An Agent-based Model of the Interbank Market: Reserve and Capital Adequacy Requirements

By

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Abstract

The purpose of this research is to study the financial contagion of the banking system in the presence of an interbank market and how do the reserve and capital adequacy requirements play a role in enhancing the stability of the system.

I develop an interbank market model that exhibits a tiering structure and study how the network characteristics and the regulatory requirements play a role on the systemic risk in the banking system. The basic model is based on Iori, Jafarey, and Padilla (2006). I advance the model in a number of aspects. Firstly, I examine an interbank market with a core-peripheral, which is found in real-world interbank markets. Secondly, I introduce a parameter that determines the difference in the size of deposit and investment opportunity in a bank, and I also limit the fluctuation in customers' deposit for small banks. In doing so, I create a banking system that shows more resemblance to the real-world market in which large banks are deposit-taking banks while small banks have proportionately more investment opportunity and that large banks are interbank borrowers while small banks are interbank lenders. Thirdly, I study the stability of the banking system with respect to both the reserve requirement and the capital adequacy requirement.

I show that the effects of the two regulatory requirements have on the stability of the system are closely related to how they affect the interbank activity. By tightening the reserve requirements, banks are forced to retain a bigger portion of deposit as liquid reserves. This act as a better insurance for individual banks against liquidity shocks. However, this action also results in a reducing the amount of surplus liquidity a potential interbank lenders has, and to some extent, it restricts these banks to transact in the interbank market completely. Therefore, this put those banks which have big fluctuation in customers' deposit in great risk because they do not have any counterparties to transact with in the interbank market when they face a liquidity shortage. Similarly, the tightening of the capital adequacy requirement also restrict interbank activity, which in turns

affects the chances of banks arranging interbank borrowings when they have a liquidity need.

Abbreviations

N	Number of simulations
C	Connectivity
T	Number of periods
M	Number of banks in the system
τ	The maturity of the investment
κ	Imbalance parameter
δ	Aggregate investment opportunity : aggregate bank size ratio
$\bar{S}$	Size of a large bank
σ_S	Fluctuation in bank size across small banks
σ_D	Fluctuation in deposit over time
σ_ω	Fluctuation in investment over time
r_D	Interest rate for deposits
r_B	Interest rate for interbank transaction
r_I	Income rate for investment
β	Reserve requirement ratio
χ	Equity requirement ratio
ϕ	Capital adequacy requirement ratio
α_L	Parameter for autoregressive process in deposit for large bank
α_S	Parameter for autoregressive process in deposit for small banks
γ	Parameter in the risky asset function
ρ_I	The weighted-risk of investment
ρ_{IL}	The weighted-risk of interbank lending

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Chapter 1

Introduction

This thesis explores the potential and severity of contagious default in the interbank market through a theoretical perspective. I develop an artificial interbank market model that mimics the features in the real-world interbank market and simulate the daily operation of banks in such a system. By introducing a channel in which banks face liquidity shortage, I study the interaction between the benefits of using of interbank market and the risk these banks are exposed to. The main objective of the research is two-fold. Firstly, it aims to understand some of the determinants that affects the stability of the banking system, in the presence of the interbank market. Secondly, it studies the interaction between these determinants and the regulatory requirements and how they contribute to the efficiency of the market and the potential contagion in the system. This chapter presents the motivation of this research, the related research background in this area, a number of research questions together with the outline of the thesis.

1.1 Background and motivation

Financial crises have always been the strongest witness to testify how susceptible the financial system can be. Although these financial crises happen rarely, when they do, they often bring a catastrophic damage to the system. Therefore, it is of interest and great importance to gain an in-depth and thorough understanding of the type of risks the system is exposed to and of any precaution measures that can be taken into consideration to prevent these risks turning into financial crises. Back in 2007 and 2008, the banking systems in the UK and U.S. experienced one of the biggest crises, which nearly brought the financial systems all down. The incident does raise the alarm again about the presence of systemic risks in the banking system and that they are capable of bringing a complete shut-down to the system.

In the literature, researchers have identified various possible channels of contagion that can be found in the banking system. On one hand, some of these affect the liability side of banks including banks runs arising from the fear of withdrawals from other depositors, see Diamond and Dybvig (1983), or a common pool of liquidity, see Aghion, Bolton, and Dewatripont (2000) and Diamond and Rajan (2005), or information contagion, see Acharya and Yorulmazer (2008). On the other hand, some of these channels of contagion have an impact on the asset side of banks and they include the payment system, as studied by Humphrey (1986) and Angelini, Maresca, and Russo (1996), or FX settlement, see Blavarg and Nimander (2002) or the interbank market, see Rochet and Tirole (1996). In the past decade, research on the study of the interbank lending market as the channel of contagion has growth extensively. This is because it is believed that a bank that transacts in the interbank market is not only exposed to counterparty risk with the other banks that it trades with, but it is also exposed to a knock-on effect through these inter-related interbank linkages.

The presence of an interbank market permits banks to exchange liquidity with others for insurance purpose; it also allows illiquid banks to have access to liquidity. In other words, an interbank market fulfils the purpose of re-distribution of liquidity among the banking system, see Allen and Gale (2000). Unfortunately, these cross holdings of interbank transactions also give rise to a channel of contagion. It has been argued that the interbank market is not only a 'shock-absorber', in the case where deficit banks use the market to take care of their liquidity issues, but it also acts as a 'shock-transmitter' when the problem in one bank is spread to the others through interbank linkages, see Nier, Yang, Yorulmazer, and Alentorn (2008). A number of questions emerge from the above argument. Do the benefits of using the interbank market outweigh the cost of the potential damage it can bring to the banking system? Under what conditions does the interbank market act as a 'shock-absorber'? Are there any ways to prevent the contagious default to take place via these interbank linkages while banks are still able to make good use of it? These lead researchers to examine the interbank market carefully.

The famous work by Allen and Gale (2000) shed light on the study of the systemic risk in the interbank market. They show that one can use network topology to represent the network structure of an interbank market and point out that the exact structure of such a market plays an important role. Since then, there have been a lot of empirical studies that investigate the exact network structure of a real-world interbank market in a particular country and/or examine the potential and severity of contagion in the market, see Blavarg and Nimander (2002), van Lelyveld and Liedorp (2006), Toivanen (2009) and many others. Upper (2011) provides a comprehensive summary and comparison of these empirical studies up to 2011. However, these empirical works have their own drawbacks. Firstly, each of them are only able to study a specific country and hence their findings and conclusions are not generalisable. Secondly, a majority of them do not actually have access to the complete bilateral interbank exposures between banks and therefore they suffer from data limitation. Thirdly, due to the data limitation problem, most of these studies choose to use the entropy maximisation (see Fang, Rajasekera, and Tsao (1997)) to estimate the bilateral interbank exposures from the aggregate exposures. Mistrulli (2011) test this methodology with a set of complete and observed bilateral exposures against the estimated bilateral exposures. The author shows that the method introduces bias into the analysis and it is likely to underestimate the potential and severity of contagion because it relies on the assumption that the interbank activity is spread evenly across the system, which is not the case in reality. Fourthly, these papers are unable to inform us how particular feature makes the banking system more prone to contagion.

In view of the drawbacks in these empirical studies, there is another strand of literature which studies the interbank market. It tackles the issue from a theoretical perspective and fills up some of the gaps that empirical studies are not able to offer. These theoretical papers attempt to understand different aspects of the interbank market through simulations in an artificial model of the interbank market. Iori, Jafarey, and Padilla (2006) run a dynamic model of an interbank market with a random network structure. They simulate a system of banks which face a fluctuation in deposit patterns and create the need of interbank borrowing. They mainly focus on the number of defaults in different scenarios and show that an increase in connectivity improves the stability of the system while an increase in heterogeneity in the system can make the system more unstable. Nier, Yang, Yorulmazer, and Alentorn (2008) build a static model of the interbank market with a random network structure. They assume interbank holdings across banks and introduce an idiosyncratic shock that randomly wipe out a bank's capital. They study the effect of a number of determinants in the banking system including connectivity, concentration, capital ratio. They extend the model by incorporating liquidity risk and also briefly look at the tiering structure. The model by Gai and Kapadia (2010) shares a lot of similarities with Nier, Yang, Yorulmazer, and Alentorn (2008), but the authors consider also an aggregate shock which reduces the capital holdings of banks. Both of these papers show a non-monotonic relationship between connectivity and the stability of the system and they also emphasise on the importance of banks being capitalised.

Although the two most common network structures found in those empirical studies for real-world interbank market are scale-free structure (see Boss, Elsinger, Summer, and Thurner (2004a), Cajueiro and Tabak (2008) and many others) and the tiering structure (see Wells (2002) and Iori, De-Masi, Precup, Gabbi, and Caldarelli (2008)), Newman, Barabási, and Watts (2006) list out the three most network structures commonly being studied for the interbank market are the random structure, the small-world structure and the scale-free structure. From a theoretical perspective, this poses a lot of questions in this line of research. For example, what are the main factors that determine the stability of an interbank market with a particular network structure? Does one network structure provide a greater stability to the system than others? Does the vulnerability of the real-world interbank markets reduce if the banks are connected in a different manner? Is there a trade-off between stability and efficiency? These motivate the research to the pursue of a thorough understanding of the interbank market and whether it exposes the banking system to financial contagion and how does this relate to the network structure of the market.

Very recently, Ladley (2013) considers an interbank market model and pays specific attentions on the effect of contagion with respect to regulatory changes. He studies three different regulatory measures and shows that a higher equity requirement brings a reduction in contagious default while a higher reserve requirement does the opposite. He also considers restricting the amount of interbank transaction and finds that it has two opposite effects, depending on the size of the shock hitting the system. This paper also motivates the research to take into account of the role of regulatory requirements in the model.

1.2 Research questions

Research question 1
If the interbank market can act as both 'shock-absorber' and 'shock-transmitter' as suggested in the literature, then can its benefits outweigh its complication? What is the trade-off in the presence of an interbank market?

Research question 2
Is it possible to achieve an optimal environment for banks to use the interbank market which gives a minimal chance of contagion?

Research question 3
Is tightening regulatory requirement the ultimate solution to the prevention of contagious default in the interbank market? What are the drawbacks?

1.3 Overview of the thesis

This thesis is organised as follows. Chapter 2 presents a literature review on the financial contagion in the interbank market. This includes the channel of contagion in the interbank lending market, the use of network topology in representing the structure of the market, the empirical studies of the network structure and the interbank contagion in real-world banking systems and the theoretical analysis of the systemic risk in some artificial models. Chapter 3 presents the implementation of Iori, Jafarey, and Padilla (2006) model, which investigate the systemic risk in the interbank market with a random network structure and in the presence of heterogeneity in the system. The implementation forms the basic model of this research. However, there are a number of problems encountered during the implementation, Therefore, Chapter 4 points out these problems and provides modifications to the model, together with some preliminary results. Chapter 5 introduces the interbank market model for this research, describing the construction of the network of an interbank market and the simulation procedure of the operation of banks. It then presents the simulation results and also provides an analysis and a discussion of the results. Chapter 6 concludes the research.

Chapter 2

Literature Review

2.1 Introduction

The interbank market is a subset of the financial system and it provides a money-exchange mechanism for banks. One of the advantages of the interbank market in the financial system is that it allows the transferral of liquidity across different banks. Liquidity transferral is particularly important to banks. This is because there is often a mismatch in the maturity of a bank's short-term liability and its long-term assets. An example of short-term liabilities is the cash that customers deposit in a bank and this deposit can be withdrawn upon customers' preference and convenience. One type of long-term assets can be in the form of investment that a bank makes, for example, loans to non-bank financial intermediaries or mortgages to individual borrowers. This mismatch in maturity can create liquidity problem for banks. Therefore, banks can use the interbank market to help manage their liquidity. Allen and Gale (2000) shows that banks can also make deposits in other banks via the interbank market as a coinsurance purpose.

Failure in managing liquidity well can cause problems, and to certain extent, it can obstruct a bank from performing its daily operation and result in default. Therefore, it is critically important for them to manage their liquidity well. Unfortunately, the liquidity of these financial intermediaries can fluctuate a lot from day to day. Therefore, it is essential for them to have access to liquidity from other means and they can do this through the interbank market. This is because the interbank market allows both liquidity-surplus and liquidity-deficit financial intermediaries to come into direct contact with each other. As a result, those with surplus can lend to those with shortage.

2.2 Network topology

A system can be characterised by the number of its components, the structure formed by these components and the interconnectedness among them, the behaviour and the functionality of the system as a whole. Therefore, in order to study the stability of a system, it is essential to gain an understanding of the network of the system, in other words, how the entities within the system are connected to each other.

Network topology can be used to represent the specific arrangement of entities within a system. Topology can be interpreted as the shape or the structure of the system. The elements within the system are represented by nodes and the connections between them are represented by links. Network topology is commonly used in computer networks, see Bird and Harwood (2005), and in biological networks, see Liao, Boscolo, Yang, Tran, Sabatti, and Roychowdhury (2003) and Klemm and Bornholdt (2005). However, there is an increase survey of the network topology in understanding financial networks, for example, Thurner, Hanel, and Pichler (2003) study the efficiency of different topologies with respect to how different agents perform in the designed risk-trading game; and Soramaki, Bech, Arnold, Glass, and Beyeler (2007) investigate the various possible network topologies of the interbank payment flows.

2.2.1 The use of network topology in the study of interbank market

The famous paper by Allen and Gale (2000) was one of the first papers that study the relationship between the network structure of the banking system and the potential of financial contagion. They consider three different structures formed by four entities and study the possibility of financial contagion in the presence of aggregate liquidity shortage. They show that some structures are more susceptible than others and in particular, they show that the 'completeness' of a structure plays an important role. The examples that they study are very basic and are difficult to make comparison to the real-world network. On a positive note, this paper shed light on the usage of network topology in studying systemic risk in the interbank market and it also shows that the use of interbank lending market can redistribute liquidity but cannot create liquidity when there is an aggregate liquidity shortage. Since then, there has been a vast growth in the literature in the analysis of the network structure of the interbank market in different countries and how does the structure pose a systemic risk to the system. Allen and Babus (2009), looking at different networks in the financial systems, suggest that the network theory together with the incorporation of economic interactions can help understand many economic phenomena. More specifically, they illustrate the use of network analysis in interpreting what has happened in the interbank market in 2008.

Bird and Harwood (2005) describe the most common network topologies found in computer networks. However, not all of the network topologies are relevant to this research. Newman, Barabási, and Watts (2006) summarise the three network structures that are either found in real-world market or used in some theoretical study of systemic risk in the interbank market. The three network structures are: the random structure, first introduced in Erdős and Rényi (1959); the small-world structure, introduced in Watts and Strogatz (1998); and the scale-free structure, introduced in Barabási and Albert (1999). A random structure, sometimes referred as the Erdős-Rényi model, is formed by introducing a link between any pair of nodes in the network with a fixed probability p. There has been some advances in creating a random network from the Erdős-Rényi model. Some of the papers that consider a random network structure include Iori, Jafarey, and Padilla (2006) and Nier, Yang, Yorulmazer, and Alentorn (2008). Although the random network structure is simply and very useful in many application, the drawback is that it does not exhibit some of the important properties and characteristics that one can usually find in real-world structure. The second structure is a small-world structure, also known as the Watts-Strogatz model. The algorithm for forming a small-world structure is by creating a ring lattice that consists of N nodes. Let k to be the mean degree of connections per node, then connect each of the node in the lattice to k of its neighbouring nodes. Next, for each pair of nodes that has a link, rewire one end of the link to another node in the lattice, chosen randomly with a probability p. The beauty of small world structure is that, if $p = 1$, then it will result in a completely random network structure.'. However, Mitchell (2006) argues that the small-world network model is not the best match to most studied real-world networks. At a similar time, Barabási and Albert (1999) designed an alternative network model, so-called the scale-free networks. A scale-free network, also referred as the Barabási-Albert model, is simply one with a power-law degree distribution. Apart these from three general model, Freixas, Parigi, and Rochet (2000) suggest another network structure, in which there are many institutions at the periphery and they are linked to the bank (or a small number of banks) at the centre. This structure is sometimes referred as the core-periphery structure or a star structure. It is later showed in some empirical research that this structure is found in some real-world interbank markets too.

2.2.2 Network structure of real-world interbank markets

Empirical studies regarding the structure of the interbank lending markets are not very common. Way before most of the existing empirical studies on interbank market were carried out, Todd and Thomson (1990) identify that these works are constrained from data limitation. This is because the interbank exposures are often only available as aggregate exposures, meaning that the bilateral interbank exposures between any two specific banks are not differentiated out from the aggregate. This fact is supported by Furfine (2003), Upper and Worms (2004), Wells (2002) and many others. There are only a few studies which managed to obtain the complete set of data on the bilateral interbank exposures for banks and they include Lubloy (2004) and Mistrulli (2011) analysing the Hungarian and the Italian interbank market respectively. In the absence of the complete set of data on the interbank counter-party loan exposures, researchers who study the interbank lending markets often have to use some assumptions in estimating the unknown data for bilateral exposures. Therefore, the results found in these empirical researches are vastly dependent on the particular system or country studied and cannot be easily generalised. It is also worth-noting that the estimation method introduces bias to the findings. Despite this shortcoming, this line of research is still extremely valuable in providing great insights into the empirical importance of interbank contagion for real-world network.

Most of the empirical studies on the network structure of the interbank market show that real-world interbank markets are often best represented by a scale-free network, in which the degree distribution follows a power law. These include Blavarg and Nimander (2002) for the Sweden interbank market; both Boss, Elsinger, Summer, and Thurner (2004a) and Boss, Elsinger, Summer, and Thurner (2004b) for the Austrian interbank market; van Lelyveld and Liedorp (2006) for the Dutch interbank market; both Cajueiro and Tabak (2008) and Santos and Cont (2010) for the Brazilian interbank market; Toivanen (2009) for the Finnish interbank market; Lubloy (2004) for the Hungarian interbank market and Inaoka, Takayasu, Shimizu, Ninomiya, and Taniguchi (2004) for the Japan banking sector. Some of the above studies work out the power law exponent and they are usually in the range between 2 - 3. Therefore, these interbank markets are characterised by a couple of large banks with many interbank connections together with many small banks with only a few interbank connections. Some other interbank markets have been found to have a network that is characterised by a degree distribution that is less than a scale-free topology, but is more than a random network. This is sometimes referred to as a tiering structure and this network topology is found in the Italian overnight money market by Iori, De-Masi, Precup, Gabbi, and Caldarelli (2008) and the UK interbank market by Wells (2002).

Other studies that analyse the network topology of some large value payment systems in a number of countries also show that those network often exhibit a scale-free topology. These include Becher, Millard, and Soramaki (2008) looking at the the CHAPS in the UK; Soramaki, Bech, Arnold, Glass, and Beyeler (2007) studying the US FedWire System and Embree and Roberts (2009) analysiing the LVTS in Canada.

2.3 Empirical studies on systemic risk

At the knowledge of using network topology to represent the interbank market structure, there are two strands of literature, summarised by Upper (2011). One is empirical, which is to use data on actual exposures to test for possibility of contagion. This shows whether a given banking system is prone to contagion, but it does not help us understand how particular features of the interbank market make it more prone to contagion. The second strand is to use the tools of network analysis to analyse complex artificial networks with the aim of detecting patterns which could make them prone to contagion.

2.3.1 Summary of findings

Some of the earliest empirical studies focus on the payment system in the interbank lending market. In these studies, the channel of financial contagion arises from the default of one participant in the clearing system triggering further defaults of at least one other participants via the payment system. Humphrey (1986) studies the Clearinghouse Interbank Payment Systems (CHIPS) in the U.S. The CHIPS, being the main clearing house in the United States, helps settle and clear interbank transactions. By simulating a settlement failure of a major participant in the CHIPS, the author studies the effect of the simulated failure has on the whole payment system. He shows that a settlement failure of a major bank has a huge effect on the system and can give rise to further rounds of settlement failures. Using a similar approach, Angelini, Maresca, and Russo (1996) studies the Italian intra-day netting arrangement system. According to their simulations, the number of systemic crises and the magnitude of the systemic crises are both small, meaning that there is little evidence to support that the interbank payment system is likely to be a channel for systemic crisis to spread. they argue that their findings were different from those of Humphrey (1986) due to a number of factors including the frequency of the interbank transaction, the size of the interbank exposures and the structure of the interbank lending market being examined. Although the two studies present opposite findings with regard to the potential of the propagation of systemic risk through payment system, the fact that these two interbank markets are different in terms of structure, size and so on shed some light on the empirical analyses for interbank market, highlighting the importance of the specifics of the market structure.

The most common approach for empirical work to study the systemic risk in real-world interbank network is based on collecting (and estimating if necessary) the complete bilateral interbank exposures in a banking system and using simulation methodologies to study the number of defaults in a banking system under different scenarios. The first step of this approach is to work out the network structure of the interbank market using banks' bilateral exposures. As mentioned above, most of the existing empirical studies suffer from data limitations with only a few exceptions. Blavarg and Nimander (2002) study the Swedish interbank market and point out that Riksbank, a non-supervisory central bank in Sweden, has the legal right to request for any information directly from the financial institutions in Sweden. As a result, complete data is available on the counterparts for four major Swedish bank. Therefore, they study the interbank exposures among these four Swedish banks and investigate the systemic risk in the Swedish interbank market. They find that the direct loss triggered by the default of a large Swedish bank is low and, in most cases, would not affect the ability of other banks to maintain their Tier 1 capital ratio with respect to the regulatory requirement. On this account, they suggest that direct contagion in the Swedish banking system is possible but unlikely. Lubloy (2004) states the fact that all banks in Hungary have to report the volume of their daily transactions, their counter-parties and the type of transactions. As a result, a unique set of data on the interbank transaction can be obtained. It shows that the potential of systemic risk in the Hungarian interbank market is low. The Italian interbank market is the other market where complete bilateral exposures data is collected. This is because, as Mistrulli (2011) points out, since January 1989, all Italian banks are required to submit their end-of-month bilateral exposures with all other banks to the Bank of Italy. The paper demonstrates that the Italian interbank market is a channel for financial contagion. However, even for high loss rates, bank failures are still not likely to end up as a systemic crisis. Only in some extreme cases, the severity of financing contagion seems considerable.

In the presence of complete bilateral interbank exposures, the three studies are suggesting that although the interbank market can be a potential channel for financial contagion, the likelihood of the contagious spread of systemic risk is low in all the countries examined. In the absence of complete data, empirical studies on bilateral interbank exposures use one of the two approaches to overcome this constraint. Firstly, empirical work can be carried out on a specific segment of the market, in which all bilateral exposures are available. This is the approach taken in Furfine (2003). Since most of the federal funds transactions are settled over Fedwire, a Real Time Gross Settlement Funds Transfer system in the United States. Therefore, he is able to obtain and use this unique set of data from the overnight federal fund transactions between U.S. banks for a two-month period in 1998 to study the contagious effect of a significant bank default. He shows that the likelihood for further rounds of default is low and that the aggregate assets of the subsequent failing banks only account for no more than 1% of the total assets of the commercial banking system in the United States. However, one pitfall of the study, as implied by the approach, is that it does not incorporate the total interbank exposures in the U.S. banking market and as a result, the study might have underestimated the likelihood of contagion within the U.S. interbank market as a whole.

The second approach has been adopted much widely because it allows researchers to include the whole interbank market. It uses a Mathematical technique called local entropy maximisation, see Fang, Rajasekera, and Tsao (1997) and Blien and Graef (1997), to estimate the unknown bilateral interbank exposures from the aggregate exposures. By using the entropy maximisation method, researchers are allowed to create a full matrix of bilateral interbank exposures between any banks. However, it relies on the assumption that the interbank borrowing and lending is as evenly as possible. Once the bilateral exposure matrix is estimated, the next step is to simulate contagion in the interbank market. A majority of the papers consider an idiosyncratic shock that hits one of the banks to the extent that the bank defaults. They then examine whether the default of the first bank will result in the default in its counter-parties through their interbank exposures. Among these papers, most of them use an exogenous parameter the 'loss-given-default' (LGD), θ, which determines the amount of asset loss when a bank defaults. Therefore, by applying an idiosyncratic shock on a random bank, together with a particular LGD, these papers aim to study the probability and severity of the systemic risk in the interbank markets. Therefore, they try to investigate how likely a financial contagion can take place as a result of a bank defaulting and when this happens, how severe the problem is. However, due to the differences in the network structure across different interbank markets; the completeness of information collected; and the variations in the methodologies applied, these studies come up with different findings and arrive at different conclusions, providing an ambiguous picture as to whether the interbank market is conducive to contagious defaults. On the positive side, the majority of these papers find that the probability of a systemic event is low and that the scope of contagion is limited. For example, Sheldon and Maurer (1998), Amundsen and Arnt (2005) and Toivanen (2009) all show that a systemic failure arising from the idiosyncratic default of one bank is unlikely for the Swiss, Danish and Finnish banking system respectively. This means that the knock-on effect caused by the initial default bank is small, even if the unexpected failure bank is a major bank of the system. But Sheldon and Maurer (1998) point out that in some rare occasions, it is not impossible for the contagious effects to affect the whole system and when it does, the effect can be quite substantial. Degryse and Nguyen (2007) also find the Belgian banking system faces very little risk of danger of contagion. However, they only consider the interbank exposures among the banks within Belgium and those only account for 15% of the total interbank exposures in the market. Therefore, it is questionable whether a default of any of those international banks with large interbank exposure will pose a much greater threat to the stability of the system. On the other hand, Upper and Worms (2004) illustrate that the German interbank market is conducive for contagion. The bankruptcy of a single bank can affect a huge part of the system through interbank exposures. They show that in some rare cases, the default of a bank in the system can wipe out 15% of the total assets in the system. Wells (2004) and van Lelyveld and Liedorp (2006) argue that although likelihood and severity of contagion is low, the failure of one of the large banks in the system result in a substantial weakening of the stability of the banking system, for example in terms of the capital holdings of banks connected to the defaulted bank. This makes the weakened banks very susceptible for further rounds attack.

Most of the empirical papers mentioned above focus on the idiosyncratic attack on a random bank in the system. Although this is feasible in real life, it is more often seen that a part of the banking system experiences a systemic shock, especially for those banks that have correlation with each other, for example investing in similar goods or industry. As Elsinger, Lehar, and Summer (2006a), Elsinger, Lehar, and Summer (2006b) and Frisell, Holmfeld, Larsson, Omberg, and Persson (2007) studying the Austrian, UK and Swedish banking system, point out that the study of systemic risk in the interbank market should not limit itself to the bilateral interbank exposures alone. This is because common exposures that some, if not all, banks face play an important role in the stability of the system. Elsinger, Lehar, and Summer (2006a) find that the correlation in banks' asset portfolios dominates contagion as the main source of systemic risk. And Elsinger, Lehar, and Summer (2006b) argue that those empirical studies that ignore the common exposures are likely to have underestimated the probability and severity of financial contagion. Upper (2011) provides a detailed summary and comparison of the methodologies, data and findings of the available empirical research on individual interbank market.

Halaj and Kok (2013) attempt to use a different approach to tackle the problem. They collect the aggregate interbank exposure of 27 national central banks of the European Union and the European Central Bank. Using all these aggregate interbank exposures, they generate many different possible interbank networks and investigate the severity of systemic risk in different network structures. They find that some network structures are less conducive to contagious default. Although these structures not necessarily coincidence with the real-world interbank network structure, their findings do raise the question of whether a shift in real-world interbank network structure can increases the resistance of the system against future attack.

2.3.2 Problems

Although the literature review on the empirical studies of the systemic risk in the interbank markets above has broadened us with the understanding on the network structures of the interbank market in various countries and the likelihood and seriousness of contagious defaults via interbank exposures, these analyses are not without their problems.

First of all, each of these analyses is based of the interbank exposure data in a specific country over a specific period of time under examination. Therefore, the results and conclusions from these empirical studies are difficult to be generalised and to be applied to other countries that were not examined. Secondly, as mentioned previously, since the bilateral interbank exposures between banks are often not available, therefore, a majority of these papers suffer from data limitation in obtaining the complete bilateral interbank exposure matrix. Some papers choose to focus on a particular segment of the market include Furfine (2003), Amundsen and Arnt (2005) and Iori, De-Masi, Precup, Gabbi, and Caldarelli (2008) and they all use the data on overnight transactions. Hence, their analyses may have underestimated the potential of contagion by ignoring the rest of the exposures. Most of the other papers use the maximum entropy techniques in estimating the complete bilateral matrix. Mistrulli (2011) test the method of maximum entropy techniques against observed bilateral interbank exposures. The author finds that the method has a tendency in underestimating the potential of contagion. This is because the maximum entropy method relies on the assumption that the banks spread their interbank borrowing and lending as widely as possible across all other banks. But the author also points out that the maximum entropy technique can occasionally overvalue the severity of contagion, depending on features like the network structure of the interbank market under examination, banks' recovery rates and their capitalisation. Overall, the paper shows that the method introduces bias into the analysis. The finding is also supported by Frisell, Holmfeld, Larsson, Omberg, and Persson (2007), in which the author shows that the bilateral interbank exposures between banks exhibit high degree of asymmetry. This contradicts with the assumption in the maximum entropy method and challenges the validity of the use of the method. Thirdly, most of these studies use a specific value for the loss-given-default (LGD) to generate sequential default. Although they run the simulations for different values of LGD, they assumed it to be constant over time. Unfortunately, Memmel and Sachs (2013) show that the assumption of a constant LGD is not realistic and may introduce bias into the analysis. They find that the assumption has two opposing effects on the system depending of its stability. They find that, for a rather stable system, the assumption of a constant LGD tends to underestimate the extent of contagion, whereas for a rather unstable system, the assumption of a constant LGD tends to overestimate the extent of contagion. Fourthly, Elsinger, Lehar, and Summer (2006b) demonstrate that the incorporation of both interbank exposure and common exposure as systemic shock affects the assessment of the potential and severity of contagion in the interbank market. The failure in capturing common exposure and correlation between banks are likely to have resulted in an under-evaluation of financial contagion. Finally, the stability of the interbank market is driven by many determinants, but not solely by its topology. Haldane and May (2011), Cont, Santos, and Moussa (2013) and Sachs (2013) argue that the number, the size and the distribution of bank's interbank linkages are important determinants and affect the stability of the interbank system. Memmel and Sachs (2013) also find that bank's capitalisation is identified as a crucial determinant. Martínez-Jaramillo, Alexandrova-Kabadjova, Bravo-Benitez, and Solorzano-Margain (2012) illustrate that the probability distribution of the initial shocks, the size of the losses and the correlation level of joint failures are also important factors.

2.4 Theoretical studies

In consequence of the restrictions and problems with the empirical work, another strand of study in the systemic risk of the interbank market is from a theoretical perspective. There are many theoretic work on the financial system. However, I will limit our attention to those focusing specifically on the interbank market. In broad, these studies create different network structures in representing the interbank linkages between banks and investigate the potential of contagion through different forms of idiosyncratic or systemic shock. One of the advantages of these studies is that they can investigate lots of different network topologies, comparing their stability and efficiency. By doing so, they can also study the effect of those determinants that govern the severity of a contagious default in the interbank system and they can arrive at some general results for any specific network.

Iori, Jafarey, and Padilla (2006) create a banking system in which banks are randomly connected. They simulate the use of interbank market as a result of a shortage in liquidity in banks. This liquidity shortage arises from the fluctuation in the customers' deposits. They first study the effect of the connectivity parameter in a random network structure with homogeneous banks and show that there is a monotonic decrease in the default of banks as the connectivity increases. They also show that increasing a reserve requirement produces a non-monotonic effect on the stability of the system and explain that the increase in reserve level interact with banks on an individual level, allowing them to be better insulated against liquidity shortage, and with the banking system as a whole, restricting the interbank activity. They later introduce heterogeneity in the system, in terms of size of bank and size of investment opportunity and illustrate that heterogeneity seems to increase the potential for contagion. Nier, Yang, Yorulmazer, and Alentorn (2008) and Gai and Kapadia (2010) also look specifically at a random network structure of the interbank market. But they differ from Iori, Jafarey, and Padilla (2006) in a number of of ways. Firstly, they consider a static model of interbank market. Secondly, they assume an amount of total interbank assets and allocate them evenly among the number of links in the system. Thirdly, they consider an idiosyncratic shock that hit one of the banks at random and remove all of its external assets. Nier, Yang, Yorulmazer, and Alentorn (2008) investigate the interaction between a number of determinants in the system and the their effects on contagion. They show that connectivity or interbank connection has two opposing effects on the system. This is because the access of the interbank market not only allow deficit banks to borrow liquidity from others, it also exposures the creditor banks to contagious risk when the lender banks default. They are unable to obtain an optimal value of connectivity that gives the least default in the system. They also show the importance of banks being capitalised, giving them more protection against the contagious default from the bank that is hit by the shock. Gai and Kapadia (2010) supports this finding and show that if banks hold a big amount of capital as their buffer, then they are more resistant against the domino-effect from the initial defaulting bank and hence, contagion rarely occurs. However, Gai and Kapadia (2010) take a step further and introduce an aggregate shock that affects the capital holdings of all banks in the system. They show that in the presence of the aggregate shock, which weakens the banks' capital buffer, banks become very susceptible to contagious default if they have interbank connections with a bank that is hit by the idiosyncratic shock.

As an extension of the model, Nier, Yang, Yorulmazer, and Alentorn (2008) also consider a tiering structure, in which there is one large bank and 24 small peripheral banks, all connecting to the large one. However, their analysis is very brief and contribute little to the understanding of the specifics of the structure and its stability against systemic risk. Georg (2013) designs a dynamic model of an interbank market with a central bank and considers three different network structures, namely the random network, the small-world network and the scale-free network, summarised by Newman, Barabási, and Watts (2006). The paper incorporates both an idiosyncratic shock arising from change in customers deposit and a common shock arising from risky investment. It illustrates that contagion effect is most severe on a random network and least severe on a scale-free network while the effect on a small-world network is somewhere in between. The findings on the effect of common shock on the system coincidences with Gai and Kapadia (2010) that knocking off part of banks' capital results in a weakening impact on the system and makes banks more vulnerable to contagion.

Unlike the above, Ladley (2013) considers a circular network structure of banks and the distance between two entities governs the ability of one bank being able to borrow from the others through the interbank market. In this study, the idiosyncratic shock is a complete wipe out of a bank's equity and reserve. It shows that the stability of the system is at its weakest at low-intermediate levels of connectivity. This coincidences with the finding from Gai and Kapadia (2010). Ladley (2013) also investigate the relationship between connectivity and contagious defaults in the presence of a systemic shock, which affects the probability of a bank's investment project to mature successfully and shows that the severity of contagion depends on the size of the systemic shock. He concludes that there is not an optimal connectivity that gives the least spread of contagion. This is because the size of the shock is unknown. The paper then turns the attention to the role of regulation in an interbank market and how can regulation be used to minimise the impact of contagion. It first shows that an increase in the level of equity ratio can reduce the contagious defaults significantly. The drawback is that it also reduces the interbank transaction. It next examines the reserve ratio and shows that tightening the reserve requirement increases the severity of contagion. The explanation is that although an increase in the reserve ratio demands banks to retain a higher proportion of their customers' deposit as reserve, this also leads to an increase in interbank borrowings for investment. Therefore, the increase in interbank activity exposes the banking system to contagious defaults.

2.5 Conclusion

This chapter surveys the literature on the study of the financial contagion in the interbank market. It first illustrates the use of network topology to represent the network structure of interbank market. It then presents the empirical and theoretical approaches in examining the financial contagion in the interbank market.

The empirical studies show that the most commonly found network structure in the real-world interbank markets are scale-free network and tiering network (or sometimes referred as money-centre network). Although the majority of these studies find that the probability of a contagion spread via interbank exposures is low, it is nevertheless possible and can be quite severe when it happens. Besides, it has been pointed out that these empirical work suffers from limitations and may underestimate the potential and severity of contagious defaults. Therefore, the second strand of literature which attempts to study the interbank market from a theoretical perspective complements the drawbacks in empirical studies. In the past decade, there has been a growth in using network topology in studying financial networks. However, when the research first began, there were not many theoretical models that study specifically on the interbank market. For example, Iori, Jafarey, and Padilla (2006) and Nier, Yang, Yorulmazer, and Alentorn (2008) both consider a random network structure for the interbank market. Therefore, this research takes Iori, Jafarey, and Padilla (2006) as the starting point.

Chapter 3

The Basic Model: Iori, Jafarey, and Padilla (2006)

3.1 Introduction

Iori, Jafarey, and Padilla (2006) simulate interbank transactions between banks to deal with the liquidity shortage. This research aims to use a similar approach to study a banking system with interbank transactions, therefore this chapter presents the implementation of Iori, Jafarey, and Padilla (2006)'s model.

3.2 Definition of liquidity

In banking, the term liquidity of a bank refers to its availability of liquid cash to meet financial obligations when they come due. Financial obligations can be of short-term and long-term. While some of them have an agreed due date, others can be exercised by the creditors at any time. Hence, banks must maintain a sufficient amount of liquidity within themselves. A bank's liquidity can be interpreted as its liquid assets. In financial accounting, the mathematical formula for balance sheet is given by

$$Equity = Assets - Liabilities$$

Liabilities of a bank refer to loans that are deposited in the bank by other entities, including individual customers, companies or other banks while assets can consist of bank's investments, loans lend to other parties and reserves. Some of these assets like investments or loans are illiquid whereas reserves are liquid. Therefore, the term 'assets' is split into two terms in the formula.

$$Equity = (Liquid\ assets + Illiquid\ assets) - Liabilities$$

Liquid assets consist of reserves and any other liquid assets that a bank holds, for example cash holdings. Therefore, liquidity refers to all these liquid assets. The mathematical formula can be refined as

$$Equity = (Liquidity + Illiquid\ assets) - Liabilities$$

By rearranging the above formula, we can express the term liquidity as

$$Liquidity = Equity + Liabilities - Illiquid\ assets$$

Therefore, the definition of 'liquidity' of a bank in the model is considered as the sum of the bank's equity and liabilities net its illiquid assets.

3.3 Overview of the model

Having defined the term 'liquidity', this section provides an overview of Iori, Jafarey, and Padilla (2006)'s model. This model simulates a system of banks that uses interbank transactions to manage their liquidity needs. It investigates both the positive and negative effects of the usage of interbank transactions to the banking system.

In this model, banks receive deposits from customers on a daily basis and the amount of deposits fluctuates from one day to another. Deposits made are short-term and customers have the right to withdraw their deposits at any time.

As a result, liquidity shortage can arise from the change in deposit and withdrawal patterns of customers. Hence, banks must maintain a certain level of liquid resources within themselves. At the same time, banks invest a portion of these funds into productive projects. In general, investment projects are usually long-term and the resources invested are often tied up with the investment, in the sense that banks are unable to retrieve those resources until the investment matures. Hence, these resources are classified as illiquid. It is of the banks' interest to maintain a balance between their liquid and illiquid resources. However, if the overall deposit is negative, meaning that there are more withdrawal than deposits, and to the extent that it is more than the liquid reserve a bank keeps, then that bank would not be able to meet its customers' demand.

In the absence of an interbank market, banks with liquidity shortage are classifed as default. However, in the presence of an interbank market, deficit banks are allowed to borrow funds from other banks through interbank transactions to deal with their liquidity shortage. If banks are unable to request and arrange enough interbank loans, then their inability to meet their customers' demands will result in default. This model studies the incorporation of interbank transactions among banks.

Figure 3.1 and 3.2 present the flowchart of how banks operate in the absence and presence of interbank transactions respectively.

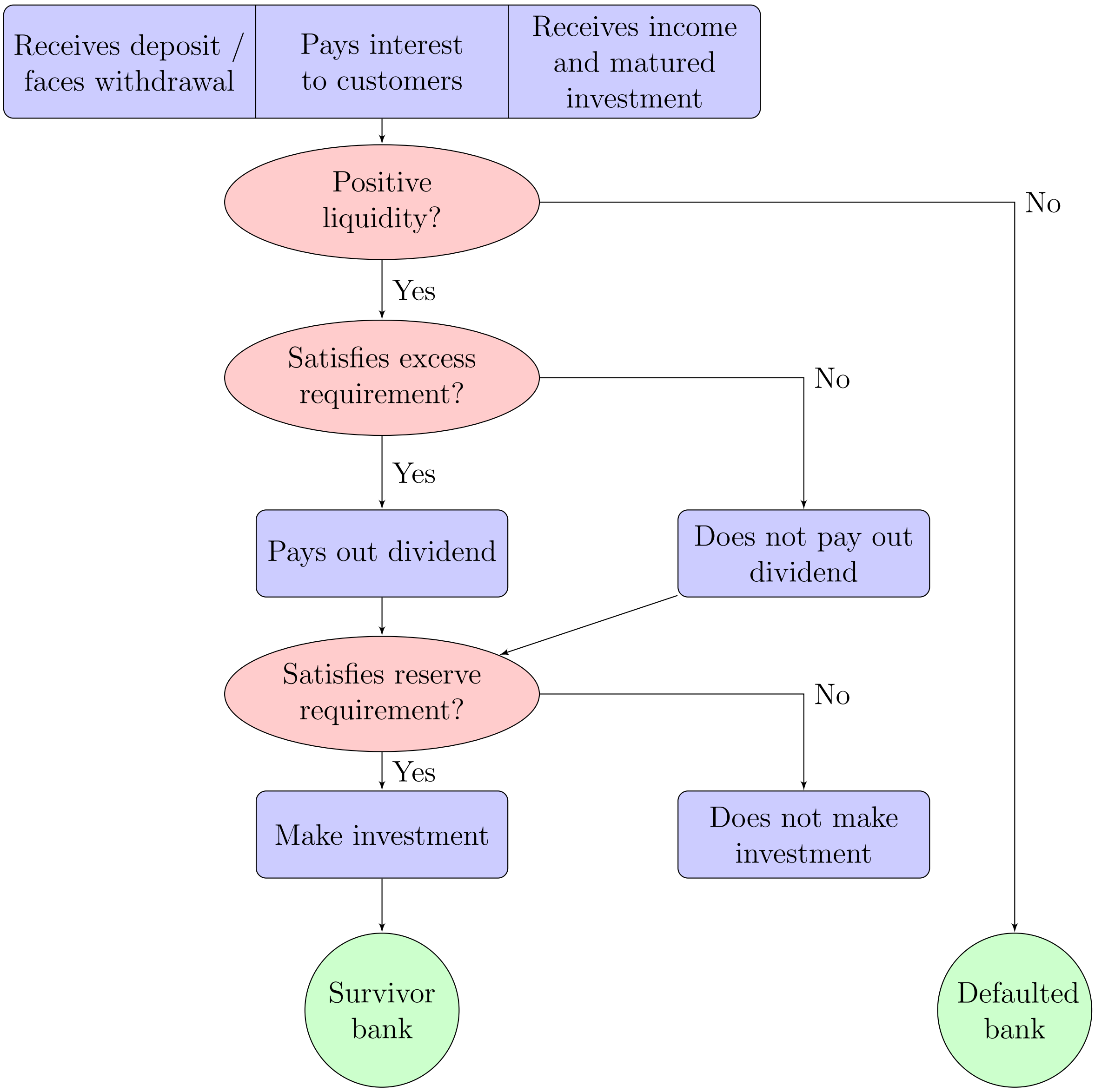


Figure 3.1: How banks operate in the absence of interbank transactions

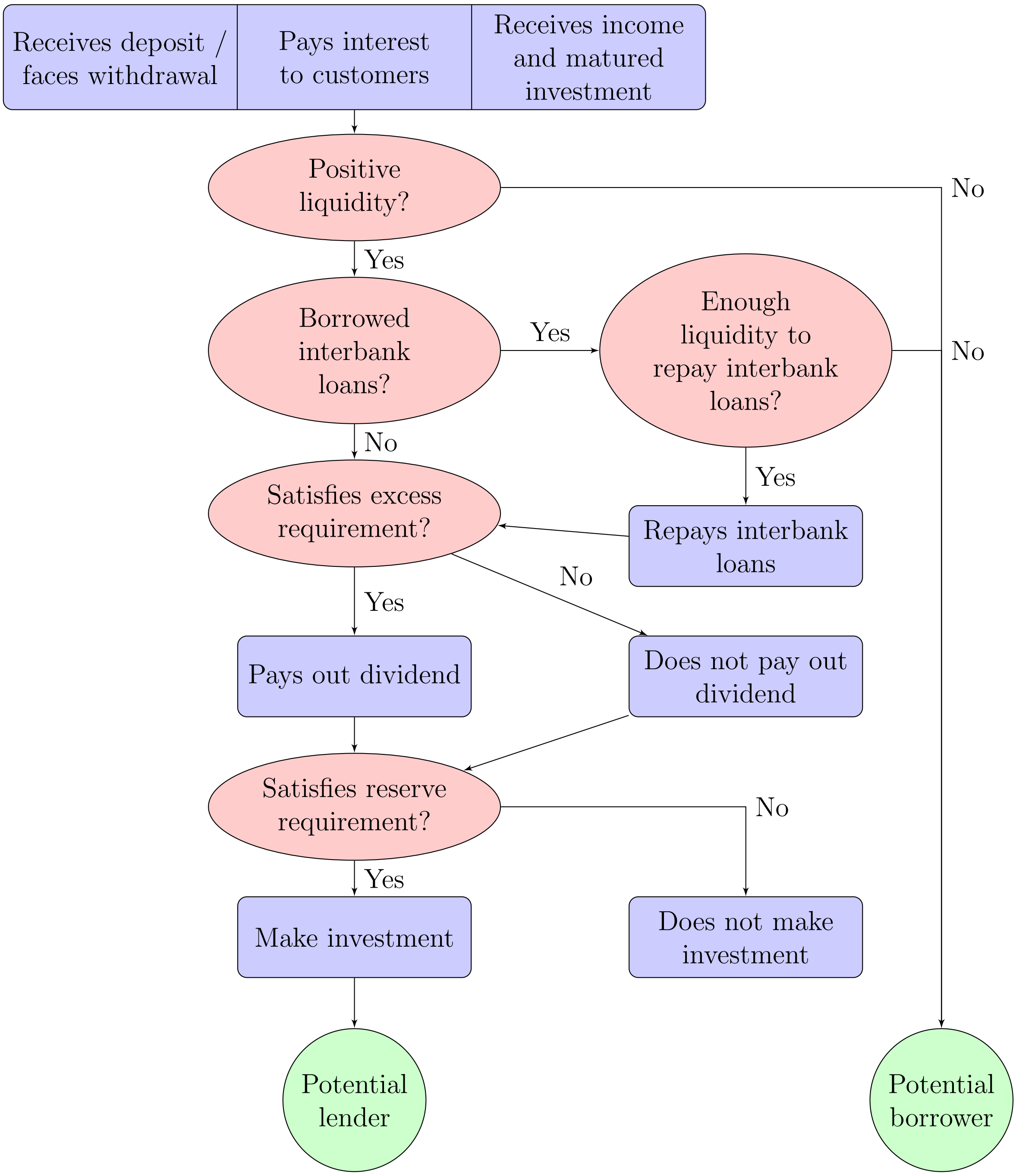


Figure 3.2: How banks operate in the presence of interbank transactions

3.4 Implementation

3.4.1 Construction of a banking system

In order to implement Iori, Jafarey, and Padilla (2006) model, a system of banks has to be constructed.

Let's consider a system of M banks. Relationships between these M banks are presented in a connectivity matrix, J_{ij} for $i = 1, 2, ..., M$ and $j = 1, 2, ..., M$. Each element in this connectivity matrix can only take one of the two values, 0 or 1 and the value it takes is used to define whether the two banks are connected to each other.

$J_{ij} = 1$ indicates that bank i and bank j are in connection and interbank transactions can be arranged between themselves while $J_{ij} = 0$ represents that bank i and bank j are not linked to each other and hence they cannot arrange interbank transactions. Below is a display of a connectivity matrix for a system of 4 banks.

$$\begin{pmatrix} J_{11} & J_{12} & J_{13} & J_{14} \\ J_{21} & J_{22} & J_{23} & J_{24} \\ J_{31} & J_{32} & J_{33} & J_{34} \\ J_{41} & J_{42} & J_{43} & J_{44} \end{pmatrix}$$

It is assumed that 'relationship' is non-directional, therefore, if two banks are connected, then both parties can be a borrower or a lender in an interbank transaction. A bank cannot arrange interbank transaction with itself, therefore, $J_{ij} = 0$ if $i = j$. Therefore, the connectivity matrix can be simplified into

$$\begin{pmatrix} 0 & J_{12} & J_{13} & J_{14} \\ J_{12} & 0 & J_{23} & J_{24} \\ J_{13} & J_{23} & 0 & J_{34} \\ J_{14} & J_{24} & J_{34} & 0 \end{pmatrix}$$

At the very beginning of the simulation, the values for the connectivity matrix J_{ij} $\forall i \neq j$ are generated randomly. The parameter C denotes the probability that $J_{ij} = J_{ji} = 1$ for any two banks i and j. For the purpose of simulations, a random number will be drawn for each pair of bank i and j. If the random number is less than or equal to C, then bank i and j are connected. On the contrary, if the random number drawn exceeds C, then bank i and j are not connected. Since matrix J is a square, symmetrical matrix, one only has to generate random numbers to determine the elements in either the upper or lower triangular matrix, excluding the diagonal, but not both.

3.4.2 Model operation in the absence of interbank market

The model operates in discrete time, which is denoted by $t = 0, 1, 2, ...$. The list of events that take place at different times of a day in the absence of interbank transaction is summarised below.

When a bank opens, it first receives deposits and withdrawals from its customers. It then pays out interest to the depositors in the previous period. After that, it receives the income from investment made in the last τ periods, together with the initial capital from the matured investment made in $t = -\tau$. When the bank closes, it pays out dividend and undertakes new investment, provided that it satisfies some regulatory requirements. In the absence of interbank transaction, a bank defaults if it has negative liquidity at the end of the day.

Initiation of bank size and average investment opportunity

At the very beginning of time, that is, $t = 0$, there are M number of banks in the system. Each of these banks is denoted by k, where $k = 1, 2, ..., M$.

The size of bank k is denoted by S^k and the heterogeneity in bank size is given by

$$S^k = |\bar{S} + \sigma_S \upsilon| \tag{3.1}$$

where $\bar{S}$ is the average bank size, σ_S is the standard deviation of bank size across banks and $\upsilon \sim N(0,1)$. The size of bank k's average investment opportunity is denoted by $\bar{O}^k$ and the heterogeneity in investment opportunity across banks is given by

$$\bar{O}^k = \delta \times |S^k + \sigma_O \nu| \tag{3.2}$$

where $\nu \sim N(0,1)$ and σ_O is the standard deviation of average investment opportunity across banks and δ is the aggregate investment opportunity : aggregate bank size ratio with $0 < \delta < 1$. The banks' sizes and their average investment opportunities are generated before the initial period.

Receipt or withdrawal of deposit

Some notations are given below: D_t^k represents the total deposit made by the customers in bank k at time t, E_t^k denotes bank $k's$ equity at time t and I_t^k denotes the total amount of investment made by bank k in the last $t - \tau$ periods.

In each period, a number of activities take place. For simplicity, these activities are assumed to follow a specific sequence. At the beginning of each period t, each bank possesses certain amount of liquid assets, depending on the deposit it received and the investment it made in the previous period $t-1$. Bank k's initial liquidity is given by

$$L_{t-1}^k = D_{t-1}^k + E_{t-1}^k - \sum_{s=1}^{\tau} I_{t-s}^k \tag{3.3}$$

Each bank k receives new deposits and withdrawals from its customers. The total amount of deposits net withdrawals is given by D_t^k. This amount changes everyday and its fluctuation is random, and according to Iori, Jafarey, and Padilla (2006), it is modelled in one of the two ways. Firstly, the fluctuation in deposits for bank k is proportional to the square root of its size and is specified by

$$D_t^k = |S^k + \sigma_D \sqrt{S^k} \epsilon_t| \tag{3.4}$$

Alternatively, the fluctuation in deposits for bank k is proportional to its size and is given by

$$D_t^k = |S^k + \sigma_D S^k \epsilon_t| \tag{3.5}$$

where $\epsilon_t \sim N(0,1)$. A bank then has to pay out interest to its depositors in the previous period. The interest rate for deposits is assumed to be fixed over time and across banks and it is denoted by r_D. Therefore, if bank k received a deposit of D_{t-1}^k at $t-1$, it has to pay out a total interest of $r_D D_{t-1}^k$ at t to its customers, regardless of withdrawal.

Receipt of investment income and matured investment

If a bank made an investment in any of the last τ periods, it receives an income from its investment. This income is assumed to be a fixed proportion of its investment, meaning that the investment is risk-free. This fixed proportion is denoted by r_I and is assumed to be constant over time and across banks. Therefore, at any time t, bank k receives an income of $r_I \sum_{s=1}^{\tau} I_{t-s}^k$. As an investment matures, a bank also receives the initial capital back. Therefore, if bank k invested $I_{t-\tau}^k$ in $t-\tau$ period ago, it receives $I_{t-\tau}^k$.

This completes the description of the events that take place when a bank opens. The liquidity at closing is given by

$$\hat{L}_t^k = L_{t-1}^k + (D_t^k - D_{t-1}^k) - r_D D_{t-1}^k + r_I \sum_{s=1}^{\tau} I_{t-s}^k + I_{t-\tau}^k \tag{3.6}$$

The equity of bank k at closing, $\hat{E}_t^k$, is given by

$$\hat{E}_t^k = \hat{L}_t^k + \sum_{s=1}^{\tau-1} I_{t-s}^k - D_t^k \tag{3.7}$$

If the change in net deposit for bank k between $t-1$ and t is negative and the magnitude is large enough, bank k can result in negative liquidity. In the absence of interbank market, the bank with negative liquidity does not have any channels to borrow funds to recover its liquidity shortage and is therefore considered as default.

Regulatory requirement

For those banks that survive this current period, they close and two events take place. If a bank is performing well and it is making profit from its investment, it is required to pay out dividend to its shareholders. However, it is crucial for the bank to satisfy some regulatory requirements. The first one that Iori, Jafarey, and Padilla (2006) consider is "excess" return, which is denoted Ex_t^k. This "excess" return is calculated by dividing its equity by its current deposit holdings and therefore Ex_t^k at any time t is given by

$$Ex_t^k = \hat{E}_t^k / D_t^k \tag{3.8}$$

In order to satisfy this requirement, the "excess" return must exceed the target capital:deposit ratio, χ. Secondly, a bank should maintain some liquid resources within itself and these resources are classified as reserves. These reserves can be used in unforeseen circumstances. At any time t, a bank's reserve requirement is calculated as a portion of its current deposits from customers and it is given as

$$R_t^k = \beta D_t^k \tag{3.9}$$

where β is the reserve requirement ratio.

Dividend payment and investment

Hence, bank k can only pay out dividend if equation (3.8) satisfies the target capital:deposit ratio. And when it does, the actual dividend payment is given by

$$Div_t^k = \max\{0, \min[r_I \sum_{s=1}^{\tau} I_{t-s}^k - r_D D_{t-1}^k, \hat{L}_t^k - R_t^k, \hat{L}_t^k + \sum_{s=1}^{\tau-1} I_{t-s}^k - (1+\chi)D_t^k]\} \tag{3.10}$$

There are three terms in the minimum bracket in the above formulation. The first term $r_I \sum_{s=1}^{\tau} I_{t-s}^k - r_D D_{t-1}^k$ refers to the net profit of the bank. The second term $\hat{L}_t^k - R_t^k$ represents the available liquidity net reserve requirement. The third term $\hat{L}_t^k + \sum_{s=1}^{\tau-1} I_{t-s}^k - (1+\chi)D_t^k]$ can be rewritten as $\hat{E}_t^k - \chi D_t^k]$ which ensures that the bank satisfies the excess return.

However, if it does not meet the regulatory requirements, it does not pay out any dividend. Therefore if $Ex_t^k \leq \chi$, then $Div_t^k = 0$. After making dividend payment, the liquidity of a bank gets updated and it is

$$\check{L}_t^k = \hat{L}_t^k - Div_t^k \tag{3.11}$$

Since a bank has to maintain a minimum reserve level, its available liquidity for investment is given by its current liquidity net its reserves, that is, $\check{L}_t^k - R_t^k$. Then, banks can consider investing in fresh investment projects. The average investment opportunity for each bank, $\bar{O}^k$, is generated at the beginning of the simulation and is given in equation (3.2). In each period t, the actual maximum investment opportunity, ω_t^k, available to each bank k fluctuates and the fluctuations are proportional to its average investment opportunity. This is given as

$$\omega_t^k = |\bar{O}^k + \sigma_\omega \bar{O}^k \eta_t| \tag{3.12}$$

where σ_ω is the standard deviation of the fluctuation in investment over time and $\eta_t \sim N(0,1)$

Undertaking new investment is a beneficial way for a bank to make profit. Ideally, a bank would invest up to its maximum investment opportunity. However, investment does not mature until τ period later and hence when resources are used for investment, they are classified as illiquid. For this reason, a bank must not use its reserve for investment. Therefore, bank k will undertake new investment based on its maximum investment opportunity and its available liquidity and this is given by

$$I_t^k = \min[\max(0, \breve{L}_t^k - R_t^k), \omega_t^k] \tag{3.13}$$

After the dividend is paid and new investment is made, it comes to the end of the period t. The liquidity and equity of bank k after dividend and investment are given respectively as

$$L_t^k = \tilde{L}_t^k - I_t^k \tag{3.14}$$

$$E_t^k = L_t^k + \sum_{s=0}^{\tau-1} I_{t-s}^k - D_t^k \tag{3.15}$$

This completes the description of how the bank system operates in the absence of interbank market. The whole cycle will re-start again for the next period, except for those banks that default.

3.4.3 Introduction of an interbank market

The introduction of an interbank market does not change the list of events that take place during the day. However, when the bank closes, the first priority of the bank is to repays its interbank loans to its lenders (if it is able to repay in full). It then pays out dividend and undertakes new investments, again assuming that it satisfies the regulatory requirements. If the bank has liquidity shortage, it can attempt to arrange and borrow through interbank market. It then repays any outstanding interbank loans from previous period.

In the presence of interbank market, a bank can borrow (lend) interbank loans from (to) its connected banks. Therefore, at the beginning of each period t, the amount of liquid assets a bank possesses not only depends on the deposit it received and the investment it made in the previous period $t-1$, but also the interbank loans it borrowed (lent) in the previous period. The total interbank borrowing and lending of bank k at any time t are denoted by IB_t^k and IL_t^k respectively. Therefore, unlike equation (3.3), at the beginning of the period, bank k's liquidity is given by

$$L_{t-1}^k = D_{t-1}^k + E_{t-1}^k + IB_{t-1}^k - IL_{t-1}^k - \sum_{s=1}^{\tau} I_{t-s}^k \tag{3.16}$$

The events that take place during the day are identical regardless of the presence of interbank transactions. Therefore, a bank receives deposit and withdrawals from its customers, pays out interest to the depositors in the previous period, receives income from investment and receives the initial capital when an investment matures. After these events, the liquidity of a bank is the same as equation (3.6).

Repayment of interbank loans

Banks close at the end of the day. In the presence of interbank transactions, banks' first priority now is to repay their interbank loans in full if there are any. If bank k arranged any interbank borrowings in the previous period, it must attempt to repay its creditors at this stage. IT_{t-1}^{kj} denotes the interbank transaction between bank k and j at $t-1$. If IT_{t-1}^{kj} is positive, it means that bank k borrowed from bank j and if IT_{t-1}^{kj} is negative, it means that bank k lent to bank j. Besides, $IT_{t-1}^{kj} \equiv 0$ if $k = j$

The interbank transactions between banks can be presented in a matrix. Below is a illustration of a system of 4 banks.

$$\begin{pmatrix} 0 & IT_{t-1}^{12} & IT_{t-1}^{13} & IT_{t-1}^{14} \\ IT_{t-1}^{21} & 0 & IT_{t-1}^{23} & IT_{t-1}^{24} \\ IT_{t-1}^{31} & IT_{t-1}^{32} & 0 & IT_{t-1}^{34} \\ IT_{t-1}^{41} & IT_{t-1}^{42} & IT_{t-1}^{43} & 0 \end{pmatrix}$$

There are always two counter-parties in an interbank transaction, therefore, if bank k borrowed from bank j, it means that bank j lent to bank k, and so $IT_{t-1}^{kj} = -IT_{t-1}^{jk}$. Hence, the interbank transaction matrix can be simplified into

$$\begin{pmatrix} 0 & IT_{t-1}^{12} & IT_{t-1}^{13} & IT_{t-1}^{14} \\ -IT_{t-1}^{12} & 0 & IT_{t-1}^{23} & IT_{t-1}^{24} \\ -IT_{t-1}^{13} & -IT_{t-1}^{23} & 0 & IT_{t-1}^{34} \\ -IT_{t-1}^{14} & -IT_{t-1}^{24} & -IT_{t-1}^{34} & 0 \end{pmatrix}$$

It is assumed that a bank can either be a creditor or a debtor, but cannot be both creditor and debtor in the same period. This means that, for a bank k, the values of all IT_t^{kj} $\forall j \neq k$ should take the same sign at any time t.

$$\sum_{j=1}^{M} IT_t^{kj} = \begin{cases} IB_t^k & \text{if } k \ \textit{borrows} \\ -IL_t^k & \text{if } k \ \textit{lends} \end{cases}$$

At this stage, it is assumed that it is the borrower banks' highest priority to repay their interbank loans from the previous period in full if possible. If they are unable to repay their debt in full, it is assumed that those banks do not repay for the time being, but instead they issue debt certificates to their creditors. These debt certificates are to be redeemed before the beginning of next period and they should still reflect in the banks' liquidity. Therefore, if bank k was a borrower, regardless of whether it can repay or it cannot repay but issue debt certificates, its liquidity is

$$\breve{L}_t^k = \hat{L}_t^k - (1 + r_B) IB_{t-1}^k \tag{3.17}$$

However, if bank k was a lender, its liquidity holding only gets updated when it receives the repayment from its debtors. For example, if bank j borrowed from bank k and it repays now, then the liquidity of bank k is given by

$$\breve{L}_t^k = \hat{L}_t^k + (1 + r_B) IT_{t-1}^{kj} \tag{3.18}$$

After this, the equity for any bank k is

$$\breve{E}_t^k = \breve{L}_t^k + \sum_{s=1}^{\tau-1} I_{t-s}^k + IB_t^k - IL_t^k - D_t^k \tag{3.19}$$

Dividend payment and investment

Banks then pay out dividend and make new investments. The description for these two events is the same as before except for formulation for dividend payment. Therefore, equations (3.8) to (3.15) remain unchanged except equation (3.10). Previously, the dividend payment is made with respect to a bank's profit, that is, its income from investment net the interest it pays to depositors, subject to the reserve requirement and the target capital:deposit ratio. However, when there exists interbank transactions, the interbank interest that a bank receives (pays) for being an interbank lender (borrower) is considered as profit (loss) and therefore should be taken into account in the formulation of dividend payment. As a result, the refined formula for dividend payment therefore is

$$Div_t^k = \max\{0, \min[r_I \sum_{s=1}^{\tau} I_{t-s}^k + r_B(IL_{t-1}^k - IB_{t-1}^k) - r_D D_{t-1}^k, \breve{L}_t^k - R_t^k, \breve{L}_t^k + \sum_{s=1}^{\tau-1} I_{t-s}^k - (1+\chi) D_t^k]\} \tag{3.20}$$

Hence, equation (3.10) is replaced by (3.20).

Arrangement of interbank transactions

In the absence of interbank transaction, the description would stop here and banks with negative liquidity would have to default. Luckily, if banks are allowed to attempt to borrow from others, they have a chance to survive if they are able to arrange enough interbank transaction(s) from one or more of its connected banks. For simulation purpose, it is assumed that a bank k with liquidity need approaches its connected banks j in a random manner. The two banks then negotiate for the amount to be transacted. This amount is assumed to be the minimum value between the liquidity need of bank k and the available liquidity of bank j net its reserve. This is given by

$$IT_t^{kj} = \min[|\breve{L}_t^k|, \breve{L}_t^j - R_t^j] \tag{3.21}$$

The arranged interbank loans do not get transacted until bank k manages to line up enough loans to cover its liquidity need. When it does, the liquidity holdings for both the borrower (bank k) and its lenders are updated accordingly. It is assumed that bank k only borrows what it needs and therefore, its liquidity becomes $L_t^k = 0$. For its creditors, their liquidity will become

$$\grave{L}_t^j = \breve{L}_t^j - IT_t^{kj} \tag{3.22}$$

There are only two ways in which a bank can end up with liquidity need at the end of the 'day time'. Firstly, if banks face a big negative change in net deposit, this can result in a negative liquidity. But if these banks manage to borrow interbank loans, they survive this current period and nothing more needs to be done.

However, the second case is that if banks have a low but positive liquidity resulting from the change in net deposit and if this prevents them from repaying their interbank loans from the previous period, they would have ended up with negative liquidity and have issued debt certificate to their creditors. In this case, when these banks manage to arrange enough interbank loans, they have to repay their interbank loans to their creditors from the previous period by redeeming these debt certificates. As a result, the liquidity holdings of these creditors should be updated. If bank j lent interbank loans to bank k in the previous period and bank k failed to repay the loans in full earlier, the liquidity of bank j is now

$$L_t^j = \grave{L}_t^j + (1 + r_B)IT_{t-1}^{kj} \tag{3.23}$$

The process of arranging interbank transaction, giving by equation (3.21) to (3.23), keeps reiterating until either there are no more banks with negative liquidity or there are no more potentials lenders available to banks with liquidity need. In the former case, all banks survive this current period and proceed to the next period whereas in the latter case, those banks which fail to borrow default and are removed from the system. For simplicity, the liquidation of a defaulted bank is not considered in the model. Therefore, a credit bank j which lent interbank loans to the defaulted bank k is assumed to suffer a loss of $(1 + r_B)IT_{t-1}^{kj}$ because they are unable to receive the interbank transaction plus interest.

3.5 Conclusion

In order to study the stability of the banking system in the presence of an interbank market and to investigate the role of the reserve requirement and capital adequacy requirement, this research has taken the approach to simulate the interactions between banks in an artificial model. When the research was first carried out, there were not many other researches that consider constructing an artificial banking system, in which banks are connected to each other via interbank transactions and aim to investigate the systemic risk in the interbank market. To my best knowledge, both Iori, Jafarey, and Padilla (2006) and Nier, Yang, Yorulmazer, and Alentorn (2008) have both taken this approach. However, Nier, Yang, Yorulmazer, and Alentorn (2008) consider a particular moment in which they randomly default one of the banks and study the knock-on effects on other banks in the system. Therefore, those banks connected to the defaulted bank either defaults too if the loss it incurs is large enough or it survives otherwise. Although this is not unheard of, the model does not allow the researchers to investigate how these banks may be able to prevent defaulting by making further interbank borrowings. In other words, the model simply assumes a particular setting of interbank transactions among banks rather than simulating the use of interbank transactions based on banks' needs. Beside, randomly defaulting a bank does not provide enough economical relevance. However, Iori, Jafarey, and Padilla (2006) consider a model of banks over a long period of time. In each period, banks face different fluctuation in customers' deposit. They are then allowed to make dividend payment and investment and transact with each other through the interbank market, all based on their liquidity position. This model allows the researchers to study the interactions of banks through the interbank market dependent on their needs and the default of banks is caused by the liquidity shortages which arise from the fluctuation in customers' deposit. Therefore, Iori, Jafarey, and Padilla (2006) model has been chosen to the first step to the research. Hence, the above sections provide a detailed description of the implementation of the Iori, Jafarey, and Padilla (2006) model.

The terminology 'liquidity' may have different meaning in different contexts. This chapter begins with a clear definition of liquidity for the purpose of this research and it will be adopted throughout. The definition of 'liquidity' of a bank in the model is considered as the sum of the bank's equity and liabilities net its illiquid assets. The chapter then proceeds in giving the description in constructing a banking system with a random network structure. It is assumed that banks are randomly connected to each other based on a parameter C. It then presents the daily operations of banks including the receipt of customers' deposit, paying out interest to customers, paying dividends to shareholders and making new investments. All the these events are assumed to take place in an order. Banks interact with each other through the interbank market when some banks face liquidity shortage. The aim of the implementation of this model is to understand how the banks can use the interbank market to deal with the liquidity problem, how frequently they use the interbank market and what are the potential consequences of the usage of such a market.

Throughout the implementation and simulation of the model, a number of problems emerged. Therefore, it is impossible to proceed with the implementation without some adjustments and modifications to the model. The next chapter first presents all of these issues together with a solution and it then shows some preliminary results.

Chapter 4

An Extended Model: Correcting Iori, Jafarey, and Padilla (2006)

4.1 Introduction

As mentioned in Chapter 3, there are a number of problems that I encountered when I tried to implement the model in Iori, Jafarey, and Padilla (2006). In this section, I discuss each other them separately with a modification solution.

4.2 Problems with the implementation

4.2.1 Fluctuation in deposits

In Iori, Jafarey, and Padilla (2006), the authors define two different ways in which the fluctuation in deposit can be modelled. As described earlier, the two different models in the fluctuation in deposits are proportional to the square root of their size and to their mean size respectively. The two fluctuations are given by equation (3.4) and (3.5) respectively and they are displayed below.

$$D_t^k = |S^k + \sigma_D \sqrt{S^k} \epsilon_t|$$

$$D_t^k = |S^k + \sigma_D S^k \epsilon_t|$$

Figure 4.1 illustrates the time series for both fluctuations given by equation (3.4) and (3.5). It shows that, if the fluctuation is proportional to the square root of the bank's size, then it simply fluctuates around the value S^k with small deviation. If this is used in the implementation to generate customers' deposit and withdrawals, then the change in daily deposits from customers would never have caused a bank to face liquidity shortage, and consequently, there would not be a need for interbank transactions and there certainly would not have any bank defaults. Therefore, in my implementation, the fluctuation in deposits is assumed to be proportional to the bank size only, that is,

$$D_t^k = |S^k + \sigma_D S^k \epsilon_t|$$

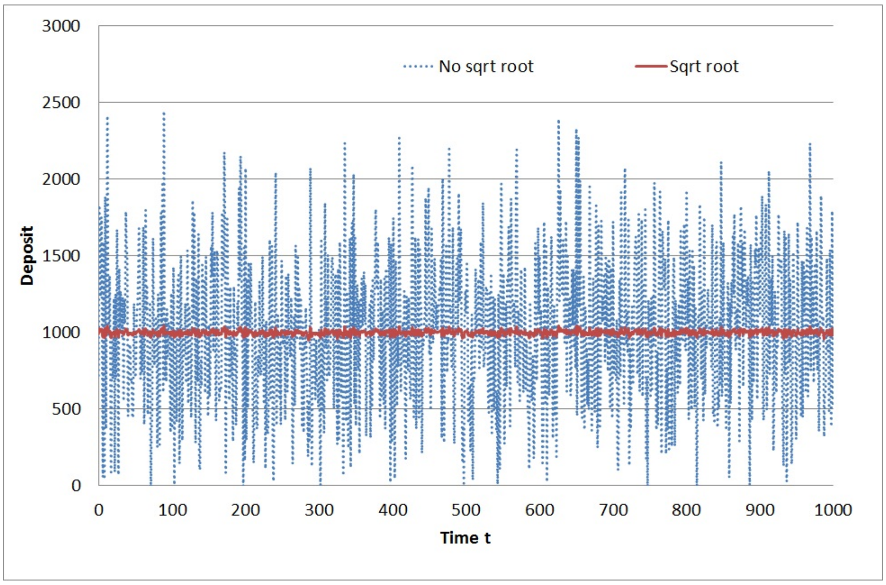
4.2.2 Capital:deposit ratio

The second problem encountered is the formula for capital:deposit ratio, which is known as the 'excess return' in Iori, Jafarey, and Padilla (2006). In the paper, the capital:deposit ratio is given by (3.8), which is

$$Ex_t^k = \hat{E}_t^k / D_t^k$$

From the above formulation, the excess return is calculated based on a bank's current equity divided by its customer deposit. Therefore, it can be interpreted as a behavioural rule for banks in determining the amount of dividend it should pay with respect to its profits. Although this formulation has its economical relevance, it does pose a problem in the simulation. The problem with this formulation is

Figure 4.1: The fluctuation in deposit proportional to either the square root of bank's size or the bank size with $T = 500$, $S^k = 1000$, $\sigma_D = 0.5$,



that when the deposit is large, the excess return is actually small and is unlikely to be able to exceed the capital:deposit ratio. As a result, the bank does not make any dividend payment. This increase in equity increases the bank size and makes it difficult to analyse and compare between banks. Therefore, it is proposed to modify the behaviour rule slightly such that it helps to keep the size of banks remain unchanged. After some considerations, the proposed modification is

$$Eq_t^k = \hat{E}_t^k / S^k \tag{4.1}$$

Therefore, the modified formulation can be interpreted as a behaviour rule that helps banks determine the dividend payment based on its excess equity from the investment income. The reasoning is as follows. In Iori, Jafarey, and Padilla (2006), they assume that, for any bank k, the initial deposit equals to its bank size, that is, $D_0^k = S^k$ and the initial equity equals to 0.3 times the initial deposit, that is, $E_0^k = 0.3 \times D_0^k$. In other words, the initial equity also equals to

0.3 times its bank size, that is, $E_0^k = 0.3 \times S^k$. Iori, Jafarey, and Padilla (2006) do not specify their choice of the value 0.3. However, this value coincides with the value they choose for χ, therefore my interpretation is that the initial equity of a bank equals to χ times its bank size. In the current model, the value for the equity at any time t only changes if one of the followings happens: a bank's equity increases if it receives income from its investment or the equity decreases if it fails to receive the interbank loan interest from its debtors.

Therefore, if the "excess return" is calculated as the ratio of the bank's current equity to its size and if this value is used to compare with χ, then at any time t, this can be used to evaluate a bank's equity, whether it is higher or lower than its initial period. One of the following cases would take place. Assuming that a bank makes some profits from its investments, then the income it receives contributes to the increase in the bank's equity. As a result, the "excess return" will be higher than χ, which means that this bank satisfies the regulatory requirement and it proceeds to make dividend payment. On the contrary, assuming that bank suffers a loss from the non-repayment of interbank loans from its debtors, then its equity decreases. This results in a value of "excess return" lower than χ, which means that the bank fails to satisfy the requirement and it does not pay out any dividend in this period. On some occasions, both factors will have an effect on the bank's equity at the same period. The two opposing effects should counteract one another and the net effect determines whether a bank pays out or retains the dividend.

This amendment manages to achieve the purpose of the regulation, that is, to allow a bank to make dividend payment when it earns income from its investment or retains dividends as bank's equity when it suffers a loss. However, for this reason, χ should no longer be named as the capital:deposit ratio. But instead it will now be called the equity requirement ratio. And this regulatory requirement will now be referred as the equity requirement. This is because it aims to monitor a bank's equity level.

The formula for dividend payment is given by (3.20) and is displayed below.

$$Div_t^k = \max\{0, \min[r_I \sum_{s=1}^{\tau} I_{t-s}^k - r_D D_{t-1}^k + r_B(IL_{t-1}^k - IB_{t-1}^k), \hat{L}_t^k - R_t^k, \hat{L}_t^k + \sum_{s=1}^{\tau-1} I_{t-s}^k - (1+\chi)D_t^k]\}$$

There are three terms in the minimum bracket in the above formulation. The first term $r_I \sum_{s=1}^{\tau} I_{t-s}^k - r_D D_{t-1}^k + r_B(IL_{t-1}^k - IB_{t-1}^k)$ refers to the net profit of the bank. The second term $\hat{L}_t^k - R_t^k$ represents the available liquidity net reserve requirement. The third term in the minimum bracket is introduced in Iori, Jafarey, and Padilla (2006) such that dividend payment does not violate the capital:deposit target. This is because it can be arranged as

$$\hat{L}_t^k + \sum_{s=1}^{\tau-1} I_{t-s}^k - (1+\chi)D_t^k = [\hat{L}_t^k + \sum_{s=1}^{\tau-1} I_{t-s}^k - D_t^k] - \chi D_t^k$$
$$\hat{L}_t^k + \sum_{s=1}^{\tau-1} I_{t-s}^k - (1+\chi)D_t^k = \hat{E}_t^k - \chi D_t^k$$

Since the regulatory requirement is now a ratio between current equity and bank size, instead of deposit, therefore, the third terms should be amended to $\hat{E}_t^k - \chi S^k$. As a result, the updated dividend payment formula is

$$Div_t^k = \max\{0, \min[r_I \sum_{s=1}^{\tau} I_{t-s}^k - r_D D_{t-1}^k + r_B(IL_{t-1}^k - IB_{t-1}^k), \hat{L}_t^k - R_t^k, \hat{E}_t^k - \chi S^k]\} \quad (4.2)$$

Therefore, the amount of dividend paid is determined by the amount of net profit a bank made in the last period, provided this amount does not violet the reserve requirement and the equity requirement.

4.2.3 Initial values for simulation

In Iori, Jafarey, and Padilla (2006), at $t = 0$, the authors choose the values of D^k_{-1}, E^k_{-1}, I^k_{-1}, I^k_{-2} and so on exogenously, but they do not specify what values they use in their simulations. However, the choice of some of these values, in particular those investments, are quite critical in initialising the simulations. The liquidity for any bank at the very beginning is given by (3.16)

$$L^k_{t-1} = D^k_{t-1} + E^k_{t-1} + IB^k_{t-1} - IL^k_{t-1} - \sum_{s=1}^{\tau} I^k_{t-s}$$

Therefore, if it is assumed that banks made lots of investments in the last τ periods at $t = 0$, then the liquidity of these banks are likely to be negative. In order to avoid this from happening, for simulation purposes, the customers' deposits in the previous period are assumed to be equivalent to the bank size, that is, $D^k_{-1} = S^k$ for all k and that all banks maintained their equity level at $t = -1$, that is, $E^k_{t-1} = \chi \times S^k$ for all k. It is also assumed that there were no interbank transactions before the initial period, that is, $IB^k_{t-1} = IL^k_{t-1} = 0$ for all k. Hence, the liquidity equation can be simplified into

$$L^k_{t-1} = S^k + \chi S^k - \sum_{s=1}^{\tau} I^k_{t-s}$$

In order to initialise the simulations, the liquidity from $t = -1$ has to be greater than or equal to zero for all banks, that is, all banks are not in default ($L^k_{t-1} \geq 0$), therefore the sum of the values for the previous τ investments must satisfy the following condition.

$$\sum_{s=1}^{\tau} I^k_{t-s} \leq (1 + \chi) S^k \tag{4.3}$$

4.3 Preliminary results

A couple of parameters will be studied. In order to be able to isolate and distinguish the individual effect of these parameters on the stability of the model, each of these parameters will be considered separately.

Firstly, a system of 400 banks (that is, $N = 400$) is studied. In the simulations presented below, unless otherwise stated, the parameters are set as follows: the size of all banks is $S^k = \bar{S} = 1000$, the maturity for investment is $\tau = 3$, the interest rate for deposit is $r_D = 0$, the investment return is $r_I = 0.01$, the equity requirement ratio is $\chi = 0.3$, the reserve requirement ratio is $\beta = 0.2$, the interbank interest rate is $r_B = 0.005$ and the aggregate investment opportunity : aggregate bank size ratio is $\delta = 0.5$.

4.3.1 The effect of connectivity C

I first examine the parameter connectivity and aim to study the benefits or drawbacks it has on the bank system. There are different possible ways to measure the advantages or disadvantages a parameter can have on the system. For the time being, I study the effect of connectivity on the stability of the bank system, in the sense of the number of banks fail to survive over a time period at different levels of connectivity.

The effect of connectivity C in the homogeneous case

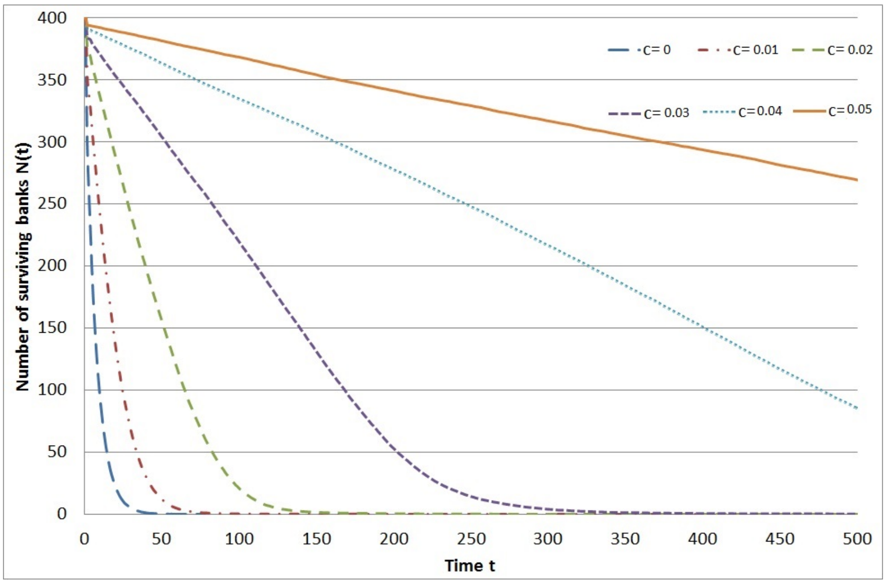
For the first set of simulations, the only fluctuation is the change in deposit and withdrawal patterns and $\sigma_D = 0.5$. These 400 banks are homogeneous in the sense that the initial bank size for each bank is identical ($\sigma_S = 0$), the average investment opportunity for each bank is identical ($\sigma_O = 0$) and that any individual bank's maximum investment opportunity at each period is the identical ($\sigma_\omega = 0$).

It is also assumed that each bank k made an investment of half of its average investment opportunity in the last τ periods, that is, $I_t^k = 0.5 \times O^k$ for $t = -1, -2, ..., -\tau$ and $k = 1, 2, ..., M$.

Figure 4.2 illustrates that, under the current scheme for the fluctuations in customers' deposit, at $C = 0$ these 400 banks all default within 50 periods. There are two reasons for this phenomenon. The obvious one is that when $C = 0$, banks are all disconnected from one another, therefore when any of them have a liquidity shortage arising from the fluctuation in customers' deposit, they immediately default as they do not have any counterparties which they can borrow interbank transactions from. The second reason is related to the fact that banks may have taken up too much investment. In the current setting, a bank is allowed to make investment as long as it has surplus liquidity above its reserve requirement and it can take up investment as much as its excess liquidity or its maximum investment opportunity. This means that there is not a channel to monitor a bank's commitment to investment opportunity relative to their funding risk. However, investment has a maturity of τ periods and hence the liquidity investment is considered as illiquid. Therefore, this causes in lots of banks defaulting as a result of liquidity shortage because most of their resources are illiquid. The situation is worsen together with the fact that they are not connected to each other through the interbank market. However, in the presence of interbank transactions, that is at $C > 0$, banks with liquidity shortage are able to borrow interbank loans and prevent themselves from bankruptcy, and hence the number of bank default reduces. More importantly, the higher the level of connectivity, the greater the number of surviving banks. The flattening part of the curve at $C = 0.03$ is a result of the averaging of 100 simulations. In the current setting, all banks will eventually default, if the simulation is run long enough. However, the fluctuation in deposit is completely random and in some rare cases, banks do not result in a negative liquidity until much later or until their connected banks are all defaulted, therefore these banks manage to survive for a long period. This phenomenon contributes to the flattened curve.

Table 4.2 shows banks' activities on investment and interbank transaction at different level of connectivity. It shows that the average percentage of investment opportunity fulfilled at different level of connectivity considered is between 64% to 67%. This will be a good reference for comparison with the heterogeneous case in average investment opportunity and the heterogeneous case in maximum investment opportunity over time. The table also shows the interbank activities between bank. It illustrates that both the average number of interbank transactions and the average volume of these transaction increase as connectivity increases. These increases have a positive effect on the stability of the system because banks are able to borrow loans via the interbank market to deal with their liquidity shortage issue. This results in the increase in the number of surviving banks as connectivity increases and supports the finding in Figure 4.2.

Figure 4.2: Number of surviving banks in the homogeneous case at different level of connectivity with $M = 400$, $N = 100$, $T = 500$, $\sigma_D = 0.5$, $\sigma_S = 0$, $\sigma_O = 0$ and $\sigma_\omega = 0$



The result presented in Figure 4.2 is the simplest scenario with only one fluctuation, that is, the change in customers' deposit and withdrawal patterns. Therefore, it can be viewed as a benchmark case and can be used as comparison with other more complicated scenarios. Obviously, the benchmark case in which all banks are homogeneous is unrealistic. Next, I introduce a couple of heterogeneous factors into the simulations. I first present the role of connectivity under each heterogeneous case. Then I study the magnitude of the effect each of these factors has on the number of surviving banks.

The effect of connectivity C on three heterogeneous cases

In the first heterogeneous case, banks differ in their bank sizes. The average bank size remains unchanged, that is, $\bar{S} = 1000$. However, the size of bank k is given by $S^k = |\bar{S} + \sigma_S \upsilon|$ where $\sigma_S = 500$. In the second heterogeneous case, banks differ in their average investment opportunity. Therefore, the size of bank k is still $S^k = 1000$ but its average investment opportunity is $\bar{O}^k = \delta|S^k + \sigma_O \nu|$ with $\sigma_O = 500$. In the third heterogeneous case, each bank faces fluctuation in its maximum investment opportunity in each period. Therefore, since $\sigma_O = 0$, its average investment opportunity is $\bar{O}^k = \delta \times S^k = 500$, but its maximum investment opportunity at any time t is given by $\omega_t^k = |\bar{O}^k + \bar{O}^k \sigma_\omega \eta_t$ where $\sigma_\omega = 0.5$. Table 4.1 displays the values of different parameters in different cases for simulations.

Table 4.1: Summary of the values for different parameters for the homogeneous case and the three heterogeneous cases

Case	σ_D	σ_S	σ_O	σ_ω
Homogeneous	0.5	0	0	0
Heterogeneous in initial bank size	0.5	500	0	0
Heterogeneous in average investment opportunity	0.5	0	500	0
Heterogeneous in maximum investment opportunity over time	0.5	0	0	0.5

Figure 4.3, Figure 4.4 and Figure 4.5 show the number of surviving banks at different level of connectivity under the three heterogeneous cases respectively.

Regarding the role of connectivity, the three cases produce qualitatively similar results, in the sense that as the level of connectivity increases, the number of bank default reduces. This is consistent with the homogeneous case. This suggest that, under the construction of the current model, in which banks are randomly connected to each others, an increase in the connections between banks within the system does strengthen the stability of the system by reducing the number of bank defaults, regardless of the heterogeneity. This relationship is monotonic.

Table 4.2: Banks' activities on investment and interbank transactions for the homogeneous case with different level of connectivity

	Connectivity					
	0	0.01	0.02	0.03	0.04	0.05
Average percentage of investment opportunity fulfilled per period	64.8	64.3	65.0	66.6	65.3	64.7
Average number of interbank transaction per period	0	16.53	36.83	57.08	75.77	92.72
Average volume of interbank transaction per period	0	460.4	1110.2	1822.2	2517.8	3242.8

Although Figure 4.3, 4.4 and 4.5 all come to the same conclusion with respect to the role of connectivity, the curve at $c = 0$ in Figure 4.4 differs from the other two heterogeneous cases or even the homogeneous case. The curve flattens at around 30 banks and is very unlikely to fall below that value regardless of however many periods the simulations are run for. This can be explained because of the nature of the heterogeneity. Since banks differ in their average investment opportunity, $\bar{O}^k = \delta|S^k + \sigma_O \nu|$, at $\sigma_O = 500$, there exists some banks that have a low average investment opportunity. At each period, these banks have a low maximum investment opportunity and as a result, they always have plenty of liquid resources in the bank, on top of their reserve requirement. This feature makes these banks extremely resistant to the fluctuation in the change in deposit and withdrawals patterns. In other words, even in the extreme case where there is a big drop in net deposit, these banks are very unlikely to end up with negative liquidity, and since $C = 0$, these banks are not connected to other banks and are not allow to lend out interbank loans, therefore, they do not face the risk of not being able to recover their loans. In the current setting, there do not exist any channels to cause these banks to default and hence they survive continuously and produce the horizontal curve in Figure 4.4.

Figure 4.3: Number of surviving banks in the heterogeneous case in initial bank size across banks at different level of connectivity with $M = 400$, $N = 100$, $T = 500$, $\sigma_D = 0.5$, $\sigma_S = 500$, $\sigma_O = 0$ and $\sigma_\omega = 0$

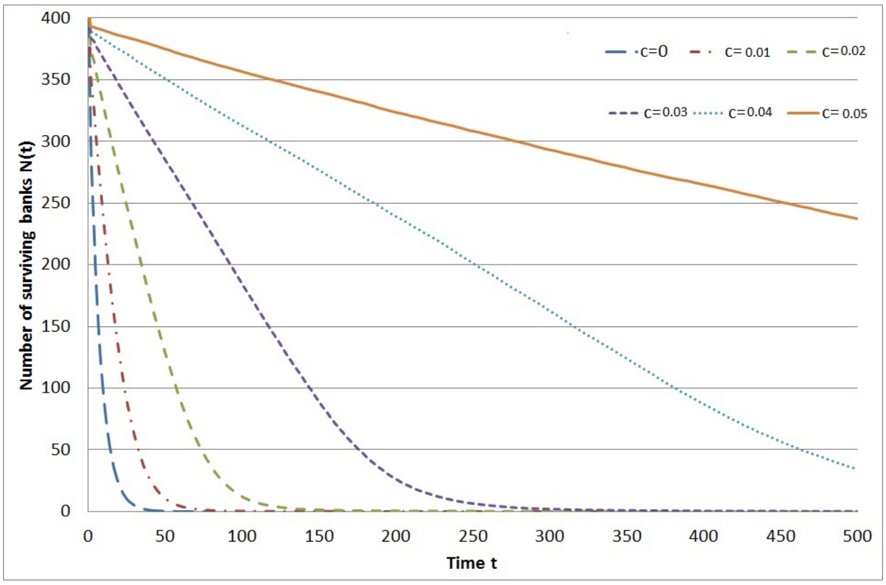


Table 4.3 , 4.4 and 4.5 show the banks' activities on investment and interbank transactions for the three heterogeneous cases respectively. A closer look at Table 4.3 shows that the all of the results are both qualitatively and quantitatively

Table 4.3: Banks' activities on investment and interbank transactions for the heterogeneous case in initial bank size with different level of connectivity

	Connectivity					
	0	0.01	0.02	0.03	0.04	0.05
Average percentage of investment opportunity fulfilled per period	65.1	64.9	65.1	65.2	65.3	65.5
Average number of interbank transactions per period	0	15.63	33.92	52.99	71.12	87.34
Average volume of interbank transactions per period	0	376.8	874.9	1507.1	2100.1	3242.8

Figure 4.4: Number of surviving banks in the heterogeneous case in average investment opportunity across banks at different level of connectivity with $M = 400$, $N = 100$, $T = 500$, $\sigma_D = 0.5$, $\sigma_S = 0$, $\sigma_O = 500$ and $\sigma_\omega = 0$

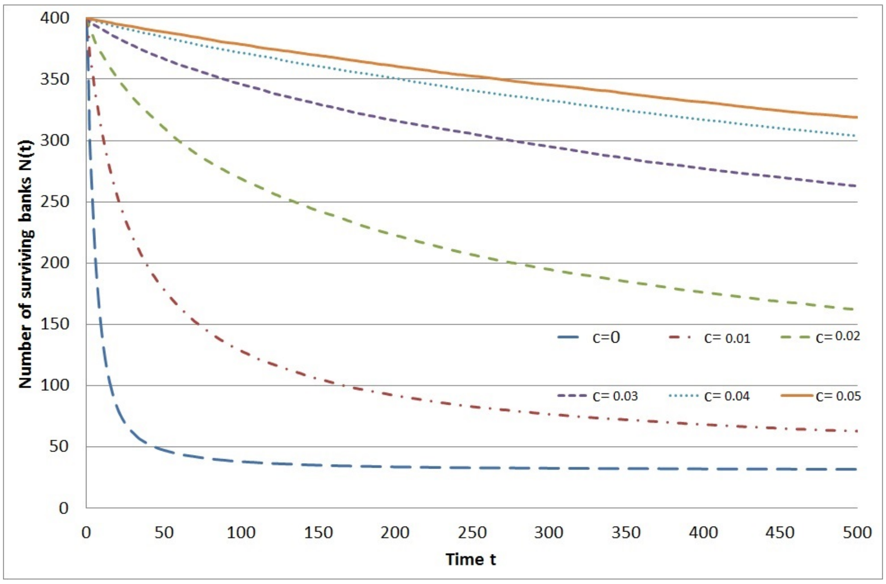


Table 4.4: Banks' activities on investment and interbank transactions for the heterogeneous case in average investment opportunity with different level of connectivity

	Connectivity					
	0	0.01	0.02	0.03	0.04	0.05
Average percentage of investment opportunity fulfilled per period	61.6	61.4	59.2	58.8	58.7	58.6
Average number of interbank transactions per period	0	94.90	139.63	156.46	163.90	166.72
Average volume of interbank transactions per period	0	3861.2	5803.2	6574.15	6931.8	7064.5

Figure 4.5: Number of surviving banks in the heterogeneous case in a bank's maximum investment opportunity over time at different level of connectivity with $M = 400$, $N = 100$, $T = 500$, $\sigma_D = 0.5$, $\sigma_S = 0$, $\sigma_O = 0$ and $\sigma_\omega = 0.5$

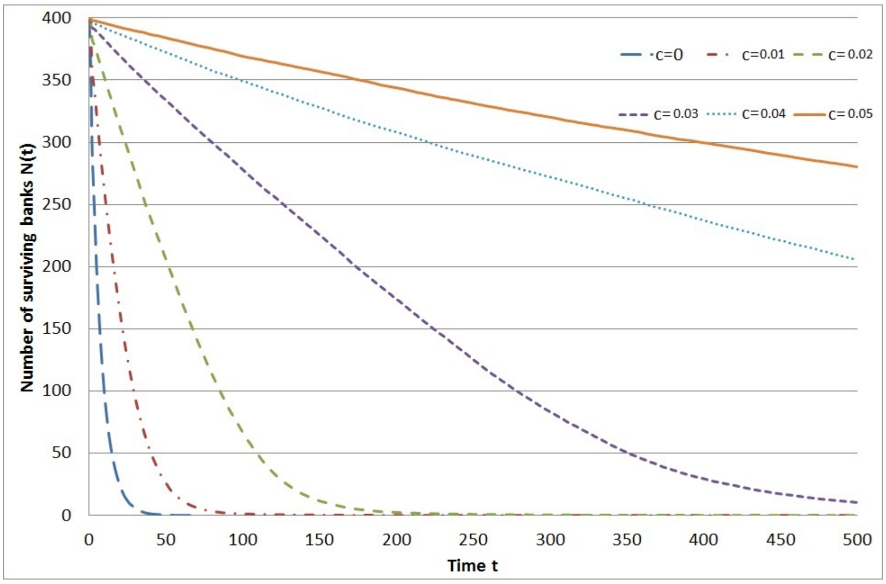


Table 4.5: Banks' activities on investment and interbank transactions for the heterogeneous case in maximum investment opportunity over time with different level of connectivity

	Connectivity					
	0	0.01	0.02	0.03	0.04	0.05
Average percentage of investment opportunity fulfilled per period	59.6	59.6	59.8	59.7	59.8	59.8
Average number of interbank transactions per period	0	69.71	117.62	136.31	140.70	143.26
Average volume of interbank transactions per period	0	2666.7	4674.6	5503.41	5702.96	5826.48

similar to results in Table 4.2. It shows that the average percentage of invest opportunity fulfilled is around 64% - 66% and both the average number of interbank transactions and their average volume increase as the connectivity increases. In table4.4, it can be seen that there is a slight down in average percentage of investment opportunity fulfilled per period for the second heterogeneous case with 61.6% in comparison with the homogeneous case. This is because when investment opportunity is not evenly distributed across banks, there exist some banks that have excess liquid reserve with low investment opportunity and others that have limited liquid reserve with investment opportunity unfulfilled. Therefore, the aggregate average investment opportunity fulfilled is lower. The table also shows that the percentage decreases as connectivity increases. This is because as banks are connected to many counterparties, there is a higher chance that they are involved in interbank activities. However, if the counterparties are unable to repay the loans in the next period, this has a impact on the ability and amount of investment opportunity the creditor bank can fulfil and this results in the decrease in the average aggregate percentage achieved. The number of interbank transactions and their average volume size are quantitatively the same as the previous two cases, in the sense that, as connectivity increases, both the number and the volume of interbank transaction increase. However the magnitude for both of these are a lot higher. The possible explanation is that those banks with high investment opportunity are likely to invest as much of as excess liquidity as possible. Unfortunately, due to the maturity of the investment, those investment resources become illiquid. Therefore, this results in them having a higher risk from liquidity shortage when their customers' withdraw their deposits. In turn, this results in them seeking for loans in the interbank market. The fact that there are banks that have low investment opportunity and have plenty surplus liquidity, these requests from the potential borrowing banks are highly likely to be fulfilled. This results in the high number and volume of interbank transactions. Lastly, the findings from Table 4.5 is quantitatively similar to Table 4.4 and the high number and volume of interbank transactions are also due to the heterogeneity in investment opportunity for banks.

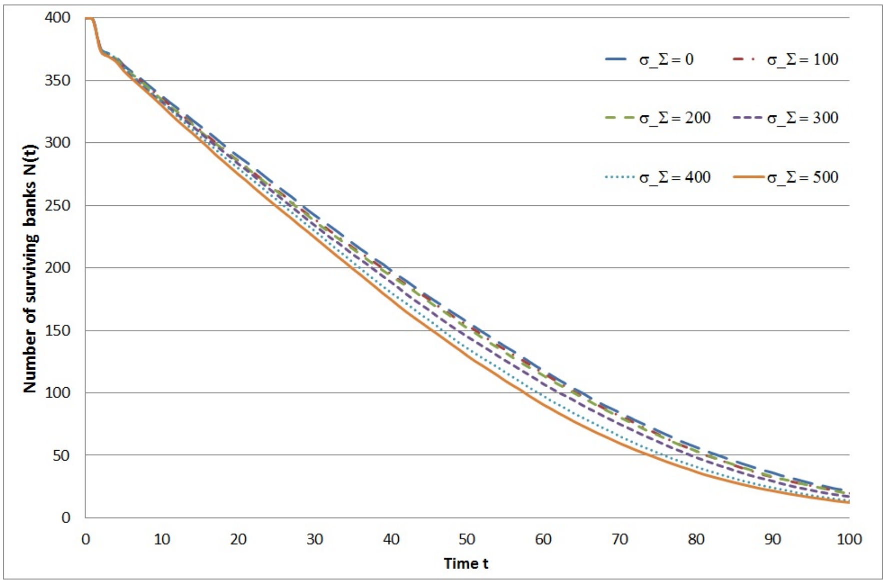
After considering the role of connectivity in the homogeneous case and the three heterogeneous cases, I now move onto studying the three heterogeneous factors individually.

4.3.2 The effect of heterogeneity

Heterogeneity in initial bank size

Previously in Figure 4.3, it shows the number of surviving banks at different level of connectivity assuming that $\sigma_S = 500$. In order to investigate the effect of the heterogeneity in initial bank size on the bank system, I study the number of surviving banks at different level of σ_S. In Figure 4.6, the value for σ_S changes from 0 to 500 at every 100 interval and the level of connectivity is assumed to be $C = 0.02$. Since in Figure 4.3, when $C = 0.02$, all 400 banks default by about 170 periods, therefore, I only present 100 period in Figure 4.6. It shows that as the value for σ_S increases, the number of surviving banks decreases, even though the difference is small for every change in σ_S in 100 interval. This suggest, although weakly, that if banks are of different sizes, the system is less stable. This finding is supported by Table 4.6, which shows the banks' activities on investment and interbank transactions. The table illustrates both the the average number and volume of interbank transactions decrease, again though weakly, as σ_S increases. The explanation is as follows: when $\sigma_S = 0$, that is when banks are homogeneous in terms of initial bank size, their average liquidity need should be more or less the same. However, as σ_S increases, the the initial bank sizes begin to differ and so do the customers' deposit and and investment opportunity. Therefore, when these 'bigger' banks suffer a liquidity shortage arising from their customers' withdrawal, the magnitude of the liquidity shortage is bigger and this reduces the chances for them to be able to borrow enough interbank loans to cover their needs. Therefore, the interbank activities decreases, as shown in Table 4.6, which in turns explains the slight increase in the number of defaults in banks for higher value ofσ_S.

Figure 4.6: Number of surviving banks in the heterogeneous case in initial bank size at different values of σ_S with $M = 400$, $C = 0.02$, $N = 100$, $T = 500$, $\sigma_D = 0.5$, $\sigma_O = 0$ and $\sigma_\omega = 0$



Heterogeneity in average investment opportunity

Similarly, I next consider the effect of the heterogeneity in average investment opportunity on the bank system. In a similar manner, I choose $C = 0.02$ and study the number of surviving banks at different level of σ_O. Since the change in values for σ_O does seem to produce a larger effect on the number of surviving

Table 4.6: Banks' activities on investment and interbank transactions for the heterogeneous case in initial bank size with different value of σ_S

	σ_S					
	0	100	200	300	400	500
Average percentage of investment opportunity fulfilled per period	65.1	65.1	65.1	65.1	65.1	65.1
Average number of interbank transactions per period	36.83	36.57	35.78	35.47	34.94	33.92
Average volume of interbank transactions per period	1100.2	1090.37	1031.33	992.2	940.6	874.9

banks, therefore I present the results for 500 periods in Figure 4.7. It shows that by increasing the heterogeneity in average investment opportunity across banks, that is as σ_O change from 0 to 500, the number of surviving banks increases dramatically. A possible explanation for this phenomenon is that some banks have a low average investment opportunity and always have surplus liquid resources in their holdings. This not only equip them well against fluctuation in deposit and withdrawal patterns, but it also allows them to have excess liquidity to offer to banks that suffer from negative liquidity. Therefore this reduces number of bank defaults in two ways. This explanation is supported by the evidence in Table 4.7. The fact that some banks have plenty of surplus liquid resources due to low investment opportunity available to them means that they can be potential interbank lenders. Table 4.7 shows that the average number and volume of interbank transactions increase substantially as σ_O increases. As a result, it is highly likely that most banks with liquidity shortage are able to borrow loans through the interbank market and hence, this increases the stability of the system and increases the number of surviving banks.

Heterogeneity in maximum investment opportunity over time

Finally, I consider the effect of heterogeneity in maximum investment opportunity over time. Again, similar to the simulations for the other two heterogeneous cases, I choose $C = 0.02$ and look at the number of surviving banks at different

Figure 4.7: Number of surviving banks in the heterogeneous case in average investment opportunity across banks at different values of σ_O with $M = 400$, $C = 0.02$, $N = 100$, $T = 500$, $\sigma_D = 0.5$, $\sigma_S = 0$ and $\sigma_\omega = 0$

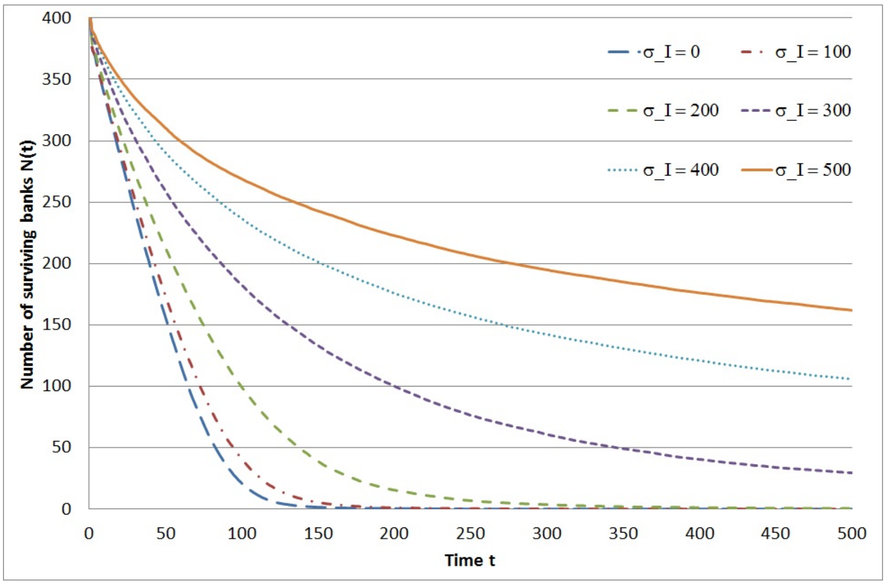


Table 4.7: Banks' activities on investment and interbank transactions for the heterogeneous case in initial bank size with different value of σ_O

	σ_O					
	0	100	200	300	400	500
Average percentage of investment opportunity fulfilled per period	65.1	66.0	66.7	65.4	62.5	59.4
Average number of interbank transactions per period	36.83	39.88	63.95	98.59	123.73	139.63
Average volume of interbank transactions per period	1100.2	1228.61	2245.5	3824.6	5034.8	5803.2

level of σ_ω. So far, as shown in Figure 4.6 and Figure 4.7, the relationship between the change in either σ_S or σ_O against the number of surviving banks is monotonic, though it is monotonically decreasing for σ_S and monotonically increasing for σ_O. However, Figure 4.8 shows that as σ_ω changes from 0 to 0.6, the number of surviving banks increases and from 0.8 onwards, the number of surviving banks actually decreases and at $\sigma_\omega = 1$, the number of surviving banks is even less than when $\sigma_\omega = 0$. This suggests that, under different conditions, there is an optimal value for σ_ω for the system to achieve a slightly more stable environment.

Figure 4.8: Number of surviving banks in the heterogeneous case in a bank's maximum investment opportunity over time at different values of σ_ω with $M = 400$, $c = 0.02$, $N = 100$, $T = 500$, $\sigma_D = 0.5$, $\sigma_S = 0$ and $\sigma_O = 0$

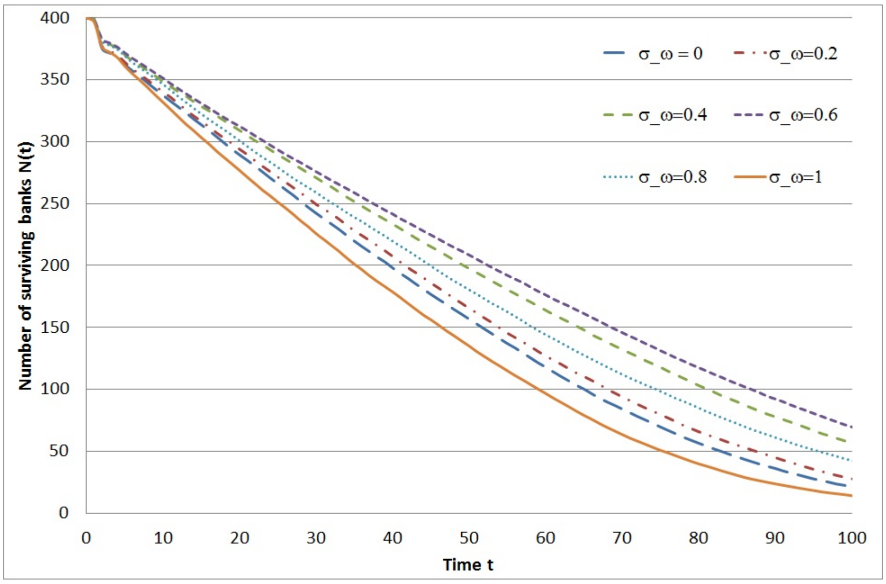


Table 4.8: Banks' activities on investment and interbank transactions for the heterogeneous case in initial bank size with different value of σ_ω

	σ_ω					
	0	0.2	0.4	0.6	0.8	1
Average percentage of investment opportunity fulfilled per period	65.1	63.9	61.7	56.9	50.4	44.7
Average number of interbank transactions per period	36.83	41.94	83.87	143.39	168.33	173.58
Average volume of interbank transactions per period	1100.2	1423.8	3210.0	5800.25	6924.4	7209.2

4.4 Conclusion

This chapter first presents the problem experienced in implementing Iori, Jafarey, and Padilla (2006)'s model. It then proposes some corrections and modifications towards the model and discusses some of the preliminary findings.

Chapter 5

A Comprehensive Interbank Market Model

5.1 Introduction

The implementation of Iori, Jafarey, and Padilla (2006) model in Chapter 3 together with some modifications in Chapter 4 form the basis of bank daily operations and the usage of interbank transactions. However, in Iori, Jafarey, and Padilla (2006), the authors consider a banking system in which banks are randomly connected to each other based on a connectivity parameter C. A number of empirical studies on the interbank market for different countries show that the actual structure of these interbank markets are not random. Some of them show that the real-world interbank markets are best represented by a scale-free network, see Blavarg and Nimander (2002) and van Lelyveld and Liedorp (2006). Others find that some interbank markets exhibit a star structure or the core-peripheral structure, see Iori, De-Masi, Precup, Gabbi, and Caldarelli (2008) and Wells (2002). Therefore, this section provides a full description of an interbank market model with a star structure.

5.2 Description of the complete model

5.2.1 Construction of a banking system

Network structure

In order to construct a star structure of banks transacting in the interbank market, I revise the construction of the banking system as follows. Let's consider a system of M banks, in which there is one large bank in the centre and $M-1$ small peripheral banks. As before, relationships between these M banks are also presented in a connectivity matrix, J_{ij} for $i = 1, 2, ..., M$ and $j = 1, 2, ..., M$. Each element in this connectivity matrix can only take one of the two values, 0 or 1 and the value it takes is used to define whether the two banks are connected to each other. $J_{ij} = 1$ indicates that bank i and bank j are in connection and interbank transactions can be arranged between themselves while $J_{ij} = 0$ represents that bank i and bank j are not linked to each other and hence they cannot arrange interbank transactions.

It is assumed that 'relationship' is non-directional, therefore, if two banks are connected, then both parties can be a borrower or a lender in an interbank transaction. A bank cannot arrange interbank transaction with itself, therefore, $J_{ij} = 0$ if $i = j$.

At the very beginning of each simulation, the values for the connectivity matrix J_{ij} $\forall i \neq j$ are generated. It is assumed that all the small banks are connected to the large bank in the centre. The parameter connectivity C is used to determine whether each of the small banks is connected among each other. For the purpose of simulations, a random number will be drawn for each pair of small-and-small bank i and j. If the random number is less than or equal to C, then bank i and j are connected. On the contrary, if the random number drawn exceeds C, then bank i and j are not connected. Therefore, each simulation generates one realisation of a banking system.

The above description has been adopted from Iori, Jafarey, and Padilla (2006) model and will only be used in the extended case. For the time being, a core-peripheral network structure will be studied in which there is only one core bank in the middle with many small banks on the peripheral. Therefore, it is assumed that every single small banks are connected to the large core bank in the middle, as well as among themselves. As a result, the parameter C will be omitted.

Bank size, size of deposit and size of investment opportunity

The size of any bank k is denoted by S^k and it is given by

$$S^k = \begin{cases} \bar{S} & \text{if } k = 1 \\ \frac{1}{M-1} \times \bar{S} & \text{if } k \neq 1 \end{cases} \tag{5.1}$$

where $\bar{S}$ is size of the large bank. For the moment, it is assumed that the aggregate bank size is split into two equal halves: one-half is allocated to the large bank and the other half is shared among the small banks. Therefore, the aggregate bank size of the system is $2\bar{S}$.

It is assumed that banks differ in terms of the level of deposit they receive and the investment opportunity they have. Previously, in the implementation of the Iori, Jafarey, and Padilla (2006) model, it has always been assumed that a bank's investment opportunity is δ times the bank size, in which the bank size equals to the average size of deposit. However, it would be more appropriate to model them in a closer resemblance to the real-world data. Firstly, large banks generally have more investment than their available liquidity while small banks are often better at receiving customers' deposit. Secondly, Cocco, Gomes, and Martins (2009) studying the lending relationships in the interbank market show that large banks are usually the net interbank borrowers while the small banks are net lenders. They also show that large banks usually borrow from the small ones. Therefore, in order for a banking system to exhibit more coherence to the reality, there should be more investment opportunity available to the large banks and the small banks should in general have excess liquidity available to lend out in the interbank market. More specifically, it will be assumed that the large banks have a size of investment opportunity that is more than δ times its average size of deposit while small banks have an investment opportunity size that is less than δ times its average deposit size. For the sake of comparison, the aggregate deposit size and investment opportunity are kept unchanged. To do so, a parameter, IM^k, is created to model this imbalance between the size of deposit and the investment opportunity for any bank k.

$$IM^k = \begin{cases} -\kappa \times (\delta S^k) & \text{if } k = 1 \\ \kappa \times (\delta S^k) & \text{if } k \neq 1 \end{cases} \tag{5.2}$$

where κ is the imbalance parameter with $\kappa \geq 0$, δ is the aggregate investment opportunity : aggregate bank size ratio. As mentioned above, the aggregate bank size is $2\bar{S}$. Therefore, the aggregate investment opportunity in the whole system is $2\delta\bar{S}$. If imbalance does not exist, that is, banks are all homogeneous in their investment opportunity : deposit ratio, then a bank's investment opportunity is simply δ times its bank size.

Once the imbalance between the size of deposit and the investment opportunity is generated, then the size of bank k's average deposit is denoted by $\bar{D}^k$

$$\bar{D}^k = S^k + \frac{IM^k}{2} \tag{5.3}$$

This average deposit size, $\bar{D}^k$, represents, over a long period of time, the average amount of deposits bank k receives from its customers. Similarly, the size of bank k's average investment opportunity is denoted by $\bar{O}^k$

$$\bar{O}^k = \delta S^k - \frac{IM^k}{2} \tag{5.4}$$

The banks' sizes, their average deposit sizes and their average investment opportunity sizes are generated before the initial period. The intuition of (5.2), (5.3) and (5.4) is to create a large bank in the centre with higher investment opportunity than its available liquidity and vice versa for the small peripheral banks.

Balance sheet of a bank

A balance sheet is a financial statement of a financial institution or organisation. It consists of the assets side and the liabilities side. Table 5.1 illustrates an example of a simplified version of balance sheet for a bank.

Assets Side	Liabilities Side
Liquidity (Cash and Reserve)	Equity
Investment	Deposit
Interbank lending	Interbank borrowing
Total assets	Total liabilities and equity

Table 5.1: An example of a simplified version of stylised balance sheet for a bank

The term 'total assets' includes everything that the bank owns whereas 'total liabilities' consists of all the debts of the banks. 'Equity' refers to the capital of the bank. In finance or accounting, since the assets side and the liabilities side must be equal, therefore, equity is given as the total assets net total liabilities.

5.2.2 Model operation

The model operates in discrete time, which is denoted by $t = 0, 1, 2, ..., T$ where $T + 1$ denotes the number of periods in each simulation.

Receipt or withdrawal of deposits

At each period t, each bank k faces new deposits and withdrawals from its customers. The total amount of deposits net withdrawals is given by D_t^k. This amount changes everyday and its fluctuation is modelled by the following process, and is given by

$$D_t^k = \begin{cases} |\bar{D}^k \times (1 - \alpha_L) + \alpha_L \times D_{t-1}^k + \bar{D}^k \times \sigma_D \epsilon_t| & \text{if } k = 1 \\ |\bar{D}^k \times (1 - \alpha_S) + \alpha_S \times D_{t-1}^k + \bar{D}^k \times \sigma_D \epsilon_t| & \text{if } k \neq 1 \end{cases} \tag{5.5}$$

where α_L and α_S are the parameters of the process and $0 < \alpha_L < 1$ and $0 < \alpha_S < 1$. σ_D denotes the standard deviation in the fluctuation in deposits and $\epsilon_t \sim N(0,1)$. This new process has been adopted to model the fluctuation in deposit. This is because the one previously used in the implementation of Iori, Jafarey, and Padilla (2006) is completely random, meaning that the deposit from one day to another can move from extreme high to extreme low or vice versa. Although this is not impossible, the random process makes this kind of change in deposit very frequently and without economical explanation. Besides, the frequent big fluctuation in deposit modelled in the previous random process also led to an unrealistic use of the interbank market for liquidity shortage. This affects the analysis on the usage of interbank market. The new process models the deposit to be dependent on the previously period, yet with a random fluctuation. This is a more realistic fluctuation in deposit with respect to the real-world phenomenon and leads to a more realistic and appropriate use of the interbank market. The interest rate for deposits is assumed to be fixed over time and across banks and it is denoted by r_D. Therefore, if bank k received a deposit of D^k_{t-1} at $t-1$, it has to pay out a total interest of $r_D D^k_{t-1}$ at t to its customers, regardless of withdrawal.

Receipt of investment income and matured investment

If a bank made an investment in any of the last τ periods, it receives an income from its investment. This income is assumed to be a fixed proportion of its investment. This fixed proportion is denoted by r_I and is assumed to be constant over time and across banks. Therefore, at any time t, bank k receives an income of $r_I \sum_{s=1}^{\tau} I^k_{t-s}$ from all its outstanding investments.

Previously, it has been assumed that investment is risk-free. In other words, when an investment matures, a bank always receive the full amount of the capital it invest. However, it is more realistic to incorporate risky assets into the model. When an investment, $I^k_{t-\tau}$ reaches it maturity, the bank faces an uncertainty that it may not be able to receive all of the capital invested. Let Z represents the loss function in which $Z \in [0,1]$ and Z has a distribution of $1 - \exp(-\lambda x)$. Then at any time t, when an investment matures, the amount of capital that a bank would receive will be given as

$$\hat{I}^k_{t-\tau} = [1 - Z] \times I^k_{t-\tau} \tag{5.6}$$

In order to generate values for Z, I will use the following theorem.

Theorem. Let $U \sim U(0,1)$ and F be a continuous and strictly increasing cumulative distribution function, then $F^{-1}(U)$ is a sample of F.

The above theorem is referred to as the inverse transform sampling method. More information can be found in Devroye (1986). The method allows the transformation of results from the uniform distribution to the distribution F. Let $F = 1 - \exp(-\lambda x)$, then $Z = F^{-1}(U)$ will have the distribution F where $U \sim U(0,1)$. However, since $F \in [0,\infty)$ while $Z \in [0,1]$, therefore only part of the uniform distribution will be considered.

$$\begin{aligned}
F(x) &= 1 - \exp(-\lambda x) \\
y &= 1 - \exp(-\lambda x) \\
\exp(-\lambda x) &= 1 - y \\
-\lambda x &= \ln(1-y) \\
x &= -\frac{\ln(1-y)}{\lambda} \\
F^{-1}(y) &= -\frac{\ln(1-y)}{\lambda}
\end{aligned}$$

Therefore the inverse function of F is $F^{-1}(U) = -\frac{\ln(1-U)}{\lambda}$. If $U \sim U(0,1)$, then $F \in [0,\infty)$, which means that $Z \in [0,\infty)$. By considering $U \sim U(0,F(1))$, then $Z \in [0,1]$. The algorithm is given below.

Algorithm 1 Inversion of the distribution function

1. Generate $U \sim U(0, F(1))$
2. $Z = F^{-1}(U)$

Output: Z has the distribution of F and $Z \in [0, 1]$

After the receipt of new deposits and withdrawals, together with investment income and the capital of the risky matured investment, the liquidity of bank k is given by

$$\hat{L}_t^k = L_{t-1}^k + (D_t^k - D_{t-1}^k) - r_D D_{t-1}^k + r_I \sum_{s=1}^{\tau} I_{t-s}^k + \hat{I}_{t-\tau}^k \tag{5.7}$$

and the equity of bank k is given by

$$\hat{E}_t^k = \hat{L}_t^k + \sum_{s=0}^{\tau-1} I_{t-s}^k + IL_{t-1}^k - D_t^k - IB_{t-1}^k \tag{5.8}$$

where IL_{t-1}^k and IB_{t-1}^k are respectively the interbank lending and interbank borrowing of bank k at period $t-1$.

Repayment of interbank loans

When banks close at the end of the day, their first priority is to repay their interbank loans in full if there are any. If they are unable to repay their debt in full, it is assumed that those banks do not repay for the time being, but instead they issue debt certificates to their creditors. These debt certificates are to be redeemed before the beginning of the next period and they should still reflect in the banks' liquidity.

The repayment procedure is as follows. At any time t, if a bank k borrowed interbank loans in the period $t-1$, then its total interbank borrowing is greater than zero, that is $IB_{t-1}^k > 0$ and the amount it has to repay to its creditor(s) is $(1 + r_B)IB_{t-1}^k$, where r_B is the interbank interest rate. If bank k's liquid cash holding is greater than the amount it should repay, that is $\hat{L}_t^k \geq (1 + r_B)IB_{t-1}^k$, then it is assumed that it does. The balance sheets of bank k and its creditor(s) should be updated. The items to be updated on the balance sheet for bank k are

$$\begin{aligned} \breve{L}_t^k &= \hat{L}_t^k - (1+r_B)IB_{t-1}^k \\ IB_{t-1}^k &= 0 \end{aligned}$$

and for bank j, assuming that it previously lent interbank loans to bank k, are

$$\begin{aligned} \breve{L}_t^j &= \hat{L}_t^j + (1+r_B)IT_{t-1}^{kj} \\ IT_{t-1}^{kj} &= 0 \\ IT_{t-1}^{jk} &= 0 \end{aligned}$$

where IT_{t-1}^{kj} denotes the actual interbank transaction between bank k and j. A positive value of IT_{t-1}^{kj} represents that bank k borrowed interbank loans from bank j in the previous period and in such a case, the value of IT_{t-1}^{jk} must be of the same magnitude but opposite sign. Although a bank can be both an interbank borrower or lender, it can only be either one of the two in any particular period. This means that, for a bank k, the values of all IT_t^{kj} $\forall j \neq k$ should take the same sign at any time t.

$$\sum_{j=1}^{M} IT_t^{kj} = \begin{cases} IB_t^k & \text{if } k \text{ } \textit{borrows} \\ -IL_t^k & \text{if } k \text{ } \textit{lends} \end{cases} \tag{5.9}$$

$IB_t^k \geq 0$ and $IL_t^k \geq 0$ for $\forall k$.

However, if bank k's liquid cash holding is less than the amount it needs to repay its creditor(s), that is $\hat{L}_t^k < (1 + r_B)IB_{t-1}^k$, then it issues debt certificate of size $(1 + r_B)IT_{t-1}^{kj}$ to each of its creditor j. All these debt certificates should be reflected on the liquidity of bank k and therefore it is $\breve{L}_t^k = \hat{L}_t^k - (1+r_B)IB_{t-1}^k$.

Upon the repayment cycle, regardless of whether a creditor bank manages to receive its interbank loans it previously lent to others, the equity of any bank k

is

$$\breve{E}_t^k = \breve{L}_t^k + \sum_{s=0}^{\tau-1} I_{t-s}^k + IL_t^k - D_t^k - IB_t^k \tag{5.10}$$

Regulatory requirement for liquidity

After the repayment of interbank loans, banks pay dividend and make new investment, provided that they satisfy a number of regulatory requirements. Therefore, before providing the description of the procedure for dividend payment and investment, I introduce these regulatory requirements. In this model, there are in total three regulatory requirements that each bank has to satisfy. The first one, which is associated with the liquidity aspect of the bank, is called the reserve requirement. The second and third one, which are both related to the equity issue of the bank, are named the equity requirement and the capital adequacy requirement respectively.

Liquidity can be understood as cash and it is classified as liquid resource because it is instantaneously available upon request. If this liquid resource is not being made good use of, then it does not bring any benefit or profit to the bank. Therefore, a bank will later make investment based on its available liquidity and aim to earn profits. However, resource assigned as investment is classified as illiquid. At any time t, a bank's reserve requirement is calculated as a proportion of its current deposits from customers and the amount of reserve, R_t^k is given as

$$R_t^k = \beta \times D_t^k \tag{5.11}$$

where β is the reserve ratio. The objective of this regulatory requirement is to make sure that a bank maintains a certain level of liquid resource, namely the reserve, within itself, and this reserve can be used against unforeseen circumstances, for example, a big customers' withdrawal in the next period. Therefore, at any particular period, if a bank is unable to satisfy its reserve requirement, it means that the bank is not allowed to commit to activities like dividend payment, making investment or lending out interbank loans to other banks. However, the bank itself should not be penalised in any ways for not reaching the required reserve level. All it means is that this bank is slightly more at risk to suffer from liquidity shortage if a big withdrawal from customers take place in the next period.

Regulatory requirements for equity
The equity requirement is given by

$$Eq_t^k = \breve{E}_t^k / S^k \tag{5.12}$$

This requirement only comes into the picture when a bank wants to pay out dividend and that Eq_t^k must be greater than or equal to the equity requirement ratio, χ. It will later be shown that the value used to determine the amount of equity each bank starts with at the very beginning of time equals to χ. In the current setting, a bank's equity is affected in one of the following five ways: it increases if a bank receives investment income; it increases (decreases) if the bank receives (pays out) interest on its interbank loans; it decreases when the bank pays out interest to its depositors; it suffers a loss if the bank doesn't receive the matured investment in full (arising from the fact that investment is not risk-free); and it decreases if the bank was an interbank lender but it is not able to receive the loans from its borrower(s). Therefore, the objective of the requirement is to check whether the bank has an overall gain or loss in equity in any particular period. If there is a net gain, meaning that the bank has an excess in equity compared to its original equity size, then the bank can consider paying out dividend to its shareholders. The amount of dividend to be paid out will be given later.

The capital adequacy requirement is considered when a bank wants to make new investment or lend interbank loans to other banks. It is given by

$$
\begin{aligned}
Cap_t^k &= \frac{Capital}{Risk\ \ weighted\ \ assets} \\
&= \frac{\breve{E}_t^k}{\rho_L \breve{L}_t^k + \rho_I \sum_{s=0}^{\tau-1} I_{t-s}^k + \rho_{IL} IL_t^k}
\end{aligned}
$$

where ρ_L, ρ_I and ρ_{IL} are the weighted-risk of cash, investment and interbank lending respectively. Since there is no risk in holding cash, therefore the weighted-risk for cash should be zero and the capital adequacy requirement can be simplified to

$$
Cap_t^k = \frac{\breve{E}_t^k}{\rho_I \sum_{s=0}^{\tau-1} I_{t-s}^k + \rho_{IL} IL_t^k} \tag{5.13}
$$

Therefore, before bank k can make further investment or lend interbank loans to others, it must be able to first satisfy the capital adequacy requirement, that is, Cap_t^k must exceed the capital adequacy requirement ratio ϕ.

Dividend payment and new investment

Dividends are paid out to shareholders when a bank makes profits. Bank k can only pay out dividend if it satisfies the equity requirement. And when it does, the actual dividend payment is the minimum value between the profit it makes, its available cash (that is, liquidity net reserve) and its excess equity. Mathematically, it is given by

$$
Div_t^k = \max\{0, \min[r_I \sum_{s=1}^{\tau} I_{t-s}^k + r_B(IL_{t-1}^k - IB_{t-1}^k) - r_D D_{t-1}^k, \breve{L}_t^k - R_t^k, \breve{E}_t^k - \chi S^k]\} \tag{5.14}
$$

However, if it does not meet the regulatory requirements, it does not pay out any dividend. Therefore, if $Eq_t^k \leq \chi$, then $Div_t^k = 0$. After making dividend payment, the liquidity and equity of a bank get updated and they respectively are

$$\grave{L}_t^k = \breve{L}_t^k - Div_t^k \tag{5.15}$$

$$\grave{E}_t^k = \grave{L}_t^k + \sum_{s=0}^{\tau-1} I_{t-s}^k + IL_t^k - D_t^k - IB_t^k \tag{5.16}$$

Then, banks can consider investing in fresh investment projects. The average investment opportunity for each bank is generated at the beginning of the simulation and is given in equation (5.4). In each period t, the actual maximum investment opportunity, ω_t^k, available to each bank k fluctuates at its average investment opportunity and it is given by

$$\omega_t^k = |\bar{O}^k + \sigma_\omega \bar{O}^k \eta_t| \tag{5.17}$$

where σ_ω is the standard deviation of the fluctuation and $\eta \sim N(0,1)$. In order to focus on the use of the interbank market as a result of liquidity shortage arising from the fluctuation of deposits, the fluctuation in the maximum investment opportunity for banks at each period will be assumed to be zero. Undertaking new investment is a beneficial way for a bank to make profit. Ideally, a bank would invest up to its maximum investment opportunity. However, investment does not mature until τ period later and hence when resources are used for investment, they are classified as illiquid. For this reason, a bank must not use its reserve for investment. Therefore, bank k will undertake new investment based on its maximum investment opportunity and its available liquidity (cash), provided that this will not violate the capital adequacy requirement. Hence, at this stage, the amount of investment bank k can make is

$$I_t^k = \min[\max(0, \grave{L}_t^k - R_t^k), \omega_t^k, \frac{\grave{E}_t^k - \phi(\rho_I \sum_{s=0}^{\tau-1} I_{t-s}^k + \rho_{IL} IL_t^k)}{\phi \rho_I}] \tag{5.18}$$

The derivation of the third term is shown below. Before a bank makes new investment, its $Cap_t^k >= \phi$. Let ΔI be the amount of investment a bank plans to make.

$$\begin{aligned}
\phi &\leq \frac{\grave{E}_t^k}{\rho_I(\sum_{s=0}^{\tau-1} I_{t-s}^k + \Delta I) + \rho_{IL} IL_t^k} \\
\grave{E}_t^k &\geq \phi[\rho_I(\sum_{s=0}^{\tau-1} I_{t-s}^k + \Delta I) + \rho_{IL} IL_t^k] \\
\phi \rho_I \Delta I &\leq \grave{E}_t^k - \phi(\rho_I \sum_{s=0}^{\tau-1} I_{t-s}^k + \rho_{IL} IL_t^k) \\
\Delta I &\leq \frac{\grave{E}_t^k - \phi(\rho_I \sum_{s=0}^{\tau-1} I_{t-s}^k + \rho_{IL} IL_t^k)}{\phi \rho_I}
\end{aligned}$$

Therefore, the amount of new investment that a bank can commit to must be smaller than or equal to the term on the right hand side of the above inequality. After the dividend is paid and new investment is made, it comes to the end of the period t. The liquidity and equity of bank k after dividend and investment are given respectively as

$$\acute{L}_t^k = \grave{L}_t^k - I_t^k \tag{5.19}$$

$$\acute{E}^k_t = \acute{L}^k_t + \sum_{s=0}^{\tau-1} I^k_{t-s} + IL^k_t - D^k_t - IB^k_t \qquad (5.20)$$

Arrangement of interbank transactions for liquidity shortage

A bank that has a negative liquidity is said to be in liquidity shortage. The changes in liquidity depend on the fluctuation in deposits and withdrawals from customers, the successfulness of investment maturing at its full value and the successfulness of receiving (paying) interbank loans from (to) its debtors (creditors). At this stage, banks can be grouped into three different categories: banks that paid out dividend and made new investment and have no excess liquidity above its reserve will do nothing; banks that have excess liquidity can be potential lenders in the interbank market; and banks that have liquidity shortage will attempt to approach their connected banks and arrange to borrow interbank loans. For simulation purpose, it is assumed that a bank k that has liquidity shortage is chosen randomly. This bank k then approaches its connected banks j one by one, again in a random manner, and try to arrange an interbank borrowing. The maximum amount that bank k can borrow from bank j depends on its liquidity need ($|\acute{L}^k_t|$), its creditor's available liquidity ($\acute{L}^j_t - R^j_t$) and the creditor's excess equity to cover the additional liabilities it is about to commit to. Mathematically, it is given as

$$\tilde{IT}^{kj}_t = \min[|\acute{L}^k_t|, \acute{L}^j_t - R^j_t, \frac{\acute{E}^j_t - \phi(\rho_I \sum_{s=0}^{\tau-1} I^j_{t-s} + \rho_{IL} IL^j_t)}{\phi \rho_{IL}}] \qquad (5.21)$$

A similar derivation for the third term is illustrated below.

In order for a bank j to lend out interbank loans, its $Cap^j_t >= \phi$. Let ΔIL be the amount of interbank loan bank j is willing to lend out.

$$\phi \leq \frac{\acute{E}_t^j}{\rho_I \sum_{s=0}^{\tau-1} I_{t-s}^j + \rho_{IL}(IL_t^j + \Delta IL)}$$

$$\acute{E}_t^j \geq \phi[\rho_I \sum_{s=0}^{\tau-1} I_{t-s}^j + \rho_{IL}(IL_t^j + \Delta IL)]$$

$$\phi\rho_{IL}\Delta IL \leq \acute{E}_t^j - \phi(\rho_I \sum_{s=0}^{\tau-1} I_{t-s}^j + \rho_{IL} IL_t^j)$$

$$\Delta IL \leq \frac{\acute{E}_t^j - \phi(\rho_I \sum_{s=0}^{\tau-1} I_{t-s}^j + \rho_{IL} IL_t^j)}{\phi\rho_{IL}}$$

Similarly, the amount of interbank lending that a bank can commit to must be less than or equal to the term on the right hand side of the above inequality. Unless the amount of $\tilde{IT}_t^{kj}$ is exactly the same as bank k's liquidity need, otherwise, the interbank transaction is only pending. Interbank loans will only be transacted if the bank with liquidity shortage can line up enough loans from its creditor(s) to avoid having a net negative liquidity. Therefore, the bank is assumed to continue approaching its connected banks for more interbank loans until either it arranges enough interbank borrowings to cover its liquidity shortage or there are no longer any other banks that are willing to lend to it. If it is the former case, then those interbank loans are transacted, whereas if it is the latter case, then the bank fails to arrange enough interbank loans to cover its liquidity shortage and hence it defaults. When banks proceed with their interbank transactions, the balance sheets of borrowing bank k and lending bank(s) j are updated.

$$\tilde{L}_t^k = \acute{L}_t^k + \tilde{IT}_t^{kj}$$
$$\tilde{IB}_t^k = \sum_{j=1}^{M} \tilde{IT}_t^{kj}$$
$$\tilde{L}_t^j = \acute{L}_t^j - \tilde{IT}_t^{kj}$$
$$\tilde{IL}_t^j = \sum_{k=1}^{M} \tilde{IT}_t^{kj}$$

For banks that manage to borrow interbank loans in this period, if their liquidity shortage arose as a result of change in deposit or a loss in the matured risky asset, then they don't have to do anything. However, if the liquidity shortage was due to the failure of repaying interbank loans for the previous period and issued debt certificate(s) to its creditor(s), then it can redeem the certificate now. Therefore, the balance sheet(s) of its creditor(s) i from the previous period get updated.

$$\tilde{L}_t^i = \acute{L}_t^i + (1 + r_B)IT_{t-1}^{ki}$$

Extension of loans

The option of extending the interbank loans will be incorporated into the model after which banks attempted to repay their interbank borrowings in full if possible. For implementation purpose, the extension of loans should come before the 'regulatory requirements for equity' paragraph. The procedure is as follows: if bank k borrowed an interbank loans from bank j in the period $t-1$ and if bank k is unable to repay the loans at t, then bank k can attempt to request to extend the loan with bank j. It is assumed that bank k will only request to extend part of the loan such that it no longer has negative liquidity. Therefore, if bank k is able to pay the interest on the interbank loans, it is assumed that it will do so and it will only request to extend the loan itself. Only if bank k is unable to pay off the interest to its credit bank, then it will request to extend the loan with interest. For the creditor bank's perspective, the incorporation of loan extension makes sense too. This is because when its debtor banks fail to repay the interbank loans, the creditor bank will suffer both liquidity and equity problem. If the effect is serious, the creditor bank may result in negative liquidity or equity and will go bankrupt itself. Therefore, this does motivate creditor banks' willingness to extend their interbank loans to its debtors. It is assumed that debtor banks are allowed to extend their loans indefinitely provided that the creditor banks are agreed to. The interbank transaction is given by the minimum between what bank k has previously borrowed from bank j and the amount it needs in order not to have negative liquidity.

$$\ddot{IT}_t^{kj} = \min(IT_{t-1}^{kj}, \max(0, R_t^k - \acute{L}_t^k)) \tag{5.22}$$

Initialisation of simulations

In order to initialise each simulation, a couple of parameters will have to be chosen exogenously. It is assumed that the deposit of each bank k at $t = -1$ to be its average deposit size; it follows that its reserve at $t = -1$ will be β multiply by its previous deposit size; the equity will be χ multiply by the bank size. It is also assumed that there were no interbank transactions between all banks at all in the period $t = -1$. Finally, it is assumed that banks have invested as much as they could in the last τ periods and that the amount is evenly spread among the last τ periods.

$$\begin{aligned}
D_{-1}^k &= \bar{D}^k \\
E_{-1}^k &= \chi S^k \\
R_{-1}^k &= \beta D_{-1}^k = \beta \bar{D}^k \\
IB_{-1}^k &= 0 \\
IL_{-1}^k &= 0 \\
I_{-s}^k &= \min[\bar{O}^k, \frac{\chi S^k + (1-\beta)\bar{D}^k}{\tau}, \frac{E_{-1}^k}{\psi \rho_I \tau}]
\end{aligned}$$

for $k = 1, 2, ..., M$ and $s = 1, 2, ..., \tau$.

The derivation of what values each of the I^k_{-s} is provided below. Since the liquidity of a bank k at the end of the period $t = -1$ is given by

$$L^k_{-1} = E^k_{-1} + D^k_{-1} + IB^k_{-1} - \sum_{s=1}^{\tau} I^k_{t-s} - IL^k_{-1}$$

If bank k made investment in the period $t = -1$, it means that its liquidity must be at least as high as its reserve requirement, that is, $L^k_{-1} \geq R^k_{-1}$. By substituting the exogenously chosen values for equity, deposit, interbank borrowing and lending into the above expression gives

$$\begin{aligned}
L^k_{-1} &= E^k_{-1} + D^k_{-1} + IB^k_{-1} - \sum_{s=1}^{\tau} I^k_{t-s} - IL^k_{-1} \\
R^k_{-1} &\leq E^k_{-1} + D^k_{-1} + IB^k_{-1} - \sum_{s=1}^{\tau} I^k_{t-s} - IL^k_{-1} \\
\beta\bar{D}^k &\leq \chi S^k + \bar{D}^k - \sum_{s=1}^{\tau} I^k_{t-s} \\
\sum_{s=1}^{\tau} I^k_{t-s} &\leq \chi S^k + \bar{D}^k - \beta\bar{D}^k \\
\sum_{s=1}^{\tau} I^k_{t-s} &\leq \chi S^k + (1-\beta)\bar{D}^k
\end{aligned}$$

By assuming that the total investment made is spread evenly across the last τ periods gives

$$\tau I_{t-s}^k \le \chi S^k + (1-\beta)\bar{D}^k$$
$$I_{t-s}^k \le \frac{\chi S^k + (1-\beta)\bar{D}^k}{\tau}$$

for all value of s.

5.3 Analysis and discussion

Unless otherwise stated, the values of all the parameters used in the simulation are given in Table 5.2.

Table 5.2: The values of all the parameters used in the simulation (unless otherwise stated)

Parameter	Representation	Value
N	Number of simulations	10000
T	Number of periods	1000
M	Number of banks	6
τ	Maturity period of an investment	3
r_D	Interest rate for deposit	0
r_I	Investment income	0.01
r_B	Interest rate for interbank lending	0.005
χ	Equity requirement ratio	0.05
β	Reserve requirement ratio	0.1
ϕ	Capital adequacy requirement ratio	0.08
$\bar{S}$	Size of large bank	5000
σ_D	Fluctuation in deposit	0.1
α_L	Parameter in the deposit fluctuation process for large bank	0.95
α_S	Parameter in the deposit fluctuation process for small bank	0.8
δ	Aggregate deposit and investment opportunity ratio	0.5
κ	Imbalance between deposit and investment opportunity	1
ρ_I	Weighted risk for investment	0.2
ρ_{IL}	Weighted risk for interbank lending	0.5

5.3.1 Network structure

Based on the description of the construction of the banking system in Section 5.2.1, in order to generate one realisation of the network structure, the two parameters needed are the number of banks in the system, M, and the imbalance parameter between bank's deposit and investment opportunity, κ. From Equation (5.1), the current network structure of the banking system can be interpreted as two segments: the large bank in the centre and the small bank(s) on the periphery. Each of these segments has the size of $\bar{S}$. It has been assumed for simplicity that there is only one large bank in the system. Therefore, the number of small banks, that is $M-1$, determines the size of each small banks. Equation (5.2) is then used to work out the imbalance between deposit and investment opportunity for each bank. This imbalance for each bank is in turn used to calculate the average deposit size and average investment opportunity size in Equation (5.3) and (5.4) respectively. Therefore, I first present the results regarding the network structure and how these two parameters play a role in the model.

The number of small banks in the system

Figure 5.1 shows the probability of banks surviving until the end of the 1000 periods with different number of 'small' banks in the system. The fluctuation in deposit for banks is given in Equation (5.5). With α_S chosen to be 0.8, small banks are modelled in the way that they do not suffer from liquidity shortage arising from the daily change in customers' deposit. Therefore, assuming that they are not connected to the 'large' bank, they do not default themselves. Hence, what Figure 5.1 shows is that the default of small banks arises as a result of transacting with the large one through the interbank market. It shows that when there are only two parties in the system, the chances of survival for both banks are the lowest. This is because the extension of interbank loans is not available and there are no other parties that the 'large' bank can borrow from. When the large bank fails to repay the loans to its creditor, the creditor bank suffers from a loss in equity and if this loss is big enough, the creditor bank defaults due to negative equity. This is supported by Figure 5.3, which shows the average number periods the large bank successfully arranges an interbank transaction before it defaults or until the end of the 1000 periods. It can be seen that, in the absence of loan extension or other interbank lenders, the large bank can only arrange 4 interbank transactions with the 'small' bank per simulation. As the number of 'small' banks increases in the system, the large one starts to have enough interbank creditors to transact with and this is reflected in Figure 5.1 in which the probability of the survival of large bank increases. The decrease in large bank default in turn reduces the probability of small banks default arising from non-repayment of interbank loans.

Figure 5.1: Probability of banks surviving until the end of the 1000 periods with different number of small banks in the system

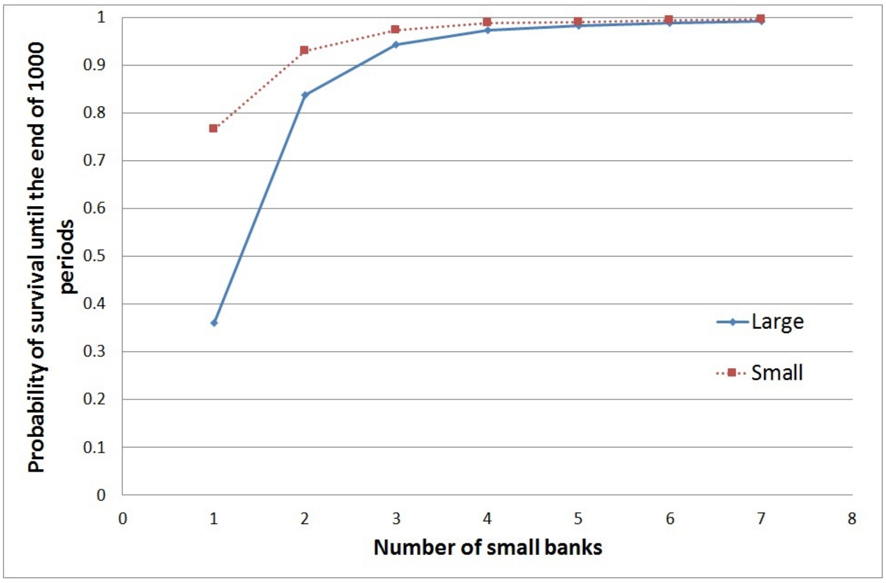


Figure 5.4 illustrates some of the features from the balance sheet of a defaulted bank. The solid line shows the average amount of negative liquidity of a bank when it defaults. There is an increase in the magnitude of negative liquidity of a defaulted bank when the number of small banks increases. This is because in the presence of more counterparties in the system, the large bank has more potential interbank creditors to borrow from. As a result, the large bank has a higher chance of surviving its liquidity shortage and hence those who default must have a high liquidity need. The fact that the dotted line is always much higher than the dash line indicates that, at all time, the immature investment is always higher than the customers' deposit. This means that, the bank should be able to pay back to its customers', even by selling off the immature investment at a discounted value.

Figure 5.2: Number of incidents of default arising from liquidity or equity issue with different number of small banks in the system

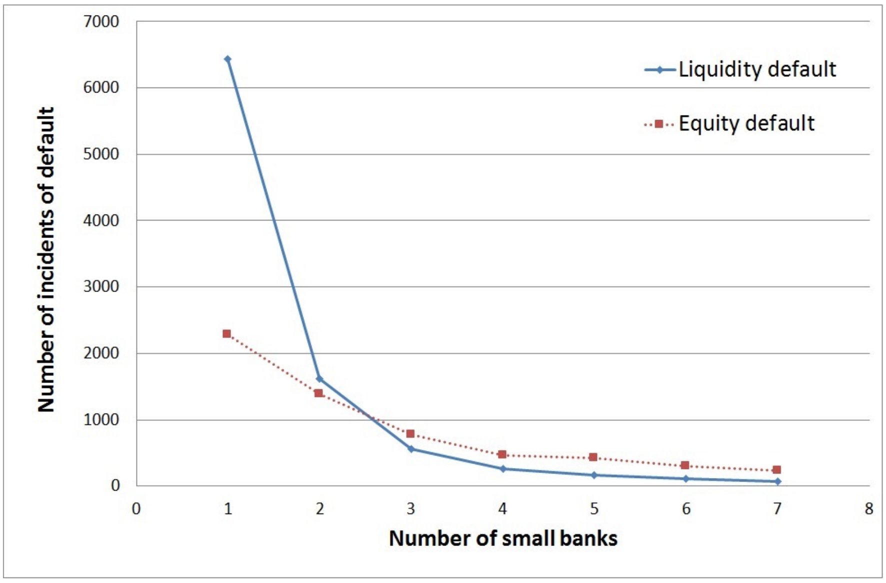


Figure 5.3: Average number of periods that the large bank successfully arranges interbank borrowings in the presence of different number of small banks in the system

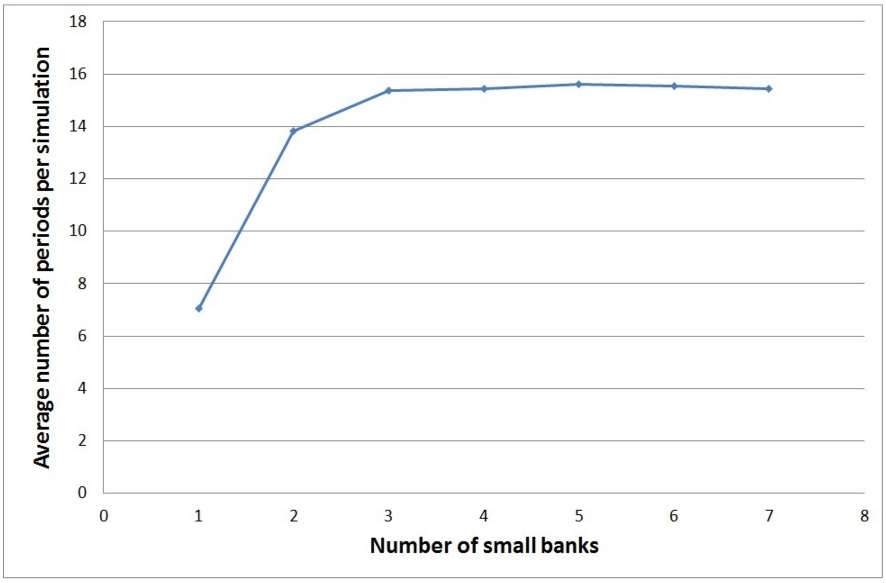


Figure 5.4: Size of customers' deposit, liquidity and immature investment when banks default, with different number of small banks in the system

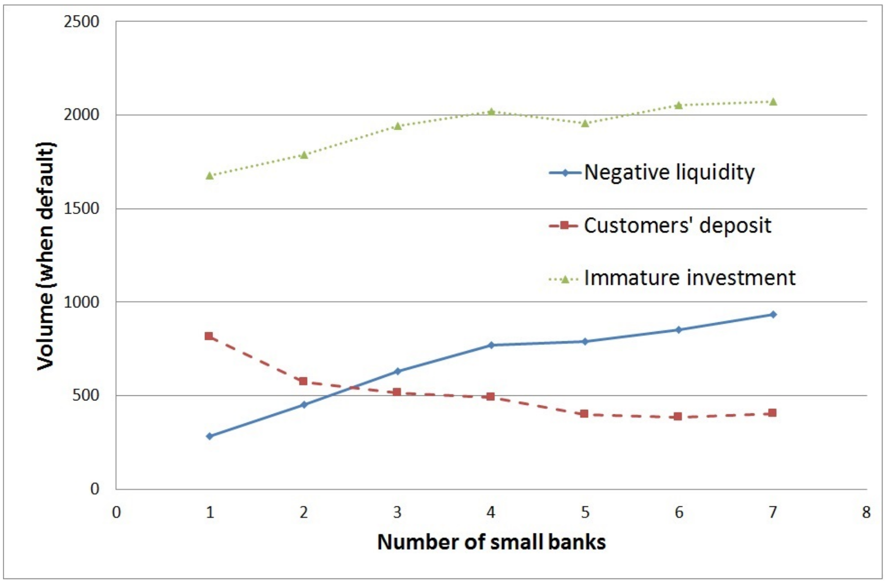
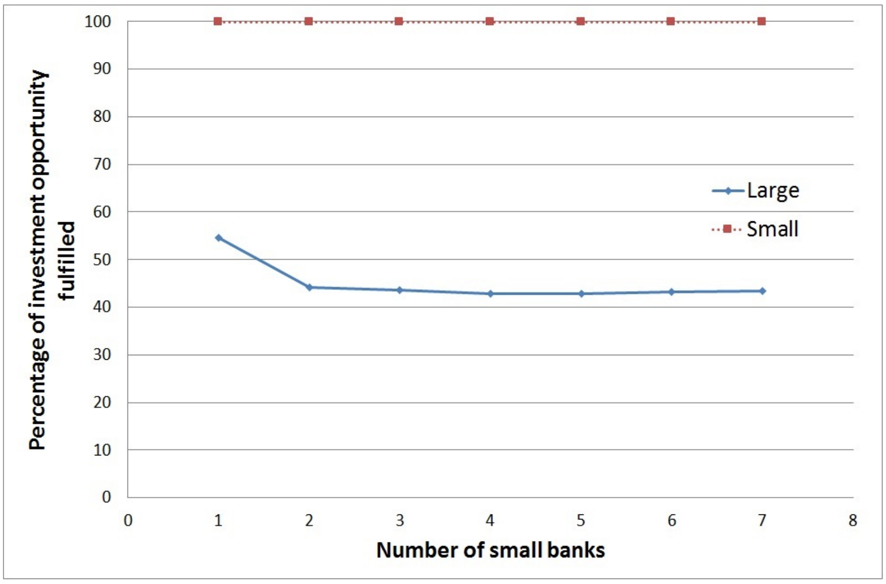


Figure 5.5 illustrates the percentage of investment opportunity achieved by banks. Due to the way the imbalance is modelled for large and small banks, it is expected that the small banks would be able to achieve close to 100% of its available investment opportunity while the large bank is only able to achieve around 40%, as indicated in Figure 5.5. It also shows that increasing the number of small banks in the system do not affect the level of investment opportunity fulfilled by small banks. Introducing more small banks in the system is equivalent to dividing the aggregate average deposit and investment opportunity in the small-bank segment equally among these small banks.

Therefore, based on the current setting, it is favourable to split the 'small-bank' sector into more small banks. This is because it increases the potential number of counterparties that the large bank can transact with. This results in an increase in the interbank activity and reduces the chances of large bank default. Since Figure 5.5 shows that the average percentage of investment opportunity fulfilled remains unchanged for both large and small banks regardless of

Figure 5.5: Percentage of investment opportunity fulfilled by banks with different number of small banks in the system



the number of small banks in the system, therefore a reduction in the default in banks also result in an overall increase in the aggregate investment opportunity fulfilled in the system.

The imbalance between deposit and investment opportunity

Next, I study the effect of this imbalance parameter on the banking system. Figure 5.6 compares the probability of bank surviving until the end of the 1000 periods, with different values for the imbalance between deposit and investment opportunity. It shows that as the value of κ increases, the probability of large bank surviving until the end of 1000 periods increases. The explanation is as follows. By definition, the imbalance parameter is used to determine the difference between size of deposit and size of investment opportunity for banks, and by assumption, only the small banks can be potential interbank lenders. Therefore, the value of κ directly affects the amount of surplus liquidity small banks have upon making investment and this in turn affects the availability of liquidity as potential interbank loans. At low value of κ, the size of imbalance is small and as a result, the amount of surplus liquidity small banks have is little. This means that the interbank activity is low. This is supported by Figure 5.8 that, the average number of interbank transactions the large bank can request is low, at low values of κ. As the value of κ increases, the interbank activity increases. This increase in interbank activity indicates that when the large bank has liquidity shortage, it is able to request interbank loans from the small banks. Hence, it results in the improved probability of survival for large bank in Figure 5.6.

While the relationship between the probability of survival against κ is monotonically increasing for large bank, the relationship between probably of survival against κ for small banks is a U-shaped curve. It can be seen that there is a slight drop in the probability of survival for small banks between $\kappa = 0.6$ to 0.8. The explanation is as follow. As mentioned before, the fluctuation in deposit for small banks is modelled to be small, therefore, the small banks alone do not suffer from liquidity shortage and hence they do not default. This means that when the small banks do not have surplus liquidity to lend out as interbank loans, that is, when the value of κ is extremely small, their probability of survival should be 1. As the value of κ gradually increases, the small banks start to lend out interbank loans to the large bank. As shown earlier, this does help the large bank in dealing with their liquidity issue and as a result, improves their chances of survival. However, in some occasions, when the large bank is unable to repay its interbank loans to its creditor, that is those small banks, then they suffer from a loss in equity and if this loss is large enough, the small banks will default. This is shown in Figure 5.7. The line which represents banks that result in liquidity default comes from the liquidity shortage of large bank and it decreases as the value of κ increases. This relates to how the interbank loans from small banks to the large one helps reducing the default of large bank. The other line which represents banks that default as a result of equity loss. The increases in the number of incidents of equity default between $\kappa = 0.6$ to 0.8 explains why there is a fall in the probability of survival for small banks in Figure 5.6.

However, Figure 5.6 shows that eventually the probability of survival for small banks increases and at large value of κ, the probability of survival for both types

Figure 5.6: Probability of banks surviving until the end of the 1000 periods with different value of κ

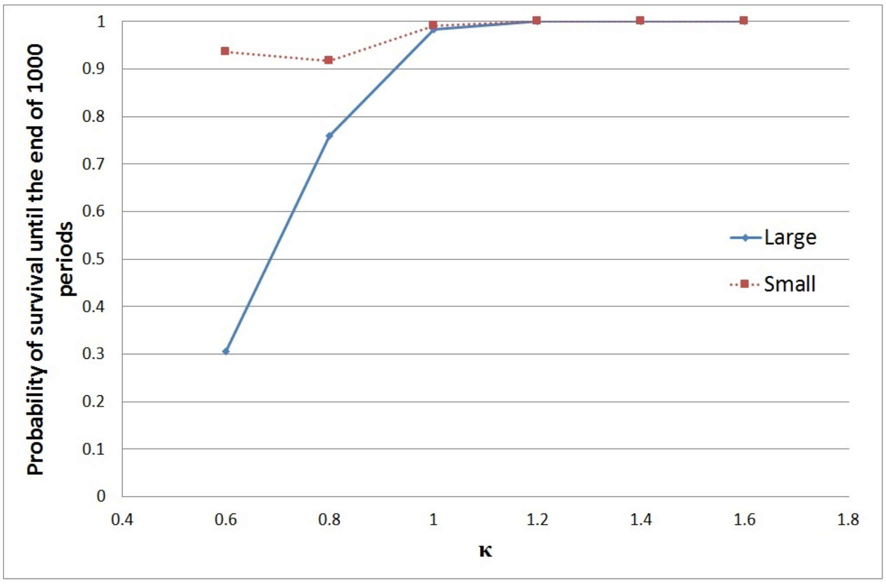
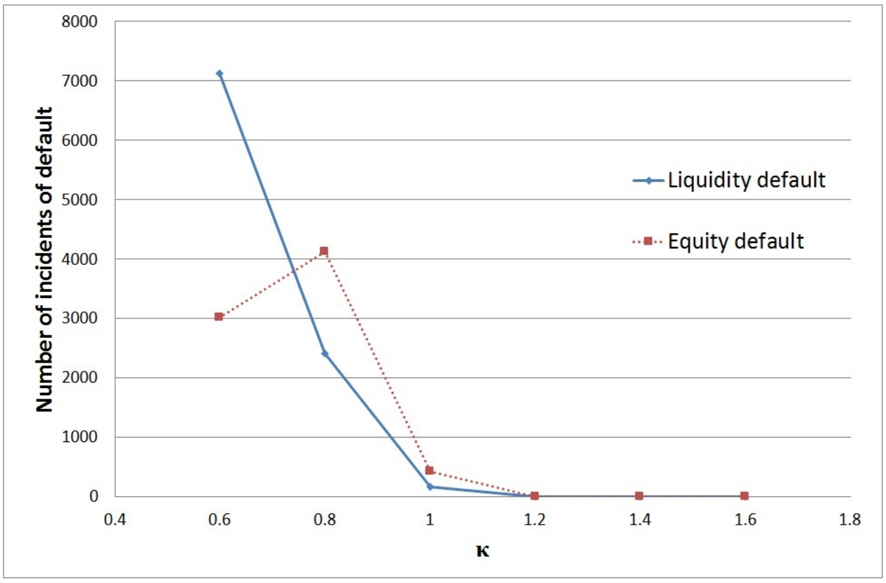
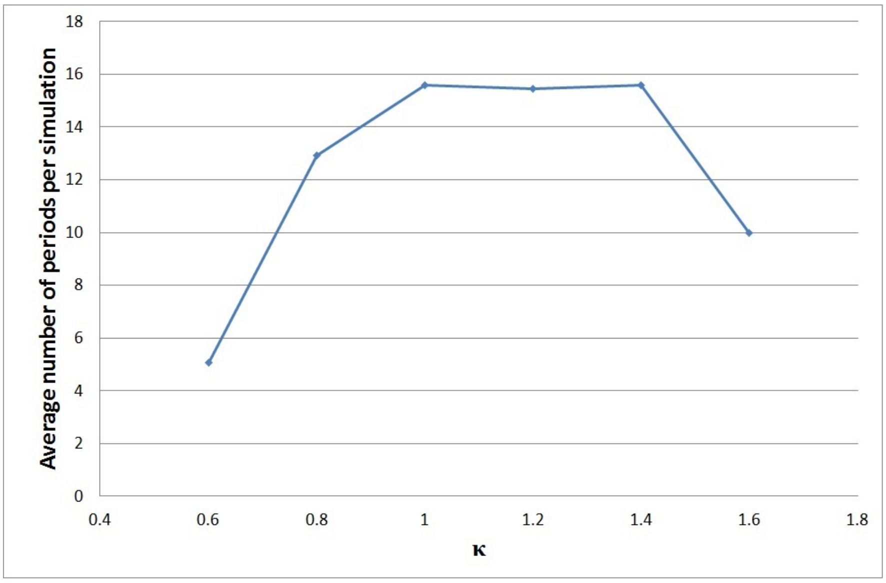


Figure 5.7: Number of incidents of default arising from liquidity or equity issue with different value of κ



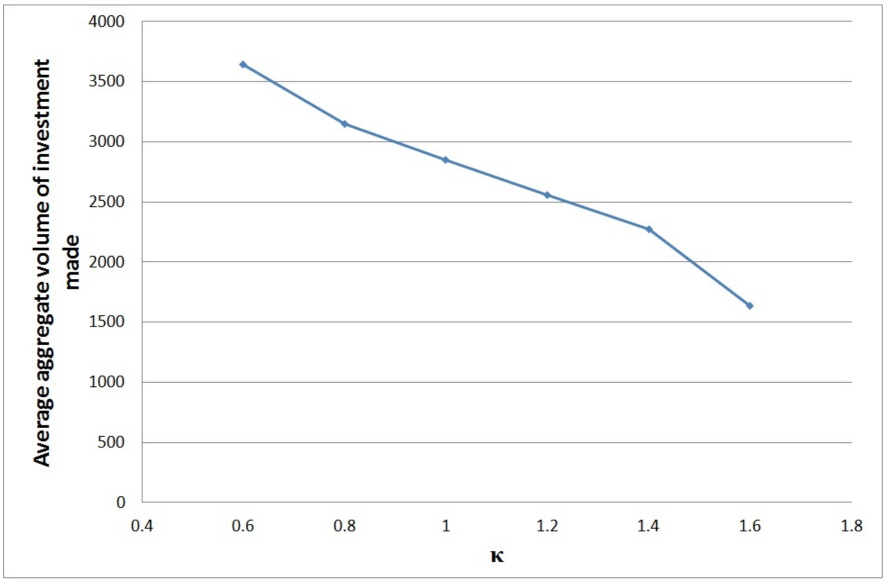
of banks is 1. This is because there are always small banks with surplus liquidity to lend out as interbank loans to the large bank when it suffers from liquidity shortage. The problem for a system with κ between 0.6 and 1 is that some of the otherwise healthy small banks default as a result of transacting in the interbank market. The trade off of using the interbank market is that on one hand, it causes small banks to suffer from equity default. On the other hand, it increases the probability of survival for large bank by nearly 50%.

Figure 5.8: Average number of periods that the large bank successfully arranges interbank borrowings with different value of κ



Although Figure 5.6 shows that the imbalance modelled has a positive effect on the stability on the banking system, it has its drawbacks. Figure 5.9 shows the average aggregate volume of investment made by all banks at different values of κ. It is of no surprise to see that the larger the imbalance, the less aggregate investment the banks are able to make. This is because a larger value of κ means that the big bank has more investment opportunity and less deposits. This results in more unfulfilled investment opportunity due to lack of available liquidity net reserve. On the other hand, a larger value of κ means that the small banks have more deposits and less investment opportunity. Therefore, there will be more surplus liquidity used while the total amount of investment fulfilled by small banks is reduced.

Figure 5.9: Average aggregate volume of investment made with different value of κ



5.3.2 Regulatory requirement

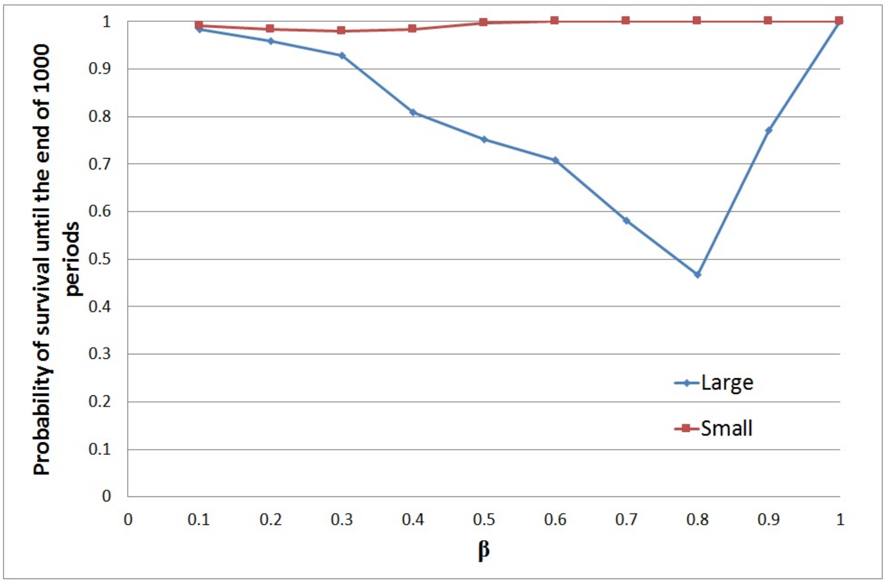
After considering the effect of the two parameters that determine the specific network structure of the banking system, I then investigate how do the regulatory requirements play a role in the model.

Reserve requirement

Figure 5.10 displays the probability of survival banks until the end of the 1000 periods with respect to different values of the reserve requirement ratio, that is β. The purpose of the reserve requirement is to ensure that banks hold a certain proportion of its customers' deposit as liquid cash. Therefore, the reserve provides insurance to banks against any unforeseen liquidity need. On the other hand, on the profitability perspective, holding reserve does not do the bank any good. Therefore, one would expect the tightening of reserve requirement, that is to increase the value of β, would bring an increase in the probability of survival for banks and in return, there would be a decrease in terms of investment opportunity fulfilled. Figure 5.13 shows the latter phenomenon that, as the value of β increases, it results in a fall in the aggregate volume of investment made. However, regarding the relationship between β and the probability of bank survival, as shown in Figure 5.10, is ambiguous. This ambiguity arises because the level of reserve a bank holds affects the likelihood and amount it can potentially lend out as interbank loans. Therefore, in the presence of an interbank market, the change in the value of β influences the stability of the system in two ways. Figure 5.10 shows that as the value of β increases, the probability of survival for the small banks increases. As mentioned previously, if small banks are not involved in any interbank transactions, the fluctuation in deposit they face is never large enough to cause any default. The reduction in interbank activity is evidenced in Figure 5.11, which shows that the average number of interbank borrowings the large bank arranges decreases as β decreases. This is accompanied by a fall in the number of incidents of equity default, as shown in Figure 5.12.

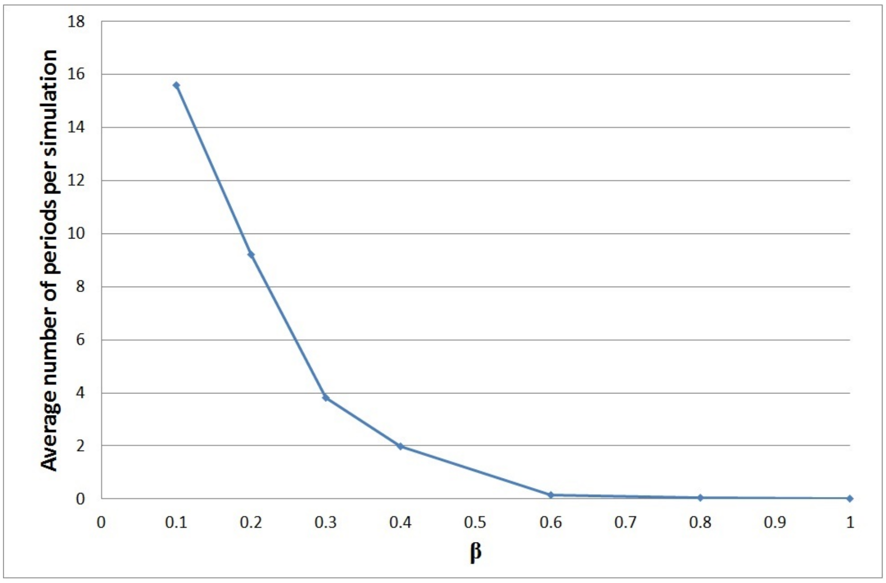
For the large bank, at low values of β, an increase of β between 0.1 to 0.3 produces a positive effect on the probability of survival for large bank. The tight-

Figure 5.10: Probability of banks surviving until the end of the 1000 periods with different values of β



ening of the reserve requirement implies that all banks are holding more reserve. However, those small banks, by assumption, are those with proportionately more deposit than their investment opportunity. Therefore, although they are retaining more deposit as reserve, they still have an abundant amount of liquid cash to be lent out as interbank loans. Therefore, this keeps the interbank market relatively active, together with the additional reserve the large bank is holding as insurance against deposit fluctuation, the probability of survival for large banks improves. Unfortunately, as the value of β continues to increases, that is between 0.3 to 0.8, the probability of survival falls rapidly. This is because the high value of reserve requirement starts to inhibit the interbank activity significantly. In the current setting, due to how the imbalance is modelled, a higher proportion of the aggregate customers' deposit is allocated to the small-bank segment. Therefore, when the value of β is increased by 0.1, the aggregate amount of addition reserve small banks hold is much greater than what the large bank holds. The change in the value of β does not affect the fluctuation in customers' deposit. In other words, the frequency and magnitude of liquidity shortage the large bank faces remain unchanged. Hence, part of liquid cash that was previously available to the large bank as interbank loans now becomes small banks' reserves. As a result, the large bank, without the aid from the interbank market, now having to rely on its own reserve against its liquidity shortage would default. Therefore, in the middle range of β, increasing the value has a stronger effect on hindering the interbank activity than on providing extra insurance for large bank.

Figure 5.11: Average number of periods that the large bank successfully arranges interbank borrowings with different values of β



However, at high value of β, that is from 0.8 onwards, increasing the value of β improves the probability of survival of large bank significantly. As shown in Figure 5.11, there are no interbank activities for β higher than 0.8. This implies that that the large bank relies solely on its reserve against the fluctuation in customers' deposit. And when $\beta = 1$, the probability of survival is 1.

Figure 5.12: Number of incidents of default arising from liquidity or equity issue with different value of β

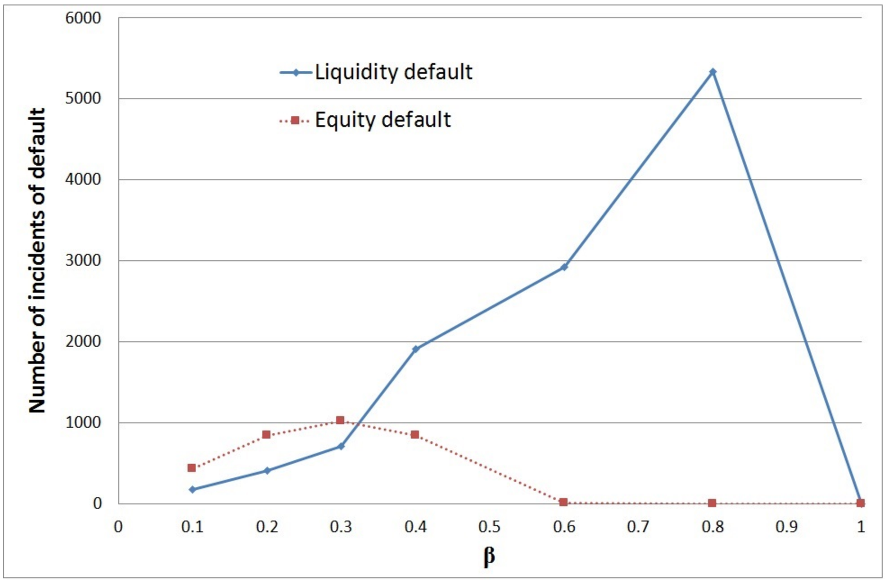
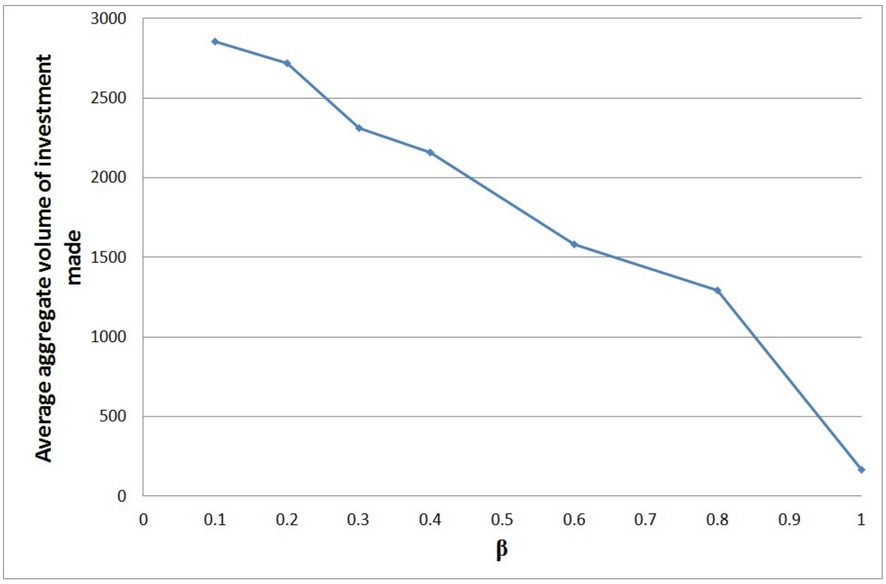


Figure 5.13: Average aggregate volume of investment made with different value of β



Capital adequacy requirement

Figure 5.14 shows the probability of survival of banks till the end of the 1000 periods with respect to different values of the capital adequacy requirement ratio, that is ϕ. Similar to the reserve requirement ratio, β, the capital adequacy requirement ratio, ϕ, affects the interbank activity directly, and hence it influences the probability of bank survivals. Figure 5.14 shows that, at low value of ϕ, increasing the value of ϕ do not have any effects on the probability of survival on both types of banks. This is because when the ratio is low, the amount of capital that a bank holds is sufficient to satisfy the capital adequacy requirement against its risk-weighted assets. In other words, when a potential interbank lender considers transacting in the interbank market, as given in Equation (5.21), the determination of the final amount to be transacted is likely to be based on the liquidity need of the borrower bank and the surplus liquidity of the lender bank. Therefore, at low values of ϕ, the interbank market functions efficiently. When ϕ is between 0.15 and 0.25, there is a fall in probability of survival for both types of banks. This is because the interbank market is still at work, but with restrictions. As shown in Figure 5.15, there is a gradual decrease in the number of interbank borrowings from large bank for $0.15 \leq \phi \leq 0.25$. This means that ϕ starts to play a role in the Equation (5.21), by either reducing the amount of interbank loan the lender bank otherwise would have lent, or preventing the lender bank to lend out any interbank loan. This exposes those banks that actually transact with the large bank to more risk.

However, as shown in Figure 5.15, the interbank activity falls considerably as ϕ exceeds 0.25. This indicates that the capital adequacy requirement ratio starts to become the main determinant in the arrangement of interbank loans and when the ratio is high enough, it completely shuts down all interbank activities. On one hand, this prevents the small banks from equity default arising from the non-repayment of interbank loans from the large bank. However, on the other hand, restricting interbank activity results in large bank defaulting immediately from its liquidity shortage.

Figure 5.14: Probability of banks surviving until the end of the 1000 periods with different values of ϕ

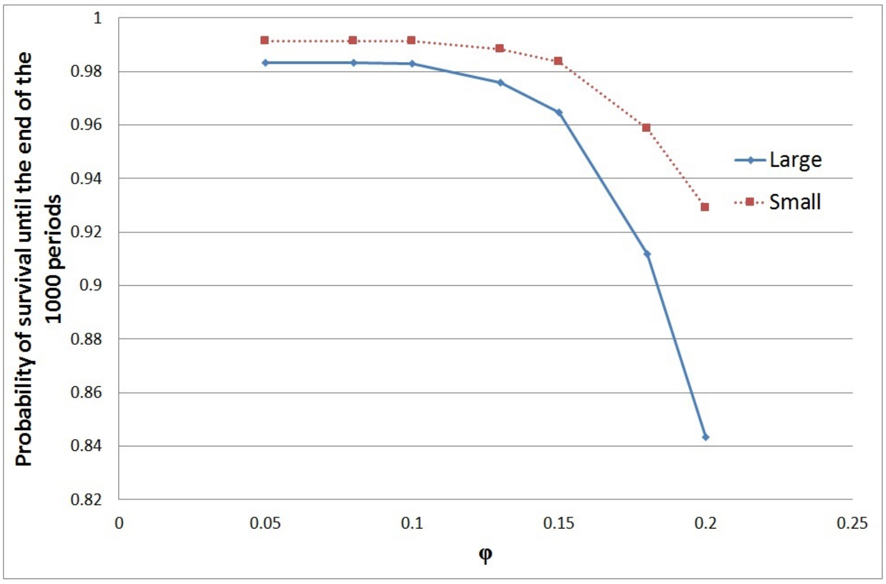
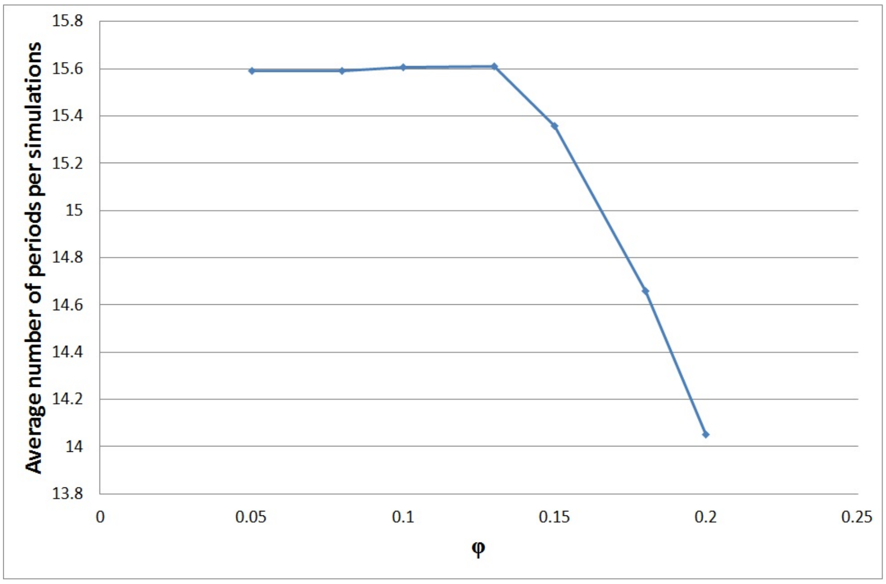


Figure 5.15: Average number of periods that the large bank successfully arranges interbank borrowings with different values of ϕ



5.4 Parameter sensitivity

In constructing the model, numerous assumptions were used together with many parameters in building the network of banking system, in the daily operations of banks and in the interaction between banks through interbank market. This section demonstrates the robustness of the results with respect to changes in the values of these parameters. The values of all parameters for the benchmark model are given in Table 5.2. Among these parameters, the reserve requirement ratio has been chosen to be 10%, according to the Boards of Governors of the Federal Reserve System[1] and the capital adequacy requirement ratio has been chosen to be 8% based on the Basel II[2]. The values for the risk weights of cash, investment and interbank lending have been chosen in accordance to U.S. Basel. III[3] Cash has 0% risk weighting, investment has 20% risk weighting and interbank lending has 50% risk weighting. Apart from these, the values of the remaining parameters are changed over a range and the results are reported below.

In order to analyse the parameter sensitivity, each of the parameters is tested separately. For each parameter, its value used in the benchmark model is taken to extend to a wide range of values. Then 100 different values are chosen from this range and for each value chosen, 1000 simulations are run to test the robustness of the parameter. δ denotes the aggregate deposit and investment opportunity ratio in the banking system. Varying the value of δ affects the liquid resource a bank holds upon making investment. This effect is two-fold. Firstly, a bank with excess liquid resource is better-protected for itself against the fluctuation in customers' deposit. Secondly, the liquid resource also affects its availability of liquidity for lending out to others through the interbank market. Therefore, it is expected that a smaller value of δ would reduce the probability and severity of a bank suffering from liquidity shortage arising from customers' deposit and would also increase the availability of liquidity for interbank transactions if needed. The value of δ in the benchmark model is 0.5. Figure 5.1 shows that, in the presence of 6 banks in the system, the probabilities of banks surviving until the end of the

[1] http://www.federalreserve.gov/monetarypolicy/reservereq.htm
[2] http://www.newyorkfed.org/education/pdf/2012/Yang_bank_capital_regulation.pdf
[3] http://www.usbasel3.com/docs/Final%20LCR%20Visual%20Memo.pdf

1000 periods are 98.31% and 99.14% for large and small banks respectively. 100 different numbers have been drawn from 0.45 to 0.55 and each value has been simulated 1000 times. Qualitatively similar results were found for this wide range of values in comparison to the benchmark model. The average value of δ in the 100000 simulation is 0.4991 and the average probability of banks surviving are 98.32% and 99.22% for large and small banks respectively. This is illustrated in Table 5.3. The finding also shows that the probabilities of survival are higher for both types of banks at a smaller value of δ and this coincides with the expectation.

α_L and α_S play a role in determining the fluctuation in deposit for large and small banks respectively. In the benchmark case, the value for α_L and α_S are 0.95 and 0.8 respectively. Both parameters were tested separately for robustness. α_L is tested against a range between 0.9 to 1 while α_S is tested against the range between 0.75 to 0.85. Again, results were found to be qualitatively the same, as shown in Table refParameter sensitivity. A decrease in the value of α_L reduces the deposit fluctuation for large bank, which is the most vulnerable bank in facing liquidity shortage. As a result, the probability of large bank surviving is higher when the value of α_L is low. On the other hand, a change in the value of α_S has a less significant effect on the bank system. This is because when $\alpha_S = 0.8$ in the benchmark model, it is low enough that small banks never experience a big enough deposit fluctuation that causes liquidity shortage in them. Therefore, varying the values of α_S does not produce a big effect on the excess resources on small banks and in turn, the availability of liquidity for interbank lending to the large bank. Overall, both parameters are found to be robust.

Similarly, R_I, R_B and σ_D are all tested for robustness and the results are all shown in Table refParameter sensitivity. All of them produce qualitatively and quantitatively similar results over a wide range of values, also given in Table ref-Parameter sensitivity.

Table 5.3: Parameter sensitivity

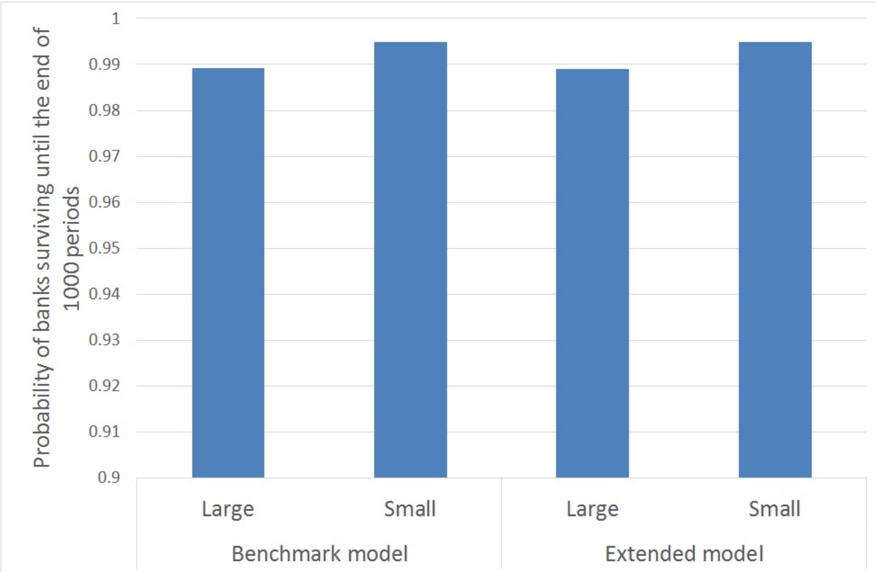
Parameter	Value	Range	Averaged value	Probability of surviving for large bank	Probability of surviving for small banks
δ	0.5	0.45 - 0.55	0.4991	0.9832	0.9922
α_L	0.95	0.90 - 1.00	0.9443	0.9928	0.9967
α_S	0.8	0.75 - 0.85	0.8010	0.9839	0.9925
r_I	0.01	0.005 - 0.015	0.010	0.9840	0.9925
r_B	0.005	0.001 - 0.010	0.005	0.9846	0.9927
σ_D	0.1	0.095 - 0.105	0.100	0.9848	0.9928

5.5 An extended model

After considering the benchmark model in the presence of 6 banks in the system, in which there are only 1 large bank in the core and 5 small banks at the peripheral, it is of interest to consider a more general case with more core banks in the system. For comparison purpose, it is assumed that for each large bank in the banking system, there are 5 other small banks present and that these 5 small banks are only connected to the same large core bank while all large banks are connected to each other. Everything else has been assumed to be the same as the benchmark model.

However, a major problem emerged when simulating this general case. Figure 5.16 shows that the probability of banks surviving until the end of the 1000 periods is the same in both the benchmark model and the extended model. First of all, it has always been modelled that large banks have plenty of investment opportunity. This means that they rarely have excess resource to lend out as interbank loans. Besides, large banks are modelled to face a bigger fluctuation in deposit such that the fluctuation is enough to cause liquidity shortage in them. In turn, this creates the need to borrow interbank transactions from other potential lenders, which happen to be the small banks. These two factors together imply that although connections exist among the large banks, they never trade with each other through the interbank market. This can be interpreted as a banking system with many subset of systems, each of which has 6 banks in total and the banks in each subset only trade with each other. Therefore, the focus will be shifted slightly to explore the effect of merging some of the small banks to form a big bank.

Figure 5.16: Probability of banks surviving until the end of the 1000 periods in the benchmark model and the extended model



Let's consider a system of 30 banks with 5 large core banks and for each of them, there are 5 small banks connected at the peripheral. Similar to what was discussed above, this can be interpreted at 5 subsets of the benchmark model and the probability of bank surviving until the end of the 1000 periods should be the same as Figure 5.16. Each time, one of the small banks from each subset will be merged together to form a large bank. Therefore, the merged bank has the same bank size of the large bank, however, it has the properties of the small banks, including the deposit fluctuation, the imbalance between deposit and investment opportunity. It is then assumed that the merged bank is now connected to all other large core banks. Therefore, after merging 5 small banks into 1, a modified banking structure is created. There are now 6 core large banks in the system, 5 of which have the same properties of the large banks as before while the remaining one functions as a small bank, despite its size. Therefore, when any of the 5 large banks face liquidity shortage, they can approach both their own connected small banks or this merged bank for interbank transaction, depending on availability. It should be noted that since one of the small banks from each subset is combined to form the merged bank, therefore, each of the large bank should now be connected to 4 other small banks only. In doing so, the aggregate bank size, customer deposit and investment opportunity all remain unchanged.

Figure 5.17: Probability of banks surviving until the end of the 1000 periods in the presence of merged banks

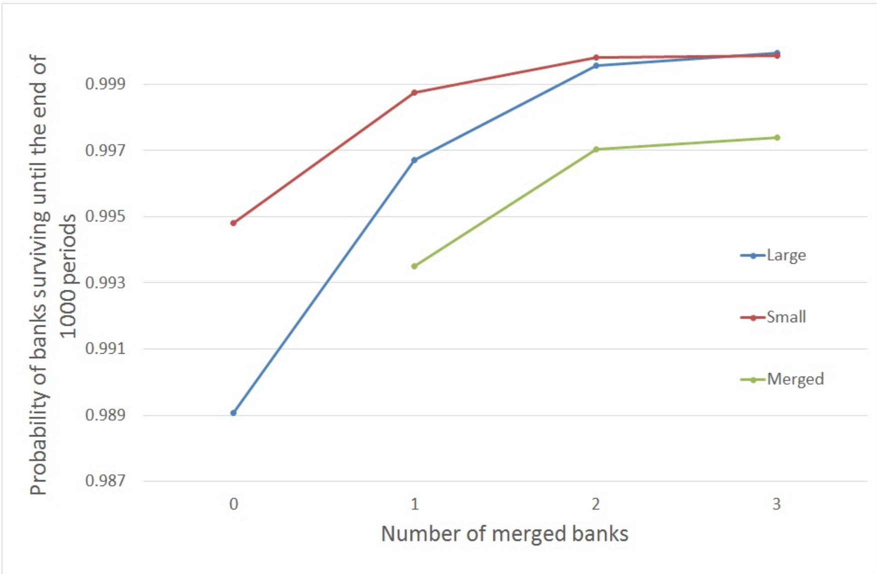


Figure 5.17 shows the probability of banks surviving until the end of the 1000 periods for large banks, small banks and merged banks. As mentioned before, in the absence of merged banks, the banking system can be interpreted as 5 subsets of the benchmark modelled connected together through the large core banks. However, based on how the banking system is designed, large banks are unable to transact with each other through the interbank market. Therefore, the probability of bank surviving is only as good as the benchmark model. But Figure 5.17 illustrates that by merging one small bank from each subset to a large core bank, the probability of banks surviving until the end of 1000 periods increases for both

large and small banks. This is because without merging some of the small banks together, some of the available liquid resource in one subset is not accessible to large core banks in another subset. This means that the merging of banks brings a better distribution of resources among the system, and as a result, it brings an increase in probability of survival for banks.

5.6 Conclusion

This chapter has presented an artificial interbank market model with a specific network structure, the tiering structure. In order for the model to exhibit higher resemblance to the real-world market, the imbalance parameter has been incorporated into the model. This imbalance parameter plays a role in determining the heterogeneity in the size of customers' deposit and investment opportunity in banks. This is because it has been empirically shown that large banks are usually the deposit-taking banks while the small banks often have more investment opportunity available to them. Besides, empirical studies also find that large banks tend to be net borrowers in the interbank market while smalls banks act as lenders. To accommodate this, the fluctuation in customers' deposit has been modelled to be large for large banks to create this higher liquidity need in large banks.

One of the main focuses of the research has been on the role of the imbalance parameter in the system. It has been shown that the stability of the banking system is closely related to the interbank activity, which in term is related to the availability of lenders and the amount of surplus liquidity they have. It has been illustrated that when interbank lenders are rare, that is when κ is low, the large bank is very vulnerable to the fluctuation of customers' deposit. This small imbalance between deposit and investment opportunity in the small banks result in them not having excess liquidity to lend out as interbank loans. However, by increasing this imbalance in banks, this makes those small banks to become potential lenders in the interbank market. The significant increase in the probability of the survival of large banks is witnessed by the increase in interbank activity. And when the heterogeneity between deposit and investment opportunity in banks become very high, the abundant availability of interbank lenders results in a very stable system. The drawback of an increase in interbank activity is that when the level of activity is mild, there are occasional default of those otherwise healthy small banks as a result of non-repayment from the large bank. Although a greater imbalance in banks does seem to improve the stability of the system, the presence of this imbalance in deposit and investment opportunity is not without its pitfall. This is because an increase in this imbalance parameter effectively reduces the aggregate investment opportunity to the 'small' banks.

The second focus of the research is on the role of regulatory requirements on the stability of the system. Similar to varying the imbalance parameter, changing the regulatory requirements has a direct effect on the availability of potential interbank lenders and the amount of funds they can provide, which in turn affect the number of defaults in the system. It has been shown that the relationship between tightening the reserve requirement and the stability of the system is non-monotonic. This is because the tightening of reserve requirement in banks has an insurance effect on individual banks but it also affects the interbank activity. When a bank has a higher reserve requirement, it has to retain a bigger portion of its customers' deposit as liquid reserve. Therefore, on an individual level, each bank is better protected against unforeseen liquidity shortages. However, holding a higher amount of liquid reserve within a bank also implies that its available surplus liquidity as potential interbank loans is reduced. On the aggregate level, this affects the successfulness of banks arranging an interbank borrowings and results iner a lower activity level of interbank trading. On the positive note, it can be seen that there is a range of values of β in which banks are better insulated against liquidity shortage, arising from the addition liquid reserve they are holding, while the interbank activity is still relatively active. Unfortunately, this range of values is highly dependent on the network structure and the characteristic of the banking system.

While tightening reserve requirement makes banks better insulated against liquidity shocks, an increase in capital adequacy requirement helps banks become better protected against equity default. When a bank lends out interbank loans, it exposes itself to counterparty risk that the borrower bank is unable to repay the loan when it is due. The capital adequacy requirement plays a role in controlling the amount of risk it takes up in risky investment and interbank lending. In other words, it regulates the chances of a bank defaulting as a result of a loss in equity. It has been shown that when the capital adequacy requirement is tightened, a liquidity surplus bank is unable to commit to as much interbank lending as it otherwise would have. On one hand, this reduces the probability of default for potential lender banks. However, it also reduces the interbank activity, which in turn results in a higher default for banks with liquidity need.

Chapter 6

Conclusion

6.1 Summary

This research studies an artificial interbank market model with a tiering network structure. It analyses some of its network structure characteristics and examines the role of regulatory requirements on the system.

Chapter 1 describes the background and the motivation of the research. It proposes a number of research questions and an overview of the thesis. Chapter 2 presents a literature review on the financial contagion in the interbank market. This includes the channel of contagion in the interbank lending market, the use of network topology in representing the structure of the market, the empirical studies of the network structure and the interbank contagion in real-world banking systems and the theoretical analysis of the systemic risk in some artificial models. Chapter 3 presents the implementation of Iori, Jafarey, and Padilla (2006) model, which investigate the systemic risk in the interbank market with a random network structure and in the presence of heterogeneity in the system. The implementation forms the basic model of this research. However, there are a number of problems encountered during the implementation, Therefore, Chapter 4 points out these problems and provides modifications to the model, together with some preliminary results. Chapter 5 introduces the interbank market model for this research, describing the construction of the network of an interbank market and the simulation procedure of the operation of banks. It then presents the simulation results and also provides an analysis and a discussion of the results.

Appendix A

Algorithm

A.1 Implementation of Iori, Jafarey, and Padilla (2006) Model

In this section, I provide all the algorithms for implementing Iori, Jafarey, and Padilla (2006) model in the presence of interbank transactions described in Chapter 3.

Algorithm 2 Connectivity matrix

Input variables: c, M
for $i = 1$ to M **do**
  for $j = 1$ to M **do**
    if $i = j$ **then**
      $J_{ij} = 0$
    else if $i > j$ **then**
      Generate a random number P such that $P \in \mathrm{U}[0, 1]$
      if $P \leq c$ **then**
        $J_{ij} = 1$
      else
        $J_{ij} = 0$
      end if
      $J_{ij} = J_{ji}$ ▷ The matrix is symmetric along the main diagonal
    end if
  end for
end for

Algorithm 3 Initial size

Input variables: $M, \bar{S}, \sigma_S, \sigma_I, \delta$
for $k = 1$ to M **do**
  Generate $\mu \sim N(0, 1)$
  $S^k = |\bar{S} + \sigma_S \mu|$

  Generate $\nu \sim N(0, 1)$
  $O^k = \delta \times |S^k + \sigma_I \nu|$
end for

Algorithm 4 Initial period

Input variables: k, χ, τ
$A^k_{-1} = S^k$
$V^k_{-1} = \chi A^k_{-1}$
for $i = 1$ to τ **do**
  $I^k_{-i} = 0.5 \times O^k$
end for
$B^k_{-1} = 0$ ▷ No interbank transaction at time $t - 1$
$L^k_{-1} = A^k_{-1} + B^k_{-1} + V^k_{-1} - I^k_{-1}$

Algorithm 5 Deposit

1: Generate $\epsilon_t \sim N(0, 1)$
2: $A_t^k = |S^k + \sigma_A \sqrt{S^k} \epsilon_t|$ or
3: $A_t^k = |S^k + \sigma_A S^k \epsilon_t|$
4: $\hat{L}_t^k = L_{t-1}^k + (A_t^k - A_{t-1}^k) - r_a A_{t-1}^k + \rho \sum_{s=1}^{\tau} I_{t-s}^k + I_{t-\tau}^k$

Algorithm 6 Repayment of interbank loans

Set $\mathbf{1} = 1$
while $\mathbf{1} \geq 1$ **do**
    Generate $k \in \mathbb{Z}_M$ ▷ Choose a bank randomly
    if $B^k_{t-1} > 0$ and $\hat{L}^k_t \geq (1 + r_b)B^k_{t-1}$ **then**
        $\hat{l}^k_t = \hat{L}^k_t - (1 + r_b)B^k_{t-1}$
        $B^k_{t-1} = 0$
        for $j = 1$ to M **do**
            if $j \neq k$ and $B^{kj}_{t-1} > 0$ **then**
                $\hat{l}^j_{t-1} = \hat{L}^j_{t-1} + (1 + r_b)B^{kj}_{t-1}$
                $B^{kj}_{t-1} = B^{jk}_{t-1} = 0$
            end if
        end for
    end if

    Set $\mathbf{1} = 0$
    for $k = 0$ to M **do**
        if $B^k_{t-1} > 0$ and $\hat{L}^k_t \geq (1 + r_b)B^k_{t-1}$ **then**
            $\mathbf{1} \ += 1$
        end if
    end for
end while

for $k = 0$ to M **do**
    if $B^k_{t-1} > 0$ and $\hat{L}^k_t \leq (1 + r_b)B^k_{t-1}$ **then**
        $\hat{l}^k_t = \hat{L}^k_t - (1 + r_b)B^k_{t-1}$
        $\hat{v}^k_t = \hat{l}^k_t + \sum_{s=1}^{\tau-1} I^k_{t-s} - A^k_t$
    end if
end for

Algorithm 7 Excess and reserve

$E^k_t = \hat{v}^k_t / A^k_t$
$R^k_t = \beta A^k_t$

Algorithm 8 Dividend payment

if $E_t^k > \chi$ **then**

$\quad D_t^k = \max\{0, \min[\rho \sum_{s=1}^{\tau} I_{t-s}^k - r_a A_{t-1}^k - r_b B_{t-1}^k, \hat{L}_t^k - R_t^k, \hat{L}_t^k + \sum_{s=1}^{\tau-1} I_{t-s}^k - (1+\chi)A_t^k]\}$

else

$\quad D_t^k = 0$

end if

$\tilde{L}_t^k = \hat{l}_t^k - D_t^k$

Algorithm 9 Investment opportunity

Generate $\eta_t \sim N(0,1)$

$\omega_t^k = |O^k + \sigma_\omega O^k \eta_t$

$\Omega_t^k = \omega_t^k$

Algorithm 10 Investment

$Inv = \min[\max(0, \tilde{L}_t^k - R_t^k), \Omega_t^k]$

$I_t^k \ += Inv$

$\Omega_t^k \ -= Inv$

$\dot{L}_t^k = \tilde{L}_t^k - Inv$

$\dot{V}_t^k = \dot{L}_t^k + \sum_{s=0}^{\tau-1} I_{t-s}^k - A_t^k - B_t^k$

Algorithm 11 Default

Set $Default = 0$
for $k = 1$ to M **do**
    if $L_t^k < 0$ or $V_t^k < 0$ **then**
        for $j = 1$ to M **do**
            $J_{jk} = J_{kj} = 0$
        end for

        $Default \ += 1$
    end if
end for

$Total - default_t = Default$
$Cumu - default_t \ += Default$

Algorithm 12 Interbank lending for liquidity

Set $\mathbf{1} = 1$
while $\mathbf{1} \geq 1$ **do**
    while $\mathbf{1} \geq 1$ **do**
        Generate $k \in \mathbb{Z}_M$ ▷ Choose a bank randomly
        if $L_t^k < 0$ and $V_t^k \geq$ **then**
            $Exchange = 0$
            while $\mathbf{1} \geq 1$ **do**
                Generate $i \in \mathbb{Z}_M$ ▷ Choose a bank randomly
                if $k \neq i$ **then**
                    if $J_{ki} = 1$ and $(L_t^k + b_t^{ik} - R_t^i) > 0$ and $B_{t-1}^{ki} \leq 0$ and $V_t^i \geq 0$ and $\Omega_t^i \leq 0$ **then**
                        $b_t^{ki} = \min[|L_t^k| - Exchange, L_t^i - B_t^{ki} - R_t^i]$
                        $b_t^{ik} = b_t^{ki}$
                        $Exchange \ += b_t^{ki}$
                    end if
                end if

                Set $\mathbf{1} = 0$
                if $Exchange < |L_t^k|$ **then**
                    for $j = 1$ to M **do**
                        if $k \neq j$ **then**
                            if $J_{kj} = 1$ and $(L_t^k + b_t^{jk} - R_t^j) > 0$ and $B_{t-1}^{kj} \leq 0$ and $V_t^j \geq 0$ and Ω_t^j **then**
                                $\mathbf{1} \ += 1$
                            end if
                        end if
                    end for
                end if
            end while

            To be continued ...

Algorithm 13 Interbank lending for liquidity - continued

if $Exchange = |L_t^k|$ **then**
  $L_t^k = 0$
  $B_{t-1}^k = 0$
  for $j = 1$ to M **do**
    if $k \neq j$ **then**
      if $b_t^{kj} > 0$ **then**
        $B_t^{kj} = b_t^{kj}$
        $B_t^{jk} = b_t^{jk}$
        $B_t^k \ += b_t^{kj}$
        $L_t^j \ += b_t^{jk}$
        $B_t^k \ += b_t^{jk}$
      end if

      if $b_{t-1}^{kj} > 0$ **then**
        $L_t^k \ += (1 + r_b)B_{t-1}^{kj}$
        Call 'Excess, dividend and investment'

        $B_{t-1}^{kj} = B_{t-1}^{kj} = 0$
      end if
    end if
  end for
else if $Exchange < |L_t^k|$ **then**
  for $j = 1$ to M **do**
    if $j \neq k$ **then**
      $b_t^{kj} = b_t^{kj} = 0$
    end if
  end for
end if

To be continued ...

Algorithm 14 Interbank lending for liquidity - continued

Set $\mathbf{1} = 0$
for $k = 1$ to M **do**
    Set $Resources = 0$
    if $L_t^k < 0$ and $V_t^k \geq 0$ **then**
        for $j = 1$ to M **do**
            if $j \neq k$ **then**
                if $J_{kj} = 1$ and $(L_t^k - R_t^j) > 0$ and $B_{t-1}^{kj} \leq 0$ and $V_t^j \geq 0$ and $\Omega_t^j \leq 0$ **then**
                    $Resources \ += (L_t^j - R_t^j)$
                end if
            end if
        end for

        if $Resources \geq |L_t^k|$ **then**
            $\mathbf{1} \ += 1$
        end if
    end if
end for
end if
end while
end while

Algorithm 15 Banking system

```
Input variables: N, T
for n = 1 to N do
    Call 'Connectivity matrix'
    Call 'Initial size'
    for t = 0 to T do
        for k = 1 to M do
            if t = 0 then
                Call 'Initial period'
            end if

            if L^k_{t-1} ≥ 0 and V^k_{t-1} ≥ 0 then
                Call 'Deposit'
                Call 'Investment opportunity'
            end if
        end for

        Call 'Repayment of interbank loans'
        for k = 1 to M do
            if L^k_{t-1} ≥ 0 and V^k_{t-1} ≥ 0 then
                Call 'Excess and reserve'
                Call 'Dividend payment'
                Call 'Investment'
            end if
        end for
        if c = 0 then
            Call ''Default'
        else if c ≠ 0 then
            Call 'Interbank lending for liquidity'
            Call 'Default'
        end if
    end for
end for
```

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