Integration of Real-time Traffic State Estimation and Dynamic Traffic Assignment with Applications to Advanced Traveller Information Systems

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Abstract

Accurate depiction of existing traffic states is essential to devise effective real-time traffic management strategies using Intelligent Transportation Systems (ITS). Existing applications of Dynamic Traffic Assignment (DTA) methods are mainly based on either the prediction from macroscopic traffic flow models or measurements from the sensors and do not take advantage of traffic state estimation techniques, which produce estimate of the traffic states with less uncertainty than the prediction or measurement alone. On the other hand, research studies highlighting estimation of real-time traffic state are focused only on traffic state estimation and have not utilized the estimated traffic state for DTA applications. This research introduces a framework which integrates real-time traffic state estimate with applications of DTA to optimize network performance during uncertain traffic conditions through traveller information system.

The estimate of real-time traffic states is obtained by combining the prediction of traffic density using Cell Transmission Model (CTM) and the measurements from the traffic sensors in Extended Kalman Filter (EKF) recursive algorithm. The estimated traffic state is used for predicting travel times on available routes in a traffic network and the predicted travel times are communicated to the commuters by a variable message sign (VMS). In numerical experiments, the proposed estimation and information framework is applied to optimize network performance during traffic incident on a two route network. The proposed framework significantly improved the network performance and commuters' travel time when compared with no-information scenario during the incident. The application of the formulated methodology is extended to model day-to-day dynamics of traffic flow and route choice with time-varying traffic demand. The day-to-day network performance

is improved by providing accurate and reliable traveller information. The implementation of the proposed framework through numerical experiments shows a significant improvement in daily travel times and stability in day-to-day performance of the network when compared with no-information scenario.

The use of model based real-time traffic state estimation in DTA models allows modelling and estimating behaviour parameters in DTA models which improves the accuracy of the modelling process. In this research, a framework is proposed to model commuters' level of trust in the information provided which defines the weight given to the information by commuters while they update their perception about expected travel time. A methodology is formulated to model and estimate logit parameter for perception variation among commuters for expected travel time based on measurements from traffic sensors and estimated traffic state. The application of the proposed framework to a test network shows that the model accurately estimated the value of logit parameter when started with a different initial value of the parameter.

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Chapter 1: Introduction

1.1   Motivation

Traffic congestion has been increasing on urban arterials and freeways as a result of steady growth in vehicular traffic at a higher pace than the increase in capacity of transportation networks. The increase in capacity of road network is constrained by the available space to add more lanes and the finances available to improve the road network capacity. Traffic congestion costs commuters in terms of fuel, extra time to commute the journey and anxiety due to extended delays to arrive at the destination. Traffic congestion also impacts on the economy and society by additional consumption of fuel and by affecting the air quality. Schrank *et al.* (2012) estimated based on INRIX traffic data that congestion on the US roads cost $121 billion due to delays and additional fuel consumption in the year 2011. The extra time spent by a commuter increased from 16 hours in 1982 to 38 hours in 2011, amounting to 5.5 billion extra hours spent on roads in 2011. Schrank *et al.* (2012) estimated that approximately 37% of the total delay is experienced outside the peak hour and day-to-day variation in travel time triggers commuters to plan their journeys for important appointments with extra time. Gordon and Pickard (2014) reported that the cost of traffic congestion to London's economy in 2013 was $8.5 billion and it is estimated to increase to $14.5 billion in 2030. The total cost of congestion to the UK economy in 2013 was $20.5 billion dollar with $2,230 per car-commuting household. With the current trend of increase in traffic congestion, the future forecasts are alarming and need appropriate measures to tackle this problem.

The effective utilization of existing road network capacity has been proven to significantly reduce the level of traffic congestion. Various Adaptive Traffic Control Systems (ATCS) have been developed and deployed in many cities around the globe. SCOOT (Hunt *et al.* 1981), SCATS (Lowrie 1982), OPAC (Gartner 1983), PRODYN (Henry 1983), SPOT (Donati *et al.* 1984), and RHODES (Mirchandani and Head 2001) are among the ATCS packages developed to reduce traffic delays. SCOOT and SCATS are the most widely used ATCS for

adaptive control of signalized traffic intersection, which adjust signal timings based on real-time observations obtained from loop detectors installed at appropriate locations in road links. The implementation of these systems has been reported to reduce the congestion in the application areas. Based on the earlier implementation of SCOOT in Glasgow and Coventry, a 12% reduction in delay due to signalized intersection is observed (Hunt *et al.* 1981). A 19% reduction in delay at signalized intersections in London is achieved by implementing ATCS using SCOOT (Chandler and Cook 1985). Similar improvements have been reported from ATCS implementations in other urban traffic networks. The adaptive traffic control systems and other research studies focusing real-time traffic management are extensively based only on the observations of traffic flow obtained from the sensors.

Real-time traffic control systems based on the measurements from traffic sensors usually use measurements of occupancy and traffic flow from inductive loop detectors. Other fixed point traffic measurement sensors include pneumatic tubes, magnetic loop, video cameras, active and passive infrared sensors and acoustic sensors. The sensors should be installed at a close distance to each other (500m~1000m) and at specific locations in road links to obtain sufficient information to perform the optimization task. The measurements obtained from the sensors are contaminated with noise and errors. The measurements are processed for filtering and smoothing before they can be used as input for any optimization task. Furthermore, the breakdown of a sensor, fault in a local controller or disruption in the communication can lead to ineffective optimization by the real-time traffic controller.

Traffic flow models have been used for offline applications of traffic planning, devising traffic management strategies and for evaluation and prioritization of various traffic management strategies. These models use historic or predicted traffic demand to perform the required task. Traffic flow models can predict the evolution of traffic flow for any study period and any size of the network. However, prediction of traffic patterns using traffic flow models based on historic traffic demand may lead to significantly inaccurate traffic management

plan when the traffic demand and/or network capacity depart from their historic trend.

Traffic flow model based *real-time traffic state estimation* combines the advantage of real-time observations from traffic sensors with the prediction power of traffic flow models. In traffic state estimation, the *state* represents the traffic flow variables modelled using the traffic flow model employed and the parameters of traffic flow model estimated in the estimation algorithm. The prediction of traffic state from traffic flow model is corrected based on the observations from the sensor in the estimation algorithm in such a way that the final estimate of the traffic state is more reliable than the prediction or the measurement alone. There has been significant amount of research in traffic flow model based traffic state estimation during last decade. However, it has not been utilized for real-time traffic management and adaptive traffic control. Thus, this research is aimed at integrating real-time traffic state estimation with dynamic traffic assignment to improve the modelling/ estimation component of dynamic traffic assignment models.

This research provides an alternative to measurement/prediction based applications of dynamic traffic assignment models for traffic management by introducing real-time traffic state estimation technique to determine the prevailing traffic conditions. This research can be useful for developers of ATCS, as they can update their measurement based optimization tools with the real-time traffic state estimation, which may significantly reduce the number of sensors required, improve the accuracy of the estimate and provide a better alternative in case of sensor breakdown. This research is also useful for traffic management authorities and city councils for real-time traffic management and deriving traveller information.

1.2 Introduction

This section provides details about the developments in the fields of real-time traffic state estimation and dynamic traffic assignment and introduces recent research studies in these two fields, as integrating these two distinguished and well developed fields is the aim of this thesis.

Real-time traffic state estimation has been an active field of research for many years. With the development of new technologies and the improvement in existing techniques for acquiring real-time traffic data, more emphasis is being given to proper utilization of such data, to obtain a more accurate and widespread picture of the state of a network. However, there are limitations in the data directly obtained from traffic sensors. Firstly, such data does not include the required parameters for devising traffic management strategies in real-time such as link/route travel times, queue lengths, level of congestion, etcetera. Another problem with obtaining real-time traffic data is that it requires a good communication infrastructure, which requires huge capital investment and continuous maintenance expenditure. On the other hand, the prediction of traffic state using only traffic flow models based on long-term historic information might contain significant error in prediction, especially when actual traffic conditions depart from their historical trend due to external factors. The external factors can affect the traffic demand such as weather, shopping events, festivals, exhibitions, sports, or due to variation in departure time of commuters. The network capacity can also be affected due to external factors such as extreme weather, traffic incident and road maintenance. To obtain complete traffic data for the whole network, traffic flow models along with measurements from sensors are used for better estimation with less uncertainty in the final estimate of traffic state compared to prediction or measurement alone. Thus, 'real-time traffic state estimation' refers to the estimation of traffic flow variables (traffic flow, density) for a segment of road or network, with an adequate time and space resolution based on limited available measurements from traffic sensors (Wang *et al.* 2008).

Recently, many research studies have focused on traffic state estimation problem (Wang *et al.* 2011; Ngoduy 2008; Ngoduy 2011; Tampere and Immers 2007; Munoz *et al.* 2006; Sun *et al.* 2004; Munoz *et al.* 2003). Of particular relevance to this research is the work of Wang and Papageorgiou (2005). They presented a methodology for estimating traffic states by combining real-time traffic data from sensors with predictions from a second-order traffic flow model. In this approach, they utilized the Extended Kalman Filter (EKF) variation on the approach originally proposed by Kalman (1960) for dynamic

systems represented by nonlinear equations. The Kalman filter operates recursively on streams of noisy input data to produce a statistically optimal estimate of the underlying system state. Wang and Papageorgiou (2005) also proposed a method for real-time estimation of the model parameters by converting these parameters into stochastic variables. They estimated unknown parameters of second order traffic flow model such as free-flow speed and traffic flow capacity. The proposed model was designed and applied for a stretch of freeway with on-ramps and off-ramps. Wang *et al*. (2011) applied the framework proposed by Wang and Papageorgiou (2005) to a sizeable freeway network of 100 km in Italy, for real-time traffic state estimation and surveillance purpose. Ngoduy (2008) proposed a framework that utilizes a particle filtering algorithm with a second-order traffic flow model to estimate traffic for a section of freeway; and in Ngoduy (2008; 2011) utilized an unscented Kalman filter algorithm with a macroscopic traffic flow model for freeway traffic state estimation. Park and Lee (2004) used a Bayesian technique to estimate travel speed for a link of an urban arterial using data from a dual loop detector.

The Cell Transmission Model (CTM) has been applied for estimation of traffic state (Tampere and Immers 2007; Munoz *et al.* 2006; Munoz *et al.* 2003) and modelling traffic flows for traffic networks. Gang *et al*. (2007) presented a traffic state estimation scheme based on the CTM and Kalman filter for a single urban arterial street under signal control. Liu *et al.* (2012) proposed a travel time estimation approach for a long corridor with signalized intersections based on probe vehicle data. Long *et al.* (2008) developed a model based on the CTM for congestion propagation and bottleneck identification in an urban traffic network. They also estimated average journey velocity for vehicles in the network. Long *et al.* (2011) applied CTM for simulating traffic jams caused due to an incident in a traffic network. They assumed that traffic flow parameters and the duration of the incident were known during the incident and used a CTM alone for traffic prediction. Zhang *et al*. (2013) compared travel-time computed using three different traffic flow models that could be used for predicting network traffic, namely the point queue model, the spatial queue model and the CTM, and concluded that the CTM is better than the other two

models for predicting travel times especially when queue spillback prevails. Sumalee *et al.* (2011) and Zhong *et al.* (2011) proposed a Stochastic CTM for network traffic flow prediction, the stochasticity intended to address uncertainties in both traffic demand and capacity supplied by the network.

Alongside the problem is that of control, by which some network controller may attempt to influence the system in some desirable way, by adjusting signal timings or speed limits, by providing information through variable message signs or in-vehicle navigation systems, or by charging tolls at some points in the network. In this case the controller may influence both the dynamic flow of traffic and the time-dependant route choices of travellers; the mutual interaction of these phenomena is the focus of Dynamic Traffic Assignment (DTA). Within this field, Kachroo and Ozbay (1998) highlighted the problem of short-term non-recurrent congestion which might be caused due to some incident, addressing this issue by dynamic traffic routing and assigning time-dependent split parameters at some diversion points. They used a feedback linearization method to obtain optimum split rate, so as to optimize network performance. In their method, they assumed availability of data from measurement sensors and only utilized these measurements, without using any kind of traffic flow model. Lo (2001) proposed a method for determining dynamic signal control timing plans based on system optimal principle using CTM based network model, which optimize network performance by keeping the density at an optimum level so as to ensure maximum flow on all links approaching a signalized intersection. The results indicated that green progression could reduce delays on the network. Smith and Mounce (2011) presented an idealized splitting rate model when travellers seek to change their route either day-to-day or within a day. This model uses splitting rates at nodes to change exit flows in such a way that Wardrop equilibrium is obtained. This approach also incorporates dynamic signal green-time reallocation to reduce delays. The model is an extension of formulation proposed by Smith (1984), which suggests that for each pair of routes joining the same O-D pair, traffic flow swaps from a more costly route to a less costly route at a rate which is proportional to the product of the flow on the more expensive route and the difference in cost between the two routes. Many other studies (e.g. Chow 2009;

Wu and Huang 2010; Carey and Watling 2012) presented DTA-based solution for improving traffic congestion without considering utilization of traffic state estimation techniques. In contrast, Ziliaskopoulos (2000) developed a CTM-based approach to compute the dynamical system optimal assignment for a network with single origin and destination, formulating the DTA problem as a linear program. In conclusion, then, DTA-based research studies into the control/optimization of networks have typically not considered the availability and reliability of real-time estimates of the traffic states. Such studies have generally assumed that all the data for the scenario is known, and there is no data available on underling changes in the traffic or road environment conditions during the time period under study.

The existing gap in the literature for utilizing real-time traffic state estimation techniques in DTA applications to improve network performance during uncertain traffic conditions is the main motivation of this research. This research is aimed at integrating real-time traffic state estimation and DTA, focusing on traveller information systems that influence route choice behaviour to improve network performance.

1.3   Research gap and expected contribution

The main contribution of this research work is to combine traffic state estimation and DTA, as the application of real-time traffic state estimation techniques for DTA has not been published before. Traffic state estimation can be considered equivalent to traffic flow prediction or traffic state reconstruction using observations from measurement sensors, as all these techniques aim to determine the state of the network (traffic flow, density, speed, travel times, etc.). Existing literature contains many studies combining traffic flow models and DTA, for example, Lo (1999), Ziliaskopoulos (2000), Lo (2001), Gomes and Horowitz (2006), Liu *et al*. (2006), Chiu *et al.* (2007). Similarly, measurements from traffic sensors have also been used for DTA and traffic management in real-time, e.g., Kachroo and Ozbay (1998), Mirchandani and Head (2001), Dotoli *et al*. (2006). However, traffic flow model based real-time traffic state estimation has not been applied to optimize network performance using DTA models. This research integrates traffic flow model

based traffic state estimation with DTA, as the estimated traffic state is considered more reliable than the prediction of traffic state using a traffic flow model or observations from traffic sensors.

This research, therefore, will focus on developing methods that combine real-time traffic state estimation with a DTA-based model of driver's route choice, with an aim to produce accurate and effective traffic management strategies. Therefore, the novelty of the research presented in this thesis is the combination of real-time traffic state estimation with DTA, as the existing literature in DTA only utilizes prediction from traffic flow models or measurements from the sensors and the literature focusing traffic state estimation problem has not utilized traffic estimation techniques for traffic management using DTA. A framework is proposed in this research in which predictive traveller information is estimated based on real-time traffic state estimates for a traffic network with unexpected variation in traffic flow conditions. The uncertainty in traffic conditions can be caused either with unexpected variation in network capacity or variation in traffic demand. The objective is that travel times predicted based on real-time traffic state estimation for available routes in a network can make travel time more reliable and travel decisions more appropriate during uncertain traffic conditions.

The proposed framework for integration of real-time traffic state estimation with DTA is applied for selected application to demonstrate the significance of the proposed framework. The approach of numerical implementation of the proposed framework allows testing the effect of various model parameters and other factors on the effectiveness of the framework in a controlled simulation environment. Furthermore, the significance of the proposed framework is tested by using an evaluation model, while knowing a complete picture of the network and traffic conditions. In this research, real-time traffic state estimation is applied for within-day and day-to-day applications of DTA models. For within-day application of the proposed framework, a traffic network affected with an incident is considered. The capacity drop due to the incident is determined using real-time parameter estimation of fundamental traffic flow diagram. The Advanced Traveller Information System (ATIS) is used to inform commuters about expected travel times and divert commuters from

the congested route to an alternative route. The expected travel time is determined by predicting the evolution of traffic flow based on current estimate of traffic state, which is based on the measurements from traffic sensors. The proposed framework is further extended for day-to-day modelling of traffic flows and route choices using real-time traffic state estimation with time varying traffic demand. With time varying traffic demand, a commuter may experience a different travel time than his experience on a given day when departing at the same time. The experienced travel time of commuters might become insignificant due to unexpected day-to-day variation in traffic demand. With time varying traffic demand, traveller information based on real-time estimated traffic state informs commuters about expected travel times on available routes.

In day-to-day DTA models for route choice, the parameters representing the behaviour of commuters play a significant role in accurate modelling of route choice behaviour and resulting traffic flows. This research utilizes observations from measurement sensors and real-time estimated traffic state to model these parameters and estimate their values. The behaviour parameters such as the weight given to the information in updating expected travel times and perception variation for expected travel time also change with uncertainty in network traffic conditions. This research proposed to estimate these behaviour parameters to improve modelling accuracy of day-to-day route choice behaviour and resulting traffic flows.

The expected contribution from this thesis can be summarized into following:

i)   The proposed framework of integrating real-time traffic state estimation and DTA is applied to a within-day application, when one of the routes in a network is affected with an incident (chapter-6).

ii)   Day-to-day variation in traffic flows and route choice of commuters are modelled using traffic flow model based traffic state estimation model when traffic demand is varying day-to-day as well as within a day (chapter-7).

iii) The parameters of DTA models representing behaviour of commuters are determined using observations from traffic sensors and real-time traffic state estimation (chapter-8).

1.4   Objectives

The main objective of this research is to integrate two distinguished and well developed fields, namely, real-time traffic state estimation and dynamic traffic assignment. The traffic state estimated in real-time is utilized to predict travel times for traveller information system, which is used to improve the network performance during uncertain traffic condition. Real-time traffic state estimation and observations from traffic sensors are applied to estimate the dynamic parameters which are significant in accurate modelling of traffic flow dynamics. To integrate the fields of real-time traffic state estimation and dynamic traffic assignment, the following aims are defined for this thesis.

i)   Improve the accuracy of traveller information by extracting predictive traveller information using real-time estimated traffic state.

ii)   Utilize real-time parameter estimation technique to detect drop in capacity during a traffic incident and route commuters to alternative routes using ATIS.

iii)   Apply real-time traffic state estimation for modelling day-to-day traffic flows and route-choices under time varying traffic demand and improve network performance using ATIS.

iv)   Estimate behaviour parameters of DTA models using observations from traffic sensors and real-time traffic state estimation.

To achieve the aims set for this research, following objective are defined:

i)   A CTM-EKF based framework is formulated to extract predictive traveller information from real-time estimated traffic state (chapter-5)

ii)   The CTM-EKF based framework for traveller information is extended to include within-day route choice using multinomial logit model (chapter-6)

iii)   A CTM-EKF based methodology is formulated to model day-to-day dynamics of traffic flow and route choice by introducing weighted average learning model, perception update model for integration of

traveller information with experienced travel time and logit model based route choice model (chapter-7).

iv) A CTM-EKF based framework is formulated to model and estimate two selected behaviour parameters in DTA models. The parameters selected for estimation include commuters' level of trust in the traveller information and perception variation among commuters about expected travel time (chapter-8).

v) Numerical experiments are carried out to test the formulated methodologies and highlight the significance of integrating real-time traffic state estimation with DTA using a simple traffic network (chapters 6, 7 , 8)

1.5 Thesis layout

The thesis highlights integration of real-time traffic state estimation techniques with application of DTA models to traveller information systems and en-route choice of commuters. All the models used as a component of the proposed framework in this research are described in detail with a comparison of alternative models and reasons for selection of a particular model. Traffic state estimation is based on prediction of traffic state from a macroscopic traffic flow model, which is corrected at each simulation time-step using observations of traffic state in an estimation algorithm. The real-time estimated traffic state is then utilized for optimizing network performance using DTA models. Thus, second chapter of the thesis provides an overview of traffic flow models, suitability of selected 'cell transmission model' and describes the selected traffic flow model for network traffic flow modelling. The prediction of traffic state from chapter-2 is forwarded to the estimation algorithm described in chapter-3. Chapter-3 highlights estimation algorithms, measurement techniques and real-time traffic state estimation model using extended Kalman filter and CTM. The DTA models and their applications are briefly described in chapter-4. Chapter-5 highlights the recent developments in DTA and ATIS, highlighting the existing gap in literature and then describing the contribution of this thesis in detail. A framework to extract predictive traveller information based on the traffic state estimation is described in chapter-5, which is

extended for applications of DTA models in the following chapters. In chapter-6, the proposed framework is applied to a within-day application of DTA for a network affected with traffic incident. The within-day application is extended to model day-to-day dynamics of route choice and traffic flows with time-varying traffic demand in chapter-7. Chapter-8 extends the day-to-day modelling framework from chapter-7 to model and estimate behaviour parameters of DTA models. Finally, chapter-9 concludes the finding of this research and highlights further research work. Figure 1.1 describes the contents and connection between different chapters in the thesis.
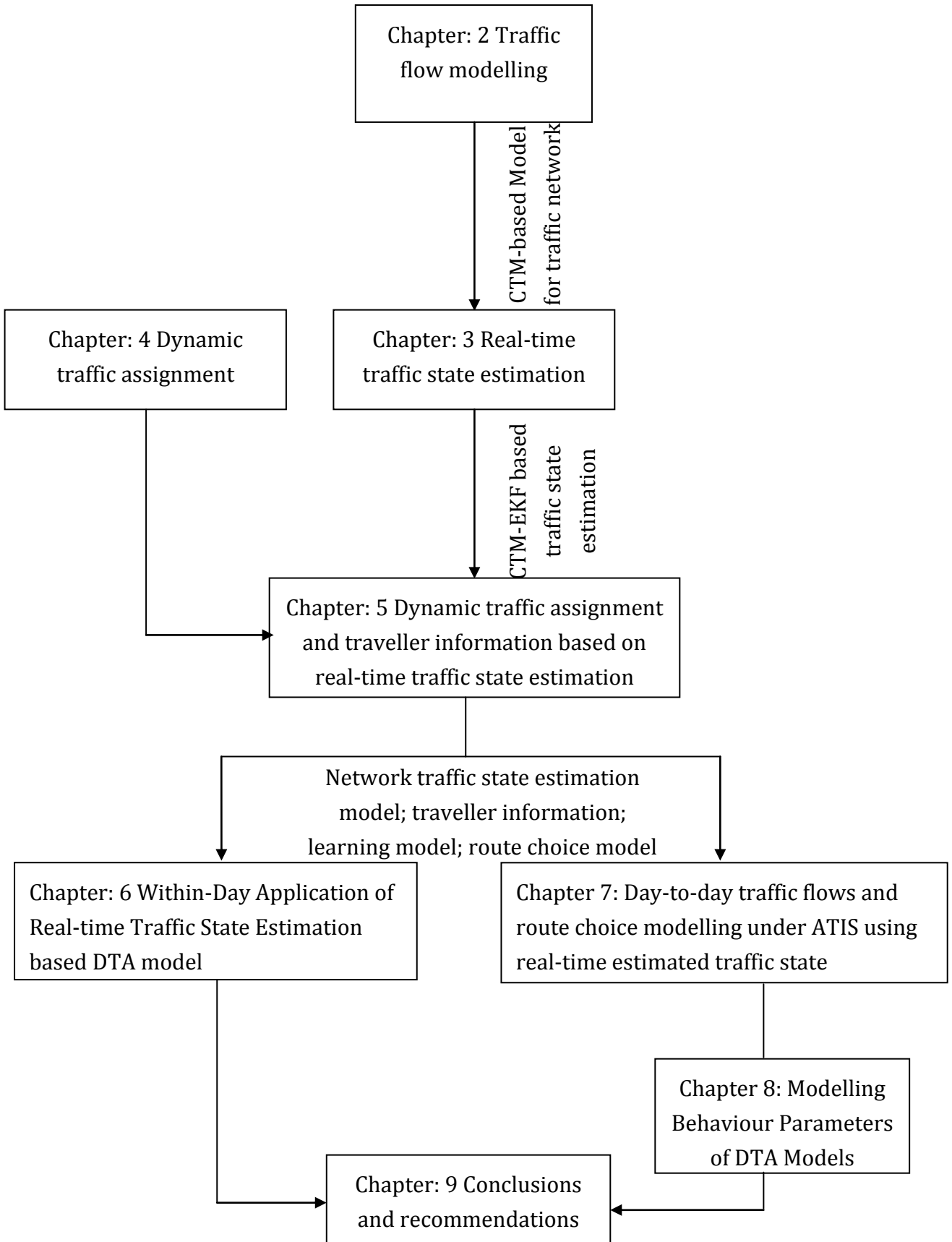
Figure 1.1 Thesis layout

Chapter 2: Traffic Flow Modelling

2.1 Introduction

Scientific mathematical models are widely used in all fields of applied and social sciences to mimic real life phenomena. The mathematical models are applied to understand a process, study the effect of various parameters and variables, and forecast the changes due to variation in some parameters and variables. The modelling of a process is generally carried out by one of the three existing methods (Papageorgiou 1998). The 'deductive' modelling approach is based on using existing laws of nature to model a process. On the other hand, 'inductive' approach uses actual data collected in past to develop a relationship between input and output variables and forecasts the output variable in future based on current observations of input variables. In 'intermediate' approach, a mathematical model is developed based on the deductive approach and then adjusted with actual observations. Advances in technology and computation efficiency have facilitated the modelling of complex processes and more emphasis is given to improve the capability of existing models and implementation of these models for large scale problems.

Rapid increase in vehicular traffic from beginning of 20th century attracted the interest of researchers to model the process of traffic flow. When compared with modelling of other stream of flows such as fluid, heat or electricity, traffic flow is more complicated as it involves human interaction. Each particle in the stream of traffic is controlled by a driver, which makes the process of traffic flow more dynamic and diverse. Furthermore, there is hardly any hope to achieve the descriptive accuracy in traffic flow modelling similar to thermodynamics or Newtonian physics. The only accurate physical law for traffic flow modelling is the conservation equation (Papageorgiou 1998). Numerous models have been proposed in past few decades to model the process of traffic flow but still no single model claims to perfectly model this phenomenon. However, with the existing limitations of these models related to either the scale of traffic network or type of traffic system, they are valuable tool to evaluate the existing traffic condition, devise real-time traffic control

strategies, forecast the impact of new infrastructure or traffic control plan, and to understand and simulate the process of traffic flow.

This chapter briefly reviews the existing models of traffic flow and implement one of the existing frameworks to model network traffic flow for real-time traffic state estimation. This chapter consists of six sections. Section-2 describes classification of traffic flow models. Macroscopic traffic flow models are discussed in section-3. The illustration of microscopic traffic flow models is provided in section-4. The model implemented for prediction of network traffic state is described in section-5 and section-6 summarizes the finding of this chapter.

2.2 Classification of traffic flow models

Based on existing literature, traffic flow models can be classified into various groups based on their distinct properties. Traffic flow models can be distinguished based on the following characteristics (Hoogendoorn and Bovy 2001):

a) Time scale
b) Type of process
c) Level of detail

2.2.1 Time scale

All traffic flow models are function of time and space, as the propagation of traffic along roads is modelled for a specific timeframe for vehicular movement from an origin to a destination. Discrete traffic flow models divide the space in small segments and time in small intervals to make the modelling process simpler and easier to obtain a unique solution of the process. Whereas, continuous traffic flow models describe the change in system as continuous over time and space. The continuous traffic flow models are impossible to solve analytically, therefore discretization in continuous traffic flow models is proposed. The LWR model proposed by Lighthill and Witham (1995) and Richards (1956) is a continuous model and it is impossible to obtain a unique solution using LWR model. However, discretization in LWR model as proposed by Daganzo (1994) makes the model simpler to implement on a larger network

and obtain a solution for traffic state. In Daganzo's model, the partial differential equation of LWR model is replaced by a simple difference equation, which makes it easier to obtain an analytical solution for any size of the network. The discretization of continuous traffic system compromises the details and dynamics in term of time and space, as traffic is assumed to be homogenously divided within a discretized segment of the road and during an interval of time.

2.2.2 Type of process

In a deterministic mathematical model input and output variables are related by a deterministic relationship, which means that if a model is processed with same input variables for many times, the output remains the same. A stochastic model contains at least one random variable, which is defined by a probability distribution or a histogram. Most of the microscopic traffic flow models are stochastic and contain more than one random variable, for example, the parameters defining reaction of drivers are stochastic. A deterministic model such as the CTM can be transformed into a stochastic model by transforming a parameter or boundary condition into a random variable. Transforming a variable or parameter from deterministic to stochastic allows incorporation of random variations in real life such as random variation in traffic flow capacity or travel demand around a mean value.

2.2.3 Level of detail

Traffic flow models can be classified into three main categories based on the level of details modelled. There are various entities in a traffic system, such as road network, traffic controls, vehicles, drivers, environment, etcetera and traffic flow models are classified based on number of entities considered in a modelling framework. Traffic flow models with higher level of details need more information and parameters to model a traffic system. On the contrary, traffic flow models with fewer entities to consider require comparatively less parameters and information about the traffic system. The level of details considered in a model is linked with the computing time and thus scale of the network feasible for implementation of the model. Traffic flow models with higher level of details are computationally more demanding and are feasible for

modelling smaller network with higher accuracy, whereas the models with lower level of details are computationally less expensive and suitable for modelling lager networks. Based on this criterion, traffic flow models can be classified as microscopic, mesoscopic and macroscopic traffic flow models.

Microscopic traffic flow models consider all the major entities in a traffic system from road network to driver behaviour. Microscopic traffic flow models consider each vehicle as a particle in the system and model the interaction of a vehicle with the road environment as well as with other vehicles and also consider dynamics in behaviour of drivers. Mesoscopic models consider medium level of details in modelling a traffic system. In mesoscopic traffic flow models, individual vehicles or driver behaviour is not considered, however these details are considered at an aggregate level and modelled using a probability distribution function. Macroscopic traffic flow models are based on aggregated traffic flow properties and describe relation between the aggregated traffic flow parameters, such as traffic density, traffic flow and mean speed. Due to fewer number of parameters in such models, the amount of information required to model a traffic system is significantly lower than other models and require fewer parameters to calibrate. Macroscopic traffic flow models do not model individual vehicles and driver behaviour, thus computationally less demanding and perform a comparatively faster simulation of the traffic system. This characteristic of macroscopic traffic flow models make them suitable for real-time traffic condition assessment and devise traffic management strategies. Macroscopic and microscopic traffic flow models are discussed in more details in the following sections of this chapter. Table 2.1 summarizes some important characteristics of macroscopic and microscopic traffic models.

Table 2.1 Some important characteristics of selected macroscopic and microscopic traffic flow models (Hoogendoorn and Bovy 2001)

Detail Level	Model name	Time scale	Process representation	Model implementation	Scope of application
Macroscopic	LWR	Continuous	Deterministic	Analytical	Aggregate lane stretches
	CTM	Discrete	Deterministic	Simulation	Motorway network
	Payne-type model	Continuous	Deterministic	Analytical	Aggregate lane stretches
	Papageorgiou Model	Discrete	Deterministic	Simulation	Motorway network
	Helbing-type models	Continuous	Deterministic	Analytical	Aggregate lane stretches
Microscopic	Safe distance model	Continuous	Deterministic	Analytical	Single-lane stretches
	Stimulus response model	Continuous	Deterministic	Analytical	Single-lane stretches
	Psycho-spacing models	Continuous	Stochastic	Simulation	Multilane stretches

2.3 Macroscopic traffic flow models

Macroscopic traffic flow models consider flow of traffic on an aggregate level and define relation between aggregated parameters of traffic flow. The aggregated parameters of traffic flow considered in modelling of traffic flow are traffic density, traffic flow rate, and mean speed. Traffic density is number of vehicles in a unit length of a road section per lane at any given time instant. Traffic flow rate is the number of vehicles passing through a point in a unit time interval. Based on the number of dynamic output equations in macroscopic traffic flow models, these models are classified as first, second or third order traffic flow models. First order traffic flow models treat traffic density or occupancy as an output variable. Second order traffic flow models also consider

the dynamics of average speed with traffic density, and third order traffic flow models consider variance of speed along with traffic density and mean speed when modelling dynamics of traffic flow.

2.3.1 First order traffic flow models

2.3.1.1 The LWR model

The first popular model in traffic flow modelling was developed independently by Lighthill and Whitham (1955) and Richards (1956), thus named as LWR model. The model became popular as it was the first model to translate the physical phenomenon of conservation from hydrodynamics for modelling of traffic flow. This is a continuous model with space s and time t as independent variables and speed $u(s, t)$, traffic density $\rho(s, t)$, and flow rate $q(s, t)$ are functions of time and space. LWR model provides a fundamental relation between these traffic flow variables and suggests that the traffic flow rate is a product of traffic density and average speed.

$$q(s,t) \; = \; \rho(s,t) \, u(s,t) \tag{2.1}$$

LWR model proposes a conservation of vehicles equation analogous to fluid dynamics in a continuous space-time scale.

$$\frac{\partial \rho(s,t)}{\partial t} + \frac{\partial q(s,t)}{\partial s} = 0 \tag{2.2}$$

Average speed of vehicles at any point depends on traffic density and it is described by a function f of traffic density.

$$u(s,t) = f(\rho(s,t)) \tag{2.3}$$

The LWR model can accurately reproduce a significant number of actual traffic flow phenomena, such as decreasing speed with increasing density, formation and dissipation of shock waves, etcetera. Furthermore, LWR model shows consistency with a class of car-following models (Papageorgiou 1998). However, it is not possible to analytically obtain an exact and unique continuous solution using partial differential equation (2.2). Another deficiency in LWR model is highlighted by Papageorgiou (1998) about the assumption in equation (2.3) that the average speed $u(s, t)$ at any point s adjusts instantaneously to current traffic density $\rho(s, t)$. This implies that a vehicle

accelerates or decelerates with a very high and unrealistic value to adjust its speed when there is change in traffic density along the road stretch. LWR-based models are adequate to model urban traffic flows where the traffic flow dynamics are governed by external factors such as stop lines and traffic signal, compared to the freeway traffic where traffic flow is affected by its inherent characteristics such as slow moving vehicles, overtaking and rapid stop-and-go waves. Despite of its above mentioned deficiencies, LWR model can achieve a certain level of accuracy when modelling freeway traffic (Papageorgiou 1998).

In conclusion, LWR was the first significant contribution to model the process of traffic flow and the model can accurately reproduce many real traffic flow phenomena. However, due to the continuous nature of the model, it becomes practically difficult to implement it for a larger road networks and it is impossible to analytically obtain a unique solution using the LWR model.

Vaughan, Hurdle and Hauer (1985) proposed a framework to address the limitation of LWR model to implement it for a multi origin-destination network with time-dependant departure times. This framework develops vehicle trajectories for each vehicle from its origin to the destination to observe its behaviour in traffic. Vaughan and Hurdle (1992) further extended their initial framework to model traffic flows on urban arterial network. However, this framework is tedious and computationally expensive to implement for the larger networks. Newell (1993) proposed a kinematic wave model based on geological erosion model proposed by Luke (1972). Newell (1993) suggested determining cumulative inflows and outflows for a link instead of determining traffic density at every intermediate point on the link. Unlike the LWR model, this model can be applied to model time-dependant multi origin-destination traffic flows. The traffic flow at a given point is influenced by the downstream traffic condition if traffic condition is saturated, whereas if the traffic condition at a point is not saturated, traffic flow is governed by upstream traffic condition. The formation and dissipation of queues is better modelled by this condition. However, using this approach traffic flows can only be determined at entry and exit point and traffic flow variables cannot be determined at any intermediate point. Furthermore, Newell (1983) model is suitable for a traffic system with triangular fundamental traffic flow diagram but it become

complicated to model a traffic system with non-homogenous relation between traffic density and traffic flow.

2.3.1.2 The cell transmission model

Daganzo (1994) proposed a simple approximation of LWR model to determine traffic state using analytical method. He discretised space into small segments called cells and time into small simulation time-steps, thus the model is named as cell transmission model (CTM). The CTM transforms differential equation from LWR model into a simple difference equation to update traffic density for each future time-step and for all the cells in a road network. This model is described in detail in section-2.5 of this chapter for modelling traffic network.

2.3.2 Second order traffic flow models

First order traffic flow models have deficiency in modelling some of the freeway traffic flow phenomena such as rapid stop and go waves. First order traffic flow models assume that traffic flow rate passing through any point along a road or from an upstream section to the downstream section only depends on traffic density and assume an equilibrium density-dependent speed function (fundamental diagram). Second order traffic flow models treat mean speed as dynamic and as an additional variable. An additional output equation is added to model mean speed dynamics. Determining mean speed along with traffic density improves the capability of model and address the deficiency of unrealistic acceleration and deceleration in first order traffic flow model. However, the addition of another variable and output equation adds more parameters to calibrate and increases the number of output variables, which leads to complex calibration and optimization, making these models computationally more demanding than the first order traffic flow models.

Payne (1971) proposed a second order traffic flow model which is based on the concept of car-following model that describes the behaviour of a vehicle when it follows a lead vehicle. Payne (1971) modelled the dynamics of average speed and proposed that the traffic flow passing through any point along a road is a function of traffic density and average speed. This model includes two differential equations, one similar to the conversation equation from LWR model and the other to determine dynamic mean speed. The model proposed

by Payne (1971) improved some of the modelling deficiencies in LWR model by introducing a partial differential equation for mean speed dynamics. However, it further complicates to obtain an analytical solution and traffic state using this model. The Payne's model shows qualitative deficiencies when modelling lane drop on freeways or merging of on-ramp traffic flows. Del Castillo *et al.* (1994) with a simulation model showed that wave characteristics resulting from Payne (1971) model are faster than average traffic speed of simulated traffic. Daganzo (1995) also criticized this deficiency of Payne's model. Phillips (1979) proposed that relaxation time in Payne (1971), which models the driver behaviour to achieve the desired speed, is dependent on traffic density. He also approximated traffic pressure term in Payne (1971) model. Kerner and Konhauser (1993) introduced viscosity term and modelled speed variance as a positive constant. Liu *et al.* (1996) summarizes the improvements in Payne-type models and implements the model to numerical schemes.

Papageorgiou *et al.* (1990) proposed discretization of Payne (1971) second order traffic flow model and addressed the issue of on-ramp merging traffic and dynamics of traffic flow with lane drop. Discretization of continuous model significantly improves computing efficiency, however this model contains a number of tuning parameters and proper tuning of these parameters is essential for proper functioning of the model. Furthermore, it requires considerable amount of real-data to calibrate these parameters and perform a simulation.

2.3.3 Higher order models

The second order traffic flow models based on Payne (1971) assume that traffic density and average speed can completely describe the existing condition of traffic. Helbing (1996) suggested that the distribution of mean speed also plays a significant role in modelling traffic flow. Helbing (1996) extended Payne-type models and introduced an additional partial differential equation to model variance of speed and proposed that traffic density and mean speed are function of speed variance. Helbing (1996) derived his model based on gas kinetic assumptions. In addition to the conservation of traffic (equation 2.2) and mean speed, another partial differential equation is added to model the

speed variance. Treiber *et al.* (1999) proposed a model based on Helbing (1996) by incorporating the effect of non-local vehicular interaction in relaxation term.

2.4 Microscopic traffic flow models

Microscopic traffic flow models have been one of the effective tools in traffic flow modelling and simulation for more than sixty years. Microscopic traffic flow models provide a powerful tool to model traffic flow and simulate the flow of traffic with a higher level of detail. These models describe the movement of individual vehicle and interaction of a vehicle with other vehicles and with the road environment. Microscopic traffic flow models also consider the behaviour of drivers in more detail compared to macroscopic traffic flow models. Macroscopic traffic flow models generally model the stream of traffic on aggregate level, whereas microscopic traffic flow models describe the behaviour of each particle of the vehicular stream.

Microscopic traffic flow models are based on two distinct groups of models. Car following models describe movement of vehicles in a single lane section of a road without any intermediate entrance or exit points. These models are based on follow-the-leader approach, where a vehicle follows its leading vehicle and anticipate and reacts to the action of leading vehicle. Pipes (1953) proposed car-following model based on safe distance principle for the following vehicle, which was further improved by Forbes *et al.* (1958), Leutzbach (1988), and Jepsen (1998). Chandler *et al.* (1958) and Gazis *et al.* (1961) proposed stimulus response approach for car following models. Wiedemann and Reiter (1992) identified deficiencies in stimulus-response models and introduced Psycho-spacing approach to model the movement of vehicles in a lane.

Lane-changing models describe lateral movement of vehicles and model lane-changing behaviour of drivers. Lane-changing models are basic component of a microscopic traffic flow model. Lane-changing is a complex phenomenon and it is performed by a driver either for downstream turning or overtaking a slow moving vehicle ahead. A Lane-changing phenomenon also impacts the flow of upstream traffic. Cassidy and Bertini (1999) and Laval and Daganzo (2006) concluded based on experiments that lane-changing attributes to the congestion at bottlenecks. Munoz and Daganzo (2004) termed the lane-

changing vehicles as a moving obstacle and impacting factor on a freeway capacity. Gipps (1986) proposed a rule-based lane-changing model for describing lane-changing decision on freeways and urban arterials. Several modifications in rule-based approach are proposed by Halati *et al.* (1997), Rickert *et al.* (1996), Kita (1999) and Hidas (2005). Ahmed *et al.* (1996) and Toledo *et al.* (2002) proposed lane-changing models based on discrete choice principle. McDonald *et al.* (1997) proposed a lane-changing model based on fuzzy-logic and Hunt and Lynos (1994) proposed a neural network based lane-changing framework. Rahman *et al.* (2013), Toledo (2007) and Moridpour *et al.* (2010) can be referred for a detailed review of lane-changing models.

2.5 The cell transmission model (CTM)

2.5.1 Suitability of CTM for this research

A detailed review of basic macroscopic and microscopic traffic flow models has been provided in earlier sections of this chapter. Based on the review of microscopic traffic flow models it can be concluded that microscopic traffic flow models provide a detailed description of traffic flow process, but these models are computationally more expensive. Thus, microscopic traffic flow models are regarded suitable for smaller networks and offline simulation of a traffic system. Furthermore, microscopic traffic flow models are only suitable to test and simulate an already devised traffic control strategy and cannot be used to develop a traffic control strategy. Whereas, macroscopic traffic flow models can be used to devise a traffic control strategy as well as to evaluate it. Therefore, a macroscopic traffic flow model is more suitable for this research as it is aimed to develop a traffic control strategy for network traffic in real-time which requires both formulation and evaluation of the control strategy.

Using second order model for traffic state prediction significantly increases number of variables and parameters compared to other first order traffic flow model such as CTM. Long *et al.* (2008) developed a model based on the CTM for congestion propagation and bottleneck identification in an urban traffic network. They also estimated average journey speed for vehicles in the network. Long *et al.* (2011) applied CTM for simulating traffic jams caused due to an incident in an urban network. They assumed that both traffic flow

parameters and the duration of the incident were known during the incident and used a CTM alone for traffic prediction. Zhang *et al.* (2013) compared travel times computed using three different traffic flow models that could be used for predicting network traffic namely the point queue model, the spatial queue model and the CTM, and concluded that the CTM is better than the other two models for predicting travel times especially when queue spillback prevails. Zhong *et al.* (2011) proposed a stochastic CTM for network traffic flow prediction, the stochasticity intended to address uncertainties in both traffic demand and capacity provided by the network.

Despite the fact that CTM is simpler to implement on a network than other higher order models, it fits well with the measurement data as discussed by Lin and Ahanotu (1995) and Lin and Daganzo (1994). In comparison to other higher order traffic flow models, CTM has fewer numbers of output variables and input parameters which qualifies CTM as a suitable model for real-time applications. CTM has been used for traffic state estimation (Munoz *et al.* 2003; Munoz *et al.* 2006; Gang *et al.* 2007; Tampere and Immers 2007, Long *et al.* 2008; Long *et al.* 2011) as well as for DTA applications of traffic network optimization (Lo 1999, Ziliaskopoulos 2000, Lo 2001, Gomes and Horowitz 2006, Liu *et al.* 2006, Chiu *et al.* 2007).

CTM has been used in recent research studies for modelling network traffic flow and estimation of real-time traffic states. The KF method is based on minimizing the square of error between the predicted and measured values for the traffic state. The EKF is an extension of KF for non-linear systems and obtaining optimal solution using EKF is not always guaranteed. CTM is a non-linear model and therefore is more suited to estimation using the EKF rather than KF. Munoz *et al.* (2003, 2006) transformed CTM into a linear model by introducing the Switch Mode Model (SMM). CTM-based SMM was derived based on five different traffic modes to avoid the non-linearity caused due to the minimum condition in the CTM. At any given time-step, one of the five modes is selected for the whole link to estimate traffic density, based on the measurement of densities at the upstream and downstream cells of the link. CTM-SMM assumes existence of a maximum of one wave front in the whole link. This means that the whole link is considered to have the same traffic flow

condition and road segments with more than one wave-front are impossible to model using CTM-SMM. Furthermore, the selection of mode for the segment requires direct measurements of traffic density at the upstream and downstream of the segment, which might not be available for every urban link or freeway stretch. Sun *et al.* (2004) used mixture Kalman filter with CTM-SMM for appropriate selection of switch mode. Tampere and Immers (2007) adapted the estimation model proposed by Wang and Papageorgiou (2005) and applied linear Kalman Filter for non-linear CTM.

2.5.2 Modelling network traffic flow using CTM

Traffic flow models for a single link may be translated into models for a network in two main ways, based either on simple node models (Zhang *et al*. 2013; Daganzo 1995) or link-node model (Zhong *et al.* 2011; Szeto *et al.* 2009). In this study we adopt the link-node model proposed by Szeto *et al.* (2009) for the CTM, which mainly comprises the methodology proposed by Daganzo (1995) to model uncontrolled merging and diverging intersections, with that proposed for signalized intersections by Lo (1999, 2001).

In the CTM representation for network traffic, the network is divided into homogeneous cells and each upstream cell is connected to a downstream cell by a connector. The traffic outflows from upstream to downstream cells and the traffic inflows to downstream from upstream cells are dictated by properties of the connector. The properties of the connectors are defined based on their location in the link and the network. To incorporate the effects of different geometries of intersections and traffic control, eight different types of connectors are defined. An 'ordinary connector' connects the upstream cell of a link to the downstream cell of the same link. A 'signalized simple connection' connects a cell of an upstream link to a downstream link controlled by a traffic signal. An 'origin connection' connects an origin dummy cell to the first cell of a link. A 'destination connection' connects the last cell of a link to the destination dummy cell. An 'unsignalized merge connection' is used to model unsignalized merging intersections. A 'signalized merge connection' models flow of traffic at a signalized merging intersection. The modelling of unsignalized diverging intersection is carried out by 'unsignalized diverge connection' and for signalized diverging intersection, 'signalized diverge connection' is used.

We suppose that the network is divided into j links such that $j=1, 2, 3...,$ and i homogeneous segments labelled $i=1, 2, 3, ...,$ with the duration of each simulation time-step Δ, measured in hours. The free-flow speed on link j is u_j and length of a cell in link j is chosen such that a vehicle can traverse the cell in one time-step if the cell is in a free flow condition, thus the length of each cell in link j is $l_j=u_j*\Delta$ km. The simulation horizon is divided into k time-steps labelled $k=1, 2, 3, ...$ We assume that each cell in the network has a maximum flow capacity of $c_i(k)$ veh/hr, a corresponding critical density of $\rho_i^c(k)$ veh/km and a jam-density represented by $\rho_i^J(k)$ veh/km based on a triangular fundamental traffic flow diagram as shown in figure 2.1.
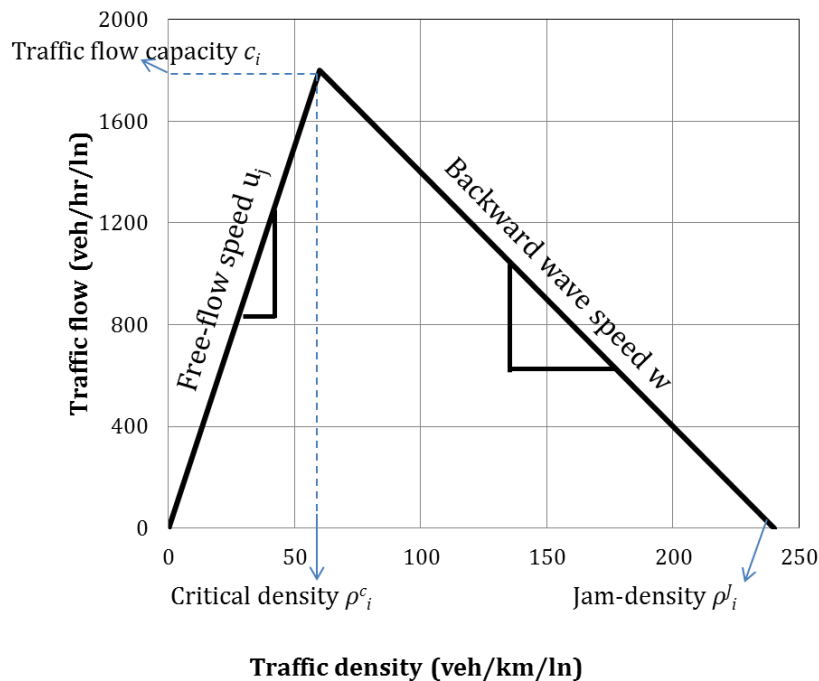
Figure 2.1 Fundamental traffic flow diagram

In this research, a triangular fundamental traffic flow diagram is assumed. However, the proposed approach is not limited to this shape of fundamental diagram and a trapezoidal fundamental diagram can also be incorporated in this research framework. Different types of connectors used to connect an upstream cell with a downstream cell are illustrated below.

2.5.2.1 Unsignalized simple connection

An unsignalized simple connection describes flow of traffic from an upstream cell to the downstream cell of a link. Consider two consecutive simple cells of a

link, A and E connected by a simple connection as shown in figure 2.2. Traffic flow using a simple connection is given by:

$$Z_A(k) = Y_E(k) = min\{u_j\,\rho_A(k),\ c_A(k), c_E(k), w[\rho_E^J(k) - \rho_E(k)]\} \qquad (2.4)$$

In equation (2.4), Z_A (k) is outflow from upstream cell A at time-step k which is equal to the inflow by downstream cell E, $Y_E(k)$, when there is a simple connection between cell A and cell E. First two terms on the right-hand side of equation (2.4) describe the amount of traffic flow which can flow from cell-A to cell-E, while last the two terms define available capacity from the downstream cell. The maximum possible flow from cell A at time-step k is $u_j\,\rho_A(k)$ which is restricted by its flow capacity $c_A(k)$. Therefore, traffic demand from cell A is the minimum of $u_j\,\rho_A(k)$ and $c_A(k)$. If traffic in cell E is in free flow condition then the available capacity from cell E is $c_E(k)$, whereas, if traffic in cell E is not in free flow condition then the available capacity from cell E equals to the space available $w[\rho_E^J(k)\text{-}\rho_E(k)]$. The term $w[\rho_E^J(k)\text{-}\rho_E(k)]$ in equation (2.4) represents the available space in cell E, when the cell is in congested condition and the flow of traffic to the downstream cell is dictated by the shockwave speed.

| A | $Z_A = Y_E$ | E |
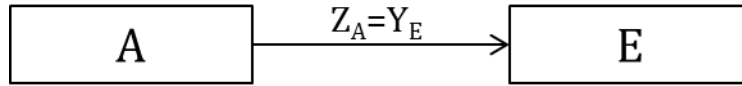
Figure 2.2 A Simple connection between two consecutive cells in a link

2.5.2.2 Unsignalized merge connection

An unsignalized merge connection describes traffic flow from two different upstream cells to a downstream cell. Figure 2.3 describes a merge connection which models merging traffic flows. Cells A and B are two different cells sending flow to the downstream cell E. If cell E has sufficient available capacity, it will accommodate flows of traffic from both the cell. If the available capacity of cell E is insufficient to accommodate traffic flows from cells A and B, then the outflows from cells A and B are based on existing traffic in those cells and priority parameters p_A and p_B associated with cells A and B, respectively. At any time instant, the sum of p_A and p_B must be one and total inflow to cell E is sum of outflows from cells A and B.
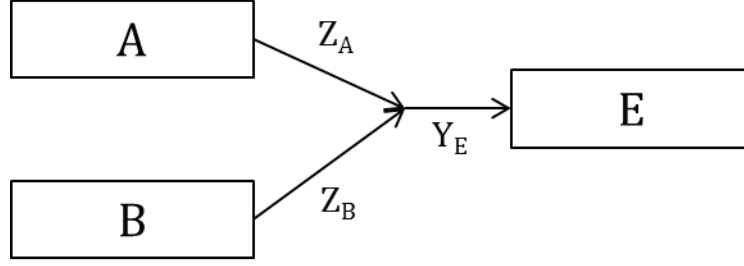
Figure 2.3 A merging intersection for CTM network model

To model traffic movements from two separate links merging into one link, some additional variables are defined. Let $S_A(k)$ and $S_B(k)$ are the maximum possible outflows from cells A and B, respectively. Maximum possible outflow from a cell depends on its local traffic demand and flow capacity.

$$S_A(k) = min\{u_j\rho_A(k),\ c_A(k)\} \tag{2.5}$$

$$S_B(k) = min\{u_j\rho_B(k),\ c_B(k)\} \tag{2.6}$$

$R_E(k)$ is the maximum possible inflow to cell E at time-step k and it is equal to flow capacity of the cell if the cell is in free flow condition or else it is equal to the available space in cell E to accommodate the traffic inflow.

$$R_E(k) = min\{c_E(k), w[\rho_E^J(k) - \rho_E(k)]\} \tag{2.7}$$

$Z_A(k)$ and $Z_B(k)$ are the actual outflows from cells A and B, respectively at time-step k and $Y_E(k)$ is the actual inflow to cell E. If sum of maximum possible flows from cells A and B is less than the receiving capacity of cell E, then the maximum possible flows from the sending cells are their actual outflows.

$$if\ R_E(k) \geq\ S_A(k) +\ S_B(k)$$

$$Z_A(k) =\ S_A(k)\ and\ Z_B(k) =\ S_B(k) \tag{2.8}$$

When receiving capacity of cell E is smaller than the sum of maximum possible flows from sending cells A and B, actual outflows are calculated as follows:

$$if\ R_E(k) <\ S_A(k) +\ S_B(k)$$

$$Z_A(k)\ =\ mid\{S_A(k), R_E(k) -\ S_B(k),\ p_B R_E(k)\} \tag{2.9}$$

$$Z_B(k)\ =\ mid\{S_B(k), R_E(k) -\ S_A(k),\ p_A R_E(k)\} \tag{2.10}$$

The actual inflow to receiving cell E is the sum of actual outflows from sending cells A and B.

$$Y_E(k) = Z_A(k) + Z_B(k) \qquad\qquad (2.11)$$

2.5.2.3 Unsignalized diverge connection

An unsignalized diverge connection describes traffic flow from the last cell of a link to first cells of two different downstream links. Figure 2.4 shows an intersection to describe the diverging flows. Traffic from cell A takes two different downstream cells, B and E, with proportion β_B of traffic demand from cell A taking cell B and β_E proportion of traffic is directed toward cell E. Daganzo (1995) assumed that these ratios are already known and can be time-varying based on route choice behaviour of commuters. This research study models the split rate as time-varying which are provided by an external component of the research framework that models route choice behaviour of travellers.
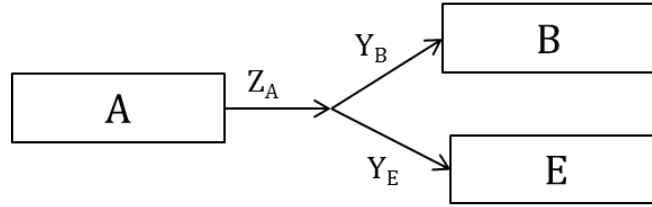
Figure 2.4 A diverge connection to model diverging traffic flows

Cell A contains traffic that will take two different paths. A proportion of the traffic in cell A passes through cell B and the remaining proportion through cell E. If the receiving capacities of cells B and E are higher than the demand for these cells then the proportions of traffic for their respective cells flow without any interruption. However, if the receiving capacity of either of the cell B or cell E is lower than the demand for these cells then the flow towards cell B or E is restricted and results in interruption of all downstream flow of traffic. This condition ensures compliance of diverge model with first-in-first-out (FIFO) principle by restricting the flow of upstream vehicles and ensuring that the traffic entered the link first, leaves the link first. Thus outflow from cell A, $Z_A(k)$ and inflows to cells B and E, $Y_B(k)$ and $Y_E(K,)$ as expressed in Daganzo (1995) and Szeto *et al.* (2009) are given by :

$$Z_A(k) = min \begin{cases} min\{u_j\, \rho_A(k),\ c_A(k)\} \\ min\{c_B(k), w\,[\rho_B^J(k) - \rho_B(k)]/\beta_B(k)\} \\ min\{c_E(k), w\,[\rho_E^J(k) - \rho_E(k)]/\beta_E(k)\} \end{cases} \qquad (2.12)$$

$$Y_B(k) = \beta_B(k)Z_A(k) \; and \; Y_E(k) = \beta_E(k)Z_A(k) \tag{2.13}$$

2.5.2.4 Origin connection

An origin connector connects an origin cell with first cell of a link. Origin connector is similar to an ordinary connector that draws traffic demand from the origin cell.

2.5.2.5 Destination connection

A destination connection connects last cell of a link to destination cell and it is also similar to ordinary link. The destination cell has infinite storage capacity and zero outflow capacity.

2.5.2.6 Conservation of traffic flow

After determining inflows and outflows for all the connectors in the network for current time step *k* using equations (2.4-2.13), traffic state for future time-step *k+1* can be predicted using discretized form of the conservation equation as follows:

$$\rho_i(k+1) = \rho_i(k) + \frac{\Delta}{l_j}\{Y_i(k) - Z_i(k)\} \tag{2.14}$$

2.6 Summary

A brief overview of fundamental traffic flow models is provided in this chapter. Macroscopic traffic flow models consider stream of traffic and model aggregate variables such as traffic density, traffic flow rate, and mean speed. Macroscopic traffic flow models are divided into three main categories, based on the number of traffic flow variables determined using a traffic flow model. First order traffic flow models determine traffic density or occupancy. The LWR model and its discretised form CTM are discussed in detail in this chapter. Second order traffic flow models also consider the dynamics of mean speed along with traffic density. A basic second order model proposed by Payne (1971) and its discretised form proposed by Papageorgiou *et al.* (1990) are described in this chapter. Higher order macroscopic traffic flow models assume that traffic density and mean speed are function of variance in speed. Microscopic traffic flow models provide a detailed description of the phenomena of traffic flow and model each vehicle and its interaction with other vehicles and road

environment. The car-following models describe movement of vehicles on a single lane stretch of a road, when a vehicle follows its lead vehicle. The lane-changing models provide description of lane-changing behaviour of drivers. Some basic car-following and lane-changing models are discussed in section 2.4 of this chapter.

The review of existing traffic flow models and their applications in different traffic systems suggests that first order traffic flow model, such as CTM can adequately model the flow of traffic for a traffic network. The number of output variables and parameters in CTM are significantly lower than the other higher order traffic flow models. This characteristic of CTM makes it feasible for real-time estimation of traffic state and its application for dynamic traffic assignment applications. Therefore, CTM is selected to model network traffic flow and the model is elaborated in detail in this chapter.

Chapter 3: Real-time Traffic State Estimation

3.1 Introduction to traffic state estimation

Real-time traffic state estimation has been an active field of research for many years. With the development of new technologies and the improvement in existing techniques for acquiring real-time traffic data, more emphasis is being given to proper utilization of such data, to obtain a more accurate and widespread picture of the state of a network. However, there are limitations in the data directly obtained from traffic sensors. Firstly, such data do not include all the required parameters for devising traffic management strategies in real-time and do not portray a complete picture of the traffic state across a network. Another problem with obtaining real-time traffic data is that it requires a good communication infrastructure and covering every part of a road network with traffic sensors requires a big capital investment and maintenance cost. On the other hand, the prediction of traffic state using only traffic flow models based on long-term historic information might contain significant error in prediction, especially when actual traffic conditions depart from their historical trend due to external factors. To obtain complete traffic data for the whole network, traffic flow models along with measurements from sensors are used for better estimation with less uncertainty in the final estimate of traffic state compared to prediction or measurement alone. Thus, 'real-time traffic state estimation' refers to estimation of traffic flow variables (traffic flow, density, speed) for a segment of road or network, with an adequate time and space resolution based on limited available measurements from traffic sensors (Wang *et al.* 2008).

Recently, many research studies have focused on such estimation problems in the particular context of freeways (Munoz *et al.* 2003; Munoz *et al.* 2006; Tampere and Immers 2007; Sun *et al.* 2004). Of particular relevance to this research is the work of Wang and Papageorgiou (2005). They presented a comprehensive methodology for estimating traffic state by combining real-time traffic data from sensors with predictions from a second-order traffic flow model. In this approach, they utilized the Extended Kalman Filter (EKF)

variation on the approach originally proposed by Kalman (1960) to combine predictions and measurements by minimizing the sum of squares of errors between the measurement and prediction. Wang and Papageorgiou (2005) also proposed a method for online estimation of the model parameters by converting these parameters into stochastic variables. The proposed model was designed and applied for a stretch of freeway with on-ramps and off-ramps. In a similar spirit, Ngoduy (2008) proposed a framework that utilizes a particle filtering algorithm with a second-order traffic flow model to estimate traffic for a section of freeway; and in Ngoduy (2008; 2011) utilized an unscented Kalman filter algorithm with a macroscopic traffic flow model for freeway traffic state estimation. Park and Lee (2004) used a Bayesian technique to estimate travel speed for a link of an urban arterial using data from a dual loop detector. Gang *et al.* (2007) presented a traffic state estimation scheme based on the Cell Transmission Model (CTM) and Kalman filter for a single urban arterial street under signal control. Liu *et al.* (2012) proposed a travel time estimation approach for a long corridor with signalized intersections based on probe vehicle data.

This chapter provides a brief account of CTM and EKF based real-time estimation framework adapted from Wang and Papageorgiou (2005). An overview of existing estimation techniques and sensor technologies to obtain real-time traffic observations is also given in this chapter. Section-2 of this chapter highlights some of the most commonly used estimation techniques. Section-3 describes the sensor technologies for observation of traffic flow parameters. The framework for traffic state estimation for this research is elaborated in section-4 and section-5 summarizes the findings of this chapter.

3.2 Overview of estimation techniques

State estimation theory has been widely used in the field of dynamic system state estimation and process control. The state of a dynamic system is estimated based on the mathematical description of the process and measurements of variables obtained from the sensors. The prediction of state of a process contains uncertainty either in modelling of the process or in the input parameters and boundary conditions. The observations obtained from

sensors are also noisy and incomplete. Hence, estimation is the process of deducing the value of a quantity of interest from indirect, inaccurate and uncertain observations (Yaakov *et al.* 2001). Estimation techniques are applied to determine a better estimate for dynamic processes in social and applied sciences for problems ranging from microscopic level to planetary orbital parameters. Estimation techniques are applied for various applications, which include (Yaakov *et al.* 2001):

- Statistical inference;
- Aviation, to determine the position and velocity of aircrafts for air traffic control system;
- stochastic control, controlling production process from a plant in presence of uncertainty;
- system identification, determination of state of a physical system or forecasting economic variables;
- Communication theory, to filter the noise from received signals;
- Image processing, to determine some parameters and characteristics of an image.

The estimation techniques were first developed to improve the estimate of positioning of celestial bodies to improve the navigation and reliance on astronomical observations. After the initial contributions by many scientists, first comprehensive estimation technique was proposed by Gauss and Legendre in early nineteen century. The method of least square proposed by Gauss and Legendre provides an optimal estimate based on noisy data. The least square method minimizes the estimated measurement error. The further advancement in the field of estimation was proposed by Norbert Wiener during World War II. The objective of estimation algorithm proposed by Norbert Wiener was to improve the prediction of target aircrafts using noisy tracking data from radar. This algorithm determines the solution based on least-mean-squared prediction error in terms of autocorrelation function of the measurement and the noise (Grewal and Andrews, 2001).

In 1960, Rudolf E. Kalman proposed a state estimation framework by introducing state variables to the Wiener filtering approach and this framework is known as Kalman filter. Kalman filter is considered as the greatest

achievement in the estimation theory during the 20th century. The Kalman filter provides an optimal estimate of a linear dynamic system. McGee and Schmidt adapted Kalman filter approach to estimate the state of non-linear dynamic systems at NASA and proposed an estimation framework which linearizes about an estimate of mean and covariance. This variant of Kalman filter is known as Extended Kalman Filter (EKF) and generally not considered as an optimal state estimator. The performance of EKF reduces with increase in the nonlinearity of the dynamic system and for such a system, the Unscented Kalman Filter (UKF) is applied. Kalman filter and EKF assume that the noises in state prediction and measurements are Gaussian and uncorrelated. In 1993, Gordon *et al.* proposed the Particle Filter (PF) approach based on online posterior density estimation by implementing Bayesian recursive equations. This approach can be used for state estimation of nonlinear and non-Gaussian dynamic systems. The above discussed models are discussed and described in detail in the following section.

3.2.1 Least square methods

The method of estimation using least square (LS) was invented by Carl Gauss in 1809 and independently by Legendre in 1806. Gauss developed the method of least square when predicting the motion of planets using observations from a telescope. This method is simple and many other estimation techniques are based on least square method. Least square method can be applied to linear and nonlinear systems with multiple input and output variables.

The least square estimator minimizes the sum of square of errors in fitting the data and determines least squares of difference between the model predictions and the observations. The observations in this method of estimation should be linearly related to the outputs and the noise is assumed to be additive zero-mean Gaussian distribution. The least square estimation is equivalent to the maximization of the likelihood function of the parameters (Yakoov *et al.* 2001).

The linear least square estimator is applied to estimate an unknown parameter by processing the entire observation set of the parameter. To implement the recursive LS estimator, an initial estimate of the parameter is required, which can be obtained by first using batch technique on a small number of initial

observations. Fletcher (1987) and Blackrnan and House (1999) introduced an iterative LS estimator which iteratively improves the current estimate using observations until the convergence criteria is achieved.

3.2.2 Kalman filter

Rudolf E. Kalman (1960) published a solution for the discrete-time linear filtering problem. The Kalman filter consists of a set of mathematical equations that provides an efficient recursive computation framework to estimate the state of a process by minimizing the mean of the squared error. Since its invention, Kalman filter has been a subject of extensive research and applications in the field of process state estimation and control. The Kalman filter can be used to determine past, current and future states of a system. Furthermore, Kalman filter is also useful in determining the state of a process when the system model is not precise.

The Kalman filter estimates the value of variable x_k in a dynamic discrete process, represented by a linear stochastic equation.

$$x_k = Ax_{k-1} + B\chi_{k-1} + \omega_{k-1} \tag{3.1}$$

where A is a n x n matrix which relates the state at time-step *k-1* with the state at current time-step *k*. The matrix B is n x l order matrix, which relates the current state of the process to optional control input χ_{k-1}. The random variable ω represents noise in modelling the process. The observations z_k are linearly related to the system state x_k and noise in measurement is represented by a random variable υ_k.

$$z_k = Hx_k + \beta\upsilon_k \tag{3.2}$$

The random variables β and α are assumed to be independent, white and described using normal probability distribution with zero mean and variance Q and R, respectively.

$$p(\omega) \sim N(0, Q) \tag{3.3}$$

$$p(\upsilon) \sim N(0, R) \tag{3.4}$$

To elaborate the computation using Kalman filter, *a priori* $\hat{x}_k^-$ and *a posteriori* $\hat{x}_k$ estimates of process state are defined. The *a priori* state estimate at time-step k is based on the knowledge of the process prior to time-step k and *a posteriori* state estimate is based on the observation obtained at time-step k. The errors in estimating *a priori* and *a posteriori* states are defined as follows:

$$e_k^- = x_k - \hat{x}_k^- \tag{3.5}$$

$$e_k = x_k - \hat{x}_k \tag{3.6}$$

The *a priori* and *a posteriori* estimate covariance is given by:

$$P_k^- = E[e_k^- e_k^{-T}] = A P_{k-1} A^T + Q \tag{3.7}$$

$$P_k = E[e_k e_k^T] = (I - K_k H) P_k^- \tag{3.8}$$

The Kalman filter estimates *a posteriori* state of the process using a linear combination of *a priori* state and a weighted difference between the actual measurement and the predicted measurement of the state.

$$\hat{x}_k = \hat{x}_k^- + K (z_k - H \hat{x}_k^-) \tag{3.9}$$

In equation (3.9), the factor $(z_k - H\hat{x}_k)$ is known as residual and it represents the discrepancy between observations and predicted observations. A factor K, known as Kalman Gain is computed so as to minimize the *a posteriori* error covariance.

$$K_k = P_k^- H^T (H P_k^- H^T + R)^{-1} \tag{3.10}$$
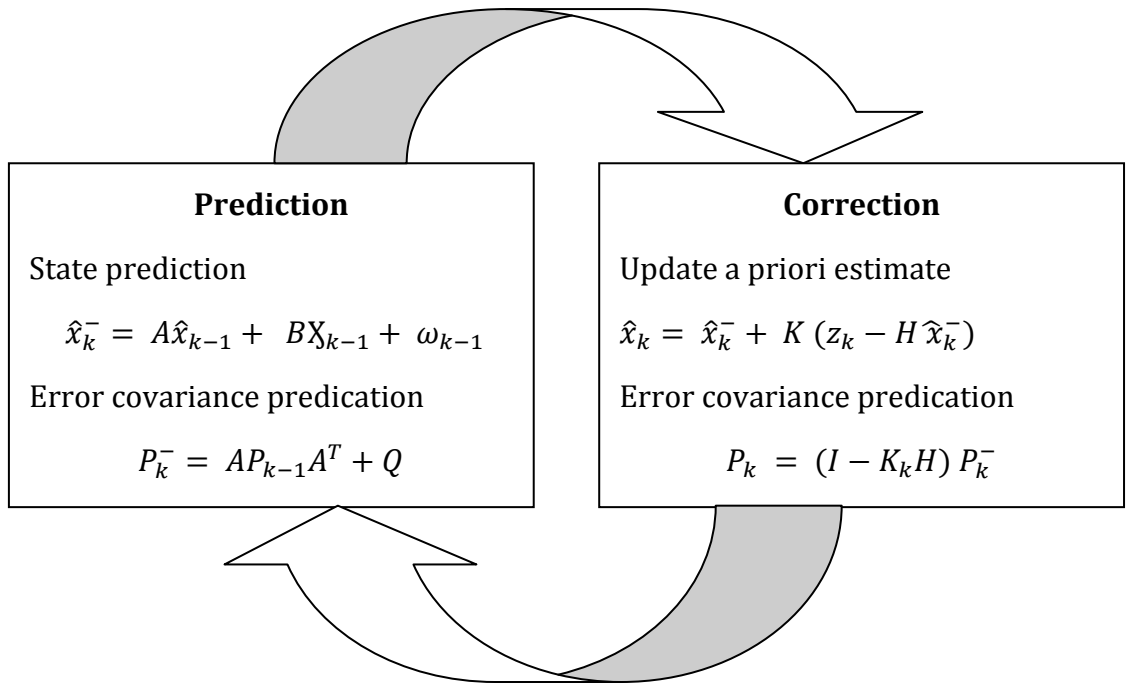
Figure 3.1 The prediction-correction feedback cycle of Kalman filter

Based on equation (3.9), the estimation process using Kalman filter can be divided into two steps as shown in figure 3.1. The first step is *prediction* and the second step is *correction*. The prediction equations in Kalman filter project the state of the system and covariance for current time-step based on the past state of the system to obtain *a priori* estimate. The correction equations provide a feedback on the prediction by incorporating a new measurement to update *a priori* estimate and obtain an improved *a posteriori* estimate of state of the process.

Kalman filter provides a computationally efficient framework to obtain optimal estimate of a process state, when the process is modelled using a linear relationship and the observations are also linearly related to the state of the system. Thus, the applications of Kalman filter are limited as it does not estimate the state of a nonlinear and non-Gaussian system.

3.2.3 Extended Kalman filter

Most of the real life phenomenon cannot be modelled using a linear relation and Kalman filter cannot be applied to estimate the state of such non-linear systems. The extended Kalman filter (EKF) was proposed to estimate the state of nonlinear dynamic systems. The EKF linearizes the estimation about the

current mean and covariance by taking partial derivatives of the process and measurement functions using *Jacobian* matrices. The EKF works on the principal that the linearized transformation of the process and measurement functions are true approximation of the actual functions. Therefore, the solution obtained using EKF is not guaranteed to be an optimal estimate of the process state.

The dynamic state of the process is governed by a non-linear stochastic difference equation represented by differentiable function *f*.

$$x_k = f(x_{k-1}, Ⅹ_{k-1}, \omega_{k-1}) \tag{3.11}$$

The measurements z_k are related to the output variable with a nonlinear stochastic function *h*.

$$z_k = h(x_k, v_k) \tag{3.12}$$

Since, it is difficult to determine the values of noise ω_k and v_{k-1} for each time-step, equations (3.11) and (3.12) can be approximated by omitting them.

$$\tilde{x}_k = f(\hat{x}_{k-1}, Ⅹ_{k-1}, 0) \tag{3.13}$$

$$\tilde{z}_k = h(\tilde{x}_k, 0) \tag{3.14}$$

The linearized transformation of process state equation (3.11) about equation (3.13) and measurement equation (3.12) about equation (3.14) is given as follows:

$$x_k \approx \tilde{x}_k + A(x_{k-1} - \hat{x}_{k-1}) + \Gamma\omega_{k-1} \tag{3.15}$$

$$z_k \approx \tilde{z}_k + B(x_k - \tilde{x}_k) + \Pi\beta_k \tag{3.16}$$

Where, x_k and z_k are actual state and measurement vectors approximated to $\tilde{x}_k$ and $\tilde{z}_k$, respectively. $\hat{x}_k$ is an *a posteriori* estimate of process state. *A*, $\boldsymbol{\Gamma}$, *B*, and Π are *Jacobian* matrices of partial derivatives.

$$A = \frac{\partial}{\partial x} f(x_{k-1}, Ⅹ_{k-1}, 0) \tag{3.17}$$

$$\Gamma = \frac{\partial}{\partial \alpha} f(x_{k-1}, Ⅹ_{k-1}, 0) \tag{3.18}$$

$$B = \frac{\partial}{\partial x} h\left(\tilde{x}_k, 0\right) \tag{3.19}$$

$$\Pi = \frac{\partial}{\partial \beta} h\left(\tilde{x}_k, 0\right) \tag{3.20}$$

The estimated state of a process using EKF can be determined using a relationship similar to KF.

$$\hat{x}_k = \tilde{x}_k + K_k\{z_k - h\left(\tilde{x}_k, 0\right)\} \tag{3.21}$$

Where K_k is Kalman gain matrix and determined for each time-step using the following relation.

$$K_k = P_k^- B_K' \left(B_k P_k^- B_K' + \Pi_k R_k \Pi_K'\right)^{-1} \tag{3.22}$$

The *a priori* error covariance matrix P_k^- and the *a posteriori* covariance matrix P_k is defined as follows:

$$P_k^- = A_k P_{k-1} A_k' + \Gamma_k Q_{k-1} \Gamma_k' \tag{3.23}$$

$$P_k = (I - K_k B_k) P_k^- \tag{3.24}$$

Prediction	Correction
State prediction $$\tilde{x}_k = f\left(\hat{x}_{k-1}, \mathsf{X}_{k-1}, 0\right)$$ Error covariance predication $$P_k^- = A_k P_{k-1} A_k' + \Gamma_k Q_{k-1} \Gamma_k'$$	Update a priori estimate $$\hat{x}_k = \tilde{x}_k + K_k\{z_k - h\left(\tilde{x}_k, 0\right)\}$$ Error covariance predication $$P_k = (I - K_k B_k) P_k^-$$
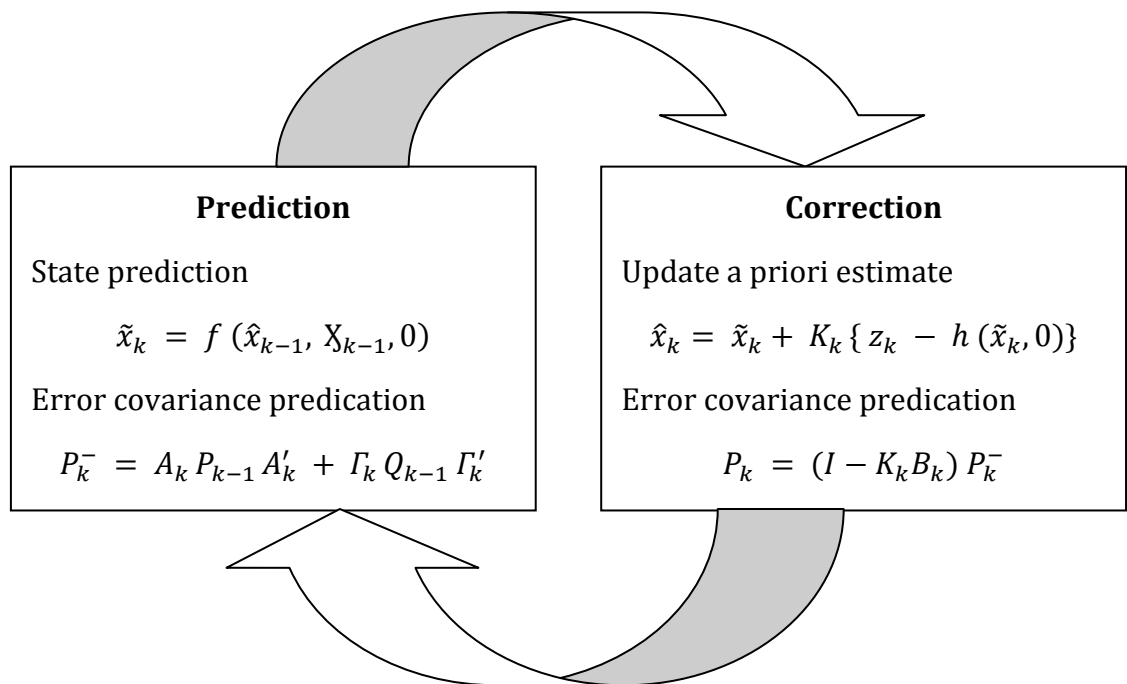
Figure 3.2 The prediction-correction feedback cycle for EKF

Similar to Kalman filter, the EKF process can also be divided into *prediction* and *correction* feedback cycle. The framework defined for EKF, linearizes the state of a non-linear process about current estimate of mean and covariance. The EKF provides a framework to estimate state of non-linear dynamic process which cannot be estimated using Kalman filter. However, the estimated state using EKF is generally not an optimal estimate due to the linear approximation. Furthermore, incorrect initial estimate of the state and inaccurate physical description of the process may cause the filter to quickly diverge.

3.2.4 Unscented Kalman filter

Kalman filter provides an optimal estimate of a process state when the process is modelled using a linear relation. The process state in extended Kalman filter is approximated by Gaussian random variable and it is analytically propagated through the linearization, which could corrupt the mean and covariance of the state estimate. Unscented Kalman filter (UKF) provides a derivative free approach to estimate the state of a nonlinear process.

Julier and Uhlmann (1997) proposed a framework to estimate state of a nonlinear process using statistical linearization technique. The UKF is developed on the principle that it is easier to approximate a probability distribution than the approximation of an arbitrary nonlinear function (Julier and Uhlmann 2004). The UKF selects a set of points to propagate them through the actual nonlinear function such that these points completely represent the mean, variance and higher order moments of the Gaussian random variable. The selection of sample points is deterministic. For a Gaussian random variable of dimension *n*, the number of selected points is *2n+1*.

Unscented Kalman filter selects a set of points deterministically to estimate the mean and covariance of nonlinear dynamic process. The basic limitation of Kalman filter and all its variants is that the noise in process and measurement should be Gaussian and the processes represented by non-Gaussian noise cannot be estimated using family of Kalman filters.

3.2.5 Particle filter

Kalman filter based estimating techniques are formulated to determine the state of a process represented by Gaussian random variable and do not address estimation of process corrupted with non-Gaussian noise. Particle filter is designed to estimate the state of nonlinear dynamic system and it is also useful to estimate the process represented by non-Gaussian random variables. Particle filter also has advantage over UKF, as the number of sigma points in UKF are based on an algorithm and are much smaller than the number of samples points selected in particle filter. The estimation error in UKF filter does not converge to zero, however in particle filter algorithm the error can converge to zero with increase in number of sample particles.

Gordon *et al.* (1993) proposed particle filter (PF) based on sequential Monte Carlo method which provides an effective solution for nonlinear and non-Gaussian problems and it has been widely applied for many important processes in science and engineering. The PF samples a number of points from hypothesized states of the process, called particles. The uncertainty and the distribution of process state are represented by a diverse set of particles. All the sampled particles are compared and weighted with the measurement of the state variable. The particles with higher weights are retained and propagated through the function while the particles with lower weights are rejected. Thus, the particles selected for estimation tend to concentrate in the region of higher probability. The PF represents the posterior density by a set of weighted particles and the state is estimated based on these particles.

Particle filter provides a sophisticated estimation technique based on simulation which is also applicable to nonlinear and non-Gaussian processes. However, it needs analysis of large number of particles and optimal number of particles cannot be determined. This deficiency makes particle filter unfeasible for real-time applications of complex dynamic systems.

3.3 Real-time traffic state measurements

The increase in traffic congestion and growth of cities needs batter traffic management strategies. Intelligent traffic systems (ITS) have been widely used

in urban and freeways traffic systems to improve network performance and optimally utilize the existing capacity of road network. Accurate observation of existing traffic condition is essential to devise, implement and evaluate traffic management strategies. With advancement in detection and communication technologies, measurement of vehicular traffic flow has been significantly improved in past few decades.

Many busy cities and freeways are covered with different traffic measuring devices to obtain measurements of traffic flow parameters. The parameters measured using traffic sensors include traffic count, vehicle classification, vehicle occupancy, space-mean speed and travel time. Traffic counts can be obtained using portable counters, permanent counters, or video recordings. The automatic counts are taken for every one hour interval for a period of 24 hours, which can be extended for a week, month or year. The traffic count data is mostly used for traffic planning and improvement in existing traffic network capacity or traffic control. The data related to different vehicle types is also collected along with vehicle count. The vehicle classification data generally classifies vehicles into cars, trucks, buses, motorcycles and HTVs (Heavy Transit Vehicles). Vehicle classification data is utilized in structure design of pavement, environmental impact analysis, revenue and toll estimation, and to estimate capacity of the highway. The sensor occupancy is an important measure of traffic congestion and road performance and it depends on the speed and length of the vehicle. The occupancy is measured in percentage of time occupied by a vehicle in the detection zone. Traffic density can be calculated based on the sensor occupancy data and traffic density observations are widely used in traffic surveillance and ITS applications. The observation of 'presence' from measurement sensors detects if the sensor is occupied with a vehicle or not. The measurement of travel time is of prime importance for commuters, traffic planners, and traffic management authorities. Travel time observations are used for congestion management, transportation planning and ATIS. Traffic management systems use performance measures which are based on travel time to evaluate and monitor traffic system performance. Some traffic management authorities provide predicted travel times using ATIS. Travel time

data can be obtained directly using test vehicle, licence plate matching, and ITS probe vehicle techniques.

Traffic observation sensors installed at a fixed location to obtain traffic measurements are defined as in-situ detectors. The location of detectors and number of detectors installed in a network play a significant role in quality of traffic measurements. All in-situ traffic detectors provide a measure of traffic flow with some additional parameter observed, depending on specific type of the detector. Table 3.1 provides a list of in-situ traffic observation technologies, with the parameters of traffic flow measured using the sensor technology.

Table 3.1 A list of different in-situ sensor technologies (Klein *et al.* 2006)

Sensor Technology	Count	Speed	Vehicle classification	Occupancy	Presence	Multilane
Pneumatic tubes	Yes	Yes	Yes	No	No	No
Magnetic loop	Yes	With dual	No	Yes	Yes	No
Inductive loop	Yes	With dual	Yes	Yes	Yes	No
Video processing	Yes	Yes	Yes	Yes	Yes	Yes
Active Infrared	Yes	Yes	Yes	No	No	Yes
Passive Infrared	Yes	Yes	No	Yes	Yes	No
Microwave Radar	Yes	Yes	Yes	Yes	Yes	Yes
Ultrasonic	Yes	No	No	Yes	Yes	No
Passive acoustics	Yes	Yes	No	Yes	Yes	Yes

3.4 Real-time traffic state estimation model

Based on the discussion on traffic flow models in chapter 2, the Cell Transmission Model (CTM) is selected to model traffic flow propagation for traffic network. CTM is a discretized form of first order LWR traffic flow model and it is considered sufficient to model network traffic flow. It is computationally less expensive than other higher order traffic flow models, which makes it more suitable for application to real-time traffic state estimation and DTA problems. CTM predicts traffic density for current time-step for the entire network based on traffic densities estimated for a previous time-step.

The Extended Kalman Filter (EKF) is selected to estimate traffic state using nonlinear CTM and linear relationship between predicted and measure state. EKF is suitable filtering technique for nonlinear traffic flow model and it is computationally feasible for real-time applications. The framework proposed by Wang and Papageorgiou (2005) for nonlinear second order traffic flow model is adapted to implement with CTM for real-time traffic state estimation. Wang and Papageorgiou (2005) applied second order traffic flow model (METANET) with non-linear EKF. Tampere and Immers (2007) applied the framework proposed by Wang and Papageorgiou (2005) for CTM with linear Kalman filter. The approach proposed in this thesis uses non-linear EKF framework proposed by Wang and Papageorgiou (2005) along with CTM for estimation of traffic state. Table 3.2 compares the approach applied in this research with the approaches used by Tampere and Immers (2007) and Wang and Papageourgiou (2005). However, applying EKF for CTM based on Wang and Papageorgiou (2005) is not the main contribution of this research. Deriving predictive traveller information and integrating route choice models from DTA is the main contribution of this research, which is highlighted in chapter-5 and applied to various applications in chapters 6, 7 and 8.

Table 3.2 Comparison of traffic state estimation approaches

Approach	Traffic flow model	Estimation algorithm
Wang and Papageorgiou (2005)	METANET	Extended Kalman Filter
Tampere and Immers (2007)	CTM	Linear Kalman Filter
Ahmed (2015)	CTM	Extended Kalman Filter

The components of traffic state estimation framework are described in this section of the chapter.

3.4.1 State-space representation of CTM for traffic state estimation

The CTM model described in section 2.5 is redefined in a compact form using state-space form for better representation and implementation of EKF. The conservation equation defined in equation (2.14) predicts traffic density for all cells in the network for current time-step based on estimated traffic state for previous time-step and it given as follows:

$$\rho_i(k + 1) = \hat{\rho}_i(k) + \frac{\Delta}{l}\{q_i(k) - q_{i+1}(k)\} \qquad (3.25)$$

Where $\rho_i(k+1)$ is predicted traffic density for all cells *i =1, 2, 3, …* at time-step *k+1*, based on estimated traffic density for current time-step $\hat{\rho}_i(k)$ and $q_i(k)$ is the inflow to cell-*i* with outflow as $q_{i+1}(k)$. The inflows and outflows are determined based on type of the cell as defined in equations (2-4-2.13). The predicted traffic densities for all the cells in the network can be represented in a vector form as follows:

$$\mathbf{z} = [\rho_1 \ \rho_2 \ \rho_3 \ …. \ \rho_N] \qquad (3.26)$$

The accuracy of traffic density predicted using equations (3.25) is highly dependent on accuracy of parameter values of the fundamental traffic flow diagram. Fundamental traffic flow diagram shown in figure 2.1 is based on initial values of traffic flow parameters and it is not constant for the simulation horizon, as it changes with the new estimate of traffic flow parameters. Accurate estimate of traffic flow parameters such as critical density, flow

capacity and jam density significantly impacts estimate of traffic state. The values of traffic flow parameters can be affected by severe weather condition, change of traffic mix, traffic incidents etc. The proposed framework is also applied to an application when traffic network is affected with an incident and the prediction model is not provided with any information about occurrence and duration of the incident. In this scenario, when estimating traffic density during incident with naive model, accurate estimation of traffic flow parameters plays a significant role. Following other research studies (Wang and Papageorgiou 2005; Wang *et al.* 2008; Ngoduy 2011) for parameter estimation, this research applies a similar approach to estimate the parameters of fundamental traffic flow diagram by converting these parameters into stochastic variables and use measurements from traffic sensors to track any changes in parameters of fundamental traffic flow diagram.

Traffic flow parameters such as critical density, jam-density and traffic flow capacity can be included in the estimation scheme for real-time parameter estimation of CTM. Other parameters of fundamental traffic flow diagram such as free flow speed and backward wave speed can be determined based on estimated values of critical density, jam-density and traffic flow capacity. However, in this research, to keep the number of variables minimum in the estimation model, only critical density is estimated for each cell with a measurement sensor. Traffic flow capacity and jam-density are calculated for each time-step using estimated critical density by assuming that free-flow speed and backward wave speed are unchanged. Furthermore, addition of all three unknown parameters in the estimation model might lead to a nondeterministic system and cause the EKF-based estimation model not converging for solution.

$$c_i(k) \ = \ \rho_i^c(k) * u \tag{3.27}$$

$$\rho_i^J(k) \ = \rho_i^c(k) \ + \ \rho_i^c(k) * u \ / \ w \tag{3.28}$$

Critical traffic density is transformed into a stochastic variable by adding a white Gaussian noise with standard deviation $\varepsilon_i^c(k)$.

$$\rho_i^c(k+1) = \rho_i^c(k) + \varepsilon_i^c(k) \tag{3.29}$$

All the parameters of fundamental traffic flow diagram and boundary conditions for CTM can be included in the estimation scheme and represented by a vector.

$$\boldsymbol{d} = [\rho_1^c \ \rho_2^c \ \rho_3^c \ \dots . \rho_a^c] \tag{3.30}$$

Where a is the number of cells in the network equipped with a sensor. The outputs from estimation model which include traffic density from CTM and unknown model parameters can be merged into an array.

$$\boldsymbol{x} = [\boldsymbol{z}^T \ \boldsymbol{d}^T] \tag{3.31}$$

If error in prediction of traffic density is $\varepsilon_i^\rho(k)$, the nonlinear function f_1 representing traffic density using CTM is given by:

$$\boldsymbol{z}(k + 1) = f_1[\ \boldsymbol{z}(k), \boldsymbol{\varepsilon}^\rho(k)] \tag{3.32}$$

The augmented state-space form of prediction function is given by:

$$\boldsymbol{x}(k + 1) = f[\boldsymbol{x}(k), \boldsymbol{\varepsilon}(k)] \tag{3.33}$$

Where

$$\boldsymbol{\varepsilon} = [\boldsymbol{\varepsilon}^{\rho T} \ \boldsymbol{\varepsilon}^{c T}] \tag{3.34}$$

Equation (3.33) represents a recursive dynamic function for prediction of traffic state.

3.4.2 State-space representation of real-time observations

Various types of traffic observation sensors collecting real-time measurements of traffic flow parameters are discussed in the previous section of this chapter. For illustrative purpose, this research work assumes that there are traffic measurement sensors installed along various links in the network, which measure traffic density and communicate it to the controller in real-time. In reality, the observations obtained from the sensors include sensor occupancy, flow rate, vehicle classification and speed. Most of the sensor technologies such as magnetic loop detector, inductive loops, video cameras, passive infrared, microwave radar and passive acoustic sensor collect measurements for traffic occupancy (Klein *et al.* 2006). Traffic density can be determined based on measurements of traffic occupancy obtained from the sensors. However, the

proposed model can be used for any other type of real observations obtained from measurement sensors such as traffic flow or speed.

In this research, we generate synthesized measurements by simulating reality using a CTM model. The CTM model to generate traffic density measurements is based on actual network capacity and provided with the values of traffic flow parameters which reflect the impact of any real-time changes on cell/route capacity. Furthermore, a white Gaussian noise is also added in the simulated reality to depict stochasticity in the measurements obtained in real-world. Let $m_i^\rho(k)$ denotes the measurement of traffic density during time period *[(k-1)t, kt]* and *Øᵢ(k)* is Gaussian white noise in measurement of traffic density. The frequency for acquiring sensor measurements is assumed equal to the CTM prediction frequency (30 seconds), but this assumption does not limit the application of proposed framework to the systems with different measurement and prediction frequencies. The measurements obtained from a traffic sensor for a given time-step is related to predicted traffic density based on the following equation:

$$m_i^\rho(k) = \rho_i(k) + \emptyset_i(k) \tag{3.35}$$

The measurements obtained from sensors can be represented using a vector $\boldsymbol{y}$.

$$\boldsymbol{y} = [m_1^\rho \ m_2^\rho \ \dots \ m_a^\rho] \tag{3.36}$$

The measurement vector $\boldsymbol{y}$ is linked with prediction vector $\boldsymbol{x}$ with a differentiable function *g* as follows:

$$\boldsymbol{y}(k) = g(\boldsymbol{x}(k), \emptyset(k)) \tag{3.37}$$

3.4.3 Extended Kalman filter for traffic state estimation

The EKF is considered as a de facto estimation algorithm for estimating state of a non-linear dynamic system and traffic flow models including CTM are nonlinear in nature. EKF has been consistently used for estimation of traffic state (Wang *et al.* 2011; Wang *et al.* 2008, Wang and Papageorgiou 2005; Meier and Wehlan 2001). Other estimation algorithms for nonlinear systems such as particle filters and unscented Kalman filter are computationally expensive algorithms when compared to EKF. The EKF is more efficient in computation and may be applied in large scale networks.

For traffic density estimation of traffic network, the framework described in Wang and Papageorgiou (2005) is adapted for CTM. The objective of EKF at each time-step k is to find a state estimate which minimizes covariance of estimation error using all available measurements till time-step k.

$$E\{[\boldsymbol{x}(k+1) - \widehat{\boldsymbol{x}}(k+1/k)]^T . [\boldsymbol{x}(k+1) - \widehat{\boldsymbol{x}}(k+1/k)]\} \qquad (3.38)$$

For any estimation problem using EKF, the following three conditions must be satisfied.

i) Noises in measurement $\emptyset(k)$ and in prediction process $\boldsymbol{\varepsilon}(k)$ are zero-mean Gaussian white random processes. For any $k>0$ and $l>0$:

$$E[\boldsymbol{\varepsilon}(k)] = 0 \qquad (3.39)$$

$$E[\emptyset(k)] = 0 \qquad (3.40)$$

$$E[\boldsymbol{\varepsilon}(k)\boldsymbol{\varepsilon}^T(l)] = \begin{cases} \boldsymbol{Q} \ if \ k = l, \\ 0 \ otherwise \end{cases} \qquad (3.41)$$

$$E[\emptyset(k)\emptyset^T(l)] = \begin{cases} \boldsymbol{R} \ if \ k = l, \\ 0 \ otherwise \end{cases} \qquad (3.42)$$

$$E[\boldsymbol{\varepsilon}(k)\emptyset^T(l)] = \begin{cases} M \ if \ k = l, \\ 0 \ otherwise \end{cases} \qquad (3.43)$$

Where, $\boldsymbol{Q}$ and $\boldsymbol{R}$ are known symmetric matrices representing variance of noise in prediction of model and noise in measurements, respectively.

ii) Initial state $\boldsymbol{x(0)}$ is a Gaussian random with known mean and covariance matrix.

$$\widehat{\boldsymbol{x}}_0 = E[x(0)] \qquad (3.44)$$

$$P_0 = E\{[\boldsymbol{x}(0) - \widehat{\boldsymbol{x}}_0] . [\boldsymbol{x}(0) - \widehat{\boldsymbol{x}}_0]\} \qquad (3.45)$$

iii) Initial state $\boldsymbol{x(0)}$ is not correlated with model prediction or measurement noise at any time instant.

The recursive equation of EKF that recursively estimates the current traffic state based on prediction of traffic state from CTM and observation of traffic state is given by:

$$\widehat{\boldsymbol{x}}(k+1/k) = f[\widehat{\boldsymbol{x}}(k/k-1), 0] + K(k) [\boldsymbol{y}(k) - g(\widehat{\boldsymbol{x}}(k/k-1), 0)] \qquad (3.46)$$

Where K is Kalman Gain Matrix and which is estimated at each time-step:

$$K(k) = [A(k)P(k/k-1)B^T(k) + \Gamma(k)M(k)\Pi(k)].[B(k)P(k/k-1)B^T(k) + \Pi(k)R(k)\Pi^T(k)]^{-1} \tag{3.47}$$

The covariance for next time-step is also predicted and given by:

$$P(k+1/k) = [A(k) - K(k)B(k)].P(k/k-1)A^T(k) + \Gamma(k)Q(k)\Delta^T(k) - K(k)\Pi(k)M^T(k)\Gamma^T(k) \tag{3.48}$$

In equation (3.47) and (3.48), *A(k)* represents first-order partial derivative of prediction function *f* with respect to the **x**, which contains all output variables. This is also known as *Jacobian* matrix. *Jacobian* matrix *B(k)* represents first-order partial derivative of function *g* with respect to vector **x**. *Jacobian* matrix $\Gamma(k)$ is first-order partial derivative of prediction function *f* with respect to prediction error *ε(k)* and *Jacobian* matrix $\Pi(k)$ is first-order partial derivative of function *g* with respect to measurement error **φ**.

$$A(k) = \frac{\partial f}{\partial x}(\hat{x}(k/k-1),0) \tag{3.49}$$

$$B(k) = \frac{\partial g}{\partial x}(\hat{x}(k/k-1),0) \tag{3.50}$$

$$\Gamma(k) = \frac{\partial f}{\partial \varepsilon}(\hat{x}(k/k-1),0) \tag{3.51}$$

$$\Pi(k) = \frac{\partial g}{\partial \emptyset}(\hat{x}(k/k-1),0) \tag{3.52}$$

The proposed CTM-EKF real-time traffic state estimation framework is applied to a simple link to elaborate the equations and matrices defined in the formulation and to demonstrate the working of the proposed framework.

3.4.4 Application of CTM-EKF model to a simple link

The CTM-based real-time traffic state estimation framework is elaborated here with an example of a simple link. A two-lane stretch of road is considered, which is equipped with a measurement sensor which provides real-time measurements of traffic density. For illustrative purpose, these measurements are generated using an independent CTM model with slightly different link demand. A Gaussian white noise is added in the simulated measurements of traffic density to depict actual noise in observations. Figure 3.3 shows the simple link considered for this example.
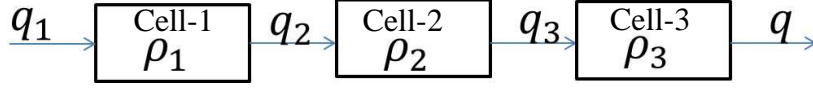
Figure 3.3 A simple link to illustrate CTM-EKF framework

3.4.4.1 Traffic density prediction based on CTM

A simple link with a length of 1500 m is considered in this application of CTM-EKF model. The link is divided into three cells of equal lengths, each of 500 m, with one dummy cell to generate traffic demand and another dummy cell to absorb traffic arriving at the destination. There is a measurement sensor in cell-2 of the link which provides real-time observations for traffic density. The free-flow speed of link is 60 km/hr and shockwave speed is 20 km/hr. All the cells in the link have traffic flow capacity of 1800 veh/hr/ln, critical density of 30 veh/km/ln and jam-density of 120 veh/km/ln. Since all the cells in the link have same parameter values which do not change during the simulation horizon, equation (2.4) to compute inflow q_i to cell i from cell $i-1$ can be simplified as follows:

$$q_i(k) = min\{u\,\rho_{i-1}(k), c,\ w[\rho^J - \rho_i(k)]\} \tag{3.53}$$

Equation (2.4) has been simplified by replacing actual outflow $Y_i(k)$ and actual inflow $Z_i(k)$ with $q_i(k)$, as inflow to a downstream cell is always equal to the outflow from the upstream cell for a simple link. Based on the calculated traffic flow for time-step k, traffic density is predicted for future time-step $k+1$ based on equation 2.14.

$$\rho_i(k+1) = \rho_i(k) + \frac{\Delta}{l}\{q_i(k) - q_{i+1}(k)\} \tag{3.54}$$

The link is simulated for a time period of 30 minutes, which is divided into 60 time-steps, each of 30 seconds. Traffic demand profiles for traffic density prediction and to generate simulated measurements are shown in figure 3.4.

The state estimation using Kalman filter based estimation frameworks assumes that there is an underlying modelling error represented by white Gaussian noise. In predicting traffic density using CTM, this error could be due to change in traffic demand, inaccurate fundamental traffic flow diagram and inappropriately calibrated CTM model.

3.4.4.2 Measurements from traffic sensors

The synthetic measurements are generated in this research using an independent CTM model based on simulated reality and provide measurements of traffic density polluted with white Gaussian noise. The CTM model to generate simulated traffic reality is provided with actual traffic demand and parameter values representing any changes due to incident, compared to naïve CTM prediction model with constant parameter values and unable to represent real-time changes in traffic demand and/or traffic flow capacity. To elaborate the process of traffic state estimation, the measurements of traffic density are generated with slightly different traffic demand compared to the demand provided to CTM model for prediction of traffic density. In this application of proposed CTM-EKF model, it is assumed that there is a measurement sensor in cell-2. Thus, traffic density from simulated reality for cell-2 is forwarded to CTM-EKF model after addition of noise.

The measurement of traffic density obtained from traffic sensor in cell-2 is mapped with the estimated output of CTM-EKF model based on equation 3.35.

$$m_1^\rho(k) = \rho_2(k) + \emptyset_2(k) \tag{3.55}$$

3.4.4.3 State-space representation of predictions and measurements

Based on methodology described in sections 3.4.1 and 3.4.2 for state-space representation, the predicted traffic density and measurements are transformed into state-space form as followed.

Traffic density prediction for three cells:

$$\mathbf{z} = [\rho_1 \ \ \rho_2 \ \ \rho_3] \tag{3.56}$$

Traffic density prediction for future time-step derived based on equation (3.54) with a prediction error defined by $\varepsilon_i^\rho(k)$.

$$\boldsymbol{\varepsilon}^\rho = [\varepsilon_1^\rho \ \ \varepsilon_2^\rho \ \ \varepsilon_3^\rho] \tag{3.57}$$

$$\mathbf{z}(k+1) = f_1[\ \mathbf{z}(k), \boldsymbol{\varepsilon}^\rho(k)] \tag{3.58}$$

In this example, no parameters are included in the estimation scheme, thus:

$$\mathbf{x} = [\mathbf{z}^T] = \begin{bmatrix} \rho_1 \\ \rho_2 \\ \rho_3 \end{bmatrix} \tag{3.59}$$

The recursive dynamic function f for traffic density prediction based on CTM is given by:

$$x(k + 1) = f[x(k), \varepsilon^\rho(k)] \tag{3.60}$$

Similarly, the measurements obtained from a sensor in cell-2 can be written in vector form.

$$y = [m_1^\rho] \tag{3.61}$$

The measurement vector y is linked with predicted traffic state x using function g based on equation (3.55).

$$y(k) = g(x(k), \emptyset(k)) \tag{3.62}$$

3.4.4.3 Traffic density estimation using CTM-EKF framework

The CTM-EKF model recursively estimates traffic density for given link based on equations (3.46-3.48). All the matrices mentioned in equations (3.46-3.48) are described here in detail for application to the simple link. For simplicity of presentation, the time-script k is removed from the description of equations.

Jacobian matrix A:

$$A = \frac{\partial f}{\partial x}(\hat{x}(k/k - 1), 0) = \frac{\partial \begin{bmatrix} \rho_1 + \frac{\Delta}{l}\{q_1 - q_2\} + \varepsilon_1^\rho \\ \rho_2 + \frac{\Delta}{l}\{q_2 - q_3\} + \varepsilon_2^\rho \\ \rho_3 + \frac{\Delta}{l}\{q_3 - q\} + \varepsilon_3^\rho \end{bmatrix}}{\partial \begin{bmatrix} \rho_1 \\ \rho_2 \\ \rho_3 \end{bmatrix}} \tag{3.63}$$

Where q is the link outflow representing outflow from cell-3 to the dummy destination cell and q_0 is traffic demand for cell-1 originating from dummy cell. Determining *Jacobian* matrix also involves partial derivatives of q_1, q_2, and q_3 with respect to ρ_1, ρ_2, and ρ_3. These partial derivatives are determined as follows:

$$\frac{\partial q_1}{\partial \rho_1} = \frac{\partial min(q_0, c_1, w(\rho^J - \rho_1))}{\partial \rho_1} = \begin{cases} -w \; if \, q_1 = w(\rho^J - \rho_1) \\ \quad 0 \; otherwise \end{cases}$$

$$\frac{\partial q_1}{\partial \rho_2} = \frac{\partial min(q_0, c_1, w(\rho^J - \rho_1))}{\partial \rho_2} = 0$$

$$\frac{\partial q_1}{\partial \rho_3} = \frac{\partial min(q_0, c_1, w(\rho^J - \rho_1))}{\partial \rho_3} = 0$$

$$\frac{\partial q_2}{\partial \rho_1} = \frac{\partial min(u \, \rho_1, c, w(\rho^J - \rho_2))}{\partial \rho_1} = \begin{cases} u \; if \, q_2 = u\rho_1 \\ 0 \; otherwise \end{cases}$$

$$\frac{\partial q_2}{\partial \rho_2} = \frac{\partial min(u\,\rho_1,c,w(\rho^J-\rho_2))}{\partial \rho_2} = \begin{cases} -w \; if\,q_2 = w(\rho^J - \rho_2) \\ \quad 0 \; otherwise \end{cases}$$

$$\frac{\partial q_2}{\partial \rho_3} = \frac{\partial min(u\,\rho_1,c,w(\rho^J-\rho_2))}{\partial \rho_3} = 0$$

$$\frac{\partial q_3}{\partial \rho_1} = \frac{\partial min(u\,\rho_2,c,w(\rho^J-\rho_3))}{\partial \rho_1} = 0$$

$$\frac{\partial q_3}{\partial \rho_2} = \frac{\partial min(u\,\rho_2,c,w(\rho^J-\rho_3))}{\partial \rho_2} = \begin{cases} u \; if\,q_3 = u\rho_2 \\ 0 \; otherwise \end{cases}$$

$$\frac{\partial q_3}{\partial \rho_3} = \frac{\partial min(u\,\rho_2,c,w(\rho^J-\rho_3))}{\partial \rho_3} = \begin{cases} -w \; if\,q_3 = w(\rho^J - \rho_3) \\ \quad 0 \; otherwise \end{cases}$$

The partial derivatives are described above to explain in detail the process of determining derivatives of components of *Jacobian* matrix A.

The *Jacobian* matrix A can be written in a compact for as follows:

$A =$

$$\begin{bmatrix} 1 + \frac{\Delta}{l}*\{(\begin{cases} -w\,if\,q_1 = w(\rho^J - \rho_1) \\ 0\,otherwise \end{cases}) - (\begin{cases} u\,if\,q_2 = u\rho_1 \\ 0\,otherwise \end{cases})\} & \frac{\Delta}{l}*\begin{cases} -w\,if\,q_2 = w(\rho^J - \rho_2) \\ \quad 0\,otherwise \end{cases} & 0 \\ \frac{\Delta}{l}*\begin{cases} u\,if\,q_2 = u\,\rho_1 \\ 0\,otherwise \end{cases} & 1 + \frac{\Delta}{l}*\{(\begin{cases} -w\,if\,q_2 = w(\rho^J - \rho_2) \\ 0\,otherwise \end{cases}) - (\begin{cases} u\,if\,q_3 = u\,\rho_2 \\ 0\,otherwise \end{cases})\} & \frac{\Delta}{l}*\begin{cases} -w\,if\,q_3 = w(\rho^J - \rho_3) \\ \quad 0\,otherwise \end{cases} \\ 0 & \frac{\Delta}{l}*\begin{cases} u\,if\,q_3 = u\,\rho_2 \\ 0\,otherwise \end{cases} & 1 + \frac{\Delta}{l}*\{(\begin{cases} -w\,if\,q_3 = w(\rho^J - \rho_3) \\ 0\,otherwise \end{cases}) - (\begin{cases} u\,if\,q = u\,\rho_3 \\ 0\,otherwise \end{cases})\} \end{bmatrix} \quad (3.64)$$

Jacobian matrix B:

$$B = \frac{\partial g}{\partial x}(\hat{x}(k/k - 1),0) = \frac{\partial [\rho_2 + \emptyset]}{\partial \begin{bmatrix} \rho_1 \\ \rho_2 \\ \rho_3 \end{bmatrix}} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \qquad (3.65)$$

Jacobian matrixΓ:

$$\Gamma = \frac{\partial f}{\partial \boldsymbol{\varepsilon}}(\widehat{\boldsymbol{x}}(k/k-1),0) = \frac{\partial \begin{bmatrix} \rho_1 + \frac{\Delta}{l}\{q_1 - q_2\} + \varepsilon_1^\rho \\ \rho_2 + \frac{\Delta}{l}\{q_2 - q_3\} + \varepsilon_2^\rho \\ \rho_3 + \frac{\Delta}{l}\{q_3 - q\} + \varepsilon_3^\rho \end{bmatrix}}{\partial \begin{bmatrix} \varepsilon_1^\rho \\ \varepsilon_2^\rho \\ \varepsilon_3^\rho \end{bmatrix}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3.66}$$

Jacobian matrixΠ:

$$\Pi = \frac{\partial g}{\partial \emptyset}(\widehat{\boldsymbol{x}}(k/k-1),0) = \frac{\partial[\emptyset_2]}{\partial[\emptyset_2]} = [1] \tag{3.67}$$

Covariance matrix Q:

If ψ is the standard deviation of Gaussian noise in prediction of traffic density using CTM, the positive diagonal matrix representing covariance matrix Q for process noise is given as follows:

$$Q = \begin{bmatrix} \psi^2 & 0 & 0 \\ 0 & \psi^2 & 0 \\ 0 & 0 & \psi^2 \end{bmatrix} \tag{3.68}$$

It is assumed that standard deviation of the process noise is same for prediction of traffic density for all three cells in the link.

Covariance matrix R:

If r is the standard deviation of measurement noise, the covariance matrix R representing noise in measurements is given by:

$$R = [r^2] \tag{3.69}$$

Cross-covariance matrix M:

The cross-covariance matrix M is given by:

$$M = \begin{bmatrix} 0 \\ \psi r \\ 0 \end{bmatrix} \tag{3.70}$$

Based on the matrices and vectors defined in equations (3.66)-(3.73), which are determined for each time-step, Kalman gain *K (3x1)* and state covariance update *P (3x3)* are determined and used in equation (3.63) for estimating traffic density.

3.4.4.3 Simulation results

The proposed CTM-EKF framework elaborated for a three cell link is simulated for a time period of 30 minutes, which is divided into 60 time-steps. The traffic demand for generating measurements for cell-2 is assumed with a different demand profile than the demand profile for CTM-EKF estimation model and naïve CTM model for traffic density prediction. The prediction from CTM model with the same noise as CTM-EKF process noise is compared with the estimated traffic density and measured traffic density to highlight the significance of estimated traffic density. Figure 3.4 compares traffic demand profiles used for measurement and CTM/CTM-EKF model.
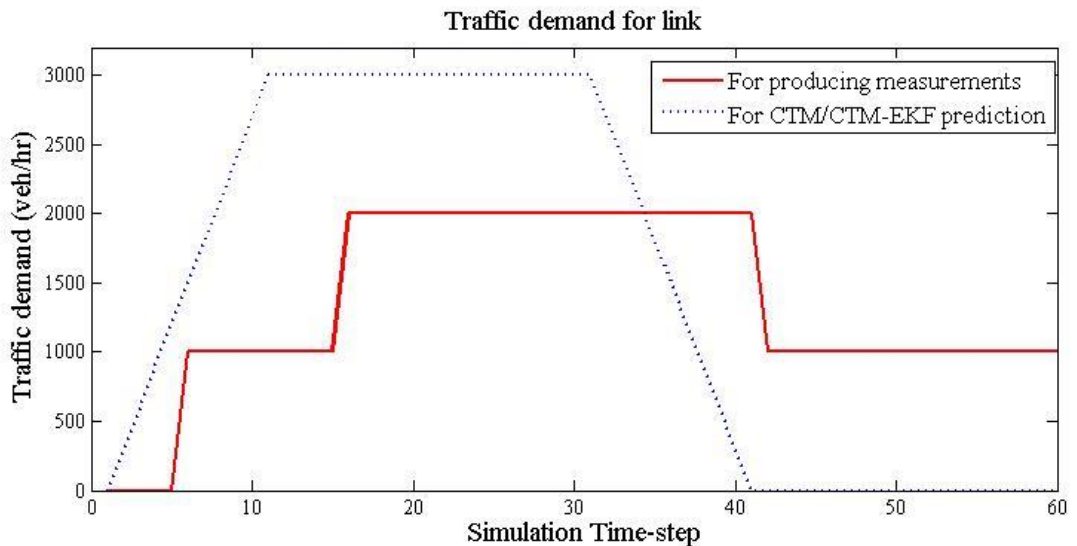
Figure 3.4 Traffic demand profiles for measurements and CTM-EKF model

The estimated traffic density using CTM-EKF model for cell-1 of the link is compared with the only CTM based prediction model, with same level of noise in the prediction. The CTM-EKF estimation model and CTM prediction model are simulated with the same demand as shown in figure 3.4. Figure 3.5 shows that CTM-EKF model estimated traffic density and CTM prediction for cell-1 are overlapping and equal for all the time-steps in the simulation horizons. Since the link is in a free-flow traffic condition throughout the simulation horizon, the traffic flow in link-1 is dictated by upstream traffic condition and unaffected by the estimated traffic density in the downstream cell. However, in congested traffic situation, when the downstream cell affects the flow of traffic from upstream cell, the CTM-EKF model will also affect traffic density estimate of the upstream cell. Figure 3.6 compares estimated traffic density from CTM-EKF

model with the prediction of traffic density from CTM and measurements from the sensor for cell-2. Figure 3.6 shows a significant improvement in estimation of traffic density with CTM-EKF model compared to prediction from CTM model. As the naïve CTM and CTM-EKF models are simulated with different demand profiles, while the measurements are reflecting actual traffic density in cell-2. CTM-EKF model recursively improves the estimated traffic density by using the measurements from the sensor, thus estimated traffic density improves significantly when compared with prediction from CTM model. A similar improvement can be observed from figure 3.7 for cell-3, as the corrected traffic density for cell-2 is propagated to the downstream cells.
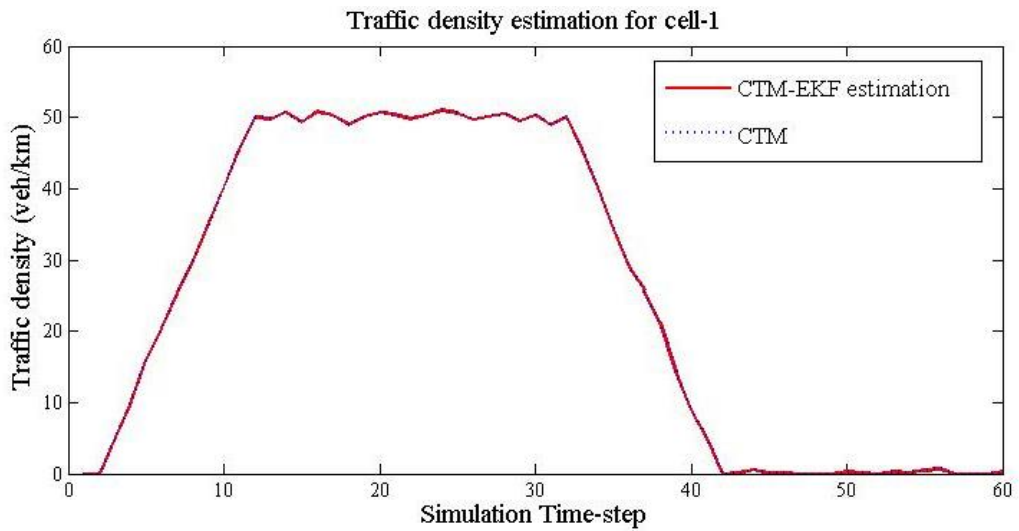
Figure 3.5 Comparison of estimated and predicted traffic density for cell-1
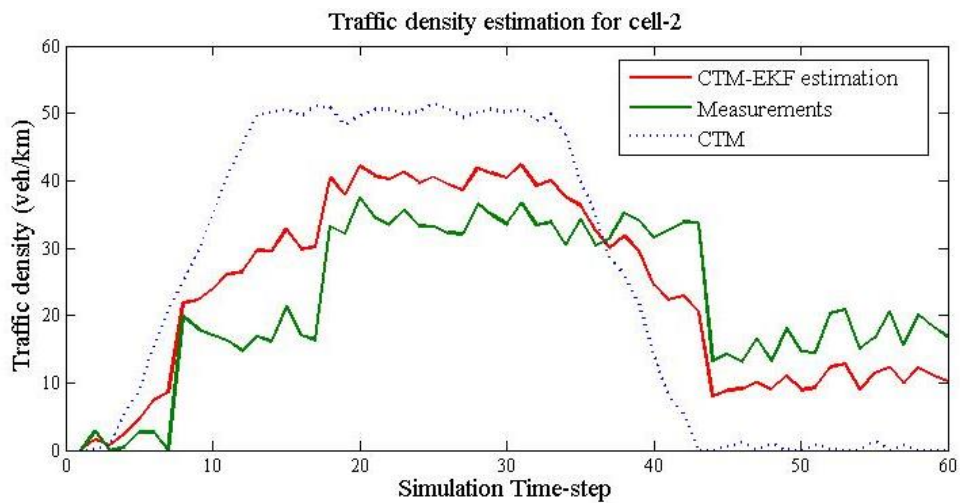
Figure 3.6 Comparison of estimated and predicted traffic density for cell-2
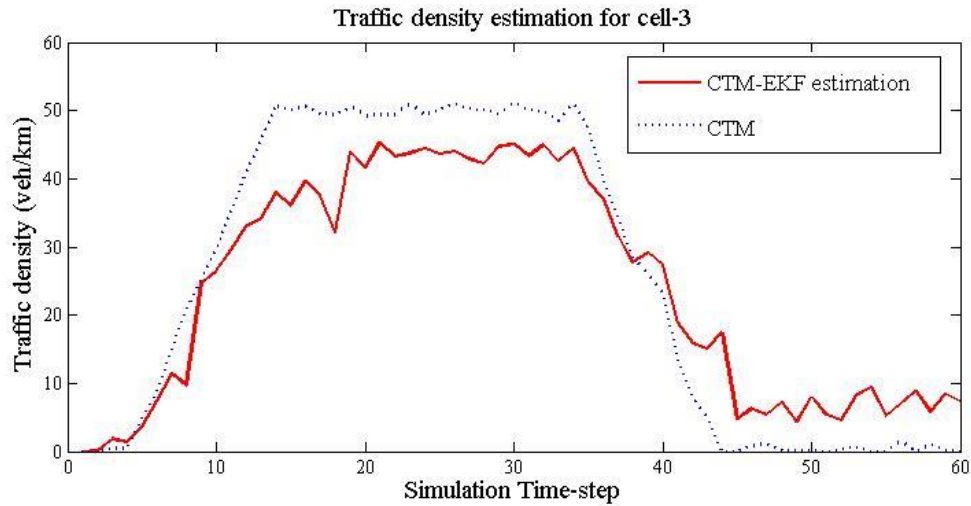
Figure 3.7 Comparison of estimated and predicted traffic density for cell-3

3.5 Summary

A brief overview of different estimation algorithms is presented in this chapter which include least square method, Kalman filter, extended Kalman filter, unscented Kalman filter, and particle filter. Kalman filter provides an optimal estimate for state of a linear dynamic system, based on unreliable model prediction and noisy measurement data. Extended Kalman filter, unscented Kalman filter and particle filter are designed to estimate the state of nonlinear dynamic systems. The EKF is more feasible for real-time applications compared to other estimation techniques, as it is computationally less expensive. The framework proposed by Wang and Papageorgiou (2005) is adapted to estimate traffic state using the cell transmission model. The parameters of fundamental traffic flow diagram are also estimated in real-time to capture any unexpected variation of these parameters due to external factors such as extreme weather, incident etcetera. A brief overview of advancement in obtaining traffic measurements is also provided in this chapter. Finally, the proposed CTM-EKF framework for real-time traffic state estimation is elaborated and applied to a simple link to demonstrate the significance of the proposed framework.

Chapter 4: Dynamic Traffic Assignment

4.1 Introduction to dynamic traffic assignment

Dynamic Traffic Assignment (DTA) models are widely used to model propagation and distribution of traffic along a network with time dependant traffic demand and network capacity. Static traffic assignment (STA) models can deal with steady-state traffic to model traffic flow based on average daily traffic or peak-hour traffic. These models are based on well-defined equilibrium principles and can provide reliable estimates for large networks, which are useful for planning purpose. On the other hand, microscopic simulation models consider the movement and interaction of individual vehicles with other vehicles as well as with its environment (roads and traffic control). Microscopic simulation models require comparatively larger dataset to model driver behaviour. Due to high computational and data requirements for microscopic simulation models, these models are well suited for analysis of intersections or small level networks (Chiu *et al.* 2010). Microscopic simulation models can be applied to evaluate various traffic control strategies on a corridor level scale. The gap between static traffic assignment models and microscopic simulation models is filled by Dynamic Traffic Assignment (DTA) models, as DTA models are useful to model time-dependent traffic demands or network capacity variations for real-size traffic networks. The variation in traffic demand or network capacity can be across the days or within a day. DTA models in contrast to STA models can represent dynamics of traffic flow in network loading component of the traffic assignment process. Different approaches to implement DTA using dynamic network loading models include Cell Transmission Model (CTM) (Lo 1999, Ziliaskopoulos 2000, Lo 2001), Deterministic Queuing Model (Cascetta and Cantarella 1991, Kuwahara and Akamatsu 1997) and Exit Flow Function (Merchant and Nemhauser 1978, Friesz *et al.*, 1989, Wie *et al.* 1995).

DTA models are used for variety of applications from long-term transportation planning and control policy evaluation to real-time traffic management. Gomes

and Horowitz (2006) and Meng and Khoo (2010) utilized DTA approach for freeway traffic congestion management using ramp-metering. Heydeccker (2002), "Brain" Park *et al*. (2009) and Park and Kamarajugadda (2007) used DTA applications to improve network performance through changing traffic signal settings. Lo and Szeto (2004) and Szeto and Lo (2005) applied within day DTA application to guide travellers using advanced traveller information systems. Zhong *et al.* (2012), Watling (2006), Heydecker and Polak (2006), and Lo and Szeto (2005) applied DTA notion for dynamic toll and road pricing for congestion management. Ukkusuri and Waller (2008) evaluated various strategies for transportation planning. Szeto *et al*. (2009) applied DTA model to forecast future traffic patterns on highways and Abdelghany *et al.* (2000) evaluated the benefits for proposed policy on congestion management in highways.

There are two possible approaches used in implementation of DTA, simulation-based and analytical-based. Simulation-based DTA models simulate probable results of a certain traffic management strategy but cannot determine a traffic management strategy. On the other hand, DTA models developed using analytical approaches are useful for both devising and evaluating traffic management strategies. To find equilibrium in DTA approaches, the following three algorithms are applied iteratively, until a convergence criteria is satisfied (Chiu *et al.* 2010).

Network Loading: In this component of DTA, a network loading model is assigned with a set of route choices and demand for all the links in the network. The network loading model determines travel times for assigned traffic demand and these travel times are assigned to the next component of DTA model to determine a new set of route choice and link demands, based on current estimate of travel times.

Path Set Update: This component of DTA model determines shortest route for each O-D pair for a given departure time, based on the travel times obtained from the network loading component. Travel times for each O-D pair and congestion pattern is analysed to determine time-dependant shortest path and it is combined with results of pervious iterations to update the path sets.

Path Assignment Adjustment: Path assignment adjustment algorithm adjusts traffic flows assigned to different routes based on updated path set. If all the traffic is assigned to the shortest path, this will not be the shortest path anymore. Thus, a portion of traffic from the path with higher travel times is assigned to the path with lower travel time. In general, this component of DTA model identifies the routes to increase and decrease traffic flows and the magnitude of assignment adjustment.

The output of path assignment adjustment algorithm is again assigned to the network loading component and these steps are performed iteratively in a sequence, until a convergence criterion is satisfied by the output. The quality of a solution obtained using DTA model is determined by the following characteristics:

i) Convergence: This quality of a solution obtained using a DTA model is determined by evaluating the deviation in flow pattern in successive iteration and comparing it with a pre-defined tolerance level. The tolerance level specifies the amount of permitted error in the final solution. A smaller value of the tolerance level is ideal; however it can significantly increase the computational time. Therefore, selection of tolerance level is a trade-off between the accuracy of the final solution and the computational time. The two commonly used approaches to define the tolerance level are the *absolute change* and *relative gap*.

ii) Sensitivity and stability: This characteristic of a DTA model output defines the robustness of a solution. Generally, a minor local change in the network should not have a significant impact on the ultimate solution. If a minor change in the network causes a big difference in the output, the solution is sensitive and non-stable.

4.2 Review of DTA models

Based on the existing literature in DTA, these models can be classified in the following categories.

4.2.1 Dynamic system optimal traffic assignment

Dynamic system optimal (DSO) traffic assignment is based on the principle that a central controller distributes the commuters on the network so that the total travel time of all the commuters is minimized and it is assumed that all the commuters adhere to the traffic assignment strategy proposed by the controller. The second principle of Wardrop (1952) for system optimal traffic assignment which states that *at equilibrium total journey time is minimized* was extended by Merchant and Nemhauser (1978) for dynamic traffic system. The DSO principle assumes that all travellers cooperate with each other in choosing their routes so that total system travel time over modelling horizon is minimized. Merchant and Nemhauser (1978) optimized departure pattern of commuters travelling from multiple origins to a destination using different links in the network to optimize the total system cost. The model is shown as a generalize form of static SO traffic assignment problem and a global solution can be obtained by piecewise linearization of the model. Carey (1987) improved DSO application by using link exit functions, which improved mathematical and algorithmic representation of the model. Chang *et al.* (1988) and Chow (2007) proposed system optimal traffic assignment for departure and route choice. Birge and Ho (1993) extended the DSO problem by treating O-D demands as stochastic variables and optimized the O-D assignment. Ran and Shimazaki (1989) utilized optimal control theory to develop a model for an urban traffic network with multiple origins and destinations. Papageorgiou Messmer (1991) proposed a framework based on feedback control using a macroscopic traffic flow model. Ziliaskopoulos (2000) formulated a DSO traffic assignment problem using CTM. The DSO model proposed by Ziliaskopoulos (2000) is linear in nature and computationally efficient but violates FIFO when applied with multiple destinations. Jahn *et al.* (2005) proposed a solution which is close to system optimal solution under route guidance traffic system. Qian *et al.* (2012) proposed a framework to approximate path marginal costs used for DSO traffic assignment problems. Carey and Watling (2012) determined and implemented system marginal costs using CTM and concluded that the DSO tolls implementation results in a solution which matches the criteria for dynamic user equilibrium solution.

In reality, system optimal does not represent actual traffic distribution along the network. However, system optimal solution provides a benchmark for system performance and traffic control measures and policies are compared with system optimal solution to evaluate the effectiveness of a particular traffic control strategy Chow (2007).

4.2.2 Dynamic user optimal traffic assignment

The user optimal principle for static traffic assignment was proposed by Wardrop (1952) and states that *the travel times on all the routes in use are equal and not greater than the travel time that would be experienced by a commuter on any unused route*. Unlike system optimal, this principle assumes that commuters do not cooperate with each other in selecting their routes and know the exact travel time of their selected routes. This principle has been extended to include generalize cost of travel which includes other components such as parking charges, tolls and fuel consumption. Friesz *et al.* (1989) extended the notion of user equilibrium for dynamic traffic systems by considering the dynamics of commuter's departure time. According to principle for dynamic user optimal (DUO) traffic assignment for route choice, the travel cost for all the routes used by commuters departing at a given departure time for each O-D pair are equal and minimal. This principle is mostly used in pure dynamic route choice applications (Szeto and Wong 2012). The DUO traffic assignment principle for departure time optimization proposed by Vickery (1969) treats route choices as fixed and commuters optimize their travel cost by selecting a departure time for each pair of O-D. The generalized DUO traffic assignment principle that combines optimization of route choice and departure time was proposed by Mahmassani and Herman (1984) and it states that *the travel costs for all the commuters departing any time and selecting any route for a given O-D are equal and minimal*.

DUO traffic assignment problem was formulated as an algorithm and mathematical program by Janson (1991). Janson (1991) used experienced travel times to determine an equilibrium solution. Smith (1993) proposed a DUE model for capacity-constrained roads in urban network. Friesz *et al.* (1993) proposed a DUO framework based on variational inequality approach. Lo and Szeto (2002a) for the first time proposed to use CTM to determine

idealized dynamic user optimal solution. The proposed approach accurately captured dynamic traffic phenomenon such as shock-waves, queue formation and dissipation, and dynamic traffic interaction across multiple links. Lo and Szeto (2002b) used CTM-network model to ensure FIFO compliance of the proposed model and used variational inequality approach to determine dynamic user optimal equilibrium. Szeto and Lo (2004) extended the CTM-based variation inequality approach for optimizing departure and route choice simultaneously. Nie and Zhang (2010) used a LWR-consistent traffic flow model for optimizing simultaneous departure and route choice by commuters. Han *et al.* (2011) proposed a CTM-based model for a single O-D pair with multiple paths to determine DUE with elastic demand. Ukkusuri *et al.* (2102) developed a framework to determine network level DUE using CTM with multiple O-D. Ukkusuri *et al.* (2102) used a path-based CTM to avoid holding-back issue in order to ensure FIFO. Formulations based on DUO traffic assignment principle assume that travellers have perfect knowledge about their expected travel times and exact travel times for a given route and departure time are known to travellers before their departure.

4.2.3 Dynamic stochastic user optimal traffic assignment

Daganzo and Sheffi (1977) extended the concept of user equilibrium in static traffic assignment to incorporate perception error in expected travel time when selecting a route. In stochastic user equilibrium (SUE), travellers select their route based on perceived travel times for the candidate routes and travellers are allowed to have different perceived travel time for the same route. Perceived travel time consists of expected travel time and a perception error defined using a probability distribution. If the perception error is removed, then SUE becomes a special case of UE problem, when all commuters select their routes based on expected travel times. Small (1982) extended the DUE principle for departure time choice to allow perception error in generalized expected travel cost. The DSUE principle for departure time and route choice states that at DSUE commuters cannot improve their perceived travel cost by unilaterally changing the combination of departure time and route choice. The route choice DUO traffic assignment principle was extended for perceived travel times by Ran and Boyce (1996) and Vythoulkas (1990) proposed

dynamic stochastic user equilibrium (DSUE) approach for simultaneous departure time and route choice optimization. Cascetta and Cantarella (1991) proposed a framework for within day and day to day traffic assignment with stochastic process. Maher and Hughes (1997) used a probit-based route choice model to determine DSUE. Watling (2002) proposed a framework that endogenously considers the variability of travel costs due to stochastic nature of traffic flows when addressing DSUE problem. Han (2003) used a deterministic queuing model to determine travel cost, logit model to find splits on links and method of successive average to obtain DSUE. Lim and Heydecker (2005) proposed a framework to obtain departure time and route choice DSUE model based on logit model. Watling (2006) introduced the later arrival penalty in considering the total travel cost.

4.2.4 Day-to-day DTA models

Day-to-day dynamic traffic assignment models deal with the route/ departure time choices of commuters in a longer study time horizon and model evolution of route flows for peak-period over a defined number of days. These models determine the route and/or departure time choice of commuters on a given day, based on the experienced travel cost in past days during the study horizon. Some DTA models also combine the pre-departure and en-route traveller information in selecting departure time/ route. Horowitz (1984) discusses the stability of stochastic equilibrium and models day-to-day route choice adjustment by introducing learning model based on weighted average approach. Cascetta (1989) discusses learning and route-choice models for day-to-day traffic assignment and introduced stochastic process model for day-to-day traffic assignment. Ben-akiva *et al.* (1991) proposed a model for day-to-day route adjustment under the traveller information. Watling and Hazelton (2003) discus the behavioural models of route-choice and impact of various factors on the day-to-day equilibrium solution obtained. A brief overview of day-to-day learning models is provided in section 4.4 and further applications of day-to-day DTA models are discussed in chapter-7 of this thesis.

4.2.5 Within-day DTA models

The within-day DTA traffic models address travel decision taken during the study horizon for a specific day without explicitly modelling the adjustment from previous days. The within-day DTA models are used for departure time optimization, route choice optimization and simultaneous optimization of departure time and route choice. These models are more extensively researched and applied in DTA compare to day-to-day DTA models. Real-time applications of ATIS and advanced traffic management systems are usually designed to implement for a given day, such as traffic management during incidents. The application of within day traffic models are briefly discussed in chapter-6 and the proposed framework of real-time traffic state estimation is applied to a within-day DTA application in chapter-6 of this thesis.

4.2.6 Doubly dynamic traffic assignment models

Day-to-day traffic assignment models only consider day-to-day dynamics in route choice and/ or departure time. On the contrary, within-day DTA models ignore day-to-day learning from previous experience and day-to-day adaption of commuters' behaviour under traveller information system. Doubly dynamic traffic assignment models fill this gap by combining within-day DTA models and day-to-day learning models. Cascetta and Cantarella (1991) proposed a doubly dynamic simulation model which determines the route flows on a given day using adaptive expectation approach proposed by Ben-Akiva *et al.* (1991). To model within-day dynamics, Cascetta and Cantarella (1991) utilized a queuing model to quantify delays on the link. Balijepalli and Watling (2005) discussed equilibrium distributions based on a stochastic doubly dynamic traffic assignment model.

The research presented in this thesis is applied for various applications for integration of real-time traffic state estimation with DTA. To achieve this objective, a within-day route choice DTA model based on CTM-EKF is applied to a small traffic network disrupted with an incident. In another application, the CTM-EKF framework for real-time traffic state estimation is used with a doubly dynamic traffic assignment model to determine route flows based on day-to-day learning model under time-varying traffic demand.

4.3 Route choice models

Route choice models are basic component of traffic assignment models. Route choice models are used in modelling the behaviour of commuters in selecting a route, based on expected/ perceived travel cost. Random utility models (discrete choice models) measure the attractiveness or utility of an alternative, when the decision maker faces more than one option to select from (Sheffi 1985). Dijkstra (1959) proposed 'all or nothing' algorithm to find the shortest path in a network for a given O-D pair. Almond (1967) proposed a framework known as 'method of successive average' (MSA) which assigned fixed weights to different links in the network. 'All or nothing' and MSA approaches are applied to determine user optimal routes in a network. This research is focusing DTA under traveller information system, which involves perception error and variation of perception among commuters. Therefore, stochastic route assignment models are more appropriate for application of the framework proposed in this research.

Multinomial Logit (MNL) model is one of the widely used random utility models to determine route choice behaviour of commuters. MNL assumes that the utility of commuters have the same error distribution, represented by Gumbel distribution. If C_i is the perceived travel cost for route i and C_j is the perceived travel cost for alternative route j, the probability of selecting route i by a commuter using MNL model is given by:

$$P_i = \frac{e^{-\theta C_i}}{e^{-\theta C_i} + e^{-\theta C_j}} \tag{4.1}$$

Where θ is the spread parameter and in DSUE assignment it reflects the variation in perception of travel cost while selecting a route. By assuming homogenous characteristic of travellers, P_i can be interpreted as the proportion of traffic from an upstream link to the downstream link. Dial (1971) proposed an algorithm, STOCH, based on MNL model to assign traffic on candidate links without explicitly defining the set of candidate links. Fisk (1980) proposed a minimization problem to determine SUE using MNL model. The limitation of MNL model because of assuming that error term is independent and identically distributed Gumbel, which ignores path overlapping, was highlighted by

Cascetta *et al.* (1996). C-logit model was proposed by Cascetta *et al.* (1996) to overcome this shortcoming of MNL model by adding another parameter in travel cost of the link to represent path overlapping. Other variations in MNL model were proposed by Ben-Akiva and Bierlaire (1999) as path-size logit model and Vovsha and Bekhor (1998) as cross-nested logit model. Huang and Li (2007) applied a multiclass logit model for stochastic traffic assignment under ATIS.

Danganzo and Sheffi (1977) proposed to model route choice behaviour using a multinomial probit model. Probit model assumes that the error in random utility term is normally distributed and the joint density function of the error terms is multivariate normal function (Sheffi 1985). Probit model is theoretically sound to describe the route choice behaviour and overcomes the deficiency of logit model. However, it is computationally expensive, enumerating routes is difficult, and convergence to the solution poses challenges in implementing probit model for traffic assignment problems (Prato 2009). Therefore, we select MNL model for route choice modelling in this research, as it has been widely used and accepted method for modelling route choice with perceived travel time.

4.4 Modelling of day-to-day learning behaviour

Most of the trips commuted by travellers are repeated on a regular pattern and travellers update their perception about the expected travel time on a given day based on travel time experienced on a route in previous days. Modelling of this day-to-day learning about expected travel time is an important aspect of day-to-day DTA models (Watling 1996). The consideration of past travel times while selecting a route becomes comparatively less significant under uncertain traffic conditions, whereas under stable traffic conditions experienced travel time is given more weight. This section provides a brief review of models used for quantifying learning behaviour of commuters while updating their perception about expected travel time.

Horowitz (1984) proposed a weighted average approach to model the day-to-day learning behaviour of commuters that has been applied by many other

researchers (Cascetta 1989, Cantarella and Cascetta 1995, Nakayama *et al*. 1999, Watling and Hazelton 2003, Zhang *et al*. 2013). This approach assigns different weights to the experienced travel times on past days based on the assumption that the more weight is assigned to the most recent experience and comparatively lesser weights are assigned to the older commuted journeys.

$$C(d) = \lambda_1 C(d-1) + \lambda_2 C(d-2) + \cdots + \lambda_m C(d-m) \qquad (4.2)$$

Where *C(d)* is the expected travel cost of a route on day d and λ is the weight assigned for a day, within a specific length of memory m such that $\sum \lambda_m = 1$. This approach has been criticized to ignore the perception about expected travel cost on a given day, which also includes information from external sources such as weather reports and ATIS. Ben-Akiva *et al*. (1991) proposed an adaptive expectation framework which also considers perceived travel time for a journey on day d, in addition to the experienced travel time. If $C_p(d)$ is the perceived travel cost for a journey on day d and *C(d-1)* is the experienced travel cost on previous day, the expected travel cost for day d is given as follows:

$$C(d) = C_p(d)\,\alpha + (1-\alpha)C(d-1) \qquad (4.3)$$

Where α is the weight given to the perceived travel time. In equation (4.3), the perceived travel time allows to include information about the journey obtained through ATIS. Other research studies (Cascettea and Cantarella 1991, Iida *et al*. 1992) have also used the convex combination model for integration of perceived travel time. Jha *et al.* (1998) utilized Bayesian algorithm to model the updating of perceived travel time. In Bayesian approach perceived travel time and traveller information from ATIS are modelled as random variables with a specified probability distribution. Pre-trip updating process combines traveller information for planned trip and experienced travel time from previous days. Post-trip updating process stores the experienced travel time by combining the experienced travel time of completed trip with the perception of travel time about the trip. Tian *et al*. (2010) used weighted sum of experienced travel times and expected travel times on previous days to establish an expectation of travel time for journey on the current day.

4.5 Summary

In this chapter, a brief overview of existing DTA models is provided. The importance of DTA models compared to STA models is highlighted. An overview of DTA model implementation and solution attributes is also provided. Traffic assignment models classified based on principle of traffic assignment such as dynamic system optimal, dynamic user optimal and dynamic stochastic user optimal, are described with some recent applications of these models. A classification of DTA models based on study horizon such as within-day and day-to-day DTA models is discussed. An overview of route choice models and day-to-day learning behaviour is also provided in this chapter. These DTA models will be employed in the following chapters with CTM-EKF based model for selected applications.

Chapter 5: Dynamic traffic assignment and traveller information based on real-time traffic state estimation

5.1 Introduction

In previous chapters, the overview of various models applied in this research along with their alternatives is provided. Chapter-2 highlighted models in traffic flow modelling and selection of CTM for this research. Chapter-3 focused on available tools in model based estimation of state of a dynamic process and described CTM-EKF based real-time traffic state estimation approach. The selected models in DTA literature, relevant to this study, are discussed in chapter-4 of this thesis. This chapter is intended to highlight the gap in existing literature of DTA, which utilize either traffic flow model for prediction of traffic state or use only measurements from traffic sensors to devise and evaluate the traffic improvement plans. Along with highlighting the contribution of this research, a framework to extract predictive traveller information based on real-time traffic state estimation is provided in this chapter. Section 5.2 reviews the existing literature in DTA along with highlighting the expected contribution of this research. Section 5.3 introduces the framework for predictive traveller information based on real-time traffic state estimation and section 5.4 summarizes the finding of this chapter. The proposed framework of extracting predictive traveller information is extended and applied to various application of DTA in the following chapters.

5.2 DTA based on real-time estimated traffic state

Research studies that focus on application of DTA for traffic management and travel time optimization in traffic networks, do not consider availability and reliability of real-time traffic estimate. Such studies have generally assumed that all the data for the scenario is known, and there is no data available on underlying changes in the traffic or road environment conditions during the time period under study. Furthermore, the impact of proposed control parameters on network traffic and compliance of road users cannot be

evaluated without availability of real-time traffic observations. Existing applications of DTA are based on either macroscopic traffic flow models or cost-flow functions, which consider historic traffic demand for implementation and analysis of proposed improvement strategies. Within this field, Kachroo and Ozbay (1998) highlighted the problem of short-term non-recurrent congestion which might be caused due to some incident, addressing this issue by dynamic traffic routing by assigning time-dependent split parameters at some diversion points. They used a feedback linearization method to obtain optimum split rate, so as to optimize network performance. In their method, they assumed availability of data from measurement sensors and only utilized these measurements, without using any kind of traffic flow model. Lo (2001) proposed a method for determining dynamic signal control timing plans based on system optimal principle using CTM based network model, which optimize network performance by keeping the density at an optimum level so as to ensure maximum flow on all links approaching a signalized intersection. The results indicated that green progression could reduce delays on the network. Smith and Mounce (2011) presented an idealized splitting rate model when travellers seek to change their route either day-to-day or within a day. This model uses splitting rates at nodes to change exit flows in such a way that Wardrop equilibrium is obtained. This approach also incorporates dynamic signal green-time reallocation to reduce delays. The model is an extension of formulation proposed by Smith (1984), which suggests that for each pair of routes joining the same O-D pair, traffic flow swaps from a more costly route to a less costly route at a rate which is proportional to the product of the flow on the more expensive route and the difference in cost between the two routes. Many other studies (e.g. Chow 2009; Wu and Huang 2010; Carey and Watling 2012) presented DTA-based solution for improving traffic congestion without considering utilization of traffic state estimate. In contrast, Ziliaskopoulos (2000) developed a CTM-based approach to compute the dynamical system optimal assignment for a network with single origin and destination, formulating the DTA problem as a linear program. In conclusion, then, DTA-based research studies into the control/optimization of networks have typically not considered the availability and reliability of real-time estimates of the traffic states.

The main contribution of this research work is to combine traffic state estimation and DTA, as it has never been published before. Traffic state estimation can be considered equivalent to traffic flow prediction or traffic state reconstruction using observations from measurement sensors, as all these techniques aim to determine the state of the network (traffic flow, density, speed, travel times, etc.). Existing literature contains many studies combining traffic flow models and DTA, for example, Lo (1999), Ziliaskopoulos (2000), Lo (2001), Gomes and Horowitz (2006), Liu *et al.* (2006), Chiu *et al.* (2007). Similarly, measurements from traffic sensors have also been used for DTA and traffic management in real-time, e.g., Kachroo and Ozbay (1998), Mirchandani and Head (2001), Dotoli *et al.* (2006). In a similar manner, this research combines model-based traffic state estimation and DTA as the estimated traffic state is considered more reliable than the prediction of traffic state using a traffic flow model or observations from traffic sensors alone.

This research work, therefore, will focus on developing methods that combine real-time traffic state estimation with a DTA-based model of driver's route choice, with an aim to produce accurate and effective traffic management strategies. Therefore, the novelty of the proposed framework is the combination of real-time traffic state estimation with DTA, as the existing literature in DTA only utilizes prediction from traffic flow models or measurements from the sensors and the literature focusing traffic state estimation problem has not utilized traffic estimation techniques for traffic management using DTA. A framework is proposed in this chapter in which predictive traveller information is estimated based on real-time traffic state estimates for a traffic network. The predictive traveller information is derived through a real-time traffic estimation model for the traffic network with online parameter estimation (In our case, this involving online estimation of the parameters of a CTM, from which future travel times are forecasted).

5.3 ATIS based on real-time estimated traffic state

Advanced Traveller Information Systems (ATIS) have been widely used to inform commuters about real-time changes in road capacity, traffic congestion and delays. ATIS help commuters to take informed decision about their route-

choice and improves network performance. Ben-Akiva *et al.* (1998) proposed a comprehensive model called DynaMIT which can be used to devise and implement real-time traffic management strategies. DynaMIT consists of various components. It stores historic demand and modifies it for any given day by applying impact of proposed guidance strategy and daily fluctuations. The demand is mapped to links using routing strategy and several iterations are performed to obtain a balance between demand and supply. This predicted O-D matrix is corrected using real-time measurements in Kalman filter. Queuing models are used in DynaMIT to model dynamics of traffic flow and further a microscopic model is used to model behaviour of drivers in ATIS. Zhang and Levinson (2008) conducted a field experiment with 115 commuters, who commuted different trips with and without having prior information about alternative routes. The commuters' vehicles were equipped with GPS to record all the journey details. The study concluded that other than travel time, route choice also depends on trip purpose, suitability of route for desire trip purpose, number of stops and perceived distance. Kusakabe *et al.* (2012) presented an experiment in which commuters were informed about incident on one of the alternate routes through VMS. The study concluded that the commuters perceived their travel time based on the incident information and considered the information in en-route choice decision. Khattak *et al.* (1995) evaluated the effect of traveller information distributed through radio and media in the city of Chicago on selection of route and departure time. Commuters through downtown Chicago area were surveyed and it revealed that commuters use travel information to reduce their anxiety even if they do not change their route. More than 60% of respondents had used this information to modify their travel route. This study also concluded that commuters will comply with the information if they perceive information as accurate and reliable. Abdel-Aty *et al.* (1997) modelled route choice behaviour under ATIS using data collected from stated preference survey. This study concluded that ATIS affects route choice behaviour of commuters and commuters prefer to choose a route where travel time information is available even travel times are higher compared to other route with shorter travel time without information. Al-Deek and Kanafani (1993) model a hypothetical network with two alternate routes, where one of the routes was affected by traffic incident. User optimal strategy is employed by

sending real-time information to vehicles equipped with ATIS. It was concluded that equilibrium is achieved when a significant number of vehicles take alternate routes, however overreaction of travellers will have negative impact on network performance. Mahmassani (1990) discussed the en-route decision making based on the information acquired during a journey. Review of above literature suggests that significant work has been carried out to study effect of traveller information on route choice behaviour and it can be concluded that accurate traveller information impacts the route choice behaviour of commuters and performance of the network.

Travellers give more weight to the information provided, if they assess the information is accurate and it can help them in improving their journey's experience. Hall (1996) criticized the traveller information provision for obtaining a system optimal solution and suggested that commuters will follow the information if they perceive it as accurate and if it optimizes their travel cost. Bonsall (1992) and Vaughn *et al.* (1993) presented evidence that travellers ignore the information if the information is found to be misleading. Bifulco *et al*. (2007) suggested that a higher number of compliance from commuters is required to improve the network performance using ATIS. Ben-Elia *et al.* (2013) proved based on an experiment that a higher compliance rate can be achieved with the provision of highly accurate traveller information. The commuters compare their experienced travel time with the information provided to assess the quality of information. Therefore, one of the objectives of this research is to develop a framework which improves the quality of information.

The CTM-EKF based model for real-time traffic state estimation described in chapter-3 is applied to derive information for ATIS, as estimated traffic state is more reliable than the prediction using traffic flow model or traffic observations from sensors. The conceptual framework of extracting the information based on real-time estimated traffic state is described in figure 5.1. In figure 5.1, $\hat{x}(k|k\text{-}1)$ represents the estimated traffic state; $x(k+n)$ is predicted traffic state based on estimated traffic state; $y(k)$ is the measurement obtained from traffic sensor and $\tau_j(k)$ is predicted travel time for link j.

The state-space model for the prediction of traffic densities and for mapping the prediction to the measurement, $\sum(x, y)$, predicts the traffic density for a future time-step, based on the estimated traffic density which was obtained using CTM-EKF model at the previous time-step. The traffic density prediction function based on CTM, $f(\hat{x}(k|k-1,0))$, is used to predict traffic density for time step k, based on all available measurements until time-step k-1. A differentiable function $g(\hat{x}(k|k-1,0))$ transforms the predicted output for time-step k into the variable measured by the traffic sensor $y(k)$. The measurement of traffic density at time-step k is compared with the predicted output and a correction factor called the Kalman Gain K is estimated using the variance in the prediction and the measurement of the traffic density. This correction is then added to the pure model-based output to obtain a final estimate of traffic density for time-step k+1. The estimated traffic density $\hat{x}(k+1|k)$ for current time-step is then forwarded to another similar CTM-network prediction model $f(x(k+n|\hat{x}(k+1))$ which predicts traffic conditions for further n time-steps, until all the vehicles entered link j at time-step k traverse the link. The predicted time taken to traverse the link j for a vehicle that entered at time-step k, $\tau_j(k|y(k); \hat{x}(k+1); x(k+n))$, thus depends on all measurements until the current time-step, the estimated traffic density for one time-step ahead and the predicted traffic density for the future n time-steps. The predicted travel times are then communicated to travellers using a VMS and this process is repeated for all the time-steps in the study horizon.

The predictive travel times for traveller information are derived based on real-time traffic state estimated using CTM-EKF model for time-step k. This is accomplished by modelling link travel time that would be experienced by last entering vehicle in link j at time-step k, as this is most likely the time taken by the next entering commuter to traverse the link. The estimated traffic densities for all the cells of link j at time-step k along with real-time estimated parameter values are assigned to another CTM model which propagates traffic entered the link at time-step k for another n number of time-steps. The additional n number of time-steps for each link is determined based on predicted traffic densities for future time-steps and n is the time-step when traffic density in all the cells in that link becomes zeros. This is achieved by taking sum of traffic densities in all

the cells in a link and determining the time-step when this sum becomes zero. In case of no traffic in the link at time-step k, free-flow travel time is assigned for that link.

$$\tau(k) = n \; when \; \textstyle\sum_{i=1}^{I} \rho_i \, (k+n) = 0 \tag{5.1}$$

Where$\sum_{i=1}^{I} \rho_i \, (k+n)$ represents sum of traffic densities in all the cells of the link.
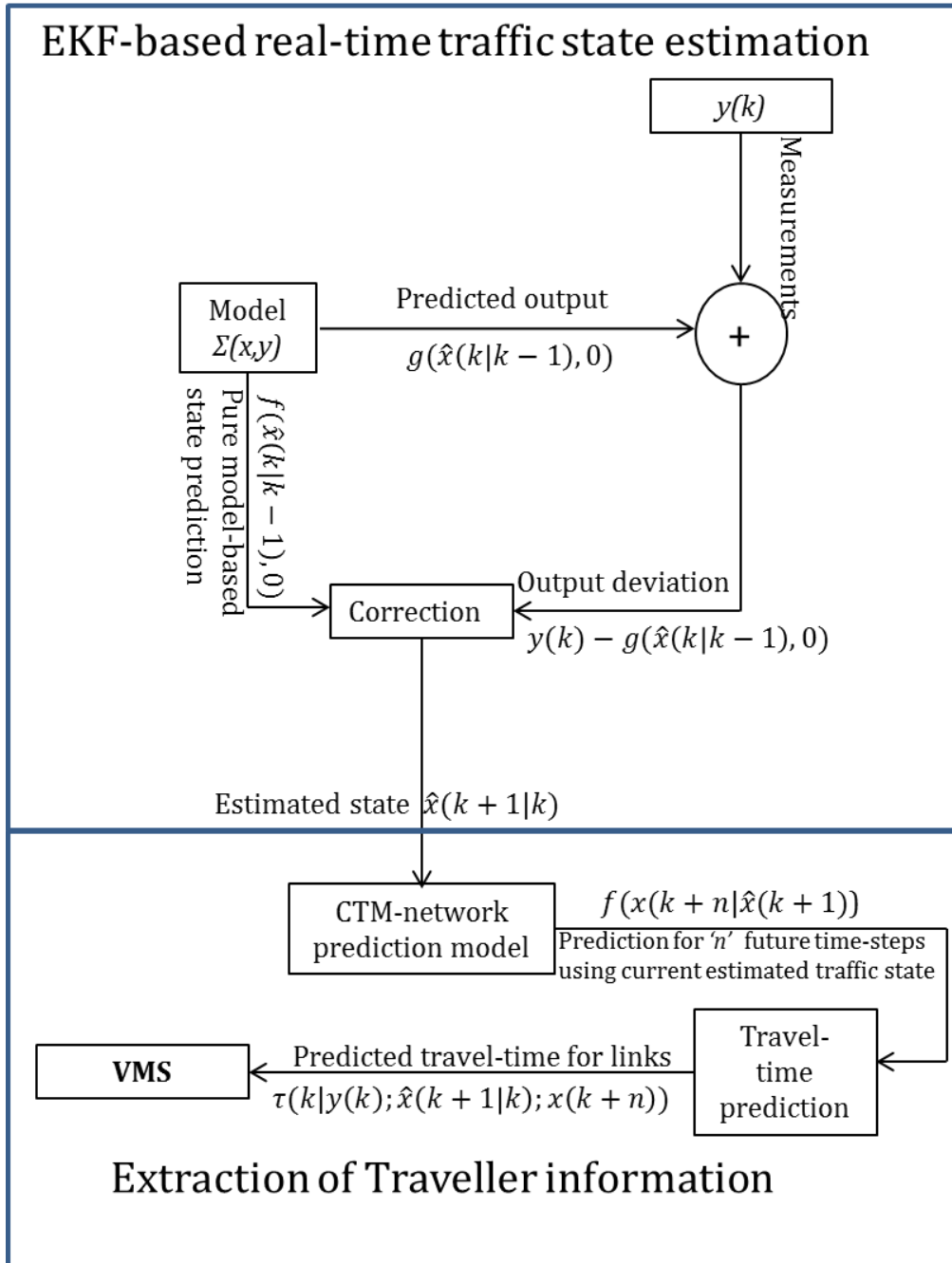
Figure 5.1 Predictive traveller information based on real-time estimated traffic state

The proposed framework for extraction of traveller information based on real-time estimated traffic state is applied to various applications in the following chapters to improve network performance by facilitating commuters with more accurate and reliable traveller information.

5.4 Summary

In this chapter, the existing gap in literature is highlighted with the contribution of this research to integrate real-time traffic state estimation and DTA models. A summary of research studies focusing DTA applications for improving traffic is provided to emphasize that the existing research in DTA has not exploited the advantage that real-time traffic state estimation offer over traffic state prediction based on traffic flow modelling or direct measurements from traffic sensors. The significance of accurate and reliable traveller information is discussed by citing the relevant literature. A framework to extract predictive traveller information from real-time traffic state estimation is provided, which will be extended in the following chapters for numerical implementation of selected DTA applications to improve network performance during disrupted traffic conditions.

Chapter 6: Within-Day Application of Real-time Traffic State Estimation based DTA model

6.1 Introduction

The existing literature for managing short-term non-recurrent traffic congestion is based on only real-time measurements from traffic sensors. This chapter formulates a within-day route choice model based on real-time traffic state estimation and implements it to a test network affected with a traffic incident. Real-time traffic state estimation including parameter estimation incorporates the effect of any unexpected changes in link capacity due to external factors such as severe weather conditions or incidents. The predictive traveller information based on real-time traffic state estimation is communicated to the commuters to inform them about prevailing traffic condition on the affected route and attract them to alternative route with lesser travel time. The methodology to formulate DTA problem based on real-time traffic state estimation is described in section 6.2. Section 6.3 describes the simulation scenario and the hypothetical network for numerical implementation of the proposed framework. The outputs from implementation of the proposed research framework are presented in section 6.4 and section 6.5 summarizes the findings of this chapter.

6.2 Methodology

The research methodology described in section 5.3 to extract traveller information based on real-time estimated traffic state is extended to integrate route choice modelling to improve network performance when the traffic network is disrupted with traffic incident. Figure 6.1, an extension of figure 5.1, describes the formulation for within-day application of DTA using real-time estimated traffic state.

A traffic network disrupted with an incident is considered to demonstrate the significance of the formulated framework. The estimation algorithm detects the

drop in capacity by using real-time parameter estimation, described in section 3.4. Predictive travel time based on prevailing traffic condition are determined as described in section 5.3 of this thesis and communicated to the commuters using a VMS, installed upstream of the diverging intersection. A discrete choice, multinomial logit model is applied to model the behaviour of drivers in adapting their route choice, when the information about expected travel times on the alternative routes is available. The multinomial logit model for route choice based on perceived travel time is described in section 4.3. If there are j number of exit links/routes emerging from a diverging intersection such that $j= 1, 2, 3...$, the number of travellers choosing exit link/route j is given by:

$$\beta_j(k+1) = e^{-\theta \tau_j(k| \mathbf{y}(k); \hat{\mathbf{x}}(k+1|k); \mathbf{x}(k+n))} \Big/ \sum_{j=1}^{J} e^{-\theta \tau_j(k| \mathbf{y}(k); \hat{\mathbf{x}}(k+1|k); \mathbf{x}(k+n))} \tag{6.1}$$

where $\tau_j(k| \mathbf{y}(k); \hat{\mathbf{x}}(k+1); \mathbf{x}(k+n))$ is the predicted travel time for link/route j at time-step k which depends on all measurements available until current time-step $[\mathbf{y}(k)]$, one step ahead estimate of traffic state using CTM-EKF model $[\hat{\mathbf{x}}(k+1|k)]$ and CTM prediction model $[\mathbf{x}(k+n)]$ that predicts travel time by simulating traffic flow for another n time-steps by using traffic that entered link/route j at time-step k such that $n=1, 2, 3, ... \tau_{max}$. θ is the logit coefficient which is specified so as to represent commuters' variation in perception of expected travel times. In practice, this coefficient could also be estimated in real-time, or by using offline data by performing logit regression with known split-rates obtained from measurements and traveller information resulting in measured split-rates.
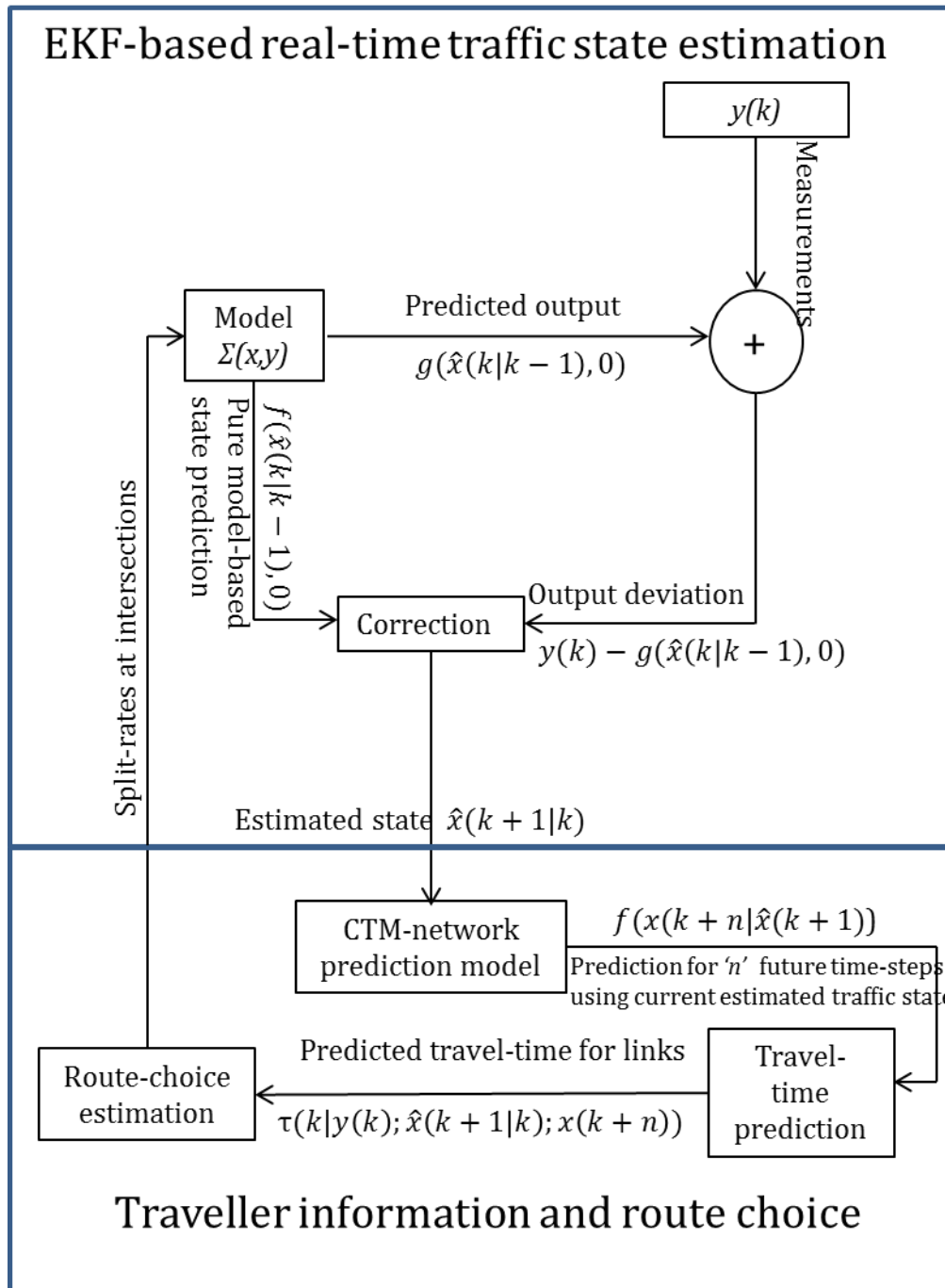
**Figure 6.1** Within-day application of DTA based on real-time traffic state estimation

6.3 Simulation scenario

The framework formulated in previous section for within-day DTA application of real-time traffic state estimation is applied to a test network to highlight the performance of the proposed framework. Real-time traffic state estimation becomes more important when actual traffic conditions depart from their

historic trend due to variation in traffic demand or network capacity. Real-time parameter estimation of fundamental traffic flow diagram enables to track any unexpected changes in traffic flow capacity of a link in the network. Similarly, any unexpected variation in traffic demand can also be identified and quantified using observations from traffic measurement sensors. For within-day application of the proposed framework, a test network as shown in figure 6.2 is simulated. The network is disrupted with an incident and commuters are informed about expected travel times on the alternative routes using ATIS to improve their travel times and network performance during disruption period.

A hypothetical diverging network is considered for numerical implementation of the proposed traffic estimation and *en-route* choice model. Figure 6.2 describes the network used for this experiment. The network consists of three links, each of length 4.5 km and divided into 10 cells of equal lengths. The first cell (cell-1) of link-1 is a dummy cell which generates traffic demand, with the last cells (cell-20 and cell-30) of link-2 and link-3 also dummies absorbing traffic arriving at the destination. There are two measurement sensors installed, one in cell-15 along link-2 and other in cell-25 along link-3, which measures traffic density in real-time and communicate to the controller.
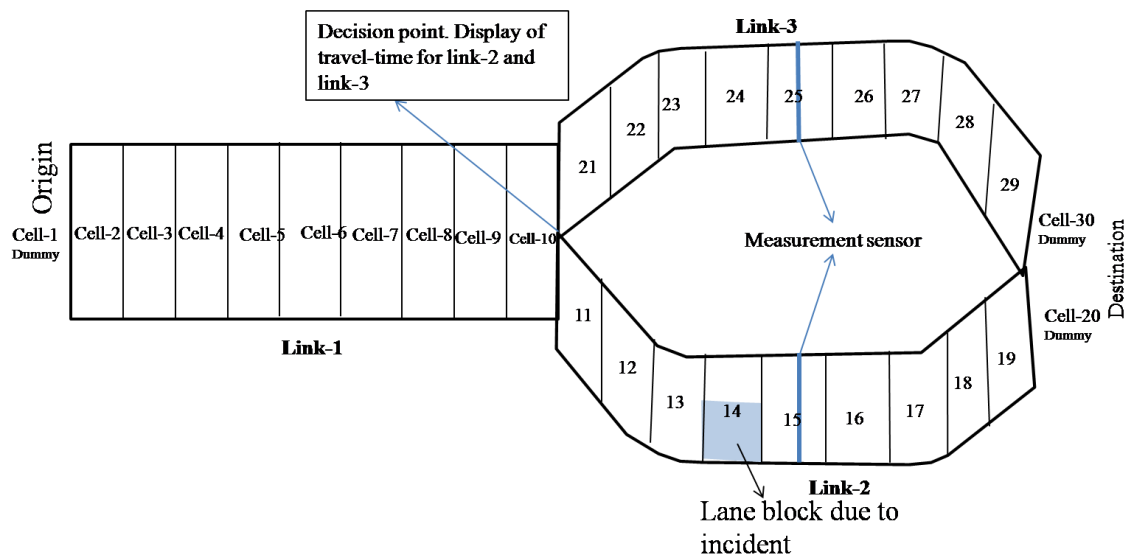
Figure 6.2 A Simple network for real-time traffic estimation and *en route* choice modelling

All the links in the network are three lane roads. The traffic demand for the network is show in figure 6.3. There is a diverging intersection at downstream

of cell-10, and traffic is diverging at this intersection on link-2 and link-3, each of these links leading traffic to the same destination. For illustrative purpose, the lengths, traffic flow capacities and speed limits of the alternative routes are considered equal, as it allows an easy base scenario for comparison of the incident scenario with the normal traffic conditions. In dynamic user equilibrium for this symmetric network, for any given departure time at the origin, the traffic will be equally divided between the routes when there is no incident. However, the methodology is general and can be applied to a network with any lengths and capacities of the alternative routes. A variable message sign is installed at link-1, at an appropriate distance before the intersection which displays the predicted travel time on the alternative routes.

This experiment was simulated for 700 time-steps of 30 seconds each, with a traffic incident occurring at time-step k=120 in cell-14 of link-2. The accident blocks two lanes of link-2 until time-step k=360, i.e. a duration of two hours. All the cells in the network have the same initial parameter values with traffic flow capacity of 5400 veh/ hr, critical density of 90 veh/ km and jam-density of 360 veh/ km. The free flow speed of all cells remains constant at 60 km/hr and backward wave speed is 20 km/hr. The characteristic of fundamental traffic flow diagram is shown in figure 2.1. The link demand profile in veh/hr is shown in figure 6.3.
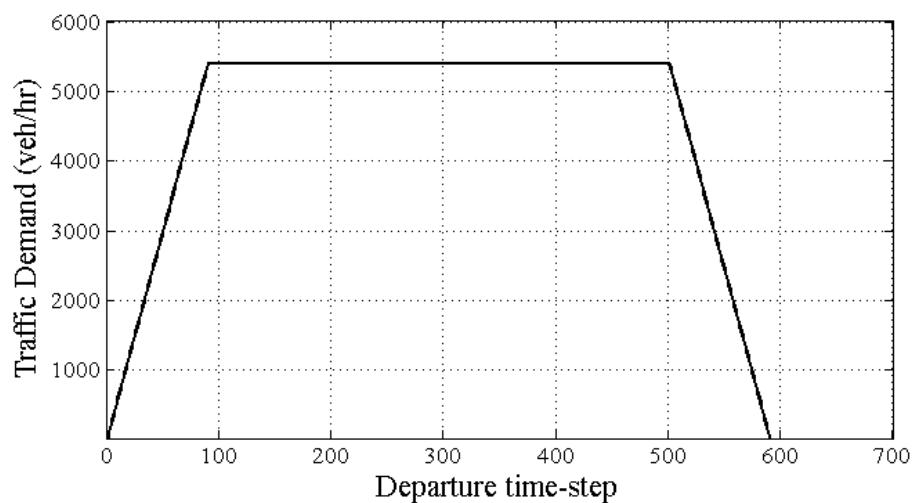
Figure 6.3 Traffic demand for the network

6.4 Simulation results

The algorithm proposed for estimation of traffic flow parameters was tested with different traffic volumes and different conditions of traffic incident along the link. The proposed algorithm was able to correctly track the drop in capacity due to the incident and also able to bring the parameter values back to their normal values once the incident is cleared. Figure 6.4 shows that reduction in traffic flow capacity due to the incident (which occurred during time-steps 120-360) was accurately identified and estimated by CTM-EKF model. The incident occurred in cell-14 of link-2, but it can only be identified and capacity can be estimated at the downstream measurement location. When there are several sensors installed along the road, any change in traffic flow parameters can be tracked and estimated at the downstream sensor of the incident location. The estimated capacity, which dropped due to the traffic incident, was subsequently brought back to its actual value by the estimation method, once the measurement of traffic flow becomes high and the link acquires its capacity flow as can be seen in figure 6.4.
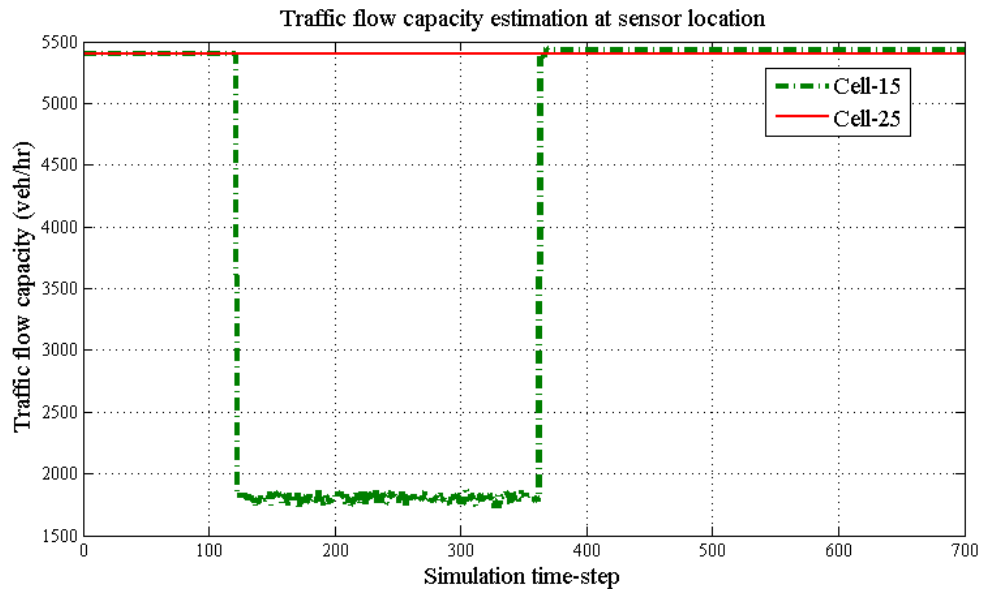
Figure 6.4. Estimation of traffic flow capacity at cell-15 and cell-25

The performance of the proposed model, which influences the dynamic route choice of commuters through the provision of predicted travel times conveyed to commuters through VMS, is compared with the case of real-time traffic state estimation model without any traveller's information. In the no information

scenario, the split rate is static since commuters are unaware of the incident and prevailing/predicted travel times on the alternative routes.

For the scenario, in which predicted travel times are provided to drivers, figure 6.5 shows the dynamic split-rate obtained through traveller information. It can be seen that, as anticipated, information provision helps alleviate congestion on the affected link by diverting traffic to the alternative route with lesser travel time. This also improves underutilization of existing network capacity and network travel time for all commuters.
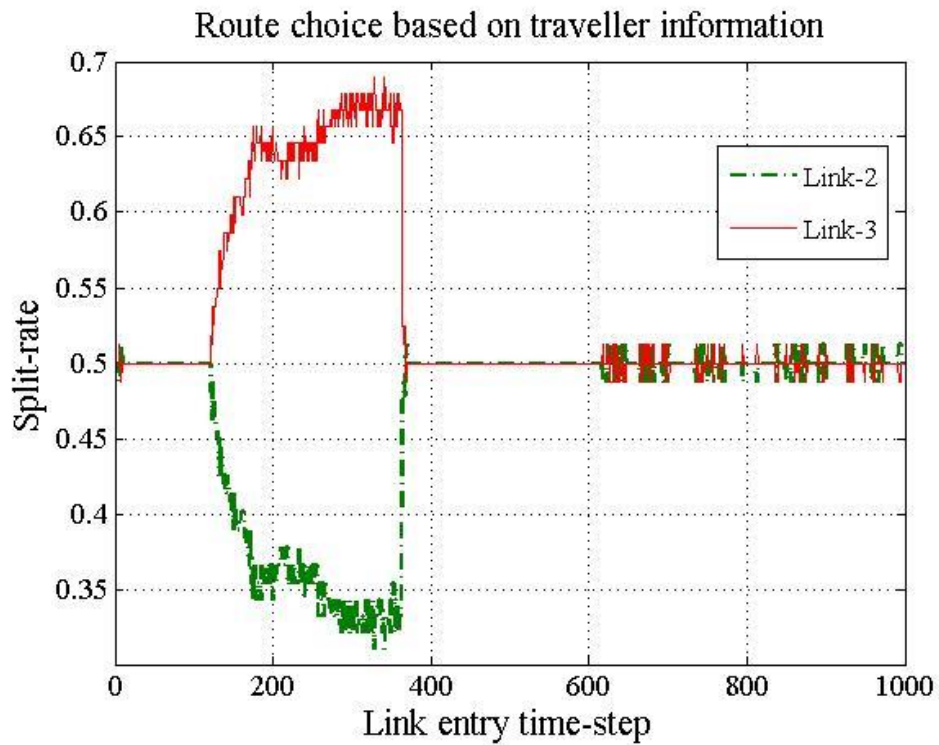
Figure 6.5 Dynamic split-rate obtained through traveller information system
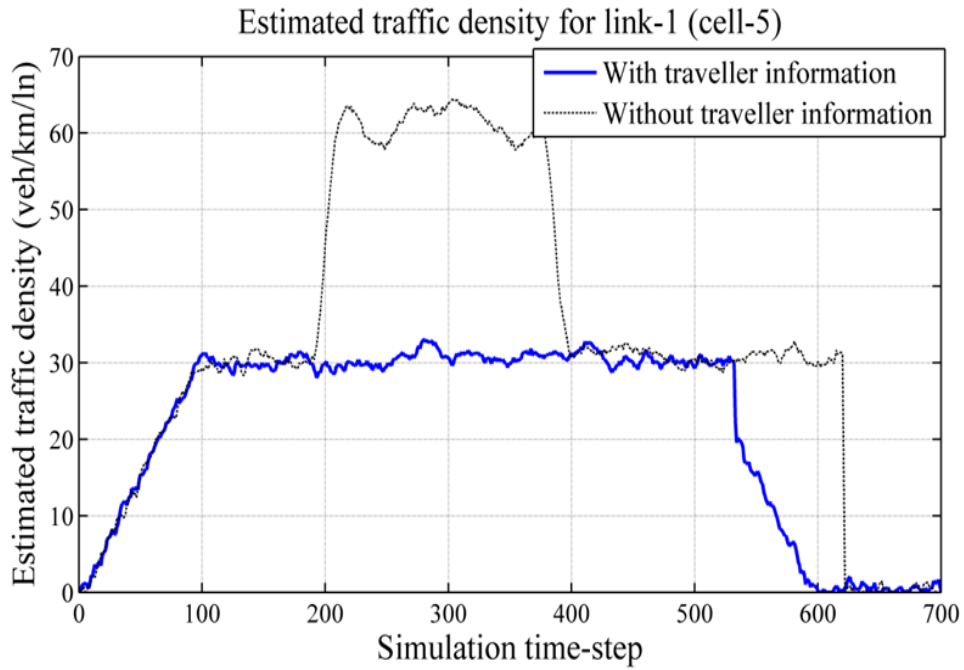
Figure 6.6 Comparison of traffic densities for link-1(cell-5) with and without traveller information
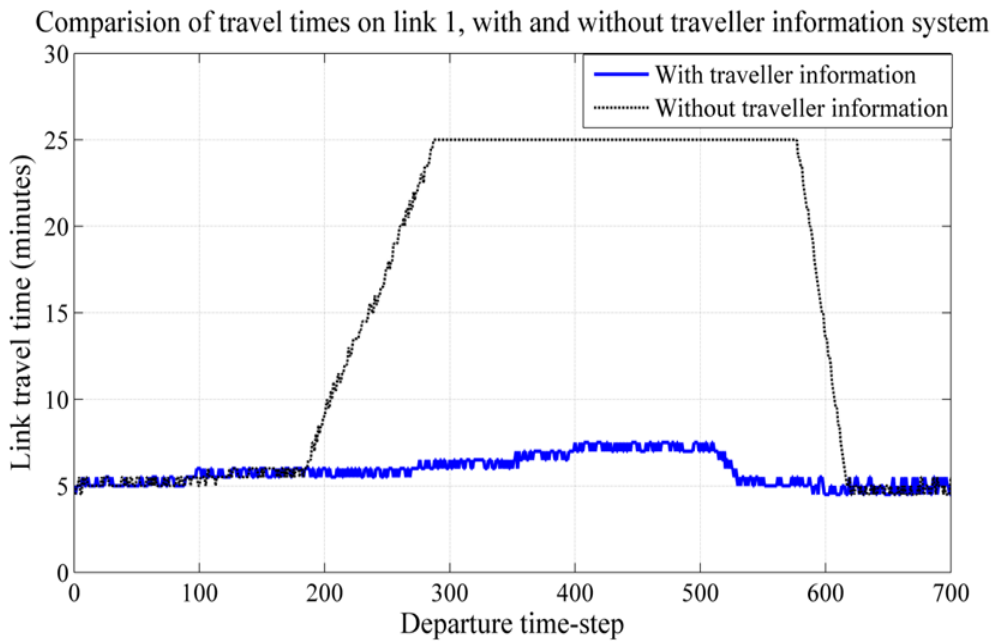
Figure 6.7 Comparison of travel times for link-1 with and without traveller information

Figure 6.6 compares the estimated traffic density for cells 5 of link-1 with traveller information and dynamic split rate with the scenario of no information. Only one cell is selected to compare the state of traffic in the link, as all other cells of link-1 show a similar profile. Figure 6.6 shows that the traffic flow in Link-1 with traveller information is most of the time either at

capacity flow or is free-flowing, but for a short period of time it also exceeds capacity flow. Whereas, in the case of no information, it can be observed that after several time-steps of the incident occurrence, congestion starts to build on link-1. This is because travellers are unaware of the incident ahead on link-2, and still the same proportion of traffic selects link-2 as a route as in normal traffic condition. Since the travellers trying to take link-2 are not able to propagate, this causes a blockage for vehicles directed towards link-3. Thus, congestion spills back to affects all upstream cells of link-1 as well. The comparison of estimated traffic density for link-1 with and without traveller information system reveals that with the traveller information system, traffic flow in link-1 was in good condition when compared with the no information scenario. All the cells of link-1 throughout simulation horizon were almost in free-flow condition with a dynamic split-rate, whereas with a constant split-rate the cells of link-1 become congested during the incident interval. Similarly, a significant improvement in travel times for link-1 can be observed from figure 6.7. The maximum experienced travel time on link-1 with traveller information was 7.5 minutes which lasted for a short interval of time, whereas without traveller information the maximum travel time on link-1 increased to 25 minutes and it remained for a longer interval of time.

Figure 6.8 Estimated traffic density (veh/km/ln) for link-2 with traveller information

Figure 6.8 shows estimated traffic density for link-2 with the traveller information and dynamic split rate and figure 6.9 shows estimated traffic density for link-2 without traveller information and constant split rate. Before occurrence of traffic incident, link-2 was in a free flow condition. After the incident, congestion starts building up in cells upstream of the incident location in link-2. The comparison of figure 6.8 and 6.9 reveals that the traffic state in link-2 shows a significant improvement with a dynamic split-rate when compared with the scenario of no information. Only Cells 13 and 14 were partly congested during the traffic incident with a dynamic split-rate, whereas without traveller information all the cells upstream of the sensor location are in a congested state for a comparatively longer interval of time. This fact is further supported by the comparison of travel times for link-2 in figure 6.9. The maximum value of travel time with a dynamic split-rate for link-2 was 12.5 minutes whereas without traveller information and dynamic split-rate, the maximum travel time increased to 22 minutes.

Figure 6.9 Estimated traffic density (veh/km/ln) for link-2 without traveller information
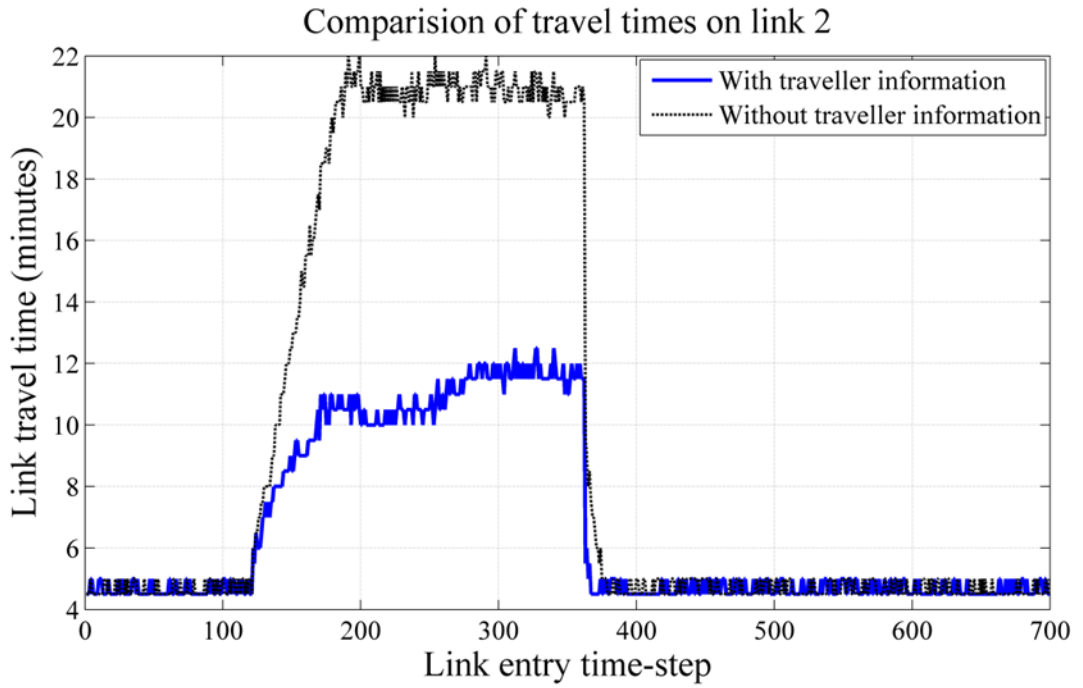
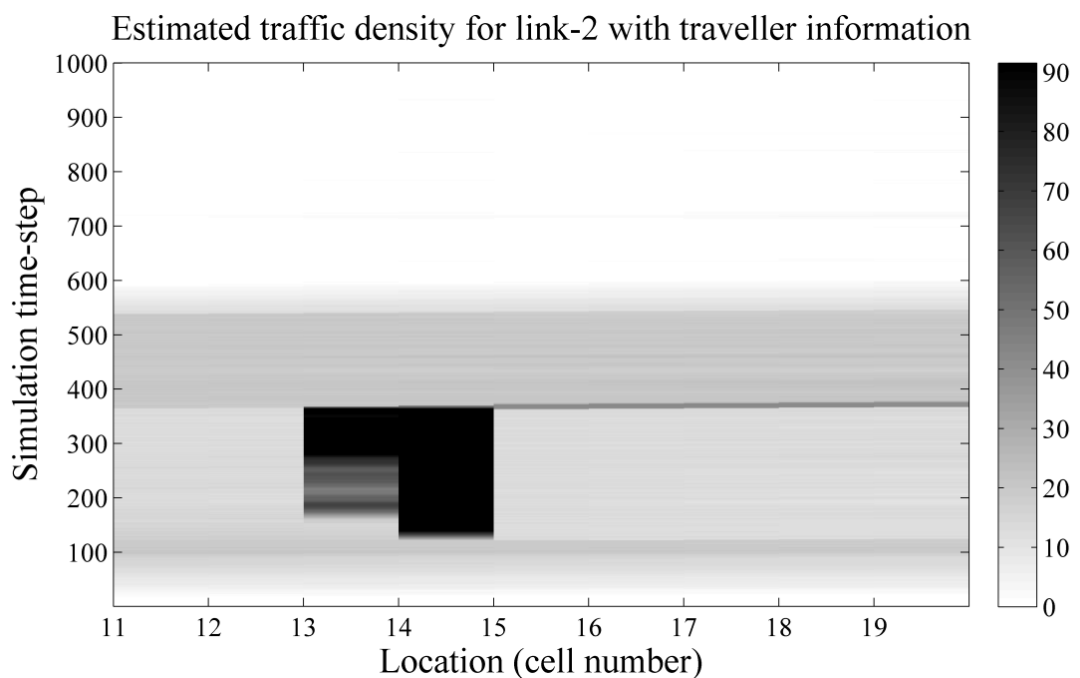**Figure 6.10** Comparison of travel times for link-2 with and without traveller information

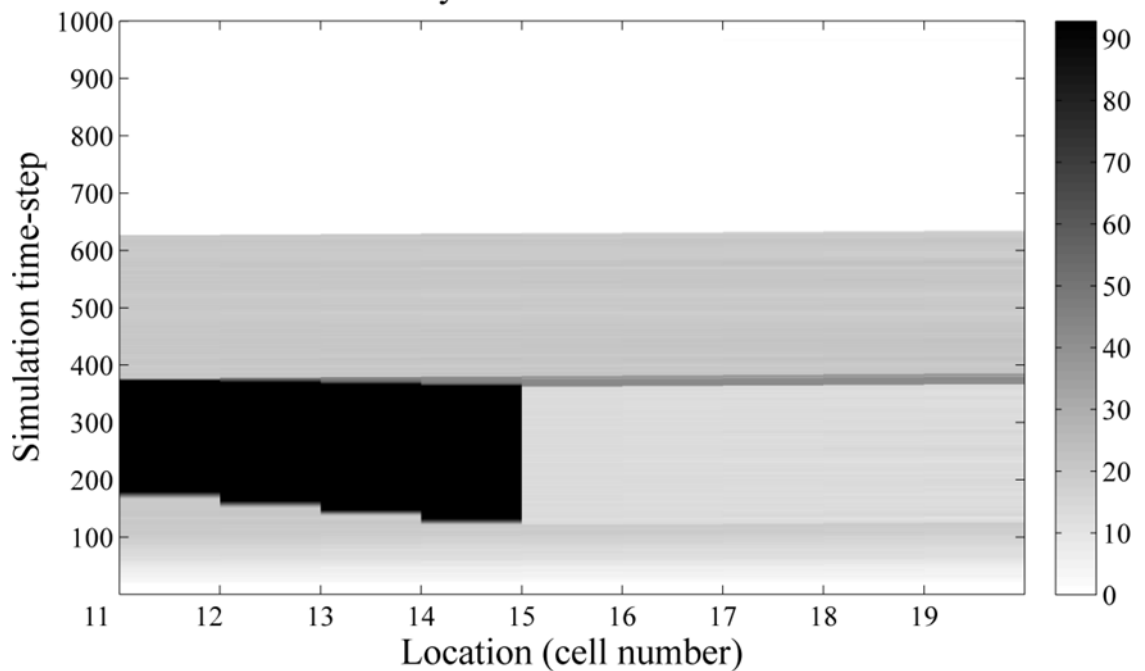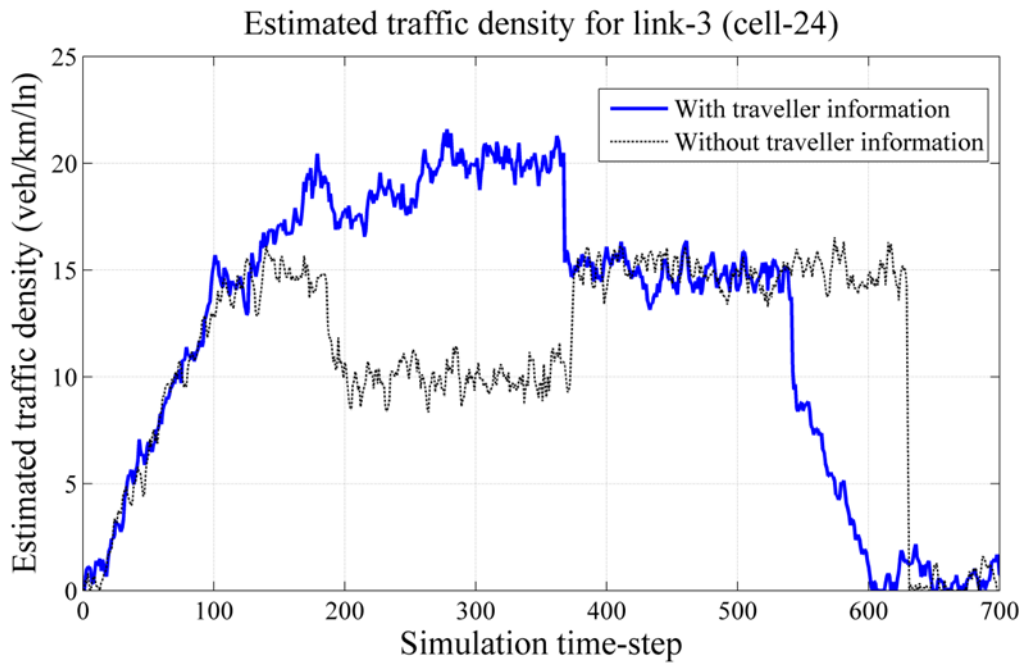**Figure 6.11** Comparison of estimated traffic density for link-3 (cell-24) with and without traveller information

Comparision of travel times on link 3
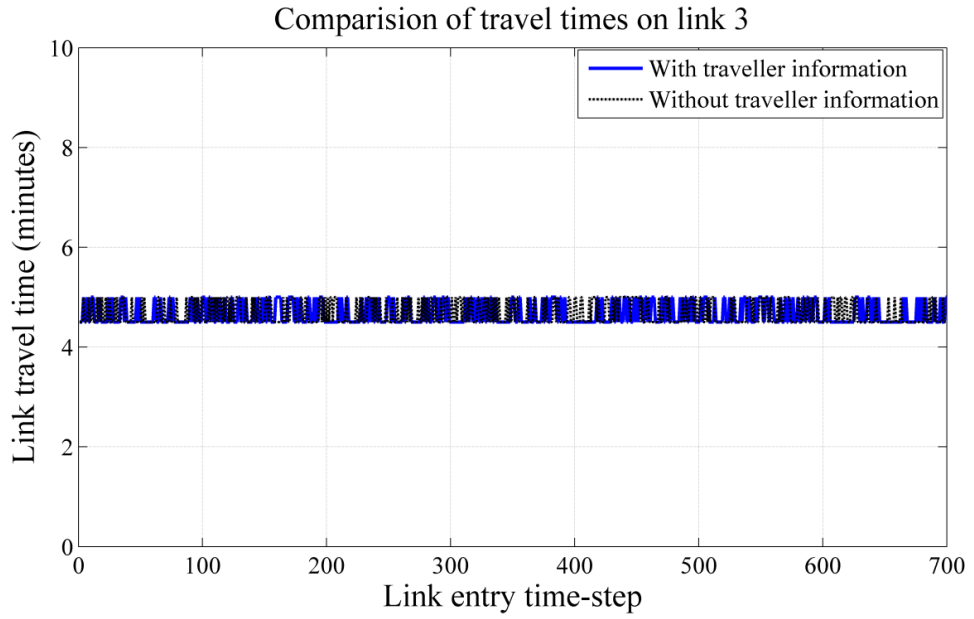
Figure 6.12 Comparison of travel times for link-3 with and without traveller information

Comparision of network delay
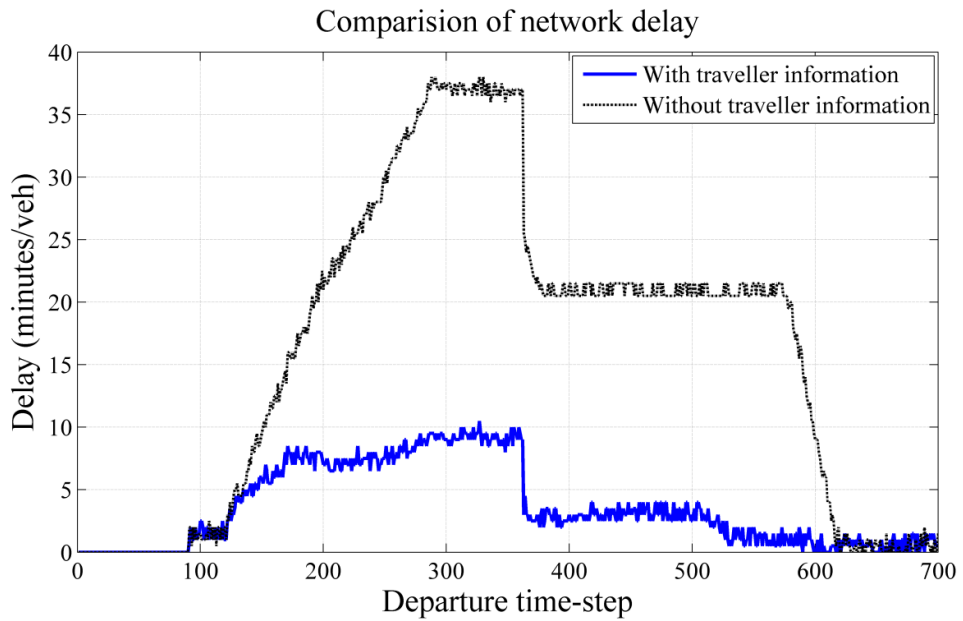
Figure 6.13 Comparison of network travel delay with and without traveller information
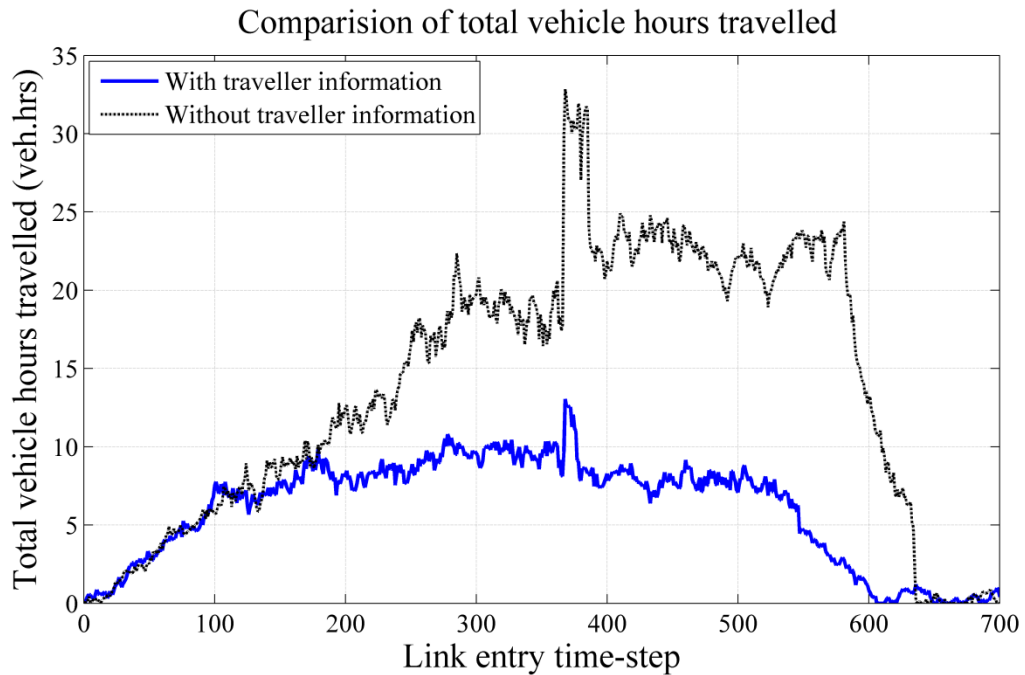
Figure 6.14 Comparison of total network travel time with and without traveller information

Figure 6.11 compares the estimated traffic density for cell-24 on link-3 with and without traveller information. All the other cells in link-3 exhibit a similar pattern; therefore, only one cell is selected for the comparison of traffic state in link-3. It can be observed from the figure 6.11 that during the interval of the incident, for no-information scenario, while other links are in a congested traffic state the available capacity on link-3 is underutilized. This is due to the fact that vehicles trying to take link-3 are blocked because traffic directed towards link-2 is unable to propagate. The comparison of the estimated traffic density with and without traveller information further confirms that the available capacity of link-3 was better utilized with the dynamic split-rate acquired through the traveller information system. The comparison of link travel times for link-3, with and without traveller information is shown in figure 6.12. It can be observed from figure 6.12 that link-3 remains in a free flow state throughout the simulation horizon, as the inflow to link-3 is not exceeding the available capacity of the link in either of the scenarios. Therefore, travel times on link-3 are unaffected with the provision of traveller information.

The overall improvement in the network performance by applying the proposed framework for integrating real-time traffic state estimation and DTA

for this application is shown in figures 14-15. Figure 14 compares total delay that each vehicle had to encounter to arrive at the destination for both the scenarios. The traffic is in free-flow condition throughout the network for the beginning of the simulation period, therefore no delay is observed till departure time-step 90. A small amount of delay can be observed from figure 14 between time-steps 90 to 120, which is same for both the scenarios. The delay in arriving at the destination increases gradually after the incident. The total delay in the case of no-information is significantly higher than the delay in the scenario with traveller information. The delay in the no-information scenario increased to 37 minutes per vehicle, whereas with the information the maximum value of delay per vehicle was recorded as 10 minutes per vehicle. Figure 15 shows total vehicle hours travelled (VHT) for the vehicles entering the links at each time-step during the simulation horizon. The total VHT is equal in both the scenarios till the occurrence of the incident. However, after the incident the VHT becomes significantly higher in the no-information scenario when compared with the scenario of traveller information.Table-2 provides a link-wise breakdown of total vehicle hours travelled for each link. An overall improvement of 5337.4 vehicle-hours (55.9%) in total VHT is obtained by the implementation of the proposed framework, which is highest for link-1 with 74.3% improvement. A higher value of VHT is observed for link-3 with the provision of traveller information when compared with no-information scenario, as number of vehicles selecting link-3 has increased with the provision of traveller information while travel time on link-3 is similar in both the scenarios.

Table 6.1 Comparison of total travel time for traffic network

	Vehicles hours travelled without ATIS (veh.hrs)	Vehicles hours travelled with ATIS (veh.hrs)	Improvement in vehicle hours travelled (veh.hrs)
Link-1	6871.1	1766.1	5105 (74.3%)
Link-2	1774.8	1114.3	660.5 (37.2%)
Link-3	899.8	1327.9	-428.1 (-47.5%)
Total	9545.7	4208.3	5337.4 (55.9%)

The proposed framework in this chapter can be extended to model large traffic systems. However, while extending the proposed methodology for larger

network, computational time might be a limiting factor as the traveller information or traffic state is updated with high temporal and spatial resolution in the proposed model. The spatial and temporal resolution of traffic state estimation and frequency of updating traveller information can be reduced to make it feasible for larger networks. A more aggregate macroscopic traffic flow model, such as Two-regime Transmission Model (TTM) by Balijepalli *et al.* (2013) can be applied for traffic state prediction to improve computation and modelling demand. Modelling of split-rates at intersections with multiple origin-destinations can be improved by pre-defining a subset of available routes that travellers can follow for each destination at any node. For a traffic network with multiple O-D flows, the CTM for multiple O-D flows can be used which distinguish traffic occupancy and flows based on origin and destination of traffic departed at each time-step by following Ukkusuri *et al.* (2012) or Carey *et al.* (2014) .

Another practical issue in implementing the proposed framework other than modelling large network using CTM is related to the information provided to the commuters and traffic management authorities can encounter these challenges. For example, the definition of destinations to which the travel time on a particular road is communicated to users could be an issue. There could be various routes leading to a destination from the location of a VMS and the consideration of communicated number of routes leading to the destination could be another implementation problem. The design of a VMS regarding the information provided can be significant and various designs of VMS can be considered while implanting the ATIS. In this research we have not addressed this issue explicitly. However, the details of implementation in real-world will depend on the nature of the problem, so a general solution is difficult to suggest and not covered in the scope of this research.

6.5 Summary

In this chapter, the proposed model for predicting traveller information based on real-time estimated traffic state is applied to a within-day application of DTA. It has been demonstrated in our numerical experiment that real-time traffic states estimated based on measurements from the sensor using EKF can

improve the reliability of the estimate. Online estimation of traffic flow parameters enables the model to track any unexpected changes in capacity of the network. The main contribution of this research work is to combine traffic state estimation with the application of DTA method, whereby the real-time estimated traffic state is utilized for influencing route choice through the provision of predicted travel time information and thus improving travel times and network performance during a traffic incident.

The proposed method has been applied to a simple, hypothetical, two-route network with one of its links affected by an incident. In our numerical experiment, the proposed model was seen to accurately identify and estimate the drop in capacity due to the incident. Predicted travel times communicated to travellers were seen to reduce demand for the affected link and helped traveller to utilize existing capacity on the alternative route. The proposed traffic management model significantly improved network performance and reduced vehicle travelled hours by 55.9% when compared with no-information scenario during the incident.

Chapter 7: Day-to-day traffic flow and route choice modelling under ATIS using real-time estimated traffic state

7.1 Introduction

In this chapter, the proposed framework to integrate real-time traffic state estimation and DTA is utilized to model day-to-day dynamics of traffic flow and route choice for traffic network under traveller information with time-varying traffic demand. The existing literature in day-to-day modelling of traffic flow and route-choice utilize either macroscopic traffic flow models or cost-flow functions using historic traffic demand. Day-to-day traffic assignment models based on macroscopic traffic flow models and historic demand are not capable of capturing unexpected variation in traffic demand or network capacity, thus the travel times obtained and used for modelling commuter's route choice could be significantly different from actual travel times.

For numerical illustration, the proposed framework is applied to a hypothetical traffic network with time-varying traffic demand. The travel times for traveller information are predicted based on real-time estimated traffic state as described in section 5.3. Section 2 of this chapter reviews existing literature in day-to-day DTA models and highlights the significance of the proposed framework. The methodology to implement the proposed framework to integrate real-time traffic state estimation and day-to-day traffic flow and route choice is described in section-3 of this chapter. Section 7.4 describes the simulation set-up for implementation of the proposed framework and section 7.5 presents the simulation results from day-to-day modelling of traffic flows and route choice. A summary of findings from this chapter is provided in section 6 of this chapter.

7.2 Modelling of day-to-day traffic flows and route choice

Travellers tend to change their route choices when travel time uncertainty increase on their preferred routes. The uncertainty in travel time is caused

when the equilibrium is disturbed either by variation in traffic demand or change in a route capacity. The change of route capacity could be due to change in traffic controls, road works or traffic incidents. Similarly, traffic demand for a route or network for a given departure time can also vary due to various possibilities such as weather, shopping events, festivals, exhibitions, sports, or due to variation in departure time of commuters. When the traffic demand is varying day-to-day for a given departure time, a commuter will experience a different travel time on the same route compared to his past experience. Similarly, if the traveller information communicated to the travellers using ATIS does not model the variations in traffic demand or network capacity, the travel time experienced by commuters can be significantly different from the one predicted by ATIS, which may cause travellers to doubt the information and ultimately start ignoring it. Therefore, this chapter highlights the significance of accurate traveller information in day-to-day route choice under traveller information when the traffic is demand is varying day-to-day and within day.

The research studies discussing day-to-day evolution of traffic pattern are mainly focused on equilibrium of system, user behaviour to adapt alternative routes or system induced day-to-day traveller's adaption of routes. Watling and Hazelton (2003) highlight the importance of day-to-day traffic assignment models, as these models allow flexibility in wide range of behaviour rules and level of aggregation. Smith (1984) proposed that travellers from routes with higher travel cost will switch to the routes with the lower cost at a rate proportional to the difference in the cost. Contrary to Smith (1984), Zhang and Nagurney (1996) modelled a projected dynamical system, which uses a minimum norm projection operator to model route flows. Friesz *et al.* (1994) proposed a model for day-to-day traffic assignment which captures dynamics in route flows and O-D demands. Bie and Lo (2010) discuss stability of user equilibrium in modelling day-to-day traffic dynamics in a dynamical traffic system. Cascetta (1989) and Hazelton and Watling (2004) proposed stochastic traffic assignment models based on Markov process. Horowitz (1984), Cantarella and Cascetta (1995) and Watling (1999) developed models which are based on weighted average of driver's experienced travel time for a selected

memory length. Smith *et al.* (2013) demonstrates a day-to-day stochastic model which combines notions from deterministic and stochastic processes. Parry *et al.* (2013) present numerical examples to demonstrate that doubly stochastic day-to-day traffic assignment models are more suitable to model the process of day-to-day traffic assignment. He *et al.* (2010) highlight the problem with path based day-to-day traffic assignment models and proposed a link-based model.

The existing literature in day-to-day modelling of traffic flow and route-choice utilize either macroscopic traffic flow models or cost-flow functions using historic traffic demand. Day-to-day traffic assignment models based on macroscopic traffic flow models and historic demand are not capable of capturing unexpected variation in traffic demand or network capacity, thus the travel times obtained and used for modelling commuter's route choice could be significantly different from actual travel times. He and Liu (2012) proposed a model to capture the dynamics of day-to-day variation in route selection when the network is significantly disrupted for a longer period of time. The proposed model by He and Liu (2012) introduced a prediction-correction process and suggested that the route choice for next day also depends on a predicted travel time component along with experienced travel time for past days. The predicted travel time is based on traveller's perception of future traffic pattern under disrupted network conditions. Cho and Hwang (2005) developed a model that combines users' behaviour with the predicted information provided by ATIS and assumed the traveller information provided by ATIS based on flows from previous days. Duong and Hazelton (2001) proposed a Markov process based model for day-to-day traffic assignment that incorporated the influence of pre-trip information in route choice of travellers. Jha *et al.* (1998) proposed a framework based on Baysian approach to update the perception of travellers based on information provided by ATIS and experienced travel time. In conclusion, significant amount of studies has been conducted to integrate the effect of traveller information with experienced travel time to update the perception of commuters about their route choice but assumptions on experienced travel time and traveller information provided by the ATIS are abstract. Furthermore, the perception update process of commuters does not

consider dynamics in reliability of traveller information and assumes the parameter reflecting this behaviour as constant.

As discussed above, extensive research has been carried out in the fields of traffic state estimation and day-to-day modelling of traffic flows. Integrating real-time traffic state estimation with day-to-day modelling of traffic flows can significantly improve the modelling accuracy, especially during disruption in network capacity or variation in traffic demand. The estimation of behaviour parameters based on estimated traffic state provides further opportunity to improve the accuracy of the modelling process. Travel times obtained based on real-time observations depict unexpected variation in demand or network capacity, whereas travel times obtained from traffic flow models or travel cost functions based on historic average, as used in existing models for day-to-day traffic assignment, are not capable of capturing these variations. The contribution of the thesis illustrated in this chapter is to combine day-to-day traffic modelling with real-time traffic state estimation by replacing the macroscopic traffic flow model/cost-flow function in network loading component with CTM-EKF based estimation model.

7.3 Methodology

This research proposes utilization of real-time traffic state estimation in modelling day-to-day evolution of traffic flows under fluctuating traffic demand. Traffic measurements from upstream traffic sensors are used to generate traffic demand for the network. The proposed model is doubly dynamic, as it considers within-day dynamics for prediction of traveller information and recording experienced travel time for each departure time-step. The day-to-day modelling component stores experienced travel time on past days for route selection on a given day.

The methodology to implement the proposed framework for day-to-day modelling of traffic flows and route choice is described in two components. The first component shown in figure 7.1 is an extension of figure 6.1 for modelling within-day dynamics, while figure 7.2 illustrates day-to-day component of the methodology to implement the proposed model for day-to-day applications.

Section 6.2 can be referred for a detailed description of figure 7.1. However, there are two additional elements in figure 7.1, when compared with figure 6.1. One element links the day-to-day dynamic component to the within day component. It stores the experienced travel time of commuters for each departure time-step for all the days in the simulation horizon and provides the experienced travel time for the selected memory length on a given day. The second additional element in figure 7.1 is about the information integration and updating of perception about expected travel time based on experienced travel time and real-time traveller information.

The within-day component of the research methodology described in figure 7.1 records the experienced travel time for commuters departing at each time-step based on estimated traffic state using EKF-CTM model. A commuter updates his perception about expected travel time based on the experienced travel time and traveller information provided through ATIS. The experienced travel time of commuters is modelled using weighted average approach, described in section 4.4. A memory length of 3 days is assumed in this application, with experienced travel time on day *d-1* given a weight of 0.5, the weight given to experienced travel time on day *d-2* is 0.3 and the weight given to experienced travel time on day *d-3* is 0.2. The average experienced travel time for route choice modelling on day *d* for departure time-step *k*, based on equation (4.2) is given as follows:

$$\bar{\tau}_e^{d,k} = 0.5 * \tau_e^{d-1,k} + 0.3 * \tau_e^{d-2,k} + 0.2 * \tau_e^{d-3,k} \qquad (7.1)$$

This research does not explicitly model day-to-day learning of individual commuters; instead day-to-day learning of experienced travel time is modelled for the group of commuters departing at one simulation time-step. In modelling day-to-day route choices based on experienced travel time, it is assumed that commuters' departure time does not change and the commuters depart at time-step *k* on day *d* departed on the same time-step on previous days. The day-to-day variation in traffic demand for a given time-step with constant departure time can be attributed to various factors which result in commuters' decision to abandon a journey on day *d*. Thus, this assumed day-to-day variation is due to commuters' flexibility in calling off a journey on some days and not on the other days. It is assumed in day-to-day modelling of commuters' average experience

travel time that they have knowledge of past travel times on the alternative routes, which they obtain from ATIS when there is a VMS and from other commuters in case of no-information.

To model the perception update, the convex combination approach proposed by Ben-Akiva *et al.* (1991) for information integration is applied. If $\bar{\tau}_e^{d,k}$ is weighted average of experienced travel time based on a selected length of memory for departure time-step k and $\tau_p^{d,k}$ is predicted travel time for the current journey on day d, the updated perception about expected travel time is given by:

$$\hat{\tau}^{d,k}(\bar{\tau}_e, \tau_p) = \tau_p^{d,k}\alpha + \bar{\tau}_e^{d,k}(1-\alpha) \tag{7.2}$$

Where, α is the parameter that represents the weight assigned to the predicted travel times and reflects the commuters' level of trust in the information provided and $\hat{\tau}^{d,k}(\bar{\tau}_e, \tau_p)$ is updated perception about expected travel time. The value of α varies from 0 to 1. A smaller value of α implies comparatively lesser trust in the information and more weight to the experienced travel time and vice versa.

For day-to-day modelling of route choice, a similar multinomial logit model is applied to model the behaviour of drivers in adapting their route choice, based on the updated perception about travel time, given in equation (7.2). The proportion of traffic selecting a link *j*, based on the updated perception about expected travel time on a given day is:

$$\beta_j^{d,k+1} = e^{-\theta\hat{\tau}_j^{d,k}(\bar{\tau}_e, \tau_p)} / \sum_{j=1}^{J} e^{-\theta\hat{\tau}_j^{d,k}(\bar{\tau}_e, \tau_p))} \tag{7.3}$$

In equation 7.3, θ is the logit coefficient which is specified so as to represent commuters' perception variation about expected travel time. The value of this parameter plays a significant role in quality of the solution obtained and in the stability of the modelling process. This research also proposes to estimate logit parameter based on the observations of traffic flow and estimated traffic state. The proposed framework for estimation of θ is described in the next chapter of this thesis.
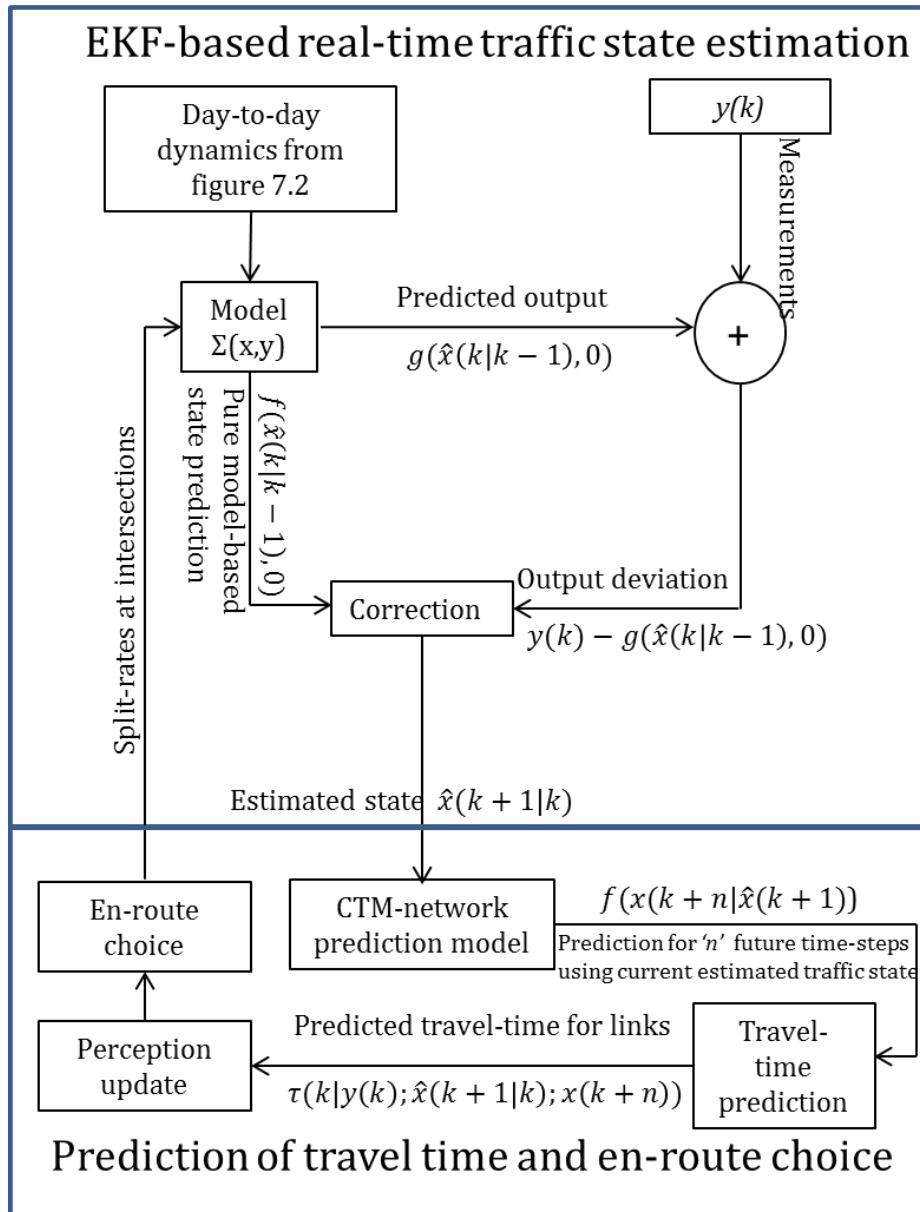
Figure 7.1 Framework for within-day modelling component of the day-to-day application
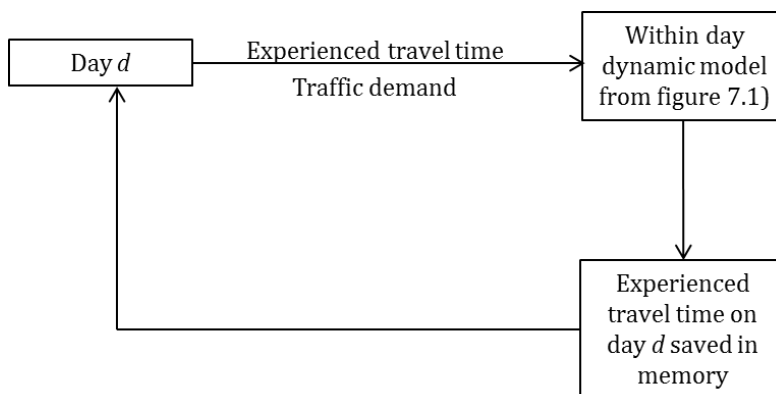
Figure 7.2 Process flow for day-to-day dynamics

7.4 Simulation setup

The proposed model for day-to-day modelling of traffic flows and route choice under influence of traveller information is applied to a hypothetical traffic network to numerically illustrate the significance of the proposed model. Figure 7.3 describes the network used for numerical illustration of proposed framework. The network is similar in topography with the network used for within-day application in chapter 6. The network consists of three links, each of length 4.5 km and divided into 10 cells of equal lengths. The first cell (cell-1) of link-1 is a dummy cell which generates traffic demand, with the last cells (cell-20 and cell-30) of link-2 and link-3 also dummies absorbing traffic arriving at the destinations. There are three measurement sensors installed, first in cell-1 of link-1, second in cell-15 along link-2 and third in cell-25 along link-3, which measures traffic density in real-time and communicate to the controller. The measurement sensor in cell-1 measures time-varying traffic demand for the network. A variable message sign is installed at the diverging point, displaying predicted travel-times for the alternative routes (links-2 and link-3) and updates it at each time-step.
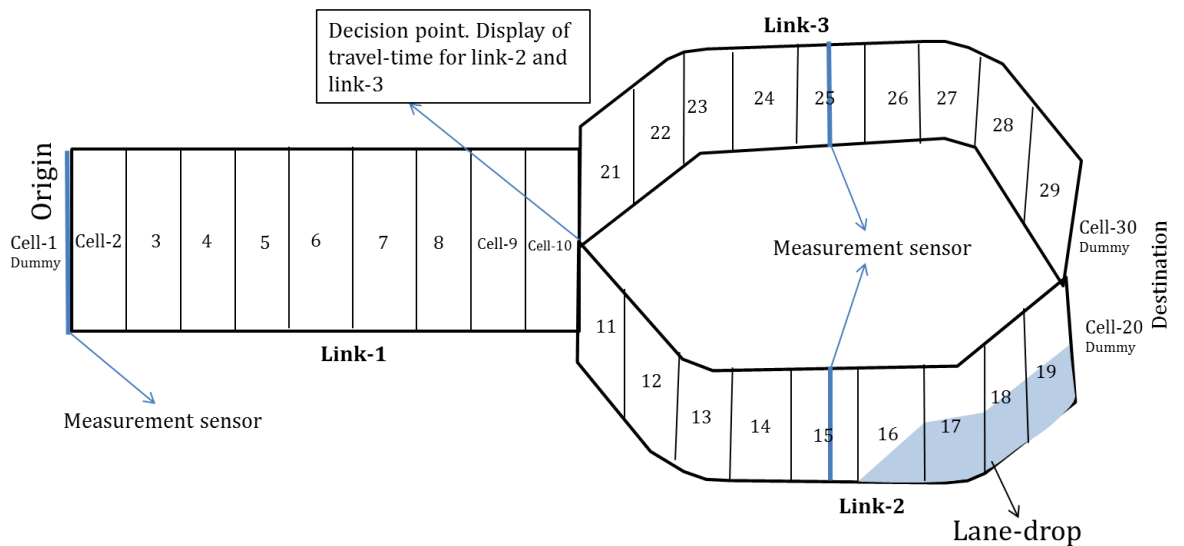
Figure 7.3 A Simple network for modelling day-to-day route choice

All the links in the network are three lanes, with two lanes of link-2 blocked from cell-16 to the end of the link. This causes asymmetry in the network and a bottleneck is developed in link-2 when the demand is higher than the downstream traffic flow capacity of link-2. The blockage of lanes in link-2 is

permanent and it does not change during entire period under study. In reality bus dedicated lanes, bus bays, infrastructure breakdown or road works can cause long-term or permanent lane drops in a road link. There is a diverging intersection at downstream of cell-10, and traffic is diverging at this intersection on link-2 and link-3, each of these links leading traffic to the same destination.

This experiment simulates peak-hour traffic which is spread over two hours for 100 days duration and traffic demand changes every day for a given departure time as well as within a day for each departure time. All the cells in the network have traffic flow capacity of 1800 veh/hr/ln, critical density of 30 veh/km/ln and jam-density of 120 veh/ km/ ln. The free flow speed of all cells remains constant at 60 km/hr and backward wave speed is 20 km/hr.

7.5 Simulation results

This section presents the outputs from implementation of the proposed research framework to the simulation scenario described in the previous section. The simulation results are divided into two subsections, according to the application. The section 7.5.1 describes simulation results from day-to-day traffic flow modelling of traffic flows and route choice under ATIS using real-time traffic state estimation. The analysis of sensitivity of the route choice behaviour to the logit model parameter is provided in subsection 7.5.2.

7.5.1 Day-to-day traffic flow modelling using real-time traffic state estimation

The proposed framework for integration of real-time traffic state estimation and DTA is applied to a test network shown in figure 7.3. The significance of proposed model under time varying traffic demand is highlighted by comparing the simulation results under ATIS based on real-time traffic state estimation with the simulation scenario without traveller information. The simulation horizon is 100 days, simulating peak-hour traffic for each day. The peak-hour is spread over 2 hours and traffic demand is varying day-to-day, as well as within day for each departure time-step. It is assumed that the traffic demand for the network is measured in real-time using the sensor installed in link-1. The scenarios with and without traveller information are simulated using the same

traffic demand profiles. Time-varying traffic demand is generated using a normal distribution, with mean gradually increasing from zero at time-step 1 to peak-hour traffic demand at time-step 60. The peak-hour traffic demand is generated with a mean of 5400 veh/hr and standard deviation of 500 veh/hr. The peak-hour traffic demand starts depleting from time-step 300 and obtains a mean of 1000 veh/hr with standard deviation of 500 veh/hr from time-step 360 onwards. Figure 7.4 shows traffic demand for three selected days; day-30, day-60 and day-90. The level of variation in traffic demand for a given time-step across days of analysis period can be observed from figure 7.4.
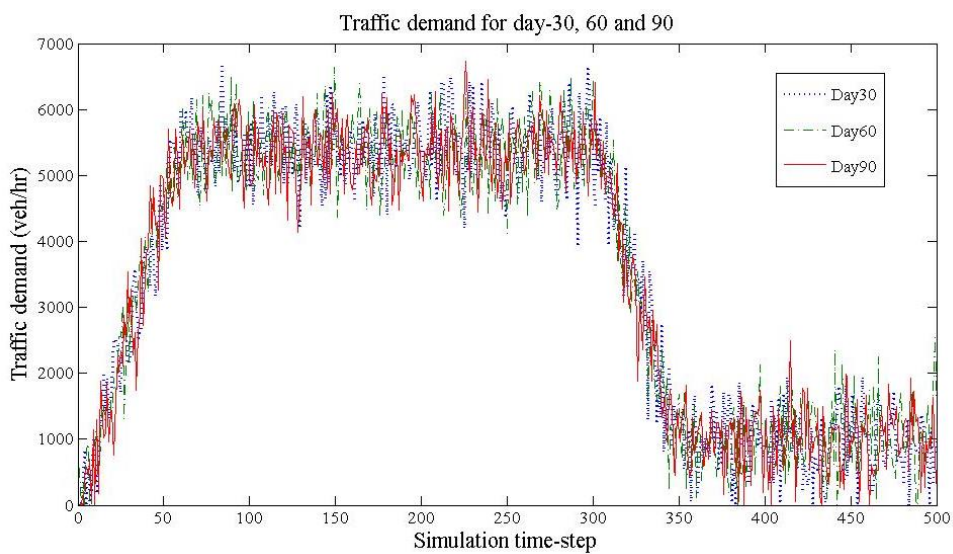
Figure 7.4 Traffic demand for some selected days

A comparison of travel times experienced on selected days for the simulation horizon of that day, with and without ATIS is provided. The split-rates at the diverging intersection are also compared for the above mentioned scenarios to assess the impact of information on route choice and stability of the system. Figure 7.5 compares experienced travel times on link-1, with and without ATIS on day 30 for all the simulation time-steps. Similarly, the comparison of experienced travel times with and without ATIS is shown in figure 7.6 and figure 7.7, for link-2 and link-3, respectively.
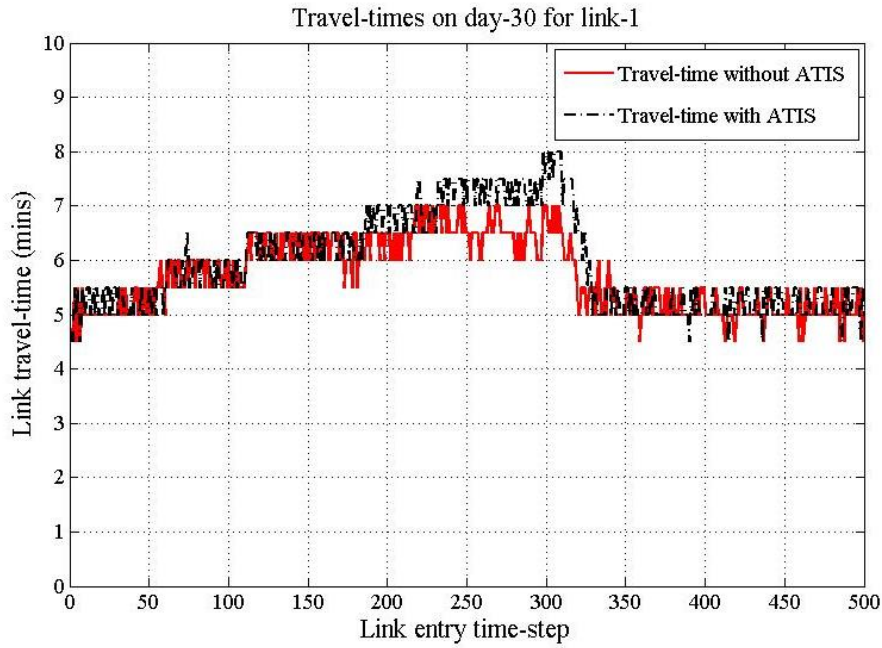
Figure 7.5 Experienced travel times on link-1 for day-30
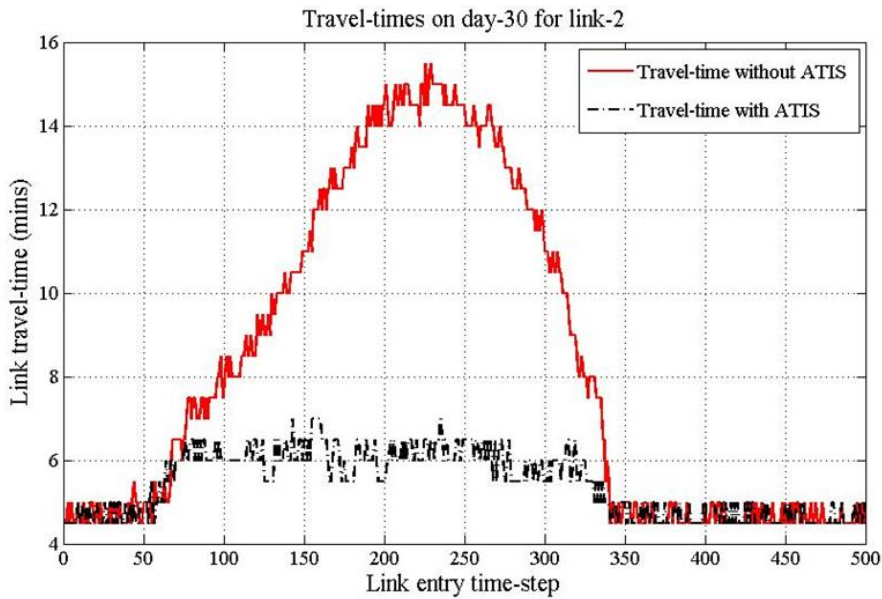
Figure 7.6 Experienced travel times on link-2 for day-30

The comparison of experienced travel with and without ATIS for link-1 on day-30 is shown in figure 7.5. The experienced travel time for link-1 also includes waiting time outside the link, in the dummy cell, when the demand is higher than the inflow/ available capacity of the link. It can be observed from figure 7.5 that travel times for link-1 were comparatively unaffected with the provision of traveller information. The link-1 is observed under free-flow traffic condition till time-step 60, as the demand for traffic link till time-step 60 is less

than the inflow capacity of the link. Traffic demand from time-step 60 to time-step 300 fluctuates around the capacity of link-1. However, due to the capacity restriction of the diverging intersection, congestion is observed to gradually increase from time-step 60. Similarly, the drop in congestion pattern can be observed from experienced travel times from time-step 300 onwards, as the traffic demand gradually reduces from time-step 300. It can also be concluded that the travel times for link-1 are not affected with the provision of traveller information. Figure 7.6 compares experienced travel times with and without traveller information for link-2 on day-30. Due to reduced number of lanes and capacity from cell-16 in link-2, a bottleneck effect is created when traffic demand is higher than the capacity at bottleneck. It can be observed from figure 7.6 that a significant improvement in travel times is observed during peak-hour traffic. The experienced travel time on link-2 under traveller information remains slightly higher than free-flow travel time (4.5 minutes) during peak-hour. Whereas, in the scenario of no-information, travel time gradually increases as the congestion increases on link-2. The maximum travel time on link-2 was observed to be 7.5 minutes with traveller information. On the other hand, the maximum travel time increased to 15 minutes without information on day-30. Furthermore, the experienced travel time with traveller information shows a steady and stable trend, whereas in no-information scenario travel times are showing higher fluctuation, which ultimately affect the route choice for the next day. Figure 7.7 shows travel times for link-3 on day-30, with and without traveller information. As there is no capacity reduction in link-3 and inflow to link-3 is always lower than the capacity flow, the link is observed in free-flow condition throughout the simulation period on day-30. The experienced travel times with and without traveller information varies around free-flow travel times on link-3.
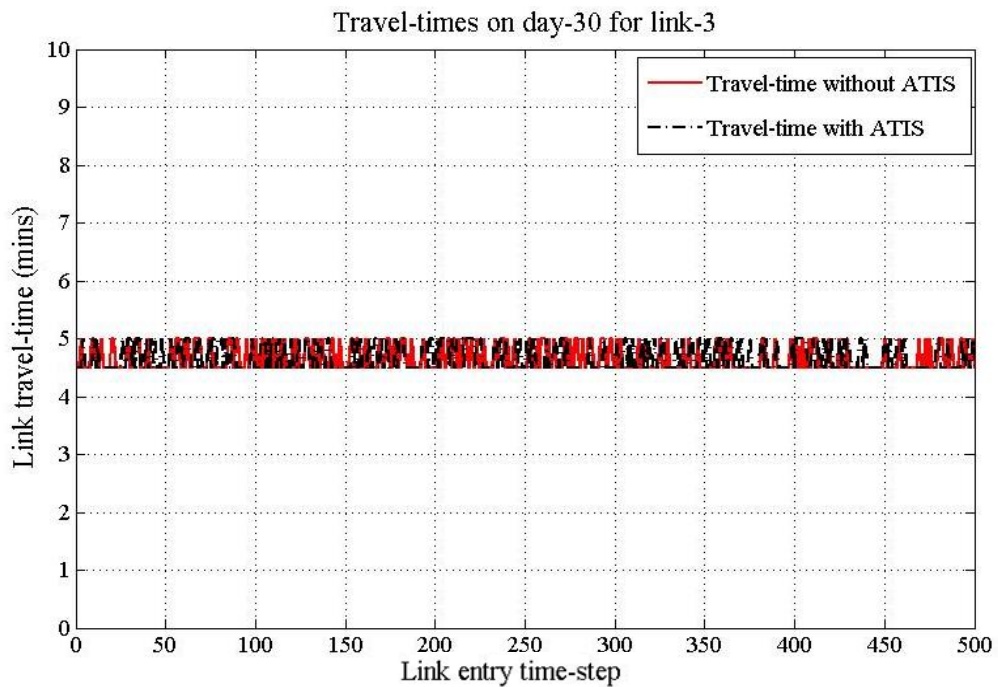
Figure 7.7 Experienced travel times on link-3 for day-30

The comparison of experienced travel on day-60 for link-1 is shown in figure 7.8. Figure 7.8 shows a trend in travel times, similar to the one shown in figure 7.5. The travel time on link-1 for day-60 fluctuates around free flow travel time for initial time-steps, when traffic demand is smaller than the link capacity. With the increase in traffic demand after time-step 60, the travel times are increased due to congestion in the link, due to downstream intersection and vehicles queuing in the dummy cell to start their journey. The travel times on link-1 gradually reduce after time-step 300, when the peak-hour traffic demand starts depleting. The analysis of figure 7.8 reveals that the experienced travel times on link-1 are unaffected with the provision of traveller information and travel times with and without traveller information are almost equal for all departure time-steps. The comparison of experienced travel times for link-2 on day-60 is shown in figure 7.9. This figure further validates the observation from figure 7.6 that traveller information has significantly improved the experienced travel times on link-2 during peak-hour traffic. Traffic is observed in congested states in both the scenarios during peak-hour traffic. However, the level of congestion, as observed from the travel times, is significantly lower when the traveller information is provided. The highest value of travel time observed in the scenario of ATIS is 7 minutes. In the scenario of no-information, travel times

experienced during peak-hour traffic are significantly higher and the maximum travel time is recorded as 13.5 minutes. Figure 7.9 indicates a significant improvement in travel times with the provision of information. The comparison of travel times on day-30 and day-60 under ATIS shows a similar pattern, which indicated stable condition of network performance across days as well as within day for peak-hour traffic. On the other hand, the comparison of figure 7.6 and figure 7.9 for travel times without information reveals the shift in pattern of congestion across days. The peak of travel time curve on day-30 is at time-step 225, whereas the peak of travel time curve for day-60 is shifted to time-step 170 on day-60. This shift of curve indicates fluctuation and instability in network performance and variation across days. The travel times for link-3 on day-60 are not shown here, as it is unaffected by the variation in traffic demand and shows a pattern similar to figure 7.7 for all the days in the simulation horizon.
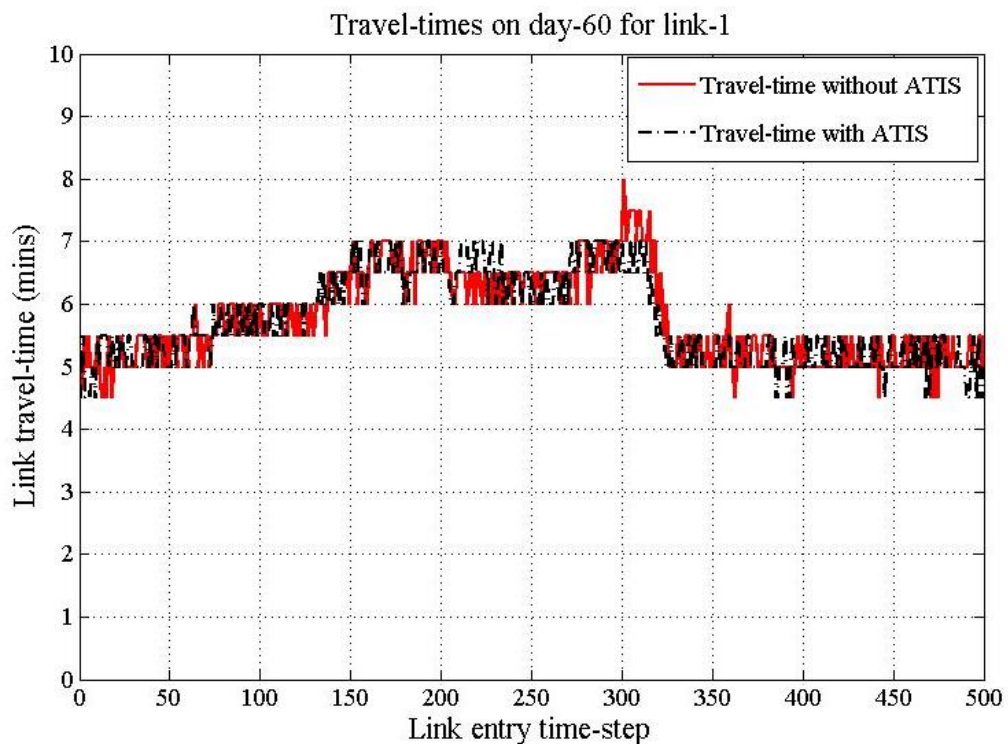
Figure 7.8 Experienced travel times on link-1 for day-60
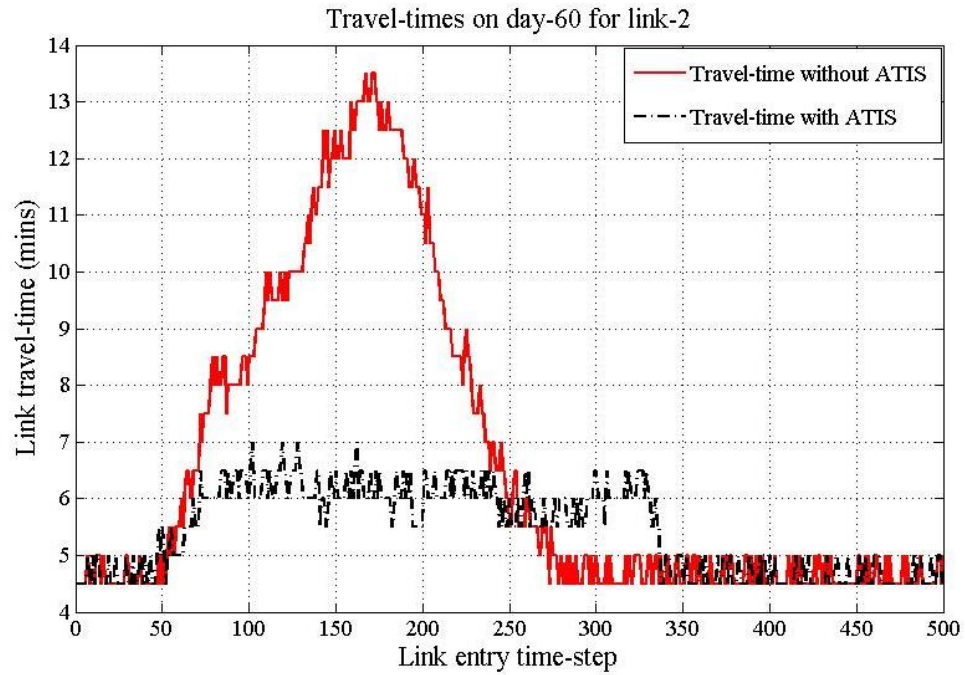
Figure 7.9 Experienced travel times on link-2 for day-60

The observations from outputs for day-30 and day-60 are further validated from the outputs for day-90. Figure 7.10 compares experienced travel times on link-1 for day-90. The comparison of figure 7.10 with figure 7.5 and figure 7.8 reveals similarity in the pattern of travel time curves for link-1. As evident from outputs from day-30 and day-60, provision of traveller information does not impact travel time on link-1. Figure 7.11 compares travel time experienced on link-2 for day-90. Figure 7.11 shows a similar trend; following figure 7.6 and figure 7.9 that traveller information significantly improves the traffic flow and alleviates traffic congestion during peak-hour traffic on day-90. The travel times with ATIS on day-90, show a similar pattern to the travel times observed on day-30 and day-60. Whereas, the curve for travel times on link-2 for day-90 without traveller information has a different shape compare to travel time curves on day-30 and day-60. The comparison of these two scenarios based on travel times for link-2 across days indicates the significance of provision of traveller information. With the provision of information, the network performance stabilizes under time varying traffic demand, whereas without traveller information, the travel time curves have different pattern for different days.
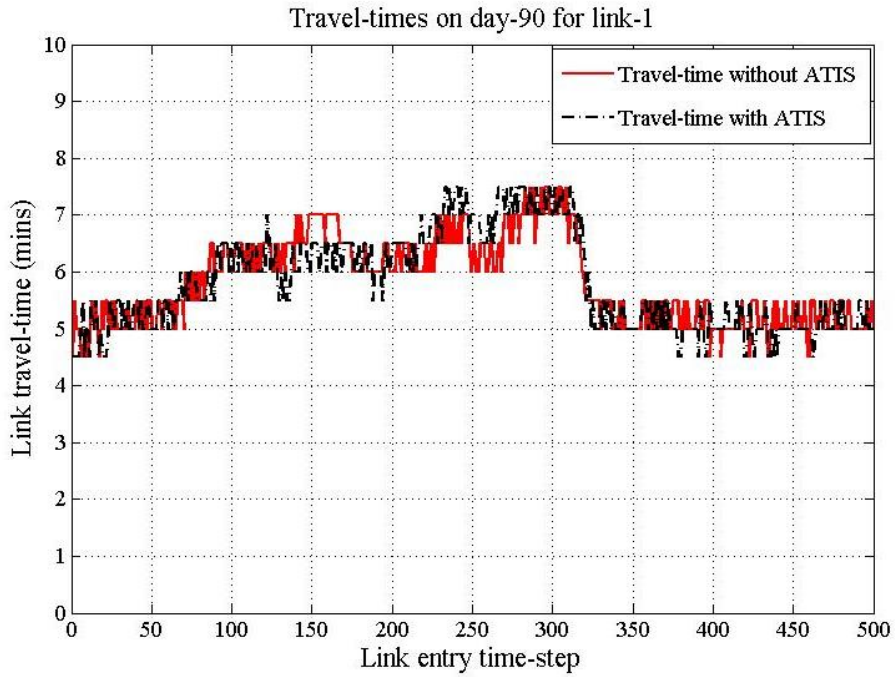
Figure 7.10 Experienced travel times on link-1 for day-90
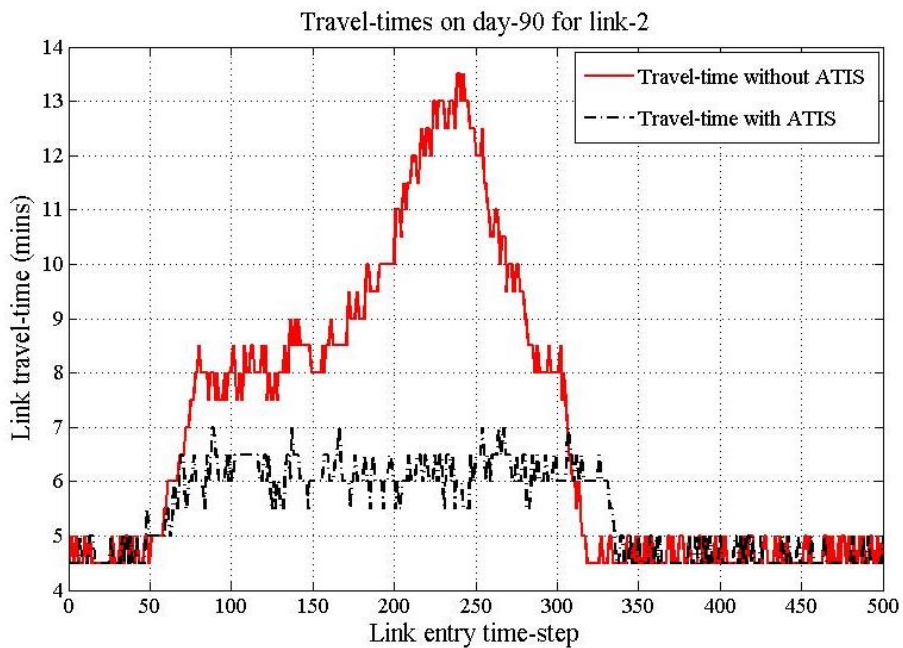
Figure 7.11 Experienced travel times on link-2 for day-90

Figures 7.5-7.11 show within day dynamics for selected days and time-step to time-step variation for peak-hour traffic flows and compare the two scenarios. The variation across days is highlighted by comparing the outputs from day-30, day-60 and day-90. However, figures 7.5-7.11 cannot represent day-to-to dynamics in travel time experienced by commuters. Thus, another set of

analysis from outputs is discussed here to appropriately highlight the day-to-day variations in travel time for selected departure/ link-entry time-steps. As evident from figures 7.5-7.11 that travel times for link-1 with and without traveller information are identical and similar across days, it is not included in the following analysis. Similarly, link-3 is always in free-flow condition in either of the scenarios, thus omitted from the day-to-day analysis. For day-to-day dynamics on link-2, the time-steps are selected so as to represent various traffic flow conditions which include free-flow, transition from free-flow to congestion, congestion and transition from congestion to free-flow. The time-steps to discuss the day-to-day dynamics are chosen based on observations from figures 7.5-7.11. Figure 7.12 compares day-to-day variation in experienced travel times for link-2 for commuters entering the link at simulation time-step 50.
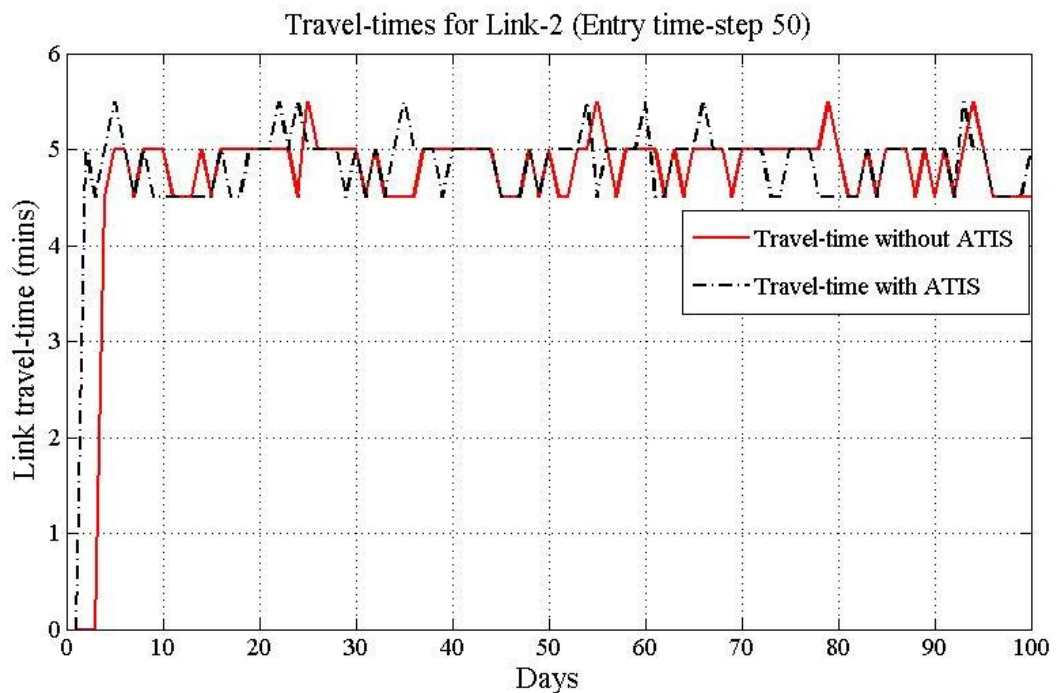
Figure 7.12 Experienced travel times on link-2 for link-entry time-step 50

Figure 7.12 shows travel time for link-2 based on link-entry time-step 50. It can be observed from figure 7.4 that traffic demand at this time-step is smaller than the available capacity and figures 7.5-7.11 endorse that the network is in free-flow state at this time-step. Figure 7.12 presents day-to-day variation in travel time under free-flow condition. It can be observed from this figure that under

free-flow condition there is a small day-to-day fluctuation in travel times. Furthermore, travel times under free-flow traffic condition are similar for both the scenarios. Time-step 150 is selected to indicate the day-to-day variation in travel times when the traffic demand is gradually increased to the level, when traffic network starts getting congested. Figure 7.13 compares travel times under ATIS with the travel times experienced without ATIS under a traffic state showing transition from free-flow to the congested one. Figure 7.13 reveals that there is stability in travel-times experienced by commuters entering the link at time-step 150 under ATIS. Whereas in the scenario of no-information, the travel time experienced are higher with a significant level of day-to-day fluctuations in travel times.
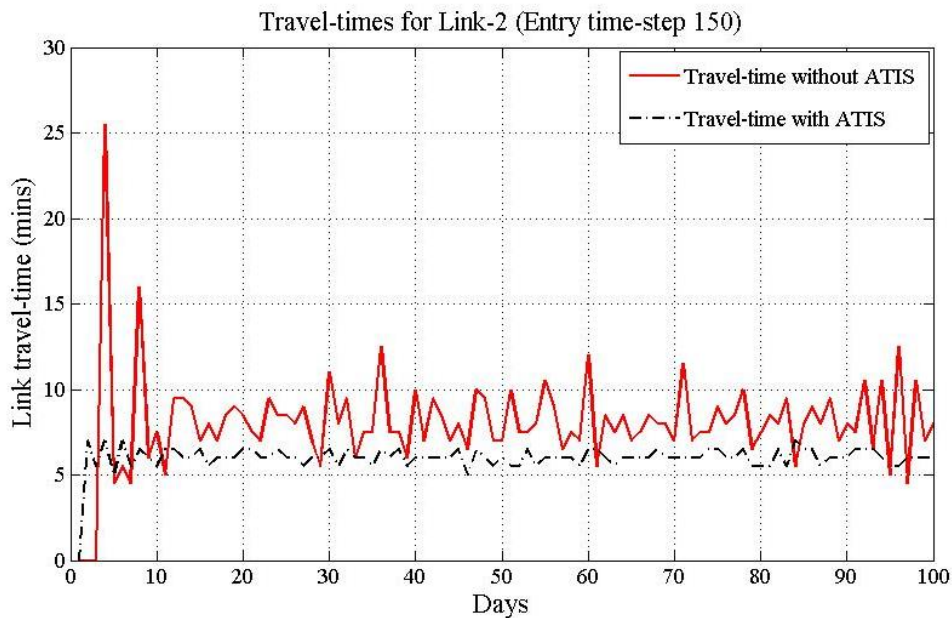
Figure 7.13 Experienced travel times on link-2 for link-entry time-step 150
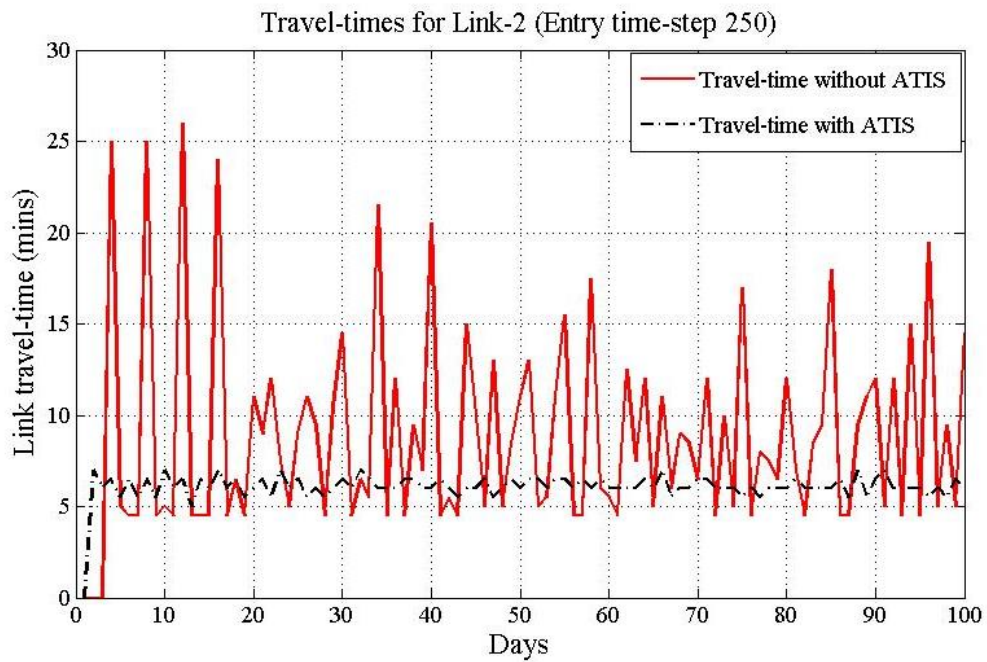
Figure 7.14 Experienced travel times on link-2 for link-entry time-step 250

The Day-to-day variation in travel times and significance of accurate prediction of traveller information under congested traffic condition with fluctuating traffic demand can be observed from figure 7.14. At time-step 250, link-2 is in congested traffic state as observed from within-day outputs for day-30, day-60 and day-90. Figure 7.14 shows that there is a significant improvement with the provision of traveller information as the travel time with information are improved and stable across the days. The predicted traveller information based on real-time traffic state estimation takes into account any unexpected variation in traffic demand or network capacity, which travellers are unaware of. This significantly improves the performance of the network under uncertain traffic conditions. Travel times without ATIS are higher and there is a significant variation in day-to-day travel times.

Another interesting output of day-to-day traffic flows and route choice modelling based on real-time traffic state estimation is the route choices represented through the split-rates at the intersections. The route choice behaviour of commuters is modelled using logit model. The logit parameter θ, which defines perception variation of commuters about expected travel time, is assumed different for both the scenarios. Huang and Li (2007) assumed

different values for perception variation based on the assumption that commuters' perception variation reduces with the provision of accurate information. Thus, the commuters without traveller information are modelled with higher perception variation as compared to the commuters with traveller information. The value of logit parameter in no-information scenario is assumed to be 0.1, reflecting higher perception variation. Whereas, in the case of traveller information, commuters are modelled with comparatively lower perception variation with logit parameter as 0.25.
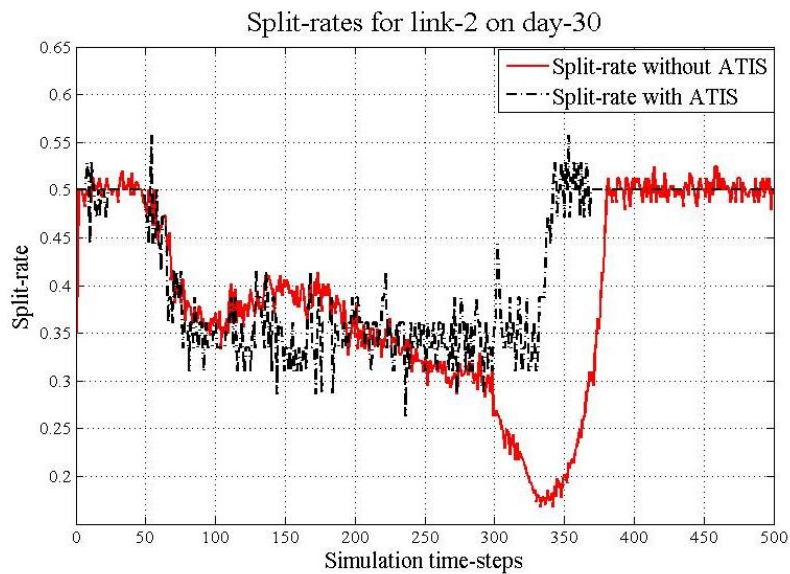
Figure 7.15 Split-rates for link-2 on day-30
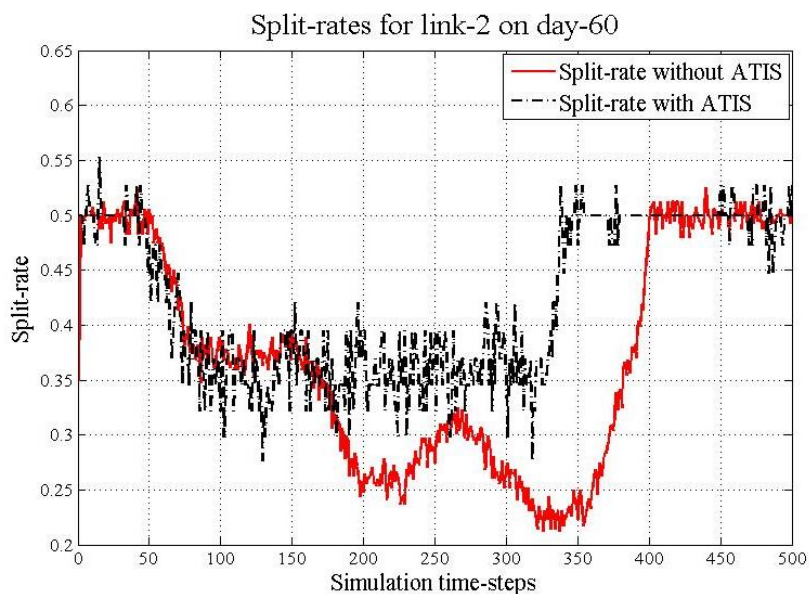
Figure 7.16 Split-rates for link-2 on day-60

Figure 7.15 compares split-rates (proportion of traffic selecting route-2) with and without ATIS on day-30. The equal split of traffic for link-2 and link-3 can be observed when traffic demand is low and smaller than the available capacity, for example, during time-steps 1 to 50. With the increase in congestion in link-2, the proportion of traffic selecting link-2 gradually reduces. The comparison of split-rates on day-30 reveals that the split-rate variation is higher with no-information scenarios, whereas with information the split-rates seem to fluctuate around a fixed value during peak-hour traffic. Furthermore, comparison of figure 7.15 and figure 7.16 indicates that the pattern of split-rate curve for day-30 and day-60 is similar under ATIS. On the contrary, the shape of the curve for split-rates significantly changes from day-30 to day-60 in no-information scenario. It can also be observed from figures 7.15-7.16 that peak-hour traffic dispersed through the network earlier when traveller information is provided. The dynamics of variation in split-rates within the day for day-30 and day-60 is described in figures 7.15-7.16. The day-to-day dynamics in variation of split-rates is highlighted by comparing the split-rates with and without information for specific link-entry time-steps. The link-entry time-steps chosen to elaborate day-to-day dynamics in travel time are used in discussion of day-to-day dynamics in route choice and resulting split-rates.
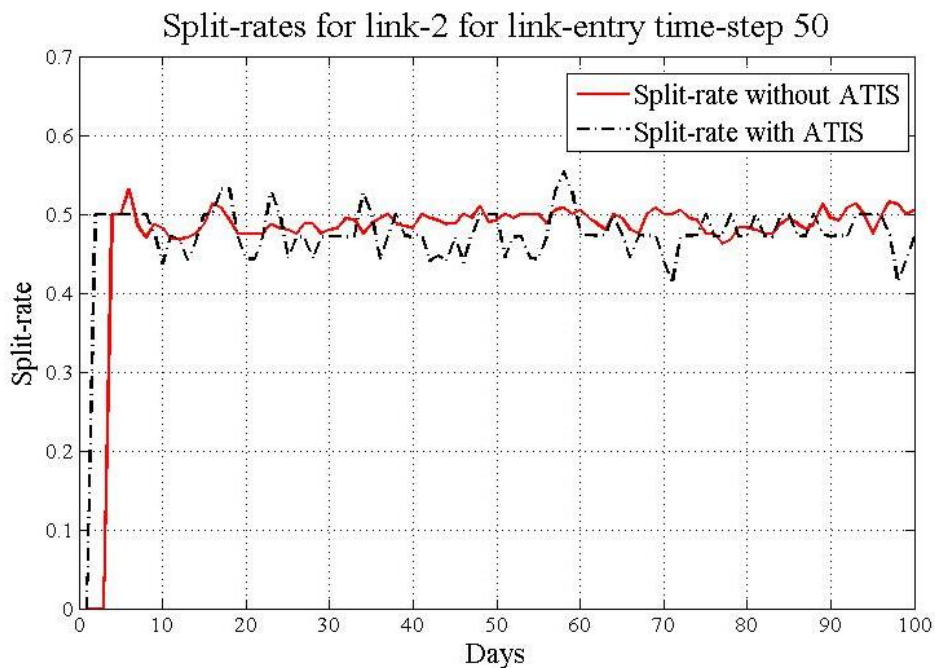
Figure 7.17 Split-rates for link-2 for link-entry time-step 50
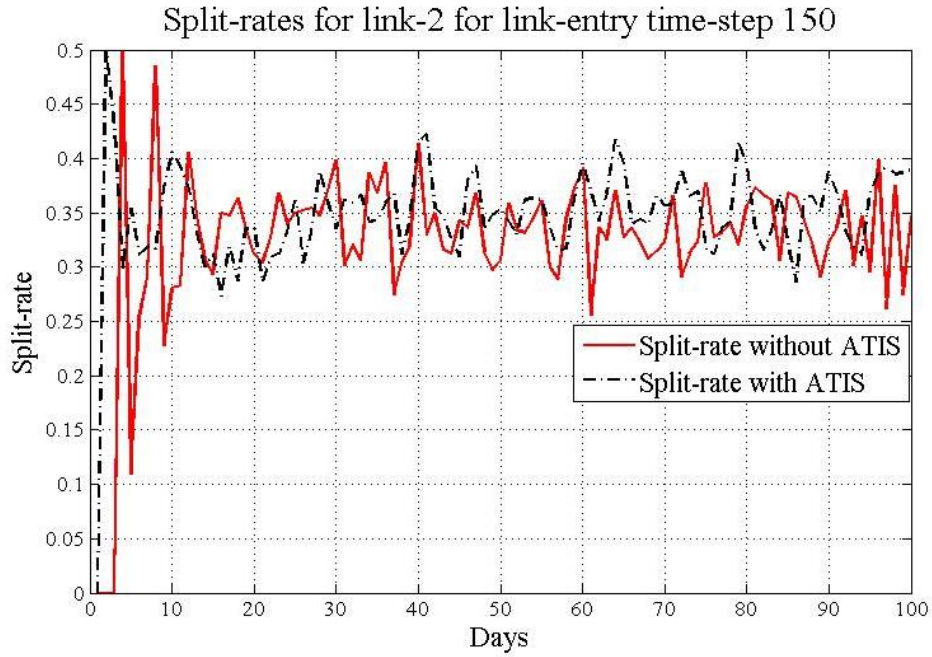
Figure 7.18 Split-rates for link-2 for link-entry time-step 150
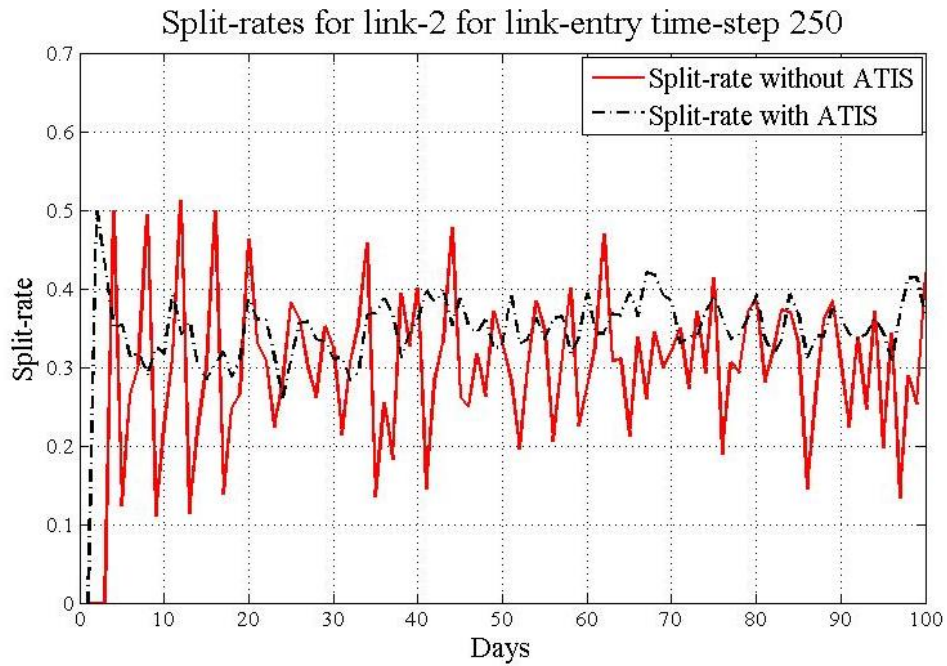
Figure 7.19 Split-rates for link-2 for link-entry time-step 250

The split-rates for departure time-step 50 are shown in figure 7.17. At time-step 50 traffic demand is smaller than the available capacity and all the links in the network are in free-flow traffic condition. It can be observed from figure 7.17 that under free-flow traffic condition, the split-rate for link-2 varies

around a mean value of 0.5, as the capacities of downstream links are higher than the traffic demand from upstream link. Figure 7.17 shows that the level of variation is higher with traveller information when compared to no-information scenario under free-flow traffic condition. This difference can be attributed to the difference of values for perception variation in both the scenario. As in free-flow traffic condition, the commuters in both the scenarios have similar experienced and expected travel times and variation in traffic demand is not affecting the free-flow travel times. With a lower perception variation in ATIS scenario, the fluctuation in route choice behaviour is comparatively higher than no-information scenario. This also indicates that the value of logit parameter greatly influences the route choice behaviour and stability of the network performance.

The day-to-day variation in route choice during congestion in peak-hour is described in figures 7.18-7.19. Figure 7.18 compares split-rates for link-2 based on link-entry time-step 150. At time-step 150, the network starts getting congested and figure 7.18 describes route-choice behaviour under mild congestion in the network. It can be observed from figure 7.18 that under mild level of congestion, the split-rates are identical in both the scenarios. However, the level of day-to-day variation in the split-rates without information is comparatively higher although the perception variation is assumed to higher in no-information scenario when compared with the scenario of traveller information. Figure 7.19 shows day-to-day variation in route choice for a departure time when traffic congestion is highest within the peak-hour period. It can be observed from figure 7.19 that the day-to-day fluctuation in route choice under time-varying traffic demand is much higher in no-information scenario, compared to route choice under traveller information. The fluctuation in route choice is caused due to day-to-day variation in travel time and time-varying traffic demand. A higher experienced travel time on a given day *d* influences the route choice behaviour on day *d+1* and consequently reduces the demand in no-information scenario, as the commuter is unaware of expected travel time on day *d+1* under fluctuating traffic demand. On the other hand, in the other scenario of route choice under traveller information the commuters are provided with the accurate information of expected travel times at

downstream links. This information accounts for any unexpected changes in network capacity or traffic demand, thus provide stability in route choice and network performance.

The route choice behaviour and resulting experienced travel times are highly dependent on logit parameter for perception variation in expected travel time. In the comparison of two scenarios, the no-information scenario is modelled using higher perception variation compared to that of with information.

7.5.2 Sensitivity analysis

7.5.2.1 Sensitivity of route choice behaviour to perception variation in travel time

The dependency of the route choices and resulting traffic flows and travel times on the logit parameter for perception variation is discussed in this section. As described earlier, that route choice behaviour of commuters is sensitive to the assumed values of the perception variation of expected travel times. This is elaborated with the help of simulation results by changing the values of the parameter and comparing the split rates obtained for the scenarios with and without traveller information. The values of logit parameter are changed from 0.1 to 0.9 with a difference of 0.1 for both the scenarios. To evaluate the sensitivity of route choice behaviour to the logit parameter, the split rates obtained for a selected link entry time-step are compared. The difference in experienced travel times on link-2 and link-3 (τ_2- τ_3) is also compared for various values of logit parameter to study the impact of split-rates on the resulting traffic flows and travel times and vice versa. The comparison of split rates and difference in travel times is shown for link entry time-step 250, as this time-step belongs to the congested period during the peak-hours.

The comparison of split rates obtained with and without traveller information for values of logit parameter from 0.1-0.9 is shown in figure 7.20. The day-to-day variation and sensitivity of route choice behaviour to the logit parameter can be evaluated from figure 7.20. With higher values of logit parameter (from 0.6 to 0.9), it can be observed from the figure 7.20 that the traffic flows on the network become more unstable and a higher day-to-day fluctuation in traffic flows is observed in both the scenarios. The comparison of split rates with and

without traveller information for all the values of logit parameter reveals that the provision of traveller information significantly improves the day-to-day fluctuation of route choices (split rates). It can be observed from figure 7.20 that even for higher values of the logit parameter, which implies that the perception variation among commuters is small and commuters react more coherently, the day-to-day variation in split-rates are significantly lower than the variation in split-rates without traveller information. A comparatively higher variation in the split rates under traveller information is observed for higher values of the logit parameter (0.6-0.9), whereas the day-to-day route choice behaviour shows stability for smaller values of the logit parameter (0.1-0.3). A significant improvement in day-to-day variation in split rates is evident from figure 7.20 with the provision of traveller information.
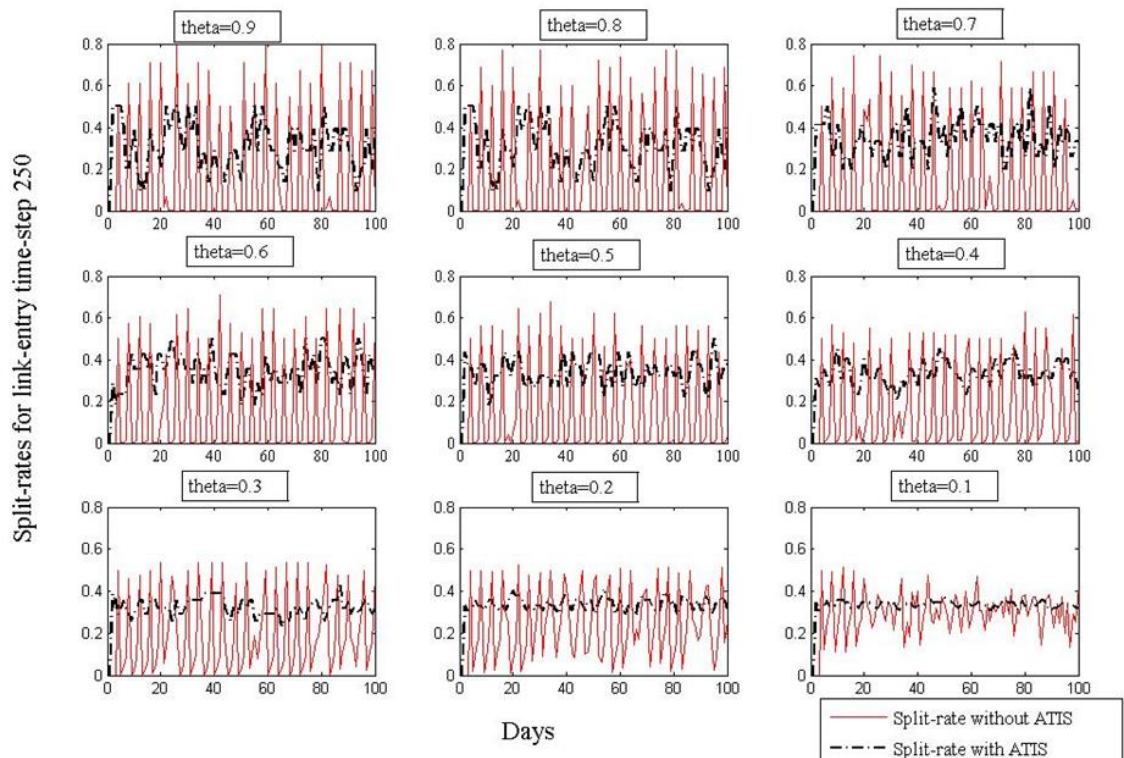
Figure 7.20 Comparison of split-rates for link entry time-step 250 for various values of logit parameter
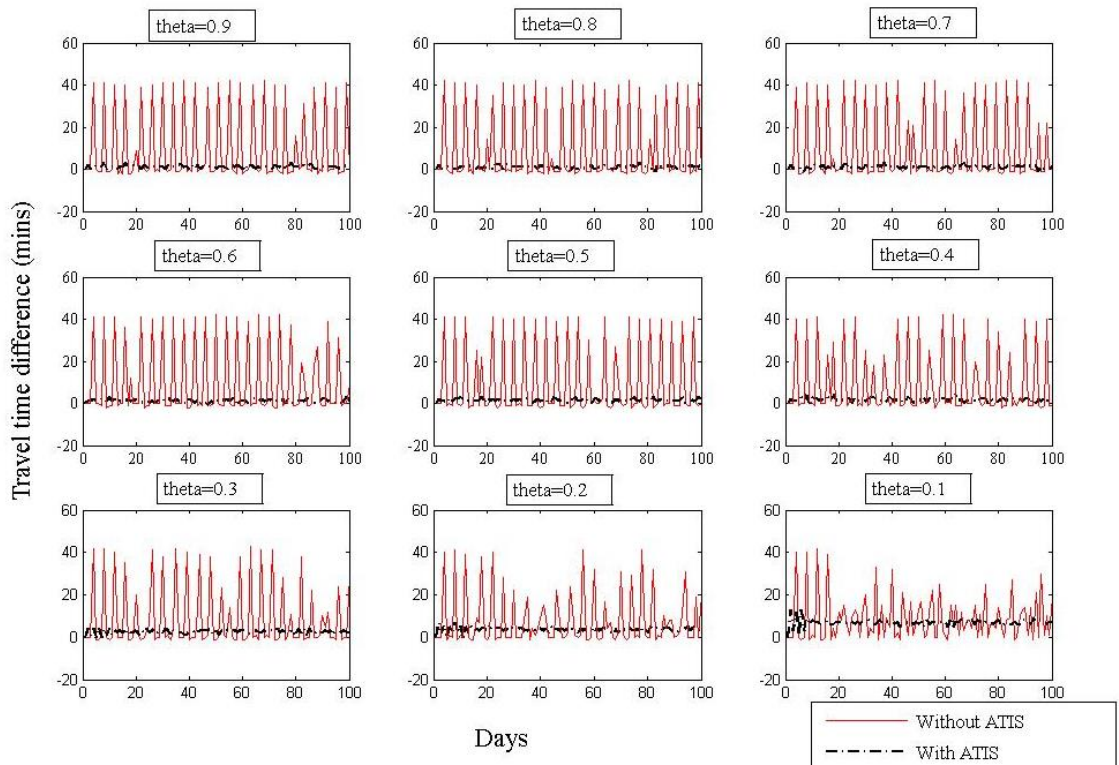
Figure 7.21 The difference in travel times on link-2 and link-3 for various values of logit parameter

The difference of travel times on link-2 and link-3 for values of logit parameter from 0.1 to 0.9, with and without traveller information is compared in figure 7.21. It can be observed from figure 7.21 that in the scenario of no-information, the difference of travel times on alternative links is very high and it fluctuates to very high level. For all higher values of logit parameter (except 0.1), the difference in travel times is unstable without traveller information. Whereas, with the provision of accurate traveller information, the difference in travel time on alternative links is very small and shows a stable trend for all values of the logit parameters. It can be concluded from figure 7.20 and 7.21 that split rates are sensitive to the perception variation among commuters for both the scenarios, with and without traveller information. However, the split rates without traveller information are comparatively more sensitive to the change in the value of logit parameter. Similarly, the difference in travel times on link-2 and link-3 is observed much higher with higher variation, without provision of the traveller information.

7.5.2.2 Sensitivity of route choice behaviour to memory length

Figure 7.14 exhibits a periodic variation in experienced travel times on link-2 without traveller information. In the case of no-information, the route choice for a given journey is purely based on experienced travel times and the length of commuters' memory and modelling assumptions play a significant role in determining the shape of travel time and resulting route choice variations. The day-to-day variation in experienced travel times for departure time-step 50 (figure 7.12) and departure time-step 150 (7.13) are comparatively smaller, as link-2 is not completely congested for these time-steps. However, for time-step 250 the link-2 is in a congested state without provision of traveller information, where the assumed memory length becomes important. It can be observed from figure7.14 that one cycle of fluctuation is spread over three days, which is assumed memory length. Based on the model described in equation (7.1) for weighted average experienced travel time, the length of memory for this application was assumed three days, with most recent experienced given the highest weight. When commuters have equal expected travel times for both the routes, the traffic demand from upstream link is equally divided for downstream alternative routes. The equal capacities of cell-11 and cell-21 result in equal inflows to link-2 and link-3 when the perceived travel times are equal for the alternative routes. The downstream capacity of link-2 is one-third of its upstream capacity which results in traffic jam in link-2, especially when the demand from upstream link-1 was higher, e.g. for departure time-step 250. On the day with equal splits on the alternative routes, the commuters who selected link-2 experienced much higher travel time than the commuters who selected link-3. This results in commuters switching to link-3 on the following day, which causes a sudden drop in experienced travel time on link-2. After few consecutive days with free-flow travel time on link-2, the effect of the journey with higher travel time becomes less relevant which again results in almost equal perceived travel times for alternative routes and the same cycle is repeated. The effect of memory length on this type of fluctuation is further validated by increasing the memory length to 10 days. Figure 7.22 shows experienced travel time for departure time-step 250 with a memory length of 10 days.
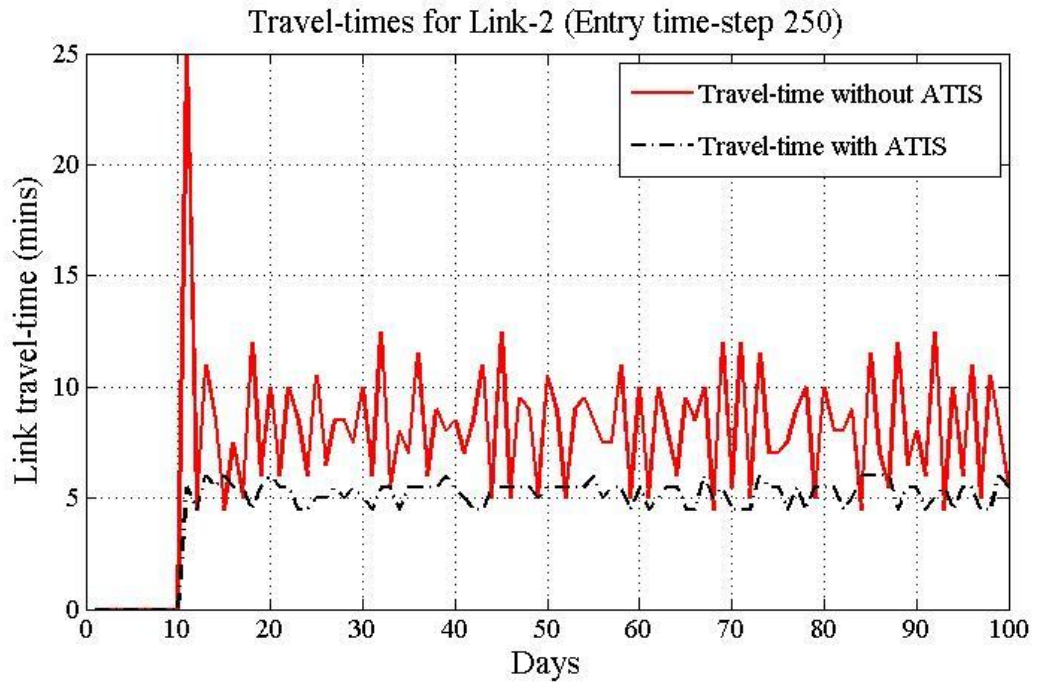
Figure 7.22 Experienced travel times on link-2 for link-entry time-step 250 with a memory length of 10 days
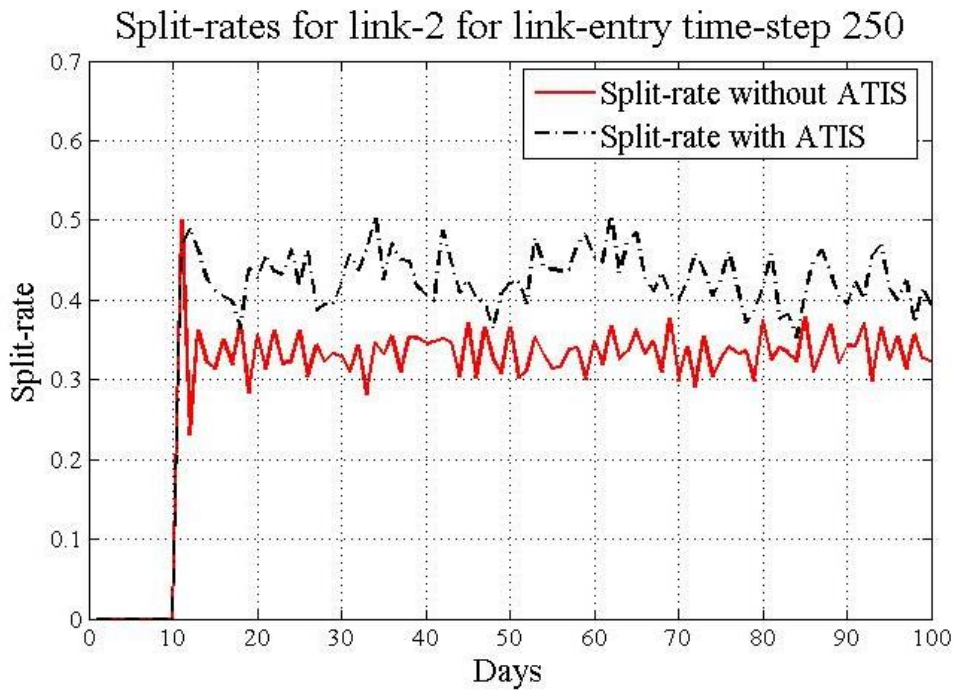
Figure 7.23 Split-rates for link-2 for link-entry time-step 250 with a memory length of 10 days

The comparison of figure 7.22 with figure 7.14 shows a significant stability in route choice behaviour with a memory length of 10-days. The experienced travel time without ATIS has improved significantly with increase in memory length. The comparison of figure 7.22 with figure 7.14 also highlights the significance of memory length assumption and selection of appropriate model to determine commuters' experienced travel times. A similar improvement in route selection is observed by comparing figure 7.19 with figure 7.23 for link-entry time-step 250. The day-to-day variation has been significantly reduced in no-information scenario by increasing the assumed memory length of commuters. Based on this comparison, it is anticipated that the simulation results for modelling route-choice in no-information scenario can be further improved if the memory length is further increased and more stability will be achieved as memory length is increased to infinity.

7.5.2.3 Sensitivity of route choice behaviour to random noises in CTM-EKF framework

There are various sources of variation in the framework employed in this research, which include demand variation, random noise in modelling of traffic flow using CTM and random noise in measurements obtained from the sensors. The effect of these noises in day-to-day variation of route choices is analysed by reducing one of the noises to zero and analyse its impact on the resulting split-rates. The results obtained from no variations are compared with the results obtained with the random variations. For this purpose, the memory length of three days is selected and logit parameter for perception variation is kept at 0.3. Thus, the results shown in this section for sensitivity of route choice behaviour can be compared with bottom-left plot in figure 7.20 (for theta=0.3 and memory length of three days).

The effect of random variation in traffic demand on split-rate is analysed by reducing the standard deviation in traffic demand from 500 veh/hr to 0 veh/hr during peak-hour traffic. Figure 7.24 shows day-to-day variation in traffic demand for departure time-step 250, with and without random variation in traffic demand.
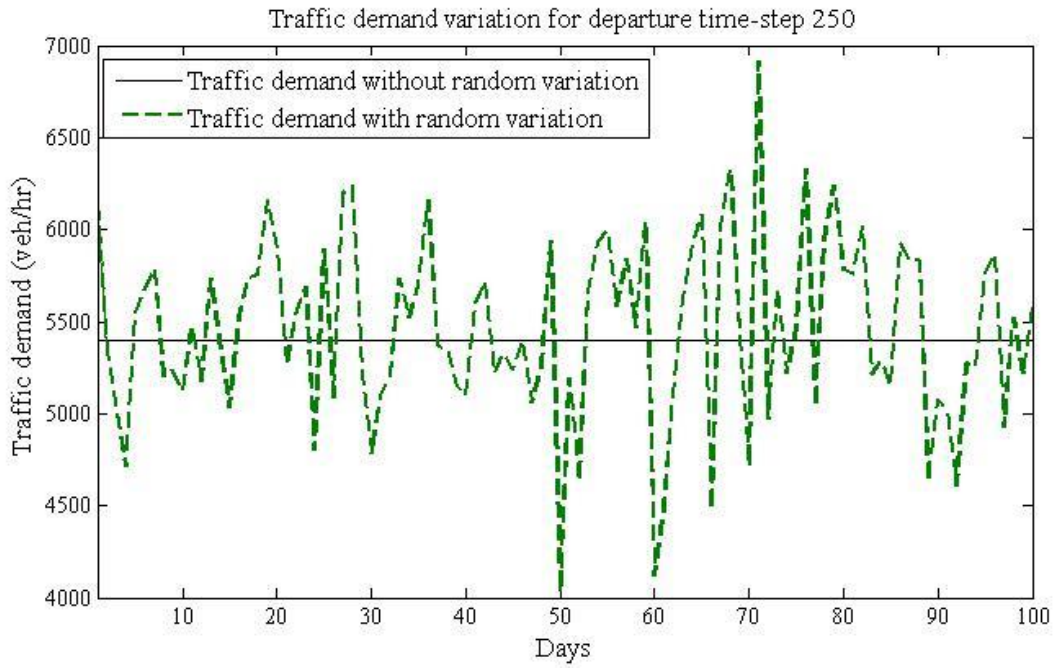
Figure 7.24 Comparison of traffic demand with and without random variation for departure time-step 250
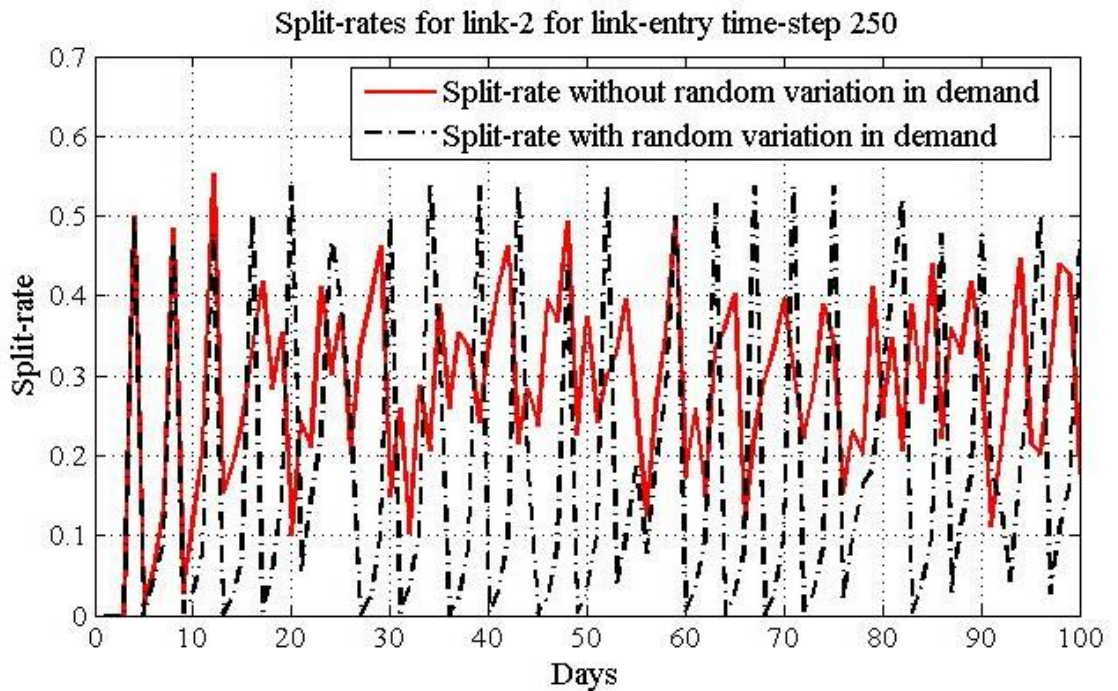
Figure 7.25 Comparison of split-rate with and without random variation in traffic demand

Figure 7.25 compares split-rates obtained with and without random variation in traffic demand. It can be observed that the variation in split-rate is comparatively lower with constant daily demand for departure time-step 250. The variation in split-rate for link-2 without demand variation can also be

attributed to the shorter memory length assumed in this research, as longer memory length results in lower variation in split-rate, as observed from figure 7.23. The impact of random noise in modelling prediction of traffic density is analysed by reducing the standard deviation of normally distributed noise in prediction to zero.
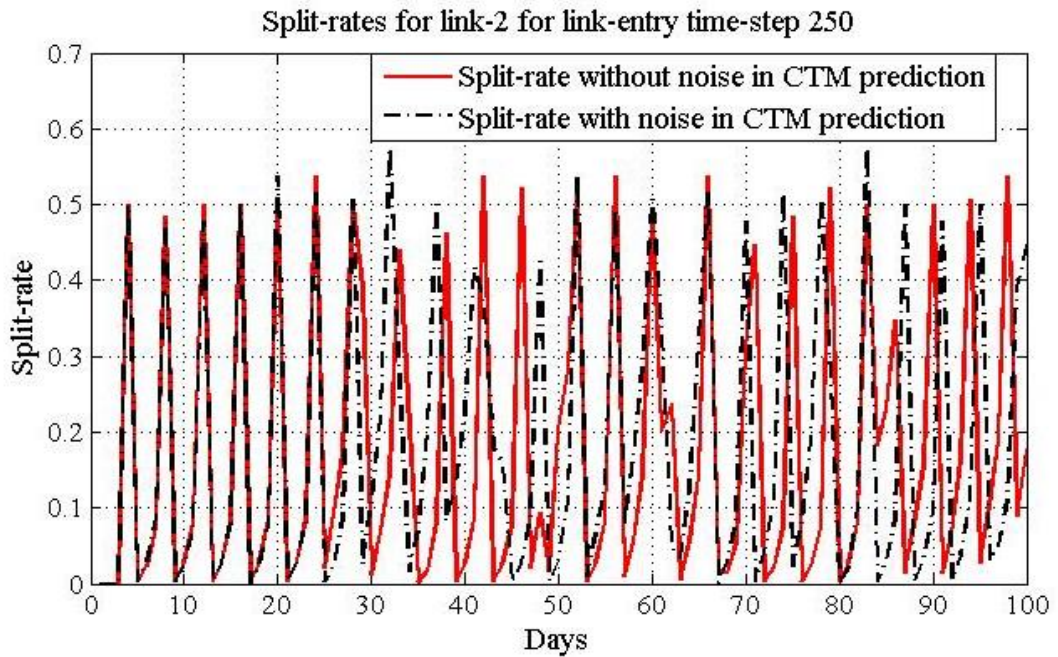
Figure 7.26 Comparison of split-rate with and without noise in prediction of traffic density

Figure 7.26 shows that there is no significant improvement in the fluctuation of split-rate by removing the random noise in prediction of traffic density using CTM. The effect of normally distributed noise in measurement is studied by removing the noise in measurement of traffic density with zero mean and standard deviation of 2 veh/km. Figure 7.27 compares the day-to-day variation in split-rates with and without random noise in measurement of traffic density. It can be concluded based on figures 7.26 and 7.27 that the noise in prediction of traffic density and noise in measurements do not affect the day-to-day variation in split-rates observed in figure 7.20. Whereas, the length of memory and variation in traffic demand significantly affects day-to-day variation in split-rates.
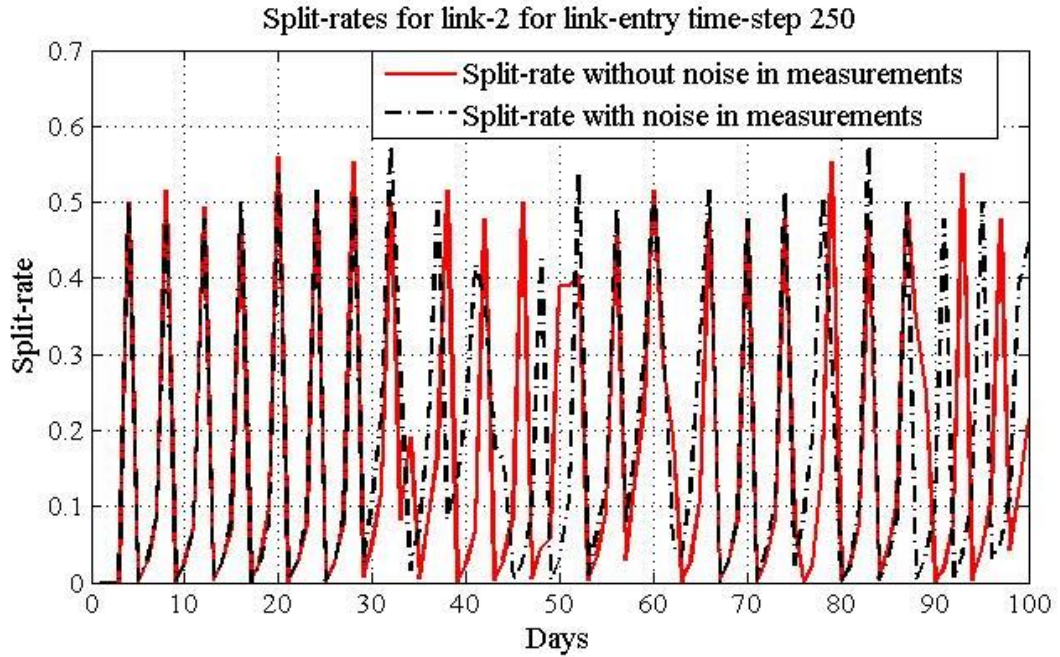
Figure 7.27 Comparison of split-rate with and without noise in measurements

7.6 Summary

In this chapter the proposed framework to integrate real-time traffic state estimation and dynamic traffic assignment is applied to model day-to-day traffic flows and dynamics in route choices. The network loading component of day-to-day traffic flow model is replaced with CTM-based real-time traffic state estimation algorithm. The real-time estimated traffic state is utilized to predict the expected travel time under fluctuating traffic demand. Commuters update their perception about expected travel time for a given journey based on their experience and the information obtained through the ATIS. The simulation results obtained by numerical implementation of the proposed framework suggest that with the reliable and accurate traveller information, commuters are facilitated to adopt a lesser congested route and avoid a congested alternative route. A significant improvement in travel times and stability of the network performance is observed by comparison of the scenarios with and without traveller information. The use of observations from traffic sensors in day-to-day modelling of traffic flows provides an opportunity to calibrate and estimate critical modelling parameters, which can improve the accuracy of the modelling process.

Chapter 8: Modelling Behaviour Parameters of DTA Models

8.1 Introduction

This chapter introduces modelling and estimation of some critical parameters of dynamic traffic assignment models used in this research. It was observed during simulation tests of scenarios presented in chapter-6 and chapter-7 that the accurate values of these parameters play a significant role in obtaining accurate results and improve the accuracy of the modelling process. Model-based estimation of traffic state provides an opportunity to model and estimate the parameters of traffic flow model, as exhibited in chapter-6, as well as the parameters of DTA models applied to model the route choice behaviour of commuters. In this research, two of the parameters from DTA models are selected for modelling and estimation using the measurements and real-time traffic state estimation. These two parameters include the parameter which assigns the weight to the traveller information in comparison to their own experience and the logit parameter which represents perception variation among commuters for expected travel time.

A methodology is proposed to model the commuters' level of trust in the information as a dynamic parameter, as commuters' level of trust in the information system changes with the accuracy of the information provided. The commuters' level of trust in the information is modelled as a dynamic parameter by comparing the experienced travel time with the travel time predicted for those commuters and communicated through ATIS. The behaviour parameter which represents perception variation in the expected travel time to model route choice behaviour is estimated based on real-time observations from the sensor and estimated traffic state. The split-rates predicted using logit model are compared with the observations after the simulation horizon for a given day and the logit model parameter θ is estimated based on the observations from the sensor using archived travellers information and resulting split-rate obtained using measurement sensors. Section 8.2 describes the proposed model and applies it to a test network to

model day-to-day variation in commuters' level of trust in the information. Section 8.3 explains the models for estimation of logit parameter and implements it to a test network to demonstrate the application of the proposed model. The findings of this chapter are summarized in section 8.4.

8.2 Commuters' level of trust in ATIS

This research proposes to model commuters' level of trust in traveller information as a dynamic parameter. With day-to-day variation in traffic demand, a commuter may experience completely different travel time for the same departure time on a given day. In contrast to his own experience, when the commuter learns that traveller information is more accurate than his past experience, the commuter gives more weight to the information in planning his next journey. Yin and Yang (2003) suggested modelling the commuters' compliance rate based on the accuracy of the traveller information. Yin and Yang (2003) considered the traveller information provision through premium devices and divided the commuters into three groups, which include equipped, unequipped and equipped but non-complying drivers. They modelled drivers' compliance as a probability of travel time experienced by complied drivers less than or equal to the travel time experienced by unequipped and non-complying users. Yin and Yang (2003) also modelled yearly penetration of ATIS devices based on the travel time saving using the information. Yang and Huang (2004) extended the model of ATIS penetration rate and suggested applying social growth and diffusion approach to model increase in usage of ATIS devices. Huang *et al*. (2008) applied the ATIS compliance rate model to determine route choice dynamics using logit model and discussed establishment of stochastic user equilibrium for different groups of commuters. Kantowitz *et al.* (1997) conducted an experiment using a simulator to quantify the threshold of accuracy to obtain compliance of commuters. The drivers were provided with the information which was 100% accurate, 71% accurate or 43% accurate. Kantowitz *et al.* (1997) concluded that the 100% accurate information resulted in best user performance and subjective opinion about the information provided. Whereas, the 71% accurate information was also accepted and utilized by the commuters but the information with 43% accuracy resulted in

significant incompliance and strong opinion against the information. The research further concluded that subsequent accurate information can improve the compliance rate, damaged due to inaccurate information. Based on the review of existing literature, this research proposes a framework to model the level of trust in the information as a function of accuracy of the information provided to the commuters. In comparison to the literature cited for this thesis, this research determines the commuters' experienced travel time based on real-time estimated traffic state and provides a complete framework to include this parameter in the modelling framework.

In the proposed framework to model commuters' level of trust in the information, a commuter compares the experienced travel time with the predicted travel time provided through ATIS after completion of his journey and evaluates the accuracy of the information. The Commuter updates the weight for the traveller information based on the difference of predicted and experienced travel time. One of the three proposed relation is selected based on the accuracy of the information to determine the weight that commuters' give to the information for a future trip. There can be three possibilities in day-to-day modelling of weight given to the information. The first possibility is that the commuter determines that the information provided is very accurate and more relevant than his experienced travel time. This causes commuter to give more weight to the information in planning the trip or en-route decision to select a route. In second case scenario, commuters' level of trust remains constant, which is the case when the information is acceptable and applied but a commuter is not completely satisfied with the accuracy of the information provided. In third possibility, a commuter decided to give less weight to the information in the next trip, when his experience significantly differs from the information provided. For the first possibility, we assume a threshold of 85% accurate information. If the difference between the predicted and experienced travel time is less than 15%, there will be an increase in the level of trust in the information which is inversely proportional to the magnitude of the difference in predicted and experienced travel times. The updated value of the weight parameter based on the learning behaviour model is given as follows:

$$\alpha^{d+1,k}\left(\bar{\tau}_e^d, \tau_p^d\right) = \alpha^{d,k}\left(\bar{\tau}_e^d, \tau_p^d\right) + \Psi(0.15 - \left|\tau_e^{d,k} - \tau_p^{d,k} \middle/ \tau_e^{d,k}\right|) \tag{8.1}$$

Where $\tau_e^{d,k}$ is the actual experienced travel time for journey on day d and Ψ is the weight assigned to the updating factor. Similarly, the level of trust in the information will be unaffected if the percentage difference in the predicted and experienced travel time is between 15%-25%.

$$\alpha^{d+1,k}\left(\bar{\tau}_e^d, \tau_p^d\right) = \alpha^{d,k}\left(\bar{\tau}_e^d, \tau_p^d\right) \tag{8.2}$$

On the other hand, if commuters find a significant difference in the traveller information and their own experience, they will give lesser weight to the information in their future journeys. The threshold value for a negative impact on level of trust is assumed to be 25%. If the difference in commuter's experience and ATIS's information is greater than 25%, there will be a decline in the commuter's level of trust in the information.

$$\alpha^{d+1,k}\left(\bar{\tau}_e^d, \tau_p^d\right) = \alpha^{d,k}\left(\bar{\tau}_e^d, \tau_p^d\right) - \Psi(0.75 - \left|\bar{\tau}_e^d - \tau_p^d \middle/ \bar{\tau}_e^d\right|) \tag{8.3}$$

The threshold values assumed for the model described in equations (6.2-6.4) are assumed based on the findings of Kantowitz *et al.* (1997), with modified threshold values. This model can be further improved by conducting surveys to obtain empirical threshold values to model the expectations of commuters from the ATIS.

8.2.1 Methodology

The methodology described in section 7.3 for day-to-day modelling of route choice and traffic flows based on real-time traffic state estimation is extended to model the commuters' level of trust in traveller information. Figure 8.1 shows modified day-to-day component of the research framework described in chapter 7. The within-day component remains same, as shown in figure 7.1. A commuter compares his experienced travel time for a journey on day *d* with the travel time communicated through ATIS to assess the accuracy of information provided. Based on the accuracy of the information, the commuter updates the weight given to the information, which is applied to the journey on day *d+1.*
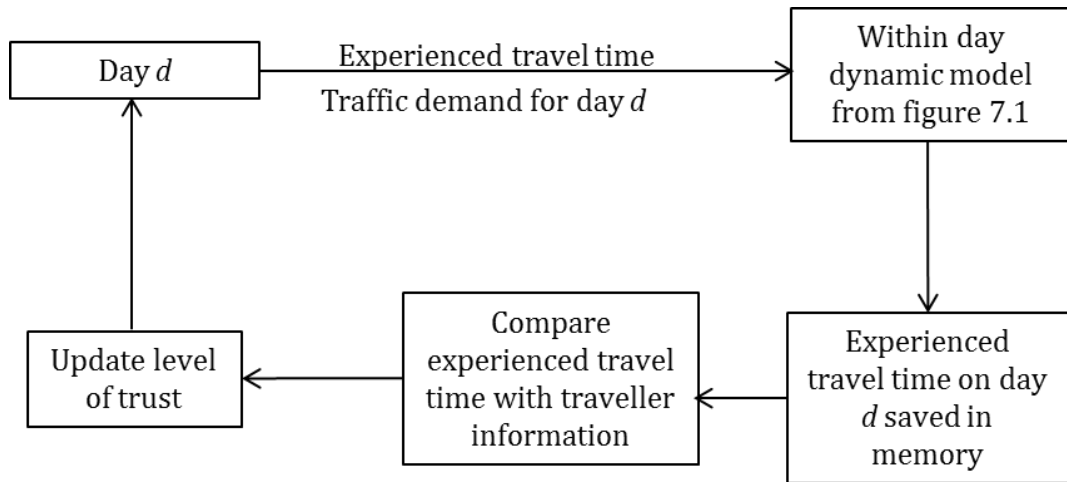
Figure 8.1 Day-to-day modelling component in modelling commuters' level of trust in the information

8.2.2 Simulation scenario

The proposed model for modelling commuters' level of trust in ATIS and weight given to the information is applied to the test network shown in figure 7.3. All the simulation conditions are similar to the conditions defined in section 7.4 for modelling day-to-day route choices and traffic flows. The traffic network is simulated for 100 days with time-varying traffic demand during the peak-hours. The variation in traffic demand causes to change their experienced travel time on the preferred route. In this application, the VMS which displays the information is assumed to be 1 km upstream of the diverging intersection. The distance between the VMS and the diverging intersection is not modelled to induce some error in the information, so that all three conditions defined in the model (equations 8.1-8.3) can be achieved.

8.2.3 Simulation results

The simulation is started with a lower bound of the parameter value, assuming that the information system is recently installed and commuters asses the accuracy of the information with time and the weight to the information becomes a dynamic parameter and a function of the accuracy of the information. The commuters' level of trust is modelled for each departure time-step for all the days in simulation horizon. To highlight within day variations in level of trust in ATIS, three representative days are discussed in the following figures.
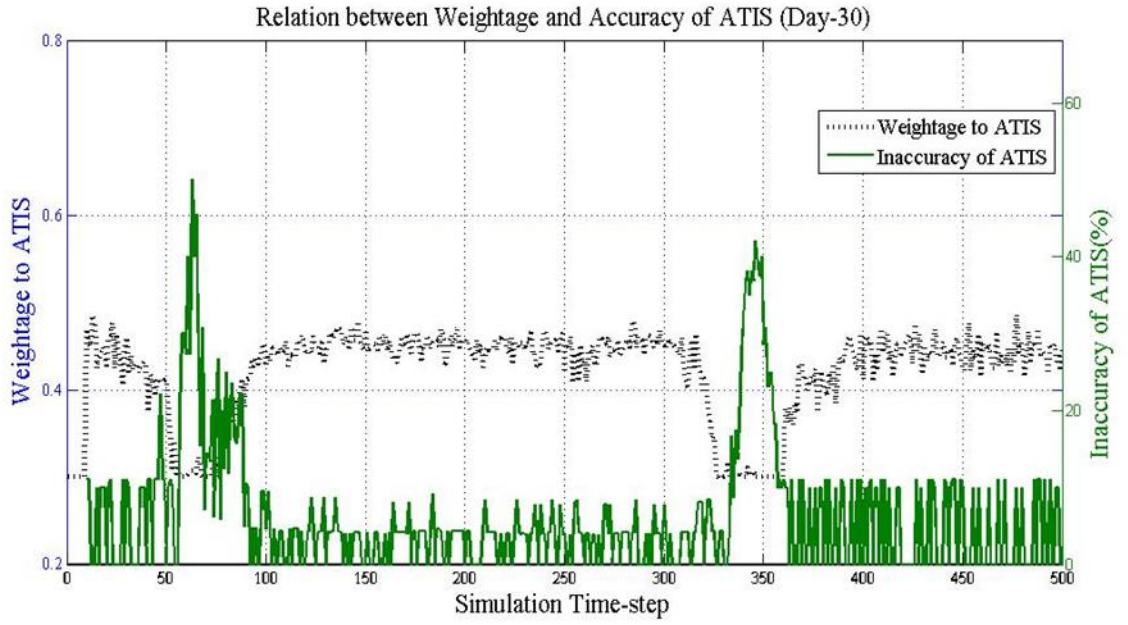
Figure 8.2 Level of trust and accuracy of traveller information on day-30
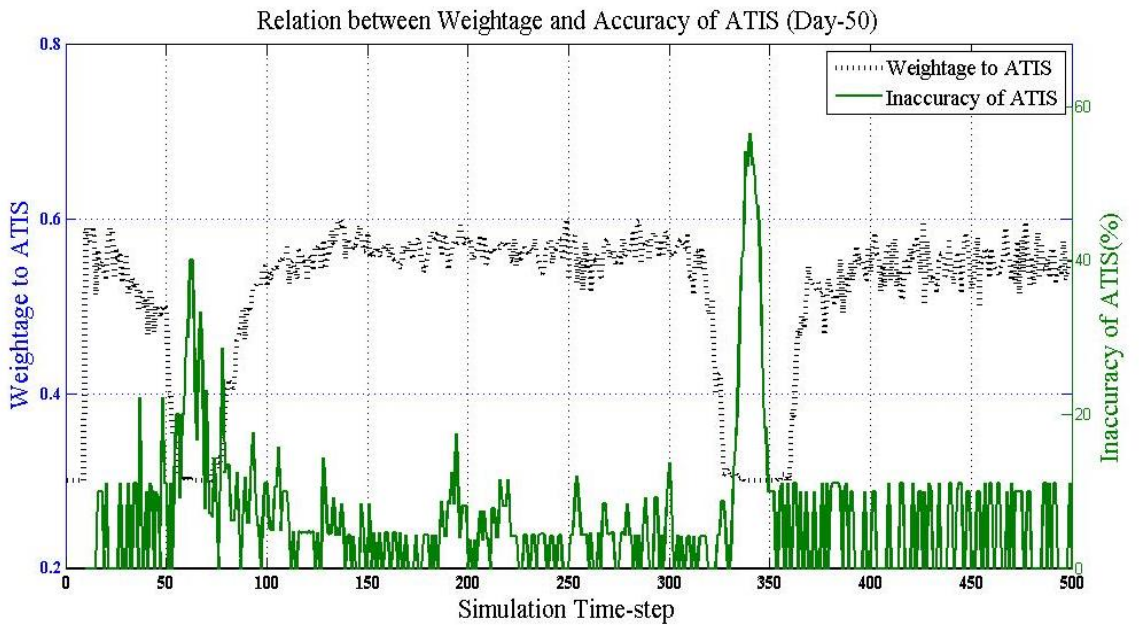
Figure 8.3 Level of trust and accuracy of traveller information on day-50
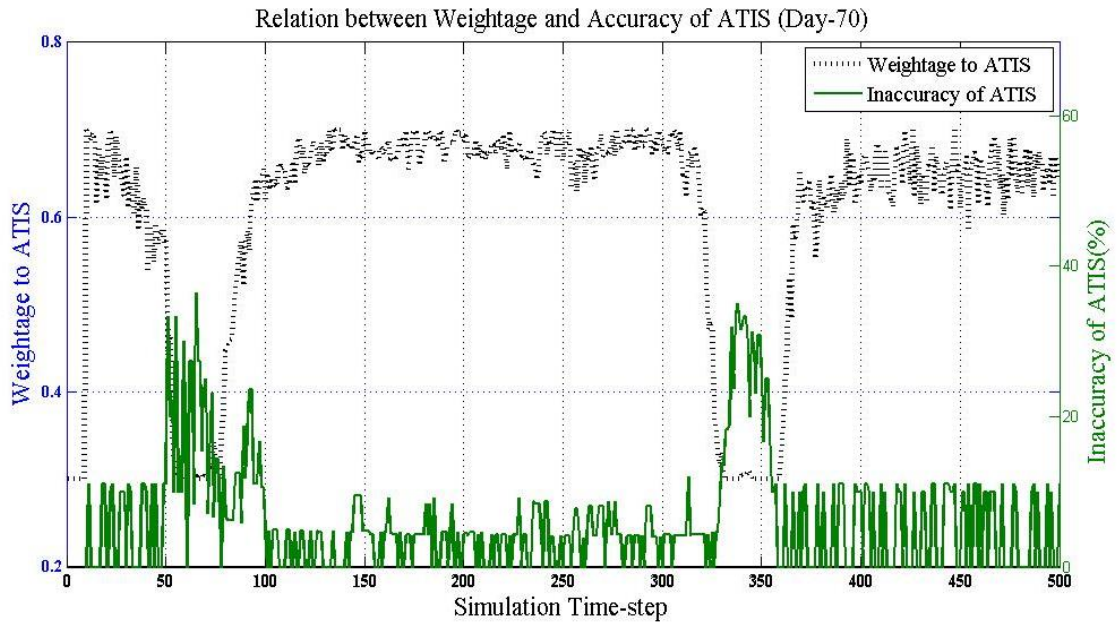
Figure 8.4 Level of trust and accuracy of traveller information on day-50

The within day variation in dynamic level of trust in information is highlighted using three representative days in figures 8.2-8.4. Figure 8.1 shows the relation between weight given to the information and the percentage inaccuracy of the information provided to the commuters on day-30. It can be observed from figure 8.2 that the information provided between time-step 60 and time-step 100 was not so accurate and the inaccuracy level was observed up to 50%. This caused the weight to the information declining to the lower bound, assumed as 0.3. During this time interval, the traffic state in link-2 changes from free-flow to congested traffic flow. The inaccuracy in prediction of traveller information is caused due to the fact that the variable message sign is assumed to be installed at a distance of 1 km from the diverging intersection. The travel times estimated to traverse link-2 at link entry time-step k are communicated to the commuters. The commuters entering link-2 at time-step $k+n$ experience a different travel time due to change in traffic condition during n number of time-steps. The number of time-steps taken to travel from the location of variable message sign to enter the link-2 is defined as n. After time-step 100, the accuracy of traveller information increases which results in a higher weight to the traveller information by commuters departing after time-step 100. Figure 8.2 further reveals that the accuracy of traveller information again reduces from time-step 330 to time-step 360, as the condition of traffic flow changes

from congested to free-flow when peak-hour traffic demand reduces. The inaccuracy in traveller information impacts commuters' trust in the traveller information and the weight given to the information. Figure 8.3 shows within day dynamics of accuracy of traveller information and commuter's level of trust in the information on day-50. The trend observed from figure 8.2 for within day dynamics of day-30 is further validated from figure 8.3. It can be observed from figure 8.3 that the average value of weight given to the information during peak-hour traffic demand increases from 0.5 on day-30 to 0.57 on day-50. The level of trust during the period of time-step 50 to 100 and 330 to 360 remains at the lower bound with 0.3, due to the quality of information provided to the commuters. Figure 8.4 illustrates variation in quality of information and weight given to the information by commuters on day-70. Figure 8.4 shows that the trend observed on day-70 is similar to the trend observed on day-30 and day-50. The average value of weight given to the information during peak-hour traffic increased to 0.66 on day-70.
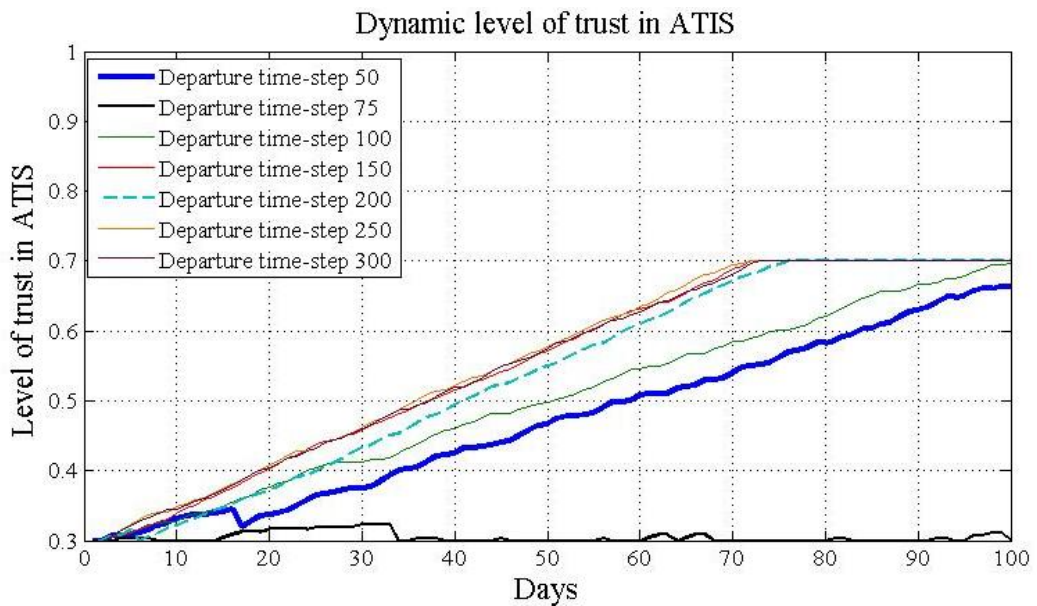
Figure 8.5 Day-to-day variations in weight to the information

The day-to-day variation in weight given to the information for selected departure time-steps is shown in figure 8.5. As observed from figures 8.2-8.4 that quality of information decreases between time-step 50 and 100, it can be observed from figure 8.5 that the weight given to the information for departure time-step 50 increases slowly compared to the commuters departing at other

time-steps. The quality of information is compromised when the traffic condition changes from free-flow to congested. This affects the commuters' level of trust in the traveller information. Day-to-day variation in the commuters level of trust for departure time-step 75 in figure 8.5 shows the trend that it remains at the lowest level of 0.3 during entire simulation horizon of 100 days. Similarly, the weight given to the information by commuters departing at time-step 100 increases with comparatively lower gradient than the commuters departing after time-step 100. However, commuters departing at other time-steps during peak-hour traffic demand show a similar behaviour in day-to-day adaption to traveller information due to the consistency in accuracy of the information, as evident from figures 8.2-8.4. In figure 8.5, the weight given to the information by commuters does not exceed 0.7, as this has been set as the upper bound for the weight given to the information.

The day-to-day modelling of level of trust in traveller information allows to model the behaviour of commuters, as they continuously asses the quality of information provided to improve their travel times. The results shown in this section suggest drop in quality of information, when the traffic state changes from free-flow to congested or vice-versa. This is caused due to not considering the time lag in covering the distance from the VMS location to the downstream link after diverging intersection. This distance is intentionally ignored to induce error in the information. This also highlights the importance of providing predicted travel times based on real-time traffic state estimation, as travel times change so rapidly that a lag of few time-steps in prediction of travel times can make the prediction significantly different when the traffic flow is changing from one condition to another.

8.3 Estimation of Logit model parameter

The knowledge about existing perception variation is critical to accurately model route choice behaviour and day-to-day dynamics of route choice. In this research, logit model is applied to model route choice behaviour of commuters. The logit model is described in section 4.3 and applied in chapter-6 and chapter-7 for modelling route choice behaviour. The logit parameter θ in logit model is very important in stability of the modelling process and obtaining

accurate outputs from the estimation model. In DTA applications for improving network performance, this parameter is also critical in implantation and evaluation of proposed changes/improvements in traffic network and control policies when the route choice is modelled using logit model. Therefore, this research proposes to estimate the value of θ, based on the observations obtained from the sensors and real-time estimated traffic state.

A smaller value of θ depicts a higher variation in perception of commuters about expected travel time, whereas a higher value of this parameter reflects smaller perception variation. In modelling traffic flow under ATIS, this parameter also reflects the quality of information provided to the commuters (Huang *et al.* 2008). When the information is perceived to be more accurate, commuters' perception variation decreases and commuters react in a more coherent way, whereas with the provision of less accurate information commuters' perception is more varied and commuters' response is more diverse. Furthermore, the uncertainty in expected travel time due to variation in traffic demand or capacity can also cause a higher perception variation among commuters. In this research, we propose to estimate this parameter based on the observations of turning movements from the sensors and traffic state estimated in real-time.

The difference between predicted and observed turning movements is evaluated and analysed. Based on the assumption that perception about expected travel time is modelled accurately, the difference in observed and predicted turning ratio is attributed to the logit parameter θ. The observed split-rates using measurements of traffic flow rates from the sensors at a given time-step are modelled backward in time to determine the time-step when the *en route* information was communicated. Based on the perceived travel times at that time and the split rates observed in result of the information, the logit parameter is estimated for each simulation time-step for a given day, after the simulation period for that day is completed. Upon completion of the simulation period for each day, the average of logit parameter values is determined to update it for the simulation of traffic flow and route choice for the following day. Using equation (7.3), the relation to determine logit-parameter based on observed split rates and traveller information is given as follows:

$$\hat{\theta}^{d,k} = \frac{\ln\{1-\tilde{\beta}_1(k)\}/\tilde{\beta}_1(k)}{\hat{\tau}_1^{d,k-\kappa}-\hat{\tau}_2^{d,k-\kappa}} \tag{8.4}$$

In equation (8.4), $\hat{\theta}^{d,k}$ is the estimated value of logit parameter for a given day d at time-step k. The average of these within-day estimated values is obtained to update the logit parameter for the simulation horizon of day $d+1$.

$$\hat{\theta}^{d+1} = \sum \hat{\theta}^{d,k}/k \tag{8.5}$$

In Equation (8.4), $\tilde{\beta}_1(k)$ is the ratio of traffic flows observed from the sensors installed along the links after a diverging intersection. In determining logit parameter for perception variation, it is assumed that the observations obtained from the sensors represent the observed split rate at the diverging intersection upstream of the sensors. This assumption will be more representative of the actual conditions if the distance of the intersection from the downstream sensor is smaller. The term $\hat{\tau}_1^{d,k-\kappa}$ in equation (8.5) represents the updated perception about expected travel time based on equation (7.2) for commuters selected route-1 (link-2) and an adjustment factor κ is estimated by determining the travel time of vehicles from the VMS to the measurement sensor. The updated perception modelled at k-κ time steps is utilized to determine the logit parameter. This estimation is carried out after a day of traffic simulation is completed and averaging of estimated values within a day minimizes the effect of any extreme value for the predicted logit parameter for upcoming day. Since, the estimation of this parameter is performed offline; it does not increase the computational demand for real-time traffic state estimation and DTA applications.

8.3.1 Methodology

For estimation of logit model parameter, the framework applied in chapter-7 for modelling day-to-day dynamics in route choice and traffic flows is extended. The within-day component, as described in section 7.3, remains unaltered. The day-to-day component for estimating logit model parameter for perception variation is shown in figure 8.6. The split-rates obtained using modelling of route choice based on logit model and pre-defined logit parameter value are compared with the split-rates obtained from the measurement sensors. The

observations of split-rates are obtained by using the measurements of traffic flows from the sensors located in alternative routes, as shown in figure 7.3. The measured split-rates along with the traveller information provided to the commuters are used to determine the logit parameters. The traffic flow is modelled backward in time to determine the time of the information, which resulted in the measured flow. The logit parameter is determined for all the time-steps of day d, and the mean value is determined to update the parameter for the next day in the modelling horizon.
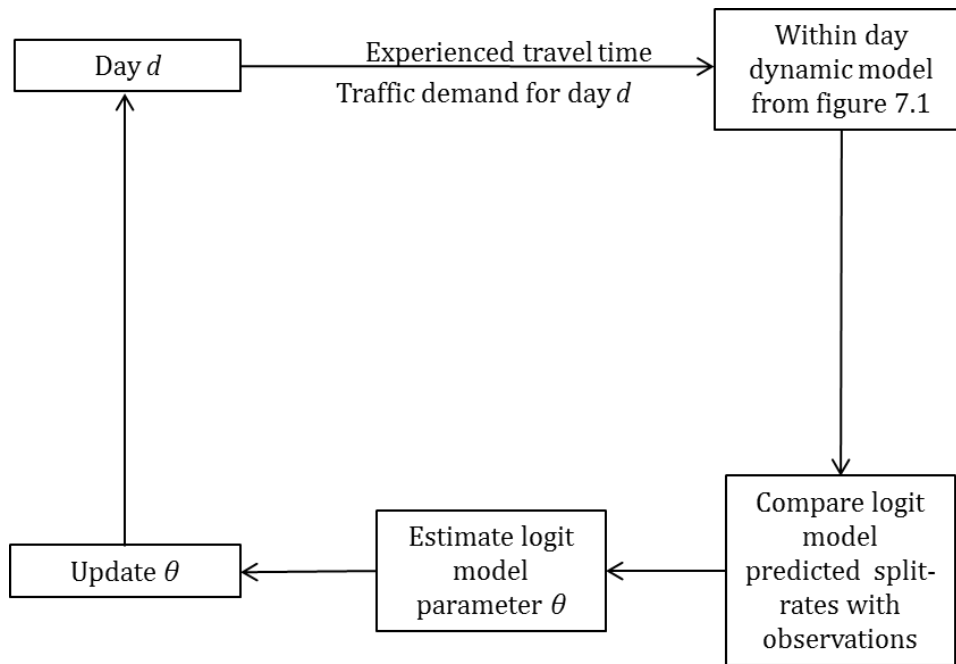
Figure 8.6 Day-to-day component of the model for logit parameter estimation

8.3.2 Simulation scenario

The logit parameter estimation model is applied to a similar scenario considered for day-to-day modelling of route choice and traffic flows in chapter-7. The network described in figure 7.3 is simulated for 100 days with time-varying traffic demand described in section 7.4.

8.3.3 Simulation results

In simulation experiment, the synthetic measurements are generated using a different value of logit parameter than the model used for prediction and estimation of traffic state. The synthetic measurements generated using logit parameter value as 0.25, reflect the existing perception variation among commuter. While the CTM-based prediction and estimation model is initialized

with a value of logit parameter as 0.1. The objective of logit parameter estimation is to estimate the prevailing perception variation using measurements from the sensor and traffic state estimation. Figure 8.7 shows day-to-day estimation of logit parameter based on the proposed estimation model.
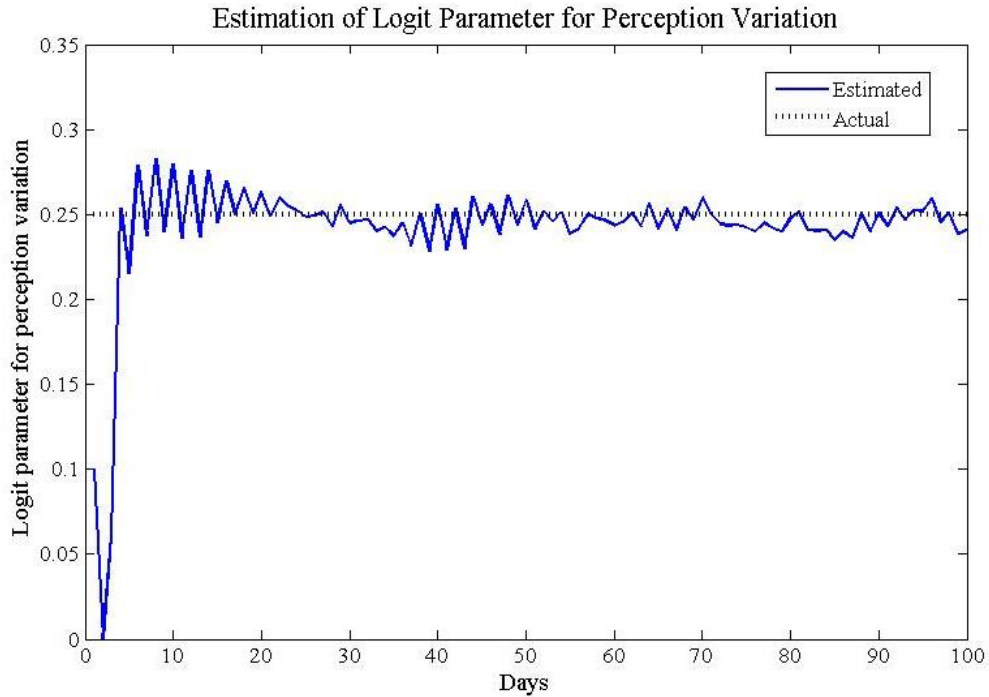
Figure 8.7 Estimation of logit parameter for perception variation

The proposed model for estimation of logit parameter was able to accurately estimate the perception variation using observations from the sensors and the traveller information provided to the commuters. The CTM-based prediction and estimation model was initialized with logit parameter as 0.1. The estimation model correctly estimated the logit parameter, after the warmup period of the simulation model on day-5. After day-5, the estimated value of logit parameter for perception variation remains very close to the actual value of the parameter. The proposed model for day-to-day estimation of the logit parameter can be more useful, when the commuters' behaviour also changes. The unexpected variation in traffic demand, network capacity, planned network improvement work, or changes in traffic control can cause uncertainty and variation in commuters' travel times. With these variations, the behaviour and

response of commuter might also change with time and their perception about expected travel time can become a dynamic parameter.

In this application of proposed model for logit parameter estimation, the model is applied not only to estimate the actual perception variation but also to determine and estimate day-to-day dynamics in perception variation. This scenario is based on the assumption that with the provision of accurate traveller information, commuters learn that traveller information is more accurate and their perception variation reduces with their day-to-day journeys. In this scenario, the synthetic measurements are generated using a dynamic logit parameter, with a small linear day-to-day increase in the parameter value. Figure 8.8 shows estimation of dynamic logit-parameter and compares the estimated values with the actual values, used for generating synthetic measurements. The proposed model for estimation of logit parameter accurately estimated the day-to-day variations in perception variation of commuters. Figure 8.8 shows that the estimated value of logit parameter is always very close to the actual value of the parameter. This also highlights the robustness of estimation model which estimated the accurate value of the logit parameter, despite a different initial value and day-to-day variation of the parameter.

The logit parameter is an important parameter in modelling route choice behaviour and day-to-day dynamics of route choice under fluctuating traffic conditions. The model proposed in this research can accurately estimate the perception variation of commuters, as well as any changes in the behaviour of commuters caused due to dynamics of travel times. The proposed model can also be applied to calibrate the perception variation of commuters using actual observations from previous days. Furthermore, the proposed framework can be extended to calibrate/estimate similar behaviour parameters in other random utility models.
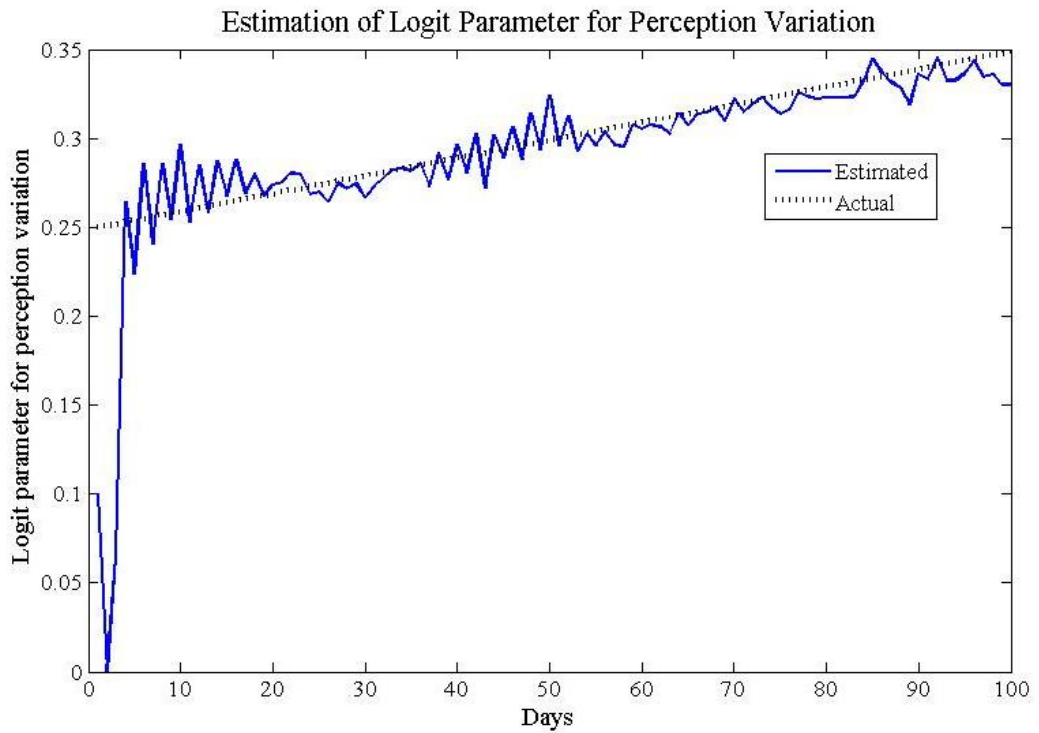
Figure 8.8 Estimation of logit parameter for perception variation

8.4 Summary

In this chapter, the framework of integrating real-time traffic state estimation and DTA is extended to estimate the parameters of DTA models based on measurements from traffic sensors and real-time traffic state estimation. The parameter of day-to-day traffic flow models which represent driver's behaviour in adapting travel choices are modelled and estimated using the proposed framework. The commuters' level of trust in the information is modelled as a dynamic parameter and as a function of the accuracy of the information provided. The logit parameter for perception variation in commuters is also estimated using the observations from the sensors and real-time traffic state estimation technique.

Chapter 9: Conclusions and recommendations

9.1 Summary

The review of existing literature in applications of dynamic traffic assignment models suggests the need of improvement in the network loading component of the models, as the existing applications of DTA models based on the prediction of traffic state using historic demand are unable to incorporate the effect of unexpected variation in traffic demand or network capacity.

Traffic flow model based traffic state estimation techniques provide an attractive alternative to the traffic flow models or cost-flow functions used in network loading of DTA models. The existing literature in traffic state estimation has not been utilized for improvement in network performance using DTA models. The existing real-time applications of DTA models for improving network performance use only measurements from traffic sensors. The model-based traffic state estimation provides a more accurate and reliable estimate of the existing traffic condition and offer promising improvement over the measurement only based estimate of actual traffic state. Thus, traffic state estimation can replace the existing estimates in DTA application for real-time traffic management from observations only to the model and observation based estimation.

The main contribution of the research presented in this thesis is to integrate two different fields of research, which are active and attract significant number of research studies. This research proposed to integrate the fields of real-time traffic state estimation and dynamic traffic assignment. The applications discussed in this thesis were based on the traveller information extracted using real-time estimated traffic state. The proposed framework for integrating real-time traffic state estimation and dynamic traffic assignment models is applied to selected applications, when there is uncertainty in prediction of traffic state due to unexpected variation in traffic demand or the network capacity.

A brief overview of traffic flow models is provided in chapter-2 of the thesis. Traffic flow models classified in macroscopic and microscopic traffic flow

models are briefly described in chapter-2 of the thesis. Macroscopic traffic flow models consider stream of traffic and model aggregate variables such as traffic density, traffic flow rate, and mean speed. The review of existing traffic flow models and their applications in different traffic systems suggests that first order traffic flow model, such as CTM can adequately model the flow of traffic for a traffic network. The number of output variables and parameters in CTM are significantly lower than the other higher order traffic flow models. This characteristic of CTM makes it feasible for real-time estimation of traffic state and its application for dynamic traffic assignment applications. Furthermore CTM has been applied in estimation of traffic state, as well as in applications of DTA models to optimize network performance. Therefore, CTM is selected to model network traffic flow and the model is elaborated in detail in chapter-2 of the thesis.

The overview of estimation algorithm is provided in chapter-3 of the thesis with overview of sensor technologies to measure traffic flow parameters. Most commonly used estimation algorithms including least square method, Kalman filter, extended Kalman filter, unscented Kalman filter and particle filter are described briefly in this chapter. Kalman filter provides an optimal estimate for state of a linear dynamic system, based on unreliable model prediction and noisy measurement data. Extended Kalman filter, unscented Kalman filter and particle filter are designed to estimate the state of nonlinear dynamic systems. The EKF is more feasible for real-time applications compared to other estimation techniques for nonlinear dynamic system, as it is computationally less expensive compared to other estimation techniques. The framework proposed by Wang and Papageorgiou (2005) is adapted to estimate traffic state using the cell transmission model. The parameters of fundamental traffic flow diagram are also estimated in real-time to capture any unexpected changes in variation of these parameters due to external factors such as extreme weather and traffic incidents. Chapter-3 also describes the estimation algorithm based on CTM and EKF.

The dynamic traffic assignments models are described in chapter-4. A classification of DTA models is discussed with brief overview of different aspects of DTA models. Chapter-5 further extends the review of DTA models

and highlights the existing gap and contribution of this research. The significance of accurate traveller information systems and their applications to improve commuters' journey times are discussed in chapter-5. Finally, the real-time traffic state estimation model based on CTM and EKF is extended to derive predicted traveller information, which includes the effect of any unexpected variation in traffic demand or network capacity.

The proposed framework for real-time traffic state estimation and predictive traveller information is applied to a within-day application of DTA in chapter-6. A small traffic network with one of its link affected with a traffic incident is simulated with the proposed framework. The parameter estimation of fundamental traffic flow diagram enables to identify and quantify any change in traffic flow capacity. The traffic state estimation using naïve prediction model with parameter estimation incorporates the impact of unexpected drop in traffic flow capacity during traffic incident. The predicted traveller information based on real-time traffic state estimation is communicated to the drivers to inform about prevailing travel times on the alternative routes in the affected traffic network. The commuters reroute themselves to avoid the congested route, which improves the network performance during the incident. The travel times and traffic densities for the proposed framework are compared with the scenario of no-information when commuters are unaware of the prevailing travel times on the downstream routes. The no-information scenario can also represent the scenario with traveller information based on historic values of travel times. The comparison of the scenarios indicates a significant improvement in traffic flow and travel time, not only on the affected link, but also for the upstream link. Furthermore, the underutilized capacity of the alternative link is better utilized by attracting commuters from affected alternative link.

The within-day application of the proposed framework is extended to day-to-day application in chapter-7 of the thesis. A network, similar to the one used in within-day application is simulated to model day-to-day variation in route choices and traffic flows with time varying traffic demand. The network is simulated for a period of 100 days, when traffic demand changes within day for all departure times as well as day-to-day for the same departure time. This time

varying traffic demand causes uncertainty in travel time and a commuter may experience a different travel time on a given day compared to the experienced travel time. This triggers commuters to give less weight to the travel times for journeys completed in previous days. The traveller information based on real-time traffic state estimation becomes more significant under such uncertainties. The simulation results obtained from implementation of the proposed framework are compared with the no-information scenario. The results are shown for selected days and selected departure times in chapter-7. The comparison of the two scenarios shows a significant improvement in travel time within a day for peak-hour traffic on a given day. Furthermore, it is also observed that the traveller information based on real-time traffic state estimation improves the stability of the network performance. The split-rates and travel times obtained using the proposed framework showed a consistent pattern, whereas in the other scenario route choices and travel times were instable and fluctuating.

The framework described in chapter-7 for modelling day-to-day route choice and traffic flows using real-time traffic state estimation is extended in chapter-8 to model and estimate parameters of DTA models. The parameters representing the behaviour of commuters in day-to-day learning behaviour and route choice are estimated based on the observations from traffic sensors and real-time estimated traffic state. The commuters' level of trust in the information is modelled as dynamic parameter and function of the accuracy of the information. It was concluded that the commuters' level of trust in the information increases as they find the predicted travel times more useful under uncertain traffic conditions. This research also models and estimates the logit parameters for perception variation in commuters. This parameter plays a significant role in modelling the route choice of commuters and simulation results obtained from the model.

9.2 Conclusions

Based on the findings of the research presented in this thesis, this section concludes the outcome of the thesis.

a) A framework to extract predictive traveller information based on real-time traffic state estimation is formulated in this thesis. The CTM to model network traffic is employed to predict traffic density for current time-step based on estimated traffic state for previous time-step. The current prediction of traffic density is corrected using measurement of traffic density in EKF. Based on the existing traffic state for a link, the link travel times are predicted that the last entering vehicle in the link at current time-step will experience. This improves the accuracy of the traveller information which ultimately results in more users complying with the information provided. In this research, the travel times for links are predicted and communicated to the commuters, while in predicting travel times the distance between the location of the VMS and the downstream link is not modelled. The inclusion of this distance in modelling predicted travel time can further improve the accuracy of the information. In reality, the travel time between the VMS to the destination should be modelled for ATIS.

b) The CTM-EKF traffic estimation model was formulated to model within-day dynamics and route choice behaviour during an incident. The drop in capacity was identified using real-time parameter estimation techniques. The utilization of real-time traffic state estimation for incident management provides an attractive alternate to existing real-time traffic management strategies based on measurements from traffic sensors, as the estimated traffic state is comparatively more reliable than the measurements from the sensors. The formulated framework has shown to significantly improve the performance of the network during the incident when compared with the scenario in which no traveller information is provided to the commuters. In the parameter estimation, only critical density is estimated, as inclusion of more parameters in the estimation scheme could lead to a nondeterministic system with the increase in number of unknown variables.

c) The within-day DTA framework based on CTM-EKF is extended to model day-to-day dynamics of route choice and traffic flows for a network with time varying traffic demand. The CTM-EKF model based predicted traveller information is provided to commuters to inform about

expected travel times on the alternative routes. The application of the formulated framework on a test network and its comparison with no-information scenario shows a significant improvement and stability in the network performance by employing the formulated methodology.

d) The use of real-time traffic observations and traffic state estimation in DTA models provides an opportunity to model, calibrate and estimate the parameters of DTA models to improve the accuracy of the modelling process. A model is formulated in chapter-8 to model commuters' level of trust in the traveller information as a dynamic parameter and function of accuracy of the information provided to the commuters. It was observed that with provision of accurate traveller information, the commuter gave more weight to ATIS in selecting their route. The logit model parameter for perception variation was accurately estimated by the application of the formulated framework.

9.3 Recommendations for future research

a) The proposed framework in this research can be extended to model large traffic systems. However, while extending the proposed framework for larger network, computational time might be a limiting factor as the traveller information or traffic state is updated with high temporal and spatial resolution in the proposed model. The spatial and temporal resolution of traffic state estimation and frequency of updating traveller information can be reduced to make it feasible for larger networks. A more aggregate macroscopic traffic flow model, such as Two-regime Transmission Model (TTM) by Balijepalli *et al.* (2013) can be applied for traffic state prediction to improve computation and modelling demand. Modelling of split-rates at intersections with multiple origin-destinations can be improved by pre-defining a subset of available routes that travellers can follow for each destination at any node. For a traffic network with multiple O-D flows, the CTM for multiple O-D flows can be used which distinguish traffic occupancy and flows based on origin and destination of traffic departed at each time-step by following Ukkusuri *et al.* (2012) or Carey *et al.* (2014) .

b) Another practical issue in implementing the proposed framework other than modelling large network using CTM is related to the information provided to the commuters and traffic management authorities can encounter these challenges. For example, the definition of destinations to which the travel time on a particular road is communicated to users could be an issue. There could be various routes leading to a destination from the location of a VMS and the consideration of communicated number of routes leading to the destination could be another implementation problem. The design of a VMS regarding the information provided can be significant and various designs of VMS can be considered while implanting the ATIS. In this research we have not addressed this issue explicitly. However, the details of implementation in real-world will depend on the nature of the problem, so a general solution is difficult to suggest and not covered in the scope of this research and can be addressed in an extension of this research.

c) In this research the objective of integrating traffic state estimation with DTA is achieved by providing predictive traveller information, which leads to stochastic user optimal traffic assignment. However, real-time traffic state estimation can also be integrated with user optimal and system optimal dynamic traffic assignment models. For example, an urban traffic network with signalized traffic controls can be optimized using real-time traffic state estimation to achieve a dynamic system optimal traffic assignment. This could be an expected improvement in existing real-time traffic management packages such as SCOOT and SCATS.

d) The CTM-EKF based framework formulated in this this research is suitable for measurement sensors collecting observations from fixed location such as inductive loop detectors. With the advancement in technologies for obtaining traffic measurements, in-vehicle devices are also being utilized in addition to in-situ traffic sensors to obtain additional information to improve the observation of traffic condition along the network. The vehicles equipped with Automatic Vehicle Location (AVL) devices provide information to the sensors installed at specific location in the network. The data acquisition from equipped vehicles includes GPS,

Bluetooth, Wi-Fi and cellular mobile data. Quiroga and Bullock (1999), D'Este *et al.* (1999), Li *et al.* (2002), and Herreraa *et al.* (2010) discussed obtaining data from GPS-based sensors for estimating travel times. The radio-frequency identification (RFID) transponders attached to the public transport vehicles which communicates to a central system by wireless data transmitted through antenna is also utilized in measuring congestion and estimating travel times (Mandal *et al.* 2011; Ban *et al.* 2009; Wright and Dahlgren 2001). . Puckett *et al.* (2010), Malinovskiy *et al.* (2010), Quayle *et al.* (2010) and many other researchers have discussed the significance and reliability issues related to observations obtained from Bluetooth devices. The data obtained from in-vehicle detection devices can also be integrated in real-time traffic management, especially when used with model based traffic state estimation.

e) The research framework proposed in this thesis should be used with actual observation from real network to assess the significance of the proposed framework. The day-to-day traffic flow models lack validation using actual observations. This research framework can be used with actual data to validate the day-to-day traffic modelling approaches.

f) The model suggested for commuters' dynamic level of trust in the traveller information can be improved by calibrating the threshold values for acceptable accuracy of the information and compliance of the commuters to the information. This can be achieved by conducting stated preference survey to obtain threshold values of accuracy of the information to be acceptable.

g) The logit parameter estimation algorithm should be used with actual observation from a real traffic network to assess the accuracy of the model. The model proposed to estimate logit parameter can be extended for estimation of parameters of other route choice models, such as probit model.

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Appendix A: List of publications and presentations originating from this resaerch

Journal Publication:

Ahmed, A., Ngoduy, D. & Watling, D. 2015. Prediction of traveller information and route choice based on real-time estimated traffic state. *Transportmetrica B: Transport Dynamics (Published online on 10 June 2015)*

http://www.tandfonline.com/doi/ref/10.1080/21680566.2015.1052110

Conference Proceeding:

Ahmed, A., Watling, D., & Ngoduy, D. 2014. Significance of Sensor Location in Real-time Traffic State Estimation. *Procedia Engineering*, *77*, 114-122.

Presentations at International Conferences and Workshops:

Ahmed, A., Watling, D., & Ngoduy, D. Significance of Sensor Location in Real-time Traffic State Estimation. *Fourth International Symposium on Infrastructure Engineering in Developing Countries,* 26-28 December 2013, Karachi, Pakistan.

Ahmed, A., Ngoduy, D. & Watling, D. Real-time urban traffic management based on estimated traffic state. *Universities Transport Study Group Conference,* 5-7 January 2014, Newcastle, United Kingdom.

Ahmed, A., Watling, D., & Ngoduy, D. Modelling day-to-day travellers' route choice using real-time estimated traffic state with time-varying traffic demand. *5th International Symposium on Dynamic Traffic Assignment,* 17-19 June 2014, Salerno, Italy.

Ahmed, A., Watling, D., & Ngoduy, D. Day-to-day variation in level of trust in advanced traveller information and the impact of improved methods for estimating traffic states. *3rd Symposium of the European Association for Research in Transportation,* 10-12 September 2014, Leeds, United Kingdom.

Ahmed, A., Watling, D., & Ngoduy, D. Modelling day-to-day travellers' route choice and level of trust in traveller information using real-time estimated traffic state. Croucher *Foundation ASI workshop: Toward a New Generation of Dynamic Network Models,* 18-20 November 2014, Hong Kong.