Essays in International Macroeconomics and Trade

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Dedication

To my parents Marysia and Włodek.

Abstract

Motivated by recent monetary expansion in the United States in the aftermath of the 2007-8 financial crisis, we use a New Keynesian three-country Center-Periphery model to define a game between policymakers in emerging economies linked to some large industrial economy via the exchange rate. We derive welfare-based payoffs to study the policy implications of monetary expansion in the US. We highlight cases in which policy coordination between emerging economies may improve their welfare. We identify cases in which countries may prefer to manipulate the exchange rate and resist currency appreciation.

We then propose a framework based on results from a Global Vector Autoregressive Model. This approach allows us to treat the emerging economies according to the observed data. We use US, Chinese and Brazilian data on consumption, output and money supply between 1994 and 2014. We perform counterfactual analysis allowing to determine the welfare outcomes of joint monetary expansions: in the US and China, in the US and Brazil, and in all three countries.

Our results are relevant for the ongoing discussion on the benefits of policy coordination and the spillover effects of monetary policy of a large industrial country on emerging economies and suggest that this framework can be used as a tool to coordinate countries on welfare-superior outcomes relative to Nash equilibrium.

In the third essay we consider an N-country two-good Cobb-Douglas model with country specific preferences and arbitrary endowments. This allows us to look at the interactions between asymmetric countries. We provide a general existence result which we then apply to our specific endowment economy, and provide conditions on the primitives of the model that ensure the existence of a pure strategy Nash equilibrium. We show that Nash equilibria with prohibitive tariffs cannot arise.

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Declaration

I declare that the work in this thesis is based on research carried out in the Department of Economics at the University of York. No part of this thesis has been submitted elsewhere for any other degree or qualification. Chapter 3 is co-written with my supervisor, Professor Subir Chattopadhyay, with a journal article based on this work forthcoming. Remaining chapters are my own work unless referenced in the text.

Introduction

This thesis is a collection of three essays about strategic interactions between policymakers in interdependent economies and is divided in two parts. Part one consists of two essays describing the effects of expansionary monetary policy of an advanced economy on the welfare of emerging economies that are its trading partners, while Part two consists of an essay about international trade in the presence of tariffs.

Existing literature typically analyzes the optimal monetary policy and monetary policy transmission mechanism between countries either in a small open economy (see Gali and Monacelli (2005)) or a two-country framework but without allowing the other country to be active (see Obstfeld and Rogoff (1995), Corsetti and Pesenti (2001), Tille (2001). Among existing examples of strategic interactions in the context of monetary policy is Canzoneri and Henderson (1992); however, their model is not appropriate for welfare analysis. Among multi-county models analysing the effect of various shocks on other countries are papers using a GVAR model (Pesaran and di Mauro (2013), Dees et al. (2007)); however, these papers do not perform the welfare analysis of such shocks and do not explicitly allow countries to behave strategically.

Given this gap in the literature, we aim to provide a framework that allows for a welfare analysis in which policymakers behave strategically in complex situations similar to those taking place recently in the global economy. We overcome the complexity by imposing game-theoretic structure in which we restrict the strategy space in a way that allows us to look at the welfare outcomes of interactions between the emerging economies.

The main innovation is in applying the game-theoretic approach that facilitates the analysis of interactions between more than two economies and their welfare consequences in various scenarios in which the emerging economies are allowed to react strategically. We explore welfare measures using two alternative approaches. In Chapter 1 we use the building blocks of a New Keynesian model developed in Corsetti et al. (2000). In Chapter 2 we make use of real world data and apply a Global Vector Autoregressive model. Both Chapters 1 and 2 have been motivated by the recent discussions between policymakers, especially in Brazil and the US, in the aftermath of the financial crisis of 2007-08 and the recent non-conventional monetary policy of the US, namely three rounds of quantitative easing (see for example Rousseff (2012), Bernanke (2013)). The ongoing debate about potential gains from coordination of international policies (see for example Ostry and Ghosh (2013), Obstfeld (2011)) or the lack thereof has motivated us to investigate this problem in the context of the policy of the emerging economies in response to monetary expansion in a large advanced economy.

One of the main contributions in Chapter 1 is defining the sufficient conditions in terms of elasticity of substitution parameters and relative size for which the resulting Nash equilibria are not optimal, when each country has only two strategies. In such cases, the so called Prisoner's Dilemma, we identify rationale for policy coordination and suggest that our framework can be used by an institution to coordinate the economies towards welfare-superior outcomes. Our sufficient conditions tell us that in the presence of large asymmetries between the emerging economies countries would not gain from policy coordination.

Since the starting motivation arose from real economic events we further extend the study using macro-level data in order to see if the empirical predictions are consistent with the theory in Chapter 1. Therefore the approach in Chapters 1 and 2 are strongly connected and the design of the welfare measure in Chapter 2 follows from Chapter 1. In both the theoretical and empirical approaches we analyze different scenarios and generate policy recommendations. As a result of the empirical analysis, we conclude that there is no need for policy coordination between Brazil and China. In each case the resulting Nash equilibrium is welfare superior to the available alternatives - there is no Prisoner's Dilemma. Chapter 2 uncovers further interesting finding: both the welfare consequences of the expansionary monetary policy in the US, as well policy recommendations for the emerging economies, vary over time. We think that the main reason for this change might reflect the way the monetary policy transmission has been affected in connection with the unconventional monetary policy of quantitative easing in the US.

Part two is joint work with Professor Subir Chattopadhyay. In this chapter, similarly to Chapters 1 and 2, we are interested in strategic interactions between policymakers in more than two economies, and work with a model that allows us to potentially analyze the role of asymmetries between countries. Although this was our initial motivation, we have identified a gap in the existing literature, namely very limited results about the conditions that ensure the existence of a pure strategy Nash equilibrium in a model like ours. Therefore we investigate the existence of a pure strategy Nash equilibrium in an N-country two-good Cobb-Douglas model with country specific preferences and arbitrary endowments. This existence result forms an essential basis for more in depth analysis of the outcomes of strategic interactions between policymakers in the presence of size and preference asymmetries. We provide conditions on the primitives of the model that ensure the existence of a pure strategy Nash equilibrium. In addition we show that Nash equilibria with prohibitive tariffs cannot arise when these conditions are satisfied. We believe that having established those results is an important step in the analysis of the outcomes of tariff wars, both for theoretical as well as numerical approaches.

Chapter 1

Strategic interactions in emerging economies and their welfare consequences: A theoretical approach

1.1 Introduction

Spillovers from domestic policies have been recognized for a long time in the macroeconomic literature. In the context of recent US expansionary monetary policy in the aftermath of the 2007-8 financial crisis (see Figure 1.1) policymakers in many, both industrial as well as emerging economies are aware of the existence of such spillovers (see Powell (2013), Sanchez (2013)). Although recognition interconnectedness is widespread, there is no consensus about the effect of such expansionary monetary policy on other countries. Often there are contradicting opinions between policymakers. On the one hand, the Governor of the Fed Powell (2013) and Fed's Chairman Bernanke (2013) consider the US expansionary monetary policy to be "enrich-thy-neighbor". On the other hand, the President of Brazil claims that such policy has a harmful effect on emerging economies and contributes to deepening of the recession (Rousseff, 2012).

These recent discussions concentrate around the role and influence of policies originating in a large industrial country such as the US on the world economy, and emerging economies in particular. However, little attention has been devoted to how this effect depends on the behaviour of individual countries (U.S. Department (of the Treasury, 2011)). This is an important strategic aspect which we want to highlight and show how

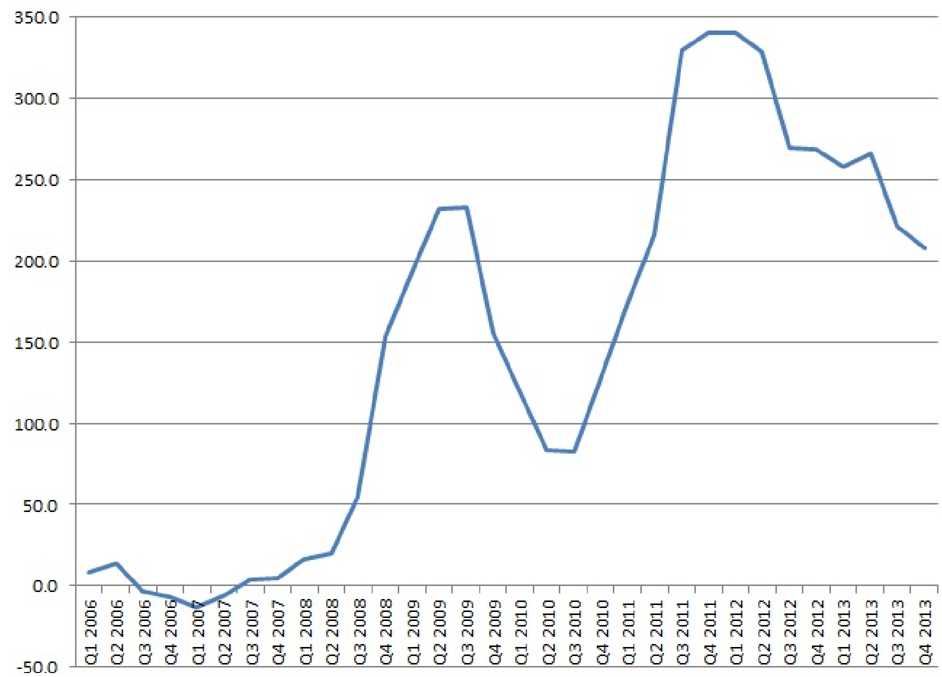

Figure 1.1: US Money supply M1 between 2006 and 2013, Quarterly, Seasonally Adjusted, Change from year ago, Billions of Dollars, Source: FRED of St. Louis (2014)

countries' welfare depends on individual actions of emerging economies. Not only do the reactions of these countries, i.e. policy decisions in response to monetary expansion in the US matter for their welfare, but also other factors such as their relative size and elasticity of substitution between goods. In this sense it becomes apparent that the same policy of the US may affect each country differently, what can be attributed to the existing asymmetries between countries. Moreover, due to these asymmetries, some of the emerging economies may play an important role in transmitting or even generating spillovers to other emerging economies. For example, Mattoo et al. (2012) have shown the impact of Chinese real exchange appreciation on boosting exports in other countries with which China competes on third markets.

With these discussions in mind, we will look at the behaviour of a large emerging economy such as China, generating spillovers to another smaller country[1], such as Brazil, that is potentially negatively affected by Chinese and US spillovers. Both countries responded to US monetary expansion with an increase in money supply (see Figure 1.2), yet the changes in their exchange rate vis-a-vis US were not the same. Complaints by Brazil's President may imply that Brazil, in allowing its currency to appreciate (see Figure 1.3), had a different experience than China, who resisted appreciation and

[1]in terms of size of trade, see Table 2.6.

initially maintained a fixed exchange rate with the US.

With the help of a game-theoretic framework we would like to understand and explain these actions and their implications on welfare of these emerging economies. We would also like to see whether alternative actions could yield welfare improvement for either or both of the countries. Using the concept of Nash equilibrium will allow us to determine the optimal response of each country.

Despite many approaches to the analysis of transmission of shocks abroad, and their influence on interconnected economies, the theoretical literature has mainly focused on two countries. Due to the important role of the strategic aspect of interactions and their influence on countries' welfare we choose to work with more than a two-country model. Among few existing three-country models is Corsetti et al. (2000). However they do not design a game, nor look at Nash equilibria. Another is Canzoneri and Henderson (1992), who however do not have the ingredients of a New Keynesian model, and they focus on coalition formation. Since countries can respond to the policy of the Center, we find it vital to include strategic interactions between policymakers in the model. We use the Corsetti et al. (2000) model and make use of their building blocks to construct a non-cooperative game[2], which in turn allows us to answer the questions of interest in a structured way. Such game-theoretic analysis has been long called for in the literature, for example by Corsetti and Pesenti (1997), who recognized that it is of interest to analyze the interactions of policymakers using game theory[3].

The study of strategic interactions between policy makers in interdependent economies using monetary policy as their instrument through a game-theoretic approach is not a new topic in the literature. This research has been pioneered by Hamada (1985). International monetary interdependence using game theory was further analyzed by Canzoneri and Henderson (1992). However, at that time microeconomic foundations of macroeconomic models were not available and thus welfare analysis using these models was not possible. The development of the New Keynesian models with nominal rigidities and monopolistic competition allows us to analyze the welfare effects of monetary policy. This chapter aims to analyze such consequences from a strategic game-

[2]We construct a game motivated by Corsetti et al. (2000) who wrote: "Yet, the next crucial step in the analysis will consist of adopting a game-theoretical approach, so as to focus more directly on strategic interactions and non-cooperative equilibria across countries. The structure of international spillovers and the micro-foundations of the mechanism of policy transmission discussed in this paper are meant to provide the building blocks for a development of the analysis in such direction." [p. 239, Corsetti et al. (2000)]

[3]"(...) we leave to further contributions the detailed game-theoretic analysis of international strategic interactions (...)" [p.26, Corsetti and Pesenti (1997)]

theoretic perspective using a welfare-based approach. Among existing games between two countries are papers by Benigno (2002), who compares competitive with monopolistic allocations, and Benigno and Benigno (2008), who look at cooperation between policymakers. However, these papers look at continuous optimal policy and its implementation in a two-country framework. Such analysis becomes extremely difficult as soon as we extend the model to three countries. This is why we will only look at discrete choices in the strategy space of policymakers in the Periphery who can choose between keeping their money supply fixed or fixing their exchange rate. In the strategies in which the Periphery countries are assumed to keep their exchange rate fixed relative to the Center, the monetary policy rule is an 'exchange-rate rule'. In such case the country's money supply has to be determined passively, by whatever is necessary to keep the exchange rate fixed. In the cases in which the Periphery countries allow their currencies to fluctuate, the monetary policy can be conducted using money supply or via following an interest-rate rule. Since the exchange rate can easily be expressed as a function of money supply in this model we choose to conduct monetary policy via the money-supply rule. Alternatively, policymakers could use interest-rate rule, which we believe will qualitatively yield similar results. However, due to the fact that the interest-rate rules involve various forms of feedback of the interest rate on inflation, output, etc., the results of the analysis are likely to be sensitive to the feedback parameters and to the choice of feedback variables. Even though we use this simplification, i.e. a discrete strategy space, the model still remains complicated since we allow for different elasticities of substitution between goods produced at home and between goods produced abroad. This approach has been introduced by Tille (2001), who looked at the effect of a monetary shock in an open economy, an extension to standard analysis assuming equal elasticities like in Obstfeld and Rogoff (1995). Our results are in terms of relative size of within- and between-country substitutabilities and relative size of countries, which allows us to investigate the role of asymmetries in welfare changes.

Our main contribution is the construction of a game-theoretic framework for the analysis of strategic interactions, shedding light on recent monetary policy developments in the United States and their effect on emerging economies, such as Brazil and China. Our framework is constructed based on payoffs derived from the building blocks of the Corsetti et al. (2000) model. In the context of recent discussions about gains from coordination and how to achieve them, we show that our framework can be used to identify the existence of such gains and possibly used as a tool for supranational authorities to coordinate countries on welfare-improving outcomes. We identify situa-

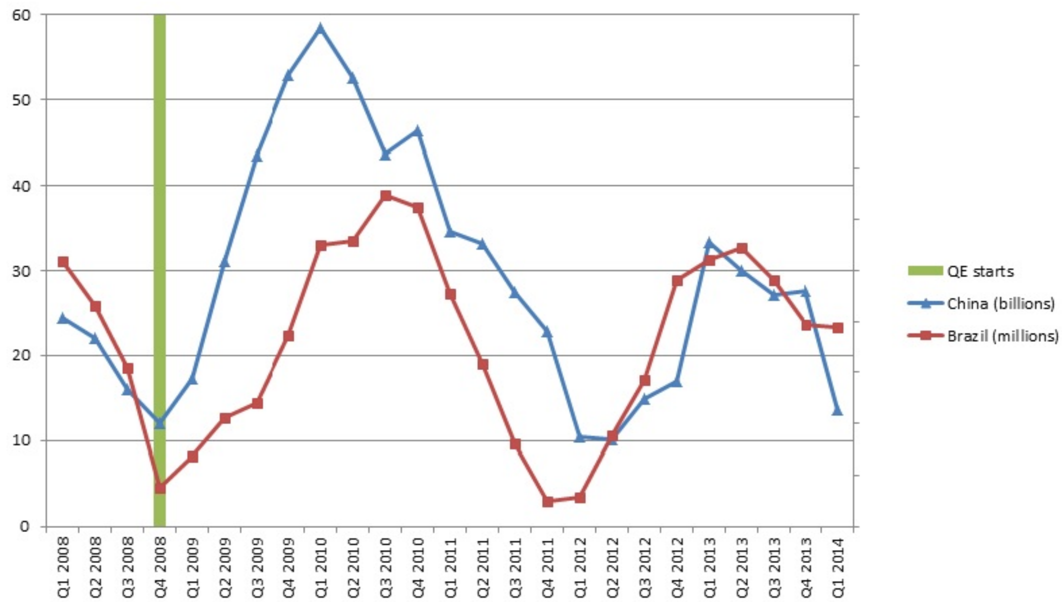

Figure 1.2: Brazil and China Money supply M1 between 2008 and 2013, Quarterly, Not Seasonally Adjusted, Change from year ago, Source: FRED (IMF) of St. Louis (2014)

tions in which self-oriented policy may lead countries to get stuck in an equilibrium with lower payoffs, the so called Prisoner's Dilemma, and show when such inferior equilibria will not arise. Need for coordination has recently received more attention, especially after the Great Moderation, due to the amplification mechanism and spillovers from uncoordinated responses of policymakers in different countries, see Taylor (2013). We believe that our findings can be used to explain why self-oriented policies may not be optimal from the point of view of global welfare, and justify coordination[4], which has been discussed in the literature for a long period of time with little support (Oudiz and Sachs (1984), Taylor (1985), Taylor (1993), Obstfeld and Rogoff (2002), Rogoff (1985), and Carraro and Giavazzi (1988)).

Rather than providing a mechanism allowing countries not to end up in Pareto-inefficient outcomes, we aim to provide a tool for policymakers and possibly for a neutral assessor, that would help in explaining the benefits of coordinated actions. "Coordination in this sense, does not require policymakers to act against their national interests, but rather to recognize that alternative policy packages - when pursued by all parties - can allow each to improve national welfare" (Ostry and Ghosh (2013), p. 5-7). Although the equilibrium with coordination improves welfare of both countries, because

[4]Typically what is meant by 'coordination' in this literature is in terms of game theory considered as 'cooperation'. We will follow the literature and point out at a later stage when the term 'coordination' is used more formally.

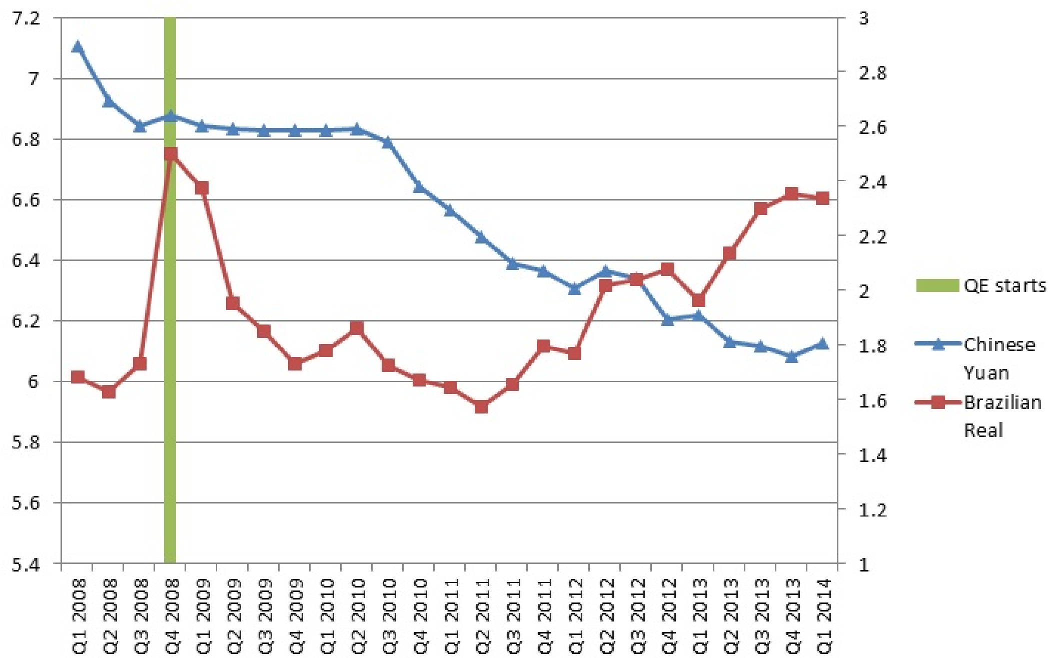

Figure 1.3: Exchange Rates of the Brazilian Real (secondary-right axis) and Chinese Yuan (primary-left axis) to the US Dollar between 2008 and 2014, Source: Banco Central do Brasil and Bank of China (via DataStream)

of the incentive to deviate, it is not a stable outcome. This is why without commitment or international sanctions, coordination is so difficult to achieve. A possible solution to that commitment problem is to allow for the possibility of side payments between all countries, which would require a change in the design of the current game.

Our main result is to show that in the absence of large size asymmetries emerging economies can benefit from coordination. This allows one to understand the common view in the ongoing debate that coordination may not always be desirable. We suggest two possible explanations why China resisted appreciation. In one of them we show that as a large emerging economy it has a dominant strategy to peg, and in the other we justify it with the presence of uncertainty about the model, a situation in which a peg yields always positive payoffs. We show that the US recommendation for emerging economies to allow their currencies to appreciate is optimal, only if these emerging economies are small and under a particular assumption on the substitutability between goods. The reasons why Brazil allowed its currency to appreciate are harder to explain. Maybe Brazil would have preferred to peg, which is an optimal strategy under certain circumstances, but the size of its monetary expansion was not large enough? After all Brazil has a flexible exchange rate and it might not be in its interest to manipulate it.

Perhaps Brazil hoped that China would do the same and then the welfare effect of US monetary expansion would not be so negative as Brazil experienced it. With the help of our model we are able to explain why Brazil had reasons to complain, and justify it with the possible spillovers generated by the policy of China.

In the next section we recall the model from Corsetti et al. (2000), and show how we derive the payoffs to construct a game between policymakers. In Section 1.3 we present our results. In Section 1.4 we conclude.

1.2 The Model

The building blocks of the model are adopted from Corsetti et al. (2000). The model is a three-country version of Obstfeld and Rogoff (1995) and Corsetti and Pesenti (2001), allowing for differences in substitutability between goods produced in different countries. Each country specializes in production of one set of goods. Producers operate in monopolistically competitive markets producing a continuum of brands and face downward sloping demand curves. The source of nominal rigidities is preset prices, that cannot be adjusted after the shocks occur in the short run. All producers must keep their prices unchanged and can adjust them in the following period, in which the long run equilibrium is reached. The only adjustment that takes place in the short run is increase in production (employment). The labour market is not modelled explicitly in this model.

Households are defined over a continuum of unit mass with the respective population's shares of the Periphery, denoted by $\gamma_P \in [0,1]$ and the Center $\gamma_C \in [0,1]$ in the world economy. The share of country A in the Periphery's populations is denoted by γ_A, where $\gamma_A \in [0,1]$, therefore the size of country A is simply $\gamma_A\gamma_P$. Similarly the size of country B is $(1-\gamma_A)\gamma_P$, since its share in the population of the Periphery is $(1-\gamma_A)$ (See Figure 1.4).

The representative household in country j, where j = A, B, C, at time t (as in Corsetti et al. (2000), p. 221) derives utility from consumption, money balances and disutility from work effort:

$$U_t^j = \sum_{s=0}^{\infty} \beta^s \left[lnC_{t+s}^j + \chi ln\left(\frac{M_{t+s}^j}{P_{t+s}^j}\right) - \frac{\kappa}{2}(Y_{t+s}^j)^2 \right] \tag{1.1}$$

where κ and χ are positive constants and β is the discount rate between 0 and 1. Household wants to maximize utility, subject to the budget constraint of the form:

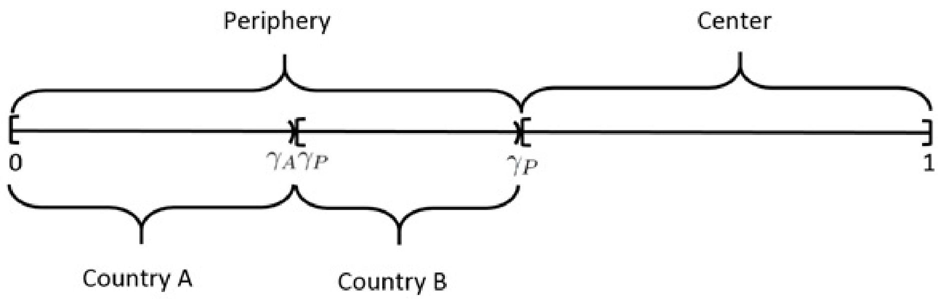

Figure 1.4: Population size in each country

$$\frac{E_t^j B_{t+1}^j}{P_t^j} + \frac{M_t^j}{P_t^j} + C_t^j = (1+i_t)\frac{E_t^j B_t^j}{P_t^j} + \frac{M_{t-1}^j}{P_t^j} + \frac{SR_t^j}{P_t^j} - \frac{T_t^j}{P_t^j} \tag{1.2}$$

where E^j is the nominal exchange rate, defined as country j's currency per unit of Center currency (i.e. $E^C = 1$), i_t is the nominal yield on the bond B_t in terms of the Center's currency; SR^j is agent's sales revenue in nominal terms, and T^j is a lump sum tax denominated in country j's currency. Each unit of household's labor, L_t produces one unit of output Y_t, so the production function is given by $Y_t = L_t$ (which linearized around the symmetric steady-state can be expressed as $y_t = l_t$).

The consumption of a representative household from country j in this model consists of goods produced in the Periphery and the Center:

$$C^j = \left[\gamma_P^{\frac{1}{\rho}}\left(C_P^j\right)^{\frac{\rho-1}{\rho}} + (1-\gamma_P)^{\frac{1}{\rho}}\left(C_C^j\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} \tag{1.3}$$

where ρ is the elasticity of substitution between the types of goods produced in the Center and the Periphery, and C_P^j is a function of goods produced in country A and country B defined as:

$$C_P^j = \left[\gamma_A^{\frac{1}{\psi}}\left(C_A^j\right)^{\frac{\psi-1}{\psi}} + (1-\gamma_A)^{\frac{1}{\psi}}\left(C_B^j\right)^{\frac{\psi-1}{\psi}}\right]^{\frac{\psi}{\psi-1}} \tag{1.4}$$

where ψ the elasticity of substitution between the types of goods produced in country A and B. In each country there is a continuum of varieties, and the consumption subindexes are given by the constant elasticity of substitution (CES) functions:

$$C_A^j = \left[(\gamma_A\gamma_P)^{-\frac{1}{\theta}}\int_0^{\gamma_A\gamma_P}(C_A^j(z))^{\frac{\theta-1}{\theta}}dz\right]^{\frac{\theta}{\theta-1}} \tag{1.5}$$

$$C_B^j = \left[((1-\gamma_A)\gamma_P)^{-\frac{1}{\theta}} \int_{\gamma_A\gamma_P}^{\gamma_P} (C_B^j(z))^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}} \tag{1.6}$$

$$C_C^j = \left[(1-\gamma_P)^{-\frac{1}{\theta}} \int_{\gamma_P}^{1} (C_C^j(z))^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}} \tag{1.7}$$

where θ is the the elasticity of substitution between the types of goods within one country (and $\theta > 1$).

Corsetti et al. (2000) assume that each country specializes in production of one type of good, which corresponds to the following restriction: $\rho \leq \psi \leq \theta$. We relax this parameter restriction to identify cases in which the expansionary monetary policy of the Center can be *beggar-thy-neighbour* for the Periphery countries, as well as a situation of *Prisoner's Dilemma* when both Periphery countries keep their exchange rates fixed.

From intertemporal optimization we obtain the following first order conditions:

- the Euler equation, which informs us about the way consumers smooth consumption over time:

$$\frac{C_{t+1}^j}{C_t^j} = \beta(1+i_{t+1})\frac{P_t^C}{P_{t+1}^C} \tag{1.8}$$

  where P_t^C is the price of goods in the currency of the Center, i.e. $P_t^C = \frac{P_t^j}{E_t^j}$.

- money demand equation, which embodies the uncovered interest parity condition, and shows that households are indifferent between consuming a unit of consumption at date t, or using the funds for real money balances, and converting them back to consumption in period $t+1$:

$$\frac{M_t^j}{P_t^j} = \chi C_t^j \frac{(1+i_{t+1})E_{t+1}^j}{(1+i_{t+1})E_{t+1}^j - E_t^j} \tag{1.9}$$

- and pricing rule in flexible price equilibrium, in which producers set the price as a markup on marginal cost, which comes form an equilibrium condition in which marginal cost of producing an extra unit equals marginal revenue generated by it:

$$\frac{P_{j,t}^j}{P_t^j} = \frac{\theta\kappa}{\theta-1} C_t^j Y_t^j \tag{1.10}$$

where $P^j_{j,t}$ is the price of good produced and consumed in country j, and P^j_t is the price level in country j.

The price indices derived from expenditure minimization problems are as follows:

$$P^i_A = \left[\frac{1}{\gamma_A \gamma_P} \int_0^{\gamma_A \gamma_P} \left(P^i_A(z) \right)^{(1-\theta)} dz \right]^{\frac{1}{1-\theta}} \tag{1.11}$$

$$P^i_B = \left[\frac{1}{(1-\gamma_A)\gamma_P} \int_{\gamma_A \gamma_P}^{\gamma_P} \left(P^i_B(z) \right)^{(1-\theta)} dz \right]^{\frac{1}{1-\theta}} \tag{1.12}$$

$$P^i_C = \left[\frac{1}{(1-\gamma_P)} \int_{\gamma_P}^{1} \left(P^i_C(z) \right)^{(1-\theta)} dz \right]^{\frac{1}{1-\theta}} \tag{1.13}$$

$$P^i_P = \left[\gamma_A \left(P^i_A \right)^{(1-\psi)} + (1-\gamma_A) \left(P^i_B \right)^{(1-\psi)} \right]^{\frac{1}{1-\psi}} \tag{1.14}$$

and

$$P^i = \left[\gamma_P \left(P^i_P \right)^{(1-\rho)} + (1-\gamma_P) \left(P^i_C \right)^{(1-\rho)} \right]^{\frac{1}{1-\rho}} \tag{1.15}$$

In the presence of monopolistic competition producers set prices above marginal cost. The markup ($\frac{\theta}{\theta-1}$) reflects the market power of producers and this results in output below competitive level. The market power of firms decreases as θ gets larger.

The current account balance equals the accumulation of net claims on abroad and income from sold units of output and payoff from riskless bonds denominated in the currency of the Center, minus households consumption:

$$\frac{E^j_t}{P^j_t}(B^j_{t+1} - B^j_t) = i_t \frac{E^j_t B^j_t}{P^j_t} + \frac{SR^j_t}{P^j_t} - C^j_t \tag{1.16}$$

and the net supply of bonds is zero in equilibrium:

$$0 = \gamma_A \gamma_P B^A + (1-\gamma_A)\gamma_P B^B + (1-\gamma_P) B^C \tag{1.17}$$

The seigniorage revenue is repayed to consumers in form of a lump sum transfer, so the government's budget constraint is of the form:

$$M^j_t - M^j_{t-1} = -P^j_t T^j_t \tag{1.18}$$

1.2.1 Solution

The model focuses on the impact of small monetary shocks and is solved for log-deviations from an initial symmetric equilibrium. We follow same log-linearization method as Corsetti et al. (2000), i.e. $x \approx \frac{X - X_0}{X_0}$. For the linearized model and solution see details in Corsetti et al. (2000) and in Appendix 1.8.

1.2.2 Macroeconomic properties of the model

The effect of unanticipated, permanent monetary expansion in one country on the rest of the world (ROW) has been analyzed in the related literature, such as Obstfeld and Rogoff (1995) and Corsetti and Pesenti (2001). These seminal models have been further extended by Tille (2001), who emphasized the role of the differences between the within- and between-country elasticities of substitution. The main difference between these approaches are the assumptions about the relative elasticities of substitution parameters. In Obstfeld and Rogoff (1995), in a two-country model, because the within-country and between-country elasticity of substitution are set equal to each other, monetary expansion in one country improves the welfare of both countries. A positive welfare change after the shock in their model is a result of the adjustment of inefficiently low output stemming from monopolistic competition, in the presence of nominal rigidities. If prices were flexible, producers could adjust prices instead of output. Corsetti and Pesenti (2001) in their model allow the within- and between-country substitutability to vary, however they fix the elasticity of substitution between goods produced in different countries to one. Fixing between-country elasticity of substitution in this way, switches off the effect coming from the so called Marshall-Lerner-Robinson (MLR) condition. The MLR condition informs us about the effect on the trade balance of a change in relative prices. If the sum of import and export elasticities (here between-country elasticity[5]) is greater than one, currency devaluation will improve the trade balance of that country where the shock has originated. As long as the between-country elasticity of substitution is different from one, consumption smoothing will take place via adjustment in the current account. This effect is absent in Corsetti and Pesenti (2001), whereas it is at the heart of Tille's (2001) paper and this chapter.

In this model the transmission mechanism is basically the same as in Tille (2001), with some exceptions, one being the consequence of allowing for more than two countries, and as a consequence introducing an additional intra-Periphery elasticity of sub-

[5] see Tille (2001), p. 443 for proof.

stitution, and the other being a consequence of countries' strategic interactions, i.e. the fact that we allow a country to react to monetary expansion, by changing its money supply. At this stage we describe the general mechanism at work in this model, treating the world as two countries (these would correspond to the strategy of an active Center and a passive Periphery) and refrain from describing the effect of monetary shock in the presence of strategic interaction, leaving them for the results section of this paper.

In an economy with monopolistic competition output is sub-optimally low. A small monetary shock raises employment and output, and in the presence of aggregate demand externality and nominal rigidity improves worldwide welfare. The distribution of welfare depends in turn on the degree of substitutability of goods between countries relative to the within-country substitutability (see equations (1.137) and (1.145)). To be more precise, the elasticity of substitution informs us at which rate exports to different countries are transformed into consumption in that country. It may happen, that welfare is decreased in some countries, which may happen even in the country where the shock originated - the so called beggar-thyself effect.

As an effect of monetary expansion in one country, call it home, its agents want to consume more of home and foreign goods due to increased wealth. Increased demand for foreign goods creates higher demand for foreign currency and therefore depreciation of home currency. The nominal exchange rate in the home country depreciates (see equations (1.130) and (1.138)) and causes a change in relative prices, in particular, the real price of home goods decreases. Depreciation generates consumption-switching towards home goods in the short run, the so called expenditure switching or consumption effect. Depending on the elasticity of substitution between goods produced in different countries this consumption switch increases, if elasticity of substitution is greater than one, or decreases sales revenue at home, if elasticity is smaller than one.

Relative output and employment at home increase in the short run (see equations (1.133) and (1.140)), and if the sales revenue increases, home agents can save part of their income in form of current account surplus (equations (1.131) and (1.142)), to smooth consumption over time.

This is a situation in which the so called MLR condition holds, meaning that goods produced at home and abroad are strong substitutes, and depreciation generates a (trade) current account surplus. Since in the short run home nominal prices are fixed we can observe terms of trade worsening in the home country[6]. Current account surplus

[6]The opposite happens, if the elasticity of substitution between home and foreign goods is less than unit elastic. The consumption at home decreases in the short term, country has a trade deficit and lends abroad to allow for higher consumption abroad.

at home implies for the foreign country, that in order to pay his debt in the long run, it has to increase production relative to home country. This in turn means that home country production in the long term will be necessary lower, causing an increase in relative price of home output. Terms of trade effect in the long term is reversed, and relative price of home output increases. This is the so called terms of trade improvement, which allows agents to afford more imports for given level of exports. As long as the goods from abroad have smaller elasticity of substitution than goods produced at home, extra consumption is too small to offset the additional effort, and welfare at home can be decreased. If the within- and between-country substitutability are the same, an increase in welfare from increased consumption is offset by the increase of disutility of work effort from increased production, and the change in utility in both countries is the same.

Increased demand in the presence of inability to adjust prices can be met by increased output allowing for higher consumption. Increased consumption increases welfare, but increased output means increased disutility from work effort. If the cost of effort is more than offset by larger consumption, the overall welfare in the country where the monetary expansion takes place increases. This happens if the between-country substitutability of goods (relative consumption of foreign goods increases) is smaller than the within-country substitutability (absolute consumption of home goods increases).

To sum up, the key mechanism relies on what happens to the home country's stock of net foreign assets. If there is a current account surplus, this stock goes up . Home agents are then wealthier than before, and this higher wealth raises their demand for leisure (if leisure is a 'normal' good). Equivalently, it lowers labour supply and hence lowers home output. A monetary expansion, if it affects the trade balance, has a permanent effect on output and the terms of trade, even though price rigidities are only temporary. This is because it has permanent effect on net foreign assets: they inhibit 'hysteresis', or 'unit root' behaviour.

1.2.3 Game setup

We analyze the effect of a positive, permanent, exogenous monetary shock originating in the big industrial economy (called the Center) on the emerging economies (called the Periphery), given that the policymakers in the Periphery have two available strategies. We assume that the Center remains passive. We consider the following sequence of events:

1. Exogenous permanent monetary shock in the Center, $\bar{m}^C > 0$,

2. Simultaneous decision about money supply by policymakers in country A and B.

We consider a game between policymakers in country A and B with a discrete set of strategies for the Periphery countries form a strategy space $\mathbb{S} = \{0, \bar{m}^C\}$[7]. In order to construct a game in normal form with strategies that have convenient real life interpretation we choose to allow policymakers to decide between two strategies. In principle, it would be possible to use a continuous strategy space, but we decide for a discrete set for two reasons. One of them is the analytical convenience that a binary choice of strategies implies, the other is the fact that in the current policy debate the binary choice corresponds to the inherent decision that the emerging economies are facing, namely whether to keep their exchange fixed or to keep their money supply fixed. On the one hand, keeping the money supply fixed is motivated by the desire to resist the inflationary pressures that arise in a country operating under fixed exchange regime after monetary expansion in the Center. Resisting inflation can be achieved by exiting the fixed exchange rate regime and allowing the currency to appreciate. On the other hand, keeping the exchange rate fixed is motivated by the desire to prevent the loss in competitiveness in the Periphery country/ies after monetary expansion in the Center. Maintaining a fixed exchange, a strategy called PEG, in the model can achieved only by an increase in money supply of the same size in the Periphery country as in the Center. Currency appreciation in the Periphery countries, a strategy called APP, can be achieved by keeping their money supply at unchanged level. Given that on average the inflationary pressures are higher in emerging economies than in advanced economies we do not consider a strategy in which abandoning fixed exchange by increasing money supply more than in the Center would result in currency depreciation in the Periphery countries.

To sum up each policymaker in country A and B maximizes the utility of a representative agent in their own country by making one of two available choices about her money supply $\bar{m}^i$, and decides whether to leave it at unchanged level or permanently increase it. The strategies of the policymaker in country $i = A, B$ result in either of these two outcomes:

1. Fixed exchange rate vis-a-vis the Center, i.e. $e^i = 0$ (see equation (1.138) in

[7] This somehow narrower and discrete strategy space enables us to overcome technical difficulties that arise as soon as we allow for a richer strategy space, which leads to achieving the same goal as in this game. For details about the alternative strategy space and its implications for the results see Remark 1.5.1 in Appendix 1.5.

Appendix 1.8), call it **PEG**, which is achieved by increasing the money supply by the same amount as the Center, i.e $m^i = \bar{m}^C$; or

2. Currency appreciation in country i, i.e. $e^i < 0$ (see equation (1.138) in Appendix 1.8), call it **APP**, which is achieved by not changing the level of money supply i.e. $m^i = 0$.

Policymakers take the strategy of the other country as given. Price setters do not form any expectations about the game, and do not expect money supply to change once the prices have been set. We could possibly explain this behaviour by assuming that, at the moment prices were set, the policymakers were not welfare maximizers but simple money supply targeters. After the prices have been set, there are unexpected changes in government in all countries, and welfare-maximizing policymakers are elected. We treat such events as having very low probability of occurrence such that the price setters were not expecting that. Policymakers maximize over available choices from the strategy space by comparing potential payoffs. Payoff construction is presented in the next section.

1.2.4 Payoffs

By comparing the payoffs from available strategies we find Nash equilibria, and give policy recommendations based on realized changes in utilities from available actions, depending on the characteristics of the economies, with particular focus on the role of asymmetries between countries. The payoffs express the changes in the utility of a representative agent after the shock, and reflect changes in short- and long-term consumption and output.

We can distinguish between three periods that play an important role in the dynamics of the game, namely $t = 0$, $t = 1$, and $t \geq 2$. In $t = 0$ each economy in an initial symmetric steady-state. This period serves as a reference and the variables of the model are log-linearized around their initial steady-state values at $t = 0$. In time $t = 1$ a permanent expansionary monetary shock takes place in the Center country. We refer to the events taking place at time $t = 1$ as short-run. Each country in the Periphery may or may not respond with a permanent monetary expansion of the same size as in the Center. Because of the presence of one-period price rigidity the producers can meet changes in demand after the shock by adjusting production/output at time $t = 1$. In the short-run, after the shock, variables can deviate from their initial equilibrium values from time $t = 0$. At $t = 2$, which we call the long-run, the economy comes back to the flexible-price equilibrium (with new permanently higher than in $t = 0$ money supply in the Center, and potentially higher/or the same money supply in the Periphery countries). Beyond $t = 2$ the economy stays in the long-run equilibrium. For notational convenience we use variables without bar to denote the log-linearized variables (around the symmetric steady-state) in the short-run (x), and variables with a bar to denote the log-linearized variables in the long-run ($\bar{x}$).

The payoffs are derived using the building blocks of the Corsetti et al. (2000) model. Following Obstfeld and Rogoff (1996) we focus only on the real components of the utility function (U_t^R), assuming that the parameter $\chi \to 0$:

$$U_t^R = E_t \sum_{s=0}^{\infty} \beta^s \left[lnC_{t+s}^j - \frac{\kappa}{2}(Y_{t+s}^j)^2 \right] \tag{1.19}$$

Since the new steady state is reached after one period (prices are fixed for one period only) we can rewrite (1.19) as the sum of the short term effect (denoted by C and Y) together with the discounted sum of long term effects (denoted by $\bar{C}$ and $\bar{Y}$):

$$U_t^R = \left[lnC^j(x) - \frac{\kappa}{2}(Y^j)^2 \right] + \frac{\beta}{1-\beta} \left[ln\bar{C}^j - \frac{\kappa}{2}(\bar{Y}^j)^2 \right] \tag{1.20}$$

We will proceed using a log-linear approximation of a variable X, where $X = \{C, Y\}$ in the utility function around its (initial) steady state X_0:

$$X = e^{logX} \approx e^{logX_0} + e^{logX_0}(logX - logX_0) \tag{1.21}$$

$$X \approx X_0 + X_0(logX - logX_0) \tag{1.22}$$

$$\frac{X - X_0}{X_0} \approx (logX - logX_0) \equiv x \tag{1.23}$$

Consider the utility function and its value at the (initial) steady state (where we omit indexes for simplicity):

$$U_t^R = \left[logC - \frac{\kappa}{2}Y^2 \right] + \frac{\beta}{1-\beta} \left[log\bar{C} - \frac{\kappa}{2}\bar{Y}^2 \right] \tag{1.24}$$

$$U_0^R = \left[logC_0 - \frac{\kappa}{2}Y_0^2 \right] + \frac{\beta}{1-\beta} \left[log\bar{C}_0 - \frac{\kappa}{2}\bar{Y}_0^{\,2} \right] \tag{1.25}$$

and subtracting equation (1.25) from (1.24) yields:

$$u^j \approx U_t^R - U_0^R = c - \frac{\kappa}{2}(Y^2 - Y_0^2) + \frac{\beta}{1-\beta}\left[\bar{c} - \frac{\kappa}{2}(\bar{Y}^2 - Y_0^2)\right] \tag{1.26}$$

Now we can log-linearize the nonlinear terms around their initial steady state value:

$$Y^2 = e^{2logX} \approx Y_0^2 + 2Y_0^2(logY - logY_0) \tag{1.27}$$

$$Y^2 - Y_0^2 \approx \underbrace{2Y_0^2(logY - logY_0)}_{\equiv y} \tag{1.28}$$

Based on that equation (1.26) can be rewritten as:

$$u^j \approx U_t^R - U_0^R \approx c - \kappa Y_0^2 y + \frac{\beta}{1-\beta}\left[\bar{c} - \kappa Y_0^2 \bar{y}\right] \tag{1.29}$$

Then we substitute the value of the initial steady state:

$$Y_0 = \sqrt{\frac{\theta - 1}{\theta\kappa}}$$

and thus

$$Y_0^2 = \frac{\theta - 1}{\theta\kappa}$$

we can express the approximation around the symmetric, flexible-price equilibrium:

$$\begin{aligned} u^j &= c^j - \kappa\frac{\theta-1}{\theta\kappa}y^j + \frac{\beta}{1-\beta}\left[\bar{c}^j - \kappa\frac{\theta-1}{\theta\kappa}\bar{y}^j\right] \\ &= c^j - \frac{\theta-1}{\theta}y^j + \frac{\beta}{1-\beta}\left[\bar{c}^j - \frac{\theta-1}{\theta}\bar{y}^j\right] \end{aligned} \tag{1.30}$$

The net present value is the sum of short term value and the discounted sum of the long term value, so:

$$x_{npv} = x + \frac{\beta}{1-\beta}\bar{x} \tag{1.31}$$

and the changes in the utility of a representative agent relative to symmetric equilibrium, following a monetary shock, will represent player's payoff of the form:

$$u^j = c_{npv}^j - \frac{\theta-1}{\theta}y_{npv}^j \tag{1.32}$$

As in Corsetti et al. (2000) we can express relative utilities in the following form (after substitution for c^j and y^j as functions of $\bar{m}^i$, see expressions in Appendix 1.8):

- Center-Periphery

$$u^P - u^C = \frac{\rho - \theta}{\rho\theta}\left(\frac{1+\rho}{1+\beta+\rho(1-\beta)}\right)(\bar{m}^P - \bar{m}^C) \tag{1.33}$$

- Intra-Periphery

$$u^A - u^B = \frac{\psi - \theta}{\psi\theta}\left(\frac{1+\psi}{1+\beta+\psi(1-\beta)}\right)(\bar{m}^A - \bar{m}^B) \tag{1.34}$$

Using equations (1.33) and (1.34), and using the fact that the change in the world utility after a positive monetary shock is:

$$u^w = \frac{1}{\theta}\bar{m}^w \tag{1.35}$$

and applying the following definitions:

$$x^P \equiv \gamma_A x^A + (1-\gamma_A)x^B \tag{1.36}$$

$$x^w \equiv \gamma_P x^P + (1-\gamma_P)x^C \tag{1.37}$$

we can derive individual payoffs.

The overall change in the utilities after the shock for country A and B can be written in terms of countries' money supply choices, and the underlying parameters of the model:

- Payoff of country A:

$$u^A = \frac{1}{\theta}[[\gamma_P\gamma_A + (1-\gamma_P)\gamma_A\Pi(\rho) + (1-\gamma_A)\Pi(\psi)]\bar{m}^A$$

$$+[\gamma_P(1-\gamma_A) + (1-\gamma_P)(1-\gamma_A)\Pi(\rho) - (1-\gamma_A)\Pi(\psi)]\bar{m}^B$$

$$+ [(1-\gamma_P)(1-\Pi(\rho))]\bar{m}^C] \tag{1.38}$$

- Payoff of country B:

$$u^B = \frac{1}{\theta}[[\gamma_P\gamma_A + (1-\gamma_P)\gamma_A\Pi(\rho) - \gamma_A\Pi(\psi)]\bar{m}^A$$

$$+[\gamma_P(1-\gamma_A) + (1-\gamma_P)(1-\gamma_A)\Pi(\rho) + \gamma_A\Pi(\psi)]\bar{m}^B$$

$$+ [(1-\gamma_P)(1-\Pi(\rho))]\bar{m}^C] \tag{1.39}$$

where $\Pi(x) = \left(\frac{x-\theta}{x}\right)\left(\frac{1+x}{1+\beta+x(1-\beta)}\right)$, where $x = \rho, \psi$.

In Lemma 1 we specify the properties of $\Pi(x)$, which will be helpful in the results section.

Lemma 1. Let $\Pi(x) = \left(\frac{x-\theta}{x}\right)\left(\frac{1+x}{1+\beta+x(1-\beta)}\right)$. Assume that $\theta > 1$. Then $\Pi(x)$ is a strictly increasing function in $x \in \mathbb{R} \setminus \{0\}$, and $\Pi(x)$ has the following properties: ***1.*** $\Pi(x) = 0$ *if* $x = \theta$, ***2.*** $\Pi(x) < 0$ *if* $0 < x < \theta$, ***3.*** $\Pi(x) > 0$ *if* $x > \theta$, ***4.*** *The Solutions to* $\Pi(x) = 1$ *are:*

$$x = \frac{\beta + \theta - \sqrt{\beta^2 + 6\beta\theta + \theta^2}}{2\beta} \quad \text{and} \quad x = \frac{\beta + \theta + \sqrt{\beta^2 + 6\beta\theta + \theta^2}}{2\beta}$$

5. $\Pi(x) > 1$ *if* $x > \frac{\beta+\theta+\sqrt{\beta^2+6\beta\theta+\theta^2}}{2\beta}$.

Proof: See Appendix 1.7.1.

Remark 1. *It follows from the definition of monotonicity that* $\Pi(\psi) > \Pi(\rho)$ *if* $\psi > \rho > 0$, *or alternatively* $\Pi(\psi) < \Pi(\rho)$ *if* $0 < \psi < \rho$, *in which case* $\Pi(\psi) = \Pi(\rho)$ *if and only if* $\psi = \rho > 0$.

$\Pi(x)$ can only be greater than 1, if x is sufficiently larger than θ (see Figure 1.6 in the Appendix). Numerical examples show that this happens, if the difference between x and θ is greater than 2. In most cases this is not an issue, because, since the within-country substitutability is greater than the one between countries usually $\Pi(x) < 0$.

Strategy specific payoffs

In this section we present payoffs specific to our game, in which each country in the Periphery, in response to monetary expansion in the Center can either adjust money supply to keep its exchange rate unchanged vis-a-vis the Center $m^i = \bar{m}^C > 0$, a strategy called **PEG**, or to allow its currency to appreciate by leaving the money supply at unchanged level $m^i = 0$, a strategy called **APP**, where $i = A, B$. We include the payoffs of the Center to see the effect of different reactions of the Periphery countries to the shock on the welfare of country C.

Definition 1. Payoff *for country* i *is denoted by* $u^i_{X,Y}$*, where* $X = APP$ *or* $X = PEG$ *is the strategy of country* A*, and* $Y = APP$ *or* $Y = PEG$ *is the strategy of country* B*. The function* $u^i_{X,Y}$ *is defined as:*

$$u^i_{X,Y} \colon S \subset \mathbb{R}^8 \to \mathbb{R}$$

where the domain S *of* $u^i_{X,Y}$ *is defined as*

$S := (1, \infty) \times (0, \infty) \times (0, \infty) \times (0, 1) \times [0, 1] \times [0, 1] \times [0, 1] \times (-\infty, +\infty)$*, with* $\theta \in (1, \infty), \psi \in (0, \infty), \rho \in (0, \infty), \beta \in (0, 1), \gamma_A \in [0, 1], \gamma_B \in [0, 1], \gamma_P \in [0, 1], \bar{m}^C \in (-\infty, +\infty)$*. The function* $u^i_{X,Y}$ *measures a change in the utility of a representative agent in country* i *after the shock, relative to initial equilibrium.*

In Table 1.1 we present a game between country A and B in normal form, with strategy specific payoffs defined below:

Table 1.1: Payoff matrix

		Country B	
		PEG	APP
Country A	PEG	$u^B_{PEG,PEG}$ $u^A_{PEG,PEG}$	$u^B_{PEG,APP}$ $u^A_{PEG,APP}$
	APP	$u^B_{APP,PEG}$ $u^A_{APP,PEG}$	$u^B_{APP,APP}$ $u^A_{APP,APP}$

- Strategy **PEG PEG** - Both Periphery countries maintain the exchange rate at unchanged level, i.e. $\bar{m}^A = \bar{m}^B = \bar{m}^C$:

$$u^A_{PEG,PEG} = \frac{1}{\theta}\bar{m}^C \tag{1.40}$$

$$u^B_{PEG,PEG} = \frac{1}{\theta}\bar{m}^C \tag{1.41}$$

$$u^C_{PEG,PEG} = \frac{1}{\theta}\bar{m}^C \tag{1.42}$$

Payoffs of countries A and B, as well as of the Center are always positive in this strategy. Equal increase in money supply in all countries must increase welfare in all of them, because of short run increase in aggregate demand, raising output and removing some of the inefficiency caused by the existence of monopoly. In such case there is no exchange rate nor terms of trade effect neither in the Periphery countries nor in the Center. This is due to proportionate increase in money supply.

- Strategy **APP APP** - Both Periphery countries allow their currency to appreciate relative to the Center, maintaining their money supply at unchanged level, i.e. $\bar{m}^A = \bar{m}^B = 0$:

$$u^A_{APP,APP} = \frac{1}{\theta}(\gamma_P - 1)(\Pi(\rho) - 1)\bar{m}^C \tag{1.43}$$

$$u^B_{APP,APP} = \frac{1}{\theta}(\gamma_P - 1)(\Pi(\rho) - 1)\bar{m}^C \tag{1.44}$$

$$u^C_{APP,APP} = \frac{1}{\theta}[1 - \gamma_P + \gamma_P\Pi(\rho)]\bar{m}^C \tag{1.45}$$

If both Periphery countries allow their currencies to appreciate, after monetary expansion in the Center, the change in their utility will be positive, if $\Pi(\rho) < 1$, and negative, if $\Pi(\rho) > 1$. The sign of the welfare change depends on the substitutability between home and foreign goods (see Lemma 1). Intuitively, after monetary expansion in the Center, Periphery experiences terms of trade improvement which makes home goods more expensive. Demand shifts away form Periphery goods and sales revenue from exports decreases. If ρ is sufficiently high (so that $\Pi(\rho) > 1$) despite of appreciation of their currency agents in the Periphery cannot afford increase in imports form the Center because their sales revenue has decreased. The net effect of these changes may be negative and therefore, in cases described above, monetary expansion in the Center may reduce welfare in the Periphery countries.

- Strategy **PEG APP** - Country A maintains the exchange rate at unchanged level, and country B allows for currency appreciation vis-a-vis the Center by leaving the money supply at unchanged level, i.e. $\bar{m}^A = \bar{m}^C$, $\bar{m}^B = 0$:

$$u^A_{PEG,APP} = \frac{1}{\theta}[1 - \Pi(\rho) + \gamma_A(\Pi(\psi) - \Pi(\rho) + \gamma_P(\Pi(\rho) - 1))]\bar{m}^C \tag{1.46}$$

$$u^B_{PEG,APP} = \frac{1}{\theta}[1 + \gamma_A(\Pi(\psi) - \Pi(\rho) + \gamma_P(\Pi(\rho) - 1))]\bar{m}^C \tag{1.47}$$

$$u^C_{PEG,APP} = \frac{1}{\theta}[1 + \gamma_P(1 - \gamma_A)(\Pi(\rho) - 1)]\bar{m}^C \tag{1.48}$$

  In case one country in the Periphery maintains money supply unchanged and the other increases it to match monetary expansion in the Center, whether their welfare will be improved or deteriorated, depends on their size and relative substitutability between home and foreign goods. We provide details on the changes in welfare of the Periphery countries in Proposition 1.

- Strategy **APP PEG** - Country A maintains the money supply and country B the exchange rate at unchanged level, i.e. $\bar{m}^A = 0$, and $\bar{m}^B = \bar{m}^C$, :

$$u^A_{APP,PEG} = \frac{1}{\theta}[1 + \Pi(\psi) - \gamma_A\Pi(\psi) + (1 - \gamma_A)\Pi(\rho) + \gamma_P(\gamma_A - 1 + \Pi(\rho) - \gamma_A\Pi(\rho))]\bar{m}^C \tag{1.49}$$

$$u^B_{APP,PEG} = -\frac{1}{\theta}[\Pi(\rho) - 1 + \gamma_A(\Pi(\psi) - \Pi(\rho)) + \gamma_P(\gamma_A - 1)(\Pi(\rho) - 1)]\bar{m}^C \tag{1.50}$$

$$u^C_{APP,PEG} = -\frac{1}{\theta}[1 - \gamma_A\gamma_P(1 - \Pi(\rho))]\bar{m}^C \tag{1.51}$$

  In case one country in the Periphery maintains money supply at unchanged and the other increases it to match monetary expansion in the center, whether their welfare will be improved or deteriorated, depends on their size and relative substitutability between home and foreign goods. We provide details on the changes in welfare of the Periphery countries in Proposition 1.

where $\Pi(x) = \left(\frac{x-\theta}{x}\right)\left(\frac{1+x}{1+\beta+x(1-\beta)}\right)$, with $x = \rho, \psi$.

We conclude this section by an observation in Remark 2, that both the strategic aspect of countries' interactions, as well as asymmetries in size and elasticity of substitution between goods, matter for their welfare, as expressed in form of derived payoffs.

Remark 2. *(Strategic aspect and role of asymmetries) Countries' payoffs depend on:*

1. *the actions taken by their trading partners,*
2. *their relative size,*
3. *relative substitutability between goods.*

1.3 Results

In this section we provide results in terms of the parameters of the model, namely the relative size of countries (γ_i) and substitutability of goods between and within countries (θ, ρ, and ψ). This particular approach allows us to deal with asymmetries in country size, and their implications on countries' welfare, and suggest some potential explanations for the behaviour of Brazil and China. We first look at the result of individual decisions of countries in the Periphery after monetary expansion in the Center, and present an array of possible welfare consequences of their reactions with particular emphasis on investigating when gains from coordination may arise.

We show what dominant strategies, and under which conditions, can arise as a result of countries interactions. We then look at the possibility of collectively suboptimal outcomes resulting from individually rational strategies, the so called Prisoner's Dilemmas. We show that, if the Prisoner's Dilemma occurs, coordination can result in higher payoffs, i.e. there are gains from coordination. We then show cases when we do not have to worry about welfare inferior equilibria. We further investigate numbers of equilibria and possibility of coordination failure. We show that multiplicity of equilibria is not an issue and therefore coordination failure will not arise.

Because of computational complexity of this model (i.e. number of parameters), we restrict our analysis to looking at extreme and intermediate cases, such as the limits of country size, and relative substitutability between- and within-countries. This approach allows us to develop an intuition about the results and highlight the role of asymmetries in size. If specific parameter values describing the economies of interest are available, these can be used to compute the exact welfare changes and to find Nash equilibria. We support our findings with illustrative numerical examples.

It is important to point out that the approach taken in this chapter, namely providing conditions on the elasticity parameters of the model when the size of the country goes to zero implies that the analysis is conducted close to the points of discontinuity.

This limitation can potentially be overcome by adopting the approach of De Paoli (2009) or Sutherland (2005).

De Paoli (2009) introduces the following parameters: home consumers' preferences for domestic goods v, and the degree of country's openness λ. Because the analysed model is a three-county model we modify the parameters in the following way. Let v_P denote the household's j preference for goods produced in the Periphery (and $(1 - v_A)$ the preference of a household in the Periphery for goods produced in country B). Households' preferences for Periphery and non-Periphery goods depend then on the relative size of the Periphery γ_P, and the degree of openness λ, such that $(1 - v_P) = (1 - \gamma_P)\lambda$ and $v_P = \gamma_P \lambda$ (and for the countries in the Periphery $(1 - v_A) = (1 - \gamma_A)\lambda$ and $v_A = \gamma_A \lambda$).

Using De Paoli (2009) approach the modified consumption equations become:

$$C^j(x) = \left[v_P^{\frac{1}{\rho}} \left(C_P^j(x) \right)^{\frac{\rho-1}{\rho}} + (1 - v_P)^{\frac{1}{\rho}} \left(C_C^j(x) \right)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}} \tag{1.52}$$

and

$$C_P^j(x) = \left[v_A^{\frac{1}{\psi}} \left(C_A^j(x) \right)^{\frac{\psi-1}{\psi}} + (1 - v_A)^{\frac{1}{\psi}} \left(C_B^j(x) \right)^{\frac{\psi-1}{\psi}} \right]^{\frac{\psi}{\psi-1}} \tag{1.53}$$

Then, if we assume that the size of country A or the Periphery as a whole goes to 0, i.e. $\gamma_A \to 0$ or $\gamma_P \to 0$, then the preference for home and foreign goods will depend solely on the degree of openness of the country: $v_A = 1 - \lambda$, and $v_P = 1 - \lambda$.

Similarly the price indices will become:

$$P_P^i = \left[v_A (P_A^i)^{(1-\psi)} + (1 - v_A)(P_B^i)^{(1-\psi)} \right]^{\frac{1}{1-\psi}} \tag{1.54}$$

and

$$P^i = \left[v_P (P_P^i)^{(1-\rho)} + (1 - v_P)(P_C^i)^{(1-\rho)} \right]^{\frac{1}{1-\rho}} \tag{1.55}$$

Since the consumption and price sub-indices will remain the same as in the model without the parameter of home consumers' preferences for domestic goods v, and the degree of country's openness parameter λ (equations 1.5-1.7, and 1.11-1.13), we conclude that introducing these parameters is not going to address the concern that some of the price indices may go to zero and production may go to infinity if we consider the limit of the country size to be zero. This is also the case in the analysis done by De Paoli (2009).

1.3.1 Enrich or Beggar-thy-neighbour?

How does monetary expansion in an industrial economy affect emerging economies? Based on the discussions between policymakers in industrial and emerging economies (Rousseff, 2012) this question does not seem to have a clear answer. In this section we show, how our framework is useful in answering this question, depending on the relative country size and substitutability of goods produced in different countries, and actions taken by individual countries in the Periphery. We show, that in terms of welfare, increase in money supply in the Center may or may not be beggar-thy-neighbour.

In the following two Propositions we look at what changes in welfare can countries in the Periphery expect from taking specific actions (APP or PEG). We focus here mainly on the outcome in terms of welfare changes, and to be more specific, whether the reaction to a shock in the Center results in welfare improvement, i.e. positive change in welfare relative to pre-shock equilibrium; or deterioration of welfare.

In Proposition 1, we show when the Periphery countries can expect their welfare to be lower, than before the shock ($u^i < 0$), conditional on the actions of the other country.

Definition 2. *Policy in the Center is said to be* beggar-thy-neighbour *for country i, if $u^i < 0$, for $i = A, B$.*

Proposition 1. Monetary expansion in the Center is beggar-thy-neighbour for the Periphery countries in the following cases:

- *for both Periphery countries in case of APP, APP, if $\rho > \frac{\beta+\theta+\sqrt{\beta^2+6\beta\theta+\theta^2}}{2\beta}$, i.e. ρ sufficiently larger than θ,*
- *for country A, if the Periphery is large, i.e. $\gamma_P \to 1$, and:*
  - *APP,PEG, if $\psi > \frac{\beta+\theta+\sqrt{\beta^2+6\beta\theta+\theta^2}}{2\beta}$, i.e. ψ sufficiently larger than θ, and country A is very small, i.e. $\gamma_A \to 0$,*
  - *PEG,APP, if $\Pi(\psi) < \frac{\gamma_A}{\gamma_A-1}$,*

- *for country B, if the Periphery is large, i.e.* $\gamma_P \to 1$, *and:*
  - *APP,PEG, if* $\Pi(\psi) < \frac{\gamma_A - 1}{\gamma_A}$,
  - *PEG,APP, if* $\psi > \frac{\beta+\theta+\sqrt{\beta^2+6\beta\theta+\theta^2}}{2\beta}$, *i.e.* ψ *sufficiently larger than* θ,
- *for country A, if the Periphery is small, i.e.* $\gamma_P \to 0$, *and:*
  - *APP,PEG, if country A is very small, i.e.* $\gamma_A \to 0$, *and* $\psi > \frac{\beta+\theta+\sqrt{\beta^2+6\beta\theta+\theta^2}}{2\beta}$, *i.e.* ψ *sufficiently larger than* θ, *or if country A is very large, i.e.* $\gamma_A \to 1$, *and* $\rho > \frac{\beta+\theta+\sqrt{\beta^2+6\beta\theta+\theta^2}}{2\beta}$, *i.e.* ρ *sufficiently larger than* θ,
  - *PEG,APP, if country A is very small, i.e.* $\gamma_A \to 0$, *and* $\Pi(\psi) - \Pi(\rho) < -1$,
- *for country B, if the Periphery is small, i.e.* $\gamma_P \to 0$, *and:*
  - *APP,PEG, if country A is very large, i.e.* $\gamma_A \to 1$, *and* $\Pi(\psi) - \Pi(\rho) < -1$,
  - *PEG,APP, if country A is very large, i.e.* $\gamma_A \to 1$, *and* $\psi > \frac{\beta+\theta+\sqrt{\beta^2+6\beta\theta+\theta^2}}{2\beta}$, *i.e.* ψ *sufficiently larger than* θ, *or if country A is very small, i.e.* $\gamma_A \to 0$, *and* $\rho > \frac{\beta+\theta+\sqrt{\beta^2+6\beta\theta+\theta^2}}{2\beta}$, *i.e.* ρ *sufficiently larger than* θ.

Proof: See Appendix 1.7.6.

Proposition 1, except for the strategy APP APP, relies on big asymmetries in country size, and often requires the within-country substitutability of goods to be smaller than between-country substitutability. This specific condition has been ruled out by Corsetti et al. (2000), who assume that each country specializes in production of single type of good, and therefore goods produced at home should be more substitutable than goods produced in different countries. If we made the same assumption, i.e. assume that $\rho \leq \psi \leq \theta$, we could conclude that **beggar-thy-neighbour** effect of expansionary monetary in the Center is likely to occur in very few cases.

Proposition 1, may help us to understand the worries expressed by Brazil's President (Rousseff, 2012) about possible harm generated by the US expansionary monetary policy on Brazilian economy. Our model allows for analyzing a scenario, which is consistent with what actually happened in Brazil after a policy of quantitative easing (QE) in the US. According to Barroso et al. (2013), QE brought about currency appreciation, accompanied by an increase in consumption and imports, and fall in industrial production in Brazil. This model predicts such a scenario accompanied by decrease in welfare

Table 1.2: Trade weights 2009

	USA	Brazil	China	Sum
USA	-	0.11	0.89	1
Brazil	0.50	-	0.50	1
China	0.88	0.12	-	1

in the emerging economy, if either the substitutability of goods produced in different countries is greater than the within-country substitutability, i.e. when $\rho > \theta$ in APP APP, independently of the size of the emerging economy; or in case of PEG APP for country B in case goods produced in the Periphery countries are more substitutable than those produced at home, i.e. when $\psi > \theta$, when country B is relatively small. If we look at the trade flows between Brazil and US relative to other emerging economies, such as China, indeed Brazil is a relatively small country[8]. Its trade is almost equally split between US and China. But trade flows from and to Brazil, from US and China are only 10% of these countries trade flows. Most of the US trade comes from China (around 90%), and most of the Chinese trade from the US (around 90%). We can summarize this observation in a Table 1.2, where we treat China, Brazil and US as a whole world, similarly as in our three-country model. This methodology is adapted from the way weight matrices are constructed for Global Vector Autoregressive Models.

The mechanism at work, which can be described with the help of our model, and is consistent with Barroso et al. (2013) is the following. After monetary expansion in the Center/US (and alternatively also in country A), the world demand shifts away from goods produced in country B (Brazil) and therefore reduces its sales revenue from exports. Because of terms of trade improvement in country B (Brazil), its agents want to substitute away from relatively more expensive (but more substitutable) foreign goods and consume more (less substitutable) home goods. The higher ρ or ψ the stronger the desire to substitute consumption of foreign goods for home goods (consumption switching). Although currency appreciation allows country B (Brazil) to afford more imports for given level of income, because of the decrease in sales revenue from exports, B (Brazil) cannot afford enough goods from abroad. This relative decrease in con-

[8]in terms of shares of import and export between countries relative to total net trade. In construction of a weight matrix expressing relative trade flows between US, Brazil and China, we have used the OECD STAN Bilateral Trade Database by Industry and End-use category (OECD, 2009), to show that Brazil is relatively small in terms of trade flows for China and US, but both China and US are important trade partners for Brazil, see Table 1.2.

sumption, together with decrease in sales revenue, and increase in the long run output necessary to repay debt (current account deficit) in the long run, can result in decreased welfare.

We can illustrate this case by constructing a numerical example in which country B (Brazil) is very small, $\gamma_B = 0.1 \times 0.4 = 0.04$, countries of the Periphery are smaller than the Center (US), $\gamma_P = 0.4$, the within- and between-country elasticities of substitution are equal $\theta = \rho = 2$, and the within-Periphery elasticity of substitution is smaller than within-country elasticity $\theta < \psi = 4$, with equal change in money supply in the Center (US) and country A (China) $\bar{m}^C = \bar{m}^A = 0.5$, and the discount factor $\beta = 0.99$. In such an economy the resulting payoffs for country A and B are: $u^A_{PEG,APP} = 0.27$ and $u^B_{PEG,APP} = -0.04$ (full payoff matrix can be found in the Appendix 1.6). If country A (China) allowed its currency to appreciate, country B (Brazil) would be better off relative to the case in which country A (China) keeps its exchange rate fixed, which is consistent with the recommendations given in the U.S. Treasury Report to Congress on International Economic and Exchange rate Policies of the Treasury (2011), and Mattoo et al. (2012). If both emerging economies managed to keep the exchange rate fixed, both would increase their welfare. Clearly, the payoff of country B (Brazil) depends on the action taken by country A (China), an observation that we made in Remark 2. The strategic aspect in the spillover mechanism of policies originating in the Center cannot be underestimated.

Interestingly, even though we treat monetary expansion in the Center as an exogenous shock, the model predicts that it may happen, that such policy can yield lower payoff for the Center than in the symmetric equilibrium. We call such a situation beggar-thyself and we define conditions in which such situation may arise.

Definition 3. *Policy in the Center is said to be* beggar-thyself, *if* $u^C < 0$.

Lemma 2. Monetary expansion in the Center is beggar-thyself, in case of:

- *APP,APP, if* $\Pi(\rho) < \frac{\gamma_P - 1}{\gamma_P}$,
- *APP,PEG, if* $\Pi(\rho) < \frac{\gamma_P \gamma_A - 1}{\gamma_P \gamma_A}$,
- *and PEG,APP, if* $\Pi(\rho) < \frac{\gamma_P(1-\gamma_A) - 1}{\gamma_P(1-\gamma_A)}$.

Proof: See Appendix 1.7.3.

Beggar-thyself outcomes for the Center may arise in every strategy except for when all countries simultaneously increase money supply. Such policy will never harm any of the countries in this model, because effectively their agents become agents of a closed economy in which monetary expansion in the presence of fixed prices brings the level of production closer to its competitive level and therefore improves welfare. In each of the remaining strategies the beggar-thyself condition depends on the relative size of the country where monetary supply increases. The right hand side in each condition is negative. For $\Pi(\rho)$ to be less than the right hand side, using Lemma 1, the elasticity of substitution between goods produced in different countries (ρ) has to be sufficiently smaller than within-country elasticity (θ). Intuitively, beggar-thyself can happen, if the worldwide impact of the monetary shock is negligible (relatively large γ_P), i.e. the Center and the country that decides to increase its money supply as well are small, and the benefit from monetary expansion is smaller than the negative terms of trade externality, which happens, when goods produced internationally are worse substitutes than those produced domestically (i.e. $\rho < \theta$).

To complete the analysis, we are now going to examine when does the policy in the Center improve welfare in the Periphery countries. For this, we will start by defining what we mean by an enrich-thy-neighbour policy.

Definition 4. *Policy in the Center is said to be* enrich-thy-neighbour *for country i, if $u^i > 0$, for where $i = A, B$.*

Proposition 2. *(Enrich-thy-neighbour)* Monetary expansion in the Center is always enrich-thy-neighbour for all countries in the Periphery in case of PEG PEG. This is independent of country size and elasticity of substitution between home and foreign goods. It can be enrich-thy-neighbour for the Periphery countries in other strategies, if the conditions in Proposition 1 do not hold.

Proof: See Appendix 1.7.7.

This result is consistent with Corsetti and Pesenti (2001) who show that monetary expansion abroad can be **"prosper-thy-neighbor"** for admissible monetary shocks, and with the view of Bernanke (2013) and Powell (2013) that US policy generated positive spillovers to the rest of the world.

Since, depending on the characteristics of the economy, the same actions of the Periphery countries can lead to opposite (in terms of sign) welfare outcomes, it is important for countries to know what are the values of the parameters describing the economy, before they decide to react to monetary expansion in the Center. Using Propositions 1 and Proposition 2 we can state a Corollary 1 highlighting the role of uncertainty in potential explanation of the behaviour of China and Brazil, who both increased money supply (a strategy which in our model leads to maintaining the exchange rate at unchanged level) after the first round of QE in the US (see Figure 1.2).

Corollary 1. *(Uncertainty)* Countries in the Periphery may prefer to intervene and resist currency appreciation to keep their exchange rate fixed, if unsure about the parameters of the model.

Proof: See Appendix 1.7.2.

If there is uncertainty about the parameters of the model (related to "uncertainty about the state of the economy" Ostry and Ghosh (2013), or Frankel (1988)), countries may prefer to stick to a strategy resulting in welfare improvement independently of the values of the parameters. This may explain why Periphery countries may prefer to keep their exchange rate fixed. In Proposition 1 and 2 we show, that pegging always results in welfare improvement, as opposed to appreciation, which depends on the elasticity of substitution between goods produced at home and abroad. As a consequence, if countries are uncertain about the underlying size and substitutability parameters, countries may prefer to choose to resist currency appreciation, as it was initially the case of China.

1.3.2 Dominant strategies

In this section we focus our attention on dominant strategies, that is strategies that yield a higher payoff for the player, for any feasible action of the other player. Finding conditions in which particular dominant strategies arise is a crucial step in finding out, whether individually rational actions may lead to collectively suboptimal outcomes, i.e. so called Prisoner's Dilemma situations.

One of the consequence of the construction of the strategy set in our game is that dominant strategies will always exist[9]. In the following section we will only look at situations resulting in symmetric dominant strategies, that is such that Nash equilibrium for both players is either APP APP or PEG PEG. However, depending on the values of elasticity parameters and relative size of countries, dominant strategy for each player may differ, and Nash equilibria such as APP PEG or PEG APP are possible. Before we move on to investigate when countries will have same dominant strategies let us define the concept first.

[9]However, it can be shown that dominant strategies may not exist, if we allow for a reacher strategy space, which we do not consider for technical reasons. For details see Remark 1.5.1 in Appendix 1.2.3.

Definition 5. Dominant strategy *for player i is her best response to any strategy of the other player, i.e. a strategy which always yields higher payoff for player i, regardless of strategy of the other player(s).*

Currency appreciation in both countries - APP APP

We first look at the possibility of one potential dominant strategy which is for both countries to prefer to allow their currency to appreciate, no matter what the other country chooses to do.

Proposition 3. *(Dominant strategy to appreciate)* Countries in the Periphery have a dominant strategy to appreciate, if the following conditions hold:

- *for Country A:*

$$\gamma_A(\Pi(\psi) - \Pi(\rho) + \gamma_P[\Pi(\psi) - 1]) - \Pi(\psi) > 0 \tag{1.56}$$

- *for Country B:*

$$\gamma_A[\Pi(\psi) - \Pi(\rho)] + (\gamma_A - 1)\gamma_P[\Pi(\rho) - 1] + \Pi(\rho) < 0 \tag{1.57}$$

Proof: See Appendix 1.7.8.

To help us see when APP APP is a dominant strategy for both countries let us have a look at these conditions for extreme and intermediate values of country size (See Table 1.5). If the Periphery is relatively large, these conditions clearly do not hold. Intuitively, since the Center is the only country where money supply increases, the overall increase in welfare generated in such case will be negligible and the Periphery countries would as well prefer to increase their money supply to generate larger increase in world welfare. Since the payoffs of country A and B are decreasing in size of the Periphery, both countries may prefer to allow their currency to appreciate, if the size of the Periphery is sufficiently small. Additionally, the following assumption on elasticities of substitution has to hold: $\rho < \psi < \theta$. Notice, that in the presence of this elasticity condition, if a country is very small, it is better off allowing its currency to appreciate. This might be the case of Brazil (see Table 1.2, in which we show that Brazil is a relatively small country for US and China) who's exchange rate vis-a-vis the US appreciated (see Figure 1.3).

We present a numerical example in Table 1.3 and describe it together with the condition for Prisoner's Dilemma in section 1.3.3.

US recommendation for emerging economies to allow their currency to appreciate (see e.g. U.S. Department of the Treasury (2011), or Obstfeld (2011)) is plausible only if we believe that these countries are sufficiently small and particular elasticity conditions hold (see Table 1.5), an observation which we make in the following remark:

Remark 3. *APP APP is a dominant strategy for both Periphery countries if* $\rho < \psi < \theta$, *and the Periphery as a whole is sufficiently small.*

Fixed exchange rate in both countries - PEG PEG

The other possibility of dominant strategy we consider is for both countries to prefer to keep their exchange rate vis-a-vis the Center fixed.

Proposition 4. *(Dominant strategy to peg)* Countries in the Periphery have a dominant strategy to adjust their money supply, such that the exchange rate vis-a-vis the Center will remain at unchanged level, if the following conditions hold:

- *for Country A:*

$$\gamma_A(\Pi(\psi) - \Pi(\rho) + \gamma_P[\Pi(\psi) - 1]) - \Pi(\psi) < 0 \tag{1.58}$$

- *for Country B*

$$\gamma_A[\Pi(\psi) - \Pi(\rho)] + (\gamma_A - 1)\gamma_P[\Pi(\rho) - 1] + \Pi(\rho) > 0 \tag{1.59}$$

Proof: See Appendix 1.7.9.

We present a numerical example in Table 1.4.

Countries in the Periphery prefer to keep their exchange rate at unchanged level, if the substitutability of goods produced between countries (ρ and ψ) is larger that the within-country substitutability (θ). This result is consistent with Tille (2001), who showed that country can be better off by increasing money supply relative to its neighbour, only if the benefit from switching consumption is larger than the increased disutility from work effort at home.

Conditions relating the size of the substitutability parameters and relative size of countries are derived in Appendix 1.7.9 and presented in Table 1.5.

Remark 4. *If one of the Periphery countries is sufficiently large and the Periphery is sufficiently large as a whole, then its dominant strategy is to peg.*

The observation in Remark 4 follows from Table 1.5, in case when $\gamma_P \to 1$ and $\gamma_A \to 0$, then B's dominant strategy is to peg.

Remark 4 in conjunction with the trade weight matrix may help us to explain why China initially chose to maintain fixed exchange vis-a-vis the US.

1.3.3 Gains from coordination in cases of Prisoner's Dilemma

When countries decide to take self-oriented actions they may end up in a Nash equilibrium that is Pareto inferior, and yield lower payoff for both players than from alternative actions. In such, so called Prisoner's Dilemma, type of situations there is a scope for welfare improvement and gains from coordination (CG)[10].

Definition 6. Gains from coordination *for country i, denoted by CG_i is the difference in country i's payoffs, given by*

$$CG_i = u^i_{-NE} - u^i_{NE}$$

where u^i_{NE} is the payoff of country i in Nash equilibrium, and u^i_{-NE} is the payoff from a strategy yielding the highest payoff for both of the players, which they choose to play, if they cooperate.

There exist many examples in the literature arguing that there are few gains from coordination, like Ostry and Ghosh (2013), Buiter and Marston (1986), Frankel (1988), or that it may even be counterproductive as argued by Rogoff (1985). Motivated by this lack of consensus we show when gains from coordination between Periphery countries may exist.

In our game, we look at two potential equilibria with Prisoner's Dilemma[11], namely APP APP and PEG PEG. In order for Prisoner's Dilemma to arise, both countries have to have dominant strategies, and the payoff from the resulting non-cooperative Nash equilibrium must be lower, than from taking alternative actions simultaneously.

[10] Whenever we use a word coordination, we actually mean cooperation. This abuse of language is common to macroeconomists and policymakers. The only time we are formally meaning coordination, as used in game theory, is in section 1.3.6.

[11] It can be shown that in this game other Nash equilibria, such as PEG APP and APP PEG, may arise.

Definition 7. Prisoner's Dilemma *is a situation in which both players have dominant strategies, and the Nash equilibrium, resulting from agents acting non-cooperatively in self interest, yields lower payoff for both, than if they could cooperate.*

In the following sections we will look at sufficient conditions for Prisoner's Dilemma to arise and investigate in which situations this might happen in our model.

Currency appreciation in both countries - APP APP

Below we present sufficient conditions for Prisoner's Dilemma with Nash equilibrium APP APP - a situation in which both countries would be better off if they coordinated and simultaneously increased money supply.

Proposition 5. *(Prisoner's Dilemma with appreciation)* If the conditions in Proposition 3 hold simultaneously with:

$$\gamma_P[1 - \Pi(\rho)] + \Pi(\rho) > 0 \tag{1.60}$$

then we call such situation Prisoner's Dilemma in which appreciation is a welfare inferior equilibrium.

Proof: See Appendix 1.7.10.

Welfare can be improved if countries coordinate and coordination gains can be computed using Definition 6.

What these conditions require in terms of relative substitutability between good and size of countries can be seen in Table 1.5 in the limiting cases. Prisoner's Dilemma is less likely to happen if there are large asymmetries in size between the Periphery countries.

These conditions do not appear to be very intuitive, therefore we present a numerical example, outline the transmission mechanism at work, and its dependence on the particular parameters in this example[12]. With help of this example we would also like to show that there is scope for policy coordination.

To describe the transmission mechanism at work contributing to changes in countries' welfare we will look at the example in Table 1.3 from a perceptive of player A.

To start with let us have a look at the overall effect of countries interactions in terms of changes in the world welfare. Welfare changes depend on countries' actions - whether

[12]In all our numerical examples the size of the shock is 0.5 units (other examples are Tille (2001), p. 438, who analyzes a numerical example normalizing the monetary expansion to $\bar{m} = 1$).

Table 1.3: Example of Prisoner's Dilemma with Nash equilibrium APP APP. Parameter values: $\theta = 3$, $\psi = 2$, $\rho = 2$, $\beta = 0.9$, $\gamma_P = 0.6$, $\gamma_A = 0.5$, $\bar{m}^C = 0.5$

		Country B: PEG	Country B: APP
Country A	PEG	A: 0.17, B: 0.17	A: 0.08, B: 0.20
	APP	A: 0.20, B: 0.08	A: **0.11**, B: **0.11**

to keep their money supply at unchanged level or to increase it. As a consequence, total world welfare change is different in different strategies and can be derived from the general expression for u^w:

$$u^w = \frac{1}{\theta}\bar{m}^w = \frac{1}{\theta}[\gamma_P\bar{m}^P + (1-\gamma_P)\bar{m}^C] = \frac{1}{\theta}\{\gamma_P[\gamma_A\bar{m}^A + (1-\gamma_A)\bar{m}^B] + (1-\gamma_P)\bar{m}^C\} \quad (1.61)$$

In particular we have:

$$u^w_{PEG,PEG} = \frac{1}{\theta}\bar{m}^C \quad (1.62)$$

$$u^w_{APP,APP} = \frac{1}{\theta}[(1-\gamma_P)\bar{m}^C] \quad (1.63)$$

$$u^w_{PEG,APP} = \frac{1}{\theta}[\gamma_P\gamma_A\bar{m}^C + (1-\gamma_P)\bar{m}^C] \quad (1.64)$$

$$u^w_{APP,PEG} = \frac{1}{\theta}[\gamma_P(1-\gamma_A)\bar{m}^C + (1-\gamma_P)\bar{m}^C] \quad (1.65)$$

In this numerical example resulting changes in world welfare are: $u^w_{PEG,PEG} = 0.17$, $u^w_{APP,APP} = 0.07$, $u^w_{PEG,APP} = 0.12$, $u^w_{APP,PEG} = 0.12$.

The redistribution of world welfare generated in each strategy between countries depends on the relative substitutability of goods, MLR conditions, as well as countries' relative size.

Let us focus on the transmission mechanism first. For simplicity and transparency this example assumes same size of the Periphery countries[13] and same within-Periphery elasticity of substitution (so that looking at A's choice will describe the decision faced by B in the alternative strategy).

In **PEG PEG** all countries increase money supply by the same amount. As a result world welfare increases proportionately and is redistributed equally among agents in all countries. In fact, this strategy is equivalent to treating the whole world as closed economy, and therefore an increase in money supply in the presence of monopolistic competition and nominal rigidities unambiguously increases welfare pushing the producers closer to perfectly competitive production level. Change in the world utility is the largest in this case.

In **APP APP** monetary expansion takes place only in the Center. Because monetary expansion takes place only in one, not too big country ($\gamma_C = 1 - \gamma_P = 0.4$), and in this strategy world utility is negatively related to the size of the Periphery, the overall change in world utility is the smallest of all four strategies. Since money supply in both Periphery countries stays at unchanged level, both countries A and B experience same changes in consumption, output and therefore welfare. The currencies of the Periphery countries appreciate relative to the Center. The size of the exchange rate adjustment depends on the degree of substitutability between goods produced in the Periphery and the Center (ρ). Because in the short run prices are fixed, an increase in money supply allows to increase consumption in the Center. Agents in the Center want to consume more of home and foreign goods, but because of exchange rate effect Center's goods become relatively cheaper for its agents, which generates consumption switching towards home goods. Short and long run consumption in the Center increases relative to the Periphery. World demand switches away from goods produced in the Periphery. In the short run Periphery countries produce less relatively to the Center. Demand switch reduces A's and B's market share of export, which causes fall in sales revenue from export. The opposite happens in the Center, where because the MLR condition holds, terms of trade worsening yields increase in sales revenue. Increase in sales revenue enables C's agents to save a share of its income that can be lend abroad for consumption smoothing purposes. In the long run, in order to repay its debt, Periphery countries will have to produce more. Long run production is therefore larger in the Periphery than in the Center. Relative change in consumption and long term output depends on the MLR condition. The relative change in welfare depends on the so called 'generalized

[13]asymmetry in size may lead to a different Nash equilibrium than APP APP (e.g. if A is very small, then APP PEG would be the Nash equilibrium).

MLR condition' defined by Tille (2001), which says that monetary expansion can raise country's welfare only if $\rho > \theta > 1$. Since in our example the opposite condition holds, the Periphery as a whole experiences increase in welfare relative to the Center. This is because an increase in consumption in the Center increases utility, but an increase in production does not generate sufficient revenue and increases disutility from work effort. In this example additionally Center's utility change is negative (see Lemma 2), because its relative size is not big enough. The adverse terms of trade in the Center does not generate large enough consumption switch towards home goods, so that higher revenue generated from producing more is not enough to compensate for the more expensive imports, away from which C cannot substitute. On the other hand Periphery countries, because of terms of trade improvement, can finance higher level of consumption for given nominal income, and because of lower demand there is a decrease in the disutility from work effort.

A's payoff in Nash equilibrium is smaller than in PEG PEG, because with lower between- ρ than within-country substitutability θ, APP APP is decreasing in size of the Periphery (here $\gamma_P = 0.6$), and the overall welfare increase generated in this strategy is the smallest. This is a case where coordination of both countries on choosing to increase their money supply would be beneficial in terms of welfare not only for the Periphery countries but also for the Center. Gains from coordination for the Periphery countries can be computed using the formula in the Definition 6: $0.17 - 0.11 = 0.06$.

In **PEG APP** both Center and country A increase their money supply, which generates a change in the world utility smaller than in PEG PEG but larger than in APP APP. Since country A takes different action than country B, this generates relative changes in consumption, output and exchange rates in the Periphery. Country B's currency appreciates relative the Center, as a consequence Periphery's currency appreciates as well but by less than in APP APP. The size of the exchange rate adjustment depends in this case on the MLR condition, i.e. the value elasticity of substitution between the Periphery countries (here $\psi > 1$). In this case agents in country A and C feel wealthier and increase consumption of both home and foreign goods. Consumption in country A increases in both short and long term relative to country B, however the overall consumption in the Periphery decreases relative to the Center. Exchange rate effect creates consumption switching in country A and C towards home produced goods, which in turn switches global demand away from goods produced in country B. While A and C's agents experience terms of trade worsening in the short term, agents in country B experience the opposite and can afford more consumption for a given level of nominal income. But because of decrease in demand for B's goods, its sales revenue decreases and B's agents borrow from abroad (from A and C) to smooth consumption. Lending to B is possible because both A and C generate additional income that can be saved from increased sales revenue due to $\rho > 1$ (MLR condition). Production increases in the short term in A relative to B, but the opposite happens in the long run. A's agents feeling wealthier prefer to substitute consumption with leisure (they can do this since they saved enough and expect repayment from abroad), while B's agents have to work more in the long run to repay their debt. Since now two countries expand money supply their relative size is big enough not to generate beggar-thyself effect and the change in welfare will be positive in all countries. However, similarly to the APP APP case, countries where monetary expansion took place will experience relatively lower welfare than their passive neighbours.

In PEG APP A's payoff is lower than in APP APP because of two reasons. Firstly, the overall welfare generated in this strategy is lower than in APP APP, secondly, A is in this case experiencing the consequences of an externality generated by the so called generalized 'MLR condition', and falling on the country that decides to increase money supply. Here country B benefits the most since its terms of trade improves in both periods and the increase in disutility of work effort is not large enough to offset an increase in utility from increased consumption.

In **APP PEG** both Center and country B increase their money supply, which generates a change in the world utility smaller than in PEG PEG but larger than in APP APP. In this case country B experiences what country A experienced in the strategy PEG APP.

The analysed example in Table 1.3 is a case that Bernanke (2013) called **"enrich-thy-neighbour"**, however we can show that there are cases, in which the same policy can generate an equilibrium with negative payoffs, where countries are stuck suffering loss in their welfare. We construct such example using Proposition 1 and present it in the Appendix 1.6. In such example the concerns by the Periphery countries that US expansionary monetary policy can generate negative spillovers may be justified.

Fixed exchange rate in both countries - PEG PEG

Prisoner's Dilemma with inferior Nash equilibrium PEG PEG happens, if both countries have dominant strategies to keep their exchange rate at unchanged level vis-a-vis the Center (see Proposition 4) and the payoffs are smaller than from allowing the currency to appreciate in both Periphery countries.

Proposition 6. *(Prisoner's Dilemma with fixed exchange rate)* If the conditions in Proposition 4 hold simultaneously with:

$$\gamma_P[1 - \Pi(\rho)] + \Pi(\rho) > 0 \tag{1.66}$$

then we call such situation Prisoner's Dilemma in which keeping exchange rate relative the Center fixed is a welfare inferior equilibrium.

Proof: See Appendix 1.7.11.

Welfare can be improved if countries coordinate and coordination gains can be computed using Definition 6.

What these conditions require in terms of relative substitutability between good and size of countries can be seen in Table 1.5 in the limiting cases. Prisoner's Dilemma is less likely to happen if there are large asymmetries in size between the Periphery countries.

Table 1.4: Example of Prisoner's Dilemma with Nash equilibrium PEG PEG. Parameter values: $\theta = 1.1$, $\psi = 6$, $\rho = 0.3$, $\beta = 0.9$, $\gamma_P = 0.1$, $\gamma_A = 0.5$, $\bar{m}^C = 0.5$

		Country B	
		PEG	APP
Country A	PEG	**0.45** **0.45**	0.28 1.32
	APP	1.32 0.28	1.14 1.14

In numerical example presented in Table 1.4 the Periphery is small as a whole and countries within the Periphery are symmetric in size. Within-country substitutability of goods are larger than between goods produced in the Center, but smaller than for goods produces in the Periphery countries. MLR conditions hold only for the Periphery countries, and not for the Center. Country A and B are stuck in Nash equilibrium with lower payoffs than from appreciation. Despite yielding higher payoffs, appreciation is not an equilibrium, because both countries have an incentive to deviate tempted by higher payoffs from alternative action. Therefore the loss in utility is $1.14 - 0.45 = 0.67$. If countries had a chance to coordinate, they could achieve improvement in their utility.

It appears that the results are sensitive to the choice of the point around which we log-linearize, which is related to the presence of an approximation error. In particular the Prisoner's Dilemma not always occurs if the variables are approximated around an alternative steady-state given the conditions on the parameters of the model specified in Propositions 5 and 6.

This has been established by comparing the results from to steady-states:

- the symmetric steady-state given by $C^2 = Y^2 = \frac{\theta-1}{\theta\kappa}$,
- and a hypothetical steady-state with a subsidy $\frac{\delta}{\delta-1}$, given by $C^2 = Y^2 = \frac{\theta-1}{\theta\kappa}\frac{\delta}{\delta-1}$, where δ can be smaller, equal or greater than θ. When $\delta = \theta$ this case would correspond to an economy with perfect competition.

To address this issue we wish to undertake the analysis in future research by using higher order approximation developed in more recent literature, such as Benigno and Woodford (2012).

1.3.4 Summary of conditions for dominant strategies and Prisoner's Dilemma in the limits

In Table 1.5 we summarize conditions for dominant strategies and Prisoner's Dilemma in limiting cases resulting from Propositions 3, 4, 5, and 6 derived in the Appendix 1.7.8, 1.7.9, 1.7.10, and 1.7.11.

Table 1.5: Summary of conditions for dominant strategies and Prisoner's Dilemma

	$\gamma_P \to 1$, $\gamma_A \to 0$	$\gamma_A \to 0.5$, $\gamma_P \to 0.5$	$\gamma_P \to 0$, $\gamma_A \to 0$	$\gamma_P \to 0$	$\gamma_A \to 0$
It is a dominant strategy to appreciate...					
...for Country A if:	$\Pi(\psi) < 0$, i.e. $\psi < \theta$	$0.5\Pi(\psi) + 0.25\Pi(\rho) < -0.25$	$\Pi(\psi) < 0$, i.e. $\psi < \theta$ $\Pi(\rho) < 0$, i.e. $\rho < \theta$	$\frac{\Pi(\psi)}{\Pi(\rho)} > \frac{\gamma_A}{\gamma_A - 1}$	$\Pi(\psi) < 0$ i.e. $\psi < \theta$
...for Country B if:	$\frac{1}{\theta}\bar{m}^C < 0$	$0.5\Pi(\psi) + 0.25\Pi(\rho) < -0.25$	$\Pi(\rho) < 0$ i.e. $\rho < \theta$	$\frac{\Pi(\psi)}{\Pi(\rho)} > \frac{\gamma_A}{\gamma_A - 1}$	$\Pi(\rho) > \frac{\gamma_P}{\gamma_P - 1}$
Condition for Prisoner's Dilemma APP APP:	$\frac{1}{\theta}\bar{m}^C > 0$	$\Pi(\rho) > -1$	$\Pi(\rho) > 0$ i.e. $\rho > \theta$	$\Pi(\rho) > 0$ i.e. $\rho > \theta$	$\Pi(\rho) < \frac{\gamma_P}{\gamma_P - 1}$
Prisoner's Dilemma with Nash equilibrium APP APP possible?[14]	NO	YES	NO	YES	NO
It is a dominant strategy to peg...					
...for Country A if:	$\Pi(\psi) > 0$, i.e. $\psi > \theta$	$0.5\Pi(\psi) + 0.25\Pi(\rho) > 0.25$	$\Pi(\psi) > 0$, i.e. $\psi > \theta$ $\Pi(\rho) > 0$, i.e. $\rho > \theta$	$\frac{\Pi(\psi)}{\Pi(\rho)} < \frac{\gamma_A}{\gamma_A - 1}$	$\Pi(\psi) > 0$ i.e. $\psi > \theta$
...for Country B if:	$\frac{1}{\theta}\bar{m}^C > 0$	$0.5\Pi(\psi) + 0.25\Pi(\rho) > 0.25$	$\Pi(\rho) > 0$ i.e. $\rho > \theta$	$\frac{\Pi(\psi)}{\Pi(\rho)} < \frac{\gamma_A}{\gamma_A - 1}$	$\Pi(\rho) < \frac{\gamma_P}{\gamma_P - 1}$
Condition for Prisoner's Dilemma PEG PEG:	$\frac{1}{\theta}\bar{m}^C < 0$	$\Pi(\rho) < -1$	$\Pi(\rho) < 0$ i.e. $\rho < \theta$	$\Pi(\rho) < 0$ i.e. $\rho < \theta$	$\Pi(\rho) > \frac{\gamma_P}{\gamma_P - 1}$
Prisoner's Dilemma with Nash equilibrium PEG PEG possible?	NO	YES	NO	YES	NO

[14] i.e. are conditions for dominant strategy for Country A and B, and condition for Prisoner's Dilemma simultaneously satisfied?

1.3.5 No need for coordination

Since the Prisoner's Dilemma is a very special case which happens in the absence of large size asymmetries between countries, we would like to show as well, when counties may be sure that it will not arise. In related literature Obstfeld and Rogoff (1995) looked at economies, where countries do not specialize in production of certain types of goods. This assumption corresponds to condition on the parameters of our economy of: $\theta = \psi = \rho$, which means that goods produced at home and abroad are equally substitutable. We can show, that in such a world with equal elasticities of substitution, Prisoner's Dilemma will never happen.

Lemma 3. *(No need for coordination)* If goods produced at home, abroad and within the region are equally substitutable ($\theta = \rho = \psi$), then PEG PEG is a unique Nash equilibrium with positive payoffs and efficient Nash equilibrium. As a consequence there is no need for cooperation.

Proof: See Appendix 1.7.4.

This result is independent of the size of the Periphery as a whole, and size of individual country.

Table 1.6: Example of Nash equilibrium PEG PEG with no need for coordination $\theta = 3$, $\psi = 3$, $\rho = 3$, $\beta = 0.9$, $\gamma_P = 0.3$, $\gamma_A = 0.8$, $\bar{m}^C = 0.5$

		Country B: PEG	Country B: APP
Country A	PEG	**0.17** (A), **0.17** (B)	0.16 (A), 0.16 (B)
Country A	APP	0.13 (A), 0.13 (B)	0.12 (A), 0.12 (B)

In Table 1.6 we present a numerical example yielding possible payoffs in all strategies, with Nash equilibrium PEG PEG. Important thing to notice in this example, is that the payoffs of both Periphery countries, country A and country B, are the same. In fact, we can show that payoffs of all countries are the same, for example in Nash equilibrium: $u^A = 0.17$, $u^B = 0.17$, and $u^C = 0.17$ (by Lemma 1, if $\Pi(x) = 0$ then $u^i = u^j$). This case is the same as in Obstfeld and Rogoff (1995) where monetary expansion increases welfare in all countries, and because the payoffs are in 'pre capita' terms, every agent regardless of the country experiences proportional increase due to the same elasticity of substitution within and between the countries.

Since countries' payoffs form APP APP are decreasing in the size of the Periphery, the smaller the Periphery, the less can the Periphery countries improve by choosing PEG PEG. The gain from choosing PEG is greater for the larger Periphery country.

However, this particular example is not the only one in which the arising Nash equilibrium is optimal. Other examples can be found, especially in the absence of size asymmetries, which is when the conditions for Prisoner's Dilemma in Proposition 5 and Proposition 6 do not hold, an observation which follows from Table 1.5, and which we summarize in the following Remark 5. This result depends on the size asymmetry rather than conditions on the elasticities of substitution.

Remark 5. *(Role of asymmetries for need of coordination) Need for coordination is not likely in cases when there are big size asymmetries between countries.*

Because of large size asymmetries between Brazil and China (see Table 1.2) we may conclude that it was not an example of Prisoner's Dilemma, and therefore there is no scope for gains from coordination.

So far the choice of size parameters for examples has been rather arbitrary. Following the adopted relative size parameters as well as the functional form of the production function ($y_t = l_t$) we can encounter a complication in establishing the relative size of different countries. The production function implies that the relative size of the countries could be measured by the population size, and this should be proportional to the output, e.g. measured using GDP of the country.

To illustrate this complication we consider China and Brazil to be the Periphery countries, and the United States to be the Center. We use the actual data of population and GDP (as a measure of production) in the year 2013 from the World Bank. Table 1.7 presents the share of each country's population and GDP in the world population and world GDP, if we treat the world to be composed of three countries, as well as the share of Brazil's and China's population and GDP in the Periphery population and Periphery GDP.

While the share of China's and Brazil's population and GDP in the population and GDP of the Periphery is not significantly different when we compare these two different measures of relative size, the proportions of country's size in the world population and GDP vary significantly between those two measures. We conclude that a reasonable value for the parameter γ_A could be between $0.8 - 0.9$ in case of China. But the size

Table 1.7: Relative size of countries in 2013. Data: World Bank

	Population share (World)	GDP share (World)
USA	72 %	24%
Brazil	11 %	6%
China	17 %	71%
	Population share (Periphery)	GDP share (Periphery)
Brazil	13 %	19 %
China	87 %	81 %

of the Periphery as a whole, depends on the measure we adopt. If we decide to use the GDP data, the parameter γ_P could be around 0.3. A numerical example presented in this section corresponds to the data, where the size of the Periphery, γ_P is equal to 0.3, and the size of country A - China, represented with γ_A is equal to 0.3. If, however we decide to use the population data, then γ_P could be around 0.7. China and Brazil are only example countries, and this parametrization will of course vary with the choice of countries.

1.3.6 Coordination failure

Coordination[15] failure is a situation, in which there are multiple equilibria, and some of them are preferred by one or more players. In such cases players would achieve mutual gains from coordination, yet there are no individual incentives to do that. In our case, potential equilibria with coordination failure would happen, when it is optimal for both countries to maintain peg, if the other country does it, and to allow their currency to appreciate when the other country does it (i.e. Nash equilibria APP APP and PEG PEG - see example in Table 1.10 in the Appendix 1.6.3 where potential Nash equilibria are in gray cells). The other potential multiple equilibria could arise when each country plays the alternative strategy - different from the other country. The resulting equilibria would then be: APP PEG and PEG APP (see Table 1.11 in the Appendix 1.6.3). If coordination failure was possible no country should have a dominant strategy, as opposed to Prisoner's Dilemma.

Definition 8. Coordination failure *is a situation with more than one pure strategy Nash equilibrium, and some of the Nash equilibria are preferred by one or more players.*

[15]here we refer to coordination as a formal game theory term.

It turns out that the consequence of the restricted strategy set, multiple equilibria will never arise. The way that the strategy set, possibly resulting in coordination failure, could be expanded is explained in Remark 1.5.1) in the Appendix 1.5. Therefore the only role for external institutions to coordinate countries on equilibria with higher welfare is in case of Prisoner's Dilemma type of situations described before.

Lemma 4. *(Coordination failure)* Given the payoff structure of this game and available strategies, coordination failure will never occur.

Proof: See Appendix 1.7.5.

We anticipate that, if we allow for a richer strategy space, coordination failure could be possible (for details see Remark 1.5.1).

1.4 Conclusions

The analysis presented in this chapter has been motivated by the recent financial crisis of 2007-08. It is intended as an in depth analysis of the welfare effects of monetary policy decisions of big industrial economies on emerging economies. While there is evidence that "when the U.S. sneezes, emerging markets catch a cold" (Mackowiak (2007), p. 2513), the role of strategic interactions of emerging economies on their own welfare and that of their trading partners has been understated. These interactions among the emerging economies are therefore the focus of this study.

In this chapter we provide a game-theoretic framework to analyze the welfare effects of monetary expansion in industrial economies on emerging markets. We highlight the strategic aspect of interactions between emerging economies and the role of existing asymmetries between countries (see Remark 2). We show when such expansionary monetary policy may not be beggar-thy-neighbour, and when countries in the Periphery actually benefit from it (see Proposition 2).

We show that US policy recommendation for emerging economies to allow their currencies to appreciate is plausible only in special circumstances (see Proposition 3 and Remark 3).

We suggest a potential explanation as to why both Brazil and China increased their money supply in response to US expansionary monetary policy (see Corollary 1 of Propositions 1 and 2). While the size asymmetry between Brazil and China might help explain why China initially kept its exchange rate fixed vis-a-vis the US (see Remark 4 and Proposition 6), it remains a challenge to explain why Brazil has not done it.

Perhaps Brazil intended to limit the appreciation by increasing money supply, yet the size of monetary adjustment was not enough to keep the exchange rate fixed.

We show when countries, as a result of self-oriented policy, may end up in an equilibrium with lower welfare - so called Prisoner's Dilemma (see Propositions 5 and 6). We show that in such cases there is need for coordination between countries in the Periphery. In such situations, our framework can be used by institutions to highlight gains from coordination to individual countries, and advice policymakers to take actions toward improvement of global welfare. However, given the existing asymmetries between the countries considered here, we believe that Prisoner's Dilemma was not a result of Brazil's and China's policy decisions (see Remark 5).

In the future, one may want to look at the nonlinear model, in which it would be possible to work with continuous space of strategies to derive reaction functions and to allow for non-equiproportionate increases in money supply. This extension would further allow us to investigate the role of asymmetries in country choices, and their implications for countries' welfare. In particular it may help to explain Brazil's behaviour, which is difficult to explain using the model used here. Another interesting direction for further research would be to test this theory empirically. What is difficult in the context of a general equilibrium model, namely including more than two or three countries, can be done empirically using Global Vector Autoregressive model. We look at such analysis in the following chapter.

1.5 Appendix: Remarks

1.5.1 Remark to section 1.2.3

In this remark we explain possible alternative strategies for country A and B. We also point out technical difficulties that arise as a consequence of expanding the strategy set by including these alternatives.

It is reasonable to assume that after monetary expansion in the Center, countries in the Periphery may want to resist further inflationary pressures and wish to keep their money supply unchanged. The strategy of pegging can be motivated by the desire of avoiding currency appreciation in the Periphery, making it less competitive and leading to lower export of the Periphery goods.

Treating APP as an anchor strategy, in which a country chooses to keep its money supply at unchanged level, i.e. $\bar{m}^i = 0$ we can show that the exchange rate stability in country i, can be achieved by adjusting money supply, such that the change corresponds to the size of the shock in the Center, i.e. $\bar{m}^i = \bar{m}^C$, but not only in this way.

Using equation (1.138), which we repeat below for convenience, we can show what alternative choices of $\bar{m}^i$ allow countries to keep their exchange rate vis-a-vis the center at unchanged level, depending on the strategy:

$$e^A - e^B = \frac{1}{\psi}\frac{1-\beta+\psi(1+\beta)}{1+\beta+\psi(1-\beta)}(\bar{m}^A - \bar{m}^B)$$

- PEG APP corresponds to B's choice of keeping money supply at unchanged level, i.e. $\bar{m}^B = 0$, and A's choice of keeping exchange rate at unchanged level, i.e. $e^A = 0$. Rearranging equation (1.138), we can show that the corresponding value of A's money supply should satisfy $\bar{m}^A = \frac{-e^B}{\frac{1}{\psi}\frac{1-\beta+\psi(1+\beta)}{1+\beta+\psi(1-\beta)}}$
- APP PEG corresponds to A's choice of keeping money supply at unchanged level, i.e. $\bar{m}^A = 0$, and A's choice of keeping exchange rate at unchanged level, i.e. $e^B = 0$. Rearranging equation (1.138), we can show that the corresponding value of B's money supply should satisfy $\bar{m}^B = \frac{-e^A}{\frac{1}{\psi}\frac{1-\beta+\psi(1+\beta)}{1+\beta+\psi(1-\beta)}}$

Since by design of the game the result of APP PEG and PEG APP in country keeping the money supply at unchanged level is appreciation of it's exchange rate relative to the Center, i.e. $e^i < 0$, money supply in the country aiming to keep its exchange rate fixed is positive, and in special case can match monetary expansion in the Center.

Allowing $\bar{m}^i$ to take values other than $\bar{m}^C$ or 0 makes the analysis of payoffs in different strategies much more complicated than the one we undertake. Recall the expressions for payoffs of the Periphery countries:

$$u^A = \frac{1}{\theta}[\alpha_A^A \bar{m}^A + \alpha_B^A \bar{m}^B + \alpha_C^A \bar{m}^C] \tag{1.67}$$

$$u^B = \frac{1}{\theta}[\alpha_A^B \bar{m}^A + \alpha_B^B \bar{m}^B + \alpha_C^B \bar{m}^C] \tag{1.68}$$

where:

$\alpha_A^A = [[\gamma_P\gamma_A + (1-\gamma_P)\gamma_A\Pi(\rho) + (1-\gamma_A)\Pi(\psi)]$, $\alpha_B^A = [\gamma_P(1-\gamma_A) + (1-\gamma_P)(1-\gamma_A)\Pi(\rho) - (1-\gamma_A)\Pi(\psi)]$, $\alpha_C^A = [(1-\gamma_P)(1-\Pi(\rho))]$, $\alpha_A^B = [[\gamma_P\gamma_A + (1-\gamma_P)\gamma_A\Pi(\rho) - \gamma_A\Pi(\psi)]$, $\alpha_B^B = [\gamma_P(1-\gamma_A) + (1-\gamma_P)(1-\gamma_A)\Pi(\rho) + \gamma_A\Pi(\psi)]$, $\alpha_C^B = [(1-\gamma_P)(1-\Pi(\rho))]$.

In order to compare payoffs in different strategies, it would now be necessary to consider different values of $\bar{m}^i$ as well as α_i^j that vary with parameter values. In this extended cases we encounter huge technical difficulty, which would require considering much more cases that we are currently looking at. However, if we know the parameters characterizing economies, as well as their choices concerning money supply, our framework is still suitable to compute the payoffs.

Our choice of discrete strategy set has consequences on the existence of dominant strategies, and the existence of coordination failure.

1.6 Appendix: Additional Examples

1.6.1 Numerical example to section 1.3.1 - Beggar-thy-neighbour

Table 1.8: Example of asymmetries in size with negative payoff for the smaller country in PEG APP. Parameter values: $\theta = 2$, $\psi = 4$, $\rho = 2$, $\beta = 0.9$, $\gamma_P = 0.4$, $\gamma_A = 0.9$, $\bar{m}^C = 0.5$

		Country B	
		PEG	APP
Country A	PEG	0.25 0.25	-0.04 0.27
	APP	0.44 0.13	0.15 0.15

1.6.2 Numerical example to section 1.3.3 - Prisoner's Dilemma with negative payoffs in Nash equilibrium APP APP

Table 1.9: Example of Nash equilibrium APP APP with Prisoner's Dilemma and negative payoffs. Parameter values: $\theta = 1.1$, $\psi = 0.2$, $\rho = 6$, $\beta = 0.9$, $\gamma_P = 0.1$, $\gamma_A = 0.5$, $\bar{m}^C = 0.5$

		Country B	
		PEG	APP
Country A	PEG	0.45 0.45	0.60 -0.67
	APP	-0.67 0.60	**-0.52** **-0.52**

1.6.3 Payoff matrices for examples of coordination failure in section 1.3.6

Table 1.10: Possible coordination failure in a game with two Nash equilibria: PEG PEG and APP APP

		Country B	
		PEG	APP
Country A	PEG	$u^B_{PEG,PEG}$ $u^A_{PEG,PEG}$	$u^B_{PEG,APP}$ $u^A_{PEG,APP}$
	APP	$u^B_{APP,PEG}$ $u^A_{APP,PEG}$	$u^B_{APP,APP}$ $u^A_{APP,APP}$

Table 1.11: Possible coordination failure in a game with two Nash equilibria: APP PEG and PEG APP

		Country B	
		PEG	APP
Country A	PEG	$u^B_{PEG,PEG}$ $u^A_{PEG,PEG}$	$u^B_{PEG,APP}$ $u^A_{PEG,APP}$
	APP	$u^B_{APP,PEG}$ $u^A_{APP,PEG}$	$u^B_{APP,APP}$ $u^A_{APP,APP}$

1.7 Appendix: Proofs

1.7.1 Proof of Lemma 1 - Π

By looking at the graph of $\Pi(x) = \left(\frac{x-\theta}{x}\right)\left(\frac{1+x}{1+\beta+x(1-\beta)}\right)$, we can see that $\Pi(x)$ is strictly increasing in x on the two intervals that conform the domain of the function. This can also be seen in figures 1.6 and 1.7.

The values at $\Pi(x) = 1$ are shown at the intersection of the solid and the dashed lines on the Figure 1.6. It is clear that since the root of $\Pi(x) = 0$ on the domain $(0, \infty)$ lies at $x = \theta$. For $\Pi(x)$ to be greater than 1, then x needs to be sufficiently larger than θ. Equivalently, for $\Pi(x) < 1$, then x requires to be sufficiently smaller than θ. Figure 1.5 shows the relationship between x and θ, while Figure 1.6 shows the graph of $\Pi(x)$ for fixed θ (here $\theta = 2$).

Notice that function $\Pi(x)$ has an asymptote at $\frac{1}{(1-\beta)}$ as it can be seen in Figure 1.7. This can also be explicitly shown by taking the limit as x tends to infinity.

$$\begin{aligned}\lim_{x\to\infty} \Pi(x) &= \lim_{x\to\infty}\left(\frac{x-\theta}{x}\right)\left(\frac{1+x}{1+\beta+x(1-\beta)}\right) = \lim_{x\to\infty}\frac{x^2(\frac{1}{x}+1-\frac{\theta}{x^2}-\frac{\theta}{x})}{x^2(\frac{1}{x}+\frac{\beta}{x}+1-\beta)}\\ &= \frac{0+1-0+0}{0+0+1-\beta} = \frac{1}{1-\beta}\end{aligned}$$

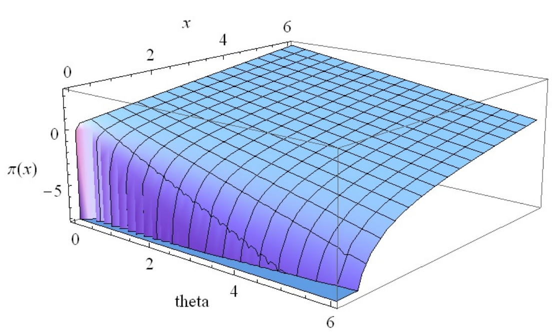

Figure 1.5: Plot of function $\Pi(x)$, for $\beta = 0.99$

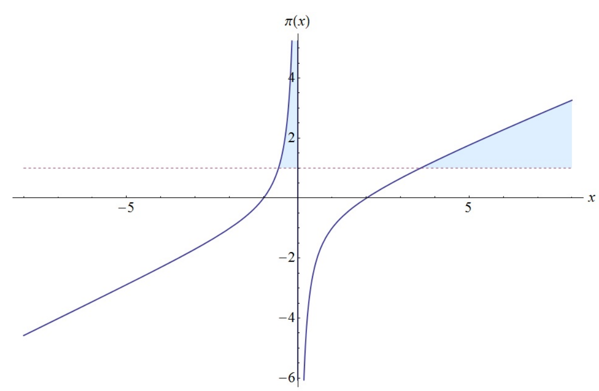

Figure 1.6: Plot of $\Pi(x)$ for fixed θ - solid line, $\Pi(x) = 1$ - dashed line ($\beta = 0.99$)

1.7.2 Proof of Corollary 1 - Uncertainty

Here we show that countries' payoffs in PEG PEG are always positive, independently of relative country size and substitutability parameters.

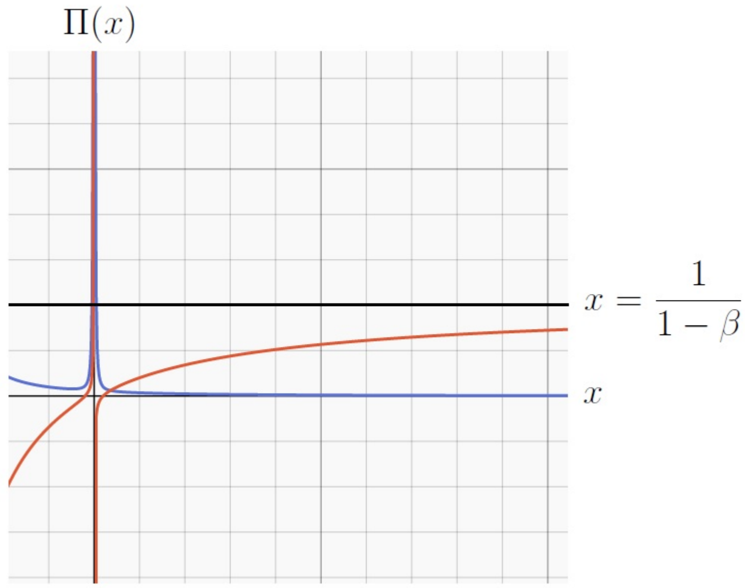

Figure 1.7: Function $\Pi(x)$ (red line), its derivative $\frac{\partial \Pi(x)}{\partial x}$ (blue line), and asymptote $x = \frac{1}{1-\beta} > 1$ (black line)

Proof. From (1.40) and (1.41) we can see that because we are only considering positive monetary expansion in the Center, i.e. $\bar{m}^C > 0$, and $\theta > 0$, $u^A_{PEG,PEG} = u^B_{PEG,PEG}$ is always positive.

Showing that other strategies may yield negative payoffs (see Proof of Proposition 1) completes the proof. $\square$

1.7.3 Proof of Lemma 2 - Beggar-thyself-C

In the following proof we show when monetary expansion in the Center can decrease C's utility depending on the action of the Periphery countries.

Proof. $u^C_{PEG,PEG} < 0 \iff \frac{1}{\theta}\bar{m}^C < 0$ which never holds, because $\theta > 0$, and $\bar{m}^C > 0$.

$u^C_{APP,APP} < 0 \iff \frac{1}{\theta}[1 - \gamma_P + \gamma_P \Pi(\rho)]\bar{m}^C < 0$, given that $\theta > 0$, and $\bar{m}^C > 0$ the condition simplifies to $1 - \gamma_P + \gamma_P \Pi(\rho) < 0 \iff \Pi(\rho) < \frac{\gamma_P - 1}{\gamma_P}$.

$u^C_{APP,PEG} < 0 \iff -\frac{1}{\theta}[1 - \gamma_A \gamma_P (1 - \Pi(\rho))]\bar{m}^C < 0$, given that $\theta > 0$, and $\bar{m}^C > 0$ the condition simplifies to $1 - \gamma_A \gamma_P (1 - \Pi(\rho)) < 0 \iff \Pi(\rho) < \frac{\gamma_P \gamma_A - 1}{\gamma_P \gamma_A}$.

$u^C_{PEG,APP} < 0 \iff \frac{1}{\theta}[1 + \gamma_P (1 - \gamma_A)(\Pi(\rho) - 1)]\bar{m}^C < 0$, given that $\theta > 0$, and $\bar{m}^C > 0$ the condition simplifies to $1 + \gamma_P (1 - \gamma_A)(\Pi(\rho) - 1)] \iff \Pi(\rho) < \frac{\gamma_P (1 - \gamma_A) - 1}{\gamma_P (1 - \gamma_A)}$. $\square$

1.7.4 Proof of Lemma 3 - No need for coordination

Proof. It is straightforward to show that given the payoffs with $\theta = \psi = \rho$, both countries have a dominant strategy to PEG, and that PEG PEG yields greater payoff than APP APP:

- Country A has a dominant strategy to PEG because, from (1.69) and (1.73): $\frac{1}{\theta}\bar{m}^C > \frac{1}{\theta}[1 + \gamma_P(\gamma_A - 1)]\bar{m}^C$, $1 > [1 + \gamma_P(\gamma_A - 1)]$, $0 > \gamma_P(\gamma_A - 1)$, which is always true since $0 < \gamma_A < 1$, and $0 < \gamma_P < 1$; and from (1.75) and (1.71): $\frac{1}{\theta}(1 - \gamma_A\gamma_P)\bar{m}^C > \frac{1}{\theta}(1 - \gamma_P)\bar{m}^C$, $(1 - \gamma_A\gamma_P) > (1 - \gamma_P)$, $\gamma_P > \gamma_A\gamma_P$, which is always true because $0 < \gamma_A < 1$,

- Country B has a dominant strategy to PEG because, from (1.70) and (1.76): $1 > (1-\gamma_P\gamma_A)$, $\gamma_P\gamma_A > 0$, and from (1.74) and (1.72): $[(1+\gamma_P(\gamma_A-1)] > (1-\gamma_P)$, $\gamma_P\gamma_A > 0$, both conditions aer always true since $0 < \gamma_A < 1$, and $0 < \gamma_P < 1$,

- PEG PEG yields greater payoff than APP APP: from (1.69), (1.70), (1.71), and (1.72) $\frac{1}{\theta}\bar{m}^C > \frac{1}{\theta}(1 - \gamma_P)\bar{m}^C$, which simplifies to $1 > 1 - \gamma_P$, and further the condition becomes $0 < \gamma_P$, which is always true, since $0 < \gamma_P < 1$.

□

Remark 6. *For convenience, we provide here the limits of expressions from section 1.2.4 used in this Proof.*

$$\lim_{\rho\to\psi\to\theta}(u^A_{PEG,PEG}) = \frac{1}{\theta}\bar{m}^C \tag{1.69}$$

$$\lim_{\rho\to\psi\to\theta}(u^B_{PEG,PEG}) = \frac{1}{\theta}\bar{m}^C \tag{1.70}$$

$$\lim_{\rho\to\psi\to\theta}(u^A_{APP,APP}) = \frac{1}{\theta}(1 - \gamma_P)\bar{m}^C \tag{1.71}$$

$$\lim_{\rho\to\psi\to\theta}(u^B_{APP,APP}) = \frac{1}{\theta}(1 - \gamma_P)\bar{m}^C \tag{1.72}$$

$$\lim_{\rho\to\psi\to\theta}(u^A_{APP,PEG}) = \frac{1}{\theta}[1 + \gamma_P(\gamma_A - 1)]\bar{m}^C \tag{1.73}$$

$$\lim_{\rho\to\psi\to\theta}(u^B_{APP,PEG}) = \frac{1}{\theta}[1 + \gamma_P(\gamma_A - 1)]\bar{m}^C \tag{1.74}$$

$$\lim_{\rho\to\psi\to\theta}(u^A_{PEG,APP}) = \frac{1}{\theta}[1-\gamma_A\gamma_P]\bar{m}^C \tag{1.75}$$

$$\lim_{\rho\to\psi\to\theta}(u^B_{PEG,APP}) = \frac{1}{\theta}[1-\gamma_A\gamma_P]\bar{m}^C \tag{1.76}$$

1.7.5 Proof of Lemma 4 - Coordination failure

Proof. Coordination failure with two equilibria APP APP and PEG PEG arises, if the following conditions hold simultaneously:

1. $u^A_{PEG,PEG} > u^A_{APP,PEG}$,
2. $u^A_{PEG,APP} < u^A_{APP,APP}$,
3. $u^B_{PEG,PEG} > u^B_{PEG,APP}$,
4. $u^B_{APP,PEG} < u^B_{APP,APP}$,

From 1. and using (1.40) and (1.49):

$$u^A_{PEG,PEG} - u^A_{APP,PEG} = \frac{1}{\theta}\bar{m}^C[\Pi(\psi) + \gamma_A(\gamma_P - \Pi(\psi) + \Pi(\rho) - \gamma_P\Pi(\rho))] > 0$$

and from 2. and using (1.46) and (1.43):

$$u^A_{PEG,APP} - u^A_{APP,APP} = \frac{1}{\theta}\bar{m}^C[\Pi(\psi) + \gamma_A(\gamma_P - \Pi(\psi) + \Pi(\rho) - \gamma_P\Pi(\rho))] < 0$$

which cannot hold simultaneously, therefore coordination failure with Nash equilibria APP APP and PEG PEG will never arise.

Coordination failure with two equilibria APP PEG and PEG APP arises, if the following conditions hold simultaneously:

1. $u^A_{PEG,PEG} < u^A_{APP,PEG}$,
2. $u^A_{PEG,APP} > u^A_{APP,APP}$,
3. $u^B_{PEG,PEG} < u^B_{PEG,APP}$,
4. $u^B_{APP,PEG} > u^B_{APP,APP}$,

From 1. and using (1.40) and (1.49):

$$u^A_{PEG,PEG} - u^A_{APP,PEG} = \frac{1}{\theta}\bar{m}^C[\Pi(\psi) + \gamma_A(\gamma_P - \Pi(\psi) + \Pi(\rho) - \gamma_P\Pi(\rho))] < 0$$

and from 2. and using (1.46) and (1.43):

$$u^A_{PEG,APP} - u^A_{APP,APP} = \frac{1}{\theta}\bar{m}^C[\Pi(\psi) + \gamma_A(\gamma_P - \Pi(\psi) + \Pi(\rho) - \gamma_P\Pi(\rho))] > 0$$

which cannot hold simultaneously, therefore coordination failure with Nash equilibria APP PEG and PEG APP will never arise.

□

1.7.6 Proof of Proposition 1 - Beggar-thy-neighbour

Proof. We want to check the following inequalities:

1. $u^A_{PEG,PEG} < 0$,
2. $u^B_{PEG,PEG} < 0$,
3. $u^A_{APP,APP} < 0$,
4. $u^B_{APP,APP} < 0$,
5. $u^A_{PEG,APP} < 0$,
6. $u^B_{PEG,APP} < 0$,
7. $u^A_{APP,PEG} < 0$,
8. $u^B_{APP,PEG} < 0$.

Conditions 1. and 2. never hold for positive monetary shock in the Center. From (1.40) and (1.41): $\frac{1}{\theta}\bar{m}^C < 0$ only, if there is negative monetary shock in the Center $\bar{m}^C < 0$. Conditions 2. and 3. hold if $\Pi(\rho) > 1$, which by property 5 of Lemma 1 happens when $\rho > \frac{\beta+\theta-\sqrt{\beta^2+6\beta\theta+\theta^2}}{2\beta}$. From (1.43) and (1.44), and using the fact that $\bar{m}^C > 0$, and $0 < \gamma_P < 1$: $\frac{1}{\theta}(\gamma_P - 1)(\Pi(\rho) - 1)\bar{m}^C < 0$, if $\Pi(\rho) > 1$.

Conditions 5.-8. are more complicated, so we look at extreme values of the country size.

First assume large Periphery, i.e. $\gamma_P \to 1$. From (1.49) we check when $[(\gamma_A - 1)(\Pi(\psi)-1)] < 0$, which for small country A simplifies to $\Pi(\psi) > 1$, which using Lemma 1 holds, if $\psi < \frac{\beta+\theta-\sqrt{\beta^2+6\beta\theta+\theta^2}}{2\beta}$. From (1.50) we check when $\frac{1}{\theta}\bar{m}^C[1+\gamma_A(\Pi(\psi)-1)] < 0$, which simplifies to $\Pi(\psi) < \frac{\gamma_A-1}{\gamma_A}$, which holds for ψ sufficiently smaller than θ (which is easier to fulfill as $\gamma_A \to 1$. From (1.46) we check when $\frac{1}{\theta}\bar{m}^C[\gamma_A + (1-\gamma_A)\Pi(\psi)] < 0$, which simplifies to $\Pi(\psi) < \frac{-\gamma_A}{1-\gamma_A}$, which holds for ψ sufficiently smaller than θ (which is easier to fulfill as $\gamma_A \to 1$. From (1.47) we check when $-\frac{1}{\theta}\bar{m}^C[\gamma_A(\Pi(\psi)-1)] < 0$, which simplifies to $\Pi(\psi) > 1$, which using Lemma 1 holds, if $\psi > \frac{\beta+\theta-\sqrt{\beta^2+6\beta\theta+\theta^2}}{2\beta}$.

Now assume small Periphery, i.e. $\gamma_P \to 0$. From (1.49) we want to check when $\frac{1}{\theta}\bar{m}^C[(1+(\gamma_A-1)\Pi(\psi)-\gamma_A\Pi(\rho)] < 0$, we can look at two extreme cases: small country A, i.e $\gamma_A \to 0$ or small country B $\gamma_A \to 1$. If $\gamma_A \to 0$, then the condition becomes $\Pi(\psi) > 1$, which by Lemma 1 holds, if $\psi > \frac{\beta+\theta-\sqrt{\beta^2+6\beta\theta+\theta^2}}{2\beta}$. If $\gamma_A \to 1$, then the condition becomes $\Pi(\rho) > 1$, which by Lemma 1 holds, if $\rho > \frac{\beta+\theta-\sqrt{\beta^2+6\beta\theta+\theta^2}}{2\beta}$. From (1.50) we want to check when $\frac{1}{\theta}\bar{m}^C[1+\gamma_A(\Pi(\psi)-\Pi(\rho))] < 0$, which does not hold for small country A, and holds for large country A, if $\Pi(\psi)-\Pi(\rho) < -1$. From (1.46) we want to check when $\frac{1}{\theta}\bar{m}^C[(1+(1-\gamma_A)\Pi(\psi)+(\gamma_A-1)\Pi(\rho)] < 0$, which does not hold for large country A, and holds for small country A, if $\Pi(\psi)-\Pi(\rho) < -1$. From (1.47) we want to check when $-\frac{1}{\theta}\bar{m}^C[\gamma_A(\Pi(\psi)-\Pi(\rho))+\Pi(\rho)-1] < 0$. If $\gamma_A \to 1$, then the condition becomes $\Pi(\psi) > 1$, which by Lemma 1 holds, if $\psi > \frac{\beta+\theta-\sqrt{\beta^2+6\beta\theta+\theta^2}}{2\beta}$. If $\gamma_A \to 0$, then the condition becomes $\Pi(\rho) > 1$, which by Lemma 1 holds, if $\rho > \frac{\beta+\theta-\sqrt{\beta^2+6\beta\theta+\theta^2}}{2\beta}$.

□

Remark 7. *For convenience, we provide here the limits of expressions from section 1.2.4 used in this Proof.*

$$\lim_{\gamma_P \to 1}(u^A_{APP,PEG}) = \frac{1}{\theta}\bar{m}^C[(\gamma_A-1)(\Pi(\psi)-1)] \tag{1.77}$$

$$\lim_{\gamma_P \to 1}(u^B_{APP,PEG}) = \frac{1}{\theta}\bar{m}^C[1+\gamma_A(\Pi(\psi)-1)] \tag{1.78}$$

$$\lim_{\gamma_P \to 1}(u^A_{PEG,APP}) = \frac{1}{\theta}\bar{m}^C[\gamma_A+(1-\gamma_A)\Pi(\psi)] \tag{1.79}$$

$$\lim_{\gamma_P \to 1}(u^B_{PEG,APP}) = -\frac{1}{\theta}\bar{m}^C[\gamma_A(\Pi(\psi)-1)] \tag{1.80}$$

$$\lim_{\gamma_P \to 0}(u^A_{APP,PEG}) = \frac{1}{\theta}\bar{m}^C[(1 + (\gamma_A - 1)\Pi(\psi) - \gamma_A\Pi(\rho)] \tag{1.81}$$

$$\lim_{\gamma_P \to 0}(u^B_{APP,PEG}) = \frac{1}{\theta}\bar{m}^C[1 + \gamma_A(\Pi(\psi) - \Pi(\rho))] \tag{1.82}$$

$$\lim_{\gamma_P \to 0}(u^A_{PEG,APP}) = \frac{1}{\theta}\bar{m}^C[(1 + (1 - \gamma_A)\Pi(\psi) + (\gamma_A - 1)\Pi(\rho)] \tag{1.83}$$

$$\lim_{\gamma_P \to 0}(u^B_{PEG,APP}) = -\frac{1}{\theta}\bar{m}^C[\gamma_A(\Pi(\psi) - \Pi(\rho)) + \Pi(\rho) - 1] \tag{1.84}$$

1.7.7 Proof of Proposition 2 - Enrich-thy-neighbour

Proof. We want to check the following inequalities:

1. $u^A_{PEG,PEG} > 0$
2. $u^B_{PEG,PEG} > 0$
3. $u^A_{APP,APP} > 0$
4. $u^B_{APP,APP} > 0$
5. $u^A_{PEG,APP} > 0$
6. $u^B_{PEG,APP} > 0$
7. $u^A_{APP,PEG} > 0$
8. $u^B_{APP,PEG} > 0$

These are exactly the exact opposite to conditions in the previous proof 2, and this follows by the same reasoning. □

1.7.8 Proof of Proposition 3 - Dominant strategies to appreciate

Proof. For country A and B to have a dominant strategy to appreciate, the following inequalities have to hold simultaneously:

1. $u^A_{PEG,PEG} < u^A_{APP,PEG}$,

2. $u^A_{PEG,APP} < u^A_{APP,APP}$,

3. $u^B_{PEG,PEG} < u^B_{PEG,APP}$,

4. $u^B_{APP,PEG} < u^B_{APP,APP}$.

From 1. and 2. we get condition (1.56) for Country A, and from 3. and 4. we get condition (1.57) for Country B.

□

Remark 8. *To look at the role of asymmetries in country size for these results, we split these cases into subcases, depending on the relative size of the countries, and consider the limits of expressions used in the inequalities above.*

"Symmetry" in size

First we consider the case, in which Periphery is half the size of the world economy and each periphery country is half the size of the Periphery, i.e. $\gamma_P = 0.5$, *and* $\gamma_A = 0.5$.

1. $\frac{1}{\theta}\bar{m}^C < \frac{1}{\theta}\bar{m}^C[(0.75 - 0.5\Pi(\psi) - 0.25\Pi(\rho)]$

2. $\frac{1}{\theta}\bar{m}^C[(0.75 + 0.5\Pi(\psi) - 0.25\Pi(\rho)] < 0.5\frac{1}{\theta}\bar{m}^C[1 - \Pi(\rho)]$

3. $\frac{1}{\theta}\bar{m}^C < \frac{1}{\theta}\bar{m}^C[(0.75 - 0.5\Pi(\psi) - 0.25\Pi(\rho)]$

4. $\frac{1}{\theta}\bar{m}^C[(0.75 + 0.5\Pi(\psi) - 0.25\Pi(\rho)] < 0.5\frac{1}{\theta}\bar{m}^C[1 - \Pi(\rho)]$

Since inequality 1. and 3., and 2. and 4. are the same we actually need to look at 2 inequalities. From 1. $0.5\Pi(\psi) + 0.25\Pi(\rho) < 0.25$, *and from 2.* $0.5\Pi(\psi) + 0.25\Pi(\rho) < -0.25$, *which is a stronger condition than 1. If both* $\Pi(\psi)$ *and* $\Pi(\rho)$ *are sufficiently negative (i.e. x sufficiently smaller than θ), then this condition holds. Also this condition might hold, if at least one* Π *is sufficiently negative, with the other being positive.*

Large Periphery - small country A

Here we consider a case of large Periphery, i.e. $\gamma_P \to 1$ *and asymmetry in size between the countries in the Periphery such that* $\gamma_A \to 0$.

1. $\frac{1}{\theta}\bar{m}^C < \frac{1}{\theta}[1 - \Pi(\psi)]\bar{m}^C$

2. $\frac{1}{\theta}\Pi(\psi)\bar{m}^C < 0$

3. $\frac{1}{\theta}\bar{m}^C < 0$

4. $\frac{1}{\theta}\bar{m}^C < 0$

Conditions 3. and 4. are violated, since both θ *and* $\bar{m}^C$ *are larger than zero. Therefore, APP APP is never a dominant strategy when the Periphery is large, and one of the Periphery countries is very small.*

Small Periphery - small country A

Here we consider a case of small Periphery, i.e. $\gamma_P \to 0$ *and asymmetry in size between the countries in the Periphery such that* $\gamma_A \to 0$.

1. $\frac{1}{\theta}\bar{m}^C < \frac{1}{\theta}[1 - \Pi(\rho)]\bar{m}^C$

2. $\frac{1}{\theta}[1 + \Pi(\psi) - \Pi(\rho)]\bar{m}^C < \frac{1}{\theta}[1 - \Pi(\rho)]\bar{m}^C$

3. $\frac{1}{\theta}\bar{m}^C < \frac{1}{\theta}[1 - \Pi(\rho)]\bar{m}^C$

4. $\frac{1}{\theta}\bar{m}^C < \frac{1}{\theta}[1 - \Pi(\rho)]\bar{m}^C$

From 1., 3. and 4. $\Pi(\rho) < 0$, *and from 2.* $\Pi(\psi) < 0$. *By Lemma 1* $\Pi(x) < 0$, *if* $x < \theta$. *Therefore, if the Periphery is small relative to the Center, and the substitutability of goods within the country is greater than between the countries, APP APP is a dominant strategy for both large and small Periphery country.*

Small country A

When country A is small, i.e. $\gamma_A \to 0$ *the conditions become:*

1. $\frac{1}{\theta}\bar{m}^C < \frac{1}{\theta}\bar{m}^C[1 - \Pi(\psi)]$

2. $u\frac{1}{\theta}\bar{m}^C[(1 + \Pi(\psi) + \gamma_P(\Pi(\rho) - 1) - \Pi(\rho)] < \frac{1}{\theta}\bar{m}^C(\gamma_P - 1)(\Pi(\rho) - 1)$

3. $\frac{1}{\theta}\bar{m}^C < \frac{1}{\theta}\bar{m}^C(\gamma_P - 1)(\Pi(\rho) - 1)$

4. $\frac{1}{\theta}\bar{m}^C < \frac{1}{\theta}\bar{m}^C(\gamma_P - 1)(\Pi(\rho) - 1)$

From 1. and 2. $\Pi(\psi) < 0$. *From 3. and 3.* $\frac{\gamma_P}{\gamma_P - 1} < \Pi(\rho)$.

Small Periphery

When Periphery as a whole is small, i.e. $\gamma_P \to 0$ the conditions become:

1. $\frac{1}{\theta}\bar{m}^C < \frac{1}{\theta}\bar{m}^C[(1+(\gamma_A-1)\Pi(\psi)-\gamma_A\Pi(\rho)]$

2. $\frac{1}{\theta}\bar{m}^C[(1+(1-\gamma_A)\Pi(\psi)+(\gamma_A-1)\Pi(\rho)] < \frac{1}{\theta}\bar{m}^C[1-\Pi(\rho)]$

3. $\frac{1}{\theta}\bar{m}^C < -\frac{1}{\theta}\bar{m}^C[\gamma_A(\Pi(\psi)-\Pi(\rho))+\Pi(\rho)-1]$

4. $\frac{1}{\theta}\bar{m}^C[1+\gamma_A(\Pi(\psi)-\Pi(\rho))] < \frac{1}{\theta}\bar{m}^C[1-\Pi(\rho)]$

From 1. and 2. $\frac{\gamma_A}{\gamma_A-1} < \frac{\Pi(\psi)}{\Pi(\rho)}$*. From 3. and 4.* $\frac{\gamma_A}{\gamma_A-1} < \frac{\Pi(\rho)}{\Pi(\psi)}$*. For example, this is true if* $\Pi(\psi)=\Pi(\rho)$*.*

1.7.9 Proof of Proposition 4 - Dominant strategies to peg

Proof. Country A and B will heave dominant strategy PEG PEG, if the following inequalities hold simultaneously:

1. $u^A_{PEG,PEG} > u^A_{APP,PEG}$,

2. $u^A_{PEG,APP} > u^A_{APP,APP}$,

3. $u^B_{PEG,PEG} > u^B_{PEG,APP}$,

4. $u^B_{APP,PEG} > u^B_{APP,APP}$.

From 1. and 2. we get condition (1.58) for Country A, and from 3. and 4. we get condition (1.59) for country B.

□

Remark 9. *To look at the role of asymmetries in country size for these results, we split these cases into subcases, depending on the relative size of the countries, and consider the limits of expressions used in the inequalities above.*

"Symmetry" in size

First we consider the case, in which Periphery is half the size of the world economy and each periphery country is half the size of the Periphery, i.e. $\gamma_P = 0.5$*, and* $\gamma_A = 0.5$*.*

1. $\frac{1}{\theta}\bar{m}^C > \frac{1}{\theta}\bar{m}^C[(0.75-0.5\Pi(\psi)-0.25\Pi(\rho)]$

2. $\frac{1}{\theta}\bar{m}^C[(0.75 + 0.5\Pi(\psi) - 0.25\Pi(\rho)] > 0.5\frac{1}{\theta}\bar{m}^C[1 - \Pi(\rho)]$

3. $\frac{1}{\theta}\bar{m}^C > \frac{1}{\theta}\bar{m}^C[(0.75 - 0.5\Pi(\psi) - 0.25\Pi(\rho)]$

4. $\frac{1}{\theta}\bar{m}^C[(0.75 + 0.5\Pi(\psi) - 0.25\Pi(\rho)] > 0.5\frac{1}{\theta}\bar{m}^C[1 - \Pi(\rho)]$

Since condition 1. and 3., and 2. and 4. are the same we actually need to look at two conditions only. From 1. $0.5\Pi(\psi)+0.25\Pi(\rho) > 0.25$*, and from 2.* $0.5\Pi(\psi)+0.25\Pi(\rho) > -0.25$*. Since condition 1. is stronger than 2., countries will prefer to PEG, if both* $\Pi(\psi)$ *and* $\Pi(\rho)$ *are sufficiently positive (i.e.* x *sufficiently larger than* θ*), or* $\Pi(\psi)$ *sufficiently larger than* $\Pi(\rho)$*.*

Large Periphery - small country A

Here we consider a case of large Periphery, i.e. $\gamma_P \to 1$ *and asymmetry in size between the countries in the Periphery such that* $\gamma_A \to 0$*.*

1. $\frac{1}{\theta}\bar{m}^C > \frac{1}{\theta}[1 - \Pi(\psi)]\bar{m}^C$

2. $\frac{1}{\theta}\Pi(\psi)\bar{m}^C > 0$

3. $\frac{1}{\theta}\bar{m}^C > 0$

4. $\frac{1}{\theta}\bar{m}^C > 0$

From 1. and 2. $\Pi(\psi) > 0$*, and 3. and 4. are always true. Therefore, if the Periphery is large and countries in the Periphery are of different size, PEG PEG is a dominant strategy when* $\psi > \theta$*.*

Small Periphery - small country A

Here we consider a case of small Periphery, i.e. $\gamma_P \to 0$ *and asymmetry in size between the countries in the Periphery such that* $\gamma_A \to 0$*.*

1. $\frac{1}{\theta}\bar{m}^C > \frac{1}{\theta}[1 - \Pi(\rho)]\bar{m}^C$

2. $\frac{1}{\theta}[1 + \Pi(\psi) - \Pi(\rho)]\bar{m}^C > \frac{1}{\theta}[1 - \Pi(\rho)]\bar{m}^C$

3. $\frac{1}{\theta}\bar{m}^C > \frac{1}{\theta}[1 - \Pi(\rho)]\bar{m}^C$

4. $\frac{1}{\theta}\bar{m}^C > \frac{1}{\theta}[1 - \Pi(\rho)]\bar{m}^C$

From 1., 3. and 4. $\Pi(\rho) > 0$, *and from (2)* $\Pi(\psi) > 0$. *By Lemma 1* $\Pi(x) > 0$, *if* $x > \theta$. *Therefore, if the Periphery is small relative to the Center, and the substitutability of goods within the country is smaller than between the countries, PEG PEG is a dominant strategy for both large and small Periphery country.*

Small country A

When country A is small, i.e. $\gamma_A \to 0$ *the conditions become:*

1. $\frac{1}{\theta}\bar{m}^C > \frac{1}{\theta}\bar{m}^C[1 - \Pi(\psi)]$
2. $\frac{1}{\theta}\bar{m}^C[(1 + \Pi(\psi) + \gamma_P(\Pi(\rho) - 1) - \Pi(\rho)] > \frac{1}{\theta}\bar{m}^C(\gamma_P - 1)(\Pi(\rho) - 1)$
3. $\frac{1}{\theta}\bar{m}^C > \frac{1}{\theta}\bar{m}^C(\gamma_P - 1)(\Pi(\rho) - 1)$
4. $\frac{1}{\theta}\bar{m}^C > \frac{1}{\theta}\bar{m}^C(\gamma_P - 1)(\Pi(\rho) - 1)$

From 1. and 2. $\Pi(\psi) > 0$. *From 3. and 4.* $\frac{\gamma_P}{\gamma_P - 1} > \Pi(\rho)$.

Small Periphery

When Periphery as a whole is small, i.e. $\gamma_P \to 0$ *the conditions become:*

1. $\frac{1}{\theta}\bar{m}^C > \frac{1}{\theta}\bar{m}^C[(1 + (\gamma_A - 1)\Pi(\psi) - \gamma_A\Pi(\rho)]$
2. $\frac{1}{\theta}\bar{m}^C[(1 + (1 - \gamma_A)\Pi(\psi) + (\gamma_A - 1)\Pi(\rho)] > \frac{1}{\theta}\bar{m}^C[1 - \Pi(\rho)]$
3. $\frac{1}{\theta}\bar{m}^C > -\frac{1}{\theta}\bar{m}^C[\gamma_A(\Pi(\psi) - \Pi(\rho)) + \Pi(\rho) - 1]$
4. $\frac{1}{\theta}\bar{m}^C[1 + \gamma_A(\Pi(\psi) - \Pi(\rho))] > \frac{1}{\theta}\bar{m}^C[1 - \Pi(\rho)]$

From 1. and 2. $\frac{\gamma_A}{\gamma_A - 1} > \frac{\Pi(\psi)}{\Pi(\rho)}$. *From 3. and 4.* $\frac{\gamma_A}{\gamma_A - 1} > \frac{\Pi(\rho)}{\Pi(\psi)}$. *For example, this is true if* $\Pi(\psi) = \Pi(\rho)$.

1.7.10 Proof of Proposition 5 - Prisoner's Dilemma with appreciation

Proof. We want to check the following conditions:

1. $u^A_{PEG,PEG} < u^A_{APP,PEG}$,
2. $u^A_{PEG,APP} < u^A_{APP,APP}$,

3. $u^B_{PEG,PEG} < u^B_{PEG,APP}$,

4. $u^B_{APP,PEG} < u^B_{APP,APP}$,

5. $u^A_{APP,APP} < u^A_{PEG,PEG}$ and $u^B_{APP,APP} < u^B_{PEG,PEG}$.

From 1. and 2. we get condition (1.56) for Country A, from 3. and 4. we get condition (1.57) for Country B, and from 5. we get condition (1.60).

Conditions 1.-4. are the same as in Proof of Proposition 3. Since payoffs in country payoffs in APP APP and PEG PEG are independent of relative size of A and B, only relative size of the Periphery as a whole, and thus symmetric, condition 5. reduces to $u^A_{APP,APP} < u^A_{PEG,PEG}$.

$\square$

Remark 10. *We split these cases into subcases, depending on the relative size of the countries to look at the role of asymmetries in size, as we did in the proof for dominant strategies.*

"Symmetry" in size

If the Periphery is half the size of the world economy and each periphery country is half the size of the Periphery, i.e. $\gamma_P = 0.5$*, and* $\gamma_A = 0.5$*, then condition 5. becomes:*

$$0.5\frac{1}{\theta}\bar{m}^C[1 - \Pi(\rho)] < \frac{1}{\theta}\bar{m}^C$$

which holds when $\Pi(\rho) > -1$*, therefore Prisoner's Dilemma is possible.*

Large Periphery - small country A

Condition 5. becomes:

$$0 < \frac{1}{\theta}\bar{m}^C$$

which always holds, but this is never a dominant strategy, therefore Prisoner's Dilemma is not possible.

Small Periphery - small country A

Condition 5. becomes:

$$\frac{1}{\theta}[1-\Pi(\rho)]\bar{m}^C < \frac{1}{\theta}\bar{m}^C$$

which holds when $\Pi(\rho) > 0$*, which by Lemma 1 holds, if* $\rho > \theta$*.*

Small country A

Condition 5. becomes:

$$\frac{1}{\theta}\bar{m}^C(\gamma_P - 1)(\Pi(\rho) - 1) < \frac{1}{\theta}\bar{m}^C$$

which holds when $\Pi(\rho) < \frac{\gamma_P}{\gamma_P - 1}$.

Small Periphery

Condition 5. becomes:

$$\frac{1}{\theta}\bar{m}^C[1-\Pi(\rho)] < \frac{1}{\theta}\bar{m}^C$$

which holds when $\Pi(\rho) > 0$*, which by Lemma 1 holds, if* $\rho > \theta$*.*

Summary of Prisoner's Dilemma conditions is presented in Table 1.5.

1.7.11 Proof of Proposition 6 - Prisoner's Dilemma with peg

Proof. For Prisoner's Dilemma with Nash equilibrium PEG PEG to arise, the following inequalities have to hold simultaneously:

1. $u^A_{PEG,PEG} > u^A_{APP,PEG}$,
2. $u^A_{PEG,APP} > u^A_{APP,APP}$,
3. $u^B_{PEG,PEG} > u^B_{PEG,APP}$,
4. $u^B_{APP,PEG} > u^B_{APP,APP}$,
5. $u^A_{APP,APP} > u^A_{PEG,PEG}$ and $u^B_{APP,APP} > u^B_{PEG,PEG}$.

From 1. and 2. we get condition (1.58), from 3. and 4. we get condition (1.59), and from 5. we get condition (1.66). Conditions 1.-4. are the same as in Proof of Proposition 3. Since payoffs in country payoffs in APP APP and PEG PEG are independent of relative size of A and B, only relative size of the Periphery as a whole, and thus symmetric, condition 5. reduces to $u^A_{APP,APP} < u^A_{PEG,PEG}$.

$\square$

Remark 11. *We split these cases into subcases, depending on the relative size of the countries, as we did in the proof for dominant strategies.*

"Symmetry" in size

If the Periphery is half the size of the world economy and each periphery country is half the size of the Periphery, i.e. $\gamma_P = 0.5$, *and* $\gamma_A = 0.5$, *then condition 5. becomes:*

$$0.5\frac{1}{\theta}\bar{m}^C[1 - \Pi(\rho)] > \frac{1}{\theta}\bar{m}^C$$

which holds when $\Pi(\rho) < -1$.

Large Periphery - small country A

Condition 5. becomes:

$$0 > \frac{1}{\theta}\bar{m}^C$$

which never holds.

Small Periphery - small country A

Condition 5. becomes:

$$\frac{1}{\theta}[1 - \Pi(\rho)]\bar{m}^C > \frac{1}{\theta}\bar{m}^C$$

which holds when $\Pi(\rho) < 0$, *which by Lemma 1 holds, if* $\rho < \theta$.

Small country A

Condition 5. becomes:

$$\frac{1}{\theta}\bar{m}^C(\gamma_P - 1)(\Pi(\rho) - 1) > \frac{1}{\theta}\bar{m}^C$$

which holds when $\Pi(\rho) > \frac{\gamma_P}{\gamma_P - 1}$.

Small Periphery

Condition 5. becomes:

$$\frac{1}{\theta}\bar{m}^C[1 - \Pi(\rho)] > \frac{1}{\theta}\bar{m}^C$$

which holds when $\Pi(\rho) < 0$*, which by Lemma 1 holds, if* $\rho < \theta$*.*

Summary of Prisoner's Dilemma conditions is presented in Table 1.5.

1.8 Appendix: Log-linearized model

In a symmetric steady state, around which the model is linearized, the real interest rate equals the discount rate:

$$\beta = \frac{1}{1+i}$$

.

In the initial steady state, both consumption and output are constant are equal:

$$C_0 = Y_0 = \sqrt{\frac{\theta - 1}{\theta \kappa}}$$

Then we can use the log-linear approximation of a variable around it's equilibrium steady-state value:

$$x \approx \frac{X - X_0}{X_0}$$

where the lowercase letter means the approximated value, X_0 - steady state value, and X - actual value of a variable. Variables with upperbars will denote long run variables and without, short run ones.

And since bond holdings are assumed to be zero:

$$B^A = B^B = B^P = B^C = 0$$

the log linear approximation in this case will be defined as:

$$\bar{b}^j = \frac{B^j}{P_0^C C_0}$$

All of the following expressions come from the Corsetti et.al. (1999) Corsetti et al. (1999b). The following notation is used: x for the short run percentage deviations from the initial steady state, and $\bar{x}$ for the long run percentage deviations from the initial steady state. More details of the model can be found in one of these: Corsetti et al. (2000), Corsetti et al. (1999a) or Corsetti et al. (1999b).

$$x^P \equiv \gamma_A x^A + (1 - \gamma_A) x^B \tag{1.85}$$

$$x^w \equiv \gamma_P x^P + (1 - \gamma_P) x^C \tag{1.86}$$

that solve for the country-specific variables we will use the following decompositions:

$$x^C = x^w - \gamma_P(x^P - x^C)$$

$$x^P = x^w + (1-\gamma_P)(x^P - x^C)$$

$$x^A = x^P + (1-\gamma_A)(x^A - x^B)$$

$$x^B = x^P - \gamma_A(x^A - x^B)$$

1.8.1 Long run

To derive the long rum solution we will start by log linearizing money demand and pricing rule equations:

$$\bar{m}^j - \bar{p}^j = \bar{c}^j \tag{1.87}$$

$$\bar{p}^j_j - \bar{p}^j = \bar{c}^j + \bar{y}^j \tag{1.88}$$

Then from price index equations we can write:

$$\bar{p}^j_P = \gamma_A \bar{p}^j_A + (1-\gamma_A)\bar{p}^j_B \tag{1.89}$$

and

$$\bar{p}^j = \gamma_P \bar{p}^j_P + (1-\gamma_P)\bar{p}^j_C \tag{1.90}$$

The log linearized output demands are:

$$\bar{y}^A = -\psi(\bar{p}^A_A - \bar{p}^A_P) + \bar{y}^P \tag{1.91}$$

$$\bar{y}^B = -\psi(\bar{p}^B_B - \bar{p}^B_P) + \bar{y}^P \tag{1.92}$$

$$\bar{y}^P = -\rho(\bar{p}^C_P - \bar{p}^C) + \bar{c}^w \tag{1.93}$$

$$\bar{y}^C = -\rho(\bar{p}^C_C - \bar{p}^C) + \bar{c}^w \tag{1.94}$$

From sales revenue and current accounts (Equations are derived as in Obstfeld and

Rogoff (1996) Obstfeld and Rogoff (1996) by integrating over time the period budget constraint of the individual, and imposing transversality condition. In this way the present value of the lifetime constraint is obtained, which after imposing money-market equilibrium reduces to $\bar{C}^A = i\bar{B}^A + \frac{\bar{P}^A_A}{\bar{P}_A}\bar{Y}^A$ then notice that $\frac{1-\beta}{\beta} = i$ and log linearize.)

$$0 = \frac{1-\beta}{\beta}\bar{b}^A - \bar{c}^A + \bar{p}^A_A - \bar{p}^A + \bar{y}^A \tag{1.95}$$

$$0 = \frac{1-\beta}{\beta}\bar{b}^B - \bar{c}^B + \bar{p}^B_B - \bar{p}^B + \bar{y}^B \tag{1.96}$$

$$0 = \frac{1-\beta}{\beta}\bar{b}^C - \bar{c}^C + \bar{p}^C_C - \bar{p}^C + \bar{y}^C \tag{1.97}$$

$$0 = \gamma_A\gamma_P\bar{b}^A + \gamma_B\gamma_P\bar{b}^B + \gamma_C\bar{b}^C \tag{1.98}$$

Center-Periphery

From pricing equation, output demand and current account equations we can write:

$$(\bar{p}^P_P - \bar{p}^C_C) - (\bar{p}^P - \bar{p}^C) = (\bar{c}^P - \bar{c}^C) + (\bar{y}^P - \bar{y}^C) \tag{1.99}$$

$$(\bar{y}^P - \bar{y}^C) = -\rho(\bar{p}^C_P - \bar{p}^C_C) \tag{1.100}$$

$$(\bar{c}^P - \bar{c}^C) = \frac{1-\beta}{\beta}\frac{\bar{b}^P}{(1-\gamma_P)} + (\bar{y}^P - \bar{y}^C) + (\bar{p}^C_P - \bar{p}^C_C) + (\bar{p}^P - \bar{p}^C) \tag{1.101}$$

And when the law of one price holds:

$$\bar{p}^C_P + \bar{e}^P = \bar{p}^P_P \tag{1.102}$$

$$\bar{p}^C + \bar{e}^P = \bar{p}^P \tag{1.103}$$

$$(\bar{p}^P_P - \bar{p}^C_C - \bar{e}^P) = (\bar{c}^P - \bar{c}^C) + (\bar{y}^P - \bar{y}^C) \tag{1.104}$$

$$(\bar{y}^P - \bar{y}^C) = -\rho(\bar{p}^P_P - \bar{p}^C_C - \bar{e}^P) \tag{1.105}$$

$$(\bar{c}^P - \bar{c}^C) = \frac{1-\beta}{\beta} + (\bar{p}_P^P - \bar{p}_C^C - \bar{e}^P) + (\bar{y}^P - \bar{y}^C) \tag{1.106}$$

And the long run solutions are:

$$(\bar{y}^P - \bar{y}^C) = -\frac{1}{2}\frac{1-\beta}{\beta}\frac{\bar{b}^P}{(1-\gamma_P)} \tag{1.107}$$

$$(\bar{c}^P - \bar{c}^C) = \frac{1+\rho}{2\rho}\frac{1-\beta}{\beta}\frac{\bar{b}^P}{(1-\gamma_P)} \tag{1.108}$$

$$(\bar{p}_P^P - \bar{p}_C^C - \bar{e}^P) = \frac{1}{2\rho}\frac{1-\beta}{\beta}\frac{\bar{b}^P}{(1-\gamma_P)} \tag{1.109}$$

Intra-Periphery

Intra-Periphery solutions are obtained as the Center-Periphery solutions.

$$(\bar{c}^A - \bar{c}^B) = \frac{1+\psi}{2\psi}\frac{1-\beta}{\beta}\frac{\bar{b}^A - \bar{b}^B}{(1-\gamma_A)} \tag{1.110}$$

$$(\bar{p}_A^A - \bar{p}_B^B) - (\bar{e}^A - \bar{e}^B) = \frac{1}{2\psi}\frac{1-\beta}{\beta}\frac{\bar{b}^A - \bar{b}^B}{(1-\gamma_A)} \tag{1.111}$$

$$(\bar{y}^A - \bar{y}^B) = -\frac{1}{2}\frac{1-\beta}{\beta}\frac{\bar{b}^A - \bar{b}^B}{(1-\gamma_A)} \tag{1.112}$$

1.8.2 Short run

In the short run prices are fixed at the following levels:

$$\bar{p}_P^A = (1-\gamma_A)(e^A - e^B) \tag{1.113}$$

$$\bar{p}_P^B = -\gamma_A(e^A - e^B) \tag{1.114}$$

$$\bar{p}_P^C = \gamma_A e^A + (1-\gamma_A)e^B = e^P \tag{1.115}$$

$$\bar{p}^A = \gamma_P(1-\gamma_A)(e^A - e^B) + (1-\gamma_P)e^A = e^A - \gamma_P e^P \tag{1.116}$$

$$\bar{p}^B = -\gamma_P \gamma_A (e^A - e^B) + (1 - \gamma_P) e^B = e^B - \gamma_P e^P \tag{1.117}$$

$$\bar{p}^C = -\gamma_P e^P \tag{1.118}$$

$$\bar{c}^j - c^j = \beta di + p^C - \bar{p}^C \tag{1.119}$$

$$\bar{m}^j - p^j = c^j + \frac{\beta}{1 - \beta}(e^j - \bar{e}^j - \beta di) \tag{1.120}$$

$$c^j - c^k = \bar{c}^j - \bar{c}^k \tag{1.121}$$

$$e^j - e^k = \bar{e}^j - \bar{e}^k \tag{1.122}$$

Output demands:

$$y^A = \psi(1 - \gamma_A)(e^A - e^B) + y^P \tag{1.123}$$

$$y^B = -\psi \gamma_A (e^A - e^B) + y^P \tag{1.124}$$

$$y^P = \rho(1 - \gamma_P) e^P + c^w \tag{1.125}$$

$$y^C = -\rho \gamma_P e^P + c^w \tag{1.126}$$

Sales revenue and current account equations:

$$\bar{b}^A + c^A = -(e^A - \gamma_P e^P) + y^A \tag{1.127}$$

$$\bar{b}^B + c^B = -(e^B - \gamma_P e^P) + y^B \tag{1.128}$$

$$\bar{b}^C + c^C = \gamma_P e^P + y^C \tag{1.129}$$

1.8.3 Center-Periphery

$$e^P = \frac{1}{\rho}\frac{1-\beta+\rho(1+\beta)}{1+\beta+\rho(1-\beta)}(\bar{m}^P - \bar{m}^C) \tag{1.130}$$

$$\frac{\bar{b}^P}{1-\gamma_P} = \frac{2\beta(\rho-1)}{1+\beta+\rho(1-\beta)}(\bar{m}^P - \bar{m}^C) \tag{1.131}$$

$$c^P - c^C = \bar{c}^P - \bar{c}^C = \frac{\rho-1}{\rho}\frac{(1-\beta)(1+\rho)}{1+\beta+\rho(1-\beta)}(\bar{m}^P - \bar{m}^C) \tag{1.132}$$

$$y^P - y^C = \frac{1-\beta+\rho(1+\beta)}{1+\beta+\rho(1-\beta)}(\bar{m}^P - \bar{m}^C) \tag{1.133}$$

$$\bar{y}^P - \bar{y}^C = -\frac{1}{2}\frac{1-\beta}{\beta}\frac{\bar{b}^P}{1-\gamma_P} \tag{1.134}$$

$$y^P_{npv} - y^C_{npv} = \frac{1+\rho}{1+\beta+\rho(1-\beta)}(\bar{m}^P - \bar{m}^C) \tag{1.135}$$

$$c^P_{npv} - c^C_{npv} = \frac{\rho-1}{\rho}\frac{1+\rho}{1+\beta+\rho(1-\beta)}(\bar{m}^P - \bar{m}^C) \tag{1.136}$$

$$u^P - u^C = \frac{\rho-\theta}{\rho\theta}(y^P_{npv} - y^C_{npv}) \tag{1.137}$$

1.8.4 Intra-Periphery

$$e^A - e^B = \frac{1}{\psi}\frac{1-\beta+\psi(1+\beta)}{1+\beta+\psi(1-\beta)}(\bar{m}^A - \bar{m}^B) \tag{1.138}$$

$$c^A - c^B = \bar{c}^A - \bar{c}^B = \frac{\psi-1}{\psi}\frac{(1-\beta)(1+\psi)}{1+\beta+\psi(1-\beta)}(\bar{m}^A - \bar{m}^B) \tag{1.139}$$

$$y^A - y^B = \frac{1-\beta+\psi(1+\beta)}{1+\beta+\psi(1-\beta)}(\bar{m}^A - \bar{m}^B) \tag{1.140}$$

$$\bar{y}^A - \bar{y}^B = -\frac{1}{2}\frac{1-\beta}{\beta}\frac{\bar{b}^A - \bar{b}^P}{1-\gamma_A} \tag{1.141}$$

$$\frac{\bar{b}^A - \bar{b}^P}{1-\gamma_A} = \frac{2\beta(\psi-1)}{1+\beta+\psi(1-\beta)}(\bar{m}^A - \bar{m}^B) \tag{1.142}$$

$$y_{npv}^A - y_{npv}^B = \frac{1+\psi}{1+\beta+\psi(1-\beta)}(\bar{m}^A - \bar{m}^B) \tag{1.143}$$

$$c_{npv}^A - c_{npv}^B = \frac{\psi-1}{\psi}\frac{1+\psi}{1+\beta+\psi(1-\beta)}(\bar{m}^A - \bar{m}^B) \tag{1.144}$$

$$u^A - u^B = \frac{\psi-\theta}{\psi\theta}(y_{npv}^A - y_{npv}^B) \tag{1.145}$$

Chapter 2

Strategic interactions in emerging economies and their welfare consequences: An empirical approach

2.1 Introduction

The debate between policymakers concerning the effects of the expansionary monetary policy in the aftermath of the financial crisis of 2007-08 on emerging economies remains present on the international agenda. While policymakers in the US assert that such policies are beneficial domestically as well as for the emerging economies (Powell, 2013; Bernanke, 2013) other policymakers, like the President of Brazil, expressed their worries about the negative impact of such policies on emerging economies (Rousseff (2012)).

Existing literature typically analyzes the optimal monetary policy and monetary policy transmission mechanism between countries either in a small open economy (see Gali and Monacelli (2005) or a two-country framework but without allowing the other country to be active (see Obstfeld and Rogoff (1995), Corsetti and Pesenti (2001), Tille (2001)). Among existing examples of strategic interactions in the context of monetary policy is Canzoneri and Henderson (1992), however their model is not appropriate for welfare analysis because it lacks the micro-foundations that have later been introduced in the New Keynesian models. Recently Pesaran et al. (2004) introduced a Global Vector Autoregressive Model (GVAR) approach, that allows for the analysis of the effects of policy shocks on other economies, regions and markets. Although there exist various applications of GVAR for counterfactual analysis (Pesaran and di Mauro (2013), Dees et al. (2007)) no one has so far used it to evaluate the welfare effects of policy spillovers between countries. Several studies have investigated the effects of quantitative easing (QE) performed by various monetary policymakers, such as the Bank of England (see Pesaran and Smith (2012), Kapetanios et al. (2012)), Bank of Japan (Hiroshi, 2010), and the Federal Reserve (Chen et al., 2012). However, the existing studies do not address the role that the QE has on the welfare of the domestic and foreign economies. Given this gap in the literature we provide a framework that allows for a welfare analysis in which policymakers behave strategically in complex situations similar to those taking place recently in the global economy. Our framework allows to address the aforementioned debate between the policymakers.

We have addressed the issues related to the welfare consequences of expansionary monetary policy of an advanced economy on its trading partners in Chapter 1 in which we borrowed the Corsetti et al. (2000) three-country New Keynesian model to provide a policy analysis of interactions between emerging economies in a strategic context.

In this chapter we investigate how our theoretical results correspond to the actual economic developments in the world economy, especially in the period after the financial crisis of 2007-08 and over the period of unconventional monetary policy of quantitative easing (QE) of the Federal Reserve. We analyze different scenarios by allowing emerging economies (EME) to respond to the monetary expansion in the US in different ways. In our counterfactual scenarios the emerging economies behave strategically and can either allow their currency to appreciate (we call it APP) or increase money supply - a strategy which is equivalent to pegging in Chapter 1 (we call it PEG).

This chapter is original in the way we provide methodology to analyse welfare changes in asymmetric economies using real world data. By establishing the link between the theory and the data we analyse the outcomes of strategic interactions between Brazil and China after monetary expansion in the US in the period before and after the financial crisis of 2007-08. Due to the fact that the post-crisis sample covers a very short period of time, we analyze the welfare effects using recursive estimation, including all available data up to the point of the crisis to isolate the effect on the pre-crisis sample, and then expanding the sample by adding an additional year up to the time of the most recent data available. In this way we compare a total of 8 samples as shown in Table 2.8. We approach the welfare analysis in a dynamic way and track the evolution of welfare effects and potential policy implications over time. We believe that the several rounds of the US quantitative easing have implications on the policy in emerging economies and we show how the policy recommendations change in connection to these events (summarized in the Table 2.8).

Motivated by the ongoing debate as well as the example presented by Obstfeld (2011) we look at the welfare impact of monetary expansion in the US on China and Brazil. Obstfeld (2011) describes a similar scenario to ours in which a large industrial economy loosens its monetary policy, and analyses an impact of such policy on EME. In his example appreciation today is seen as more costly (loss in one payoff unit) than inflation tomorrow (loss in 0.9 payoff unit). When one country allows its currency to appreciate it has to pay an appreciation cost (-1), and the country whose currency does not appreciate has to pay the cost of inflation (-0.9) which is partially offset by an effective nominal depreciation due to the other country's appreciation relative to the dollar (and measured by the extent of intra-EME trade parameter $\alpha > 0.1$). In his game the ad-hoc payoffs for the emerging economies are always negative (see Figure 2.13 in Appendix 2.8.2) and the emerging economies are treated symmetrically. Countries' interactions in his game result in a Prisoner's Dilemma[1] (Obstfeld calls it a coordination failure). In this case the Nash equilibrium in which both countries continue to peg their currencies to that of the advanced economy results in lower payoff (of (-1)) than if both would allow their currencies to appreciate (of $-(1-\alpha)$). Thus Obstfeld's policy recommendation for EME is to appreciate. We put his example into a theoretical context in Chapter 1 and into an empirical one in this chapter.

We use an empirical approach, in particular a Global Vector Autoregressive (GVAR) model to derive payoffs using data from US, China and Brazil. Since GVAR performs well in forecasts, we make use of the forecasted values of consumption and output as well as generalized impulse response functions to construct payoffs in a non-cooperative game between China and Brazil. The advantages of applying a GVAR is the ability to relax unrealistic assumptions from the theory and 'let the data speak for itself'. We assess the welfare effects of individual country shocks on themselves and their trading partners, and by using the linear combination of shocks, we assess the impact of regional shocks (shock taking place simultaneously in more than one economy) on the interdependent economies. The analysis of impulse response functions enables us to evaluate the short- and long-term effects of shocks on consumption and output, which are the main components in our welfare functions.

We construct a data set consisting of consumption, output, exchange rate and money

[1]Prisoner's Dilemma is a situation in which both players have dominant strategies, and the Nash equilibrium, resulting from agents acting non-cooperatively in self interest, yields lower payoff for both, than if they could cooperate.

supply (M1) variables for China, Brazil and the US. We believe to have achieved several improvements relative to the analysis conducted in Chapter 1. Among them are the ability to avoid the calibration of the elasticity of substitution and relative size parameters of the model. We construct country-specific payoffs using data that reflect the existing asymmetries between countries. In performing this analysis we did not have to assume the nature of nominal rigidities, which is considered a strong assumption in most of the New Keynesian models.

We are motivated to address the following questions: Are the welfare consequences quantitatively and qualitatively the same for both emerging economies as in Obstfeld (2011) and in Chapter 1[2]? Can we expect the empirical results to differ from these predictions due to asymmetries between EME which are not modelled in Chapter 1, such as, for example, different current account positions? What is the sign of the welfare effect of the monetary expansion in the US in emerging economies? The theory in Chapter 1 suggests that when both countries increase the money supply, this strategy always results in positive payoffs, while according to Obstfeld (2011) all strategies result in negative payoffs. Is Prisoner's Dilemma a possible outcome of strategic interactions between China and Brazil? If yes, would appreciation in both countries improve their welfare relative to the Nash equilibrium as recommended by the U.S. Department of the Treasury (2011) and Obstfeld (2011)? Or is there actually no benefit from policy coordination because of the large asymmetries between them as suggested by the findings in Chapter 1? Is the policy of the US beggar-thy-neighbour or enrich-thy-neighbour for EME?

As a result of this analysis we identify optimal strategies and give policy recommendations for China and Brazil. Our framework can serve as a tool for policymakers and international organizations to analyze the welfare impact of various policies in different scenarios in the short- and long-term and help to shed light on the potential role that quantitative easing in the US had on the monetary policy transmission mechanism between the advanced and the emerging economies. By constructing our framework we provide a methodology that can be used to assess the welfare effects of other policy actions other than those discussed here on the trading partners and domestically. According to Obstfeld (2011) an institution such as the International Monetary Fund (IMF) can provide advice to emerging economies and advise on taking actions which are optimal not only from an individual point of view but also from a global perspective. We believe that our framework can serve policymakers and institutions such as

[2]For details see Appendix 2.8.2.

the IMF in evaluating the potential benefits from coordination, an important topic on the international policy agenda. We show that China and Brazil would not benefit from policy coordination. Our results reveal a novel finding; the unconventional monetary policy of the US can have an impact on the monetary policy transmission mechanism in the emerging economies. We find that the effects of the expansionary monetary policy in the US on emerging economies in the period before the financial crisis is beggar-thy-neighbour, while it improves their welfare in the period after QE. We find that the policy recommendations for EME vary depending on the forecasting horizon. Our framework shows that, potentially due to existing asymmetries between Brazil and China, the effect of consumption and output, and therefore welfare, differs substantially between these economies, especially in the period before QE. After QE, we observe convergence in the consumption and output responses in China and Brazil. Our analysis reveals that the effect of expansionary monetary policy in the US is much smaller domestically than in the emerging economies.

This Chapter is organized as follows: In Section 2.2 we introduce the econometric methodology; in Section 2.4 we explain how we relate the theory to the empirical approach and data; in Section 2.5 we present the data; in Section 2.6 we present the results; and in Section 2.7 we conclude.

2.2 Methodology - Cointegrated Global Vector Autoregressive Model

In this section we present the approach we take to obtain the components used in the construction of individual countries' payoffs. In the modelling process we are using the Smith and Galesi (2011) GVAR Toolbox.

2.2.1 Variables choice

The choice of variables is motivated by the theoretical structure of the Corsetti et al. (2000) model, in which the change in the utility after a monetary shock can be analysed by looking at changes in consumption and output, and the parameters of the model, such as within-country substitutability $\theta > 1$, and discounting factor $\beta \in (0, 1)$. Therefore we choose consumption C_{it}, output Y_{it}, and money supply M_{it}, where i denotes the number of countries in the model.[3] Since, we want to analyse the effect of monetary

[3] Among existing studies who used these variables in VARs are Cochrane (1994, 1998).

shocks originating in various countries on the welfare of the country itself and its trading partners, we are including both home and foreign variables into our model. In order to express the data in US dollars, the data also contains the exchange rates of Brazil and China relative to the US dollar. Since the data is expressed in real terms using the deflator there is no need for inclusion of prices as one of the variables. Variables construction is presented in Section 2.4.1 Variables.

The vector of country-specific home variables is denoted by X_t:

$$X_t = \begin{bmatrix} C_{it} \\ Y_{it} \\ M_{it} \end{bmatrix}$$

Vector of country-specific foreign variables, which is constructed using trade weights, is denoted by X_t^F:

$$X_t^F = \begin{bmatrix} C_{it}^F \\ Y_{it}^F \\ M_{it}^F \end{bmatrix}$$

where $X_{it}^F = \sum_{j=0}^{N} w_{ij} X_{jt}$, with non-negative trade weights w_{ij}, such that $\sum_{j=0}^{N} w_{ij} = 1$, and $w_{ii} = 0$ for all i. For the construction of trade weights we use bilateral trade data, and treat world as consisting of only i countries. For details see Section 2.5.

We can illustrate the construction of home and foreign variables in a three country example, considering the countries of interest, namely the US, China (CH) and Brazil (BR).

Let X_t:

$$X_t = \begin{bmatrix} x_{USt} \\ \\ \\ X_{CHt} \\ \\ \\ X_{BRt} \\ \\ \\ \end{bmatrix} = \begin{bmatrix} C_{USt} \\ Y_{USt} \\ M_{USt} \\ C_{CHt} \\ Y_{CHt} \\ M_{CHt} \\ C_{BRt} \\ Y_{BRt} \\ M_{BRt} \end{bmatrix}$$

The foreign variables are constructed for the three country case in the following way:

$$X^F_{CHt} = w_{CH,BR}X_{BRt} + w_{CH,US}X_{USt}$$

$$X^F_{BRt} = w_{BR,CH}X_{CHt} + w_{BR,US}X_{USt}$$

$$X^F_{USt} = w_{US,BR}X_{BRt} + w_{US,CH}X_{CHt}$$

where $w_{CH,BR}$ is the share of trade with Brazil in China's total trade.

2.2.2 Constructing individual country models

Unit root test

We proceed to investigate the properties of the variables, in particular if these are stationary or follow a unit-root process. We use two tests: augmented Dickey-Fuller test (ADF) and weighted symmetric (WS) estimation of ADF. WS test statistic is considered to perform better than the standard ADF test measured by a smaller mean square error, because it exploits the time reversibility of stationary autoregressive process (Park and Fuller, 1995). Lag order in the tests can be selected using Akaike information criterion (AIC) or the Schwartz Bayesian Criterion (SBC).

In Table 2.1 we report the results from the ADF and WS tests for home variables in the full sample[4]. Since the WS test performs better we use it to conclude about the order of integration of the variables. First we look at the values of the test for the variables in levels. We cannot reject the null hypothesis of unit root in case of all variables consumption, output and money supply in all countries. Yet the first difference of these variables, except for output in China, where the null hypothesis of unit root cannot be rejected (although only marginally), is stationary.

Since the data is non-stationary, but difference stationary (See Table 2.1) to avoid spurious regression we can either work with the model with differences, or we can test if the linear combination of these variables is stationary (for formal test see Table 2.3). If the linear combination of variables is stationary we say that variables are cointegrated and can use the model in levels to investigate the long-run relationships between the variables.

[4]The results for foreign variables as well as other sample are available upon request.

Table 2.1: Unit Root Tests for the Domestic Variables at the 5% Significance Level for 1994 Q4 - 2014 Q1, C-V - critical value, ADF - augmented Dickey-Fuller test, WS - weighted symmetric test

	ADF				WS			
Variables	C-V	USA	China	Brazil	C-V	USA	China	Brazil
Y (with trend)	-3.45	-1.48	-1.67	-1.73	-3.24	-0.78	-2.67	-1.64
Y (no trend)	-2.89	-2.75	0.07	-1.84	-2.55	0.95	0.08	-1.57
DY	-2.89	-3.51	-1.78	-6.07	-2.55	-3.69	-2.00	-6.25
DDY	-2.89	-7.75	-35.75	-9.32	-2.55	-7.92	-35.39	-9.61
C (with trend)	-3.45	-1.54	-1.69	-1.69	-3.24	-0.94	-1.94	-1.68
C (no trend)	-2.89	-2.79	1.88	-1.88	-2.55	0.88	0.34	-1.62
DC	-2.89	-4.10	-2.55	-6.35	-2.55	-4.30	-2.55	-6.49
DDC	-2.89	-8.74	-25.15	-7.91	-2.55	-8.88	-25.69	-8.02
M (with trend)	-3.45	-0.41	-1.40	-2.52	-3.24	0.28	-1.74	-1.62
M (no trend)	-2.89	2.73	-1.27	-2.55	-2.55	1.18	1.33	1.56
DM	-2.89	-2.80	-5.72	-3.29	-2.55	-2.88	-5.92	-3.61
DDM	-2.89	-7.62	-9.89	-9.30	-2.55	-7.84	-10.18	-9.17

Lag order

Consider a VARX(p_i, q_i) model where p_i is the lag length of domestic and q_i of foreign variables.

$$X_{it} = a_{i0}+a_{i0}t+\Phi_{i1}X_{i,t-1}+...+\Phi_{i2}X_{i,t-p_i}+\Lambda_{i0}X_{it}^F+\Lambda_{i1}X_{i,t-1}^F+...+\Lambda_{i2}X_{i,t-q_i}^F+\varepsilon_{it} \quad (2.1)$$

We select the lag order of the variables in VARX by using the highest value provided by the Akaike information criterion (AIC).

In Table 2.2 we report the AIC and BSC Criteria for Selecting the Order of the VARX Models in the pre-crisis and the full sample.

We decide to use AIC criterium to select the lag length for the endogenous variables in Brazil and China in all samples. Since the AIC criterium is maximized for both USA and China for two lags we decide to choose this lag length. To remain consistent in the comparison between countries we decide to select the same lag length in Brazil in all samples.

Table 2.2: AIC and BSC Criteria for Selecting the Order of the VARX Models

			1994 Q4 - 2007 Q2			1994 Q4 - 2014 Q1		
	p	q	AIC	SBC	logLik	AIC	SBC	logLik
USA	1	1	681.9	650.7	714.9	1015.0	976.6	1048.0
	2	1	684.0	644.2	726.0	1021.2	972.3	1063.2
China	1	1	461.0	429.8	494.0	698.6	660.1	731.6
	2	1	469.7	430.0	511.7	705.1	656.2	747.1
Brazil	1	1	351.2	320.0	384.2	523.5	485.0	556.5
	2	1	347.1	307.4	389.1	520.0	471.1	562.0

Individual country models

Once we have selected the variables and their order we are ready to estimate individual country Vector Autoregressive Models with foreign variables treated as exogenous (VARX)(see equation (2.1)).

We can use the Granger representation theorem that states (cp. Verbeek (2008), p.332), that if a set of variables is cointegrated there exists a valid error correction representation of the data. In that case we can express the model in the following error correction form:

$$\Delta X_{it} = c_{i0} - \Pi_i[Z_{i,t-1} - \gamma_i(t-1)] + \Lambda_{i0}\Delta X^F_{i,t} + \Gamma_i Z_{i,t-1} + \varepsilon_{it} \tag{2.2}$$

where:

- $\Pi_i = \alpha_i \beta_i^{'}$
- $Z_{it} = (x_{it}^{'}, x_{it}^{F'})'$ is a $k_i \times r_i$ matrix of rank r_i,
- α_i is $k_i \times r_i$ loading matrix of rank r_i,
- β_i is $(k_i + k_i^F) \times r_i$ matrix of cointegrating vectors of rank r_i.

We can see from equation (2.2) that country specific models allow for cointegration between home and foreign, as well as within home variables. The rank of the matrix Π_i, which we denote by $r_i = rank(\Pi_i) \leq k_i$ informs about the number of cointegrating relationships and is determined using the error-correction forms of the individual country equations.

Table 2.3: Cointegration Results for Trace Statistic at the 5% Significance Level in sample (1) 1994 Q4 - 2007 Q2 and (2) 1994 Q4 - 2014 Q1

				Trace statistic	
Country	Null hypothesis	Alternative	Critical value	(1)	(2)
USA	$r = 0$	$r = 1$	64.54	72.18	68.85
	$r \leq 1$	$r = 2$	41.03	33.96	28.21
	$r \leq 2$	$r = 3$	20.98	9.17	9.86
China	$r = 0$	$r = 1$	64.54	94.36	77.47
	$r \leq 1$	$r = 2$	41.03	34.85	33.92
	$r \leq 2$	$r = 3$	20.98	13.84	10.70
Brazil	$r = 0$	$r = 1$	64.54	92.90	62.91
	$r \leq 1$	$r = 2$	41.03	31.97	37.79
	$r \leq 2$	$r = 3$	20.98	9.25	17.43

The VARX models are expressed in error correction form and estimated separately for each country to obtain the number of cointegrating relations r_i, the speed of adjustment coefficients α_i, and the cointegrating vectors β_i (See Table 2.7: VECMX Estimation: Cointegrating Vectors). The remaining parameters of the VARX models are then estimated using ordinary least squares regressions, conditional on the estimate of β_i from equation 2.3 containing the vector error correction terms $ECM_{i,t-1}$ corresponding to the r_i cointegrating relations of the i^{th} country model (cp. Smith and Galesi (2011) p.87-88):

$$\Delta X_{it} = c_{i0} + \delta_i ECM_{i,t-1} + \Lambda_{i0} \Delta X_{i,t}^F + \Gamma_i Z_{i,t-1} + \varepsilon_{it} \tag{2.3}$$

The corresponding cointegrated VARX models are then estimated.

To test for cointegration we use trace statistic to test hypothesis about the rank of Π_i, as proposed by Johansen (1988).

In Table 2.3[5] we report the Cointegration Results for Trace Statistic at the 5% Significance Level which in small samples has better power properties than the maximal eigenvalue statistic.

The trace statistic indicates one cointegrating vector in each country in the pre-crisis sample, and one cointegrating vector in the US and China in the full sample. To remain consistent in the comparison of all the samples that we use in the study (the number

[5]The results for the remaining samples are available upon request.

of cointegrating vectors suggested by the trace statistic in 5 out of 8 samples is one in each country) we decide to analyze the data choosing one cointegrating vector in each country in each sample.

Weak exogeneity

In each model we treat home variables as endogenous, and foreign variables as weakly exogenous. By weak exogeneity we mean that there is no long-run feedback from domestic to foreign variables. In such case foreign variables are said to be 'long-run forcing' for domestic variables. The implication of weak exogeneity is that the error correction terms do not enter in the marginal model of foreign variables. We test for weak exogeneity using a test of joint significance (F-test) of the coefficient on the error correction terms in the marginal model of the form:

$$\Delta X_{it,l}^{F} = a_{il} + \sum_{j=1}^{r_i} \zeta_{ij,l} E\hat{C}M_{ij,t} + \sum_{k=1}^{s_i} \phi_{ik,l}^{'} \Delta X_{i,t-k} + \sum_{m=1}^{n_i} \psi_{im,l}^{'} \Delta \tilde{X}_{i,t-m}^{F} + \eta_{it,l} \tag{2.4}$$

and the joint null hypothesis is $H_0 : \zeta_{ij,l} = 0$, where:

- $E\hat{C}M_{ij,t}$, $j = 1, 2, ..., r_i$ are the estimated error correction terms corresponding to r_i cointegrating relations,
- s_i - is the lag order of the lag changes for the home variables,
- n_i - is the lag order of the lag changes for the foreign variables,.

In Table 2.4 we report the results from the F-Test for weak exogeneity. In case of the pre-crisis sample we cannot reject the null hypothesis of weak exogeneity in case of all foreign variables but foreign consumption in China, and we conclude that these variables are weakly exogenous. The foreign variables in case of the full sample are all weakly exogenous.

2.2.3 GVAR - dynamic analysis

GVAR Solution

Once we have specified and estimated individual country models we proceed to solve the GVAR model for the world as a whole making use of the trade weight matrix, an

Table 2.4: Test for Weak Exogeneity at the 5% Significance Level (For the sample 1994 Q4 - 2007 Q2 the critical value is 4.11, and for 1994 Q4 - 2014 Q1 it is 3.99)

	Foreign variables					
Country	Y^F	C^F	M^F	Y^F	C^F	M^F
	1994 Q4 - 2007 Q2			1994 Q4 - 2014 Q1		
USA	1.04	0.38	2.43	0.02	0.22	0.02
China	0.33	0.60	1.35	0.22	0.07	0.38
Brazil	2.91	6.65	0.76	0.02	0.28	1.21

information which we use to construct the $(k_i + k_i^F) \times k$ link matrix W_i, where k_i is the number of home, and k_i^F is the number of foreign variables, and $k = \sum_{i=1}^{N} k_i$. W_i is such that $Z_t = (X_{it}', X_{it}^{F'})' = W_i X_{it}$. Individual country models are stacked and solved simultaneously.

In the case of our three-country model with three variables, the link matrix for each country is of dimension 6×9:

$$W_{US} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \omega_{US,CH} & 0 & 0 & \omega_{US,BR} & 0 & 0 \\ 0 & 0 & 0 & 0 & \omega_{US,CH} & 0 & 0 & \omega_{US,BR} & 0 \\ 0 & 0 & 0 & 0 & 0 & \omega_{US,CH} & 0 & 0 & \omega_{US,BR} \end{pmatrix}$$

$$W_{CH} = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \omega_{CH,US} & 0 & 0 & 0 & 0 & 0 & \omega_{CH,BR} & 0 & 0 \\ 0 & \omega_{CH,US} & 0 & 0 & 0 & 0 & 0 & \omega_{CH,BR} & 0 \\ 0 & 0 & \omega_{CH,US} & 0 & 0 & 0 & 0 & 0 & \omega_{CH,BR} \end{pmatrix}$$

and

$$W_{BR} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \omega_{BR,US} & 0 & 0 & \omega_{BR,CH} & 0 & 0 & 0 & 0 & 0 \\ 0 & \omega_{BR,US} & 0 & 0 & \omega_{BR,CH} & 0 & 0 & 0 & 0 \\ 0 & 0 & \omega_{BR,US} & 0 & 0 & \omega_{BR,CH} & 0 & 0 & 0 \end{pmatrix}$$

We can rewrite equation (2.1) in the following way:

$$A_{i0}Z_{it} = a_{i0} + a_{i1}t + \sum_{l=1}^{p} A_{il}Z_{it-l} + \varepsilon_{it} \tag{2.5}$$

where $p = max_i(p_i, q_i)$, $A_{i0} = (I_{k_i}, -\Delta_{i0})$, and $A_{il} = (\Phi_{il}, \Delta_{il})$ for $l = 1, ..., p$; which can be rewritten using the weight matrix as:

$$A_{i0}W_iX_{it} = a_{i0} + a_{i1}t + \sum_{l=1}^{p} A_{il}W_iX_{it-l} + \varepsilon_{it} \tag{2.6}$$

These individual country models are then stacked to obtain:

$$G_0X_{it} = a_{i0} + a_{i1}t + \sum_{l=1}^{p} G_lX_{it-l} + \varepsilon_t \tag{2.7}$$

where

$$G_0 = \begin{pmatrix} A_{10}W_1 \\ A_{20}W_2 \\ \vdots \\ A_{N0}W_N \end{pmatrix}$$

and

$$G_l = \begin{pmatrix} A_{1l}W_1 \\ A_{2l}W_2 \\ \vdots \\ A_{Nl}W_N \end{pmatrix}$$

and since G_0 is a non-singular matrix of weights and parameters, we can multiply equation (2.7) by G_0^{-1} from the left to obtain:

$$X_t = b_{i0} + b_{i1}t + \sum_{l=1}^{p} F_l X_{t-l} + v_t \tag{2.8}$$

where:

- $b_{i0} = G_0^{-1} a_{i0}$
- $b_{i1} = G_0^{-1} a_{i1}$
- $F_l = G_0^{-1} G_l$
- $v_{it} = G_0^{-1} \varepsilon_t$
- X_t is the vector of endogenous variables (where in solving the model all variables are treated as endogenous).

Equation (2.8) is then solved recursively.

The program reports the eigenvalues of the GVAR model and corresponding moduli. The eigenvalues are computed for the companion coefficient matrix F for the model given by equation (2.8) in a matrix form:

$$\begin{bmatrix} X_t \\ X_{t-1} \\ \vdots \\ \vdots \\ X_{t-l+1} \end{bmatrix} = \begin{bmatrix} F_1 & F_2 & \cdots & 0 & 0 \\ I_k & 0 & \cdots & 0 & 0 \\ 0 & I_k & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I_k & 0 \end{bmatrix} \begin{bmatrix} X_{t-1} \\ X_{t-2} \\ \vdots \\ \vdots \\ X_{t-l} \end{bmatrix} + \begin{bmatrix} v_t \\ 0 \\ \vdots \\ \vdots \\ 0 \end{bmatrix}$$

which are denoted by $\lambda_{eig} = a \pm bi$ and are computed from the equation involving the following determinant: $|I_{kp}\lambda_{eig} - F| = 0$.

Persistence profiles

If the cointegrating vector is valid the time profile of an effect of a system or variable-specific shock will go to zero as the time goes to infinity. We obtain the persistence profiles in the following way. We can express a GVAR model written by equation (2.8) as a moving average representation:

$$X_t = d_t + \sum_{s=0}^{\infty} A_s v_{t-s} = v_t + A_1 v_{t-1} + A_2 v_{t-2} + ... \tag{2.9}$$

with a deterministic component d_t and

$$A_s = F_1 A_{s-1} + F_1 A_{s-1} + ... + F_1 A_{s-1} \tag{2.10}$$

with $s = 1, 2, ...$ and $A_0 = I_m,\ A_s = 0$, for $s < 0$.

To look at the persistence profile of country-specific cointegrating vectors we express equation 2.9 using $Z_i = W_i X_{it}$ as:

$$Z_{it} = W_i d_t + v A_0 v_t + \sum_{s=1}^{\infty} W_i A_s v_{t-s} \tag{2.11}$$

Then the persistence profiles PP of $\beta_{ji}' Z_{it}$ with respect to shock v_i are given by:

$$PP(\beta_{ji}' Z_{it}; v_t, n) \frac{\beta_{ji}' W_i A_n \Sigma_v A_n' W_i' \beta_{ji}}{\beta_{ji}' W_i A_0 \Sigma_v A_0' W_i' \beta_{ji}} \tag{2.12}$$

where Σ_v is the covariance matrix of v_t, and n is the time horizon.

We present the persistence profiles of the pre-crisis and full sample on Figure 2.2.[6] We conclude that the effect of a system-wide shock dies out quickly in all samples and therefore the cointegrating vectors are valid.

Generalized impulse responses

In our model we investigate the effect of shocks to money supply on individual-country variables consistent with four scenarios:

1. a one standard error positive money supply shock (exogenous expansionary monetary) in the US, a strategy we call APP APP,
2. a one standard error positive money supply shock in the US and China, a strategy we call PEG APP,
3. a one standard error positive money supply shock in the US and Brazil, a strategy we call APP PEG,
4. a one standard error positive money supply shock in the US, China and Brazil, a strategy we call PEG PEG.

Notice that cases (2),(3), and (4) are analysed as a linear combination of shocks.

The generalized impulse response functions (GIRFs) are in general defined as:

[6] Persistence profiles of the remaining samples are very similar and available upon request

$$GIFR(X_t; \varepsilon_{ilt}, n) = E(X_{t+n}|\varepsilon_{ilt} = \sqrt{\sigma_{ii,ll}}, \Omega_{t-1}) - E(X_{t+n}|\Omega_{t-1}) \tag{2.13}$$

where $\sqrt{\sigma_{ii,ll}}$ is the diagonal of the variance covariance matrix of Σ_u, and Ω_{t-1} is the information set at time $t-1$.

Since the shocks across countries may be positively correlated we are looking at the general and not orthogonal impulse (OIR) responses. As opposed to orthogonal impulse responses the ordering of the variables in the generalized impulse responses does not matter. We discuss the GIRFs specific to our model in more detail in Section 2.6.2.

2.2.4 Forecasting

Because GVAR performs well in forecasts we use the forecasted values of consumption and output in each country in the construction of the payoffs. Using the GVAR Toolbox we obtain point forecasts which are computed recursively for a model given by equation (2.8):

$$\mu_h = \hat{b}_0 + \hat{b}_1(T + l) + \sum_{l=1}^{p} \hat{F}_l \mu_{l-T} + v_t \tag{2.14}$$

where the initial values are: $\mu_0 = X_T$ and $\mu_{-1} = X_{T-1}$.

2.3 Econometric approach and the economic application

The econometric analysis of the impact of shocks to individual country money supply and the effect of joint shocks on the country and its trading partners' consumption and output serves as a first stepping stone in a direction of developing an empirical apparatus designed for welfare evaluation of joint monetary policies in interdependent economies. This analysis aims to extract information from the data by constructing individual country vector autoregressive models. Given the data is non-stationary and the variables are cointegrated (the linear combination of variables is stationary and there exist and long-run relationship between them) we estimate the vector error correction models with foreign variables (VECMX) for each country to extract information about the number of cointegrating relations, r_i, the speed of adjustment of the variables to the long-run equilibrium, α_i, and the cointegrating vectors, β_i. Having determined the cointegrating vectors we can then estimate the parameters of the individual country models with foreign variables (VARX). Then, in order to analyze the impact of the monetary expansion in one country on the variables in another country we construct a Global Vector Autoregressive Model (GVAR) in which we stack the individual country models (VARX). In order to obtain a GVAR in home variables X_i' we need to 'remove' the foreign variables by using an identity $Z_i = (X_i', X_i^{F'})' = W_i X_{it}$ which collects all home variables and the weights used to construct the foreign variables (for more details see section 'GVAR Solution'). The GVAR can be then used to assess the impact of the monetary expansion (shock) on individual country variables in one country using the Generalized Impulse Response Functions (GIRFs), and in case of simultaneous monetary expansion in more than one country, using a linear combination of shocks. This counterfactual analysis allows to see how the variables behave immediately in response the shock as well as determine the time path of the variables by looking at the forecasted variables obtained from the GVAR. The GIRFs together with forecasted values of the variables in each country are considered to be the key components in assessing the welfare impact of (joint) monetary expansion in interdependent economies.

It is important to mention that the econometric results themselves are a contribution and that the presented building blocks extracted from the econometric analysis and intended for a welfare evaluation constitute only a first attempt in the direction of a more detailed welfare analysis using an empirical approach. Further work needs to be done to develop and improve this approach.

2.4 Bridge between the theory and the data

In this section we explain the theoretical approach to payoffs derivation taken in Chapter 1 and based on the Corsetti et al. (2000) model. Then we suggest an alternative approach to construct the payoffs using data. Before we do that we introduce the treatment of the variables in the theoretical approach and explain the difficulties we encounter in trying to relate the empirical approach to the theory.

In the context of the Corsetti et al. (2000) model we analyze the effect of a monetary shock in the Center on welfare in the Periphery countries considering the following sequence of events:

1. The economy is in initial equilibrium at time $t = 0$ at which all exogenous variables are constant. We call it the **initial steady-state**.

2. There is an exogenous permanent monetary shock in the Center, at time $t = 1$.

We call it the **short-run**.

3. Simultaneous decision about money supply by policymakers takes place in country A and B at time $t = 1$. Because prices are fixed in the short-run after the shock variables other than prices will adjust. The adjustment takes one period only and the economy jumps to the **new steady state** level in time $t \geq 2$.

4. The economy reaches a new equilibrium at time $t \geq 2$. We call it the **long-run**.

In Corsetti et.al. (2000) the model is linearized around the symmetric steady-state. Corresponding to the time when the events occur we apply the following notation for the variables:

1. In time $t = 0$ the level variables are denoted with subscript "0", for example, the initial level of money in country i is denoted as M_0^i. We denote the steady-state values by an upper bar $\bar{M}^i$, and because the system is initially in a steady-state we can write the initial steady-state value of money supply in country i as $\bar{M}_0^i$.

2. In time $t = 1$ the level variables are denoted with subscript "1", for example, the level of money in country i at the time of the shock is denoted as M_1^i. To study the linear system we use the following log-linear approximation of variables around the symmetric steady-state, for example in case of money supply in general: $m^i \equiv \frac{dM_t^i}{M_0^i} \approx \frac{M_t^i - \bar{M}_0^i}{\bar{M}_0^i}$ denotes the deviation of money supply in country i at time t from its initial steady-state level. Whenever the time subscript is omitted in the approximation we refer to a short-run in a variable without bar, and the long variable with bar.

3. In time $t \geq 2$ the level variables are denoted with subscript "2" until infinity. Because the adjustment takes place one period only, at time 2 and beyond the variables are at their new steady-state level, which we denote with bar: $\bar{M}_2^i$. We denote the new steady-state values by an upper bar $\bar{M}^i$, and express them as deviations from the initial steady-state in time $t = 0$ in the following way: $\bar{m}^i \equiv \frac{d\bar{M}_t^i}{\bar{M}_0^i} \approx \frac{M_t^i - \bar{M}_0^i}{\bar{M}_0^i}$.

Using this notation we define the monetary shock at time t in the Center (US) as:

$$M_t^{US} - \bar{M}_0^{US} > 0 \tag{2.15}$$

in levels, and as:

$$m^{US} \equiv \frac{dM_t^{US}}{\bar{M}_0^i} \approx \frac{M_t^{US} - \bar{M}_0^{US}}{\bar{M}_0^{US}} > 0 \tag{2.16}$$

for $t = 1$, and:

$$\bar{m}^{US} \equiv \frac{d\bar{M}_t^{US}}{\bar{M}_0^{US}} \approx \frac{\bar{M}_t^{US} - \bar{M}_0^{US}}{\bar{M}_0^{US}} > 0 \tag{2.17}$$

for $t \geq 2$, in log-linear approximation. In fact because the change in money supply in the Corsetti et.al. model is permanent we can write that: $m^{US} = \bar{m}^{US}$.

The Corsetti et al. (2000) model assumes that countries in the Periphery, in our case Brazil (BR) and China (CH), have a fixed exchange rate relative to the Center (US). As a consequence in order to manipulate the exchange rate countries undertake adjustments in money supply. It can be seen from the relative equations for the exchange rates in terms of money supply changes and underlying parameters:

$$e^P = \frac{1}{\rho}\frac{1 - \beta + \rho(1 + \beta)}{1 + \beta + \rho(1 - \beta)}\left(\bar{m}^P - \bar{m}^{US}\right) \tag{2.18}$$

$$e^{CH} - e^{BR} = \frac{1}{\psi}\frac{1 - \beta + \psi(1 + \beta)}{1 + \beta + \psi(1 - \beta)}\left(\bar{m}^{CH} - \bar{m}^{BR}\right) \tag{2.19}$$

where P - Periphery, $e^i \approx \frac{E^i - E_0^i}{Ei_0}$ is the deviation of an exchange rate after the shock from its initial equilibrium, $\bar{m}^i \approx \frac{M^i - M_0^i}{Mi_0}$ is the deviation of money supply after the shock from its initial equilibrium, $\psi > 0$ is the substitutability between goods produced within Periphery, $\rho > 0$ is the substitutability between goods from the Periphery and the Center, $\beta \in (1, 0)$ is the discount factor, and $i = P, CH, BR, US$.

Expressions (2.18) and (2.19) are derived using the following definitions of individual variables and their approximations:

$$x^W = \gamma_P x^P + (1 - \gamma_P)x^C \tag{2.20}$$

and

$$x^P = \gamma_A x^A + (1 - \gamma_A)x^B \tag{2.21}$$

where $W = World$, $P = Periphery$, $C = US$, $A = China$, and $B = Brazil$, $\gamma_A \in [0, 1]$ is the share of country A in the Periphery's populations, $(1 - \gamma_A) = \gamma_B$ is the share of country B in Periphery's population, and $\gamma_P \in [0, 1]$ is the share of country A and B together in the world's populations.

Equations (2.18) and (2.19) serve as a reference for the strategies in our game between policymakers who have as a goal to maximize the utility of a representative household in their countries. It can be seen from equation (2.18) that in order to maintain fixed exchange relative to the Center the size of adjustment in the money supply in the Periphery has to be the same as the change in money supply in the Center, so that $e^P = 0$.

The actions of the policymaker in country $i = A, B$ result in either of these two outcomes:

1. Fixed exchange rate vis-a-vis the Center, i.e. $e^i = 0$, call it **PEG**, which is achieved by increasing the money supply by the same amount as the Center, i.e $\bar{m}^i = \bar{m}^C$; or

2. Currency appreciation in country i, i.e. $e^i < 0$, call it **APP**, which is achieved by not changing the level of money supply i.e. $\bar{m}^i = 0$.

In our game in which countries are not cooperating and act only in self-interest, we look at the following four situations:

1. **APP APP** - money supply increases in the USA only $\bar{m}^{US} > 0$, and China and Brazil remain passive. As a result of maintaining the money supply at unchanged level relative to symmetric equilibrium, namely $\bar{m}^{CH} = 0$, $\bar{m}^{BR} = 0$, the exchange rate in the Periphery appreciates. We can show that using equations (2.18) and (2.21)[7]:

$$\bar{m}^P = \gamma_{CH}\bar{m}^{CH} + (1 - \gamma_{CH})\bar{m}^{BR} = 0$$
$$\Rightarrow e^P = \frac{1}{\rho}\frac{1 - \beta + \rho(1 + \beta)}{1 + \beta + \rho(1 - \beta)}\left(0 - \bar{m}^{US}\right) < 0$$

2. **PEG APP** - money supply increases in the USA and China by the same amount, i.e. $\bar{m}^{CH} = \bar{m}^{US} > 0$, and Brazil keeps the money supply at unchanged level $\bar{m}^{BR} = 0$, which results in currency appreciation of Brazil's exchange rate relative to the US: $e^{BR} < 0$, while Chinese exchange remains the same $e^{CH} = 0$, which we can show using equations (2.19) and (2.21):

[7] It is worth to mention that appreciation of currency in country i is also possible when the country adjusts its money supply by a small amount, however not larger than the size of change in the money supply in the Center.

$$e^{CH} = 0$$

$$0 - e^{BR} = \frac{1}{\psi}\frac{1-\beta+\psi(1+\beta)}{1+\beta+\psi(1-\beta)}\left(\bar{m}^{US} - 0\right)$$

$$\Rightarrow e^{BR} = -\frac{1}{\psi}\frac{1-\beta+\psi(1+\beta)}{1+\beta+\psi(1-\beta)}\bar{m}^{US} < 0$$

$$\Rightarrow e^{P} = \gamma_{CH}e^{CH} + (1-\gamma_{CH})e^{BR} = (1-\gamma_{CH})e^{BR} < 0$$

3. **APP PEG** - money supply increases in the USA and Brazil by the same amount, i.e. $\bar{m}^{BR} = \bar{m}^{US} > 0$, and China keeps the money supply at unchanged level $\bar{m}^{CH} = 0$, which results in currency appreciation of China's exchange rate relative to the US: $e^{CH} < 0$, while Brazil's exchange remains the same $e^{BR} = 0$, which we can show using equations (2.19) and (2.21):

$$e^{BR} = 0$$

$$e^{CH} - 0 = \frac{1}{\psi}\frac{1-\beta+\psi(1+\beta)}{1+\beta+\psi(1-\beta)}\left(0 - \bar{m}^{US}\right)$$

$$\Rightarrow e^{CH} = -\frac{1}{\psi}\frac{1-\beta+\psi(1+\beta)}{1+\beta+\psi(1-\beta)}\bar{m}^{US} < 0$$

$$\Rightarrow e^{P} = \gamma_{CH}e^{CH} + (1-\gamma_{CH})e^{BR} = \gamma_{CH}e^{CH} < 0$$

4. **PEG PEG** - money supply increases in the USA, and both China and Brazil increase money supply by the same amount, i.e. $\bar{m}^{CH} = \bar{m}^{BR} = \bar{m}^{US} > 0$, which results in unchanged level of exchange rate in the Periphery (China and Brazil) relative to the Center (US):

$$\bar{m}^{P} = \gamma_{CH}\bar{m}^{CH} + (1-\gamma_{CH})\bar{m}^{BR} = \bar{m}^{BR} = \bar{m}^{CH} = \bar{m}^{US}$$
$$\Rightarrow e^{P} = \frac{1}{\rho}\frac{1-\beta+\rho(1+\beta)}{1+\beta+\rho(1-\beta)}\left(\bar{m}^{US} - \bar{m}^{US}\right) = 0$$

and

$$e^{CH} = e^{BR}$$

2.4.1 Theoretical derivation of payoffs

By comparing the payoffs from available strategies we find Nash equilibria, and give policy recommendations based on realized changes in utilities from available actions. The payoffs express the changes in the utility of a representative agent after the shock, and reflect changes in short- and long-term consumption and output.

We repeat for convenience the way the payoffs are derived in Chapter 1 in this section. The payoffs are derived using the building blocks of the Corsetti et al. (2000) model. Following Obstfeld and Rogoff (1996) we focus only on the real components of the following utility function U_t^i of a representative household in country i:

$$U_t^i = \sum_{s=0}^{\infty} \beta^s \left[lnC_{t+s}^i + \chi ln\left(\frac{M_{t+s}^i}{P_{t+s}^i}\right) - \frac{\kappa}{2}[Y_{t+s}^i]^2 \right] \tag{2.22}$$

where κ and χ are positive constants and β is the discount rate between 0 and 1, assuming that the parameter $\chi \to 0$ we denote the real component as, denoting it with the superscript $U_t^{R,i}$ [8] :

$$U_t^{R,i} = \sum_{s=0}^{\infty} \beta^s \left[lnC_{t+s}^i - \frac{\kappa}{2}[Y_{t+s}^i]^2 \right] \tag{2.23}$$

where, C_t^i is consumption of a representative agent in country i, Y_t^i is output of a representative household in country i, which enters negatively in the utility function because it generates disutility from work effort.

Since the new steady-state is reached after one period (prices are fixed for one period only) we can rewrite (2.23) as the sum of the short-term effect (denoted by C and Y) together with the discounted sum of long-term effects (denoted by $\bar{C}$ and $\bar{Y}$):

$$U_t^{R,i} = \left[lnC^i - \frac{\kappa}{2}[Y^i]^2 \right] + \frac{\beta}{1-\beta}\left[ln\bar{C}^i - \frac{\kappa}{2}[\bar{Y}^i]^2 \right] \tag{2.24}$$

We will proceed using a log-linear approximation of a variable X, where $X = \{C, Y\}$ in the utility function around its (initial) steady state X_0:

[8] Following Obstfeld and Rogoff (1996) (p.662) assume that the disutility from work effort l is $-\phi l$, and the production function is given by $y = Al^{\alpha}$, where A denotes productivity, and $\alpha > 0$. Assuming that $\alpha = 0.5$ we get $\kappa = \frac{2\phi}{A^2}$, which helps to interpret how κ changes with productivity. If productivity increases κ falls.

$$X = e^{logX} \approx e^{logX_0} + e^{logX_0}(logX - logX_0) \tag{2.25}$$

$$X \approx X_0 + X_0(logX - logX_0) \tag{2.26}$$

$$\frac{X - X_0}{X_0} \approx (logX - logX_0) \equiv x \tag{2.27}$$

Consider the utility function and its value at the (initial) steady state (where we omit indexes for simplicity):

$$U_t^R = \left[logC - \frac{\kappa}{2}Y^2\right] + \frac{\beta}{1-\beta}\left[log\bar{C} - \frac{\kappa}{2}\bar{Y}^2\right] \tag{2.28}$$

$$U_0^R = \left[logC_0 - \frac{\kappa}{2}Y_0^2\right] + \frac{\beta}{1-\beta}\left[log\bar{C}_0 - \frac{\kappa}{2}\bar{Y_0}^2\right] \tag{2.29}$$

and subtracting equation (2.29) from (2.28) yields:

$$u^j \approx U_t^R - U_0^R = c - \frac{\kappa}{2}(Y^2 - Y_0^2) + \frac{\beta}{1-\beta}\left[\bar{c} - \frac{\kappa}{2}(\bar{Y}^2 - Y_0^2)\right] \tag{2.30}$$

Now we can log-linearize the nonlinear terms around their initial steady state value:

$$Y^2 = e^{2logX} \approx Y_0^2 + 2Y_0^2(logY - logY_0) \tag{2.31}$$

$$Y^2 - Y_0^2 \approx \underbrace{2Y_0^2(logY - logY_0)}_{\equiv y} \tag{2.32}$$

Based on that equation (2.30) can be rewritten as:

$$u^j \approx U_t^R - U_0^R \approx c - \kappa Y_0^2 y + \frac{\beta}{1-\beta}\left[\bar{c} - \kappa Y_0^2 \bar{y}\right] \tag{2.33}$$

Then we substitute the value of the initial steady state:

$$Y_0 = \sqrt{\frac{\theta - 1}{\theta\kappa}}$$

and thus

$$Y_0^2 = \frac{\theta - 1}{\theta\kappa}$$

we can express the approximation around the symmetric, flexible-price equilibrium:

$$\begin{aligned} u^j &= c^j - \kappa\frac{\theta-1}{\theta\kappa}y^j + \frac{\beta}{1-\beta}\left[\bar{c}^j - \kappa\frac{\theta-1}{\theta\kappa}\bar{y}^j\right] \\ &= c^j - \frac{\theta-1}{\theta}y^j + \frac{\beta}{1-\beta}\left[\bar{c}^j - \frac{\theta-1}{\theta}\bar{y}^j\right] \end{aligned} \tag{2.34}$$

The net present value is the sum of short term value and the discounted sum of the long term value, so:

$$x_{npv} = x + \frac{\beta}{1-\beta}\bar{x} \tag{2.35}$$

and the changes in the utility of a representative agent relative to symmetric equilibrium, following a monetary shock, will represent player's payoff of the form:

$$u^j = c^j_{npv} - \frac{\theta-1}{\theta}y^j_{npv} \tag{2.36}$$

Policymakers in each country use money supply as their policy variables. Since both consumption and output change after the monetary shock, these can be expressed in terms of money supply. This is the approach we took in Chapter 1 where we derived the following expressions for utilities depending on the strategy chosen by each country in the Periphery:

$$u^A_{PEG,PEG} = \frac{1}{\theta}\bar{m}^C \tag{2.37}$$

$$u^B_{PEG,PEG} = \frac{1}{\theta}\bar{m}^C \tag{2.38}$$

$$u^A_{APP,APP} = \frac{1}{\theta}(\gamma_P - 1)(\Pi(\rho) - 1)\bar{m}^C \tag{2.39}$$

$$u^B_{APP,APP} = \frac{1}{\theta}(\gamma_P - 1)(\Pi(\rho) - 1)\bar{m}^C \tag{2.40}$$

$$u^A_{PEG,APP} = \frac{1}{\theta}[1 - \Pi(\rho) + \gamma_A(\Pi(\psi) - \Pi(\rho) + \gamma_P(\Pi(\rho) - 1))]\bar{m}^C \tag{2.41}$$

$$u^B_{PEG,APP} = \frac{1}{\theta}[1 + \gamma_A(\Pi(\psi) - \Pi(\rho) + \gamma_P(\Pi(\rho) - 1))]\bar{m}^C \tag{2.42}$$

$$u^A_{APP,PEG} = \frac{1}{\theta}[1+\Pi(\psi)-\gamma_A\Pi(\psi)+(1-\gamma_A)\Pi(\rho)+\gamma_P(\gamma_A-1+\Pi(\rho)-\gamma_A\Pi(\rho))]\bar{m}^C \quad (2.43)$$

$$u^B_{APP,PEG} = -\frac{1}{\theta}[\Pi(\rho)-1+\gamma_A(\Pi(\psi)-\Pi(\rho))+\gamma_P(\gamma_A-1)(\Pi(\rho)-1)]\bar{m}^C \qquad (2.44)$$

where $\Pi(x) = \left(\frac{x-\theta}{x}\right)\left(\frac{1+x}{1+\beta+x(1-\beta)}\right)$, and $x = \rho, \psi$.

The payoffs can be summarized in the Payoff matrix in Table 2.5. Countries compare their payoffs in two different strategies APP and PEG given the choice of strategy of the other country. In **Nash equilibrium** no country has an incentive to deviate from the chosen strategy as they would get a lower payoff from such action.

Table 2.5: Payoff matrix

		Country B	
		PEG	APP
Country A	PEG	$u^B_{PEG,PEG}$ $u^A_{PEG,PEG}$	$u^B_{PEG,APP}$ $u^A_{PEG,APP}$
	APP	$u^B_{APP,PEG}$ $u^A_{APP,PEG}$	$u^B_{APP,APP}$ $u^A_{APP,APP}$

2.4.2 Payoffs construction using data and the GVAR

The purpose of this chapter is to analyse the welfare implications of monetary expansion in the Center in different strategies and compare the payoffs derived from a theoretical model in Chapter 1 with payoffs constructed using data and econometric method. Although we try to stay as close as possible to the model in Chapter 1 we encounter various complications in trying to map the payoffs derived in Chapter 1 to payoffs derived in this chapter. It is important to point out the main differences, which we consider to be a necessary compromise in order to make progress at this stage. These should be kept in mind while comparing the empirical with the theoretical results.

- In Chapter 1 the payoffs are expressed in terms of the policy variable, namely money supply and the underlying parameters of the Corsetti et al. (2000) model. Since the consumption and output data, which are the variables in the utility function measuring the welfare of agents in each economy are observable we use them in this chapter instead to avoid calibration of all parameters but one (θ) which is necessary to quantify the welfare changes in Chapter 1.

- In Chapter 1 we assume that the net-asset positions are zero in all countries in the symmetric equilibrium, an assumption which we relax for the purpose of the empirical analysis (in order to keep the model as simple as possible we do not include current account as a variable in the VAR).

- In the model in Chapter 1 the variables in levels in the theoretical model do not grow, while the data shows a clear time trend in the variables. We take logs of the variables so the growth over the forecasted horizon is not too large.

- While the money supply adjustment in the Periphery countries necessary to maintain the exchange rate at unchange level vis-a-vis the Center is obtained by matching the size of the monetary shock in the Center in Chapter 1, the strategy PEG is modelled as one standard deviation change in money supply in each country in the empirical model. This may not correspond to the same size of change in money supply of each country. We solve this problem by normalizing the size of the shock by the inverse of the standard deviation. However, it is important to keep in mind that such monetary adjustment, although true in the theoretical model, may not correspond to the true exchange adjustment in reality.

- While the shock in the model in Chapter 1 is permanent, the shock in the empirical approach is transitory (one-off). However, due to the non-stationarity of the variables the shocks may have permanent effect.

- In Chapter 1 we assume one-period price rigidity, while in this Chapter we do not make any assumptions about the type of nominal rigidities prevailing in the economies of interest.

We suggest three alternative approaches to the construction of the welfare measure using data from the econometric analysis:

1. An approach in which we need to calibrate the within-country substitutability parameter θ, and allow to substitute for a symmetric steady-state value of output in each period. In this approach we make use of the forecasted values of consumption and output, as well as the generalized impulse response functions of these variables.

2. An approach in which we need to calibrate the parameter in the disutility from work effort κ. In this approach there is no need to make use of the symmetric equilibrium values of consumption and output, as the forecasted values of consumption and output, together with the impulse response functions used to compute the welfare changes.

3. An approach in which we use the log-linearized version of the utility function, as derived in Chapter 1. We need to calibrate the within-country substitutability parameter θ and make use of the generalized impulse response functions for consumption and output.

Below we present the way in which welfare changes can be evaluated using data and results from the GVAR estimation, as well as the predictions in terms of short- and long-run Nash equilibria in each of the aforementioned methods.

First approach to welfare evaluation using data and GVAR

We proceed with the derivation of the payoffs using the **first approach**.

In order to derive payoffs we use the approach explained in Section 2.4.1 and adjust it using an information from the data, such as the forecasted values of consumption C^i_{t+s} and output Y^i_{t+s} in each period after the shock, as well as the effect of the shock on these variables as represented by the GIRFs ($GIRF(C^i_t)$ and $GIRF(Y^i_t)$).

Let us repeat equation (2.23) for convenience:

$$U^{R,i}_t = \sum_{s=0}^{\infty} \beta^s \left[lnC^i_{t+s} - \frac{\kappa}{2}[Y^i_{t+s}]^2 \right]$$

The total differential of equation (2.23), which shows the total change in the real component of the utility of an individual i is then:

$$dU^{R,i}_t = \sum_{s=0}^{\infty} \beta^s \left[\frac{1}{C^i_{t+s}(x)} dC^i_{t+s} - \kappa Y^i_{t+s}(x) dY^i_{t+s} \right] \quad (2.45)$$

we can the multiply and divide each period by $Y^i_{t+s}(x)$ to obtain:

$$dU_t^{R,i} = \sum_{s=0}^{\infty} \beta^s \left[\frac{1}{C_{t+s}^i} dC_{t+s}^i - \kappa \left[Y_{t+s}^i \right]^2 \frac{dY_{t+s}^i}{Y_{t+s}^i} \right] \tag{2.46}$$

We then follow Obstfeld and Rogoff (1996) and substitute for Y its equilibrium steady state-value[9]:

$$Y = \sqrt{\frac{\theta - 1}{\theta \kappa}} \Rightarrow Y^2 = \bar{Y}^2 = \frac{\theta - 1}{\theta \kappa}$$

we obtain:

$$\begin{aligned} dU_t^{R,i} &= \sum_{s=0}^{\infty} \beta^s \left[\frac{1}{C_{t+s}^i} dC_{t+s}^i - \kappa \frac{\theta - 1}{\theta \kappa} \frac{dY_{t+s}^i}{Y_{t+s}^i} \right] \\ &= \sum_{s=0}^{\infty} \beta^s \left[\frac{dC_{t+s}^i(x)}{C_{t+s}^i} - \frac{\theta - 1}{\theta} \frac{dY_{t+s}^i}{Y_{t+s}^i} \right] \end{aligned} \tag{2.47}$$

We then use expression (2.47) to derive the approximate change in the utility of a representative agent in country i in different strategies, as described in Section 2.4. In each strategy we divide the terms in equation (2.47) by the change in the money supply in the US (dM_t^{US}) in case of the strategy APP APP, in China and the US ($dM_t^{US,CH}$) in case of the strategy PEG APP, in Brazil and the US ($dM_t^{US,BR}$) in case of the strategy APP PEG, and in all three countries ($dM_t^{US,CH,BR}$) in case of the strategy **PEG PEG**. We illustrate this procedure of payoff construction in the strategy **APP APP** first:

$$u_{APP,APP}^i \approx \frac{dU_t^{R,i}}{dM_t^{US}} =$$

$$\sum_{s=0}^{\infty} \beta^s \left[\frac{dC_{t+s}^i}{dM_t^{US}} \frac{1}{C_{t+s}^i} - \frac{\theta - 1}{\theta} \frac{dY_{t+s}^i}{dM_t^{US}} \frac{1}{Y_{t+s}^i} \right] \tag{2.48}$$

In this case we do not need to define short- and long-term[10] as in Corsetti et al. (2000), Obstfeld and Rogoff (1996) and Chapter 1, instead we sum each period, and

[9] We are aware that this is a vary strong assumption, which postulates the same equilibrium value of output in every period - even in the periods after the shock and even with the positive trend in output present in the data. Despite these limitations, we consider it a crucial step in order to progress at this stage of the analysis and given the small size of the shocks and changes in the logarithm of the output in Brazil and China over time (for example 10.24 in 2014 Q2 and 10.81 in 2026 Q3 in China) we consider our results to be approximations.

[10] An alternative approach to derive payoffs by constructing short- and long-term from the data is explained in Appendix. Although qualitatively this approach yields similar policy recommendations in terms of Nash equilibria, the results are not quantitatively equivalent.

make use of the GIRFs[11] and the forecasted values of consumption and output:

$$\frac{dC^i_{t+s}}{dM^{US}_t} = GIRF(C^i_t; \varepsilon_{M^{US}_t}, s) = E(C^i_{t+s} | \varepsilon_{M^{US}_t} = \sqrt{\sigma^{US}}, \Omega_{t-1}) - E(C^i_{t+s} | \Omega_{t-1}) \quad (2.49)$$

and

$$\frac{dY^i_{t+s}}{dM^{US}_t} = GIRF(Y^i_t; \varepsilon_{M^{US}_t}, s) = E(Y^i_{t+s} | \varepsilon_{M^{US}_t} = \sqrt{\sigma^{US}}, \Omega_{t-1}) - E(Y^i_{t+s} | \Omega_{t-1}) \quad (2.50)$$

so the adjusted payoff (over the horizon of the forecast S) becomes:

$$u^i_{APP,APP} \approx \frac{dU^{R,i}_t}{dM^{US}_t} = \sum_{s=0}^{S} \beta^s \left[GIRF(C^i_t; \varepsilon_{M^{US}_t}, s) \frac{1}{C^i_{t+s}} - \frac{\theta - 1}{\theta} GIRF(Y^i_t; \varepsilon_{M^{US}_t}, s) \frac{1}{Y^i_{t+s}} \right] \quad (2.51)$$

The remaining payoffs can be constructed in the same way and are given by the following formulas:

- in case of **PEG APP**:

$$u^i_{PEG,APP} \approx \frac{dU^{R,i}_t}{dM^{US,CH}_t} = \sum_{s=0}^{\infty} \beta^s \left[\frac{dC^i_{t+s}}{dM^{US,CH}_t} \frac{1}{C^i_{t+s}} - \frac{\theta - 1}{\theta} \frac{dY^i_{t+s}}{dM^{US,CH}_t} \frac{1}{Y^i_{t+s}} \right] \quad (2.52)$$

where for $X = C, Y$

$$\frac{dX^i_{t+s}}{dM^{US,CH}_t} = GIRF(X^i_t; \varepsilon_{M^{US}_t} = \varepsilon_{M^{CH}_t}, s) \quad (2.53)$$

- in case of **APP PEG**:

$$u^i_{APP,PEG} \approx \frac{dU^{R,i}_t}{dM^{US,BR}_t} = \sum_{s=0}^{\infty} \beta^s \left[\frac{dC^i_{t+s}}{dM^{US,BR}_t} \frac{1}{C^i_{t+s}} - \frac{\theta - 1}{\theta} \frac{dY^i_{t+s}}{dM^{US,BR}_t} \frac{1}{Y^i_{t+s}} \right] \quad (2.54)$$

where for $X = C, Y$

[11] Later in Section 2.6.2 we make a remark on the way we construct weighted GIRFs in order to analyze an impact of monetary shocks of the same size in each country.

$$\frac{dX^i_{t+s}}{dM^{US,BR}_t} = GIRF(X^i_t; \varepsilon_{M^{US}_t} = \varepsilon_{M^{BR}_t}, s) \tag{2.55}$$

- and finally in case of **PEG PEG**:

$$u^i_{PEG,PEG} \approx \frac{dU^{R,i}_t}{dM^{US,CH,BR}_t} = \sum_{s=0}^{\infty} \beta^s \left[\frac{dC^i_{t+s}}{dM^{US,CH,BR}_t} \frac{1}{C^i_{t+s}} - \frac{\theta - 1}{\theta} \frac{dY^i_{t+s}}{dM^{US,CH,BR}_t} \frac{1}{Y^i_{t+s}} \right] \tag{2.56}$$

where for $X = C, Y$

$$\frac{dX^i_{t+s}}{dM^{US,CH,BR}_t} = GIRF(X^i_t; \varepsilon_{M^{US}_t} = \varepsilon_{M^{CH}_t} = \varepsilon_{M^{BR}_t}, s) \tag{2.57}$$

Second approach to welfare evaluation using data and GVAR

We proceed with the derivation of the payoffs using the **second approach**.

To derive the welfare change of agent in economy i we start by totally differentiating the agent's utility function:

$$dU^{R,i}_t = \sum_{s=0}^{\infty} \beta^s \left[\frac{1}{C^i_{t+s}} dC^i_{t+s} - \kappa Y^i_{t+s} dY^i_{t+s} \right] \tag{2.58}$$

In the strategy **APP APP**, only the US changes money supply and Brazil and China keep it unchanged. In that case the change in the utility in country i, where $i = \{China, Brazil\}$ can be defined as:

$$\frac{dU^{R,i}_{APP,APP,t}}{dM^{US}_t} = \sum_{s=0}^{\infty} \beta^s \left[\frac{1}{C^i_{t+s}} \frac{dC^i_{t+s}}{dM^{US}_t} - \kappa Y^i_{t+s} \frac{dY^i_{t+s}}{dM^{US}_t} \right] \tag{2.59}$$

The change in a variable (consumption and output) after monetary shock in the US $\left(\frac{dX^i_{t+s}}{dM^{US}}\right)$ at time $t+s$, where $X = \{C, Y\}$ can be defined using the generalised impulse response functions (GIFRS) in the following way:

$$\frac{dC^i_{t+s}}{dM^{US}_t} = GIRF(C^i_t; \varepsilon_{M^{US}_t}, s) = E(C^i_{t+s} | \varepsilon_{M^{US}_t} = \sqrt{\sigma}, \Omega_{t-1}) - E(C^i_{t+s} | \Omega_{t-1}) \tag{2.60}$$

$$\frac{dY^i_{t+s}}{dM^{US}_t} = GIRF(Y^i_t; \varepsilon_{M^{US}_t}, s) = E(Y^i_{t+s} | \varepsilon_{M^{US}_t} = \sqrt{\sigma}, \Omega_{t-1}) - E(C^i_{t+s} | \Omega_{t-1}) \tag{2.61}$$

where $\varepsilon_{M_t^{US}} = \sqrt{\sigma}$ is the size of the shock to money supply in the US, i.e. one standard deviation, Ω_{t-1} is the available information up to the period before the shock occurs; and so the change in the utility at time $t+s$ can be expressed as:

$$\frac{dU^i_{APP,APP,t+s}}{dM_t^{US}} = \frac{1}{C^i_{t+s}}\left[E(C^i_{t+s}|\varepsilon_{M_t^{US}} = \sqrt{\sigma}, \Omega_{t-1}) - E(C^i_{t+s}|\Omega_{t-1})\right]$$
$$- \kappa Y^i_{t+s}\left[E(Y^i_{t+s}|\varepsilon_{M_t^{US}} = \sqrt{\sigma}, \Omega_{t-1}) - E(Y^i_{t+s}|\Omega_{t-1})\right] \quad (2.62)$$

From here we proceed to obtain the expression for the sum of short- and long-term changes, which we can treat as an approximation of the net present value expression given by equation (2.34). The main difference is that the long horizon in Chapter 1 is defined over an infinite time, while here we define the long run over the horizon of the forecast.

Short and long run are defined in the following way:

- Short - immediate effect.
- Long - remaining quarters up to period 50 of the forecast.

Assuming that in time 0 the system is in equilibrium, and that the shock happens in time t, the short run change in the utility can be then given by the following formula:

$$u^i_{APP,APP,Short} = \left[\frac{1}{C^i_t}\frac{dC^i_t}{dM_t^{US}} - \kappa Y^i_t \frac{dY^i_t}{dM_t^{US}}\right] \quad (2.63)$$

where $t = 1$, and the long-run:

$$u^i_{APP,APP,Long} = \sum_{s=2}^{50} \beta^s \left[\frac{1}{C^i_{t+s}}\frac{dC^i_{t+s}}{dM_t^{US}} - \kappa Y^i_{t+s}\frac{dY^i_{t+s}}{dM_t^{US}}\right] \quad (2.64)$$

which we add to obtain the payoff of country i, which already takes the discount factor into account, and corresponds to the utility constructed using the sum of net present value of changes in consumption and output in Chapter 1:

$$u^i_{APP,APP} = u^i_{APP,APP,Short} + u^i_{APP,APP,Long} \quad (2.65)$$

In the strategy **PEG PEG**, after the expansionary monetary shock in the US, both China and Brazil adjust their money supply correspondingly. In that case the change in the utility in country i, where $i = \{China, Brazil\}$ can be defined as:

$$
\begin{aligned}
\frac{dU^{R,i}_{PEG,PEG,t}}{dM_t^{US+CH+BR}} &= \sum_{s=0}^{\infty}\beta^s\left[\frac{1}{C^i_{t+s}}\left\{\frac{\partial C^i_{t+s}}{\partial M_t^{US}}+\frac{\partial C^i_{t+s}}{\partial M_t^{CH}}+\frac{\partial C^i_{t+s}}{\partial M_t^{BR}}\right\}\right] \\
&- \sum_{s=0}^{\infty}\beta^s\left[\kappa Y^i_{t+s}\left\{\frac{\partial Y^i_{t+s}}{\partial M_t^{US}}+\frac{\partial Y^i_{t+s}}{\partial M_t^{CH}}+\frac{\partial Y^i_{t+s}}{\partial M_t^{BR}}\right\}\right] \qquad (2.66)
\end{aligned}
$$

The change in the utility depends on the linear combination of changes in consumption and output after monetary shock in each country $\left(\frac{\partial X^i_{t+s}}{\partial M^{US}}\right)+\left(\frac{\partial X^i_{t+s}}{\partial M^{CH}}\right)+\left(\frac{\partial X^i_{t+s}}{\partial M^{BR}}\right)$, where $X=\{C,Y\}$ can be defined using the generalised impulse response functions (GIRFS) in the following way, at time $t+s$:

$$
\left\{\frac{\partial C^i_{t+s}}{\partial M_t^{US}}+\frac{\partial C^i_{t+s}}{\partial M_t^{CH}}+\frac{\partial C^i_{t+s}}{\partial M_t^{BR}}\right\}=GIRF(C^i_t;\varepsilon_{M_t^{US}}=\varepsilon_{M_t^{CH}}=\varepsilon_{M_t^{BR}},s)
$$

$$
=\varphi_{US}E(C^i_{t+s}|\varepsilon_{M_t^{US}}=\sqrt{\sigma},\Omega_{t-1})-E(C^i_{t+s}|\Omega_{t-1})
$$

$$
+\varphi_{CH}E(C^i_{t+s}|\varepsilon_{M_t^{CH}}=\sqrt{\sigma},\Omega_{t-1})-E(C^i_{t+s}|\Omega_{t-1})
$$

$$
+\varphi_{BR}E(C^i_{t+s}|\varepsilon_{M_t^{BR}}=\sqrt{\sigma},\Omega_{t-1})-E(C^i_{t+s}|\Omega_{t-1}) \qquad (2.67)
$$

where φ_i $i=\{US,BR,CH\}$ is a proportionality factor, and

$$
\left\{\frac{\partial Y^i_{t+s}}{\partial M_t^{US}}+\frac{\partial Y^i_{t+s}}{\partial M_t^{CH}}+\frac{\partial Y^i_{t+s}}{\partial M_t^{BR}}\right\}=GIRF(Y^i_t;\varepsilon_{M_t^{US}}=\varepsilon_{M_t^{CH}}=\varepsilon_{M_t^{BR}},s)
$$

$$
=\varphi_{US}E(Y^i_{t+s}|\varepsilon_{M_t^{US}}=\sqrt{\sigma},\Omega_{t-1})-E(Y^i_{t+s}|\Omega_{t-1})
$$

$$
+\varphi_{CH}E(Y^i_{t+s}|\varepsilon_{M_t^{CH}}=\sqrt{\sigma},\Omega_{t-1})-E(Y^i_{t+s}|\Omega_{t-1})
$$

$$
+\varphi_{BR}E(Y^i_{t+s}|\varepsilon_{M_t^{BR}}=\sqrt{\sigma},\Omega_{t-1})-E(Y^i_{t+s}|\Omega_{t-1}) \qquad (2.68)
$$

and so the change in the utility at time $t+s$ can be expressed as:

$$
\begin{aligned}
\frac{dU^i_{t+s}}{dM_t^{US+CH+BR}} &= \frac{1}{C^i_{t+s}}GIRF(C^i_t;\varepsilon_{M_t^{US}}=\varepsilon_{M_t^{CH}}=\varepsilon_{M_t^{BR}},s) \\
&- \kappa Y^i_{t+s}GIRF(Y^i_t;\varepsilon_{M_t^{US}}=\varepsilon_{M_t^{CH}}=\varepsilon_{M_t^{BR}},s) \qquad (2.69)
\end{aligned}
$$

Assuming that in time 0 the system is in equilibrium, and that the shock happens in time t, the short run change in the utility can be then given by the following formula:

$$u^i_{PEG,PEG,Short} = \left[\frac{dU^i_{t+s}}{dM_t^{US+CH+BR}}\right] \tag{2.70}$$

where $t = 1$, and the long-run:

$$u^i_{PEG,PEG,Long} = \sum_{s=2}^{50} \beta^s \left[\frac{dU^i_{t+s}}{dM_t^{US+CH+BR}}\right] \tag{2.71}$$

which we add to obtain the payoff of country i, to obtain a counterpart of the payoff constructed using the sum of net present value of changes in consumption and output in Chapter 1:

$$u^i_{PEG,PEG} = u^i_{PEG,PEG,Short} + u^i_{PEG,PEG,Long} \tag{2.72}$$

In the strategy **PEG APP**, after the expansionary monetary shock in the US, China increases its money supply while Brazil maintains it at the pre-shock level. In that case the change in the utility in country i, where $i = \{China, Brazil\}$ can be defined as:

$$\begin{aligned}\frac{dU^{R,i}_{PEG,APP,t}}{dM_t^{US+CH}} &= \sum_{s=0}^{\infty} \beta^s \left[\frac{1}{C^i_{t+s}} \left\{\frac{\partial C^i_{t+s}}{\partial M_t^{US}} + \frac{\partial C^i_{t+s}}{\partial M_t^{CH}}\right\}\right] \\ &- \sum_{s=0}^{\infty} \beta^s \left[\kappa Y^i_{t+s} \left\{\frac{\partial Y^i_{t+s}}{\partial M_t^{US}} + \frac{\partial Y^i_{t+s}}{\partial M_t^{CH}}\right\}\right]\end{aligned} \tag{2.73}$$

The change in the utility depends on the linear combination of changes in consumption and output after monetary shock in each country $\left(\frac{\partial X^i_{t+s}}{\partial M^{US}}\right) + \left(\frac{\partial X^i_{t+s}}{\partial M^{CH}}\right)$, where $X = \{C, Y\}$ can be defined using the generalised impulse response functions (GIRFs) in the following way, at time $t + s$:

$$\begin{aligned}&\left\{\frac{\partial C^i_{t+s}}{\partial M_t^{US}} + \frac{\partial C^i_{t+s}}{\partial M_t^{CH}}\right\} = GIRF(C^i_t; \varepsilon_{M_t^{US}} = \varepsilon_{M_t^{CH}}, s) \\ &= \varphi_{US} E(C^i_{t+s} | \varepsilon_{M_t^{US}} = \sqrt{\sigma}, \Omega_{t-1}) - E(C^i_{t+s} | \Omega_{t-1}) \\ &+ \varphi_{CH} E(C^i_{t+s} | \varepsilon_{M_t^{CH}} = \sqrt{\sigma}, \Omega_{t-1}) - E(C^i_{t+s} | \Omega_{t-1})\end{aligned} \tag{2.74}$$

and

$$\left\{\frac{\partial Y^i_{t+s}}{\partial M^{US}_t} + \frac{\partial Y^i_{t+s}}{\partial M^{CH}_t}\right\} = GIRF(Y^i_t; \varepsilon_{M^{US}_t} = \varepsilon_{M^{CH}_t}, s)$$

$$= \varphi_{US} E(Y^i_{t+s} | \varepsilon_{M^{US}_t} = \sqrt{\sigma}, \Omega_{t-1}) - E(Y^i_{t+s} | \Omega_{t-1})$$

$$+ \varphi_{CH} E(Y^i_{t+s} | \varepsilon_{M^{CH}_t} = \sqrt{\sigma}, \Omega_{t-1}) - E(Y^i_{t+s} | \Omega_{t-1}) \tag{2.75}$$

and so the change in the utility at time $t+s$ can be expressed as:

$$\frac{dU^i_{t+s}}{dM^{US+CH}_t} = \frac{1}{C^i_{t+s}} GIRF(C^i_t; \varepsilon_{M^{US}_t} = \varepsilon_{M^{CH}_t}, s)$$
$$- \kappa Y^i_{t+s} GIRF(Y^i_t; \varepsilon_{M^{US}_t} = \varepsilon_{M^{CH}_t}, s) \tag{2.76}$$

Assuming that in time 0 the system is in equilibrium, and that the shock happens in time t, the short run change in the utility can be then given by the following formula:

$$u^i_{PEG,APP,Short} = \left[\frac{dU^i_{t+s}}{dM^{US+CH}_t}\right] \tag{2.77}$$

where $t = 1$, and the long-run:

$$u^i_{PEG,APP,Long} = \sum_{s=2}^{50} \beta^s \left[\frac{dU^i_{t+s}}{dM^{US+CH}_t}\right] \tag{2.78}$$

which we add to obtain the payoff of country i, to obtain a counterpart of the payoff constructed using the sum of net present value of changes in consumption and output in Chapter 1:

$$u^i_{PEG,APP} = u^i_{PEG,APP,Short} + u^i_{PEG,APP,Long} \tag{2.79}$$

In the strategy **APP PEG**, after the expansionary monetary shock in the US, Brazil adjusts its money supply while China maintains it at the pre-shock level. In that case the change in the utility in country i, where $i = \{China, Brazil\}$ can be defined as:

$$\frac{dU^{R,i}_{APP,PEG,t}}{dM^{US+BR}_t} = \sum_{s=0}^{\infty} \beta^s \left[\frac{1}{C^i_{t+s}} \left\{\frac{\partial C^i_{t+s}}{\partial M^{US}_t} + \frac{\partial C^i_{t+s}}{\partial M^{BR}_t}\right\}\right]$$

$$-\sum_{s=0}^{\infty}\beta^s\left[\kappa Y^i_{t+s}\left\{\frac{\partial Y^i_{t+s}}{\partial M_t^{US}}+\frac{\partial Y^i_{t+s}}{\partial M_t^{BR}}\right\}\right] \tag{2.80}$$

The change in the utility depends on the linear combination of changes in consumption and output after monetary shock in each country $\left(\frac{\partial X^i_{t+s}}{\partial M^{US}}\right)+\left(\frac{\partial X^i_{t+s}}{\partial M^{BR}}\right)$, where $X=\{C,Y\}$ can be defined using the generalised impulse response functions (GIRFS) in the following way, at time $t+s$:

$$\left\{\frac{\partial C^i_{t+s}}{\partial M_t^{US}}+\frac{\partial C^i_{t+s}}{\partial M_t^{BR}}\right\}=GIRF(C^i_t;\varepsilon_{M_t^{US}}=\varepsilon_{M_t^{BR}},s)$$

$$=\varphi_{US}E(C^i_{t+s}|\varepsilon_{M_t^{US}}=\sqrt{\sigma},\Omega_{t-1})-E(C^i_{t+s}|\Omega_{t-1})$$

$$+\varphi_{BR}E(C^i_{t+s}|\varepsilon_{M_t^{BR}}=\sqrt{\sigma},\Omega_{t-1})-E(C^i_{t+s}|\Omega_{t-1}) \tag{2.81}$$

and

$$\left\{\frac{\partial Y^i_{t+s}}{\partial M_t^{US}}+\frac{\partial Y^i_{t+s}}{\partial M_t^{BR}}\right\}=GIRF(Y_t;\varepsilon_{M_t^{US}}=\varepsilon_{M_t^{BR}},s)$$

$$=\varphi_{US}E(Y^i_{t+s}|\varepsilon_{M_t^{US}}=\sqrt{\sigma},\Omega_{t-1})-E(Y^i_{t+s}|\Omega_{t-1})$$

$$+\varphi_{BR}E(Y^i_{t+s}|\varepsilon_{M_t^{BR}}=\sqrt{\sigma},\Omega_{t-1})-E(Y^i_{t+s}|\Omega_{t-1}) \tag{2.82}$$

and so the change in the utility at time $t+s$ can be expressed as:

$$\begin{aligned}\frac{dU^i_{t+s}}{dM_t^{US+BR}}&=\frac{1}{C^i_{t+s}}GIRF(C^i_t;\varepsilon_{M_t^{US}}=\varepsilon_{M_t^{BR}},s)\\&-\kappa Y^i_{t+s}GIRF(Y^i_t;\varepsilon_{M_t^{US}}=\varepsilon_{M_t^{BR}},s)\end{aligned} \tag{2.83}$$

Assuming that in time 0 the system is in equilibrium, and that the shock happens in time t, the short run change in the utility can be then given by the following formula:

$$u^i_{APP,PEG,Short}=\left[\frac{dU^i_{t+s}}{dM_t^{US+BR}}\right] \tag{2.84}$$

where $t=1$, and the long-run:

$$u^i_{APP,PEG,Long}=\sum_{s=2}^{50}\beta^s\left[\frac{dU^i_{t+s}}{dM_t^{US+BR}}\right] \tag{2.85}$$

which we add to obtain the payoff of country i, to obtain a counterpart of the payoff constructed using the sum of net present value of changes in consumption and output in Chapter 1:

$$u^i_{APP,PEG} = u^i_{APP,PEG,Short} + u^i_{APP,PEG,Long} \tag{2.86}$$

Third approach to welfare evaluation using data and GVAR

We proceed with the derivation of the payoffs using the **third approach**.

To derive the welfare change of agent in economy i we start by recalling the expression for change in the utility of a representative agent derived before:

$$u^i = c^i - \frac{\theta-1}{\theta}y^i + \frac{\beta}{1-\beta}\left[\bar{c}^i - \frac{\theta-1}{\theta}\bar{y}^i\right]$$

Consider strategy **APP APP** first. We define the change in consumption in the short-run as the immediate effect of a monetary shock on consumption, i.e $t=1$:

$$\begin{aligned} c^i_{APP,APP} &= GIRF(C^i_t;\varepsilon_{M^{US}_t}) \\ &= \left[E(C^i_t|\varepsilon_{M^{US}_t} = \sqrt{\sigma}, \Omega_{t-1}) - E(C^i_t|\Omega_{t-1})\right] \end{aligned} \tag{2.87}$$

and long-term (between s and S, which in our case is between period 2 and 50):

$$\begin{aligned} \bar{c}^i_{APP,APP} &= \frac{1}{S-s}\sum_{s=5}^{S} GIRF(C^i_t;\varepsilon_{M^{US}_t}, s) \\ &= \frac{1}{S-s}\sum_{s=2}^{S}\left[E(C^i_{s+S}|\varepsilon_{M^{US}_t} = \sqrt{\sigma}, \Omega_{t-1}) - E(C^i_{t+s}|\Omega_{t-1})\right] \end{aligned} \tag{2.88}$$

the change in output in the short-run is given by:

$$\begin{aligned} y^i_{APP,APP} &= GIRF(Y^i_t;\varepsilon_{m^{US}_t}, s) \\ &= \left[E(Y^i_{t+s}|\varepsilon_{M^{US}_t} = \sqrt{\sigma}, \Omega_{t-1}) - E(Y^i_{t+s}|\Omega_{t-1})\right] \end{aligned} \tag{2.89}$$

and the change in the long-term output (between $s=2$ and $S=50$):

$$\bar{y}^i_{APP,APP} = \frac{1}{S-s}\sum_{s=2}^{S} GIRF(Y^i_t;\varepsilon_{M^{US}_t}, s)$$

$$= \frac{1}{S-s} \sum_{s=5}^{S} \left[E(Y^i_{s+S} | \varepsilon_{M^{US}_t} = \sqrt{\sigma}, \Omega_{t-1}) - E(Y^i_{t+s} | \Omega_{t-1}) \right] \tag{2.90}$$

Then the payoff of a representative agent in country i is defines similar to the net present value equation:

$$u^i_{APP,APP} = c^i_{APP,APP} - \frac{\theta-1}{\theta} y^i_{APP,APP} + \frac{\beta}{1-\beta} \left[\bar{c}^i_{APP,APP} - \frac{\theta-1}{\theta} \bar{y}^i_{APP,APP} \right] \tag{2.91}$$

The payoffs in the remaining strategies are defined in a similar manner, using the impulse reposes after monetary expansion in more than one country (linear combination of shocks).

2.5 Data

In this section we describe the data and the sources we have chosen for the analysis and explain the transformations we have done.

2.5.1 Variables

The choice of variables for the GVAR is motivated by the theoretical model by Corsetti et al. (2000). Since our motivation is to compare changes in welfare in emerging economies resulting after monetary expansion in an industrial economy we select the variables which are used in construction of the payoffs, such as consumption ($CONS_{it}$) and output (GDP_{it}), as well as money supply ($M1_{it}$), which is the policy instrument in each economy (where i = country, t = time).

We obtain the nominal variables in home currency $CONS_{it}$, GDP_{it}, and $M1_{it}$ from the data sources. As a source of the data we use national statistics via DataStream and Federal Reserve Economic Data (FRED). For Brazil we use Brazilian ipeadata macroeconomic database provided by the Institute of Applied Economic Research (Ipea) and available online. We use quarterly data. If the variables are not seasonally adjusted, we do the adjustment using EViews using the adjustment method of moving average (Ratio to moving average - Multiplicative). Since variables come from different sources we make necessary adjustments to express the variables in common units (millions or billions). To facilitate the comparison as well as be able to analyze the impact of monetary shocks of the same size in all countries we use we use an exchange rate (EX_{it})

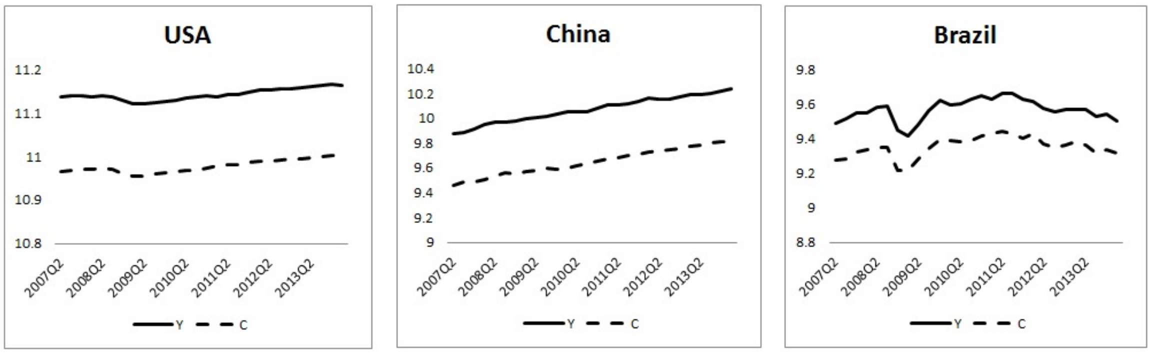

Figure 2.1: Consumption and output in US, Brazil and China, Natural logarithm of the level, US dollars (2005)

and express the variables in the same currency, namely the US dollar. In order to look at real variables we deflate the consumption and output with the use of GDP deflators (country specific D_{it}). If necessary we re-base the GDP deflators to a common base year, for example "2005=100". We use natural logarithms of each variable.

Each variable is transformed into a real seasonally adjusted variable according to the following formula:

$$C_{it} = ln(c_{it}) = \frac{CONS_{it}}{D_{it}} \times EX_{it}$$

$$Y_{it} = ln(y_{it}) = \frac{GDP_{it}}{D_{it}} \times EX_{it}$$

$$M_{it} = ln(m_{it}) = M1_{it} \times EX_{it}$$

The summary statistics for the full sample 1994 Q4 - 2014 Q1 are available in the Appendix 2.8.4 in Table 2.11. We provide the plots of consumption and output in each country on Figure 2.1.

In order to analyse the effect of joint shocks in more than one country we are constructing regions, which are aggregating the country variables making use of the PPP GDP data from the World Development Indicators Data Set of the World Bank.

2.5.2 Trade weights

In order to construct the global model we make use of the trade weights. In construction of weight matrices (as well as regional aggregation procedure) we follow Dees et al. (2007). We construct trade weights by using the data on imports and exports from the

Table 2.6: Trade weights 2009

	USA	Brazil	China	Sum
USA	-	0.11	0.89	1
Brazil	0.50	-	0.50	1
China	0.88	0.12	-	1

OECD STAN Bilateral Trade database by Industry (grand total) and End-use category (Total trade in goods), assuming that the world consists of the number of countries that we include in our model. In case of our three-country model these are USA, Brazil and China. We sum imports and exports of each country and compute what are the shares of individual countries trading partners in their total trade. We are using trade weights for year 2009.

Trade wights are used to construct foreign variables as well as to solve the global VAR. To construct trade weights for United States, Brazil and China we collect data on bilateral exports and imports in thousand USD, in this particular case for 2009. We sum the value of bilateral export and import and treat the world as if it was composed of only those three countries. This allows us to compute the share of trade coming from each trade partner in total trade of a country, values which are presented in Table 2.6. Notice that each column adds up to one. So for example, only 11% of total US trade comes from Brazil, while 0.89% of its trade comes from China. Half of the total trade of Brazil comes from China, while the other half from the US, and 88% of total trade in China comes from the US, while only 0.12% of its trade comes from Brazil.

2.5.3 Regional aggregation

In the construction of regions we use PPP GDP which is a measure of gross domestic product using purchasing power parity rates. We use data from the World Bank.

We then use these weights together with country specific variables to construct regional variables using the following formula:

$$X_{REGIONt} = \Sigma_l^{N_i} \omega_{il}^0 X_{ilt} \tag{2.92}$$

where $X_{REGIONt}$ is the regional variable in a region, ω_{il}^0 is a weight used in aggregation (here PPP GDP), and X_{ilt} is a country-specific variable (country l in region i), i is the number of countries in the region.

In particular, in order to analyse linear combination of money shocks, we construct the following regions:

- REGION1 consisting of two countries in the strategy PEG APP, namely US and China, and the regional variable becomes:

$$M_{REGION1t} = PPP, GDP(US_t)M_{USt} + PPP, GDP(CH_t)M_{CHt} \tag{2.93}$$

- REGION2 consisting of two countries in the strategy PEG APP, namely US and Brazil, and the regional variable becomes:

$$M_{REGION2t} = PPP, GDP(US_t)M_{USt} + PPP, GDP(BR_t)M_{BRt} \tag{2.94}$$

- REGION3 consisting of three countries in the strategy PEG PEG, namely US, China and Brazil, and the regional variable becomes:

$$M_{REGION3t} = \\ PPP, GDP(US_t)M_{USt} + PPP, GDP(CH_t)M_{CHt} + PPP, GDP(BR_t)M_{BRt} \tag{2.95}$$

Similarity as in the case of constructing trade weights one can use values of PPP GDP from a particular year, or the average over few years. We have decided to chose the latter, yet we have checked and the size of the weights do not vary significantly over time.

2.6 Results

In this section we present the results from the GVAR for selected samples. We show emerging countries' payoffs, which we construct using the results from the GVAR and the resulting Nash equilibria. The way we approached the estimation procedure allows us to analyse the evolution of the Nash equilibria over time. The findings are presented within the relevant subsections.

2.6.1 Cointegrating Vectors and Persistence Profiles

We estimate 8 samples in annual intervals of the final period over the period beginning in 1994 Q4 until 2007 Q2 through 2014 Q1. In this section we present the estimation

Table 2.7: VECMX Estimation: Cointegrating Vectors - t-values in brackets

	1994 Q4 - 2007 Q2			1994 Q4 - 2014 Q1		
	USA	China	Brazil	USA	China	Brazil
Y	1.00	1.00	1.00	1.00	1.00	1.00
	-	-	-	-	-	-
C	-0.92	-1.87	-1.07	-1.13	-1.57	-1.05
	(5.67)	(5.88)	(41.41)	(6.63)	(4.84)	(14.67)
M	0.13	-0.53	0.06	0.06	-0.19	-0.02
	(-1.46)	(2.11)	(-1.64)	(-1.29)	(1.02)	(0.13)
Y^*	-0.16	2.17	1.48	-0.10	-3.47	0.77
	(2.74)	(-2.02)	(-3.07)	(1.57)	(2.27)	(-0.61)
C^*	0.15	-2.70	-0.91	0.00	4.88	0.20
	(-1.48)	(2.15)	(1.97)	(-0.02)	(-2.35)	(-0.27)
M^*	-0.04	-1.28	-0.94	0.13	0.62	0.33
	(0.83)	(5.10)	(3.39)	(-2.26)	(-2.21)	(-1.01)
trend	0.00	0.02	0.00	0.00	0.00	-0.01
	(-0.47)	(-2.91)	(-0.44)	(0.51)	(0.28)	(1.18)

results from regressions using the full sample, i.e. 1994 Q4 - 2014 Q1, and the pre-crisis sample, i.e. 1994 Q4 - 2007 Q2. The estimation for the periods between these interval are available upon request.

We decide to impose the same normalization in all samples in order to facilitate the comparison. Table 2.7 summarizes the estimated coefficients together with the t-values in brackets for the full sample and the pre-crisis sample. In all samples we normalize the domestic output in each country. Under this normalization we expect the coefficient on consumption to be negative and close to one, implying one to one relationship between consumption and output.

The coefficient on consumption in China is -1.56 and in Brazil -1.05, in the full sample, and -1.87 in China and -1.07 in Brazil in the pre-crisis sample. The coefficients are of expected sign and size, and statistically significantly different from zero, and except from China imply one to one relationship between consumption and output.

Since we are interested in the effect of changes in foreign money supply as well as domestic money supply we want to look at the estimated coefficients on M^* and M in both samples. The coefficients on foreign money supply M^* in China and Brazil in the pre-crisis sample are statistically significantly different from zero (at 0.05 significance level) and negative, implying an increase in output, after monetary expansion abroad.

The estimated coefficient on domestic money supply M in both China and Brazil and in both samples are less significant and much smaller. In the full sample, the estimated coefficients on the foreign money supply have the opposite sign relative to the pre-crisis sample, and are less significant. We expect these differences in estimated coefficients to generate different welfare consequences of strategic interactions in the pre-crisis and full samples.

We present the persistence profiles of the cointegrating vectors in each country for the pre-crisis and full sample in Figure 2.2.[12] These persistence profiles indicate that the system comes back to an equilibrium after the shock in both emerging economies very quickly. The adjustment takes place between 8 quarters for the samples ending in the pre-crisis period, through 2009 Q2 and then reduces to 4-5 quarters in the samples finishing after 2010 Q2.

2.6.2 Generalized Impulse Response Functions

The Generalized Impulse Response Functions (GIRFs) inform us about the effect of a one standard deviation shock on individual variables over time. Together with the forecasted values of consumption and output the GIRFs are the main variables we use in construction of the payoffs in the non-cooperative game between emerging economies.

Since in Chapter 1 strategy PEG corresponds to an increase in money supply in an emerging economy of the same size as in the US we need to make sure that the size of the shock to money supply in each country is the same. Since GVAR is constructed from individual country models, the size of the the one standard deviation of the residual in money equation may in general not be the same. In order to measure an impact of a monetary shock of the same size in each country we weight the GIRFs for consumption and output by the size of the shock. We use GIFRs from regional shocks, that is the linear combination of shocks originating in different economies, and normalize them by the weights constructed from the standard deviation of the residuals σ^i in the money equations. We define the weight for country i by the inverse of the standard deviation in each country: $\frac{1}{\sigma^i}$.

Let the weights in our three-country model be $\frac{1}{\sigma^{US}}$, $\frac{1}{\sigma^{CH}}$, and $\frac{1}{\sigma^{BR}}$, and the GIRF of a variable X, where $X = C, Y$ in country i at time t after s quarters, in this case a linear combination of shocks in all three countries (that is the regional shock) be denoted by $G\hat{IRF}(X_t^i; \varepsilon_{M_t^i}, \varepsilon_{M_t^{CH}}, \varepsilon_{M_t^{BR}}, s)$. Then the weighted GIRF of variable X which reflects

[12]The persistence profile in the remaining samples are very similar and available upon request.

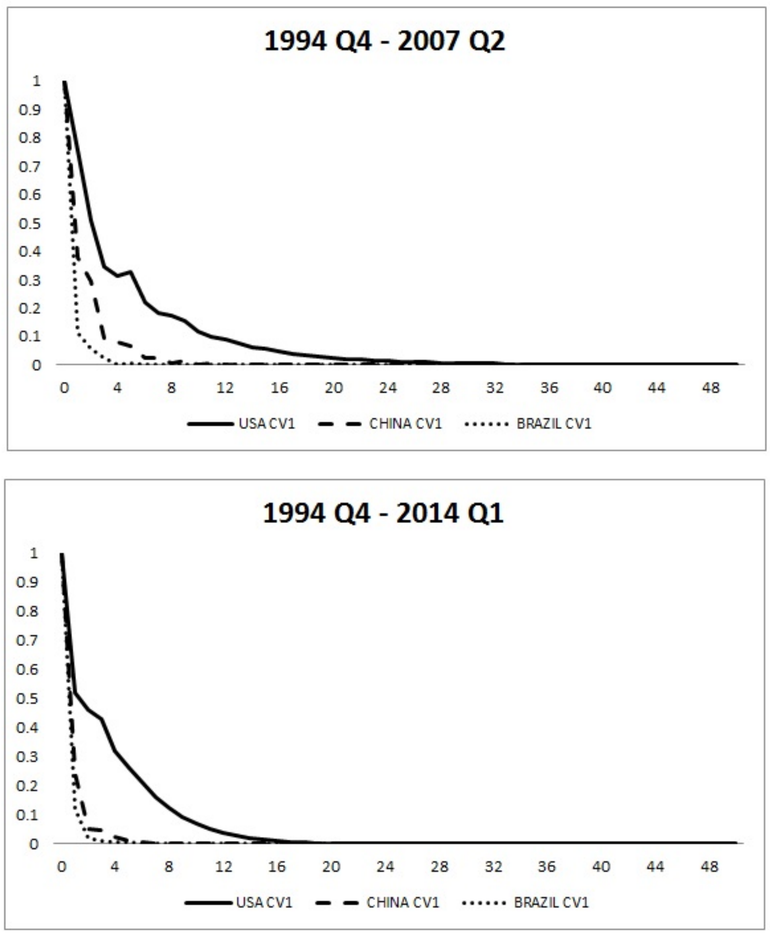

Figure 2.2: Persistence profiles of the cointegrating vectors in the pre-crisis and full sample

the shock of the same size in each country, in the case of strategy **PEG PEG**, is given by the following formula:

$$GIRF(X^i_{t,PEG,PEG}) = GIRF(X^i_t; \varepsilon_{M^{US}_t}, \varepsilon_{M^{CH}_t}, \varepsilon_{M^{BR}_t}, s)$$

$$= \frac{1}{\sigma^{US}} G\hat{I}RF(X^i_t; \varepsilon_{M^{US}_t}, \varepsilon_{M^{CH}_t}, \varepsilon_{M^{BR}_t}, s)$$

$$+ \frac{1}{\sigma^{CH}} G\hat{I}RF(X^i_t; \varepsilon_{M^{US}_t}, \varepsilon_{M^{CH}_t}, \varepsilon_{M^{BR}_t}, s)$$

$$+ \frac{1}{\sigma^{BR}} G\hat{I}RF(X^i_t; \varepsilon_{M^{US}_t}, \varepsilon_{M^{CH}_t}, \varepsilon_{M^{BR}_t}, s) \tag{2.96}$$

In a similar way we construct the GIRFs in the remaining strategies:

- in case of **PEG APP**:

$$GIRF(X^i_{t,PEG,APP}) = GIRF(X^i_t; \varepsilon_{M^{US}_t}, \varepsilon_{M^{CH}_t}, s)$$

$$= \frac{1}{\sigma^{US}} G\hat{I}RF(X^i_t; \varepsilon_{M^{US}_t}, \varepsilon_{M^{CH}_t}, s) + \frac{1}{\sigma^{CH}} G\hat{I}RF(X^i_t; \varepsilon_{M^{US}_t}, \varepsilon_{M^{CH}_t}, s) \tag{2.97}$$

- in case of **APP PEG**:

$$GIRF(X^i_{t,APP,PEG}) = GIRF(X^i_t; \varepsilon_{M^{US}_t}, \varepsilon_{M^{BR}_t}, s)$$

$$= \frac{1}{\sigma^{US}} G\hat{I}RF(X^i_t; \varepsilon_{M^{US}_t}, \varepsilon_{M^{BR}_t}, s) + \frac{1}{\sigma^{BR}} G\hat{I}RF(X^i_t; \varepsilon_{M^{US}_t}, \varepsilon_{M^{BR}_t}, s) \tag{2.98}$$

- and in case of **APP APP**:

$$GIRF(X^i_{t,APP,APP}) = GIRF(X^i_t; \varepsilon_{M^{US}_t}, s) = \frac{1}{\sigma^{US}} G\hat{I}RF(X^i_t; \varepsilon_{M^{US}_t}, s) \tag{2.99}$$

For example in the case of a full sample the standard deviations of the residuals in the money equations are: $\sigma^{US} = 5\%$, $\sigma^{CH} = 1\%$, and $\sigma^{BR} = 2.7\%$, so the weighted GIRFof a variable X ,for country i in time t in the strategy PEG PEG is given by:

$$GIRF(X^i_{t,PEG,PEG}) = \frac{1}{0.005} G\hat{IRF}(X^i_{t,PEG,PEG})$$

$$+\frac{1}{0.01} G\hat{IRF}(X^i_{t,PEG,PEG}) + \frac{1}{0.027} G\hat{IRF}(X^i_{t,PEG,PEG})$$

and the constructed GIRF shows an effect of 1% change in money supply in each country on the variable X such as consumption and output in country i.

In Chapter 1 we analyze the effect of monetary expansion in different strategies on consumption and output. However, the effects in Chapter 1 are presented not as absolute changes in the variables, but as relative changes, i.e. relative to the other emerging economy. This is due to the algebraic complexity that arises as soon as we extend the model beyond two countries. Such approach makes it difficult to formulate predictions as for the direction of change in the individual variables. This complication does not arise as far as the data analysis using GVAR methodology is concerned, and we can use this approach to analyse the responses in the components of the utility of each country in different strategies.

On Figures 2.3, 2.4, 2.5, and 2.6 we present the weighted GIRFs reflecting the change in output and consumption in both China and Brazil in different samples (multiplied by 100)[13].

The response of consumption and output in China depends strongly on the strategy of Brazil, and less so on China's own strategy in all sample periods. On the other hand, the change in Brazil's consumption and output depends more on its own strategy and less on the strategy of China. This responses are rather counterintuitive given the trade linkages between those countries.

By performing the analysis over various sample periods in time we can see how the pattern of changes in response of consumption and output in China and Brazil evolves, see Figures 2.3 - 2.6.

In the period before the financial crisis of 2007-08, the US monetary expansion affects negatively both output and consumption in Brazil. This effect remains in the sample period finishing in 2008 Q2. In the sample period finishing in 2009 Q2 the effect on the long-term consumption and output becomes positive in the strategy PEG APP, while the immediate effect is still negative. No matter when the shock in the US occurs[14], and no matter what Brazil is doing, its output after the US monetary expansion falls in the short-term in all analysed sample periods. Short-run negative effect on consumption can

[13]Remaining samples GIRFs are available upon request

[14]by construction the shock happens at the end of each sample period

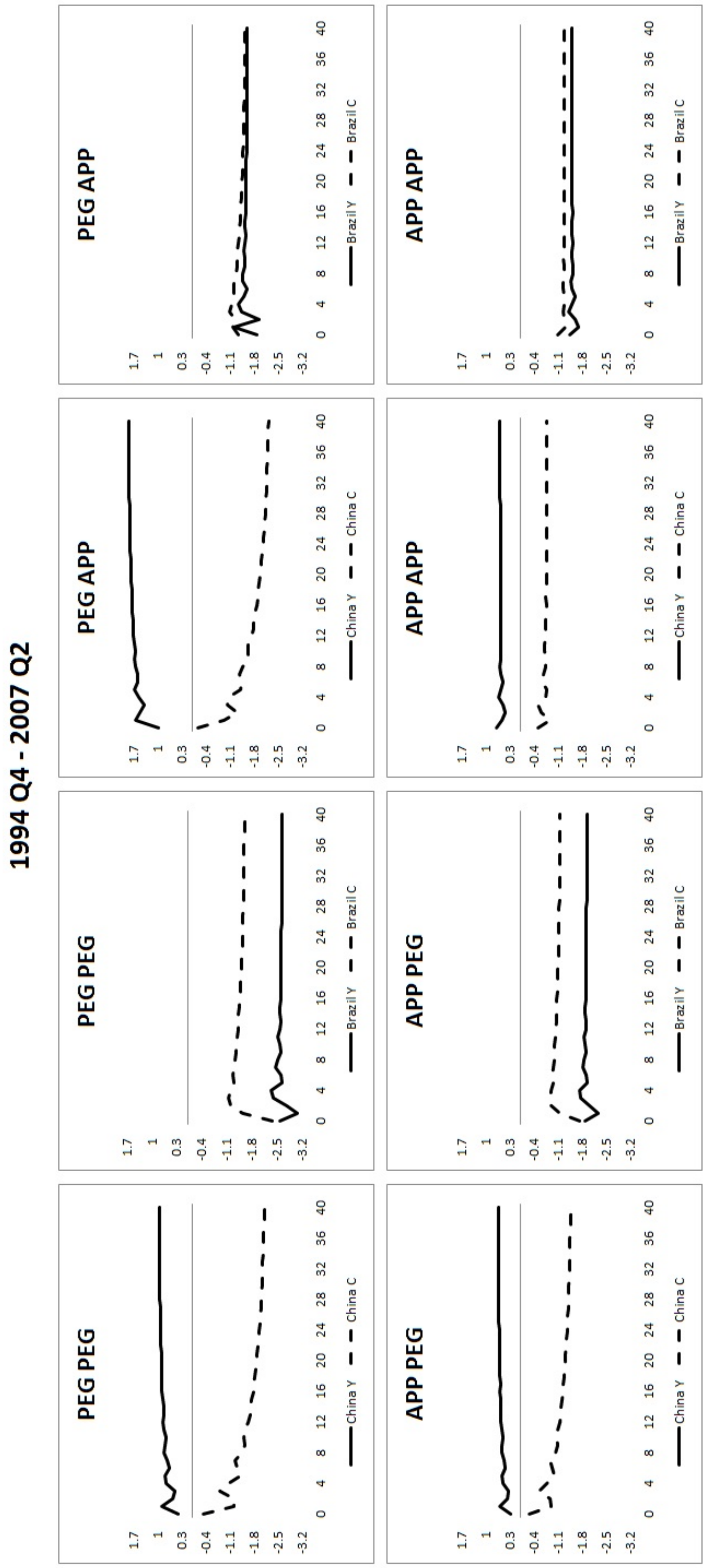

Figure 2.3: GIRFs of consumption (C) and output (Y) 1994 Q4 - 2007 Q2

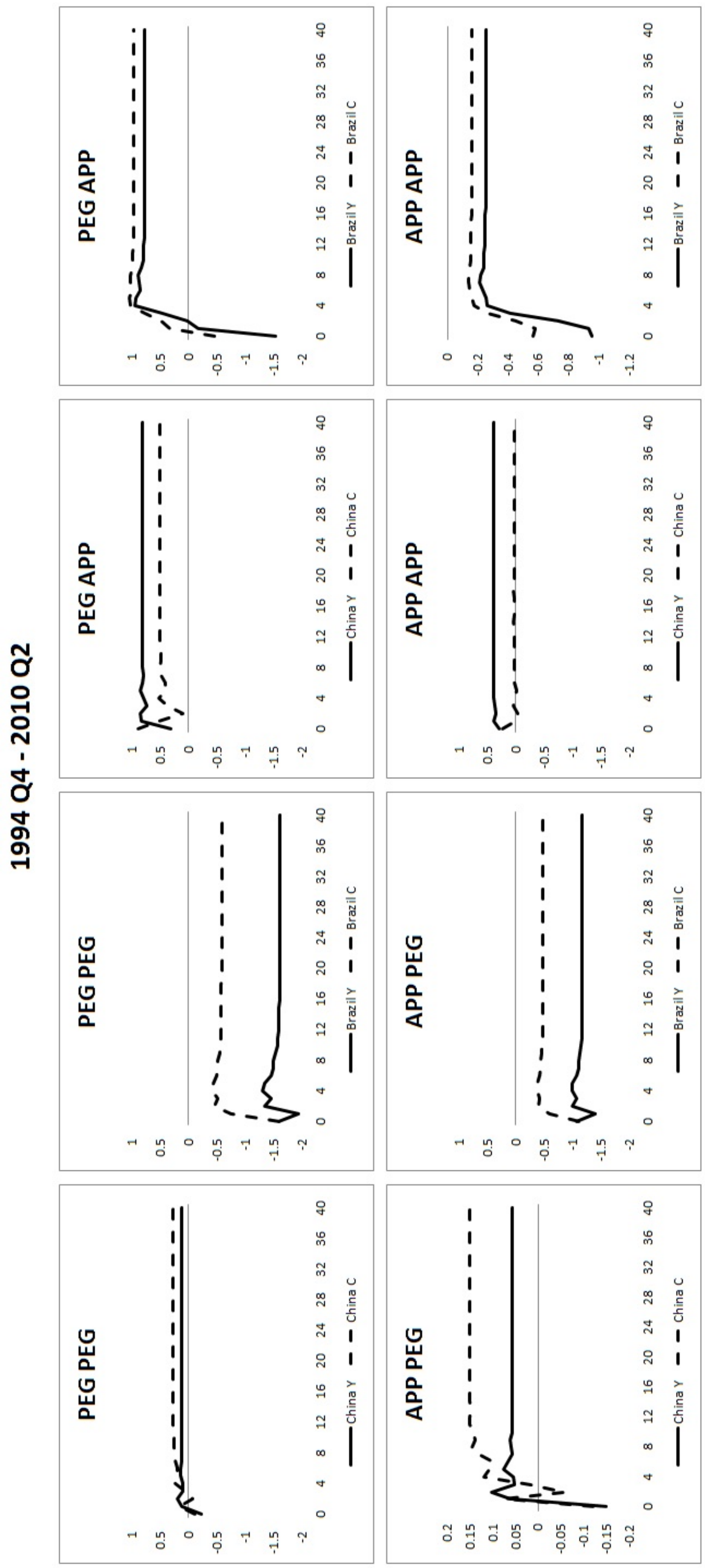

Figure 2.4: GIRFs of consumption (C) and output (Y) 1994 Q4 - 2010 Q2

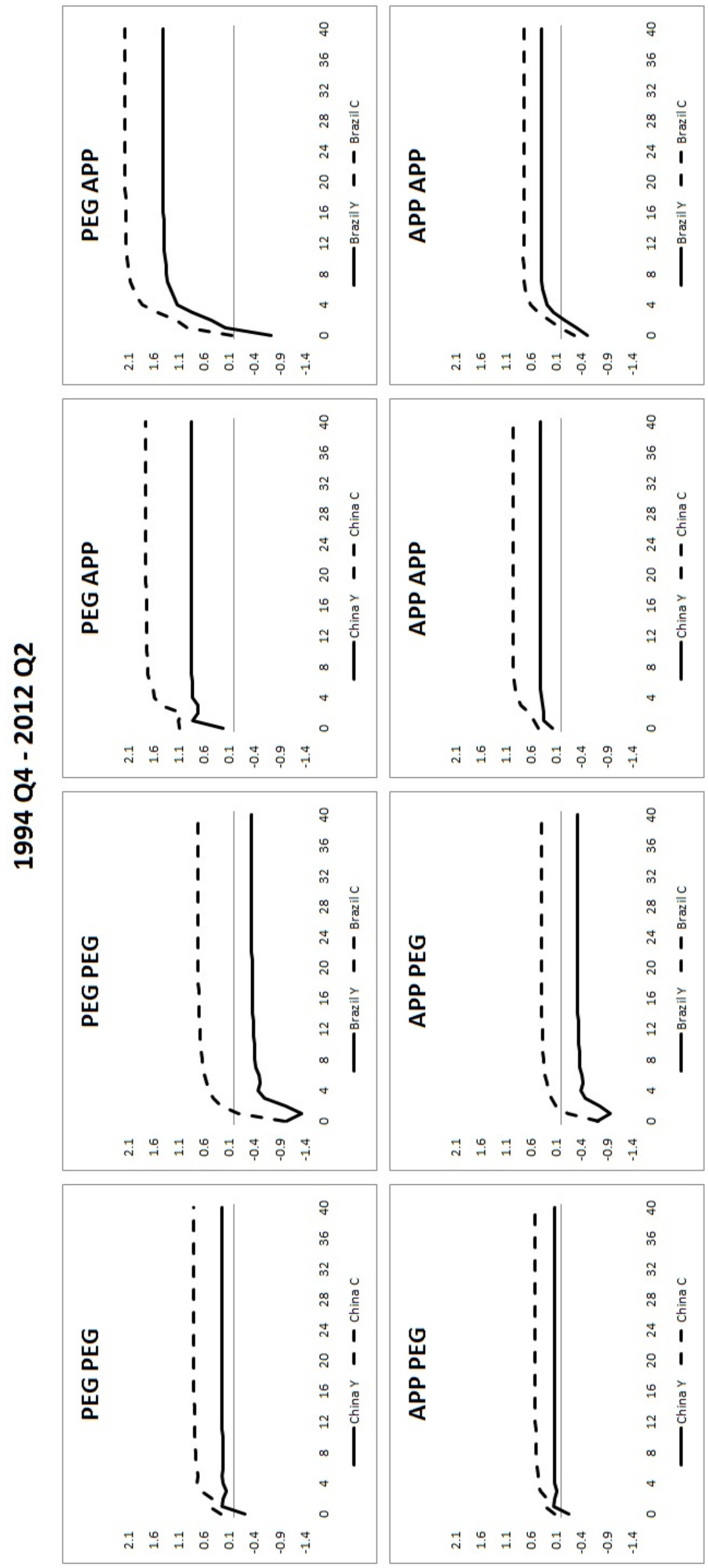

Figure 2.5: GIRFs of consumption (C) and output (Y) 1994 Q4 - 2012 Q2

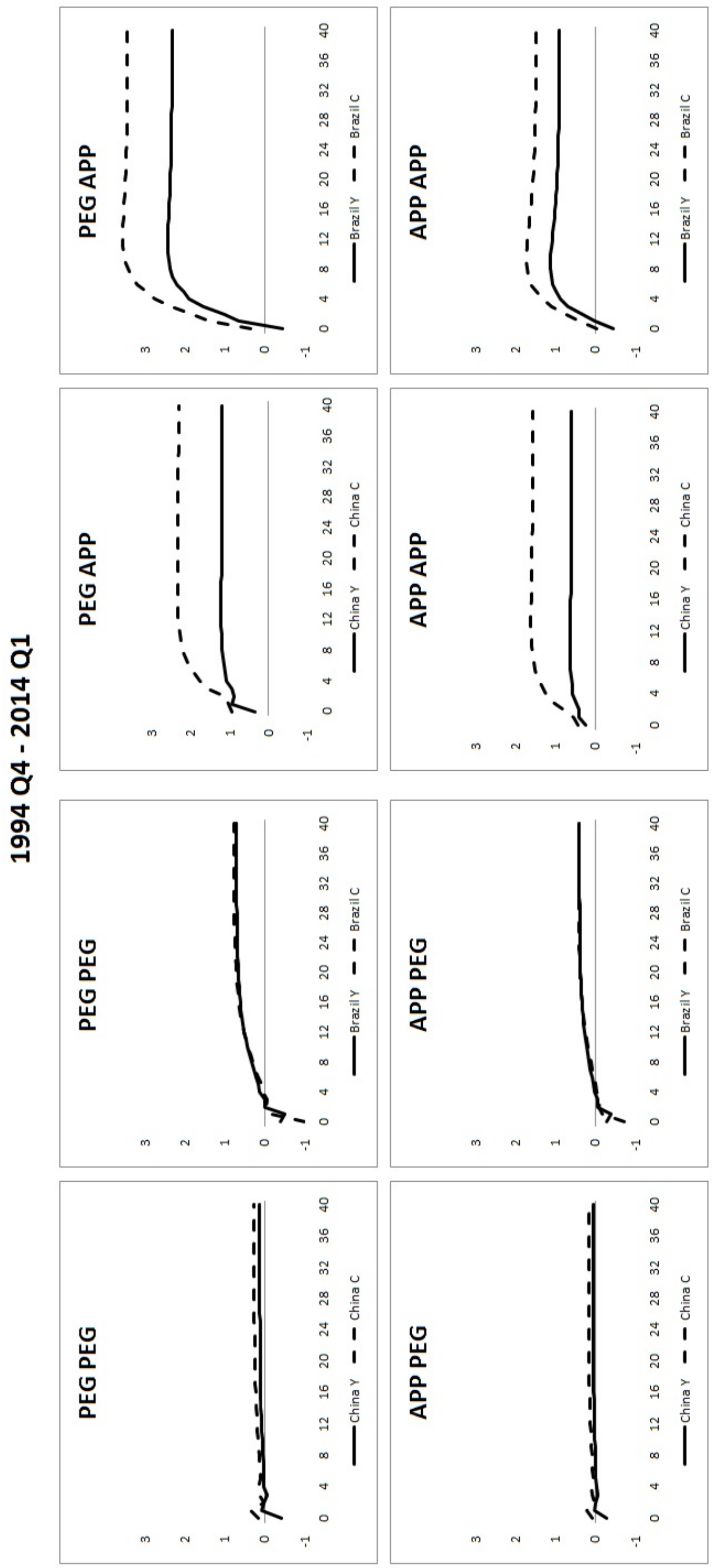

Figure 2.6: GIRFs of consumption (C) and output (Y) 1994 Q4 - 2014 Q1

be observed in the sample periods until 2011 Q2 and becomes positive in all analysed samples thereafter. Beginning in the sample period in 2011 Q2 the effect of monetary expansion in the US on consumption and output turns positive in the long-run in the strategy PEG APP and APP APP and in all strategies after the sample period finishing in 2013 Q2.

The decrease in Brazilian output in the samples until around 2010 is consistent with the traditional monetary policy transmission mechanism that predicts a relatively lower output in the short- and in the long-run in the country with no monetary shock (which in our model is equivalent to allowing its currency to appreciate), if the so called Marshall-Lerner-Robinson (MLR) condition[15] does not hold. The fact that the same output response is present in all strategies in Brazil, and that the GIRFs of both consumption and output are similar in different strategies within the same sample (except for the period of 2009 Q2 - 2011 Q2) could possibly be attributed to the steady appreciation of real that took place in Brazil since 2003 (Mourougane, 2011), except for the period around 2008-2010, which persists in the data.

In the period before the financial crisis of 2007-08 and up to approximately 2008 Q2 the effect of a monetary expansion in the US is negative for Chinese consumption, both in the short- and in the long-run in all strategies. This is in contrast to the response of output in China which increases in all strategies in both short- and long-run in all analysed sample periods. The initially negative effect on consumption in China becomes positive over time, especially in the long-run in all sample periods finishing after 2010 Q2.

Relatively higher output and relatively lower consumption in China in all strategies might reflect the fact that during the period until 2009 China was pegging (which is reflected in the data). According to the standard monetary transmission mechanism, if the MLR condition does not hold, in a country where monetary expansion takes place (which in our model is equivalent to maintaining fixed exchange rate) the consumption is relatively lower and the output relatively higher. This effect may prevail over a longer time horizon, even after China abandoned peg since we estimate the model recursively

[15]The MLR condition informs about the effect of a change in relative prices after monetary shock on the trade balance. If the sum of import and export elasticities is greater than one (when this is the case we say that the MLR condition holds, and that it does not if the sum is smaller than one), the currency devaluation (caused by monetary expansion) will improve the trade balance of that country where the shock has originated. As long as the between-country elasticity of substitution is different from one, consumption smoothing will take place via adjustment in the current account. This condition is relevant for the theory predictions of consumption and output responses after a monetary expansion.

(the information in the data from the previous periods influences the results).

The reaction of output in EME is similar to Chen et al. (2012) in terms of their shape of impulse responses of the GDP growth in China and Brazil. Although our analysis relates the effect of QE on EME only indirectly, our findings are very similar to Chen et al. (2012) who attempt to assess the direct impact of the first rounds of the quantitative easing on advanced and emerging economies by analysing the effect of reduction in the US term spread on such variables as money growth, GDP, inflation, stock prices, etc. Unfortunately there analysis does not look at the effect of such policy on consumption nor welfare.

Finding 1. *The effect of the US monetary expansion on consumption in China and Brazil, and on output in Brazil is significantly different in the periods before the financial crisis of 2007-08 and the first round of QE, and after those events. The initially negative short- and long-run effect on consumption in China turns positive after 2010 Q2 and the initially negative effect on output and consumption in Brazil turns positive after 2012 Q2 in the long-run.*

The different response of consumption and output in the period before and after the first round of QE potentially reveals a change in the monetary transmission mechanism over time. This change might be associated with the increase in global linkages and globalization, or with the effect of the unconventional monetary policy in the US on the transmission mechanism. Considering the theory in Chapter 1, this variation could possibly be caused by a change in the underlying parameters of the model such as the substitutability between goods, which in turn are related to the so called MLR condition.

The inspection of the Figures 2.3 - 2.6 reveals another finding:

Finding 2. *The long-run response of output in China and Brazil after the US monetary expansion converges over time after the policy of QE in the US.*

The convergence of output and long-run consumption effects in China and Brazil over time could possibly be explained by the fact that after 2009 China became the major trading partner of Brazil, while the role of the US decreased over time (Mourougane, 2011).

The reaction of consumption and output to the US monetary expansion in the emerging economies plays a crucial role in the evaluation of welfare changes expressed using the constructed payoffs. We describe how these changes drive the evolution of payoffs over time in Section 2.6.3 where we show that the change in the response of consumption and output in China and Brazil corresponds to the period in which the Nash equilibrium changes, that is the period until 2010 Q2 and 2011 Q2 (see Finding 4).

2.6.3 Payoffs

Using the computed forecasts for output and consumption, together with weighted GIFRs we construct the payoffs in the way described in Section 2.4.2. The results presented here are discussed using the first of three suggested approaches. The results from the remaining two approaches are presented in the Appendix 2.8.1.

The payoff matrices that contain the payoffs from different samples are the normal-form representations of the non-cooperative game between China and Brazil, and are presented on Figure 2.7. On Panel A we present the short-term payoffs, representing the immediate change in the utility after the shock in each emerging economy for the three samples: pre-crisis sample, i.e. 1994 Q2 - 2007 Q2, the sample until 2011 Q2 and the full sample, i.e. 1994 Q4 - 2014 Q1 for the elasticity of substitution parameter $\theta = 6$. This is the most used value of θ in the macroeconomics literature (for example Gali and Monacelli (2005)). On Panel B we present the long-run effects in form of cumulative payoffs over the period of 50 quarters for the corresponding samples as in the short-term for the same elasticity of substitution parameter $\theta = 6$. Nash equilibria are in bold. We have identified a pattern in the way Nash equilibria evolve over time and present it in Table 2.8. Because the resulting Nash equilibria are the same in 'adjacent' samples[16], we decided to present the short- and long-run results only for the representative samples. The remaining payoff matrices are available upon request.

Table 2.8 presents the summary of the results obtained from the payoff matrices in all samples over time for the elasticity of substitution parameter $\theta = 6$. However, since in Chapter 1 the within-country elasticity of substitution θ is assumed to be greater than 1 we perform robustness checks to see if the Nash equilibria vary with the values of this elasticity parameter between 1 and 6. We conclude that the results are quite robust in terms of the chosen values of the elasticity of substitution parameter θ.

As far as the immediate adjustment of consumption and output in each sample is concerned, the short-run policy recommendation for China is to peg and for Brazil to allow its currency to appreciation. PEG APP is the short-run Nash equilibrium in all samples, with one exception of the sample ending in 2010 Q2. Similar policy has

[16] For example the long-run Nash equilibrium is the same in samples finishing in 2007 Q2 and 2008 Q2.

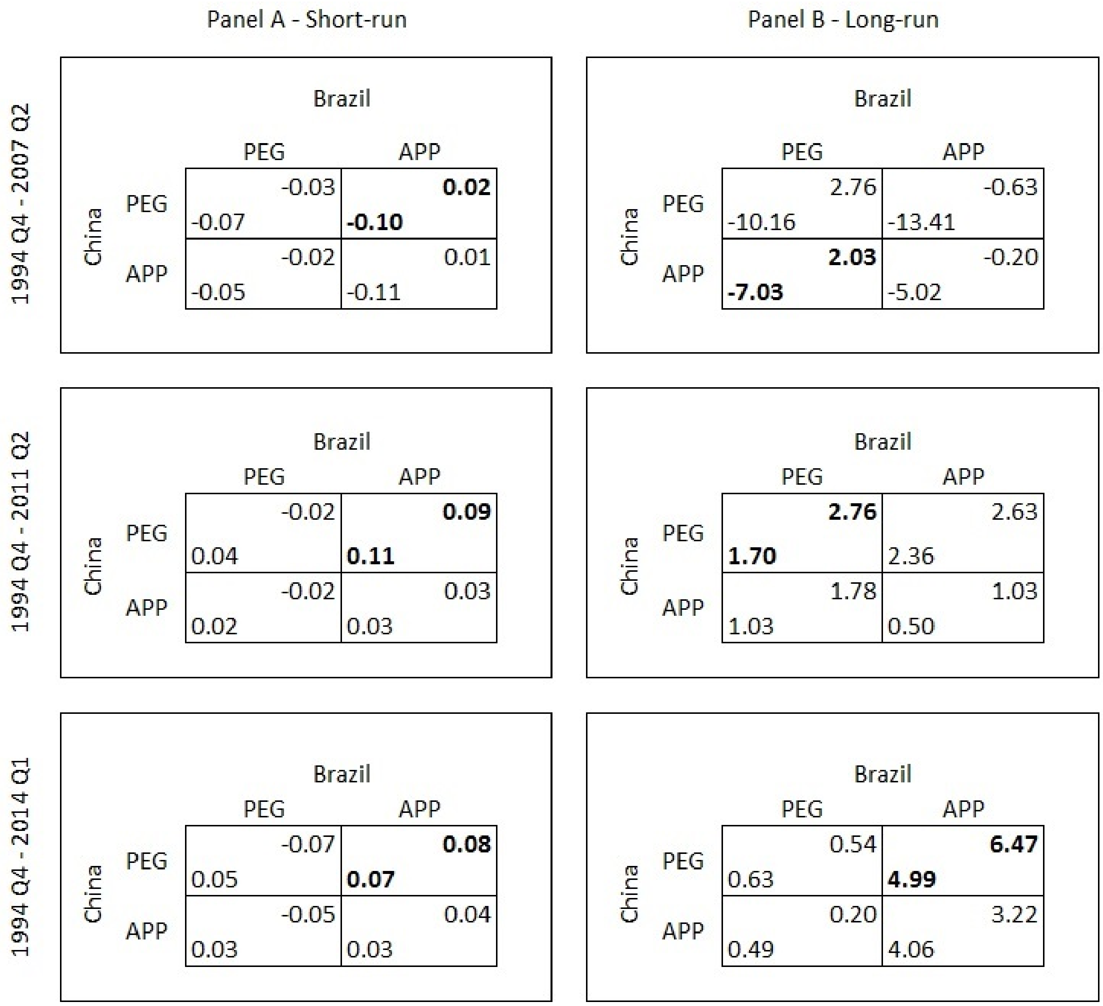

Figure 2.7: Short and long-run Nash equilibria in selected samples

been observed after the the first round of the quantitative easing in the US and may explain that it was optimal for China, from the short-run point of view, to maintain fixed exchange rate relatively to the US.

Finding 3. *The policy recommendations for the emerging economies after monetary expansion in the US vary depending on the forecasting horizon. In the short-run it is optimal to peg for China, and for Brazil to allow its currency to appreciate. In the long-run the policy recommendations are history-dependent (see Finding 4).*

Although in the short-run PEG APP results in more beneficial welfare effect for Brazil and China, it is not a long-run Nash equilibrium in all samples. Based on the computed payoffs, it is optimal for China to allow its currency to appreciate in the periods just before, during the period of the financial crisis, and during the first round of

Table 2.8: Nash equilibria (NE) over time in the estimated samples in the context of the US policy of quantitative easing (QE)

		Nash equilibria	
Sample period	**Economic event**	**Short-run**	**Long-run**
1994 Q4 - 2007 Q2	Pre-crisis period	PEG APP	APP PEG
1994 Q4 - 2008 Q2	Financial crisis, QE I begins	PEG APP	APP PEG
1994 Q4 - 2009 Q2	QE I	PEG APP	APP PEG
1994 Q4 - 2010 Q2	QE I ends	APP PEG	PEG PEG
1994 Q4 - 2011 Q2	QE II	PEG APP	PEG PEG
1994 Q4 - 2012 Q2	Between QE II and QE III	PEG APP	PEG APP
1994 Q4 - 2013 Q2	QE III	PEG APP	PEG APP
1994 Q4 - 2014 Q1	Exit from the QE	PEG APP	PEG APP

quantitative easing (until between 2009 Q2 and 2010 Q2[17]), while it is optimal for Brazil to increase money supply over that period from the long-run perspective. Between the period around 2010 Q2 and 2011 Q2 welfare of both China and Brazil increases relative to the pre-shock level in all strategies, and the resulting Nash equilibrium changes to PEG PEG, in cases of high elasticity of substitution (e.g. 6). This is the only period in which the payoffs are sensitive to the choice of values of the elasticity of substitution θ. For low values of θ the Nash equilibrium is then PEG APP, which is the same as the optimal strategy from the short-run perspective. The increase in the utilities can possibly be explained by the fact that the economy was in a recovery period after the crisis. Alternatively, we could attribute this change in payoffs to the way the monetary transmission mechanism has been altered over time by the US policy of quantitative easing. One can see this effect in the way consumption and output respond differently to monetary expansion in the US over time on the GIRFs Figures (e.g. Fig 2.3 - 2.6).

Finding 4. *The policy recommendations for the emerging economies vary depending on when the monetary expansion in the US is applied (i.e. is history-dependent). Our framework suggests that in the period finishing in 2009 Q2, i.e. had the monetary expansion in the US taken place in 2009 Q2, it would have benn optimal for China to allow its currency to appreciate and for Brazil to peg. In the period approximately between 2010 Q2 - 2011 Q2 it would have been optimal for both emerging economies to increase their money supply and keep their exchange rate vis-a-vis the US fixed, while*

[17] In order to identify an exact moment of the policy change it would be helpful to perform recursive estimation using quarterly intervals instead of annual ones.

after 2011 Q2 it would have been optimal for China to keep the exchange rate fixed while for Brazil to allow its currency to appreciate. We attribute this time variation to potential changes in the monetary transmission mechanism over time, possibly due to the effect of the unconventional monetary policy of quantitative easing in the US.

It is interesting to point out that despite the fact that no matter what Brazil is doing, that is in every strategy, its consumption and output decrease in the samples finishing in 2009 Q2, while its payoff is positive and larger than the one of China. At the same time in the aforementioned period the model predicts that Chinese consumption will decrease and output will increase. But because the output enters the utility function with a negative sign, and more output means more disutility from work effort, Chinese payoffs are initially negative. The way the payoffs are constructed should be kept in mind while analyzing the results and interpreting the policy recommendations. We summarize this important remark below (Remark 12).

Remark 12. *By construction, a payoff of country i is positive if the increase in consumption is larger than the increase in output (weighted by $\frac{\theta-1}{\theta}$ where the weight is decreasing in θ), or if the decrease in output (weighted) is larger than the decrease in consumption.*

The payoff matrices are snapshot representations of welfare changes over particular time horizon. Although the long-term changes in the utility computed as cumulative payoffs may be positive, the welfare effect in the emerging economies may vary over time, and even be negative in the short-run. To illustrate that, we plot the per-period changes in the utility over time in China and Brazil in various samples on Figures 2.8, 2.9, 2.10, and 2.11[18]. For example, the initially negative welfare effect in Brazil in the strategy PEG PEG in sample 1994 Q2 - 2010 Q2 (see Figure 2.9) turns positive over time, and this strategy is in fact the Nash equilibrium.

This framework has been designed to analyze how the welfare of emerging economies is affected after the monetary expansion in the US, nevertheless our approach allows to show the impact of such policy of the US on itself in all analysed strategies. The payoffs, representing welfare change in the US are presented relative to Chinese and Brazilian payoffs on Figures 2.8 - 2.11. These figures illustrate that a monetary expansion in the US has much stronger welfare effect in both China and Brazil than domestically. The welfare effect of monetary expansion in the US (even when the monetary expansion

[18]Remaining samples are available upon request.

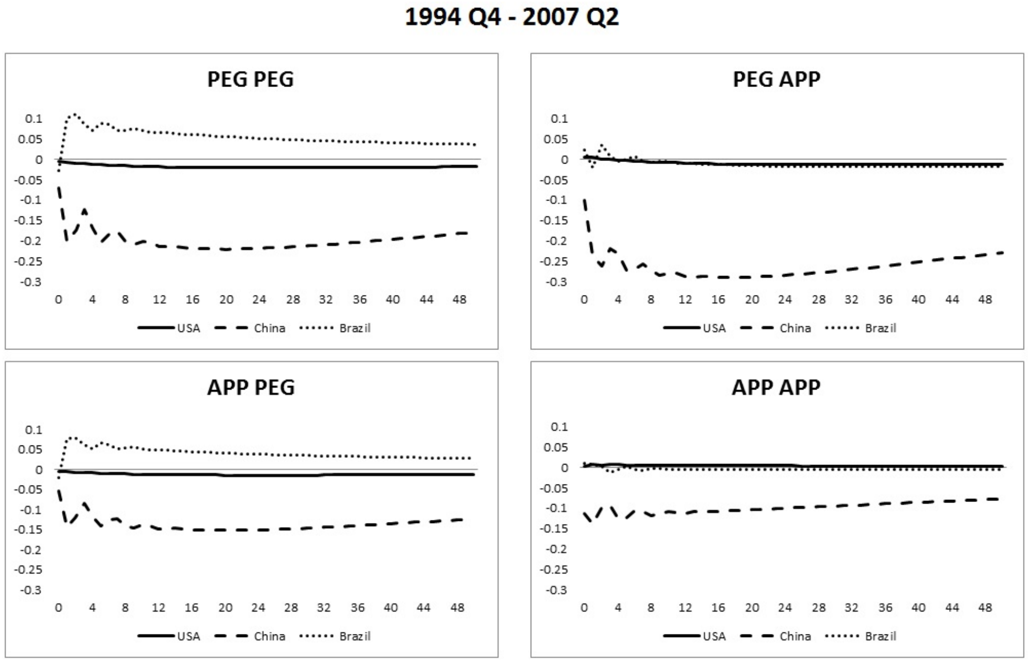

Figure 2.8: Payoffs for the sample 1994 Q4 - 2007 Q2 with $\theta = 6$ - Nash equilibrium APP PEG

takes place simultaneously in the emerging economies) is close to zero in the US in all strategies. We summarize this observation below.

Finding 5. *Welfare in the emerging economies which are trading partners of the US reacts more to the expansionary monetary policy of the US than the welfare of the US itself.*

Although both Chen et al. (2012) and Bernanke (2013) do not explicitly analyze the welfare effect of the QE on EME, their simulations suggest that there are spillovers from the US policy to EME. These spillovers however, may not be very negative and may vary between economies. Indeed the Figures 2.8 - 2.11 show that the welfare impact of expansionary monetary policy in the US varies initially between Brazil and China and becomes more positive after QE in all countries, including the US over time. This brings us to another finding:

Finding 6. *Initially in the sample periods finishing in 2007 Q2 and 2008 Q2 the policy of the US is* ***beggar-thy-neighbour*** *for EME in most cases regardless of the strategy employed by the EMEs. This result is consistent with Obstfeld's (2011) predictions that the emerging economies lose no matter what they do. The payoffs turn positive for Brazil and China in the sample finishing in 2010 Q2 in all strategies but APP APP.*

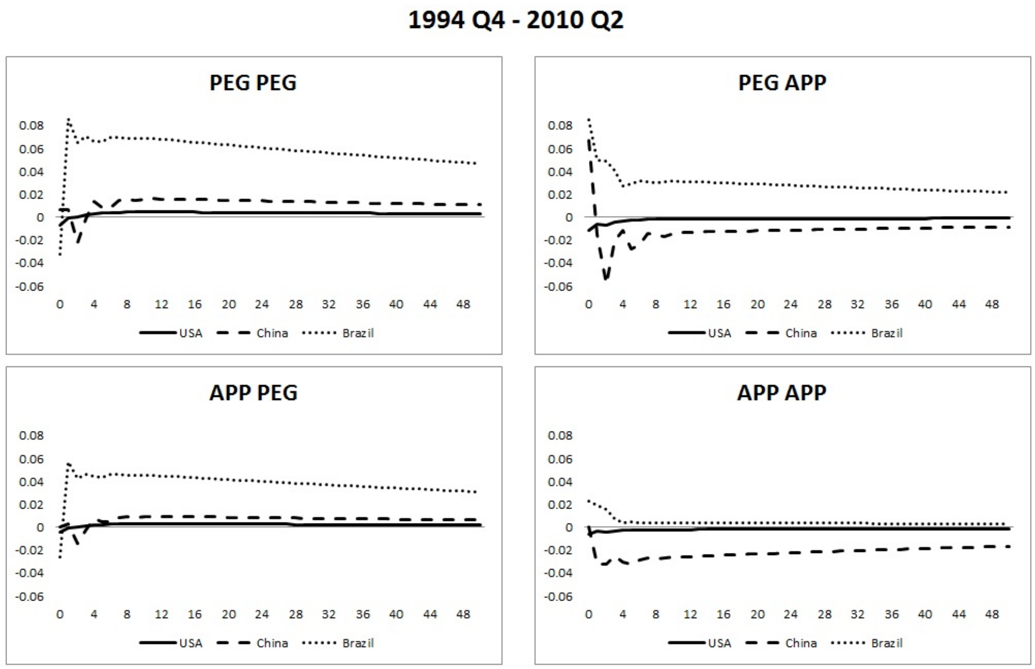

Figure 2.9: Payoffs for the sample 1994 Q4 - 2010 Q2 with $\theta = 6$ - Nash equilibrium PEG PEG

This result is in contrast to Obstfeld (2011), who suggested that EME's loss can be reduced if both countries allow their currencies to appreciate. With the sample finishing in 2011 Q2 the policy of the US becomes ***enrich-the-neighbour*** *in all strategies, which is consistent with Chapter 1 for particular combination of the size and substitutability parameters (see Proposition 2).*

Our Finding 6 sheds light on the existing debate between policymakers about the effect of the monetary policy expansion in the US after the crisis (Bernanke (2013), Rousseff (2012)). Our analysis suggests that after the first round of QE such policy had a positive long-term welfare effect on China and Brazil.

2.6.4 Policy implications

In this Section we relate our findings and resulting Nash equilibria to the economic developments in the world economy, and summarize potential policy recommendations for China and Brazil.

The resulting short-run Nash equilibrium is PEG APP in all analyzed samples except for the one ending in 2010 Q2. In fact after the crisis this is exactly the policy followed by China and Brazil. China maintained fixed exchange relative to the US until 2010.

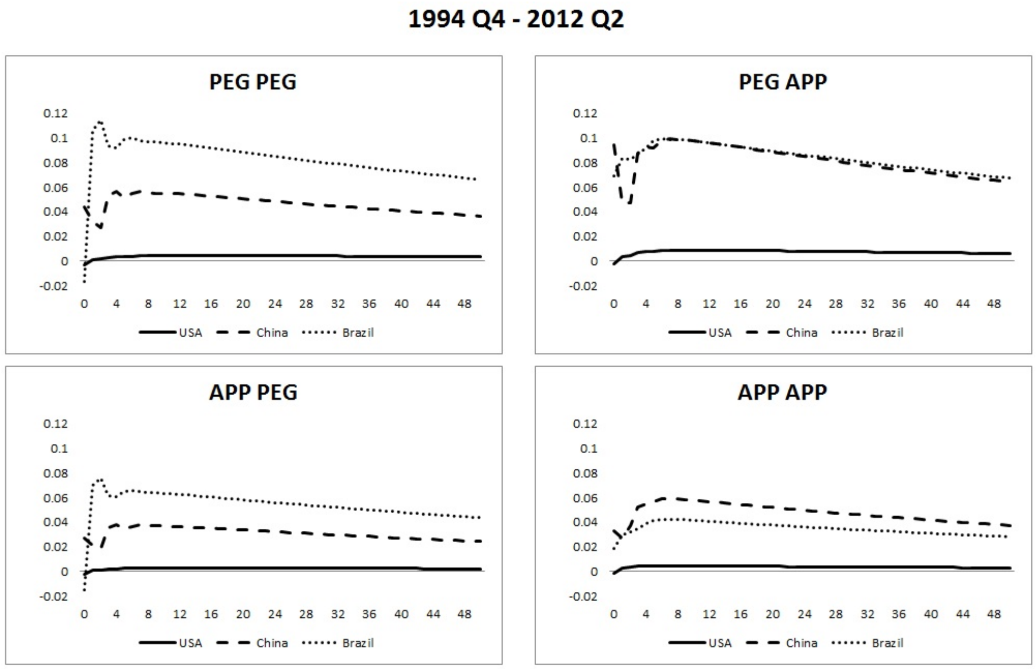

Figure 2.10: Payoffs for the sample 1994 Q4 - 2012 Q2 with $\theta = 6$ - Nash equilibrium PEG APP

Brazil has allowed real to appreciate continuously since 2003 (Mourougane, 2011). Since the policy followed by China and Brazil is a Nash equilibrium only in the short-run, this might suggest that the policymakers in those countries might be shortsighted.

Our framework suggests that in the long-run in the period before the QE, both economies would be better off in the long-run by pursuing the opposite strategy, namely APP PEG. We believe that this policy suggestion at that moment would help to improve welfare in China and Brazil. At that time inflation in China was high and growing (food inflation in 2007 was 12.3% and in 2008 14.3% in 2008, OECD) and further monetary expansion would only contribute to even greater increase in prices. Brazil by allowing its currency to appreciate worsened its competitiveness and monetary expansion could potentially help to increase exports. In practice Brazil appreciation did not continue over the period of 2008-2010, which is consistent with our policy recommendations for Brazil. Our framework however suggest that Brazil should stop allowing its currency to appreciate a year earlier and a year after than it actually did that.

In the period of policy change (i.e. in the samples finishing in 2010 Q2 and 2011 Q2) both countries would be recommended to increase their money supplies. Although PEG PEG strategy is a Nash equilibrium, the presented payoff matrix shows that the alternative strategy PEG APP do not differ significantly in terms of changes in welfare.

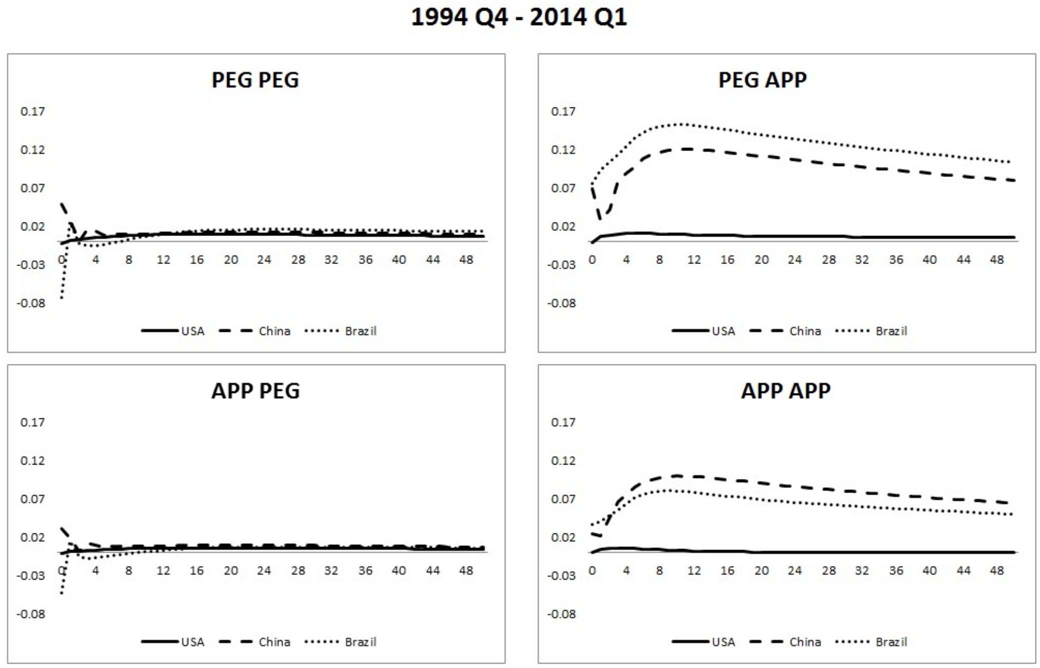

Figure 2.11: Payoffs for the full sample with $\theta = 6$ - Nash equilibrium PEG APP

The policy recommendation changes significantly in the period after the QE and the long-term Nash equilibrium becomes the same as the short-term one.

The different pre-QE and post-QE long-run welfare effects as well as the convergence in the consumption and output responses in China and Brazil might as well be influenced by changing expectations over time, these in turn could be related to the policy of the QE in the US. At this stage we cannot be sure that this is a direct evidence of the US policy of the quantitative easing but it would be interesting to investigate this connection in more detail. Another explanation might be that since there are significant differences between these economies, such as different ongoing policies, stage of development, trade linkages and exchange rate regimes, and consumption and output responses to crisis (see Figure 2.1) it is plausible to expect differences in welfare effects between them over time.

Our results are not consistent with the theory which predicts quantitatively the same payoffs for both emerging economies in the strategy PEG PEG and APP APP. We can possibly explain this departure from the theoretical predictions by differences between the Brazilian and Chinese economies which are not captured in the theoretical model, yet are observed in the data. Similarly to the findings in Chapter 1 we identify no need for policy coordination in the results form the empirical analysis for Brazil and China. We do not observe so called Prisoner's Dilemma in the data, perhaps due to the same reason as in Chapter 1, i.e. large asymmetries between Brazil and China.

Our Finding 5, which says that the effects of expansionary monetary policy in the US has stronger welfare effect on EME than domestically is important to be considered by the policymakers in the US in their future policy decisions.

The Nash equilibria on Figure 2.7 convey an interesting information related to Pareto optimality. Outcomes of strategic interactions between players in a non-cooperative game are considered to be Pareto optimal, if there is no combination of strategies yielding a higher payoff for both countries, than the resulting Nash equilibrium. If however, strategies that allow to improve upon existing Nash equilibrium exist, coordinating countries towards those strategies would result in gains from coordination. In Chapter 1 we have shown sufficient conditions for such cases, which are called Prisoner's Dilemmas.

Finding 7. *According to our framework, short- and long-run Nash equilibria for China and Brazil after monetary expansion in the US are Pareto optimal. Therefore, there is no need for these EME to coordinate on alternative policies.*

This result is consistent with Chapter 1, in which we explain that in the presence of large country-size asymmetries between EME there are no gains from coordination, and in contrast to the suggestions of the IMF (Lagarde (2014)).

It is important to point out that the fact that China and Brazil adopted various policy tools during the period of the analysis has not been explicitly dealt with in the current approach. The change in the exchange rate regime in China could potentially be isolated by undertaking counterfactual analysis allowing to evaluate how the economy would behave in the absence of the regime switch. The approach of modeling regime changes suggested by Hamilton (1989) could be useful. For example, the fact that China abandoned the fixed exchange rate relative to the US dollar in 2009 could potentially be controlled for by forecasting the variables in the model and applying the monetary shocks to the hypothetical data assuming the exchange rate remained fixed. We are aware that our approach could be improved in this sense, and allowing to extract the effect of the actual policy component on the analyzed variables could lead to different welfare evaluation than in the current approach.

To assess the effect of other than exchange rate and monetary policy tools, such as capital controls and reserve accumulation would require a model including more variables. It might therefore be worth to be considered in the future research.

2.7 Conclusions

We propose a novel framework to analyse welfare consequences of monetary expansion in the US on China and Brazil. We find that the effect of the expansionary monetary policy on welfare of EME is time-dependent and varies with the policy horizon used for evaluation of the effects of such policy (Finding 3 and Finding 4). The Nash equilibria are different in the samples before the financial crisis of 2007-08, and in the period before the QE than the Nash-equilibria in the post-crisis samples. We identify optimal strategies which can be used as policy recommendations for EME. In particular, we find that before QE China could have increased its welfare by easing the inflationary pressures via currency appreciation, and Brazil could have gained from monetary expansion which could have improved its competitiveness and increased its exports. In the period after QE our policy recommendations are the same in the long- and in the short-run, and suggest monetary expansion in China and currency appreciation in Brazil. We shed light on the ongoing debate about the effects of the expansionary monetary policy of the US on its trading partners. We find that the welfare effect of such policy is initially negative for Brazil and China, which is consistent with the analysis in Obstfeld (2011). However, over time, in particular after the first round of QE, the welfare effect on EME becomes positive, as asserted by Bernanke (2013), or Powell (2013) (Finding 6). This time-varying welfare effect is a consequence of the change in the response in consumption in China and Brazil in the periods before the QE and after, and the change in the response in output in Brazil in those periods (Finding 1). Although the reaction of output to monetary expansion in the US initially differs significantly between Brazil and China (decrease in of output in Brazil vs. an increase in output in China), we observe convergence in those responses over time, especially after the policy of QE (Finding 2). Among possible explanations of this change over time might be the effect that QE potentially has on the transmission mechanism of monetary policy internationally. We find that the welfare effect of expansionary monetary policy in the US is much weaker in the US than in Brazil and China. In fact, it is close to zero in the US and over time becomes more positive in all countries, including the US (Finding 5). We find, possibly due to the asymmetries between Brazil and China, that there is no need for policy coordination and the so called Prisoner's Dilemma do not arise in any of the analysed samples and analysed horizons, which is consistent with the findings in Chapter 1 (Finding 7).

We believe that our framework can serve as a tool for international organizations to identify welfare-superior strategies of the EME and to highlight the welfare improvement coming from coordination of policymakers in EME towards such policies.

By addressing the existing gap in the literature pointed out in the introduction, we hope to have provided a stepping stone for future research investigating the welfare effects of self-oriented policies on individual economies and their trading partners. We believe that the introduced framework could be improved or amended in several ways. Including more countries could enrich the analysis and draw more realistic conclusions for the global economy. Among other improvements one might perform the analysis on stationary data, consider different specifications, and apply time-varying weights, to name just a few. In order to analyze the change over time in a more accurate way one could consider quarterly recursion instead of the annual one applied here. An additional extension could be to include the US as an active player to identify the so called Nash-Nash equilibria of a global game. This would allow us to investigate if the US can benefit from 'bribing' the emerging economies using transfer payments to change their policy actions for the benefit of the US. Addressing these issues is part of our future research agenda.

Table 2.9: Nash equilibria (NE) over time in the estimated samples in the context of the US policy of quantitative easing (QE) - Alternative 2

Sample period	Economic event	Nash equilibria Short-run	Nash equilibria Long-run
1994 Q4 - 2007 Q2	Pre-crisis period	APP PEG	APP PEG
1994 Q4 - 2008 Q2	Financial crisis, QE I begins	APP APP	APP PEG
1994 Q4 - 2009 Q2	QE I	no NE	APP APP
1994 Q4 - 2010 Q2	QE I ends	PEG PEG	APP PEG
1994 Q4 - 2011 Q2	QE II	PEG PEG	APP PEG
1994 Q4 - 2012 Q2	Between QE II and QE III	PEG PEG	APP PEG
1994 Q4 - 2013 Q2	QE III	PEG PEG	APP PEG
1994 Q4 - 2014 Q1	Exit from the QE	APP APP	APP PEG

2.8 Appendix

2.8.1 Nash equilibria in alternative methods

Table 2.9 presents the predicted short- and long-run equilibria arising using second method.

Results are sensitive to the choice of the the parameter in the disutility from work effort κ in the sense that it (κ) acts as a scaling factor but the Nash equilibria stay the same.

Table 2.10 presents the predicted short- and long-run equilibria arising using third method.

The results from Alternative 1 are qualitatively and quantitatively different in most cases from the Alternative 1 presented in the main body of the Chapter.

The results from Alternative 3 are qualitatively similar (same Nash equilibria in most periods) but quantitatively different from the Alternative 1 presented in the main body of the Chapter.

2.8.2 Related theory and predictions

Predictions from the theory based on the Corsetti et al. (2000) model

In Chapter 1 we show that depending on the relative values of the substitutability parameters, size asymmetry between the countries, and their actions the change in welfare after monetary expansion in the Center the change in welfare varies.

Table 2.10: Nash equilibria (NE) over time in the estimated samples in the context of the US policy of quantitative easing (QE) - Alternative 3

		Nash equilibria	
Sample period	**Economic event**	**Short-run**	**Long-run**
1994 Q4 - 2007 Q2	Pre-crisis period	PEG APP	APP PEG
1994 Q4 - 2008 Q2	Financial crisis, QE I begins	PEG APP	APP PEG
1994 Q4 - 2009 Q2	QE I	PEG APP	APP PEG
1994 Q4 - 2010 Q2	QE I ends	PEG APP	PEG PEG
1994 Q4 - 2011 Q2	QE II	PEG APP	PEG PEG
1994 Q4 - 2012 Q2	Between QE II and QE III	PEG APP	PEG PEG
1994 Q4 - 2013 Q2	QE III	PEG APP	PEG APP
1994 Q4 - 2014 Q1	Exit from the QE	PEG APP	PEG APP

In case all of the three countries increase money supply (by the same amount, which is equivalent with maintaining the exchange rate vis-a-vis the center at unchanged level), a strategy that we call PEG PEG, the change in individual's country welfare will always be **positive**.

The results of such policy can however be **negative** and bring about deterioration of welfare in emerging economies (EME). This can happen for example in the strategy APP APP in case the substitutability between goods produced between the countries in the Periphery and the Center (ρ) is larger that the within country substitutability (θ).

The exact conditions informing about then the expansionary policy if an advanced economy is positive and when negative in terms of welfare of the emerging economies are presented in Proposition 1 and Proposition 2 and we repeat them for convenience in Appendix 2.8.3.

In case of PEG APP the negative change in welfare of emerging economies can occur:

- when the Periphery countries are large as a whole - for the smaller country if the elasticity of substitution between the goods produced in the Periphery (ψ) is larger than the within country substitutability (θ), and when the following condition hold $\Pi(\psi) < \frac{\gamma_A}{\gamma_A - 1}$,
- when the Periphery countries are small as a whole - in case the substitutability of goods produced between the countries in the Periphery and the Center (ρ) [for the larger country] or the elasticity of substitution between the goods produced

in the Periphery (ψ)[for the smaller country] is larger that the within country substitutability (θ).

Negative change in the utility of EME in case of APP PEG occurs in similar cases to PEG APP (see PEG APP with mirror image results for one of the Periphery countries).

In general the following two scenarios could result from policy interactions in EME. We are interested if similar result can be obtained using the GVAR analysis.

1. **Scenario 1**: All payoffs positive, when the following condition on the relative elasticity of substitution parameters is satisfied: $\theta \geq \psi \geq \rho$.

2. **Scenario 2**: Positive payoffs in PEG PEG, negative in APP APP, and negative in PEG APP and APP PEG for the country that pegs, if either $\theta \leq \psi$ or $\theta \leq \rho$ (sufficiently smaller).

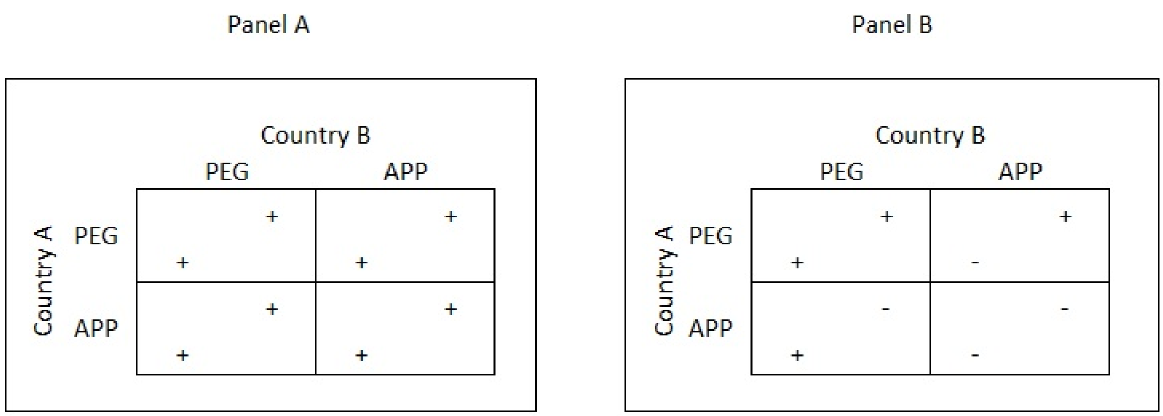

Figure 2.12: Potential scenarios Chapter 1

Obstfeld's (2011) predictions

In this section on Figure 2.13 we present the payoff matrix suggested by Obstfeld (2011), where $\alpha > 0.1$ is the share of intra-EME trade and -0.9 is the cost of inflation.

		Brazil	
		Fix	Appreciate
China	Fix	-0.9 -0.9	-1 -(0.9-α)
	Appreciate	-(0.9-α) -1	-(1-α) -(1-α)

Figure 2.13: Payoff matrix suggested by Obstfeld (2011)

2.8.3 Propositions - Welfare changes

Proposition 1. *(Beggar-thy-neighbour)* Monetary expansion in the Center is beggar-thy-neighbour for the Periphery countries in the following cases:

- for both Periphery countries in case of APP, APP, if $\rho > \frac{\beta+\theta+\sqrt{\beta^2+6\beta\theta+\theta^2}}{2\beta}$, i.e. ρ sufficiently larger than θ,
- for country A, if the Periphery is large, i.e. $\gamma_P \to 1$, and:
  - APP,PEG, if $\psi > \frac{\beta+\theta+\sqrt{\beta^2+6\beta\theta+\theta^2}}{2\beta}$, i.e. ψ sufficiently larger than θ, and country A is very small, i.e. $\gamma_A \to 0$,
  - PEG,APP, if $\Pi(\psi) < \frac{\gamma_A}{\gamma_A - 1}$,
- for country B, if the Periphery is large, i.e. $\gamma_P \to 1$, and:
  - APP,PEG, if $\Pi(\psi) < \frac{\gamma_A - 1}{\gamma_A}$,
  - PEG,APP, if $\psi > \frac{\beta+\theta+\sqrt{\beta^2+6\beta\theta+\theta^2}}{2\beta}$, i.e. ψ sufficiently larger than θ,
- for country A, if the Periphery is small, i.e. $\gamma_P \to 0$, and:
  - APP,PEG, if country A is very small, i.e. $\gamma_A \to 0$, and $\psi > \frac{\beta+\theta+\sqrt{\beta^2+6\beta\theta+\theta^2}}{2\beta}$, i.e. ψ sufficiently larger than θ, or if country A is very large, i.e. $\gamma_A \to 1$, and $\rho > \frac{\beta+\theta+\sqrt{\beta^2+6\beta\theta+\theta^2}}{2\beta}$, i.e. ρ sufficiently larger than θ,
  - PEG,APP, if country A is very small, i.e. $\gamma_A \to 0$, and $\Pi(\psi) - \Pi(\rho) < -1$,
- for country B, if the Periphery is small, i.e. $\gamma_P \to 0$, and:
  - APP,PEG, if country A is very large, i.e. $\gamma_A \to 1$, and $\Pi(\psi) - \Pi(\rho) < -1$,
  - PEG,APP, if country A is very large, i.e. $\gamma_A \to 1$, and $\psi > \frac{\beta+\theta+\sqrt{\beta^2+6\beta\theta+\theta^2}}{2\beta}$, i.e. ψ sufficiently larger than θ, or if country A is very small, i.e. $\gamma_A \to 0$, and $\rho > \frac{\beta+\theta+\sqrt{\beta^2+6\beta\theta+\theta^2}}{2\beta}$, i.e. ρ sufficiently larger than θ.

where γ_A and γ_B are country A and B population shares in the population of the Periphery, γ_P is the population's share of the Periphery in the world economy, β is the discount factor, θ is the within-country elasticity of substitution between types of goods produced domestically, ψ is the elasticity of substitution between the types of goods produced in the Periphery countries, ρ is the elasticity of substitution between

Table 2.11: Summary statistics in the full sample 1994 Q4 - 2014 Q1

	Mean	Median	Max	Min	Std. dev.	Mean	Median	Max	Min	Std. dev.
	Home output					Foreign output				
USA	11.1	11.1	11.2	11.0	0.1	9.7	9.6	10.2	9.4	0.2
China	9.7	9.7	10.2	9.3	0.3	10.9	10.9	11.0	10.8	0.0
Brazil	9.5	9.5	9.8	9.1	0.2	10.4	10.4	10.7	10.1	0.2
	Home consumption					Foreign consumption				
USA	10.9	10.9	11.0	10.8	0.1	9.4	9.3	9.8	9.1	0.2
China	9.4	9.3	9.8	9.0	0.2	10.7	10.7	10.8	10.6	0.1
Brazil	9.3	9.3	9.6	8.9	0.2	10.1	10.1	10.4	9.9	0.1
	Home money supply					Foreign money supply				
USA	12.1	12.1	12.4	12.0	0.1	12.7	12.7	13.3	12.1	0.4
China	12.9	13.0	13.5	12.3	0.4	12.0	12.0	12.3	11.8	0.1
Brazil	11.0	11.0	11.5	10.2	0.4	12.5	12.5	13.0	12.2	0.2

the types of goods produced in the Center and the Periphery, and $\Pi(\rho)$ and $\Pi(\psi)$ are defined as $\Pi(x) = \left(\frac{x-\theta}{x}\right)\left(\frac{1+x}{1+\beta+x(1-\beta)}\right)$, where $x = \rho, \psi$.

Proposition 2. *(Enrich-thy-neighbour)* Monetary expansion in the Center is always enrich-thy-neighbour for all countries in the Periphery in case of PEG PEG. This is independent of country size and elasticity of substitution between home and foreign goods. It can be enrich-thy-neighbour for the Periphery countries in other strategies, if the conditions in Proposition 1 do not hold.

2.8.4 Summary statistics

Chapter 3

Pure Strategy Nash Equilibrium in Tariff Games in an N-Country Trade Model

Małgorzata M. Mitka joint with Professor Subir Chattopadhyay

3.1 Introduction

Despite many initiatives to facilitate free trade, such as the General Agreement on Tariffs and Trade in 1994, tariffs remain widely used in many countries. Customs duties on merchandise imports, called tariffs, change both the relative price and volume of trade between economies, as well as generate revenue for the importing country. It is well recognized that a country imposing tariffs can benefit from such protectionist measures even in case of retaliation, that is when trading partners impose tariffs in return. When countries engage in retaliatory practice we call such a situation a tariff war. However, retaliation may result in lower welfare relative to a situation in which not all countries impose tariffs. It is therefore crucial to understand when retaliation is beneficial and how should the optimal tariffs be set. We are motivated by recent interactions between advanced and emerging economies, such as Brazil adopting tariffs on goods from China and the United States (see Colitt (2011)), and United States imposing tariffs on Chinese steel products, solar panels and other goods (see Evans (2014)), and would like to understand how the differences between such economies drive the outcomes of tariff wars.

There is a clear tendency in the way tariffs are set depending on the origin of the imported goods. Data on tariffs and bilateral imports from the "World Tariff Profiles 2013" (UNCTAD, 2013) shows that the share of duty-free imports in both emerging and advanced economies for a trading partner, which is an emerging economy, is significantly lower than for the trading partners with advanced economies. For example 3.9% of Brazil's tariff lines for imports of agricultural products from China is duty free, while it's 74.7% with the United States. Similarity in India: 6.2% of Indian tariff lines for imports of agricultural products are duty free for imports from China, but 69.1% for imports from the US. When we look at the tariff pattern in advanced economies, we can see again that the emerging economies have the least share in duty-free tariff lines, for example 6.4% in case of agricultural products from China, but 92.3% in case of agricultural products from Canada.

Another regularity in the way tariffs are set concerns the type of products, such as agricultural and non-agricultural goods. The value of the duty-free imports from the emerging economies is low for agricultural and very high for the non-agricultural goods (for example Brazil's value of duty-free imports of agricultural goods from China is worth 0.0%, while it is worth 90% in case of non-agricultural goods). It is also lower for advanced economies for agricultural than for non-agricultural goods coming from emerging economies (for example US's value of duty-free imports of agricultural goods from China is 0.8% and 35.9% of non-agricultural goods).

Although these are bilateral statistics, countries trade with more than one country, and it is well recognized that the tariffs are chosen depending on the decisions of the trading partners and countries who are considered to be competitors on third markets. Most of the literature on tariff wars uses two-country models (for example Kennan and Riezman (1988), Syropoulos (2002), Wong (2004)), making it impossible to analyze interactions between countries competing on a third market. Motivated by the scarcity of appropriate models which could be used for a formal analysis of tariff wars we extend the standard two-county two-good model to many countries, and because we want to investigate tariff patterns depending on the characteristics of an economy, in particular the inherent asymmetries between them, we allow for asymmetries in country preferences. Although our goal is to understand how asymmetries drive the outcomes of tariff wars in a multi-country environment, before we address this issue we need to investigate when the existence of a Nash equilibrium is guaranteed. Therefore our primary focus in this chapter is to establish whether extending the two-country two-good model prevailing in the literature to a multi-country one generates consequences in terms of characterisation of equilibria. We want to establish if such equilibria always exist. Even though a relatively large body of literature investigating trade with tariffs exists, it is difficult to identify a general existence result in such models and answer our questions of interest. Some of the early papers are silent about the existence and analyse the problem only graphically (Johnson, 1953-54). These are followed by papers which provide solutions for optimal tariffs (Kennan and Riezman, 1988).

More recently Wong (2004) addresses the existence problem and specifies conditions in which the equilibrium exists, and providing examples of non-existence. Huang et al. (2013) investigate the existence of multiple equilibria using numerical methods. We provide more details concerning related literature in Section 3.2.

We consider a multi-country two-good model[1] with tariffs and provide conditions on the primitives of the model that guarantee existence of a pure strategy interior Nash equilibrium in a tariff game. We show that no-trade equilibria in such a game do not exist, and provide an example in which there is no Nash equilibrium no matter what tariff rate the country chooses.

The structure of this chapter is as follows: In Section 3.2 we provide a discussion of related literature; in Section 3.3 we present the model and results; in Section 3.4 we conclude.

3.2 Related literature

Since the paper of Johnson (1953-54), who showed that even with retaliation a country can benefit by imposing a tariff on imports, there has been a large body of research investigating the conditions under which a country can win a tariff war and achieve a higher welfare from imposing tariff relative to the free trade equilibrium. Johnson (1953-54) argues that winning a tariff war depends on the elasticity of import demand. Using a graphical example in a two-country two-good case, he shows that it is the country with the higher elasticity of import demand that wins the tariff war, but only if the other country's elasticity of import demand is very low. Intuitively a country wins a tariff war when the improvement in its terms of trade offsets the decrease in the trade volume.

Subsequently, Gorman (1958) imposed more formal structure on the analysis of tariff games with retaliation, but investigated only cases in which offer curves, that is the quantity of one good that a country exports for each quantity of the good that it

[1]We believe that a two-good model can be a good representation of the world if we consider these two goods to be agricultural and non-agricultural goods, in relation to the aforementioned aggregate statistics from the "World Tariff Profiles".

imports (also called the reciprocal demand curve), have constant elasticity of import demands and therefore optimal tariffs are independent of the tariffs imposed by other countries.

If, however, tariffs depend on each other, then one can construct reaction functions, that is best responses to tariff decisions of other country (or counties in the case of more than two-country models). The reaction functions depend on the primitives of the model and are informative about the existence and number of equilibria. In a two-country model, for example, the number of intersections of the reaction functions (when continuous) determines the number of pure strategy Nash equilibria. If there are discontinuities in the reaction functions this may lead to non-existence of an equilibrium (see Wong (2004) discussed below).

Among authors who consider best responses dependent on the tariff choice of the relevant trading partner is Kuga (1973). Using a multi-country and multi-good model with factors of production he proves the existence of a Nash equilibrium, but only in mixed strategies. He does not consider the existence of a pure strategy Nash equilibrium. To prove the existence of a mixed strategy equilibrium Kuga restricts the strategy set (set of possible tariffs) to be finite, and assumes cardinal utility functions.

A further contribution is Kennan and Riezman (1988), in which the authors show that, in equilibrium, optimal tariffs can be expressed in terms of countries' endowments. They use a two-country two-good endowment economy with symmetric Cobb-Douglas preferences. They provide a condition in terms of endowments, determining who wins a tariff war. Although they did not prove it, in their simple structure Nash equilibria always exist and countries reaction functions are well defined. After relaxing the symmetry of preferences (see Kennan and Riezman (1984)), the problem of finding Nash equilibria becomes more complicated.

Several extensions of the two-country two-good model have been done, mostly with the purpose of showing that trade agreements such as customs unions can improve trading partners' welfare. The welfare improvement is a result of choosing a common tariff on good of which both countries forming the customs union are importers, because this common tariff is lower relative to Nash equilibrium tariff, see for example Kennan and Riezman (1990). These papers make an explicit assumption on the trade pattern and assume the existence of a Nash equilibrium. Their numerical examples usually impose symmetry of endowments and preferences in order to simplify the analysis.

More recent papers, such as Syropoulos (2002), using a two-country two-good model with production, provides conditions on existence of a Nash equilibrium and shows how a country size matters for a tariff war looking at the limiting cases. Syropolous analysis relies on the characteristics of the price elasticities of demand and their elasticities.

Wong (2004) addresses the existence problem in a more rigorous way and provides conditions in terms of the elasticity of "the elasticity of import" that guarantee the convexity of the offer curves. He provides conditions for homothetic and quasi-linear preferences that satisfy those conditions and that guarantee the existence of an equilibrium. He showed that non-convexity of offer curves can generate discontinuities[2] in the reaction functions, and lead to non-existence of Nash equilibria. He illustrates his finding with an analytical example, which we reproduce here (see Figure 3.1).

Figure 3.1 illustrates Wong's (2004) non-existence result in a two-country two-good economy. The picture shows the endowment point ω, the home F, and foreign F^* offer curves, and home I and foreign I^* indifference curves. The best response correspondences are represented using dotted lines. The area enclosed by the foreign offer curve F^* is non-convex for some tariff rates. Home country maximises utility subject to the foreign's country offer curve, and because of this non-convexity there are more than one tangency points of the home indifference curve with the foreign offer curve. Both points x^b and x^c solve the maximization problem faced by home country under the constraint specified by the foreign offer curve F^*. This multiplicity of solutions leads to the discontinuity in the best response correspondence of the home country. Therefore there is no trading Nash equilibrium in such economy. Wong's counterexample indicates that the key ingredient in the general existence result in Otani (1980) is the assumption (Assumption 11b, p. 649) which assures the required convex-

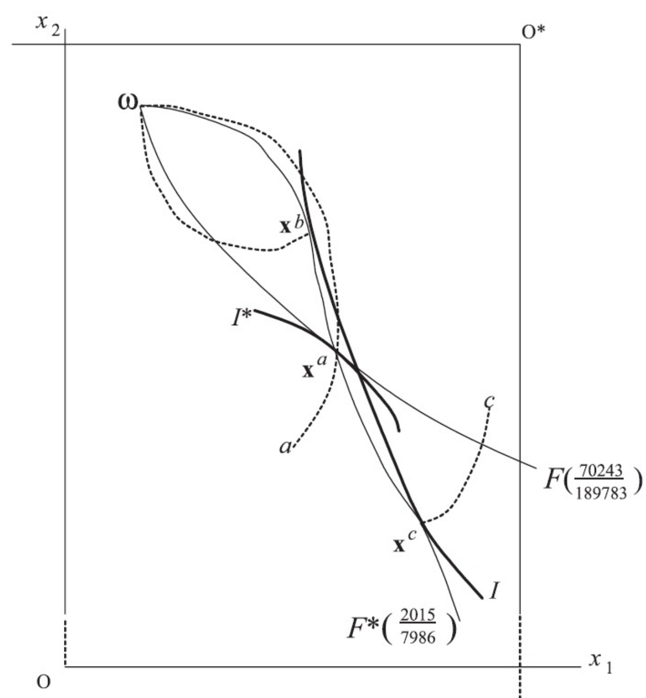

Figure 3.1: Non-existence of trading Nash equilibrium, Wong (2004), p. 380

[2]This problem has already been recognized by Johnson (1953-54), who however did not analyse it further.

ity, although Otani justifies it with the subjective conjecture of the government (about the vector of parameters characterizing the class of preferences, without specifying them explicitly).

Among existing papers there have also been several numerical approaches to the tariff games. One of them is by Hamilton and Whalley (1983) in which they calculate optimal tariffs in a two-country two-good model using various functional forms in an exchange model as well as model with production. They find that optimal tariffs vary with import price elasticities. They recognize the computational difficulties that arise as soon as a model is extended beyond two countries and two goods.

In a paper by Abrego et al. (2006) the authors consider a three-county three-good pure exchange economy with constant elasticity of substitution preferences, and analyze how trade patterns may change depending on the equilibrium concept. Their computational evidence shows that pattern of trade is more likely to change in economies with three or more countries and three or more goods with asymmetric preferences. They find that when comparing free trade and customs union the possibility of reversing the trade flows happens in 35% of cases, while in case of customs unions and Nash equilibria the pattern changes in 40% of cases.

Among more recent ones is Huang et al. (2013) who compute multiple Nash equilibria in a model with constant elasticity of substitution (CES) preferences. In contrast to the prevailing view, that what matters in the tariff war are the elasticities of import demand, the authors find that these are the elasticities of substitution that matter. They show that even in a two-country two-good economy with CES preferences when the substitution elasticities are low it is possible for multiple competitive and Nash equilibria to arise.

Our approach is slightly different in that we do not relay on the elasticity assumptions and look at the first and second derivatives of the objective function. We consider an N-country two-good Cobb-Douglas model with country specific preferences and arbitrary endowments. This allows us to look at the interactions between asymmetric countries. We provide conditions on the primitives of the model that ensure the existence of a pure strategy Nash equilibrium without restricting the trade pattern. Moreover, we show that Nash equilibria with prohibitive tariffs cannot arise.

3.3 The Model

Consider a world with two goods and a set $\mathcal{I} = \{1, 2, \cdots, I\}$ of countries. The goods are traded in international markets at prices $p_1 > 0$ and $p_2 > 0$. The government in each country sets a tariff on the second good;[3] the gross tariff rate is denoted τ_i for $i \in \mathcal{I}$. We impose the restriction that the gross tariff rate is always positive, $\tau_i > 0$. Trade is free if $\tau_i = 1$ in every country. The tariff τ_i induces a vector of domestic prices $(p_1, \tau_i p_2)$ in country i. The tariff generates revenue if $\tau_i > 1$ and country i is an importer of good 2 or if $\tau_i < 1$ and country i is an exporter of good 2. The proceeds from the tariff are redistributed to consumers in country i in the form of a lump-sum.[4]

The representative consumer in each country has an endowment, denoted $\omega_i \in R^2_+/\{0\}$ for $i \in \mathcal{I}$, and behaves competitively when faced with the vector of domestic prices $(p_1, \tau_i p_2)$ in country i. The quantities of each good demanded by the consumer in country i are denoted x_{i1} and x_{i2}. The income available to the consumer is denoted w_i and

$$w_i = p_1 \omega_{i1} + \tau_i p_2 \omega_{i2} + (\tau_i - 1) p_2 (x_{i2} - \omega_{i2}) \tag{3.1}$$

and the budget constraint faced by the consumer is

$$p_1 x_{i1} + \tau_i p_2 x_{i2} \leq w_i \tag{3.2}$$

We shall assume that the consumer in country i has a utility function u_i of the Cobb-Douglas form, so $u_i(x_{i1}, x_{i2}) = {x_{i1}}^{\alpha_i} {x_{i2}}^{1-\alpha_i}$, with parameter $\alpha_i \in (0, 1)$.

We also make the nondegeneracy assumptions: $\sum_i \omega_{i1} > 0$ and $\sum_i \omega_{i2} > 0$.

In Section 3.3.7 we solve the demand problem faced by the consumer in country i and show that:

$$x_{i1}(p_1, p_2, \tau_i) = \frac{\alpha_i \tau_i}{\alpha_i \tau_i + (1 - \alpha_i)} \frac{p_1 \omega_{i1} + p_2 \omega_{i2}}{p_1} \tag{3.3}$$

and

[3] In Section 3.3.7 we consider the more general framework in which each government can set a tariff rate on each good, and we show that the ratio of the tariff rates on the two goods determines demand, international prices, etc., so that the specification adopted is without loss of generality.

[4] We do not impose the condition "$\tau_i > 1$ if and only if country i is a net importer of good 2". In doing so, we follow much of the literature, e.g. Otani (1980) and Kennan and Riezman (1988); Wong (2004) does impose the restriction.

$$x_{i2}(p_1, p_2, \tau_i) = \frac{1-\alpha_i}{\alpha_i \tau_i + (1-\alpha_i)} \frac{p_1 \omega_{i1} + p_2 \omega_{i2}}{p_2} \tag{3.4}$$

Since demand is homogeneous of degree zero in international prices, we may normalize international prices and set $p_1 = 1$, and solve for the Walrasian equilibrium price vector. This is done in Section 3.3.7 where we show that

$$p_2^*(\vec{\tau}) = \frac{\sum_i [1 - A(\alpha_i, \tau_i)] \omega_{i1}}{\sum_i A(\alpha_i, \tau_i) \omega_{i2}} \tag{3.5}$$

where $\vec{\tau} = (\tau_1, \tau_2, \cdots, \tau_I) \in R^I_{++}$ and $A(\alpha_i, \tau_i) := \frac{\alpha_i \tau_i}{\alpha_i \tau_i + (1-\alpha_i)}$.

Utility at the Walrasian equilibrium can now be calculated. $v_i : R^I_{++} \to R$ denotes the indirect utility function that is induced for country i. We shall specify suitable values $\underline{\tau}$ and $\overline{\tau}$, where $\infty > \overline{\tau} > \underline{\tau} > 0$, and restrict the tariff rate set by country i to satisfy $\overline{\tau} \geq \tau_i \geq \underline{\tau}$. We are now in a position to analyse a game in strategic form where the strategies are tariff rates chosen by each government and the payoff functions are $(v_1(\vec{\tau}), \cdots, v_I(\vec{\tau}))$.

3.3.1 Properties of the payoff functions in the tariff game

Given a vector of tariff rates, the utility achieved by each country at the market clearing international relative price is given by the following equation:

$$v_i(\vec{\tau}) = (x_{i1}(p_2^*(\vec{\tau})))^{\alpha_i} (x_{i2}(p_2^*(\vec{\tau}))^{1-\alpha_i} \tag{3.6}$$

We turn to a study of the function v_i. Lemma 5 below provides an explicit form for v_i and its first derivative. This shows that v_i is well defined and continuously differentiable.

Lemma 5. *The utility function v_i is:*

(i) well defined:

$$v_i(\vec{\tau}) = (A(\alpha_i, \tau_i))^{\alpha_i} (1 - A(\alpha_i, \tau_i))^{1-\alpha_i} \left[\omega_{i1} (p_2^*(\vec{\tau}))^{\alpha_i - 1} + (p_2^*(\vec{\tau}))^{\alpha_i} \omega_{i2} \right] \tag{3.7}$$

(ii) and continuously differentiable

$$\frac{\partial v_i(\vec{\tau})}{\partial \tau_i} = v_i(\vec{\tau}) \cdot \frac{\partial A}{\partial \tau_i}(\alpha_i, \tau_i) \cdot \{M(\alpha_i, \tau_i) + N(\alpha_i, \vec{\tau})\} \tag{3.8}$$

where: $M(\alpha_i, \tau_i) = \frac{\alpha_i - A(\alpha_i,\tau_i)}{1-A(\alpha_i,\tau_i)} \frac{1}{A(\alpha_i,\tau_i)}$ *and* $N(\alpha_i, \vec{\tau}) = \frac{(1-\alpha_i)\omega_{i1}}{\sum_j [1-A(\alpha_j,\tau_j)]\omega_{j1}} - \frac{\alpha_i \cdot \omega_{i2}}{\sum_j A(\alpha_j,\tau_j)\omega_{j2}}$.

Next, we provide conditions that ensure that at a low enough value of the tariff rate set by country i, the payoff function is increasing in τ_i, and that the payoff function is decreasing in τ_i at a high enough value of the tariff rate. These boundary tariff rates are denoted $\underline{\tau}$ and $\overline{\tau}$ and specify the strategy set of each country. That is the content of Lemma 6 and it ensures that for every profile of actions $\tau_{-i} := ((\tau_j)_{j \neq i})$, there is at least one value $\tilde{\tau}_i \in (\underline{\tau}, \overline{\tau})$ such that the equation $\frac{\partial v_i}{\partial \tau_i}(\tilde{\tau}_i, \tau_{-i}) = 0$ is satisfied. $\underline{\tau}$ and $\overline{\tau}$ do more than just restrict the strategy set to be compact; they are consistent bounds since they ensure that any solution to the first order condition is interior. The alternative would be a strategy set where a maximizer exists but is on the boundary, and the derivative of the objective function at the maximizer is not zero. Such a bound would be artificial in that changing it would change the solution and hence the putative Nash equilibrium.

Lemma 6. *(i) If* $\frac{\omega_{i2}}{\sum_j \omega_{j2}} < 1$ *for all* $i \in \mathcal{I}$ *then there exists* $\underline{\tau} \in (0,1)$ *such that* $\frac{\partial v_i}{\partial \tau_i}(\underline{\tau}, \tau_{-i}) > 0$ *for* $\tau_{-i} \in [\underline{\tau}, \infty)^{I-1}$.
(ii) If $\frac{\omega_{i1}}{\sum_j \omega_{j1}} < 1$ *for all* $i \in \mathcal{I}$ *then there exists* $\overline{\tau} > 1$ *such that* $\frac{\partial v_i}{\partial \tau_i}(\overline{\tau}, \tau_{-i}) < 0$ *for* $\tau_{-i} \in (0, \overline{\tau}]^{I-1}$.

In Lemma 7 we provide a means to exactly evaluate the sign of $\left.\frac{\partial^2 v_i}{\partial \tau_i \partial \tau_i}(\vec{\tau})\right|_{\frac{\partial v_i}{\partial \tau_i}(\vec{\tau})=0}$. Define the function $sign : R \to \{-1, 0, 1\}$ by $sign(x) = -1$ if $x < 0$, $sign(x) = 0$ if $x = 0$, and $sign(x) = 1$ if $x > 0$.

Lemma 7. *The sign of the second derivative at which the first order condition is satisfied, i.e.* $sign\left\{\left.\frac{\partial^2 v_i}{\partial \tau_i \partial \tau_i}(\vec{\tau})\right|_{\frac{\partial v_i}{\partial \tau_i}(\vec{\tau})=0}\right\}$ *is given by the following expression:*

$$sign\left\{-2\alpha_i\tau_i\omega_{i2} - [1 - 2\alpha_i(1-\tau_i)]\left[\sum_{j\neq i} A(\alpha_j, \tau_j)\omega_{j2}\right]\right\}. \tag{3.9}$$

Finally, in Lemma 8 we show two properties: that $\left.\frac{\partial^2 v_i}{\partial \tau_i \partial \tau_i}(\vec{\tau})\right|_{\frac{\partial v_i}{\partial \tau_i}(\vec{\tau})=0} \neq 0$, so any solution to the first order condition is robust rather than this being a generic property, and that if v_i is increasing at $\underline{\tau}$, a sufficient condition for which is provided in Lemma 6 (i), then $\left.\frac{\partial^2 v_i}{\partial \tau_i \partial \tau_i}(\vec{\tau})\right|_{\frac{\partial v_i}{\partial \tau_i}(\vec{\tau})=0}$ is always negative.

Lemma 8. *(i)* $\frac{\partial^2 v_i}{\partial \tau_i \partial \tau_i}(\vec{\tau})\Big|_{\frac{\partial v_i}{\partial \tau_i}(\vec{\tau})=0} \neq 0.$

(ii) If $\alpha_i \leq 1/2$ *or if* $\frac{\omega_{i2}}{\sum_j \omega_{j2}} < 1$ *then* $\frac{\partial^2 v_i}{\partial \tau_i \partial \tau_i}(\vec{\tau})\Big|_{\frac{\partial v_i}{\partial \tau_i}(\vec{\tau})=0} < 0.$

The proofs of Lemma 5-8 are available in the Appendix 3.5.

3.3.2 Equilibrium

Our objective is to investigate the conditions for the existence of a pure strategy Nash equilibrium in the tariff game. Lemma 5 in Section 3.3.1 established that the function v_i is well defined and continuously differentiable; however, v_i typically fails to be concave. In Proposition 7 in Section 3.3.3 we provide a set of conditions on the graph of i's payoff function that ensure the existence of an interior Nash equilibrium in a general setting with one dimensional strategy sets. The remaining properties of the function v_i established in Section 3.3.1 lead to Proposition 8 in Section 3.3.4 which identifies conditions on the primitives of the model that ensure that the graph of v_i is such that Proposition 7 applies. In Section 3.3.5 we provide an example to show that the conditions in Proposition 8 are tight. Finally, in Section 3.3.6 we show that under very mild conditions, in any interior Nash equilibrium there is trade.

3.3.3 An existence result

Consider the following game. The set of players is $\mathcal{I} = \{1, 2, \cdots, I\}$ with generic element i. S is the strategy set of each player. The choice made by player i is denoted s_i. Payoffs are given by the functions $\pi_i : S^I \to R$. Let $s_{-i} := ((s_j)_{j \neq i})$ denote a profile of actions for all but agent i, and write payoffs as $\pi_i(s_i, s_{-i})$. We have[5]

Proposition 7. *Assume that* $S := [\underline{s}, \overline{s}] \subset R$*, and that, for every* $i \in \mathcal{I}$*, the function* π_i *is twice continuously differentiable on* $(\underline{t}, \overline{t})^I$ *where* $[\underline{s}, \overline{s}] \subset (\underline{t}, \overline{t})$*. Suppose that for each* $i \in \mathcal{I}$ *and every profile* s_{-i} *the following conditions hold:*

(i) $\frac{\partial \pi_i}{\partial s_i}(\underline{s}, s_{-i}) > 0$ *and* $\frac{\partial \pi_i}{\partial s_i}(\overline{s}, s_{-i}) < 0,$

(ii) if $\tilde{s}_i$ *is such that* $\frac{\partial \pi_i}{\partial s_i}(\tilde{s}_i, s_{-i}) = 0$ *then* $\frac{\partial}{\partial s_i}\left(\frac{\partial \pi_i}{\partial s_i}(\tilde{s}_i, s_{-i})\right) < 0.$

Then there exists $(s_1^*, \cdots, s_I^*) \in S^I$*, with* $s_i^* \in (\underline{s}, \overline{s})$ *for each* $i \in \mathcal{I}$*, such that for each* $i \in \mathcal{I} \quad s_i \in S \quad \Rightarrow \quad \pi_i(s_i^*, s_{-i}^*) \geq \pi_i(s_i, s_{-i}^*)$.

[5]We state and prove this intuitive result since we were unable to find a suitable reference.

We provide the proof of Proposition 7 below.[6]

The result says that when the strategy space is an interval, and player i's payoff function is (i) increasing in i's choice at the left boundary and decreasing in i's choice at the right boundary and (ii) has a negative second derivative at every point at which the first derivative is zero, the game has an interior pure strategy Nash equilibrium. Clearly, there is a unique solution to the first order condition; the key point in the proof is to show that, therefore, the payoff function π_i is quasiconcave in s_i.

Proof of Proposition 7. Fix a profile s_{-i} and consider the problem of identifying $\hat{s}_i := \text{argmax}_{s_i \in S} \pi_i(s_i, s_{-i})$. Since S is a compact set and π_i is a continuous function of s_i, such a value $\hat{s}_i$ must exist.

Note that $\frac{\partial \pi_i}{\partial s_i}(s_i, s_{-i})$ is a continuously differentiable function of s_i. Since $S = [\underline{s}, \overline{s}]$, by condition (i), continuity, and the Intermediate Value Theorem, there is a value $\tilde{s}_i \in (\underline{s}, \overline{s})$ at which $\frac{\partial \pi_i}{\partial s_i}(\tilde{s}_i, s_{-i}) = 0$, i.e. the function has a zero in the interior of the set S. By condition (ii), and continuity of the second derivative, it can have only one zero; furthermore, since $\frac{\partial}{\partial s_i}\left(\frac{\partial \pi_i}{\partial s_i}(\tilde{s}_i, s_{-i})\right) < 0$, the sufficient condition for $\tilde{s}_i$ to be a local maximum is met.

By condition (i), $\hat{s}_i \notin \{\underline{s}, \overline{s}\}$, i.e. the solution to the maximization problem cannot be at either boundary point. But then $\hat{s}_i = \tilde{s}_i$ since the necessary condition for an interior point to be a maximizer is satisfied only at $\tilde{s}_i$.

We have shown that given a profile s_{-i}, i's best response always exists, is an interior point, and is a single value. But then, as we now show, the function π_i must be quasiconcave in s_i. If not then for some profile s_{-i} and some p, the upper set is not convex, i.e. there are values s_i^1, s_i^2, and s_i^3 in the set S such that $s_i^1 < s_i^2 < s_i^3$ and $\pi_i(s_i^1, s_{-i}) \geq p$, $\pi_i(s_i^3, s_{-i}) \geq p$ but $\pi_i(s_i^2, s_{-i}) < p$. Since π_i is continuously differentiable, there would exist $s_i^4 \in (s_i^1, s_i^3)$ such that $\frac{\partial \pi_i}{\partial s_i}(s_i^4, s_{-i}) = 0$ and $\frac{\partial}{\partial s_i}\left(\frac{\partial \pi_i}{\partial s_i}(s_i^4, s_{-i})\right) > 0$, where the latter follows from the fact that $\pi_i(s_i^1, s_{-i}) \geq p$, $\pi_i(s_i^3, s_{-i}) \geq p$ but $\pi_i(s_i^2, s_{-i}) < p$. But that contradicts condition (ii) in the statement of the proposition.

Since the set S is compact and convex, and for each i the payoff functions are quasiconcave in s_i for a given profile s_{-i}, the existence of a pure strategy Nash equilibrium follows (see, for example, Theorem 3 in Debreu (1982)). Interiority has already been established and is maintained since S is compact and π_i is continuous so π_i is uniformly continuous.

$\square$

[6]Furthermore, since S is compact and π_i is continuous, all Nash equilibria are interior, $s_i^* \in (\underline{s}, \overline{s})$.

3.3.4 Pure strategy equilibria in the tariff game

We are now in a position to apply Proposition 7 to show that interior pure strategy Nash equilibria exist in the tariff game played by countries whose fundamentals satisfy the conditions specified in Proposition 8. The conditions restrict the distribution of endowments by ruling out extreme cases in which a single country's endowment of a good is equal to the world's endowment of that good. The example in Section 3.3.5 illustrates that the conditions cannot be relaxed.

Proposition 8. *Assume that for all $i \in \mathcal{I}$ (i) $\frac{\omega_{i2}}{\sum_j \omega_{j2}} < 1$ and (ii) $\frac{\omega_{i1}}{\sum_j \omega_{j1}} < 1$. The tariff game of such an economy has an interior pure strategy Nash equilibrium.*

The proof of Proposition 8 follows immediately by applying Proposition 7 and Lemma 5-8 and is omitted.

3.3.5 An example

In this example we show what happens when the conditions on the primitives of the model specified in Proposition 8 do not hold.

Example 1. *Consider a two country world. Let country 2's endowment of the second good be 0, $\omega_{22} = 0$, in particular $\omega_2 \notin R^2_{++}$. For given tariff rates the Walrasian equilibrium is always well-defined. Yet, the tariff game does not have an interior Nash equilibrium. To see this, use Lemma 5 (ii) to obtain the sign of $\frac{\partial v_1}{\partial \tau_1}(\vec{\tau})$ by evaluating the expression within braces:*

$$\frac{\alpha_1 - A(\alpha_1,\tau_1)}{1 - A(\alpha_1,\tau_1)} \frac{1}{A(\alpha_1,\tau_1)} + \frac{(1-\alpha_1)\omega_{11}}{\sum_j [1 - A(\alpha_j,\tau_j)]\omega_{j1}} - \frac{\alpha_1 \cdot \omega_{12}}{\sum_j A(\alpha_j,\tau_j)\omega_{j2}}$$

$$= \frac{\alpha_1 - A(\alpha_1,\tau_1)}{1 - A(\alpha_1,\tau_1)} \frac{1}{A(\alpha_1,\tau_1)} + \frac{(1-\alpha_1)\omega_{11}}{\sum_j [1 - A(\alpha_j,\tau_j)]\omega_{j1}} - \frac{\alpha_1}{A(\alpha_1,\tau_1)}$$

$$= \frac{\alpha_1 - 1}{1 - A(\alpha_1,\tau_1)} + \frac{(1-\alpha_1)\omega_{11}}{\sum_j [1 - A(\alpha_j,\tau_j)]\omega_{j1}}$$

$$= (1-\alpha_1) \left\{ \frac{\omega_{11}}{\sum_j [1 - A(\alpha_j,\tau_j)]\omega_{j1}} - \frac{1}{1 - A(\alpha_1,\tau_1)} \right\}$$

$$= (1-\alpha_1) \frac{\omega_{11} \sum_j [1 - A(\alpha_1,\tau_1)] - ([1 - A(\alpha_1,\tau_1)]\omega_{11} + [1 - A(\alpha_2,\tau_2)]\omega_{21})}{\sum_j [1 - A(\alpha_j,\tau_j)]\, \omega_{j1} [1 - A(\alpha_1,\tau_1)]}$$

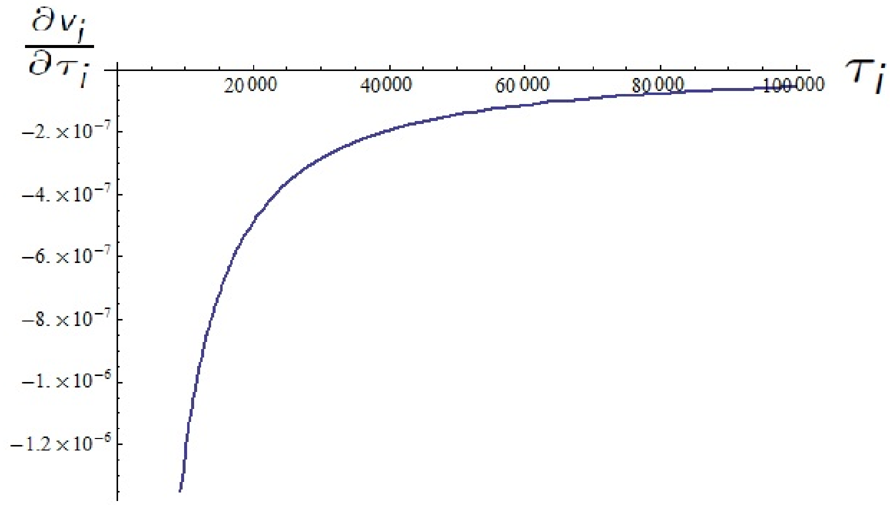

Figure 3.2: First derivative of country i in Example 1

$$(1-\alpha_1)\frac{-[1-A(\alpha_2,\tau_2)]\omega_{21}}{[1-A(\alpha_1,\tau_1)]\sum_j[1-A(\alpha_j,\tau_j)]\,\omega_{j1}}<0$$

since $\omega_{21} > 0$ *necessarily as* $\omega_{22} = 0$ *and* $\omega_i \in R^2_+/\{0\}$. *This shows that regardless of the vector of tariffs chosen, country's* 1*'s first order condition can never have an interior solution.*

It is important to mention that he example does not restrict ω_1 or preferences. We present the graph of the first order condition of country i on the following Figure 3.2.

3.3.6 On Nash equilibria that eliminate trade

In this subsection we show that if initial endowments are such that in the absence of tariffs there is trade, i.e. the endowment is not a Pareto optimal allocation, and no country's endowment of the first good is equal to the world's endowment of the first good, then in any interior Nash equilibrium of the tariff game there is trade. This result appears to contradict statements in the literature that suggest that a tariff regime with prohibitive rates that eliminate trade is always a Nash equilibrium.[7]

Proposition 9. *Assume that the vector of endowments is not a Pareto optimal allocation and that* $\frac{\omega_{i1}}{\sum_j \omega_{j1}} < 1$ *for all* $i \in \mathcal{I}$. *Then at every interior Nash equilibrium there is trade.*

[7] Dixit (1987) appears to be the first to make the claim; it is repeated and used in Syropoulos (2002).

The proof of Proposition 9 can be found after the proofs of Lemma 5 - 8.

3.3.7 Model equivalence

We start with the optimization problem faced by the representative consumer in country i. She faces international prices $p_1 > 0$ and $p_2 > 0$, and the government in country i sets gross tariff rates $\tau_{i1} > 0$ and $\tau_{i2} > 0$ on the net trade in each good (as we show below, the more general specification with a tariff on each good does not provide greater flexibility than a tariff on a single good). Tariff revenues are redistributed in the form of a lump-sum and so income in country i, denoted w_i, is given by

$$w_i = \tau_{i1}p_1\omega_{i1} + \tau_{i2}p_2\omega_{i2} + (\tau_{i1}-1)p_1(x_{i1}-\omega_{i1}) + (\tau_{i2}-1)p_2(x_{i2}-\omega_{i2}) \tag{3.10}$$

Evidently,

$$w_i = (\tau_{i1}-1)p_1x_{i1} + (\tau_{i2}-1)p_2x_{i2} + p_1\omega_{i1} + p_2\omega_{i2} \tag{3.11}$$

The optimization problem faced by i taking p_1, p_2, τ_{i1}, and τ_{i2} as given is

$$\max\, {x_{i1}}^{\alpha_i}{x_{i2}}^{1-\alpha_i} \tag{3.12}$$

subject to

$$\tau_{i1}p_1x_{i1} + \tau_{i2}p_2x_{i2} \leq w_i \tag{3.13}$$

The constraint holds with equality, and the first order necessary and sufficient conditions for x_i to solve the problem are that there is a $\lambda_i > 0$ such that

$$\alpha_i{x_{i1}}^{(\alpha_i-1)}{x_{i2}}^{1-\alpha_i} = \tau_{i1}p_1\lambda_i \tag{3.14}$$

$$(1-\alpha_i){x_{i1}}^{\alpha_i}{x_{i2}}^{(-\alpha_i)} = \tau_{i2}p_2\lambda_i \tag{3.15}$$

$$\tau_{i1}p_1x_{i1} + \tau_{i2}p_2x_{i2} = w_i \tag{3.16}$$

Simplifying

$$\frac{\alpha_i{x_{i1}}^{(\alpha_i-1)}{x_{i2}}^{1-\alpha_i}}{\tau_{i1}p_1} = \lambda_i = \frac{(1-\alpha_i){x_{i1}}^{\alpha_i}{x_{i2}}^{(-\alpha_i)}}{\tau_{i2}p_2} \tag{3.17}$$

$$\iff x_{i2} = \frac{(1-\alpha_i)\tau_{i1}p_1}{\alpha_i \tau_{i2} p_2} x_{i1} \tag{3.18}$$

Demand $x_{i1}(p_1, p_2, \tau_{i1}, \tau_{i2})$ can now be calculated as that value that satisfies

$$p_1 x_{i1} + p_2 \frac{(1-\alpha_i)\, p_1}{\alpha_i \left(\tau_{i2}/\tau_{i1}\right) p_2} x_{i1} = p_1 \omega_{i1} + p_2 \omega_{i2} \tag{3.19}$$

$$\iff p_1 \left\{ \frac{\alpha_i \left(\tau_{i2}/\tau_{i1}\right) + (1-\alpha_i)}{\alpha_i (\tau_{i2}/\tau_{i1})} \right\} x_{i1} = p_1 \omega_{i1} + p_2 \omega_{i2} \tag{3.20}$$

$$\Rightarrow \qquad x_{i1} = \left\{ \frac{\alpha_i (\tau_{i2}/\tau_{i1})}{\alpha_i \left(\tau_{i2}/\tau_{i1}\right) + (1-\alpha_i)} \right\} \frac{[p_1 \omega_{i1} + p_2 \omega_{i2}]}{p_1} \tag{3.21}$$

World markets will clear at prices (p_1^*, p_2^*) if and only if $\sum_i x_{i1}(p_1^*, p_2^*) = \sum_i \omega_{i1}$

$$\iff \qquad \sum_i \left\{ \frac{\alpha_i (\tau_{i2}/\tau_{i1})}{\alpha_i \left(\tau_{i2}/\tau_{i1}\right) + (1-\alpha_i)} \right\} \frac{[p_1^* \omega_{i1} + p_2^* \omega_{i2}]}{p_1^*} = \sum_i \omega_{i1} \tag{3.22}$$

Clearly, we can normalize prices and set $p_1 = 1$. Also, the tariff rates set by each country affect prices only through the ratios of the tariffs on the two goods. Therefore, we may work with the variables $\tau_i = \tau_{i2}/\tau_{i1}$. Let $\vec{\tau} = (\tau_1, \tau_2, \cdots, \tau_I)$. For $\alpha_i \in (0,1)$ and $\tau_i > 0$, set $A(\alpha_i, \tau_i) := \frac{\alpha_i \tau_i}{\alpha_i \tau_i + (1-\alpha_i)}$. $A(\alpha_i, \tau_i) \in (0,1)$. We have

$$p_2^*(\vec{\tau}) \cdot \sum_i A(\alpha_i, \tau_i) \omega_{i2} = \sum_i \left[1 - A(\alpha_i, \tau_i)\right] \omega_{i1} \tag{3.23}$$

$$\iff p_2^*(\vec{\tau}) = \frac{\sum_i [1 - A(\alpha_i, \tau_i)] \omega_{i1}}{\sum_i A(\alpha_i, \tau_i) \omega_{i2}} \tag{3.24}$$

We have obtained the market clearing international relative price as a function of the tariff rates set by each of the countries and the parameters specifying economic fundamentals $((\alpha_i, \omega_i)_{i \in \mathcal{I}})$. Also, consumption of good 1 in country i at equilibrium prices is $x_{i1}\left(p_2^*\left(\vec{\tau}\right)\right) = A(\alpha_i, \tau_i)[\omega_{i1} + p_2^*\left(\vec{\tau}\right)\omega_{i2}]$.

3.4 Conclusions

Motivated by economic developments in the world trade as well as lack of theoretical existence results concerning strategic interactions between countries in a multi-country world we extend a two-country two-good model to facilitate such analysis. We provide conditions on the primitives of the model that ensure the existence of a pure strategy Nash equilibrium and showed that Nash equilibria with prohibitive tariffs cannot arise. We consider these two important results as a good starting point for conducting further analysis that would investigate the role of asymmetries between countries on the outcome of tariff wars. Preliminary steps have been taken by looking at numerical examples. In the future we would like to undertake a quantitative evaluation of the results in the light of the stylized facts mentioned in the introduction. However, given the complex nature of the problem in the presence of asymmetries in models with more than two countries, we are convinced that an analytical approach would help to establish more clear cut results. We leave this for future research.

3.5 Appendix: Proofs

In what follows, we will find it easier to work with the reciprocal of the price $p_2^*(\vec{\tau})$. So define the function f by

$$f(\vec{\tau}) = \frac{\sum_i A(\alpha_i, \tau_i)\omega_{i2}}{\sum_i [1 - A(\alpha_i, \tau_i)]\omega_{i1}}. \tag{3.25}$$

Lemma E1 is our first supplementary result; it provides the evaluation of two partial derivatives that will be used later.

Lemma E1. *The functions A and f are differentiable on their domains and*

$$\frac{\partial A}{\partial \tau_i}(\alpha_i, \tau_i) = \frac{\alpha_i(1-\alpha_i)}{[\alpha_i\tau_i + (1-\alpha_i)]^2} > 0$$

$$\frac{\partial f}{\partial \tau_i}(\vec{\tau}) = \frac{\frac{\partial A}{\partial \tau_i}(\alpha_i, \tau_i)}{\sum_j [1 - A(\alpha_j, \tau_j)]\omega_{j1}} \left\{ f(\vec{\tau})\omega_{i1} + \omega_{i2} \right\}.$$

Proof. We compute each of the expressions.

$$\frac{\partial A}{\partial \tau_i}(\alpha_i, \tau_i) = \frac{\partial}{\partial \tau_i}\left[\frac{\alpha_i\tau_i}{\alpha_i\tau_i + (1-\alpha_i)}\right] = \frac{\alpha_i[\alpha_i\tau_i + (1-\alpha_i)] - \alpha_i\tau_i\alpha_i}{[\alpha_i\tau_i + (1-\alpha_i)]^2} = \frac{\alpha_i(1-\alpha_i)}{[\alpha_i\tau_i + (1-\alpha_i)]^2} > 0.$$

$$\frac{\partial f}{\partial \tau_i}(\vec{\tau}) = \frac{\partial}{\partial \tau_i}\left[\frac{\sum_j A(\alpha_j, \tau_j)\omega_{j2}}{\sum_j [1 - A(\alpha_j, \tau_j)]\omega_{j1}}\right]$$

$$= \frac{\frac{\partial A}{\partial \tau_i}(\alpha_i, \tau_i)\omega_{i2}\left\{\sum_j [1 - A(\alpha_j, \tau_j)]\omega_{j1}\right\} - \left\{\sum_j A(\alpha_j, \tau_j)\omega_{j2}\right\}\frac{\partial A}{\partial \tau_i}(\alpha_i, \tau_i)(-1)\omega_{i1}}{\left[\sum_j [1 - A(\alpha_j, \tau_j)]\omega_{j1}\right]^2}$$

$$= \frac{\frac{\partial A}{\partial \tau_i}(\alpha_i, \tau_i)\left[\omega_{i2}\left\{\sum_j [1 - A(\alpha_j, \tau_j)]\omega_{j1}\right\} + \left\{\sum_j A(\alpha_j, \tau_j)\omega_{j2}\right\}\omega_{i1}\right]}{\left[\sum_j [1 - A(\alpha_j, \tau_j)]\omega_{j1}\right]^2}$$

$$= \frac{\frac{\partial A}{\partial \tau_i}(\alpha_i, \tau_i)}{\sum_j [1 - A(\alpha_j, \tau_j)]\omega_{j1}}\left\{f(\vec{\tau})\omega_{i1} + \omega_{i2}\right\}.$$

□

We can now proceed to prove Lemma 5 which we repeat below for convenience:

Lemma. *(5) The utility function v_i is:*

(i) well defined:

$$v_i(\vec{\tau}) = (A(\alpha_i,\tau_i))^{\alpha_i}(1-A(\alpha_i,\tau_i))^{1-\alpha_i}\left[\omega_{i1}(p_2^*(\vec{\tau}))^{\alpha_i-1} + (p_2^*(\vec{\tau}))^{\alpha_i}\omega_{i2}\right]$$

(ii) and continuously differentiable:

$$\frac{\partial v_i}{\partial \tau_i}(\vec{\tau}) = v_i(\vec{\tau})\cdot\frac{\partial A}{\partial \tau_i}(\alpha_i,\tau_i)\cdot\left\{\frac{\alpha_i - A(\alpha_i,\tau_i)}{1-A(\alpha_i,\tau_i)}\frac{1}{A(\alpha_i,\tau_i)} + \frac{(1-\alpha_i)\omega_{i1}}{\sum_j\left[1-A(\alpha_j,\tau_j)\right]\omega_{j1}} - \frac{\alpha_i\cdot\omega_{i2}}{\sum_j A(\alpha_j,\tau_j)\omega_{j2}}\right\}$$

Proof.

$$\text{(i)} \qquad v_i(\vec{\tau}) = (x_{i1}(p_2^*(\vec{\tau})))^{\alpha_i}(x_{i2}(p_2^*(\vec{\tau})))^{1-\alpha_i}$$

which, upon using the first order condition, becomes

$$v_i(\vec{\tau}) = (x_{i1}(p_2^*(\vec{\tau})))^{\alpha_i}\left(\frac{(1-\alpha_i)}{\alpha_i\tau_i p_2^*(\vec{\tau})}x_{i1}(p_2^*(\vec{\tau}))\right)^{1-\alpha_i}$$

$$= (x_{i1}(p_2^*(\vec{\tau})))\left(\frac{(1-\alpha_i)}{\alpha_i\tau_i p_2^*(\vec{\tau})}\right)^{1-\alpha_i} = A(\alpha_i,\tau_i)\left[\omega_{i1} + p_2^*(\vec{\tau})\omega_{i2}\right]\left(\frac{(1-\alpha_i)}{\alpha_i\tau_i}\right)^{1-\alpha_i}\left(\frac{1}{p_2^*(\vec{\tau})}\right)^{1-\alpha_i},$$

where we incorporate the explicit form of the demand function $x_{i1}(p_2^*(\vec{\tau}))$. By rewriting the expression $\frac{(1-\alpha_i)}{\alpha_i\tau_i}$ in terms of $A(\alpha_i,\tau_i)$, we obtain

$$v_i(\vec{\tau}) = A(\alpha_i,\tau_i)\left[\omega_{i1}\left(\frac{1}{p_2^*(\vec{\tau})}\right)^{1-\alpha_i} + \left(\frac{1}{p_2^*(\vec{\tau})}\right)^{-\alpha_i}\omega_{i2}\right]\left(\frac{1}{A(\alpha_i,\tau_i)} - 1\right)^{1-\alpha_i}$$

$$= (A(\alpha_i,\tau_i))^{\alpha_i}(1-A(\alpha_i,\tau_i))^{1-\alpha_i}\left[\omega_{i1}(p_2^*(\vec{\tau}))^{\alpha_i-1} + (p_2^*(\vec{\tau}))^{\alpha_i}\omega_{i2}\right].$$

(ii) Recall that $f(\vec{\tau}) = \frac{1}{p_2^*(\vec{\tau})}$ so that (i) may be rewritten as

$$v_i(\vec{\tau}) = (A(\alpha_i,\tau_i))^{\alpha_i}(1-A(\alpha_i,\tau_i))^{1-\alpha_i}\left[\omega_{i1}(f(\vec{\tau}))^{1-\alpha_i} + (f(\vec{\tau}))^{-\alpha_i}\omega_{i2}\right].$$

We proceed to differentiate the function.

$$\frac{\partial v_i(\vec{\tau})}{\partial \tau_i} = \left\{\alpha_i(A(\alpha_i,\tau_i))^{\alpha_i-1}\frac{\partial A}{\partial \tau_i}(\alpha_i,\tau_i)(1-A(\alpha_i,\tau_i))^{1-\alpha_i}\right.$$

$$\left.+(A(\alpha_i,\tau_i))^{\alpha_i}(1-\alpha_i)(1-A(\alpha_i,\tau_i))^{-\alpha_i}(-1)\frac{\partial A}{\partial \tau_i}(\alpha_i,\tau_i)\right\}\cdot\left\{(f(\vec{\tau}))^{1-\alpha_i}\omega_{i1} + (f(\vec{\tau}))^{-\alpha_i}\omega_{i2}\right\}$$

$$+(A(\alpha_i,\tau_i))^{\alpha_i}(1-A(\alpha_i,\tau_i))^{1-\alpha_i}\left\{(1-\alpha_i)(f(\vec{\tau}))^{-\alpha_i}\frac{\partial f}{\partial \tau_i}(\vec{\tau})\omega_{i1}+(-\alpha_i)(f(\vec{\tau}))^{-\alpha_i-1}\frac{\partial f}{\partial \tau_i}(\vec{\tau})\omega_{i2}\right\}.$$

We group the terms in the first two lines in the expression above to obtain the first two lines below; we also substitute for $\frac{\partial f}{\partial \tau_i}(\vec{\tau})$ from Lemma E1 and collect some common terms in the last line above to obtain the third and fourth lines below. We have

$$\frac{\partial v_i(\vec{\tau})}{\partial \tau_i}=\left\{(A(\alpha_i,\tau_i))^{\alpha_i}(1-A(\alpha_i,\tau_i))^{1-\alpha_i}(f(\vec{\tau}))^{-\alpha_i}\right\}$$

$$\cdot\frac{[\alpha_i\,(1-A(\alpha_i,\tau_i))-(1-\alpha_i)\,A(\alpha_i,\tau_i)]}{A(\alpha_i,\tau_i)[1-A(\alpha_i,\tau_i)]}\frac{\partial A}{\partial \tau_i}(\alpha_i,\tau_i)\cdot\{(f(\vec{\tau}))\omega_{i1}+\omega_{i2}\}$$

$$+\left\{(A(\alpha_i,\tau_i))^{\alpha_i}(1-A(\alpha_i,\tau_i))^{1-\alpha_i}(f(\vec{\tau}))^{-\alpha_i}\right\}$$

$$\cdot\frac{\frac{\partial A}{\partial \tau_i}(\alpha_i,\tau_i)}{\sum_j[1-A(\alpha_j,\tau_j)]\omega_{j1}}\{f(\vec{\tau})\omega_{i1}+\omega_{i2}\}\cdot\left\{(1-\alpha_i)\omega_{i1}+(-\alpha_i)(f(\vec{\tau}))^{-1}\omega_{i2}\right\}.$$

We collect terms and use the expression for $v_i(\vec{\tau})$ obtained in (i) to simplify the expression to

$$\frac{\partial v_i(\vec{\tau})}{\partial \tau_i}=v_i(\vec{\tau})\cdot\frac{\partial A}{\partial \tau_i}(\alpha_i,\tau_i)\cdot\left\{\frac{\alpha_i-A(\alpha_i,\tau_i)}{A(\alpha_i,\tau_i)[1-A(\alpha_i,\tau_i)]}+\frac{(1-\alpha_i)\omega_{i1}+(-\alpha_i)(f(\vec{\tau}))^{-1}\omega_{i2}}{\sum_j[1-A(\alpha_j,\tau_j)]\omega_{j1}}\right\}$$

$$=v_i(\vec{\tau})\cdot\frac{\partial A}{\partial \tau_i}(\alpha_i,\tau_i)\cdot\left\{\frac{\alpha_i-A(\alpha_i,\tau_i)}{1-A(\alpha_i,\tau_i)}\frac{1}{A(\alpha_i,\tau_i)}+\frac{(1-\alpha_i)\omega_{i1}}{\sum_j[1-A(\alpha_j,\tau_j)]\omega_{j1}}-\frac{\alpha_i\cdot\omega_{i2}}{\sum_j A(\alpha_j,\tau_j)\omega_{j2}}\right\},$$

where we incorporate the explicit form of the function $f(\vec{\tau})$. □

The next supplementary result distills the key implication obtained so far.

Lemma E2. *sign* $\left\{\frac{\partial v_i}{\partial \tau_i}(\vec{\tau})\right\}=$ *sign* $\left\{\frac{\alpha_i-A(\alpha_i,\tau_i)}{1-A(\alpha_i,\tau_i)}\frac{1}{A(\alpha_i,\tau_i)}+\frac{(1-\alpha_i)\omega_{i1}}{\sum_j[1-A(\alpha_j,\tau_j)]\omega_{j1}}-\frac{\alpha_i\cdot\omega_{i2}}{\sum_j A(\alpha_j,\tau_j)\omega_{j2}}\right\}$

Proof. Since $A(\alpha_i,\tau_i)\in(0,1)$ and $\omega_i\in R^2_+/\{0\}$, from Lemma 5 (i) we have $v_i(\vec{\tau})>0$. Also, by Lemma E1, $\frac{\partial A}{\partial \tau_i}(\alpha_i,\tau_i)>0$. The result follows directly from Lemma 5 (ii). □

For notational ease, we define

$$M(\alpha_i,\tau_i):=\frac{\alpha_i-A(\alpha_i,\tau_i)}{1-A(\alpha_i,\tau_i)}\frac{1}{A(\alpha_i,\tau_i)};\qquad N_i(\vec{\tau}):=\frac{(1-\alpha_i)\omega_{i1}}{\sum_j[1-A(\alpha_j,\tau_j)]\omega_{j1}}-\frac{\alpha_i\cdot\omega_{i2}}{\sum_j A(\alpha_j,\tau_j)\omega_{j2}}.$$

Remark 13. *Evidently, $\frac{\partial v_i}{\partial \tau_i}(\vec{\tau})$ has the same sign as $M(\alpha_i, \tau_i) + N_i(\vec{\tau})$.*

Our next supplementary result provides an evaluation of $M(\alpha_i, \tau_i)$.

Lemma E3.

$$M(\alpha_i, \tau_i) = \frac{(1-\tau_i)[\alpha_i \tau_i + (1-\alpha_i)]}{\tau_i}.$$

Proof. Since

$$\frac{\alpha_i - A(\alpha_i, \tau_i)}{1 - A(\alpha_i, \tau_i)} = \frac{\alpha_i - \frac{\alpha_i \tau_i}{\alpha_i \tau_i + (1-\alpha_i)}}{1 - \frac{\alpha_i \tau_i}{\alpha_i \tau_i + (1-\alpha_i)}} = \frac{(\alpha_i - 1)\alpha_i \tau_i + \alpha_i(1-\alpha_i)}{(1-\alpha_i)} = \alpha_i(1-\tau_i)$$

we have

$$M(\alpha_i, \tau_i) = \frac{\alpha_i - A(\alpha_i, \tau_i)}{1 - A(\alpha_i, \tau_i)} \cdot \frac{1}{A(\alpha_i, \tau_i)} = \frac{\alpha_i(1-\tau_i)[\alpha_i \tau_i + (1-\alpha_i)]}{\alpha_i \tau_i} = \frac{(1-\tau_i)[\alpha_i \tau_i + (1-\alpha_i)]}{\tau_i}.$$

□

We turn to the proof of Lemma 6 which establishes conditions under which there is an interior solution to the first order condition $\frac{\partial v_i}{\partial \tau_i}(\vec{\tau}) = 0$. We repeat Lemma 6 below for convenience:

Lemma. *(6) (i) If $\frac{\omega_{i2}}{\sum_j \omega_{j2}} < 1$ for all $i \in \mathcal{I}$ then there exists $\underline{\tau} \in (0,1)$ such that $\frac{\partial v_i}{\partial \tau_i}(\underline{\tau}, \tau_{-i}) > 0$ for $\tau_{-i} \in [\underline{\tau}, \infty)^{I-1}$; (ii) if $\frac{\omega_{i1}}{\sum_j \omega_{j1}} < 1$ for all $i \in \mathcal{I}$ then there exists $\overline{\tau} > 1$ such that $\frac{\partial v_i}{\partial \tau_i}(\overline{\tau}, \tau_{-i}) < 0$ for $\tau_{-i} \in (0, \overline{\tau}]^{I-1}$.*

Proof. By Lemma E2 and E3, the sign of $\frac{\partial v_i}{\partial \tau_i}(\vec{\tau})$ is determined by the sign of the expression

$$\frac{(1-\tau_i)[\alpha_i \tau_i + (1-\alpha_i)]}{\tau_i} + \frac{(1-\alpha_i) \cdot \omega_{i1}}{\sum_j [1 - A(\alpha_j, \tau_j)]\omega_{j1}} - \frac{\alpha_i \cdot \omega_{i2}}{\sum_j A(\alpha_j, \tau_j)\omega_{j2}}.$$

In (i) below, we ignore the positive second term and show that even so the sum of the remaining terms is positive for $\underline{\tau}$ sufficiently small. In (ii) we ignore the last term, which is negative, and show that for $\overline{\tau}$ sufficiently large the sum of the remaining terms is, nonetheless, negative.

(i) By hypothesis $\frac{\omega_{i2}}{\sum_j \omega_{j2}} < 1$ for all $i \in \mathcal{I}$; so there must exist $\underline{\tau} \in (0,1)$ such that

$$\forall\, i \in \mathcal{I} \quad \frac{\omega_{i2}}{\sum_j \omega_{j2}} < (1-\underline{\tau}) \qquad \Longleftrightarrow \qquad \frac{\alpha_i}{A(\alpha_i,\underline{\tau})} \frac{\omega_{i2}}{\sum_j \omega_{j2}} < (1-\underline{\tau}) \frac{\alpha_i \underline{\tau} + (1-\alpha_i)}{\underline{\tau}}.$$

But then, since $A(\alpha_j, \tau_j)$ is increasing in τ_j, for $\tau_j \geq \underline{\tau}$ we must have

$$0 < (1-\underline{\tau}) \frac{\alpha_i \underline{\tau} + (1-\alpha_i)}{\underline{\tau}} - \frac{\alpha_i \cdot \omega_{i2}}{\sum_{j \neq i} A(\alpha_j, \tau_j)\omega_{j2} + A(\alpha_i, \underline{\tau})\omega_{i2}}.$$

That verifies the sign of the expression.

(ii) By hypothesis $\frac{\omega_{i1}}{\sum_j \omega_{j1}} < 1$ for all $i \in \mathcal{I}$; so there must exist a $\overline{\tau} > 0$ sufficiently large such that

$$\forall\, i \in \mathcal{I} \quad \frac{\omega_{i2}}{\sum_j \omega_{j2}} < \left(1 - \frac{1}{\overline{\tau}}\right) \qquad \Longleftrightarrow \qquad \frac{1-\alpha_i}{[1-A(\alpha_i,\overline{\tau})]} \frac{\omega_{i2}}{\sum_j \omega_{j2}} < \left(1 - \frac{1}{\overline{\tau}}\right) [\alpha_i \overline{\tau} + (1-\alpha_i)].$$

But then, since $A(\alpha_j, \tau_j)$ is increasing in τ_j, for $\tau_j \leq \overline{\tau}$ we must have

$$\frac{(1-\overline{\tau})[\alpha_i \overline{\tau} + (1-\alpha_i)]}{\overline{\tau}} + \frac{(1-\alpha_i) \cdot \omega_{i1}}{\sum_{j \neq i}[1 - A(\alpha_j, \tau_j)]\omega_{j1} + [1 - A(\alpha_i, \overline{\tau})]\omega_{i1}} < 0.$$

That verifies the sign of the expression. $\square$

We now prove six supplementary results that prepare the groundwork for the proof of Lemma 7. The first one, Lemma E4, provides an evaluation of $\frac{\partial M}{\partial \tau_i}(\alpha_i, \tau_i)$.

Lemma E4.

$$\frac{\partial M}{\partial \tau_i}(\alpha_i, \tau_i) = -\alpha_i - \frac{1-\alpha_i}{(\tau_i)^2}.$$

Proof.

$$\frac{\partial M}{\partial \tau_i}(\alpha_i, \tau_i) = \frac{\partial}{\partial \tau_i} \left\{ \frac{(1-\tau_i)[\alpha_i \tau_i + (1-\alpha_i)]}{\tau_i} \right\}$$

$$= \frac{\tau_i \{-[\alpha_i \tau_i + (1-\alpha_i)] + (1-\tau_i)\alpha_i\} - (1-\tau_i)[\alpha_i \tau_i + (1-\alpha_i)]}{(\tau_i)^2}$$

$$= \frac{-\alpha_i (\tau_i)^2 - (1-\alpha_i)\tau_i + \tau_i(1-\tau_i)\alpha_i - (1-\tau_i)\alpha_i\tau_i - (1-\tau_i)(1-\alpha_i)}{(\tau_i)^2}$$

$$= -\alpha_i - \frac{1-\alpha_i}{(\tau_i)^2}.$$

□

In Lemma E5 we provide an evaluation of $\frac{\partial N_i}{\partial \tau_k}(\vec{\tau})$.

Lemma E5.

$$\frac{\partial N_i}{\partial \tau_k}(\vec{\tau}) = \frac{\frac{\alpha_k(1-\alpha_k)}{[\alpha_k\tau_k+(1-\alpha_k)]^2}}{\left[\sum_j A(\alpha_j,\tau_j)\omega_{j2}\right]^2}\left\{(1-\alpha_i)\left(f(\vec{\tau})\right)^2\cdot\omega_{k1}\omega_{i1}+\alpha_i\cdot\omega_{i2}\cdot\omega_{k2}\right\}.$$

Proof.

$$\frac{\partial N_i}{\partial \tau_k}(\vec{\tau}) = \frac{\partial}{\partial \tau_i}\left\{\frac{(1-\alpha_i)\omega_{i1}}{\sum_j[1-A(\alpha_j,\tau_j)]\omega_{j1}} - \frac{\alpha_i\cdot\omega_{i2}}{\sum_j A(\alpha_j,\tau_j)\omega_{j2}}\right\}$$

$$= \frac{(-1)(1-\alpha_i)\omega_{i1}[-\frac{\partial A}{\partial \tau_k}(\alpha_k,\tau_k)\omega_{k1}]}{\left[\sum_j[1-A(\alpha_j,\tau_j)]\omega_{j1}\right]^2} - \frac{(-1)\alpha_i\cdot\omega_{i2}[\frac{\partial A}{\partial \tau_k}(\alpha_k,\tau_k)\omega_{k2}]}{\left[\sum_j A(\alpha_j,\tau_j)\omega_{j2}\right]^2}$$

$$= \frac{\partial A}{\partial \tau_k}(\alpha_k,\tau_k)\left\{\frac{(1-\alpha_i)\omega_{i1}\omega_{k1}}{\left[\sum_j\left[1-A(\alpha_j,\tau_j)\right]\omega_{j1}\right]^2} + \frac{\alpha_i\cdot\omega_{i2}\omega_{k2}}{\left[\sum_j A(\alpha_j,\tau_j)\omega_{j2}\right]^2}\right\},$$

$$= \frac{\frac{\alpha_k(1-\alpha_k)}{[\alpha_k\tau_k+(1-\alpha_k)]^2}}{\left[\sum_j A(\alpha_j,\tau_j)\omega_{j2}\right]^2}\left\{(1-\alpha_i)\left(f(\vec{\tau})\right)^2\cdot\omega_{k1}\omega_{i1}+\alpha_i\cdot\omega_{i2}\cdot\omega_{k2}\right\},$$

where we use the expression for $\frac{\partial A}{\partial \tau_i}(\alpha_i,\tau_i)$ obtained in Lemma E1 and the explicit form of the function $f(\vec{\tau})$.

□

Our objective is to pin down the sign of $\frac{\partial^2 v_i}{\partial \tau_i\,\partial \tau_i}(\vec{\tau})$ at $\vec{\tau}$ at which $\frac{\partial v_i}{\partial \tau_i}(\vec{\tau}) = 0$. The next two supplementary results provide evaluations that are very useful in reaching our objective.

Lemma E6. $\frac{\partial v_i}{\partial \tau_i}(\vec{\tau}) = 0$ *if and only if*

$$(1-\alpha_i)\cdot\left(f(\vec{\tau})\cdot\omega_{i1}\right)^2 = -\frac{(\alpha_i)^2}{1-\alpha_i}\cdot(\omega_{i2})^2 + 2\alpha_i\cdot f(\vec{\tau})\cdot\omega_{i1}\cdot\omega_{i2}$$

$$+\frac{1}{1-\alpha_i}\left[\frac{\alpha_i - A(\alpha_i,\tau_i)}{1-A(\alpha_i,\tau_i)}\right]^2 \left[\frac{\left[\sum_j A(\alpha_j,\tau_j)\omega_{j2}\right]}{A(\alpha_i,\tau_i)}\right]^2.$$

Proof. From Lemma E2 we know that

$$\frac{\partial v_i}{\partial \tau_i}(\vec{\tau}) = 0 \quad \Longleftrightarrow \quad \frac{\alpha_i - A(\alpha_i,\tau_i)}{1-A(\alpha_i,\tau_i)}\frac{1}{A(\alpha_i,\tau_i)} + \frac{(1-\alpha_i)\omega_{i1}}{\sum_j\left[1-A(\alpha_j,\tau_j)\right]\omega_{j1}} - \frac{\alpha_i\cdot\omega_{i2}}{\sum_j A(\alpha_j,\tau_j)\omega_{j2}} = 0.$$

Move the first term in the latter expression to the right, multiply each term by $\left[\sum_j A(\alpha_j,\tau_j)\omega_{j2}\right]$, then use the definition of the function $f(\vec{\tau})$ and rearrange the expression to obtain

$$(1-\alpha_i)\cdot f(\vec{\tau})\cdot\omega_{i1} - \alpha_i\cdot\omega_{i2} = -\frac{\alpha_i - A(\alpha_i,\tau_i)}{1-A(\alpha_i,\tau_i)}\frac{\left[\sum_j A(\alpha_j,\tau_j)\omega_{j2}\right]}{A(\alpha_i,\tau_i)}$$

$$\Longleftrightarrow \quad (1-\alpha_i)^2\cdot\left(f(\vec{\tau})\cdot\omega_{i1}\right)^2 + (\alpha_i)^2\cdot(\omega_{i2})^2 - 2(1-\alpha_i)\alpha_i\cdot f(\vec{\tau})\cdot\omega_{i1}\cdot\omega_{i2}$$
$$= \left[\frac{\alpha_i - A(\alpha_i,\tau_i)}{1-A(\alpha_i,\tau_i)}\right]^2 \left[\frac{\left[\sum_j A(\alpha_j,\tau_j)\omega_{j2}\right]}{A(\alpha_i,\tau_i)}\right]^2$$

and the result follows. □

Lemma E7. $\frac{\partial v_i}{\partial \tau_i}(\vec{\tau}) = 0$ *if and only if*

$$(1-2\alpha_i)(\omega_{i2})^2 + 2(1-\alpha_i)\cdot f(\vec{\tau})\cdot\omega_{i1}\cdot\omega_{i2}$$
$$= (\omega_{i2})^2 - 2\omega_{i2}\left\{\frac{(1-\tau_i)[\alpha_i\tau_i + (1-\alpha_i)]}{\tau_i}\right\}\left[\sum_j A(\alpha_j,\tau_j)\omega_{j2}\right].$$

Proof. From Lemma E2 we know that

$$\frac{\partial v_i}{\partial \tau_i}(\vec{\tau}) = 0 \quad \Longleftrightarrow \quad \frac{\alpha_i - A(\alpha_i,\tau_i)}{1-A(\alpha_i,\tau_i)}\frac{1}{A(\alpha_i,\tau_i)} + \frac{(1-\alpha_i)\omega_{i1}}{\sum_j\left[1-A(\alpha_j,\tau_j)\right]\omega_{j1}} - \frac{\alpha_i\cdot\omega_{i2}}{\sum_j A(\alpha_j,\tau_j)\omega_{j2}} = 0.$$

Move the first term in the latter expression to the right, multiply each term by $\left[\sum_j A(\alpha_j,\tau_j)\omega_{j2}\right]$, and use the definition of the function $f(\vec{\tau})$ to obtain

$$(1-\alpha_i)\cdot f(\vec{\tau})\cdot\omega_{i1} - \alpha_i\cdot\omega_{i2} = -\frac{\alpha_i - A(\alpha_i,\tau_i)}{1-A(\alpha_i,\tau_i)}\frac{\left[\sum_j A(\alpha_j,\tau_j)\omega_{j2}\right]}{A(\alpha_i,\tau_i)}.$$

Multiply each term by $2\omega_{i2}$ and rearrange to obtain

$$(1-2\alpha_i)(\omega_{i2})^2+2(1-\alpha_i)\cdot f(\vec{\tau})\cdot\omega_{i1}\cdot\omega_{i2} = (\omega_{i2})^2-2\omega_{i2}\left\{\frac{\alpha_i - A(\alpha_i,\tau_i)}{1-A(\alpha_i,\tau_i)}\frac{\left[\sum_j A(\alpha_j,\tau_j)\omega_{j2}\right]}{A(\alpha_i,\tau_i)}\right\}.$$

Use Lemma E3 in the expression above to get the desired result

$$\begin{aligned}&(1-2\alpha_i)(\omega_{i2})^2+2(1-\alpha_i)\cdot f(\vec{\tau})\cdot\omega_{i1}\cdot\omega_{i2}\\ &= (\omega_{i2})^2-2\omega_{i2}\left\{\frac{(1-\tau_i)[\alpha_i\tau_i+(1-\alpha_i)]}{\tau_i}\right\}\left[\sum_j A(\alpha_j,\tau_j)\omega_{j2}\right].\end{aligned}$$

□

Next we use Lemma E6 and E7 in LemmaE5.

Lemma E8. *If* $\frac{\partial v_i}{\partial \tau_i}(\vec{\tau}) = 0$ *then*

$$\frac{\partial N_i}{\partial \tau_i}(\vec{\tau}) = \frac{\frac{(\alpha_i)^2}{[\alpha_i\tau_i+(1-\alpha_i)]^2}}{\left[\sum_j A(\alpha_j,\tau_j)\omega_{j2}\right]}\left\{\frac{(\omega_{i2})^2}{\left[\sum_j A(\alpha_j,\tau_j)\omega_{j2}\right]}-2\omega_{i2}\frac{(1-\tau_i)[\alpha_i\tau_i+(1-\alpha_i)]}{\tau_i}\right\}+\alpha_i\left\{\frac{(1-\tau_i)}{\tau_i}\right\}^2.$$

Proof. Consider the expression obtained in Lemma E5 when $k = i$. We have

$$\frac{\partial N_i}{\partial \tau_i}(\vec{\tau}) = \frac{\frac{\alpha_i(1-\alpha_i)}{[\alpha_i\tau_i+(1-\alpha_i)]^2}}{\left[\sum_j A(\alpha_j,\tau_j)\omega_{j2}\right]^2}\left\{(1-\alpha_i)\left(f(\vec{\tau})\cdot\omega_{i1}\right)^2+\alpha_i\cdot(\omega_{i2})^2\right\}.$$

Use Lemma E6 to substitute for the first term within braces to obtain

$$\begin{aligned}\frac{\partial N_i}{\partial \tau_i}(\vec{\tau}) &= \frac{\frac{\alpha_i(1-\alpha_i)}{[\alpha_i\tau_i+(1-\alpha_i)]^2}}{\left[\sum_j A(\alpha_j,\tau_j)\omega_{j2}\right]^2}\left\{-\frac{(\alpha_i)^2}{1-\alpha_i}\cdot(\omega_{i2})^2+2\alpha_i\cdot f(\vec{\tau})\cdot\omega_{i1}\cdot\omega_{i2}\right.\\ &\left.+\frac{1}{1-\alpha_i}\left\{\frac{(1-\tau_i)[\alpha_i\tau_i+(1-\alpha_i)]}{\tau_i}\right\}^2\left[\sum_j A(\alpha_j,\tau_j)\omega_{j2}\right]^2+\alpha_i\cdot(\omega_{i2})^2\right\}\\ &= \frac{\frac{\alpha_i(1-\alpha_i)}{[\alpha_i\tau_i+(1-\alpha_i)]^2}}{\left[\sum_j A(\alpha_j,\tau_j)\omega_{j2}\right]^2}\left\{\frac{\alpha_i(1-2\alpha_i)}{1-\alpha_i}\cdot(\omega_{i2})^2+2\alpha_i\cdot f(\vec{\tau})\cdot\omega_{i1}\cdot\omega_{i2}\right\}\end{aligned}$$

$$+\frac{\alpha_i(1-\alpha_i)}{[\alpha_i\tau_i+(1-\alpha_i)]^2}\frac{1}{1-\alpha_i}\left\{\frac{(1-\tau_i)[\alpha_i\tau_i+(1-\alpha_i)]}{\tau_i}\right\}^2$$

$$=\frac{\frac{(\alpha_i)^2}{[\alpha_i\tau_i+(1-\alpha_i)]^2}}{\left[\sum_j A(\alpha_j,\tau_j)\omega_{j2}\right]^2}\left\{(1-2\alpha_i)\cdot(\omega_{i2})^2+2(1-\alpha_i)\cdot f(\vec{\tau})\cdot\omega_{i1}\cdot\omega_{i2}\right\}+\alpha_i\left\{\frac{(1-\tau_i)}{\tau_i}\right\}^2.$$

Now use the result in Lemma S.7 E7 to obtain

$$\frac{\partial N_i}{\partial\tau_i}(\vec{\tau})=\frac{\frac{(\alpha_i)^2}{[\alpha_i\tau_i+(1-\alpha_i)]^2}}{\left[\sum_j A(\alpha_j,\tau_j)\omega_{j2}\right]}\left\{\frac{(\omega_{i2})^2}{\left[\sum_j A(\alpha_j,\tau_j)\omega_{j2}\right]}-2\omega_{i2}\frac{(1-\tau_i)[\alpha_i\tau_i+(1-\alpha_i)]}{\tau_i}\right\}+\alpha_i\left\{\frac{(1-\tau_i)}{\tau_i}\right\}^2.$$

□

Our penultimate supplementary result is used in Lemma 7 and 8.

Lemma E9. *If $\vec{\tau}$ is such that $\frac{\partial v_i}{\partial\tau_i}(\vec{\tau})=0$ then $\sum_{j\neq i}A(\alpha_j,\tau_j)\omega_{j2}>0$.*

Proof. Since $A(\alpha_j,\tau_j)>0$, $\omega_{j2}\geq 0$ and $\sum_{i\in\mathcal{I}}\omega_{i2}>0$, we have (i) $\sum_{j\neq i}A(\alpha_j,\tau_j)\omega_{j2}\geq 0$ and (ii) $\sum_{j\neq i}A(\alpha_j,\tau_j)\omega_{j2}=0$ if and only if $\omega_{j2}=0$ for all $j\neq i$ and $\omega_{i2}>0$.

If $\omega_{j2}=0$ for all $j\neq i$ and $\omega_{i2}>0$ then, using Lemma E2,

$$sign\left\{\frac{\partial v_i}{\partial\tau_i}(\vec{\tau})\right\}=sign\left\{\frac{\alpha_i-A(\alpha_i,\tau_i)}{1-A(\alpha_i,\tau_i)}\frac{1}{A(\alpha_i,\tau_i)}+\frac{(1-\alpha_i)\omega_{i1}}{\sum_j[1-A(\alpha_j,\tau_j)]\omega_{j1}}-\frac{\alpha_i}{A(\alpha_i,\tau_i)}\right\}$$

$$=sign\left\{\frac{\alpha_i-A(\alpha_i,\tau_i)-[1-A(\alpha_i,\tau_i)]\alpha_i}{1-A(\alpha_i,\tau_i)\cdot A(\alpha_i,\tau_i)}+\frac{(1-\alpha_i)\omega_{i1}}{\sum_j[1-A(\alpha_j,\tau_j)]\omega_{j1}}\right\}$$

$$=sign\left\{(1-\alpha_i)\left\{-\frac{1}{1-A(\alpha_i,\tau_i)}+\frac{\omega_{i1}}{\sum_j[1-A(\alpha_j,\tau_j)]\omega_{j1}}\right\}\right\}<0$$

unless $\omega_{j1}=0$ for all $j\neq i$ and $\omega_{i1}>0$ in which case the expression takes the value zero. So if $\omega_{j2}=0$ for all $j\neq i$ and $\omega_{i2}>0$ and $\vec{\tau}$ is such that $\frac{\partial v_i}{\partial\tau_i}(\vec{\tau})=0$ then, necessarily, $\omega_{j1}=0$ for all $j\neq i$ and $\omega_{i1}>0$. But that contradicts our assumption that for every $i\in\mathcal{I}$, $\omega_i\in R^2_+/\{0\}$. We conclude that if $\vec{\tau}$ is such that $\frac{\partial v_i}{\partial\tau_i}(\vec{\tau})=0$ then $\sum_{j\neq i}A(\alpha_j,\tau_j)\omega_{j2}>0$. □

We use Lemma E4, E8 and E9 to prove Lemma 7 which we repeat below for convenience:

Lemma. *(7)*

$$sign\left\{\frac{\partial^2 v_i}{\partial\tau_i\,\partial\tau_i}(\vec{\tau})\bigg|_{\frac{\partial v_i}{\partial\tau_i}(\vec{\tau})=0}\right\} = sign\left\{-2\alpha_i\tau_i\omega_{i2} - [1-2\alpha_i(1-\tau_i)]\left[\sum_{j\neq i} A(\alpha_j,\tau_j)\omega_{j2}\right]\right\}.$$

Proof. From Lemma 6 (ii), E2, and the definitions, we have

$$\frac{\partial v_i}{\partial\tau_i}(\vec{\tau}) = v_i(\vec{\tau})\cdot\frac{\partial A}{\partial\tau_i}(\alpha_i,\tau_i)\cdot\{M(\alpha_i,\tau_i)+N_i(\vec{\tau})\}\,.$$

By Remark 13, $\frac{\partial v_i}{\partial\tau_i}(\vec{\tau}) = 0$ if and only if $M(\alpha_i,\tau_i)+N_i(\vec{\tau}) = 0$. Since $\frac{\partial^2 v_i}{\partial\tau_i\,\partial\tau_i}(\vec{\tau})$ can be written as the sum of three terms, it follows that at any $\vec{\tau}$ such that $\frac{\partial v_i}{\partial\tau_i}(\vec{\tau}) = 0$ two of the terms must be zero. Furthermore, as in the proof of Lemma E2, $v_i(\vec{\tau}) > 0$ and $\frac{\partial A}{\partial\tau_i}(\alpha_i,\tau_i) > 0$; evidently,

$$sign\left\{\frac{\partial^2 v_i}{\partial\tau_i\,\partial\tau_i}(\vec{\tau})\bigg|_{\frac{\partial v_i}{\partial\tau_i}(\vec{\tau})=0}\right\} = sign\left\{\frac{\partial M}{\partial\tau_i}(\alpha_i,\tau_i)\bigg|_{\frac{\partial v_i}{\partial\tau_i}(\vec{\tau})=0} + \frac{\partial N}{d\tau_i}(\vec{\tau})\bigg|_{\frac{\partial v_i}{\partial\tau_i}(\vec{\tau})=0}\right\}.$$

From Lemma E4 and E8, we have

$$\frac{\partial M}{\partial\tau_i}(\alpha_i,\tau_i)\bigg|_{\frac{\partial v_i}{\partial\tau_i}(\vec{\tau})=0} + \frac{\partial N}{d\tau_i}(\vec{\tau})\bigg|_{\frac{\partial v_i}{\partial\tau_i}(\vec{\tau})=0} = -\alpha_i - \frac{1-\alpha_i}{(\tau_i)^2}$$

$$+\frac{\frac{(\alpha_i)^2}{[\alpha_i\tau_i+(1-\alpha_i)]^2}}{\left[\sum_j A(\alpha_j,\tau_j)\omega_{j2}\right]}\left\{\frac{(\omega_{i2})^2}{\left[\sum_j A(\alpha_j,\tau_j)\omega_{j2}\right]} - 2\omega_{i2}\frac{(1-\tau_i)[\alpha_i\tau_i+(1-\alpha_i)]}{\tau_i}\right\}+\alpha_i\left\{\frac{(1-\tau_i)}{\tau_i}\right\}^2.$$

Since

$$-\alpha_i - \frac{1-\alpha_i}{(\tau_i)^2} + \alpha_i\left\{\frac{(1-\tau_i)}{\tau_i}\right\}^2 = -\frac{1-2\alpha_i+2\alpha_i\tau_i}{(\tau_i)^2} = -\frac{1-2\alpha_i(1-\tau_i)}{(\tau_i)^2},$$

it follows that

$$sign\left\{\frac{\partial^2 v_i}{\partial\tau_i\,\partial\tau_i}(\vec{\tau})\bigg|_{\frac{\partial v_i}{\partial\tau_i}(\vec{\tau})=0}\right\} = sign\left\{\mathcal{K}_i\right\}$$

where

$$\mathcal{K}_i = \frac{\frac{(\alpha_i)^2}{[\alpha_i\tau_i+(1-\alpha_i)]^2}}{\left[\sum_j A(\alpha_j,\tau_j)\omega_{j2}\right]}\left\{\frac{(\omega_{i2})^2}{\left[\sum_j A(\alpha_j,\tau_j)\omega_{j2}\right]} - 2\omega_{i2}\frac{(1-\tau_i)[\alpha_i\tau_i+(1-\alpha_i)]}{\tau_i}\right\}-\frac{1-2\alpha_i(1-\tau_i)}{(\tau_i)^2}.$$

We have

$$\mathcal{K}_i \left[\sum_j A(\alpha_j,\tau_j)\omega_{j2}\right]^2 = (\omega_{i2})^2 \frac{(\alpha_i\tau_i)^2}{[\alpha_i\tau_i + (1-\alpha_i)]^2} - \left[\sum_j A(\alpha_j,\tau_j)\omega_{j2}\right]^2 [1-2\alpha_i(1-\tau_i)]$$

$$-2\omega_{i2}\frac{(1-\tau_i)[\alpha_i\tau_i + (1-\alpha_i)]}{\tau_i}\frac{(\alpha_i\tau_i)^2}{[\alpha_i\tau_i + (1-\alpha_i)]^2}\left[\sum_j A(\alpha_j,\tau_j)\omega_{j2}\right].$$

Introduce the notation $\mathcal{A}_i := \sum_{j\neq i} A(\alpha_j,\tau_j)\omega_{j2}$, and simplify the expression to obtain

$$\mathcal{K}_i \left[\sum_j A(\alpha_j,\tau_j)\omega_{j2}\right]^2 = (\omega_{i2})^2 \frac{(\alpha_i\tau_i)^2}{[\alpha_i\tau_i + (1-\alpha_i)]^2} - 2\omega_{i2}\frac{(1-\tau_i)(\alpha_i)^2\tau_i}{[\alpha_i\tau_i + (1-\alpha_i)]}\left[\mathcal{A}_i + A(\alpha_i,\tau_i)\omega_{i2}\right]$$

$$-\left[\mathcal{A}_i + A(\alpha_i,\tau_i)\omega_{i2}\right]^2 [1-2\alpha_i(1-\tau_i)].$$

Now expand the terms within brackets and collect terms to obtain

$$\mathcal{K}_i \left[\sum_j A(\alpha_j,\tau_j)\omega_{j2}\right]^2$$

$$= -[1-2\alpha_i(1-\tau_i)]\left[A(\alpha_i,\tau_i)\omega_{i2}\right]^2 + (\omega_{i2})^2\frac{(\alpha_i\tau_i)^2}{[\alpha_i\tau_i + (1-\alpha_i)]^2} - 2\omega_{i2}\frac{(1-\tau_i)(\alpha_i)^2\tau_i}{[\alpha_i\tau_i + (1-\alpha_i)]}\left[A(\alpha_i,\tau_i)\omega_{i2}\right]$$

$$-[1-2\alpha_i(1-\tau_i)]\left[2\mathcal{A}_i \cdot A(\alpha_i,\tau_i)\omega_{i2}\right] - 2\omega_{i2}\frac{(1-\tau_i)(\alpha_i)^2\tau_i}{[\alpha_i\tau_i + (1-\alpha_i)]}\left[\mathcal{A}_i\right]$$

$$-[1-2\alpha_i(1-\tau_i)](\mathcal{A}_i)^2,$$

which, upon recalling the definition of $A(\alpha_i,\tau_i)$, may be simplified to

$$\mathcal{K}_i \left[\sum_j A(\alpha_j,\tau_j)\omega_{j2}\right]^2 = \frac{(\alpha_i\tau_i)^2}{[\alpha_i\tau_i + (1-\alpha_i)]^2}\left\{-[1-2\alpha_i(1-\tau_i)] + 1 - 2(1-\tau_i)\alpha_i\right\}(\omega_{i2})^2$$

$$-\frac{2\alpha_i\tau_i}{[\alpha_i\tau_i + (1-\alpha_i)]}\left\{[1-2\alpha_i(1-\tau_i)] + (1-\tau_i)\alpha_i\right\}\left[\mathcal{A}_i\omega_{i2}\right]$$

$$-[1-2\alpha_i(1-\tau_i)](\mathcal{A}_i)^2.$$

Evidently, the coefficients in the first term on the right add up to zero and the second term can be simplified to obtain

$$\mathcal{K}_i\left[\sum_j A(\alpha_j,\tau_j)\omega_{j2}\right]^2 = -\frac{2\alpha_i\tau_i\left[1-\alpha_i(1-\tau_i)\right]}{\left[\alpha_i\tau_i+(1-\alpha_i)\right]}\left[\mathcal{A}_i\omega_{i2}\right]-\left[1-2\alpha_i(1-\tau_i)\right](\mathcal{A}_i)^2$$

$$= -\mathcal{A}_i\left\{2\alpha_i\tau_i\omega_{i2}+\left[1-2\alpha_i(1-\tau_i)\right]\mathcal{A}_i\right\}.$$

By Lemma E9, at $\vec{\tau}$ such that $\frac{\partial v_i}{\partial \tau_i}(\vec{\tau})=0$, $\sum_{j\neq i} A(\alpha_j,\tau_j)\omega_{j2}>0$, i.e. $\mathcal{A}_i>0$, and so, at such a $\vec{\tau}$,

$$sign\left\{\frac{\partial^2 v_i}{\partial\tau_i\,\partial\tau_i}(\vec{\tau})\,\bigg|_{\frac{\partial v_i}{\partial\tau_i}(\vec{\tau})=0}\right\} = sign\left\{-2\alpha_i\tau_i\omega_{i2}-\left[1-2\alpha_i(1-\tau_i)\right]\left[\sum_{j\neq i}A(\alpha_j,\tau_j)\omega_{j2}\right]\right\}$$

where we recall that $\mathcal{A}_i := \sum_{j\neq i} A(\alpha_j,\tau_j)\omega_{j2}$. □

Our last supplementary result is used in Lemma 8 to claim that $\frac{\partial^2 v_i}{\partial\tau_i\,\partial\tau_i}(\vec{\tau})\,\Big|_{\frac{\partial v_i}{\partial\tau_i}(\vec{\tau})=0}\neq 0$.

Lemma E10. *The equations*

$$2\alpha_i\tau_i\omega_{i2}+\left[1-2\alpha_i(1-\tau_i)\right]\left[\sum_{j\neq i}A(\alpha_j,\tau_j)\omega_{j2}\right]=0$$

$$\frac{\alpha_i-A(\alpha_i,\tau_i)}{1-A(\alpha_i,\tau_i)}\frac{1}{A(\alpha_i,\tau_i)}+\frac{(1-\alpha_i)\omega_{i1}}{\sum_j\left[1-A(\alpha_j,\tau_j)\right]\omega_{j1}}-\frac{\alpha_i\cdot\omega_{i2}}{\sum_j A(\alpha_j,\tau_j)\omega_{j2}}=0$$

cannot hold simultaneously.

Proof. Set $\mathcal{A}_i := \sum_{j\neq i} A(\alpha_j,\tau_j)\omega_{j2}$, and rewrite the first equation as

$$-2\alpha_i\tau_i\omega_{i2}-\left[1-2\alpha_i(1-\tau_i)\right]\mathcal{A}_i=0 \qquad\Longleftrightarrow\qquad \alpha_i(1-\tau_i)\mathcal{A}_i=\alpha_i\tau_i\omega_{i2}+\frac{\mathcal{A}_i}{2}. \qquad (*)$$

Now observe that

$$\frac{(1-\tau_i)\left[\alpha_i\tau_i+(1-\alpha_i)\right]}{\tau_i}\left[\sum_j A(\alpha_j,\tau_j)\omega_{j2}\right]-\alpha_i\omega_{i2}=\frac{\alpha_i(1-\tau_i)}{A(\alpha_i,\tau_i)}\left[\mathcal{A}_i+A(\alpha_i,\tau_i)\omega_{i2}\right]-\alpha_i\omega_{i2},$$

where we use the definition of $A(\alpha_i, \tau_i)$,

$$= \frac{\alpha_i \tau_i \omega_{i2}}{A(\alpha_i, \tau_i)} + \frac{\mathcal{A}_i}{2A(\alpha_i, \tau_i)} + \alpha_i(1 - \tau_i)\omega_{i2} - \alpha_i \omega_{i2},$$

where we use $(*)$,

$$= \frac{\alpha_i \tau_i \omega_{i2}}{A(\alpha_i, \tau_i)} + \frac{\mathcal{A}_i}{2A(\alpha_i, \tau_i)} - \alpha_i \tau_i \omega_{i2} = \alpha_i \tau_i \omega_{i2} \left[\frac{1}{A(\alpha_i, \tau_i)} - 1\right] + \frac{\mathcal{A}_i}{2A(\alpha_i, \tau_i)} > 0$$

if $\omega_{i2} > 0$; this is because $1 > A(\alpha_i, \tau_i)$ and $\mathcal{A}_i \geq 0$. Recall Lemma E3 to conclude that if $\omega_{i2} > 0$ then

$$\frac{\alpha_i - A(\alpha_i, \tau_i)}{1 - A(\alpha_i, \tau_i)} \frac{1}{A(\alpha_i, \tau_i)} \left[\sum_j A(\alpha_j, \tau_j)\omega_{j2}\right] - \alpha_i \omega_{i2} > 0.$$

Since $\omega_i \neq (0, 0)$, it is evident that

$$\frac{\alpha_i - A(\alpha_i, \tau_i)}{1 - A(\alpha_i, \tau_i)} \frac{1}{A(\alpha_i, \tau_i)} \left[\sum_j A(\alpha_j, \tau_j)\omega_{j2}\right] + (1 - \alpha_i)\omega_{i1} \frac{\left[\sum_j A(\alpha_j, \tau_j)\omega_{j2}\right]}{\sum_j \left[1 - A(\alpha_j, \tau_j)\right]\omega_{j1}} - \alpha_i \omega_{i2} > 0$$

and the result follows since, under our maintained assumptions, $\left[\sum_j A(\alpha_j, \tau_j)\omega_{j2}\right] > 0$. □

Below we provide the proof of Lemma 8 which we repeat below for convenience:

Lemma. *(8)*

$$\frac{\partial^2 v_i}{\partial \tau_i \partial \tau_i}(\vec{\tau}) \Big|_{\frac{\partial v_i}{\partial \tau_i}(\vec{\tau}) = 0} \neq 0$$

If $\alpha_i \leq 1/2$ *or if* $\frac{\omega_{i2}}{\sum_j \omega_{j2}} < 1$ *then* $\frac{\partial^2 v_i}{\partial \tau_i \partial \tau_i}(\vec{\tau}) \Big|_{\frac{\partial v_i}{\partial \tau_i}(\vec{\tau}) = 0} < 0$.

Proof. Lemma 7, Lemma E2 and Lemma E10 directly imply that $\frac{\partial^2 v_i}{\partial \tau_i \partial \tau_i}(\vec{\tau}) \Big|_{\frac{\partial v_i}{\partial \tau_i}(\vec{\tau}) = 0} \neq 0$.

If $\alpha_i \leq 1/2$ then $0 < 1 - 2\alpha_i(1 - \tau_i)$ for all $\tau_i > 0$, and the required result follows directly from Lemma 7.

We turn to the case where $\alpha_i > 1/2$. Let $\hat{\tau}_i$ be the unique value such that

$$\omega_{i2} = \frac{[2\alpha_i(1-\hat{\tau}_i) - 1]}{2\alpha_i\hat{\tau}_i} \cdot \left[\sum_{j\neq i} A(\alpha_j,\tau_j)\omega_{j2}\right].$$

Observe that

$$sign\left\{-2\alpha_i\tau_i\omega_{i2} - [1-2\alpha_i(1-\tau_i)]\left[\sum_{j\neq i} A(\alpha_j,\tau_j)\omega_{j2}\right]\right\} = \begin{cases} 1 & \text{if} \quad \tau_i \in (0,\hat{\tau}_i) \\ 0 & \text{if} \quad \tau_i = \hat{\tau}_i \\ -1 & \text{if} \quad \tau_i \in (\hat{\tau}_i,+\infty)\,. \end{cases}$$

Since $\frac{\omega_{i2}}{\sum_j \omega_{j2}} < 1$, By Lemma 6 (i), there exists $\underline{\tau} \in (0,1)$ such that $\frac{\partial v_i}{\partial \tau_i}(\underline{\tau},\tau_i) > 0$ for $\tau_{-i} \in [\underline{\tau},\infty)^{I-1}$. Notice that, since $\frac{\partial v_i}{\partial \tau_i}(\underline{\tau},\tau_{-i}) > 0$, if $\vec{\tau}$ is such that $\frac{\partial v_i}{\partial \tau_i}(\vec{\tau}) = 0$ then $\tau_i \neq \underline{\tau}$.

We have three cases to consider. (i) If $\hat{\tau}_i < \underline{\tau}$ then the result follows from the observation above, which assures us that the sign of the expression in brackets is negative, and Lemma 7. (ii) If $\hat{\tau}_i = \underline{\tau}$, then, since $\frac{\partial v_i}{\partial \tau_i}(\underline{\tau},\tau_{-i}) > 0$, if $\vec{\tau}$ is such that $\frac{\partial v_i}{\partial \tau_i}(\vec{\tau}) = 0$ then, necessarily, $\tau_i > \underline{\tau} = \hat{\tau}_i$, and the argument given in (i) can be used. (iii) Consider the case where $\underline{\tau} < \hat{\tau}_i$ and let $\vec{\tau}$ be such that $\frac{\partial v_i}{\partial \tau_i}(\vec{\tau}) = 0$. If for such a $\vec{\tau}$ we have $\tau_i > \hat{\tau}_i$ then we can use the argument given in (i). We turn to the case that remains in which $\vec{\tau}$ is such that $\underline{\tau} < \tau_i \leq \hat{\tau}_i$ as we have already shown that $\tau_i \neq \underline{\tau}$.

So consider first $\vec{\tau}$ such that $\frac{\partial v_i}{\partial \tau_i}(\vec{\tau}) = 0$ and $\tau_i \in (\underline{\tau},\hat{\tau}_i)$. By the observation above, which assures us that the sign of the expression in brackets is positive, and Lemma 7, we must have $\frac{\partial^2 v_i}{\partial \tau_i\,\partial \tau_i}(\vec{\tau})\Big|_{\frac{\partial v_i}{\partial \tau_i}(\vec{\tau})=0} > 0$. Since $\frac{\partial v_i}{\partial \tau_i}(\vec{\tau})$ is a continuous function, and $\frac{\partial v_i}{\partial \tau_i}(\underline{\tau},\tau_{-i}) > 0$, the Intermediate Value Theorem implies that there must exist $\vec{\tilde{\tau}}$ with $\tilde{\tau}_i \in (\underline{\tau},\hat{\tau}_i)$ with $\frac{\partial^2 v_i}{\partial \tau_i\,\partial \tau_i}(\vec{\tilde{\tau}})\Big|_{\frac{\partial v_i}{\partial \tau_i}(\vec{\tilde{\tau}})=0} < 0$; Here we use the fact that the value $\hat{\tau}_i$ is unique so by the observation above and Lemma 7, $\frac{\partial^2 v_i}{\partial \tau_i\,\partial \tau_i}(\vec{\tau})\Big|_{\frac{\partial v_i}{\partial \tau_i}(\vec{\tau})=0} \neq 0$. But then, by Lemma 7, the sign of the expression in the observation made earlier must also be negative. But that contradicts the observation above, since the sign of the expression must be positive for all $\tau_i \in (0,\hat{\tau}_i)$.

The remaining case where $\vec{\tau}$ is such that $\frac{\partial v_i}{\partial \tau_i}(\vec{\tau}) = 0$ and $\tau_i = \hat{\tau}_i$ is ruled out since $\frac{\partial^2 v_i}{\partial \tau_i\,\partial \tau_i}(\vec{\tau})\Big|_{\frac{\partial v_i}{\partial \tau_i}(\vec{\tau})=0} \neq 0.$ $\square$

Below we provide a proof of Proposition 9, which we repeat below for convenience:

Proposition. *(9) Assume that the vector of endowments is not a Pareto optimal allocation and that $\frac{\omega_{i1}}{\sum_j \omega_{j1}} < 1$ for all $i \in \mathcal{I}$. Then at every interior Nash equilibrium there is trade.*

Proof. Let $\vec{\tau}$ be an interior Nash equilibrium profile of tariff rates. It follows $\frac{\partial v_i}{\partial \tau_i}(\vec{\tau}) = 0$ for each $i \in \mathcal{I}$. By Lemma E2, for each $i \in \mathcal{I}$,

$$\frac{\alpha_i - A(\alpha_i, \tau_i)}{1 - A(\alpha_i, \tau_i)} \frac{1}{A(\alpha_i, \tau_i)} + \frac{(1-\alpha_i)\omega_{i1}}{\sum_j [1 - A(\alpha_j, \tau_j)]\omega_{j1}} - \frac{\alpha_i \cdot \omega_{i2}}{\sum_j A(\alpha_j, \tau_j)\omega_{j2}} = 0$$

$$\iff \quad \frac{\alpha_i - A(\alpha_i, \tau_i)}{1 - A(\alpha_i, \tau_i)} \frac{\sum_j [1 - A(\alpha_j, \tau_j)]\omega_{j1}}{A(\alpha_i, \tau_i)} + (1-\alpha_i)\omega_{i1} - \frac{\alpha_i \cdot \omega_{i2} \sum_j [1 - A(\alpha_j, \tau_j)]\omega_{j1}}{\sum_j A(\alpha_j, \tau_j)\omega_{j2}} = 0 \qquad (*)$$

since, under our maintained hypotheses, $\sum_j [1 - A(\alpha_j, \tau_j)]\omega_{j1} > 0$.

Suppose that the interior Nash equilibrium has the additional property that there is no trade. It follows that at $p_2^*(\vec{\tau})$, $(x_{i1}(p_2^*(\vec{\tau})), x_{i2}(p_2^*(\vec{\tau}))) = \omega_i$ for all $i \in \mathcal{I}$. So i's first order condition in the domestic market for the two goods, $x_{i2} = \frac{(1-\alpha_i)}{\alpha_i \tau_i p_2^*(\vec{\tau})} x_{i1}$, must hold at $(x_{i1}, x_{i2}) = \omega_i$. To summarize, for each $i \in \mathcal{I}$, in addition to $(*)$, the following must be true:

$$p_2^*(\vec{\tau}) = \frac{1}{\tau_i} \frac{(1-\alpha_i)/\omega_{i2}}{\alpha_i/\omega_{i1}} \qquad p_2^*(\vec{\tau}) = \frac{\sum_i [1 - A(\alpha_i, \tau_i)]\omega_{i1}}{\sum_i A(\alpha_i, \tau_i)\omega_{i2}}.$$

It follows that, for all $i \in \mathcal{I}$,

$$\frac{\alpha_i - A(\alpha_i, \tau_i)}{1 - A(\alpha_i, \tau_i)} \frac{\sum_j [1 - A(\alpha_j, \tau_j)]\omega_{j1}}{A(\alpha_i, \tau_i)} + (1-\alpha_i)\omega_{i1} - \alpha_i \cdot \omega_{i2} \frac{1}{\tau_i} \frac{(1-\alpha_i)/\omega_{i2}}{\alpha_i/\omega_{i1}} = 0$$

$$\iff \quad \frac{\alpha_i - A(\alpha_i, \tau_i)}{1 - A(\alpha_i, \tau_i)} \frac{\sum_j [1 - A(\alpha_j, \tau_j)]\omega_{j1}}{A(\alpha_i, \tau_i)} + \left(1 - \frac{1}{\tau_i}\right)(1-\alpha_i)\omega_{i1} = 0,$$

which, upon using Lemma E3, becomes

$$\iff \quad \left(1 - \frac{1}{\tau_i}\right) \left\{ -[\alpha_i \tau_i + (1-\alpha_i)] \sum_j [1 - A(\alpha_j, \tau_j)]\omega_{j1} + (1-\alpha_i)\omega_{i1} \right\} = 0,$$

so that at least one of the two expressions within brackets must be zero.

If $\left(1 - \frac{1}{\tau_i}\right) = 0$ for each $i \in \mathcal{I}$, then $\vec{\tau} = (1, \cdots, 1)$ and from i's first order condition in the domestic market for the two goods, at the interior Nash equilibrium under consideration, marginal rates of substitution are equalized across countries. This can happen only if the endowment distribution is also a Pareto optimal allocation, a situation which we have ruled out. It follows that for each $i \in \mathcal{I}$

$$[\alpha_i \tau_i + (1-\alpha_i)] \sum_j [1 - A(\alpha_j, \tau_j)]\omega_{j1} = (1-\alpha_i)\omega_{i1}$$

$$\iff \quad \sum_j [1 - A(\alpha_j, \tau_j)]\omega_{j1} = [1 - A(\alpha_i, \tau_i)]\omega_{i1},$$

where we use the fact that $\alpha_i \tau_i + (1-\alpha_i) > 0$ together with the definition of $A(\alpha_i, \tau_i)$. These are I equations with $1 - A(\alpha_i, \tau_i) > 0$ and $\omega_{i1} \geq 0$ for each $i \in \mathcal{I}$, and $\omega_{i1} > 0$ for at least two countries. It is evident that the equations cannot hold simultaneously.

We have shown that assuming that there is no trade at the interior Nash equilibrium results in a contradiction. $\square$

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