Modelling Motivic Processes in Music

A Mathematical Approach

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Art is different from science.

While science requires systematically all characteristic cases, art is satisfied with a lesser number of interesting ones: as many as fantasy demands in order to produce for itself an image of the whole, in order to dream about it.

For this reason even development should never be understood here to mean that all cases must come into being, but rather just a few, the interesting ones –

"More of this another time," the artist can say – "for today enough of it."

Arnold Schoenberg[1]

Mathematics is in a way like the production of feelings in the listener: It occurs in all the arts, but only in the case of music is a big fuss made about it.

Eduard Hanslick[2]

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[1] Arnold Schoenberg, *The Musical Idea and the Logic, Technique, and Art of Its Presentation*, ed. and trans. by Patricia Carpenter and Severine Neff (New York: Columbia University Press, 1995), p. 93.
[2] Eduard Hanslick, 'From *Vom Musikalisch-Schönen*', ed. and trans. by Geoffrey Payzant, in *Source Readings in Music History*, ed. by Oliver Strunk, edn rev. by Leo Treitler, 7 vols (New York: Norton, 1998), VI: *The Nineteenth Century*, ed. by Ruth A. Solie, pp. 160–69 (first publ. in *On the Musically Beautiful: A Contribution towards the Revision of the Aesthetics of Music*, ed. and trans. by Geoffrey Payzant (Indianapolis: Hackett, 1986), pp. 32–35, 38–44) (pp. 166–67).

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Abstract

This thesis proposes a new model for motivic analysis which, being based on the metaphor of a web or network and expanded using the mathematical field of graph theory, balances the polar concerns prevalent in analytical writing to date: those of static, out-of-time category membership and dynamic, in-time process. The concepts that constitute the model are presented in the third chapter, both as responses to a series of analytical observations (using the worked example of Beethoven's Piano Sonata in F minor, Op. 2, No. 1), and as rigorously defined mathematical formalisms. The other chapters explore in further detail the disciplines and methodologies on which this model impinges, and serve both to motivate, and to reflect upon, its development. Chapter 1 asks what it means to make mathematical statements about music, and seeks to disentangle mathematics (as a tool or language) from science (as a method), arguing that music theory's aims can be met by the former without presupposing its commonly assumed inextricability from the latter. Chapter 2 provides a thematic overview of the field of motivic theory and analysis, proposing four archetypal models that combine to underwrite much thought on the subject before outlining the problems inherent in a static account and the creative strategies that can be used to construct a dynamic account. Finally, Chapter 4 applies these strategies, together with Chapter 3's model and the piece's extensive existing scholarly literature, to the analysis of the first and last movements of Mahler's Sixth Symphony. The central theme throughout – as it relates to mathematical modelling, music theory, and music analysis – is that of potential, invitation, openness, and dialogic engagement.

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List of Abbreviations

3MR *Mathematical and Computational Approaches to Music: Three Methodological Reflections*, ed. by Anja Volk and Aline Honingh (=*Journal of Mathematics and Music*, 6.2 (July 2012))

The following two volumes contain excerpts of historical analyses edited, translated, and introduced by Ian Bent. They are cited using Bent's standardised chapter titles (square brackets removed) and with full original publication details (as supplied by Bent) given in the bibliography. Page ranges are given for each analysis and its introduction, and references to a page from an introduction are suffixed by 'ed' to signal that they have been written by Bent (e.g. 'Hoffmann, p. 141ed').

MA19I *Music Analysis in the Nineteenth Century*, ed. and trans. by Ian Bent, Cambridge Readings in the Literature of Music, 2 vols (Cambridge: Cambridge University Press, 1994), I: Fugue, Form and Style

MA19II *Music Analysis in the Nineteenth Century*, ed. and trans. by Ian Bent, Cambridge Readings in the Literature of Music, 2 vols (Cambridge: Cambridge University Press, 1994), II: Hermeneutic Approaches

S&I *Style and Idea: Selected Writings of Arnold Schoenberg*, ed. by Leonard Stein, trans. by Leo Black (London: Faber and Faber, 1975; rev. repr. Berkeley: University of California Press, 1984)

Other Citation Conventions

The *Stanford Encyclopedia of Philosophy* is a dynamic online resource that is archived four times each year; this thesis adopts its recommended practice of citing, for each article, the earliest archived edition containing the article's most recent version.

Schoenberg's *Coherence, Counterpoint, Instrumentation, Instruction in Form* and *The Musical Idea and the Logic, Technique, and Art of Its Presentation* (see bibliography for full details) are facing-page translations of unpublished manuscripts: page citations therefore give translated (odd-numbered) pages only, with the original German being available on the previous (even-numbered) page.

Glossary of Mathematical Symbols

Logical operators

$\Rightarrow$	'implies'/'if…then…'
$\Leftrightarrow$	'implies and is implied by'/'is equivalent to'/'if and only if'
$\forall$	'for all'/'for every' (e.g. $x + 0 = x, \forall x$)
$\exists$	'there exists' (e.g. $\forall x, \exists y$ such that $x + 1 = y$)
$\neg$	'not'/'negation of'; if $A \Rightarrow B$ then $\neg B \Rightarrow \neg A$
$\blacksquare$	End of proof
$\nRightarrow$, $\nexists$, etc.	'does not imply', 'there does not exist', etc.

Sets

$\{a, b, c\}$	Unordered set (i.e. $\{a, b, c\} = \{b, c, a\}$ etc.) containing a, b, and c. Upper-case letters usually denote sets, lower-case letters elements, and script letters (e.g. $\mathcal{M}$) sets of sets.
(a, b, c)	Ordered set (in this example, a 'triple'); $(a, b, c) \neq (b, c, a)$ etc.
$\in$	(is)'a member of the set'/'in'/ 'drawn from', e.g. $a \in \{a, b, c\}$, $x \notin \{a, b, c\}$
$\|X\|$	The number of elements in the set X e.g. $\|\{a, b, c\}\| = 3$
…	Used in listing integers; so $i = 1, \dots, n$ means 'for integer values of i from 1 to n'
$\emptyset$	The empty set; $\|\emptyset\| = 0$
$\mathbb{Z}^+$	The set of positive integers: $\{1,2,3, \dots\}$
$\mathbb{Q}^+$	The set of positive rational numbers i.e. all $^a/_b$ where $a \in \mathbb{Z}^+$ and $b \in \mathbb{Z}^+$
$\mathbb{R}^+$	The set of positive real numbers (i.e. $\mathbb{Q}^+$ and irrational numbers like $\sqrt{2}$ and π)
$\{ \ \mid \ \}$	'the set of all … such that…'; so $\mathbb{Q}^+ = \left\{ ^a/_b \,\middle\|\, a, b \in \mathbb{Z}^+ \right\}$
$\cap$	The intersection operator; $A \cap B$ is the set of all objects in both A and B
$\cup$	The union operator; $A \cup B$ is the set of all objects in A, B, or both. This can be iterated, so $\bigcup_{i=1}^{n} X_i = X_1 \cup X_2 \cup \dots \cup X_n$ and $\bigcup_{i=1,3} X_i = X_1 \cup X_3$
$\uplus$	The disjoint union operator; $\{a, b\} \cup \{b, c\} = \{a, b, c\}$ but $\{a, b\} \uplus \{b, c\} = \{a, b, b, c\}$
$\setminus$ or $-$	The complement operator; $A \setminus B$ is the set of all objects in A but not B
$\subset$ and $\subseteq$	(is) 'a subset of' and (is) 'a subset of, or equal to'; so $\{a, b\} \subset \{a, b, c\}$
$MAX\{ \quad \}$	The maximum value of the given set; $MIN\{ \quad \}$ is similarly the minimum value

Other

$f(x, y)$	Function of x and y which outputs a single value; $f(x)$ is a function of x only
$f: X \rightarrow Y$	'the function f maps members of set X to members of set Y'
$\sum_{i=1}^{n} x_i$	The sum of all x_i for $i = 1$ to n: $x_1 + x_2 + \dots + x_n$
$\lfloor x \rfloor$ and $\lceil x \rceil$	The integer floor and integer ceiling functions: $\lfloor 1.8 \rfloor = 1$ and $\lceil 1.8 \rceil = 2$, for example
$[a , b)$	The closed–open interval from a to b: $[a, b) = \{x \in \mathbb{R} \| a \leq x < b\}$

Introduction

E. T. A. Hoffmann's 1810 review of Beethoven's Fifth Symphony – '[a]rguably the most celebrated document in the history of music criticism', according to Ian Bent – begins its analysis of the first movement with a casual reference to 'the main idea [*Hauptgedanke*] consisting of only two bars, which subsequently appears again and again in a variety of forms'.[1] The observation seems innocuous, even obvious; but it establishes the conceit that what is seen and heard in Beethoven's Fifth Symphony is the constant reappearance and reinterpretation of a single 'idea' in different 'forms': 'the endless reshaping of a basic shape', in Arnold Schoenberg's phrase.[2] This understanding is not limited to Hoffmann and Schoenberg – Jonathan Dunsby notes that Hoffmann's review 'is often cited as the first torch in [a] phalanx of critical illumination deriving from Goethean organicism and marching on to this day' – and the implied locus of the 'main idea', 'basic shape', or 'motive' (a term not actually used by Hoffmann) varies between the members of this phalanx.[3] Some consider it to be a concrete musical segment (for example, the first two bars) while others (explicitly or implicitly) construct a more abstract archetype; all, however, agree that it is underwritten by the central concept of recurrence. This presents a paradox: that to apply a label such as 'motive' to a singular object is actually to tacitly assert a set of relationships between a plurality of objects.

A certain fluidity between individual musical segments on one hand, and the abstract types or groupings that they realise or define on the other, is therefore apparent in most analytical writings on the subject (as discussed in further detail in Sections 2.1 and 2.2). As Bent argues, '[w]e have been conditioned (by twentieth-century analysts writing largely about eighteenth-century music) to look for unifying forces', even when each segment in a unified category 'is a cleanly incised utterance with its own distinct properties'.[4] The assumption that motives are sets of segments can therefore be a misleading one, as sets depend on a property known as transitivity: if a and b are in the same set, and b and c are in the same set, then a and

[1] Ernst Theodor Amadeus Hoffmann, 'Review: Beethoven's Symphony No. 5 in C Minor', trans. by Martyn Clarke with David Charlton and Ian Bent, in *MA19II*, pp. 145–60 (introduction pp. 141–44) (pp. 141ed, 147). The German term in square brackets is included in Bent's edition; see the list of abbreviations on p. x for an explanation of how this volume and its companion *MA19I* are cited in this thesis.
[2] Arnold Schoenberg, 'Linear Counterpoint', in *S&I*, pp. 289–95 (p. 290).
[3] Jonathan Dunsby, 'Thematic and Motivic Analysis', in *The Cambridge History of Western Music Theory*, ed. by Thomas Christensen (Cambridge: Cambridge University Press, 2002), pp. 907–26 (p. 909). Bent lists the terms that Hoffmann uses, and notes that *Motiv* is not among them, in his introduction to Hoffmann's review (p. 141ed). I use 'motive' throughout this thesis (spelled to accord with the Germanic *Motiv* rather than the French *motif*) as an umbrella concept (see Section 2.1).
[4] Ian Bent, 'Introduction [to Part I: Elucidatory Analysis]', in *MA19II*, pp. 31–38 (p. 37).

c are in the same set. If one motive is said to develop into another, or combine with another, or liquidate, or share parallels with another category that the analyst ultimately wishes to keep separate (as is the case when correspondences between first and second subjects are noted), the transitivity condition is violated and the set model breaks down.

This thesis argues that a better model for defining the relationship between musical segments and motivic categories is provided by the image of a network: a series of links or bridges between individual points (known as nodes), aggregating into a structural picture of the target phenomenon. The idea of a motivic network or web is not uncommon in music theory (although it is not as prevalent as the hybrid set/segment model): Rudolph Reti, for example, speaks of a work's 'thematic and contrapuntal web' (which can turn in to a 'vague, amorphous mass' to be replaced with 'characteristic melodies' in listeners' memories), Stephen Davies of 'an unbroken web of unobvious relationships […] although there is no element or set of elements common to all parts of the work', and Kofi Agawu of 'the dispersal of motifs in a kind of network claim[ing] a significant part of [a] movement's teleology'.[5] The metaphor seems to be particularly popular in discussions of Wagner's music: the composer himself considers a 'web of basic themes [which] intertwine with one another' to be definitive of 'symphonic unity', while Thomas Grey writes of his 'associative motivic networks', Bent of his 'complex circuitry', and Richard Taruskin of his 'formally free (or ad hoc) web[s] of motifs'.[6] Similar phrases tend to surface whenever Mahler's thematic process is discussed, as examined in further detail below.

Suggestive as these comments may be, however, they have not been comprehensively pursued to their music-analytical ends; this is particularly surprising given the preponderance of

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[5] Rudolph Reti, *The Thematic Process in Music* ([New York: Macmillan, 1951; repr.] London: Faber and Faber, 1961), p. 283; Stephen Davies, 'Attributing Significance to Unobvious Musical Relationships', *Journal of Music Theory*, 27 (1983), 203–13 (p. 205); Kofi Agawu, *Music as Discourse: Semiotic Adventures in Romantic Music* (Oxford: Oxford University Press, 2009), p. 273.
[6] Thomas Grey, '*...wie ein rother Faden*: On the Origins of "Leitmotif" as Critical Construct and Musical Practice', in *Music Theory in the Age of Romanticism*, ed. by Ian Bent (Cambridge: Cambridge University Press, 1996), pp. 187–210 (p. 188); Hans von Wolzogen, '"Prelude", "Act I Scene I", *Parsifal: A Thematic Guide through the Poetry and the Music*', in *MA19II*, pp. 93–105 (introduction pp. 88–92) (p. 90ed); Richard Taruskin, *The Oxford History of Western Music*, 6 vols (New York: Oxford University Press, 2005), IV: *The Early Twentieth Century*, p. 354. Wagner's 'Gewebe' is rendered 'web' in Alfred Clayton's translation (quoted here); alternatives include 'network' (by Anthony Newcomb), 'fabric' (by Grey), and 'tissue' (by William Ashton Ellis): see Richard Wagner, *Gesammelte Schriften und Dichtungen*, 4th edn, 10 vols (Leipzig: Siegel; Linnemann, 1907), X, 185; Carl Dahlhaus, 'The Music', trans. by Alfred Clayton, in *Wagner Handbook*, ed. by Ulrich Müller and Peter Wapnewski, translation ed. by John Deathridge (Cambridge, MA: Harvard University Press, 1992), pp. 297–314 (p. 304); Anthony Newcomb, 'Those Images That Yet Fresh Images Beget', *Journal of Musicology*, 2 (1983), 227–45 (p. 233, n. 16); Thomas S. Grey, *Wagner's Musical Prose: Texts and Contexts* (Cambridge: Cambridge University Press, 1995), p. 90; and *Richard Wagner's Prose Works*, trans. by William Ashton Ellis, 8 vols (London: Kegan Paul, Trench, Trübner, 1892–99), VI: *Religion and Art* (1897), p. 183.

networks in recent music theory.[7] Introduced by David Lewin in his influential monograph *Generalized Musical Intervals and Transformations*, the concept of a transformation network aims to put the analyst '*inside* the music' by reconceptualising an 'interval' between two points as a 'characteristic gesture', or motion, from one to the other.[8] The focus therefore shifts from individual pitches, pitch-class sets, or consonant triads (for example) to the relational structures between them, which come to behave in quasi-motivic ways: to take one of Lewin's analyses as an example, he argues that the openings of the first and third movements of Beethoven's First Symphony realise the same underlying transformation network.[9] A related idea, reinvigorated in Lewin's work, is that of a spatial harmonic map or 'game board' (Lewin's phrase), the most well-known of which is the Neo-Riemannian *Tonnetz*: similar maps have proliferated since Lewin, a major recent development being Dmitri Tymoczko's argument that if we wish to use such maps to measure musical distance, then they need to be understood in terms of their geometry.[10] Networks are abstract sets of relationships, and while they are often represented "spatially" or visually, it can be misleading to read too much into the printed layout of a particular network (as anyone who has chosen to take three Tube trains, rather than the five-minute walk, to get from Warren Street to Great Portland Street will attest).[11]

While Lewin does construct several networks in which the nodes themselves are motives, these are often treated like pitch-class sets (i.e. with rhythm and note order playing secondary roles) and their scope can be seen to support Lewinian theorist Steven Rings's observation that '[t]ransformational theory is at its most powerful in the pluralistic exploration of phenomenologically rich local passages'.[12] The vignettes offered by Lewin and Rings,

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[7] Figures that look like motivic networks appear in David L. Montgomery, 'The Myth of Organicism: From Bad Science to Great Art', *Musical Quarterly*, 76 (1992), 17–66, but on closer inspection these are essentially paradigmatic charts (the problematic assumptions of which are discussed in Chapter 2) with superfluous arrows added.

[8] David Lewin, *Generalized Musical Intervals and Transformations* (New Haven: Yale University Press, 1987; rev. repr. with a foreword by Ed Gollin New York: Oxford University Press, 2007; repr. 2011), p. 159.

[9] Ibid., pp. 169–74.

[10] Ibid., p. 21. The literature is sizeable, but good overviews may be found in Richard Cohn, 'Introduction to Neo-Riemannian Theory: A Survey and a Historical Perspective', *Journal of Music Theory*, 42 (1998), 167–80 and *The Oxford Handbook of Neo-Riemannian Music Theories*, ed. by Edward Gollin and Alexander Rehding (New York: Oxford University Press, 2011). Tymoczko sets out his geometric argument concisely in Dmitri Tymoczko, 'Geometrical Methods in Recent Music Theory', *Music Theory Online*, 16.1 (2010) <http://www.mtosmt.org/issues/mto.10.16.1/mto.10.16.1.tymoczko.html> [accessed 17 December 2014]; see also Chapter 2, notes 78 and 82 below.

[11] See Bill Bryson, *Notes from a Small Island* (London: Doubleday, 1993; repr. 1999), p. 52 for more on the Tube map's geographical oddities.

[12] Steven Rings, *Tonality and Transformation* (New York: Oxford University Press, 2011), p. 38. An example of a motivic analysis by Lewin may be found in *Generalized Musical Intervals*: here he

sensitive and nuanced though they are, do not tend to concern themselves with the complex and comprehensive webs of motivic relationship that stretch across entire movements. My own master's dissertation began to address this lacuna, analysing Liszt's B minor Piano Sonata by grouping its melodic segments into a hierarchy of motives, types, and variants, and using these groupings as nodes in transformational networks.[13] The arrows between the nodes were labelled with well-defined functions, making my networks Lewinian in method but, in their static categorisations of each node's contents, not fully transformational in spirit. This thesis tilts that balance in the other direction, doing away with neat functions between abstracted categories in favour of contextually influenced derivation relationships that obtain directly between musical segments. This not only respects my argument that morphological identity of pitch and rhythm is neither a guarantee of, nor a pre-requisite for, motivic relationship (see Section 2.3, point six), but imbues the resultant networks with an increased dynamism and sense of temporal unfolding.[14]

The centre of gravity of this thesis is Chapter 3, which sets out a proposed mathematical network model for motivic analysis alongside a worked example (the first movement of Beethoven's Piano Sonata in F minor, Op. 2, No. 1).[15] The other chapters serve to underwrite this model, expanding on its methodological and theoretical underpinnings, clarifying what it does and does not purport to say, and suggesting how it might be analytically applied. These other chapters should not, however, be understood as simply prefatory or supplementary: the arguments and insights that they propose have wider ramifications for mathematical music theory, motivic analysis, and Mahler's Sixth Symphony respectively, to an extent thereby standing for themselves.

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uncovers retrograde-inverted forms of motives from *Parsifal* chained together in the opera's Schenkerian middleground (Figure 7.4, p. 162) and represents them in a transformation network (Figure 8.3, p. 181). For an empirical study which suggests that arrhythmic pitch-class set descriptions may be inadequate for defining piece-specific motives that are salient, distinctive, and significant, see David Huron, 'What is a Musical Feature? Forte's Analysis of Brahms's Opus 51, No. 1, Revisited', *Music Theory Online*, 7.4 (July 2001) <http://www.mtosmt.org/issues/mto.01.7.4/mto.01.7.4.huron.html> [accessed 17 December 2014]; the same Brahms quartet is also the subject of the papers in *Computational Music Analysis*, ed. by Christina Anagnostopoulou and Chantal Buteau (=*Journal of Mathematics and Music*, 4.2 (July 2010)).

[13] Daniel Holden, 'A Mathematical Model of Motivic Transformation' (unpublished master's dissertation, University of Leeds, 2011).

[14] Lewin was constantly aware of the tension between music's unfolding in time and its analytical representations in space: this tension becomes thematic in his analysis of Stockhausen's *Klavierstück III*, found in David Lewin, *Musical Form and Transformation: Four Analytic Essays* (New Haven: Yale University Press, 1993), pp. 16–67.

[15] A full segmented score of this movement is provided as Figure 3.22, and also as a PDF on the supplementary CD to allow side-by-side comparison with the discussion.

Chapter 1 addresses the question of what it means to make mathematical statements about music, seeking to disentangle mathematical music theory from the scientific pretensions read into it by some of its critics (as well as, it must be said, some of its proponents). The relationship between mathematics and science is not fixed and well-defined, but nor is that between science and music theory, a field which exhibits what Nicholas Cook has called '[e]pistemological slippage' in its desire to suggest new hearings of music as much as to describe existing ones.[16] Theory's ability to influence perception and observation, seen as scientifically problematic in the concepts of theory-ladenness, coherentism, and pseudo-science, becomes a positive asset in the humanities, where the goal of theory is typically to provide a flexible orienting framework – a 'good comparison', in Schoenberg and Cook's preferred phrase – rather than a predictive explanatory system.[17] In this context, mathematics can be understood as a language for the formulation of rich and detailed metaphors about music, these metaphors constituting a natural amplification of the creative and metaphorical nature of all mathematical modelling.

One of Chapter 1's important arguments, advanced in Section 1.5 especially, is that it can be a mistake to replace or ignore traditional music theory: not only, within a humanities paradigm, are its insights valid as alternative lenses through which to view a complex musical phenomenon, but existing music theory can take on a mediating role in influencing how composers, performers, and listeners experience music. Many mathematical models of music are therefore better characterised as models of music theories, and Chapter 2 examines the concept of a motive in detail to make the considerations underwriting Chapter 3's model more explicit. Departing from a more detailed exploration of the paradox of repetition outlined above (Section 2.1), four archetypal conceptual models of motivic structure are presented in Section 2.2 and shown to underpin, in various combinations, four important motivic theories. If the archetypal models represent alternative resolutions of the tension between static/category-based and dynamic/process-based understandings, then Sections 2.3 and 2.4 explore these understandings' inherent "internal" problems (and opportunities) in detail; Section 2.3 in particular identifies six major barriers to formal motivic modelling, contributing to an explanation of mathematical music theory's tendency to avoid contextually defined motives in favour of abstracted universes of chords and scales. The final section of the chapter reintroduces the concept of a network – or graph, in a mathematical sense distinct from that of

[16] Nicholas Cook, 'Epistemologies of Music Theory', in *The Cambridge History of Western Music Theory*, ed. by Christensen, pp. 78–105 (p. 102).
[17] See Arnold Schoenberg, *Theory of Harmony*, trans. by Roy E. Carter (London: Faber and Faber, 1978), p. 11 and Nicholas Cook, 'Music Theory and "Good Comparison": A Viennese Perspective', *Journal of Music Theory*, 33 (1989), 117–41.

an $x - y$ plot – to provide a pivot between the thematic overview of music-theoretical literature presented in Chapter 2 and the formalised model presented in Chapter 3.

Chapter 3 can be understood as two parallel chapters arranged in three alternating blocks. The first chapter explains the model's main concepts informally and illustrates them with references to the Beethoven piece where possible, while the second defines the same concepts in a more formal mathematical language. Cross-references allow the second to be read as a summary of the ideas arising in the first and an effort to make them more precise, generalisable, and rigorous; alternatively, they allow the first to be read as exegesis and exemplification of the informationally dense mathematical language found in the second. The decision to separate out the mathematical content was taken in order to stop the main thrust of the argument getting clogged with potentially off-putting technical detail, though even those readers with little mathematical background are invited to follow the cross-references and, in doing so, to glean a more rounded picture of the model being presented. (A similar invitation applies to analytical comments: references to bar numbers and labelled figures are provided, and while the general contours of the argument can be traced without following these up, a more concrete picture emerges when they are).[18] Graph theory is a relatively self-contained mathematical field, so most of Chapter 3's mathematical content should be understandable through reference to the glossary of standard mathematical symbols on p. xi, the fundamental concepts presented at the end of Section 2.5, and the terms defined as the chapter proceeds. As well as the definitions, theorems, and proofs, the mathematical sections contain further figures and prose explanations relating to the mathematical content.

The final chapter uses the ideas developed in the rest of the thesis to construct analyses of the first and last movements of Mahler's Sixth Symphony. Mahler's motivic process has been frequently discussed – John Williamson considers it 'widely accepted' that his motivic technique 'is different in kind from that of his most distinguished predecessors' – and a central theme in this reception history has been the idea that his motives are defined as pluralities.[19] Theodor Adorno, for example, argues that they 'are too independent, too evidently living beings in process, to […] sink their identity in a *seamless* web' (my italics); Erwin Stein that they are not 'the kind of unalterable ideas which make suitable music examples for programme notes'; Paul

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[18] I would recommend, in particular, reading Chapter 4 alongside a score of Mahler's Sixth Symphony (the reductions presented in Seth Monahan's thesis are excellent for this purpose: see Seth Monahan, 'Mahler's Sonata Narratives', 2 vols (unpublished doctoral dissertation, Yale University, 2008), II: *Supplements* (pp. 303–532), pp. 386–413, 490–532). The Mahler analyses repeat the process of the Beethoven analysis on a much larger scale: a firm grasp of the manageable detail of Chapter 3 therefore helps when reading Chapter 4.

[19] John Williamson, 'New Research Paths in Criticism, Analysis and Interpretation', in *The Cambridge Companion to Mahler*, ed. by Jeremy Barham (Cambridge: Cambridge University Press, 2007), pp. 262–74 (p. 269).

Bekker that Mahler's 'new kind of thematic form […] no longer knew of incessant striving from a creative spiritual centre but, on the contrary, had first to gather strength in the multiplicity of its appearances'; and Agawu that 'in a complex work like the first movement of Mahler's Ninth, giving a single label – as opposed to a multitude of labels in a network formation – may seem to do violence to thematic interconnectedness and the numerous allusions that constitute its thematic fabric'.[20] Mahler himself, according to Josef Foerster, thought in terms of themes 'already embellished, developed, and in many ways linked to secondary thematic permutations'.[21]

Yet, as always with Mahler, the true picture is more complex. He saw himself (at times) as a Beethovenian organicist: he told Natalie Bauer-Lechner that 'music is governed by the law of eternal evolution, eternal development', conceived in 'in one grand sweep'; he also told Anton von Webern, in comments reminiscent of Schoenberg's viewpoint discussed in Chapter 2, that '[j]ust as in nature the entire universe has developed from the primeval cell, […] so also in music should a larger structure develop from a single motive in which is contained the germ of everything that is yet to be'.[22] The Schoenbergian *Grundgestalt* (see Section 2.2) appears to lurk, too, behind his injunctions to Max Marschalk: '*Themes* – these must be clear and *plastic*, so that they can be clearly recognized at any stage of modification or development – and then *varied* presentation, holding the attention above all through the logical *development* of the inner

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[20] Theodor W. Adorno, 'Mahler: Centenary Address, Vienna 1960 and Afterthoughts, 1961', in *Quasi una Fantasia: Essays on Modern Music*, trans. by Rodney Livingstone (London: Verso, 1992; repr. 1994), pp. 81–110 (pp. 94–95); Erwin Stein, 'Mahler the Factual', trans. by Hans Keller, in *Orpheus in New Guises* (London: Rockliff, 1953), pp. 15–18, p. 17; Paul Bekker, *Gustav Mahlers Sinfonien* (Berlin: [Schuster & Loeffler], 1921), pp. 17–18, quoted in Theodor W. Adorno, *Mahler: A Musical Physiognomy*, trans. by Edmund Jephcott (Chicago: University of Chicago Press, 1992; repr. 1996), p. 65; Agawu, *Music as Discourse*, p. 271. On a personal note, the kernel of inspiration for this research was an undergraduate analysis of the first movement of the Second Symphony: traditional letter-name designations seemed inadequate when faced not only with the sheer number of motivic ideas, but also with the comprehensive web of inter-relationship binding them all together.

[21] Josef Bohuslav Foerster, *Der Pilger: Erinnerungen eines Musikers*, trans. by Pavel Eisner (Prague: [Artia], 1955), p. 356, quoted (and trans.) in Stephen E. Hefling, '"*Ihm in die Lieder zu blicken*": Mahler's Seventh Symphony Sketchbook', in *Mahler Studies*, ed. by Stephen E. Hefling (Cambridge: Cambridge University Press, 1997), pp. 169–216 (p. 214).

[22] Natalie Bauer-Lechner, *Recollections of Gustav Mahler*, trans. by Dika Newlin, ed. by Peter Franklin (London: Faber Music, 1980), pp. 130, 29; Webern's diary, 3 February 1905, quoted in Hans Moldenhauer, with Rosaleen Moldenhauer, *Anton von Webern: A Chronicle of His Life and Work* (London: Gollancz, 1978), p. 75. Williamson, arguing that Mahler's themes do not behave like 'cells in a unified organism', concedes that this 'may, however, have been how Mahler saw them himself' (John Williamson, 'Mahler, Hermeneutics and Analysis', *Music Analysis*, 10 (1991), 357–73 (p. 360)).

idea, but also by the *genuine opposition* of contrasting motives.'[23] Plurality creeps in as he discusses the 'inexhaustible wealth of variations' that this process produces: while he rails against the practice of 'playing around with some poor little scrap of a theme', at other times he is reported to have 'filled a pile of sketch-sheets and his pocket music notebooks with a hundred variants of a motif or a modulation', considering it 'a superhuman labour and waste of energy […] to have to create everything you need on the spot, without provision in advance, or any "collection"'.[24] The result – whether understood organically, pluralistically, compositionally, or in terms of an extensive field ('network', again, for Agawu) of quotation, self-quotation, and allusion – is a complex motivic language ripe for analytical discussion.[25] The interconnections and allusions that blur motives into each other and into other movements, possibly by other composers, must be treated with care lest, as Laura Hedden warns, '[m]otives that resemble one another in shape, but not in function [are] mistakenly […] grouped under the same semantic interpretation': Mahler considered the 'profound logic' of motivic connection to be secondary to an overriding need to 'embrace [the world]' in his famous conversation with Sibelius.[26]

The tension between motivic homogeneity and affective delineation is nowhere more apparent than in the Sixth Symphony, making this work an ideal testing ground for a model that privileges heterogeneously constructed parallels between specific musical passages over crisp morphological categorisations. The Sixth Symphony has also been chosen for another reason, one that would at first seem to place it at the back of the queue: it is one of Mahler's most frequently discussed symphonies, and its first and last movements in particular have received recent detailed analytical attention from Seth Monahan in three articles, two thesis chapters, and

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[23] Letter to Max Marschalk, 12 April 1896, in *Selected Letters of Gustav Mahler*, ed. by Alma Mahler and Knud Martner, trans. by Eithne Wilkins and Ernst Kaiser with Bill Hopkins (London: Faber and Faber, 1979), p. 182 (no. 160). Analyses of the first movements of Mahler's Ninth and Tenth symphonies from explicitly Schoenbergian perspectives may be found in Jonathan Mann, '"Nothing but the Theme Itself": Developing Variation in the Andante Comodo of Symphony No. 9, by Gustav Mahler' (unpublished master's thesis, Wayne State University, 2008) and Jack Boss, 'Mahler's Musical Idea: A Schenkerian-Schoenbergian Analysis of the Adagio from Symphony No. 10', in *Analyzing the Music of Living Composers (and Others)*, ed. by Jack Boss and others (Newcastle upon Tyne: Cambridge Scholars Publishing, 2013), pp. 115–31.
[24] Bauer-Lechner, p. 29 'inexhaustible', 'scrap'; p. 61 'filled'; p. 170 'superhuman'.
[25] Agawu, *Music as Discourse*, p. 47. For a comprehensive study of quotation in Mahler, see Henry-Louis de La Grange, 'Music About Music in Mahler: Reminiscences, Allusions, or Quotations?', in *Mahler Studies*, ed. by Hefling, pp. 122–68.
[26] Laura Hedden, 'The Motivic Code: Defining an Element of Mahler's Style' (unpublished doctoral dissertation, Princeton University, 2009), p. 28; Karl Ekman, *Jean Sibelius: His Life and Personality*, trans. by Edward Birse with a foreword by Ernest Newman (New York: Knopf, 1938; repr. New York: Tudor, 1946), p. 191.

two chapters of his forthcoming monograph.[27] Any analysis is necessarily a re-imagining, a performative act of individual engagement (as argued in Section 1.3), and as such a given work will always bear the weight of more interpretation. But further to this, Chapter 4 hopes to show that dialogue with existing analyses can be a strength rather than an obligation, inflecting certain insights, challenging others, and taking yet others as routes into a motivic labyrinth that can lead to a number of different endpoints (Adorno argues that Mahlerian themes are always provisional: 'the listener should not cling to themes, but instead should propose them to himself, and await events').[28] Monahan actively invites others to use his work in this way: 'I would be pleased if this study served as the starting point for any extended dialogue that brings us into a closer engagement with [Mahler's] music. With Mahler, there is always so much more to say.'[29]

The tools developed in this thesis can be understood similarly: they prompt, suggest, facilitate, offset, and nuance insights, but do not determine or circumscribe them, and are not always insightful in and of themselves. Chapter 4 is the result of an interaction between the score of Mahler's Sixth Symphony, the scholarship surrounding it, the tools developed in this thesis, and the mind of an analyst, with the aim of demonstrating the kind of insights that such an interaction can produce. In the spirit of Monahan's dialogue, I have included all the data used in my analyses as Microsoft Excel spreadsheets on a supplementary CD, and I discuss in Section 4.3 how these data relate to the eventual analyses as presented (portions of the Beethoven data appear in Chapter 3, but the Mahler data are so numerous that to reproduce them in the thesis itself would be impractical, overwhelming, and even tangential). Each spreadsheet includes the macro that implements the formal ideas of Chapter 3, and so the interested reader can track the structural consequences of competing interpretations by making small changes to my judgements and running the macro again; a blank spreadsheet is also included to be used as a template for new analyses.

The thesis thereby ends, in the Afterword and on the supplementary CD, near to where it begins: with Section 1.3's portrait of analysis as an open invitation rather than a closed answer. The material in between – just as with a pair of motivic segments – affects not only the understanding of the second, but, retrospectively, the memory of the first.

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[27] See Seth Monahan, '"Inescapable" Coherence and the Failure of the Novel-Symphony in the Finale of Mahler's Sixth', *Nineteenth-Century Music*, 31 (2007–08), 53–95; Seth Monahan, '"I Have Tried to Capture You …": Rethinking the "Alma" Theme from Mahler's Sixth Symphony', *Journal of the American Musicological Society*, 64 (2011), 119–78; Seth Monahan, 'Success and Failure in Mahler's Sonata Recapitulations', *Music Theory Spectrum*, 33 (2011), 37–58 (pp. 51–54); Monahan, 'Narratives', I: *Text* (pp. i–302), pp. 104–49, 240–88 and II, 310–28, 367–83; and Seth Monahan, *Mahler's Symphonic Sonatas* ([New York]: Oxford University Press, forthcoming).
[28] 'Centenary Address', p. 100.
[29] '"Inescapable" Coherence', p. 94.

1 Mathematics and Music Theory: An Unscientific Defence

1.1 Introduction: Oppositions and Conflations

In a recent article decrying 'creationist' music theory (that which reifies musical documents as 'God's creations' rather than 'creations of God's creatures'), Richard Taruskin ends by singling out two exemplary theorists who are 'becoming aware of the pitfalls of creationism and acting on that awareness'.[1] The theorists are Robert Gjerdingen and Lawrence Zbikowski, and Taruskin sees it as 'no accident, of course, that both Gjerdingen and Zbikowski have taken inspiration […] from the work of Leonard B. Meyer (1918–2007), the great pragmatic exception among the founders of American music theory'. Meyer, according to Taruskin, 'never lost sight of real worlds' in his work, chiefly because he 'based his music-theoretical ideas on messy psychological premises rather than tidy mathematical systems'.

The general criticisms of mainstream American music theory evoked by Taruskin are nothing new, either in his own work or in musicology more generally.[2] Having their *locus classicus* in Joseph Kerman's programmatic injunctions to 'get out' of formalism in the 1980s, such criticisms are now orthodox and have done much (although, Taruskin would argue, not enough) to bring about a greater degree of methodological self-awareness amongst music theorists.[3] What is

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[1] Richard Taruskin, 'Catching Up with Rimsky-Korsakov', *Music Theory Spectrum*, 33 (2011), 169–85 (p. 181 'becoming aware'; p. 180 other quotes in this sentence; p. 182 other quotes in this paragraph). The passage about 'God's creations' also appears, in a slightly altered form, in Richard Taruskin, 'Reply to Brown and Dempster', *Journal of Music Theory*, 33 (1989), 155–64 (p. 162); see next note.

[2] Kofi Agawu, 'Taruskin's Problem(s)', *Music Theory Spectrum*, 33 (2011), 186–90 (pp. 186–87) tracks how Taruskin has 'repeat[ed] himself […] loudly and unapologetically' (p. 187) in articles spanning thirty-two years.

[3] See Joseph Kerman, 'How We Got Into Analysis, and How to Get Out', *Critical Inquiry*, 7 (1980–81), 311–31 and Joseph Kerman, *Musicology* (London: Fontana Press, 1985), especially chapter 3 (pp. 60–112). We must be careful, however, of narrativising a generational shift away from "scientific" music theory and towards "poetic" music theory, a view which David Lewin claims 'is not exactly wrong, but [which] is very far from adequate to engage the critical issues at hand, issues which it hopelessly trivializes' (see David Lewin, 'Music Theory, Phenomenology, and Modes of Perception', in *Studies in Music with Text* (New York: Oxford University Press, 2006; repr. 2009), pp. 53–108 (first publ. in *Music Perception*, 3 (1985–86), 327–92) (p. 102); further references cite the version in *Studies in Music with Text*). In a 1997 article, Agawu cites a number of music-theoretical writings from 1983–94 that already explicitly engage issues characteristic of the so-called "New Musicology", including phenomenology, hermeneutics, symbolism and semiotics; questions of unity, narrative, and meaning; the contingency and constructedness of analyses; and the work of Jacques Derrida, Roland Barthes, and Harold Bloom. See Kofi Agawu, 'Analyzing Music under the New Musicological Regime', *Journal of Musicology*, 15 (1997), 297–307.

particularly interesting about Taruskin's latest critique is how he associates 'tidy mathematical systems' with what he terms musical creationism, and Meyer's 'messy psychological premises' with what he terms musical evolutionism: he seems to be suggesting that, in the domain of music theory at least, mathematical and scientific modes of thinking are antithetical.

Similarly arresting juxtapositions are presented in Cook's 2002 article 'Epistemologies of Music Theory'. In tracing the history of music theory '[b]etween art and nature' (p. 85), he associates Milton Babbitt's calls for a more "scientific" music theory in the 1960s with a turn away from the idea that music is firmly rooted in nature and therefore natural science: partly for Babbitt to legitimise his own music by recasting consonance as a culturally conditioned (rather than innate) phenomenon, and partly as a reaction against the Nazi 'rhetoric of natural origins and the drawing from them of universal and unchangeable criteria of value' (p. 90). In this climate, Cook labels as 'striking' not only that the idea of performativity (see Section 1.3 below) entered the theoretical mainstream thanks to Lewin, 'the leading contemporary exponent of a formalized approach to music that would appear, more than any other, to embody the strictly scientific epistemology that Babbitt adumbrated in 1961' (p. 97), but also that the associated pragmatic emphasis on the importance of 'useful, useable, relevant, or significant characterizations' was called for by Babbitt himself in 1965.[4]

These examples highlight a series of oppositions (science and mathematics, psychology and music theory, pragmatism and "mainstream" music theory, natural science and scientific music theory, performativity and formalism, formalism and musical relevance) and conflations (science and formalism, mathematics and mainstream theory) that appear by turns familiar, naïve, or self-contradictory, and imply a rift between music theory and "real" science (psychology for Taruskin and natural science for Cook). Part of the problem is that several of these terms have taken on special shadings in relation to music theory, and these can be traced back to Babbitt's attempt to

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[4] This chapter cites three important methodological papers by Babbitt, all available in (and cited from) *The Collected Essays of Milton Babbitt*, ed. by Stephen Peles and others (Princeton: Princeton University Press, 2003). They are (with page numbers from *The Collected Essays* and original publication details):

- 'Past and Present Concepts of the Nature and Limits of Music', pp. 78–85 (first publ. in *International Musicological Society: Report of the Eighth Congress, New York, 1961*, [ed. by Jan LaRue, 2 vols (Kassel: Bärenreiter, 1961–62), I: Papers,] pp. 398–403);
- 'The Structure and Function of Musical Theory', pp. 191–201 (first publ. in *College Music Symposium*, 5 (1965), 49–60);
- 'Contemporary Music Composition and Music Theory as Contemporary Intellectual History', pp. 270–307 (first publ. in *Perspectives in Musicology: The Inaugural Lectures of the Ph.D. Program in Music at the City University of New York, ed. Barry S. Brook, Edward O. Downes, and Sherman van Solkema* (New York: Norton, 1972), pp. 151–84).

The first two have been reprinted in *Perspectives on Contemporary Music Theory*, ed. by Benjamin Boretz and Edward T. Cone (New York: Norton, 1972), as pp. 3–9 and 10–21 respectively. For 'useful, useable', see Babbitt, 'Structure and Function', p. 194, quoted in Cook, 'Epistemologies', p. 97.

bind the discipline into the unusually tight weave of science and mathematics proposed by the philosophy of logical positivism (explored in more detail in Section 1.2). It is, however, problematic to assume too much about the relationship between mathematics and science: even outside the domain of music theory, this area is one of intense enquiry in the contemporary philosophy of mathematics.[5] For while mathematics is popularly referred to as "the language of science" or even (by Gauss) 'the queen of the sciences', in Mark Colyvan's phrase 'she would seem to be an eccentric and obstinate queen'.[6] Mathematics deals with abstract rather than physical entities, proceeds *a priori* by rational deduction rather than *a posteriori* by empirical induction, and deals in provable theorems rather than revisable theories. Even so, the so-called 'indispensability argument' – that our best scientific theories cannot be stated without mathematics, thus entailing an ontological commitment to mathematical objects – retains a persuasive force and gives pause to anyone seeking to link physical phenomenon, scientific theory, and mathematical model in a neatly demarcated chain of cause and effect.[7]

This chapter unfolds an argument, fully realised in Section 1.10, that mathematical models are essentially rich and detailed metaphors which can therefore fulfil the suggestive, performative ends of music theory (as set out in Section 1.3) without presupposing a scientific epistemology. Such an argument requires careful consideration of mathematics' role in science and, especially, science's role in music theory: while Section 1.4 identifies a decisive incompatibility of aims in respective attitudes toward overturning sedimented knowledge, Section 1.5 problematises this in terms of both music theory's mediating role in psychological experiments, and science's capacity to enrich the descriptive capabilities of music theory. These issues are reframed in relation to the 'two cultures' of the sciences and the humanities in Section 1.6, and the expectations that each culture places on a 'theory' – in particular, of the balance between conceptual elegance and goodness-of-fit – are examined in more detail in Sections 1.8 and 1.9. Under this sharpened understanding of what it means to make scientific claims about music, mathematics is free, after a methodological

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[5] A recent introduction to the field devotes four of its seven central chapters to this area, and the issues raised permeate much of the rest of the book: see Mark Colyvan, *An Introduction to the Philosophy of Mathematics*, Cambridge Introductions to Philosophy (Cambridge: Cambridge University Press, 2012), pp. 36–117.

[6] Colyvan, p. 1. Gauss's phrase was (reportedly) 'die Königin die Wissenschaften' (see Wolfgang Sartorius von Waltershausen, *Gauss zum Gedächtniss* (Leipzig: Hirzel, 1856; repr. Wiesbaden: Sändig, 1965), p. 79): but as Geraint Wiggins points out, while *Wissenschaft* is normally translated as 'science', it actually means something closer to 'scholarship' (see Geraint A. Wiggins, 'Response to Marsden and Mazzola: On the Correctness of Imprecision and the Existential Fallacy of Absolute Music', in *3MR*, pp. 93–101 (pp. 93–94)). The next sentence in the Gauss source explicitly makes reference to *Naturwissenschaften* (natural sciences), while the previous holder of the title 'queen of the sciences' was theology, further advising caution against a literal interpretation of the standard Gauss translation.

[7] See Colyvan, especially pp. 41–51, for an outline of the indispensability argument and its main criticisms.

discursion into how we should understand mathematical definitions in Section 1.7, to emerge from the chimerical mode of music theory often referred to as 'formalism'. This role for mathematics – in particular as set into relief against science – is what drives and underpins this thesis as a whole.

1.2 A "Scientific" Music Theory

The delicacy of the relationship between mathematics and science is nowhere more apparent than in the philosophy of logical positivism. David Huron has argued that positivism, despite being 'the preeminent target of postmodernist critiques', actually only exercised an influence 'almost exclusively restricted to American psychology' between 1930 and 1965.[8] It was, however, a central component in Babbitt's methodological call for a more "scientific" music theory:

> [Q]uestions of musical theory construction attend and include all matters of the form, the manner of formulation, and the signification of statements about individual musical compositions, and the subsumption of such statements into a higher-level theory, constructed purely logically from the empirical acts of examination of the individual compositions.[9]

Although he never labels his philosophy explicitly, there are clear echoes of positivism in Babbitt's writing: he frequently cites, and borrows conceptual terminology from, philosophers associated with positivism such as Rudolf Carnap, Carl Hempel, Hans Reichenbach, Gustav Bergmann, W. V. O. Quine, and Paul Oppenheim. In particular, Babbitt endorses the characteristic positivistic principle of *verificationism*: that *observational* concepts carry empiric content and thus are distinct from *theoretical* or *logical* concepts, which must nevertheless be empirically verifiable through the observational statements that they imply.[10] Babbitt gives 'a simple and yet immensely practical and powerful music-theoretical example' of this distinction by contrasting an interval number (the difference between two pitches) with what he calls an index number (the sum of two pitches).[11] The former is 'a direct musical perceptible' (p. 298), audible and identifiable even when transposed. The latter, conversely, is 'theoretical' (p. 299): the musical meaning of D+E=F+C♯, the exact value of which is dependent on the choice of pitch-class 0, is unclear. It turns out that just as intervals relate to transposition, index numbers relate to inversion (p. 299): if there are n occurrences of

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[8] David Huron, 'The New Empiricism: Systematic Musicology in a Postmodern Age' (lecture 3 of the Ernest Bloch Lectures, University of California, Berkeley, 1999), available at <http://www.musiccog.ohio-state.edu/Music220/Bloch.lectures/3.Methodology.html> [accessed 17 December 2014] (paras. 24 and 25).

[9] Babbitt, 'Structure and Function', p. 191.

[10] Verificationism is a problematic concept and as such has seen a number of definitions; mine draws on A. J. Ayer's and Carnap's (adding the term 'theoretical' from Babbitt – see following example) as summarised in Richard Creath, 'Logical Empiricism', in *The Stanford Encyclopedia of Philosophy*, ed. by Edward N. Zalta, Spring 2014 edn <http://plato.stanford.edu/archives/spr2014/entries/logical-empiricism/> [accessed 17 December 2014] (section 4.1).

[11] 'Contemporary Music Composition', p. 298 (further citations follow in parentheses in the text); see also 'Structure and Function', pp. 196–97.

interval a in pitch-class set X, then X shares n of its pitch-classes with the transposition of X by a; similarly, n occurrences of sum a in set X entails $2n$ shared pitch-classes between X and its inversion about $a/2$.[12] Index numbers therefore have a variety of compositional and analytical consequences: this, for Babbitt, constitutes their empirical verification.

Two observations about this philosophy may be noted: first, that a call for positivism is not the same thing as a call for scientific method; and second, that empirical "verification" (broadly construed) is a necessary foundation for formal theorising. Babbitt himself underlines the first observation when he answers the hypothetical charge that music is not a science by asserting that '[t]his, naturally, is not the point, not even a point'.[13] The point, as he argues in a famous passage, is terminological rigour:

> [T]here is but one kind of language, one kind of method for the verbal formulation of "concepts" and the verbal analysis of such formulations: "scientific" language and "scientific" method. […] Our concern is not whether music has been, is, can be, will be, or should be a "science," whatever that may be assumed to mean, but simply that statements about music must conform to those verbal and methodological requirements which attend the possibility of meaningful discourse in any domain.[14]

The scare quotes around "scientific" resound throughout his work, as do calls for 'an adequately reconstructed terminology' satisfying 'the shared standards of rational discourse'.[15] Unless a concept can be defined in terms of musical observables, Babbitt argues, intersubjective understanding breaks down to be replaced by a set of personal opinions that can be neither corroborated nor refuted, thereby curtailing discussion and paralysing musical scholarship. At a time when music theory was trying to establish itself as an intellectual discipline in the American academy, such paralysis was potentially fatal, as Babbitt was acutely aware. He worries that 'musical theory is not a theory in any sense in which the term ever has been employed', that there is a possibility of 'student[s] of contemporary philosophy and science [dismissing] the theory and – therefore, probably – the music, as immature and irresponsible', that those 'of predominantly literary orientation [may] transplant mistakenly the prevalent verbiage of that domain to our, at least, more modest area of activity', and that non-specialist writers on music are frequently

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[12] There are two slight modifications to these rules that have been omitted from the above formulation for the sake of brevity. The number of common pitch-classes under tritone transposition is actually double the number of tritones in the set (since two pitch-classes a tritone apart map onto each other when transposed by a tritone); similarly, since each pitch-class in a pair that sums to a maps onto the other under inversion about $a/2$, the count of common pitch-classes increases by one if the set contains $a/2$ itself.

[13] 'Structure and Function', p. 192.

[14] Babbitt, 'Past and Present Concepts', p. 78. Huron sees positivism's main methodological legacy today as the fact that it 'drew attention to the issue of language and meaning in scientific discourse' ('The New Empiricism', para. 24).

[15] 'Structure and Function', p. 191; 'Contemporary Music Composition', p. 294.

misinformed partly due to specialists' failure to use terminology clearly.[16] His concerns are echoed by Gjerdingen (one of Taruskin's evolutionists) nearly thirty years later when he argues that music theory's 'premisses of discourse' should be open to 'translation into domains where inexactitude is never mistaken for subtlety'.[17]

Clarity of definition is therefore a central component of Babbitt's '"scientific" language', but it is not its only component. If a good definition fixes an observational concept, it can then be used as the basis of a logical system of theoretical concepts which reduces to 'a connected set of axioms, definitions, and theorems, the proofs of which are derived by means of an appropriate logic […] when uninterpreted predicates and operations are substituted for the terms and operations designating musical observables'.[18] Babbitt is suggesting that any work we require a scientific theory to do (such as generalisation, explanation, and prediction) can be accomplished by pouring appropriate definitions into a mathematical theory (that is, a logically connected set of ideas – see Section 1.10), ensuring via the deductive certainty of mathematics that all statements retain their moorings in empirical observation. Although this separation of theory and definition has been critiqued (since definitions are theory-laden – see Section 1.4 – and tied up in the deductive process – see Section 1.7), Babbitt's unusually tight relationship between scientific and mathematical methods has coloured the terms' implications in the context of music theory ever since.

As Marion Guck has convincingly argued, emphasis on Babbitt's '"scientific" language' (and on formal mathematics in particular) has overshadowed his complementary call for '"scientific" method'.[19] This is not a scientific method of rigorous data analysis and falsification, but a broader interpretation of empiricism (as reliance on sense experience) that can be seen in the verificationist insistence that all concepts are either directly observable or capable of implying observational statements. Versions of this requirement can be seen at work in a variety of contexts as a means of justifying abstraction: Matthew Brown and Douglas Dempster suggest that everything in a music theory should 'contribute to an explanation of musical events that are hearable' but need not itself be hearable; Cook proposes that 'no analytical model should be more abstract than it has to be in order to communicate the analyst's interpretation of a work'; and

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[16] 'Structure and Function', p. 193 'employed'; p. 200 'student[s]', 'literary'; see also 'Contemporary Music Composition', pp. 270–74. For more on music theory's entry into the academy see Patrick McCreless, 'Contemporary Music Theory and the New Musicology: An Introduction', *Journal of Musicology*, 15 (1997), 291–96 (pp. 292–93), and for more on the requirements of the label 'theory' see Section 1.8 below.
[17] Robert Gjerdingen, 'An Experimental Music Theory?', in *Rethinking Music*, ed. by Nicholas Cook and Mark Everist (Oxford: Oxford University Press, 1999; repr. 2001), pp. 161–70 (p. 169).
[18] Babbitt, 'Past and Present Concepts', p. 79.
[19] Marion A. Guck, 'Rehabilitating the Incorrigible', in *Theory, Analysis and Meaning in Music*, ed. by Anthony Pople (Cambridge: Cambridge University Press, 1994), pp. 57–73.

Colyvan argues that the indispensability argument still applies to highly abstract mathematics '[s]o long as the chain of applications eventually bottoms out in physical science'.[20] What is interesting about Babbitt's brand of verification is that it foregrounds music theory's status as a performative, an utterance that is itself an action rather than a representation: for Babbitt, this makes theory-led composition the apotheosis of music theory. It is no coincidence that 'useful' and 'useable' come first in the list of desirable features that so intrigued Cook (cited in Section 1.1). In his discussion of index numbers, Babbitt sees the assignment of observational content to a theoretical concept as a creative act – 'What *could be* meant or designated by a sum of pitch numbers?' (italics added) – and then justifies the utility of a property of certain twelve-tone sets in what would appear to be strikingly circular terms:

> This property is explanatory in that it explains the compositional use of such related sets, and – by extension – suggests more general applications […]. This property also functions as predictive in determining possible attributes of future works concerned with exposing this property.[21]

1.3 Suggestive Description and Performative Perception

The "scientific" and performative aspects of Babbitt's theory appear to point in different directions, the former resting on the problematic notion of replicable and direct musical observables, the latter almost glorying in the arbitrariness and, therefore, creative potential of musical theory-building. Underlying this superficial tension are deeper questions concerning purpose and epistemology: that is, the kind of knowledge that music theory and analysis concern themselves with. These are difficult questions to address in general, not only because a wide variety of individualised practices fall under the rubric of 'music theory', but also because those practices in themselves exhibit Cook's '[e]pistemological slippage': each work of music theory 'says how things are, it suggests how you might hear things, it recaptures historical conceptions, and each register merges imperceptibly into the next'.[22] Cook's prime example is Fred Lerdahl and Ray Jackendoff's *Generative Theory of Tonal Music*, which draws on Heinrich Schenker's quasi-metaphysical music theory, the same theory's American post-war formalisation, Meyer and Cooper's work on rhythm (influenced by Gestalt psychology), and structural linguistics (a partially empirical discipline). Lerdahl's further work adds in a spatial mathematical model which overlaps both with Schoenberg's harmonic theory (itself drawing on work by Arthur von Oettingen and Hugo Riemann) and Carol Krumhansl's empirical

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[20] Matthew Brown and Douglas J. Dempster, 'The Scientific Image of Music Theory', *Journal of Music Theory*, 33 (1989), 65–106 (p. 96); Cook, 'Music Theory and "Good Comparison"', p. 140; Colyvan, p. 44.
[21] 'Structure and Function', pp. 196, 199.
[22] 'Epistemologies', p. 102; the rest of the present paragraph summarises pp. 99–102, where full references to the numerous works mentioned in passing here may be found. For the theory at the centre of Cook's discussion, see Fred Lerdahl and Ray Jackendoff, *A Generative Theory of Tonal Music* (Cambridge, MA: MIT Press, 1983; repr. 1985); see also Chapter 2, n. 92.

experiments on the perception of pitch relations. The resultant writing veers between the scientific and the literary, and also finds a performative outlet in Lerdahl's work as a composer.

Broadly speaking, we might follow David Temperley in recognising two basic modes of music theory: the *descriptive* mode, which 'attempts to describe listeners' unconscious mental representations of music', and the *suggestive* mode, which aims 'to find and present new ways of hearing pieces'.[23] Each, he argues, is valid on its own terms, and a given theory may differ in purpose across different structural levels, for different people, and at different times: a Schenkerian analysis might describe listeners' experiences of local directed motion whilst also suggesting a binding *Ursatz* structure, for instance (p. 75), and a theory intended to be descriptive might turn out to be suggestive if its 'assumption of perceptual uniformity is false' (p. 78). Temperley argues that 'remaining noncommittal between them, in the hope that something like this might happen' is a methodological problem since we need to be 'clear in our own minds about what we are claiming' (p. 78); this is convincing insofar as a given theory cannot pretend to be describing experiences at the same time as it seeks to impart new ones. However, there are dangers in both unchecked description (stating the obvious, making problematic truth claims about how the music "is", or too readily attributing a particular technical "cause" to a supposed experiential "effect") and unchecked suggestion (prioritising theories over pieces of music via anything-goes "formalism").

Most music theorists seek to retain both modes in their writing and, in doing so, endorse a kind of modified verificationism: analysis must latch onto description but ultimately go further in what it proposes, all without straying from the realms of plausibility. This is a delicate balance, with Cook noting that 'the perceptual reality of what the music theorist is saying seems to become parlous just at the point that it becomes of the greatest aesthetic interest'.[24] Consider the following analytical maxims:

> **Agawu**: An analysis works best if it proceeds in a two-way stream, making explicit some of the listener's intuitions (a confirmatory function) while at the same time firing the imagination that contemplates the music (an exploratory function).
>
> **Guck**: If a description is *evocative*, this constitutes strong empirical evidence in its favour.
>
> **Schoenberg**: Efforts to discover laws of art can then, at best, produce results something like those of a good comparison […]. In making a comparison we

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[23] David Temperley, 'The Question of Purpose in Music Theory: Description, Suggestion, and Explanation', *Current Musicology*, 66 (1999), 66–85 (pp. 68, 70); further page references to this article follow in parentheses in the text. 'Description' is a broad category: as well as encompassing personalised analysis ('I hear…'), which may or may not assume shared perceptions with the reader ('…is heard as…'), it can also include perceptual or cognitive studies of music, or speculation regarding compositional processes. It can even describe music in a way that avoids the composer and listener altogether, and hence buy into a form of musical creationism.
[24] 'Music Theory and "Good Comparison"', p. 120.

> bring closer what is too distant, thereby enlarging details, and remove to some distance what is too close, thereby gaining perspective.
>
> **Cook**: [I]nterpretation means transforming potential meaning ['tensional or energetic qualities' present in the music] into actualized meaning ['that depends on words for its formulation and communication'] […] It is for this reason that there is a kind of sleight of hand in the impression [writers] give of simply describing how the music is, when in reality they are in the business of proposing interpretations and so constructing actualized meaning.
>
> **Babbitt**: [I]n addition to the crucial point of susceptibility to a perceptually feasible interpretation, the potential fruitfulness of [a music theory] is obviously contingent upon its degree of avoidance of "triviality" both formally and interpretively, in the sense of containing musical interpretations of the logical entailments of the formal system.[25]

All of these writers show a clear concern that theory and analysis should remain anchored in, but not circumscribed by, musical "common sense"; to borrow another phrase from Cook, such analyses aim towards 'the fusion of aural perception and imagination that we refer to as "hearing"'.[26] Otherwise, we have no way of choosing between the infinite number of theoretical lenses that could be used to view a given piece: the only thing that elevates the meaningless to the suggestive, in other words, is the presence of the descriptive.

Under this paradigm of music theory, it is easy to see how an analysis becomes a creative aesthetic response to a musical composition, just as any 'evocative' poetic description both describes and enhances its target experience. Lewin links this idea to literary theorist Harold Bloom's proposal that poems can be understood as responses to other poems, but broadens 'poem' to include any act of *poiesis* (making), including criticism, analysis, and the very act of perception itself.[27] For Lewin, this feature of music theory is more important than any superficial differences between "scientific" and "poetic" writing styles such that 'the writings of Babbitt are as much poems, in the broad interpretation of the post-Bloomian view, as are the writings of [critical musicologist James] Randall'.[28] Agawu has compared analysis to both performance and

[25] V. Kofi Agawu, *Playing with Signs: A Semiotic Interpretation of Classic Music* (Princeton: Princeton University Press, 1991), p. 79; Guck, p. 63; Schoenberg, *Theory of Harmony*, p. 11; Nicholas Cook, 'Theorizing Musical Meaning', *Music Theory Spectrum*, 23 (2001), 170–95 (p. 186; p. 185 'tensional'); Babbitt, 'Past and Present Concepts', p. 82.
[26] 'Music Theory and "Good Comparison"', p. 129.
[27] Lewin, 'Phenomenology', p. 100; see Harold Bloom, *The Anxiety of Influence: A Theory of Poetry* (New York; London: Oxford University Press, 1973; repr. 1975). Bloom's ideas have also been applied to music in Joseph N. Straus, *Remaking the Past: Musical Modernism and the Influence of the Tonal Tradition* (Cambridge, MA: Harvard University Press, 1990); see pp. 12–20 especially.
[28] 'Phenomenology', p. 102. Tymoczko has suggested that Guerino Mazzola's highly technical 1335-page book *The Topos of Music* (Guerino Mazzola and others, *The Topos of Music: Geometric Logic of Concepts, Theory, and Performance* (Basel: Birkhäuser, 2002)) could be understood 'as a radical and challenging artwork that adopts the guise of scholarship, but whose ultimate goal is beauty rather than truth': see Dmitri Tymoczko, 'Mazzola's Model of Fuxian Counterpoint', in *Mathematics and Computation in Music: Third International Conference, MCM 2011: Paris, France, June 15–17, 2011:*

composition, and recently August Sheehy has likened it to improvisation.[29] Taking this idea one step further, Alan Marsden has proposed a circular model of music's ontology in which music as a psychological entity in a composer's mind becomes, via the behaviour it influences, the sounds the behaviour creates, and the patterns and properties of the sounds, a set of concepts which almost – but not quite – lines up again with music as a psychological entity.[30] Marsden then goes on to argue that this disconnect creates 'potential music', either in the form of compositions which have not yet been performed and therefore only exist in the composer's mind, or in patterns and properties which are not accepted as hearable concepts. The model therefore relates a piece's conception to its reception extremely closely, and implies that every analysis is, in effect, a new piece of potential music in the conceptualisation it proposes.

Analysis, then, is *doing* something to a piece rather than finding something out about it: as Schoenberg comments in a letter to Reti, 'even had your analysis not told me anything new about my piece, still it was, above all, a deed. And a deed is to be valued above all as it springs from a productive urge'.[31] It therefore does not terminate in an article, book, presentation, or lesson (artefacts that Lewin describes as 'ski tracks on the hill behind'), but must continue with the reader and listener; a good analysis, argues Agawu, 'must always make discovery possible; if it seems closed, if it provides answers rather than further questions, it betrays its most potent attribute'.[32] The reader of an analysis is attempting to relive the analytical process, shuttling between score, prose, diagram, and (performed, imagined, and/or recorded) sound, so that its insights may be, in Cook's words, '"seen" in the same sense as a joke: that is to say, not conceptualised in analytical terms, but grasped through what could be called a kind of inner performance'.[33] An analysis lecture enacts and externalises this performance, but can leave the audience with little more than 'a positive "aura"' bereft of the ability to 'review, criticise and go beyond': while this stymies the 'cumulative advancement of knowledge' for Jean-Jacques Nattiez, it also robs listeners of the opportunity to perform analyses in their own unique ways.[34]

Proceedings, ed. by Carlos Agon and others, Lecture Notes in Artificial Intelligence, 6726 (Berlin: Springer, 2011), pp. 297–310 (p. 306).

[29] See Kofi Agawu, 'How We Got Out of Analysis, and How to Get Back In Again', *Music Analysis*, 23 (2004), 267–86 and August Sheehy, 'Improvisation, Analysis, and Listening Otherwise', *Music Theory Online*, 19.2 (June 2013) <http://mtosmt.org/issues/mto.13.19.2/mto.13.19.2.sheehy.php> [accessed 17 December 2014].

[30] Alan Marsden, 'Final Response: Ontology, Epistemology, and Some Research Proposals', in *3MR*, pp. 161–67 (p. 162).

[31] Rudolph Reti, *Tonality – Atonality – Pantonality: A Study of Some Trends in Twentieth-Century Music*, rev. edn (London: Barrie and Rockliff, 1960), p. 48 and plate II.

[32] Lewin, 'Phenomenology', p. 95; Agawu, *Music as Discourse*, p. 319.

[33] 'Music Theory and "Good Comparison"', p. 128.

[34] See Jean-Jacques Nattiez, 'Varese's *Density 21.5*: A Study in Semiological Analysis', trans. by Anna Barry, *Music Analysis*, 1 (1982), 243–340 (p. 244). As Nattiez's argument suggests, this creates

1.4 A Scientific "Music Theory"

Most analysis – whether describing, suggesting, or performing the behaviour of composers, listeners, or analysts – retains as its primary concern the human experience of sound.[35] The best route to a scientific music theory therefore seems to be through the study of perception and cognition: but, like any interdisciplinary enterprise, the meeting of methodologies can become a zone of negotiation or even conflict, each side remaining sceptical about the guiding assumptions of the other. Gjerdingen, wishing to take '[c]areful measurements of what is truly measurable' and to situate these within general theories of human perception, denounces music theorists' retention of inadequate, ill-defined, and aesthetically loaded terminology as a yardstick to judge psychological research: 'Why don't psychologists study *real* musical issues?', a hypothetical interlocutor asks; 'Why, for example, can't they measure how long a tonic can be prolonged?'.[36]

The scientific freedom to escape assumptions 'grounded in, and hobbled by, the narrow aesthetic philosophy of an earlier century', as Geraint Wiggins puts it, engages the issues of coherentism and foundationalism, two of Michel Foucault's 'epistemes' on which Cook bases his epistemological and historical survey of music theory.[37] Although broadly historical in origin, epistemes apply across all of human thought and their deployment as 'epistemological options' (p. 80) continues today; particularly, Cook argues, in music theory. Coherentism is an episteme characteristic of pre-seventeenth century thought and justifies knowledge by assessing its compatibility with existing knowledge: it therefore accumulates observations based on judgements of similitude, emulating the 'great chain of being' stretching from God to the soil via angels, humans, animals, and plants.[38] This is an approach that, in Gjerdingen's phrase, 'precludes fundamental revisions, major discoveries, or even accidental breakthroughs', shortcomings that

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certain institutional problems since analysis-as-performance fits uncomfortably within traditional models of research; cf. Babbitt's anxieties concerning music theory's respectability cited in Section 1.2, and see also Agawu, 'How We Got Out of Analysis', pp. 274–77.

[35] See Cook, 'Music Theory and "Good Comparison"', p. 117 for some qualifications: he cites Carl Dahlhaus's comment that this is largely a product of the last two centuries, Lewin's argument (in 'Phenomenology') that music theory can focus too much on listening at the expense of other modes of response, and articles suggesting that some non-Western musics are structured around gesture rather than sound. However, the basic observation as it applies to music theory in the contemporary West, he argues, still stands.

[36] Gjerdingen, 'Experimental Music Theory', pp. 166, 163.

[37] Wiggins, 'Response to Marsden and Mazzola', p. 97; Cook, 'Epistemologies', in particular pp. 80–84. Cook draws on Michel Foucault, *The Order of Things: An Archaeology of the Human Science*s (London: Tavistock, 1970), particularly pp. 22–61.

[38] There is also a sense in which modern scientific theories are coherentist, in that they "add" but never "take away": Einsteinian gravitation subsumes Newtonian gravitation as a special case, and the observation that the Earth is a sphere does not contradict the understanding that the planet's surface is locally flat (see Isaac Asimov, 'The Relativity of Wrong', *Skeptical Inquirer*, 14.1 (Fall 1989), 35–44).

foundationalism aims to overcome by 'sweep[ing] away sedimented knowledge' in the spirit of the Enlightenment.[39] It therefore takes as little as possible for granted and builds theories in a systematic, rational way on a few fundamental "certainties" (which may or may not be empirical, and which in turn may or may not be established through a scientific method).

The wholesale rejection of traditional music theory can, as argued in the remainder of this chapter, be a highly problematic move. Some writers, like Temperley, try to limit foundationalism's ambit by separating music theory's entangled modes of discourse in a manner reminiscent of Stephen Jay Gould's argument for the 'non-overlapping magisteria' of science and religion: in this understanding, the two magisteria (teaching authorities) answer fundamentally different types of question (broadly speaking, science answers 'how?' while religion answers 'why?'), and so neither should pronounce on the activities of the other.[40] We therefore find Temperley drawing the descriptive–suggestive divide and implying that descriptive questions are best answered by music psychologists, Eugene Narmour distinguishing cognitive scientists' interest in 'natural' modes of listening from music theorists' active construction of '*denaturalized*' ones, Ian Cross arguing that science can 'underpin' but not '*replace* elements of analytical or hermeneutic accounts', and Gjerdingen similarly arguing that music theory should not 'become', but rather 'embrace experimental science' so as not to lose its 'important historical and art-critical components'.[41] While the prevailing picture is one of two broadly independent projects continuing alongside each other, Cross and Gjerdingen in particular appear to imply that the results of scientific enquiry can safely be passed to music theory for 'hermeneutic' or 'art-critical' interpretation as shown in Figure 1.1, just as one might explore the theological implications of Darwinian evolution.

The major problem with this strategy is that descriptions and observations are always *theory-laden*: they are inseparable from their guiding theories, which always underpin the choice, execution, and interpretation of measurements. Even a basic musical statement such as 'the violin plays a C' embeds a whole host of ideas about sound, tuning systems, notational systems, scales, and the primacy of the individual pitch as the unit of theorising; the phrase 'the violin plays pitch-class 0' is an alternative description of the same event, but one which rests on a different set of assumptions and suggests a different musical and theoretical context (it is argued in Section 1.10 that this can be enough to make it a description or model of a *different* event). This not only makes strict foundationalism as an escape from theory impossible (it is better understood as the sweeping away

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[39] Gjerdingen, 'Experimental Music Theory', p. 162; Cook, 'Epistemologies', p. 84.
[40] See Stephen Jay Gould, 'Nonoverlapping Magisteria', *Natural History*, 106.2 (March 1997), 16–22 and 60–62.
[41] Temperley, 'Question of Purpose'; Eugene Narmour, 'Our Varying Histories and Future Potential: Models and Maps in Science, the Humanities, and in Music Theory', *Music Perception*, 29 (2011–12), 1–21 (p. 11); Ian Cross, 'Music Analysis and Music Perception', *Music Analysis*, 17 (1998), 3–20 (pp. 6, 16); Gjerdingen, 'Experimental Music Theory', p. 169.

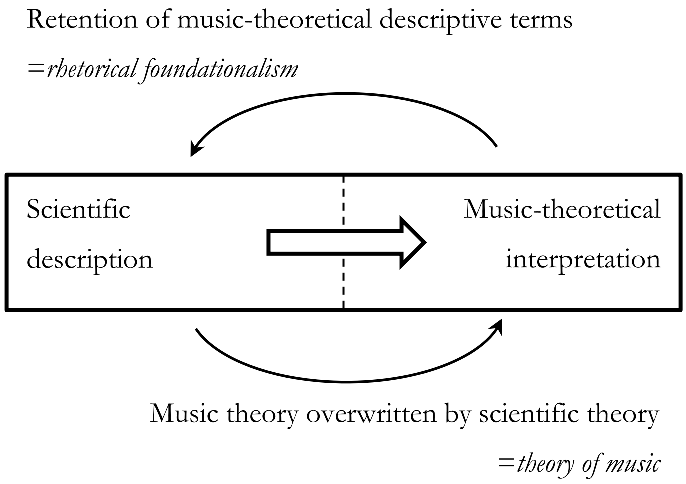

Figure 1.1: A simple trajectory from description to theory is problematised by the theory-ladenness of observational terminology.

of accumulated knowledge *by* another theoretical outlook; a paradigm shift in the sense of Thomas Kuhn), but also confounds any simple attempt to 'embrace' experimental science within music theory, to proceed from scientific description to hermeneutic gloss, since the latter comes with its own set of descriptive concepts and the former its own set of theoretical implications.[42] It is in these traditional descriptive concepts that a suggestive music theory trades, making use of a shared language to construct and communicate its insights: by retaining its stake in description and its attachment to the historical or aesthetic value of certain concepts, the walls of its magisterium become porous.[43]

Music theory therefore embeds a certain coherentist bias: Agawu's two-way stream, Guck's evocative description, Schoenberg's good comparison, Cook's actualised meaning, and Babbitt's non-trivial interpretation all make appeals to existing conceptualisations and draw their strength from not contradicting them.[44] These existing conceptualisations are the ground truths that new

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[42] See Thomas S. Kuhn, *The Structure of Scientific Revolutions*, 4th edn with an introductory essay by Ian Hacking (Chicago: University of Chicago Press, 2012), pp. vii–xxxvii for Hacking's introduction to Kuhn's thought (pp. xvii–xxv discuss 'paradigm' in particular). Guck (pp. 71–72) has argued that a narrative reading similarly does not simply gloss analytical "facts", but '*begins* life as an analysis' (p. 71), with observation and interpretation inseparably intertwined.

[43] Cross argues that authors should be explicit about whether their theory 'employ[s] either a folk-psychological theory of musical perception or a cognitive-scientific one' (p. 18), a distinction that at first seems to mirror Temperley's separation of suggestion (traditional theory) and description (cognitive science) but in fact suggests that *both* types of theory engage the descriptive. The concept of a folk psychology is defined at n. 52 below.

[44] Coherentism can be said to underlie certain twentieth-century analyses in a more direct way too: Cook cites Reti's method of 'piling up of resemblance upon resemblance' ('Epistemologies', p. 82), a tactic also apparent in certain pitch-class set analyses. Such analysis at its worst is simply 'commentary, endlessly reiterated' (p. 82) on the prevailing truth that "this work of music is

music theories, even purportedly foundationalist ones, seek to model (an argument which is expanded in Sections 1.5, 1.7, and 1.9 below). Cook recounts Rameau's attempts to derive the "laws" of harmony from the overtone series of a *corps sonore*, focusing in particular on the logical contortions that he must perform in order to reconstruct the minor triad.[45] Clearly, any theory that ignores music in the minor key is at best incomplete and at worst inadequate – judgements that are based on existing theory and also existing practice (which is in turn shaped and mediated by theory) – and so, by 'adopting the rhetoric of foundationalism but in reality synthesizing received knowledge within a more or less unified framework', Rameau's theory shades into coherentism.[46]

The trope of rhetorical foundationalism is a common one in music theory, usually proceeding (as in Rameau's writings) from the overtone series as the prime "certainty" rooting music in the natural world. The trope began, apocryphally, with Pythagoras's stroll past a blacksmith who was striking metal with hammers of different weights: consonant intervals were produced by hammers with weights in small consecutive whole-number ratios (the unison (1:1), octave (2:1), fifth (3:2) and fourth (4:3)), an idea which tied in (coherently) with the Pythagorean worldview that numbers were the 'constituent elements of reality', icons of the harmonious design of the Universe.[47] This Pythagorean cosmology persisted into the Renaissance: major and minor thirds and sixths were defined and legitimated (by, for example, Bartolomeo Ramis de Pareia in his *Musica practica* of 1482) not through strict acoustical consistency with the perfect intervals, but by using the next ratios in the sequence, 5:4 (=80:64≈81:64) and 6:5 (=162:135≈160:135).[48] A shift in this thinking was marked at the end of the sixteenth century by a subtle reconceptualisation of mathematics from building block of the universe to tool of physics: consonance not as the product of whole-number ratios *per se*, but of the overtone series produced by a vibrating object (although Catherine Nolan notes that Rameau persisted with a mathematical explanation in terms of string length ratios as late as 1722).[49] From there, Cook traces a general historical trend – via Helmholtz, Schenker, Schoenberg, and Babbitt, whose rejection of the "given-ness" of consonance was referred to in Section 1.1 – away from the idea that the overtone series can motivate and justify all the contents of a given theory.[50] The urge to begin afresh from the first principles of acoustics and

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unified", identifying more and more occurrences of a particular pattern without concern for how these patterns change through time, or how they function structurally, rhetorically, or affectively.

[45] Cook, 'Epistemologies', pp. 84–87.

[46] Ibid., p. 85.

[47] See Catherine Nolan, 'Music Theory and Mathematics', in *The Cambridge History of Western Music Theory*, ed. by Christensen, pp. 272–304 (pp. 272–73; p. 273 'reality').

[48] Ibid., p. 276. If a tone=a fifth–a fourth=3:2/4:3=9:8, then a major third=two tones=$(9:8)^2$=81:64 and a minor third=a fifth–a major third=3:2/81:64=32:27(=160:135).

[49] Ibid., p. 278.

[50] Schenker, for example, saw the major triad as an abbreviation of the overtone series, and the minor triad as an artificial imitation of the major triad; he claimed that Nature provides the 'hint'

psycho-acoustics has continued in modern theory, however, with Tymoczko citing papers on the perception of consonance in animals to support the idea of its innateness, and Wiggins piecing together psycho-acoustical studies to sketch a theory rooted in near-universal human perception.[51]

It was argued above (see Figure 1.1) that an attempt to proceed from scientific description to music-theoretical interpretation is bound to be problematic since each has a stake in the other's language: in Rameau's case, his foundationalist project failed because he allowed the traditional concepts of the latter to influence the method of the former. The complementary solution – giving scientific concepts the theoretical breathing-space to overwrite traditional music theory if necessary – is advocated by some cognitive scientists like Wiggins or historians like Taruskin, who disavow a "separate-but-equal" coexistence or indeed ignore the possibility of an alternative magisterium altogether. Wiggins argues that music theory, being 'a descriptive model of music *perception* as collectively learned in a particular culture' (known as a folk psychology), becomes a pointless exercise unless it diverts all of its efforts to the study of the cognitive activities of those who produce and receive it; anything else is 'to study (or model) an effect without considering (or modelling) its cause', reaching empirical conclusions (about listeners' or composers' mental activities) through rational introspection.[52] The 'folk psychology' of music theory should therefore, he argues, be entirely superseded by a scientifically rigorous cognitive music theory.[53]

Such an approach is likely to produce an account that is more scientific than Rameau's; it is also likely to produce something that stands outside what would normally be termed 'music theory', given the understanding proposed in Section 1.3. As Colyvan has argued, those seeking to understand (or even change) the methodology, epistemology, and philosophy of a particular field should not attempt to redefine it to such a radical extent that it no longer bears any relation to practice: '[t]he job of philosophers of science and mathematics is to help make sense of, and contribute to, science and mathematics *as practised* [and] not to overrule the pronouncements of mathematics and science on philosophical grounds' (p. 78). Wiggins himself considers 'a quick look over the music theory section of any good library' sufficient reason to reject an overly restrictive

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and art does the rest (see Cook, 'Epistemologies', p. 89, and pp. 85–91 more generally). The term 'hint' is from Heinrich Schenker, *Harmony*, ed. by Oswald Jonas, trans. by Elisabeth Mann Borgese (Chicago: University of Chicago Press, 1954; repr. 1972), p. 20.

51 See Dmitri Tymoczko, *A Geometry of Music: Harmony and Counterpoint in the Extended Common Practice* (New York: Oxford University Press, 2011), p. 5, n. 5 and Geraint A. Wiggins, 'Position Paper: Music, Mind and Mathematics: Theory, Reality and Formality', in *3MR*, pp. 111–23 (p. 118).

52 Wiggins, 'Position Paper', p. 120; Geraint A. Wiggins, 'Final Response: The Future of (Mathematical) Music Theory', in *3MR*, pp. 135–44 (p. 137). For Taruskin, talk about music is meaningless 'unless its agents can be specified; and *agents can only be people*' (*Oxford History*, I: *The Earliest Notations to the Sixteenth Century*, p. xxvi).

53 Cross similarly distinguishes between folk psychology and cognitive science in the realm of music theory (see n. 43 above), but does not argue that the latter should always supersede the former (see n. 41 above).

definition of music theory, and so it is problematic for him to dismiss the heterogeneous reality of music-theoretical practice as (among many other things) 'the intellectual self-satisfaction of the lone scholar', and argue that it 'is both a proxy and approximate', when such an account also entails dismissing the books on his shelves, the professional practice of the writers of those books, and the historical-theoretical environments surrounding the subjects of those books.[54]

The writing of music theory is a suggestive, performative act that nevertheless must 'slip' into the descriptive mode to anchor its insights in empirical observation. If an account of music is framed in terms of scientifically supported theories of perception, then it is better characterised as a *theory of music* rather than a music theory (a *theory about music*); the latter might still use the paraphernalia of scientific method (including statistical tests and mathematical models), but must retain as its primary purpose the performance and communication of suggestive insights in a shared conceptual language.[55] On the face of it, this reframing is mere wordplay, retaining ideas and definitions whilst simply swapping terms around on the surface: but one of the terms – 'music theory' – is central to the present discussion and has a rich sedimented meaning that cannot be ignored.

1.5 Mediation

As is the case with any binary categorisation, the dividing line between theories of and theories about music is not always clear, and this is an even more pressing concern when the second term is defined by its slippage into areas of the first. An argument against dispensing with traditional music theory in terms of respecting the reality of practice is presented above, but there is also a more concrete methodological reason why a theory of music cannot afford to ignore theories about music: the phenomenon of mediation. Countering Wiggins's assertion that music theory is mostly descriptive, an optional extra for composers, Marsden argues that the practices of composing and listening are unavoidably affected by music theory: either directly through training, or indirectly through acculturation in the results of that training.[56] An alternative interpretation might place theory, performatively, as *post facto* composition, such that the response to Wiggins's insistence that '[s]omeone, somewhere, was the first person to use [a perfect cadence], and it was only after the event that it was theorized' is that they *hadn't* actually used a perfect cadence until someone else,

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[54] 'Position Paper', p. 112; 'Final Response', p. 140; 'Response to Marsden and Mazzola', p. 99.
[55] The theories-of/theories-about terminology comes from Wiggins, 'Position Paper', p. 119 and p. 122, n. 8, but I do not agree with his cursory dismissal of the latter as irrelevant in an exploration of the meaning of 'music theory'.
[56] Wiggins, 'Response to Marsden and Mazzola', p. 98; Alan Marsden, 'Response to Geraint Wiggins', in *3MR*, pp. 125–28 (pp. 125–26).

somewhere else, theorised what a perfect cadence was, thus turning a collection of pitches into a theoretical entity.[57]

An example of theoretical mediation is provided by Marsden in his discussion of the gap-fill principle: that is, the music-theoretical idea that a melodic leap is usually followed by motion in the opposite direction (Figure 1.2a).[58] A statistical investigation by Huron and Paul von Hippel has suggested that this is only partially accurate, and that the phenomenon can be better modelled by regression to the mean (Figure 1.2b): the two principles agree if the leap spans the melody's centre point, but where they disagree, regression to the mean provides the better fit with the observed data.[59] Independently of these findings, Wiggins and Marcus Pearce have created a learning-based computational model of melodic expectation in which '*no hard-wired, music-theoretic rules are necessary*': the model makes accurate predictions of listeners' expectations in a given context based on statistical analysis of a large number of melodies it "knows".[60] If regression to the mean characterises a large number of melodies, then, and if listeners' expectations are conditioned by the melodies they know, then it might seem reasonable to suggest that listeners expect regression to the mean rather than gap-fill. However, this is not the case: an experiment by von Hippel has shown no preference for either principle among non-musically trained subjects, and a preference for gap-fill among the musically trained, leading Huron to posit the possibility that listeners (especially those who have received some training in music theory) use the gap-fill principle as a heuristic, an efficient (if occasionally inaccurate) rule of thumb.[61] 'Clearly,' argues Temperley in a recent study comparing rule-based and statistical models of interval probability, 'humans have the capacity to learn general rules; they also have the capacity to absorb large amounts of statistical information',

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[57] Wiggins, 'Final Response', p. 138. Cf. Jerrold Levinson's argument that, since '[a] sound event conforming to the sound structure of Beethoven's Quintet, Opus 16 logically could have occurred in the Paleozoic era', an ontology of musical works must take into account the composer's role in indicating a particular sound structure: see Jerrold Levinson, 'What a Musical Work Is', *Journal of Philosophy*, 77 (1980), 5–28 (repr. in *Music, Art, and Metaphysics: Essays in Philosophical Aesthetics* (Ithaca: Cornell University Press, 1990; repr. 1996), pp. 63–88) (p. 7).
[58] Alan Marsden, 'Position Paper: Counselling a Better Relationship Between Mathematics and Musicology', in *3MR*, pp. 145–53 (pp. 146–49); the present paragraph summarises Marsden's narrative.
[59] Paul von Hippel and David Huron, 'Why Do Skips Precede Reversals? The Effect of Tessitura on Melodic Structure', *Music Perception*, 18 (2000–01), 59–85.
[60] Wiggins, 'Response to Marsden and Mazzola', p. 95; see also Marcus T. Pearce and Geraint A. Wiggins, 'Expectation in Melody: The Influence of Context and Learning', *Music Perception*, 23 (2005–06), 377–405. Their model (known as IDyOM) is therefore a theory of music, predicated on theoretical ideas about human learning in general rather than on traditional music theory.
[61] Paul von Hippel, 'Melodic-Expectation Rules as Learned Heuristics', in *Seventh International Conference on Music Perception & Cognition: Conference Proceedings*, ed. by Catherine Stevens and others (Adelaide: Causal Productions, 2002), pp. 315–17 [on CD]; David Huron, *Sweet Anticipation: Music and the Psychology of Expectation* (Cambridge, MA: MIT Press, 2006; repr. 2007), pp. 94–97 (these pages of Huron's book outline Marsden's gap-fill narrative in similar terms).

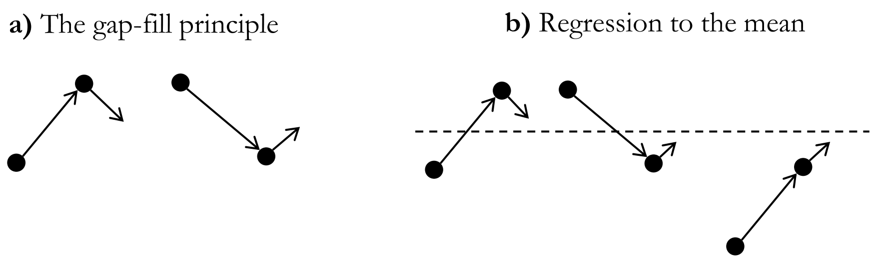

Figure 1.2: The gap-fill principle agrees in part with the principle of regression to the mean.

necessarily inviting a trade-off (discussed in more detail in Sections 1.8 and 1.9) between the conceptual simplicity and goodness-of-fit of a proposed model.[62]

The argument for statistical learning underpins a research methodology – christened 'musical corpus research' in the introduction (by Temperley and Leigh VanHandel) to a recent pair of issues of *Music Perception* devoted to the topic – in which the lines between theories of and about music are at their most blurred.[63] As its name suggests, this methodology examines corpora of musical data (usually suitably encoded scores) using computational and statistical methods and so, according to Temperley and VanHandel, 'becomes an important part of the field of music perception: gathering statistical information from music simulates the listener's learning process, and provides parameters needed for the modeling of expectation and other aspects of perception' (p. 1). The leap is thereby easily made from a 'creationist' account of the regularities found in musical objects (note that at no point is it necessary to conduct any psychological tests on human subjects, despite tests of this type unexpectedly saving the notion of gap-fill) to an 'evolutionist' account of the underlying perceptual mechanisms of composers and listeners. There is a subtle but significant difference, however, between (for example) Yuri Broze and Huron's suggestion that higher music tends to be faster due to the physical properties of instruments and performers (as well as the perceptual limitations of listeners), and Jon Prince and Mark Schmuckler's supposition 'that composers correlate the tonal and metric hierarchies in their compositions' consciously as a way of manipulating listeners' expectations and therefore 'communicating musical emotion'.[64] Both are cognitive hypotheses about compositional behaviours: but while the former constitutes a

[62] David Temperley, 'Probabilistic Models of Melodic Interval', *Music Perception*, 32 (2014–15), 85–99 (p. 98; see also p. 96).
[63] The special issues are numbers 1 (September 2013), 1–95 and 3 (February 2014), 191–301 of *Music Perception*, 31 (2013–14); all items cited in notes 64 to 66 below are drawn from these. For the introduction, see David Temperley and Leigh VanHandel, 'Introduction to the Special Issues on Corpus Methods', *Music Perception*, 31 (2013–14), 1–3 (p. 1 'musical corpus research').
[64] Yuri Broze and David Huron, 'Is Higher Music Faster? Pitch–Speed Relationships in Western Compositions', *Music Perception*, 31 (2013–14), 19–31 (pp. 29–30); Jon B. Prince and Mark A. Schmuckler, 'The Tonal-Metric Hierarchy: A Corpus Analysis', *Music Perception*, 31 (2013–14), 254–70 (p. 265).

theory of music, offering acoustical and physiological explanations for its observed phenomena, the latter only makes sense under the assumption that the composers being discussed (and the listeners that those composers presupposed) had some concept of tonal theory (statistically learned or otherwise). Conclusions of the second type are strengthened by what Gjerdingen terms '"historically informed" corpus studies' in his contribution to the special issues of *Musical Perception*: he argues that, for example, rather than tallying Roman-numeral progressions in Bach, it might be more enlightening to look for the *cadenza doppia*, a scale-degree archetype 'taught to every apprentice musician at the conservatoires in Naples' in Bach's time.[65]

The normative or regulative music theories that shape musical practice (and therefore mediate the musical "object") can incorporate a variety of preoccupations (for example, the quasi-theological justification of the 'just' thirds 5:4 and 6:5), but are often themselves responses to perceived musical patterns: in other words, they can be folk psychologies, or pseudo-theories, of music. Some music theories are therefore amenable to rigorous empirical testing (John Paul Ito measures how well Koch's theories describe Mozart's music), or else can be treated as records of cognitive or perceptual responses to music: when Joshua Albrecht and Daniel Shanahan report a new key-finding algorithm which matches piece titles and the judgements of independent analysts with 95.1% accuracy, what they have really constructed is a model of how composers and analysts form a theory of a piece of music's key.[66] But even if a music theory is found to be wanting as an empirical description of a musical corpus (Ito (p. 220) concludes that 'the patterns Koch describes […] are not prevalent enough to justify taking them as givens'), this is not grounds for its rejection as a mediating heuristic for listeners or composers (as in the case of gap-fill) or a good comparison (as in Ito's insistence (ibid.) that 'Koch's tools can provide an extremely rich perspective on the music of Mozart and Haydn') – even if the comparison is not as good as first thought. Certain pieces of musical corpus research therefore produce what Gjerdingen characterises as 'complexified dogma': they establish the domain of applicability of, or the regularities that may have given rise to, a particular music theory, but do not seek to question its basic premises.[67]

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[65] Robert O. Gjerdingen, '"Historically Informed" Corpus Studies', *Music Perception*, 31 (2013–14), 192–204 (p. 197). The same argument is advanced from the perspective of the corpus itself, rather than the theories used to mine it, in Justin London's contribution (Justin London, 'Building a Representative Corpus of Classical Music', *Music Perception*, 31 (2013–14), 68–90), which aims to assemble a corpus of classical works reasonably familiar to a typical modern listener.
[66] John Paul Ito, 'Koch's Metrical Theory and Mozart's Music: A Corpus Study', *Music Perception*, 31 (2013–14), 205–22; Joshua Albrecht and Daniel Shanahan, 'The Use of Large Corpora to Train a New Type of Key-Finding Algorithm: An Improved Treatment of the Minor Mode', *Music Perception*, 31 (2013–14), 59–67 (pp. 62, 66).
[67] Gjerdingen, 'Experimental Music Theory', p. 162.

1.6 Two Cultures

If we take the goals of music theory to align with the goals of the humanities more broadly (as Section 1.8's discussion of theory in the humanities argues we can), some of the tensions between science and music theory can be understood as a clash of C. P. Snow's 'two cultures' (the sciences and the humanities).[68] As suggested above, each culture has different requirements of a theory, different expectations of a definition, and different concepts of empirical adequacy, and these three crucial aspects of theory-building are examined in more detail in the next three sections. But nearer the surface than these epistemological and methodological concerns are matters of institutional culture and professional training that, as argued in a recent issue of the *Journal of Mathematics and Music* devoted to methodological debate, frequently lead to misunderstandings between domains.[69] To give a sketch of the relevant straw men: musicologists are vague, quick to make unsupported empirical claims, and anxious about the scientific need for abstraction and generalisation, with those that do dabble in science and mathematics frequently lacking the relevant training 'to meet the specific standards required of researchers in established experimental sciences' or in high-level mathematics 'far from [that] conceived by music theorists'.[70] Scientists, meanwhile, are locked in a meaningless and arrogant quest for "precise" definitions of nuanced and sedimented terminology (Guerino Mazzola, encountering 'a set of completely incoherent, fuzzy, and fragmentary "theories"', in his own words 'decided to rethink musicology in a rigorous way'), and 'are often unwilling to discuss their work in terms intelligible to the uninitiated'.[71] They are also guilty of deficiencies in training, with Marsden warning against the 'mathematician who believes that he or she understands music simply because he or she is good at playing it and has heard lots' (echoing Babbitt's characterisation of 'the light-hearted impudence of the scientist vis-à-vis music'), and arguing that it is possibly easier for a musician to learn a circumscribed set of scientific methods and concepts than it is for a scientist 'to learn the subtleties of music analysis'.[72] Huron frames

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[68] See C. P. Snow, *The Two Cultures and the Scientific Revolution: The Rede Lecture 1959* (Cambridge: Cambridge University Press, 1959; repr. 1960).
[69] See *3MR*, and in particular (for a summary), Anja Volk and Aline Honingh, 'Introduction: Mathematical and Computational Approaches to Music: Challenges in an Interdisciplinary Enterprise', in *3MR*, pp. 73–81.
[70] Gjerdingen, 'Experimental Music Theory', p. 169; Guerino Mazzola, 'Position Paper: Thinking Music with Precision, Depth, and Passion', in *3MR*, pp. 83–91 (p. 88).
[71] Guerino Mazzola, 'Mazzola's Response to Marsden's Text', in *3MR*, pp. 155–59 (p. 156); Raymond Erickson, 'Music Analysis and the Computer', *Journal of Music Theory*, 12 (1968), 240–63 (p. 241), quoted in Alan Marsden, '"What Was the Question?": Music Analysis and the Computer', in *Modern Methods for Musicology: Prospects, Proposals, and Realities*, ed. by Tim Crawford and Lorna Gibson, Digital Research in the Arts and Humanities (Farnham: Ashgate, 2009), pp. 137–47 (p. 137) with the observation that 'little has changed' in the intervening forty years.
[72] Marsden, 'Position Paper', p. 152; Babbitt, 'Contemporary Music Composition', p. 272; Marsden, 'What Was the Question?', p. 147. Mazzola's proposed musicological pre-requisites for interdisciplinary work seem basic and incomplete alongside his seven advanced mathematical topics

these differences in more equitable terms by arguing that they reflect the different training priorities attached to research methodology and epistemology, respectively, in the sciences and the humanities.[73]

When mathematics, which often unwillingly serves as an icon of scientific epistemology, appears within the context of music theory, it can, then, be subject to criticism from both cultures: on the one hand as an inadequate or even manipulative attempt to impart an illusive sense of scientific rigour, and on the other as an essentialising "explanation" of the heterogeneous, culturally situated practice of music. But if mathematics is the 'language' of science, and not itself science, then there is no reason why it cannot be – alongside written analyses, verbal analyses, diagrams, and performances – a language of music theory. One way it might fill this role, as suggested in Section 1.5, is as a tool for what Cook describes as 'doing traditional musicology better': statistical and computational studies allow rich observations to be made about large quantities of data without sacrificing the underlying nature of the questions asked, and indeed Marsden has argued that all testable claims made about music should be underpinned by such studies.[74] But mathematics can also serve the suggestive ends of good comparison through the process of modelling, as discussed in Section 1.10: once ideas have been conceptually clarified (which 'is not the same', warns Marsden, 'as making them easier to understand'), finding where these 'tidy mathematical systems' (see n. 1) break down when faced with musical reality can be instructive in revealing what individuates a particular piece or practice.[75] The next three sections explore what it means for a mathematical model to 'clarify' sedimented terminology, to serve as a 'comparison' within the context of humanities theory, and to be described, under certain criteria, as 'good'.

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and five sizeable computational ones: 'Sonata theory, serialism, large forms, instrumentation and orchestration, music semiotics, acoustics.' ('Position Paper', p. 90)

[73] 'The New Empiricism', para. 10.

[74] Nicholas Cook, 'Towards the Compleat Musicologist?' (paper presented at the International Society for Music Information Retrieval Conference, London, 2005), available at <http://ismir2005.ismir.net/keynote.html#cook> [accessed 17 December 2014], p. 6; Alan Marsden, 'Music, Mathematics, Morality and Motion' (paper presented at the RMA Study Day on Music and Mathematics, Leeds, 2014). Huron advances a similar argument in rhetoric reminiscent of Babbitt's call for terminological rigour ('The New Empiricism', para. 155): 'For the new empiricist, an interest in quantitative methods has nothing to do with science. It has everything to do with becoming a more observant music scholar.'

[75] Alan Marsden, 'Response to Guerino Mazzola', in *3MR*, pp. 103–06 (p. 104).

1.7 Theory-Building I: Definition

The *Stanford Encyclopedia of Philosophy* gives a number of terms that can be used to characterise different types of definition, of which three will be particularly useful for the following discussion:[76]

1) A *stipulative definition* 'imparts a meaning to the defined term, and involves no commitment that the assigned meaning agrees with prior uses (if any) of the term'.
2) A *descriptive definition* 'aim[s] to be adequate to existing usage'.
3) An *explicative definition* 'aims to respect some central uses of a term but is stipulative on others'.

Marsden associates stipulative definitions with rationalism and descriptive definitions with empiricism; broadly speaking, we might therefore equate the former with mathematics (as the logician Gottlob Frege has argued we should) and the latter with science.[77] The third type of definition was proposed by Carnap, one of Babbitt's chief influences, and we can see embedded within it the twin positivist concerns for empirical validity and formal manipulability. It also reflects the idea of 'evocative' description in music theory since an analysis must simultaneously describe and stipulate.

In practice, stipulative and descriptive definitions exist on a spectrum, and most definitions are explicative to some degree. No descriptive (or dictionary) definition can ever completely capture a term's full range of uses and resonances and so must impose a stipulative closure, and most stipulative concepts are attached to words with analogous informal meanings (such as the mathematical sense of 'set'). This spectrum must be negotiated in any work of mathematical music theory (or, more generally, any work of applied mathematics): when Chantal Buteau and Mazzola define a motive as 'a non-empty finite set of notes [with no two notes sharing the same onset time]', they are clearly trying to be faithful to the concept of melody whilst stipulating which of its aspects are to shape its role as a formal object.[78]

Stipulative definitions in mathematics are therefore determined not (or not only) by a drive towards "better" or more precise concepts, but by an invisible hand of theory similar to that which guides the process of rhetorical foundationalism. Imre Lakatos has argued that the traditional, strictly deductive, definition–theorem–proof format of mathematical writing 'hides the adventure':

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[76] Anil Gupta, 'Definitions', in *The Stanford Encyclopedia of Philosophy*, ed. by Edward N. Zalta, Fall 2014 edn <http://plato.stanford.edu/archives/fall2014/entries/definitions/> [accessed 17 December 2014]; the three quotes are from sections 1.3, 1.4, and 1.5 respectively.
[77] Marsden, 'Final Response', p. 163; Gottlob Frege, 'Logic in Mathematics', in *Posthumous Writings*, ed. by Hans Hermes, Friedrich Kambartel, and Friedrich Kaulbach, trans. by Peter Long and Roger White with Raymond Hargreaves (Oxford: Blackwell, 1979), pp. 203–50 (e.g. pp. 203, 211).
[78] Chantal Buteau and Guerino Mazzola, 'Motivic Analysis According to Rudolph Réti: Formalization by a Topological Model', *Journal of Mathematics and Music*, 2 (2008), 117–34 (p. 119).

definitions 'frequently look artificial and mystifyingly complicated' and theorems 'are loaded with heavy-going conditions', but '[o]ne is never told how these complications arose'.[79] The process of mathematical reasoning is therefore closer to the picture painted by Colyvan (p. 148):

> We start out with some intuitive mathematical concepts, derive some results using these concepts, refine our definitions in light of the results, and revisit the derivations. Neither definition nor derivation is logically or temporally prior to the other. This is in stark contrast to the received view of mathematics being a deductive science, cranking out theorems by agreed-upon rules and using well-defined concepts given in advance.

To illustrate this, Lakatos presents a dialogue on Euler's polyhedron formula: that is, for any polyhedron, the number of vertices plus the number of faces minus the number of edges equals two.[80] It is easy to find counter-examples to this conjecture (for example, a cube with another cube hollowed out from its centre, or two pyramids that touch at a vertex), and so we are faced with a choice: consider the conjecture disproved, revise our definition of a polyhedron to exclude the (admittedly odd) counter-examples (known as 'monster-barring'), or make a strategic retreat and admit that the conjecture only applies to certain polyhedra which agree with the assumptions made in the proof. The initial descriptive notion of a polyhedron (a solid bounded by polygonal faces) is thereby refined by additional stipulation (either in the definition or in the theorem) and becomes explicative.

Stipulative definitions hence do some of the work of monster-barring, but may not be judged on their descriptive adequacy in isolation: the "missing" parts of a definition can be scattered throughout the rest of the theory and contribute to an emerging model of the phenomenon under consideration. Buteau and Mazzola's definition of a motive bars infinite, empty, or simultaneous sets of notes, but otherwise seems hopelessly general: it encompasses any set of any number of notes which need not be contiguous and may never appear again within a piece. However, they go on to define, for instance, a motive's *gestalt* as the set of other motives derivable from it via an agreed group of transformations (including some acting on its intervals or contour), and a motive's *ϵ-neighbourhood* as the set of motives whose gestalten are within distance ϵ of its own (pp. 121–22; note my italics, commonly used to signify that a stipulative definition is being imposed). Ideas of variation and similarity thus appear in the theory and reflect back on the meaning of the original term 'motive'.

The process of monster-barring is what gives a mathematical model its conceptual clarity: it ensures that if an explicit set of conditions holds, then the model's conclusions are deductively valid. But, as elaborated in Section 1.10, since every assumption, condition, stipulation, and logical

[79] Imre Lakatos, *Proofs and Refutations: The Logic of Mathematical Discovery*, ed. by John Worrall and Elie Zahar (Cambridge: Cambridge University Press, 1976), p. 142.
[80] See Lakatos, pp. 6–105; his argument is also summarised in Colyvan, pp. 145–47.

deduction closes off portions of the messy and potentially inconsistent phenomenon that is being modelled, no model is a perfect translation of its object. It is therefore problematic for Mazzola to claim that he and Buteau have 'succeeded in turning motif theory into a valid topological theory', as this would suggest that the ill-defined field of 'motif theory' can impart no insights outside those portions of Reti's theory which Buteau and Mazzola have managed to model adequately.[81] There also seems to be some confusion over what, exactly, the object of Mazzola's modelling is: despite his foundationalist complaint that '[i]n musicology and more generally in the humanities […] [a] definition must always fulfil a grown content [which we cannot be sure] is adequate for understanding the facts', his practice here and elsewhere (including a formalisation of the rules of Fuxian counterpoint) is heavily mediated by the traditional concerns of music theory.[82] Yet Mazzola sees his project as scientific, endorsing a Roger Penrose-derived ontology in which mathematics is 'the innermost ontological kernel of physical reality' that is not applied 'for fun, but because the objective situation requires it': he therefore dismisses criticisms that his mathematical methods are obsfuscatingly complex as 'psychologically motivated, but not scientifically acceptable'.[83]

1.8 Theory-Building II: Theory

The scientific and musical relevance of formalisation for its own sake has been questioned by many writers: 'no amount of formalism can ever transform a description into an explanation' for Brown and Dempster, 'formalizations alone do not make theories better' for Anja Volk and Aline Honingh, Cook warns against analyses that simply create 'a new kind of score', Marsden against music theories adopting mathematical 'veneers', and, perhaps most damningly, Wiggins argues: 'Music theory as it stands may well *describe* musical works usefully, and mathematical music theory may do so more formally, and more precisely. Frankly, though, if that were really all music theory can be, I would rather just listen.'[84] These warnings are salutary, especially within the context of claims to scientific acceptability, but they risk overlooking the extent to which "mere" descriptions of music, formal or otherwise, can serve as good comparisons: all discourse about music is necessarily metaphorical, and all metaphors necessarily suggest as much as they describe (see Section 1.10).

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[81] Mazzola, 'Position Paper', p. 84.
[82] Mazzola, 'Response to Marsden's Text', p. 157 (see also Tymoczko, 'Mazzola's Model of Fuxian Counterpoint'). Mazzola (ibid.) goes on to contrast the idea of 'grown content' with the standard mathematical procedure of revising definitions in the light of counter-intuitive examples, but does not explicitly refer to Lakatos.
[83] Guerino Mazzola, 'Mazzola's Response to Wiggins' Position Paper', in *3MR*, pp. 129–33 (p. 130) 'kernel'; Mazzola, 'Position Paper', p. 89 'fun', 'motivated'.
[84] Brown and Dempster, p. 81; Volk and Honingh, p. 79; Cook, 'Towards the Compleat Musicologist?', p. 3; Marsden, 'Position Paper', p. 151; Wiggins, 'Final Response', p. 140.

Schoenberg's notion of good comparison is connected more broadly to the meaning of the term 'theory' within the humanities. When Cook notes that, for the coherentist, 'commentary, endlessly reiterated, is accorded the same epistemological status as empirical observation', he hints at his later distinction between the humanities and the sciences in that the former seek 'not certainty but understanding, and the means by which it is to be achieved is not explanation but elucidation'.[85] He cites Bent's characterisation of idealised scientific explanation as a linear path from general to particular, while human-scientific elucidation is a circular process between part and whole; crucially, in the humanities neither domain has 'ontological priority'.[86] This accords with Narmour's definition of what 'theory' (in the sense of postmodern theory, gender theory, or Marxist theory) means in a humanities context (p. 4):

> *Theory* in the humanities means any kind of more or less fixed and hypothetically identifiable top-down approach to a text or a score, where the target of the literary or musical analysis is mapped onto the phenomenon in accordance with the chosen approach.

The very purpose of theory in the humanities, then, is to load observations; to put the general at the service of the particular rather than subsume the particular as a case of the general. 'The scientist wishes to make it unnecessary to know each in order to know all', summarises Benjamin Boretz, while 'the musician wishes to make it impossible to know all without knowing each. […] To learn to hear a unique thing as a categorical thing is a net loss for musical experience'.[87]

In using humanities theory, Narmour speaks of 'the intellectual promise of an atypical mapping' (p. 4), and it is this mapping – including (perhaps especially) its weak points, monsters, omissions, and *ad hoc* patches – that sheds light on the phenomenon, setting a piece's unique features into relief. Again, this is at the heart of Cook's interpretation of 'good comparison': the frequent criticism of Schenkerian analysis that it treats a piece's most interesting and individualising moments as superficial foreground features is turned on its head, with their striking effect being *explained* through their inability to be part of the flat, generic background structure.[88] In such a

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[85] Cook, 'Epistemologies', pp. 82, 93.
[86] Ian Bent, 'General Introduction', in *MA19II*, pp. 1–27 (p. 9), quoted in Cook, 'Epistemologies', p. 93.
[87] Benjamin Boretz, 'Musical Cosmology', *Perspectives of New Music*, 15.2 (Spring–Summer 1977), 122–32 (p. 130). According to Cook, music theory's shift towards theories that are chiefly valued for the insights they can provide into individual works is dated by Dahlhaus to the close of the eighteenth century (see 'Epistemologies', p. 80, which draws on Thomas Christensen, 'Review[: Carl Dahlhaus, *Die Musiktheorie im 18. und 19. Jahrhundert: Grundzüge einer Systematik*]', *Music Theory Spectrum*, 10 (1988), 127–37, as acknowledged in Cook, 'Epistemologies, p. 78, n. 2). Cook also notes that the post-World War II decline of comparative methods in musicology (such as style analysis) ironically parallels the rise of the computer, the very device that would allow accurate and rapid processing of large amounts of data (Cook, 'Towards the Compleat Musicologist?', pp. 4–5).
[88] Cook, 'Music Theory and "Good Comparison"', p. 132. As Huron points out, '*ex planum*' carries a literal sense of '*making flat*' ('The New Empiricism', para. 130).

context, refining the theory to better fit the phenomenon can risk doing a disservice to both. Lewin takes as an example the famous duck/rabbit optical illusion: although one could create a new term to describe the figure, the perceptual tension between the duck and rabbit is what characterises the image – '[w]ho *cares* if you see a dubbit?'.[89]

It would be a misrepresentation to say that humanities theories are therefore free of the need for empirical justification, as discussed in further detail in the next section. However, owing to their intentional and unbreaking flexibility, many humanities theories do not conform to the scientific standard of *falsifiability*, which requires theories to be testable through being open, in principle, to refutation through empirical observation (in the classical example, no number of white swans can prove the idea that all swans are white, but a single black swan can disprove it).[90] Empirical observations that seem to refute a music theory can be explained as individuating features set into relief by good comparison, or defined circularly to stand outside the theory's domain of applicability: fitting a Schenkerian archetype to a piece of music, for example, depends on the ingenuity of the analyst; if it were to prove impossible, Schenker would take this as proof that the piece does not, therefore, constitute a true *Meisterwerk*. We might, following Karl Popper's definition of the term, uncharitably therefore consider music theory to be a pseudo-science; but such a characterisation is unhelpful as it connotes pretensions to a scientific method that music theory as a whole, and as practised, does not necessarily entail.[91]

Huron has proposed the more neutral term 'theory-conserving' to describe those fields in which false negatives (missing something) are seen as more serious than false positives (making incorrect assertions), and so which guard against discarding theories prematurely (with the

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89 Lewin, 'Phenomenology', p. 91.
90 The falsificationist position is most strongly associated with Karl Popper (see next note); for a summary of its main criticisms see Huron, 'The New Empiricism', para. 22. Certain music theories can be phrased in falsifiable terms, especially if they make cognitive claims – Cross singles out Lerdahl and Jackendoff's generative theory and Narmour's melodic theory as 'the two most highly developed' (p. 6) – and, unsurprisingly, these are the ones that are most frequently subjected to psychological testing (see Cross, pp. 6–13; this frequent testing is what facilitates Marsden's discussion – summarised in Section 1.5 above – of gap-fill, a concept which appears prominently in Narmour's work). Ito transforms Koch's theory into a series of empirical predictions, which he tests using corpus methods: but the results as they relate to falsification are moot, as he acknowledges (see Section 1.5), since such theories can always be rescued through appeals to the suggestive mode.
91 See Karl R. Popper, *Conjectures and Refutations: The Growth of Scientific Knowledge*, 4th edn (London: Routledge and Kegan Paul, 1972; repr. 1981), pp. 33–37. For an argument that a necessary component of a pseudo-scientific practice is its presentation of itself as scientific, see Sven Ove Hansson, 'Science and Pseudo-Science', in *The Stanford Encyclopedia of Philosophy*, ed. by Edward N. Zalta, Spring 2014 edn <http://plato.stanford.edu/archives/spr2014/entries/pseudo-science/> [accessed 17 December 2014] (section 3).

attendant risk of provisionally accepting them prematurely).[92] This desire to retain theories which have not yet been falsified, but which are largely unfalsifiable, is a contributing factor to a coherentist outlook. The split between theory-conserving and theory-discarding methodology is often drawn between the sciences and the humanities, but there are more fundamental factors at play including the availability of data, the possibility of generating more data (and therefore testing predictions; see Section 1.9), the political implications of closing off debate in data-poor fields, the choice between holism via pluralism or via generalisation, and the relative ethical costs of false positives versus false negatives (for example in the legal preference for false acquittals over false convictions: 'beyond all reasonable doubt').[93] To take one of Popper's examples of a pseudo-science (see n. 91): a theory-conserving scholar might value Marxist theory as one perspective among many contributing to an understanding of a complex, interrelated, and finite set of historical events (or a work of literature, or a piece of music), and not as the "best" current predictive, explanatory, or unificatory model of a relatively circumscribed and ongoing phenomenon.[94]

Retaining an "inadequate" yet familiar theory can therefore be more informative, in the context of the humanities, than rejecting it and constructing a "better" one: coherentism yet again emerges as an important strategy in music theory. This carries an ideological component: if we agree with Julian Johnson that '[a]nalysis, properly understood, is a critical activity in that it destroys the illusion of seamless unity which is the ideology of every work of art', then this critical power is forfeited by 'an analytical approach which appeals only to its readers' sense of satisfaction'.[95] Cook argues for an understanding of musical works as 'fugitive amalgams of […] potentially meaningful attributes' rather than 'authorized wholes stabilized by dominant interpretations', and there is a certain resonance here with an idea that Marsden highlights as one of the foremost strengths of computational analysis: that its end result is often not a preferred interpretation, but 'a mapping of the terrain of possible interpretations'.[96] Unable to supply the interaction between human and sound so essential to the act of analysis, a computer must simply enumerate possibilities under

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[92] See Huron, 'The New Empiricism', paras. 70–79.
[93] These factors and others are discussed in depth in Huron, 'The New Empiricism', paras. 66–134.
[94] Cf. Cohn: 'If we still invoke formal archetypes, it is not because we agree with their designers that they embody a unifying spiritual principle, or even because they are functional imperatives, but rather because we find them heuristically useful in individual cases.' (Richard Cohn, 'The Autonomy of Motives in Schenkerian Accounts of Tonal Music', *Music Theory Spectrum*, 14 (1992), 150–70 (p. 170)).
[95] Julian Johnson, 'Analysis in Adorno's Aesthetics of Music', *Music Analysis*, 14 (1995), 295–313 (p. 311); Cook, 'Epistemologies', p. 98.
[96] Cook, 'Theorizing Musical Meaning', p. 189; Marsden, 'What Was the Question?', p. 145. Marsden's argument accords with Roman Frigg and Stephan Hartmann's point that computer simulations facilitate 'systematic exploration of a model's parameter space': see Roman Frigg and Stephan Hartmann, 'Models in Science', in *The Stanford Encyclopedia of Philosophy*, ed. by Edward N. Zalta, Fall 2012 edn <http://plato.stanford.edu/archives/fall2012/entries/models-science/> [accessed 17 December 2014] (section 3.1).

certain conditions and in doing so effectively explode the idea of a privileged, innate reading of a work; it shatters the artistic illusion in the most comprehensive and systematic way possible.

1.9 Theory-Building III: Verification

While its roles as suggestive heuristic (Section 1.5) and humanities theory (Section 1.8) relax the need for music theory to make empirically testable, statistically significant claims, Sections 1.2 and 1.3 follow Babbitt and Guck in arguing that a form of empirical verification is at work whenever a description is considered to be evocative (or a comparison to be good, or a theory to be compositionally fruitful, or an observation to be "musical"); Section 1.4 argues, moreover, that this verification is underpinned by a sense of coherentism. Given the impossibility of isolating a musical "object" from its music-theoretical mediations, mathematical models of music usually end up as models of music theories and so have a degree of coherentism guaranteed: but to return to Mazzola's problematic claims of 'success' in Section 1.7, not only is the translation between music-theoretical target and mathematical model never perfect, but the target phenomenon itself is seldom well-defined.

To illustrate this, Marsden refers to a computer program designed by Kemal Ebcioğlu to harmonise melodies in the style of a Bach chorale.[97] How is its success to be evaluated? Scientific theories are usually tested on their predictive ability, but the Bach program cannot predict what Bach will write next as the data set is retrospective and finite; in order for predictions to be tested, argues Huron, the possibility for the generation of new data must be open since the same data cannot be used both to formulate and test a theory.[98] Of course, some chorales could be set aside as "unseen" by the program; but once the program's harmonisation of these melodies has been tested against the real thing, the theory is unfalsifiable since in order to determine whether a new harmonisation "sounds like Bach", we need 'something which is itself effectively a theory of Bach-style chorale harmonisation, which is circular'.[99] The same problems plague computer programs that seek to produce analyses, since there is no standard ground truth but instead a coherentist urge to produce something that looks convincingly "musical"; one reason why such programs often shy away from proposing preferred interpretations and instead enumerate possible analytical conclusions under given parameter settings.

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[97] 'Response to Mazzola', p. 104. The system under discussion is outlined in Kemal Ebcioğlu, 'An Expert System for Harmonizing Four-part Chorales', *Computer Music Journal*, 12.3 (Fall 1988), 43–51.
[98] See Huron, 'The New Empiricism', paras. 99–109; he also argues here that causation (as opposed to simple correlation) may only be established for phenomena that can be manipulated in controlled experiments.
[99] Marsden, 'Response to Mazzola', p. 104.

Care must be taken not to advance too naïve or restrictive a view of prediction. Brown and Dempster have argued that even our best scientific theories are incapable of predicting the behaviour of individual cases, and instead generalise over 'classes of individuals and not individual events *per se*' (p. 92). Formulating classes of potential phenomena with the same characteristics as the one under consideration permits prediction of what is possible in principle, with research into the Big Bang providing an extreme example as it must necessarily generalise over a class containing a single member. Babbitt frames this idea in terms of the linguistic analogy that music theory "predicts" new pieces of music (cf. his circular statement quoted at the close of Section 1.2) in the way that grammar predicts sentences in a language.[100] Marsden proposes an alternative view: that a music theory is essentially a prediction about 'the musical effect of a particular configuration of notes' on a listener, an observation that Wiggins takes as support for the idea that music theory can only predict as part of a wider theory of cognitive behaviour (i.e. a theory of music).[101]

A program that can generate the finite set of Bach chorale harmonisations given their melodies can therefore tell us something about Bach's harmonic syntax, or predict which new harmonisations listeners might judge to be particularly Bach-like; but some critical examination is required in order to determine what is useful (and, indeed, possible), in this context. Marsden states that it is desirable for a theory to be 'smaller' than its target phenomenon – it shouldn't, for instance, simply consist of a database of all the harmonisations and a search function on the melody input – but that the 300+ rules in Ebcioğlu's Bach program amount to a figure 'strikingly close' to the number of chorales (371) in the Riemenschneider edition.[102]

What is to be gained from a music theory that fits the data well but fails to communicate its underlying principles clearly? Although John Rahn has argued that the 'apotheosis' of theory is a system 'that is not only capable of generating the piece it explains in all its particularity and richness […], but is capable of generating *only* that piece', Edward Pearsall recognises that a good theory 'accounts for some of [a piece's] complexity […] without building unnecessary complexity into the theoretical model itself'.[103] Even within the broader context of scientific computer simulation, where it is reasonable to pursue predictive accuracy over theoretical elegance, Roman Frigg and Stephan Hartmann have warned against:

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[100] Babbitt, 'Contemporary Music Composition', p. 301.
[101] Marsden, 'Response to Wiggins', p. 126; Wiggins, 'Final Response', p. 138.
[102] Marsden, 'Response to Mazzola', p. 104. In another sense, a theory should also be "bigger" by not being limited to its initial data set; Marsden elsewhere argues that theories should '*encompas*[*s*]' rather than 'simply correspon[d] to' their targets ('Position Paper', p. 150; cf. the mapping account of applied mathematics discussed in Section 1.10).
[103] John Rahn, 'Aspects of Musical Explanation', *Perspectives of New Music*, 17.2 (Spring–Summer 1979), 204–24 (p. 207); Edward Pearsall, 'Transformational Streams: Unraveling Melodic Processes in Twentieth-Century Motivic Music', *Journal of Music Theory*, 48 (2004), 69–98 (p. 92).

> [I]ncreasingly complex but conceptually premature models, involving poorly understood assumptions or mechanisms and too many additional adjustable parameters [which] may lead to an increase in empirical adequacy […] but not necessarily to a better understanding of the underlying mechanisms.[104]

Such a model brings to mind Lewin's 'dubbit', and its attendant warning is surely even more pertinent to fields in which 'empirical adequacy' (e.g. in some sense "predicting" new Bach chorales) is less important.

Within a scientific methodology, there is a professional reticence to summarise or essentialise, to put words in the data's mouth lest it is forced to say more than it is trying to (hence, as Huron notes, wordings such as '*the data are consistent with*' X and not '*the data* "*support*"' X in scientific journals).[105] The individual rules of a computational model are left to speak for themselves, and not associated with problematic coherentist terms. Conversely, in sensitive computational music analysis, Marsden argues that:

> [A]n extra step is needed after the computer has done its work to make the connections back to the world of personal listening experience, to illustrate how conclusions drawn from a study of […] many pieces influence our understanding of individual pieces, and to explain how [imperceptible details] do nevertheless have an impact on what we hear.[106]

Similar injunctions could be made about theories, such as Mazzola's, which make use of advanced mathematical methods since as the complexity of a music theory increases, its utility as a good comparison decreases. Tymoczko (no mathematical dilettante himself) has observed that Mazzola's work is incapable of comprehension and critique by music theorists outside his own circle of collaborators.[107] His theoretical insights, too superficial to be of music yet too opaque to be about it, therefore end up empirically unsupported or self-supporting; they fall in the woods and, since there is no-one around to hear them, cease to be music theories.

One way to return a complex theory about music to the realm of good comparison is to use it to generate a theory about, or a model of, a particular piece, and then to use this model as an analytical tool for setting the piece into relief. For example, a harmonisation of a real Bach melody by Ebcioğlu's program, or of pared-down versions of it possibly including some rules drawn from harmony textbooks, could be compared with the actual chorale data with the intention of highlighting the points where Bach "breaks the rules" as worthy of special analytical attention. The program could also be used as a measuring tool for a corpus study, testing how well the models generated by the program fit with a variety of corpora (including the Bach chorales, music by Bach in other genres, chorales by other composers including student exercises, other sacred music, and

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104 Frigg and Hartmann, section 3.1.
105 Huron, 'The New Empiricism', para. 23.
106 Marsden, 'What Was the Question?', p. 143.
107 Tymoczko, 'Mazzola's Model of Fuxian Counterpoint', p. 298.

broader "control sets" of tonal music) in order to suggest which of its rules seem to characterise the Bach chorales in particular. Both of these suggestions clear the opacity of the theory and enhance the potential for a good comparison that influences the way individual works or groups of works are heard; they use empirical, computational, and mathematical methods, yet remain within the epistemological orbit of music theory.

1.10 Models as Metaphors

The arguments presented above project a certain interchangeability between the term 'model' and the term 'theory': the relationship between the two is not well-defined, but a common understanding casts models in a mediating role between theories and phenomena, as specific manifestations of the former and simplifications of the latter.[108] The motion of a pendulum, for example, can be modelled by an equation derived from Newton's second law under certain simplifying assumptions about the physical setup (e.g. that the string is massless and inextensible); analogously, we might see a Schenkerian analysis as a model of a piece of music derived from Schenker's theory under certain assumptions about the piece (e.g. that it is more-or-less tonal, and that pitch is a stronger organising factor than rhythm). Fields in which overarching, unifying theories are rare sometimes, however, use models to stand in for theories: Frigg and Hartmann cite biology and economics, while Narmour notes the preponderance of 'mappings or models without explicit goals of theoretical unity' in the humanities.[109] Target phenomena in either case can be individual objects (a Meccano bridge models a bridge, a computer harmonisation of a Bach chorale melody models Bach's harmonisation), or more general theories and processes that produce such objects (Meccano-building models the engineering process, Ebcioğlu's program models (a theory of) Bach's harmonic syntax). When the encompassing theory is mathematical but not scientific, this relationship changes slightly as 'theory' in mathematics denotes something more like 'subdiscipline', a consistent and broadly self-contained collection of logically related ideas (such as graph theory, group theory, or number theory). A mathematical theory 'of' or 'about' something therefore really behaves more like a model, a matching between one of these subdisciplines and a target phenomenon.

To construct a mathematical model of a phenomenon is to take an existing structural archetype 'from the shelf' and modify or extend it to fit the situation at hand, an understanding known as the *mapping account* of applied mathematics.[110] The choice of structure and choice of correspondence (or 'mapping') 'is an art and not a mechanical procedure', with the fitting of phenomenon to theory via model constituting, according to Nancy Cartwright, an explanation of

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[108] See Frigg and Hartmann, section 4 and n. 125 below for more on this relationship.
[109] Frigg and Hartmann, section 4.2; Narmour, p. 4.
[110] See Colyvan, pp. 106–09; p. 108 'shelf'.

the phenomenon in question.[111] Some features of the target system are preserved and some are omitted. Importantly, some are *added* – or suggested – by the mathematical theory or by other applications of the same structure (as, for example, in the concept of 'ecological inertia' arising from an application of the mathematics used in celestial mechanics to the problems of population dynamics), thus emphasising the role of structural over material correspondence in mathematical thinking. For the British pure mathematician G. H. Hardy, 'a mathematical idea is "significant" [to both mathematics and other sciences] if it can be connected, in a natural and illuminating way, with a large complex of other mathematical ideas'; in slogan form, 'mathematical objects are places in structures', or 'mathematics is the art of giving the same name to different things'.[112]

This crucial addition of structure – which only happens because the fit between model and object is imperfect – can be broken down, as argued by R. I. G. Hughes, into a three-stage process: first a correspondence is *denoted* between the target system and a mathematical structure, then the mathematical conclusions within this structure are *demonstrated*, and finally the results are *interpreted* back in terms of the target system.[113] All three stages must be active, even creative, processes since, argue Frigg and Hartmann, 'we do not learn about [a model's] properties by looking at it'.[114] With echoes of positivism (although with more emphasis on the comparison of two broadly self-contained entities than the internal reconstruction of a mathematical theory from empirical primitives), this determination of what is necessary, possible, and impossible within a given setup and under certain constraints can form an explanation of an observed phenomenon.

A music-theoretical example of this kind of thinking may be found in Tymoczko's *A Geometry of Music*.[115] Expanding on previous collaborative work with Ian Quinn and Clifton Callender, Tymoczko builds a basic model of voice-leading by representing a motion between two n-note chords as an arrow between two points on an n-dimensional grid.[116] Considering the two-

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[111] Frigg and Hartmann, section 4.2; Nancy Cartwright, *How the Laws of Physics Lie* (Oxford: Clarendon Press; New York: Oxford University Press, 1983), p. 152.
[112] G. H. Hardy, *A Mathematician's Apology* (Cambridge: Cambridge University Press, 1940; repr. 1941), p. 29. The first slogan is given without attribution in Colyvan, p. 40, and the second may be found in Henri Poincaré, *Science and Method*, trans. by Francis Maitland with a preface by Bertrand Russell (London: Nelson, [1914]), p. 34. For more on ecological inertia, see Colyvan, p. 114.
[113] R. I. G. Hughes, 'Models and Representation', in *Proceedings of the 1996 Biennial Meetings of the Philosophy of Science Association: Part II: Symposia Papers*, ed. by Lindley Darden (=*Philosophy of Science*, 64.4 suppl. (December 1997)), S325–36.
[114] Frigg and Hartmann, section 3.1.
[115] The following condenses some of the main ideas found on pp. 65–99.
[116] See Clifton Callender, Ian Quinn and Dmitri Tymoczko, 'Generalized Voice-Leading Spaces', *Science*, n.s., 320 (2008), 346–48. For a simple example of voice-leading motion (a short two-voice passage on a two-dimensional grid), see Figure 3.1.3 in *A Geometry of Music*, p. 67; this figure is also available, with audio, on the book's companion website <http://www.oup.com/us/companion.websites/9780195336672/examples/chapter3/figure_313> [accessed 17 December 2014].

dimensional example (i.e. when the chords are in fact dyads), Tymoczko then observes that the grid repeats itself in two ways: firstly, each 12×12 block is repeated an octave higher, and secondly, each 12×12 block contains two copies of each pair of notes (one for each possible ordering). The general idea is then that the space is "folded" twice (into a Möbius strip) so that, for example, the point (C4,E4) matches up with both (C5,E5) and (E4,C4) (among others). The choice to make these folds (and therefore omit certain information) was motivated by the original purpose in building the model: to study how individual voices move in progressions between chords. Under such a framework, (C4,E4)→(B3,F4) is equivalent to (E5,C6)→(F5,B5) since the voices behave in exactly the same way.[117]

Having imported a geometric structure, Tymoczko can then reframe musical questions in terms of geometry, or ask about the musical ramifications of certain geometric properties. In particular, he observes that unisons lie at the edges of the space, while chords that divide the octave evenly (tritones in two-dimensional space, augmented triads in three-dimensional space, and diminished sevenths in four-dimensional space) lie at the centre. This means that nearly-even chords (such as fifths, major triads, dominant sevenths, and nearly all other common chords) are all gathered around the centre and are therefore close to each other in terms of voice-leading: more specifically, it transpires that nearly-even n-note chords are particularly close to their transpositions by $12/n$ semitones.[118] This gives rise to an empirical hypothesis: that if composers wish to avoid parallel motion whilst moving between chords of the same type, then we would expect root motions of a *major* third (12/3=4 semitones) between triads and of a *minor* third (12/4=3 semitones) between seventh chords. A statistical analysis of a large number of pieces by Schubert and Chopin supports this claim: 39% of Schubert's major triad progressions move by a major third (as opposed to 26% by minor third) and 42% of his dominant seventh progressions move by a minor third (as opposed to 3% by a major third).[119] Denoting multiple musical voices as points on a co-ordinate system, demonstrating the mathematical conclusions of this, and interpreting these conclusions as musical possibilities therefore shows how mathematical modelling can determine

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[117] I adopt Callender, Quinn, and Tymoczko's notation here (see supporting online material <http://www.sciencemag.org/content/320/5874/346/suppl/DC1> [accessed 17 December 2014] (p. 1)): voice-leadings are represented as arrows pointing between two ordered sets (signified by round brackets), mapping the first element to the first element (i.e. C4 to B3), the second to the second (E4 to F4), and so on.

[118] Of course, every chord is close to its transpositions via voice-leadings that move in strict parallel motion: the voice-leadings of interest here are those which feature contrary motion too. Consider, for example, moving the voices of (C4,G4) inwards: the result is the fourth (C♯4,F♯4), an interval equivalent (in this space) to the fifth (F♯,C♯), which is the 12/2=6-semitone transposition of (C,G).

[119] See *A Geometry of Music*, p. 99; for Chopin, the analogous figures are 25% as opposed to 15%, and 20% as opposed to 9%. Tymoczko admits that his methodology is 'crude but, hopefully, unbiased' (p. 97, n. 29); it lacks, for example, a formal hypothesis, significance testing, and a proposed function from voice-leading distance to frequency of occurrence.

what constraints apply within a certain setup – and that these theoretical constraints are consistent with empirical data.

It is important to stress that the constraints demonstrated in a particular mathematical system are not regulative compositional demands but rather "paths of least resistance" in a certain formal sense; modulation is similarly "constrained" by the fact that a major scale shares six of its seven pitch-classes with the major scale formed on its fifth degree. The leap from here to an empirical hypothesis – that composers will tend to prefer such paths of least resistance – introduces a cognitive element and echoes the epistemological short-circuiting of musical corpus research, slipping from a formal theory about music to a statistical analysis bearing on a theory of music (whilst still being mediated by, accepting the basic premises of, and remaining relevant to, music theory).

Constructing a mathematical model is therefore akin to constructing a metaphor: in Tymoczko's case, the metaphor allows us to see harmonic patterns tracing paths through an abstract space, and to inflect our hearing based on associated concepts such as distance, smoothness, progress and scenic detour (which is why his work remains a music theory, albeit an epistemologically slippery one). Such elisions between mathematics and metaphor are embodied perhaps most clearly and repeatedly in Lewin's work. Even at a theoretical rather than analytical level, *Generalized Musical Intervals and Transformations* foregrounds the metaphorical nature of mathematical modelling by developing two distinct mathematical systems (each occupying half of the book) from two interpretations of a single schematic picture (an arrow between two points): one emphasises interval and distance, and the other motion and transformation (see p. xxix). The conflation of formalism and performativity that 'surprised' Cook (see Section 1.1) also arises in his review of Lewin's book of analytical essays *Musical Form and Transformation*: despite *Generalized Musical Intervals* containing mathematics that 'frightened [him] off', Cook praises 'Lewin's enthusiastic and infectious espousal of the metaphorical and indeed fictive nature of all analytical writing'.[120]

Marsden, however, marks an important distinction between mathematical theories and metaphors:

> A mathematical theory should not 'break down' at any point […] Another mathematical theory of the same phenomenon would either have correspondences with different aspects of the music, and so be strictly a theory of a different phenomenon, or be equivalent to the first theory, and so in a sense be the same theory. […] There are not alternative equally valid but essentially

[120] Nicholas Cook, 'Review[: David Lewin, *Musical Form and Transformation: Four Analytic Essays*]', *Music & Letters*, 77 (1996), 143–46 (pp. 143, 146).

> different mathematical theories of the same phenomenon, whereas there are different equally 'valid' (in the sense of facilitating understanding) metaphors.[121]

It is important to note that Marsden is *not* implying the existence of a single unified mathematical theory of music: his contention is that any attempt to construct one necessarily selects only a few of music's many and varied 'aspects', defining a simplified (object) model (or a particular 'phenomenon') which behaves in a way that has essentially only one mathematical description. This, taking 'mathematical theory' to mean 'scientific theory expressed mathematically', and assessing validity in terms of prediction, is accurate: either two theories predict the same thing and so are equivalent, or they differ, in which case at least one is wrong (although, as argued in Section 1.9, prediction can be a problematic enterprise in music theory). There is also significant work done by the qualifier 'essentially different', covering a range of scenarios from a simple terminology change to the case of one theory subsuming or overlapping the other: Einsteinian and Newtonian theories of gravitation predict the same motion for a falling apple and so are 'the same' in that case even if not more generally.[122]

However, on the understanding of mathematical theories-of/about (i.e. mathematical models) advanced above, the idea of 'equivalent' models becomes as problematic as the idea of 'equally "valid" […] metaphors' of 'the same phenomenon'. A metaphor is an invitation to compare one domain (e.g. the world) with another (e.g. a stage), carrying no guarantee that two readers (or the author and a reader) will draw the same correspondences between the same aspects of these domains (some might take away the message that the world is deterministic, others that societies require their members to fulfil different roles, others that an element of pretence is involved, and others that the scenery is pretty). Just like Marsden's characterisation of modelling, every reading of every metaphor therefore technically deals with a different phenomenon: but in practice we see these readings 'facilitating' different 'understanding[s]' of underlying phenomena which actually come to be *defined by* the overlapping and incomplete patchwork of metaphors used to describe them (my concept of 'the world' will be different to yours, partly dependent on whether and how we each consider it to be stage-like).

Mathematical models necessarily work in the same way, framing the object through the lens of the theory and therefore changing not only the way that the object is conceptualised, but also the questions that can be asked of it and the directions that future developments can take. This phenomenon is embedded in mathematics as deeply as in notation itself: consider the symbols '80', 'eighty', '*quatre-vingts*', and 'LXXX'. As 'places in structures', or models of the number-line, they are as equivalent as one is likely to get since they are merely different names for the same quantity: but whether one understands this quantity as 'eight tens', 'four twenties', or 'fifty and ten

[121] 'Position Paper', p. 150.
[122] See also n. 38 above.

and ten and ten' is significant. Without wishing to indulge in counter-factualism, it seems unlikely that the Romans would have invented the computer since binary numbers require an understanding of place value, which is a structure added by other numeral systems (binary numbers are in base 2; 'eighty'=80 in base 10, while '*quatre-vingts*'=40 in base 20).[123] So, returning to Marsden's categorisation: are the different symbols models of different phenomena, or are they essentially equivalent? Are the differences in added structure and "feel", and the attendant differing potentials for future exploration, enough to assure the former? If so, can two models *ever* be equivalent or is one of Marsden's categories empty? What about if we transpose the question back to an explicative, evocative, perception-modifying music theory: do the differences in feel become even more important than the formal correspondences that two models might share?[124]

The differences between metaphors and mathematical models therefore subsist in degree and not in kind: mathematical models, insofar as they invoke near-"pure" structural archetypes, leave less room for interpretative variety ('all the world's an oblate spheroid'), and are generally more explicit about the aspects they address and exclude ('if one neglects friction and assumes spherical bodies with even mass distribution, then when studying collisions, all the world's a snooker table'). In a scientific context, this facilitates a programme of reductionism (i.e. breaking a larger problem into several smaller ones), but humanities theories and musical analyses tend to aim directly at messy wholes rather than beginning with a series of simplifying assumptions. We therefore understand individual pieces of music through the heterogeneous patchwork of (mathematical or non-mathematical) models, metaphors, and humanities-style theories used to analyse them – Schenkerian, Neo-Riemannian, harmonic-functional, formal, topical, semiotic, textual, narratological (sometimes, indeed often, in the same analysis) – with this patchwork constituting a "theory" of the piece in question.[125]

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[123] For more on the importance of notation in mathematics, including more advanced examples of notation influencing development, see Colyvan, pp. 132–48; for a music-theoretical example, see Babbitt's discussion of index numbers as extensions of interval numbers (outlined in Section 1.2 above). These ideas also bear on the indispensability argument (see Section 1.1) in their problematisation of the idea that mathematics is merely a convenient notation for some truth that lies "behind", and independent of, the language used to express it.

[124] A possible difference between metaphors and mathematical models is the locus of this "feel": while metaphors are inextricably tied to the words that form them, models, as argued by Frigg and Hartmann (section 2.4), do enjoy some degree of independence from their individual descriptions. Analyses are situated somewhere between these two, as they are shaped strongly by words but ultimately only 'grasped' (as models are) by "performing" them (see Section 1.4 above).

[125] This is a *semantic* view of the relationship between models and theories: that is, that a theory is no more or less than a collection of models rather than an abstract formal framework that is realised by its models (the *syntactic* view; see Frigg and Hartmann, section 4). In the semantic view, non-formal models (such as analogies from other domains, or other modifications of the same structural archetype) are treated seriously in their own right, and not as mere differing interpretations of a fundamentally equivalent formal structure. "Tonal theory" can be seen as a

The similarity between metaphors and mathematical models is apparent in Marsden's example of a musical metaphor, which is strikingly scientific: he corresponds musical notes to bodies in motion, explaining that certain properties of the latter (such as inertia) have musical interpretations while others (such as collision) do not, thereby causing the metaphor to break down.[126] Cook notes that this intertwining of scientific and metaphorical discourse is common in music theory, for example in the trope of likening tonal motion to gravity: some writers see tonal gravity as a kind of natural law, others intend the comparison as a metaphor, and yet others blend these two in a manner befitting music theory's characteristic epistemological slippage. 'But', argues Cook, 'their performative effect, their impact on perception or belief, remains the same'.[127]

1.11 Conclusions: Of and About Music

The roles that science and mathematics play in music theory, like the very music analyses they facilitate, resist neat aphoristic summary. Some of the oppositions and conflations with which this chapter opened are in fact complex relations of co-dependence at arms' length (namely the oppositions of science with mathematics and psychology with music theory), while others (such as formalism's conflation with science and opposition to performativity and musical relevance, or mainstream theory's conflation with mathematics and science and opposition to pragmatism) are stereotypes resulting from a misreading of the nature of analytical knowledge. A central chimera – the "scientific music theory" – is usually either not conventionally scientific or not conventionally a music theory. Any analytical utterance, mathematical or not, is necessarily metaphorical: mathematical models and statistical conclusions of and about music should therefore not pretend (or be perceived to be pretending) to be explanatory "improvements" or natural laws.

Mathematical methods do not, in themselves, give scientific answers: but the epistemology of music theory as a humanities subject permits, in its slippage between the descriptive and the suggestive, the incorporation of scientific methods to serve the music-theoretical end of achieving a

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semantic music theory; likewise, "scale theory", for all its mathematical rigour, is actually a fairly heterogeneous collection of approaches that each choose to formalise and generalise a different property of the diatonic scale, from the fact that it is formed by taking a tritone shortcut across the circle of fifths, to the fact that each interval appears within it a unique number of times, to the fact that each n-note set produces n different chromatic forms when transposed along the scale. A slightly different take on this idea is provided by Cohn and Dempster: rather than arguing that an analysis is a "theory" of a piece formed from a patchwork of systems, they suggest that a piece might be seen as a (potentially unique) "solution" arising from a number of simultaneously-deployed generative procedures (a Schenkerian tonal hierarchy overlaid with a topical narrative and a motivic process of expansion, for example). See Richard Cohn and Douglas Dempster, 'Hierarchical Unity, Plural Unities: Toward a Reconciliation', in *Disciplining Music: Musicology and Its Canons*, ed. by Katherine Bergeron and Philip V. Bohlman (Chicago: University of Chicago Press, 1992), pp. 156–81 (pp. 174, 176).

[126] 'Position Paper', p. 150.

[127] See Cook, 'Epistemologies', p. 98.

good comparison. Mathematics, if it avoids pseudo-scientific opacity, can provide extremely powerful conceptual frameworks, facilitating rich and detailed metaphors for the musical object (however defined) under consideration: the goal of the rest of this thesis is to do this with respect to the music-theoretically mediated "object" of motivic structure. And while 'messy psychological premises' (see n. 1) might take us closer to a scientific understanding of music, the practice of devising, exploring, modifying, applying, stretching, breaking, and untidying 'tidy mathematical systems' retains its powerful intellectual, aesthetic, creative, and performative value.

2 Motive(s): Towards a Mathematical Model

2.1 The Problem of Repetition

Despite subtle distinctions in usage (even amongst occurrences of the same term), the concepts 'motive', 'theme', 'figure', 'idea', 'germ', 'cell', and 'paradigm' (among many others including, perhaps, 'pitch-class set class') all share a single defining characteristic: they are all predicated on the idea of recurrence or repetition. Schoenberg and Reti, the two writers whose influence on today's motivic thinking is arguably the most significant, respectively stress that '[*t*]*he most important characteristic of a motive is its repetition*' and define a motive as 'any musical element […] which, by being constantly repeated and varied throughout a work or a section, assumes a role in the compositional design somewhat similar to that of a motif in the fine arts'.[1] Nicolas Ruwet's paradigmatic method begins by searching for 'sequences – the longest possible – which are repeated in their entirety', and Schenker defines a motive as 'a recurring series of tones' forming 'the basis of music as an art' in his earlier writing (a viewpoint echoed more recently by John Rothgeb's observation that 'Schenkerian thought recognizes only one imperative for thematic content: the necessity of repetition').[2] The idea of repetition persists despite Schenker's radical redefinition of both a motive's material (as a linear progression rather than a rhythmic series of pitches) and its significance within a piece's organisation (a negative assessment not borne out, argues Richard Cohn, by Schenkerian practice), meaning that Pieter van den Toorn is right to assert that '[m]otives are inseparable from patterns of repetition'.[3]

As aphorised by George Kubler, however: 'It is in the nature of being that no event ever repeats, but it is in the nature of thought that we understand events only by the identities we imagine among them.'[4] If, then, musical repetition can never strictly occur, these imagined identities must be constructed by prioritising certain musical parameters and ignoring others: this is 'the neutralisation of differences considered to be negligible' in Nattiez's phrase.[5] This

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[1] Arnold Schoenberg, *Coherence, Counterpoint, Instrumentation, Instruction in Form*, ed. by Severine Neff, trans. by Charlotte M. Cross and Severine Neff (Lincoln: University of Nebraska Press, 1994), p. 31; Reti, *Thematic Process*, pp. 11–12, n. 1.
[2] Nicolas Ruwet, 'Methods of Analysis in Musicology', trans. by Mark Everist, *Music Analysis*, 6 (1987), 3–36 (p. 18); Schenker, *Harmony*, pp. 4, 5; John Rothgeb, 'Thematic Content: A Schenkerian View', in *Aspects of Schenkerian Theory*, ed. by David Beach (New Haven: Yale University Press, 1983), pp. 39–60 (p. 39).
[3] See Cohn, 'Autonomy' and Pieter C. van den Toorn, 'What's in a Motive? Schoenberg and Schenker Reconsidered', *Journal of Musicology*, 14 (1996), 370–99 (p. 370).
[4] George Kubler, *The Shape of Time: Remarks on the History of Things* (New Haven: Yale University Press, 1962), p. 67.
[5] '*Density 21.5*', p. 253.

prioritisation is often left implicit, usually resting on characteristics of relative or absolute pitch and rhythm whilst allowing other parameters such as timbre and dynamics to vary; however, the arbitrariness (at least in theory) of these choices is a recurrent theme in writings espousing a paradigmatic method.[6] This idea can be taken to two opposing extremes. Firstly, we can adopt the Heraclitean dictum that it is impossible to step in the same river twice: even under the strictest identity conditions, there is an unavoidable experiential and therefore structural distinction between the first and second hearings of "the same" pattern. This lies at the heart of Kierkegaard's characterisation of '[t]he dialectic of repetition': 'what is repeated has been, otherwise it could not be repeated, but precisely the fact that it has been gives to repetition the character of novelty'.[7] A motive is therefore an inherently plural entity as it can only exist as a relationship between two or more musical events: a relatively uncontentious conclusion that nevertheless seems to challenge what Zbikowski has recognised as 'our intuition that there is just one main form of the opening motive, despite evidence to the contrary'.[8]

The second possible strategy is to acknowledge that, since quasi-arbitrary criteria must be applied to define repetition, the term can be broadened to include instances of variation, elaboration, transformation, or development (or, again, a variety of other similar terms). The first conclusion casts repetition as a special kind of development, the second casts development as a special kind of repetition: in practice, the conclusions are therefore largely equivalent in that they speak to the impossibility of separating the two categories out. However, the term 'development' carries more than a simple implication of 'change': it also suggests teleology, growth, or evolution and, in its musical sense, a certain kind of process, argument, or plot. In a cautionary article, Peter Hoyt notes that "developing" originally carried a sense of "uncovering", "bringing out", or "de-enveloping" what is already present (as in "to develop a photograph"), and as such was actually contrasted with ideas of wilful change and restlessness (since introspective character development

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[6] See, for example, Nattiez, '*Density 21.5*', pp. 248–49 and Ruwet, p. 17, which includes the somewhat unusual claim that 'segments, variable as to pitch and duration, can be considered as repetitions as long as they are identical in other respects'.

[7] S[øren] Kierkegaard, *Repetition: An Essay in Experimental Psychology*, trans. by Walter Lowrie (London: Oxford University Press; Princeton: Princeton University Press, 1942), p. 34. The understanding that 'musical time, unlike architecture, permits no simple relationships of symmetry' is characteristic of Adorno's thought (*Mahler*, p. 52), and also inflects Agawu's paradigmatic method (see *Music as Discourse*, pp. 200, 268, for example).

[8] Lawrence M. Zbikowski, 'Musical Coherence, Motive, and Categorization', Music Perception, 17 (1999–2000), 5–42 (p. 24). This tension is also acknowledged by Cohn, who notes that '[a]lthough a *motive* is a singular abstract entity, the term normally implies the existence of a plural set of realizations, and some relationship between them' ('Autonomy', p. 165), and Emilios Cambouropoulos, who suggests that musical similarity is not about connecting entities, but rather 'the emergence of the core musical entities themselves' which exist 'primarily by virtue of self-reference via repetition and variation' (Emilios Cambouropoulos, 'How Similar Is Similar?', in *Discussion Forum 4B: Musical Similarity*, ed. by Petri Toiviainen (=*Musicae Scientiae*, 13.1 suppl. (March 2009)), pp. 7–24 (p. 8)).

requires a slowing of the action).[9] This original sense of development remains embedded in the word and defines its most problematic aspect: its association with the organicist idea that, as expressed metaphorically by Schoenberg, '[i]n an apple tree's blossoms, even in the bud, the whole future apple is present in all its details'.[10]

In understanding the term 'development' and its organicist implications, we must be careful, however, not to confuse temporal with logical growth, or necessity with possibility. Immediately following his apple metaphor, Schoenberg goes on to clarify its subject (ibid.): 'Similarly, a real composer's musical conception, like the physical, is one single act, comprising the totality of the product.' He is therefore not suggesting that the "seed" of a piece of music is its first few bars, but rather the abstract 'idea' (*Gedanke*) which the piece realises; the confusion between background-to-foreground compositional growth and left-to-right "narrative" growth has been likened by Ruth Solie to the discredited biological theory that ontogeny (the growth process of an individual) recapitulates phylogeny (the evolution of a species).[11] And while many writers argue that a short melodic phrase can imply certain variations or methods of development, this is often followed by a reference to the critical role of the composer in selecting and ordering which of these possible continuations to use. Johann Christian Lobe, writing in 1850, argues that 'invention, strictly speaking, thus resides purely in the model, for the extension is a continuation of what already exists', but then that 'the possibilities for transformation […] cannot be wholly exhausted' by an individual piece.[12] Similarly, A. B. Marx, writing in 1856, remarks that no phrase 'actually exhausts its content' in terms of variation, and Riemann, writing in 1890, sketches some possible variations of a Bach theme and then notes: 'We find none of these possibilities taken up in the finished piece. It is for us to concentrate on what Bach has selected from the wealth of possibilities'.[13] Distinguishing compositional from temporal growth helps to resolve the apparent contradiction between Schoenberg's apple metaphor and his argument, in a draft for his unfinished book *The Musical Idea and the Logic, Technique, and Art of its Presentation*, that 'the usual understanding of the <u>motive as germ</u> of the piece out of which it grows' is inadequate since 'if this conception

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[9] See Peter A. Hoyt, 'The Concept of *Développement* in the Early Nineteenth Century', in *Music Theory in the Age of Romanticism*, ed. by Bent, pp. 141–62 (pp. 151–53, 159).
[10] Arnold Schoenberg, 'Folkloristic Symphonies', in *S&I* , pp. 161–66 (p. 165).
[11] See Ruth A. Solie, 'The Living Work: Organicism and Musical Analysis', *Nineteenth-Century Music*, 4 (1980–81), 147–56 (pp. 153–54) for more on this distinction, and for examples of its confusion in writings by Schenker and Reti.
[12] Johann Christian Lobe, 'First-movement Form in the String Quartet: Beethoven: Op. 18 no. 2 in G: Allegro', in *MA19I*, pp. 201–20 (introduction pp. 197–200) (pp. 211, 215).
[13] Adolf Bernhard Marx, 'Form in Music', in *Musical Form in the Age of Beethoven: Selected Writings on Theory and Method*, ed. and trans. by Scott Burnham, Cambridge Studies in Music Theory and Analysis, 12 (Cambridge: Cambridge University Press, 1997), pp. 55–90 (p. 73); Hugo Riemann, 'Prelude and Fugue in B♭ minor, Bk II No. 22 from J. S. Bach's *Well-tempered Clavier*', in *MA19I*, pp. 114–24 (introduction pp. 108–13) (p. 114).

were correct, only one single piece could arise from one motive. As is well known, such is not the case'.[14]

A melodic segment's course is not predestined, therefore, by any inherent properties of its musical material; the same could be said of a piece of music's first chord. But there is a crucial difference between the motivic and harmonic domains, and that is in the existence of a syntax which identifies a universe of possible objects, describes which objects are more or less likely to follow each other, and provides a means for understanding those transitions hierarchically. It therefore makes sense to speak of a piece's harmonic or tonal course, but the idea of a motivic course is more ambiguous: as Carl Dahlhaus argues, degree of change and temporal separation within the piece are two independent dimensions, but 'the concept of development misleadingly suggests that the one coincides as a matter of principle with the other'.[15] Agawu similarly contrasts '*logical*', or 'simple-to-complex', ordering with '*chronological*' ordering, arguing that the 'dissonance between [these] domains is far more widespread and fundamental than has so far been recognized'.[16]

There is therefore no theory of motivic development – no 'specific and independent structural/processive principle' that 'transcend[s] the taxonomic [and] explains, rather than describes, the diachronic ordering of variants', to use Meyer's list of desiderata.[17] This is attributed by van den Toorn to 'the closeness with which motives are bound to individual contexts and their makings' (p. 383), whilst Meyer proposes a socio-historical argument: that structures 'based upon classlike identity rather than on learned constraints' helped to fulfil 'the needs of the novices in the bourgeois audience' during the nineteenth century.[18] Both Schoenberg and Reti acknowledge this theoretical vacuum but demur from claiming to fill it: Reti's opening pages, for example, argue that

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[14] *Musical Idea*, p. 151n (see n. 1 above for information on citations of this volume). I have preserved Schoenberg's (and the editors') idiosyncratic formatting as much as possible.
[15] Carl Dahlhaus, 'What Is "Developing Variation"?', in *Schoenberg and the New Music: Essays by Carl Dahlhaus*, trans. by Derrick Puffett and Alfred Clayton (Cambridge: Cambridge University Press, 1987), pp. 128–33 (p. 133).
[16] *Music as Discourse*, p. 167.
[17] Leonard B. Meyer, 'A Pride of Prejudices; Or, Delight in Diversity', *Music Theory Spectrum*, 13 (1991), 241–51 (pp. 245, 246).
[18] Meyer, 'A Pride of Prejudices', p. 243; see Leonard B. Meyer, *Style and Music: Theory, History, and Ideology* (Philadelphia: University of Pennsylvania Press, 1989), pp. 163–217 for a more detailed examination of romanticism's 'elite egalitarian[ism]' (p. 163). Meyer's argument is problematised in a perceptual study by Alexandra Lamont and Nicola Dibben, which finds that listeners are more likely to base similarity judgements on '"surface" features such as dynamics, articulation, texture, and contour rather than on "deeper" features such as motivic or harmonic relationships' (Alexandra Lamont and Nicola Dibben, 'Motivic Structure and the Perception of Similarity', *Music Perception*, 18 (2000–01), 245–74 (p. 245)) – although, of course, these sets of features may coincide in any given example, and motivic resemblances pointed out by analysts exist at a variety of depths from the obvious to the unconvincing.

'"thematic structure" has become an almost fundamental *term* in music, yet its full meaning and content have never been realized concretely' in a manner 'analogous to, and complementing, the old disciplines of harmony, counterpoint, and the general schemes of form'; his later analyses then make the point that 'thematic phenomena are so manifold and complex that in a sense they evade academic tabulation. Though they can perhaps be described, they can hardly be comprised in an actual "system." They are too intimately connected with the creative process itself'.[19] Schoenberg, who 'did not claim to be a theorist' according to the editors of his draft of *The Musical Idea*, was cautious of the distinction between theory and explanation on one hand and description and presentation (both of a piece through an analysis and a musical idea through a piece) on the other.[20] Such accounts, however, undersell the importance of "mere" description to the theoretical enterprise and expect too much from theory in terms of prediction and explanation (see Sections 1.8 and 1.9); these issues come into even sharper focus within the context of a music theory of good comparison.

The motive-concept in music is torn between static, synchronic, category-dependent repetition and dynamic, diachronic, syntax-dependent development, and each of these poles embeds its own inherent problems which are explored in detail in Sections 2.3 and 2.4. The tension between the poles is the subject of Section 2.2, which proposes four archetypal models of how it might be negotiated and shows how the interactions of these models can be seen to underpin four important motivic theories. Before this, however, it is necessary to establish the musical phenomena that I am taking the term 'motive' to include. The definition produced will lie at the descriptive end of the explicative spectrum (see Section 1.7); but as some stipulative closure is unavoidable, it will mark out the relevant literature to be discussed in this chapter (in particular the four important motivic theories of Section 2.2), and will also define the object for which a mathematical model is sketched in Section 2.5 and expanded in Chapter 3. While Definition 1 seeks to capture existing usage, a definition cannot be assessed in isolation from its theory (as argued in Section 1.7): it assumes and implies properties and relationships that, strictly speaking, lie beyond its object. In particular, Definition 1 fixes the term ***motivic segment***, which does not make sense without understanding that these segments relate to each other in certain ways to form motives. Unpacking these relationships is the goal of this chapter, while modelling them mathematically is the goal of the next.

The term 'motive' is used in this thesis as an umbrella for a variety of concepts (some of which are listed in the first sentence of this chapter) that are neither wholly interchangeable nor wholly distinct (the crucial commonality being, of course, repetition). Writing over 150 years ago,

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[19] Reti, *Thematic Process*, pp. 3, 5, 233.
[20] Editors' preface to *Musical Idea*, pp. xv–xxi (p. xvi).

A. B. Marx complains that the term 'motive' is 'lost […] in slippery vagueness (it designates a melody, a fragment of a melody, a phrase built out of melody and harmony – anything one wants)', and this situation largely persists today, although with certain common strands emerging.[21]

Firstly, a motive is usually a melodic entity, a segment of what Leopold Mozart referred to as a composition's *filo* ('thread'), but it can also permeate other areas of texture: Schoenberg elevates an 'accompanying voice' to a 'subordinate voice' if it follows its own independent process of developing variation.[22] Its content is usually defined in terms of pitch, but rhythm has a role to play too: William Drabkin's *Grove* entry acknowledges the possibility of independent rhythmic motives 'capable of recognizable performance on unpitched percussion', Adam Ockelford and others argue the importance of rhythmic similarity in creating coherence among dissimilar pitch patterns, and, according to his editors, '[t]he importance of rhythm to Schoenberg's concept of motive cannot be overemphasized.'[23] Harmony and tonality normally assume secondary roles (although they may, argues Drabkin, 'contribute potently to a composite motif', and Schoenberg suggests that harmonic change is an important category of variation) unless a pitch sequence is abstracted away from a concrete appearance and used to explain a particular simultaneity or succession of key areas.[24] This strategy bears the influence of pitch-class set theory (which treats a motive as an unordered set of intervals) and Schenkerian theory (which treats a motive as a linear progression that may appear at any hierarchical level).[25]

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21 'Form in Music', p. 66.
22 Schoenberg, *Musical Idea*, pp. 267–69. For uses of the thread metaphor in musical writing, see Grey, '*...wie ein rother Faden*'; Leopold Mozart is discussed on pp. 197–98, n. 23.
23 William Drabkin, 'Motif', in *Grove Music Online* <http://www.oxfordmusiconline.com> [accessed 17 December 2014] (para. 2); Adam Ockelford, 'Similarity Relations Between Groups of Notes: Music-Theoretical and Music-Psychological Perspectives', in *Discussion Forum 4B*, ed. by Toiviainen, pp. 47–98 (p. 74); editors' commentary to Schoenberg, *Musical Idea*, pp. 1–86 (p. 28). The idea associated with Ockelford above is also found in Arnold Schoenberg, *Fundamentals of Musical Composition*, ed. by Gerald Strang with Leonard Stein (London: Faber and Faber, 1967; repr. 1983), p. 8; see Chapter 3, n. 34 below. Similarly Hoffmann, in his review of the Fifth Symphony, admires Beethoven's ability 'to relate all the secondary ideas [*Nebengedanken*] and episodes by their rhythmic content [*rhythmischer Verhalt*] to [a] simple theme' (p. 153; German terms in square brackets are included in Bent's edition).
24 Drabkin, para. 2; Schoenberg, *Fundamentals*, p. 10.
25 See, for example, van den Toorn, pp. 374–79, which takes the opening motto of Brahms's Third Symphony as the source of the first movement's modal mixture, common-tone pivots and modulations to the lowered submediant; van den Toorn also points out that the sequential repetitions of the motto in the movement's transition generate the 'celebrated hexachord' set 6-20 (p. 379), an observation which comes complete with a Freudian-slip reference to Allen Forte's *The Structure of Tonal* [sic] *Music* (n. 29). Reti, by treating motives as abstract collections of pitches and intervals, approaches this viewpoint in a number of his analyses, but remains open to charges of arbitrariness since he cannot justify the selection of non-contiguous notes through recourse to a Schenker-style hierarchical theory (see, for example, Rothgeb, pp. 41–42).

Secondly, motives are usually understood as *units*, what Steven Jan refers to as 'discrete, "digital" patterns within the fluid, "analog" continuity of the sound stream', distinguished from each other and possibly from "non-motivic" material (termed 'connective tissue' by Jan).[26] Even without imposing a higher-level motivic theory, the concept of breaking a melody up into units constitutes an important and unavoidable perceptual mechanism that relies heavily on patterns of repetition. Temperley has argued that metrically parallel units (note the importance of rhythm) are perceived in a so-called 'modular' way that is fast, automatic, and nearly impossible to override conceptually (just as the "trick" of an optical illusion – e.g. that two lines are the same length – can never be *seen*, even if it is known): this automatic segmentation also influences, as much as it is influenced by, the perception of meter, and its effect is therefore lessened if metric and pitch patterns do not align.[27] The issue of segmentation comes to the fore in paradigmatic or computational approaches, as discussed in more detail in Sections 2.2 and 3.1; but it is not a universal music-theoretical preoccupation, and it is particularly secondary in the continuous and overlapping generative process of Schoenberg's developing variation (examined in Section 2.2 and exemplified in Figure 2.2). However, the term 'gestalt' – as a 'characteristically articulated' and more-or-less "closed" unit – retains its place in Schoenberg's terminological armoury, most famously in his concept of the *Grundgestalt* (see Section 2.2).[28]

Finally, the notion of a motive being 'the shortest complete idea in music' (for Parry), 'the shortest subdivision of a theme or phrase that still maintains its identity as an idea' (for Drabkin), or 'the smallest *recognizable* part of a musical work' (for Zbikowski) is pervasive but exists in tension with the concept of variation, since if a' is understood as a variation of a then the two must share some smaller subset of features.[29] Schoenberg attempts to resolve this problem by defining a motive as 'at any one time the smallest part' of a gestalt (a concept analogous to what is referred to here as a 'motive'): his definition then continues by referring to a motive as 'a complex of interconnected features', thereby invoking a still smaller entity (the 'feature').[30] Nevertheless, given that it is unlikely that an entire formal section would be referred to as a motive, we might include

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[26] Steven Jan, 'Meme Hunting with the Humdrum Toolkit: Principles, Problems, and Prospects', *Computer Music Journal*, 28.4 (Winter 2004), 68–84 (p. 68).
[27] David Temperley, 'Motivic Perception and Modularity', *Music Perception*, 13 (1995–96), 141–69. For a recent experimental study testing the effects of misalignment between metric grouping and motivic pattern, see Stefanie Acevedo, David Temperley, and Peter Q. Pfordresher, 'Effects of Metrical Encoding on Melody Recognition', *Music Perception*, 31 (2013–2014), 372–86.
[28] Schoenberg, *Musical Idea*, p. 171.
[29] See Parry's 1906 article on 'Figure' for *Grove's Dictionary*, quoted in Dunsby, p. 907; Drabkin, para. 1; and Zbikowski, p. 13. Although recognition of variation is not straightforwardly predicated on similarity, as I argue in Section 2.3, point six below, connected segments do usually share at least some of the same outward features.
[30] *Musical Idea*, pp. 169–71.

some notion of brevity (although more along the lines of a musical molecule than a musical atom) to arrive at a descriptive definition of a motivic segment:

Definition 1

A ***motivic segment*** is a short, recognisably recurrent melodic gestalt.

2.2 Four Archetypal Models and Four Motivic Theories

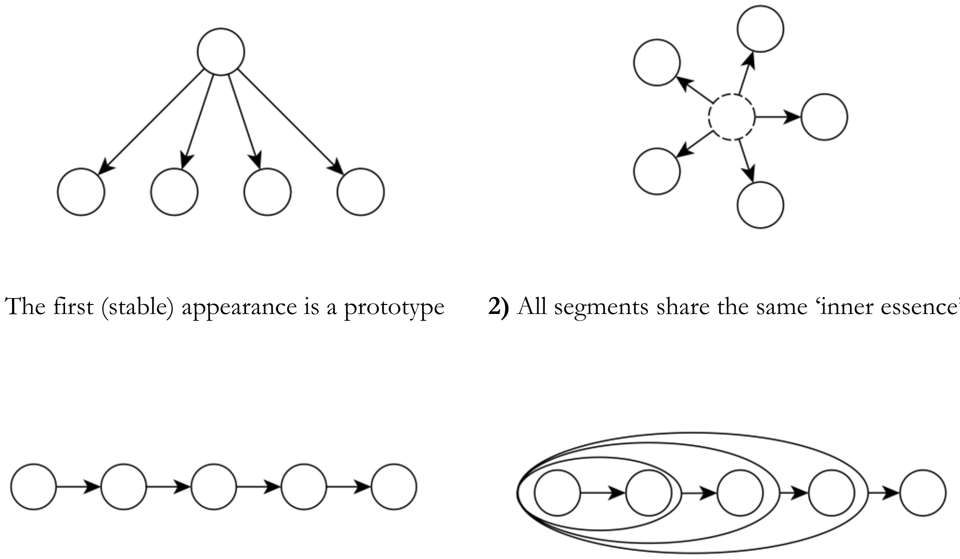
Since motivic segments are concrete sequences of notes but recurrence is also an important part of Definition 1, a given motivic segment implies the existence of a set, $\mathcal{M}$, of related motivic segments drawn from the same piece of music. The relationships between the segments in this set must negotiate the tension between repetition and development discussed above, and this negotiation is realised in a slightly different way by each of the four archetypal models proposed below and illustrated schematically in Figure 2.1. Real motivic theories and analyses frequently blend aspects of all four; but it is useful, following Schoenberg's aim of good comparison, to separate out these distinct conceptual and formal underpinnings in order to sharpen an assessment of how that blending functions in specific cases. No assumptions are made at this stage concerning:

a) whether or not every note in the piece is contained in a segment contained in $\mathcal{M}$;
b) whether any material outside $\mathcal{M}$ is non-motivic connective tissue, or whether it falls into other sets that may partly overlap with $\mathcal{M}$.

The four models, then, might be characterised as follows:

1) The first ("stable") segment of $\mathcal{M}$ to appear in the music serves as a prototype against which all the others are compared.[31] This is seldom an explicit part of any theory; but since, as Zbikowski notes, 'it is often the case that significance accrues to the first events in any psychological process' (p. 23), it is usually a feature of analyses. It is implied by analytical writing which labels a musical excerpt as, say, motive x, and uses phrases such as 'x is declaimed by the trombones in bar 17' and 'a tune based on an inversion of x is heard in bar 27'.

[31] The qualifier "stable" is intended to account for the fact that the first appearance of a given motivic category is not always considered to be its most analytically pertinent: the classic case is that of a movement's introduction "foreshadowing" its main themes, a device which Meyer considers to be a rare genuine example of motivic syntax (see Section 2.4 at n. 106) and which Edward Cone designates an instance of reverse derivation (see Section 2.3, point six). Trying to identify which appearances "state" and which merely "foreshadow" is a problem which recalls the *sorites* paradox (see Section 2.3, point one), so while the term is admitted as a heuristic in this descriptive archetype, no such explicit formal distinction is made in the model developed in this thesis (see, nevertheless, Chapter 3, n. 16, which explores how passages which "foreshadow" others can give rise to particular formal structures).



1) The first (stable) appearance is a prototype **2)** All segments share the same 'inner essence'

3) 'Derivational unity' vs. 'uniform substance' **4)** A category emerges through time

Figure 2.1: Four archetypal models of the negotiation between repetition and development.

2) Each member of $\mathcal{M}$ is a realisation of the same abstract type, so all share the same '*inner essence*' (Reti's phrase) and display what Solie terms 'organic unity'.[32] For some writers, this makes the members of $\mathcal{M}$ equivalent and interchangeable; but for others, this inner essence is a deeply buried compositional principle that unifies otherwise contrasting passages (such as first and second subjects). A. B. Marx, for example, writes 'that every [phrase] is one-sided, that one can glean other, even opposed, sides from its content, giving the [phrase] a consequent or an opposing phrase. These two (or more) phrases belong to each other, in accordance with their related content, and can form an internally unified whole'.[33]

3) Each member of $\mathcal{M}$ varies the previous member in a continuous process of development (Solie's 'organic growth', to be contrasted with the 'organic unity' of archetype 2) so that, in Dahlhaus's words, 'while one may sense a derivational unity, it is not always possible to speak of a uniform substance'.[34] This process may trace or thematise a directed progression, such as the "resolution" of a particular motivic interval (see Section 2.4); there

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[32] See Reti, *Thematic Process*, p. 13 and Solie, pp. 148–52.

[33] 'Form in Music', p. 73. Here his term *Satz*, which Burnham retains in the original German, has been replaced with [phrase]; Burnham's parenthetical original German terms have also been excised here. '*Satz*' and 'phrase' are not equivalent concepts, but the substitution clarifies the quotation without the need for a lengthy terminological digression: Burnham notes elsewhere that *Satz* may refer to 'any closed structure' ranging from phrase to theme to movement (Adolf Bernhard Marx, 'From *The Theory of Musical Composition*', ed. and trans. by Scott Burnham, in *Source Readings in Music History*, ed. by Strunk and Treitler, VI, 181–89 (p. 181n)).

[34] See Solie, pp. 152–54 and Dahlhaus, 'Developing Variation', pp. 132–33.

may also be an implication that this trajectory is predetermined (as discussed in Section 2.1).

4) The set $\mathcal{M}$ unfolds with the music, as an evolving category, with each new member of $\mathcal{M}$ being weighed against, and ultimately contributing to, a combination of all the previous members. This 'cognitive re-alignment of material as perceived over time' is considered to be a hallmark of Kierkegaardian 'dynamic repetition' (as opposed to developing variation) by Tim Howell.[35] It differs from archetype 2 in that it has an implied directionality and respects Adorno's argument that '[u]nity […] is undermined as soon as it ceases to unify a plurality': its path from heterogeneity to unity has been, for Dahlhaus, 'neglected in analytical practice' as it traces 'the opposite process' to that of development.[36] If combined with 3, it adds Dahlhaus's uniform substance to his derivational unity and so underpins interpretations in which we are invited to see "the same" object, such as a character in a narrative, changing through time. This archetype, being closely related to the idea of category formation, often arises in approaches influenced by cognitive science.[37]

These archetypes are just that: archetypes. To illustrate their utility as conceptual tools for understanding the assumptions which underlie specific motivic theories, analyses, and models, the remainder of this section is given over to detailed explorations of four styles of analysis that have proved to be influential on today's motivic thought: Schoenbergian, Retian, paradigmatic, and leitmotivic.

Schoenberg's concept of *developing variation* – as a series of changes to motivic segments that 'proceed more or less directly toward the goal of allowing new ideas to arise' – is largely underpinned by archetype 3, and his understanding of development as a structural, generative process leads him to consider a sonata's development section to be misnamed since it 'seldom lead[s] to the "development" of anything new'.[38] At the same time, however, terms such as 'variant' (a segment changed in a way that has 'little or no influence on the continuation') and 'motive-form' suggest the static, category-based conception of archetype 2: the inner essence in this

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[35] Tim Howell, 'Brahms, Kierkegaard and Repetition: Three Intermezzi', *Nineteenth-Century Music Review*, 10 (2013), 101–17 (p. 111).
[36] Adorno, 'Centenary Address', p. 94; Dahlhaus, 'Developing Variation', p. 133. The idea of earned unity also underpins Cone's notion of epiphany; see Section 2.3, point six.
[37] See, for example, Cambouropoulos (quoted in n. 8 above) and Zbikowski.
[38] *Coherence,* p. 39; *Fundamentals*, p. 200n. The following discussion is largely informed by *Musical Idea*, including the editors' preface, pp. xv–xxi, commentary, pp. 1–86, and useful 'Concordance of Terms', pp. 351–401, which brings together occurrences of selected key terms in Schoenberg's writings. Also useful as a summary of Schoenberg's thought, complete with a speculative analytical example of his ideas in practice, is Severine Neff, 'Schoenberg and Goethe: Organicism and Analysis', in *Music Theory and the Exploration of the Past*, ed. by Christopher Hatch and David W. Bernstein (Chicago: University of Chicago Press, 1993), pp. 409–33.

case would be the motive.[39] Motive-forms are chained together into gestalten (more complete units such as phrases and themes), and a work's gestalten can be traced back to their *Grundgestalt*, a prototype suggestive of archetype 1 (although this understanding is complicated by Schoenberg's stipulation that a twelve-tone work's *Grundgestalt* is its tone row, relating the term to a more abstract entity).[40] The chaining together of motive-forms saturates the texture to the point that the Schoenbergian answer to a) above would be affirmative: the 'musical prose' generated by developing variation, exemplified for Schoenberg above all by the music of Brahms, depends on the overlapping of motive-forms (which in some cases are simply intervals) with phrase boundaries and each other to create a texture in which every note is motivic.[41] This technique is illustrated most clearly in Schoenberg's analysis of the main theme of the Andante of Brahms's String Quartet, Op. 51, No. 2 in his essay 'Brahms the Progressive' (reproduced here as Figure 2.2). Such an account can conflict with the automatic modular perception of metrically parallel motives (see Temperley's explanation cited at n. 27 above) through 'the unexcelled freedom of its rhythm and [its] perfect independence from formal symmetry'.[42]

The mediation between the different archetypes at work in Schoenberg's thought is provided by the central concept of the *idea* (*Gedanke*), 'the essence of the work, […] its total dynamic, the balance of forces within the whole'.[43] According to Schoenberg, every *Grundgestalt* embeds a 'tonal problem':

> Every tone which is added to a beginning tone makes the meaning of that tone doubtful. If, for instance, G follows after C, the ear may not be sure whether this expresses C major or G major, or even F major or E minor; and the addition of other tones may or may not clarify this problem. In this manner there is produced a state of unrest, of imbalance which grows throughout most of the piece, and is enforced further by similar functions of the rhythm. The method by which balance is restored seems to me the real *idea* of the composition.[44]

The purpose of developing variation is therefore to 'elaborat[e] the *idea* of the piece' by presenting and solving the problem; Schoenberg likens the process to that of an architect who cannot begin without a plan, but must ultimately commence the building work by joining two bricks together.[45] In this way Schoenberg evokes archetype 4 through the implication that the stages of 3, the various motive-forms and gestalten, are in fact the same *Grundgestalt* (type 1) reinterpreted in new contexts and incrementally progressing towards a goal – a goal that is not embedded in the first segment as such, but in the overall movement design (of which the first segment is nevertheless a crucial

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[39] *Fundamentals*, p. 9.
[40] *Musical Idea*, pp. 169, 259.
[41] Arnold Schoenberg, 'Brahms the Progressive', in *S&I*, pp. 398–441 (p. 415).
[42] Schoenberg, 'Brahms the Progressive', p. 416.
[43] Editors' preface to *Musical Idea*, p. xix.
[44] Arnold Schoenberg, 'New Music, Outmoded Music, Style and Idea', in *S&I*, pp. 113–24 (p. 123).
[45] Arnold Schoenberg, 'Bach', in *S&I*, pp. 393–97 (p. 397); *Musical Idea*, p. 151.
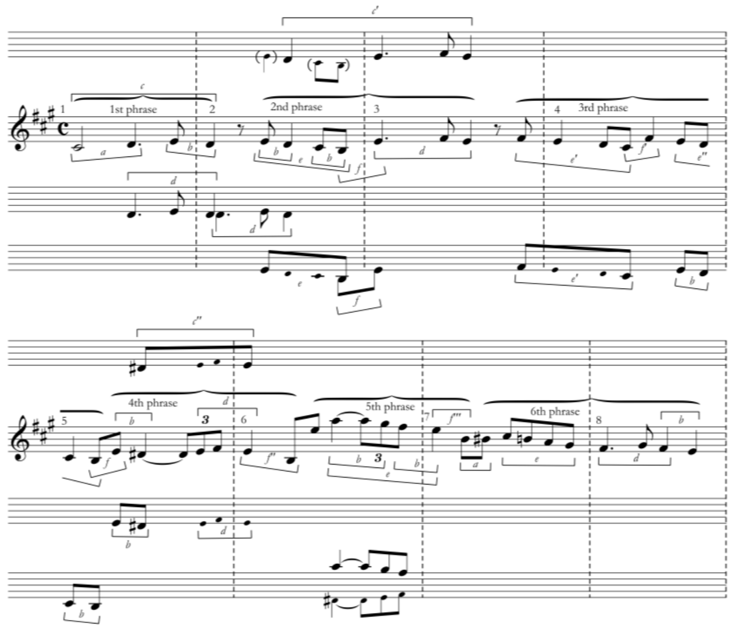
Figure 2.2: An example of developing variation.
Schoenberg's analysis of the opening theme of the Andante from Brahms's String Quartet, Op. 51, No. 2.[46]

initiating part: Schoenberg's translators bring out the resonances between 'motive', 'motion', and 'motor').[47]

Reti, despite the title of his theoretical monograph (*The Thematic Process in Music*), is more concerned with 'uniform substance' than 'derivational unity', and therefore tends to adhere more closely to archetype 2; recall that the phrase 'inner essence' is his. However, the full context of this phrase reveals that it is not meant to imply stasis or repetitiveness: '[The composer] strives toward *homogeneity in the inner essence* but at the same time toward *variety in the outer appearance*.'[48] Similarly to Schoenberg, then, he sees his 'process' as a means to generate new material from old whilst retaining a deep structural unity; but '[w]hether or not the inner identity is *demonstrable* is a matter of entire indifference to the composer' (p. 243). It also appears to be a matter of indifference to the performer and the listener, as Reti explains when elucidating the meaning of a particular analytical observation (p. 47):

> It does not imply by any means that this B, G, E, should be artificially accented by the performer, nor that it must be heard and understood as a motivic utterance by the listener. The unnoticeable influence that it may exert on the listener as a

[46] Processed from Ex.46 in 'Brahms the Progressive', p. 430.
[47] *Coherence*, p. 27; cf. p. 26 '*Motiv*', '*Bewegung*', and '*Motor*'.
[48] *Thematic Process*, p. 13; further references are given in parentheses in the text.

> passing subconscious recollection – in fact, *its theoretical existence in the piece* – suffices. It constitutes a symbol of the ever recurring idea, nothing more.[49]

Reti's perspective is therefore (re-)compositional, asking 'not whether shapes are "similar" but whether the composer in forming them endowed them with qualities that assure some bond between them' (p. 353): this is why he disavows the term 'theory', since this 'compositional inspiration' does not rely on 'any specific device, old or new, that can be formulated' (p. 67), and indeed need not even be a conscious process of manipulation.[50] His analytical narratives are frequently compositional narratives, aiming 'to retrace the compositional process' (p. 117; perhaps this is the *Process* of his title?) using (re-)constructive language with pretensions to explanation: to take one example, 'the resulting theme would have seemed too short had the composer immediately annexed segment III. Therefore he prolonged the theme by inserting bars 4 and 5' (p. 167).

The other archetypes are, however, by no means absent from Reti's work. Archetype 1 is frequently implicit in the musical excerpts Reti chooses to illustrate a particular motive. Tracing thematic connections among the pieces of Schumann's *Kinderszenen*, Reti describes the first as 'the thematic source and sample for all the following ones' (p. 32), thus invoking archetype 1 at the level of movements rather than motives. He then organises the cycle into groupings of three to five movements and traces a 'progression formed by the keys or, perhaps more adequately expressed, *by the pitches and key qualities* of [the movements'] thematic material' (p. 53), culminating when 'the poet himself steps before the curtain' (p. 50) and states the main motive with its last note as part of the globally stable tonic for the first and only time in the work.[51] The resonances with archetypes 3 and 4, and with Schoenberg's tonal problem and musical idea, are clear, and their implications are realised more fully in chapters 5 (pp. 109–38) and 6 (pp. 139–92) of Reti's book. These are devoted to the phenomenon of 'thematic resolution', 'the problem of how a theme *moves by transformation*

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[49] Reti explicitly contrasts 'theoretical, analytical similarity' (p. 143) with 'musical affinity' (p. 242) at least twice more; discovery of the former does not change the fact that '[a]s an actual musical utterance [a second theme] does truly form a new thought' (p. 143). This sits uneasily with his insistence, near the opening of the book, that '[i]n order to comprehend the full meaning of the following analytic deductions, with all their structural and esthetic implications, the examples quoted must be understood, indeed, *heard*, as musical utterances' (p. 6). It is possible to address this apparent double standard through Temperley's distinction between metrically parallel, 'phenomenologically direct' relationships – which are perceived modularly – and those of a more abstract nature – which can be perceived eventually, but only in a 'slow, deliberate, phenomenologically indirect way' (see 'Modularity', p. 167; cf. 'descriptive' and 'suggestive' theories, Section 1.3). To 'artificially accent' a relationship of the second type, or expect its 'musical affinity' to be heard and understood straight away, would be to misrepresent it as an instance of the first – which is not to say that it cannot ever be heard in principle.

[50] On balance however, 'confronted with an abundant variety of different and irrefutable proofs' (p. 233), and after devoting an entire chapter to the question (pp. 233–47), he convinces himself that the process must be essentially conscious.

[51] The last piece is therefore a 'moment of narration'; see Section 2.4.

toward a goal and how in this process the dramatic development of the work and its thematic course are intertwined' (p. 139); this renewed emphasis on process leads Walter Frisch to single out the analysis of Mozart's Symphony No. 40 in this section as Reti's 'most persuasive'.[52] Reti even directly associates the various occurrences of a theme to 'the same hero' (p. 136): 'The new environments influence the "hero," who in turn influences them, and the final outcome is the result of this interplay, both in a symbolic and in a technical, that is, thematic sense.'

A letter from Schoenberg to Reti, reprinted in facsimile in Reti's second monograph, shows that the former considered the latter to be 'a person who stands very close to my sphere of thought'; the two do indeed share certain similarities of outlook.[53] However, to return to Solie's two types of musical organic growth (that is, background-to-foreground construction and left-to-right progress), Reti largely leaves the relationship between them unexplained. This stands in sharp contrast to Schoenberg's understanding, which relates concept to unfolding exceptionally tightly: developing variation is the means through which the story of the musical idea is told.

By its very nature, **paradigmatic analysis** rests on archetype 2: its goal is to abstract segments away from a piece of music's syntagmatic chain (left-to-right progress) and into categories.[54] This is, however, an inherently provisional separation since syntagmatic analysis 'uses information and segmentation established from a paradigmatic point of view', and paradigmatic segmentation frequently relies on syntagmatic criteria such as rests or sudden changes of register.[55] To use Nattiez's word (ibid.), the point of this style of semiotic analysis is to 'thematis[e]', rather than reinforce, the distinction between the syntagmatic and the paradigmatic.

Another thematised opposition within the paradigmatic method is that of analytic versus synthetic procedures: in other words, the methodological balancing act between the bottom-up empirical project 'to derive the units, classes of units, and rules of their combination which together constitute the code', and the top-down theoretical desire to describe this code 'uniformly with

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52 Walter Frisch, *Brahms and the Principle of Developing Variation*, California Studies in Nineteenth-Century Music, 2 (Berkeley: University of California Press, 1984), p. 22.
53 Reti, *Tonality – Atonality – Pantonality*, p. 48 and plate II. Frisch considers Schoenberg's legacy to lie not in a rigorous theoretical system but a 'critical or analytical tradition' (p. 18) whose major contributors include Reti, Adorno, Erwin Ratz, Josef Rufer, Hans Keller, Alan Walker, Karl Wörner, Arno Mitschka, and Dahlhaus (see pp. 18–29). I would also add David Epstein: see David Epstein, *Beyond Orpheus: Studies in Musical Structure* (Cambridge, MA: MIT Press, 1979).
54 I am using the term 'paradigmatic' in preference to 'semiotic' as the latter can refer to a variety of approaches to meaning and signification in a variety of musics running the gamut from "absolute" to audiovisual. My focus here is on the Ruwet–Nattiez approach (adopted, in part, by Agawu) to what Agawu terms 'introversive semiosis': signs that signify in reference to each other and 'not necessarily by referential or extramusical association' (*Playing with Signs*, p. 51).
55 Nattiez, '*Density 21.5*', p. 252 'uses'; Ruwet, p. 14. See also Chapter 3, n. 14.

maximum internal coherence and simplicity'.[56] The aim, therefore, is to cover and "explain" as much of the analysed piece as possible in terms of a code (a grammar or means of interpretation), and to keep the connective tissue to a minimum.[57] In order to do this whilst keeping the criteria for segmentation and repetition as explicit as possible, thereby avoiding the contortions on display in Figure 2.2, a piece of music will generally have more than one *paradigmatic block* (each of which is roughly equivalent to what is referred to as an $\mathcal{M}$-set above).

In theory, units grouped along the same *paradigmatic axis* (column) are 'equivalent from a given point of view': note that 'this does not mean that they are homogeneous' since the units can be arranged in many different ways corresponding to different criteria of identity, and units on the same axis in one arrangement might lie on different axes in others.[58] The notion of a *block*, 'a more or less homogeneous group constructed by the analyst on the basis of one or several criteria, *dominant* and convergent' is intended to mediate between these conflicting arrangements through analytical assessment of which criteria are most important at a given point; the existence of this feature is what makes paradigmatic analysis impossible to implement computationally, according to Nattiez, since '*explicit* is not synonymous with *algorithmic* or *mechanical*'.[59] Blocks are therefore intended to represent classes of "grammatically" interchangeable units, with the code of syntagmatic succession understood to act directly on these. Paradigmatic analyses can, however, slip into language which suggests syntactically significant change occurring within a block, thereby invoking archetypes 1 ('the head of the paradigm'), 3 ('paradigmatic link with [1] *through* the intermediary [38]') and 4 ('references to the entire substance of what has transpired').[60] Nattiez

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[56] Ruwet, p. 11 'to derive'; p. 12 'uniformly'. These issues are discussed in relation to music theory more generally in Sections 1.8 and 1.9.
[57] Ruwet is, in general, much more explicit about connective tissue than Nattiez. He advocates finding strictly repeated sequences of the greatest possible length, then provisionally labelling everything that is left as a 'remainder'. These remainders are then examined and possibly assimilated on a higher level so that, for instance, (A+x)+(A+y) becomes A+A' (see pp. 18–19). Nattiez, on the other hand, typically skips the intermediary step and assigns varied units, or units only related through distribution (e.g. B and X in A+B+A'+X), to the same block (see, for example, '*Density 21.5*', p. 268).
[58] Nattiez, '*Density 21.5*', p. 248.
[59] Nattiez, '*Density 21.5*', p. 257 '*dominant*'; p. 255 '*explicit*'. Nattiez's comments have not prevented researchers from trying to implement paradigmatic analysis computationally: see, for example, Kamil Adiloglu, Thomas Noll and Klaus Obermayer, 'A Paradigmatic Approach to Extract the Melodic Structure of a Musical Piece', *Journal of New Music Research*, 35 (2006), 221–36, and note Christina Anagnostopoulou and Chantal Buteau's characterisation of Nattiez's supposed belief in 'the neutrality, objectivity and scientific nature of music analysis' (Christina Anagnostopoulou and Chantal Buteau, 'Can Computational Music Analysis Be Both Musical and Computational?', in *Computational Music Analysis*, ed. by Anagnostopoulou and Buteau, pp. 75–83 (p. 76)).
[60] Nattiez, '*Density 21.5*', p. 284 'head'; p. 303 'through'; Agawu, *Music as Discourse*, p. 268 'substance'. The italics have been added, but the square brackets are present in the original and refer to units used in Nattiez's analysis.

even criticises Harry Halbreich's archetype 1 remark that '[e]verything is born of the first bars' by noting that 'not all the relationships between units are *transitive*' (i.e. that not everything explained using archetype 3 of Figure 2.1 can be rephrased in terms of archetype 1); in doing so, he therefore inadvertently implies a major criticism of his own approach, as discussed in point four of Section 2.3 below.[61]

Finally, although the concept was not intended to be applicable beyond the repertoire of Wagner's music-dramas, the **leitmotivic** approach has proved to be more widely influential; one need only call to mind the symphonies with so-called 'Fate' motives (such as Beethoven's Fifth, Tchaikovsky's Fourth and Fifth, and Mahler's Sixth) to see that motives with symbolic or narrative implications are not confined to operatic works. The leitmotive-concept is strongly characterised by its allowance, and even expectation, of both connective tissue and multiple $\mathcal{M}$-sets, an approach which contrasts with the generative, unifying, and texture-saturating models of Schoenberg and Reti. Grey has written on 'the agency of the *Leitfaden* or guiding thread image that underwrites this term', following Theodor Uhlig in comparing leitmotivic structure to the red thread used in weaving and braiding to provide 'a visual means of orientation within a complex fabric'.[62] The thread can disappear from view, beneath 'large swathes of often fairly neutral musical recitation or else semantically "unmarked" (non-recurring) arioso melody' (p. 199), but it is'imaginarily construed as a continuous but subconscious presence, somewhere beneath the threshold of our perceptions or beneath the surface of the work' (p. 204). This stands in contrast to the ever-visible thread of Schoenberg's developing variation as exemplified in Figure 2.2, but the two models can co-exist: Grey refers to 'ordinary, "unmarked" musical motives in other musical contexts' (p. 188), and Reti maintains that although 'thematic structure' and 'thematic symbolism' are 'separate in principle', Wagner lets leitmotives 'emerge as parts of the organic thematic design' and 'endow[s] ordinary thematic phrases with leitmotivic effects', thereby developing a single 'convincing entity' from the two disparate phenomena.[63]

To create a 'leitmotivic effect' and signal when the red thread re-emerges from the 'unmarked' texture, leitmotives need to be clearly recognisable: the first sentence of Arnold Whittall's *Grove* entry on the subject therefore duly insists that a leitmotive must be 'clearly defined

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[61] Nattiez, '*Density 21.5*', p. 331; for Halbreich's remark see Harry Halbreich, 'Etude de l'oeuvre d'Edgard Varèse', in Georges Charbonnier, *Entretiens avec Edgard Varèse* (Paris: Belfond, 1970), pp. 121–67 (p. 151), quoted in Nattiez, '*Density 21.5*', p. 330.
[62] Grey, '*...wie ein rother Faden*', pp. 196, 199 (further references given in parentheses in the text); see also Theodor Uhlig, 'Drei Tage in Weimar', in *Musikalische Schriften*, ed. by Ludwig Frankenstein (Regensburg: Bosse, 1913), p. 333, quoted in '*...wie ein rother Faden*', p. 198.
[63] *Thematic Process*, p. 337.

so as to retain its identity if modified on subsequent appearances', suggesting archetype 2.[64] Archetype 1 is again never far from analytical practice, however, with the dramatic context of a leitmotive's first appearance frequently determining its name – even though this original association is 'almost always multivalent' and can lead to alternative labellings (e.g. the 'Flight'/'Redemption through Love'/'Glorification of Brünnhilde' leitmotive of the *Ring*).[65] But Grey warns against 'reading motives as fixed musical-semantic tags, static signifiers that betray the true, fluid, semantically indistinct or labile nature of musical signification', advocating instead a sensitivity 'to the fluid, evolving realities of their musical-dramatic implementation'.[66] Bent similarly asserts that 'each [leit]motif must develop progressively, usually doing so in several directions at once', which invokes the directionality of archetype 3, and elsewhere characterises Hans von Wolzogen's 1882 understanding of a leitmotive as '[a] cluster of definable separate motifs […] linked by organic mutation', tinging the idea of unity from plurality (archetype 4) with some archetype 3-like organicism in the form of non-teleological chance 'mutation'.[67]

Archetypes 1 to 4 represent one way to break complex analytical and theoretical statements into their constituent conceptual underpinnings, and these in turn rest on the balance between static, repetition-based and dynamic, development-based models. A slightly different decomposition is proposed by Vera Micznik, who splits a motive's 'multi-levelled semiotic meanings' into morphological, syntactic, and semantic dimensions by exploring the material similarity between musical units, the distribution of those units in time, and their connotative topical content (finding, for instance, echoes of 'Viennese salon music' in Mahler's Ninth Symphony).[68] Schoenberg arrives at a similar categorisation when listing the ways that musical ideas can cohere, grouping his initial list of sixteen under three headings: 'musical content', 'other types of spiritual content' and 'something formal' (although the extent to which form and syntax are related will be discussed below).[69] The following two sections examine the natures of motivic morphology and motivic syntax independently and in detail; the issue of semantics is dealt with at various points throughout, although it is a particular feature of the discussion of musical narrative in Section 2.4.

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64 Arnold Whittall, 'Leitmotif', in *Grove Music Online* <http://www.oxfordmusiconline.com> [accessed 17 December 2014] (para. 1).
65 See Whittall, para. 5.
66 '*...wie ein rother Faden*', p. 188.
67 Ian Bent, 'General Introduction', in *MA19I*, pp. 1–17 (p. 16); von Wolzogen, p. 91ed.
68 Vera Micznik, 'Music and Narrative Revisited: Degrees of Narrativity in Beethoven and Mahler', *Journal of the Royal Musical Association*, 126 (2001), 193–249 (pp. 203, 213). For an introduction to topic theory, see *The Oxford Handbook of Topic Theory*, ed. by Danuta Mirka (Oxford: Oxford University Press, 2014).
69 Schoenberg, *Coherence*, p. 63.

2.3 Similarity and Categorisation

One of the fundamental goals of any motivic analysis is to extract one or more $\mathcal{M}$-sets from a piece of music. Although this can be done explicitly (as in a paradigmatic or computational study), these $\mathcal{M}$-sets usually arise implicitly through a series of analytical observations often presented as a hypothetical enquiry: a shape or small set of shapes is noticed near the beginning of the piece, then actively searched for in what follows. (Of course, the hypothesis is seldom rejected since once a shape is proposed it is difficult to avoid seeing it everywhere – and 'seeing' rather than 'hearing' is apposite.) Even analyses foregrounding archetype 3, and therefore shying away from questions of categorisation, are not simple segmentations of the syntagmatic chain with arrows interspersed but meaningful links between segments considered to be associated: they therefore require the same kind of critical similarity judgements to be made.

Once a small number of putative $\mathcal{M}$-sets has been established, the question facing the analyst is how the rest of the piece can be understood in relation to them. This inevitably guides segmentation (especially if derivatives of a single $\mathcal{M}$-set are assumed to cover the entire piece – again, see Figure 2.2) and requires a judgement to be made regarding which set(s), if any, a particular passage belongs to. Although Schoenberg uses six letters and up to three dashes to distinguish the (ultimately related) segments of Figure 2.2, category judgement may be more a matter of degree than kind: as Dunsby and Whittall note with respect to a particular example, labels are often 'based more on the closeness of the "a" statements to each other than on the principle that "b" and "c" are totally independent of "a"'; a corollary of this is that certain passages might be judged sufficiently dissimilar to all three to be labelled connective tissue.[70]

This idea of similarity and distance along a continuous spectrum of segments stretching from identical to unrelated is a tempting one, and is certainly an improvement to the naïve view that it is possible to separate a piece of music into neatly reified, non-overlapping, and independent $\mathcal{M}$-sets. Nevertheless, the notion of categorisation based on a similarity or distance measure is a problematic one for a number of reasons. Some of these are intrinsic to the exercise of similarity-matching, and are therefore challenges that must be negotiated by the field of music information retrieval (MIR); others stem from the fact that motivic analysis, as a theory about music rather than a theory of music (see Section 1.4), has a different set of goals to those that typify MIR. Finding a way to match segments in building a musical search engine can be a very different task to tracing the "story" of a particular motive throughout a single piece of music, even though each can feed in to the other – for example, if a search engine is used as an analytical tool, or a motivic analysis as a record of perceptual judgements to be modelled (see Section 1.5). The following six problems,

[70] Jonathan Dunsby and Arnold Whittall, *Music Analysis in Theory and Practice* (London: Faber Music, 1988), p. 160.

while constructed with music theory in mind, can therefore be understood to apply, in part, to the enterprise of MIR.

First, extracting a collection of sets from a collection of distance measures is not straightforward. A solution is suggested by fuzzy set theory, which proposes that traditional set theory's 'in' and 'out' states are actually the two endpoints (1 and 0 respectively) of a continuous spectrum: similarity or distance measures can therefore be re-appropriated as so-called membership functions, such that a similarity value of 0.82 means that one segment is 82% "in" the other's set. Thresholds can then be defined to convert fuzzy sets into 'crisp' sets, mapping, say, anything above 70% to 'in' and everything else to 'out'. Applying this to the case of motivic categorisation, one can take a putative $\mathcal{M}$-set, find some suitably defined (possibly in terms of the similarity measure) "average" of its members, and then test every other segment in the piece against it, admitting into the set those above the threshold. A more sophisticated version of this process would recalculate the average every time a new member is admitted, turning an archetype 1-based model into an archetype 4-based model: both strategies are used in Quinn's study of the different contour types in Steve Reich's *The Desert Music*, a piece that is admittedly atypical due to its high degree of near-exact repetition.[71] An alternative method would be to use statistical cluster analysis to determine which segments, given a specified threshold or clustering parameter, appear to group together into sets; such an approach would find "the best" solution without the need for the user to propose possible $\mathcal{M}$-sets.

The problem, however, lies in setting the similarity threshold or clustering parameter: in an article surveying the subject, Olivier Lartillot and Petri Toiviainen highlight the fact that 'no heuristic for precisely fixing this value has been proposed' and so it 'relies entirely on the user's intuitive choices' (Quinn suggests adopting the mean similarity of each set member to the "average", but then immediately lowers this – intuitively – to avoid filtering out 99.994% of candidate segments).[72] It is a mistake to assume that between all 'definitely in' cases and 'definitely

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[71] Ian Quinn, 'Fuzzy Extensions to the Theory of Contour', *Music Theory Spectrum*, 19 (1997), 232–63. Most of Quinn's article centres around the problem of how to judge whether or not a new segment should be included in a pre-defined 11-member set, but pp. 260–63 model an 'in-time' process that measures each segment's similarity to the "average" member of a single growing $\mathcal{M}$-set containing all of the piece's previous segments. Quinn's work builds on the contour theory of Robert Morris, Elizabeth West Marvin, Paul A. Laprade, Michael Friedmann, Larry Polansky, and Richard Bassein; while this field is reminiscent of pitch-class set theory and serial theory in its use of sets and matrices, Quinn later reinterprets contours as regions of n-note space in Callender, Quinn, and Tymoczko, supporting online material, pp. 15–16, 37–38.

[72] See Olivier Lartillot and Petri Toiviainen, 'Motivic Matching Strategies for Automated Pattern Extraction', in *Discussion Forum 4A: Similarity Perception in Listening to Music*, ed. by Petri Toiviainen (=*Musicae Scientiae*, 11.1 suppl. (March 2007)), pp. 281–314 (p. 286) and Quinn, pp. 256–57. See also Cambouropoulos, pp. 16–18 (and the rest of this section) for a critique of the argument that a 50% threshold at least ensures that segments are more similar than dissimilar.

out' cases lies a crisp borderline, as illustrated by the classical *sorites* paradox: in various formulations, it asks when a heap of wheat stops being a heap as single grains are taken away, when a man becomes bald if he loses one hair at a time, and when a tadpole becomes a frog given a series of photographs taken at one-second intervals during its life. The concepts involved are all *vague* (in the philosophical sense) and the paradox can only be resolved (inadequately) by redefining them and retreating to extreme cases: so a heap is redefined as two or more grains, a man is only bald if he lacks a single hair on his head, a tadpole only becomes a frog in the last second of its life, and a motive is a single segment.

Linking these ideas back to more traditional motivic theories, we are often confronted with the idea that development involves "more" change than variation, and that the two exist on a spectrum: Reti, for example, proposes a 'gamut' from 'imitation' to 'varying' to 'transformation' to 'indirect affinity', terminating, at least in theory, in 'nonrelationship' which is 'virtually unknown in great compositional literature'.[73] Reti is aware that 'there is no clear border line between these different principles, and it is by no means always distinguishable when a change ceases to be mere "varying" and begins to be "transformation"': he considers this terminological problem to be 'rather unimportant', however, especially when placed next to the importance of recognising the 'different principles' at work.[74] The idea that variation and development can be compared in terms of degree, but differ fundamentally in kind, is a common one: Lisztian thematic transformation for van den Toorn and Frisch, 'leitmotiv and programmatic content' for Adorno, and popular song composition and repetitive sequence for Schoenberg are all devices which are separate from, but comparable (sometimes unfavourably) to, "true" developing variation, a label which itself would be redundant were its component terms not conceptually distinct yet capable of adjectivally modifying each other.[75] A motivic similarity threshold predicated solely on degree of change therefore runs the risk of over-simplification.

Second, any measure of motivic similarity would have to deal regularly with segments of different sizes, a formal problem with no consistently successful practical solution. In Schoenberg's list of twelve elementary ways to vary the intervals and rhythms of an initial motive, seven

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[73] *Thematic Process*, p. 239 'gamut'; p. 240, n. 1 'unknown'; p. 240 the rest.
[74] Ibid., p. 61.
[75] See van den Toorn, p. 384; Frisch, pp. 42–52; Theodor W. Adorno, *Philosophy of Modern Music*, trans. by Anne G. Mitchell and Wesley V. Bloomster (New York: Seabury Press; London: Sheed & Ward, 1973), pp. 57–58; Severine Neff, 'Schoenberg as Theorist: Three Forms of Presentation', in *Schoenberg and His World*, ed. by Walter Frisch (Princeton: Princeton University Press, 1999), pp. 55–84 (pp. 67–72); and Arnold Schoenberg, 'Criteria for the Evaluation of Music', in *S&I*, pp. 124–36 (p. 131). Frisch cites Mitschka, 'apparently unfamiliar with the writings of Schoenberg and his followers', as the originator of the phrase *variierende Entwicklung*, or 'varying development' (p. 24; see Arno Mitschka, 'Der Sonatensatz in den Werken von Johannes Brahms' (unpublished inaugural dissertation, University of Mainz, 1961)).

(including 'addition of upbeats', 'repetition of features', and 'filling up intervals with ancillary notes') involve changing the number of notes in the segment; the uniform segment sizes in Quinn's Reich study are therefore, again, atypical of usual motivic practice.[76] Together with Callender and Tymoczko, Quinn has written on the problems of measuring distances between chords of different sizes (as the resultant geometric spaces are infinite-dimensional); but since the article's focus is ultimately on voice-leading progressions, and numbers of voices do not tend to change mid-passage, the writers claim that the problems can usually be worked around by simplifying to a finite-dimensional case.[77] For example, the abstract progression $V^7 \rightarrow I$ links a four-note chord to a three-note chord, but any four-part realisation will involve doubling or omitting notes to turn it into a progression between two four-note chords (e.g. (G,G,F,B) → (C,G,E,C)), hence describing a measurable distance in four-dimensional space.

This voice-leading is a *non-bijective mapping* between the two sets: some pitch-classes in the first chord are sent to unique pitch-classes in the second (F to E), but some are sent to two different pitch-classes (G to both C and G), some to the same pitch-class (G and B to C), and some are not mapped to anything (D; analogously unused members of the second set would also be permitted under the definition of a non-bijective mapping).[78] Finding such mappings in the motivic domain – that is, pattern-matching between strings of different sizes – is a non-trivial problem, and is indeed a central concern of MIR: solutions to finding an appropriate alignment or embedding include fitting curves and measuring distances or correlations, testing all possible alignments to find the one that gives the highest similarity, measuring the edit distance (i.e. the number of insertions and deletions) between two strings, and applying reductive (quasi-Schenkerian) algorithms until the segments are comparable at the same size – and each of these methods must decide whether to work with pitch, pitch-class, interval, scale-degree, contour, duration, metric level, and/or harmonic encoding.[79] But even if an alignment is found, there seems to be no natural way to relate this back to a similarity measure: taking the similarity between the

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[76] Schoenberg, *Fundamentals*, p. 10.
[77] See Callender, Quinn and Tymoczko, supporting online material, pp. 6–7, 14.
[78] Strictly speaking, Callender, Quinn, and Tymoczko's voice-leadings are not mappings but line segments in a continuous geometric space: Tymoczko explores this distinction, and critiques Lewin's implication that mappings may adequately model 'directed measurements, distances, or motions' (*Generalized Musical Intervals*, p. 16), in Dmitri Tymoczko, 'Lewin, Intervals, and Transformations: A Comment on Hook', *Music Theory Spectrum*, 30 (2008), 164–68 and Dmitri Tymoczko, 'Generalizing Musical Intervals', *Journal of Music Theory*, 53 (2009), 227–54. The first of these articles is a response to Julian Hook's generalisation of Lewin's bijective theoretical constructs to incorporate non-bijective mappings in Julian Hook, 'Cross-Type Transformations and the Path Consistency Condition', *Music Theory Spectrum*, 29 (2007), 1–40.
[79] A useful summary and bibliography of recent approaches may be found in Alan Marsden, 'Interrogating Melodic Similarity: A Definitive Phenomenon or the Product of Interpretation?', in *Creativity Rethinks Science*, ed. by Federico Avanzini, Giovanni De Poli, and Davide Rocchesso (=*Journal of New Music Research*, 41.4 (December 2012)), pp. 323–35 (pp. 326–27).

aligned subsets implies that adding notes does not decrease similarity, but taking the average across all possible alignments would unfairly penalise simple cases such as added passing notes. How do we quantitatively relate the operations 'changing a note' and 'adding a note' on the same numeric scale? The subtle conceptual migration from similarity and distance to embedding and mapping that underlies this question leads to the next problem with motivic similarity measures.

Third, to use terms borrowed from Kamil Adiloglu, Thomas Noll and Klaus Obermayer, there is an important distinction to be made between 'similarity by proximity' and 'similarity by symmetry'.[80] Broadly speaking, the first concerns itself with "analogue" similarity or distance measures, and the latter with "digital" (and inherently non-fuzzy) functions or mappings, considering two objects (say, pitch-class sets) to be similar if one can be reached from the other by some permitted combination of functions (say, transposition and inversion). Both types of similarity are important: although retrograde forms, such as those frequently identified by Reti, can be derided as imperceptible, Ockelford argues that in a different light Reti's 'position makes good sense, since retrogression demands, in logical terms, a minimal degree of change'.[81] A lot also rests on encoding: if we consider a segment to be a set of pitches then it is symmetrically close but proximally distant to its transpositions (given a unit of measurement defined as a change of one note by one semitone), whereas if we consider it to be a set of intervals then it is similar (in fact, identical) to its transpositions in both cases.

While each type of similarity corrects oversights in the other, combining both types in a single model can be problematic. Pearsall's theory of 'transformational streams', for example, introduces 'transformational communities' (sets defined by combinations of retrograde, inversion, and cyclic permutation; p. 72), which are linked by 'motivic transmutations' (changes to a motive's interval sizes and/or directions; pp. 73–74). Changing too many intervals at once can lead to a

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[80] Adiloglu, Noll and Obermayer, p. 224. These broadly align with the geometric and mapping-based approaches referred to in n. 78 above, and also with Marsden's distinction (following Amos Tversky's) between geometric similarity measures and those based on sets of features ('Interrogating Melodic Similarity', pp. 325–27). Note that mathematically speaking, a symmetry is simply a function that preserves some feature of an object, and does not necessarily imply the musical baggage that has built up around the term (such as periodic phrasing, balance, or mirror-relationship).

[81] Ockelford, p. 84. The perceptibility of serial transformations (retrograde and inversion) seems to be a disputed matter, with Lerdahl citing four studies implying 'that permutational structures are hard to learn and remember' (Fred Lerdahl, 'Cognitive Constraints on Compositional Systems', in *New Tonality*, ed. by Paul Moravec and Robert Beaser (=*Contemporary Music Review*, 6.2 (1992)), pp. 97–121 (p. 116)). However, the Dowling study that he cites reaches the conclusion that these transformations can be 'recognized with better than chance accuracy' (W. Jay Dowling, 'Recognition of Melodic Transformations: Inversion, Retrograde, and Retrograde Inversion', *Perception & Psychophysics*, 12 (1972), 417–21 (p. 417)), and a more recent study similarly reports subjects' 'classification accuracy of mirror forms' to be 'above chance' (Carol L. Krumhansl, Gregory J. Sandell, and Desmond C. Sergeant, 'The Perception of Tone Hierarchies and Mirror Forms in Twelve-Tone Serial Music', *Music Perception*, 5 (1987–88), 31–77 (p. 31)).

disintegration of the similarity relationship, so Pearsall only considers transmutations that reverse up to half of a motive's directions and change its interval sizes by up to two semitones (p. 75). This threshold is misleading, however, since any sequence of directions can be obtained by changing up to half of the directions of an original shape or its inversion: once the halfway point has been reached, transmutations actually head back towards their original community.

Any concept of distance in music theory cannot, therefore, rest on a symmetry- or mapping-based account and must be understood geometrically: this is an argument which has been advanced by Tymoczko on several occasions.[82] His injunction applies even when the mappings involved seem closely related to a concept of distance: for instance, the Neo-Riemannian operations P (for parallel), L (for leading-tone exchange) and R (for relative) represent the three possible ways of moving from one consonant triad to another by moving a single voice by one or two semitones – illustrated by example, $P(\text{C,E,G}) = (\text{C,E}\flat\text{,G})$, $L(\text{C,E,G}) = (\text{B,E,G})$, and $R(\text{C,E,G}) = (\text{C,E,A})$. The progression below shows one way to get from C major to F minor:

$$\begin{pmatrix}\text{G}\\\text{E}\\\text{C}\end{pmatrix}\overset{R}{\rightarrow}\begin{pmatrix}\text{A}\\\text{E}\\\text{C}\end{pmatrix}\overset{L}{\rightarrow}\begin{pmatrix}\text{A}\\\text{F}\\\text{C}\end{pmatrix}\overset{P}{\rightarrow}\begin{pmatrix}\text{A}\flat\\\text{F}\\\text{C}\end{pmatrix}$$

At each stage, the moves are locally minimal in the sense described above, and so it is tempting to use them to form a distance measure: under this measure, C major is closer to F major than F minor. However, $(\text{C,E,G}) \rightarrow (\text{C,F,A})$ involves $1 + 2 = 3$ semitones of total voice-leading motion, whereas $(\text{C,E,G}) \rightarrow (\text{C,F,A}\flat)$ involves $1 + 1 = 2$: the locally efficient moves do not aggregate into a globally analogous measure of distance, and it can indeed be seen that the top voice in the example – i.e. G → A → A♭ – changes direction to move back towards its starting-point. Functions, mappings, and symmetries, even music-theoretically pertinent ones, can therefore produce anomalies in relation to proximity-based distance measures unless these functions are understood in relation to the geometric spaces they work within.

Fourth, the relationships that hold between motivic segments are normally of a different type to those required to form coherent sets and distance measurements. Related to the ideas of similarity by symmetry and similarity by proximity, consider the two mathematical concepts defined below, the first of which seeks to generalise the notion of equality, and the second that of distance:

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[82] See Tymoczko, *A Geometry of Music*, pp. 412–17, which expands Tymoczko, 'Geometrical Methods'. The sources cited in n. 78 above critique Lewin's use of group-theoretical mappings to measure intervals, whereas the sources cited here deal with mapping-based graphs constructed by a wider range of authors (see also the present Introduction at notes 10 and 11).

A relation $\sim$ between two objects is called an *equivalence relation* if:

e1) $x \sim x$

e2) If $x \sim y$ then $y \sim x$

e3) If $x \sim y$ and $y \sim z$ then $x \sim z$

A function $d(x, y)$ is called a *metric* if:

m1) It is never negative

m2) $d(x, y) = 0$ only when $x = y$

m3) $d(x, y) = d(y, x)$

m4) "Direct" routes are never longer:

$$d(x, z) \leq d(x, y) + d(y, z)$$

Following these definitions, equality ($=$) is an equivalence relation whereas inequality ($\neq$) is not (condition e3 fails), and straight-line distance is a metric whereas journey time is not (since condition m3 fails if, for example, points x and y lie at the top and bottom of a hill). An important consequence of an equivalence relation is that it can be used to generate what are known as *equivalence classes*: starting with a pair $x \sim y$, one can add objects such as $w \sim x$ and $y \sim z$ whilst ensuring that, thanks to e3, if a new object is related to one of the members, then it is related to all of them (a property known as *transitivity*). Condition e2 ensures that it is irrelevant which order this is done in, and e1 ensures that an element unrelated to anything else can still form an equivalence class on its own: for a given equivalence relation, every object in a set can therefore be placed in exactly one (non-overlapping) equivalence class, an arrangement known as a *partition* of the set.[83]

Equivalence classes are a particularly strong kind of grouping, formalising, as they do, the idea of sorting objects into discrete pigeonholes, boxes, or folders; the meaning behind the label 'motive x' in an archetype 1-inspired analysis seems to be 'member of the equivalence class also containing x, given a suitable equivalence relation $\sim$' (written 'member of $[x]_\sim$' for short). If $\sim$ is *intransitive* (i.e. if condition e3 fails) then the result is either overlapping sets (such as $\{x, y\}$ and $\{y, z\}$) or a larger set in which not everything is related ($\{x, y, z\}$ in which $x \nsim z$). Similarly, as outlined above, if a proposed distance measure is not a true metric then it cannot be interpreted geometrically and therefore can give rise to anomalies.[84] It is natural, therefore, to ask whether a suitable equivalence relation and/or metric could be defined on motivic segments.

[83] A numeric example: suppose we take the set of all non-simplified fractions $^a/_b$ and the equivalence relation '='. Each fraction falls into a single class, such as $\{^1/_2, ^2/_4, ^3/_6, \dots\}$, and is equal to everything in its class and nothing outside it. Interpreting x and y as pitch-class sets and $\sim$ as 'can be transformed by transposition or inversion into' also shows that pitch-class set classes are equivalence classes.

[84] Counting the shortest Neo-Riemannian path between two triads does actually define a metric; but since this metric does not respect voice-leading distance or number of common tones (F minor and E♭ minor are both three moves away from C major but share one and zero tones with it respectively), 'it is an open question whether there is any intuitive notion of musical distance that is being modelled here' (Tymoczko, *A Geometry of Music*, p. 412). It seems the best we can do is observe, tautologically, that the length of the shortest Neo-Riemannian path between two triads

Conditions e1 and m1 are straightforward to satisfy in the way that the relation and distance function are defined. Condition m2 would require careful negotiation since segments which are strictly distinct are often held to be identical (if, for example, one is an exact transposition of the other): the distance function must therefore either be defined to act on *classes* of segments rather than segments themselves, or must take account of temporal separation such that no two segments can ever be truly identical. The symmetry conditions e2 and m3 might again be satisfied through careful definition, but Marsden has noted that familiarity and relative length can make melodic similarity judgements asymmetrical: an unfamiliar melody is more likely to be judged similar to a familiar one than vice versa, and a short melody might call to mind a longer one without the converse being true (Marsden illustrates this with the extreme example of a symphony being represented by a ringtone).[85]

Conditions e3 and m4 prove to be more problematic in their underlying assumption that pairwise relationships are directly comparable to each other and therefore imply conclusions about sets with more than two members. The example of Neo-Riemannian voice-leading distance above illustrates why this cannot be assumed in general: each stage moves one of three voices by one or two semitones up or down, so the "unit" of distance is actually three distinct units which cannot be treated as equivalent.[86] This is why steps away from the initial point can actually end up moving back towards it (as the top voice in the example shows), and the reason for this is intransitivity: $7 \neq 6 \neq 7$ is an expression that makes sense reading from left to right, but which is clearly invalid if we attempt to skip the intervening step and conclude that $7 \neq 7$. Motives are amalgamations of several musical dimensions (absolute and relative chromatic and diatonic pitches, durations, onsets within a metric framework, contours, and so on), with any given motivic change moving, fixing, or going backwards in any or all of these directions at once. Irrespective of the number of these

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measures the minimum number of moves it takes to get from one to the other where each step only moves one voice by one or two semitones, and where the intervening steps must also be consonant triads.

[85] 'Interrogating Melodic Similarity', p. 324; Alan Marsden, 'The Plurality of Melodic Similarity', in *Proceedings of the Eighth Sound and Music Computing Conference* (Padova: [n. pub.], 2011), pp. 5–10 <http://www.smcnetwork.org/smc_papers-2011179> [accessed 17 December 2014] (p. 5): the former paper expands the latter, which is therefore only cited if it contains ideas not present in the former.

[86] Lora Gingerich's theory of motivic functions runs into similar problems: while she defines fifteen different transformations (such as sharpening or flattening individual notes, inverting or expanding individual intervals, and permuting the note order), there are often multiple ways to get from one segment to another (for example, *UP12=SHARP*^{12}, *FLAT=SHARP*$^{-1}$). This lends the theory a degree of interpretative flexibility (although the cumbersome notation can cloud the good comparison in places), but leads to formal redundancy in its failure to identify a set of independent dimensions that could form the basis of a geometric motivic space (or even a coherent system for labelling motivic transformations). See Lora L. Gingerich, 'A Technique for Melodic Motivic Analysis in the Music of Charles Ives', *Music Theory Spectrum*, 8 (1986), 75–93; the fifteen transformations are defined on pp. 77–82.

dimensions that a given metric can account for, however, it must still output a single number and therefore presuppose the problematic one-dimensional spectrum from identity to non-relationship that is under critique in this section.

Attempting to distil differences in a variety of dimensions down to a single number can run the risk of assuming that the relative importance of these dimensions is fixed and independent of context. Interpretations are frequently modified to maximise similarity, with different dimensions taking precedence at different times (Nattiez's 'dominant criteria'); in such contexts, Marsden argues, 'we should *expect* [condition m4] to be violated: that melody *b* can be interpreted in different ways to be similar to both *a* and *c* does not imply that there is any way to interpret *a* to be similar to *c*'.[87] Reti, for example, speaks of a theme that 'shows no kinship' to a predecessor until the appearance of 'another thematic shape *between* these two themes which is readily recognized as related to both'; elsewhere he hears 'the first theme unmistakeably resounding from the Interlude theme, which itself is a clear copy of the second theme'.[88] His strategy in both cases is to find a *b*-shape that links otherwise unrelated *a*- and *c*-shapes; couching the same observation in re-compositional language, he claims that 'one of the favourite means in the technique of classical composers' is 'the building of a thematic shape from a blending of two previous ones', creating an overlap between $[a]_{\sim}$ and $[c]_{\sim}$ (in the form of *b*) without implying any relationship between *a* and *c* themselves.[89] This phenomenon can be a source of awkwardness in paradigmatic analysis which, despite permitting 'oblique' relationships that fit between or across paradigms, still largely functions on the pigeonhole model: Agawu, for instance, is uncomfortable with the seemingly forced decision to place a *b*-like segment in a new paradigm that 'is, technically, not a paradigm since it occurs only once and clearly grows out of Paradigms E and F'.[90]

Fifth, and implicit in the discussion so far, is a problem that is inherent in most music-theoretical formalisations (as discussed in Section 1.9 above), but perhaps most of all in motivic analysis: the lack of an unambiguous and coherent ground truth. It is comparatively easy to fault a proposed formalism by finding musical examples (or examples of analyses) that expose some oversight in the model; consider the above criticisms that a distance-based model cannot take account of the (under-defined or possibly non-existent) border between variation and development, or of the conceptual 'closeness' afforded by symmetry operations such as inversion. That these counter-examples (Lakatos's 'monsters'; see Section 1.7) exist is only fatal for an approach based on strict falsificationism (a position untenable within music theory anyway; see Section 1.8): they do

[87] 'The Plurality of Melodic Similarity', p. 8; for Nattiez's dominant criteria, see Section 2.2 above.
[88] *Thematic Process*, p. 240 'kinship', 'between'; pp. 144–45 'unmistakably'. See also the discussion of Cone's 'epiphany' in point six below.
[89] *Thematic Process*, p. 25.
[90] See *Playing with Signs*, p. 68; oblique relationships are discussed in Nattiez, '*Density 21.5*', p. 251.

not necessarily stop a model from being a useful heuristic or good comparison, and so need not always be barred (although it is my contention that the problems with motivic distance are so numerous that the idea ceases to function as a good comparison – or at least that a better comparison can be found).

However, when one begins to ask exactly what phenomenon is being modelled by the notion of motivic distance, one finds, like Marsden, that it appears to have no 'stable underlying cognitive functions'; as the discussion in Section 2.2 shows, its analytical definitions are similarly heterogeneous.[91] Marsden contrasts this with the idea of tonal pitch hierarchy: if one conducts a perceptual experiment asking listeners to rate how well each of the twelve chromatic pitch-classes fits into a passage in, say, C major, and then takes a passage in C major and counts the frequency distribution of notes, then the two resulting profiles will be similar. Moreover, these profiles correlate well with a reasonably coherent and consistent set of traditional music-theoretical ideas (i.e. that the tonic is the most important note, followed by the dominant, and so on).[92] Listeners' motivic similarity ratings, on the other hand, seem to vary depending on the experimental paradigm used, some of which invoke "real" musical situations and some of which do not: in his survey of the topic, Marsden notes that subjects have been variously asked to numerically rate the similarity between two melodies, rank all or some of them in relation to a comparison melody, choose the most and least alike pairs in a set of three, sort melodies into categories, hum a melody from memory, or deliberately vary a given melody; some of these tasks have also been modified by instructions to, for example, explicitly consider a particular feature (such as contour) or rate melodies (asymmetrically) as if one was 'a student's attempt to reproduce a teacher's melody'.[93] Different contexts require the listener to creatively *construct* different kinds and degrees of similarity (variation form, for instance, invites the listener to maximise similarity judgements) and so, argues Marsden, 'it is probably safer to consider melodic similarity to be a family of possibly related phenomena'.[94]

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[91] 'Interrogating Melodic Similarity', p. 325.
[92] See Carol L. Krumhansl, *Cognitive Foundations of Musical Pitch*, Oxford Psychology Series, 17 (New York: Oxford University Press, 1990); for a music theory built on these empirically derived tonal hierarchies, see Fred Lerdahl, *Tonal Pitch Space* (Oxford: Oxford University Press, 2001; repr. 2005). Of course, as discussed in Section 1.5, it is simplistic to say that traditional theoretical ideas about tonality were "right all along", since they undoubtedly influenced the composers who created the frequency distributions and (directly through training or indirectly through a musical corpus) the experimental subjects who rated the musical examples.
[93] 'Interrogating Melodic Similarity', pp. 324–25; p. 325 'attempt'.
[94] Ibid., p. 325. For a comparative study which reaches a different conclusion, namely 'that a subgroup of music experts has a reliable and consistent notion of melodic similarity, and that this notion can be measured with satisfactory precision', see Daniel Müllensiefen and Klaus Frieler, 'Modelling Experts' Notions of Melodic Similarity', in *Discussion Forum 4A*, ed. by Toiviainen, pp. 183–210 (p. 183).
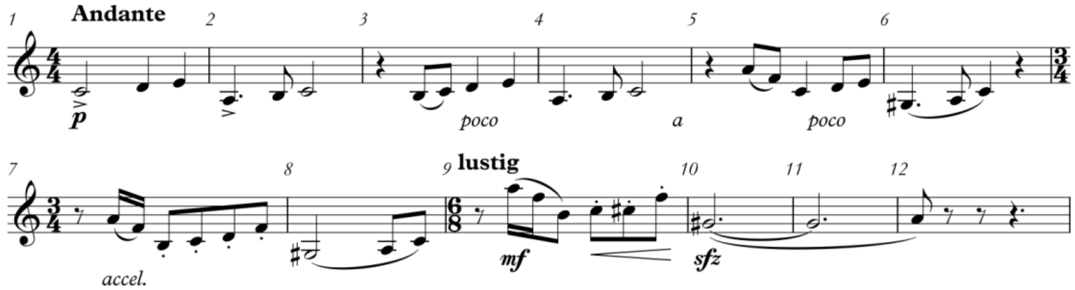
Figure 2.3: Similarity is strongly dependent on rhetorical context.
Charles Seeger's 'logical' linking of the opening of Schubert's 'Great' C major Symphony and a passage from Strauss's *Till Eulenspiegels lustige Streiche*.[95]

Sixth and finally, any notion of morphological similarity between independent, free-floating segments is often challenged, constructed and overwritten by the rhetorical disposition of those segments within a piece of music. The strong effect of phrase beginnings on similarity judgements (between antecedent and consequent pairs, for example) has been implied above (see n. 57) and has proved to be influential on at least one formal similarity model which considers a motive's first intervals to be crucial in identifying temporally distant repetitions.[96] More heterogeneous local processes play a part too: Figure 2.3 reproduces a musical excerpt not drawn from a real piece, but constructed artificially by Charles Seeger. It creates a convincing connection between two disparate segments in a way that also illustrates how this sixth problem plays on the concerns of the previous five: the point at which the first segment becomes the last is either near-impossible to identify or completely arbitrary; each segment is difficult to align with its predecessor as the added notes can be matched in terms of either pitch or rhythm (is the C in bar 5 aligned with the B or C in bar 7?); closeness can be understood both as semitone distance (A in bar 4 becomes G♯ in bar 6) and conceptual parsimony (bar 9 is almost an octave transposition of bar 7 – 'almost' of course referring to the semitone distance between C♯ and D); the relationships between segments are intransitive (as the first and the last are unrelated directly); and the musical context invites the listener to maximise the similarity between adjacent pairs of segments.

To draw an example from a real piece of music – indeed, the piece of music most often held up as a paradigmatic example of motivic organisation (see Introduction at n. 3) – consider Figure 2.4, a passage from the development section of the first movement of Beethoven's Fifth

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[95] Processed from Charles Seeger, 'On the Moods of a Music-Logic', in *A Musicological Offering to Otto Kinkeldey upon the Occasion of His 80th Anniversary*, ed. by Charles Seeger (=*Journal of the American Musicological Society*, 13 (1960)), pp. 224–61 (p. 233, Ex. 5). I am grateful to Ed Venn for bringing this example to my attention.
[96] See Lartillot and Toiviainen, p. 289.

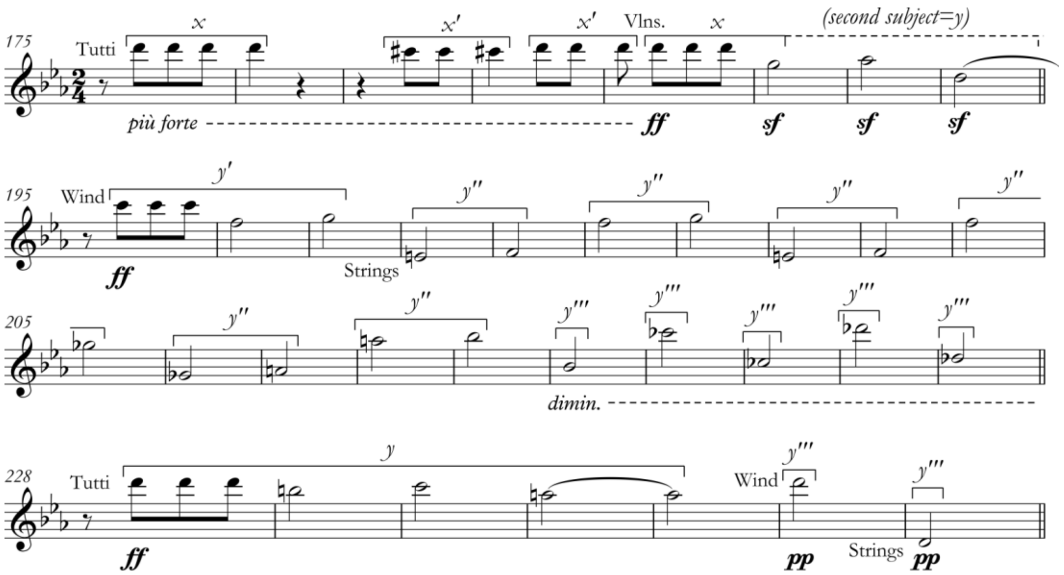

Figure 2.4: Context can intransitively link distinctive and undistinctive segments.
Beethoven's Fifth Symphony, first movement, bars 175–234 (excerpts).[97]

Symphony. This has been identified by Edward Cone as an example of 'intransitive derivation' and in my own earlier work as an example of 'continuous development'.[98] The movement's opening motive (x) is shortened to a three-note version (x') and also used to initiate the second subject (y). This second subject is then progressively truncated to five (y'), two (y''), then one (y''') note(s), such that when the complete phrase (y) is heard again in bar 228, the individual notes (y''') that follow are heard in relation to it: a link has been forged, through the second subject, between the distinctive opening motive and an undistinctive single note. This particular process – that is, 'gradually depriving the motive forms of their characteristic features and dissolving them into uncharacteristic forms, such as scales and broken chords' – is termed 'liquidation' by Schoenberg, its purpose being to 'neutralize the obligations' of motivic material and keep runaway development in check.[99]

Liquidation is not the only process covered by Cone's term 'derivation', and in a relatively short article he examines a wide range of rhetorical devices that complicate the idea that 'if y, which resembles x, follows x, then we normally accept y as derived from x'.[100] Initially (pp. 240–44), his

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[97] Processed from Ludwig van Beethoven, *Symphonie Nr. 5 in c-moll, op. 67* [Score], ed. by Jonathan Del Mar (Bärenreiter: Kassel, 1999).
[98] Edward T. Cone, 'On Derivation: Syntax and Rhetoric', *Music Analysis*, 6 (1987), 237–55 (pp. 242–43); Holden, pp. 13–14.
[99] Editors' commentary to Schoenberg, *Musical Idea*, p. 53. The obligations of the material are related to the musical idea and the tonal problem: according to Schoenberg, the problem being resolved in this movement is the G–E♭ interval in the opening motive, which could imply E♭ major or C minor (see 'Folkloristic Symphonies', p. 164).
[100] Cone, 'On Derivation', p. 240; further references are given in parentheses in the text.

discussion activates many of the same issues encountered in Section 2.2 above: the Beethoven example relates to archetype 3; variation form is seen as motivated by archetype 1 but frequently incorporating sub-1s or -3s in the way that variations group into sections; and something like archetype 4 is proposed to model variations derived 'through' their predecessors (an understanding implicit, Cone argues, in archetype 3 itself). Cone then considers units formed from multiple sources stated successively, simultaneously, or in more subtle 'portmanteau' (p. 245) forms (taking, for example, the rhythm of one and intervals of another): these syntactic devices share the common 'rhetorical aim' of 'epiphany […,] designed to compel the listener to realise a previously unsuspected – or at most unconfirmed – relationship' (p. 246) in the manner of Reti's examples cited in relation to point four above.[101] This is followed by a discussion of 'rederivation' (p. 248), a device by which a passage of development culminates in an existing theme and can therefore symbolise such notions such as 'home-coming' or 'self-discovery' (p. 249); it tends, therefore, to occur at the boundary between development and recapitulation sections.[102] The rest of the article (pp. 250–54) is then devoted to the question of whether or not derivation can work in reverse. Cone first examines derivatives that might be seen as simultaneous with their sources (such as complete canons by retrograde or diminution), then sources that (from a composer's, and possibly listener's, point of view) stand outside the temporal succession of the composition (songs recycled as sonata movements, chorales, *cantus firmi*, folk-songs, or tone rows); he then moves onto introductory 'foreshadowing', narrative quotation marks (the opening of Strauss's *Till Eulenspiegel* announcing, for example, 'I'm going to tell you about a rogue named Till' (p. 251)), and Schenkerian derivation from the background, before finally arriving at his 'true case of reversed

[101] One of Cone's examples is a discussion of the 'tangled lines of inheritance' (p. 246) leading to Brünnhilde's Immolation scene, illustrating that leitmotives need not be simple static symbols but can participate in complex, rhetorically-deployed networks of allusions.
[102] Adorno sees this process as tautologous, since the outcome of the development's extensive motivic working turns out to be 'the affirmation and justification of what has been, what was there in any case' (*Mahler*, p. 94). Mahler's structures, Adorno claims, usually manage to escape this tautology; it is therefore somewhat surprising to see Cone's most extensive example of rederivation being drawn from Mahler's Fifth Symphony. The passage under question is the final peroration which, being a successful culmination of the Finale's material, succeeds where the previous 'insufficiently motivated' appearance of the same music near the end of the second movement failed (Cone, 'On Derivation', p. 249). Cone's reading implies that the peroration is a delayed case of Adornian *Erfüllung* (fulfilment); for an argument that it is an Adornian *Durchbruch* (breakthrough; Section 2.4 explains these terms in further detail) see William Kinderman, *The Creative Process in Music from Mozart to Kurtág* (Urbana: University of Illinois Press, 2012), pp. 102–37 (p. 134 in particular). Kinderman argues that Adorno considers the chorale's first appearance, as 'failed teleology' (Kinderman, p. 118), to be more authentic than the second, as 'mere conviction' (Adorno, *Mahler*, p. 137): but the very fact that the same musical material can point in radically different affective directions should give pause to the notion that rederivation is always a tautologous reinforcement of the status quo.

temporality [...] the potpourri overture' (p. 254), in which the themes are heard as derived from the following opera and not vice-versa.[103]

The syntactical and rhetorical devices of epiphany, rederivation and reverse derivation do more than simply obscure or clarify existing motivic relationships: they are integral to the experience of the piece as it unfolds. Cone discusses this briefly, but elaborates in more detail in his earlier article 'Three Ways of Reading a Detective Story – Or a Brahms Intermezzo', which uses the example of Conan Doyle's 'The Adventure of the Speckled Band' to examine the role of analysis in hearing (or reading).[104] A first reading, argues Cone, simply follows the succession of events as written with the primary aim of finding out what happens next. A second reading is an analysis, an out-of-time study of 'an object abstracted or inferred from the work of art' (p. 80): in a Sherlock Holmes story, this object is a chronological ordering of the plot events, which are narratively revealed out of sequence (a story typically begins with a client visiting Holmes and Watson to recount what they know about a past crime, and ends, after following Holmes's investigative process in the "present", with Holmes revealing his reconstruction of the events leading up to that crime to Watson and the reader). A third reading is then an idealised first or an enacted second: it is informed by analysis, but in a sense aims to suppress synoptic insights by focusing on 'the strategies of concealment and disclosure by which the author controls the process' (p. 81). A good performance, being a temporally projected analysis, is therefore a third reading, since the performer is in a position to anticipate surprises (like Holmes) rather than experience them with the audience (like Watson; see pp. 90–93). This carries suggestions of archetype 4, being a combination of an "in-time" first reading (archetype 3) and an "out-of-time" second reading (archetypes 1 and 2). Cone also argues that a good analysis is a third reading as it should connect spatial, abstract, synoptic perspectives to the music as it unfolds in time (p. 86), thereby encouraging explorations of phenomena (such as ambiguity) that tend to get flattened by a second reading.[105] If a is linked to c through b, then, their intransitivity as abstracted segments tells only part of the story: equally, if not more, important is their rhetorical disposition in time since $a \to b \to c$ and $a - c - \ldots - b$ describe two significantly different musical processes.

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103 The Strauss example is a 'moment of narration' – see Section 2.4.
104 Edward T. Cone, 'Three Ways of Reading a Detective Story – Or a Brahms Intermezzo', in *Music: A View from Delft: Selected Essays*, ed. by Robert P. Morgan (Chicago: University of Chicago Press, 1989), pp. 77–93 (first publ. in *Georgia Review*, 31 (1977), 554–74); further references to the version in the *Selected Essays* are given in parentheses in the text.
105 Agawu argues that '[o]nce a specified context and a specific metalanguage intervene, [...] the interpretation of a musical event as ambiguous in the strict sense becomes untenable', implying that ambiguity must disappear in a third reading; see Kofi Agawu, 'Ambiguity in Tonal Music: A Preliminary Study', in *Theory, Analysis and Meaning in Music*, ed. by Pople, pp. 86–107 (p. 90).

2.4 Syntax, Form, and Narrative

Section 2.1 agrees with Meyer that a 'structural/processive principle' (see n. 17) of motivic development comparable to that which describes the norms of harmonic progression must remain largely illusive: not only is the universe of possible objects much larger and more heavily piece-specific, but even when the concerns of determinism, organicism, and unity are stripped away, the assumption that Agawu's 'simple-to-complex' ordering even exists (in the light of Section 2.3), let alone in some necessary relationship with 'chronological' ordering, is a problematic one. When such a relationship does hold, it is often easily identified: complex-to-simple ordering is Schoenberg's liquidation, and the simple-to-complex ordering often found generating a movement's main theme in its introduction is identified by Meyer as one of the very few 'clear and unproblematic' instances of motivic syntax.[106] Yet, as shown above, categorisation cannot exist completely independently of syntagmatic influence, whether this is interpreted as development, syntax, rhetoric, grammar, process, narrative, developing variation, or distributional assimilation; to hold the extreme stance that the temporal organisation of motivic segments cannot be comprehensively theorised and therefore does not warrant serious discussion rests on a misunderstanding of music "theory" (see Section 1.8) and hence closes down a number of analytically fruitful heuristic approaches.

One of these approaches is a consideration of how motives relate to form. 'The idea of a form-generating motive – almost as if it actually had a will of its own,' notes David Montgomery, 'has survived into the present day as a commonplace doctrine of theoretical musicology'.[107] His case is perhaps overstated in the light of the crucial distinctions between temporal and logical growth and necessity and possibility made in Section 2.1 ('[a] work of art is created as a whole, all at once,' insists Schoenberg, 'not the motive toward it'), but he is perceptive to note 'our own willingness to experience a work through all its contrasting formal sections as a predetermined journey of its motivic and thematic elements'.[108] This not only shifts the focus from dubious speculation concerning a piece's organic generation to what Nattiez has termed the listener's 'narrative impulse', but also sets up a suggestive dynamic between formal sections and motivic journeys, and invites the question of whether the latter are indeed 'predetermined' by the former.[109]

Schoenberg argues that they should not be: that any passage serving a formal function (such as a transition or codetta) 'must be an idea which had to take this place even if it were not to

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106 'A Pride of Prejudices', p. 245. It also appears in Cone's list of reverse derivations (Section 2.3, point six).
107 Montgomery, p. 25.
108 Schoenberg, *Musical Idea*, p. 149; Montgomery, p. 25.
109 Jean-Jacques Nattiez, 'Can One Speak of Narrativity in Music?', trans. by Katharine Ellis, *Journal of the Royal Musical Association*, 115 (1990), 240–57 (p. 243).

serve for this purpose or meaning or function; and this idea must look in construction and in thematic content as if it were not there to fulfil a structural task'.[110] The demands of the musical idea come first, with Reti similarly arguing that formal archetypes 'can only describe the more outward, ephemeral attributes of the complex and mysterious process through which "form" manifests itself in music'.[111] It might, therefore, be seen as strange that the results of this 'complex and mysterious process' frequently look like traditional formal archetypes (Erwin Ratz speaks of Mahler's 'almost somnambulistic sureness of formal conception'), and it is precisely this reconstruction and re-affirmation that Adorno sees as conformist and tautologous (see n. 102).[112] Others hold that the revivification of a supposedly static formal mould is a mark of compositional quality: Bent's commentary on Theodor Helm's 1885 analysis of a Beethoven string quartet notes that motivic and sonata-form analyses 'cohabit' without conflicting, showing that Beethoven has 'raised [sonata form] to the new and higher dynamic principle of motivic association; thus in the analysis the terminology of sonata form is sublimated into that of motivic analysis'.[113]

Just as prevalent as the idea that generic, modular forms require specific, continuous processes to breathe life into them is the equal but opposite conceit that forms (as static frameworks) prevent motivic processes from becoming aimless or overgrown. Bent notes that in the nineteenth century, the prevailing compositional theory became 'one of a potentially infinitely expanding structure triggered by an initial motivic inspiration and perpetuated by the laws of organic musical procreation, ultimately to be controlled and governed by one of a small number of underlying formal archetypes', even though the archetypes could be left changed as a result.[114] Indeed, when discussing how nineteenth-century writers were more prepared to engage in criticism than their modern counterparts, he points to Hermann Kretzschmar's 1898 guide to Bruckner's Fourth Symphony as a prime example, which argues that a 'large number of themes and motifs […] is not a sign of fecundity and abundance; rather it is the weakness of the composition, the consequence of insufficient control and mastery of its material'; Bent adds that form and motive

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[110] 'Brahms the Progressive', p. 407.
[111] *Thematic Process*, p. 105.
[112] Erwin Ratz, 'Musical Form in Gustav Mahler: An Analysis of the Finale of the Sixth Symphony', trans. by Paul Hamburger, *Music Review*, 29 (1968), 34–48 (p. 41). *Pace* Adorno's interpretation of his works, Mahler seems to acknowledge his own sleepwalking when discussing his Third Symphony: 'I see that without my having planned it, this movement – just like the whole work – has the same scaffolding, the same basic groundplan that you'll find in the works of Mozart and, on a grander scale, of Beethoven.' (Bauer-Lechner, p. 66)
[113] Theodor Helm, 'Beethoven: String Quartet in A Minor, Op. 132', in *MA19II*, pp. 242–66 (introduction pp. 238–41) (p. 241ed). Bent also notes Helm's use of 'vaguely Wolzogenish affective or descriptive tags: "scurrying semiquaver figure *a*", "heroic figure d", and so forth' (p. 241ed), demonstrating the reach of the leitmotive idea (which has its *locus classicus* in Hans von Wolzogen's thematic guides) beyond Wagner's music-dramas.
[114] Bent, 'General Introduction', *MA19I*, p. 16.

'become decoupled', with motivic growth becoming 'disproportionately prolific' and leading to 'formal incoherence'.[115] The recurrent metaphor of control in this perspective is nowhere sharper than Adorno's gloss of the 'Tragic' subtitle of Mahler's Sixth Symphony as '[t]he totality that sanctions for its own glory the destruction of the individual', and Monahan's detailed analytical exploration of this idea will be discussed in Chapter 4.[116] It suffices to note here that Monahan's Sonata Theory-inspired analysis frequently construes motivic goals as manifestations of the sonata principle (such that, for example, the second-theme elements "desire" recapitulation in the home key), yet maintains an important tension between the generic, order-imposing demands of the sonata and the freer, more individualised processes of 'novelistic' development (see Section 4.2) that Adorno saw as characteristic of Mahler's music.[117]

Momentarily setting aside metaphors of self-determined growth or generic functionality, a third approach to motivic form attempts, in the spirit of paradigmatic analysis, to pare form back to its fundamentals as 'the traces left by the work of repetition'.[118] Repetition is the crucial process that converts 'linear' into 'circular' time, to use Howell's Jonathan Kramer-inspired terminology, and so one way to view form is simply as the sum of its repetitions.[119] It is useful here to distinguish, as Rothgeb does, between 'temporally distant repetitions of form-defining significance and the more immediate and local repetitions from bar to bar and phrase to phrase'; expanding his 'local repetitions' to include chains of derivation yields the distinction between 'recall' and 'continuous development' made in my master's dissertation.[120] A refinement is made by Meyer's three-part model (interpreting work by Roger Schank and Robert Abelson in a musical context), which reintroduces traditional notions of form by distinguishing between *scripts* (familiar or archetypal schemas) and *plans* (work-specific patterns of organisation); these are again contrasted with moment-to-moment 'standard causal chain expansion[s]'.[121]

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115 Bent, 'Introduction [to Part I]', *MA19II*, pp. 37–38; Hermann Kretzschmar, 'Anton Bruckner: Symphony No. 4 in E♭ ("Romantic")', in *MA19II*, pp. 109–17 (introduction pp. 106–08) (pp. 117, 107ed).
116 Adorno, *Mahler*, p. 97; Monahan, '"Inescapable" Coherence'.
117 The classic text for Sonata Theory is James A. Hepokoski and Warren Darcy, *Elements of Sonata Theory: Norms, Types, and Deformations in the Late-Eighteenth-Century Sonata* (New York: Oxford University Press, 2006; repr. 2011).
118 Agawu, *Music as Discourse*, p. 166.
119 See Howell, p. 104 and Jonathan D. Kramer, *The Time of Music: New Meanings, New Temporalities, New Listening Strategies* (New York: Schirmer, 1988).
120 Rothgeb, p. 39; Holden, p. 13. Howell somewhat surprisingly considers 'local, more immediate instances of repetition' to be 'less dynamic' than 'larger-scale gestural recurrence' (p. 113); here he appears to be using 'dynamic' in the Kierkegaardian sense of 'dynamic repetition' (see n. 7 above) rather than to signify a foregrounded sense of motion or process.
121 Roger C. Schank and and Robert P. Abelson, *Scripts, Plans, Goals and Understanding: An Inquiry into Human Knowledge Structures* (Hillsdale, NJ: Erlbaum, 1977), p. 70, quoted in Meyer, *Style and Music*, p. 245.

Formal functions – whether construed as external, internal, or merely the result of repetition – are therefore important for any consideration of motivic structure. But can segments fulfil functions that do not necessarily serve such generic formal ends as closing or transitioning, and hence signify independently of their temporal and formal locations within a piece of music? Agawu has written extensively on the conventional gestures that characterise beginnings, middles, and endings in classic music and argues that these gestures have become independent of their structural locations and thereby allow composers to 'play' with their meanings, for example by beginning a piece with a closing gesture.[122] The syntactic functions of liquidation and its generative inverse ('solidification', perhaps?) are discussed above, and Cone's epiphany and rederivation could also be considered non-formal functions (that is, answers to the question 'what is this segment doing?' that do not refer to its role within a more-or-less normative formal script). To this list we might add piece-specific categories such as those constructed by Nattiez in his analysis of Varèse's *Density 21.5*: '*flights* lead to a climax', '*permutation* is stagnant, delaying the appearance of a new note which is generally a semitone higher', *progression* to new pitch levels is facilitated by oblique paradigms, and *descents* serve to keep the progressive expansion of the melodic range in check.[123] These ideas approach Adorno's repeated calls for a 'material theory of form' which would characterise sections of music through their affective meanings 'beside or below' their formal functions: Adorno's three 'essential genres' are *Durchbruch* ('breakthrough', in which an element from "outside" the work intrudes and changes its course), *Suspension* (in which time is arrested and an "outside" is implied but 'without positively asserting the presence of the Other'), and *Erfüllung* ('fulfilment', which 'achieve by form […] what the breakthrough promised itself from outside').[124]

Motives may also be seen to fulfil tonal functions, whether in the generation and resolution of a tonal problem (as in Schoenberg's concept of the musical idea), or under a quasi-Schenkerian paradigm of 'composing-out' between structural levels (as illustrated in n. 25 above with an example from the work of van den Toorn). Reti even devotes chapter 8 of *The Thematic Process in Music* to 'Thematic Key Relations' (pp. 219–30): although this is the section of the book where he is least sure of himself, repeatedly qualifying this part of his theory as 'somewhat fragmentary' (p. 229), 'less manageable' (ibid.), or 'lack[ing a] binding, lawlike quality' (p. 219), he cannot resist pointing out

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122 See *Playing with Signs*, especially pp. 51–79.

123 See Nattiez, '*Density 21.5*', p. 283 for the first three of these (italics added) and p. 289 for the last; the entire piece is mapped out in relation to the four categories on pp. 290–94.

124 Adorno sketches some characteristics of his proposed material theory in *Mahler*, pp. 44–46 and Theodor W. Adorno, 'On the Problem of Musical Analysis', trans. by Max Paddison, *Music Analysis*, 1 (1982), 169–87 (p. 185); the definitions quoted here are from *Mahler*, p. 43 (p. 41 'essential genres', p. 45 'beside or below'). His proposed theory never received a full theoretical exposition (perhaps due to its inherent resistance to abstraction), but Max Paddison, *Adorno's Aesthetics of Music* (Cambridge: Cambridge University Press, 1993; repr. 1997) devotes an entire chapter (pp. 149–83) to its discussion. Note 102 above gives a brief example of *Durchbruch* and/or *Erfüllung* at work in Mahler's Fifth Symphony.

that C–D♭–C motives permeate the texture of Beethoven's 'Appassionata' sonata up to and including the level of the three movements' focal pitches (pp. 222–26), and even argues that the movements of the 'Moonlight' sonata, being in the same key and arranged in a long-short-long pattern, echo the work's opening melody (p. 226).

Non-formal functions, particularly those serving no apparent tonal purpose, necessarily open up the somewhat vexed question of musical narrative; this is especially pertinent here since, according to Agawu, 'thematic or motivic process' is '[a] favourite dimension' for the construction of narrative readings (often through an analogy between recurrent themes and characters).[125] Although in a recent review Matt BaileyShea considers 'the most common objections to narrative analysis' to have been 'rigorously counter[ed]', it remains important to identify the locus (as BaileyShea admits) and nature of any proposed musical narrative.[126] In Carolyn Abbate's oft-quoted phrase, 'musical works have no ability to narrate in the most basic literary sense; that is, to posit a narrating survivor of the tale who speaks of it in the past tense'.[127] This lack of ability to establish a narrator and a past tense, together with the absence of other narrative features such as causal relationships, agents, semantic referents, and an essential tension between chronological story and the discourse that unfolds it (as in a Sherlock Holmes tale), seems to place sustained narration out of music's reach. This is not, however, to deny that these criticisms can be problematised, nor that individual 'moments of narration' can be found within musical works.[128] Writers sympathetic to the idea of musical narrative have cited literary theorists to show that many of these supposedly essential features frequently do not appear in literary narratives: causality is constructed by the reader, the past tense is often transformed into an imagined present or replaced with ungrammatical constructions (as in Hemingway's '[i]t was now lunch time'), and the notion that a story can exist independently of its discursive constructions has come under attack.[129]

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125 Agawu, *Music as Discourse*, p. 103; see, for example, Adorno, *Mahler*, p. 72: 'the thematic figure is no more indifferent to the symphonic flow than are the characters in a novel to the dimension of time within which they act'. The notion of a straightforward correspondence between themes and characters is problematised below.

126 Matt BaileyShea, 'Review of Michael L. Klein and Nicholas Reyland, eds., *Music and Narrative since 1900* (Indiana University Press, 2013) and Peter Kivy, *Antithetical Arts: On the Ancient Quarrel between Literature and Music* (Clarendon Press, 2009)', *Music Theory Online*, 19.3 (August 2013) <http://mtosmt.org/issues/mto.13.19.3/mto.13.19.3.baileyshea.php> [accessed 17 December 2014] (para. 1; see also para. 12).

127 Carolyn Abbate, 'What the Sorcerer Said', *Nineteenth-Century Music*, 12 (1988–89), 221–30 (p. 230). A modified form of this article also appears as chapter two in Carolyn Abbate, *Unsung Voices: Opera and Musical Narrative in the Nineteenth Century* (Princeton: Princeton University Press, 1991), pp. 30–60.

128 The phrase is Abbate's; see *Unsung Voices*, p. xi.

129 See Byron Almén, 'Narrative Archetypes: A Critique, Theory, and Method of Narrative Analysis', *Journal of Music Theory*, 47 (2003), 1–39 (pp. 5–8) for causality; Gerald Prince, *Narratology: The Form and Functioning of Narrative*, Janua Linguarum, 108 (Berlin: Mouton, 1982), pp. 28–29,

Under the premise that narratives must be constructed rather than simply discovered in the musical text, the question of whether music can narrate then becomes one of how it is, or can be, *made* to narrate or project a musical plot by listeners and analysts (an instance of music theory's status as a performative; see Section 1.3). Some writers (such as Abbate, Micznik, and Lawrence Kramer) seek musical moments of narration, devices which suggest features of literary narrative such as those listed above: Micznik, for example, considers changes of style, external intrusions, quotations or allusions, and topical references to increase the strength of an implied narrator's voice and hence lend the piece a high 'degree of narrativity'.[130] Those works which move further away from "pure" story and therefore seem non-narrative at first (such as *Finnegan's Wake*) actually have an enhanced sense of narrative mediation, and therefore a higher degree of narrativity. Other writers such as Byron Almén, Gregory Karl, and Fred Maus explore how narrative readings can produce interesting analyses of music that does not seem, as in Adorno's memorable description of Mahler's Fourth Symphony, to be 'composed within quotation marks', but activates in some other way the human narrative impulse.[131] Abbate argues that under this latter strategy, a 'critical methodology […] becomes a mere machine for naming any and all music'; it is therefore important to distinguish between those analyses that propose narratives (or narrative features), and those that propose plots, a distinction not intended as a strict taxonomy but as a characterisation of the different strands of thought that may even be active within a single piece of writing.[132]

According to Maus, the transformation of sound to plot is dependent on the crucial imaginative leap of construing musical events as musical actions: action implies intention and therefore agency, and any interpretation seeking to understand such intention 'will normally situate it within a relatively extended sequence of events'.[133] Actions can be determined through establishing (arguably arbitrary) symbolic, "leitmotivic" associations, topical or intertextual

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quoted in Micznik, p. 196, n. 18 for the imagined present; Wallace Martin, *Recent Theories of Narrative* (Ithaca: Cornell University Press, 1986; repr. 1994), pp. 136–42, summarised in Almén, 'Narrative Archetypes', p. 9 for ungrammatical constructions; and Barbara Herrnstein Smith, 'Narrative Versions, Narrative Theories', in *On Narrative*, ed. by W. J. T. Mitchell (Chicago: University of Chicago Press, 1981), pp. 209–32 (first publ. in *Critical Inquiry*, 7 (1980–81), pp. 213–36), paraphrased in Fred Everett Maus, 'Music as Narrative', *Indiana Theory Review*, 12 (1991), 1–34 (pp. 23–24) for stories independent of discourse.

130 Micznik, p. 247. See also Abbate, *Unsung Voices* and Lawrence Kramer, 'Musical Narratology: A Theoretical Outline', *Indiana Theory Review*, 12 (1991), 141–62. Maus argues that musical narrators are perhaps more 'akin to the silent, invisible intelligence that guides the montage of a film rather than a vividly dramatized speaker like Huck Finn or a moralizing commentator like George Eliot' ('Music as Narrative', p. 34).

131 Adorno, *Mahler*, p. 96. See Byron Almén, *A Theory of Musical Narrative* (Bloomington: Indiana University Press, 2008) and 'Narrative Archetypes'; Gregory Karl, 'Structuralism and Musical Plot', *Music Theory Spectrum*, 19 (1997), 13–34; and Fred Everett Maus, 'Music as Drama', *Music Theory Spectrum*, 10 (1988), 56–73 and 'Music as Narrative'.

132 Abbate, *Unsung Voices*, p. xi.

133 'Music as Narrative', p. 7.

references, or what Micznik refers to as 'gestural connotations': parallels between musical shapes and 'structures or processes from other domains of reality' (p. 226). Although this sounds fanciful at first, the metaphorical nature of analytical language places every utterance, for Maus, on a spectrum 'from technical to emotive or anthropomorphic' such that, for example, Nattiez's flights, Cone's epiphany, and Adorno's breakthrough all carry affective connotations alongside their more specific technical (i.e. stipulative – see Section 1.7) meanings.[134]

Agency and temporality can be fluid, however, even within a plot-focused (as opposed to a narrativity-focused) enquiry. Micznik refuses to anthropomorphise musical events, 'as if music as "action" or "predicate" needed an external agency as the subject who performs its actions' (p. 243), considering them to be 'stative' (that is, 'constitut[ing] a state') rather than 'active'; the states are determined by their semantic topical connotations, and the narrative is supplied by the listener in response to the changes of state (as when, in the first movement of Mahler's Ninth Symphony, 'an old-fashioned Biedermeier world' gives way to 'a modernistic, stormy, disturbed world' (p. 218)).[135] Karl, following Cone, interprets events not as actions, but as 'forces and impressions of mental life' (p. 17) experienced by a posited compositional persona 'in the present tense, though not in objective time, each musical moment embodying indeterminate but almost invariably longer spans of experience' (pp. 16–17); this persona is not to be confused with the composer, nor with any narrator that the work might imply.[136] Even if coherent groupings of motivic segments can be found, a single one of these groupings can embed 'simultaneous multiple agency', with Maus arguing that in any plot-driven analysis 'it seems best to give an account […] that leaves the determination of character vague'.[137]

Emplotted analyses tend to favour a structuralist position – maintaining, that is, that the (usually binary) relationships between events are more important than the events themselves – and Almén uses this defence against Nattiez's argument that music cannot specify its objects of reference and therefore cannot narrate.[138] The idea of the dramatic archetype is a structuralist one:

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134 Maus, 'Music as Narrative', p. 20.

135 The distinction between stative and active events is Gerald Prince's (pp. 62–63, quoted in Micznik, p. 218, n. 64). A similar emphasis on transition, although between action sequences rather than states, is apparent in Rick Altman's theory of narrative (see Rick Altman, *A Theory of Narrative* (New York: Columbia University Press, 2008), especially pp. 21–27). This describes the 'following-patterns' created when different 'following-units' ('portion[s] of the text where a character […] is followed continuously' (p. 22)) are linked by different types of 'modulation' (for example, 'metonymic', in which 'direct spatial contact' between one unit and the next is established (p. 24), or 'hyperbolic', which presents a discontinuity and leaves the audience to infer the link).

136 See also Micznik, p. 247 and Edward T. Cone, *The Composer's Voice*, The Ernest Bloch Lectures (Berkeley: University of California Press, 1974).

137 Karl, p. 25 'simultaneous'; Maus, 'Music as Narrative', p. 14 'vague'.

138 See Almén, 'Narrative Archetypes', pp. 10–11, which responds to Nattiez, 'Narrativity', pp. 257 and 249. This sense of 'structuralism', which follows Karl's usage, has recently been identified by

Maus quotes Aristotle's understanding of character as 'subsidiary to the action' and mainly constructed 'with a view to the action', while certain novels can be understood as relationships between ideas rather than stories or character portraits for their own sake.[139] Of course, from the standpoint of narrativity, moments of narration rely on the disruption of normative structural procedures and forms, such that for Lawrence Kramer 'narrative elements in music represent, not forces of structure, but forces of meaning' (p. 161). Kramer notes a reluctance amongst structuralist writers to 'leav[e] the safe haven of form' (p. 142), a criticism echoed by Abbate when she observes that if a motive with symbolic connotations appears in a situation that conflicts with the analytical plot being proposed, it is explained away as a product of a strictly musical process and therefore conveniently 'stripped of its symbolic meaning'.[140] But from the perspective of emplotment, the search for moments of narration can easily lapse into an exercise in plugging the gaps of formal ambiguity and can therefore underplay the more holistic role of plot in music: 'one is not driven to a dramatic or narrative approach by the failure of more conventional approaches', argues Maus.[141]

As noted above, a structuralist plot-based analysis usually relies on some form of binary opposition (a subtrope of which is the search for a particular "desired" end-state): this paradigm generates weak narrativity according to Micznik (p. 248), especially when compared to the heterogeneous richness of topical reference. Karl argues that topical references and gestural parallels are indeed semantically rich but, by their very nature, function *generically*, and that for any specific semantic meaning to be generated they must interact in 'relations of identity and opposition' (p. 19) – a point that is actually tacitly supported by Micznik's analysis of the first movement of Mahler's Ninth cited above. Karl's analytical method hinges on the concept of the literary foil, a character whose similarity to another sets into relief (like a good comparison) the more significant differences between the two in order to yield insights into the characters, plot, or author.[142] The main agents (or, more precisely, mental states of the piece's persona) in a typical musical plot are therefore the protagonist ('the persona's will to action and the seat of its identity'), the antagonist ('the persona's mental representation of an extrapersonal force or […] some aspect

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Michael Klein as an American 'synonym for formalism' that does not presuppose the implications of semiotic thought that the term carries in Europe (posting to SMT-talk mailing list, 10 July 2014; the archives of this list are freely available at <http://lists.societymusictheory.org/pipermail/smt-talk-societymusictheory.org> [accessed 17 December 2014]).

139 Aristotle, *Poetics*, trans. by Samuel Henry Butcher (New York: Hill and Wang, 1961), pp. 62–63, quoted in Maus, 'Music as Drama', p. 72.

140 Abbate, 'What the Sorcerer Said', p. 224.

141 'Music as Narrative', p. 18.

142 See Karl, pp. 17–18. Karl's examples are Raskolnikov and Razumikhin from Dostoyevsky's *Crime and Punishment*: both are poor, yet the former turns to crime while the latter remains honest. Each character serves as a "control" for the other: without Razumikhin, for example, we might be tempted to read Raskolnikov as Dostoyevsky's comment on the effects of poverty.

of the self perceived as foreign or inimical to the persona's interests'), and the goal state (usually figured as an idealised protagonist); these may interact through various archetypal 'functional sequences' (such as 'enclosure', 'subversion', and 'counteraction') which have an effect on the strength or fortunes of each of the persona's states.[143] This approach, argues Karl, avoids atomistic observations with no sense of directedness and allows for the construction of coherent (yet flexible) plots that may span whole movements (or even whole works).

A similar theory is proposed by Almén who, in addition to emphasising the importance of relationships of conflict, explicitly draws attention to 'the listener's identification with one pole or the other, the initial condition, and the outcome' as constitutive elements of musical plot.[144] Drawing on the work of Northrop Frye and James Jakób Liszka, Almén identifies four narrative archetypes based on the interaction of the two binary dimensions of order and transgression and victory and defeat (pp. 11–20). It is in the second of these binaries (i.e. the emphasis on either victory or defeat) that the listener's identification with one of the poles becomes definitive: the 'victory of order over transgression' characterises the Romance archetype (as evidenced by a typical *Superman* story, for example), whereas the 'defeat of transgression by order' is characteristic of Tragedy (as in Orwell's *1984*).[145] Elements representing order, transgression, and empathy are found amongst musical characteristics functioning topically, conventionally, or gesturally; a minor-key funeral march that is closed harmonically might be considered to represent fatalistic order, for example, while an ascending phrase suggesting a major key might represent optimistic transgression, setting up a dynamic from which it is possible to read an entire movement.[146]

Once the listener's likely empathetic leanings have been established, it is possible to deduce what a composition's "desired" end-state might be. Since order is usually represented by the musical material that opens a work, one popular Romance trope is the quest to return to a particular "pure" motivic form heard near the opening.[147] Elgar's First Symphony, for example, opens with a diatonic theme that Elgar described as a 'sort of *ideal* call – in the sense of persuasion,

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[143] See pp. 20–22 for a detailed typology of functional sequences and, in Figure 1 (p. 21), a graphical emplotment of the first movement of Beethoven's 'Appassionata' Sonata with a key that helpfully summarises Karl's theory. The quoted definitions of protagonist and antagonist are found on p. 23.

[144] 'Narrative Archetypes', p. 34, n. 34. I refer to this source throughout this paragraph; see also the same author's *Theory of Musical Narrative*.

[145] See p. 18. The other two archetypes (ibid.) are 'Comedy' (the 'victory of transgression over order') and 'Irony/Satire' (the 'defeat of order by transgression', perhaps more clearly characterised as a dystopian denial of the possibility of heroic figures).

[146] This is exactly what Almén does in relation to Chopin's C minor Prelude on pp. 20–27.

[147] Maus proposes an alternative interpretation inflected with a narrative story/discourse split: 'When a movement begins with a cadential gesture, for instance, one might regard it as the end of a story; the piece might be heard, then, as reaching into the past to show the events that led up to that cadence.' ('Music as Narrative', pp. 28–29).

not coercion or command' before plunging into an hour of agitated chromatic music: when the opening theme does return in its key of A♭ at the end of the symphony, the apparent hollowness of its victory suggests that the symphony may be an example of the Ironic archetype (figured as failed Romance).[148] This interpretation is, however, dependent on the stability of the opening material: Schoenberg's tonal problem represents a quest for stability starting from an inherently unstable source, and Cone's concept of the 'promissory note', a pitch such as a prominent leading note which 'strongly suggest[s] an obligation that it [fails] to discharge' similarly invokes an external "desired" order that is implied, but not stated, by the musical material.[149] Depending on the piece and the interpretation, Romance can shade into Comedy if the unstable element is seen as a transgressor striving to establish a *new* order (in Karl's language, if the protagonist figures the goal state as an idealised version of itself), as in the conventional *per aspera ad astra* symphonic plot: Beethoven's Fifth Symphony is a Romance if read as an attempt to purge its initial motive of instability, or a Comedy if read as an individual's journey from darkness to light. Narratives of desire can also focus on particular pitch-classes: Nattiez's reading of *Density 21.5* centres around the forces which serve to expand and contract its melodic range and thus work towards a melodic high-point, and Maus notes that certain pitch-classes (especially lowered sixth degrees) can function as 'obtrusive' tonal problems that the piece's narrative must deal with in order to move on.[150]

Musical plots, while often resting firmly on the interpretation of motivic connections, can therefore not be touted as answers to the search for a "purely" motivic syntax. It is difficult to imagine how agency and empathy could be constructed without recourse to topical, gestural, or

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148 Letter to Ernest Newman, 4 November 1908, in *Edward Elgar: Letters of a Lifetime*, ed. by Jerrold Northrop Moore (Oxford: Clarendon Press, 1990), p. 200. The ambiguous status of the work's ending has been a common theme in Elgar scholarship. Elgar himself describes the ideal call's return as 'conquering (subduing)' in the above-cited letter, a description that contrasts markedly with the theme's originally non-coercive nature and therefore hints that the victory may be Pyrrhic (or even that the listener's sympathies do not, or no longer, lie with order – Almén does not explicitly consider the possibility that a listener's sympathies might shift during the course of a piece). For a particularly sophisticated analytical interpretation see J. P. E. Harper-Scott, '"A Nice Sub-Acid Feeling": Schenker, Heidegger and Elgar's First Symphony', *Music Analysis*, 24 (2005), 349–82, especially pp. 372–73.

149 Edward T. Cone, 'Schubert's Promissory Note: An Exercise in Musical Hermeneutics', *Nineteenth-Century Music*, 5 (1981–82), 233–41 (p. 235).

150 'Music as Narrative', p. 20. J. P. E. Harper-Scott offers a psychoanalytical reading of desire in music, in which the Lacanian '*objet a*', the much longed-for missing "it", can never provide true satisfaction once acquired: see J. P. E. Harper-Scott, *The Quilting Points of Musical Modernism: Revolution, Reaction, and William Walton* (Cambridge: Cambridge University Press, 2012). Pages 117–29 present an analysis of Wagner's Prelude to *Tristan und Isolde* in terms of its missing A minor: if the longed-for chord were to appear, argues Harper-Scott, 'we would recognize at once that it was not "it", not the *objet a* we were seeking' (p. 118), and therefore that the true object of our desire was 'desire itself' (p. 120). This position is sympathetic to a structuralist plot analysis since the semantic importance of the *objet a* is determined by its relationship to the other elements of the piece, and not to any inherent properties it might have.

conventional connotations, or desire without reference to harmony (broadly construed to include both the tonal tendencies of promissory notes and the quest for 'missing' pitch-classes within a post-tonal framework).[151] It would also be a stubbornly blinkered analysis that systematically avoids the narrative implications of formal or tonal processes in, for instance, setting up expectations for the return or arrival of particular keys or themes. Finally, although agency need not be construed naïvely as the representation of particular characters by particular recurrent themes, plot constructions do require more-or-less coherent segment groupings in order to describe how "a" certain agent changes over time.[152] It is therefore perhaps surprising that this leaves archetype 3 of Figure 2.1 the least well-suited to describe musical emplotment – despite its implications of Agawu's 'speech' mode and the rhetoric of motivic 'discourse' or 'argument' surrounding its manifestations (see Figure 2.3 and Figure 2.4, for example).[153] Its interpretation hinges on whether we are willing to see the final segment as a transformed version of the first, or whether it is better understood as "new" material (albeit with well-traceable origins); this in turn may depend on whether the final segment appears to initiate a new process of its own.

However, despite (or perhaps because of) the fact that the motivic dimension cannot provide its own 'specific and independent structural/processive principle' and so must draw on others, it retains its power as an entry point into a particular piece of music's unique features. To deny the possibility of motivic plot altogether is to jettison the variety of suggestive approaches and techniques outlined above, just as to admit defeat in the face of the problematic nature of categorisation is to neglect the important tension between circular and linear time that characterises musical experience. The final section of this chapter sketches a mathematical approach to the problem that endeavours to accept and model this tension, with the remainder of the thesis being devoted to its fuller development, exploration, and exemplification. In particular, the strategies for

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151 A post-tonal piece might, for example, conspicuously avoid a certain pitch-class or set whilst freely employing others, or a process of gradual registral expansion might set up, deny, or achieve the expectations of a further-expanding range; Miloš Zatkalik also notes that the gradual deployment of all possible pitch-class sets displaying a certain property, or of all possible textural combinations within an instrumental ensemble, can set up musical processes with clear desired goals (see Miloš Zatkalik, 'Reconsidering Teleological Aspects of Nontonal Music', in *Music Theory and its Methods: Structures, Challenges, Directions*, ed. by Denis Collins, Methodology of Music Research, 7 (Frankfurt a.M.: Lang, 2013), pp. 265–300). These ideas also appear in a recent monograph by Jack Boss, which uses Schoenberg's concept of the musical idea, and in particular the 'problem, elaboration, solution' paradigm, to analyse his twelve-tone music: see Jack Boss, *Schoenberg's Twelve-Tone Music: Symmetry and the Musical Idea* (Cambridge: Cambridge University Press, 2014).

152 See Almén, 'Narrative Archetypes', p. 8. This observation may be recast in terms of a compositional persona through Cone's suggestion that '[f]ormal repetitions are often best interpreted as representations of events rehearsed in memory' ('Schubert's Promissory Note', p. 240): in both cases, we are seeing "the same" thing at different times. Howell follows Kierkegaard in distinguishing repetition, or experiencing-again, from recollection, or re-experiencing (see pp. 103–04).

153 See Agawu, *Music as Discourse*, pp. 98–102.

emplotted analysis discussed above will underwrite not only certain aspects of the formal model developed in Chapter 3, but also the analytical approach taken in Chapter 4.

2.5 A Graph-Theoretic Model

Defined mathematically, a ***graph*** is a set, V, of elements called ***vertices*** or ***nodes*** and another set, E, whose members (called ***edges***) are pairs of elements from V.[154] It is commonly represented visually, with the vertices as points or shapes and the edges as lines between those points, but it is important to remember that a graph is not a visual or geometric entity as such: rather, it is a set of abstract relationships between objects, independent of any visual presentation. Indeed, it is possible to shift the focus in the definition of a graph slightly and understand E not as a set, but as a relation: two vertices from V are related if there is an edge between them (symbolically: for $u, v \in V, uEv \Leftrightarrow \{u, v\} \in E$). The edge relation is symmetric (so if u is related to v, v is related to u: $uEv \Leftrightarrow vEu$), but not reflexive (since in the basic definition above, a vertex cannot be paired with itself – but see Definition 2.1) or transitive, and so it is not an equivalence relation (see Section 2.3, point four). It does, however, give rise to an equivalence relation: that of connectivity, where two vertices are ***connected*** if a ***path*** exists between them (i.e. a sequence of vertices $u, w_1, w_2, w_3, \ldots, w_n, v$ such that $uEw_1, w_1Ew_2, \ldots, \; w_nEv$). This relation is reflexive (we can assert, by definition, that a vertex is connected to itself), symmetric (since a path from u to v implies a path from v to u), and transitive (since if paths from u to w and w to v exist, then a path from u to v exists), so the relation induces a partition on the set of vertices: the equivalence classes are then known as ***connected components***. If we remove the symmetry of the edge relation then the result is a ***directed graph*** or ***digraph***: instead of the edges being unordered pairs $\{u, v\}$ they become ordered pairs (u, v) known as ***arrows***, which are visually represented as arrows rather than lines. The consequence for the notion of connectivity is that it splits into three types: we say that a component is ***simply connected*** if the replacement of all directed edges with undirected ones produces a connected graph; we say that it is ***weakly connected*** if for every pair of vertices u, v there is a path from u to v and/or a path from v to u; and we say that it is ***strongly connected*** if there is a path from u to v and a path from v to u for every possible pair.

[154] The concepts outlined in this paragraph, as well as some others to be used in the next chapter, are defined formally at the end of this section. The terms and results are standard, but I have used two texts in particular as reference works: Reinhard Diestel, *Graph Theory*, Graduate Texts in Mathematics, 173, 4th edn (Heidelberg: Springer, 2010; repr. 2012) is a recent, formal, and comprehensive survey of the field, while Frank Harary, Robert Z. Norman, and Dorwin Cartwright, *Structural Models: An Introduction to the Theory of Directed Graphs* (New York: Wiley, 1965) focuses on how directed graphs might be used in the context of empirical modelling.

The four archetypes represented in Figure 2.1 are all digraphs (archetype 4 requires a slight extension of the definition as some of its arrows point between groups of vertices rather than individuals).[155] The dotted central vertex in archetype 2 shows that vertices and edges can have attributes assigned to them, often represented through some visual means such as line style or colour (in this example, the central vertex is the only one with the attribute 'is an abstraction from the piece'). Graphs, and digraphs in particular, can therefore be used to model the relations between motivic segments, taking what was referred to as $\mathcal{M}$ in Section 2.2 as the vertex set. We can, in fact, go further: given that motivic relationships are typically intransitive (see Section 2.3, point four), we can drop the stipulation that the members of $\mathcal{M}$ must all be mutually related to each other, simply requiring instead that each element of $\mathcal{M}$ is either the initial or terminal point of at least one arrow (as otherwise the element does not recur and so is not motivic). This means that different $\mathcal{M}$-sets do not need to be assumed *a priori*, but instead can be extrapolated from the intransitive mass of motivic relations present within one large set of all the piece's motivic segments: the simultaneous intransitivity of the arrow relation and transitivity of the connectivity relation lends each category a coherence of identity, yet also a rich internal structure of arrow relationships that overwrites assumptions of mutual interchangeability. The stratified blocks of a paradigmatic analysis constructed from overriding criteria are therefore replaced by dynamic, non-linear, and interconnected groupings arising from local, segment-to-segment associations.

What do the arrows in such a model actually represent? Linking, as they do, concrete segments on the musical time-line, they are the agents by which linear time is converted into circular time, and so can represent a whole range of musical relationships, often simultaneously. In a simple case, they might represent (varied or unvaried) repetition by linking a segment to a predecessor which is particularly morphologically similar. They might also represent the form-building role of repetition, linking an antecedent to its consequent or a recapitulation to its exposition, and they can do this individually or as part of larger thematic groupings. They might also represent derivation, transforming one segment into the next, or epiphany, combining two segments into one; they might alternatively acknowledge the lasting influence of a particularly strong motivic statement (typically near the beginning of a piece) and link certain segments back to a specific paradigmatic example. The different ways in which motivic segments can relate to each

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[155] Archetype 4 could be understood as the result of an iterated process of grouping applied to archetype 3. Similar to the concept of condensation (see Definition 2.5), this process would create a partition of the vertex set in which every element lies in its own part, except for the two leftmost vertices, which would be grouped together. An arrow would then be drawn between new vertices A and B if and only if there was an arrow between a and b for some $a \in A$ and $b \in B$. This means that arrows would not point from sets to vertices, but to sets containing single vertices: the last element in the chain in archetype 4 of Figure 2.1 should not, for example, be v_5, but $\{\{\{v_5\}\}\}$, with the arrow pointing into it originating from $\{\{\{v_1, v_2\}, \{v_3\}\}, \{\{v_4\}\}\}$.

other are therefore well-served by a non-geometric digraph structure: far from being a shortcoming, it is argued that this captures the heterogeneity of motivic relationship better than the problematic similarity spectrum model. This approach is broadly structuralist – it treats the links between segments, rather than the musical material of the segments themselves, as primary – and so the symmetry-based morphological model presented in Section 3.3 is to be understood as an abstracted 'second reading' to be used in constructing an emplotted third.

The possibilities for nuanced interpretation are extensive; however, any model must necessarily emphasise some aspects of the target phenomenon at the expense of others, and this one runs the risk of downplaying categorisation in its focus on temporal process. Of course, the connected components "fall out" of the pairwise segment-to-segment associations, but these components can run the risk of being too coarse-grained: a single passage such as that presented in Figure 2.3 can forge two reasonably independent and coherent categories into one component. Within the constraints of digraph structure, a segment cannot be linked to an inner essence or an entire category, meaning that an arbitrary decision is sometimes forced between concrete segments; evolving categories like that portrayed in archetype 4 retain an important role in the analytical process, but can only exist as formal objects *after* the analytical event, deduced *a posteriori* from the resultant graph structure. The next chapter, after formalising the model sketched above, outlines a number of ways in which substructures between the level of the individual link and the entire component can be found within an analytically constructed graph.

Graph Theory: Some Fundamentals

Definition 2.1

A ***graph*** $G = (V, E)$ is a set V of ***vertices*** or ***nodes*** and a set E of ***edges*** (we assume from now on that both of these sets are finite). Each edge is defined as a pair of vertices from V, known as the edge's ***endpoints***, which the edge is said to ***join***. The ***complete graph*** on n vertices is that non-multigraph (see below) in which every vertex is joined to every other; its edge set therefore has $\frac{1}{2}n(n-1)$ members. (The complete graph on one vertex is simply that vertex).

If E is a multiset (i.e. contains at least one vertex pair more than once) then G is a ***multigraph***, and any edges sharing the same endpoints are said to be ***parallel***. If an edge is itself a multiset (i.e. joins a vertex to itself) then it is known as a ***loop***, and G is again said to be a multigraph.

If each edge in E has one of its vertices chosen to be an ***initial point*** and the other to be a ***terminal point***, then the set of edges E becomes a set of ***arrows*** A and the resulting structure $D = (V, A)$ is known as a ***directed graph*** or ***digraph***. The digraph D is said to be an ***orientation*** of the ***underlying graph*** G. Whilst an edge is written as an unordered set $\{a, b\} = \{b, a\}$ (often written ab or ba) an arrow is written as an ordered pair (a, b) or (b, a): parallel edges in E therefore only map to parallel arrows in A if they are oriented in the same direction; otherwise they are known as ***antiparallel***.

Definition 2.2

The ***degree*** of a vertex v in a graph G, written $d_G(v)$, is the number of times v appears as an endpoint in E (if G is not a multigraph, then this is equal to the number of vertices in V joined to v; if G is a multigraph, then note that each loop contributes 2 to $d_G(v)$). The ***indegree*** and ***outdegree*** of a vertex v in a digraph D, $ind_D(v)$ and $outd_D(v)$ respectively, are the numbers of times v appears as a terminal and initial point in A respectively (so each loop contributes 1 to each). Clearly, $ind_D(v) + outd_D(v) = d_G(v)$.

Definition 2.3

The graph $G' = (V', E')$ is said to be a ***subgraph*** of $G = (V, E)$ if $V' \subseteq V$ and $E' \subseteq E$. It is said to be an ***induced*** subgraph if E' is the set of all edges in E whose endpoints both lie in V'. This definition extends to directed graphs in the natural way (i.e. by replacing E with A and G with D).

Definition 2.4

A ***path*** P is a subgraph of G such that $V' = \{a_1, a_2, \dots, a_n\}$ and $E' = \{a_1a_2, a_2a_3, \dots, a_{n-1}a_n\}$ (where all the a_i are distinct). If a_na_1 is added to E', then the path becomes a ***circuit***. For brevity, paths can be written $a_1a_2 \dots a_n$.

In a digraph, a ***directed path*** is a path in the underlying graph oriented in the direction of its vertex ordering – so $A' = \{(a_1, a_2), (a_2, a_3), \dots, (a_{n-1}, a_n)\}$. In this case, a_1 is said to ***reach*** a_n, and a_n is said to be ***reachable*** from a_1 (trivially, each vertex is said to reach itself). A path in the underlying graph which is not necessarily oriented as a directed path is known as an ***undirected path***. Each vertex v in an oriented path is known as a ***transmitter*** if $ind_P(v) = 0$, a ***receiver*** if $outd_P(v) = 0$, and a ***carrier*** if $ind_{P(v)} = outd_{P(v)} = 1$; directed paths are therefore precisely those which have a transmitter at one end, a receiver at the other, and carriers in between. A ***directed circuit*** is a directed path with (a_n, a_1) added to A'.

A graph or digraph which contains no circuits is said to be ***acyclic***. An acyclic graph is a ***tree***, and an acyclic digraph is a ***DAG*** – a directed acyclic graph.

When paths or circuits are referred to in the context of a digraph, they are assumed to be directed unless specified otherwise.

Definition 2.5

In a graph, two vertices are said to be ***connected*** if a path exists between them. As this is an equivalence relation (we specify that a vertex is connected to itself, even if it does not have a loop), the vertex set is partitioned into sets known as ***connected components***: two vertices lie in the same connected component if and only if there is at least one path between them.

In a digraph, two vertices are ***connected*** or ***simply connected*** if they are connected in the underlying graph; they are ***weakly connected*** if one reaches the other; and they are ***strongly connected*** if each reaches the other (i.e. if they lie on a directed circuit). Simple and strong connection are equivalence relations inducing partitions into ***simple components*** and ***strong components*** respectively; the set of ***weak components*** does not partition the vertices since weak connection is asymmetric and therefore not an equivalence relation.

The ***condensation*** $\boldsymbol{D^* = (V^*, A^*)}$ of a digraph $D = (V, A)$ takes the set of strongly connected components V^* as its vertices and includes an arrow (V_1, V_2) in A^* if and only if there is an arrow in A from a member of V_1 to a member of V_2 ($V_1 \neq V_2$).

Proposition 2.6

i. Every vertex in a DAG is reachable from a vertex of indegree 0 and reaches one of outdegree 0. Every simple component of a DAG therefore includes at least one vertex of indegree 0 and at least one vertex of outdegree 0.

ii. If $a, b \in V$ are members of strong components $a^*, b^* \in V^*$ respectively, then there is a directed path from a to b in D if and only if there is a directed path from a^* to b^* in D^*.

iii. Condensations are always DAGs.

Proofs

i. We prove that every vertex reaches a vertex of outdegree 0; a symmetrical argument shows that every vertex is reachable from a vertex of indegree 0. If vertex a_1 does not reach a vertex of outdegree 0, then we can construct an arbitrarily long path $a_1 a_2 a_3 \ldots$ by following an arrow out of each vertex. Since D is finite, one of the a_i must be repeated; but this contradicts the fact that D is a DAG. So a_1 must, in fact, reach a vertex of outdegree 0.

ii. Every pair of consecutive vertices on a directed path from a to b either belong to the same strong component, or they do not. In the former case, the vertices map to the same vertex in D^*; in the latter case, an arrow must exist between their images in D^*, meaning that a path must always exist between a^* and b^*. Conversely, paths in both directions must exist between any pair of vertices that lie in the same strong component, so any path (i.e. sequence of strong components) between a^* and b^* is easily converted to a path between a and b.

iii. Suppose, aiming for a contradiction, that D^* contains a circuit, and that a and b are two distinct vertices in this circuit. Then there is a directed path from a to b and another from b to a, so they must lie in the same strong component; but then they cannot be distinct vertices in D^*. So D^* is acyclic. ■

3 A Formalised Model and an Analytical Example

The previous chapter outlined the rationale behind choosing a graph-theoretic model, and in its final section sketched the general correspondences between the target phenomenon (the connectedness of musical segments) and the chosen mathematical concept (a directed graph). The aim of this chapter is to explore that relationship in more detail: to define formal concepts that capture the pertinent characteristics of particular musical examples in generalisable ways, and to use graph theory as a means of organising and interpreting musical observations. The concepts that arise are thus fed by both musical and formal concerns, embedding a certain tension between the intuitive clarity of musical examples and the mathematical rigour necessary to ensure that such examples can be generalised successfully (including barring all monsters – see Section 1.7) and therefore successfully implemented algorithmically.

Anticipating a mixed readership, the decision has been taken to use a worked analytical example to motivate, clarify, and illustrate the mathematical material: this example will be the first movement of Beethoven's Piano Sonata in F minor, Op. 2, No. 1 (reproduced and segmented in full in Figure 3.22 and on the supplementary CD), and the formal ideas arising from its analysis will be summarised and made rigorous in mathematical language at the end of each section.[1] Technical terms defined stipulatively, or common terms re-defined explicatively (see Section 1.7), appear in bold italic typeface where they are defined informally, with mathematical sections being cross-referenced as appropriate within the main text. Mathematical proofs are included for completeness, but are not essential to the understanding of the musical arguments (or even the mathematical definitions and theorems) proposed. Aside from any stylistic considerations of exposition, it is hoped that the continuous presence of an evolving musical analysis will serve as a reminder that the ideas proposed are not theories of music: they are elements of a framework that can be used to interpret and guide a series of analytical decisions.

3.1 Analytical Input: Segmentation, Derivation, and Succession

In an important sense the formalisms outlined here do not model pieces of music: they model musical analyses. In other words, their basic inputs are analytical decisions, and their basic outputs are the logical implications of those decisions (notwithstanding the possibility that decisions can be revised explicitly once their implications have been revealed, or implicitly in the way that all

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[1] The example has been processed from Ludwig van Beethoven, *Pianoforte-Sonaten* [Score], Ludwig van Beethovens Werke, 16, 3 vols (Leipzig: Breitkopf & Härtel, 1862–90; repr. New York: Kalmus, [n.d.]), I (1862), 1–4.

analytical observations are necessarily theory-laden). The model therefore outlines a three-stage process which loops round into a cycle: some primary observations are made, these observations are subject to formal manipulation, and the resulting conclusions are analytically interpreted back within their musical context (possibly giving rise to a new set of input interpretations). The middle stage of this process concerns itself with the formal structures outlined in the subsequent sections of this chapter, implemented via a computer algorithm – specifically, a VBA macro running from a NodeXL spreadsheet.[2]

Conversely, no computational methods are proposed to carry out the first stage, but nor is it claimed that such methods are irrelevant; they have been avoided here partly for reasons of scope (as Section 2.3 shows, the study of computational segmentation and pattern-matching is a burgeoning field with many open problems) and time (since it would require the encoding of entire movements rather than selected segments), but there is also a deeper methodological component to be considered. The interaction between human mind and (imagined) sound is crucial to the act of analysis – the editors of a special issue of the *Journal of Mathematics and Music* on computational music analysis state that 'in our opinion, no analysis can ever be fully automated', and that 'sometimes approaches with the least computational complexity can reveal the most musically relevant results' – and so one of the best uses for a computer program, argues Marsden, is as 'an analyst's assistant, […] finding relationships from which a human analyst might choose'.[3] The role of the human therefore remains substantial in the absence of a "ground truth", and so the labour-saving benefits to this project of constructing and adopting sophisticated segmentation and pattern-matching programs (to carry out what are, after all, the first steps) are possibly negated by the inevitable need to manually sift the individual results (especially since motivic analysis and similarity matching are not synonymous – see Section 2.3, point six). Marsden's critique of pitch-class set theory rests on the idea that the process of segmentation itself is the analysis, and he argues that traditional analytical concepts (such as Nattiez's 'dominant criteria' – see Section 2.2) are powerful 'specifically in their allowance for expert knowledge and experience'.[4] The first steps in this model are therefore left for the analyst to carry out manually, a process which personal experience has

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[2] NodeXL 'is a free, open-source template for Microsoft® Excel® 2007, 2010 and 2013' that is designed to permit the construction and exploration of network graphs within the familiar Office software; it can be downloaded at <http://nodexl.codeplex.com> [accessed 17 December 2014]. The use of Excel is inspired by Anthony Pople's in his *Tonalities* project: see Anthony Pople, 'Using Complex Set Theory for Tonal Analysis: An Introduction to the Tonalities Project', *Music Analysis*, 23 (2004), 153–94.

[3] Anagnostopoulou and Buteau, p. 82; Marsden, 'Position Paper', p. 150. Marsden goes on to state, however, that he 'know[s] of no case in which such an approach has been used in this manner'. See also Chapter 1 at n. 96.

[4] 'What Was the Question?', pp. 139, 144. The principal difference between pitch-class set theory and my own model is that the analysis continues after the 'trivial calculation and collation' (p. 139) has been carried out by the computer.

Label	Parents	Successors	Bar	[Pitches]-[Time-points]							
032|C6	032|B5	033|D6	32	C6	Eb6	Db6	-	2.5	3	3.5	4

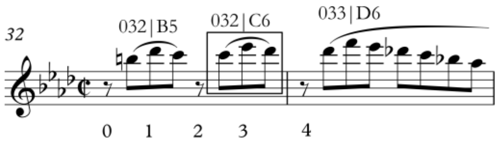

Figure 3.1: The encoding of segment 032|C6 from Figure 3.22.

shown to be an excellent way to get to know the score; it is worth reiterating, however, that there is no reason why proposed computational 'assistants' cannot be slotted into the higher-level framework proposed here.[5]

The initial task facing the analyst, then, is to produce a spreadsheet populated with rows like that depicted in Figure 3.1 (drawn from the 'Chapter 3 – Beethoven' spreadsheet included on the supplementary CD). The first thing to note about this figure is the way it encodes the musical segment itself: as a succession of pitches and time-points. Factors such as harmony, instrumentation, articulation, and dynamics are not encoded explicitly, chiefly because they are secondary to the definition of a motivic segment as a melodic entity (Definition 1) and so introduce unnecessary complexity to the tasks that the algorithm outlined here is designed to carry out. Whilst it may be interesting to track, say, the various articulations applied to a particular figure throughout a piece (as can be done once the algorithm has given its results), encoding articulation as a motive-defining factor independent of pitch and rhythm seems to be a strategy that would only bear fruit in a limited number of specialised (and probably obvious) cases. I adopt, however, Cook's defence against the charge that Schenkerian analysis only deals with pitch structures: 'no good Schenkerian analysis ignores [register, rhythm and other surface features]; instead, it presents the *results* of a careful consideration of these features, though it does so silently'.[6] In a similar way, the segments identified in these analyses can be, and often are, shaped and delineated by factors that they do not explicitly encode.

Each segment is encoded in a single row as a sequence of pitches followed by a sequence of time-points, separated by '-'; the bar in which the segment begins is also specified, and this is automatically (via an Excel formula) turned into a three-digit form and combined with the first pitch to yield a segment ***label*** (segments beginning with the same pitch in the same bar are highlighted automatically in the spreadsheet, and must then have numeric suffixes manually

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[5] In particular, a visual interface using optical character recognition to encode user-highlighted segments of a PDF score would drastically speed up these first steps. The resulting annotated score could be integrated with the other input processes (by allowing the analyst to draw arrows on the score itself) and also the output method (by transforming a PDF into an interactive graph incorporating musical excerpts and even audio playback).

[6] Nicholas Cook, *A Guide to Musical Analysis* (London: Dent, 1987; repr. 1992), p. 61.

assigned – ‘032|C6(1)’ and ‘032|C6(2)’, for example; see Definition 3.1.6).[7] Each ***pitch*** is encoded with a ***letter name***, optional ***accidental***, and ***octave number*** which applies to the letter name such that C4 = middle C and each octave runs from C to the B above (Definition 3.1.1). Commensurate with Definition 1, these pitches are understood to constitute a single melodic line, and so no time-point may be duplicated within a single row (a direct result of Definition 3.1.4 and Definition 3.1.6). The time-point of each pitch’s onset (and the final offset) is encoded with respect to a local 0 – the initial bar-line of the entry in the ‘Bar’ column – which means that the segment is in its ***bar form*** (Definition 3.1.6). The encoding of grace notes and other less definite forms of metric notation is, as in performance, a matter of case-by-case interpretation, partly based on how integral they are to the shaping of a segment as a recurrent melodic unit: the grace notes of 007|C6 are here interpreted to be melodically more important than the arpeggiated chord; the grace notes of 004|C5 and similar segments are interpreted as before-the-beat quavers; and a basic form of reduction removes the trills of 085|C5 and 087|C6 to make them more directly comparable to 081|C5 and 083|C6, for example.[8]

The unit of a single beat stays fixed throughout a movement, but this global conceptual beat need not be identical to a single beat as notated in the score: the number of global beats in each bar needs specifying in the movement’s ***time signature map***, which should list, on a separate spreadsheet within the same workbook, the points at which this number changes (starting with ***bar*** 1, or 0 if the movement begins with an anacrusis, and ending with the final bar to confirm the length of the movement; see Definition 3.1.2). If, for instance, $\frac{4}{4}$ changes to $\frac{3}{4}$ in the score, the indication ♩=♩ suggests a change from 4 to 3, while 𝅗𝅥=♩ suggests a change from 4 to 6 or 2 to 3, and 𝅝=𝅗𝅥. suggests that the “time signature” should remain constant while each “beat” after the change has length $\frac{4}{3}$ (such that three of these add up to the length of a single four-beat bar). The global beat is primarily a notational convention, however, and so these suggestions should be balanced with the need to keep encoding manageable and rhythms comparable: not only is it tiresome or impossible to try and encode beats of length $\frac{4}{3}$ or every (notated or un-notated) tempo nuance in the piece (see n. 12), but the algorithm searches for exact duration matches, meaning that if the tempo doubles but the notation remains the same (for example), it is usually better not to record a change. In the Beethoven example, which has 152 bars, begins with an anacrusis, features no time signature changes, and is easier to encode with four beats in a bar (despite the fact that its time signature is ¢), the table simply consists of two rows: 0|4 and 152|4.

While bar form undoubtedly facilitates encoding (by using local zero-points and salient beat units), it can be difficult to work with once input is completed since identical onsets can be

[7] Further label references in this chapter relate to Figure 3.22.
[8] The encodings of these segments, and all others, are available on the supplementary CD.

encoded in different ways (for example, the onset of C5 in bar 15 is recorded as both ‘0 beats from bar 15’ and ‘4 beats from bar 14’). Converting each ***time-point*** to a distance from a global zero (the beginning of bar 1 or 0) is a good first step, but making use of the time signature information to express each beat as a fraction of a bar – so ‘time-point 130.5’ becomes ‘time-point 32.625’, the fifth quaver of bar 32 – is even better, making references more intuitive and metric parallels more immediate (this is discussed in more detail in Section 3.3, particularly Definition 3.3.3 and Theorem 3.3.4). Interpreted geometrically, this introduces a notion of periodicity to the piece’s ***time-line***, twisting it into a helix (spring shape) in which metrically equivalent positions (e.g. all the second beats in four-beat bars) line up vertically and bars with fewer beats have shorter loops since beat length is constant (see Definition 3.1.3). Such a model brings to mind Howell’s characterisation of the relationship between circular and linear time (p. 112):

> While repetition patterns, *in themselves*, may be indicative of circular time […] they actually articulate a temporal framework within which a subtle process of developing variation can be perceived. It is only by having a constant against which we can measure change, that a sense of ongoing, linear time can be assimilated.

To understand how the chosen method of time-point encoding deals with duration, and how this imposes certain conditions on the first and last time-points of each row, a brief examination of Eytan Agmon’s theory of musical durations, which treats durations as mathematical intervals with associated ***lengths***, will prove instructive.[9] Departing from the question of when a piece of music ends (it cannot be at its last attack, as this initiates a duration), Agmon argues that musical ***durations*** are inherently asymmetric: that is, they include their ‘left extreme[s]’ (***onsets*** or attacks), but not their ‘right extreme[s]’ (***offsets*** or releases; Definition 3.1.4), which are ‘ordinarily’ defined by ‘the attack of another duration […] that immediately follows’.[10] Durations are therefore ‘*uniquely associated* with a single moment in time’, a cognitively efficient strategy that is reflected in William Rothstein’s observation that ‘the word “beat” […] signifies both a single point in time and the temporal distance between two such points’.[11] In this understanding, rests are ‘pseudo-durations at best’ since they lack well-defined attacks and so easily blur with the offsets that precede them: in the typically Mahlerian rhythm  for example, ‘[d]o we really hear a *duration* between

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[9] See Eytan Agmon, ‘Musical Durations as Mathematical Intervals: Some Implications for the Theory and Analysis of Rhythm’, *Music Analysis*, 16 (1997), 45–75. Unlike a musical interval, a mathematical interval is not just a distance but a portion of the number-line.
[10] Agmon, p. 48. Computational and empirical studies usually make use of ‘inter-onset intervals’ rather than durations, with Lartillot and Toiviainen, for example, arguing that ‘[i]nter-onsets might be considered as more prominent than note durations because note onsets are perceptually more salient than offsets’ (p. 285). This strategy also has benefits in encoding and computational efficiency (see next sentence).
[11] Agmon, p. 48; William Rothstein, *Phrase Rhythm in Tonal Music* (New York: Schirmer, 1989), p. 58, quoted in Agmon, p. 49. Rothstein notes in particular that upbeats are often understood as durations and downbeats as time-points (p. 58).

[…] the release of the first note and the attack of the second?', or is the rest better understood 'in terms of texture, articulation, and even timbre'?[12] Given these ideas, and the unpredictability of reverberation, the release of a piece's final note is therefore 'more mental than physical in character': candidates for the endpoint of Figure 3.22 include the second beat of the last bar, the third beat of the last bar (including the *fermata*) or, if one recognises a two-bar hypermetrical phrasing pattern, the downbeat of a hypothetical *extra* bar.[13] And although these concepts arose through consideration of endings, they can also apply to beginnings: pieces starting with an incomplete bar, for example, might be "felt" (by performers especially) to begin with the preceding downbeat.

In a ***motivic segment***, the onset of each pitch also serves as the offset of the preceding ***note*** (Definition 3.1.5 and Definition 3.1.6; this is also the defining characteristic of a ***subsegment***); the exception to this is, of course, the final offset, which must be encoded directly to give one extra time-point in each row when compared to the number of pitches (i.e. one more than the segment's ***size***). This lends rests a pseudo-durational or articulatory status, justified under the assumption that segments must have a gestalt-like character (Definition 1): since 150|F5, for example, has been identified as a segment, it may be safely assumed that the rests within it do not create enough of a discontinuity to split the segment into three, and are therefore matters of articulation.[14] The direct encoding of the final offset is subject to a degree of interpretative freedom and so it is not considered part of a segment's rhythm proper (Definition 3.3.3 excludes it from the assessment of rhythmic equivalence): its primary purpose is to fix a segment's ***duration*** on the time-line (Definition 3.1.6) and hence permit the construction of chains of succession (defined later in this section and in Definition 3.1.8). While a segment may therefore incorporate a rest within its final duration, its first duration must begin with an attack: in addition, this first attack must occur within the bar specified in the 'Bar' column (otherwise it is not in bar form as defined in Definition 3.1.6). This means that the first time-point of each row takes a value that is greater than

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[12] Agmon, p. 54 '[d]o we really', 'in terms of'; p. 55 'pseudo-durations'. Rhythm thus shades into articulation just as meter shades into tempo; to pay too much heed to rests within rhythms or tempo relationships across time signatures necessarily leads to thorny questions of how these encodings relate to *staccato* rhythms without rests, or changes from *andante* to *poco meno mosso* within the same time signature. It is therefore necessary to keep the definitions of rhythm and meter fairly simple and restrictive in order to ensure that encodings are consistent.

[13] Agmon, p. 49.

[14] In other cases, as noted by both Ruwet and Nattiez, rests can play a vital role in delineating segments from each other on the syntagmatic chain: see, for example, Ruwet, pp. 14, 19 and Nattiez, '*Density 21.5*', pp. 272–73. James Tenney and Larry Polansky's computational method of segmentation, critiqued by Nattiez (ibid., pp. 324–29), identifies a segment boundary wherever the interval between two onsets is '*greater than those immediately preceding and following it*' (James Tenney with Larry Polansky, 'Temporal Gestalt Perception in Music', *Journal of Music Theory*, 24 (1980), 205–41 (p. 208)).

or equal to 0, but strictly less than the number of beats in its bar (since bars, like other durations, include their left but not right extremes).

The '***Parents***' column records the label, or labels separated by commas, of any segment(s) that the analyst considers the current row to be more-or-less directly derived from (if such parents exist). If a segment represents a completely new idea in the piece, then this cell is left blank and the segment is a ***source*** (Definition 3.2.9).[15] This decision should not be seen as separate from the process of segmentation, but intimately tied up with it: as Emilios Cambouropoulos argues, humans do not begin the process of categorisation 'with an accurate description of entities and properties' but instead 'alter their representations of entities concurrently with emerging categorizations and similarity judgements' (p. 11). Not only, therefore, should a motivic segment not be recorded if it is not 'recognisably recurrent' (Definition 1, reflected in Definition 3.1.7), but that recurrence should help to determine how related segments are encoded. For example, the grace note of 005|C5 is encoded to be consistent with the grace note of 004|C5, and the decision to include the initial C4 of 035|C4 is made through analogy with the phrasing of 020|Fb5. Whilst the 'Parents' column can include relationships of the kind that Cone labels '***derivation***' (see Section 2.3, point six), here the term is used to cover a broader range of devices, with Cone's 'derivation' being relabelled 'progression' for reasons explained in Section 3.2. The penultimate paragraph of the previous chapter included an indicative discussion of the various factors that may underpin the choice of a particular parent, and a considerable portion of the algorithm's work lies in determining how these factors apply to each individual derivation arrow (as discussed below); the analyst does not, therefore, need to explicitly distinguish between different types of arrow at this stage. Beyond the basic formal stipulations that a segment may not be derived from itself, or from the same parent more than once, the following constraint applies: given a proposed derivation $A \to B$ (i.e. an entry of 'A' in B's row), A must begin before B ends (Definition 3.1.7). This rules out strict reverse derivation, but still permits the variety of formations represented in Figure 3.2, many of which can be used to describe certain contrapuntal structures (number 10, for example, could represent a canon, and numbers 8 and 9 might represent canons by diminution in which the augmented version is seen to give rise to the shorter version and vice-versa, respectively).[16]

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[15] Technically speaking, Definition 3.2.9 only defines sources with respect to condensed, and therefore acyclic, graphs (see Definition 2.5 and Proposition 2.6iii), so the term strictly applies to strong components of the derivation digraph (Definition 3.2.1) containing no member which is derived from outside the component.

[16] Although reverse derivation as an arrow pointing backwards in time is not permitted, it may still manifest itself as an interpretative analytical category in a number of ways. If A "foreshadows" B, for example, then one might expect segments related to both but appearing after B to be connected to the more "stable" B in preference to A: this would appear in the graph as a relatively small

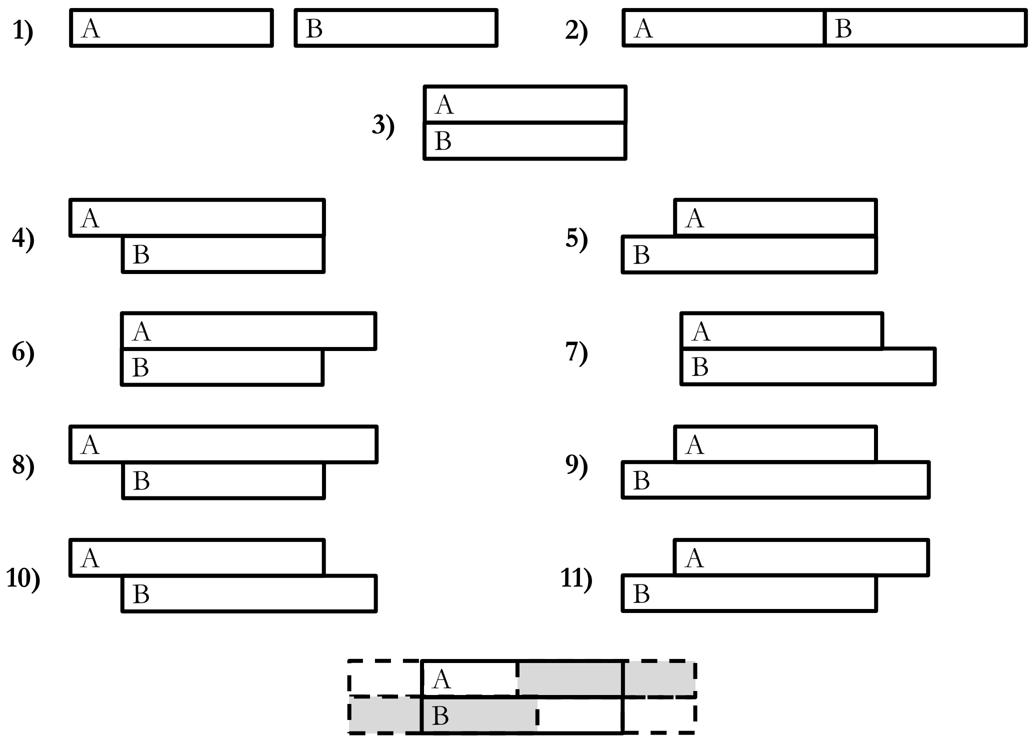

Figure 3.2: The eleven possible configurations of the derivation $A \to B$.
These are generated by moving the onsets and offsets of A and B within the marked zones of the schematic at the bottom of the figure.

It is important to remember that parent choice is necessarily selective, and indeed should not aim to be comprehensive. Whilst the analytical observation that a certain segment serves as a symbol of an entire category might force a partially arbitrary choice of parent, the power of the graph-theoretic interpretation means that if the analyst wishes to place segment A in category C, it is sufficient merely to assign an existing member of C as one of A's parents. Of course, a little knowledge of how the algorithm forms its categories is useful here, and A may ultimately not be a member of C at all hierarchical levels (it is only guaranteed to be part of the same simple component): these issues are explored in more detail in Sections 3.2 and 3.3.

The final piece of information required for each segment is a list of its '***Successors***' in the syntagmatic chain. This column is designed to allow the model to deal with non-monophonic music by clarifying how segments are part of longer melodic lines, streams, or voices: this in turn allows the identification of recurrent groupings (themes) in the manner outlined in Section 3.2 (see Definition 3.2.7 and Theorem 3.2.8 in particular). Succession relationships are subject to the constraint that if B is a successor to A, then B must begin after A begins and end after A ends (Definition 3.1.8): this means that the only derivation configurations in Figure 3.2 that are permitted as succession configurations are numbers 1 (e.g. 018|Eb5–020|Fb5), 2 (e.g. 020|Fb5–

collection of segments appearing in a relatively limited range of bars pointing into a larger collection of more widely distributed segments.

022|Fb5), and 10 (e.g. 014|Db5–015|C5).[17] In addition, if B begins before A ends (configuration 10) then the two segments must ***overlap***: that is, the final pitches and time-points of A must match the initial pitches and time-points of B.[18]

The algorithm can assign an ***assumed successor*** to A (Definition 3.1.8) by first considering all those segments that begin after A's initial onset and before its final offset: if exactly one of these overlaps with A, then it is chosen as A's assumed successor (so, for example, 015|C5 is the assumed successor to 014|Db5). If none overlap, then A's assumed successor is the segment whose initial onset has the lowest value greater than or equal to A's final offset (so 003|G4 is the assumed successor to 000|C4): if this segment is not unique, or if A overlaps with more than one segment, then the segment has no assumed successor and so the analyst must settle the tie. Usually this involves making a choice between the candidates, but it is also possible to list several segments separated by commas (for example if a melodic line splits into a melody and countermelody). A segment may also be assigned multiple successors if there is no particular sense of continuation: if the third voice in a passage of three-voice counterpoint ends before the others, for example, there might be no reason to prefer either of the remaining voices as a continuation, and so both are listed. Only those segments that end the piece are permitted to have no successors at all.

Beyond settling ties, the analyst is free to ignore or overwrite assumed successors, subject to the constraints on succession relationships outlined above (and in Definition 3.1.8) and commensurate with the idea that a succession chain is a slightly more analytically mediated concept than a simple ordered list (entries in the 'Successors' column always take precedence over the algorithm's choices). Segment 032|C6, for example, is clearly continued by 033|D6, even though 033|C3 is technically the next to begin (note that the positioning of final offsets can be used to force the algorithm to behave in a certain way without the need to explicitly list a successor: in this example, changing the final time-point of 032|C6 from 4 to 4.5 to incorporate the rest would make 033|D6 its assumed successor). Finally, it is worth observing that no problems arise from listing successors that duplicate the algorithm's choices: it is sometimes convenient, for example, to encode segments in the order of a particular succession chain, and then simply copy and paste the 'Label' column's values into the 'Successors' column, shifting the list up by one cell.

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[17] Here and elsewhere, '–' is used informally to denote a span of successive segments: this is usually a single chain between the two specified endpoints, but may also include branches pointing into or out of that chain (particularly in the discussion of themes in Section 3.2).

[18] The nature of this matching is detailed in Figure 3.4 and Definition 3.1.8: in particular, the initial onset of B and final offset of A are granted some flexibility, and pitch identity is defined enharmonically (so the pitches must map to the same value under c^+: see Definition 3.3.1).

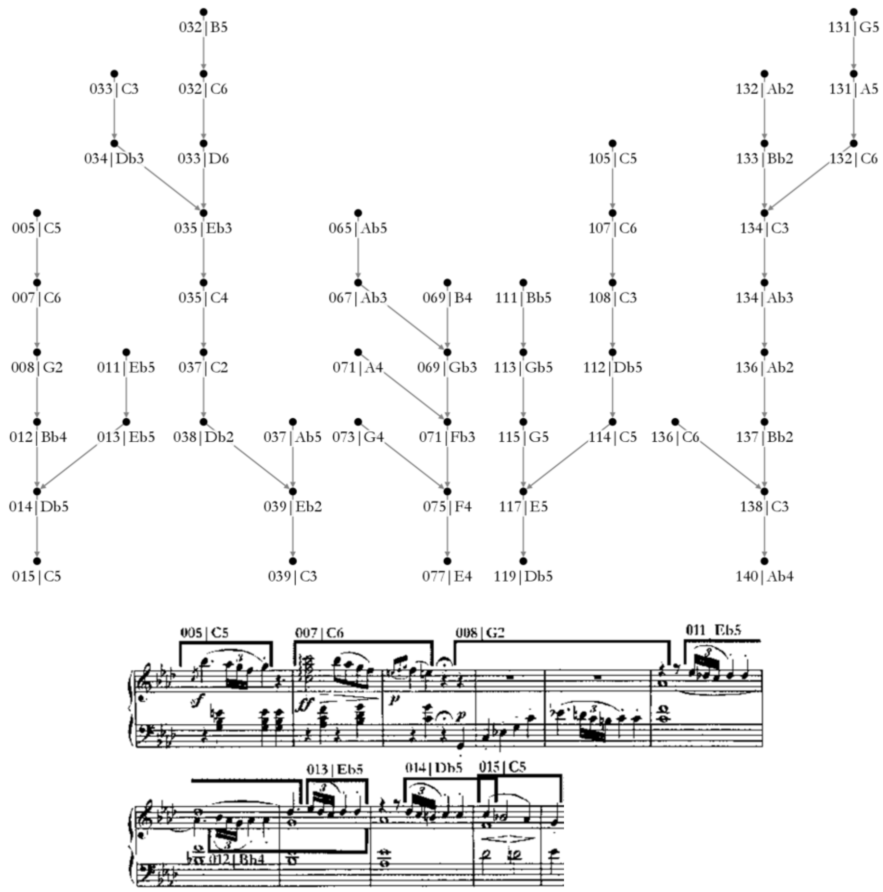

Figure 3.3: The simultaneous succession chains of Figure 3.22.
The score for the first of these is reprinted below the figure; note that 008|G2 ends with a semibreve and so is simultaneous with 011|Eb5.

It is not uncommon for large sections of a piece to trace out a single syntagmatic chain (Leopold Mozart's melodic *filo*, or 'thread' – see Chapter 2 at n. 22), and when the succession relationships are represented as a digraph (see Section 3.2, in particular Definition 3.2.2) the formal structure that results is a single directed path $A \rightarrow B \rightarrow C \rightarrow \cdots$. Figure 3.3 shows the sections of the Beethoven example which do not fall into this structure: these are the imitative section at bars 10–13 (reproduced below the figure), with 014|Db5 representing the point at which the two lines merge back into one (an extended recapitulation, bars 110–117, is shown as the figure's fourth graph); the passage in the development which repeats a cadential pair of segments (067|Ab3 and 069|B4) twice (as 069|Gb3 & 071|A4 and 071|Fb3 & 073|G4; note that an alternative analysis could have 069|B4–071|A4–073|G4 forming a parallel chain rather than collapsing back to a single voice after each cadence); and the passage discussed above in which a new melody enters in the bass (033|C3) as an ongoing process of ascent reaches its summit in 033|D6 (repeated four bars later; again, 033|D6 could be legitimately succeeded by 037|Ab5 rather than rejoining the melody after the segment somewhat arbitrarily ends in bar 34). The fifth graph of Figure 3.3, which represents the recapitulation of the second, is in fact identical to the second when considered as an abstract set of succession relationships independent of visual representation: the two graphs are therefore said to be *isomorphic*.[19] The visual difference is an incidental by-product of the layout algorithm used by NodeXL, and so no significance should be attached to any chain that might appear to be the "main" one, or to any instances of such a chain deviating from the vertical (for example in the third graph, where the *filo* enters at the left with 065|Ab5 but continues at the right with 077|E4). Visual representation does play an important part in later figures, but Figure 3.3 is presented in a "neutral" arrangement to reinforce the concept that graphs are primarily sets of relationships, and not geometric maps.

Mathematical Formalisation

Definition 3.1.1

A ***letter name*** $\boldsymbol{l}$ is an element of the set $L = \{$C,D,E,F,G,A,B$\}$, an ***accidental*** $\boldsymbol{a}$ is an element of the set $A = \{$z,b,Ø,#,x$\}$, and an ***octave number*** $\boldsymbol{o}$ is an integer such that $0 \leq o \leq 9$. A ***pitch*** $\boldsymbol{p}$ is an ordered triple (l, a, o), usually written lao; CØ4 $=$ C4 is middle C, and is enharmonically equivalent to B#3.

Definition 3.1.2

Given a finite set B of sequential integers starting from 0 or 1 (i.e. for fixed $\alpha \in \{0,1\}$ and $\beta \in \mathbb{Z}^+, B = \{b \in \mathbb{Z}^+ \cup \{0\} | \alpha \leq b \leq \beta\}$) known as ***bars***, a ***time signature map*** $\boldsymbol{s}$ is a surjective function from B to $\mathbb{Q}^+$. After specifying $s(0)$ if $0 \in B$ or $s(1)$ if $0 \notin B$, s is usually described by listing b and $s(b)$ whenever $s(b) \neq s(b-1)$.

A ***time-point*** $\boldsymbol{t}$ is a member of $\mathbb{R}^+ \cup \{0\}$ such that $MIN(B) \leq t < MAX(B) + 1$. It may be alternatively notated as an ordered pair $(\lfloor t \rfloor, s(\lfloor t \rfloor)(t - \lfloor t \rfloor))$.[20]

Definition 3.1.3

A composition's ***time-line*** can be understood as a helix in three dimensional space parameterised by the following equations:

$$x(t) = \frac{s(\lfloor t \rfloor)}{2\pi}\left(1 + cos2\pi\left(t + {}^1\!/_2\right)\right)$$

$$y(t) = \frac{s(\lfloor t \rfloor)}{2\pi} sin2\pi\left(t + {}^1\!/_2\right)$$

$$z(t) = t$$

where $MIN(B) \leq t < MAX(B) + 1$. Each coil of the helix represents one bar, with a distance of 1 round the $x - y$ projected circumference (i.e. not the helix arc length) representing one beat. Since this beat is held constant throughout the piece, longer bars have larger loops, but on the z-axis passing through the start of each bar, the bars are spaced equally.

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[19] That is, one can find a way to match each vertex in one graph with a partner in the other in a way that preserves all the arrow relationships (so that the partner of A's successor is a successor of A's partner).

[20] If t is an integer, then $\lfloor t \rfloor = t$ and $\lceil t \rceil = t + 1$. In practice, t is always a member of $\mathbb{Q}^+ \cup \{0\}$, but it is useful for the helix of Definition 3.1.3 to be defined as a continuous space.

Definition 3.1.4

A ***duration*** $\boldsymbol{d} = [\boldsymbol{t_1}, \boldsymbol{t_2})$, where $t_1 < t_2$, is a closed–open interval of a time-line; t_1 is known as an ***onset*** and t_2 an ***offset***. A duration's ***length*** is given by $t_2 - t_1$.

Definition 3.1.5

A ***note*** is a pitch–duration pair (p, d).

Definition 3.1.6

A ***motivic segment*** $\boldsymbol{M}$ ***of size*** $\boldsymbol{n}$ (i.e. $|M| = n$) is a set of notes $\{(p_1, d_1), (p_2, d_2), \ldots, (p_n, d_n)\}$ such that $d_1 = [t_1, t_2)$, $d_2 = [t_2, t_3)$, …, $d_n = [t_n, t_{n+1})$. It can therefore be specified by listing $p_1, p_2, \ldots, p_n, t_1, t_2, \ldots t_{n+1}$, and we may write $p_i \in M$ and $t_i \in M$ for brevity (even though pitches and time-points are technically not members of M, but members of members of M).

A motivic segment's ***duration*** is $[t_1, t_{n+1})$, so we may write $\boldsymbol{on(M)} = t_1$ and $\boldsymbol{off(M)} = t_{n+1}$.

A ***subsegment*** is a contiguous subset of M i.e. $\{(p_i, d_i) \in M | c_1 \leq i \leq c_2\}$ for some fixed c_1 and c_2, $1 \leq c_1 < c_2 \leq n$ (trivially, M is a subsegment of itself).

A motivic segment can be rewritten into its ***bar form*** $(\boldsymbol{t_0}, \boldsymbol{M})$, where $t_0 = \lfloor t_1 \rfloor$ and $t_0^* = 0$, by replacing each t_i of M with t_i^* as follows:

$$t_i^* = \begin{cases} t_{i-1}^* + s(\lfloor t_i \rfloor)(t_i - t_{i-1}) & \lceil t_{i-1} \rceil > \lfloor t_i \rfloor \\ t_{i-1}^* + s(\lfloor t_{i-1} \rfloor)(\lceil t_{i-1} \rceil - t_{i-1}) + s(\lfloor t_i \rfloor)(t_i - \lfloor t_i \rfloor) & \text{if } \lceil t_{i-1} \rceil = \lfloor t_i \rfloor^{21} \\ t_{i-1}^* + s(\lfloor t_{i-1} \rfloor)(\lceil t_{i-1} \rceil - t_{i-1}) + s(\lfloor t_i \rfloor)(t_i - \lfloor t_i \rfloor) + \sum_{j=\lceil t_{i-1} \rceil}^{\lfloor t_i \rfloor - 1} s(j) & \lceil t_{i-1} \rceil < \lfloor t_i \rfloor \end{cases}$$

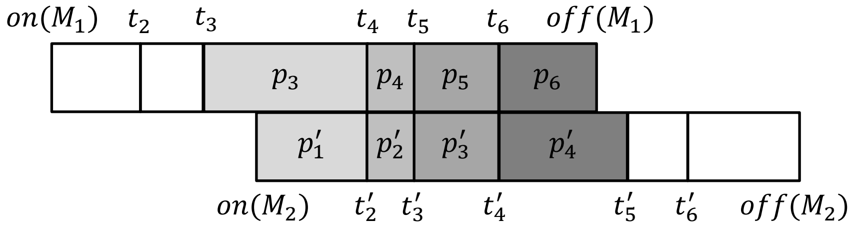
A segment's ***label*** is given by $t_0 | p_1$, or $t_0 | p_1(1)$, $t_0 | p_1(2)$, etc. if more than one segment beginning with p_1 starts in bar t_0.

The collection of all the motivic segments listed for a given time-line (i.e. piece) is denoted $\mathcal{M}$.

Definition 3.1.7

A ***derivation*** relationship, $\boldsymbol{M_1 \delta M_2}$, is a distinct ordered pair of motivic segments (M_1, M_2) such that $on(M_1) < off(M_2)$; M_1 is said to be a ***parent*** of M_2. Each element of $\mathcal{M}$ must be δ-related to at least one other element of $\mathcal{M}$.

[21] Note that for each d_{i-1}, $t_{i-1} < t_i$ (Definition 3.1.4), so if $\lceil t_{i-1} \rceil > \lfloor t_i \rfloor$ then $\lfloor t_{i-1} \rfloor = \lfloor t_i \rfloor = \lceil t_{i-1} \rceil - 1$.



$$\{t_4, t_5, t_6\} = \{t'_2, t'_3, t'_4\}$$
$$|M_1| = 6,\ \alpha = 4$$

Figure 3.4: An illustration of overlapping as defined in Definition 3.1.8iii.
Shaded pitches map to the same value under c^+ (see Definition 3.3.1).

Definition 3.1.8

A ***succession*** relationship, $\boldsymbol{M_1 \sigma M_2}$, is a distinct ordered pair of motivic segments (M_1, M_2) such that:

i. $on(M_1) < on(M_2)$;

ii. $off(M_1) < off(M_2)$;

iii. If $on(M_2) < off(M_1)$, the segments must ***overlap***: that is, $\{t_i \in M_1 | on(M_2) < t_i < off(M_1)\} = \{t'_i \in M_2 | on(M_2) < t'_i < off(M_1)\}$ and $c^+(p'_i) = c^+(p_{|M_1|-\alpha+i})$ for $i = 1$ to α
where $p'_i \in M_2, p_j \in M_1$ and $t'_\alpha = MAX\{t'_i \in M_2 | t'_i < off(M_1)\}$.
The function c^+ is defined in Definition 3.3.1, and Figure 3.4 illustrates the concept of overlapping schematically.

If $\Sigma(M_1) \subset \mathcal{M}$ is defined as the set of all possible $M_2 \in \mathcal{M}$ satisfying i to iii above for a fixed M_1, then if there is a segment $M_s \in \Sigma(M_1)$ such that $on(M_i) > on(M_s) \forall M_i \in \Sigma(M_1) \backslash M_s$ and no member of $\Sigma(M_1) \backslash M_s$ overlaps with M_1, M_1 has an ***assumed successor*** – namely, M_s.[22]

Segment M_1 does not have a successor at all if and only if $\Sigma(M_1)$ is empty.

[22] Here Σ is used simply as a symbol denoting a set, without meaning summation.

3.2 The Derivation and Succession Digraphs

Once the input is complete, the first task carried out by the algorithm is to construct the ***derivation digraph*** (Definition 3.2.1) and the ***succession digraph*** (Definition 3.2.2), which collate all the pairwise derivation and succession relationships respectively (the first of these digraphs is shown in full for the Beethoven example in Figure 3.23; again, its visual layout is secondary in defining its meaning).[23] As the phrase 'distinct pair' in Definition 3.1.7 implies, the derivation digraph may not include parallel arrows or loops; the second sentence of Definition 3.1.7 also ensures that isolated vertices are not permitted (Corollary 3.2.4). Unlike the succession digraph (see, again, Corollary 3.2.4), the derivation digraph may contain directed circuits (including antiparallel arrows). The arrows in these circuits are still subject to the condition that prohibits strict reverse derivation in Definition 3.1.7, and so any circuit segment may end only after its parent has begun, and must have begun before its child ends. Circuits within single succession chains are therefore very unusual (though technically possible, albeit by introducing a high degree of encoding redundancy due to the amount of overlapping needed); this, and their tendency to work against the flow of time, makes circuits somewhat rare (given the general preference for a single compositional *filo*), but they cannot be completely dismissed *a priori*. Consider, for example, a canon by diminution (represented as configuration 8 from Figure 3.2): one reading of this might have B beginning as a derivation of A, but then overtaking such that the second half of A is derived from B, thus creating the pair of antiparallel arrows (A, B) and (B, A). Lemma 3.2.5 and Theorem 3.2.6 further explore some of the conditions that give rise to derivation circuits.

The interactions of the two digraphs (which share the same vertex set) define a relationship inspired by Agawu's terminological differentiation of 'succession as distinct from progression'.[24] One segment is therefore said to progress to another (i.e. they are in a relationship of ***progression***) if the latter both succeeds and is derived from the former (Definition 3.2.3; these arrows are coloured green in the NodeXL output). This is a straightforward renaming of Cone's derivation and my continuous development (as described and exemplified in Section 2.3, point six) in order to bring into sharper focus the nature of this device as simultaneous derivation and succession. The term covers quite a broad range of analytical categories, including generation of a responding phrase (000|C4→003|G4; see also the definition of archetype 2 in Section 2.2), exact, truncated, or extended repetition at the same (011|Eb5→013|Eb5, 015|C5→018|Eb5, 020|Fb5→024|Fb5, 115|G5→117|E5) or a different (095|Ab4→096|Eb6) pitch level, liquidation (145|E5→150|F5), sequence (013|Eb5→014|Db5, 032|B5→033|D6, 033|C3→035|Eb3, 065|Ab5→071|Fb3,

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[23] See Definition 2.1 to Proposition 2.6 for an introduction to the general graph-theoretical terminology used in this chapter.
[24] *Music as Discourse*, p. 93.

073|G4→077|E4, 088|F6→089|Db6, 111|Bb5→113|Gb5, 112|Db5→ 114|C5), a freer but still directed series of focal pitches (A♭5–B♭5–C6 in 004|C4→007|C6,[25] C5–F5–C6–F6 in 081|C5→084|F6), or a combination of several of the above (026|G4→030|G5).[26] All the examples of progression in this movement are included in the above taxonomy of informally defined classes, but note that the converse is not true: for example, the pair 101|F4–103|G4 is not a progression despite the fact that it is almost identical to 000|C4→003|G4. This is a direct result of an input decision to treat the second pair as a repetition of a previously generated grouping rather than a retracing of the progression process.

The notion of segment grouping is formalised in Definition 3.2.7 and Theorem 3.2.8 through a concept labelled ***thematic repetition***. At the core of this concept is a relationship known as ***thematic joining***: a derivation arrow (A, C) is thematically joined to (B, D) if B is a successor to A and D is a successor to C (or, since the relationship is symmetric, if A is a successor to B and C is a successor to D). The arrows (000|C4,101|F4) and (003|G4,103|G4), for example, are thematically joined, as are (003|G4,103|G4) and (004|C5,104|C5): this creates a pair of proto-***themes***, since the succession chain 101|F4–103|G4–104|C5 is derived, in order, from the succession chain 000|C4→003|G4–004|C5. By continuing to add as many thematically joined derivation arrows as possible, the full themes extend as far as 012|Bb4 and 112|Db5 respectively before 014|Db5 and 114|C5 (being unconnected by a derivation arrow for analytical reasons discussed in more detail below) break the pattern. Note also that while these themes are single succession chains, not all themes need to be: the general requirement is that they are connected subgraphs of the succession digraph (Theorem 3.2.8i), so may include branches.

As the lengthy Definition 3.2.7 shows, some monster-barring is necessary to exclude structures we might not think of as themes but which satisfy the conditions of thematic joining: the overlapping ABC and BCD in the progression chain $A \to B \to C \to D$ and the counter-intuitive $B_1C_1A_2$ and $B_2C_2A_3$ in $A_1B_1C_1A_2B_2C_2A_3B_3C_3$, for example. We may forbid segments from being included in both the first and second themes of the same thematic repetition, and insist that there is a preferred resolution of passages like that shown in the second example: the theme pairs $A_1B_1C_1/A_2B_2C_2$ and $A_2B_2C_2/A_3B_3C_3$ should obviate the need for $C_1A_2B_2/C_2A_3B_3$ and A_1B_1/A_2B_2, for instance. With suitable conditions in place (as illustrated in Figure 3.11), it follows

[25] Cone notes that this directed motion, extended backwards to include the F of bar 1 and the G of bar 3, is repeated in diminution to yield the bass notes of bars 5–8 (i.e. F–G–A♭–B♭–C in which the first two inter-onset intervals are twice the size of the second two; see Edward T. Cone, *Musical Form and Musical Performance* (New York: Norton, 1968), pp. 75–76).
[26] Here '→' is used as a version of the succession marker '–' (see n. 17) which indicates that all of its succession relationships are also progression relationships; it refers only to single chains, and not branches, of the succession digraph.

Exposition	Recapitulation
000|C4–012|Bb4	101|F4–112|Db5
011|Eb5→013|Eb5	111|Bb5→113|Gb5
020|Fb5–039|Eb2	119|Db5–138|C3
041|D4–046|G5	140|Ab4–145|E5

Exposition	Development
020|Fb5–027|D5	055|Gb5–062|B4

Simultaneous	
011|Eb5→013|Eb5	012|Bb4–014|Db5
111|Bb5–115|G5	112|Db5–117|E5
069|B4–069|Gb3	071|A4–071|Fb3

Immediate	
000|C4→003|G4	004|C5→005|C5
033|C3–035|C4	037|C2–039|C3
041|D4–042|G4	043|D4–044|G4
043|D4–044|G4	045|D5–046|G5
081|C5→084|F6	085|C5–088|F6
088|F6→089|Db6	090|F6–091|Db6
095|Ab4→096|Eb6	097|F4–098|Db6
097|F4–098|Db6	099|Eb4–100|C6

026|G4→027|B4	031|G5–032|B5
055|Gb5–057|Gb5	063|Ab5–065|Ab5

Figure 3.5: The 18 thematic repetitions in Figure 3.22 (given Figure 3.23 and Figure 3.3).

that each derivation arrow can be part of at most one thematic repetition (expressed in Definition 3.2.7 as connected components of size greater than 1 in a graph in which the derivation arrows are the *vertices*), and that this is incompatible with the idea of disjoint themes only when a derivation circuit is present (Theorem 3.2.8iv): in this case, when the symmetrical nature of the circuit prohibits a preferred interpretation, the thematic repetition is known as ***circular***.[27]

Figure 3.5 lists the 18 thematic repetitions (grouped into categories) that 78 of the Beethoven example's 126 derivation arrows identify. It is instructive, both in terms of piece and method, to examine these 18 theme pairs in more detail; in particular, the crucial roles played by segmentation and development are brought to the fore in such an examination. Firstly, note that a recapitulation realised as an exact reprise of the exposition (and segmented in the same way) would manifest itself as a single long thematic repetition. That is not the case here: putting aside the extra final four segments, the recapitulatory deletion of 039|C3 and replacement of 014|Db5–018|Eb5 with 114|C5–117|E5 splits the exposition "theme" into shorter sections.[28] A great deal therefore rests on how variation is interpreted: had, for example, 039|Eb2 and 039|C3 been combined into a single segment, or the repeated Cs in bar 139 understood as a form of 039|C3, the second split

[27] Lartillot and Toiviainen show a similar concern for rooting out redundant repetition in pattern recognition: rather than establishing segmentation preference rules, however, they condense 'all the possible rotations of [a] periodic pattern […] into one single cyclic pattern' (p. 295).
[28] Note that although there are only two discontinuities here, this splits the recapitulation into four. This is because, as Figure 3.3 and Figure 3.7 show, 014|Db5 is a successor to *both* 013|Eb5 and 012|Bb4, so its removal not only disconnects what follows it from what precedes it, but also isolates the two chains that precede it from each other.

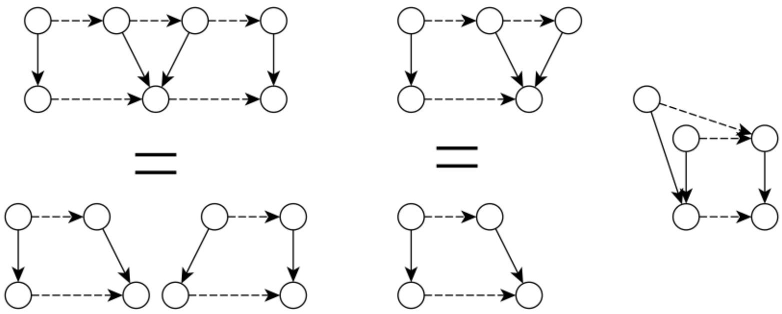

Figure 3.6: The effects of different types of merging on thematic structure. Succession arrows are shown dashed and derivation arrows solid. Similar examples can be constructed to show the effects of the inverse process of splitting.

would not have occurred. The themes as they are identified thus direct attention towards certain critical points in the music, and invite alternative interpretations that would lead to different formal conclusions.

An alternative (abstract) situation forcing a thematic split is shown in Figure 3.6. In the diagram on the left, the merging of two successive segments into one necessitates a split into two thematic repetitions since no derivation arrow in the first is thematically joined to an arrow in the second. A similar situation obtains in the central picture: this time, however, the second repetition consists of a single arrow and so is not a thematic repetition at all (Definition 3.2.7). These examples suggest that the segments in the first and second themes must be related via a bijection (that is, a one-to-one matching of each segment in the first with a unique partner in the second), but this is not the case as the picture on the right shows: the first two arrows are both thematically joined to the third without the two themes overlapping with each other. Thematic repetitions including this permitted merging structure (or its inverse, in which one segment breaks into two) are known as ***non-isomorphic***, and are prefixed with * in the NodeXL output (Definition 3.2.7).

Returning to the Beethoven example, bars 11–14 and 111–18, as well as marking the principal point of difference between the exposition and recapitulation (due to their transitional functions), appear again in Figure 3.5 as examples of ***simultaneous*** thematic repetition: that is, they are thematic repetitions in which the first onset of the second theme occurs before the final offset of the second (Definition 3.2.7).[29] Figure 3.7 shows the relationships of derivation and succession between the relevant segments: it can be seen that, as per the criteria for thematic repetition stated above, the shaded second theme is distinct from the unshaded first theme and derived from it in a way that preserves its succession relationships. However, each repetition displays an additional succession (in fact, progression) relationship: the final unshaded segment

[29] These same segments appear as parallel succession chains in Figure 3.3 and, looking ahead, as epiphanies in Figure 3.8.

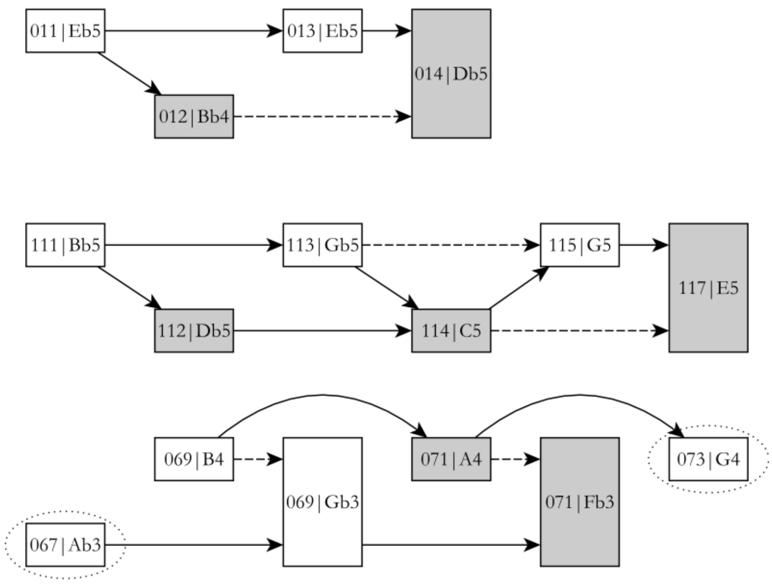

Figure 3.7: The three examples of simultaneous thematic repetition from Figure 3.5. The first and second themes in each case are indicated through shading (segments in dotted rings included for context), while solid arrows represent derivation and horizontal arrows represent succession (so solid horizontal arrows represent progression). Each tall shaded segment lies in two succession chains and hence undermines the separation between the two themes.

progresses into the final shaded segment. This undermines the sense of continuation between the final two shaded segments, and therefore the clear imitative pattern set up by, for example, 111|Bb5→113|Gb5 and 112|Db5→114|C5.

The third example of simultaneity in Figure 3.7 emphasises the "end-weighted" nature of thematic repetition in this model. Again, it can be verified that the shaded and unshaded segments (ignoring those ringed by dotted lines) fall into groups related internally by succession and to each other by derivation, but examining bars 67–74 in context shows that this grouping is somewhat counter-intuitive: as the discussion of Figure 3.3 above shows, the segments fall more naturally into the cadential pairs 067|Ab3 & 069|B4, 069|Gb3 & 071|A4, and 071|Fb3 & 073|G4. These pairs are not themes in the present technical sense as they are not held together by succession: they cannot be, as the second member of each pair begins before the first has ended, and without any overlapping notes. Again, the progression relationships *between* the themes undermine the sense of succession *within* each theme, and the instability produced here seems to be more pronounced than in the previous two examples. Their interpretation being so context-dependent, simultaneous thematic repetitions found by the algorithm are therefore prefixed 'SIM' so that they may be easily isolated and examined on a case-by-case basis.

The final column of Figure 3.5 lists those thematic repetitions in which an earliest segment of the second theme is a successor to a latest segment of the first: in other words, in which the repetition is ***immediate*** (Definition 3.2.7; these are prefixed with 'IMM' in the NodeXL output). Many of these are straightforward: note, in particular, the double repetition of 041|D4–042|G4 as 043|D4–044|G4, then 043|D4–044|G4 as 045|D5–046|G5. The second of these pairs serves as a reminder that the term 'repetition' can be misleading, since themes are based purely on relationships of succession and derivation and not morphology: segments 085|C5–088|F6, for example, are morphologically quite distinct from 081|C5→084|F6, but the second theme can be seen to share a certain structural organisation (four segments, rising C5–F5–C6–F6) with the first; this parallel between wholes but not necessarily parts mirrors Adorno's variant concept, which is discussed in more detail in Section 4.2. The final two rows in this column list themes that are repeated after brief passages of intervening material and so are not strictly immediate: note in particular that 055|Gb5–062|B4 has already been identified as a thematic unit, repeating 020|Fb5–027|D5, and so 063|Ab5–065|Ab5 may be understood as a truncated immediate repetition of this material rather than a non-immediate full repetition of 055|Gb5–057|Gb5.

In some of the rows of Figure 3.5, both themes are progression chains. This means that all the second-theme segments after the first have at least two arrows pointing into them, corresponding to two alternative methods of derivation: progression-then-repetition or repetition-then-progression (note that this equates to choosing both the 'repetition of a previously generated grouping' and 'retracing of the progression process' interpretations discussed above). Any segment with indegree greater than or equal to two is labelled an ***epiphany*** (Definition 3.2.10), adapting Cone's terminology explicatively and adding to it stipulatively (see Section 2.3, point six and Section 1.7). Whilst this definition respects Cone's central concept – two or more processes culminating in a single point, and hence retrospectively connecting their origins – it (like the present use of 'derivation') covers a broader range of scenarios than those Cone uses the term to describe. It is therefore useful to distinguish two subclasses: an epiphany is ***strong*** if all the processes leading into it cannot be traced back to a single point, and ***weak*** otherwise. The former is closest to Cone's original usage, while the latter describes something like the process of rederivation (or progression–repetition/repetition–progression), here recast as a kind of "false" epiphany: consider, for example, the moment of recapitulation in a sonata movement, which retrospectively connects the final segments of the development to the initial segments of the exposition, creating a sense of culmination akin to a "true" epiphany – despite the fact that the former set of segments is ultimately derived from the latter.

All four epiphanies in the Beethoven example, shown in Figure 3.8, are weak, and so describe simultaneous processes that begin and end at the same points as each other. Segment 005|C5, for example, is both a quasi-sequential progression from 004|C5 and a truncation (with the same harmonic underpinning) of 003|G4. The square is completed by considering the relationships between these segments and 000|C4: 003|G4 is its response phrase (or quasi-

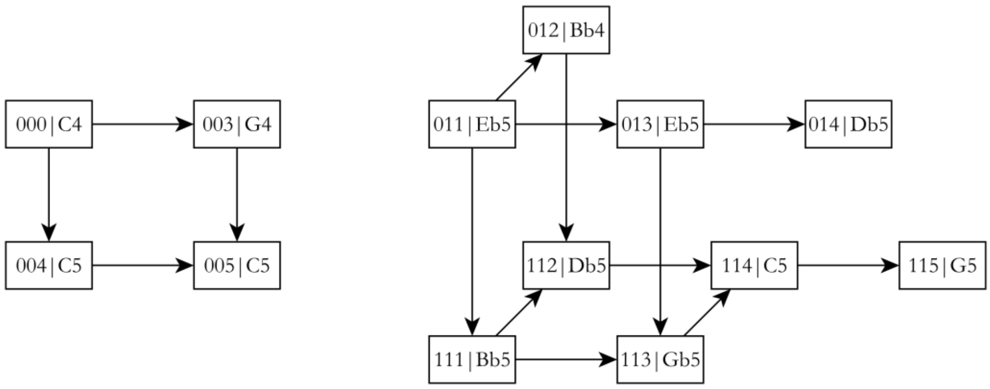

Figure 3.8: The moments of epiphany in Figure 3.23.
In the digraph on the right, voices (i.e. progression chains) run horizontally, and the upper and lower halves represent the exposition and recapitulation respectively.

sequential progression), and 004|C5 is its truncation. We might therefore see 005|C5 as a truncation of 000|C4's response, or as a progression from its truncation; alternative interpretations (created by pairing off the sides of the square in different ways) might see a progression (000|C4→003|G5) and its truncated repetition (004|C5→005|C5), or the splitting (003|G4 keeps the shape, 004|C5 keeps the harmony) and re-integration (keeping the harmony of 003|G4 and shape of 004|C5) of 000|C4 into a transformed version of itself.

The other three epiphanies in the Beethoven example (112|Db5, 113|Gb5, and 114|C5) interrelate to create a total of four paths from 011|Eb5 to 114|C5. They have been arranged in Figure 3.8 such that progressions run horizontally and arrows linking segments in the exposition to segments in the recapitulation run vertically. This shows, among other things, that 112|Db5 branches off from 111|Bb5 just as 012|Bb4 branches off from 011|Eb5; that the progression 111|Bb5→113|Gb5 repeats the progression 011|Eb5→013|Eb5; and that the progression 112|Db5→114|C5 imitates the progression 111|Bb5→113|Gb5 (see also Figure 3.7). Just as in the simpler example, alternative interpretations are possible and may be joined together in various ways: 114|C5 could be the outcome of a two-stage derivation initiated by a repeat of 011|Eb5, for example, or of a single-stage progression initiated by a repeat of 011|Eb5's imitator 012|Bb4.

At the close of Chapter 2, it was argued that while the connected components of a derivation digraph represent a good first level of motivic categorisation, reasonably independent and coherent categories can be forced into the same connected component through passages of progression (as in the Schubert-to-Strauss example of Figure 2.3). Given that this idea of two becoming one is enacted above all at a piece's points of epiphany, it seems natural to propose a subdivision of the connected components that retraces the merging process in order to split the components at such points. The basic underlying idea is that a segment's identity is derived chiefly

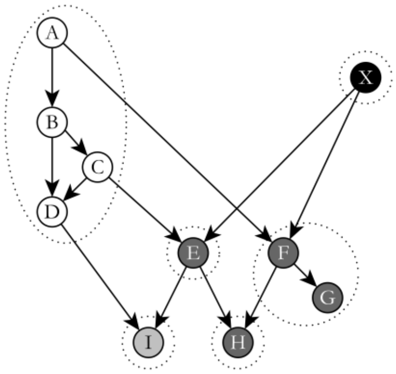

Figure 3.9: An example of family formation.

Families are marked with dotted circles, and siblings are the same shade. Segments A and X are sources, D is a weak internal epiphany, E and F are strong epiphanies, and H and I are weak external epiphanies (see also Figure 3.12).

from the blend of segments it is ultimately derived from: to see how this is applied in practice, consider Figure 3.9.

We might imagine A and X, the two sources (Definition 3.2.9) in Figure 3.9, emitting blue and yellow dyes respectively along the derivation arrows. Segments B, C, and D would then be coloured blue to constitute one ***family*** (Definition 3.2.12) while X would be the only member of the yellow family. At E and F – points of strong epiphany that hence derive their identities from two families (A's and X's) to create new identities dependent on both – the dyes would independently mix into the same shade of green; but since the process, and not just the result, of the blending is crucial, the two segments lie in separate ***sibling*** families (Corollary 3.2.15). Contrast this with segment G, which has F as its only parent and so lies in the same family as it: the rationale for separating E and F rests on the supposition that if they truly were members of the same family, one would have been derived from the other rather than both from first principles. Segment H, a weak epiphany with parents in different families, marks the start of another new family: being the same shade of green as E and F (and G), H's family is a sibling to theirs. Finally, green meets blue again at I to create a new family which is a different shade of green to the others; it is therefore a ***cousin*** (Corollary 3.2.15) to them.

The strong epiphanies E and F function in exactly the same way as described by Cone: A and X begin the piece as independent motives, are repeated and varied to form two stable families, and then are brought together to create a relationship between them – or to emphasise through rhetoric a latent relationship that the two families had shared from the outset. The weak epiphanies in Figure 3.9 function in quite different ways, however: H and I, being parented by segments in different families, initiate new families, while D does not. The class of weak epiphanies is therefore subdivided into ***external*** and ***internal*** types (Definition 3.2.10) depending on whether or not the paths from a shared origin pass through different families or, equivalently, other strong or weak external epiphanies (compare the paths from B to D with the paths from A or X to H or I; see also Lemma 3.2.11). The crucial segments that initiate new families are therefore the sources, strong epiphanies, and weak external epiphanies: every family contains exactly one of these (Corollary 3.2.14), and the families may be defined as the simple components of the digraph which removes every arrow pointing into a strong or weak external epiphany – the so-called ***inter-family arrows*** (Definition 3.2.12; see Figure 3.12 for an illustration). This formulation is proved to arrive at the same set of families as the dyeing process outlined above (formalised as a labelling algorithm) in Theorem 3.2.13, and the colour of each family can be formalised as a ***profile*** vector $P(a)$ (Definition 3.2.9) which has as many components as there are sources and records the percentage of each segment derived from each source via an equally-weighted average of its parents' profiles. For Figure 3.9, $P(A) = P(B) = P(C) = P(D) = (100,0)$, $P(X) = (0,100)$, $P(E) = P(F) = P(G) = P(H) = (50,50)$, and $P(I) = (67,33)$. It can be seen that siblings have identical profile vectors, and cousins have profile vectors with nonzero entries in the same places (Corollary 3.2.15); siblings are therefore also cousins.[30]

The algorithm labels families with ordinal numbers carrying no real significance beyond identification (they are assigned using a breadth-first search – see n. 46 – which fans out from each source, yielding an approximate ordering by first onset): families containing sources (***source families***) are prefixed with 'S', and others (***composite families***; see Corollary 3.2.14) have three-part hierarchical labels dependent on cousin grouping, sibling grouping, and finally family (such that families sharing the same first part are cousins and those sharing the same second part are siblings). In Figure 3.9, $A/B/C/D$ and X might belong to S1 and S2 respectively, then E to 1.1.1, F and G to 1.1.2, H to 1.1.3, and I to 1.2.1. In the Beethoven example, since every epiphany is a weak internal epiphany the five families S1 to S5 are identical to the connected components: as can be seen in Figure 3.23, the five sources are 000|C4 (which has 96 descendants), 015|C5 (which has 2), 042|G4 (which has 9), 069|B4 (which has 4), and 081|C5 (which has 11). The profile of each segment therefore consists of a single '100' in one of the five slots of the vector; to reiterate, this is not because every segment is identical to a source, but because every segment is derivationally related to only one source.

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[30] The above concepts (sources, profiles, epiphanies, families, and inter-family arrows) are well-defined only with respect to DAGs, so it is necessary in general to find a graph's condensation (Definition 2.5) before applying these ideas. The properties of each vertex in the condensation (i.e. strong component of the original) can then be translated to apply to the original digraph in a natural way (see Definition 3.2.9 to Corollary 3.2.15 for a formalisation). As the first paragraph of this section suggests, it is reasonable to expect derivation digraphs to be nearly acyclic.

The claim made here is that families form relatively self-contained units in which differences between segments cannot be explained through appeals to any new or foreign ideas in the piece, but must derive from "internal" processes, logics, or narratives – which nevertheless retain the potential to be globally significant. Each family is therefore roughly equivalent to a paradigmatic block: but here the awkwardness of oblique relationship is replaced by the concept of epiphany in a way that obviates Agawu's concern regarding one-off hybrid segments (see Chapter 2 at n. 90). Having refined a basic formal categorisation based on derivational connection into one that accounts for rhetorical disposition (specifically through considering epiphany as a digraph structure), the next section examines the morphological relationships of musical material that hold between the segments themselves.

Mathematical Formalisation

A note on notation: In Section 3.1, motivic segments were understood as sets (of notes) and so were labelled using upper-case letters. In the following, we are chiefly interested in segments as vertices, and so lower-case letters are used as labels; this does not change the definition of a motivic segment, but does allow upper-case letters to be used as labels for sets of segments (and of arrows). This convention is retained in Section 3.3, which brings together the set-focused and vertex-focused perspectives. The script letter $\mathcal{M}$ is retained throughout for the set of all segments.

Definition 3.2.1

A ***derivation digraph*** $\mathcal{D} = (\mathcal{M}, D)$ is a finite digraph taking $\mathcal{M}$ as its set of vertices and including (m_1, m_2) in its arrow set D if and only if $m_1 \delta m_2$.

Definition 3.2.2

A ***succession digraph*** $\mathcal{S} = (\mathcal{M}, S)$ is a finite digraph taking $\mathcal{M}$ as its set of vertices and including (m_1, m_2) in its arrow set S if and only if $m_1 \sigma m_2$.

Definition 3.2.3

Two segments m_1 and m_2 are said to be in a relationship of ***progression*** if $m_1 \delta m_2$ and $m_1 \sigma m_2$. Taking the derivation and succession digraphs $(\mathcal{M}, D)$ and $(\mathcal{M}, S)$ respectively, the ***progression digraph*** is described as $(\mathcal{M}, D \cap S)$.

Corollary 3.2.4

Derivation and succession digraphs have the following basic properties:

i. Neither digraph may contain loops or parallel arrows.
ii. Succession digraphs may not contain circuits.
iii. Derivation digraphs may not contain isolated vertices (i.e. connected components of size 1).

Proofs

i. This follows directly from Definition 3.1.7, Definition 3.1.8, Definition 3.2.1, and Definition 3.2.2.

ii. Since all onsets (and offsets) in a succession chain must be strictly increasing (i.e. $on(m_1) < on(m_2) < \cdots < on(m_n)$ – see Definition 3.1.8), it is necessarily true that $on(m_n) > on(m_1)$ and so m_1 may not be a successor to m_n.

iii. This follows directly from the second sentence of Definition 3.1.7.■

Lemma 3.2.5

A path of derivations $m_1 m_2 \dots m_n$ may always be converted to a circuit by adding (m_n, m_1) if, for all i from 1 to $n-1$, and for some fixed value of k from 1 to $n-1$:

$off(m_i) \geq off(m_{i+1})$ if $i < k$;

$off(m_k) \geq on(m_{k+1})$;

$on(m_i) \geq on(m_{i+1})$ if $i > k$,

and at least one of these relationships is not one of equality.

Equivalently, if each (m_i, m_{i+1}) is configured with reference to Figure 3.2 as:

3, 4, 5, **6**, **8**, or **11** if $i < k$;

2, **3**, **4**, **5**, **6**, **7**, **8**, **9**, **10**, or **11** if $i = k$;

3, **5**, 6, 7, **9**, or **11** if $i > k$,

including at least one of the bold configurations, a circuit is always possible.

Proof

For each derivation arrow (m_1, m_2), a number of relationships between its onsets and offsets must hold according to Definition 3.1.4, Definition 3.1.6, and Definition 3.1.7. It is useful to model these as a digraph where (x, y) represents $x > y$ (Figure 3.10a), since any path within this digraph corresponds to a conclusion concerning the relation between its endpoints (for these purposes we may model equality as a double-headed arrow). When each bold edge of Figure 3.10a is assigned a value of $<$, $=$, or $>$, the eleven configurations of Figure 3.2 arise as follows:

	$off(m_1) < off(m_2)$	$off(m_1) = off(m_2)$	$off(m_1) > off(m_2)$
$on(m_1) < on(m_2)$	1/2/10	4	8
$on(m_1) = on(m_2)$	7	3	6
$on(m_1) > on(m_2)$	9	5	11

If a $>$ or $=$ relationship is assigned to at least one of the bold edges, the dashed edge will always point $(off(m_1), on(m_2))$ since there will be a path from $off(m_1)$ to $on(m_2)$ via $on(m_1)$ or $off(m_2)$ (for example: $off(m_1) > on(m_1) = on(m_2) \Rightarrow off(m_1) > on(m_2)$). If both bold edges are assigned a $<$ relation, then the dashed edge remains unspecified, and can take any of the three relation values corresponding to configurations 1, 2, and 10 respectively. (If we remove the requirement of Definition 3.1.7 – i.e. the arrow $(off(m_2), on(m_1))$ – we find that a symmetrical situation holds for two $>$ relations, such that the bottom-right cell of the table would hold three configurations: 11, and configurations akin to 2 and 1 with the letters A and B reversed.)

Taking the digraph model above and applying it repeatedly to build a path of derivations, the goal of the proof is to understand what conditions necessarily lead to the diagonal arrow $(off(m_1), on(m_n))$ of Figure 3.10b, permitting the derivation $m_n \delta m_1$ and thereby closing the

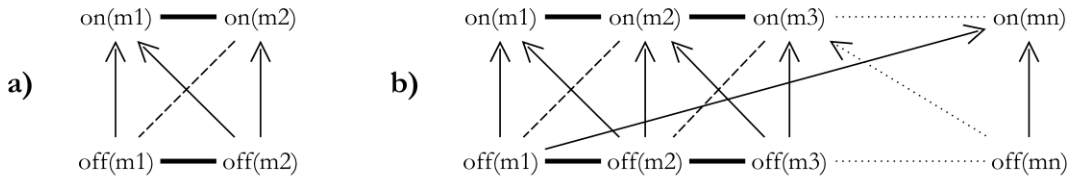

Figure 3.10: Modelling onset–offset inequalities as digraphs in the proof of Lemma 3.2.5. Bold edges may point in either (or both) direction(s), and in some circumstances determine the dashed arrows' orientations. Dotted edges and arrows represent a repeating structure.

circuit. Firstly, note that passing from the top to the bottom of the figure is only possible if $(on(m_i), off(m_{i-1}))$ is included as an arrow for some i, and that this in turn is only possible if $off(m_{i-1}) < off(m_i)$ and $on(m_{i-1}) < on(m_i)$. This means that the start of the downwards arrow, $on(m_i)$, is unreachable from $off(m_1)$, and therefore that any path from $off(m_1)$ to $on(m_n)$ must pass from the bottom half to the upper half of the figure exactly once.

The most general way to do this is via one of the dashed arrows: any path including a vertical arrow followed or preceded by a left-to-right arrow can be shortened to one including a left-to-right diagonal arrow (as the discussion of the table of configurations above shows), and any path including a bottom-right-to-top-left diagonal arrow can be shortened to one including a vertical arrow ($off(m_i)on(m_{i-1})on(m_i)$ becomes $off(m_i)on(m_i)$). Since $on(m_n)$ must be strictly greater than $off(m_1)$, at least one of the arrows in the path must not be double-headed. The paths so described (starting with a path of offsets, then jumping upwards to a path of onsets, even if this jump is the first or last move) are formalised as stated in the theorem, and can be rephrased in terms of configuration numbers by considering the inequalities in relation to the table of configurations above. ■

(Note that the lack of a path between $off(m_1)$ and $on(m_n)$ does not imply that $off(m_1) \leq on(m_n)$, only that the relation $off(m_1) > on(m_n)$ does not follow as a direct consequence of the pattern of onsets and offsets. This is analogous to the dashed-arrow relationship between $off(m_1)$ and $on(m_2)$ above, since it is fixed under certain onset–offset patterns, but may vary under others.)

Theorem 3.2.6

Consider the following statements concerning the derivation circuit $m_1 \dots m_n m_1$ and some $i \geq 1$:

A. The circuit is not constructed as described in Lemma 3.2.5.

B. (m_i, m_{i+1}) is configured as 1.

C. (m_h, m_{h+1}) is configured as 2, 7, 9 or 10 and for some $i > h \geq 1$, (m_i, m_{i+1}) is configured as 2, 4, 8, or 10.

D. $on(m_{i+1}) > off(m_1)$.

E. $i < n - 1$ and for some $j > i$, (m_j, m_{j+1}) is configured as 5, 9, or 11 such that $on(m_{j+1}) < off(m_1)$.

Then D ⇒ A ⇔ B or C, and D ⇒ E. Note that this means that D can *only*, but does not *always*, hold when A is true.

Proofs

To prove that **D ⇒ A** it is sufficient to prove that ¬A ⇒ ¬D. If the circuit is constructed as in Lemma 3.2.5 (i.e. ¬A), then there is a path between $off(m_1)$ and $on(m_n)$. If $i+1>k$ then $on(m_{i+1})$ must be part of this path, whereas if $i+1 \leq k$ then $off(m_{i+1})$ must be part of the path. But $off(m_{i+1}) > on(m_{i+1})$, so a path can always be found from $off(m_1)$ to $on(m_{i+1})$. This path may be comprised entirely of double-headed arrows without violating Lemma 3.2.5 (provided $on(m_p) > on(m_{p+1})$ for some p, $i+1 \leq p < n$), so $off(m_1) \geq on(m_{i+1})$ (i.e. ¬D).

It is easy to check that B and C conflict with the configurations outlined in the statement of Lemma 3.2.5, so **B or C ⇒ A**.

Since Lemma 3.2.5 describes all the situations in which a path of length >1 between $off(m_1)$ and $on(m_n)$ (but not vice-versa) arises, if we assume A to be true, then no such path exists. This means that $off(m_h) < off(m_{h+1})$ for some h, and either $off(m_h) < on(m_{h+1})$ or $on(m_i) < on(m_{i+1})$ for some $i > h$. In the first of these cases, (m_h, m_{h+1}) must be arranged in configuration 1; in the second, (m_h, m_{h+1}) must be arranged in configuration 2, 10, 7, or 9 whilst (m_i, m_{i+1}) must be arranged in 1, 2, 10, 4, or 8. So **A ⇒ B or C**.

For a circuit to form, $on(m_n) < off(m_1)$, so if D is true then $i \neq n-1$. Moreover, if $on(m_{i+1}) > off(m_1)$, then $on(m_n) < on(m_{i+1})$, which means that $on(m_{j+1}) < on(m_j)$ for some j, $i < j < n$. The arrow (m_j, m_{j+1}) therefore needs to be configured as 5, 9, or 11 with $on(m_{j+1}) < off(m_1)$ to ensure that $on(m_n) < off(m_1)$. So **D ⇒ E.■**

Definition 3.2.7

For distinct $a, b, c, d \in \mathcal{M}$, two derivation arrows (a, c) and (b, d) are said to be ***thematically joined*** if $(a, b), (c, d) \in S$ or $(b, a), (d, c) \in S$.

Let D be the vertices of a graph in which the set of edges J consists precisely of those pairs which are thematically joined, then define:

$\mathcal{J}_1$ as the set of all paths in (D, J) between any pair $a_1 = (x_2, x_1)$ and $a_2 = (x_1, x_0)$ chosen such that a_1 is not connected to (x_3, x_2) for any x_3;

J_1 as the set of all edges $\{b, a_2\}$ concluding paths in $\mathcal{J}_1$; and

J_1^- as the set of all $a \in J_1$ such that there is no $b \in J_1$ satisfying, $\forall P \in \mathcal{J}_1$, $a \in P \Rightarrow b \in P$ but $b \in P \nRightarrow a \in P$.

If J_i^- is obtained from $(D, J \setminus \bigcup_{r=1}^{i-1} J_r^-)$ in a similar way, and n is the largest value of i for which $J_i^- \neq \emptyset$, then the ***thematic repetitions*** in the piece are defined by the connected components of $(D, J \setminus \bigcup_{r=1}^{n} J_r^-)$ having two or more vertices. The initial points of the D-arrows constitute the first ***theme*** $\mathcal{M}_1$ of each component, and the terminal points of the D-arrows constitute the second theme $\mathcal{M}_2$ (see Figure 3.11).

If the earliest onset of $\mathcal{M}_1$ occurs before the latest offset of $\mathcal{M}_2$, the themes are ***simultaneous***. If a segment with an earliest onset in $\mathcal{M}_2$ is a successor to a segment with a latest offset in $\mathcal{M}_1$, the thematic repetition is ***immediate***. If the D-arrows define a bijection between $\mathcal{M}_1$ and $\mathcal{M}_2$, then the themes are ***isomorphic***.

Theorem 3.2.8

If D' is the vertex set of a connected component of $(D, J \setminus \bigcup_{r=1}^{n} J_r^-)$, then:

i. $\mathcal{M}_1$ and $\mathcal{M}_2$ form connected regions of $\mathcal{S}$;

ii. $D' \subseteq \{(a,b) \in D | a \in \mathcal{M}_1, b \in \mathcal{M}_2\}$;

iii. $(D, J \setminus \bigcup_{r=1}^{n} J_r^-)$ contains no complete path from any $\mathcal{J}_i$ for $i = 1$ to n;

iv. $\mathcal{M}_1 \cap \mathcal{M}_2 \neq \emptyset$ if and only if some subset of D' forms a circuit in $\mathcal{D}$; in this case, the thematic repetition is said to be ***circular***.

Proofs

i. If two D-arrows are joined by an edge in $(D, J \setminus \bigcup_{r=1}^{n} J_r^-)$, then by definition their initial points are joined by an arrow in $\mathcal{S}$. Since D' is connected in $(D, J \setminus \bigcup_{r=1}^{n} J_r^-)$, the set of initial points in D' (i.e. $\mathcal{M}_1$) must be simply connected in $\mathcal{S}$. A similar argument holds for $\mathcal{M}_2$.

ii. Clearly every member of D' points from $\mathcal{M}_1$ to $\mathcal{M}_2$ by definition. But we cannot, in general, assume the converse (and therefore that the sets are always equal) since not every arrow that points from $\mathcal{M}_1$ to $\mathcal{M}_2$ is thematically joined to another.

iii. We seek to prove that every path in $\mathcal{J}_i$ contains at least one edge from J_i^- for a given i; from this it follows that $(D, J \setminus J_i^-)$ cannot contain any of the paths from $\mathcal{J}_i$, and therefore that statement iii is true.

   Any path in $\mathcal{J}_i$ must contain a member of J_i by definition so assume, aiming for a contradiction, that some $P \in \mathcal{J}_i$ contains some $a \in J_i \setminus J_i^-$ but no $a' \in J_i^-$. Since $J_i \setminus J_i^-$ may be defined as the set of all $a \in J_i$ such that there is some $b \in J_i$ appearing in all the same paths (in $\mathcal{J}_i$) as a (but in at least one without a), $b \in P$. So either $b \in J_i^-$ (a contradiction) or $b \in J_i \setminus J_i^-$ and there exists some c appearing in all the same paths as b but not vice-versa (we know that $a \neq c$ since b appears in at least one path without a). Iterating this argument we reach an inevitable contradiction: since D is finite and (D, J) is not a multigraph, J_i must be finite, and so every path in $\mathcal{J}_i$ contains at least one edge from J_i^-.

iv. The latter statement implies the former since any vertex in a circuit is both an initial and a terminal point: if $(x_2, x_1), (x_1, x_0) \in D'$, then $x_1 \in \mathcal{M}_1 \cap \mathcal{M}_2$.

   Conversely, if $\mathcal{M}_1 \cap \mathcal{M}_2 \neq \emptyset$ then there must be some vertex $x_1 \in \mathcal{M}$ which is both an initial and terminal point in D' (i.e. $(x_2, x_1), (x_1, x_0) \in D'$). There must also be some vertex $x_3 \in \mathcal{M}$ such that $(x_3, x_2) \in D'$: otherwise, (x_2, x_1) and (x_1, x_0) would satisfy the conditions for a_1 and a_2 respectively in Definition 3.2.7, and would hence be disconnected in $(D, J \setminus \bigcup_{r=1}^{n} J_r^-)$ (as proved in part iii above). Repeating this argument, we find that $(x_4, x_3), (x_5, x_4), \ldots \in D'$: but since $\mathcal{M}$ is finite, one of the x_i must be repeated, thereby creating a circuit of D-arrows.■

Some of the main concepts from Definition 3.2.7 and Theorem 3.2.8 are illustrated in Figure 3.11. The patterns of derivation (solid arrows) and succession (dashed arrows) shown in the first column are converted to (D, J) graphs in the next: in row a, for example, the thematically joined pairs of derivation arrows $(a, c), (b, d)$ and $(a', c), (b, d)$ become edge-joined pairs of vertices $\{ac, bd\}$ and $\{a'c, bd\}$ (the shared vertex c makes the resultant themes non-isomorphic). Row a contains a pair of arrows that satisfy the conditions for a_1 and a_2 in Definition 3.2.7, and so the path between them in (D, J) is a member of $\mathcal{J}_1$ (dashed in the diagram) and its final edge

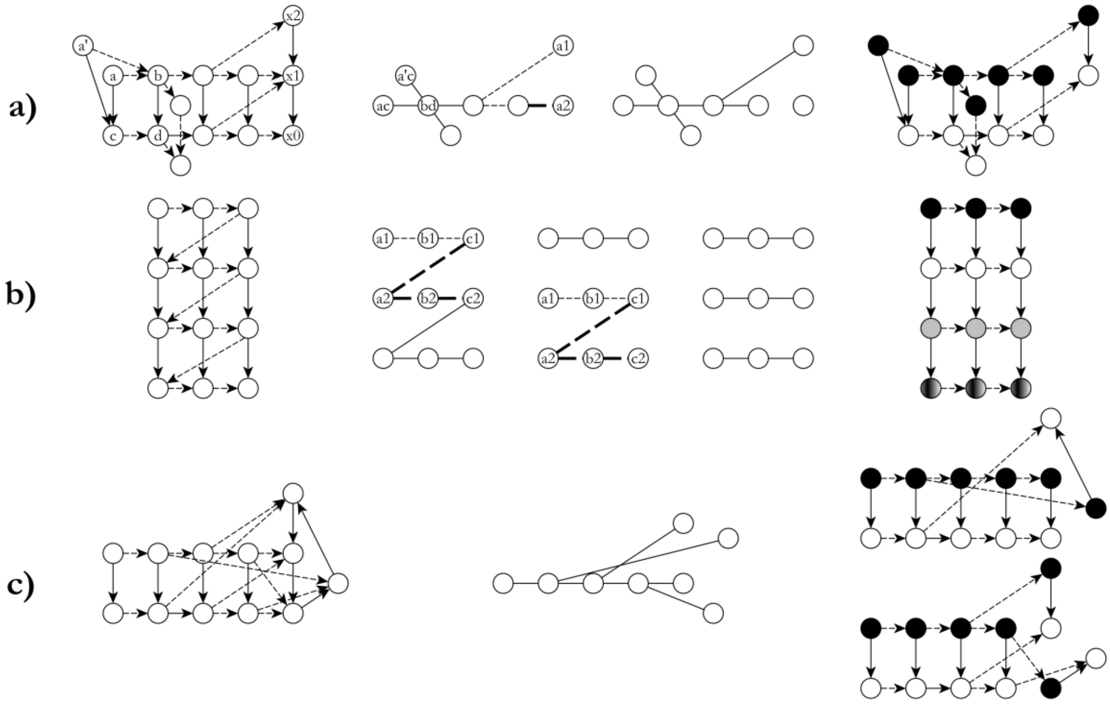

Figure 3.11: Some examples illustrating Definition 3.2.7 and Theorem 3.2.8.
The first column shows three different arrangements of derivation (solid) and succession (dashed) arrows; the second column shows the successive $(D, J\backslash \bigcup_{r=1}^{i-1} J_r^-)$ graphs for each of these (with edges in $\mathcal{J}_i$ paths dashed and members of J_i bold); and the third column shows the resulting divisions of the original vertices into themes. See text following Theorem 3.2.8 for further explanation.

(pointing into a_2) is a member of J_1 (bold in the diagram). As it is the only member of J_1, it is also a member of J_1^-, and so is removed from the graph to form $(D, J\backslash J_1^-)$: as this contains no a_1, a_2 pairs, it defines a thematic repetition which gives rise to $\mathcal{M}_1$ (black vertices) and $\mathcal{M}_2$ (white vertices) in the final column (which, for clarity, omits all succession arrows from the first column that do not point between members of the same theme).

Row b is an example of immediate thematic repetition: the twelve vertices in the first column form a single succession chain in which each segment is derived from the one occurring three places before. There are three possible a_1, a_2 pairs (the second and third are labelled with b and c), and again the paths between them (i.e. a_1 to a_2, b_1 to b_2, and c_1 to c_2) are shown dashed with the final edges in bold. This time, however, the paths overlap, and it can be checked that $\{c_1, a_2\}$ is the only edge in J_1^- (since, for example, $\{a_2, b_2\}$ appears in the b and c paths but so does $\{c_1, a_2\}$, which also appears without $\{a_2, b_2\}$ in the a path). The graph $(D, J\backslash J_1^-)$ opens up the second layer so the process can be repeated: this could not have been done in stage one since a_1 in stage two functions as a_2 in stage one. Stage three – the graph $(D, J\backslash J_1^- \cup J_2^-)$ – is the final stage, giving rise to the three thematic repetitions shown in the third column. Note that the white and grey themes each appear in two thematic repetitions, first as $\mathcal{M}_2$ and then as $\mathcal{M}_1$, analogous to B

and C in the derivation chain $A \rightarrow B \rightarrow C \rightarrow D$. Whilst the above process seems to be a laborious way to arrive at a fairly instinctive partition of the twelve segments into themes (that produced by starting at the beginning of the chain and including derivation arrows until a segment from $\mathcal{M}_2$ appears in $\mathcal{M}_1$), it has the advantage that it is generalisable to other situations – not least those portrayed in rows a and c, but also including variations of row b in which, for example, the succession order of a_1 and b_1 is reversed.

Row c gives an example of a circular thematic repetition, and this can be resolved into disjoint themes in two ways as shown in the third column. Although the structure is similar to that shown in row a, the crucial difference is that the symmetry created by the circuit does not allow any one of its arrows to be prioritised for inclusion: $\mathcal{J}_1$ is therefore empty here. Choosing to break the circuit by removing one of its four arrows leads to one of the two alternative interpretations as shown in the third column; more complex arrangements than c featuring several circuits sharing edges and vertices yield a wider range of interpretations, some of which cannot be fixed through the removal of a single arrow.

Definition 3.2.9

A vertex of $\mathcal{D}^*$ (see Definition 2.5) is a ***source*** if it has indegree 0; the set of all sources in a given $\mathcal{D}^*$is denoted $S^* = \{s_1, s_2, \ldots, s_r\}$.

The ***profile*** of each s_i, $P(s_i)$, is given by an r-component vector with an entry of 100 in the ith position and 0s everywhere else. If the n parents of a given vertex a are listed arbitrarily as $m_1, m_2, \ldots, m_n$, then the profile of each $a \in \mathcal{M}^*$ is given by $P(a) = \frac{1}{n}\sum_{i=1}^{n} P(m_i)$. If $m \in \mathcal{M}$ is in strong component m^* then $P(m) = P(m^*)$.

Definition 3.2.10

Any segment $m_1 \in \mathcal{M}^*$ with $ind_{\mathcal{D}^*}(m_1) \geq 2$ is a moment of ***epiphany***. If there exists a vertex $m_0 \in \mathcal{M}^*$ such that every parent of m_1 is reachable from m_0 then m_1 is a ***weak epiphany***; otherwise, it is a ***strong epiphany***. If m_1 is a weak epiphany, then it is categorised as ***external*** if at least one of the vertices on the paths from m_0 to m_1 is a strong epiphany or weak external epiphany; otherwise, it is categorised as ***internal***.

Lemma 3.2.11

If a condensed derivation digraph contains a weak external epiphany, then it must contain at least one strong epiphany.

Proof

Assume, for a contradiction, that $\mathcal{D}^*$ contains at least one weak external epiphany and no strong epiphanies. Then for a given weak external epiphany w_1, Definition 3.2.10 states that there must be a path leading from some other weak external epiphany w_2 to w_1, and so in turn from w_3 to w_2, w_4 to w_3, and so on. Since derivation digraphs must be finite, at least one of the w_i must be repeated in this list, say as w_{i+k}; but then we have a circuit $w_{i+k}w_{i+k-1} \ldots w_{i+1}w_i = w_{i+k}$, thus contradicting the definition of $\mathcal{D}^*$ as a condensation (Proposition 2.6iii). Therefore, $\mathcal{D}^*$ must contain at least one strong epiphany.■

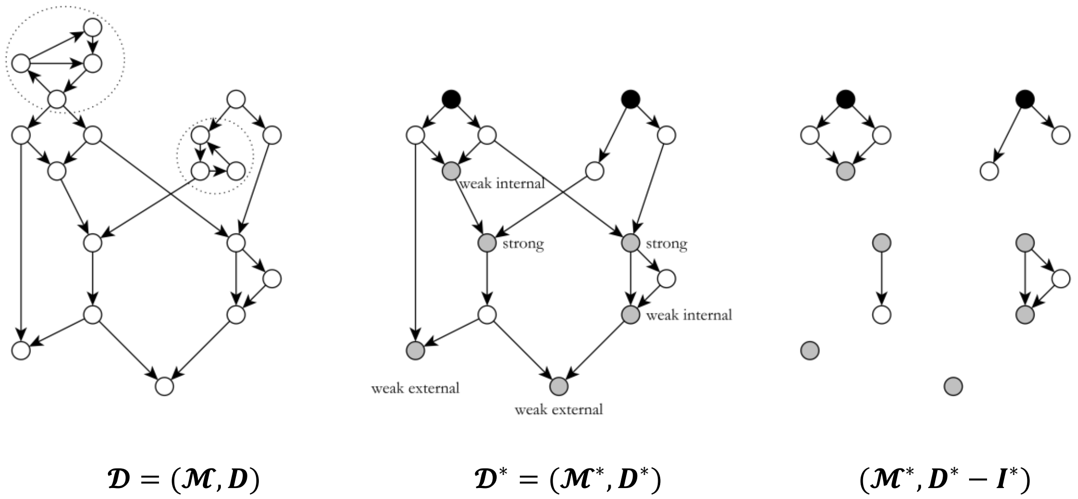

Figure 3.12: The condensation and partitioning into families of a derivation digraph. The first digraph features two strongly connected components (in dotted circles), which are shrunk to single vertices in the other two digraphs. The categorisation of epiphanies in the second digraph (coloured grey, with sources in black) induces the partition into families seen in the third; the family categorisation can then be extended back into the original vertex set by re-expanding the strongly connected components.

Definition 3.2.12

Let $E^* \subset \mathcal{M}^*$ be the set of all strong and weak external epiphanies in $\mathcal{D}^*$, and let I^* be the set of arrows which point into E^* (i.e. $I^* = \{(m_1, m_2) \in D^* | m_2 \in E^*\}$). Then the ***families*** of $\mathcal{D}^*$ are the simple components of the digraph $(\mathcal{M}^*, D^* - I^*)$ (see Figure 3.12 for an example).

If $m_i \in \mathcal{M}$ is in strong component m_i^*, then m_1 and m_2 are members of the same family in $\mathcal{D}$ if and only if m_1^* and m_2^* are members of the same family (or are equal) in $\mathcal{D}^*$. Those arrows in D whose endpoints lie in different families are known as ***inter-family arrows***, the set of which is denoted I.

Theorem 3.2.13

The following labelling process partitions the vertex set of $\mathcal{D}^*$ into families:

1. Label each source with its own name.
2. Label every vertex that is not a source only when all of its parents have been labelled:
   a. If its parents all have the same label, assign this label to the vertex.
   b. Otherwise, label the vertex with its own name.
3. Repeat step 2 until all vertices have been labelled.

Proof

Let $\mathcal{D}^*$ be labelled as described above, and denote the label of vertex a as $l(a)$. First we note that the proposed labelling is indeed possible (and partitions the vertices) since a DAG induces a partial ordering on its vertices: if we tried to run the process on $\mathcal{D}$, which in general may contain circuits, we would reach a stalemate at every strongly connected component while each vertex in each circuit waits for its parent to be labelled. We also note that, since any vertex a may only have label x if all of its parents have label x (or if $a = x$), the collections of vertices all having the same label must be (simply) connected regions of $\mathcal{D}^*$. In other words, if a and b both have label x, then there must exist paths from x to a and x to b in which every vertex also has label x. Note that this is a stricter condition than simple connectivity (since the x–a and x–b paths must be directed), but a looser condition than weak connectivity (since a and b need not be joined by a directed path in either direction, unless $x = a$ or $x = b$).

We now seek to prove that the set of arrows whose endpoints are given non-identical labels are precisely those arrows that make up I^*. From this it follows directly that the arrows in $D^* - I^*$ are precisely those arrows of D^* that point between identical labels, and hence that the connected regions defined by label equality coincide exactly with the components of $(\mathcal{M}^*, D^* - I^*)$ (i.e. the families).

Consider an arrow (a, b) in which $l(a) \neq l(b)$. Clearly $ind_{\mathcal{D}^*}(b) \geq 2$ (since if $ind_{\mathcal{D}^*}(b) = 1$, b would have inherited a's label), so b is a moment of epiphany. In addition, we know that at least one other parent of b, say a', does not have label $l(a)$ (as otherwise b would again inherit $l(a)$), and therefore that $\nexists x$ such that there are label-preserving paths from x to a and x to a'. If $\nexists x$ such that there are *any* paths from x to a and x to a', then b is a strong epiphany. If $\exists x$ such that non-label-preserving paths exist from x to a and a', then there must be at least one non-label-preserving arrow in these paths, and therefore at least one other b-like epiphany. This reasoning can be applied recursively until, as in the proof of Lemma 3.2.11, we must reach a strong epiphany; retracing our steps, the other b-like epiphanies can then be classified as weak external. Arrows with non-identically labelled endpoints are therefore members of I^*.

A similar argument running in the opposite direction shows that strong and weak external epiphanies necessarily have different labels to their parents. Consider a strong epiphany b with parents a and a'; by definition, there is no vertex x such that paths to b via both a and a' exist, so a and a' must have different labels to each other and hence to b. Now if a digraph contains at least one weak external epiphany then, by Lemma 3.2.11, the digraph must contain a weak external epiphany b with paths $x \dots ab$ and $x \dots a'b$ such that at least one of these vertices is a strong epiphany. Since strong epiphanies necessitate a label change, this means that b cannot have the same label as both a and a', and therefore that all other weak external epiphanies in the digraph necessitate label changes. So all the members of E^* have different labels to their parents, and hence all members of I^* are non-label-preserving arrows. Since we have proved above that all non-label-preserving arrows are members of I^*, it follows that the two sets are equal.■

Corollary 3.2.14

Every family contains exactly one member of $S^* \cup E^*$. If it contains a member of S^*, it is a ***source family***; otherwise, it is a ***composite family***.

Proof

When the vertices of $\mathcal{D}^*$ are labelled as described in Theorem 3.2.13, all the members of a single family are given the same label. Moreover, each vertex may only be labelled with its own name or that of its parents (if they all share the same label); as the proof of Theorem 3.2.13 shows, those vertices in the former category are precisely the sources and the strong and weak external epiphanies. If two vertices in the same family are labelled with their own names, then clearly not every vertex in the family can have the same label, giving rise to a contradiction.■

Corollary 3.2.15

Members of the same family always have the same profile, but segments with the same profile do not always belong to the same family.

Distinct families with identical profiles are known as ***siblings***; families ultimately derived from the same sources (i.e. with nonzero entries in the same components of the profile vector) are known as ***cousins***.

Proof

Definition 3.2.9 and Theorem 3.2.13 show that there is a certain correspondence between assigning profiles and assigning labels (and therefore families): each source is assigned a brand new value of its own, and a profile/label is passed from parent to child if all of the other parents of the same child have the same profile/label. However, while this is the only way to pass a label from parent to child, it is not the only way to pass a profile from parent to child since a vertex may share a profile with only one of its parents (for example if their profiles are (50,50), (75,25), and (25,75)). In addition, two vertices with the same profile need not be connected via profile-preserving paths: a vertex with parent profiles (75,25) and (25,75) will have profile (50,50), as will other vertices with parent profiles (100,0) and (0,100). Label equivalence is thus a stricter form of profile equivalence. ■

3.3 Morphology

The discussion up to this point has focused on the structural, graph-theoretic relationships that hold between motivic segments modelled as vertices, making use of the derivation and succession (and occasionally segment onset and offset) information from the analytical input. The final section of this chapter now looks "inside" the vertices to examine how the morphological characteristics of pitch and rhythm inflect the structure of the digraph. To ignore this crucial aspect would be long-sighted, but it is worth reiterating that morphology is of interest to the present model only insofar as it can be used to enrich a structural digraph interpretation. The ideas outlined in this section are not, therefore, proposed as solutions to the more general problems of melodic similarity (outlined in Section 2.3), not least because motivic analysis and melodic similarity matching are not identical goals (although they are, of course, related), but also because their theory-discarding (see Section 1.8) emphasis on unambiguous equivalence excludes certain more context-dependent relationships (such as patterns amongst non-contiguous notes). The algorithm outputs a relatively small set of confidently identified relationships rather than a potentially enormous set of possible ones: as the following makes clear, however, the former can be used to establish indirect connections that equate, in many cases, to instances of the latter.

The approach taken here is also atypical when set alongside models of melodic pattern-matching in that, formally speaking, segmentation and similarity assessment are kept separate rather than existing in a symbiotic mutual dependence (although they do still implicitly influence each other here). The morphological concepts introduced are intended, then, to organise the members of $\mathcal{M}$ into a logical, abstract, and atemporal structure (Cone's second reading – see Section 2.3, point six), which can then be used as an orienting framework to chart the piece's unfolding in time (Cone's third reading). Metaphors that treat a piece of music as a path through an abstracted space are nothing new in music theory, from the major scale and the circle of fifths to the transformation network of pitch-class sets used in Lewin's analysis of Stockhausen's *Klavierstück III* and the four-dimensional cube used by Tymoczko in his analysis of two piano pieces by Chopin.[31] The principal difference between these harmonic spaces and the motivic ones outlined here is that the latter are piece-specific and therefore inherently incomplete: like Mendeleev's periodic table, they organise known elements into a logical arrangement, leaving appropriate gaps for those whose existence is theoretical.

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[31] Lewin, *Musical Form and Transformation*, pp. 16–67 (see, in particular, Examples 2.5 and 2.6 on pp. 34 and 38–39); Tymoczko, *A Geometry of Music*, pp. 284–93. Tymoczko reaches the fascinating conclusion that 'Chopin's F minor Mazurka [Op. 68, No. 4] and E minor Prelude [Op. 28, No. 4] are related by way of a tritone substitution' (p. 291, Figure 8.5.11).

Given two motivic segments of the same size, A and B, we begin by defining three equivalence relations that can hold between them (Definition 3.3.1 to Definition 3.3.3).[32] If the sequence of directed semitone intervals between consecutive notes in A matches the corresponding sequence in B (or, equivalently, if A and B are exact transpositions of each other when note spelling is ignored), then A and B are chromatically equivalent or ***C-equivalent***, written $A \sim_C B$. If the sequences of diatonic intervals (i.e. intervals between letter names – up a third, down a fourth, and so on) in A and B are identical, then A is diatonically equivalent or ***D-equivalent*** to B: $A \sim_D B$.[33] Finally, if the time-points of A can be mapped to the time-points of B by adding a constant integer to each, and if the pattern of time signatures across the bars they occupy is the same, then they are rhythmically equivalent or ***R-equivalent***, $A \sim_R B$ (this definition is recast in terms of the time-line helix of Definition 3.1.3 in Definition 3.3.3 and Theorem 3.3.4). Instead of separately listing each type of equivalence that holds between a given pair, the letters C, D, and R can be combined for the sake of brevity: in particular, if a pair is CDR-equivalent, $A \sim_{CDR} B$, then this is often referred to simply as equivalence. Another particularly pertinent type of composite equivalence, CD-equivalence, occurs when a pair of segments has the same sequence of spelled pitch intervals (e.g. up a major third, down a minor ninth, repeated perfect unison).

In the language of Section 2.3, point three, these are function-based categorisations by symmetry and so cannot be used to measure distance.[34] Each type of equivalence is necessarily transitive with itself (from the definition of an equivalence relation), so the statements 033|C3 $\sim_{DR}$ 034|Db3 and 034|Db3 $\sim_{DR}$ 137|Bb2 together imply that 033|C3 $\sim_{DR}$ 137|Bb2. Note, however, that like the three Neo-Riemannian transformations P, L, and R, the different types of equivalence are not transitive with each other as they each fix different dimensions while the others move freely: if 130|F5 $\sim_D$ 131|G5 and 131|G5 $\sim_{CD}$ 131|A5, then while we can safely conclude that 130|F5 $\sim_D$ 131|A5, we must observe directly that, in addition, 130|F5 $\sim_R$ 131|A5 (and therefore that 130|F5 $\sim_{DR}$ 131|A5).

Musically, these equivalence relations correspond to the Schoenbergian description of variation as 'changing some […] features and preserving some [others]': once an identical pattern has been identified in one dimension (say, rhythm), it is assumed that the others may vary quite

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32 See Section 2.3, point four, for the definition of an equivalence relation.
33 The assumption here is that note spelling functions as an adequate indicator of scale degree.
34 Recall that a symmetry is a function that preserves a certain set of features. We can define equivalence relations and equivalence classes either in terms of the common features that the class members share (e.g. pitch-integer, letter-name, or inter-onset intervals) or in terms of the functions that preserve those features (e.g. 'A is equivalent to all those segments that can be reached by a chromatic/diatonic/rhythmic transposition'). Therefore, even though some of the definitions above are expressed in terms of shared features, they are still essentially function-based (rather than proximity-based).

widely without disintegrating the proposed similarity relationship.[35] The concept has been applied to the problem of motivic pattern extraction by Lartillot and Toiviainen, who outline a model based on 'exact matching along multiple musical dimensions' rather than on allowing a fuzzy tolerance in every dimension at once: the resultant so-called 'heterogeneous patterns' switch between levels of specificity (for example, "an ascending major sixth ending on a strong beat followed by three quavers descending by second and two notes ascending to an A in any octave") and form a hierarchy based on shared characteristics (so the example given above would be a subclass of the more general "up–down–down–down–up–up").[36] The present model requires exact *complete* matching in *one* dimension, but the principle of holding certain elements constant while others vary freely is the same in both cases.

In addition to the three basic equivalence relations (and the four composite ones), the symmetric but intransitive relation of embedding, or ***E-relation***, is defined between segments of different sizes: $A{\sim_E}B$ if A is CDR-equivalent to a subsegment of B or if B is CDR-equivalent to a subsegment of A (Definition 3.3.5; certain conditions create transitive E-relations as specified in Corollary 3.3.7ii). Not only does this notion extend the concept of exact matching plus free variation to allow comparison between motives of different sizes, but it does so in a way that keeps a check on the tendency of mathematical and computational approaches to uncover large numbers of matches that are often judged analytically insignificant.[37] Shorter segments, for example, have a propensity to appear everywhere – notice that 071|A4 is contained in 117|E5 (as E–F) – but this problem would be compounded if individual C-, D-, and R-embedding were permitted among subsegments: in that case, 071|A4 would be related to every segment containing an ascending semitone, ascending second, or second–third beat crotchet rhythm.

Certain C-, D-, or R-embeddings *are* analytically significant (for example, it is unsatisfactory that segments 000|C4 and 003|G4 are morphologically unrelated) and the concept of indirect relation through embedding proves to be a powerful analytical tool for the identification of such relationships. To explore the indirect connections produced by an intransitive relation, a graph-theoretic model naturally suggests itself; again, however, the temptation to treat this graph as a means to measure distance must be avoided. Taking $\mathcal{M}$ as the set of vertices, the set of edges is defined as the union of four smaller sets corresponding to C-, D-, R-, and E-relation respectively

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[35] Schoenberg, *Fundamentals*, p. 8; he goes on to state explicitly (cf. Chapter 2, n. 23 above) that '[p]reservation of rhythmic features effectively produces coherence'.
[36] See Lartillot and Toiviainen, p. 286; see also Olivier Lartillot, 'Taxonomic Categorisation of Motivic Patterns', in *Discussion Forum 4B*, ed. by Toiviainen, pp. 25–46 for more on pattern representation and hierarchical description.
[37] See, for example, Marsden, 'Position Paper', p. 149: 'Commonly, […] mathematical and computational approaches find many more motives and many more relationships between fragments than traditional motivic analysis.'

(Definition 3.3.6): taking the disjoint union produces the ***type multigraph*** (in which a CD-related pair, for example, is joined by both a C-edge and a D-edge), whereas taking the usual union produces the ***type graph*** (in which a CD-related pair is joined by a CD-edge). In the subgraphs of the type multigraph formed by including all the edges of one equivalence relation (say, all the C-edges, or all the pairs joined by both C- and D-edges) the connected components will be complete graphs (Definition 2.1) since equivalence relations are transitive: note that this is not always the case in the type graph since some C-equivalent pairs, for instance, may be joined instead by CD-, CR-, or CDR-edges (Corollary 3.3.7i). The only subgraph of the type graph guaranteed to produce complete connected components is that formed from the set of CDR-edges, and any two vertices in one of these components will stand in exactly the same C-, D-, R-, and E-relationships to all the other vertices in the graph (Corollary 3.3.7i and iii).

Figure 3.24 shows the full type graph for Figure 3.22. The large number of edges (599 as opposed to the derivation graph's 126 arrows) makes the graph difficult to read directly, but the difference in general shape when compared to Figure 3.23 is clear: note, in particular, the clusters of vertices forming complete graphs.[38] It is the connections between these clusters (especially via E-edges) that establish intransitive and indirect links: in particular, with reference to the example cited above, segment 101|F4 (the start of the recapitulation) is both R-equivalent to 003|G4 and embedded in 000|C4. It can be seen from Figure 3.24 that certain vertices create "bottlenecks" in the graph and are therefore crucial in establishing indirect links. Calculating a quantity known as betweenness centrality for each vertex, x, is one way to find these bottlenecks: included as a built-in feature of NodeXL, this measure looks at all the shortest paths between every pair of vertices in the graph and counts how many include x. For the Beethoven example, the most central vertex is 012|Bb4, followed by the CDR-equivalent two-note minor second figures 082|F5 and 092|F6, and then by the CDR-equivalent 095|Ab4, 096|Eb6, and 100|C6 (as suggested by Corollary 3.3.7iii, CDR-equivalent segments must have identical betweenness centralities). These segments, of which the turn figure 095|Ab4 is particularly analytically salient, isolate as independent gestalten the shared subsegments of, for example, 000|C4, 004|C5, 011|Eb5, 052|Bb4, 053|Db5, and 115|G5 (all of which are unrelated directly to each other): they therefore function like epiphanies in that they confirm concretely a morphological relationship that would otherwise remain theoretical.

[38] As implied above, complete subgraphs of the type multigraph will always correspond to complete subgraphs of the type graph, albeit with potentially different labelling. The vertices in the third-largest component of Figure 3.24, for instance, are all R-equivalent and so form a complete R-subgraph of the type multigraph; as shown here, however, some are also C- or D-equivalent, meaning that not every edge in the component is a pure R-edge.

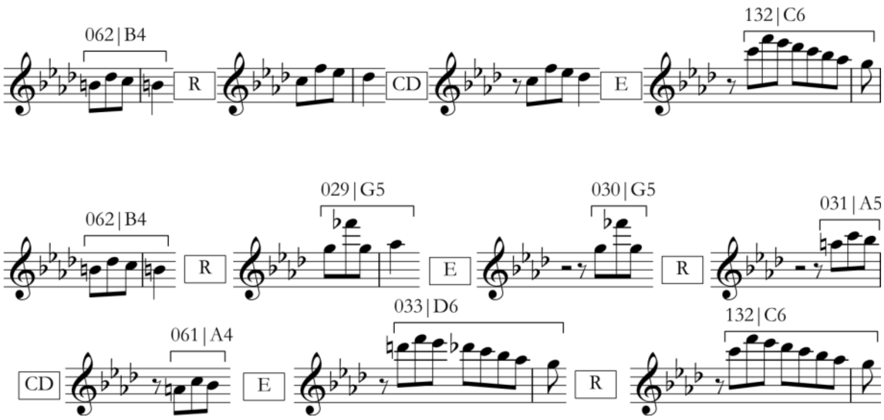

Figure 3.13: Theoretical connections are often shorter than paths in the type graph. The top half of the figure shows a theoretical link between 062|B4 and 132|C6, while the bottom half shows a shortest path of concrete segments linking the two in the type graph. Note that this second path is not, and need not be, in temporal order.

The insistence that shared subsegments must be stated independently to create a connection necessarily involves a trade-off: while it limits the proliferation of potentially insignificant relationships and anchors a piece's morphological structure in the concrete rhetorical processes of the music, it is highly sensitive to slight changes in segmentation and can leave segments which are intuitively similar completely unconnected, even indirectly (this gives the concept a theory-discarding, rather than theory-conserving, bias; see Section 1.8). Even those segments which *are* connected can delineate shortest paths which seem unnecessarily convoluted: segments 062|B4 and 132|C6, for example, are straightforwardly related through near-embedding which can be realised by the path of two theoretical segments shown in Figure 3.13 (or even more directly by a three-note subsegment spanning the bar-line), but in the type graph they are separated by at least five other vertices. Again, this shows that path length is inadequate as a measure of similarity; even theoretical path length is misleading, since at most three extra segments are needed to connect any pair of vertices, no matter how dissimilar they might be (Corollary 3.3.7iv).

The type graph provides an abstract theoretical map of the morphological relationships (understood as combinations of C-, D-, R-, and E-relations) between a piece's segments: as argued in the previous section, however, segments derive their identities primarily from derivational, rather than morphological, correspondences. Motivic segments are not simply patterns of intervals: each segment carries the sequence of derivations that led to it as part of its identity, and a significant difference in this domain can overwrite any surface similarity that two segments may share. Two segments (such as 083|C6 and 149|Ab5) might therefore be morphologically identical but derivationally distinct (if they lie in different families), and so those edges in the type (multi)graph with endpoints in different families are designated ***cross-family edges*** (identified by a suffix of 'x' in their labels). Endpoints of cross-family edges that are CDR-equivalent still stand in identical C-, D-, R-, and E-relationships to the graph's other vertices (Corollary 3.3.7iii), but at least one of each

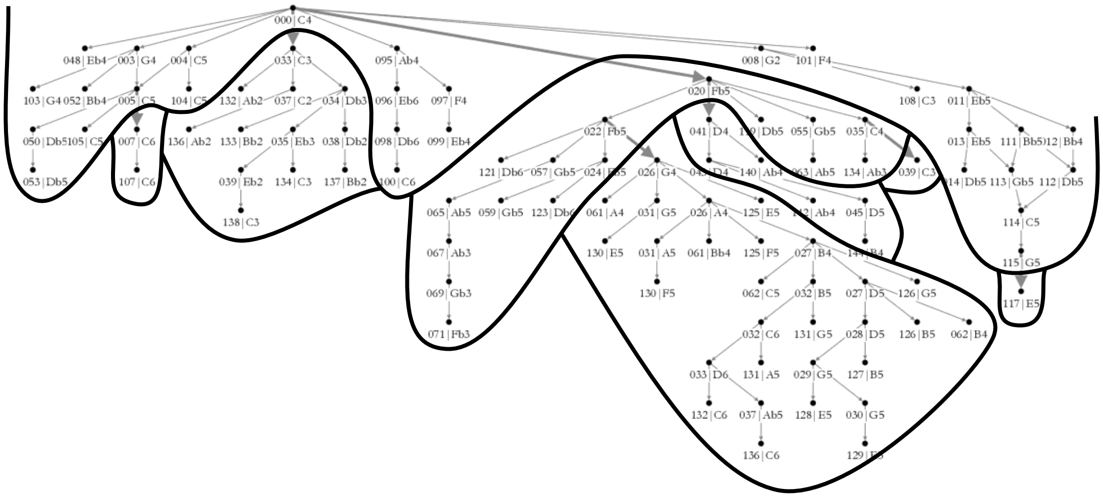

Figure 3.14: Family S1 of Figure 3.23 partitioned into type regions.
Arrow labels have been omitted from the NodeXL output for clarity, and developing arrows are indicated by increased thickness.

pair of edges joining these endpoints to any other vertex must also be a cross-family edge (Corollary 3.3.9i). When all the cross-family edges are removed from the type graph, the resultant connected components are known as ***types***: these subdivide the original type graph components, known as ***cross-family types*** (Definition 3.3.8; these are the boxed subgraphs in Figure 3.24). Cross-family edges might be expected to be more frequent amongst shorter segments, since the likelihood of two derivationally unrelated segments "accidentally" arriving at the same morphological structure decreases as the number of notes increases: every one of the 49 cross-family edges in Figure 3.24, for example, has a size 2 segment as at least one of its endpoints.

To ground these concepts even more concretely in the musical (or, more precisely, analytical) reality of the music as it unfolds, a third hierarchical level of morphological categorisation is proposed: the ***type region*** (Definition 3.3.8). Type regions, while also subdividing the types in the type graph, are defined as subdivisions of the families in the derivation digraph: specifically, if all the ***cross-type arrows*** (or ***developing arrows***) which join members of different types are removed, the type regions are the resultant simple components. Put a slightly different way: two segments are part of the same type region if there is an undirected path of derivation arrows between them, and if every vertex on this path belongs to the same type (Corollary 3.3.9ii and iii). Figure 3.14 shows the largest family (also the largest component) of Figure 3.23 partitioned into type regions with curved lines crossing the developing arrows (which are drawn thicker): the same partition is shown using vertex colour in the NodeXL output.

It is not unusual for all the members of one type to be contained in one type region (as is the case with every type in the present example): types are, after all, intended to model relationships of morphological closeness, and so it is to be expected that a given segment will often have a

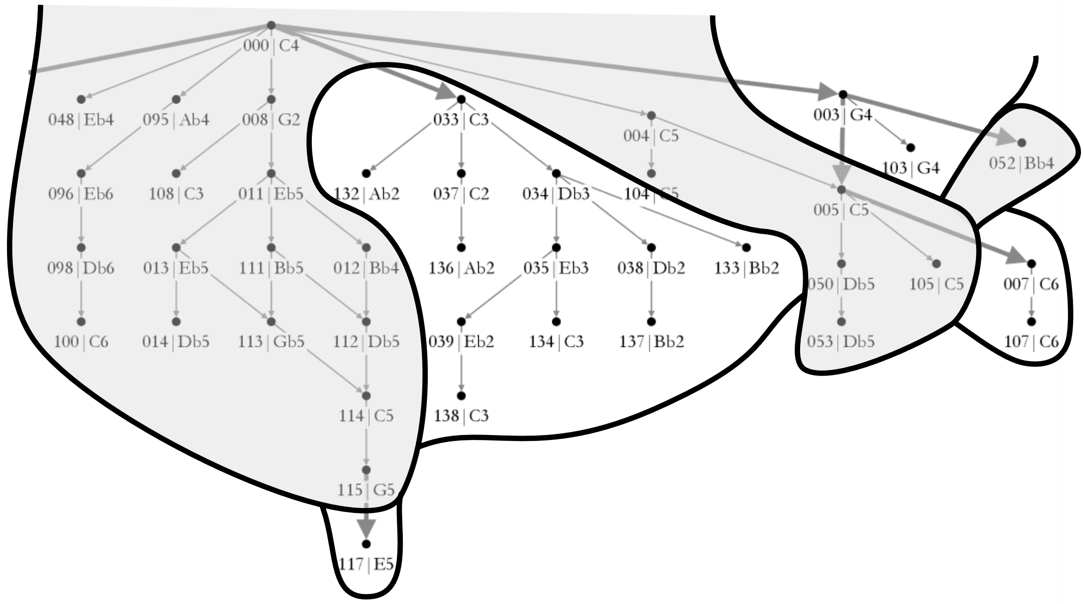

Figure 3.15: The type region containing 003|G4 is a subtype of the shaded type. This figure removes 097|F4, 099|Eb4, and 101|F4 from the analysis: cf. the type region headed by 000|C4 in Figure 3.14. The shaded type regions belong to the same type.

member of the same type (if one exists) chosen as a parent. Treating this expectation as normative, the types identified by the algorithm can be adjusted with respect to the analytical input to produce more flexible morphological categories – specifically by grouping together distinct types which seem, in the way the analysis has been constructed, to behave as one. Suppose the only way to get into type Y in the derivation digraph is through type X, and that every developing arrow leaving type Y returns to type X: this suggests that the arrows in and out of Y do not really 'develop' at all, and that Y is therefore a simple, interchangeable variation – a ***subtype*** – of X (Definition 3.3.11). This assumption, however, needs to be balanced with the alternative interpretation – that XYX represents a genuine circular process rather than a mere interchangeability of types – and so structures with extra stages which therefore carry an enhanced sense of process (such as the nested $XYZYX$ or the extended $XYZX$, where the grouping $Y + Z$ behaves like a single subtype of X but maintains some internal structure) are not included in the present definition (see Corollary 3.3.12, which also shows that two types may not be subtypes of each other; note also that Z remains, however, a subtype of Y in the first example above).

The concept of subtypes can rehabilitate some of the relationships rejected by the theory-discarding trade-off discussed above, in which subsegment similarities are only recognised if the subsegment appears independently at some point in the piece. Suppose, for example, that segments 097|F4, 099|Eb4, and 101|F4 are deleted from the Beethoven piece, severing, therefore, the

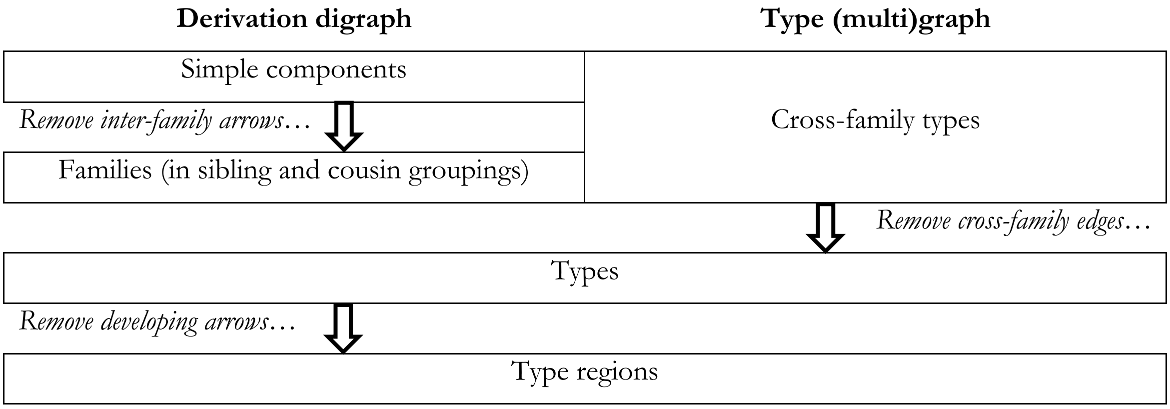

Figure 3.16: A summary of the model's hierarchy of categories.
Subtypes are not shown: these use type region information to create groups of types.

indirect connections between 000|C4 and 003|G4 (discussed above), and between 003|G4 and two of its children 005|C5 and 052|Bb4. Given that 000|C4, 005|C5, and 052|Bb4 remain connected in the type graph (through, for example, 095|Ab4), a portion of Figure 3.14 changes as shown in Figure 3.15: this places 003|G4 and the CDR-equivalent 103|G4 in a subtype of the larger type region, creating a relationship between the type of 003|G4 and the type of 000|C4 that is weaker than identity, but stronger than the mere existence of a theoretical linking segment.[39] The subtype concept can also be used, on an *ad hoc* interpretative basis, to informally describe some of the smaller types in Figure 3.14 (which contains no true subtypes): the theoretical segments needed to link 117|E5 and 039|C3 to their respective parents 115|G5 and 035|C4 (unlike those needed to link 007|C6 to 005|C5) suggest themselves straightforwardly enough for the former to behave like subtypes of the latter.

Type regions are labelled in a similar manner to families: with a three-part label dependent on cross-family type, type, and type region. If one type is found to be a subtype of another, the latter adopts the two-part label of the former with the addition of a lower case letter; if the two types belong to different cross-family types, the subtype is suffixed with the label '(x[A])', where [A] is the number of its cross-family type (if 3.1 was found to be a subtype of 2.4, for example, it would be renamed 2.4a(x3), with type regions 2.4a(x3).1, 2.4a(x3).2, and so on; the next subtype of 2.4 would be 2.4b). The hierarchy of categorisation is summarised in Figure 3.16, which shows how categories defined in one graph carry over to the vertices of the other and help define the next level: following each column downwards traces a path of ever-finer subdivisions (note that the families do not subdivide the cross-family types since each family may appear in several cross-family types and vice-versa). The nesting of these categories inside each other is fixed formally in

[39] In general, for one type to be classed as a subtype of another, all the regions of the former must be surrounded by regions of the latter: in this example, the type of 003|G4 has only one region.

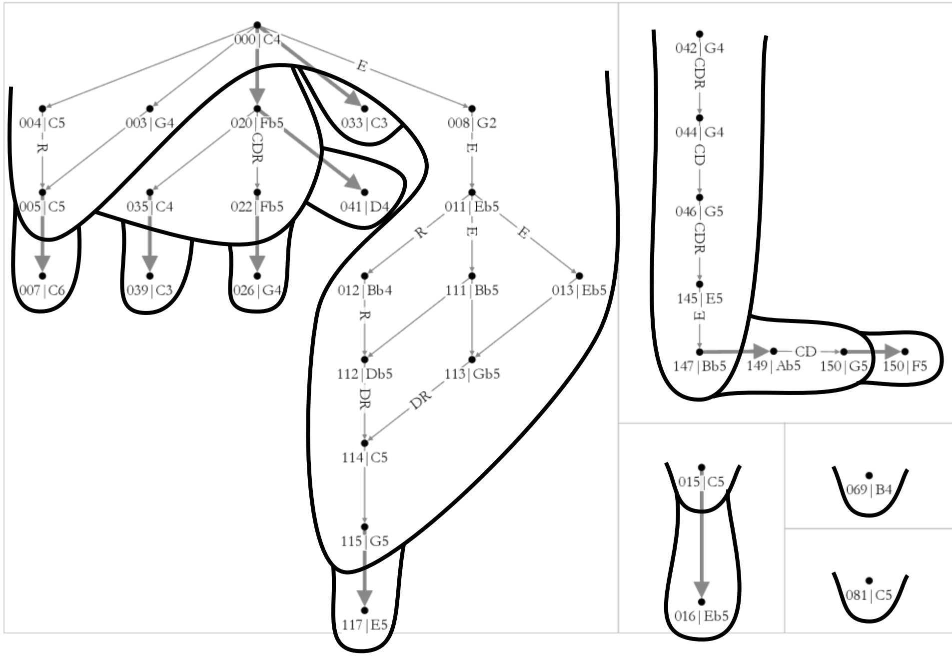

Figure 3.17: The motivic process digraph for Figure 3.23 partitioned into type regions.

Theorem 3.3.10, which also shows that the set of developing arrows includes the set of inter-family arrows (labelled 'Inter' in the NodeXL output), the set of developing arrows with endpoints in different cross-family types ('x-Developing'), the intersection of these two sets ('x-Inter'), and the remaining developing arrows ('Developing'). All the other derivation arrows are labelled with the C-, D-, R-, or E-relation that holds between their endpoints (suffixing an 'x' if these are in different families), or left blank if they are indirectly related members of the same type region.

Using these concepts, a sketch of the movement's essential motivic processes can be formed from the derivation digraph as shown in Figure 3.17: this ***motivic process digraph*** (Definition 3.3.13) is built around the piece's sources and developing arrows and includes all the paths between them.[40] Every type region is represented (Corollary 3.3.14v) and so it can be useful to think of the motivic process digraph as a "collapsed" version of the derivation digraph, replacing each type region with a single vertex in the first instance, then re-expanding it to include concrete derivation chains where necessary. Compare, for example, Figure 3.14 with the largest component of Figure 3.17: the type regions and the arrows between them are the same, and some type regions are represented by single vertices; it would be contradictory to the intransitive nature of development, however, to connect 000|C4 to 007|C6 directly, and so all the derivation paths between these vertices (via 003|G4, 004|C5, and 005|C5) are included. Another way to think of

[40] This is another concept technically defined only for DAGs and therefore requiring a given derivation digraph's condensation to be found first: full details of the translation between condensation and original can be found in Definition 3.3.13.

the motivic process digraph is as the result of a successive "pruning" of dead-ends within each type region (Theorem 3.3.15).

The motivic process digraph aims to summarise the derivational history of every "new" segment in the piece: given that the first occurrence of every type region (and therefore every strong and weak external epiphany too; Corollary 3.3.14iii) is included, any segment in $\mathcal{M}$ may be derived from, and linked by a path of C-, D-, R-, and E-edges to, some vertex in the motivic process digraph without changing type (or, therefore, family).[41] The NodeXL output categorises the vertices of the motivic process digraph based on the three sets used in Definition 3.3.13: initial and terminal points of developing arrows are labelled 'Ai' and 'At' respectively, with sources labelled 'At' and vertices that fulfil both roles at once labelled 'Ait'; vertices reachable from two distinct 'A' vertices are labelled 'B'; and all other vertices reaching 'A' or 'B' vertices are labelled 'C'.

As Figure 3.14 and Figure 3.17 show, two type regions can have substantially different sizes, temporal distributions, and degrees of internal variation: not every category at the same hierarchical level, therefore, necessarily has the same analytical significance. Whilst no *a priori* method for determining significance is proposed here, the temporal distribution (and therefore size) of each type region can be displayed in a map of the movement similar to a paradigmatic chart as shown in Figure 3.18. With bars on the x-axis and type regions (grouped into families) on the y-axis, a cell is filled if a member of its row's type region occupies (at least a part of) its column's bar. Note that the x-axis is split into three blocks – exposition, development, and recapitulation – for the sake of display and to facilitate proportional comparison: these top-level sectional breaks, and the subsectional breaks that determine the placement of the vertical lines, can be entered as part of the analytical input into the same worksheet as the time signatures. They are intended as no more than guides to make the chart easier to navigate, but the form that they describe may bear directly on certain choices in the analytical input: segments in the section that the analyst has chosen to label 'Recapitulation', for example, are likely to be derived from segments in the section labelled 'Exposition'.[42]

Whilst full inventories of each category can be found by applying Excel's built-in filters to the 'Vertices' spreadsheet, an approximate inventory of each type region can be gleaned from Figure 3.18 by reading along the shaded cells in each row (this is only approximate since the cells represent bars, which are not in one-to-one correspondence with the segments). Inventories of larger categories can be sketched too, if necessary by applying Excel filters to the first column of

[41] 'First occurrence' is here used informally to stand for sources or terminal points of derivation arrows; such vertices need not be the first to appear chronologically in a given type region due to the temporal flexibility of the derivation relationship (Definition 3.1.7 and Figure 3.2).
[42] Navigation is also aided by the fact that clicking in any shaded cell displays its bar number in the Excel formula bar, preceded by 'n.' if it is the first member of a type region.

Figure 3.18: The distribution of type regions throughout Figure 3.22.
Bars are plotted on the horizontal axis, which is marked out with sectional divisions (and split into three blocks corresponding to the exposition, development, and recapitulation), while type regions (demarcated into families by horizontal lines) are shown on the vertical axis (and labelled in the form [cross-family type].[type].[type region]). The first appearance of each type region is shaded grey.

the map itself: families correspond to blocks of rows, and cross-family types to rows sharing their first label number (it can be seen clearly that no type in this example contains more than one type region, not only because no two rows have the same first and second numbers, but also because no third number is greater than one). The algorithm also automatically produces a similar, but "collapsed", map in which each row represents one family: this retains, however, the convention of shading the first appearance of each type region (grey in Figure 3.18 and orange in NodeXL).

It is clear that the bulk of this movement's activity occurs in family S1, with the first and second subjects forming distinct and independent type regions within this group and making the piece, in this analysis, essentially monothematic (in other words, there is a derivational link but a structurally significant morphological break between 000|C4 and 020|Fb5 – see Figure 3.17). Other categories have more limited reach: the three type regions of the second largest family, for instance, are contained entirely within bars 42–48 and 141–52 (reach is not always a measure of significance since this family has an important closing function, and indeed is the only family heard in bars 146–end). Observations such as these feed into a consideration of the piece's motivic processes and categories, and can be read alongside its harmonic, tonal, and formal structures, as well as its more heterogeneous gestural or "poetic" qualities, to produce an analytical plot.

Reading Figure 3.18 and Figure 3.17 in this way, the movement therefore unfolds as follows. The opening idea over a tonic harmony gives rise to a responding phrase on the dominant, and this pair is then repeated in a truncated form and brought to a cadence via a liquidated (and therefore dissimilar) phrase followed by a pause. The opening idea then returns and is developed to form the bridge section, which again culminates in a dissimilar motive (although it does bear certain similarities to the opening idea through the weak-to-strong minor second fragment heard in bars 12–13).[43] The principal motive of the second subject, a loose inversion of the opening idea, is heard two-and-a-half times before an extended sequential passage of hesitant quaver segments based around the ascending third shape at the end of the second subject motive. This reaches a climax in bar 33, the start of the codetta, and liquidates into generic scale passages while a rising arpeggio figure (related, therefore, to the opening idea) is heard sequentially in the bass; the phrase is rounded off with a descending shape similar to the second subject motive, and subsequently repeated. The exposition closes with a pair of alternating segments, one based around the rhythm and third at the end of the second subject motive, and the other a new idea using a rising sixth and descending fifth.

The development begins by following a similar blueprint to the opening of the piece: there is no liquidation, however, and no bridge, and the forward-driving quaver sequence that previously led to the codetta is cut off by a periodic return to the beginning of the second subject (one tone higher). The second subject motive is then treated sequentially in the bass to cadence in a number of different keys; these cadences are reinforced by an ascending minor second pattern in the treble, which detaches itself from its original context (bars 73–78) and dissolves into two bars which are devoid of motivic content (bars 79–80). After this total liquidation, a new motive (related rhythmically to the ascending minor second) is treated quasi-sequentially over a behind-the-beat

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[43] More directly, the segment 015|C5 fulfils the process of the previous bars in two ways: it completes the E♭–D♭–C line in bars 13–15, and extends the descending third of the triplet semiquavers to a descending fourth rather than turning back on itself as all the previous segments have done. These relationships were not noted in the original analytical input, and so their discovery during the process of emplotted analysis is an example of the cyclical process of revision set out at the opening of Section 3.1. (If the derivation 014|Db5δ015|C5 – see Definition 3.1.7 – were to be included, the two type regions of family S2 would become type regions of family S1.)

pedal C to construct a four-bar phrase, which is then repeated in a decorated form; its final segment, however, gets stuck, and is repeated neurotically until the treble cuts out and the pedal C moves to the strong beat. Fragments of the opening motive return, leading to a strong restatement of the opening in the home key of F minor at bar 101.

The recapitulation proceeds mostly as in the exposition, except that the bridge refuses to coalesce on the descending idea of bars 15–20; the fragment of the opening idea is therefore left to wander for longer until it is capped off by a (repeated) segment characterised by a diminished fourth and augmented fifth. The bridge-concluding segments in both the exposition and recapitulation therefore not only differ from each other, but are unique within the movement. The remainder of the movement is an almost exact transposition of the corresponding passage in the exposition: bar 139, however, omits the descending second-subject-derived pattern in favour of a strong bassline reinforcement of the final structural perfect cadence. The (harmonically closed) periodic alternating pattern is also extended and liquidated at the close of the movement as the harmony ventures away from F minor: first into a sequence of descending fifths (bars 147–50), and then into an archetypal cadential melodic pattern. The tonal closure is hence undermined by a harmonic excursion and a linear (rather than periodic) motivic process, thus leaving a certain open-endedness in preparation for the second movement, or for the repeat of the development and recapitulation that is marked in some editions.

The foregoing analytical plot is an example (albeit a somewhat extreme one) of interpretation, the third stage of the denotation–demonstration–interpretation paradigm outlined in Section 1.10: it translates the technical language of digraphs, families, type regions, and segment labels underlying Figure 3.17 and Figure 3.18 into more traditional music-theoretical terminology arranged in a conventional analytical narrative. Indeed, some of its observations bypass the technical structures entirely, so the question is unavoidable: could the preceding three paragraphs of prose have been written straight from the score, without any recourse to mathematical formalisation at all?

I would argue that the answer is potentially yes, but offer three important qualifications in defence of the formalised framework set out in this chapter. Firstly, as suggested in Section 1.1, the question of the indispensability of mathematics – that is, whether or not scientific theories can be expressed perfectly without any recourse to mathematics – is an active topic of debate in the philosophy of science. The model outlined here makes no claims towards scientific validity, but any questioning of whether its mathematical elements are strictly necessary opens up a whole field of arguments and counter-arguments regarding the necessity and utility of *any* work of applied mathematics, from Tymoczko's geometry of music to Einstein's theory of relativity; certain apparently specific concerns about the role of mathematics in the present model are therefore, in reality, more general concerns about the epistemological basis of mathematics itself.

Secondly, just as any denotation (i.e. translation from target phenomenon to mathematical concept) is imperfect, so is the inverse process of interpretation. The analysis above uses ill-defined terms such as 'motive' and 'idea' to stand in (and not always consistently) for precisely defined concepts such as 'type region', so there is inevitable loss in translation. Sometimes 'motives' are liquidated into other 'motives', sometimes they are liquidated into generic material, sometimes they 'develop' into new types, subtypes, or even simply segments of the same type. The traditional language, whilst intuitively graspable to those with some musical training and adequate for the expression of many important insights, actually serves here to mask a richer and more detailed analysis that clarifies the range and nature of relationships that the term 'motive' encapsulates; this expansion of the term is, after all, the very purpose of this thesis. This is also why, as an example of interpretation, the analysis above is somewhat extreme: usually the aim is to use the formal concepts to illuminate the target phenomenon, and not to assiduously filter out all technically defined language as though it never existed.

Thirdly, in an argument that recalls the discussion of "accidental" morphological similarities that have no derivational underpinnings (as modelled by cross-family types): whether or not the analysis above *could* arise without the mathematical concepts, it *did* arise through the interaction of Figure 3.17, Figure 3.18, the musical score, and the mind of an analyst. Theoretical lenses focus on some features and blur some others (hence the fairly limited discussion of harmony and tonality above): would the uniqueness of the bridge-closing segments, or the liquidation of the second subject into motives only heard in the development, have been remarked upon in an analysis produced without any conceptual map similar to Figure 3.18? It is possible, but seems unlikely.

The ideas set out in this chapter comprise one particular toolkit for building models of pieces of music. Each model can be revised and, following Frigg and Hartmann's assertion that 'we do not learn about [a model's] properties by looking at it', actively engaged with: the rich suite of tools built into both Excel and NodeXL permit detailed examinations of the properties of a given analysis (which is why all files are included on the supplementary CD).[44] Some ideas are suggested throughout this chapter, but NodeXL's layout, grouping, and metrics algorithms, and Excel's filtering and statistical tools (including PivotTable summaries like those found in the CD's spreadsheets) all bear further exploration: particularly useful for the construction of emplotted analyses is NodeXL's 'Dynamic Filters' feature, which allows onsets to be filtered with sliding scales to build up a picture of how the various graphs grow with time. The opportunities for further programming in VBA are also extensive.

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[44] Frigg and Hartmann, section 3.1.

The following, and final, chapter is a case study in producing a more extended analysis of two substantial movements from the same work using this toolkit. The toolkit is used, like the work of other analysts (especially Monahan), as a source of starting-points for exploration; as a "way in" to a work that is too complex to admit a single overarching plot or explanation. The plot fragments and other insights that result are presented in prose, incorporating figures and technical language when doing so increases the richness and nuance of the interpretation; whilst they shed new light (or refract old light) onto the piece, however, they never do, and never can, reveal the full story.

Mathematical Formalisation

Definition 3.3.1

Let L and A be defined as in Definition 3.1.1. The functions $\boldsymbol{d}: L \to \{0,1,2,3,4,5,6\}$, $\boldsymbol{c}: L \to \{0,2,4,5,7,9,11\}$ and $\boldsymbol{n}: A \to \{-2,-1,0,1,2\}$ are defined respectively through the sets of ordered pairs $\{(\mathrm{C},0),(\mathrm{D},1),(\mathrm{E},2),(\mathrm{F},3),(\mathrm{G},4),(\mathrm{A},5),(\mathrm{B},6)\}$, $\{(\mathrm{C},0),(\mathrm{D},2),(\mathrm{E},4),(\mathrm{F},5),(\mathrm{G},7),(\mathrm{A},9),(\mathrm{B},11)\}$, and $\{(\mathrm{z},-2),(\mathrm{b},-1),(\emptyset,0),(\#,1),(\mathrm{x},2)\}$.

The domains of the first two of these functions can be extended to the set of pitches:

$$\boldsymbol{d}^{+}(l,a,o) = d(l) + 7(o-4)$$
$$\boldsymbol{c}^{+}(l,a,o) = c(l) + 12(o-4) + n(a)$$

Definition 3.3.2

Given a motivic segment m of size $n \geq 2$ as defined in Definition 3.1.6, the $2 \times n-1$ ***interval matrix of*** $\boldsymbol{m}$, $\boldsymbol{I}(\boldsymbol{m})$, has its ith column $I(m)_{C_i}$ given by $\begin{pmatrix} c^{+}(p_{i+1}) - c^{+}(p_i) \\ d^{+}(p_{i+1}) - d^{+}(p_i) \end{pmatrix}$. Its ith row is denoted $I(m)_{R_i}$.

Definition 3.3.3

The following equivalence relations may hold between two motivic segments m_1 and m_2 of size $n \geq 2$:

C-equivalence: $m_1 \sim_C m_2 \Leftrightarrow I(m_1)_{R_1} = I(m_2)_{R_1}$

D-equivalence: $m_1 \sim_D m_2 \Leftrightarrow I(m_1)_{R_2} = I(m_2)_{R_2}$

R-equivalence: $m_1 \sim_R m_2 \Leftrightarrow (x(\tau_i), y(\tau_i), z(\tau_i)) = (x(t_i), y(t_i), z(t_i) + \alpha)$ for all $i \in \{1, \dots, n\}$ where t_i and τ_i are the time-points of m_1 and m_2 respectively and α is a constant (x, y, and z are defined in Definition 3.1.3).

If $m_1 \sim_C m_2$ and $m_1 \sim_D m_2$, then write $m_1 \sim_{CD} m_2$ for brevity; CR-, DR-, and CDR-equivalence are defined similarly.

Theorem 3.3.4

For two time-points t_i and τ_i, if $x(\tau_i) = x(t_i)$ and $y(\tau_i) = y(t_i)$, then $z(\tau_i) - z(t_i)$ must be an integer.

For any pair $m_1 \sim_R m_2$, α must also therefore be an integer corresponding to a translation along the time-line by an integer number of bars.

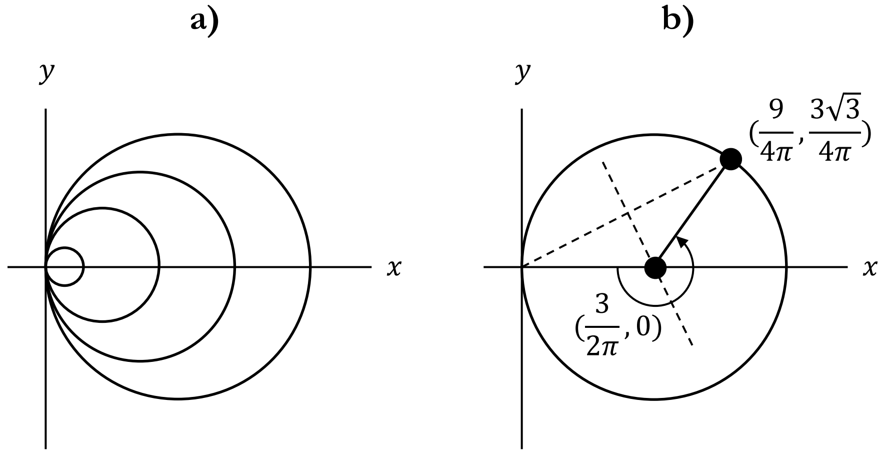


Figure 3.19: a) The x–y projections of different-sized coils all pass through $(0,0)$; b) Calculating the properties of the unique coil that passes through a given point. See the proof of Theorem 3.3.4 and the discussion which follows.

Proof

First note that for any integer value of t (that is, any time-point that is the first beat of a bar), $2\pi(t + {}^{1}/_{2})$ will be an odd multiple of π and so $x(t) = \frac{s(\lfloor t \rfloor)}{2\pi}(1-1) = 0$ and $y(t) = \frac{s(\lfloor t \rfloor)}{2\pi} \times 0 = 0$. Since every bar must have a first beat, the $x - y$ projection (or "shadow") of every coil of the helix must pass through $(0,0)$, with the radius of the coil being determined by $\frac{s(\lfloor t \rfloor)}{2\pi}$; Figure 3.19a shows the projections of a number of coils of different sizes.

Every point $(x(t), y(t))$ on the half-plane $x > 0$ lies on the circumference of exactly one of these coil projections, thus determining the time signature of the bar t occurs in (from the radius of the coil) and the beat of the bar t occurs on (from the anticlockwise angle its radius makes from the radius to $(0,0)$). If t_i and τ_i map to the same (x, y) co-ordinates, they must therefore have the same fractional parts (i.e. $t_i - \lfloor t_i \rfloor = \tau_i - \lfloor \tau_i \rfloor$), and so $\tau_i - t_i$ (and therefore $z(\tau_i) - z(t_i)$) must be an integer. The result for α follows directly.■

To illustrate the proof of Theorem 3.3.4 with a worked example (sketched in Figure 3.19b), take the point $\left(\frac{9}{4\pi}, \frac{3\sqrt{3}}{4\pi}\right)$. Since both this point and $(0,0)$ lie on the circumference of the coil we seek, the centre of the coil must be equidistant from both. The line joining the two points directly is given by $y = \frac{x}{\sqrt{3}}$; its perpendicular bisector must therefore have gradient $-\sqrt{3}$ and pass through the point $\left(\frac{9}{8\pi}, \frac{3\sqrt{3}}{8\pi}\right)$, meaning its equation is $x = \frac{3}{2\pi} - \frac{y}{\sqrt{3}}$. All the coil centres lie on the x-axis (i.e. the line $y = 0$), so the centre of this particular coil is $\left(\frac{3}{2\pi}, 0\right)$, giving the coil a circumference of 3 – that is, three beats in a bar. The angle from the horizontal is given by $tan^{-1}\left(\frac{3\sqrt{3}}{4\pi} \div \left(\frac{9}{4\pi} - \frac{3}{2\pi}\right)\right) = tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$, which means that the full angle working round the coil anticlockwise from $(0,0)$ is $\frac{4\pi}{3}$, or two-thirds of a full turn. Any t such that $\left(x(t), y(t)\right) = \left(\frac{9}{4\pi}, \frac{3\sqrt{3}}{4\pi}\right)$ must therefore be the

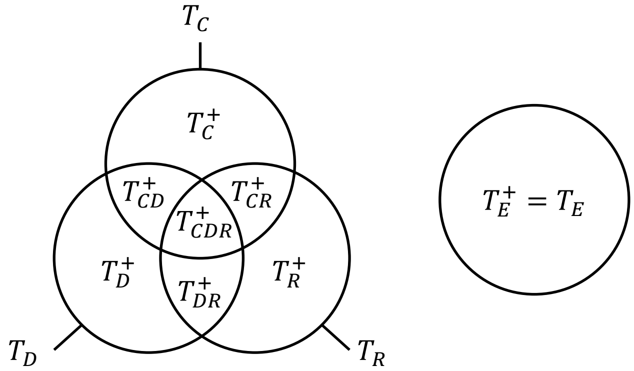


Figure 3.20: The edge set of the type graph, T, is partitioned into up to 8 disjoint sets (each denoted T_X^+) by the intersection relationships between the four T_X sets.

second beat of a three-beat bar, and so must have fractional part $\frac{2}{3}$; the difference between any two such points is therefore always an integer.

Definition 3.3.5

Two motivic segments m_1 and m_2, $2 \leq |m_1| < |m_2|$, are **E-related** ($m_1 \sim_E m_2$ and $m_2 \sim_E m_1$) if and only if there exists a value c such that $\{(p_i, d_i) \in m_2 | c \leq i \leq c + |m_1| - 1\} \sim_{CDR} m_1$ – that is, if m_1 is CDR-equivalent to a subsegment (see Definition 3.1.6) of m_2.

Definition 3.3.6

Given a set of motivic segments $\mathcal{M}$, let $T_X = \{\{a, b\} \subseteq \mathcal{M} | a \sim_X b\}$ for $X = C, D, R, E$, and let $T_{XY} = T_X \cap T_Y$. Then the ***type multigraph*** is $(\mathcal{M}, T^+)$ where $T^+ = \uplus_{X=C,D,R,E} T_X$. The ***type graph*** is $(\mathcal{M}, T)$, where $T = \cup_{X=C,D,R,E} T_X$; the edge set T is naturally partitioned into up to 8 disjoint sets (some of which may be empty) denoted T_X^+ for $X = C, D, R, CD, CR, DR, CDR$ or E as shown in Figure 3.20.

Corollary 3.3.7

The type multigraph and type graph have the following properties:

i. For $X, Y \in \{C, D, R\}$, the connected components of $(\mathcal{M}, T_X)$, $(\mathcal{M}, T_{XY})$, $(\mathcal{M}, T_{CDR})$ and $(\mathcal{M}, T_{CDR}^+)$ are complete graphs.

ii. If $\{a, b\}, \{b, c\} \in T_E$, and either $|a| < |b| < |c|$ or $|c| < |b| < |a|$, then $\{a, c\} \in T_E$.

iii. If a and b are members of the same connected component of $(\mathcal{M}, T_{CDR}^+)$, then $\{a, y\} \in T_X^+ \Leftrightarrow \{b, y\} \in T_X^+$ for all $y \in \mathcal{M} \setminus \{a, b\}$.

iv. For any two segments $a, b \in \mathcal{M}$, at most three new segments $x, y, z \notin \mathcal{M}$ are needed to ensure a path from a to b in the type (multi)graph for $\mathcal{M} \cup \{x, y, z\}$.

Proofs

i. This follows directly from the transitivity property of equivalence relations. Note that since $T_X^+ \subseteq T_X$ and $T_{XY}^+ \subseteq T_{XY}$, $T_{CDR}^+ = T_{CDR}$ is the only part in the partition of T that always defines an equivalence relation.

ii. If a is equivalent to a subsegment of b and b is equivalent to a subsegment of c, then a is equivalent to a subsegment of c. This implies, but is not implied by, the statements $a \sim_E b$ and $b \sim_E c$ since $\sim_E$ is symmetric (so b might be equivalent to a subsegment of a and c equivalent to a subsegment of b instead). If the sizes of a and c are both greater or both less than the size of b, then one is not necessarily equivalent to a subsegment of the other (they may, in fact, be the same size); the transitivity of the $\sim_E$ relation therefore only holds if b is the segment in the middle.

iii. Since a and b are CDR-equivalent, anything C-, D-, and/or R-equivalent to a or one of its subsegments must also be C-, D-, and/or R-equivalent to b or one of its subsegments, and anything with a subsegment that is CDR-equivalent to a must also have a subsegment that is CDR-equivalent to b.

iv. Assuming that a and b are not already connected in $(\mathcal{M}, T)$ (whereupon x, y and z can be chosen arbitrarily), it is always possible to choose x and z, $|x| = |z|$, as arbitrary subsegments of a and b respectively. Segment y can then be chosen as, say, C-equivalent to x and R-equivalent to z, creating a path $a \sim_E x \sim_C y \sim_R z \sim_E b$. Segments a and b can be linked with only two additional segments if a subsegment of a is C-, D-, or R-equivalent to a subsegment of b ($a \sim_E x \sim_{C/D/R} y \sim_E b$), or with only one if a and b are the same size ($a \sim_{C/D/R} x \sim_{C/D/R} b$) or embed CDR-equivalent subsegments ($a \sim_E x \sim_E b$).■

Definition 3.3.8

The connected components of the type graph $(\mathcal{M}, T)$ are known as ***cross-family types***.[45]

An edge $\{a, b\} \in T$ is known as a ***cross-family edge*** if a and b are in different families. The set of cross-family edges is denoted by X.

The connected components of the graph $(\mathcal{M}, T - X)$ are known as ***types***.

An arrow $(a, b) \in D$ is known as a ***cross-type arrow*** or a ***developing arrow*** if a and b are in different types. The set of developing arrows is denoted by N; a subset of this is the set N_X of arrows $(a, b) \in D$ such that a and b are in different cross-family types.

The connected components of the digraph $(\mathcal{M}, D - N)$ are known as ***type regions***.

Corollary 3.3.9

i. For any $\{a, b\} \in X$ and $\{a, c\}, \{b, c\} \in T$, with a, b, and c distinct, $\{a, c\}$ and/or $\{b, c\}$ are members of X.

   Combining this result with Corollary 3.3.7iii, if $\{a, b\} \in X \cap T^+_{CDR}$ then $\{a, c\}$ and $\{b, c\}$ are both members of the same part of the partition of T, and at least one is a member of X.

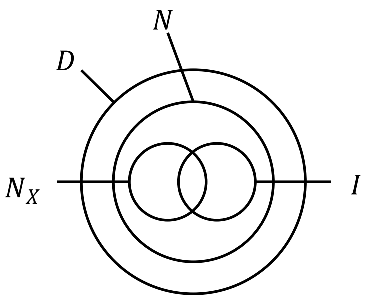
ii. For any pair of segments $a, b \in \mathcal{M}$ and an undirected path between them in $(\mathcal{M}, D)$:

   a. a and b are members of the same type region if and only if some such path contains no member of N;

   b. a and b are not members of the same type if the path contains exactly one member of N.

iii. For any undirected circuit $\mathcal{C} \in (\mathcal{M}, D)$ with arrow set C, $|C \cap N| \neq 1$.

[45] These correspond to the components of the type multigraph; the following definitions refer to the type graph alone for simplicity but can easily be adapted to apply to the type multigraph.



	Family	Cross-family type	Type
$D\backslash N$	Same	Same	Same
$N\backslash N_X \cup I$			Different
$N_X\backslash I$		Different	[⇒Different]
$I\backslash N_X$	Different	Same	[⇒Different]
$N_X \cup I$		Different	[⇒Different]

Figure 3.21: Every arrow of D falls into one of five sets depending on whether its endpoints' families, cross-family types, and types are the same or different.

Proofs

i. Assume, for a proof by contradiction, that $\{a, c\} \notin X$ and $\{b, c\} \notin X$. Then c is in the same family as b and a and so b and a are in the same family as each other, contradicting the assumption that $\{a, b\} \in X$. The combination with Corollary 3.3.7iii follows straightforwardly, noting that a and b are members of the same connected component of $(\mathcal{M}, T^+_{CDR})$ if and only if $\{a, b\} \in T^+_{CDR}$ by Corollary 3.3.7i.

ii. The type regions are defined as the simple components of $(\mathcal{M}, D - N)$. Saying that two segments are members of the same type region is therefore equivalent to saying that they can be joined by an undirected path in $(\mathcal{M}, D)$ that does not cross an edge in N.

   If an undirected path contains exactly one arrow from N, its endpoints can be labelled x and y such that the (undirected) subpaths from a to x and y to b include only arrows whose endpoints belong to the same type. This means that a and x are members of the same type, y and b are members of the same type, and x and y are members of different types; a and b are therefore members of different types.

iii. Since a circuit by definition includes two paths between every pair of vertices, a contradiction arises if one of these paths contains a member of N and the other does not (parts a and b of ii above are incompatible due to the 'TR⇒T' part of Theorem 3.3.10). A circuit may not, therefore, contain only one member of N.■

Theorem 3.3.10

Let XT, T, TR, and F represent the statements 'Segments $a, b \in \mathcal{M}$ belong to the same cross-family type/type/type region/family' respectively. Then TR ⇒ T ⇒ XT, T ⇒ F, and if $(a, b) \in D$, then T ⇒ TR. A corollary (¬F ⇒ ¬T) is that every inter-family arrow is a developing arrow, so $I \subseteq N$.

The intersections of the D-subsets I, N, and N_X therefore partition D as shown in Figure 3.21.

Proofs

By definition, the graph $(\mathcal{M}, D - N)$ contains no arrows whose endpoints lie in different types. Two vertices linked by an undirected path in this graph (i.e. two members of the same connected component) must therefore belong to the same type – so **TR ⇒ T**.

If two vertices are joined by a path in $(\mathcal{M}, T - X)$, then they must be joined by a path in $(\mathcal{M}, T)$ – so **T ⇒ XT**.

The graph $(\mathcal{M}, T - X)$ contains no edges whose endpoints lie in different families; any two vertices in the same connected component (i.e. type) must therefore lie in the same family, so **T ⇒**

F (its corollary follows directly: if two vertices are in different families, then they must be in different types, so if there is a derivation arrow between them it is both an inter-family arrow and a developing arrow).

If the endpoints of (a, b) lie in the same type, then $(a, b) \in D\backslash N$ and so A and B lie in the same component of $(\mathcal{M}, D - N)$ – so **T** ⇒ **TR**.■

Definition 3.3.11

If X and Y are the vertex sets of two distinct types, then Y is a ***subtype*** of X if and only if:

i. no vertex in Y is a member of a strong component in $\mathcal{D}$ which is a source in $\mathcal{D}^*$;

ii. every developing arrow joined to a member of Y is joined to a member of X;

iii. at least one developing arrow points from Y to X.

Corollary 3.3.12

If Y is a subtype of X, then Y has no subtypes; in particular, X cannot be a subtype of Y.

Proof

If Z is a subtype of Y then, by condition ii of Definition 3.3.11, every developing arrow joined to a member of Z is joined to a member of Y. By the same condition, every developing arrow joined to a member of Y must be joined to a member of X: this means that Y cannot have a subtype Z unless $Z = X$.

However, every vertex in $\mathcal{D}$ must be reachable from a vertex whose strong component is a source in $\mathcal{D}^*$ (by Proposition 2.6i, ii, and iii). If X and Y are subtypes of each other, then neither type contains such a vertex (by condition i of Definition 3.3.11), meaning that every vertex in X or Y must be reachable from a vertex *not* in X or Y: this contradicts condition ii.■

Definition 3.3.13

Let:

$A^* = \{a^* \in \mathcal{M}^* | ind_{\mathcal{D}^*}(a^*) = 0\} \cup \{a^* \in \mathcal{M}^* | (a, b) \text{ or } (b, a) \in N, b \in \mathcal{M}\}$
(that is, the set of all strong components of $\mathcal{D}$ that are sources and/or contain endpoints of developing arrows);

$B^* = \left\{b^* \in \mathcal{M}^*\backslash A^* \middle| \exists a_1^*, a_2^* \in A^*, a_1^* \ldots b^*, a_2^* \ldots b^* \in D^*,\ a_1^* \ldots b^* \cap a_2^* \ldots b^* = \{b^*\}\right\}$
(that is, the set of vertices in $\mathcal{M}^*\backslash A^*$ that are reachable from two distinct members of A^* by two disjoint paths in $\mathcal{D}^*$); and

$C^* = \{c^* \in \mathcal{M}^* | \exists m^* \in A^* \cup B^*, c^* \ldots m^* \in \mathcal{D}^*\}$
(that is, the set of all vertices in $\mathcal{M}^*$ that reach any member of A^* or B^* in $\mathcal{D}^*$).

The ***motivic process digraph*** (C, N^+) is the subgraph of $(\mathcal{M}, D)$ induced by $C = \{c \in \mathcal{M} | c^* \in C^*\}$.

Corollary 3.3.14

The motivic process digraph has the following properties:

i. $A^* \subseteq C^*$ and $B^* \subseteq C^*$, but $A^* \cap B^* = \emptyset$;

ii. $N \subseteq N^+$;

iii. $E^* \subset A^*$ (where E^* is the set of strong and weak external epiphanies; see Definition 3.2.12);

iv. a vertex is included in C^* if and only if it lies on an undirected path $a_1^* \ldots a_2^* \in \mathcal{D}^*$, $a_1^* \neq a_2^* \in A^*$, that contains no other members of A^* and no subpath $m_1 \to x \leftarrow m_2$ that can be replaced with a subpath $m_1 \leftarrow y \to m_2$ from $\mathcal{D}^*$, where if $m_1 \in a_1^* \to x$, $m_2 \notin a_2^* \to x$ ($\to$ indicates a directed path);

v. the mapping that sends every component of $(C, N^+ - N)$ to the type region of its members is a bijection; moreover, each component of $(C, N^+ - N)$ contains at least one vertex whose strong component is a member of A^*.

Proofs

i. By Definition 2.4, every vertex reaches itself, so all members of A^* and B^* are included in C^*, whereas B^* and A^* are distinct by definition.

ii. The strong components of the endpoints of every developing arrow lie in A^* (by definition) and therefore C^* (by part i). Since every vertex in a strong component in C^* lies in C, and N^+ is the set of all arrows whose endpoints lie in C, the set of developing arrows lies in N^+.

iii. Since the strong components of endpoints of arrows in N lie in A^*, and $I \subseteq N$ (Theorem 3.3.10), $E^* \subset A^*$ since every strong or weak external epiphany is a strong component that must contain the terminal point of at least one inter-family arrow (I^*, which bridges families by Definition 3.2.12, is related to I through Proposition 2.6ii). However, $E^* \neq A^*$ as $\mathcal{M}$ must include at least one vertex whose strong component is a source in $\mathcal{D}^*$ (Proposition 2.6i).

iv. First, if $c^* \in C^*$ then c^* reaches some $a^* \in A^*$ and/or $b^* \in B^*$; we may also note that every vertex is reachable from a vertex of indegree 0 (Proposition 2.6i) and therefore a member of A^*. If c^* reaches a^*, since it is also reachable from some $a^{*\prime} \neq a^* \in A^*$ ($\mathcal{D}^*$ is acyclic), it lies on a path as described. If c^* reaches b^*, c^* must be reachable from some a^* and b^* from some $a^{*\prime} \neq a^*$ (since b^* must be reachable from at least 2 distinct members of A^*), so $a^* \to c^* \to b^* \leftarrow a^{*\prime}$ is the required undirected path (which, containing only one receiver, has no replaceable subpaths).

Now suppose c^* lies in P, an undirected path as described: we aim to show that c^* must reach a member of A^* and/or a member of B^*. Given that it is impossible to have a path of carriers between two transmitters or two receivers, transmitters and receivers must alternate in any oriented path, and since each path endpoint is adjacent to only one path arrow, it must be either a transmitter or a receiver: every oriented path therefore takes the form $a_1^* = T_1 R_1 T_2 R_2 \ldots$ or $a_1^* = R_1 T_1 R_2 T_2 \ldots$, terminating with $a_2^* = T_n$ or R_n. If $a_1^* = R_1$ and c^* is a carrier between T_1 and R_1 then clearly a path exists from c^* to a member of A^*; similarly, if $a_2^* = R_n$ and c^* lies between T_n (or T_{n-1} if $a_1^* = R_1$) and R_n then a path exists from c^* to a member of A^*. We may restrict our attention, therefore, to paths of the form $T_1 R_1 \ldots R_{n-1} T_n$, which means that no vertex in the path (except, trivially, the endpoints) can reach a member of A^*. Given that each R_i is reachable by precisely those vertices between T_i and T_{i+1} inclusive, if it can be shown that every R_i is a member of B^*, then every $c^* \in P$ reaches a member of A^* and/or B^* as required.

Every transmitter in P that is not one of the two endpoints must be reachable in $\mathcal{D}^*$ from a member of A^* (of indegree 0; Proposition 2.6i) but may not be a member of A^* itself (by definition of P): we label an arbitrary member of A^* that reaches T_i without passing through any other members of A^* as $a^*(T_i)$. From the definition of P, we know that for each subpath $T_i \to R_i \leftarrow T_{i+1}$ there exists no $y \in \mathcal{M}^*$ such that $T_i \leftarrow y \to T_{i+1} \in \mathcal{D}^*$ (provided $n > 2$; if $n = 2$ then T_1 and T_2 are distinct members of A^* by definition, so R_1 is a member of B^*). Vertices $a^*(T_i)$ and $a^*(T_{i+1})$ must therefore be distinct, so $a^*(T_i) \to T_i \to R_i$ and

$a^*(T_{i+1}) \rightarrow T_{i+1} \rightarrow R_i$ are two distinct paths from two distinct members of A^* leading to R_i, meaning $R_i \in B^*$ as required.

v. First note that since $C \subseteq \mathcal{M}$ and $N^+ \subseteq D$, any component of $(C, N^+ - N)$ must be entirely contained within a component of $(\mathcal{M}, D - N)$ i.e. a single type region. The function that sends each component to the type region(s) of its members is therefore single-valued; we seek to prove that it is also injective (i.e. that no two components lie in the same type region) and surjective (i.e. that at least one vertex from every type region may be found in $(C, N^+ - N)$).

For the first of these, assume for a contradiction that m_1 and m_2 lie in two different components of $(C, N^+ - N)$ but the same type region. Then an undirected path exists between them in $(\mathcal{M}, D - N)$ but not $(C, N^+ - N)$, so every undirected path between m_1^* and m_2^* in $\mathcal{D}^*$ contains at least one member of $\mathcal{M}^* \backslash C^*$ (this conclusion about the condensation can be made due to Proposition 2.6ii). Since m_1^* and m_2^* must be members of C^*, they each lie on an undirected path (whose other vertices are also in C^*) between two members of A^* as described in part iv: these four members of A^* must be distinct otherwise an undirected path with all vertices in C^* is created between m_1^* and m_2^*. Now consider the path formed by taking the undirected path $a_1^* \dots m_1^* \dots m_2^* \dots a_2^*$ and replacing every subpath forbidden by part iv with its alternative (so $x \rightarrow z \leftarrow y$ becomes $x \leftarrow w \rightarrow y$) until no such subpaths remain. If this path contains no members of A^* then we have reached a contradiction since m_1^* and m_2^* are joined by a path in which every member is also a member of C^* (by part iv); if the path does contain members of A^* then a contradiction is also reached since $a_1^* \dots m_1^* \dots a_3^*$, $a_3^* \dots a_4^*$, …, $a_n^* \dots m_2^* \dots a_2^*$ are undirected paths as described in part iv that link up to give an undirected C^*-path between m_1^* and m_2^*.

For the proof that every type region has at least one vertex in $(C, N^+ - N)$, it is sufficient to show that every type region contains at least one vertex whose strong component is in A^*. For a given type region, if one of its vertices, a, is an endpoint of an arrow in N, then $a^* \in A^*$. Otherwise, the type region is not only a component of $(\mathcal{M}, D - N)$, but also of $(\mathcal{M}, D)$: it therefore must contain a strong component that is a source in $\mathcal{D}^*$ (Proposition 2.6i) and hence a member of A^*.■

Theorem 3.3.15

The motivic process digraph can be constructed from $\mathcal{D}^*$ with the following algorithm:

1. Set $C^* = \mathcal{M}^*$, let A^* be fixed as in Definition 3.3.13, and set every vertex in C^* as unmarked with $O(c) = outd(c)$.
2. If an unmarked vertex c exists in C^* such that $O(c) = 0$:
    a. If c has one parent, b, and $c \notin A^*$, remove c from C^* and subtract 1 from the value of $O(b)$; otherwise mark c with 'xx' if $c \in A^*$ or 'x' otherwise, and return to step 2. If now $O(b) = 0$ set $c = b$ and repeat a. Otherwise, return to step 2.

    Otherwise, go to step 3.
3. If C^* contains a vertex c marked 'x':
    a. Trace a path *backwards* along the arrows from c until a vertex $a \in A^*$ is reached. Again starting from c, perform a reverse depth-first search until all paths leading back

from c to an already-visited vertex have been tested; if at any point a vertex $a' \neq a \in A^*$ is reached, stop the search, mark c with 'xx', and return to step 3.[46] Otherwise, if P_c is the set of parents of c, remove c from C^* and subtract 1 from $O(p)$ for each $p \in P_c$. If $O(p) = 0$ for any $p \in P_c$, go to step 2; otherwise go to step 3.

Otherwise, go to step 4.

4. Let C and N^+ be constructed from C^* as in Definition 3.3.13.

Proof

First, note that in any DAG where all the vertices of outdegree 0 are members of C^*, all the other vertices must also be members of C^*. This is because every vertex in a DAG reaches a vertex of outdegree 0 (Proposition 2.6i), and every vertex that reaches a member of C^* must also be a member of C^* (since a vertex that reaches a vertex that reaches a member of $A^* \cup B^*$ must also reach that member of $A^* \cup B^*$). The algorithm therefore works by successively removing vertices of outdegree 0 (i.e. $O(a) = 0$) from $\mathcal{M}^*$ until all the vertices of outdegree 0 that remain fulfil the criteria for membership in C^* (in fact, they must fulfil the criteria for membership in $A^* \cup B^*$ since no vertex in $C^* \backslash A^* \cup B^*$ can have outdegree 0). Once C^* has been found, it is straightforward to list the members of each of its strong components, and to find the arrows in D that have both their endpoints in this list (step 4).

Step 2a sorts any vertex of outdegree 0 into one of three categories: those marked 'xx' are members of A^* and must therefore be retained in C^* (Corollary 3.3.14i); those marked 'x' have indegree greater than or equal to 2 and are therefore potential members of B^* (note that membership of A^* takes precedence over potential membership of B^*); and all those that remain cannot be in C^* since (having outdegree 0) they do not reach any member of A^*, B^*, or C^*. The members of this final category must have indegree 1 (since vertices of indegree 0 are members of A^* by definition) so each vertex that is removed reduces the outdegree of one other vertex by 1; if this creates a vertex of outdegree 0, then it can only be retained in C^* subject to the categorisation procedure of 2a, and so the process loops.

Once the process reaches step 3, every remaining vertex of outdegree 0 is either a member of A^* or a potential member of B^*. Those in the latter category must be checked to see if two disjoint paths from two distinct members of A^* can be found, and this is the function of the reverse depth-first search in step 3a: if such paths exist, then the vertex is marked 'xx' to be retained alongside the vertices in A^*, but otherwise it is removed and its parents' outdegrees altered accordingly (possibly necessitating a return to step 2). Once every vertex of outdegree 0 has either been removed or marked 'xx', the vertices that remain will be precisely the members of C^* as required. ■

[46] Primogeniture succession is an example of a *depth-first search*, which finds a longest path before backtracking to the most recent junction (Queen Elizabeth II, Prince Charles, Prince William, Prince George, Prince Harry, etc. – as of 17 December 2014). Compare with a *breadth-first search*, which fans out to examine each generation before progressing to the next (Queen Elizabeth II, Prince Charles, Prince Andrew, Prince Edward, Princess Anne, Prince William, etc.).

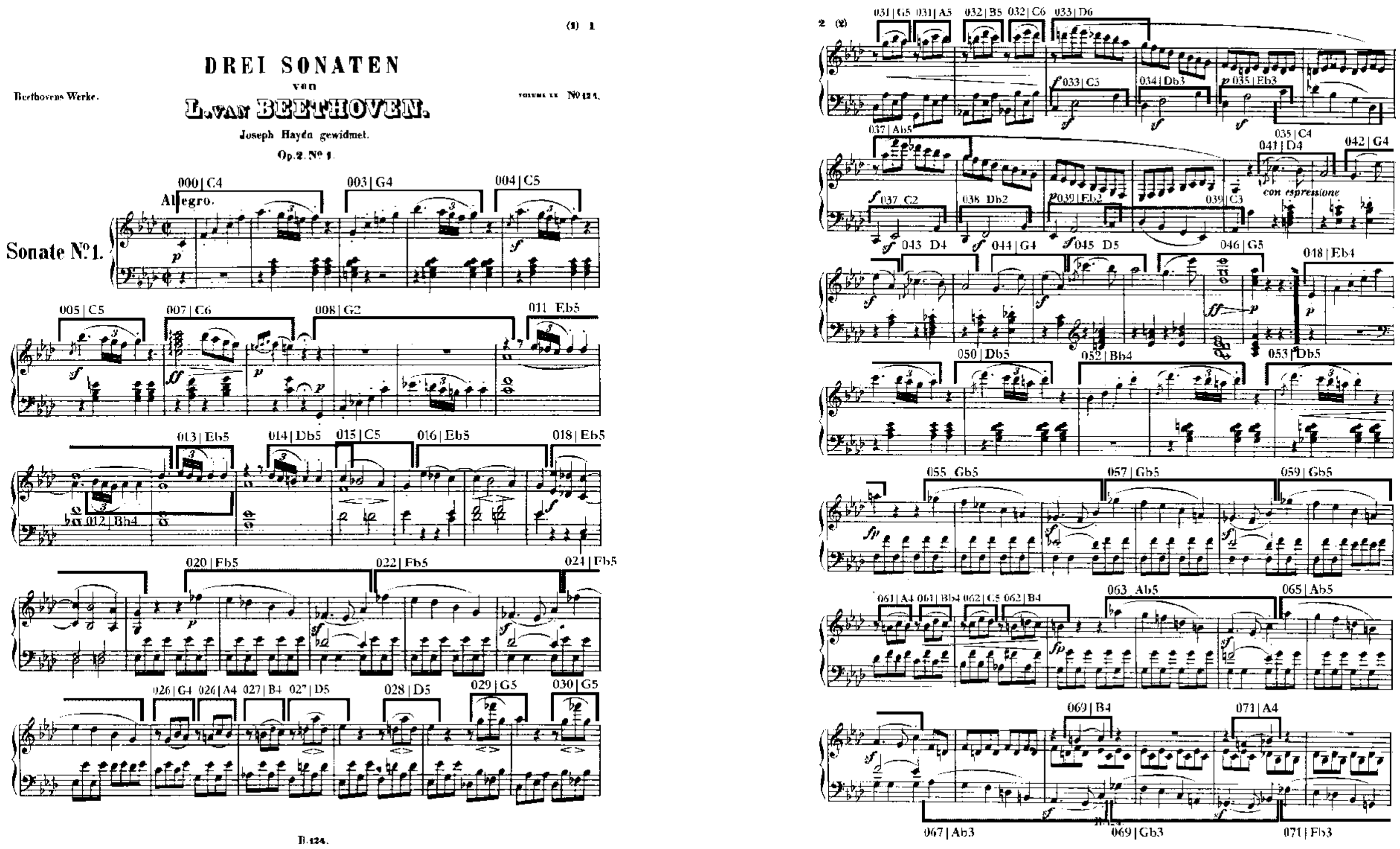



Figure 3.22: A segmentation of the first movement of Beethoven's Piano Sonata in F minor, Op. 2, No. 1.
See n. 1. This figure is also included on the supplementary CD to facilitate side-by-side comparison with the in-depth discussion in the text.

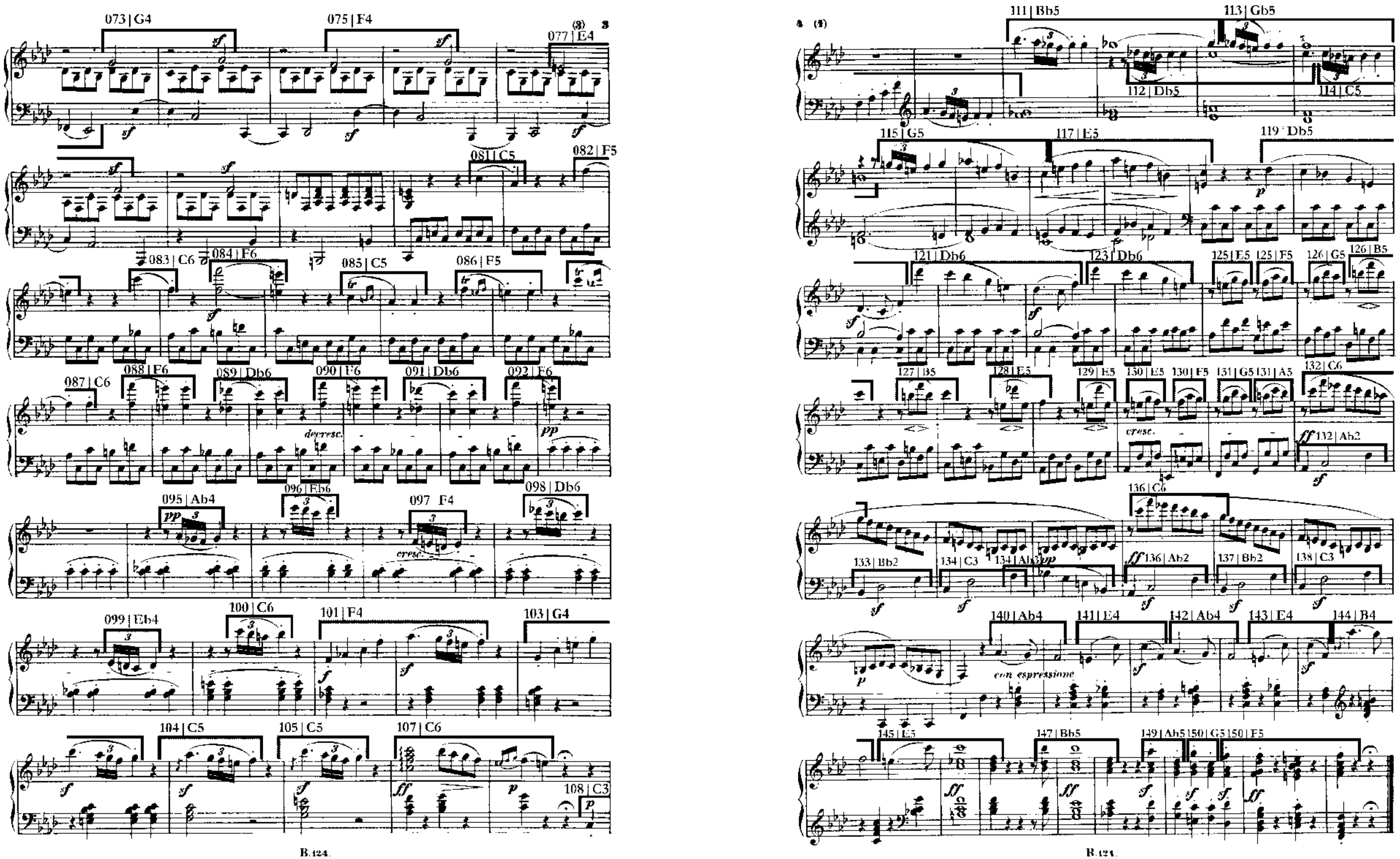



Figure 3.22 continued.

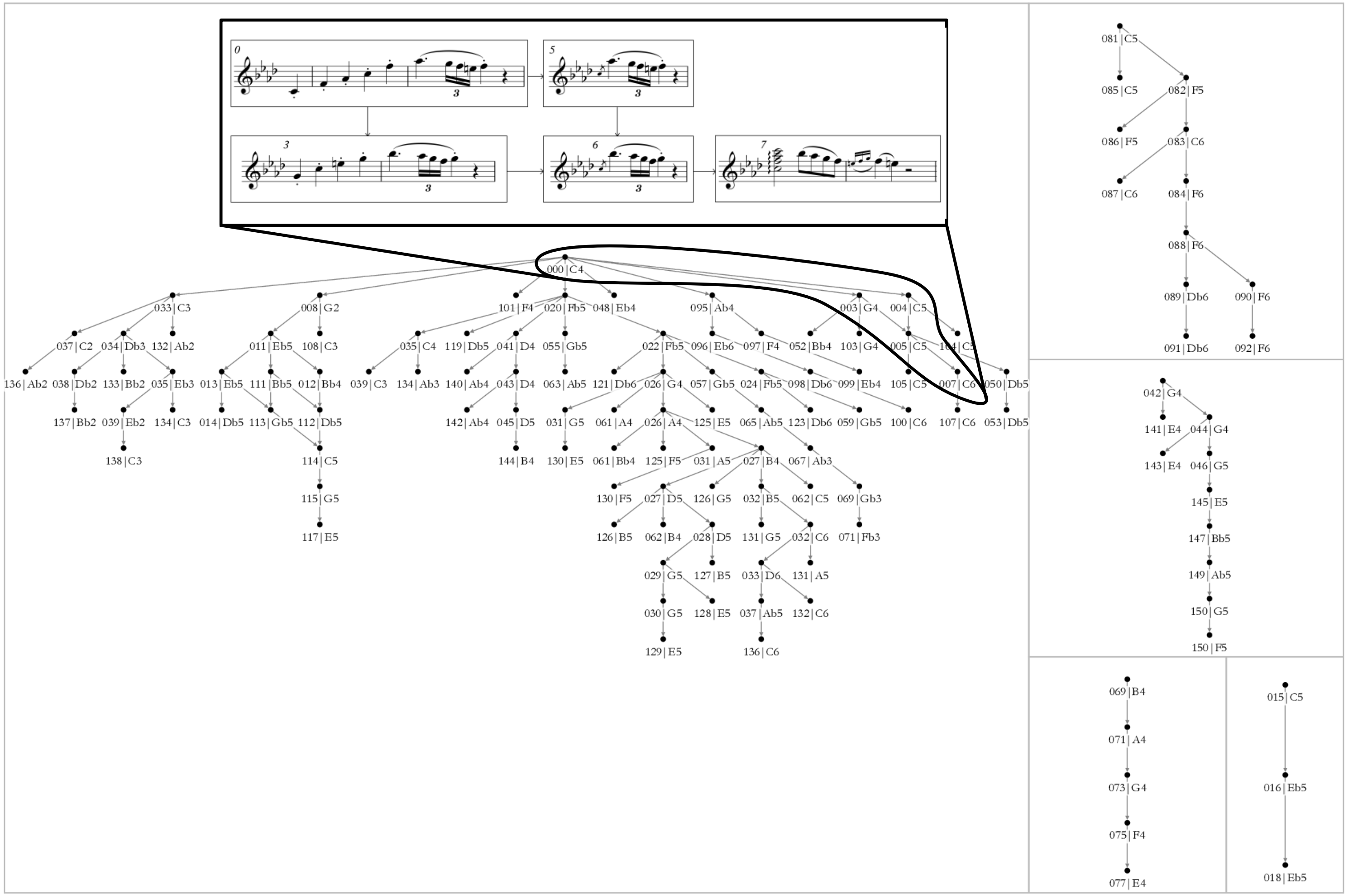


Figure 3.23: The full derivation graph for Figure 3.22 with the first five segments magnified.

The segments shown in bars 5 and 6 have their grace notes encoded to begin before the beat and so have labels beginning 004… and 005… respectively.

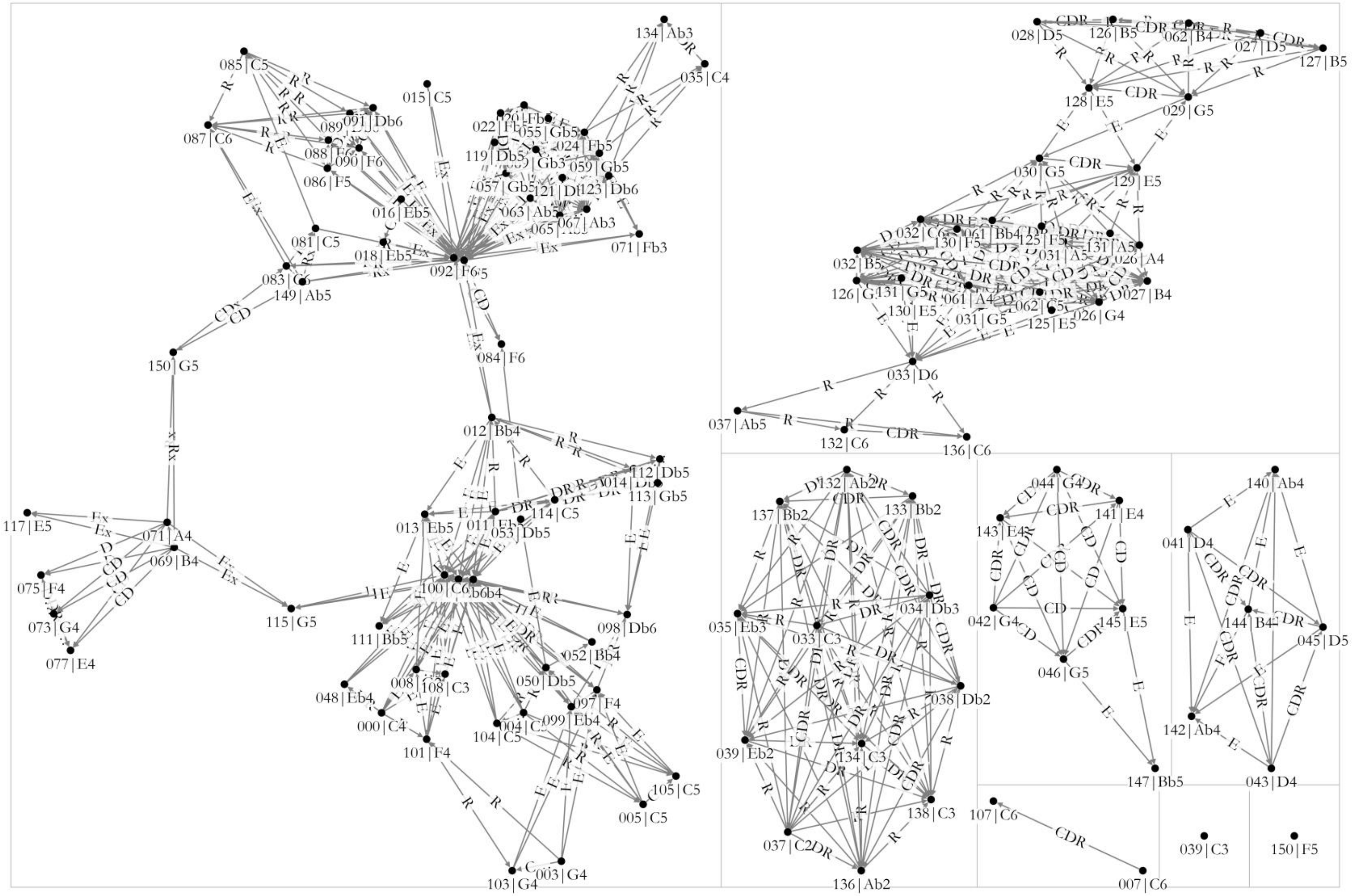


Figure 3.24: The full type graph for Figure 3.22.

Connected components (cross-family types) are separated into boxes.

4 Mahler's Sixth Symphony

Amongst all of Gustav Mahler's symphonies, there seems to be something about the Sixth that analysts and musicologists find irresistible. Three book-length studies and an appellation of 'pre-eminence in Mahler's ouevre' from the unavoidable figure of Adorno ('around whom analytical programmes have hovered with varying degrees of frustration and veneration', according to Jeremy Barham) crown countless chapters, articles, and substantial discussions in Mahler biographies.[1] To cite here only the most recent of these: two of the four case studies in Monahan's 2008 thesis (and forthcoming monograph) are 46- and 49-page discussions (plus musical examples) of the symphony's first and last movements respectively; and although the second of these was published as a 43-page double-column article in 2007, Monahan's 60-page 2011 article on the first movement actually advances a significantly *new* perspective when compared to that which is presented in his thesis.[2]

In such a crowded marketplace, it seems natural to ask what this chapter hopes to add. As part of this thesis, its primary purpose is to suggest how the model developed in previous chapters can be applied in the context of an extended analysis; it aims, therefore, more towards insights about the model's potential than new ground in the field of Mahler studies. The final section of this chapter reflexively examines how the proposed formalisms shift between foreground and background in the various analytical modes deployed in Sections 4.1 and 4.2, and so some patience is requested if they momentarily seem overwhelmingly close or vanishingly distant (cf. the discussion closing the previous chapter). That said, any analysis (especially one constructed in sight of a new theory) always says something new (as the discussion of "equivalent" models and metaphors in Section 1.10 shows) and seeks to confirm, challenge, extend, and nuance a reader's existing conceptualisation of a piece of music. While this involves, according to Agawu, 'a fresh engagement, a re-enactment, not an aggregation of facts about previous enactments', this chapter's broader aim is to show that such previous enactments need not be obstacles around which new analyses should steer to avoid accusations of duplication: they can, and should, instead be read as links in an ongoing chain of suggestive description and performative perception (see Section 1.3) to

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[1] The three book-length studies are Norman Del Mar, *Mahler's Sixth Symphony: A Study* (London: Eulenburg, 1980); Hans-Peter Jülg, *Gustav Mahlers Sechste Symphonie*, Freiburger Schriften zur Musikwissenschaft, 17 (Munich: Katzbichler, 1986); and Robert Samuels, *Mahler's Sixth Symphony: A Study in Musical Semiotics*, Cambridge Studies in Music Theory and Analysis, 6 (Cambridge: Cambridge University Press, 1995). Adorno's label (which he assigns to the last movement in particular) is found in *Mahler*, p. 135, while Barham's comment is from Jeremy Barham, 'Introduction', in *Perspectives on Gustav Mahler*, ed. by Jeremy Barham (Aldershot: Ashgate, 2005), pp. xxiii–xxx (p. xxviii).

[2] See Introduction, n. 27, which also lists Monahan, 'Success and Failure', pp. 51–54 as a discussion of the outer movements of the Sixth.

be engaged with dialogically and extended analytically.[3] Monahan's recent, extensive, and persuasive analyses are particularly well-suited to this purpose, especially given both his stated desire that they be used this way (cited at n. 29 in the Introduction above), and his own predilection for reading with and against the observations of his predecessors. The results of my dialogical method – in particular the choice to adopt several of Monahan's category labels throughout my own analysis – are evaluated in Section 4.3.

The Sixth Symphony's fecund potential for reinvention, which makes its performative analytical chain a fruitful one to examine, seems to be tied to the work's relative interpretative openness: unlike many of Mahler's other symphonies, the Sixth was never programmatically glossed by its composer, and nor does it include any sung texts or substantial reworked passages from previous compositions (Monahan identifies one brief quotation: the chorale in the Finale is derived from St Peter's penitential cry 'Und sollt' ich nicht weinen, du gütiger Gott' in the *Wunderhorn* setting that comprises the fifth movement of the Third Symphony).[4] Nevertheless, there are certain biographical accretions that have assumed canonical status: the '*Tragische*' subtitle under which Mahler conducted the work at least once; the 'three blows of fate' represented by the hammer-blows in the Finale and purportedly anticipating events in Mahler's life; and the portraits of the Mahlers' young children in the 'arhythmic [*sic*] games' of the Scherzo, and of his wife Alma in the 'great soaring theme' of the first movement.[5] Of these, it is the 'Tragic' reading that has come to define one of two standard critical tropes used to approach the symphony. Whilst it is undoubtedly supported by the brutal effect of the hammer-blows and the affective bleakness of the symphony's ending, its adoption as a quilting point for the work's multifaceted complexity can

[3] Agawu, 'How We Got Out of Analysis', p. 274.
[4] See Monahan, '"I Have Tried"', p. 161. Monahan also suggests that the Sixth's popularity with critics and audiences may stem, in part, from its concordance with 'the Mahler that postwar audiences have constructed in their own image: cynical, knowing, internally conflicted, and immune to the untenable promises of fast-fading Romantic ideologies' ('"Inescapable" Coherence', p. 54).
[5] See Henry-Louis de La Grange, *Gustav Mahler*, 4 vols (Oxford: Oxford University Press, 1995–), III: *Vienna: Triumph and Disillusion (1904–1907)* (1999), p. 814, n. 44 for the first of these. The others all originate in (and are quoted here from) Alma's memoirs (Alma Mahler, *Gustav Mahler: Memories and Letters*, ed. by Donald Mitchell, trans. by Basil Creighton, 2nd edn (London: Murray, 1968), p. 70) and persist as interpretative strategies despite having been seriously questioned by several authors. The symbolic import of the three hammer-blows is undermined by the fact that Mahler initially intended five, and later two; de La Grange also argues that the third blow – Mahler's heart diagnosis – was actually a relatively manageable condition peripheral to the infection that killed him, and in any case pales next to the blow of Alma's admission of infidelity in 1910 (see de La Grange, *Mahler*, III, 813–14, which still maintains that Mahler may have deleted the third hammer-blow for superstitious reasons). That Mahler's resignation (not dismissal) from the Vienna State Opera was a 'blow' also seems debatable (as suggested in, for example, Jonathan Carr, *The Real Mahler* (London: Constable, 1997), p. 135). Regarding the other 'portrait' myths, the Mahlers had only one very young child while Gustav was composing the Scherzo in the summer of 1903, and it is not entirely unproblematic that Mahler should depict his new wife in a 'Tragic' symphony (Monahan's 2011 reading of the first movement seeks to reconcile the 'Tragic' and 'Alma' myths in order to pursue their hermeneutic and 'psychobiographical' ends; see '"I Have Tried"', p. 123).

damage the hermeneutic potential of the second trope, which is nevertheless frequently advanced at the same time as the first.[6]

This second trope is one of polarity, of extreme contrast and even conflict between (usually two) affective zones.[7] The narrative potential of binary opposition is by no means unique to this work (it is a central component of Karl's theory of musical narrative, discussed in Section 2.4), but the 'sharp contours' of the outer movements' two-block expositions (mirrored by the contrasting inner movements) invites an interpretation of the first trope that hinges on conflict and ultimate defeat (recall that Almén's 'Tragedy' archetype, again discussed in Section 2.4, is defined as the 'defeat of transgression by order').[8] The 'two-block exposition' (sometimes called a *Dutchman*-exposition after the paradigmatic example of Wagner's overture) is a term coined by James Hepokoski to describe those sonata expositions which dramatise the opposition between their two subject areas: the first subject is usually restless, minor-key, and "masculine", whilst the second (occurring after a fairly minimal transition, 'a mere panning from one tableau to another') is redemptive, major-key, "feminine", and often apotheosised in the recapitulation or coda.[9]

While Monahan pursues an interpretation in terms of gender politics in his 2011 reading of the first movement of Mahler's Sixth (using an earlier article by Hepokoski as a springboard), it is worth noting that Hepokoski suggests a number of other binaries that a two-block exposition might enact: the examples he gives are 'tormented hero/redemptive agent; active struggle/withdrawal into the erotic; tyrannical oppression/projected political emancipation'.[10] The last of these certainly has its resonances with the Sixth: it chimes with the uniquely twentieth-century subjectivity ascribed to Mahler's character by postwar audiences (see n. 4), and respects Adorno's near-canonical portrait of the Sixth as '[t]he totality that sanctions for its own glory the destruction of the individual'.[11] Monahan neatly surveys the specific interpretations that have been

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[6] 'Quilting point' is a term from the work of Jacques Lacan (literally *point de capiton*, 'upholstery button'; see Harper-Scott, *Quilting Points*, p. 7) referring to a single 'master signifier' (p. 8) adopted as a means of explaining other, 'floating' (p. 8), signifiers: 'lines painted on the ground and a scattering of nets, balls, and rackets are meaningless until quilted by the idea "tennis", for instance' (p. 7).
[7] Monahan also identifies two tropes, but these both fall under my first: the symphony as 'consummate essay in negativity or cynicism' ('Narratives', I, 104), or as 'the downfall of some tragic hero' (ibid., p. 105; see also '"Inescapable" Coherence', pp. 53–54). Samuels notes Constantin Floros's implied refusal to resolve the symphony's polarities: 'since all human life is here, there is no need to see the negative episodes as having the last word' (p. 80).
[8] See Almén, 'Narrative Archetypes', p. 18. The phrase 'sharp contours', as it applies to 'symphonic formalism', is Adorno's (*Mahler*, p. 93).
[9] James Hepokoski, 'Beethoven Reception: The Symphonic Tradition', in *The Cambridge History of Nineteenth-Century Music*, ed. by Jim Samson (Cambridge: Cambridge University Press, 2001; repr. 2002), pp. 424–59 (p. 448).
[10] 'Beethoven Reception', p. 448. See also Monahan, '"I Have Tried"', p. 146, which cites James Hepokoski, 'Masculine–Feminine', *Musical Times*, 135 (1994), 494–99.
[11] Adorno, *Mahler*, p. 97. Some of the more explicitly Adorno-inspired analyses of the Sixth include Monahan, '"Inescapable" Coherence'; de La Grange, *Mahler*, III, 808–41; Samuels; and Bernd

attached to the symphony's suggestive polarity: 'I have read the Sixth as the ascension of the masculine over the feminine, but we can just as easily hear it as a triumph of death over life (with [Donald] Mitchell), of unfreedom over freedom (with Adorno), of aggression over compassion (with [Stefan] Hanheide), of fate over defiance (with Alma) or alienation over acceptance (with [Henry-Louis] de La Grange), and so on.'[12]

This concept of heightened contrast or polarity has implications beyond the intra-movement plots set in motion by the two-block expositions: it serves as a binding agent for the symphony in both a direct way and a more abstract way. First, this symphony is unusual in Mahler's output in that it includes an unambiguous "motto" theme, first heard at bar 57 of the first movement and repeated with very little variation a further 21 times (according to Robert Samuels's tabulation) throughout the symphony.[13] It embeds a crucial polarity in that its only melodic material is a shift from the third of a major triad to the third of a minor triad; on its last appearance (the final sonority of the piece and an excellent illustration of just how important single-note motives can be), the major triad is absent completely. This somewhat crude example of a standard "cyclic" technique is merely, observes de La Grange, 'the most obvious' in 'a list of parallels between the opening Allegro and the Finale [that] is impossible to draw up exhaustively'; these parallels, coupled with similarities in the movements' affective and formal profiles, invite a comparison that leads to a second, more dialectical, understanding of the function of polarity in the symphony as a whole.[14]

In this second understanding, the clarity of the first movement's sonata form (complete with repeat marks at the end of the exposition)[15] invites a reading of the Finale in which, as

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Sponheuer, *Logik des Zerfalls: Untersuchungen zum Finalproblem in den Symphonien Gustav Mahlers* (Tutzing: Schneider, 1978) (the last of which is 'more or less adopted wholesale' by Jülg, according to Samuels, p. 79).

12 "'I Have Tried'", p. 172. These are all examples of writers transforming similar understandings of potential meaning into actualised meanings: see Cook, 'Theorizing Musical Meaning', pp. 181–86, and Chapter 1 at n. 25 above.

13 See Samuels, p. 160 and notes 42, 45, 92, 93, and 102 below. Peter Brown considers this symphony to be 'Mahler's most blatantly cyclic effort' (A. Peter Brown, *The Symphonic Repertoire*, 5 vols (Bloomington: Indiana University Press, 2002–), IV: *The Second Golden Age of the Viennese Symphony* (2003), p. 655).

14 See de La Grange, *Mahler*, III, 817. Julian Horton has argued that, in 'all of his nine completed symphonies', Mahler's motivic technique shuns superficial cyclic repetition but constantly references other movements and other works in more mutable ways, 'question[ing] the notion of inter-movement unification even as it references its guiding method' (Julian Horton, 'Cyclical Thematic Processes in the Nineteenth-Century Symphony', in *The Cambridge Companion to the Symphony*, ed. by Julian Horton (Cambridge: Cambridge University Press, 2013), pp. 190–231 (p. 219)). This self-negating aspect of Mahler's approach to repetition is characteristic of Adorno's thought, and in particular his concept of the *variant*, those 'always different yet identical figures' that saturate Mahler's music (*Mahler*, p. 86; see also Sections 4.2 below and 2.1 above).

15 The repeat marks not only generate potential for dialectic engagement, but also for confusion over bar numbering: I treat the first-time bracket as bars 123–27, such that bar 128 coincides with

Samuels puts it, 'form is in some sense primary [to its] semiotic process'; more colourfully, Adorno writes that the Finale 'melts the crust of form, which the first movement had hardened dialectically, as if whole geographic regions were glowing volcanically and their settlements were pitching into each other in a river of fire'.[16] The Finale's complexities are therefore offset (dialectically) against the relative simplicity of both the opening movement's form and the generic sonata model it evokes; but when gestures which can only be read as structural markers are undermined by their voice-leading contexts and thereby 'simultaneously insis[t] on and den[y] a reading', the Symphony 'confront[s] the institution within which it situates itself' and the result, for Adorno, is no less than 'the end of the symphonic sonata, or […] the end of the order that bore the sonata'.[17]

4.1 The First Movement

How do these polarities – thematic, affective, tonal, and formal – play themselves out in the motivic structures of the symphony's outer movements?[18] A convenient starting-point is Monahan's motivic segmentation of the first movement's primary theme, reproduced alongside my own in Figure 4.1.[19] These two interpretations are in broad agreement as segmentations of a syntagmatic chain; the major difference here is one of hierarchical level, with Monahan tending to group similar successive segments together rather than making their internal derivational relationships more explicit (Monahan's $P^{1.1}$, for example, refers to the same music that I split into

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rehearsal number 14, and have amended all cited bar references to match this scheme. This is Monahan's practice in his thesis, but his 2011 articles assign bar 123 to RN14 (in the figures and examples, at least; a correction to his 'Alma' article had to be issued in the subsequent issue of the journal to ask readers to subtract 5 from all numbers above 122 in the text (see *Journal of the American Musicological Society*, 64 (2011), 470)). Given that the first-time bracket is nearly identical to the opening, I have treated it as part of the repeat and therefore not included any of its segments in my analysis; nor have I listed every segment in the repeated section twice.

16 Samuels, p. 65; Theodor W. Adorno, 'Mahler Today', trans. by Susan H. Gillespie, in *Essays on Music*, ed. by Richard Leppert (Berkeley: University of California Press, 2002), pp. 603–11 (p. 609).

17 Samuels, p. 86 'insis[t]', 'confront[s]'; Adorno, 'Mahler Today', p. 609 'the end'.

18 Throughout this and the following section, every effort has been made to include figures and annotated score excerpts for local processes discussed in detail: these have been processed from Gustav Mahler, *Symphonie Nr. 6 in vier Sätzen für grosses Orchester* [Score], ed. by Erwin Ratz, Sämtliche Werke: Kritische Gesamtausgabe, 6, rev. edn (Lindau: Kahnt, 1963), but Monahan's annotated short scores have also proved invaluable (see 'Narratives', II, 386–413 and 490–532). Dynamic markings and most cautionary accidentals have been omitted, but articulation marks have been retained where possible; and needless to say, a selective attitude towards the reduction of Mahler's complex textures has been necessary. Discussions of longer-ranging processes necessarily cite bar numbers (see n. 15 above), and the reader is urged to follow these up to fully comprehend the arguments being made and to situate them within the symphony as a whole. Finally, the graphs, tables, and charts used in constructing these analyses are available on the supplementary CD.

19 Monahan's chief theoretical orientation is Hepokoski and Darcy's Sonata Theory, which uses P(rimary) and S(econdary) as labels for what might traditionally be called first and second subjects/key areas. One should be aware, however, that P and S are defined stipulatively (such that, for example, the two areas must be separated by a mid-exposition break called a medial caesura, which can take a variety of different cadential forms): see Hepokoski and Darcy, especially pp. 16–18 and 23–29.
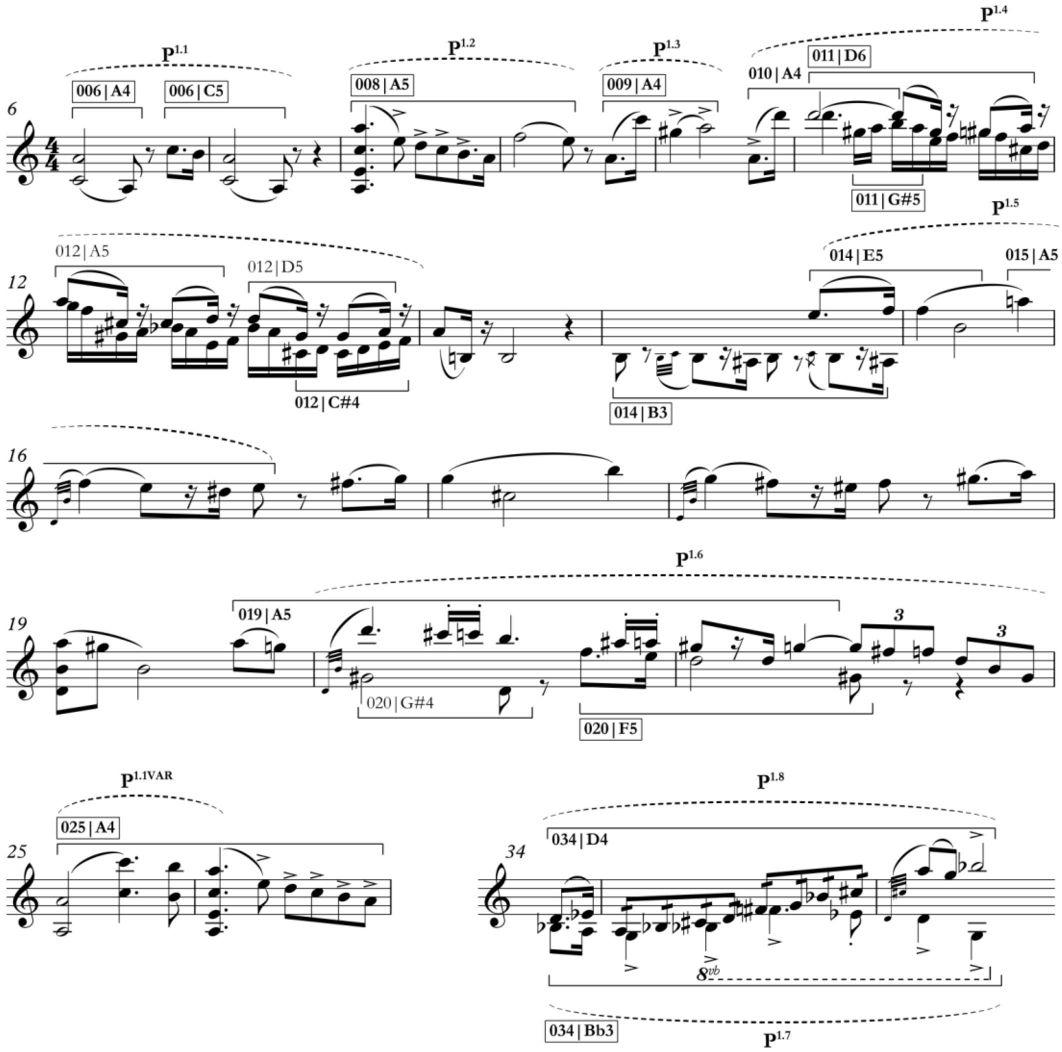
Figure 4.1: The primary theme area of the first movement in my segmentation (square brackets) and Monahan's (dotted slurs, $P^{1.1}$–$P^{1.8}$).[20]

Only segments which fall in one of Monahan's motives and/or begin a type region are included; the latter are in bold typeface, and are surrounded by a box if they also begin a new family. See also Figure 4.3.

the progression 006|A4→006|C5). This kind of informal grouping is heuristically useful, but can hide aspects of the motivic story being told: this, in turn, can affect an assessment of the music's formal and tonal processes.

To illustrate this concretely, consider the movement's brief introduction, reproduced in Figure 4.2. Its stubborn insistence on a tonic pedal and lack of any clear dominant harmony make it difficult to read as a structural upbeat to the arrival of the A minor primary theme at bar 6. As such, existing analyses tend to see it doing little more than establishing an affective ground (Adorno speaks of the movement's ever-present 'soundless march rhythm') for the movement to unfold against: for Norman Del Mar it 'sets the march-like character'; for de La Grange it 'symbolizes the

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[20] Adapted from Figure 4.3 in Monahan, 'Narratives', II, 314; he labels his figure 'Primary Theme (P) Motive Index'.
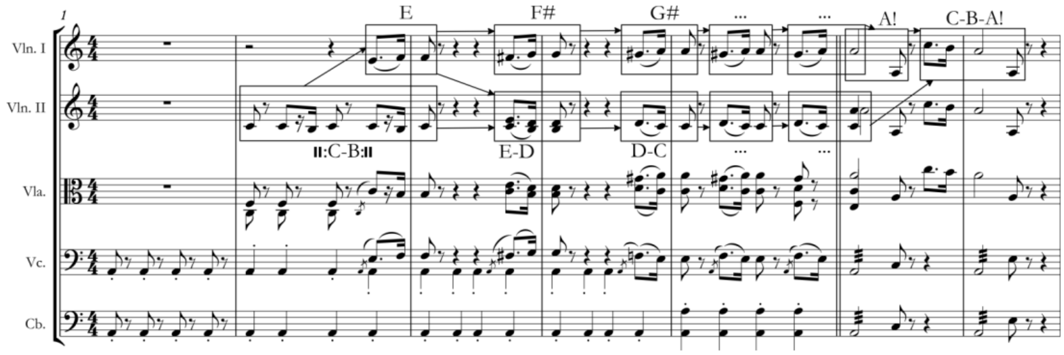
Figure 4.2: Motivic processes in the introduction to the first movement (bars 1–5).

determination of the "hero'" whose story we are about to hear; and for Egon Gartenberg it opens up a vista of 'a march of symphonic breadth'.[21] This static, scene-setting quality is certainly borne out by the aforementioned tonic pedal and segment 002|C4, which remains centred on the chordal third of A minor and touches on $\hat{2}$ only very briefly. An escape bid is made by 002|E4, which introduces the chordal fifth, ascends rather than descends, and, crucially, does not return to its opening note, becoming melodically open rather than closed. This open ascent becomes sequential transposition, outlining a melodic minor $\hat{5}$–$\sharp\hat{6}$–$\sharp\hat{7}$ in its chromatic climb; the energy of this is briefly dammed again by threefold repetition in bars 4–5 (accompanied by a crescendo) before finally discharging onto the A of bar 6.

Simultaneously, the ascending 002|E4 is inverted to 003|E4, which seeks to mirror the upwards motion towards the tonic through the exertion of a downwards pressure on 002|C4 in order to complete a $\hat{3}$–$\hat{2}$–$\hat{1}$ descent. Again through transposition E–D becomes D–C by bar 5, but the crucial C–B and B–A are missing until the octave-leap fulfilment of the ascending pattern assists its descending counterpart by prefixing C–B–A to create 006|C5. The stalled downwards tendency of $\hat{2}$ in 002|C4 is thus finally fulfilled by 006|C5, and this is made possible by 006|A4 as the fulfilment of the "escaped" 002|E4. In this way, the harmonically static introduction motivically reinforces a sense of arrival on the octave-leap As in bar 6 (one approached from above, and the other from below, starting from C4). This is an introductory function which is not immediately apparent in a purely harmonic reading, nor in a reading which smoothes over the progression from 006|A4 to 006|C5.

If heuristic groupings of motivic segments can obscure local processes, possibly with significant formal implications, then we might ask what effect these groupings have on the formation of larger-scale categories. Figure 4.3 lists every new type region to appear in bars 6–21, adds in any of Monahan's motives not already included, and gives the percentage of the movement's segments that belong to each new category at all applicable hierarchical levels (recall

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[21] Adorno, *Mahler*, p. 161; Del Mar, p. 34; de La Grange, *Mahler*, III, 821; Egon Gartenberg, *Mahler: The Man and His Music* (London: Cassell; New York: Schirmer, 1978), p. 307.

		C.S.F	C	S	F	XT.T.TR	XT	T	TR
$P^{1.1}$	006|A4	S2	-	-	4.2%	1.2.1	[81.9%]	3.8%	3.8%
	006|C5	4.1.1	6.9%	4.3%	4.3%	1.3.1		3.3%	2.6%
$P^{1.2}$	008|A5	S3	-	-	7.4%	1.49.1		1.8%	0.8%
$P^{1.3}$	009|A4	S4	-	-	1.3%	1.5.1		0.5%	0.3%
$P^{1.4}$	010|A4					1.6.1		0.7%	0.7%
	011|D6	S5	-	-	3.9%	1.7.1		3.9%	3.9%
	012|A5								
	012|D5								
	011|G#5	S6	-	-	4.4%	2.1.1	3.5%	3.5%	3.5%
	012|C#4					3.1.1	0.9%	0.9%	0.8%
	014|B3	[S1]	-	-	[21.2%]	[1.1.1]		[1%]	[0.4%]
$P^{1.5}$	014|E5					[1.4.1]		[10.2%]	[4.6%]
	015|A5					1.18.1		1.2%	1.2%
$P^{1.6}$	019|A5					4.1.1	0.1%	0.1%	0.1%
$P^{1.1*}$	020|G#4								
	020|F5	4.2.1		2.6%	0.5%	1.8.1		0.5%	0.5%
$P^{1.1VAR}$	025|A4	5.1.1	11%	0.3%	0.3%	1.50.1		0.3%	0.3%
$P^{1.7}$	034|Bb3	[5.2.1]		[8.9%]	[8.9%]	1.13.1		8.1%	7.8%
$P^{1.8}$	034|D4					1.11.1		0.1%	0.1%

** not explicitly labelled by Monahan*

Figure 4.3: Category sizes for Monahan's first-movement P-motives (see Figure 4.1) and any other segments in bars 6–21 that begin new type regions.

The type region label for each segment that begins a new type region is listed in the 'XT.T.TR' column. Percentages then indicate how many of the movement's segments belong to this type region (final column), and to any new categories at higher levels that the segment also initiates (the 'T' count includes subtypes, and the 'C.S.F' column lists the labels of new families; see Section 3.2, Section 3.3, and Figure 3.16 for an explanation of this labelling hierarchy). Square brackets indicate categories that have already appeared in the piece, but in segments not listed in the table, and dashes indicate that source families cannot, by definition, have cousins or siblings.

that a new family or type necessitates a new type region, that a new cross-family type or set of siblings necessitates a new type or family respectively, and that a new set of cousins necessitates a new set of siblings; see Theorem 3.3.10 and Corollary 3.2.15). The first row tells us, for example, that 006|A4 is the source of a family incorporating 4.2% of the movement's segments; of these, 3.8% (of the movement, equivalent to 90.6% of the family) lie in its type region (the only one in its type), and 81.9% of the movement lies in its cross-family type (which is actually begun by 002|C4, not shown in the table – hence the square brackets).

Figure 4.3 clarifies an understanding (suggested above) of Monahan's 'motive index' as a segmentation of the first subject area rather than an efficient listing of the movement's most recurrent motivic ideas.[22] Some of his motives betray a degree of internal self-similarity ($P^{1.4}$ can be

[22] This is not how Monahan himself sees the index, which he claims identifies the 'Spartan stock of basic motives' underlying 'a tour-de-force of Mahlerian developing variation' ('Narratives', I, 119); in the light of the discussion below, his claim seems untenable.

split into an $abbb$ format – as it has been in my analysis, with all the bs belonging to the same type region), hiding, as demonstrated with respect to $P^{1.1}$=006|A4→006|C5 above, part of the movement's motivic story. If we accept that Monahan's labels as categories are adequately modelled by my own formal structures at some level – in other words, that the set of segments in the piece that Monahan would label '$P^{1.2}$' is close enough to some category (say, the type region, or the family) formed around my 008|A5 – then we can see that their levels of recurrence vary quite considerably.[23] The families built around the two segments that make up $P^{1.1}$ constitute roughly equal proportions of the movement, but the type regions do not; $P^{1.7}$ initiates the largest type region in the movement, while the type region containing $P^{1.3}$ appears only once again (as segment 402|A4); and certain category-establishing segments in bars 6–21 are not included in Monahan's list. Neither Monahan's categories nor my own therefore demonstrate evenness of size or significance (properties which are not always correlated: the family of the motto rhythm's first appearance, 057|A2, contains only 2.2% of the movement's segments); the differences between the two interpretations lie in the explicitness of their membership criteria, and the sharpness with which categories are distinguished from individual segments.

The label '$P^{1.2}$' is a convenient shorthand which, in the manner of archetype 1 from Section 2.2, indexes a whole cluster of related segments using the paradigmatic example in bars 8–9. While this makes it more readily apprehensible to a reader than 'type region 1.49.1', such a label blurs the line between segment and category and stamps all of its members with connotations of the primary subject area. These connotations, however, are not always appropriate, since while much of the motivic material introduced here is certainly 'primary' to the movement, it is not necessarily restricted to, or even particularly strongly associated with, those areas of the movement's structure allotted to the first subject.

To illustrate this, Figure 4.4 and Figure 4.5 construct versions of Figure 4.1 and Figure 4.3 for the movement's secondary theme. Perhaps their most striking property is the lack of square brackets in Figure 4.5: this means that all the motivic material in the paradigmatic statement of S (and, indeed, the entire S-section, bars 76–128) is either new, or derived from the paradigmatic statement of P in bars 6–21. Of the former, it is only Monahan's $S^{1.1}$ (my 076|A5) and $S^{1.2}$ (my 078|G5) that initiate new categories of any significance; of these, the latter is the S-theme's only

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[23] It is difficult to reconstruct Monahan's categories entirely due to the problems outlined in Section 2.3: does $P^{1.1VAR}$ (in bar 25 of Figure 4.1) belong to the same category as $P^{1.1}$, for example, or is Monahan simply pointing out a freer parallelism? This reaches its familiar *ad absurdum* conclusion when a dotted line under $S^{1.1}$ in his secondary theme index (see Figure 4.4) indicates a parallelism with $P^{1.2}$ – to which category, then, does that segment belong? The approach taken here – essentially to extrapolate from Monahan's segment labels using my own model – is not perfect, but seems preferable to indulging in the highly speculative exercise of guessing how Monahan would segment the entire movement.

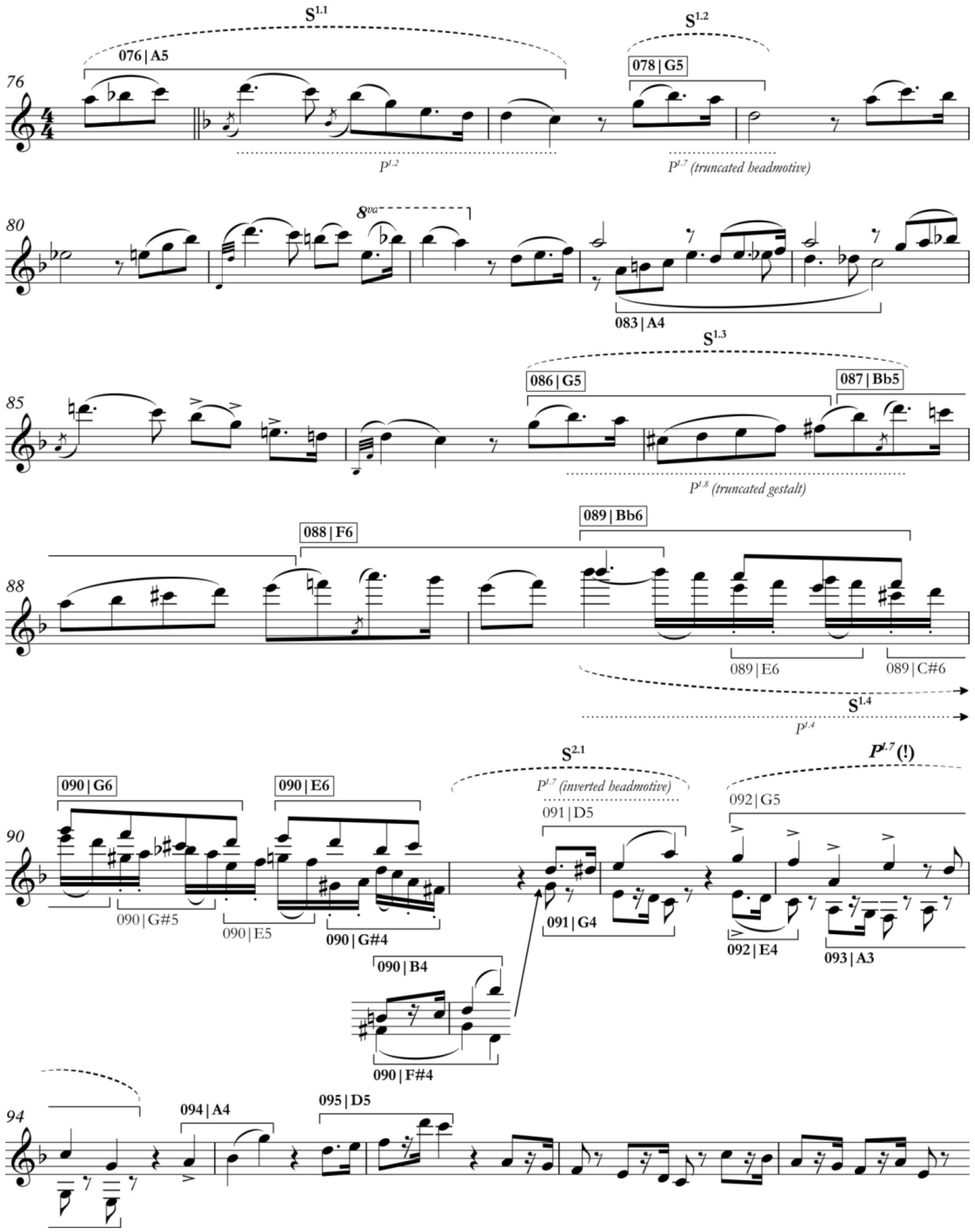

Figure 4.4: Segmentations of the first movement's secondary theme (cf. Figure 4.1).[24]
In addition to the conventions of Figure 4.1, Monahan's parallels to the P-theme are marked with dotted lines.

source. (Nine of the fifteen other new type regions contain a single segment, and the other six contain between two and four.) The head-motive of the "Alma" theme, the motive that one would therefore expect to function most clearly as an icon for the second subject throughout the movement, is, as Monahan points out, closely related to P1.2. While its status as a head-motive does tip its associative balance in the favour of the second, rather than the first, subject, it is interesting

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[24] Adapted from Figure 4.6 in Monahan, 'Narratives', II, 319.

		C.S.F	C	S	F	XT.T.TR	XT	T	TR
$S^{1.1}$	076|A5					1.9.2		3.4%	2.7%
$S^{1.2}$	078|G5	S12	-	-	5.1%	1.20.1		4.2%	4.2%
	083|A4					1.21.1		0.1%	0.1%
$S^{1.3}$	086|G5	8.1.1	1.2%	0.8%	0.4%	1.22.1		0.4%	0.4%
	087|Bb5	8.1.2			0.1%	1.23.1		0.1%	0.1%
	088|F6	8.2.1		0.4%	0.4%	1.24.1		0.4%	0.4%
	089|Bb6	7.1.1	1.2%	0.1%	0.1%	1.25.1		0.1%	0.1%
$S^{1.4}$	089|E6								
	089|C#6								
	090|G#5								
	090|E5								
	090|G#4					3.1.2			0.1%
	090|G6	7.3.1		0.1%	0.1%	1.26.1		0.1%	0.1%
	090|E6	7.4.1		0.1%	0.1%	1.27.1		0.1%	0.1%
	090|F#4					1.28.1		0.4%	0.4%
$S^{2.1}$	090|B4					1.29.1		0.7%	0.5%
	091|D5								
	091|G4					1.3a.1			0.5%
	092|E4					1.3.2			0.1%
“$P^{1.7}$”	092|G5								
	093|A3					1.30.1		0.1%	0.1%
	094|A4					1.29a.1			0.1%
	095|D5					1.31.1		0.3%	0.3%

Figure 4.5: New categories, Monahan’s S-motives, and associated percentages for bars 76–98 of the first movement (cf. Figure 4.3).
All other categories within this range appear for the first time in Figure 4.3.

to note that 15 of its family’s 57 members (26%) occur outside those sections of the movement that Monahan designates to be based on S material.[25]

The exposition’s transitional section (TR in Sonata Theory nomenclature – not to be confused with the abbreviation for ‘type region’ in Figure 4.3 and Figure 4.5) has frequently attracted attention for its detached aloofness from the sonata processes unfolding around it. Its chorale topic ‘cannot lead anywhere’ according to Adorno, and its failure to adequately prepare the tonal ground for the second subject (it remains resolutely in A minor) enhances the second subject’s ‘sensation as character’; for Monahan (citing a remark by Hepokoski), the music ‘seems merely to “compose out” the silence of the medial caesura’, preserving the two-block exposition,

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25 I follow the formal overview given as Figure 1 in “‘I Have Tried’”, p. 125, which mostly replicates Figure 4.1 from ‘Narratives’, II, 312 (subject to the change in bar numbering outlined in n. 15). When Monahan reproduces the figure again, albeit with the development omitted, as Example 8 in ‘Success and Failure’, p. 52, the final section (bars 426–end) is labelled ‘S’ rather than ‘Episode Two(S/TR)’, further emphasising the “S-ness” of this music. The 15 non-S segments occur in P-based sections – so the motive never appears in the purely TR(ansitional) bars 61–77 and 341–53.

and for Samuels, 'the use of a clearly non-symphonic genre (the chorale) precisely at this moment, which is so crucial to the symphonic process, foregrounds the artificiality of the choice'.[26]

Monahan reads the chorale's inertia as the fallout of a rupture created by the first appearance of the motto; in particular, he observes that '[o]nly *eight* of the thirty-two notes in the soprano voice are not involved in some semitonal descent' (i.e. that they take on the motto's melodic shape).[27] This association between a symbol of fate or tragedy (here the motto) and a transitional chorale theme is writ large in the Finale by the hammer-blows; the third blow, that which 'fells [the hero]' (as Gustav via Alma has it), is the only one to coincide with the motto, purging its major chord and initiating the symphony's dying utterances.[28] Aside from the motto and the *pizzicato* remnants of the first subject's opening segments, two other families corresponding to the first two two-bar phrases of the chorale are found here; originally, three were identified, but the third segment (bars 67–68) remained unconnected to anything else in the movement in my analysis and so was found to be non-motivic (see Definition 3.1.7).[29] These two source families, and all of the composite families derived from at least one of them, occur during six passages in the

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[26] Adorno, 'Centenary Address', pp. 108, 109; Monahan, '"I Have Tried"', p. 147, n. 76; Samuels, p. 145. Other comments include de La Grange's label of 'odd, expressionless' (*Mahler*, III, 821) and Johnson's 'dissociated' (Julian Johnson, *Mahler's Voices: Expression and Irony in the Songs and Symphonies* (Oxford: Oxford University Press, 2009), p. 192); Floros's claim that it 'rests […] within itself' (Constantin Floros, *Gustav Mahler: The Symphonies*, trans. by Veron Wicker (Portland: Amadeus Press, 1993; repr. Aldershot: Scolar Press, 1994), p. 167); and Del Mar's assertion that it 'plays no part in the argument' (p. 35). Topically, Del Mar sees it symbolising 'calm faith even under severest adversity' (ibid.) and Hans Eggebrecht as an 'intrusion from above' (Hans Heinrich Eggebrecht, *Die Musik Gustav Mahlers* (Munich: Piper, 1982), p. 113, quoted in Monahan, 'Narratives', I, 123, n. 34), but these interpretations contrast sharply with Williamson's 'secularized' (John Williamson, 'The Development of Mahler's Symphonic Technique with Special Reference to the Compositions of the Period 1899–1905' (unpublished doctoral thesis, University of Oxford, 1975), p. 115, quoted in Monahan, 'Narratives', I, 123) and Monahan's 'archaic […] whiff of the sacred' in a marriage ritual that joins the two gendered subject areas only nominally ('"I Have Tried"', p. 147, n. 76). This is a good example of how taking the idea of tragedy as a quilting point can affect a reading of a passage that others might see as hopeful or innocuous, potentially neglecting the work's defining contrasts and polarities by seeking to resolve them into an overarching scheme.

[27] 'Narratives', I, 123, n. 35.

[28] Alma Mahler, p. 70; it was this third hammer-blow, in bar 783 of the Finale, that was deleted by a supposedly superstitious Mahler after the publication of the first edition (see n. 5 above). The symbolic status of the motto is canonical: to cite just a few examples, its rhythm and major–minor fall are 'Fate motifs' for Del Mar (p. 24), 'tragic symbols' for Floros (*Symphonies*, p. 167), 'the "tragic" theme' for Gartenberg (p. 307), and 'the tragedy motif' for Michael Kennedy (Michael Kennedy, *Mahler*, The Master Musicians Series (London: Dent, 1974; repr. 1978), p. 118); Peter Brown also sees the fall as an allusion to the coda of the '*Todtenfeier*' first movement of the Second Symphony (p. 656). The allegiances between the motto, chorale, and hammer-blow (and the chorale-derived hammer-blow theme) are explored further in Section 4.2, where it is argued that they together represent a crucial third agent suppressed in Monahan's reading.

[29] Of course, 'unconnected' is relative, in this symphony in particular; the three notes C–A–B are prominent in this movement (cf. 006|C5's C–B–A) and the Finale (cf. 017|B3's B–C–A, a retrograde inversion; see also n. 67). Still, they fall short of being 'a short, recognisably recurrent melodic gestalt' (Definition 1) within the context of this movement.

movement: one during the exposition, three during the development, one during the recapitulation, and one during the coda.

The five post-expositional passages might be read as an attempt for this music to fulfil the transitional role that it failed to play in the exposition. It serves in the development as both the gateway to (bars 210–20) and from (bars 244–47) the idealised 'fantasy projection' of the S-theme (bars 222–43), which 'unfolds within a finely-spun contrapuntal texture […] unspoiled by Strauss's high-industrial orchestra'.[30] Monahan has the first of these passages 'sleepwalking' through a dreamlike re-enactment of the exposition, and the second attempting to take 'some critical "insight" […] back down from the Alpine heights' after an all-too-fleeting 'confrontation with eternity'.[31] Within the context of a Mahlerian *Klangfläche* ('sound-plane' usually connoting nature; the term is Dahlhaus's), the chorale makes a virtue of the very stasis that seemed so out-of-place in the exposition.[32] The celesta countermelody that accompanies and then extends the Moses-like return from the mountain (bars 244–55; the three source families introduced appear nowhere else in the movement) suggests that the chorale has indeed been changed by its experience.[33] Sure enough, when the chorale appears for the fourth time to lead into the recapitulation it is subjected to basic development: diminution (bars 276–77), sequential iteration (F♯ in bar 270, D in bar 276, F in bar 279, and F♯ in bar 283), interval change, and liquidation as the first phrase is heard independently of the second (bars 279 and 283).

It is, however, the chorale's appearance in the recapitulation, as shown in Figure 4.6, that truly realises its developmental potential.[34] The skeletal *pizzicati* that accompanied the chorale in

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30 Monahan, 'Narratives', I, 138; here Monahan borrows the term 'fantasy projection' from Warren Darcy, 'Rotational Form, Teleological Genesis, and Fantasy-Projection in the Slow Movement of Mahler's Sixth Symphony', *Nineteenth-Century Music*, 25 (2001–02), 49–74.

31 'Narratives', I, 139 'sleepwalking'; p. 140 'insight', 'confrontation'. The 'Alpine heights' are suggested by the cowbells and Mahler's comments on them (albeit in relation to their role in the Seventh Symphony), symbolising 'the loneliness of being far away from the world' and evoking 'a sound of nature, echoing from a great distance […] as though [one] stood on the highest peak, in the face of eternity' (see Constantin Floros, *Gustav Mahler*, 3 vols (Wiesbaden: Breitkopf & Härtel, 1977–85), I: *Die geistige Welt Gustav Mahlers in systematischer Darstellung*, pp. 323, 430, quoted in Floros, *Symphonies*, p. 165).

32 See Carl Dahlhaus, *Nineteenth-Century Music*, trans. by J. Bradford Robinson, California Studies in Nineteenth-Century Music, 5 (Berkeley: University of California Press, 1989), p. 307; Robinson translates the term as 'sound-sheet'. Adorno cites this same passage as an example of 'extra-territorial', time-arresting *Suspension* (*Mahler*, p. 41; see also Chapter 2 at n. 124 above).

33 The melody is, in fact, suggested in bars 216–18; but it is unmarked by the accents of bars 244–46 and is not stated independently (as it is at bars 250–55), suggesting that it is part of the general sonic background here and not yet strictly motivic. Perhaps it, too, is changed by its journey up the mountain?

34 This reading is hinted at by de La Grange when he considers the recapitulation's transition to be 'influenced by the "nature" episode in the development' (*Mahler*, III, 824), but he does not pursue the relationship any further than a correspondence in texture and orchestration. Interestingly, when comparing recapitulation to exposition, de La Grange discusses the transition's modifications 'in particular' (ibid.) and devotes only half a sentence to the second subject, whereas Del Mar
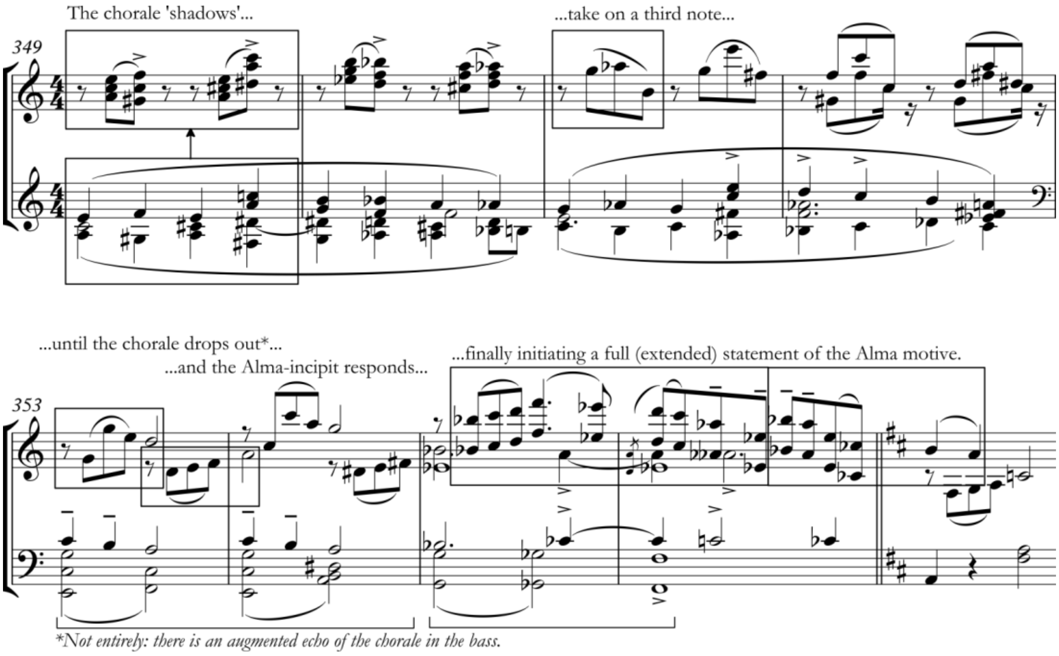
Figure 4.6: The 'developing' transition in the first-movement recapitulation, bars 349–56.
Only the most relevant segments are highlighted here.

the exposition are still present, but their thematic material shadows the chorale rather than representing the aftershock of the first subject. After a brief intervening passage unconnected to anything else in the movement (bars 343–48), these two-note shadows take on a third note as the chorale incipit is repeated; in bar 353, the shadows are substantial enough to stand alone, forming the call to which the first few notes of the second subject are the response. Perhaps disoriented by the transition's newly-found effectiveness, the second subject does not 'burs[t] in' as before (in Del Mar's phrase (p. 39)) but gradually assembles a substantially truncated repetition (in the "wrong" key) characterised variously as 'protracted, groping', 'linger[ing] to reflect on its experiences in the development', 'an act of passive-aggressive retaliation' against 'its role as a mere functionary in the sonata's master plan', or 'still distracted by fantasy' – all by Monahan alone.[35]

The final appearance of the chorale, in the movement's closing peroration (bars 458–61), is a little more cryptic. Monahan reads the close of the development as a 'field of open conflict'

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reverses this and considers the transition to be 'virtually identical' when set alongside the 'major change' undergone by the second subject (p. 39).

[35] 'Narratives', I, 143 'groping'; '"I Have Tried"', p. 152 'linger[ing]'; p. 53 the rest. Del Mar notes that the second subject 'emerges most subtly out of the repeating phrases of [the chorale]' (p. 39), and Samuels that the orchestration and motivic material serve to smooth rather than delineate the transition's boundaries in the recapitulation (p. 146; see n. 43 below). He also argues that this constitutes 'the longest "developmental" passage in the movement' (p. 146) after a largely modular development section, and that while reserving development of the second subject for the recapitulation is nothing new, 'it is usually part of the integration of the material into the tonic' (p. 147). The section, in its proposal and subsequent denial of a sonata-form reading, is therefore another instance of the symphony's self-negating formal dialectic.

between TR and P (in particular $P^{1.7}$) as the latter abruptly rebukes the former for attempting to reverse the normative expositional order (labelled, by Sonata Theory, the 'rotational order'): P (bar 128) – TR (bar 201) – S (bar 222) is reflected around S and becomes the path back down the mountain, TR (bar 239) – P (bar 256).[36] Once the chorale has been 'overwhelmed' and 'submerg[ed]' (Del Mar's terms; see p. 39), the recapitulation can continue in its expected order; beginning, of course, with the triumphant P. The coda (as a 'recomposition of the [development]') further underlines this "correction" by presenting the elements involved in the conflict in their normative order: this revokes their connotations of a journey away from S, which has become 'liberat[ed]', in part, by the mercenary $P^{1.7}$.[37]

The chorale as it appears in the coda – three-part trombones, two-bar phrasing, both halves together with no sequential treatment – seems to have reverted to its pre-mountain form, and thus works against a reading which sees it emerging unscathed by the retransitional battle to finally fulfil its intended purpose in the recapitulation. But perhaps its very triumph as a transition *is* its tragedy, since the purpose of a transition is to lose its identity to the themes on both sides: its skeletal countermelodies in the exposition and recapitulation point backwards to P and forwards to S respectively, and its submergence by P in the retransition is matched by its burial as an almost negligible contrapuntal detail in the fevered excitement of S in the coda. Its wish was granted on its journey up the mountain – it learned how to become a transition – but only through experience have the tragic consequences of this made themselves clear.[38] If we read P as an antagonistic reinforcer of hegemony and S as a bid for freedom (as it seems reasonable to do: they represent Gustav and Alma respectively in Monahan's 2011 'marriage' reading of the first movement, and totality and individualism respectively in his analysis of the Finale) then, as Monahan points out, the re-establishment of the rotational order is good for both P and S (as the former achieves its goal while the latter is given 'the final word').[39] The suppression of the chorale's attempt at reversal is

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36 See Monahan, 'Narratives', I, 139–41 (p. 141 'field'). The concept of form as a continual recycling of large-scale 'rotational' blocks is explored in relation to this symphony (and to the Andante in particular) by Darcy, 'Rotational Form'.
37 See Monahan, 'Narratives', I, 141'liberat[ed]'; p. 144 'recomposition'; p. 146; and II, 320.
38 The darker side of this music is hinted at by Schoenberg, who writes of its 'cool, icy comfort from a height which is reached only by one who soars to resignation; only he can hear it who understands what heavenly voices whisper without animal warmth' (Arnold Schoenberg, 'Gustav Mahler', trans. by Dika Newlin, in *S&I*, pp. 449–72 (p. 457)). The passage's double-faced nature is examined in more detail in the next section.
39 See Monahan, 'Narratives', I, 147 and '"I Have Tried"'; his characterisation of the Finale's themes is painted in broad strokes here (but unpacked further in Section 4.2 below) and agrees with other authors' readings of their affective and topical profiles (see, for example, Stephen E. Hefling, 'Song and Symphony (II). From *Wunderhorn* to Rückert and the Middle-Period Symphonies: Vocal and Instrumental Works for a New Century', in *The Cambridge Companion to Mahler*, ed. by Barham, pp. 108–27 (p. 123)). Johnson provides an interesting inverted counterpoint to this interpretative theme by contrasting the 'open-ended developmental tendency of the symphony' with the 'self-contained strophic form of the Lied' in relation to first and second subject groups (see *Mahler's*

thus imperative for both, which perhaps explains the uneasy alliance of $P^{1.7}$ and S: TR's assumption that S will support its anti-hegemonic stance explains the care with which it prepares the ground in the recapitulation before it comes to its horrible realisation and is ultimately swallowed. The *fortissimo* appearance of TR in the coda is, then, an act of defiance, an attempt to un-wish its wish and return to its state in the exposition, aloof and unentangled in sonata politics; as Stephen Hefling notes, 'the "chorale", now in the brass, finally sounds like one'.[40]

Of the remaining sources in the movement which have not been discussed above, three are introduced in the exposition and two in the development.[41] The first of these is an undistinctive two-note oscillating figure (the family contains a single type region) which liquidates to lead into the motto in the exposition and recapitulation (bars 47–56 and 331–38) and appears nowhere else; the second and third are the tattoo rhythm and major–minor fall of the motto itself. If the motto families, and any composite families that are derived from them, are tracked through the movement, we see that the two elements become decoupled in the development: the tattoo underscores tranches of march-like music in the first episode (bars 128–36, 157–66, and 171–78) while the major–minor fall resounds as a ghostly echo separating to two chorale phrases in the sleepwalk up the mountain (bars 208–14). Eager to be reunited in the recapitulation, the major–minor fall weakly attempts to assert itself too soon (so weakly, in fact, that Samuels fails to notice it) in bars 319–20.[42] When the appropriate recapitulatory moment arrives (just before the chorale), it is the rhythm that jumps in too soon: it overlaps with the liquidating figure and catches the major–minor fall by surprise, weakening the impact of the latter and paving the way for the transfigured chorale (bars 337–40).[43] In the remainder of the movement, the two halves are not only decoupled but varied: the rhythmic patterns in bars 395–400 refer back to the development's first episode rather than the motto *per se* (they constitute a new pair of cousin families in my reading), and the major–minor shift at the movement's *Höhepunkt* nine bars before the end is, as Monahan argues,

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Voices, p. 171). The co-dependence of P and S suggests a sinister nuance to their unfolding drama (especially departing from Monahan's domestic reading): namely, that since S strives for freedom but does all it can to remain within P's hegemony, their situation is not unlike that of an abusive relationship. Accordingly, P reneges on their truce in the reversed recapitulation of the Finale, giving itself (what seems like) the last word (see Section 4.2 below).

[40] 'Song and Symphony (II)', p. 120.

[41] It does make sense to speak of sources in this graph even though they are strictly defined in the condensation only (see Definition 3.2.9): this is because no source in this graph is strongly connected to any other vertex, so their strong components contain only themselves. In fact, there is only one strong component of size greater than one in this piece: a pair of antiparallel edges between the simultaneous octave-separated segments in bar 485. Normally doublings at the octave are not encoded separately, but these represent the convergence and conclusion of two hitherto parallel succession chains (cf. flutes and trumpets, bars 481–84; see further discussion at the end of this section). The convergence of motivic processes back towards their points of origin is, I argue below, a prominent tragic narrative device in the Finale.

[42] Bars 319–20 are not included in Samuels's tabulation of the motto's appearances (p. 160).

[43] Samuels notes the 'softening of the orchestration […] which removes the sense of abrupt disjuncture' in this passage (p. 146).

'semiotically "defused"', relating more strongly to Wagnerian 'redemptive peroration' than this work's contextual "Fate" motive (the final tonic major chords in *Tristan* and the *Ring*, the moment of arrival in the grail temple in *Parsifal*, and a significant structural moment in Strauss's *Tod und Verklärung* are all prepared by similar $\hat{6} \rightarrow \flat\hat{6}$ motions).[44] After a few appearances in the Scherzo, destabilised by the triple-time meter (Samuels identifies these at bars 261–66, 419–26, and 432–39), and a hint of major–minor interchanging in the Andante, the motto is next heard again at full strength in bar 9 of the Finale (the point at which, it seems, Mahler originally intended to place the first hammer-blow).[45]

The two remaining sources in the development occur at bars 155 (a four-note descending scale figure repeated in the following bar) and 162 (a melodic turn figure that is spun out into a melody in bars 162–66 and 171–74; this is motive $P^{2.1}$, the first half of theme P2, for Monahan).[46] A segment combining the scale shape with the turn rhythm is heard in bars 177–78 (and again, with the turn figure itself, in bars 191–95), while the sequential treatment of the scale which opens the coda (on B3 in bar 383, then C4 in bar 385) culminates, on D4, in a complementary combination of the turn *shape* and scale *rhythm* that Adorno labels 'an irruption of the horrible'.[47] The turn melody also echoes, and hybridises with, a liquidated form of $P^{1.7}$ in bars 176–77; this is then extended to form an ascending version of the latter (bars 177–80) which features heavily in the bass, and later the melody, of the retransition (bars 259–62, 269–75, and 278–81).

The affective qualities of aloofness and disjuncture associated with both the chorale and the motto not only suggest certain narrative interpretations as explored above, but also facilitate those very interpretations by disassociating themselves from the rest of the movement's motivic material to the point that they are recognisable as independent entities. We must tread more carefully in reading the movement's chief formal and narrative polarity (that between the first and second subject areas) since, as shown in Figure 4.5, the degree of motivic correspondence between the two themes is high. This is not unique to this piece: indeed, for Donald Mitchell, one of Reti's most radical insights is in showing how masterworks' contrasting first and second subjects betray a

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[44] See Monahan, 'Narratives', I, 148–49; the quotes are from p. 149. Given that these bars relate to the motto only morphologically, and not derivationally or 'semiotically', they remain outside the motto families and their composites in my reading. (The motto's syntactic function is also absent: Monahan notes (ibid., p. 148) that this is the only time that it does not signal a change of mode.)
[45] See Samuels, p. 160 and de La Grange, *Mahler*, III, 809, 813 ('Introduction to the first movement (bar 9)' on p. 813 appears to be a mistake).
[46] See Figure 4.5 in Monahan, 'Narratives', II, 323.
[47] *Mahler*, p. 125. Definition 1 is again invoked here since scale and turn figures are ubiquitous in almost any music. In particular, the bar that separates the second and third occurrences of the scale-in-turn-rhythm family (bar 194) includes a turn-in-scale rhythm segment; but this occurs at the same time as a more prominent developmental motive in the horns, and is cast into the shade by the flutes, which pick out the scale-in-turn-rhythm segments on each side whilst falling silent for this bar. It thus does not make a significant enough impression on its first appearance (especially when compared to its 'irruption' form) to be treated as motivic.

deeper organic unity and so necessarily belong to the same piece as each other.[48] But Monahan has written on the Finale's 'metastasiz[ing] […] organicism run amok', making it 'exceedingly difficult to know, to retain in the mind's ear'; his 2011 reading of the first movement also hinges on the idea that Mahler's 'idealized image of the feminine Other [in the Alma theme] was so thinly actualized […] that it could so easily and so tellingly shade into narcissistic self-depiction [in its echoes of the P-theme]'.[49] The first three source families to appear in the work (the dotted pattern that saturates the introduction, the octave leap, and the "Alma" motive $P^{1.2}=S^{1.1}$), together with the composite families made up solely from this set of three, contain 51.2% of the work's segments. Moreover, 81.9% of the work's segments lie in the same cross-family type, meaning that for any two segments picked at random, there is a 67.3% chance that a path of C, D, R, and E relationships exists between them such that every interim segment appears concretely in the piece.[50]

Our way in to this impenetrable thicket of motivic relationships seems to be the S-theme's only source: the inauspicious four-note segment 078|G5 (labelled $S^{1.2}$ by Monahan). The secondary theme unfolds in periods alternating between $S^{1.1}$ (the "Alma" motive) and $S^{1.2}$ (see Figure 4.7), and this local motivic polarity (and quest for integration) enacts in miniature the broader affective polarity of the movement's two subject areas. Comparing the first four notes of 076|A5 (henceforth the 'Alma-incipit') and the entirety of 078|G5, we see that while both are upbeat figures, stepwise motion and straight quavers are associated with the former, and leaping motion and dotted rhythms with the latter. We might also note that the Alma-incipit is the chief characteristic that distinguishes 076|A5 from its ancestor in the P-theme, 008|A5, so the story that can be read through this section is that of $S^{1.2}$ – the only "true" S-motive – trying to rehabilitate its partner's "S-ness" (and in doing so to transfigure itself) by driving for an independent statement of the Alma-incipit.

This process begins in the second period, where $S^{1.2}$ mimics the melodic shape of the Alma-incipit; this is reinforced by a "straight" phrase in the winds which fills the silences between the two occurrences of $S^{1.2}$. In the next period, $S^{1.2}$ tries a different tack and makes a bid for freedom, co-opting the rushing motive from P ($P^{1.8}$; see Figure 4.1) and initiating three, rather than two, segments: but this results in a collapse similar to that seen in the P-theme (although the angular quaver figure has been replaced by an augmented version of the semiquaver figure; see Figure 4.4), followed by a 'grotesque' march that Monahan reads as a significant problematisation

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48 See Mitchell's 'Prefatory Note' to Reti, *Thematic Process*, pp. v–vii (p. vi).
49 '"Inescapable" Coherence', p. 75 'amok'; p. 80, n. 92 'to know'; '"I Have Tried"', p. 148 'idealized'. He also warns against 'overemphasiz[ing] "*materiale Einheit*" at the expense of rhetorical differentiation (which is more crucial to the musical argument)' as this can quite easily lead to an overly homogenous view of the movement ('Narratives', I, 135, n. 57); the model presented in this thesis, being built on the considerations of Chapter 2, also seeks to respect the role of rhetoric and derivation in motivic analysis.
50 There is a 66.9% chance that any two random segments will lie in the biggest cross-family type.
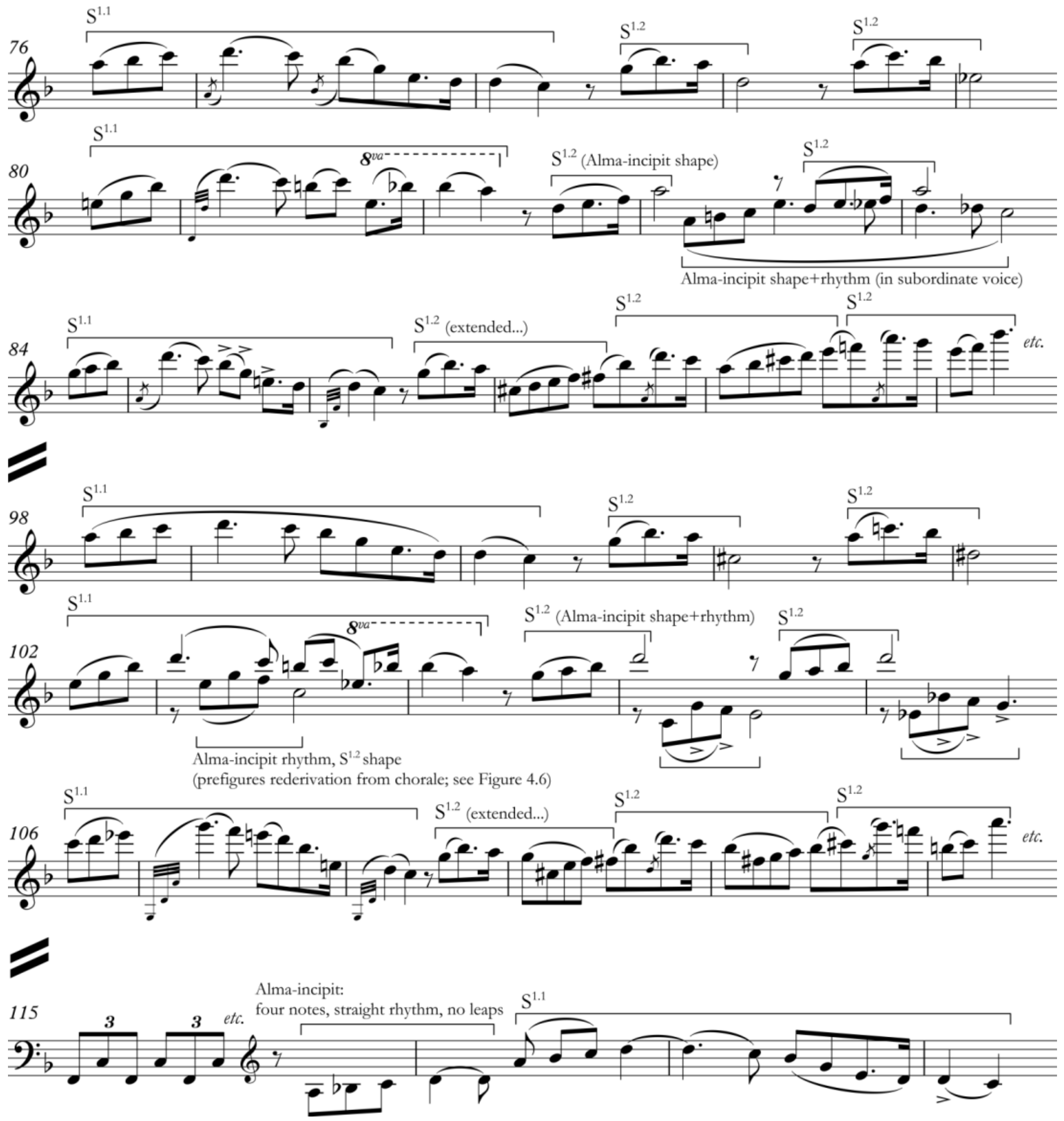
Figure 4.7: The first-movement S-theme's periods (bars 76–89, 98–111, and 115–18). The segmentation is mine, using Monahan's labels (see Figure 4.4): only those segments most relevant to the in-text discussion are included here.

of a naïve 'Alma-portrait' reading.[51] The periodic S-phrases then resume; in the second of these (i.e. the fifth iteration of the $S^{1.1}$–$S^{1.2}$–$S^{1.2}$ pattern), we are conditioned to listen for $S^{1.2}$ after $S^{1.1}$ and so the repeated Alma-incipit that we hear is understood as the first straight appearance of $S^{1.2}$ in the main melody.

This is a moment of strong epiphany between two families which have been separated thus far, but the integration of the two motives is not yet complete as $S^{1.2}$ still contains a melodic leap; this leap also seems to have spread back to the Alma-incipit, which leaps in the same place as $S^{1.2}$ in all but the first and fourth periods (we might also note the straight leaps in bars 103, 105, and 106, which prefigure the chorale's recapitulatory rederivation of the S-theme). The bid for freedom therefore again results in collapse, but this is significantly rhetorically softened as the scurrying

[51] See Monahan, '"I Have Tried"', pp. 134, 136; 'grotesque' is his term.

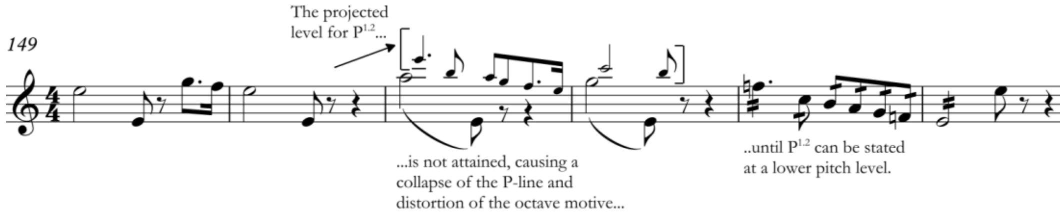

Figure 4.8: The collapse of the attempted *tutti* statement of P near the beginning of the first movement's development section (bars 149–54).

semiquaver figure becomes a rippling textural filler for the celesta, triangle, and winds. Finally, just as the tonic is achieved and the codetta opens, we hear an independent Alma-incipit arising in a moment of weak external epiphany: $S^{1.1}$, having asserted its independence from its ancestor $P^{1.2}$ through rederivation from the redemptive $S^{1.2}$, is now free to unfold on its own terms and in its own time (bars 115–22). Note that in the foregoing analysis the agents or narrative functions $S^{1.1}$ and $S^{1.2}$ are at first identified with one family each (S3 and S12), but are then read to continue through or across the composite families that arise as the analysis proceeds: an interpretative leap is required, for example, to read the composite family 8.1.1 (which combines $S^{1.2}$ with the rushing motive) as a "transfigured" $S^{1.2}$. This points to an important difference between Monahan's labels (as agents) and my own (as categories) – even as each inflects the other's reading.

The development begins as P regroups, aided by the motto rhythm. Re-appropriating the rushing $P^{1.8}$ to gain higher melodic ground (bars 138–42), a *tutti* statement of the entire primary theme in E minor is attempted at bar 149, but $S^{1.1}/P^{1.2}$ refuses to comply: stubbornly retaining its exposition pitch level of A5 rather than the required E6 (bar 151), it causes the P-line simply to give up and crumple back down to E4, and in doing so distorts the octave motive into an eleventh (Figure 4.8). The motive slowly regains its shape – outlining a tenth, G5–E4, in bar 152 – until it finally has the strength to summon up $P^{1.2}$ on F5, almost a full octave short of its required pitch level. A suggestion of $S^{1.2}$ is heard in bars 166–70 in a motive labelled $P^{2.2}$ by Monahan; although its augmented reference to $S^{1.2}$ is not immediately obvious, I would argue that it suggests a certain redemptive escapism in its ascending shape, turn towards major harmony, and ability to halt the relentless motto rhythm.[52]

The return of the four-note Alma-incipit is, however, anything but escapist: although it retains the pitch-classes of its emancipatory appearance in bars 115–16, it has dropped another octave, regained its dotted rhythm, and is tossed around in dialogue with its inversion (bars 178–182) to usher in its 'disfigurement' in 'a lumbering march', '[e]lephantine and inert'.[53] Once the march proper begins at bar 183, the disfigured incipit is re-joined with $S^{1.1}$ and the pitch level gradually sinks through A♭ (bars 186–87), G (bar 188), and F (bar 190). Despite an ascending

[52] See Monahan, 'Narratives', II, 396: here he interprets $P^{2.2}$ as related to $P^{1.7}$, the '"free agent"' that assists in the liberation of S in his reading (ibid., I, 141).
[53] Monahan, '"I Have Tried"', p. 150.

tendency in bars 190–91, the descending shapes go on to proliferate: a trumpet segment strips all the leaps from $S^{1.1}$ in a moment of weak epiphany (bars 192–93), and its descent is echoed in the merging of the scale and turn motives (bars 193 and 195), the rattling version of $P^{1.6}$ (bars 194 and 196–97), and two disfigured forms of $S^{1.1}$ (bars 195 and 197–98; the former absent-mindedly tails off chromatically, while the latter's incipit decouples itself in terms of pitch, register, and instrumentation in bar 196). P then marshals its forces for an ascent mirroring its unsuccessful attempt in bars 144–48, but gets stuck in bar 200 as the movement 'tear[s] open from within' and the cowbell music begins.[54]

It is interesting to note that the segments at bars 201–04 and 208–10, whilst both recognisable as $S^{1.1}$ (albeit abandoned by their incipits), both begin new cross-family types; that is to say, no other segment has the same chromatic, diatonic, or rhythmic structure, and they neither embed nor are embedded by any other segment. The affective sense of Otherness is thus very subtly supported motivically, and this continues into the weak external epiphany that constitutes the first phrase of the Utopian vision (bars 222–23).[55] The most significant morphological relationships between this phrase and the other members of its cross-family type are its embedding of the Alma-incipit from bar 115–16 (not as an incipit, but at its end), and, ironically, the fact that its rhythm has only so far occurred at two other points in the movement: once in bars 25–26 and again in bars 153–54, both during statements (or attempted statements) of the P-theme.

The Utopian music itself falls into two strophes that each loosely retread the $S^{1.1}$–$S^{1.2}$–$S^{1.2}$ pattern, once in G major and again in E flat major (the key of the Symphony's Andante). The first of these features two "Utopian" versions of $S^{1.1}$ (one ascending and one descending) and three statements of $S^{1.2}$, and is rounded off by a hybrid of the "Utopia" motive with the angular quavers from $P^{1.4}$ in bars 228–29 (the only moment in bars 222–43 that does not lie in a family entirely derived from $S^{1.1}$ and/or $S^{1.2}$). The five appearances of $S^{1.2}$ in this section each make use of a different initial interval – perfect fourth, perfect fifth, minor seventh, diminished fourth, and then finally the usual minor third (bars 225–27 and 232–33) – subtly echoing the more radical development being undergone by $S^{1.1}$ as the solo violin spins out a counterpoint to the familiar periodic structure in the solo horn (bars 230–36). The two instruments swap roles for the final phrase in bars 237–38 before the ascending and descending versions of $S^{1.1}$ are heard in the bass clarinet to lead to the transfigured chorale. This passage of subtle development, flanked by *Klangflächen* and adopting their affective stasis, is the means by which the chorale learns its lesson;

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[54] Monahan, 'Narratives', I, 135.
[55] The phrase 'Utopian vision' is Monahan's, and he uses it to designate three episodes in the Finale discussed below (see '"Inescapable" Coherence', p. 62); other writers (Peter Brown, for example) also see the cowbell episodes as 'foil[s] to the work's predominantly tragic character' (Peter Brown, p. 662). Bruno Walter, however, considers 'that other world' to be completely outside of this symphony's 'field of vision' (Bruno Walter, *Gustav Mahler*, trans. by James Galston (London: Kegan Paul, Trench, Trübner, 1937), p. 123).

but the Utopian kind of development on show here is not that of forward-driving symphonic motivic working, but rather the free and even heterophonic spinning-out of contrapuntal lines that Monahan so admires in Mahler's earlier symphonies (see above at n. 30) and which Johnson sees projecting 'a fragile sense of forward motion and arrival thus far lacking' in the movement.[56] Given that Monahan considers the S-theme's primary flaw to be that it is 'precariously overextended', 'stretching its scant melodic/motivic resources to the limits of good taste', it seems that the S-theme itself also has something to learn from its Utopian transfiguration.[57]

The story of the S-motives resumes at the start of the recapitulation in bar 291, which is in A major until $P^{1.2}/S^{1.1}$ has been heard; its echo in the bass ends on E, which derails the recapitulation (just as the same double-faced motive derailed the development; see Figure 4.8) by persisting throughout melodic material which was heard in the exposition over A (bars 297–306). The expected harmony is resumed at bar 307; but the appearance of 309|A4 (the recapitulation of 025|A4, which combines $P^{1.1}$ and $P^{1.2}$) re-opens the fissure between the melodic and bass strata (this time the pedal drops by a fifth to D in bar 315). The melody begins to sink to compensate for this – by a semitone in bars 318–20, then to a fourth below the original in bars 320–21 – and once the D pedal quits, the bass adopts a level to match the melody. But as we enter the liquidation, the fissure opens up again at a point where $S^{1.1}$ was heard in the exposition (compare bars 46 and 330): the bass stays on E, but the "melodic" parts flounder around and cannot settle on a particular level. It is into this fractured environment that the motto prematurely enters and the chorale makes an unhindered attempt to effect a transition.

The second subject area attempts to forget the humiliation inflicted on the Alma-incipit in the developmental march, and so picks up where the exposition left off: four-note segments abound in bars 353–59 (rederived as an answer to the chorale counterpoint – see Figure 4.6 – and demonstrating the porous boundaries between $S^{1.2}$ and the Alma-incipit), but we don't hear the Alma-incipit in its undotted leap-free form until bars 360–61, where it serves as the opening of three overlapping statements of $S^{1.1}$ (residual heterophony from the Utopian episode, perhaps?). Its independent form appears in bar 364 as a pun (if we permit augmented seconds and augmented unisons as diatonic steps): the horns pick out A♯–C♯–D–D♯ from the extended "bid-for-freedom" version of $S^{1.2}$, respelled as B♭–C♯–D–D♯. The Alma-incipit proper is heard again in the codetta: this time twice, setting up both iterations of the luxuriating $S^{1.1}$. Its indignity in the development has been overcome by its Utopian transformation, its preceding fresh start in the opening up of new but closely related type regions, and its successful derailing of the P-theme's recapitulation.

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56 Johnson, *Mahler's Voices*, p. 70. Johnson later describes this motion as 'gently tangential to the main trajectory of the movement' (ibid., p. 222), and considers it ultimately unfulfilled (ibid., p. 70).
57 Monahan, '"I Have Tried"', p. 134.

The wheel of fortune must turn once more, however, as the distant thumping pedal of P returns to begin the coda (labelled by Monahan 'the Allegro's darkest, most toxic music').[58] When a march that mirrors the development's begins in bar 395, its new accompaniment figure is derived from the Utopian vision (bars 396–402); deprived of its affective support, this figure does little more than hopelessly reiterate the same reified CDR-class at the same pitch level. The reappearance of the subtly $S^{1.2}$-derived ascending figure ($P^{2.2}$; bars 400–02 and 413–18) is similarly stripped of the features that marked it out as redemptive in bars 166–70; instead it gets carried along with the general sweep, taking on an association with a $P^{1.1}$-derived motive that comes to 'spread malignantly through the symphonic tissue' of the Finale (first heard in bar 393, trombones; Monahan considers this to be derived from $P^{1.1VAR}$, but it is prepared more directly by the stretching and inversion of the second half of $P^{1.1}$ by the basses in bars 391–92).[59] The redemptive motive does seem to have had some effect, however, as in bar 402 the accompaniment motive shifts up to F2 and changes its intervals slightly; while it drops back down to E2 in bar 410, it then jumps up to G2 for its final appearance in bar 416. This ascent is coupled with an increasing density of $P^{1.2}/S^{1.1}$ quotations (bars 391–92, 411–12, 414–16, and 418–19) until the incipit returns and 'the Alma melody buoys up from the contrapuntal depths in A *major*' in bars 419–21, pointing forwards to the key of the movement's apotheosis, and backwards to that of the recapitulation's opening.[60]

The next section (bars 426–33) resists straightforward amalgamation into an overarching narrative. On one hand, it is in E♭ minor (a tritone away from the triumphant A major, and the modal inverse of the Utopian E♭ major); it reinstates the march topic, including the disfigured dotted Alma-incipit; its primary motivic material is drawn from the most P-reminiscent part of S (bars 90–98, omitted in the recapitulation); and it is presented as the culmination of the P music cut off by the rupture in bar 201 (compare bars 199–201 with bars 423–25).[61] On the other hand, it comes between $S^{1.1}$'s reassertion of A major and its apotheosis in the same key, and carries a certain air of triumph rather than parody in its dotted Alma-incipit and $S^{1.2}$ quotations (bars 426–29); Hefling considers the passage to be the first stage in the second subject's 'replenish[ment]'.[62] Monahan explains this as the 'exil[e]' of the strongly P-derived S-section, 'sustain[ing] the galloping tension' of bars 423–25; if, as suggested in n. 61 above, the rupture is interpreted as the "true" continuation of these bars, then the E♭ minor section may even be seen as a radical recomposition

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58 'Narratives', I, 144.
59 Monahan, 'Narratives', I, 145. The role that this new motive – Monahan's 'G2' – plays in the Finale is discussed in more detail in Section 4.2.
60 Monahan, '"I Have Tried"', p. 154.
61 See Monahan, '"I Have Tried"', pp. 154–55 and Figure 2 on p. 157. Monahan reads the parallel to bars 199–201 slightly differently: for him, bars 423–25 employ 'the same spastic figure that triggered the development's rupture' ('Narratives', I, 145), suggesting that rupture, and not culmination, is the more natural response to this material.
62 'Song and Symphony (II)', p. 120.

of the Utopian vision.[63] This reading is borne out by the return of the shimmering sonorities and ascending-fourth dotted fanfares in the ensuing transition to the apotheosis (bars 434–48); a passage which, Monahan notes, also incorporates the liquidated motives from the retransition that led to A major.[64] Add in a strongly foregrounded "Alma" statement (complete with incipit, bars 440–43: although there is a slight fracture as the incipit's last note slips up a semitone and becomes "de-elided" with the start of the $P^{1.2}$-derived descent), juxtaposed with a return of the 'galloping tension' music and 'malignant' Finale motive (bars 453–46), and we find a passage that seems to heighten, rather than resolve, the symphony's polarities just as it approaches a moment of ostensible closure.

By bar 446, however, it seems clear in which direction the movement will go: the dominant of A major established, the Alma-incipit is heard in augmentation on *fortissimo* brass. Although it is cut off before its final note (in a gesture of the kind that almost always leads to catastrophe in the Finale), the phrase is picked up again on the same pitches, but this time supported by A major harmony and continuing into an augmented $S^{1.1}$ that is CD-equivalent to its first appearance in bar 76. What follows is a passage in which every segment (excluding the two that constitute the chorale quotation in bars 458–61) is related to at least one of the two S-motives in some way, and the type regions duly proliferate: the Alma-incipit attaches itself to $P^{1.7}$ and the octave leap (bars 453–56), the 'malignant' motive gives way to $S^{1.1}$ as a tail (bars 470–71, tuba), and the violins freely counterpoint $S^{1.1}$ in a figure that echoes and rehabilitates the ascending $P^{2.2}$ from the development (bars 463–65). As the movement draws to a close, $S^{1.2}$ and the Alma-incipit draw into an even tighter weave, falling over each other in versions that neutralise and reclaim the leaps and dotted rhythms that were construed as antagonistic in the exposition. In the final bars, $S^{1.2}$ provides itself as an incipit to the Alma motive while the Alma-incipit figure sheds its leap and does the same (see also n. 41). The conflict of the movement is resolved; but after two inner movements that provide extended character portraits of the symphony's poles (A minor hegemony and E♭ major Utopia), the threads are untied and picked up again in the Symphony's gargantuan, and tragic, Finale.[65]

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[63] Monahan, 'Narratives', I, 145; see also '"I Have Tried"', p. 155.
[64] See 'Narratives', I, 145–46. The correspondence between this passage and the vision is also noted by Peter Brown (p. 666) and Del Mar (p. 41).
[65] Samuels suggests that the first movement's sense of resolution is illusive: 'the motivic activity is maintained right up to the double bar line' as forward-driving motives are reified through obsessive repetition and harmonic stabilisation (p. 152). Again, this signals a desire to resolve the symphony's contrasts onto the quilting point of 'tragedy'; to explain away its outwardly positive passages as "not really" positive at all.

4.2 The Finale

Although there are of course striking differences between the two movements (most significantly in their respective codas), as discussed above it can be productive to read the Finale against its first-movement predecessor – a movement which, for Adorno, the Finale 'heightens […] and negates'.[66] This heightening is certainly apparent in the Finale's recycling of motivic stock – the octave leap and the minor third are the most basic elements – to produce the 'asphyxiating' sense of 'inescapable coherence' that gives Monahan his chapter and article title: 93.3% of the movement's segments belong to its largest cross-family type, and this takes the odds of a CDRE-path existing between any two randomly selected segments up to 87.1% (and the odds of a direct C, D, R, or E relationship between any two segments up to 3.3%).[67] Perhaps more important (as acknowledged by Adorno and Constantin Floros) is the reappearance of distinctive timbral and topical areas from the first movement – the march, the chorale, the cowbells and their attendant Utopian distance – as these features bring otherwise subterranean games of motivic correspondence to the unavoidable discursive surface.[68]

We therefore find another structural polarity: the motivic material serves to unify and homogenise, while the affective regions (both those from the first movement, and "internal" gestures such as the hammer-blows or the 'I[ntroductory]-complex' of strings, harp, and celesta) serve to distinguish and delineate.[69] As in the first movement, however, two affective regions carry motivic material that remains relatively independent from the rest of the Finale: the motto and the chorale (which comes to be associated with the hammer-blows through its transformation into the hammer-blow theme). Read this far as tracing two parallel stories (despite the fact that their first appearances are yoked together to form the exposition's transition), their roles here are more closely intertwined: not so much in terms of their points of occurrence on the music's time-line, but in the convergence of the narrative functions that they discharge (they also share a morphological

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[66] *Mahler*, p. 138. This dialectical relationship makes the outer movements an ideal pair to analyse when space does not permit a detailed examination of the entire symphony; I restrict myself to noting here that the melodic-motivic lens, in particular as it relates to form, has been a popular one through which to view the Andante: see Schoenberg, 'Mahler', pp. 460–62; Samuels, pp. 18–63; Darcy; and James Buhler, 'Theme, Thematic Process and Variant Form in the Andante Moderato of Mahler's Sixth Symphony', in *Perspectives on Gustav Mahler*, ed. by Barham, pp. 261–94.
[67] Del Mar (p. 53), de La Grange (*Mahler*, III, 817), and Monahan ('"Inescapable" Coherence', p. 71) identify the octave leap as an important unifying factor both within the Finale and across the other movements; Ratz (p. 37) recognises its prominent "internal" role but does not make its connection to the first movement explicit. Floros (*Symphonies*, p. 184) and de La Grange (*Mahler*, III, 817) also single out the "Alma" motive, while Monahan (ibid.) highlights the minor third and Adorno notes that the A–B–C–A motive of the Finale (e.g. bars 16–22, tuba) retrogrades the A–C–B–A of $P^{1.1}$ ('Centenary Address', p. 105). Monahan first discusses the concept of inescapable coherence on p. 75 of his eponymous article, and the term 'asphyxiating' is Adorno's (*Mahler*, p. 103).
[68] See Adorno, 'Centenary Address', p. 105 and Floros, *Symphonies*, pp. 164–65.
[69] 'I-complex' is Monahan's term, and he reads it as a structural marker delineating the movement's four main blocks from each other (see '"Inescapable" Coherence', pp. 61–63 and below at n. 117).

correspondence in that the chorale begins in the same way that the motto rhythm ends: with three repeated notes of equal duration).[70] The motto is, in Floros's phrase, an 'iron clamp', recurring virtually unchanged throughout the symphony as the antithesis of development; the chorale, having learned precisely what development means for a transitional theme, has come to occupy a similar antagonistic position.[71]

The dialectic of stasis and development dramatised in the first movement (and seemingly resolved by the idealised passage atop the mountain, which managed to project both at once) is thus re-opened in the Finale; it is, however, somewhat suppressed in Monahan's reading of the movement's competing formal forces. He approaches his analysis through the Adornian concept of the novel-symphony, a structure in which '[t]he listener must abandon himself to the flow of the work, from one chapter to the next, as with a story when you do not know how it is going to end'.[72] The concept is a dialectical one, defined against, but remaining dependent on, ossified and pre-determined formal models which 'supply critical points of reference and are thus indirectly constitutive of the work's meaning'.[73] Monahan seeks 'a more integrated view of emplotment' (p. 60) by departing from Adorno's characterisation of sonata form as the static antithesis (variously labelled the Classical, formalistic, architectonic, or dramatic impulse) to novelistic construction: he uses Sonata Theory's descriptions of 'the genre's built-in teleologies' (p. 60) to conclude '[t]hat *the sonata itself* might act as an agent' (p. 92).[74] This lends a sense of dynamism to both of the main characters (i.e. the formal and the novelistic paradigms) in Monahan's plot, but each still behaves antagonistically towards the other: in Adorno's 'single suggestive comment' (p. 55) on totality and individuality quoted above at n. 11, Monahan sees the sonata 'momentarily suffused with agency' (p. 92), defining an image of 'the monolithic work pitted against its own constituent elements' (p. 55).

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[70] Opinions are split regarding how 'yoked together' the two areas in the first movement are. Samuels (p. 144), Del Mar (p. 34), and Peter Brown (p. 662) consider the motto (bars 57–60) to be part of the transition or bridge (Brown even goes as far as to label it *1T* and the chorale *2T*); de La Grange (*Mahler*, III, 821) and Monahan (see n. 25 above) have the transition beginning at bar 61; and Floros (*Symphonies*, p. 166) puts the motto and chorale in two *sui generis* sections (although he suggests that the chorale takes 'the position that is normally occupied by the transition' (ibid., p. 167)). Mahler himself places a rehearsal number (7) at the start of the chorale, but not at the start of the motto. These formal disagreements contextualise Agawu's implication that the first and third chords of the chorale reverse the A major to A minor motion of the motto (Kofi Agawu, 'Prolonged Counterpoint in Mahler', in *Mahler Studies*, ed. by Hefling, pp. 217–47 (p. 227)).
[71] Floros, *Symphonies*, p. 164.
[72] Adorno, 'Centenary Address', p. 87.
[73] Monahan, '"Inescapable" Coherence', p. 57; in the present section, parenthetical page numbers in the text which relate to Monahan refer to this article. Ratz's analysis is also informed by the Adornian formal dialectic discussed here (see p. 35 especially).
[74] The term 'dramatic' in this context is opposed to 'narrativistic' in referring to the temporal structure of plays rather than novels; cf. Maus's pair of articles 'Music as Drama' and 'Music as Narrative'.

This competition partly takes place in the arena of thematic and motivic process. According to Adorno, the 'technical formula' on which the novel-symphony depends is the *variant* technique, a device which allows the inner motivic materials of fixed, gestalt-like themes to shift like components of a mobile sculpture, 'revis[ing] their nuances, their lighting' without damaging the identity of the theme as a whole.[75] The inescapable coherence of this movement's motivic substance (in particular the P-theme, 'a ragtag group of undistinguished motives and fanfares' for Monahan (p. 76)) is therefore read by Monahan as an anti-novelistic force, one which 'collude[s] with the sonata and its A-minor agenda' (p. 75) in bringing about total integration. The eventual 'Failure of the Novel-Symphony' alluded to in Monahan's title is clinched in the movement's closing gesture, stripped of all melody and reinforcing a single static harmony; but, following Samuels's interpretation, to view this moment as a victory for sonata form is surely mistaken. Monahan implicitly agrees in a later article, quoting Adorno on this movement's dramatisation of 'the end of the symphonic sonata' (see n. 17 above) and agreeing that after the 'self-destructive Finale' of the Sixth, the sonata plot paradigm 'loses much of its explanatory power' with respect to Mahler's *oeuvre*.[76] The sonata, then, has over-reached itself (Samuels speaks of 'the suicide of the symphony'): its determination to impose homogeneity on the movement's novelistic materials has resulted in a negation of the very temporality and dynamism which allowed it to be read as an agent in the first place (and which Adorno acknowledges: 'epic composition was never the mere antithesis of the dramatic but also close to it, like the novel, in its onward momentum, its tensions and explosions').[77] This destruction of both sonata and novelistic paradigms creates a power vacuum into which the static third agents – the non-symphonic chorale and the non-novelistic (i.e. unchanging) motto – rush: when the critic Robert Hirschfeld attacks Mahler's use of the hammer by commenting that '[s]peakers whose words fail them at the decisive moment beat the table with their fists', he inadvertently hints at the twin failures of the movement's discursive frameworks (brought about, in part, by the agency of the hammer).[78] The mutual origin of the chorale and the

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[75] Adorno, *Mahler*, pp. 86, 87. Buhler uses the variant technique to orient his formal analysis of the symphony's Andante (see 'Theme, Thematic Process and Variant Form').

[76] 'Success and Failure', p. 54; he makes a similar point at the end of his '"Inescapable" Coherence' thesis chapter (see 'Narratives', I, 286).

[77] Adorno, *Mahler*, p. 97; Samuels's fifth chapter (pp. 133–65) is entitled 'Musical Narrative and the Suicide of the Symphony'.

[78] Robert Hirschfeld, '[Review of the Vienna Performance of Mahler's Sixth Symphony]', *Wiener Abendpost*, 10 January 1907, quoted in *Mahler: A Documentary Study*, ed. by Kurt Blaukopf with contributions by Zoltan Roman, trans. by Paul Baker and others (London: Thames and Hudson, 1976), p. 246. Hirschfeld, 'that remarkably astute but wholly ambivalent contemporary critic of Mahler' (Buhler, p. 264), has something of a reputation for writing 'with more perspicacity than he knew' (Julian Johnson, 'Mahler and the Idea of Nature', in *Perspectives on Gustav Mahler*, ed. by Barham, pp. 23–36 (p. 28)).

motto in the liminal space of the transition rather than as main (P or S) characters in the sonata plot therefore takes on added significance in this context.[79]

The chorale makes its first Finale appearance (followed by the motto's second) in bar 49; recalling the first movement, it introduces four new source families as two-bar phrases which are repeated and varied to make up a second half of equal length.[80] Contrasted with the 'vacuum clouded only with elementary particles' that precedes it, the effect of this regularity is striking: but while for de La Grange (and Adorno), 'this hostile, reactionary, and unyielding element unquestionably has a negative meaning and a negative role', for Monahan, it is a Freudian attempt to 'repeat actions as a means of mastering the past' (p. 71), 'a rebeginning, a new start of the sort necessary to find the "ideal ending"' (p. 73).[81] Its attempt to establish an alternative C minor sonata is considered by Monahan to be one of the movement's two 'broadly defined and largely independent forces of resistance against the sonata's A-minor hegemony' (p. 81; the other is S2, beginning at bar 205); but as suggested above and argued below, this characterisation forms only part of the movement's complex and shifting network of oppositions and allegiances.

After a foretaste of the chorale's later incarnation as the hammer-blow theme in bars 82–85 (followed by a swarm of what Monahan calls 'G-motives' (see pp. 79–80): generic segments related to the motto rhythm that are not associated with any particular sonata zone and tend to bring about liquidation and collapse), the chorale next appears in the transitional section of the thematic exposition.[82] Here, it has not only taken on the distended shape of the G2-motives (the 'malignant' chains of octave leaps first heard just prior to the chorale in bars 42–43), but also appears in a

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[79] Ratz notes the transition's importance in his analysis: being 'twice as long' as the first subject, he argues that '[o]ne might equally well call it the second main subject group' (p. 39).
[80] As discussed at n. 29 above, the first movement's four chorale phrases initiate only two families: the third segment is derived from the first, and the fourth is non-motivic.
[81] Monahan, '"Inescapable" Coherence', p. 68 'vacuum'; de La Grange, *Mahler*, III, 833 'hostile'. Adorno considers the chorale 'dark' (in de La Grange's translation; see *Mahler*, III, 833, n. 149) or 'sombre' (in Jephcott's; see Adorno, *Mahler*, p. 97): the original is 'düstere' (see Theodor W. Adorno, *Gesammelte Schriften*, ed. by Rolf Tiedemann and others, 20 vols (Frankfurt a.M.: Suhrkamp, 1970–86), XIII: *Die musikalischen Monographien*, ed. by Gretel Adorno and Rolf Tiedemann (1971), p. 245).
[82] The Finale's sectional boundaries are notoriously ambiguous, as discussed most extensively by Samuels (pp. 64–90; see also n. 117 below) and also by Peter Brown (pp. 671–73). My own interpretation (which mostly follows Monahan's as given in '"Inescapable" Coherence', p. 62) takes bars 1–113 to be the motivic exposition (laying out the movement's principal melodic material) and bars 114–228 to be the thematic exposition (establishing the complete themes and tonal areas that set the sonata in motion; the idea that the movement enters and exits "sonata space" follows Monahan, '"Inescapable" Coherence', p. 65). While slow generative introductions are nothing new, witnessing the very derivation and assembly of supposedly gestalt-like themes takes on added significance in the context of the 'failure of the novel-symphony' and lends the first section a truly expository – and not merely introductory – function. Other writers who place bars 1–113 in the exposition are Floros (*Symphonies*, p. 182) and Hans Redlich (Hans Ferdinand Redlich, 'Analysis', in Gustav Mahler, *Symphony VI: A Minor* [Score], ed. by Hans Ferdinand Redlich (London: Eulenburg, 1968), pp. viii–xxiii (p. xxii)), although almost all analyses acknowledge these bars' generative role.

shuffled order: bars 141–44 are based on the chorale's second quarter (bars 53–56) and further stretched in bars 145–148, before bars 149–57 vary the chorale's second half (bars 57–64) and 160–67 its first (bars 49–56). The third quarter is then treated sequentially (bars 168–71) before the doubly-stretched second quarter returns and finally snaps into a whirlwind of G-motives (see Figure 4.11).

The segments caught in this whirlwind suggest a suspicion which the S-themes later confirm: that the label 'G' is not entirely adequate since, while retaining a degree of narrative distinctness, these motives easily shade into segments found within the P- and S-themes themselves (de La Grange even goes as far as to consider the transition 'a development within the exposition', an idea which exists in a certain tension with the chorale's static nature).[83] This is discussed in further detail below, but for present purposes it suffices to note that the incipit of S2 is closely related to the octave-leaping G2 (which has a tendency to appear simultaneously with the chorale – or hammer-blow theme, as it becomes). This parallel is strengthened by chorale-type gestures in the S2 theme leading to ultimately disrupted cadences (bars 216 and 299–300) which set the template for the first hammer-blow itself (bar 336), preceded by a passage in which a rather distant relative of the chorale's first quarter (oboes, bars 319–22) gets stuck going round in liquidated circles (bars 322–27).

The chorale's liquidation here and shuffled distension in the thematic exposition are what the hammer-blow and its associated theme seek to reverse. The theme restratifies the motivic material into two blocks: the first consists of the first two segments of the chorale (still not in their original form, but the leap of a seventh in bar 83 has become a more manageable fifth) accompanied by a G2-chain and dotted, P-reminiscent strings (bars 336–43); the second consists of double-speed G-motives accompanied by dotted thirds and rushing semiquavers (bars 344–47). Under encouragement from the shorter rhythmic values in the strings, the chorale and G2 motives return to their original notated rhythms (although still in a higher tempo) for the repeat of the first block, while the repeat of the second morphs the G-motives into the shape of the chorale's second segment; these are batted around the orchestra before being rederived, by bar 357, into the emancipatory fanfare from bar 217. This time the bid succeeds (last time it careered into the return of the I-complex) and leads to the Finale's first 'Utopian vision' (Monahan's term; see p. 62).

What, exactly, is this a vision of? In tonal terms, Monahan reads its glimpse of A major as an idealised projection of the sonata's outcome that is tenable only as long as the G-motives are kept at bay; when they begin to reappear in bar 372, he argues, the tonality slips and C minor regains control (p. 84). This is the key of the chorale's alternative sonata, and while the chorale and the S-themes share a common enemy in the A minor P-theme, '[f]or the chorale, whose aim is to control an orderly and efficient sonata, the S-themes' formally disruptive short-cuts to

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[83] See de La Grange, *Mahler*, III, 834.

transcendence are unwelcome' (p. 82; recall also that S made an enemy of TR by subsuming it in the first-movement recapitulation). I find this reading problematic for two main reasons, the most immediate of which is the motivic material of the Utopian vision: there are unambiguous G2 quotations in bars 368 (horns) and 371 (bass instruments), and the first phrase (bars 364–66) is related to the first appearances of G1 (if such a stable category can be identified) in bars 86ff. (Ratz (p. 44) even considers G1 to be 'worked out jointly' with the second subject here). Moreover, it seems odd to be advocating a chorale as the instigator of a sonata (alternative or otherwise) when its very effect (in the first movement especially) is one of non-symphonic disjuncture or stasis; Monahan even draws attention to the last-movement chorale as 'a voice from beyond the Sixth Symphony entirely' (p. 73, n. 70) in pointing out that it is a *Wunderhorn* quotation (see also above at n. 4).

The chorale topic, redemptive key, merger with the "generic" (i.e. less symphonic-teleological) motives, and S1-quotations suggest a reading which looks back to the Utopian music of the first movement. There the virtues of stasis were extolled: even though the plot proposed above was one of the chorale's search for developmental capacity, the central Utopian episode sought to bestow this through free, almost heterophonous, variation, drifting some distance from its start to its end in a way uninfluenced by the goal-directed pressures of symphonic development. This kind of stasis is the opposite face to that imposed by the 'iron clamp' of the motto (hence the motto quotations in the first-movement *Klangfläche*, bars 208–09 and 213–14), but both kinds work to undermine the authority of the sonata.[84] The dynamic interactions between the symphony's three main characters function on zones of conflict and agreement: P-as-sonata-agent seeks to impose order on S-as-novelistic-freedom, but the latter is ultimately dependent on the former for its generic desires (i.e. resolution in the tonic major) and so both resist TR/motto-as-non-symphonic-stasis. At the same time, both TR and P tend towards homogeneity and unity (leading the latter to negate itself) while TR and S tend towards the vision of "another way" (which would, of course, strip S of its animating force). To take just one of these zones of conflict, stasis is presented both as a liberating force for subjectivity in a world driven by the demands of a hegemonic system, and as simply an alternative (and ultimately even more suffocating) system itself; the refusal to resolve these two pictures into one respects the symphony's polarity and, in its denial of easy answers, its real tragedy.[85]

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[84] Redlich links, as 'sign-posts' of the symphony's 'philosophical message' (p. viii), the cowbells, deep bells, and motto: they are all 'fundamental stationary sounds, the first two of which are off pitch while all three are impervious to thematic or rhythmic development' (p. ix).
[85] In Almén's terminology, this denial might make Mahler's Sixth his 'Ironic' symphony; see 'Narrative Archetypes', p. 18 and Chapter 2, n. 145 above. In an article examining his four narrative archetypes (or 'mythoi') in relation to Mahler's early symphonies, Almén chooses the second and third movements of the Second Symphony as examples of an ironic mythos (see 'The Sacrificed Hero: Creative Mythopoesis in Mahler's *Wunderhorn* Symphonies', in *Approaches to Meaning*

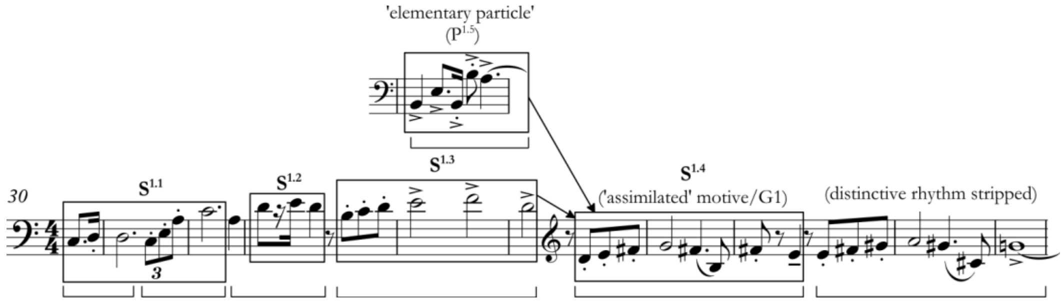

Figure 4.9: The forging of theme S1 and the origin of G1 in the Finale, bars 30–39. Monahan's segmentation is shown using boxes (with corresponding labels) and my own using brackets.[86]

Within this interpretative context we may revisit the so-called 'G-motives'. The motivic exposition immediately sets up a contrast between two source families: the sombre dotted minor third cell (to which the single occurrence of the octave-leap family in bar 16 may be appended), and the more rhapsodic clarinet gesture in bar 19. These call across the chasm to each other until the deep bells make their first appearance and theme S1 begins to be forged; during this, a simultaneous 'elementary particle' (Monahan's term; see n. 81 above) drifts into view in the bassoon and bass clarinet (bar 34; see Figure 4.9). The gravitational pull of the more massive S1 fuses the two gestures of bar 34 into the gesture beginning at bar 35 (Monahan's $S^{1.4}$), which then has its final distinctive two-note rhythm stripped away in bars 37–39. In response to this display of assimilation, the oboe mocks the malevolent dotted celesta and tuba in diminution before the horns retaliate by fusing the octave and minor third cells together to create what was referred to above as G2.

Following the chorale, the elementary particle makes another bid to become a theme in its own right, developing into an ascending fanfare (bars 69–73) before being answered (or 'opposed', according to Ratz (p. 38)) by its assimilated version, the prefigured hammer-blow theme, and a writhing mass of melodically restricted assimilated motives (these are the first appearances of G1 referred to above).[87] The final section of the motivic exposition is a gathering of momentum for the P-theme's march: reiteration of the minor third motive in bars 103–07 creates a homogenous aural background that gathers energy in rhythmic diminution in bars 108–09. 'Rhapsodic' motives fall over each other in an attempt to escape what they have already realised, confirmed in a

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in Music, ed. by Byron Almén and Edward Pearsall (Bloomington: Indiana University Press, 2006), pp. 135–69 (pp. 157–60)).

[86] Monahan's labels for this and the following figure are extrapolated from Example 2 in '"Inescapable" Coherence', p. 71.

[87] Ratz also sees G1, a hybrid of P and S motives, taking on a 'pronounced transitional function' (p. 40); this reactivates the questions of transition identity discussed in Section 4.1 and supports a reading of G1 as a homogenising force. Del Mar similarly sees the motives in these bars as 'composite but salient' (p. 54), and both he and Ratz (pp. 40–41) remark on the importance of rhythm in binding together this disparate category (cf. Chapter 2 at n. 23 above; Ratz even makes direct reference to Schoenberg).

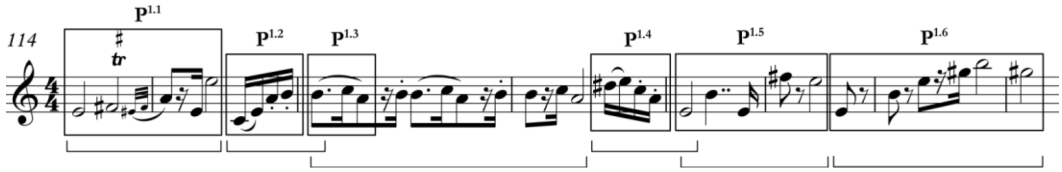

Figure 4.10: The Finale's P-theme in Monahan's segmentation (boxes) and my own (brackets), bars 114–21.

dramatised moment of epiphany in bars 112–14: the $(+2, -3)$ pattern of the hegemonic minor third has been embedded in the descending form of their gesture of individualism since bar 25, and they are finally assimilated just as A minor is confirmed for the start of the thematic exposition. Parading its trophies, the P-theme (Figure 4.10) unfolds a minor third/dotted rhythm segment ($P^{1.1}$), the ascending ($P^{1.2}$) and descending ($P^{1.4}$) forms of the rhapsodic motive, the homogenous background that allowed their capture ($P^{1.3}$), and not only the 'elementary particle' itself ($P^{1.5}$), but also its fanfare-like attempt at initiating its own theme ($P^{1.6}$).[88]

The above plot demonstrates that motives G1, G2, $P^{1.5}$, and $S^{1.4}$, and all the segments related to them, do indeed work across thematic zones, but that their meanings are not entirely 'generic' and interchangeable: they are related not only in a static, paradigmatic, morphological sense, but also share a more concrete common backstory that can illuminate plot processes later in the movement. Take, for example, the 'whirlwind' (referred to above) that appears after the distended and shuffled chorale/hammer-blow theme; this is taken by both Del Mar and Floros to constitute the transition proper (bars 176–90; Figure 4.11).[89] The elementary particle ($P^{1.5}$) again tries to bring some melodic coherence to the liquidated mass of semiquavers, its initial melodic interval widening from a fifth (bar 180) to a sixth (bar 182). At the same time, the rhythmically similar but kinetically opposed G1/$S^{1.4}$, which tends towards stepwise descent rather than leaping ascent, outlines a descending scale in the trumpets from E5 to F3 over 8 bars. This drags $P^{1.5}$ down into a G2 chain (recall that this is the 'malignant' motive that combines the octave leap and minor third, the two strongest symbols of P-hegemony); but the distinctive ♪𝄾𝅗𝅥. tail that was originally the property of $P^{1.5}$ passes into this voice from the G1-scale and is ultimately translated from a descending strong–weak figure to an ascending weak–strong one in the first segment of S1. Staggering on through identity-stripping bombardment, $P^{1.5}$ thereby finally instigates a new theme thanks to the very motive that dissolved it into a G2 chain (its descendant G1) – although is victory is somewhat Pyrrhic given that this not only happens in a transition (the illusive freedom of which

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88 Figure 4.10 shows that my own segmentation of the Finale's P-theme (in particular, my third segment) does not respect the kind of internal derivation pattern that Monahan was critiqued for overlooking in Section 4.1. I defend this position in this particular case by arguing that the wider consequences are less significant than those arising from the segmentation of $P^{1.1}$ in the first movement, and also that it is analytically suggestive to view this gesture as a roll of aural wallpaper (rather than a modular progression chain of $P^{1.3}$-motives) that can be cut off to various lengths during the movement.

89 Del Mar, p. 55; Floros, *Symphonies*, p. 183.

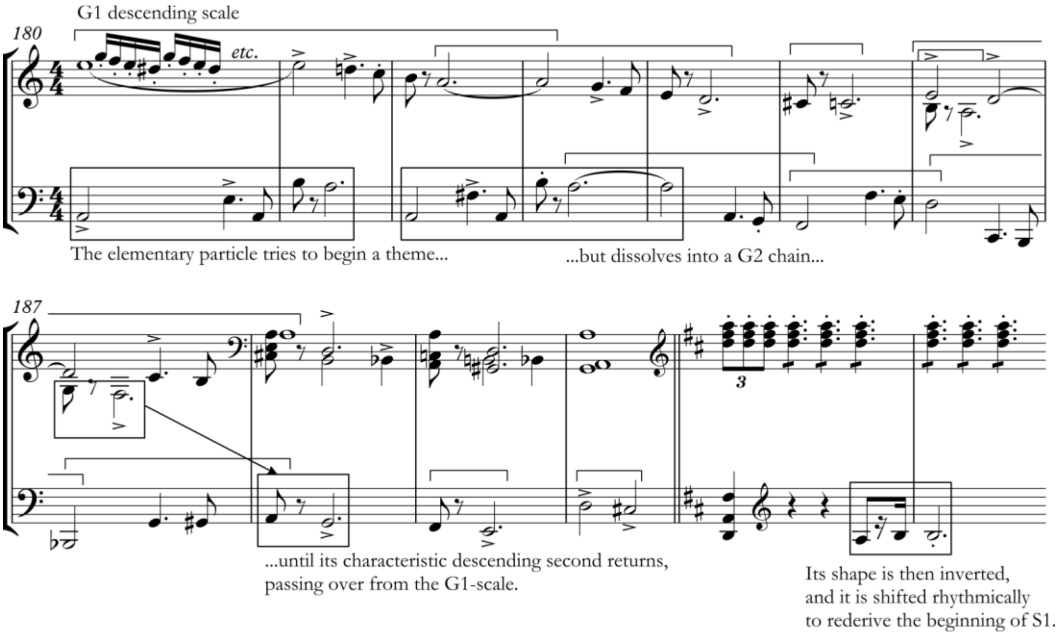

Figure 4.11: The 'whirlwind' of G-motives after the collapse of the chorale in the Finale, bars 180–92.
All segments in the passage are marked with brackets, while boxes pick out key stages in the motivic process as discussed in the text.

is discussed in Section 4.1), but also merely rederives the only P- or S-theme already heard in a complete form in the motivic exposition.

The thematic inclinations of $P^{1.5}$ are entirely absent from the first Utopian vision which, as argued above, makes a virtue of stasis and flaunts its unificatory credentials through the gentle stratified repetitions of a three-note (two-pitch) fragment. To recap the historical thematic processes embedded in this tiny phrase (as shown in Figure 4.12): it is an augmented version of the emancipatory fanfare first heard in bar 217, understood there as a derivative of $S^{1.2}$ which itself varies the opening of theme S1 (traceable back to the dotted minor-third cell in the motivic exposition, and rederived, as shown in Figure 4.11, from $P^{1.5}$ in the thematic exposition). In the Utopian vision, it is figured more immediately as the outcome of G2's adoption of the shape of the second segment of the chorale in the repetition of the hammer-blow theme; it then shades back into a G1 motive in bar 372. A hint of the complexity of its resonances is given in bars 366–69, shown at the bottom of Figure 4.12. The $S^{1.3}$ motive, heard simultaneously with, and ultimately incorporating, $P^{1.5}$ in the motivic exposition, is here accompanied by the 'malignant' G2 in the horns. The adoption of the three-note fragment as a tail to $S^{1.3}$ is continued by the final three notes of G2: this gesture, fractured across two octaves, reconstitutes the dotted "G2" version of the chorale's second segment from which the three-note fragment was derived.

Towards the end of the vision, a snatch of the I-complex's descending broken chord (itself related to the first movement's Alma theme) is heard in the first violins in bar 374. When the Utopian space is opened again in bar 458, it is the varied I-complex from the opening of the development (bar 229) that provides its thematic material, prompting Monahan to remark that 'the

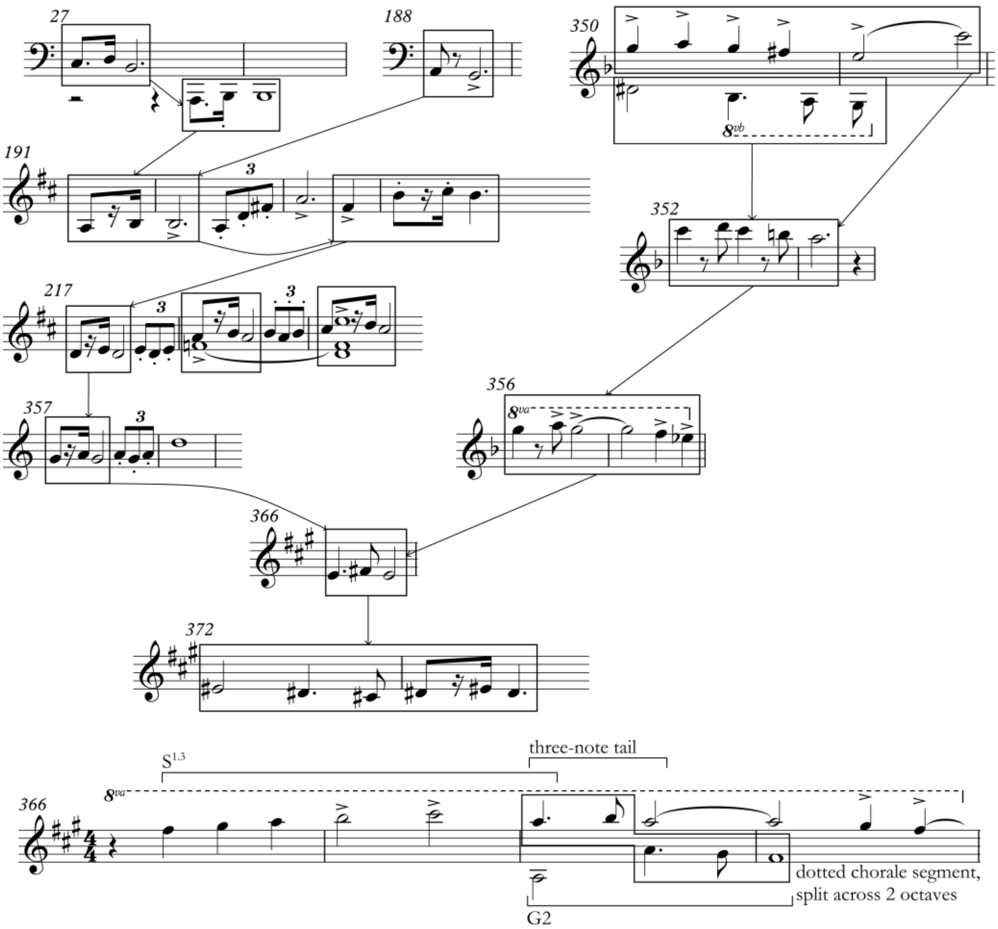

Figure 4.12: The complex resonances of the three-note motive from the Finale's first Utopian vision.

relation between thematic materials and the character-impulses they bear is more fluid than we might have assumed' (p. 86); here the vision 'insists that if the tonic major is to become a reality, the I-complex – which harbours the major-minor motto – must be transfigured' (ibid.).[90] Elements of the first Utopian vision ($S^{1.3}$ and the three-note fragment) begin to surface amidst the undulating scales of the I-complex until a *forte* statement of the three-note motive in the flutes and oboes serves as a reminder of its multivalent and therefore fickle nature: the music suddenly darkens as the writhing G1-motives from the motivic exposition return to lead into the second hammer-blow (compare bars 469–78 with bars 86–95, where they led into the motto).

The music following this hammer-blow at first unfolds in a similar way to that following its predecessor: the first major point of divergence is that the second block is truncated from four bars to two. The G2-motives having been so curtailed (the principal melodic line even loses its characteristic octave leap: compare the woodwind in bars 487–88 to the F trumpets in bars 344–45), the second chorale segment does not take on a dotted rhythm as expected in bar 493. A

[90] Floros similarly takes the I-theme as an example of 'how one and the same theme can totally change its character' (*Symphonies*, p. 184).

furious triple statement restores the octave leap to the motivic foreground (bars 497–99) while the bass instruments chain together G2-motives in diminution and the horns state them at their original rhythmic value, dragging the chorale segment down until it finally takes on the dotted rhythm again in bar 504. Alternating G2 and dotted-chorale motives tumble into the recapitulation, their constituent leaps and dotted figures not so much liquidating as being thrown off as collision debris, until the I-complex (back in its non-transfigured form) once more applies the iron clamp of the motto. At the very moment in the symphonic structure that Adorno identifies as containing its greatest artifice, the moment of recapitulation's tautologous affirmation of 'a celebratory "That is it" in repeating what has already existed in any case', the sonata order (represented by the non-chorale materials) makes a desperate attempt to remake the chorale in its own image; but it is clamped on one side by the hammer, and on the other by the I-complex (which Del Mar sees as going through the motions of recapitulation, entering 'willy-nilly […] regardless of the tonal conflict').[91]

The restatement of thematic material is thrown off course by the fact that A minor is too weak to wrest control of the motto (as it did in bar 9), which is now heard in C minor (the key of the chorale) for the first time. (Tracking the motto to this point, we hear it on A in bar 9, on G following the chorale in bar 65, then the triad alone on C in bars 96, 395, and 401).[92] Monahan reads this as C minor 'abandon[ing] any image of a finale that "might have been" and mak[ing] its last bid for authority simply by imitating its A-minor rival' (p. 86); in my own reading, this gesture is a bald statement of alliance between the motto and the chorale, both representing stasis. But, of course, change of key is still change and so is anathema to the motto, which mostly reinstates the pitch level of A for its seven further appearances (at bars 622, 668, 686, 754, 783, 798, and 820).[93]

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[91] Adorno, *Mahler*, p. 63 (see also Chapter 2, n. 102); Del Mar, p. 60. For more on the tonal conflict of this passage, see John Williamson, 'Dissonance Treatment and Middleground Prolongations in Mahler's Later Music', in *Mahler Studies*, ed. by Hefling, pp. 248–70 (pp. 253–54).
[92] In his inventory of motto appearances, Samuels (p. 160) misses the one in bar 96 and leaves that in bar 395 implicit (through analogy with bar 401, which he does list, and which almost exactly repeats bar 395). He also includes an extra appearance of the triad, in G, at bar 338, but I do not consider this to be a Definition 1-commensurate instance since the chord is in first inversion and its rhythm does not match the hypermeter, coinciding as it does with the third and fourth (rather than first and third) notes of the hammer-blow theme. Monahan's annotated short score ('Narratives', II, 490–532) agrees almost exactly with my listing (the only disagreement is at bar 798; see next note).
[93] Samuels (p. 160) adds an extra appearance (on A) at bar 696, but again this does not coincide with the hypermeter; it is omitted from both mine and Monahan's inventories. The occurrences at bars 622 and 754 are listed as '[r]hythm only' and given no pitch level by Samuels; while the triad is missing in both cases, the latter simultaneously reinforces the tonic and dominant of A in the timpani, and the former the dominant in an ecstatic (and therefore anathemic) passage of prevailing A tonality. The occurrence at bar 798 is listed by neither Samuels nor Monahan and actually occurs on D: while this is the key of the thwarted S-themes, its barely audible appearance (lacking the fifth of the triad or the rhythm) has very little effect on the key or the mood of the coda. It might be read as a final subtle reminder of the positive side of stasis.

It is perhaps the greatest irony of this symphony that its furthest-reaching and most obvious narrative-teleological act of symbolism – major–minor in bar 57 of the first movement becomes minor–minor in bar 820 of the last – is carried out by the very gesture that represents stasis, atemporality, and homogeneity.[94]

One of the principal axioms of the novel-symphony is that 'musical time, unlike architecture, permits no simple relationships of symmetry. […] What happens must always take specific account of what happened before'.[95] The I-complex that begins the recapitulation overwrites this, skipping back over its transfiguration in the development to its expositional form: but as, in its 'dissolution field', C minor 'takes on [A minor's] entropic inertia', elements of S (as avatars of the novel-symphony) and the transfigured I-complex (as an unavoidable memory) disrupt the progress of a near-exact motivic recapitulation.[96] Of the 45 segments in bars 520–74, 35 (including the first 30 in the section) are derived from bars 3–40 (including thematic repetitions of sizes 29, 3, and 2); 4 are versions of the Alma motive ($S^{1.5}$ in the context of this movement); 3 repeat a version of the transfigured I-theme (bars 566–68); and the remaining 3 echo a phrase from the development's dissolution field (compare bars 561–65 with bars 250–53 and 278–80; the Alma motive is also prominent in this section).[97]

As the novel paradigm gains strength, theme S1 returns to undergo a similar kind of "static development" to that enjoyed by S2 in the Utopian episode of the first movement. Although not quite as improvisatory or contrapuntally limpid, there is a similar freedom of motivic development, proliferation of quasi-heterophonic entries, and lightness of scoring (including the solo violin); Del Mar observes that here, 'suddenly the air has cleared' (p. 61). The augmented $S^{1.3}$ that follows in bar 601 finally fulfils the versions at bars 93 and 476 (which were cut off by the motto and hammer respectively) in a passage reminiscent of the lead-in to the first movement apotheosis at bars 446–48 (compare the timpani in IV, bar 601 and I, bar 449).[98] This is followed by a passage which seems to relive that movement's closing peroration in its mood, key, and ecstatic motivic invention: Monahan notes that it 'sweep[s] up all of the movement's thematic characters in a manic A-major

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[94] In addition, two of the final three segments – the single pitch and the second iteration of the motto rhythm – actually begin new type regions.
[95] Adorno, *Mahler*, p. 52.
[96] Monahan, '"Inescapable" Coherence', p. 86 'entropic inertia'. The term 'dissolution field' is Samuels's (p. 78) and Monahan's (p. 63, n. 46) preferred translation of Adorno's *Auflösungsfeld*, usually rendered 'disintegration field' by Jephcott (e.g. *Mahler*, p. 99); however, Jephcott uses 'field of dissolution' and 'dissolution field' on p. 98 and p. 159 respectively (cf. Adorno, *Gesammelte Schriften*, XIII, 245 and 302). It refers to those 'inert and inchoate expanses' (Monahan, '"Inescapable" Coherence', p. 63, n. 46) that follow the I-complex in this movement; Floros labels those same expanses 'music from far away' (*Symphonies*, p. 180).
[97] The fortunes of the Alma motive in the Finale are not tracked exhaustively here, but provide a suggestive avenue for further exploration.
[98] Only Del Mar seems to note this correspondence (p. 62), but he does not extend the parallel backwards to the S1 episode or forwards to the ecstatic section.

whirlwind' (p. 88), 20 of the 29 type regions used in bars 610–41 being new. Particularly significant are $S^{1.4}$ and its echoes of the first movement's $P^{1.7}$ (bars 618–21); the chorale/G2/motto quotation in bars 622–27 (paralleling, but certainly not as submerged as, the chorale in the first movement, bar 458); two appearances of the Alma motive with its incipit (bars 630–34); and, in bar 635, the first appearance of an 'elementary particle' type region since its attempt to salvage the Utopian vision at bar 383 after crashing into the I-complex in bars 224–28.[99] Getting carried away in the excitement and resuming its drive to begin a theme of its own, it has forgotten that it (and its ascending fanfare descendant, bars 636–41) is already part of a theme – the P-theme – and so it becomes the weak link that allows the A minor sonata to finally perform its recapitulatory function. And 'perform' is an apposite word: Monahan notes that 'P actually progresses toward an increasingly exact reprise as it unfolds; it does not just embody repetition mindlessly, it *achieves* it, even flaunts it' (p. 88) in its 'undoing of novelistic time' (ibid.). Of the 44 segments in bars 642–66, 34 (including the last 25) are derived from bars 114–37 (including themes of sizes 31 and 2), with 7 of the remaining 10 being derived within the section itself.

In Monahan's reading, this is a moment of terrible victory for P as it establishes an 'architectonic order' and 'tonal singularity' (p. 88) from which the novelistic materials can make no recovery. On the contrary, I argue that this is the beginning of the end for P, the point at which the sonata overreaches and ultimately collapses into itself, clearing the field for the novelistic and static paradigms. The key event is, once more, the transition (bar 668ff.), in which the TR and S1 themes are 'den[ied] […] even the dignity of an autonomous reprise', 'paraded *en masse* like prisoners of war'.[100] This action is one that is ultimately antithetical to sonata form, which rests on the functional, sectional, and temporal differentiation of its themes; P, having 'undo[ne] novelistic time' in the previous section, now undoes its own sonata temporality ('[t]he recapitulation becomes an apparition', in Adorno's words), opening the way for the black hole of the motto to collapse the form into a singularity.[101] The initial time-warping effects of this are demonstrated by the shifting motivic layers in the first four bars (see Figure 4.13): when compared to bars 139–42, the dotted bass pattern and chorale/G2 figure are in the same place (although the latter has been inverted and the pitch pattern of the former shifted forward by a beat), while the repeated trombone triads and

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99 The fusion of $P^{1.5}$ and $S^{1.3}$ into $S^{1.4}$ is the only motivic element of bars 1–38 missing from bars 520–66; it marks the point at which the recapitulation goes awry (although $S^{1.4}$ and the start of its following segment are heard, in augmentation, in bars 561–66).
100 Monahan, '"Inescapable" Coherence', p. 88. Del Mar advances a different reading: that the S1 motives spill over as a result of the previous passage's unstoppable 'momentum' (p. 62). He still notes, however, the close correspondence between the recapitulation of P and its parallel passage in the exposition.
101 Adorno, *Mahler*, p. 93.

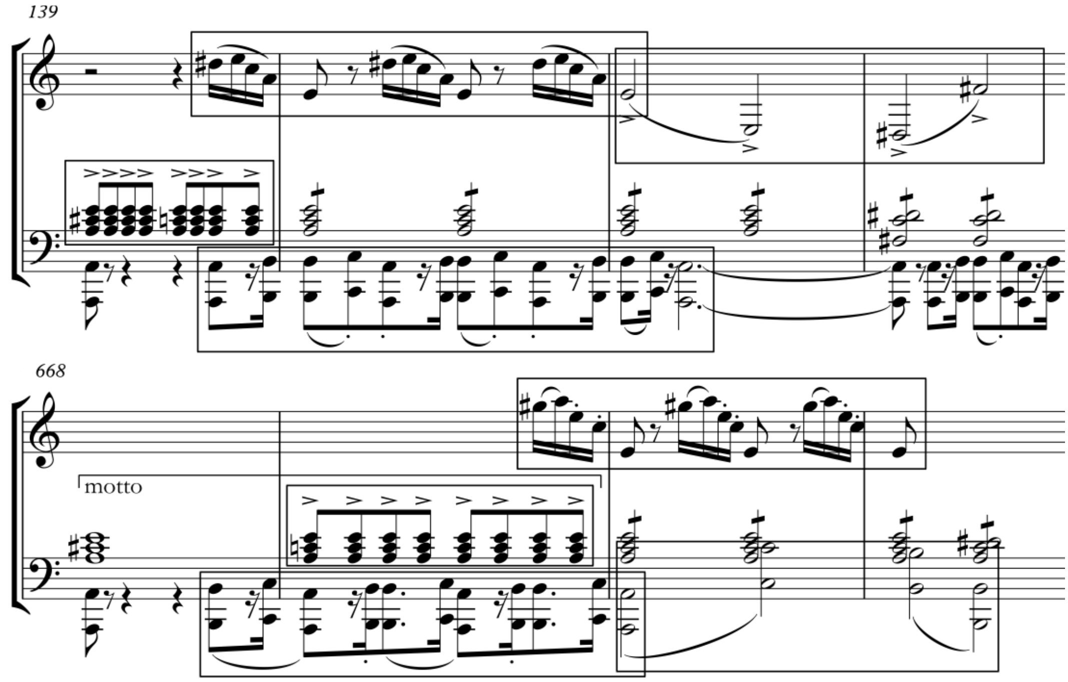

Figure 4.13: Shifting motivic layers in bars 139–42 and 668–71 of the Finale.
Some segments from the second passage have been omitted for the sake of clarity.

rhapsodic violin motives have been moved later by one bar to coincide with the motto's minor triad; this lends it, in Samuels's phrase, an 'emphasise[d] corporeality'.[102]

Aside from the omission of the first 'distended' chorale, which is overwritten by S1 (compare bars 674–77 to 145–48), the transitional sections in the exposition and recapitulation largely bear bar-by-bar comparison: bars 678–85 are based on bars 149–56 (albeit with "Alma" fragments replacing the accompanying octave leaps, and the chorale's final segment reverting to its descending bar 62 form rather than the optimistic bar 154 form), while bars 688–705 even more closely retrace bars 160–77 (25 of the 38 segments in the latter are derived from distinct segments of the former's 27, with a further 9 in the latter being derived within the section itself).[103] The G2-motives and dotted minor thirds which link the two halves of the chorale in bars 157–59 are replaced by another motto in bars 686–87, and this functions as a temporal short-circuit back to the previous motto shown in Figure 4.13 (bars 668–70): an almost exact repetition, it retains the violin semiquavers (which are *not* present in bars 157–59) and again leads into the inverted chorale incipit (which *is* present, as a G2-like bass, in bar 160; the source of the inverted chorale after the motto at

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[102] Samuels, p. 162. In our motto inventories, Samuels (p. 160), Monahan ('Narratives', II, 496), and I do not list the echo of the motto in the trombone triads' first appearance at bar 139.
[103] These observations presuppose an almost Adornian 'variant' understanding since a derivation relationship does not necessarily imply morphological identity. It is natural to compare segments in this section with segments in the exposition's transition (a reading which Samuels argues is 'reliant on a formal model fixed in advance' in its 'suppression' of the 'arguably prior' theme S1 (p. 76)), and similar thematic groupings can be found even if individual melodic shapes (to say nothing of orchestration and harmony) change: morphological identity alone, as argued in Chapter 2, is unable to give a full picture of a piece's motivic processes.

bar 670 is therefore revealed after the motto at bar 688). Two distinct passages in the exposition's time-line (bars 139–42 and 157–59) are thereby implicitly fused in the recapitulation (bars 668–71 and 686–8), anti-novelistically revoking everything in between them and tightening the clamp around the symphony's temporality.[104] A similar glitch is projected by the tragic detail of the completely unnecessary bar 667: after a bar-by-bar recounting of the primary theme up to bar 666 (cf. bar 138), bar 667 inserts itself just before the transition (from bar 668/139) as a nod towards recapitulatory recomposition. But it does not go anywhere: it uses the same motivic material and drives towards the same key as the previous bar, and then is cut off by the motto before the transition proceeds, once again, via bar-by-bar repetition.

When the chorale finally snaps, the whirlwind of G-motives from the exposition (including the rederivation of $S^{1.1}$ from $P^{1.5}$) is replaced by something even more catastrophic. Octave leaps from G2 and the distended chorale (plus the chorale's second segment in bars 710–11) give way to triumphant, chorale-like G-motives (bars 712–15) before the sonata flatly challenges the second motto's recapitulatory excisions by hyperbolically asserting the missing bars. But here they liquidate down to the symphony's 'submotivic *Urformen* […], the octave leap and the $\hat{2}$–$\hat{3}$–$\hat{1}$ minor-third cell'; the passage that follows (720–24) is devoid of motivic content completely, losing even the repeated $\hat{2}$–$\hat{3}$–$\hat{1}$ semiquavers that threaded through the similar section in the development (bars 385–94).[105] The octave leap and the minor third (and their combination in G2) are the principal icons of sonata homogeneity: in the Schoenberg–Reti tradition, submotivic features such as these are the means by which the surface contrasts and narratives of sonata form are bound together into an organic whole. But here, after one of the sonata's most ingenious unifying ideas (that TR and S1 can be controlled through almost-simultaneous recapitulation), the tautologous conclusions of such a reductive quest for uniformity make themselves clear as the topmost branches of the great motivic tree that was rooted at the beginning of the symphony are cut back or grafted onto one another. Even the celebratory music in bars 712–15 makes use of motivic material that is by now so

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[104] Samuels sees a similar process at work in the way that the I-complex and hammer-blow are 'shared out between the double bar lines at bar 229 and bar 336, as if deliberately to complicate or frustrate the reading of the form' (p. 88). On a grander scale, Adorno notes that in the development '[t]he great march is installed between the concrete piers of the [chorale]', again invoking the idea of a static clamp around a more dynamic section (*Mahler*, p. 100); more generally, he sees the development divided into four by gestures which rein in excessive motivic working (these are the hammer-blows, the general pause at bar 394, and the return of the I-complex; see *Mahler*, pp. 98–100).

[105] Monahan, '"Inescapable" Coherence', p. 88. Del Mar considers 'the absence of that motivic link' (i.e. the missing semiquavers in bars 720–24) to be 'terrifying', precipitating a crisis which allows second-subject material to emerge in much the same way as it did from the exposition's whirlwind (p. 63). See also n. 114 below for more on this passage's "a-motivic" status.

homogenised that it is difficult to trace its origins: the sonata has succeeded and, at the same time, failed, dragging the sonata-dependent S1 and chorale themes with it.[106]

Into this vacuum rush five heterophonous statements of the Alma-derived motive to begin the third and final Utopian vision, an attempt by the novelistic S-music to turn paralysis into a form of stasis (recall that the novel-symphony functions on the variant, which earns its temporality 'not from the pent-up, onward-driving force of Beethoven, but from the amplitude of a hearing encompassing the far distance').[107] The motivic material in this section is mostly based on what Monahan (p. 88) terms the 'emancipation motive' (the transfigured I-theme that began the development; a hint of the original I-theme is given in the cellos, bars 732–35): this blossoms in the familiar 'Utopian' manner in its free unfolding of heterophonous, long-breathed melodies.[108] As its confidence increases, the music even tries the same tactic to reanimate the defeated G2-motive (bars 741–43); its potential for such treatment is limited, however, and it is abruptly thrown to one side as the key shifts to F major (the key of the original statement of the Alma theme in the first movement). An intertextual foreshadowing of the 'pushing forward' motive from the first movement of the Ninth Symphony (bars 750–51) introduces a parade of G1-motives and the motto rhythm 'in full thematic regalia'; but unlike Monahan, I see this parade as a mockery made *by* the tonic major, and not *of* the tonic major, as the wind and strings weave S2-incipits, emancipation motives, and Alma motives around the now 'thematic' (i.e. novelistic) G1-motives (note also that this theme makes heavy use of the multivalent three-note idea from the first Utopian vision shown in Figure 4.12).[109]

In Del Mar's reading, 'even Motto theme A has lost its ferocity, as the horns declaim a victory march' (p. 63); but while the G-motives (as symbols of sonata homogeneity) are safely defeated and can be reanimated within the novelistic paradigm, the attempt to rehabilitate the motto rhythm goes too far: it cannot be excised as its suffocating stasis is ingrained too deeply in the novel-symphony's non-linear, non-teleological temporality. Tragedy is therefore imminent: but

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106 Although motives from S1 have an important role to play in the next section, the S1 theme as a whole (the artificial sonata-dependence of which is discussed below) does not appear again. The chorale has had an uneasy relationship with the sonata since its first-movement quest to become a transition; its parading and ultimate collateral demise here bear the scars of that quest and establish the motto as an autocratic symbol of stasis. The motto is therefore able to co-opt, in bar 783, the chorale/hammer-blow theme's most direct affective symbol: the hammer-blow itself.

107 Adorno, *Mahler*, p. 87.

108 Monahan, '"Inescapable" Coherence', pp. 86, 88.

109 Monahan, '"Inescapable" Coherence', p. 90 'regalia'. The motive from the first movement of the Ninth (first heard prominently in the trumpets, bars 44–45) is identified by Micznik as 'hav[ing] the function of "pushing forward" or "leading to" other events' (p. 218; see also p. 225). In the Finale of the Sixth, the segment at bar 750 bears a clear relationship to G1 (in particular, it is an augmentation of the 'full regalia' form at bars 754–55): however, the rhythm, static harmony leading quasi-cadentially into a new section, use of homophonic brass, and predominance of descending semitones cannot help but connote intertextually for those familiar with the Ninth.

it is difficult to hear the motives in the final bars of this section (bars 765–72) as 'warlike' (with de La Grange), or 'boisterous', 'emblematic of the sonata's triumph', and 'betray[ing a] nihilistic glee' (with Monahan).[110] These bars are as redemptive as anything in the Second Symphony, and as accepting as anything in the Ninth or *Das Lied von der Erde*; this affective surface notwithstanding, they also represent the culmination and apotheosis of one of the movement's 'red thread' motivic stories.[111]

To retrace the story up to this point: motive $P^{1.5}$, the ultimate ancestor of the movement's G1-motives, was introduced in bar 34 as an independent element that quickly got pulled into theme S1 (Figure 4.9). It tried to form its own theme again in bar 70, but by bar 118 had become embedded in the P-theme, its octave leap widened to a ninth. After the collapse of the chorale in the transition (bar 180), it made its most "thematic" appearance in the movement, being treated quasi-sequentially and liquidated to re-derive S1 (Figure 4.11) before attempting a codetta (brusquely cut off by the I-complex) in bar 224. Its only appearance in the development was a single-segment attempt to claw back the first Utopian vision as it slipped away (bar 383), and its overexcitement in the 'retransition' was what permitted the return of the A minor P-theme as its leap dropped back down to an octave (bars 638–39) then a seventh (bars 640–41), before being restored to a ninth in the P-theme itself (bars 646–47).[112] Its apotheosis, bars 765–72, not only completes the intervallic process by starting with a ninth (bars 765–66) and twice iterating a radiant tenth (bars 767–70), but also combines two of its memories in a gesture of self-immolating sublimation: it liquidates to a two-note cell in the manner of its "thematic" appearance (Figure 4.11), and unfolds a complete descent from $\hat{7}$ to the $\hat{1}$ of the I-complex (its first and only appearance in A minor), walking defiantly into the flames of order that have cut off its impulses throughout the movement. For Adorno, quoted by de La Grange, '[t]hese bars contain the feeling of "despite everything!", of success in the face of a doom which the [coda] can do nothing to diminish'; for Ratz, they 'affir[m] the belief that perseverance in an apparently lost cause may be, morally speaking, of the greatest value for the development of the individual as well as of mankind'.[113]

The flames of order alluded to above warrant some unpacking. Of the eight appearances of $P^{1.5}$ (or P1$^{.5}$-based passages) throughout the movement, two are subsumed within P-themes, two

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[110] See de La Grange, *Mahler*, III, 837 and Monahan, '"Inescapable" Coherence', p. 90.
[111] The 'red thread' metaphor underwriting leitmotivic analysis is discussed in Section 2.2.
[112] 'Retransition' is Monahan's label for bars 612–41 since bar 642 marks the 'Recapitulation Proper' (the return of P and A minor); the I-complex at bar 520 (following what I refer to as the retransition above) opens the 'Pre-Recapitulatory Space' and therefore the 'Recapitulatory Block' as a whole (see '"Inescapable" Coherence', p. 62 and below at n. 117).
[113] Adorno, 'Centenary Address, p. 103, quoted in de La Grange, *Mahler*, III, 837; Ratz, p. 47. Ratz even implies that Mahler's decision to remove the third hammer-blow, signalling that 'death is not the end, but the ascent to higher spheres' (p. 48), may have been prompted by the redemptive quality of this passage.

cut off by I-complexes, two pulled into S1, one overgrown by G-motives and the chorale, and one buried in the a-motivic avalanche that later kills the sonata. Only two of these five agents have unambiguous allegiances: the P-themes shore up the sonata, while the a-motivic passages attack it with inertia (the a-motivic passage in the development liquidates to a general pause in bar 394 – it 'wipes the slate clean' for Del Mar – before a P-based march, the development's most coherent episode, asserts its dominance, despite the motto's protestations in bars 395 and 401).[114] At first, it seems unusual for an S-theme to represent order: but this is the same supposedly novelistic gestalt (for Adorno, 'the most novel-like constellation anywhere in Mahler') that we have seen meticulously assembled in the introductory dissolution field, and which Monahan has argued represents 'a mere mutation of its nemesis P' (p. 77); S1 is therefore an intruder in S-space, the P-theme in an S-theme's clothing.[115] The G-motives, being ultimately descended from $P^{1.5}$, are a negative homogenous projection of the very element that wishes to transfigure itself by breaking through from the sonata into novelistic space; they are its most direct antithesis, its shadow.[116]

The immolation of $P^{1.5}$ is finally accomplished, however, by the I-complex, whose double-faced nature leads Samuels to single it out as one of the most concrete manifestations of the symphony's self-negation (see above at n. 17). On the one hand, it functions as a formal marker and so acts as an enforcer of the sonata: for Monahan, its four appearances initiate the expositional, developmental, recapitulatory, and coda blocks (although the first three of these blocks consist of 'anticipation phases' before the sonata proper takes place in their 'accomplishment phases').[117] On the other hand, since a formal marker relies on being immediately recognisable on each appearance,

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[114] Del Mar, p. 58. In his *Theory of Harmony* (which is dedicated to Mahler), Schoenberg argues that the hammered chord in bars 385ff. is melodic in origin (p. 330): while it is distinctive enough to suggest the comparison with bars 720ff., it is not a melodic gestalt in the spirit of Definition 1.
[115] Adorno, *Mahler*, p. 98. Monahan's Example 5 (p. 78) shows how S1 simply varies the elements of P in order, and de La Grange even argues that the former retains the latter's 'character [of] tension and defiance' (*Mahler*, III, 834), despite then going on to describe S1 as 'the most "positive" element in the Finale' (ibid., p. 835). Adorno, in a characteristic self-contradiction (and possibly referring to the entire theme complex S1+S2), maintains that the theme 'strings together heterogeneous components' which are nevertheless 'organically intertwined', allowing it 'to be utilized equally well as a unity, to be selected and spun out like an individual component' (*Mahler*, p. 98).
[116] Ratz follows the *Dutchman* archetype to read the P and S themes as 'hero' and 'point of rest' respectively (see this chapter's introduction), but then reads the entire transition as an "internal" threat to the hero: 'the idea is suggested to us that threats to our existence reside, properly speaking, within ourselves' (p. 43). This makes an interesting counterpoint to Del Mar's interpretation of the first-movement chorale, cited in n. 26 above, as 'suggesting calm faith even under severest adversity' (p. 35).
[117] See '"Inescapable" Coherence', p. 62. In the six analyses of the Finale that Samuels compares, the only unanimously-agreed formal boundary is the start of the coda (marked by an I-complex) at bar 773 (see Samuels, pp. 71, 74–75). Both Samuels (p. 86) and Monahan ('"Inescapable" Coherence', p. 64, n. 52) identify self-standing formal markers like the I-complex to be typically Mahlerian devices: other examples that they list include the openings of the Second, Third, and Fourth symphonies, the *Schreckenfanfaren* in the last movement of the Second, and the outbursts in the second movement of the Fifth.

it must include an element of stasis: not only does the I-complex explicitly and anti-novelistically revoke its transfigured second appearance at the start of the recapitulation, but three of the four “model” statements of the motto (major–minor fall plus rhythm) occur within I-complexes (the fourth occurs after the chorale in bars 65–66). The final I-complex at bar 773 sets up the expectation of a fourth block which the sonata is now too weak to fulfil: the coda block lasts only 49 bars as against 220+ for each of the other three, and includes none of the expected post-I rotational elements. The section labelled ‘epilogue’ by most writers (bars 790–819) picks over the remnants of the shattered sonata and novel-symphony – the octave and dotted minor third motives and their various combinations including G2, S2, the emancipation motive, and the dotted chorale second segment – before the final motto concludes the work. This conclusion, argues Samuels, has less to do with tonality and voice-leading (it is difficult to identify the structural arrival of A amidst the I-complex and the chromaticism of the epilogue) than it has to do with motivic, formal, and narrative process (p. 163):

> We know that the work has ended, not so much because of a cadence, as because all the developmental material has been worked through, and because the motive signals closure through its narrative coding. Where on its first appearance ([at bar 57 of the first movement]) it was integrated into the voice-leading succession, its ‘closural’ significance arising solely from its position in the form, here this state of affairs is reversed: the motive stands outside linear succession, which simply accepts the bass note A as conclusive. This constitutes a brilliant solution to the problem of making the work end: there is no comforting (but false) cadence or culminatory resolution, yet the work truly ends, and ends with a corporeal violence which completes the narrative scheme rather than just breaking off the progress of the music.

With the sonata and the novel-symphony dead, the forces of stasis finally, then, give way to the forces of silence.

4.3 Epilogue: Coherence and Categorisation

As argued at the end of the previous chapter, the concepts developed in this thesis are not intended to stand as analyses in themselves, but rather to stimulate and facilitate analytical insights; in particular, they serve to focus attention on the domain of motivic process. This much should be clear from the analyses above, which zoom in and out on the musical detail – from the close examination of the first-movement introduction in Figure 4.2 to the exhaustive lists of all motto appearances – and deploy the technical language of type regions, families, and epiphanies only when it is helpful to do so (usually when quantifying the level to which a given category saturates a given passage). The chapter’s trajectory may be read as one in which the formal model gradually moves further into the background, beginning with the explicit tallying and tracking of emerging categories in Section 4.1 and ending with the freer narrative ‘third reading’ (see Section 2.3, point six) of the Finale in Section 4.2 (which nevertheless was still constructed in extensive consultation with the spreadsheets included on the supplementary CD). Throughout, morphological novelty and correspondence have been used to offset, but not overwrite, affective or syntactic differentiation; themes P and S1 in the Finale, for example, are treated independently despite their close motivic relationship. The complete digraphs themselves, having 766 and 1127 vertices respectively, do not feature explicitly as good comparisons (commensurate with the arguments made in Section 1.9): several digraph-like figures included in the chapter (such as Figure 4.12) are really schematics, informal reductions of processes uncovered during the analysis rather than strict subgraphs of a derivation digraph.

Also apparent throughout the chapter is the use of (frequently borrowed) descriptive labels for heuristic categories such as 'the motto rhythm', '$P^{1.4}$', or 'the elementary particle'. These not only enhance readability and ease of cross-reference (compared to 'family S6', 'type region 1.6.1', or 'type region 1.16.1'), but attribute narrative roles ('gesture of stasis'), formal functions ('part of the P-theme'), or motivic histories ('subsumed into S1') to musical segments; as argued in Section 4.1, they are often best understood as agents or functions rather than strict segment groupings. Such labels also facilitate dialogue with other analyses (Monahan's in particular), and can be useful heuristically even if questioned at some level (the 'elementary particle' $P^{1.5}$ is not uncomplicatedly a force of P, for example, and nor are the G-motives wholly generic).

As in all motivic analyses, a certain tension underlies an unwritten assumption: that motivic events in the music represent the same character changing through time. Strictly speaking, the chorale may not appear or recur, nor may it be modified or developed into something else, since what we conceive as a gradual re-moulding of a single object (for example in the first-movement transition depicted in Figure 4.6) is actually a stream of objects. This is the principle on which categorisation (and its associated problem of intransitivity) hinges, and which the graph-theoretical model seeks to address. Without apologising for the use of labels that facilitate dialogue or arise heuristically from consideration of the musical data, it is interesting to examine the extent to which these heuristic categories coincide with the categories proposed by the formal model.

Figure 4.14 lists the principal motivic categories or characters used in my analysis of the Finale above, and tallies the number of times that each is used to refer to a particular type region. For example, the phrase 'the elementary particle makes another bid to become a theme in its own right, developing into an ascending fanfare (bars 69–73)' refers to the segments 069|G2, 070|G2, and 072|D4; the first (the elementary particle) belongs to type region 1.16.1 and the others (the ascending fanfares) to 4.1.1 (all three lie in family S9), so these figures are tallied in the appropriate sections of the table. No segment has been listed twice (such that, for example, each 'chorale/G2 motive' is placed in just one category), and I have tried to limit extra detail which is not explicitly mentioned in the analysis. For example, when 'the wind and strings weave S2-incipits, emancipation motives, and Alma motives around the […] G1-motives', or when one section repeats another already discussed in motivic detail, I have itemised and tallied the relevant segments; but when the text refers to 'S1' I have simply counted the ordered segments that constitute that theme, and not the derivative versions that frequently counterpoint it. A total of

532 of the movement's 1127 segments (47%) have been listed, spanning 13 of the 19 source families, 29 of the 77 composite families, and 115 of the 281 type regions.

The only "perfect" category – in which the text uses a single label to refer to every member of a single family and no others – is the motto (which is actually two perfect categories: the triad and the rhythm). This very separation from the rest of the movement's motivic material is what makes the complete listing of its appearances possible, and underlines its affective qualities of separateness and resistance to development. Slightly less perfect are the first and fourth chorale segments and 'rhapsodic' motive, which each refer to two families never given another label, but do not mention every single occurrence of them. All of the rest overlap in some way (such that certain families are referred to by at least two different names); whilst this could be seen as a criticism of the formal model and/or the heuristic categories, it is my contention that this mismatch tells us something about the movement's motivic processes themselves – in particular, their inescapable coherence.

It was argued above that one of this movement's primary narrative-structural devices is the liquidation and re-integration of the branches of the motivic tree, and this can be seen most clearly in the action of type region 1.2.1 from family 5.1.1. This composite family is formed in bar 42 as a hybrid of source families S1 and S3 (broadly speaking, the octave leap and minor third), and comes to be associated most strongly with the category G2 (70 of the 93 appearances of 1.2.1 listed in the table – out of a total of 151 – are given this label). This 'malignant' type region is one which metastasises throughout the other categories – invading the territory of the G1-motives (bars 94–95), influencing the shape of the third chorale segment in the transition (bars 141–42), generating the incipit of the S2-theme (bars 205–06), and infecting the emancipation motive at the start of the epilogue (bars 790–92) – until it actually spreads back into the octave leap itself: the passage of liquidation in bars 712–20 prepares the ground for the phrases of the epilogue, which bear similarities to G2 but suggest segmentation patterns that re-isolate the octave leap.[118] By then the damage is done: once an octave-leap segment has been derived from a single G2 segment (say, 716|D5 from 708|E4, which lies in type region 1.2.1), any further octave leaps derived from that one will carry a trace of the "G2" type region 1.2.1. While the term 'octave leap', then, can be used to describe morphologically identical pre- and post-G2 segments, the fact that their type regions are different betrays the fact that their identity has changed; perhaps the alternative order proposed by homogeneity and stasis is closer to the novelistic paradigm than first thought.

Similar stories can be told about the other overlaps in the table: the derivational interchanges between the elementary particle and G1 (family S9), or between the dotted chorale segment and the three-note motive (type region 1.81.1); the merging and re-isolation of the second

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118 The term 'metastasize' is used by Monahan ('"Inescapable" Coherence', p. 75) to describe the action of the anti-novelistic themes in the movement.

chorale segment with and from the first (family S11); the inclusion of the octave leap and dotted chorale segment within the I-complex and emancipation motive (type regions 1.1.1 and 1.80.1), or of the Alma motive and G1 within the S1-theme (families 9.1.1 and 21.1.1, and type region 1.4.1); and the pervasive homogenising effect of dotted three-note motives within G1, S1, the three-note fragment, and the infected emancipation motives (type region 1.20.1). Monahan ends his study of the Sixth Symphony's Finale by remarking: 'With Mahler, there is always so much more to say', an assessment with which I heartily concur.[119] But I would add the caveat that the same applies, to a greater or lesser extent, to any piece of music by any composer. This thesis as a whole, its formalisations and its analyses as well as its reconceptualisations of mathematical and motivic analysis, should be seen as a contribution towards some of it being said.

[119] '"Inescapable" Coherence', p. 94.

Label		Family	Type Region	No. of references	Total segments in this type region
Motto	Triad	**S5**	**1.3.1**	8	8
			1.181.1	2	2
			53.1.1	1	1
				11	11
	Rhythm	**S6**	**1.180.1**	16	16
			2.1.1	3	3
			54.1.1	1	1
				20	20
Elementary particle/$P^{1.5}$		S9	1.16.1	16	17
			3.1.1	1	1
	Asc fanfare		4.1.1	4	5
				21	54
G2		5.1.1	1.2.1	70	151
			1.124.1	1	1
			1.125.1	1	1
			1.115.2	3	5
			38.1.1	1	1
				76	169
		5.2.3	**1.127.1**	1	1
				1	1
		16.2.1	**1.84.1**	1	1
				1	1
	S2-incipit	5.1.1	1.2.1	3	151
				3	169
Chorale/ Hammer-blow	First segment	**S10**	**1.8.1**	6	6
			25.1.1	2	2
			1.163.1	1	1
				9	9
		11.2.1	**1.79.1**	2	2
				2	4
	Second segment	S11	1.14.2	14	17
			1.14.1	3	3
			1.13.1	1	1
			1.36.1	2	2
				20	29
	Third segment	**S12**	**1.9.1**	4	4
			1.37.1	2	2
				6	6
		5.1.1	1.2.1	6	151
				6	169
	Fourth segment	S13	**1.10.1**	4	4
			1.15.1	2	2
			8.1.1	1	1
				7	9
		19.1.1	**1.35.1**	2	2
			1.35a.1	1	1
				3	4
	Merged first and second	S11	**1.76.1**	6	6
				6	29
		1.1.1	**1.38.1**	1	1
			9.1.1	1	1
			1.166.1	1	1
			1.167.1	1	1
				4	4
		11.1.1	**1.74.1**	1	1
				1	1
Dotted chorale second segment (=G2+chorale 2)		16.1.1	1.80.1	8	16
				8	18
		16.1.2	**1.126.1**	4	4
			1.126a.1	1	1
			1.126.2	1	1
				6	6
		16.2.2	1.81.1	4	6
				4	6

Figure 4.14: Table comparing the heuristic categories used in Section 4.2 with families and type regions.

Shaded categories appear more than once in the table, and bold categories have all of their segments referred to by the given label.

Octave leap	S1	1.1.1	3	40
			3	40
	5.1.1	1.2.1	11	151
		1.168.1	2	2
			13	169
	29.3.1	1.75.1	3	21
			3	21
(Dotted) (minor) third/ repeated $\hat{2}$–$\hat{3}$–$\hat{1}$	S3	1.20.1	43	158
		1.20a.1	1	1
		1.96.1	1	1
			45	180
	4.1.2	**1.97.1**	1	1
			1	5
	4.4.1	1.95.1	7	17
		27.1.1	1	1
			8	18
	16.3.1	**1.187.1**	3	3
			3	3
	16.5.1	**1.188.1**	1	1
			1	1
"Alma" motive	21.1.1	1.41.1	5	16
		17.1.1	1	1
		1.173.1	2	2
			8	36
	21.2.1	**1.153.1**	1	1
		47.1.1	1	1
			2	2
	9.1.1	**1.162.1**	4	4
		1.43.2	1	1
		1.164.1	1	1
		1.169.1	1	1
		1.171.1	1	1
			8	20

	9.2.1	**1.131.1**	1	1
			1	2
	18.2.1	**1.176.1**	2	2
		1.177.1	1	1
		1.178.1	1	1
		1.182.1	1	1
		1.183.1	1	1
		23.2.1	1	1
			7	8
	13.2.1	**1.170.1**	1	1
		1.172.1	1	1
			2	2
'Rhapsodic' motive	S7	1.6.1	19	85
			19	90
	4.2.2	1.28.1	11	36
			11	43
G1/'G'	5.2.1	1.19.1	11	19
		1.19.2	4	7
		1.88.1	1	1
		1.174.1	1	1
			17	34
	5.1.1	1.2.1	2	151
			2	169
	S3	1.20.1	5	158
		1.20b.1	1	1
			6	180
	S9	1.16.2	5	8
		1.16a.1	1	1
		1.71.1	1	2
			7	54
	7.1.1	1.4.1	4	13
			4	24

Figure 4.14 continued.

	23.1.1	**1.21.1**	2	2
		1.123.1	1	1
			3	7
	23.1.2	**1.22.1**	2	2
			2	2
	31.4.1	**1.90.1**	1	1
		1.93.1	1	1
		1.94.1	1	1
			3	3
I-complex/emancipation motive	S1	1.1.1	28	40
			28	40
	S2	1.11.1	27	30
		1.11a.1	1	1
		1.11.2	1	1
			29	32
	S3	1.20.1	15	158
			15	180
	S4	**1.17.1**	3	3
		1.128.1	3	3
		1.42.1	3	3
		1.12.1	3	3
		1.7.1	3	3
			15	15
	18.1.1	**1.87.1**	1	1
			1	2
	29.2.1	**1.120.1**	1	1
			1	1
	29.2.2	**1.119.1**	1	1
			1	1
	16.1.1	1.80.1	7	16
		52.1.1	1	1
			8	18

	5.1.1	1.2.1	1	151
			1	169
	5.5.1	**1.186.1**	1	1
			1	1
S1 (and independent references to $S^{1.3}$ and $S^{1.4}$)	S3	1.20.1	6	158
		1.122.1	7	9
			13	180
	S8	1.141.1	3	11
			3	12
	7.1.1	1.4.1	6	13
		1.5.1	4	4
		1.151.1	1	1
		1.151.2	1	3
		1.130.1	1	1
		41.1.1	1	1
			14	24
	16.4.1	1.83.1	1	3
		1.121.1	2	2
		1.121.2	1	1
			4	6
	9.1.1	**1.132.1**	1	1
		1.52.2	1	1
			2	20
	21.1.1	**42.1.1**	1	1
			1	36
Three-note fragment	S3	1.20.1	1	158
			1	180
	16.2.2	1.81.1	2	6
			2	6
	7.1.2	**1.118.1**	1	1
			1	1

Figure 4.14 continued.

Afterword

In closing, we might return – in a Kierkegaardian spirit of dynamic repetition – to Schoenberg's epigraph to this thesis. His distinction between science's systemisation of 'all characteristic cases' and art's interest in 'just a few, the interesting ones' takes on new light when read, through Chapter 1, as a sketch of science's desire to extrapolate the general from the particular, and art's desire to view the particular through the prism of the general. Read through Chapter 4, the same distinction can be understood as a comment on the inherent incompleteness of any analysis; refracted again through Section 1.10, we might ask where this leaves the process of mathematical modelling.

The model presented in Chapter 3 is founded on the image of a motivic network: its central argument is that motives can be fruitfully understood as internally-structured, non-homogenised categories that emerge from pairwise, time-directed, and rhetorically constructed relationships between individualised musical segments; it leans, therefore, towards archetype 4 of Figure 2.1, but can also incorporate the non-linearity of archetype 1, the informal heuristic 'cores' of archetype 2 (as shown in my use of Monahan's categories in Chapter 4), and the intransitive temporal process of archetype 3. However, to borrow another phrase from Schoenberg (cited in n. 2 of the Introduction), the 'basic shape' of this model still has room for 'reshaping'. The limits of automated motivic analysis are discussed in Section 2.3, and the justifications for manual segmentation in Section 3.1, but there is nothing in principle precluding the adoption of more sophisticated computational methods for segmentation, encoding, or the identification of derivational and successional relationships (n. 5 of Chapter 3 in particular gives an indicative list of additions that could make the input method more user-friendly).

The model and its macro also have potential for modular extension, adding new processes (possibly tailored to the requirements of particular analyses) to the existing program. To list a few features, the desire for which arose during the writing of Chapter 4: the generation of figures such as Figure 4.3 and Figure 4.5 could be automated; the relationships between families could be made clearer by explicitly listing each's parent and descendant families; the "reach" of each node (i.e. the temporal distance between its offset and the latest of its children's onsets) could be taken as a measure of its importance as a referential example; and a derivation digraph spelling out the relationships between themes could be defined to create the process $\mathcal{M}_1 \to \mathcal{M}_2 \to \mathcal{M}_3$ from the pairs $(\mathcal{M}_1, \mathcal{M}_2)$ and $(\mathcal{M}_2, \mathcal{M}_3)$ (this is exactly what the derivation digraph does for segments). The decision was taken not to implement these processes computationally in order to illustrate that model-building is always open-ended: there will always be more functionality to add, and so the focus of this thesis has been to establish a sound conceptual architecture rather than an exhaustive toolkit of features.

The generality of this architecture therefore allows it, with some modification, to be applied to different repertories. The Beethoven and Mahler examples were found to be different not only in scale, but also in structure (the former, for example, contained no composite families): indeed, one of the strengths of this model is that different pieces (or different interpretations of the same piece) are capable of producing different structures (unlike in Schenkerian and pitch-class set analyses, which generate their insights mainly from the process of reduction and not from the eventual structural product). A comparative study of motivic practice using this model could therefore prove insightful (what do Strauss's motivic networks look like, for example? Or Schoenberg's? Or a jazz improvisation's?), but care would need to be taken to avoid making empirical generalisations about musical practices from small data sets (i.e. single pieces). There are also potential compositional applications, taking a network derived from an analysis (or by some other means) and filling its nodes with musical material to produce a (possibly non-linear or interactive) piece of music; this potential for interactivity also underwrites a wide range of possible artistic responses, educational strategies, or workshop-style activities to engage non-academic audiences.

The creative act of composition, as argued in Chapter 1, is not too distant from that of analysis: in both cases, the model chosen influences the thinking about the music. The path from analytical input to formal structure is well-defined (by Chapter 3), and one could argue that the inputs themselves (i.e. the segmentation and derivation relationships) are fairly uncontentious (Figure 4.1 and Figure 4.4 show, for example, that my segmentations broadly coincide with Monahan's). The path from formal structure to analysis, however, travels along freer lines, inviting diversions into analytical territories off the motivic track (as the occasional appearance of formal and tonal considerations in Chapter 4 attests). At the end of Chapter 3 it was argued that the starting-point (i.e. the spreadsheets on the supplementary CD) and the route taken are significant in understanding the final destination: transposing this to the path between input and formal structure, one might then ask whether the mathematical details of Chapter 3 need to be fully understood by anyone wishing to use the model. They are by no means essential: but the more one delves into the workings of the model, the more one understands its assumptions, strengths, and blind spots, and the sharper, therefore, the detail of its good comparison becomes.

The foregoing has presented one way of modelling motives mathematically: its definitions have been by turns descriptive ('motivic segment' in Definition 1), explicative ('motivic segment' in Definition 3.1.6), and stipulative ('strong' and 'weak' in Definition 3.2.10), making liberal use of the sedimented meanings or metaphorical resonances of certain terms ('epiphany', in Cone's sense, in Definition 3.2.10; 'family' in Definition 3.2.12). These sedimented meanings, even in stipulative definitions, have been embraced rather than bemoaned; they contribute to the richness of the mathematical metaphor by latching onto existing music theories (and aesthetics – cf. the 'great motivic tree' of Section 4.2) and making the model's insights more evocative. But equally, I might have considered a motivic segment to be a point (or contour region) in Tymoczko's geometric space, a pattern with expectational probabilities that change as its category grows, a message sent down a noisy channel as its variations proliferate, or a function (or "colouring") of a particular alphabet (such as a chord or a scale). Returning to the thesis's other epigraph, the differences between 80 and LXXX become, when situated within the context of a suggestive, perception-changing music theory, something certainly worth making a 'big fuss' about.

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