Development of a Combined Activity Scheduling Model for Tours

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Intellectual Property and Publication Statement

The candidate confirms that the work submitted is his own, except where work which has formed part of jointly-authored publications has been included. The contribution of the candidate and the author to this work has been explicitly indicated below. The candidate confirms that appropriate credit has been given within the thesis where reference has been made to the work of others.

During the course of this research two publications were produced, one of them was jointly-authored with the research supervisors. The candidate was the leading author in all publications and his contribution was significant. Chapter 6 partially includes the work reported in Adnan et al (2009). This paper is titled as "*Model for integrating home-work tour scheduling with time-varying network congestion and marginal utility profiles for home and work activities*", and have appeared in the Transportation Research Record, Journal of the Transportation Research Board, No. 2134, pp: 21-30. The candidate contribution was significant in carrying out the reported research presented in this paper, which includes; development of a generalised model for scheduling of the home-work tour, numerical and analytical illustration of the property of the model when time-of-day preference are considered, and implementation of the model for the numerical experiments. The other two authors were contributed in a way that they helped candidate to further refine the candidate original thoughts regarding the developed model. They have also provided necessary guidance to better formulate the content and structure of the paper.

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ABSTRACT

The relationship between travel growth, increased congestion and effectiveness of traffic management measures can be better understood by examination of change in people's travel patterns due to congestion and its mitigation policies. The studies suggested that combined models are vital to accurately foresee the impact of policies on travel behaviour, as they integrate the effect of congestion on the scheduling of activities through feedback mechanism. Models within the Activity-based approach predict an individual activity-agenda and its schedule but they lack in representing congestion as an endogenous variable. In contrast, combined models are limited as they tend to incorporate fewer scheduling dimensions for a part of the activity-travel pattern (e.g. home to work trip). Based on this, the primary objective of this thesis is to contribute towards improvements and extensions of the existing combined models.

This thesis presented a combined model that integrates the modelling of activity scheduling dimensions (for daily and weekly activity-travel patterns) with the dynamic representation of congestion under the framework of the fixed point problem. Modelled scheduling dimensions include: departure time, activity duration, activity sequence and route choice. The essential aspect of the model is based on the trade-off between the utility of participating in various activities, which contain time-of-day preference and satiation effects, and the disutility of travel. The development process presented for the model is generalised and it can accommodate any operational model within the demand and supply sides. However, the model application in this thesis is limited to the simplified network which can be extended for a real network by following the notions of model development. A variety of numerical experiments were performed in order to assess the model working and the implications of a range of policies. Results obtained from all the numerical experiments are plausible and these are explained well in the thesis.

Contents

Tables

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Abbreviations

AB:	Activity-Based
SEM:	Structural Equation Models
CPM:	Computational Process Models
MNL:	Multinomial Logit
NL:	Nested Logit
PAT:	Preferred Arrival Time
TDM:	Travel Demand Management
ALBATROSS:	A Learning-based Transportation Oriented Simulation System
AMOS:	Activity-Mobility Simulator
BB:	Bowman and Ben-Akiva, Day Activity Schedule Model System
CEMDAP:	Comprehensive Econometric Microsimulation of Daily Activity Pattern
PCATS:	Prism-Constrained Activity-Travel Simulator
PETRA:	Danish Activity based travel demand model by Mogens Fosgerau
SCHEDULER:	known also as Scheduler
SMASH:	Simulation Model of Activity Scheduling Heuristics
STARCHILD:	Simulation of Travel / Activity Responses to Complex Household Interactive Logistic Decisions
STGP:	Synthetic Travel Pattern Generator
TA:	The Tel-Aviv Activity based Model System
TASHA:	Toronto Area Scheduling Model with Household Agents
TRANSIMS:	TRansporation ANalysis and SIMulation System
VISTA:	Visual Interactive System for Transportation Algorithm
SDF:	Schedule delay formulation
MUF:	Marginal utility formulation
SDE:	Schedule delay early
SDL:	Schedule delay late
LP:	Fixed late penalty
OGEV:	Ordered generalised extreme value
MNP:	Multinomial probit
IID:	Independently and identically distributed
IIA:	Independence of irrelevant alternatives

DTA:	Dynamic traffic assignment
FP:	Fixed point
SDUE:	Stochastic dynamic user equilibrium
SQP:	Sequential quadratic programming
BFGS:	Broyden-Fletcher-Goldfarb-Shanno
MSA:	Method of Successive averages
MATLAB:	Matrix Laboratory, Scientific language package
ATIS :	Advanced Traveller Information System
DNL:	Dynamic network loading
PST:	Preferred start time

Notations

a	Value of time parameter having units as utils/minute
$\mathrm{m_e}$	Early arrival penalty parameter having units as utils/minute
$\mathrm{m_l}$	Late arrival penalty parameter having units as utils/minute
M	Fixed Late arrival Penalty parameter having units as utils
$R(t)$	Travel time at time *t* having a unit as minutes
t_w	Time window across both side of *PAT* with a unit as minutes
α	Represents maximum utility point of the marginal utility curve for a particular activity on the time axis with units as minutes
β	Represents steepness of the marginal utility curve for a particular activity around maximum utility point with a unit of 1/minutes
γ	Represents skew ness of the marginal utility curve for a particular activity
$U^{\max}$	Area under the marginal utility curve in utils
h_0	Represents marginal utility in order to reverse the bell shaped marginal utility curve with a units as Utils/minutes
a	Marginal utility at time of day t_a for a particular activity with a units as Utils/minute
b	Marginal utility at time of day t_b for a particular activity with a units as Utils/minute
c	Marginal utility at time of day t_c for a particular activity with a units as Utils/minute
t_{at}	Arrival time at work activity location in minutes past midnight
t_d	Departure time corresponding to arrival time t_{at} in minutes past midnight
$V_h{}'(t)$	Marginal utility at time of day *t* for home activity in Utils/minute
$V_w{}'(t)$	Marginal utility at time of day *t* for work activity in Utils/minute
$V_a{}'(\tau_a)$	Marginal utility at time of day *t* for a particular activity in Utils/minute
η_a	Scaling parameter representing utility individual getting at the first minute of an activity in Utils
τ_a	Activity duration in minutes
$P_n(i)$	Probability for an individual *n* choosing alternative *i*
V_{in}	Systematic utility for an individual *n* choosing alternative *i* in utils
σ^2	Variance of random error term in utility expression with units as utils^2
μ	Scaling parameter related as a inverse of a variance of random error term in utility expression in 1/utils
μ_d	Scaling parameter in utility expression, representing for destination alternatives in 1/utils
μ_m	Scaling parameter in utility expression representing for different modes in

	1/utils
$U(d,m)$	Utility of an individual for choosing destination *d* and mode *m* in utils
V_d	Component of a systematic utility that represent variables specific only for choice of destination *d* in utils
V_{dm}	Component of a systematic utility that represents variables that combinely effect choice of destination and mode in utils
V_d^*	Log sum variable in utils
ΔCS_n	Change in individual n consumer surplus with units as £ or $
R	Revenues from policy in £ or $
ΔW	Change in socio-economic benefits in £ or $
κ	Marginal utility of income, in utils/£ or utils/$
$e(t)$	Inflow rate at time *t* in vehicles/minute
$o(\varphi(t))$	Outflow rate at time $\varphi(t)$ in vehicles/minute
$\varphi(t)$	Exit time of a vehicle who entered into the link at time *t* in minutes
$\dot{\varphi}(t)$	Rate of change of exit time with respect to time *t*
ϕ	Free flow travel time in minutes
C	Outflow capacity of the link in vehicles/minute
$z(t)$	Number of vehicles at the end of the link
$x_2(t+\phi_1)$	Number of vehicles in the second part of the link at time $(t+\phi_1)$
ϕ_1	Free flow travel time for the first part of the link in minutes
ϕ_2	Free flow travel time for the second part of the link in minutes
δ	Time increment for supply side model implementation (minutes)
L_1	Inflow limit which first causes the start of the queue at the end of the link in vehicles/minute
L_2	Inflow limit which first causes outflow rate equals Capactiy of the link in vehicles/minute
n	Parameter of the Adnan-Fowkes model
t_i	Time of day, discretised with subscript *i*
x^*	Vector containing the solution flow rates
$F(\mathrm{x})$	Function of vector flow rates
$G(\mathbf{x})$	Gap function, a scalar quantity with units as (Vehicles/minute)^2
ε	Error in solution of fixed point problem, a scalar quantity with units as (Vehicles/minute)^2
s_n	Step size of a solution algorithm at *n*th iteration
$\mathbf{H}_n^{-1}$	Inverse of a Hessian matrix at nth iteration
$\nabla G(\mathbf{x_n})$	Gradient of a gap function with respect to x at nth iteration

τ_w	Duration of a work activity in minutes
τ_h	Duration of a home activity in minutes
i	Departure time from home to work in the morning in minutes past midnight
j	Deprture time from work to home in the evening in minutes past midnight
Δ	Duration of a departure period in minutes
D	First departure period time for the morning commute in minutes past midnight
Y	First departure period time for the evening commute n minutes past midnight
$\mathbb{K}$	Set of combination of morning and evening departure periods available to an individual for its decision making
V_{ij}	Systematic utility for choosing *i*th departure period for the morning commute and *j*th departure period for the evening commute in utils
R_i	Travel time at time *i* in minutes
R_j	Travel time at time *j* in minutes
P_{ij}	Probability of choosing *i*th departure period for the morning commute and *j*th departure period for the evening commute
$\underline{\omega}$	Vector of parameters required for using NL model
Q	Number of individuals or vehicles
q_{ij}	Individuals who have chosen *i*th departure period for the morning commute and *j*th departure period for the evening commute
q_i	Individuals who have chosen *i*th departure period
q_j	Individuals who have chosen *j*th departure period
ψ	Inverse of capacity of the link in minutes/vehicle
$\zeta(\Theta)$	Function representing number of vehicles on the link or number of vehicles in the queue at the end of link at respective time for respective DNL models
R_M	Vector containing R_i as its elements
$\mathbf{R}_E$	Vector containing R_j as its elements
$\mathbf{P}$	Matrix containing P_{ij} as its elements
$\mathbf{V}$	Matrix containing V_{ij} as its elements
$\mathbf{q}$	Matrix containing q_{ij} as its elements
$\mathbf{R}$	Vector containing R_M and $\mathbf{R}_E$ as its elements
λ	In-vehicle travel disutility parameter in utils/minute
η_w	Scaling parameter representing utility individual getting at the first minute

	of a work activity in utils
r^1	Route for individuals performing home-work tour
k	Departure time from an additional activity location to home or work
s	Sequence of performing three activity tour
l	Representing link in a route
r^2	Route for individuals performing three-activity tour
V_{ijr^1}	Systematic utility for choosing *i*th departure period from home, *j*th from work and route r^1 for travelling between home and work activity locations in utils
$R^{hw}_{ir^1}$	Travel time at time i for route r^1 when going from home activity location to work activity location in minutes
$R^{wh}_{jr^1}$	Travel time at time j for route r^1 when going from work activity location to home activity location in minutes
$\xi^{hw}_{lr^1}$	Link-route indicator variable, 1 when link l is a part of route r^1 when going from home activity location to work activity location otherwise 0
$\xi^{wh}_{lr^1}$	Link-route indicator variable, 1 when link l is a part of route r^1 when going from work activity location to home activity location otherwise 0
R_{il}	Travel time on link l at time i in minutes
Φ	Functional parameter which relates travel times with systematic utility of conducting home-work tour
Q_1	Number of individuals performing home-work tour
P_{ijr^1}	Probability for choosing *i*th departure period from home, *j*th from work and route r^1 for travelling between home and work activity locations
V_{ijksr^2}	Systematic utility for choosing *i*th departure period from home, *j*th from work, *k*th from additional activity location, when performing a three activity tour with a sequence s using route r^2 for travelling between three activity locations in utils
Ω	Functional parameter which relates travel times with systematic utility of conducting three activity tour
R_{isr^2}	Travel time at time i for route r^2 with a sequence s in minutes
Ψ	Functional parameter that forms fixed point formulation for two user's class problem in which one performing home-work tour and other performing three-activity tour
$\hat{\mathbf{Q}}$	Matrix containing elements q_{ijr^1} and q_{ijksr^2}
$\hat{\mathbf{R}}$	Matrix containing elements R_{il}, R_{jl} and R_{kl}
q_{ijr^1}	Number of individuals who have chosen *i*th departure period for leaving from home, *j*th departure period for leaving from work and with a route r^1
q_{ijksr^2}	Number of individuals who have chosen *i*th departure period for leaving from home, *j*th departure period for leaving from work and *k*th departure

	period for leaving from third activity with a sequence s and route r^2
V_{tw}	Systematic utility for tele-work alternative in utils
T_w	Representing extent of utility an individual obtained by choosing a tele-work alternative in utils
τ_w^{fxd}	Fixed duration of work activity in minutes
V_{td}	Systematic utility for an individual for typical days of the week in utils
V_{atd}	Systematic utility for an individual for an atypical day of the week in utils
$(\tau_{wr^1})_t$	Duration of work activity of an individual with route r^1 on typical days of the week in minutes
$(\tau_{wr^1})_a$	Duration of work activity of an individual with route r^1 on an atypical day of the week in minutes
$\hat{\mathrm{Q}}_{\mathrm{week}}$	Matrix containing elements q_{ijr^1} and q_{ijksr^2} for weekly activity scheduling problem
$\hat{\mathrm{R}}_{\mathrm{week}}$	Matrix containing elements R_{il}, R_{jl} and R_{kl} for weekly activity scheduling problem

Chapter 1

INTRODUCTION

1.1 MOTIVATION

The relationship between travel growth, increased congestion and effectiveness of traffic management measures can be better understood by examining how people actually change their travel patterns in order to cope with congestion and policies (e.g increased travel costs, reduced parking spaces and flexible working hours etc) that are implemented to increase transport efficiency. Empirical studies (Small 1982, Kitamura et al 1997, Ettema and Timmermans 2003, Ye et al 2007) suggested that individuals change their activity schedules in response to these policies (e.g. adjustment in departure times, activity durations, adjustments in sequencing their activities, change in their mode and route choice etc) and also instead of performing shorter and simpler tours (e.g. tours consist of two activities) they tend to chain their activities in more complex activity patterns. This suggests that models which integrate the effect of congestion on the scheduling of activities through a feedback mechanism (i.e. combined models) are vital to accurately foresee the impact of congestion management policies on travel behaviour.

Within the transport modelling literature, sophisticated models are presented that model the complete activity-travel pattern of an individual within an Activity-Based (AB) approach (Bowman and Ben-Akiva 2000, Arentze and Timmermans 2004, Bhat et al 2004, Shiftan et al 2004). These models examined the interaction between household members in order to form an individual daily activity agenda and also to model different scheduling dimensions of daily activity patterns, treating level of service (network performance indicator) as an exogenous variable. On the other hand, continuous research in the transport network assignment area has delivered analytical and micro simulation models in which traffic on the network can be assigned dynamically with incorporation of departure time and route choice as scheduling dimensions (Peeta and Ziliaskopoulos 2001, Lam and Huang 2002, Heydecker and Addison 2005). The premise of existing traffic assignment models is still based on the trip-based approach, i.e. these models cannot integrate morning and evening commute together let alone the complete activity pattern of individuals. Integration of AB models with traffic assignment models in a unified framework seems natural and inevitable as foreseen by prominent researchers in transport modelling (Ben-Akiva et al 2008, Vovsha 2009). Pursuing the same line of action, the focus of this thesis is to form the basis for formulation of a combined analytical model (that integrate scheduling of individual's activity-travel pattern with representation of network congestion) though on a limited scale at this point in time. The analytical modelling approach, in which average behaviour of population is modelled, is followed in this thesis in order to exploit the advantages it offers such as less data requirements, estimation of fewer parameters, mathematical tractability of the models and faster run times. Furthermore, the analytical approach seems more appropriate as the basic goal of the combined models is to provide an assessment of broader and long term policies of congestion management which require examination of average behaviour of population.

Modelling literature which focuses on analytical combined models is very limited. The models presented so far are based on the scheduling of the morning commute and having the choice of departure time, route and duration at the intermediate stops as modelling dimensions with representation of congestion (Abdelghany and Mahmassani 2003, Lam and Huang 2002). These models can be taken as an extension of the seminal work of Vickrey (1969). Fewer efforts, such as Zhang et al (2005) and Heydecker and Polak (2006) have also been presented that incorporate the entire day activity pattern (home-work tour) of individuals with departure time choice and activity duration as the scheduling dimensions. This thesis examines the issues within combined modelling, and based on understanding these issues, a model is presented that incorporates scheduling of daily activity-travel patterns of individuals with more scheduling dimensions (departure times, activity duration, activity sequencing, route) while maintaining the dynamic representation of congestion on the road network. Furthermore, in this thesis a model is presented that attempts to represent the *weekly* scheduling of activities in a combined modelling framework. This is important because substantial evidence exists which suggests that individuals vary their activity-travel pattern on day-to-day basis. Therefore, the results from the models which are based on the *daily* notion may be misleading in this circumstance.

1.2 RESEARCH OBJECTIVES

The aim of this thesis is to model and analyse the equilibrium between the benefits gained by participation in activities and losses incurred during the resulting travel (travel in this thesis is considered as an activity which renders disutility). This is developed within a framework that not only enables the model to explicitly capture the impact of congestion management schemes and policies on different scheduling dimensions of individual activity-travel patterns but transfers the effects of changed activity-travel pattern onto network performance indicators. It is obvious that the development of this model requires comprehensive efforts and a longer period of study duration; therefore, this research is focused on a generalised model that can potentially be extended through future endeavours. The major objectives formulated for this research are as follows

(1) To establish a state of the art review of activity scheduling models, relevant issues and modelling considerations within the combined modelling framework.

(2) To develop a combined activity scheduling model that embodies a simple daily activity-travel pattern with dynamic traffic assignment over a simplified network in a generalised manner that can be easily extendable.

(3) To carry out a variety of numerical experiments in order to investigate functionality of the model and to suggest potential arenas for meaningful extensions of the developed model in (2).

(4) To systematically extend the framework of the developed model to represent weekly scheduling of activities which is in line with (2) and (3) and incorporate more activity scheduling dimensions.

(5) To conduct numerical experiments to show working of the extended model and demonstrate the implications of a congestion mitigation policy.

1.3 SCOPE AND LIMITATIONS OF RESEARCH

The scope and limitations of this research are derived from the level of complexity of the research problem at hand. Furthermore, the time period available to conduct the research also plays a vital role in conjunction with problem complexity. The theme of this research spans many sub-topics of transport modelling e.g. Demand side models, Supply side models and optimisation of the combined problem, therefore, it is required to carefully devised a strategy to accomplish the objectives of the research to a best degree of satisfaction.

The development of a comprehensive activity scheduling model with a dynamic representation of network congestion, which represents all the scheduling dimensions of the activity-travel pattern, is obviously too ambitious for this limited period of research. Therefore, the choices of mode and location are not considered in this research. Also, individual activity-travel patterns are considered which comprise home-based tours involving only three activities i.e. home, work and shopping or leisure. Additionally, it is assumed that individuals have prior information regarding the daily agenda of their activities i.e. which activities they will perform in a given day. The model is constructed in a manner that all the choice decisions are made at one point in time and prior to the execution of the activity-travel pattern. Despite all these limitations, it should be noted that the model in this research is devised in a generalised manner and it would be easily extendable to a variety of dimensions.

1.4 THESIS PLAN

The following paragraphs along with figure 1.1 illustrate the thesis composition in different chapters along with their brief description. These paragraphs also mention the methodology adopted for reporting various aspects of the research.

Chapter 2 provides a detailed review of activity scheduling modelling approaches. Models based on activity-based (AB) approach are reviewed on the basis of scheduling dimensions they incorporate and the principles on which individuals are taking decisions. A further review of models was carried out for combined models, in which scheduling of the morning commute is modelled with network congestion. A smaller number of modelling attempts based on the scheduling of a simple home-work tour with congestion are also comprehensively discussed. At the end of the chapter, the discussion is summarised by identifying gaps in the literature and directions for more research which are in line with the objectives set out for this thesis.

Chapter 3 presents examination of the modelling considerations required at the demand side of the combined model. This includes comparison of approaches used for individual decision-making in regard to their compatibility for the development of an analytical combined model. This chapter also presents detailed description for the specification of the utility function in order to identify essential components of the utility of activity engagement for activities such as: home, work, shopping and leisure etc.

Chapter 4 discusses modelling considerations required at the supply side of the combined model. This chapter examines the necessity of the dynamic representation of congestion on the road network when activity scheduling dimensions (especially time related) are modelled in combination. In addition to this, macroscopic dynamic network loading models are discussed in detail and also a new loading model (Adnan-Fowkes model) is also discussed in detail along with its properties. This model was developed with a joint effort of the author and Dr. Anthony Fowkes during the course of this research.

Chapter 5 defines the scope of the study and describes the framework and basis of the combined model developed in this research after exploring modelling approaches and considerations in chapter two, three and four. In this chapter, a formulation process of the fixed point problem is discussed in detail for the integration of the demand and supply side. Furthermore, two solution algorithms are also discussed for solving the optimisation problem.

Chapter 6 reports the development of the basic combined model for modelling home-work tour scheduling with a refined definition of the utility function necessary to combine the morning and evening commute together. The refined definition of the utility function for integration of home-work tour is supported with a demonstration of numerical and analytical proofs. This chapter also reports the development process of the extended version of the daily scheduling model which incorporates two user classes carrying out different tour types and also includes more scheduling dimensions.

Chapter 7 reports the results obtained for several numerical experiments conducted in order to investigate the functionality of the model described in chapter 6. The lessons learned from these numerical experiments are discussed in detail.

Chapter 8 describes the development of the extended model for the representation of weekly activity scheduling with network congestion. The performance of weekly activity scheduling model is analysed by conducting some numerical experiments which include a policy test as well. This chapter also presents comprehensive discussions on the obtained results of these experiments. Further straightforward extensions of the weekly model are also presented in this chapter by relaxing some of its assumptions.

Chapter 9 concludes the research by thoroughly examining the degree of achievement of research objectives. This chapter also discusses recommendation for further research in order to enhance model capabilities.

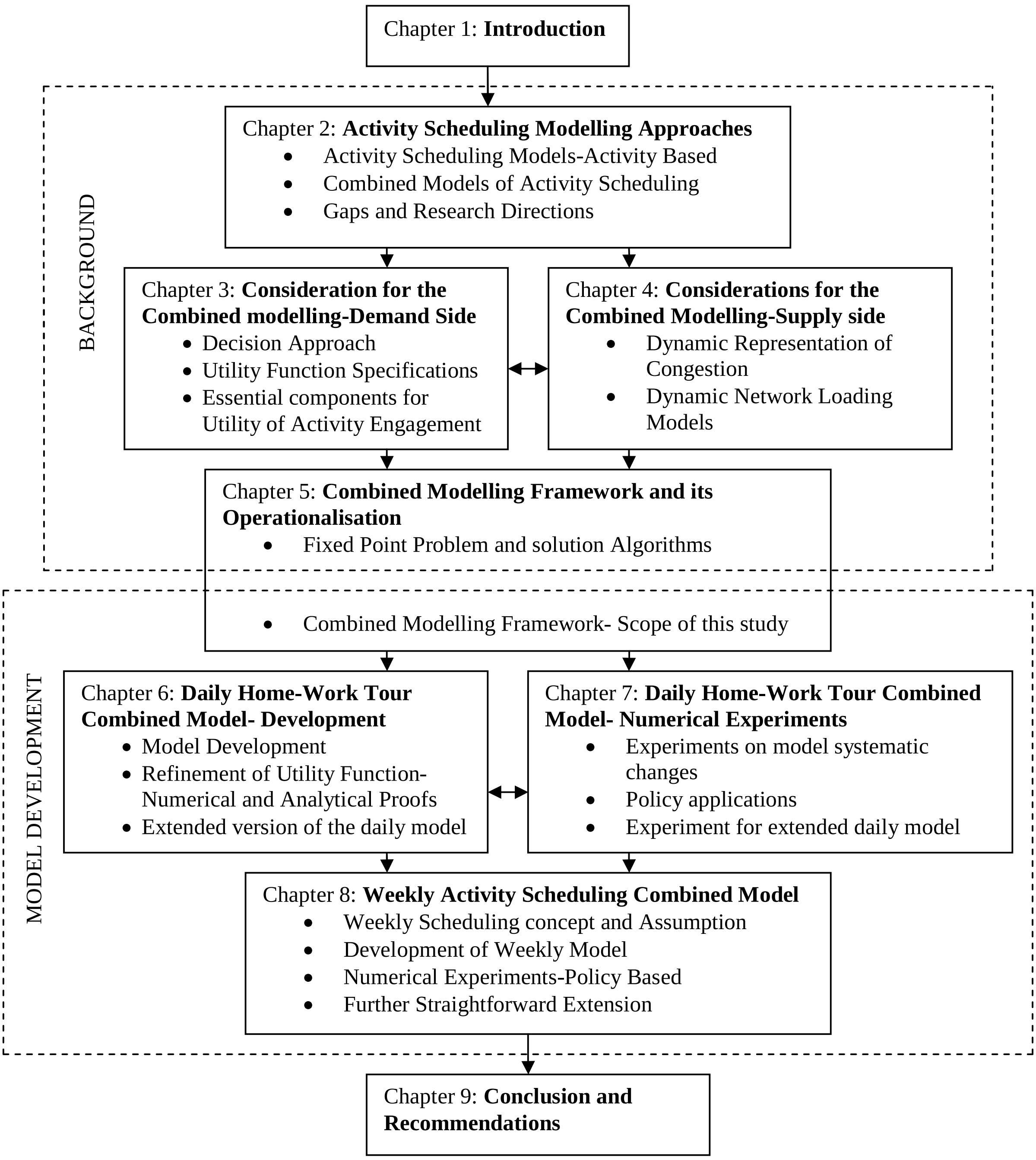


Figure 1.1: **Thesis Plan**

Chapter 2

ACTIVITY SCHEDULING MODELLING APPROACHES

2.1 GENERAL

The literature review carried out in this chapter is mostly focused on building an understanding of different approaches used to model the activity scheduling process. It has been observed that the literature in this area has grown dramatically over the last three decades. Advancements in the activity-based approach have resulted in the development of scheduling models that are focused on the entire daily activity travel pattern not just commute travel. Within this approach, activity schedules have been modelled using econometrics and rule-based (heuristic) techniques. Additionally, some researchers (as illustrated in section 2.2) have empirically examined causal relationships between various *dimensions of activity scheduling such as duration, timing, sequence, location and modes.* On the other hand, some researchers (shown in section 2.3) have focused only on the scheduling of the morning commute based on the Vickrey (1969) concept of trade-off between the schedule delay penalties and travel time. Attempts (presented in section 2.4) have also been made to combine the activity scheduling models with congested networks in order to address the impacts of congestion on the scheduling of activities. In the following sub-sections, different scheduling models are reviewed in order to identify gaps that are required to be bridged with further research.

2.2 ACTIVITY-BASED APPROACH AND ACTIVITY SCHEDULING MODELS

2.2.1 Activity Based Approach

The theory behind the activity-based (AB) approach is summarised from the following three points (Bowman and Ben-Akiva 2000, Arentze and Timmermans 2004):

- The derived nature of travel i.e. participation of an individual in an out-of-home activity gives rise to travel.
- Spatio-temporal constraints an individual faces to gain utility by participation in an activity.
- The role and interaction of different household members through which household needs are transformed into individual activities.

In this approach, the decision for travel of an individual is modelled as a part of modelling the demand for activities (Shiftan et al 2004). Therefore, this approach offers a wider framework to view complex behaviour of an individual's travel decisions rather than simply concentrating on trips as in the traditional approach. For example, travel in the AB approach is viewed as an action which does not provide direct benefits to individuals, but its bridging nature i.e. linking activities together, provides so much attraction that individuals do travel in order to gain overall benefits. This travel-activity trade-off helps widen the overall framework of this approach in which not only trips are important but activities are also important as they control the demand for travel.

Kitamura (1997) suggested that the travel demand forecasting required a significant enhancement of the abstract representation of behaviour evident in the traditional trip-based approach. He argued that people would not think about how many trips to make when developing a plan for a day; rather one would think about what to do, where to go and how to get there, and trips come into the picture in response to these questions. Kim et al (2006) pointed out that extensive use of the disaggregate modelling technique within the trip-based modelling framework has induced behavioural notions in the overall demand forecasting procedure, but the analysis focus remains on individual *trips* and their attributes. As the trip-based approach relies heavily on considering trips as an independent entity for analysis, therefore it is not capable of addressing correlated aspects of an individual's decision for series of trips in a given day. In some instances, the forecasts of trip-based approaches have proved to be inaccurate due to this mis-specification: an inappropriate representation of travel behaviour relationships (Kitamura et al 1995, Lam and Yin 2001). This can be understood by considering an example of a home-based tour that contains two or more trips: the four-step procedure (trip-based approach) investigates each trip independently and often fails to recognise the existence of linkages among trips. However in reality, if a private car is chosen for one trip, this choice would always influence an individual towards using the same mode for successive trips of the same tour. Additionally, the assessment of policies, which are inevitable in order to address issues (such as growth in the information technology, general aging of the population, sustainability of cities and transport systems) requires distinguishable analysis of the impacts of traffic that is diverted (by adjusting routes, mode, locations, departure times, activity durations) and that is induced (new trips and activities) in the system. The representation of this phenomenon is explicitly tied to a modelling framework that fully encompasses the travel behaviour of individuals. This requires an approach which offers wider framework for analysis of travel behaviour than the conventional trip-based approach (McNally and Rindt 2008).

In earlier attempts to address the weaknesses of the trip-based approach, tour-based models were developed in the early 1980s in the Netherlands (Daly et al 1983), and are being used extensively in Europe after their further refinement (Algers et al 1995). Tour-Based models are often categorised as a basic representation of the AB approach as it reflects some, but not all, of the tenets of the AB approach. Additionally, the models following a tour-based approach actually serve as the basis for the development of a generalised tour-based or full-scale AB models (Bowman and Ben-Akiva 2000, McNally and Rindt 2008). In the tour-based models, trips are grouped in such a fashion that all travel can be viewed in terms of round-trip journeys based on either home or work. This is because a tour is considered as a basic unit of analysis in these models. Figure 2.1 explains the notion of home based and work based tours as home and work activities are taken as base activities in this approach. This was done in order to classify the complexity of the activity patterns, which also provides ease in the development of the tour-based models. For example, separate models are developed for the home-based and work-based tours, because of the requirement of different explanatory variables and also this helps reduce the number of combinations of the modelled alternatives (Algers et al 1995).

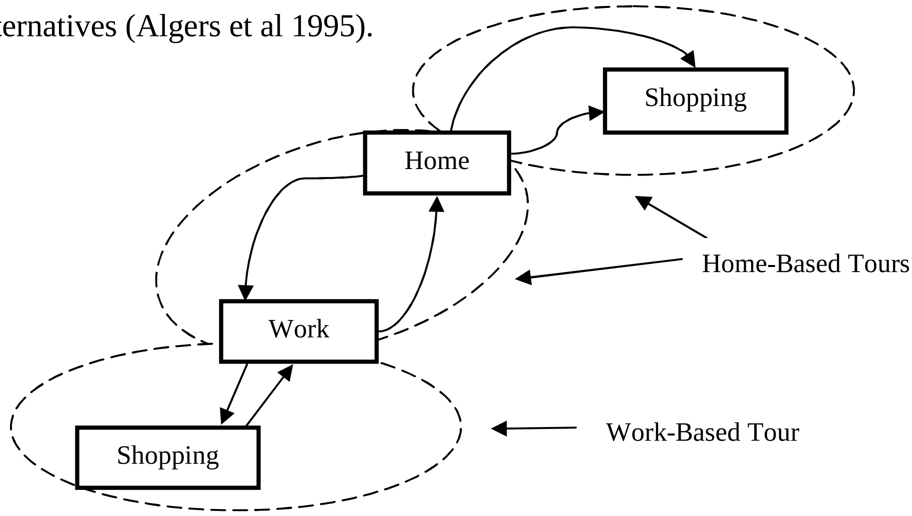


Figure 2.1: Home-based and Work-Based Tours Representation in the Tour-Based Models (Jovicic 2001, p. 12)

In this approach, each tour is viewed independently from the other tours performed in the same day by an individual. This fact introduces a weakness with no connection or linkage among multiple tours taken on the same day by an individual; therefore, the inter-tour temporal and spatial constraints are not explicitly addressed from this approach (Bowman and Ben-Akiva 2000, Jovicic 2001). In sub-section 2.2.2, models developed under the AB approach (or generalised tour-based approach) are discussed in detail along with their characteristics.

2.2.2 Modelling Considerations within AB models

The modelling systems developed within the AB approach can be classified on various bases, such as the employed decision making methodology within the modelling system and activity scheduling dimensions considered within these systems. This section discusses these issues in detail and then comparison is made for different AB modelling systems.

2.2.2.1 Decision Making Methodology

The AB models to date have usually employed both or either of the two distinctive decision making methodologies, which actually lead to the determination of an individual daily activity schedules. These are as follows

- *Econometric Modelling*
- *Rule-based or Computational Process Modelling (CPM)*

The *econometric modelling* technique involves using systems of equations to capture relationships among the macroscopic indicators of activity and travel, and to predict the probability of a decision outcome (Bhat et al 2004). The models based on the econometric principles are developed on the rationale that this technique allows the examination of alternative hypotheses in the form of causal relationships between the attributes of activities, travel, socio-demographics and land use. Within this technique, models are developed utilising the discrete choice modelling methodology, the hazard duration based models and the structural equation models (Bhat et al 2004, Buliung 2005). Discrete choice modelling methodology assumes decisions are made as a process wherein a decision maker (e.g. individual or household), faced with a set of alternatives, chooses to maximise the utility. Multinomial logit (MNL) and nested-logit (NL) model forms are among the operational examples that are most widely used in travel behaviour modelling (Bhat et al 2004, Jovicic 2001). Hazard duration models, which are helpful in examining the impacts of temporal aspects of activity-travel behaviour (e.g. temporal constraints in the form of timing and duration of activities and associated travel), are also used to understand the concept of dependency on durations. These models predict the likelihood of an activity ending, dependent on the time already dedicated to the pursuit (Bhat 1996, Buliung 2005). Hazard based models have been used within the AB modelling systems as a part of properly describing the behaviour mainly for the duration aspects, whereas other aspects of behaviour are usually modelled with the discrete choice models. Structural equation models (SEM) require understanding of *direct, indirect* and *total* effects for the model specification and interpretation. When two variables affect one another without intervening variables, this effect is termed as a direct effect, while the indirect effects involve mediation by at least one other variable. The sum of these effects is known as the total effects. This can be better understood with the following example mentioned by Buliung (2005, p. 14) in the context of activity travel behaviour:

> *One might specify socio-demographic characteristics as direct exogenous predictors of activity participation. Socio-demographics could also impact travel-behaviour (e.g. trip frequency) indirectly through activity participation. That is longer duration in a particular activity could mediate the frequency of other activities. Socio-demographic variables could also be simultaneously specified as direct predictors of travel measures (e.g. trip frequency). For instance, age could be specified to have a direct effect on trip frequency. The total effect of socio-demographics on travel behaviour in this case would be the sum of the direct and indirect effects of specified socio-demographic predictors.*

The SEM methodology effectively provides the means for a systematic assessment of the inter-relationship across individuals, their time and spatial constraints and other variables and therefore, it is generally regarded as a *descriptive tool and does not have direct forecasting applications*. However, the estimated relationships among different variables render a promising background for the systematic development of AB models (Golob 2001, McNally and Rindt 2008). Within econometric modelling techniques, discrete choice modelling methodology has been used extensively in the development of AB models because of its well-established theoretical basis, professional familiarity and forecasting applications. The other two modelling procedures serve as fillers for representing various aspects of activity-travel behaviour. A main criticism of the econometric based models is that they do not explicitly model the behavioural mechanism underlying activity engagement and travel which often yields satisficing outcomes rather than optimal decisions based on utility maximisation (Arentze and Timmermans 2004, Bhat et al 2004, Lee and McNally 2006). These satisficing outcomes are due to the limitation in the cognitive capability of the decision makers. Rule based or CPM methodology answers this criticism and is explained below.

The *rule-based or Computational Process Models* (CPM) uses a set of heuristic rules in the form of a condition-action (If-Then) structure in order to solve a particular task at hand. These models utilize search processes that explicitly account for the cognitive limitations by incorporating decision rules in the computational process (Kitamura et al 1995; Arentze and Timmermans 2004). CPM places most attention on explaining how individuals think when building schedules by employing a learning mechanism in the modelling structure. The learning mechanism is responsible for reinforcing future behaviour through positive experiences of past and then gradually these experiences are transformed into refined heuristics which are applied in specific choice situations (Arentze and Timmermans 2004, Jovicic 2001). CPM are characterised as flexible in representing the complexity of travel decision making and explicitly capturing schedule constraints but issues in statistical estimation and calibration of these models are yet to be defined and resolved (Bhat 2002). This induces the drawback that they cannot be checked for statistical properties. Additionally, some heuristic rules that were incorporated in the CPM models are unproven and have not been verified with real data. This weakens the claim that CPM technique properly models decision-making behaviour (Lee and McNally 2006).

The above discussion suggests that both of the above mentioned decision making methodologies, i.e. econometric and CPM, have some limitations in addressing the complexity involved in decisions related to activity scheduling. This makes it entirely subjective in terms of the purpose of the study within the AB approach (i.e. which methodology meets the specific requirements of the study being carried out). For example, if statistical checks of the obtained results are demanded then econometric methodology is preferred over CPM (Jovicic 2001).

2.2.2.2 Activity Scheduling Dimensions

Scheduling of activities is a major component within AB models (Jovicic 2001). This component actually derives the individual's daily activity-travel pattern taking into account their daily agenda of activities, socio-economic characteristics and spatio-temporal constraints (Bowman and Ben-Akiva 2000). Activity scheduling is defined by Axhausen (1995) as "*the joint choice of the time, duration, location, mode and route for a sequence of activities drawn from a given set of aware activity needs*". This definition has been adopted by Kitamura (1997) and Lee and McNally (2006). However, some studies within the AB approach also recognise the importance of the choice of sequence for the activities that are planned for a given day, which results in the formation of simple or multiple tours for the entire daily activity-travel pattern (Bowman and Ben-Akiva 2000, Shiftan et al 2004, Arentze and Timmermans 2004). Apart from that, Bhat et al (2004) also added the joint participation element (i.e. involvement of two or more persons of the same household in an activity at the same time) in the activity scheduling models with several other dimensions of activity scheduling. The discussion below emphasises the importance of each scheduling dimension considered in the AB models.

- *Departure time choice*; as congestion is not a uniform phenomenon and varies over the day, therefore, some travellers adjust their departure times to avoid the worst congestion periods. Most of the modelling studies examined choice of departure time for a morning trip (i.e. trip from home to work), by formulating a choice problem in a finite number of discrete time periods and modelled the choice using random utility maximisation theory (Small 1982 and 1987, Polak and Jones 1994, Bradley et al 1998, Bhat 1998). These studies typically employed the Vickrey (1969) approach of schedule delay for quantifying the trade-offs between time varying travel times and cost with inherent preferences for undertaking activities at certain time-of-day. Consideration of the choice of departure time is vital for analysing travel behaviour as it is found (in joint studies of mode with time-of-day choice and route with time-of-day choice) that time-of day choice is more sensitive than mode and route choice (Hendrickson and Plank 1984, Hess et al 2004).

- *Choice of activity duration;* this represents another important temporal aspect of activity-travel behaviour. The models that include choice of activity duration are able to answer, how long the activity is pursued. This is vital for the AB models because in these models the entire daily activity-travel pattern is modelled with explicit consideration of the daily time budget for an individual (Bhat and Misra 1999). Therefore, choice of duration for one activity may affect the choice of duration or other scheduling dimensions for the earlier or subsequent activities in the daily activity pattern. Duration models have successfully explained the dependency of travel behaviour on activity durations. For example, Bhat (1996) developed a model for the duration of shopping activity. He found that longer duration of work activity has a negative impact on the duration of after work shopping activity; on the other hand, departing before 4 pm from work significantly increases the duration of shopping activity as an individual would have more time and opportunities than in the former case.

- *Route choice;* the choice of route available to an individual is examined mostly with the help of generalised travel cost, constituting travel time and travel cost, and is embedded within the demand-supply equilibrium framework, with the aim of minimising generalised travel cost of road users (user equilibrium) or minimising generalised travel cost for overall population (social equilibrium) (Ortúzar and Willumsen 2001). The literature within the traffic assignment modelling suggested that route choice is always an integral part of the modelling system, whether the system is based on a static (e.g. SATURN and EMME/2) or a dynamic environment (e.g. CONTRAM and DYNAMIT). The wide acceptability (within the academic world and in the practitioner community as well) of the comprehensive commercial traffic assignment packages could be the main cause of a limited incorporation of the route choice dimension within AB models (Vovsha 2009). Furthermore, incorporation of route choice demands the representation of the road network of the area being studied, inclusion of which limits the application of the particular AB modelling system, and also generalisation of this requires too much effort which has already being done through rigorous research efforts in the form of the above discussed traffic assignment packages. Therefore, most of the AB modelling systems avoid incorporation of route choice as the scheduling dimension, because these models usually rely on traffic assignment packages to model route choice along with the prediction of traffic on the roads. This is based on the aggregation of outputs from AB models in the form of time-of-day based trip matrices. It is worth mentioning here that the traffic assignment packages are developed on the basis of the trip-based approach and do not consider the daily activity-travel pattern, therefore, relying on them to produce a final output (traffic volumes) may cause some loss of behavioural richness gained by using sophisticated AB models (McNally and Rindt 2008). Proper integration of the AB models with traffic assignment packages, is one of the major focuses of the current research within the AB approach, and is evident through the future development programs of the US Federal Highway Authority (Vovsha, 2009).

- *Mode choice;* in the trip-based modelling approach, this dimension has been considered explicitly before assignment of traffic on the road network. There have been many modal split studies because of the key role played by public transport (in the form of buses, rail, tube, etc) in policy making. The modal split models are largely developed using random utility theory and use attributes of the trip maker, type of journey and transport facility as the main determinant for the choice of mode (Ortúzar and Willumsen 2001). Mode choice incorporation is important in AB models because it is not only representing its direct effect on travel behaviour but also reflects secondary effect, which results in more accurate assessment of any measures of travel demand management (TDM). For example, a transit subsidy may result in commuters changing the mode of travel for the home-to-work trip, from drive alone to transit; this is a primary effect of TDM. However, because of such a situation it is not possible for a person to stop on the way home to buy groceries. Therefore, when the person now returns home by transit, it is now necessary to take the car and drive to a nearby store. This is a secondary effect and in such cases the advantages of TDM may be at least partially offset by the reduction of the work auto trip being replaced by a new shopping auto trip.

- *Activity location choice;* this is another dimension within the activity-travel pattern which reflects the spatio-temporal aspects of travel behaviour. Traditionally, destination choice has been modelled using synthetic models (i.e. gravity models) within which a deterrence function of generalised cost between the zones is employed (Ortúzar and Willumsen 2001). Location choice has been modelled more explicitly using random utility theory. For example, Kitamura et al (1998) studied the effect of time-of-day dependency, activity duration and the origin of trip on the choice of location/destination. They concluded that the deterrence effect of travel time increases towards the end of the day as time constraints tighten, and participation in activities for a longer period tend to a selection of the farther locations for these activities. Within AB models, the choice of location of activity participation is vital as it spatially pegs the daily activity-travel pattern of individuals (Sivakumar and Bhat 2006).

- *Choice for the sequencing of activities;* it has been an established fact that in order to gain maximum satisfaction (utility) within a limited time budget and because of the various other intervening aspects, travellers are inclined to arrange activities in a chained pattern (Adler and Ben-Akiva 1979). As demand for participation in activities derives travel, thus for the sake of attending more activities individuals have to reduce their travel time because the dwell time (activity duration) can only be compressible to a short extent (Liu et al 2008). This results in complex chained patterns of activities. The traditional approach, which has its basis in single *trips,* cannot adequately depict the effects of choice of activity sequence on the travel behaviour (Liu et al 2008, Ashiru et al 2004). Very recently, Liu et al (2008) studied the impact of activity chaining on travel behaviour, they found that the earlier the commuter departs for work, or the later he reaches work, it is more likely for him to link non-work activities. This indicates that work activity is playing a key role for the choice of sequencing activities.

- *Joint choice of activity participation;* this scheduling dimension represents the involvement of two or more persons of the same household or different households in an activity at the same time and location. Bhat et al (2004) emphasised that this dimension is important to incorporate in the analysis of travel behaviour as it links the travel pattern of different individuals. For instance, it is possible that changes in an individual travel pattern in response to a certain policy measure may affect the activity-travel pattern of his/her companion. Furthermore, empirical evidence also suggests that joint participation in activities with the family members and friends, tend individuals to travel farther and pursue activities for a longer duration (Vovsha et al 2004). Recent empirical analysis of joint participation in activities carried out by Srinivasan and Bhat (2008) concludes that there is a need to incorporate inter-household and intra-household interactions in the activity-travel analysis for representing the implications of joint participation. For example, high fractions of joint leisure-type of activities undertaken at a particular location imply that individuals may not be entirely flexible in their scheduling choices for the pursuit of discretionary activities.

The above discussion on a*ctivity scheduling dimensions* provides a useful base to understand the role of each scheduling dimension on the daily activity-travel pattern. Furthermore, it renders the proper footing to compare different AB modelling systems presented in the literature in terms of the complexity they represent in the daily activity-travel pattern of the individual.

2.2.3 AB Modelling Systems-Properties and Considerations

Table 2.1 illustrates properties of the different AB modelling systems presented in the literature. Properties includes the employed decision making methodology, scheduling dimensions incorporated within these systems, nature of the output of these modelling systems and some general characteristics of these modelling systems. The table in overall show the level of complexity these AB models are able to represents.

The development of ALBATROSS (Arentze and Timmermans 2004) is taken as a significant contribution in the AB approach, as it models the complete activity scheduling process in a microsimulation environment. This system incorporates household interactions to generate a set of activities that an individual needs or wishes to carry out. The scheduling process involves adding flexible activities, such as shopping to the initial schedule skeleton that is composed of the fixed activities with their start time and location as known. The modelling system SAMS is a broader system that contains land use and vehicle transaction model (i.e. a model which considers decisions to acquire, dispose and replace vehicles and the choice of vehicle types) in addition to AMOS (an activity-based component of SAMS). AMOS includes a baseline activity analyser, a TDM response generator and rescheduling and evolution modules. SCHEDULER and STARCHILD were recognised as early examples of rule based models (McNally and Rindt 2008), however, their framework and pattern of heuristic rules have been used in the construction of recently developed SMASH and ALBATROSS.

Econometric models, for example, BB system, PETRA and TA System have a slight dissimilarity in their modelling structures with each other, however, they are estimated entirely for different populations i.e. BB system is estimated for Portland and Boston, PETRA is estimated for Denmark, and TA is developed for Tel-Aviv data set.

Table 2.1: Activity Based Modelling Systems and their Characteristics

Modelling System	Model Base	Modelled Dimensions and Characteristics	Output	Reference
ALBATROSS	Rule Based (CPM)	Decision rules derived directly from activity-travel data. Household interactions are considered. Scheduling decisions are carried out in dynamic setting (i.e. during execution). Activity duration is considered as fixed, Joint participation and route choice are not modelled)	Daily Activity Pattern	Arentze and Timmermans (2004)
SAMS and AMOS	Econometric and Rule Based (CPM)	SAMS is an integerated simulation model system comprising land use and vehicle transaction models along with AMOS (activity based component of SAMS). AMOS takes base activity travel pattern and generate modified pattern for individuals in response to TDM strategies. Route and Joint Activity participation choice is not modelled.	Daily Activity Pattern	Kitamura et al (1995)
BB System	Econometric	Generate activities and model activity schedules for individuals through hierarchical logit structure. Activity Pattern comprised of primary and secondary tours (Joint participation, route choice are not incorporated)	Daily Activity Pattern	Bowman and Ben-Akiva (2000)
CEMDAP	Econometric	Generate activities and model activity schedules for individuals with incorporation of Joint participation, and duration. location and mode choice (Route choice is not incorporated)	Daily Activity Pattern	Bhat et al (2004)
PCATS	Econometric and Rule Based (CPM)	Time-space prism is used for representing spatio-temporal constraints with block periods (fixed activities) and open periods (flexible activities) for modelling scheduling dimensions. (Joint participation and route choice are not modelled)	Daily Activity Pattern	Kitamura et al (2000)
PETRA	Econometric	Nested logit based structure, includes car ownership, home based tours with complex patterns, destination and mode choice.	Daily Activity Pattern	Jovicic (2001)
SCHEDULER	Rule Based (CPM)	Theoretical scheduling framework, Feedback mechanism between scheduling and execution. Scheduling is conceptualised as the insertion of non-routine activities around routine activities.	Daily Activity Pattern	Garling et al (1994)
SMASH	Econometric and Rule based (CPM)	Deals only with activity scheduling in sequential way. Adjust schedules during execution through add, delete, substitution and termination functions. Joint participation and route choice are not modelled.	Daily Activity Pattern	Ettema et al (1996)

STARCHILD	Econometric and Rule Based	Treated as an earlier example of rule based model. Generate activities and model activity schedules for individuals. Joint Participation, Mode, route choice are not incorporated.	Daily Activity Pattern	Recker et al (1986)
STGP	Econometric	Simulator assumes a sequential history and time of day dependent structure, activity type, duration, location, mode choice are incorporated (Joint Participation and route choice not modelled)	Daily Activity Pattern	Kitamaura et al (2000)
TA System	Econometric	Consider Auto ownership and model up to two complex tours per day for each individual i.e. Primary and secondary with intermediate stops destination choice. Nested logit Model structure is used for modelling Activity scheduling. Joint participation and route choice are not considered.	Daily Activity Pattern	Shiftan et al (2004)
TASHA	Rule Based (CPM)	Utilised concept of the project to organise activity episodes, Joint participation and location are considered with other scheduling dimensions (Mode and route choice are not incorporated)	Daily Activity Pattern	Miller and Roorda (2003)

ALBATROSS: A Learning-based Transportation Oriented Simulation System
AMOS: Activity-Mobility Simulator
BB System: Bowman and Ben-Akiva, Day Activity Schedule Model System
CEMDAP: Comprehensive Econometric Microsimulation of Daily Activity Pattern
PCATS: Prism-Constrained Activity-Travel Simulator
PETRA: Danish Activity based travel demand model developed by Mogens Fosgerau 2001
SCHEDULER: known also as Scheduler
SMASH: Simulation Model of Activity Scheduling Heuristics
STARCHILD: Simulation of Travel / Activity Responses to Complex Household Interactive Logistic Decisions
STGP: Synthetic Travel Pattern Generator
TA System: The Tel-Aviv Activity based Model System
TASHA: Toronto Area Scheduling Model with Household Agents

The modelling systems that employed econometric decision making methodology utilised an adapted sample enumeration method for predicting the individual activity schedules for an entire day because of the fairly large number of alternatives in these models. So the outcome is not in the form of probabilities but in the form of a activity-travel pattern for a particular individual. This adapted sample enumeration method can be further illustrated by the following example; in the BB system, the sample from the census data is divided into four income levels, four age classes of the head of the household and four household size, therefore, in total 64 groups (cells) are defined. Now, say there are 13 individuals in the forecasting year in cell number 5 of zone 10, then 13 respondents were drawn from the corresponding cell by applying a Monte Carlo simulation. For each of the 13 drawn individuals the activity schedule is calculated in the model in the form of calculated probabilities based on the known characteristics of these individuals. For each modelled outcome a random number between 0 and 1 is then drawn in order to simulate a particular outcome according to the calculated probabilities from the model. This method is more elaborately stated in Ben-Akiva and Lerman (1985, pp 147). The modelling system CEMDAP differs from the other econometric models in a way that it not only models the activity pattern of workers but also models non-workers as well. The output of almost all the AB models shown in table 2.1 has focused on deriving individual daily activity pattern.

The AB approach has established its strength as a framework for travel demand analysis. However, with all its analytic strengths and the underlying tenets, it has not delivered a fully operational practical tool to the practitioner community. Furthermore, the models developed within the AB approach have not incorporated route choice as an integral dimension within their modelling system as is evident from table 2.1. This suggests that these models are basically relying on traffic assignment models, which have their roots in the trip-based approach, to predict flow on the road network. The behavioural realism which is gained by using sophisticated AB models in the form of the output, i.e. individual daily activity pattern, can be lost by again aggregating the trips at different times-of-day from these individual patterns, in order to provide input to traffic assignment models. The joint effect of congestion on route choice and other scheduling dimensions cannot be incorporated using this sequential process to find the final output (vehicles) on the road network. For example, due to congestion on the road, some persons may change their mode and route jointly or some persons may change their departure times and route jointly or some person may change their entire pattern of activity-travel based on the route they have selected. Therefore, it is entirely necessary that route choice is also jointly integrated with other scheduling dimensions of the individual daily activity-travel pattern.

Another weakness within the AB models, because of not having the integrated framework with the assignment models, is that these models are only able to reflect the first-order effect of policy on the behaviour. The second-order effect of the policy, because of the changed behaviour on the traffic conditions, cannot be transmitted to the behavioural side as there is no linking mechanism between the two sides (Lam and Huang 2002). For example, flexible working hour policy may induce behavioural change such as; some commuters may shift their travel mode; some commuters may undertake a non-work activity during the commuting trip. Consequently, the flow distribution on the network will change temporally and spatially and the effect of these changes in flow distribution cannot again transmit back to the behavioural side as no linking mechanism exists. Therefore, a *two-way link* is crucial between demand and supply sides to better assess the effect of policies (Abdelghany and Mahmassani 2003).

Integration of the models based on the AB approach with the models of the supply side has the potential to substantially improve the current level of travel demand analysis. Therefore, it is necessary to gain understanding about the issues involved when demand and supply sides are integrated with each other. In the next sections, some modelling efforts are discussed, starting from the studies that focused on the morning commute and then the modelling attempts are discussed that represents daily activity travel patterns.

2.3 COMBINED MODELLING -MORNING COMMUTE SCHEDULING

Boyce and Bra-Gera (2004), in their review paper for the combined modelling systems, highlighted the fact that to predict travel choices on a congested urban road network, travel times must be endogenous to the model. This thought was first presented in the seminal work of Beckmann et al (1956). Boyce and Bra-Gera (2004) further pointed out that with the strong hold of the sequential four step forecasting approach in the decades of the sixties, seventies and eighties, this notion was not considered in the main stream modelling studies, and when it was realised again, modellers then began to ask how to *combine* these steps with a more consistent method i.e. a re-emergence of the combined modelling issue. A model based on the combined modelling approach can answer criticism on the models that are based on the trip-based and the AB approaches, as combined models can successfully represents the interplay between the travel time and activity/travel schedules. Furthermore, results from these models represent the secondary effects of the congestion mitigation policies on the demand and supply sides through the feedback mechanism, therefore, these models are potentially better tools for the investment appraisals. In the following sub-sections, modelling efforts are discussed that combines the morning commute scheduling considering the effects of network congestion.

2.3.1 The Seminal Work of Vickrey

Vickrey (1969) introduced a concept of individual's departure time decision for the morning commute trip between a single origin-destination pair as a trade-off between schedule delay penalties and time spent in travelling. The model considered a single bottleneck connecting a residential area with the city centre and derived a departure time profile for the morning peak trip based on the optimisation of schedule delay penalties taking into account of the Preferred Arrival Time (PAT) and travel time. It is assumed in this model that at equilibrium, no individual could modify his/her departure time choice in order to (strictly) decrease his/her travel cost. Furthermore, it is also assumed that the travellers are aware of the amount of congestion and its impact on travel times (e.g. from daily experience) and that they may respond to this by changing their departure times. This deterministic formulation of the model embeds a rather strong assumption in a sense that no unmeasured interpersonal variations are accounted (Small 1982, Small 1987). Therefore, empirical studies of departure time choice which used Vickrey's approach re-formulated the underlying continuous departure time choice problem as a choice problem involving a finite number of discrete time periods and modelled the choice between these periods within the framework of random utility theory. For example, Small (1982) and (1987), Abkowitz (1981), Chin (1990) etc.

The Vickrey model has been used as a template to construct more realistic descriptions; as it is extended to a great extent. The analytical extensions envisaged so far are mentioned below: (1) Simple network with several routes for one or two O - D pairs (Ben-Akiva et al 1986, Arnott et al 1990). (2) Heterogeneous values of the unit cost parameters, i.e. parameters attached to schedule delay penalties and travel time (Arnott et al 1988). (4) Incorporation of more scheduling dimensions within the Vickrey framework along with the elastic demand (Tabuchi 1993, Huang and Yang 1996, Arnott et al 1997).

2.3.2 Complex Combined Morning Commute Scheduling Models

Since Vickrey's theoretical and Small's empirical papers, many modelling attempts have been made with the introduction of increased complexity in order to simulate real world scenarios. The significance of them is to model departure time and route choices in a *dynamic* combined modelling framework. Time-varying or dynamic representation of congestion is necessary because it is a fundamental requirement for modelling departure time choice. These research efforts can be traced back to the key work of Mahmassani and Herman (1984) and Arnott et al (1990) for a single O-D pair. In order to solve this problem for larger networks, several models have been proposed by various researchers using different approaches on dynamic traffic networks. Examples are; Friesz et al (1993), Smith (1993), Ran et al (1996), Chen and Hsueh (1998), Ziliaskopoulos and Rao (1999), Heydecker and Addison (1998) and Huang and Lam (2001) These studies are different with each other in the aspects of formulating the problem (i.e. route-based or link-based), representation of the traffic stream (i.e. through macroscopic functions or microscopic simulations), the equilibrium type (i.e social optimal or user equilibrium), incorporation of the randomness (i.e. deterministic or stochastic) and representation of the demand as elastic or inelastic. Fewer modelling efforts are also presented in the literature which incorporates more complexity in the morning commute scheduling models. These are illustrated in table 2.2, the term *complex*, indicates here that the morning commute is modelled incorporating departure times and route choice with one or more other scheduling dimensions in a combined modelling framework that captures time-varying travel times.

The model proposed by Abdelghany and Mahmassani (2003) experimented with three types of morning commuters having different trip chains as shown in figure 2.2, i.e. Home-Work, Home-Intermediate-Work and Home-Intermediate-Intermediate-Work. The duration of activities at the intermediate stops and PAT at the intermediate and final destinations (i.e. Work activity location) are considered as exogenous in the model. Departure time choice at the origin (home activity location) and route choice are modelled for all the three trip chains. Choice of sequence is only modelled for the commuters who are performing their morning commute with two intermediate stops. The model developed by Lam and Huang (2002) considers only two types of trip chains i.e. home-work and home-intermediate-work with an argument based on empirical evidence that these trip chains constitute 99% of the total trip-chains in the morning commute. Therefore, they have not considered the choice of sequence in their modelling framework; instead they considered choice of location as another vital scheduling dimension, which is active only in the home-intermediate-work pattern. The generalised disutility which has to be minimised by each commuter is composed of; constant time dependent home utility, schedule delay penalty of arrival (time-early or time-late) at work location, the fixed positive utility at intermediate stop and the cost of travel time. Similar to Abdelghany and Mahmassani (2003), duration of activity at the intermediate stop in this model was assumed exogenously.

Table 2.2 Complex morning commute combined scheduling models

Modellers	Modelled Dimensions	Traffic Performance model	Deterministic /Stochastic	Network Characteristics
Abdelghany and Mahmassani (2003)	Departure time, Route choice and Sequencing	DYNASMART (Traffic simulator having capability of simulating traffic with trip-chaining)	Stochastic	The network consists of 22 nodes and 68 directed links. The network has 16 origin nodes and is divided into six zones with 6 nodes serving as destinations.
Lam and Huang (2002)	Location, Departure times and Route Choice	Deterministic Queue Model (Point-Queue Model)	Deterministic	Single O-D, with 7 other nodes in a grid fashion. Within a grid network 3 nodes are treated as choice of location.
Ramadurai and Ukkusuri (2008)	Duration, departure time, route choice and activity location	Cell-based Transmission Model	Stochastic	Double Diamond Network-Containing 8 nodes (home and work activity nodes along with four non-work activity nodes

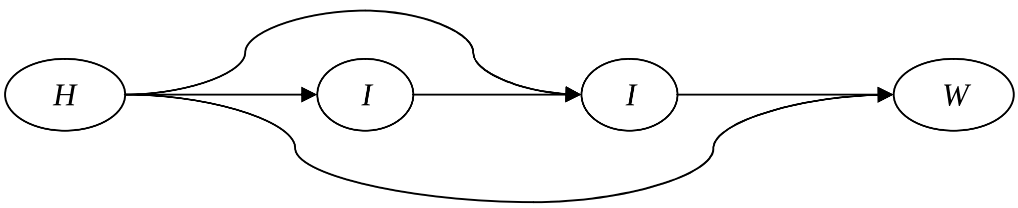


Figure 2.2: Home (*H*) to Work (*W*) trip with Intermediate (*I*) stops

The recent extension of the both above discussed models is presented by Ramadurai and Ukkusuri (2008). They incorporate similar trip chains as suggested by Lam and Huang (2002) and model activity duration along with the location, departure time and route choice for the morning commuters. It should be noted that location and duration choices are only active for the trip chain that constitutes the home-intermediate-work pattern. The generalised disutility function is also similar to Lam and Huang (2002); however, to accommodate duration choice, instead of using fixed positive utility value of intermediate locations, they employed duration dependent utility of these intermediate locations. The common feature of these complex scheduling models and the models that incorporate departure time and route choice is that they are only modelling the scheduling dimensions involved in the morning commute. Therefore, the repercussions of morning commute congestion effects on the scheduling dimensions of other subsequent trips and activities individual performed in a given day are ignored in these models. In section 2.4, some combined modelling efforts are discussed in which entire day activity-travel pattern is modelled.

2.4 COMBINED MODELLING – DAILY ACTIVITY-TRAVEL PATTERN SCHEDULING

In this section, those models are discussed that attempt to model scheduling of the complete daily activity-pattern of the individual with network congestion. Few models are developed so far under this notion, which can be categorised as the *analytical models* that incorporate simple activity-travel pattern i.e. home-work tour, and *microsimulation models* that incorporate an activity-based model integrated with the traffic microsimulation package.

2.4.1 Simple Activity-Travel Pattern Scheduling with Congestion

Table 2.3 illustrates and characterises the modelling efforts with simple activity-travel pattern in the combined modelling framework. The model proposed by Lam and Yin (2001) is based on the premise that the predetermined time-of-day dependent utility profiles of each activity type is responsible for deriving the activity participation of each individual dynamically in three activities that require travelling i.e. home, office and lunch. In this model, they have divided the study time horizon in equal time slices (i.e one hour), and the individuals are supposed to choose activity type at each time slice/period while staying at a particular location considering the utility of activity type at that period and travel time required to reach at the other activity locations. For example, if an individual is staying at home at a particular time period, then he will choose to join work, lunch or stay at home for the next time period based on the utility gains available to join these activities along with the disutility of travel to reach other activity locations. Because of the choices of each activity type available at all time periods, this model does not consider the duration of each activity explicitly for each individual. Furthermore, many studies have pointed out that different activities of a daily activity-travel pattern are connected through the people's decision on how to allocate their time over the course of the day (de Palma and Lindsey 2002 and Zhang et al 2005). In the simple home-work tour context, arrival time at work may affect the time spent at work location and/or the desired departure time in the evening. This has been tested empirically by Wang (1996). This suggests that the consideration of the duration of activities is vital in order to explain the essential linkages among the trips for multiple sequential activities that form the daily activity-travel pattern of an individual.

Zhang et al (2005) following the comments regarding consideration of the duration of activity, developed the model which investigate the choice of departure time and duration for the work activity through the nested logit model for the home-to-work tour within a combined modelling framework. Similar to Lam and Yin (2001), they also incorporated time-of-day dependent utility for the measurement of the utility of activity participation. Their generalised cost function includes: utility for home activity participation; utility for work activity participation; and travel cost. Heydecker and Polak (2006) proposed the model which is similar to Zhang et al (2005) in various aspects (can be seen in Table 2.3). However, they assumed that individuals are perfectly aware of the amount of congestion and its impact on travel times (i.e. deterministic). Furthermore, they also investigated the effect of introducing congestion elimination tolls on the departure times of individuals and then the amount of time spent at home and work locations. The model presented by Kim et al (2006) is somewhat different to the models discussed above in this sub-section. This model can be viewed as a microsimulation model in which an individual can insert or delete activities (that are considered flexible such as shopping or leisure) in between the fixed activities (i.e. home and work) in order to form an optimal activity chain through maximisation of overall utility of activity-travel pattern. The generalised cost is based on the time-of-day dependent utility profiles of activities (which captures utility of participation of activities) and disutility of travel through the traffic microsimulation package DYNASMART-P.

Table 2.3 Daily activity-travel pattern scheduling models with Network Congestion

Modellers	Modelled Dimensions	Traffic Performance model	Deterministic/ Stochastic	Network Characteristics
Lam and Yin (2001)	Departure times with activity type choice and route choice	Time variant BPR type function	Stochastic	Simple Network consisting of thee activity type choice, having 6 one-way links
Zhang et al (2005)	Departure time choice for the morning commute and Duration of work activity	Point Queue Model	Stochastic	Single O-D network with one two-way route
Heydecker and Polak (2006)	Departure time choice for the morning and evening commute	Point Queue Model	Deterministic	Single O-D network with one two-way route
Kim et al (2006)	Destination, Departure time, duration and route choice	DYNASMART-P	Microsimulation (Deterministic)	Network contains 33 zones with several links (Can be applied for general network)

The common feature of all the models discussed in this sub-section is that they employ the time-of-day dependent utility profiles of activities in order to represent utility gained through participation in activities. This has been criticised by many researchers (Ettema and Timmermans 2003, Ashiru et al 2004, Yamamoto 2000) that time-of-day dependent utility profiles are not able to capture activity satiation effects, which suggests that the marginal utility of activity decreases with the increase in the duration of that activity, a notion that is in-line with the principles of economics. Chapter 3 will discuss this issue in further detail and investigate the implications of using only time-of-day dependent utility profiles for measurement of utility of activity participation. In the next sub-section, modelling efforts are discussed that combines activity-based models with traffic microsimulation packages.

2.4.2 AB Scheduling and Traffic Microsimulation Models

There are only two comprehensive modelling efforts found in the literature which can be classified under this sub-section. These are discussed as follows:

2.4.2.1 TRANSIMS

TRANSIMS (TRansporation ANalysis and SIMulation System) is developed in order to provide a comprehensive model system that replaces the entire current transportation modelling paradigm. This system is in a continuous development since

1995, the initial version of this system was developed by Los Alamos National Laboratories under the US Department of Transportation and Environment Protection Agency support. The modelling system is composed of a series of modules that produce synthetic households, activities for individuals within these households, the choice of routes for movements among these activities, and the microsimulation of these movements to create traffic dynamics on the network. The module names itself specify their role within the system; these are Population Synthesizer, Activity Generator, Route Planner, Traffic Microsimulation and Feedback Controller. McNally and Rindt (2008) mentioned that the activity-based model which is mentioned as BB system in Table 2.1 is a central feature of TRANSIMS. This system creates a synthetic population for an urban area using census and survey data, while generating daily activities and its scheduling for each individual it maintains the individual identities during route planning and traffic microsimulation on the transport network. However, it has been noted that this system is dependent on extensive data defining the area being studied and has been very limited in application (McNally and Rindt 2008).

2.4.2.2 CEMDAP-VISTA Interaction

Lin et al (2008) presented the modelling system which is developed by integrating the activity-based modelling system CEMDAP (which stands for Comprehensive Econometric Model for Daily Activity Pattern) developed by Bhat et al (2004), and simulation-based dynamic traffic assignment module VISTA (which stands for Visual Interactive System for Transportation Algorithm) developed by Waller et al (1999). A fixed point problem is formulated to integrate both systems and criteria for measuring convergence are also discussed which are based on travel time and number of trips. This system also works on the generation of synthetic population based on census data, this synthetic population then feeds into CEMDAP in order to produce individual daily activity-travel pattern. These activity-travel patterns are then converted into trip tables by time-of-day and then feed to VISTA. VISTA, through its three main modules (i.e. Optimal routing, Path Assignment, Traffic Simulation) produces output in terms of vehicles and travel time, per interval and road segment, which is then converted into level-of-service states and feedback to CEMDAP. The process continuously iterates until some convergence of travel times and trip tables is achieved. Similar to TRANSIMS, this system also demands extensive data for the area being studied which limits its application.

2.5 GAPS IN COMBINED ACTIVITY SCHEDULING MODELLING

It has been established through the review of different activity scheduling models that the *combined modelling* approach in which traffic performance indicators are treated endogenously provides a better framework for the analysis of travel behaviour. This is because, this framework ensures consistency within the demand and supply sides, and the effect of any congestion mitigation policy can be examined on both sides together due to the employment of a feedback mechanism. Furthermore, it is evident that significant advancements have occurred on the demand and supply sides. As comprehensive modelling systems were developed within the AB approach, which model almost all the activity scheduling dimensions of the daily activity-travel pattern of an individual except route choice, on the other hand, within supply side analytical and simulation models are developed which dynamically assigns traffic on the road network. This shows that the progress in the two streams was achieved relatively independently, which is also evident from the fewer modelling efforts described in section 2.4. The following observations are made for the analytical or macroscopic combined models that represents simple daily activity-travel pattern (e.g. home-work tours).

- It has been noted that very few activity scheduling dimensions are considered. For example, out of three studies, two of them focused on modelling departure time choice and activity duration for home and work activity, the another attempt which consider route choice and departure time along with activity choice, did not considered activity duration. Activity-travel pattern that involves three or more activities are not considered within these models. Additionally, the choice of activity sequence, departure time, duration, route, location and travel mode are also not considered jointly.

- These models are not developed under the viewpoint of generalisation, because of which the effects of different modelling considerations cannot be investigated and therefore, comparison of the results cannot be made. For example, the effects of different demand models, effect of different supply models, effect of different time discretisation at both sides etc.

- The utility specification in all the models include time-of-day variant travel time and marginal activity utility, for representation of disutility of travel and the measurement of benefits obtained through activity participation respectively. The role of activity satiation effects in activity participation is completely ignored. This suggests that the implications of considering only time-of-day dependent utility of activity participation are not explored.

- It has also been noted that the Vickrey formulation of schedule delay penalty (which has been significantly used in the literature) is not incorporated in these models, instead time-of-day dependent utility formulation is employed without explaining the similarities, dissimilarities and advantages it offers over the Vickrey formulation.

- The impact of policies such as road capacity expansion, time variant tolls, time variant parking fee and flexible working hour's scheme on different activity scheduling dimensions are not investigated in detail.

The development of microscopic combined models by integrating AB model and traffic simulation models may address some issues (especially incorporation of fewer activity scheduling dimensions) which are observed for the macroscopic combined models. However, extensive requirement of data for operationalisation (which is not only for the population synthesis but for the calibration of the underlying AB model) significantly limits the application of these models particularly where the goal is to analyse the impact of broader policies. Furthermore, it has been noted that all the models discussed in different sections of this chapter are focused on the modelling of the *daily* activity-travel pattern of individuals with the assumption that all the weekdays are similar to each other. This is to say that, an individual for which an activity-travel pattern is predicted for a given working day, he/she follows the same activity-travel pattern for all other working days of the week. This might be not important in the context of morning commute because the impacts of intra weekday variations are not too significant on the morning peak spread (Pendyala 2003). However, in the context of daily activity-travel pattern the implications of this can be better explained with the following example: if an activity-travel pattern of an individual for a given day involves three activities, for example, home-work-shopping-home, it is then entirely infeasible for that individual to involve in the same pattern for the next day of the week provided that need for shopping activity has already been satisfied. In these circumstances, current models may significantly overestimate the number of shopping trips. No such modelling study to date has been reported which deals with intra-weekdays variations in the activity-travel pattern of individuals within combined modelling framework despite of significant empirical evidences, which are as follows.

- Hanson and Huff (1986) and Huff and Hanson (1986) present detailed discussions regarding the habitual and variable behaviour of individuals over time. They pointed out that when travel behaviour is examined in a 'disjointed' framework (say, a work trip examined in isolation from the overall daily activity-travel pattern); the observed variability is not significant on a day-to-day and week-to-week basis. However, when instead of using a disjointed framework, the overall daily activity-travel pattern is examined they found that variability is significant to a great extent. This suggested that individuals are performing different activity-travel patterns over the entire week days (e.g. on a given day they are involved in home-work tour but on some other day they are involved in home-work-shopping tour).

- Kitamura and van der Hoorn (1987) investigated the timing with which an individual replicates its travel pattern using a Dutch National Mobility panel data collected for two consecutive weeks which were six months apart. They found that about 30 percent of the male workers and 41 percent of the female workers had daily patterns of shopping participation on four or less of the days within two weeks (on other days they are performing simple home-work tour). Furthermore, other workers perform shopping activity more frequently but not all the days of the week.

- The recent empirical study of the weekly activity pattern conducted by Buliung et al (2008) using the Toronto Travel Activity Panel Survey, concludes that there exists a day-to-day variability in the activity-travel pattern of individuals. They found that individual activity scatter (measure of the activity participation in a day) dropped to a very low value on Wednesday and Thursday, however, in the initial part of the week (i.e. Monday and Tuesday) individual's activity scatter was significantly high. This suggests that individuals in the latter part of the week conduct a simplistic activity-travel pattern (i.e. home-work tour) while in the initial part of the week they are involved in more activities. They also suggested that this variation within weekdays may also raise questions concerning the extent to which the weekday/weekend distinction is particularly useful and meaningful with respect to activity-travel behaviour.

These studies made clear that day-to-day variability in activity-travel behaviour exists and is substantial. Therefore, incorporation of this notion in the combined modelling framework would significantly improve the current combined models.

2.6 WAY FORWARD

Arising from the issues mentioned in this chapter, especially in section 2.5, this research focuses on the development of a combined model for daily activity-travel patterns. This research will also explore those issues that are needed to be comprehensively examined, as observed within an analytical combined model. In addition to this, the model will be developed in such a manner that it can easily be extended to incorporate the weekly activity-travel pattern. The next three chapters discuss the issues and modelling considerations for the combined model. Chapter 6 then demonstrates the development of the model for a simple activity patterns and chapter 7 examines the properties of the developed model through numerical experiments. Chapter 8 extends the model in a way that it incorporates the weekly activity-travel pattern and demonstrates some numerical experiments as well. Chapter 9 concludes the research reported in this thesis.

Chapter 3

CONSIDERATIONS FOR THE COMBINED MODEL-DEMAND SIDE

3.1 GENERAL

The literature review reported in the previous chapter regarding the activity scheduling modelling approaches establishes that the combined modelling framework is more appropriate for the analysis of activity-travel behaviour of individuals in response to any measure of travel demand management. The development of a model within the combined modelling framework requires a thorough understanding of its different components and issues within each. These components in their simplistic terms can be represented as follows:

- *Demand side*
- *Supply side*
- *Integration of demand and supply sides.*

The demand side component within the combined modelling framework usually deals with the overall setting of the problem, considering the employed underlying decision making methodology. This includes, what type of activity scheduling dimensions are required to illustrate (given with the known activity agenda of an individual) in order to form a problem and which decision making methodology will render a suitable framework for the problem analysis. Although each decision making methodology has its own rationale, their selection for the combined model can also be depend on the available resources to fulfil the study objectives and the nature of other components of the combined model in terms of their compatibility for the integration. The aim of this chapter is to explore the demand side of the combined model within the circumference of the objectives formulated for this study, and then to put forward a profound base at the demand side for the development of a combined model. The rest of this chapter is organised as follows.

The next section examines the decision making methodologies in more detail, which has already been discussed to an extent in chapter 2, in order to justify the employment of one of them for this study. After that an extended analysis is carried out of the methodology, which has been found appropriate for this study in earlier sections. The analysis includes a rigorous examination of the relevant concepts used in the previous studies in order to determine the extent to which they represent the dependence of individual decisions regarding their activity scheduling dimensions. Finally, the models required for operationalise the demand side are discussed followed by the summary of the chapter that highlights the conclusions drawn from the work reported in this chapter.

3.2 DEMAND SIDE- SELECTION OF DECISION MAKING METHODOLOGY

In chapter 2, two distinctive decision making methodologies have been discussed briefly. In this section these methodologies are further elaborated and compared with each other within the notion of the objectives of this research. Finally, the decision is made regarding the suitability of a particular methodology in order to form a premise for the development of a combined model.

3.2.1 Rule-based or Computational Process Model

The premise of this decision making methodology is the notion that individuals search a solution space only partially because of their limited cognitive ability. Their search is based on heuristics that often yield satisficing outcomes, which are not necessarily optimal (Gärling et al 1995). The models based on this methodology use a set of empirically derived rules in the form of a condition-action (If-Then) structure in order to reach a particular decision. The model system ALBATROSS (illustrated in Table 2.1) is a fine example of a fully operational CPM. In this model rules or heuristics are described in a descriptive format and no mathematical or algebraic functions are used to evaluate the final outcome. For example, Arentze and Timmermans (2004) shows that heuristics based on space-time constraints for determining the set of the location choices for a particular activity in a given schedule *S* is as follows:

A location *l* is considered feasible if the following two constraints are met:

$$\exists\ g \in G_l,\ g \in G\{a(\tau)\} \tag{3.1}$$

$$T_{l_g}^{f\max}(\tau) - T_{l_g}^{s\min}(\tau) \geq v^{\min}(\tau) \tag{3.2}$$

where, τ is an index of activities in a given schedule S, G_l is the set of known facility type at location *l*, $G\{a(\tau)\}$ is the set of facilities compatible with the activities of type $a(\tau)$, *g* is the type of facility which is required to perform a particular activity, $v^{\min}(\tau)$ is the minimum duration of an activity and $T_{l_g}^{f\max}(\tau)$ and $T_{l_g}^{s\min}(\tau)$ are defined as follows

$$T_{l_g}^{f\max}(\tau) = \max\left\{{}_d t_{l_g}^{\min}, T^{f\min}(\tau - 1) + t_l^t(\tau)\right\} \tag{3.3}$$

$$T_{l_g}^{S\min}(\tau) = \max\left\{{}_d t_{l_g}^{\max}, T^{s\max}(\tau + 1) - t_l^t(\tau + 1)\right\} \tag{3.4}$$

where, ${}_d t_{l_g}^{\min}$ and ${}_d t_{l_g}^{\max}$ are the known opening and closing times of facilities of type *g* at location *l* on day *d*, $T^{f\min}$ is the earliest end time and $T^{s\max}$ the latest start time of the pervious and next activity respectively and t_l^t is travel time to the activity location using the mode chosen in a previous step. After the definition of the set of location choices through equations 3.1 to 3.4, another set of heuristics is required to illustrate the different ways of trading-off required travel time with the attractiveness of locations. Accordingly, the final choice of activity location is then made. This example suggests that the development process of the set of heuristic rules for different dimensions of scheduling of activities requires rigorous examination of data, so that some sort of generality in the rules can be represented. This introduces the limitation in terms of application of these models, as the set of proposed rules (heuristics) that suits the sample does not necessarily cover all cases that might occur in the forecast considering the population or another sample of respondents.

It has been suggested in many studies of CPMs that these models are able to represent the heuristic and context dependent nature of choice behaviour of individuals compared to the utility maximisation framework (Ettema et al 1996, Arentze and Timmermans 2004). However, due to the lack of ability of the CPM structure to incorporate a framework within which models are statistically estimated and calibrated, these models are not often used in practice. The existence of these models provides a test bed for the alternative methodologies especially for the models which are based on random utility theory (McNally and Rindt 2008, Buliung 2005, Arentze and Timmermans 2004). However, so far to the best of the author's knowledge, there is no comprehensive study exists which compares the outcome obtained from the rule-based and an equivalent random utility maximisation based models.

3.2.2 Random Utility Maximisation

The models based on econometric principles are developed on the rationale that individuals maximise the utility for the selection of their choices. Discrete choice analysis methods which employ random utility theory have played a prominent role in the development of the econometric activity-based modelling systems (Ben-Akiva and Bowman 2000, Bhat et al 2004). The framework of discrete choice models is such that it provides the output as probabilities of choosing each of the available alternatives, and in doing so the individuals can maximise their perceived utility only and predicted behaviour is not entirely representative of the optimal behaviour. This is evident from the following expression (equation 3.5) which represents two components of the utility (U_{in}, utility of an alternative *i* for an individual *n*) i.e. systematic or observable part (V_{in}) and random or unobservable part (ε_{in}).

$$U_{in} = V_{in} + \varepsilon_{in} \tag{3.5}$$

Questions can be raised when random utility theory is operationalised through some assumptions for the random component (i.e. pre-specified behaviour of the random component in the utility through random distributions). For example, the fundamental model (i.e. Multinomial Logit model) is derived on the basis of distribution of the random component as an extreme value Gumbel distribution with independent and identical error structures across alternatives and individuals. However, recent advancements in the area of discrete choice modelling allow for the relaxation of certain strict assumptions of the random component of the utility (e.g. relaxation of independent and identically distributed error structure across alternatives, relaxation of response homogeneity and error variance-covariance homogeneity) render a more sophisticated structure of the operational models. These models are known as Nested logit model, Ordered generalised extreme value model, Mixed logit model and Heteroscedastic multinomial logit model etc. The comprehensive discussion on this area has been provided by Bhat (2002).

The interesting notion regarding this methodology is that it incorporates any number of explanatory variables to represent the systematic part of the utility, and there are methods available (i.e. maximum likelihood) through which this systematic part of the utility is actually calibrated (estimation of the parameters attached to explanatory variables) from data. Furthermore, on some occasions especially in the models of scheduling of activities, the specification of the systematic part of the utility is based entirely on some generalised linear or non-linear functions (e.g. Schedule delay penalty formulation (Vickrey 1969), time-of-day and duration dependent marginal utility functions for different activities (Ettema and Timmermans 2003, Joh et al 2003). However, it is noted that non-linear functions induce complexity (such as non-convexity in the optimisation problem) in the estimation of the discrete choice models. To overcome this, some studies presented alternative algorithms to solve this estimation problem (Ettema and Timmermans 2003, Joh et al 2003). Sometimes, these generalised functions are found to provide a relatively profound base for the representation of the observed component (systematic utility) in the utility. This is evident from the wide use of the Vickrey (1969) schedule delay formulation for modelling the departure time choice of an individual.

3.2.3 Study Objectives and Decision Making Methodology

In the above two sub-sections (3.2.1 and 3.2.2), properties and different considerations of the two distinctive decision making methodologies are discussed in detail. It has been found that both methodologies have their respective merits and demerits relative to the premise on which they are based. This sub-section illustrates what type of characteristics and features are necessary for a decision making methodology which fulfil the objectives set out for this study, and based on this a decision will be made regarding the selection of a particular methodology. The following points represent the key features underlying the objectives of this study:

- *Development of an analytical combined model;* this will be achieved using a macroscopic representation of the behaviour of a population on the demand and supply sides.

- *Methodological nature of the study;* this suggest that the focus of the study should be on the development of the model and its analysis through numerical experiments rather than the collection of data and model estimation. This study aims to combine demand and supply sides not only for the single trips but for the daily and weekly activity-travel patterns (home-based tours), the model development exercise and its analysis itself a huge task and require considerable efforts in terms of time. Therefore, the collection of data and estimation of the model are not included in the objectives of this study, however, this can be done in the form of a completely separate study using the reported research in this thesis as a base.

- *Easy and ready availability of the operational models;* this suggests that the operational models required within the demand and supply sides can be easily accommodated (integrated) within the framework of a combined model in order to perform numerical experiments which are necessary for the analysis of the developed combined model.

The above three points represents the underlying features of this study, they suggest that decision making methodology which is based on the random utility is more appropriate for this study. This is because this methodology is able to support the macroscopic representation of the behaviour of a population (i.e. a group of individuals selecting a particular alternative from the given choice set). In its principle, the random utility models are estimated with disaggregate data (i.e. based on individuals) but the incorporation of probabilistic notion within the framework of these models make the output obtained from them as expectations on an individual level (i.e. the output at an individual level does not indicate which alternative is selected). This output needs to be aggregated across all individuals to provide an expected total usage (market shares) which maintains a macroscopic representation. On the other hand, rule-based models always use microsimulation environment within their modelling framework (i.e. each individual is modelled separately) and the output from these models are always on an individual level and a separate program need to be run to form an output which is aggregated across alternatives for the macroscopic representation.

The second issue further favours the selection of random utility methodology, because in the absence of a data set one cannot formulate heuristic rules for scheduling of activities. Furthermore, there is no such study exists which presents the set of heuristic rules for all the scheduling dimensions of an activity-travel pattern, so that these can be exploited in this study. In addition to this, formulation of a computer programme in order to operationalise the model which is based on heuristics also requires extensive efforts at the demand side let alone the integration of the demand and supply side in order to develop a combined model. Random utility theory in this regard, gives a flexibility to use generalised functions for the representation of systematic utility. Furthermore, there are many modelling studies exist which utilise these generalised functions and have estimated their parameters. On the third issue, again the random utility theory offers an advantage over rule-based models. Because several operational models (i.e. Multinomial logit and Nested logit models etc) are available in the literature which are well researched and documented and the mathematical construction of these models is such that lesser efforts are required to program them.

3.3 SPECIFICATION OF SYSTEMATIC UTILITY FOR ACTIVITY SCHEDULING

This section represents an analysis of the utility functions used in the discrete choice models for modelling activity scheduling dimensions. The section starts with description of utility specifications in the econometric AB modelling systems. Some generalised functions are also discussed which are used for modelling different scheduling dimensions. After these descriptions, an analysis is presented in order to differentiate the primary features and characteristics of these generalised functions with the aim to identify a specification that is more appropriate for the development of a combined model.

3.3.1 Utility Specification in Econometric AB Modelling Systems

Within econometric Activity-based (AB) modelling systems, the BB system (Bowman and Ben-Akiva system) has achieved the status as the first true econometric AB model system. PETRA and TA (Tel-Aviv) systems which are developed afterwards have very similar features (in terms of the model structure i.e. hierarchical) to those found in the BB system (refer to table 2.1). The specification of the systematic utility in these models is based on the variables that represent: alternative-specific constants, alternative-pecific level of service variables, socio-economic variables and the variable that represent logsum (i.e. this is included in order to link the upper level dimensions of scheduling with a lower level scheduling dimension in a hierarchical model structure). To understand the variables involved in the specification of systematic utility of these AB modelling systems an example is illustrated in the following paragraphs which discusses the specific application of the BB system for a Portland region.

In the BB system apart from other scheduling dimensions (i.e. tour type choice, time-of-day choice), destination and mode choice are modelled jointly for primary and secondary tours. Primary tours are those in which the most important activity of the day is included (i.e. work or school), and all other tours in a same day conducted by the same individual are categorised as secondary tours. However, the choice of mode and destination for the primary tours is placed on a higher level in the nested structure than the choice of mode and destination for the secondary tours. The time-of-day choice for primary tours is also modelled and is placed on a higher level than the choice of mode and destination for primary tours. Bowman and Ben-Akiva (2000) shown an example of the application of the BB system for the Portland region, they estimated MNL models for the choice of mode and destination for each tour type and time-of-day choice for the primary tour. For the mode and destination model, the choice set contains 48 alternatives representing 8 possible geographic zones as destinations and 6 modes available for each destination. For the time-of-day choice model, the choice set comprised of 16 alternatives representing the combination of 4 time periods (Morning peak, Midday, Evening Peak and Other) in which the whole day is divided. Each of the 16 alternatives comprised of 1 of 4 time periods for departure from home to the primary destination and 1 of 4 time periods for departure from the primary destination returning home. The variables that were found significant in representing the systematic utility are described in Table 3.1.

It should be noted that there is no logsum variable in the secondary tour model of mode and destination choice because this dimension is placed at the last level in the nested structure. From table 3.1, it is clear that alternative-specific constants capture the significant share in representing the individual’s behaviour regarding their choices. However, some socio-economic variables and level of service variables also show their significances.

Table 3.1: Significant variables in the Time-of-day, Mode and Destination Choice Models within the BB System

Time-of-day choice model	Mode and Destination choice models	
Primary Tour	**Primary Tour (Work as Primary activity)**	**Secondary Tour**
Basic alternative specific constants: Midday to Midday (travel to and from primary destination) Before AM peak to AM Peak Before AM Peak to Midday Before AM peak to PM Peak AM peak to AM Peak AM peak to Midday AM peak to PM peak AM peak to after PM Peak Midday to PM peak Midday to after PM peak PM peak to PM peak PM peak to after PM peak After PM peak to after PM peak	*Mode Constants:* Drive alone (da) Shared ride (sr) Transit with auto (ta) Transit with walk (tw) Walk alone (wa) Bicycle (bi)	*Mode Constants*: Drive alone (da) Shared ride (sr) Transit with auto (ta) Transit with walk (tw) Walk alone (wa) Bicycle (bi)
	Level of Service Variables: Cost for motorised modes Cost for persons with employer incentive (da) Cost for persons with employer transit incentive (ta) Cost for persons with employer transit incentive (tw) Cost /Income, motorised modes In-vehicle time (auto) In-vehicle time (transit) Out-of-vehicle time (auto) Out-of-vehicle time (transit) Distance , walk Distance, bicycle	*Level of Service Variables:* Cost /Income, motorised modes In-vehicle time (auto) In-vehicle time (transit) Out-of-vehicle time (auto) Out-of-vehicle time (transit) Distance , walk Distance, bicycle
Activity Pattern dummy variables Work purpose, alternatives involving at least 1 peak period Work purpose, alternative is AM peak to PM peak Work purpose, alternative is before AM peak to before PM peak, or after AM peak to after PM peak Work purpose, alternative is after PM peak to after PM peak	*Socio-economic Variables:* Autos per driver, (sr) Autos per driver, (ta) Autos per driver, (tw) Autos per driver, (wa) Autos per driver, (bi)	*Socio-economic Variables:* Autos per driver, (sr) Autos per driver, (ta) Autos per driver, (tw) Autos per driver, (wa) Autos per driver, (bi) Household income, tw Household income, wa Household income, bi
Logsum Expected maximum utility from primary tour mode and destination choices	*Alternative Specific Dummies:* Age under 20 , (bi)	*Alternative Specific Dummies:* Mode matches primary tour mode, (da) Mode matches primary tour mode, (sr) Mode matches primary tour mode, (bi) Work tour, destination matches primary tour destination
	Size Variables: Employement in CBD Zones Employment in non-CBD Zones	*Size Variables:* Employement in CBD Zones Employment in non-CBD Zones
	Logsum Variables: Expected maximum utility from secondary tour mode and destination choices	*Logsum Variables:* No Logsum variable because this choice dimension lies at the last level of the nested model

Utilization of the variables (presented in Table 3.1) for the representation of systematic utility may demand collection of data that represent activity-travel dairy of individuals. The underlying features of the objectives of this study suggest that data collection exercise should be avoided in order to focus entirely on the development of the combined model, it is therefore necessary to look for potential alternatives such as generalised functions for representation of systematic utility. Some generalised functions are discussed in the following sub-sections which are used for modelling activity scheduling dimensions with a comprehensive illustration of their properties.

3.3.2 Schedule Delay Formulation (SDF)

This formulation is based on the key works of Vickrey (1969) and Small (1982) for modelling departure time choice related to the morning commute. The schedule delay formulation (SDF) is widely used within the framework of discrete choice modelling. This is evident from the numerous studies (small, 1982 and 1987; Abkowitz, 1981; Bates et al, 1990; Daly et al, 1990), which either uses the same formulation proposed by Vickrey or improved it further to a smaller extent for better representation of the context of their study. Figure 3.1 represents the piece-wise linear SDF which is similar to that used in most of the above mentioned studies.

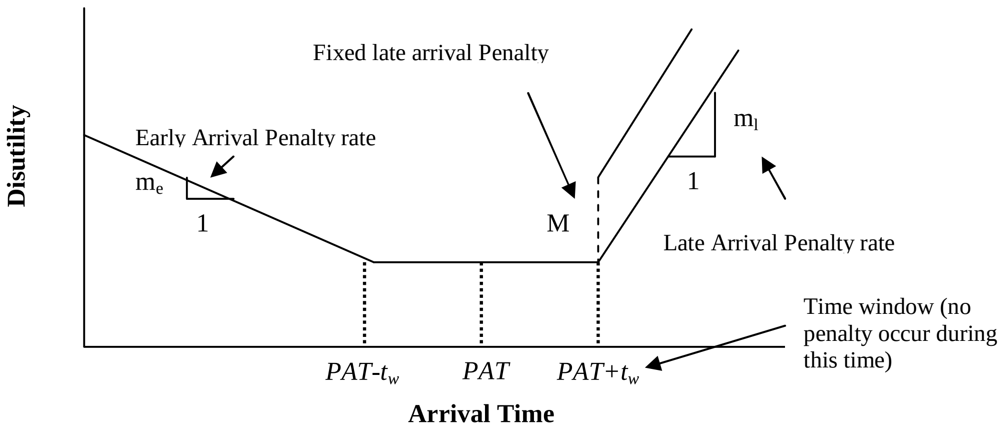


Figure 3.1: Piece-wise Linear Schedule Delay Formulation

The SDF presented in figure 3.1 can be expressed mathematically along with the time-of-day variant travel time for the representation of the systematic utility for the departure time choice modelling. This is as follows

$$V(t) = a\,R(t) + \mathbf{m_e}.SDE(t) + \mathbf{m_l}.SDL(t) + \mathbf{M}.LP(t) \tag{3.6}$$

where, $V(t)$ represents deterministic utility component of individual utility at departure time t and $R(t)$ shows travel time at departure time t. $a, \mathbf{m_e}, \mathbf{m_l}, \mathbf{M}$ are the negative parameters associated with travel time, early time, late time and fixed late penalty respectively. The early and late schedule-delay attributes $SDE(t)$ and $SDL(t)$ are defined as

$$SDE(t) = \max\left(0, PAT - t_w - \left(t + R(t)\right)\right) \tag{3.7}$$

$$SDL(t) = \max\left(0, \left(t + R(t)\right) - \left(PAT + t_w\right)\right) \tag{3.8}$$

where, *PAT* represents preferred arrival time of an individual to participate in an activity, t_w represents time window within which no schedule delay penalty occurs. *LP*(*t*) in equation 3.6 gives a fixed late penalty, which is defined as follows

$$LP(t) = \begin{cases} 1 & \text{if } t + R(t) > PAT + t_w \\ 0 & \text{otherwise} \end{cases} \tag{3.9}$$

Figure 3.1 and expressions (3.6 to 3.9) suggest that an individual who wants to depart at time *t* for participation in an activity (say work activity) from the current location of his stay (say home), will make a trade-off between the disutility associated with the travel time and the disutility associated with the schedule delay penalties governed by the *PAT*. The *SDE* cost component of the penalty structure highlights the fact that if the individual arrives early at the work location (i.e. earlier than *PAT* - t_w) then he may incur some loss of utility at the previous location (i.e. at the origin which in this case is home). Although an individual may gain some minimal utility by being at its destination early but this gain of utility is lower in comparison to the loss of utility at its previous location. This is true in the case of work activity at the destination, due to its anchoring nature relative to a particular time-of-day i.e. work start time. The marginal loss of this utility is considered constant with respect to time and represented through $\mathrm{m_e}$ in equation (3.5). This suggests that if a person arrived 10 minutes earlier than the *PAT* - t_w, then he may incur loss of utility of around “10. $\mathrm{m_e}$ ”, and if a person arrived 30 minutes earlier than the *PAT* - t_w, this loss is around “30. $\mathrm{m_e}$ ”. The representation of loss of utility in this manner may be true in a simplistic sense, but it is necessary to examine the nature of the activity at the previous location and its importance for an individual. For example, a loss of 10 minutes in home activity participation would be expected to be of little significance among the individuals; however, if the same is increased up to 30 minutes, individuals may consider it as a significantly high loss (as many household or individual related chores can be performed which may cause tremendous amount of effect on daily life). Therefore, the value of $\mathrm{m_e}$ and its linear relationship with time-of-day, can vary for different activities at the previous location. The provision of time window (t_w) across *PAT* within which individuals are not subjected to any penalty, may explain this notion to some extent but the assumption of linearity regarding the marginal schedule delay cost m_e can be challenged as it only able to provide a naïve representation.

The components of the SDF which represents late arrival penalties are *SDL* and *LP*. The *SDL* cost is representing the loss of utility an individual incurs by being late in arrival at a particular location with respect to the *PAT*. Similar to m_e, the marginal cost of lateness m_l is also considered as a constant for simplicity in the SDF. The step function *LP* in the form of the fixed late arrival penalty over *SDL* cost represents the more severe repercussions to individuals on being late for participation in an activity which is highly constrained by time-of-day (i.e. work activity). This is based on the empirical findings of Small (1982). It is worth mentioning here that the parameter m_l in the late arrival penalty structure that includes in *SDL* is interpreted as a representation of schedule delay cost which carries the combined effect of utility of activities at the origin and destination (Small 1982 and Small 1987). The same interpretation of schedule delay costs is also followed in this thesis.

Arising from the above discussion, *SDE* and *SDL* represents the schedule delay early and late costs which carries the combined effect of utility of activities at the origin and destination. In section 3.3.3, a concept of time-of-day dependent marginal utility of an activity is discussed in detail in order to form a basis for comparison of this concept with the SDF, because both of these formulations are somehow related to time-of-day.

3.3.3 Time-of-day based Marginal Utility of an Activity

The concept of time-of-day based marginal utility of an activity was more formally introduced by Polak and Jones (1994) in the context of activity scheduling. This concept is further refined by Ettema and Timmermans (2003), who proposed specific functional forms for the marginal utility of activities. The central theme of this concept is that *for each time-of-day t, there exists a marginal utility (which may vary over time), expressing the utility gained from one additional time unit of activity participation*. The functional forms introduced so far in the literature (Joh et al 2002, Ettema and Timmermans 2003, Heydecker and Polak 2006) are based on bell-shaped and piece-wise constant profiles. These profiles assume that the marginal utility of an activity is high at a certain time-of-day and decreases as one moves away from that time-of-day. Figure 3.2 shows some examples of marginal utility profiles based on certain mathematical functions for different activities. For example, in the case of home activity, the marginal utility of stay-at-home is considered higher in the early morning, late evening and at night than the day time because people prefer to stay at home for regular home activities in these times such as having a family breakfast, family dinner, watching TV and sleeping. Similarly, for work activity high marginal utility is considered during the core working hours i.e. 9am to 4pm.

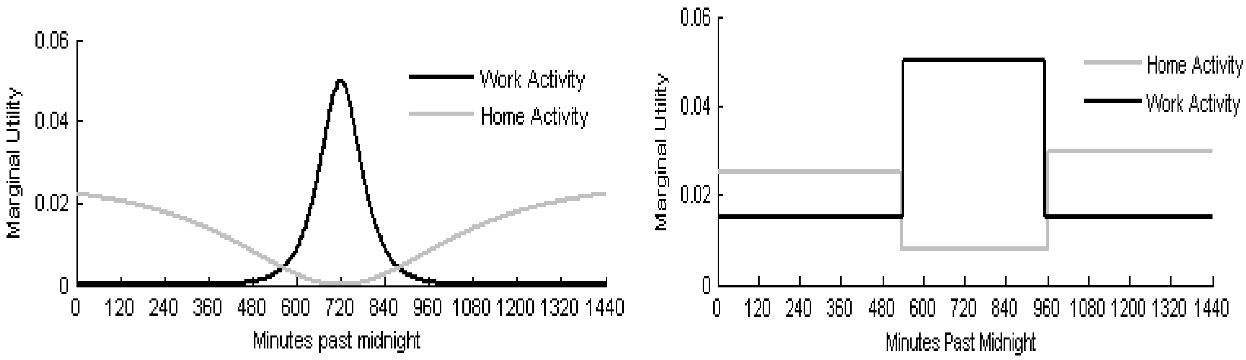


Figure 3.2: Marginal utility profiles for home and work activities

The mathematical representations of the marginal utility profiles shown in Figure 3.2 are given in Table 3.2 along with the values of the parameters through which profiles represented in Figure 3.2 are drawn.

The utility of an activity can be derived by integrating the marginal utility function over a certain time period. For example, if an individual participates in an activity for a certain amount of time i.e. with activity starting time as t_s and the ending time as t_e, then the utility of activity participation $V_a(t_s, t_e)$ is given by

$$V_a(t_s, t_e) = \int_{t_s}^{t_e} V_a^{'}(x)\, dx \tag{3.10}$$

Table 3.2: Mathematical definitions of marginal utility profiles

Nature of profile	Mathematical Functions	Parameters Values used for Different Activities in figure 3.2
Bell-Shaped	$V_a^{'}(t)=\left(\dfrac{\gamma\,\beta U^{\max}}{\exp\left(\beta[t-\alpha]\right)\left(1+\exp\left(-\beta\left[t-\alpha\right]\right)\right)^{\gamma+1}}\right)$ (Ettema and Timmermans 2003, Joh et al 2002)	Work Activity: $\alpha=720$ minutes past midnight, representing the highest marginal utility point on time-of-day axis $\beta=0.02$ per min, representing the steepness of the increase/decrease around the highest marginal utility point $\gamma=1$, controls the symmetry of the profile $U^{\max}=5$ Utils, represent the area under the curve
Inverse Bell-Shaped	$V_a^{'}(t)=h_o-\left(\dfrac{\gamma\,\beta U^{\max}}{\exp\left(\beta[t-\alpha]\right)\left(1+\exp\left(-\beta\left[t-\alpha\right]\right)\right)^{\gamma+1}}\right)$ (Zhang et al 2005)	Home Activity: $\alpha=720$ minutes past midnight, $\beta=0.04$ per min, $\gamma=1$, $U^{\max}=12.5$ Utils $h_0=0.025$ Utils/min ; This parameter is responsible for reversing the bell-shaped curve and also reversing the effect of $U^{\max}$.
Piece wise constant	$V_a^{'}(t)=\begin{cases} a & 0<t\le t_a \\ b+a & t_a<t\le t_b \\ b+a+c & t_b<t\le 1440 \end{cases}$ (Heydecker and Polak 2006)	Work Activity: $a=0.015$ Utils/min, $b=0.035$ Utils/min, $c=-0.035$ Utils/min Home Activity: $a=0.025$ Utils/min $b=-0.017$ Utils/min, $c=0.022$ Utils/min

where, $V_a^{'}(t)$ represents time-of-day t dependent marginal utility in Utils/min of an activity, α, β, γ, $U^{\max}$, h_0, a, b, c are parameters that generally controls the shape of the profile.

Equation (3.10) suggests the fact assumed in this approach that one unit of activity engagement at time-of-day *t* will always yield utility based on the specification of the time-of-day based marginal utility function. The effect of "history" or more specifically the duration for which an activity is performed is not taken into account in this approach. For example, 1 min of work done at 2:00 pm may yield a different utility if one started working at 7:00 am than if one started working at 11:00 am. Therefore, a satiation or activity fatigue effect needs to be considered along with the time-of-day effect for measuring the utility of activity participation. The next section provides a comparison between the SDF and time-of-day based marginal utility formulation (MUF) as these are related to each other in the aspect of representing time-of-day effect on activity participation.

3.3.4 Comparison of MUF with SDF for home-to-work trip context

In figure 3.1, the schedule delay formulation (SDF) is presented for the direct measurement of disutility; it should be noted that constant marginal disutilities ($\mathrm{m_e}, \mathrm{m_l}$) in the SDF (Vickrey and Small formulation) represent the marginal disutility of arriving early and late at work activity location with respect to a set PAT. The marginal disutility rate of arriving early ($\mathrm{m_e}$) is usually interpreted as the difference in the marginal cost associated with the previous activity (in this case home) and current activity (in this case work), as an individual arrives more early at work than his *PAT* then he is losing greater utility by not being at home than the amount of utility he is gaining from being at work earlier. Similarly, the marginal disutility rate of arriving late ($\mathrm{m_l}$) is usually interpreted as the difference between the marginal cost associated with the work activity and home activity, as individual arrives more late at work activity than his PAT then he is losing more utility at work than the utility gained by being at home. This suggests that disutility rates in the SDF are defined in such a manner that it contains the effect of both activities (i.e. previous activity (home) and current activity (work)) together with respect to the *PAT*. The discussion in this section presents *how a time-of-day based marginal utility formulation (MUF) can be interpreted in terms of SDF* (described in figure 3.1 and through equation 3.6). Figure 3.3 present marginal utilities of being at home and work activities against the plotted clock time (i.e. from 360 to 720 minutes past midnight) following the bell-shape curve expression shown in table 3.2.

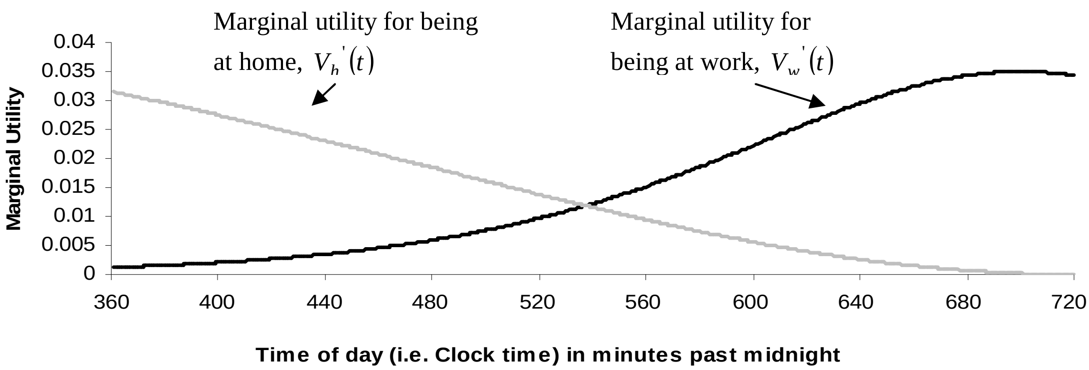


Figure 3.3: Marginal utility profiles for home and work activities with respect to clock time

Figure 3.3 represents marginal utilities against the *clock time*; however, figure 3.1 which represents SDF is plotted as a disutility against *arrival time* at work activity location. In order to compare MUF and SDF, it is desirable to convert figure 3.3 in such a manner that it represent the same axes shown in figure 3.1 (i.e. disutility against arrival time). This has been done by assuming constant travel time at all clock times, and measuring the areas under the two curves shown in figure 3.3 which in combination represent the utility by an individual in order to reach at work activity location at time t_{at} (i.e. arrival time). The mathematical expression for measuring these areas is given as follows:

Utility for arriving work activity location at time t_{at} is given by equation (3.11), time t_d is the corresponding departure time for arrival at t_{at} ,

$$V\left(t_{at}\left(t_d\right)\right)_g = \int_{360}^{t_d} V_h{}'(x)dx + \int_{t_{at}(t_d)}^{720} V_w{}'(x)dx + \lambda\, R\left(t_{at}\left(t_d\right)\right) \tag{3.11}$$

Utility from equation (3.11) is actually representing different areas under the marginal utility curves for home and work activities. If these are measured for the marginal utility curves shown in figure 3.3, then the following figure is obtained.

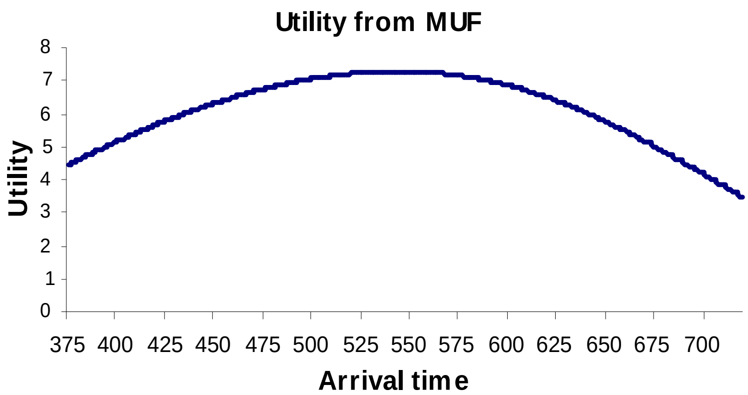


Figure 3.4: Utility from MUF for the morning commute against arrival time at work activity location

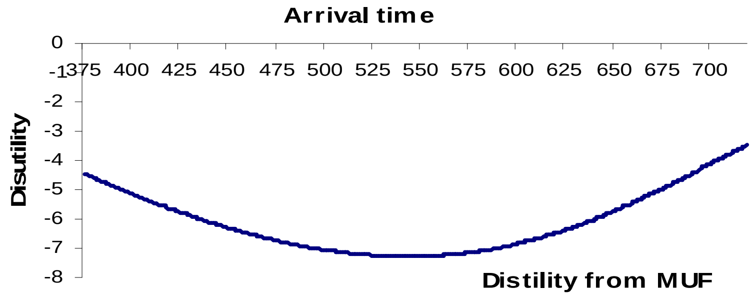


Figure 3.5: **Disutility from MUF for the morning commute against arrival time (in minutes past midnight)**

When utility (as shown in figure 3.4) is multiplied with -1 in order to represent disutility and plotted against arrival time then figure 3.5 is obtained which is shown above. Figure 3.3 and figure 3.5 have different things on the horizontal axis (i.e. clock time and arrival time). The disutility plot against arrival time from MUF (figure 3.5) can be directly compared with the plot in which SDF was shown (refer to figure 3.1). If an individual's *PAT* is assumed as a time of day at which two curves (marginal utility profiles of home and work activities) meets in figure 3.3 (i.e. around 540 minutes past midnight), then figure 3.5 suggests the similar anecdote as presented for SDF in section 3.3.2 (i.e. an individual incurs higher disutility if his arrival time at work activity location is away from either side of his *PAT*). The rate of change of disutility from MUF before and after *PAT* is dependent on the manner in which two marginal utility curves (for home and work activities) are defined, this is illustrated as follows:

Differentiating –ve of equation (3.11) (which represents disutility from MUF) with respect to arrival time t_{at} (i.e. for small increase in arrival time Δt_{at}), then the following is obtained.

$$\frac{d\left(-V\left(t_{at}\left(t_d\right)\right)\right)}{d\,t_{at}} = \left[-\int_{360}^{t_{at}(t_d)-R(t_{at}(t_d))+\Delta t_{at}} V_h'(x)dx - \int_{t_{at}(t_d)+\Delta t_{at}}^{720} V_w'(x)dx\right] - \left[-\int_{360}^{t_{at}(t_d)-R(t_{at}(t_d))} V_h'(x)dx - \int_{t_{at}(t_d)}^{720} V_w'(x)dx\right]$$

Since $t_{at}\left(t_d\right) - R\left(t_{at}\left(t_d\right)\right) = t_d$

and $R\left(t_{at}\left(t_d\right)\right)$ is assumed constant therefore its derivative is zero.

$$\frac{d\left(-V\left(t_{at}\left(t_d\right)\right)\right)}{d\,t_{at}}=\left[-\int_{360}^{t_{at}(t_d)-R(t_{at}(t_d))+\Delta t_{at}} V_h{}'(x)dx+\int_{360}^{t_{at}(t_d)-R(t_{at}(t_d))} V_h{}'(x)dx\right]+\left[\int_{t_{at}(t_d)}^{720} V_w{}'(x)dx-\int_{t_{at}(t_d)+\Delta t_{at}}^{720} V_w{}'(x)dx\right]$$

As Δt_{at} is a small time interval, therefore the above expression can be written as:

$$\frac{d\left(-V\left(t_{at}\left(t_d\right)\right)\right)}{d\,t_{at}}=-V_h{}'\left(t_{at}\left(t_d\right)\right)+V_w{}'\left(t_{at}\left(t_d\right)\right) \qquad (3.12)$$

Equation (3.12) represents the disutility rate from MUF in terms of marginal utility curves of home and work activities. According to equation (3.12) and the marginal utility curves shown in figure 3.3, the rate of change of disutility from MUF is –ve before the time-of-day when these two curves intersect each other (i.e. at 540 minutes past midnight) and it is +ve afterwards. This is similar to SDF, as early arrival disutility rate ($\mathbf{m_e}$) and late arrival disutility rate ($\mathbf{m_l}$) in SDF are defined in the same manner. However, in SDF, *PAT* is exogenously defined which is in contrast to MUF where shape of the marginal utility curves for activities implicitly considers this notion (Ettema et al 2007 and Ettema and Timmermans 2003). Despite these characteristics (both of these formulations are dependent on time-of-day), the effect of activity history or duration on the scheduling of activities is ignored in both formulations.

3.3.5 Duration based Marginal Utility of an Activity

Activity satiation effect implies that the utility derived from one additional time unit of activity participation diminishes with increasing duration. Many activities are likely to be subject of fatigue or satiation effects; therefore, it is necessary to adopt a formulation which ensures that utility of activity participation along with time-of-day dependency also show dependency on the *duration* of the activity.

Following the above, Yamamoto et al (2000) presented a logarithmic function of activity duration for measuring utility of an activity and when this function is differentiated it gives the following

$$V_a^{'}(\tau_a) = \eta_a \ \frac{1}{\tau_a} \qquad \qquad \forall \tau_a > 0 \tag{3.13}$$

where, $V_a^{'}(\tau_a)$ represents the duration based marginal utility of an activity *a*, τ_a represents the duration or time allocated for an activity (which cannot be negative and should be greater than zero) and η_a represents the scale parameter with units as utils. The specification presented in equation (3.13) assumes that marginal utility of an activity is immediately starts diminishing as soon as the activity starts, however, there may be some activities for which an optimal duration exists, before which their duration dependent marginal utility is increasing and after that optimal point their marginal utility may start declining. This can be observed in cases where activity duration is not constrained; there are chances that the individual allocate more time to an activity than its optimal duration. In such situation, logarithmic function is not appropriate. Joh et al (2005) utilised a similar mathematical function as discussed earlier for time-of-day dependent marginal activity presented by Ettema and Timmermans (2003), however, instead of time-of-day dependency their function is dependent on activity duration. This function form assumes that too little time to be involved in an activity will imply a low utility and too much time may lead to boredom and satiation. This can be given as follows, with the same parameters (having the similar role in defining the shape of the curve) as used in time-of-day dependent marginal utility.

$$V_a^{'}(\tau_a) = \left(\frac{\gamma \, \beta U^{\max}}{\exp\left(\beta[\tau - \alpha]\right)\left(1 + \exp\left(-\beta\left[\tau_a - \alpha\right]\right)\right)^{\gamma+1}} \right) \tag{3.14}$$

Figure 3.6 show both functional forms (i.e. logarithmic and bell-shaped) of the duration based marginal activity utility.

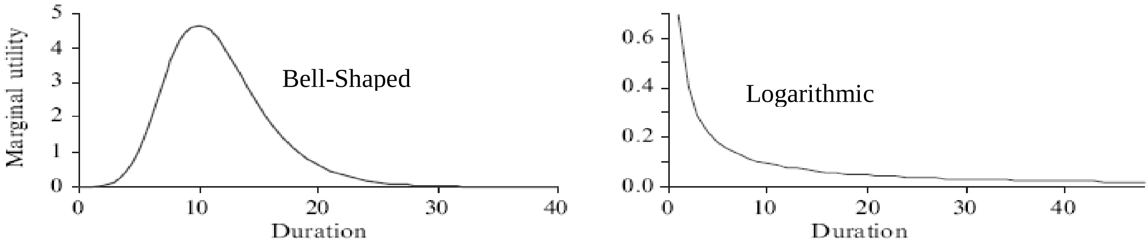


Figure 3.6: Duration based marginal activity utility profiles

The use of duration based marginal activity utility together with the time-of-day dependent marginal activity utility provides a framework in which scheduling costs arising due to time reallocations to other activities in the schedule can be considered. For example, departing late from the origin may have an effect on the end time of the activity at the destination and indirectly on the time spent on other activities. However, relying entirely on the duration based marginal activity utility for activity scheduling renders a formulation, which ignores the time-of-day preferences of individuals for conducting certain activities. Therefore, representation of time-of-day preferences and satiation effects of activities are necessary ingredients for the models of activity scheduling.

3.3.6 Lessons learned from generalised systematic utility formulations

In this section, a summary is presented of the three generalised formulations discussed in sections 3.3.2 to 3.3.5. Based on the key features of these three formulations, an appropriate way forward is proposed that helps specify the systematic utility component of the utility framework for the development of the combined model aimed at in this thesis.

Schedule delay formulation: This formulation well represents the anchored points on the time-of-day axis that exists for certain activities as their desired or preferred start time, before which they do not render any significant benefits to individuals. For example, start times of work and school activities and opening hours of stores and facilities. The formulation is well documented in the literature and used extensively for modelling departure time choice of the morning commute, however, the parameters associated with the late arrival penalties do not consider the effect of utility gains from the activity at the origin. Furthermore, this formulation does not explicitly include the valuation of utility of activity participation and satiation effects of an activity.

Time-of-day dependent marginal activity utility: This formulation addresses some of the weaknesses of the schedule delay formulation. For example, the scheduling costs can be expressed for the activity at the origin as well as activity at the destination and provide a framework in which utility of activity participation can be valued. Therefore, this formulation renders an approach within which scheduling of the entire day activities and the associated travel can be modelled. Due to the continuous marginal utility functions associated with activities, this formulation does not exhibit strict timing constraints in the form of anchor points on the time-of-day axis which are required for the daily activities like work and school. In addition to this, because of the purely time-of-day dependency, this formulation does not consider the effects of activity satiation for the valuation of utility of activity participation.

Duration dependent marginal activity utility: This formulation explicitly includes activity satiation effects but due to the dependency on activity duration, time-of-day preferences are completely ignored. Therefore, relying entirely on this formulation for activity scheduling will cause serious misspecification in the valuation of utility of activity participation.

The combined model aimed in this thesis considers the activity scheduling dimensions, which include departure time choice, activity duration, activity sequence and route choice for a given activity pattern. Departure time choice, activity duration and to an extent activity sequence choice (provided that an agenda of activities is known with their locations and mode i.e what type of activities are required to perform with which mode and location) are usually considered as temporal dimensions of activity scheduling and route choice is usually considered to be dependent on path travel times. Therefore, the combination of the above discussed generalised formulations in addition with the representation of travel times may provide a suitable framework for modelling the considered activity scheduling dimensions for a population representing a single user class. However, types of activities (i.e. home, work, shopping) can effect the choice of the formulation for proper representation of temporal constraints. For example; for Work and School activities schedule delay formulation is more appropriate, and for home, shopping and leisure activities time-of-day based marginal utility formulation is more suitable.

3.4 OPERATIONAL MODELS WITHIN DEMAND SIDE

In section 3.3, specification of the systematic utility is presented with a comprehensive discussion of some generalised formulations. This section considers the representation of the random part of the utility framework through some assumptions which lead into the development of models that help operationalising the modelling framework within the demand side. The recent advancements in this area render a variety of sophisticated models by relaxing some of the key assumptions of the fundamental multinomial logit (MNL) model. For example; Ordered generalised extreme value model (OGEV) developed by Small (1987) has been used for departure time choice for the morning commute because this model allows incorporation of correlation of alternatives which are in close proximity by order, Multinomial Probit (MNP) model mostly used in route choice modelling provide a framework in which errors structure of the alternatives can be correlated in a more flexible way using multivariate normal distribution (Sheffi 1985). Most of the activity scheduling models which combines more than one scheduling dimensions (e.g. departure time and route choice or departure time and mode choice) used either MNL or Nested logit models. This is evident in BB system, PETRA and TA system as well as all these activity scheduling models used Nested logit models in order to preserve the hierarchical notion in the individual decision regarding different scheduling dimension. Therefore, this thesis focused on the use of the fundamental MNL and relatively advanced Nested logit (NL) model for operationalisation of the demand side. Characteristics and functional form of these models are discussed in sub-sections 3.4.1 and 3.4.2 respectively.

3.4.1 Multinomial Logit Model

The model form reported in equation (3.15) is derived under the framework of random utility maximisation theory with certain assumptions regarding the random component (error term) of the utility framework (see equation (3.5)). These assumptions as mentioned in (Bhat 2002) are as follows:

- Error terms are independent and identically distributed (IID) with Gumbel distribution across alternatives.
- Error variance-covariance structure of the alternatives is identical across individuals (i.e. an assumption of error variance-covariance homogeneity)
- An assumption of response homogeneity (i.e. same value of the parameters of the observed attributes across individuals)

From the assumptions it follows that:

$$P_n(i) = \frac{\exp(\mu V_{in})}{\sum_{j \in c_n} \exp(\mu V_{jn})} \tag{3.15}$$

where, V_{in} is the systematic utility of alternative i for individual n, $P_n(i)$ represents probability of an alternative being chosen by individual, μ represents the logit scale parameter which is inversely proportional to the variance (σ^2) of the Gumbel distributed error term (i.e. $\mu = \pi / \sigma^2 \sqrt{6}$, π = 3.14159) and normally considered equal to 1, and C_n represents the set of alternatives for an individual.

The above mentioned three assumptions together lead to a well-known property (or limitation) within MNL model (Ben-Akiva and Lerman 1985, p. 108). This property is termed as independence of irrelevant alternative (IIA) which is illustrated in Ortuzar and Willumsen (1994, p. 215) as

> *Where any two alternatives have a non-zero probability of being chosen, the ratio of one probability over the other is unaffected by the presence or absence of any additional alternative in the choice set*

This property of the MNL model was first considered as an advantage, as a new alternative can easily be accommodated if not present at the calibration stage (given that its attributes are known). However, later this property has been perceived as a limitation of the model in the case where alternatives are correlated to each other. For example, the literature often gives an example of choice between car, bus and rail, in this case due to any improvement in the attributes of rail alternative, it is a general perception that the "bus" share will suffer the most (due to the inherent correlation between bus and rail). However, the MNL model possesses IIA property by which "car" share will suffer in the same proportion as bus share.

In the case when departure time choice is modelled (as the case with this thesis), alternatives (i.e. departure periods) are in natural order which suggests some correlation between nearby alternatives. When MNL model is used for modelling departure time choice, it might be argued that the IIA property of the model may cause dubious results, for example if one alternative become expensive then despite the expectations of shifting of individuals to nearby alternatives, the MNL model will shift in a proportionate manner among all available alternatives. Due to this reason, Small (1987) suggested that use of MNL model for naturally ordered choices is unreasonable because the error terms of the nearby alternatives may be correlated. On the similar issue, Batley et al (2001) studied the application of 5 different choice models which includes (MNL, nested logit model(NL), ordered generalised extreme value (OGEV), mixed logit (ML) model, multinomial probit (MNP) model) for a morning trip departure time choice. They concluded that ML and MNP models are performing better in comparison with MNL, OGEV and NL but within MNL, OGEV and NL models there are no significant differences observed. In this thesis use of demand side operational models are limited to MNL and NL model as the main goal of this thesis is not to study which model is best for modelling departure time choice but to render a generalised framework for the combined model through which more scheduling dimensions are modelled for different type of tours individuals perform over a day and a week. In future, however, this generalised framework can be used to incorporate more sophisticated operational models such as MNP and ML which avoid the implications of IIA property in the MNL model.

3.4.2 Nested Logit Model

The IID assumption of the error term in the MNL model for different alternatives can be relaxed in several ways; one of them could be to allow for error terms to be correlated while maintaining the assumption that they are identically distributed (i.e. identical, but non-independent random components). Nested logit (NL) model is based on this way of relaxation of IID assumption. However, this model permits covariance in error term only among subsets (or nests) of alternatives (each alternative can be assigned to one and only one nest). Each level of nest in the NL model has associated with it a dissimilarity (or logsum) parameter that determines the correlation in the unobserved components among alternatives in that nest (Bhat 2002).

The model for the two levels of nesting structure, for the problem of destination *d* and mode *m* choice in which mode choice is nested under destination choice is reported in equation (3.16) and (3.17). This is with a definition of utility as

$$U(d,m)=V_d+V_{dm}+\varepsilon_d+\varepsilon_{dm} \tag{3.16}$$

$$P(d,m)=\frac{\exp\{\mu_d(V_d+V_d^*)\}\exp(\mu_m\ V_{dm})}{\sum_{d'}\exp\{\mu_d(V_d+V_d^*)\}\sum_{m'}\exp(\mu_m\ V_{dm})} \tag{3.17}$$

with $$V_d^*=\left(\frac{1}{\mu_m}\right)\ln\sum_{m'}\exp(\mu_m V_{dm'}) \tag{3.18}$$

where, $U(d,m)$ represents the total utility, V_d+V_{dm} represents the systematic part of the utility in which the first term representing the factors associated only with destination and second term represents the factors associated with both choice dimensions, $\varepsilon_d+\varepsilon_{dm}$ represents the error terms which are separately assumed as IID through Gumbel distribution. μ_d and μ_m are the scale parameters belongs to their relevant nest, however, only the ratio μ_d/μ_m can be estimated with the assumption that μ_m equals 1. For internal consistency of the model with the theory it is required that $\mu_d \le \mu_m$. If the ratio μ_d/μ_m comes 1 than model collapses to MNL model, which suggests that correlation among the same nest alternatives does not exist. The correlation of the utilities among the same nest alternatives can be given as $Corr=1-(\mu_d/\mu_m)^2$.

The term V_d^* represented through equation (3.18) is termed as the logsum and is often referred as a measure of consumer surplus as it is a scalar summary of the expected "worth" of a set of travel alternatives in a lower level nest (Ben-Akiva and Lerman 1985, p. 301). In the literature of economic welfare the term consumer surplus is defined as the excess of valuation of product over the price actually paid. Williams (1977) first advocated the use of logsum as a measure of consumer surplus. Taking the random utility formulation into account (as presented in equation 3.5), the expected consumer surplus is given by:

$$E(CS_n)=\frac{1}{\kappa}\ E\left[\max\ (U_{in}\quad \forall i)\right] \text{ or}$$

$$E(CS_n)=\frac{1}{\kappa}\ E\left[\max\ (V_{in}+\varepsilon_{in}\quad \forall i)\right]$$

where V_{in} is the systematic component of the overall utility and the expectation is over all possible values of the ε_{in} 's (random component of the overall utility). The error term (random component of the utility) could be interpreted in two very different ways. It could be assumed to represent the modeller's error in excluding some important attributes that affect the travellers' utilities and therefore decisions. Alternatively, it could be assumed to represent the uncertainty of the travellers when choosing between alternatives. In this thesis it is assumed that the error is all of the second type, with no modeller error, in order that the logsum can be properly interpreted as a measure of consumer surplus. κ represents marginal utility of income, usually a price or cost variable enters the systematic utility and, in case that happens in a linear additive fashion, the negative of its co-efficient is κ by definition (De Jong et al 2005, Train 2003 p. 61). If each ε_{in} is IID extreme value and utility is linear in income (that is κ is constant with respect to income), then the expectation becomes:

$$E(CS_n) = \frac{1}{\kappa} \cdot \frac{1}{\mu} \ln\left(\sum_{i=1}^{I} \exp(\mu V_{in}) \right) + \mathrm{C}$$

where C is an unknown constant that represents the fact that the absolute value of utility cannot be measured. The term in parentheses in this expression is the denominator of a multinomial logit choice probability (see equation 3.15). Aside from the division and addition of constants, expected consumer surplus in a logit model is simply the log of the denominator choice probability. In case of nested logit model as depicted in equation (3.17) for two-level of choices (i.e. destination and mode) the expected consumer surplus is given by:

$$E(CS_n) = \frac{1}{\kappa} \cdot \frac{1}{\mu_d} \ln\left(\sum_{d'} \exp\{\mu_d (V_d + V_d^*)\} \right) + \mathrm{C}$$

where, V_d^* represents the logsum term already defined in equation (3.18), it is representing the expected maximum value of utility for the mode choice (lower level nested choice). Consumer surplus defined for MNL and NL model has been used as a measure of evaluation in many studies for a particular policy by determining the change in the consumer surplus before and after scenarios of policy application (De Jong et al 2005). The change in the consumer surplus ΔCS_n for logit case for an individual *n* can be given by

$$\Delta CS_n = \frac{1}{\kappa}\left[\frac{1}{\mu}\ln\left(\sum_{i=1}^{I^1}\exp\left(\mu V^1{}_{in}\right)\right) - \frac{1}{\mu}\ln\left(\sum_{i=1}^{I^0}\exp\left(\mu V^0{}_{in}\right)\right)\right] \tag{3.20}$$

where, superscript 0 and 1 refer to before and after the change. According to De Jong et al (2005) and de Palma and Lindsey (2006), the change in socio-economic benefits obtain from implementing any transport policy can be given as

Change in socio-ecnomic benefits (ΔW) = Change in total consumer surplus ($Q*\Delta CS_n$)

+ Revenues from policy (R) (3.21)

where, Q represents total number of individuals. In chapters 7 and 8, where some policy tests have been described for the model developed in this thesis, the socio-economic benefits are determined using the above equations in order to present a single comparable summary measure of performance for different policy scenarios.

The NL model has been applied to multidimensional choice context as well as one-dimensional contexts where subsets of the available alternatives share common unobserved component of utility. A problem with this model is that it requires a *priori* specification of the nesting structure, which suggests that the number of different structures is required to be estimated in a search for the best structure, and the number of alternative nesting structure increases rapidly with the increase in the number of choices (Bhat 2002).

3.5 SUMMARY

This chapter illustrates the issues and modelling consideration within the demand side of the combined model which will help the development of the model in chapter 6. Section 3.2 and its sub-sections explains the underlying mechanism within the two distinctive decision making methodologies used for modelling activity scheduling dimensions. This section then highlights the key features of the study objectives formulated in chapter 1 and based on that a decision is made regarding the use of random maximization utility theory for this study. In section 3.3 and its sub-sections, a number of formulations for the specification of the systematic component of the utility framework are discussed in detail. These include: the schedule delay formulation, time-of-day based marginal activity utility and duration based marginal activity utility. Within this section, it is also concluded that combination of these formulation will render an appropriate framework for the development of the combined model in which departure times, duration, activity sequencing and route choice dimensions of activity scheduling will be considered. In the final section (section 3.4) some operational models (MNL and NL) within the demand side are discussed which will used in the development of the combined model. The next chapter focuses on the issues within supply side of the combined model.

Chapter 4

CONSIDERATIONS FOR THE COMBINED MODEL-SUPPLY SIDE

4.1 GENERAL

This chapter demonstrates modelling considerations within the supply side of the combined modelling framework. The supply side is usually characterised as a method for the representation of flow of traffic on the road network. The main issues within the supply side may be classified as follows

- Macroscopic or microscopic representation of traffic
- Representation of time

The flow of traffic on the road network can be considered either as a macroscopic (i.e. group of vehicles) or microscopic (i.e. each vehicle separately) for a given time horizon. On the other hand, representation of the time dimension if considered in the modelling framework then the model is termed as dynamic and if the time dimension is not considered then those models are termed as static. In dynamic models, the time dimension is represented in various ways (some models consider this at 1 minute intervals or even shorter than that and some models consider this using larger intervals such as 15 minute intervals or even 30 minute intervals). However, in the static models there is no explicit consideration of time dimension but implicitly these models assume the representation of the time as an hour or multiples of hours and because of this representation these models further assumes that there are no interaction effects that exist between these time intervals.

The issue of the representation of the time dimension is interlinked with the representation of traffic in a manner that one needs the other for taking full advantage from the chosen representation. For example, if a microscopic representation is considered for the traffic, it naturally requires consideration of short intervals of time in order to fully exploit the advantages it offers over the macroscopic representation (i.e. policies like: real-time driver information systems, ramp-metering and responsive traffic signal systems etc, can be examined for short-term forecasting). This suggests that a trade-off exists regarding the selection of a particular representation of traffic and time dimension over the benefits and costs (in terms of resources) for fulfilment of the study objectives. This chapter first discusses the selection of an appropriate representation of traffic and time, based on the study objectives. Subsequent sections then present an analysis of the existing operational models within the supply side that fulfil the adopted representation of traffic and time.

4.2 SUPPLY SIDE-MODELLING CONSIDERATIONS

4.2.1 Dynamic Representation of Network Congestion

The focus of this study is to model activity scheduling dimensions such as departure time, activity duration, activity sequence and route choices together within a framework of a combined model. These scheduling dimensions require time-varying representation of network congestion (i.e. time-varying travel times) in order to perceive relative attractiveness of available alternatives with time. Therefore, it is necessary to employ a dynamic representation at the supply side. Furthermore, long intervals of time (i.e hour or multiples of hours) are not appropriate. This is because the representation of utility for activity participation is considered in this study as a continuous function of time-of-day and duration, which is established in chapter 3. This suggests that the value of positive utility is changing with respect to time (i.e. significantly short intervals of time). Therefore, the consideration of the travel time in long intervals (average travel time for an hour) will not provide a coherent modelling framework.

The dynamic representation of network congestion through considerably short time intervals render a basis within which travel times of the entering vehicle at a particular time on the link is estimated based on the amount of traffic already exist on the link at that time. Furthermore, this representation also illustrates the propagation of traffic on the link. This is in contrast with the static representation of time, in which it is assumed that flow is constant over each route from its origin to destination and the volume on the link is computed by adding up the volumes on the routes that go through it (Mun 2007, Ben-Akiva et al 2007).

4.2.2 Micro and Macro Representation of Traffic

This is an important distinction among the dynamic traffic assignment models, the term microscopic usually refers to those models which deal with vehicles, and macroscopic models are those which are flow-based. A microscopic representation of traffic enables the treatment of many traffic phenomena through detailed vehicle-vehicle and vehicle-infrastructure interactions. For example, capacity-reducing effects of lane changing, impacts of heavy vehicles and interactions of vehicles at intersections (effects of left turning vehicles, intersection blocking) can be examined. Due to the same reason, this representation demands much more detailed information about both the networks and vehicles which often require high manpower and computational costs. On the other hand, a macroscopic representation of traffic involves a small set of time-dependent variables (i.e. inflow rate, link volume etc) which are meant to represent average behaviour of traffic. Therefore, comparatively they are simple to implement and efficient to compute even for larger road networks but at the same time do not provide flexibility to capture complex traffic phenomena, such as lane changing, heavy vehicle interactions etc. The advantages and disadvantages of both these representations are complementary; favouring one over the other is entirely based on the application environment and its goals. Given the objectives formulated for this research within which an analytical model is the aim, which requires average behaviour of vehicles, a macroscopic representation of traffic seems more appropriate.

Various forms of macroscopic operational models (discussed in section 4.3 and 4.4) within supply side are presented in the literature of dynamic traffic assignment (DTA). These models help estimating link time-varying travel times whilst maintaining the time-varying propagation of flow on the network. This is in contrast with the static treatment where flow propagation is not at all an issue. These models are developed with a view point of providing a simplistic framework, through a relatively simple mathematical construction and low manpower costs, for implementation of macro-scale planning applications. However, their proper and efficient use requires that these models should comply with certain properties. This is necessary because if these models do not possess these properties then their use in DTA may provide misleading results. This is explained in the following sub-section where these properties are also described.

4.2.3 Desirable properties for macroscopic dynamic models

There are several requirements identified from the literature that appropriate dynamic loading models (macroscopic traffic performance models) should meet for their application to DTA. A comprehensive review of these requirements has been provided by Mun (2007), Heydecker and Addison (2006), and Mun (2001). In this section a brief review of these requirements is provided. These are as follows

- Flow Conservation
- Flow Propagation
- First-in-First-out (FIFO)
- Causality
- Reasonable Outflow behaviour
- Positivity, existence and uniqueness

The following paragraphs discuss the above mentioned requirements in detail

- *Flow Conservation*; this is an important requirement which ensures that any vehicle that enters in to the link will exit as well. In other terms, total inflow to the link at any time t should be equal to the total outflow and the vehicles which are traversing the link at that time. Mathematically this can be expressed as

$$E(t) = O(t) + x(t) \tag{4.1}$$

where, $E(t)$ and $O(t)$ are accumulated inflow and accumulated outflow at times t. If we consider that at an initial time t_0 the link is empty, then the above equation ensures that difference between the cumulative inflow and outflow at any time t is the amount of vehicles on the link at that time, which is represented as $x(t)$.

- *Flow Propagation;* this requirement ensures that the flow on the link should propagate in a manner consistent with the speed of the vehicles. Total inflow to the link at time t should be equal to total outflow to the link at an exit time φ, which is also a function of time t. This suggests that the minimum time a vehicle experiences while traversing on the link is equal to free-flow travel time of the link. Mathematically this can be expressed as

$$E(t) = O(\varphi(t)) \tag{4.2}$$

Differentiating (4.2) with respect to time *t*, gives the following

$$e(t) = o(\varphi(t))\dot{\varphi}(t) \tag{4.3}$$

where, $e(t)$ is the inflow rate at time *t*, $o(\varphi(t))$ is the outflow rate at an exit time $\varphi(t)$ and $\dot{\varphi}(t)$ is the rate of change of exit time which is responsible for variation in the outflow rate compared to inflow rate. For example, if rate of change of exit time $\dot{\varphi}(t)$ is constant, then outflow rate at time $\varphi(t)$ exactly matches with inflow rate at time *t*. Equation (4.3) is termed as time-flow consistency equation as it ensures the consistency between the three important ingredients i.e. inflow, outflow and travel time.

- *First-in-First-out* (FIFO)*;* for macroscopic loading models it is necessary that the FIFO condition is not violated. This is because macroscopic models deal with a group of vehicles having similar characteristics, therefore they should take a similar amount of time for traversing a link i.e. vehicles that entered a link at the same time should take the same time to traverse on the link. If FIFO is violated then it suggests that the rate of exit time for some vehicles can be negative (i.e. $\dot{\varphi}(t) < 0$ which means that vehicles that entered a link earlier (later) than those who entered at time *t* may exit the link later (earlier) than these vehicles). Based on this, equation (4.3) which can be rearranged for estimating outflow rate (as inflow rate is usually given), may give negative outflow rate (see equation 4.4).

$$o(\phi(t)) = e(t)/\dot{\varphi}(t) \tag{4.4}$$

The term $\varphi(t)$ can also be expressed in travel time $R(t)$, which is given as:

$$\varphi(t) = t + R(t) \tag{4.5}$$

therefore, to hold FIFO intact it is necessary that $\frac{dR(t)}{dt} \geq -1$

- *Causality;* In the DTA literature, causality is termed as the dependency of the upstream vehicles on the downstream vehicles when travel time is estimated for the upstream vehicles. The dynamic loading model is required to meet this condition, as it is unacceptable and far away from reality that travel times of the vehicles which are at the downstream of the link is affected by vehicles upstream of the link. It has been shown in the literature that outflow models, in which outflow rate is taken as a function of vehicles on the link irrespective of their location, exhibit violation of the causality condition (Astarita 1996, Mun 2001). This is because this outflow rate is then used in calculating travel time through equation (4.4); therefore, travel time of a vehicle downstream can be affected by vehicles upstream.

- *Reasonable outflow behaviour;* this requirement is described as it is generally accepted that the outflow rate increases as the amount of traffic on the link increases until it reaches the outflow capacity of the link, provided that there is no capacity constraints on the following links. Mun (2001) mentioned that some non-linear travel time models behave unreasonably when the traffic on the link exceeds certain levels, i.e. the outflow rate decreases as the amount of traffic on the link increases.

- *Positivity, existence and uniqueness;* it is required for the DTA that the three important terms should be positive i.e. Inflow rate which is the given quantity, amount of traffic on the link and outflow rate.

$$e(t) \geq 0, \quad x(t) \geq 0, \quad o(t) \geq 0 \qquad \forall t$$

Existence means that for any pattern of inflows and outflow it is always possible to obtain a travel time for vehicles entering at time *t*. Uniqueness here means that travel time is unique with respect to entry time. In addition, computational efficiency of the loading model is also considered as an important requirement, because computational efficiency is directly related to the amount of time a model required for its successful run. Therefore the model that has higher computational efficiency would be more preferable than others.

The next section discusses some properties and characteristics of some macroscopic traffic performance models that will be utilised in this research. Furthermore, some numerical experiments are also conducted based on assumed time-varying inflow profiles to support the characteristics and properties mentioned for each model.

4.3 MACROSCOPIC DYNAMIC NETWORK LOADING MODELS

The literature of DTA presents a number of different macroscopic dynamic loading models having different features and characteristics. Some of them have found to be inconsistent with the desirable properties for DTA. For example, non-linear travel time models, outflow models (Mun 2001, Mun 2007), whole-link models within which inflow rate and outflow rate are used to calculate travel time (Daganzo 1995). In this section, Point-queue, linear travel-time and divided linear travel time models are discussed as it has been established in the literature that these models are consistent with desirable properties for DTA, relatively simpler in their mathematical construct and require less computational efforts compared to other existing models (Nie and Zhang 2005a). Sub-section 4.3.1, 4.3.2 and 4.3.3 describe the details of the three loading models considered in this thesis.

4.3.1 Point-Queue Model

This model is often termed in the literature as the bottleneck model, and most widely used in DTA because of its simplicity. The model is given by

$$o(t)=\begin{cases} e(t-\phi) & z(t)=0 \text{ and } e(t-\phi)<C \\ C & \textit{otherwise} \end{cases} \tag{4.6}$$

and

$$\frac{d\,z(t)}{dt}=\begin{cases} 0 & z(t)=0 \text{ and } e(t-\phi)<C \\ e(t-\phi)-C & \textit{otherwise} \end{cases} \tag{4.7}$$

where, $e(t-\phi)$ is the inflow rate at time $t-\phi$; and $z(t)$ is the number of vehicles in the queue at time *t*, *C* is the capacity of bottleneck (exit capacity of the link in this case); and ϕ is the free flow travel time.

Vehicles that enter a link at time *t* have travel time $R(t)$ given by:

$$R(t)=\phi+\frac{z(t+\phi)}{C} \tag{4.8}$$

Equations (4.6) to (4.8) represent this model and according to these, vehicles that enter a link at time t are allowed to traverse it with free-flow travel time ϕ if there is no queue on the link downstream at time t and if the inflow rate at that time does not exceed capacity. If the inflow rate exceeds capacity, a queue forms at the end of the link, but it does so vertically without occupying any space on the link this is why this model is termed as a point-queue model. From this model, travel time is estimated as the free-flow travel time whenever the inflow rate does not exceed capacity. This suggests that this model does not describe the network behaviour properly and travel time is clearly underestimated in the situation where the link is busy but not overloaded. This involves a major simplification of reality, since increasing congestion will cause increasing travel times before full capacity of outflow is reached (Heydecker and Addison 1998). In addition to this, Mun (2007, p. 240) mentioned the oversimplification considered in the second state of this model (which suggests that outflow rate equals capacity of the link when inflow rate equals or exceeds capacity) by giving empirical findings illustrated in the US Highway Capacity Manual. According to these findings, if *inflow rate is equal to capacity* over a longer period of time then the level of service of the link is in state E, which indicates that the operation in this facility is unstable, i.e. speed and flow rates fluctuate. However, in this circumstance this model always provides outflow equals to the capacity of the link and the link operate under free flow condition (i.e. no queue at the end of the link). Despite these oversimplifications, this model has been extensively used in the DTA literature (for example, Ben-akiva et al 1986, Arnott et al 1990, Heydecker and Polak 2006) in order to represent flow of traffic on a link. The extensive use of this model is based on the fact that the model is fulfilling all the desirable properties required for use in DTA and also it requires less effort in terms of computation and implementation (Nie and Zhang 2005a).

4.3.2 Linear Travel Time Model

This model is originally proposed by Friesz et al (1993) for use in DTA. In this model the travel time $R(t)$ for vehicles that entered the link at time t is estimated as a linear function of the number of vehicles on the link at that time. The model is given by

$$R(t) = \phi + \frac{x(t)}{C} \tag{4.9}$$

where, ϕ and C are represent free-flow travel time and out-flow capacity of the link respectively. Traffic on the link $x(t)$ is calculated using the flow conservation and flow propagation functions described in equation (4.1) and (4.3).

Several properties of this model were explored by many researchers (Astarita, 1996; Carey and McCartney, 2002; Mun, 2001; Nie and Zhang, 2005b) and it has been established in the literature that this model fulfils all the desirable conditions for DTA like, positivity, causality, FIFO, flow conservation and propagation etc (Mun 2007). Nie and Zhang (2005b) suggested a discretisation approach for this model which is based on cumulative arrival and departure curves. This approach makes the implementation of this model easier and simpler. Some of the drawbacks are also pointed out within the model as it tends to overestimate travel time because all vehicles downstream are considered in the estimation of travel time. This has been termed as a *double-counting effect* in the literature (Mun, 2007; Nie and Zhang, 2005a). Further to that, the extent of over-estimation of travel time i.e. degree of over-estimation is not explored explicitly with real traffic data. It can be easily seen that equation (4.8) and (4.9) are similar to each other but queuing delays are defined very differently. In equation (4.8), queuing delays are evaluated at time $t+\phi$ i.e. when a vehicle reaches the end of the link, however, in equation (4.9) queuing delays are evaluated at time *t*. Additionally, in equation (4.8) only those vehicles are considered that form the queue at the end of the link, not the entire existing traffic on the link.

This model is also extensively used in DTA and it has been shown in the literature that this model respects all the desirable properties for DTA (Friesz, 1993; Nie and Zhang, 2005b; Carey and McCartney, 2002; and Mun, 2007). However, the assumption of linearity in the travel time function of this model can be challenged as it has been generally accepted that travel time increases non-linearly in congested conditions (Jang et al 2005).

4.3.3 Divided Linear Travel Time Model

Mun (2001) proposed a divided linear travel time model, which can be considered as an extension of the model discussed in section 4.3.2, in order to address drawbacks in that model, such as overestimation of travel time and smoothness of outflow profile (this is necessary because that outflow profile will serve as inflow for the next link). According to Mun (2001), the link is divided into two parts, one is the area where traffic can propagate with free-flow speed and the other is the one where the linear travel time model is applied. He found out that when the linear travel time model is discretised for its implementation, if the ratio of the length of analysis time interval (Δt) to free flow travel time (ϕ) is in the range of 0.8 ~1 then the outflow profile obtained from this model is much smoother. Therefore, he suggested that in the second part of the link, the free flow travel time is equivalent to length of the analysis time interval. This can be better understood from figure 4.1.

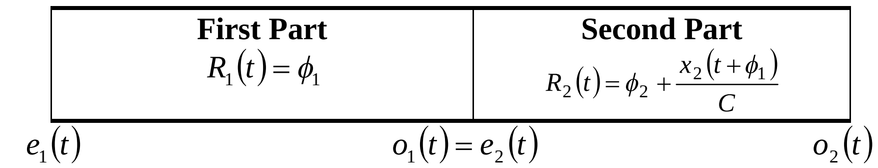


Figure 4.1: Schematic representation of a divided linear travel time model

Travel time of the first part of the link: $R_1(t) = \phi_1 = \phi - \Delta t$

Travel time of the second part of the link: $R_2(t) = \phi_2 + \frac{x_2(t+\phi_1)}{C} = \Delta t + \frac{x_2(t+\phi_1)}{C}$

where, ϕ_1 and ϕ_2 (i.e. $\phi = \phi_1 + \phi_2$) are the free flow travel time of the first and second part of the link respectively, $x_2(t+\phi_1)$ is the amount of traffic on the second part of the link and Δt is the length of analysis time interval (discretised time step, e.g. 1 min or 0.5 min). Accordingly, the total link travel time is then,

$$R(t) = R_1(t) + R_2(t) = \phi + \frac{x_2(t+\phi_1)}{C} \tag{4.10}$$

For the determination of number of vehicles and outflow, this model also utilises flow conservation and propagation equations (4.1) and (4.3) for the second part of the link. The model respects FIFO principle and consistent with all other requirements of DTA (Mun 2001).

Similar to the linear travel time model, this model also follows the assumption of linearity in the estimation of travel time, thus non-linear behaviour of travel time (i.e. increase in travel time with congestion) is not addressed. However, the overestimation problem of the linear travel time model in uncongested condition is successfully addressed to an extent by using only that proportion of traffic on the link for measuring queuing delay which exists in the second part of the link. In the next section, a novel model (Adnan-Fowkes model) is presented that is developed during the course of this research in order to address some of the drawbacks of point-queue and linear travel time models (Adnan and Fowkes, 2009). However, this thesis utilised all the above discussed models together with the Adnan-Fowkes model for representation of the supply side in the combined modelling framework. This general framework of the combined model allows comparisons of the results for different supply side models.

4.4 ADNAN-FOWKES MODEL

4.4.1 Model Formulation and its behaviour

This model is first proposed in Adnan and Fowkes (2009), but later it has been realised that it needs further correction (i.e. compatibility of equations in terms of units), which is now corrected and presented in this section in its discretised form. This model was developed with an aim that it addresses the drawback of underestimation of travel time of the point-queue model and overestimation problem of the linear travel time model. The model follows a piece-wise linear travel time function (controlled through three states of outflow rate) which approximates a non-linear travel time function anticipated in reality when a link is congested. This model can be viewed as an extension of the point-queue model, because instead of two states (free-flow and fully-congested flow) three states are proposed (free-flow, partially-congested flow and fully-congested flow) within the model. In addition to that, two outflow controlling parameters are used which constrain the behaviour of the model in such a manner that it not only removes the overestimation error in the linear travel time model under less congested environment but also removes the underestimation error in the Point-Queue model when the link is moderately congested but has not yet reached at its full capacity. Figure 4.2 illustrates the behaviour of all the three models discussed in section 4.3, along with the Adnan-Fowkes model. The x-axis in the three plots of figure 4.2 represents the respective variables through which these models calculate queuing delays.

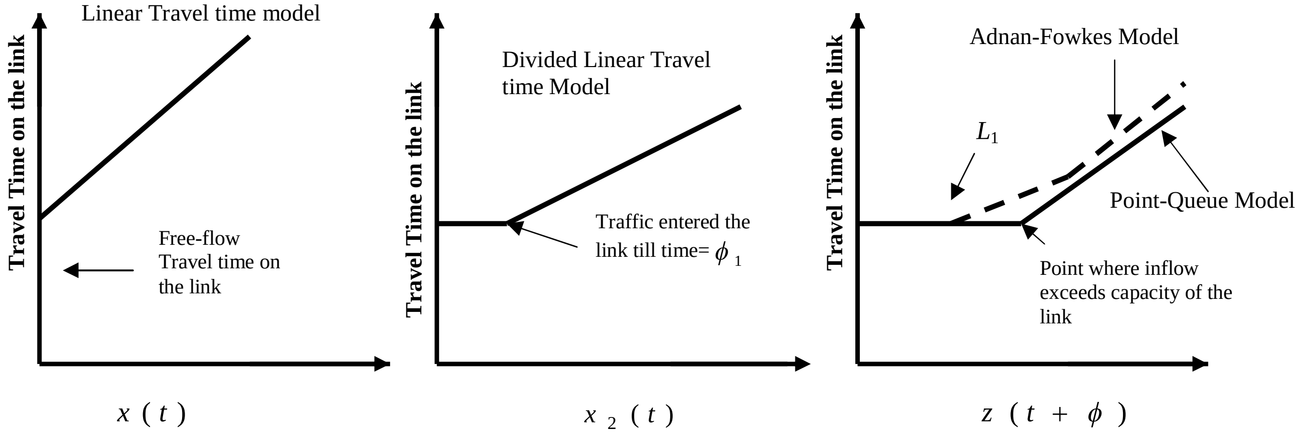


Figure 4.2: Behaviour of different Loading Models

The discretised version of the Adnan-Fowkes model is given as follows:

$$o(t_i, t_i+\delta) = \begin{cases} e(t_i-\phi, t_i+\delta-\phi)+z(t_i) & e(t_i-\phi, t_i+\delta-\phi)+z(t_i) < L_1\cdot\delta \\ \dfrac{L_1\cdot\delta+(n-1)\left[e(t_i-\phi, t_i+\delta-\phi)+z(t_i)\right]}{n} & L_1\cdot\delta \le e(t_i-\phi, t_i+\delta-\phi)+z(t_i) < L_2\cdot\delta \\ C\cdot\delta & e(t_i-\phi, t_i+\delta-\phi)+z(t_i) \ge L_2\cdot\delta \end{cases} \quad (4.11)$$

and

$$L_2 = \frac{nC - L_1}{n-1} \quad (4.12)$$

or equivalently, $L_1 = nC - (n-1)L_2$ (4.12a)

$$\Delta z(t_i) = \begin{cases} -z(t_i) & e(t_i-\phi, t_i+\delta-\phi)+z(t_i) < L_1\cdot\delta \\ \dfrac{e(t_i-\phi, t_i+\delta-\phi)-(n-1)z(t_i)-L_1\cdot\delta}{n} & L_1\cdot\delta \le e(t_i-\phi, t_i+\delta-\phi)+z(t_i) < L_2\cdot\delta \\ e(t_i-\phi, t_i+\delta-\phi)-C\cdot\delta & e(t_i-\phi, t_i+\delta-\phi)+z(t_i) \ge L_2\cdot\delta \end{cases} \quad (4.13)$$

where, $e(t_i-\phi, t_i+\delta-\phi)$ is the *number of vehicles* entering in the link within time segment $t_i-\phi$ and $t_i+\delta-\phi$, ϕ represents free flow travel time (in minutes) on the link, $z(t_i)$ represents the *number of vehicles* in the queue at the end of the link at time t_i, δ is the time increment for model implementation in minutes, $o(t_i, t_i+\delta)$ represents *number of vehicles* coming out from the link during time segment t_i and $t_i+\delta$. n is the calibration parameter and should be greater than unity (n >1), L_1 represents the link inflow in number of vehicles per unit time $(L_1 \le C)$ below which travel time on the link equals free flow journey time and L_2 represents the link inflow in number of vehicles per unit time that first causes outflow to reach the capacity level of the link. Equation (4.13) is also a part of the model and is responsible to conserve flow on the link, as it is derived from flow conservation equation (see section 4.4.2 for further details on it). Equations (4.11) and (4.13) are now consistent in terms of units of quantities used in them. For estimation of travel time for the vehicle enter at time t_i , equation (4.8) of the point-queue model is *retained* in this model. Of course, because the amount of queuing is different in the two models (point-queue and Adnan-Fowkes), actual values of $R(t_i)$ will differ between the two models.

The three states proposed in Adnan-Fowkes model are consistent to each other as they join correctly together with the help of equation (4.12) or (4.12a). This can be shown as follows using the state boundary conditions:

when $e(t_i-\phi, t_i+\delta-\phi)+z(t_i)=L_1\cdot\delta$,

$$\textbf{State 2} = \frac{e(t_i-\phi, t_i+\delta-\phi)+z(t_i)+(n-1)[e(t_i-\phi, t_i+\delta-\phi)+z(t_i)]}{n}$$
$$= e(t_i-\phi, t_i+\delta-\phi)+z(t_i) = \textbf{State 1}$$

and when $e(t_i-\phi, t_i+\delta-\phi)+z(t_i)=L_2\cdot\delta$, using equation (4.12a)

$$\textbf{State 2} = \frac{\{nC\cdot\delta-(n-1)[e(t_i-\phi, t_i+\delta-\phi)+z(t_i)]\}+(n-1)[e(t_i-\phi, t_i+\delta-\phi)+z(t_i)]}{n} = C\cdot\delta = \textbf{State 3}$$

A special property of this model is that when L_1 is assumed equal to C, equation (4.12) gives L_2 equal to C as well regardless of the value of n and the model collapses into Point-Queue model. It would be interesting to suggest a value of n for which the model provides plausible results; however, its true value needs to be calibrated through examination of real data. Table 4.1 suggests that the model is flexible in the selection of values for L_1 or L_2 and n within their stipulated limits to obtain the desirable behaviour. The next section presents some discussion which highlights the behaviour of the Adnan-Fowkes model against the important desirable properties for DTA i.e. flow conservation, flow propagation, FIFO and causality. It is very obvious to see that the model respects the requirement of reasonable outflow as outflow rate is defined as function of inflow rate (see equation 4.11).

Table 4.1: **Model behaviour with different values of L_1 and *n***

L_1	*n*	L_2 (from equation 4.12)	$o(t_i, t_i+\delta)$ (from equation 4.11)	*Comments*
C	*>1*	*C*	$\begin{cases} e(t_i-\phi, t_i+\delta-\phi)+z(t_i) \\ C\cdot\delta \end{cases}$	2nd state in equation (4.11) is inactive, and the model collapses to Point-Queue model.
0.5C	*2*	*1.5C*	$\begin{cases} e(t_i-\phi, t_i+\delta-\phi)+z(t_i) \\ 0.25C\cdot\delta+[e(t_i-\phi, t_i+\delta-\phi)+z(t_i)] \\ C\cdot\delta \end{cases}$	All three states are active and model may gives behaviour as half-way between linear travel time and Point-Queue models.
0.5C	*3*	*1.25C*	$\begin{cases} e(t_i-\phi, t_i+\delta-\phi)+z(t_i) \\ 0.167C\cdot\delta+0.667[e(t_i-\phi, t_i+\delta-\phi)+z(t_i)] \\ C\cdot\delta \end{cases}$	All three states are active here as well and again it behaves half-way between linear travel time and Point-Queue model.
0.5C	*5*	*1.125C*	$\begin{cases} e(t_i-\phi, t_i+\delta-\phi)+z(t_i) \\ 0.1C\cdot\delta+0.8[e(t_i-\phi, t_i-\phi)+z(t_i)] \\ C\cdot\delta \end{cases}$	All three states are active here as well and again it behaves half-way between linear travel time and Point-Queue model. Showing the range of *n* in which model is behaving plausibly.
0.5C	*100*	*1.005C*	$\begin{cases} e(t_i-\phi, t_i-\phi)+z(t_i) \\ 0.005C\cdot\delta+0.99[e(t_i-\phi, t_i+\delta-\phi)+z(t_i)] \\ C\cdot\delta \end{cases}$	All three states are active here but range between L_1 and L_2 is squeezed with increase in *n*. Model again collapsing towards the Point-Queue model as initial two states tend to become similar to each other.

4.4.2 Examination of the Model for Desirable Properties

Adnan-Fowkes model is examined here regarding the desirable properties for DTA. The following paragraphs discuss this in detail.

- *Flow Conservation;* the flow conservation equation (4.1) can be formulated to represent the traffic at the end of the link which forms the vertical queue as

$$E\left(t_i - \phi\right) = O\left(t_i\right) + z\left(t_i\right) \tag{4.14}$$

It should be noted that cumulative number of vehicles entered in the link up till time ($E(t_i - \phi)$) in equation (4.14) is considered at time $t_i - \phi$, this suggests that vehicles which are traversing on the link during time $t_i - \phi$ and t_i are not considered in this representation. Adnan-Fowkes model in which equation (4.13) is included for estimation of the change in the queuing traffic $\Delta z(t_i)$, is actually derived by using equation (4.16) which is given by taking the difference of the quantities in equation (4.14) at $t_i - \phi$ and $t_i + \delta - \phi$. This is represented as follows:

$$E\left(t_i + \delta - \phi\right) - E\left(t_i - \phi\right) = O\left(t_i + \delta\right) - O\left(t_i\right) + z\left(t_i + \delta\right) - z\left(t_i\right) \tag{4.15}$$

The equation (4.15) can be written as

$$\Delta z\left(t_i\right) = e\left(t_i - \phi, t_i + \delta - \phi\right) - o\left(t_i, t_i + \delta\right) \tag{4.16}$$

- *Flow Propagation;* similar to the point-queue model, Adnan-Fowkes model also uses a free-flow travel time ϕ, as the minimum travel time that is required to traverse the link. Therefore, the model is able to describe the spatial propagation of the flow on the link.

- *FIFO;* this condition suggests that the model gives travel time in such a fashion that it always respect the following expression.

$$\frac{\Delta R\left(t_i\right)}{\delta} \geq -1$$

In the Adnan-Fowkes model, equation (4.8) is used for estimation of travel time, and differentiation of this equation for small amount of time increment δ gives the following:

$$\frac{\Delta R(t_i)}{\delta} = \frac{1}{C}\frac{\Delta z\left(t_i + \phi\right)}{\delta} \tag{4.17}$$

For showing that the model fulfils FIFO, it requires that for all states of the model in equation (4.13) should be greater than or equal to – $C.\delta$. Equation (4.13) which represents the change in the queue at time t_i ,can be reformulated to represent the same change of rate at time $t_i + \phi$ for consistency of time dimension. This can be given as

$$\Delta z(t_i + \phi) = \begin{cases} -z(t_i + \phi) & e(t_i, t_i + \delta) + z(t_i + \phi) < L_1 \cdot \delta \\ \dfrac{e(t_i, t_i + \delta) - (n-1) z(t_i + \phi) - L_1 \cdot \delta}{n} & L_1 \cdot \delta \le e(t_i, t_i + \delta) + z(t_i + \phi) < L_2 \cdot \delta \\ e(t_i, t_i + \delta) - C \cdot \delta & e(t_i, t_i + \delta) + z(t_i + \phi) \ge L_2 \cdot \delta \end{cases} \tag{4.18}$$

If state 1 is considered in equation (4.18), which is constrained by the inflow L_1 and by definition this should be less than or equal to *C*, therefore, boundary condition for state 1 should follow $e(t_i, t_i + \delta) + z(t_i + \phi) < C \cdot \delta$, which suggests that $z(t_i + \phi) < C \cdot \delta$, this can be written as $-z(t_i + \phi) > -C \cdot \delta$. So, FIFO is respected in the State 1. The proof for State 3 is also very simple to illustrate for this property, i.e. number of vehicles entered in the link during time segment $(t_i, t_i + \delta)$ should always follow $e(t_i, t_i + \delta) \ge 0$, which suggests that the minimum possible value of state 3 is $-C \cdot \delta$, so FIFO is maintained here as well. The state 2 of equation (4.18) is constrained with the boundary condition; i.e. $e(t_i, t_i + \delta) + z(t_i + \phi) < L_2 \cdot \delta$, therefore, if the proof is illustrated for this boundary condition then it can be said that all states in equation (4.18) always greater than $-C \cdot \delta$. The proof for state 2 is as follows:

The State 2 of the model is given by

$$\Delta z(t_i + \phi) = \frac{e(t_i, t_i + \delta) + z(t_i + \phi) - L_1 \cdot \delta - n\ z(t_i + \phi)}{n} \tag{4.19}$$

Susbstituing the value of L_1 from equation (4.12a) gives the following

$$\Delta z(t_i + \phi) = \frac{e(t_i, t_i + \delta) + z(t_i + \phi) - (nC - (n-1)L_2) \cdot \delta - n\ z(t_i + \phi)}{n}$$

$$= \frac{e(t_i, t_i + \delta) + z(t_i + \phi) - L_2 \cdot \delta}{n} - C \cdot \delta + L_2 \cdot \delta - z(t_i + \phi)$$

$$= \frac{e(t_i, t_i + \delta)}{n} - \frac{L_2 \cdot \delta - z(t_i + \phi)}{n} + L_2 \cdot \delta - z(t_i + \phi) - C \cdot \delta$$

$$= \frac{e(t_i, t_i + \delta)}{n} + \left[L_2 \cdot \delta - z(t_i + \phi) - \frac{L_2 \cdot \delta - z(t_i + \phi)}{n} \right] - C \cdot \delta$$

$$= \frac{e(t_i, t_i + \delta)}{n} + \left(L_2 \cdot \delta - z(t_i + \phi) \right) \left(1 - \frac{1}{n} \right) - C \cdot \delta \tag{4.20}$$

The boundary condition of the State 2 i.e. $e(t_i, t_i + \delta) + z(t_i + \phi) \le L_2 \cdot \delta$, suggesting that $z(t_i + \phi) \le L_2 \cdot \delta$, this means the quantity $(L_2 \cdot \delta - z(t_i + \phi)) \ge 0$, furthermore it is known that $e(t_i, t_i + \delta) \ge 0$ and $n \ge 1$. Therefore, the first two terms in the R.H.S of equation (4.20) are always positive or equal to zero. This suggests that equation (4.20) can be written as $\Delta z(t_i + \phi) \ge -C \cdot \delta$. Thus, FIFO is preserved for the State 2.

- *Causality;* the model also respects causality, as travel time of the vehicle entering at time *t* is taken as a function of vehicle already entered in the link and have joined the queue at the end of the link. In addition to this, outflow rate defined through equation (4.11) used only that traffic that already joined the queue (at the end of the link). This is in contrast to *outflow models* [$o(t) = f\ (x(t))$], where outflow rate is taken as a function of vehicles on the link irrespective of their location, and due to this travel time estimated for the vehicle downstream is effected by the vehicles upstream (Mun 2007, p.239).

In the next sub-section, numerical analysis is carried out for all the models discussed above in order to support the features mentioned for each of the model. This is also necessary to evaluate different models behaviour as shown in figure 4.2.

4.5 NUMERICAL EXPERIMENTS

This section represents the results obtained after the numerical implementation of all the four supply-side models discussed in sections 4.3 and 4.4. For numerical evaluation it is required to represent the model in discretised time units. *Appendix*-I illustrates the discretised solution methods (algorithms) of these model which are developed in such a manner that they approximately provide the solution of these models. For linear travel time and point-queue model the solution methods proposed by Nie and Zhang (2005a, 2005b) are used in this thesis, and for divided linear travel time model, the solution method for liner travel time model proposed by Nie and Zhang (2005b) is modified according to the model illustration given in Mun (2001, 2007). For Adnan-Fowkes model, the solution method of the point-queue model proposed by Nie and Zhang (2005a) is modified according to the model illustration shown in section 4.4. These solution methods can be viewed in *Appendix-I* of this thesis. The models are evaluated numerically for four different scenarios (i.e. using four different inflow profiles) which were first used by Nie and Zhang (2005a) in their study for the comparison of different loading models. The first inflow profile represents piece-wise constant inflow in light traffic congestion, second profile represents piece-wise constant inflow in heavy traffic, third profile represents slowly varying inflow in moderately-congested traffic, and the last one represents fast varying inflow in moderately congested traffic. The last two inflow profiles will able to capture the transition from light to heavy congested or vice versa. Figure 4.3 shows the four different inflow profiles used to evaluate model behaviour. The capacity (C) of the link is assumed equal to 1000 vehicles/hour (16.67 vehicle/minute), free flow travel time (ϕ) is assumed equal to 10 minutes and one time step is considered equal to 1 minute.

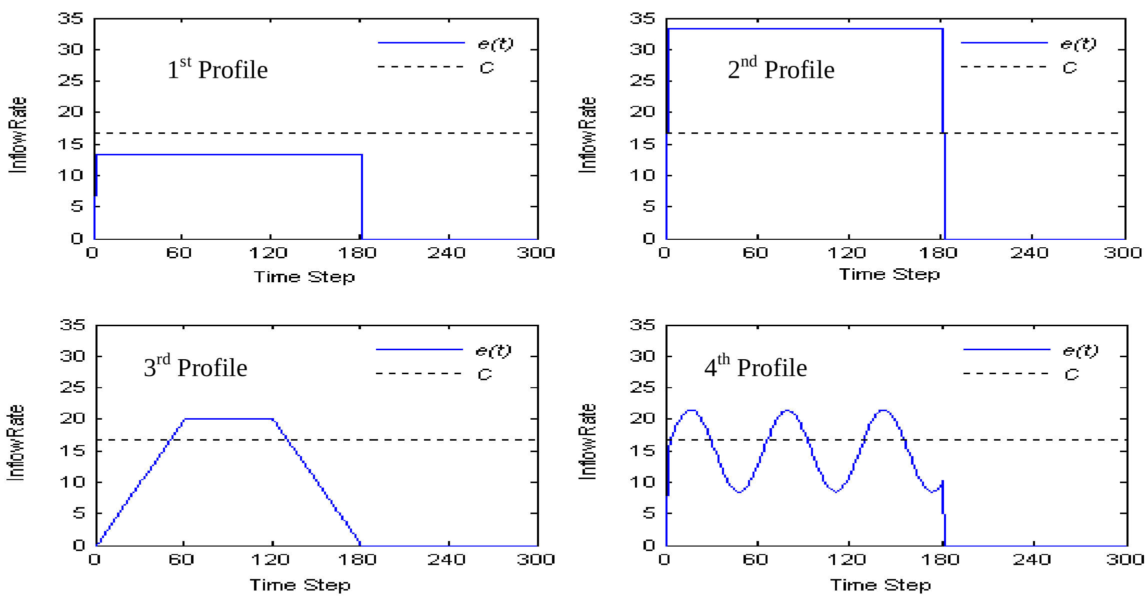


Figure 4.3: Four Inflow Profiles Scenarios

4.5.1 Constant Inflow with Light Traffic Congestion

This inflow profile (see figure 4.3; upper-left plot) is selected in order to analyse the behaviour of the above discussed four loading models in light traffic congestion. It is more likely in this case that traffic on the link will traverse with a link's free-flow travel time while independent of inflow variation over time. Furthermore, this inflow profile (due to its constant nature) clearly reveals the variation in the resulting outflow profile for each model. The results obtained for the four different models are shown in figure 4.4.

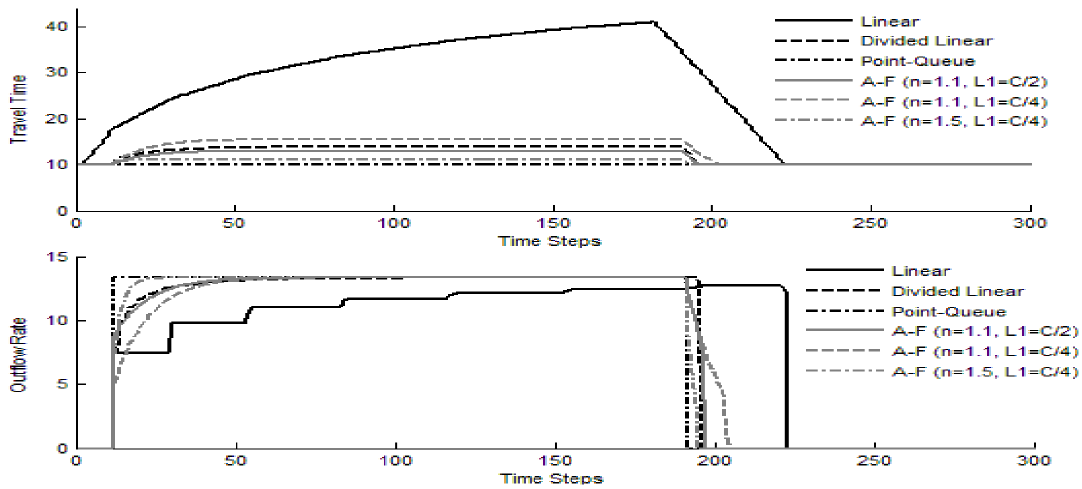


Figure 4.4: Travel Time and Outflow Profiles for 1st Inflow Profile

Figure 4.4 clearly reflects the overestimation behaviour of the linear travel time model, as this model immediately incorporates the effects of congestion for vehicles upstream caused by the vehicles down stream. On the other hand, point-queue model shows that the link is always at a free-flow state (link traverse time is always equal to free-flow travel time i.e.10 minutes), suggesting underestimation of travel time. Divided linear travel time model, which is developed to overcome the overestimation problem in linear travel time model, is behaving well and successful in overcoming the overestimation problem. This is because only part of the traffic existing on the link is considered for estimating congestion effects. Furthermore, this model is not showing congestion effects for the first few initial time steps, this is due to the assumption of the vacant link at the start of simulation and also the manner in which this model works i.e. dividing the link into two parts. So, the vehicles which first entered the link have to traverse with a free-flow speed in the first part of the link. The component responsible for consideration of the congestion effects is active at the time when vehicles reach at the second part of the link. Adnan-Fowkes model (presented as A-F model in the figure 4.4) is experimented with three different combinations of values of L_1 and *n*. As inflow rate in this case is always under capacity, therefore, only two initial states of this model are active dependent on the chosen value of L_1 . If L_1 is considered greater than 0.8 *C* (i.e. constant inflow rate of this inflow profile), then in this circumstance only the first state of the model will be active. Adnan-Fowkes model, which is developed to overcome the underestimation error in the point-queue model, behaves according to expectations.

It can be seen from figure 4.4 that the divided linear travel time model and Adnan-Fowkes model are behaving similar to each other but there is a fundamental difference in the construction of these two models. Divided linear travel time model can only avoid consideration of congestion effects up to the time steps (tick of clock) equivalent to the free-flow travel time for the first part of the link irrespective of the amount of inflow rate. This suggest that if inflow rate is considerably lower (lower than L_1) over a period of time longer than free flow time, then this model also incorporates the congestion effects (similar to linear travel time model) and therefore travel time for the vehicles upstream is increased (greater than free flow travel time) which is not desirable. However, Adnan-Fowkes model follows a more appropriate approach in this regard as it uses the mechanism in which inflow rate of the link is the main factor for controlling the consideration of congestion effects. This suggests that if inflow rate is considerably lower (lower than L_1) then this model always predicts travel time equal to free flow travel time of the link.

The outflow profiles shown for the models give further insights into the behaviour of these models. In case of linear travel time model it has been noted that outflow profile is increasing with time but in periodic steps whose extent is damped out over time. Carey and McCartney (2002 and 2003) explored this accidental by-product of linear travel time model and explained the occurrence of this in detail through analytical illustration. They found that when there is a sudden step increase in the inflow rate, this causes an infinite sequence of steps or jumps in the outflow profile which gradually damp over time. The same phenomenon has happened in the current experiment, as inflow rate in this experiment has increased from zero to 0.8C in almost no time. It has been further suggested by Carey and McCartney that if inflow profile varies slowly the pseudo-periodic jumps in the outflow become insignificant. Based on this they recommended the use of this model in a situation where inflow rate is not rapidly changing. They have also pointed out that if link length is substantially small then outflow profile obtained from this model is much smoother, this has been the basis of the divided linear travel time model proposed by Mun (2001). The outflow profile of divided linear travel time model obtained for this experiment is much smoother which confirms the above arguments. The outflow profile obtained for the Point-Queue model is a replication of the inflow profile with a time lag; this is according to the expectations because there are no vehicles in the queue at the end of the link for this model as inflow rate at all time is lower than the capacity of the link. The outflow profiles obtained for the cases of Adnan-Fowkes model are such that outflow rate is increases with the increase in the amount of queue at the end of the link (as suggested by second state of the model, the model uses its second state because inflow rate assumed here is 0.8C which is greater than L_1), but due to the constant inflow rate, after certain time the amount of vehicle in the queue at the end of link will become constant and outflow rate matches inflow rate suggesting link in a steady state (constant travel time over time). When inflow rate dropped to zero at the end of simulation experiment, then queue start dissipating, Adnan-Fowkes model first uses its second state for determination of outflow rate due to the higher amount of vehicles in the queue (higher than $L_1 \cdot \delta$) which render outflow rate according to the queuing vehicles, but over time when queuing vehicles are lower than $L_1 \cdot \delta$, then first state of the model is active and all the queuing vehicles are exited from the link in a next time step. This is the reason why at the end of simulation experiment outflow rate is decreasing at a different rate and suddenly it becomes zero.

4.5.2 Constant Inflow with Heavy Traffic Congestion

This constant piece-wise inflow profile (see upper-right plot of figure 4.4) in which a heavily congested condition is simulated through the inflow rate that is always twice as great as the capacity of the link up till 180 time-steps. This case is simulated in order to see the travel time and outflow behaviour of the models under consideration which would be mainly due to the queues at the bottleneck and less dependent on the variation of inflow rate.

The results obtained for this experiment are shown in figure 4.5. It can be seen that again linear travel time model is overestimating the travel time compared to other models. However, the degree of overestimation is significantly less in this case compared to the light traffic congestion case (shown in section 4.5.1). This suggests that the impact of double counting effect in estimating travel time from this model is much less in heavy congestion conditions. This can be explained through the outflow rate profile, as in this case the outflow rate increases very rapidly, thus causing less traffic on the link for measurement of travel time.

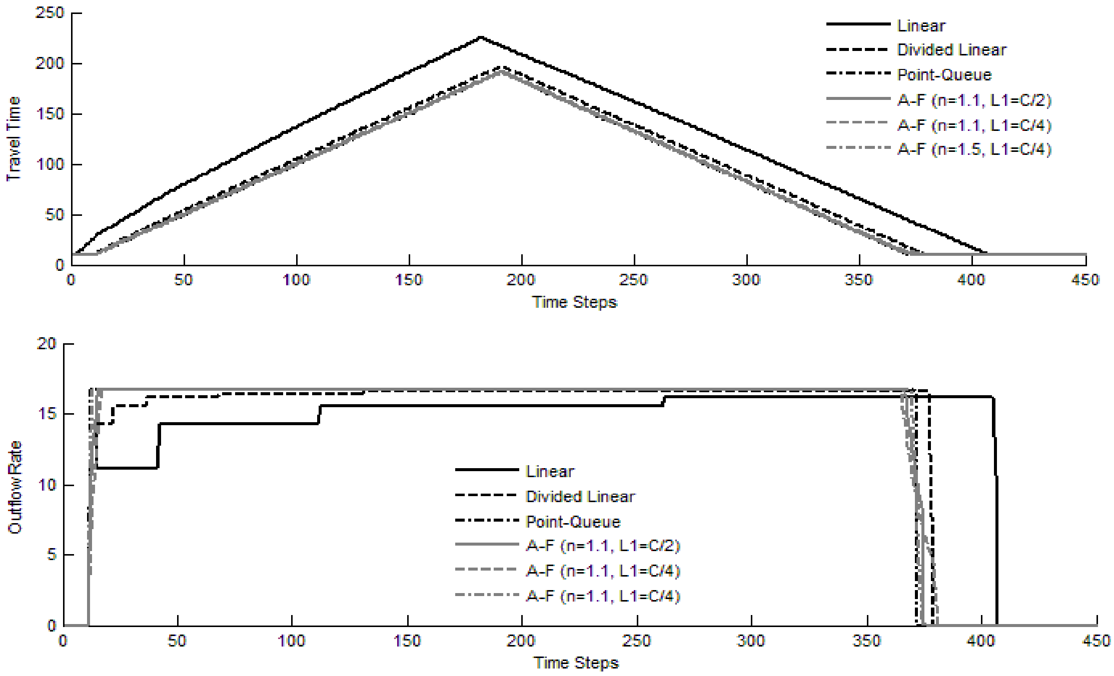


Figure 4.5: Travel Time and Outflow Profiles for 2nd Inflow Profile

Divided linear travel time, point-queue and Adnan-Fowkes models are again producing very similar results in this case. In the point-queue model, under this inflow profile case, the 2nd state is always active which says that outflow from the model equals the capacity of the link. The same behaviour is noted for Adnan-Fowkes model as well, even variation in the values of L_1 and n are not causing any difference. This is because, inflow rate is taken here as double of the capacity and all combinations of L_1 and n examined here gives value of L_2 lower than the $2C$. As a result of this, the first and second state of Adnan-Fowkes model is always inactivated and the model behaves equivalent to the point-queue model. Therefore, for the point-queue and Adnan-Fowkes models, the link is at the free-flow state only up to the few initial time steps (i.e. equivalent to free flow travel time), which is the notion on which divided linear travel time model is built. This is the main reason for the similar behaviour of these three models. The outflow profiles obtained for linear travel time and divided linear travel time models confirms the findings of Carey and McCartney (2002 and 2003) about the pseudo-periodic jumps. It should be noted that for constant inflow rate above capacity, these models are not able to provide steady state travel times, because in the linear travel time and divided linear travel time models vehicles on the link continue to increase at all times and in the point-queue and Adnan-Fowkes models queue at the end of the link is continue to increase at constant rate. This results in increase in travel times at all times until vehicles are continue to entered into the link as depicted in figure 4.5 (travel times is increases up till 180 time steps).

4.5.3 Slowly Varying Inflow with Moderate Traffic Congestion

This inflow profile (shown in the bottom-left plot of figure 4.3) is investigated in order to show the behaviour of the models for peak hour traffic. The inflow is gradually increases in this case and reaches at $1.2C$ in 60 time steps, it remains constant for next 60 time steps and then decreases to zero in further 60 time steps. The results obtained for this experiment are shown in figure 4.6.

It is very clear from figure 4.6 that the linear travel time model again produces significantly higher travel times and in this case the degree of overestimation of travel time is significantly higher compared to travel times obtained from other models. Point-queue model has also shown free-flow state and consideration of congestion effects, suggesting both of its states are active in this situation. However, the underestimation problem of this model in initial stages (i.e. travel time is equal to link's free-flow travel time) is clearly evident. Divided linear travel time model and Adnan-Fowkes models show reasonable estimation of travel times. It has been noted that outflow profile of the divided linear travel time model never reaches capacity (C) of the link at any time (similar to linear travel time model). This situation may raise the question regarding the meaning of the term C used in these models (i.e. linear and divided linear travel time model) because inflow exceeds C at some points in time (see inflow profile for this case) but outflow never reaches C. Adnan-Fowkes and Point-Queue models do not raise this question as outflow from the link reaches capacity (C) of the link at some points in time. Adnan-Fowkes model is more flexible with the introduction of two more parameters (i.e. L_1 and n) in their modelling framework, that certainly provide more ease for adjustment of travel time profile obtained from this model with real data. The interesting point here is that the value of n is playing a major role in defining the degree of convexity of the travel time profile, while L_1 defines the starting point after which effect of congestion is considered for the incoming vehicles. This trend can be seen in the outflow profile as well.

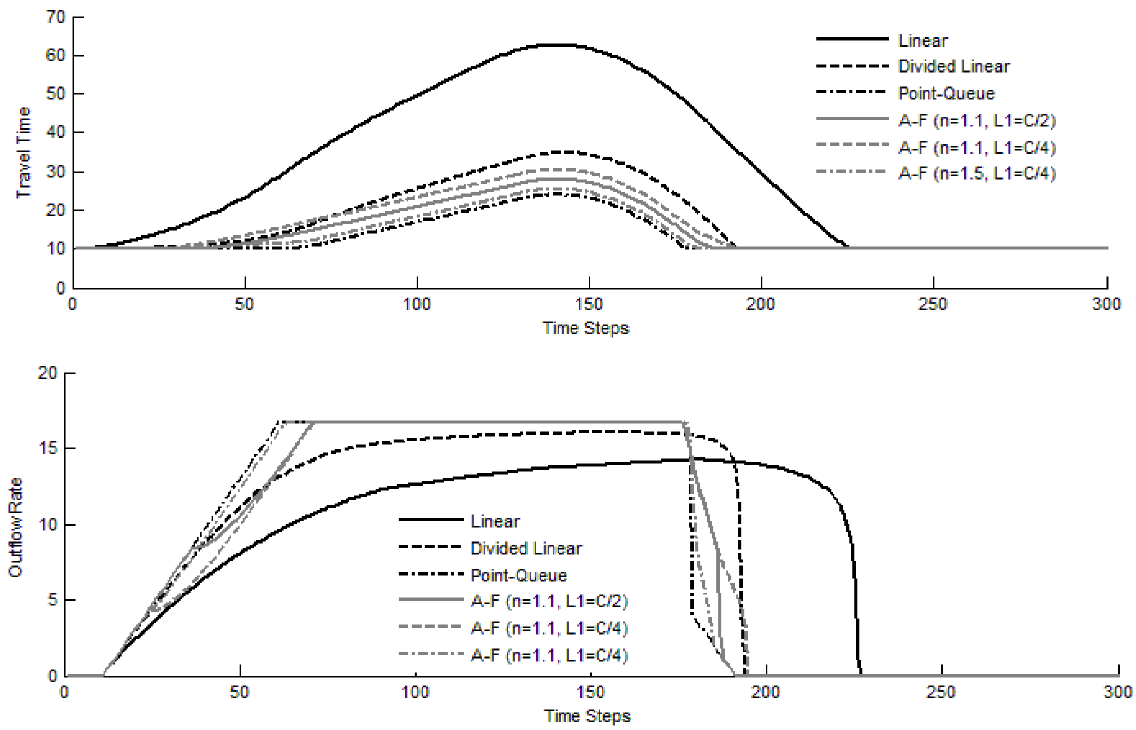


Figure 4.6: Travel Time and Outflow Profiles for 3rd Inflow Profile

4.5.4 Fast Varying Inflow with Moderate Traffic Congestion

In this case behaviour of the models is analysed for the fast varying inflow profile (shown in bottom-right plot of figure 4.3). Inflow profile is based on sinusoidal function and it is varied in such a manner that it fluctuates across the capacity of the link i.e. at some instant inflow is under capacity and at some other instant inflow is over capacity. The highest value inflow can take is 1.3*C* and the lowest value of inflow is around 0.48*C*. The results obtained for this case are summarised in figure 4.7.

Figure 4.7 reveals another important feature of the linear travel time model apart from its overestimation problem. This is regarding the nature of the outflow profile obtained for this model, which is significantly unsmooth compared to the other outflow profiles. This unsmooth nature of the outflow profile may cause problems when used as inflow rate for the subsequent links of the network. Other three models show similar behaviour as obtained for other inflow profiles. Further to that it has been noted that the travel time profiles and outflow profiles try to replicate the features of inflow profiles (i.e. travel time and outflow profiles fluctuates with fluctuation of the inflow profile). However, the degree of fluctuation of profiles obtained for the linear travel time model is much higher compared to the results obtained for other models.

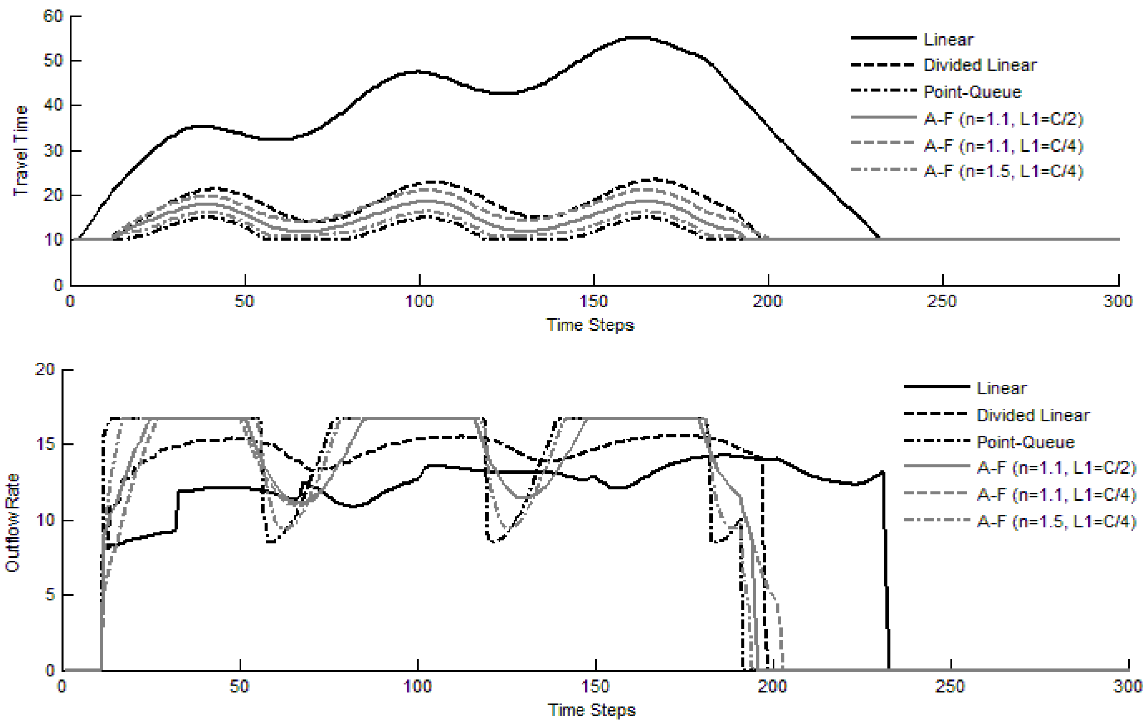


Figure 4.7: Travel Time and Outflow Profiles for 4th Inflow Profile

These numerical experiments clearly suggest that examination of real data is necessary in order to justify the selection of a proper model. Further to that this examination allows calibration and estimation of parameters involved in the models. As no such study exists that is focused on the examination of these models with real data, in this research these models will be used with the assumption of parameter values through which plausible results are obtained.

4.6 SUMMARY

This chapter discusses the important modelling issues at the supply side of the combined modelling framework. It is decided that macroscopic representation of traffic and a dynamic representation of the time dimension at the supply side are appropriate for the development of a combined model aimed at in this research. In relation to this, desirable properties required for the macroscopic dynamic network loading models for proper behaviour of these models on the road network are discussed in detail. It has been suggested in the literature that models in which travel time is taken as a function of vehicles on the link provide a framework that fulfils all the desirable properties. Based on that, Point-queue, linear travel time and divided linear travel time models are discussed in this chapter which not only fulfils the desirable properties but their mathematical construction is such that they required less computation effort for their implementation. However, some problems exist in these models which are primarily due to the oversimplification of the model representation. A novel model (Adnan-Fowkes model) is also comprehensively discussed in this chapter along with the illustration of proofs regarding the fulfilment of important desirable properties. This model was jointly developed during the course of this research and first presented in Adnan and Fowkes (2009), the further corrected version of this model is described in section 4.4 of this chapter. Finally, a section that illustrates numerical comparison of these models is presented for four different set ups of inflow profiles. This helps identifying the difference in the behaviour of these models in the considered conditions.

The next chapter illustrates another important paradigm in the combined modelling framework. This is regarding the operational framework of the combined model through which demand and supply sides are joined together.

Chapter 5

DEMAND-SUPPLY INTEGRATION AND COMBINED MODELLING FRAMEWORK

5.1 GENERAL

This chapter demonstrates the framework of the combined model through which demand and supply sides are integrated with each other. Generally, a fixed point formulation has been adopted in most of the studies that deals with the integration of demand and supply sides especially when the demand side is dealing with the scheduling of a complete activity pattern for an individual. This can be viewed in the combined models, for example: CEMDAP-VISTA (Li et al 2008), TRANSIMS (McNally and Rindt 2008), models presented by Zhang et al (2005) and Kim et al (2006). Based on this, fixed point formulation is discussed in this chapter, and then methods are discussed through which a fixed point problem is normally solved to bring the system in equilibrium.

In the subsequent section, a conceptual modelling framework is presented in order to form the basis for the development of a combined model. This conceptual framework formulates the scheduling problem for a given activity agenda of an individual (list of activities in which an individual needs to participate). The scheduling problem is formulated in which an individual takes a decision regarding their choice of scheduling dimensions (such as departure times, duration, activity sequence and route choice) by making a trade-off of the utility obtained by activity participation against the disutility of network congestion. The network congestion is endogenous in this framework, suggesting a combined model. A fixed point problem formulation approach is used to integrate the demand side with the supply side, the solution of that renders the system in equilibrium.

5.2 DEMAND AND SUPPLY INTEGRATION

This section describes the mathematical approach followed in this thesis for the integration of demand and supply sides. This integration is the main component of the combined model, as it provides a mechanism through which these two sides not only interact with each other, but also brings a complete system in the equilibrium (i.e. demand and supply sides are consistent with each other). The interaction in the demand and supply sides is necessary to establish because both these sides are formulated in such a manner that they require the output of the other as a key input for their own progression. Figure 5.1 explains this notion in more detail.

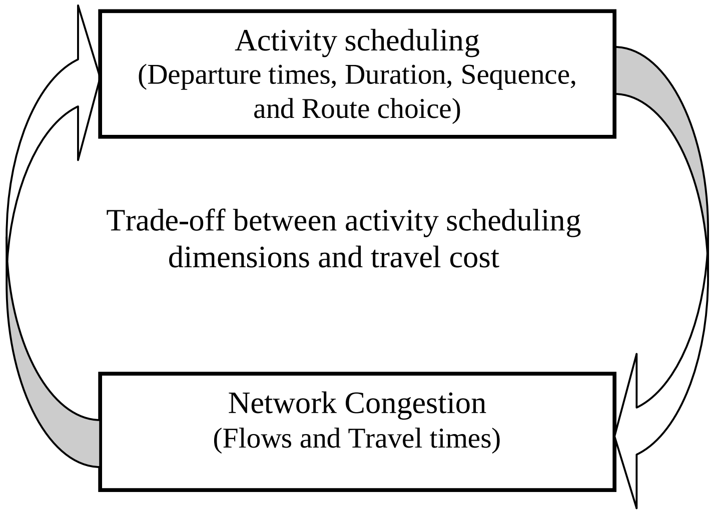


Figure 5.1: Interaction of the demand and supply sides

Figure 5.1 suggests that within the demand side an individual needs to make a decision regarding scheduling of their complete activity pattern (i.e. departure times, duration of activities, sequencing of activities and route choice). An individual does so on the basis of the benefits he/she gains by participating in activities and the cost he/she incurs by travelling on the network based on the scheduling choices. So, the demand side requires inputs that represent travel costs at different times on the network for the prediction of individual scheduling choices. In the same manner, the supply side requires the input in the form of demands at different times of the day in order to predict time varying travel costs based on the underlying supply model. This interactive framework ensures that any changes in the road network, would not only affects travel costs but the effect of this is transferred to the demand side through the changed travel costs, resulting in changed scheduling behaviour of individuals. This changed scheduling behaviour would then result in different travel costs at the supply side. Thus first and second order effects of any policy can be captured through this combined framework. This suggests that the demand and supply sides should be consistent to each other at some point, otherwise this cycle goes on and on and no combinations of plausible and understandable results are obtained at both sides. Mathematical approaches, such as equivalent optimisation (EO), fixed point (FP), variational inequality (VI) or non-linear complementary problems (NCP), render a framework in which most of the transport network related problems are brought in equilibrium (i.e. consistency of demand and supply side is achieved) (Patricksson 1994, p.74). This thesis follows a fixed point problem formulation to bring the system in equilibrium. This formulation along with its solution approaches are discussed in detail in the next sub-sections.

5.2.1 Fixed Point Problem Formulation

A fixed point (FP) problem is based on a mapping F: $\mathbb{R}^n \rightarrow \mathbb{R}^n$ assumed to be continuous. A FP is a vector $\mathrm{x}^* \in \mathbb{R}^n$ such that

$$F\left(\mathbf{x}^*\right) = \mathbf{x}^* \tag{5.1}$$

Equivalently, a FP is the solution of a system of non-linear equations

$$L\left(\mathbf{x}^*\right) = 0 \tag{5.2}$$

where, $L\left(\mathbf{x}^*\right) = F\left(\mathbf{x}^*\right) - \mathbf{x}^*$

Transport network equilibrium models utilised the FP approach in many studies. For example: Daganzo (1982) presented and analysed a FP model for stochastic and deterministic equilibrium assignment for inelastic demand, based on the work of Daganzo and Sheffi (1977); Cantarella (1997) presented a FP formulation of multi-mode, multiuser equilibrium assignment with elastic demand; Bar-Gera and Boyce (2004) proposed a FP formulation to formulate general combined models (mode and route choice together); Li and Huang (2005) presented a FP model for studying the morning commute behaviour in stochastic and time dependent transport networks. The models which are based on daily activity travel patterns, as discussed in chapter 2 (see sections 2.4.1 and 2.4.2) are also formulated as a FP problem such as CEMDAP-VISTA, TRANSIMS and Zhang et al (2005) model. The wide use of FP formulation is based on the fact that it imposes the least stringent conditions on the involved functions (i.e. the supply and demand models). The above discussion suggested that FP formulation has been used across wide variety of transport network problems especially in the cases where stochasticity is involved in the problem (Ben-Akiva et al 2007, Centarella 1997).

There are several ways in which the FP problem can be solved; this thesis follows a standard method of reformulating the FP as the minimisation of an equivalent gap function, G(x), so that G(x) = 0 corresponds to a solution of the FP. Standard solution algorithms can be applied to this minimisation problem. Many studies have used a similar method to solve the FP problem by using the gap function mentioned in equation (5.3) for example (Ben-Akiva et al 2001, Bottom et al 1999, Bierlaire and Crittin 2006).

$$G(\mathbf{x}) = \| \mathbf{x} - F(\mathbf{x}) \|_2^2 \tag{5.3}$$

Equation (5.3) can be interpreted as an inconsistency in the fixed point solution for the norm defined above, with G(x) having units (vehicles/time period)^2. The approximate solution is obtained when the value of $G(\mathrm{x}) < \varepsilon$, where, ε is the tolerance limit and in this thesis it is assumed equal to 10^{-5}. The minimisation of the gap function presented in equation (5.3) *may* render the solution of the FP problem presented in equation (5.1) in cases when the value of gap function is considerably small (i.e. 10^{-5}). Convergence of this algorithm is guaranteed in the case that the objective function is both convex and differentiable, so that a unique FP solution exists. In this thesis however, the existence and uniqueness of a FP are not proved for the problems formulated in chapter 6 and chapter 8. Nevertheless, various numerical experiments are performed and results are illustrated in chapters 7 and 8, which suggest the above method does find an approximate solution of the FP problem in the cases illustrated. However, the recommendation is that when this heuristic is used for solving the problem, that it be tested with respect to different initial conditions and seed values, and gap value monitored in all situations.

5.2.2 Solution Algorithms

There are many standard algorithms that can be used for minimisation of the gap function shown above for solving the FP problem, classical ones are those which are based on averaging methods such as method of successive averages (MSA) and most efficient are those which are based on gradients (Newton methods) such as sequential quadratic programming (SQP). The problem which involves scheduling choices of complete activity pattern along with dynamic representation of congestion on the network is clearly representing a case where evaluation of function itself is a tedious task. Therefore, in this thesis Newton's gradient method is not used. However, a more efficient method than MSA, which is known as quasi-Newton method is also utilised in this thesis. A recognized algorithm within quasi-Newton method is known as BFGS for unconstrained optimisation. MSA and BFGS algorithms are utilised in this thesis for solving the FP problem formulated in chapter 6. These two algorithms are briefly discussed below.

5.2.2.1 Method of Successive Averages (MSA)

This method is useful for large-scale problems because of the insignificant amount of linear algebra associated with the generation of each iterate, as well as its moderate memory requirement as only the last two iterates have to be stored (Sheffi 1985, p.326). This method can be summarized as follows: choose a starting point $\mathbf{x}_0$ and generate a succession of points of the form

$$\mathbf{x}_{n+1} = \mathbf{x}_n + s_n \left(F\left(\mathbf{x}_n\right) - \mathbf{x}_n \right) \tag{5.4}$$

where, s_n step-size at *n*th iteration, and the sequence of step-sizes $s_1, s_2, \ldots$ is determined prior to the start of the algorithm. Sheffi (1985, p. 324) mentioned that for this algorithm to converge, it is required that the objective function need to be continuous and convex in shape and the sequence of step-sizes has to satisfy the following two conditions:

$$\sum_{n=1}^{\infty} s_n = \infty \ , \qquad \sum_{n=1}^{\infty} s_n^2 < \infty$$

In view of the above conditions, the simplest step-size sequences is, $s_n = 1/n$. This algorithm performs well for initial iterations but converges very slow near solution, because step-size becomes sub-optimal. It can be seen that this method does not require gap function for actually solving the FP problem, however, when this method is used gap function is also evaluated and its value is monitored. Considerably small value of gap function (G(x) < 10^{-5}) may suggest that convergence is achieved.

5.2.2.2 BFGS Algorithm

The BFGS method (named after Broyden-Fletcher-Goldfarb-Shanno) is derived from Newton's method in optimization (Walsh 1975). Newton's method assumes that the function can be locally approximated as a quadratic Taylor expansion and uses the Jacobian (first order derivative) and Hessian (second order derivative) matrices to find the stationary point. In quasi-Newton method, the *Hessian matrix* is updated at each iteration with the formula proposed in various methods (i.e. BFGS, DFP etc.). The Jacobian information can be supplied either through analytical or finite difference techniques. The BFGS method proceed as follows: Given $\mathbf{x}_n$, the next successive point is

$$\mathbf{x}_{n+1} = \mathbf{x}_n + s_n \cdot \left(- \mathbf{H}_n^{-1} \cdot \nabla G(\mathbf{x_n})\right) \tag{5.5}$$

where, $\mathbf{H}_n^{-1}$ is the inverse of Hessian matrix of the objective function which is updated iteratively through a proposed formula, and $\nabla G(\mathbf{x_n})$ is the gradient of the objective function (gap function in equation 5.3) evaluated at $\mathbf{x}_n$. s_n is the step size at each iteration. The step by step procedure of this algorithm can be seen in Walsh (1975). This algorithm is already programmed in the MATLAB optimisation tool box under *fminunc* function. When this algorithm is employed in experiments of chapter 7 and 8 for minimisation of the constructed gap function for solving the FP problem, the gradient of the objective function required for its implementation has been worked out implicitly by *fminunc* through finite difference method. Chapter 7 also discusses and compares the speed of convergence of the two discussed algorithms for the FP problem formulated in Chapter 6 of the developed combined model.

5.3 COMBINED MODELLING FRAMEWORK

This section describes the combined modelling framework by revisiting chapter 3, chapter 4 and section 5.2 of chapter 5. Furthermore, two more issues regarding the problem formulation are discussed in this section. This forms the basis for the development of a mathematical formulation of the combined model presented in Chapter 6.

5.3.1 Conceptual Framework

The demand side of the combined model discussed in chapter 3 presented two distinctive issues of the combined model. The first one is regarding the underlying specification of the systematic utility function based on the concept of time-of-day dependent and duration dependent marginal utility of activities. It has been summarised that these concepts are necessary to incorporate for modelling scheduling choice of activity travel pattern. The next issue was regarding the operational models such as MNL, NL etc. that can be used to predict the choice probabilities. The supply side of the combined model discussed in chapter 4, suggested a range of dynamic traffic performance models that can be utilised for representation of the dynamic network congestion. Section 5.2 in this chapter, described the approach suitable for the formulation of the combined model and discussed standard algorithms which are available for obtaining the solution. Integration of the above discussed issues in a unified framework renders a conceptual modelling framework for the combined model. This can be viewed from figure 5.2. The figure shows that the modelling framework assumes that the individual daily or weekly activity agenda is given (i.e. set of activities an individual needs to perform in a given day or week, *e.g, home-work tour on daily basis or daily home-work along with shopping activity once in a week*).

It should be noted that the model framework is such that it can model scheduling choices of daily activity agenda as well as weekly activity agenda of an individual. Additionally, the road network and its properties (such as link free-flow travel times and capacities) along with the location of activities are given. As choice of mode is not modelled here, therefore, it is assumed that all individuals are travelling in their private vehicles having vehicle occupancy equals unity. The demand side predicts the scheduling choices of an individual through its operational model and underlying utility specification and finally provides departure rates from an origin (say home) to any destination (say work). On the other hand, these departure rates (inflows) serve as an input for the supply side and based on any traffic performance model (mentioned in chapter 4) time varying travel times for different routes of the network are predicted. These time varying travel times are required at the demand side to feed into the utility specification. Thus, a fixed point problem is formed, and can be solved using a heuristic procedure discussed in section 5.2.1. The solution of the FP problem gives rise to stochastic dynamic user equilibrium (SDUE).

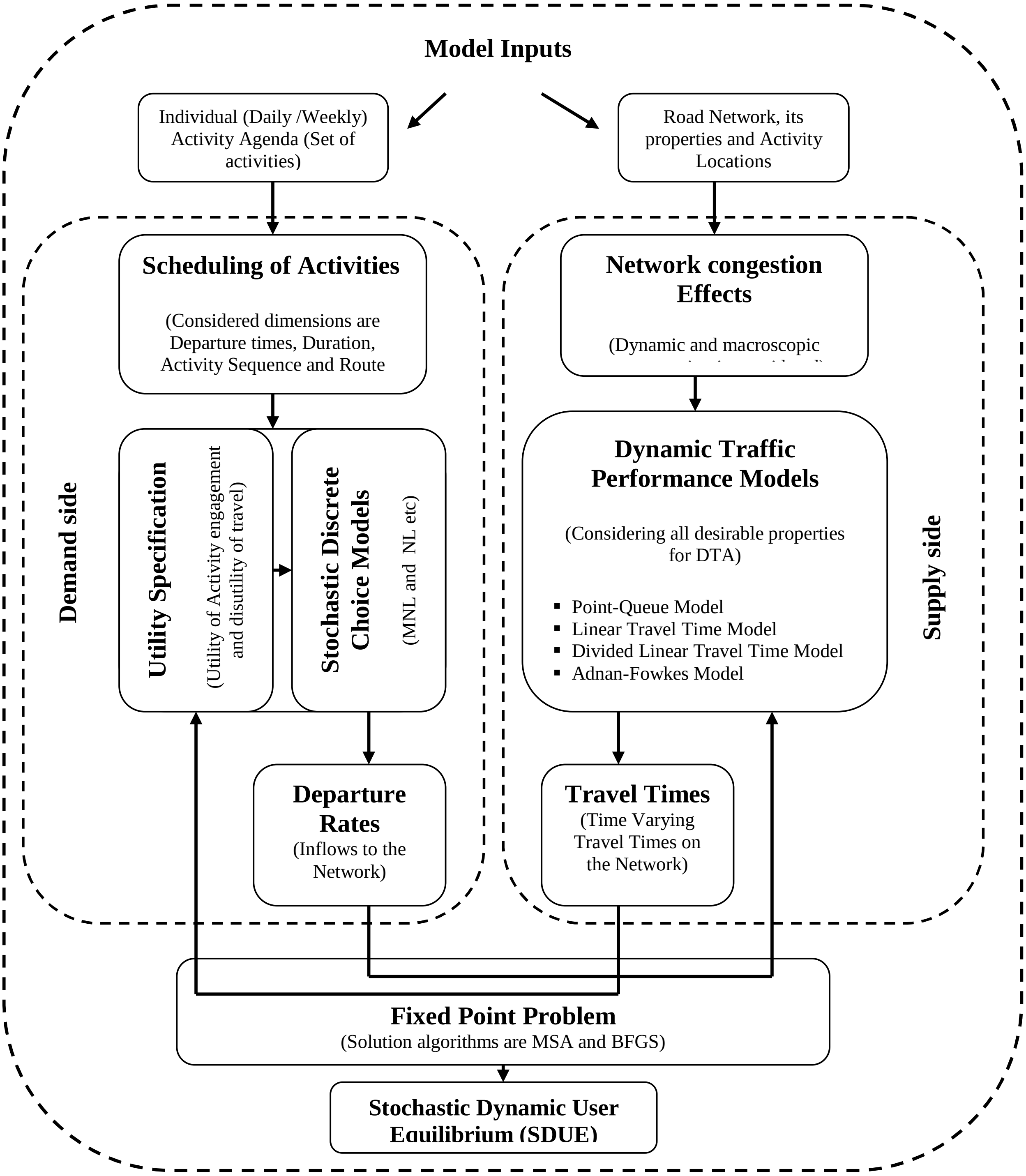


Figure 5.2: Conceptual Modelling Framework for the Combined Model

5.3.2 Path-based Formulation of the Model

While chapter 3 and 4 mainly discuss issues within the demand and supply sides respectively, it is important at this point now to discuss important issues regarding the mathematical formulation of the combined model.

Usually, assignment models (static or dynamic) are formulated using a link-based formulation, however, from the point of view of dynamic modelling and its application to Advanced Traveller Information System (ATIS) path-based assignments are ideal because the controller needs to provide paths to trip-makers. These paths can be obtained from link-based formulation as well, but the problem is that when paths are formed from the link-based formulation using expressions containing the link-path incidence matrix, the uniqueness is not guaranteed (i.e. many combinations of paths flows that satisfy the link-flows are obtained using the link-based formulation for traffic assignment). This is explained through an example in Sheffi (1985, p.68). It is also pointed out that path-based formulations are computationally not efficient as in this formulation enumeration of paths for each origin-destination pairs is required before loading of the flows on the network, however, path-based formulations are useful where the utility function involves variables which are strictly specific to paths. The incorporation of marginal activity utility concept for modelling scheduling dimensions (which is used in this thesis) requires path-based travel times for the valuation of activity utility, thus suggesting that the scheduling problem of the combined model should be formulated as path-based. Therefore, the scheduling problem formulated in chapter 6 and 8 uses the path-based formulation. This is also useful for application of the combined model in such situation where path-based variables are involved in the utility function. Gabriel and Bernstein (1997, p.338-339) discussed situations where variables in the utility function are path-based. Furthermore, incorporation of travel time reliability notion also requires path-based formulation of the problem.

5.3.3 Temporal Issues

Another issue in the problem formulation is the representation of the time dimension at the demand and supply side, because implementation of the solution procedure usually requires discretisation of time. This discretisation is often based on availability of input data, accuracy requirements and consideration of computational efforts. Usually, at the demand side larger intervals of time (order of several minutes) are considered compared to the supply side where intervals of time are of order of less than a minute. Larger intervals of time at the demand side are considered because of the fact that the behaviour of people does not change appreciably over smaller time intervals considered at the supply side. Furthermore, consideration of large intervals prevents the correlation among the time based alternatives (departure times) and provides justification for the use of a basic operational model at the demand side (i.e. MNL). The mathematical formulation of the combined model presented in chapter 6 considered the same notion.

5.4 SUMMARY

This chapter first discussed the mathematical approach (i.e. FP problem), appropriate for the integration of the demand and supply sides in order to bring the system in equilibrium. Two standard solution algorithms are also discussed through which the constructed gap function for the FP problem can be minimised in order to heuristically find a solution of the FP problem. Secondly, the chapter presented a conceptual modelling framework for the combined model developed by revisiting the issues and the modelling considerations discussed in chapter 3 and 4. This conceptual modelling framework renders a profound basis for the development of the combined model (presented in chapter 6 and 8). In addition to this, the chapter discusses two important issues regarding the mathematical formulation of the problem i.e. path-based formulation and time discretisation. Both of them are vital when the mathematical formulation of the combined model is practically implemented.

Chapter 6

SCHEDULING OF DAILY HOME-BASED TOURS IN A COMBINED MODELLING FRAMEWORK

6.1 GENERAL

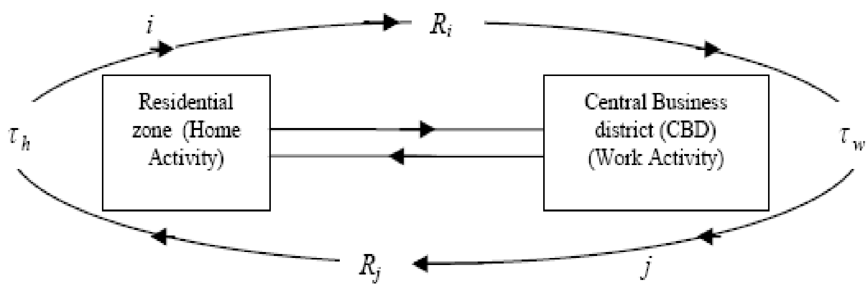
The work presented in previous chapters is regarding the discussion of potential issues, approaches and operational models within the three individual components (demand side, supply side and demand-supply integrator) of the combined modelling framework. The discussion of various issues in these chapters has built an understanding for the development of a combined model aimed at in this thesis. Based on this extensive background, chapters 6, 7 and 8 presents the body of work carried out in this research related to the model development, its extensions and applications.

In this chapter development of a combined model for the scheduling of daily home- based tours is presented. This comprises the simple home-work tour and home-work tour with an additional activity, these two activity patterns are chosen as it has been found out that these are the most common patterns among the urban resident individuals (Bowman and Ben-Akiva (2000) and Shiftan et al (2004). At a preliminary stage, the model considers departure time and activity duration choices as scheduling dimensions for the home-work tour (which is presented in Adnan, 2009). The route choice and activity sequence choice is not considered in this preliminary model. This has been done in order to understand the role of time-of-day and duration dependent marginal utility of activities for connecting morning and evening commute together, which is explained in section 6.3 through numerical and analytical proofs. The findings obtained in this section are crucial as these contradict with the earlier attempts of scheduling morning and evening commute together with network congestion. For home-work tour there is no question regarding the choice of sequence, however, route choice is important and how it can be incorporated for general networks is illustrated in section 6.5. This section also represents the extension of the preliminary model presented in section 6.2 for simple home-work tour. The extended model represents different types of tours (other than home-work tour) carried out in a day by individuals in a simplified network with all four scheduling choices (i.e. departure times, duration, activity sequence and route choice). The last section summarises the work reported in this chapter.

6.2 MODEL DEVELOPMENT

6.2.1 Modelling Assumptions

A home-work tour is considered between the home and work activity locations which are connected with a single two-way divided link. The scheduling dimensions involved here are the choice of departure times for work and from work. Durations of work and home activities are also considered in this structure, which can be derived from departure times along with the travel time to get to work and home. Figure 6.1 further explains this framework in detail. Activity scheduling for this tour can be defined by a pair of discrete departure times from home and work activity denoted by *i* and *j* respectively.



i and *j* are departure times (i.e. clock-times) from home and work location respectively
Ri and *Rj* are the travel times on the link at their respective departure times for the morning and evening commute respectively
τ_w and τ_h are the duration of work and home activity and are given by
$\tau_w = j-(i-R_i)$, $\tau_h = 1440-(\tau_w + R_i + R_j)$
Time unit is taken in minutes and a full day is considered that comprises 1440minutes

Figure 6.1: Home-Work Tour time cycle

Scheduling for the home-work tour = (departure time from home, departure time from work)

$$= (i, j) \in \mathbb{N}^2$$

where, $i = T, T+1\cdot\Delta, T+2\cdot\Delta, \ldots, T+(D-1)\cdot\Delta$; $j = Y, Y+1\cdot\Delta, Y+2\cdot\Delta, \ldots Y+(D-1)\cdot\Delta$; and these departure periods belongs to $\mathbb{N}$ that represents the set of integer numbers. The duration of each departure time period Δ can be considered as one minute, 10 minutes or 30 minutes. *D* represents the total number of departure periods considered for each commute. *T* represents the time-of-day of the first departure period in minutes past midnight (e.g. 6:00 am in the morning is 360 minutes) for the morning commute. Similarly, *Y* represents the time-of-day of the first departure period for the evening commute. Suppose that $\mathbb{K}$ is the set of all possible combinations of (*i, j*) i.e:

$$\mathbb{K} = \{(i, j):\ i \in \mathrm{N},\quad j \in \mathrm{N},\quad T \le i \le T + (D-1)\cdot\Delta,\ Y \le j \le + (D-1)\cdot\Delta\}$$

The above definition of departure times for the morning commute and the evening commute is such that not all combination of *i* and *j* are feasible, especially when $T = Y$, as individuals always allocate some time to work activity and they also consider some time for travelling between home and work activity locations. The definition of activity utilities and travel disutility, therefore, play a major role, and individuals decide about there departure times from home and to home by maximising their total utility of the tour.

6.2.2 Demand Side of the Model

The overall utility for this tour, according to the utility maximisation framework of Ettema and Timmermans (2003), which is also used in Zhang et al (2005), can be expressed as

$$U_{ij} = V_{ij} + \varepsilon_{ij} \tag{6.1}$$

where, $V_{ij} = \left(V^A + V^T\right)$ represents the systematic utility and ε_{ij} represents the random term associated with each alternative. V^T is the total utility derived from travelling and V^A is the total utility derived from participating in activities. V^T and V^A are themselves the sum of utilities of *M* number of trips and *N* number of activities respectively and are given by

$$V^T = \sum_{m=1}^{M} V^{T_m} \tag{6.2}$$

$$V^{A} = \sum_{n=1}^{N} V^{A_n} \tag{6.3}$$

In the above specification, the utility of a trip made at time *t* is characterized by the travel time and travel cost. The utility derived from activity participation is described in the next section. The above three equations can be termed as a generalized utility framework that can accommodate all types of individual daily activity patterns. For the home-work tour, we can write as:

$$V_{i\,j} = V^{h} + V^{w} + V^{T_{h-w}} + V^{T_{w-h}} \tag{6.4}$$

where, V^{h} represents the utility gained at home, V^{w} represents the benefits obtained at work and where, $V^{T_{h-w}}$ and $V^{T_{w-h}}$ are the negative utilities (disutility) of travelling from home to work and work to home and are dependent on travel times R_i and R_j respectively. Individuals will trade-off between the overall travel cost of the two trips and benefits gained through participating in home and work activities, when taking their decision of scheduling for the tour. It should be noted that activity participation utility terms are based on predetermined marginal utility profiles which are also dependent on travel times at departure time *i* and *j*. Therefore, the utility of home-work tour scheduling is dependent on $V^{T_{h-w}}$ and $V^{T_{w-h}}$ which are the functions of R_i and R_j. Based on this, the systematic utility of the tour can be written as

$$V_{i\,j} = f\left(R_i\,, R_j\right) \tag{6.5}$$

For operationalisation of the above utility framework, operational models within the demand side can be used for the calculation of the probabilities for the alternatives. The sum of these probabilities across individuals represents the market share for each alternative. For example, for the Multinomial logit (MNL) model,

$$P_{i\,j} = g\left(V_{i\,j}\right) \qquad \forall\left(i,\ j\right) \in \mathbb{K} \tag{6.6}$$

and for other discrete choice models such as nested logit (NL)

$$P_{i\,j} = g\left(V_{i\,j};\underline{\omega}\right) \qquad \forall\left(i,\ j\right) \in \mathbb{K} \tag{6.7}$$

where, $P_{i\,j}$ = Probability of choosing alternative (*i*, *j*) and $\underline{\omega}$ = Vector of additional parameters for a particular model form other than MNL. Suppose that *Q* is the total number of individuals in the residential zone, then the choice rate of individuals who will depart from home and work at time *i* and *j* respectively is given by:

$$q_{i\,j} = Q\,P_{i\,j} \tag{6.8}$$

The number of trips at departure time *i* from home to work q_i can be determined by summing over all the combined choices $q_{i\,j}$ over the departure time j and similarly, q_j can be worked out, as given in equation (6.9). These inflows then feed into the supply side to determine time varying travel times.

$$\begin{aligned} q_i &= \sum_j q_{i\,j} \\ q_j &= \sum_i q_{i\,j} \end{aligned} \tag{6.9}$$

6.2.3 Supply side of the Model

It has been mentioned already that the supply side of the model represents time-varying congestion through time-dependent travel times. For this purpose, different dynamic network loading (DNL) models can be utilised which require inflow profiles (outcome of the demand side) as discussed in chapter 4. The DNL models considered in this thesis assume that the travel time of a vehicle entering at time *i* is a linear function of the number of vehicles existing on the link or the number of vehicles forming a vertical queue at the end of the link at time $i+\phi$. Therefore, for the morning trip travel time R_i is given by

$$R_i = \phi + \psi \cdot \zeta\left(\Theta\right) \tag{6.10}$$

where, ϕ is the free-flow travel time on the link and ψ represents the inverse of the exit capacity of the link. Function $\zeta(\Theta)$ represents the functional form for the measurement of the queuing delay in the different DNL models. For example, in linear travel time model this functional form is equivalent to $\zeta(\Theta) = x_i$, where x_i represents number of vehicles on the link at time *i*. In the point-queue model is equivalent $\zeta(\Theta) = z_{i+\phi}$, which represents vehicles at the end of the queue. Similarly, the Mun (2001) and the Adnan-Fowkes models (see chapter 4, section 4.3 and 4.4) require a different interpretation of the function $\zeta(\Theta)$. The functional form $\zeta(\Theta)$ depends on the flow conservation and propagation equation specified for a particular model and these equations are the function of inflow profiles, Therefore, it can be written as

$$\zeta(\Theta) = w(\mathbf{q}) \tag{6.11}$$

where, w is the functional parameter that ensures the compatibility of (6.11) with flow conservation and propagation equations for each DNL model explained in chapter 4, and **q** represents a matrix containing elements q_{ij}. From (6.11), it can be shown that the travel time profiles from a particular DNL model is a function of inflows

$$\mathbf{R}_M = s(\mathbf{q}) \tag{6.12}$$

where, $\mathbf{R}_M$ is the vector (having R_i as their elements) that represents profile of travel times for trips to work in the morning. Similarly $\mathbf{R}_E$ can be worked out, which contains R_j as their elements, and representing profile of travel time for trip to home in the evening.

6.2.4 Fixed Point Problem Formulation

From the above sections, a fixed point problem can be formulated as:

$$\mathbf{q} = Q \cdot \mathbf{P}\left(\mathbf{V}(\mathbf{R}(\mathbf{q}))\right) \tag{6.13}$$

where, **P** and **V** are two dimensional vectors containing elements P_{ij} and V_{ij} respectively. and $\mathbf{R}$ is a vector containing elements $\mathbf{R}_M$ and $\mathbf{R}_E$

The solution of equation (6.13) results in an SDUE allocation of schedules of the home-work tour which, may be defined as follows: At SDUE no motorist can improve his/her perceived utility of scheduling the tour by unilaterally changing schedules. This follows directly from the interpretation of the choice probability as the probability that the perceived utility of the chosen schedule for the tour is the highest of all the schedules for the tour. The minimisation of the gap function presented in equation (6.14) by using a standard solution algorithm may solve the above formulated fixed point problem in a case when gap value is very small (i.e. $G(\mathbf{q}) < \varepsilon$). This is a heuristic method (which is also used in many studies as pointed out in section 5.2.1) as in this method it cannot be guaranteed that this method always solves the fixed point problem presented in equation (6.13).

$$min \; G(\mathbf{q}) = \| \; \mathbf{q} \; - \; Q \;\; \mathbf{P} \; (\mathbf{V}(\mathbf{R}(\mathbf{q}))) \|_2^2 \qquad (6.14)$$

6.3 UTILITY OF AN ACTIVITY PARTICIPATION – EFFECT OF ONLY TIME-OF-DAY REPRESENTATION

The essential aspect of the model presented in section 6.2 lies within the utility specification, which suggests that the utility of scheduling the home-work tour contains two components; the utility of activity engagement and the utility of travel, represented in equation (6.4). This equation is elaborated here in the following sub-sections.

6.3.1 The Utility of Activity Engagement

In chapter 3 comprehensive discussions has been carried out regarding ways to measure the utility of activity engagement. It has been shown that the scheduling theory of Vickrey (1969) and Small (1982), which is based on the preferred arrival time (PAT), is capable of representing time-of-day preference for a particular activity. It uses a form that anchors a time-of-day axis at a particular point in time (i.e. PAT), which can (if needed) be considered as a preferred activity start time. This suggests that this concept is appropriate for representing the *time-of-day effect* for fixed-in-time activities. However, this concept does not provide a valuation of activity utility. An alternative approach which is based on the time-of-day dependent marginal utility of an activity is capable of providing a framework through which the utility of an activity can be valued (utility an individual gains by participating in an activity). Researchers have proposed functional forms for the marginal utility of different activities, most common are bell-shaped and piece-wise constant profiles as shown in chapter 3. These profiles assume that the marginal utility of an activity is high at a preferred time-of-day and decreases as one move away from that time-of-day.

Earlier works on scheduling of activities for the home-work tour context e.g. Zhang et al (2005), Ettema and Timmermans (2003), Kim et al (2006) and Heydecker and Polak (2006), have considered marginal utility profiles for an activity as a function of time-of-day only, and stated that their model integrates both the morning and evening commute. If this is considered as true then the activity engagement related components in equation (6.4) can be written as

$$V^h = \int_0^i V'^h(t)\,dt + \int_{j+R_j}^{1440} V'^h(t)\,dt \qquad \text{and} \qquad V^w = \int_{i+R_i}^{j} V'^w(t)\,dt$$

where V'^h and V'^w are marginal utility functions for the home and work activities respectively. These marginal utility functions may follow any form provided that they are dependent on time-of-day. The marginal utility functions for home and work activities are integrated over the time duration individuals have spent performing these activities. So, the overall utility of activity engagement is given by

$$V^h + V^w = \int_0^i V'^h(t)\,dt + \int_{i+R_i}^{j} V'^w(t)\,dt + \int_{j+R_j}^{1440} V'^h(t)\,dt \tag{6.15}$$

The next sub-section illustrates that if the marginal utility of activities is taken only as a function of time (individual time-of-day preference is only considered in measuring utility of activity engagement), then this utility specification does not integrate the two commute trips together (Adnan et al 2009). That is to say there is no difference in results between the cases where the two commute trips are modelled in combination or in separation.

6.3.2 Numerical Illustration

The travel component in the utility formulation, which represents the disutility of travel, can be measured through travel times. Therefore, the following can be written:

$$V^{T_{h-w}} + V^{T_{w-h}} = \lambda R_i + \lambda R_j \tag{6.16}$$

where λ is a negative parameter representing the pure in-vehicle disutility experienced while travelling. This should not be confused with the value of travel time parameter used in many modelling studies for representation of the value-of-time, which contains disutility from other factors that are present in our model as part of the utility of activity engagement (see chapter 3, section 3.3.4). Therefore, the total utility can be given as

$$V_{ij} = \int_{0}^{i} V'^{h}(t)\,dt + \int_{i+R_i}^{j} V'^{w}(t)\,dt + \int_{j+R_j}^{1440} V'^{h}(t)\,dt + \lambda R_i + \lambda R_j \tag{6.17}$$

With the above utility function, the following assumptions are made for this experiment. Suppose that there are in total Q = 5000 commuters who will conduct home-work tour and the morning departure time starts from 0600hours (i.e. T = 0600 hours or 360 minutes past midnight). In total, D = 8 departure periods each of Δ =30 minute duration are considered for each of the morning and evening commute. Similarly, it is assumed that departure times for the evening commute starts from Y = 1400hours. The in-vehicle disutility parameter is assumed as λ = -0.08 £/minute. Free-flow travel time (ϕ) on the link is considered as 10 minutes with an exit capacity (C) of 1800 vehicles/hour. At the supply side for this experiment, the point-queue model has been utilised with an analysis time interval (δ) as 1 minute and MNL model was used at the demand side. It is required to feedback the travel times into the utility specification of the model; therefore, travel times obtained from supply model at each minute, were averaged for 30 minutes duration before feeding into the utility specification as each departure period considered is of 30 minutes duration. For the home activity, an inverse bell-shaped time-of-day dependent marginal utility function is assumed (this has already explained in Chapter 3). This represents that the utility of stay-at-home is higher in the early morning and evening than the day time, because people prefer to stay-at-home for activities such as having a family dinner, watching TV and sleeping. The functional form of this marginal utility function follows from Ettema and Timmermans (2003) and Zhang et al (2005) and is given by

$$V'^{h}(t) = h_0 - \frac{\beta\,\gamma\,U^{\max}}{\exp\left[\beta\,(t-\alpha)\right]\left[1+\exp\left(-\beta\,(t-\alpha)\right)\right]^{\gamma+1}}$$

For work activity, the bell-shaped time-of-day dependent marginal utility profile is assumed which provides high utility at mid-day. This represents that workers start to warm up after arrival at their office and work efficiently around mid-day and after this period worker's efficiency keeps declining until one leaves office. Similar specification has been assumed in other studies e.g. Zhang et al (2005), Heydecker and Polak (2006) and Ettema and Timmermans (2003). This is given by:

$$V^{'w}(t) = \frac{\beta\,\gamma\,U^{\max}}{\exp\left[\beta\,(t-\alpha)\right]\left[1+\exp\left(-\beta(t-\alpha)\right)\right]^{\gamma+1}}$$

where, h_0, α, β, γ, U_0 are the parameters that controls the shape of the marginal utility profiles. For home activity the values assumed for these parameters are; h_0=0.025, α=720, β=0.04, γ=1, U_0=12.5 and for work activity these parameter values are taken as; α=720, β=0.02, γ=1, U_0=5 which is assumed in Zhang et al (2005). Figure 3.2 in chapter 3 showed the shape of these marginal utility functions with the above parameter values. With the above mentioned assumption the combined morning and evening commute is modelled and the results obtained at equilibrium are shown in figure 6.2.

For the separate modelling case, to find out the utility for the morning commute; the marginal utility of home and work activities are integrated over the half-day period, starting from midnight and ending at 12 noon. For the evening commute utility, the remaining day is considered. The utility for both these commutes is given as

$$V_i = \int_0^i V^{'h}(t)\,dt + \int_{i+R_i}^{720} V^{'w}(t)\,dt + \lambda R_i \quad \text{, for morning commute} \tag{6.18a}$$

$$V_j = \int_{720}^{j} V^{'w}(t)\,dt + \int_{j+R_j}^{1440} V^{'h}(t)\,dt + \lambda R_j \quad \text{, for evening commute} \tag{6.18b}$$

For both these commutes two fixed point problems are solved independently, unlike the combined home-work tour. The results obtained are shown in figure 6.2 as demand and travel time profiles. The figure shows that there are absolutely no differences in the results of separate and combined modelling cases (i.e. demand and travel time profiles are exactly the same for combined and separate treatment of morning and evening commutes). The explanation of this phenomenon lies within the marginal utility profiles. This is illustrated analytically in the next sub-section.

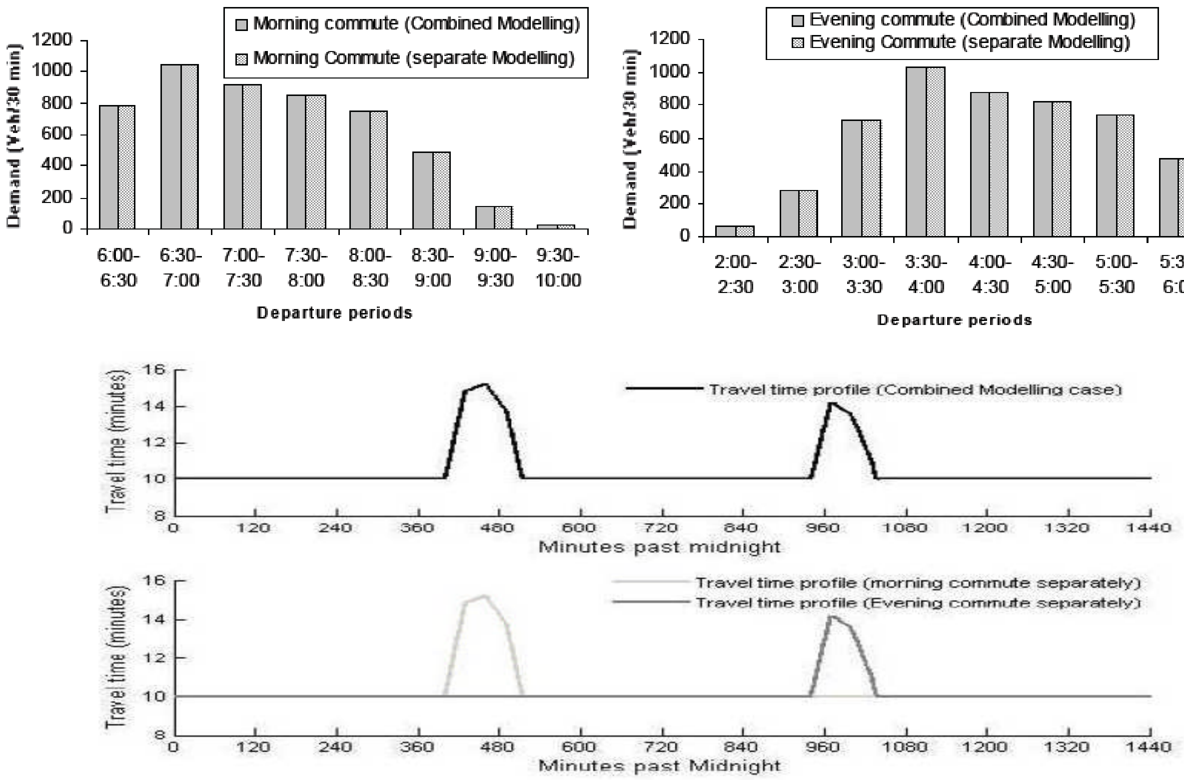


Figure 6.2: Demand and travel time profiles for combined and separate modelling cases

6.3.3 Analytical Illustration

The analytical illustration starts with equations (6.17), (6.18a) and (6.18b), with the assumption that the marginal utility of home and work activities may follow any form keeping their time-of-day dependency. Rewriting these equations gives:

For overall utility of scheduling of the tour:

$$V_{ij} = \int_{0}^{i} V'^{h}(t)\,dt + \int_{i+R_i}^{j} V'^{w}(t)\,dt + \int_{j+R_j}^{1440} V'^{h}(t)\,dt + \lambda R_i + \lambda R_j \qquad (6.19)$$

For morning commute:

$$V_i = \int_0^i V'^h(t)\,dt + \int_{i+R_i}^{T} V'^w(t)\,dt + \lambda\, R_i \qquad (6.20a)$$

For evening commute:

$$V_j = \int_T^j V'^w(t)\,dt + \int_{j+R_j}^{1440} V'^h(t)\,dt + \lambda\, R_j \qquad (6.20b)$$

where T is an arbitrary time that follows $((i+R_i) \le T \le j)$. If (6.19) is compared with (6.20a) and (6.20b) then it can be written as:

$$V_{i\,j} = V_i + V_j \qquad (6.21)$$

Now it can be demonstrated that when the marginal utility of home and work activities are taken as a function of time-of-day, then there is no difference in modelling morning and evening commute separately or jointly, provided that equation (6.21) holds. Mathematically it is equivalent to say that:

$$\sum_{j=Y}^{Y+\Delta\cdot(D-1)} q_{i\,j} = q_i \qquad (6.22)$$

where, $q_{i\,j}$ is the demand predicted for an alternative (*i, j*) using (6.19), and its sum across the *j*th dimension represents the demand for the *i*th departure period of the morning commute. q_i is the demand predicted from the separate modelling of the morning commute for an alternative *i* using (6. 20a). Equation (6.22) can be written in probabilistic terms as:

$$\sum_{j=Y}^{Y+\Delta\cdot(D-1)} P_{i\,j} = P_i \qquad (6.23)$$

where, $P_{i\,j}$ is the probability calculated for an alternative (*i*, *j*) and P_i is the probability calculated from the separate modelling of the morning commute for an alternative *i*. If MNL model is used to calculate the probabilities shown in (6.23), then it can be written as follows:

$$\sum_{j=Y}^{Y+\Delta\cdot(D-1)} \frac{\exp(V_{ij})}{\sum_{i=T}^{T+\Delta\cdot(D-1)} \sum_{j=Y}^{Y+\Delta\cdot(D-1)} \exp(V_{ij})} = \frac{\exp(V_i)}{\sum_{i=T}^{T+\Delta\cdot(D-1)} \exp(V_i)} \tag{6.24}$$

Proof: Using (6.21) we can write down the left side of the equation (6.24) as:

$$\sum_{j=Y}^{Y+\Delta\cdot(D-1)} \frac{\exp(V_{ij})}{\sum_{i=T}^{T+\Delta\cdot(D-1)} \sum_{j=Y}^{Y+\Delta\cdot(D-1)} \exp(V_{ij})} = \sum_{j=Y}^{Y+\Delta\cdot(D-1)} \frac{\exp(V_i + V_j)}{\sum_{i=T}^{T+\Delta\cdot(D-1)} \sum_{j=Y}^{Y+\Delta\cdot(D-1)} \exp(V_i + V_j)}$$

By using properties of *exp*, this can be written as

$$= \sum_{j=Y}^{Y+\Delta\cdot(D-1)} \frac{\exp(V_i)\ .\ \exp(V_j)}{\sum_{i=T}^{T+\Delta\cdot(D-1)} \sum_{j=Y}^{Y+\Delta\cdot(D-1)} \exp(V_i)\ .\ \exp(V_j)} = \exp(V_i)\left[\sum_{j=Y}^{Y+\Delta\cdot(D-1)} \frac{\exp(V_j)}{\sum_{i=T}^{T+\Delta\cdot(D-1)} \sum_{j=Y}^{Y+\Delta\cdot(D-1)} \exp(V_i)\ .\ \exp(V_j)}\right]$$

$$\text{since} \quad \sum_{v}^{V}\sum_{w}^{W} a_v \cdot b_w = \sum_{v}^{V} a_v \sum_{w}^{W} b_w$$

$$= \exp(V_i)\left[\frac{\sum_{j=Y}^{Y+\Delta\cdot(D-1)} \exp(V_j)}{\sum_{i=T}^{T+\Delta\cdot(D-1)} \exp(V_i) \cdot \sum_{j=Y}^{Y+\Delta\cdot(D-1)} \exp(V_j)}\right] = \frac{\exp(V_i)}{\sum_{i=1}^{X} \exp(V_i)} = \text{Right side of (6.24)}$$

The above analytical illustration shows that equation (6.21) plays a vital role in detaching the morning and evening commute in the combined model. This equation also suggests that the utility of choosing departure time for the morning and evening commutes is independent from each other, *which is the consequence of using time-of-day specific marginal utility for home and work activities*. This is because these marginal utility functions assume that one unit of activity engagement at time-of-day *t* will always yield the same utility, irrespective of the activity start and end times. The same results can be obtained for the Vickrey (1969) and Small (1982) preferred arrival time based framework, because it also captures only the time-of-day representation and the two commutes utilities are easily separable in the form of equation (6.21). This suggests that the representation of only a time-of-day specific component of activities is not enough to model scheduling of the home-work tour. This finding contradicts with the previous works carried out for integrating morning and evening commutes together with the network congestion, such as Heydecker and Polak (2006), Zhang et al (2005) and Kim et al (2006), as these studies only used a time-of-day representation in the systematic utility specification of their modelling framework. The next section discusses about the inevitable refinement in the utility specification in order to appropriately model scheduling of the home-work tour.

6.4 REFINEMENT-ACTIVITY UTILITY SPECIFICATION

The time-of-day dependent marginal utility has been criticised by various authors (e.g. Ettema et al 2007 and Joh et al 2005), as it does not incorporate the activity satiation effect, which implies that the utility derived from one additional time unit of activity participation diminishes with increasing duration. If the marginal utility of an activity is taken as a function of *duration*, then it is obvious that it interlinks the utility of morning and evening commutes, as both utilities are then dependent on each other. Therefore, equation (6.21) would not hold in this case.

6.4.1 Role of Duration based Marginal Utility

Yamamoto et al (2000) and Bhat and Misra (1999) presented a duration based utility profile that follows a logarithmic function. According to them, the utility of an activity, (for example, work) is given by:

$$V^{w}(\tau_w)=\eta_w \ln(\tau_w) \tag{6.25}$$

which gives a marginal utility function for the work activity of:

$$V^{'w}(\tau_w)=\eta_w \frac{1}{\tau_w} \qquad (\tau_w > 0) \tag{6.26}$$

where, τ_w denotes the duration of work activity and is given according to the home- work tour modelling framework in section 6.2 as $\tau_w = j-(i+R_i)$, and where, η_w represents a scaling parameter.

It should be noted that relying *entirely* on a duration based marginal utility for modelling scheduling of the home-work tour is not realistic, as in that case an individual's time-of-day preferences for participating in activities are completely ignored. Therefore, *both* of these ingredients are important: that is to say, we need both a time-of-day element and a duration element.

6.4.2 Provision for Fixed in Time and Flexible Activities

Ettema et al (2007) argued that time-of-day dependent marginal utility functions are continuous in their nature. These functions neglect the fact that most every day activities are not flexible in terms of time-of-day, e.g. work and school arrangement and opening hours of stores are the constraints that play a vital role in determining the schedule. Therefore, these activity types require a formulation in which start times of these activities are anchored on the time-of-day axis, and any deviation from that time results in a utility loss. Fortunately, the schedule delay formulation presented by Vickrey (1969) and Small (1982) is sufficient to deal with such discontinuities, as in this formulation there exists a certain preferred start time of each activity, and deviations from that time result in a negative utility. Section 3.3 already discussed this notion comprehensively and presented its comparison with time-of-day dependent marginal utility functions.

Moreover, Ettema et al (2007) estimated a model that contains: time-of-day dependent marginal utility function, duration dependent marginal utility function and schedule delay formulation for scheduling of an activity pattern consisting of home, work and after-work activities. They showed that the parameters of schedule delay are found significant only for the work activity due to its relatively less flexible nature than other activities.

6.4.3 Recommendations for Activity Utility Function

Arising from the above, we can conclude that the scheduling of the whole-day activity pattern is dependent on the types of activities actually involved. It is due to their nature that different components show their significance in the total utility measurement, e.g. the non-flexible nature of the work activity causes the significance of schedule delay parameters and the fatigue-less nature of the home activity causes irrelevance to the duration component. Therefore, for the home-work tour scheduling model, the following is proposed:

- **For the home activity,** the use of time-of-day dependent marginal utility will be plausible. This is because this formulation not only captures individual time-of-day preferences but also renders a framework through which home activity utility can be evaluated for the time an individual stays at home. Furthermore, individual time-of-day preferences for this activity are rather flexible compared to work and school activities as being late in reaching home after work (for an hour or less than this) is usually not considered as a significant amount of loss for an individual. Moreover, the nature of this activity is such that it usually exhibits significantly less effect of satiation, as individuals may engage themselves in various sorts of different works (such as watching television, sleeping, eating etc.) during their stay at home. So, the home utility can be given as

$$V^h = \int_0^i V'^h(t)\,dt + \int_{j+R_j}^{1440} V'^h(t)\,dt$$

- **For work activity,** a specification that contains duration based marginal utility function and schedule delay formulation (representing the time-of-day element) will be plausible. This is given as:

$$V^w = \left(\int_0^{\tau_w} V'^w(\tau)\,d\tau \right) + g\left(i + R_i - PST\right)$$

where $V'^w(\tau)$ is the duration dependent marginal utility function, $\tau_w = j - i - R_i$ and $g\left(i + R_i - PST\right)$ represents the scheduling cost imposed on an individual in the form of a late-arrival penalty. Here *PST* represents the preferred start time of an activity. There are three advantages of using a duration based marginal utility for the work activity. The first advantage is that when the home-work tour scheduling model uses the above mentioned specification of the utility for home and work activity, the duration based marginal utility of work activity ensures that morning and evening commutes are appropriately integrated with each other, i.e. equation 6.21 would not hold. The second advantage is that the satiation effects of work activity are incorporated. The third advantage is that the valuation of utility of work activity is possible, since the time-of-day element for this activity is based on the schedule delay formulation which is required to incorporate the strict fixed-in-time notion of this activity. It should be noted that an early-arrival penalty is not considered explicitly here. This is because when home and work activity utilities are joined together for modelling the home-work tour, the early arrival scheduling cost which is usually representing cost, associated with the trip origin (in this case it is the home activity) is already considered through time-of-day dependent marginal utility of the home activity.

A summary of the above discussion is that in order to represents the systematic utility for scheduling of the home-work tour, we would therefore require the following form:

$$V_{i\,j} = \left(\int_0^i V'^{\,h}(t)\; dt \right) + \left(\int_{0_i}^{\tau_w} V'^{\,w}(\tau)\;\; d\tau \right) + g\left(i + R_i - PST\right) + \left(\int_{j+R_j}^{1440} V'^{\,h}(t)\; dt \right) + \lambda\, R_i + \lambda\, R_j \qquad (6.27)$$

6.5 COMBINED MODEL FOR COMPLEX DAILY ACTIVITY-TRAVEL PATTERN

6.5.1 Definitions and Assumptions

This section presents an development of the combined model which incorporates a more complex activity-travel pattern than the home-work tour. The complexity is introduced in a sense that two types of population or user classes are considered. The first population segment is carrying out a simple home-work tour and the second one is carrying out a home-work tour with an additional activity (after work or before work). In addition to this, route choice is also considered for both population segments. For the second population segment activity sequence choice is also incorporated, because there are three activities involved in their activity pattern and choice of activity sequencing may play a significant role in the overall scheduling of their travel pattern. The model development is presented for the network shown in figure 6.3, which contains three locations: home, work and an additional activity place which could be a shopping activity location. It should be noted that 6 links are used to connect these activity locations together. Table 6.1 indicates the scheduling choices for each population segment according to the network shown in figure 6.3.

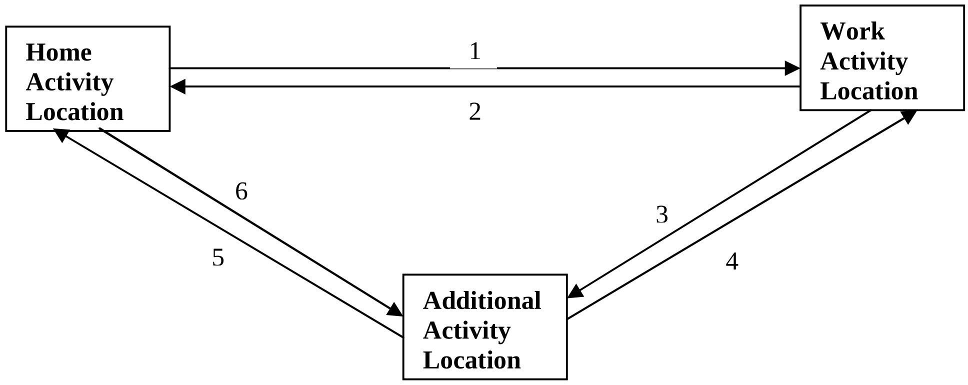


Figure 6.3: A simple example network with three activity locations

Table 6.1: Scheduling choices for population segments according to figure 6.3

Population Segment	Departure times	Duration	Activity Sequence	Routes
Home-Work Tour	**Active** Number of choices depends upon consideration of T, D , Y and Δ	**Active** Considered implicitly for home and work activities by considering the choice of departure times for evening commute.	**Not Active** This is not active because only two activities are considered in the whole day.	**Active,** According to figure 6.3 it is active as for home-work tour there are four possible straight forward routes **1**. Link1-Link2 **2**. Link1-Link3-Link5 **3**. Link6-Link4-Link2 **4**. Link6-Link4-Link3-Link5
Home-Work with an additional activity	**Active** Number of choices depends upon consideration of T, D , Y , Z and Δ	**Active** Considered implicitly for home and work activities by considering the choice of departure times for evening commute. For an additional activity a choice of duration is considered in the modelling framework by introducing another choice of departure time starting from time Z	**Active** Two choices for activity sequence are possible if it is assumed that every activity will be performed once **1**.Home-Work-Add.Activity-Home **2**. Home-Add.Activity-Work-Home	**Active**, According to figure 6.3, for 1st choice of activity sequence the following routes are available **1**. Link1-Link3-Link5 **2**. Link6-Link4-Link3-Link5 For second choice of sequence, the following routes are available **1**.Link6-Link4-Link2 **2**.Link1-Link3-Link4-Link2

6.5.2 Model Development

The scheduling problem can be defined for the two population segments as

Population segment 1: home-work tour;

Scheduling of the tour = (i, j, r^1)

Population segment 2: home-work with an additional activity;

Scheduling of the tour = (i, j, k, s, r^2)

where, i , j, k are the departure time choices from home, work and an additional activity respectively, where r^1 and r^2 represent the choices of routes available respective to each tour types according to the considered network and s represents the choices for sequencing of activities. Consideration of route choice requires an additional step i.e. enumeration of paths/routes for the particular combination of other scheduling dimensions in order to fully illustrate the total alternatives available to an individual. The scheduling dimensions k and s are active only when a population is considered which will carry out an activity-travel pattern containing home, work and an additional activity.

The systematic utility of the home-work tour can be given by using equation (6.27) which can be generalised in order to accommodate route choice. This is as follows

$$V_{i\,j\,r^1} = \left(\int_0^i V'^h(t)\,dt\right) + \left(\int_0^{\tau_{wr^1}} V'^w(\tau)\,d\tau\right) + g\left(i + R^{hw}_{i\,r^1} - PST\right) + \left(\int_{j+R^{wh}_{j\,r^1}}^{1440} V'^h(t)\,dt\right) + \lambda R^{hw}_{i\,r^1} + \lambda R^{wh}_{j\,r^1} \qquad (6.28)$$

where $R^{hw}_{i\,r^1}$ is the travel time between home to work at departure time i for route r^1 and $R^{wh}_{j\,r^1}$ is the travel time between work to home at departure time j for route r^1. $\tau_{wr^1} = j - i - R^{hw}_{i\,r^1}$, is the duration of work activity. The route r^1 is defined as the combination of all links that connects home to work in a cyclic path (i.e. according to figure 6.3, the first route is a combination of link 1 and link 2, the second route is a combination of link1, link3 and link 5 and the third route is a combination of link6, link4 and link2). $R^{hw}_{i\,r^1}$ and $R^{wh}_{j\,r^1}$ can be given as follows

$$R^{hw}_{i\,r^1} = \sum_l R_{il} \cdot \xi^{hw}_{l\,r^1} \quad \text{and} \quad R^{wh}_{j\,r^1} = \sum_l R_{jl} \cdot \xi^{wh}_{l\,r^1} \tag{6.29}$$

where, $\xi^{hw}_{l\,r^1}$ or $\xi^{wh}_{l\,r^1}$ are link-route indicator variables (0-1 integer variables) between the two activity locations, in this case between home to work and work to home respectively. These variables are equal to 1 if link l is a part of route r^1 and are equal to 0 otherwise. R_{il} is the travel time on link l at time i. Their sum across all the links exists between home and work activity locations which forms part of the route r^1, which will give travel time between home to work at a particular time. According to figure 6.3 for the first route, there is only 1 link (link 1) for the travel from home to work. For the second route, again there is only 1 link (i.e. link 1), however, for the third route there are two links (i.e. link 6 and link 4) for the travel from home to work activity locations, so in this case $R^{hw}_{i\,r^1}$ is a combination of $R_{i6} + R_{i4}$. This is generally shown in equation (6.29) as the systematic utility presented in equation (6.28) requires travel times at time i for the complete route r^1.

Equation (6.28) suggests that the utility of the home-work tour is a function of travel times $R^{hw}_{i\,r^1}$ and $R^{wh}_{j\,r^1}$ as a time-of-day dependent marginal utility of home activity and duration dependent marginal utility of work activity, along with the preferred start time of work activity is predetermined and usually given as an input. So, it can be written as:

$$V_{i\,j\,r^1} = \Phi\left(R^{hw}_{i\,r^1}\, ,\, R^{wh}_{j\,r^1}\right) \tag{6.30}$$

Assuming that Q_1 is the total population that performs a home-work tour in a given day, then the rate of departure flows for a particular route $q_{i\,j\,r^1}$ is then given by using any operational model at the demand side (e.g. MNL model). This is as follows:

$$q_{i\,j\,r^1} = Q_1 \cdot P_{i\,j\,r^1} = Q_1 \cdot P_{i\,j\,r^1}\left(V_{i\,j\,r^1}\right) \tag{6.31}$$

where $P_{i\,j\,r^1}$ is the probability of choosing departure time *i* and *j* for travel from home and work respectively using route r^1.

Now consider the second population segment which is performing a home-work tour with an additional activity. This tour has a choice of activity sequence as well along with other scheduling dimensions. The overall utility of this tour can be given according to the individual sequence choice of activities. This is because the choice of sequence actually renders which activity will be performed prior to the next activity. This is illustrated in the following equations:

when *s* =1, activity sequence is home-work-additional activity-home with work activity duration as $\tau_{w\,r^2} = j - i - R^{hw}_{i\,r^2}$ and additional activity duration as $\tau_{a\,r^2} = k - j - R^{wa}_{j\,r^2}$:

$$
\begin{aligned}
V_{i\,j\,k\,1\,r^2} = & \int_0^i V'^h(t)\,dt + \left(\int_0^{\tau_{w\,r^2}} V'^w(\tau)\,d\tau \right) + g\left(i + R^{hw}_{i\,r^2} - PST\right) \\
& + \int_{j+R^{wa}_{j\,r^2}}^{k} V'^a(t)\,dt + \int_0^{\tau_{a\,r^2}} V'^a(\tau)\,d\tau + \int_{k+R^{ah}_{k\,r^2}}^{1440} V'^h(t)\,dt \\
& + \lambda\, R^{hw}_{i\,r^2} + \lambda\, R^{wa}_{j\,r^2} + \lambda\, R^{ah}_{k\,r^2}
\end{aligned}
\qquad (6.32a)
$$

when *s* = 2, activity sequence is home-additional activity-work-home with work activity duration as $\tau_{w\,r^2} = j - k - R^{aw}_{k\,r^2}$ and additional activity duration as $\tau_{a\,r^2} = k - i - R^{ha}_{i\,r^2}$

$$
\begin{aligned}
V_{i\,j\,k\,2\,r^2} = & \int_0^i V'^h(t)\,dt + \int_{i+R^{ha}_{i\,r^2}}^{k} V'^a(t)\,dt + \int_0^{\tau_{a\,r^2}} V'^a(\tau)\,d\tau \\
& + \left(\int_0^{\tau_{w\,r^2}} V'^w(\tau)\,d\tau \right) + g\left(k + R^{aw}_{k\,r^2} - PST\right) + \int_{j+R^{wh}_{j\,r^2}}^{1440} V'^h(t)\,dt \\
& + \lambda R^{ha}_{i\,r^2} + \lambda R^{aw}_{k\,r^2} + \lambda R^{wh}_{j\,r^2}
\end{aligned}
\qquad (6.32b)
$$

The above two equations (6.32a and 6.32b) can be expressed as follows, which is similar to equation (6.30):

$$
V_{i\,j\,k\,s\,r^2} = \Omega\left(R_{i\,s\,r^2}, R_{j\,s\,r^2}, R_{k\,s\,r^2}\right) \qquad (6.33)
$$

where when s =1 then $R_{i\,s\,r^2} = R^{hw}_{i\ r^2}$, $R_{j\,s\,r^2} = R^{wh}_{j\ r^2}$ and $R_{k\,s\,r^2} = R^{ah}_{k\ r^2}$ and

when s = 2; $R_{i\,s\,r^2} = R^{ha}_{i\ r^2}$, $R_{j\,s\,r^2} = R^{wa}_{j\ r^2}$ and $R_{k\,s\,r^2} = R^{aw}_{k\ r^2}$

and $R^{hw}_{i\ r^2} = \sum_l R_{i\,l} \cdot \delta^{hw}_{l\,r^2}$. In a similar way, $R^{ha}_{i\ r^2}$, $R^{wh}_{j\ r^2}$, $R^{wa}_{j\ r^2}$, $R^{ah}_{k\ r^2}$ and $R^{aw}_{k\ r^2}$ can be defined.

With the above specification of utility and its dependence on travel times, the departure rates ($q_{i\,j\,k\,s\,r^2}$) can easily be determined using an operational model at the demand side (e.g. MNL) with the assumption that Q_2 number of individuals will perform this tour in a given day. This is as follows:

$$q_{i\,j\,k\,s\,r^2} = Q_2 \cdot P_{i\,j\,k\,s\,r^2} = Q_2 \cdot P_{i\,j\,k\,s\,r^2}\left(V_{i\,j\,k\,s\,r^2}\right) \tag{6.34}$$

Equations (6.31) and (6.34) suggest that departure rates $q_{i\,j\,r^1}$ and $q_{i\,j\,k\,s\,r^2}$ are a function of utilities $V_{i\,j\,r^1}$ and $V_{i\,j\,r^1}$, which are a function of travel times (see equations 6.30 and 6.33). These travel times which are specific to a particular trip between locations (say home to work), time-of-day, and routes are actually based on the travel time at a particular link l at a particular time-of-day, either *i, j or k* (see equation 6.29 and explanation after equation 6.33). These time dependent link travel times, either R_{il} or $R_{j\,l}$ or $R_{k\,l}$, are calculated using any DNL model (discussed in chapter 4) and are again dependent on both departure rates $q_{i\,j\,r^1}$ and $q_{i\,j\,k\,s\,r^2}$, which are responsible to provide time-dependent inflows to the links. Overall, this constitutes a fixed point problem which can be represented as follows

$$\hat{\mathbf{Q}} = \Psi\left(\hat{\mathbf{R}}\left(\hat{\mathbf{Q}}\right)\right) \tag{6.35}$$

where, $\hat{\mathbf{Q}}$ is a matrix containing elements $q_{i\,j\,r^1}$ and $q_{i\,j\,k\,s\,r^2}$, and $\hat{\mathbf{R}}$ is also a matrix containing elements R_{il}, $R_{j\,l}$ and $R_{k\,l}$. The solution of the above fixed point problem represents stochastic dynamic user equilibrium for the two user classes which are performing a home-work tour and home-work tour with an additional activity in a given *day*.

The above presented model can be extended to represent many user classes in a more general network with various locations of home, work and other activities, though the formulation is not given here. The combined model presented above in section 6.2 (for simple home-work tour) and section 6.5 (for complex tours), are numerically solved and their results are presented in chapter 7 under various different scenarios.

6.6 SUMMARY

This chapter demonstrated the step-by-step development of the combined model for scheduling of daily activity-travel patterns. In section 6.2, a combined model is formulated as a fixed point problem, represented a simple daily activity-travel pattern (i.e. home-work tour) with scheduling choices of departure times and duration. Section 6.3 presented an analysis of the utility function of the model in a situation where the activity utility is represented only as a function of time-of-day. It has been shown numerically and analytically, that when activity utility is considered as a function of time-of-day only, the two commutes in a home-work tour (home to work trip and work to home trip) are not appropriately joined with each other, because both commute's utilities are independent of each other. Section 6.4 then presented the role of duration based marginal utilities in connecting the two commute together, and also shows which ingredient (time-of-day dependency or duration dependency) is a better representation of utility of an activity in what circumstances. This section finally recommends a utility specification for the scheduling of a home-work tour, which will be analysed in detail through numerical experiments in chapter 7. Section 6.5 presented development of the combined model for a more complex home-based tour (i.e. home-work tour with an additional activity) along with home-work tour, as two different user classes. The scheduling choices incorporated are departure times, activity durations, activity sequence and route choice. The model is formulated as a fixed point problem and its solution renders two-user classes based stochastic dynamic user equilibrium. This model can be extended to incorporate multiple user classes (i.e. each of them having different daily activity-travel patterns) with many combinations of home, work and other activity locations in a general network. Chapter 7 presents results of some numerical experiments which show the application of the model for various policies i.e. time-dependent tolls, flexible working hour schemes and telecommuting schemes.

Chapter 7

COMBINED MODEL FOR DAILY TOURS- NUMERICAL EXPERIMENTS

7.1 GENERAL

The previous chapter demonstrated the development of a combined model for the scheduling of daily tours. This chapter reports results and findings of the numerical experiments conducted to achieve two main goals. The first goal is to assess the plausible working of the model after its implementation through a computer program by making some systematic changes in the model framework. For example, changing of the operational models within the demand and supply sides, examining the convergence pattern of the model using different solution algorithms and investigating the changes in the model predictions when different analysis time-interval are used at the demand and supply sides. The results and findings from these experiments are important in order to understand and recognise the role of these systematic changes in each of the components of the combined model. Moreover, these experiments will provide a profound basis for the further extension of the model, so that the model predictions represent much more realistic behaviour. These are for example, incorporation of more scheduling dimensions, incorporation of more user classes (multiple-user classes), incorporation of weekly activity-travel pattern of individuals and extending the model for the general road networks.

The second goal is to apply the developed model in order to assess the implication of some policies. The numerical experiments performed for the achievement of the second goal includes: introduction of dynamic tolls, flexible working hour scheme with respect to time-of-day and work activity duration and the effect of availability of tele-work (home-based-work) option for the commuters. The results and findings of these experiments are vital because these will render a basis on which the dimensions of further extension of the model can be prioritised. Furthermore, a single comparable summary measure of performance is also evaluated for different policy scenarios using the logsum term (i.e. natural log of the denominator of logit model). This helps identify the strength of a particular policy scenario in terms of its overall socio-economic benefits to the society. Section 7.2 discusses the results and findings of numerical experiments conducted to achieve the first goal. Section 7.3 presents the results of numerical experiments which are conducted to achieve the second goal. Section 7.4 demonstrates the findings from the results of the numerical experiments using multiple-user classes and incorporation of more scheduling dimensions. The last section summarises the discussion carried out in this chapter.

7.2 NUMERICAL EXPERIMENTS-ASSESSING MODEL PLAUSIBILITY

The results and findings of the numerical experiments shown in this section are discussed with the aim that it reflects the working of the model under different circumstances. These circumstances are as follows:

- The use of different solution algorithms (discussed in chapter 5) with different initial starting values to examine the model convergence and uniqueness of the solution.
- The use of different operational models under demand and supply sides (discussed in chapter 3 and 4) in order to examine the effects of these changes in the model predictions.
- The variation of analysis time interval at demand and supply sides in order to investigate the role and significance of this issue on the model predictions.

The mathematical illustration of the model discussed in chapter 6 (section 6.2) for the scheduling of the home-work tour (i.e. departure time choices from home and work activities) is flexible enough to incorporate the above listed changes in the demand and supply sides. The systematic utility specification for the home-work tour scheduling model discussed in section 6.4 as equation (6.27) is adopted in all the experiments in this section. This utility specification ensures that morning and evening commutes in the home-work tour are held together with each other and two important ingredients (such as time-of-day and duration elements) for measuring the utility of an activity engagement are incorporated. To conduct the numerical experiments some assumptions are made regarding the values of the parameters of the marginal utility curves for the home and work activities. As has already been discussed, the goal of this thesis is not the estimation of parameters for these marginal utility functions, however, if a practical case study is derived, it is believed that the parameters required for the marginal utility functions for home and work activities can easily be estimated. For example, Ettema and Timmermans (2003) and Ettema et al (2007) have utilised the state sponsored activity-travel diary data of the Netherlands to estimate these marginal utility curves using the framework of utility maximisation for home and work activities. Studies carried out by Joh et al (2002 and 2005) reported estimation of various forms of non-linear marginal utility functions which are based on time-of-day and duration, these marginal utility functions were estimated for activities like work, home and shopping by segmenting the population in three different groups based on the combination of their characteristics i.e. gender, age and work-orientation. The values of the parameters for the marginal utility functions assumed for conducting the numerical experiments are consistent with the magnitude of the values found in these studies.

For home activity, an inverse bell-shaped marginal utility function is assumed which is dependent on clock-time. The functional form as presented in chapter 3 (table 3.2) is given by:

$$V'^{h}(t) = h_0 - \frac{\beta \gamma U_0}{\exp\left[\beta\left(t-\alpha\right)\right]\left[1+\exp\left(-\beta\left(t-\alpha\right)\right)\right]^{\gamma+1}} \tag{7.1}$$

where, h_0 = 0.03 utils/min, α = 720 minutes past midnight , β =0.01 per minute, γ =1, U_0 =10 utils, are the parameters that control the shape of the marginal utility profile. For work activity, the marginal utility is assumed as a function of duration and also for incorporating time-of-day preference, schedule delay approach is used with a preferred work activity start time considered as *PST* = 0900 hours. Late arrival penalty parameter is taken as m_l = -0.04 utils/min. The duration dependent marginal utility of work activity follows the following functional form with a scaling parameter η_w = 5 utils:

$$V'^{w}(\tau)=\eta_w\left(\frac{1}{\tau}\right) \qquad (\tau>0) \tag{7.2}$$

It is also assumed that the free-flow travel time on the links is considered as ϕ =10 minutes with a capacity C =1800 vehicles/hour for each link. The in-vehicle travel time parameter is assumed as λ = -0.08 utils/min. The following sub-sections present and discuss the results obtained from the numerical experiments.

7.2.1 Convergence efficiency of different solution algorithms

This experiment examines the convergence efficiency of the two solution algorithms by heuristically solving a FP problem through the constructed gap function presented in chapter 6 (see equation 6.14). The same gap function is presented here as equation 7.3, which suggested that its minimisation is required to be carried out and it represents the sum of the squares of the differences of the solutions at successive iterations.

$$min\ G(\mathrm{q})=\| \mathrm{q} - Q\ \mathrm{P}\left(\mathrm{V}(\mathrm{R}(\mathrm{q}))\right)\|_2^2 \tag{7.3}$$

The following assumptions are also made to practically apply the combined scheduling model for the home-work tour explained in section 6.2 of chapter 6. It is assumed that the departure time for the morning commute (home to work trip) starts from T = 0800hours. In total, 4 departure periods (D) each of 30 minutes duration (Δ) are considered for each of the morning and evening commutes. Similarly, it is assumed that the departure time for the evening commute starts from Y = 1600hours. At the demand side, a MNL model was used as an operational model and the supply side employed point-queue model with an analysis time interval (δ) of 1 minute. The total number of commuters carrying out the home-work tour are assumed equal to Q = 3000 for the experiments shown in table 7.1.

Table 7.1 shows the results of the experiments when different starting values were used to run the minimisation algorithm (BFGS) in order to minimise gap function (i.e. through this heuristic method the approximate solution of the formulated FP problem can only be achieved when the value of gap function is near zero). The results shown in table 7.1 suggest that for this particular setup of the model parameters, gap function reaches to approximately zero and provide almost identical final solutions for different starting values used in this experiment. This indicates that BFGS algorithm (which requires gradients of the gap function and is supplied through finite differences method) can also be used for solving the FP problem through its constructed gap function; however, it is not sure that the gap function used here is differentiable. The small differences in the solutions are because of the stopping criterion (i.e. $G(\mathrm{q}) \leq 10^{-5}$) used for the minimisation program which is evident from slightly different values of the gap function at equilibrium obtained for each experiment. Another thing which is revealed from table 7.1 is (as would be expected) the faster convergence of the problem when the pattern of the starting values provided is close to the problem solution. This is evident from the results of experiments 3 and 4 shown in table 7.1 as the problem reaches at its equilibrium solution in less number of iterations compared to experiment 1. These findings may not be true for other cases (other setups of the problem) and therefore, these cannot be generalised; therefore each time this algorithm was run for different setup / values of the parameters it was tested in this way (i.e. value of gap function is small and there are no multiple solutions) to confirm that the solution obtained is reasonable.

Table 7.1: Examination of solution using different starting values

Experiment No.	Starting values pattern	No. of iterations used	Solutions	Gap function value
1	$\begin{bmatrix} 187.5 & 187.5 & 187.5 & 187.5 \\ 187.5 & 187.5 & 187.5 & 187.5 \\ 187.5 & 187.5 & 187.5 & 187.5 \\ 187.5 & 187.5 & 187.5 & 187.5 \end{bmatrix}$	32	$\begin{bmatrix} 410.58 & 632.86 & 865.92 & 968.51 \\ 0.18 & 0.34 & 0.55 & 0.73 \\ 1.32 & 2.53 & 4.18 & 5.55 \\ 10.10 & 19.65 & 32.94 & 44.05 \end{bmatrix}$	5.8823×10^{-7}
2	$\begin{bmatrix} 0.00. & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 \end{bmatrix}$	21	$\begin{bmatrix} 410.58 & 632.86 & 865.93 & 968.51 \\ 0.18 & 0.33 & 0.55 & 0.73 \\ 1.32 & 2.53 & 4.18 & 5.55 \\ 10.11 & 19.66 & 32.93 & 44.05 \end{bmatrix}$	3.8112×10^{-6}
3	$\begin{bmatrix} 500 & 500 & 500 & 500 \\ 0.00 & 0.00 & 0.00 & 0.00 \\ 100 & 100 & 100 & 100 \\ 100 & 100 & 100 & 100 \end{bmatrix}$	20	$\begin{bmatrix} 410.59 & 632.86 & 865.91 & 968.52 \\ 0.18 & 0.34 & 0.55 & 0.73 \\ 1.32 & 2.53 & 4.18 & 5.55 \\ 10.10 & 19.65 & 32.94 & 44.04 \end{bmatrix}$	2.1238×10^{-6}
4	$\begin{bmatrix} 1000 & 500 & 500 & 1000 \\ 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 \end{bmatrix}$	19	$\begin{bmatrix} 410.58 & 632.86 & 865.92 & 968.51 \\ 0.19 & 0.34 & 0.55 & 0.73 \\ 1.32 & 2.54 & 4.18 & 5.55 \\ 10.12 & 19.65 & 32.93 & 44.05 \end{bmatrix}$	1.8977×10^{-6}

Figure 7.1 shows the results of an experiment in which different solution algorithms were used for minimising the gap function. When the results shown in figure 7.1 are analysed using efficiency indicators of the two solution algorithms given in table 7.2, it is revealed that in this example, the gradient based solution algorithm (i.e. BFGS) is a significantly more efficient and faster algorithm compared to the Method of successive averages (MSA). Figure 7.1 suggests that MSA is a better algorithm in terms of its smoothness and monotonic decreasing nature, however, this algorithm is very slow in reaching the desired stopping criteria set for the convergence. This is evident from table 7.2 in which it is shown that even after 1000 iterations, the value of gap function is not reached at its desired minimum (i.e. 10^{-5}) . This is due to the use of the pre-determined step-size in MSA algorithm because step-size is sub-optimal when algorithm reaches near the solution. Of course, these results are only illustrative and are not meant to be general conclusion about the algorithm, but suggest that the BFGS heuristic is potentially a reasonable algorithm for solving the problem

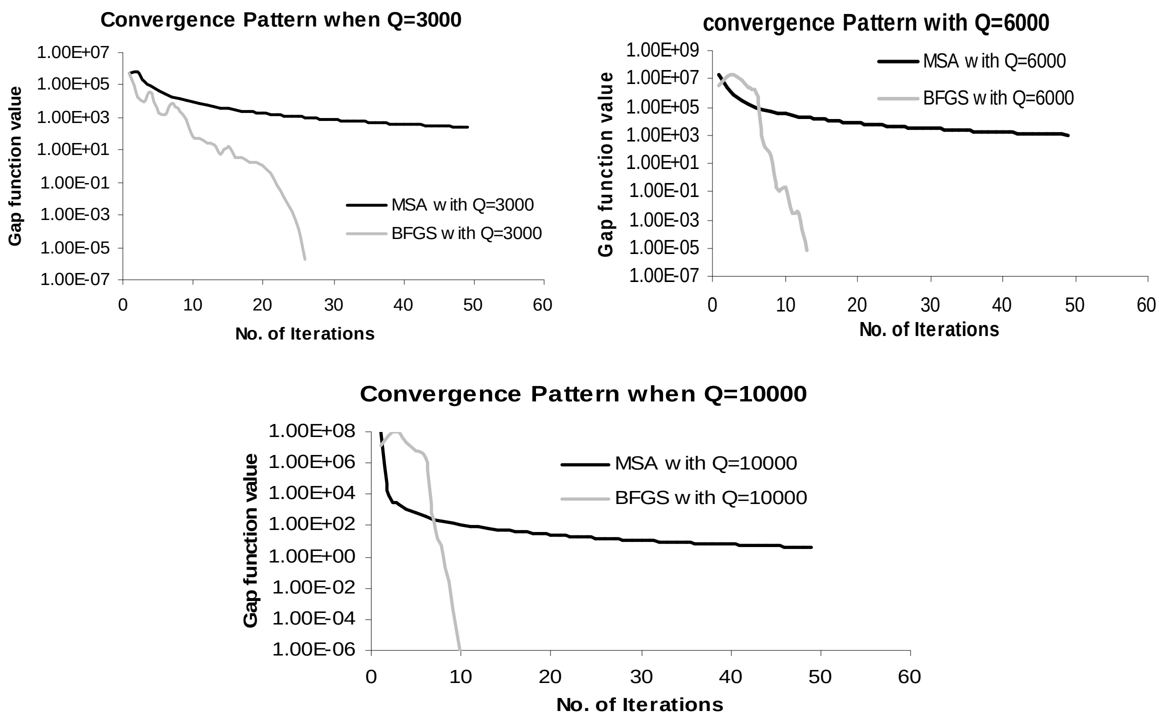


Figure 7.1: Convergence Pattern of the constructed gap function in (vehicles/30minutes)^2 with different values of *Q*

Table 7.2: Convergence efficiency indicators of the two solution algorithms

Solution Algorithms	Demand Levels (Q)	Final Gap Value (vehicle/30 mintues)^2	Computing Time (minutes)*	Function Evaluations Required
MSA	3000	1.0441	40	1000 function evaluations
	6000	2.3052	44	1000 function evaluations
	10000	0.0094	52	1000 function evaluations
BFGS (using fminunc function in the MATLAB)	3000	1.8977×10^{-6}	19	26 iterations with 469 function evaluations
	6000	7.7903×10^{-6}	9	13 with 221 function evaluations
	10000	5.1175×10^{-6}	7	10 with 170 function evaluations

*Computing time is based on a Desktop PC: Intel Pentium 4, 3.00GHz, 1GB RAM

Figure 7.1 along with indictors shown in table 7.2 suggests that BFGS is an efficient algorithm to converge the problem at its equilibrium solution point but it takes a few initial iterations to settle down before giving smooth and sharp decreasing convergence pattern. This initial unsmooth nature might be because of the use of infeasible starting values supplied to run the minimisation program.

Main Findings:

- Solution of the problem reported in this experiment is exists and it is unique as well.
- BFGS heuristic has been found an efficient algorithm than MSA for the experiments reported in this sub-section. On this basis other experiments reported in this chapter utilised BFGS heuristic as the solution algorithm.

7.2.2 Different operational models at the demand side

This experiment was performed to examine the effects of changing the operational model at the demand side. This was achieved using two different operational models of the demand side (i.e. Mulitnomial logit (MNL) model and Nested logit (NL) model). The following assumptions are made to practically apply the combined scheduling model for the home-work tour explained in section 6.2. It is assumed that the departure time for the morning commute (home to work trip) starts from T = 0600hours. In total, 10 departure periods (D) each of 30 minute duration (Δ) are considered for each of the morning and evening commute. Similarly, it is assumed that departure time for the evening commutes is start from Y = 1400hours. At the supply side for this experiment, point-queue model has been utilised with an analysis time interval (δ) of 1 minute. The total Q = 6000 commuters are assumed to carry out the home-work tour for this experiment. This experiment uses the same parameter values of marginal utility functions as suggested in section 7.2.

7.2.2.1 Explanation of the model predictions using MNL model at the demand side

This sub-section provides the detailed explanation of the results obtained for the combined model using MNL model at the demand side. This is provided in order to understand the results (model predictions) of the combined model when basic operational model is incorporated, so that a clear appreciation of the differences in the results is made when other operational models are considered. The results shown in figure 7.2 represents demand and travel time profiles with respect to time-of-day, obtained at the equilibrium.

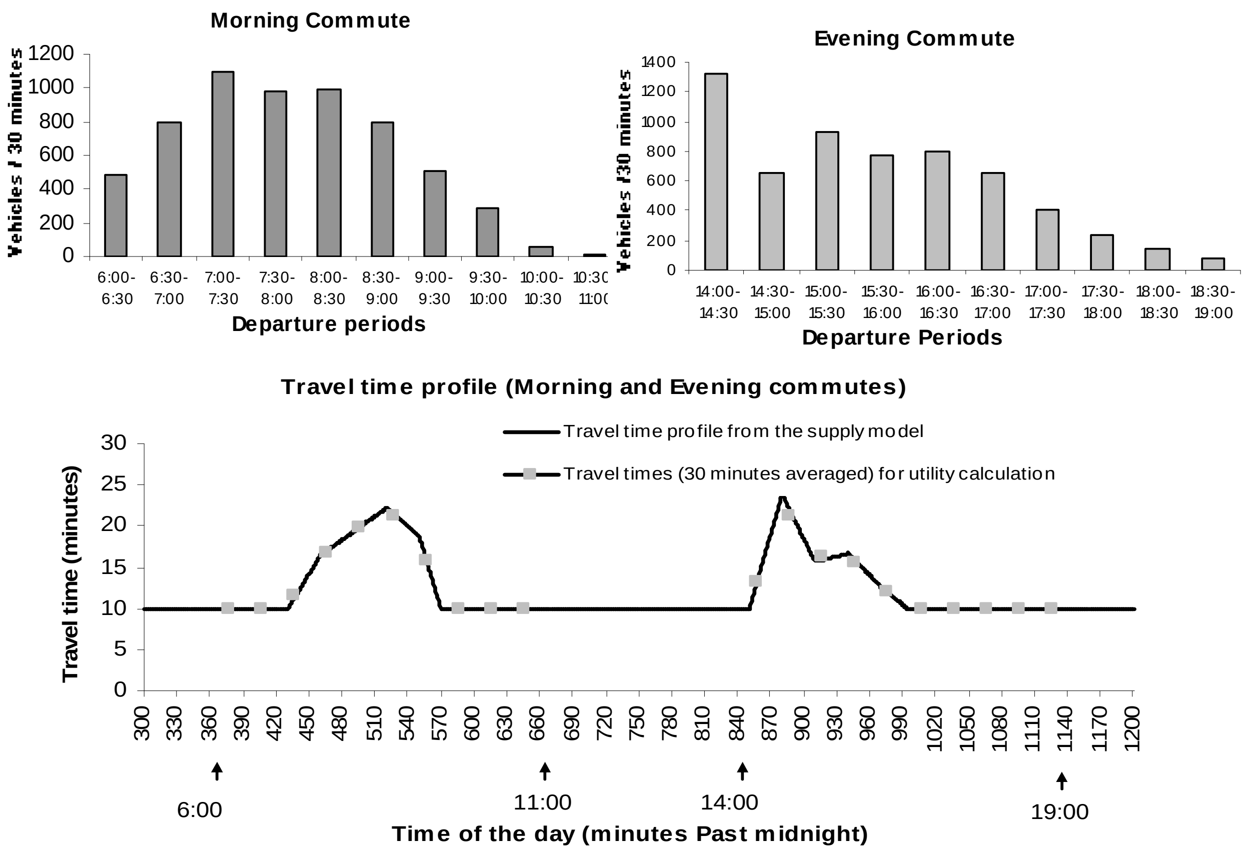


Figure 7.2: Demand and Travel time profiles with respect to time of day

Figure 7.2 reveals that in the morning commute, demand is gently increases with respect to time and then after reaching its peak it is decreases in the later departure periods. The same trend is obtained for the travel time profile of the morning commute. The demand profile obtained for the evening commute is of a different nature than the morning commute, as the first departure period in the evening commute has a significantly higher demand than later departure periods and as a result of this the travel time profile is immediately increased to its peak value (23.8 minutes) for the evening commute. However, in the later departure periods of the evening commute the demand profile is smoothly decreasing. There are two main reasons for the particular spread of the demand profile (or travel time profile) obtained for the morning and evening commutes. The first one is related to the involved stochasticity in the problem (individuals are not fully aware of the maximum utility alternative), as it is known that the random error component is involved in the total utility obtained for each alternative, therefore, the less attractive alternatives also receive some of their shares from the market. This is one of the reasons why every departure period in the morning and evening commutes share some demand from the total of 6000 commuters. Another reason of getting the particular spread (most of the individuals have chosen departure periods earlier than 9:00 am) in the morning commute demand profile is the use of the late arrival penalty at the work location (as preferred start time of work activity is assumed as 9:00 am). The unusual spread of the evening commute profile can be better understood by examining each of the ingredients involved in the utility calculations and parameter values used for this experiment. Figure 7.3 shows the systematic utility values for participation in the home and work activities along with the total systematic utility for each of the alternatives of the tour.

The comparison of figure 7.3 (a) and (b) reflects the role of travel time disutility which is evident from the unsmooth nature of the profile obtained for the total systematic utility of the tour. The earlier departure periods combination of the morning and evening commutes (departure periods 1 to 4 for both commutes) have higher utilities compared to their later departure periods combination. For the evening commute, it has been noted that highest utility is always obtained for the departure period 1 (i.e. 14:00 to 14:30) irrespective of the morning departure period. That is why, more individuals have chosen the first departure period for their return to home trip, more individuals for the first departure of the evening commute means higher level of congestion in later time periods, therefore, the demands suddenly drop in the second departure period of the evening commute, and when the congestion level decreases again higher demands are obtained for third, fourth and fifth departure periods of the evening commute. It is interesting to see why utility values are higher for the initial combinations of the departure periods in the morning and evening commute, for this purpose figure 7.3 (b) is further decomposed into the utility profiles of home and work activities. Figure 7.4 shows utility profiles of the work and home activities.

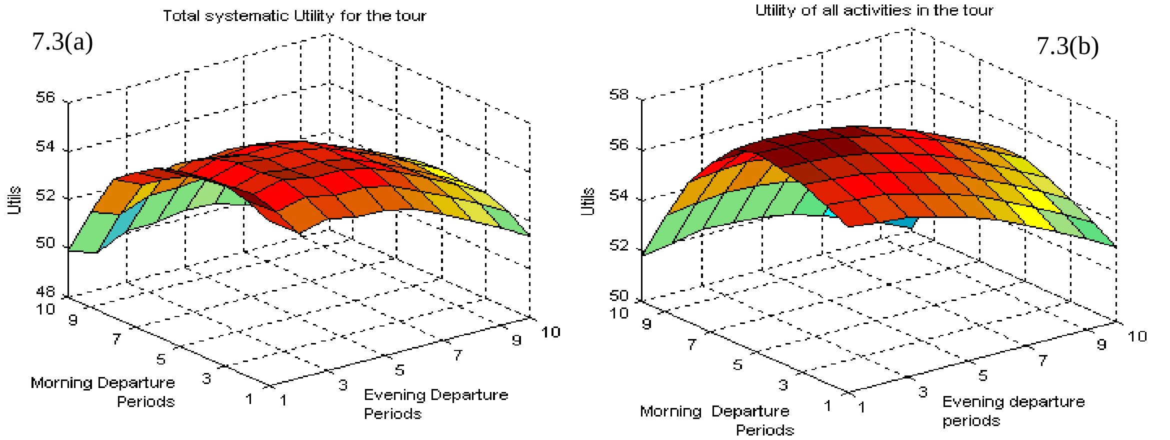


Figure 7.3: (a) Total Systematic utility for each alternative of the tour (V_{ij}), (b) Utility of all activities in the tour for each alternative ($V^h + V^w$)

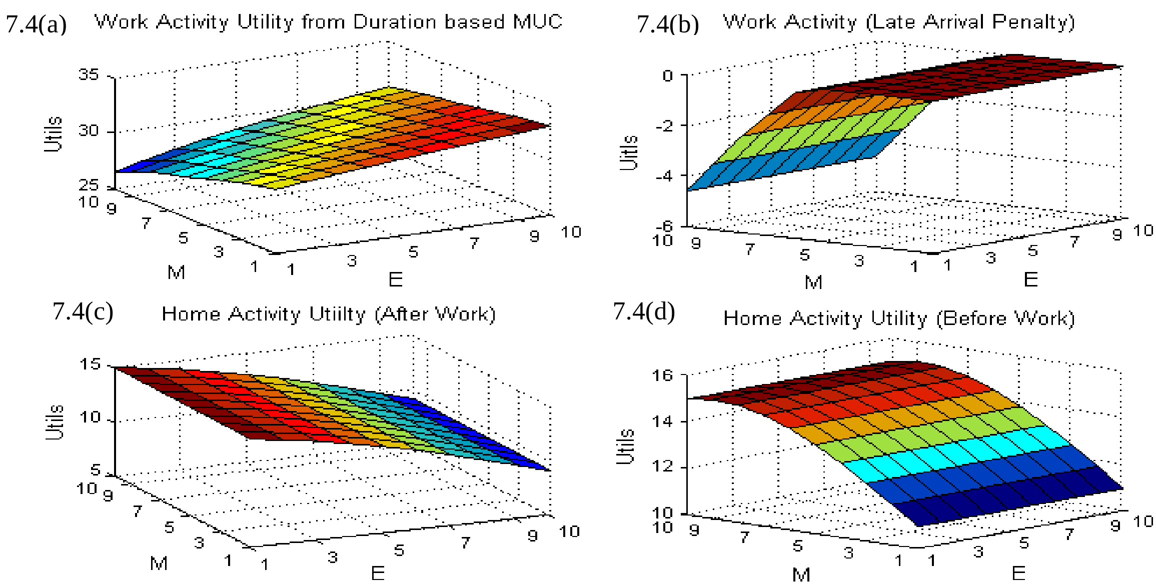


Figure 7.4: Work and Home Activity utility for each alternative
(M represents morning departure periods and E represents evening departure periods)

Figure 7.4(a) presents the work activity utility of each alternative from the duration based marginal utility function. This figure shows that the work utility is higher for a combination of departure periods (alternative) which include the first morning departure period and last evening departure period. This combination gives the maximum available duration of work and therefore marginal utility function for work activity which is duration based render higher utility. Figure 7.4 (b) represents the late arrival penalty for each alternative, it has a lower value (equal to zero) for the alternatives which combine the initial six departure period of the morning commute with all departure periods of the evening commutes. This late arrival penalty profile has played a vital role in the spread of the morning commute demand profile (see figure 7.2). Figure 7.4(c) and (d) represents the home activity utility after-work and before-work respectively. Before-work home utility is varying along the morning departure periods with higher utility values for the later departure periods, and after-work home activity utility is varying along evening departure periods with higher utility values for the initial departure periods.

The higher utility value of the after-work home activity in the initial evening departure periods is the key factor for the prediction of higher demand in the initial evening departure period. This is because an individual obtains 15 utils by selecting the first departure period of the evening commute from after-work home activity participation, and before-work home activity participation gives him around 10.9 to 15 utils depending upon his selection of the morning commute departure period. The duration based work activity utility gives him around 30.8 to 26.5 utils depending upon his selection of the morning commute departure period. If the individual selects later departure periods of the morning commute (i.e departure periods 7 to 10), he may lose some of his utility in the form of late arrival penalty. However, later departure periods in the evening commute with the selection of initial departure periods of the morning commute gives an individual a chance to earn up to 3 more utils from the duration based work activity utility but in doing so an individual may loose up to 6 utils from the after-work home activity participation, as later departure periods of the evening commute may only provide 9 utils. So, the first departure period in the evening commute along with the initial morning departure periods provides him the highest activity participation utility, and among those the alternative which combines the third departure period of the morning commute and the first departure period of the evening commute has the maximum utility (i.e. 55.4 utils). Furthermore, the travel time disutility in this circumstance is always lower in the first departure period than the later departure periods of the evening commute. This is because the supply model used in this experiment provides an average 30 minutes travel time based on the free-flow travel time (10 minutes) for the first ten minutes (as this model starts working at time 14:00 pm) and then incorporates congestion effects in the later 20 minutes resulting in lower values of average travel time than later departure periods. Therefore, the difference of 3 utils between the after-work home utility and the duration based work utility is the main reason of the attractiveness of the first departure period of the evening commute among the individuals. The above discussion signifies the role of the parameter values of the home and work activities marginal utility functions used in this experiment because the utility profiles shown in figure 7.4 are the direct function of these parameter values.

7.2.2.2 Comparison of the model predictions for MNL and NL models

Compared to the MNL model, the NL model requires an assumption about the value of an additional parameter (i.e. a dissimilarity parameter (θ)), therefore, several runs of the combined model were carried out using different values of this parameter in order to make a systematic comparison of results obtained using NL and MNL models. The structure of the NL model assumed for this experiment is defined as follows. First an individual selects a departure period for his/her morning commute and then selects the departure period for his/her return trip to home. The schematic figure of this structure is shown in Figure 7.5.

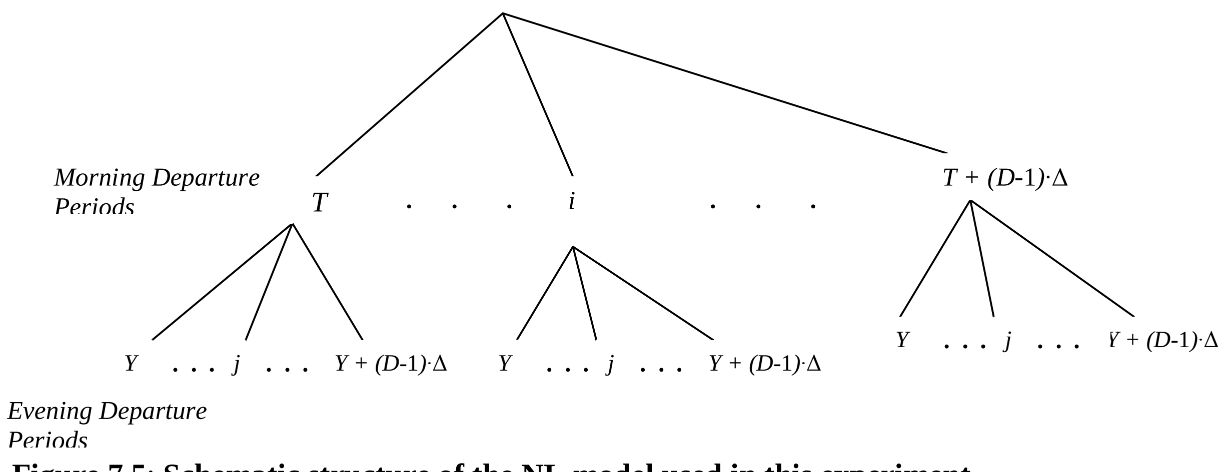


Figure 7.5: **Schematic structure of the NL model used in this experiment**

The NL model is usually used in order to see the effect on the results when correlations among the same nest alternatives exist. The additional parameter ρ reflects the correlation among the same nest alternatives as Corr = 1-θ^2. Usually the dissimilarity parameter ($\theta = \mu_i / \mu_j$) is estimated from the estimation software with the assumption that μ_i =1 (it can be greater than 1 but the condition $\mu_i \leq \mu_j$ should always followed which is necessary for the consistency of the model), this suggests that θ is always ranging between 0 and 1. If its value turns out as 1, the NL structure then collapses to MNL (i.e. no correlation). It should be noted that the correlation between the same nest alternatives is higher if lower values of θ is considered.

Figure 7.6 revealed that the use of NL model with a structure specified in figure 7.5, does not make any significant changes in the demand and travel time profiles of the morning commute. It is the property of the NL model that, when μ_i =1 with $\mu_i \leq \mu_j$ (i.e. $\mu_j \geq 1$), the marginal choice probability of the dimension at the highest level of nesting structure P_i (in this case it is the morning commute departure periods) is the same as the marginal choice probability of that dimension when the MNL model is used provided that utility functions and the parameter values are same as mentioned by Ben-Akiva and Lerman (1985, p.288). This is also shown in table 7.3.

Table 7.3: Marginal choice probabilities in Joint MNL model and NL models

	MNL Model	NL Model	Comments
Marginal choice probability for the morning departure periods	$P_i^{MNL} = \dfrac{\exp\left(\ln\sum_j \exp(V_{ij})\right)}{\sum_i \exp\left(\ln\sum_j \exp(V_{ij})\right)}$	$P_i^{NL} = \dfrac{\exp\left[\left(\frac{1}{\mu_j}\cdot\ln\sum_j \exp(V_{ij}\cdot\mu_j)\right)\cdot\mu_i\right]}{\sum_i \exp\left[\left(\frac{1}{\mu_j}\cdot\ln\sum_j \exp(V_{ij}\cdot\mu_j)\right)\cdot\mu_i\right]}$	When μ_i =1 and $\mu_j \geq 1$ then $P_i^{MNL} = P_i^{NL}$
Marginal choice probability for the evening departure periods	$P_j^{MNL} = \dfrac{\exp\left(\ln\sum_i \exp(V_{ij})\right)}{\sum_j \exp\left(\ln\sum_i \exp(V_{ij})\right)}$	$P_j^{NL} = \dfrac{\exp\left[\left(\frac{1}{\mu_i}\cdot\ln\sum_i \exp(V_{ij}\cdot\mu_i)\right)\cdot\mu_j\right]}{\sum_j \exp\left[\left(\frac{1}{\mu_i}\cdot\ln\sum_i \exp(V_{ij}\cdot\mu_i)\right)\cdot\mu_j\right]}$	When μ_i =1 and $\mu_j \geq 1$ then $P_j^{MNL} \neq P_j^{NL}$

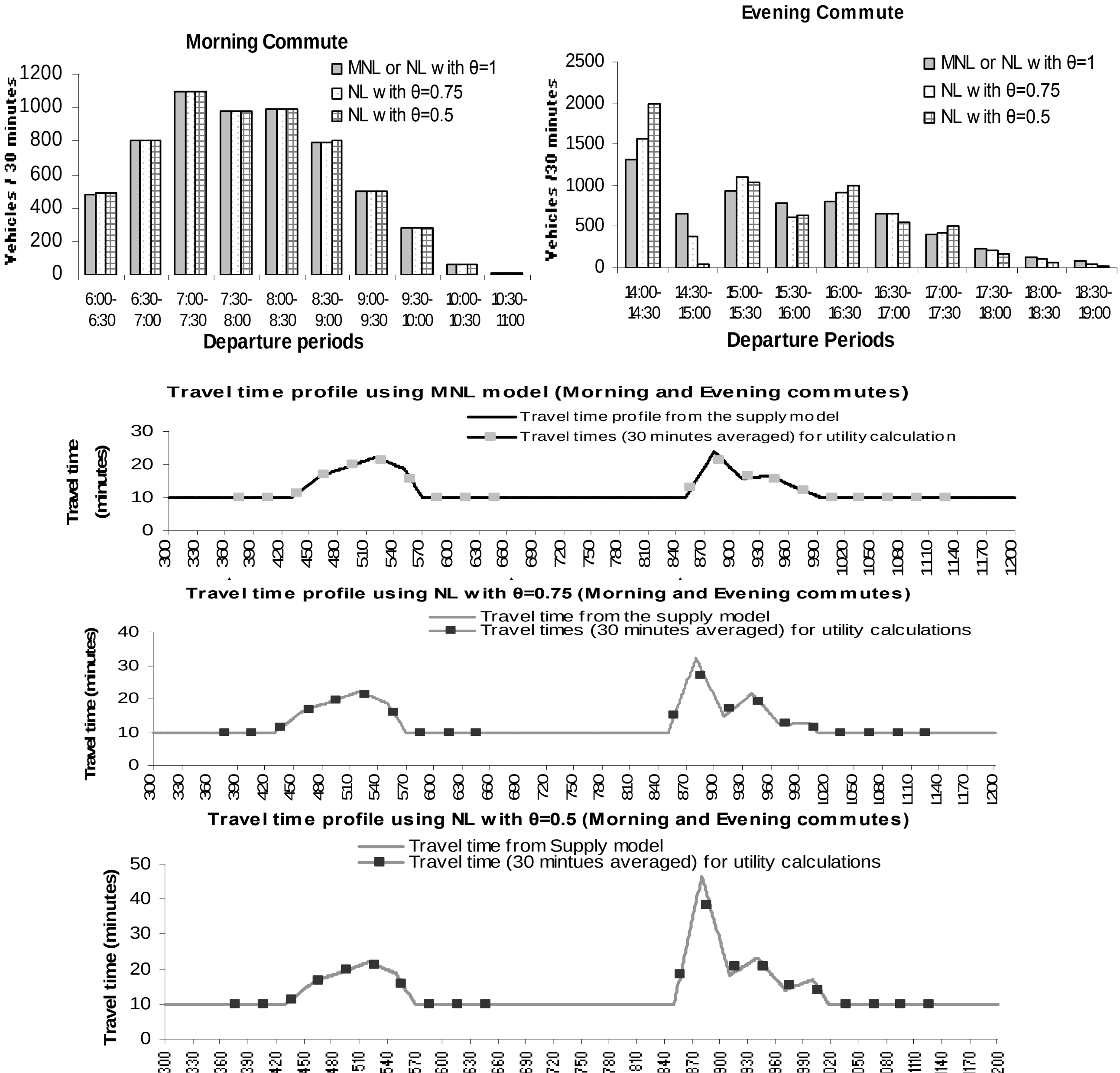


Figure 7.6: Demand and Travel time profiles with respect to time-of-day

Figure 7.6 suggests that there are some minor changes in the demand and travel time profiles of the morning commute when NL model is used, which clearly suggests that the overall systematic utility of the alternatives (V_{ij}) has been changed. These changes in

V_{ij} are due to the feedback of the consequences of the change in the marginal choice probability of the dimension at the lower level of the nesting structure P_j, which is because of the assumed correlation among the alternatives sharing a common morning departure period. For example, the change in the marginal choice probability of the evening commute departure periods brought changes in the demand and travel time profiles of the evening commute, and due to the feedback mechanism of the combined model these changes in the demand and travel profiles of the evening commute have caused variations in the overall systematic utility. More specifically, only those ingredients of the overall systematic utility are changed which are dependent on the evening commute travel time (i.e. R_j), such as after-work home utility and disutility of the in-vehicle evening commute travel. It has been noted that the nature of the after-work home utility profile (can be seen from figure 7.4(c)) is such that it is varying across the evening departure periods for a particular morning departure period but there is no variation across the morning departure periods for a particular evening departure period. The same trend is observed for the in-vehicle travel time disutility of the evening commute. This is why there are very insignificant changes obtained for the morning commute demand and travel time profiles but significant changes are obtained for the evening commute demand and travel time profiles. The another reason is that duration of the work and home activities are considered flexible in the model, so any changes in the evening commute are absorbed by changing the duration of these activities, and therefore, the effects of the evening commute are not fully transferred to the morning commute.

Figure 7.6 further reveals that lower values of θ (i.e. higher values of μ_j, which means higher correlation among the same nest alternatives), is causing more change in the marginal choice probability of the evening commute departure periods, this is also evident from its expression in table 7.3. The higher values of μ_j also suggests that the systematic utility component of the total utility gets higher weightage than its random error term, and therefore, the overall results from the model are further moving towards deterministic predictions (i.e. higher utility alternatives would attract significantly higher demand). This is the reason why the demand profile obtained for the lower values of θ are such that higher utility alternatives are attracting more demand as shown in figure 7.6 for the first departure period of the evening commute. The second departure period in the evening commute is shown to have lower demand because of the higher travel time due to shifting of more individuals in the first departure period.

The above comparison of the model predictions using MNL and NL model at the demand side suggests that the developed combined model is behaving plausibly with different operational models at the demand side. However, selection of the particular model is entirely dependent on the relationship between the modelled scheduling dimensions, which can only be examined through real data and its analysis regarding existence of the particular decision hierarchy. For example; individuals either choose morning departure period first and based on that they decide about the evening departure period for their return journey (NL case with the structure shown in figure 7.5), or they may choose their morning and evening departure periods jointly (MNL model case) or their decision is completely different from both these approaches.

Main Findings:

- The developed combined model can accommodates different operational models of the demand side. However, the experiment reported in this sub-section compared the results for the two models (i.e. MNL and NL models) which were found plausible based on the underlying assumptions of these models.
- The selection of the particular operational model of the demand side is entirely dependent on the envisaged relationship between the modelled scheduling dimensions, which require examination and the analysis of the real data.

7.2.3 Different operational models at the supply side

This sub-section presents the results of the experiments that were conducted in order to see the effect of using different operational models at the supply side of the combined model. The experiments were performed using the linear travel time model, divided-linear travel time model, Point-Queue model and Adnan-Fowkes model. All these models are comprehensively discussed in chapter 4. The experimental setup used for the experiments in section 7.2.2 was also adopted in this case, which means that there are in total 100 alternatives representing the combination of 10 departure periods for the morning and 10 for the evening commute. It is already known that Adnan-Fowkes model requires the assumption about two additional parameters for its numerical implementation, i.e L_1 and *n*. In order to make systematic comparison of the results of these experiments with different supply models, three different combinations of L_1 and *n* were assumed for the Adnan-Fowkes model. At the demand side of the combined model, MNL model was employed. Figure 7.7 shows demand profiles obtained of this experiment at equilibrium.

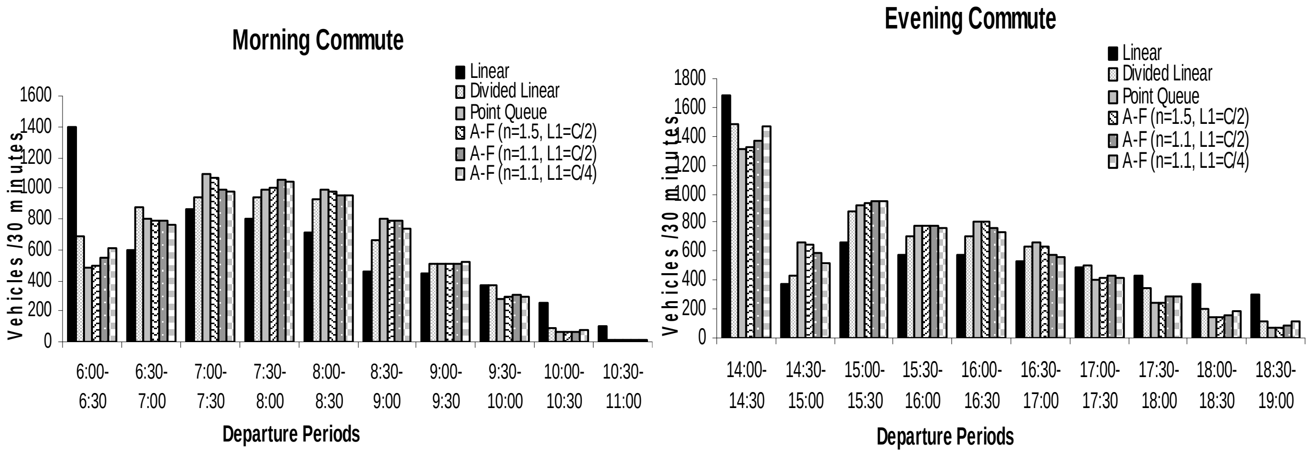


Figure 7.7 Demand profiles of morning and evening commute using different supply models

It is revealed from Figure 7.8 that travel times obtained from the point-queue model are lower than other three models. Higher travel times are obtained when the linear travel time model is used. Divided linear travel time model and Adnan-Fowkes provides moderate values of travel times. This is due to the inherent properties of these models, as it was already mentioned in chapter 4, namely that the point–queue model underestimates travel time when there is no congestion on the road because this model always gives travel time equal to free flow travel time of the link unless inflow to the link exceeds its capacity. Linear travel time model estimates the higher value of travel times because the structure of the model is such that it calculates the travel time for the incoming vehicle at a particular time by considering all the existing vehicles on the link at that time, even when there are few vehicles on the link. Therefore, liner travel time model overestimates travel time when there is no congestion on the link and this effect propagates further which results in higher values of travel time. This property of the linear travel time model is termed as a double counting effect in the DTA literature. Divided linear travel time model presented by Mun (2001) is a result of the modification proposed in the linear travel time model. This model addresses the overestimation problem existing in the linear travel time model, as in this model the link is divided into two sections and traffic is supposed to propagate onto the first section with the free flow speed. When traffic reaches the second section of the link whose free flow travel time is recommended to be equal to the time interval at the supply side, the flow propagates according to the linear travel time model. This results in consideration of congestion effects of the vehicles only in the second section which is the limiting part of the whole link. Adnan-Fowkes model which is already illustrated in chapter 4 also produces the travel times in between point-queue model and linear travel time depending upon the values of n and L_1.

Increase in travel times due to the inherent properties of supply side models other than point-queue model results in the higher weightage of the systematic part of the utility compared to the random error part. This suggests that the higher value of travel times as obtained in the case of linear, divided linear and Adnan-Fowkes model are causing more attractiveness of the higher utility alternatives and therefore, model predictions are moving towards the deterministic side (as shown in figure 7.7). This is the reason why more demand has been observed for the first departure period of the morning and evening commutes when the linear travel time model was used. The above results suggest that the preference made for the use of a particular supply model over others may considerably change the predictions obtained from the combined model. Therefore, it is entirely necessary to examine and calibrate the particular supply model with the real data. This is beyond the scope of this research; however, in future a study could be devised using this as a main objective. The supply model whose behaviour is in close proximity with the reality should be employed in the combined model in order to obtain better predictions.

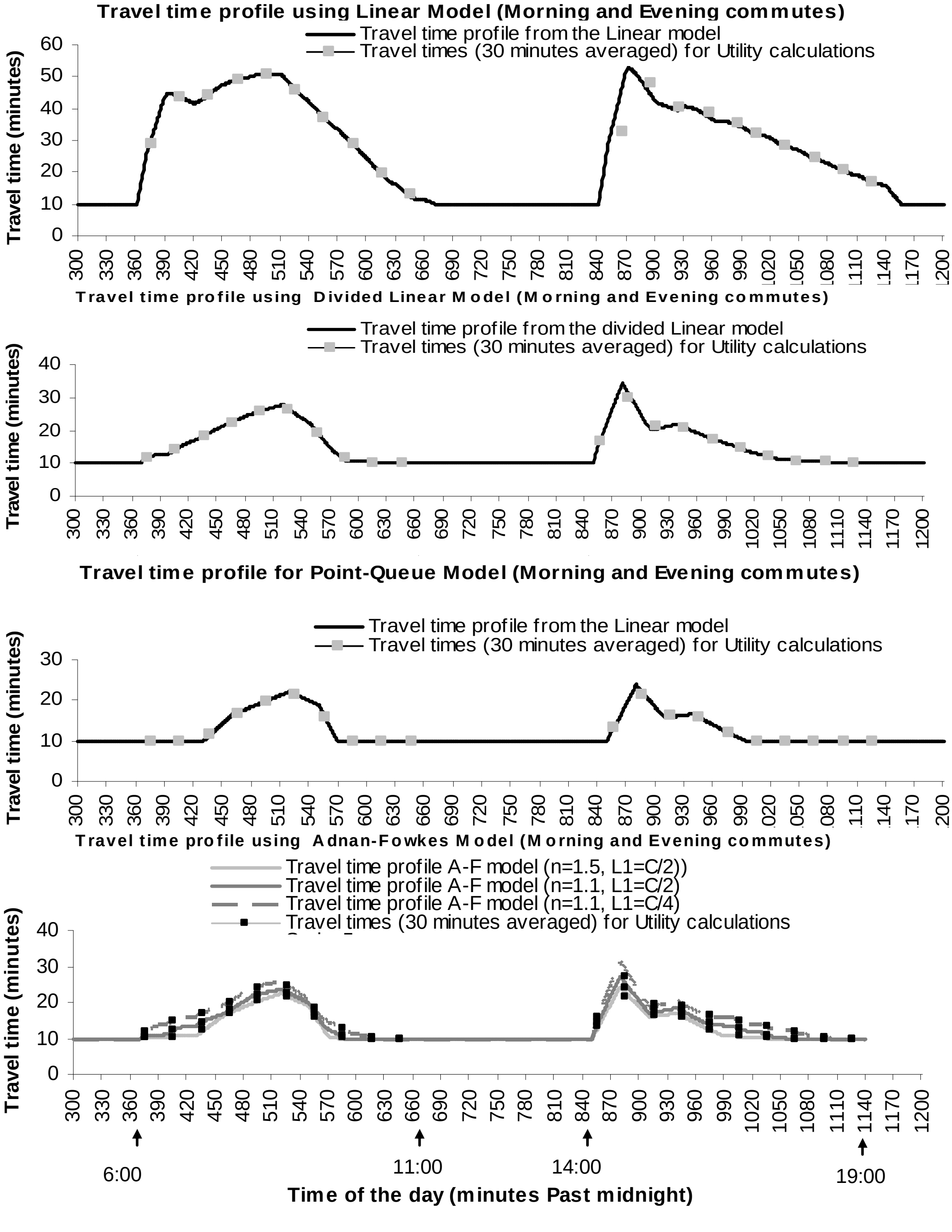


Figure 7.8: Travel time profiles from different supply models

Main Findings:

- The developed combined model can accommodates different operational models of the supply side. However, the experiment reported in this sub-section compared the results for the four models (i.e. linear travel time, divided linear travel time, Point-Queue and Adnan-Fowkes models) which were found plausible based on the underlying assumptions of these models.

- The selection of the particular operational model of the supply side is dependent on the proximity of its behaviour with the reality. This requires examination of real data which can be done in future research.

7.2.4 Effect of variation in analysis time interval-Demand side

The results of the numerical experiments presented in this sub-section show the effect on model predictions of varying the analysis time interval at the demand side. The analysis time interval at the demand side is the duration (Δ) of each departure period considered in the experiment. In all the experiments whose results are shown in the above sections, this duration was considered as 30 minutes. This sub-section presents results of experiments in which this duration was varied as 10, 20 and 50 minutes. The change of the duration (or analysis time interval at the demand side) brought the change in the experimental setup as well because of the change in the number of optimisation variables. The time horizon considered for this experiment was the same as selected in the previous experiments i.e. 5 hours for the morning and 5 hours in the evening commute, the Δ =10 minutes means 900 alternatives, Δ =20 minutes means 225 alternatives, Δ =30 minutes means 100 alternatives and Δ =50 minutes implies 36 alternatives are required to be analysed. At the demand side, MNL model was used as an operational model and at the supply side Point-Queue model was employed with supply side analysis time interval of 1 minute. The results of the experiments are shown in figures 7.9 and 7.10.

Results shown in figures 7.9 and 7.10 revealed that finer values of Δ (lower values) are producing much smoother and accurate prediction of demand at a particular instant of time. This is more clearly evident in figure 7.9, as from figure 7.9 (a) one can observe an average demand for each of the 10 minutes and also those 10 minutes of the day (in the morning and in the evening) are easily identifiable in which demand has the highest value. This is not possible in the case where Δ was assumed as 20, 30 and 50 minutes, as the model then predicts an average demand for each of the 20, 30 and 50 minutes period.

However, finer values of Δ demand higher computational time because number of optimization variables increases significantly with the decrease in the Δ.

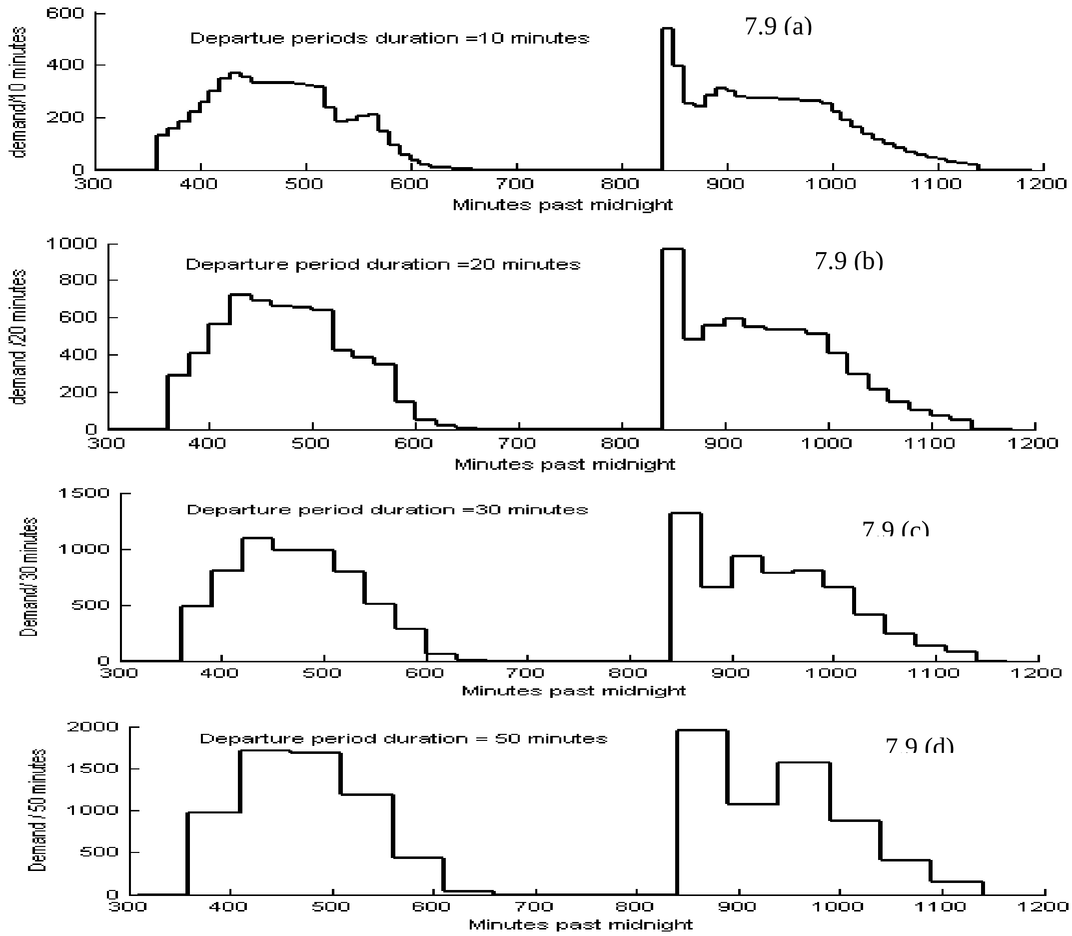


Figure 7.9: Demand profiles using different values of Δ

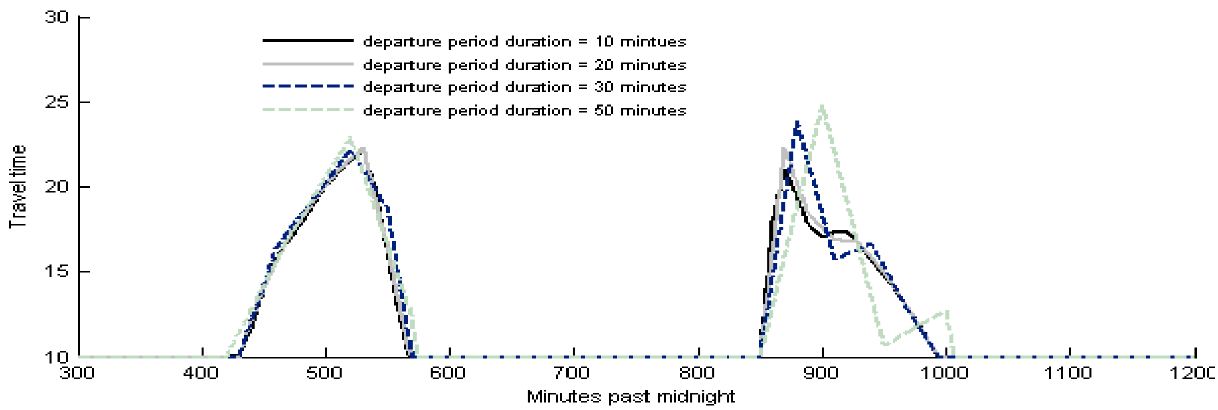


Figure 7.10: Travel time profiles using different values of Δ

Figures 7.9 and 7.10 further suggest that when demand is under capacity there is not much change in the results with the change in Δ value. However, when demand is above capacity in a certain departure periods, there are significant changes in the results noted with a change in Δ value. This is evident from the travel time profile of the morning and evening commutes, as in the morning commute demand is under capacity for the few initial and last departure periods and therefore travel time profiles obtained at different values of Δ are quite similar except in the middle departure periods of the morning commute where demand is over capacity. The same trend is observed for the evening commute travel time profile as well, however, the effect is more here because demand is significantly over capacity in the first departure period and therefore, travel times are significantly higher and the effects of this are transferred to subsequent departure periods as well. The changes in the travel time profiles because of the over capacity demands at respective departure period is due to the fact that point-queue model was used at the supply side which only incorporates a congestion effect when the link inflow is equal or over capacity.

Main Findings:

- Use of the finer values of Δ increases the accuracy of the model predictions but at the same time require higher computational time because of the significant increase in the optimisation variables.
- The significance of the finer values of Δ increases with the increase in the network congestion.

7.2.5 Effect of variation in analysis time interval-Supply side

The results shown in this sub-section are from the numerical experiments carried out using different analysis time interval at the supply side. In all the previous experiments whose results are shown in earlier sub-sections, the analysis time interval (δ) was considered equal to 1 minute. In this sub-section, results are shown for the experiments in which this parameter (i.e. δ) was varied as 30 seconds, 1 minute, 5 minutes and 10 minutes. The analysis time interval at the demand side was considered as 30 minutes and the experimental setup employed here considered 10 departure periods in each of the morning and evening commutes with morning commute start at 0600 hours and evening commute start at 1400 hours. The Point-Queue model was used as an operational model at the supply side and at the demand side MNL model was employed. One of the motivation for carrying out this experiment is to analyse a trade-off that may exists regarding the suitable value of δ over the computational cost, similar to the experiment with different values of departure period duration (Δ) .

Results shown in figure 7.11 reveals that there are no significant changes in the model predictions when supply side analysis time- interval is varied. However, finer values of δ ensure that 30 minute averaged travel time used at the demand side for the utility calculation is accurate. This is evident form the demand profiles of the morning and evening commutes as demand profiles obtained for δ equal 1 minute and 30 seconds (0.5 minute) are very close to each other, however, there are some changes noted (not of very significant nature) when δ is higher (i.e. 5 minutes and 10 minutes). It has been noted that finer values of δ require more computational time than higher values of δ. This suggests that a trade-off exists and suitable value of δ may be subjected to the available resources and time.

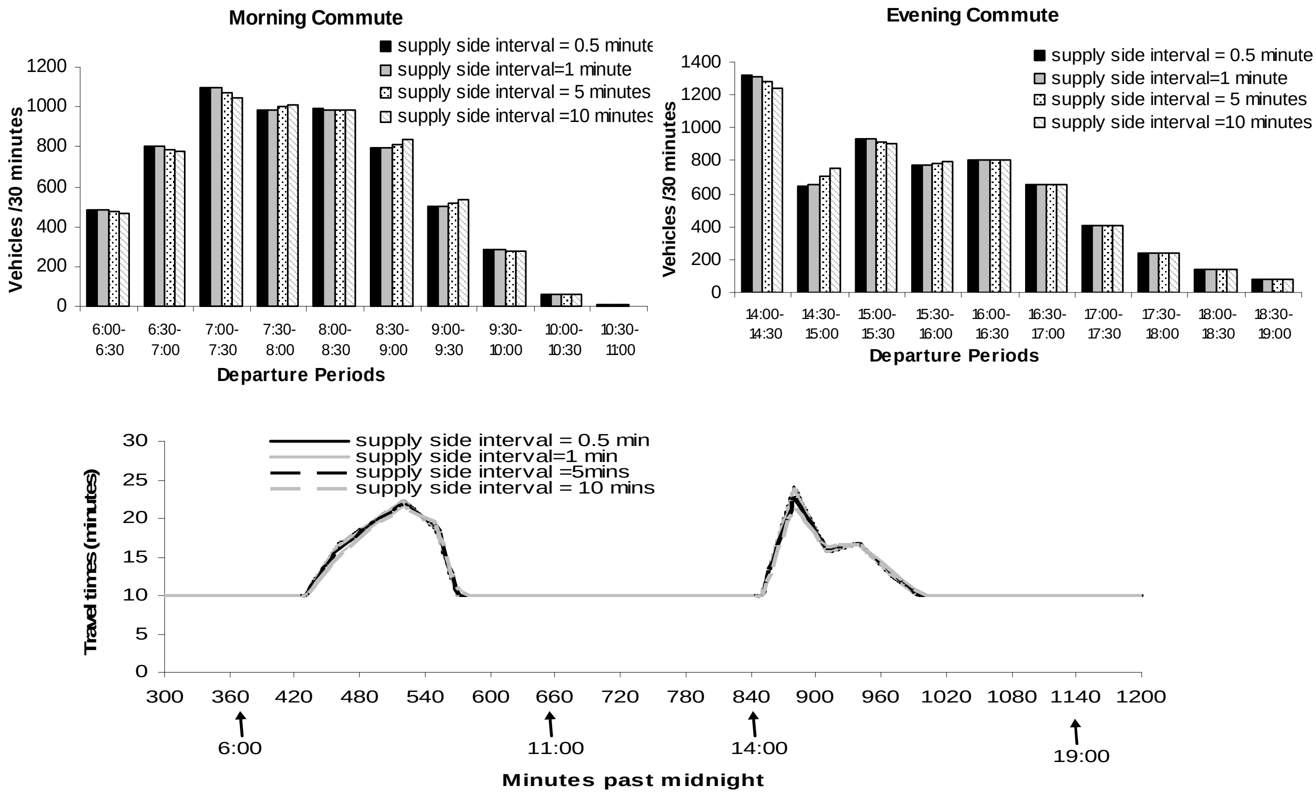


Figure 7.11: Demand and Travel time profiles for different values of δ

The results of several experiments shown in this section are clearly depicting that the developed combined model is behaving according to the expectations. Furthermore, it is flexible enough to incorporate changes either in the demand and supply sides. In addition to that, results of all different experiments shown in this section suggest that not only solutions of the combined model exist but these solutions are unique as well. This is evident form the comparative analysis of the results mentioned for the different experiments. The next section discusses the results of the experiments obtained by employing certain policy schemes.

7.3 Numerical Experiments-Model Applications for Policies

The results and finding of the numerical experiments reported in this section presents the implications of certain demand management policies on the model predictions. These policies are as follows:

- Incorporation of dynamic tolls in order to reduce congestion on the links.
- Incorporation of tele-work option in the model framework in order to examine the implication of this policy option on link congestion.
- Implementation of flexible working hour scheme with respect to time-of-day and duration of the work activity.

To conduct these experiments the same values of parameters were utilised as assumed in section 7.2. However, some minor modifications in the model structure were assumed in order to implement the above mentioned policies. The details of these minor modifications with respect to a particular policy are illustrated in the following sub-sections, where results are also explained for that policy scenario.

7.3.1 Experiments Incorporating Dynamic Tolls

To conduct this experiment, it is assumed that dynamic tolls are induced to reduce congestion. This has been done by adding two more terms in the systematic utility specification of the overall utility of alternatives (i , j) i.e. in equation 6.27. The modified utility expression can be given as follows

$$V_{ij} = \int_0^i V'^h(t)\,dt + \left(\int_{0_i}^{\tau_w} V'^w(\tau)\,d\tau\right) + g\left(i + R_i - PST\right) + \int_{j+R_j}^{1440} V'^h(t)\,dt + \lambda R_i + \lambda R_j + \omega \cdot (toll_i) + \omega \cdot (toll_j) \tag{7.4}$$

where, $toll_i$ and $toll_j$ represent the implemented tolls in money units (e.g. GBP or US$) on the link for the morning departure period i and for the evening departure period j respectively. ω is a negative parameter with the unit as utils/£ or utils/$, so that the overall systematic utility unit remains as utils.

The same setup of the problem is followed for this experiment was illustrated in the previous sections. The analysis time interval at the demand side was considered as 30 minutes with 10 departure periods in each of the morning and evening commutes with the morning commute starting at 0600 hours and the evening commute starting at 1400 hours. The values of the parameter are also assumed as illustrated in section 7.2, however, an additional parameter ω is considered here as equal to 0.95 utils/£ (so that the value of in-vehicle travel time obtained is around 8 pence/minute ($\lambda / \omega = 0.08 / 0.95 = 8.42$ pence/minute) suggested by Wardman (1997)). The effects of arbitrary dynamic tolls on the model predictions are examined using two different strategies of tolls. The first strategy assumed similar tolls for the middle departure periods of the morning commute only and the second strategy assumed dynamic tolls in both commutes. The second tolling strategy is based on the demand profile of the no toll case which is considered here as a base case (i.e. experiment results illustrated in section 7.2.2.1), higher demand departure periods have higher value of tolls and the lower demand departure periods have lower value of tolls. Figure 7.12 shows these two tolling strategies.

Results obtained after the implementation of the tolling strategies are shown in figure 7.13. The figure also presents the results of the no toll case (i.e. results of the experiment shown in section 7.2.2.1), so that a systematic comparison can be made and the effect of the tolling strategies can be clearly appreciated

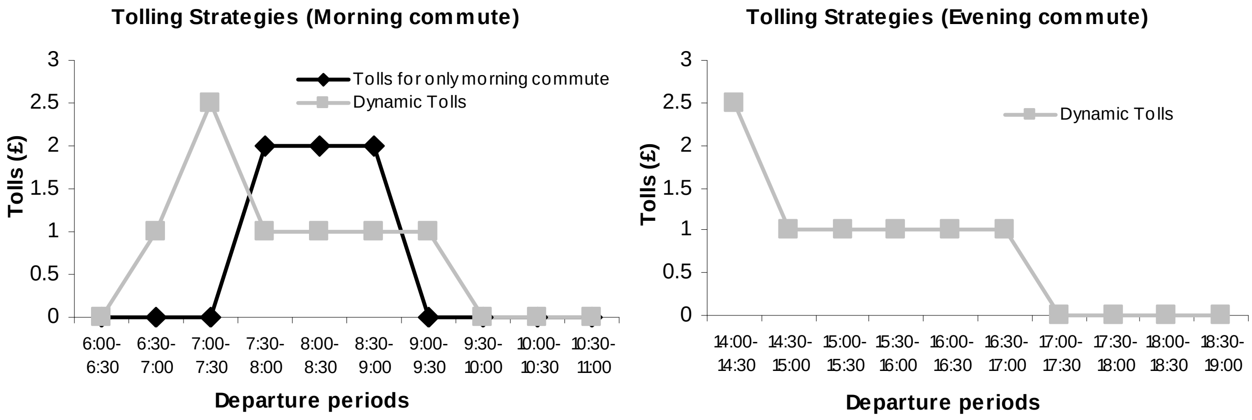


Figure 7.12: Dynamic Tolling strategies for the numerical experiment

Figure 7.13 shows that the tolling strategy which assumes tolls in the morning commute only have significant impact in changing the demand and travel time profiles of the morning commute compared to the no toll case. There are some changes noted in the actual demand and travel time profiles of the evening commute because of this tolling strategy but these changes are very insignificant and cannot be appreciated from shown figure because of its scale. This suggests that the extent of the amount of tolls is not enough to bring any significant changes in the evening commute demand and travel time profiles due to the underlying notion incorporated in the model regarding the flexible duration of the activities. Furthermore, it has been noted that the demand in the morning commute at the time-of-day for which tolls are also assumed, has considerably moved to both in the earlier and in the later departure periods, resulting in a very low demand in those times-of-day. This is the reason why demand and travel time profiles of the morning commute show two peaks. In continuation of the above point, it is further noted that for those of the later departure periods (i.e. departure periods 7, 8, 9 and 10) demand has been moved only in the departure period 7 (9:00am-9:30am). This is because of the assumed late arrival penalty at work place which is active from 9:00 am, as further moving from this time-of-day may cause more late arrival penalty (disutility). As most of the demand is moved toward earlier departure periods in the morning commute and no significant change is observed for the evening commute, it is likely that the duration of the work activity becomes longer compared to the no toll case. Figure 7.14 shows the work activity duration frequency distribution along with the indication of the weighted average duration of work activity in both cases. For the no toll case this is around 7.56 hours and for the morning commute toll case this value is around 7.6 hours.

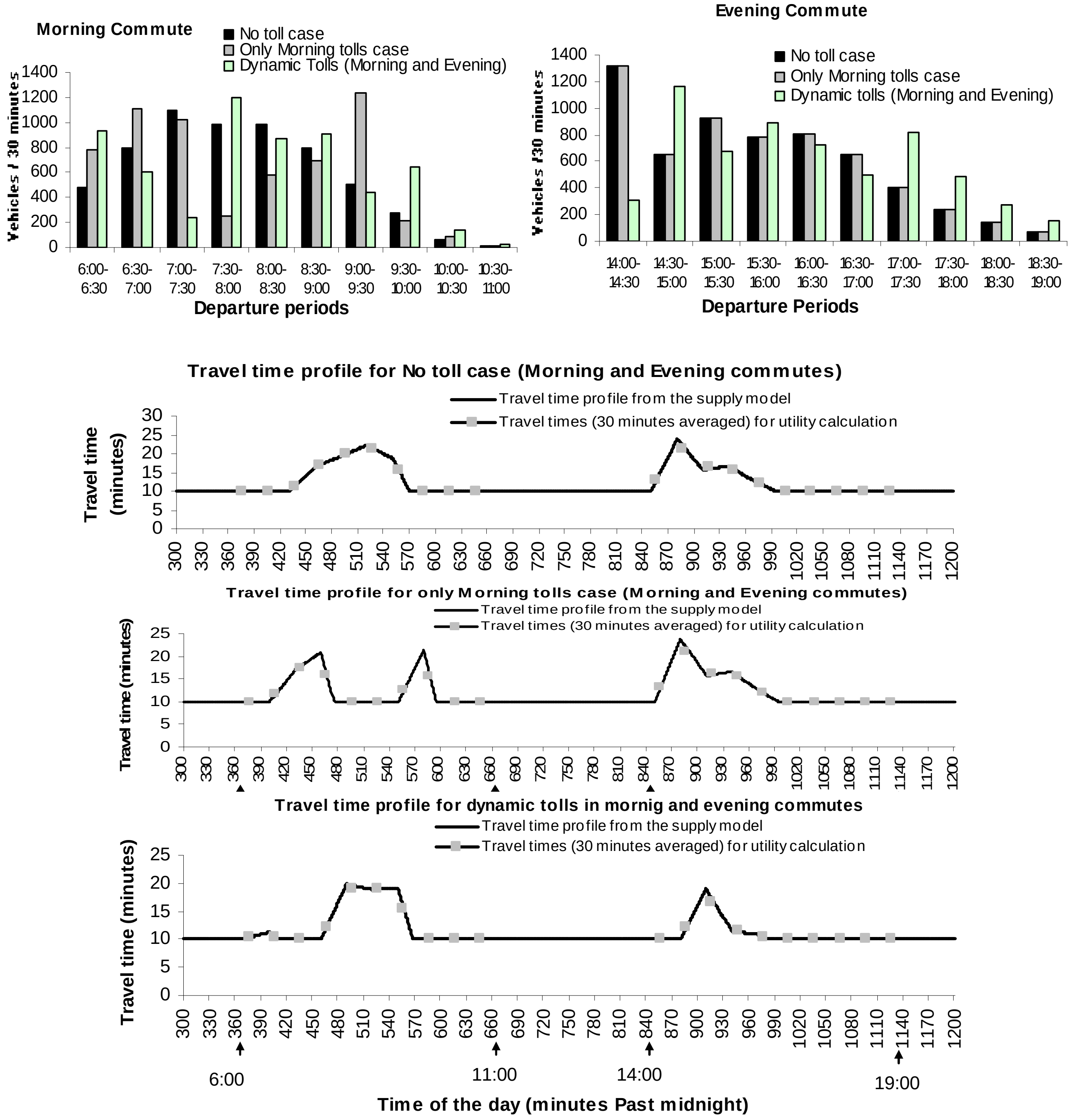


Figure 7.13: Demand and Travel time profiles for different tolling strategies

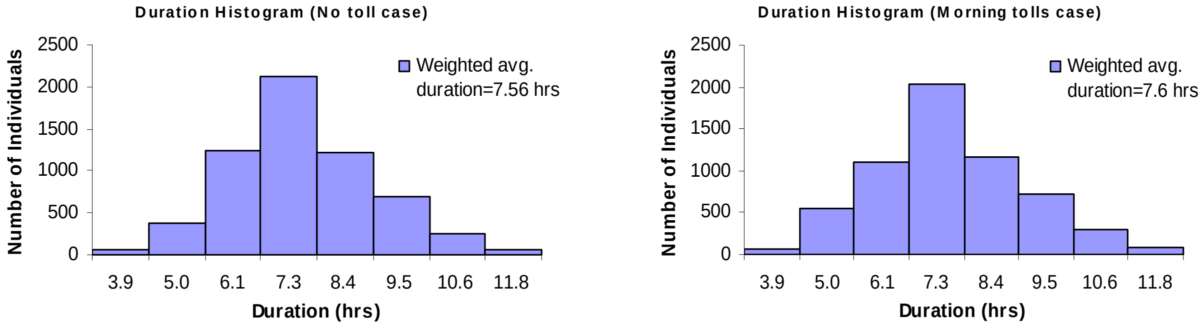


Figure 7.14 Work Activity Duration for no toll and morning commute tolls cases

The implementation of the second tolling strategy, which is based on the demand profile of the no toll case and induce tolls on both commutes (i.e. morning and evening), results in demand and travel time profiles which are significantly different compared to the no toll case as shown in figure 7.13. These results suggest that the peak is dispersed significantly due to the introduction of tolls. This is the consequence of a new balance of trade-off between travel cost, with additional cost in terms of tolls and benefits gained through participation in activities. It is therefore useful to examine the change in the components of the utility function to better understand the complicated trade-offs involved in the process. Figure 7.15 presents the total systematic utility profiles for the two cases (i.e. no toll case and a case which employed the 2nd strategy of tolls). The comparison of these utility profiles clearly depicts the effect of tolls on the total systematic utility i.e. the higher utility alternatives in the no toll case now have considerably lower values of utility when tolls are employed.

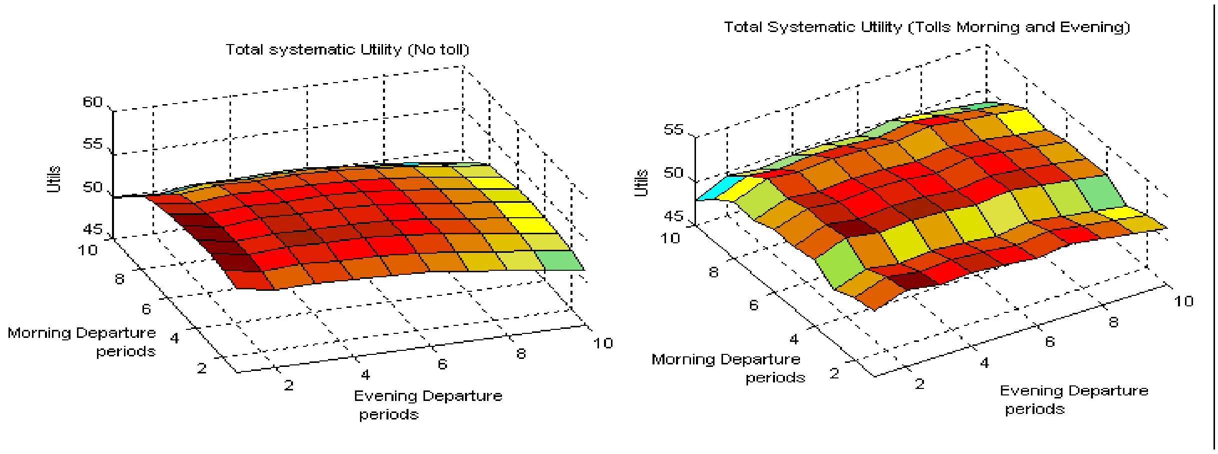


Figure 7.15: Total systematic utility profiles for the no toll and 2nd toll strategy cases

It has been noted that incorporation of tolls not only results in an increase in the disutility of alternatives but due to the feedback mechanism of the model, these tolls then results in a change in the demand at those times-of-day, resulting in changed travel time profiles. This suggests that the change in the overall systematic utility profile of the toll case (compared to no toll case) is because of the increase in the disutility (direct impact of tolls) and also due to the changes in those components of systematic utility which are dependent on the travel times R_i and R_j. Figure 7.16 shows the further decomposition of the total systematic utility of the two cases (i.e. no toll case and 2nd toll strategy) in order to investigate which component of the systematic utility has a significant role in causing these model predictions when tolls are employed.

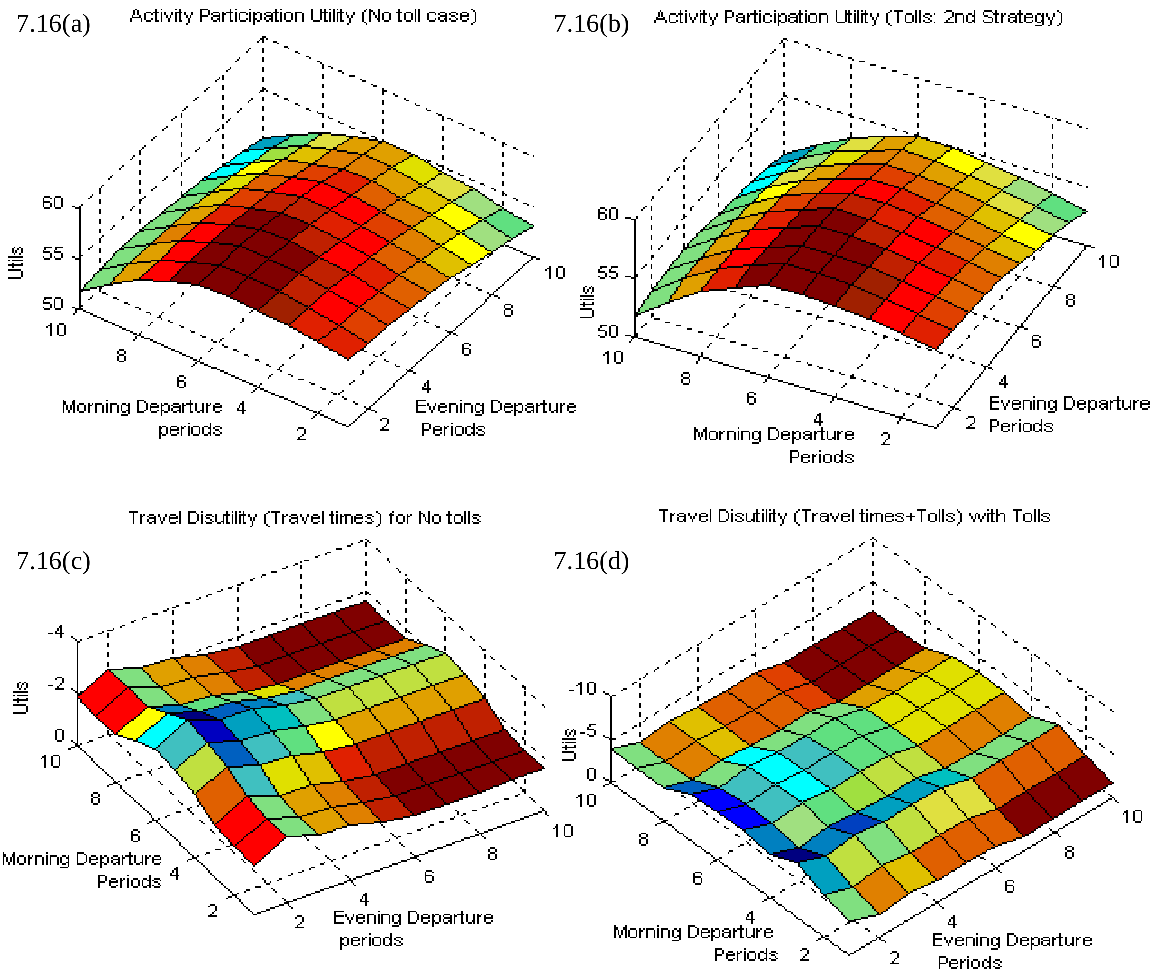


Figure 7.16: Activity Utility and Travel Disutility profiles

Figure 7.16 shows that the incorporation of tolls is not causing any significant change in the overall utility of activity participation as figure 7.16 (a) and (b) are similar to each other. There are some differences noted in the components of activity participation utility which are dependent on travel times (e.g. After-work home activity utility, work utility and late arrival penalty), but these differences are very small and cannot be appreciated from figure 7.16. This suggests that the role of the different components of the activity participation utility is not of a significant nature in the predicted demand and travel time profiles, however, the role of the total travel disutility (travel time + tolls) is very obvious as figure 7.16 (c) and (d) are significantly different from each other. From this one may conclude that the direct effects of tolls are of a more significant nature than its indirect effects in the current setting of the problem. However, it is important to note that the indirect effects are entirely dependent on the relationship incorporated in the model between the direct and indirect effects. In the current setting of the problem, travel times R_i and R_j are the key factors which link the direct effects with indirect effects of policies. These two factors are attached to activity utility components in such a manner that they may either increase or decrease the limits of integrals of the marginal utility of activities. It now entirely depends on the shape of these marginal utility curves (or in other words on the parameters which are responsible for the shape of these marginal utility curves), because the shape of the marginal utility of activity will actually lead to the changes in the utility of activity participation due to the change in the limits of the integrals (due to changes in travel times).

Similar to the point noted for the morning toll case regarding work activity duration, in this case it is also noted that the duration of the work activity has been increased considerably. Figure 7.17 shows the duration frequency distribution of the work activity, with a weighted duration of work activity of *7.8 hours* (relatively higher than the no toll case). The higher duration of work activity when tolls are incorporated is due to the fact that more demand has been shifted towards earlier departure periods in the morning and later departure periods in the evening commute as this combination of departure periods now offers higher utility.

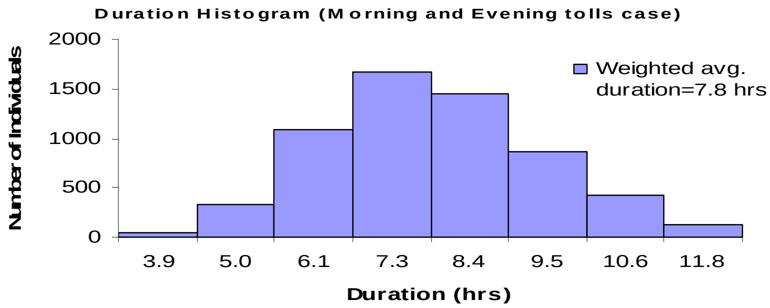


Figure 7.17 Work Activity duration for the 2nd strategy tolls

The trend that the duration of the work activity increases with the incorporation of tolls may suggest that an increase in the generalised travel cost causing individuals to stay longer at the work place. But this trend is only observed when generalised travel cost is increased systematically as executed in the experiments whose results are shown in this section. Similar results were reported in Heydecker and Polak (2006) when they have analyzed the effects of *congestion elimination* tolls on the work duration. If tolls are incorporated in the departure periods where demand is already lower in the no toll case, then it may result in a demand profile in which demand is higher for a combination of departure periods which results in the lower duration of work activity. This is because the duration of work activity is not constrained (i.e. flexible) in both model (i.e. the daily tour model reported in this research as well as in Heydecker and Polak, 2006), which allows individuals to nullify the effect of tolls by participating more in such activities from which they could gain more benefits.

Table 7.4 presents the summary of the social benefits obtained for different strategies of tolling assumed in this experiment. The measure of consumer surplus (logsum) not only accounts for the increased travel cost (tolls) but also incorporates the effects of decreased travel time due to tolls and accordingly increased utility at the home and work activity due to this decrease in travel times. It may be seen in table 7.4 that increase in travel cost (by inducing tolls) decreases the consumer surplus as expected. This is because the choices (choice of departure times) individuals have become more expensive due to the introduction of tolls. The positive benefits observed here suggest that the tolling strategies assumed in this experiment are good enough as on one side they are able to reduce travel times and on the other side revenue is generated from them. However, these benefits need to be compared with the cost required to construct the infrastructure and the manpower needed for toll collection (i.e. operating costs) which is not taken into account here.

Table 7.4: Summary of Benefits from different strategies of tolling

Tolling Strategy	Consumer surplus (logsum) in £ A*	Total Consumer surplus (TCS) in £ B**	Change in total Consumer surplus (ΔTCS) in £, w.r.t base case ΔTCS	Total Generated Revenue from Tolls in £ R = $\sum q_i.(toll)_i + \sum q_j.(toll)_j$ R	Benefits in £ ΔW=ΔTCS+R ΔW
Without tolls (base case)	61.697	370182	-----	0	---
1st Tolling strategy (morning tolls)	61.196	367176	-3006	3076.88	73
2nd Tolling strategy	60.194	361164	-9018	9359.535	347.7

$* A = \frac{1}{\omega}[\log \ sum], ** B = Q \cdot A$

Main Findings:

- The developed model is able to incorporate dynamic tolls through minor changes in the utility expression, and the results obtained are plausible.
- The effects of the first tolling strategy (i.e. tolls on the morning commute only) are not transferred significantly on the evening commute, which is due to flexibility incorporated in the model regarding durations of activities.
- Systematic increase in the generalised travel cost (i.e. for congestion elimination) results in the longer duration of work activities for models in which duration of work activity is considered flexible.

7.3.2 Experiments Incorporating Tele-Work scheme

This experiment was performed in order to see the effects on the demand and travel time profiles of the home-work tour when the tele-work option was available to all individuals (i.e. commuters). The individuals who choose tele-work option will remain at home and do their work activity while staying at home. So, there is no need to commute between the home and work activity location and therefore these individuals do not take part in the formation of congestion on the links. This suggests that the total demand (Q) that is used in this experiment is an elastic demand. This is because the demand (Q) is now based on the trade-off between the satisfaction individuals obtain by choosing the tele-work option and overall benefit they gain choosing any departure period combination for the home-work tour. This experiment does not need a modification in the systematic utility expression of the model, but it requires an additional systematic utility expression. This expression provides the utility to an individual who choose tele-work option, suggesting that the tele-work option should be incorporated as an additional alternative. In all previous experiments individuals are choosing alternatives which are a combination of the morning and evening departure periods, however, in the present experiment besides these departure periods an additional alternative of tele-work is available to all individuals. The systematic utility expression for this experiment is given as follows:

$$V_{ij} = \left(\int_0^i V'^h(t)\, dt \right) + \left(\int_{0_i}^{\tau_w} V'^w(\tau)\ d\tau \right) + g\left(i + R_i - PST\right) + \left(\int_{j+R_j}^{1440} V'^h(t)\, dt \right) + \lambda R_i + \lambda R_j \tag{7.5}$$

$$V_{tw} = T_w \tag{7.6}$$

Equation (7.5) is the same as equation (6.27) and it provides the utility of the home-work tour. Equation (7.6) for simplicity is based on the constant (T_w), which gives the utility of remaining home and performing the tele-work activity while staying at home (V_{tw}). The experiment is performed using different values of the constant (T_w) along with the similar settings of the problem as used in section 7.3.1 (i.e. 10 departure periods in each of the morning and evening commute with MNL model at the demand side and Point-queue model at the supply side). The results obtained from these experiments are shown in figures 7.18 and 7.19.

The results shown in figures 7.18 and 7.19 reveal that tele-work option may be quite an effective policy for elimination of the congestion on the links; however, much is dependent on the satisfaction (utility) individuals have obtained by using this option. If the results of the experiment for the without tele-work are compared with the experiment in which T_w = 50 utils, then it is noted that there are no significant changes in the demand and travel time profile. This suggests that the utility individuals obtain by being involved in the tele-work option is less than the utility individuals are getting by involvement in the home-work tour as the highest utility alternative in the without tele-work option experiment is 55.5 utils. In the experiment in which T_w = 60, a significantly high number of individuals have chosen the tele-work option, the reason that tele-work option now becomes the highest utility alternative. The fact that some individuals are still performing the home-work tour in this case, this is due to stochastic nature of the model (i.e. due to the random error associated with the systematic utility of each alternative, individuals are not fully aware of their highest utility alternative).

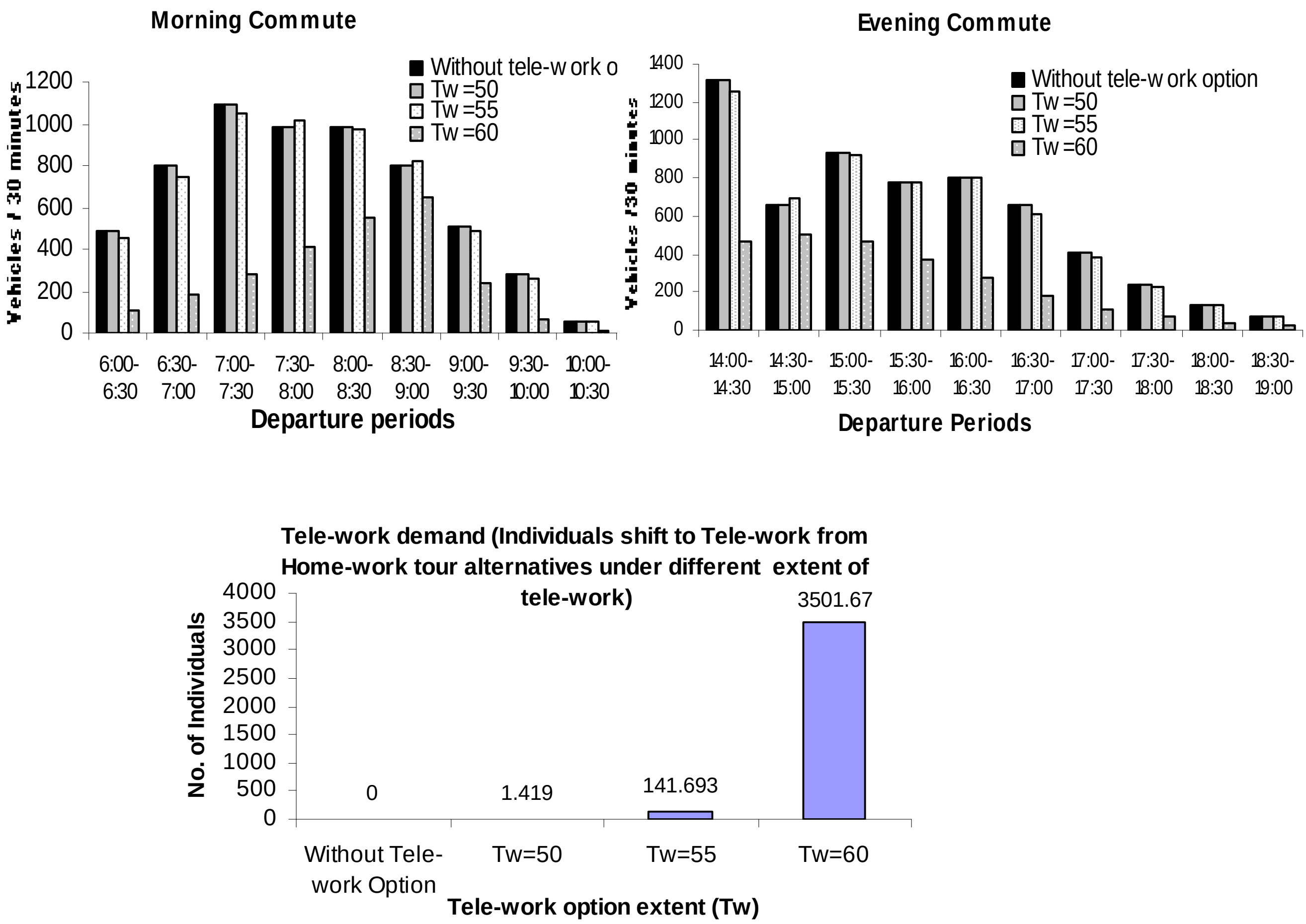


Figure 7.18: Demand profiles for home-work tour and tele-work option with different values of T_w

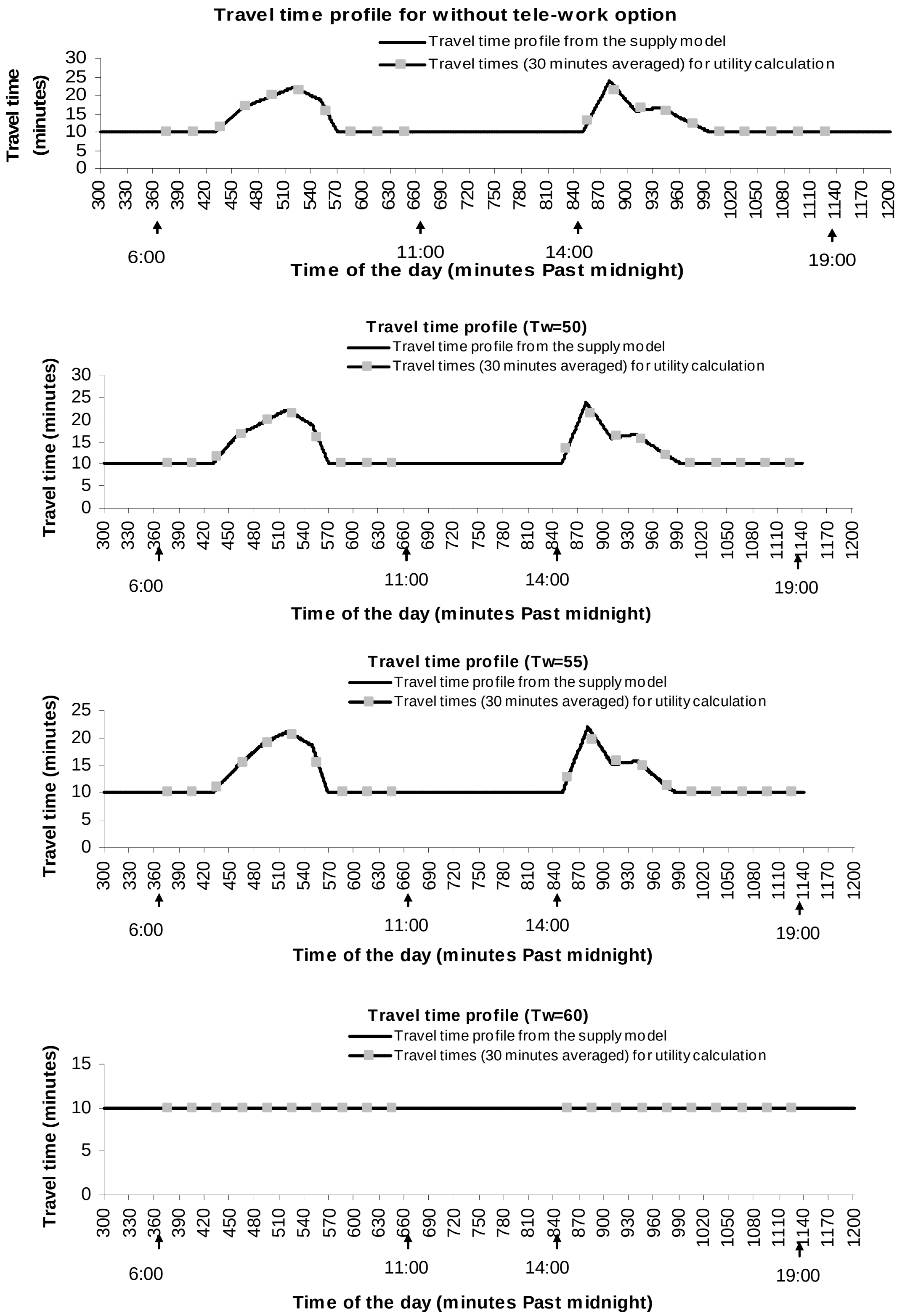


Figure 7.19: Travel time profiles for home-work tour with different values of T_w

As in these experiments the MNL model was used at the demand side, this is the reason why it has been observed that the presence of an additional alternative (tele-work option) is not affecting the ratio of the probabilities between other alternatives (i.e. IIA property of the model) because all alternatives are assumed to be independent to each other. In this case, as tele-work option is significantly different from other alternatives (departure time combinations of the morning and evening commute), the use of NL model may provide better predictions compared to the MNL model with the consideration of structure in which some correlation is assumed for the alternatives which represent combination of morning and evening departure periods. However, this is something which may only be confirmed by the examination of real data.

Main Findings:

- The developed model is able to incorporate tele-work option as an alternative, and the results obtained are plausible suggesting that this may be effective policy for congestion elimination.
- This experiment further suggests that the developed model can also deal with elastic demand.

7.3.3 Experiments Incorporating Flexible Work Hour schemes

This experiment was performed in order to see the effects on the demand and travel time profiles of the home-work tour when the work activity is considered as flexible with respect to time-of-day and with respect to its duration. The duration of activities in the developed model is already flexible, as an individual may choose different durations of home and work activities by choosing different departure times for their morning and evening commute trips. However, the model is rigid to an extent with respect to time-of-day, because for the work activity the model incorporates a late arrival penalty (time-of-day ingredient for work activity). The flexibility with respect to time-of-day may be introduced in the model by relaxing the extent of the late arrival penalty at work location. This may be termed as flexibility with respect to work start time. This experiment does not need modification in the systematic utility expression of the model but it requires change in the parameter (m_l) value which is attached with the late arrival penalty for work activity. The experiment was performed using lower values of m_l (such as -0.02 and 0) then the one used in the previous experiments. The lower values of m_l indicate lower late arrival penalty for the work activity and therefore provide higher flexibility with respect to work start time. The results are reported in figure 7.20.

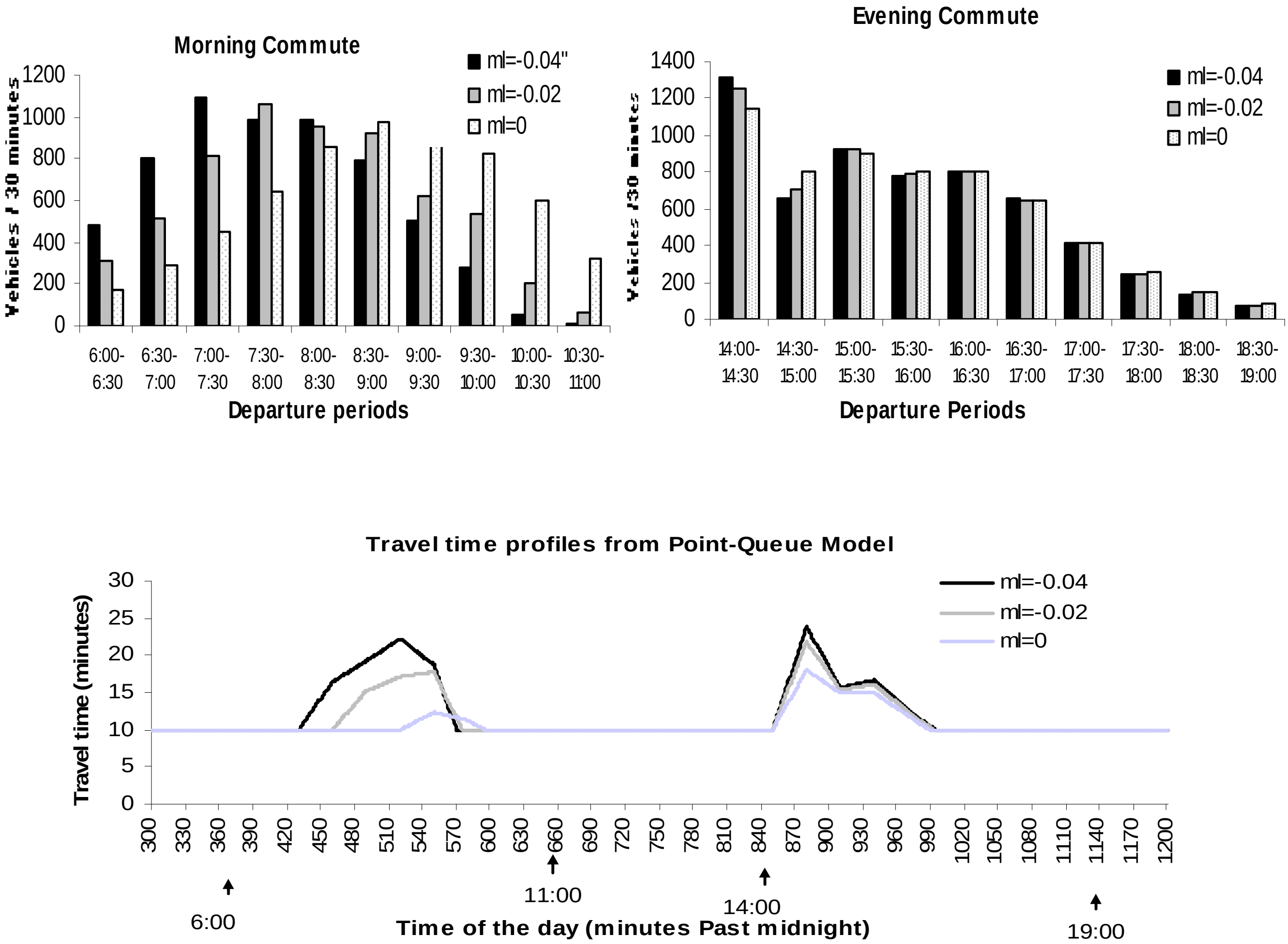


Figure 7.20: Demand and travel time profiles for time-of-day flexibility of work activity

Figure 7.20 reveals that the introduction of flexibility in the work activity start time is helpful in reducing congestion on the link. In the morning commute the demand is shifted towards the later departure periods with the lower values of m_l. This suggests that the lower late arrival penalty at the work location is giving an individual a chance to obtain more benefits from before-work home activity, but in doing do so there may be some reduction in his/her work activity utility (because of the change in duration) which may be taken care by choosing the later departure periods in the evening commute. It has been noted that in the evening commute where the first departure period share the highest demand in normal condition (non-flexible work start time) is now getting lower share of the demand compared to the cases where m_l is higher.

The case where m_l is considered to have a zero value indicates that the work activity start time is entirely flexible (i.e. no restriction on work activity start time) and because of that considerable number of individuals have chosen later departure periods in the morning commute. But there are some individuals who still chose earlier departure periods in the morning commute and also in the evening commute no significant changes are noted. There may be two reasons for that, one is the stochastic nature of the problem (as individuals are not aware of their maximum utility alternative) and also on the interaction between the marginal utility curves of the home and work activities. As the lower late arrival penalty at the work location allows individuals to choose later departure periods in the morning commute (and they are getting more benefits from the before-work home activity) but individuals are not significantly changing their departure periods for the evening commute may be because the longer duration of work activity is not rendering as much benefits as going home earlier and staying at home (after-work home activity benefits). Figure 7.21 also supports the above arguments as weighted work activity duration when m_l =-0.02 and m_l =0.0 is noted as 7.28 hrs and 6.78 hrs respectively which is lower than the one (7.56 hrs) which is noted in the case when m_l =-0.04. Figure 7.21 presents work activity duration histograms for all three experiments presented in this sub-section with different values of late arrival penalty parameter (m_l).

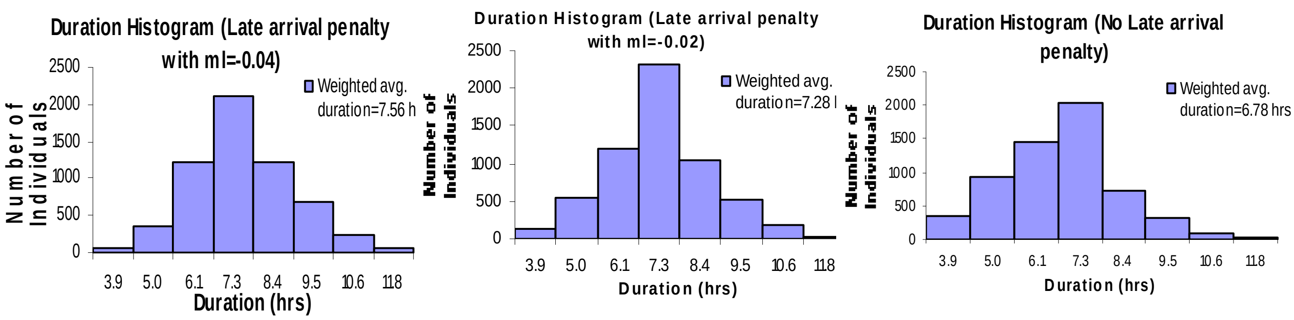


Figure 7.21: Work Activity Duration for different values of late arrival penalty parameter

Table 7.5 presents the summary of the socio-economic benefits obtained when experiments are performed using different extents of the work start time flexibility (i.e. by lowering late arrival penalty parameter). It may be seen that consumer surplus has increased with the decrease in the extent of late arrival penalty parameter. This indicates that a decrease in the late arrival penalty gives individuals an opportunity to choose from the wider range of attractive alternatives (as implementation of late arrival penalty significantly decreases the utility of departure periods after *PAT*), and as a result travel times are decreased which give rise to higher values of consumer surplus. Table 7.5 suggests incorporating flexibility in the work start time is an effective policy in terms of overall socio-economic benefits but this is dependent on the nature of work activity. If the work activity is such that its start time at a particular time-of-day is not a significant issue then introduction of this policy may render significant benefits, otherwise decrease in the late arrival penalty may cause some other costs which will reduce the benefits (e.g. at a production lines, where a minimum number of staff need to be in attendance).

Table 7.5: Summary of Benefits from different extent of late arrival penalty parameter

Late Arrival Penalty Parameter	Consumer surplus (logsum) in £ A	Total consumer surplus (TCS) in £ B	Change in total consumer surplus or Benefits w.r.t. base case in (£) ΔW=ΔTCS
m_l=-0.04 (base case)	61.697	370182	-----
m_l=-0.02	62.201	373206	3024
m_l=0.0	62.925	377550	7368

The above results of the experiments regarding flexibility of work activity start time already incorporated a notion that the duration of work activity is flexible i.e. individuals are allowed to choose different duration of work activity based on the trade-off between the activity participation utility and travel disutility. It is useful to examine the results of experiments in which work activity duration is considered as fixed. This suggests that in this condition individuals are only allowed to choose their morning commute departure times, their evening commute departure times can easily be given by adding morning travel times (R_i) and fixed work activity duration (τ_w^{fxd}) to their morning commute departure times (i.e. $j = i + R_i + \tau_w^{fxd}$). The systematic utility expression is then given by:

$$V_i = \left(\int_0^i V'^h(t)\, dt \right) + \left(\int_0^{\tau_w^{ffxd}} V'^w(\tau)\, d\tau \right) + g\left(i + R_i - PST\right) + \left(\int_{i+R_i+\tau_w^{fxd}+R_{i+R_i+\tau_w^{fxd}}}^{1440} V'^h(t)\, dt \right) + \lambda R_i + \lambda R_{i+R_i+\tau_w^{fxd}} \qquad (7.7)$$

The experimental arrangement when equation (7.7) was incorporated contains 10 morning departure periods (similar to the previous experiments) and the demand and supply sides incorporated MNL model and point-queue model respectively. The results of the experiments are shown in figure 7.22, where demand and travel time profiles are shown for various fixed work activity durations and also for the case in which work activity duration is considered as flexible (i.e. individuals are allowed to choose different work duration on the basis of trade-off between the activity participation utility and travel disutility).

The results shown in figure 7.22 suggest that when the duration of work activity is considered as fixed, the demand and travel time profile of the evening commute entirely replicates those of the morning commute. The only difference between the morning commute and evening commute demand and travel time profile is the amount of time lag between the two commuting trips which is equivalent to the addition of the morning commute travel times and the fixed work activity duration. Figure 7.22 further suggest that an increase in the fixed duration of the work activity causes individuals to choose earlier departure periods for their morning commute. This may be because of the way in which home and work activity utilities are defined. It is known that an increase in the duration of work activity always increases the benefits an individual gets from the work activity participation, but the marginal utility of work activity is always diminishing with an increase in its duration. Therefore, the morning departure period which gives maximum utility in the case where work activity duration is 6 hours would not be able to provide the maximum utility in the case where work activity duration is changed from 6 to 7 or 8 hours. Therefore, some individuals are moved to other morning departure periods, and the effect of this can be seen from the travel time profiles.

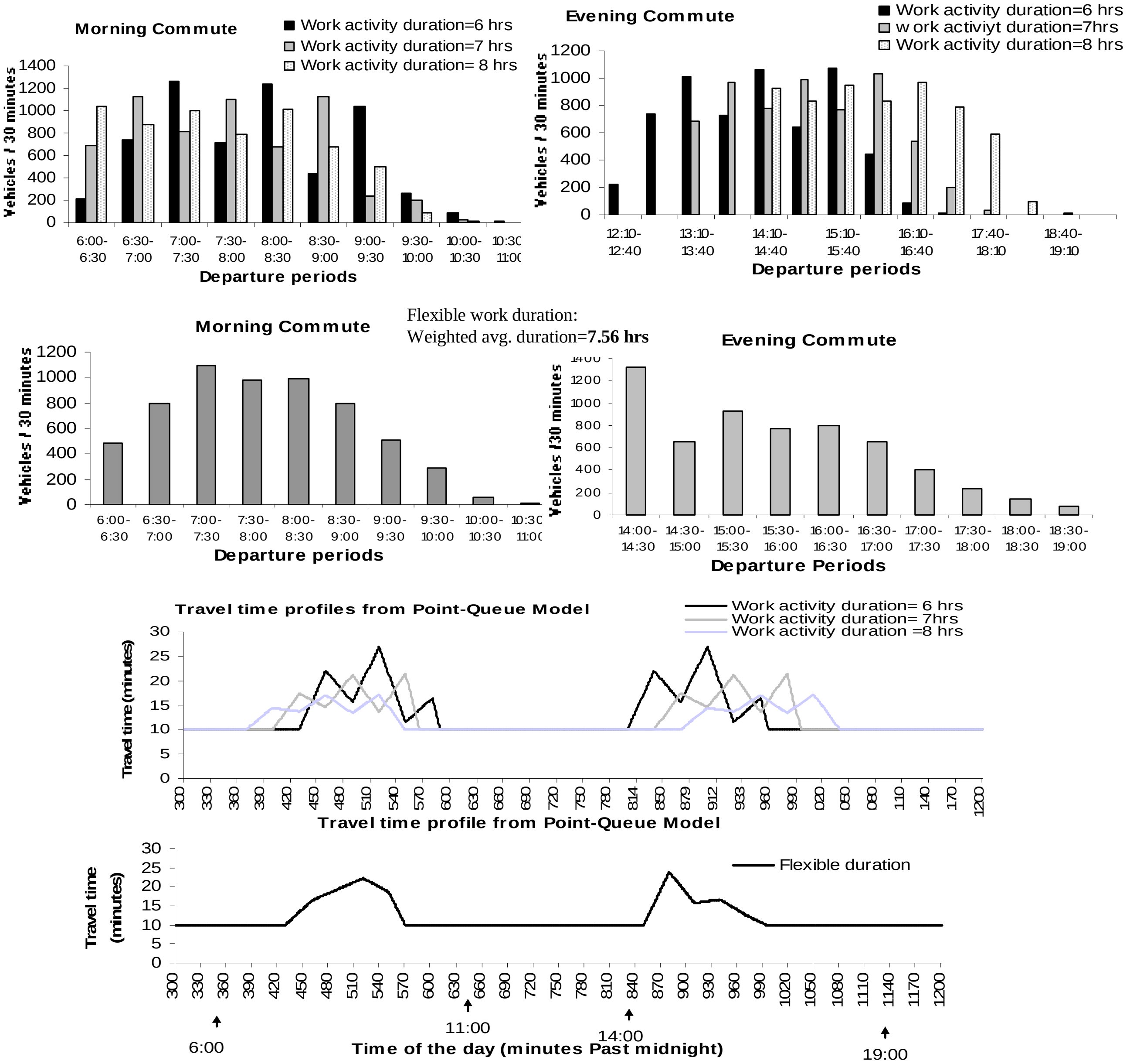


Figure 7.22: Demand and travel time profiles for flexible and fixed work duration

When the duration of work activity is considered as fixed, then in this situation individuals are only able to change their departure times of the morning commute, However, flexibility in the duration of work activity gives the individual an opportunity to make changes in the choice of the combination of the morning and evening departure periods. This is the reason why morning and evening commute departure time and travel time profiles are significantly different to each other when the work activity is considered as flexible (as evident from figure 7.22). Thus, flexibility in the duration of the work activity provides an individual a chance to better cope with the congestion effects, however, when modelling a single day only with the assumption that other days of the week are a replicate of the modelled day then this flexible duration notion may raise questions. This is because in the reality, the duration of work activity if not fixed on a single day then it may be constrained on some another time horizon e.g. a week or a month. Chapter 8 discuss this issue in more detail.

Table 7.6 presents the summary of the net socio-economic benefits when different fixed work activity durations are considered in comparison with the flexible work activity duration. The results shown in table 7.6 are according to the property of the logsum term as described in Ben-akiva and Lerman (1985, p. 301), which suggests that this term has a monotonic relationship with respect to choice set size provided that all other things remains the same. When work activity duration is considered flexible there are in total 100 alternatives (combination of 10 departure period in the morning and 10 in the evening), however, when this duration is considered as fixed then individuals are left with 10 alternatives (morning departure periods) as evening departure time is determined exactly by morning departure time. Table 7.6 reveals that introduction of fixedness in the work activity duration causes a decrease in the consumer surplus, as individuals are left with the limited choice set and whatever travel cost they are bearing in the morning time, the same needs to borne in the evening as well. Furthermore, fixedness in the work activity duration for higher or lower amount of time also causes some disutility, as individuals may want to work for less amount of time or vice versa, as in the earlier case individuals may lose some utility at home location or in later case they may lose some utility at work location. This is the reason why consumer surplus is higher in the case when work activity duration is 7 hours in comparison when this is fixed to 8 or 6 hours.

Table 7.6: Summary of benefits from different fixed work activity duration

Work Activity Durations	**Consumer surplus (logsum) in £** **A**	**Total Consumer surplus in £** **B**	**Change in total consumer surplus or benefits in £ w.r.t. base case** **ΔW=ΔTCS**
Flexible (Base case)	61.697	370182	-----
Fixed to 8 hours	59.627	357762	-12409.4
Fixed to 7 hours	59.771	358626	-11548.5
Fixed to 6 hours	59.481	356886	-13287.6

Main Findings:

- The developed model is able to provide plausible results when work activity start time is considered flexible (i.e. lower late arrival penalty).
- The experiment with fixed duration of work activity replicates the demand and travel time profiles of the morning commute in the evening commute.
- The notion of flexibility of the duration of activities incorporated in the model may seems unreasonable when the modelling horizon is just a single day, as many jobs require individuals to stay agreed number of hours at work place on a time horizon of a week or a month (This point is explained well in chapter 8).

7.3.4 Lessons Learned from Model Applications

The developed model has been applied successfully in order to represent the implications of different congestion mitigation policies. For all the policies tested in sections 7.3.1 to 7.3.3, the model predictions are found plausible as these policies are able to reduce congestion on the links and the change in the departure time profiles is in line with the plausible behaviour. The following are the general key points which may be considered as the lessons learned from the model applications:

- A methodical change in the systematic utility of home-work tour in order to reduce congestion on the link is causes a change in the duration of home and work activities. It has been noted that for a policy like dynamic tolls (in which the systematic utility is methodically decreasing) results in the increase of the duration of work activity, however, policies like flexible work start time and tele-work scheme (in which systematic utility of home-work tour is increased), results in the decrease of the duration of work activity. This is mainly because the durations of activities are not constrained in the model.

- The experiment which involves incorporation of the tele-work scheme suggested that the developed model can also deal with an elastic demand case. Results suggest that the tele-work option may provide an effective policy for elimination of congestion on the road networks.

- The developed model provides a flexible tool to test different policies by making minor modifications in the parameters or in the utility specification. This is evident from the application of three altogether very different policies within the same model framework. Policies like time-based parking charges and link capacity improvements are also easily tested from the model. Furthermore, the model in its current stage considered two activities and analysed only simple tours between these two activities. The model is not only applicable for the home-work tour, but it can also be utilised when it is needed to analyse other simple tours e.g. home-shopping tour, work-shopping tour, home-leisure tour or any tour which involves two activities.

7.4 NUMERICAL EXPERIMENTS- EXTENDED MODEL

In sections 7.2 and 7.3 the developed model has undergone comprehensive testing and application. It has been noted that the home-work tour version of the model (which only incorporates two scheduling dimensions such as departure times and activity duration) is successful in providing plausible results under different circumstances. The present secion extends this testing, by illustrating the working of the extended version of the model which is described in chapter 6, section 6.5. The extended version of the model incorporates four scheduling dimensions, which include: departure times, activity duration, activity sequence and route choice. Further to that it not only considers simple tours (tours based on two activities) but other types of tours, e.g tours based on three activities, can also be examined, as the extended version of the model is able to incorporate multiple user classes. One class or group is assumed to perform a home-work tour and other class is assumed to perform a travel pattern which consists of three activities i.e. home-work-shopping-home. The following section presents the results of the numerical experiments performed for the extended model.

7.4.1 Numerical Experiment 1-Moderate Congestion

The network shown in figure 6.3 that contains six (uni-directional) links and three activity locations is presented here again for more clear illustration of the considered scheduling dimensions for this experiment. This is shown in figure 7.23. The individuals who are performing home-work tour are considered with the set-up shown in table 7.7, and individuals who are performing a three activities tour are considered with the set-up shown in table 7.8.

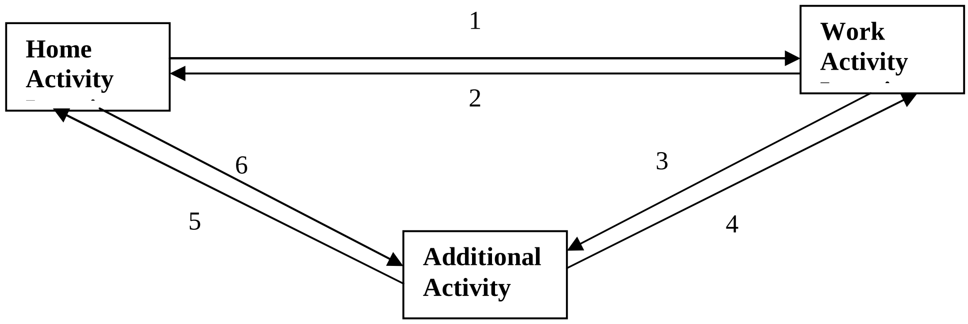


Figure 7.23: Example Network with three activity locations and six uni-directional links

Table 7.7: Home-work tour class experimental setup

Scheduling dimensions	**Illustration of Scheduling dimensions**	**Home-Activity Parameters**	**Work-Activity Parameters**	**Other Parameters**	**Demand and Supply models**
Departure times (Activity durations)	T=0700 hours, D =4, Δ=30 and Y=1500 hours (4 departure periods in each of the morning and evening commutes)	h_0 = 0.03utils/min, α = 720 minutes past midnight , β =0.1, γ =1, U_0 =10 utils,	PST = 0900hours m_l =.04utils/min η_w = 5 utils	ϕ =10minutes, C =1800veh/hr, λ = -0.08 utils/min, δ=1 minute, Q_1=3000	MNL and Point-Queue models
Routes	**1**. Link1-Link2 **2**. Link1-Link3-Link5 **3**. Link6-Link4-Link2 **4**. Link6-Link4-Link3-Link5				

Tables 7.7 and 7.8 explain the setup of the numerical experiment; it has been shown that for the home-work tour, 4 choices of routes are considered along with 4 departure time choices for each of the morning and evening commutes. The considered 4 routes are those in which the home-work tour can be performed without travelling on the same link twice (i.e. these are acyclic routes for performing home-work tour and represent the top four choices among the individuals). For the three activities tour, two routes are considered for each sequencing option. There are other options available for the routes (in accordance with the network shown in figure 7.23) but in this experiment only two are chosen for each sequencing option, in order to include the presence of route choice along with departure time and activity sequence choices for this user class, but keeping the number of choice dimension to a manageable size. The systematic utility expression shown in chapter 6 as equation 6.32 was utilised in this experiment. The results obtained for this experiment are shown in figures 7.24, 7.25 and 7.26.

Table 7.8: Three activities (home, work and additional activity) tour class experimental setup

Scheduling dimensions		Illustration of Scheduling dimensions	Home-Activity Parameters	Work-Activity Parameters	Additional-Activity Parameters	Other Parameters	Demand and Supply models
Activity Sequence		**1.** Home-add-activity-work-home **2.** Home-work-add-activity-home	h_0 =0.03utils/min, α = 720 minutes past midnight , β =0.01,γ =1, U_0 =10 utils,	PST = 0900hours, m_l =.04utils/min η_w = 5 utils	α_a = 750 minutes past midnight , β_a =0..005 ,γ_a =1, U_{0a} =15 utils, η_a =1utils	λ = -0.08 utils/min, ø = 10minutes, C =1800veh/hr, δ=1 minute,Q_2=*3000*	MNL and Point-Queue models
Departure times (Activity durations)		D =4, Δ=30 T=0700 hours (from home activity location) Z_1=1000 hours (from add-activity location: sequence is home-add. activity-work-home) Y=1500 hours (from work location) Z_2 =1900 hours (from add. activity location: sequence is home-work-add. activity-home) (4 departure periods for each of the commute trips)					
Routes	1st Sequence	**1.** Link1-Link3-Link5 **2.** Link 6-Link4-Link3-Link5					
	2nd Sequence	**1.** Link6-Link4-Link2 **2.** Link1-Link3-Link4-Link2					

See table 3.2 for definitions of the symbols of home and work activity parameters

Figure 7.24 shows the results obtained for the user class who was involved in performing home-work tour. Route numbers and activity sequence numbers referred as below are defined in tables 7.7 and 7.8. Most of the individuals of this user class are using route 1 (i.e. link 1 - link 2) and route 3 (i.e. link 6- link 4- link 2) for travelling between the two activity locations. The other two routes in which link 3 was involved in the return trip to home, are avoided by the individuals of this user class due to the significantly higher congestion in the evening times (see figure 7.26), as higher travel times on link 3 have significantly reduced the utility of using routes 2 and 4. The higher travel times on link 3 is caused by other user class who are travelling from the locations of the work activity to the additional activity *at similar times*. These are around 3000 in total and all of them are departing in the first departure period which is from 15:00-15:30 hours (see figure 7.25, third row). Furthermore, for the home-work tour user class it has been noted that route 1 was used by a higher number of individuals (1810 out of 3000) than those who used route 3. This is because route 1 is the most direct route for travelling between home and work locations as it only contains two links which have lower travel times than 3 links of the route 3. It is interesting to note that the amount of disutility from travelling on link 6 and link 4 in order to reach the work location in the morning is higher compared to link 1, even when link 6 and link 4 are operating in free flow conditions.

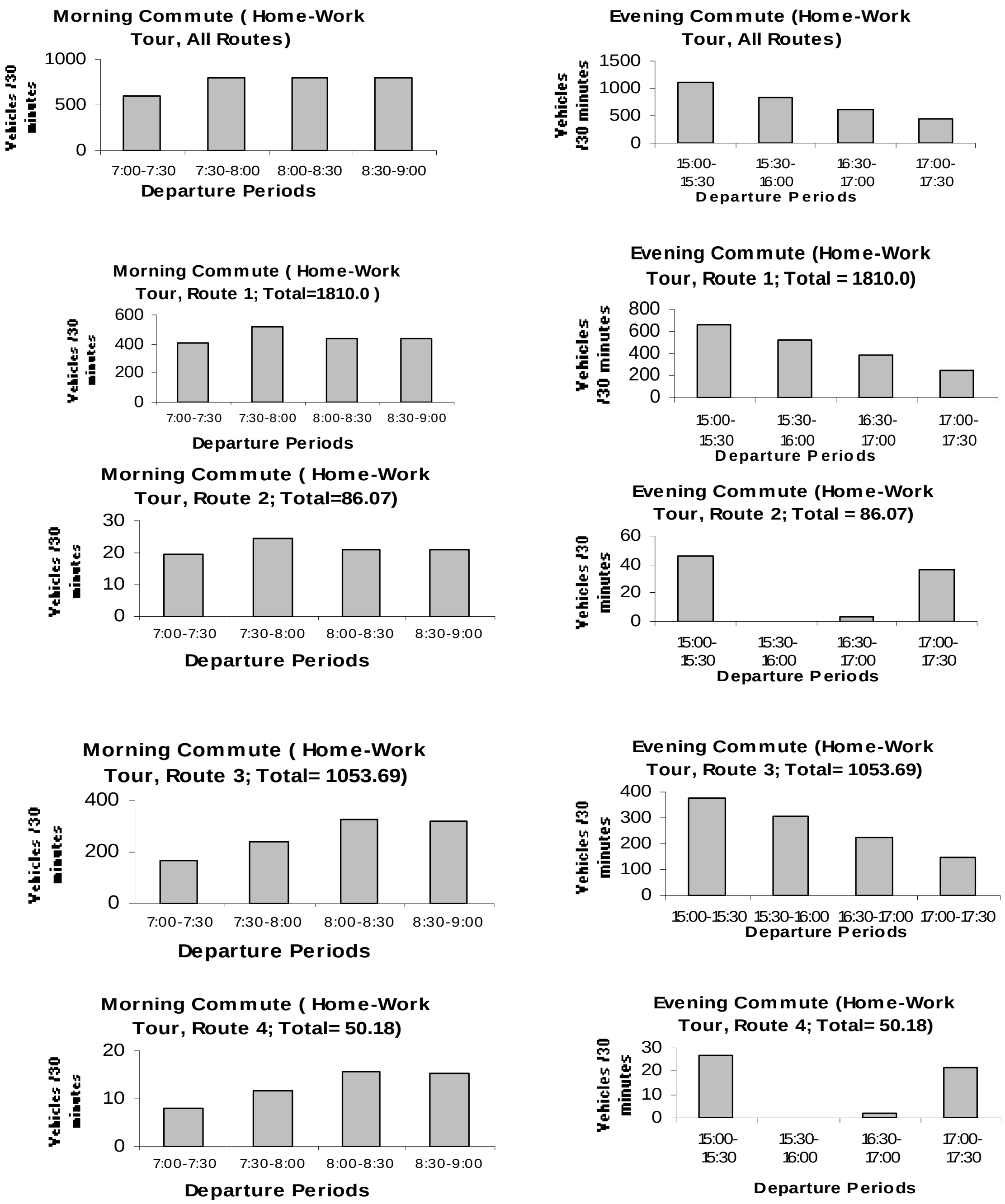


Figure 7.24: Home-Work Tour Demand Profiles with departure time and route choices for the Population Segment Performing Home-Work Tour

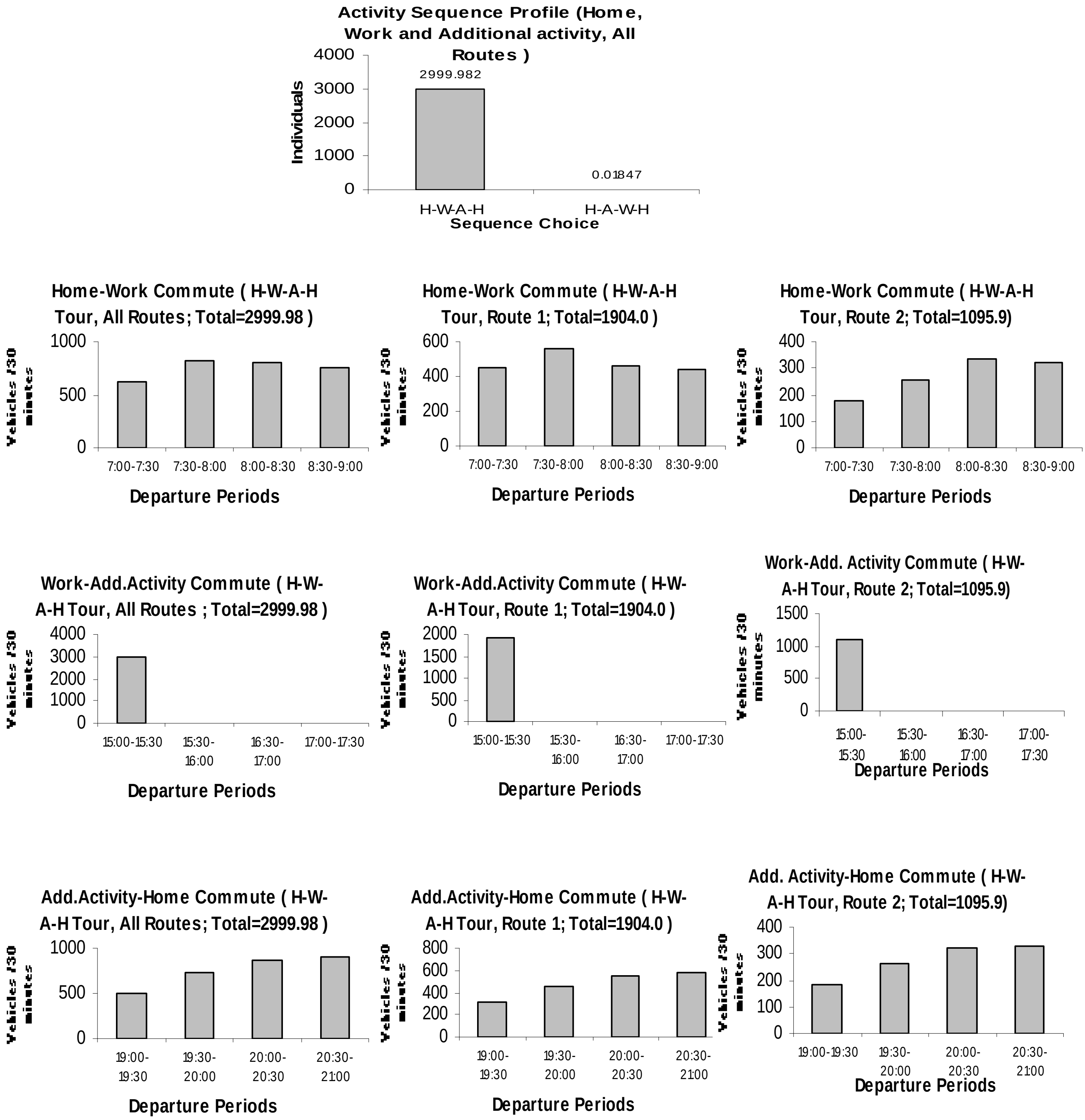


Figure7.25: Home-Work-Add. Activity Tour Demand Profiles with Departure times and route choices for Population Segment Performing Three-Activity tour

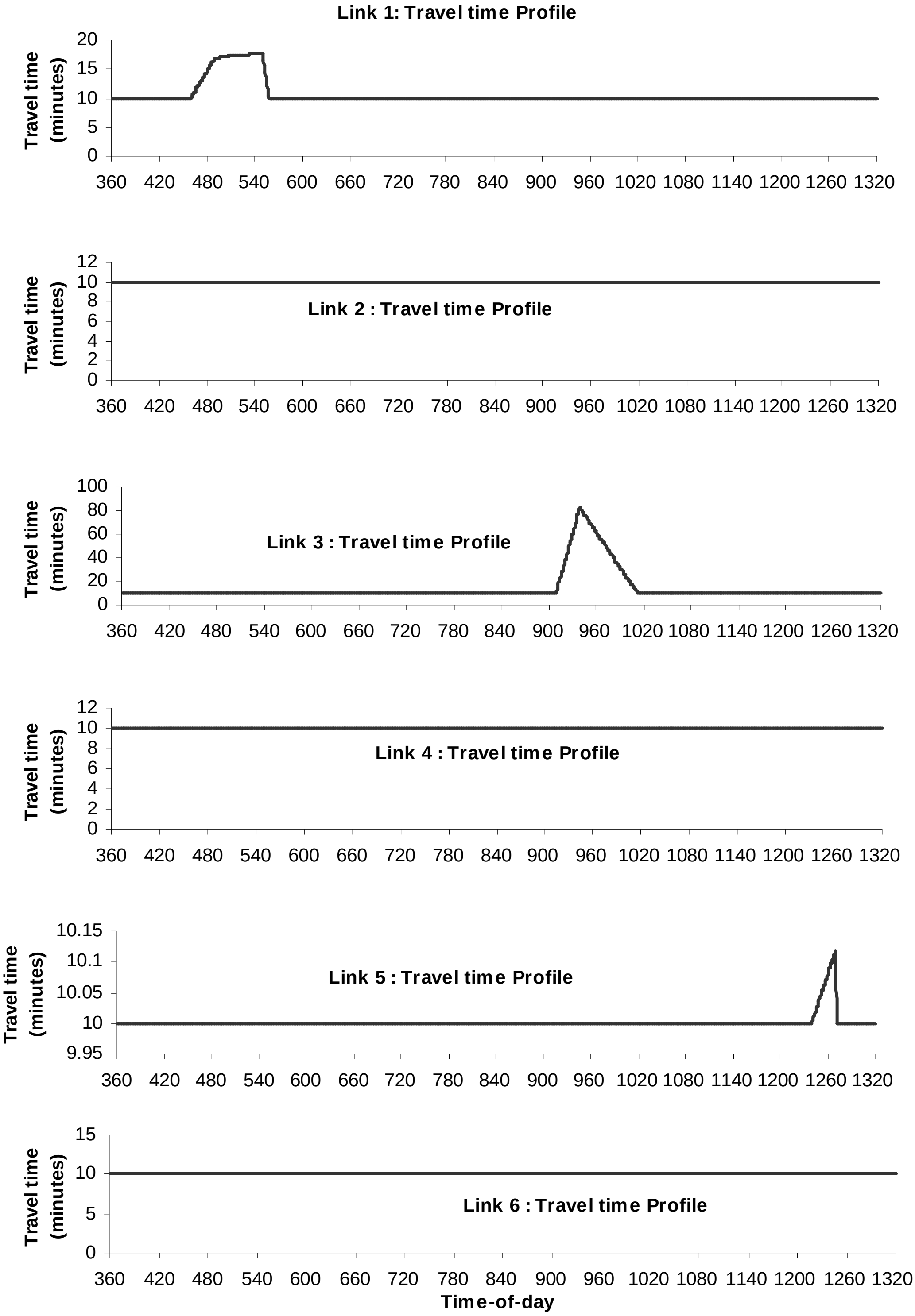


Figure 7.26: Travel time profiles for the links of the network shown in figure 7.23 for all user classes

Figure 7.25 shows the results obtained for the user class involved in performing the three-activity tour. Almost all individuals in this user class have chosen activity sequence 1 in which the work activity is performed prior to the additional activity (i.e. H-W-A-H tour, see first plot of figure 7.25). This is because this sequence offers significantly higher utility to individuals than the 2nd sequence. The reason behind the low utility of the 2nd sequence is the implication of the late arrival penalty at work location as *PST* was set as 9:00 hours. In the set up of the problem individuals are not able to depart from the additional activity location prior to 10:00 hours. Therefore, when individuals perform additional activity prior to the work activity the effect of the late arrival penalty is much more significant. This effect of the late arrival penalty is more significant than the utility an individual gains through participation in the additional activity prior to the work activity and the disutility they bear from the congestion on link 3 in the evening times when they depart from the work activity in order to perform the additional activity. It has been further noted that route 1 has been preferred by the individuals of this user class (who have chosen the 1st sequence option) over route 2. This is because route 1 requires travelling on 3 links, in which total travel disutility is less compared to route 2 which requires travelling on 4 links. However, if a route had been included in the choice set for this sequence option which does not contain link 3 (e.g. link1-link2-link6-link5), then the model predictions might have been very different altogether, as individuals then could certainly avoid travelling on link 3 and as a result of this link 3 may not appear as a highly congested link. This experiment contained limited choices of routes for each sequencing option; this has been done intentionally in order to reduce the computational costs because a higher number of alternatives will certainly lead to higher run time.

7.4.2 Numerical Experiment 2-High Congestion

The experimental setup for this experiment was the same as the setup shown in tables 7.7 and 7.8 for experiment 1 except for a change in the number of individuals performing home-work and three-activity tours. The number of individuals performing home-work tour was assumed equal to Q_1 = 5000 and the same number of individuals were considered for the three-activity tour i.e. Q_2 = 5000. This experiment was performed in order to see the changes in the model predictions when congestion on the link is higher compared to the previous case. Results are reported in figures 7.27, 7.28, 7.29 and 7.30.

Figure 7.27 shows the results obtained for the user class who was involved in performing the home-work tour. It has been noted that in terms of route choice again route 1 and 3 are more preferred among individuals than route 2 and 4. The reason is same as route 2 and 4 involves link 3 which has higher travel times in the evening and causing more disutility in return to home trip than link 2. Comparison of figures 7.24 and 7.27 suggests that increase in the number of individuals causes selection of only first departure period in the morning commute. This is because increase in the demand from 3000 to 5000 significantly increases the travel times in the morning commute either link1 or link 6 + link 4 is used for reaching at the work location from home. Furthermore, not only this user class's individuals are travelling on these links but the individuals of another user class (three-activity tour) have also used the same links in the morning times. The same trend is noted for the demand profiles of three-activity tour (sequence 1) because of the higher travel times on link 1, link 3 and link 6.

It has been noted that route 3 for home-work tour class and also route 2 of sequencing option 1 of three-activity tour class both require individuals to travel on link 4 along with link 6 in order to reach the work activity location. But surprisingly link 4 is noted as operating under free-flow condition; however, link 6 is under a heavily congested condition. This is despite the fact that almost the same number of individuals are travelling on these links, except those few individuals who have chosen route 1 in sequence option 2 as they have to participate in the additional activity after travelling on link 6. This free-flow condition on link 4 is because of the fact that this experiment utilised the Point-Queue model at the supply side. The outflow obtained from the Point-Queue model for any inflow profile is such that at a certain time interval it is either equal to capacity or less than the capacity, it will never go over capacity. When this outflow profile is loaded on the next successive link (of similar capacity) as an inflow profile, the Point-Queue model will always yield free-flow travel times because inflow is either capacity or lower than the capacity of the link. This same phenomenon has happened in this experiment: since the outflow profile from link 6 which will act as an inflow profile for link 4 is such that, for each time interval, flow is never exceeding capacity. This phenomenon may not be noted if other supply models were utilised such as linear travel time, Divided Linear travel time or Adnan-Fowkes models which are described in chapter 4. These models also yields outflow which is either equals capacity or less than the capacity, but also incorporate congestion effects for inflow lower than the capacity.

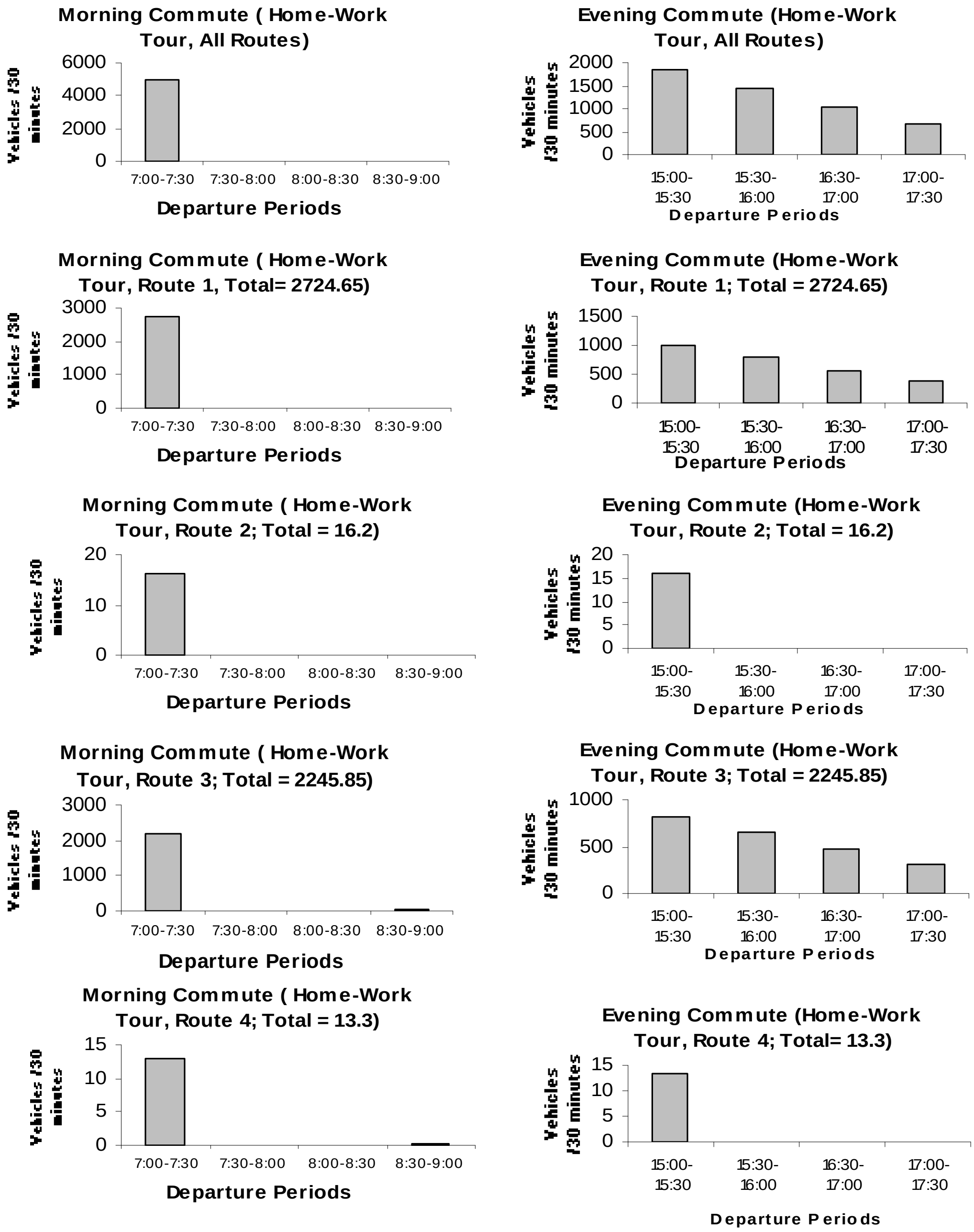


Figure 7.27: Home-Work Tour Demand Profiles with departure time and route choices for the Population Segment Performing Home-Work Tour

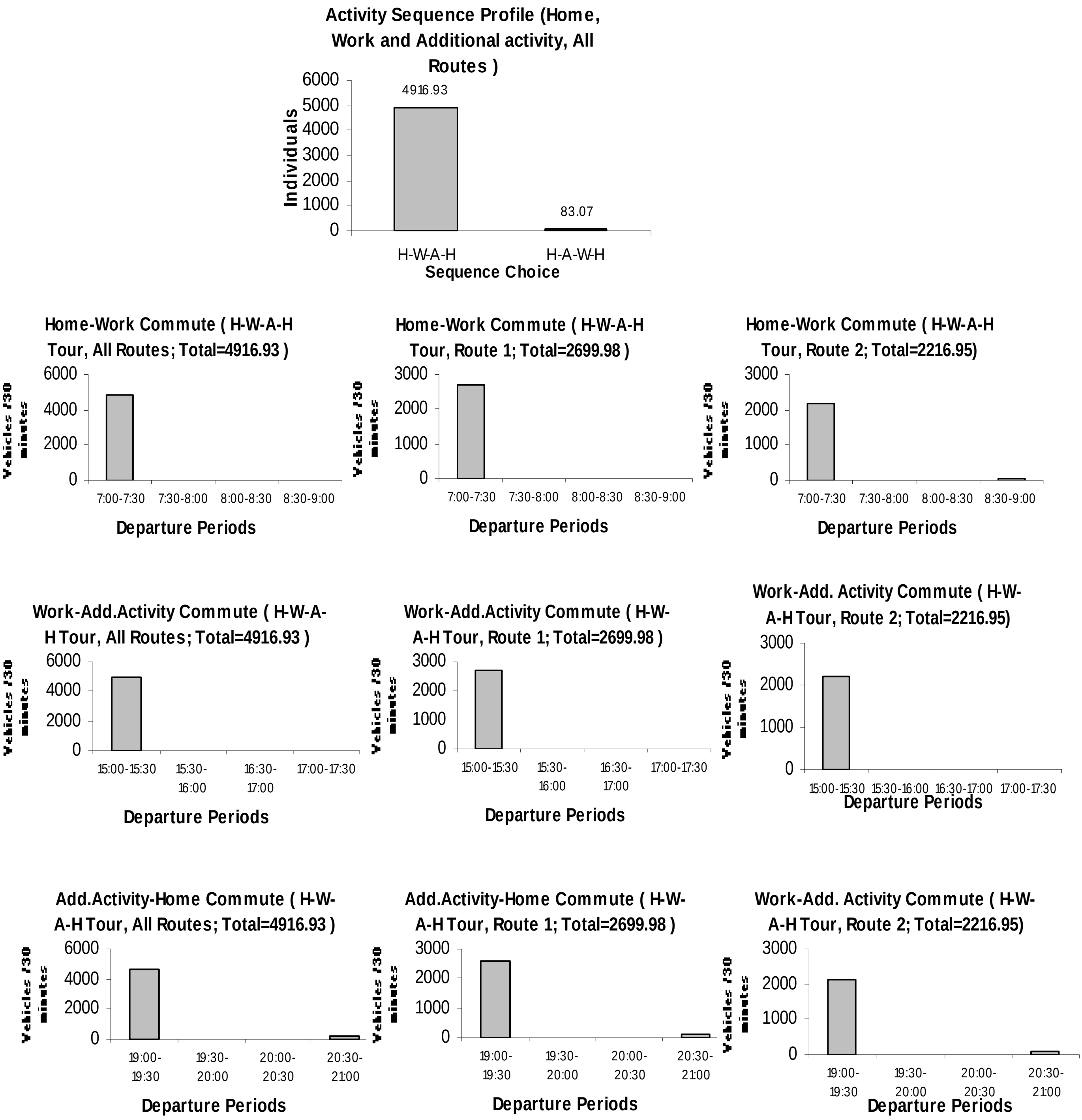


Figure 7.28: Home-Work-Add. Activity Tour Demand Profiles with departure time and route choices for Population Performing Three-Activity Tour with Sequence 1

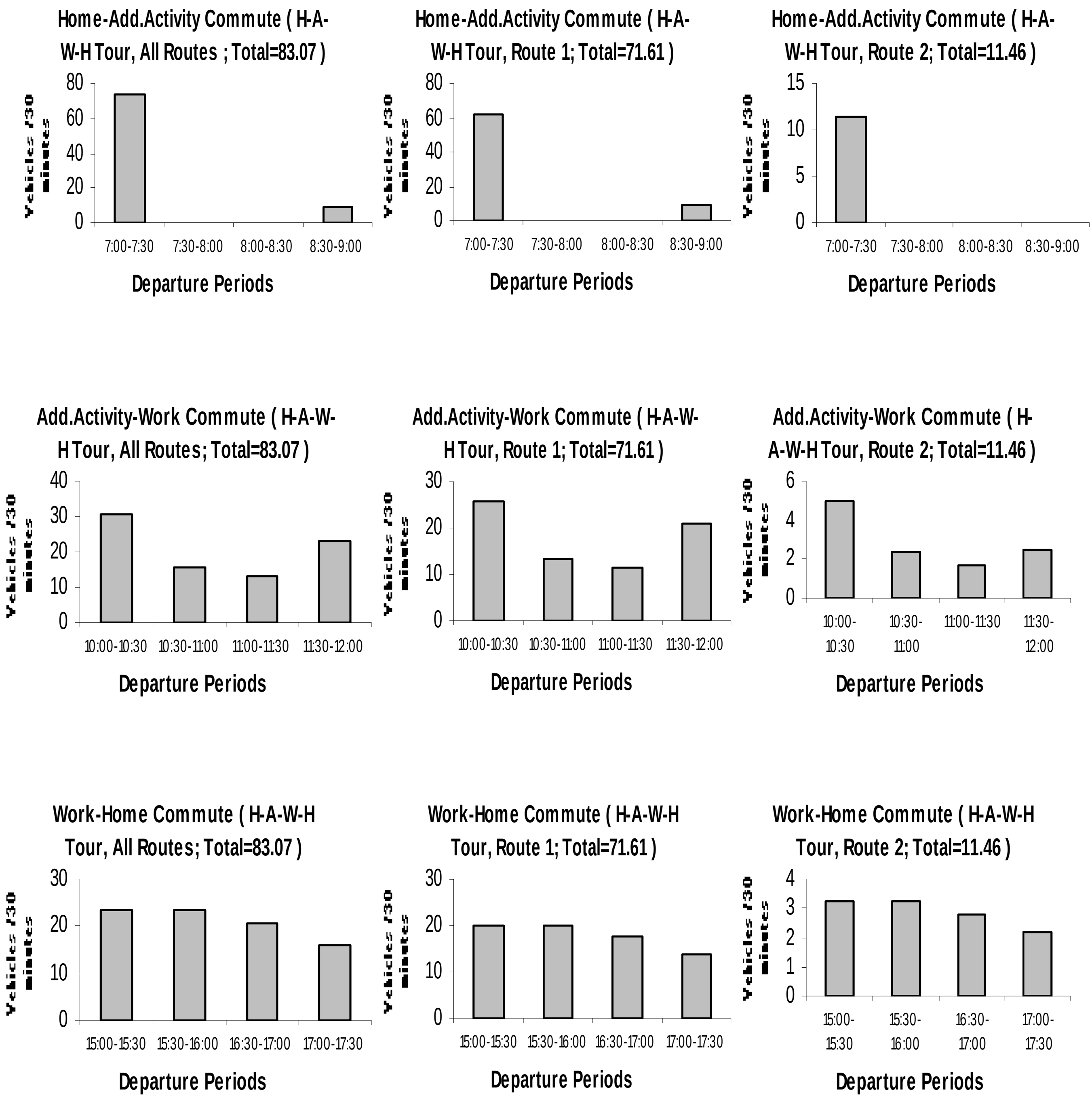


Figure 7.29: Home-Add. Activity-Work Tour Demand Profiles with departure time and route choices for Population Performing Three-Activity Tour with Sequence 2

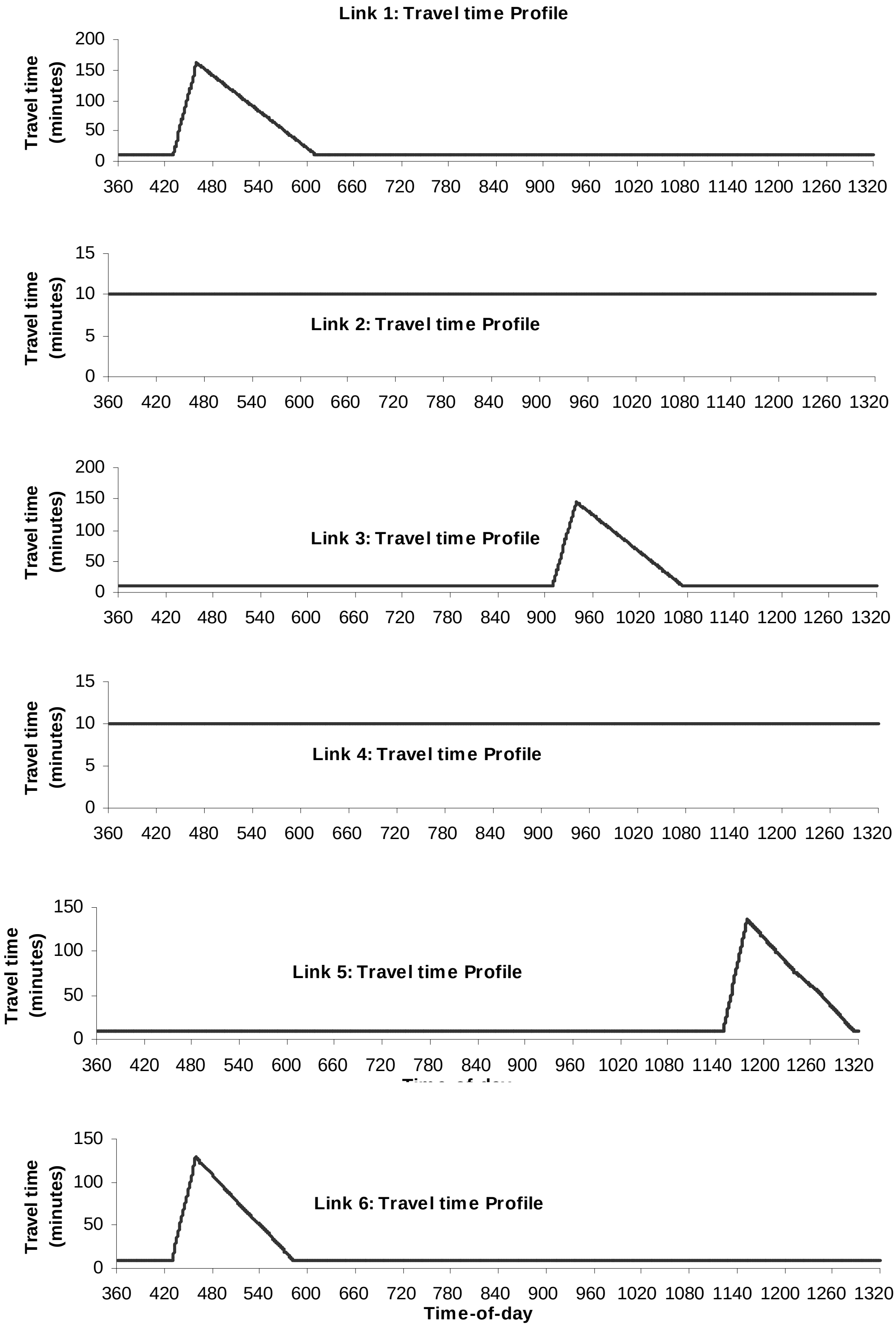


Figure 7.30: Travel time profiles for the links of the network shown in figure 7.23 for all user classes

All other results observed for this experiment are similar to those explained in sub-section 7.4.1 except for a slightly higher percentage of individuals who have chosen sequence 2 for performing their three-activity tour. This is because of the stochastic nature of the model. The results of the two experiments performed for the extended model show that the extended model is providing plausible predictions for the multiple user classes' case, along with the incorporation of scheduling dimensions such as route and activity sequence choice. Following similar notions as those shown in the development of the extended model, it can be further extended for more general networks, and not only the degrees of freedom of the incorporated scheduling dimensions can be increased but other scheduling dimensions can be incorporated, such as activity location choice and mode choice to reflect the more complex nature of daily activity-travel patterns.

7.5 SUMMARY

This chapter demonstrated comprehensive testing and assessment of the developed model through various numerical experiments. Some numerical experiments were also performed for examining the implication of certain congestion mitigation policies. It was concluded from the obtained results that the model predictions are plausible and can be explained under all circumstances that are shown above.

The results of the two numerical experiments were also illustrated comprehensively for the extended model version which not only incorporates more scheduling dimensions but also incorporates two user classes with respect to their type of tours. It was again concluded that the extended version of the model provides plausible results. It has been suggested that using the similar notions as explained under the extended model development process, the model can be further extendable for more general networks and other scheduling dimensions. The next chapter will presents the extension of the developed model for incorporation of tours on a *weekly* basis along with its examination through various numerical experiments.

Chapter 8

DEVELOPMENT OF A COMBINED MODEL FOR WEEKLY ACTIVITY SCHEDULING

8.1 GENERAL

Chapter 6 demonstrated the development of a combined model for the daily tours and in relation to this, chapter 7 demonstrated the application of the developed model through various numerical experiments. The daily activity scheduling model was developed in a sense that it considers the scheduling dimensions (such as departure time, route and sequencing choice) for each activity in the tour with a notion that scheduling of each activity in the tour is also dependent on the benefits and costs associated with other activities. The consideration of the departure time choice for each activity in the tour implicitly incorporates the fact that the duration of each activity is flexible (i.e. individuals are choosing different durations of activity by choosing different departure time combination for successive activities in the tour). It is because of the presence of this notion in the model that any systematic changes, which are required for a particular policy application (e.g. increase in the travel disutility in terms of tolls) result in a significant change in the duration of each activity in the tour. This has been observed in the results of numerical experiments shown under section 7.3.

The combined model presented in chapter 6, which incorporates daily tours (e.g home-work tour, home-work-additional activity tour), is only meant for modelling scheduling dimensions of these tours on a time frame of a single day. The flexibility notion especially regarding the duration of the work activity may seem unreasonable in this case. This is because, for a single day modelling case, the flexibility in the duration of the work activity may not render desirable results unless a constraint on the duration of the work activity is incorporated. This is necessary because the nature of the most of the jobs is such that in the end individuals need to equate a particular number of hours with the cumulative time they have spent over a week or month. For example, in some jobs individuals are required to perform 40 hours of work per week regardless of the work activity duration on a single day. This suggests that the reported model in chapter 6 and 7 is only applicable for work jobs which are based on the idea that on a single day whatever time an individual spent at the work location, he will gain utility accordingly without considering the weekly work hour requirements. However, most jobs do not have this nature, as in these jobs there is a mutual agreement between the employer and employee to work a given number of hours each week. For example, a worker in a post office is committed to stay at the work location for around 40 hours each week. If in a case due to some circumstances on a given day, he may leave early from the work, so in order to fulfil his agreement with the employer he need to compensate his early going from the work location on some other days of the week. This reflects the notion that the nature of work activity is not entirely flexible, which is in contrast to the developed model for daily activity scheduling.

This chapter presents development process for further extension of the model presented in chapter 6 by incorporating a weekly time horizon, based on the arguments presented above. The weekly activity scheduling model (presented in this chapter) not only constrains the weekly work activity duration but at the same time also presents a framework through which an individual may carry out different tours over an entire week. For example, on a particular given day an individual is carrying out home-work tour; however, on another day the same individual is involved in carrying out three-activity tour. This is useful because many empirical studies reported that majority of the individuals are involved in different activity-travel pattern over the entire week (Section 2.6 reported extract from some of these studies). Section 8.2 discusses some concepts and assumptions of the weekly activity scheduling modelling framework. Section 8.3 presents model development process, based on this; section 8.4 illustrates results of some numerical experiments. Section 8.5 discusses the way forward for further improvement in the model, followed by a concluding section.

8.2 WEEKLY ACTIVITY SCHEDULING-CONCEPTS AND ASSUMPTIONS

This section discusses several key points in order to form a basis for the development of a conceptual framework for the weekly activity scheduling model. Furthermore, some assumptions are also discussed in detail which helped in formulating a mathematical illustration of the weekly activity scheduling model. The focus of this section is based on the two major points; the first one is regarding the linking mechanism of the duration of the work activity through which a flexible duration of the work activity on a single day may become fixed on a weekly basis (*a week here is defined as workweek which contains five days i.e. from Monday to Friday*). The second point is based on the discussion of the method through which different activity-travel patterns are incorporated in an entire week for the same individual. This point also discusses the framework through which weekly patterns are modelled together under different assumptions of the similarity of the week days. The following sub-sections discuss these two issues in more detail.

8.2.1 Weekly Duration of Work Activity As a Constraint

It has been already mentioned that the developed daily activity scheduling model of chapter 6 considers the duration of involved activities in the tour as flexible and due to this fact it has been observed that the effects of any systematic changes (e.g. introduction of tolls) results in a changed durations of activities. In the context of modelling scheduling of a tour in the time horizon of a single day, the flexibility of the work activity duration may be questionable. This is because there are many jobs that do not possess the nature of fully flexible work activity duration. It is possible that for a given day work activity duration is flexible but in comparison to the entire week or month an individual has to perform a certain amount of work. This suggests that if an individual changed his time at work (e.g. increased the time at work) on a given day, then he would have to decrease his time at work on some other day and vice versa. The daily activity scheduling model cannot therefore apply to all days of the week. The weekly activity scheduling model which is presented in the later sections of this chapter is based on this background. The linking mechanism between the single day work activity duration and an entire week is based on the incorporation of a constraint which represents the fixed work activity duration on a weekly basis (e.g. 40 hours per week, including lunch break).

The total weekly work activity duration may be different for each individual and dependent on many factors such as type of job, nature of an agreement between the employer and employee and qualification and experience of the individual etc. A separate study can be devised to estimate the amount of weekly duration of the work activity for the individuals. For the sake of simplicity and the development of the model, in this chapter it is assumed that every individual is required to stay a fixed amount of hours (say *40 hours*) at the work place each week. This assumption can be relaxed if individuals tend to work a similar number of hours each week (e.g. average 40 hours but with a small spread).

8.2.2 Different Tours For Each Individual In A Week

This sub-section provides the details regarding the incorporation of different tours for each individual in a week. This is based on the argument that some individuals do not exhibit similar activity-travel pattern (tours) on all week days. Empirical studies (reported in section 2.6) which are based on weekly activity-travel pattern of individuals have presented significant evidence that there are some activities in which individuals are involved which are performed on a 3-days, 4-days and 5-days basis, even some activities are performed on a monthly basis. These findings provide enough evidence to believe that individuals carry out different tours on different days of the week. Based on this background, the weekly activity scheduling model, which is presented in the later sections, incorporates a notion that each individual is involved in two different types of tours within a week. The first tour comprises of two activities i.e. *home and work activities* and the second tour comprises of three activities i.e. *home-work and an additional activity (three-activity tour)*. There may be more than two types of tours in which individuals are involved within a week, but here for the sake of simplicity and model development purpose only two types of tours are considered. The same method and principles which is shown in this chapter can be utilised for incorporating other types of tours. So, the weekly activity pattern includes home and work activities on a daily basis and an additional activity (either shopping or leisure activity) as once in a week.

Arising from the above discussion, it is assumed that for every individual, there are four *typical days* within the week in which the commuter only follows a home-work tour and for an atypical day an individual follow a pattern in which he/she perform an additional activity along with the participation in home and work activities. On *an atypical day* of the week the duration of the work activity is already known because of the weekly work activity duration constraint. It is further assumed that all the days of the week are similar to each other. That is to say that on a given day a pre-specified proportion of individuals are performing home-work tour and the remainder are performing three-activity tour, the same proportion of individuals are performing these tours on the other four days of the week. This assumption gives an advantage that it requires to model a single day, however, if it is assumed that week days are not similar to each other then all the five days must need to be considered. The composition of commuters (individuals) for typical day and atypical day tours can be found by keeping a particular day total to 100% and atypical day tour commuters total to 100% across the five weekdays. This is because on a given day all individuals are involved in a manner that some of them are performing typical day tour and remaining are performing atypical day tour, but at the same time those individuals who are performing atypical day tour should not be involved in this tour on any other day of the week as atypical day tour should be performed only once in a week. This is illustrated in table 8.1.

Table 8.1: Composition of commuters for typical day and atypical day tours under different assumptions of weekdays similarity

Week days		Monday	Tuesday	Wednesday	Thursday	Friday	Total
All days are similar to each other	% of individuals for a Typical day tour	80%	80%	80%	80%	80%	4 x 100%
	% of individuals for an Atypical day tour	20%	20%	20%	20%	20%	100%
	Total	100%	100%	100%	100%	100%	
All days are not similar to each other	% of individuals for a Typical day tour	80%	90%	65%	70%	95%	4 x 100%
	% of individuals for an Atypical day tour	20%	10%	35%	30%	5%	100%
	Total	100%	100%	100%	100%	100%	

Modelling these two types of tours together in a week renders such a framework that all the activity scheduling dimensions considered in this research can easily be incorporated in the weekly activity scheduling model. Dimensions such as departure times, duration and routes choices can be incorporated if only home-work tour is considered but activity sequence choice can only be incorporated if three-activity tour is involved. Section 8.3 presents the mathematical illustration of the weekly activity scheduling model based on the notions described in this section.

8.3 DEVELOPMENT OF THE WEEKLY MODEL

The model presented in this section considers a week-based scheduling of daily home-work tour along with the weekly additional activity. The scheduling problem is based on the choice of departure times and route for four typical days of the week given that every individual has a car and location of home and work activity is known. For an atypical day, scheduling problem is based on the choice of departure time for every commute, route choice and sequence of performing activities. The similar network is used here for the model development as used in chapter 6 and 7 for the multiple user class experiments. The network is presented here again for ready reference.

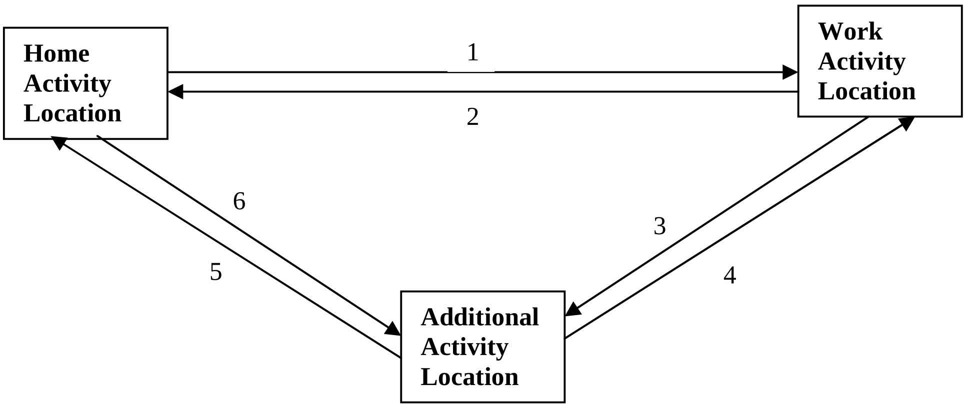


Figure 8.1: Network containing three activity centres i.e. home, work and an Additional Activity

The scheduling problem in accordance with the assumptions and the above figure can be defined as

Scheduling dimensions of the weekly tours are = (i, j, k, r^1, s, r^2)

where, i, j and k are the departure times from home, work and an additional activity locations, r^1 is the route choice for home-work tour on typical days of the week, s represents the choice of sequence for the activities and r^2 is the route choice for the tour performed on an atypical day of the week. The next sub-section describes the utility specification of the weekly activity scheduling model based on the above discussed scheduling problem.

8.3.1 Utility Specification of the weekly activity scheduling model

The total weekly utility of the daily home-work tour combined with a weekly additional activity is given by:

$$U_{(i,j,k,r^1,s,r^2)} = V_{(i,j,k,r^1,s,r^2)} + \varepsilon_{(i,j,k,r^1,s,r^2)} \tag{8.1}$$

where, $V_{(i,j,k,r^1,s,r^2)} = 4 \cdot V_{td} + V_{atd}$, representing the systematic utility (based on the assumption that all week days are similar to each other) and $\varepsilon_{(i,j,k,r^1,s,r^2)}$ represents the random term associated with each alternative. V_{td} represents a typical day utility for a simple home-work tour which would be performed by an individual throughout the four days of the week. V_{atd} represents the utility of an atypical day of the week in which individual have to involved in an additional activity. The typical day utility V_{td} is similar to equation (6.28) for the home-work tour having the choice of departure times and routes, this is given as follows:

$$V_{td} = \left(\int_0^i V'^{\,h}(t)\,dt\right) + \left(\int_0^{(\tau_{wr^1})_t} V'^{\,w}(\tau)\,d\tau\right) + g\left(i + R^{hw}_{ir^1} - PST\right) + \left(\int_{j+R^{wh}_{jr^1}}^{1440} V'^{\,h}(t)\,dt\right) + \lambda R^{hw}_{ir^1} + \lambda R^{wh}_{jr^1} \tag{8.2}$$

In order to work out the expression for an atypical day utility V_{atd} it is necessary to first work out the duration of the work activity on an atypical day. This is because the duration of the work activity for the entire week is kept constant among all the individuals, however, for typical days of the week the work activity duration is flexible. The duration of the work activity (in minutes) on a given typical day $\left(\tau_{wr^1}\right)_t$ for different routes r^1, is given by

$$\left(\tau_{wr^1}\right)_t = j - \left(i + R^{hw}_{ir^1}\right) \tag{8.3}$$

Using equation (8.3) and assuming the entire week work activity duration as *40 hours per week*, the duration of work activity (in minutes) on an atypical day $\left(\tau_{wr^1}\right)_a$ of the week is given by

$$\left(\tau_{w\,r^1}\right)_a = 2400 - 4 \cdot \left(j - \left(i + R^{hw}_{ir^1}\right)\right) \tag{8.4}$$

With the use of equation (8.4) the utility of the atypical day tour V_{atd} can be worked out but because of the incorporation of the sequence choice it can be given according to the ways in which activities on this tour can be sequenced. This is illustrated in the following equations.

when s =1, activity sequence is home-work-additional activity-home,

$$\begin{aligned} V_{atd} = & \left(\int_0^i V'^h(t)\,dt\right) + \left(\int_0^{\left(\tau_{wr^1}\right)_a} V'^w(\tau)\,d\tau\right) + g\left(i + R^{hw}_{ir^2} + \left(\tau_{wr^1}\right)_a - PST\right) \\ & + \left(\int_A^k V'^a(t)dt + \int_0^{\tau_{ar^2}} V'^a(\tau)d\tau\right) + \left(\int_{k+R^{ah}_{kr^2}}^{1440} V'^h(t)\,dt\right) \\ & + \lambda R^{hw}_{ir^2} + \lambda R^{wa}_{\left(i+R^{hw}_{ir^2}+\left(\tau_{wr^1}\right)_a\right)r^2} + \lambda R^{ah}_{kr^2} \end{aligned} \tag{8.5}$$

where, $A = i + R^{hw}_{ir^2} + \left(\tau_{wr^1}\right)_a + R^{wa}_{\left(i+R^{hw}_{ir^2}+\left(\tau_{wr^1}\right)_a\right)r^2}$ and duration of additional activity $\tau_{ar^2} = k - A$

when s = 2, activity sequence is home-activity-work-home

$$\begin{aligned} V_{atd} = & \left(\int_0^i V'^h(t)dt\right) + \left(\int_{i+R^{ha}_{ir^2}}^k V'^a(t)dt + \int_0^{\tau_{ar^2}} V'^a(\tau)d\tau\right) + \left(\int_0^{\left(\tau_{wr^1}\right)_a} V'^w(\tau)\,d\tau\right) + g\left(k + R^{aw}_{kr^2} + \left(\tau_{wr^1}\right)_a - PST\right) \\ & + \left(\int_B^{1440} V'^h(t)\,dt\right) + \lambda R^{ha}_{ir^2} + \lambda R^{aw}_{kr^2} + \lambda R^{wh}_{\left(k+R^{aw}_{kr^2}+\left(\tau_{wr^1}\right)_a\right)r^2} \end{aligned} \tag{8.6}$$

where, B = $k + R^{aw}_{kr^2} + \left(\tau_{wr^1}\right)_a + R^{wh}_{\left(k+R^{aw}_{kr^2}+\left(\tau_{wr^1}\right)_a\right)r^2}$ and duration of additional activity $\tau_{ar^2} = k - i + R^{ha}_{ir^2}$

It can be seen that the systematic utility $V_{\left(i,j,k,r^1,s,r^2\right)}$, of the weekly activity scheduling is always a function of travel times given that marginal utility functions for home, work and additional activities are known, therefore it can be written as

$$V_{\left(i,\,j,\,k,\,r^1,\,s,\,r^2\right)} = \Omega \begin{cases} \left(R^{hw}_{ir^1}, R^{wh}_{jr^1}, R^{hw}_{ir^2}, R^{wa}_{\left(i+R^{hw}_{ir^2}+\left(\tau_{wr^1}\right)_a\right)r^2}, \; R^{ah}_{kr^2} \right) & when \;\; s=1 \\ \left(R^{hw}_{ir^1}, R^{wh}_{jr^1}, R^{ha}_{ir^2}, R^{aw}_{kr^2}, R^{wh}_{\left(k+R^{aw}_{kr^2}+\left(\tau_{wr^1}\right)_a\right)r^2} \right) & when \;\; s=2 \end{cases} \tag{8.7}$$

8.3.2 Formulation of the fixed point problem

Equation (8.7) suggests that the systematic utility of the weekly activity scheduling model is dependent on the travel times on the network. These travel times can be worked out from the link-route indicator variables and link travel times at a particular time as described in sub-section 6.5.2 and equation (6.29). Travel time on a particular link of the given network at a particular time is given by the use of a particular supply model (i.e. Point-queue, Linear travel time and Adnan-Fowkes models). These models require inflow profiles (i.e. amount of vehicles that will enter on the link at a particular time) which can be worked out using equation (8.8).

Suppose that Q individuals are involved in performing this weekly tour, based on the total systematic utility of the weekly tour $V_{\left(i,j,k,r^1,s,r^2\right)}$ and the use of the MNL model (demand side operational model) provides the departure rates $q_{\left(i,j,k,r^1,s,r^2\right)}$, which is given by:

$$q_{\left(i,j,k,r^1,s,r^2\right)} = Q \cdot P_{\left(i,j,k,r^1,s,r^2\right)} = Q \cdot P_{\left(i,j,k,r^1,s,r^2\right)}\left(V_{\left(i,j,k,r^1,s,r^2\right)}\right) \tag{8.8}$$

The departure rates $q_{\left(i,j,k,r^1,s,r^2\right)}$, which are shown as a function of utility basically constitute the inflow profiles to the links through which travel times on the links are determined. As it is already assumed that all days are considered similar to each other, this gives an advantage that only a single day is required to model, however on that single day 80% of the individuals are performing their typical day tour and remaining 20% of the individuals are performing their atypical day tour. The departure rates belong to the typical day tour $q_{\left(i,j,r^1\right)}$ can be worked out as:

$$q_{(i,j,r^1)} = \sum_k \sum_s \sum_{r^2} \left(0.8 \cdot q_{(i,j,k,r^1,s,r^2)} \right) \tag{8.9}$$

The departure rates belong to the atypical day tour $q_{(i,k,s,r^2)}$ can be given as:

$$q_{(i,k,s,r^2)} = \sum_j \sum_{r^1} \left(0.2 \cdot q_{(i,j,k,r^1,s,r^2)} \right) \tag{8.10}$$

Equation (8.7) shown that the systematic utility is a function of travel times, determination of which require departure rates which is a function of systematic utility (see equation 8.8, 8.9 and 8.10). This dependence of travel times on departure rates and dependence of departure rates on travel times constitutes a fixed point problem. This can be represented as follows:

$$\hat{\mathrm{Q}}_{\text{week}} = \Psi \left(\hat{\mathrm{R}}_{\text{week}} \left(\hat{\mathrm{Q}}_{\text{week}} \right) \right) \tag{8.11}$$

where, $\hat{\mathrm{Q}}_{\text{week}}$ is a matrix containing elements $q_{(i,j,r^1)}$ and $q_{(i,k,s,r^2)}$, and $\hat{\mathrm{R}}_{\text{week}}$ is also a matrix containing elements as travel times on the network at a particular link at a particular time. The solution of the above fixed point problem (equation 8.11) represents stochastic dynamic user equilibrium for the weekly activity scheduling of a daily home-work tour along with a weekly additional activity.

8.4 NUMERICAL EXPERIMENTS-RESULTS EXPLANATION

8.4.1 Experimental Setups and Assumptions

The model framework and its mathematical illustration presented in section 8.3 are very general because it encompasses several dimensions of activity scheduling for an entire workweek. However, for presenting that generalised illustration several assumptions were made which themselves give an indication of the complexity of the problem. In this section, results of the four simplified numerical experiments are reported in order to show the workability and the application of the model. The first two experiments considered scheduling dimensions regarding typical days only and assume that atypical day scheduling dimensions are dependent on the typical day scheduling dimensions. The last two experiments considered sequence choice as well (i.e. atypical day scheduling dimension) along with the typical day scheduling dimensions.

1st setup:

The setup of the first two experiments is as follows, for scheduling of an entire workweek according to the network shown in figure 8.1, it is assumed here that individuals have only choice of departure times and routes for their home-work tour which is performed by an individual during the four typical days of the week (i.e. scheduling problem is based on i, j and r^1, and all other dimensions such as k, s and r^2 equal 1 in this case). Link 4 and link 6 of the network are assumed non-operational in these first two experiments, as this helps reduce the choice of routes r^1 for the home-work tour from 4 to 2. On an atypical day, individuals will depart from home at the same time as they are departing in typical days, then perform the work activity in order to complete their 40 hours of weekly work activity duration and then they will depart from the work activity location in order to perform an additional activity for the fixed amount of duration (i.e. one hour). This suggests that k (departure time from the work activity location to an additional activity location on an atypical day) in equation (8.5) is replaced by $\left(k = i + R_{ir^2}^{hw} + \left(\tau_{wr^1}\right)_a + R_{\left(i + R_{ir^2}^{hw} + \left(\tau_{wr^1}\right)_a\right)}^{wa} + 60\right)$. After performing an additional activity individuals will depart for home. Therefore, on an atypical day no scheduling dimension is modelled explicitly which is equivalent to say that the entire scheduling of an atypical day is dependent on the typical day scheduling dimensions. Departure times start from T=0700 hours with D = 4 and Δ=30 minutes for the morning commute (home to work trip) and the departure times for the evening commute (work to home trip) for typical day tour are start from Y=1600 hours with similar values of D and Δ. Free-flow travel time on all links is considered as 10 minutes with a link capacity equals 1800 veh/hr. The parameters for measuring utility of typical day and atypical day tour are assumed same as considered in the experiments shown in chapter 7. The second experiment was performed with the similar setup but with the consideration of tolls on the link 2.

2nd setup:

The two other experiments were conducted using a slightly different setup. A scheduling dimension which represents the choice of the sequence is also considered for an atypical day tour. In addition to this, link 6 and link 4 are also considered operational in these experiments, but route choice for typical day tour (home-work tour) is again limited to 2, first route is composed of *link 1 and link 2*, and the second route include *link 6, link 4, link 3 and link 5*. The choice of sequence is considered in such a manner that on an atypical day, individuals who have chosen 1st activity sequence (i.e. home-work-additional activity and home) will follow the route that contains link 1, link 3 and link 5. The individuals with 2nd activity sequence (i.e. home-additional activity-work-home) will follow the route that includes link 6, link 4 and link 2. The duration of additional activity on atypical day tour is again considered fixed here in these experiments as well for the amount of 1 hour. The second experiment with this setup incorporates dynamic tolls on link 1 of the network. All other assumptions were considered similar to the experiments with the 1st setup discussed earlier.

8.4.2 Discussion on Results

Experiments under 1st setup:

The results related to the 1st setup are discussed in this sub-section. The first experiment under this setup was performed without the consideration of tolls on any link of the network. The results of this experiment are reported in figure 8.2, 8.3 and 8.4.

Figure 8.2 represents the demand profile based on the departure times and available routes for individuals performing a typical day tour (home-work tour) on a given day. Link 1 is common in the two routes (i.e. Route 1: link1-link2, and Route 2: link1-link3-link5) which are available to individuals who are performing home-work tour, so in the morning commute all the individuals have to travel on link 1 in order to reach at the work activity location. Individuals who are performing their typical day routine are selecting route 1 because this route is the direct route and even after the loading of the most of the demand on this route, the travel time at all times of the day on link 2 are lower than total free-flow travel times on link 3 and 5. It should be worth noting that those individuals who are performing their atypical day tour on a given day are also travelling on link 3 and link 5, but these individuals are just 600 in total, which is far below the capacity of these links. This is the reason why on link 3 and link 5 free-flow travel condition is prevailing at all times of the day (see figure 8.3).

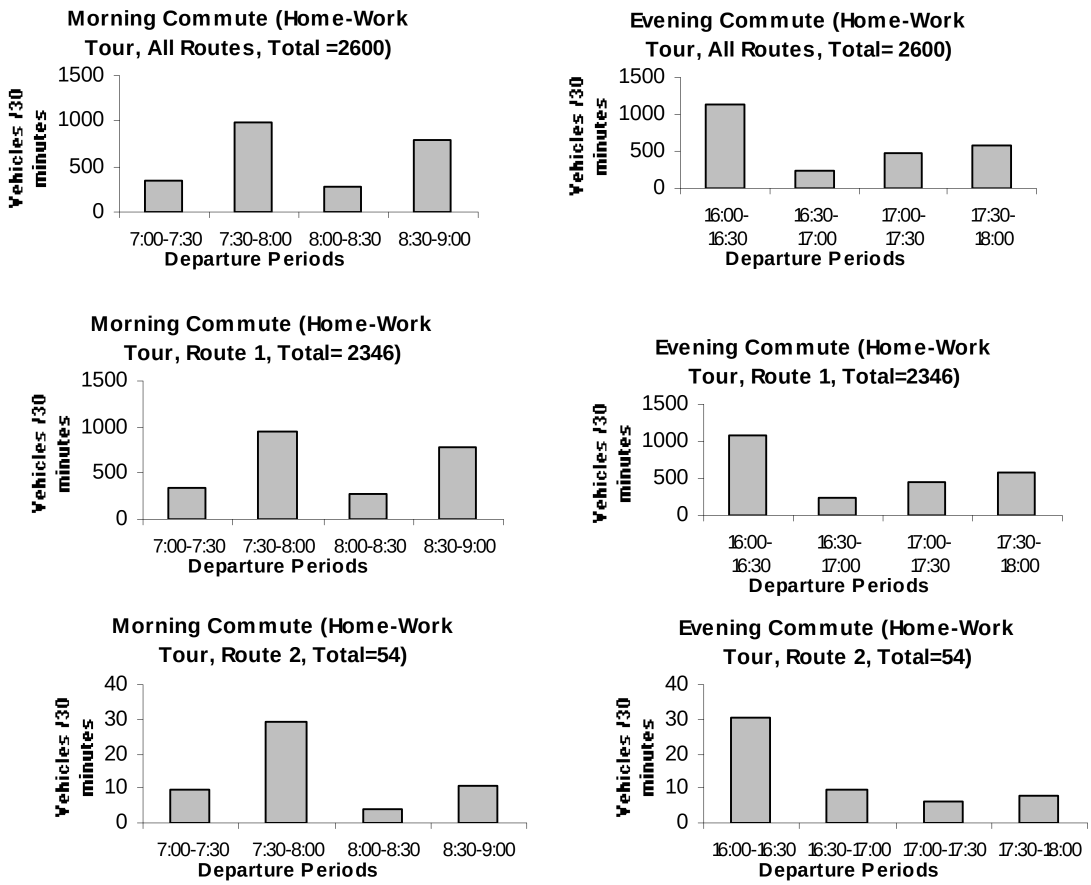


Figure 8.2: Demand profiles based on departure times and routes for individuals performing typical day tour on a given day

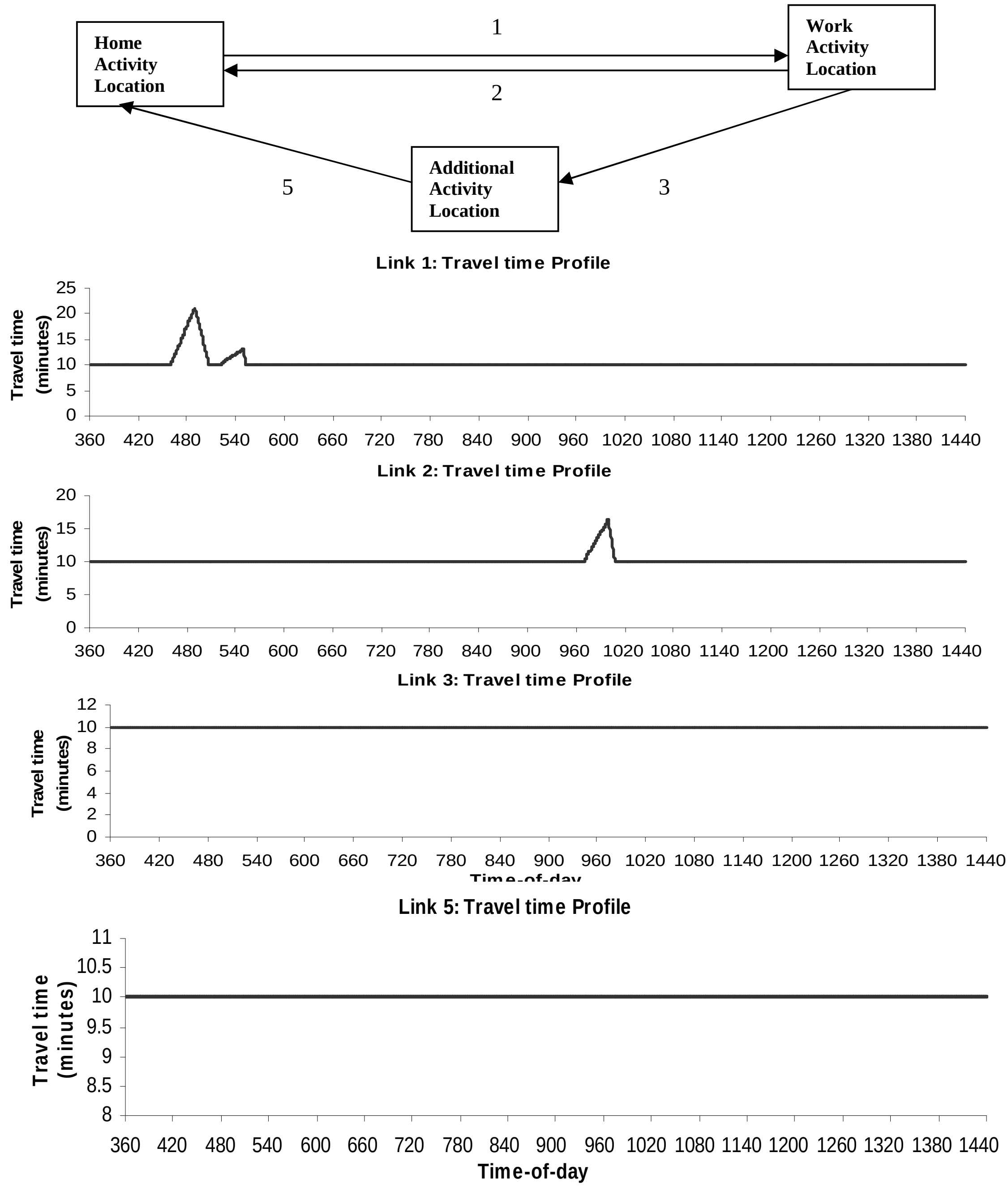


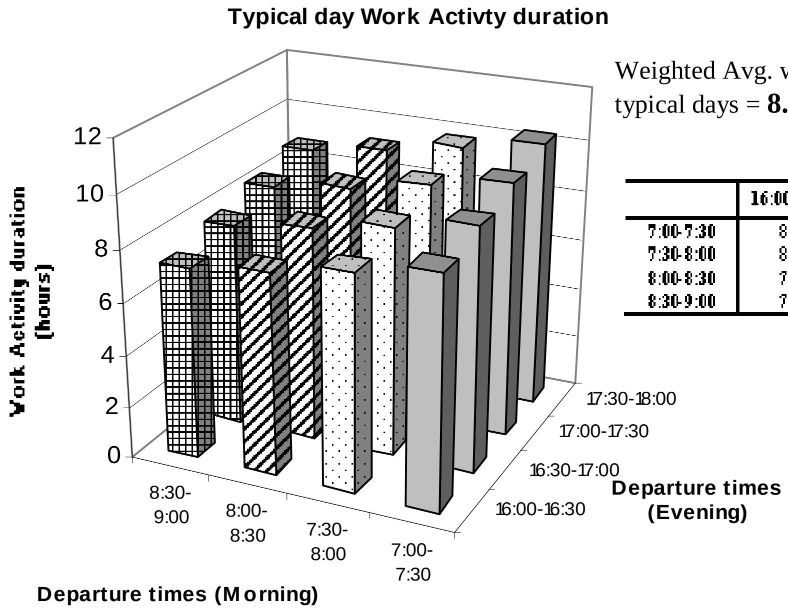
Figure 8.3: Considered Network and Travel time profiles on each link for individuals performing typical day and atypical day tour on a given day

The shape of the morning commute and evening commute demand profiles can be explained with the help of travel time profiles of link 1 and link 2 presented in figure 8.3.

The higher demand in the departure periods 7:30-8:00 and 16:00-16:30 is because of the fact that this combination of departure periods (i.e. an alternative) represents the highest utility alternative. The highest utility of this alternative is partly because of lowest disutility of travel in these times and getting the maximum advantage of the duration based work activity utility as longer work activity duration will not render as much utility as an individual loses from the time-of-day based home activity utility.

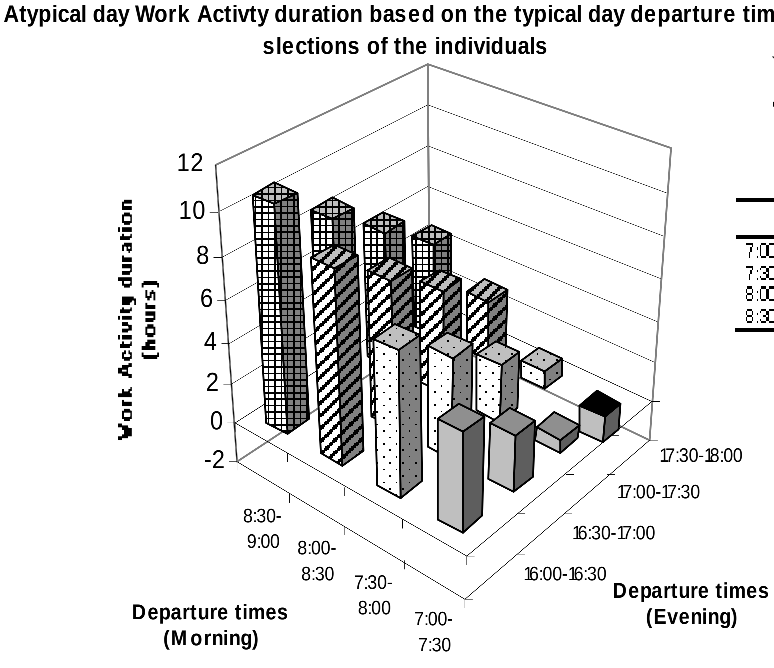
Figure 8.4 indicates that if individuals have chosen later departure periods in the morning commute and earlier departure periods in the evening commute then on typical days of the week their work activity duration is around 7.3 to 8 hours, and in order to complete 40 hours of work activity duration in a week then on an atypical day an individual need to stay at work activity location for much longer period of time. In doing so an individual loses much of his home activity utility on an atypical day because on an atypical day an individual also need to stay at an additional activity location for an hour after performing work activity. In the similar manner if individuals have chosen earlier departure period in the morning and later departure periods in the evening then on typical days of the week their work activity duration is around 10 hours, which results in no obligation towards an individual to perform work activity on an atypical day. However, in doing so an individual loses much of his home activity utility on typical days (because of staying longer at work place). Both these circumstances are infeasible for an individual, so alternatives which are providing work activity duration between 8 to 9 hours on a typical day are most attractive among individuals.

The shape of the work activity duration profiles as shown in figure 8.3 indicating the fact that travel time on the link 1 is playing a major role. The higher travel times in the morning commute results in the lower duration of the work activity in the later departure periods, thus causing higher duration of work activity on an atypical day. So, in order to understand the results it is required that all the three figures i.e. 8.2, 8.3 and 8.4 should be analysed together.



Weighted Avg. work activity duration on typical days = **8.59 hours**

	16:00-16:30	16:30-17:00	17:00-17:30	17:30-18:00
7:00-7:30	8.83	9.33	9.83	10.33
7:30-8:00	8.29	8.79	9.29	9.79
8:00-8:30	7.73	8.23	8.73	9.23
8:30-9:00	7.32	7.82	8.32	8.82



Weighted Avg. work activity duration on an atypical day = **5.64 hours**

	16:00-16:30	16:30-17:00	17:00-17:30	17:30-18:00
7:00-7:30	4.67	2.67	0.67	-1.33
7:30-8:00	6.84	4.84	2.84	0.84
8:00-8:30	9.08	7.08	5.08	3.08
8:30-9:00	10.72	8.72	6.72	4.72

Figure 8.4: Work Activity duration on typical and atypical day of the week

The second experiment within the 1st setup assumed that tolls are introduced on link 2 of the network shown in figure 8.3. The tolling strategy was based on the demand profile obtained for link 2 in the without toll case (i.e. demand profile for route 1 of typical day routine), as higher demand departure periods have higher levels of toll. The results for this experiment are reported through figure 8.5 and 8.6.

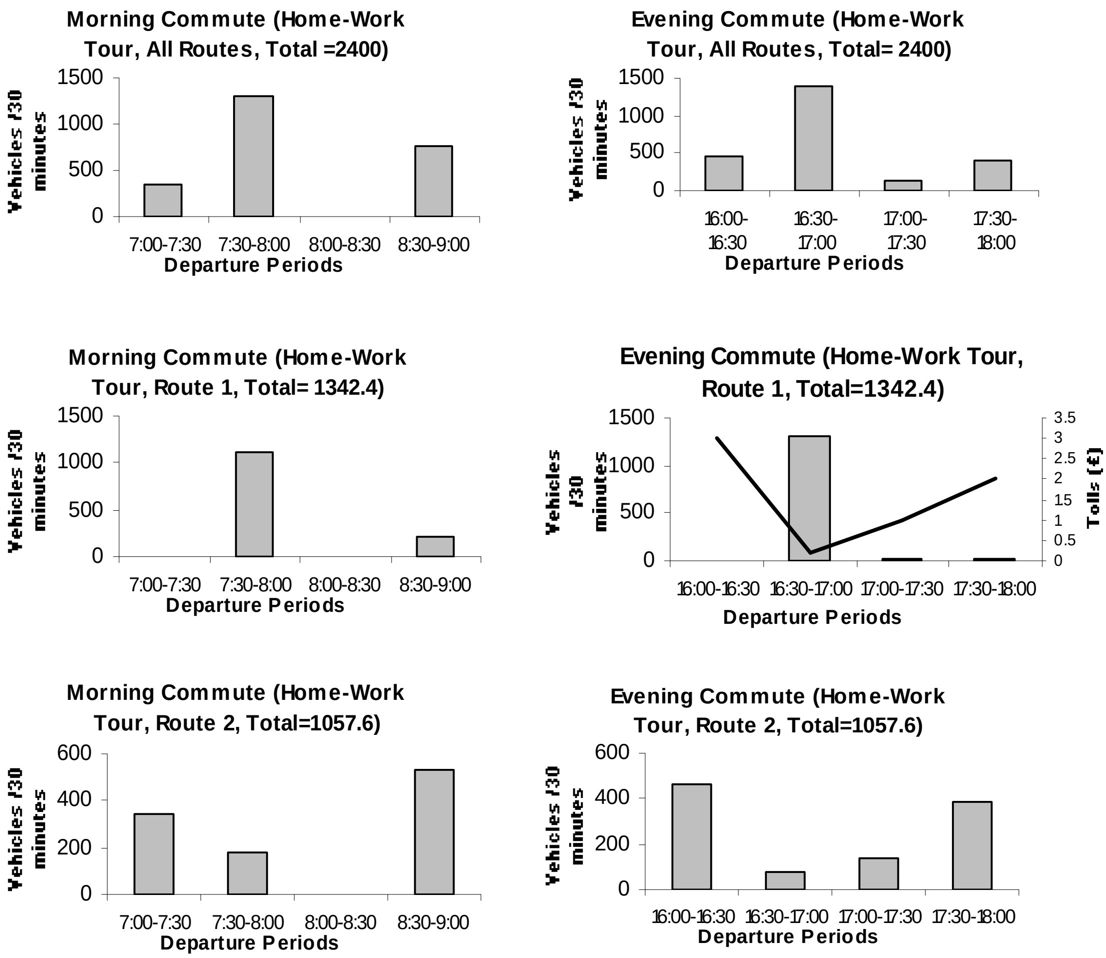


Figure 8.5: Demand profiles for individuals performing typical day tour on a given day when tolls introduced on link 2

Figure 8.5 shows that due to the introduction of toll on link 2, which is the part of route 1 (Route 1: Link 1- Link 2), a considerable amount of individuals have moved to route 2 (Route 2: Link 1- Link 3- Link 5). However, in the morning commute all of the individuals need to travel on link 1 but in the evening commute those who have switched to route 2 have used link 3 and link 5 to reach home after work. The evening commute demand profile for individuals who are using route 1, has been changed considerably in comparison with without toll case, as all the individuals in route 1 are now using departure period 16:30-17:00 (which is the departure period where no toll was considered, see 4th plot in figure 8.5). For route 2, in the morning commute most of the individuals have chosen last departure period, this is partly due to the avoidance of congested departure period (i.e. 8:00-8:30) and partly due to gain some more benefits from the home activity. For route 2, evening commute demand profile is very similar to what have been observed in without toll case. Travel time profiles for each link are shown in figure 8.6.

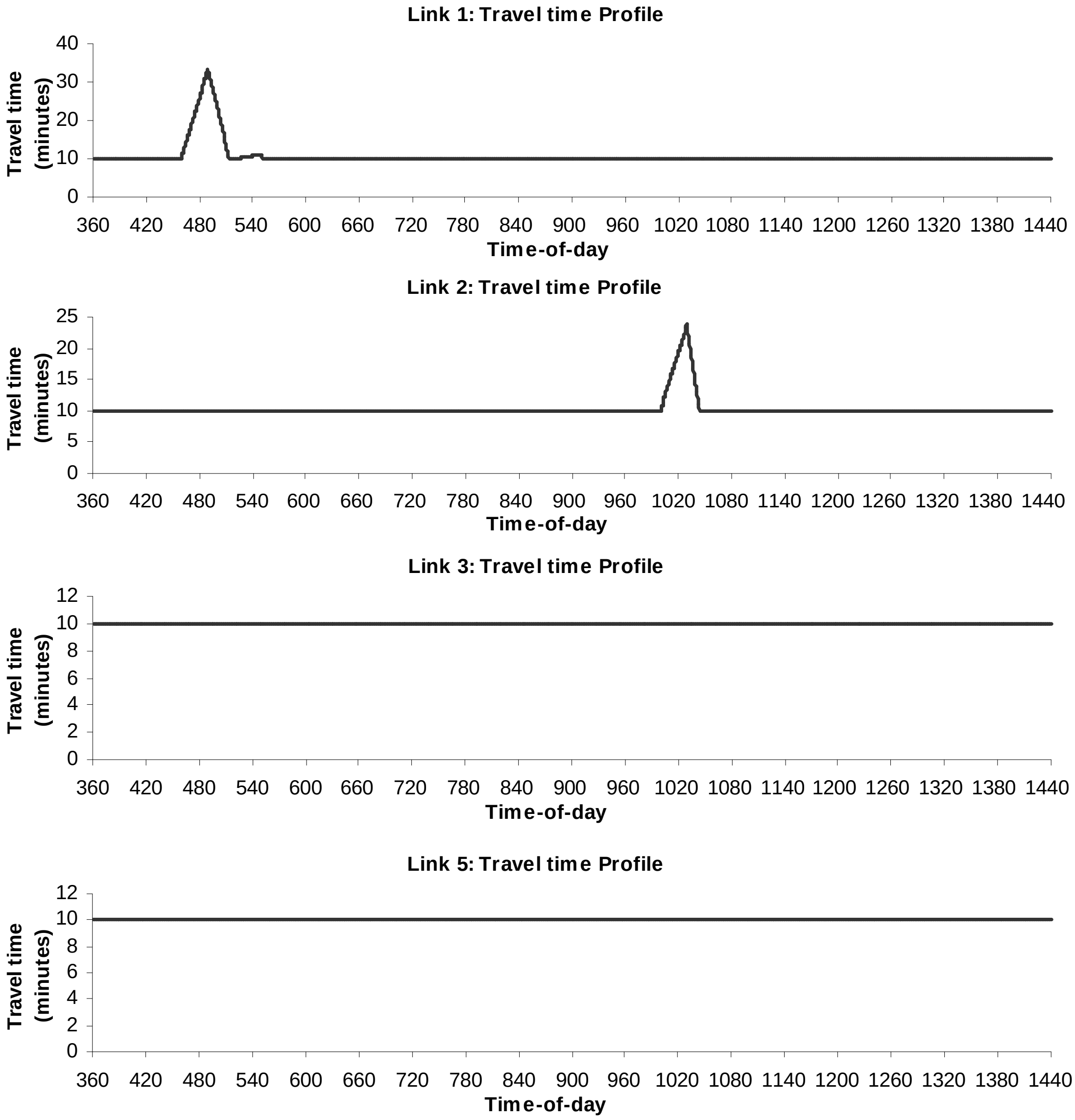


Figure 8.6: Travel time profiles on each link for individuals performing typical day and atypical day tour on a given day when tolls are introduced on link 2

Link 3 and link 5 are still under free-flow travel condition even considerable amount of individuals are now using route 2. This is because of the fact that this considerable amount of individuals is distributed in such a manner that demand in a particular departure period is well below the capacity of these links. In this experiment it has been noticed that weighted duration of work activity for typical and atypical day has slightly changed from the previous experiment. As noted in experiments in chapter 7, introduction of tolls caused changed duration of activities in the tour. Here as well, weighted average typical day work activity duration is increased from *8.59 to 8.64 hours* and in relation to this weighted average atypical day work activity duration has been decreased from *5.64 to 5.44 hours.* This suggest that the increase in the disutility of travel causes the change in the duration of work activity on typical days (as duration of work activity on a given day is flexible) but as a consequence of this duration of work activity on an atypical day is also effected. This is the notion on which weekly activity scheduling model was developed.

Table 8.2 presents the summary of the socio-economic benefits evaluated using logsum term for the tolling strategy assumed in this experiment. It has been revealed from the table that the consumer surplus is decreased with the introduction of tolls, which is expected. However, the revenue generated from the tolls is not significant in order to provide positive benefits. This clearly suggests that tolling strategy assumed in this experiment is not viable in terms of overall benefits. The significant decrease in the consumer surplus is primarily due to the manner in which dynamic tolls are assumed on the link 2 which causes all the demand to squeeze into the second departure period of the evening commute of route 1 (see figure 8.5). This not only causes an increase in travel times on links 1 and 2 in comparison with the without tolls scenario, but also renders significantly lower revenue. The next sub-section represents the results of the experiments in which sequence choice is also considered with all other scheduling dimensions.

Table 8.2: Summary of benefits from the tolling strategy

Tolling Strategy	Consumer surplus (logsum) in £ A	Total consumer surplus in £ B	Change in total consumer surplus in £ w.r.t base case ΔTCS	Total Generated Revenue from Tolls in £ R = $\sum (link\,2demand)_i.(link\,2\,tolls)_i$ R	Benefits in £ ΔW= ΔTCS+R ΔW
Without tolls (base case)	532.095	1596285	-----	0	----
Tolls on link 2	527.368	1528104	-14181	305.1032	-13875.89

$* A = \frac{1}{\omega}[\log\ sum]$, $**B = Q \cdot A$,

Experiments under 2nd setup:

The first experiment under the 2nd setup was performed without the consideration of tolls and choice dimension considered are as follows; departure time, activity durations and route choice for typical day routine and choice of sequencing of activities for atypical day routine. The results are reported in figures 8.7, 8.8, 8.9 and 8.10.

Figure 8.7 shows demand profiles based on the departure times and routes available to individuals for performing home-work tour (i.e. typical day tour). As already mentioned, Route 1 in this experiment is composed of link 1 and link 2 and Route 2 contains link 6, link 4, link 3 and link 5. This suggests that if a free-flow condition prevails on all links of the network (as all links are assumed to have similar properties) then individuals choose only route 1 for their travelling between home and work activity location. Figure 8.7 confirms this, as from 2400 individuals around 2386 individuals have chosen route 1 for their typical day routine. This can be explained very easily in conjunction with travel time profiles for each link of the network shown in figure 8.8, as link 1 and link 2 are moderately congested but travel times on these links at those times where demands are higher for route 1 are always lower in comparison with the free-flow travel times on link 6, link 3, link 4 and link 5.

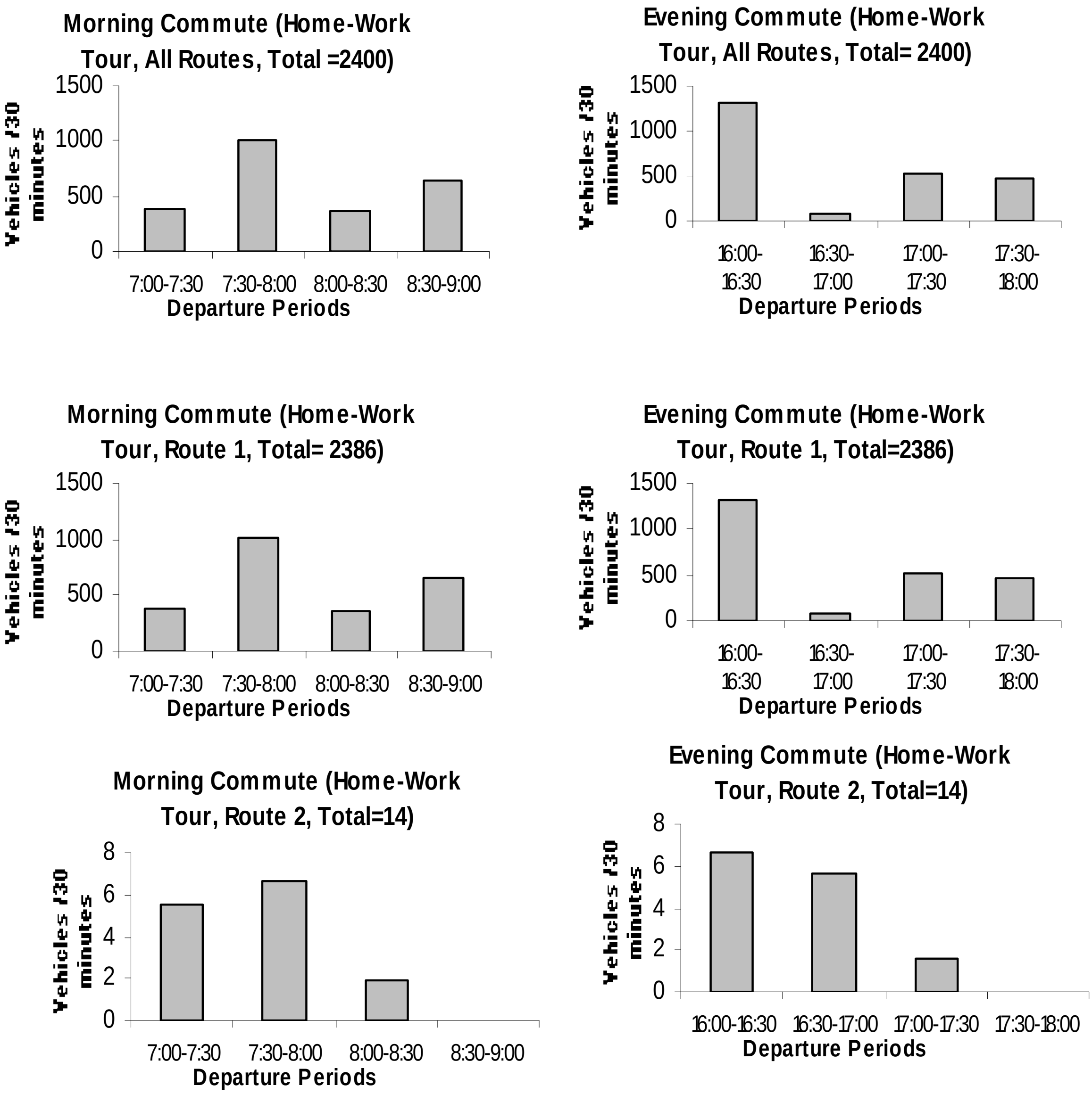


Figure 8.7: Demand Profiles based on departure times and routes for individuals performing typical day tour

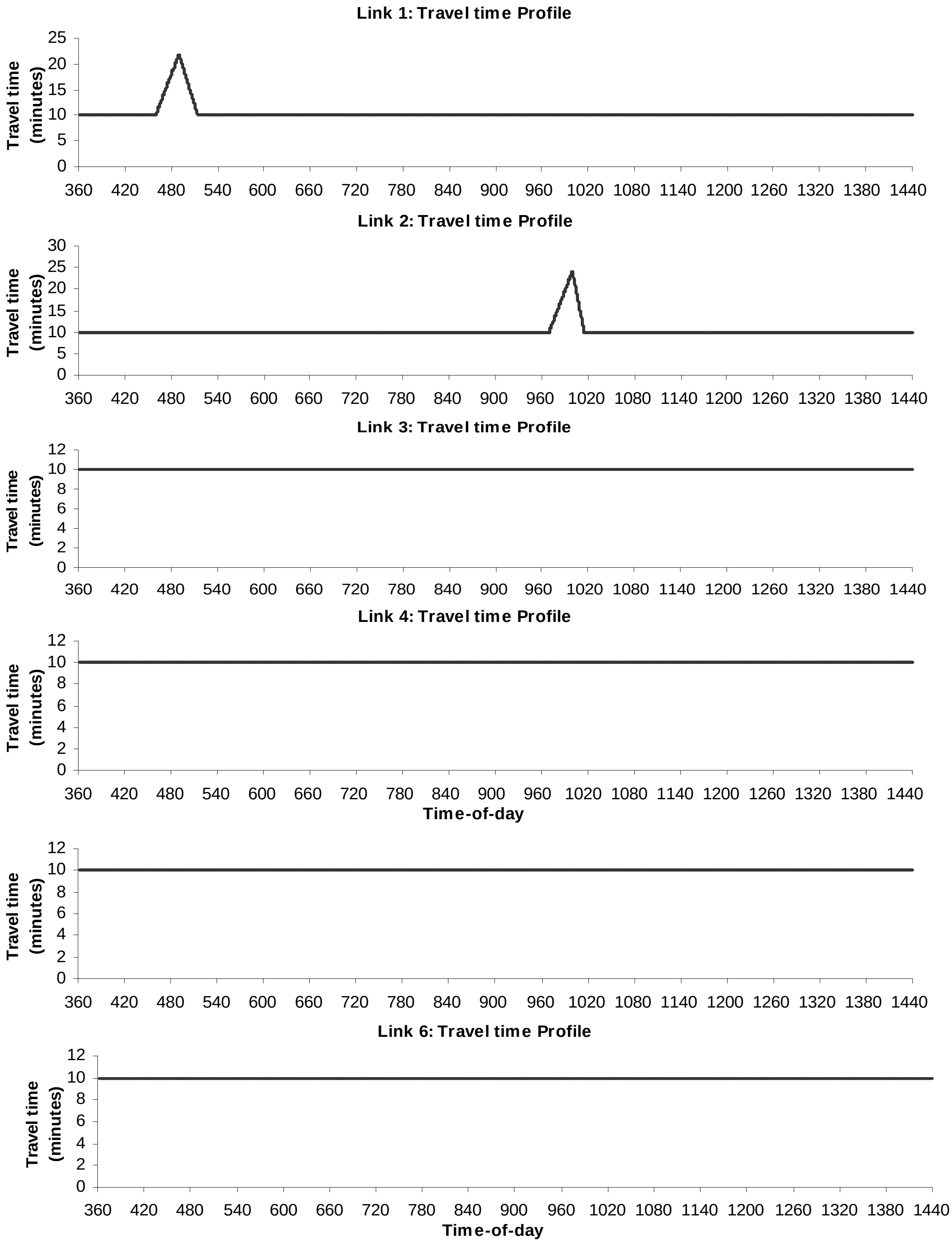
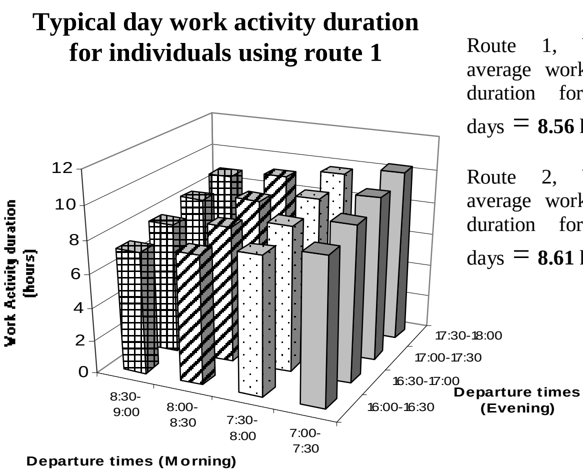
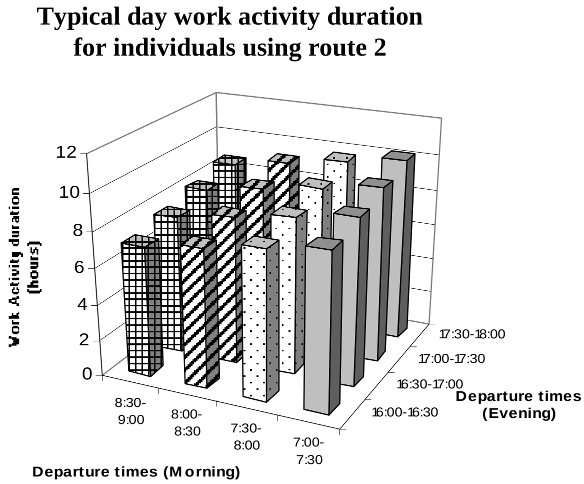


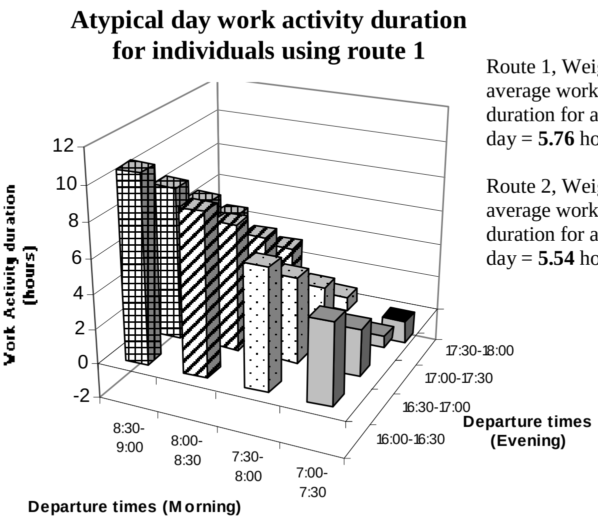
Figure 8.8: Travel Time profiles on each link of the considered network



Route 1, Weighted average work activity duration for typical days = **8.56** hours

Route 2, Weighted average work activity duration for typical days = **8.61** hours





Route 1, Weighted average work activity duration for atypical day = **5.76** hours

Route 2, Weighted average work activity duration for atypical day = **5.54** hours

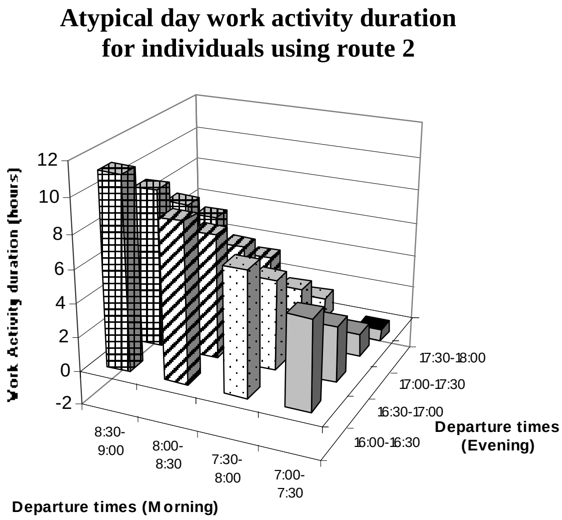


Figure 8.9: Work activity duration profiles on Typical and atypical day of the week

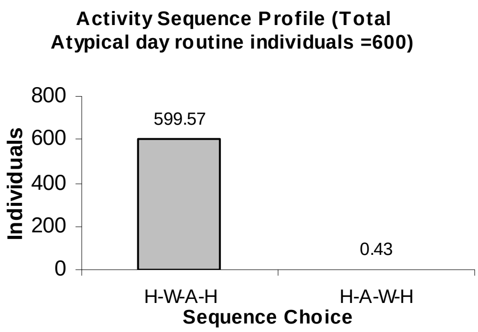


Figure 8.10: Demand profile based on activity sequence for individuals performing atypical day tour

Figure 8.7, further reveals that the alternative which combines departure period (i.e. 7:30-8:00) in the morning and departure period (i.e. 16:00 -16:30) in the evening commute is the highest utility alternative and that is why demands in these periods are higher (see 3rd and 4th plot in figure 8.7). This is partly due to the lower disutility of travel time in these departure periods (can be seen from figure 8.8) and partly due to obtain reasonable amount of work activity duration on typical days of the week (can be seen from figure 8.9). This is because of the use of duration based marginal utility function for the work activity, as staying longer at the work place rendering some utility but that utility is not as much as an individual loses by not participating in the home activity. Furthermore, extreme longer and lesser durations of work activity on typical days are infeasible for an individual because as a consequence of this individual need to stay for respective shorter and longer amount of durations at work place on an atypical day of the week. This is illustrated in figure 8.9.

Figure 8.10 reveals that individuals who are performing their atypical day routine are choosing the first activity sequence (i.e. home-work-additional activity-home). This is not because of the late arrival penalty based time-of-day component of the work activity utility as on an atypical day work activity start time is assumed flexible in these experiment (i.e. no late penalty). The fact that individuals are choosing 1st sequence is because of the definition of the time-of-day based marginal utility function for an additional activity, which is defined in such a manner that it provides higher utility in later part of the day. This assumption is reasonable in a sense that if an additional activity is assumed as a shopping activity (i.e. buying groceries etc) then it would be infeasible for an individual to carry the bought stuff with him to perform work activity in the case of 2nd activity sequence. Furthermore, individuals are also bounded with the same departure periods choices for leaving from home on an atypical day as they have for typical days of the week. This is again in favour of choosing 1st activity sequence.

Figures 8.11, 8.12 and 8.13 presents the results of the experiment under 2nd setup when tolls are introduced on link 1 of the network shown in figure 8.1. The tolling strategy is based on the demand profile obtained for link 1 in without toll case experiment. The tolls are assumed in such a manner that higher demand departure periods (in the case of without toll experiment) are considered with higher levels of toll. The profile of the toll strategy based on the departure periods is shown in 3rd plot of figure 8.11.

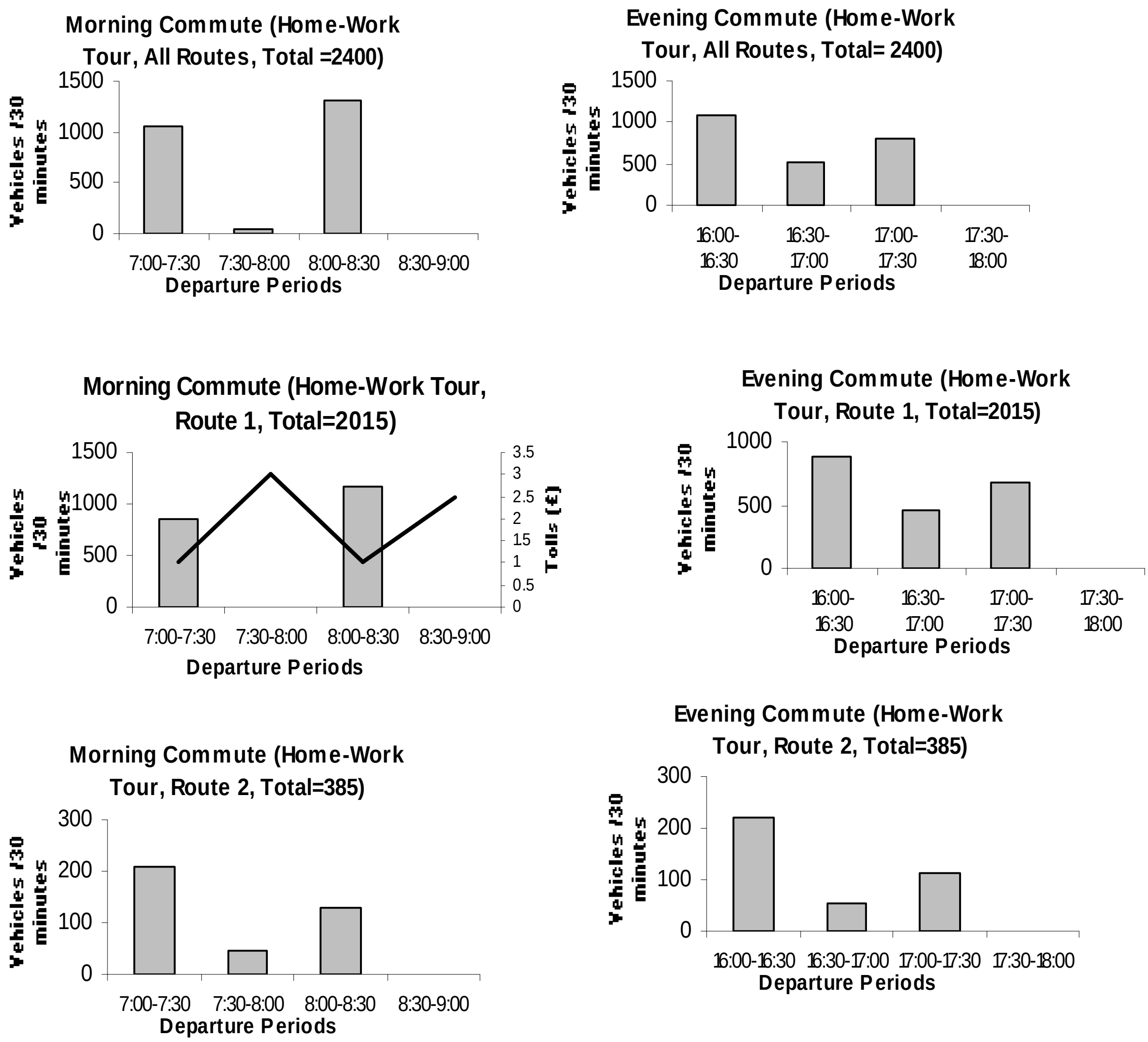


Figure 8.11: Demand Profiles based on departure times and routes for individuals performing typical day tour when tolls introduced on link 1

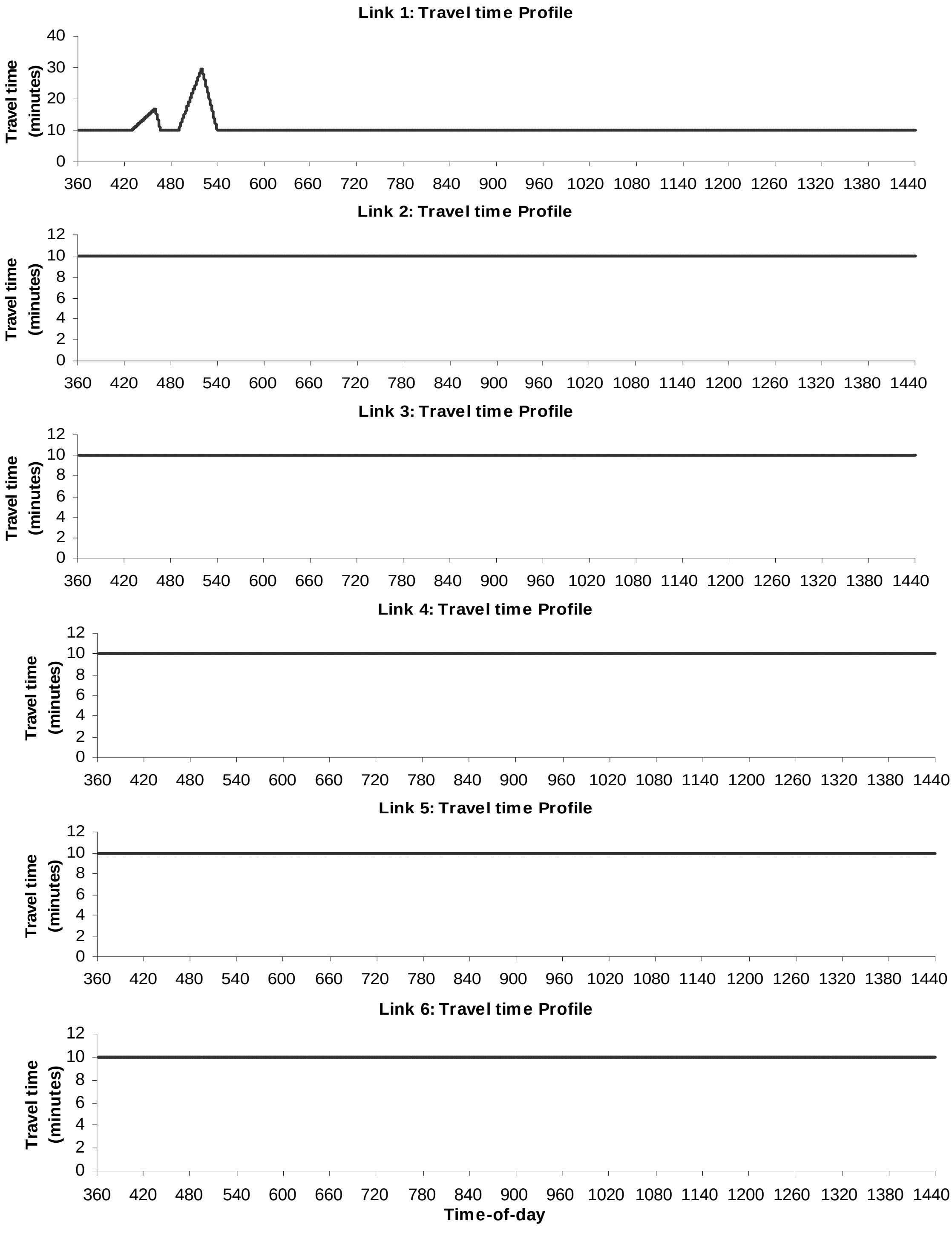


Figure 8.12: Travel Time profiles on each link of the considered network when tolls introduced on link 1

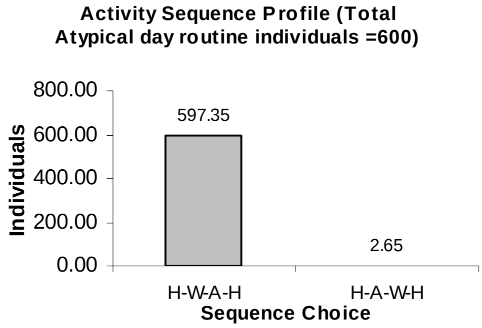


Figure 8.13: Demand profile based on activity sequence for individuals performing atypical day tour when tolls introduced on link 1

Figure 8.11 represents that due to the introduction of tolls on link 1, considerable amount of individuals have changed their route as around 2015 individuals are now using route 1 (Route 1: link 1-link2) and around 385 individuals are performing their typical day routine using route 2 (Route 2: link 6-Link 4-Link3-Link 5). Additionally, tolls on the link 1 have changed the choice of departure times of those individuals who are still using route 1. Most of the individuals are now using departure period 1 and 3 as in these departure periods level of toll is lower. Figure 8.12 reveals that except link 1 all other links of the network are operated under free-flow condition. Furthermore two peaks are noted in the travel time profile of link 1. The morning commute demand profile for route 1 suggests that demand is lower than the capacity in the first period, but travel time profile of link 1 suggests that there is some congestion at earlier times as well. The earlier small peak is because of combined effect of the individuals who are performing their typical day routine and individuals who are performing their atypical day routine through sequence 1 (see figure 8.13). The combination of both these category of individuals result in a demand at 1st departure period which is greater than the capacity of link 1.

Introduction of tolls on link 1 significantly changed the evening commute profile on route 1 in comparison to the without toll case. First of all demand is distributed over the departure period in such a manner that it is always under capacity, this is the reason why the travel time profile of link 2 shows free-flow travel condition. Furthermore, later departure periods of the evening commute are also sharing some demand which is unlike without toll case. This is partly due to the free-flow travel condition on link 2 at all times of the day, and partly because much of the demand has been shifted in the 3rd departure period of the morning commute, so in order to obtain reasonable amount of work activity duration on typical days of the week individual need to stay slightly longer at the work place in the evening. As a result of this duration based marginal utility curve for work activity has now moved to later times, and therefore individuals are getting much more utility then they could get by being at home in these times.

In this experiment as well, due to tolls the average work activity duration on typical days of the week has been increased from *8.56* and *8.61* to *8.65* and *8.63* for route 1 and route 2 respectively. Higher change in the duration of work activity for route 1 is due to the change in the travel time of link 1 as tolls are basically introduced on link 1. As a consequence of this on an atypical day the average duration of work activity is decreased from *5.76* and *5.54* to *5.40* and *5.47* for route 1 and route 2 respectively. This indicates that the increase in the disutility of travel causes the change in the duration of work activity on typical days (as duration of work activity on a given day is flexible) but as a consequence of this duration of work activity on an atypical day is also affected.

Table 8.3: Summary of benefits from the tolling strategy

Tolling Strategy	**Consumer surplus (logsum) in £**	**Total consumer surplus in £**	**Change in total consumer surplus in £ w.r.t. base case**	**Total Generated Revenue from Tolls in £ C =** $\sum (link1demand)_i \cdot (link1tolls)_i$	**Benefits in £ ΔW=ΔTCS+R**
	A	**B**	**ΔTCS**	**R**	**ΔW**
Without tolls (base case)	530.907	1592721	----	0	----
Tolls on link 1	525.881	1577643	-15078	2014.895	-13063.105

The summary of the socio-economic benefits for this experiment with the assumed tolling strategy are shown in table 8.3. The table reveals that the assumed tolling strategy is not rendering any positive benefits; therefore, the application of this tolling strategy is not viable. The significant decrease in the consumer surplus is due to an increase in the travel times on link 1 compared to no tolls case. It is interesting to state that in this experiment sequence choice is incorporated (alternatives are increased), but in comparison to table 8.2 the logsum value obtained for without tolls case in this experiment is lower, this is due to the fact that other alternatives (alternatives other than sequence choice) are not similar in this experiment in comparison to the experiment results shown in table 8.2. So, both experiments entirely represent different scenarios and cannot be compared together as just an increase in the number of alternatives.

Main Findings:

- The weekly activity scheduling model reported in this chapter is behaving plausibly and the results obtained from different experiments can be explained.

- The obtained results are effectively reflecting the notion incorporated regarding the fixed duration of work activity on a weekly time horizon.

- The significance of the choice of sequence cannot be well appreciated with the obtained results because of the assumptions regarding the time-of-day based marginal utility of an additional activity and the setup of the numerical experiment, however, if an additional activity is defined as an activity which is related to bank visit or some other activity which need to be performed in the middle of the day, then the obtained result would be different. Therefore, sequence choice will be more important in the case where there are two or three additional activities of different nature are included in the tour.

8.5 RELAXING WEEKLY MODEL ASSUMPTIONS -DISCUSSION

8.5.1 Incorporating different work activity duration constraint

The model developement explained in section 8.3 is based on the assumption that all individuals are required to perform 40 hours (2400 minutes) of work activity duration in the entire week. This constraint represents the inflexible nature of the work activity in a very simple way, with the assumption that on a single day the duration of the work activity is considered flexible, however, for an entire week the work activity duration is fixed. It is possible that weekly duration of work activity for different individuals is different because of its dependence on many factors as mentioned in section 8.2.1, which require a separate study. The focus here is to describe the way in which this different weekly activity duration for each individual can be incorporated in the model.

The simple way to incorporate different weekly work activity duration by dividing the total number of individuals into different user classes, each user class will perform certain amount of weekly work activity duration which is different from other user classes. The step where duration of work activity is calculated for atypical day tour (as shown in the model development, section 8.3) will be important because for each user class separate duration of work activity will be obtained. All other steps which involve atypical day duration of work activity will then be adjusted according to the different user classes. This can be done by following the similar notion in which daily model (shown in section 6.4) for different user classes was developed.

8.5.2 Treating differences in week days

The model development process explained in section 8.3 is based on the assumption that all week days are similar to each other. This is the reason why the systematic utility shown in equation (8.1) contains typical day and atypical tours utilities in such a manner that typical day tour utility is multiplied with 4 and an atypical day utility is multiplied with unity. The assumption of similar weekdays gives an advantage that only a single day is required to model. This sub-section highlights the fact that what types of changes are required in the model if all days are considered different to each other.

Table 8.1 (shown under section 8.2) indicate two different scenarios, first one assumes that all days of the week are similar to each other and in relation to this assumption it indicate that how individuals are allocated themselves for typical day and an atypical day tour on a given single day. The second scenario assumes that all days are different to each other, and in relation to this assumption the table indicated allocation of individuals for typical day and an atypical day tours. It should be noted that the allocations of individuals are not unique in the second scenario, as there are many possible ways in which individual can perform these tours in an entire week keeping the fact that each individual need to perform home-work tour on four days and on the fifth day he/she is required to perform an atypical day tour (three-activity tour). Table 8.4 indicates some examples of the composition of individuals for these tours with the assumption that all days are different to each other.

Table 8.4: Possible illustrations of composition of commuters for typical day and atypical day tours

Week days		Monday	Tuesday	Wednesday	Thursday	Friday	Total
Example 1	Typical day tour	70%	60%	85%	90%	95%	4 x 100%
	Atypical day tour	30%	40%	15%	10%	5%	100%
	Total	100%	100%	100%	100%	100%	
Example 2	Typical day tour	80%	90%	65%	70%	95%	4 x 100%
	Atypical day tour	20%	10%	35%	30%	5%	100%
	Total	100%	100%	100%	100%	100%	
Example 3	Typical day tour	75%	65%	85%	95%	80%	4 x 100%
	Atypical day tour	25%	35%	15%	5%	20%	100%
	Total	100%	100%	100%	100%	100%	

When it is assumed that all weekdays are different to each then the weekly model is to run for five weekdays together. The systematic utility when all weekdays are different to each other in the case for *Example 1* shown in table 8.2 is given by:

$$\begin{aligned} V_{\left(i,\, j, k, r^1, s, r^2\right)} = &\left(0.7 \cdot V_{td}^{Monday} + 0.3 \cdot V_{atd}^{Monday}\right) + \left(0.6 \cdot V_{td}^{Tuesday} + 0.4 \cdot V_{atd}^{Tuesday}\right) \\ &+ \left(0.85 \cdot V_{td}^{Wednesday} + 0.15 \cdot V_{atd}^{Wednesday}\right) + \left(0.9 \cdot V_{td}^{Thursday} + 0.1 \cdot V_{atd}^{Thursday}\right) \\ &+ \left(0.95 \cdot V_{td}^{Friday} + 0.05 \cdot V_{atd}^{Friday}\right) \end{aligned} \tag{8.12}$$

In equation (8.12) the factors attached with typical day and atypical day tours utilities are reflecting the degree in which these days are different to each other. This equation further indicates that when *all days are similar to each other* equation (8.12) simply collapses to the equation 8.13, which was used in the model illustration shown in section 8.3.

$$\begin{aligned} V_{\left(i,\, j, k, r^1, s, r^2\right)} = &\left(0.8 \cdot V_{td}^{Monday} + 0.2 \cdot V_{atd}^{Monday}\right) + \left(0.8 \cdot V_{td}^{Tuesday} + 0.2 \cdot V_{atd}^{Tuesday}\right) \\ &+ \left(0.8 \cdot V_{td}^{Wednesday} + 0.2 \cdot V_{atd}^{Wednesday}\right) + \left(0.8 \cdot V_{td}^{Thursday} + 0.2 \cdot V_{atd}^{Thursday}\right) \\ &+ \left(0.8 \cdot V_{td}^{Friday} + 0.2 \cdot V_{atd}^{Friday}\right) \\ = &\ 0.8 \cdot \left(V_{td}^{Monday} + V_{td}^{Tuesday} + V_{td}^{Wednesday} + V_{td}^{Thursday} + V_{td}^{Friday}\right) \\ &+ 0.2 \cdot \left(V_{atd}^{Monday} + V_{atd}^{Tuesday} + V_{atd}^{Wednesday} + V_{td}^{Thursday} + V_{atd}^{Friday}\right) \\ = &\ 4 \cdot V_{td} + V_{atd} \end{aligned} \tag{8.13}$$

The duration of the work activity within the typical day tour is also required to calculate for each day when it is assumed that all days are different to each other. This can be worked out as follows:

$$\left(\tau_{wr^1}\right)_t^{Monday} = j - \left(i + \left(R_{ir^1}^{hw}\right)^{Monday} \right) \tag{8.14}$$

Equation (8.14) suggested that travel time from home to work for the morning commute will be different for each day and due to this reason typical day work activity duration is also different for each day. This is because on each day different number of individuals are loaded on the network for performing typical and atypical day tours. Similar to the equation (8.14), the duration of work activity can be determined for other days of the week. The duration of the work activity for an atypical day tour can be given by using equation (8.14) and the constraint representing total duration of the work activity for an entire week. This is represented as follows:

$$\left(\tau_{wr^1}\right)_a = 2400 - \begin{bmatrix} 0.7 \cdot \left(\tau_{wr^1}\right)_t^{Monday} + 0.6 \cdot \left(\tau_{wr^1}\right)_t^{Tuesday} \\ + 0.85 \cdot \left(\tau_{wr^1}\right)_t^{Wednesday} + 0.9 \cdot \left(\tau_{wr^1}\right)_t^{Thursday} + 0.95 \cdot \left(\tau_{wr^1}\right)_t^{Friday} \end{bmatrix} \tag{8.15}$$

Equations (8.14) and (8.15) will help measuring the systematic utility of the typical day and atypical day tour for each day, and then finally total systematic utility which is shown in equation (8.12) can be obtained. Following the same process, as shown in model illustration for the formulation of the fixed point problem (section 8.3), this problem can also be solved.

8.5.3 Incorporating more tours within a week

Development of the model shown in section 8.3 is based on the assumption that there are only two types of tours individuals can perform in an entire week. The first tour type contains only two activities (i.e. home-work tour) which need to be performed for four days of the week, and the second tour type contains three activities (home-work and an additional activity tour) which need to be performed on a single day within a week. There may be some other types of tours individuals may perform within a week (e.g. tour which contains four activities or more). Additionally, individuals may want to perform home-work tour for three days of the week, and three activity tour on the two remaining days of the week. These two conditions (which relax the above discussed assumptions of the model) can be easily incorporated in the model.

The assumption regarding performing home-work tour for three days and three-activity tour for other two days can be incorporated in a way that instead of assuming 100% of the individuals in an entire week for performing atypical day tour, it should be assumed that 200% of individuals are involved in an atypical day tour. This suggests that if all days are considered similar then on a single day 60% of individuals are performing typical day tour and 40% of individuals are performing atypical day tour. Furthermore, the duration of work activity for an atypical day can be calculated using the following equation.

$$\left(\tau_{wr^1}\right)_a = \left(2400 - 3 \cdot \left(\tau_{wr^1}\right)_t\right) \cdot 0.5 \tag{8.16}$$

Equation (8.16) suggested that duration of the work activity is distributed equally for the remaining two days of atypical day tour. In order to calculate systematic utility for an entire week the following expression should be used.

$$V_{\left(i,j,k,r^1,s,r^2\right)} = 3 \cdot V_{td} + 2 \cdot V_{atd} \tag{8.17}$$

The relaxation of the assumption regarding incorporation of more tours further complicates the model structure but it can be done within the framework of the developed model. Suppose that individuals are involved in three types of tours within a week. The first tour (i.e. the home-work tour) will be performed by each individual for three days of the week. The second tour which contains three activities (i.e. home, work and an additional activity) will be performed on one of the remaining two days of the week and the last type of tour which contains four activities (i.e. home, work, 1st additional activity and a 2nd additional activity) will be performed on the remaining day of the week. If it is considered that all days are similar to each other, then on a single day 60% of individuals are performing their typical day tour (i.e. home-work tour), 20% of individuals are performing 1st atypical day tour (three-activity tour) and remaining 20% of individuals are performing their 2nd atypical day tour (four activity tour). The duration of work activity for 1st and 2nd atypical day tours can be calculated using equation (8.16) provided that work activity duration is assumed to be distributed equally for 1st and 2nd atypical day tours. Incorporation of four-activity tour as a 2nd atypical day tour will further increase the considered scheduling dimensions, the dimensions which represents the departure times for the two additional activities and the sequence and route choice dimensions. The total systematic utility expression can be given as

$$V_{\left(i,j,k,l,m,r^1,s_1,s_2\,r_1^2,r_2^2\right)} = 3\cdot V_{td} + 1\cdot V_{atd_1} + 1\cdot V_{atd_2} \tag{8.18}$$

where, V_{atd_1} and V_{atd_2} representing systematic utility of the 1st and 2nd atypical day respectively. $V_{\left(i,j,k,l,m,r^1,s_1,s_2\,r_1^2,r_2^2\right)}$ representing the total systematic utility of the entire week in which three types of tours are considered. The scheduling dimensions *l* and *m* represents the departure times for the 1st and 2nd additional activity on a 2nd atypical day tour which contains two additional activities along with the home and work activities. s_2 and r_2^2 represents sequence and route choice for 2nd atypical day tour. With the use of equation (8.18) and following the other similar notions as explained in the model illustration, the weekly scheduling problem with three types of tours can also be modelled.

8.6 SUMMARY

This chapter reported the development process of the weekly activity scheduling model along with its application through some numerical experiments. The development of the weekly activity scheduling model was primarily based on the daily activity scheduling model reported in chapters 6 and 7. The weekly activity scheduling model developed on the notion that on a given day, the duration of work activity is flexible but in an entire week an individual need to stay at the work location for a stipulated weekly duration of the work activity. This is more reasonable because in the daily model context the change in the duration of work activity due to change in the circumstances (e.g. introduction of tolls) may become questionable. Another main point because of which weekly activity scheduling model was developed is to incorporate different tour types for each individual in an entire week. This is to say that on one day individual is performing a tour containing two activities and on the other day the same individual is performing three-activity or four-activity tour. Section 8.2 discussed these two main points in a detailed manner, and the following section (section 8.3) presents the development process of the weekly model.

The numerical experiments are reported for the weekly model in section 8.4, the results of these experiments suggests that the model is behaving plausibly. The constraint that the weekly work activity duration was fixed to 40 hours has played a significant role in keeping the work activity duration in the reasonable limits on a given single day even when tolls are introduced on some links of the network. Section 8.5 presented some meaningful extensions of the model by relaxing some of the assumptions made while demonstrating the mathematical illustration of the weekly activity scheduling model. The next chapter conclude this thesis and put forward some recommendations for carrying out further research work for the model improvement.

Chapter 9

CONCLUSIONS AND RECOMMENDATIONS

9.1 GENERAL

This thesis presented a combined model that integrates the modelling of activity scheduling dimensions (for daily and weekly activity-travel patterns) and a dynamic representation of congestion on the network. The essential aspect of the model is based on the trade-off between the utility of participating in various activities and the disutility of travel between the activity locations. The modelling framework developed for the daily and weekly activity scheduling models is such that it can encompass a range of random utility models at the demand side and on a similar notion a range of dynamic network loading models can also be used at the supply side. The numerical implementations of the model presented for the daily and weekly models is such that it can only be used for the hypothetical network considered in this thesis (see figure 8.2); however, using the principles mentioned under the mathematical illustration of these models, their numerical implementation can be extended to incorporate a real size network. A variety of numerical experiments were performed in order to assess the working of the models and also the implications of a range of policies. It has been noted that results obtained from all the numerical experiments are plausible and explainable. Section 9.2 further elucidates the degree of achievement of the objectives set out for this research and section 9.3 demonstrates the recommendations for further improvement of the model.

9.2 DEGREE OF ACHIEVEMENT OF RESEARCH OBJECTIVES

This research had in total five objectives which are described in chapter 1. The following sub-sections discuss the degree of achievement of each objective.

9.2.1 Objective 1

To establish a state of the art review of activity scheduling models, relevant issues and modelling considerations within the combined modelling framework.

This objective was achieved by reviewing the already developed activity scheduling models under the combined modelling framework, and by performing a rigorous analysis of the issues related to the demand and supply sides of the combined modelling framework along with their integration. The models developed under the activity-based (AB) approach were considered first in order to understand various activity scheduling dimensions and their role in the daily activity travel pattern of the individuals. It has been noted that the model development paradigm of the AB models is exclusively based on the demand side. The supply side is considered exogenously in these models. Due to this, the behavioural realism incorporated in the AB models significantly loses its credibility when a sequential process is adopted to predict flows on the network (i.e. use of a traffic assignment model). The literature within the combined modelling focuses more on scheduling of the morning commute (home to work trip) only, however, a smaller number of models are also reported which attempt to model scheduling of the simple daily activity travel pattern (home-work tour). An extensive review of all these models along with their properties is provided in chapter 2 with the identification of the observed gaps.

Chapter 3 extensively discussed the issues involved at the demand side of the combined modelling framework. This chapter analysed issues related to the different decision making methodological frameworks. Additionally, the measurement of the utility of activity participation is discussed in detail and based on that various functional forms are presented which are dependent on time-of-day and activity duration for measurement of utility according to different activity types. Furthermore, operational models at the demand side are also discussed with their properties and limitations. Modelling considerations involved at the supply side of the combined modelling framework are comprehensively examined in chapter 4. These include the representation of traffic on a macro or micro scale and the representation of time dimension. In addition to this, four operational models are comparatively discussed on the basis of their behaviour and their confirmity with the desirable properties for dynamic traffic assignment. The issues regarding the integration of the demand and supply sides in the combined modelling framework are presented in chapter 5, which also demonstrate that the suitability of the fixed point problem formulation for the scheduling problems based on stochastic user equilibrium. Two solution algorithms are also investigated along with their properties and requirements.

The above two paragraphs suggested that the work reported in chapter 2, chapter 3, chapter 4 and chapter 5 shows the rich background work that has been done for the achievement of the first objective. This background work not only fulfils the first objective but also provides a profound base for the achievement of other objectives set out for this research.

9.2.2 Objective 2

To develop a combined activity scheduling model that embodies a simple daily activity-travel pattern with dynamic traffic assignment over a simplified network in a generalised manner that can be easily extendable.

This objective has been achieved by the development of a combined model for scheduling of the home-work tour using a single two-way link between the home and work activity location. Departure time to and from work are modelled as the only scheduling dimension for the home-work tour, and the duration of the involved activities in the tour are considered implicitly in the modelling framework. Chapter 6 presented a generalised development of this simplified model. The model is general in a sense that it can accommodate any operational models within the demand and supply sides. This model is different from the previous reported models in the literature because of the incorporation of two essential ingredients (i.e. time-of-day and duration dependent marginal utility functions) for the measurement of activity participation utility. The inclusion of these ingredients not only ensures that the model incorporates time-of-day preference for activity participation and satiation effects of the activity, but also guarantees that the two commute trips involved in the home-work tour are held together. This point is discussed in detail with the help of numerical experiments and a mathematical illustration is also presented as an analytical proof.

Chapter 6 also presented development process of the model which is as an extension of the simplified model. This extended model incorporates two user classes which are carrying out different types of tours in a given day (i.e. home-work tour and three-activity tour) and include scheduling dimensions such as route and activity sequence choice along with the already considered departure times and duration choices. This clearly suggests that the simplified model can be easily extendable to a variety of dimensions.

9.2.3 Objective 3

> *To carry out a variety of numerical experiments in order to investigate the functionality of the model, and to suggest potential arenas for meaningful extensions of the developed model under objective 2.*

This objective has been met with the development of a generalised computer program using MATLAB as a tool. The computer program was developed with utmost care and its development process involved step by step assessment of the each component of the combined model. For example, working of the different operational models of the demand and supply sides was assessed using the reported behaviour of these models in the literature. This was indeed the most time consuming activity of this research. After development of the computer program, a variety of tests were performed. Chapter 7 reported results of the numerical experiments which were performed in order to achieve the two main goals. The first goal was to assess the model plausibility by inducing some systematic changes in the model. The experiments carried out under this goal include: model convergence behaviour using two solution algorithms, the use of different operational models at the demand and supply sides and the use of different time discretisation schemes for the demand and supply sides. The second goal was to show the application of the model for various congestion mitigation policies and their implications on the model predictions. The experiments carried out under this goal include: application of dynamic tolls, incorporation of tele-work scheme and incorporation of time-of-day and duration based flexibility for the work activity. It was concluded that the model predictions are plausible and explicable for all such circumstances reported above. Chapter 7 also reported results of two experiments which were performed for the extended version of the daily activity-travel pattern model which incorporates two user classes with different tour types.

It was noted that the combined model framework was based on the notion that the durations of the activities are considered flexible, and due to this fact any changes in the model input caused changes in the duration of the involved activities in the daily tours. This flexibility notion seems unreasonable especially in the case where work activity is involved. This is because most of the jobs in real life have a nature that the employer and employee are mutually agreed on a given number of hours of work specified on a weekly or a monthly basis. This suggests that the model reported in chapter 6 along with its numerical illustration in chapter 7 is only applicable for jobs which are based on the idea that on a single day whatever time an individual spent at the work location, he will gain utility accordingly. This limitation of the model provide a profound base for further extension of the model which is more meaningful than simply extended the model for incorporating more scheduling dimensions.

9.2.4 Objective 4

To systematically extend the framework of the developed model to represent weekly scheduling of activities which is in line with objectives 2 and 3 and incorporate more activity scheduling dimensions.

This objective was achieved by extending the daily activity-travel pattern model into a weekly activity-travel pattern model. As is shown in sub-section 9.2.3, a constraint for the work activity duration is necessary to incorporate in order to reflect the appropriate representation of the work activity in the model. With this in mind, the daily model was extended in such a manner that it provides the framework that on a given day work activity is flexible but on a weekly basis individuals need to spend an agreed number of hours at work location. Additionally, it was also noted in the literature that individuals do not normally perform similar tours on all days of the week i.e. their tour type may change across the week days. This notion is also introduced in the weekly activity scheduling model, which suggests that for example; on a given day an individual performs a home-work tour but on some other day of the week the same individual is performing a three-activity tour (i.e. home-work and an additional activity).

Chapter 8 presented the development of the weekly activity scheduling model with the assumptions that, (i) the weekly work activity duration is 40 hours; (ii) there are four typical days in the week in which individuals follow a home-work tour and (iii) the fifth day is an atypical day in which individuals perform a three-activity tour. Furthermore it was further assumed that all days are similar to each other and based on this the population is distributed in such a manner that, on a given day, a given proportion of individuals are involved in a home-work tour and the remainder are performing a three-activity tour. This point was well elaborated in Chapter 8 with an illustration involving some examples. The weekly activity scheduling model reported in chapter 8 incorporates four scheduling dimensions such as departure times, activity durations, activity sequence and route choice on a given day.

9.2.5 Objective 5

To conduct numerical experiments to show working of the extended model and demonstrates the implications of a congestion mitigation policy

This objective was met by extending the compute program developed for the daily activity scheduling model. The extension of the computer program in order to represents a weekly activity scheduling model was done along with the step by step assessment of each component. Chapter 8 also reported results of some numerical experiments that showed the working of the model. Numerical experiments reported for the weekly activity scheduling model are based on two experimental setups; the first setup involved three scheduling dimensions namely departure time, duration and route choice, and the second setup considered activity sequence choice as well along with the scheduling dimension considered in the first setup. Each of the experimental setups reported the results of the two numerical experiments, the first experiment under each experimental setup considered no tolls (extra cost) on the links and the second experiment considered dynamic tolls (extra cost) on the specified links. This has been done in order to compare the model predictions for without and with tolls scenario under each experimental setup. The predicted results render enough indications that the model is behaving plausibly and yielding results according to the expectations.

9.3 DIRECTIONS FOR FUTURE RESEARCH

There are several dimensions in which this research can be extended to enhance and improve the modelling methodology. Availability of restricted resources in terms of time and funds, limits the scope of this research and due to this various assumptions were made to simplify the overall scheduling problem. Furthermore, some complex issues were avoided in each of the component of the combined modelling framework in order to develop a model within the stipulated time budget. The following are some specific areas of further research through which the research reported in this thesis may be improved and extended.

9.3.1 Model extension for a real road network

The model reported in this thesis along with its numerical illustration was based on the hypothetical network in which there are three activity locations connected with each other with 6 uni-directional links. As already mentioned, due to the complex nature of the problem, the scope of the model development process and its application was limited to a simplified network, however, based on the principles mentioned during the model development process the model can be easily extendable to represent a real road network. The only problem for the model development for the real road network is that the degree of each scheduling dimension will increase tremendously, and it may require a significant amount of time to obtain the converged solution. It may be possible that certain rules (assumptions) are adopted in order to limit the overall number of alternatives to a manageable size. For example, some procedure can be adopted to limit the number of routes (possible paths) between the activity locations using a criterion based on the total distance and free-flow travel time. Furthermore, the use of advanced and super computers which possess high processing speed can be used to minimise the program run time.

9.3.2 Application of a More Sophisticated Operational Models

This research utilised MNL and NL models as operational models within the demand side of the combined modelling framework, and a similarly limited number of models (such as four models namely, linear travel time, Point-Queue, Divided linear travel time and Adnan-Fowkes models) are studied or used within the supply side of the model.

These operational models have some limitations and there are more sophisticated models reported in the literature which could be utilised. Within the demand side of the combined modelling framework instead of using MNL and NL models, Probit and Mixed logit models can be utilised which provide greater flexibility to represent correlation of the error terms (random component of the utility expression) between the alternatives and between individuals. In the case of the supply side, important traffic phenomena such as the representation of queue spill-back and shock waves are completely ignored by using the above-listed four dynamic link loading models. Cellular Transmission model is the most likely candidate; however, use of the sophisticated models brings more complexity in the combined model because it requires more computational time. The main aim of this research is to develop a model which is based on a generalised notion, and this research is successful in reporting that model in this thesis, therefore, future applications of the model can easily accommodate any operational models within the demand and supply sides.

9.3.3 Analytical Illustration of Equilibrium Properties

This thesis has numerically shown that for the given values of the parameters the model is behaving plausibly and its solution exists and is unique. It might be that there are some cases (for some parameter values) where standard solutions algorithms may not provide converged solution (equilibrium solution), or it may be possible that in some cases the model solution is not unique. To examine these equilibrium properties a rigorous analysis is required for the model in order to establish that solution of the model exists and it is unique as well for all cases, and if not then what would be the possible reasons. The analytical illustration of the existence and uniqueness of the model solution in a general manner is much more demanding. However, it is helpful for not only increasing the model's credibility, but it also reveals in what situation the model may not render plausible predictions.

9.3.4 Incorporation of more scheduling dimensions

This thesis focuses only on four scheduling dimensions within different tour types; however, in reality there are other important scheduling dimensions as well which need to be considered. This includes mode choice, activity destination choice, choice of different activity-travel patterns and choice of joint activity-participation etc. There are many studies available which have their focus on these scheduling dimensions but those studies are conducted in isolation with other scheduling dimensions (e.g. Bhat 2007). These studies may lead to a way forward to develop a framework in which all the scheduling dimensions are considered in a combined modelling framework. This framework would integrate the activity generation process which requires consideration of different dynamic processes within the household, such as household needs generation, household interactions and task allocations processes. This framework would lead towards the development of a comprehensive combined activity modelling system.

9.4 CONCLUSIONS

This work has made a significant contribution to the improvement and extension of the already existing analytical models developed under the notion of the combined modelling framework for a daily activity-travel pattern. In summary, the research:

- Critically reviews the existing models of activity scheduling which are based on the combined modelling framework. Based on that review, gaps are identified such as extension of the modelling framework by incorporating more scheduling dimensions, improvement in the measurement of utility of an activity participation and incorporation of weekly scheduling of activities;
- Identifies two essential ingredients in order to measure utility of activity participation i.e. individual time-of-day preference and activity satiation effects. It has been numerically and analytically proved in the thesis that duration based marginal utility function (which represents activity satiation effects) played a vital role in combining different commute trips of the tour;
- Reports a development of a new dynamic link loading model which addresses the weakness of the already existing and widely used point-queue and linear travel-time models;
- Develops a generalised home-work tour combined model considering a single link between the given home and work activity locations. The model is generalised in a sense that it can accommodate any operational model within the demand and supply sides of the combined model. Furthermore, the model is developed in a way that it is easily extendable for a range of scheduling dimensions. This model has undergone rigorous testing and application for congestion mitigation policies;

- Reports the development of an extended daily activity-travel pattern combined model which incorporates two user classes performing different tours in a given day and models departure time choices, activity duration, activity sequence choice and route choice;
- Develops a weekly activity scheduling combined model with the incorporation of a constraint on weekly work activity duration to represent the constrained nature of this activity. Additionally, this model allows individuals to indulge in different tours across a week.

The research reported in this thesis will paves a way forward for the development of a more holistic framework for modelling scheduling dimensions of the complex tours and activity-travel patterns. The research in this dimension will continue to grow and it will render important and promising avenues for the improvement of applied travel models.

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Appendix-I

DISCRETISED ALGORITHMS FOR DYNAMIC LINK LOADING MODELS

1 Point-Queue Model

The algorithm is taken from Nie and Zhang (2005a), and is as follows:

Step 0 Initialization: set $i = \phi$; $z = 0$; $o_1 = o_2 = \cdots = o_\phi = 0$.

Step 1 Move forward: $i = i + 1$; $z = z + e_{(i-\phi)}$.

Step 2 Calculate o_i and update z. If $z > C \cdot \delta$, then $o_i = (C \cdot \delta)$ and $z = z - (C \cdot \delta)$; otherwise, $o_i = z$ and $z = 0$.

Step 3 Calculate R_i through equation (4.8), If $i < (T+(D-1) \cdot \Delta) / \delta$, go to Step 1; otherwise, stop.

2 Linear Travel time Model

The algorithm is taken from Nie and Zhang (2005b), and is as follows:

Step 0 Initialization: $x_1 = 0$; $o_j = 0$; for $j \in \{1,2,\ldots \phi/\delta\}$; $\varphi_1 = \phi$; k = ϕ/δ; RES= 0; $i = 1$.

Step 1 Move. Set $i = i + 1$; calculate $x_i = x_{i-1} + e_{(i-1)} - o_{i-1}$; $R_i = \phi + (x_i/(C \cdot \delta))$; $\varphi_i = (i-1).\ \delta + R_i$. Set NIT $= (\varphi_i/\delta)$- k. If NIT < 1 go to Case a; otherwise go to Case b.

Case a: Calculate RES = RES + $e_{(i-1)}$.

Case b: Set k = k + 1; calculate $\rho_i = e_{(i-1)}/(\varphi_i - \varphi_{(i-1)})$; o_k = RES + [(k$\cdot\delta$)- φ_{i-1}] $\cdot\rho_i$
For n = 2 to NIT: set k = k + 1, $o_k = \delta \cdot \rho_i$. Calculate, RES = $(\varphi_i - (k \cdot \delta)) \cdot \rho_i$.

Step 2 If $i < (T+(D-1) \cdot \Delta) / \delta$, go to Step 1; otherwise, stop.

3 DIVIDED LINEAR TRAVEL TIME MODEL

The algorithm is obtained by modifying the algorithm presented for the linear travel time model (as discussed above) in accordance with the definition of the divided linear travel time model proposed by Mun (2001). The algorithm is as follows:

Step 0 Initialization: x_{21} = 0; o_j = 0; for $j \in \{1,2,...\phi/\delta\}$; $\varphi_1 = \phi$; ϕ_1 = δ ; k = ϕ/δ ; RES= 0; i = 1; $e_{2(m)}$=0; for $m \in \{1,2,...((\phi-\phi_1)/\delta)\}$; set $M = (\phi-\phi_1)/\delta$

Step 1 Inflow profile preparation for 2nd part of the link: For s = 1 to (T+(D-1) ·Δ), $e_{2(M+s)} = e_{(s)}$

Step 2 Move. Set $i = i + 1$; calculate $x_{2i} = x_{2_{(i-1)}} + e_{2(i-1)} - o_{i-1}$; $R_{2i} = \phi_1 + (x_{2i}/(C\cdot\delta))$; φ_i = (i -1). δ + R_{2i} . Set NIT $=(\varphi_i/\delta)$- k. If NIT < 1 go to Case a; otherwise go to Case b.

Case a: Calculate RES = RES + $e_{2(i-1)}$.

Case b: Set k = k + 1; calculate $\rho_i = e_{2(i-1)}/(\varphi_i - \varphi_{(i-1)})$; o_k = RES + [(k·δ)- φ_{i-1}] ·ρ_i
For n = 2 to NIT: set k = k + 1, $o_k = \delta\cdot\rho_i$. Calculate, RES = $(\varphi_i-(k\cdot\delta))\cdot\rho_i$.

Step 3 Calculate $R_i = (\phi - \phi_1) + R_{2i}$; If i < (T+(D-1) ·Δ) / δ, go to Step 2; otherwise, stop.

4 ADNAN-FOWKES MODEL

The algorithm is obtained by modifying the algorithm presented for the point-queue model (discussed above) in accordance with the definition of the link loading model proposed in the thesis (section 4.4). The algorithm is as follows:

Step 0 Initialization: set $i = \phi$; z = 0; $o_1 = o_2 = \cdots = o_\phi$ = 0; set $L_2 = \dfrac{nC - L_1}{n-1}$

Step 1 Move forward: $i = i + 1$; $z = z + e_{(i-\phi)}$.

Step 2 Calculate o_i and update z. If $z > L_2$ · δ, then o_i = (C · δ) and $z = z - o_i$; else If $z > L_1$ · δ, then $o_i = \dfrac{L_1\cdot\delta + (n-1)\cdot z}{n}$ and z = z-o_i; otherwise, $o_i = z$ and z = 0.

Step 3 Calculate R_i through equation (4.8), If i < (T+(D-1) ·Δ) / δ, go to Step 1; otherwise, stop.