium, *Econometrica*, 52, 1-20.

A.K.Dixit(1992). The Art of Smooth Pasting, S.T.I.C.E.R.D. discussion paper.

D.Duffie(1990). Money in General Equilibrium Theory, in Handbook of Monetary Economics, volume 1.

J.Dutta and H.Polemarchakis(1990). Asset Markets and Equilibrium Processes, *The Review of Economic Studies, 57, 229-254.*

A.Drazen(1981). Inflation and Capital Accumulation Under a Finite Horizon, *Journal of Monetary Economics*, 8, 247-260.

R.E.A.Farmer(1993). The Macroeconomics of Self Fulfilling Prophecies, MIT Press.

R.E.A.Farmer and M.Woodford(1984). Self Fulfilling Prophecies and the Business Cycle, CARESS working paper.

J.Faust(1989). Supernovas in Monetary Theory: Does the Ultimate Sunspot rule out Money? *American Economic Review*, 79, 872-881.

M.Feldman and C.Gilles(1985). An Expository Note on Individual Risk without Aggregate Uncertainty, *Journal of Economic Theory*, 35, 26-32.

S.Fischer(1979). Capital Accumulation on the Transition Path in a Monetary Optimising Model, *Econometrica*, 47, 1433-1439.

T.S.Fuerst(1992). Liquidity, Loanable Funds and Real Activity. *Journal of Monetary Economics*, 29, 3-24.

M.Friedman(1969). The Optimum Quantity of Money and Other Essays

D.Gale(1973). Pure Exchange Equilibrium of Dynamic Economic Models, *Journal of Economic Theory*, 6, 12-36.

J.Geanakoplos and A.Mas-Colell(1989). Real Indeterminacy with Financial Assets, *Journal of Economic Theory*, 47, 22-38.

J.Geanakoplos and H.Polemarchakis(1985). Walrasian Indeterminacy and Keynesian Macroeconomics, *Review of Economic Studies*, 53, 755-779.

J.Geanakoplos and H.Polemerchakis(1991). Overlapping Generations, in Handbook of Mathematical Economics, volume 4, eds. W.Hildenbrand and H.Sonnenschein, North - Holland.

J.M. Grandmont(1983). Money and Value, Cambridge University Press.

J.M. Grandmont(1985). On Endogenous Competitive Business Cycles, *Econometrica*, 53, 995-1045.

F.Hahn(1965). On Some Problems of Proving Existence of Equilibrium in a Monetary Economy, in The Theory of Interest Rates, eds. F.Hahn and F.Brechling, Macmillan.

F.Hahn(1971a). Equilibrium with transactions costs, *Econometrica*, 39, 417-439.

F.Hahn(1971b). Professor Friedman's Views on Money, *Economica*, 38, 61-80.

F.Hahn(1973). On Transactions Costs, Innessential Sequence Economies and Money, The *Review of Economic Studies*, 40, 449-461.

N.H.Hakansson(1971). Optimal Investment and Consumption Strategies under Risk for a Class of Utility Functions, *Econometrica*, 38, 587-607.

M.Hellwig(1982). Precautionary Money Holdings and the Payment of Interest on Money. University of Bonn Discussion Paper.

A.Imrohoroglu(1992). The Welfare Costs of Inflation Under Imperfect Insurance, *Journal of Economic Dynamics and Control*, 16, 79-91.

K.L.Judd(1985). The Law of Large Numbers with a Continuum of Random Variables, *Journal of Economic Theory*, 35, 19-25.

T.J.Kehoe(1992) Intertemporal General Equibrium Models, in the Economics of Missing Markets, Information and Games. ed F.Hahn.

T.J.Kehoe, D.K.Levine and M.Woodford(1992). The Optimum Quantity of Money Revisited, in Economic Analysis of Markets and Games, Eds. P.Dasgupta et al., M.I.T. Press.

R.G.King and C.I.Plosser(1984). Money, Credit and Prices in a Real Business Cycle, *American Economic Review*, 74, 363-380.

A.Kitagawa(1994). Risky Storage and the Value of Money, *Economics Letters*, 45, 73-77.

N. Kiyotaki and R.Wright(1989). On Money as a Medium of Exchange, *Journal of Political Economy*, 97, 927-954.

K.Kultti(1995). A Finite Horizon Monetary Economy, *Journal of Economic Dynamics and Control,* 19, 237-251.

D.Laidler(1988). Taking Money Seriously, *Canadian Journal of Economics,* 21, 687-713.

A.Leijonhufvud(1973). Effective Demand Failures, *Swedish Journal of Economics*, 75, 27-48.

D.K.Levine(1985). Liquidity and Random Market Closure, University of Cambridge Discussion Paper.

D.K.Levine(1989). Efficiency and the Value of Money, *The Review of Eco-*

nomic Studies, 56, 77-88.

D.K.Levine(1991). Asset Trading Mechanisms and Expansionary Policy, *Journal of Economic Theory*, 54, 148-164

R.E.Lucas(1980). Equilibrium in a Pure Currency Economy, in Models of Monetary Economies, eds. J.H.Kareken and N.Wallace, Federal Reserve Bank of Minneapolis.

R.E.Lucas and N.Stokey(1987). Money and Interest in a Cash in Advance Economy, *Econometrica*, 55, 491-513.

R.E.Lucas(1992). On Efficiency and Distribution, *Economic Journal*, 102, 233-247.

A.L.Marty(1994). What is the Neutrality of Money? *Economics Letters*, 44, 407-409.

P.Mehrling(1995). A Note on the Optimum Quantity of Money, *Journal of Mathematical Economics*, 24, 249-258.

R.Merton(1969). Lifetime Portfolio Selection Under Uncertainty: The Continuous Time Case, *Review of Economics and Statistics*, 51, 247-57.

R.Merton(1971). Optimum Consumption and Portfolio Rules in an Continuous Time Model, *Journal of Economic Theory*, 3, 373-413.

R.Merton(1990). Continuous Time Finance, Blackwell.

A.Orphanides and R.Solow(1990). Money, Inflation and Growth, in Handbook of Monetary Economics, volume 1, eds B.Friedman and F.Hahn, North Holland.

J.M.Ostroy(1973). The Informational Efficiency of Monetary Exchange, *American Economic Review*, 63, 597-610.

J.M.Ostroy and R.M.Starr(1974). Money and the Decentralisation of Exchange, *Econometrica*, 42, 1093-1113.

D.Pantinkin(1965). Money, Interest and Prices, Harper and Row.

E.Phelps(1973). Inflation in the Theory of Public Finance, *Swedish Journal of Economics*, 75, 67-82.

R.Radner(1972). Existence of Equilibrium of Plans, Prices and Price Expectations in a Sequence of Markets, Econometrica 40, 289-303.

N.Rankin.(1994). Monetary Uncertainty in Discrete Time Money in the Utility Function Models, *Economics Letters*, 44, 127-132.

R.Reichlin(1986). Equilibrium Cycles in an Overlapping Generations Economy with Production, *Journal of Economic Theory*, 40, 89-102.

D.Romer(1986). A Simple General Equilibrium Version of the Baumol-Tobin Model, *Quarterly Journal of Economics*, 101, 663-686.

P.A.Samuelson(1958). An Exact Consumption Loan Model of Interest with or without the Social Contrivence of Money, *Journal of Political Economy*, 66, 467-482.

T.J.Sargent(1987). Dynamic Macroeconomic Theory, Harvard University Press.

J.Schectman and V. Escudero(1977). Some Results on an Income Fluctuation Problem, *Journal of Economic Theory* 16, 151-166.

J.A. Scheinkman and L.Weiss(1986). Borrowing Constraints and Aggregate Economic Activity, *Econometrica,* 54, 23-45.

K.Shell(1971). Notes on the Economics of Infinity, *Journal of Political Economy*, 79, 1002-1012.

D.C. Shimko(1993). Finance in Continous Time, A Primer. Kolb Publishing Company.

M.Sidrauski(1967). Rational Choice and Patterns of Growth in a Monetary Economy, *American Economic Review*, 57, 534-544.

B.D.Smith(1989). Legal Restrictions, Sunspots and Cycles, *Journal of Economic Theory*, 47 369-392.

R.M.Starr(1972). The Structure of Exchange in Barter and Monetary Economies, *Quarterly Journal of Economics*, 86, 290-302.

R.M.Starr(1974). The Price of Money in a Pure Exchange Monetary Economy with Taxation, *Econometrica*, 42, 45-54.

A.Stockman(1981). Anticipated Inflation and the Capital Stock in a Cash in

Advance Economy, *Journal of Monetary Economics*, vol 8, no. 3 387-393.

B.Taub(1988). Efficiency in a Pure Currency Economy with Inflation, *Economic Inquiry*, 26:no.4, 567-583.

B.Taub(1994). Currency and Credit are Equivalent Mechanisms, *International Economic Review*, 35, 921-956.

J.Tirole(1985). Asset Bubbles and Overlapping Generations, *Econometrica*, 53, 1499-1528.

J.Tobin(1958). Liquidity Preference as Behavior Towards Risk, *The Review of Economic Studies*, 25, 65-86.

J.Tobin(1980). Discussion, in: J.Karaken and N.Wallace, eds, Models of Monetary Economies, Federal Reserve Bank of Minneapolis.

R.Townsend(1980). Models of Money with Spatially Separated Agents, in Models of Monetary Economies, eds. J.H.Karaken and N.Wallace, Federal Reserve Bank of Minneapolis.

N.Wallace(1980). The Overlapping Generations Models of Fiat Money, in Models of Monetary Economics, eds. J.H.Karaken and N.Wallace, Federal Reserve Bank of Minneapolis.

P.Wang and C.Yip(1993), Alternative Approaches to Money and Growth, *Journal of Money, Credit and Banking*, 1992, 24, 553-562.

P.Weil(1989). Overlapping Families of Infinitely Lived Agents, *Journal of Public Economics*, 38, 183-198

P.Weil(1991). Is Money Net Wealth? *International Economic Review*, 32, 37-53.

L.Weiss(1980). The Effects of Money on Welfare in the Steady State, *Econometrica* 48, 565-576.

C.Wilson(1979). An Infinite Horizon Model with Money, in J.Scheinkman and J.Green, eds, General Equilibrium, Growth and Trade, Academic Press.

M.Woodford(1988). Expectations, Finance, and Aggregate Instability, in M.Kohn and S.C.Tsiang eds. Finance Constraints, Expectations and Macroeconomics, Oxford University Press.

M.Woodford(1990). The Optimum Quantity of Money, in Handbook of Monetary Economics, Volume 2, eds. B.Friedman and F.Hahn, North-Holland.