

LAMP SPECTRUM AND RELATIVE SPATIAL BRIGHTNESS AT PHOTOPIC LIGHT LEVELS

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ABSTRACT

This thesis proposes that the spectral power distribution (SPD) of lighting can be modified to enhance spatial brightness. Energy saving is then possible by using SPD that allows illuminance to be reduced whilst maintaining the same level of spatial brightness. The Akashi and Boyce study demonstrates an energy saving of 33% by using lamps of higher correlated colour temperature but it is widely known that this is not a good metric for predicting spatial brightness.

The aim of this study was to identify a metric for predicting spatial brightness. The first approach followed the method of Cowan and Ware: use the results of past experiments to test potential metrics. 65 studies of spatial brightness and SPD were found. Initially, these lead to different conclusions as to whether SPD affects spatial brightness. The reasons for this are that they used different methodologies and hence review of method was used to screen the credible data from within these 65 studies: only 19 of them were considered to be credible. This thesis focussed on the category rating procedure. The review of methods included an experiment comparing rating scales with different response ranges and a meta-analysis comparing results gained when either brightness or visual clarity were the objective of the experiment. Two potential metrics for spatial brightness are the scotopic to photopic (S/P) luminance ratio and the area of the colour gamut (GA). Results from the credible studies were used to test these models: while both models suggest a reasonable prediction, it was found that they were not independent for this set of data and it was therefore not possible to discriminate between them.

Hence an experiment was carried out to directly test these metrics. The experiment employed full field sequential evaluation of stimulus pairs, with matching and discrimination procedures. Three SPDs were compared, these chosen to isolate the S/P and GA effects. Following Berman et al, one pair had identical chromaticity but different S/P ratios: a second pair had identical S/P ratio but different gamut area; the third pair had different S/P and gamut area. The two procedures led to similar results: null condition trials confirmed that doubt about interval bias in the Berman et al data was unwarranted. It was found that lighting of higher S/P or higher GA enhance spatial brightness: it was also found that their effects appear to be additive.

When the final remodelling was done by adding the data points from the new experiment to the data set, the models of the difference of S/P ratio and the log ratio of GA had the best fits with spatial brightness. Their correlations were equally plausible with mean illuminance ratio of the data set.

This thesis demonstrates that SPD affects spatial brightness, allowing lower illuminances to be used when using lighting of higher S/P ratio and gamut area.

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LIST OF ABBREVIATIONS

ССТ	Correlated colour temperature
CRI	Colour rendering index
CSA	Cone surface area
GA	Gamut area
ipRGC	Intrinsically photosensitive retinal ganglion cells
LWS	Long-wavelength sensitive
MC	Magnocellular ganglion cells
MWS	Medium-wavelength sensitive
Р	Photopic luminance
PC	Parvocellular ganglion cells
S	Scotopic luminance
S/P ratio	Scotopic luminance / Photopic luminance ratio
SWS	Short-wavelength sensitive
V'(_)	CIE Standard Scoropic Observer
V(_)	CIE Standard Photopic Observer

Chapter 1

1 Introduction

- 1.1 Electrical Lighting and Work
- 1.2 Aims and Objectives of This Study

1.1 Electrical Lighting and Work

Commercial buildings, such as office buildings are defined as presenting high-energy consumption, 20-45% of this consumption is constituted by lighting. It is possible to reduce lighting energy consumption up to 50% with the new technologies and efficient use of electrical lighting (Dubois and Blomsterberg, 2011).

There are three possibilities for reducing the electricity consumption of lighting community (Boyce, 2010).

To use daylight more efficiently in combination with better control on electrical lighting.
According to the research, using daylight with lighting control systems has potential to reduce the electrical energy consumption in office building by 30-60% (Dubois and Blomsterberg, 2011). However, Boyce (2010) suggested that this solution is too slow to achieve: creating new gaps on outer shells of the buildings in order to take in more sun light or constructing new buildings with more windows require long time.

2. To develop more energy efficient lighting technology. As scientist and researchers are developing new technologies and systems with low energy consumption every day, how and when it is going to be possible to have an ideal technology is still uncertain. Since it costs large amounts to replace old systems with new ones and most of the purchasers of the technology do not seem to be convinced to replace their luminaires or systems with them, this approach is not that practical at the moment.

3. Reducing the illuminances used in new and existing installations.

Lighting for offices in the UK tends to be designed to achieve an average horizontal illuminance of 500 lux (Dubois and Blomsterberg, 2011). It has been suggested that this could be reduced by, say, 100 lux, providing up to 20% reduction in energy consumption without significantly reducing the visibility of the task (Boyce et al, 2006). Task performance though is more than visibility: if the reduced illuminance led to an environment that was considered to be gloomy this may affect people's mood and thus their motivation to work (Boyce et. al., 2003; Knez, 2001). Visibility is one of the factors of visual performance highly related with illuminance, and the contrast, colour and size of the task. Increasing the illuminance improves visual performance up until some level, i.e. with gradually decreasing returns. As shown in Figure 1.1 a larger improvement can be achieved by changing either the size or the contrast of the task than by increasing the illuminance.



Figure 1.1 Visual performance and task illuminance (after Tregenza and Loe, 2014).

However, the reason that these high levels of lighting are still being used in the offices is mostly related with the users' preferences. According to Boyce (2014), there are two possibilities for why office users prefer to have higher levels of light. One might be related with what they are used to; for the last generation of workers, illuminances in offices were arranged approximately to 500 lux. When there is no obvious effect on visibility and/or comfort, people prefer the light level that they are used to. Another possibility is that office users think there is enough light in the room to see whatever they are expected to do in there. As one of the consistently evaluated dimensions of the lighting is brightness, the amount of light in the workspace, which can be described as spatial brightness, is highly related with the users' expectations. And so, it is important to fulfil the expectations in order not to negatively affect the mood of the worker as task performance involves visual performance and mood in relation with motivation (Boyce, 2014). One way to adjust the spatial brightness is to use characteristics of lamp spectral power distribution (Akashi & Boyce, 2006).

It is well known that photometry based on the standard photopic observer does not fully account for visual response. For example, it has been demonstrated that lamp spectral power distribution affects the perception of brightness, so that two different lamps providing the same illuminance can produce different spatial brightnesses. In the study by Boyce (1977), two symmetrically arranged booths were presented to the subjects with three different lighting conditions. When matched for equal visual appearances the required illuminances depended on the spectral power distributions (SPD) of the lamps. Similarly, in the brightness discrimination study by Berman et. al. (1990), an effect of SPD on brightness was obtained. These findings suggest it should be possible to select a lighting spectrum to offset a reduction in illuminance and so maintain the same level of brightness. Such an approach could allow the mood and motivation of the worker to be influenced more positively while reducing energy consumption. However, there is as yet no accepted means by which to characterise the influence of a spectrum on spatial brightness. Previous studies exploring this effect of SPD on spatial brightness used different experimental methods. These methods establish different relations with the environment and include different interactions with the participant. Consequently they provide different and sometimes contrasting results on how lamp SPD affects spatial brightness.

1.2 Aims and Objectives of This Study

The principle aims of this study are;

- To investigate the effects of lamp SPD on spatial brightness in the photopic viewing conditions experienced by the user in interior spaces;
- To find out how different experimental conditions affect the assessment of SPD effects on spatial brightness, and thus promote procedures with reduced systematic bias.
- To identify a lighting metric to help predict the effect of lamp SPD on spatial brightness, with a focus on simple metrics that are easier to use and more likely to be accepted.

The resulting objectives for the study are disclosed in the structure of this thesis: The information on current lighting practice in office areas and the visual needs of office workers in the previous sections of the current chapter provides the necessary context for this study. The first half of Chapter 2 describes the human visual system and how the amount of light is being defined. In the second half of the chapter, the metrics related with SPD and how the changes in light spectrum relevant to affect spatial brightness at photopic light levels are discussed. This discussion is needed in order to establish that SPD effects have already been validated, accordingly the research questions for the current study specified at the end of Chapter 2. A classification of past studies of SPD and spatial brightness according to the experimental method that has been used is described in Chapter 3 and credible studies are defined. A detailed investigation on one of the experimental methods, category rating task is presented in Chapter 4. In the first half of Chapter 4, a new experiment on the number of response categories is described. In the second half of the chapter, a meta-analysis on terminology used in brightness studies is explained. A new laboratory experiment testing the effects of potential metrics on spatial brightness is explained in Chapter 5. Details of the participants and the apparatus are given. This chapter also describes three different experimental methods used to measure the lamp spectrum effects and the validation of the results gained from different methods. An approach to develop spatial brightness models with potential metrics of SPD using credible data from past studies that is gathered together is described in Chapter 6. Chapter 7 provides the further discussion on the findings gained from all of the analyses. Chapter 8 provides the overall conclusions and recommendations for further work. Additional information is provided in five appendices: Appendix A includes details of the studies using category rating method with the presented environmental conditions and the questions asked in the experiment; Appendix B contains the questionnaire used in the

experiment of number of response categories; Appendix C presents the SPD values of the lamps used to predict brightness; Appendix D covers information sheet and consent form for the new experiment; Appendix E shows examples of tabulations used to test the normality of the distributions in the experiments. A list of referenced work is placed at the end of the thesis.

Chapter **2**

2 SPD and Visual Response

- 2.1 Introduction
- 2.2 Visual system
- 2.3 Measuring Light
- 2.4 Measures of Colour
- 2.5 Spectral Power Distribution (SPD) and Spatial Brightness
- 2.6 Summary

2.1 Introduction

This chapter presents a summary of standard physical photometry and considers some basic physiology to demonstrate why an effect of spectral power distribution on spatial brightness is expected. Then, potential metrics that were proposed to have an effect on spatial brightness at photopic light levels are discussed. This leads to the research questions of this thesis.

2.2 Visual system

The visual system processes an image with eye and brain working together. A cross-sectional diagram of the eye can be seen in Figure 2.1. Firstly, light enters the eye through the transparent area in the front called the cornea. This layer becomes white while curling to the back of the eye creating an outer layer, which maintains the circular shape of the eye. The next layer is known as ciliary muscles and it becomes the iris in front of the eye creating the circular opening called the pupil. Behind the pupil, light passes through to the lens, which is flattened or fattened by ciliary muscles to vary the refraction. After light passes through the lens, it reaches the retina where it is absorbed by photoreceptors and converted into neural signals. These signals pass to the visual cortex via ganglion cells in the retina to continue processing in the brain (Boyce, 2014).





2.2.1 Rod and cone photoreceptors

The last layer of retina holds four different visual photoreceptors divided into two groups which are rods and cones. Rods are active at low light levels and are not involved in colour vision. Cones are active at higher light levels and they have three types. The sensitivity of three cones varies and their greatest sensitivity lies at 450, 525 and 575 nm wavelengths for short-, medium- and long-wavelength sensitive cones respectively as shown in Figure 2.2 (Boyce, 2014). Cones provide colour vision.



Figure 2.2 Spectral sensitivity of the eye for rods and cones (after Hunt and Pointer, 2011).

The region of the retina where cones are most densely packed is the fovea. Medium- and longwavelength sensitive (MWS and LWS) cones are mostly present in the central fovea. Shortwavelength sensitive (SWS) cones have higher density with an increasing eccentricity from the central fovea. The proportion of LWS, MWS and SWS cone density in the fovea is approximately 32:16:1. There are no rods in the fovea; they are located outside the fovea and their maximum density is at about 20[°] eccentricity from the fovea (Boyce, 2014). Figure 2.3 shows distribution of rod and cone photoreceptors across the retina. There are many more rods in the retina than cones. As the fovea is where the resolution of details occurs and other fine discriminations take place, cones play an important part in human vision even though their number is less than rods. However, rods are more sensitive to the light than cones.



Figure 2.3 The distribution of rod and cone photoreceptors across the retina (after Sekuler and Blake, 2014).

2.2.2 Colour vision

Human colour vision is trichromatic characterized by three cone photoreceptors. Figure 2.4 shows how colour and light-dark input is received through different combinations of three cone photoreceptors, directed to one achromatic non-opponent and two chromatic opponent channels to create visual response. Signals from cone photoreceptors are transmitted to ganglion cells and the output from different cone photoreceptors are compared to gain colour vision. According to Figure 2.4, MWS and LWS cones which provide the input for achromatic channel to be transmitted to the visual cortex by magnocellular (MC) ganglion cells. MC cells are concentrated in periphery and are faster to respond the changes in light levels. The two chromatic channels used the opponent inputs: MWS vs. (LWS + SWS) signals for red-green and SWS vs. (MWS + LWS) signals for blue-yellow channels. This information transmitted to the visual cortex by parvocellular (PC) ganglion cells. PC cells are dominant in fovea and parafovea, they are better at resolving details than MC cells and sensitive to colour (Boyce, 2014).





The intrinsically photosensitive retinal ganglion cells (ipRGC) shown in Figure 2.4 were only recently discovered and are a special type of ganglion cell using melonopsin as photopigments and having maximum sensitivity at 480nm to short-wavelengths. They are evenly distributed in

the retina outside the fovea. They have slower response to light than rods and cones. These types of photoreceptors are not included in the image processing of human vision. However, it has been found that they use the input from rod and cone photoreceptors. It is believed that the ipRGCs control pupil size (Berman, 2008).

2.2.3 Photopic, scotopic and mesopic vision

The sensitivity of four photoreceptors in retina changes depending on the light level. For luminance levels higher than approximately 5 cd/m², photopic vision operates with cone photoreceptors. Meaning that, at photopic light levels there is colour vision occurs and eye is able to refine the details with good resolution (Hunt and Pointer, 2011). At the luminance levels lower than approximately 0.005 cd/m², scotopic vision functions with only rod photoreceptors. Rods in scotopic vision only allow the shades of greys to be seen without any colour information and with low resolution of details. Mesopic vision is in between these two visions, functioning between 0.005 and 5 cd/m². Both rod and cone photoreceptors are active in mesopic vision (Boyce, 2014). Figure 2.5 shows the relationship of luminance and photoreceptors in photopic, scotopic and mesopic vision. Recommended office illuminances are between 300-500 lux in many countries. These levels lie in photopic region and so involve cone photoreceptors.



Figure 2.5 Relationship between luminance and photoreceptor function in the scotopic, mesopic and photopic regions (after Purves & Lotto, 2003).

2.3 Measuring Light

2.3.1 Terms used to define light

Section 2.2 explained how the human visual system works. Physiological reactions in the visual system start with light entering the eye. Light varies in many different ways. This section describes light properties and how they are measured.

Light is a flow of electromagnetic radiation. This radiation is visible to human at wavelengths between about 380 and 780nm. Spectral power distribution (SPD) describes the spread of radiation power within the visible spectrum. A graphical representation of the relative power at each visible wavelength is called SPD (IES, 2014). Information from the SPD of a light source can be used to determine the colour characteristics of the lit environment. Each light source has its own SPD depending on changes in power at different wavelengths. Figure 2.6 presents SPDs of full spectrum fluorescent (FS) and high pressure sodium (HPS) lamps.



Figure 2.6 Spectral power distributions of two light sources (SPD data from Fotios and Levermore, 1997).

The amount of light falling on a unit surface area is called illuminance (Tregenza and Loe, 2014). Illuminance measured in lumens per square metre (Im/m²), also known as lux. Light falling on surface will be transmitted, absorbed or reflected. The amount of light reflected in a given direction from a unit surface area is known as luminance, measured in candelas per square meter (cd/m²) (Hunt and Pointer, 2011). These two parameters defining 'how much light' are objective and repeatable measures as is essential for comparing light sources. The evaluations of both illuminance and luminance are precise, while some subjective and not precise measures of light also exist. A subjective, perceived evaluation of luminance depending on light-dark adaptation of human eye is called brightness. Brightness is an "attribute of a visual perception according to which an area appears to emit, or reflect, more or less light" (CIE-eILV, 2014). The main focus of this study is on the amount of light in a space rather than a localised area of an object. In such cases, the spatial brightness of the interior spaces is considered.

A draft definition of spatial brightness was developed by the Illuminating Engineering Society (IESNA), Visual Effects of Lamp Spectral Distribution committee. This committee has not yet published the definition, however it has been appeared in four publications (Fotios and Cheal, 2011; Fotios and Atli, 2012; Fotios et al, 2013; CIE, 2014) and it serves as a useful description for what is meant by *spatial brightness* in the current study:

"Spatial brightness describes a visual sensation to the magnitude of the ambient lighting within an environment, such as a room or lighted street. Generally the ambient lighting creates atmosphere and facilitates larger visual tasks such as safe circulation and visual communication. This brightness percept encompasses the overall sensation based on the response of a large part of the visual field extending beyond the fovea. It may be sensed or perceived while immersed within a space or when a space is observed remotely but fills a large part of the visual field. Spatial brightness does not necessarily relate to the brightness of any individual objects or surfaces in the environment, but may be influenced by the brightness of these individual items."

2.3.2 How much Light: Luminance and Brightness

The emitted energy in the form of radiation called radiant flux (Hunt & Pointer, 2011). Visual response to a radiant flux is measured with luminous flux which is providing a measure to light output from a source. Radiant flux is weighted, wavelength by wavelength, by the relative spectral sensitivity of the human visual system and luminous flux is obtained. The relative spectral sensitivity curve was first presented by Commission Internationale de l'Eclairage (CIE) in 1924 as Standard Photopic Observer, represented by V(λ). CIE Standard Photopic Observer described by the measurements using flicker photometry and step by step brightness matching methods in the central 2° of the fovea (CIE, 1978). In flicker photometry, the quantity of the chromatic light is adjusted to match with a reference light. The reference and adjustable light presented alternating temporally and the adjustment done until the minimum flicker is obtained. The step by step brightness matching, the observer matches two light sources in a bipartite field until they will have the same brightness (CIE, 1978). As these data collected with 2° in the central fovea, it is mostly using the responses from LWS and MWS cone photoreceptors (Lennie, Pokorny and Smith, 1993) and sensing the light level. However, it does not represent colour vision properly.

The CIE Standard Scotopic Observer was adopted in 1951. This one is dependent on the responses coming from rod photoreceptors, in which different colours are not seen, only a sense of the light level. Maximum sensitivities for standard photopic and scotopic observers can be seen in Figure 2.7 occurring at 555 nm and 507 nm, respectively.



Figure 2.7 Relative spectral sensitivity curves of photopic V(λ) and scotopic V'(λ) vision (spectral sensitivity data from CIE, 1978).

The relationship of power input and light output (luminous flux) is called luminous efficacy (K), measured in lumens/Watt (Tregenza & Loe, 2014). This function arises from at 555 nm 1W of radiant flux produces 683 lm, for both photopic and scotopic conditions. As 555 nm is corresponding to maximum luminous efficacy (K_m) for CIE Standard Photopic Observer is 683 lm/W staying unchanged. It equals to 1699 lm/W for CIE Standard Scotopic Observer.

Luminance is defined as the luminous intensity per unit projected area in a given direction and it is presented as the photometric measure of radiance by CIE. Thereby, an integrated radiance of a source ($L_{e,\lambda}$) weighted by the spectral luminosity V(λ) of the CIE Standard Observer (CIE, 1978), for photopic luminance (K_m=683 lm/W):

$$L_{V} = K_{m} \int_{360nm}^{830nm} L_{e,\lambda}(\lambda) \cdot V(\lambda) d\lambda$$
 Equation 2.1

It is also adapted to scotopic luminance (K'_m=1699 lm/W) as:

$$L'_{V} = K'_{m} \int_{380nm}^{780nm} L_{e,\lambda}(\lambda) \cdot V'(\lambda) d\lambda$$
 Equation 2.2

This luminance function is based on an addition in spectral efficiency functions. This additivity law is known as Abney's Law and it has good compatibility with flicker photometry. However, it doesn't work well with the colour sensitivity of human vision. What Abney's Law tells us is one light which appears yellow as a mix of red and green has the same luminance with sum of the luminances of the mixed red and green. When the brightness matching method will be used to test this law, the lights of red and green having equal brightness with a reference white light would be mixed to have yellow. As a result, yellow light would not have twice the amount of original reference white light (CIE, 1978). This shows that the method being used to obtain luminous efficiency function influences the function itself and can minimise human eve sensitivity to chromatic channels. As it was mentioned in Section 2.2.2 human vision uses the information from chromatic channels as well as achromatic channel which are mostly considering the light level. According to this limitation, it can be said that luminance which is defined by $V(\lambda)$ cannot always be representative of how bright a stimulus looks. Here occurs a difference of how something is actually bright (according to its luminance) and how bright a person perceives it, which is related with different characteristics of either SPD of a light source or the environment.

The phenomenon known as Helmholtz-Kohlrausch effect describes how the chromatic channels contribute to perceived brightness. As the colourfulness of a colour increases, there is a tendency to see it brighter in photopic vision, e.g. if a red light compared side by side with a white light of the same luminance, the red light looks brighter. This considers the chromatic adaptation of human eye and can be related with the trichromatic theory of cone photoreceptors which transmit the information not just with achromatic channels and also with two opponent chromatic channels of blue-yellow and red-green (Yaguchi and Ikeda, 1983).

As a result of incompatibility between additivity essential for CIE Standard observer and nonadditive nature of trichromatic channels in human visual system, there is this difference occur between luminance and brightness. With the knowledge of this difference, this thesis will focus on the perceived brightness, spatial brightness in particular. The next section explains how a light source of different SPD affects spatial brightness and proposed effects of metrics that can be derived from SPD on spatial brightness.

2.4 Measures of Colour

2.4.1 Chromaticity

While moving from photometric quantities to colorimetry system providing predicting perceptual matches of colour, there are again measurements from colour matching involved. In CIE colorimetry system, there are three colour matching functions which can be considered as another form of standard observer. These functions are mathematical calculations to identify the position of a colour in the CIE colorimetry system with x, y and z chromaticity coordinates so that colours having same spectral sensitivity are positioned in the same point. These x, y and z

values can be obtained using spectral power distribution. By multiplying three colour matching functions $x(\lambda)$, $y(\lambda)$ and $z(\lambda)$ with spectral distribution of a light source, wavelength by wavelength, tristimulus values of X, Y and Z can be obtained. Then, the individual value divided by the sum of all three will give the chromaticity coordinates of the light source (e.g. x = X / (X+Y+Z)). Two of the chromaticity coordinates are enough to position the colour in the chromaticity diagram such as x and y. As spectral power distribution of a light source considers colour information and can be used to supply luminance, it appears to be a source to quantify spatial brightness. Using spectral power distribution (SPD), some metrics to identify the characteristics of a light source are defined to discard the complexity of CIE colorimetry system. Some of these metrics do not fully describe light source SPD, as they reduce a complex spectral distribution to a single index, however they are widely known characteristics and frequently being used by the manufacturers, researchers and lighting designers.

2.4.2 Correlated Colour Temperature and Colour Rendering Index

The CIE colorimetry system is the most complete method to quantify colour. However, this system is too complex to be used in lighting industry. Instead there are two widely used singlenumber metrics using CIE colorimetry system known as correlated colour temperature (CCT) and colour rendering index (CRI). The colour appearance of a white light source having chromaticity coordinates close to the Plankian locus is quantified with its CCT. This coordinate originates from spectral emission of a black body and its radiant function represents its temperature. By using the isotemperature lines, which are plotted from blackbody locus to the chromaticity coordinates of the source, its CCT can be obtained. High CCT values (e.g. 6500 K) appear cool and low CCT values (e.g. 2700 K) appear warm. Two light sources can have same CCT however; they may have different chromaticities and thus may appear very different to the eye. Table 2.1 shows CCT and CRI values of the FS and HPS lamps shown in Figure 2.6.

Lamp	CCT (Kelvin)	CRI
Full spectrum fluorescent	5900	92
High pressure sodium	1800	-2.5

Table 2.1 CCT and CRI values of FS and HPS lamps (Fotios and Levermore, 1997).

CIE defines CRI with 14 standard test colours to find out the effect of a light source on surface colour in comparison with a reference light source of the same CCT. How well are these 14 standard test colours are rendered with a light source is defined according to a reference light source. The calculations are done with defining the position of a surface colour in colour space under the reference light and the light source of interest, and then the difference between these two positions is expressed (CIE, 1995). The smaller this difference, higher the CRI. It can be told as the light sources ability to show object colours 'natural' when compared to the reference

source. The maximum CRI value is 100 and CRI value decreases, as the colour rendering of the object gets unnatural under the light source.

2.5 Spectral Power Distribution (SPD) and Spatial Brightness

2.5.1 Potential metrics of Spatial Brightness

This section focuses on defining these potential metrics of spatial brightness, which were proposed in past studies. According to the information gained above on human eye physiology, these metrics can be explained under two categories; the ones located toward the models which short-wavelength (blue) contribution to brightness with rods and SWS cones (e.g scotopic luminance/photopic luminance (S/P)) and the other group considering colour contribution (e.g. gamut area (GA) and trichromaticity).

2.5.1.1 Models of short-wavelength contribution

The S/P ratio is the ratio of the photopic (P) and scotopic (S) luminances (Equation 2.2 / Equation 2.1) of a source and this ratio was proposed by Berman et al (1990) as a metric for brightness at photopic levels. The concern of Berman et al (1990) was the potential contribution of rod photoreceptors to photopic vision. As it was explained in Section 2.2.4 CIE Standard Photopic Observer consists of information from cones in central 2° of the fovea neglecting any contribution of either rod or SWS cone photoreceptors in the periphery. According to the findings of higher perceived brightness under higher S/P in their study, Berman et al (1990) determined that there occurs a scotopic vision with brightness in full field view. Thus, they proposed to consider sensitivity of scotopic vision with brightness lumens model in order to gain information on brightness perception in addition to LWS and MWS cones. Spaces lit by two different lamps of equal brightness lumens (Equation 2.3) would appear equally bright.

Brightness Lumens =
$$P(S/P)^{0.5}$$
 Equation 2.3

Later on, this effect of scotopic vision on brightness perception presented by Berman et al (1990), was discussed as a potential SWS cone contribution in Fotios and Levermore (1998) intended to be more consistent with physiological framework of photopic vision and proposed to be considered using SWS cone/photopic luminance (SWS/P) as an alternative to S/P. In Rea, Radetsky & Bullough (2011), it was suggested that mesopic brightness can be modelled by the sum of V(λ) and the SWS cone response.

Recently, with the new photoreceptor type called intrinsically photosensitive retinal ganglion cells (ipRGC) being discovered outside the central fovea with a peak sensitive around 480 nm, this scotopic component of S/P reported as a proxy for the response of the ipRGC rather than the rods (Berman, 2008). In their study done by both mice and human Brown et al (2012)

reported the melanopsin photopigment that are in ipRGC contributed to perceived brightness. Mice which were rodless and coneless were picking up green light to be brighter than red light when they arranged to have equal luminance. Additionally, in the experiment done with human beings, reference stimulus with melanopsin 0%, there were no differences in brightness distinguished. These results of Brown et al (2012) were providing evidence of the scotopic effect would be originating from ipRGC cells. Berman (2008) determined the circadian regulated effect of ipRGC as a replacement of S/P with C/P ratio. He mentions about their relationship as:

$$S/P = (0.66C/P)^{0.74}$$
 or $C/P = (1.37S/P)^{1.35}$ Equation 2.4

2.5.1.2 Models of chromatic contribution

In the second group of potential metrics, CCT and CRI are well known descriptors of the colour appearance of illumination and illuminated surfaces as explained previously. Fotios (2001) suggested a simultaneous application of CCT and CRI in order to gain a reliable prediction of which of two stimuli is brighter.

Gamut area (GA) was suggested in a previous study as another metric to correlate better with judgements of visual appearance of a lit scene using a matching task than did CCT or CRI (Boyce, 1977). Gamut area is a measure of the colour differences between a range of coloured surfaces, with a larger gamut area implying greater saturation of surface colours, and thus that the lighting is brighter (Boyce, 1977). Figure 2.8 shows gamut areas of two light sources (FS and HPS) as an example.



Figure 2.8 The colour gamut areas of two lamps (SPD data shown in Figure 2.6 from Fotios and Levermore, 1997).

Gamut area is derived from the area contained within the irregular octagon enclosed by the chromaticity coordinates of the eight colour samples used in the CIE General Colour Rendering Index (Equation 2.5). Although Boyce (1977) originally used u,v chromaticity from the 1960 Uniform Chromaticity Scale (UCS) diagram to determine gamut area he subsequently (Boyce,
2003) suggested using u', v' chromaticity from the CIE 1976 UCS diagram, and that is what was used in the current study.

GA = 0.5
$$[(u'_1v'_2 - u'_2v'_1) + (u'_2v'_3 - u'_3v'_2) + ... + (u'_7v'_8 - u'_8v'_7) + (u'_8v'_1 - u'_1v'_8)]$$
 Equation 2.5 where u'_n, v'_n = chromaticity of colour sample n

Another metric was developed by Fotios and Levermore (1998) as cone surface area (CSA) an extension of gamut area. It is suggested that the volume of a colour solid would correlate with visual clarity assessments, the 3-dimensional nature of which would correspond with the 3-dimensional nature of colour appearance, i.e. three tristimulus values or three descriptors (hue, saturation, brightness) needed to describe colour appearance. CSA is the total surface area of a regular cone having its base on the CIE 1976 u'-v' uniform chromaticity diagram (Equation 2.6). The base of this cone is assumed circular, to simplify the calculation of surface area, and of equal area to the octagonal colour gamut in the u'-v' diagram. The perpendicular height of the cone is given by the w' chromaticity of the light source. Since w' changes over the chromaticity diagram, CSA varies with chromaticity, and is therefore sensitive to both the colour rendering and colour appearance properties of a spectrum.

	CSA	= area of base + curved surface area	Equation 2.6
		= GA + πrL	
where	r	= radius of base of cone = $\sqrt{(GA/\pi)}$	
	L	= length of slope of cone = $\sqrt{(r^2 + (w')^2)}$	
	w'	= perpendicular height of cone = 1 - (u' + v')	

There were two equivalent brightness equations were developed by Cowan and Ware (1983) and the CIE (2011) the supplementary system of photometry were also considering chromatic contribution.

The Cowan and Ware equation is shown in Equation 2.7. This equation was determined from those of the 29 brightness matching studies they collected which met criteria including photopic adaptation and field sizes of 0.5° to 2.0° with data obtained using larger or smaller fields being excluded. Clearly this does not match the data suggested to be pertinent for spatial brightness, for which a field of at least 20° degrees has been proposed (CIE, 2014). The data used by Ware and Cowan to derive Equation 2.7 included results from Alman (1977), Alman et al (1983), Booker (1978) and Thornton et al (1980): of these, the first three have been identified as inappropriate evidence for spatial brightness which is explained in Chapter 3.

$$\begin{array}{rl} {\sf L}_{\sf eq} & = {\sf L}.10^{\sf C} & {\sf Equation}\ 2.7 \\ {\sf where}\ {\sf C} & = 0.256 - 0.184y - 2.527xy + 4.656x^3y + 4.657xy^4 \\ {\sf x},y & = {\sf CIE}\ 1931\ {\sf chromaticity}\ {\sf coordinates} \end{array}$$

CIE Equivalent luminance (CIE, 2011) (Equation 2.8) was developed to describe the brightness of a light or an object at any level including mesopic levels for a 10° field centrally fixated, so is

not ideal for spatial brightness, and for monochromatic reference light with a frequency of 555nm.

	L_{eq}	$= (L)^{a}.(L')^{1-a}.10^{C}$ Equation 2.8	
where	L	= CIE 2° photopic luminance	
	Ľ'	= CIE scotopic luminance	
	а	= achromatic adaptation coefficient.	
	С	$= a_c.f(x,y)$	
	ac	= chromatic adaptation coefficient	
	f(x,y)	$= 0.5 \log (-0.0054 - 0.21x + 0.77y + 1.44x^{2} - 2.97xy + 1.59y^{2} - 2.11zy^{2}) - \log (-0.0054 - 0.21x + 0.77y + 1.44x^{2} - 2.97xy + 1.59y^{2} - 2.11zy^{2})$	g y

One last metric examined to find out about chromatic effect on brightness perception was trichromaticity. The three spectral channels were already mentioned on Section 2.2.2. In their study Houser, Tiller and Hu (2004) hypothesized that a SPD having closer match with the spectral input of human vision can improve brightness perception. They compared lamps with SPDs having peak –wavelength at 450, 545 and 610 nm (naming them as prime-colours) and reported perceived brightness differences originated from chromaticity differences.

2.5.2 Brightness: Evidence for Effects of Lamp SPD

65 studies have investigated SPD and spatial brightness. 52 of these reported that SPD effects spatial brightness, 6 reported no effect, and 7 studies were not clear about their findings. In all of these studies which investigated SPD and spatial brightness, a range of different metrics have been used to quantify the magnitude of any effect. Most commonly considered metric in these studies was CCT. Akashi and Boyce (2006) had 33% of illuminance reduction by providing higher CCT in a field study of office lighting. Similarly, perceived dimness was decreased with increasing CCT from 2700K to 6300K at a constant illuminance in Boyce and Cuttle (1990). In contrast, Davis and Ginther (1990) found no CCT effect on brightness in a full size laboratory study using same experimental method with these studies. Hu, Houser and Tiller (2006) were using another experimental method to examine CCT effect on spatial brightness, and they reported that even though lamp SPD was related to brightness, CCT was too limited to characterize this relationship with sufficient accuracy. Vienot et al (2009) proposed a model of brightness for photopic levels that uses lamp CCT to quantify the effect of lamp SPD and demonstrated a trade-off with decreasing illuminance and increasing CCT. Besides, they conducted their study with lamps of high CRI which implied that they considered CRI as a predictor of spatial brightness as well. However, they focused on only one metric as in most of these studies. This attempt might be inconvenient as it may give false impression of that one metric described the response of the participant, whereas another feature of the lighting condition was the main reason of the effect.

Berman et al (1990) also focused on one metric of spatial brightness, S/P ratio. In their study a trade-off to reduce the room illuminance with high S/P lamp was obtained. The lamp having higher S/P ratio was perceived as the brighter in spite of lower luminance. However, another

study using one of the lamps with both high CCT and high S/P ratio to compare with another lamp of low CCT and low S/P ratio, found no effect of these metrics on spatial brightness (Houser, Fotios and Royer, 2008). Boyce (1977) also tested multiple metrics of CCT and CRI with a different experimental method and he reported CCT as not a good predictor of brightness. He proposed GA to be used as a predictor of spatial brightness instead of CCT and CRI. He reported that lamps with large GA appeared more saturated and thus were perceived as brighter.

In one of the two studies conducted by lamp spectrum having three peaks in wavelengths (the peak-wavelengths were called prime-colours) an effect was reported on spatial brightness (Houser, Tiller and Hu, 2004). The lamp having higher peak at long-wavelength perceived brighter by the participants in Houser, Tiller and Hu (2004). In Royer and Houser (2012), they confirmed that light stimuli measured to be identical according to CIE photometry and colorimetry do not appear equally bright or the same colour. They also found that S/P ratio, Cirtopic to Photopic ratio, prime colour theory, correlated colour temperature, photometry, colour quality metrics (including gamut area), linear brightness models, and colour appearance models all failed to predict or correctly order the difference in the participants' perception of brightness.

There are other studies, which didn't specifically focus on a metric but compared different lamp spectra. Vrabel et al (1998) compared 5 different lamps, high grade halophospor (HGHP) and T8 lamps were reported to be brighter than the other 3 lamps. Similarly, Fotios and Gado (2005) indicated SPD effect on spatial brightness, in which Verivide lamp was brighter than warm white fluorescent (WW). According to previous studies, there is evidence that SPD effects spatial brightness. However, there are some studies which showed contradicting results. One reason for this difference is the methods used in these studies. These studies were using different methods and modes of experiment to explore the SPD effect on spatial brightness. Chapter 3 focused on the experimental methods used in brightness tests with a detailed review of SPD and spatial brightness studies.

2.6 Summary

Until now, how light affects the human visual system and how the visual information gained through photoreceptors to process the image were determined. Its representation in photometry and colorimetry were identified in order to understand why the perceived brightness and the measured illuminance differ from each other. Then, the potential interaction of SPD and spatial brightness (brightness perception defined as the amount of light in a space) were explained with potential tools for predicting how these two were linked with each other. Although, appearance models known as opponent-colour theory (Hunt and Pointer, 2011) are likely to be an accurate one for spatial brightness, it is too complex for practical use, for this reason the current study

didn't include this model. It is based on the characteristics of the photoreceptors over a limited range of conditions and the most complicated to implement. For the current study, complexity is an important criterion because it may be a barrier to implementation. Similarly, prime-colour theory of Houser, Tiller and Hu (2004) was not an easy one to use metric for the current study as it does require more than one index to be applied. Therefore, the metrics used in lighting practice today that give simple descriptions of an SPD, such as CCT, CRI, GA, and S/P (Berman, 1990; Boyce, 1977 and Fotios and Levermore, 1998) will be explored in the current study. While these do not fully describe light source SPD as they reduce a complex spectral distribution to a single index, they are established and widely known characteristics and so would be simple to implement.

A range of previous studies exploring these potential metrics will be examined in Chapter 3 as classified according to the experimental method they used. As mentioned earlier and also can be seen in the review on Chapter 3 that GA is considered to be a more precise metric for judgements of spatial brightness than CCT and CRI. Besides, CSA appear to provide a little improvement to the information gained from GA and since its proposal would require establishment of a new metric rather than adopting a widely used existing one, it will be dropped from further analyses. Similar situation occurs with SWS/P when it is set with S/P ratio at photopic vision. Therefore, S/P ratio will be kept to continue with further analysis. The equivalent brightness equation by Cowan and Ware included data of at most 2° visual field and ignoring either scotopic or ipRGC contribution. Besides, from Chapter 3 it can be seen that there were data from some unreliable studies included in their analysis; this is also leaving if the equivalent brightness equation can predict spatial brightness questionable. Both Cowan and Ware and CIE equivalent brightness equations considered colour appearance, however they didn't include CRI to their models.

Accordingly, the questions for this study may be summarized as:

1. Is reducing light level possible while maintaining the spatial brightness? The existing evidence of electrical lamps, suggests SPD can be an influencing factor to have a trade-off with luminance levels and still provide sufficient brightness. Different models with characteristics of SPD and luminance were indicated in the past studies, however the details about how these metrics relate with brightness demonstrates differences in different studies according to the experimental method and the environmental features used.

2. Do the different techniques commonly used to assess spatial brightness yield comparable results?

Results from the past studies had different experimental methods using different presentation techniques, environmental conditions and stimuli yield different results and sometimes proposed different metrics to be effective on spatial brightness (Davis and Ginther, 1990; Berman, 1990).

Detailed investigations on how these methods used and what are the outcomes are discussed in Chapter 3 and 4 to figure out how the future experiment of this study will be structured.

3. *Can S/P and GA be used to predict a model of spatial brightness?* From potential metrics proposed in the past studies, these two metrics appear to provide a good estimate of brightness even though they are reducing the whole SPD to one index. Berman et al (1990) demonstrated S/P effects on spatial brightness. In a different study Houser et al (2008) indicate S/P ratio as not effecting spatial brightness. Gamut area was proposed as a predictor of spatial brightness by Boyce (1977). No other known studies indicating GA is not a good predictor for spatial brightness at photopic levels. All the other proposed metrics had their own limitation to estimate the spatial brightness as ignoring short-wavelength and chromatic contribution and visual field size. S/P and GA were indicated to have their bases in physiology of human eye and can be worth further investigation to find out their applications for spatial brightness.

4. Do the effects of lamp S/P and GA interact with each other to predict spatial brightness? As previously mentioned, there are many components that are understood to contribute to shaping spatial brightness in human vision. Fotios (2001) mentioned that when CCT and CRI are considered together, they may give a reliable prediction of brightness. Similarly, with scotopic contribution from S/P and chromatic contribution from GA to the model, their interaction may give a reliable estimate of spatial brightness as well as operating individually.

Chapter **3**

3 Brightness: Evidence for effects of lamp SPD

- 3.1 Introduction
- 3.2 Experimental Methods
- 3.3 Category Rating Studies
- 3.4 Matching Studies
- 3.5 Discrimination Studies
- 3.6 Adjustment Studies
- 3.7 Summary

3.1 Introduction

This chapter examines the existing evidence for lamp SPD effects on spatial brightness in photopic conditions. The studies reviewed use one of four psychophysical methods namely category rating, matching, discrimination and adjustment. This review of past studies was carried out by giving attention to experimental design, the criteria which suggest whether the data are considered credible (e.g. by counterbalancing spatial position in a side-by-side test) and the factors which place the results in a context (e.g. evaluation mode).

3.2 Experimental Methods

In past studies there are unique sets of experimental conditions including the SPDs, experimental procedures, evaluation modes, visual scenes and field sizes. What it is necessary to know is whether these differences matter. For example, the results from the discrimination study of Berman et. al. (1990) disagree with the results in the category rating study of Boyce and Cuttle (1990) which was indicating an effect of SPD on spatial brightness. As shown in Table 3.1 there were experimental design differences between these two studies including lamp SPDs and evaluation modes. The question is which of these differences led to different conclusions about the relationship of lamp spectrum and spatial brightness.

Table 3.1 A comparison of psychophysical methods used in Berman et al (1990) and Boyce and Cuttle (1990)

Design factor	Boyce and Cuttle (1990)	Berman et. al. (1990)
SPDs	3 SPD (different CCTs)	2 SPD (equal chromaticity) (different S/P ratios)
Procedure	Procedure Category rating	
Evaluation mode	Separate	Rapid sequential
Visual scene	Real office room	Room
Field size	Full field	Full field
Effect of SPD	NO	YES

A wide range of past studies were analysed in order to get information about how these experimental designs really work. These past studies of spatial brightness are discussed according to the experimental procedures that were used and requirements for a controlled and reliable study are listed. Following the work of Commission Internationale de l'Eclairage (CIE) TC 1-80, there are four basic types of procedures mentioned for spatial brightness studies: adjustment, matching, discrimination and category rating. The relationships between these procedures are shown in Figure 3.1. Further possible methods for evaluating visual scenes, such as magnitude estimation (assigning a number to the stimuli to describe how intense it is or so; there might not be any limit to the range of numbers, whole numbers, decimals or fractions

can be used) (Stevens, 2008), have been used rarely if at all in past research of spatial brightness. Therefore, these rarely used experimental methods are not included in this study.

	Absolute measurement (No external reference present)	Relative measurement (Presence of an external reference)
Passive No interaction with stimulus	Category Rating	Discrimination
Active Interaction with stimulus	Adjustment	Matching

Figure 3.1 Basic procedures for measurement of spatial brightness (CIE, 2014).

Before a detailed explanation of these four experimental methods, four modes of observing the stimuli in brightness experiments should be clarified; separate, simultaneous, (rapid) sequential and in succession. In separate presentation, stimuli are observed and evaluated individually. One stimulus is isolated from any other stimuli or any reference standard. In simultaneous presentation, there are more than one stimuli presented at the same time in adjacent spatial locations. Generally, it is limited to two stimuli in which one of them is being compared with the other. When it is presented as rapid sequential each stimulus is shown one by one with short periods generally in 3-5 seconds and they are being compared with each other. In rapid sequential mode, each stimulus can be presented more than once, for the participants to complete the comparison. In the last mode, the stimuli are being presented in succession at the same place and each stimulus is observed and evaluated separately (no comparison with any other stimulus exists). The main differences between these modes are the chromatic and light adaptations (Fotios, 2006).

While applying these experimental methods and modes, in order to process the changes in the level and colour of the illumination a process of adaptation occurs in participants visual system (Hunt, 1998). The sensitivity to adapt to the changing illumination differs with the spectral power distribution, which is called chromatic adaptation. As a result of this adaptation, despite the changes in illuminant SPD the colours of objects will tend to appear constant. This colour constancy is due to the limits of the level of adaptation, although the illuminant SPD has an effect on the perception of brightness. According to this, by matching two different light conditions simultaneously, the colour appearance will give different results than evaluating the conditions separately. When two stimuli differ from each other in side by side matching,

participants will not have complete adaptation to either of them. The cone photoreceptors will reach their maximum sensitivity in 10-12 min while 60 min is needed for rod photoreceptors to become fully adapted (Boyce, 2003). According to Fairchild and Reniff (1995), 60% of adaptation is reached after 5 sec. In most of the rapid sequential studies, stimuli are presented in 5 sec or less which means that the participants are not fully adapted to the presented lighting conditions (Berman et al, 1990; Houser et al, 2009; Royer & Houser, 2012; Vrabel et al, 1998). Significant difference between rapid sequential presentation and simultaneous side-by-side presentation is found by Foster et. al. (2001). A higher degree of colour constancy and lower variance between participants in rapid sequential evaluation than simultaneous evaluation were obtained. When the stimuli presented separately, the adaptation duration can be longer than the other modes. Participant might be observing the lighting condition for minutes, hours or even days depending on the study. In such cases of longer observation of lighting condition, a full adaptation to both colour and light level will occur. Accordingly, the brightness effects might be expected to differ less, however, there are studies which had 15-20 minutes of adaptation and found significant effects of SPD on spatial brightness (Boyce and Cuttle, 1990).

3.3 Category Rating Studies

The focus of this section is the category rating method used in SPD and spatial brightness studies. The category rating method is explored here in detail because there is already an ample amount of reviews on matching, discrimination and adjustment methods in the current literature whereas there is not sufficient reviews on category rating method in the literature (Fotios, Houser and Cheal, 2008, Logadottir, Christoffersen and Fotios, 2011). Category rating studies were reviewed by Fotios and Houser (2009) and the recommended measures were tentative and required further validation: current section presents a critical review of the study of Fotios and Houser (2009) and chapter 4 presents two pilot studies of category rating issues.

In category rating studies an illuminated space is presented to the subject in order to evaluate it by using rating scales. The presentation can be either in succession, or as separate conditions (Vienot et al, 2009; Akashi and Boyce, 2006). As shown in Figure 3.2 two types of scales can be used to evaluate the scenes namely semantic differential scale or Likert scale. In semantic differential scale, the brightness is evaluated along the bright-dim axis which is specifying an evaluation range between the end points. On the other hand, Likert scale is asking for the agreement of the participant with perceived brightness of the room along the given scale. In such cases, end points of the given range are not defined as properly as semantic differential scale. For instance in Figure 3.2, Boyce and Cuttle (1990) gave the statements of *very much* and *not at all* for opposite end points and they were asking for the evaluations of the brightness and dimness of the room. The paradox with such evaluation is that while they were supplying a concurrent validity of the experiment by two separate but related evaluations of the same

stimulus, they allowed participants to make ratings with undefined categories like very much and/or not at all.

As it can be seen from Figure 3.2, the rating categories were used with a neutral point in the middle. Another way to apply a category rating procedure can be without a neutral point so that the participant wouldn't be allowed to give equally bright answer, which is referred as forced-choice method.

Vrabel, E	Berneck	er & Mis	strick, 19	98					
Question	n: Rate t	he scene	accordir	ng to the	scale giv	en.			
	Bright	1	2	3	4	5	6	7	Dim
(Semantic differential scale)									
Boyco 8	Cuttle	1000							
	· Mark		assion o	f the light	ing of thi	s room o	n the foll	wing sca	
Question	I. Walky		6331011 0	i ule ligiti	ing of th	5 10011 0		wing sca	103.
		Verv			Ν	lot at			
		Much							
		wuch				all			
	Bright	1	2	3	4	5			
	Dim	1	2	3	4	5			
(Likert scale)									

Figure 3.2 Examples of semantic differential and Likert rating scale used in past studies of spatial brightness.

Two types of experimental designs, which are repeated measures and independent samples, can be used in category rating studies. In repeated measures, more than one stimulus is presented in succession and the participant rates each condition separately. Independent sample procedure involves just one condition to be evaluated with the given rating scale.

Category rating studies can be conducted either in booths or in real life conditions like field and full size lab studies. Using different visual scenes may provide more realistic environments and different field sizes.

Following review considers the data collection, analysis methods and their presentation in the studies with sufficient details. Six criteria for data collection proposed by Fotios and Houser (2009) in order to reduce bias:

(1) Randomised or counterbalanced stimulus order (repeated measures only)

(2) Equalized number of stimulus magnitudes with the number of points in the response range

- (3) Valid data analysis with precise data reporting
- (4) Stimulus range anchored to the response range
- (5) Even number of response categories
- (6) A null condition trial

3.3.1 Randomised or counterbalanced stimulus order

When the stimulus is presented in repeated measures the subject evaluates the scene according to both present and previous stimuli creating the order effect (Poulton, 1979, Fotios and Houser, 2009). Which means that ratings awarded to a stimulus may be biased by ratings awarded to stimuli observed earlier in the experiment (Flynn et. al., 1979, Gescheider, 1997). This effect generally originates from the subject's desire to be consistent all through the questions or the scenes that were being evaluated (Schuman and Presser, 1996). In order to avoid possible biases, either a randomized and counterbalanced order while presenting the stimuli is required or the independent samples method can be used (Fotios and Houser, 2009, Poulton, 1989). In the case of independent samples, different groups of test participants will be assigned to different stimuli and each subject evaluates only one stimulus, which avoids the order effect (Akashi and Boyce, 2006). However, while using independent samples, it must be kept in mind that the differences in perception might be caused by the participant not just by the stimuli. In such kind of experiments, different groups of participants evaluate different stimulus, therefore any effect detected might originate from the diversity of participants not the variable factor of the visual scene. In the current study using either randomised order or independent samples is one of the essential requirements.

3.3.2 Number of response points

Grouping Bias

According to Miller (1956), the human brain starts to have error while distinguishing between more than six items, and after eight items it starts to recode them by grouping to facilitate remembering the items. Thereby, the items that are similar in some important aspects are grouped together and the minor differences between them cannot be found with more than eight stimuli. Besides, as shown in Figure 3.3, the range of the stimuli magnitudes being unequal with response range also causes minor differences not to be distinguished and to be grouped, especially when the response range is smaller than the stimuli magnitude (Fotios and Houser, 2009, Poulton 1989). A point raised by Poulton (1989) and Green and Rao (1970) on response categories was the number of stimuli and rating points of response scales covering identical ranges in order to avoid grouping bias. This makes the subjects' task more precise and may help to detect the differences more accurately. According to the analysis of Green and Rao (1970), the response range should cover at least six points and increasing it to more than six points provided a little more information. For the current study, using equal numbers of stimulus



magnitudes with the number of points in the response range is included as another essential requirement.

Figure 3.3 Illustration of Grouping Bias. In illustration b, reducing the response range from 5 to 3 forced participants to group 2 different sizes of stimuli into same response point.

Even or Odd Number of Categories

Poulton (1989) also suggests that with a middle point in the response range, contraction bias may occur (see Figure 3.4), i.e. when people tend to select a response category which is too close to the centre of the response range (Poulton, 1989). In such cases, participants' response is either too small for the stimulus which is above the centre of the range or it is too large for the stimulus that is below the centre. In the studies in which a middle value is explicitly offered, people are much more likely to select the middle and have a tendency to avoid using the ends of the scale (Nowlis et. al., 2002, Bishop, 1987, Presser and Schumann, 1980). This choice generally arises from the ambivalent attitude of the participant towards the other alternatives (Nowlis et. al., 2002). Even or odd numbered category ranges affect the mean rating and distribution of the judgements. In some cases even numbered categories supplied significant results towards one end of the scale, while odd numbered categories produced neutral results for the same questions (Dawes, 2008, Nowlis et. al., 2002). Besides, different results occurred by odd and even numbers of response categories according to the issue that has been asked to the participant (Moors, 2008).



Figure 3.4 Illustration of Contraction Bias. a) Stimuli assessment spread across whole response range. b) Stimuli assessment converges towards central region of response range.

In order to find out if the response range creates any difference in the subject's judgement, a new study was carried out (see Chapter 4). In this study it is found that while the distribution of the responses have changed, the number of response categories did not affect the central tendency of opinion. However, according to the results of Akashi and Boyce (1990), an effect of SPD was detected with one experiment using two category rating points but no effect was found with a second experiment having five rating points.

All these evidences indicate an ambiguity whether the category numbers effect the participant judgements. Therefore, in order to further investigate whether an even numbered category range makes a difference in brightness evaluations or not, this criterion is considered in the desirable requirements for the current study, which are specified below. This new study on response categories is explained in detail in Chapter 4.

3.3.3 Anchoring

The method of defining the response range by displaying some of the stimuli before the experiment is mentioned as anchoring (LeBoeuf and Shafir, 2006; Poulton, 1989). People tend to underestimate the stimuli with high intensities, which create the contraction bias (Gescheider, 1997). Anchoring the stimulus range provides a reference in evaluating the items and avoids contraction bias (Fotios and Houser, 2009, Schumann and Presser, 1996). Anchoring can be done either by presenting all the response levels in the beginning of the experiment or

presenting the stimuli beyond the two ends of the stimulus set (Poulton, 1989). The method of anchoring carries great importance, especially for the couple of initial judgments by providing initial frame of reference as well as the entire experiment. This criterion is covered in desirable requirements for the current study.

3.3.4 Valid data analysis and reporting

For a credible study, in which there is data analysis and statistical calculations, quantitative information needs to be provided in the reports. Ideally, presenting the mean ratings with standard deviations provides some useful information by enabling to run some statistical tests in order to understand the effects. Reporting all these information in a structure with an understandable manner such as mentioning mean ratings with the name and the results of suitable statistical analysis, which was applied to the data, also helps to draw conclusions about the SPD effect on brightness. The clear interpretation of the data analysis and reporting is the third essential requirement for the current study. Other than that, the data from null-condition (if there is any) is also important for the internal reliability of the study.

3.3.5 Null-condition trial

The last criterion is a null-condition trial, which acts like a control group for the whole study by giving information about the internal reliability. According to the design of the study, a condition can be repeated during the experiment as null condition. For instance, Akashi and Boyce (2006) asked participants to evaluate the light settings of four rooms three times with varying illuminance and CCTs. One of the four rooms had the same light setting for all three evaluations as control group. However, most of the reviewed studies do not include any null-condition trials; therefore it is in desirable requirements for the current study.

When the list of previous category rating studies was done and the requirements were considered, almost none of the studies meet half of these criteria. Therefore, two sets of requirements were assigned as essential and desirable (see Table 3.2).

Table 3.2 Essential and desirable requirements suggested to determine credible data in category rating (Fotios and Houser, 2009).

Essential Requirements	Desirable Requirements
randomised or counterbalanced stimulus order	anchored stimulus range
equalized number of stimulus magnitudes with	even number of response
the number of points in the response range	categories
 appropriate data analysis and informative reporting 	null condition trial

The credible data from past category rating studies are determined as those meeting the proposed essential requirements. Three criteria are assigned as desirable since just Boyce (1977), Davis & Ginther (1990) and Flynn & Spencer (1977) have included null condition trials; Akashi & Boyce (2006) have included both null-condition and even numbers of response range and only Vrabel et al (1998) had anchored stimulus range. If these criteria would essentially be considered for the review, only 5 studies would be included and less data would be available for the review.

3.3.6 Studies Using Category Rating Method

This section of the research will review the studies using category rating method to evaluate lighting conditions under different types of lamps at photopic levels. The requirements of category rating method that has been mentioned earlier in section 3.3 are considered in order to identify credible studies, which give robust evidence on the effects of lamp spectrum on spatial brightness.

Thirty category rating studies were evaluated regarding the three essential requirements (randomised or counterbalanced stimulus order, equalized stimulus magnitudes with response range and quantitative data). Table 3.3 shows ten of the thirty studies meet these requirements and supply robust evidence to investigate the relationship between light spectrum and spatial brightness (Akashi and Boyce, 2006; Boyce, 1977; Boyce and Cuttle, 1990; Boyce, Akashi, Hunter and Bullough, 2003; Davis and Ginther, 1990, Flynn and Spencer, 1977; Han and Boyce, 2003; Piper, 1981; Vienot, Durand and Mahler, 2009; Vrabel, Bernecker and Mistrick, 1998). Appendix A shows the whole list of studies included in the review.

	Essential requirements			Desirable requirements		
Study	Stimulus order randomised	Stimulus magnitudes equalized	Quantitative data reported	Stimulus range anchored	Nb response categories even	Null condition (control
	or counterbalan ced	with response range				group) trial
Akashi &	N.A	\checkmark	\checkmark	Х	\checkmark	\checkmark
Boyce, 2006	N.A	\checkmark	\checkmark	х	х	\checkmark
Boyce, 1977	\checkmark	\checkmark	\checkmark	х	х	\checkmark
Boyce et al, 2003	\checkmark	\checkmark	\checkmark	х	х	х
Boyce & Cuttle, 1990 (exp 2)	\checkmark	\checkmark	\checkmark	х	х	х
Davis & Ginther, 1990	\checkmark	\checkmark	\checkmark	х	х	\checkmark
Flynn & Spencer, 1977	\checkmark	\checkmark	\checkmark	х	х	\checkmark
Han & Boyce, 2003	N.R	N.A	\checkmark	х	N.A	х
Piper, 1981	\checkmark	\checkmark	\checkmark	\checkmark	х	х
Vienot et al, 2009	$\mathbf{\mathbf{\mathbf{\mathbf{\mathbf{\mathbf{\mathbf{\mathbf{\mathbf{\mathbf{\mathbf{\mathbf{\mathbf{\mathbf{\mathbf{\mathbf{\mathbf{\mathbf{$	1	$\mathbf{\mathbf{\mathbf{\mathbf{\mathbf{\mathbf{\mathbf{\mathbf{\mathbf{\mathbf{\mathbf{\mathbf{\mathbf{\mathbf{\mathbf{\mathbf{\mathbf{\mathbf{$	x	x	x
Vrabel et al, 1998	\checkmark	\checkmark	\checkmark	\checkmark	х	х

Table 3.3 Ten category rating studies evaluated according to the credible data requirements.

Table 3.4 summarises the methods used in these credible studies. One of the ten studies is a field study (Akashi and Boyce, 2006), six of them are full size laboratory studies (Boyce and Cuttle, 1990; Boyce, Akashi, Hunter and Bullough, 2003; Davis and Ginther, 1990; Flynn and Spencer, 1977; Piper, 1981; Vrabel, Bernecker and Mistrick, 1998) and three of them (Boyce, 1977; Vienot, Durand and Mahler, 2009; Han and Boyce, 2003) are studies done in booths. In all three of the studies done in booths, an SPD effect was reported. However, the trends in full size lab or field studies did not appear to show any specific trends on how lamp characteristics

affect spatial brightness. Hence, no specific effect of experimental environment can be generalised.

Study	Stimuli	Scale	Environment	Results
	3 x CCT	2-points	Real office	With high CCT, high
Akashi & Boyce,	(3500K, 5000K, 6500K)	Likert scale	Real Office	brightness
2006	2 x Illuminance	5-point semantic	Pool office	No offect
	(3 lamps, 2 lamps)	differential scale	Real Office	No ellect
Boyce 1977	5 fluorescent lamps	7-point semantic		SPD effect might be
Doyce, 1977	2 x illuminance	differential scale	Office mock-up	related with CRI and
	(350 & 600 lux)	unerential scale		GA
	2 x fluorescent			
Boyce et al. 2003	(3000K, 1.3 S/P & 6500K,	7-point	Full size lab with	
	2.1 S/P)	l ikert scale		No effect
	2 x illuminance	Liken Scale	onice environment	
	(344 & 500 lux)			
	4 x CCT (2700K,		Full size office lab	
Boyce & Cuttle, 1990	3500K, 4200K, 6300K)	5-point	with achromatic or	With high CCT, high
(exp 2)	1 x Illuminance	Likert scale	chromatic	brightness
	(225 lux)		environment	
	2 x CCT			
Davis & Ginther,	(2750K, 5000K)	7-point semantic	Full size lab with	No effect
1990	3 x Illuminance	scale*	office environment	
	(~270, 590, 1345 lux)			
Flynn & Spencer	5 lamps	7-point semantic		
1977	(3 x fluorescents, HPS,	scale	Full size lab	SPD effect
	Warm delux mercury)	codio		
	3 x CCT			
Han & Bovce, 2003	(3000K, 4100K, 6500K)	Continues rating line	Office mock-up	With high CCT, high
······	3 x Illuminance	(8.4 cm)		brightness
	(100, 500, 1000 lux)			
	2 x lamps			
Piper, 1981	(CW fluorescent, HPS)	7-point semantic	Full size lab with	No effect
	1 x Illuminance	scale	office environment	
	(~540 lux)			
	3 x CCT			
Vienot et al, 2009	(2700K, 4000K, 6500K)	7-point semantic	Booth	CCT effect
	3 x Illuminance	scale		
	(150, 300, 600 lux)			
	5 lamps			
Vrabel et al, 1998	(3 x fluorescents, MH,	7-point semantic	Full size lab with	SPD effect
	HPS)	scale	office environment	
	1 x illuminance			
	(~538 lux)			

Table 3.4 Summary of methods used in category rating studies considered to be credible.

*Authors reported it as continuous rating line; however there were 7 boxes specified to evaluate the setting.

In the ten credible studies eight of them used odd numbered response categories; seven of them are 7-point scales and one was 5-point Likert scale (Boyce and Cuttle, 1990). All of the 7-point scales used a semantic differential format (Boyce, 1977; Davis and Ginthner, 1990; Flynn and Spencer, 1977; Piper, 1981; Vienot, Durand and Mahler, 2009; Vrabel, Bernecker and Mistrick, 1998) except Boyce et al (2003), which used a Likert scale. In Akashi and Boyce (1990), two category formats were used as even and odd. Odd category had 5-point semantic scale and even category had 2-point Likert scale. As a result, a CCT effect was found with the 2-point Likert scale, however no effects were determined with the 5-point semantic scale. This was the only difference specified on scale format from ten credible studies. Only study, which didn't include any specific points in the response range, was Han and Boyce (2003). They used a continuous rating line in 8.4 cm and mentioned about CCT effect on brightness.

Six of the ten studies mentioned SPD effects on spatial brightness (Akashi and Boyce, 2006; Boyce, 1977; Boyce and Cuttle, 1990; Han and Boyce, 2003; Vienot, Durand and Mahler, 2009; Vrabel, Bernecker and Mistrick, 1998). Four of these studies (Akashi & Boyce, 2006; Boyce & Cuttle, 1990; Han & Boyce, 2003; Vienot et.al. 2009) stated CCT effects on brightness perception. In all of these studies the environment was perceived brighter as the CCT increased. In Vienot, Durand and Mahler (2009) a trade-off was obtained by reducing the illuminance and increasing CCT. For instance, the perceived brightness with a light setting of 4000K at 300 lux was similar to that with a setting of 6500K at 150lux; likewise the perceived brightness of 2700K at 600lux was similar with that of 4000K at 300lux. In Han and Boyce (2003) the perceived brightness difference depending on CCTs was getting more distinct as the light level increases. Similarly, in Boyce and Cuttle (1990) the room was perceived brighter under higher CCT levels with constant illuminance of 225 lux except 6300K, which appeared dimmer than 3500K and 4200K. There was some uncertainty with CCT effects on brightness in Akashi and Boyce (2006) in which two sets of experiments were conducted with different rating scales. The experiment carried out with 2-point rating scale indicated CCT effects. In this part of the experiment, perceived brightness for the participants was the same with 6500K when the light level reduced by 1/3 of the other condition with a 3500K lamp. However, in the second experiment that was using 5-point rating scale there were no CCT effects on perceived brightness. This might be referring to an impact of the number of response points as such an even or odd response range giving different results of perceived brightness.

Other than CCT, in Boyce (1977) effect of CRI and GA on spatial brightness was indicated. Results of Boyce (1977) showed significant difference between lamps of Natural and Kolor-rite with White fluorescent. The light settings with Natural and Kolor-rite lamps were perceived more satisfactory than White lamp. Boyce pointed out that these results might be correlated with CRI or GA of the lamps. Vrabel, Bernecker and Mistrick (1998) mentioned a SPD effect on spatial brightness. In their study, T8 fluorescent and HGHP lamps were rated high on all the scales including brightness. These two had high CCT and CRI levels; however which metric of the SPD generated this result was not certain.

In the study done by Flynn and Spencer (1977) brightness results were grouped under visual clarity with three more scaled features (stimulating, distinct and hazy) with factor analysis. According to the results, an effect of SPD was reported on visual clarity. As the category of visual clarity covers an overall evaluation of the features, it is not certain that if the mentioned effect was dependent on brightness or any other factor. Similarly, the results from Piper (1981) were not certain about the SPD effect. Piper reported that the HPS lighting was perceived to be slightly dimmer than the CW lighting according to the mean ratings. When an analysis using t-test were done with the mean ratings and standard deviations provided by Piper (1981), this difference was not significant.

In Boyce et al (2003), two age groups of subjects (ages between 18-28 and ages between 61-78) participated in the experiment. When a 3000K lamp at 500 lux was compared with a 6500K lamp at 344 lux, there was no significant difference between mean pupil areas of young participants. This might be indicating a trade-off between CCT and illuminance as the light level decreased and the CCT increased and no difference was perceived. However, Boyce et al (2003) reported that these results were dominated by the illuminance rather than SPD of the lamp.

The most precise statement asserting that lamp type does not affect brightness was mentioned only by Davis and Ginthner (1990), in which the two adjective pairs (bright/dim and stimulating/relaxing) were grouped under brightness category based on the Pearson correlation test. As the results were not directly related with bright-dim evaluations of the participants, Davis and Ginther's statement might not be giving an answer on relationship of SPD with spatial brightness. Since the mean ratings and standard deviation values were not reported for the adjective pair bright/dim, no control tests were likely to be done.

Twenty studies did not present reliable evidence of SPD and spatial brightness. These studies will not be taken into consideration in further analysis due to not following the specified essential requirements. The reasons for omitting these studies are presented in Table 3.5.

Missing requirements	Studies found to be not credible
failure to randomise, or report whether presentation sequence were randomised	Fleischer, Krueger and Schierz, 2001; Ishida, Ikeyama and Toda, 2007; Ju, Chen & Lin, 2012; Lin et al, 2007; Oi and Takahashi, 2007; Oi and Takahashi, 2013
having a large number of stimuli relative to the number of response options thus leading to a suspect grouping bias	Boyce and Cuttle, 1990 (experiment 1); Ishida, Ikeyama and Toda, 2007; Lin et al, 2007; Oi and Takahashi, 2007; Oi and Takahashi, 2013, Rea, 1982; Takahashi et al, 2013
not reporting sufficient quantitative data or procedural design	Baron, Rea and Daniels, 1992; Bartholomew, 1975; Cockram, Collins & Langdon, 1970; DeLaney et. al., 1978; Fleischer, Krueger and Schierz, 2001; Ishida, Ikeyama and Toda, 2007; Ju, Chen & Lin, 2012; Knez, 1995; Knez, 2001; Lin et al, 2007; McNelis et. al., 1985; Oi and Takahashi, 2007; Oi and Takahashi, 2013; Rea, 1982; Rubinstein and Kirschbaum, 2003; Takahashi et al, 2013; Tiller and Rea, 1992; Wake et. al., 1977; Zhan et al, 2003
and not reporting clearly the precise items	Fleischer, Krueger and Schierz, 2001; Rubinstein and Kirschbaum, 2003; Zhan et al. 2003
ion minori ratingo word bought	

Table 3.5 Category rating studies found to provide insufficient data and reasons to be omitted.

In conclusion, there is some evidence from studies using category rating that it is possible to reduce the illuminance and maintain brightness by choice of lamp spectrum. Some of the credible category rating studies presents significant SPD effects on spatial brightness. However, not all studies agree with this statement. On one hand some of these studies stated CCT and CRI effects on brightness. On the other hand, Boyce (1977) mentioned that CCT is not a good predictor of brightness. To understand this effect more precisely, studies using other experimental methods will be reviewed in the following sections

3.4 Matching Studies

In matching studies there are two stimuli to be compared. One of the stimuli is the reference and the other one is the adjusted stimulus. In this method, participants are given the reference brightness level and asked to adjust the amount of light of the second stimulus until it has the nearest possible brightness match with the reference. In some cases, the experimenter might make the adjustment according to commands from the subject. The visual scene that is defined as reference is lit with a constant luminance. When the brightness of the second visual scene is adjusted to match the reference, the final light levels are recorded. The output is the ratio of luminances of the two final visual scenes at perceived equal brightness. In some studies output ratios can be at equal clarity or equal appearance. Following Fotios & Gado (2005) it is assumed that these results are a suitable proxy for judgements of equal brightness. A detailed investigation of terms and scales used in spatial brightness studies is going to be reviewed and presented in Chapter 4.

Although most of the matching studies of photopic light levels were carried out side by side in the current review (Boyce, 1977; Fotios and Levermore, 1997; Fotios and Gado, 2005; Hu, Houser and Tiller, 2006), it can also be applied in sequential mode (Fotios and Cheal, 2010). According to Uchikawa and Ikeda (1986), simultaneous matching is more accurate since it avoids the biases that can occur because of memory limitation. In the experiment that was compared simultaneous and sequential matching test by Fotios and Cheal (2010), similar estimates of illuminances were required for equal spatial brightness. Therefore, they suggest that both modes for evaluation have equal validity.

Five criteria for data collection and reporting in order to reduce bias in matching tasks proposed by Fotios et al (2008) is summarized:

- (1) Balanced stimulus position
- (2) Illuminance control applied to both stimuli
- (3) Starting illuminance balanced
- (4) Valid data analysis with precise data reporting
- (5) A null condition trial

Fotios et al (2008) reported that in side-by-side matching experiments, both lamps must be used to illuminate left-hand side and right-hand side spaces for an equal number of trials in order to avoid positional bias. When the stimuli presented after each other and the same side of the visual field kept as reference all through the experiment, participants learn which stimulus of a pair is the standard. After a while they start to evaluate each stimulus against the range of stimuli presented and avoid comparing the stimulus with the reference (Poulton, 1977).

Similarly, conservative adjustment bias occurs if dimming is applied to only one of the stimulus in a matched pair. In Houser et al (2003), side by side matching task was applied using identical lamps in both rooms. A significant difference found with the variable stimulus to be set at a higher illuminance than the reference. In contrast, Fotios and Gado (2005) reported that the participant set the illuminance of variable stimulus below that of the reference. In the study on linear measurements, LaBoeuf and Shafir (2006) also found that the participants tend to underestimate the target and were matched the shorter length stimulus to the original one. This might also be related with the initial length of the variable stimulus. When participants were asked to match a higher brightness level with the reference, variable stimulus tended to be adjusted to a higher value than the reference and when the dimming started from a lower level than the reference, the result tended to be the opposite because of the conservative adjustment. Therefore, the adjustment should be applied to both stimuli in each pair for an equal number of trials and a precaution can be taken by counterbalancing the initial illuminance of the

variable stimulus (Fotios et al 2008). As Fotios et al (2008) had contrary evidence on the application of starting illuminance; it was not possible to give conclusive direction, therefore this precaution grouped under desirable requirement for the current study.

Reporting the quantitative data is one of the primary requirements of credible studies for all of the methods used in spatial brightness and in any research. In matching studies reporting the numeric data to show the central tendency like illuminance ratio at equal brightness, a measure of dispersion and sample size are important. To determine whether an apparent difference is real, statistical analysis is needed and sufficient data should be provided to enable such analysis.

The other criterion is applying a null condition trial in the experiment. For category rating studies this criterion was kept in desirable requirements due to very few applications in the studies. Although, it is a desirable requirement all the credible studies except Hu, Houser and Tiller (2006) included null condition trial in their matching experiment to identify any biases that occur in the application of the experimental method.

Twenty one studies using a matching method to explore the SPD effects on brightness at photopic light levels were reviewed in this section. Five requirements to avoid biases in matching studies that are mentioned above were applied to identify credible data. As shown in Table 3.6, four studies (Boyce, 1977; Fotios & Gado, 2005; Fotios & Levermore, 1997; Hu, Houser & Tiller, 2006) using matching procedure is suggested to provide credible estimates of the illuminance ratio for equal brightness (Fotios et al, 2013).

	Essential requirements			Desirable requirements		
Study	Stimulus	Illuminance	Quantitative	Starting	Null-	
Clady	position	control applied	data	illuminance	condition trial	
	balanced	to both stimuli		balanced		
Boyce, 1977	\checkmark	\checkmark	\checkmark	N.A*	\checkmark	
Fotios &			. /	NR		
Gado, 2005						
Fotios &						
Levermore,	\checkmark	**	\checkmark	Х	\checkmark	
1997						
Hu, Houser						
& Tiller,	\checkmark	\checkmark	\checkmark	\checkmark	X	
2006						

Table 3.6 Four matching studies evaluated according to the credible data requirements

*Both stimuli started from a constant reference illuminance.

**Fotios and Levermore (1997) applied the dimming correction factor to offset the effect of conservative adjustment as found in their null condition trials.

Hu, Houser and Tiller (2006) and Fotios and Gado (2005) asked participants to match brightness. However, Fotios and Levermore (1997) and Boyce (1977) asked for visual equality and equal satisfaction respectively, in order to obtain evaluations of brightness of the environment. Fotios and Gado (2005) had five other visual objectives to match other than equal brightness, including equal satisfaction and visual equality. They obtained similar results in all of these visual objectives that were exploring the relation of SPD and brightness. Therefore, equal satisfaction and visual equality were referred as spatial brightness.

Table 3.7 summarises the methods used in these credible studies. One of the four studies was full sized lab study (Hu, Houser and Tiller, 2006) and the other studies were conducted in adjacent booths (Boyce, 1977; Fotios & Gado, 2005; Fotios & Levermore, 1997). All three of these booth studies reported a spectrum effect on perceived brightness. The only study that couldn't find an effect on brightness was Hu, Houser and Tiller (2006), this result might be originated from the environment of experiment being a full size lab. But also the metric examined in the experiment was CCT and this metric of lamp found not be a good predictor of brightness by Boyce (1977) as well. Instead of CCT or CRI, Boyce (1977) indicated that perceived brightness fits best with GA. Fotios and Levermore (1997) also reported an effect of different spectrum on visual equality dependent on colour quality. In their study, lower light levels were required with lamps of higher colour quality than the lamps of poorer colour quality with full spectrum fluorescent lamp having approximately 20% less illuminance than warm white fluorescent.

Study	Stimuli	Method	Environment	Results
Boyce, 1977	5 fluorescent lamps 2 ref. illuminances (300 & 600 lux)	Side-by-side matching	Office mock-up	SPD effect might be related with GA
Fotios & Gado, 2005	2 x fluorescent (2950K, 52 CRI & 6500K, 98 CRI) Ref. illuminance (320 lux)	Side-by-side matching	Booths with achromatic or chromatic combined environment	SPD effect
Fotios & Levermore, 1997	5 fluorescent lamps 3 x illuminance (filters with 70, 50 and 25% transmission)	Side-by-side matching	Booths with achromatic or chromatic combined environment	With high colour quality, high brightness
Hu, Houser & Tiller, 2006	2 x CCT (3500K, 6500K) Ref. illuminance (538 lux)	Side-by-side matching	Full size lab with office environment	No CCT effect

Table 3.7 Summary of methods used in matching studies considered to be credible.

Sixteen studies did not present reliable evidence of SPD and spatial brightness. These studies will not be taken into consideration in further analysis due to not following the specified essential requirements. The reasons for omitting these studies are presented in Table 3.8.

Missing requirements	Studies found to be not credible
	Alman, 1977; Alman, Breton & Barbour, 1983; Aston &
	Bellchambers, 1969; Bellchambers & Godby, 1972; Booker,
failure to balance stimulus position and	1978; Hashimoto & Nayatani, 1994; Houser & Hu, 2004; Ju,
application of dimming	Chen and Lin, 2012; Vidovsky-Németh and Schanda, 2012;
	Vandhal, Gudd and Schierz, 2009; Worthey, 1985;
	Zheleznikova & Myasoedova, 1995
not reporting sufficient quantitative data or	Chee, Yi & Cho, 2005; Harrington, 1954; Lemons &
	Robinson, 1976; Thornton, Chen, Morton & Rachko, 1980;
procedural design	Thornton & Chen, 1978

Table 3.8 Matching studies for	und to provide insufficient	t data and reasons to be omitted
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In conclusion Hu, Houser & Tiller (2006) supported Boyce (1977) findings of CCT not being a good predictor of brightness. There was some evidence that a trade off with lamp spectrum and light level can be obtained. Features of colour quality and GA of the lamp were indicated as effective metrics on spatial brightness (Boyce, 1977; Fotios and Levermore, 1997). Next sections will continue to explore the SPD effects studied in other experimental methods.

3.5 Discrimination Studies

In discrimination studies, generally two stimuli are presented for participant to evaluate the spatial brightness of the visual scenes (booths, rooms or light patches). In this kind of tasks, spatial and temporal juxtaposition has been used for the stimuli presentations referring to simultaneous (side by side) and sequential or successive (after each other), respectively. The difference between two temporal juxtapositions is in the sequential mode each stimulus is alternated back and forth by refreshing the memory whereas in successive mode the judgement is made after only one presentation of the stimulus (Fotios and Houser, 2013). During the stimuli presentation the luminance of the lamps are kept constant and the participants are asked to report which scene is brighter. Mostly, they are not allowed to respond with 'equally bright' option, this is being forced choice procedure.

Yeshurun et. al. (2008) suggested that two-interval forced choice procedure (i.e. temporal juxtaposition) needs to be applied with caution and testing for bias as it is potentially difficult to interpret. For instance, the cases like participants have no idea about their preference of the stimuli and they answer the questions just by guessing and/or choosing one of the intervals randomly. Similarly, most of the time interval bias occurs depending on either the presentation order or the duration. However, Yeshurun et al (2008) was not able to find a specific pattern to

explain the reason for this bias. Fotios and Houser (2013) also mentioned the biases that can occur in the application of forced choice discrimination tests. Four criteria for data collection and reporting in order to reduce bias in discrimination tests suggested by Fotios and Houser (2013) are summarized:

- (1) Counterbalance the spatial and/or temporal location of the stimuli
- (2) Compare all possible pairs
- (3) Randomized stimulus order
- (4) Valid data analysis with precise data reporting

In simultaneous evaluations the scenes are juxtaposed either in left-right or top-bottom spatial locations, in order to avoid positional bias counterbalancing needed to be applied. In some studies even though the luminances are equal in both locations or a higher luminance exists in left-hand side, right side is judged brighter (Rea, Radetsky and Bullough, 2011, Stephens and Bolander, 2005). Accordingly, in sequential or successive presentation of stimuli an interval bias may occur if counterbalancing is not applied. In such cases, two or more stimuli are presented in temporal intervals (interval 1 and then interval 2 and so on) and the order of the stimuli presented may affect the brightness judgement. One reason is that the participant cannot record the sensory intensity of the first stimulus to compare with the next one and a memory limitation occurs. Thus a potential advantage of using successive presentation is that the stimuli displayed more than one by making back and forth in between the visual scenes (Fotios & Houser, 2013, Yeshurun et. al, 2008). In such cases how many times the repetition can be done is an important question. Berman et al (1990) applied 3 times alteration of two intervals of a pair. At least 72% of the participants evaluated second interval as brighter in two of the comparisons even though it had lower luminance than the first interval. There was a possible interval bias in their study and it might be related with the number of alteration between intervals. Hence, participants' first reaction to the scene can still be guessing about which one appeared brighter or answering as having 'no idea' (Yeshurun et. al, 2008). Therefore, counterbalancing the spatial and/or temporal location of the stimuli can be a good prevention. A more detailed study on using forced choice evaluation is explained in Chapter 4.

In some of the discrimination studies a reference stimulus is assigned to be compared with the rest of the stimuli (Fotios & Cheal, 2008; Uchikawa & Ikeda, 1986). To compare with the reference the experimenter specifies range of other stimuli and their distribution in the specific range. If the distribution of the magnitude of the test stimuli above and below than that of the reference are not equalized a frequency bias occurs. Fotios and Houser (2013) explained this with the example of displaying 100, 200, 300, 400 and 500 lux to compare with reference of 400 lux. In which just one stimulus higher than and three stimuli lower than reference light condition are presented. Comparing all possible pairs of stimuli instead of identifying one reference stimulus may avoid this bias. Similarly, range bias arising from the selected range by experiment can be prevented by comparing all possible pairs. Using just one range of

illuminance to compare the brightness may give misleading results if there were no alternative method that has been used to confirm the effect. Besides, the order of these stimulus pairs should be in randomized order (Poulton, 1977).

Lastly, the method that is used in the study, the results with numeric data showing central tendency and statistical analysis must be reported clearly. The reporting must include sufficient information to enable statistical analysis to determine whether the apparent difference is significant or not. Besides, a null-condition trial will make it easier to find out if any possible biases occurred in the experiment.

Eleven studies using discrimination method to explore the SPD effect on brightness at photopic light levels were reviewed in this section. Four requirements to avoid biases in discrimination studies that are mentioned above were applied to identify credible data. As shown in Table 3.9 five studies (Berman et.al, 1990; Houser, Tiller & Hu, 2004; Houser, Fotios & Royer, 2009; Royer & Houser, 2012; Vrabel et. al., 1998) using discrimination procedure provide reliable estimates of the illuminance ratio for equal brightness (Fotios et al, 2013).

	Essential requirements				Desirable requirement
Study	Stimulus position or order balanced	All pairs compared	Stimulus order randomised	Quantitative data	Null- condition trial
Houser, Tiller & Hu, 2004	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark
Royer & Houser, 2012	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark
Houser, Fotios & Royer, 2009	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark
Vrabel, Bernecker & Mistrick, 1998	\checkmark	\checkmark	\checkmark	\checkmark	Х
Berman, 1990	✓*	\checkmark	\checkmark	\checkmark	\checkmark

Table 3.9 Five discrimination studies evaluated according to the credible data requirements

* Potential position bias was tested in null-condition in new experiment that was explained in Chapter 6.

As shown in Table 3.10, four of the five credible studies were conducted in rooms (Houser, Tiller & Hu, 2004; Houser, Fotios & Royer, 2009; Vrabel, Bernecker and Mistrick, 1998; Berman et. al., 1990) and one of them was with booth (Royer & Houser, 2012). Three of them used sequential discrimination mode (Vrabel, Bernecker and Mistrick, 1998; Berman et. al., 1990; Royer & Houser, 2012), one used side-by-side mode (Houser, Tiller & Hu, 2004) and one used both of the modes (Houser, Fotios & Royer, 2009).

Study	Stimuli	Method	Environment	Results
Houser, Tiller & Hu, 2004	4 x fluorescent 2 x CCT (3500K, 6500K) 3 x peak wavelength (different levels at 450, 545, 610 nm) 1 x illuminances (538 lux)	Side-by-side discrimination	Full size lab with office environment	Prime-colour theory
Royer & Houser, 2012	8 x SPD (4 lamps different peaks at blue wavelength & 4 lamps different peaks at red wavelength) 1 x illuminance (555 lux)	Rapid sequential discrimination	Booth	Prime-colour theory
Houser, Fotios & Royer, 2009	2 x light setting (2900K, 1.7 S/P & 7200K, 2.6 S/P) 2 x luminance (24, 30 cd/m ²)	Rapid sequential and side-by-side discrimination	Full size lab	No SPD effect
Vrabel, Bernecker & Mistrick, 1998	5 x lamps (3 x fluorescents, MH, HPS) 1 x illuminance (~538 lux)	Rapid sequential discrimination	Full size lab with office environment	SPD effect
Berman, 1990	2 x metamer fluorescent 2 x S/P ratios (0.85, 2.43) 2 x photopic luminance ratio (1.3, 2.2)	Rapid sequential discrimination	Room	S/P ratio effect

Table 3.10 Summary of methods used in discrimination studies considered to be credible.

Two of these studies mentioned prime-colour theory to effect spatial brightness (Houser, Tiller & Hu, 2004; Houser, Fotios and Royer, 2009). In the study of Houser, Fotios and Royer (2009) two parameters of SPD; CCT and S/P ratio were reported to be unrelated with spatial brightness. CCT had already been reported as not being a good predictor of brightness perception with studies using other experimental methods (Boyce, 1977; Hu, Houser and Tiller, 2006) and prime-colour theory proposed to be used in order to predict brightness. On the other hand, S/P ratio, which was determined as not affecting the perceived brightness by Houser,

Fotios and Royer (2009), reported as an effective metric by Berman et al (1990). The lamp having higher S/P ratio was perceived brighter even though it had lower luminance than the other lamp. Thereby, Berman et al (1990) interpreted SPD effect of S/P ratio on brightness perception. In Berman (1995) this effect of S/P mentioned to be related with the new photoreceptor of intrinsically photosensitive retinal ganglion cells (ipRGC). Vrabel et. al (1998) also indicated SPD effect related with any of the metrics mentioned in previous studies, was Royer and Houser (2012). According to their results, they were suggesting to develop a new metric to predict spatial brightness.

Six studies are considered as not providing appropriate evidence for lamp spectrum and spatial brightness due to not following the specified essential requirements. The reasons why they will not be taken into consideration in further analysis are presented in Table 3.11.

Missing requirements	Studies found to be not credible
failure to balance stimulus position	Stephens & Bolander, 2005
failure to compare all the possible pairs	Pracejus, 1967
not reporting sufficient quantitative data	Cockram, Collins & Langdon, 1970; Harper, 1974; Navaab,
or procedural design	2001; Manav, 2007; Pracejus, 1967

Table 3.11 Discrimination studies found to provide insufficient data and reasons to be omitted

Similar with the results of two other experimental methods (category rating and matching) used in brightness tests; some effect of SPD is also found in discrimination studies. Different metrics like CCT, S/P ratio and prime-colour theory were suggested to be influential on perceived brightness. There were contrary results on S/P ratio effect in Houser, Fotios and Royer (2009) with Berman (1990) and prime-colour theory effect in Royer and Houser (2012) with Houser, Tiller and Hu (2004). As the main focus of this study is on S/P ratio, further analysis to define the effect of S/P ratio on spatial brightness was presented in Chapters 5 and 6.

3.6 Adjustment Studies

In adjustment studies there are no external references presented. Participants are instructed to adjust the amount of light either with direct control of the dimmer or by leading the experimenter to do it for them. In such cases, there are no specific scales for participants to evaluate the stimulus; however the stimuli range is limited with the experimenter's choice. In this type of experiment, the stimuli are presented either in succession or separately. Accordingly, each visual stimulus is evaluated in isolation of any other stimuli. The output is the preferred or optimum light level according to the participant.

There are not many studies using the adjustment method. Even though a couple of studies that are using adjustment method (Juslen, 2006; Logadottir et al, 2011; Qiao, 2007) did not directly

ask for evaluating the brightness of the stimulus, still they can be considered as a proxy for preferred or optimum brightness (Fotios & Gado, 2005). Four criteria for data collection and reporting in order to reduce bias in adjustment tests suggested by Logadottir et al (2011) and Fotios et al (2013) are summarized:

- (1) Take extra care to choose presented stimulus range
- (2) Randomised presentation order
- (3) Anchor the stimulus range
- (4) Valid data analysis with precise data reporting

Stimulus range and anchoring biases were investigated in adjustment studies. In the studies, adjustments were available for participants in some illuminance ranges specified by experimenter. The reported mean of the preferred illuminances tended to fall near the centre of the available illuminance range. Therefore, different ranges of stimuli give different preferred light levels, creating a stimulus range bias (Fotios & Cheal, 2010). Another reason that we cannot be certain whether these studies provide reliable estimate of illuminance for equal brightness under lighting of different SPD is the anchor effect. In Logadottir et al (2011), it is demonstrated that low anchors lead to low estimates of preference and high anchors lead to high estimates. Such conservative bias was presented in the study done by LaBoeuf and Shafir (2006), in which the experiment is done with stimuli in different lengths. Similarly, target was estimated shorter in short anchored stimulus than the long anchored stimulus. As a result, there is some doubt as to whether the adjustment method provides reliable evidence to compare preferred brightness under lighting of different SPD.

3.7 Summary

A review of experimental methods used in spatial brightness and the studies using these methods was done. List of requirements defined in order to present an informative work using four experimental methods which avoids possible biases. The review focused more widely on category rating while existing reviews were used for other methods; matching, discrimination and adjustment. Credible data from past studies according to the identified essential and desirable requirements were investigated. 19 of the 65 reviewed studies on spatial brightness were found to be credible. 15 of the 19 credible studies reported SPD effect on spatial brightness.

According to the results of the past credible data, it is possible to have lower levels of illuminance while maintaining brightness depending on the spectrum of the lamp. Most of the studies analysed CCT effects on spatial brightness and some of them present significant results (Han & Boyce, 2003; Vienot et.al. 2009). However, not all of the studies agree on this particular effect (Boyce, 1977; Houser, Fotios and Royer 2009; Houser, Tiller and Hu, 2004). Although, an effect of CRI on brightness was indicated by Boyce (1977), there are not enough past experiments to discuss these effects of CRI on brightness. Contrary results on S/P ratio was

presented by Berman et al (1990) and Houser, Fotios and Royer (2009). There were no effects of S/P ratio on spatial brightness found in both side by side and sequential discrimination tests done by Houser, Fotios and Royer (2009). On the other hand, the setting, which had lower luminance with higher S/P ratio perceived brighter in the study of Berman et al (1990). Similarly, contrary findings presented on prime-colour theory using discrimination method in Houser, Tiller and Hu (2004) and Royer and Houser (2012).

Even though the hints of an effect of SPD on spatial brightness exist in these past studies, it is still not possible to name the precise lamp characteristic that causes this effect. Some further work is done with a new experiment in Chapter 5 and with an analysis in Chapter 6 which is using the past credible data and the data from new experiment to explore the potential metrics that have an effect on brightness. Before that, Chapter 4 will include additional examination of methodology used in category rating tests of brightness.

Chapter **4**

4 Category Rating: Further Analysis of Methodology

- 4.1 Introduction
- 4.2 Detailed Investigations on Number of Response Categories in Rating Studies
- 4.3 Defining visual response in Category Rating Tests

4.1 Introduction

This chapter presents two studies carried out to better understand the category rating method. These were an experiment exploring the number of response categories and a critical review of past studies to investigate definitions of spatial brightness and visual clarity.

4.2 Detailed Investigations on Number of Response Categories in Rating Studies

Many previous studies have used category rating to evaluate spatial brightness and other aspects of the visual environment in order to compare the effectiveness of different lighting conditions such as the spectral power distribution of the light source. There are suggestions in the literature stating that the number of response categories in a semantic differential rating scale can affect judgments. For example, whether or not the response range includes a neutral (or, middle) category (i.e. an odd or even number of response categories) affects the response recorded: there is evidence that the presence of neutral categories can enhance response contraction bias and this reduces the ability to discriminate between stimuli. Scale format has not been extensively examined for appraisals of the visual environment.

The 7-point scale is commonly used to define the semantic differential rating task (Tiller & Rea, 1992; Houser et al., 2002). Of 21 previous studies of SPD and spatial brightness using category rating, 12 used 7- point rating scales (Flynn & Spencer, 1977; Wake, Kikuchi, Takeichi, Kasama & Kamisasa, 1977; Piper, 1981; Rea, 1982; Davis & Ginthner, 1990; Tiller & Rea, 1992; Vrabel, Bernecker & Mistrick, 1998; Houser, Tiller, Bernecker & Mistrick, 2002; Boyce, Akashi, Hunter & Bullough, 2003; Ishida, Ikeyama & Toda, 2007; Oi & Takahashi, 2007; Vienot, Durand & Mahler, 2009), for example a scale ranging from 1=dim to 7=bright. Other brightness studies have used different response ranges; 2-point (Akashi & Boyce, 2006), 5-point (Boyce & Cuttle, 1990; Akashi & Boyce, 2006; Bartholomew, 1975; Knez, 1995; Knez 2001), 8-point (Fotios & Cheal, 2007), 9-point (Boray et al., 1989) and 10- point (Houser et al., 2002). In two other studies it is not clear what rating scales were used (Fleischer et al., 2001; Rubinstein & Kirschbaum, 2003). There is, however, a growing awareness that rating questions may be vulnerable to response style behaviours causing non-random response errors (Moors, 2008) which led Fotios and Houser (2009) to suggest that response range is one issue to be considered when screening previous studies of spatial brightness.

A key question is whether there is an optimal number of response categories, from both cognitive and statistical considerations: what is needed is a sufficient number of response categories that optimizes reliability yet does not cause unnecessary burden upon a respondent (Moors, 2008). In their review of category rating Fotios and Houser (2009) suggested that a response scale of around seven points is about right. This was based largely on Miller (1956)

who stated that more than seven categories can lead to greater confusion for respondents. Alwin (1992) found the 2-point scale to measure attitude direction as reliably as other response scales, and thus, if the purpose of measurement is to assess only the direction of attitudes the 2-point scale will do as well or better than other forms; longer response scales add information regarding intensity as well as direction but may also cause rating scale biases. Dawes (2008) presented previous work to demonstrate that changing the number of response categories can affect the relative mean rating and the distribution of judgements. His ratings of price consciousness with a Likert scale used three scale formats, 5-, 7- and 10-point response ranges, and even though the overall mean ratings had slight differences for three of the response ranges (6.9, 6.9, and 6.6, respectively) with 10-point response scale the evaluations were found to be significantly lower than 5 and 7-point response. There were no significant difference in the results of 5 and 7-point responses. In that case, although the distribution (skewness and kurtosis) of the assessments in three of the response scales was not different from each other, there occurred an effect of odd and even numbers of responses on the mean evaluations. In contrast Parducci and Perrett (1971) compared ratings of the physical size of squares using semantic differential rating (very large to very small) with either 6 or 9 categories and concluded there were no significant differences in the information gained.

Response ranges may offer odd or even numbers of categories. A bi-polar response range with an odd number of categories allows respondents the option of choosing the middle (or neutral) category and not committing to a positive or negative response as they would with an even number of points. The presence or absence of the middle category in a survey question can make a significant difference in the conclusions that would be drawn about the distribution of public opinion on an issue, because such alternatives usually attract a substantial number of people who may be ambivalent about other alternatives offered to them (Bishop, 1987). Most of the literature discussing response range format refers to social issues (Bishop, 1987) so further data are needed to examine any effects of response range on lighting perception. In order to set up a valid and reliable research experiment the ambiguity about effects of different response ranges on lighting perception should be cleared out. For this reason an experiment is conducted to decide on which response range to employ in the further research which is subject to this thesis.

4.2.1 Method

Evaluations of a lecture theatre were sought using a questionnaire and this asked for ratings of four items, addressing loudness, thermal comfort, brightness and visual clarity (Figure 4.1). A written definition of the intended limits of the response scale was given for each question to anchor the response scale: for brightness this was 'Assume the brightest is represented by the light level in an outdoor sports area (when all the floodlights are on) and the dimmest is the light level of an outdoor parking lot at night' which was the definition used by Vrabel, Bernecker and Mistrick (1998, p.33) (See Appendix B for an example of the questionnaire).

Q1. Please evaluate the **loudness** of this room from 1 (very quiet) to *X* (very loud).

Q2. Please evaluate the **thermal comfort** of this room from 1 (very cool) to *X* (very warm).

Q3. Please evaluate the **brightness** of lighting in this room from 1 (very dim) to X (very bright).

Q4. Please evaluate the **clarity** of lighting in this room from 1 (very hazy) to X (very clear).

Figure 4.1 The four survey questions. The upper limit (X) of each range was either 5, 6, 7 or 8, with the same upper limit for all four questions on the questionnaire.

A group of 84 university students were asked to provide individual evaluations of environmental aspects of their lecture room. The questionnaire was administered on two separate days, approximately one month apart, to the same class of students. Although this was nominally the same sample it is likely that these were not identical groups, and questionnaires with different response scales were distributed randomly on both days. A warm air system provided heating and ventilation; the room had no daylight and was illuminated by electrical lighting, this being set to the dimmed level to enhance visibility of the projector screen. The lighting was switched to the same setting for both evaluation sessions.

Four different versions of the questionnaire were used and these differed only in the number of response categories, i.e. either 5, 6, 7 or 8 categories. Each response scale thus ranged from 1 to either 5, 6, 7 or 8. All four questions on a particular questionnaire used the same number of response points, and the questionnaires were distributed randomly. The 84 participants received and completed the questionnaire simultaneously; discussion was not permitted during this task and the lecturer did not receive any comments that different rating scales were used. The students were asked to do this as an example of environmental rating during a lecture on thermal comfort and were not informed about the objective of the study.

It should be noted that this questionnaire was used specifically to compare results obtained with different response ranges. An alternative design would be used if the primary intention was to evaluate the environment, including reversing the polarity of some response ranges to counter repetitive response ticking, repeated questions addressing the same issue to provide alternate-form reliability (Litwin, 1995), and, in the case of repeated measures, ensuring the number of response categories allowed the opportunity to distinguish between stimuli (Fotios & Houser, 2009).

4.2.2 Results

Table 4.1 shows the median and mean responses, the standard deviations and sample size for the evaluations of environmental characteristics.
Response range		Q1		Q	2	Q	3	Q4	
		(loudness)		(thermal comfort)		(brightness)		(clarity)	
		Day 1	Day 2	Day 1	Day 2	Day 1	Day 2	Day 1	Day 2
	Median rating	2	3	3	3	2	3	3	3
	IQR	1	1	1	1	1	1	2	1.5
5- point	Mean rating	2.41	2.71	2.65	3.25	2.51	2.71	3.03	3.09
	Std Dev	0.68	0.71	0.61	0.85	0.63	0.84	1.11	0.88
	n	29	21	29	21	29	21	29	21
	Median rating	3	3	3	3	3	3	3	3
	IQR	1	0.25	1	1	0	1	1	2
6-point	Mean rating	2.56	2.86	3.26	3.40	3.00	3.13	3.56	3.81
	Std Dev	0.78	0.63	0.68	0.66	0.60	0.71	1.19	1.00
	n	23	22	23	22	23	22	23	23
	Median rating	3	3	4	4	3	3	4	4
	IQR	1	1	0.5	1	0	0.75	2	2
7- point	Mean rating	3.22	3.20	3.76	4.20	2.95	3.25	4.04	4.30
	Std Dev	0.68	0.76	0.72	0.69	0.78	0.78	1.49	1.34
	n	22	20	22	20	22	20	22	20
	Median rating	3.5	3	3.5	5	4	4	4.5	4
8- point	IQR	1	1	2	1.5	1	2	3	1
	Mean rating	3.50	3.71	3.70	4.71	3.50	4.00	4.50	4.42
	Std Dev	0.84	0.90	1.05	1.14	0.70	1.04	1.50	0.97
	n	10	21	10	21	10	21	10	21

4.2.2.1 First and second evaluation sessions

Initially, the results were analysed to determine whether there were differences between the first and second evaluation sessions (Day 1 vs. Day 2). Table 4.1 reveals that mean ratings on the second day were slightly higher than the first day in 13 of the 16 cases. Figure 4.2 shows the distribution of responses for ratings of the four environmental items using the 5-point response scale on the two evaluation days.

The data were assumed to be independent samples as different participant groups evaluated the room in separate days. Thus the Mann- Whitney and the two-sample Kolmogorov- Smirnov tests were employed (Field, 2005). This analysis compared responses gained from the same scale types on both days e.g., comparing ratings of brightness using the 5-point scale on Day 1 with ratings of brightness using the 5-point scale on Day 2, thus there were in total 16 analyses (4 questions x 4 rating scales). To reduce the incidence of capitalising on chance (increasing the probability to get low significant results than applying only one test) when carrying out multiple statistical analyses a decision was taken to adopt p \leq 0.01 as the critical value for determining significant differences. In that case, the chance to obtain a difference, where there is no actual difference, was decreased by searching a difference of 1% instead of 5%.



Figure 4.2 Results of evaluations of the four environmental items using the 5- and 6-point response scales on the two evaluation days. The 6-, 7- and 8-point scales suggested lesser differences between the evaluation days than did the 5-point scale. Graphs with 5- and 6-point response scales are presented here as the example.

The Mann-Whitney test suggests differences between the two evaluation sessions only in two of the 16 cases, which are loudness (Q1; p=0.008) and thermal comfort (Q2; p=0.011) with the 5-point scale. The Kolmogorov-Smirnov test does not suggest any differences to be significant.

Field (2005) suggests The Kolmogorov-Smirnov test tends to have better power than the Mann-Whitney test for sample sizes of less than 25 per group: Table 4.1 shows that all groups in the current data had samples of less than 25 except for 5-point ratings on Day 1. It was concluded that similar responses were gained on both evaluation sessions, thus it was decided to combine the results gained from the two sessions into a single data set for subsequent analyses.

4.2.2.2 Graphical comparisons

Figure 4.3 shows the distribution of responses for the four rating scales across the four evaluation items. To assist direct comparison of the different rating scales these were converted to a common scale: a 10-point range was chosen so that all four original response ranges were subjected to transformation. Following Dawes (2008) the transformation was carried out such that the lowest rating (1) remained unchanged, the highest rating was set to 10, and middle categories were uniformly spaced in between these two end points (see Table 4.2).



Figure 4.3 Distribution of environmental evaluations. The original rating scales were converted to a 10-point range: numbers above the bars show the original category number. Percentage values show the distribution of responses to the polar positions, with judgements for the neutral

category in the odd scales equally divided.

Table 4.2 Original response categories and re-scaled values when converted to a ten-point range

Response scale	Original response scales and values when rescaled to a 1-10 scale										
5-point	1	2				3			4		5
	1.0	3	.25		5.50				75	10.0	
6-point	1	2			3		4			5	6
	1.0	2.8			4.6		6.4			8.2	10.0
7-point	1	2		3		4		5		6	7
	1.0	2.5		4		5.5		7		8.5	10.0
8-point	1	2	3		4		5		6	7	8
	1.0	2.29	3.57		4.86		6.14		7.43	8.71	10.0

Figure 4.3 does not suggest a strong tendency to choose the neutral value available in the odd ranges as the middle category is the mode response in only three of the eight cases of the 5-point and 7-point ranges. For the eight cases with an even scale and for the five remaining odd cases the mode response is the category just below the middle of the range.

Following Bishop (1987), one approach to comparison is to compare the percentage of judgements above and below neutral category after the division of opinion between the polar positions. These are shown in Figure 4.3; responses for the neutral category in the 5- and 7- point ranges were divided equally between the two sides. Comparison of the percentages of judgements for the polar positions does not suggest any consistent trends.

According to Figure 4.3, in questions 2 and 4 the middle value is the most evaluated point for odd categories 5 and 7. The tendency percentages with all four categories are close to 50 except the response category 6 in question 2, this shows a high consistency in between even and odd numbers of response categories. For questions 1 and 3, even though the most frequently chosen response point is lower than the middle value for all four response categories, there is a high tendency to choose the middle value in odd categories. This trend might confirm the results of Dawes (2008) by implying that with even numbers of categories participants forced to apply their actual preferences, whereas they can be acting ambivalent in odd numbers of categories.

4.2.2.3 Statistical analysis

The data were considered to be independent samples and were not considered to be drawn from a normally distributed population using statistical analysis (Shapiro-Wilks, Kolmogorov-Smirnov). According to both Shapiro-Wilks and Kolmogorov-Smirnov analysis results of four questions were significantly non-normal at p<0.05 except thermal-comfort question with 8-point response categories p=0.09.

The Kruskal-Wallis test was applied to the results of each question to examine the effect of response range: this did not suggest any differences to be significant. Paired comparisons were also carried out. The Mann-Whitney test suggests the difference to be significant ($p \le 0.01$) in only one of the 24 cases (4 evaluation items x 6 response scale pairs), and this was between ratings made using the 6-point and 7-point ranges for brightness (Q3). Parametric tests tend to be better at detecting differences than non-parametric tests (Coolican, 1994) and therefore the analysis was repeated using the t-test: this also did not suggest the effect of response range on ratings to be significant.

What the Mann-Whitney test does is to determine whether there are differences in the location (i.e. central tendency) of two samples by using the difference between mean ranks of the two samples as the statistic. An alternative test for unrelated, non- parametric samples is the two-sample Kolmogorov-Smirnov (K-S) test. The K-S test compares cumulative distributions (distribution of running total of the mean ratings): if the two samples have been drawn from the same population then these distributions may be expected to be fairly close to each other (Siegel & Castellan, 1988). If the two samples are too far apart at any point, which refers to the maximal distance between cumulative frequency distributions of the two samples, this suggests the samples come from different distributions. Thus the K-S test is sensitive to the dispersion of data (e.g. skewness) in the two samples as well as location.

The K-S test suggests significant differences between rating scales as shown in Table 4.3. For ratings of loudness, thermal comfort and brightness, differences between response scales are significant in several cases, whereas for ratings of clarity, differences between ratings are not suggested to be significant. Where the differences between ratings are significant, these suggest differences mainly between the 6-point range and the rest three ranges.

Table 4.3 Level of significance for differences between pairs of response scales as determined using the two- sample Kolmogorov-Smirnov test. Differences considered to be significant (p<0.01) are highlighted in bold.

Response	Q1	Q2	Q3	Q 4
scale pairs	(loudness)	(thermal comfort)	(brightness)	(clarity)
5-6	≤ 0.001	0.004	0.015	0.144
5-7	0.006	0.046	0.006	0.063
5-8	\leq 0.001	0.133	0.001	0.037
6-7	0.016	≤ 0.001	≤ 0.001	0.026
6-8	0.005	0.001	0.009	0.061
7-8	0.019	0.036	0.001	0.447

The difference in conclusions drawn from the Mann-Whitney test and the K-S test arise because the two samples (different response scales) yield the same central tendency of judgement (e.g. whether an item is considered to be too much or too little) but may affect the distribution profile (e.g. whether the response pattern indicates a pointy or heavy-tailed distribution; or the responses building up towards positive values or negative values) (Dawes, 2008; Siegel & Castellan, 1988).

That these data suggest response range affects the dispersion of data but not the central tendency is in contrast to Dawes (2008) findings using Likert scale ratings of price consciousness, which suggested significant effects on the mean rating but not on dispersion. There was agreement between the Mann-Whitney and K-S tests when analysing the Day 1 vs. Day 2 data, which implies that ratings made using the same response scale and evaluation item but on different days yield the same distribution of responses; the different distribution profiles in the results were caused by the response scale format and the evaluation item rather than being an effect of the respondents.

4.2.2.4 Ignoring neutral ratings

To compare ratings recorded using their 4- and 5-point scales, Nowlis et al (2002) used a procedure in which judgements awarded to the middle category of the 5-point scale were ignored and they compared the four remaining points directly with the points of the 4-point response scale. Ratings of 4 or 5 in the 5-point scale were thus shifted to ratings of 3 or 4 respectively in the quasi 4- point scale. For the current data, this provides a means of comparing results accumulated from 6- and 7-point scales. To compare 5-point scale using this method would need data with 4-point scale and 8-point scale would need results from 9-point scale to ignore middle point for this type of comparison. Therefore, it was only possible to use this method with 6- and 7-point scales with current data. Following Nowlis et al (2002), all neutral responses in the 7-point scale (i.e. all judgements at category point 4) were ignored, and ratings of 5, 6 and 7 were shifted to ratings of 4, 5 and 6. Figure 4.4 shows ratings gained using the 6-point range and the transformed 7-point range.

Figure 4.4 does not suggest that removal of the neutral ratings affects the distribution profile except for the ratings of thermal comfort (Q2) where the mode rating has moved from slightly below neutral with the 6-point range to slightly above neutral with the transformed 7-point range.



Figure 4.4 Evaluations of questions Q1-Q4 in 6-point and 7-point response ranges. These graphs show responses gained with the original 6-point scale and also the 7-point scale with the neutral responses omitted, the higher categories shifted, and the remaining frequencies normalised to 100%.

For thermal comfort (Q2) the mode response with the 7- point range was the neutral category (4) but it was the category below neutral (3) for the other three questions. The neutral category of Q2 accounted for 57% of judgements and thus removal of these and normalisation of the remaining data to 100% forced more attention to be paid to the tails which were previously far less significant.

Neither the Mann-Whitney test nor the K-S test suggest any significant difference between the 6-point scale and the transformed 7-point scale. This finding is different to the findings of comparison of the original 7-point scale with the 6-point scale, where the Mann-Whitney test suggests a difference in Q3 and the K-S test suggests a difference in Q2 and Q3: the transformation has not affected the central tendency of the data but has reduced differences in data dispersion. Although, Presser and Schuman (1980) reviewed studies, which used ignoring the neutral ratings approach and concluded that there was not a significant change in distributions when the middle responses were excluded. The change between the results of original comparison and ignoring the neutral ratings was somehow expected as distribution of two point scales (6-point scale and 7-point scale) became similar when the neutral point is omitted in the 7-point scale.

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These results suggest that omitting the neutral category in a semantic differential response scale does not affect the conclusion drawn from the data. Explicitly offering a middle position significantly increases the size of middle category, but tends not to affect univariate distributions (Presser & Shuman, 1980).

4.2.3 Conclusion: Number of points in response range

This study was carried out to determine whether the number of response categories in a semantic differential scale would affect conclusions drawn from the data about evaluations of acoustic, thermal and visual comfort of a room. In order to test this, an experiment using 5, 6, 7 and 8 response points were conducted.

It was concluded that:

(1) The different scale formats did not lead to significant differences in central tendency. In other words the same conclusion as to population opinion about the environment would be drawn with either of these scales.

The traditional view suggests that results between odd and even scales will be unaffected since if the respondents are truly neutral then they will randomly choose one or other side of the issue, so forcing them to choose should not bias the overall results (Nowlis et al, 2002). The current data support this opinion.

(2) The different scales led to different distribution profiles, and this may be associated with whether or not scales offer a middle, neutral category. Whether this is of importance may depend on the questions to be asked of the data.

Then, choosing whether or not a scale should allow a neutral opinion becomes a critical question. There is some advice from Payne (1951): if the direction in which people are leaning on an issue is the type of information wanted, it is better not to offer the middle category, but if it is desired to sort out those with more definite convictions on the issue then it is better to offer the middle category. Thereby, using even number of response points defined to be a desirable requirement in Chapter 3, in order to provide more information on how to apply category rating method in brightness studies.

Besides, data of this experiment were collected from independent samples using a semantic differential rating scale. Further data are needed to examine whether evaluations of lighting using repeated measures judgements and Likert scales are affected by the response range. Besides sampling method, greater difference in scale ranges that are compared should also be taken into consideration for further research. It may be that a greater difference in scale range

would lead to significant differences since large response ranges have not been used in previous studies of spatial brightness.

4.3 Defining visual response in Category Rating Tests

This section investigates definitions of spatial brightness and visual clarity by experts and also how these terms are understood by participants in the context of an experiment carried out to find out the SPD effects on visual preference using category rating method.

The basis of the complication in psychophysical experiments lies under the fact that individuals mainly share information through spoken and written words and the information gained through visual patterns are mostly consist of cultural background, personal experience of vision and recognition (Flynn et. al., 1979). When the modes, patterns and colours of the lit environment altered the impression of the space differ unintentionally, depending on previously gained information through different means. Therefore, an adequate specification of the stimulus and how it is questioned is needed. When this is not done, inaccurate answers might be collected from the participants like Rea (1982) had: participants focusing on target brightness (contrast) instead of evaluating the overall brightness of the room in which getting answers on spatial brightness was the main purpose of the experiment.

Tiller and Rea (1992) discovered that a few dimensions used in the category rating task would potentially refer to scalable aspects of the luminous environment: clear-hazy, visually warm-visually cool, no eye discomfort-great eye discomfort, bright-dim, focused-unfocused, colourful-colourless, nonspecular-specular, focused-blurred and glare-nonglare. Although, two of these aspects, clear-hazy (visual clarity) and bright-dim (spatial brightness) were interpreted separately, there are some intentions by the participants to use these two features in place of each other. In this section these two features of lit environment are going to be compared to address two questions: do lighting researchers think there is a difference between these two scales, and do naïve test participants indicate a difference through their judgments?

4.3.1 Definitions from Lighting Researchers

One way to determine whether lighting researchers consider spatial brightness and visual clarity to be different phenomena is to compare the definitions they report for these items.

Brightness is defined as the attribute of a visual sensation according to which a given visual stimulus appears to be more or less intense; or, according to which the area in which the visual stimulus is presented appears to emit more or less light (Wyszecki & Stiles, 1982). The current study is concerned with *spatial* brightness, a relatively new expression that relates to the perceived amount of light in a space; it is the ambient lighting of a space rather than lighting of a

task, object or surface (Fotios and Cheal, 2011). Previous expressions for spatial brightness have included general lighting and room brightness (SLL, 2002), building lighting (Loe, 1999) and environmental brightness (Oguichi, Ishida & Hokoi, 1999). A draft definition of spatial brightness was described in Chapter 2, Section 2.3.1.

Brightness and spatial brightness are two clearly defined terms. However, this is not the case for visual clarity. Although many previous studies claim to have investigated visual clarity (Aston and Bellchambers, 1969; Bellchambers and Godby, 1972; Thornton and Chen, 1978; Worthey, 1985; Hashimoto and Nayatani, 1994; Vrabel et al, 1998), it is not a well defined term, in fact the comments reported in this section and in Table 4.4 are the only ones that exist in the literature.

Study	Meaning of Visual Clarity					
Aston and	"The estisfaction gained by you personally discounting as far as possible					
Bellchambers, 1969	The satisfaction gamed by you personally, discounting as far as possible					
(p.260)	any obvious differences in colour and brightness					
DeLaney et al, 1978	"At present the meaning of visual clarity is not clear. There are no objective					
(p.74)	criteria for understanding the concept visual clarity."					
	One of the most important characteristics of the colour rendering properties					
Hashimoto et al, 2000	of light sources, and that visual clarity is caused by the feeling of contrast					
	between coloured objects under illumination					
IES Lighting						
Handbook, 1984	An abstract concept, usually defined as a combination of colour rendering,					
(cited by Vrabel et al,	colour discrimination, colour preference, and border sharpness					
1998)						
Lyness 1006 (n.64)	"for a given illuminance, lamps having good colour rendering properties					
Lyness, 1990 (p.04)	tend to make an interior look brighterThis effect is known as visual clarity."					
Thornton and Chen	"Distinctness of detail" and					
1079 (p 95 96)	"The perceived brightness of an illuminated space may be closely					
1976 (p.65-66)	related to visual clarity."					
	"Clear can be thought of as how a distant mountain will look during a clear					
Vrabel et al, 1998	sunny day. Individual trees can be seen and small clearings in the forest are					
(p.33)	visible. On an overcast day, with some fog, individual trees might not be					
	distinguishable, and the clearings are not as easily seen."					
Worthey 1985	Suggests a link with the apparent contrast between colours, in particular					
	red and green and for display screens					

Table 4.4 Definitions and explanations for visual clarity used in past studies.

Thus, according to reported comments, it appears that Aston and Bellchambers (1969) consider spatial brightness and visual clarity to be different phenomena. Also, Vrabel et al (1998) gives different definitions to visual clarity and brightness. The visual clarity definitions can be seen Table 4.4 and for brightness this definition was:

"Bright is represented by the light in an outdoor sports area (when all the floodlights are on). Dim is the level of an outdoor parking lot at night" (p.33).

This is not, however, a universal opinion: Hashimoto and Nayatani (1994) suggested that the term brightness sensation has the same meaning as visual clarity; Flynn et. al. (1973) used factor analysis to group their rating data and suggested that their *perceptual clarity* factor could also have been named spatial brightness since it seemed to relate to variations in illuminance and the factor included ratings of clear-hazy and bright-dim; and, as noted in Table 4.4, Thornton and Chen (1978) suggested that the brightness of an illuminated space may be closely related to visual clarity.

4.3.2 Participant Response to Visual Environment Questions

4.3.2.1 Participant response to Open Questions

Participant response to brightness and clarity questions can be estimated from the responses of naïve test participants when making judgements of spatial brightness and clarity. Firstly, consider that when Boyce and Cuttle (1990) asked test participants to describe the lighting in a room in their own words, they found out that participants used mainly terms of brightness and clarity. This suggests that clarity is not an unfamiliar percept when making visual judgments, or at least that the term is considered to be relevant for describing lighting. What is not known is whether individual respondents used only one or both of these terms. In order to find out participants' usage of the terms, next section reviews tests which employ category rating method and in this procedure test participants are free to make separate evaluations of brightness and clarity.

4.3.2.2 Participant Response in Category Rating

In the following section previous studies, which used category rating to evaluate spatial brightness and visual clarity judgements were analysed. Note that in these previous studies the term brightness is used but the visual fields and test procedures suggest judgements of spatial brightness rather than object brightness (Boyce and Cuttle, 1990, Flynn and Spencer, 1977; Fotios and Cheal, 2007). In all of these studies, the lamps are presented either in full size rooms (Bartholomew, 1975; Boyce and Cuttle, 1990, Flynn and Spencer, 1977; Piper, 1981; Rea, 1982; Vrabel, 1998) or as representations of room in smaller sizes (DeLaney et al, 1987; Fotios and Cheal, 2007; Vienot et al, 2009) aiming to have full field visual scenes.

Three different approaches are used to compare spatial brightness and visual clarity judgements in these past studies according to the quality and quantity of data reported. Firstly, some studies reported a statistical analysis by which judgements were compared. Secondly, some studies report mean ratings and standard deviations which permits simple post-hoc

analysis using the *t*-test. Lastly, some studies report only the mean (or median) rating and these data were used to draw graphs to enable visual comparison. These three approaches are listed in the order of robustness. There are some studies in which either the data is reported insufficiently to permit any of these approaches to comparison or the report reveals a weakness that suggests the results are not reliable.

4.3.2.2.1 Studies reporting statistical analysis

Table 4.5 shows previous studies, which presented statistical analysis to compare spatial brightness and visual clarity judgements. These studies were reporting correlation results of the two environmental judgements.

Comparison	Study	Items rated	Method of	Additional	Agreement between ratings
method			comparison by	method of	of brightness and clarity?
			study author(s)	comparison	
	DeLaney et al	14 items including	Principal	Graph of	The Principal Components
	1978	bright/dim and	component factor	mean ratings	Factor analysis suggests that
		clear/hazy using a 7-	analysis		brightness and clarity ratings
		point semantic			are not similar but the factor
		differential scale			groupings are not as expected.
					Comparison of their mean
					ratings suggests similarity.
	Flynn &	19 items including	Principal	Graph of	Yes
	Spencer,	bright/dim, hazy/clear	component factor	mean ratings	
	1977	using a 7-point semantic	analysis		
		differential scale.			
Studios	Rea, 1982	8 items including	Pearson product-	Graph of	It is not known whether or not
roporting		bright/dim, hazy/clear	moment	mean ratings	the reported correlations are
etatistical		using a 7-point semantic	correlation		statistically significant. The
analveie		differential scale.	coefficient (r)		mean ratings are almost
anarysis					identical in 3 of the 6 cases.
	Vrabel et al	8 items including	Correlation	Graph of	Reported to be not similar but
	1998	bright/dim and		mean ratings	there is no justification for the
		clear/hazy using a 7-			threshold value of correlation
		point semantic			used.
		differential scale.			
	Study from	4 items including	Wilcoxon signed	Graph of	Reported to be similar
	Section 4.2	bright/dim and	rank	mean ratings	
		clear/hazy using one of			
		the 5, 6, 7 or 8-point			
		semantic differential			
		scale.			

Table 4.5 Past studies	using category	v rating to eva	aluate spatial brightn	ess and visual claritv
	uoning outogoi		naato opatiai brightii	ooo ana vioaai olanty

Flynn and Spencer (1977) used 7-point semantic differential scales to rate 19 items including bright-dim and clear-hazy. They analysed their data using principal component factor analysis and this suggested that observers tended to use clear-hazy and bright-dim scales in similar way. These two ratings scales were grouped along with distinct-vague and stimulating-subduing scales in their *visual clarity* factor. Similarly, Flynn et al (1979) found that clear-hazy, distinct-vague, bright-dim and faces clear-faces obscure rating scales were used in similar ways and these were grouped in a visual clarity factor.

Figure 4.5 shows the mean clarity and brightness ratings reported by Flynn and Spencer (1977) for their two experiments: It can be seen that brightness and clarity ratings tend to follow the same trend for different lamps. Flynn and Spencer (1977) did not report the standard deviations

for these ratings but they stated that the difference between mean ratings would be significant, and this was 0.47 in experiment 1 and 0.67 in experiment 2. Using these critical differences suggests that in at least 10 of the 13 cases the differences between brightness and clarity rating are not significant.





Figure 4.5 Mean ratings of brightness and clarity from Flynn and Spencer (1977); experiment 1 (top) and experiment 2 (bottom). These data were taken from Table V (experiment 1) and Table VII (experiment 2) of Flynn and Spencer. The endpoints of the original scales were 1 (bright, hazy) and 7 (dim, clear): the brightness scale has been reversed in this Figure so that for both scales a rating of 7 represents bright and clear. The lamp types in each graph are arranged in order of descending brightness ratings.(Fotios and Atli, 2012)

Vrabel et al (1998) used 7-point semantic differential scale to rate items including bright-dim and clear-hazy. Their initial analysis of variance revealed that rating scale had a significant effect, but that is unsurprising since ratings of items such as colourfulness, naturalness, visually warm/cool are likely to evoke different responses. Following a correlation analysis they reported that judgements of brightness and clarity were not similar because the correlation between these items (0.66) is less than their reported critical value (0.80). Significant correlation was

reported between brightness and colourfulness scales, and between clarity, likeness, pleasantness, naturalness and edge sharpness scales. Two items are not clear in this report; the method of correlation and determination of the critical value (0.80). Figure 4.6 shows the mean brightness and clarity ratings from Vrabel et al. It can be seen that the trends for brightness and clarity ratings show some difference. In the absence of variance data it is not possible to test whether the differences between lamp pairs are significant.





Figure 4.6 Mean ratings of brightness and clarity from Vrabel et al (1998). The lamp types are arranged in order of descending brightness ratings. These data were taken from Figure 8 of Vrabel et al. In the original data a rating of 1 identified the bright and clear ends of the scales: for consistency with other Figures in the current review, the polarities of both scales have been reversed and thus a rating of 7 identifies the bright and clear ends of the scales. (Fotios and Atli, 2012)

Delaney et al (1978) sought judgements of 14 evaluation items including clear and bright using 7-point semantic differential rating scales. This study used side-by-side booths to present lamps to observers using either simultaneous, separate or sequential-haploscopic evaluations. This was an ambitious project to investigate the relationship between lamp type, illuminance method of stimulus presentation and visual scene on the visual assessment of illuminated interiors, however, the paper is confusing and only partial results are given. Too many comparisons are discussed with insufficient data for each. Only the few key comparisons reported by DeLaney et al are permitted, and these fail to completely describe the conditions under which the comparisons were made. There is no evidence of a balanced design and there is no null condition data reported to identify the size of differences other than lamp type. Two principal component factor analyses were reported. The first analysis concerned separate evaluations of a scene containing coloured rectangles. This suggested bright/dim ratings to be part of a factor labelled brightness/colour that contained also ratings of colourfulness and colour contrast. Ratings of clear/hazy were placed in the factor labelled coolness that contained also ratings of colourfulness, and colour contrast.

distinctness and saturation but not clear/hazy. These groupings are not as might be expected from the rating scale labels, i.e. it might be expected that clear/hazy ratings would contribute to a clarity factor rather than a coolness factor. The second analysis was for observation of a cave scene; ratings of bright/dim were this time included in their clarity/brightness factor but ratings of clear/hazy were again in the coolness factor. Figure 4.7 shows mean ratings from Delaney et al (1978) in these seven lamp pairs, brightness and clarity judgements appear to have similar mean ratings other than for lamp pair 1.





Figure 4.7 Results of brightness and clarity ratings from DeLaney et al (1978). These data were taken from Figures 7 – 13 of DeLaney et al. In trials for these data test participants observed two stimuli simultaneously and used response scales to report whether the stimuli appeared to be equal in brightness and clarity, or to give ratings of 1-3 to the stimulus appearing brighter and/or clearer. For a given lamp pair, the mean brightness and clarity ratings were always in the same direction, i.e. suggested lighting from the same lamp in the pair to provide the greater brightness and visual clarity. (Fotios and Atli, 2012)

Rea (1982) investigated the effects of viewing direction and polarisation (but not SPD) on room evaluation using semantic differential rating scales including hazy-clear and dim-bright. Tiller and Rea (1992) provided further analyses of these data. For the six combinations of viewing direction and polarisation the correlation (r) between the brightness and clarity ratings ranged from 0.102 to 0.521, but it is not reported whether these correlations are statistically significant or not. Figure 4.8 shows the mean ratings of brightness and clarity for each of the six lighting conditions (Rea, 1982). For last three conditions there are almost identical ratings, and the other three appear to be comparable, but it is not possible to perform a statistical test since the original paper did not report standard deviations. Lighting conditions 1-3 in Figure 4.8 suggest slightly higher ratings of brightness than clarity whereas in conditions 4-6 the mean ratings are almost identical. The environmental difference between these two groups is the direction of view of the test participant relative to the light source, being 0° for conditions 1-3 and 90° for conditions 4-6.





Figure 4.8 Mean ratings of brightness and clarity from Rea (1982). Note that the six different test conditions were variations in viewing direction and polarity. These data were taken from Table 1 of Rea. Following the original work, the end points of the rating scales are 1 (dim, hazy) and 7 (bright, clear). (Fotios and Atli, 2012)

4.3.2.2.2 Results of response range study in section 4.2

In the study explained in Section 4.2, judgements of a lecture room of 4 evaluation items including clarity and brightness were done using four different response ranges (5-, 6-, 7- and 8- point response scale). Figure 4.9 shows the mean ratings of brightness and clarity for each response scale. Wilcoxon signed rank test suggests these ratings are not significantly different from each other for all four response ranges.



Figure 4.9 Mean ratings of brightness and clarity from response range study explained in Section 4.2. Note that the four different evaluations are variations in response scales. Following the original work, the end points of the rating scales are 1 (dim, hazy) and 5, 6, 7 or 8 (bright, clear).

4.3.2.2.3 Post-hoc statistical analysis of studies not comparing brightness and clarity

Table 4.6 shows the studies that didn't present any statistical analysis to compare results gained from spatial brightness and visual clarity judgements. The reported mean ratings and standard deviations of these judgements were used to make comparisons with Post-hoc analysis.

Comparison method	Study	Items rated	Method of comparison by study author(s)	Additional method of comparison	Agreement between ratings of brightness and clarity?	
Post-hoc statistical	Fotios & Cheal 2007	8 items including hazy, clear, dim, dark and bright using an 8-point response scale with end points labelled not-at-all-so and very-much-so. 7 items including	None	Wilcoxon test and t-test applied to original data.	Yes	
analysis of difference	Piper 1981	dim/bright and hazy/clear using a 7-point response scale	None	t-test applied to mean ratings	Yes	
	Vienot et al 2009	9 items including dark/bright and crepuscular/clear using a 7-point response scale	None	t-test applied to mean ratings	Yes (7 of 9 cases suggest similar ratings)	

Table 1.6 Dact studies	ucing optogony	rating to avaluate a	natial brightness and	l vieual clarity
	using calegory	rating to evaluate s	pallal phyritiess and	i visual ciality

Piper (1981) sought observers' responses to lighting in a small room lit alternately by high pressure sodium (HPS) and cool white fluorescent (CW) lamps using semantic differential rating scales with these sources giving an illuminance of 538 lux on the desk. Table 4.7 compares mean ratings of brightness and clarity: note that Piper reports ratings for the HPS lamp relative to the CW lamp, and for the red filtered HPS lamp relative to the standard HPS lamp, rather than giving separate ratings for each light source. The standard deviations are large compared with the difference between the mean ratings: the *t*-test does not suggest these differences to be significant (p>0.05).

		HPS lamp	HPS lamp with red
Rating scale		relative to CW	filter relative to HPS
		lamp	lamp
Bright/dim	Mean	-1.7	-2.3
	Std Dev	2.8	4.6
	n	24	24
Clear/hazy	Mean	-2.0	-3.7
	Std Dev	2.5	1.5
	n	24	24
Difference between clarity and brightness ratings (two-tailed t-test)		n.s	n.s

Table 4.7 Comparison of brightness and clarity ratings reported by Piper (1981).

Note: data taken from Figures 6 and 9 of Piper.

Vienot et.al (2009) used semantic differential scales to evaluate nine combinations of three illuminances and three CCT. This study was carried out in French, and the translated ratings included dark-bright and crepuscular-clear. Crepuscular relates to low light levels at dusk (Hornby, 2010) and does not provide the expected opposite of clear: it is assumed that this is an error of translation and the current analysis assumes the rating scale was hazy/clear. Vienot et al provide the means and standard deviations for all nine of their rating scales under each of the nine lighting conditions (Table 4.8). The two-tailed *t*-test does not suggest these ratings to be significantly different in seven of the nine cases; the difference is close to being significant at p=0.05 in one case (4000K, 300 lux) and is significant (p<0.05) in one case (2700K, 600 lx).

Rating scale	e	2700K	4000K	6500K	2700K	4000K	6500K	2700K	4000K	6500K
		150 lx	150 lx	150lx	300lx	300lx	300lx	600lx	600lx	600Ix
Dark-Bright	Mean	3.85	3.50	5.25	4.35	5.20	5.60	5.05	5.95	6.16
	Std Dev	1.27	1.10	1.52	1.04	1.28	1.10	1.05	0.83	0.69
Crepuscular	Mean	3.55	3.35	5.10	4.00	4.50	5.70	4.25	6.05	6.21
-Clear	Std	1.32	1.31	1.55	1.34	1.40	1.42	1.37	1.00	0.63
	Dev									
Difference b	etween									
brightness and										
crepuscular ratings		n.s.	n.s.	n.s.	n.s.	p≈0.05	n.s.	p<0.05	n.s.	n.s.
(two tailed t-	test)									

Table 4.8 Comparison of brightness and clarity ratings reported by Vienot et al (2009). Note; n=30 in all cases.

Note: data taken from Table 4 of Vienot et al.

Evaluations of spatial brightness and clarity using category rating were reported by Fotios and Cheal (2007) for lighting at mesopic levels. Eight items were rated (bright, dim, dark, clear, hazy, pleasant, warm and cool) along an 8-point scale with end points labelled *very much so* (1) and *not at all so* (8) and this was done under ten combinations of light source and illuminance by 47 test participants. The original data for these tests are available in the thesis presented by Cheal, (2007) and the individual ratings were used for the current analyses. Table 4.9 shows the mean results.

Table 4.9 Mean (and standard deviation) of brightness and clarity ratings as carried out by Fotios and Cheal (2007).

Lamp	Rating scale										
	Bright	Clear	Dim	Dark	Hazy						
<u>15 lux</u>											
LPS	5.1 (1.96)	5.4 (1.92)	3.6 (2.03)	3.4 (1.79)	3.4 (1.99)						
HPS	6.4 (1.23)	6.3 (1.54)	2.4 (1.28)	2.1 (1.15)	2.7 (1.68)						
CFL	7.1 (0.98)	6.9 (1.27)	1.9 (1.29)	1.5 (0.85)	1.9 (1.23)						
MH1	7.1 (0.85)	7.0 (1.41)	1.9 (1.07)	1.6 (1.08)	1.9 (1.29)						
MH2	7.2 (1.32)	7.4 (0.79)	1.6 (0.92)	1.3 (0.63)	1.7 (0.95)						
<u>2 lux</u>											
LPS	1.8 (0.72)	2.4 (1.38)	6.7 (1.57)	6.3 (1.50)	6.0 (1.75)						
HPS	2.2 (1.20)	2.5 (1.07)	6.3 (1.64)	6.1 (1.54)	6.3 (1.34)						
CFL	2.8 (1.42)	3.1 (1.49)	5.7 (1.95)	5.3 (1.78)	5.8 (1.74)						
MH1	2.8 (1.70)	2.9 (1.62)	6.2 (1.66)	5.7 (1.76)	5.7 (1.89)						
MH2	3.2 (1.50)	3.8 (1.61)	5.6(1.48)	4.8 (1.84)	5.0 (1.96)						

Note: original data available from Cheal (2007). Note that the end points of the response ranges for these ratings were labelled *very much so* (1) and *not at all so* (8).

These data were analysed using the Wilcoxon test for each of the ten combinations of lamp type and illuminance separately. For each combination, the results of ratings were compared for four pairs: bright-clear, dim-hazy, dark-hazy and dim-dark. Ratings of clear-bright and dim-hazy were not suggested to be different except for ratings made under the LPS lamp at 2 lux (p=0.032 and p=0.012 respectively). In only one case were ratings of hazy and dark suggested to be different (CFL, 15 lux, p=0.043) and in no cases were ratings of dim and dark suggested to be different. According to two-sample t-test, clear-bright are again suggested to be different and hazy-dim are close to being considered different under the LPS lamp at 2 lux (p=0.007 and p=0.060 respectively). Ratings of hazy and dim are close to being different under the MH2 lamp at 2 lux (p=0.072). Hazy-dark are close to be different under HPS lamp and different under MH1 and MH2 lamps under 15 lux (p=0.059, p=0.046 and p=0.045, respectively).

Thus of the forty comparisons of ratings from the Fotios and Cheal data, the Wilcoxon test suggests only three to be statistically different and the t-test suggests five to be different with two further cases close to significant.

4.3.2.2.4 Graphical analysis of studies not reporting variance data

Table 4.10 shows the studies that didn't present any statistical analysis and variance data (standard deviation) to statistically compare judgements of spatial brightness and visual clarity. The reported mean ratings of these judgements were used to draw graphs in order to investigate any trends between these judgements.

Comparison method	Study	Items rated	Method of comparison by study author(s)	Additional method of comparison	Agreement between ratings of brightness and clarity?
	Bartholomew, 1975	20 items including dull/bright, dark/light and blurry/clear using a 5 point response scale.	None	Graph of mean ratings	Yes for lamp 1; not certain for lamp 2
Comparison of mean ratings	Boyce & Cuttle, 1990	19 items including bright, dim, hazy, and clear using a 5-point response scale with end points labelled not-at-all-so and very- much-so.	None	Graph of mean ratings	Inconclusive

Some studies report the mean ratings for brightness and clarity and comparison of the trends enables similarity to be judged. In the absence of variance data such as standard deviations, comparison of mean results is an imprecise approach by which to draw conclusions. Bartholomew (1975) evaluated two lamps using 5-point response scales. Figure 4.10 shows the mean ratings of dark-light, dull-bright and blurry-clear. For lamp 1 these ratings are almost identical, while for lamp 2 the blurry-clear and dark-light ratings are also almost identical but the dull-bright rating is slightly different.



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Figure 4.10 Results of brightness and clarity ratings from Bartholomew (1975). These data were taken from Figure 1 of Bartholomew. In the original 5-point response ranges a rating of 5 identified the bright, light and clear ends of the scales. (Fotios and Atli, 2012)

Boyce and Cuttle (1990) used a 5-point response range of *very much so* (1) to *not at all so* (5) to evaluate room lighting using 19 items including bright, dim, clear and hazy. In the current review, the ratings of bright and clear have been reversed so that all scales have the same polarity. A stimulus considered to be bright would have a rating of 5 on both the bright and dim scales, and thus the two scales provide a measure of internal consistency. Similarly a rating of 5 on the clear and hazy scales denotes a stimulus considered to have high visual clarity. Boyce and Cuttle presented mean ratings using a graph from which were estimated the data used below, but the report did not include standard deviations.

Figure 4.11 shows mean ratings at the four different CCT used in experiment 1, and these are averaged across the four levels of illuminance. For the 2700K lamp the ratings of clear and hazy, and also dim and bright, are almost identical and this suggests good internal consistency. That these ratings coincide so precisely suggests that the difference between the bright/dim and clear/hazy ratings, being larger, may be significant; unfortunately there are insufficient data to test this. Ratings for the remaining three CCTs do not suggest such a separation.





Figure 4.11 Mean ratings at the four different CCT used in experiment 1: these are averaged across the four levels of illuminance (1990). These data were taken from Table 10 of Boyce and Cuttle. Mean ratings for the bright and clear scale were reversed so that a rating of 5 denotes a stimulus that appeared bright and clear using all four scales. (Fotios and Atli, 2012)

Figure 4.12 shows mean ratings at the three different CCT used in experiment 2, and these were carried out at only one illuminance. Similarly to the 2700K lamp in Figure 4.11, the data for all three lamps on Figure 4.12 show good internal consistency for ratings along the clear and hazy scales, and also the bright and dim scales, and a clear difference between these two groups of ratings.



Lighting condition (ccr)



Figure 4.13 shows mean ratings at the four different illuminances in experiment 1 from Boyce and Cuttle (1990): these ratings are averaged across the four levels of CCT. All four rating scales demonstrate that with an increase in illuminance there is a concomitant increase in both brightness and visual clarity. The differences between ratings of brightness and clarity appear to be smaller at high illuminances than at low illuminances. The Vienot et al (2009) and Fotios and Cheal (2007) studies also included ratings at more than one level of illuminance: while the Fotios and Cheal data (2.0 and 15.0 lux) suggest a very slight difference between bright and clear ratings at low illuminance than at high illuminance, the Vienot et al data (150, 300 and 600 lux) do not. Similarly, the data reviewed in the current study do not suggest that CCT has a consistent effect on the relationship between ratings of brightness and clarity.





Figure 4.13 Mean ratings at the four illuminances used in experiment 1: these are averaged across the four levels of CCT (Boyce & Cuttle, 1990). These data were taken from Table 9 of Boyce and Cuttle. Mean ratings for the bright and clear scale were reversed so that a rating of 5 denotes a stimulus that appeared bright and clear using all four scales. (Fotios and Atli, 2012)

Thus the data presented by Boyce and Cuttle do not provide a conclusive opinion to agreement between ratings of brightness and clarity, with some data suggesting similarity and other data suggesting they are not the same. The absence of mean ratings for individual stimuli (ratings in experiment 1 are averaged across four levels of CCT or illuminance) and the absence of variance indices hinder analyses of these data.

Boyce (1977) used questions of "*How satisfactory is the lighting level in the office?*" and "*How visually distinct are the details in the office?*" to relate with brightness and clarity respectively in the category rating parts of his experiments. His study is of interest because a large number of stimuli were used in the two experiments. Experiment 1 used three types of lamps, two illuminances and three levels of interior colourfulness, giving 18 stimulus combinations. Experiment 2 used four types of lamps, two illuminances and either a coloured or achromatic environment, giving 16 stimulus combinations. The mean ratings are shown in Figure 4.14 and

these were gained using 7-point response scales. For the satisfaction with light level question the scale ranged from 1 (very satisfactory) to 7 (very unsatisfactory), and for the distinct details question the scale ranged from 1 (very unclear) to 7 (very clear). In Figure 4.14 the rating scale for the satisfaction with light level question has been reversed so that a rating of 7 indicates a very satisfactory light level, and the scale polarity is therefore consistent with the previous figures. It can be seen that mean ratings tend to follow the same relationship for different stimuli – a stimulus which is considered to be satisfactory in light level would also be considered very clear in distinctness of details. Linear regression between the 34 mean ratings of satisfaction with light level and distinction of details has a correlation of R^2 =0.82.



--+-- distinctness of detail

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Figure 4.14 Mean ratings from Boyce (1977) for *How satisfactory is the lighting level in the office?* and *How visually distinct are the details in the office?*. These are Boyce's experiment 1 (top) and experiment 2 (bottom) and the data were taken from Table 4 (experiment 1), Table 9 (experiment 2, coloured) and Table 11 (experiment 2, achromatic) of Boyce. The original ratings of satisfaction with light level have been reversed so that a rating of 7 indicates lighting considered to be very satisfactory in light level and details are very clear. Note that the stimuli in

each graph are arranged in order of descending ratings of satisfaction with light level. (Fotios and Atli, 2012)

4.3.2.2.5 Overall Spatial Brightness and Visual Clarity Correlation

Figure 4.15 shows the correlation between 43 pairs of mean clarity ratings and mean brightness ratings from five previous studies (Boyce, 1977; Flynn and Spencer, 1977; Fotios and Cheal, 2007; Rea, 1982; Vienot et al, 2009; Vrabel et al 1998). These are studies in which both brightness and clarity scales were used and in which more than two stimuli were used to ensure meaningful correlation analysis: thus the results from Piper (1981) and Bartholomew (1975) are not included. Also excluded are the results from Boyce and Cuttle (1990) as these show mean ratings averaged across multiple levels of lamp type and illuminance but not for each combination of lamp type and illuminance separately. The brightness scales for Flynn and Spencer were reversed so that these had the same positive polarity as did their brightness ratings. For the Fotios and Cheal study Figure 4.15 uses their results for two ratings scales, bright and clear. Linear regression suggests a coefficient of linear determination R²= 0.86 for the complete set of 77 pairs of mean ratings which demonstrates a trend for stimuli receiving high ratings of brightness to also receive high ratings of clarity.



Figure 4.15 Mean brightness ratings plotted against mean clarity ratings as reported in previous studies.

4.3.3 Conclusion: Defining visual response

In four studies, the data do not suggest that brightness and clarity ratings are different (Boyce, 1977; Flynn & Spencer, 1977; Fotios & Cheal, 2007; Piper, 1981; Vienot et al, 2009). In the DeLaney et al (1978) study, it is reported that brightness and clarity lead to different ratings but

comparison of the mean ratings suggest otherwise. In the Rea (1992) and Bartholomew (1975) studies the ratings are almost identical for half of the stimuli examined and similarity for the remainder cannot be determined. Data from the Boyce and Cuttle (1990) study are inconclusive as to whether the ratings are different. Thus results from the majority of studies indicate that ratings of brightness and clarity lead to similar judgements.

In one study (Vrabel et al, 1998) the ratings of brightness and clarity were reported be different. What is interesting about the Vrabel et al study is that they provided test participants with written definitions of brightness and clarity prior to trials. This gives rise to a possible explanation for the findings of this review: when naïve test participants are provided with definitions of brightness and clarity then this encourages different judgements, but they do not discern a difference when these terms are undefined. Note however that Fotios and Cheal (2007), who used illuminances in the mesopic region, also provided definitions of their rating items to test participants at the commencement of each test session: this was a visual demonstration of brightness and written definition of clarity (and as reported by Cheal (2007) this written definition was taken from Vrabel et al.) and their results suggest a high degree of correlation between ratings of brightness and clarity. It should also be noted that further clarification is required as to the statistical basis for the decision made by Vrabel et al and hence that the reported differences may in fact be by chance.

A further caveat is that while the results from these tests do not tend to suggest a difference between spatial brightness and visual clarity, this does not mean that individual test participants did not perceive and intend to convey a difference. It may be that there is a difference in their judgements but that this is lost in the variance. It is unfortunate that many studies did not report the standard deviations or other measures of variance.

In many cases this review concludes that there is no difference between brightness and clarity ratings for a particular stimulus; "no difference" is almost always a weak conclusion. Further work is required to provide a higher standard of evidence to demonstrate that the existing indifference between clarity and brightness phenomena is not due to weaknesses in the way the experimental questions were asked, the interpretation of the experimental questions by the test participants, and/or the methods of analysis.

4.3.4 Summary

For the design of future experiments involving evaluations of the visual environment this review suggests that experimenters need to take further effort to define to test participants the nature of their rating items, as has previously been recommended (Houser and Tiller, 2003; Tiller and Rea, 1992). The items rated should be something that can be defined (e.g. spatial brightness) rather than asking about a fuzzy concept such as visual clarity. There is also a need for caution when interpreting the results of such tests: just because the test instructions requested

judgement of a certain parameter it does not mean that the results gained from test participants are for the same visual phenomena as the experimenter assumed.

Besides the definitions of the rating items, number of response categories might also affect the answer of the participants. Even though the mean rating of the stimuli wouldn't change, providing a middle response might result in different distributions than an even response category. Therefore, the tendency to choose the middle response may show an increase.

Chapter 5

5 New Experiment of Spatial Brightness

- 5.1 Introduction
- 5.2 Lamps
- 5.3 Apparatus
- 5.4 Test Procedure
- 5.5 Results and Analysis
- 5.6 Discussion
- 5.7 Summary

5.1 Introduction

This chapter describes an experiment conducted to investigate spatial brightness at photopic levels under lighting of different S/P ratio and gamut area. This was done to provide additional examination of S/P ratio and GA. Two experimental methods were used, from studies by Berman et al (1990) and Fotios and Cheal (2011). Berman et al (1990) considered S/P effects on spatial brightness at photopic light levels and used rapid sequential discrimination method. Fotios and Cheal (2011) conducted an experiment with both discrimination and matching methods to examine the relation of different lamp SPDs with spatial brightness at mesopic light levels. These two procedures (matching and discrimination) were adopted following review of methodology (chapters 3 and 4) which suggested that the adjustment and category rating methods were not sufficiently unbiased or not appropriate for gaining information about the magnitude and direction of the SPD-illuminance relationship for spatial brightness.

Two different methods of experiment used to understand if two methodologies used to test the same features give similar answers and can be used to validate the gained results. Besides, questions rose about if three alterations of two stimuli to discriminate between their brightness in sequential task are sufficient and if colour surfaces in the space effects spatial brightness were investigated.

As the study utilised human participants, steps were taken to ensure the treatment of each person met an approved ethical standard. This approach was a requirement within the University which gave general guidelines to protect the rights and interests of participants. The main principle accepted for this type of research can be summarized as not to harm, keep the confidentiality of personal information and to have informed consent. Copies of the participant information sheet and consent form (see Appendix D) were approved by the ethics committee at the University before the experiments were conducted. Participants were accommodated by word-of-mouth and they attended the experiment voluntarily.

5.2 Lamps

Three SPDs were generated for these trials using an LED array as shown in Figure 5.1. This comprised two identical, linear arrays of LEDs, with each array containing six clusters of four types of LED having different chromaticities (Table 5.1). The control system allowed the intensity of each type of LED to be independently modulated, thus allowing a wide range of unique spectra to be set. Of particular note for the current work, the four-LED system allowed for S/P ratio to be varied whilst maintaining a constant chromaticity. The LED arrays were fitted to the test booth above the position of the observer's head, and thus there was no direct sight of the source.



Figure 5.1 One of the LED arrays with six clusters of four types of LED.

Primary LED	x	У
Red	0.698	0.302
Green	0.154	0.666
Blue	0.146	0.036
Amber	0.592	0.407

Table 5.1 Chromaticities (CIE 2°) of the LEDs used in the array.

These were supplied by John Barbur and colleagues at City University London.

The current work required three different SPDs, following Berman et al two having identical chromaticity but different S/P ratios (SPDs A and B), and a third (SPD C) having similar S/P ratio but different chromaticity to SPD B. While choosing SPDs A and B to have chromaticities similar to that of Berman et al, main criterion was to seek the highest and lowest S/P ratios possible with the LED array. For SPDs B and C, the highest difference for GA of the two SPDs were searched while keeping S/P ratios as similar as possible. These three SPDs are identified in Figure 5.2 and Table 5.2 displays their chromaticity, S/P ratio and gamut area. The values in Table 5.2 were derived from spectral power distributions measured from the observers' viewpoint, and are thus the lamp SPDs as modified by internal reflection in the test apparatus. Measurements were recorded using a Konika-Minolta CS1000 spectroradiometer, calibrated immediately prior to this experiment. In Berman et al chromaticities were reported in CIE 10° and this study tried to obtain similar chromaticities with Berman et al, therefore the values were reported in CIE 10° in Table 5.2.



Figure 5.2 Spectral power distributions of the SPDs. These were measured from the observers view point and hence include modification by the test apparatus, and are normalised for a peak response of 1.0.

Table 5.2 Description of the LED spectra and blended fluorescent lamps used in brightness assessments in Berman et al (1990).

l ight setting	10° chromaticity		S/P	Gamut Area			
Light ootting	Х ₁₀	y 10	0/1	Cullut Arcu			
SPD used in current work							
А	0.49	0.40	1.02	0.0017			
В	0.49	0.40	1.77	0.0041			
С	0.44	0.36	1.81	0.0069			
Lamps used by Berman et al, 1990							
R213	0.46	0.42	2.40	N.R			
WWG	0.48	0.41	0.85	N.R			

For the current work, all properties were derived from SPD measured from observer's view of test apparatus. Note: Berman et al did not report S/P ratios: these were determined from photopic and scotopic luminances reported in their Table 2. N.R = Not Reported.

5.3 Apparatus

This experiment was carried out using the single booth shown in Figure 5.3, a similar apparatus to that used by Royer and Houser (2012). The viewing chamber of the booth was of approximate dimensions 900 mm deep, 1000 mm wide and 1150 mm high. Test participants sat at the front of this booth, a distance approximately 700 mm from the rear wall and thus the sides extended behind their head, giving full field stimulation of the retina.



Figure 5.3 Drawing (not to scale) of the test apparatus. Dimensions are in mm.

The interior surfaces were painted with a matt white paint having a reflectance of approximately 0.8, this being uniform across the visible spectrum as shown in Figure 5.4. This environment was purposefully neutral, following Berman et al. Colour was introduced for some trials using a Mondrian array covering the back wall of the booth as presented in Figure 5.5. This array contained three colours (red, yellow and blue) of approximately equal proportions. Similarly, coloured areas and achromatic areas were arranged to have approximately equal proportions. Table 5.3 shows chromaticity coordinates of the coloured papers under Lamp A at 67 cd/m².



Figure 5.4 Spectral reflectance of interior of cabinet (measurements done at 83 cd/m² with x,y of 0.312, 0.324 and S/P of 2.8) Note: The deviation away from a spectral ratio of 1.0 occurs at the shortest wavelengths where the relative power of the SPD was low (tending towards zero). This is not considered as a problem since it is associated with a very low luminances.



Figure 5.5 Photography of the test booth and Mondrian pattern. Note: for clarity in this photograph the test lighting is switched off and the laboratory lighting is switched on.

Colour	2° chromaticity			
Coloui	X ₂	У 2		
Black	0.319	0.334		
Blue	0.248	0.253		
Red	0.501	0.323		
Yellow	0.448	0.436		
White booth	0.321	0.341		

Table 5.3 Chromaticity coordinates of coloured papers used in Mondrian like pattern.

Past results of Fotios and Cheal (2011) suggest that the degree of colourfulness does not significantly affect the results of spatial brightness judgements according to the brightness assessments completed with four field designs and four lamp pairs using matching procedures. One of the four conditions was an achromatic interior of the side-by-side booths; two of the chromatic fields included either coloured objects or coloured surface. The last condition was a uniform field covering the front opening of the achromatic booths with a neutral and uniform sheet. Their results suggested negligible difference between these four fields. Thus, there was no a-priori reason for the selection of these particular colours.

5.3.1 Luminance distribution

It was important for the distribution of surface luminances to be stable under changes of SPD and luminance, i.e. that luminances measured at various points around the cabinet interiors varied proportionally. Differences were not expected since all SPDs examined were provided by the same LED array. To assess the stability of the relative luminance distribution between different SPDs and luminance settings, luminances were measured at a grid of 26 points across the rear and side walls and floor of the booth, with the luminance meter (Konica-Minolta LS100 calibrated prior to this experiment) aimed from the participant's viewpoint (Figure 5.6). With all SPDs and luminances, the luminance distribution was approximately constant along the horizontal direction and varied 20% from ceiling to floor.



Figure 5.6 Luminance measurement points (1-26 in red font) in booth. Measurements were done with angles of 20° and 45° with top and bottom LED arrays, respectively. Distances of reference points from the edges were shown with arrows in centimetres.

5.4 Test Procedure

Tests with each participant were completed in a single two-hour session. Lighting for the initial test session was provided by SPD A set to 67 cd/m^2 . In this time the participant was given instructions for the test procedure and completed a distraction test of attempting to place FM-100 colour chips into correct order, each participant finished this session in their own timing which was at least 10 min. The first lighting condition for the experiment was presented following this adaptation.

For a given SPD pair, six steps were carried out (three of them with achromatic and the other three with chromatic environment):

Achromatic environment

Berman et al discrimination task Fotios & Cheal discrimination task Fotios & Cheal matching task

Chromatic environment

Berman et al discrimination task Fotios & Cheal discrimination task Fotios & Cheal matching task
Participant 1	Berman (Ach.) Berman (Ch.) F&C Disc. (Ch.) F&C Disc. (Ach.) F&C Matching (Ach.) F&C Matching (Ch.)
Participant 2	F&C Matching (Ach.) F&C Matching (Ch.) Berman (Ch.) F&C Disc. (Ch.) Berman (Ach.) F&C Disc. (Ach.)
Participant 3 etc.	Berman (Ch.) F&C Disc. (Ch.) Berman (Ach.) F&C Disc. (Ach.) F&C Matching (Ach.) F&C Matching (Ch.)

Table 5.4 Presentation order of the six steps for first three participants

Note: F&C Disc. refers to Fotios and Cheal Discrimination method. Ch:Chromatic Ach: Achromatic

As shown in Table 5.4 the order of the six steps was counterbalanced between test participants. Within a test session, the matching and discrimination trials were carried out as separate blocks, the order of these being balanced. Within the discrimination block, the Berman et al procedure and Fotios and Cheal procedure were carried out in a balanced order. The three procedures were used with both achromatic and coloured interior surfaces, the order of which being balanced. SPD pairs were presented in an order that was randomised between participants.

The Berman et al task was carried out using two SPDs (A and B), the aim being to replicate their work while testing the S/P ratio effect on spatial brightness, which was one of the metrics found to correlate well with brightness both in linear and stepwise regression tests (see Chapter 5). For the Fotios and Cheal task the third SPD was introduced (C) and the three SPDs were presented in all three possible pairs (i.e. A/B, A/C and B/C). This set of SPD pairs was selected to test S/P ratio and GA effect on spatial brightness.

Luminance was adjusted using two mechanisms. For the experimenter, this was done by using the control software to set a previously determined luminance. For test participants during the matching task, adjustment was carried out using a rotary dial, this having three 360° turns from minimum to maximum to reduce the chance of a positional cue.

5.4.1 Participants

28 test participants were used and were confirmed as having colour-normal vision using the Ishihara test were used. Fourteen were male and 14 were female of 28 samples, and their ages were in the range of 22 to 42 years.

The results of Fotios and Cheal (2011) mesopic brightness matching data suggested an effect size of 0.79. The calculation of the effect size was done by Equation 5.1 using the mean ratings (0.89) and standard deviations (0.14) of Fotios & Cheal (2011) results:

effect size= $(\mu^1 - \mu^2) / \sigma$ (Faul et al, 2007) Equation 5.1 μ^1 =population mean (0.89) μ^2 =no effect of spectrum (1.00) σ =standard deviation in population (0.14) effect size= 0.79

An effect of 0.8 was suggested to be a large effect by Cohen (1992) for which a sample size of 28 is sufficient to detect the standard level of probability of mistakenly rejecting the null hypothesis (α =0.05) with a common assumption of power of 0.80 (Field, 2005). When these values of effect size, α -level, and statistical power applied in *G*Power*, which was recommended by Field (2005) to be a powerful tool to calculate sample size, required sample size was found to be 21 (Faul et. al, 2007). Besides, the demands of the variance stable rank sums method for analysing data from the Fotios and Cheal discrimination procedure and judgements, which will be used to analyse the data collected from current study, required 17 test participants to insure the possibility of the three SPD being significantly different at an alpha level of 0.01 (Dunn-Rankin et al, 2004). This is a slightly larger sample than used in previous works of Berman et al (n=12) and Fotios and Cheal (n=21).

5.4.2 Procedure I: Berman et al

This part of the experiment used the Berman et al (1990) procedure to compare the brightness of two sources of identical chromaticity using full field stimulation. Berman et al conducted a rapid sequential discrimination test using four comparisons of two SPDs in a room. They selected two SPDs with identical chromaticity but different S/P ratios. Three of the comparisons had same luminance ratios: two with different SPD, one with the same SPD and fourth comparison had higher luminance ratio than the other three comparisons. In their study, comparisons 1 and 2 used to have two different typical interior light levels. Three comparisons with two SPDs were used to test brightness lumens model (Equation 2.3). The comparison with the same SPD was a null condition.

The task used by Berman et al (1990) was followed as near as possible. The key differences were:

 In Berman et al test participants were located inside a small room (2 m deep, 2 m wide and 2.3 m high). The LED array available did not offer sufficient power to light this environment to the same luminance and uniformity as in Berman et al and hence a smaller space was used, as shown in Figure 5.3. This allowed full field stimulation, the critical requirement of Berman et al.

- An LED array was used rather than blended fluorescents lamps.
- The LED array did not enable the identical chromaticity to that used by Berman et al, but it is close (see Table 5.2). What was done was to ensure that the two SPDs used (A and B) were of the same chromaticity.
- The LED array did not permit as great a difference in S/P ratio between the two sources as did Berman et al. This was accounted for by using Brightness Lumens (see Equation 5.2) to predict the luminances required for equal brightness and resulted in a smaller luminance difference being used in trials than were used by Berman et al.
- A null condition with comparison between settings of the same SPD and luminance was added in order to better validate the procedure. In particular, whether the three successive presentations of each SPD was sufficient to offset interval bias (Fotios & Houser, 2013).

Berman et al used a sequential discrimination procedure to compare two SPDs. These were not compared on an equal luminance basis but with the luminances presented in four specific conditions (Table 5.5). In comparison 1, SPD B (high S/P ratio) was presented at a lower luminance (40 cd/m²) than was SPD A (low S/P ratio: 47 cd/m²) to demonstrate that test participants would tend to report SPD B as brighter than A despite the lower luminance. Comparison 2 repeated comparison 1, using the same ratio of photopic luminances, but at a higher absolute luminance (67 and 57 cd/m² for A and B respectively), thus to examine spatial brightness at two typical interior light levels. Note that luminances reported here were as measured on the rear wall of the booth, at the centre point 700 mm above the floor, approximately the observer's view point if looking straight ahead.

	Comparison	Comparison	Comparison	Comparison	Comparison
	1	2	3	4	5
				Control	Null
	A / B	A / B	A / B	B / B	A / A
Photopic					
luminances	47 / 40	67 / 57	67 / 40	67 / 57	47 / 47
(cd/m ²)					
Scotopic					
luminances	48 / 71	68 / 101	68 / 71	119 / 101	48 / 48
(cd/m ²)					
Luminance ratio	1 1 9	1 1 9	1.69	1 1 0	1 00
(higher /lower)	1.18	1.18	1.08	1.18	1.00
Predicted brighter	В	В	А	Higher	Equal
setting	_	_		luminance	- 4
	1	1	1	1	

Table 5.5 Lighting conditions examined in the stages of experiment repeating Berman et al (1990) and additional null condition (comparison 5).

The luminances at which SPDs A and B would appear equally bright were predicted using Brightness Lumens (Equation 2.3, Chapter 2), a tentative metric for the effect of lamp spectrum on spatial brightness (Berman, 1995). Spaces lit by two different lamps of equal brightness lumens would appear equally bright.

SPD A with a photopic luminance of 67 cd/m² was chosen as the reference, this being the luminance as used by Berman et al for their low S/P source in comparison 2. According to brightness lumens, SPD B requires a photopic luminance of 51 cd/m² for equal brightness (a photopic luminance ratio of 67/51 = 1.31):

Brightness Lumen	s	= F	P _A (S/P) _A ^{0.8}	${}^{5} = P_{B} (S/P)_{B}^{0.5}$	Equation 5.2
		=	67 (1) ^{0.5}	$= x (1.77)^{0.5}$	
Accordingly,	х	=	51		

To promote a tendency for SPD B to be identified as brighter, this was presented in comparison 2 at a luminance of 57 cd/m^2 , slightly above that was needed for equal brightness but still a lower photopic luminance than SPD A.

In comparison 3 SPD B (high S/P ratio) was presented at a much lower luminance (40 cd/m²) than was SPD A (low S/P ratio: 67 cd/m²), a luminance ratio of 1.68 compared with the ratio of 1.18 used in comparisons 1 and 2. In this situation it was expected that test participants would

tend to identify SPD A as brighter, the higher luminance of SPD A now outweighing the higher S/P ratio of SPD B. In comparison 4 the two stimuli compared were identical in spectra (SPD B) but of different luminances, the ratio (1.18) being similar to that as used in comparisons 1 and 2. This is a control comparison which examines whether the luminance differences used in comparisons 1 and 2 are discriminable. For this study an additional comparison was added, comparison 5, a null condition in which both settings had identical SPD and luminance.

Comparison 1						
*A47 / B40						
A47	presented for 5 sec					
dark	presented for 100 millisec					
A40	presented for 5 sec					
dark	presented for 100 millisec					
A47	presented for 5 sec					
dark	presented for 100 millisec					
A40	presented for 5 sec					
dark	presented for 100 millisec					
A47	presented for 5 sec					
dark	presented for 100 millisec					
A40	presented for 5 sec					
particip	ant asked to choose the brighter lamp					
	A47 dark A40 dark A47 dark A47 dark A40 dark A47 dark A47 dark A47 dark A47 particip					

Table 5.6 An example of presentation order of the paired lighting conditions in Berman discrimination task.

* A47 denotes SPD A with a luminance of 47 cd/m².

A rapid sequential evaluation mode was applied during the experiment. As shown in Table 5.6, in a trial, each source was presented for 5 s, with three presentations of each source separated by a 100 ms dark interval. For each of the five comparisons, the two stimuli were compared ten times. Presentation order was counterbalanced, with each SPD presented first for five of the ten evaluations. Hence this required a test participant to provide 50 evaluations each for the chromatic and achromatic conditions. The five comparisons were carried out randomly within the block of 50 evaluations.

Following Berman et al, the two sources being compared were identified by the experimenter to the test participant by giving each source a random number (from within the range 1 to 9) and test participants were informed of each source using this number, e.g. 'Here is number 3, here is number 7' repeated three times. The question was then asked '*Which one appeared brighter?*' Responses were recorded by the experimenter with bespoke software to display the stimuli and record the answers. The last presentation remained on while the question was asked and until the next sequence began, approximately 6-7 seconds later. The instructions

were clarified as follows: 'By brightness we mean the amount of light in the booth, ignoring any colour differences between lights and surfaces. The two different number labels for the lights in each pair are taken randomly from the range 1 to 9. When a number comes up again it does not mean the same light as in a previous pair; judge the current lights only; try to avoid being influenced by memory of previous lights.'

5.4.3 Procedure II: Fotios & Cheal matching

The second procedure follows that used by Fotios and Cheal (2011) who examined spatial brightness at mesopic levels of adaptation. Fotios and Cheal (2011) compared five different SPDs, for each comparison one SPD was defined as reference and the other one was adjusted by the participant to have the same spatial brightness with the reference in side-by-side booths. For validation of the matching results, discrimination task with both booths at reference illuminance lit by different SPDs were conducted with forced choice procedure. Same experimental methods applied in this study at photopic levels of adaptation with sequential mode in one booth instead of simultaneous presentation in side-by-side booths. Since, experiments conducted by Fotios and Cheal (2010) using both matching and discrimination tasks with side-by-side and sequential modes presented same results at mesopic light levels, it was not expected that this difference would significantly affect the results. For concurrent validation of the matching results a brightness discrimination task was also included within the procedure, this being the third procedure of the current study.

Light settings were seen in pairs, presented sequentially. Each source was presented for 5 s, with at least three presentations of each source separated by a 100 ms dark interval. Participants were able to see the stimuli as many times as they need, before the experimenter recorded the final evaluation. The first presented SPD was always at the reference luminance (50 cd/m^2) and the participant adjusted the luminance of the second SPD until the two appeared, as near as possible, equally bright. As above, brightness was described as the amount of light in the whole scene, which could be judged independently from any other visual differences such as colour.

A/B	B/C	A/C	A/A
			Null-condition
 *A50 / B75	B50 / C75	A50 / C75	A50 / C75
A50 / B25	B50 / C25	A50 / C25	A50 / C25
B50 / A75	C50 / B75	C50 / A75	
B50 / A75	C50 / B75	C50 / A75	

Table 5.7 Fourteen lamp pairs presented in Fotios and Cheal Matching task.

*A50 denotes SPD A with a luminance of 50 cd/m².

Table 5.7 shows that each test participant provided four brightness matches for each of the three SPD pairs and two matches for the null condition of SPD A at 50 cd/m2, counterbalancing both the initial luminance of the variable stimulus (set by the experimenter to a level clearly higher or lower than the reference, luminances of 75 cd/m² and 25 cd/m² respectively) and application of dimming to both sources. These trials were carried out in a random order and all 14 pairs were evaluated in both chromatic and achromatic environment by each participant, which was making a total of 28 lamp pairs.

5.4.4 Procedure III: Fotios & Cheal discrimination

For discrimination judgements, two SPDs were presented sequentially and test participants instructed to state which was the brighter, a forced-choice procedure with the equally bright response option not permitted. Each source was presented for 5 s, with three presentations of each source separated by a 100 ms dark interval. Both SPDs provided the same luminance, 50 cd/m^2 . The SPD sequence (first or second) was random within the eight evaluations in each of the achromatic and chromatic environments. The null condition trial was with SPD A at 50 cd/m^2 .

5.5 Results and Analysis

5.5.1 Procedure I: Berman et al

The results of trials carried out using the Berman et al procedure are shown in Table 5.8 as achromatic and Table 5.9 as chromatic. These data presents the number of times the participant chose the given condition as brighter. In total, 280 trials (28 subjects x 10 repeats) were evaluated. Thus, 140 votes per SPD in a given pair would indicate equal brightness. This is the result found with one of the null condition trials (comparison 5), which was having the luminance and SPD the same with each other, suggesting negligible interval bias. These data are repeated measures and are not drawn from a normally distributed population. Analysis using the Wilcoxon signed-ranks test did not suggest differences between the first and second intervals to be significant, for trials with either the chromatic or achromatic environments.

	Compa	rison 1	Compa	rison 2	Compa	rison 3	Compa	omparison 4		Comparison 5	
Test					Control Null			ull			
Participant	A47*	B40	A67	B57	A67	B40	B67	B57	A47	A47	
									(1 st)	(2 nd)	
1	5	5	6	4	10	0	10	0	4	6	
2	3	7	3	7	10	0	6	4	3	7	
3	4	6	2	8	10	0	10	0	6	4	
4	4	6	2	8	10	0	10	0	3	7	
5	7	3	7	3	10	0	10	0	6	4	
6	1	9	1	9	10	0	10	0	4	6	
7	9	1	9	1	9	1	9	1	9	1	
8	10	0	4	6	10	0	10	0	10	0	
9	6	4	5	5	6	4	4	6	6	4	
10	8	2	6	4	10	0	10	0	3	7	
11	3	7	0	10	10	0	10	0	6	4	
12	3	7	1	9	10	0	9	1	4	6	
13	4	6	1	9	10	0	10	0	6	4	
14	0	10	1	9	10	0	10	0	4	6	
15	7	3	4	6	9	1	10	0	5	5	
16	7	3	2	8	9	1	10	0	6	4	
17	2	8	4	6	10	0	10	0	5	5	
18	7	3	6	4	10	0	10	0	5	5	
19	1	9	0	10	10	0	10	0	8	2	
20	0	10	0	10	10	0	10	0	5	5	
21	8	2	5	5	10	0	10	0	1	9	
22	8	2	3	7	10	0	10	0	3	7	
23	5	5	6	4	10	0	10	0	6	4	
24	9	1	4	6	10	0	10	0	6	4	
25	0	10	0	10	1	9	10	0	4	6	
26	6	4	4	6	10	0	10	0	6	4	
27	2	8	3	7	10	0	10	0	6	4	
28	6	4	8	2	10	0	10	0	6	4	
TOTAL	135	145	97	183	264	16	268	12	146	134	
Mean	4.8	5.2	3.5	6.5	9.5	0.6	9.6	0.4	5.2	4.8	
Std Dev	2.96	2.99	2.55	2.55	1.83	1.83	1.35	1.35	1.89	1.89	
Median	5	5	3.5	6.5	10	0	10	0	5.5	4.5	

Table 5.8 Results of achromatic discrimination trials following the Berman et al procedure.

These data are the frequencies by which each of a pair of stimuli was considered to be brighter. *A47 denotes SPD A with a luminance of 47 cd/m².

	Compa	arison 1	Compa	rison 2	Compa	parison 3 Comparison 4		Comparison 5		
Test							Cor	ntrol	N	ull
Participant	A47*	B40	A67	B57	A67	B40	B67	B57	A47	A47
									(1 st)	(2 nd)
1	7	3	6	4	10	0	10	0	5	5
2	6	4	4	6	10	0	10	0	4	6
3	5	5	3	6	10	0	10	0	5	5
4	10	0	7	3	10	0	10	0	7	3
5	9	1	6	4	10	0	10	0	5	5
6	0	10	1	9	10	0	9	1	6	4
7	10	0	10	0	10	0	10	0	4	6
8	1	9	0	10	8	2	10	0	6	4
9	8	2	4	6	10	0	9	1	4	6
10	10	0	8	2	10	0	10	0	6	4
11	4	6	6	4	9	1	8	2	6	4
12	4	6	3	7	9	1	10	0	5	5
13	2	8	2	8	9	1	10	0	6	4
14	3	7	1	9	10	0	10	0	3	7
15	8	2	9	1	10	0	10	0	5	5
16	8	2	4	6	10	0	10	0	7	3
17	4	6	1	9	7	3	9	1	4	6
18	9	1	10	0	10	0	10	0	5	5
19	10	0	3	7	10	0	10	0	6	4
20	0	10	1	9	0	10	10	0	4	6
21	6	4	7	3	10	0	10	0	5	5
22	9	1	9	1	10	0	10	0	2	8
23	7	3	9	1	10	0	10	0	5	5
24	3	7	1	9	10	0	10	0	3	7
25	0	10	0	10	8	2	10	0	9	1
26	7	3	4	6	10	0	10	0	4	6
27	6	4	4	6	10	0	10	0	4	6
28	3	7	4	6	7	3	10	0	6	4
TOTAL	159	121	127	152	257	23	275	5	141	139
Mean	5.68	4.32	4.54	5.43	9.18	0.82	9.82	0.18	5.04	4.96
Std Dev	3.29	3.29	3.16	3.14	2.02	2.02	0.48	0.48	1.43	1.43
Median	6	4	4	6	10	0	10	0	5	5

Table 5.9 Results of chromatic discrimination trials following the Berman et al procedure.

These data are the frequencies by which each of a pair of stimuli was considered to be brighter. * A47 denotes SPD A with a luminance of 47 cd/m².

In the control condition trial (comparison 4), one of the stimuli was at higher luminance than the other and the SPDs were equal. Almost 100% of the participants evaluated the stimulus with higher luminance to be brighter than the other interval. This difference was confirmed to be

significant for both the chromatic and achromatic environments using the Wilcoxon test (p<0.01). The result of comparison 4 is of interest because the luminance ratio presented is identical to that used in comparisons 1 and 2 where the SPDs of the two settings were different, with the SPD of higher S/P ratio (B) being presented at the lower luminance.

For three of the cases (comparisons 1 with achromatic and comparison 2 both with chromatic and achromatic settings), SPD B was voted to have brighter environment. However, in chromatic comparison 1, SPD A was considered to be brighter. Even though there was small difference between SPD A and B in achromatic comparison 1, the results were close to indicate equal brightness. Comparisons 1 and 2 provided the same ratio (higher/lower) of photopic luminances and the same SPD pairs, but comparison 2 was carried out at a higher absolute luminance than comparison 1. For both the achromatic and chromatic data, at the higher luminance (comparison 2) there was a higher frequency of reports that the high S/P ratio setting (B) was brighter: at the lower luminance, the two settings were of near equal brightness in the achromatic environment and the high S/P ratio source was considered to be the dimmer in the chromatic environment. Differences between comparison 1 and comparison 2 are suggested to be significant (p<0.01) using the Wilcoxon test.

These data suggest that the relative luminances required for equal brightness varies with absolute luminance. Although, this finding disagrees with past studies (Fotios and Levermore, 1997; Boyce, 1977), there are some recent publications suggesting that spectral sensitivity for spatial brightness might differ for different light levels (Rea et al, 2011).

Following Berman et al, the Wilcoxon test was applied to comparison 1 with comparison 4 and similarly to comparison 2 with comparison 4 in order to investigate an SPD effect. The differences were confirmed to be significant for both the chromatic and achromatic environments for all four cases (p<0.01). For comparisons 1, 2 and 4 the luminance ratios were the same, the difference was in the compared SPDs. In comparisons 1 and 2 SPD A was compared with SPD B, however in comparison 4 SPD B was compared with itself. Therefore, significant difference of these comparisons demonstrate a SPD effect, showing that the higher S/P ratio of setting B led to fewer reports that setting A was the brighter.

In comparison 3, the SPD of higher S/P ratio (B) was again presented at the lower luminance, but the difference was much larger than in comparisons 1 and 2. The results demonstrate a near 100% frequency for the SPD of higher luminance to be brighter. What comparison 3 shows is that if higher S/P ratio does lead to higher spatial brightness, there is a limit to the effect, as at some point the majority of responses are for the setting of higher luminance regardless of the S/P ratio. Figure 5.7 shows the proportion of votes for a particular source to be brighter plotted against the ratio of photopic luminances, and these six points are for comparisons 1, 2 and 3 for the achromatic and chromatic environments. A response proportion of 0.5 indicates the two

SPDs were considered equally bright, and in Figure 5.7 this would be a ratio of photopic luminances of A/B = 1.22. If the two environments were considered separately then this ratio would be 1.26 for the achromatic environment and 1.16 for the chromatic environment.



Figure 5.7 The proportion of votes for SPD A to be brighter than SPD B plotted against the ratio of photopic luminances. Note: "1C" indicates comparison 1 with the chromatic environment.

Brightness Lumens (Equation 5.2) predicted that these two sources would be equally bright with a luminance ratio of 1.31, a slightly higher ratio than found in these results. For these data, changing the index in Equation 5.2 from 0.5 to 0.36 (i.e. Brightness lumens= $P(S/P)^{0.36}$) provides the prediction of luminances for equal brightness. It is not claimed that 0.36 is the more correct value, and when fitting brightness lumens to the results of past studies found that 0.56 was the optimum value (See Chapter 6, Table 6.12. Instead, this difference indicates the variability found in brightness responses and that S/P ratio alone may be insufficient to predict relative spatial brightness.

5.5.2 Procedure II: Fotios & Cheal Matching

5.5.2.1 Null Condition Results

Within the matching procedure there were four null condition trials. Two identical SPDs (A) were matched with the variable SPD starting from either a higher (75 cd/m²) or lower (25 cd/m²) luminance than the reference (50 cd/m²), and this with the achromatic and chromatic environments. As shown in Table 5.10, the mean illuminance ratios (fixed/variable) ranged from 0.98 to 1.02 in these four cases.

	Achro	omatic	Chro	matic	
	*Start high	**Start low	Start high	Start low	
Mean luminance ratio	0.98	1.02	1.00	1.01	
Std. Dev	0.06	0.06	0.08	0.10	
Ν	28	28	28	28	
Difference from unity (t-test)	n.s.	n.s.	n.s.	n.s.	

Table 5.10 Results of null condition trials in the matching procedure. Note: n.s. = not statistically significant, p>0.05.

* Lamp A dimmed from 75cd/m² to have same brightness with Lamp A at 50cd/m².

** Lamp A dimmed from 25cd/m² to have same brightness with Lamp A at 50cd/m².

These data were considered to be normally distributed following analysis using measures of central tendency, dispersion, graphical presentation and statistical analysis (Shapiro-Wilks, Kolmogorov-Smirnov). Normality checking process is demonstrated in Appendix E. Application of the t-test does not suggest these mean illuminance ratios to depart significantly from unity in any of the four conditions and thus that interval bias was negligible.

The results demonstrate an anchor effect: when the variable source started from a low luminance the equal brightness luminance is lower than that found when starting from a high luminance. While a similar anchoring effect has been found in some past studies (Fotios & Cheal, 2007; Fotios & Levermore, 1997), a significant effect in the opposite effect has also been found (Houser, Tiller & Hu, 2003). The t-test suggests the anchor effect is near significant (p=0.064) for the achromatic environment but for the chromatic environment did not suggest the difference to be significant (p=0.63). The initial luminances were balanced in trials to offset the effect of anchoring.

5.5.2.2 Mixed-SPD Results

In trials, each of the three SPD pairs (A/B, A/C and B/C) was matched four times by each test participant, in order to balance which of the pair was the variable source and whether this started from a higher or lower luminance than that of the reference. This was repeated for the achromatic and chromatic environments. The results of these trials are shown in Table 5.11, these data being the mean illuminance ratio at equal brightness.

Test condition			Achromatio	2		Chromatic	
		A/B	A/C	B/C	A/B	A/C	B/C
1 st dimmed,	Mean illuminance	1 17	1 1 1	1 1 1	1 21	1 / 2	1 17
start high	ratio	1.17	1.44	1.11	1.21	1.42	1.17
	Std. Dev.	0.21	0.24	0.17	0.23	0.25	0.21
1 st dimmed,	Mean illuminance	1.18	1.32	1.11	1.16	1.40	1.13
start low	ratio						
	Std. Dev.	0.17	0.21	0.17	0.19	0.19	0.18
2 ^{na} dimmed,	Mean illuminance	1.19	1.23	1.14	1.18	1.39	1.19
start high	ratio						
	Std. Dev.	0.14	0.15	1.20	0.17	0.22	0.25
2 ^{na} dimmed,	Mean illuminance	1.19	1.30	1.16	1.22	1.45	1.20
start low	ratio						
	Std. Dev.	0.12	0.20	0.16	0.17	0.30	0.29

Table 5.11 Results of the brightness matching tests: mean illuminance ratios at equal brightness. (n=28, all cases).

Analyses of these distributions revealed 10 outlier values from within the 672 data points. These being: Achromatic: <u>A</u>/B #4; B/<u>C</u> #3, 12, 17: Chromatic: <u>A</u>/B #8, 10, 13, 21; B/<u>C</u> #6, 8, 9; A/<u>C</u>, #8, 8. Note: underlined lamp is the one which was dimmed during trials. Analysis of the distributions with outlying values omitted suggested they were drawn from normally distributed populations. Two-way repeated measures ANOVA was applied to examine the effect of SPD order (e.g. whether A of the pair A/B was the first or second to be presented in the sequence) and the effect of initial luminance (i.e. luminance of the variable SPD set to a high or low level prior to the trial) with the outlier values omitted and treated as missing values. ANOVA does not suggest that starting luminance (high or low) led to significant differences in luminance ratio at equal brightness. In only two of the six cases, ANOVA suggests SPD order (1st or 2nd in the sequence) to be significant (A/B chromatic and A/C achromatic). In any case, starting luminance and SPD presentation order were counterbalanced within trials to offset the effects of any such bias.

For each test participant, the mean of these four trials was therefore used as the best estimate of their luminance ratio at equal brightness for each combination of SPD pair and interior colour. These data are shown in Table 5.12. Analysis of these merged distributions suggested they were drawn from a normally distributed population, the one outlier found in this set being retained. According to the one-sample t-test, these illuminance ratios depart significantly from unity in all 6 cases (p<0.01), thus demonstrating that SPD has a significant effect on spatial brightness.

Table 5.12 Results of the brightness matching te	st: after all four conditions of balanced position
and starting illuminance were combined.	

Test condition			Achromatic		Chromatic		
		A/B	A/C	B/C	A/B	A/C	B/C
	Mean illuminance ratio	1.18	1.32	1.13	1.19	1.41	1.17
Overall	Std. Dev.	0.10	0.14	0.10	0.13	0.17	0.14
	Difference from unity (t-test)	p<0.01	p<0.01	p<0.01	p<0.01	p<0.01	p<0.01

5.5.3 Procedure III: Fotios & Cheal Discrimination

Results of the brightness discrimination trials are shown in Table 5.13. These show the frequency of responses in which one lamp in the pair was considered to be brighter.

Table 5.13 Results of brightness discrimination tests: judgements of brighter SPD when presented at equal illuminance.

		Frequency for first SPD in each pair to be judged as brighter									
		Achro	omatic		Chromatic						
	A/B	A/C	B/C	Null	A/B	A/C	B/C	Null			
Forward order	4	1	6	13	6	0	2	14			
(n=28)			-	-	-	-					
Reverse order	2	3	6	14	7	1	3	17			
(n=28)	-	Ũ					-				
Overall (n=56)	6	4	12	27	13	1	5	31			
Frequency											
Percentage	10.7%	7.3%	21.4%	49.1%	23.2%	1.8%	8.9%	55.4%			
Brighter lamp	В	С	С	=	В	С	С	=			
Difference (Dunn Rankin)	p<0.01	p<0.001	p<0.05	n.s.	p<0.05	p<0.001	p<0.001	n.s.			

Image: Image:

In null condition trials, identical SPDs were compared at equal luminance, and test participants responded whether the first or second interval was the brighter. The results indicate almost equal frequencies for the first and second intervals (the first interval was reported to be brighter in 49.1% and 55.4% of trials for the achromatic and chromatic tests respectively) and thus that interval bias was negligible. The binomial test did not suggest interval bias to be significant in either case.

It can be seen in Table 5.13 that the frequency of votes for SPD A in the pair A/B is similar for both presentation orders (i.e. A/B and B/A), and this is also the case for pairs A/C and B/C. Of the 335 discrimination trials (i.e. 336 trials with one missing case) test participants gave different responses in their two trials per SPD pair (i.e. A/B and B/A) on only 22 occasions. This suggests that presentation order had negligible effect and in any case this was balanced.

Differences between SPDs were examined using Dunn-Rankin Variance Stable Rank Sums (Dunn-Rankin et al, 2004). This analysis suggests that SPD B is brighter than SPD A (p<0.01, achromatic; p<0.05 chromatic); SPD C is also brighter than SPD A (p<0.001, achromatic and chromatic); SPD C is brighter than SPD B (p<0.001 chromatic, p<0.05 achromatic).

5.5.4 Results of Chromatic Environment

In Berman et al discrimination task, Table 5.10 shows that the proportions of test participants considering SPD A to be the brighter are similar for the achromatic and chromatic environments in comparison 3 with a larger difference for comparisons 1 and 2. The Wilcoxon test suggested differences between the chromatic and achromatic environments to be near significant for comparison 1 (p=0.059) and significant for comparison 2 (p=0.05) but did not suggest differences in comparisons 3, 4 or 5 to be significant (p \approx 0.50). This pattern may be as expected: when the settings are of identical SPD (comparisons 4 and 5) or when the difference in luminance is large (comparisons 3 and 4) then addition of the coloured Mondrian pattern made little difference, but when the judgement was made more difficult by using settings of different SPD and little difference in brightness, then the coloured surface had an effect.

As shown in Figure 5.5, the trials carried out with the coloured surfaces inside the booth in matching task led to illuminance ratios, which depart further from unity and with a higher variance than trials with the achromatic surfaces. The effect of adding the coloured surface was examined using the paired samples t-test. For SPD pair A/B the t-test did not suggest a significant difference (p=0.64). The differences were significant for SPD pairs B/C (p<0.05) and A/C (p<0.01), with the coloured environment leading to illuminance ratios that depart further from unity than with the achromatic surface.

As shown in Figure 5.8 proportions of votes for SPD B and C to be brighter were slightly differed for two environments in Fotios & Cheal discrimination test. However, the lamps preferred to be brighter in paired comparisons for chromatic and achromatic environments were same. When SPD A compared with SPD B, SPD B voted to be brighter. SPD C was the brightest in between these three lamps for both achromatic and chromatic environments.





5.6 Discussion

This experiment was carried out to identify whether the effect of SPD on spatial brightness is predicted by S/P ratio and GA, and to compare different experimental procedures.

The results are summarised in Table 5.14. It can be seen that the three procedures concur as to which of a pair of SPDs would be considered the brighter at equal luminance. For the A/B pair, luminance ratios for equal brightness were determined using either the matching procedure or interpolated from the Berman et al discrimination procedure. According to the one sample t-test these are significantly different (p<0.01) for the achromatic environment but are not suggested to be different for the chromatic environment. Further evidence is needed to compare these methods and to determine which provides the more accurate response. What we can confirm is that the matching procedure can be completed in less time, which is why in the current study we did not use the Berman et al procedure for the remaining two lamp pairs.

Procedure	Finding		Achromatio	;	Chromatic		
		A/B	A/C	B/C	A/B	A/C	B/C
Berman et al	Brighter SPD	В	-	-	В	-	-
discrimination	Luminance ratio for equal	1.26	-	-	1.16	-	-
	brightness						
	Brighter SPD	В	С	С	В	С	С
Fotios & Cheal, matching	Luminance ratio for equal brightness	1.18	1.32	1.13	1.19	1.41	1.17
	Std. dev.	0.10	0.14	0.10	0.13	0.17	0.14
Fotios & Cheal, discrimination	Brighter SPD	В	С	С	В	С	С

	Table 5.14	Comparison	of the results	gained from	different test	procedures
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This study aimed to repeat, as near as possible, the experiment reported by Berman et al (1990). One reason for this replication was that Berman et al did not include a null condition trial sufficient to evaluate interval bias associated with sequential evaluations - the potential tendency for test participants to consistently report one interval (e.g. the second) as being the brighter regardless of the stimuli observed. As reported in Chapter 3, this is particularly expected in procedures where two stimuli are observed only once each, with the judgement made during observation of the second: there is a tendency for memory to recall the first interval as being darker than it was (LaBoeuf and Shafir, 2006; Uchikawa and Ikeda, 1986) thus enhancing the frequency by which the second interval is reported to be brighter. Berman et al used a sequential evaluation where each stimulus was presented three times, and review (Fotios & Houser, 2013) of these data asked whether this repeated sequential presentation was sufficient to counter interval bias. In the current study, comparison 5 was included to examine this, being a null condition where both settings were of equal luminance and SPD. Analysis of these data did not find a difference between the two intervals which suggests that three sequential observations of each SPD in alternation is sufficient to offset the interval bias associated with successive evaluation.

SPD pair A/B were of equal chromaticity but different S/P ratio. Interpolation of the results suggest significant difference in spatial brightness at equal luminance, thus confirming that in this case the higher S/P ratio led to higher spatial brightness. SPD pair B/C were of similar S/P ratio but different gamut area, and the results demonstrate that the source of higher gamut area was significantly brighter at equal luminance.

For sources of equal chromaticity, the S/P ratio matters, confirming the conclusion drawn by Berman et al (1990). If instead the S/P ratio is held constant, then gamut area matters. SPD pair

A/C presented differences in S/P ratio and gamut area, and here the source of higher S/P ratio and gamut area was found to be significantly brighter. Pair A/C indicates that both S/P ratio and gamut area matter when neither is held constant, and thus that better prediction of relative spatial brightness would be found by considering both metrics simultaneously. What is interesting here is that transitivity holds: within the achromatic and chromatic results individually, the product of A/B and B/C provides good agreement for the finding of A/C. If one effect (i.e. S/P ratio or gamut area) were dominant, then assumption of transitivity from A/B and B/C would tend to over-estimate the result for A/C.

Clearly these results are not confirmation that S/P ratio and gamut area are the optimum metrics. It may be found that the s-cone or ipRGC response is more appropriate than the scotopic component of the S/P ratio, and current activity regarding colour rendering may establish a better metric than gamut area.

The results suggest some differences between brightness evaluations made in the chromatic and achromatic environments. According to the matching test, the difference is significant for SPD pairs A/C and B/C, these having different chromaticities, but not for A/B which had similar chromaticity. This disagrees with the findings of past experiments that the colour of surfaces in an environment did not affect evaluations of spatial brightness (Fotios & Cheal, 2011; Boyce, 1977; Boyce & Cuttle, 1990; Han and Boyce, 2003). One reason may be that in the current study the test participant was placed a relatively short distance from the booth surfaces and this may have led to evaluations of the surface rather than of the illuminated volume. For future work, this study can be repeated with an experiment using a larger test environment.

5.7 Summary

This chapter describes an experiment carried out to investigate the influence of lamp SPD on spatial brightness using three different procedures.

The discrimination procedure used by Berman et al (1990) was validated through inclusion of an additional null condition to evaluate interval bias and through parallel use of alternate procedures, the matching and discrimination procedures used by Fotios and Cheal (2011). These different procedures provided converging evidence as to which of a pair of SPDs is the brighter, and provided similar estimates as to the magnitude of the effect.

This study provides further support for the conclusion reported by Berman et al that for two lights of equal chromaticity and equal luminance, the one of higher S/P ratio will appear brighter. Berman et al used only two SPDs to test this proposal. In the current work a third SPD was added in order to evaluate the impact of a chromatic contribution to spatial brightness for two lights of equal S/P ratio. The results suggest that both the S/P ratio and chromatic contribution

are important and that considering both metrics simultaneously enables a better prediction of spatial brightness under different SPDs. In other words, the results suggest that consideration of either S/P ratio or gamut area alone would be insufficient.

Further analysis is done with credible past data and data from new experiment to investigate more on S/P ratio and GA effect on brightness. Regression test is applied to this data set in order to predict a brightness model with two metrics (S/P ratio and GA).

Chapter **6**

6 Testing Models of Spatial Brightness Using Past Data

- 6.1 Introduction
- 6.2 Past Studies of Spatial Brightness
- 6.3 Establishing Mean Illuminance Ratios
- 6.4 Establishing Lamp SPD
- 6.5 Potential metrics of Spatial Brightness
- 6.6 Predicting Illuminance Ratios
- 6.7 Stepwise Regression
- 6.8 Category Rating Studies
- 6.9 Summary

6.1 Introduction

This chapter presents an investigation of potential metrics (S/P and GA for the current study) for spatial brightness by using the results of studies considered to provide credible evidence (as explained in Chapter 3) and data from new experiment to test predictions. A similar method was used in Cowan and Ware (1983) with 29 studies supplying brightness matching data being used to develop a model of predict equal brightness. The aim of this chapter was also to develop a model that is understandable and easy to use for both researchers and designer following Cowan and Ware's method; therefore correlation and stepwise regression analysis were applied to data collected from past studies. This analysis requires three stages of work:

(1) Establishing reliable and appropriate evidence of SPD and spatial brightness.

(2) Identifying the SPD for these lamps hence to establish their characteristics.

(3) Comparing predictions made using these metrics with correlation and stepwise regression analysis.

6.2 Past Studies of Spatial Brightness

Empirical evidence of the relationship between lamp SPD and spatial brightness were reviewed from over sixty studies in Chapters 3. Each study used different combinations of independent variables and experimental procedures such as lamp SPD, response task, stimulus size, illuminance and evaluation mode. The first step in interpreting these data was exploration of research methodologies to identify how these differences in methodology matter and hence those studies giving credible estimates of lamp SPD effects on brightness. Credible used in here is intended to mean that the data used in the experimental procedure, is unbiased or at least the direction and magnitude of bias is reasonably well known. A list of the credible evidence determined according to analysis in Chapter 3 is presented in Table 6.1.

		SPI	D char	acteris	ation i	n origi	nal rep	ort ²		
Study	Method ¹	Data	Graph	ССТ	CRI	Chromaticity	S/P	GA	Conclusion: does SPD affect brightness?	Reported metric for spatial brightness (if any) ³
				Studi	es usir	ng a ma	atching	g proc	edure	
Boyce, 1977	Simultaneous evaluation in side-by- side booths; 3 levels of surface colourfulness	x	x	\checkmark	\checkmark	x	х	х	Yes	Gamut area
Fotios & Gado, 2005;	Simultaneous evaluation in side-by- side booths; achromatic surfaces	x	\checkmark	\checkmark	\checkmark	x	х	x	Yes	Lamp type⁴
Fotios & Levermore, 1997	Simultaneous evaluation in side-by- side booths; achromatic surfaces with coloured objects	х	\checkmark	X ⁵	X ⁵	\checkmark	\checkmark	X ⁵	Yes	Cone surface area (3D colour gamut) and S- cone contribution. ⁵
Hu et al, 2006	Simultaneous evaluation of side-by- side full scale rooms; achromatic surfaces. Parallel trials also using discrimination task.	x	x	\checkmark	\checkmark	\checkmark	\checkmark	x	No	None provided. Suggested that any derived measures, such as CCT, are inadequate to predict relative brightness perception.
Atli (New experiment)	Rapid sequential evaluation in booth, using both chromatic and achromatic environment.	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark	Yes	S/P ratio and Gamut area
				Studie	s using	a disci	riminati	on pro	cedure	
Berman et al, 1990	Sequential evaluation of two intervals in single room; achromatic surfaces	x	x	x	x	\checkmark	х	х	Yes	S/P ratio (as a proxy for the ipRGC). ⁶
Houser et al, 2004	Simultaneous evaluations of side-by- side full scale rooms. Rooms were furnished as private offices and contained a range of colourful objects.	x	\checkmark	/	\checkmark	/	\checkmark	x	Yes	Prime colour theory supported. CCT and S/P ratio theories not supported.
Houser et al, 2009	Study 1: Simultaneous evaluation when facing two side-by- side rooms. Study 2: Rapid-sequential evaluations when immersed in one room. The rooms were empty and achromatic.	x	\checkmark	\checkmark	х	x	\checkmark	x	Yes	None provided. ⁷
Royer & Houser, 2012	Sequential evaluation of a single booth that enveloped participants to give a full field; the booth was empty and achromatic.	х	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark	Yes	Prime colour theory supported. CCT and S/P ratio theories not supported.

Table 6.1 Summary of studies considered to provide credible evidence of lamp spectrum and spatial brightness by using procedures that meet suggested recommendations for best practise.

Vrabel et al, 1998	Sequential evaluations in a room; achromatic surfaces (white walls and ceiling, grey floor)	х	\checkmark	\checkmark	\checkmark	х	х	х	Yes	Lamp type
—				Studies	using	a cateo	gory rat	ing pro	ocedure	
Akashi & Boyce, 2006	Separate evaluations in workplace offices; achromatic room surfaces, greyish-blue furnishing, coloured desk-top objects	x	x	\checkmark	x	x	x	x	Yes	ССТ
Boyce et al, 2003	Separate evaluations in a room; white surfaces and desks but one unpainted brick wall; diffuse lighting	x	x	\checkmark	\checkmark	x	\checkmark	x	No ³	A trend mentioned about CCT, S/P ratio, but not supported
Boyce, 1977	Separate evaluations in in side-by-side booths; 3 levels of surface colourfulness	x	x	\checkmark	\checkmark	x	x	x	Yes	Gamut area
Boyce & Cuttle, 1990 (experiment 2)	Separate evaluations in a room; 2 types of surface colour and presence/absence of coloured objects.	х	\checkmark	\checkmark	\checkmark	х	х	х	Yes	ССТ
Davis & Ginther, 1990	Separate evaluations in a room; room surface colours not stated; artwork on wall and coloured fruit on table.	x	~	\checkmark	\checkmark	х	x	х	No ⁸	-
Flynn & Spencer, 1977	Separate evaluations in a room; removed coloured objects and displays to surfaces of light beige or natural wood.	x	x	\checkmark	х	х	x	x	Yes	Lamp type
Han & Boyce, 2003	Separate evaluations in a booth; 3 levels of surface colourfulness.	х	x	\checkmark	\checkmark	х	х	х	Yes	ССТ
Piper, 1981	Separate evaluations in a room; surface colours not reported.	х	\checkmark	x	х	х	х	х	No ⁹	A trend mentioned about CCT, S/P ratio, but not significant
Vienot et al, 2009	Separate evaluations in a booth, surface colours.	x	\checkmark	\checkmark	\checkmark	x	x	x	Yes	ССТ
Vrabel et al, 1998	Separate evaluations in a room; achromatic surfaces (white walls ceiling, grey floor).	x	\checkmark	\checkmark	\checkmark	х	х	х	Yes	Lamp type

Notes:

1 All studies supplied diffused lighting except Vienot et al (2009) in which the distribution of light was uncertain. All studies were done by evaluating whole environment except Berman et al (1990) had flat surface (wall) in front of the participant, Vrabel et al (1998) used head rests looking towards wall and desk surfaces ahead (this may have restricted observation of whole environment) and in Piper (1981) the task was looking at a sheet and reading the letters (this may have restricted observation of whole environment).

- 2 Some additional information for SPD characterization was mentioned in Flynn and Spencer (1977) and Boyce (1977) as lamp names, Boyce (1977) also mentioned GA of two lamps out of six of them.
- 3 Boyce et al (2003) report a trend but the effect is not significant: they suggest it to be "an effect masked by noise".
- 4 In some studies stimuli were different types of lamps having different SPDs without any certain metric specified or controlled.
- 5 Fotios and Levermore (1997) reported evaluation of metrics in subsequent articles (Fotios and Levermore, 1998a, 1998b) and his thesis (Fotios, 1997).
- 6 Berman et al (1990) originally promoted a rod contribution to spatial brightness, and hence the S/P (scotopic to photopic) ratio. Following new findings in vision this was amended to a contribution from the intrinsically photosensitive retinal ganglion cells (ipRGC) (Berman, 2008).
- 7 Results of Royer & Houser (2012) showed that S/P, prime-colour theory, CCT, V(λ), colour quality metrics, linear brightness models, and colour appearance models could all fail to predict or correctly order perceptions of brightness.
- 8 Davis and Ginther (1990) reported not significant differences between lamps, however the analysis was including stimulating/relaxing ratings with bright/dim.
- 9 Piper (1981) reported a trend but the effect was not significant according to the post-hoc analysis done with t-test using mean ratings and standard deviation reported in original article.

6.3 Establishing Mean Illuminance Ratios

The current study intends to screen metrics for predicting the illuminance ratio needed for equal brightness, and thus required results gained using either a brightness matching procedure or a two-sample brightness discrimination procedure carried out at multiple levels of illuminance. Results from matching and discrimination studies will be used as these give the magnitude and direction of relative spatial brightness. List of the studies providing this data is in Table 6.2. After the metric results screened from matching and discrimination studies, it was compared with data from rating studies as these were providing only the direction, not the magnitude.

	Poforonco		Mean	644	Included
Study		Lamp pair (A/B)	illuminance	Day	in
	light level(s)		ratio (A/B)	Dev.	Data Set
	Studies u	sing a matching procedu	ire		
Boyce 1977	350 lx and	Natural/ White	0.75	0 13	A & B
Doyce, 1977	600 lx		0.75	0.15	
		Kolor-rite/ White	0.76	0.14	A & B
		Kolor-rite/ Natural	1.05	0.13	A & B
		Northlight/ Kolor-rite	1.09	0.30	A & B
		Northlight/ Daylight	0.85	0.24	A & B
		Kolor-rite/ Daylight	1.07	0.29	A & B
		Natural/Grolux	1.46	0.22	Only A
Fotios &	320 lv	\/eri\/ide/\//\/	0.89	0.38	A & B
Gado, 2005	520 1	venvide/www	0.89	0.50	
	approx. 100 to	LPS/ WW	2.27	1.54	Only A
	800 lx (3	HPS/ WW	2.11	0.96	Only A
	reference	CW/ WW	0.94	0.19	A & B
Fotios &	levels gained	FS/ WW	0.80	0.25	A & B
Levermore,	using neutral				
1997	density filters				
	of 25%, 50%	BG/GLS	0.75	0.27	A & B
	and 75%				
	transmittance)				
	538 lx	CV35/ CV65	1.00	*	A & B
Hu et al, 2000		VT35/ VT65	0.98		A & B
	50 cd/m ²	A/B	1.19	0.12	A & B
Aur (new		A/C	1.37	0.16	A & B
experiment)		B/C	1.15	0.12	A & B
	Studies usin	g a discrimination proce	dure		
Berman et. al.,	30-67 cd/m ²	R213/M/M/C	0.61	*	Δ&R
1990	50–67 Cu/m	11210/00000	0.01		
Houser et. al.,	24 cd/m ²	2900K/7200K	1 08	*	A & R
2009	and 30 cd/m ²	200010720010	1.00		

Table 6.2 Results from past studies of spatial brightness which used matching and discrimination procedures. These values were used to screen metrics of spatial brightness.

* standard deviation of illuminance ratio at equal brightness not known for these studies.

Some of the studies reported the illuminance ratios of the lamp pairs in their articles (Boyce, 1977; Fotios and Gado, 2005; Fotios and Levermore, 1997; Hu, Houser and Tiller, 2006) and average illuminance ratios were calculated using the reported values. For two other studies

(Berman et al, 1990; Houser, Fotios and Royer, 2009) illuminance ratios were calculated using reported luminance values in the original articles. Table 6.3 shows from which values the illuminance ratios of the lamps for Boyce (1977), Fotios and Gado (2005), Fotios and Levermore (1997) and Hu, Houser and Tiller (2006) studies were calculated. Table 6.3 References of mean illuminance ratios calculations.

Study	Lamp pair	Reference
Berman, 1990	R213/ WWG	Reported in captions of the original study (ratio of the reported luminance levels of the lamps at equal brightness) (p.40)
	Natural/ White Kolor-rite/ Natural	Table 3 of the original study (average of all 6 illuminance ratios of low, medium and high colourfulness at both 350 and 600 lux) (p.13)
Boyce, 1977	Northlight/ Kolor-rite Northlight/ Daylight Kolor-rite/ Daylight	Table 8 of the original study (average of all 4 illuminance ratios of achromatic and chromatic at both 300 and 600 lux) (p.18)
	Kolor-rite/ White	Table 3 and 8 of the original study (average of 10 illuminance ratios) (p.13, 18)
	Natural/ Grolux	Reported on p.16 of the original article
Fotios & Gado, 2005	Verivide/ WW	Table 5 of the original study (average of two experiment results of overall mean illuminance ratio for equal brightness) (p.128)
Fotios & Levermore, 1997	LPS/ WW HPS/ WW CW/ WW FS/ WW	Table 4 of the original study (p.167)
	BG/GLS	Table 8.1 of Fotios (1997) PhD thesis (p.227)
Houser et al, 2009	Lamp A/ Lamp B	Table 6 of the original study (calculated with the pooled percentage of side-by-side and rapid sequential results) (p.131)*
Hu et al, 2006	CV35/ CV65 VT35/ VT65	Figure 2 of the original study (ratio of the reported mean illuminances of the lamps) (p.78)
Atli (new experiment)	A/B A/C B/C	Table 5.14 of this thesis(average of chromatic and achromatic results)

^{*}Explained in details below.

Houser, Fotios and Royer (2009) compared two CCTs with each other at 24 and 30 cd/m² in both side-by-side and rapid sequential discrimination test with two participant groups (expert and naïve). In the results, they reported how many times a lamp evaluated brighter in percentages. In order to calculate mean illuminance ratio for the lamp pair A/B the combined results from two participants group were used. The main idea of these calculations was to find out what would be the luminance levels when two lamps were at equal brightness. Calculations were done for each lamp both at 24 and 30 cd/m² and using both of the percentages results from side-by-side and rapid sequential tests. An example of the calculations with Lamp B (7200K) at 24 cd/m² can be explained. Two lamp pairs which had Lamp B at 24 cd/m² (comparison 1 done by Lamp A at 24cd/m² with Lamp B at 24cd/m² and comparison 2 was done with Lamp A at 30cd/m² and Lamp B at 24cd/m²) were included in the calculations as shown in Table 6.4. As shown in Figure 6.1 luminance levels for 2900K lamp were plotted in the graph and luminance levels for 50% were determined by calculating x when y=50.

Table 6.4 Brightness results from Houser, Fotios and Royer (2009) for lamp pairs of $2900K/24cd/m^2$, $7200K/24cd/m^2$ and $2900K/30cd/m^2$, $7200K/24cd/m^2$

I	Side by side (%) Lamp B 24cd/m [*]	2	R	apid sequenti (%) amp B 24cd/m	al 1 ²
Lamp A 24cd/m ²	Lamp A 30cd/m ²	Calculated luminance for 50%	Lamp A 24cd/m ²	Lamp A 30cd/m ²	Calculated luminance for 50%
30	77	26.6	41	96	25



Figure 6.1 Results of side by side and rapid sequential brightness discrimination tests from (Houser, 2009) to show interpretation of luminance for equal brightness. Data for Lamp B at 24 cd/m².

Using the calculated luminance levels for 50%, two luminance ratios for Lamp A and Lamp B were found as 1 and 1.1. Same calculations were repeated for Lamp B at 30cd/m² and Lamp A at both 24 and 30cd/m². Final luminance ratios used in current study were the average of 8 calculated luminance levels for 50%.

6.4 Establishing Lamp SPD

For some studies, the S/P ratio and gamut area of test lamps were not reported. The spectral power distributions of lamps used in these studies are therefore required in order to calculate the S/P ratio and GA. However, none of the studies in Table 6.2 presented their lamp SPD in numeric form. This section is a discussion of how lamp SPD were obtained and validated.

SPD were determined for the range between 380nm to 780 nm as is recommended by CIE (2005). Whilst intervals of 5nm are considered to be acceptable (CIE, 2005) the current study used 1nm intervals. Where data were provided at 5 nm intervals, it was interpolated to an interval of 1nm.

Two studies were carried out by Houser and his colleagues. Hu, Houser and Tiller (2006) reported the CCT, CRI, S/P ratios and x,y chromaticity of their four lamps custom-made fluorescent lamps, VT35, VT65, CV35 and CV65. The second experiment reported them is the side by side test which is included in the current analysis. Houser, Fotios and Royer (2009) presented graphs of the SPDs of their lamps, and reported the CCT and the S/P ratio of their lamps, these being LED lamps of two different CCTs and S/P ratios. The SPDs for the lamps used in both studies were supplied by Kevin Houser to the author in spread sheet format (personal communication to D Atli, 15/02/2012).

For the Hu et al (2006) study these were provided at 0.25 nm intervals and were reduced to 1nm intervals for the current analysis. As shown in Figure 6.2 comparison graphs of these two SPDs were drawn using 0.25nm and 1nm data do not suggest any differences. As shown in Table 6.5 values of chromaticity, CCT and Ra calculated using the 1nm data show reasonable agreement with the values presented in the original publication (Hu, Houser and Tiller, 2006).



Figure 6.2 An example for lamp SPD graphs of Hu, Houser and Tiller (2006) in 0.25 nm and 1 nm intervals. Note: SPD values arranged to have highest value as 1 and for 1 nm interval wavelength 1 added to those values to present both SPDs separately.

	Values reported in Hu et al.			Values calculated using				
Lamp	np (2006)				provided SPD			
	ССТ	CRI	x	У	ССТ	CRI	x	У
VT35	3244	81.8	0.4151	0.3863	3221	81.2	0.417	0.386
VT65	6361	82.5	0.3143	0.3369	6583	82.4	0.311	0.332
CV35	3276	84.1	0.4157	0.3910	3349	83.8	0.413	0.392
CV65	6149	74.0	0.3177	0.3425	6393	73.8	0.314	0.339

Table 6.5 Comparison of reported and calculated CCT, CRI and chromaticity of light sources used by Hu et al. (2006).

For the Houser et al (2009) study the SPDs were provided at 1 nm intervals. Graphs drawn using these data match those presented by Houser et al: As shown in Table 6.6, S/P ratios and CCT calculated using these data are similar to those reported by Houser et al. (2009).

Table 6.6 Comparison of reported and calculated CCT and S/P ratios of light sources used by Houser et al. (2009).

Lamp	Values reported in Houser et al. (2009).		Values calculated using provided SPD		
	ССТ	S/P	ССТ	S/P	
А	2900	1.7	2890	1.7	
В	7200	2.6	7453	2.6	

Two studies were carried out by Fotios and his colleagues. Fotios and Levermore (1997) provide the graphs of SPD for their lamps. They used seven lamps: LPS, HPS, WW fluorescent, CW fluorescent, FS fluorescent, blue-glass and GLS tungsten. The SPD were presented in Fotios' PhD thesis (Fotios 1997) at 4nm intervals and subsequently interpolated to 1nm intervals. Values of CCT, CRI, x,y chromaticity and the S/P ratio calculated using the 1nm data are similar to those reported in Fotios PhD thesis (1997,p.167, 232, 233, 239, 246). Graphs of SPD drawn using the 1nm and 4nm data appear to match precisely. In Fotios's second study, Fotios and Gado (2005) presented graphs of the SPDs of the two lamps they used, which are warm white and Verivide D65 fluorescent lamps, and reported their CCT and R_a. These lamps were available in the laboratory where the author of the current study is working. To measure the SPDs, these lamps were placed in the apparatus used in the experimental work (side-by-side booths) reported by Fotios and Gado and their SPD measured at 1nm intervals using a KonicaMinolta CS1000a spectroradiometer focused on a reference white in the floor of the booths. Graphs drawn using these data, and calculated values of CCT and R_a matched the data presented by Fotios and Gado (2005). Table 6.7 presents these values.

Table 6.7 Comparison of reported and calculated CCT and CRI values of light sources used by Fotios and Gado (2005).

Lamp	Values reported in Fotios and Gado (2005)		Values calculated using measured SPD		
	ССТ	CRI	ССТ	CRI	
А	2950	52	3034	51	
В	6500	98	6423	98	

For two studies (Berman et al, 1990; Boyce, 1977) the SPDs were not available from the original authors, these being older studies. Berman et al (1990) used two light sources, named R213 and WWG, and these were each combinations of two types of fluorescent lamps. The WWG lamp comprised a blend of light from a warm white and a gold fluorescent lamp, and the R213 lamp comprised a blend of light from a red fluorescent lamp and a fluorescent lamp using phosphor 213. The x,y chromaticity (10 degree observer) were reported, and there were data (Table 2 of Berman et al, 1990) to enable calculation of the S/P ratios, but the article did not include the SPD. Berman provided graphs of the SPD of the four lamps (personal communication to Fotios, 12th October 2000). These graphs were digitised and the SPD estimated at 1nm intervals. To check the accuracy of these estimates of SPD the x,y chromaticities (10 degree) were compared with the values reported by Berman et al (Table 6.8). These appear to be reasonably similar and thus it was concluded that the digitised spectra were reasonable estimates.

Table 6.8 Chromaticity (10 degree) of individual lamps used by Berman et al (1990): comparison of values reported by Berman with values calculated using SPD digitised from graphs supplied by Berman.

	Chromaticit by Berm	ies reported nan et al	Chromaticities calculated using digitised estimate of SPD			
Lamp	("visually	matched")				
	X 10	y 10	X ₁₀	y ₁₀		
WW	0.457	0.387	0.470	0.389		
G	0.541	0.457	0.554	0.435		
R	0.686	0.314	0.669	0.309		
213	0.133	0.571	0.136	0.568		

The spectra of the combination lamps used by Berman et al were estimated by adding weighted combinations of the two constituent lamps in order to match their S/P ratios. For the R213 source, Table 2 of Berman et al 1990 shows that this source had a scotopic luminance of 73 cd/m^2 and a photopic luminance of 30 cd/m^2 , giving an S/P ratio of 2.4. This was achieved using a blend of 57% lamp R and 43% lamp 213. For the WWG source, Table 2 of Berman et al 1990 shows that this source had a scotopic luminance of 40

cd/m², giving an S/P ratio of 0.85. This was achieved using a blend of 78% WW and 22% G. Table 6.9 shows the values of chromaticity and S/P ratio for these estimated SPD compared with the values reported by Berman et al. These values are in reasonably close agreement. Figure 6.3 shows the spectra of these two lamps.

	Values reported	d by Berman et 990	Values calculated using estimated SPD		
	R213	WWG	R213	WWG	
x ₁₀	0.460	0.479	0.446	0.492	
y ₁₀	0.419	0.406	0.418	0.401	
S/P ratio	2.4	0.85	2.4	0.85	

Table 6.9 Comparison of reported and calculated chromaticities and S/P ratios of light sources used by Berman et al (1990).



Figure 6.3 Estimated SPD of the R213 and WWG blended light sources used by Berman et al (1990). SPD values arranged to have highest value as 1 and for R213 lamp 1 added to those values to present both SPDs separately.

Boyce (1977) used six different fluorescent lamps: Natural, White, Kolor-rite, Daylight, Northlight and Grolux. He reported the CCT, R_a and gamut area (calculated using the 1960 CIE-UCS chromaticity diagram) but did not report the SPDs. Boyce's article was published in 1977: estimates of SPD were obtained by matching the lamp name and CCT with the typical fluorescent lamps described in the 1972 edition of Lamps and Lighting (Henderson and Marsden, 1972) which provided graphs of SPDs (Figure 12.8 and 12.12 of their book). These graphs were digitised and the SPD estimated at 1nm intervals. CCT and R_a determined using these estimated SPD appear to be reasonably close to the values reported by Boyce for five of the lamps, therefore it was considered that the estimated SPDs are reasonable to be used (see Table 6.10).

Lamp	Values reported in Boyce V (1977)		Values calci	Values calculated using	
			estimated SPD		
	ССТ	CRI	ССТ	CRI	
Natural	4000	85	3980	84	
White	3500	56	3380	56	
Kolorite	4000	92	3970	97	
Daylight	4300	65	4320	65	
Northlight	6500	95	6620	91	
Grolux	5400	9	19970	36	

Table 6.10 Comparison of CCT and R_a for lamps used by Boyce: comparison of values reported by Boyce with values calculated using estimated SPD.

For the Grolux lamp the CCT and R_a do not match those reported by Boyce. In subsequent analyses the CCT and R_a reported by Boyce were used and the estimated SPD used to determine S/P ratios, gamut area, and x,y chromaticity. Due to the uncertainty this lamp was placed in the category of uncertain data.

Similar to Boyce's Grolux lamp, there was some uncertainty with LPS and HPS lamps reported by Fotios and Levermore, (1997). Their LPS/WW and HPS/WW lamp pairs had relatively high standard deviations as presented in Table 6.2 and thus the mean illuminance ratio reported (median for the LPS/WW) is not a precise estimate. Furthermore, because of the narrow SPDs of LPS and HPS lamps, their spectral properties can be extreme relative to the other sources, leading to a strong anchor on regression.

Table 6.2 shows the twenty data points available from the seven studies considered to give reliable estimates of illuminance ratio at equal brightness. As, three of these were considered to be uncertain (the LPS/WW, HPS/WW and Natural/Grolux), all analyses were carried out for two sets of data: one set contained all 20 data points (Data set A), and a second set omitted the three lamp pairs leaving 17 data points (Data set B).

Using the numeric values of SPDs as established above, S/P ratio and GA calculations were carried out for the range 380nm to 780 nm at 1nm intervals. SPD values for all the lamps were presented in Appendix C. GA values of the lamps were calculated by using Equation 2.5 in Chapter 2. S/P ratios is the scotopic luminance/photopic luminance of the lamps as already been explained in Chapter 2. Table 6.11 shows the values of the S/P ratio and GA for the lamps identified in Table 6.2.

Study	Lamp	GA	S/P
Hu, Houser & Tiller, 2006	VT35	0.005958	*1.23
	VT65	0.007951	*2.01
	CV35	0.005211	*1.16
	CV65	0.006602	*1.89
Fotios & Gado, 2005	WW (measured)	0.003086	1.02
	Verivide (measured)	0.007103	2.44
Houser et al, 2009	3000K	0.005768	*1.71
	7500K	0.010573	*2.62
Boyce, 1977	Grolux	0.012442	3.18
	Natural	0.006043	1.67
	White	0.003577	1.18
	Kolorite	0.006276	1.73
	Daylight	0.004660	1.59
	Northlight	0.007099	2.35
Fotios & Levermore, 1997	WW	*0.002860	*0.99
	FS	*0.006950	*2.30
	CW	*0.006540	*2.07
	LPS	*0.000004	*0.24
	HPS	*0.000390	*0.44
	GLS	*0.003190	*1.28
	BG	*0.004080	*1.55
Berman et al, 1990	WWG	0.002480	0.85
	R213	0.004569	2.40
Atli (new experiment)	A	*0.0017	*1.02
	В	*0.0041	*1.77
	С	*0.0069	*1.81

Table 6.11 Summary of lamp characteristics explored as metrics for spatial brightness.

* These values of S/P and/or GA were reported in the original studies: all other values were calculated from estimated SPD.

6.5 Potential metrics of Spatial Brightness

The main focus of the calculations included in this chapter is on S/P and GA which were proposed previously to provide improvements on brightness perception by Berman et al (1990) and Boyce (1977). These two metrics are main interest to the current study because they have their bases in physiology. S/P embodies the receptive property of human eye and GA can be related to chromatic activity and may thus offer a simple proxy for chromatic contribution to brightness.

Firstly, it was questioned whether these two metrics are independent of each other. When the linear regression of each pair of the two metrics for the 26 lamps identified in Table 6.11 was calculated, there is a strong correlation between GA and S/P ratio (r^2 =0.81, n=26, p<0.0001) (See Figure 6.4). This was indicating that high S/P lamp also had high GA. Therefore, one would not expect the current analysis to discriminate between GA and S/P ratio.



Figure 6.4 Linear regression between GA and S/P ratio for lamps in Table 6.11.

Most of the past studies considered only one metric in their studies and the findings were related with the values and effects of this single metric (Berman et al, 1990; Boyce and Cuttle, 1990; Davis and Ginther, 1990; Han and Boyce, 2003; Piper, 1981; Vienot et al, 2009). For example, Vienot et al (2009) tested CCT effect on brightness and they did not consider whether CRI or some other metric was also highly correlated with brightness. However, if more than one metric would be used it would be possible to make a comparison in between and may be that one metric found to be influencing the results previously wouldn't be the actual reason. Hence, false support for a metric may occur. In order to avoid this limitation, the analyses are carried out with both metrics by focusing on do they work for predicting brightness.

6.6 Predicting Illuminance Ratios

Values of S/P ratio and GA for the credible data were then manipulated to seek a precise and accurate prediction for illuminance ratios at equal brightness. In order to do the predictions, functions were designated and regression tests were applied to the data set, which was generated by using data from credible past studies and the new experiment.

The two basic functions are ratios and differences. Two lamps identified as Lamp 1 and Lamp 2, for which the mean illuminance ratio at equal brightness is E_1/E_2 . For metric ratios, correlations
were sought with GA_2/GA_1 and $(S/P_2)/(S/P_1)$, and for the difference correlations they were sought with GA_1 - GA_2 and (S/P_1) - (S/P_2) .

Berman et al (1990) suggested that the S/P ratio provides an estimate of brightness, proposing the metric Brightness Lumens which uses the square-root of the S/P ratio, i.e. $(S/P)^{0.5}$. This leads to the question as to whether the prediction ability of other potential metrics would be improved if raised to a non-unity power. Optimum powers were determined for each metric, and for both data sets of A and B to reduce to a minimum total of square root of difference between illuminance ratios and the value predicted by the metric for all lamp pairs. For example, $[(E_1/E_2)-(GA_2/GA_1)^x]^2$ for all 20 lamp pairs calculated and an optimum power (x) was computed in order to obtain minimum total of all lamp pairs for data set A (it was a total of 17 lamp pairs for data set B). These optimum powers are shown in Table 6.12. The main reason of using the difference of illuminance ratio and the predicted metric was to get these two values as close as possible to each other. For S/P ratio with the data set A, the optimum index was found to be 0.56: this was reduced to 0.50 partly to match the value proposed by Berman et al and partly because the square-root function provides a more elegant solution. Similar application is also done for couple other values as shown in Table 6.12.

Metric	Optimised Power		Optimised Power		
motrio	(Data set A)		(Data	et B)	
	Lamp2 ^x /Lamp1 ^x	Lamp1 ^x -Lamp2 ^x	Lamp2 ^x /Lamp1 ^x	Lamp1 ^x -Lamp2 ^x	
S/P	0.56*	-0.79	0.24	0.51*	
GA	0.14	-0.12	0.25	0.18	

Table 6.12 Optimised power index for S/P and GA for data sets A and B.

*values were rounded up to 0.50 for the further analysis

In addition to ratio and differences of S/P ratios and GAs, this study also considered differences and ratios of logarithmic values. Logarithmic values were determined because previous work (Boyce, 1977) suggested log (gamut area ratio) as in Equation 6.1 to provide a good model for lamp spectrum effects. Boyce (1977) was suggesting:

Following Boyce (1977), simpler log equations of ratio and difference were also added to model predictions such as Log lamp2 /Log lamp1 and Log lamp1 – Log lamp2. The reason to keep both of the ratio function as Lamp2/Lamp1 was assumed that metric ratios were inversely proportional with luminance ratio of two lamps. This is meaning, if a Lamp 1 has higher S/P ratio than Lamp 2, the equal brightness will be obtained with lower light levels of Lamp 1 than Lamp 2.

Table 6.13 shows the results of linear regression between illuminance ratios at equal brightness as found in experiments and values predicted by S/P ratio and GA. These data are the regression coefficient (r^2) and the slope and intercept of the regression equation. Ideally, r^2 is high: for ratio functions the slope and intercept of the equation should approach unity and zero respectively, (and zero and unity for difference equations).

If two lamps of identical GA (or other metric) are compared then it is expected that an illuminance ratio of unity would be found for equal spatial brightness. An appropriate equation would also predict an illuminance ratio of unity when lamps of identical GA are input, and this is shown by the null value column in Table 6.13.

Metric Eunction	r ²		Equation			
		slope	intercept	null value		
	Analysi	s with data set A		I		
S/P2 ^{0.5} / S/P1 ^{0.5}	0.765	0.914	0.130	1.044		
S/P1 ^{-0.79} - S/P2 ^{-0.79}	0.821	0.650	1.054	1.704		
LogS/P1 - LogS/P2	0.670	-1.238	1.104	-0.134		
LogS/P ₂ / LogS/P ₁	0.022	0.007	1.083	1.090		
GA2 ^{0.14} / GA1 ^{0.14}	0.686	1.002	0.017	1.019		
GA1 ^{-0.12} - GA2 ^{-0.12}	0.694	0.625	1.023	1.648		
Log GA1 - Log GA2	0.777	-0.527	1.020	0.493		
LogGA ₂ / LogGA ₁	0.807	-2.197	3.247	1.052		
	Analysi	s with data set B				
S/P2 ^{0.24} / S/P1 ^{0.24}	0.534	1.181	-0.183	0.998		
S/P1 ^{0.5} - S/P2 ^{0.5}	0.507	-0.421	1.005	0.584		
LogS/P ₁ - LogS/P ₂	0.531	-0.629	1.005	0.376		
LogS/P ₂ / LogS/P ₁	0.461	0.014	0.901	0.915		
GA2 ^{0.25} / GA1 ^{0.25}	0.694	0.966	0.004	0.970		
GA1 ^{0.18} - GA2 ^{0.18}	0.664	-3.696	0.984	-2.712		
$Log GA_1 - Log GA_2$	0.680	-0.578	0.984	0.406		
LogGA ₂ / LogGA ₁	0.646	-1.357	2.350	0.993		

Table 6.13 Regression coefficients (r^2) between illuminance ratios at equal brightness and functions of the proposed metrics.

Observations about which metrics and functions appeared to correlate well with illuminance ratios for equal brightness were drawn with regard to the limitations of these data. These limitations include the small sample size (n=20), that some metrics were determined using estimates of lamp SPD, and that metrics reported in original articles may have been manufacturers reported values and did not account for modification by reflectances in the test apparatus. According to Table 6.13 the ratio function (Lamp2^x/Lamp1^x) provides the slopes close to '1', intercept close to '0' and the null value close to '1' for both S/P ratio and GA in both

sets. For data set A, r² for S/P ratio and GA are 0.765, 0.686, respectively. This r² value is smaller for S/P ratio in data set B as being 0.534 however it is even higher for GA 0.694. Hence, ratio of the metrics seems to provide better prediction of brightness than does the difference or log functions. Figure 6.5 shows a high and a low correlation graph of S/P ratio with illuminance ratio. The top graph of Figure 6.5 with the equation of ratio of S/P presents a more proportionate trend than the bottom graph of log ratio equation with S/P in which mean illuminance ratio changes only slightly with the log ratio equation meaning that not well correlated with each other.



Figure 6.5 Examples of correlations of illuminance ratio at equal brightness and ratios and log ratio functions of S/P. a) example of high correlation with data set A. b) example of low correlation with data set A.

6.7 Stepwise Regression

Stepwise regression was employed to determine whether a model comprising both S/P ratio and GA would be of benefit when predicting spatial brightness. There is some reason to suspect this. Although, they were testing S/P ratio in Berman et al (1990), they also stated that if their light conditions would differ in chromaticity, it would introduce a brighter result with the condition having greater chromaticity relative to achromatic white. Therefore, adding a second term characterising differences in chromatic properties might be useful. It has been suggested that two metrics are required to give a more complete characterisation of the colour rendering properties of a lamp and at mesopic levels it has been proposed that the relationship between lamp type and illuminance is characterised by S/P ratio and R_a (Fotios & Goodman, 2012).

Rules of thumb suggest that 10-15 data points are required per term in an equation determined by regression (Field, 2005). There are only 20 data points in data set A and 17 data points for set B, and thus it was determined to explore models with at most two-terms. At first each metric and function were used to predict a model. Then, a systematic approach was used in which each combination of metric and function was paired with every other combination, and this was repeated for both data sets individually.

In only two cases it was found that adding the second term increased the correlation (r^2) , and these two cases are shown in Table 6.14. For data set A, correlation (r^2) increased slightly from a value of approximately 0.69 as found for the individual metric to approximately 0.79 for the models with two metrics. For data set B, no advantage of adding the second metric was found.

Model	Equation	" 2	r²	Null
Model	Equation		change	value
1	E ₁ /E ₂ =0.017 + (GA ₂ ^{0.14} /GA ₁ ^{0.14})	0.69	—	—
2	$E_1/E_2 = 0.466 + 0.604(GA_2^{0.14}/GA_1^{0.14}) - 0.682(logS/P_1 - logS/P_2)$	0.78	0.09	0.388
3	E ₁ /E ₂ =1.021 + 0.624(GA ₁ -0.12-GA ₂ -0.12)	0.70	—	_
4	$E_1/E_2 = 1.052 + 0.382(GA_1^{-0.12}-GA_2^{-0.12}) - 0.668(logS/P_1 - logS/P_2)$	0.79	0.09	0.766

Table 6.14 Results of stepwise regression analysis with two metrics for data set A.

Except the equation presented in Table 6.14, the calculations done with two metrics in stepwise regression analysis always determined a model only with one of the metrics which was giving the same values of slope and intercept as presented in Table 6.13. According to these regression results, the single metrics of difference of S/P ratio and log ratio of GA were providing higher correlations than the models presented in Table 6.14. As shown in Figure 6.6, it was possible to have better predictions of brightness with only one metric instead of using both of the metrics in a model.



Figure 6.6 Graphs of models using one metric for data set A. a) difference of S/P ratio model. b) log ratio of Gamut area model. Data points from new experiment are shown in red

Therefore, for data set A it was concluded that the results were best modelled using these two s of difference of S/P and log ratio of GA:

$$E_1/E_2 = 1.052 + 0.648(S/P_1^{-0.79} - S/P_2^{-0.79})$$
 Equation 6.2

$$E_1/E_2 = 3.23 - 2.182(logGA_2 / logGA_1)$$
 Equation 6.3

Equation 6.3 with log ratio of GA was aiming to use a similar function with what Boyce (1977) proposed. The correlation of 0.81 obtained from this equation showed that Boyce (1977) proposal can result to a good prediction.

None of the equations were providing a model with both metrics when the data set B was used. For this data set, most of the correlations appeared to be lower than data set A except the ratio of GA. According to the results of data set B GA had better predictions than S/P ratio.

For data set B, it was concluded that the results were best modelled using this equation with the ratio of GA ($r^2=0.69$):

$$E_1/E_2 = 0.034 + 0.934(GA_2^{0.25}/GA_1^{0.25})$$
 Equation 6.4

As a result, the best fits of the potential metrics with mean illuminance ratio were obtained with difference of S/P ratio (r^2 =0.82) and log ratio of GA (r^2 =0.81) with 20 data points (Figure 6.6). Accordingly, both metrics were equally plausible to predict spatial brightness. Both metrics had similar correlation with Fotios and Levermore (1998) (r^2 =0.80) model of log ratio of GA. Besides, these were higher correlation values than Boyce (1977) which was presenting an 'r' value of 0.80 (r^2 =0.64) with GA (Equation 6.1) in Figure 2 of the original article. According to this figure, 12 data points were used, in the current study a larger sample size was used. This was indicating a better fit of the model with spatial brightness.

6.8 Category Rating Studies

A further test of the potential brightness models was to compare predictions of brighter light source with those gained using a category rating procedure. Past studies using category rating have tended to use only a single reference illuminance, and thus the results identify the direction of difference in brightness (which is brighter) but there is no estimate as to the magnitude of the difference.

Requirements for reliable data when using a category rating procedure include that the stimuli are presented in a random order and that the number of stimuli do not greatly exceed the number of rating points; it is also desirable that stimulus ranges and response ranges are anchored but few studies have done this as explained in Chapter 3. In Table 6.15 credible studies which tested three or more SPDs (i.e. for discrimination when testing predictions) were identified to continue analysing. Two studies meeting these criteria and the ones likely to get the SPDs are those by Vrabel et al (1998) and the second experiment reported by Boyce & Cuttle (1990). Firstly, it was required that the SPD of lamps used in these studies were established in order to calculate values of S/P and GA.

Study	Variables	SEEKING 3 OR MORE SPD	If yes, likelihood of getting SPD
Akashi & Boyce, 2006	Field study in 4 open plan offices 2 x illuminance; 3 x SPD	Y	Х
Boyce, Akashi, Hunter & Bullough, 2003	Lab study 2 x illuminance 2 x SPD	Х	-
Boyce, 1977	Lab study 2 x illuminance 3 x SPD (exp1) 4 x SPD (exp2)	Y	Y (ALREADY USED SAME AS BRIGHTNESS MATCHING)
Boyce & Cuttle, 1990 (experiment 2)	Lab. study 1 x illuminance 4 x SPD (8 cases with object and wall colour variations)	Y	Y (ESTIMATED SPD DID NOT GIVE SAME CCT, CRI AS THE ORIGINAL ARTICLE)
Davis & Ginther, 1990	Lab. study 3 x illuminance 2 x SPD	Х	-
Flynn & Spencer, 1977	Lab. study 1 x illuminance 8 cases incl. 4 x SPD)	Y	Х
Han & Boyce, 2003	Lab study 3 x illuminance 3 x SPD 3 x décor	Y	Х
Piper, 1981	Lab study 1 x illuminance 2 x SPD	Х	-
Vienot, Durand & Mahler, 2009	Lab study 3 x illuminance 3 x SPD	Y	Х
Vrabel, Bernecker & Mistrick, 1998	Lab study 1 x illuminance 5 x SPD	Y	Y

Table 6 15 Summary	of credible rating	studies and the or	nes using 3 or more	SPDs
rabic 0.15 Ourninary	of cicubic rating		nes using 5 or more	01 D3.

Graphs of the SPD for the lamps used by Boyce and Cuttle (1990) were presented in Figure 3 of their article and these were digitised to get SPD values. Values of CCT and CRI obtained using these estimated SPD do not compare well with the values reported by Boyce and Cuttle (1990) as shown in Table 6.16 which suggests the estimated SPD are not reasonable and therefore these data were not used in analysis.

Lamp	Values reported in Boyce and Cuttle (1990)		Values calco estimat	ulated using ed SPD
	ССТ	CRI	ССТ	CRI
Lamp A	2700	82	2225	56
Lamp B	3500	85	3224	69
Lamp C	4200	85	2817	76
Lamp D	6300	85	5514	83

Table 6.16 Comparison of CCT and CRI calculated using estimated SPD with values reported by Boyce & Cuttle (1990).

Graphs of the SPD for the lamps used by Vrabel et al (1993) were obtained from Vrabel's thesis (Vrabel, 1993) and digitised. Values of CCT and CRI obtained using these estimated SPD compare well with the values reported by Vrabel et al (1998) as shown in Table 6.17 which suggests the estimated SPD are reasonable. Table 6.18 shows the values of S/P and GA as calculated for these lamps.

Table 6.17 Comparison of CCT and CRI calculated using estimated SPD with values reported by Vrabel et al (1998).

1	Values repor	rted in Vrabel	Values calc	ulated using
Lamp	(19	198)	estimated SPD	
	ССТ	CRI	ССТ	CRI
CW	4100	62	4189	66
HGHP	5000	91	4826	89
MH	4200	60	4085	61
Т8	4100	82	3921	81
WHPS	2700	80	2765	85

Table 6.18 Characteristics of lamps used by Vrabel et al (1998) as determined from estimated SPD.

Lamp	S/P	GA
CW	1.558	0.00476
HGHP	1.946	0.00640
MH	1.570	0.00409
Т8	1.584	0.00572
WHPS	1.183	0.00363

Table 6.19 compares the results from Vrabel et al (1998) with predictions made using S/P and GA. Vrabel et al reported differences in brightness ratings (their Figure 8) with an apparent significance level of p=0.10, although this is not clear. Vrabel et al also used a discrimination

task, and in two cases the results were different to that found using the rating task according to Table 3 in their report. In Table 6.19 the identification of brightest lamp is guided by the discrimination results for these two lamp pairs: for the WHPS-CW lamp pair the rating conclusion of WHPS being brighter was changed to no difference and for the WHPS-MH lamp pair, the rating conclusion that WHPS was brighter was changed to no difference.

Test results		Prediction of brighter lamp		Predictions match results? (1 = yes, 0 = no)				
Lan	nps	Mean br rat	ightness ing	Brighter Iamp	S/P	GA	S/P	GA
1	2	1	2					
MH	Т8	3.2	4.8	Т8	Т8	Т8	1	1
Т8	CW	4.8	3.8	Т8	Т8	Т8	1	1
MH	HGHP	3.2	4.6	HGHP	HGHP	HGHP	1	1
CW	HGHP	3.8	4.6	HGHP	HGHP	HGHP	1	1
WHPS	Т8	4.5	4.8	Т8	Т8	Т8	1	1
MH	CW	3.2	3.8	CW	MH	CW	0	1
WHPS	CW	4.5	3.8	ns	CW	CW	0	0
WHPS	HGHP	4.5	4.6	ns	HGHP	HGHP	0	0
Т8	HGHP	4.8	4.6	ns	HGHP	HGHP	0	0
WHPS	MH	4.5	3.2	ns	МН	MH	0	0
					Т	otal (Yes)	5	6

Table 6.19 Testing predictions of the brightness ratings of Vrabel et al (1998).

Mean brightness ratings that originally reported as bright (1) / dim (7) converted to dim (1) / bright (7) in this table. ns = difference is not significant. Prediction of brighter lamp was chosen according to the metric values in Table 6.18.

In Table 6.19, the predictions of brighter lamp were established by comparison of the differences in values of S/P and GA as presented in Table 6.18 for each lamp. A zero difference would suggest no difference in brightness. In Table 6.19 the six lamp pairs for the test results suggesting a difference are grouped: for these, S/P ratio and GA tend to identify the differences but there is little difference between GA and S/P. Neither of the two models is able to consistently predict the results of the four lamp pairs for which the test results do not suggest differences in brightness to be significant.

6.9 Summary

Two metrics for spatial brightness, S/P ratio and GA, were examined in this chapter using data from past studies. They both appeared to give equally good predictions according to stepwise regression test; however it was not possible to conclude that any one model is better than the others. Analysis done by using rating results of Vrabel et al (1998) also seemed not to indicate that neither of the two metrics tended to predict any difference.

There were some limitations for the model predictions that were conducted by using data from past studies. The most definite one was the data points that were used to predict the models. Due to a small amount of availability of credible data, the set including all the lamps had 20 data points and for data set B, it was even smaller with 17 illuminance ratios. This might be the main reason of not being able to achieve any models consisting two of the metrics with data set B and similarly, obtaining higher correlations with models of one metric instead of two metric models with data set A. Another limitation of this data set was, the SPD information was not provided in all the studies. Even with the studies presenting SPD graphs or the metric information, it was not certain that the measurements were taken from actual experiment set up or just reporting the information of what the manufacturer provides.

Chapter **7**

7 Summary

- 7.1 Work carried out for this study
- 7.2 Research Methodology
- 7.3 Does SPD affect spatial brightness?
- 7.4 Model analysis
- 7.5 Analysis for Future Work

7.1 Work carried out for this study

This research investigates the spatial brightness response to lighting from lamps of different spectral power distribution (SPD) at photopic levels of illumination. There were three stages of work. First, past experimental studies were reviewed in the search for credible data of the relationship between SPD and illuminance. This review used consideration of methodology to determine which studies were credible, and these requirements were shown in Table 7.1. As part of this process, further study was carried out of the response range and test instructions in the category rating procedure. The main requirements for credible data include a balanced or randomised stimulus presentation order and clear reporting of the study and the analysis used. Of the 65 studies reviewed it was concluded that 19 provided credible data. Desirable requirements like having a null-condition trial in the experiment were also specified, however they were not included in the essential requirements to identify the credible studies as very few of them included these requirements, like Akashi & Boyce (2006), Boyce (1977), Fotios & Gado (2005) and Houser, Tiller & Hu (2004).

Requirement	Procedure					
Requirement	Category rating	Matching	Discrimination	Adjustment		
Randomised or counterbalanced stimulus order	✓	~	~	~		
Appropriate data analysis and informative reporting	✓	\checkmark	~	~		
The number of points in the response range and the number of stimulus magnitudes are approximately equal	✓					
Counterbalance the spatial and/or temporal location of the stimuli		~	~			
Compare all possible pairs of the test stimuli		\checkmark	~			
Illuminance control applied to both stimuli		\checkmark				
Choose stimulus ranges and starting points with consideration to range bias and anchor bias				~		

Table 7.1 Essential requirements for each experimental method.

Another stage was to conduct a new experiment to evaluate different brightness under lamps of different SPDs, with lighting specifically chosen to compare S/P ratio and GA. The results suggested the visual mechanism underlying these responses may have an additive effect.

These new data were added to the set of credible data from past studies and the brightness modelling applied with S/P ratio, GA.

This set of data used to screen potential metrics for spatial brightness, an approach similar to that used by Cowan and Ware (1983) who established a metric for the chromatic contribution to small field (2 degree) brightness. This thesis reports examination of two metrics, S/P ratio and gamut area (GA), following evidence in previous studies (Berman et al, 1990; Boyce, 1977) that these might be suitable. Of the 19 credible studies, this analysis used only those providing a quantitative relationship between illuminance and SPD, i.e matching and discrimination studies. To use this set of studies required that the SPD were estimated, this being rarely reported. It was found that, for these data, S/P ratio and GA were not independent – for a particular lamp, S/P and GA would both be high or both be low and this meant it was not possible to discriminate between them.

Finally, as a guide to future work two new metrics, intrinsically photosensitive retinal ganglion cell (ipRGC) and Bullough (in press 2014) were also analysed in regression tests with this set of data.

7.2 Research Methodology

7.2.1 Number of response categories

26 past studies of brightness perception out of 30 were using odd numbers of response categories and more specifically 17 of them had 7 point range. Accordingly, review of past studies using the category rating procedure required further consideration of two issues: (i) does the response range matter, and (ii) do evaluations of brightness and visual clarity give the same or different results?

Further tests on category rating task applied to evaluate environmental condition including brightness and visual clarity of lighting in the room was carried out with semantic differential scale and independent samples. Main idea while conducting this experiment was to compare even and odd numbers of response categories in order to investigate the recommendation done by Fotios and Houser (2009) suggesting to use even rating scales in brightness rating tasks. According to the results, there was no significant difference in mean ratings between 5-, 6-, 7- and 8- point ranges. However, the distribution of the brightness evaluation suggested significant differences depending on the response categories used for all the point ranges except 5- and 6-point ranges. There were no significant differences between number of response categories used to evaluate visual clarity. Accordingly, the traditional view might be followed with the idea that truly neutral respondents will randomly choose one or the other side of the issue when there was no neutral point provided (Nowlis et al, 2002); therefore forcing them to choose shouldn't bias the overall results.

The findings of the current study were the opposite of study done by Dawes (2008) on cost of items in a store and money saving. In Dawes (2008) 5-, 7- and 10- point ranges were compared and there were little difference between mean ratings of 5- and 7- point ranges found. When the combined results of 5- and 7- point ranges were compared with 10-point range the difference between mean ratings were significant. However, when the distribution of the evaluations depending on the response scale was tested no significant differences were obtained both for skewness and kurtosis. These different findings between different evaluation items raised the question of the results of rating studies being related with the topic that was asked to the participants. There were other examples of different results of even and odd response categories depending on the topic, like studies done on education, social security and consumer attitude (Bishop, 1987; Nowlis et al, 2002; Moors, 2008). In that case, Payne (1951) suggested using even number of categories is better if the purpose is to understand which direction people are leaning on an issue. Otherwise, offering a middle category may give more definite convictions on the issue. Accordingly in this study, using even number of response categories were classified under desirable requirements. It was suggested that using even numbers of response categories in the future may provide more information to understand how it affects the results of brightness studies.

7.2.2 Defining visual response

There were different visual objectives being used to evaluate lit environment. Some visual objectives were used in place of each other in category rating studies which lead the participants to give similar responses to different visual objectives. Therefore, this part of the study focused on the visual objectives that were evaluated by the participants in the lighting experiments. Visual clarity and brightness were two visual objectives that were widely used in the lighting studies. Although, brightness had more defined explanations, visual clarity had varying descriptions. Some researchers identify visual clarity as different objective than brightness (Aston and Belchambers, 1969). Especially, Vrabel et al (1998) gave two different definitions for brightness and visual clarity. There were other researchers who provide definitions of visual clarity relating with brightness and metrics that affect brightness like CRI. When the responses of the participants to brightness and visual clarity in the past studies were analysed, it was demonstrated that the two objectives were similar to the participants. The most explicit difference between the results of these two visual objectives and visual clarity. In this study, it was concluded that there was no difference between brightness and clarity ratings.

According to this information, explaining the features of the experiment in detail and provide clear instructions about rating items to the participants carry an important role on their decision

making. Therefore, instructions given to the participants need to be considered carefully for future research.

7.2.3 Procedures

Most of the past studies that do not meet the criteria for credible data were having missing information; thus they were not clear about the procedural design and generally the results were included the trends of the participant evaluations without sufficient quantitative data. In such cases, the studies lost their reliability. One of the purposes in this thesis was to apply these requirements in a new experiment and compare if different procedures give different or similar results. Therefore, two experimental methods were used in the new experiment.

Matching and discrimination methods were applied with both chromatic and achromatic environments in order to test S/P ratio and GA effects on spatial brightness. Discrimination task applied in two different ways, one of them used brightness lumens (Equation 2.3) approach suggested by Berman et al (1990), which was focusing on S/P ratio differences of two SPDs. The second approach had the main purpose to validate matching test using similar procedure as Fotios and Cheal (2011) in sequential form instead of side-by-side. To apply these experimental methods, a booth set to provide fulfilled vision and lamps, which had different SPDs were presented sequentially.

When the results of Berman et al discrimination and Fotios and Cheal matching compared, the trends were in the same direction. The effect of S/P ratio was bigger in Berman et al discrimination procedure in achromatic environment than the matching test. However, there was no significant difference between two procedures in chromatic environment. The findings of Fotios and Cheal discrimination task also confirmed the results gained in both Berman et al discrimination and Fotios and Cheal matching tests. According to the results, the lamps with higher S/P ratio and GA in the compared pairs perceived brighter which was indicating an SPD effect on spatial brightness.

Besides, the doubt about Berman et al method of sequential discrimination was tested. According to null condition results, presenting 2 stimuli with 3 alterations in order to compare their brightness was found to be a valid application for this method.

7.3 Does SPD affect spatial brightness?

15 of the 19 credible studies supported the SPD of the interior lighting effect on spatial brightness. Similar with the results of Berman et al (1990), S/P ratio effect on spatial brightness was investigated in the new experiment. In this experiment higher spatial brightness were reported by the participants with achromatic environment under lamp SPD having high S/P ratio, even though it had lower luminance, same as Berman et al (1990). In contrast with the

results of Berman et al (1990), this difference between SPDs was increased when the luminance levels increased, which was indicating an effect of absolute luminance on equal brightness. The same effect also existed in chromatic environment evaluations. When colour added with a Mondrian like surface to the visual environment, lamp SPD, which had low S/P ratio and high luminance perceived brighter in one of the comparisons. In the second comparison, the difference between SPDs was smaller than achromatic environment, however, lamp having high S/P ratio at lower luminance was still perceived brighter. The discrimination method was applied to chromatic environment as an additional approach to Berman et al (1990) and a significant difference between chromatic and achromatic environment was investigated.

In the second part of the experiment three SPDs were included; one SPD had lower S/P ratio and same chromaticity with second SPD, third SPD had higher GA and similar S/P ratio with second SPD. Hence, it was possible to investigate S/P ratio and GA effects both separately and together by evaluating different SPD pairs.

In the matching test each of the three SPDs were compared with each other to examine S/P ratio and GA effect on spatial brightness. Each pair found to be significantly different from each other, indicating a SPD effect. Lamps A and B had S/P ratio difference with similar chromaticity and Lamp B, which had higher S/P ratio was evaluated to be brighter. Lamps B and C had GA difference with similar S/P ratio, Lamp C, which had higher GA was perceived brighter. According to these results, both S/P ratio and GA are good predictors of spatial brightness individually. In the last comparison of Lamps A and C, Lamp C had both higher S/P ratio and GA than Lamp A and it was evaluated to be brighter. This result demonstrated that their effects were additive.

7.4 Model analysis

The effect of SPD on spatial brightness was supported with the new experiment in the previous section. Stepwise regression tests were done to predict this effect using data from previous studies and the new experiment. Past studies providing either illuminance ratio at equal brightness or multiple levels of illuminance in two-sample discrimination tests were specified to test S/P ratio and gamut area (GA). Two groups of data set were arranged to do stepwise regression tests to predict a model of illuminance ratio with S/P ratio and GA; one set of data had 20 (data set A) and the other one had 17 (data set B) data points.

With data set A, two models for spatial brightness including both of the metrics were obtained using log difference of S/P ratio together either with ratio of GA ($r^2=0.78$) or difference of GA ($r^2=0.79$). However, these models didn't provide higher correlations than the models using only one metric. According to the results, both of the metrics provided high correlations with

illuminance ratio individually (e.i correlation with ratio of log GA, r^2 =0.81), indicating reliable predictions of spatial brightness (Equation 7.1).

$$E_1/E_2 = 3.23 - 2.182(logGA_2 / logGA_1)$$
 Equation 7.1

In data set B, there were no models calculated with using two metrics. However, for this set of data models with GA had explicitly higher correlations than S/P ratio. The best prediction was obtained using the ratio of GA (r^2 =0.69) (Equation 7.2).

$$E_1/E_2 = 0.034 + 0.934(GA_2^{0.25}/GA_1^{0.25})$$
 Equation 7.2

According to these results, GA seemed to predict better brightness models by itself than using it in combination with S/P ratio. This finding was contrary to the results in new experiment (Chapter 5). The reason for this difference could be arising from the data set that was used in model analysis. The data points used for this type of analysis were too small and in data set B it got even smaller. This might be the reason not to predict a model with two metrics in regressions test using data set B. Another limitation was with the SPD establishing. Since none of the studies provided the exact SPD values, which were measured in the actual experiment environment, the metric values used to run regression tests accommodate some uncertainty.

Therefore, GA and S/P ratio can be used together to predict spatial brightness, however, past data didn't provide the conditions to find out this effect, as GA and S/P ratio values of the lamps were not independent. In order to develop a model with GA and S/P ratio, more experiments needs to be done with lighting sources which have independent values of GA and S/P ratio.

7.5 Analysis for Future Work

As it was mentioned in Chapter 2, Berman (1995) indicated the rod photoreceptors' effect on spatial brightness with brightness lumens, which included an equation of square root of S/P ratio. Then, a new photoreceptor named as intrinsically photosensitive retinal ganglion cells (ipRGC) was discovered having peak sensitivity around 480 nm. Therefore, Berman (2008) found this metric to be applicable to predict spatial brightness and highly correlated with S/P ratio. As ipRGC is related with spectral response of circadian system, it was also found to have effect on alertness and hence on task performance (Boyce, 2014). Therefore, further correlation analysis were done with ipRGC/P using data set B which didn't include the comparisons using lamps of Grolux, HPS and LPS.

Similarly, Bullough (in press 2014) demonstrated an equation to predict brightness including ipRGC and short-wavelength sensitivity (Equation 7.3).

 $B(\lambda) = V(\lambda) + 0.5 \text{ Mel}(\lambda) + 1.5 \text{ S}(\lambda)$

Equation 7.3

 $B(\lambda)$ = predicted brightness

 $Mel(\lambda)$ = spectral distribution of ipRGC

 $S(\lambda)$ = spectral distribution of S-cone

Table 7.2 shows the results of the correlations of ipRGC/P and Bullough brightness model with illuminance ratios of data set B.

Table 7.2 Regression coefficients (r^2) between illuminance ratios at equal brightness and functions of the proposed metrics.

Metric Eurotion	r ²			
metric runction	I	slope	intercept	null value
	Analysis w	ith data set B		
ipRGC/P2 ^{0.53} / ipRGC/P1 ^{0.53}	0.441	-0.546	0.996	0.450
ipRGC/P1 ^{0.18} - ipRGC/P2 ^{0.18}	0.511	1.262	-0.269	0.993
$Log(ipRGC/P_1) - Log(ipRGC/P_2)$	0.020	0.006	0.882	0.888
Log(ipRGC/P ₂) / Log(ipRGC/P ₁)	0.508	-0.509	1.001	0.492
Bullough2 ^{-0.10} / Bullough 1 ^{-0.10}	0.161	5.235	0.928	6.163
Bullough $1^{0.18}$ - Bullough $2^{0.18}$	0.164	2.654	-1.724	0.930
Log Bullough $_1$ - Log Bullough $_2$	0.161	1.868	-0.941	0.927
Log Bullough $_2$ / Log Bullough $_1$	0.164	-0.436	0.930	0.494

Powers for difference and ratio equations were optimised.

According to Table 7.2 correlations of Bullough brightness model with illuminance ratio of data B were too low. However, difference of ipRGC/P and log ratio of ipRGC/P provided closer correlation indexes to S/P ratio and GA results of data set B as in Equation 7.2. According to these results, it will be a good proposal to consider ipRGC/P as brightness metric for future work.

Chapter **8**

8 Conclusion

- 8.1 General conclusions
- 8.2 Recommendations for further work

8.1 General conclusions

The aim of this study was to identify a metric for predicting spatial brightness. The first approach was to use the results of past studies testing potential metrics following the method used by Cowan and Ware (1983). 65 past studies investigating SPD effects on spatial brightness were reviewed; a limitation of drawing conclusions from these data is that they tended to use different experimental methods. Experimental investigations of the effects of lighting spectrum on the perception of spatial brightness are susceptible to misleading results owing to the methods used. A significant number of past studies cannot be considered credible owing to systematic bias in experiments, poor explanation of methods, or lack of quantitative data. The spectral power distribution (SPD) of interior lighting influences the level of spatial brightness for occupants. This conclusion is supported by 15 of 19 previous studies that are considered credible, together with the results of experiments carried out for the current study.

Review of the methods included an experiment focusing on even and odd numbers of response scales used in category rating tests and a meta-analysis comparing results of brightness and clarity rating judgements. It was concluded that using response ranges having 5-, 6-, 7- and 8-points resulted in the same mean ratings (when converted to a common scale) of brightness while leading to different distribution profiles (section 4.2.2). Past researchers have mixed opinions as to whether spatial brightness and visual clarity describe the same, or different, visual responses. While brightness is reasonably well defined, visual clarity has varying descriptions, some suggesting similarities with brightness and some identified differences between brightness and clarity. An analysis was carried out using data from past category rating studies to compared judgements of brightness and clarity: it was found that they lead to very similar responses to a given lighting condition (section 4.3.2.2). This study provides evidence for judgements of spatial brightness and visual clarity made by naïve test participants can lead to the same outcome even the researchers define these terms differently.

One approach was to carry out a new experiment to test these metrics. This experiment designed with three SPDs chosen carefully to isolate and identify the effects of S/P and GA on spatial brightness. The experiment enabled full field vision and applied with sequential evaluations of stimulus pairs with matching and discrimination methods. The two methods (matching and discrimination) lead to similar results. Null-condition trials assured that three times alteration between the stimuli in a pair was sufficient to discriminate their brightness difference. Besides, the doubt about interval bias in Berman et al (1990) was unwarranted. The results comparing different SPDs show that higher S/P and higher GA enhanced spatial brightness. When the chromaticities of the lamps were same, higher S/P improved spatial brightness. Moreover, when both S/P and GA of the lamps were different, lamp having higher S/P and higher GA provided higher spatial brightness, showing that their effect was additive. Same experiment was conducted with chromatic environment. Spatial brightness enhanced with

addition of chromatic Mondrian like surface except for the lighting conditions having similar chromaticity.

The focus of this study was to investigate two potential metrics for spatial brightness, S/P ratio and gamut area (GA). Results from the past studies considered to be credible were used to test these models. Both of the metrics provided high correlations with illuminance ratio individually, indicating reliable predictions of spatial brightness, as might be used in studies considering only one metric to demonstrate success of that metric. However, they were found not to be independent for this data set and thus it was not possible to discriminate between the two metrics.

After data from the new experiment was added, remodelling with 20 data points was performed. As a result, the models of the difference of S/P ratio and log ratio of gamut area had the best fits with spatial brightness. This was indicating the same effect of S/P ratio and GA on spatial brightness. Furthermore, with some more future works that use lamps having independent values of GA and S/P ratio, additive effect of these two metrics can be explored.

8.2 Recommendations for further work

Knowledge gained from the following work would support to provide higher spatial brightness and correct use of experimental methods:

- Models with SWS-cone and different alternatives of chromatic contribution can provide good spatial brightness predictions. An application of prime-colour theory proposed by Houser, Tiller and Hu (2007) and SWS/P metric proposed by Rea, Radetsky and Bullough (2011) can provide accurate applications for predicting spatial brightness.
- A study to observe how these metrics work together with ipRGC and provide pupil response to brain can improve the knowledge of spatial brightness.
- Collect or try to find more credible data to gain bigger sample size to predict a spatial brightness model from past data.
- A field study of spatial brightness with both chromatic and achromatic environment. It will be necessary to consider lower light levels than the ones originally used to compensate brightness with high S/P and GA.
- An investigation of even and odd rating scales with repeated measures and Likert scale.

The analysis in this thesis of methodology in the category rating procedure suggests three precautions that should be considered when using this approach in further work:

- Include null condition trials to give information about bias that occurred during the experiment. In category rating this might be the repeated evaluation of identical stimuli to allow comparison of responses gained. In discrimination trials this might be the use of identical stimuli in both intervals.
- Test instructions should accurately identify the visual response sought in an evaluation and should include steps to check that the participant's understanding matches that of the experimenter.

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A Category Rating Studies

A.1 List of Category rating studies of Spatial Brightness A.1 List of Category rating studies of Spatial Brightness

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Table /

Study	Method						Essential Re	equirements		_	Desirable Requirem	ients		Comments
	Method	Number	Age	Adaptation	Type of	Stimulus	Stimulus	Stimulus	Quantita	Conclusion	Stimulus range	No. of	Null	(biases and
		of		time	scale	variations	order	magnitudes	tive data	: Reliable	anchored?	response	condition?	lamp SPD)
		subjects				(independe	randomis	equalized	reported	evidence?		categories	(control	
						nt samples	ed or	with				even? (forced	group)	
						or repeated measures)	lanced?	range?				(iorceu choice)		
Studies I	meeting requiremen	its for good	procedure	in the category r	ating task									
Akashi	Field study in 4	20-90	N.R	3-9 months of	Agree or	Independe	N.A	Yes	Yes	Yes	No	Yes	Yes	SPD effect
8	open plan			working	disagree	nt samples		(3 stimuli, 2				(2 category		exists
Boyce,	offices to find			under same	I			rating				test)		
2006	out CCT effect on brightness			condition				points)						
	perception and			2 0 months of				V.c.	Vec		QA			0000000
	visual	08-07	Y.Z	3-9 months of	-2/2 (very	Independe	N.A	Yes	Yes	Yes	NO	NO	Yes	Kesponse
	performance in			working	gloomy/ver	nt samples		(3 stimuli, 5 ratiar						contraction
	relation to				y bright)			raung						Dids, Uruer
	energy saving.			condition	-2/2 (too			points)						errect.
					dark/too									
					bright)									effect.
Boyce,	Experiment	144	N.R	N.R	1/7 (very	Repeated	Yes	Yes (5	Yes	Yes	No	No	Yes	SPD effect
1977	with booths. 5				satisfactory	measures		stimuli, 7						exist
	fluorescent				/very			rating						
	lamp. 2				unsatisfact			points)						
	illuminance levels.				ory)									
Boyce,	Full size lab.	N.R	2	~10 min	1/7	Repeated	Yes	Yes (4	Yes	Yes	No	No	No	No SPD effect
Akashi,	Study. 2 lamps		groups		(agreement	measures		stimuli, 7						
Hunter	with different				evalution)			points)						
જ	spectrum. 2		Z8,											
Bulloug	illuminance		Q/-10											
h, 2003	levels													
Boyce	Full size lab.	10	20-55	Fully adapted	1/5 (very	Repeated	Yes	Yes (8	Yes	Yes	No	No	No	Response
જ	study			(took 2 colour	much	measures		stimuli, 5						contraction
Cuttle,	investigates the			discriminatio	so/not at			rating						bias.

SPD effect exist	No SPD effect. However, the evaluations were done according to factor analysis.	Contraction bias. Not certain about SPD effect because of the evaluations according to factor analysis.	SPD effects.	No SPD effect exist	SPD effect exist
	Yes	Yes.	N	No	No
	°Z	9Z	A.N	No	٥N
	٥	S	N	Yes (2 conditions 1st one is the neutral)	No
	Yes	Yes	Yes	Yes	Yes
	Yes	Yes	Yes	Yes	Yes
points)	Yes (6 stimuli, 7 rating points)	Yes (8 stimuli, 7 rating scales)	N.A (continues rating line)	Yes (2 stimuli, 7 rating scales)	Yes (6 stimuli, 7 rating
	Yes	Yes	N.R	Yes	Yes
	Repeated measures	Repeated measures	Repeated measures	Repeated measures	Repeated measures
all so) (for bright, dim, hazy, clear)	1/7 Bright/dim Continues rating line.	Bright/dim & hazy/clear (no numerical expressions used)	Continous line, dim/bright	Dim/bright	1/7 (dark/brigh t and
n tests before evaluating the lighting source)(20-30 min.)	60 sec. Not fully adapted.	60 sec. Before each setting	N.R	Fully adapted more than 13 min.	Not fully adapted (3 min. of
	20-45	Early 20's	20-40	Avera ge 30	18-36
	40	48	21	20	30
CCT effect on interior perception of the individuals.	Full sized lab. experiment with 2 types of fluorescent lamp and 3 illuminance	Full size lab. test with constant illuminance by diff. CCTs and several light sources	Booth experiment. Fluorescents with 3 different CCTs, 3 illuminance levels.	Full size lab test with 2 lighting conditions.	Study done in booths with LEDs.
1990 (exp. 2)	Davis & Ginthe r, 1990	Flynn & Spence r, 1977	Han & Boyce, 2003	Piper, 1981	Vienot, Durand &

Mahler , 2009				adaptation for each illuminance)	crepuscular /clear)			points)						
Vrabel, Bernec ker & Mistric k, 1998	Full size lamp study with 5 different types of lamps which has different SPDs.	29	Avera ge 21	Not reported (Fotios (2006) suggested to 1 min.)	1/7 (bright/dim and clear/hazy)	Repeated measures	Yes	Yes (5 stimuli, 7 rating points)	Yes	Yes	Yes (Anchoring by description)	°Z	°N N	SPD effect exist
Studies ¹	NOT meeting require	ements fo	r good proc	cedure in the categ	ory rating task									
Baron, Rea & Daniels	Lab. study done by 2 illuminance	64 M, 27 F	Under gradst udents	Adapted, ta- ook 3 other tasks before	Dim/bright, clear/hazy	Independe nt samples	N.A	A.N	No. No mean ratings	2	ON	oN N	N.N	Didn't report the rating scale.
				cvariating the env. (Knez suggests 20 min.)					dev.					No SPD effect
	Lab. study done by 2 illuminance and 2 CCTs.	59 M, 13 F	Under gradst udents	Adapted, took 5 other tasks before evaluating the env.	Dim/bright, clear/hazy	Independe nt Samples	A.N	N.A	No. No mean ratings and st. dev.	Ŷ	ON	°Z	N.N N.N	Didn't report the rating scale. No SPD effect
Barthol omew, 1975	Field study done in classrooms with two different types of light sources with different colours.	58	Under grad studen ts	During the whole class same lighting condition	1/5 (dull/bright & dark/light). Continues rating line.	Repeated measures	Yes	Yes (2 stimuli, 5 rating points)	No. No statisti cal data.	Ŷ	Yes	°Z	л. Л.	Response contraction bias, no null condition trial. SPD effects

Response contraction	bias, grouping bias.	NO SPD effect.	Not certain	No seperate data and results of brightness. SPD effects.	Has too many missing information.	No quantitative data, not clear about the process.
No			Yes	N.N	х. Х	٥N
No			٥	Q	Q	N.R
<u>o</u>			9	I.R.	R.	0
No			Ŷ	°Z	Ŷ	on on one of the second
Yes (mav	have error type 1)		No (mean rrating ed, no ed, no st. dev. and stats)	°N	Yes	No. No statisti cal data
No (22 stimuli. 5	rating points)		Yes (5 stimuli, 9 points)	Yes (7 stimuli, 7 rating points)	Not certain about how many stimuli were used?	N.N
Yes			Yes?	Yes	N.R	No
Repeated measures			Repeated measures	Repeated measures	Repeated measures	Repeated measures
1/5 (very much	so/not at all so) (for bright, dim,	nazy, ciear)	9 point scale(No numbers given, not bright enough/to o bright)	1/7 (highly satisfying/h ighly disterbing)	Equal +3	N.N
Fully adapted			N.R	2-3 sec.		Fully adapted (3 weeks for each lighting condition)
20-55			л. Х	N.R		N.R
15			40	16		40
Full size lab. studv	investigates the CCT effect on interior	perception of the individuals.	Field study with 3 fluorescent lampsand daylight.	Exp. done by booths includes brightness rating with 7 lamp types differing in CCT.		Field and full size lab. study, with different lighting scenarios.
Boyce &	Cuttle, 1990 (exp. 1)		Cockra m, Collins & Langdo n, 1970	DeLane y et. al., 1978		Fleisch er, Kruege r & Schierz , 2001

Response contraction bias, grouping bias, no null condition. SPD effects depending on graphs.	SPD effects.	SPD effects	Response contraction bias, no null- condition trial. SPD effect exist	SPD effects.	Does not include results for brightness.
° Z	0 N	٥N	ŶŹ	NO	N.N.
°Z	°Z	Q	9Z	Yes	N
х Х	Ŷ	No	Ŝ	N	N
Ŷ	oN	٥N	٥	oN	No
No. No c data c data	No. No statisti cal data, mean ratings , st. dev.	No. No mean ratings and st. dev.	No. No ratings and st.dev.	No	No
No (25 stimuli, 7 rating points)	Yes (9 stimuli, 7 rating points)	Yes (4 stimuli, 5 rating points)	Yes (3 stimuli, 5 rating points)	No (12 stimuli, 4 rating points)	Yes (5 stimuli, 7 rating points)
х. Х	No (balance d only starting from top or bottom) 1-9, 9-1	Y.N	A.N	N.N	Yes
Repeated measures	Repeated measures	Independe nt samples	Independe nt samples	Repeated measures	Repeated measures
Very/neith er/very (dark/brigh t & clear/vague)	1/7 (very dim-very bright)	Dim, bright (one way rating)	Little or not at all/very much (for bright, dim)	1/4 (very good evaluation/ worst evaluation)	1/7 (not satisfying/s atisfying)
Not mentioned (Probably, first inspiration, because 25 stimulus showed to one participant)	20 min dark adaptation in the beginning, 5 min between each condition	After 120 min. the lighting evaluated	After 105min. the lighting evaluated	N.R	2-3 sec.
N.R	18-25	18-55	N.N	20-25	N.R
۵	10	96	108	10	2 groups :36 and 49
Experiment done by booth. 5 different CCTs and 5 different illuminances were used in combinations.	In booth. 9 conditions (3 CCTs, 3 illuminance)	Full size lab. study with different illuminance, CCT, CRI conditions.	Full size lab. study investigates CCT effect on room light estimation under 500 lux.	A lab study done with T8 fluorescent lamps, having 3 CCTs.	Exp. done by two adjacent booths.
Ishida, Ikeyam a & Toda, 2007	Ju, Chen and Lin, 2012	Knez, 1995	Knez, 2001	Lin, Ju, Chen, Chen & Wang, 2007	McNeli s et. al., 1985

It has too many lacking details in the article. Grouping bias. Doesn't include brightness data and results.	Doesn't mention about the brightness results.	Grouping and contraction biases.	SPD effects.
N.N	No	N.N	0 Z
0Z	oZ	0 Z	Yes
Я. Х	07	9	9
0	2	0	2
No No No (includ es graphs not cal data)	No. No mean ratings and stats.	ž S	No No (includ es graphs no numeri cal data)
No (20 stimuli, 7 rating points)	No (18 stimuli, 7rating scale)	No(24 stimuli, 7 rating points)	Not Clear
х. Z	N.N	Yes	Yes
Repeated measures	Repeated measures	Repeated measures	Repeated measures
Preference: not at all preferred/v preferred	7 rating scales	Bright/Dim (No numerical expression used)	х. Z
N.N.	10 sec	N.N	N.R (Probably, not fully adapted because 40 min for 20 cases)
Stude nts	22-39	N.N.	18-56
ω	Ø	σ	160
1:10 scale models prepared with 5 diff. illuminance and 4 CCTs considering 6 diff. activities.	1:10 scale models used to evaluate 5 different CCT lamps.	Full sized lab. Experiment with constant, 2 different positions (0 and 90 deg.) of desk arranged, 3 different polarization.	Full size lab. Experiment on visual satisfaction in the offices, done by 5 illuminance levels and 4 CCTs.
Oi & Takaha shi, 2007	Oi & Takaha shi, 2013	Rea, 1982	Rubinst ein & Kirschb aum, 2003

Tiller &	Full sized lab.	48	N.R	N.R	Bright/Dim	Repeated	Yes	Yes (4	No	No	No	No	No	It doesn't
Rea,	experiment in 2				(No	measures		stimuli, 7						include
1992	phase. First				numerical			rating						proper
	phase covers				expression			points)						quantitative
	the experiment				used)									data.
	of Rea (1982),													Grouping
	second phase													bias.
	covers 4													
	different type													No SPD
	of lighting.													effect.
Takashi	Experiment	N.R	N.R	10 sec for	-3/3 (7	Repeated	Yes	No (28	No.	No	No	No	No	SPD effects
et. al.,	done with			each light	rating	measures		stimuli, 7	No					according to
2013	booth having 7			cond.	scale,			rating	mean					the graphs.
	CCTs				dark/bright			scales)	ratings					
	provided by				(and					
	filters in front							_	stats.					
	of LEDs.													
Wake,	1:5 scale	48	15-37	Waited for	1/7	Repeated	Yes	Yes (6	No.	No	No	No	No	Contraction
Kikuchi	model box is			several min.	(fitness or	measures		stimuli, 7	No					bias.
	used to			In order to	unfitness)			rating	raw					
Takeic	experiment 6			adapt the	(for bright,			points)	data					SPD effects
hi.	kinds of lamps			light	clean)				exists					
kasam	in order to			,										
	examine diff.							_						
d dilu Vamica	SPDs.													
5a,														
11/CT	A living soom	•				Denocted		~ 12		No.	No.	V N	No.	Not clock in
711011,		2			U.N	Nepealed	ß							
Hao,	set used to					measures		_	No					reporting.
Kand &	evalate 7							_	mean					
Hajimu	fluorescent							_	ratings					
, 2003	lamps with 4								and					
	CCTS.								stats.					
	0													

N.R : Not Reported N.A: Not Applicable

B Number of Response Categories

B.1 Questionnaire used in the experiment: example with 6 rating categories

B.1 Questionnaire used in the experiment: example with 6 rating categories



Environmental Satisfaction Survey

Question 1: Please evaluate the loudness of this room from 1 (very quiet) to 6 (very loud).

Assume the loudest is the sound of the music in a night club with fully equipped loud speakers and the quietest is the sound level in the house in which you are staying alone at night. Please circle the number:

very quiet



Question 2: Please evaluate the thermal comfort of this room from 1 (very cool) to 6 (very warm).

Assume the warmest is the hot sunny summer day on the beach and the coolest is the snowy day with frozen slippery roads.

Please circle the very cool	number;				very warm
1	2	3	4	5	6

Question 3: Please evaluate the brightness of lighting in this room from 1 (very dim) to 6 (very bright).

Assume the brightest is represented by the light level in an outdoor sports area (when all the floodlights are on) and the dimmest is the light level of an outdoor parking lot at night. Please circle the number;

ery	dim				very	bright
	1	2	3	4	5	6

Question 4: Please evaluate the clarity of lighting in this room from 1 (very hazy) to 6 (very clear).

Assume the clearest is how a distant mountain will look during a clear sunny day, individual trees • can be seen and small clearings in the forest are visible and the haziest is on a overcast day, with some fog, individual trees might not be as distinguishable, and the clearings not as easily seen Please circle the number:

very hazy					very clear
1	2	3	4	5	6

Please return to Deniz Atlı, PhD student, School of Architecture, University of Sheffield, room nb: BS 18.

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C Lamps Used in this Study

C.1 Spectral power distribution (SPD) values of the lamps

C.1 Spectral power distribution (SPD) values of the lamps

Table C.1 List of SPD values of lamps used in past studies and in the new experiment

All SPD normal)s are lised max	(=1																								
			Fotios&	Lever						Houser	&Fotio	Hu,Hou:	ser&Til			Berm	-	Boyc								
	Fotios&	Gado	more							s		ler				an		a)						Atli		
Uni	Veriv									3000	7500					٨W	-	Grol I	Vatu V	Whit	Kolor I	Dayli	Northl			
t	ide	MM	LPS	HPS	GLS	BG	ΜM	CV	FS	¥	×	VT35	VT65	CV35	CV65	ŋ	R213 I	- ×r	al		te	ght	ight	A	В	υ
	0,06	00'0	00'0	00'0	00'0	0,01	00'0	00'0	0,06	0,00	00'0	00'0	0,00	0,00	0,00	0,01	0,00	0,02	0,18	0,05	0,04	0,08	0,0774	00'0	0,00	00'0
380	240	902	187	282	890	392	645	632	737	645	874	092	165	515	219	850	886	439	243	000	651	696	9	002	000	001
	0,06	00'0	00'0	00'0	00'0	0,01	00'0	00'0	0,06	0,00	0,01	00'0	00'0	0,00	0,00	0,01	0,00	0,02	0,10	0,05	0,04	0,09	0,0774	00'0	0,00	00'0
381	219	955	172	242	906	439	695	600	463	826	014	000	232	194	247	948	886	439	135	000	651	179	9	011	000	005
	0,06	0,00	00'0	00'0	00'0	0,01	00'0	00'0	0,06	0,00	00'0	00'0	0,00	0,00	0,00	0,01	0,00	0,02	0,10	0,05	0,04	0,09	0,0845	00'0	0,00	00'0
382	192	932	157	206	923	486	746	568	196	819	966	089	235	364	225	948	886	787	135	417	651	179	1	059	000	000
	0,06	0,01	0,00	00'0	00'0	0,01	00'0	0,00	0,05	00'0	00'0	00'0	0,00	0,00	0,00	0,01	0,00	0,03	0,10	0,05	0,05	0,09	0,0845	00'0	0,00	00'0
383	237	012	142	175	942	533	801	538	940	781	842	226	263	360	406	948	886	136	811	417	233	662	1	110	053	000
	0,06	0,01	0,00	00'0	00'0	0,01	00'0	0,00	0,05	0,00	00'0	00'0	0,00	0,00	0,00	0,01	0,00	0,03	0,10	0,05	0,05	0,10	0,0915	00'0	0,00	00'0
384	207	072	127	154	962	582	861	511	702	765	700	044	313	180	181	948	886	484	811	833	233	145	5	015	000	900
	0,06	0,01	0,00	00'0	00'0	0,01	00'0	0,00	0,05	0,00	00'0	00'0	0,00	0,00	0,00	0,02	0,00	0,03	0,11	0,05	0,05	0,10	0,0915	00'0	0,00	00'0
385	180	125	111	162	984	628	948	488	493	788	778	019	336	195	229	298	886	833	486	833	814	145	5	004	000	021
	0,06	0,01	0,00	00'0	0,01	0,01	0,01	0,00	0,05	0,00	00'0	00'0	0,00	0,00	0,00	0,02	0,00	0,04	0,11	0,06	0,05	0,10	0,0985	00'0	00'00	00'0
386	234	211	095	178	600	678	036	469	310	706	680	058	372	392	349	298	886	181	486	250	814	628	6	000	004	003
	0,06	0,01	0,00	00'0	0,01	0,01	0,01	00'0	0,05	00'0	00'0	00'0	0,00	0,00	0,00	0,02	0,00	0,04	0,12	0,06	0,05	0,11	0,1056	00'0	0,00	00'0
387	237	334	081	197	037	733	122	452	160	605	590	126	332	159	311	201	886	530	162	250	814	111	с	024	012	005
	0,06	0,01	0,00	00'0	0,01	0,01	0,01	0,00	0,05	0,00	00'0	00'0	0,00	0,00	0,00	0,02	0,00	0,04	0,12	0,06	0,06	0,11	0,1056	00'0	0,00	00'0
388	349	369	690	215	070	795	201	439	044	483	522	017	278	262	314	201	886	878	162	250	395	594	m	000	022	000
	0,06	0,01	00'0	00'0	0,01	0,01	0,01	00'0	0,04	00'0	00'0	00'0	0,00	00′0	0,00	0,02	0,00	0,05	0,12	0,06	0,06	0,11	0,1126	00'0	00'0	00'0
389	367	450	059	212	109	878	224	429	987	411	520	060	250	249	205	298	886	226	162	250	977	594	∞	025	600	000
	0,06	0,01	0,00	00'0	0,01	0,01	0,01	00'0	0,04	00'0	00'0	00'0	0,00	0,00	0,00	0,02	0,00	0,05	0,13	0,06	0,06	0,12	0,1126	00'0	0,00	00'0
390	403	554	052	204	153	968	246	422	965	463	558	271	284	234	297	201	886	575	514	667	977	077	∞	012	000	000
	0,06	0,01	0,00	00'0	0,01	0,02	0,01	0,00	0,04	0,00	00'0	00'0	0,00	0,00	0,00	0,02	0,00	0,05	0,13	0,07	0,06	0,12	0,1197	00'0	0,00	00'0
391	530	620	049	193	200	065	279	419	975	456	503	000	389	142	256	201	886	923	514	083	395	077	2	045	000	600
	0,06	0,01	0,00	00'0	0,01	0,02	0,01	0,00	0,05	0,00	00'0	00'0	0,00	0,00	0,00	0,02	0,00	0,06	0,14	0,07	0,18	0,26	0,3098	00'00	0,00	00'0
392	590	209	049	181	249	165	336	420	011	358	511	147	329	168	206	201	886	272	189	500	023	570	9	018	000	003
	0,06	0,01	0,00	00'0	0,01	0,02	0,01	0,00	0,04	0,00	00'0	0,00	0,00	0,00	0,00	0,02	0,00	0,06	0,14	0,20	0,28	0,34	0,4014	00'0	0,00	00'0
393	679	749	059	170	297	257	316	366	935	303	762	080	146	267	248	298	886	620	189	833	488	783	1	000	000	012
	0,06	0,01	0,00	00'0	0,01	0,02	0,01	0,00	0,04	0,00	00'0	00'0	0,00	0,00	0,00	0,02	0,00	0,06	0,43	0,28	0,28	0,39	0,4929	00'0	0,00	00'0
394	750	853	071	160	348	354	389	341	932	157	706	013	350	233	237	298	886	696	919	333	488	130	9	005	000	900
	0,06	0,01	0,00	00'0	0,01	0,02	0,01	0,00	0,05	0,00	00'0	00'0	0,00	0,00	0,00	0,02	0,00	0,07	0,43	0,27	0,29	0,39	0,4929	00'0	0,00	00'0
395	797	873	083	153	403	458	613	369	052	013	611	138	335	214	323	649	886	317	919	917	070	130	9	000	002	001
u c	0,07	0,02	0,00	0,00	0,01	0,02	0,02	0,00	0,05	0,00	00'0	0,00	0,00	0,00	0,00	0,02	0,00	0,07	0,43	0,27	0,29	0,39	0,4929	0,00	0,00	00'0
390	ζζU	203	760	0ct	403	7/5	043	4/5	340	710	907	036	C52	107	780	049	988	999	ATA	A1/	0/0	130	٩	600	n	900

00'0	014	00'0	000	00'0	000	00'0	000	00'0	000	00'0	000	00'0	000	00'0	000	00'0	000	00'0	000	00'0	000	00'0	000	00'0	000	00'0	000	00'0	000	00'0	000	0,00	000	00'0	000	00'0	014	00'0	031	00'0	041	00'0	082	00'0	129
0,00	000	0,00	000	0,00	000	0,00	000	0,00	000	0,00	000	0,00	000	0,00	030	0,00	015	00'0	000	0,00	000	00'0	000	0,00	000	00'0	000	00'0	000	00'0	000	0,00	000	0,00	000	0,00	010	0,00	024	00'0	022	00'0	012	0,00	037
00'0	900	00'0	002	00'0	019	00'0	002	00'0	023	00'0	000	00'0	000	00'0	005	00'0	008	00'0	000	00'0	000	00'0	000	00'0	000	00'0	000	00'0	001	00′0	008	00'0	000	00'0	000	00'0	000	00'0	032	00'0	012	00′0	022	00'0	071
0,4929	٥	0,4929	9	0,4929	9	0,4929	9	0,4929	9	0,4929	9	0,4929	9	0,4929	9	0,4929	9	0,4929	9	0,4929	9	0,4929	9	0,4929	9	0,4929	9	0,4929	9	0,4929	9	0,5000	0	0,5000	0	0,3802	8	0,3098	9	0,2816	6	0,2887	£	0,2957	7
0,39	130	0,39	130	0,39	130	0,39	130	0,39 (130	0,39 (130	0,39 (130	0,39 (614	0,39 (614	0,39 (614	0,39	130	0,39 (614	0,39 (614	0,39 (614	0,38 (647	0,29 (469	0,23	671	0,20	290	0,20	290	0,20	773	0,20	773	0,21	256	0,21	739
0,29	0/0	0,29	070	0,29	070	0,28	488	0,28	488	0,29	070	0,29	070	0,29	070	0,29	070	0,29	070	0,29	070	0,29	070	0,29	070	0,29	070	0,29	070	0,29	070	0,28	488	0,20	930	0,16	279	0,12	791	0,12	791	0,13	953	0,13	953
0,27	91/	0,28	333	0,27	917	0,27	917	0,27	917	0,27	917	0,27	917	0,27	917	0,27	917	0,27	917	0,27	917	0,27	917	0,27	917	0,27	917	0,27	917	0,27	917	0,11	667	0,11	250	0,11	667	0,11	667	0,11	667	0,12	083	0,12	083
0,43	919	0,44	595	0,44	595	0,44	595	0,44	595	0,43	919	0,43	919	0,43	919	0,43	919	0,44	595	0,44	595	0,43	919	0,43	919	0,43	919	0,43	919	0,43	919	0,43	919	0,43	919	0,43	919	0,43	919	0,43	919	0,43	919	0,25	000
0,08	014	0,08	362	0,08	711	0,31	010	0,31	010	0,31	010	0,31	010	0,31	010	0,31	010	0,31	010	0,31	010	0,31	010	0,31	010	0,31	010	0,31	010	0,31	010	0,31	010	0,15	331	0,15	679	0,16	376	0,16	725	0,17	422	0,18	118
0,00	886	0,00	886	0,00	886	0,00	886	0,00	886	0,00	886	0,03	139	0,14	654	0,17	658	0,01	386	0,01	386	0,01	386	0,00	886	0,00	886	0,00	886	0,00	886	0,00	886	0,00	886	0,00	886	0,00	886	0,00	886	0,00	886	0,00	886
0,02	649	0,02	649	0,02	649	0,02	649	0,02	649	0,02	649	0,03	000	0,03	000	0,03	350	0,32	102	0,23	687	0,04	753	0,04	753	0,05	454	0,03	350	0,03	350	0,03	350	0,03	350	0,03	350	0,03	350	0,03	350	0,03	350	0,03	350
0,00	343	0,00	277	0,00	344	0,00	257	0,00	423	0,00	871	0,03	171	0,06	201	0,07	353	0,05	903	0,05	298	0,04	992	0,04	120	0,01	916	0,01	204	0,01	218	0,01	128	0,01	288	0,01	488	0,01	535	0,01	842	0,02	002	0,02	288
0,00	1/9	0,00	181	0,00	960	0,00	236	00'0	155	0,00	806	0,03	243	0,06	403	0,07	628	0,06	131	0,05	478	0,05	159	0,04	007	0,01	590	0,00	716	0,00	069	0,00	557	0,00	648	0,00	650	0,00	733	0,00	771	00'0	817	00'0	903
0,00	662	0,00	249	0,00	324	0,00	231	00'0	344	0,00	983	0,03	567	0,06	925	0,08	089	0,06	648	0,06	900	0,05	570	0,04	575	0,01	994	0,01	186	0,01	214	0,01	186	0,01	294	0,01	434	0,01	673	0,01	892	0,02	184	0,02	453
0,00	124	0,00	120	0,00	074	0,00	084	0,00	109	0,00	540	0,02	542	0,05	048	0,05	989	0,04	827	0,04	227	0,03	973	0,03	144	0,01	156	0,00	469	0,00	467	0,00	345	0,00	408	0,00	389	0,00	491	0,00	584	0,00	631	0,00	660
00'0	690	0,00	674	00'0	751	0,00	816	00'0	944	00'0	696	00'0	992	0,01	076	0,01	010	0,01	170	0,01	383	0,01	562	0,01	574	0,01	705	0,01	702	0,01	865	0,01	965	0,02	102	0,02	293	0,02	537	0,02	830	0,03	003	0,03	393
0,00	064	0,00	127	0,00	237	0,00	340	0,00	385	0,00	269	0,00	200	0,00	253	0,00	327	0,00	485	0,00	603	0,00	651	0,00	522	0,00	486	0,00	457	0,00	436	0,00	472	0,00	529	0,00	553	0,00	628	0,00	676	0,00	655	0,00	687
0,05	893	0,06	702	0,07	812	0,09	259	0,11	790	0,14	453	0,17	003	0,19	195	0,20	032	0,20	323	0,20	124	0,19	490	0,18	178	0,16	663	0,15	121	0,13	727	0,13	201	0,12	958	0,12	953	0,13	145	0,13	476	0,13	924	0,14	455
0,00	۲۱/	0,01	690	0,01	546	0,02	159	0,03	198	0,04	283	0,05	313	0,06	187	0,06	491	0,06	563	0,06	429	0,06	112	0,05	503	0,04	815	0,04	128	0,03	521	0,03	313	0,03	247	0,03	303	0,03	465	0,03	705	0,04	018	0,04	391
0,02	//8	0,03	818	0,05	206	0,06	982	0,10	097	0,13	320	0,16	332	0,18	809	0,19	391	0,19	213	0,18	371	0,16	959	0,14	737	0,12	270	60'0	788	0,07	520	0,06	435	0,05	728	0,05	330	0,05	176	0,05	128	0,05	218	0,05	407
0,02	703	0,02	847	0,03	004	0,03	174	0,03	362	0,03	563	0,03	777	0,04	004	0,04	244	0,04	494	0,04	754	0,05	020	0,05	297	0,05	577	0,05	857	0,06	134	0,06	389	0,06	641	0,06	893	0,07	148	0,07	420	0,07	969	0,07	975
0,01	05c	0,01	603	0,01	684	0,01	774	0,01	876	0,01	988	0,02	107	0,02	234	0,02	370	0,02	511	0,02	657	0,02	807	0,02	958	0,03	111	0,03	262	0,03	412	0,03	552	0,03	069	0,03	828	0,03	996	0,04	110	0,04	257	0,04	405
00'0	156	00'0	167	00'0	183	00'0	202	00'0	227	00'0	255	00'0	282	00'0	310	00'0	329	00'0	346	00'0	363	00'0	380	00'0	400	00'0	421	00'0	440	00'0	458	00'0	465	00'0	472	00'0	480	00'0	491	00'0	519	00'0	549	00'0	580
0,00	084	0,00	074	0,00	063	0,00	053	00'0	054	00'0	058	00'0	063	0,00	067	00'0	064	00'0	061	0,00	056	0,00	052	00'0	050	00'00	048	00'0	048	0,00	048	0,00	052	0,00	057	0,00	090	0,00	062	0,00	055	0,00	047	0,00	038
0,02	106	0,04	382	0,07	015	0,09	699	0,12	289	0,15	507	0,20	122	0,27	499	0,36	346	0,34	302	0,28	034	0,18	859	0,10	084	0,06	478	0,05	620	0,04	886	0,04	556	0,04	657	0,04	787	0,04	921	0,05	079	0,05	200	0,05	358
0,07	547	0,08	671	0,10	477	0,12	337	0,14	193	0,16	462	0,19	647	0,24	683	0,30	724	0,29	428	0,25	294	0,19	181	0,13	502	0,11	352	0,10	985	0,10	789	0,10	863	0,11	130	0,11	507	0,11	880	0,12	260	0,12	678	0,13	040
1	397		398		399		400		401		402		403		404		405		406		407		408		409		410		411		412		413		414		415		416		417		418		419

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00'0	164	00'0	228	00,00	00'0	390	00'0	495	00'0	618	00'0	759	00'0	944	0,01	143	0,01	380	0,01	655	0,01	966	0,02	379	0,02	201	269	0,03	836	0,04	469	0,05 211	0,06	031	0,06	972	0,07	986	0,09	114	0,10 344	0,11	647
00'0	049	0,00	029	0,00	0,00	092	00'0	132	0,00	149	0,00	177	0,00	221	0,00	254	0,00	320	0,00	385	0,00	460	0,00	540	0,00		770	0,00	893	0,01	054	0,01	0,01	393	0,01	605	0,01	847	0,02	COT C	0,UZ 389	0,02	689
00'0	090	00'0	093	0,00	00'0	163	00'0	188	00'0	241	00'0	290	00'0	362	00'0	449	0,00	4 2 2 4	00'0	661	00'0	789	0,00	936	0,01	111	306	0,01	508	0,01	773	0,02	0,02	391	0,02	758	0,03	171	0,03	020	0,04 096	0,04	607
,3098	9	,3098 ,	9	,3239 4	,3309	6	,3380	æ	,5704	2	,6831	0	,0000	0	,0000	0	,0000	D	,0000	0	,0000	0	,0000,	0	,0000		0	,0000	0	,9929	9	,9929 6	,9929	9	,9929	9	,9929	9	,9929 6	0,000	6766'	,9366	2
0,21 C	739	0,22 C	705	0,23 188	0,23 C	188	0,34 C	783	0,58 C	937	0,76 C	812	0,76 1	812	0,76 1	812	0,76 1	212	0,76 1	812	0,76 1	812	0,76 1	812	0,76 1	0 7E 1	812	0,76 1	812	0,76 C	812	0,76 C 812	0,76 C	812	0,76 C	812	0,68 C	116	0,53 C	140	ט,30 נ 918	0,30 0	435
),13	953),14	535),14 535),36	628),55 (233),62	791),62	791),61	628),61	628),62	507),62	209),61	628),61	628),61	020	047),61	628),61 (628),61 628),61	628),61 (628),61	628),61	14/),61 628	,61	628
,12	8	,12	000	,12 (50	1 17	,50 (1 17	,50 0	333	,50	333	,50 (417 412	,50 (117	,50	533	20	†17	.50	417 	,50	±17	50		117	,50 (117	,50 (333	,50	,50 (t17	,50 (117	,50	t17	20	533	,50 117	,16 (250
25 0	8	25 0	8	5 8	25 0	76 2	25 0	76 2	27 0	27 8	87 0	62	87 0	62 4	87 0	62 4	87	20	87	62 4	87	62 62	87	62 4	87	207 20	65	87 0	62 4	87 0	62 8	62 0	87 0	62 4	87 0	62 4	87 0	62 4	87	2 2	87 62	87 0	62
18 0,	67 0	19 0,0	64 0	61 0, 0, 0,	20 0	57 6	21 0,	54 6	21 0,	51 0	22 0,	97 1	23 0,	93 1	24 0,	90	, 0, 25	- T	64	08	64 0,	08	64 0, 0,	08	64 0, 0,		5 8	64 0,	08	64 0,	08	64 08 0, 0	64 0,	08 1	64 0,	08 1	64 0,	08 1	64 0, 2	 8 3	64 08 1 v	64 0,	08 1
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0,02	505	0,0	96	0,0	0,0	742	0'0	17	0,0	761	0,0	297	0,0	927	0,0	603	0,0	73(0,0	92(0, j	678	0,0	45	0,10	4 C	1.00	0,32	923	0,48	84	0,4	0,37	454	0,24	979	0,12	737	0,12	0	U, 1 ⁴ 842	0,15	086
00'0	936	0,01	090	0,01	0,01	197	0,01	314	0,01	445	0,01	663	0,01	743	0,01	887	0,02	560	0,02	200	0,02	450	0,02	676	0,03	160 0	107	0,23	233	0,34	767	0,29 684	0,24	233	0,13	064	0,04	238	0,03	202	0,u3 917	0,03	922
0,02	880	0,03	250	0,03 674	0,04	166	0,04	715	0,05	310	0,05	949	0,06	653	0,07	402	0,08	18/	0,08	858	60'0	678	0,10	514	0,11	71 U	252	0,38	506	0,53	482	0,47 437	0,41	043	0,27	342	0,16	276	0,16	141	0,10 479	0,16	704
00'0	732	0,00	822	00,0	0,00	943	0,01	146	0,01	236	0,01	356	0,01	516	0,01	674	0,01	01.8	0,01	950	0,02	141	0,02	332	0,02	17	0,0 932	0,18	648	0,27	443	0,23 464	0,19	292	0,10	640	0,03	632	0,03	055	0,U3 355	0,03	481
0,03	854	0,04	428	0,04 876	0,05	432	0,06	056	0,06	902	0,07	831	0,08	921	0,10	151	0,11	400	0,12	947	0,14	364	0,15	977	0,18		186	0,22	496	0,24	987	0,27	0,30	534	0,33	589	0,36	787	0,40	0.80	0,43 707	0,47	365
00'0	178	0,00	974	0,01 061	0,01	182	0,01	340	0,01	463	0,01	643	0,01	821	0,02	013	0,02	724	0,02	522	0,02	708	0,02	922	0,03	007	606 606	0,03	945	0,04	260	0,04 720	0,05	241	0,05	765	0,06	300	0,06 785	C8/	0,U/ 402	0,08	038
0,15	031	0,15	492	0,15 975	0,16	496	0,17	067	0,17	150	0,17	531	0,18	445	0,20	127	0,23	/79	0,28	036	0,33	262	0,39	211	0,47	0.56	084	0,63	255	0,68	403	0,66 789	0,62	924	0,57	463	0,51	063	0,45	707	0,39 226	0,33	802
0,04	810	0,05	208	0,05 645	0,06	129	0,06	699	0,06	666	0,07	508	0,08	313	0,09	529	0,11	C 80	0,14	321	0,17	391	0,20	846	0,25	105 0	351	0,34	473	0,37	568	0,37 237	0,35	746	0,33	419	0,30	577	0,27	808	0,25 145	0,22	634
0,05	656	0,05	768	0,05 976	0,06	156	0,06	480	0,06	179	0,06	322	0,07	231	60'0	228	0,13	098	0,19	737	0,26	696	0,34	568	0,45	10,56	492	0,65	602	0,71	891	0,68 824	0,62	693	0,54	447	0,45	034	0,36	020	0,28 478	0,21	003
0,08	254	0,08	532	0,08 807	60'0	076	60'0	338	0,09	585	0,09	822	0,10	050	0,10	268	0,10	40/	0,10	662	0,10	856	0,11	053	0,11 0770	011	495	0,11	725	0,11	961	0,12 199	0,12	441	0,12	687	0,12	937	0,13	192	0,13 451	0,13	714
0,04	554	0,04	708	0,04 861	0,05	012	0,05	160	0,05	296	0,05	427	0,05	553	0,05	675	0,05	/89	0,05	006	0,06	011	0,06	123	0,06	141	363	0,06	487	0,06	614	0,06 745	0,06	879	0,07	017	0,07	157	0,07	105	0,U/ 448	0,07	597
0,00	610	00'0	627	0,00 641	00'0	656	00'0	672	00'0	701	00'0	732	00'0	764	00'0	798	0,00	824	00'0	851	00'0	880	00'0	912	0,00	000	006	0,01	052	0,01	094	0,01 126	0,01	149	0,01	162	0,01	163	0,01 11F	110	0,01 066	0,01	028
0,00	031	0,00	030	0,00	00'0	034	0,00	038	0,00	045	0,00	051	0,00	057	00'00	062	0,00	Tqn	0,00	058	0,00	055	0,00	053	0,00		055	0,00	056	0,00	056	0,00	00'0	046	0,00	041	0,00	036	00'00	U34	0,00 032	0,00	031
0,05	488	0,05	660	0,05 809	0,05	966	0,06	119	0,06	318	0,06	509	0,07	015	0,08	240	0,11	4 T 8	0,17	103	0,24	269	0,32	735	0,43	101	010	0,82	094	0,91	764	0,91 668	0,73	629	0,46	264	0,27	422	0,14 5 48	040	0,09 404	0,08	668
),13	493	0,13 225	935	0,14 437),14	956	J,15	457	J,15	663),16	676),17	577),19	113	0,22 524	175	0,28	221	J,35	320	0,43	535),54 157	717	382	06'(214	66'C	555	1,00), 84	015),59	579),42	942),31 777	/32	J,27 562),27	438
	420	-	421	,	1	123		124		125		126	<u> </u>	427		428	-	- 	-	130	-	431	-	432	-	,	134		135	`	436	137		138	-	139		440	-	, 1 #1	142		143
	*		7	4	<u>.</u>	4		7		4		4		7		7		1		7		7		7		.	ব	<u>.</u>	~	<u>.</u>	7	4	.i	4		4		7		4	4	<u>.</u>	4

0,13	083	0,14	556	0,16	690	0,17	622	0,19	094	0,20	483	0,21	753	0,22	839	0,23	650	0,24	153	0,24	392	0,24	339	0,23	964	0,23	283	0,22	342	0,21	239	0,20	031	0,18	778	0,17	503	0,16	256	0,15	025	0,13	931	0,12	956
0,03	021	0,03	374	0,03	725	0,04	074	0,04	407	0,04	735	0,05	021	0,05	275	0,05	460	0,05	574	0,05	646	0,05	632	0,05	539	0,05	386	0,05	175	0,04	947	0,04	677	0,04	418	0,04	140	0,03	867	0,03	612	0,03	384	0,03	193
0,05	189	0,05	747	0,06	369	0,06	976	0,07	542	0,08	107	0,08	609	60'0	600	60'0	347	60'0	540	60'0	633	60'0	596	0,09	458	60'0	187	0,08	807	0,08	371	0,07	887	0,07	404	0,06	896	0,06	382	0,05	927	0,05	489	0,05	120
0,7394	4	0,5140	∞	0,5070	4	0,5070	4	0,5211	æ	0,5281	7	0,5352	1	0,5422	S	0,5493	0	0,5563	4	0,5563	4	0,5704	2	0,5704	2	0,5774	9	0,5845	1	0,5915	ß	0,5915	2	0,6056	3	0,6056	с	0,6126	∞	0,6197	2	0,6267	9	0,6267	9
0,30	435	0,30	435	0,30	918	0,31	401	0,31 (401	0,31 (884	0,32 (367	0,32 (367	0,32	367	0,32	850	0,33	333	0,33	816	0,33	816	0,34 0	300	0,34 0	300	0,34 (783	0,34	783	0,35	266	0,35	266	0,35	749	0,35 (749	0,35 (749	0,36	232
0,61	628	0,60	465	0,37	209	0,22	093	0,22	093	0,22	674	0,23	256	0,23	256	0,23	837	0,23	837	0,24	419	0,24	419	0,25	000	0,25	581	0,25	581	0,26	163	0,26	163	0,26	744	0,27	326	0,27	326	0,27	326	0,27	907	0,28	488
0,16	250	0,16	667	0,17	083	0,17	083	0,17	083	0,17	500	0,17	500	0,17	500	0,17	500	0,17	917	0,18	333	0,18	333	0,18	333	0,18	333	0,18	333	0,18	750	0,18	750	0,18	750	0,18	750	0,18	750	0,18	750	0,18	750	0,19	167
0,87	162	0,87	162	0,87	162	0,87	162	0,87	162	0,36	486	0,36	486	0,36	486	0,37	162	0,37	838	0,38	514	0,38	514	0,38	514	0,39	189	0,39	189	0,39	865	0,39	865	0,40	541	0,40	541	0,40	541	0,41	216	0,41	216	0,41	892
0,32	056	0,32	056	0,32	404	0,32	753	0,32	753	0,32	753	0,33	101	0,33	101	0,33	101	0,33	449	0,33	449	0,33	449	0,33	449	0,33	449	0,33	449	0,33	449	0,33	449	0,33	449	0,33	449	0,33	101	0,33	101	0,33	101	0,33	101
0,00	886	0,00	886	0,00	886	0,00	886	00'0	886	0,00	886	00'0	886	0,00	886	0,00	886	0,00	886	0,00	886	0,00	886	0,00	886	0,00	886	0,00	886	00'00	886	0,00	886	0,00	886	0,00	886	0,00	886	0,00	886	0,00	886	0,00	886
0,04	753	0,04	753	0,04	753	0,05	103	0,05	103	0,05	103	0,05	103	0,05	103	0,05	103	0,05	103	0,05	103	0,05	103	0,05	103	0,05	103	0,05	103	0,05	454	0,05	454	0,05	454	0,05	454	0,05	454	0,05	454	0,05	454	0,05	454
0,15	278	0,15	427	0,15	443	0,15	338	0,15	291	0,15	046	0,14	914	0,14	674	0,14	377	0,14	035	0,13	727	0,13	330	0,12	914	0,12	428	0,12	007	0,11	490	0,11	052	0,10	554	0,10	152	0,09	775	0,09	358	0,08	946	0,08	507
0,03	915	0,03	989	0,03	982	0,03	989	0,03	923	0,03	867	0,03	822	0,03	752	0,03	692	0,03	634	0,03	510	0,03	398	0,03	307	0,03	161	0,03	040	0,02	976	0,02	842	0,02	726	0,02	594	0,02	524	0,02	499	0,02	403	0,02	329
0,16	924	0,17	020	0,17	960	0,17	039	0,16	919	0,16	671	0,16	525	0,16	318	0,15	957	0,15	578	0,15	224	0,14	779	0,14	284	0,13	826	0,13	283	0,12	763	0,12	306	0,11	799	0,11	309	0,10	779	0,10	382	60'0	887	60'0	420
0,03	493	0,03	561	0,03	521	0,03	520	0,03	489	0,03	483	0,03	428	0,03	360	0,03	312	0,03	227	0,03	163	0,03	600	0,02	945	0,02	830	0,02	726	0,02	585	0,02	492	0,02	448	0,02	322	0,02	251	0,02	145	0,02	101	0,02	041
0,50	950	0,54	392	0,57	691	0,60	703	0,63	320	0,65	651	0,67	695	0,69	129	0,69	989	0,70	384	0,69	989	0,68	842	0,67	121	0,65	041	0,62	603	0,59	842	0,56	723	0,53	352	0,49	875	0,46	504	0,42	954	0,39	799	0,37	038
0,08	520	0,09	001	0,09	481	0,09	918	0,10	249	0,10	537	0,10	777	0,10	887	0,10	912	0,10	924	0,10	796	0,10	556	0,10	286	0,09	918	0,09	575	0,09	248	0,08	813	0,08	306	0,07	832	0,07	421	0,06	842	0,06	330	0,05	973
0,29	199	0,27	299	0,26	350	0,26	111	0,26	347	0,26	365	0,26	560	0,26	877	0,27	257	0,27	506	0,27	759	0,28	013	0,28	266	0,28	512	0,28	754	0,28	066	0,29	219	0,29	439	0,29	650	0,29	852	0,30	043	0,30	221	0,30	388
0,20	523	0,19	815	0,19	579	0,19	687	0,20	016	0,20	208	0,20	462	0,20	745	0,21	025	0,21	179	0,21	299	0,21	389	0,21	452	0,21	501	0,21	524	0,21	519	0,21	483	0,21	407	0,21	301	0,21	166	0,21	900	0,20	828	0,20	625
0,14	667	0,11	852	0,10	310	0,09	723	0,09	771	0,09	532	0,09	532	60'0	969	60'0	948	0,10	023	0,10	106	0,10	195	0,10	287	0,10	379	0,10	470	0,10	557	0,10	640	0,10	708	0,10	769	0,10	824	0,10	875	0,10	923	0,10	967
0,13	981	0,14	260	0,14	540	0,14	819	0,15	094	0,15	350	0,15	600	0,15	847	0,16	092	0,16	335	0,16	579	0,16	827	0,17	081	0,17	355	0,17	634	0,17	916	0,18	198	0,18	469	0,18	738	0,19	007	0,19	277	0,19	559	0,19	842
0,07	749	0,07	904	0,08	090	0,08	217	0,08	373	0,08	525	0,08	676	0,08	826	0,08	976	0,09	122	0,09	269	0,09	418	0,09	571	0,09	737	0,09	906	0,10	078	0,10	251	0,10	419	0,10	588	0,10	759	0,10	932	0,11	113	0,11	296
0,01	013	0,01	095	0,01	201	0,01	319	0,01	439	0,01	545	0,01	631	0,01	686	0,01	702	0,01	595	0,01	457	0,01	305	0,01	158	0,01	046	00'0	970	00'0	945	0,00	983	0,01	178	0,01	432	0,01	725	0,02	038	0,02	390	0,02	209
0,00	030	00'00	030	00'00	030	00'0	030	00'0	030	0,00	029	0,00	028	0,00	027	0,00	025	0,00	025	0,00	025	0,00	025	0,00	026	0,00	028	0,00	030	0,00	032	0,00	034	00'00	036	0,00	038	0,00	039	0,00	041	0,00	044	0,00	046
0,08	637	0,08	660	0,08	695	0,08	748	0,08	815	0,08	852	0,08	934	0,09	016	0,09	106	0,09	169	0,09	253	0,09	333	0,09	423	60'0	503	0,09	545	60'0	641	60'0	664	60'0	721	0,09	780	0,09	820	0,09	866	0,09	910	60'0	950
0,27	957	0,28	532	0,29	131	0,29	745	0,30	308	0,30	872	0,31	465	0,32	088	0,32	651	0,33	185	0,33	808	0,34	342	0,34	935	0,35	469	0,35	973	0,36	507	0,36	951	0,37	426	0,37	841	0,38	345	0,38	701	0,39	057	0,39	472
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0,03	014	0,02 837	0,02	707	0,02 596	0,02	516	0,02	453	0,02	414	0,02		367	0,02	388	0,02	419	0,02 500	0,02	600	0,02	0.00	888	0,03	082	0,03	315 0.03	589	0,03	872	0,04	700	604	0,05	028	0,05 500	0,05	991
0,04	718	0,04 353	0,04	690	0,03 802	0,03	571	0,03	347	0,03	133	0,02	600	798	0,02	651	0,02	536	0,02 464	0,02	391	0,02	0 0 0	331	0,02	329	0,02	38U 0.07	438	0,02	527	0,02	0.03	782	0,02	953	0,03 140	0,03	360
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60'0	081	0,08 618	0,08	223	0,07 730	0,07	347	0,07	076	0,06	727	0,06	0.06	130	0,05	780	0,05	507	0,05 2.26	0,05	063	0,04	0 0 0	691	0,04	482	0,04	298	239	0,04	076	0,04	0.03	958	0,03	984	0,03 971	0,03	931
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0,30	543	0,30 686	0,30	812	0,30 928	0,31	035	0,31	136	0,31	182	0,31 745	0.21	348	0,31	513	0,31	734	0,32	0,32	560	0,33	0.34	335	0,35	600	0,36	9/1 038	393	0,39	893	0,41		529	0,43	490	0,43 788	0,43	767
0,20	398	0,20 146	0,19	863	0,19 558	0,19	234	0,18	893	0,18	442	0,18	0.17	660	0,17	411	0,17	290	0,17 367	0,17	693	0,18	010	615	0,21	179	0,22	929 0.74	785	0,26	812	0,28	0.30	380	0,31	640	0,31 874	0,31	628
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60'0	394	0,10	119	0,10	886	0,11	719	0,12	594	0,13	569	0,14	547	0,15	603	0,16	689	0,17	811	0,18	978	0,20	119	0,21	267	0,22	397	0,23	519	0,24	572	0,25	592	0,26	531	0,27	350	0,27	778	0,27	778	0,30	556	0,30	556
0,06	544	0,07	153	0,07	812	0,08	502	0,09	218	60'0	975	0,10	781	0,11	625	0,12	488	0,13	375	0,14	288	0,15	193	0,16	082	0,16	979	0,17	544	0,19	298	0,19	298	0,21	053	0,21	053	0,21	053	0,21	053	0,22	807	0,22	807
0,03	620	0,03	880	0,04	196	0,04	511	0,04	844	0,05	227	0,05	604	0,06	000	0,06	433	0,06	862	0,07	324	0,07	771	0,08	220	0,08	671	60'0	960	0,09	516	60'0	904	0,10	284	0,10	604	0,10	907	0,11	153	0,11	327	0,11	467
0,6478	6	0,6478	6	0,6478	6	0,6478	6	0,6408	S	0,6338	0	0,6267	9	0,6267	9	0,6267	9	0,6267	9	0,6267	9	0,6126	∞	0,6197	2	0,6197	2	0,6056	m	0,5985	6	0,5985	6	0,5915	5	0,5915	S	0,5845	1	0,5845	1	0,5774	9	0,5704	~
0,37	198	0,37 (198	0,36 (715	0,36 (715	0,36 (715	0,36 (715	0,36 (232	0,36 (232	0,36 (232	0,35 (749	0,35 (266	0,35 (266	0,35 (266	0,34 0	783	0,34 (783	0,35 (266	0,35	266	0,35	266	0,34 0	300	0,34 0	300	0,33	816	0,33 (816	0,33	816
0,36	047	0,36	047	0,36	047	0,36	628	0,36	628	0,37	209	0,37	209	0,37	791	0,37	791	0,38	372	0,38	372	0,38	372	0,39	535	0,39	535	0,40	116	0,40	116	0,40	698	0,40	698	0,41	279	0,41	279	0,42	442	0,42	442	0,43	0.73
0,17	917	0,17	917	0,17	917	0,17	917	0,17	917	0,17	917	0,17	500	0,17	083	0,17	500	0,17	917	0,17	917	0,17	917	0,17	917	0,17	917	0,18	333	0,18	333	0,18	333	0,18	333	0,18	750	0,18	750	0,18	750	0,19	167	0,19	583
0,47	297	0,47	297	0,47	297	0,47	297	0,47	297	0,47	297	0,47	297	0,47	297	0,47	973	0,47	973	0,47	973	0,47	973	0,47	973	0,48	649	0,47	973	0,47	973	0,47	973	0,47	973	0,47	973	0,47	973	0,47	973	0,47	973	0,47	973
0,26	132	0,25	784	0,25	436	0,25	087	0,24	739	0,24	390	0,24	042	0,23	693	0,23	345	0,22	997	0,22	648	0,22	300	0,21	951	0,21	603	0,21	254	0,20	906	0,20	557	0,20	209	0,19	861	0,19	512	0,19	164	0,18	815	0,18	467
0,26	920	0,29	924	0,34	931	0,38	937	0,42	191	0,43	943	0,46	947	0,50	452	0,53	206	0,55	208	0,57	461	0,59	214	0,60	215	0,61	467	0,61	717	0,61	717	0,60	996	0,60	215	0,59	214	0,58	029	0,55	459	0,53	206	0,51	954
0,05	805	0,05	805	0,05	805	0,05	805	0,06	155	0,05	805	0,05	805	0,05	805	0,05	805	0,05	805	0,05	805	0,05	805	0,05	805	0,05	805	0,05	805	0,05	805	0,05	805	0,05	805	0,05	805	0,05	902	0,05	902	0,05	902	0,05	602
0,17	531	0,16	188	0,14	727	0,13	370	0,11	583	0,10	649	0,10	370	60'0	422	0,07	551	0,05	733	0,04	414	0,03	563	0,02	976	0,02	609	0,02	258	0,02	065	0,01	894	0,01	782	0,01	806	0,01	774	0,01	695	0,01	674	0,01	617
0,11	090	0,10	217	60'0	159	0,08	280	0,07	053	0,06	480	0,06	342	0,05	708	0,04	469	0,03	328	0,02	379	0,01	741	0,01	365	0,01	154	0,00	914	0,00	779	00'00	730	0,00	664	0,00	705	00'00	706	00'0	693	00'00	707	0,00	707
0,04	080	0,04	240	0,04	073	0,04	145	0,04	266	0,04	430	0,04	590	0,04	870	0,05	159	0,05	565	0,05	850	0,06	126	0,06	562	0,07	048	0,07	539	0,08	079	0,08	698	60'0	383	0,10	148	0,10	921	0,11	646	0,12	443	0,13	283
0,01	201	0,01	342	0,01	192	0,01	278	0,01	416	0,01	522	0,01	649	0,01	800	0,02	023	0,02	298	0,02	398	0,02	536	0,02	755	0,03	015	0,03	278	0,03	594	0,03	925	0,04	263	0,04	740	0,05	112	0,05	482	0,05	889	0,06	340
) 60,C	613) 60,C	824	0,10 (158),10 (624),11 (137),11 (789	0,12	496),13 (317	0,14 (234),15 (224	0,16 (336),17 (537	0,18 (863),20 (305),21 (807),23 (467	0,25	213	0,27	063),29 (000),30 (925	0,32 (833),34 (883	0,36 (967
0,03	817	0,04 (064),04 (368	0,04 (665),04 (981	0,05 (413	0,05 (852), 06 (341), 06 (904), 07 (535), 08 (173), 08 (845) 60'C	529	0,10 (313),11 (196	0,12 (129	0,13 (690	0,14 (064	0,15 (111	0,16 (151	0,17 (135	0,18 (221	0,19 (344
),43 (461),42 (904	0,42 (122),41 (165),40 (072),38 (884),37 (640),36 (379),35 (141),33 (965),33 (000),32 (133),31 (359),30 (674),30 (060), 29 (580),29 (135),28 (743),28 (394),28 (077),27 (784),27 (505),27 (205
),30 (955),29 (913),28 (542),26 (917),25 (095),23 (139),21 (660),19 (046),17 (045),15 (156),13 (610),12 (237),11 (035),10 (000) 60'(174),08 (492),07 (934),07 (480),07 (060),06 (771),06 (509),06 (292),06 (09.2
),11 (640),11 (640),11 (616),11 (586),11 (553),11 (521),11 (497),11 (479),11 (469),11 (469),11 (491),11 (523),11 (563),11 (608),11 (647),11 (691),11 0	744),11 (808),11 (889),11 (984),12 (095),12 (224),12 (369
),26 0	294),26 (564),26 (842),27 0	124),27 0	410),27 0	669),27 0	988),28 (279),28 (573),28 C	867),29 (181),29 (489),29 (784),30 (058),30 (275),30 (468),30 0	642),30 0	802),30 (950),31 (092),31 (232),31 (376),31 0	536
),16 C	128),16 C	352),16 C	578),16 C	808),17 C	044),17 C	286),17 C	539),17 C	798),18 C	062),18 C	329),18 C	605),18 C	879),19 C	148),19 C	409),19 C	640),19 C	862),20 C	075),20 C	282),20 C	489),20 C	693),20 C	897),21 C	102),21 C	500
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0,30	556	0,30	556	0,30	556	0,30		556	0,27	778	0,27	778	0,27	778	0,27	375	0,26	636	0,25	836	0,24	986	0,24	117	0,23	214	0,22	0.21	419	0,20	531	0,19 636	0.18	758	0,17	897	0,17	067	0,16	242	0,15 458	0,14	694
0,22	807	0,22	807	0,22	807	0,22 807	0.00	807	0,22	807	0,21	053	0,21	053	0,21	053	0,21	053	0,19	298	0,19	298	0,19	298	0,17	544	0,17	0.16	426	0,15	754	0,15	0,14	398	0,13	730	0,13	093	0,12	442	0,11 860	0,11	267
0,11	571	0,11	618	0,11	607	0,11 529	011	418	0,11	253	0,11	067	0,10	833	0,10	569	0,10	287	60'0	971	60'0	624	60'0	296	0,08	949	0,08	0.08	271	0,07	907	0,07 551	0.07	236	0,06	868	0,06	580	0,06 777	2/3	לט,ט 967	0,05	682
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),43	605	,44	186	,44	186	767		930 930),45	930),46	512),46	512),47	093),47	674),47	674	0,47	674),47	674),48	256),48	148	256),48	837),48 827	0,48	837),48	837),48	837	,49	4 LV	884	0,72	674
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,47 C	973	,47 C	973	,47 C	973	,47 C		, ^{4,}	,47 C	973 8	,47 C	973	,47 C	973	,47 C	973 (,47 C	973 (,47 C	973	,47 C	973	,47 C	973	,47 C	973	,48	48	549	,49 C	324	,49 C	49 0	324	,50 C	800	,50 C	576	16, 0	272	,91 192 192	,91 0	392
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0,14	071	0,14	834	0,15	691	0,16	201 L	119	0,17	783	0,18	408	0,18	913	0,19	358	0,19	768	0,20	860	0,20	353	0,20	520	0,20	599	0,20	02.0	540	0,20	400	0,20	0.20	186	0,21	002	0,20	507	0,19	202	0,12 848	0,18	738
0,06	758	0,07	122	0,07	577	0,07		291 291	0,08	658	0,08	995	60'0	226	60'0	462	0,09	675	0,09	845	60'0	964	0,10	086	0,10	192	0,10	0.10	138	0,10	960	0,10	0.10	094	0,11	048	0,10	700	0,09	040	0,09 421	60'0	553
0,39	010	0,41	060	0,43	205	0,45	970	0,40 934	0,48	620	0,50	233	0,51	775	0,53	101	0,54	141	0,55	038	0,55	683	0,56	185	0,56	472	0,56	0.56	472	0,56	185	0,55 647	0.54	894	0,54	105	0,53	137	0,51	0520	טל,ט 878	0,49	480
0,20	450	0,21	608	0,22	712	0,23		0,24 743	0,25	680	0,26	549	0,27	373	0,28	104	0,28	676	0,29	133	0,29	522	0,29	774	0,29	911	0,29	62.0	865	0,29	728	0,29 176	0,29	110	0,28	653	0,28	173	0,27	8/c	0,2b 961	0,26	206
0,26	907	0,26	612	0,26	317	0,26	0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	733 733	0,25	437	0,25	137	0,24	795	0,24	459	0,24	145	0,23	864	0,23	660	0,23	505	0,23	401	0,23	0.23	216	0,23	194	0,23 338	0.23	706	0,23	845	0,24	523	0,26	700	0,28 541	0,33	143
0,05	917	0,05	758	0,05	607	0,05		266	0,05	101	0,04	942	0,04	780	0,04	632	0,04	501	0,04	395	0,04	304	0,04	253	0,04	255	0,04	1.2c 0.04	259	0,04	365	0,04	0,05	459	0,05	982	0,07	308	0,09	184	0,13 779	0,21	287
0,12	536	0,12	727	0,12	943	0,13	1010	457	0,13	765	0,14	113	0,14	509	0,14	949	0,15	435	0,15	968	0,16	551	0,17	183	0,17	863	0,18	0.19	367	0,20	194	0,21	0,22	003	0,22	832	0,23	780	0,24	710	0,26 288	0,27	310
0,31	705	0,31	883	0,32	071	0,32 784	0.37	502 502	0,32	723	0,32	940	0,33	114	0,33	291	0,33	478	0,33	684	0,33	952	0,34	245	0,34	559	0,34	0.35	228	0,35	578	0,35	0.36	309	0,36	681	0,37	063	0,37	400	0,37 866	0,38	319
0,21	519	0,21	735	0,21	958	0,22 196		441	0,22	691	0,22	944	0,23	184	0,23	429	0,23	683	0,23	949	0,24	249	0,24	561	0,24	881	0,25	0.25	524	0,25	840	0,26 152	0.26	456	0,26	739	0,27	015	0,27 787	197	0,27 558	0,27	851
0,01	363	0,01	502	0,01	580	0,01	F 20	251	0,01	027	00′0	798	0,00	660	00'0	545	00'0	453	00'0	384	0,00	351	00'0	337	00'0	336	00'0	0.00	339	00′0	339	00'0	00,0	344	00'0	351	0,00	360	0,00	2/1	0,UU 382	0,00	385
0,00	032	0,00	030	00'00	030	0,00	800	047	0,00	055	0,00	090	0,00	051	0,00	040	0,00	028	0,00	017	0,00	015	0,00	016	0,00	019	0,00	0.00	028	0,00	035	0,00	00,0	046	0,00	050	0,00	052	0,00	004	0,UU 055	00'0	056
0,10	390	0,10	520	0,10	705	0,10 894	510	131	0,11	366	0,11	684	0,11	997	0,12	337	0,12	729	0,13	147	0,13	633	0,14	120	0,14	657	0,15 261	0.15	897	0,16	597	0,17 336	0.18	122	0,18	996	0,19	855	0,20	16/	0, 21 823	0,22	893
0,37	989	0,37	752	0,37	574	0,37 367	720	159	0,36	951	0,36	714	0,36	507	0,36	269	0,36	062	0,35	824	0,35	587	0,35	320	0,35	113	0,34	0.34	638	0,34	431	0,34 103	0,33	986	0,33	778	0,33	600	0,33	422	0,33 393	0,33	452
	514		515		516	517	17	518		519		520		521		522		523		524		525		526		527	570	070	529		530	531	1	532		533		534	с. 2 С. 2 С.	C5C	536		537
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0,13	956	0,13	247	0,12	561	0,11	914	0,11	286	0,10	692	0,10	111	60'0	553	60'0	019	0,08	528	0,08	042	0,07	589	0,07	169	0,06	769	0,06	411	0,06	058	0,05	753	0,05	464	0,05	217	0,04	978	0,04	772	0,04	603	0,04	450
0,10	698	0,10	153	0,09	623	0,09	112	0,08	639	0,08	165	0,07	723	0,07	291	0,06	874	0,06	482	0,06	960	0,05	737	0,05	402	0,05	077	0,04	774	0,04	489	0,04	223	0,03	974	0,03	733	0,03	505	0,03	300	0,03	121	0,02	940
0,05	402	0,05	136	0,04	873	0,04	653	0,04	431	0,04	200	0,03	663	0,03	807	0,03	624	0,03	456	0,03	302	0,03	162	0,03	044	0,02	944	0,02	860	0,02	782	0,02	744	0,02	718	0,02	713	0,02	740	0,02	798	0,02	856	0,02	976
0,8591	S	0,8591	ъ.	0,8662	0	0,8662	0	0,8591	S	0,8662	0	0,8662	0	0,8591	S	0,8591	5	0,8591	5	0,8591	5	0,8591	5	0,8662	0	0,8450	7	0,6831	0	0,6126	8	0,5281	7	0,5211	m										
0,88	406	0,88	406	0,88	406	0,88	406	0,88	406	0,88	406	0,88	406	0,88	406	0,88	406	0,88	406	0,88	406	0,88	406	0,88	406	0,88	406	0,88	889	0,88	889	0,78	744	0,78	744	0,70	048	0,71	014	0,71	981	0,73	430	0,74	396
0,73	256	0,73	256	0,73	256	0,73	256	0,72	674	0,72	674	0,73	256	0,73	256	0,72	674	0,72	674	0,72	674	0,72	674	0,72	674	0,73	256	0,73	256	0,73	256	0,73	256	0,73	256	0,73	256	0,51	163	0,50	000	0,50	000	0,50	000
0,73	750	0,73	750	0,73	750	0,73	750	0,73	750	0,73	750	0,73	750	0,73	750	0,73	750	0,73	750	0,73	750	0,73	750	0,73	750	0,73	750	0,73	750	0,73	750	0,73	333	0,71	250	0,72	083	0,72	500	0,74	167	0,76	250	0,77	917
0,91	892	0,91	892	0,91	892	0,91	892	0,91	892	0,91	892	0,91	892	0,91	892	0,92	568	0,92	568	0,91	892	0,91	892	0,91	892	0,91	892	0,91	892	0,91	892	0,91	892	0,91	892	0,91	892	0,91	892	0,92	568	0,70	270	0,70	946
0,11	847	0,11	498	0,11	498	0,31	010	0,31	010	0,31	010	0,31	010	0,31	010	0,31	010	0,31	010	0,31	010	0,31	010	0,31	010	0,31	010	0,31	010	0,31	010	0,31	010	0,31	010	0,08	014	0,08	014	0,07	666	0,07	666	0,07	317
0,10	398	60'0	647	0,08	896	0,08	646	0,07	895	0,07	645	0,10	649	0,22	665	0,28	422	0,40	756	0,04	891	0,04	708	0,04	458	0,04	207	0,03	639	0,03	389	0,03	139	0,02	888	0,02	888	0,02	638	0,02	638	0,02	388	0,02	388
0,12	930	0,13	476	0,14	021	0,14	917	0,15	462	0,16	358	0,17	000	0,18	344	0,18	986	0,19	979	0,20	622	0,34	507	0,63	584	1,00	000	0,46	192	0,26	132	0,26	931	0,28	118	0,28	761	0,29	403	0,30	844	0,31	487	0,32	577
0,09	055	0,14	200	0,22	077	0,40	960	0,82	145	1,00	000	0,96	117	0,86	340	0,92	679	0,43	307	0,25	586	0,27	709	0,24	118	0,19	932	0,19	180	0,15	836	0,09	831	0,05	363	0,03	498	0,02	776	0,02	362	0,02	046	0,01	778
0,06	833	0,10	059	0,15	286	0,27	799	0,56	982	0,69	309	0,66	602	0,61	397	0,70	685	0,31	753	0,17	774	0,19	251	0,16	832	0,14	036	0,13	464	0,11	211	0,07	034	0,03	788	0,02	430	0,01	922	0,01	579	0,01	371	0,01	208
0,18	787	0,17	827	0,16	606	0,16	347	0,15	823	0,15	326	0,14	866	0,24	483	0,58	842	0,25	525	0,13	232	0,12	381	0,11	986	0,11	666	0,11	199	0,10	864	0,10	453	0,09	638	0,09	143	0,08	680	0,08	298	0,07	921	0,07	599
60'0	800	60'0	072	0,08	434	0,08	154	0,07	884	0,07	645	0,07	426	0,13	394	0,34	810	0,14	149	0,06	663	0,06	258	0,06	124	0,06	025	0,05	765	0,05	651	0,05	489	0,04	981	0,04	563	0,04	337	0,04	128	0,03	946	0,03	810
0,48	010	0,46	576	0,45	213	0,43	707	0,42	237	0,40	731	0,39	118	0,37	504	0,35	927	0,34	353	0,32	850	0,31	431	0,29	978	0,28	620	0,27	325	0,25	973	0,24	708	0,23	474	0,22	334	0,21	176	0,20	100	0,19	043	0,18	075
0,25	452	0,24	697	0,23	942	0,23	119	0,22	316	0,21	422	0,20	567	0,19	969	0,18	879	0,18	059	0,17	313	0,16	579	0,15	840	0,15	150	0,14	473	0,13	757	0,13	149	0,12	520	0,11	919	0,11	299	0,10	741	0,10	144	0,09	616
0,39	029	0,46	164	0,54	512	0,67	114	0,79	625	06'0	778	66'0	305	0,99	235	0,95	886	0,89	873	0,81	812	0,72	053	0,61	580	0,51	113	0,41	371	0,35	337	0,30	563	0,26	864	0,24	055	0,21	770	0,20	078	0,18	867	0,18	024
0,30	348	0,40	652	0,51	891	0,66	277	0,79	970	0,91	650	1,00	000	0,98	017	0,92	340	0,83	923	0,73	722	0,63	771	0,53	512	0,43	467	0,34	160	0,27	746	0,22	460	0,18	170	0,14	744	0,11	926	0,09	809	0,08	208	0,07	062
0,28	975	0,31	614	0,35	557	0,44	074	0,53	383	0,62	638	0,70	995	0,75	455	0,78	190	0,79	219	0,78	559	0,74	455	0,69	407	0,64	144	0,59	393	0,58	221	0,58	080	0,58	762	0,60	057	0,61	473	0,63	199	0,65	144	0,67	211
0,38	779	0,39	235	0,39	678	0,40	055	0,40	418	0,40	775	0,41	134	0,41	526	0,41	928	0,42	341	0,42	765	0,43	215	0,43	699	0,44	123	0,44	572	0,44	993	0,45	404	0,45	806	0,46	200	0,46	607	0,47	000	0,47	372	0,47	717
0,28	140	0,28	420	0,28	685	0,28	868	0,29	860	0,29	294	0,29	492	0,29	719	0,29	958	0,30	208	0,30	470	0,30	754	0,31	047	0,31	349	0,31	657	0,31	957	0,32	261	0,32	573	0,32	892	0,33	231	0,33	577	0,33	929	0,34	285
00'0	392	00'0	407	00'0	432	00'0	479	00'0	541	00'0	618	00'0	711	00'0	832	00'0	996	0,01	108	0,01	255	0,01	398	0,01	540	0,01	677	0,01	807	0,01	899	0,01	992	0,02	094	0,02	215	0,02	076	0,02	091	0,02	384	0,03	081
00'0	056	00'0	056	0,00	055	00'0	053	00'0	050	00'0	048	00'0	046	00'0	046	00'0	047	00'0	049	00'0	052	00'0	057	00'0	063	00'0	068	00'0	071	00'0	067	00'0	061	00'0	056	00'0	052	00'0	049	00'0	050	0,00	059	0,00	078
0,24	116	0,25	664	0,27	766	0,31	148	0,36	404	0,43	818	0,67	437	0,93	312	1,00	000	0,98	873	0,82	821	0,58	055	0,49	322	0,46	188	0,45	347	0,45	863	0,47	124	0,48	691	0,50	296	0,51	844	0,53	373	0,54	940	0,56	469
0,33	719	0,34	431	0,35	913	0,38	820	0,43	327	0,49	377	0,69	690	0,89	680	0,93	120	0,89	680	0,73	280	0,49	555	0,40	125	0,35	617	0,33	960	0,31	762	0,31	050	0,30	635	0,30	338	0,30	101	0,29	923	0,29	834	0,29	775
	538		539		540		541		542		543		544		545		546		547		548		549		550		551		552		553		554		555		556		557		558		559		560

0.04	325	0,04	222	0,04	178	0,04 147	0.04	156	0,04	211	0,04	297	0,04	444	0,04	642	0,04	897	0,05	222	0,05	631	0,06	106	0,06	697	0,07	400	0,08	225	0,09 156	0,10	242	0,11	519	0,12	950	0,14	572	0,16	400	0,18 503	0,20	811
0.02	765	0,02	600	0,02	453	0,02 319	0.02	200	0,02	072	0,01	963	0,01	870	0,01	775	0,01	687	0,01	616	0,01	553	0,01	494	0,01	445	0,01	398	0,01	380	0,01 2.48	0.01	329	0,01	334	0,01	335	0,01	354	0,01	201	0,U1 429	0,01	444
0.03	124	0,03	296	0,03	531	0,03 791	0.04	131	0,04	540	0,05	007	0,05	562	0,06	224	0,06	978	0,07	867	0,08	878	0,10	080	0,11	462	0,13	073	0,14	902	0,16 073	0.19	340	0,22	071	0,24	444	0,28	889	0,33	555 7 5 7	778 / 178	0,42	222
5281	70201	,5281	7	,5281	7	,5281 7	5281	7	,5633	8	,5774	9	,6267	9	,6267	9	,6267	9	,6267	9	,6267	9	,6267	9	,6267	9	,6267	9	,6267	9	,6267 6	.6267	9	,6267	9	,6267	9	,6267	9	,6267 6	0	,020, 6	,6267	9
0 77 0	879	0,76 0	329	0,76 0	812),77 0 778 0	0.78 0	261	0,79 0	710),80 0	193	0 68'(855	0 66'(034	0 66'(034	l,00	000	0 66'(517	0 66'(517	0 66'(517	0 66'(517	0 66'(517),99 () 5 1 7	0 66.0	517	0 66'(517	0 66'(517	0 66'(517	0 0 0		ט לצי, 17 ט	0 66'0	517
02	200	,50 0	000	,50 (8	50	.50	581	,50 (581	,50 (581	,56 (176	,62	509	,62	509	,62	509	,62 (509	,63 (372	,63 (372	,63 (372)63	372	,63	,62	509	,62 (509	,62	791	,63	372	63	2/2	70,	,62 (509
78 0	,50	,80 0	333 (,82 0)83 (8, 83	0 66	83	00,	000	0 66'	83	0 66	83	0 0	00	0	00	0	000	00,	00	0 66'	83	00,00	00	00	8	8	00		0 66	83	00,00,	00	0 0	00	00	8	8,8	25		0 66	83
71 0	22	72 0,	97 8	73 0,	49 C	74 24 0	75 0	0	76 1,	51 C	76 0,	51 5	0	03	78 1,	78 C	99 1	24 C	99 1,	24 C	99 1,	24 C	0 00	00	00 1	0	99 1,	24 C	1	24 C	-1 	, 0 0	24 5	99 1,	24 C	99 1	24 C	00	0	98	35	38	00	00
0	17 6	J6 0,	59 2	J6 0,	59 6	, 0, 0,	0.0	20	J6 0,	20 3	JG 0,	72 3)6 O,	72 7)6 0,	72 3)5 O,	23 33	12 0,	92 3	12 0,	44 3	12 1,	4	12 1,	92 0	12 0,	92 3	12 0,	92 3	° ()	, 0. 12	92 3	12 0,	92 3	12 0,	92 3	12 1,	92 0	17 17	; ‡	- 0 - 1 2 2	12 1,	44 0
00	3.0	0,0	10 9(0,0	37 9(11 0,0	0.0	87 6.	0,0	37 6:	0,0	87 2.	0,0	36 2	11 0,0	37 2.	0,0	.6 9	0, 11 0,	36 89	0,:	36 5 [,]	0,:	36	0,:	36 8 <u>9</u>	0,	8.	0 0	80 01	0°.	5 0.	12 8)3 O,:	8 68	0, 10	54 89	10,	54 89	5 S	n 5		, <u>0</u>	5.
00	4 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7	5 0,0	1 13	6 0,0	2 88	0,0	8 0.0	8	9'0 6	5 63	0,0	9 63	т 0'0	4 38	2 0,0	5 63	0,0 E	8	4 0,0	5 38	5 0,0	6 35	6 0,0	8	6 0,0	7 35	8 0,0	4	0'0	0 14	0,0 0,7	0.0	5 64	0 0,0	38	2 0,0	7 45	0,0	0 45	4 0 0,0		4 C,C	1 0,0	9 45
1 0.2	- 10 8 2 8 2 8	1 0,3	3 01	1 0,3	6 45	1 0,3	0.3	18	0 0,3	2 27	0,4	6 61	0,4	1 16	0,4	1 25	0,4	1 24	0,4	5 08	1 0,4	9 52	1 0,4	4 16	1 0,4	2 96	2 0,4	3 15	4 0,4	8 05	8 0,4	0.5	2 33	1 0,5	1 78	2 0,5	1 12	1 0,5	1 56	9,0 		0,0 0,0 0,0	0,0	8 99
00	7 52	1 0,0	1 38	0,0	9 20	0,0	0.0	92	0'0 0	8 84	0'0 C	3 78	0,0	4 79	0'0	5 76	0,0	8 80	0'0	5 89	0′0 C	2 09	1 0,0	3 45	1 0,0	3 70	1 0,0	1 10	0,0	1 66	7 0,0 6.3	0.0	1 60	9 0,1	3 16	1 0,1	60	0,1	5 75	1 0,1	5 5 5	τ 90 τ γ	0,2	1 46
ò c	11	0,0	07:	0,0	946	0,0(0.0	768	0,0(728	0,0(643	0,0	737	0,0	75(0,0	. 658	0,0	65(0,00	823	0,0	048	0,0	25	0,0	00	0,0	92.	0,0	0.0	23	50 ' 0	53	0,1	- 23(0,1(525	0,1,	Ϋ́, Ϋ́,	106 106	0,15	12
20.0	351	0,07	133	0,06	857	0,06	0.06	201	0,05	922	0,05	663	0,05	470	0,05	248	0,04	932	0,04	585	0,04	332	0,04	182	0,03	866	0,04	005	0,06	227	0,10	0.07	083	60'0	918	0,08	429	0,06	730	0,06	040 190	cv,v 046	0,03	821
0.03	721	0,03	673	0,03	546	0,03	0.03	209	0,03	037	0,02	915	0,02	915	0,02	787	0,02	493	0,02	294	0,02	210	0,02	079	0,02	027	0,02	061	0,03	657	0,06	0,04	243	0,06	398	0,06	203	0,05	280	0,04	200	cu,u 741	0,02	584
017	157	0,16	181	0,15	249	0,14 335	0.13	417	0,12	650	0,11	879	0,11	201	0,10	606	0,09	966	0,09	369	0,08	817	0,08	333	0,07	874	0,07	530	0,07	150	0,06	0,06	529	0,06	196	0,05	930	0,05	708	0,05	70C	cu,u 317	0,05	213
	097	0,08	536	0,08	054	0,07	0.07	048	0,06	606	0,06	206	0,05	840	0,05	559	0,05	262	0,04	958	0,04	718	0,04	516	0,04	327	0,04	178	0,04	004	0,03 8.4.4	0,03	689	0,03	531	0,03	407	0,03	355	0,03	C+2	u,u 163	0,03	131
0 17	277	0,16	738	0,16	359	0,16 090	0.15	716	0,15	424	0,15	231	0,15	157	0,15	142	0,15	314	0,15	721	0,16	413	0,17	784	0,19	401	0,21	175	0,23	019	0,24 765	0.26	436	0,27	974	0,29	324	0,30	238	0,30	720 100	405 405	0,31	969
0.06	203	0,05	642	0,05	314	0,05 156	0.04	946	0,04	841	0,04	838	0,04	937	0,05	034	0,05	270	0,05	685	0,06	316	0,07	336	0,08	598	0,10	088	0,11	792	0,13	0,16	022	0,18	255	0,20	470	0,22	653	0,24	042	υ,∠0 361	0,27	694
0.69	117	0,71	034	0,72	945	0,74 833	0.76	607	0,78	352	0,80	078	0,81	797	0,83	466	0,85	173	0,86	951	0,88	833	0,91	317	0,93	785	96'0	086	0,98	066	0,99 175	66'0	817	1,00	000	66'0	731	0,98	699	0,97	302	597 797	0,94	279
0.48	049	0,48	331	0,48	546	0,48 678	0.48	645	0,48	524	0,48	326	0,48	061	0,47	714	0,47	332	0,46	936	0,46	546	0,46	239	0,45	959	0,45	706	0,45	481	0,45 317	0.45	166	0,45	012	0,44	843	0,44	557	0,44		0,45 976	0,43	722
0 34	643	0,35	001	0,35	355	0,35 704	0.36	041	0,36	370	0,36	689	0,36	666	0,37	295	0,37	581	0,37	858	0,38	126	0,38	380	0,38	628	0,38	875	0,39	124	0,39	0.39	629	0,39	929	0,40	200	0,40	460	0,40	17/	0,40 983	0,41	249
0.04	401	0,06	336	0,08	973	0,12 400	0.18	746	0,25	238	0,31	145	0,35	736	0,35	303	0,33	283	0,30	138	0,26	325	0,22	495	0,18	843	0,15	756	0,13	619	0,13 601	0.15	134	0,17	984	0,22	276	0,29	020	0,36	880 7 AF	0,45 526	0,54	578
	107	0,00	152	0,00	216	0,00 304	0.00	479	00'0	660	00'0	826	00'0	955	0,00	946	0,00	890	0,00	800	00'0	687	0,00	535	0,00	394	00'00	284	00'0	228	0,00	00,00	000	00'0	000	0,00	983	0,02	971	0,06	108 201	0,115 035	0,22	232
0 57	997	0,59	488	0,60	959	0,62 373	0.63	788	0,65	163	0,66	463	0,67	743	0,68	928	0,70	055	0,71	106	0,72	119	0,73	170	0,75	062	0,80	126	0,85	209	0,91	0.94	917	0,92	547	0,88	764	0,84	426	0,80	401	0,78 693	0,78	081
0 J G	715	0,29	647	0,29	606	0,29 561	0.29	543	0,29	517	0,29	514	0,29	534	0,29	540	0,29	564	0,29	579	0,29	656	0,29	804	0,30	724	0,34	312	0,37	960	0,42 378	0,45	255	0,43	149	0,40	900	0,36	507	0,33	555 7 2 7	0,32 028	0,31	613
	561		562		563	564		565		566		567		568		569		570		571		572		573		574		575		576	577		578		579		580		581	601	785	583	1	584
ι		L									L			l.		1									L			l						l							l			l

0,23	406	0,26 247	0.30	556	0,33	333	0,36	111	0,38	889	0,41	667	0,41	667	0,44	444	0,41	667	0,41	667	0,38	889	0,36	111	0,30	556	0,27	367	0,23	983	10,01	0.18	842	0,17	167	0,15	266	0,15	278	0,14	1011	0,14 914
0,01	520	0,01	0.01	702	0,01	807	0,01	935	0,02	960	0,02	270	0,02	481	0,02	689	0,02	939	0,03	177	0,03	491	0,03	830	0,04	200	0,04	626	0,05	0/0	د0,0 11ء	100	195	0,06	863	0,07	584	0,08	393	0,09 07.0	010	0,10 242
0,46	667	0,53 333	0,60	000	0,66	667	0,73	333	0,80	000	0,86	667	0,93	333	0,97	778	1,00	000	0,97	778	0,95	556	0,88	889	0,80	000	0,68	889	0,60	000	0,48	040	000	0,33	333	0,26	667	0,22	222	0,18	1 C C C	013 013
,6267	9),6267 6	.6267	9	,6126	8	,5774	9	,5704	2	,5281	7	,5211	m	,5211	m	,5140	∞	,5140	∞	,5140	∞	,5070	4	,5070	4	,5070	4	,5000	0 0	0005,0	1850	2	,4788	7	,4788	7	,4718	m),4647 0	C 1617	,4047 9
0,93 (720	0,93 (0.87 (923	0,87 (440	0,86 (957	0,86 (473	0,85 (066	0,84 (541	0,83 (575	0,82 (126	0,81 (159	0,80 (676	0,79 (227	0,78 (261	0,77 (295	0,76 (329	0,74 (0.72	913	0,72 (947	0,71 (981	0,70 0	531	0,70	040	599 v
0,62	209	0,62 209	0,62	209	0,62	209	0,62	209	0,59	302	0,55	814	0,55	814	0,55	814	0,55	814	0,55	814	0,55	814	0,56	395	0,56	395	0,56	395	0,56	395 0 75	0,56 770	0 56	977 977	0,57	558	0,57	558	0,57	558	0,58	140 0 E0	0,00 140
0,99	583	1,00 000	0.91	250	06'0	417	0,89	583	0,89	167	0,88	333	0,87	500	0,87	083	0,86	250	0,85	000	0,84	583	0,84	167	0,82	500	0,81	250	0,80	833	0,80	070	167	0,77	917	0,77	083	0,75	833	0,75	000	c/,u 750
1,00	000	0,99 324		324	0,99	324	0,99	324	0,99	324	66'0	324	66'0	324	0,91	892	0,91	892	0,92	568	0,92	568	0,93	243	0,93	243	0,93	243	0,93	243	0,93 242	600	243	0,92	568	0,92	568	0,91	892	0,91	10.0	1,91 892
0,12	544	0,12 544	0.12	544	0,04	878	0,04	878	0,05	226	0,05	226	0,05	226	0,05	226	0,05	226	0,05	575	0,05	575	0,05	575	0,05	923	0,05	923	0,06	7/7	0,06 77,0	100	620	0,06	696	0,06	696	0,07	317	0,07	000	0,00 014
0,01	454	0,01 203	0.01	771	0,01	771	0,01	771	0,01	521	0,01	839	0,01	839	0,02	156	0,02	474	0,02	792	0,03	109	0,03	745	0,04	062	0,05	015	0,05	968 202	0,06 604	500	875	60,0	145	0,10	416	0,12	640	0,14 EAE	040 740	0, 10 452
0,55	280	0,54 249	0.54	346	0,54	794	0,54	794	0,55	339	0,55	436	0,55	436	0,55	086	0,55	183	0,55	183	0,55	183	0,55	280	0,54	832	0,54	735	0,54	384	0,54 787	0 53	6839	0,53	391	0,52	943	0,52	496	0,52	040	10,01 502
0,17	456	0,15 970	0.19	006	0,20	070	0,15	576	0,12	977	0,10	735	60'0	617	60'0	691	0,07	046	0,04	414	0,03	478	0,03	188	0,03	607	0,04	705	0,04	د08 د08	0,03	100	376	0,02	089	0,02	193	0,02	346	0,02	00/	cu,u 332
0,12	938	0,13 121	0.19	166	0,18	542	0,12	548	0,10	245	60'0	333	0,10	477	0,12	879	0,09	692	0,05	684	0,04	480	0,04	300	0,05	209	0,07	369	0,07	638 0 0 1	20,0 222	200	642	0,03	084	0,03	293	0,03	676	0,04	300	cu,u 443
0,03	676	0,05 388	0,10	868	60'0	578	0,04	754	0,03	753	0,04	841	0,07	835	0,11	667	0,09	320	0,05	780	0,04	734	0,04	209	0,05	684	0,07	907	0,08	191	20,0 710	5	318	0,03	743	0,03	957	0,04	284	0,04	100	cu,u 938
0,02	539	0,04 275	0.09	917	0,08	547	0,03	700	0,02	840	0,03	953	0,06	961	0,10	867	0,08	572	0,04	987	0,04	065	0,04	072	0,05	052	0,07	306	0,07	643 0.01	د0,0 201		724	0,03	206	0,03	424	0,03	837	0,04		516
0,05	081	0,04 973	0,04	962	0,04	955	0,04	991	0,05	045	0,05	124	0,05	206	0,05	332	0,05	464	0,05	597	0,05	809	0,06	063	0,06	379	0,06	648	0,07	870	0,07 26.0	200	716	0,08	118	0,08	602	0,09	168	60'0	010	0,10 592
0,03	690	0,03 078	0.03	192	0,03	275	0,03	352	0,03	474	0,03	602	0,03	739	0,03	942	0,04	137	0,04	352	0,04	606	0,04	880	0,05	125	0,05	454	0,05	84/	0,06 250	900	0,00 666	0,07	132	0,07	661	0,08	232	0,08		595 595
0,31	912	0,31 940	0.31	762	0,31	358	0,30	535	0,29	521	0,28	365	0,27	120	0,25	885	0,24	645	0,23	433	0,22	280	0,21	154	0,20	179	0,19	413	0,18	91/	0,1/ /10	210	840	0,17	771	0,20	803	0,30	109	0,41	7 53	434
0,28	274	0,28 392	0,28	064	0,27	304	0,25	888	0,24	165	0,22	246	0,20	239	0,18	417	0,16	665	0,15	026	0,13	548	0,12	358	0,11	386	0,10	644	0,10	146	0,09	800	564	0,09	041	0,10	834	0,16	404	0,23		00,0 597
0,93	305	0,92 457	0.91	722	0,91	085	06'0	590	06'0	143	0,89	707	0,89	245	0,88	610	0,87	919	0,87	180	0,86	399	0,85	629	0,84	813	0,83	942	0,83	004	0,81		819 819	0,79	624	0,78	369	0,77	023	0,75	0.74	0,74 246
0,43	539	0,43 4 <i>7</i> 2	0,43	383	0,43	435	0,43	676	0,44	002	0,44	392	0,44	826	0,45	239	0,45	673	0,46	126	0,46	596	0,47	094	0,47	599	0,48	102	0,48	596 22	0,49		493	0,49	914	0,50	315	0,50	069	0,51	040	390 390
0,41	515	0,41 789	0,42	077	0,42	380	0,42	730	0,43	094	0,43	466	0,43	838	0,44	179	0,44	518	0,44	859	0,45	207	0,45	586	0,45	971	0,46	356	0,46	/36	0,47	100	438	0,47	778	0,48	114	0,48	451	0,48	00/0	0,49 116
0,63	671	0,72 479	0.80	657	0,87	860	0,93	236	0,97	148	66'0	450	1,00	00	0,97	580	0,93	548	0,88	190	0,81	791	0,74	447	0,66	602	0,58	938	0,51	497	726 726		631	0,36	149	0,32	256	0,28	866	0,26		695 695
0,42	664	0,64 100	0.84	016	0,99	891	1,00	000	0,94	704	0,85	163	0,72	535	0,57	868	0,42	525	0,27	608	0,14	340	0,08	219	0,04	409	0,02	377	0,01	593 0 00	0,00		166	00,00	128	0,00	326	0,00	239	0,00		u,uu 226
0,77	737	0,77 527	0.77	241	0,76	992	0,76	591	0,76	132	0,75	578	0,75	024	0,74	374	0,73	705	0,72	979	0,72	157	0,71	355	0,70	418	0,69	501	0,68	582 5	0,6/ 51/	0.66	482	0,65	450	0,64	323	0,63	214	0,62	t 1	00'n 866
0,31	495	0,31 495	0.31	524	0,31	465	0,31	435	0,31	228	0,31	139	0,31	079	0,31	020	0,30	902	0,30	783	0,30	694	0,30	664	0,30	724	0,30	753	0,30	/83	0,30		694	0,30	664	0,30	694	0,30	694	0,30		991 991
	585	586		587		588		589		590		591		592		593		594		595		596		597		598		599	}	009	601	3	602		603		604		605	505	8	607

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0,15	211	0,15	778	0,16	586	0,17 642	1010	975 975	0,20	575	0,22	458	0,24	678	0,27	117	0,30		0,33 222	950	ac,u 111	0,41	667	0,44	444	0,50	000	556	0,61	111	0,66	0.72	222	0,77	778	0,83	333	0,86		1 <i>6</i> '0	0,94 444	1111
0,11	346	0,12	567	0,13	893	0,15 370	5, C	023 023	0,19	298	0,21	053	0,22	807	0,26	0T2	0,28	220	U,31 570	0 35 U	088	0,40	351	0,43	860	0,49	123	386 386	0,59	649	0,64	0.70	175	0,75	439	0,80	702	0,85	00 0	474	0,94 737	101
0,12	329	0,10	131	0,08	320	0,06 847	500	cn'n	0,04	636	0,03	842	0,03	160	0,02	047	70'N	022	10'0	000	10,0	0,01	370	0,01	176	0,01	500	0,00 882	00'0	742	0,00	0.00	594	00'0	537	0,00	486	00'0	401	436	0,00 406	400
,4647	6	,4577	S	,4507	0	,4507 0	1507	, uct,	,4507	0	,4507	0	,4507	0	,4507		,45// 7		,4/18 c	c 8871	7	,5070	4	,5140	∞	,5352	I E177	277 10	,5422	5	,5422 5	5352	1	,5352	1	,5422	2	,5493	0	c coc, 8	,5704 2	7
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72 0	17	71 0	67	70 0	17 8	69 28		333	67 0	83 (65 0	33	65 0	8	63		0 20				12	59 0	67 6	57 0	17 7	56		2 0 7 88	54 0	83 2	23	52 0	8	51 0	50	50 0	8	48	- CC	83	45 0 33 (30
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0,51	055	0,50	503	0,50	061	0,49 516		971 971	0,48	075	0,47	530	0,47	082	0,46	DOT	0,43	T+0 0	0,44		908 908	0,43	460	0,42	915	0,41	776	0,40 676	0,39	936	0,39	160 180	495	0,37	853	0,36	860	0,36		575 575	0,34 776	
0,04	729	0,0	010	0,25	192	0,57		ac'n 204	0,31	411	0,21	800	0,17	825	0,13	600	U,11	070	U, LI 1 ()	4.50	0, U,	0,15	979	0,16	457	0,14	469	922	0,10	314	0,09	70.0	188	0,05	882	0,05	444	0,05	220	308 308	0,09 253	nc7
0,07	867	0,15	359	0,43	519	1,00	000	06,0 694	0,53	701	0,36	589	0,28	451	0,19	010	0,13 641	140	U, LL GE G	011	001	0,13	299	0,13	579	0,12	382	031	0,10	237	0,0	0.08	592	0,07	895	0,07	706	0,08	t 1 1 1	496	0,15 161	TOT
0,08	276	0,15	684	0,43	590	1,00	000	0,90 363	0,53	285	0,35	992	0,27	117	0,16	002	60,0	77 V	0,Ub		40,04 897	0,04	229	0,03	981	0,04	332	963	0,05	516	0,05	0.06	187	0,06	562	0,06	840	0,07	cc /	022	0,14 719	CT /
0,07	921	0,15	208	0,43	226	1,00	0.06	0,90 279	0,52	978	0,35	209	0,26	833	0,16	000	0,09 675	0/0	0,Ub	1700	0,04 715	0,03	949	0,03	753	0,04	780	760	0,05	290	0,05	0.05	985	0,06	352	0,06	676	0,07	010	0, 10 811	0,14 586	noc
0,11	406	0,12	227	0,13	159	0,14 080	0.15	0,15 142	0,16	307	0,17	637	0,19	028	0,20	420	12/0	105	0,23 AAG	400 0.25	771	0,27	049	0,29	046	0,31	133	478	0,35	666	0,38	0.41	305	0,44	281	0,47	401	0,50	1/1	213 213	0,57 942	742
0,10	400	0,11	260	0,12	241	0,13	0 11	0,14 244	0,15	356	0,16	636	0,17	946	0,19	3/1	0,20	62.0	U, 22 E 11	14C	0,24 285	0,26	092	0,27	066	0,29	756	220	0,34	069	0,37	0.39	812	0,42	625	0,45	529	0,48	16/	2c'n	0,55 614	4TO
0,65	771	0,77	704	0,88	018	0,95 766	8 6	000	0,94	392	0,85	527	0,74	611	0,62	040	0,53 775	C / C	0,45 CCC	75.0	789 789	0,31	419	0,27	760	0,25	310	835	0,23	860	0,22	0c/ 0.22	695	0,22	790	0,22	838	0,22	502 150	0,41 337	0,20 231	TC7
0,38	242	0,45	835	0,52	557	0,57 878	0.61	063 063	0,58	477	0,53	971	0,48	244	0,41	734	/3/ /2/	404	0,33		711 711	0,25	739	0,23	259	0,21	247	0,13 622	0,18	300	0,17	124 0.16	115	0,15	223	0,14	393	0,13 E03	200	0,12 598	0,11 654	+CD
0,72	853	0,71	574	0,70	302	0,69 071	120	712 712	0,66	282	0,64	823	0,63	348	0,61	2/0	0,60		620	057	0,27 656	0,56	237	0,54	737	0,53	231	736	0,50	266	0,48	c./o	527	0,46	223	0,44	963	0,43 760	60/	0,42 614	0,41 490	470
0,51	720	0,52	047	0,52	363	0,52 667	050	959	0,53	239	0,53	504	0,53	754	0,53	70/	0,54 202	102	0,54	050	0,34	0,54	733	0,54	820	0,54	889	950	0,55	016	0,55 125	0.55	266	0,55	403	0,55	543	0,55 675	2/0	604 804	0,55 930	טכע
0,49	443	0,49	767	0,50	086	0,50 400		0c'n	0,51	600	0,51	305	0,51	596	0,51	004	2C,U	1/7	70,U	0,50	754	0,53	029	0,53	266	0,53	005	739	0,53	991	0,54	0.54	621	0,54	956	0,55	297	0,55 620	1000	063 963	0,56 297	167
0,21	886	0,20	787	0,20	089	0,19 694		ст'л 505	0,19	438	0,19	375	0,19	212	0,18	844 0.11	0,17 655	CCD	0,15 75 0	011	0,14 756	0,13	250	0,12	052	0,10	968	013	60'0	206	0,08	coo 0.08	291	0,08	000	0,07	778	0,07	100	u,u/ 315	0,07 122	777
00'C	268	0,00	285	00'0	311	0,00 3,47		377	00'0	421	00'0	460	00'0	491	0,00	100	00'r	400		4 20	387	00,0	332	00'0	283	0,00	700	196	00'0	161	00,00	747 0.00	130	00'0	124	00'0	123	00,00	124	129 129	0,00 135	CCT
),59 (813	0,58	666	0,57	481	0,56 316		131 cc,u),53 (984	3,52 (819	0,51	634	0,50	445	0,49	C07	1.27	1 97 (0,40 971),45 (805), 44 (678	0,43	י כער י	442),41 (372	0,40	,39 (213	J,38 (143	J,37	092	0,36		047	0,34 053	ccN
),31 (524),32 (770),34 (282	35 (3000	, 206 706	,35 (142),34 (075) 33 (155),32 (100	1,32	204) 32 (, cc,	114	,32 (551),32 (888) 33 (155 20	574),33 (333) 33 (-33 (126) 33 (960),33 (185) 33 (7 CC -	260),333 (749 (/47
C	. 80	ں 	60	<u>ں</u>	310	ی ۲		12	0	13	C	14	0	15	ن ن ب	010	، ر ۲	, T	، ر و	0	ہ ر 10	0	20	0	21		0 770	23	0	24	، ن ب	0	26	C	.27	0	528	` ن م		30 30	3. C	151
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0,97	222	1,00	000	1,00	000	0,97	222	0,94	444	0,91	667	0,86	111	0,80	556	0,72	222	0,63	889	0,58	333	0,50	000	0,44	444	0,38	889	0,33	333	0,27	139	0,22	667	0,18	864	0,15	633	0,12	950	0,10	747	0,08	908	0,07	383
0,96	491	0,98	246	1,00	000	0,98	246	0,96	491	0,92	982	0,87	719	0,82	456	0,75	439	0,68	421	0,59	649	0,52	632	0,45	614	0,40	351	0,33	333	0,28	070	0,24	561	0,21	053	0,16	800	0,13	923	0,11	568	0,09	604	0,07	940
00′0	380	00'0	360	00'0	348	00'0	328	00'0	308	00'0	274	00'0	260	00'0	228	00'0	222	00'0	187	00'0	160	00'0	135	00'0	112	00'0	107	00'0	089	00'0	077	00'0	690	00'0	067	00'0	065	00'0	059	00'0	058	00'0	044	00'0	090
0,5704	2	0,5704	2	0,5422	5	0,5281	7	0,4788	7	0,4436	9	0,4225	4	0,4014	1	0,3873	2	0,3802	8	0,3732	4	0,3732	4	0,3732	4	0,3732	4	0,3873	2	0,4014	1	0,4929	9	0,5915	5	0,6831	0	0,7183	1	0,7183	1	0,7112	7	0,7042	m
0,39 (614	0,38 (647	0,37 (681	0,36 (232	0,35 (749	0,34 (783	0,33 (333	0,32 (850	0,32 (367	0,31 (401	0,30 (435	0,29 (469	0,28 (502	0,28 (502	0,27 (536	0,26 (570	0,26 (087	0,26 (087	0,25 (121	0,24 (155	0,23 (671	0,23 (188	0,22 (705
0,73	256	0,72	093	0,69	767	0,68	023	0,66	279	0,66	279	0,66	279	0,66	279	0,66	860	0,66	860	0,66	860	0,66	860	0,67	442	0,68	023	0,69	186	0,71	512	0,74	419	0,78	488	0,82	558	0,84	884	0,85	465	0,87	791	0,88	953
0,44	583	0,42	917	0,42	500	0,40	417	0,39	167	0,37	917	0,36	667	0,35	000	0,33	750	0,32	500	0,31	667	0,30	833	0,29	583	0,29	167	0,27	917	0,27	500	0,26	667	0,25	833	0,24	583	0,24	167	0,23	333	0,22	917	0,22	500
0,77	703	0,76	351	0,75	676	0,75	000	0,74	324	0,73	649	0,72	973	0,72	297	0,70	946	0,70	946	0,69	595	0,68	919	0,67	568	0,67	568	0,66	892	0,65	541	0,65	541	0,64	189	0,63	514	0,62	838	0,62	162	0,60	811	0,60	811
0,31	359	0,30	314	0,28	920	0,27	526	0,26	481	0,25	436	0,24	739	0,24	042	0,23	693	0,23	345	0,23	345	0,23	345	0,23	693	0,24	390	0,25	436	0,32	056	0,43	554	0,54	007	0,65	505	0,78	746	0,88	850	0,97	561	0,98	955
0,87	293	0,89	834	0,91	105	0,93	329	0,93	964	0,95	553	0,96	506	0,98	094	0,98	729	0,98	729	0,99	365	0,99	682	0,99	682	1,00	000	1,00	000	0,99	682	0,99	365	0,98	729	0,98	412	0,98	094	0,96	188	0,95	870	0,94	600
0,34	231	0,33	394	0,32	751	0,31	758	0,31	213	0,30	668	0,30	220	0,29	480	0,28	487	0,27	591	0,27	046	0,26	501	0,25	956	0,24	963	0,24	320	0,23	775	0,23	230	0,22	782	0,21	789	0,21	244	0,20	669	0,20	153	0,19	706
0,06	916	0,03	368	0,02	020	0,01	577	0,01	406	0,01	303	0,01	178	0,01	144	0,01	115	0,01	178	0,01	210	0,01	363	0,01	507	0,01	617	0,01	652	0,01	587	0,01	672	0,02	051	0,02	868	0,03	158	0,02	292	0,01	813	0,01	584
0,11	142	0,05	206	0,02	827	0,02	237	0,01	972	0,01	800	0,01	666	0,01	529	0,01	471	0,01	480	0,01	564	0,01	559	0,01	648	0,01	736	0,01	869	0,01	643	0,01	687	0,02	258	0,03	882	0,04	592	0,03	148	0,02	413	0,02	017
0,10	808	0,04	872	0,02	654	0,02	138	0,01	920	0,01	680	0,01	551	0,01	435	0,01	347	0,01	340	0,01	268	0,01	256	0,01	187	0,01	169	0,01	156	0,01	056	0,01	086	0,01	658	0,03	250	0,04	001	0,02	716	0,02	073	0,01	650
0,10	656	0,04	802	0,02	492	0,01	987	0,01	728	0,01	582	0,01	426	0,01	349	0,01	237	0,01	216	0,01	158	0,01	125	0,01	109	0,01	114	0,01	052	00'00	924	0,00	951	0,01	509	0,03	167	0,03	891	0,02	587	0,01	985	0,01	530
0,61	922	0,65	938	0,69	953	0,74	077	0,78	272	0,82	252	0,86	160	0,89	925	0,93	152	0,95	733	0,97	741	66'0	175	1,00	000	66'0	677	0,98	638	96'0	845	0,94	227	06'0	929	0,86	985	0,82	395	0,77	483	0,72	320	0,66	870
0,59	433	0,63	252	0,67	185	0,71	255	0,75	372	0,79	465	0,83	490	0,87	446	0,91	013	0,93	894	0,96	433	0,98	445	0,99	703	1,00	000	0,99	611	0,98	331	0,96	387	0,93	689	0,90	258	0,86	188	0,81	523	0,76	561	0,71	438
0,18	881	0,16	940	0,14	887	0,12	856	0,10	981	0,09	796	0,08	877	0,08	198	0,07	734	0,07	451	0,07	336	0,07	368	0,07	525	0,07	837	0,08	211	0,08	604	0,08	972	0,09	191	0,09	334	0,09	393	0,09	359	60'0	100	0,08	781
0,10	646	0,09	308	0,07	954	0,06	656	0,05	487	0,04	742	0,04	180	0,03	784	0,03	536	0,03	427	0,03	427	0,03	516	0,03	672	0,03	889	0,04	125	0,04	354	0,04	549	0,04	621	0,04	630	0,04	574	0,04	450	0,04	167	0,03	847
0,40	390	0,39	281	0,38	192	0,37	126	0,36	087	0,35	115	0,34	161	0,33	214	0,32	261	0,31	212	0,30	166	0,29	142	0,28	161	0,27	314	0,26	520	0,25	770	0,25	053	0,24	345	0,23	658	0,22	066	0,22	338	0,21	697	0,21	067
0,56	051	0,56	150	0,56	248	0,56	349	0,56	459	0,56	610	0,56	770	0,56	930	0,57	086	0,57	203	0,57	314	0,57	425	0,57	542	0,57	668	0,57	810	0,57	974	0,58	165	0,58	426	0,58	710	0,59	008	0,59	312	0,59	588	0,59	860
0,56	631	0,56	953	0,57	278	0,57	609	0,57	950	0,58	326	0,58	712	0,59	100	0,59	487	0,59	853	0,60	213	0,60	566	0,60	912	0,61	233	0,61	556	0,61	888	0,62	238	0,62	661	0,63	860	0,63	540	0,63	975	0,64	368	0,64	744
0,06	946	0,06	765	0,06	593	0,06	432	0,06	279	0,06	136	0,06	001	0,05	875	0,05	756	0,05	650	0,05	549	0,05	451	0,05	354	0,05	244	0,05	136	0,05	030	0,04	930	0,04	845	0,04	767	0,04	669	0,04	639	0,04	601	0,04	567
0,00	142	0,00	147	0,00	152	0,00	156	0,00	158	0,00	158	0,00	155	0,00	149	0,00	139	0,00	118	0,00	960	0,00	074	0,00	056	0,00	053	0,00	054	0,00	058	0,00	064	0,00	067	0,00	071	0,00	073	0,00	075	0,00	073	0,00	071
0,33	079	0,32	123	0,31	168	0,30	269	0,29	333	0,28	473	0,27	594	0,26	734	0,25	932	0,25	110	0,24	345	0,23	581	0,22	874	0,22	167	0,21	479	0,20	810	0,20	065	0,19	453	0,18	792	0,18	175	0,17	600	0,17	032	0,16	482
0,33	541	0,33	037	0,32	384	0,31	702	0,31	168	0,30	813	0,30	605	0,30	546	0,30	486	0,30	368	0,30	219	0,30	042	0,29	953	0,30	012	0,30	249	0,30	753	0,31	495	0,32	384	0,33	155	0,33	630	0,33	778	0,33	719	0,33	749
	632		633		634		635		636		637		638		639		640		641		642		643		644		645		646		647		648		649		650		651		652		653		654

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0,06	231	0,05	614	0,04	/83	0,04 081	0,03	494	0,03	019	0,02	613	0,02	290	0,02	4 TO	10,0	600 100	573	0,01	400	0,01	255	0,01	126	0,01	00.0	922	00′0	843	0,00	00'0	729	00'0	672	00'0	640	0,00	1000	567	00'0	548
0,06	702	0,05	921	0,05	100	0,04 253	0,03	626	0,03	112	0,02	670	0,02	318	0,02	870 0	10'0	50	567	0,01	377	0,01	229	0,01	860	0,00	00.0	903	0,00	808	0,00	00'0	681	0,00	641	0,00	574	0,00 5.41		508	00'0	475
00'0	690	00'00	257	0,00	204	u,uu 248	00'0	240	00'0	246	0,00	225	0,00	238	0,00	000	0,00 216	000	223	0,00	217	00'0	211	00'0	195	0,00	00.0	201	00'0	188	0,00 186	00′0	195	00'0	186	00'0	188	0,00 181		ں,ت 168	00′0	174
7464	∞	7816	6	8591 5	C 100	0343 7	8943	7	8943	7	8169	0	7605	و و	/183	T	00/00	6408	р С	5845	1	5422	ъ	4788	7	4577 E	4084	ъ	3802	8	3521	3309	6	3098	9	2957	7	2746 5	ר אר	9	2464	∞
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6 ' 0	308	0,96	65	1,0 0,1	Š,	, 8	1,0(000	0,99	65	6 ' 0	30	0,96	16.	2 2 2 2 2		0,0		7 <u>9</u> 7	0,68	66	0,62	02	0,55	749	0,48	0.4	46	0,35	54(0,2,	0,23	648	0,14	63/	0,1	58;	0,1	5 C	1.51	0,1(105
0,93	647	0,92	376	06'0	/8/	152 152	0,89	199	0,88	246	0,87	928	0,86	340	0,84	70/	0,0 181		50%	0,80	622	0,78	398	0,77	127	0,76	0.73	633	0,72	680	0,71	0,69	503	0,68	550	0,65	169	0,65	272 0.63	150	0,61	561
0,19	063	0,18	518	0,18	0,0	0,1/ 525	0,17	077	0,16	629	0,16	181	0,15	539	0,15	1441	0,14 806	77 U	448	0,14	000	0,13	806	0,13	358	0,12	0.12	813	0,12	365	0,12 268	0,11	820	0,11	372	0,10	924	0,10	010	379	0,10	281
0,01	206	0,01	026	0,00	1/6	029 029	00'0	006	00'0	785	0,00	982	0,01	009	0,01	022	10'0	800	371	0,01	455	0,01	651	0,01	788	0,01	0.01	425	0,01	303	0,01	0,01	252	0,01	189	0,01	135	0,01	100	334	0,01	439
0,01	450	0,01	179	0,01	707	10,UI 387	0,01	162	00'0	938	0,01	271	0,02	311	0,02	7/0	0,UZ	100	446	0,01	345	0,01	493	0,01	626	0,01	0.01	328	0,01	212	0,01	0,01	129	0,01	100	0,01	005	0,01	100	- 20'n	0,01	191
0,01	137	0,00	926	0,00	065	180 180	00'0	953	0,00	743	0,01	047	0,02	016	0,02	\$ 5 5		ξ, OOO	940	0,00	698	00'0	663	0,00	642	0,00	0.00	566	0,00	566	0,00	0,00	490	0,00	412	0,00	400	0,00 358		391	0,00	389
0,01	046	0,00	888	0,00	883	110 110	0,00	879	00'0	675	0,00	940	0,01	938	0,02	4/9	TU,U		908	0,00	630	0,00	604	00'0	628	0,00	00.0	483	0,00	527	0,00	0,00	422	0,00	405	0,00	380	0,00		321 321	0,00	304
0,61	456	0,56	900	0,50	1//	0,40 002	0,41	556	0,37	289	0,33	299	0,29	616	0,26	242	278 278		728	0,18	437	0,16	468	0,14	701	0,13	0.11	538	0,10	312	0,09 136	0,08	028	0,07	092	0,06	393	0,05	20.0	030	0,04	389
0,66	179	0,60	759	0,55	160	uc,u 675	0,45	895	0,41	390	0,37	046	0,33	112	0,29	475	0,20 161	103 0 73	119	0,20	519	0,18	164	0,16	032	0,14 154	0.12	483	0,11	043	0,09 716	0,08	518	0,07	594	0,06	831	0,06 090	200	436	0,04	807
0,08	441	0,08	122	0,08	170	096	0,07	913	0,07	860	0,07	729	0,07	565	0,07	202	10,0	0.06	783	0,06	418	0,06	048	0,05	693	0,05	0.05	187	0,04	979	0,04 798	0,04	637	0,04	500	0,04	389	0,04 304	500	240	0,04	203
0,03	525	0,03	235	0,03	171	061	0,03	045	0,03	062	0,03	108	0,03	163	0,03	700		0.03	205	0,03	137	0,03	052	0,02	957	0,02	0.02	776	0,02	689	0,02 607	0,02	545	0,02	491	0,02	447	0,02 413	110	390 390	0,02	375
0,20	444	0,19	825	0,19	194 0 1 0	о, то 568	0,17	948	0,17	339	0,16	739	0,16	155	0,15	715		14	562	0,14	095	0,13	650	0,13	223	0,12	0.12	405	0,12	015	0,11 636	0,11	263	0,10	668	0,10	540	0,10 186		823	60'0	467
0,60	130	0,60	397	0,60	0 50	003 903	0,61	170	0,61	454	0,61	798	0,62	159	0,62	270	70/0	200	225	0,63	553	0,63	892	0,64	249	0,64 657	0.65	092	0,65	554	0,66 042	0,66	566	0,67	112	0,67	675	0,68 750	250	830 830	0,69	415
),65	103),65	443	0,65	/4/	040),66	330),66	623),66	996),67	312	7.97	100	10'1	890	235	0,68	481),68	722),68	970),69	69.0	562),69	878	207	0,70	558	0,70	915	0,71	275),71 633	171	973	0,72	305
,04 (531	,04	489	04	101	319	,04 (232	,04 (151	,04	D91	04)45 2.5	0,04	0TC	4 C		5 6 5 C	04	121	,04 (188	,04 (254	04) 40.	328	,04 (334	,04 10 10 10	,04 (235	,04 (127	,03	993	,03 224	0.02	540	,03 (433
0 00	68	0	e7 <i>i</i>	0		2 0	000	74	0 00	. 26	0	72 (0	68 68	85	5 5 5			64 2	0	67	000	71	0 00	74	0 0	00	12	00	89	00	00	59	0 00	54	000	49	85	, c	, 64	0 00	54 4
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	65		65		6	65		65		99		99		99		В	99	3	99	}	99		99		99	22	3	67		67	67		67		67	(9	57	3	67	<u> </u>	67

00'0	515	00'0	495	00'0	469	00'0	444	00'0	429	00'0	419	00'0	413	00'0	395	00'0	388	00'0	371	00'0	364	00'0	374	00'0	356	00'0	353	00'0	349	00'0	341	00'0	330	00'0	335	00'0	347	00'0	345	00'0	333	00′0	302	0,00	319
0,00	448	0,00	429	0,00	411	0,00	398	0,00	378	0,00	343	0,00	354	0,00	329	0,00	333	0,00	326	0,00	311	0,00	316	0,00	310	0,00	284	0,00	279	0,00	292	0,00	288	0,00	293	0,00	287	0,00	277	0,00	276	00'0	268	00'00	266
00'0	161	00'0	152	00'0	148	00'0	154	00'0	148	00'0	126	00'0	139	00'0	144	00'0	137	0,00	143	00'0	123	00'0	119	00'0	132	00'0	133	00'0	121	00'0	132	00'0	133	00'0	128	00'0	121	00'0	119	00'0	118	00′0	960	00'0	109
0,2323	6	0,2183	1	0,2112	7	3,2042	ŝ	0,1901	4	0,1831	0	0,1831	0	0,1760	9	0,1619	7	0,1619	7	0,1549	æ	0,1478	6	0,1408	S	0,1338	0	0,1338	0	0,1338	0	0,1197	2	0,1126	8	0,1126	∞	0,1126	8	0,1056	Э	0,0985	6	0,0985	6
0,12 (077	0,11 (111	0,11 (111	0,11 (111	0,10 (628	0,10 (628	0,10 (145	0,09	662	0,09	662	0,09	179	0,09	179	0,09	179	0,09	179	0,08 (696	0,08 (696	0,08	213	0,08	213	0,08	213	0,07	729	0,07	729	0,07 (246	0,06 (763	0,06	763
0,43	605	0,42	442	0,41	860	0,40	116	0,39	535	0,38	372	0,37	791	0,37	209	0,36	628	0,36	047	0,34	884	0,34	884	0,33	721	0,33	140	0,32	558	0,31	977	0,31	395	0,30	814	0,30	233	0,29	651	0,29	651	0,28	488	0,29	070
0,10	833	0,10	417	0,10	417	0,10	000	60'0	583	60'0	167	0,08	750	0,08	333	0,08	333	0,07	917	0,07	500	0,07	500	0,07	083	0,07	083	0,06	667	0,06	250	0,05	833	0,05	833	0,05	833	0,05	417	0,05	000	0,05	000	0,05	000
0,42	568	0,41	892	0,41	216	0,41	216	0,40	541	0,39	189	0,39	189	0,38	514	0,37	838	0,37	162	0,36	486	0,36	486	0,35	811	0,35	135	0,34	459	0,33	784	0,33	784	0,33	108	0,32	432	0,31	757	0,31	081	0,30	405	0,30	405
60'0	408	0,08	711	0,08	014	0,07	317	0,06	696	0,06	272	0,05	923	0,05	575	0,04	878	0,04	530	0,04	181	0,03	833	0,03	484	0,03	136	0,02	787	0,02	439	0,02	091	0,01	742	0,01	394	0,01	394	0,00	697	0,00	697	0,00	000
0,60	291	0,59	020	0,57	749	0,56	479	0,55	208	0,54	255	0,52	984	0,51	396	0,49	490	0,48	469	0,47	516	0,45	928	0,44	089	0,43	704	0,43	690	0,41	481	0,41	798	0,41	163	0,38	054	0,37	033	0,36	080	0,35	445	0,35	445
60'0	834	0,09	736	60'0	288	60'0	288	60'0	191	0,08	743	0,08	743	0,08	295	0,08	198	0,07	750	0,07	750	0,07	653	0,07	556	0,07	205	0,07	108	0,06	660	0,06	660	0,07	108	0,06	660	0,06	115	0,06	115	0,06	017	0,06	017
0,01	621	0,01	585	0,01	531	0,01	491	0,01	286	0,00	923	0,00	747	0,00	857	0,01	315	0,01	578	0,01	283	0,00	806	0,00	677	0,00	570	0,00	899	0,01	230	0,00	837	0,00	455	0,00	362	0,00	325	0,00	256	0,00	267	0,00	346
0,01	343	0,01	279	0,01	244	0,01	206	0,01	115	0,00	868	0,00	896	0,01	181	0,02	025	0,02	636	0,02	003	0,01	199	0,00	806	0,00	785	0,01	415	0,01	918	0,01	326	0,00	649	0,00	437	0,00	407	0,00	373	0,00	386	0,00	444
0,00	362	0,00	391	00'0	385	00'00	370	0,00	404	0,00	361	0,00	490	0,00	906	0,01	919	0,02	487	0,01	880	0,01	043	0,00	770	0,00	720	0,01	320	0,01	865	0,01	185	00'00	553	0,00	388	00'00	309	0,00	322	0,00	307	0,00	427
0,00	353	0,00	346	00'0	307	0,00	312	0,00	337	0,00	415	0,00	474	0,00	863	0,01	863	0,02	503	0,01	803	0,01	600	0,00	655	0,00	682	0,01	318	0,01	842	0,01	199	0,00	581	0,00	351	0,00	307	0,00	263	0,00	330	0,00	348
0,03	833	0,03	553	0,03	093	0,02	837	0,02	657	0,02	492	0,02	310	0,02	106	0,01	868	0,01	715	0,01	681	0,01	550	0,01	716	0,01	726	0,01	464	0,01	345	0,01	140	0,01	136	0,01	122	0,01	070	00'0	869	00'0	689	0,00	445
0,04	269	0,03	780	0,03	343	0,03	030	0,02	724	0,02	502	0,02	383	0,02	244	0,02	083	0,01	926	0,01	856	0,01	772	0,01	728	0,01	686	0,01	576	0,01	364	0,01	021	00'0	921	00'00	876	00'00	850	00'00	750	0,00	700	0,00	559
0,04	196	0,04	220	0,04	303	0,04	409	0,04	531	0,04	629	0,04	791	0,04	910	0,05	001	0,05	054	0,05	021	0,04	940	0,04	812	0,04	637	0,04	327	0,04	011	0,03	727	0,03	513	0,03	437	0,03	496	0,03	719	0,04	132	0,04	987
0,02	367	0,02	365	0,02	371	0,02	379	0,02	385	0,02	386	0,02	378	0,02	360	0,02	329	0,02	282	0,02	208	0,02	116	0,02	008	0,01	884	0,01	702	0,01	524	0,01	369	0,01	256	0,01	206	0,01	234	0,01	358	0,01	596	0,02	100
60'0	125	0,08	799	0,08	508	0,08	237	0,07	987	0,07	756	0,07	554	0,07	367	0,07	192	0,07	023	0,06	866	0,06	704	0,06	531	0,06	339	0,06	068	0,05	784	0,05	503	0,05	237	0,05	045	0,04	878	0,04	734	0,04	609	0,04	500
0,70	000	0,70	584	0,71	142	0,71	669	0,72	259	0,72	827	0,73	420	0,74	024	0,74	638	0,75	261	0,75	892	0,76	529	0,77	171	0,77	815	0,78	466	0,79	113	0,79	755	0,80	386	0,81	001	0,81	599	0,82	179	0,82	739	0,83	273
0,72	629	0,72	945	0,73	239	0,73	528	0,73	817	0,74	110	0,74	424	0,74	747	0,75	079	0,75	421	0,75	779	0,76	145	0,76	515	0,76	888	0,77	245	0,77	604	0,77	696	0,78	343	0,78	750	0,79	163	0,79	578	0,79	988	0,80	380
0,03	222	0,03	013	0,02	837	0,02	673	0,02	520	0,02	379	0,02	248	0,02	130	0,02	027	0,01	939	0,01	886	0,01	844	0,01	809	0,01	776	0,01	717	0,01	657	0,01	603	0,01	557	0,01	541	0,01	535	0,01	535	0,01	538	0,01	534
00'0	058	0,00	062	00'0	064	00'0	064	00'0	063	00'0	061	00'0	053	0,00	045	0,00	039	00'0	034	00'0	039	00'0	046	00'0	055	00'0	064	00'0	070	0,00	076	00'00	080	0,00	081	0,00	075	0,00	067	0,00	059	0,00	053	0,00	057
0,07	309	0,07	088	0,06	849	0,06	623	0,06	419	0,06	239	0,06	021	0,05	861	0,05	685	0,05	570	0,05	551	0,05	534	0,05	421	0,05	160	0,04	869	0,04	632	0,04	451	0,04	281	0,04	177	0,04	059	0,03	916	0,03	782	0,03	667
0,19	662	0,19	401	0,19	122	0,18	835	0,18	544	0,18	241	0,17	992	0,17	740	0,17	515	0,17	289	0,17	088	0,16	895	0,16	669	0,16	343	0,15	993	0,15	655	0,15	442	0,15	252	0,14	976	0,14	644	0,14	270	0,13	974	0,13	722
	679		680		681		682		683		684		685		686		687		688		689		069		691		692		693		694		695		969		697		698		669		700		701

00'0	309	00'0	322	0,00	315	0,00 313	00'0	327	00'0	332	00'0	316	0,00	324	00'0	321	00'0	309	0,00	314	0,00	304	00'0	305	00'0	301	00'0	308	0,00	314	0,00 294	00'0	296	00'0	297	00'0	667	00'0	794	0,00	00.0	311	00'0	306
00'0	269	0,00	267	00'00	265	0,00 273	00'0	252	00'0	270	00'00	247	00'00	272	00'00	253	00'0	258	00'00	269	00'00	253	0,00	240	0,00	241	00'00	259	00,00	239	0,00 258	00'0	251	00'00	243	0,00	248	0,00	867	0,00	00.00	247	0,00	262
00'0	114	00'0	121	0,00	112	0,00 102	00'0	117	00′0	108	00'0	103	0,00	118	00'0	087	00'0	111	0,00	105	0,00	123	00'0	118	00'0	104	0,00	107	0,00	160	0,00 074	00'0	102	0,00	105	0,00	094	0,00	TOT	0,00 105	0.00	094	00′0	118
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0,00	338	0,00	448	0,00	663	0,01 162	0,02	305	0,03	519	0,04	113	0,04	277	0,03	343	0,02	425	0,03	053	0,03	064	0,01	642	0,00	625	0,00	282	0,00	141	u,uu 248	00'0	102	0,00	173	0,00	202	0,00	677	0,00	00.0	188	00'0	203
00'0	547	0,00	787	0,01	190	0,02 025	0,04	080	0,06	191	0,07	064	0,07	375	0,05	690	0,04	140	0,05	304	0,05	317	0,02	813	0,01	017	00'0	462	0,00	383	u,uu 276	00'0	303	00'0	304	0,00	787	0,00	330	0,00 264	0.00	195	00'0	262
0,00	498	0,00	710	0,01	118	10,0 996	0,04	048	0,06	143	0,06	971	0,07	333	0,05	669	0,04	165	0,05	359	0,05	285	0,02	871	0,01	007	0,00	415	0,00	309	0,00 306	00'0	277	0,00	286	0,00	310	0,00	708	0,00	0.00	245	0,00	225
0,00	484	0,00	644	0,01	118	10,0 937	0,03	981	0,06	117	0,06	931	0,07	343	0,05	651	0,04	107	0,05	290	0,05	302	0,02	853	00'0	995	00'0	401	0,00	351	0,00 301	00'0	230	00'00	262	0,00	243	0,00	741	00,0	00.0	197	00'0	240
00'0	356	00'0	395	0,00	326	0,00 337	00'0	331	00'0	242	00'0	152	0,00	028	00'0	043	00'0	117	00'0	185	0,00	306	00'0	301	00'0	461	00'0	566	00'0	039	0,00 805	0,01	137	0,01	280	0,01	1/7	0,01	7/7	0,01 044	0.00	845	00′0	862
0,00	493	0,00	493	00'00	516	0,00 481	00'0	470	0,00	423	00'00	326	00'00	313	00'0	358	0,00	418	0,00	379	00'00	334	0,00	415	00'00	777	00'00	805	0,00	828	019 019	0,01	008	0,01	034	0,00	942	0,01	014	0,00 968	00.0	625	00'0	423
0,05	966	0,07	100	0,08	230	0,09 431	0,10	492	0,11	309	0,11	775	0,11	257	0,10	391	60'0	284	0,08	045	0,06	929	0,05	835	0,04	809	0,03	901	0,03	340	0,UZ	0,02	606	0,02	382	0,02	1/2	0,02 01F	CIU	0,01 904	0.01	830	0,01	776
0,02	669	0,03	357	0,04	038	0,04 770	0,05	428	0,05	950	0,06	273	0,06	050	0,05	621	0,05	038	0,04	355	0,03	663	0,02	962	0,02	290	0,01	686	0,01	319	043 U,UI	0,00	846	0,00	715	00'0	/79	0,00	6/c	0,00	0.00	560	0,00	548
0,04	402	0,04	311	0,04	222	0,04 131	0,04	035	0,03	927	0,03	806	0,03	633	0,03	450	0,03	265	0,03	088	0,02	949	0,02	826	0,02	719	0,02	628	0,02	505	0,UZ 511	0,02	467	0,02	428	0,02	3/8	0,02	329	0,02 779	0.02	229	0,02	167
0,83	782	0,84	265	0,84	720	0,85 129	0,85	513	0,85	876	0,86	221	0,86	567	0,86	006	0,87	218	0,87	521	0,87	813	0,88	087	0,88	338	0,88	564	0,88	/40	0,88 892	0,89	028	0,89	152	0,89	767	0,89	473	0,89 542	0.89	644	0,89	736
0,80	762	0,81	130	0,81	483	0,81 808	0,82	120	0,82	421	0,82	715	0,83	014	0,83	309	0,83	599	0,83	884	0,84	171	0,84	449	0,84	715	0,84	996	0,85	1/1 201	371 c8/U	0,85	560	0,85	747	0,85	757	0,86	тол	0,86 382	0.86	595	0,86	801
0,01	530	1,01	524	0,01	516	1,01 501	0,01	485	0,01	468	0,01	452	1,01	442	0,01	433	0,01	425	0,01	417	1,01	402	0,01	389	0,01	378	0,01	373	0,01	380	403	10,C	421	10,0	439	0,01	448	0,01	403	0,01 455	10.0	455	10,C	451
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0,00	267	0,00	254	00'0	253	00'0	275	00'0	264	00'0	266	00'0	282	00'0	264	0,00	281	00'0	265	00'0	279	00'0	299	00'0	306	00'0	310	00'0	314	00'0	273	0,00	323	0,00	309	0,00	321	0,00	335	0,00	331	0,00	320	00'0	342
00'0	094	00'0	960	00'0	124	00'0	121	00'0	106	00'0	079	00'0	094	00'0	107	00'0	118	00'0	126	00'0	093	00'0	660	00'0	115	00'0	660	00'0	093	00'0	126	00'0	106	00'0	092	00'0	119	00'0	113	00'0	077	00′0	114	00'0	100
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0,19	186	0,18	605	0,18	605	0,18	023	0,18	023	0,18	023	0,17	442	0,16	860	0,16	279	0,16	279	0,16	279	0,15	698	0,15	698	0,15	116	0,15	116	0,15	116	0,14	535	0,13	953	0,13	953	0,13	953	0,13	372	0,13	372	0,12	791
0,02	083	0,02	083	0,02	083	0,02	083	0,01	667	0,01	667	0,01	667	0,01	667	0,01	667	0,01	667	0,01	667	0,01	250	0,01	250	0,01	250	0,01	250	0,01	250	0,01	250	0,01	250	0,01	250	0,01	250	0,01	250	0,01	250	0,00	000
0,18	919	0,18	243	0,18	243	0,18	243	0,17	568	0,16	892	0,16	892	0,16	216	0,16	216	0,15	541	0,15	541	0,14	865	0,14	865	0,14	189	0,14	865	0,00	000	0,00	000	0,00	000	0,00	000	0,00	000	0,00	000	0,00	000	0,00	000
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0,16	702	0,16	067	0,15	431	0,15	114	0,14	478	0,14	478	0,14	161	0,13	843	0,13	208	0,12	572	0,11	937	0,12	438	0,11	619	0,11	302	0,10	666	0,10	349	0,10	349	0,11	302	0,15	431	0,10	031	0,09	396	0,09	078	0,08	693
0,03	389	0,03	389	0,03	389	0,03	389	0,03	389	0,03	291	0,03	291	0,03	291	0,03	291	0,03	194	0,03	291	0,02	843	0,02	843	0,02	941	0,02	941	0,02	843	0,02	746	0,02	746	0,03	389	0,03	972	0,03	583	0,02	746	0,02	746
00'0	227	0,00	112	00'0	189	00'00	491	00'0	788	00'0	621	00'0	308	00'0	245	00'0	235	00'0	034	00'0	053	00'00	100	00'0	167	00'0	159	00'0	048	0,00	120	00'00	183	00'0	208	0,00	168	0,00	123	0,00	120	0,00	112	0,00	100
00'0	268	0,00	267	00'0	235	00′0	595	0,01	124	00'0	864	00'0	471	00'0	336	00'0	228	00'0	133	00'0	165	00′0	168	00'0	143	00′0	165	00′0	223	0,00	208	00'0	311	00'0	373	0,00	244	0,00	208	0,00	193	0,00	220	0,00	081
0,00	250	0,00	197	00'0	264	0,00	538	0,00	889	00'0	756	00'0	464	00'0	356	0,00	167	0,00	175	0,00	094	0,00	055	0,00	211	0,00	160	0,00	171	0,00	151	0,00	288	0,00	286	0,00	281	0,00	134	0,00	172	0,00	139	0,00	172
0,00	195	0,00	207	0,00	261	0,00	507	0,00	801	0,00	720	0,00	431	0,00	332	0,00	194	0,00	129	0,00	060	0,00	176	0,00	163	0,00	203	0,00	172	00'00	253	0,00	298	0,00	324	0,00	210	0,00	147	0,00	146	00'0	160	00'00	154
0,01	032	0,01	146	00'0	958	00'0	894	00'0	845	00'0	768	00'0	661	00'0	525	0,00	518	0,00	299	00'0	191	00'0	052	00'0	036	00'0	016	00'0	000	00'0	080	0,00	323	0,00	084	00'0	133	00'0	032	00'0	900	00′0	000	00'0	025
0,00	492	0,00	451	0,00	397	0,00	407	0,00	391	0,00	382	0,00	337	0,00	265	0,00	326	0,00	229	0,00	211	0,00	128	0,00	234	0,00	202	0,00	102	00'0	168	0,00	289	0,00	205	0,00	249	0,00	232	00'0	263	00'0	144	00'0	210
0,01	750	0,01	748	0,01	769	0,01	831	0,01	901	0,01	968	0,02	019	0,01	992	0,01	948	0,01	895	0,01	842	0,01	816	0,01	801	0,01	799	0,01	813	0,01	871	0,01	937	0,02	002	0,02	059	0,02	071	0,02	066	0,02	048	0,02	016
0,00	540	0,00	532	00'0	522	0,00	498	0,00	467	00'0	429	00'0	385	0,00	316	0,00	247	0,00	184	0,00	134	0,00	128	0,00	138	0,00	162	0,00	195	0,00	233	0,00	276	0,00	322	0,00	369	0,00	408	0,00	447	0,00	486	0,00	525
0,02	109	0,02	090	0,02	025	0,02	041	0,02	067	0,02	960	0,02	120	0,02	114	0,02	094	0,02	056	0,01	666	0,01	888	0,01	768	0,01	649	0,01	543	0,01	499	0,01	474	0,01	462	0,01	459	0,01	440	0,01	427	0,01	424	0,01	433
0,89	801	0,89	834	0,89	827	0,89	735	0,89	605	0,89	447	0,89	270	0,89	064	0,88	863	0,88	686	0,88	548	0,88	526	0,88	552	0,88	621	0,88	726	0,88	843	0,88	988	0,89	163	0,89	365	0,89	599	0,89	860	06'0	149	06'0	464
0,87	005	0,87	205	0,87	402	0,87	594	0,87	783	0,87	970	0,88	155	0,88	321	0,88	495	0,88	682	0,88	891	0,89	174	0,89	476	0,89	786	06'0	092	06'0	339	06'0	580	06'0	822	0,91	073	0,91	372	0,91	684	0,92	002	0,92	322
0,01	444	0,01	434	0,01	422	0,01	405	0,01	385	0,01	365	0,01	344	0,01	324	0,01	305	0,01	287	0,01	271	0,01	255	0,01	243	0,01	237	0,01	239	0,01	273	0,01	309	0,01	342	0,01	363	0,01	322	0,01	273	0,01	226	0,01	192
0,00	125	0,00	132	00'0	137	0,00	132	00'0	125	00'0	117	00'0	109	0,00	106	0,00	104	0,00	102	00'00	101	0,00	093	0,00	087	0,00	085	0,00	087	0,00	110	0,00	135	0,00	160	0,00	181	0,00	179	00'0	171	00'0	158	0,00	143
0,01	878	0,01	835	0,01	794	0,01	795	0,01	733	0,01	700	0,01	652	0,01	640	0,01	590	0,01	548	0,01	566	0,01	535	0,01	545	0,01	520	0,01	456	0,01	446	0,01	391	0,01	401	0,01	363	0,01	377	0,01	332	0,01	303	0,01	334
0,08	909	0,08	464	0,08	307	0,08	129	0,07	942	0,07	791	0,07	577	0,07	364	0,07	301	0,07	171	0,07	174	0,07	325	0,07	364	0,07	233	0,06	963	0,06	527	0,06	278	0,06	100	0,06	002	0,05	851	0,05	697	0,05	584	0,05	679
	726		727		728		729		730		731		732		733		734		735		736		737		738		739		740		741		742		743		744		745		746		747		748

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0,00	324	0,00	320	0,00	352	0,00	0,00	340	0,00	381	0,00	388	0,00	408	0,00	399	0,00	418	0,00	391	0,00	431	0,00	388	0,00	421	0,00	436	0,00	457	u,uu 466	0,00	472	0,00	454	0,00	460	0,00	489	0,00		458	0,00	481
00'0	123	00'0	166	00'0	117	0,00 104	00'0	120	00'0	120	00'0	109	00'0	117	00'0	142	00'0	143	00'0	165	00'0	111	00'0	100	00'0	147	00'0	118	0,00	181	0,00 118	00'0	116	00'0	178	00'00	120	00'0	120	0,00 185		170	00'0	112
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0,01	918	0,01	832	0,01	784	0,01 798	0,02	051	0,02	353	0,02	664	0,02	948	0,03	067	0,03	122	0,03	114	0,03	045	0,02	841	0,02	608	0,02	376	0,02	0/T	171 171	0,02	206	0,02	260	0,02	309	0,02	247	0,02		089	0,02	015
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0,01	430	0,01	456	0,01	525	0,01	0,01	984	0,02	349	0,02	705	0,03	014	0,03	127	0,03	157	0,03	108	0,02	986	0,02	727	0,02	430	0,02	126	0,01	84/	10,U 727	0,01	653	0,01	615	0,01	601	0,01	586	0,01	50	587	0,01	596
06'0	808	0,91	175	0,91	564	0,91 973	0,92	408	0,92	856	0,93	311	0,93	768	0,94	213	0,94	653	0,95	083	0,95	502	0,95	904	0,96	291	0,96	662	0,97	at n	0,97 349	0,97	663	0,97	958	0,98	233	0,98	484	0,98 715	800	927	66'0	120
0,92	627	0,92	930	0,93	230	0,93	0,93	819	0,94	109	0,94	401	0,94	969	0,95	900	0,95	320	0,95	636	0,95	953	0,96	273	0,96	588	0,96	894	0,97	/81	0,97 452	0,97	700	0,97	932	0,98	150	0,98	348	0,98 534		709	0,98	875
0,01	238	0,01	295	0,01	348	0,01 385	0,01	317	0,01	237	0,01	163	0,01	112	0,01	172	0,01	261	0,01	366	0,01	476	0,01	545	0,01	608	0,01	667	0,01	c7/	10,0 817	0,01	896	0,01	950	0,01	965	0,01	850	0,01	500	544	0,01	389
0,00	127	0,00	111	0,00	098	0,00	0,00	084	00'0	086	00'0	860	0,00	122	0,00	168	0,00	226	0,00	296	0,00	377	0,00	475	0,00	578	0,00	682	0,00	787	0,00 894	00'0	986	0,01	045	0,01	058	00'00	930	0,00	80	586	00'0	414
0,01	365	0,01	329	0,01	373	0,01 347	0,01	254	0,01	174	0,01	169	0,01	260	0,01	609	0,02	417	0,03	272	0,03	497	0,03	243	0,02	650	0,01	910	0,01	489	0,UI 336	0,01	296	0,01	359	0,01	529	0,01	600	0,01 538	500	436	0,01	261
0,06	035	0,06	400	0,06	406	0,06 213	0,05	733	0,05	130	0,04	840	0,04	674	0,04	588	0,04	502	0,04	404	0,04	345	0,04	582	0,05	311	0,05	738	0,05	/72	347	0,04	463	0,03	977	0,03	713	0,03	645	0,03		051	0,04	179
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00'0	552	00'0	593	00'0	511	00'0	551	00'0	600	00'0	592	00'0	576	00'0	613	
0,00	526	0,00	517	00'0	530	0,00	512	0,00	546	0,00	546	00'0	549	0,00	589	
00'0	113	0,00	159	00'0	171	00'0	134	00'0	133	00'0	131	00'0	156	00'0	152	
0,0000	0	0,0000	0	0,0000	0	0,0000	0	0,0000	0	0,0000	0	0,0000	0	0,0000	0	
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0,03	745	0,03	745	0,03	745	0,03	745	0,03	745	0,12	640	0,04	698	0,03	109	
0,02	396	0,02	045	0,02	045	0,02	045	0,02	045	0,02	045	0,02	396	0,03	583	
0,00	000	0,00	000	00'0	000	0,00	000	00'0	000	00'0	000	00'00	000	00'0	000	
0,00	000	0,00	000	00'0	000	0,00	000	00'0	000	00'0	000	0,00	000	00'0	000	
0,00	000	0,00	000	0,00	000	0,00	000	0,00	000	0,00	000	0,00	000	0,00	000	
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0,00	595	0,00	443	00'0	368	0,00	411	0,00	591	0,00	675	00'0	475	0,00	398	
0,01	998	0,01	995	0,02	002	0,02	014	0,02	018	0,02	022	0,02	027	0,02	032	
0,00	717	0,00	709	00'0	695	00′0	676	00'0	658	00'0	637	00'0	615	00'0	592	
0,01	601	0,01	604	0,01	604	0,01	598	0,01	575	0,01	546	0,01	512	0,01	477	
0,99	297	0,99	455	0,99	595	0,99	715	0,99	803	0,99	877	0,99	941	1,00	000	
0,99	039	0,99	196	0,99	347	0,99	490	0,99	624	0,99	752	0,99	877	1,00	000	
0,01	304	0,01	244	0,01	207	0,01	194	0,01	219	0,01	261	0,01	315	0,01	375	
0,00	334	0,00	278	0,00	243	0,00	226	0,00	219	0,00	223	0,00	233	0,00	246	
0,01	225	0,01	179	0,01	155	0,01	106	0,01	148	0,01	092	0,01	120	0,01	139	
0,04	173	0,03	953	0,03	541	0,03	238	0,03	126	0,03	084	0,02	995	0,02	849	
	773		774		775		776		777		778		779		780	

D Ethical Considerations

- D.1 Information sheet for new experiment
- D.2 Participant consent form of new experiment

D.1 Information sheet for new experiment

Information Sheet

1. Research Project: Lamp spectrum and spatial brightness

You are being invited to take part in a research project. Before you decide it is important for you to understand why the research is being done and what it will involve. Please take time to read the following information and discuss it with others if you wish. Ask us if there is anything that is not clear or if you would like more information. Take time to decide whether or not you wish to take part.

2. What is the project's purpose?

This phd project will investigate different types of electric lamp that are used for interior lighting. The lamps differ in the spectrum or colour of the light that they produce. The research will examine some of the visual effects caused by these differences, especially the effects on the perception of brightness. We aim to gather evidence that can be used to improve the quality of interior lighting.

3. Why have I been invited to participate?

We are looking for the participation of a diverse group of people in order to identify average judgements. There are just a few personal requirements: minimum age of 18 years, normal colour vision, and no serious visual disabilities.

4. Do I have to take part?

Taking part in this research is entirely voluntary. If you do decide to take part you will be given this information sheet to keep if you wish and be asked to sign a consent form. You can still withdraw at any time without giving a reason.

4. What will I have to do if I take part?

You will need to come to the lighting laboratory on the 19th floor of the University Arts Tower on up to six separate occasions over the space of a few days or weeks depending on other commitments. Each session will take about two hours. The lighting tests require judgements of brightness. This is done by comparing two sources of light and then adjusting a dimmer switch to match their brightnesses. You will also be asked to give some verbal responses, for example to express a preference for one of two separate lighting conditions.

None of the main tests is designed to measure the state of a person's eyesight because the focus of this research is the quality of the lighting. Although participation in this research is not thought to be difficult the sessions can be quite repetitious and so require a reasonable amount of patience.

5. What are the possible disadvantages and risks of taking part?

The lighting conditions used for this research are similar to those found under normal interior lighting. This means orange or white illuminations and normal daylight levels. In the unlikely event that you experience any discomfort you can stop the procedure. There will be no flashing lights. On your first visit there will be a brief test for normal colour vision. There is only a small chance that this simple test could reveal a colour vision abnormality that a person was previously unaware of.

6. What are the possible benefits of taking part?

Whilst there are no immediate benefits for participants in the research, it is hoped that the knowledge gained will contribute to changes in interior lighting that improve visibility for users while consuming less electrical energy.

7. What if something goes wrong?

If you are unhappy with the way you have been treated, or with anything that has happened during or following your participation, then please contact Dr. Steve Fotios (Tel. 0114 2220371) who is leading the project. If you feel your complaint has not been dealt with satisfactorily then please contact the University's Registrar and Secretary (Tel. 0114 2220399).

8. Will my taking part in this project be kept confidential?

The limited amount of personal information that we collect for the project will be kept strictly confidential. Your test results will contribute to the average results for a group of participants and will not be analysed individually. No person will be identified in any reports or publications.

9. What will happen to the results of the research project?

The results of this research are likely to be published in lighting journals and presented at lighting conferences in the two years following your participation as well as being published in a PhD.

10. Who is organizing and funding the research?

This research is being carried out within the School of Architecture at the University of Sheffield. I am an independently funded student.

11. Who has ethically reviewed the project?

This project has been ethically approved via the School of Architecture's ethics review procedure. The University's Research Ethics Committee monitors the application and delivery of the University's Ethics Review Procedure across the University.

12. Contact for further information

Deniz Atli (PhD student) School of Architecture University of Sheffield Telephone: 07979926640 Email: d.atli@sheffield.ac.uk

Prof. Steve Fotios School of Architecture University of Sheffield Telephone: 0114 2220371 Email: steve.fotios@sheffield.ac.uk

D.2 Participant consent form of new experiment

Participant Consent Form

[personal information will be kept strictly confidential]

Titl	e of Project: <u>Lamp S</u> (An invest	Spectrum and Spatial Brig igation of light sources for	<u>htness</u> interior lighting)
			Please tick box
1.	I have read the information s and have had the chance to	sheet for the above study ask questions.	
2.	I understand that my particip free to withdraw at any time <u>d.atli@sheffield.ac.uk</u> , tel:07	bation is voluntary and tha without giving a reason (C '979926640).	t I am Contact: Deniz Atli,
3.	I understand that my respon I give permission for member anonymised responses. I un the research materials, and report or reports that result f	ises will be kept strictly con ers of the research team to iderstand that my name w I will not be identified or id from the research.	nfidential. have access to my ill not be linked with entifiable in the
4.	I agree for the data collecte	ed from me to be used in f	uture research.
Yo Thi	ur signature will certify that yo ank you.	ou have voluntarily decided	d to participate in this study.
			<u> </u>
Na	me of Participant	Date	Signature
	Deniz ATLI		
Re	esearcher	Date	Signature

E Analysing the results of the experiment

E.1 Tabulating normality profile

E.1 Tabulating normality profile

		AA-A high Achromatic	AA-A low Achromatic	AA-A high Chromatic	AA-A low_	Chromatic
Ocurtural	Maar	0.00	-	1.00	1.01	1.00
Tendency	Mean	0.98	1.02	1.00	1.01	1.03
	Median	0.98	1.02	1.01	1.00	1.04
NORMALITY?		Normal	Normal	Normal	Normal	Normal
Graphical	Histogram	\checkmark	Х	Х	\checkmark	
	Box Plot	\checkmark	Near	\checkmark	X + 2 outliers*	X + 1 outlier
	Q-Q plot	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark
NORMALITY?		Normal	Near	Normal	Normal	Normal
Measures of	Skewness	-0.057	0.075	0.185	-0.564	0.762
aispersion	(within ±0.5)					
	Kurtosis	-0.330	-1.322	-0.369	1.870	0.678
	(within ±1.0)					
NORMALITY?		Normal	Near	Normal	Not Normal	Near
Statistical tests						
Shapiro-Wilks	statistic	0.992	0.073	0.683	0.120	0.138
	level of significance					
Kolmogorov- Smirnov	statistic	0.200	0.111	0.200	0.033	0.044
	level of significance					
NORMALITY?		Normal	Normal	Normal	Near	Near
OVERALL ASS OF NORMALIT	ESSMENT Y	NORMAL	NORMAL	NORMAL	NORMAL	NORMAL

Table E.1 Normality profile for null-condition of matching test in new experiment

* Data Number:8,9