The Strong Theory of Relative Identity

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0.1 Abstract

This dissertation considers a theory of numerical identity, first presented by P.T. Geach (1962). I label this theory 'the strong theory of relative identity'. I suggest that the strong theory of relative identity involves three theses, which I name 'GT', 'RI', and 'SRI'. I argue that each of these theses is logically independent. I consider arguments for and against each of these theses in turn. I conclude that none of the arguments for GT, RI, or SRI are conclusive. However, I also argue that the arguments against GT, RI and SRI are unsuccessful. I argue, further, that the strong theory of relative identity, and GT in particular, is incompatible with classical semantics and classical first-order logic with identity. I consider alternative non-classical logical systems and semantics which might be compatible with the strong theory of relative identity. Finally, I consider the philosophical applications of the strong theory of relative identity. I focus on one area, specifically philosophical theology, and I argue, with respect to the logical problem of the Trinity, that either GT is true or orthodoxy is false.

CONTENTS

0.2 *Acknowledgements*

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0.3 Declaration

I hereby declare that this submission is entirely my own work except where due acknowledgement is given. This work has not previously been presented for an award at this, or any other, University.

0.4 Introduction

Natural languages such as English contain a multiplicity of constructions expressing apparent relations of identity. Some of these relations seem, from the sentences in which they are expressed, to be dyadic. Others seem to be triadic. Apparent dyadic relations of identity in natural language are often found in clauses involving the expressions '...is identical with...', '...is the same as...', '...is equal to...', or simply '... is ...'. Apparent triadic relations of identity are normally expressed in English with the construction '...is the same ... as ...'. This may be completed by a count noun or a mass term. To complicate matters, the expression '...is the same ... as ...' seems to function differently depending on which noun fills the empty place. It seems, *prima facie*, as if a different relation is involved in the sentence 'Clark Kent is the same man as Superman' from that involved in the sentence, 'Your eyes are the same colour as the sea'. The latter difference is often characterized by distinguishing between relations of 'qualitative identity', such as '... is the same colour as...', and relations of 'numerical identity', such as '... is the same man as ...'. The number and nature of relations of identity will be the subject matter of this dissertation.

According to one influential view of identity, the appearance of multiplicity is deceptive, and there is really only one genuine relation of identity. However, according to an alternative view, made famous by the late Peter Geach (1962, 1967, 1980), there is not one relation of identity, but many. Moreover, the supposed single 'genuine' relation of identity is, in fact, incoherent. This dissertation is concerned with the second of these views. More particularly, I will consider a theory which I will call 'the strong theory of relative identity'. I will identify this theory with the conjunction of three theses. In what follows, I will define these three theses and consider the arguments that have been presented both for and against them. I will conclude that none of the arguments either for or against any of the theses is entirely compelling. The following discussion will be split into seven chapters, the content of which I will briefly outline here.

0.4.1 Chapter 1: The Strong Theory of Relative Identity

In Chapter 1, I will set the stage by distinguishing between two sets of theories about the logic of identity. We begin with theories of absolute identity. Defenders of theories of absolute of identity disagree amongst themselves on a wide range of issues; however, for convenience sake I will treat this set as a single theory. The theory of absolute identity is the orthodox theory of numerical identity. According to absolute identity, relations of numerical identity can be characterized as 'the strongest equivalence relations'. A relation is an equivalence relation if and only if it is symmetric, transitive, and reflexive in its field. The strongest equivalence relations are those equivalence relations which guarantee the indiscernibility of any pair jointly satisfying them. That is, for any x and y, if x bears relation R to y, x is indiscernible from y. According to absolute identity, all relations of numerical identity are characterizable in this way. In what follows, I will call the relations posited by the theory of absolute identity, 'relations of absolute identity'.

Thus, the theory of absolute identity entails:

> (0.1) Every genuine relation of identity satisfies the following four features: reflexivity, symmetry, transitivity, and the principle of the indiscernibility of identicals (to be defined formally in due course).

In addition, many, but not all, philosophers who ascribe to absolute identity also hold the following:

> (0.2) Satisfaction of these four formal features is a sufficient condition for some relation, R's, being a relation of numerical identity.

> (0.3) Statements involving apparent triadic relations of numerical identity of the form 'x is the same F as y', where F is a schematic letter replaceable by some sortal term, are logically equivalent to statements of the form 'x is F, y is F, and x is identical with y'.

> (0.4) There is exactly one relation of identity.

The second family of theories about the logical structure of identity relations are the theories of relative identity. Theories of relative identity have their source in the work of Geach, particularly his *Reference and Generality* (1962, 1980). In Chapter 1, I will expound Geach's views on identity. I will argue that Geach's theory involves three central theses, which I will name 'GT', 'RI', and 'SRI'. I argue that none of these theses entails either of the others. Therefore, each would need to be defended separately. GT is the thesis that there are no relations of absolute identity. RI is the thesis that there are true statements of the form: x is the same F as y, x is not the same G as y, and x or y is a G, where 'F' and 'G' are sortal terms. SRI is the thesis that every relation of identity involves a sortal and is not logically reducible to a relation of identity without a sortal.

Collectively these positions form Geach's theory of relative identity. As we will see, various philosophers apart from Geach have espoused views resembling Geach's in one respect or another, though no one else has explicitly taken on board all of Geach's commitments.

SRI, the thesis that statements involving relations of identity are relative to some sortal, bears a *prima facie* similarity to theses which have been defended by, among others, such philosphers as Anthony Kenny (1963), Leslie Stevenson (1972, 1975), Michael Dummett (1973, 1991), Harold Noonan (1980, 1997), E.J. Lowe (1983, 2009), and David Wiggins (1980, 2001). However, I draw a distinction between the views of these philosophers and SRI. These philosophers all believe that 'x is identical with y' implies that 'For some F, x is the same F as y'. None of these philosophers would subscribe to the stronger theses that Geach advances, that 'x is identical with y' is incomplete.

RI has been a particularly controversial thesis. Apart from Geach, RI (or a variant of it) has been defended by Douglas Odegard (1972), Eddy Zemach (1973, 1982, 1991), E.J. Borowski (1975), Nicholas Griffin (1977, Griffin rejects SRI, however), A.P. Martinich (1978, 1979), Anil Gupta (1980), George Myro (1985), Peter van Inwagen (1988, 2003), James Cain (1989)[1], Harry

[1] Martinich, Van Inwagen, and Cain are primarily concerned with the theological implications of theories of relative identity. For more on this topic, see Chapter 7.

Deutsch (1998)[2] and Uwe Meixner (2005). It has been criticized by Quine (1964), Dummett (1973, 1991), Lowe (1989, 2009), and Wiggins (2001), amongst others.

The most controversial of all Geach's claims is GT. This has been rejected even by most philosophers who describe their own theories of identity as theories of 'relative identity'. Zemach (1973: 211) and Griffin (1977: 142-154) both reject GT, while van Inwagen declares himself neutral on the existence of absolute identity (van Inwagen 1988: 257).[3] As I have said, there are several philosophers who align themselves with Geach, in so far as they describe their own theories as 'theories of relative identity'. Some of these accept none of the three theses that I have attributed to Geach (for example, Stevenson 1972, 1975); others accept some but not all of SRI, RI, and GT.[4] In order to clarify the different positions that go under the title of 'relative identity', I will categorize theories of relative identity by dividing them into two groups.

The first group I will call 'weak theories of relative identity'. A theory is to count as a weak theory of relative identity if and only if it includes thesis RI but not the thesis GT. According to this view, Griffin, Zemach and van Inwagen all endorse theories of weak relative identity.[5]

The second group I will call 'strong theories of relative identity'. A theory is a strong theory of relative identity if and only if it includes both the theses RI and GT. Geach's theory is a strong theory of relative identity, as he is committed to each of GT, RI, and SRI. In fact, Geach's is the only developed

[2] Odegard, Borowski, Myro and Deutsch are all primarily interested in diachronic identity. Odegard and Myro, for example, defend a temporal version of RI, which is consistent with a form of LL where the substitution instances of the second-order variable or schematic letter is restricted to non-temporal properties. For objections to the termporal version of RI, see Fredrick Doepke (1982). For a response to Borowski's defence of diachronic relative identity relations, see Roland Puccetti (1978). I will not discuss the temporal form of RI in this dissertation.

[3] Some parallels might be drawn between Geach's position and some views held by Locke (*Essay Concerning Human Understanding*, Book II, Chapter XXVII: 15. For a discussion of Locke's views and an argument that Locke would have rejected relative identity, see Conn 2003: Chapter III) and the the Wittgentein of the *Tractatus* (5.5301). However, any such connections are of primarily historical interest and of no use in supporting GT.

[4] Cain (1989) is the only philosopher, of whom I am aware, who seems to defend all three theses, apart from Geach.

[5] This classification follows that provided by Michael Rea (2003: 433-437).

strong theory of relative identity to be found in the literature.[6,7,8]

Having classified different theories of identity, I briefly consider how to choose between them. I argue that a successful defence of the strong theory of relative identity must involve an *a priori* argument for the truth of GT. I will try to provide such an argument in Chapter 2.

0.4.2 Chapter 2: Geach's Argument Against Absolute Identity

In Chapter 2, I consider Geach's arguments for GT. Geach's presents his argument in a paper entitled 'Identity' (1967). The deductive structure of Geach's argument is unclear, so a major task of the chapter is expository. I identify the premises of Geach's argument and note the objections to them. I suggest several alterations to Geach's argument which would allow it to escape the objections that have been raised.

For example, Geach claims that there is no criterion for a relation R's expressing absolute identity (Geach 1967: 6). Geach is open to the accusation that he is attacking a straw man, since he only considers a limited number of possible criteria. I argue that Geach's argument, if it is to be compelling, must be expanded to show that all the proposed alternatives in the literature fail to provide a criterion of application for the general term 'relation of absolute identity'. Another alteration to Geach's argument, which I propose, involves an apparently central component of Geach's argument, which is, in fact, irrelevant to the issue. It has been claimed that the satisfaction of the four formal features of absolute identity provides a criterion for a relation's expressing absolute identity, so long as the interpretation of any given theory T coincides with the quotient-structure for T. Geach argues at length that this proposal leads to unwanted ontological consequences. Geach's objections

[6] Geach also commits himself to several additional theses, the significance of which I will consider only briefly. These include that derelativization thesis (Geach 1973: 287-302) and the counting thesis (for example, Geach 1962: 63, also see Noonan 1997: 639-640).

[7] Van Inwagen suggests that most relative identity theorists follow Geach in rejecting the existence of relations of absolute identity (van Inwagen 1988: 256). I cannot find any support for this view, however.

[8] According to this classification, Stevenson's theory of identity is not a theory of relative identity, in spite of the fact he classifies it as such himself. This is because he rejects RI (Stevenson 1972: 158).

to the proposed procedure are unsuccessful. I argue that the objections are unnecessary in any case, since the interpretation strategy would refute Geach's argument only if we, in fact, always did reinterpret the sentences of T in this way. However, we do not do so, so the interpretation strategy fails as an objection to Geach's argument. This allows me to provide a simpler version of Geach's argument.

Having suggested various modifications, I then set out a charitably reconstructed version of Geach's argument for GT. I consider each of the premises and I conclude that several of the premises have not been conclusively established.

0.4.3 Chapter 3: RI

Having considered GT in Chapter 2, I turn to the remaining theses involved in Geach's strong theory of relative identity. Chapter 3 will involve a consideration of Geach's arguments for SRI. Geach's 'river and waters argument' (Geach 1962: 150-151) and his 'men and heralds argument' (Geach 1980: 183-184) aim to establish SRI. Both these arguments have been criticised by Lowe (1989: 43-63). I conclude that these arguments fail to establish the thesis. I also consider arguments for RI from Griffin (1977) and Zemach (1973, 1982). I argue that these also fail. I conclude that RI is unproven.

I then consider objections against RI, from Lowe (1989a) and Wiggins (2001). I argue that if GT is false, these objections succeed; however, if GT is true, these objections fail. I conclude that RI is not disproved.

0.4.4 Chapter 4: SRI

In Chapter 4, I turn to SRI. I consider an argument from William Alston and Jonathan Bennett (1984) which, if sound, would support SRI. Alston and Bennett argue that if Frege's Cardinality Thesis (henceforth CT) is true, then all relations of identity must involve some sortal term. Alston and Bennett conclude from this that CT must be false. However, there are good reasons to think CT is true, so the alleged entailment from CT to SRI would provide support for the latter. I attempt to reconstruct an argument to this effect.

Such an argument would require a claim about the relation between statements involving predications of cardinality and statements involving identity relations. However, I argue that the relation that does hold between these two classes of statements is such that CT does not entail SRI. I conclude that the appeal to CT fails to provide support for SRI. SRI, then, is also unproven.

0.4.5 Chapter 5: Objections to Relative Identity

In Chapter 5, I consider the objections that have been raised against GT. I will consider objections from Quine (1964), Dummett (1973: 547-583, 1991), and John Hawthorne (2003). These objections show that GT is incompatible with classical semantics. Therefore, if GT is true, classical semantics would need to be replaced by a non-classical semantics. I consider some of Geach's suggestions for solving these problems. I further note an objection raised by James Cain (1985), which suggests that the apparatus, which Geach introduces to solve the earlier objections, itself entails the failure of the syllogisms. I put off any attempt to resolve this objections until the following chapter.

0.4.6 Chapter 6: The Logic and Semantics of Relative Identity

I begin this chapter by considering how Geach might respond to Cain's objection. I suggest that Geach's most promising response is to deny that the cases, which Cain suggests Geach is committed to, are genuine counterexamples to the syllogisms.

I then consider the kind of logic and semantics which might be compatible with the strong theory of relative identity. There are several systems that have been developed for theories of relative identity, as well as several others which, though not developed with relative identity in mind, are compatible with it. For example, classical first-order logic without identity is compatible with strong theories of relative identity. However, I argue that there is a benefit to developing a more expressive logical language than first order logic without identity. Such a language would involve inference rules for relations of relative identity. I consider the strengths and weaknesses of various systems to be found in the literature.

I then turn to the semantics for relative identity. I suggest that a semantics compatible with the strong theory of relative identity must differ from classical semantics in a number of respects, but that these do not show that such a semantics is incoherent.

0.4.7 Chapter 7: Applications of Relative Identity

In Chapter 7, I consider the implications of the strong theory of relative identity for other areas of philosophy. Relative identity may be used to solve various puzzles in metaphysics. However, I will focus on one area of applied philosophy in this chapter. This is the field of philosophical theology, on which theories of relative identity have made the most notable impact. I will consider what light might be shed on this field by the conclusions reached in this dissertation. I argue that relative identity, if true, would provide a resolution to the logical problem of the Trinity. More particularly, I consider the different responses to the logical problem of the Trinity in the literature and argue that either GT is true, or the orthodox doctrine of the Trinity is false.

1. THE STRONG THEORY OF RELATIVE IDENTITY

I will start by considering some of the things that are typically said by philosophers about identity relations. I will classify these into theories of identity. I will begin with the orthodox view, before turning to Geach's theory of relative identity. Having sketched the alternative positions I will make some observations about the sort of argument which would be necessary to support the theory of relative identity.

A distinction is often drawn between sentences which are said to express 'numerical identity' and those that are said to express 'qualitative identity'. Examples of the kind of sentences said to express numerical identity include 'Clark Kent is the same man as Superman', 'Hesperus is Phosphorus', and 'One plus one equals two'. Examples of the kind of sentences said to express qualitative identity include 'Your eyes are the same colour as the sea', 'These two boys are as tall as one another', and 'Jane and John are equally clever'. Cashing out the distinction between numerical identity and qualitative identity is a surprisingly challenging task, however. One option is that a statement of the form 'x is the same F as y', where 'x' and 'y' are singular terms and 'F' is a sortal term, expresses numerical identity if and only if it licenses the conclusion that x and y are one and the same thing. By contrast, a statement of the form 'x is the same F as y' is a statement of qualitative identity if and only if it licenses merely the weaker conclusion that x and y have some one property in common (note that, throughout this dissertation, I intend the widest sense of the term 'property', that is, an abundant account of properties, whereby there is some property for every predicate. However, I do not think any of the claims I defend in this dissertation depend on this notion of properties, and so I will not try to resolve the various paradoxes that accompany this notion).[1] We are interested only in the former class, that is, relations of numerical identity.

1.1 Theories of Identity

1.1.1 The Theory of Absolute Identity

It is a key thesis of the orthodox view of identity that there exists a first-order dyadic relation, R, satisfying the following four formal features:

> **Strong Reflexivity**: $\vDash \forall x R(x, x)$
>
> **Symmetry**: $\vDash \forall x \forall y (R(x, y) \rightarrow R(y, x))$
>
> **Transitivity**: $\vDash \forall x \forall y \forall z ((R(x, y) \wedge R(y, z)) \rightarrow R(x, z))$
>
> **The Indiscernibility of Identicals**: If x bears relation R to y, then everything true of x is true of y, and everything true of y is true of x.

I express the last of these features informally because a precise formulation of the principle will involve issues which I wish to postpone for the moment. We will return in Chapter 2 to consider the various formulations of the principle of the indiscerniblity of identicals, which can be phrased either as an axiom-schema or as a non-schematic second-order principle. We will also need to consider whether the expression 'everything true of x' is to be understood as including all possible predicates (or properties) or, if not, how to limit its application. Once we have agreed upon a formulation for the indiscerniblity of identity (also known, somewhat misleadingly, as 'Leibniz's Law', henceforth to be shortened to 'LL'),[2] we will be in a position to reduce our four formal features to two, because symmetry and transitivity can be derived from

[1] Namely, the property of being a particular F, which can be cashed out in terms of common inclusion in some equivalence class, which is itself a partition on the class of all Fs.

[2] Note that the principle here going under the name of 'LL' is not the much more controversial 'principle of the identity of indiscernibles', about which I will have very little to say. The latter states that, for any x and any y, if everything true of x is true of y, then x and y are absolutely identical.

reflexivity and any of the mainstream formulations of LL.[3] There has been a great deal of debate over whether the formal features of absolute identity amount to a definition of absolute identity, or whether some other definition can be provided. We shall consider some of the alternatives in due course. In many formal systems, a primitive symbol is introduced to express absolute identity. Some philosophers hold that identity is, by its nature, indefinable, because any definition would involve circularity (for example, Savellos 1990, McGinn 2000: 7-9). Other philosophers are not concerned by the worry of circularity and provide various definitions, usually either involving some version of LL (for example, Russell and Whitehead 1970 [1910]: 168 or, more recently, Krause and French 2006: 15) or a set-theoretic definition (for example, Enderton 2000: 5). It is often said that absolute identity is first-order indefinable but second-order definable, with the caveat that a second-order definition will involve absolute identity in the *definiens*[4]. Again, we will leave the issue of definitions of identity for further discussion in Chapter 2.

It is also sometimes said that the relation expressed in such sentences as 'Your eyes are the same colour as the sea' is simply a roundabout way of saying that a particular colour is numerically identical with itself (McGinn 2000: 2-3). In other words, relations of qualitative identity can be eliminated in favour of a relation of numerical identity. This view supports another

[3] Proof of symmetry: assume

(1) $P(x, y)$

where P is a two-place predicate expressing absolute identity. By reflexivity, it is the case that

(2)$P(x, x)$

By LL, any property of x is a property of y, this includes the property expressed by the symbols '$P(\ , x)$'. Applying this to (2), we get

(3)$P(y, x)$

Proof of transitivity: assume

(1) $P(x, y) \wedge P(y, z)$

By conjunction elimination, we get both

(2) $P(x, y)$

and

(3) $P(y, z)$

By LL and (2) we can replace y with x in (3), giving

(4) $P(x, z)$

[4] A proof of the indefinability of identity in first-order logic can be found in Hodges 1983, we shall consider possible second-order definitions of identity in Chapter 2.

important thesis concerning identity, namely that there is really only one such relation, and that the very notion of different identity relations is incoherent (Frege 1903: 254, McGinn 2000: 1-7).

The relation just introduced is to be called 'the absolute identity relation'[5] and will, henceforth, be represented by the symbol '=' when appearing in formulae. According to philosophical orthodoxy, this relation holds between everything and itself and never holds between a thing and something other than itself.

The theory of absolute identity is often supplemented by a claim concerning statements involving apparent triadic numerical identity relations, such as that expressed by the sentence 'Clark Kent is the same man as Superman'. The thesis is first found in the work of Frege, and in his honour I will call the 'Fregean Analysis'.

> **Fregean Analysis**: All statements of the form 'x is the same F as y', in which '...is the same F as...' expresses a relation of numerical identity, are necessarily equivalent with statements of the form 'x is an F and y is an F' and 'x is absolutely identical with y'.

The significance of this thesis is that, if it is true, all triadic relations of numerical identity can be eliminated and replaced with simple predications and a dyadic relation of absolute identity without loss of expressiveness. It follows from the Fregean Analysis that all such triadic relations of numerical identity satisfy LL, so there are no cases of the following form:

$$x =_F y \wedge x \neq_G y \wedge (G(x) \vee G(y))$$

where the symbols '$=_F$' and '$=_G$' represent the numerical identity relations '... is the same F as...' and '... is the same G as ...' respectively. A formal proof of this result may be given, with 'FA' (for 'Fregean Analysis')

[5] The theory of absolute identity is compatible with the existence of different relations of absolute identity. For example, it is compatible with absolute identity that the relations expressed by '... is the same man as ...' and '... is the same mouse as...' are both relations of absolute identity but, perhaps, not the same relation.

standing here for the inference rule

$$\frac{x =_F y}{F(x) \wedge F(y) \wedge x = y}$$

and 'substitution of identicals'[6] here standing for the inference rule

$$\frac{x =_F y, G(x)}{G(y)}$$

Proof 1		Rule of Inference	Assumption Set
1	$x =_F y \wedge x \neq_G y \wedge (G(x) \vee G(y))$	Assumption	1
2	$x =_F y$	Conjunction Elimination 1	1
3	$x \neq_G y$	Conjunction Elimination 1	1
4	$G(x) \vee G(y)$	Conjunction Elimination 1	1
5	$F(x) \wedge F(y) \wedge x = y$	FA 2	1
6	$\neg(G(x) \wedge G(y) \wedge x = y)$	FA 3	1
7	$x = y$	Conjunction Elimination 5	1
8	$\neg G(x) \vee \neg G(y) \vee x \neq y$	De Morgan's 6	1
9	$\neg G(x)$	Additional Assumption	1, 9
10	$\neg G(y)$	Substitution of Identicals 7, 9	1, 9
11	$\neg G(x) \wedge \neg G(y)$	Conjunction Introduction 9, 10	1, 9
12	$\neg(G(x) \vee G(y))$	De Morgan's 11	1, 9
13	$G(y)$	Discharge Assmuption 9 ($\perp$ 4, 12)	1
14	$\neg G(x) \vee x \neq y$	Modus Tollens 8, 13	1
15	$\neg G(x)$	Additional Assumption	1, 15
16	$\neg G(y)$	Substitution of Identicals 7, 15	1, 15
17	$\neg G(x) \wedge \neg G(y)$	Conjunction Introduction 15, 16	1, 15
18	$\neg(G(x) \vee G(y))$	De Morgan's 17	1, 15
19	$G(y)$	Discharge Assumption 15 ($\perp$ 4, 18)	1
20	$x \neq y$	Modus Tollens 14, 19	1
21	$\neg(x =_F y \wedge x \neq_G y \wedge (G(x) \vee G(y)))$	Discharge Assumption 1 ($\perp$ 7, 20)	1

The Fregean Analysis is not entailed by the theory of absolute identity and a shorter proof with the same result could be given with suitable inference rules introduced governing relations of the form $\ulcorner x =_F y \urcorner$, which makes no appeal to the Fregean Analysis. The important point is that all orthodox theories of identity entail that it can never be the case that 'x is the same F as y, but not the same G as y', when '...is the same F as ...' and '... is the same G as...' express relations of numerical identity, and when either x or y is (a) G.

[6] Some philosophers use the term 'substitution of identicals' to mean the clearly false thesis that any two co-referring expressions are replaceable in any sentence *salva veritate*. Similarly, some philosophers use the term 'indiscernibility of identity' to refer to the following inference rule. As I have said, I use the latter term synonymously with 'Leibniz's Law', while the former names the inference rule provided.

Yet natural language seems to provide many counterexamples to this result.

1.1.2 Apparent Counter Examples to Orthodoxy

Wiggins, in his classic work on identity *Sameness and Substance* (1980 23-44, 2001: 29-50), tries to classify apparent counter-examples to philosophical orthodoxy about identity by dividing into five categories all the possible cases in which it can be false that 'x is the same G as y'.

Wiggins's five categories are as follows:

> Type 1: $x =_F y \wedge x \neq_G y \wedge \neg G(x) \wedge \neg G(y)$
>
> For example: 'The evening star is the same planet but not the same star as the morning star.'
>
> Type 2: $x \neq_F y \wedge x \neq_G y$
>
> For example: 'Venus is not the same star as Mars nor the same anything as Mars.'
>
> Type 3: Cases involving phased sortals. The logical structure of such examples is disputed.
>
> For example: 'John Doe, the boy whom they stupidly took for a dunce, is the same human being as Sir John Doe, the Lord Mayor of London, yet not the same boy.'[7]
>
> Type 4: $x =_F y \wedge x \neq_G y \wedge \neg G(x) \wedge G(y)$

Wiggins does not believe that there are any true cases of Type 4, but gives the following examples that seem to be plausible candidates:

[7] This case is slightly altered for simplicity.

(1.1) ‘This pile of fragments is the jug you saw last time you were here.’

(1.2) (The fragment having been reconstructed into a coffee pot) ‘This coffee pot is the jug (or fragments) you saw last time.’

(1.3) ‘Cleopatra’s Needle in 2014 is not the same stone as it was in 1900 (the stone having been gradually replaced by concrete), but it is the same landmark.’

Type 5: $x =_F y \wedge x \neq_G y \wedge G(x) \wedge G(y)$

Once again, Wiggins thinks that purported examples of Type 5 must either be false or misunderstood. Nevertheless plausible examples are as follows:

(1.4) ‘I moor my vessel in the river Scamander. The next day, it is the same river as the previous night but not the same water.’

(1.5) ‘The boy John Doe is the same human being as John Doe the mayor but not the same collection of cells.’

(1.6) ‘The old church is the same church as the rebuilt church (which has none of the same masonry) but not the same building/stonework.’

(1.7) ‘The train from London to Bristol is the same train in 1962 as it was in 1911 but not the same collection of coaches and locomotive.’

(1.8) ‘You may see the same official today as you did yesterday, without his being the same man.’

(1.9) 'Dr Jekyll and Mr Hyde are not the same person/personality though they are the same man.'

(1.10) 'The doctrine of the Trinity (e.g. the Son is the same divinity as the Father without being the same person).'

Of these, it is clear that Types 1 and 2 pose little philosophical difficulty for the Fregean Analysis and the theory of absolute identity, and I will have no more to say about them.

Wiggins thinks that 'John Doe, the boy whom they stupidly took for a dunce, is the same human being as Sir John Doe, the Lord Mayor of London, yet not the same boy' does not express the disputed form. The example, and all similar cases involving phased sortals, such as 'boy', is ambiguous between tensed and tenseless statements, in Wiggins's view (Wiggins 2001: 29-33). If it is tensed, then the second identity relation ought to be phrased in the past tense, in which case the sentence is false. For John Doe was the same boy as Sir John Doe was (even if he had not been knighted when he was a boy). If the statement is to be understood tenslessly, then at least according to one popular semantics for tenseless statements, the sentence must be understood as saying 'John Doe is, was, or will be the same human being Sir John Doe and it is not the case that John Doe is, was, or will be the same boy as Sir John Doe.' But once again, this is false, for Sir John Doe was a boy, and he was the same boy that John Doe was. So much for phased sortals.

Type 4 examples are those where the referring expression on one side of an identity relation falls under some sortal which the referring expression on the other side of the identity relation does not fall under.

Wiggins considers each case in turn and attempts to show that they are all subject to alternative translations, according to which they do not instantiate the disputed logical form (Wiggins 2001: 34-43). Examples (1.1) and (1.2) illustrate the problems of material constitution. Can an entity falling under a sortal, say 'statue', also fall under another sortal term derived from the material with which it is made, say 'clay'? This would seem to imply that the entity (the statue/clay) will have contradictory properties, for example, different persistence conditions. Wiggins argues that this is impossible. In his view, the statement ‘The statue is clay’ does not involve any identity relation at all, but rather the copula expresses the ‘is’ of constitution. Thus, ‘the statue is clay’, in Wiggins's view, means ‘the statue is made of clay’, rather than ‘the statue is identical with some clay’.[8]

We move to case (1.3). Wiggins asks what it is that ‘Cleopatra's Needle’ refers to. A stone? Or a landmark? If ‘Cleopatra's Needle’ is the name for the stone in 1900, then it is not the name for anything in 2014 because that stone is no more in 2014. However, if ‘Cleopatra's Needle’ is the name for a landmark, then the claim ‘Cleopatra's Needle is not the same stone in 2014 and 1900’ is just the claim that Cleopatra's Needle is not made of the same material at these two dates, and this, once again, can be explained in terms of the ‘is’ of constitution and is not an instance of the disputed form. Wiggins, thus, concludes that there can be no genuine Type 4 cases. He contends that either apparent Type 4 cases collapse into time-indexed Type 3 cases, or they turn out to be cases of constitution relations rather than the genuine identity relation.

Wiggins's response to Type 5 cases adds little to his responses to Type 4 cases (Wiggins 2001: 43-50). The various examples turn out either to involve the ‘is’ of constitution (examples (1.4) and (1.5)), a qualitative identity relation ((1.6) and (1.7)), or a simple ambiguity in the denotation of the singular terms ((1.6), (1.7), (1.8), and (1.9)). In Wiggins's view, only one example seems to involve unambiguous expressions and two identity relations. This is example (1.10). Wiggins concludes that this statement is necessarily false, as it is self-contradictory, assuming all relations of numerical identity satisfy LL. I leave discussion of this example to Chapter 7. I will move instead to consider a theory which accepts the possibility of Type 4 and Type 5 cases.

[8] Wiggins' appeal to an ‘is’ of constitution is controversial and even some philosophers committed to an orthodox theory of identity would reject that there is any such things as an ‘is’ of constitution (for example see Noonan 1976, who tries to show that none of the cases Wiggins considers are cases of RI without appealing to an ‘is’ of constitution).

1.1.3 An Alternative Theory

It seems, then, that absolute identity can be supplemented by a series of claims about the logical structure of statements in natural language, which explain the apparent counterexamples. Wiggins has provided us with a way of construing each of the apparent examples such that they are not counterexamples to absolute identity. However, we are not duty bound to accept Wiggins's translations. I will now consider a rival theory, which challenges both the Fregean Analysis and the theory of absolute identity.

I have already given a rough sketch of the relative identity view, first advocated by Geach. I will now begin to set out this view in detail. We will begin with Geach's own words:

> (1.11) "x is the same A as y" does not "split up" into "x is an A (and y is an A) and x is the same as y." (Geach 1962: 39)
>
> (1.12) "Being the same water" cannot be analysed as "being the same (something -or-other) and being water." (Geach 1968: 151)
>
> (1.13) When one says "x is identical with y" this, I hold, is an incomplete expression; it is short for "x is the same A as y" where "A" represents some count noun understood from the context of utterance–or else it is just a vague expression of a half-formed thought. (Geach 1967: 3)
>
> (1.14) On my own view of identity I could not object in principle to different As being one and the same B... (Geach 1968: 157)
>
> (1.15) No criterion has been given, or, I think, could be given for a predicable's ⟨predicate's⟩ [9] being used in a language L to

[9] Geach uses the term 'predicable' in place of the more familiar 'predicate'. Geach feels that the latter is ambiguous in the following way. The sentence 'The man who broke the bank at Monte Carlo dies in misery' has, according to one use of the term 'predicate', only a single predicate, namely '... died in misery'. According to an alternative use of 'predicate', that same sentence has, at least, two predicates, the one noted above, but also '... broke the bank at Monte Carlo'. That the latter is a predicate can be seen in the sentence

express absolute identity. The familiar axiom schemata for identity could at most guarantee, if satisfied, that the relative term under investigation will be true in L only of pairs that are indiscernible by descriptions framed in terms of the other predicables of L. This cannot guarantee that there is no proper extension of L, with extra predicables, that makes possible the discrimination of things which were indiscernible by the resources of L. (Geach 1991: 297)

(1.16) [I]t seems clear that on my general view of numbers and identity it is quite useless, indeed nonsensical, to characterize a proper name as a name whose sense restricts it to naming only *one thing*; instead, what has to be explained is a name's being a proper name *for an* A; and such a name may be a shared name of several Bs, so long as each such B is the same A as any other. I thus came to accept the view of Lesniewski and other Polish Logicians that there is no distinct syntactical category of *proper* names. (Geach 1980: 15)

1.1.4 Theses

It is clear from the above excerpts that Geach rejects both the Fregean Analysis (Quotes (1.11) and (1.12)) and absolute identity (Quotes (1.15) and (1.16)). As far as positive theses go, the following three suggest themselves:

SRI: All identity relations have the form '... is the same F as...',

'John broke the bank at Monte Carlo'. The term 'predicate' is thus ambiguous between these two uses. Geach's solution is to use 'predicable' to replace 'predicate' when used in the second sense and to reserve 'predicate' for the first sense of the term only (Geach 1980: 50). Although Geach's point is well taken, I will use the traditional terminology throughout, except when quoting.

where F is replaceable by a sortal term. (Quote (1.13))[10,11]

RI: There exists some true statement of the form: $\ulcorner x =_F y \wedge x \neq_G$

[10] In the vast literature on identity relations, many philosophers can be found advocating some principle similar to SRI. Geach has more explicitly defended the claim that 'x is identical with y' is true only if there is some general term, A, such that x is the same A as y. However, this statement of the thesis remains ambiguous, and the different philosophers who would agree to this claim, would also attach different interpretations to this thesis. I therefore suggest a more specific form of this thesis in my statement of SRI, which I think better reflects the thesis that Geach intends.

Geach distinguishes between two kinds of general terms, on the one hand, substantival and, on the other hand, adjectival (Geach 1980: 63). Geach attributes this distinction to Aquinas (IA q 39). On occasion, Geach seems to suggest that any general term can complete the expression '... is the same ... as ...' (Geach 1969: 558). This position is open to a number of objections, not least of which is that it would then be impossible to distinguish between relations of numerical and qualitative identity (Geach/Feldman 1969: 558-559). In later works, Geach is careful to specify that the only multi-place predicates that express identity are those that involve substantival (sortal) terms (Geach 1980: 63-64). This is so because only sortal terms provide a criterion of identity. In the absence of such a term and the corresponding criterion, grammatical relations of identity are incomplete and, assuming no sortal term and criterion of identity can be supplied from the context of utterance, then the sentences in which such grammatical relations are found do not express propositions. Therefore, SRI involves sortals rather than simply general terms. Moreover the claim is not simply that statements involving identity entail statements involving sortals, but rather that statements involving identity relations involve sortals as a part of their semantic content.

[11] One further important point about SRI is in order. It is tempting to think that SRI entails the thesis that there exists a relation of identity which is logically triadic. After all, the expression '... is the same F as ...' would need to be filled with three separate word-tokens to express a determinate proposition, and by SRI, every relation of identity instantiates this form. Geach, however, is keen to reject this view, at least if it is taken to mean that there is just one relation of identity, but a triadic one, which can be turned into a proposition by appropriate assignment of terms. Geach appeals to an insight of Wittgenstein's:

> Some reader perhaps thinks that 'the same' is always the same, and criteria of identity are just a matter of psychology; if so, I may here quote Wittgenstein's parody: 'High pitch is high pitch' it's merely a psychological matter whether you hear it' (like the pitch of a scream) 'or see it' (like the pitch of a roof). The relation expressed by 'of higher pitch than' has in fact the same logical properties in both cases; but that does not mean there is just one relation, which we happen to learn about by two different avenues of sense; and the like, I maintain, holds for identity relations. (Geach 1972: 249)

For Geach, then, the general form '...is the same ... as ...' does not itself express a particular relation. Rather, his view is that, for every assignment of a sortal to fill the middle gap, there is a particular dyadic relation of identity. However, this does not, in Geach's view, mean that any two such relations are instances of the same triadic relation.

$y \wedge (G(x) \vee G(y))\urcorner$. (Quotes (1.14) and (1.16))

GT: There exists no absolute identity relation. (Quote (1.15))

I will devote the rest of this chapter to considering some of the relationships between the theses I have now introduced. However, before I can do this, I will try to shed a bit more light on two notions that are important for what follows. These are the notions of a sortal and a criterion of identity.

1.1.5 Sortal Terms

The term 'sortal' is one that, until now, I have been taking for granted. It is worth providing a more rigorous explanation of how this term is to be understood, particularly as there are several different uses of the term to be found in the literature. The term is originally due to Locke (*Essay Concerning Human Understanding*: III, III: 15), but the contemporary usage of the term is due, in part, to a distinction drawn by Geach, in *Reference and Generality*, between two kinds of general term. Geach does not use the term 'sortal' but rather distinguishes between 'substantival' and 'adjectival' terms. Geach's distinction is as follows:

> [S]ubstantives have (singular or plural) number on their own account, whereas adjectives have a number determined by the nouns they qualify. (Geach 1980: 63)

Geach continues,

> [O]nly in connection with some terms can the question be asked how many so-and-so's there are. For example, although we have the phrase "the seven seas", nobody could set out to determine any division to the water area in the world into seas in the way that the term "letter" (in the typographical sense) does determine a division of the printed matter in the world into letters.
>
> This ... ground of distinction between terms was recognized by Frege and Aquinas. Frege said that only such concepts as

> "sharply deliminated" what they applied to, so that it was not "arbitrarily divisible", could serve as units for counting; to link this up with what I have been saying, we need only observe that for Frege a concept was what a language represented by a general term. Frege cagily remarks that in other cases, e.g. "red things", no finite number was determined. But of course the trouble about counting the red things in a room is not that you cannot make an end of counting them, but that you cannot make a beginning; you never know whether you have counted one already, because "the same red thing" supplied no criterion of identity. Aquinas similarly mentions the grammatical fact that, in Latin, substantives have (singular or plural) number on their own account, whereas adjectives have a number determined by the nouns they qualify; I shall follow him in distinguishing general terms as substantival and adjectival. Grammar is of course only a rough guide here: 'sea', for example, could be an adjectival term, although grammatically a substantive. (Geach 1980: 63)

The distinction Geach is drawing is, so far, between countability and non-countability. The distinction that Geach draws in this passage has become common for distinguishing between sortal and non-sortal terms.[12] This, then, is one of the contemporary uses of the term 'sortal'. According to this distinction, roughly speaking, a general term is a sortal term if and only if it provides a principle for counting its instances. In other words, sortal terms are just those terms that have the logical status of count nouns (Lowe 1989a: 11).

However, Geach goes on:

> Countability is a sufficient condition for considering a term as substantival; this is so because we (logically) cannot count As unless we know whether the A we are now counting is the same A as we counted before. But it is not necessary, in order that "the

[12] Variations on it can be found in P.F. Strawson 1959: 168, John R Wallace 1965: 9, Griffin 1977: 43, and Dummett 1981: 547-648.

> same A" shall make sense, for the question "How many As?" to make sense, we can speak of the same gold as being first a statue and then a great number of coins, but "How many golds?" does not make sense; the "gold" is a substantival term, though we cannot use it for counting. (Geach 1980: 64)

When Geach clarifies his distinction, we see that it is not between those terms which provide a principle for counting the number of their instances and those terms that do not. The former are a proper subset of substantival terms. For Geach, mass terms may also serve as substantival terms. Geach's distinction, then, is between any term, F, for which the following construction makes sense, '... is the same F as...' and those terms for which the construction does not make sense. This way of phrasing the distinction is not sufficiently rigorous, however, because it does not exclude dummy-sortals. The expression '... is the same thing as...' makes good grammatical sense, and yet, 'thing' is not a sortal/substantival term (whether '...is the same things as...' expresses a genuine relation without the involvement of some sortal is at issue between those philosophers who accept SRI and those who do not). What Geach means is that any general term, F, is a sortal if and only if it provides a criterion of identity for Fs. The notion of a 'criterion of identity' is also in need of explication, which I will provide in the next section. For the moment, a criterion of identity is a principle according to which it is possible to determine the truth of sentences of the form 'x is the same F as y'.

This, then, is the second notion of sortal to be found in the literature.[13] Note that the criteria of identity and principles of counting are not equivalent (I will have more to say about the relationship between identity and cardinality in Chapter 4). A criterion of identity for Fs may provide a principle by which it is possible to determine an answer to the question 'how many Fs' or to the question 'how much F'. A principle for counting may provide a

[13] Lowe agrees with Geach that mass terms may be sortal terms. However, Lowe does not think that association with a criterion of identity is a sufficient condition for a term's being a sortal, because he allows some basic sortals that lack criteria of identity (Lowe 1989a: 20-21). Apart from basic sortals, Lowe's distinction runs parallel to Geach's, though Lowe uses the term 'sortal' in place of Geach's 'substantival', (Lowe 1989a: 10, Lowe 2007: 515).

determinate answer only to the former question (Lowe 1989a: 11). For what follows, I will be using the term 'sortal', rather than 'substantival'. However, the use of the term 'sortal' in the rest of this dissertation corresponds to Geach's use of 'substantival'. I therefore permit mass terms to be sortal terms, and I distinguish sortal terms from non-sortal terms on the basis of whether or not they provide a criterion of identity.

1.1.6 Criteria of Identity

We have distinguished between those general terms which are sortals and those that are not by appealing to the notion of criteria of identity. This is another notion which has several different uses. I will try to clarify how I intend to use the expression.

Geach makes a distinction between two kinds of criteria. One of these he describes as 'the standard we judge by'. The other is that according to which we recognize something (Geach 1973: 288). The latter are merely principles which we happen to use in practice for the purposes of identification. For example, we recognize a man by his face, in the latter sense of the term 'criterion'. But it is not a criterion of being that man that he have a particular face (Geach 1973: 288). A principle of recognition is not what Geach has in mind when he introduces the notion of criteria of identity. However, Geach does not make it clear what is meant by 'standard by which we judge', if it is not a principle by which we recognize.

We may come to a better grasp of the relevant notion of criteria of identity by considering some of the more recent work that has been done on the notion. The most sustained examination of criteria of identity has been undertaken by Lowe.

> One thing I should emphasize is that a "criterion of identity", as I am now using the expression, is not to be conceived of as a heuristic or evidential or in any other sense purely epistemic principle, but rather as a semantic rule (though, obviously, questions of knowledge and meaning cannot be wholly separated). That is to say, it is not a requirement of a criterion of identity in my sense

> that it should necessarily provide us with an effective means of coming to know whether or not a given identity statement, "x is identical with y", is true: rather it should tell us, so to speak, what it takes for x and y to be that same or different or, in terminology drawn from Locke, "wherein their identity or diversity consists". In other words, it should specify ... in an informative way, be it added ... the truth-conditions of the statement "x is identical with y?" (Lowe 1989a: 15-16)

For Lowe, then, criteria of identity are semantic criteria rather than epistemic criteria. Although Geach's use of the expression 'standard by which we judge' is ambiguous, I suggest that he intends to make the same distinction as Lowe is making. For what follows, I will also assume that a criterion of identity is a semantic principle, as Lowe has explained that notion. Note that, if criteria of identity are semantic rather than epistemic criteria, any given criterion of identity, say the criterion of identity for Fs, may fail to provide a guide to establishing whether, in fact, x and y are the same F. A semantic criterion can establish what it means for x to be the same F as y, without setting conditions under which we could ever actually establish the truth of any such sentence.

What, then, will a criterion of identity look like? We may first consider a highly influential example of a criterion of identity, to be found in the work of Frege:

> The judgement "line a is parallel to line b?" can be taken as an identity. If we do this, we obtain the concept of a direction, and say: "the direction of line a is identical with the direction of line b." (Frege [1884] 1968: 74)

In other words, the direction of some line is the same direction as that of another line if and only if the two lines are parallel. We may note an interesting feature of this criterion. If we take the left-hand of the biconditional, we may construe it as a relation of numerical identity only if we take the *relata* to be the direction of the lines. If we do this, then the criterion for the identity of directions is a relation that holds between lines.

In other words, the Fregean criterion of identity has the form:

$$(1.16)\ \forall x \forall y (f(x) =_F f(y) \leftrightarrow Rxy)^{14}$$

Where '$f(\ )$' stands for a function (in the above case, a function from lines to the direction of lines). This sets the identity conditions for Fs as a relation that holds between Gs.

This form of criteria of identity is in contrast to the simpler form favoured by Lowe:

$$(1.17)\ \forall x \forall y ((F(x) \wedge F(y)) \rightarrow (x =_F y \leftrightarrow Rxy))^{15}$$

According to (1.17), the conditions for the identity of x and y involve a particular relation holding between x and y. (1.17) faces a circularity worry that does not face (1.16). (1.17) involves quantification over Fs in both sides of the biconditional. In other words, comprehension of the right hand side of the biconditional depends on prior understanding of the expression $F(x)$. Some philosophers hold that we can only sensibly assert $F(x)$ if we understand the criterion of identity for Fs (for example, P.F. Strawson 1959: 168). Geach, for one, certainly holds this; Geach thinks that all statements of the form $F(x)$ are 'derelativizations' of sentences of the form $\ulcorner x =_F y \urcorner$ (Geach 1973: 287-302).[16] Geach's notion seems to involve a sort epistemic priority. Therefore, if this thesis is true, the right hand side of the biconditional in

[14] The formulae (1.16) and (1.16) are proposed as forms for criteria of identity in Lowe 1989b.

[15] (1.16) and (1.17) are sometimes called 'first-level' and 'second-level' criteria of identity respectively.

[16] Noonan (1997: 637-639) suggests that a key move in Geach's writings is one that we have not, till now, considered. This is what Noonan calls 'The derelativization thesis'. According to this thesis, statements of the form 'x is an F' are 'derelativizations' of statements of the form 'x is the same F as x' Where 'F' is some sortal term (Geach 1960: 106). What this seems to mean is that relations of relative identity, involving a sortal term 'F', are, in some way, more fundamental than simple predications of the sortal term 'F', such that the latter are derived from and entailed by the former. In other words, it does not make sense to say 'x is F' unless it makes sense to say 'x is the same F as x'. Indeed, it seems that Geach thinks that we cannot understand the former utterance unless we understand the latter.

Noonan seems to think that the derelativization plays a key role in supporting RI. Consider the following purported case of RI. 'a is the same word-type as b, but a and b are different word-tokens, and either a or b are word-tokens.' A major objection to cases

(1.17) is only comprehensible to someone who already understands the criteria of identity for Fs. (1.17), then, would be viciously circular as a criterion of identity for Fs. Other philosophers reject the claim that comprehension of $F(x)$ depends on understanding the criterion of identity for Fs. If this is true, then statements of the form (1.17) can provide informative criteria of identity.[17]

Geach, then, can only accept criteria of identity having the form (1.16). However, for what follows, I do not propose to assume that Geach is right that predications involving sortals presupposes grasping the criteria of identity associated with the relevant sortals. Therefore, I will accept criteria of identity of either form (1.16) or form (1.17), so long as they cannot be shown to presuppose a prior understanding of the criteria of identity that they are intended to assert. However, given that criteria of identity are semantic rules, in Lowe's sense, we must reject the possibility of explicitly impredicative criteria of identity, that is, criteria of identity which involve the notion for which they provide a criterion in the criterion itself.

Having made the notions of sortal and criterion of identity sufficiently clear for my purposes, I will now consider the relationships between the

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of this type is that, it is not clear that a and b are being used unambigiously throughout. Are they word-types? Or word-tokens? If they are word-tokens, according to absolute identity, then to say that they are the same word-type does not entail that they are word-types at all, but simply that they are equiform word-tokens. Geach, however, thinks that words can be both word-types and word-tokens at the same time. Geach may appeal to the derelativization thesis for support for this view. If the dereletivization thesis is true, then, it might be thought, the statement 'a is a word-type' is just a derelativization of the statement 'a is the same word-type as a'. Thus suggest that the latter entails the former. If this entailment does hold, then, in the disputed case, a and b are word-types, and the disputed case does turn out to be a case of RI.

However, this appeal to the derelativization thesis to support RI does not seem very strong to me. Even if the derelativization thesis is true, it can only be true for a limited number of statements of the form 'x is the same F as y'. If it held for all such cases, then we could infer from 'x is the same colour as y' to 'x and y are colours'. Naturally this is not a valid inference, and Geach would reject it. Therefore only some relations of the form '... is the same F as...' can be derelativized. Naturally, in the type-word/token-word case, the theory of absolute identity entails that the relation '... is a type-word as...' is not one of these relations. To assume that it is would simply beg the question in favour of RI. In light of this, whatever the significance of the derelativization thesis, I do not think it provides support to RI and I will not consider it further in this dissertation.

[17] for a more complete discussion of this issue, see Lowe 1989b.

various theses involved in Geach's theory of relative identity.

1.2 Relationship Between Theses

Although Geach provides numerous arguments for his theory of relative identity, it is far from clear which of the component theses need to be argued for separately and which are entailed by the others. Therefore, before I consider the arguments themselves I wish to briefly map out the relations between the theses, and the sort of argumentative support that the strong theory of relative identity requires. I will argue that each of that theses is logically independent.

1.2.1 Independence of SRI, RI, and GT

I will look at the relationships between each of the three theses, beginning with the relationship between SRI and the other theses. Consider the following case. If there can exist no identity relation without reference to some specific sortal term, then surely Geach is right when he says '"x is the same A as y" does not "split up" into "x is an A (and y is an A) and x is the same as y"' (Geach 1962: 39). I take this to follow immediately from SRI. It is easy to demonstrate that SRI and the Fregean Analysis are incompatible. Assume that the Fregean Analysis is true. Thus, if 'x is the same F as y' is true, then 'x is the same as y' is also true. However, according to SRI, sentences of the form 'x is the same as y' do not express propositions (assuming that context does not convey sufficient information to determine a sortal-relative relation, for which '... is the same as...' is an abbreviation) and so can never be true. So it turns out that, given SRI, either all statements of the form 'x is the same F as y' are false, or it is false that 'x is the same F as y' splits up into predication(s) and absolute, non-relativized identity statements. Naturally there are some true statements of numerical identity, and so, SRI and the Fregean Analysis are incompatible. So, if SRI is true, then Geach is certainly right to say that relativized identity statements do not split up. But what follows from this? Does this provide any support for either RI or GT?

SRI does not support GT. GT is true if and only if there are no relations of absolute identity. SRI is true if and only if all relations of numerical identity involve sortal terms. But it is, *prima facie*, possible that some relation is a relation of absolute identity and also involves a sortal term. For example, it is compatible with absolute identity, that '... is the same man as ...' be a sortal-relative absolute relation of identity. So, it seems that SRI may be true and GT false. So SRI does not entail GT.

Moreover, as Noonan points out 'it may be that whenever a term "A" is interpretable as a sortal term in a language L the expression (interpretable as) "x is the same A as y" in L will be satisfied by a pair $\langle x, y \rangle$ only if the I-predicate of L is satisfied by $\langle x, y \rangle$', where an I-predicate is a predicate which expresses indiscernibility relative to L (Noonan 1997: 637). In other words, even if all relations of identity involve sortals, nevertheless all of these relations may satisfy reflexivity and LL. If all relations of identity satisfy LL, then RI is false. So SRI does not entail RI either.[18]

Similarly, there is no entailment relation in the opposite direction. RI does not entail SRI for the simple reason that RI requires only that there be some relations of identity which do not satisfy LL. RI, unlike SRI, is compatible with the existence of some unrelativized and absolute identity relations. For the same reason, RI does not entail GT, either.[19]

It might be that Geach assumes that SRI follows from GT. This at least would explain Geach's otherwise surprising lack of argument for SRI. I will consider Geach's defence of SRI, such as it is, in due course, but SRI does not follow trivially from GT. It is compatible with the truth of GT that there

[18] For an informal model of this possibility, imagine a language in which there is no word corresponding to the English 'same' or 'identical', but rather only a prefix which attaches to sortal terms, to generate sortal-relative relations. Moreover, in this hypothetical language it is a grammatical rule, followed without exception, that this prefix is only used if the relation thereby formed satisfies the four laws of absolute identity relations. This entails that relations of identity are all relative to sortal terms, so SRI is true, but that all such relations are relations of absolute identity, which entails the falsity of both RI and GT.

[19] Weak theories of relative identity generally accept RI but reject GT and SRI. Griffin (1977) is representative of this view, he thinks that there are true cases of RI and therefore some relations of numerical identity which do not satisfy LL. At the same time, he thinks that there are some genuine relations of absolute identity, and that these need not involve sortal terms.

be non-relativized relations of numerical identity, so long as these are not relations of absolute identity. A dyadic relation of numerical identity may fail to be a relation of absolute identity, if, for example, it did not satisfy LL. Given this, GT on its own does not entail that there are no genuine dyadic relations of identity; it entails merely that, if there are dyadic relations of numerical identity, they are not relations of absolute identity. So GT is in fact compatible with the falsity of SRI.[20]

Moreover, GT does not entail RI, either. GT, being a negative existential claim, does not entail anything about what sorts of identity relations do exist. RI, by contrast, entails that there are relation of identity which do not satisfy LL.[21]

It seems, then, that SRI, RI, and GT are three mutually independent

[20] Curiously, Wiggins claims there is an entailment between these two theses (Wiggins 1967: 27). However, he is wrong, as is shown by the development of consistent formal systems that involve SRI but reject RI (see Griffin and Routley 1979).

[21] More can be said about the relationship between GT and RI, however this raises some issues which I have not yet introduced. I will, therefore, confine this discussion to a footnote. We will see that there are different versions of LL. Of particular importance to the present issue, is the distinction between those versions of LL which involve quantification over the properties of all possible languages and those that involve quantification over a fixed stock of properties. We will see, further, that Geach's argument for GT, which I will defend, entails that there are no relations of the former kind, and that relations of the latter kind are not absolute identity relations. Thus, taken one way, GT entails that there are no relations satisfying the four formal features of absolute identity. However, taken another way (that is, where LL involves quantification over a fixed stock of predicates) GT does not entail that there are no relations with these formal features. It merely entails that such relations are not relations expressing identity. In fact, I think the view that there are no relations satisfying the weaker version of the four formal features is simply incoherent. If it is true, then there would be no relations which satisfy reflexivity and fixed-stock-LL. We can, in fact, go further than this. For any relation, R, that does satisfy fixed-stock-LL in a language, there is a restriction of the domain of that language, which involves all and only those elements that satisfy R. In this fragment, R would satisfy both fixed-stock-LL and reflexivity. This shows that there would be no relations at all that satisfy fixed-stock-LL.

This, it seems to me, is incoherent. Consider a theory with a domain involving only one element, x. Presumably, it is necessarily true that it is not the case that $\phi(x)$ and not $\phi(x)$, where ϕ is replaceable by any predicate from the fixed-stock. But from this, it follows necessarily that, if it is the case that $\phi(x)$, then $\phi(x)$. From this, naturally, we can derive, necessarily $\phi(x)$ if and only if $\phi(x)$. Finally, for any relation whatever, simply by adding an antecedent to a necessary truth, we get the following: necessarily, if x R's x, then $\phi(x)$ if and only if $\phi(x)$. It, therefore, seems to me that there must be some relations which satisfy the fixed-stock version of LL.

theses. Thus, a convincing proof of a Geachean theory of strong relative identity would require separate arguments for each of these three theses. Having determined that all three theses require support if the strong theory of relative identity is to be defended, I will make a few observations on the kind of support that is required, before I consider the arguments in the literature.

1.2.2 How to Evaluate Theories of Identity

Noonan suggests that there is an asymmetry involved in evaluating the two classes of identity theories, in that theories of relative identity cannot be supported by an appeal to examples, while the truth of a theory of absolute identity can be (Noonan 1980: 3).

This claim comes in the context of Noonan's discussion of the thesis that I have termed 'GT', and it is not clear that this goes for each of Geach's theses. I will, therefore, consider in turn the form a sound argument for each of these theses would take.

GT

Noonan is certainly right that the thesis GT cannot be proved with an example. GT makes a negative existential claim, that there are no absolute identity relations. Naturally, such a claim cannot be established by an example. Nor can the claim be established by considering every single relation to establish whether or not it is a relation of absolute identity. Instead, a sound argument for GT must involve a conceptual claim about the properties a relation of absolute identity would have and a further claim that it is *a priori* impossible for any object to exist having these properties. We will consider such an argument in Chapter 2.

RI

RI involves the rejection of LL. More particularly, it entails that there exists some relation of numerical identity which does not satisfy LL.[22,23] However, this is not to say that an example will be easy to come by. We have already considered a series of apparent examples of identity relations which do not satisfy LL. In each case, however, we saw that there is some plausible interpretation of the terms involved such that the antecedent does not involve a relation of numerical identity, so the sentences did not provide counterexamples to LL. Moreover, we might think that it will be possible to interpret all apparent counterexamples to LL in this way. As Noonan points out,

> [I]t is hard to see how such a denial ⟨of LL⟩ could be argued for; if a case is described in which objects x and y differ in their properties is this not the best reason there could be for saying they are distinct? If someone insists on a counterexample to Leibniz's Law why is this not simply evidence that he means by the expression for identity something different from the rest of us? (Noonan 1980: 3).

[22] Some relative identity theorists advocate 'modifying' LL rather than rejecting it (For example, Griffin 1977: 140-141, Wiggins 2001: 46). When I come to discuss LL in detail, I will consider several alternaitve renderings of the principle.

[23] Noonan claims that, if GT is true, there are no counter-examples to LL, but equally, there are no relations which satisy LL (Noonan 1980: 3). Noonan's argument for this point is not clearly articulated, but I think his idea is as follows. The thesis which Noonan is calling 'Leibniz's Law' is expressible as a material conditional, which involves a relation of absolute identity in the antecedent. In other words, Noonan has the following schema in mind:

(LL′) $\forall x \forall y(x = y \rightarrow (F(x) \rightarrow F(y)))$ (Noonan 1980: 1)

If GT is true, then there are no relations of absolute identity, and so, (LL′) is ill-formed, since the meaningless symbol, '=', is present in the antecedent. A counterexample to LL would involve an instance of a true antecedent and false consequent. But if the formula '$x = y$' is ill-formed, then there is no possibility of a true antecedent, and therefore, there is no possible counter-example to (LL′).

This is all true of course, but I did not express Leibniz's Law with the schema (LL′) but rather with the informal schema LL. LL did not involve the primitive symbol '=' but the schematic letter 'R'. I have also claimed that a relation is a relation of absolute identity only if it satisfies LL. There are of course many relations for which 'R' can be substituted such that LL is false. A counterexample to Leibniz's Law would consist of a relation of numerical identity which was not replaceable for 'R' *salva veritate*. It seems, then, that Noonan's judgement that RI cannot be proved with an example was premature.

In other words, if the notion of numerical identity involves LL, then surely there could not be a compelling counterexample to the principle (we will have more to say about the conceptual claim that the notion of a relation of numerical identity involves that relation satisfying LL in Chapter 3, Section 2). However, there is the possibility that the intuitive connection which, Noonan claims, holds between numerical identity and LL, will conflict with some equally strong intuition that a proposed counterexample is both a relation of numerical identity and does not satisfy LL. In other words, the conceptual claim that all relations of numerical identity satisfy LL is not safe from revision in light of a sufficiently motivated example of an apparent relation of numerical identity that does not satisfy LL. For example, Zemach argues that the identity conditions for vague objects depend on the existence of identity relations that do not satisfy LL (Zemach 1974, 1983, 1991). If this is true, and if the existence of vague objects is a thesis which is better established than the thesis that all relations of identity satisfy LL, then the latter may give way.

We therefore must leave open the possibility of an example which supports the thesis RI. In Chapter 3, Section 1, we will look at several arguments which propose such examples.

SRI

SRI, like GT, makes a universal claim, namely that all relations of numerical identity involve sortals. Again, it is clear that such a thesis cannot be proved with an example, for examples of numerical identity relations involving sortals would only licence a conclusion about some relations of numerical identity. This thesis could only be proved by an argument showing the *a priori* impermissibility of relations of numerical identity which do not involve sortals. I will consider one such argument in Chapter 4.

Thus, the strong theory of relative identity cannot be proven with examples alone. At least some of the theses will require *a priori* argument. I wish to consider now how matters stand with the theory of absolute identity. *Prima facie*, absolute identity is in no different a position than the strong theory of relative identity. For absolute identity involves both universal claims, for example that all relations of numerical identity satisfy LL (the rejection of RI), and existential claims, that there is at least one relation of absolute identity (the rejection of GT).[24] It would seem, at first glance, that the first is not a claim that can be established by an example, and that the second claim can be. In fact, I think this is deceptive, and that the rejection of RI can, in principle, be proved with an example, while the rejection of GT cannot. Again, we will consider each of these in turn.

¬RI

To see how an example might support the falsity of RI, consider once again Noonan's claim that non-satisfaction of LL is the best reason for thinking that some relation is not a numerical identity relation. If this is so, then all relations that do not satisfy LL are not relations of numerical identity. The only worry with this policy is that it might result in unacceptable consequences for our ontology, if for example, Zemach is right in thinking that it would require the abandonment of vague objects. Worries like this can be avoided if enough examples of numerical identity relations which do satisfy LL can be introduced to provide identity conditions for all the objects in the domain of discourse. An example of a relation which satisfies LL and which is (strongly) reflexive would therefore provide strong support for the falsity of RI. If such a relation does exist, then defenders of absolute identity, following Noonan, may take this relation as the only relation of numerical

[24] We may note at this point that, of all the theses involved in the strong theory of relative identity, only one of them is compatible with, though not entailed by, absolute identity. This is SRI. The falsity of SRI could be proved with an example, if a compelling example of a relation of numerical identity which does not involve a sortal can be found.

identity. This would entail the falsity of RI. Similarly, a series of examples of weakly reflexive absolute identity relations, such that the union of the sets of their *relata* is co-extensive with the domain of discourse, would, again, support the rejection of RI.

¬GT

It might be supposed that the falsity of GT can be proved with one compelling example of a relation of absolute identity. However, things will turn out to be more complicated than this. We will see that the argument for GT turns on the claim that there cannot be relations of absolute identity because there is no criterion for a predicate's expressing absolute identity. If this argument is sound, then no example would suffice. If this argument fails, then the falsity of GT can be established with an example, and such an example will not be difficult to find. I will conclude by noting several examples of apparent relations of absolute identity to which defenders of absolute identity have appealed in the past to demonstrate the falsity of GT:

> (1.18) 'x is essentially a man and y is essentially a man and x is the same man as y' (Anscombe in Noonan 1980: IX).[25,26]

> (1.19) The relation between a thing and itself and a thing and nothing other than itself.[27]

[25] Elizabeth Anscombe, from whom this example is borrowed, seems committed to the rejection of Geach's Thesis.

[26] Notice that this example only seeks to establish an absolute identity relation holding between men, this would presumably not give us sufficient ontology to establish the truth of absolute identity generally. As such, it serves (if the example is accepted) only to defeat strong relative identity, not weak relative identity.

[27] This relation (if such a relation exists) would prove absolute identity. Naturally, it is a relation which is bound to be rejected by the strong relative identity theorist (and almost certainly by the weak absolute identity theorist also). The real significance of this example is that it shows just how much the relative identity theorist is forced to give up. In particular, by denying that this relation is an absolute identity relation, the relative identity theorist must give up a metaphysics of subsisting objects, distinguishable from one another absolutely. This shows that this kind of metaphysics is incompatible with the strong theory of relative identity. This sort of worry comes across most clearly in the objections to relative identity from Wiggins 2001.

(1.20) The relation between the referent of a name and the referent of the same name.[28]

1.2.4 Conclusion

In Chapter 1, I have presented the three theses that are central to Geach's strong theory of relative identity. I have argued that each would need to be supported independently. The first thesis I will consider is GT. A successful argument for GT would need to show that the very notion of absolute identity is such that there cannot be any absolute identity relations. In the next chapter, I will consider such an argument from Geach.

[28] This example shows how theories of relative identity differ from more orthodox views, with respect to philosophy of language. To explain why (1.20) is not a relation of absolute identity would require a very complex theory of naming. Geach attempts to solve this problem with his distinction between names for and names of (Geach 1980: 70). Once again, if this view of language is false, then so too is GT. These sorts of worries come across most clearly in Dummett's attacks on relative identity (Dummett 1973, 1991).

2. GEACH'S ARGUMENT AGAINST ABSOLUTE IDENTITY

In his various writings on identity, Geach provides several arguments in favour of his theory of relative identity. In *Reference and Generality*, Geach claims, but does not try to prove, that the orthodox theory of identity is incoherent because it implies the existence of one or more relations of absolute identity, and no such relations exist (Geach 1962: 38-39). In a subsequent article, simply entitled 'Identity' (1967), Geach attempts to provide an argument for this position, that is, for GT. Geach continues to defend his view in further papers (1969, 1973, 1991). In each case, Geach alters his attack on the orthodox theory of identity. The deductive structure of the argument presented in Geach's papers is therefore, at times, unclear. It has been criticized on a number of separate grounds, and the received wisdom is that the argument fails to establish the conclusion that no relation of absolute identity exists (Noonan 1997: 642-645, Hawthorne 2003: 120-123). In what follows, I will revisit Geach's argument. Section 1 will involve a close analysis of the argument, as presented by Geach. I will consider the major objections against Geach's argument, and I will propose the emendations that are required to answer these objections. In Section 2, I will charitably reconstruct Geach's argument. I will consider each of the premises of the reconstructed argument in turn, and I will suggest that several of the premises are likely to be rejected. I will consider how Geach might further motivate these premises. I will conclude that the argument is ultimately inconclusive and that its force ultimately depends on a series of further philosophical issues that lie beyond the scope of this dissertation.[1]

[1] Geach's argument encouraged a long list of responses and replies. Wading through the literature has therefore become a difficult task. I will provide a brief chronology here. The

2.1 Geach's Argument

In the following section, I will present the key claims of Geach's case against absolute identity. I will outline the apparent deductive structure of Geach's argument and consider the objections that have been raised against the argument. I will note where the objections succeed and how Geach's argument might be amended to avoid these objections. The central adjustments that I propose will be as follows:

> (2.1) elaborate the inference from the absence of a criterion for a predicate's expressing absolute identity to the non-existence of relations of absolute identity,
>
> (2.2) replace reference to Wang's Schema with reference to the axiom schemata reflexivity and LL,
>
> (2.3) consider what grounds there might be for rejecting set-theoretic definitions,
>
> (2.4) provide an explicit argument from the proposed second-order criteria to Grelling's paradox,

and

> (2.5) abandon Geach's attempt to show that Quine's proposal for theory reinterpretation is incoherent.

2.1.1 Geach's Thesis

Geach begins his argument against absolute identity with the following thesis:

argument appears first in the aforementioned paper 'Identity' in 1967. An early reply from Feldman can be found in *The Review of Metaphysics*, alongside a response from Geach and a further rejoinder from Feldman (Feldman/Geach 1969). Next published, is an insightful reply to Geach by Nelson (1970). This is followed by an elaboration of the argument by Geach (1973), a response from Dummett (1991), and a final contribution from Geach, in which he responds to his critics generally (Geach 1991: 276-299). All of this is in addition to the discussions of the argument that can be found in most publications which consider theories of relative identity in any depth, such as Noonan (1997) and Hawthorne (2003).

> I am arguing for the thesis that identity is relative. When one says 'x is identical with y', this, I hold, is an incomplete expression; it is short for 'x is the same A as y', where 'A' represents some count noun understood from the context of utterance–or else, it is just a vague expression of a half-formed thought. (Geach 1972: 238)

In light of what follows, however, this seems an odd way for Geach to phrase his intended conclusion. This passage suggests that the central aim of the paper is to establish the thesis that relations of identity involve general terms as a part of their content, in other words, a thesis akin to SRI. While it is certain that Geach holds these theses to be true, it is equally clear that the argument he presents in this paper is aimed at another conclusion altogether.

Geach concludes his argument with the following:

> We thought we had a criterion for a predicable's ⟨predicate's⟩ expressing strict identity; but the thing has come apart in our hands; and no alternative rigorous criterion that could replace this one has thus far been suggested. I urged initially on intuitive grounds that there just is no such notion as unqualified identity; it now looks as though my intuition was reliable. I might say: the prosecution rests. (Geach 1972: 241)

Here Geach reaches the conclusion that there is no notion of 'unqualified identity', though he really means that there is no relation of unqualified (or absolute) identity.[2] His grounds for concluding that there is no such thing as absolute identity are simply that there is no criterion for any predicate's expressing absolute identity.

Geach's further clarifies his position in subsequent papers:

> I can state once more the point I have often made. No criterion has been given, or, I think, could be given for a predicable's being

[2] As we will see, Geach's argument depends on there being a concept of absolute identity, but a concept which necessarily has the null class as its extension.

> used in a language L to express absolute identity. (Geach 1991: 297)
>
> ...if we say "Whatever is true of x is true of y, and conversely" without restricting "true of" to the predicables of some language L, it is not clear that we have managed to say anything. The absolute identity that was opposed to merely numerical difference is a chimera; absolute indiscernibility is a will-o'-the-wisp that we pursue in vain. (Geach 1973: 298)

Geach speaks of a predicate's expressing strict identity. 'Expressing strict identity', then, names a meta-linguistic property that predicates may or may not have. Whether any predicate does in fact express absolute identity depends partly on whether there is, in fact, any such property and partly on whether or not the extension of the concept corresponding to that property is the null class.[3] As I understand Geach, he means to claim that the extension of the concept '... is a relation of absolute identity' is the null class. Although some of Geach's comments seem to suggest the former contention, I cannot credit this to Geach, as it would not make sense of his deductive strategy. Geach takes for granted that we have a particular idea of what a relation of absolute identity would be, as we will see. He uses expressions like 'strict identity', 'absolute identity', and 'unqualified identity' regularly and clearly intends his reader to understand something by them. He, therefore, cannot mean literally that there is no such notion, but rather that there are no relations corresponding to such a notion. This is an issue to which I will return later in the chapter when considering an objection from Hawthorne (2003). For now, having identified Geach's intended conclusion, we will look at how he tries to get there.

[3] Noonan makes a similar sounding point, which must be distinguished from mine. He argues that Geach cannot mean that there are no predicates expressing absolute equivalence, but must instead mean that all predicates that do express absolute equivalence must have the null class as their extension (Noonan 1980: 3). Noonan thinks that Geach must intend the second meaning, rather than the first, because some predicates are bound to express absolute equivalence, as he defines the expression. This is a result of Noonan's, rather confusing, definition of 'absolute equivalence'. My own definition does not have this result. In my view, Geach does intend that there are no predicates that express absolute equivalence relations.

2.1.2 Criteria

As we have seen, a central premise of Geach's is that there is no criterion for a predicate's expressing absolute identity. Geach does not explain why a criterion is needed, he also does not explain what sort of criterion he has in mind.

Geach is explicit that he is not demanding a definition of identity, although he has occasionally been interpreted as making this demand (Feldman/Geach 1969: 549, Hawthorne 2003: 122). In response to one of his critics, Geach responds:

> I did not suggest Feldman's formula (6) as a definition of identity; nor did I argue that if a man cannot define a word, then he has no concept answering to it.
>
> What I did put into the mouth of an absolute-identity man was a *thesis*:
>
> (T) x is identical with y iff whatever is true of x is true of y and conversely. This differs from Feldman's (6) by all the difference between a statement and a convention, which comes out in the difference between "iff" and "$=_{df}$". (Feldman/Geach 1969: 557)

Geach, perhaps, fails to make the distinction between definitions and criteria sufficiently clear, especially as it is by no means agreed amongst philosophers what the relation '$=_{df}$' signifies. It seems, though, that Geach intends his demand for a criterion to be, in some sense, weaker than a demand for a definition. It might be that Geach takes the latter but not the former to involve intentional equivalence. Geach does tell us, however, that the form of a criterion must involve material equivalence.

Geach cannot be demanding a criterion of identity in this instance. A criterion of identity is relative to some sortal term, F, and provides conditions for the truth of sentences of the form 'x is the same F as y'. It seems rather that, in this case, we are looking for the conditions under which it is true to say, of some predicate P, that 'P expresses absolute identity'.

There is, however, another kind of criterion very closely related to criteria of identity to which Geach attributes importance. A criterion of application (alternatively, 'conditions for application') is a criterion for some x's falling under some general term, 'A'. Geach thinks all general terms, both sortal and non-sortal, have criteria of application (see the discussion of Geach's views in Dummett 1973: 547-548). These provide us with truth conditions for statements of the form 'x falls under the general term "A"'.

We saw, when we considered the notion of criteria of identity, that these were semantic principles rather than epistemic ones, which is to say, in Lowe's words 'semantic principles whose grasp is essential to an understanding of ... general term⟨s⟩' (Lowe 1989b: 13). Similarly, criteria of application are semantic, not epistemic. Which suggests that, if there is no criterion of application for Fs, then 'for some x, x is an F' does not express a determinate proposition.

Although Geach is nowhere explicit that the criterion he is interested in is a criterion of application for the general term 'relation of absolute identity', this would make sense of his argumentative strategy. Should we, then, accept Geach's claim that all general terms have criteria of application?

If this claim is true, it would follow that, if there is no criterion of application for the expression 'relation of absolute identity', there are no true sentences of the form 'P is a relation of absolute identity'.

Some philosophers may reject this claim, however. Lowe, for example, in the context of criteria of identity, thinks that some concepts are so basic that they are exceptions to the otherwise general demand for criteria (Lowe 1989a: 20-21). Perhaps this goes for criteria of application as well, moreover, perhaps identity is one of these ultimately basic concepts.

If identity is this kind of concept, then Geach's demand, and, indeed, the argument which follows, will prove unavailing. Geach, naturally, is unlikely to be impressed by this response. Geach holds that there is no relation of absolute identity. As such, he is clearly not disposed to agree that the concept is semantically privileged in this way.

It is not clear how this sort of dispute can be resolved. In any case, I will not try to resolve it here. If there are concepts that do not require criteria of application, and if absolute identity is one of these concepts, Geach's argument will not work. In the interests of pursuing Geach's line of though, however, we shall assume that the concept of absolute identity does require a criterion of application.

Given this assumption, if Geach wishes to show that there is no notion of absolute identity, he could do this by showing that there is no criterion of application for the term 'relation of absolute identity'. However, as I have said, I cannot credit this view to Geach. Instead, I think his claim is that there is no non-contradictory criterion of application. Note that there is nothing wrong with a contradictory criterion of application for a general term. It means merely that, necessarily, the corresponding concept has as its extension the null set. This, then, is Geach's strategy, as I understand it.

With a rough notion of a criterion for a relation's expressing absolute identity in hand, we can now consider the structure of Geach's argument.[4]

2.1.3 The Deductive Structure of Geach's Argument against Absolute Identity

Geach begins by introducing what he takes to be the orthodox theory of identity. In Geach's view, the orthodox theory of identity consists of a single thesis which can be framed as an axiom schema and appended to FOL to pro-

[4] In fact, Geach seems to reject the universal claim that all general terms requires criteria of application elsewhere in his writings. In his textbook *Reason and Argument*, Geach has the following to say, which sheds light on his attitude to terms of disputed application:

> One useful way of coming to understand the meaning of an imperfectly clear term is to produce some good example where the term plainly applies. Plato represents Socrates as objecting to this procedure: unless we already know quite well what the term means, there will be no unexceptional examples to show us; so example are useless anyhow. The truth is that if misunderstanding arises it may be resolved either by producing criteria for using a term or by giving good clear examples: we can work from examples to get criteria that will fit them, and we can use criteria to apply the term to new examples. (Geach 1976: 40)

In fact, Geach must be talking about epistemic principles here, rather than the semantic criteria of application that we are interested in. For the latter, it is clear that Geach thinks there are no exceptions. Every general term must have a corresponding semantic criterion of application in his view.

duce FOL with identity (henceforth, FOL$^=$). The schema Geach introduces on behalf of the absolute theory of identity is as follows:

(2.6) $\vdash Fa \leftrightarrow Vx(Fx \wedge x = a)$[5]

Geach calls this 'Wang's Schema' and references Quine's *Set Theory and its Logic* (1963: 13) as its source. In that work, the Schema appears as:

(2.7) $\vdash Fy \leftrightarrow \exists x(x = y \wedge Fx)$

In both versions, 'F' is ranging over all possible predicates, rather than just sortal terms. From such a schema, the four traditional laws of identity can all be derived as theorems.[6,7] Having identified his target, Geach goes on:

> In face of these well-known results, it may seem an enterprise worthy of a circle-squarer to challenge the classical theory of identity. All the same, it has an Achilles' heel, as I hope to show. (Geach 1972: 239)

So what, then, is the supposed 'Achilles heel' of the theory of absolute identity? Geach's next move is to introduce the notion of an I-predicate (in Geach's own vocabulary, 'an I-predicable'), defined as any two-place predicate, P, such that 'P' can replace the occurrence of '=' in (2.6)/(2.7) where the resulting schema is truth-preserving. Geach then raises a problem for I-predicates:

> However, if we consider a moment, we see that an I-predicable in a given theory T need not express strict, absolute, unqualified identity; it need mean no more than that two objects are indiscernible by the predicables that form the descriptive resources of the theory.' (Geach 1972: 240)

[5] Note that 'V' is a symbol that Geach uses for existential quantification (Geach 1972: 239).

[6] Proof of reflexivity: Taking (2.7), Replace 'F' with $\ulcorner x \neq \urcorner$, we get $y \neq y \leftrightarrow \exists x(y = x \wedge y \neq x)$. We can of course prove that $\neg \exists xy = x \wedge y \neq x$ and, given the biconditional, we get $\neg(y \neq y)$.

[7] Proof of LL: (2.7), we can derive, by weakening the biconditional, $((\exists x)(x = y \wedge Fx)) \rightarrow Fy$. Note the scope of the quantifier, this entails that $\forall x(x = y \rightarrow (Fx \rightarrow Fy))$. Note that this last forumla is frequently used as a formulation of LL. My version, with a biconditional in place of the final conditional is derivable from it.

Geach concludes from this that I-predicates do not express absolute identity. This is the supposed 'Achilles' heel' of the theory of absolute identity. The thought seems to be this: satisfaction of Wang's Schema (in the sense of replaceability *salva veritate* of 'P' for '$=$') might be thought to provide a criterion for some P's expressing absolute identity. However, relations which satisfy Wang's Schema do so relative to a given theory, T. There is no guarantee that a given I-predicate-relative-to-T also satisfies Wang's Schema relative to some other theory, T'. This, Geach seems to think, is good reason for concluding that satisfying Wang's Schema does not provide a sufficient condition for being an absolute identity relation. Thus, Geach concludes, Wang's Schema does not provide a criterion for a relation's expressing absolute identity.

I have suggested that we must understand 'criterion' here to mean 'criterion of application'. In other words, Geach's concern here is whether Wang's Schema can provide the expression 'relation of absolute identity' with a criterion of application. It seems, then, that Geach's attack on absolute identity has the following structure:

2.1.4 Argument 1

P1. The theory of absolute identity proposes the following criterion of application for the term 'relation of absolute identity': Any predicate, P, is a relation of absolute identity if and only if P satisfies Wang's Schema.

P2. Every criterion of application provides a sufficient condition.

C1. If the theory of absolute identity is true, then the proposed criterion provides a sufficient condition.

P3. The proposed criterion does not provide a sufficient condition for a predicate's falling under the term 'relation of absolute identity'.

C2. The theory of absolute identity is false.[8]

Let us consider Geach's premises in turn.

2.1.5 P1

The theory of absolute identity proposes the following criterion of application for the term 'relation of absolute identity': Any predicate, P, is a relation of absolute identity if and only if P satisfies Wang's Schema.

Geach does not argue at any length for P1, and the premise might be thought to be a straw man. This charge has some substance as there are alternative and more popular criteria for a relation's expressing absolute identity that Geach does not consider. In fact, few logicians make use of Wang's schema (this was so even at the time of Geach's first paper on identity) to characterize identity in a first-order language. It is more common to introduce reflexivity and LL individually, by appending them as axiom schemata to FOL, as was done in Chapter 1, or by simply adding the corresponding rules of inference.[9]

It should also be noted that the conjunction of reflexivity and LL is introduced as a *characterization* of absolute identity in first-order logic, not as a criterion of a predicate's expressing absolute identity. Moreover, the first-order characterization of absolute identity is not introduced as a definition of identity, either. As we saw in Chapter 1, absolute identity is first-order indefinable but is thought by many to be definable in either a second-order language, with LL serving as *definiens*, or by using the tools of set theory, for example as follows:

'$x = x$'$=_{def} < x, x > | x \in A$[10,11]

[8] My Argument 1 resembles Nelson's reconstructed version of Geach's argument (Nelson 1970: 243). Other responses to Geach's argument invest too little effort into establishing the intended deductive structure of Geach's case against absolute identity.

[9] In Geach's final discussion of his argument, he does focus on the two axiom-schemata rather than Wang's Schema (Geach 1991: 296-299).

[10] Enderton 2000: 5.

[11] Informally, we may say: Where A is a set, the expression '$I(A, \zeta, \eta)$'$=_{def}$ the partition

or, alternatively,

Geach does not explicitly argue for the indefinability of absolute identity, and Geach is not arguing that we can infer from the indefinability of absolute identity to the non-existence of absolute identity (Geach 1969: 556-557), though he has occasionally been misinterpreted as doing so (Geach/Feldman 1969: 549, Hawthorne 2003: 122). Nevertheless, if identity is definable in second-order logic or in set theory, a criterion of application for a relation's expressing absolute identity can be formed by stipulating that a relation falls under the term 'relation of absolute identity' if and only if the relation has the property of satisfying the *definiens*. In other words, if absolute identity is second-order definable or definable set-theoretically, then there is an apparent criterion for a relation's expressing absolute identity which can be provided in either second-order logic or set theory.

Given this, Geach is attacking a straw man unless either his critique of Wang's Schema can extend to the alternative characterizations of identity, or independent reasons can be provided for rejecting these characterizations as criteria of application. Given this, my charitable reconstruction of Geach's argument will target the axiom-schemata LL and reflexivity, rather than Wang's Schema. I will also consider what grounds Geach might have for thinking that the other characterizations of absolute identity in the literature fail to provide criteria of application for the term 'relation of absolute identity'. Geach does, in fact, have something to say about definitions of absolute identity, other than those involving set theory. Geach dispenses very quickly with informal definitions in his 1967 article, his objection to a typical informal definition of absolute identity runs as follows:

> "For real identity", we may wish to say, "we need not bring in the ideology of a definite theory T. For real identity whatever is true of something identical with a is true of a and conversely, regardless of which theory this can be expressed in; and a two-place predicable signifying real identity must be an I-predicable no matter what other predicables occur along with it in the the-

of A by $I(A, \zeta, \eta)$ gives the set of ordered pairs $\langle x, x \rangle$

> ory." But if we wish to talk this way, we shall soon fall into contradictions; such unrestrained language about "whatever is true of a", not made relative to the definite ideology of a theory T, will land us in such notorious paradoxes as Grelling's and Richard's. (Geach 1972: 240)

So Geach rejects the informal definition of the absolute identity relation as 'whatever is true of something identical with a is true of a and conversely', on the grounds that talk of 'whatever is true of a' leads to Grelling's (1936) and Richard's paradoxes (1905). Geach does not provide an explicit argument from the semantic paradoxes to the inadmissability of expressions such as 'whatever is true of a', and some critics have disputed whether he is right about this (Geach/Feldman 1969, Nelson 1970). A satisfactory reconstruction of his argument will certainly need to fill this gap. I will provide such an argument in Section 2. I will consider only Grelling's paradox, as Richard's would complicate matters without shedding additional light.[12,13]

Grelling's paradox is normally framed as a contradiction derived by adding to a language certain stipulatively defined adjectives. However, the paradox can be framed just as easily using stipulatively defined predicates rather than

[12] Although all the paradox's of self-reference share a similar structure, Geach's appeal to Grelling's and Richard's paradoxes is chosen for good reason. Geach could not have called on the more famous paradoxes of self-reference, like Russell's or the liar paradox, because the standard solutions to these do not suggest that there exists, in any given language, some property which is inexpressible. We will see that this is o ne possible response to Grelling's paradox, which might provide support to GT.

[13] Richard's paradox is closely related to Cantor's diagonalization argument, but, as in the Grelling paradox, incoherence arises from our apparent ability to construct certain predicates in ordinary language, which lead to contradictory results. In brief, this paradox arises by running a Cantor-style diagonal argument with real numbers defined in a natural language. Consider the following. It seems that we can define real numbers in a language such as English. Moreover, it seems that if we were to compile a list of such definitions, that list would be infinite. We order the corresponding list of real numbers by the length of their English definitions. It seems that we now have an ordered list of all the defined real numbers. However, we can introduce a new definition as follows: a real number $<$ 1, for each decimal place, n, n is occupied by 2 if the nth decimal place of the nth real number on the list of real numbers is 1, otherwise n is occupied by 1.

This definition is guaranteed of defining a real number which is not on the list. But it must be on the list, because this contains all real numbers defined in English. Thus we arrive at a paradox. Notice that our definition was entirely arbitrary, we could have chosen any number of ways of defining a number that was on the list (I.J. Good 1966) adjectives. Let us attempt to add to the English language two second-order predicates, defined as follows:

'... is heterological' $=_{def}$... is a predicate which is not true of itself.

'... is homological' $=_{def}$ ... is a predicate which is true of itself

We run into the paradox when we attempt to predicate '... is heterological' of itself. Are we to say that '...is heterological' is itself heterological? Or is it homological?

The problem is this. If we attempt to say '"... is heterological" is homological', then we are saying that '... is heterological' is true of itself. This is of course just to say that '"... is heterological" is heterological'. Thus, we have contradicted ourselves. We have said that '... is heterological' is both heterological and homological. But if we begin by saying that '" ...is heterological" is heterological', then clearly we have just said of '... is heterological' that it describes itself, in which case we must admit that '... is heterological' is homological, and again, we reach a contradiction. It seems, then, that introducing the stipulatively defined predicate '... is heterological' into a language leads to inconsistency. It is Geach's contention that talk of 'whatever is true of x' will have the same consequence.

In his final work on identity, Geach makes explicit that his argument against the informal definition, by appealing to Grelling's paradox, is intended to serve as an argument against any criterion of a relation's expressing absolute identity which involves quantification over all predicates (Geach 1991: 297). On account of this paradox, Geach thinks that the standard formal second-order definition of identity,

'$x = y$'$=_{def}$ $\forall P(P(x) \leftrightarrow P(y))$[14]

leads to Grelling's paradox if we interpret the third quantifier as ranging unrestrictedly.[15] In Section 2, we will consider if an argument from Grelling's

[14] Russell and Whitehead [1910] 1970: 168

[15] It seems that Geach's argument, if sound, would undermine unrestricted second-order objectual quantification generally. It is not clear if Geach intending this further result.

paradox can be provided, which shows that a criterion of application involving second-order definitions of absolute identity would entail that the extension of the concept *relation of absolute identity* is the null class.

2.1.6 P2

Every criterion of application provides a sufficient condition.

P2 hardly needs defending. If C is a criterion of application for 'P', then C has the form 'x is (an) P if and only if ϕ'. So the right-hand side of C provides a sufficient (and necessary) condition for the satisfaction of the general term for which C provides a criterion of application. This is simply what it is to be a criterion of application.

2.1.7 P3

The proposed criterion does not provide a sufficient condition for a predicate's falling under the term 'relation of absolute identity'.

Geach argues for P2 at much greater length then he does for P1. Geach argues that the alternative to taking the schematic letter in Wang's Schema to be replaceable by any possible predicate, prohibited, Geach contends, by the semantic paradoxes, is to take it as ranging over just those predicates found in the ideology of a given theory, T. However, this does not guarantee that any x and y jointly satisfying some I-predicate really are identical, claims Geach.

> We can never so specify what we are quantifying over that we are secure against an expansion of our vocabulary enabling us to discriminate what formerly we could not.(Geach 1973: 301)

In other words, it is possible that there is some extension of T, T', with a larger vocabulary, such that for some pair, $\langle x, y\rangle$ jointly satisfying some I-predicate in T, x and y are found to be discernible relative to T'. This means, of course, that x and y do not satisfy LL relative to T' and therefore that the relation that holds between them does not satisfy Wang's Schema relative to T'. Geach attempts to bolster his case with an example.

2.1.8 Example

Take the two-place predicate:

> '...is equiform with...'

There is a theory, T, the ideology of which involves sufficient predicates to distinguish one word-type from another but insufficient predicates to distinguish two different word-tokens of the same type. Take the sentence '"horse" is equiform with "horse"'. Relative to T, the two-place predicate '... is equiform with...' is an I-predicate because T lacks the resources to distinguish any two equiform words. Now imagine an addition to the ideology of T of sufficient predicates to distinguish between two tokens of the same word-type. We will call the expanded theory 'T''. If the occurrences of 'horse' are interpreted as referring to token words, then, relative to T' the predicate is no longer an I-predicate. Thus, claims Geach, the predicate never expressed an absolute identity relation at all. It seems that this goes for any theory-relative I-predicate, for nothing can prevent the addition of new predicates to a theory. So long as there exists the possibility of adding new predicates to the ideology of a theory, the possibility of discriminating what we were previously unable to discriminate will remain.[16]

Intuitively, it seems that if x and y are absolutely identical, they are one and the same thing, and this does not seem to be a theory-relative matter. On the basis of this intuition, if a predicate is an I-predicate relative to T and not an I-predicate relative to T', then it expresses absolute identity in

[16] Geach remarks, in response to one of his critics, that it is confused to set up the relationship between theories in temporal terms (Feldman/Geach 1969: 557). Rather, claims Geach, the distinction must be made in timeless, set-theoretic, terms. Apart from the fact that Geach himself does not follow his own advice consistently, Geach's concern is motivated by a desire to try to demonstrate claims which I will ultimately reject anyway, so I will continue to set out the relationship between theories and their sub-theories in temporal terms, as this way of expressing the issue makes it easier to grasp.

neither T nor T'. Or rather, to be more precise, if a predicate, P, is an I-predicate relative to a theory T, but for some pair of objects, $\langle x, y \rangle$ satisfying P in T, x and y are discernible relative to T', an extension of T, then P did not express an absolute identity relation in T. It is this intuition which gives force to Geach's initial argument. Geach's word-type/word token example is intended to provide us with a demonstration of the truth of the second premise of Argument 1. I will call this defence of P2 'Argument 2'. The structure of Geach's Argument 2 seems to be as follows:

2.1.9 Argument 2

P2.1. For any objects x and y, if x and y are discernible relative to the resources of any theory whatever, x and y are not absolutely identical in any sub-theory of that theory.

P2.2. Wang's Schema is theory-relative, in the sense that the schematic letters range over those predicates that form the ideology of a given theory. So, for any pair $\langle x, y \rangle$, it is possible that $\langle x, y \rangle$ satisfies Wang's Schema relative to some theory, T, but that x and y are discernible relative to some other theory T', where T' is an extension of T.

C2.1. For any x and y, it is possible that x and y satisfy Wang's Schema and are not absolutely identical. (from P2.2)

C2.2. Therefore, Wang's Schema does not provide a sufficient condition for some relation's being a relation of absolute identity. (*Modus Ponens* P2.1, C2.1)

Once again, we will consider the truth of the premises in turn.

2.1.10 P2.1

For anything x and y, if x and y are discernible in any theory whatever, x and y are not absolutely identical in any sub-theory of that

theory.

Geach neither defends P2.1 nor even makes explicit that he is asserting it. However, the validity of Geach's argument requires P2.1. More particularly, I think that Geach is assuming that the notion of absolute identity is incompatible with the following state of affairs: x is absolutely identical with y and there is some model of a theory, T, in which x has some property that y lacks.

This premise, however, has been challenged. Hawthorne (2003: 122-123) has argued that Geach's claim here is self-defeating. Geach's claim is that the discernibility of x and y in any theory whatever entails the non-absolute identity of x and y. However, Hawthorne argues, this entailment is only true if absolute identity is characterized by a version of LL which ranges over the predicates of all possible languages. But Geach himself has argued that this version of LL entails a contradiction. In other words, Hawthorne thinks that P2.1 derives its plausibility from a notion of absolute identity which Geach cannot accept in the first place. A successful argument will need to show how this objection can be avoided. I will do this in Section 2 of Chapter 2.

2.1.11 P2.2

Wang's Schema is theory-relative, in the sense that the schematic letters range over those predicates that form the ideology of a given theory. So, for any pair $\langle x, y\rangle$, it is possible that $\langle x, y\rangle$ satisfies Wang's Schema relative to some theory, T, but that x and y are discernible relative to some other theory, T', where T' is an extension of T.

As we have seen, Geach thinks that, in order to avoid the semantic paradoxes, we must interpret the schematic letter used in expressing Wang's Schema as being relative to a given language. This, claims Geach, guarantees that cases like the word-type/word-token case introduced above are always possible.

Moreover, as Geach says,

> It is useless to protest that this need not happen; that such a situation can arise is already enough to show how inadequate a test it is for a predicable's expressing absolute identity in a language L, that when it is used in L the familiar axiom schemata for identity are satisfied. (Geach 1991: 298)

P2.2 has been heavily criticized by Dummett. One of Dummett's criticisms is that Geach's position 'depends upon an oscillation between taking a language or a theory, to be interpreted and taking it to be uninterpreted' (Dummett 1991: 165). I will now identify the supposed oscillation and show how it can be eliminated.

Responding to the possibility of the word-type/word-token cases that Geach suggests, Dummett argues as follows:

> Whether this can happen or not depends not only upon how the domain has been specified, but also on the interpretation of the predicates of L. If, in M, they were so interpreted that $=_L$ does not come out as the identity relation over the domain D, then the situation envisaged by Geach is possible. If, however, the predicates of L were so interpreted that $=_L$ came out as the identity relation over D_M, then no such possibility exists. (Dummett 1991: 164)

In other words, Dummett thinks that word-type/word-token cases are impossible so long as we are careful to interpret the terms of the theory in a certain way. Moreover, Dummett thinks that this fact is insufficient to establish Geach's thesis that Wang's Schema fails as a criteria of a relation's expressing absolute identity.

In Dummett's view, Geach's argument is targeting a thesis, numbered '(iv)' in Dummett's paper, which Geach attributes to the absolute theory of identity. That is: 'For any language L, we can so construe L that, if $=_L$ holds between x and y, then x and y are absolutely identical.' (Dummett 1991:

164) Dummett then argues that the mere possibility of interpretations of a theory which allow for word-type/word-token cases is not enough to establish the falsity of (iv). As Dummett puts it,

> Given a model M of T (formulated in L), by restriction we obtain a model M^0 of T^0 (formulated in L^0) having the same domain as M. In the case imagined, $=_{L^0}$ will not, in M^0, denote identity. But that does not disprove thesis (iv), which merely stated that there would be some model of T^0 in which $=_{L^0}$ denoted identity. If we consider such a model, say M^1, it need not be a submodel of M. (Dummett 1991: 165)

If Dummett is right in taking (iv) to be the position that Geach is attacking, then he has shown that Geach's argument fails. However, Geach nowhere ascribes (iv) to the absolute theory of identity, and the soundness of his argument does not depend on the falsity of (iv). Geach's conclusion, as we have seen, is that the satisfaction of Wang's Schema does not provide a sufficient condition for a relation's expressing absolute identity. This conclusion will be true so long as there are even some interpreted theories which involve relations which do satisfy Wang's Schema and do not express absolute identity.

The issue may be brought into sharp relief by considering the alternative possible formulations of the premise P2.2. P2.2 is ambiguous between the following:

> (2.8) There is no interpretation of the terms of T' such that x and y are indiscernible.

> (2.9) There is at least one interpretation of the terms of T' such that x and y are discernible.

> (2.10) We take T and T' to be interpreted theories, which is to say that the terms must be interpreted in T' the same way that they are interpreted in T.

To put each of these more accurately:

> **P2.2′**: For any pair $\langle x, y\rangle$ in a theory T, the following is true. $\langle x, y\rangle$ may satisfy Wang's Schema relative to T, but there exist extensions of T, T' such that the truth of T', under any interpretation, demands that x and y are discernible.

> **P2.2″**: For any pair $\langle x, y\rangle$ in a theory T, the following is true. $\langle x, y\rangle$ may satisfy Wang's Schema relative to T, but there exist interpretations of a theory T', an extension of T, such that the truth of T' demands that x and y are discernible.

and

> **P2.2‴**: Given a structure, M, of a theory, T, some pair $\langle x, y\rangle$ may satisfy Wang's Schema relative to T, while given, M', an extension of M and a structure of an extension, T', of T, x and y are discernible relative to T'.

P2.2′ and P2.2″ take T and T' as uninterpreted theories, allowing x and y to be interpreted differently in T' from their interpretation in T. P2.2‴ takes the theories T and T' as interpreted theories, thus interpreting x and y differently in T' from how they are interpreted in T is ruled out.

Dummett thinks that P2.2′ is obviously false. He argues that P2.2″ is true, but presuming we read P2.1 along the same lines as P2.2, P2.1 would then be false. For we cannot conclude anything merely from the fact that x and y can be interpreted in such a way that they are discernible. So which of these did Geach intend? Moreover, can any one of them be used to generate a sound argument for Geach's conclusion?

Let us begin with option P2.2′. Dummett is right that P2.2′ is simply false. There must be at least one interpretation of the terms of T' which is such that all the pairs jointly satisfying Wang's Schema in T also satisfy Wang's Schema in T'. A charitable interpretation of Geach's argument will reject P2.2′ as the correct reading of P2.2.

We turn, then, to option P2.2$''$. Taking P2.2$''$ as the intended meaning of P2.2 would require a new version of P2.1 as well. The argument would depend on the claim that Wang's Schema is a necessary and sufficient condition for a predicate's expressing a relation of identity if and only if that relation were such that there were no interpretation of its *relata* such that any pair were discernible. If P2.1 in this form were to be accepted, then Geach's argument would be valid. But the mere statement of the revised P2.1 shows that it is also false. Of course there are some interpretations of x and y in any theory such that x and y are discernible (Dummett 1991: 162-163). This will be true just so long as the domain of discourse contains any two things that are discernible. But the mere fact that, according to some interpretation, x and y are discernible does not show us that x and y are not identical. Geach cannot intend P2.2$''$ as the correct version of P2.2, because he cannot accept the corresponding version of P2.1.

Finally, we turn to option P2.2$'''$. Reading P2.2 as P2.2$'''$, Geach is claiming that there is a least one case where, for some x and y, x and y are indiscernible with respect to a theory, T, and discernible with respect to T', an extension of T, and crucially, where all the common terms of T and T', including of course x and y, have the same interpretation in both T and T'.

Geach fails to adequately clarify his position between options P2.2$'$, P2.2$''$, and P2.2$'''$, however he undoubtedly holds P2.2$'''$, even if he seems to defend the other claims at times as well. Moreover, option P2.2$'''$ is the only one which is not obviously false and thus has a hope of supporting a sound argument for P2. Therefore, I advocate option P2.2$'''$, and, in my charitably reconstructed version of the argument which follows, I will assume that option P2.2$'''$ is the intended meaning of P2.2.

At this point Geach tries to defend P2 from what he considers to be a possible objection. The objection is based on a proposal of Quine's. Since this plays an apparently large role in Geach's argument and is the source of much of the criticism of Geach's argument, it would be well to set it out in depth. The relevance of the objection may not be clear at first, and ultimately I will argue that Geach would have been better to ignore it altogether, as it does not in fact threaten any of the required premises of Geach's argument.

2.1.12 Quine's Proposal

Quine defends the following thesis:

> In general we might propound this maxim of the identification of indiscernibles: Objects indistinguishable from one another within the terms of a given discourse should be construed as identical for that discourse. More accurately: the references to the original objects should be reconstrued for purposes of the discourse as referring to other and fewer objects, in such a way that indistinguishable originals give way each to the same new object. (Quine 1953: 71)

What Quine intends is best understood by considering another example:

> Suppose a discourse about person stages, and suppose that whatever is said about any person stage, in this particular discourse, applies equally to all person stages which make the same amount of money. Our discourse is simplified, then, by shifting its subject matter from person stages to income groups. Distinctions immaterial to the discourse at hand are thus extruded from the subject matter. (Quine 1953: 71)

Thus, let us take T' to be the discourse discussed by Quine, which by hypothesis, takes person stages as the elements of the domain of discourse. Now let us consider the set of sentences in that theory in which the only property of person stages relevant for judging the truth of the sentence is how much money each person stage possesses. This set of sentences is of course a sub-theory of T'. We will call this sub-theory 'T'. Quine is suggesting that we construe the range of the quantifiers in T as income groups rather than person stages, as in T'. This suggestion also has the apparent result that a two-place predicate, such as '...is as wealthy as...', which is an I-predicate relative to T' is also an I-predicate relative to T.

Consider, in T' we may find the statement 'x is as wealthy as y'. It is clear that this is not expressing a relation of absolute identity between x and

y. By hypothesis, the terms 'x' and 'y' map on to person stages. It does not follow from the fact 'x is as wealthy as y', where x and y are person stages, that x is exactly the same thing as y. They could be different person stages with the same net worth. Moreover, the predicate '... is as wealthy as...' does not guarantee indiscernibility between person stages, and so '...is as wealthy as...' is not an I-predicate relative to T'.

Things stand differently with regards to T, however, if we adopt Quine's proposal. The quantifiers in T range over income groups. Again consider the sentence 'x is as wealthy y'. It is clear that if x and y are income groups, and if they are equally wealthy, then they are simply one and the same income group. So it seems that '... is as wealthy as...' is an I-predicate relative to T.

If this policy is always followed, we will never find a relation that satisfies Wang's Schema relative to one theory but does not satisfy Wang's Schema relative to an extension of that theory. This is because for any pair, $\langle x, y\rangle$ indiscernible relative to T, in any extension of T, T', and featuring a statement of the form $\ulcorner F(x) \wedge \neg F(y)\urcorner$, Quine's procedure demands that x and y be interpreted differently in T' from how they were interpreted in T. This proposal may seem to cause a problem for Geach, because his defence of P2 depends on the possibility of cases of the word-type/word-token variety. Consistently following Quine's procedure would act to prevent any such case from occurring.

This procedure is familiar to contemporary logicians. The structure of T in which 'x' and 'y' are interpreted as income groups is called the 'quotient-structure' of T. Indeed, many responses to Geach are willing to accept that satisfaction of Wang's Schema (or reflexivity and LL) does not, on its own, provide a necessary and sufficient condition for a relation's expressing identity (Nelson 1970), at least not in the sense that it guarantees that all interpretations of a given I-predicate must construe it as an absolute identity relation (Geach 1991: 162-163). Rather, the claim is that a given I-predicate can always be interpreted as expressing absolute identity, so long as we follow Quine's procedure, and that this is enough to refute Geach's argument against the existence of absolute identity relations.

2.1.13 Geach's Objections to Quine

Geach says of Quine's response 'As you might expect, I have no knock-down logical answer to this; Quine is hardly going to be caught out in a straightforward logical mistake.' (Geach 1972: 243) Nevertheless, Geach thinks that Quine's suggestion is not one that ought to be adopted. Geach, in different versions of his argument against absolute identity, gives two related reasons for rejecting Quine's proposal. These are first, indecent ontological expansion, and second, the existence of objects that, he claims, are 'incoherent'. I will consider each of these in turn.

Quine's suggestion relies on our ability to construe the range of the quantifiers in such a way that all statements constructible in theory T have the same truth-conditions in an expanded theory, T'. Geach accepts that we can construe the range of the quantifiers in such a way that no contradiction arises but thinks that doing so 'involves a sin against a highly intuitive methodological programme' (Geach 1972: 243). The 'highly intuitive methodological programme' to which Geach refers is that we should maintain the same ontological commitments while allowing expansion of our ideology. This is important, says Geach, referencing Quine himself, because it is implicit in our understanding of the quantifiers that their range remain stable.[17]

Geach explains how Quine's proposal would offend against the programme of maintaining a stable ontology:

> There are many ways of counting ... words: as John Austin remarked, in a rare flash of perceptiveness, type-words and token-words are just two among many ways of counting words. We may, for example, count the dictionary-entry words in a book ... If now we choose to follow Quine, there will be in *rerum natura* ever so many different domains of words, just in one volume on my shelves at Leeds. (Geach 1972: 244)

[17] Geach himself describes his objection to Quine's proposal as '*ad hominem*'(Geach/Feldman 1969: 557). Quine expresses a well known love for 'desert landscapes' in philosophy, and Geach thinks that this is incompatible with the ontological expansion that he thinks is entailed by the proposal. So, concludes Geach, neither Quine nor anyone with the same philosophical preferences as Quine ought to adopt the above proposal.

The point may be summed up as follows. Take again our theory T', expressed in a language with descriptive resources sufficient to distinguish two different tokens of the same word-type, and T''s sub theory T, which is the fragment of T' expressible in a language capable only of distinguishing word-types. The two-place predicate '... is equiform with ...' is an I-predicate relative to T but not an I-predicate relative to T'. In accordance with Quine's proposal, we construe the quantifiers in T as ranging over word types and thus conclude that the predicate '... is equiform with ...' expresses an absolute identity relation. However, when we consider the expanded theory T', we construe the quantifiers as ranging over word tokens. So the truth of 'x is equiform with y', does not entail that x and y are identical. Thus, relative to T', '... is equiform with...' does not express absolute identity. Geach points out that there is yet another sub theory of T', we will call it 'T''', which requires that we construe the quantifiers as ranging over 'dictionary words' (one dictionary word, x, is identical with a dictionary word, y, if and only if they have the same entry in the *Oxford English Dictionary*).

A key claim of Geach's is that, if a theory is committed to the existence of some entity, then any expansion of that theory is committed to the existence of that entity as well. In other words, theory T' is committed to the existence of all the entities quantified over in theories T and T''. Geach considers this to be problematic for Quine's proposal. For surely we are not committed to the existence of dictionary words in ordinary discourse? Still less of the increasingly bizarre kinds of words that we would be required to recognize because they are the assignments of the variables in various sub-theories of T'? And yet, if there is a restriction of the language that we speak, such that the quotient-structure of that language fragment takes dictionary words as the semantic values of some of our referring phrases, then, Geach claims, we must admit dictionary words into our ontology. Geach believes that such a situation would be intolerable.[18]

[18] Quine is, in fact, well aware that his proposal would entail an enlarged ontology. He justifies this as follows:

> Note, however, that from an over-all or absolute point of view the expedient is quite opposite to Occam's razor, for the multiple entities a, b, etc., have not been dropped from the universe; the Cayster has simply been added.

2.1.14 Surmen

We have seen that a very large number of objects might have to be quantified over in order to fulfil Quine's policy of construing the quantifiers in such a way as to allow I-predicates to express identity while guaranteeing that every statement retains the same truth-conditions in each sub theory. In a later paper, Geach adds a second worry. Geach attempts to show that some of the objects that a follower of Quine's proposal would be committed to will be incoherent. Geach argues for this conclusion by introducing his notion of 'absolute surmen'. x and y are the same surman if and only if x and y are male and have the same last name. An 'absolute surman' is a surman that exists independently of any particular man. If the quantifiers range over men, then the two-place predicate '... is the same man as...' is an I-predicate relative to theories which do not have sufficient predicates to distinguish between different men with the same last name. Given what we have said above about an expanded theory carrying over all the ontological commitments of each of its sub theories, it seems that any reasonably expressive theory will have to recognize the existence of both men and surmen. But a surman, by stipulation, is a man. As Geach puts it, 'he has brains in his skull and a heart in his breast and guts in his belly' (Geach 1972: 245). But what, then, is the population of Leeds, for example? Is it the combined total of men and surmen? Every surman is a man by stipulation, so if we are not to count them separately when we count the number of men in Leeds, then we must say that each surman is identical with some (ordinary) man. But which of the men named 'Smith' is Smith (the surman)? Surely none of them. So it seems that the surmen are entities independent of the men. Thus we are committed to the existence of absolute surmen. Geach concludes from this that following Quine's procedure would entail the existence of incoherent objects (Geach 1973: 299-300).

There are contexts in which we shall still need to speak differentially of a, b, and others rather than speaking indiscriminately of the Cayster. Still the Cayster remains a convenient addition to our ontology because of the contexts in which it does effect economy. (Quine 1953: 70)

It may not be clear that this result does, even *prima facie*, amount to an incoherence. However, Geach's point may be expressed as a dilemma arising from the Quinean procedure. The horns of the dilemma are as follows. Either every surman is identical with some individual men, in which case Smith, the surman, is one of the men named 'Smith'. However, the choice of which man seems entirely arbitrary. Or each surman is numerically distinct from each individual man, in which case, the total male population of Leeds is the total of individual men and surmen. This is not an incoherent result, but if Quine's proposal cannot be supplemented by an alternative to these options, the dilemma is, at least, unpalatable.

2.1.15 Responding to the Objections

Every published response to Geach's argument has involved some attack on Geach's objections to Quine (Geach/Feldman 1969 552-554, Nelson 1970: 250-255, Dummett 1991: 172-180, Noonan 1997: 642-645 and Hawthorne 2003: 116-117). Geach seems to present his case against the Quinean suggestion as a key component of his argument against absolute identity, and so, it is not surprising that commentators have assumed that Geach needs to show that the Quinean suggestion is misguided in order to defend his own case. However, I think that this is a mistake.

The arguments against Geach's objections to Quine's proposal are strong. One major point of contention is this: if theory T is a proper sub-theory of theory T', is the latter theory committed to all of the ontological commitments of the former? Both of Geach's objections to Quine's proposal depend on it being the case that the ontological commitments of sub-theories carry over to the extended theory. Several critics of Geach, however, have claimed that this is not the case. Feldman, for example, sees no reason to think that Quine should be committed to the existence of surmen as well as men in a theory which is capable of differentiating two different men who have the same surname. In his view, the move from the smaller theory to the larger involves the rejection of a domain of discourse including the former entities in favour of a domain including only the latter (Geach/Feldman 1969: 553-

554). Dummett argues that ontological commitments carry over only if we take the terms of T as already interpreted. That is, to use our former example, if the term 'x' in a structure, M of T refers to a word-type, then the structure M', an extension of M, of T', an extension of T, will involve reference to word-types. Merely extending an existing structure is contrary to Quine's intentions, claims Dummett. The whole point of the reinterpretation strategy is that 'x' maps on to an entity in T' which it did not map on to in T. This would involve a structure of T' which was not an extension of the structure of the sub-theory T. In other words, Quine's proposal takes theories to be uninterpreted (See Dummett's discussion in 1991: 165-166).

Even if Geach were right in his claim that the ontological commitments of the sub-theory carry over to the extended theory, his argument seems to me to be weak. As Nelson points out, Quine should be happy to grant the existence of creatures such as surmen, and this need not conflict with his taste for desert landscapes, just so long as he can say that a surman is the set of all men with the same last name (Nelson 1970: 253). This would not involve a radical reinterpretation because the terms of T which were interpreted as referring to surmen would still do so. The population of Leeds would remain stable; we count the number of men and women and ignore the infinite number of sets of men and women. There simply is no fact of the matter which man Smith is, any more than there is a fact of the matter which man the set of all black-haired-males is. It seems, then, that Quine has the resources to avoid the problematic conclusions that Geach tries to draw out from his proposal.

So it seems that Geach's case against the Quinean proposal is not very strong. What consequences does this have for Geach's argument against absolute identity? It has been assumed by many of the contributors to the debate (the assumption seems most obvious in Feldman/Geach 1969 and Nelson 1970) that if Geach fails to undermine Quine's proposal, then the latter can provide a response to Geach's attack on absolute identity. It is not at all clear to me that this is so.

I do not think that Geach's argument against absolute identity depends on his ability to show that Quine's proposal is unworkable. Geach's claim is that Wang's Schema does not provide a sufficient condition for a predicate's expressing absolute identity. Wang's Schema fails to provide a sufficient condition for a predicate's expressing absolute identity just in case there is some predicate, P, which satisfies Wang's Schema and does not express absolute identity. Geach has provided an example of one such predicate, namely '... is equiform with...'. Quine's proposal offers a procedure for construing all such relations in such a way that there is no *a priori* objection to interpreting them as absolute identity relations. But John Perry points out that Quine's claim is compatible with Geach's:

> In order to be in the position of denying Geach's position, Quine would have to hold that we must interpret "$E(x, y)$" $\langle$'E' here stands for any equivalence relation$\rangle$ in T as expressing identity. If he holds only that we may do so, he is allowing the possibility of there being an I-predicable that does not express identity and is not in disagreement with Geach. (Perry 1968: 38)

In other words, for Quine's procedure to serve as a counter-example to Geach's conclusion, it would have to be the case that we must follow Quine's procedure. But we have been given no reason to think that we always do happen to follow this procedure in practice.

In fact, Quine seems to grant that we can, in practice, avoid adopting the proposed procedure. It seems rather that Quine has a normative claim in mind. When considering counter-examples to the proposed reinterpretation involved in his group-stages/income groups example, Quine says,

> In cases of this kind we could protest that the interpretation of the universe and predicates has been ill chosen, and that it might better be so rectified as to construe the members of the universe as whole income groups. (Quine 1969: 15)

However, Quine does not follow this up with an argument showing why one interpretation is better than the other or what constitutes an 'ill-chosen' interpretation of a theory. Quine gives us no reason to think that adopting an interpretation of a theory apart from the quotient-structure for that theory violates some norm. It is not even clear what such a norm would be.

Consider a speaker of English who is unfamiliar with the notions either of a word-token or a word-type (surely not an uncommon occurrence). When this speaker says '"horse" is the same word as "horse"', she is not in a position to say whether she means 'horse' the word-type or 'horse' the word-token. Further assume that this speaker's other predicates would be sufficient to distinguish one word-type from another but not one word-token from another. Thus, at the time of utterance, the quotient-structure for our speaker's theory involves word-types. When she learns what word-tokens and word-types are (along with the predicates to distinguish different instances of the former), she is in a position to determine whether her earlier utterance of the phrase '"horse" is the same word as "horse"' involved reference to word-tokens, or word-types, or to neither. What if she were to tell us that, on reflection, her earlier utterance of the expression '"horse" is the same word as "horse"' simply did not convey the thought that the word-type 'horse' was self-identical? Is she wrong to do so? If so, why?

One line of thought runs as follows. Assume that the speaker can only have been talking about word-tokens or word-types. She could not have been talking about word-tokens, since the fragment of English which she spoke at the time did not include sufficient predicates to distinguish any word-token from any other. Therefore, she must have been talking about word-types. This argument might be bolstered by an appeal to Russell's principle. According to Gareth Evans's version of the principle, in order to make a judgement about a, we must be able to distinguish a from all other things (Evans 1982: 89). Evans defends the principle as follows:

> [T]he idea people express by saying that one cannot possess the concept of being F, and be able, for example, to entertain the though that some G is F, without knowing what it is for a particular G to be F. (1982: 109)

This is a claim I would hardly want to dispute. Moreover, Geach's claim that any general term, A, requires a criterion of application is motivated by the idea that we must know what it is for something to be (an) A. Similarly, some degree of understanding of a singular term is a prerequisite to expressing propositions involving that term. However, I deny Evans's stronger claim that we must be able to distinguish a from *all* other things. In particular, that, to understand what it is for a to be an F, a speaker must understand what it would be for a to be a G, for any G that the speaker has a conception of (Evans 1982: 104). Although it is true that to refer to a we must be able to distinguish a from at least some things that are not a, the stronger claim is highly contentious and there are many plausible counter-examples.

Geach thinks that it is false that terms like 'horse' must either be interpreted as word-tokens or as word-types. Geach thinks that such expressions do not fall neatly into one or the other of the two categories, but rather, are simply words, which can have different criteria of identity supplied for them. Responding to objections against his view, Geach says:

> I dismiss the protest that this result if incoherent because the entity in question must be of only one of these three ⟨Geach also allows dictionary-entry words⟩ kinds; there is no *must* about it. We have in view an entity that belongs to the field of those different equivalence relations, and therefore comes under three different counts using different count nouns; each of the count nouns applies–that is how count nouns are used. It is on the contrary the question "But which is it really?" that is incoherent and unintelligible. (Geach 1973: 294)

Note though, even if we do think that words must be interpreted as either word-types or word-tokens, we are not committed to the Quinean procedure. We do not need to accept Geach's ontological flexibility in order to show that Quine's strategy does not save the criterion for expressing identity. What if the speaker insists that she was talking about word-tokens, when she said '"horse" is the same word as "horse"'? The reason that was given for rejecting this as impossible is that the speaker cannot have been talking about a kind of entity which she was unable to distinguish even in principle. But this reason is unsatisfactory. It is false that we are unable to talk about things simply because we are unable to distinguish one from another.

One case to be found in the literature that requires reference to entities that cannot be distinguished in principle are Max Black style universes composed of different spheres having the same properties (Black 1952). As the spheres have no properties to distinguish them, the quotient-structure for a theory about Black's universe would take the domain of discourse as composed of one object, rather than two. We cannot distinguish two spheres, and so, given Quine's proposal, our referring expressions do not map onto two different spheres. There are, of course, philosophers who would accept this, most notably in the recent literature, perhaps, is Hawthorne, who thinks that the sphere(s) is a single object multiply located (O'Leary-Hawthorne 1995). But of course, this is counter-intuitive and runs against the general response to Black's thought experiment.

In addition, at least one view of quantum physics involves the claim that we can quantify over entities which are indiscernible in theory (Krause and French 2006). If Quine's procedure is followed without exception, this is not merely false but logically impossible.

Following Quine's procedure in all circumstances therefore seems to rule out acts of reference and quantification which are in fact possible. For it seems that it is possible to talk about entities that we cannot distinguish. By this same standard, it seems that there exists the possibility of adopting an interpretation of a given theory, which does not coincide with the quotient-structure of that theory.

The result of all this is that, even if we grant that there is a quotient-structure for any theory whatever, and that Geach's arguments against ever adopting Quine's proposal fail, nevertheless Quine's proposal still does not provide a guarantee against objects being indiscernible relative to one theory but discernible relative to an expansion of that theory.

It might be wondered why Geach expends so much effort in attacking Quine's procedure (Geach 1967, 1972: 241-247, Geach/Feldman 1969: 557-558, Geach 1973: 298-302)[19], if, as I contend, the attack was unnecessary

[19] Geach perhaps came to realize the irrelevance of his objections to Quine's procedure

to establish Geach's central hypothesis. I suspect that Geach mistakenly believed that if Quine's procedure was coherent, it would undermine the argument against absolute identity, because it would provide a procedure for avoiding interpretations of theories which involve counter-examples to Wang's Schema. If Geach did think this, it might also explain a curious ambiguity in his argument. On one hand, Geach, in some passages, claims to show merely that the orthodox theory of identity fails because Wang's Schema fails as a criterion for a predicate's expressing absolute identity. On the other hand, in several places Geach claims to be arguing for a stronger thesis; that there is no possible criterion which could be proposed that would salvage the orthodox theory of identity. I suspect that Geach's attacks on Quine's procedure are related to his attempts to prove the stronger thesis. At the same time, the fact that he occasionally commits himself merely to the weaker thesis may suggest that he himself was not convinced of his objections against Quine's procedure.

In any case, Quine's procedure cannot provide a counter-example to either the weaker or the stronger claims. Geach's argument for GT does, in fact, depend on a defence of the stronger claim and he must, consequently, reject the possibility that any satisfactory criterion might be proposed in the future. The weaker claim would provide, at best, inductive support for the conclusion that there are no relations of identity (Calvert 1973: 16). The stronger claim would provide deductive support for the conclusion that there are no relations of absolute identity. My reconstructed version of Geach's argument will be framed accordingly.

We may, therefore, simplify Geach's argument by abandoning the attempts to show that Quine's procedure leads to unwanted results. The other modifications we have noted are as follows. Geach must provide a justification for the inference from the lack of a criterion for a predicate's expressing absolute identity to the conclusion that there are no relations of absolute identity. Geach's argument must be extended to show that all candidate

as a defence of his views on identity. This at least would explain the otherwise surprising absence of this issue in his final writings on identity. His only reference to his earlier discussions of Quine's procedure is to concede the failure of the 'surman' example (Geach 1991: 276-299).

criteria fail. Finally, an explicit argument showing the second-order quantification leads to Grelling's paradox must be provided. The extent to which all these things can be achieved will be considered in Section 2.

2.2 A Charitably Re-constructed Geachean Argument Against the Existence of Absolute Identity Relations

In this section, I will present an argument, charitably reconstructed from the various arguments provided by Geach. I will not defend the soundness of the argument, as I shall note several possible objections to which Geach does not have compelling answers. The conclusion of the argument, about which I will remain neutral, is that the only characterization of absolute identity which might provide a criterion of application for the term 'relation of absolute identity' is a characterization involving LL, where the latter involves quantification over all possible predicates (or, alternatively, properties) which might be added to a language. Moreover, any such version of LL entails a contradiction; therefore the term 'relation of absolute identity' has as its extension, necessarily, the null class. The argument will therefore focus very much on LL. In order to justify this focus, I will begin by making a few remarks about different characterizations of absolute identity to be found in the literature and about the different versions of LL.

We have seen that Geach does not consider all the characterizations of identity which might serve as a criterion of application for the term 'relation of absolute identity'. We may begin by dividing these into four categories:

(2.11) informal characterizations of identity:

For example, The relation that everything has to itself and to nothing else.

(2.12) meta-linguistic characterizations:

For example, 'P expresses absolute identity' is true if and only if P is an absolute identity relation, where '... is an absolute identity relation' is a predicate of the meta-language.

(2.13) set-theoretic characterizations:

For example, P expresses an absolute identity relation on a set A if and only if the following is true:

$I(P) = \langle x, x \rangle | x \in A$, where '$I$' is an interpretation function,

or, alternatively,

for some pair, ρ, ρ satisfies P if and only if $\rho = \langle x, x \rangle$, for some x.[20]

(2.14) characterizations involving LL.[21]

Any of these, in order to characterize the absolute identity relation, must involve some guarantee of indiscernibility. A relation that guarantees the indiscernibility of its *relata* relative to any predicates/properties whatever will be said to guarantee the 'absolute indiscernibility' of its *relata*. A relation that merely guarantees the indiscernibility of its *relata* relative to a fixed-stock of predicates/properties will be said to guarantee the 'relative indiscernibility' of its *relata*. We may now consider the alternatives.

To begin with, informal criteria are simply English translations of one of some version of (2.12)-(2.14). The example characterization, for instance, is an informal translation of (2.13), and so, these also stand or fall together. We can therefore turn to our next category.

[20] I am grateful to my examiner, Ian Rumfitt, for suggesting this alternative to me.

[21] I will not separately consider Wang's Schema, because all different versions of Wang's Schema stand or fall with different versions of criterion (2.14) to which they are logically equivalent.

Turning, then, to (2.12). It is commonplace to take the symbol '=' as a logical primitive in FOL$^=$. The meaning of the symbol '=' may be provided by given by providing truth-conditions for statements of the form $\ulcorner x = y \urcorner$. For example, if the meta-language is English, we might say $\ulcorner x = y \urcorner$ is true if and only if 'x is absolutely identical with y'. Geach, however, has a response to this:

> It is a particularly futile semantic assent to stipulate that a predicable of a language shall express this sort of identity ⟨i.e. absolute identity in the meta-language⟩, and then call this "a complete semantical characterization in the metatheory". (Geach 1973: 297)

It is a 'futile semantic assent' because the expression 'absolute identity' in the meta-language, in Geach's view, would still need a criterion for its application. It is, perhaps, not clear that this is a sufficient reason for rejecting these types of criteria. I have already noted that if there are particular concepts which are so basic to our conceptual framework that the corresponding predicates cannot be provided with a criterion of application, and if absolute identity is such a concept, then Geach's argument will not be sound. This same point can be extended to the current proposal. If there are some predicates that can only be provided with truth-conditions by appealing to the same predicate in the meta-language, and if this process is justified in virtue of the basicness of the concept, then Geach cannot exclude these types of criteria. Once again, it is very difficult to resolve this sort of issue. It is not clear how we might determine which concepts are basic in this way. Once again, I will note that this response, if identity is basic in this way, would render Geach's argument unsound.

We turn, then, to (2.13). There are a number of proposed characterizations of absolute identity to be found in the literature using the tools of set-theory. Take, for example, the following:

> for some pair, ρ, ρ expresses absolute identity if and only if $\rho =< x, x >$, for some x.

The question I will consider, then, is whether this formulae can be used as a criterion for a predicate's expressing a relation of absolute identity. How might Geach respond the candidate criteria? There is a set-theoretic version of Grelling's paradox, but it will not help Geach, as the appeal to Grelling's depends on unrestricted second-order quantification, and the proposed set-theoretic criteria do not involve second-order quantification. Geach, then, must have some other reason for rejecting these possible criteria.

In the absence of any explicit argument from Geach, I suggest that his best case for rejecting set-theoretic characterizations of absolute identity is to try to show that the proposed characterizations fail to rule out counter-examples to absolute indiscernibility, such as the type-word/token-word case.

A first objection to this response is that we cannot find out that x is discernible from x when we add a new predicate to our language, because that would be simply contradictory, it would entail, for some P, that both $P(x)$ and $\neg P(x)$. However, Dummett (1991: 168) when discussing the occurrence of such cases involving proper names, grants that such cases do not entail a contradiction, 'If we discover that some name ,"*a*",... is really a shared name, we shall, presumably, replace it as quickly as possible by two or more new names, say "*b*" and "*c*".' Why, then, can the same not happen with variables? When we discover that the variable, x, has been assigned ambiguously, does that not simply show that we must now use new variables to distinguish what formerly we could not, and is the process one which might, in principle, go on *ad infinitum*? If it can, once again, Geach will infer that the characterization has failed to rule out cases of the name-type/name-token variety and, therefore, failed to uniquely characterize absolute identity which, as we have seen, he takes to involve absolute indiscernibility.

It might be argued that this response misses the point of using multiple tokens of the same variable, x. The very notion of an assignment for variables depends, it might be thought, on value assigned to a variable type, being identical to the value assigned to any other token of the same variable type.

This is a strong objection to Geach's view, and it is unclear how he might respond to it. However, he might begin to respond by pointing out that what this shows is that the proposed characterization must involve a particular interpretation of the correct use variables, one which involves the stipulation that multiple tokens of the same variable type must be assigned exactly the same object as value. This is an interpretation which Geach himself will view as incoherent because he does not believe that the there are such things as absolutely identical and non-identical assignments for variables. He may therefore claim that the proposed characterization depends on an interpretation which must have the notion of absolute identity built into it, and will therefore be, by his lights, incoherent.

This response is unlikely to sway the absolute identity theorist, who, naturally, has no objection to use of absolute identity in understanding the assignments of variables. As with previous cases, the debate has reached an impasse, where Geach's rejection of a proposed criterion depends on his view that absolute identity is, in fact, incoherent. Again, this reasoning will not be compelling for those who do not already agree with Geach. Once more, I will pursue Geach's argument, putting this issue to the side.

Turning, finally, to (2.14). If there is a characterization of identity which can provide a criterion of application for the term 'relation of absolute identity' it will, therefore, involve some version of LL. But what version? In Chapter 1, I noted that there are various formulations of LL but put off discussion of the alternatives. It is now time to say something about this issue.

There are at least three different issues that are raised in trying to express LL. They are as follows. First, should LL be expressed as a first-order schema or as a second-order theorem? Second, should the expression 'everything true of ...' be understood as ranging over properties or predicates? Third, should the expression 'everything true of...' be understood as ranging over all possible properties/predicates or just those of a specified language? The third issue, as we have seen, is addressed in Geach's argument. We will leave it for the moment. We will consider the first two issues here.

First, then, we must decide whether to express LL as a first-order schema or a second-order theorem. The following two criteria of application present themselves:

(2.14a) a second-order criterion:

P expresses absolute identity if and only if the following is true

$\forall x \forall y (P(x, y) \leftrightarrow \forall Q(Q(x) \leftrightarrow Q(y)))$

(2.14b) a schematic first-order criterion:

P expresses absolute identity if and only if it satisfies the following axiom-schemata

$\forall x P(x, x)$

$\forall x \forall y P(x, y) \rightarrow (\phi(x) \leftrightarrow \phi(y))$

Note that the distinction between the proposed first-order and second-order criteria for a relation's expressing absolute identity cross-cuts the distinction between relative indiscernibility and absolute indiscernibility. The first-order axiom schema LL would guarantee the absolute indiscernibility of its *relata* if the schematic letter, 'Q', is replaceable by any predicate expressing a property of the *relata*. The first-order axiom schema LL would guarantee the relative indiscernibility of its *relata* if the schematic letter, 'Q', is replaceable only by the predicates of a given language.

The second-order principle of the indiscernibility of identicals would guarantee the absolute indescerniblity of the *relata* if the domain of the quantifier binding the predicate variable included all predicates expressing properties of the *relata*. The second-order principle would guarantee relative indescernibility if the domain of the quantifier binding the predicate variable were restricted to a fixed-stock of predicates.

Having noted this, I will take the first-order axiom schemata to guarantee relative indiscernibility and the second-order principle of the indiscernibility of identicals to guarantee absolute indiscernibility. This will avoid needless repetition. Everything I say regarding the first-order axiom schemata would also be true of the second-order principle with a restricted domain of predicates. Similarly, everything I say about the second-order principle with an unrestricted domain of predicates would also be true of the first-order axiom schemata when unrestricted substitution is permitted. Finally, we must decide whether to take LL to involve properties or predicates.

Richard Cartwright (1971) argues in favour of the property over the predicate version of LL, on the grounds that only the property version can escape the apparent counterexamples to LL involving intentional contexts.

Quine (1986) thinks that we can get rid of the counterexamples simply by specifying that grammatical predicates are only really being predicated if the subject term is being used referentially. Moreover, Quine argues, if Q is replaceable by all predicates, then LL is stronger, because there are some predicates which are not properties but should be included in the range of the variable Q. So, Quine concludes, the predicate version of LL is to be preferred.

Following the contemporary trend, I shall assume for what follows that quantification over properties is a genuine possibility, and will therefore follow Cartwright. With this in mind, we can now consider a reconstructed version of Geach's argument.

2.2.1 Argument 3

P3.1 If the criterion of application for the general term 'relation of absolute identity' entails a contradiction, then the extension of that general term is the null class.

P3.2 If there is a criterion of application for the general term 'relation of absolute identity', then it either guarantees that the arguments of any two-place predicate, P, falling under that general term are indiscernible with respect to all possible properties, or it guarantees merely that the arguments of P are indiscernible relative to a fixed-stock of properties.

P3.3 For any proposed criterion of application for the general term 'relation of absolute identity', C, which guarantees merely, for every two-place predicate, P, falling under that general term, that the arguments of P are indiscernible relative to a fixed-stock of properties, the following is possible: for some two-place P satisfying C in a structure M of a theory T, there is a pair $\langle x, y\rangle$ satisfying P in M of T, but not satisfying P in M' of T', where M' is an extension of M, and T' is an extension of T.

P3.4 For any two-place predicate, P, P falls under the general term 'relation of absolute identity' only if, for any pair $\langle x, y\rangle$, jointly satisfying P in M of T, in which P guarantees indiscernibility, it is not the case that there is some extension of M, M', in an extension T' of T, in which x and y are discernible.

C3.1 There is no criterion of application for the general term 'relation of absolute identity' which guarantees merely that the arguments of a two-place predicate, P, falling under that general term, are indiscernible relative to a fixed-stock of properties. (*Calemes* Syllogism from P3.3, P3.4.)

C3.2 If there is a criterion of application for the general term 'relation of absolute identity', then it guarantees that the arguments for any two-place predicate, P, falling under that general term, are indiscernible with respect to all possible properties. (*Modus Tollens* P3.2, C3.1)

P3.5 A criterion of application for the general term 'relation of absolute identity' which guarantees that, for any two-place predicate, P, falling under that general term, the arguments of P are indiscernible with respect to all possible properties entails a contradiction.

C3.3 The extension of the general term 'relation of absolute identity' is the null class. (*Modus Ponens* C3.1, C3.2)[22]

Argument 3 is valid. We therefore consider each premise in turn.

2.2.2 P3.1

If the criterion of application for the general term 'relation of absolute identity' entails a contradiction, then the extension of that general term is the null class.

It is, of course, possible to talk about contradictory notions. By saying a contradictory concept has the null class as its extension, I simply mean that there are no true contradictions. We can consider other, parallel, cases. The terms 'true contradiction', 'square circle', and 'married bachelor' are all general terms. If they were not, it would not make sense to say, for example, 'There are no unmarried bachelors'. Moreover, the terms themselves are not meaningless or ambiguous. It is because we understand the terms that we are able to say that there cannot be any instances of them. The extensions of each of these concepts ('concept', here, is being used in a very broad sense, parallel to 'predicate') must be the null class because there are no unmarried bachelors, square circles, or true contradictions. So too, in Geach's view, for relations of absolute identity. I will return to this issue when discussing P3.4.

2.2.3 P3.2

If there is a criterion of application for the general term 'relation of absolute identity', then it either guarantees that the arguments

[22] Carl Calvert, in his unpublished thesis on Geach's argument for relative identity (1973: Chapter II), provides a reconstruction of Geach's argument which resembles mine in some respects. One major difference is that Calvert splits Geach's position up into several separate arguments; whereas I attempt to present his case as a single argument. Moreover, we do not attribute to Geach exactly the same premises. I am very grateful to Dr. Calvert for permitting me to cite his work.

of any two-place predicate, P, falling under that general term are indiscernible with respect to all possible properties, or it guarantees merely that the arguments of P are indiscernible relative to a fixed-stock of properties.

The notion of absolute identity is intimately bound up with indiscernibility. A relation that did not guarantee the indiscernibility of its *relata* is certainly not a relation of absolute identity. The various criteria that are to be found in the literature either guarantee the indiscerniblity of their *relata* relative to all possible predicates or merely to those of a given language. A criterion of a relation's expressing absolute identity must involve one of these features.

2.2.4 P3.3

For any proposed criterion of application for the general term 'relation of absolute identity', C, which guarantees merely, for every two-place predicate, P, falling under that general term, that the arguments of P are indiscernible relative to a fixed-stock of properties, the following is possible. For some two-place P satisfying C in a structure M of a theory T, there is a pair $\langle x, y\rangle$ satisfying P in M of T, but not satisfying P in M' of T', where M' is an extension of M, and T' is an extension of T.

P3.3 is the premise which Geach devotes most effort to defending. We have seen that he attempts to do so by appealing to the word-type/word-token case described above. Several critics have attacked this case. We have already seen that Geach's own defence of the claim that there is some relation which jointly satisfies the four formal features of absolute identity is both intricate and confused by his drawn out and futile campaign against Quine's procedure to systematically interpret any given theory, T, so that the model of T corresponds to the quotient-structure of T.

Notwithstanding the debate over Quine's procedure, some of Geach's critics have been happy to grant at least part of this premise. It is a fact recognized by Quine himself (1986 :63), that logic can provide no guarantee that, because a predicate satisfies both reflexivity and LL relative to theory T, that predicate must also satisfy reflexivity and LL relative to theory T', an extension of T. In fact, as Krause and French point out, this feature of identity relations is logically commonplace (2006: 252).[23] Consider again the case of the woman who learns, late in life, to distinguish between word-types and word-tokens. I argued that there is no reason to reject the possibility that she discovers that she had been talking about different word-tokens, even when it had been impossible for her to distinguish them. This serves as an example of some x and some y satisfying an I-predicate relative to a theory, T, but being discernible relative to T', an extension of T.

2.2.5 P3.4

For any two-place predicate, P, P falls under the general term 'relation of absolute identity' only if, for any pair $\langle x, y\rangle$, jointly satisfying P in M of T, in which P guarantees indiscernibility, it is not the case that there is some extension of M, M', in an extension T' of T, in which x and y are discernible.[24]

The thought that motivates P3.4 is that the very notion of absolute identity is incompatible with a pair $\langle x, y\rangle$ satisfying the relation and yet being discernible. Hawthorne (2003: 122-123), however, presents an objection to Geach's argument, which challenges this intuition. Hawthorne's objection may be presented as a dilemma. Either unrestricted second-order quantification does not entail a contradiction (i.e. P3.5 is false) in which case the argument against absolute identity is unsound, or unrestricted second-order

[23] In fact, Krause and French think that even the unrestricted second-order characterizations of identity can provide no guarantee against coming to discern what has been indiscernible. This is a result of their views on how we evaluate interpretations (Krause and French 2006: 254-258).

[24] If this premise is false, then absolute identity becomes a theory-relative notion. This is a proposal suggested by Feldman and rejected by Geach (Geach explicitly rejects language-relative identity in Geach/Feldman 1969: 559).

quantification does entail a contradiction (i.e. P3.5 is true). However, if unrestricted second-order quantification entails a contradiction, then we cannot express the thought that a predicate, P's, expressing absolute identity is incompatible with the arguments of P being discernible (i.e. P3.4 is false). This is because indiscernibility in this context is absolute indiscernibility, but the very notion of absolute indiscernibility depends for its expression on unrestricted second-order quantification, which, by hypothesis, leads to paradox. In other words, if unrestricted second-order quantification is impossible, the worry that some x and y satisfying a relation of absolute identity will turn out to be discernible is not even an expressible worry. Once again the argument against absolute identity would turn out to be unsound.[25]

A more general concern, related to Hawthorne's objection, may be pressed. It seems that P3.4 is making a conceptual claim about absolute identity. But given that Geach claims that his argument shows 'there is no such notion as absolute identity' (Geach 1972: 24), what could possibly support P3.4? The answer to the latter worry is simply that Geach is wrong to claim that there is no notion of absolute identity. It is key to the argument against absolute identity that there is such a notion.

With this in mind, we may respond to Hawthorne's specific objection. Let us assume for the moment that Geach is right to claim that unrestricted second-order quantification, of the kind expressed by 'everything true of ...', leads to paradox. What follows from this? It follows that, when such predications are asserted, the proposition is contradictory. However, it does not follow that all propositions involving quantification over all possible predicates entail a contradiction. For example, second-order quantification over all possible predicates is coherent, in the sense that statements involving second-order quantification are possibly true, if the bound variables are within the scope of a connective. To make this point clear, consider a parallel case. The sentence 'x is F and not F' entails a contradiction. The sentence 'if x is a G, then x is F and not F' entails no contradiction, but merely that x is not a G. This is the difference between Geach's use of second-order quan-

[25] Geach points out that this is a general problem with pseudo-concepts (Geach 1991: 296).

tification in P3.4 and the use of second-order quantification to characterize actual relations of numerical identity. The second entails a contradiction, the first does not. For Geach, as I interpret him, is merely claiming that if there is a relation of absolute identity, then, as a matter of conceptual analysis, it would involve quantify over all possible properties. We may add that P3.4, therefore, depends on there being a notion of absolute identity, which involves absolute indiscernibility. That is to say, P3.4 depends on LL providing the criterion of application for the general term 'relation of absolute identity'. We will return to the conceptual link between numerical identity and indiscernibility in Chapter 3.

2.2.6 P3.5

A criterion of application for the general term 'relation of absolute identity' which guarantees that, for any two-place predicate, P, falling under that general term, the arguments of P are indiscernible with respect to all possible properties entails a contradiction.

We have already seen that Geach claims that a second-order criterion for a predicate's expressing absolute identity will soon run afoul of semantic paradoxes such as Grelling's. Geach concludes that expressions like 'whatever is true of x' are incoherent (Geach 1972: 240). To clarify Geach's reasoning, I will suggest an argument against the proposed second-order criterion of a predicate's expressing absolute identity. I will then consider the responses to Grelling's paradox. I will argue that there are various ways of responding to the argument, only some of which support P3.5.

The following argument, if it is to provide support for P3.5, must show that the following criterion of application for the general term 'relation of absolute identity':

> (2.15) For any predicate P, P expresses absolute identity if and only if it is the case that

$$\forall x \forall y (P(x,y) \leftrightarrow \forall Q(Q(x) \leftrightarrow Q(y)))$$

results in contradiction if we assume that the third quantifier is unrestricted.

Argument 4[26]

Begin with the supposed criterion of a relation's expressing identity. We may render (2.15) into English as follows:

> (2.15*) P expresses absolute identity if and only if, for any x and for any y, ($P(x,y)$ if and only if, for any Q, (Q is true of x if and only if Q is true of y)).

Our argument proceeds by assuming the right hand side of (2.15*) and deriving from it a contradiction.

> (2.16) For any x and for any y, $P(x,y)$ if and only if, for any Q, (Q is true of x if and only if Q is true of y).

From (2.16) we can derive:

> (2.17) For any x, $P(x,x)$ only if for any Q(Q is true of x if and only if Q is true of x).

By the truth-conditions for bi-conditionals, we can derive:

> (2.18) For any x, $P(x,x)$ only if, for any Q, it is not the case that (Q is true of x and it is not the case the Q is true of x).

Now we stipulatively define the following predicate: 'x is heterological' $=_{def}$ 'it is not that case that x is true of itself'. This predicate is intended to stand for the property of heterologicality, under the assumption that there is such a property. Assigning the newly defined predicate for the variable 'Q', we get:

[26] Feldman (Feldman/Geach 1969: 549-550) provides a reconstructed argument. However, mine is different in some respects. I do follow not Feldman in taking the word 'heterological' as the subject of the predicate '... is heterological', in order to derive a contradiction. Rather, I consider whether the property heterologicality satisfies '... is heterological'.

(2.19) For any x, $P(x, x)$ only if it is not the case that (x is heterological, and it is not the case that x is heterological).

Assign the property heterologicality as the value of the bound variable x. Thus,

(2.20) P(heterologicality, heterologicality) only if it is not the case that (heterologicality is heterological, and it is not the case that heterologicality is heterological).

We make an additional assumption,

(2.21) heterologicality is heterological.

From (2.21), and the stipulative definition of 'x is heterological' we arrive at:

(2.22) It is not the case that heterologicality is true of itself,

which is of course to say that,

(2.23) It is not the case that heterologicality is heterological.

This contradicts our additional assumption, made at (2.21), so we can infer that (contrary to the discharged assumption):

(2.24) It is not the case that heterologicality is heterological.

From (2.24) and the definition of 'x is heterological', we arrive at:

(2.25) It is not the case that it is not the case that heterologicality is heterological.

But of course, in classical logic, it follows from (2.25) that

(2.26) heterologicality is heterological.

Again we arrive at a contradiction ((2.24), (2.26)).

If the stipulatively defined expression 'x is heterological' genuinely designates the property of heterologicality, then, assuming bivalence, contradiction is unavoidable in any theory which involves statements with expressions like 'everything true of x', where 'everything true of...' is replaceable *salva veritate* with a predicate for any property. If the predicate does designate a property, therefore, there is a *prima facie* case for rejecting unrestricted second-order quantification. It is on this assumption that Geach's argument rests.

But, of course, Grelling's paradox is a problem quite aside from its relationship to the proposed criterion. Philosophers have proposed numerous responses to the paradox.[27] If the best available responses involve rejecting some other premise of Argument 4, apart from (2.15*), the proposed criterion might not need to be rejected. In other words, there might be responses to the paradox which are compatible with the proposed second-order criteria of a relation's expressing absolute identity.

So, how are we to respond to the paradox? Geach claims that the 'classical responses to the paradox involve rejecting talk of 'whatever is true of x ...', when unrelativized to the resources of a particular language (Geach 1969: 557).

2.2.7 Responding to Grelling's Paradox

Presumably, when Geach speaks 'the classical responses' to the semantic paradoxes he has in mind those responses inspired by Tarski's (1933, Geach 1991: 298) semantic conception of truth, which can be used to respond to the liar's paradox, as well as Russell's (1905, 1944) type-theory, which can provide a response to Russell's set-theoretic paradox. Tarski's 'hierarchies of

[27] For example, Martin 1968, whose response provided at least some of the inspiration for Kripke's ground-breaking work (Kripke 1975: 698), Laurence Goldstein (2003), and Newhard (2005). Newhard, in the most recent response to be worked out in detail, argues that '... is heterological' can apply to most words, but not to the words that are sensitive to semantic context, such as 'heterologicality'. However, it is not clear that Newhard's account can respond to all versions of the paradox.

language' and Russell's type-theory are structurally similar[28] and dominated the literature on the semantic and set-theoretic paradoxes at the time of Geach's original presentation of his argument against absolute identity.[29] We will first consider whether these kinds of responses really do support Geach's contention that unrestricted second-order quantification is 'dubiously intelligible' (Geach 1969: 557).

A Tarski/Russell-style heirarchical approach requires that the metalanguage and object language be carefully distinguished. For any predicate of level n, all arguments for that predicate are of level n-1 or lower. We specify the language-level for each term in the definition of 'x is heterological'.

We thus with replace the following definition:

'x is heterological' $=_{def}$ 'it is not that case that x is true of itself'

with

H: 'x'$_j$ is heterological$_k$ $=_{def}$ x_i is not true of 'x_j'[30]

Definition H is itself a statement of level k. As we have said, the hierarchical approach to the semantic paradoxes involves the claim that predicates can only be satisfied by terms of a lower level than the predicate itself. So "x", the variable ranging over predicates, is of level j. However, a problem arises when we look at the right-hand side of the definition operator. 'x' is a free variable standing for the property which is expressed by the name "x". On one hand 'x' must be of a lower level than "x", because "x" is the name of 'x'. On the other hand, 'x' must be of a higher level than "x", because 'x' is being predicated of "x" in the *definiens* of H, and we have said that predicates can only be satisfied by terms of a lower level. So, 'x is heterological'

[28] As indeed both the liar paradox and Russell's paradox are with Grelling's paradox. For discussion of the relationship between Grelling's paradox and the liar's and Russell's paradoxes, see Newhard 2005 and Martin 1968.

[29] We may note, first of all, that Geach is certainly right of his own time that Russell's and Tarski's hierarchies of types and languages respectively were widely accepted as providing the best available responses to the paradoxes of self-reference generally.

[30] Taken from Newhard 2005: 10.

is not well-defined by H (Newhard 2005: 10). This problem is inescapable regardless of the values of j, k, and i.[31]

An alternative response to the semantic paradoxes has increased in popularity since Geach's original paper was published.[32] According to those philosophers who follow Saul Kripke's (1975) work on definitions of truth, a statement such as 'heterologicality is heterological' is neither true nor false but rather has a third truth-value, 'undefined'. This avoids the need to posit different levels of language and might seem to offer the prospect of permitting predicates like '... is heterological' as the possible interpretations of second-order predicate variables. The result of so doing would not be contradiction but rather a truth-value gap. Some second-order sentences would therefore be neither true nor false. But this is perfectly acceptable to those philosophers who endorse Kripke-style responses to the semantic paradoxes.

Thus, there exists a strengthened version of Grelling's paradox which leads to contradiction even if we allow for truth-value gaps. Such 'revenge problems' are a well-known feature of Kripke-style responses to the semantic paradoxes.[33] The standard response to the strengthened version of Grelling's paradox is to take the position that '... is undefined' is a predicate of the meta-language, not the object language. This suggests that the resources of the two languages must still be kept separate. In Kripke's own words, 'The ghost of the Tarski hierarchy is still with us' (Kripke 1975: 714).

What does all of this show us? Several possible conclusion might be drawn. First, there is the conclusion that Geach would have us draw. Namely, that there exists a property that cannot be quantified over in a sufficiently

[31] The ideology of the meta-language may well include the whole ideology of the object language but not the other way around (Martin 1966: 322-324).

[32] Along these lines, Feldman claims that Geach's reasoning depends on the following, he thinks dubious, principle: 'For any predicable, F, and any thing, x, if it is not the case that F is true of x, then it is the case that F is false of x' (Feldman 1969: 550). Feldman follows this by claiming that 'at least some philosophers who have grappled with the paradox would prefer to reject this, so as to leave open the possibility that some predicables are neither true of, nor false of certain things' (Feldman 1969: 550). In fact, the argument I have provided makes no reference to the 'dubious principle' which Feldman attributes to Geach, though it does make an inference which would be invalid in systems without the law of excluded middle, namely the inference from (2.25) to (2.26).

[33] Newhard (2005), for example, considers a strengthened version of Grelling's paradox.

expressive language, on pain of contradiction. This is similar to Quine's response to the paradox:

> In view of Grelling's paradox we know a set which is determined by no sentence of the object language; namely, the set of all sentences of the object language that do not satisfy themselves. If a sentence determined this set, the sentence would be "$\neg(x$ satisfies $x)$" or an equivalent; and Grelling's paradox shows that no such sentence is admissible in the object language. (Quine 1970: 53)[34,35]

If this response is adopted, then quantification over all possible properties is ruled out. We may note, however, that Quine's way of putting the issue seems related to his assumption, noted earlier, that all second-order quantifiers range over predicates rather than properties. If Quine were right about this, then it would follow from the fact that we can identify a property which we cannot name without contradiction directly to the conclusion that unrestricted second-order quantification is incoherent (because, in this case,

[34] Quine points out that the moral of Grelling's paradox is exactly the inverse of the moral of Russell's paradox. While the former shows that there is some property (or set) which is inexpressible by any predicate in the object language, the latter shows that there is some predicate in the object language which does not denote a set (Quine 1970).

[35] Note that Grelling's paradox can be reintroduced as a set-theoretic paradox. Where A is the set of all word-types and we define the relation $I(A, x, y)$ as follows: For x and y in A, $I(A, x, y)$ if and only if, for each subset X of A, either x and y are both elements of X or neither is an element of X, we can introduce a partition on A: $\forall x(x \in A \rightarrow (x \in H$ if and only if x is not true of $x))$. From this a paradox is derivable.

A contradiction can be derived so long as we allow the expression 'any subset of A' to include a subset of A generated by a partition of A by the definition of 'x is heterological'. If there is a set of all objects not true of themselves, and if we can map some predicate of our second-order language onto that set, then a contradiction is unavoidable.

This version can also be used to show that set-theoretic versions of unrestricted second-order LL cannot be used to provide a criterion of application for the term 'relation of absolute identity'. For example, Quine's attempted set-theoretic definition of identity, for a language with four predicates:

> D^3'$x = y$'$=_{def}$ $(Ax \equiv Ay) \wedge (\forall z)((Bzx \equiv Bzy) \wedge (Bxz \equiv Byz) \wedge (Czx \equiv Czy) \wedge (Cxz \equiv Cyz) \wedge (\forall z')((Dzz'x \equiv Dzz'y) \wedge (Dzxz' \equiv Dzyz') \wedge (Dxzz' \equiv Dyzz')))$, where '$A$' is a one-place predicate, 'B' and 'C' are both two-place predicates and 'D' is a three-place predicate (Quine 1963: 13-15).

Quine's definition is, in any case, generally rejected (see Savellos 1990 and Beziau 2003).

predicates are the things being quantified over). If we assume second-order quantification ranges over properties, however, this inference is not immediate, because the fact that a stipulative definition for a real property leads to paradox might be evidence for rejecting the definition, but not for rejecting that the property can be quantified over.

There is, however, an alternative response that is possible. Specifically, we might take the argument to show that there is no property of heterologicality at all (Calvert 1973: 17-19). This would provide no support to P3.5. Geach might object that there surely are concepts which are not true of themselves, for example long, unspeakable, and cacophonous.

However, it might further be proposed, in response, that the definition of heterologicality can be modified for different languages, in a way similar to how the traditional accounts treat the truth predicate. There are, then, properties of heterologicality, but not the one, single, unnameable property which Geach's view entails. Geach might object that reflection on those things that are not true of themselves, suggests that they all have one thing in common, and that there is therefore only one property of heterologicalty, and not many.

Which of these alternative responses[36] to the argument is the most appropriate is not something I will try to sort out here. I will restrict myself to concluding that Geach has not demonstrated that Grelling's paradox support P3.5. One response to the paradox can be used to that end, but it is one of several, and it is by no means clear that the conclusion Geach draws is more appropriate than the alternatives.

[36] A further alternative response is to hold that quantification over all properties is permissible, but that the argument merely shows that the universal quantifier cannot be replaced *salve veritate* by a predicate designating one of the properties over which the quantifier ranges. This position, in other words, entails that the property of heterologicality cannot be designated by a predicate in an expressive language but can, nonetheless, be quantified over. This response would not lend support to P3.5. A compelling argument for P3.5 would, therefore, show that this response does not work. Geach provides no guidance on this and I can think of no argument, except to note that this involves quantifying over something that cannot be named.

2.2.8 Conclusion

In the preceding chapter, I have presented a version of Geach's argument that there are no relations of absolute identity. I considered each of the premises and concluded that they are not all conclusively proven. I argued, in Chapter 1, that RI and SRI require separate arguments, and I will now move to a consideration of those arguments.

3. RI

In the previous chapter, I considered an argument for the thesis GT, which states that there are no relations of absolute identity. GT is one of the three component theses of Geach's strong theory of relative identity which I identified in Chapter 1. The other central theses of Geach's strong theory of relative identity are RI and SRI. RI is the thesis there are true statements of the form 'x is the same F as y and x is not the same G as y', where '... is the same F as...' and '... is the same G as...' express numerical identity relations, and where either x or y is a G. SRI is the thesis that all relations of identity have the structure '... is the same F as ...' where F is a sortal term. In this chapter, I will consider the extant arguments for and against RI; in the following chapter I will consider arguments for and against SRI.

Geach provides an argument in the first edition of *Reference and Generality* (1962: 150-151), which he alters considerably for the 3rd edition of that work (1980: 183-184) that has often be taken as a defence of RI. I will consider both versions of this argument and conclude that both fail. In addition, Griffin (1977) argues for RI on the grounds that it provides the advantage of simplicity, while Zemach (1974, 1986) argues that the existence of vague objects entails the truth of some cases of RI. I will argue that both Griffin and Zemach fail to establish their conclusions.

I then turn to objections to RI. I consider Lowe's claim that no object can fall under different sortals. I argue that Lowe fails to sufficiently support his claim. Finally, I consider Wiggins's objection that cases of RI, that is, true instances of the form $\ulcorner x =_F y \wedge x \neq_G y \wedge (G(x) \vee G(y)) \urcorner$, are ruled out *a priori* because the very notion of numerical identity involves indiscernibility. I argue that Wiggins' objection is compelling only if one assumes, as Wiggins does, that GT is false. I, therefore, conclude that RI is, as yet, neither proved nor disproved, but that the prospects for weak theories of relative identity are poorer than the prospects for strong theories of relative identity.

3.1 Arguments for RI

RI is true if and only if there is a true statement of the form $\ulcorner x =_F y \wedge x \neq_G y \wedge (G(x) \vee G(y))\urcorner$. There are many sentences which might,*prima facie*, be thought be instances of this form. We have seen some examples in Chapter 1.

We also noted that, if '$=_F$' and '$=_G$' are interpreted as relations with the traditional formal features of absolute identity, then all cases of RI are demonstrably inconsistent. This was demonstrated with Proof 1, in Chapter 1. Therefore, if RI is true, there is some inference involved in Proof 1 that must be invalid. The inferences that are invalid according to most relative identity theorists occur at lines (10) and (16) of the proof.[1] However, these inferences are valid so long as the inference rule which I named 'the substitution of identicals' is truth-preserving. So RI generally involves the rejection of this inference rule. In addition to this, RI entails the falsity of the related thesis, LL.

> **Proof:**
>
> Assume the truth of a case of RI. This entails that some x and y jointly satisfy some relation of numerical identity, but there is some other relation of numerical identity which either x or y jointly satisfies with something z, but which x and y do not jointly satisfy. Call this relation '$=_G$'. This entails that there is a property, for some z, expressed by the symbols $\ulcorner =_G z\urcorner$, which is satisfied by one of, x or y, but not both. So x and y are discernible. By LL, no x and y which jointly satisfy some

[1] Note that most relative identity theorists also reject the rule of inference which I titled 'the Fregean Analysis'. This inference occurs at lines (5) and (6) of the proof. In fact, neither the Fregean Analysis nor the substitution of identicals is strictly incompatible with RI. RI does, however, entail that one of them must be rejected.

numerical identity relation are discernible. So LL is incompatible with RI.

Thus, one result which follows from RI is that there are numerical identity relations which do not imply the indiscernibility of their *relata*. Should we accept the possibility of true cases of RI, at the cost of rejecting LL? I will consider four arguments for doing so. Geach attempts to prove RI with two examples, his 'river and waters' case (1962: 150-151) and his 'men and heralds' case (1980: 183-184). I will consider each of these in turn. I will look at objections to these cases by Lowe (1989a), and I will conclude that both of Geach's arguments fail. I will then consider an argument which appeals to simplicity from Griffin and an argument which appeals to vagueness from Zemach. I will argue that both of these arguments fail to establish the truth of RI.

3.1.1 Two Geachean Arguments, River and Waters/Men and Heralds

In the first two editions of *Reference and Generality* (1962, 1968), Geach provides an argument which has often been viewed as an attempted defence of his theory of relative identity. I will call this argument 'the river and waters argument'. It is not at all clear, however, what Geach intends the river and waters argument to establish, for it does not appear alongside anything more than a vague outline of the theory relative identity. Rather, from the context in which the argument appears, it seems that Geach's more immediate concern is to make a point about quantification.[2] In the third and final[3] edition of *Reference and Generality*, Geach replaces the river and

[2] Dummett (1973: 551-558) notes that Geach's deductive strategy at this point is ambiguous. It is not clear if Geach is trying to use this example to defend RI, or whether he is assuming the truth of RI, and he is using the example to defend his views on quantification. I will put the questions of exegesis to the side and will simply consider whether the example can be used to provide an argument for RI.

[3] I have undertaken an, as yet, unsuccessful search for the 4th edition of *Reference and Generality* (1980), which Geach apparently wrote or, at least, intended to write. My search took me to the Balliol College archive, to which Geach left many of his papers. Edited copies of the 2nd and 3rd editions can be found there. With them can be found several letters by Geach to the then archivist. In several of these, Geach advises destroying the archived papers on the grounds that these do not reflect his final opinions, and that

waters argument with an argument based around a new example. I will call this second argument 'the men and heralds argument'. The argument retains the same structure in both versions, and the context and apparent conclusion remain unchanged, yet the objections raised against the river and waters argument on one hand and the men and heralds argument on the other are different, and so I will consider them as separate arguments.

I will begin with the river and waters argument. I will show how the example might be turned into an argument for the rejection of LL, and that this in turn might appear to give support to RI. I then consider the objections to the river and waters argument from Wiggins and Lowe. I argue that Geach's argument fails to establish that LL fails to hold for some relations of numerical identity and, therefore, provides no support to RI.

I will then turn to Geach's second argument. Following the same method, an argument from the men and heralds example to the rejection LL can be given. The argument escapes the objections raised against the river and waters argument. However, new objections against this argument, from Lowe (1989a), show that it, too, fails. A possible reply to the objections is considered and rejected on the grounds that, if the suggestion is followed, the men and heralds argument becomes susceptible to the same objections as the river and waters argument.

I conclude that Geach fails to provide an argument for rejecting LL and thus fails to provide any support for RI.

3.1.2 River and Waters

Turning, then, to the river and waters argument. The relevant passage may be quoted in full.

> According to this accepted view, we may treat the proposition:

they may confuse future generations of researchers. Geach says that his final opinions can be found in the 4th edition of *Reference and Generality* published by the University of Warsaw in 2004. I contacted that institution only to find out that the 2004 publication was not a new edition but, rather, a Polish translation of the 3rd edition with only minor changes. My thanks are due to Professor Joanna Odrowaz-Sypniewska, who helped Geach prepare the 2004 Polish translation of *Reference and Generality.*

(3.1)[4] Heraclitus bathed in some river yesterday and bathed in the same river today

as equivalent to:

(3.2) Something (or other) is a river, and Heraclitus bathed in it yesterday, and Heraclitus bathed in it today

or, using "bound variable" letters, as equivalent to:

(3.3) For some x, x is a river, and Heraclitus bathed in x yesterday, and Heraclitus bathed in x today.

Now by parity of reasoning we may analyse:

(3.4) Heraclitus bathed in some water yesterday and bathed in the same water today

as equivalent to:

(3.5) Something (or other) is water, and Heraclitus bathed in it yesterday, and Heraclitus bathed in it today

or again to:

(3.6) For some x, x is water, and Heraclitus bathed in x yesterday, and Heraclitus bathed in x today.

...

we may assert the additional premise "whatever is a river is water" or "For any x, if x is a river, x is water". Now given this premise, (3.5) or (3.6) is inferable from (3.2) or (3.3). But clearly this premise would not warrant us in inferring (3.4) from (3.1): it is notorious that (3.1) could be true and (3.4) false. Hence the above analyses of (3.1) and (3.4), which stand or fall together, must both be rejected.

It is easy to see what has gone wrong; (3.5) or (3.6) tells us that Heraclitus bathed in the same something-or-other on two successive days and that this something-or-other "is" water. This does indeed follow from (3.2) or (3.3), and therefore from (3.1), but it is a much weaker proposition than (3.4). "Being the same water" cannot be analysed as "being the same (something-or-

[4] All quotes in this dissertation that involve numbered theses have had the numbers altered to correspond to my own numbering system.

other) and being water". (Geach 1968: 174-176)

From the final sentence, it seems that Geach's conclusion is to reject the Fregean Analysis, namely that statements like 'x is the same water as y' are always logically equivalent to statements of the form 'x is the same as y and x is a water'. This rejection is an important component of Geach's theory of identity. However the theory involves much more than this claim, as we have seen. In the quoted passage, Geach makes no reference to any of the component theses of his theory of strong relative identity: GT, SRI, and RI. Yet the river and waters example has often been taken as an argument for the last of these theses.

The immediate context of Geach's river and waters example is a point Geach is trying to establish about quantification. Namely, it is Geach's contention that restricted grammatical quantification, for example expressions like 'all cats...', is not reducible to unrestricted grammatical quantification, for example, statements like 'all things that are cats...'. This position, judging from its place in the dialectic is, at least in part, motivated by Geach's view that 'thing that is a cat' is not a 'logically simple sign', analogous to a complex name. He concludes from this that 'Something that is a cat' cannot be analysed as consisting of two parts, the quantifier, 'some', and the referring expression 'thing that is a cat' (Geach 1968: 149).[5,6] From this, Geach

[5] Geach concludes, 'We cannot, then, accept the conventional way of reducing the restricted quantification of (4) to the unrestricted quantification of (5) or (6).' (Geach 1968: 153)

Geach's own view is that,

(5) Something (or other) is water, and Heraclitus bathed in it yesterday, and Heraclitus bathed in it today

and

(6) For some x, x is water, and Heraclitus bathed in x yesterday, and Heraclitus bathed in x today.

Are true if and only if the following is true

(7) Some A is water, and Heraclitus bathed in that (same) A yesterday, and Heraclitus bathed in the same A today

This view entails that, for some A, 'There is some x such that ϕ' is true if and only is 'There is some x such that x is an A and ϕ.' (Geach 1968: 150-156) It is clear from the context that Geach intends for 'A' to range over sortals. In other words, everything satisfies some sortal.

[6] The reasons Geach gives for this position is as follows, first, whereas the relation between a name and its referent is tenseless, the relation between 'thing that is a cat...'

concludes as follows 'we have failed in our attempts to explain "anything" and "something" in terms of "any" and "some"; is the converse sort of explanation feasible?' (Geach 1968: 149) Geach then proceeds into the quoted passage.[7]

Geach's river and waters argument can, however, be turned into an argument for RI in the following way. First, we will remove one possible source of confusion. Geach has been challenged (Quine 1963, Helen Cartwright 1965) for using the term 'water' in such a way that it is ambiguous whether he intends it to be understood as a count noun or mass term. I shall use it as a count noun, and define it as follows. For some river, x, a water is n gallons of water, where n varies over time such that, at t_m, n is just the same as the same number of gallons of water which is in x at t_m. In other words, a water is just the amount of water that fills a given river. Next, assume the truth of the premises. Thus, whatever is a river is a water. Moreover, Heraclitus bathed in some river yesterday and bathed in the same river today. Now further assume that Heraclitus bathed in some water yesterday, and it is not the case that Heraclitus bathed in the same water today. Given certain rules for translating sentences of English into FOL, this can be formalized as follows:

3.1.3 Argument 6

Using the following dictionary: R: ... is a river, W: ... is a water, B: ... bathes in...., h: Heraclitus

Assumption:

and the cat of which the statement including that expression is true is not. Second, a name supplies a criterion of identity, and 'thing that is a cat...' does not.

[7] Geach notes, but sidesteps, Quine's view that names are logically redundant, as they are always reducible to quantified expressions. Geach seems to provisionally agree, though he thinks Quines doctrine is an empty one. The point for Geach is that proper names have the same referential function as substantival terms (i.e. sortals). As Geach says 'both in acts of naming and within propositions, use for example of "cat...the same cat...the same cat..."closely corresponds in its referential force to repeated use of the proper name "Jemima"; I hold that recognition of proper names as logical subjects stands or falls with recognition of an irreducible subject role for substantival general terms.' (Geach 1968: 150).

(3.7) $\exists x(Rx \wedge Bhx \text{ at } t_1 \wedge \exists y(Ry \wedge Bhy \text{ at } t_2 \wedge x = y))$

Assumption:

(3.8) $\forall x(Rx \leftrightarrow \exists y(Wy \wedge x = y))$

Assumption:

(3.9) $\exists x(Wx \wedge Bhx \text{ at } t_1 \wedge \exists y(Wy \wedge Bhy \text{ at } t_2 \wedge x \neq y))$

Let us add the further assumption that it is only possible to bathe in one water at any one time:

(3.10) $\forall x \exists y \forall z(Wx \wedge Byx \text{ at } t_1) \rightarrow ((Wz \wedge Byz \text{ at } t_1) \rightarrow x = z)$

From these we can derive the following. By replacing the variables in (3.10) with names we get:

(3.11) $Ra \wedge Bha \text{ at } t_1 \wedge Rb \wedge Bhb \text{ at } t_2 \wedge a = b$

From the conjunction of (3.11) and (3.8) we get:

(3.12) $\exists x(Wx \wedge x = a)$

and

(3.13) $\exists x(Wx \wedge x = b)$

we can replace the variables in (3.12) and (3.13) to get:

(3.14) $Wc \wedge c = a$

and

(3.15) $Wd \wedge d = b$

At this juncture we can either assume that the relations which hold between c and a and between d and b, are non-Leibnizian and grant that LL does not characterize all relations of numerical identity, or we can assume that they do satisfy Leibniz's Law. We will take the latter course, which means we can derive the following from (3.11), (3.14), and the substitution of identicals:

(3.16) Bhc at t_1

By parity of reasoning we can derive, following from (3.11), (3.15) and the substitution of identicals:

(3.17) Bhd at t_2

From (3.9), (3.10), (3.16), and (3.17) we can prove:

(3.18) $c \neq d$

At this point, assuming the truth of (3.7)-(3.9), we have no option but to accept that the relation that holds between a and b is non-Leibnizian, on pain of inconsistency. For to assume that LL held for this relation would allow the following, from (3.11), (3.15), and the transitivity of identity:

(3.19) $a = d$

At the same time, from (3.14), (3.18), and the substitution of identicals, we can establish that:

(3.20) $a \neq d$

Thus, it seems that we can establish the existence of non-Leibnizean relations of absolute identity from a set of three sentences, which seem (at first glance) to be capable of being simultaneously true. This in itself is not an immediate proof of the thesis RI, but cases of RI seem to be plausible instances of a non-Leibnizean identity relations. If non-Leibnizean realtions of identity are possible, it would seem highly appealing to conclude that a is the same river as b, but that a is not the same water as b, although both a and b are waters. Thus it seems that Geach's example of waters and rivers does provide at least some *prima facie* support for RI. The immediate question of course is whether the assumptions Geach makes about the interpretation of the English expressions are well-founded.

3.1.4 Possible Alternatives: Wiggins and Lowe

We have already seen examples structurally similar to our reconstruction of Geach's river and waters case. Wiggins provides a series of apparent counter-examples to absolute identity. We considered the following case in Chapter 1:

> Type 5: $a =_F b \wedge a \neq_G b \wedge G(a) \wedge G(b)$
>
> For example, I moor my vessel in the river Scamander. The next day, it is the same river as the previous night but not the same water.

We saw at the time that Wiggins thinks that purported examples of Type 5 must be false and that sentences which apparently instantiate this logical form must be interpreted either as involving ambiguous referring expressions, an 'is' of constitution, or a qualitative identity relation in place of an 'is' of identity. Thus, Geach would have us interpret (3.7)-(3.9) as involving the 'is' of identity, but Geach's premise (3.8), 'every river is water', involves an 'is' of constitution in Wiggins's view. With this interpretation, Argument 2 is unsound, for the crucial premise (3.8) is false. In other words, it is not the case that every river is identical with some water. Without this premise, the argument does not go through, and no support is lent by the example to RI.

Lowe adopts Wiggins's strategy in attacking Geach's argument. He adds the following justification to Wiggins's contention that it must be false that a river is numerically identical with some water:

> This, at bottom, is because rivers and waters have different criteria of identity, and an individual of one sort or kind cannot also belong to another sort or kind with a different criterion identity from that of the first. (Lowe 1989a: 53)

By adopting the Lowe and Wiggins interpretation, it is possible to accept the truth of each of the following sentences, (3.7a) Heraclitus bathed in some river yesterday and bathed in the same river today.

(3.8a) Whatever is a river is water.

(3.9a) Heraclitus bathed in some water yesterday and bathed in a different water today.

while rejecting the existence of non-Leibnizean identity relations. This involves interpreting 'Whatever is a river is water' as involving the 'is' of constitution rather than the 'is' of identity.

According to this view, as soon as we make determinate what the subject of the sentences is, we can see which occurrences of 'is' express identity and which express constitution. Let us assume that, in each of (3.7)-(3.12), we are talking about the river Thames. Once we have interpreted the referential expressions in that way, it is clear that all occurrences of 'is' in premises (3.7)-(3.9) are examples of the 'is' of identity, while all the premises (3.10)-(3.12) as well as the additional premise that everything that is a river is water involve the 'is' of constitution. Given this way of interpreting (3.7a)-(3.7c), no support is offered to RI. Rather than argue in favour of his own interpretation of the disputed sentences, Geach instead provides an new example, which he seems to think is a more compelling case of RI.

3.1.5 Men and Heralds

In the third edition of *Reference and Generality*, Geach replaces the river and waters case with a new one (Geach 1980: 174-184). Geach neither disowns the original example, nor does he explain what the new example is supposed to add. We will see that, though it is not so easily disposed of, it, too, fails to establish GT.

The structure of the new example remains unchanged. Now, however, we begin with the following statement:

(3.21) Lord Newriche discussed armorial bearings with some herald yesterday and discussed armorial bearings with the same herald again today.

Geach again argues that this statement cannot be logically equivalent to:

(3.22) Something (or other) is a herald, and Lord Newriche discussed armorial bearings with it yesterday and discussed armorial bearings with it again today.

Once again, this is because if (3.21) were logically equivalent to (3.22), then, with the additional premise,

(3.23) Whatever is a herald is a man,

(3.21) and (3.23) would entail:

(3.24) For some x, x is a man, and Lord Newriche discussed armorial bearing with x yesterday and discussed armorial bearing with x again today.

However, claims Geach, (3.21) and (3.23) do not entail (3.24), because it is logically possible that 'with a change of personnel in the Heralds' College, Lord Newriche might have seen a different man on Monday and Tuesday but the same herald, namely Bluemantle' (Geach 1980: 176).

Geach's example trades on the fact that, at the College of Arms, the official positions are given fanciful names, such as 'Bluemantle', and that different men may occupy the post of Bluemantle on different days. Yet all the heralds are men.[8]

Once again, an argument may be found, parallel to Argument 6, starting with the three assumptions:

(3.25) Lord Newriche discussed armorial bearings with some herald yesterday and discussed armorial bearings with the same herald again today.

[8] Still true apparently; the College of Arms has yet to appoint its first female herald.

(3.26) Lord Newriche discussed armorial bearings with some man yesterday and discussed armorial bearings with the same man again today.

and

(3.27) Whatever is a herald is a man

to the conclusion that there are non-Leibnizean relations og numerical identity. Of course, this argument will also be valid only if the copulas are all interpreted as expressing numerical identity. Note however, that the objection to Argument 6, that the 'is' in 'Whatever is a herald is a man' expresses the relation of constitution rather than identity, is far less persuasive in this case. It does not seem that men 'constitute' heralds in the way that water constitutes a river. Some other response must be provided for the men and heralds argument.

3.1.6 'Is' of Instantiation

Lowe has a separate objection to the men and heralds argument (1989a: 43-50). Lowe suggests that there is another use of the word 'is', namely an 'is' of instantiation. The 'is' of instantiation is a sub-class of the 'is' of predication. An important logical feature of the relation of instantiation is the one of the *relata* is a concrete entity and the other is abstract.[9] An example of the 'is' of instantiation can be found in the statement 'this animal is a horse'.

[9] The important difference, in Lowe's view, is that, in the first case, the copula is redundant, while in the second it is not. It is not redundant in the second case because the addition of the copula is necessary to distinguish between occurrences of a sortal as a subject and occurrences as a predicate. Thus, 'horses are animals' involves 'horses' as a logical unit. However, 'these are horses' does not involve 'horses' as a logical unit but rather '... are horses'. This is because predicates and subjects are very different kinds of things, and a single logical unit, like 'horses', cannot play both roles without what Lowe calls 'systematic ambiguity' (he accuses Geach of this, because Geach thinks that the 'is' in statements involving sortals is redundant). All this of course holds for proper names as well in Lowe's view: i.e. there is a difference between the 'i's of attribution, where the 'is' is redundant and the 'is' of identity where it is not (Geach's position is ambiguous in the case of proper names, specifically, whether the 'is' is redundant or not). I am unconvinced of Lowe's argument here, particularly because I do not see that the charge of 'systematic ambiguity' is a knock-down objection and why the presence of the word 'is' should be assumed to be the only way of getting rid of that supposed ambiguity. Lowe, however,

Lowe then argues that the expression '... is a herald' involves the 'is' of instantiation. Because to say, of any concrete entity, that it is Bluemantle, is simply to say that it occupies a particular office. Geach's argument would only be valid if we interpreted each of the copulas as expressing numerical identity, and we can only do that if we interpret the *relata* as being concrete entities. As Lowe points out 'what Geach is doing, in effect, is to reject talk of these (abstract entitites) in favour of talk of a (hitherto unrecognized!) kind of concrete entity' (Lowe 1989a: 49). Lowe goes on to point out the drawbacks of recognizing such things as concrete heralds (they would seem to lead gappy existences, for one thing, Lowe 1989: 50). The important point, however, is that Geach has not succeeded, with the men and heralds argument, in providing a case which establishes that there are non-Leibnizean relations of identity, because there exists a plausible interpretation of the component sentences of the example which is compatible with the falsity of that claim. It seems, then, that neither of Geach's arguments has provided any support to RI. Several philosophers have nonetheless followed Geach in advocating RI, though providing different arguments in its favour.

3.1.7 Griffin's Argument

Next, I will consider an argument presented by Griffin (1977: 204-212). Griffin grants that sentences which are apparent cases of RI can plausibly be interpreted in some other way without affecting their truth-value and thus do not serve as counter-examples to the theory of absolute identity. We are, therefore, faced with a decision. We can accept that all apparently true cases of RI in fact instantiate some other logical form and thus preserve a classical theory of identity, or we can accept that there are genuine cases of RI. Griffin thinks that we ought to prefer an interpretation of such sentences which does involve non-Leibnizean identity over an interpretation which does not, on the grounds that the former allows us greater theoretical simplicity.

We will return to a case of Geach's:

proceeds to employ these distinctions to distinguish between individuals and sorts in an interesting and, I think, plausible way. However, this has little bearing on his attack on Geach's example, so I will put it to one side (Lowe 1989a: 28-42).

(3.28) Smith is the same herald as Jones, but they are not the same man.

In Griffin's view, the first conjunct of (3.28) may be interpreted either as a relative identity statement or as a 'common property statement'. Griffin's use of the term 'common property statement' is closely related to the more familiar notion of 'qualitative identity'. The notion of a common-property statement can be cashed out as follows:

(3.29) A statement, $\boldsymbol{P}$, of the form 'a is the same F as b', is a common-property statement if and only if it entails that 'a is an F and b is the same F' but does not entail that 'a is the same thing as b'.[10]

If (3.28) is interpreted as involving a relative identity relation, then it is a case of RI. If it is interpreted as involving a common property statement, no such consequence follows. Griffin argues that it is preferable to interpret all such statements, indeed all statements of the form 'a is the same F as b', as involving relations of relative identity, rather than interpreting them as common-property statements. Griffin tries to show that there are certain norms of theory selection given which, he claims, it would be preferable to interpret (3.28) as involving relative identity.

The norms of theory selection that Griffin highlights are two: first, simplicity and second, non-arbitrariness (Griffin 1977: 211-212). In Griffin's view, it seems, a theory which provides a single account of the logical structure of a class of statements is, *ceteris paribus*, to be preferred over a theory that involves two different accounts of the logical structure of the same class of statements. Griffin thinks that accepting genuine cases of RI offers simplicity because it dispenses with a theoretical distinction between statements of numerical identity and common-property statements. Thus, it is simpler with respect to the theoretical apparatus rather than the ontological commitments, that is, it achieves theoretical elegance, rather than ontological parsimony.

[10] For a discussion of the criterion for some statement's being a common property statement, see Griffin 1977: 204-212.

Griffin (1977: 204-212) also thinks that, wherever the line between statements of numerical identity and common-property statements is drawn, the resulting distinction is bound to be an arbitrary one.

Griffin's argument becomes somewhat hard to follow at this point because he gives very little by way of sustained defence of the claim that the these norms are the only or even the most relevant norms according to which our decision ought to be made. The absolute identity theorist may object that, while accepting Griffin's claim to elegance and non-arbitrariness, the other theoretical benefits of absolute identity are such that these considerations are defeasible in light of more pressing considerations.

However, I do not think the absolute identity theorist is under any compulsion to show even this much. I think that any absolute identity theorist can give a much simpler response to arguments like Griffin's. As we will shortly see, Wiggins holds that the very notion of numerical identity involves indiscernibility. If Wiggins is right about the supposed conceptual link between numerical identity and indiscernibility, then the existence of true cases of RI is ruled out *a priori*. If this is the case, then the elegance that is achieved by RI is simply not a relevant consideration. We will return to this issue later in this chapter, where I will argue that no defence of RI can escape Wiggins's conceptual response, unless it is supplemented with a successful argument for GT.

3.1.8 Zemach's Argument

Zemach (1982, 1991) provides another argument for RI by appealing to an example which seems to instantiate the disputed form. In fact, if Zemach is right, cases of RI would include most statements involving numerical identity relations.

To understand Zemach's argument, consider the following case:

(3.30) Samuel Clemens was Mark Twain.

We will take it that (3.30) is a true statement, and that it expresses an numerical identity relation. Now consider the following statement:

(3.31) Samuel Clemens on July 4, 1885 was partly composed of the semi-digested food in his stomach.[11]

Is (3.31) true or false? Surely, suggests Zemach, the concept of such things as men are vague with respect to such properties as '...partly composed of the semi-digested food in his stomach'. Given this, Zemach thinks there simply is no fact of the matter about whether or not Samuel Clemens on July 4, 1885 was partly composed of the semi-digested food in his stomach.

Yet, there is certainly an object in the vicinity of Samuel Clemens on the day in question which is partly composed of the semi-digested food in his stomach. Let us designate the entity composed of the skin, hair, teeth, internal organs, etc, as well as the semi-digested food, man '*s*'. Let us designate the skin, hair, teeth, internal organs, etc, without the semi-digested food, man '*c*'.

Given LL, *s* is not absolutely identical with *c*. *s* has the property of being partly composed of semi-digested food. *c* lacks this property. Which of these is Samuel Clemens? It would seem that both *s* and *c* have equal claim to be the referent of 'Samuel Clemens'. In this case, the name seems simply ambiguous. Even if it is a shared name, assuming there are such things, the token of 'Samual Clemens' in (3.31) must have referred uniquely to either *s* or *c*. But surely it does not, because there is no reason for the name to refer to one entity rather than the other. Moreover, the name could not refer to both, because, given (3.30) and the fact that *s* and *c* are non-identical, this would violate the transitivity of identity, as it would imply that each of the different referents of 'Samuel Clemens' was identical with Mark Twain, but they were not identical with one another.

This has a number of unwanted consequences. It would seem that 'Samuel Clemens' either does not refer at all or is a shared name, whose use is systematically ambiguous. Given this, the sentence (3.31) does not express a proposition. Moreover, when we count the number of men who wrote *Adventures of Huckleberry Finn*, we will have to count *s* and *c* separately, as they are not absolutely identical. So it turns out that *Adventures of Huckleberry*

[11] This example is based on one from Zemach 1982: 297.

Finn was co-authored. Moreover, the same problem affects almost all the utterances of ordinary language, claims Zemach (1982: 295). If each of our apparently referring expressions is, in fact, indeterminate in reference, the way that 'Samuel Clemens' is indeterminate between c and s, all the sentences involving these referring expressions will turn out to be false. This is an unacceptable consequence.

All these consequences are avoided if we accept that s bears a relation of non-Leibnizian numerical identity to c. s and c are the same man, but they are not the same extended body. But this last statement has the following form, $\ulcorner s =_M c \wedge s \neq_B c \wedge (B(s) \wedge B(c))\urcorner$, where '$=_M$' stands for '... is the same man as...', and '$=_B$' stands for '... is the same extended body as...' and 'B' stands for '... is a body'. From this, of course, we can derive $\ulcorner s =_M c \wedge s \neq_B c \wedge (B(s) \vee B(c))\urcorner$, which is a case of RI.

Though Zemach's work constitutes a novel contribution to the literature on vagueness, this argument fails to establish RI. Once again, as with both of Geach's arguments, the theory of absolute identity has the resources to avoid the unacceptable consequences that Zemach derives without appealing to non-Leibnizian identity relations. There are a series of alternatives for escaping the consequences.

We could, for example, reject that 'Samuel Clemens' is indeterminate with respect to the truth of '... is partly composed of the semi-digested food in his stomach'. Notably Gareth Evans (1978) and Timothy Williamson (1994), amongst others, argue for this response. This is, of course, compatible with never being in a position to know whether or not Samuel Clemens is partly composed of the of the semi-digested food in his stomach. It is simply the claim that all such statements are either true or false.

But let us assume that there are genuinely vague terms. Even so, Zemach has failed to establish that there are true cases of RI. For it might be the case that the truth of the sentence 's is absolutely identical with c' is itself vague and lacks a truth-value.

Let suppose, further, that there are genuinely vague terms, and that sentences like 's is absolutely identical with c' must be either true or false. Even still, Zemach has not proved the existence of true cases of RI. For we might say that the sentence 's is absolutely identical with c' is false, but that this does not license the conclusion that s and c are two different men. We might say that s and c bear the relation of 'sameness without identity' to one another.[12]

There are of course still more alternatives. With the exception of the first suggestion, Zemach does not argue against any of these alternative proposals. Therefore, he has not shown that the truth of sentences like (N) depends on the truth of RI. Zemach's argument, therefore, fails to prove RI.

This completes my survey of arguments for RI. I have argued that none of the four arguments that I considered provide the necessary support for RI. Next, I will consider arguments that are intended to show that RI must be false.

3.2 Arguments Against RI

3.2.1 Lowe's objection

I will begin with an objection from Lowe (1989a: 53-63). Lowe thinks it can never be the case that an individual falls under two different sortals at the same time, which have different criteria of identity. Lowe therefore proposes a rule, which states that an individual, x, may be a member of two sorts F and G if and only if either one of F and G are related as species to genus, or there is another sort, H, which is a subspecies of both F and G, and x is a member of H (Lowe 1989a: 53-54). If this rule were to be violated, the individual may no longer satisfy the persistence conditions for one sortal, while continuing to satisfy the persistence conditions of another sortal. It would seem, in this case, that the individual both exists and does not exist, and this is incoherent. If Lowe is right in this claim, then he is right that individuals cannot be of different sortals, for it is overwhelmingly plausible that individuals may not both persist and have ceased to exist at the same time. This principle entails the falsity of RI.

[12] For a recent examination of the notion of 'sameness without identity', see Graf Fara 2008.

Yet this is hardly a knock-down objection. According to the strong theory of relative identity, it is possible that, for some x, x is an F and a G at some time, t_1, and for x to be an F but not a G at t_2. This does not entail, though, that x both exists and does not exist at t_2, rather x exists but is no longer a G.[13] The persistence conditions for G's tell us, not when x continues or ceases to exist, but simply when any given G continues or ceases to exist. Lowe's objection, then, fails to show that cases of RI are impossible.

3.2.2 Wiggins' Conceptual Objection

I will next consider a claim which I have noted earlier and which I think provides the strongest objection against RI. We have seen that Wiggins[14] argues that cases of RI are impossible, and it is to his argument that I now turn.

Wiggins's primary target is the claim that there exist non-Leibnizian relations of identity (1980, 2001). Wiggins makes the following claim:

> Leibniz's Law marks off what is peculiar to real identity and it differentiates it in a way in which transitivity, symmetry and reflexivity (all shared by exact similarity, weighing the same, having exact equality in part, etc.) do not. (Wiggins 2001: 27)

For Wiggins, then, indiscernibility is inherent in our notion of numerical identity. In his view, talk of 'non-Leibnizian identity relations' is really talk of mere equivalence relations.

Several philosophers who defend weak theories of relative identity have, at least in part, rested their case on occurrences of the expression 'identity' in natural language which violates Leibniz's Law (Zemach 1974, 1982, 1983,

[13] Lowe responds that such a move entails a relativization of the notion of existence (Lowe 1989a: 57), in the sense that the situation envisioned would entail that x exists *qua* F, but had ceased to exist *qua* G. I do not see why Lowe thinks that a *qua* strategy is the only alternative here or, for that matter, why he thinks it is so problematic.

[14] The conceptual objection, alongside semantic objections of the kind discussed in Chapter 5 of this dissertation, is the one of the most common challenges raised against theories of relative identity. Apart from Wiggins, this objection can be found in Stevenson 1972, Noonan 1980, Doepke 1982: 12-17, Lowe 1989a, and McGinn 2000: 4-5.

Deutsch 1998, and Garbacz 2002). The empirical point, however, hardly needs elaboration. After all, we talk of national identity, sexual identity, gender identity, cultural identity and so on, and none of these satisfy LL. But, for Wiggins, this is simply to employ a different use of the term 'identity'. No example of a non-indiscernible relation need worry Wiggins, who is always permitted to rename any such relation 'a mere equivalence relation', withholding the term 'identity' for only those relations which guarantee indiscernibility. Wiggins's point is about the concept *identity* and is not to be solved empirically.

Is Wiggins's conceptual claim correct? I think it quite obviously is, in the sense that there is a use of the term 'identity' which does necessarily involve indiscernibility. Perhaps this use of the term is more common amongst philosophers than non-philosophers, but nevertheless Wiggins is perfectly right to insist that, according to one concept of identity, non-Leibnizian identity relations are ruled out *a priori*.

It seems to me, however, that whether or not this conceptual point can be useful as an objection against RI depends entirely on whether or not GT is true. Assume, for the moment, that GT is false. In this case, there are relations of absolute identity which do satisfy LL. Wiggins, then, will insist that these are the only relations of numerical identity, properly speaking. And, if we take him to mean that these are the only relations that fall under the philosopher's notion of identity, characterized by the four laws of identity, then he is right. Unless some reason can be given why the distinction between numerical identity relations and equivalence relations *ought* not to be drawn, Wiggins's conceptual claim is sufficient to rule out non-Leibnizian relations of identity. We considered two arguments, Griffin's argument from simplicity and Zemach's argument from vagueness. However, we saw that neither of these arguments is particularly compelling. Unless some further argument can be provided, the prospects for supporting RI, while assuming the falsity of GT, seem to me to be poor. That is to say, Wiggins's conceptual objection suggests that weak theories of relative identity are false.

What if GT is true? In this case, Wiggins's conceptual claim is a claim about a concept whose extension is the null-class. The claim, of course, is still true but trivially so. There are no relations of non-Leibnizian numerical identity because there are no relations of numerical identity at all, at least not in Wiggins's sense of the term. Moreover, no distinction is possible between predicates which express genuine numerical identity and those that merely express equivalence relations, because there are no relations that express 'genuine numerical identity'.[15]

So where does all of this leave the thesis RI? I think all of the extant arguments in its favour fail. For any example apparently having the disputed form, there is always some interpretation of that example which does not involve a case of RI. At the same time, I think that, if GT is true, the coherence of interpretations which do logically imply true cases of RI cannot be ruled out *a priori*. RI, then, is neither proven nor disproven.

[15] Regarding the distinction between relations of identity and relations of mere equivalence, some philosophers attached important to a thesis which I will not discuss at length in this dissertation. Noonan (1997: 639-640) draws attention to this thesis, which he attributes to Geach, and which has been called 'the counting thesis': that we can count be a relation weaker than absolute identity. That is, there is some I-predicate, which is not, and does not entail a relation of absolute identity, such that, if $I(x, y)$, then x and y are to be counted as one. Certainly, Geach's theory of identity does depend on the truth of this claim. Note that this is different from claiming that we can count by relations that are not identity relations. I think that Geach would reject the latter claim. It seems to me that Geach's persistence in using the term 'identity', when discussing relations that do not satisfy the formal features traditionally associated with identity, results from an implicit assumption that any relation that we can count by is, *ispo facto*, a relation of numerical identity.

4. SRI

We have seen that the arguments for GT and RI are indecisive. We now turn to the final component of Geach's strong theory of relative identity, SRI. Although there are several arguments in the literature advocating theses similar to SRI, for example, that a sentence with the grammatical form 'x is (numerically) identical with y' is true only if a sentence of the form 'x is the same A as b' is true, where A is a general term (for example, Wiggins 1980: 55-76), the truth of such theses gives no support to SRI. SRI is the stronger thesis that all atomic sentences involving a relation of numerical identity are of the logical form 'x is the same F as b' where F is a sortal. It is possible that SRI is false and similar but weaker theses are true.[1] So the truth of SRI needs to be established by an argument different from those presented by philosophers like Wiggins.

4.1 An Argument for SRI

Geach does not provide anything like a clear-cut argument in favour of SRI, though we have seen that this thesis is independent of both RI and GT. Instead, what we find in Geach's writings are several passing comments on a supposed relationship between relative identity and Frege's views on the logical structure of statements involving predications of cardinality. These reflections have been built into an argument by Alston and Bennett (1984). I will consider their argument and conclude that it fails to establish SRI. I will then consider an argument against SRI from Le Poidevin (2009). I will conclude that Le Poidevin's argument also fails. My final judgement on SRI,

[1] Each non-relativized relation may be semantically complete and still entail some relativized identity relation.

then, is that, like RI, it is neither proven nor disproven.

4.1.1 Two 'Parallel' Theses

In *Reference and Generality*, Geach claims that Frege's cardinality thesis supports his own view on identity (Geach 1980: 64, 176). Geach provides little argument except to note that both theses involve relativization of certain classes of statements to general terms (concept-terms or count-nouns). Alston and Bennett (1984: 557) defend Geach's assertion, providing an argument that the two theses are 'parallel'. Alston and Bennett do not, however, say exactly what it means for the two theses to be parallel. In responding to Alston and Bennett, Sacchi and Carrara (2006: 547) suggest that the theses would be parallel 'if the reasons which justify the involvement of the general term "A" in the first case would also justify a parallel involvement in the second case'. The central concern of this section is to show to what extent this condition is satisfied by the cardinality thesis and SRI.

Geach suggests that SRI is entailed by a central Fregean thesis. Geach writes,

> Frege emphasized that "x is one" is an incomplete way of saying "x is one A, a single A", or else has no clear sense; since the connection of the concepts one and identity comes out just as much in the German "*ein und dasselbe*" as in the English "one and the same", it has always surprised me that Frege did not similarly maintain the parallel doctrine of relativized identity. (Geach 1980: 64)

To adjudicate this claim, we will have to consider Frege's argument.

4.1.2 Frege's Relativity Argument

Frege argues that numbers are not predicated of individuals or collections of individuals, but, rather, numbers are predicated of concepts, where such predications have the form '... has the extension n', and the empty argument place is filled by a first-level concept (Frege [1884] 1968: 59). This thesis is considered by some, including Frege himself, to be a profound discovery about the nature of number and about statements involving numbers. The argument which leads Frege to this conclusion has come to be called 'the relativity argument'. In this section, I wish to examine one basic move of the relativity argument, namely the arguments provided by Frege for thinking that number cannot be a property of individuals or groups of individuals. Frege presents two arguments for this claim. It is this conclusion, an interim conclusion in the relativity argument, which seems to bear some relation to SRI.

4.1.3 The Thesis

Frege's Cardinality Thesis can be stated as follows:

> 'The content of a statement of number is an assertion about a concept.' (Frege 1968: 59)[2]

Frege's argument for the cardinality thesis proceeds by considering and rejecting existing accounts of the nature of numbers. Frege considers the abstractionist accounts from Cantor, Schroeder, and Mill (Frege 1968: 27-33), subjectivist accounts from Kant and Schloemilch (Frege 1968: 33-38), a set theoretic account from Thomae (Frege 1968: 38-39), and finally definitions of the number one in terms of the concept of unity, from Schroeder, Leibniz, and Bauman (Frege 1968: 39-44). In the course of examining the views mentioned above, Frege presents several arguments against the view that numbers are themselves concepts. I therefore introduce, as an interim conclusion of the Relativity Argument:

CT*: Numbers are not first-order concepts.

We will next consider the support for CT*.

[2] I use the Austin translation throughout, Mike Beaney translates this passage as 'a statement of number *contains* an assertion about a concept' (Beaney 1997: 99, my italics).

4.1.4 The First Argument

Frege provides a well-known argument for CT*. The argument can be extrapolated from the following passage (beginning with a quote from Baumann).

> "[E]xternal things do not present us with any strict units; they present us with isolated groups or sensible points, but we are at liberty to treat each one of these itself again as a many." And it is quite true that, while I am not in a position, simply by thinking of it differently, to alter the colour or hardness of a thing in the slightest, I am able to think of the *Iliad* either as one poem, or as 24 Books, or as some large Number of verses. Is it not in totally different senses that we speak of a tree as having 1000 leaves and again as having green leaves? The green colour we ascribe to each single leaf, but not the number 1000. If we call all the leaves of a tree taken together its foliage, then the foliage too is green, but it is not 1000 . . . if I place a pile of playing cards in (someone's) hands with the words: find the number of these, this does not tell him whether I wish to know the number of cards, or of complete packs of cards, or even, say, of honour cards at skat. To have given him the pile in his hands is not yet to have given him completely the object he is to investigate; I must add some further word–cards, or packs, or honours. (Frege 1968: 28-29)

It seems that concepts like *colour* and *hardness* have the following features:

> (4.1) For any object, x, of which the concept-word is predicated, that concept-word can be truly predicated of at least some part of x.

> (4.2) The concept-word can be truly predicated of x, no matter how x was conceived of.

The foliage example shows that the number one does not have feature (4.1). The pack of cards example shows that the number one does not have feature (4.2).

4.1.5 SRI and CT: The Differences

Frege's thesis concerns the subject of propositions involving identity. SRI concerns the predicates themselves. Frege's strategy for treating the ambiguity of sentences involving cardinality, that is, CT* above, is to construe the subject of these sentences as being a concept and the predicate as something like 'having the extension n'. As Perry (1970) points out, if Frege's thesis really were the parallel of SRI, Frege would have held that there were a multiplicity of cardinalities, one for each of the general terms which we are able to count. According to this view, which Perry calls 'the relative number thesis', a group of individuals has no absolute cardinality, but instead can be counted in several different ways. In other words, there is no such thing as having the number two *simpliciter*. Rather, there are a series of number-properties: having the pack-number two; having the card-number two; having the honours-at-skat-number two and so forth.

But it seems that this is not Frege's view. So it may at first glance seem that the two theses are not parallel. Alston and Bennett (1984), however, argue that there is a parallel, nonetheless. Firstly, they claim, the difference between the cardinality thesis and the relative number thesis is merely notational. Frege could have accepted either on the basis of the arguments he provides for CT*. Given this, Alston and Bennett argue that

> [J]ust as we have constructed a variant on Frege's doctrine of cardinality which does make it run parallel to the relative identity thesis, we could instead modify the latter so as to make it parallel to what Frege actually held about cardinality. Having become convinced that 'a=b' won't do as it stands, and that a general concept must be lurking somewhere in the vicinity, Geach might, in closer emulation of Frege, have gone on to construe identity as a relation between concepts. Instead of insisting that all state-

> ments of numerical identity are of the form 'a is the same F as b' he might instead have opted for "The concept *a which is F* is uniquely coextensive with the concept *b which is F*." (1984: 557)

They go on to say,

> Do these complications blunt the force of Geach's appeal to the close connection of identity and cardinality? We think not. It seems clear that for both topics we can move freely between the "changing the subject" version and the "relativizing the predicate" version, that the two versions are motivated by the same considerations, and that they accommodate the same range of data. Thus Geach can still ask: if we adopt one of these "generality" theses for number, how can we refuse to adopt some generality thesis for identity? (1984: 557)

Thus, Alston and Bennett conclude, the cardinality thesis and relative identity are parallel after all.

Alston and Bennett do not, however, provide an explicit argument showing that either the relative number thesis or the cardinality thesis entails any version of relative identity. We will need to find such an argument if the case for SRI is to progress any further.

4.1.6 Finding the Principle of Equivalence

In what follows I intend to introduce a principle which elucidates the relationship between statements involving identity and statements involving cardinality.

Introductory logic textbooks and first year courses alike teach new students that if they would make a statement involving cardinality, using the vocabulary of formal logic, they must do so using identity relations. Thus:

(4.3) Two students failed the exam.

is written

(4.4) $\exists x \exists y(S(x) \wedge S(y) \wedge F(x) \wedge F(y) \wedge x \neq y)$, where S: '... is a student' and F: '... failed the exam'.

However, it is never divulged what the relationship between (4.3) and (4.4) actually is. Obviously, there is some principle of translation between English and FOL$^{=}$ at work here. But besides this, there must be some other principle, for if (4.3) is to be written as P2, then surely some important relationship holds between (4.3) and the English translation of (4.4): 'There is some x and some y such that x is a student, and y is a student, and x failed the exam, and y failed the exam, and x is not identical with y'. Are these two sentences which assert the same proposition? This might be conceivable if identity and cardinality are taken to have the same propositional content. At the very least it seems that there must be a special relationship between (4.3) and the English translation of (4.4), which allows (4.3) to be translated as (4.4). If this is the case, then it seems that this principle, on which depends the validity of such a fundamental use of formal logic ought to be made explicit.

Before I attempt to formulate the principle, I will restrict the sentences that I am interested in. First of all, we need to exclude sentences which involve the relevant relations in an irrelevant way. We are not interested in all sentences with a cardinal number in them or with the word 'identical'. We are only interested in each atomic sentence with an identity relation as the main connective, or the negations of such sentences, or conjunctions of the above (otherwise we would only be able to include the cardinal numbers one and two). Similarly, we are only interested in atomic sentences which involve grammatical predications of cardinal numbers. Having set these provisos, we may turn once more to the relation.

I propose the following principle:

Equivalence of cardinality and identity thesis:

> For every statement, $\boldsymbol{P}$, which involves an identity relation, there is some statement, $\boldsymbol{Q}$, involving cardinality and just the same subject and mass terms and count nouns as $\boldsymbol{P}$ such that, necessarily, $\boldsymbol{P}$ is logically equivalent to $\boldsymbol{Q}$, and for any statement,

R, involving cardinality, there exists at least one statement, ***S***, involving an identity relation and just the same subject and mass terms and count nouns as ***R*** which is such that necessarily ***R*** is logically equivalent to ***S***.

With this equivalence thesis in hand, we are at last in a position to suggest an argument on behalf of Alston and Bennett which might suggest an entailment from the cardinality thesis to SRI, before considering objections to it.

If we assume the truth of the cardinality thesis and the falsity of SRI, then, Alston and Bennet might argue, we run into a contradiction. The idea is this, if SRI is false, then there is some statement of identity which does not involve specification of some sortal term. By the equivalence thesis, if there is some statement of identity not involving a sortal term, then there is some statement involving cardinality not involving a sortal term. So there is some statement of cardinality not involving a sortal term, but, by the cardinality thesis, it might be claimed, it is not the case that there is some statement of cardinality not involving a sortal term. So our initial assumption might seem to be inconsistent. If this is right, the truth of the cardinality thesis would be incompatible with the falsity of SRI. In other words, the former entails the latter. This, I think , is the best case that can be made for Alston and Bennett's conclusion. However, objections have been raised to which I now turn, and which, I think, ultimately show that Alston and Bennett's argument is unsound.

4.1.7 Sacchi and Carrara's Objection

So it seems that there might be a *prima facie* case for thinking that the cardinality thesis implies some version of SRI. In light of the conceptual connections between cardinality and identity, we might expect that, if sentences of one kind necessarily involve sortal terms, then so do sentences of the other. However, Sacchi and Carrara argue, in response, that the cardinality thesis implies absolute identity and is therefore incompatible with SRI. Their argument rests on the claim that there are some meaningful sentences which, if the cardinality thesis is true, must include an absolute identity relation as a logical component. Carrara and Sacchi give the following example: 'What is said in sentences of the form "x is one" is that the concept *being identical with x* has the property of having a singular exemplification.' (Carrara and Sacchi 2007: 549)

Sacchi and Carrara claim that, following the cardinality thesis, this sentence must be understood as predicating a number of a concept rather than an object. Carrara and Sacchi suggest that the only possible concept that can be derived from this sentence would be 'identical to Solon'. Thus the sentence is to be understood as '"being identical to Solon" has extension one'. But the translation of the sentence involves an unrelativized relation of numerical identity. Thus, they conclude that the cardinality thesis is incompatible with SRI.

Strong claim: CT entails the falsity of SRI

Sacchi and Carrara have argued that if we take sentences involving cardinality to be statements about concepts, then some sentences must be statements about concepts involving unrelativized identity relations.

It might at first glance seem odd to interpret 'Solon is one' as saying something about a concept involving an identity relation. After all, there is no obvious reference to the notion of identity in the original, pre-theoretic rendering of the statement. No doubt Sacchi and Carrara will respond that the introduction of an identity relation is a trivial move, since the concept '... being identical with Solon' is already implicit in the use of the name 'Solon' in the sentence. If this is right, then the same should go for all sentences involving cardinality with names as grammatical subjects. Thus, 'Tommy and Timmy are two cats' would be interpreted as 'the concept *cats identical to Tommy or Timmy* has extension two' (Sacchi and Carrara 2007: 549-550).

Sacchi and Carrara take themselves to have shown that the cardinality thesis implies the existence of relations of identity which do not involve sortal terms because the conceptual specification required for some, apparently meaningful, predications of cardinality, must itself involve a relation of identity which lacks any sortal.

However, Sacchi and Carrara are wrong about this. We can still provide Frege-style reconstructions of such sentences along the lines required by the cardinality thesis without making reference to any identity relation at all. For example, the sentence 'Tommy and Timmy are two cats' could be reconstructed as 'the concept under which fall the cats Timmy and Tommy, and nothing else, has extension two'.

This kind of reconstruction of sentences involving names and cardinality is compatible with the cardinality thesis because the subjects of these sentences are still concepts. These sentences remain compatible with SRI because they do not involve absolute identity relations. Thus, *contra* Sacchi and Carrara, the cardinality does not imply that SRI is false.[3]

[3] Another published paper, by Patricia Blanchette (1999), also targets Alston and Bennett's argument from the cardinality thesis to SRI. However, Blanchette's objection depends on the incompatibility of all versions of the cardinality thesis with Geach's strong theory of relative identity as a whole, most particularly RI, rather than simply the inference from the cardinality thesis to SRI. Since I wish to consider only the arguments for and against SRI in this chapter, I will restrict my discussion of Blanchette's paper to this footnote.

Like Sacchi and Carrara, Blanchette argues that Geach's theory of identity is actually incompatible with the cardinality thesis (Blanchette 1999: 212-213). Blanchette argues that Geach's view entails that it is possible for a domain to consist of elements all of which are Fs and Gs but for the cardinality of the domain to be n Fs and m Gs where $n > m$. Note that SRI does not, in fact, have any such consequence. Clearly, Blanchette has in mind RI here, which does have the consequence that Blanchette claims. Blanchette claims that this is incompatible with the cardinality thesis, because the cardinality of a concept (presumably a Z-F set, in contemporary versions of the cardinality thesis) has, in Frege's view, a single determinate cardinality (Blanchette 1999: 213). So it seems that the conjunction of SRI and RI is incompatible with the cardinality thesis.

Moreover, though Blanchette accepts Alston and Bennett's claim that the relative number thesis (which she names 'RN') is a variant of the cardinality thesis (Blanchette 1999: 217), Blanchette argues that the relative number thesis is similarly incompatible with the strong theory of relative identity.

> RN is the doctrine that a given pile has different "relative cardinalities" in the sense that it can be divided into parts in different ways; Geach's doctrine is that a given pile has different "relative cardinalities" because even given a particular way of dividing it into parts, these parts are only "relatively" identical or non-identical with one another. (Blanchette 1999: 211)

To see what Blanchette has in mind, consider the following. Take Frege's example of a pile of cards. Imagine the set of individual cards in this pile. We have now specified a particular set, corresponding to the concept 'cards in the pile'. According to Blanchette, However, even if the cardinality thesis and SRI are not incompatible, Sacchi and Carrara think that they have shown that we can adopt the former without adopting the latter. Consider that the argument from the cardinality thesis to SRI rests on the relationship between the class of sentences involving cardinality and those involving identity, specifically that we seem to be able to translate sentences of one class into sentences of the other. This seemed to suggest that the cardinality thesis implies SRI. This is so because a sentence which involved an identity relation has no corresponding sentence involving a predication of cardinality to translate into. However, if we follow Sacchi and Carrara's proposal, the relationship between the two classes of statement will still hold even if SRI is false. Take the example: 'Solon is identical with Solon'. This sentence does not involve a sortal. However, it does translate into the sentence suggested by Sacchi and Carrara, namely '*being identical with solon* has extension one'. In this case, the statement predicating cardinality does not involve a sortal term, so neither does the latter. Yet the latter is admissible according to the cardinality thesis. In other words, the cardinality thesis does not entail the sortal-relativity of all statements involving numerical identity relations.[4,5] I conclude, then, that the Alston-

the cardinality thesis and the relative number thesis both entail that this set has a determinate cardinality, but Geach's view entails that this set does not have a determinate cardinality, because it is a relative matter whether or not any x and y in the set are to be counted as one or two.

In fact, Blanchette fails to establish her case on several points. It is not clear why the relative number thesis should entail that some concepts should admit of multiple cardinalities, for example the pile which is n packs and m cards, but that the concept 'cards in the pile' should only admit of one cardinality, namely the number of individual cards. Surely it makes sense to say 'The cards in this pile number n packs'.

Similarly, it is not clear that Geach's view really does entail that any set must admit of multiple cardinalities. This will depend heavily on the set theory which is adopted. I talk more about the set theory which best fits the strong theory of relative identity in Chapter 6, but the consequences of this set theory with regard to the cardinalities of sets have still to be worked out in full. Therefore, a final resolution to this problem must await future work.

[4] Wiggins, in the course of arguing on behalf of the Fregean thesis that existence is a second-level concept, suggests the a very similar strategy. (Wiggins 1995: 95-96).

[5] It might be objected, on behalf of Alston and Bennett, that this strategy for avoiding their conclusion is incompatible with some of the passages from *Grundlagen*, for example

Bennett argument for SRI fails. Next, I will consider an argument from Le Poidevin against SRI.

4.1.8 Le Poidevin's Objection to SRI

Of the numerous objections that target theories of relative identity, almost all are aimed at theses that I have called 'GT' and 'RI'. Since, as we have seen, very little independent argument has been advanced for 'SRI', it is, perhaps, not surprising that there are very few objections against that thesis in particular. There is however, one objection to relative identity in the literature, which I take to affect SRI primarily. The objection comes from Le Poidevin (2009). Le Poidevin argues that relative identity entails statements of the form:

(4.5) $x =_F y \land x \neq_G y$

However, argues Le Poidevin, (4.5) entails the existence of relations of absolute identity, because it involves multiple tokens of the same variable. But relative identity, in Le Poidevin's view, rejects the existence of absolute identity. To avoid inconsistency, Le Poidevin thinks the relative identity theorist must replace (4.5) with the following,

(4.6) $x_1 =_F y \land x_1 =_F x_2 \land x_2 \neq y$

But this generates a new problem. Christopher Hughes Conn explains Le Poidevin's next move:

> If our goal is to replace affirmations of absolute identity with affirmations of relative identity, then it should be clear that we are no better off than before. For (4.6) succeeds in avoiding one affirmation of absolute identity only by committing itself to another, namely, the assertion that the very thing (x_2) which is

1968: 40, in which Frege seems to reject 'all-embracing' concepts. However, these passages may well be a simple mistake on Frege's part, as they conflict with other, and more significant passages of the same work, for example Frege's definition of zero. I am grateful to my examiner, Ian Rumfitt for pointing this out to me.

> the same F as x_1 is not the same G as y. It is only too clear, moreover, that by substituting a suitably relativized version of this identity-statement, we will be committing ourselves to yet another statement of absolute identity. (Conn 2012: 63-64)[6]

The worry, then, is that true statements instantiating form (4.5) somehow lead to an infinite regress. Le Poidevin's argumentative strategy is to raise a preliminary objection: that statements of the form (4.5) are incompatible with some feature of relative identity, then to offer an apparent escape route for relative identity, namely, indexing the variables. Finally, he tries to show that the attempted escape would lead to a regress.

However, the attempted escape route, the repeating and subscripting of variables, is not required to defend SRI. Naturally, Geach does not think that the referents of two tokens are absolute identical, because there is no such relation. However, it will be the case that, for any x, x is relatively indiscernible (indiscernible with respect to a fixed-stock of properties) from x. This will be a condition on the assignment of the variable x. Given this, why would the relative identity theorist need to specify some relative identity relation holding between the two tokens of x, in order to avoid inconsistency? There is no such need, and the regress does not get started.

We have therefore considered the arguments for and against SRI, and I have argued that they all fail. Once again, I remain neutral about the thesis. However, there are a class of objections that do not clearly target any particular one of the three theses in particular which I have not yet considered, but which are potentially problematic for the strong theory of relative identity. It is to these that I will turn in the next chapter.

[6] Dummett raises the same concern, though in a less developed argument than Le Poidevin's. Dummett (1973) argues that Geach has inadvertently introduced a double relativity in his thesis and in so doing has ensured its incoherence. For Geach holds that there are true sentences of the form 'x is the same F as y and x is not the same G as y and either x or y is a G'. But if what has just been said is correct, then this is wrong, argues Dummett, the correct form of identity statements ought to be 'x (the F) is the same F as y (the F)'. We must relativize the names before we use them. But this in turn seems to rule out cases of RI, because if we are limiting ourselves to discussing only x (the F) and y (the F), then it is not possible for them to disagree over their properties, because they are the same F. A satisfactory response to Le Poidevin's argument will answer this concern as well.

5. OBJECTIONS TO RELATIVE IDENTITY

In the previous chapters, I have distinguished three theses which are involved in Geach's strong theory of relative identity. I have considered arguments for and against these theses and argued that, so far, they are neither proven nor disproven. However, I have so far avoided a large class of objections to theories of relative identity generally, to which I will now turn. These are objections which have been raised against the semantic and logical consequences of relative identity. In particular, the following three charges stand out. Theories of relative identity are incompatible with singular reference (Alston and Bennett 1984: 557-560, Dummett 1991: 167, Hawthorne 2003: 113). Singular terms are incompatible with the very notion of quantification (Quine 1964, Dummett 1991), and Geach's theory of relative identity entails the failure of the syllogisms (Cain 1988).

One of the difficulties in assessing these objections is that those philosophers who have raised the objections are rarely careful to distinguish the different theses involved in theories of relative identity. Consequently, it is not always clear what thesis is believed to lead to which unwanted conclusion. The objections are most often phrased as attacks on the thesis I have dubbed 'RI'; however, it will become clear that the first two objections are really problematic only for those theories of identity involving GT, while the third is directed at further views of Geach's that we shall consider shortly. I will argue that the incompatibility between relative identity, specifically GT, and features of classical semantics can be traced to a more general incompatibility than has hitherto been recognized, between the set-theoretic assumptions of the strong theory of relative identity and classical semantics, respectively. Moreover, I suggest that all of the semantic objections show that the theory of relative identity needs to be supplemented by a non-classical semantics. I also draw several conclusions about what such non-classical semantics would need to be like. Specifically, if GT is true, the appropriate semantics is one in which there is no possible characterization of the domain of discourse in terms of the most fine-grained elements of the domain. I consider such a semantics in Chapter 6. Finally, I will show how the apparent failure of the syllogisms derives from Geach's own attempts to escape the semantic objections. I will put off attempting to resolve this last problem until the next chapter.

5.0.9 A Preliminary Objection

Wiggins, who, as we have seen, thinks there is a conceptual connection between relative identity and indiscernibility, poses the following question as a challenge to RI:

> How, if *a* is *b*, could there be something true of the object *a* which was untrue of the object *b*? They are the same object. (Wiggins 2001: 27)

Note, however, that RI does not entail that there are cases of *a* and *b* that are the same object and have different properties. Cases of RI involve some x and y that jointly satisfy some sortal-dependant identity relation and have different properties. However, it is not the case that if x and y jointly satisfy some sortal dependent identity relation, then x and y are the same object. So RI may be true without there being any instances of the kind Wiggins is objecting to. Moreover, if SRI is also true, then such instances are impossible, because there is no relation '... is the same object as...'.

However, this does not entirely dispel the general worry. To put this objection into focus, let us consider a case of RI. First, stipulate that 'Smith' names a surman, that 'John Smith' names a man born on March 18th, and 'Sam Smith' names a man born on March 19th. Further assume the truth of the following.

> (5.1) John Smith is the same surman as Sam Smith, but John Smith is a different man than Sam Smith (further stipulate that

(5.1) is a case of RI).

(5.2) John Smith is the same surman as Smith.

(5.3) Sam Smith is the same surman as Smith.

(5.4) Every surman is a man.

(5.5) Every man has a birthday.

From these, we can derive the following:

(5.6) Smith is a man.

and, from thence, we can derive:

(5.7) Smith has a birthday.

At this point, an objection can be raised. When is Smith's birthday? March 18? March 19? Both? Neither? We have already seen that RI is incompatible with LL. But even if LL is rejected, a problem remains. For we have said that Smith has a birthday, but it does not seem that there is any particular day on which Smith was born. We could of course simply reject (5.5), but this would be to allow a class of birthday-less men.

Wiggins's analysis of this problem is that RI entails certain unwanted semantic consequences. As Wiggins explains,

> Suppose there were terms t^1 and t^2 both designating z, one and the same donkey, and suppose there were a context $\phi(\ )$ such that the result of supplying t^1 to it was true and the result of supplying t^2 was false. What ought we to say if it were suggested that the open sentence $\phi(x)$ determined a property? Call the putative property Q. We ought to ask: How can the donkey both have and lack the property Q? The question is unanswerable.

> Let the R⟨relative identity⟩-theorist note that this argument can be stated, as it is stated here, without showing any special favour between '=' and '$=_F$.' It supports Leibniz's Law for both of these relations. In order to counter it, the R-theorist will have to uncover much more complexity than appears to be present in the innocuous locution 't^1 designates z and t^2 designates z.' Nor is that enough. R-theorists will need to deny the very possibility of there being such a relation as simple designation. (Wiggins 2001: 28)

The charge here is that any semantics for the expression 't^1 designates z' that is compatible with RI will be excessively complicated and will not involve the notion of simple designation. To consider how far this objection succeeds, let us briefly note Geach's view of referring expressions. The metaphysical issue, how it is possible for some x and y to have different properties and yet be identical relative to some sortal, is answered by Geach, with an account of how referring expressions work. Geach introduces a distinction between two kinds of names, which he calls 'names of' and 'names for', respectively. As Geach puts it,

> A proper name carrying as part of its sense the criterion of identity expressed by "the same cat" may be called a name for a cat. . .a proper name is a name of a cat if it is not an empty name but does actually name a cat. (Geach 1980: 70)[1]

[1] Hawthorne objects to Geach's contention that names have semantic content which involve criteria of identity:

> Current wisdom about proper names would also need rethinking were Geach's approach to be accepted. According to Geach, in order for a proper name to have a legitimate place in the language, it must have a criterion of identity associated with it-given by a relative identity predicate. The popular view (see Kripke 1971) that a name can be cogently introduced by either demonstration-'Let "Bill" name that thing (pointing)'-or else by reference-fixing description (that need not encode a sortal in Geach's sense)-'Let "Bill" name the largest red thing in Alaska'-is thus anathema to Geach. (Hawthorne 2003: 113)

Hawthorne is singling out neither RI nor GT, but rather a further thesis of Geach's as the point of incompatibility with the popular view of proper names. Hawthorne thinks that

This response is open to all the usual objections raised against descriptive theories of proper names, given that the distinction is cashed out in terms of the descriptive content involved in the sense of a name. However, we will put these concerns to one side. If a name, '*a*', is a name for a cat, then the sense of '*a*' will include all sufficient descriptive content such that only one cat is named by '*a*'. If '*a*' is a name of a cat, the referent will have the property of being a cat, but the sense of the name will not necessarily include sufficient information to determine which cat. We can now see how the name of/name for distinction serves to resolve the problem.

If we stipulate that the substitution instances of the quantified expression 'every man' in 'every man has a birthday' are all those names for men and none of the names of men that are not names for men, the inference from (5.5) and (5.6) to (5.7) is invalid. It does not follow that Smith has a birthday and the problem disappears.

If Geach's notions of names of and names for are coherent, the distinction seems to provide a resolution to the problem of surmen apparently having multiple birthdays. However, Dummett's second concern remains a major worry on this account. According to the standard view, names are singular terms. It is not clear that any sense can be made of the very notion of a 'singular' term, if relative identity is true.

Geach's view that each name corresponds to some criterion of identity, is incompatible with the plausible view that the reference of a term can be fixed ostensively or via a description lacking a sortal. Hawthorne is right on this score.

However, it is by no means clear how damaging this is to Geach's case. Even if the strong theory of relative identity does require Geach's further thesis that names involve criteria of identity, Hawthorne has not shown that Geach is unable to adequately tweak the notion of ostensive and descriptive reference-fixing in such a way that they are made compatible the rest of the theory. For example, we could restrict reference-fixing descriptions to just those that involve sortals, and we could further stipulate that all legitimate cases of ostensive reference-fixing implicitly involve sortals knowable by the context of the ostension. These would surely not be wholly implausible alterations to the popular theory of reference-fixing. I think, then, that, though Hawthorne is right in claiming that Geach's views are incompatible with some aspects of orthodox semantic theory, the examples that he points too are not ones which undermine the strong theory of relative identity.

5.0.10 Relative Identity and Singular Reference

Several different reasons have been provided for thinking that relative identity is incompatible with the classical semantics for proper names, as we shall see.[2] I intend to establish what features of relative identity, in particular, are incompatible with classical semantics. I will argue that both RI and GT are incompatible with singular reference.

Alston and Bennett argue against relative identity as follows:

> We not infrequently succeed in picking out particular items ... physical objects, events, experiences, properties, persons, institutions ... by the use of proper names, definite descriptions, and indexical expressions of various sorts. Given that we have succeeded in picking out something by the use of 'a' and picking out something by the use of 'b' it is surely a completely determinate proposition that $a = b$, that is, it is surely either true or false that the item we have picked out with 'b' is the item we have picked out with 'a'; nor do we have to range a and b, covertly or overtly, under a common concept in order to form an identity proposition with a determinate truth-value. If a is the number 15 and b is Sally's new hat, it is clearly false that $a = b$, and no question 'Aren't they the same what?' is left dangling. (Alston and Bennett 1984: 558)

Alston and Bennett are here suggesting that the mere occurrence of apparent singular reference (by naming, definite description, etc.) in natural language is sufficient proof of the existence of absolute and unrelativized identity relations. Thus, in Alston and Bennett's view, singular reference entails both the falsity of GT and also the falsity of SRI. Alston and Bennett do not provide much by way of sustained argument to back up their intuition that

[2] This is denied by Griffin (Griffin 1977: 156-157). In fact, in this regard, Griffin appeals to Dummett for support. Claiming that Dummett successfully argues that there is no formal entailment from Geach's relative identity thesis to a non-classical interpretation of the quantifiers. The reference is to an early work of Dummett (1973: 562), and in later work, in which Dummett argues that there is such an entailment and which I quote at length in what follows, Dummett's criticisms of Geach seem to be expressible formally.

the presence of certain kinds of expressions in natural language entail the existence of an absolute identity relation, still less that such a relation must be dyadic. I think their position depends not simply on the existence of certain features of natural language but also on a particular view of how those features work. We can get a better grasp of the supposed problem by considering some additional thoughts from van Inwagen.

Van Inwagen argues:

> The philosopher who eschews classical, absolute identity must also eschew singular terms, for the idea of a singular term is ... at least in currently orthodox semantical theory, bound to the classical semantical notion of reference or denotation; and this notion, in its turn, is inseparably bound to the idea of classical identity. It is a part of the orthodox semantical concept of reference that reference is a many-one relation. And it is a part of the idea of a many-one relation–or a one-one relation, for that matter–that if x bears such a relation to y and bears it to z, then y and z are absolutely identical. (That's what it says on the label.) (van Inwagen 1988: 259)

Van Inwagen is arguing that relative identity is incompatible with the classical semantics for singular terms on the grounds that the relation of singular reference is many-one, but that relative identity is incompatible with the notion of a single referent for a given term. Indeed, Geach would disagree with the view, which van Inwagen attributes to classical semantics, that naming is a many-one relation (one or more names referring to one object). Geach countenances the existence of 'shared names', that is, names with more than one bearer (Geach 1991: 17-18). In other words, Geach thinks that naming is a many-many relation. Moreover, the name of/name for distinction involves a conception of proper names that are not related many-one to elements in the domain of discourse. The whole point is that any given proper name bears different kinds of naming relations to entities which are not absolutely identical with one another. The name 'Bluemantle', for example, refers to a herald at the College of Arms (as a name for) and coincidentally to John Smith, the man who is performing the duties of Bluemantle at the time (as a name of). The herald and the man are not absolutely identical, so the name 'Bluemantle' does not refer solely to one absolutely individuated object. Moreover, the name of/name for distinction (or something similar) is required to make sense of the thesis RI. The truth of statements of the form: for some x and y, x is the same F as y but not the same G, depends on there being at least two different kinds of name. This shows that RI is incompatible with the thesis that naming is always many-one. Any theory of identity involving RI, therefore, must deny that all names are singular terms. Van Inwagen, then, is right. The strong theory of relative identity is incompatible with the many-one view of names. However, it is not simply a matter of supplanting the many-one view of names with a many-many view, for, it turns out that the latter is also incompatible with the strong theory of relative identity.

Although Geach claims to accept the existence of shared names (1980: 216), he is not, in fact, free to make the distinction between names that are singular terms and shared names. The difference between these categories is that the first refer to just one object, while the latter may refer to more than one. But of course, this very way of talking presupposes that we can distinguish between a single entity and a multiplicity of entities singled out only as 'objects', without further specification. But to do this would require absolute identity. If classical semantics is appropriate for apparent singular terms in natural language, then there exists a relation, which we call 'reference', between every name and exactly one individual element in the domain of discourse. But, if we allow ourselves talk of 'one individual element in the domain of discourse', then we must have access to an absolute identity relation in order to individuate any given element from the others. For this reason, GT is also incompatible with classical semantics. It seems, then, that relative identity is incompatible with naming being a many-one relation in at least two respects. If RI is true, there are counter-examples to the many-one view of naming. If GT is true, the very terms in which the relationship is expressed lack sense.

GT and RI, then, are both incompatible with the classical semantics for proper names. Next, we will consider if the same is true for the classical semantics for quantified expressions. I will argue that it is.

5.0.11 Quantification

We began the discussion of the semantics of proper names with an intuitive objection to relative identity. The same worry can, of course, be phrased using devices of quantification, as in the following objection to relative identity from Michael Burke,

> If a cat and one of its proper parts are one and the same cat, what is the mass of that one cat? (Burke 1994: 138 footnote 19)

Once again, if we assume that all cats have some weight, and that proper parts of cats weigh less than the whole cats of which they are parts, then it seems that the cat in Burke's example has two different weights. If RI is true, '... is the same cat as ...' is not a relation of absolute identity. It is not the case that some x and y that are the same cat are exactly the same thing. So why ought we to think that they should have the same weight in the first place? What gives this example its force, is the intuition that every cat has just one weight or at least just one weight at any given time.

Cases like this show that RI entails a non-classical view of quantificational expressions, parallel to the view of proper names entailed by RI. Indeed, Geach seems to have been aware of this consequence, as his treatment of the quantifiers is exactly the parallel of his name of/name for distinction. We have already briefly noted Geach's view that restricted quantification over Fs is not reducible to unrestricted quantification over things that are Fs. We are now in a position to consider this component of Geach's views in more depth.

Geach's advocates a substitutional account of the quantifiers. As he says, 'I think the best way to understand applicatives like 'some', 'every', 'most', and the like, is to consider first their use in harness, not with substantival general terms, but with lists of proper names' (Geach 1980: 191). Such a list of proper names can characterize the contents of the domain of discourse for a language, in the sense that there is nothing in the domain which is not named by a name on the list. Geach does not mean that every competent speaker of a given language is in possession of such a list of names which characterizes the domain for that language (1980: 184, 205-206), but rather, that the contents of a domain are so structured that there must be a characterizing list of names even if unknown, and that truth-conditions for our statements about the contents of a domain can always be provided which appeal to such a list. In particular, the truth-conditions Geach provides for the quantifiers, 'some' and 'many' are provided in terms of substitution from a list of names. Given that names come in two varieties, names of and names for, two different kinds of quantification are possible, in Geach's view, depending on which names can be substituted *salve veritate*. On one hand, 'some F is a G' is true if and only if there is a name of and for an F, a, which is such that $G(a)$ (Geach 1980: 206). While 'some x is an F and a G' is true if and only if it is possible to introduce some name, a such that $F(a)$ and $G(a)$, without the further stipulation that a be a name for either an F or a G, though it will, of course, be a name of both (Geach 1980: 184). Similar truth-conditions hold for the quantifier 'any'. These truth-conditions have the result that it is possible that 'some x is an F and a G' is true, while 'some F is a G' is false, so the latter (restricted) quantification is not reducible to the former (unrestricted).

We may get a better grasp of Geach's views here by considering an example. First, stipulate that 'Tibbles' is a name for a cat, while 'Tibbles' and 'Tib' are both names of one and the same cat. In other words, the sense of the name 'Tibbles' includes sufficient content to identify Tibbles as a particular cat. 'Tibbles', then, can never name something which is not a cat, nor something which is a different cat from Tibbles at t_1. Matters stand differently with those names of cats which are not at the same time names for cats. I will stipulate that 'Tib' be such a name. 'Tib' names a cat on the mat at t_1 (in fact it names the same cat as 'Tibbles'). Tib by t_2, however, may have ceased to be the same cat as Tibbles, may have ceased to be the same anything as Tibbles, and may have ceased to be a cat at all.

The important point is that the possible truth-preserving substitutions for the quantifier in:

(5.8) Every cat has a tail.

are names for cats, and the possible truth-preserving substitutions for the quantifier in:

(5.9) Everything that is a cat has a tail.

are names of but not necessarily for cats, as we have just seen. Any name for a cat is a name of a cat, so under assumption that 'Tibbles' is the only name for the cat on the mat at t_1, (5.8) is true if and only if Tibbles has a tail. (5.9) is true if and only if the names of all things that are cats and are on the mat at t_1 have tails. As it happens of course (5.9) turns out to be false because 'Tib' names one thing that is a cat on the mat at t_1 and does not have a tail.

In Burke's case, the first quantified expression, 'a cat' involves existential restricted quantification over cats. The second quantified expression, 'its proper part', is elliptical for 'some proper part of the aforementioned cat', involves unrestricted quantification over things that are cats. The rule that all cats have exactly one weight involves restricted quantification over all cats. Since restricted quantification is not reducible to unrestricted quantification, this rule does not entail that all things that are cats, and are the same cats as one another, have the same weight. In Burke's case, then, the cat has just one weight. The fact that there are things which are cats, have different weights, and are the same cat as the aforementioned cat, does not mean that the cat has more than one weight.

5.0.12 An Objection: Cain

It seems, then, that the name of/name for distinction and the thesis of the irreducibility of restricted quantification play an important role in responding to some of the objections to the theory of relative identity. However, there is a cost involved. A major objection to Geach's views on the quantifiers has been presented by James Cain (1988). Cain argues that Geach's thesis that restricted quantification over Fs is not reducible to unrestricted quantification over things that are Fs, leads to the failure of the syllogisms. Cain has in mind cases like the following:

(5.10) All men are mortal

(5.11) Bluemantle is a man

But

(5.12) Bluemantle is not mortal

'Bluemantle', recall, is the name for a herald at the College of Arms. It seems, Geach's position commits him to the truth of all three of these statements. Bluemantle is a herald, and is not mortal but continues so long as there is a man (or woman) to perform the associated functions. Yet, surely it is the case that all men are mortal. Moreover, it seems that Geach is not in a position to deny the minor premise either, since he has appealed to this quite explicitly in presenting his argument for RI (as we saw in Chapter 3). This, in turn, entails the following counterexample to the *Darii* syllogism:

(5.13) All men are mortal

(5.14) Something is a man

But it does not follow logically that

(5.15) Something is mortal

So it seems that Geachean cases of relative identity lead to the failure of at least one of the syllogisms.[3]

This is a serious worry, to which we will return at the beginning of the next chapter. If it turns out that Geach's irreducibility thesis does lead directly to the failure of the syllogisms, then Geach's theory of relative identity would involve a significantly more substantial diversion from orthodoxy than hitherto expected. This would cast serious doubt on the feasibility of Geach's irreducibility thesis, but without this thesis, we have seen that the strong theory of relative identity faces other objections.

[3] Cain shows that many of the syllogisms will be affected. I will focus on only one, but what I say will generalize.

I will put off consideration of the ways in which Geach might respond to Cain for the moment, because there remain more general worries about the semantics of the strong theory of relative identity which must be identified first. In particular, it is not clear that the irreducibility thesis can provide a suitable account of the quantifiers even if it escapes Cain's objection. For, several philosophers, namely, Quine (1964) and Dummett (1991), have objected that relative identity is incompatible with the very notion of quantification.

5.0.13 Further Objections: Quine and Dummett

This point is first raised by Quine:

> This doctrine [Geach's denial of the existence of an absolute relation of identity] is antithetical to the very notion of quantification, the mainspring of modern logic. Quantification depends upon there being values of variables, same or different absolutely; grant quantification and there remains no choice about identity, not for variables. For a language with quantification in it there is but one legitimate version of "$x = y$". (Quine 1964: 101)

This point is expanded upon by Dummett,

> If we are engaged in giving a verbal statement of the interpretation of the object-language, we have first to specify the domain of the variables. To give the interpretation of the non-logical constants, we have to be able to refer in the metalanguage to elements of that domain, or to pick out subsets of it. In order to know whether a given interpretation is admissible, that is, intelligible, we must know when two terms of the metalanguage pick out the same element of the domain, since the requirement that one and the same element, considered as picked out by each of two distinct singular terms of the metalanguage, should behave differently in respect of the satisfaction of some predicate of the object-language will render the interpretation contradictory and

> so inadmissible. Hence, to give an interpretation relative to a domain presupposes a relation of identity defined over it. And, according to the present argument, the same applies to the case in which we have only a mental apprehension, rather than a verbal statement, of the interpretation. (Dummett 1991: 294)

Given the incompatibility just noted between the strong theory of relative identity and the classical semantics for proper names, these objections will, perhaps, come as no great surprise.

Geach recognizes that the domain of discourse for a language cannot be a set defined by elements distinguished from one another by an absolute identity relation, if no such relation exists. This leaves Geach needing to explain how the domain of discourse is so structured that we can quantify over entities when those entities are not distinguished from one another absolutely.

We have already seen that Geach adopts a substitutional account of quantification, according to which the contents of the domain are characterized by a list of names. Whereas a classical semantics would introduce an 'interpretation function' defined such that, for each name as argument, some element in the domain of discourse is returned as value. In Geach's view, we cannot say what the contents of a domain of discourse for a language are, apart from using the referring expressions of that language. But this is not to say that the names of a language lack *denotata*, non-lingistic entities to which the names refer; merely that the *denotata* are not individuated from one another absolutely. This picture of the domain of discourse either requires us to grant that the domain is not itself a set or to adopt a broader view of what constitutes a set.

As Cantor puts it, 'a set is a collection into a whole of definite, distinct elements of our intuition or of our thought' (as quoted in Fraenkel 1966: 9). That the intended interpretation of the formal system of set theory involves absolute identity is further evidenced by considering a further passage from Cantor:

> [A] variety (an aggregate, a set) of elements that belong to a certain conceptual subject is well defined if by virtue of its definition

> and of the Principle of Excluded Middle it must be determined as *internally determined* whether an element of such a conceptual subject is an element of the variety, *so as* if two objects belonging to the set, despite the formal diversity by means of which they are given, are identical or not. (As quoted, E. Casari 1976: 22.)

The trouble for the strong theory of relative identity is not over, as Dummett has another objection against the semantics of relative identity. Dummett has the following to say about Geach's proposal concerning the use of a list of names to specify a domain of discourse:

> These remarks hang oddly together with the thesis to which Geach subscribes, as do I, that to every proper name is associated some determinate criterion of identity, given that he acknowledges that a list is a string of proper names; indeed, without affecting the issue, we could allow the list to contain singular terms other than proper names. It is not that Geach has gone back on this thesis concerning proper names, since he reiterates it in the very article, "Ontological Relativity and Relative Identity", under discussion. If for two proper names occurring in the list there are associated conflicting criteria of identity, it would not appear to make sense to say that they denoted the same object; and, when the same criterion is associated with two names, that criterion determines the sense of asking whether a repetition is involved, without the need for any arbitrary choice. Given an assignment, to each entry on the list, of an appropriate criterion of identity, it seems that an identity relation over the elements of the domain specified by the list had thereby been determined, one that will, as before, set bounds to the fineness of the interpretation of any $=_L$. (Dummett 1991: 298)

Dummett is arguing *ad hominem* that Geach thinks that every name on the list has some criterion of identity. If this is so, then surely the relation that holds between the referent of a name for an object and the referent for another name for the same object is a relation that expresses absolute identity.

Dummett is right about this, up to a point. If it is true that every name in any given language corresponds to some criterion of (absolute) identity, then, of course, specifying the domain of discourse by a list of names entails that a relation of absolute identity can be defined over that domain. The referent of any name will be absolutely identical with the referent of the same name and the associated criterion of identity will give us a principle for determining which names co-refer. Geach, therefore, cannot consistently hold that all names are associated with criteria of (absolute) identity. Rather, Geach (1991: 292) intends quite a different claim, though one which is not strictly entailed by his other views, that all names come associated with a criterion of identity relative to some sortal, F. This is just to say that all names are names for some F. Such a view, at any rate, would escape the charge of inconsistency.

Dummett has a final, and perhaps more challenging, objection:

> [I]t would seem that the list itself, taken together with those constraints, if any, sets a bound to the fineness of any equivalence relation expressed by some $=_L$. For, even if there are no constraints, we surely cannot get any finer equivalence relation than that which holds between the object corresponding to each entry and itself, but not between any such object and the object corresponding to any other entry. (Dummett 1991: 167-168)

In other words, Dummett suggests that no matter what constraints you put on what names are allowed onto the list, there is no more fine-grained relation than that which holds between the referent of one name on the list and the referent of that same name. The relation that holds between the referent of a name and the referent of the same name is surely a relation of absolute identity. So long as we have adequately specified the domain of discourse for the language, it seems impossible to distinguish two different entities going by the same name. If this were to happen, it would simply show that we had not adequately specified the domain in the first place.

This is a major difficulty for Geach's proposal, as it casts doubt on GT. It casts doubt on GT because the existence of a most fined-grained characterization of the domain entails that the cases envisioned by Geach, of previously indiscernible objects becoming discernible, are impossible. That is, it suggests that P3.3 of the argument presented in Chapter 2 is false. Geach is committed to P3.3, and therefore if Dummett is right that Geach's views on specifying a domain entail the existence of a most fine-grained characterization of the domain of discourse, then Geach's views on specifying a domain are incompatible with GT.[4] So GT entails that there is no 'most fine-grained' characterization of any domain of discourse.

To dispose of this objection against Geach's proposed theory of quantification, it will be necessary to develop a semantics for quantifiers, based on Geach's theory and to show that this semantics escapes Dummett's objections. I will make some suggestions in this regard in Chapter 6.

5.0.14 Conclusion

In this chapter, we saw that relative identity has been challenged on account of its supposed incompatibility with classical semantics. First, it seems that GT is incompatible with singular reference. Second, it seems that GT is incompatible with the traditional notion of quantification.

It seems, then, that if the strong theory of relative identity is true, then singular reference is impossible, at least in the conventional sense. But more importantly, it seems that GT is incompatible with the assumption that a domain of discourse for a language can be thought of as composed of discrete, absolutely discriminated elements. Quine has said 'grant quantification and

[4] As Dummett puts it,

> ... there is compelling fear of incompatibility between the picture that we are accustomed to form of the classical interpretation of the quantifiers and the picture evoked by Geach's doctrine on identity ... ⟨according to the former⟩ the *picture* we have of what constitutes a domain of objects which can serve as the range of the individual variables is such that it is impossible to see how there could be any objection to suppose an absolute relation of identity defined on it: the elements of the domain are thought as being, in Quine's words, the same or different absolutely. (Dummett 1973: 562-563)

there remains no choice about identity, not for variables' (Quine 1964: 101). It seems to me it would be more accurate to say that if one grants a traditional set-theoretic framework for the semantics of a language, then there remains no choice about identity.

The strong theory of relative identity is left, then, with a series of problems. It is not clear whether some account of names and quantifiers can be provided which is compatible with GT. It is also unclear if some suitable explanation can be given of what the domain of discourse for a language is, if it is not a set as traditionally understood. Moreover, it is not clear that any such account can be given which does not involve an ultimately-fine-grained characterization of the contents of the domain, such that the word-type/word-token style cases which are so important for Geach are possible. This worry is made more acute by Geach's further contention that all names come with criteria of identity.

In addition to all this, we have seen that the apparatus that Geach introduces in his account of proper names and quantification appears to lead to the failure of the syllogisms. This is *prima facie* a serious problem for the semantics of relative identity, and we will begin our consideration of the possible responses to the various problems raised in this chapter by confronting this one.

6. THE LOGIC AND SEMANTICS OF RELATIVE IDENTITY

We established in the last chapter that the strong theory of relative identity is incompatible with classical semantics, and the proposals that Geach has made to address these problems seem to entail some odd logical consequences. In this chapter, I will consider the logical systems and the kind of semantics which would be compatible with theories of relative identity. I will begin by considering the challenge pressed by Cain, to the effect that Geach's theory entails the failure of the syllogisms. I will then consider the semantics implied by relative identity. First, the kind of semantics compatible with GT, before considering how it would have to be altered to incorporate RI and SRI as well, as well as how these might escape the various worries raised in the last chapter. I finish the chapter by considering the various logical systems that might be suitable for theories of relative identity.

6.0.15 Geachean Quantification and the Syllogisms

We will begin, then, by considering the apparent failure of the syllogisms. We saw that Cain provides examples of the following kind:

(6.1) All men are mortal

(6.2) Bluemantle is a man

But

(6.3) Bluemantle is not mortal

The major premise involves universal quantification. For this case to serve as a counterexample to the syllogism, this quantifier must be interpreted as involving restricted quantification over men, rather than unrestricted quantification over all things that are men. If it the quantifier were unrestricted, the premise would be straightforwardly false, because of course there are things that are men but are not mortal– Bluemantle, for example. If the first premise were false, then naturally the case would not serve as a counterexample. The key to the second premise is that involves a name of a man, but not a name for a man, which is then repeated in the conclusion. If, instead of 'Bluemantle', a name for a man had been chosen, then no counterexample could be generated. Any name for a man, will satisfy the predicate '... is mortal', and will do so as a matter of necessity. Similarly, we could not create a counter-example to the *Darii* syllogism using a restricted existential quantifiers in the minor premise and conclusion.

Cain's counterexamples, then, depend on a particular mismatch between the quantifier in the major premise and the referring expressions/quantifiers in the minor premise and conclusion. Let us consider how Geach might respond to these cases.

I think the preliminary point that needs to be made on Geach's behalf is that Geach has the resources to provide an account of the syllogisms according to which there are no counterexamples. This is because Geach's account of the syllogisms will rule out the kind of mismatching that Cain's counterexamples depend upon. To see this, consider the following.

Having made a distinction between two kinds of quantification, Geach will of course not permit this distinction to be ignored in a formal statement of his system. The symbols '$\forall$' and '$\exists$', then, can stand for restricted quantification, or restricted quantification, but not both. Let us assume, first that it stands for unrestricted quantification. The *Darii* Syllogism can now be phrased as follows:

$\forall x Fx \to Px$

$\exists x Fx$

$\exists x Px$

However, none of Cain's cases can provide a counterexample to this rule. For the second quantifier is a restricted quantifier, and we have seen that the Cain cases all require that the quantifier in the minor premise and conclusion be unrestricted.

Similarly, if we assume that '$\forall$' and '$\exists$' represent unrestricted quantification, then, again we cannot find a counterexample, because, in this case, the major premise of any proposed counterexample will turn out to be false. As we saw, 'All men are mortal', when interpreted as involving an unrestricted quantifier is straightforwardly false.

All of this goes for cases where the minor premise and conclusion involve names rather than quantifiers. A Geachean formal system will stipulate that the names in the minor premise and conclusion must be names for the sortal type to which the quantifier in the major premise is restricted. Having built this stipulation into the formal system, no counterexamples can be constructed.

The objection then, cannot target the coherence of a Geachean formal system, which preserves the syllogisms. Rather, the objection targets informal cases, and the fact that the formal systems will exclude cases by stipulation and therefore, perhaps, fail to recognize some genuine instances of the syllogisms, as found in ordinary language.

In other words, Cain (1988: 88) objects that (6.1)-(6.3) is manifestly a counterexample to the syllogisms and that, if Geach's formal system does not recognize it as such, so much the worse for the formal system. The question then is, is it obvious the truth of the premises (6.1) and (6.2) entail the falsity of the conclusion? Or, to put the point in another way, is it obvious that cases with the grammatical structure of counter-examples to the syllogisms, need to be recognized as such? Geach may answer 'no' to these questions.

Note that there are plenty of grammatical counterexamples to the syllogisms. Consider the following:

(6.4) All water evaporates

(6.5) The Thames is water

But

(6.6) The Thames does not evaporate

Naturally, philosophers can provide simple explanations of why this is not, in fact, a counterexample to one of the syllogisms. But this does not change the fact that the three intuitively true statements involved in this case can be put into the grammatical form of a counter-example to one of the syllogisms. Why should this case not be a problem for philosophers generally, but (6.4)-(6.6) should be a problem for Geach? Perhaps, Cain may say, simply because the intuition that (6.1)-(6.3) genuinely does instantiate the form which entails a counterexample to one of the syllogisms is stronger in that case. It is not clear how compelling this answer is though. Geach would no doubt respond that, once a speaker of ordinary English has become sufficiently well versed in the distinction between restricted and unrestricted quantification and the parallel distinction between names of and names for, it will become apparent to him or her that (6.1)-(6.3) does not, in fact, instantiate a counterexample to the syllogisms.

I will not pass a final judgement on how compelling these responses to Cain's objection are. I have presented what I take to be the best response open to Geach, whether this response is sufficient, I leave an open question. If they are not, a defender of the strong theory of relative identity is left with the unpalatable dilemma of either biting the bullet and granting the invalidity of the syllogisms or finding some suitable replacement for the irreducibility of restricted quantification thesis and the corresponding name of/name for distinction. I shall not follow up this issue any further in this dissertation, but turn instead to a consideration of whether the other objections raised in the last chapter can be answered.

6.1 Semantics

Returning, then, to the semantic worries that we raised in the last chapter. To recapitulate, these are as follows. First, whether a semantics for proper names and the quantifiers can be provided if GT is true, given that that thesis is incompatible with the notion of singular reference and individuated assignments for variables. Second, whether an account of domains of discourse can be provided which is open to evermore fine-grained characterizations of the contents of the domain. I shall divide the following discussion into two parts. First, I will consider what a semantics compatible with GT would need to be like, given the objections raised in the last chapter. Second, I will expand the discussion by considering how Geach's further semantic apparatus (names of, names for, and the irreducibility of restricted quantification thesis) might be useful in providing a semantics compatible with RI and SRI as well.

6.1.1 Prospects for a Semantics for GT

Recall that classical semantics stipulates that the interpretation of any singular term in a language is an element in the domain of discourse. Formally, this can be written:

> (6.7) If $N \in C$ is the set of all names in a language, L, and S is a structure, $\langle D, I\rangle$, then S must be such that for every element $n \in N$, $I(n) \in D$, where I is an interpretation function and D is the domain for L.

This is to say, all names in L have referents in the domain of discourse. An assumption underlying (6.7) is that there is a collection of entities, which we have called D, which are the possible *denotata* of the singular terms of the language for which a semantics is being provided. Moreover, it is the standard assumption that D is a Z-F set, specifically, that D is the set of all possible objects nameable in the language L. We have already seen that the traditional interpretation of the formal language of set theory (both naive and Z-F) is incompatible with the strong theory of relative identity because the traditional interpretation of an Z-F set presupposes that any set contains a number of elements distinguished from one another absolutely.

The other requirements for a complete semantics are rules for interpreting the quantifiers and variables, predicate letters, complete propositions and the logical vocabulary. The semantics for propositions and the logical vocabulary (apart from identity relations themselves, about which I shall say more in the next section) do not pose any special problems for the strong theory of relative identity. However, it should be noted that the incompatibility between relative identity and singular reference, which van Inwagen and Alston and Bennett have drawn our attention to, and the incompatibility between relative identity and the traditional notion of quantification which Dummett and Quine have drawn our attention to, are in fact only two instances of a wider incompatibility between the strong theory of relative identity and classical semantics. The same considerations which show that GT is incompatible with the classical semantics of singular terms also show that GT is incompatible with the classical semantics of predicates as well. A statement of the classical semantics for predicates is as follows:

> (6.8) If $F_n \in C$ $(n \geq 1)$ is the set of all n-place predicates in L, then S must be such that for every element $\in F_n$, the set of all n-tuples such that $\langle I(t_1), ..., I(t_n)\rangle$ satisfy F is a subset of D.

(6.8) says that there is a function from any n-place predicate in a language L which returns as value a set of n-tuples, such that each n-tuple is a subset of D. In other words, the value of a predicate is the set of its arguments. If the arguments of a given predicate, F, are collectively a subset of D, and D, as before, is assumed to be Z-F set, as traditionally understood, then distinguishing one subset of D from another is a matter of distinguishing the elements of one subset from the elements of another; again (given the identity conditions for Z-F sets) this entails absolute identity.

Classical semantics, therefore, assumes that there is a definable function from referring expressions and predicates in a language to elements and sets of elements in the domain of discourse for that language. For a function to be well-defined it must return individual values for its arguments, always returning the same values for the same arguments. But this implies that the set of names for a language L maps onto a set of individuated objects, which are picked out by these names. Similar considerations apply to the account of the assignment function for variables. This is precisely the picture of naming which must be rejected according to the strong theory of relative identity.

Does this mean that a semantics cannot be provided for proper names if GT is true? I think not, however, such a semantics will involve the rejection of certain assumptions that underlie classical semantics.

GT is compatible with stipulations like (6.7) and (6.8) so long as the contents of D are not understood as involving entities the same or different absolutely, and so long as the interpretation and assignment functions are not understood as mapping terms of a language onto such elements.

We have seen that Dummett and Quine take these features of a semantics to be ruled out *a priori*. But it is not clear why this should be so. Indeed, if we take D to be a set, and understand that notion along the lines suggested by Cantor, then Dummett and Quine are right. However, it does not seem obvious that we do need to understand the notions of a domain and a mapping onto a domain along these lines. We could, for example, continue to use the formal apparatus of set-theory, while rejecting the those elements of the traditional interpretation of the formal system which are in conflict with GT.[1]

It might be objected that, shorn of its classical interpretation, stipulations like (6.7) and (6.8), which involve such set-theoretic notation of set inclusion, are simply meaningless, and cannot shed any light on how to interpret the singular terms of a language. If this were true, and the formal systems of orthodox set-theories cannot be provided with a non-standard interpretation which does render them compatible with GT, then one alternative for the strong theory of relative identity would be to frame the intended semantics in some non-classical set-theory which is more amenable to the strong theory of relative identity. This possibility has not been explored in detail, but at least one logician has passed the following judgement:

> The prospects for developing logics of RI (relative identity) in a nonstandard set theory are rather poor, but I think that Blizzard's theory of multisets or the Krause conception of quasi sets

[1] The axiom of extensionality might have to be modified as well, as it involves a relation of absolute identity. However, this is perhaps less implausible than it may sound. Stevenson (1972: 155) notes that Geach proposes the following case of RI in private correspondence: A is the same set as B, but not the same ordered set. If this were a case of RI, this would suggest that the axiom of extensionality provides a criterion for the relative identity relation, '...is the same set as ...'. For an objection to this example, see Lowe 1989a: 18. Lowe's central claim is that no set is also an ordered set.

may be useful in this respect. (Garbacz 2002: 28)

I admit that I do not see why he thinks the prospects are poor, especially given one of the alternatives he recommends seems to me to offer great promise, namely quasi-set theory. I will turn, then, to quasi-set theory, an alternative set theory originally developed in Krause's paper 'On a Quasi-Set Theory' (Krause 1992) and subsequently, in more detail by Krause together with French (2006, 2010). The theory of quasi-sets involves sets with elements of two different kinds: one labelled 'M-atoms'; objects for which all the traditional laws of logic (including identity) apply. These are no different than the Ur-elements of Z-F set theory. The other entities are called 'm-atoms'. As Krause explains:

> The atoms of the other kind (m-atoms) may be intuitively thought of as elementary particles of modern physics, and we will suppose, following Schrodinger's ideas, that identity is meaningless with respect to them (Schrodinger 1952: 16-68). Then we will admit that the Traditional Theory of identity (TTI) does not apply to the m-atoms. These facts enable us to hold, with regard to the m-atoms, that the concepts of indistinguishability and identity may not be equivalent. Therefore, roughly speaking we can say that a q-set (quasi-set) is a collection of objects (called elements) such that to the elements of one of the species (the m-atoms), the notion of identity (ascribed by classical logic and mathematics) lacks sense. (Krause 1992: 402-403)

Krause and French, therefore, accept two kinds of entities into their domain of discourse: M-atoms, to which the notion of absolute identity does apply, and m-atoms, to which the notion of absolute identity does not apply. Or, more specifically, for any m-atom, x, it is not the case that x is absolutely identical with x. This makes quasi-set theory attractive as a groundwork for a semantics compatible with GT. It suggests that a q-set of indiscernible m-atoms cannot be thought of as composed of ultimately-fine-grained elements distinguished from one another absolutely. A formal semantics in which the notion of a domain, and interpretation and an assignment are thought of as involving q-sets and m-atoms may, therefore, escape the objections raised against a semantics for GT. I shall not try to work out the details of such a semantics here, but simply note that this may provide an alternative to conceiving of the notions of a domain, interpretation, and assignment, which do not presuppose the existence of absolute identity relations.[2]

[2] Krause and French (1992: 404) provide a dictionary involving, amongst others, the two following definitions:

(a) $Q(x) =_{def} \neg(m(x) \vee M(x))$ Where 'Q' stands for '... is a quasi-set' (henceforth, 'q-set').

(b) $x = y =_{def} \neg m(x) \wedge \neg m(y) \wedge x \equiv y$

From the definitions that they provide, Krause and French are able to prove the following theorems:

Theorem 1: The defined relation '=' has all the usual properties of classical equality.

Theorem 2: $\forall x \forall y(\neg m(x) \wedge \neg m(y) \rightarrow (x \equiv y \rightarrow x = y))$

This says that for, anything which is not an m-atom, indistinguishability implies identity.
Krause (1992: 404) then provides the following axioms of quasi-set theory:

(A1) $\forall x(\neg(m(x) \wedge M(x)))$

(A2) $\forall x \forall y(x \in y \rightarrow Q(y))$

(A3) $\forall x(Z(x) \rightarrow Q(x))$

(A4) $\forall_Q x \exists_m y((y \in x) \rightarrow \neg Z(x))$

(A5) $\forall_Q x \forall y((y \in x \rightarrow D(y)) \rightarrow Z(x))$

(A6) $\forall x \forall y((m(x) \wedge x \equiv y) \rightarrow m(y))$

The theorems that follow from (A1)-(A6) and are relevant to the present project include:

(3b) For all x and for all y, $[x]$=$[y]$ if and only if $x \equiv y$.

Needless to say, (a), (b), and Theorems 1, 2, and (3b) are all incompatible with GT. Quasi-set theory must therefore be modified if it is to rendered compatible with Geach's views on identity.

I propose the following modifications:

Firstly, we reject the existence of M-atoms altogether. This of course means that we abandon axiom (A1). Secondly, we replace definition (a) with (a)*:

Definition (a)*: $Q(x) =_{def} \neg m(x)$, where Q stands for '...is a q-set'.

Thirdly, we replace definition (b) with (b)*:

Definition (b)*: $x =_Q y =_{def} \neg m(x) \wedge \neg m(y) \wedge \forall z(z \in x \rightarrow z \in y)$ Where '$=_Q$' is to be read '... is the same q-set as...'.

This definition provides an extensional criterion of identity for q-sets. Note, however, that this definition does not imply the indiscernibility of some x and y which are the same q-set. This is intentional, for a plausible case of RI is the following: for q-sets x and y, it is possible for x to be the same q-set as y, but, for example, x to be a different ordered-q-set as y. x may have the same elements as y and, therefore, by (b)*, x and y are the same q-set, yet x and y may be differently ordered and, therefore, discernible.

Given this alteration, neither Theorem 1 nor Theorem 2 of quasi-set theory is a theorem of modified quasi-set theory. However, the following is derivable as a theorem:

Therefore, the prospects for a semantics compatible with GT depend on either a non-standard interpretation of the formal language of traditional set theory or an unorthodox system of set theory being found that can be used to frame the definition of a structure for a semantics compatible with GT, which can provide an account of the interpretation of names, predicates, and assignments for variables that does not presuppose absolute identity. If this can be found, the answer raised in Chapter 5 can be answered.

6.1.2 Prospects for a Semantics for GT, RI, and SRI

Even if some notion of a set can be provided which is compatible with GT, things become more complicated if RI and SRI are true. This would require a semantics which does not presuppose the existence of relations of absolute identity but also provides a semantics for relations of sortal relative identity. We saw, in Chapter 5, that Geach introduces the notions of 'names of' and 'names for' and the thesis of the irreducibility of restricted quantification, all of which are helpful in understanding RI. I will show how these notions might find a place in a semantics for strong relative identity.

In order to show how cases of RI are possible, we will also need to say something about the semantics for statements involving identity. To make sense of locutions such as 'a is the same F as b', we will introduce a new

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Theorem 1*: $\forall x \forall y(\neg m(x) \wedge \neg m(y) \rightarrow (x \equiv y \rightarrow x =_Q y))$

Moreover, several of Krause's additional theorems are not derivable in modified quasi-set theory. (3b), for example, is not a theorem of modified quasi-set theory. The following, however, is now a theorem:

(3b)* For all x and for all y, if $[x] =_Q [y]$, then $x \equiv y$

In other words, for any x and y, where the q-sets $[x]$ and $[y]$ are the same q-set, the m-atoms x and y are indistinguishable. The modifications to quasi-set theory that I am proposing are thus two-fold. First of all, modified quasi-set theory will eliminate all reference to M-atoms, and secondly, it will involve the replacement of the relation '$=$', when taking expressions for q-sets as arguments, with the relation '$=_Q$', a relation which does not guarantee indistinguishability. With these two alterations, I believe quasi-set theory can be purged of any commitment to absolute identity. Krause and French (2010: 116-121) provide a proof of the consistency of quasi-set theory that can be easily adapted to modified quasi-set theory. Therefore, I think it provides an appropriate framework for providing a semantics which will be compatible with the strong theory of relative identity.

syntactical category into our semantics, that of relative-identity relations.

First a bit of ground work. The notion of a sortal will be as explained in Chapter 1. We introduce a set of sortals, S, into the vocabulary for L, the language for which we are providing a semantics. Each sortal partitions the domain of discourse into disjoint subsets, a set of those things that are Fs and a set of those things that are not.

We may then introduce the following definition:

Definition 1:

> If 'a', 'b' $\in T_L$ (where T_L is the set of terms of the language) and $F \in P_L$ (where P_L is the of predicates of the language), then '$a =_F b$' $\in \mathbf{P}_L$ (where $\mathbf{P}_L$ is the set of all formulae of the language).[3]

We may further stipulate that every relation $=_F$ partitions the set of things that are F into equivalence classes, that is sets of things that are the same F.

The structure for a semantics incorporating the name of/name for distinction will be more complex than in classical semantics. Rather than a pair, involving a domain and an interpretation function, the structure of a semantics for GT, RI, and SRI will involve several different types of interpretation function. First, there will be a function which takes names as arguments and returns as values that for which they are names for. We might further stipulate that the name-for-value of a name of will be one of the subsets of the domain generated by one of the relative-identity relations. If the name-for referent of 'a' is a subset generated on a domain, D, by the relation '$=_F$', then 'a' is the name for an F. Implicit in this stipulation is that every name is a name for, and never both a name for an F and a name for a G. This, I think, is what Geach had in mind when defending the thesis that every name comes with a criterion of identity.

[3] Adapted from Garbacz 2002. Garbacz's semantics for the formal language in which his logical systems are expressed is, in fact, incompatible with the strong theory of relative identity, more particularly with GT, in various respects, so I will not consider it further here.

In addition to this, the structure of a semantics for GT, RI, and SRI will involve a further set of interpretation functions, where for every sortal, F, there is some I^F. These also take names as arguments and return as values subsets of D. For every x, where x is an element in the value of an I^F function, it will be the case that $F(x)$. However, unlike the name-for function, it will not necessarily be the case that they are all the same F, just as 'Bluemantle' may be a name of several different men.

As far as the quantifiers are concerned, we have already noted that Geach's views suggest a close connection between the name for/name of distinction and the irreducibility of restricted quantification thesis. In particular, we saw that Geach seems (1980: 184, 206-207) to have in mind some account of substitutional quantification, such that the unrestricted quantifiers, when ranging over things that are F are replaceable *salve veritate* by any name. In other words:

> $\langle \forall x F(x) \rangle$ is true if and only if every name is such that $F(a)$.

and

> $\langle \exists x F(x) \rangle$ is true if and only if there is some name a, such that $F(a)$.

Similarly, restricted quantifiers are replaceable by names for.

> $\langle \forall_F x G(x) \rangle$ is true if and only if, for any name a of and for an F, $G(a)$.

and

> $\langle \exists_F x G(x) \rangle$ is true if and only if there is at least one name, a of and for an F such that $G(a)$. (Geach 1980: 2006)

This completes my overview of the possibilities for the semantics for theories of relative identity. Before I bring this chapter to a close, I will make some reflections on the sort of logical system which is best suited to the strong theory of relative identity, by considering the various systems developed for relative identity that are to be found in the literature.

6.2 *Logical Systems for Strong Theories of Relative Identity*

I will begin my survey of the extant systems compatible with strong theories of relative identity by considering the *desiderata* for such a logical system. This will, of course, depend on which theory of relative identity we are interested in. A weak theory of relative identity is compatible with a logical system which licenses some inferences involving absolute identity, while a strong theory of relative identity would be incompatible with any such logic. A strong theory of relative identity is therefore incompatible with FOL$^{=}$. A strong theory of relative identity is compatible with FOL without identity. However, to adopt FOL without any further rules of inference would mean some intuitively valid inferences were not underwritten by the logical system. Given that the strong theory of relative identity is compatible with FOL, it is only those inferences which involve relations of identity that are under threat. But take the following as an example:

Argument 8

P8.1 a is the same person as b.

P8.2 a is the son of John Smith.

Therefore

C8.1 b is the son of John Smith.

Given FOL, the inference drawn in argument 8 is invalid. Of course, it might be that the most appropriate logical system for the strong theory of relative identity does not license the inference in Argument 8. Perhaps it is a substantive, non-logical, maybe even contingent, claim that, if a is the same person as b and a is the son of John Smith, then b is the son of John Smith. However, this clearly conflicts with some widely held intuitions. It would be good to know if there is a logical system compatible with the strong theory of relative identity which could underwrite the inference in Argument 8 and other similar inferences. However, determining which inferences are valid and which are not is not a simple task. I have remained neutral about the thesis RI; however, a principled decision about the most appropriate logical system would require either accepting or rejecting that thesis. For example, the following case might be thought to be as intuitively obvious as Argument 8.

Argument 9

P9.1 *a* is the same person as *b*.

P9.2 *a* is 5'8".

Therefore,

C9.1 *b* is 5'8".

However, It has been argued that relative identity provides the best solution to the problems associated with change over time (in this regard, see Odegard 1972, Borowski 1975, Gupta 1980, and Deustch 1998). If this is so, then instances of change over time can be expressed with cases of RI. For example:

> R″: The baby is the same person as the grown man but not the same temporal slice.

Of course, most philosophers deny that sentences such as R″ genuinely a case of RI. But, if we assume for the moment that it is, then we would seem to have a counterexample to the validity of Argument 9. In this case, the grown man is the same person as the baby, but surely it is possible for the grown man to be 5'8", and for the baby to be some other (presumably shorter) height. So, if sentences like R″ are possibly true and are cases of RI, then inferences like that involved in Argument 9 are invalid.

Moreover, the choice of logic does not depend only on the truth or falsity of RI. Even if RI is true, where to draw a principled distinction between valid and invalid inferences remains problematic. It could be that cases of RI are possible, but that sentences like R″ are not, in fact, examples of them. RI, then, is compatible both with the validity and the invalidity of Argument 9. The problem, then, is to determine not just the truth of RI but also what instances of RI are possibly true.

For any theory of identity, therefore, discerning the appropriate logical system depends on some system satisfying at least the three following *desiderata*:

> (6.13) The system must be compatible with the various theses involved in the theory (in the case of theories of relative identity, these are some combination of GT, RI, and SRI).
>
> (6.14) The system must be consistent and complete.
>
> (6.15) The system must not involve an *ad hoc* distinction between valid and invalid inferences.

Several attempts have been made to develop logical systems for theories of relative identity, notably, by Griffin and Routley (1979), van Inwagen (1995), and Garbacz (2002).[4] In what follows I will consider these systems. I cannot endorse any given one of these systems, as I intend to remain neutral concerning the various component theses of relative identity. However, we will see that each of the systems is compatible with a certain view of the relationship between identity and logic. The choice between these systems is left for the future.

I will consider these systems from the weakest to the strongest.

[4] To be sure, there are other alternative systems of first-order logic which may be compatible with strong relative identity; for example Wehmeier's Wittgensteinian-inspired logic without objectual identity (Wehmeier 2012) and Krause and French's Schrodinger logics (Krause and French 2006, first developed by da Costa 1980). However, none of these are tailor-made for relative identity, and so I will not consider them further here.

6.2.1 *Stevenson and Van Inwagen's Logics*

Chronologically, the first system relevant to theories of relative identity is Stevenson's axiomatized system (1975). The motivation for Stevenson's system is to shed light on the behaviour of sortals. Stevenson's logic has several benefits, relative identity versions of reflexivity, transitivity, and symmetry are all theorems (1975: 194). However, Stevenson's logic is incompatible with GT and also with RI. Not only is '=' a primitive symbol in the language of Stevenson's logic, but the following is a theorem:

$\vdash (t^1 =_F t^2)$ if and only if $(F(t^1) \wedge t^1 = t^2)$

In other words, the Fregean Analysis is provable (1975: 195). The Fregean Analysis is, of course, incompatible with both GT, since it implies that there exists a relation of absolute identity and with RI because it implies the indiscernibility of the *relata* of any relative identity relation.

In spite of this, Stevenson's logic may cast some interesting light on the appropriate logic for theories of relative identity generally. In particular, Stevenson develops a notion which is important to understanding the logic of sortals. This is the notion of 'an ultimate sortal' (1975: 195). A sortal, F, *dominates* a sortal, G, if and only if $\ulcorner x =_F y \urcorner$ entails $\ulcorner x =_G y \urcorner$. For example, plausibly, if Clark Kent is the same man as Superman, then Clark Kent is the same human being as Superman. F is an *ultimate sortal* if and only if, for everything x, F dominates every sortal G such that $G(x)$. The existence of dominating and ultimate sortals is, of course, important for determining which cases of RI are possible (see Griffin 1977: 76-92). These notions have been incorporated into the subsequent systems that have been developed, and it is to these that I will now turn.

I will begin, then, with van Inwagen's, provably consistent, system of natural deduction (van Inwagen 1988: 248-260). The language of van Inwagen's logic involves a constant, 'I', standing for relations of relative identity. However, it involves neither the symbol '=' nor any singular terms. As we have seen, van Inwagen believes that naming and relative identity are incompatible. In the language of van Inwagen's logic, therefore, all reference is carried out by quantified expressions (van Inwagen 1988: 260). If the strong theory of relative identity can be provided with a semantics for proper names, then the vocabulary of the language of van Inwagen's logic is unnecessarily restricted.

Van Inwagen's system of natural deduction involves adding just two new inference rules to the system of natural deduction for FOL. These are as follows:

Symmetry$_{vI}$:

$$\frac{Ix, y}{Iy, x}$$

and

Transitivity$_{vI}$:

$$\frac{Ix, y, Iy, z}{Ix, z}$$

These are, of course, relative-identity versions of symmetry and transitivity. However, van Inwagen does not provide a relative-identity version of reflexivity and provides nothing similar to LL. No inferences of the form:

$$\frac{Ix, y, \varphi(x)}{\varphi(y)}$$

are valid in van Inwagen's system. Van Inwagen, naturally, accepts that many inference of this kind are highly intuitive. He suggests that some relative-identity relations 'dominate' certain predicates. However, in van Inwagen's view, all claims concerning what relations dominate what properties are substantive metaphysical theses and not guaranteed by logic (van Inwagen 1988: 256). This may mean that van Inwagen's system fails to satisfy *desideratum* (6.14). Note that van Inwagen logic does not abandon identity as a logical notion all together. We have seen that it does involve two laws of identity. However, the absence of any form of reflexivity means that it is possible that there are no objects that satisfy any relation *I*. The absence of any form of LL means that, even if there are entities that do satisfy identity relations, they may not share any other properties in common. In other words, whether or not van Inwagen's logic is complete depends on whether or not it is a logical truth that entities have determinate identity conditions and whether these identity conditions involve some principle of substitutivity.

6.2.2 Routley and Griffin's Logics

Stronger alternatives to van Inwagen's system can be found in the form of a series of second-order logics for relative identity developed by Routley and Griffin (Routley and Griffin 1979). Routley and Griffin's first system is closely connected to van Inwagen's system; the major difference is that Routley and Griffin build the notion of predicate domination into their logic. Routley and Griffin (1979: 70) introduce a new symbol, '$\mathit{\Delta}_F$',[5] which is a function on the sortal term F, and designates the set of properties which F dominates. Routley and Griffin's first system is arrived at by adding the following definition to FOL (without identity):

D1: $x =_F y =_{def} (\forall P \in \mathit{\Delta}_F)(P(x) \leftrightarrow P(y))$

With this definition, Routley and Griffin are able to prove the consistency of their system (1979: 73-74), amongst other theorems. In addition to theorems corresponding to symmetry and transitivity, from D1 and the definition of the expression '$\mathit{\Delta}_F$', they are able to prove the following version of reflexivity and LL:

Reflexivity$_{RG}$: $\vdash x =_F x$

LL$_{RG}$: $\vdash (x =_F y \wedge \mathit{\Delta}_F) \rightarrow (\phi(x) \leftrightarrow \phi(y))$

LL$_{RG}$ tells us that any pair jointly satisfying some relative-identity relation are indiscernible with respect to the predicates dominated by that relative

[5] In fact, Routley and Griffin use '$\mathit{\Phi}$' as a schematic letter for sortals, rather than 'F'. Here and throughout this section, I make small alterations in order to use the same symbols through my dissertation.

identity-relation. This, of course, does not mean that arguments like 9 are valid, for it is still not a logical matter *which* predicates are dominated by a given relative-identity relation. Nevertheless, the system is stronger than van Inwagen's system in this respect.

Reflexivity$_{RG}$, however, poses a problem for Routley and Griffin's (1979: 76-77) system, as they are willing to admit. For this theorem tells us that any x is the same ϕ as itself. But the schematic letter, ϕ is replaceable by any predicate whatever. In other words, x is the same horse as x and x is the same person as x, and so on, regardless of what x is. Routley and Griffin also note that, in their system we cannot infer from '$x =_F y$' to '$F(x)$' or to '$F(y)$'. This is surely counter-intuitive.

To solve both these problems, Routley and Griffin (1979: 74-77) offer another alternative. This replaces D1 with D1*:

D1*: $x =_F y =_{def} (F(x) \wedge F(y) \wedge (\forall P \in \Delta_F)(P(x) \leftrightarrow P(y)))$

From D1*, it is possible to infer from '$x =_F y$' to $F(x)$ and $F(y)$. It also entails a different version of reflexivity.

Reflexivity$_{RG*}$: $\vdash F(x) \rightarrow x =_F x$

Surely Reflexivity$_{RG*}$ is more plausible than Reflexivity$_{RG}$. Given this, the modified version is the better of Routley and Griffin's alternatives to van Inwagen's system. The salient differences between Routley and Griffin's modified system and van Inwagen's system are two-fold. Firstly, the former involves weak reflexivity, while the latter does not. Secondly, the former entails that relations of relative identity guarantee indiscernibility with respect to a certain class of predicates, while the latter does not. If these sorts of inferences are logically valid, then Routley and Griffin's modified logic is the appropriate logic for relative identity. If all substitutivity claims are properly issues for metaphysics rather than logic, then van Inwagen's logic is the appropriate system.

As we noted however, Routley and Griffin's system does not underwrite any particular inferences involving dominated properties. It might be thought that some such inference ought to be underwritten by logic. For even stronger systems, we must look to the most recent work that as been done on logics for relative identity.[6]

6.2.3 Garbacz's C1

Garbacz provides no fewer than 11 different logical systems compatible with various theories of relative identity (Garbacz 2002). Each is defined by a different set of assumptions. The one that I will begin with is his 'minimal monadic logic of relative identity', which he names 'C1'. C1 is the weakest of the eleven systems Garbacz devises, and indeed, all the other systems are extensions of C1. C2 - C11 differ from C1 for the most part in licensing versions of LL.

C1 is defined by the following assumption:

C1: If $A^1, A^2 \in \Im(F)$ and $A^1 \neq A^2$, then $A^1 \cap A^2 = \varnothing$

where '$\Im(F)$' represents any partition on the domain of discourse by a relation of the form '... is the same F as...'. In other words, the resulting sets of a partition on the domain by any relative identity relation are disjoint. From C1 and the semantics Garbacz provides, five sequents follow:

(6.15) $\vDash_{C1} (x = x \wedge \phi(x)) \rightarrow x =_{\phi} y$

---

[6] The system to which Routley and Griffin devote the greatest amount of effort is a three-valued logic of significance. I have not argued in this dissertation for a third truth-value and I don't not think that any of the component thesis of the strong theory of relative identity give support to the existence of a third truth-value. Given this, I think that unless a good reason can be given for adopting a three-valued logic, the appropriate logic for theories of relative identity will be bivalent.

Regardless, Routley and Griffin's, as they stand, logics are all incompatible with RI, and therefore with the strong theory of relative identity. Routley and Griffin provide the following truth conditions for statements involving relative identity relations.

> $a =_F b$ if and only if $F(a) \wedge F(b)$ and (if $F' \in D_F$, then $F'(a)$ if and only if $F'(b)$), where D_F is the set of all the predicates dominated by the sortal 'F'.

In other words, a bears some relative identity relation to b if and only if a and b satisfy all the same sortals. This, of course, rules out true cases of RI. This account of relative identity relations will, therefore, have to be rejected in order to make the Routley and Griffin systems compatible with Geach's views.

(6.16) $\vDash_{C1} x =_\phi y \rightarrow \phi(x)$

(6.17) $\vDash_{C1} \phi(x) \rightarrow x =_\phi x$

(6.18) $\vDash_{C1} x =_\phi y \rightarrow y =_\phi x$

(6.19) $\vDash_{C1} (x =_\phi y \wedge y =_\phi z) \rightarrow x =_\phi z$

Of these, sequents corresponding to 1-4 are true in $\text{FOL}^=$.

With these facts established, Garbacz (2002: 33-36) proceeds to give his sequent calculus for C1, which he calls 'SQ1'. He provides a series of rules (see Appendix A), R1-10, which are familiar from the standard calculus for $\text{FOL}^=$. These are supplemented by four additional rules of inference, which he calls 'R11A-R11D'. Informally, these are:

(6.20) From 'x is identical with y' and 'x is (an) F', infer 'x is the same F as y'.

(6.21) From 'x is the same F as y', infer 'x is an F'.

(6.22) From 'x is the same F as y', infer 'y is the same F as x'.

(6.23) From 'x is the same F as y' and 'y is the same F as z', infer 'x is the same F as z'. (Garbacz 2002: 34)

With these rules, Garbacz is able to prove the completeness of SQ1.[7] That is, all the sequents which are semantically entailed by the assumption C1 are provable, given the rules of SQ1. (6.15)-(6.19) are, therefore, theorems. These theorems give us versions of weak reflexivity (6.17), symmetry (6.18),

[7] Garbacz is able to prove Theorem 9.1: 'If $\Phi \vDash_{Cn} \varphi$, then $\Phi \vdash_{SQn} \varphi$', which simply states the completeness of each of the Garbacz sequent calculi. The proof, which I will not repeat here, is a modification of Henkin's proof of the completeness of the standard first order system (Garbacz 2002: 38-41).

and transitivity (6.19), as well as the intuitive principle that if x and y are the same F, then they are Fs. (6.15), however, is incompatible with GT.

In fact, C1 is incompatible with GT several times over. Garbacz's system presupposes the existence of absolute identity by involving a semantics formulated using the tools of Z-F set theory.[8,9] Moreover, statements of the form $\ulcorner x = y \urcorner$ are well formed in the language Garbacz uses to express C1. More importantly for our present concerns though, C1 entails theorems involving absolute identity relations:

$$(6.24) \vDash_{C1} (x = y \wedge \phi(x)) \rightarrow x =_{\phi} y$$

All of this is quite at odds with the central tenet of the strong theory of relative identity.

In addition to this, the assumption C1 itself involves an absolute relation of identity, as do the rules of inference R9 and R10. C1, then, is a logic which may be appropriate for weak theories of relative identity but inappropriate for Geach's views. As it stands, C1 does not satisfy *desideratum* (6.13). Having said this, I think that Garbacz's work may provide an important stepping stone for understanding the logical properties of the strong theory of relative identity.

C1 can be modified in such a way that it does not presuppose the existence of relations of absolute identity, if it is possible to replace the absolute identity relations in C1 with suitable relativizations. A system compatible with the strong theory of relative identity would involve (6.16)-(6.18) as theorems, but would not involve (6.15). From SQ1, the inference rules corresponding to those of FOL (i.e. R1-R8) still form a necessary component of the sequent

[8] See ⟨Assumption 1.4⟩, Garbacz 2002: 28. Moreover, the existence of objects with determinate absolute identity conditions is explicitly built into Garbacz's semantics.

> 'If $\ulcorner x = y \urcorner$, then x and y will be called AI-objects ... the definition of an RI-object is bound to refer to AI-objects; namely, and $RI(\delta)$ (that is, $RI(F)$) is the set of AI-objects that are the same δ (that is, F).' (Garbacz 2002: 28)

[9] Garbacz, no doubt, would grant that his semantics is incompatible with the strong theory of relative identity. Providing a semantics for the strong theory of relative identity falls outside the scope of Garbacz's project, as he is interested only in weak theories of relative identity.

calculus for a system compatible with the strong theory of relative identity, while the inference rules involving absolute identity must be rejected. As for Garbacz additional rules, R11A must be rejected, as it involves an absolute identity relation, while R11B-R11D, are all compatible with C1*.

Note though, that even if it can be modified to be compatible with the strong theory of relative identity, C1 is a very weak system and, as it stands, does not add much of interest to the van Inwagen and Routley and Griffin systems, except for being more technically developed.[10] I am interested in stronger logical systems for relative identity. This is why Garbacz's extensions of C1 are of value

For each of the extensions of C1, Garbacz adds further sequents to create more powerful logical systems, which underwrite a larger number of inferences. Some of these have intuitive appeal and are worth considering here. I do not have the space to consider all the extension. I will list them and possible objections to them in Appendix A. I will, however, look at two examples here.

SQ2, the sequent calculus for system C2 involves all the inference rules of SQ1, plus the following:

R12

$$\frac{\Phi\forall\ x(F(x) \leftrightarrow G(x)), \Phi a =_F b}{\Phi a =_G b}$$

While, SQ3 adds the following:

R13

$$\frac{\Phi\forall x(F(x) \to G(x)), \Phi(a =_F b) \wedge (b =_G c)}{\Phi a =_G c}$$

[10] Indeed, Garbacz (2002: 47) claims that C1 is the appropriate system for van Inwagen's views on identity. In fact, he is wrong about this, given that C1 assumes the existence of relations of absolute identity and van Inwagen remains neutral on their existence. Moreover, C1 is stronger than the C1 involves a version of relative-identity-reflexivity (fact 4.1, number 3, Garbacz 2002: 30), while, as we have seen, van Inwagen's system does not.

Each of these rules has at least some intuitive appeal and would license many apparently valid inferences. However, among these are the following inferences

Argument 10

> **P10.1** Everything that is a human being is a persona and *vice versa*.
>
> **P10.2** Clark Kent and Superman are the same human being.

Therefore,

> **C10.1** Clark Kent and Superman are the same persona. (By R12)

Argument 11

> **P11.1** Every river is water.
>
> **P11.2** The river that Heraclitus bathed in yesterday is the same river as the river he bathed in today.
>
> **P11.3** The river Heraclitus bathed in today is the water of yesterday's rainfall.

Therefore,

> **C11.1** The river yesterday is the same water as yesterday's rainfall. (By R13)

But of course, these two cases could be taken as plausible cases of RI and, therefore, counter-examples to the proposed rules of inference. A logic which involves rules of inference concerning the domination of properties by relative identity relations will be bound to rule out some cases of RI. In short, if sortal domination is a properly logical notion, the choice of logic will depend not merely on the truth of RI but on what particular kinds of cases of RI are possible. This is not an issue I can address here, but it does represent an important future field of research for theories of relative identity.[11]

In the final analysis, there are several possible logical systems compatible with the theory of strong relative identity. GT is compatible with any system that does not involve an absolute identity relation. The choice of logic depends, then, on the truth of RI as well as whether relative identity is a properly logical notion or not. As we have seen, there are at least three options here. firstly, the structure of identity relations generally is a matter for logic, but it is a metaphysical issue whether anything, in fact, satisfies such relations and in what such satisfaction consists (van Inwagen's position). Secondly, it is a logical matter that satisfying an identity relation involves satisfying a certain set of predicates, that is, it is a fact of logic that, if x satisfies some particular predicates, then x bears some relation of identity to x (weak reflexivity), and if x and y jointly satisfy an identity relation, then x and y must have certain properties in common, but the properties that entail and are entailed by the satisfaction of identity relations are a question for metaphysics (Routley and Griffin's position). Thirdly, all identity relations are properly logical notions and, therefore, the predicates that entail and are entailed by the satisfaction of any given identity relation are matters for logic to pass judgement upon (Garbacz's C2-c11).

A further option is to abandon the search for a new logical system for relative identity and rest content with classical FOL without identity. This would be appropriate if the notion of identity is entirely non-logical, and if even the most basic structural features of the relations are metaphysical hypotheses. I have not tried to presented a sustained argument to the effect that identity relations and the corresponding inferences ought or ought not to be incorporated into a predicate logic. This issue is outside of the scope of my work, and so I pass on with it unresolved.

[11] For more on this issue the systems developed by Gupta (1980) and Deutsch (1998) may be useful.

6.2.4 Conclusion

In Chapter 6, considered how Geach might respond to the objections raised in the previous chapter. First, I considered possible responses to Cain's claim that Geach's views entail the failure of the syllogisms. Second, I considered the prospects for answering the semantic objections to relative identity. I argued that these depend on the development on a non-standard set-theoretic framework in which to frame the notions of an interpretation, an assignment and a domain of discourse. Finally, I have briefly considered the various possibilities for first-order systems of predicate logic which would be compatible with the theory of strong relative identity. In the next chapter I will consider some of the philosophical consequences the strong theory of relative identity will have.

7. APPLICATIONS OF RELATIVE IDENTITY

If GT, RI, or SRI are true, many avenues for further research are opened as a result. In particular, the metaphysical consequences of strong theories of relative identity have yet to be worked out in full.[1] The metaphysical issues that are raised are far too numerous to deal with sufficiently in this dissertation, so I will consider only one issue in depth, namely, the relationship between theories of identity and the logical problem of the Trinity. However, before I do this, I would like to briefly note some of the other metaphysical issues which are raised by the positions taken in this dissertation.[2]

[1] Relative identity theorists are sometimes challenged for paying too little attention to the metaphysical picture to which their views on identity commit them (Dummett 1973: 561-567, Richard H. Feldman, 1981: 373)

[2] To begin with, much more can be said about how the picture I have sketched fits into more general metaphysical debates. That is, how the metaphysics entailed by GT compares to existing theories. Dummett characterizes the metaphysics entailed by Geach's views on identity as 'an amorphous lump of reality, in itself not articulated into distinct objects' (Dummett 1973: 563.) However, Matti Eklund is of the opinion that there are a number of different versions of amorphous lump ontologies (Eklund 2008). Following Eklund, we may call these 'theories of ontological pluralism'. But which theory of ontological pluralism is compatible with GT?

One influential contemporary view is Hilary Putnam's (1994) 'conceptual relativity'. Putnam outlines this theory as follows:

> All situations have many different correct descriptions, and ... even descriptions that, taken holistically, convey the same information may differ in what they take to be "objects" – There are many usable extensions of the notion of an object – The logical primitives themselves, and in particular the notions of object and existence, have a multitude of different uses rather than one absolute "meaning". (Putnam 1994: 300)

Putnam is here suggesting that there might exist multiple accurate descriptions of the world. The difference between these descriptions is the different ways in which words like 'object' or 'exists' are being used. From the claim that 'exists' has several different possible uses, comes Eli Hirsch's (2002) theory of quantifier variance. That is, that the quantifiers of a language are ambiguous and may range over different kinds of entities, providing different domains of discourse for the same language.

There are many traditional metaphysical problems that must be reconsidered in light of GT. As I have argued, if GT is true, the traditional arguments against RI lose their force. Further, if RI is true, possible instances of RI may include:

(7.1) The male infant is the same human being as the grown man, but not the same boy, though the male infant is a boy. (From an example in Geach 1957: 69)

(7.2) Cleopatra's Needles in 2014 is not the same stone as it was in 1900 (the stone having been gradually replaced) but it is the same landmark. (Wiggins 2001: 34)

The Putnam/Hirsch view is compatible with GT. However, once again, the compatibility of the Putnam/Hirsch view of ontology with a theory involving GT will depend on whether that theory also involves RI, as RI is incompatible with the Putnam/Hirsch view.

What separates the metaphysics of the strong theory of relative identity from that of Putnam and Hirsch (who do not accept RI) is that for the latter there are different uses of words like 'object' and 'exists', and one's ontology will depend on which use of the word is being employed. Once that is established the objects can be distinguished from one another absolutely. The strong theory of relative identity is different. If there are true cases of RI, the ontological entities which are named on either side of the relation must exist in just the same sense as one another. Thus, any similarity between the strong theory of relative identity and quantifier variance, and its relatives, is only *prima facie*. The metaphysics implied by the strong theory of relative identity is, perhaps, closer to an ontological nihilism than to the conceptual relativity of Putnam or Hirsch. Ontological nihilism is the rejection of ontology altogether. This radical view has been sketched by Cortens and Hawthorne, in their article 'Towards Ontological Nihilism'. Cortens and Hawthorne (1995) note that any defender of ontological nihilism face the major task of describing the world, given human languages which are clearly built on the assumption of ontological entities. There are two possible responses. One response would be to follow Bradley, who holds 'that the concept of an object is indeed indispensable to our thinking but nevertheless ill-suited to characterise ultimate reality as it is in itself' (Cortens and O'Leary-Hawthorne 1995: 148). Though, as Cortens and Hawthorne point out, this response 'forced Bradley to deny that we can ever think or say anything straightforwardly true, and to embrace his notorious degrees of truth doctrine' (Cortens and O'Leary-Hawthorne 1995: 148). The other response would be to attempt to show that there is some description of the world which is in fact 'ontologically innocent'. It is the latter that Cortens and Hawthorne attempt to achieve in their paper, showing that the world is describable without reference to objects or 'stuffs'. Thus, for Cortens and Hawthorne, sentences likes 'there is a pebble' do not reflect ultimate reality as well as sentences like 'it is pebbling' (O'Leary-Hawthorne and Corten 1995: 148). It is, as yet, unclear to me whether the metaphysics implied by the strong theory of relative identity is closer to Bradley or the Cortens-Hawthorne view.

(7.3) Tibbles is the same cat as Tibbles-minus-Tibbles's Tail, but Tibbles is not the same collection of feline tissue as Tibbles-minus-Tibbles's Tail.

Therefore, if RI is true, there may be solutions to the problems of change over time, (7.1), material constitution, (7.2), and the problem of the many, (7.3). Of course, whether (7.1)-(7.3) really are cases of RI is a matter of debate, even if RI is true. So further work remains to be done on these issues on what cases of RI are possible, if any, given the truth of GT.

One further metaphysical problem to which relative identity has been applied, and which I intend to reconsider in light of the truth of GT, is the logical problem of the Trinity. That is, the Christian doctrine that three distinct persons are wholly divine and yet there is exactly one divinity. Although several attempts have been made to solve this problem, including several that involve theories of relative identity, I will argue that all of these fail unless they involve the thesis GT.

7.0.5 The Doctrine of the Trinity

In recent philosophical theology, various accounts of the mystery of the Trinity have tried to steer a middle path between two heresies. On one hand, orthodoxy is threatened by tritheism, the heretical view that there are three Gods, while on the other hand, avoiding tritheism runs the risk of falling into modalism, the heretical view that the individual persons of the Trinity are merely modes of the same entity. Social Trinitarianism tries to avoid modalism by stressing the distinctness of the divine persons. In so doing, it faces accusations of tritheism. The Latin Trinitarian approach, by contrast, stresses the unity of God, at the expense of the distinctness of the persons and therefore runs the risk of modalism.[3]

I shall argue that Social Trinitarianism does entail tritheism. However, I shall also argue that most versions of Latin Trinitarianism also entail tritheism. I shall argue that tritheism is avoided only by an account of the Trinity

[3] See McCall and Rea 2009: 1-15 and Moreland and Craig 2003 for good recent accounts of the difference between these two approaches.

involving GT.

7.0.6 Social and Latin Trinitarianism

The Athanasian Creed involves 44 theological theses concerning the doctrines of the Trinity and the Incarnation. Amongst these theses are the following four:

> (7.4) 'And the Catholic faith is this, that we worship one God in Trinity, and Trinity in unity.'
>
> (7.5) 'Neither confounding the persons nor dividing the substance.'
>
> (7.6) 'So the Father is God, the Son is God, and the Holy Spirit is God.'
>
> (7.7) 'And yet they are not three Gods, but one God.'

The interpretation of (7.4)-(7.7), however, is the issue at stake between Social Trinitarianism and Latin Trinitarianism.

The central *desiderata* for interpretations of (7.4)-(7.7) is that they avoid both modalism and tritheism. I shall be arguing that the only coherent interpretation of (7.4)-(7.7) which avoids tritheism is an interpretation which involves GT, so I shall be targeting all those accounts of the Trinity which reject GT. An account of the Trinity, T, which rejects GT, is tritheistic if and only if it satisfies the following:

> (7.8) T entails that there are three persons which are not absolutely identical with each other and that are each absolutely identical with some God.

I shall argue that all accounts of the Trinity which reject GT satisfy (7.8).

We will begin by considering the Social Trinitarian interpretations of (7.4)-(7.7). The Social Trinitarian view is best summed up by McCall and Rea:

> ST (Social Trinitarianism) is usually associated ... with the claims that it "starts" with threeness and moves toward oneness, that the divine persons are numerically distinct, and that the unity of the Trinity can be understood by way of a "social analogy": the divine persons are relevantly like a family, a supremely unified community of monarchs, or three human persons whose interpersonal relationships are so strong as to be unbreakable. (McCall and Rea 2009: 2)

We can get a better handle on this by considering one influential version of Social Trinitarianism. Richard Swinburne sets out his views as follows:

> On the account which I have given, the three divine individuals taken together would form a collective source of the being of all other things; the members would be totally mutually dependent and necessarily jointly behind each other's acts. This collective would be indivisible in its being for logical reasons-that is, the kind of being that it would be is such that each of its members is necessarily everlasting, and would not have existed unless it had brought about or been brought about by the others ... The claim that 'there is only one God' is to be read as the claim that the source of being of all other things has to it this kind of indivisible unity.
>
> But then how is the claim that each of the individuals is "God" to be understood? Simply as the claim that each is divine-omnipotent, perfectly good, etc. (Swinburne 1994: 27)

Swinburne's account involves the claim that God is 'a collective'. The persons are distinct entities, which, taken together, compose the divine collective. The relationship between each of the persons and the Godhead, then, is a relation of constitution. Other versions of Social Trinitarianism involve some other general term in place of 'collective'. For example, David Brown (1985, see also Leftow 1999: 217-221) sees the Godhead as something like a group mind, composed of three constituent minds. While C. Stephen Layman

(1988) holds that the Godhead is the bearer of the sum of the properties of the three persons. So, while each of the persons of the Trinity are not omnipotent, the three persons taken together are omnipotent. All of these versions of Social Trinitarianism are structurally alike in the following respect. The Godhead is something over and above any one of the persons which constitute it, and the truth of statements of the form 'x is God' is grounded in x's possession of certain divine properties rather than any identification of x with the Godhead. Note that, throughout this chapter I will use the term 'divine'/'divinity', in place of 'God', as the latter is too often ambiguous between is various possible uses as a proper name and a count noun. When I intend to use a proper name, I shall use the term 'the Godhead'.

All of this is in contrast to Latin Trinitarianism. According to Latin Trinitarianism, none of the three persons is a proper part of the Godhead; rather each person is wholly divine. This is motivated, in part, by the rejection on the part of some of the Church Fathers of the view that the persons in any way *compose* the Godhead. This has lead some Latin Trinitarians to assert that some relation of numerical identity holds between each of the persons and the Godhead, as we will see.

Consider another statement of Trinitarian theology:

> (7.9) The Father is the same divinity as the Son, and the Son is the same divinity as the Holy Spirit, and the Holy Spirit is the same divinity as the Father.

According to both Social Trinitarianism and Latin Trinitarianism, (7.9) is true. If it were false, then, given (7.7), there would be three distinct divinities. However, Social Trinitarinism would interpret (7.9) differently from Latin Trinitarianism. According to Social Trinitarianism (7.9) asserts that, for any two persons of the Trinity, they are both members of the same divine collective or group mind, etc. In other words, we can translate (7.9) as:

> (7.9′) The Father is a member of the same F as the Son, and the Son is a member of the same F as the Holy Spirit, and the Holy Spirit is a member of the same F as the Father.

If (7.9′) is an accurate translation of (7.9), then the latter is composed of the conjunction of three common-property statements.[4]

But in fact, we can be more specific than this. It is not simply that the Social Trinitarianism involves rejecting the inference from 'a is a member of the same F as b' to 'a is the same thing as b'; Social Trinitarianism is in fact committed to the claim that the Father and the Son are numerically distinct. (7.5′), then, is not simply a conjunction of common-property statements; it is a conjunction of what I will call 'mere common-property statements', where a mere common-property statement is defined as follows:

> (7.10) A statement, $\boldsymbol{P}$, of the form 'a is the same F as b,' is a mere common-property statement if and only if it entails 'a is an F and b is the same F' and if it is the case that 'a is not the same thing as b'.

However, this leads to a serious problem for Social Trinitarianism. Leftow has argued, convincingly, that the Godhead and the three persons of the Trinity must be divine in the same sense of the term 'divine' (Leftow 1999: 221). The alternative is the heresy of Arianism, roughly the view that one or more of the persons of the Trinity are divine only derivatively. Indeed the doctrine that each of the persons of the Trinity is wholly God is confirmed by the eleventh council of Toledo in the seventh century.

If we take the claim that the Godhead and the persons are divine in just the same sense at face value, this entails the following:

> (7.11) If 'the Godhead is divine' entails 'the Godhead is numerically identical with one divinity', then, for any person of the Trinity, x, 'x is divine' entails 'x is numerically identical with one divinity'.

[4] More particularly, it involves a relation of constitution. The Father (partly) constitutes God. As William Lane Craig, a prominent Social Trinitarian says, 'it seems undeniable that there is some sort of part/whole relation obtaining between the persons of the Trinity and the Godhead' (Craig 2003: 590). In fact, as we shall see, I not only think that this is deniable, but that the failure to deny it results in tritheism.

Any Trinitarian theory which involves a relation of absolute identity accepts the antecedent of this conditional. Therefore, the consequent follows. In other words, if the Son is divine, then the Son is absolutely identical with one divinity. Naturally, all forms of Trinitarianism accept that the Son is divine. The Father, similarly, is divine and, therefore, absolutely identical with one divinity. So, too, for the Holy Spirit. However, by the Social Trinitarian interpretation of (7.9) as a conjunction of three mere common property statements, the Father, Son, and the Holy Spirit are not absolutely identical with one another. By the transitivity of absolute identity, there are, at least, three divinities which are not absolutely identical with one another.[5,6]

For these reasons, Social Trinitarianism entails tritheism. However, I will argue that most forms of Latin Trinitarianism, including Leftow's, entail tritheism for similar reasons.

7.0.7 Latin Trinitarianism and the Logical Problem of the Trinity

For what follows, I will consider some of the various versions of Latin Trinitarianism that have been proposed. Latin Trinitarianism is sometimes divided between those theories which adopt RI and those that do not, the former being described as 'relative Trinitarianism' (McCall and Rea eds. 2009: 9-14). Leftow (2004) and Brouwer and Rea (2005), for example, reject RI, while Martinich (1978, 1979), van Iwagen (1988, 2003), Cain (1989), Christopher Hughes (2009), and Conn (2012) accept it. The central objection to all versions of Latin Trinitarianism is that the proposed interpretations of the sentences (7.1)-(7.5) lead to what has become known as 'the logical problem of the Trinity'. The problem can be stated in many different ways. But I will set it up as follows:

Translate (7.5) as:

[5] In fact, there are four. The Godhead is absolutely identical with some God which is not absolutely identical with any of the other three (Leftow 1999: 218).

[6] Defenders of Trinity Monotheism hold that there are two different ways to be divine (Craig 2009: 95-96). However, I think that Leftow is right that this position is simply a form of Arianism (Leftow 1999: 218).

(7.12) The Father is not identical with the Son, and the Father is not identical with the Holy Spirit, and the Son is not identical with the Holy Spirit, and there is only one divinity.

Translate (7.6) as:

(7.13) The Father is identical with some divinity, and the Son is identical with some divinity, and the Holy Spirit is identical with some divinity.

Translate (7.7) as:

(7.14) There is some divinity, x, and for any divinity y, x is identical with y.

For the argument that follows we will take (7.12), (7.13), and (7.14) to be theological axioms. From this we may derive the following:

Proof 2

1.	The Son is identical with some divinity.	Omitting Conjunction: axiom (7.13)
2.	The Father is identical with some divinity.	OC: axiom (7.13)
3.	The divinity that the Son is identical with is identical with the divinity that the father is identical with.	axiom 14: 1,2
4.	The Son is identical with the divinity that the Father is identical with.	Transitivity: 1, 3
5.	The Son is identical with the Father.	Transitivity: 4, 2
6.	The Father is not identical with the Son.	OC: axiom 31.9
		$\perp 4, 5$

Thus, assuming classical logic with identity, the theological axioms we began with are inconsistent, therefore false. It seems that the claim that each of the three persons of the Trinity are identical with some divinity leads, via the classical logic of identity, to contradiction. It seems our options are either to abandon the Athanasian Creed, to maintain the Creed in spite of its inconsistency, to interpret the theological statements differently than (7.12)-(7.14), or to replace classical logic with some other logic, given which, Argument 1 is invalid.

I am interested in the prospects for an orthodox doctrine of the Trinity, so I will assume the truth of the Athanasian Creed. Our second option would be to say that the Creed is inconsistent and yet true. The result would seem to amount to Fideism, roughly the view that articles of faith are not answerable to reason (in this case, to logic). However, we might avoid the accusation of Fideism in the following way: reject classical $FOL^{=}$ in favour of some logical system which permits true contradictions. That is, adopt a dialethic logic. This is an alternative which has not yet been developed in any depth as a response to the logical problem of the Trinity.[7] Recent defences of dialetheism, most notably by Graham Priest (1983), perhaps make this a worthwhile avenue for further research. However, accepting true contradictions is undoubtedly a high price to pay, and any resolution to the logical of the problem of the Trinity which offers consistency is to be preferred. There remain two alternative strategies for resolving the logical problem of the Trinity, either by reinterpreting the theological statements or by adopting a deviant logic. I will first argue that those versions of Latin Trinitarianism that do not involve adopting a deviant logic entail tritheism. I will focus on Leftow's time-travel analogy (2004), but I suggest that the point generalizes.

7.0.8 Time Travel and the Trinity

Leftow's account of the Trinity depends on an analogy with time travel. If time travel is possible, then it is apparently possible for one and the same person to be multiply instantiated simultaneously in different spatial locations. For example, it is possible for a person to journey through time and join his or her earlier self. Presumably, if this is possible, it is possible for three instantiations of the same person to be present in the same room at one time. Leftow suggests that this possibility serve as a model for the Trinity. Each of the persons is like one of the instantiations; just as the three instantiations are all one and the same person, even though they do not have all the same properties, so too all three persons are one and the same divinity, and that single divinity is to be identified with a single divine substance persisting through time. It is easy to see why this account is generally considered a version of Latin trinitarianism. The most apparent worry here is modalism rather than tritheism. Quite apart from the general worries about the possi-

[7] Though its possibility as a response to the logical problem of the Trinity has been noted by David Efird (2012: 191).

bility of time travel, it is not clear that the Father, Son and Holy Spirit are genuinely different persons on this account. However, I have another worry. I think that this account is still tritheistic.

Note that Leftow's account does not involve GT. Let us assume, then, that there is a relation of absolute identity. Next, consider the relation that holds between any two of the persons. If we take 'the Father' and 'the Son' both to name the Godhead, that is, the single divine substance that persists through time, then we have clearly reached straightforward modalism. This cannot be what Leftow has in mind. Rather, when we say 'the Father is not the Son', we are referring to two instantiations of the divine substance, rather than the divine substance itself. Leftow seems to conceive of these instantiations as temporal phases of the divine substance. This suggests that Leftow assumes perdurantism. Given this, the Father, Son, and Holy Spirit are not absolutely identical with each other or with the Godhead. At the same time, each is divine. By (7.11), which is supported by Leftow's own contention that the persons are divine in just the same sense as the Godhead, this seems to entail tritheism. To see this, once again, we can assume that the Godhead is absolutely identical with some divinity, so, by (7.11), each of the persons is absolutely identical with the some divinity. Yet none of the persons is absolutely identical with any of the other persons, so each of the divinities with which the persons are identical must similarly be absolutely non-identical with each other.

Leftow may want to reject this application of (7.11) because he thinks that the relation between the persons and the Godhead is something like the relation of identity through time. After all, the boy may have different properties from the grown man, but this does not prevent the boy and the man from being the same person, even when '...same person as...' is construed as a relation of absolute identity. However, this appeal is illegitimate. As we have seen, Leftow is committed to perdurantism in order to avoid modalism. But, on this account, a temporal instance of a substance *instantiates* a substance, rather than being absolutely identical with that substance. The persons therefore are divine in the sense of instantiating the Godhead, while the Godhead is divine in the sense of being absolutely identical with some divinity. This contradicts Leftow's assertion that the persons and the Godhead are divine in just the same sense. The appeal to the relation of identity over time therefore fails, and again we arrive at tritheism.

Note that this form of argument suggests that any account fo the Trinity that holds that there is only one kind of divinity, that the Godhead is absolutely identical with some divinity, and that the persons are not absolutely identical with some divinity, entails tritheism. Next, we turn to accounts of the Trinity according to which there is an identity relation that holds between each person and some divinity. To begin with, we may set out the general structure of relative Trinitarian accounts.

7.0.9 The Relative Identity Solution to the Logical Problem of the Trinity

Geach attributes a theory of relative identity to Thomas Aquinas, in particular with respect to Aquinas's views on the doctrine of the Trinity. We can get a better grasp the significance of strong relative identity for Latin Trinitarianism, if we consider Geach's account of Aquinas.

> A few remarks on the logic of "there is but one God" and "the one and only God." On Russell's theory of descriptions "the one and only God is X" would be construed as meaning:
>
> "For some y, y is God, and for any z, if z is God, z is the same as y, and y is X"; And this, shorn of the final clause "and y is X", would also give the analysis of "there is but one God." Aquinas would certainly have objected, on general grounds, to the clause "z is the same as y"; the sameness, as we saw, must for him be specified by some general term signifying a form of nature. Now the general term that we need to supply here is clearly "God"; so "there is but one God" will come out as:
>
> "For some y, y is God, and, for any z, if z is God, z is the same God as y." It is important to notice that this would leave open the possibility of there being several Divine Persons; there would still be but one God, if we could truly say that any Divine Person

was the same God as any other Divine Person. (Anscombe and Geach 1961: 118)

Geach, therefore, holds that the sentence ‘The Father is identical with God’ must be completed with the general term ‘God’. I, however, will continue my practice of replacing ‘God’ with ‘divinity’, and therefore, we arrive at ‘The Father is the same divinity as the Godhead.’ We will, henceforth, represent this relation with the symbol ‘$=_D$’. We can also say that any of the persons is the same divinity as any other persons. So we get:

(7.14) The Father $=_D$ the Son $\wedge$ the Son $=_D$ the Holy Spirit $\wedge$ the Holy Spirit $=_D$ the Father.

But Geach tells us that this leaves open the possibility of the three persons remaining distinct. We can therefore translate our theological axiom (7.2) as:

(7.15) The Father $\neq_P$ the Son $\wedge$ the Son $\neq_P$ the Holy Spirit $\wedge$ the Holy Spirit $\neq_P$ the Father $\wedge$ there is only one divine substance. Where the symbol ‘$=_P$’ stands for the relation ‘... is the same person as...’.

Geach intends that we take (7.14) and (7.15) as interpreted, where the relations ‘... is the same divinity as...’ and ‘... is the same person as...’ are relations of numerical identity. The possibility of (7.14) and (7.15) being true together entails that the relation ‘... is the same divinity as...’ does not satisfy LL.

Proof: Assume ‘... is the same divinity as...’ does satisfy LL. It follows that everything true of the Father is true of the Son. This entails that, if the Father is the same person as the Father, then the Son is the same person of the Father. The Father is the same person as the Father, so, by LL, the Son is the same person as the Father. By omitting the conjunction on (7.15) it is also the case that the Father is not the same person as the Son. And we have reached a contradiction.

The relation '... is the same divinity as...' is therefore non-Leibnizian. This solves the logical problem of the Trinity, because a contradiction cannot be derived when (7.2) is interpreted as (7.15) and (7.5) is interpreted as (7.14). Argument 1 depended on the inference from 'The Father is identical with some divinity' and 'The Son is identical with some divinity' to 'The Father is identical with the Son'. If we follow Geach, however, we must say 'The Father is the same divinity as the Godhead' and 'The Son is the same divinity as Godhead', but this only licenses the conclusion that 'The Father is the same divinity as the Son', and this is consistent.

Geach's account of the Trinity escapes the logical problem of the Trinity by appealing to RI. To see that (7.14) and (7.15) entail the truth of RI, consider the following.

By omitting the conjunction on (7.14) we get:

(7.16) The Father is the same divinity as the Son.

By omitting the conjunction on (7.15) we get:

(7.17) The Father is not the same person as the Son.

Finally, all versions of the doctrine of the Trinity are committed to:

(7.18) The Father is a person.

By introducing the conjunction of (7.16)-(7.18), we get:

(7.19) The Father is the same divinity as the Son, and The Father is not the same person as the Son, and The Father is a person.

Thesis (7.19) is an instance of the cases of relative identity posited by RI.

Geach's strategy has been a very influential amongst Latin Trinitarians, and many philosophers have adopted similar strategies, as we have noted. These accounts can be divided into those versions that involve a weak theory of relative identity, that is, that reject GT, and those that involve a strong theory of relative identity, accepting GT. It might be noted that the above account of Geach's strategy for avoiding the logical problem of the Trinity has not involved any reference to GT. Given this, it is not immediately obvious that GT is required in order to escape the logical problem of the Trinity. Next, I will consider theories of the Trinity that involve weak theories of relative identity. I will argue that these, too, entail tritheism.

7.0.10 Weak Theories of Relative Identity

Weak relative identity is compatible with (7.14) and (7.15), and, given this, that the relation '... is the same divinity as...' does not satisfy LL. This is sufficient to escape the logical problem of the Trinity, as was shown above.

Weak relative identity is represented in the literature on Latin trinitarianism by van Inwagen (1988, 2003) and Conn (2012).[8] Van Inwagen, in

[8] Another version of Latin Trinitarianism is the 'sameness without identity' view defended by Brouwer and Rea (2005). Their defence of the coherence of the Trinity consists of two parts. First a logical/semantic claim, that we can define some relation which is not an identity relation but which permits the inference: if $R(x, y)$, then x and y are one.

Second, there is a metaphysical claim. The metaphysical claim is that 'The familiar particulars of experience must be conceived of as hylomorphic compounds–that is, as matter-form structures related to other things sharing their matter by the relation of accidental sameness'. In the case of the Trinity, we are not concerned with a compound of matter and form but rather of essence and form. Thus, there is one divine essence and three 'distinctive Trinitarian properties' which serve as the form for each of the three Persons. Brouwer and Rea further note that 'As in the case of matter, moreover, we can regard the divine essence not as an individual thing in its own right but rather as that which, together with the requisite "form", constitutes a person' (Brouwer and Rea 2005: 77). Interesting though the metaphysical claim is, for the moment I wish to focus only on the logical claim.

The claim that there is a relation of 'sameness without identity' can, apparently, be used to escape the logical problem of the Trinity in the following way. If there is such a relation of numerical sameness, and if that relation is sometimes named in ordinary language by the English word 'is', then the sentence 'The Father is divine, and the Son is divine, and the Holy Spirit is divine, and there is only one divinity', may be true without entailing that the Son is strictly identical to the Father, or the Father or Son to the Holy Spirit. Each of the persons has the 'numerical sameness' relation to each other (at least *qua* divinity) and to the Godhead. Thus, they are not strictly identical but they are to be counted as one.

Structurally this is similar to the Social Trinitarian responses, and the same objections apply. As with the Social Trinitarian accounts, the persons are not absolutely identical with one another. Again, like the Social Trinitarian accounts, the Godhead is absolutely identical with some divinity. By (7.11), this entails that each of the persons is absolutely fact, declares himself agnostic about GT (1988: 257), while Conn (2012) advocates rejecting GT because, he claims, it is subject to an unanswerable objection.[9]

According to the weak relative identity account of the Trinity, the Father and the Son are not absolutely identical, but there is another, weaker, relation of numerical identical which they jointly satisfy. Once again, the same simple argument may be deployed. If we take seriously Leftow's contention that there is only one way of being divine, then we are committed to (7.11). But, as we have seen, this leads to tritheism.

The general worry can be brought into sharper focus in the case of weak relative identity accounts in particular. Leftow's contention that there is only one way of being divine is motivated by the desire to avoid Arianism, the heretical view that one of the persons is only divine in some derivative sense. However, this worry recurs in one form or another in each of the accounts of the Trinity we have so far considered. The only way of avoiding this worry is to assert that the relation that holds between each person and some divinity is the same relation that holds between the Godhead and some divinity, and this leads to tritheism. The same is true with the weak relative identity account of the Trinity. As we have seen, the weak theory of relative identity faces a serious challenge from Wiggins's conceptual objection. Once again, the idea was that satisfying LL is just what it is to be a relation of identity. Given this, relations of non-Leibnizian 'identity' are not really relations of identity at all. Moreover, if an absolute identity relation is admitted, then the domain of discourse for a language admits of a most-fine-grained

identical with some divinity and therefore, by transitivity, there are, at least, three divinities. Brouwer and Rea hold that we count the number of divinities as one, even if the persons and the Godhead have different properties. I will not pass judgement on the coherence of a relation of sameness without identity. However, note that, even if we do sometimes count by a relation of sameness without identity, we also count by identity relations. Brouwer and Rea would, of course, accept that the godhead is absolutely identical with a divinity and is not absolutely identical with any other divinity, naturally, this entails that the Godhead is one divinity. So, divinities can be counted according to relations of absolute identity. By this method, we can still count to three divinities given that, as we have seen, each of the persons is absolutely identical with some divinities, and they are not absolutely identical with each other.

[9] He has in mind Le Poidevin's (2009) objection, which I discussed in Chapter 4.

characterization, specifically that generated by partition of the domain by the absolute identity relation. Given this, the proposition 'the Father is not absolutely identical with the Son' involves a more fine-grained relation than does 'the Father is the same divinity as the Son'. If all this is true, then the Godhead, on the weak relative identity view, is no more than an equivalence class generated by a less-than-maximally fine grained equivalence relation. The persons represent a more-fine-grained description of the contents of the world. This suggests that a more fundamental description of the world, on the weak relative identity view, involves three divine beings, rather than one. Once again, this sounds like tritheism.

7.0.11 Strong Relative Identity

With the failure of weak relative identity to provide an adequate Latin Trinitarian response to the logical problem of the Trinity, we will naturally wonder how strong relative identity fares. We might expect to find the same problem here.

In fact, we do not. All of the above accounts of the Trinity turn out to be tritheistic under the assumption that the persons and the Godhead are divine in the same sense. If we take this claim seriously then (7.11) follows if and only if there exists some relation of absolute identity. Strong relative identity, by contrast, avoids this problem. If strong relative identity is true, there are no relations of absolute identity. This means that, even assuming that it is true that the persons and the Godhead are divine in the same sense, we cannot derive tritheism, because the key premise, (7.11), is ill-formed. The general schema still holds. Any relation that the Godhead bears to any divinity, the persons also bear to any divinity. However, the Godhead is not absolutely identical with any divinity, and neither are any of the persons. The argument to tritheism is thus rendered invalid.

7.0.12 Conclusion

In this chapter, I have considered several accounts of the Trinity. I argued that all of these accounts entail tritheism. I argued that the only way of avoiding is by adopting GT.

Conclusion

In this dissertation, I have looked at the strong theory of relative identity. In the first half of the thesis, I considered arguments in favour of the strong theory of relative identity. In the second half of this dissertation, I considered objections to the strong theory of relative identity and how these might be answered. In this conclusion I will briefly outline the results of each chapter of the foregoing dissertation in turn.

In Chapter 1, I classified theories of identity. First, I distinguished between theories of absolute identity and theories of relative identity. A theory is a theory of absolute identity if and only if it involves the claim that all relations of numerical identity are relations of absolute identity. A theory is a theory of relative identity if and only if it involves the thesis that there are true sentences instantiating the form: $\ulcorner \exists x \exists y x =_F y \wedge x \neq_G y \wedge G(x) \vee G(y) \urcorner$.

I then subdivided theories of relative identity into two further categories; strong theories of relative identity and weak theories of relative identity. This distinction was drawn in terms of the relation between the theory and a further thesis, which I named 'GT'. GT is the thesis that there are no relations of absolute identity. A theory is a weak theory of relative identity if and only if it is a theory of relative identity, and it rejects GT. A theory is a strong theory of relative identity if and only if it is a theory of relative identity and it includes GT. The rest of the dissertation focuses on strong theories of relative identity. More particularly, I considered Geach's theory of identity, which is the only strong theory of relative identity which has been developed in the contemporary literature. Aside from the above mentioned theses, Geach is committed to a thesis, which I named 'SRI', which states that all relations of identity involve sortal terms. I argued that each of the three theses involved in the strong theory of relative identity needs to be defended separately, as none entails either of the others. I also considered whether any of the three theses, RI, GT, or SRI, can be proved with an example. I concluded that GT and SRI cannot be proved with examples but that, in principle, RI can. I further argued that the alternative to relative identity, AI, cannot be proved with an example.

Chapter 2 considered Geach's argument for GT. The chapter was divided into two sections. The first involved an exposition of Geach's argument. In Section 1, I argued that the objections raised against Geach do succeed against his formulation of the argument, but that they might, perhaps, be avoided by a series of alterations to Geach's argument. The alterations which I suggested are as follows: to focus the argument on the existence of relations of identity characterized by reflexivity and LL, rather than those characterized by Wang's Schema. To defend the inference from the absence of a criteria for a predicate's expressing absolute identity to the non-existence of relation of absolute identity. To provide an explicit argument from the proposed second-order criteria to Grelling's paradox. Finally, to abandon Geach's attempts to show that Quine's proposal for reinterpretation is incoherent.

In Section 2, I incorporated the proposed alterations to arrive at a charitably reconstructed version of Geach's argument. I considered each of the premises in turn and concluded that not all of them have been conclusively established. Therefore GT is unproven

In Chapter 3, I considered the arguments that have been advanced in favour of and against RI. I considered four arguments that are intended to support the thesis that there are possibly true cases of RI. I argued that each of these fails. The first argument, Geach's 'river and waters' argument, fails because Geach fails to establish the key second premise, that every river is numerically identical with some water. A similar problem, though involving slightly different issues, confronts Geach's second attempt, with his 'men and heralds' argument. Specifically, Geach fails to show that heralds are concrete entities. Without this, once again, the second premise, that all heralds are numerically identical with some man, is unproven. We then considered two further arguments for RI from two defenders of weak theories of relative identity, Griffin and Zemach. Griffin argues that a theory allowing the existence of non-Leibnizian relation of numerical identity is simpler than a theory which distinguishes between relations of numerical identity and common-property relations, based on satisfaction of LL. Zemach argues that the existence of vague objects entails the truth of some cases of RI. I argued that both Griffin and Zemach fail to establish their claims. I then considered objections to RI. I argued that the strongest objection against RI is Wiggins's conceptual objection. I holds that this objection can only be answered if GT is true, not if it is false. For this reason, I concluded that all weak theories of relative identity are false.

In Chapter 4, I considered the only developed argument in the literature in favour of SRI. This argument, from Alston and Bennett, is that Frege's cardinality thesis entails SRI. I considered the relationship between the two theses, but conclude, following Saachi and Carrara, that CT does not entail SRI. I therefore concluded that both RI and SRI are unproven. I considered an objection against SRI, from Le Poidevin, I argue that this too fails.

In Chapter 5, I turn to the remaining objections against theories of relative identity. I suggested that several of these objections focus on the same apparent weakness. They show that the strong theory of relative identity is incompatible with classical semantics. In fact, it is incompatible with any semantics which takes the domain of discourse for a language to be a set as traditionally understood. I also noted an objection from Cain, to the effect that Geach's views entail the failure of the syllogisms.

In Chapter 6, I consider how the objections raised in the previous chapter might be answered. I started by suggesting the best response open to Geach against Cain's objection. I then considered the kind of semantics which might be compatible with the strong theory of relative identity and escape the objections raised against that theory. Finally, I considered the logical systems that have been developed, which might be compatible with relative identity.

In Chapter 7, I considered one area of philosophy to which the results of the previous discussion might be fruitfully applied. I considered the relative identity responses to the logical problem of the Trinity in philosophical theology. While most versions of the relative identity response involve RI and either reject GT or are neutral concerning it, I argued that either GT is true or Christian orthodoxy is false.

There are potentially many more areas of philosophy to which the strong theory of relative identity may be fruitfully applied, but these must remain tasks for the future.

Appendix A: List of possible rules of inference for logics of relative identity, with counterexamples.

R1

$$\frac{\Phi_1\varphi}{\Phi_2\varphi}$$

if $\Phi_1 \subseteq \Phi_2$, where Φ is a set corresponding to any monadic property, and *phi* is any singular term.

R2

$$\frac{}{\Phi\varphi}$$

if $\varphi \in \Phi$

R3

$$\frac{\Phi\varphi_1\varphi_2, \Phi\neg\varphi_1\varphi_2}{\Phi\varphi_2}$$

R4

$$\frac{\Phi\neg\varphi_1\varphi_2, \Phi\neg\varphi_1\neg\varphi_2}{\Phi\varphi_1}$$

R5

$$\frac{\Phi\varphi_1\varphi_3, \Phi\varphi_2\varphi_3}{\Phi(\varphi_1 \vee \varphi_2)\varphi_3}$$

R6 (i)

$$\frac{\Phi\varphi_1}{\Phi(\varphi_1 \vee \varphi_2)}$$

(ii)

$$\frac{\Phi\varphi_1}{\Phi(\varphi_2 \vee \varphi_1)}$$

R7

$$\frac{\Phi\varphi[\beta/\alpha]}{\Phi\exists\beta\varphi}$$

R8

$$\frac{\Phi\varphi_1[\beta_1/\beta_2]\varphi_2}{\Phi\exists\beta_{1\varphi_1\varphi_2}}$$

if β_2 is not free in the sequent $\Phi\exists\beta_1\varphi_1\varphi_2$

R9

$$\frac{}{\alpha = \alpha}$$

R10

$$\frac{\Phi\varphi[\beta/\alpha_2]}{\Phi\alpha_1 = \alpha_2\varphi[\beta/\alpha_2]}$$

R11-R20 are the rules of inference that distinguish Garbacz systems from classical logic. All Garbacz systems involve R1-R11. SQ1, the sequent calculus for C1, involves those and no others. SQ2, the sequent calculus for C2, also involves R12. SQ3 also involves R13, but not R12. SQ4 also involves R14 but not R12 or R13, and so on.

R11A

$$\frac{\Phi\alpha_1 = \alpha_2, \Phi\delta(\alpha_1)}{\Phi\alpha_1 =_\delta \alpha_2}$$

Note that R11A involves a premise which is ill-formed if SRI is true. R11A, therefore, is not compatible with Geach's strong theory of relative identity.

R11B

$$\frac{\Phi\alpha_1 =_\delta \alpha_2}{\Phi\delta(\alpha_1)}$$

This rule is compatible with all of GT, RI and SRI. Interestingly, Zemach rejects this rule.

R11C

$$\frac{\Phi\alpha_1 =_\delta \alpha_2}{\Phi\alpha_2 =_\delta \alpha_1}$$

This rule of inference is what I have called ‘relative symmetry’ and is compatible with all theories of relative identity.

R11D

$$\frac{\Phi\alpha_1 =_\delta \alpha_2, \Phi\alpha_2 =_\delta \alpha_3}{\Phi\alpha_1 =_\delta \alpha_3}$$

This rule of inference is what I have called ‘relative transitivity’ and is compatible with all theories of relative identity.

R12

$$\frac{\Phi\forall\beta(\delta_1(\beta) \equiv \delta_2(\beta)), \Phi\alpha_1 =_{\delta_1} \alpha_2}{\Phi\alpha_1 =_{\delta_2} \alpha_2}$$

Counterexample: Everything that is a human being is a persona and every persona is a human being. Superman is the same human being as Clark Kent. Superman is not the same persona as Clark Kent.

R13

$$\frac{\Phi(\delta_1(\beta) \to \delta_2(\beta), \Phi(\alpha_1 =_{\delta_2} \alpha_2) \wedge (\alpha_2 =_{\delta_2} \alpha_3)}{\Phi\alpha_1 =_{\delta_2} \alpha_3}$$

Counterexample: Every river is water. The river that Heraclitus bathed in yesterday is the same river he bathed in today, and the river he bathed in today is the same water as yesterday’s rainfall. The river Heraclitus bathed in yesterday was not the same water as yesterday’s rainfall.

R14

$$\frac{\Phi(\delta_1(\beta) \to \delta_2(\beta), \Phi(\alpha_1 =_{\delta_2} \alpha_2) \wedge (\alpha_2 =_{\delta_2} \alpha_3)}{\Phi(\alpha_1 =_{\delta_1} \alpha_3)}$$

Counterexample: Every river is water. The river that Heraclitus bathed in yesterday is the same river he bathed in today, and the river he bathed in today is the same water as yesterday's rainfall. The river Heraclitus bathed in yesterday was not the same river as yesterday's rainfall.

R15

$$\frac{\Phi(\delta_1(\beta) \rightarrow \delta_2(\beta), \Phi(\alpha_1 =_{\delta_2} \alpha_2) \wedge (\alpha_2 =_{\delta_2} \alpha_3)}{\Phi(\alpha_1 =_{\delta_1} \alpha_3 \wedge \alpha_1 =_{\delta_2} \alpha_3)}$$

Counterexample: Every river is water. The river that Heraclitus bathed in yesterday is the same river he bathed in today, and the river he bathed in today is the same water as yesterday's rainfall. It is not the case that the river Heraclitus bathed in yesterday was the same river and the same water as yesterday's rainfall.

R16

$$\frac{\Phi(\alpha_1 =_{\delta_1} \alpha_2) \wedge (\alpha_2 =_{\delta_2} \alpha_3)}{\Phi(\alpha_1 =_{\delta_1} \alpha_3) \vee (\alpha_1 =_{\delta_2} \alpha_3)}$$

Counterexample: The river that Heraclitus bathed in yesterday is the same river he bathed in today, and the river he bathed in today is the same water as yesterday's rainfall. The river Heraclitus bathed in yesterday was neither the same river, nor the same water, as yesterday's rainfall.

R17

$$\frac{\Phi(\alpha_1 =_{\delta_1} \alpha_2), \Phi\delta_2(\alpha_1)}{\Phi\delta_2(\alpha_2)}$$

Counterexample: Superman is the man as Clark Kent. Superman is a superhero. Clark Kent is not a superhero.

R18

$$\frac{\Phi(\alpha_1 =_{\delta_1} \alpha_2), \Phi(\delta_2(\alpha_1) \wedge \delta_2(\alpha_2))}{\Phi\alpha_1 =_{\delta_2} \alpha_2}$$

Counterexample: John Smith is the same surman as Sam Smith. John and Sam Smith are both men. John and Sam Smith are not the same man.

R19

$$\frac{\Phi(\alpha_1 =_{\delta_1} \alpha_2), \Phi\delta_2(\alpha_1)}{\Phi(\alpha_1 =_{\delta_2} \alpha_2)}$$

Counterexample: John Smith is the same surman as Sam Smith. John Smith is a man. John and Sam Smith are not the same man.

R20

$$\frac{\Phi(\alpha_1 =_{\delta} \alpha_2)}{\Phi(\alpha_1 = \alpha_2)}$$

If SRI is true, the conclusion is ill-formed. R20 is therefore incompatible with Geach's strong theory of relative identity.

List of Abbreviations

GT: There exists no relation of absolute identity.

RI: There are possibly true instances of the form $\ulcorner x =_F y \wedge x \neq_G y \wedge (G(x) \vee G(y))\urcorner$.

SRI: All relations of numerical identity involve sortal terms as a part of their content.

FOL: First-order logic.

FOL$^=$: First-order logic with the (classical) laws of identity.

CT*: Numbers are not first-order concepts.

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