THE EFFECT OF METACOGNITIVE TRAINING ON THE MATHEMATICAL WORD PROBLEM SOLVING OF SINGAPORE 11-12 YEAR OLDS IN A COMPUTER ENVIRONMENT

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The candidate confirms that work submitted is her own and that appropriate credit has been given where reference has been made to the work of others.

ABSTRACT

This study aims to establish the extent to which metacognitive training plays a part in Singapore primary students' word problem solving in a computer environment. The study involved 142 Singapore 11 to 12-year-old students from two primary schools.

The study adopts a two-phase design, combining a quasi-experimental design and a case study design. For the quasi-experimental design, analysis of students' mathematical achievement test data is used to investigate the relationship between metacognitive training, students' level of mathematical achievement and their mathematical word problem solving performance. For the case study design, analysis of the think aloud protocol data during word problem solving of eight pairs of students is used to explore the role of metacognition in mathematical word problem solving in a computer environment. In addition, student questionnaire and teacher interview data provide descriptive accounts of students' metacognitive knowledge during mathematical word problem solving.

The findings from the analysis of mathematical achievement test and think aloud protocol data reveal that metacognitive training results in improvement in mathematical word problem solving performance, and that lower achievers appear to show the full benefit from metacognitive training only after a period of time. The findings of the think aloud protocol data also reveal that i) generating metacognitive behaviours, and knowing when and how to use them during word problem solving are important determinants for successful word problem solving, and ii) students have distinctive progressions of word problem solving activity which can be represented by five types of cognitive-metacognitive word problem solving models. These progressions of word problem solving activity seem to relate to students' success in word problem solving. It is also proposed that there is a relationship between affect, students' ability to develop metacognitive awareness, and word problem solving. In addition, effective pair collaboration is influenced by students' mathematical beliefs, and how students are paired according to their metacognitive knowledge.

The educational and pedagogical implications of these findings are discussed, particularly in relation to the Singaporean context.

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[a] *Barnabas or 'Son of Encouragement' was given to Joseph, a Levite from Cyprus (NIV,Acts 4:36), from the Jerusalem Christians. He was nicknamed Barnabas because he was drawn to people he could encourage and was a great help to those around him.*

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LIST OF ABBREVIATIONS

The following abbreviations have been used in the thesis.

S1	School 1
S2	School 2
HA	Higher achievers
LA	Lower Achievers
T	Treatment
C	Control
PSLE	Primary School Leaving Examination
MOE	Ministry of Education
IT	Information Technology

Chapter One
Introduction

1. Introduction and Summary of Study

This study undertakes to explore and investigate the effect of metacognitive training on mathematical word problem solving of Singapore 11-12 year olds in a computer environment. It is hoped that the findings and conclusions of this study may lead to raising awareness of the role of metacognition in word problem solving and in a computer environment; and to enable teachers and teacher educators in Singapore to rethink and re-examine the issue of metacognition along lines of pedagogical enquiry in mathematics word problem solving in Singapore primary schools.

The intended mathematics curriculum in Singapore, the Revised Mathematics Syllabus implemented in 1992 (see Appendix A), aims to enable students to develop their ability in mathematical problem solving (Curriculum Planning Division, 1995), and metacognition (see Chapter Two, section 2) is identified as one of the attainments of problem solving ability (see Appendix A). Besides the emphasis of problem solving in mathematics classrooms, the Information Technology Masterplan in Education (Ministry of Education, 1998) was launched in 1997 whereby all primary schools need to focus 30% of their curriculum on the use of information technology (IT), basically the computer, by the year 2002. It appears appropriate that research should focus on students' metacognition during mathematical problem solving in a computer environment. In addition, the study is also designed to contribute to the development of the Thinking programme (Mok, 1997) for the primary school.

There are two main aims in this study. The study aims to investigate the effect of explicit metacognitive training on upper primary students' mathematical word problem solving performance. The study also aims to address the role of metacognition in word problem solving in a computer environment. This phenomenon of the impact of metacognitive training on students' mathematical word problem solving will be studied in the light of current literature.

The overall design of this study is a two-phase design (Lee, 1999) combining, a quasi-experimental design which is 'scientific' (Robson, 1993, p. 18) in nature and a case study design which is exploratory-interpretive (Cohen & Manion, 1994, p. 259) in nature. The study attempts to test hypotheses related to the effect of metacognitive training and cognitive apprenticeship instruction amongst 142 students across two schools. It also attempts to explore the phenomenon of 8 pairs of students' use of metacognition in mathematical word problem solving across the two schools. The main methods of data collection are mathematical achievement tests in the form of pre-test, post-test and delayed post-test; think-aloud protocols; student questionnaire; and teacher interview. The analysis of the mathematical achievement test data is intended to provide some empirical information on the effect of metacognitive training on students' mathematical word problem solving performance, and the differential effect of metacognitive training on students of different mathematical achievement levels. The analysis of the think aloud protocol data is aimed at exploring the role of students' metacognition during mathematical word problem solving. The student questionnaire and teacher interview schedule provide descriptive accounts of students' metacognitive knowledge (see Chapter Two, section 2) during mathematical word problem solving.

2. Rationale for doing the Study

The research literature as well as my own experience as a mathematics teacher in a Singapore primary school provide the background for the rationale of this study. The rationales for this study are set out in the aims as stated in sections 2.1 and 2.2.

2.1 To Investigate the Effect of Metacognitive Training on Students' Mathematical Word Problem Solving Performance

The Revised Mathematics Syllabus (see Appendix A) was first implemented in Singapore primary schools in 1992. Since then, guidance in the actual use of metacognition to teach mathematics has never been made explicit to teachers. Although it is true that many students develop some metacognitive awareness and engage in some control on their own, and that most teachers do give students some guidance in regulating their word problem solving process (e.g. encourage students to check their work), mathematics instruction is still focused primarily on mathematical content. Wong's (1997) study reveals that during problem solving (see Chapter Two, section 3), Singapore students would focus on verification of solutions but placed less emphasis on the use of monitoring strategies in planning, executing and orientation. He also notes that Singapore teachers do not usually introduce metacognition as a topic in their lessons but subsume the concept of metacognition within the lesson content. As a result, metacognition during problem solving is not perceived as important since it is not taught explicitly and the concept of metacognition appears to be lost amongst the more important subject matter (Wong, 1997). In fact, there is sufficient evidence in Yeap and Kaur's (1996) study that Singapore students tend to view mathematics as a collection of tools and algorithms to be used to obtain definite quantitative answers to given word problems. Most students would also tend to believe that mathematics is an individual endeavour.

In 1997, the Prime Minister launched 'The Thinking Schools, Learning Nation' programme where he spelled out his vision to make Singapore a nation which learns all the time (The Straits Times, 3 June 1997, p. 22). 'Thinking Schools' ensures that Singapore students are equipped with skills and knowledge; and values and instincts to face future challenges, while 'Learning Nation' aims to promote a culture of continual learning beyond the school environment (Ministry of Education, 1998, p. 16). Fundamentally, this programme is an effort by the Government to improve Singapore's education system and to prepare Singaporeans for the challenge of the 21st century. It is in the recognition that in this information age, the ability to think and to innovate will be a key factor in sharpening Singapore's competitive edge in the next stage of her development (Tay-Koay, 1999).

In 1997, three innovative programmes were implemented in the Singapore primary and secondary schools in order to achieve the goals of 'Thinking Schools, Learning Nation'. One of them was the Thinking programme, which affected all secondary schools. Its aim is to help Singapore students become better thinkers and learners (Yee, 1998). One of the objectives of this programme is to enable students to develop positive habits which would help them become critical, creative and self-regulated, thinking learners (Yee, 1998). The Singapore Government appears to have recognised and given prominence to the explicit teaching of Singapore students' thinking skills, and has set a certain percentage of the curriculum time devoted to teaching these skills to secondary students. Teachers have to teach these general thinking skills, based on the Dimensions of Learning framework (Marzano et al, 1988). Metacognition is identified as one of the components to become a better thinker. From these general thinking skills lessons, subject teachers from different subject domains are to elicit specific skills from the programme and incorporate them into their lessons. For example, in Mathematics, all mathematics teachers, trained in conducting the Thinking programme, are given a Mathematics Infusion Package which contains sample lesson plans related to the revised mathematics syllabus. The lesson plans aim to help students develop mathematical thinking; develop a better understanding of mathematical concepts; learn problem solving heuristics explicitly; and apply the acquired mathematical concepts, skills and heuristics to solve problems (Yee, 1998). As Mok (1997, foreword) remarks:

'The skills and processes of thinking (taught in the programme) *will help them not only to learn more proficiently, but also guide them to make important decisions in life, to solve problems, to respond to circumstances and to exercise judgement responsibly.'*

Mayer and Wittrock (1996) posit that the main idea in such thinking programmes is that with direct instruction in thinking skills (behaviours and thoughts), the problem solver is influenced in their representation of a problem and the planning and monitoring of problem solving solutions.

The Thinking programme is not accessible to primary school students yet but I believe that making young learners become aware of their thinking, especially in metacognition, is crucial. As an erstwhile mathematics teacher in a primary school, my main concern was that all my students not only were able to understand what was taught, but was also able to excel according to their mathematical ability in the Primary School Leaving Examination or PSLE (see Appendix A). This notion of 'mathematical success' stems from my belief that the more experience students have of solving different mathematics word problems, the more experience they would have of using different heuristics which might help them tackle new word problems. I was diligent not only in preparing different types of word problems for my students to solve each week, but also in marking them, ensuring that the method they used made sense. I was also conscious that my students needed to understand the heuristics used, so a focus on heuristics was the order of the day in my lesson planning. However, this 'formula' used to help my students improve in their mathematical word problem solving performance was more successful for the higher achievers than the lower achievers. My hypothesis is that higher mathematical achievers have within themselves a self-regulatory system that enables them to monitor their word problem solving process. Hence, they are more successful. However, the lower mathematical achievers are often not conscious of these self-regulatory skills during word problem solving, hence, they often are unsuccessful. This is not just a 'gut feeling', but has roots in theoretical backing. Evidence from Schoenfeld (1985) suggests that compared with an 'expert' problem solver, 'novices' lack essential metacognitive monitoring, assessing and decision making skills. These, according to Schoenfeld, are essential elements that determine one's success or failure in problem solving. The present study also aims to identify the difference between higher and lower mathematical achievers' metacognitive behaviours. Identifying their differences may help teachers understand how to assist lower mathematical achievers focus on the processes they lack, and how to assist higher mathematical achievers become more aware of their already potentially effective processes so that they know and understand their strengths.

Lesh (1982), Silver (1982) and Schoenfeld (1982) regard metacognitive actions as the 'driving forces' in problem solving, influencing cognitive behaviour at all phases of problem solving. Some research studies have also reported success in making young children become more aware of their regulation during problem solving (Clements, 1990). Specifically, there is evidence that students trained in learning to monitor and control their own cognitive processes for solving mathematics problems do better than untrained students (Cardella-Elawar, 1992). Studies which have attempted to train metacognitive strategies in mathematical problem solving tend to focus on college students (Schoenfeld, 1985) and secondary school students (Mevarech, 1999; Oladunni, 1998; Maqsud, 1998) but there are few studies which focus on metacognitive training with upper primary school students. In addition, although there has been some research done in identifying the role of metacognition in students' problem solving in the Singapore context (Yeap, 1997), there remains an existing gap in research on the effect of metacognitive training and the role of metacognition in the context of Singapore primary students' mathematical word problem solving. It is hoped that this study will, in some measure, fill the research gap in this area where the effect of metacognitive training on mathematical word problem solving performance and the role of metacognition in mathematical word problem solving in the context of the Singapore primary mathematics classroom can be revealed.

2.2 To Explore the Role of Metacognition During Mathematical Word Problem Solving in a Computer Environment

In 1997, the Masterplan for IT in Education (Ministry of Education, 1998) was also launched. Its aim is '*to provide a blueprint for the use of IT in schools and access to an IT-rich school environment for every child*' (Ministry of Education, 1998, p. 17). The target of the Masterplan, to be achieved by the year 2002, is for students to have hands-on use of computers for 30% of their curriculum time; to provide sufficient computer facilities in order to attain the ratio of every two students to a computer in the primary school; and for students in Primary 4 (9-10 years old) and above to get free e-mail accounts. The Thinking programme has a direct relevance for promoting students' creativity and thinking and the computer is considered as a useful tool for the development of these 'high level' skills (Williams, 1999). Of major importance to these initiatives is the interim measure adopted by the Ministry of Education to provide teachers with the time that they need to implement these initiatives. The Ministry of Education, in 1999, embarked on an exercise to reduce the standard curriculum content by up to 30%. The time that is freed by the content reduction is intended to be used to infuse thinking skills and incorporate the use of information technology in the classrooms (Curriculum Planning & Development Division, 1998). Based on the government's move to implement major initiatives to encourage the use of information technology in the classroom, it appears critical that research should not only explore the development of appropriate software to be used in a computer-rich environment, but also explore the role of teachers and effective pedagogy that can maximise students' learning in such a powerful learning environment.

Looi and Tan (1997) developed a powerful computer learning environment, WordMath, that is modeled according to the instructional approach of cognitive apprenticeship (see Chapter Two, section 5.1). It is designed to teach word problem solving to 9-12 year old students in Singapore primary schools using the model approach (see Appendix B). In their pilot study, Looi and Tan (1997) conclude that WordMath harnesses the power of computers to empower the students to learn and explore word problem solving because it provides multiple instructional modes and stepwise-tutoring of word problem solving. However, Looi and Tan's (1997) study did not explicitly address how the students' cognitive skills in the cognitive apprenticeship mode of instruction aided them in their word problem solving attempts. The present study hopes to delineate the levels of students' cognition while solving word problems in WordMath environment and address the role of metacognition in this computer environment.

There have been attempts to employ metacognitive training within computer environments (Clements, 1990; Mevarech & Kapa,1996; Kramarski & Mevarech, 1997; Mevarech, 1999). In these computer environments, different metacognitive training strategies have been used. Kramerski and Mevarech (1997), and Mevarech and Kapa (1996) used a problem solving strategy called SOLVE when students were programming in Logo. SOLVE is the acronym of the problem solving stages suggested by Polya (1973). King (1994) also designed a guided questioning strategy which functioned as a metacognitive strategy to help students pay attention to their problem solving process and monitor their progress. The questions evoke metacognitive processes such as planning, monitoring and decision making during problem solving. As far as I know, no major study has been carried out to explore the effect of training students with a metacognitive strategy and explore the role of primary students' metacognition during mathematical word problem solving in a computer environment in a Singapore context. This study is an initial attempt to explicitly train Singapore students with a metacognitive strategy, and then explore the relationship between the role of metacognition and mathematical word problem solving in a computer environment.

In most of the above studies employing metacognitive training within computer environments, the students were working and discussing their tasks in small groups or in pairs. Pirie (1991) defined discussion to be purposeful talk on a mathematical subject in which there are genuine pupil contributions and interaction. Hoyles et al (1991) identified four interrelated aspects of the potential role of discussion in learning. They are distancing, conflict, scaffolding and monitoring. Specifically, Hoyles et al (1991) suggest that the role of monitoring in discussion is significant in the regulation and direction of the activity of the group. Hence, such talk facilitates metacognition which is the students' internal self-regulation and reflection upon the state of understanding. For example, students working in twos or in small groups need to develop plans through the exchange of ideas; monitor and check each other's actions; to keep track in relation to the goal of the activity; and finally to explain and convince others in the group. Empirical research (e.g. Bangert-Drowns, 1993; Kramarski & Mevarech, 1997) and those mentioned above consistently indicate the importance of training students to monitor, control and regulate their learning as they are using computers, and allowing students to work in small groups or in pairs appears to maximise that potential (Artzt & Armour-Thomas, 1992). Current Singapore policy in the teaching of mathematics (Curriculum Planning Division, 1995) also emphasises the importance of teaching mathematics in more reflective ways that will encourage students to activate metacognitive processes when they engage in problem solving activities. In the framework (see Appendix A) of the mathematics curriculum, communication is identified as one of the sub-components in the Skills of mathematical problem solving and one way of promoting communication is to have group/pair discussion during mathematics lessons. In this study, all students are made to work in pairs and are encouraged to solve the word problems collaboratively (see Chapter Two, section 7) during the training sessions and the video-recording sessions. It is hoped that arranging students in pairs will encourage them to work collaboratively. This, in turn, is hoped to allow pairs to generate metacognitive processes when they engage in word problem solving.

3. Theoretical Framework and Aims of the Research

Figure 1.1 shows the theoretical framework of the research.

The study focuses on training students to activate metacognitive processes while solving word problems in a WordMath environment. Its main aim is to investigate the effect of this training on students' ability to solve word problems using the model approach and reflect on their learning. The study also aims to examine the role of metacognition in word problem solving in a computer environment.

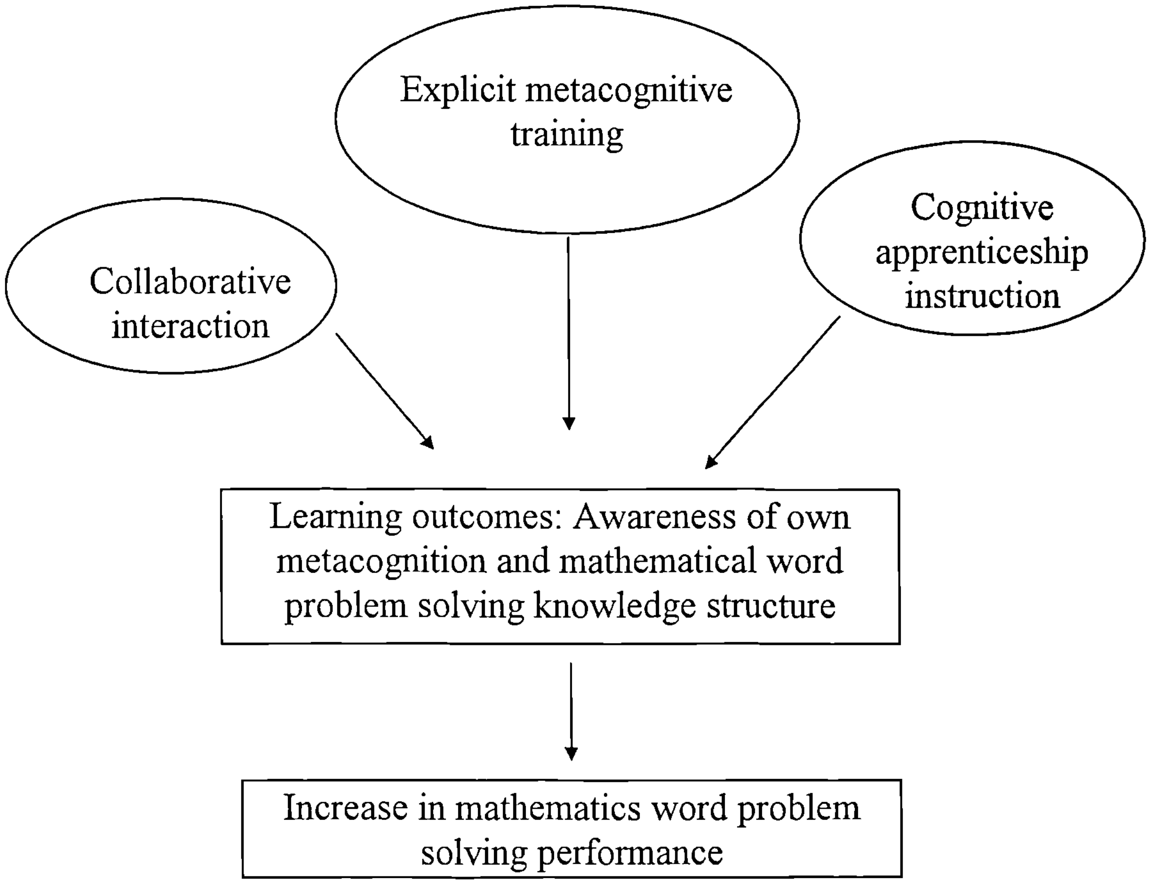


Figure 1.1: Theoretical Framework of the Research

4. **Research Questions**

Four research questions have been identified as follows:

Research Question 1: What effect will metacognitive training have on performance on mathematical word problem solving?

Research Question 2: To what extent is improved performance on mathematical word problem solving (as a result of cognitive apprenticeship instruction) related to the mathematical achievement of the student?

Research Question 3: Do the effects of metacognitive training on performance on mathematical word problem solving vary with the mathematical achievement of the student?

Research Question 4: What is the role of metacognition and its influence on word problem solving performance in a computer environment?

Research questions 1, 2 and 3 are addressed quantitatively (see Chapter Five) while research question 4 is addressed qualitatively (see Chapter Six). The quantitative method includes statistical analysis of mathematical achievement tests data while the qualitative aspect includes analysis of think aloud protocols using video-taped data, student questionnaires and teacher interviews (see Chapters Three and Four).

5. **Overview of Thesis**

Following Chapter One, the second chapter provides the theoretical background to the study by discussing some of the theories related to the area under investigation. Chapter Two begins by providing some broad perspectives with regard to metacognition and problem solving in the context of the Singapore mathematics syllabus and moves on to provide definitions of the terms, 'metacognition', 'cognition', and 'word problem solving', used in this study. This is followed by a discussion of literature and related research carried out on metacognition in the arena of Mathematics Education. This literature is organised under five broad headings, namely: metacognition in relation to mathematics performance; mathematics instruction including cognitive apprenticeship instruction; computer environments; collaboration; and verbal reports.

Chapters Three and Four deal with the methodology of the study. Chapter Three focuses on planning and piloting the study. It discusses the design, methodology and procedure of the study. The nature and rationale of the research design is discussed and this is followed by a description of the techniques that were used for the generation and collection of data. There are two parts to Chapter Four. The first part deals with the implementation of the study. It begins by providing a detailed description of the sampling and rationale for the choice of schools and students. Then, it provides an account of the actual procedure involved in the intervention process. In the second part, the approaches to data analysis used for both the quasi-experimental data and case study data are discussed.

The fifth chapter is a presentation and description of the two statistical techniques, repeated measures three-way analysis of variance where students are blocked according to the school factor and repeated measures two-way analysis of covariance, which are used to analyse the data from the two quasi-experimental designs described in Chapter Four. The use of these two statistical techniques are appropriate because they are able to eliminate, or at least try to reduce, the specific threats to validity arising from the design of the study. This chapter also attempts to investigate the hypotheses developed from research questions 1 to 3, and provide possible relationships between metacognitive training, students' level of mathematical achievement and their mathematical word problem solving performance.

The sixth chapter is a presentation and description of the case study data. It attempts to explore the role of metacognition during word problem solving in a computer environment and examine its influence on word problem solving performance as specified in research question 4. In the first part of the chapter, I seek to present the outcome of the analysis of the think aloud protocol data which includes descriptions of the students' progression of word problem solving activity with the help of timeline representations and data display tables. At the end of the first part of the chapter, I highlight and describe some patterns of students' progression of word problem solving activity which emerged from the think aloud protocol analysis and display tables. In the second part of the chapter, there is a descriptive account of the students' metacognitive knowledge during word problem solving based on the analysis of student questionnaire and teacher interview data. These accounts provide a historical context to the analysis of the students' word problem solving process.

In Chapter Seven, I seek to discuss and interpret the findings from the quasi-experimental and case study data. The findings of this study are discussed under three main headings, namely: (1) learning environment and mathematical word problem solving performance; (2) cognitive perspective and word problem solving; and (3) affect and word problem solving.

The final chapter of this thesis is the conclusion of the study. In this chapter, I begin by summing up the findings of this study and go on to discuss the educational and pedagogical implications for the effect of metacognitive training and the role of metacognition in Singapore mathematics classroom, especially in a computer environment. A critique, showing some of the limitations of the study, is made before identifying the contributions that this study has made to the existing field of research in metacognition. The final section of Chapter Eight concludes with some suggestions for further research.

Chapter Two
Theoretical Background and Review of Literature

1. Introduction and Overview of Chapter

The aims of the study are to investigate the effect of metacognitive training on students' mathematical word problem solving performance and to explore the role of metacognition during mathematical word problem solving in a computer environment in Singapore primary schools. Section 2 begins by providing a broad understanding of metacognition and then narrows down to focus on the distinction between metacognition and cognition. This distinction will be used as a starting point in the present study. Section 3 also begins with a broad perspective of what problem solving in the context of Singapore mathematics syllabus entails and then focuses on the type of 'problem solving', word problem solving, used in the present study. A working definition for word problem solving in the study will be provided in this section.

In order to provide a basis to operationalise the general research aims, it is necessary to turn to areas of literature that are related to the topic under investigation. Hence, the next five sections review the literature and research on metacognition, with a particular focus on Mathematics Education. As the main focus of this study is on the effect of metacognitive training on students' mathematical word problem solving performance, some recent research carried out on training students in metacognitive strategies for problem solving and how this training influences the students' mathematical problem solving performance will be discussed in sections 4 and 5. Before moving on to the next section, section 5 concludes with a brief discussion on the relationship between affect and promoting students' metacognitive awareness. As mentioned earlier, this study also aims to understand the role of students' metacognition in a computer environment by exploring the levels of students' cognition during word problem solving. Since research on computer environments has shown that they are powerful influences on students' learning, section 6 of this chapter reviews some of the literature and research on metacognitive training in a computer environment. This review begins by providing a description of the computer environment used in research focusing on metacognitive training. The computer environment is supposed to be different from the traditional classroom environment and this difference is claimed to enable students bring about higher level of thought processes during problem solving (Clements, 1990, p. 147). The review also includes a discussion of some of the metacognitive strategies researchers have used to promote students' metacognitive awareness during problem solving in a computer environment. Section 7 reviews literature that focuses on the processes of collaboration during word problem solving. In section 8, there is a review of the emerging metacognitive strategies in students' verbal report when they collaborate. The review of the above five areas of literature will form the theoretical background for this study.

2. Metacognition

Lesh (1982), Silver (1982) and Schoenfeld (1982), influential researchers in the arena of metacognition in mathematical problem solving, regard metacognitive actions as the 'driving forces' in problem solving, influencing cognitive behaviour at all phases of problem solving[1]. If metacognition is so important, there is a need to explain what metacognition is, its importance and its impact in mathematics education. This section 2 and sub-section 2.1 will attempt to answer these questions.

In a meta-analysis, Lester (1994) draws out two results that have come to be generally accepted between the relationships of metacognition and mathematical problem solving. They are:

1. Effective metacognitive activity during mathematical problem solving requires knowing not only what and when to monitor but also how to monitor; and
2. Teaching students to be more aware of their cognition and be better monitors of their mathematical problem solving actions should take place in the context of learning specific mathematics concepts and techniques. This is because there is evidence that general metacognitive instruction is likely to be less effective.

[1] Defining 'problem solving' is difficult and complex. Section 3 provides what problem solving entails in the Singapore mathematics curriculum.

However, there is still much confusion as to precisely what the term metacognition means. Schoenfeld (1992) posits that metacognition is primarily concerned with those human reasoning processes that are necessary to solve problems for which no completely developed or automated procedures are available. In Flavell's (1976, p. 232) terms, metacognition refers to '*knowledge concerning one's own cognitive processes*' that is used in '*monitoring and consequent regulation and orchestration of those processes*'. In this study, Flavell's definition of the term metacognition is used as a starting point.

In the domain of learning and instruction, mathematics education in particular, there seems to be an acceptance that metacognition involves three separate but related aspects, namely:

1. metacognitive knowledge: knowledge about own thought processes or cognition (e.g. What does a person know about his/her own thinking?) (Lester, 1994);
2. executive control: control or self-regulation of activity during problem solving (e.g. How well does a person keep track of what (s)he is doing and how well does (s)he use those observations to guide his/her problem solving actions?) (Lester, 1994); and
3. metacognitive beliefs: beliefs and intuitions (e.g. What ideas a person brings to his/her work in mathematics and how this shapes the way in which (s)he does mathematics?) (Schoenfeld, 1985; 1992).

With regard to the first aspect, metacognitive knowledge involves knowing about the strengths and weaknesses of one's own cognitive abilities, strategies and resources in relation to the performance of specific cognitive tasks. Some examples include the awareness of limits of short-term memory and knowing that memory is fallible but that aids (e.g. mnemonics) are helpful for retaining information (De Corte et al, 1996). Garofalo and Lester (1985) categorise metacognitive knowledge into three categories: person category, task category and strategy category. The person category consists of what (s)he believes about himself/herself and others as cognitive beings. For example, Markman (1979) concludes that young children predict their performance on motor tasks much better than on recall tasks. The task category includes knowledge about the depth and requirements of tasks, as well as knowledge about factors and conditions, that make some tasks more difficult than others. For example, Canney and Winograd (1979) conclude that younger readers and poor comprehenders view reading as a decoding activity whereas better comprehenders view reading as a search for meaning. Finally in the strategy category, it includes knowledge of general and specific cognitive strategies. For example, Kreutzer et al (1975) found that older children can think of more mnemonics and retrieval strategies and also exhibit more planning strategies than younger children.

However, from research findings, metacognitive knowledge is not so finely categorised. Many of the examples are placed in more than one category. This implies that metacognitive knowledge involves the interaction of person, task and strategy categories during problem solving. For example, with regard to mathematics tasks, person-by-task interactions include the students' ability to estimate the task's difficulty and preference for a particular type of task; person-by-strategy interactions include the students' ease and familiarity with, and confidence in using useful strategies; and task-by-strategy interactions involve an awareness that a particular class of problems can be solved using a certain heuristic. These interactions of person, task and strategy knowledge can have an influence on the students' decision to regulate his/her activity. For example, if the student is aware that (s)he is prone to making computation errors when working fast, (s)he is likely to monitor and is more likely to check the results more diligently (Garofalo & Lester, 1985).

The second aspect of metacognition is control or self-regulation. In brief, it refers to the processes in planning, controlling and reflecting. Schoenfeld (1992) identifies awareness, monitoring and assessing progress 'on line' and acting in response to the assessments of online progress as important aspects of self-regulation. De Corte et al (1996, p. 506) define self-monitoring and self-regulating mechanisms as '*the executing control structure that organises and guides our learning and thinking processes*'. In the domain of mathematics, Lester and Garofalo (1982), Kruteskii (1976) and Nelissen (1987) report differences between more and less able elementary and secondary pupils with respect to self-monitoring and self-control. At the college level, Schoenfeld (1985) found that in comparison with an expert problem solver, students lacked essential metacognitive monitoring, assessing and decision making skills though they did not lack conceptual knowledge. From these research studies, one can conclude that skilled problem solvers appear to have a high level of control, and a systematic and persistent orientation towards their goal.

In the third aspect, metacognitive beliefs, Schoenfeld (1992) found in classroom observations that many students come to believe that school mathematics consists of mastering formal procedures that are completely different from real life, from discovery and from problem solving. They also believe that all problems can be solved within a short span of ten minutes or less; and only geniuses are capable of discovering mathematics. These beliefs have unfortunate behavioural consequences. For example, students who appear to believe that all problems can be solved in ten minutes or less will simply stop working on a problem after a few minutes, even if they would have been able to solve it with more effort. Students who believe that mathematical understanding is simply beyond ordinary people like themselves are likely to simply accept and memorise what is given to them without attempting to make sense of it on their own (Schoenfeld, 1987). Schoenfeld believes that instructional programmes should be designed to change students' conception of mathematics. Research in this aspect is still in its infancy and researchers have agreed that more work is needed to clarify the role of metacognitive beliefs in problem solving (Lester, 1994), and its influence in metacognition.

In this present study, students' metacognitive knowledge and their executive control are elicited so that the role of metacognition during word problem solving in a computer environment can be examined. In the following sub-section, there is an attempt to distinguish between metacognition and cognition. Identifying the difference allows the delineation of the levels of cognitive behaviours in students' word problem solving in a computer environment (see Chapter Four, section 5).

2.1 Metacognition and Cognition

In relation to distinguishing between 'metacognitive' and 'cognitive', Garofalo and Lester (1985) suggest thinking of cognition as involved in doing whereas metacognition as involved in choosing and planning what to do and monitoring what is being done.

Artzt and Armour-Thomas (1992), and Montague and Bos (1990) distinguish between 'metacognitive' and 'cognitive' by looking at cognitive and metacognitive strategies, also building on Flavell's (1976) contribution to metacognition. In the area of mathematics, cognitive strategies refer to either specific heuristics or general procedures such as reading the problem, paraphrasing, visualising, hypothesising, estimating, computing and checking the problem (Montague & Bos, 1990). In contrast, metacognitive strategies refer to the knowledge of and being aware of the problem solving strategies, use of these strategies and having control over these strategies for purposes of regulating and monitoring performance (Montague & Bos, 1990). Hence, cognitive strategies are invoked to make cognitive progress but metacognitive strategies are to monitor it. In Artzt and Armour-Thomas' (1992) framework (see Appendix G), in order to differentiate cognitive and metacognitive behaviours, they distinguish metacognitive behaviours as behaviours which could be exhibited by statements made about the problem or about the problem solving process whereas cognitive behaviours are behaviours which could be exhibited by verbal or non-verbal actions that indicated actual processing. For example, when I work on a mathematical word problem, I might realise that the problem is more complex than I had thought at first (metacognitive). I may suppose that the best thing I could do would be to start over again (metacognitive), so I read the problem to ensure that I fully understand it (cognitive). In the midst of this mentally demanding task, I might keep track on how well things are going (metacognitive). If things appear to be proceeding well, I could continue along the same path towards my goal (cognitive); if they appear to be problematic, I would consider seeking alternative paths to my goal (metacognitive).

I propose that the distinction between metacognition and cognition in word problem solving situations will be defined as follows: cognitive behaviours are indicated by the actual processing of the word problems and metacognitive behaviours are indicated by the knowledge of and being aware of the word problem solving strategies, use of the strategies and having control over these strategies for purposes of regulating and monitoring word problem solving performance. This distinction will be used in the analysis of students' think aloud protocols during word problem solving (see Chapter Four, section 5 and Chapter Six, sections 2, 3 and 4).

3. Problem Solving in the Context of Singapore Mathematics Syllabus

In this section, I will provide an account of what problem solving entails in the context of Singapore mathematics syllabus, the Revised Mathematics Syllabus (see Appendix A), and, drawing from literature, define the type of problem solving approach that is recommended by the Revised Mathematics Syllabus. Then, in the following sub-section, I shall redefine the problem solving recommended in the Revised Mathematics Syllabus as word problem solving. This definition of word problem solving will be used in the study context.

In Appendix A, there is a brief description of the Revised Mathematics Syllabus which was implemented in 1992. One of the central foci in this Revised Mathematics Syllabus is on problem solving (Lim, 1990). According to the report on the Revised Mathematics Syllabus, problem solving is not a distinct topic but a process or activity that should permeate the entire mathematics programme and provide the context in which concepts, skills and processes can be learnt. In Teong (1997), I recommended four areas of concern that might affect how teachers teach and Singapore students learn using the Revised Mathematics Syllabus. They are the new approaches to teaching, the language factor, pupils' readiness, and the assessment methods. In this section, I will only focus on the concern where new approaches to teaching are recommended in the Revised Mathematics Syllabus.

The problem solving approach recommended by the Ministry of Education (Singapore) is 'teaching for problem solving' (Schroeder & Lester, 1989, p. 39). According to Schroeder and Lester (1989), this approach engages the students to learn mathematics so that they are able to use the gained knowledge to solve other problems. Problem solving might be viewed as an activity students engage in only after a new concept is introduced or following work on a computational skill or algorithm. The purpose of the 'problem solving activity' is to give students 'practice' to 'apply' recently acquired concepts and skills to 'real-life' problems. An example would be problems under the heading 'Using addition to solve problems' where a 'model' solution of a simple story problem is provided for solving other very similar problems. Often, by simply following the pattern established in the 'model' solution, students often can obtain the solution but when students encounter problems that do not follow the sample, they often feel at a loss. When taught in this way, Schroeder and Lester (1989) caution that students often simply pick out numbers in each problem and apply the given operation(s) to them without regard for the problem's context, and often they get the correct solutions. In their view, this practice is not problem solving, for it does not even require mathematical thinking. Schroeder and Lester's (1989) concern is that this is a dangerous way to learn which would lead to a long term side effect, that is to give students a false picture that all mathematics problems can be solved quickly and relatively effortlessly without the need to understand how the mathematics they are using relates to real situations.

Unfortunately, the approach described to problem solving instruction, 'teaching for problem solving' is recommended in Singapore Mathematics textbooks and the Teacher's Guide books developed by the Curriculum Planning Division. Teachers treat problem solving in isolation, only introducing the 'strategies' after the basic computational algorithms have been taught. This method is also encouraged by the extra activities labeled as 'Mathematical Thinking', 'Problem Solving' and 'Mathematical Investigation' found in the mathematics resources such as the Teacher's Guide book and the Challenging Mathematics problems package. It might be that the teachers are uncertain about what problem solving entails and by following the official Teacher's Guide books which contain very detailed advice and 'real' lesson plans for teachers, these teachers might be heading in the 'wrong direction' in terms of helping their students develop thinking strategies in problem solving. It appears that the recommended approach in the Revised Mathematics Syllabus is not used in the true spirit of problem solving which Schroeder and Lester (1989, p. 39) define as 'teaching via problem solving'. I propose that the 'problem solving' recommended by the Ministry of Education should be redefined as 'word problem solving'. In the following sub-section, what word problem solving entails and an example of a word problem that is used during word problem solving will be provided.

3.1 Word Problem Solving

The word problems used in the present study are routine, multistep problems (Reed, 1999, p. 62) that exist in the textbooks after the concepts of a topic in the mathematics syllabus have been taught. The following is an example of such word problems:

A bottle weighs 2.5 kg when it is $\frac{1}{3}$ *filled with cooking oil. It weighs 3.3 kg when it is* $\frac{1}{2}$ *full. Find the weight of the empty bottle.*

These word problems consist of three parts: givens, a goal and obstacles (Davidson & Sternburg, 1998). The givens are the elements, their relations and the conditions that compose the initial form of the word problem. The goal is the desired outcome or solution. When students work on the word problem, they actively try to transform the given state of the situation into a desired or goal state. Obstacles are the characteristics of both the word problem and the difficulty involved in changing the given state of the word problem into the desired one or the difficulty involved in recognising when a correct transformation has occurred (Davidson & Sternburg, 1998).

The word problems defined above are the types of word problems Singapore students are engaged in during word problem solving. They are taught to use the model approach (see Appendix B) to solve these word problems. In the present study, these word problems will be used. Then the relationship between the students' metacognitive behaviours, their differential mathematical achievement and their mathematical word problem solving performance will be examined when the students are engaged in word problem solving on the word problems as described above.

I have provided a broad understanding of what metacognition is and distinguished between metacognition and cognition. This distinction will be used as a starting point to develop the framework to delineate students' levels of cognitive behaviours in word problem solving in Chapters Four and Six. I have also introduced the 'problem solving' approach stated in the Revised Mathematics Syllabus and provided a rationale for redefining the approach as word problem solving. This definition will be used as a working definition to the approach students use to solve word problems as described above.

In the following sections, evidence relevant to the use of metacognition in mathematics education is elaborated. Factors that specifically contribute to the success in students' word problem solving are also highlighted and discussed.

4. **Metacognition and Mathematical Performance**

In the 1980s, mathematics educators began studying the role of metacognition in the performance of mathematical tasks (Garofalo & Lester, 1985; Schoenfeld, 1987; Silver, 1985). They argue that what one knows or believes about oneself, as a learner and doer of mathematics, and how one controls and regulates one's behaviours, while working through mathematical tasks, can have powerful effects on one's performance. Schoenfeld (1985) also argues that metacognitive processes such as assessing one's knowledge, formulating a plan of attack, selecting strategies and monitoring and evaluating progress play a central role in mathematical performance by enabling effective decisions to be made regarding the allocation of time, energy and knowledge resources. According to Mayer and Wittrock (1996, p. 51),

'An instructional implication of the metacognitive view is that pupils need to learn when to use various cognitive processes, including being aware of their processes, monitoring their cognitive processes and regulating their cognitive processes.'

Cardelle-Elawar (1992) provided classroom instruction to low-performing sixth grade children on how to use linguistic, strategic and procedural knowledge to solve mathematical story problems. The teacher helped the students learn to recognise when they did not understand the meaning of the word, did not have all the necessary information to solve the problems, did not know how to break the problem into steps or did not know how to carry out a computation. Trained students showed large pretest-to-posttest gains in mathematics achievement and attitudes toward mathematics whereas control students did not. Apparently, learning to monitor and control one's cognitive processes for solving mathematics problems in the classroom transferred positively to solving other types of mathematics problems in a written test (Mayer & Wittrock, 1996). The above study and others (Chinnapan & Lawson, 1996; McCrindle & Christensen, 1995; Delclos & Harrington, 1991) provide evidence that providing metacognitive training has a significant impact on students' mathematical performance and this raises the issue of how the instruction should be effectively implemented to bring forth this impact. The following section will examine how research has looked into ways to make students become aware of their thinking process during problem solving.

5. Metacognition and Mathematics Instruction

Lester (1983) and Schoenfeld (1983) believe that the failure of most efforts to improve students' problem solving performance is largely due to the fact that instruction has overemphasised the development of heuristic skills and has virtually ignored the managerial skills necessary to regulate one's activity (Garofalo & Lester, 1985). Callahan and Garofalo (1987) highlight that typical in mathematics instruction, class work and homework are designed to increase students' knowledge about concepts and procedures and usually do not direct attention to developing students' metacognitive behaviours. Although, students might develop some awareness and engage in some control on their own and that most teachers give guidance in regulating their mathematical tasks, mathematics instruction is still primarily focused on mathematical content. There is evidence (Mayer & Wittrock, 1996) that it is ineffective to divorce metacognitive instruction from subject content. Hence, metacognitive training must be incorporated into the teaching of the subject itself (Lester et al, 1989). Researchers like King (1991, 1994), Kramaski and Mevarech (1997), Mevarech and Kapa (1996), and Schoenfeld (1985) design metacognitive training that reinforces conscious reflection and regulation of the problem solving processes. Loper and Murphy (1985) note that central to the studies that employ metacognitive training in problem solving, there is an emphasis on two features: an awareness of one's own cognitive skills and abilities and the efficient use of this self-awareness to self-regulate cognitive activity. According to Loper and Murphy (1985, p. 224), metacognitive training is defined as '*a systematic attempt to induce enhanced levels of cognitive performance in a child through the training of self-awareness and/or self-regulatory skills*'.

Chinnapan and Lawson (1996) report that training which focuses on the use of executive strategies concerned with the management of problem solving activities directs students' attention to planning of the solution path, checking calculations and reviewing of their solution. This training results in an improvement in both high- and low-achieving students in geometry problem solving. In Singapore, Yeap and Menon (1996) observed ten students solving non-routine mathematical problems. They conclude that the ability to monitor and regulate one's problem solving process is central to success in problem solving. Specifically, they noticed that though students demonstrated metacognition during problem solving, their strategies were not always efficient and successful. As a result, Yeap and Menon (1996) highlight the need for mathematics instruction to incorporate the development of metacognitive strategies and emphasise systematic thinking.

The following sub-section provides a description of a specific instruction, cognitive apprenticeship instruction, with metacognitive training. This form of instruction is claimed to be effective in making students become aware of their metacognition during problem solving (Schoenfeld, 1985).

5.1 Metacognition and Cognitive Apprenticeship Instruction

Schoenfeld's (1985) study provide evidence that a classroom approach which encourages students to internalise metacognitive processes can improve problem solving performance as well as providing useful approaches to the investigation of awareness and self-control. Schoenfeld designed an instructional programme intended to promote students' ability to regulate during problem solving. His programme models the instructional framework of the cognitive-apprenticeship model of instruction (Collins et al, 1989; 1991). This model is derived from the metaphor of the apprenticeship working under the master craftsperson in traditional societies and from the way people seem to learn in everyday informal environments (Lave, 1988). The cognitive apprenticeship model rests on the conception of the 'ideal' apprenticeship as a method of becoming a master in a complex domain. In contrast to the classroom context, Collins and Brown (1988) recommend establishing settings where 'worthwhile' problems can be worked out and solved. According to De Corte et al (1996), the key instructional strategies or components of the cognitive apprenticeship model are that: instruction focuses on the content; is situated in the students' learning experience; models and explains how a process works; coaches students by observing them and providing help when needed; provides opportunities for students to articulate their thoughts; provides ample opportunities for students to reflect on what they have done; encourages students to explore different strategies and hypotheses and observe their efforts; and is sequenced in a chronological order so that concepts are taught in increasing complexity.

In the present study, WordMath is intended to be used as a tool which the students can use in their collaborative word problem solving. It is designed on the instructional approach of cognitive apprenticeship methodology (Collins et al, 1991) and its aim is to help students learn to solve word problems (Looi & Tan, 1997), word problems that are particular to the Singapore mathematics curriculum (see section 3.1). According to the developers (Looi & Tan, 1997), there are six pedagogical domains to promote understanding and expertise. They are the modeling and explaining domain; the coaching domain; the scaffolding domain; the articulation domain; the reflection on performance domain; and the exploration domain. This software was tested on thirty-six 11 to 12-year-olds from two Singapore schools. The researchers conclude that '*WordMath is a useful software for pupils to learn word problem solving*' (Looi & Tan, 1997, p. 15). WordMath, though a 'tutoring engine' (op cit p. 15), provides pupils some form of control in their learning. For example, pupils can choose which mode to solve word problems, from a 'didactic' style in the 'Coaching' mode to a more 'exploratory' style in the 'Practice and Reflection' mode. In addition, some of the pedagogical domains such as the reflection on performance domain, the articulation domain and the exploration domain, allow pupils to explore and encourage them to exhibit metacognitive behaviour. Looi and Tan (1997) highlight and note that '*when two students worked together in this way, there was always much discussion between the students on how to respond to the system*' (Looi & Tan, 1997, p. 14). The researchers feel that creating an environment where pupils work in pairs with WordMath helps to promote discussion amongst pupils and even teachers. In the present study, metacognitive training is also incorporated into the use of WordMath. This combination of learning, according to Schoenfeld (1985), is able to encourage students to become more aware of their metacognition during problem solving.

Researchers like Jarvela (1998), Leo and Galloway (1996), Mayer (1998), Schoenfeld (1987), and Vauras et al (1999) argue that there is a need to address the affective issues such as metacognitive beliefs (see section 2) and the motivational aspect of cognition in the arena of metacognitive training. These affects, according to McLeod (1992), play a significant role in mathematical learning and instruction. Furthermore, Reeve (1996) argue that the ability students monitor and regulate their own learning, and internalise these processes is influenced by motivational issues. For example, Leo and Galloway (1996) suggest that the failure of some children to respond to the instructional method in Cognitive Acceleration in Science Education (CASE) might be due to their motivational style of learning instead of their failure to learn CASE strategies. Others, like Jarvela (1998) and Mayer (1996), suggest that a cognitive apprenticeship and technology rich environment may maintain the students' tendency towards task orientation, which is indicated by the students' intrinsic motivation on the task and their persistent strive for mastery. However, Vauras et al (1999) argue that in their study, there exist conflicts that make students resist behaving according to the (real and imagined) norms of their peers. In addition, little is known how these students overcome these conflicts by gradually gaining a higher sense of competence and self-efficacy, which may help them towards more independent cognitive functioning. The researchers (Vauras et al, 1999) feel that in studies that promote strategic learning, there is a need not only to train students in the use of cognitive and metacognitive strategies, but also in emotional coping and motivation strategies that promote self-regulation.

In the present study, due to time constraints, I have not trained students to deal with the affective issues. However, these issues will be taken into consideration in the discussion chapter (see Chapter Seven, sections 2.2.1 and 4). The following section provides accounts of research that focus on the common features in promoting metacognitive awareness in a computer environment, and the types of metacognitive strategies researchers use to encourage students become more aware of their thinking process in the computer environment.

6. Metacognition and Computer Environments

There have been several attempts made by researchers to employ metacognitive training within computer environments (Clements, 1990; Mevarech & Kapa, 1996; Kramarski & Mevarech, 1997; Mevarech, 1999). Clements (1990), and Mevarech and Kapa (1996) highlight two distinct features of the computer environments which they claim would enhance information processing during problem solving, namely completeness and explicitness. For example, Clements (1990) described the Logo programming environment as complete in that (a) children engaged in all problem solving process, (b) both general knowledge and domain specific knowledge were addressed, (c) there was a comprehensive set of pedagogical approaches used, and (d) social and emotional aspects of learning were considered. In addition, the explicitness arose when the 'homunculi' (Clements, 1990, p. 144) instructional device was introduced to the children. This 'homunculi' instructional device was made up of cartoon anthropomorphisms of the metacomponential processes which were represented as the problem decider, the representer, the strategy planner, and the debugger (Clements, 1990, p. 144). Then the children were asked to verbalise their goals and solution procedures, as well as their metacomponential processes, before overtly attempting a solution. To Clements (1990, p. 147), such attention to explicit awareness of metacomponential processes in a Logo programming environment stands in contrast with the traditional pedagogical emphasis on conveying a large corpus of factual knowledge, which often obfuscates higher-thought processes.

Other researchers have also devised their own metacognitive training strategies which add to the explicitness of the computer environments. Kramarski and Mevarech (1997), and Mevarech and Kapa (1996) used a problem solving strategy called SOLVE with students programming in Logo. SOLVE is the acronym of the problem solving stages suggested by Polya (1973): Systematic analysis; Overall planning; Linking together the partial solutions; Verification; and Evaluation of the overall solution. Mevarech and Kapa's (1996) study revealed that knowledge acquisition and certain aspects of creativity and metacomponents (e.g. correctness) were strongly affected by the problem solving based Logo, Logo-Stat. Like Clements (1990), the researchers noted that using the SOLVE strategy further served to focus the children's attention on their information processing. Kramarski and Mevarech (1997) reported positively that students who were exposed to metacognitive treatment with SOLVE tended to construct graphs better and were able to reflect on their learning better compared to their counterparts who were not exposed to such treatment. They noted that the study raised the question of the differential effects of metacognitive training on lower and higher achieving students. They suggest that there is reason to suppose that students with different prior knowledge would benefit differently from metacognitive training and this issue is important for heterogeneous classrooms where students with different backgrounds and aptitudes learn together.

In all these studies, there is an underlying notion that increasing students' conceptual and procedural knowledge is not sufficient to enhance students' mathematical problem solving performance. Students need to be aware of their metacognitive processes and schools need to establish explicit training to develop this awareness. On the other hand, this awareness will only be effective if metacognitive training is focused on specific subject domains for there is evidence (Mayer & Wittrock, 1996) that metacognitive training only fosters 'transfer' within specific subject domains.

In the present study, students solved word problems in the WordMath environment with a metacognitive strategy, CRIME (see Appendix F), which I had developed. Its aim is to promote students' metacognitive awareness during word problem solving. CRIME is another acronym of the word problem solving stages: Careful Reading; Recall Possible Strategies; Implement Possible Strategies; Monitor; and Evaluation. At each stage of the word problem solving, there are questions to direct the students to regulate and monitor their solution. It was piloted on two students in Leeds (see Chapter Three, section 4 2) and CRIME was found to be an effective strategy to make students become more aware of their thinking process in word problem solving.

As mentioned in Chapter 1, section 2.2, in all studies employing metacognitive training within computer environments, the students were working on and discussing their tasks in small groups. As highlighted in section 5.1, Looi and Tan (1997) also note the importance of articulation when solving word problems in WordMath as it encourages students to exhibit metacognitive behaviour. In the following section, I will consider the importance of collaborative talk during word problem solving because it is relevant to the present study.

7. Metacognition and Collaboration

Dillenbourg (1999, p. 12) states that the first criteria for an interaction to be characterised as collaborative is that it should be '*interactive*'. The degree of interactivity among peers is not defined by the frequency of interactions, but by the extent to which these interactions influence the peers' cognitive processes. Crook (1995) identifies three processes in social interaction which are collaborative: articulation; conflict; and co-construction.

Articulation occurs when learners work closely in problem solving and are required to make their thinking public and explicit (Crook, 1995). This involves peers articulating their opinions, predictions, and interpretations which might contribute to the tasks. According to Crook (1995), there are two reasons why articulation promotes learning. One reason is that peer articulation is linked to self-reflective processes arising from the responsibility of justifying and declaring your own ideas to a collaborator. As a result, Damon and Phelps (1989, p. 152) declare that

'In order to work productively with their partners, children must publicly recapitulate their own emerging understanding of the task. This, we believe is a process that strongly facilitates intellectual growth because it forces the subjects to bring to consciousness the ideas that they are just beginning to grasp intuitively. The responsibility that children feel for communicating well with their peer partners induces them to gain greater conceptual clarity for themselves'.

The other reason is that when ideas are publicly articulated, the talk of one participant serves to create for the other exemplars of strategic moves that might lead to successful problem solving. This latter suggestion is thought to be linked to the interpretation of effective practice in the 'zone of proximal development' (Crook, 1995, p. 134). According to Crook (1995), homogeneous pairs can also give rise to a socially defined cognitive system of the same sort: one that is comparable to that traditionally discussed for novices working with more expert partners. The cognitive benefits arising from peer articulation would be associated with the processes of internalisation, that is '*the opportunity to participate in the processes of coordinating a problem solving strategy creates the conditions for transfer from Vygotsky's inter-mental plane to his private intra-mental plane*' (Crook, 1995, p. 134). For example, as a learner is pressurised to make ideas public, the learner may have to slot into an externally located cognitive system that implicates a partner's contributions also. As a result, participation in such systems may then be internalised.

The presence of a peer encourages peer articulation and sometimes conflict arises. Conflict arises when peers disagree with the other in the way the problem situation is interpreted or the strategy used to solve the problem. The benefit of conflict is supposed to occur in the context of disagreements between peers and their efforts to resolve them. For example, a partner might not agree with his/her peer's strategy. This often forces the other to defend his/her strategy or reasoning and this makes him/her think more deeply about what (s)he has proposed. Sometimes, the defender can convince his/her peer and sometimes (s)he cannot. When (s)he cannot, (s)he may re-evaluate his/her own reasoning and that of his/her peer, and moves on to a new strategy. This may be related to the Piagetian perspective of cognitive development as a collaborative process (Crook, 1995). Piaget speculated that individual development is facilitated by co-operation between peers in resolving cognitive conflicts provided by their differing perspectives (Rogoff, 1998). Such conflicts were observed in many of the students' word problem solving in this study (see Chapter Six, section 2).

Co-construction is a notion which often arises in discussions of peer interaction more influenced by Vygotsky's socio-cultural thinking (Wertsch, 1990). Theorists in this domain focus on how children take individual responsibility for complementary cognitive functions while solving a problem. This might enable them to be organised within the context of some overall converging discussion about the task. Co-construction existed when students L and JK (see Appendix H and Chapter Six, section 2. 5), took responsibility to solve the eight word problems together. As pointed out by Crook (1995, p. 137), '*strategies of sharing responsibility may serve to accelerate the* *participants' joint construction of some worthwhile convergence - a common object of some sort'.*

I have discussed the significance of collaboration during word problem solving and the processes that may lead to collaborative talk. From the above argument, it appears that peer collaboration facilitates metacognition. In the present study, the students were made to solve word problems in peers. It is hoped that putting them in pairs will encourage collaborative talk. The next section identifies the procedure used to 'capture' this talk.

8. **Metacognition and Verbal Reports**

Most major research studies in the area of metacognition use a quantitative methodology to analyse and draw conclusions on metacognitive components. Schoenfeld (1985), and Goos and Galbraith (1996) 'capture' and analyse students' covert metacognitive strategies during problem solving using the think aloud procedures which are used to give researchers a glimpse at covert strategy activity that is not accessible in normal circumstances.

Kail and Bisnaz (1982) describe any strategy as a sequence of activities rather than a single event. This means that, among other things, students need to acquire both the component processes and a routine for organising the processes. Another significant aspect of any strategy is that it is largely under the control of the learner (Garner, 1988). This implies that though certain subroutines may be learned to a point of being automatic, strategies are generally deliberate, planned, and consciously manifested in activities. Paris et al (1983, p. 285) call them '*skills under construction*'. This means that strategies require 'on-the-spot' resources that are not limitless and that strategies can be examined, reported and modified. As a consequence, the think aloud procedures are means to externalise metacognitive and cognitive strategies in instances where they are not readily observable because they produce the concurrent verbalisations about the activity the students are engaged in (Ercisson & Simon, 1993). In Schoenfeld (1985), and Goos and Galbraith's (1996) studies, the researchers use the think aloud protocols to capture the essential elements in the problem solution while students solve mathematical problems. Both researchers were apparently successful in identifying the impact of the (presence or absence of) assessment and consequent decision-making of the solution as a whole in the problem solving process when they analysed the think aloud protocols.

Artzt and Armour-Thomas' (1992) study used think aloud protocols to delineate the levels of cognitive behaviours of students' problem solving in groups. They were apparently successful in identifying the impact of the levels of cognition on groups' problem solving. Based on Green and Gihooly's (1996, p. 54) claim, '*The most useful reports are straightforward verbalisations of ongoing thoughts as it happens, without either elaboration or explanation. Such direct concurrent reports are generally accurate and reasonably complete and have little reactive effect beyond slowing of performance*'. Ericsson and Simon (1993) have identified concurrent reports as a form of verbal report which are produced when people are doing a task at hand. At the extreme end of the spectrum is the Level 1 verbalisation which is simply the verbalisation of verbal working memory content that is ordinarily heeded in doing the task. At the other end of the spectrum is the Level 3 verbalisation which involves changes in working memory content, making inferences, interpretation, a shift in attention, or some other additional processing. According to Ericsson and Simon (1993), Level 3 verbalising is expected to alter task performance. This usually occurs when students are given verbalising instructions such as 'Explain what you are thinking?' which directs the students to their own thought processes. Level 2 verbalisation involves the person verbalising content ordinarily heeded in doing the problem task, describing what (s)he is doing which requires no explanation. According to Ecrisson and Simon (1993), in all studies reviewed, when the instructional procedures conformed to the Level 1 and Level 2 verbalisations, the studies gave no evidence that the verbalisation changes the course of the students' thought structure. At this point, I argue that the concurrent reports and the instruction used in the case study design to analyse students' think aloud protocols are aimed at Level 1 and Level 2 verbalisations. It is because I made an attempt to adhere to the principles listed by Green (1995) in order to maximise the validity of the verbal report. First, appropriate instructions (see Chapter Three, section 3.2.1i) were used to guide the production of concurrent reports aimed at Level 1 and Level 2 verbalisations where the students were not encouraged to explain or rationalise their thoughts. Second, the concurrent reports were video-recorded as the task was being carried out with minimum intervention from me (see Appendix I). However, I would like to add that in the present study, when the students worked collaboratively, they might engage in Level 3 verbalisation, involving explanations of their own thoughts to their peer. Such verbalising is metacognitive.

The present study uses a think aloud procedure to 'capture' metacognitive strategies during word problem solving. Green (1995) and other researchers' (Ericsson & Simon, 1993) advice were heeded and the technique of using this procedure will be described in Chapter Three, section 3.2.1 and the analysis of the protocols using the think aloud procedure will be presented in Chapter Six, section 2.

To sum up, I have discussed some studies carried out on how metacognitive training was used in mathematical problem solving and in computer environments, the findings of which showed the positive effect of such training on students' mathematical problem solving performance. I have also discussed studies that adopt the cognitive apprenticeship instruction that encourage students in their use of metacognition during problem solving (Collins et al, 1989). The findings of these studies have relevance for this present study based on how it can help to illuminate and interpret findings which emerge from this study about the effect of metacognitive training in a computer environment using cognitive apprenticeship instruction. However, there are differences between the contexts of some of these studies (i.e. those of college students solving non-routine word problems, those using SOLVE in Logo environment, and others) and those of Singapore primary students solving routine word problems. It is hoped that the present study is able to fill the research gap in the arena of metacognitive training in word problem solving in a computer environment.

Chapter Three
Research Design and Issues

1. Overview of Chapter

The previous two chapters provided the background for the study in terms of its aims, context and its theoretical framework. This chapter discusses the research paradigm and design adopted for the realisation of the aims of the study. It also reports on the planning of the study at various stages of the research. The next section, section 2, deals with the nature and rationale of the research design, followed by the selection of data generation and data collection techniques in section 3. A brief account of the initial preliminary studies and a pilot study carried out amongst students in a Singapore primary school and a Leeds secondary school are provided in section 4. A summary of the results generated during these studies is also reported in this section. The planning of the design, methodology and procedure as well as the piloting were carried out in the ten months prior to the fieldwork. In Chapter Four, there is a more detailed description of the implementation of the research study; the sampling and the rationale for the choice of students and schools for the data collection; and finally, how the data was processed and analysed.

2. Nature and Rationale of Research Design

This section begins by providing the nature and rationale of the research design that is used in this study. First, there is a brief overview of the overall research design. Then it goes on to discuss the approaches that are used to collect the data. Finally, it highlights the advantages and limitations in adopting a design of this nature for the intended study.

The design of this study is a two-phase design (Lee, 1999), combining a quasi-experimental design which is scientific (Robson, 1993, p. 18) in nature and a case study design which is largely exploratory-interpretive (Cohen & Manion, 1994, p. 259) in nature. As I am interested in investigating the relationships (Robson, 1993, p. 79) between metacognitive training, students' level of mathematical achievement and their mathematical word problem solving performance, a quantitative approach was thought to be most suitable for this purpose. As stated in Chapter One, this study also sets out to explore the role of metacognition of Singapore 11-12 year old students' word problem solving in relation to their levels of cognition. Hence, a qualitative analysis (Stake, 1995, p. 41) of students' word problem solving data in the form of think aloud transcripts is intended to provide some information as to the extent the levels of cognitive behaviours interact and influence the outcome of word problem solving. In addition, analysis of the teacher interview and student questionnaire data allows me to identify the metacognitive knowledge (Chapter Two, section 2) and this provides a rich account of the type of knowledge students bring with them to word problem solving.

The advantage of a two-phase design is that it capitalises on the particular strengths of two traditionally separate research orientations (Lee, 1999). In particular, a quantitative study's quasi-experimental design (see section 3.1) may inform me as I hypothesise on the effect of metacognitive training on students' mathematical word problem solving performance before, immediately after and then a prolonged period after the metacognitive training. In a complementary fashion, the qualitative study may also allow me to obtain a deeper or richer sense of how these metacognitive decisions, during mathematical word problem solving, influence students' mathematical word problem solving performance where pair think aloud protocols are used to elicit qualitative information.

However, such designs are without limitations. In the design of the study, care was taken to maximise the quality of the research. The first precaution was to ascertain construct validity (Robson, 1993, p. 68), i.e. establishing correct operational measures for the concepts being studied. Yin (1989) suggests that to achieve construct validity, the researcher must select the items to be studied, and then to demonstrate that the measures employed actually measure the items or the behaviour being studied. In the quantitative component, the students' 1998 end-of-the-year mathematics examination scores were identified as a good measure of students' level of mathematical achievement, and the mathematical achievement test data were identified as a good measure of students' mathematical word problem solving performance. In the qualitative component, students' word problem solving behaviours, and students' word problem solving performance were thought to be good measures to show the relationship between the role of metacognition and its influence on word problem solving performance. However, for reasons given below, one cannot completely guarantee construct validity for qualitative and quantitative work.

The second consideration taken into account was external validity (Cohen et al, 2000, p. 109). This has to do with whether the results can be generalised to the wider population, cases or situations. Both results of the quantitative and qualitative studies cannot be generalised to a population. However, the findings from the quasi-experimental design can represent 'realistic' conclusions in educational settings, and the findings from the case study design can be used to generate theory as is the tradition in qualitative research. In the present study, the qualitative study is used to generate insights into the theory of the role of metacognition in word problem solving.

The third consideration is reliability (Cohen et al, 2000, p. 117). This aspect deals with reproducibility of the results over time, over instruments and over groups of respondents. The goal of reliability is to minimise errors and biases in a study. In a research study, the researcher is encouraged to document the procedures used as detailed a manner as possible so that another researcher repeating the same procedures would get similar results. However, in the quasi-experimental design and the think aloud protocol in the case study design for the present study, reproducibility is neither possible nor desirable. In the former, there are selection effects (see Chapter Four, section 5.1) due to my lack of control in selecting the appropriate schools and classes based on the criteria for this study (see Chapter Four, section 2.2), and in the latter each think aloud protocol is unique. Although data collected in the qualitative study is richer, the following shortcomings could arise:

1. There might be a limitation in my interpretation of students' think aloud protocol because of the difficulty in carrying out think aloud methods. According to Goos and Galbraith (1996) even in a 'dyad' (see Chapter Four, section 2.1), thoughts are not always verbalised and, therefore, are not easily accessible to the observer. In this study, the only behaviours that could be categorised as metacognitive were those that were audible to me. If the students were typing during the episodes of exploration,

implementation, and/or verification, without verbal explanation, their behaviours were most likely to be categorised as cognitive. It is possible that students were monitoring their work at these moments and that their metacognitive behaviours were overlooked because they had not articulated their thoughts;

2. There is a limitation in the motivation for some students to engage seriously in the study because the learning strategy I had presented to them was not examined and graded. There were also other distractions. For example, in School 1, there were badminton and soccer tournaments, and case study students had to miss these tournaments in order to attend the video-recording sessions. In School 2, the training sessions were held during the school vacation, and some students had to forego their holiday activities in order to attend my training sessions. The issue of motivation will be revisited in Chapter Seven, section 2.2.1; and
3. There is a limitation in my ability to maintain a position of neutrality because of my own bias towards different 'dyad's' (see Chapter Four, section 2.1) word problem solving. This bias may have encouraged or even hampered interactions with different 'dyads', and may also have played an important part in the analysis, generation of findings and their interpretation.

I, however, am aware of these limitations, and have made a conscious effort to reduce their effect to an extent which could make the findings valid and reliable. This section has discussed the design of the study, the advantage of adopting such a design and its limitations. The following section will discuss how data was generated and describe the various data collection techniques used in this study.

3. Selection of Data Generation and Data Collection Techniques

This section considers the various research techniques used for data generation and data collection. It also seeks to give rationale for the use of each of these methods and explains the nature of the methods used as well as who the students will be. Finally, it discusses how the appropriate instruments were used in the data collection.

3.1 **Quasi-experimental Design**

In this study, it was not possible for me to undertake true experiments (see Chapter Four, section 2.2). It was best to employ the quasi-experimental design (Cohen & Manion, 1994, p. 169) whose methodology is a 'compromise' of the true experimental design because the random assignment of classes in this study was inappropriate. Cook and Campbell (1979, p. 6) define quasi-experiment as

'experiments that have treatments, outcome measures, and experimental units, but do not use random assignment to create the comparisons from which treatment-caused change is inferred. Instead the comparisons depend on nonequivalent groups that differ from each other in many ways other than the presence of a treatment whose effects are being tested'.

Leik (1997) cites 'field experiment' as an example of a quasi-experiment. According to him, the major advantage of doing a field experiment as compared to conducting a true experiment in a laboratory is that it has to do with studying a social phenomenon in its natural setting rather than in the artificiality of the laboratory. Moreover, a field experiment is much more closely tied to ongoing social reality. As a consequence, it is more likely to satisfy criteria of external validity. With respect to factors that may affect the internal validity of data obtained in field experiments, Leik (1997) has identified three, namely: manipulation, control and random assignment. Leik (1997) highlights the difficulties involved in satisfying internal validity for field experimentation. For example, can we really deduce that the change in Y was brought about by the change in X? This is because field experiment rules out random assignment and the question is whether the observed effects should be attributed to some manipulation or to group member composition. Though field experiments cannot approach very closely to the ideal manipulation, control and random assignment, it should be recognised that it is often very difficult or almost impossible to achieve this ideal set of conditions, especially in educational research. Leik (1997) advises that quasi-experiments, especially field experiments, should not be thought of as some sort of poor relative of laboratory experiments. According to him, each type of research contributes an important part to an overall understanding of how things really work. For example, if there are strong hunches from field experiment but not sufficient evidence to enable us to understand the underlying causal process, then a well-designed experiment is in order. The two are complementary approaches to understanding the reality we wish to study (Leik, 1997).

In the present study, the main technique used to collect data for the quasi-experimental design is the word problem items (see Appendix C) in the mathematics achievement tests (e.g. pretest, posttest 1 and delayed post-test 1). These word problems from the topics Whole Number and Fraction were selected on the basis that i) mathematical word problem solving is developed in these topics in the Revised Mathematics Syllabus (MOE, 1995) (see Appendix A), and ii) Singapore 11-12 year old students are familiar with word problem solving involving Number and Fraction since they have been exposed to the Number and Fraction concepts and word problem solving involving these concepts from the time they started attending primary schools. Hence, the word problems for the quasi-experimental design are the kinds of tasks students are expected to 'master' in school and on which their mathematical performance is systematically monitored and evaluated. They are also designed to reflect the type of word problems used in the Primary School Leaving Examination or PSLE (see Appendix A) for the topics on Whole Number and Fraction. These word problem items 'passed' the item analysis test (Kubiszyn & Borich, 1990) which indicate that they are within the students' level of difficulty (see section 4.3) before they were administered to the students (see Chapter Four, section 4). See Appendix C for the mathematics achievement test items for the pre-test, post-test 1 and delayed post-test 1.

3.2 **Case Study Design**

Like Robson's (1993, p. 146) description of case studies, the present case study is '*a strategy for doing research which involves an empirical investigation of a particular contemporary phenomenon within its real life context using multiple sources of evidence*'. According to Stake's (1995, p. 3) definition, the present case study design is an instrumental case study. With regard to the selection of cases, Stake (1995) cautions that case study is not a sampling research and, in the instrumental case study, selecting cases with our purposes in mind is likely to lead us to understandings, to assertions, and perhaps to modification of generalisations. In addition, careful consideration is to be made with regard to the uniqueness and contexts of the alternative selections for these may aid or restrict our understanding.

A limitation to case study research is that it is often seen as a poor basis for generalisation. So, to gain the needed confirmation, to increase credence in the interpretation, to demonstrate commonality of an assertion, data source triangulation (Stake, 1995, p. 112) is used in this present study. Data source triangulation is defined as a protocol to see if the phenomenon or case remains the same at other times, in other spaces, or as persons interact differently (op cit p. 112). Stake (1995, p. 113) asserts that it is an effort to see if what we are observing and reporting carries the same meaning when found under different circumstances. In this study, two schools have been selected so that the cases observed in the first school, the descriptions and eventually the interpretation may be triangulated with the second school. In the case study design, methodological triangulation (Cohen & Manion, 1994, p. 236), involving student questionnaire and teacher interview schedule, is also used to check on the validity of the students' metacognitive knowledge.

The techniques used to collect data for the case study design were: simple observation to elicit students' think aloud protocols; semi-structured teacher interview; student questionnaire; and use of school documents and students' records. In the following section, issues with regard to these techniques will be discussed

3.2.1 Simple Observation and Think-Aloud Protocols

Observations of students' mathematical word problem solving form an integral part of the research methodology. In this part of the study, where it is qualitative in nature, simple observation allows for significant features of the phenomenon under study to emerge. This involves observing behaviours which are observable. In the case of this study, video-recording students' think-aloud protocols helps to establish an account of the extent of students' metacognition as well as the patterns in which it is used.

i) Nature and Rationale

Think-aloud procedures were used to 'capture' and analyse students' covert metacognitive strategies during word problem solving. They were used to provide a glimpse at covert strategy activity that is not accessible in normal circumstances. In Chapter Two, section 8, I have provided examples of how researchers have used the think aloud procedure to delineate students' metacognitive strategies during problem solving. In this section, I will describe the technique used to 'capture' these strategies and present how I have taken into consideration Green and other researchers' (Ericsson & Simon, 1993; Green & Gihooly, 1996) advice to maximise the validity of video-recorded concurrent verbal reports (see Chapter Two, section 8).

Green and Gihooly (1996) define protocols as detailed records of behaviour during a task. In particular, verbal protocols, like think aloud protocols, are transcriptions that are derived from recording students' speech while they are carrying out a task under thinking aloud instructions. Instructions should encourage the students to verbalise overtly all thoughts and actions that would normally be silent. In such instances, students are not asked to explain or justify what they are doing and they are not asked to report their strategies. Unlike one person protocol, students in pair or small group protocols naturally tend to explain or justify their strategies to their peer(s) and this helps the researcher to make inferences about cognitive strategies. In the present study, students worked in pairs, and the instructions and probes of think aloud protocols were restricted to '*Tell me what you are thinking while solving the word problem*' at the start of students' word problem solving and '*keep talking*' when students paused for more than 10 seconds while solving the word problem. The sustained verbalisations and non-verbal behaviour records were video-recorded. The students were told to use ordinary, everyday terms and concepts. The students were also encouraged to think aloud as if they were talking to their peer and I remained out of view to avoid cueing the student. This was crucial because during the preliminary study with two students (see section 4.2), a confused or puzzled look from my facial expression had prompted the student(s) to provide extra explanations and this, according to Green and Gihooly (1996), distorts the student's normal thought processes. Any instrument used to collect data is fallible, hence, one needs to weigh the pros and cons of an instrument and choose the most appropriate one based on one's research questions. Some cautionary steps were planned and taken into consideration to prevent the pitfalls suggested by Green (1995), Green and Gilhooly (1996), Garner (1988), and Ericsson and Simon (1993). For example, all pairs were provided with warm-up sessions to familiarise themselves with overtly verbalising all their thoughts. In addition, as mentioned, the students were grouped in twos to produce two-person protocols. This arrangement appeared to ease the pressure on the students, for the burden of being uncomfortable in verbalising aloud, of having to speak into a microphone, and of having to be observed by me while solving word problems was shared between them. The conversation within the 'dyad' (see Chapter Four, section 2.1) was able to make certain covert decision-making behaviours overt.

For the case study design, the word problem solving tasks were used to elicit students' word problem solving behaviours. The word problem selection was guided by the following criteria:

1. The word problems were selected from the topics Number and Fraction. Criteria for choice of these topics were similar to those for the mathematical achievement test items (see section 3.1);
2. The word problems have to be challenging enough to require and elicit metacognitive behaviours while at the same time being within the students' (i.e. lower achievers) capacity; and
3. The word problems need to contain a mixture of familiar word problems and genuine 'process' word problems so that initial success on the former can help put the students at ease at the start of each think-aloud session.

The selected mathematics word problems for think-aloud sessions are listed in Appendix C. All the word problems are carefully selected to prevent students from relying on routine and automising procedures which do not need regulatory processes, and all of them are arranged according to their level of difficulty.

ii) **Students and Implementation Procedure**

The students solving the word problems in WordMath were video-recorded to provide a permanent record of word problem solving data and to allow for repeated study and scrutiny of the data. Transcripts of students' word problem solving were analysed in terms of students' levels of cognition during word problem solving. Provision was made for the students observed to use a relatively sensitive boundary microphone attached to a cam-corder so that their 'talk' could be recorded clearly. I am aware that recordings of any nature are difficult as there is the interference of noise to contend with. The position where the video recorder was to be placed also needed consideration. I was able to try it out in one of my pilot observation sessions with Singapore students (see section 4.2) and the quality of recording was found to be relatively good. I also decided that the cam-corder should be positioned behind the students, where students were using WordMath, so that the diagram drawn and the solution typed into the working space provided in WordMath could be captured (see Appendix I for some illustrations of students' word problem solving). The students were also informed that they were given about 25 minutes to solve each word problem. At the end of the 25 minutes, they would be told to either stop working or provide a final statement to indicate their solution. This imposed time limit was not to encourage the students to rush through solving the word problems but to set a time limit for them to work towards solving the word problem. Based on my observation of students solving word problems during the pilot study in Singapore (see section 4.2), most students were able to solve the word problems in less than 25 minutes. For those students who were not successful in their word problem solving, giving them more time had not helped them in being successful in their word problem solving. Hence, the time limit of 25 minutes to solve each word problem was a reasonable period for each 'dyad' (see Chapter Four, section 2.1).

3.2.2 Teacher Interview and Student Questionnaire

For case study design, Creswell (1998) and Stake (1995) recommend that the researcher should provide a detailed description of the case(s) and its/their setting in order to make meanings in the analysis and interpretation of case(s). Hence, the analysis of the teacher interview and student questionnaire data is an attempt to provide descriptive accounts of the type of metacognitive knowledge students' bring to word problem solving sessions.

According to Robson (1993, p. 228), interview is a kind of conversation, a conversation with a purpose. But this conversation, according to Cohen and Manion (1994, p. 271) is one '*initiated by the interviewer for the specific purpose of obtaining research-relevant information and focused by him on content specified by research objectives of systematic description, prediction or explanation*'. In the present study, a semi-structured teacher interview (Robson, 1993, p. 231) was used, where I had worked out a set of questions (see Appendix D) but modified their order based on my perception of what seemed appropriate at that time. This instrument was used on four teachers in two primary schools, three teachers from School 1 and one teacher from School 2 (see section 3.2.2 ii). For the student questionnaire, due to time constraints, instead of interviewing the target students in the case study design, a self-completed questionnaire (see Appendix E) was administered to target students (see Chapter Four, section 2.2.2) from each school after the pretest was administered.

i) **Nature and Rationale**

The main aim of the teacher interview schedule (see Appendix D) and the student questionnaire (see Appendix E) in this case study design is to elicit information about the students' metacognitive knowledge (see Chapter Two, section 2). The student questionnaire contains items drawn from Callanhan and Garofalo's (1987) suggestions for questions which teachers might use to help students develop awareness. The student questionnaire is linked to the word problems in the pre-test which the students would have taken before answering the questionnaire. The questionnaire was designed:

1. To generate data about student awareness across the interactions of person, task and strategy categories during word problem solving (Callanhan & Garofalo, 1987). For example, person-by-task interactions are probed by asking students to rate in terms of some of the word problem's familiarity and difficulty, together with their confidence in the correctness of their solution. The person-by-strategy interactions are probed by asking students to list useful strategies (i.e. reading the word problem carefully, checking the solution, writing down all the steps with the relevant working statements) which may ensure their success in word problem solving. Finally, the task-by-strategy interactions are probed by asking students to identify the type of heuristics (i.e. drawing models) they would usually use in solving the kind of word problems in the Singapore mathematics syllabus; and
2. To provide additional data on the students' word problem solving ability, enabling inferences drawn from the think aloud protocols to be checked.

In addition, interviews with the students' mathematics teachers 'triangulate' (Cohen & Manion, 1994, p. 233) and provide support for the students' account of their word problem strategy factors during word problem solving. The items from the teacher interview schedule were drawn from the student questionnaire but rephrased in order to elicit information from the teachers with regard to the target students' metacognitive knowledge. These two instruments provide rich accounts of the students' metacognitive knowledge of their word problem solving behaviours before the metacognitive training sessions.

ii) **Students, Teachers and Implementation Procedure**

The target students (see Chapter Four, section 2.2.2) were chosen on the basis of their 1998 end-of-year Mathematics examination results. In School 1, the Mathematics teachers and also form teachers, who taught the selected students in 1998 were interviewed in December 1998. Two out of the three teachers were new to the school and had only taught the students for less than four months. These two teachers admitted that they did not know the students very well but would try to answer the interview items as honestly as possible. The third teacher had taught the students for a year and would be teaching the students again in 1999. In School 2, the Mathematics teacher was interviewed in May 1999. She is the head of the mathematics department of the school and had taught one of the classes since 1997 and the other class since 1998. All the interviews were audio-recorded to provide permanent records so that they could be transcribed and analysed.

The student questionnaire was conducted before the training sessions but after taking the pre-test. This was because some of the items from the student questionnaire required the students to refer to the word problem items in the pretest. The students were given an hour to answer all the items in the questionnaire. Instructions began by explaining what each item required them to write and how to fill in the columns below each item (see Appendix E). The students were told to write as much as they could for each questionnaire item.

3.2.3 School Documents and Students' Records

The reports from the school documents provided an intellectual history to the study. The students' academic records (i.e. the 1998 Mathematics end-of-the-year result) allowed me to plan and allocate students to their respective pairs, higher achievers and lower achievers (see Chapter Four, section 2.2.1a)), for the metacognitive training sessions before the actual data collection proper.

4. Three Preliminary Studies and a Pilot Study

Wilson (1996) defines a pilot investigation as a small scale trial before the main investigation. It is intended to assess the adequacy of the research design and of the instruments to be used for data collection. He emphasises that piloting the data collection instruments is essential. Preliminary studies of the student questionnaire (see Appendix E), the metacognitive strategy (see Appendix F) used during the metacognitive training sessions, and the mathematical achievement test items (see Appendix C) were carried out before the pilot study.

4.1 The First Preliminary Study

The first preliminary study was implemented in May 1998. A student questionnaire was sent to a former colleague with clear instructions on how the questionnaire should be conducted. She conducted the session on seventy-eight 11-12 years old students from two EM2 classes (see Appendix A). The purpose of this preliminary study was to ascertain that the students understood the questions, as initially phrased, and that the language used was appropriate; that Singapore 11 to 12-year-olds were able to identify and articulate their awareness of their word problem solving processes in the written form; to discern if they needed a particular type of prompting to aid them in answering the items; and to ascertain that the items were ordered appropriately. The responses from the questionnaire in the preliminary study on the whole yielded rich data with regard to the students' perception of their own metacognitive knowledge (see Chapter Two, section 2) and they were able to give appropriate reasons for their behaviours. I was also able to describe most students' metacognition by extracting the students' word problem solving strategy factors (person-by-task, person-by-strategy and task-by-strategy interactions) during word problem solving (see section 3.2.2 i) with reference to the items in the questionnaire. Hence, this exercise indicated that the self-completed questionnaire was a viable tool to elicit descriptive accounts of Singapore 11-12 years old students' awareness of their metacognitive knowledge during word problem solving. In the actual study, data from the teacher interview schedule complement the students' description of their metacognitive knowledge. The following is an account of S's metacognitive knowledge (from the first preliminary study) written by me based on the data drawn from the student questionnaire items.

S believes (questionnaire data) that a major source of his errors is carelessness and loss of concentration. His strategic awareness appears to be well developed, for he is aware of the usefulness and relevance of strategies for careful reading and organising information in the form of a diagram and systematic presentation of a solution. Checking his written work is also constantly mentioned as crucial for successful word problem solving. He is also aware that losing focus creates barriers to solving word problems. In summary, the person and strategic components of S's metacognitive knowledge (Garfalo & Lester, 1985) appear to be fairly well developed as he appreciates his abilities and weaknesses and has some understanding of the reasons.

4.2 **The Second Preliminary Study**

Metacognitive training using CRIME (see Appendix F) was conducted with two Year 7 higher achievers in Leeds in May 1998. They were trained to focus on their metacognition using CRIME while solving word problems with paper and pencil. The students were given two training sessions, each lasted for an hour, before they were video-recorded while solving four word problems. The purpose of this second preliminary study was to explore if CRIME was a conducive strategy to promote students' metacognitive awareness during word problem solving; and to see if students' cognitive processes during word problem solving could be detected and categorised using Schoenfeld's (1985) episodic framework.

This preliminary study helped me confirm the virtue of letting students solve word problems collaboratively. This was because the students, J and A, were engaging in the three processes, articulation, conflict and co-construction identified by Crook (1995) in social interaction during collaborative word problem solving (see Chapter 2, section 7). I also observed that changes needed to be made in CRIME. For example, under the 'RECALL possible strategies' category, there is a need to extend the possible strategies. Hence, possible strategies were included in the 'RECALL possible strategies' category, namely: Draw (solve by drawing models/diagrams); Small (simplify problem using small numbers); Parts (solve part(s) of the problem first); Before (use before-after concept); and Backwards (solve by working backwards) (see Appendix F). Then, the heading for CRIME was modified to a nonsensical sentence that read: 'It is a CRIME to draw small parts before backwards' where it is hoped that the 'draw small parts before backwards' would remind students of the possible strategies during word problem solving.

In the second preliminary study, the analysis of the students' think aloud protocol data also revealed that these students' word problem solving enterprise could be detected and categorised in Schoenfeld's episodic framework. The exercise to categorise the students' protocol data into episodes was eventful, and interesting results emerged from this exercise. It was observed that the students' word problem solving process was similar to the descriptions provided by Schoenfeld's (1985) study of an 'expert' problem solver. The students' timeline representation while solving the BOTTLE (see Appendix C, number 2, F2) word problem is shown in Figure 3.1. The descriptions of the episodes (Schoenfeld, 1985) in Schoenfeld's episodic framework are summarised in Table 3.1. See Schoenfeld (1985, p. 297 to 301) for a fuller account of the definitions of the episodes.

Episode	Descriptions
Reading	This episode begins when a student starts to read the problem statement aloud. It includes: 1. silent rereading of the problem; 2. vocal rereading of the problem; and 3. verbalisations of parts of the problem statement.
Analyse (well structured, stick closely to the conditions or goals of the problem)	In this episode, the student makes an attempt to understand the problem fully; to select an appropriate perspective and to reformulate the problem in those terms; and to introduce for consideration whatever principles or mechanisms that might be appropriate.

Exploration (less structured and is further removed from the original problem)	In exploration, the student makes a tour through the problem space to search for relevant information that can be incorporated into the analysis-plan-implementation sequence. The student may also use a variety of problem solving heuristics - the examination of related problems, and the use of analogies. In this episode, students might engage in local or global assessments. They are identified as New Information/New Procedure and Local Assessment. New Information points are subdivided into two types: 1. points where previously overlooked or unrecognised information came to light; and 2. points where the possibility of using a new procedure is mentioned. Local Assessment include: 1. assessing the current state of the problem solver's knowledge (what is known/not known); 2. procedure (checking accuracy of execution, assessing relevance or usefulness); and 3. result (assessing accuracy or reasonableness).
Planning-Implementation	In this episode, the student overtly makes a structured plan and the implementation of the plan is orderly. The student also monitors his/her process with feedback to planning and assessment at local and/or global levels. These are the same as New Information/New Procedure and Local Assessment.
Verification	In this episode, the student reviews partial or final solution; tests his/her solution in some ways; and assesses his/her solution, either with an evaluation process, or an assessment of confidence in the result.

Transition	Transition is a juncture between episodes where the metacognitive decisions (or absence) will make or break a solution. In this episode, the student makes an assessment of the current solution state. If a solution path is abandoned, the student makes an attempt to salvage or store things that might be valuable in it. There might be an attempt to identify the local and global effects on the solution of the presence or absence of assessment as previous work is abandoned. The student might also make an assessment of the short and/or long term effects on the solution of the new direction.

Table 3.1: Summary Table of Schoenfeld's Episodic Framework

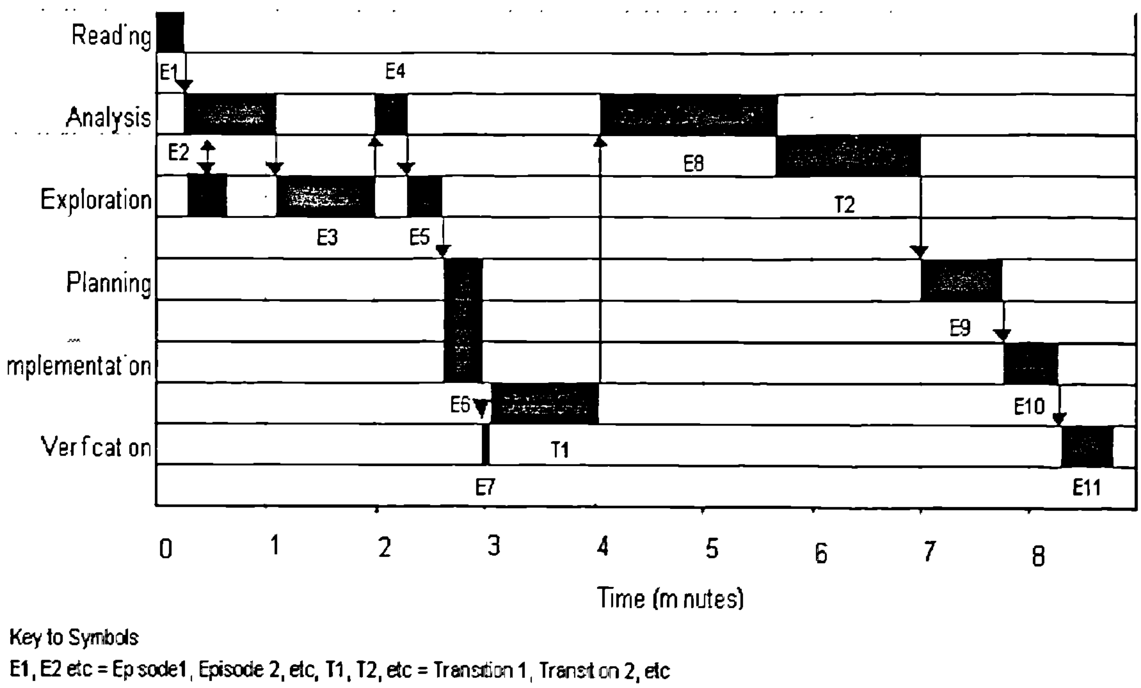


Figure 3.1: A Timeline Representation of J and A solving the BOTTLE Word Problem

4.3 The Third Preliminary Study

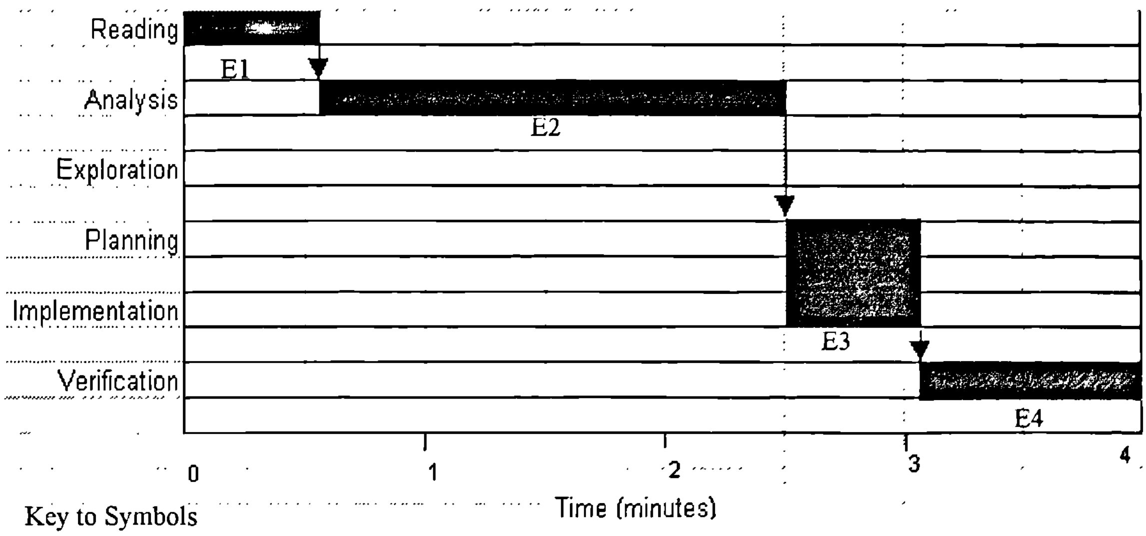
The final preliminary study was conducted at the end of the 1998 academic year in Singapore. The purpose of the third preliminary study was to elicit suitable word problem items for the mathematical achievement tests in the quantitative study. One hundred 11-12 year old students solved twenty word problems on four separate occasions. These word problems were randomly ordered. The students were given half an hour to solve five word problems during each session. I conducted these sessions and similar instructions were given to these students on the four occasions. During each session, the students were told the amount of time given to solve the five word problems; they were encouraged to solve all the word problems; they were also encouraged to use the model approach to solve the word problems; and they were told to proceed to the next word problem if they had difficulties on the previous one and returned to it when all the word problems were solved. I found this third preliminary study fruitful as analysis of the items revealed that nine out of twenty word problems were suitable for the quantitative study. This is because the values of the Facility index for the suitable nine word problems lie between 0.3 and 0.7 while those for the Discrimination index were above 0.3, and according to Kubiszyn and Borich (1990), these values are acceptable. Due to time and logistic constraints, I had to take the tenth word problem from the students' Mathematics workbook (Curriculum Development Institute of Singapore, 1996). This constituted the ten word problem items for the mathematics achievement tests in the quasi-experimental design. See Appendix C for the ten word problems in the pre-test, post-test 1 and delayed post-test 1 for the quantitative study.

4.4 The Pilot Study

The pilot study was implemented in November 1998 in a Singapore primary school. The purpose of the pilot study was three-fold. I wanted to observe how Singapore 11 to 12-year-old higher achievers and lower achievers used CRIME (see Appendix F) to solve word problems with and without WordMath. I also wanted to see if I could identify the Singapore students' metacognitive behaviours during word problem solving using Schoenfeld's episodic framework in the same way as in the second preliminary study I had conducted in Leeds (see section 4.2). Finally, I also wanted to observe if there really existed a difference in the metacognitive behaviours between students of different levels of mathematical achievement. Four pairs of higher achievers and lower achievers participated in this pilot study in Singapore, samples which in some ways represented the population which was to be used in the actual study.

The instruments used in the piloting were the CRIME strategy (see Appendix F) and word problem items (see Appendix C) in WordMath. The results of this pilot study raised three issues. First, CRIME was again observed to be a suitable strategy which enabled students to become aware of their metacognition during word problem solving. It was observed that students, higher and lower achievers, who were explicitly trained to use CRIME, demonstrated more occasions of monitoring activities compared to those who were not using CRIME. This observation appeared crucial, but Schoenfeld's episodic framework appeared to have limitations in demonstrating this difference. This limitation was also addressed by Artzt and Armour-Thomas (1992) who noted that Schoenfeld's framework was not able to identify statements made about the problem (the more 'local' indications of metacognitive behaviour). As a result, as Schoenfeld (1985, p. 293) admitted, the framework could not address the important role that consistent monitoring and evaluation of solutions play in the problem solving process. Next, it was interesting to observe from the think aloud protocol data that the higher achievers, with or without metacognitive training, had problem solving behaviours which were similar to Schoenfeld's description of an expert's problem solving activity, in that they analysed the word problems thoroughly before implementing their strategies. However, the Singapore higher achievers demonstrated a more systematic progression of word problem solving activity (i.e. Read → Analyse → Plan ↔ Implement → Verify) throughout their word problem solving compared to Schoenfeld's (1985, p. 312) description of the 'expert's' problem solving activity. The word problem solving activity of the BOTTLE (see Appendix C, number 2, F2) word problem by a pair of Singapore higher achievers is shown in Figure 3.2.

From the analysis of the students' protocol data, I again observed that there appeared to exist a difference in the length of time word problem solvers devoted to the different episodes, and it appeared that this difference influenced their success in word problem solving. At the end of the pilot study, my dilemma was how to explicitly demonstrate this interesting phenomenon which I felt Schoenfeld's episodic framework would not be able to do so.



Key to Symbols
E1, E2, etc = Episode 1, Episode 2, etc

Figure 3.2: A Timeline Representation of J and KY solving the BOTTLE Word Problem

4.5 **Summary of the Three Preliminary Studies and the Pilot Study**

The three preliminary studies and the pilot study were useful activities in that they helped me identify some of the possible issues to focus in the actual field work and I was able to redefine, modify and refine the instruments for the actual study. Yin (1989) says that pilot studies help investigators to redefine their data collection plans with respect to both the content of the data, as well as the procedures to be followed. The four issues that needed attention in the actual fieldwork were as follows:

1. In terms of the metacognitive strategy with CRIME, I realised that I needed to be more explicit in my instructions and use more examples to lead students to understand and see the importance of being aware of their own thinking process during word problem solving;
2. I also realised that training students to think aloud was important if I wanted to elicit observable data. Hence, training to think-aloud was reconceptualised as important and should take precedence, unlike initial planning to make think-aloud sessions short and concise;
3. I was also challenged to re-think the way higher achievers in Singapore primary school students solve word problems. The progression of their word problem solving activities appeared to be different from those of the 'experts' described by Schoenfeld and the students in Leeds; and

4. I had to reconsider an alternative method of analysis to demonstrate that the length of time students devoted to each episode in Schoenfeld's episodic framework was critical to their word problem solving outcome.

5. **Summary**

In this chapter, I have discussed the nature and rationale for adopting a scientific and exploratory-interpretive approach for this study. As I am interested in testing hypotheses to investigate the effect of metacognitive training on students' mathematical word problem solving performance and exploring the role of metacognition in word problem solving, I found that the approaches suited the purpose of my research. I then went on to discuss some of the main methods or techniques of data collection planned for the fieldwork, which included: mathematical achievement tests during pre-test, post-test and delayed post-test; simple observations to elicit students' think aloud protocol data; teacher interviews; and student questionnaires. I also gave my rationale as to why I chose to use these instruments. While the mathematical achievement tests were used to test hypotheses and make generalisations of students' mathematical word problem solving performance, simple observations were aimed at eliciting data which would be useful for the process of comparing students' word problem solving behaviours, and the teacher interview schedule and student questionnaire were aimed mainly to provide a historical context of students' metacognitive knowledge before the treatment. This was followed by a report of the three preliminary studies conducted in a Leeds secondary school and in a Singapore primary school, and a pilot study conducted in a Singapore primary school, and the issues for the actual study were raised.

Chapter Four documents the implementation process which involves the sampling method, the intervention period and data collection during the five months of fieldwork carried out in two Singapore primary schools. The issues with regard to the approach of data processing for the quasi-experimental study and case study will also be discussed. This includes how the mathematical achievement test data is analysed using two statistical techniques; and how the students' think-aloud protocol data is coded, categorised and analysed. The results of these data analyses will be presented in Chapters Five and Six. This will be followed by the discussion of findings of the mathematical achievement test, think aloud protocol, teacher interview and student questionnaire data in Chapter Seven.

Chapter Four
Data Collection and Data Analysis

1. Outline of the Chapter

This chapter documents the data collection during the five months of fieldwork (from January to May 1999) in two Singapore primary schools and how the data is analysed. Chapter Three provided the rationale for the quasi-experimental and case study designs and the rationale for the types of instruments used. This chapter continues with a description of how the data was collected and briefly describes the issues with regard to how the data was analysed and processed. In section 2, I will discuss the sampling issues with regard to the quasi-experimental and case study designs. Then, in section 3, a review of the instruments for the actual data collection will be reported. This is followed, in section 4, by a description of the intervention process and the data collection used in the quasi-experimental study and the case study. Finally the issue of analysing the data will be discussed in section 5.

2. Sampling and Rationale for Choice of Schools and Students

This section gives a brief account of the nature of sampling, i.e. the selection of schools and classes as well as the students involved. It also seeks to give a rationale for the choice of sampling. In this study, it was not possible to have a pre-structured approach to sampling as the selection finally depended on the availability of classes and students and my own resources.

2.1 Rationale

According to Cohen and Manion (1994, p. 89), '*the correct sample size depends upon the purpose of the study and the nature of the population under scrutiny*'. The quantitative study is inferential in nature, so the sample should be large enough so that empirical generalisations can be made from the sample selected to the population from which it comes (Robson, 1993, p. 154). However, due to constraints of time and logistic considerations, I had to base my research on two Singapore primary schools. Hence, sampling of students in the two schools in the quantitative study is stratified (Cohen & Manion, 1994, p. 87) where the students from the classes were stratified into similar academic profile groups, each group containing either higher achievers or lower achievers. On the other hand, the qualitative study is broadly exploratory in nature, so it was not my intention to go for a large sample as the concern is with depth and not breadth. Moreover, my concern for the qualitative study is with theoretical inferences from the data collected. Hence, the sampling for the case study design is purposive (Cohen & Manion, 1994, p. 89) where the sample size is limited to four pairs of higher achievers and four pairs of lower achievers. From this point onwards, two students who worked collaboratively on word problems will be referred to as a 'dyad'.

2.2 Sampling: schools, classes and students

The scope of this study covers Singapore primary schools. The schools and classes chosen were based on the following criteria:

1. homogeneous school population in terms of the gender composition of students; and
2. EM2 classes (see Appendix A).

It was extremely difficult to enlist schools for this study because of the commitment needed. Ten schools which met the above criteria were approached about taking part in the research. Some declined because they felt they could not accommodate the request of using the computer resources for two consecutive weeks; some because the target students needed to spend time preparing for the PSLE (see Appendix A). Eventually, two schools, labeled School 1 and School 2, consented to participate in the research. However, School 2 did not meet the above criteria, as it is a co-education school, and only two classes were able to be involved in the study, one of which was an EM2 class and the other an EM1 class (see Appendix A). According to the head of department, the EM1 class was the 'second best' in the school. Given these two classes, I anticipated that having an EM1 class in the study would contribute to the threats of internal validity (Cook & Campbell, 1979, p. 51).

In performing the analysis on quantitative data in quasi-experimental designs, Cook and Campbell (1979) recommend that the researcher should explicitly try to rule out as many potential biases as possible. According to them, this usually entails multiple analyses of the data, with each analysis aimed at estimating the effects of different patterns of potential biases (Cook & Campbell, 1979, p. 200). Robson (1993) also sees the virtue of 'bracketing' the effect of a treatment by using a variety of different but reasonable techniques of analysis. According to him (Robson, 1993, p. 366), the purpose is to seek to eliminate, or at least try to reduce, the effect of selection and other threats through the design of the study rather than relying on the statistical analysis removing their effects. Hence, I propose that in the present study, two forms of analyses, (i) repeated measures three-way analysis of variance (Cook & Campbell, 1979, p. 175) where students are blocked (see Appendix K) according to the school factor, and (ii) repeated measures two-way analysis of covariance (Cook & Campbell, 1979, p. 153) should be employed to analyse the quantitative data in order to address the issues of reliability and validity. These forms of analyses will be described more fully in section 5.1. The following sub-sections will first describe the two quasi-experimental designs which use the two techniques of analyses as mentioned above to analyse the quantitative data, and then the last sub-section will describe the case study design which will be used to analyse the qualitative data.

2.2.1 The Quasi-experimental Designs

A total of 142 Primary 6 (11-12 years old) students from School 1 and School 2 participated in the study. The students from the two Primary 6 EM2 classes from School 1 and the two Primary 6 classes, one EM1 and one EM2, from School 2 were assigned to two conditions:

1. **Treatment:** students who received metacognitive training, underwent cognitive apprenticeship instruction in a computer environment, and were engaged in collaborative learning during mathematical word problem solving; and
2. **Control:** students who did not receive metacognitive training but underwent cognitive apprenticeship. instruction, and were engaged in collaborative learning during mathematical word problem solving.

All students from the treatment and control classes were ranked and stratified (Cohen & Manion, 1994, p. 87) into similar academic profile groups, labeled as higher achievers and lower achievers, based on their 1998 end-of-the-year Mathematics examination result. Higher achievers comprised of students with Mathematics score of 75% and above, while lower achievers comprised of students with scores between 50% and 74%. This was in accordance to the categories of the scores in the PSLE grading system (see Appendix A). These pairs worked collaboratively during the metacognitive training sessions.

The following sub-sections describe how the students were assigned to the two quasi-experimental designs for data analysis purposes. The first quasi-experimental design will employ the repeated measures three-way analysis of variance (Chapter Five, section 3) where the students are blocked according to the school factor, while the second quasi-experimental design will employ the repeated measures two-way analysis of covariance (Chapter Five, section 4) to analyse the quantitative data.

a) **Quasi-experimental Design 1**

In the first quasi-experimental design, the students are blocked according to the school factor (see Appendix K). By blocking the students according to School 1 or School 2, I am presuming that there is a school effect. Since I had no control on the selection of school and I had anticipated the potential biases that might occur with the presence of the EM1 students in School 2 (see section 2.2), blocking the school factor is an attempt to separate the treatment effect from the effect of selection differences. For example, there is a possibility of a significant difference in mathematical word problem solving which is connected with the school factor. However, I would probably not be able to make definitive causal statements about the effect. This is because the school factor represents a variety of variables, from institutional policy to cultural factors, and it correlates with many other variables. Hence, blocking the samples according to the school factor is a way to increase the statistical power (Newton & Rudestam, 1999, p. 70) of the analysis.

Consequently, quasi-experimental design 1 first involved blocking the students according to the school factor, School 1 or School 2. The students from the treatment class in School 1 were ranked and then stratified (see section 2.2.1) into similar academic profile groups, labeled as higher achievers and lower achievers, based on their 1998 end-of-the-year Mathematics examination result as mentioned above. This process was repeated for the students in the control class in School 1. The stratification technique, to separate the students into higher and lower achievers in School 1, was again carried out on students from the treatment and control classes in School 2. Hence, the eight groups, treatment (T) with higher achievers (HA) and lower achievers (LA), and control (C) with higher achievers and lower achievers in School 1 (S1); and treatment with higher achievers and lower achievers, and control with higher achievers and lower achievers in School 2 (S2), form the samples for the quasi-experimental design 1 (see Appendix K). However, throughout the whole study, other students who were not involved in the quasi-experimental design 1 were also included in the metacognitive training. These students also took the mathematical achievement tests: pre-test, post-test 1 and delayed post-test 1, but their results were not considered during the data analysis for design 1. This was to minimise the Hawthorne effect (Jaeger and Bond, 1996, p. 882): situations in which the treatment group might perform better than was typical because of the novelty of the treatment and the special attention they received. In the matrix table (Table 4.1) shown below, five students were assigned to the different conditions, making a total of 40 students for the quasi-experimental design 1.

	School 1		School 2		
	Treatment	Control	Treatment	Control	Total
HA	5	5	5	5	20
LA	5	5	5	5	20
Total	10	10	10	10	40

Table 4.1: Quasi-Experimental Design 1

Repeated measures three-way analysis of variance where students are blocked according to the school factor will be used to analyse the quantitative data of quasi-experimental design 1 (see Chapter 5, section 3).

b) **Quasi-experimental Design 2**

The students from the two Primary 6 EM2 classes from School 1 and the two Primary 6 classes, one EM1 and one EM2, from School 2 were assigned to either treatment or control classes as described above. The students from the treatment classes in School 1 and School 2 together were ranked and then stratified as a whole into higher and lower achiever groups based on their 1998 end-of-the-year Mathematics examination result as described above. This stratification technique on the treatment students was repeated for the students in the control classes in School 1 and School 2 together. Hence, the four groups, treatment (T) with higher achievers (HA) and lower achievers (LA), and control (C) with higher achievers and lower achievers, form the samples for the quasi-experimental design 2 (see Appendix K). Like the students in the quasi-experimental design 1, there were other students who were not involved in the quasi-experimental design 2 who were also included in the metacognitive training and took the mathematical achievement tests. In the matrix table (Table 4.2) shown below, twenty-five students were assigned to the different conditions, making a total of 100 students for the quasi-experimental design 2.

	Treatment	Control	Total
HA	25	25	50
LA	25	25	50
Total	50	50	100

Table 4.2: Quasi-experimental Design 2

Repeated measures two-way analysis of covariance will be used to analyse the quantitative data based on quasi-experimental design 2 (see Chapter 5, section 4).

2.2.2 The Case Study Design

For the case study design, a 'dyad' (see section 2.1) of higher achievers and a 'dyad' of lower achievers from the treatment and control classes in each school were selected and their metacognitive behaviours during word problem solving compared. This meant that there were two dyads of higher achievers and two dyads of lower achievers from each school participating in each condition. The case study design is shown in Table 4.3.

	Students in School 1		Students in School 2		
	Treatment	Control	Treatment	Control	Total
HA	2	2	2	2	8
LA	2	2	2	2	8
Total	4	4	4	4	16

Table 4.3: Case Study Design

3. Data Collection Approaches

Chapter Three, section 3, described the rationale for the choice of instruments used for the data collection. These instruments include:

1. Word problem tests (see Appendix C) where the student performance in completing word problem items were used to test hypotheses generated in the quasi-experimental design;
2. Video-recordings of word problem solutions in WordMath (see Appendix C) where the students in the case study design would solve word problems and their think aloud protocols would be video-recorded and analysed to elicit students' word problem solving activities; and
3. Audio-recordings of teacher interview (see Appendix D) and self-completed student questionnaire items (see Appendix E) where the information from these two instruments collected would provide an account of the students' (in the case study) metacognitive knowledge (see Chapter Two, section 2) before the intervention.

These instruments were used in the main data collection and the procedure for implementing these instruments is described in the following section.

4. The Intervention Process

The intervention process started in January 1999 and ended in May 1999, a period of five months. In the following sub-sections, an account of the study context, the sequence of the intervention and the specifics with regard to metacognitive training amongst the treatment classes and groups for the quasi-experimental and case study designs will be described.

4.1 The Setting

The study was located within groups of students preparing for their PSLE (see Appendix A). Highly-structured textbooks and workbooks used the model approach to demonstrate how mathematical word problems were solved. Students would have been exposed to this model approach since Primary 2 (8 to 9 years old). The mathematics teachers used a detailed Teachers' Handbook to teach the mathematics syllabus. Again, the use of the model approach to solve mathematics word problems was recommended and demonstrated in all these handbooks. Working in small groups might not be the norm in a mathematics lesson. However, students would have experience of interpersonal collaboration in English and Science classes. (See Appendix A for a description of the Singapore mathematics curriculum)

4.2 A Chronological Sequence to the Study

After choosing the students based on the students' end-of-the-year (1998) mathematics examination results for the case study design, the mathematics teachers (see Chapter Three, section 3.2.2 ii) of the students from School 1 were interviewed in December 1998. The Mathematics head-of-department (see Chapter Three, section 3.2.2 ii) in School 2 was interviewed in March 1999.

In the new academic year, January 1999, the students in the treatment and control classes in School 1 were given a pretest (see Appendix C) before the training sessions. The student questionnaire (see Appendix E) was also administered to the treatment and control groups in the case study design to probe their metacognitive knowledge. Both the treatment and control classes in the quasi-experimental design were taught how to use WordMath but the control class was trained separately from the treatment class. The differences between the training sessions between the treatment and control classes lay in the implementation of the metacognitive training. The treatment class was instructed in the model approach using WordMath with metacognitive training whereas the control class was instructed in the model approach with the same software without being exposed to the metacognitive training. The specific differences between the two instructional methods are described in section 4.3. During the training sessions, all students worked collaboratively with their partner. As mentioned in section 2.2.1, the students were paired according to their 1998 end-of-the-year Mathematics examination results. Students worked in pairs throughout the study except when they sat for the mathematical achievement tests. These training sessions also served as 'warm-up' sessions for students in the treatment and control groups to talk during word problem solving. After the training sessions, posttest 1 was administered to the students in the treatment and control classes for the quasi-experimental design. The students were tested individually.

After two weeks of training for the treatment and control classes in School 1, the students chosen for the case study design had an additional two sessions of training in the following two weeks. Each session lasted for about an hour whereby the students again collaboratively solved four word problems. During these sessions, all students, especially from the treatment groups, would slowly take over the full responsibility of solving word problems from WordMath by themselves drawing on the training they had had. I was present but ceded control to the students. Following this, there were two further WordMath sessions, for about an hour, in which the students solved four word problems (one topic per session). These sessions, which only involved students in the case study design, will be labeled as posttest 2. The treatment and control students' think-aloud protocols and their work on the computers were video-recorded. The solutions the students keyed into the computer were also printed. The treatment and control groups were not provided with any paper and pencil. This was to ensure that these students did their working on the space provided in WordMath on the screen.

Two delayed posttests, 1 and 2, were administered about six weeks after posttests 1 and 2 were administered. Delayed post-test 1 was administered to all the students in the treatment and control classes in the quasi-experimental design while delayed posttest 2 was administered only to the treatment groups in the case study design. Delayed posttest 2 was administered to determine if the students from the treatment groups in the case study design had retained the metacognitive training behaviours for word problem solving. In delayed posttest 1, students in the quasi-experimental design solved the word problems on an individual basis and in delayed posttest 2, the treatment students in the case study design worked in pairs and their 'think-aloud' protocols were again video-recorded. The research study in School 1 ended in mid April 1999.

In brief, the pretest was administered to the treatment and control classes for the quasi-experimental design before the intervention, posttest 1 was administered to the treatment and control classes for the quasi-experimental design immediately after the intervention and delayed posttest 1 was administered six weeks after posttest 1. On the other hand, posttest 2 was administered to the treatment and control dyads for the case study design after the intervention and two extra training sessions, and delayed posttest 2 was administered to the treatment dyads six weeks after posttest 2. Table 4.4 below demonstrates the work schedule of the students and teachers involved in the research.

	Week 1	Weeks 2 & 3	Week 4	Week 5	Weeks 6 to 9	Week 10	Week 11
Quasi-experimental and Case Study Students	Pretest	Four training sessions	Post-test 1			Delayed post-test 1	
Case Study Students	Student question-naire		Two training sessions	Post-test 2			Delayed posttest 2 (only for treatment dyads)
Teachers	Inter-view						

Table 4.4: Work Schedule of Students and Teachers involved in the Research

The above procedure was again applied to the treatment and control classes in School 2. However, their training sessions started in mid March 1999, during the one-week school term break, and the research study ended at the end of May 1999. During the school term break, each class had two training sessions. Each session lasted for two hours.

4.3 Training and Practice

The treatment and control classes in School 1 participated in four 60-minute training sessions on the use of WordMath over a period of two weeks. In School 2, the students had two 2-hour training sessions on two consecutive days. Students were told that the materials used were going to help them become better word problem solvers. This served as a motivation for students to co-operate with me. In order to control for effects due to differences in students' previous metacognitive knowledge, all classes received information on metacognition. Students were told that 'thinking out loud' (see Chapter Two, section 5.8) was a way of making their thinking clearer, that it provided them with an opportunity to check for errors in each other's logic and understanding and that it helped them to control their thinking and manage their word problem solving. This message was reviewed and emphasised before each training session. Presumably, this information would encourage students to verbalise during word problem solving so that their covert reasoning and strategies used could be made overt to their partners. This was also an advantage for the later analysis of video-recorded word problem solving sessions.

The treatment classes were exposed to a detailed explanation to the acronym, CRIME (see Appendix F), which comprises of the word problem solving stages. The acronym served as a means for students to become explicitly aware of their word problem solving processes. Students in the treatment classes were told that thinking of CRIME while solving word problems was a way of managing and checking their thinking and word problem solving. They were also told that keeping themselves aware of what they were doing during word problem solving and monitoring the effectiveness of their direction towards a solution would also improve their word problem solving performance. Each training session with the treatment classes began by making the students recall what CRIME entailed. The control classes were told the importance of monitoring their word problem solving process as described above but using CRIME was not identified as a way of doing so.

During the first training session, the treatment and control classes were taught how to use WordMath and encouraged to think aloud. During the second training session, the treatment classes were provided with direct explanation, explicit cognitive modeling and 'scaffolded' (Bruner, 1985) practice in using CRIME while solving word problems. First, the students and I classified the types of word problems encountered and discussed the different word problem solving strategies in which students used to solve word problems. I then summarised the discussion by introducing CRIME. Each dyad in the treatment classes was given the acronym card and each stage used in word problem solving was read and explained. Next, the process of applying these stages during word problem solving was modeled for the students in conjunction with a word problem using the model approach (see Appendix B). In the demonstration, I used 'think aloud' to verbalise my thought processes during word problem solving while the students followed the processes by observing their acronym cards. I asked myself questions from the set of guiding questions in each stage and responded appropriately or thought of the possible strategy appropriate to model the word problem. This continued until a solution was reached and evaluation was demonstrated. Following my modeling of the strategy, the students practised the process with another word problem, with feedback and help from me. During the next two training sessions, the students practised using CRIME in conjunction with WordMath. During every training session, not only did I give the acronym card to each dyad and remind them to think about the stages and questions while solving the word problems, but I also encouraged volunteers to demonstrate to the class how they had used CRIME to solve word problems.

The control students also worked in pairs for four training sessions with WordMath. In contrast, the control students received no training, modeling or instructions regarding the use of CRIME. The control students discussed concepts only in the context of solving WordMath word problems.

During the training sessions, I used the same approach to familiarise the treatment and control students to the WordMath user-interface. I also monitored all the students and gave reminders to them to work together. My classroom instruction to the treatment and control classes were video-recorded during the first training sessions. This was to check whether I had been consistent and unbiased in my instruction towards all the classes (surface analysis suggests that I was).

In sections 2 to 4, I have described the research process, mainly.the data collection and the intervention procedure. I have addressed sampling and how my sampling affected the way the students were allocated in the quasi-experimental and case study designs. I have also reviewed the instruments used in this research. Finally, I concluded this section by giving an account of the procedure of the intervention process. The following sections provide an account of how the data was analysed.

5. Data Analysis

Robson (1993, p. 372) summarises the purpose of analysis as '*the major task is to find answers to your research questions*'. This has a major influence on the kinds of analysis that are needed to answer research questions. According to Robson (1993), in order to come up with trustworthy answers, the analysis has to treat the evidence fairly and without bias, and the conclusions drawn must be compelling, not least in ruling out alternative interpretations. The following subsections discuss issues that needed to be addressed when the data collected was analysed.

5.1 Quantitative Analysis

As noted in Chapter Three, section 3.1, I have to explicate the specific threats to valid causal inference in quasi-experimental designs. Cook and Campbell (1979) list the threats of internal validity (p. 38) and external validity (p. 37) that might exist in quasi-experimental designs. They include 'history', maturation, testing, and instrumentation (Cook and Campbell, 1979, p. 51-55). These threats were considered in the present study and care was made to avoid these threats. However, some of the threats were beyond my control, some of which I have acknowledged in the stages of the data analysis. For example, I have used two schools in order to provide external validity to the interpretation of the quantitative and qualitative analysis (see Chapter Three, section 3.2). However, the selection of schools and the classes involved (see section 2.2) were beyond my control and these contribute to possible selection effects that may bias the results. Consequently, I proposed that two statistical techniques of analyses should be used to rule out these potential biases. They are the repeated measures three-way analysis of variance where the students are blocked according to the school factor and the repeated measures two-way analysis of covariance (see section 2.2). Both methods make an attempt to separate the effect of the independent variables on the dependent variable from the effect of selection differences (Robson, 1993, p. 366). For example, in quasi-experimental design 1 (see section 2.2.1 a), the students are blocked (see section 2.2.2 a) according to the school factor and repeated measures three-way analysis of variance is employed to analyse the quantitative data. According to Ferguson & Takano (1989, p. 391), the block design is a way of reducing the error variance due to an extraneous variable or variables. It is a method to ensure that the pretest means of the students in the two conditions do not differ substantially. In quasi-experimental design 2, however, the pretest means of the students from the different conditions differ substantially (see Chapter Five, section 4.2, Table 5.4). According to Cook and Campbell (1979, p. 153), repeated measures two-way analysis of covariance with the pretest as a covariate will be able to adjust for the initial differences between the different groups in the quasi-experimental design 2. I used the statistical software, SPSS 9.0 (Norušis, 1999), to analyse the quantitative data in both quasi-experimental designs. The testing of hypotheses and the findings of the analysis on the quantitative data from the two quasi-experimental designs, quasi-experimental design 1 and quasi-experimental design 2, will be reported in Chapter Five.

5.2 Qualitative Analysis

As mentioned in Chapter Three, section 3.2.2, a detailed description of the case(s) and its/their setting should be provided in a case study in order to make meanings in the analysis and interpretation of case(s) (Creswell, 1998; Stake, 1995). Thereafter analysing and interpreting the collected data can begin. Stake (1995) advocates four forms of data analysis and interpretation in case study research: categorical aggregation; direct interpretation; patterns; and naturalistic generalisations. According to Stake (1995, p. 74), case study relies on the two methods, direct interpretation and categorical aggregation, to reach new meanings about cases. In the former method, the researcher concentrates on the single instance, trying to pull the case apart and put it back together again in order to draw meaning from it (Stake, 1995, p. 75), while the latter method involves examining a collection of instances, and expecting from the aggregate that issue-relevant meanings to emerge (Stake, 1995, p. 75). The next form of analysis and interpretation is the search for patterns, for consistency within certain conditions which Stake (1995, p. 78) calls ‘correspondence’. There are times when the patterns may be known in advance, drawn from the research questions, and serve as a template for the analysis. At other times, the patterns may emerge unexpectedly from the analysis. However, during this stage of analysis the researcher will look for ‘correspondence’ in the data which might take the form of a table showing the relationship between two or more categories. If there exist strong patterns (correspondence) in the data, the researcher will use these patterns to make assertions in his/her report. The final form of analysis and interpretation is the 'naturalistic generalisation' (Stake, 1995, p. 85), which are conclusions arrived at through the researcher's personal engagement in the research's affairs or by vicarious experience so well constructed that the researcher feels as if it has happened to himself/herself. In this study, some of the stages of the think aloud protocol analysis reported in Green (1995, p. 127) and those recommended by Stake (1995) are described below.

The initial stage of analysis involved transcription of the material of the video-tapes as well as incorporation of any non-verbal (i.e. using the mouse or typing statements into the computer screen) and para-linguistic (i.e. colloquial expressions used amongst Singaporeans) communication. The second stage involved partitioning the protocols into macroscopic chunks of consistent behaviours called episodes. According to Schoenfeld (1985, p. 292), an episode is a period of time during which an individual or problem solving group is engaged in one large task or a closely related body of tasks in the service of the same goal. These episodes were taken from the modified Artzt and Armour-Thomas' (see section 5.2.1 and Appendix G) framework. The third stage involved composing a summary of the 'story' being told by the episodes in the think aloud protocol of the N3 word problem, with the help of timeline representations (see Chapter Six, section 2). The next stage was delineating the units of meaning. This involved organising and compressing an assembly of information that permits conclusion drawing and/or action taking (Huberman and Miles, 1998). Dyads' word problem solving behaviours were mapped onto the modified Artzt and Armour-Thomas' cognitive-metacognitive word problem solving model (see Figure 4.1) and display tables, that tabulated the time (and %) dyads devoted to word problem solving activities (see Chapter Six, section 3 and 4), were developed to '*see a reduced set of data as a basis for thinking about its meanings*' (Huberman and Miles, 1998, p. 180).

Determining the patterns arising from clusters of students' progression of word problem solving activity marked the start of the second level of analysis (see Chapter Six, section

4). Related patterns were then clustered into five relevant types of cognitive-metacognitive word problem solving models that emerged from the think aloud protocol analysis and display tables. The final stage was writing a composite summary of dyads' progression of word problem solving activity for the N3 word problem (see Chapter Six, section 4). This involved a third level of analysis. It involved providing a description of the students involved, including an account of their metacognitive knowledge; the setting where the intervention was carried out; the intervention procedure; and the instruments used in the case study (see Chapter Three, section 3.2; Chapter Four, sections 2, 3 and 4; and Chapter Six, section 5). As mentioned in Chapter Three, section 2, I am aware that the five types of cognitive-metacognitive word problem solving models that represent students' progression of word problem activity cannot be generalised to the population but they provide a theoretical inference to be made with regard to the role of metacognition in word problem solving.

The reminder of this section consists of subsections. First, in subsection 5.2.1, the modified Artzt and Armour-Thomas' framework (see Appendix G) used in analysing the think aloud protocol data is introduced. This is followed by an introduction to the modified Artzt and Armour-Thomas' word problem solving model (see Figure 4.1) whereby the phases of students' word problem solving will be classified. Then, in subsection 5.2.2, there is a detailed account of how the students' think aloud protocol data was processed and categorised into episodes based on the modified Artzt and Armour-Thomas' framework. An example of this process will be illustrated. Subsection 5.2.2 also describes how the inter-coder reliability check was established during data analysis. Finally, subsection 5.2.3 discusses the rationale for using numerical data as part of the analysis for the case study design in the form of data display tables.

5.2.1 Modification of Artzt and Armour-Thomas' Framework for Analysing Think Aloud Protocol Data

Schoenfeld (1983) devised a scheme of parsing protocols into episodes (see section 5.2) and executive decision points (i.e. transition) (see Chapter Three, section 4.2). The executive decision points served as the mechanisms by which the problem solving process was kept on track. Although his framework focuses on the points at which metacognitive decisions may be considered and on their importance in the problem solving process, Schoenfeld does not specify in the analyses of protocols the cognitive levels of the episodes. As mentioned in Chapter Three, sections 4.4 and 4.5, Schoenfeld's episodic framework is limited in that it is not able to explicitly delineate the levels of cognitive behaviours; and it is not able to demonstrate the length of time students devote to their word problem solving behaviours in different episodes, which appears to be critical in their word problem solving outcome. In this present study, in order to examine the influence metacognitive training has on students, it appears crucial to be able to delineate explicitly the type and level of cognitive processes higher achievers and lower achievers use and to understand the mechanisms by which these processes facilitate word problem solving. This subsection describes a framework, a modified version of Artzt and Armour-Thomas' framework (1992), which is able to analyse the think aloud protocol data in that specific manner. See Appendix G for the descriptions of the categories in Artzt and Armour-Thomas' original framework and the modified framework which is used in the study.

In this study, a modified version of Artzt and Armour-Thomas' (1992) framework for the think aloud protocol analysis of word problem solving in mathematics was used to analyse the students' word problem solving data. The Artzt and Armour-Thomas' framework was designed to differentiate explicitly between cognitive and metacognitive problem solving behaviours observed within different episodes of problem solving in small group settings. The framework is a synthesis of the problem solving steps identified in mathematical research by Garofalo and Lester (1985), Polya (1973) and Schoenfeld (1985), and of cognitive and metacognitive levels of problem solving behaviours studied within cognitive psychology, in particular, by Flavell (1981). In the original Artzt and Armour-Thomas' framework (1992), Schoenfeld's scheme was used as a foundation for the development of the framework. According to Artzt and Armour-Thomas (1992), Schoenfeld's scheme aims to highlight major strategic decisions, suggest when they should have been made (if absent) and assess the quality of the decisions per se. Each protocol in Schoenfeld's scheme is parsed into macroscopic episodes, representing periods of time during which the subjects are engaged in distinctive types of problem solving behaviour. They are reading, analysis, exploration, planning/implementation, verification, and transition. As noted in Chapter Three, section 4.4, Schoenfeld's framework has its limitations. Its critical limitation is that the framework would not allow Schoenfeld (1985, p. 293) to address the important role that consistent monitoring and evaluation of solutions play in the problem solving process. Artzt and Armour-Thomas amended Schoenfeld's framework to address this issue.

The first modification to Schoenfeld's framework in Artzt and Armour-Thomas' framework was to separate the episode of plan/implement into two distinct episodes. This was because the two episodes did not always occur sequentially. According to Artzt and Armour-Thomas (1992), a student quite often proposed a plan that was immediately rejected by his/her peer. In these cases, implementation did not occur. Second, Artzt and Armour-Thomas (1992) expanded Schoenfeld's framework to include 'understanding the problem' and 'watching and listening'. According to them, the frequent comments students made regarding the conditions of the problem, recognised by Polya as important, provided a valid reason for its inclusion in the framework. In addition, as in small group settings, verbal interaction taking place implied that at certain times students were watching and listening to one another. This resulted in having eight problem solving episodes in the Artzt and Armour-Thomas' framework: read, understand, analyse, explore, plan, implement, verify, and watch and listen. The third modification from Schoenfeld in Artzt and Armour-Thomas' framework involved categorising the eight problem solving episodes as cognitive or metacognitive. As mentioned in Chapter Two, section 2.1, a definition to distinguish between cognition and metacognition can be taken from Garofalo and Lester's (1985, p. 164) definition, '*Cognition is involved in doing, whereas metacognition is involved in choosing and planning what to do and monitoring what is to be done*'. This means that, according to Artzt and Armour-Thomas (1992, p. 141), '*metacogitive behaviours could be exhibited by statements made about the problem or about the problem solving process while cognitive behaviours could be exhibited by verbal or non-verbal actions that indicated actual processing of information*'. This distinction between cognitive and metacognitive actions concurs with those of Flavell (1981). Table 4.5 shows the categorisation of the episodes according to their predominant cognitive level (Artzt and Armour-Thomas, 1992, p. 142). A rationale for these categories follows.

Episode	Predominant Cognitive Level
Read	Cognitive
Understand	Metacognitive
Analyse	Metacognitive
Explore	Cognitive or Metacognitive
Plan	Metacognitive
Implement	Cognitive or Metacognitive
Verify	Cognitive or Metacognitive
Watch and Listen*	

* Level not assigned

Table 4.5 : Categorisation of Episodes

According to Artzt and Armour-Thomas (1992), episodes of reading are predominantly cognitive because they exemplify instances of doing; episodes of understanding are predominantly metacognitive because this category is assigned only when students made comments that reflected attempts to clarify the meaning of the problem; and episodes of analysing and planning are, by their nature, metacognitive behaviours. This is because according to Schoenfeld (1985, p. 298),

'In analysis an attempt is made to fully understand the problem, to select an appropriate perspective and reformulate the problem in those terms, and to introduce for considerations whatever principles or mechanisms might be appropriate. The problem may be simplified or reformulated.'

Statements made about the problem or problem solving process reveal such thought processes, thereby making them metacognitive. Similarly, episodes of planning are revealed by statements made about how to proceed in the problem solving process. Behaviours coded as exploring, implementing and verifying can sometimes be categorised as cognitive and sometimes as metacognitive. An example of exploration which was cognitive was described by Schoenfeld (1987) where he documented that exploration at the cognitive level alone often resulted in an unchecked 'wild goose chase' (op cit p. 210). When exploration is guided by the monitoring of either oneself or peer, that behaviour could be categorised as exploration with metacognition. As a consequence of such monitoring either self or peer regulation of the exploration process could occur thereby keeping the exploration controlled and focused. The same analysis applies to implementation and verification, which may occur with or without monitoring and regulation.

Amendments were made to Artzt and Armour-Thomas' framework to serve the purpose of the present study. According to Artzt and Armour-Thomas' original framework, there is a distinction between 'understanding the problem' and 'analysing the problem' and their distinctiveness is explicated in Appendix G. However, I want to draw on Schoenfeld's observation - that is, in analysing a problem, '*an attempt is made to fully understand the problem*' (Schoenfeld, 1985, p. 298). This appears to imply that in analysis, students also make attempts to understand the word problem. Hence, the descriptions for the categories 'understanding the problem' and 'analysing the problem' in Artzt and Armour-Thomas' framework are recategorised as descriptors for the category 'analysing the word problem' in the modified Artzt and Armour-Thomas' framework (see Appendix G). In the present study, students were paired to elicit think aloud protocol data during word problem solving. It appeared that continual verbal interactions were evident between students during word problem solving. Each student was also involved in either typing or controlling the mouse. Hence, the category 'watch and listen' was not appropriate in pair protocols.

Figure 4.1, a modification of Artzt and Armour-Thomas' (1992, p. 143) cognitive-metacognitive model, illustrates the variety of sequences of behaviour that could occur during word problem solving. This model will be used to examine the word problem solving sequence of students' word problem solving behaviour in Chapter Six, section 4.

5.2.2 Processing the Think Aloud Protocol Data

As presented in section 5.2, the data analysis of the think aloud protocols in the case study involved some of the stages recommended by Green (1995) and Stake (1995). The following is a presentation of the stages I took while analysing the think aloud protocol data.

After video-recording each dyad's word problem solving on eight word problems (see Appendix C), the think aloud protocols were transcribed and then analysed. I adopted the transcribing conventions suggested by Silverman (1993, p. 118) to transcribe the students' word problem solving think aloud protocols.

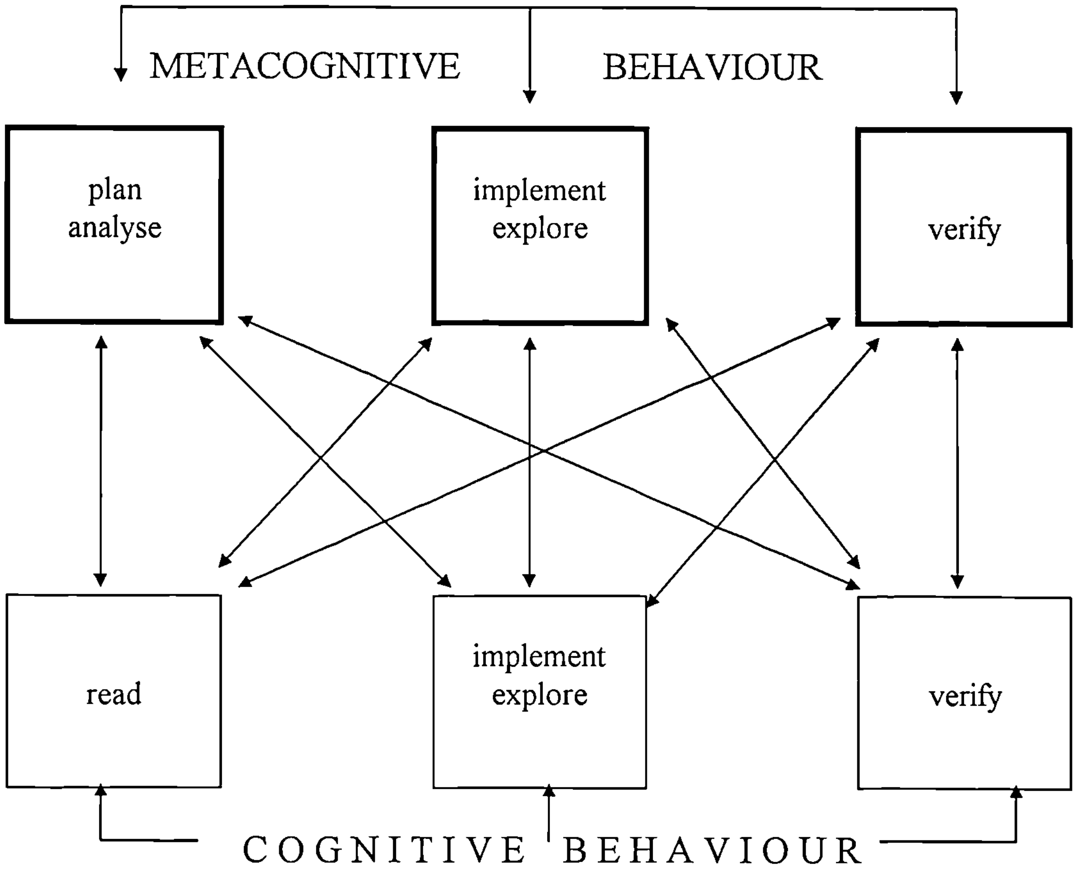


Figure 4.1: A Modified Artzt and Armour-Thomas' Cognitive-metacognitive Word Problem Solving Model

After that, the students' word problem solving transcripts were segmented and encoded (Green and Gilhooly, 1996, p. 60) according to the episodes in the modified Artzt and Armour-Thomas' framework as described in section 5.2.1 (see also Appendix G). Coding reliability (Green and Gilhooly, 1996, p. 62) was checked by a research assistant from the School of Education, who coded 11 protocols from higher achievers and lower achievers of the treatment and control dyads (see Chapter Four, section 2.1), chosen at random. The level of inter-rater agreement on the category validation was 86% which is within acceptable limits (Green and Gilhooly, 1996, p. 62). The purpose of the check was to give confidence that the categories from the modified Artzt and Armour-Thomas' framework are viable and that I was consistent in applying them. I was then entitled to use my judgement on all the transcripts for the purpose of the analysing the think aloud protocol data.

After the think aloud transcripts were coded into episodes (see section 5.2.1), diagrams (see Chapter Six, section 2) called timeline representations were drawn to map the flow of dyad's thinking in word problem solving against the episodes in the viable framework (see Appendix G). These diagrams provided a starting point for describing the dyad's full word problem solving think aloud protocol (see Chapter Six, section 2 and Appendix H). In addition, each dyad's word problem solving behaviour from the four word problems, N2, N3, N4 and F2 (see Appendix C), were mapped onto the modified Artzt and Armour-Thomas' cognitive-metacognitive word problem solving model (see Figure 4.1). The purpose of mapping the dyad's word problem solving behaviour onto the model is to examine possible emerging patterns that might arise from each dyad's progression of word problem solving activity. Only four of the eight word problems, the N2, N3, N4 and F2 (see Appendix C) were used for analysis because of the following reasons. The word problems N1 and F1 were not chosen because their purpose was to make the dyads feel at ease at the beginning of each video-recording sessions in posttest 2 (see Chapter Four, section 4.2) before they moved on to solve more 'difficult' word problems. The word problems F3 and F4 were also not chosen based on my observation that, while dyads solved these word problems, they appeared to have many difficulties and most of them exceeded the 25 minutes' time limit. Hence, the word problems N2, N3, N4 and F2 are appropriate for analysing students' word problem solving behaviours as they appear to be more representative of how dyads would normally solve word problems during word problem solving. Data display tables were then tabulated to compare the time taken devoted to metacognitive and cognitive behaviours by dyads from different conditions (see Chapter Six, section 3). With the help of the modified Artzt and Armour-Thomas' cognitive-metacognitive word problem solving models (Figure 4.1) and tabulated display tables based on the four word problems, I was able to establish a unique progression of word problem solving activity for each dyad (see Chapter Six, section 4). Focusing on each dyad's unique progression of word problem solving activity, I was able to cluster dyads, who had similar progression of word problem solving activity, into five different clusters and labeled them as Type P, Type Q, Type R, Type S and Type T cognitive-metacognitive word problem solving models (see Appendix L and Chapter Six, section 4). These emergent progressions of students' word problem solving activity from students' think aloud protocol data shed insights to the relationship between the role of metacognition and its influence on students' word problem solving performance (see Chapter Seven, section 3.3).

The following example shows the analysis procedure applied to the word problem solving protocols. Here and in Chapter Six, section 2, analysis of the N3 word problem has been chosen because, in my opinion, it best illustrates the students' typical metacognitive behaviours and collaborative style. The word problem context is as follows:

N3 *Joe Ee, Mun Fai and Jing Hao shared 400 marbles amongst themselves. Joe Ee received 28 marbles. Jing Hao received seven times the total number of marbles Joe Ee and Mun Fai received. How many more marbles did Jing Hao receive than Mun Fai?*

Although this word problem context resembles some textbook questions, it has an added complication, that is to find the number of marbles Jing Hao and Mun Fai each had, based on the condition that Jing Hao received seven times the total number of marbles Joe Ee and Mun Fai received. A successful solution requires careful identification of the goal, and working backwards to establish sub-goals and suitable strategies for achieving them. Before describing the word problem solving behaviours of the students and giving actual protocol examples, using the framework described in section 5.2.1 (see Appendix G), the following is a presentation of an outline of several approaches that could be used (many of which the students used) to solve this word problem. The following presentation also serves to illustrate the different episodes and cognitive levels which might be categorised in the students' think-aloud protocols. In the actual transcripts (see example in Appendix H), 'items' (see also section Chapter Six, section 2) are used to define complete statements or phrases students articulate when solving the word problem.

Episode 1: Reading the problem (cognitive).

The student read or listen to his/her peer read the word problem.

Episode 2: Analysing the problem (metacognitive).

Student attempts to analyse the word problem can be done in many ways. For example:

1. The meaning of the word problem statement '*Jing Hao received seven times the total number of marbles Joe Ee and Mun Fai received*' can be clarified.

2. A model diagram can be drawn to represent all the word problem conditions. In this case, the number of Jing Hao's marbles will be represented by 7 units and the total number of Joe Ee and Mun Fai's marbles will be represented by 1 unit. Within the 1 unit, Joe Ee has 28 marbles. All the 8 units represent 400 marbles.
3. They can make an attempt to simplify the word problem by making subgoals. For example, they might see that it is essential to find the total number of Joe Ee and Mun Fai's marbles (1 unit) first before they can find the number of Jing Hao's marbles.

Episode 3: Planning (metacognitive).

If the students attempt to plan an approach for solving the word problem, there are many ways it can be done. For example:

1 a) Divide 400 into 8 parts to find the value of 1 part.
   b) Subtract the value of 1 part and 28 to find the number of Mun Fai's marbles.
   c) Multiple the 1 part by 7 to find the number of Jing Hao's marbles.
   d) Take the difference of Jing Hao and Mun Fai's marbles.

2 a) Multiple 28 by 8.
   b) Subtract 400 and the value of a)
   c) Take the value of b) and divide by 8 to find the number of Mun Fai's marbles.
   d) Subtract 400 and the sum of Mun Fai and Joe Ee's marbles to find the number of Jing Hao's marbles.
   e) Take the difference of Jing Hao and Mun Fai's marbles.

3. Make a chart using headings of number of Jing Hao's, Joe Ee's (28) and Mun Fai's marbles and the total number of marbles (400). Start with one heading first and then continue adding the number of marbles until Jing Hao has 7 times the total number of Joe Ee and Mun Fai's marbles and their total number of marbles equals 400.

Episode 4: Exploration (cognitive and metacognitive)

If the students monitor or regulate the progress of their attempted actions, which are further removed from the original word problem, they are exploring (metacognitive). If, however, the student is merely making a series of random guesses or continuously using a flawed strategy, the student is embarking on unmonitored exploration (cognitive) that is unlikely to result in a solution.

Episode 5: Implementation (cognitive and metacognitive)

If the students have devised a plan for solving the word problem, they are likely to try to implement the plan. If they do this systematically by monitoring and regulating the implementation (metacognitive), the student is likely to find that the plan either is appropriate and will lead them towards a solution or is not appropriate and will lead them to terminate the implementation and try to devise another plan. If the implementation is not monitored (cognitive alone) but is focused on calculations, it may be successful. However, they might become too involved in a poor plan that is unlikely to lead to a solution.

Episode 6: Verification (cognitive and metacognitive)

For an effective verification to take place, the students must be able to take their final solution and check that the total number of Jing Hao, Joe Ee and Mun Fai's marbles is 400, and that the number of Jing Hao's marbles is seven times the number of Joe Ee and Mun Fai's marbles. This process entails manipulating the numbers (cognitive) and monitoring the results to check that they meet the conditions of the word problem (metacognitive).

5.2.3 Rationale for Using Numerical Data in Analysing Think Aloud Protocol Data

After the students' think aloud protocols were categorised into episodes (see section 5.2.2), the time in seconds (and %) students devoted to the cognitive and metacognitive behaviours in the modified Artzt and Armour-Thomas' framework was tabulated in data display tables (Huberman & Miles, 1998). This sub-section discusses the rationale for using numerical data as part of the qualitative analysis. It aims to show that numerical data represented by data display tables adds an important dimension to the broader description of patterns of students' word problem solving behaviours.

Debates in approaches to educational research appear to assume that there are two major paradigms between 'quantitative' and 'qualitative' research. One distinction made between quantitative and qualitative approaches has to do with the type or nature of data, the kind of procedure used for data collection and data analysis. The most commonly used procedures associated with a hypothesis-testing approach is that of experimental methods, surveys and questionnaires which tend to yield 'hard, reliable' data which are numeric in nature and are often reported in descriptive and inferential statistics. On the other hand, an exploratory, hypothesis-generating approach is associated with observations, interviews and other more naturalistic procedures of data collection which tend to yield 'rich, deep', non-numeric data and are often reported in verbal protocols.

I wish to argue that these distinctions are not as clear cut as they appear to be and that it is not helpful to divide these two approaches based on numeric or non-numeric statistical data. Silverman (1993), Miles and Huberman (1998), Behrens and Smith (1996) and others argue that whether one uses numbers or words to ascribe properties to the data is not the main issue because numbers or words are symbols that have underlying referents, and in both cases a whole series of inferences and arguments connect the referents to the symbol. Instead, it is the actual properties we are interested in. Qualitative researchers like Miles and Huberman (1998), and Silverman (2000) assure us that the presentation of numbers does not disqualify a study from being qualitative in nature. Silverman (2000, 185) argue that simple counting techniques based on categories can offer a means to survey the whole corpus of data ordinarily lost in intensive qualitative research. Instead of taking the researcher's word for it, the reader has a chance to gain a sense of the flavour of the data as a whole. Hence, quantification can supplement, extend and enhance qualitative analysis (Ely et al, 1997, p. 194). Ely et al (1991) succinctly summarises the whole issue, and argue that the crux of the issue is whether the qualitative/quantitative researcher has managed to shed light on his/her decision-making process in establishing findings.

In exploring the role of metacognition during students' word problem solving and investigating how the differences in higher and lower achievers' word problem solving behaviour relate to their mathematical word problem solving performance, the main procedure used in this study was students' think-aloud protocols. Video-recording students' think aloud protocols while solving word problems constitute the data collection. A careful, rigorous and detailed analysis of the think aloud protocols using the modified Artzt and Armour-Thomas' word problem solving framework is used in order to trace and describe the trends and typicality of the phenomenon under investigation. This was accomplished across the dyad's word problem solving. In Chapter Three, section 4.5, I noted that my overall impression was that the length of time students devoted to episodes in Schoenfeld's episodic framework appeared to relate to their word problem solving outcome. This apparent relationship is crucial in the field study in identifying how episodes that are metacognitive and cognitive in nature relate to students' word problem solving using the present modified Artzt and Armour-Thomas' framework. The 'crude' quantitative data I am going to record (see Chapter Six, section 3) will not allow any real test to make major conclusions. Nonetheless, it will offer a summary measure and to shed insights on the phenomenon being studied. The results of data display tables are presented and described in Chapter Six, section 3.

6. **Conclusion**

This chapter has provided a picture of how the research was implemented in two Singapore schools over a period of five months and how subsequently the data from the research was analysed. The various issues with regard to sampling were recognised and documented and they will be taken into account when the results of this study are presented and discussed. This chapter has also raised the issue of the data analysis methods used in the quantitative and qualitative studies. For the quantitative study, I have proposed that two statistical techniques are appropriate to analyse the quantitative data because these techniques appear to be able to reduce potential biases caused by selection differences. For the qualitative study, a systematic procedure recommended by Stake (1995) and Green (1995) is felt to be appropriate for the case study design in this research. In addition, I have argued that using numerical data to analyse the qualitative data is legitimate because it is also a procedure to present in-depth understanding of students' word problem solving. I have also argued that Schoenfeld's episodic framework initially used in the analysis of the data in the preliminary and pilot studies is not sufficient and have presented a modified version of Artzt and Armour-Thomas' framework and their modified cognitive-metacognitive model of phases of word problem solving to be more suitable in the present study. In Chapters Five and Six, the results of the quantitative and qualitative data analysis will be presented and described, and these findings will be discussed in Chapter Seven.

Chapter Five
Presentation and Description of Quasi-Experimental Data

1. Introduction and Overview of Chapter

In Chapter Four, the quasi-experimental designs (see Chapter Four, Tables 4.1 and 4.2) were explained how the hypotheses related to metacognitive training, students' levels of mathematical achievement and mathematical word problem solving performance were tested. The rationale for using these two quasi-experimental designs to analyse the quantitative data was provided (see Chapter Four, section 2.1). An account was also provided for the sampling method, the collection of quantitative data in the form of mathematical achievement tests, the processing of the quantitative data before analysis, and the techniques used to analyse the mathematical achievement test scores during pretest, posttest 1 and delayed posttest 1 (see Chapter Four, section 4.2). In this chapter, the results of the quantitative analysis will be presented in three main sections. In section 2, the research questions and their hypotheses for the study are listed. Section 3 then describes in detail how the quantitative data in quasi-experimental design 1 (see Chapter Four, section 2.2.1 a) was processed and analysed using repeated measures three-way analysis of variance. This is followed in section 4, with a detailed description of how the quantitative data in quasi-experimental design 2 (see Chapter Four, section 2.2.1 b) was processed and analysed using repeated measures two-way analysis of covariance. In section 5, Pearson product-moment correlation was employed to provide possible answers for research question 2 (see section 2.1). Finally, section 6 provides a summary of the analysis of the quantitative data in the quasi-experimental designs based on the research questions 1, 2 and 3 (see section 2.1).

2. Summary Description of the Research Design

The sampling method was described in Chapter Four. This section is a review of the sampling method, and how the quantitative data was collected, processed and analysed. A brief account of the intervention process will also be given.

Two quasi-experimental designs were developed so that statistical techniques could be employed to test the hypotheses (see Chapter Five, section 2.1) generated for the study despite threats to validity arising from sampling (see Chapter Four, section 2.2). The first quasi-experimental design (see Chapter Four, section 2.2.1 a, and Appendix K) involved blocking the students according to *the school factor. The students in the treatment and* control classes in School 1 and School 2 were ranked and stratified into higher and lower achiever groups. A total of 40 students were assigned to the different conditions in quasi-experimental design 1 (see Chapter Four, Table 4.1).The second quasi-experimental design (see Chapter Four, section 2.2.1 b, and Appendix K) involved assigning the students from the two classes in each school to either treatment or control conditions. The students from the treatment classes in School 1 and School 2 together were stratified into higher and lower achiever groups. The stratification process was repeated for the students from the control classes in School 1 and School 2 together. A total of 100 students were assigned to the different conditions *in quasi-experimental* design 2 (see Chapter Four, Table 4.2).

The intervention process (see Chapter Four, section 4 and Table 4.4) started in January 1999. Students in the treatment and control classes in School 1 were given a pretest before the training sessions. The training session lasted for two weeks. After the training sessions, posttest 1 was administered to the treatment and control classes. Six weeks after posttest 1, delayed posttest 1 was administered to the treatment and control classes. In both posttest 1 and delayed posttest 1, the students solved the word problems individually. The research study in School 1 ended in mid March 1999. The above procedure was also applied to the treatment and control classes in School 2, starting in mid March 1999 and ending at the end of May 1999.

2.1 Research Questions and their Hypotheses

As stated in Chapter Three, one of the objectives in this study was to investigate possible relationships between metacognitive training, students' level of mathematical achievement and their mathematical word problem solving performance, and a quantitative approach was considered suitable for this purpose. The following questions and hypotheses were developed to provide possible answers for the relationships between metacognitive training, students' level of mathematical achievement and their mathematical word problem solving performance.

Research Question 1: What effect will metacognitive training have on performance on mathematical word problem solving?

Hypothesis A: Metacognitive training will result in a greater improvement in performance. (It is assumed that performance will increase as a result of instruction.)

Research Question 2: To what extent is improving performance on mathematical word problem solving (as a result of cognitive apprenticeship instruction) related to the mathematical achievement of the student?

Hypothesis B: Lower achievers will benefit more from cognitive apprenticeship instruction (i.e. lower achievers in the treatment and control groups will increase their mathematical word problem solving performance more compared to the higher achievers in either groups).

Research Question 3: Do the effects of metacognitive training on performance on mathematical word problem solving vary with the mathematical achievement of the student?

Hypothesis C: Metacognitive training will improve the relative performance of lower achievers more than higher achievers (i.e. the improvement in performance allowing for the differential effects of instruction on the two groups.)

A repeated measures three-way analysis of variance and a repeated measures two-way analysis of covariance were employed to test the above hypotheses. In each case, the null hypothesis is tested by forming an F-ratio (Henkel, 1976, p. 64), and the null hypothesis will be rejected if the probability of the null hypothesis being true is less than 0.05.

In the following two sections, sections 3 and 4, the statistical analyses employed to test the above hypotheses will be described and the results generated will be presented. From here on in this chapter, 'posttest 1' is referred to as 'posttest' and 'delayed posttest 1' is referred to as 'delayed posttest'.

3. Analysis of Quasi-experimental Design 1

3.1 Theoretical Considerations

According to Ferguson and Takane (1989, p. 251), in its simplest form, the analysis of variance is used to test the significance of the differences between the means of a number of different populations. For example, I may wish to test the effects of *k* treatments. A different treatment is applied to each of the *k* samples, each sample having *n* members. In an ideal experiment, the members are assigned to treatments at random. The means of the *k* samples are calculated. The null hypothesis is formulated that the samples are drawn from populations having the same mean. Assuming that the treatments applied are having no effect, some variation due to sampling fluctuation is expected between means. If the variation cannot reasonably be attributed to sampling error, I would reject the null hypothesis and accept the alternative hypothesis that the treatments applied are having an effect.

In this study, a repeated measures three-way analysis of variance, employed in quasi-experimental design 1, is generated to analyse the mathematics achievement test data and draw conclusions from them for comparing the effect of students' learning of explicit metacognitive training using CRIME, against students' learning without explicit metacognitive training. It involves three independent variables where one of the variables, the school variable, is blocked (see Chapter Four, section 2.2.1 a). The three independent variables are:

1. School: School 1 (S1) versus School 2 (S2);
2. Metacognitive training:
   a) treatment, where students had metacognitive training in a computer environment (T) versus
   b) control, where students did not have metacognitive training in a computer environment (C); and
3. Mathematics achievement: higher achievers (HA) versus lower achievers (LA).

The dependent variable is the number of correct answers on a mathematical achievement test (see Appendix C) in the different conditions measured before, immediately after and six weeks' after the experimental treatment. For the repeated measures three-way analysis of variance, the assumptions are that the distribution of the dependent variable, the mathematical achievement test scores, is normal within students of each condition, and the variation in word problem solving performance on the tests comes from the following sources:

- 'error' plus
- the main effect of school, plus
- the main effect of metacognitive training, plus
- the main effect of mathematical achievement, plus
- the interaction of school and metacognitive training, plus
- the interaction of school and mathematical achievement, plus
- the interaction of metacognitive training and mathematical achievement, plus
- the interaction of school, metacognitive training and mathematical achievement.

The formal model, model 1, for a repeated measures three-way analysis of variance (Iversen & Norpoth, 1976, p. 33) of the *l*-th observation in the cell defined by the i-th row for the school factor, the j-th column for the metacognitive training factor, and the k-th layer for the mathematics achievement factor, y_{ijkl}, can be written as the following sum

$$y_{ijkl} = \mu + a_i + b_j + c_k + d_{ij} + e_{ik} + f_{jk} + g_{ijk} + \epsilon_{ijkl} \qquad [1]$$

This equation can be read as follows. The test score for student *l* in group i,j,k, y_{ijkl}, equals the mean score of all students in the entire population, μ, plus three main effects, a_i, b_j, and c_k, plus three 2-way interaction effects, d_{ij}, e_{ik}, and f_{jk}, plus a 3-way interaction effect, g_{ijk}, plus an error element, ϵ_{ijkl}, that represents the idiosyncrasies of the particular student or errors arising from uncertainties in the measurement.

3.2 Results

Table 5.1 shows the means and standard deviations of the quasi-experimental design 1 described in Chapter Four, section 2.2.1 a.

	Metacognitive Training	Mathematical Achievement		Pretest Scores	Posttest Scores	Delayed Posttest Scores
School 1	Treatment	HA	Mean	2.00	5.20	6.60
			N	5	5	5
			Std Deviation	2.12	1.64	2.07
		LA	Mean	1.20	1.6	2.40
			N	5	5	5
			Std Deviation	1.10	2.07	2.07
	Control	HA	Mean	2.60	3.60	5.00
			N	5	5	5
			Std Deviation	1.67	1.82	1.58
		LA	Mean	0.60	0.80	0.60
			N	5	5	5
			Std Deviation	0.89	0.45	0.55
School 2	Treatment	HA	Mean	8.00	9.20	9.60
			N	5	5	5
			Std Deviation	1.58	0.84	0.55
		LA	Mean	6.00	6.80	7.80
			N	5	5	5
			Std Deviation	1.58	1.30	0.84
	Control	HA	Mean	7.40	7.80	7.00
			N	5	5	5
			Std Deviation	0.89	1.48	1.00
		LA	Mean	2.40	3.00	3.40
			N	5	5	5
			Std Deviation	1.67	1.22	2.07

Table 5.1: Summary Table of the Means and Standard Deviations of Quasi-experimental Design 1

Visual inspection of these results show all groups improving in word problem solving as a result of cognitive apprenticeship instruction (training in WordMath), and almost all groups continue to improve after the instruction is ended – that is there is an increase in success between the posttest and delayed posttest. Statistical techniques will be applied to determine whether the improvements in word problem-solving performance are significantly greater for some groups compared with others.

The Kolmogorov-Smirnov test confirmed that the distribution of the quasi-experimental design 1 test scores may be regarded as being normally distributed: pretest (p = 0.206), posttest (p = 0.721) and delayed posttest (p = 0.509). Table 5.2 is a summary table of the output from the repeated measures three-way analysis of variance employed in quasi-experimental design 1.

Source of Variation	Sums of squares	Degrees of freedom	Mean square	F-ratio	Significance
School (Block)	444.675	1	444.675	114.755	0.0001
Metacognitive training	102.675	1	102.675	26.497	0.0001
Mathematical achievement	291.408	1	291.408	75.202	0.0001
School by Metacognitive training	23.408	1	23.408	6.041	0.020
School by Mathematical achievement	0.675	1	0.675	0.174	0.679
Metacognitive training by Mathematical achievement	12.675	1	12.675	3.271	0.080
School by Metacognitive training by Mathematical achievement	9.075	1	9.075	2.342	0.136
Error	124.000	32	3.875		

Table 5.2: Summary Table of the Repeated Measures Three-way Analysis of Variance

The interpretation of the repeated measures three-way analysis of variance (shown in Table 5.2) is as follows:

1. School (as anticipated by blocking) has a significant influence on the students' word problem solving performance (F = 114.755, p<0.05).

2. Metacognitive training has a significant influence on the students' word problem solving performance (F = 26.497, p<0.05). That is metacognitive training results in a greater improvement in mathematical word problem solving performance.

3. The level of students' mathematical achievement has a significant influence on the students' word problem solving performance (F = 75.202, p<0.05). Visual inspection

of the means of the higher and lower achievers in the pretest, posttest and delayed posttest scores in Table 5.1 indicates that the pretest, posttest and delayed posttest mean scores for the lower achievers, 2.55, 3.05 and 3.55 respectively, were lower than those for the higher achievers' pretest, posttest and delayed posttest mean scores which were 5.00, 6.45 and 7.05 respectively. Figure 5.1 below shows the difference between the higher and lower achievers' pretest, posttest and delayed posttest mean scores.

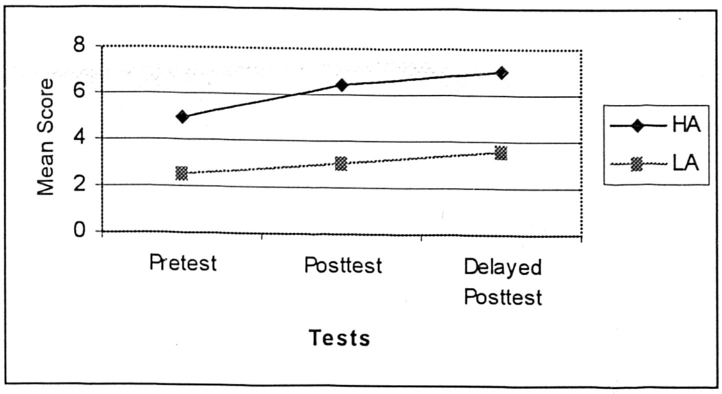


Figure 5.1: Graph of Higher and Lower Achievers' Mean Test Scores

Figure 5.1 indicates that higher achievers appear to outperform lower achievers in mathematical word problem solving. To examine whether higher achievers benefit more from cognitive apprenticeship instruction compared with lower achievers, a correlation, the Pearson product-moment correlation (Howitt & Cramer, 1997, p. 62), between the students' level of mathematical achievement and their gain mean score between pretest and posttest, pretest and posttest, and posttest and delayed posttest are determined. This account will be presented and described in section 5.

4. The School by Metacognitive training interaction is also significant ($F = 6.041$, $p<0.05$). Figure 5.2 illustrates the School by Metacognitive training interaction. Figure 5.2 reinforces interpretation 1, in that it suggests that the treatment and control students in School 2 tend to outperform the treatment and control students in School 1. This result was anticipated since the school factor was blocked. It also suggests that students in School 2 benefited more from metacognitive training than students in School 1.

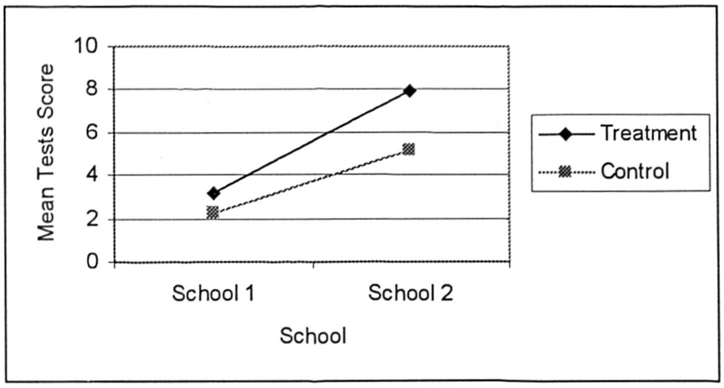


Figure 5.2: Graph of School by Metacognitive Training Interaction

5. There is not significant interaction between school and mathematical achievement ($F = 0.174$, $p > 0.05$). That is the difference in performance in mathematics word problem solving between higher and lower achievers in School 1 was not significantly different from the difference between higher and lower achievers in School 2.

6. There is not significant interaction at the 5% level between metacognitive training and mathematical achievement ($F = 3.271$, $p = 0.08$). The interaction is significant at the 0.1 level, however, and merits further consideration. Table 5.3 shows the average gain score percentage of the students' mathematical word problem solving performance.

	Treatment			Control		
	|P1-P2|	|P1-P3|	|P2-P3|	|P1-P2|	|P1-P3|	|P2-P3|
HA	44%	62%	18%	14%	20%	6%
LA	12%	30%	18%	8%	10%	2%

P1 : pretest mean; P2 : posttest mean; and P3: delayed posttest mean

Table 5.3 : Summary Table of the Average Gain Score Percentage

Visual examination of the cells in Tables 5.1 and 5.3 suggests that

a) higher achievers who had metacognitive training outperformed higher achievers who did not have metacognitive training (there is an average gain of 44% and 62% compared with 14% and 20%, see Table 5.3);
b) lower achievers who had metacognitive training also outperformed lower achievers who did not have metacognitive training (there is an average gain of 12% and 30% compared with 8% and 10%, see table 5.3);

c) higher achievers appeared to benefit more from metacognitive training than lower achievers based on the difference between the pretest and posttest means and the difference between the pretest and delayed posttest means. Figures 5.3, 5.4 and 5.5 illustrate the relationships between the gain in the mean scores (see Table 5.1) of the students in different conditions.

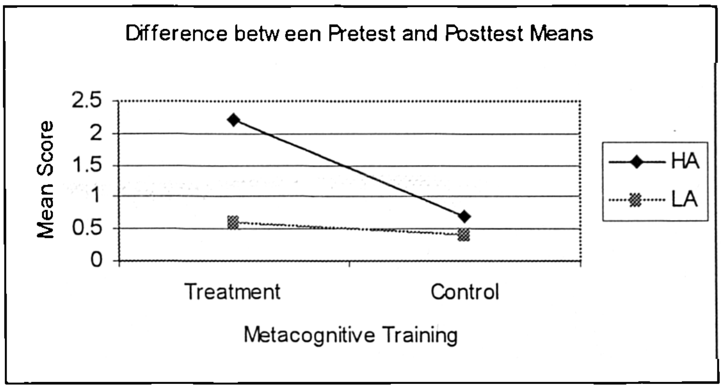


Figure 5.3 : Gain Mean Score of Students based on Pretest and Posttest Means

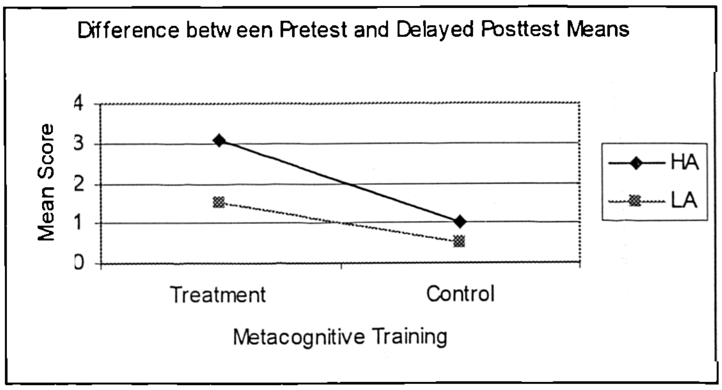


Figure 5.4: Gain Mean Score of Students based on Pretest and Delayed Posttest Means

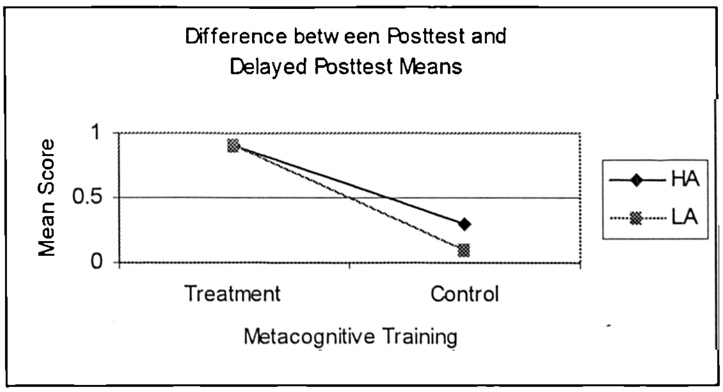


Figure 5.5 : Gain Mean Score of Students based on Posttest and Delayed Posttest Means

d) lower achievers appeared to show a greater delayed benefit from metacognitive training than do higher achievers, that is, the improvement between the posttest and delayed posttest in the treatment group was greater for the lower achievers. The observation, based on Figure 5.5, appears to be similar to the observation in the qualitative analysis whereby lower achievers who had metacognitive training showed an improvement in their mathematical word problem solving performance only after a prolonged period (see Chapter Six, section 3.2). This phenomenon will be discussed in Chapter Seven, section 2.2.

7. There is not significant interaction between school, metacognitive training and mathematical achievement ($F=2.342$, $p>0.05$).

4. **Analysis of Quasi-experimental Design 2**

4.1 **Theoretical Considerations**

Analysis of covariance is a statistical procedure that addresses priori differences between groups in an experiment (Jaeger & Bond, 1996, p. 893). Unlike blocking techniques, analysis of covariance controls extraneous variation through statistical adjustments, by adjusting the dependent variable for the effects of one or more continuous independent variables, the covariates, which usually are measured priori to the administration of the treatments. See Jaeger & Bond (1996, p. 893 to 894) for a fuller account of the analysis of covariance.

In this study, a repeated measures two-way analysis of covariance, employed in quasi-experimental design 2, is generated to analyse the mathematics achievement test data and draw conclusions from them for comparing the effects of students' learning of explicit metacognitive training using CRIME, against students' learning without explicit metacognitive training. It involves two independent variables (see Chapter Four, section 2.2.1 b)). The two independent variables are:

1. Metacognitive training:
   a) treatment, where students had metacognitive training in a computer environment (T) versus

b) control, where students did not have metacognitive training in a computer environment (C); and

2. Mathematics achievement: higher achievers (HA) versus lower achievers (LA).

The dependent variable is the number of correct answers on a word problem solving test (see Appendix C) in the different conditions measured before, immediately after and six weeks after the experimental treatment. In the repeated measures two-way analysis of covariance, the pretest is used as a covariate in the analysis of the quantitative data because it is a bias due to selection differences (see Chapter Four, section 2.2). The purpose of using the pretest as a covariate is to provide an adjustment for initial differences between the groups. According to Newton and Rudestam (1999, p. 222), analysis of covariance yields a more accurate estimate of treatment effects by correcting posttest scores using the regression between pretest and posttest. By reducing the variance of the dependent variable after the treatment effect is accounted for, analysis of covariance can improve statistical power and precision. Hence, the formal model, model 2, for the two-way analysis of covariance (Wildt & Ahtola, 1978, p. 19) with one covariate is represented as:

$$\begin{aligned} y_{ij} = \mu + \beta_{1j}(\text{training effect}) + \beta_{2j}(\text{achievement effect}) + \\ \beta_{3j}(\text{training by achievement interaction}) + \beta_{4j}(\text{covariate}) + \epsilon_{ij} \end{aligned} \qquad [2]$$

where y_{ij} is the observed value of the dependent variable for the j-th observation within the i-th treatment level, μ or intercept is the overall mean, β is the regression coefficient representing the average effect of a one unit change in the covariate on the dependent variable and ϵ_{ij} is the idiosyncrasies of the particular student or errors arising in uncertainties from measurements. This means that the observed value of dependent variables, y_{ij}, is equal to the intercept, μ, plus the effect of the metacognitive training factor, plus the effect of the mathematical achievement factor, plus the interaction effect of the metacognitive training and mathematical achievement factors, plus the effect of the covariate, plus the residual, ϵ_{ij}.

4.2 **Results**

Table 5.4 shows the unadjusted means and standard deviations of the quasi-experimental design 2 described in Chapter Four, section 2.2.1 b. Table 5.5 shows the adjusted means and their respective standard errors, of quasi-experimental design 2 after repeated measures two-way analysis of covariance was generated to adjust for the initial difference caused by selection differences.

The Kolmogorov-Smirnov test confirmed that the distribution of the residuals in quasi-experimental design 2 may be regarded as being normally distributed (Cook and Campbell, 1979, p. 153): (posttest mean = 4.396E-9, std deviation = 0.9847, p = 0.891); (delayed posttest mean = -1.06E-9, std deviation = 0.9847, p=0.833) and it is therefore reasonable to assume that the residuals are from the same population and the model (see model 2) satisfies the assumptions for analysis of covariance.

Metacognitive Training	Mathematical Achievement	Unadjusted	Pre-test Scores	Post-test Scores	Delayed Post-test Scores
Treatment	HA	Mean	4.56	5.88	7.12
		N	25	25	25
		Std Deviation	2.83	2.42	2.32
	LA	Mean	2.36	3.16	4.12
		N	25	25	25
		Std Deviation	2.41	2.58	2.55
Control	HA	Mean	3.40	4.04	5.04
		N	25	25	25
		Std Deviation	2.50	2.52	1.84
	LA	Mean	2.76	3.12	3.88
		N	25	25	25
		Std Deviation	1.54	1.67	1.74

Table 5.4: Summary Table of the Unadjusted Means and Standard Deviation of Quasi-experimental Design 2

Meta-cognitive Training	Mathematical Achievement	Adjusted	Post-test Scores	Delayed Post-test Scores
Treatment	HA	Mean	5.01	6.33
		N	25	25
		Std Error	0.35	0.33
	LA	Mean	3.77	4.68
		N	25	25
		Std Error	0.35	0.32
Control	HA	Mean	3.95	4.96
		N	25	25
		Std Error	0.34	0.32
	LA	Mean	3.46	4.19
		N	25	25
		Std Error	0.34	0.32

Posttest and delayed posttest means evaluated at covariate of pretest = 3.27

Table 5.5: Summary Table of the Adjusted Means and Standard Error of Quasi-experimental Design 2

Table 5.6 is a summary table of the output from the repeated measures two-way analysis of covariance with the pretest as a covariate.

Source of Variation	Sums of squares	Degrees of freedom	Mean square	F-ratio	Signi-ficance
Pretest	446.806	1	446.806	108.383	0.001
Metacognitive training	32.205	1	32.205	7.812	0.006
Mathematical achievement	49.011	1	49.011	11.889	0.001
Metacognitive training by Mathematical achievement	8.078	1	8.078	1.960	0.165
Error	391.634	95	4.122		

Table 5.6: Summary Table of the Repeated Measures Two-way Analysis of Covariance

The interpretation of the repeated measures two-way analysis of covariance shown in Table 5.6 is as follows:

1. Metacognitive training has a significant influence on the students' word problem solving performance ($F = 7.812$, $p<0.05$). That is metacognitive training results in a greater improvement in mathematical word problem solving performance (see also section 3.2, number 2).

2. The level of mathematical achievement has a significant influence on the students' word problem solving performance (F = 11.889, p<0.05). Visual inspection of the adjusted means of higher and lower achievers in the posttest and delayed posttest in Table 5.5 suggests that the adjusted posttest and delayed posttest mean scores for the lower achievers, 3.62 and 4.44 respectively, were lower than those for the higher achievers' adjusted posttest and delayed posttest mean scores which were 4.48 and 5.64 respectively. This suggests that the higher achievers would outperform lower achievers in mathematical word problem solving (see also section 3.2, number 3). Figure 5.6 below illustrates the difference between the higher and lower achievers' adjusted posttest and delayed posttest mean scores.

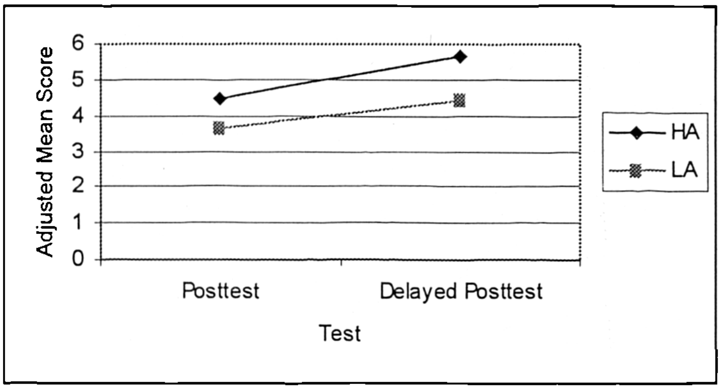


Figure 5.6: Graph of Higher and Lower Achievers' Adjusted Mean Test Scores

Figure 5.6 indicates that higher achievers appear to outperform lower achievers in mathematical word problem solving. To examine whether the higher achievers benefit more from cognitive apprenticeship instruction compared with lower achievers, a correlation between the students' level of mathematical achievement and their gain mean score between pretest and posttest, pretest and delayed posttest, and posttest and delayed posttest must again be established. This account will be presented and described in section 5.

3. There is not significant interaction at the 5% level between metacognitive training and mathematical achievement (F = 1.96, p>0.05).

5. **Pearson Product-Moment Correlation**

Tilley (1996, p. 125) states that correlation is primarily concerned with describing the strength and direction (positive and negative) of a relationship between variables. This strength and direction of the relationship can be expressed by means of a correlation coefficient. A correlation coefficient can take on values that range from -1, indicating a perfect negative relationship, through 0, indicating no relationship, to +1, indicating a perfect positive relationship. The size of the correlation coefficient indicates the strength of the relationship. The closer the correlation coefficient is to -1 or +1, that is, the farther away from 0, the stronger the relationship.

In section 3.2, number 3 and section 4.2, number 2, the findings suggest that higher achievers outperform lower achievers in mathematical word problem solving. Here, I want to examine the strength and direction of the relationship between the students' level of mathematical achievement and their gain mean score between pretest and posttest, pretest and delayed posttest, and posttest and delayed posttest. Table 5.7 is a summary of Pearson product-moment correlation coefficients (Howitt & Cramer, 2000, p. 62) and their statistical significance for a sample size of 142 students from School 1 and School 2. The following graphs, Figures 5.7, 5.8 and 5.9, demonstrate the relationship between the students' level of mathematical achievement and their gain mean score.

Gain Score Mean	N	Pearson Correlation Coefficient	Significance (2-tailed)
Pretest and posttest	142	0.086	0.31
Pretest and dclayed posttest	142	0.183	0.03
Posttest and delayed posttest	142	0.036	0.67

Table 5.7 : Summary Table of the Pearson Product-Moment Correlation Coefficients and their Statistical Significance

The findings in Table 5.7 and Figures 5.7, 5.8 and 5.9 indicate that

a) though the Pearson product-moment correlation coefficients are positive for the gain mean score between pretest and posttest, posttest and delayed posttest, 0.086 and 0.036 respectively (see Table 5.7), the correlation of these variables are not significant at the 5% level of significance; and

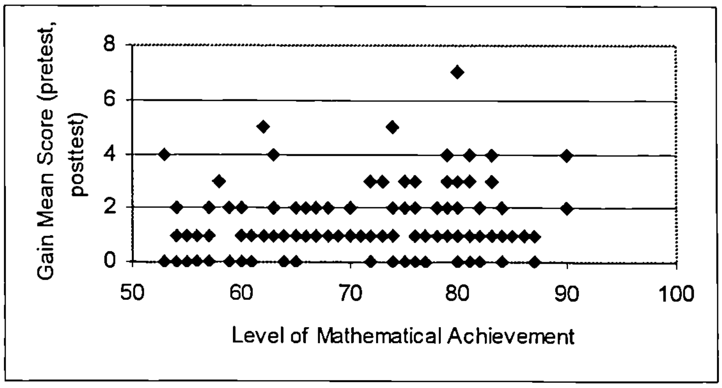


Figure 5.7 : Relationship between Level of Mathematical Achievement and Gain Mean Score (Pretest and Posttest)

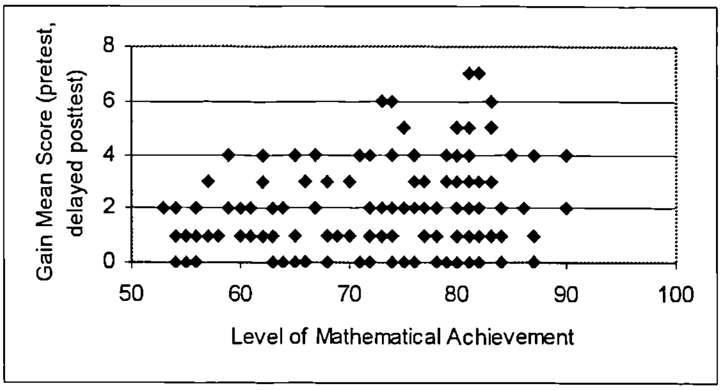


Figure 5.8 : Relationship between Level of Mathematical Achievement and Gain Mean Score (Pretest and Delayed Posttest)

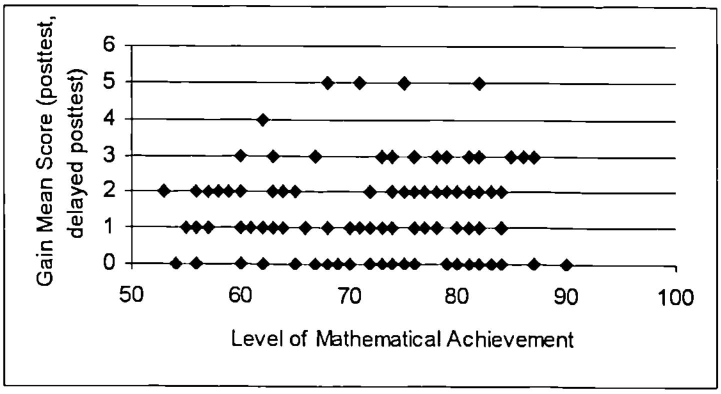


Figure 5.9 : Relationship between Level of Mathematical Achievement and Gain Mean Score (Posttest and Delayed Posttest)

b) the Pearson product-moment correlation coefficient is positive for the gain mean score from pretest to delayed posttest ($r = +0.183$) and it is statistically significant ($p = 0.03$) at the 5% level with sample size of 142. According to Newton and Rudestam (1999, p. 264), this relationship between higher achievers and their gain mean score from pretest to delayed posttest is weakly positive. Further inspection is made by tabulating (see Table 5.8) the gain mean score for the higher and lower achievers' mathematical word problem solving performance in order to examine this relationship.

HA			LA		
|P1-P2|	|P1-P3|	|P2-P3|	|P1-P2|	|P1-P3|	|P2-P3|
1.42	2.04	1.43	1.33	1.73	1.29

P1 : pretest mean; P2 : posttest mean; and P3 : delayed posttest mean

Table 5.8 : Summary Table of the Gain Mean Score of Higher and Lower Achievers

The indicative findings in Table 5.8 appear to indicate that higher achievers benefit more from cognitive apprenticeship instruction compared with lower achievers based on the difference between the pretest and posttest means, pretest and delayed posttest means, and posttest and delayed posttest means. However, the correlation between the level of mathematical achievement and the gain mean score is only statistically significant at the 5% level for the gain mean score from pretest to delayed posttest as indicated in Table 5.7. The above findings suggest that higher achievers show a greater overall benefit from cognitive apprenticeship instruction compared with lower achievers from pretest to delayed posttest.

6. **The Quantitative Component: A Summary**

In this chapter, the possible relationships between metacognitive training, students' level of mathematical achievement and mathematical word problem solving performance were investigated. Two statistical techniques, a repeated measures three-way analysis of variance where students were blocked according to the school factor and a repeated measures two-way analysis of covariance, were employed to test the hypotheses in section 2.2. The following is an attempt to combine the possible relationships observed by the results generated from the two statistical techniques, and using these findings to provide answers to the research questions 1, 2 and 3 in section 2.1.

i) Research Question 1 : What effect will metacognitive training have on performance on mathematical word problem solving?

The findings from the two statistical techniques concur for this research question. That is metacognitive training results in a greater improvement in mathematical word problem solving performance (see section 3.2, number 2 and number 4, and section 4.2, number 1).

ii) Research Question 2 : To what extent is improving performance on mathematical word problem solving (as a result of cognitive apprenticeship instruction) related to the mathematical achievement of the student?

The findings from the two statistical techniques (see section 3.2, number 3 and section 4.2, number 2) show that higher achievers outperform lower achievers. However, findings from section 5 appear to indicate that higher achievers only demonstrate an overall benefit from cognitive apprenticeship instruction compared with lower achievers based on their gain mean score from pretest to delayed posttest.

iii) Research Question 3 : Do the effects of metacognitive training on performance on mathematical word problem solving vary with the mathematical achievement of the student?

The findings from the two statistical techniques (see section 3.2, number 6c and section 4.2, number 3c) do not offer statistical significant evidence that the benefits from metacognitive training on mathematical word problem solving performance varies with the level of students' mathematical achievement. However, visual examination of the pretest, posttest and delayed posttest means for quasi-experimental design 1 appears to suggest that higher achievers benefit more from metacognitive training than do lower achievers.

The findings also imply that lower achievers only show the full benefits from training after a period of time (see section 3.2, 6d) from posttest to delayed posttest.

The above findings based on the two statistical techniques will contribute to the discussion of the effect of metacognitive training on the mathematical word problem solving of Singapore 11-12 year olds in WordMath environment in Chapter Seven.

Chapter Six
Presentation and Description of Case Study Data

1. Introduction and Overview of Chapter

In Chapter Four, I described how the pretest, posttest 1 and delayed posttest 1 data, think aloud protocol, student questionnaire and teacher interview data were processed and analysed. The outcome of the analyses of the pretest, posttest 1 and delayed posttest 1 data for the quasi-experimental design was presented and described in Chapter Five. This chapter seeks to present the outcome of the analysis of the think aloud protocol, student questionnaire and teacher interview data for the case study design in order to address research question 4 in Chapter One, section 4.

The case study design was described in Chapter Four. 8 dyads from the treatment and control classes in School 1 and School 2 were selected (see Chapter Four, Table 4.3). The intervention process for the dyads was also described in Chapter Four, section 4. The case study design seeks to explore the role of metacognition in word problem solving (see Chapter One, section 4) in a computer environment and its influence on students' word problem solving outcome, using the analysis of students' think aloud protocol data during word problem solving in posttest 2 and delayed posttest 2 (see Chapter Four, section 4.2). The main purpose of the teacher interview and student questionnaire data is to provide an account of the students' metacognitive knowledge (Chapter Two, section 2) before the intervention. Therefore, analysis of student questionnaire and teacher interview data is used to identify the students' metacognitive knowledge before the intervention.

The chapter consists of four main sections. Section 2 presents and describes the students' full think aloud protocol data during word problem solving of the N3 word problem, with the help of diagrams called timeline representations. Section 3 goes on to display the length (and %) of time students devote to metacognitive and cognitive behaviours in word problem solving. This is followed by a comparison of the length (and %) of time treatment and control groups devote to metacognitive and cognitive behaviours during word problem solving. Finally, there are display tables of the specific length (and %) of time students devote to episodes (cognitive and metacognitive) in the modified Artzt and Armour-Thomas' framework (see Appendix G) during posttest 2 and delayed posttest 2. These data display tables will provide insights to the relationship between cognitive and metacognitive behaviours and students' word problem solving.

Section 4 goes on to highlight and describe five types of cognitive-metacognitive word problem solving models that emerged from the analysis of students' think aloud protocols presented in section 2, and the display tables in section 3. It attempts to identify the distinctive progression of students' word problem solving activity and how they relate to their word problem solving performance.

Finally, section 5 presents and describes the students' metacognitive knowledge (case study design) from the teacher interview and student questionnaire data. This section seeks to identify the metacognitive knowledge each student brings to word problem solving before the intervention.

2. Presentation and Description of Dyads' Think Aloud Protocol Data

As mentioned in Chapter Four section 5.2, the episodes from the modified Artzt and Armour-Thomas' framework were used to categorise think aloud protocol data. The framework was able to delineate students' metacognitive and cognitive behaviours during word problem solving. The following sub-sections are detailed descriptions of the students' word problem solving behaviour on the N3 word problem (see Appendix C and Chapter Four, section 5.2.2). Appendix I (Compact Disc) illustrates S2 students L and JK, HM and XY, K and SJ, and E and XF solving the N3 word problem and the full think aloud protocol of these dyads' solutions are given in Appendix H. A further four dyads' (from S1), A and CC, ES and J, SM and B, and B and P, description of their think aloud protocols are also given here.

2.1 The Full Analysis of A and CC's (T/HA) Think Aloud Protocol

The overall structure of the solution analysis for the N3 word problem protocol is shown in Figure 6.1. This involves 185 'items' (Chapter Four, section 5.2.2) by the interacting students. Note that the words 'lah' and 'leh' in the students' talk are common colloquial expressions used in conversations amongst Singaporeans.

Episode 1: Reading (Item 1)

They began by reading the problem aloud.

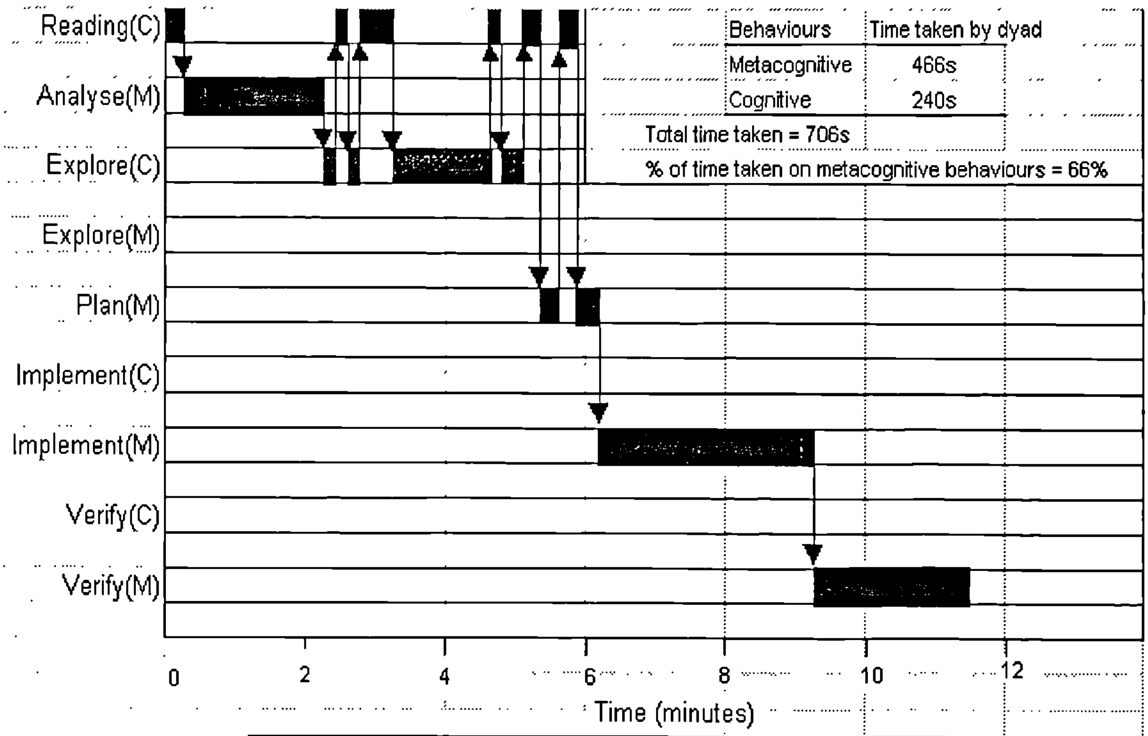


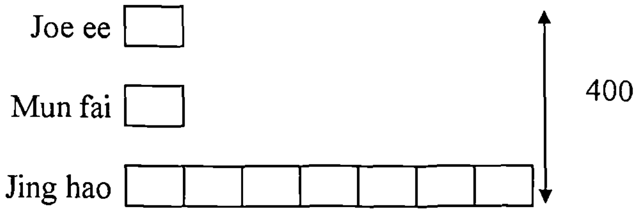
Figure 6.1: A timeline representation of A and CC (T/HA) solving 1N3

Episode 2: Analysis (Items 2 - 33)

A and CC made an attempt to represent the word problem situation by drawing a diagram with the aid of WordMath facilities. At one point, they appeared to be uncertain on how many units Mun Fai should be represented by.

CC: [no Mun Fai
Mun Fai first / Mun Fai first
Mun Fai is (3)
Mun Fai cannot be /might be the same and cannot be the same

They decided to draw the following diagram, labeling Mun Fai as 1 unit. Then, they proceeded to the next episode.



Episode 3: Exploration (cognitive) (Items 34 - 37)

A suggested that *'may be take 400 minus 28 divided by 1 or 2'*. However, CC appeared to be unconvinced by the idea and also appeared to be uncertain on how to proceed. He decided to reread the word problem.

Episode 4: Reading (Item 38)

In this episode, CC focused on reading the statement, *'Jing Hao received seven times the total number of marbles Joe Ee and Mun Fai received'*.

Episode 5: Exploration (cognitive) (Items 39 - 42)

A interrupted CC and repeated the idea he had suggested in Episode 3. He tried to convince CC by showing him that his idea will *'..find 1 unit for this'*.

Episode 6: Reading (Items 43 - 48)

However, CC still appeared to be unconvinced that A's idea was appropriate. He devoted 27 seconds to reading the word problem again, slowing down when he approached *'Jing Hao received seven times ... '*. He appeared to be trying to make sense of this word problem statement and was trying to connect this condition to the diagram drawn.

Episode 7: Exploration (cognitive) (Items 49 - 68)

In this episode, they engaged in a trail-and-error strategy.

CC:	try the 400 minus 28 one lah divided by 8 / equals to /	{typing: [400-28]/8= }
A:		{calculator: press 400-28}
CC:	372 / 372 divided by 8 / equals to 46 cannot be what that means wrong leh may be this one is 9	{calculator: press /8}
A:	wait	{calculator: press 400-28}

CC: try 372 first
divided by 9 / divided by 9
A: also wrong {calculator: press /9}
CC: 10 huh?
Cannot be what / 10

Though they made no assessments, they were able to estimate that their answer on the calculator was inappropriate, evidenced by their utterances '*cannot be what*' and '*also wrong*'. However, they gave no reasons for the inappropriateness of these results. After the third attempt to divide the result of '400-28' by 10, A decided to read the word problem again.

Episode 8: Reading (Items 69 - 70)

A's reading of '*Jing Hao received seven times the total number of marbles ...*' appeared to suggest that they had not understood what this statement meant.

Episode 9: Exploration (cognitive) (Items 71 - 81)

CC suggested taking '*400 divide by 28*'. He did not make any assessment of his suggestion. A's reluctance to implement CC's suggestion caused CC to return to reading the word problem.

Episode 10: Reading (Items 82 - 84)

CC devoted 12 seconds to rereading the whole word problem.

Episode 11: Planning (Items 85 - 89)

On A's request, CC repeated his approach in Episode 9. A considered CC's suggestion and challenged him to consider '*how you know 1 unit is 28*?' This challenge caused CC to pause and think of another approach.

Episode 12: Reading (Items 90 - 94)

They appeared to have reached an impasse and both decided to read the word problem again. They devoted 27 seconds to reading.

Episode 13: Planning (Items 95 - 102)

CC suddenly had a 'brain wave' - '*I know already*'. He described to A what needed to be done.

CC: 28 times 7 lah
after that you minus lah
[minus both lah then you get this one lah

Episode 14: Implementation (metacognitive) (Items 103 - 147)

In this episode, CC appeared to be the leader, directing A in the implementation of his plan. A, on the other hand, consistently directed CC's attention to the goal of the word problem, ensuring that CC understood what was required.

A: no / because we only find some of the parts here /
but they say the total number of Joe Ee and Mun Fai (2)

Episode 15: Verification (metacognitive) (Items 148 - 185)

In this final episode, they checked the solution of the word problem by testing that the solution satisfied the conditions of the word problem.

A: Jing Hao has 350 marbles
CC: 350 marbles [Jing Hao has / minus
A: [28
[plus 22
CC: [plus 22
50
350 minus 22
A: Mun Fai and Joe Ee had 50 marbles altogether / so
CC: 350 plus 50 equals 400

A and CC were able to provide the number of Mun Fai and Jing Hao's marbles (see the above exchange), but they typed in the final answer as 174. This was a carelessness on their part. After the computer had provided them with the feedback that the answer they had entered into the box was incorrect, they returned to their working and immediately discovered that they had subtracted incorrectly. On this account, I have accepted that they had provided a correct solution and were successful in their word problem solving.

The protocol of A and CC (T/HA) could be summarised as:

1. Reading and rereading parts of the word problem;

2. Exploration (cognitive), in which they used the trail-and-error strategy in an attempt to reduce the discrepancies between the givens and the goal of the word problem. When this did not work, they read the word problem again, hoping that some insights would be illuminated by rereading the word problem. This strategy usually worked for them whereby reading the word problem many times seemed to direct them to a clearer understanding;
3. Planning, in which they devised a clear plan that would lead them to the solution;
4. Implementation (metacognitive), in which they followed their plan but monitored each step they took; and
5. Verification (metacognitive), in which they used a test to check that the solutions of the calculated unknowns satisfied the conditions of the word problem.

2.2 The Full Analysis of ES and J's (T/LA) Think Aloud Protocol

The overall structure of the solution analysis for the N3 word problem protocol is shown in Figure 6.2. This involves 146 'items' (Chapter Four, section 5.2.2) by the interacting students.

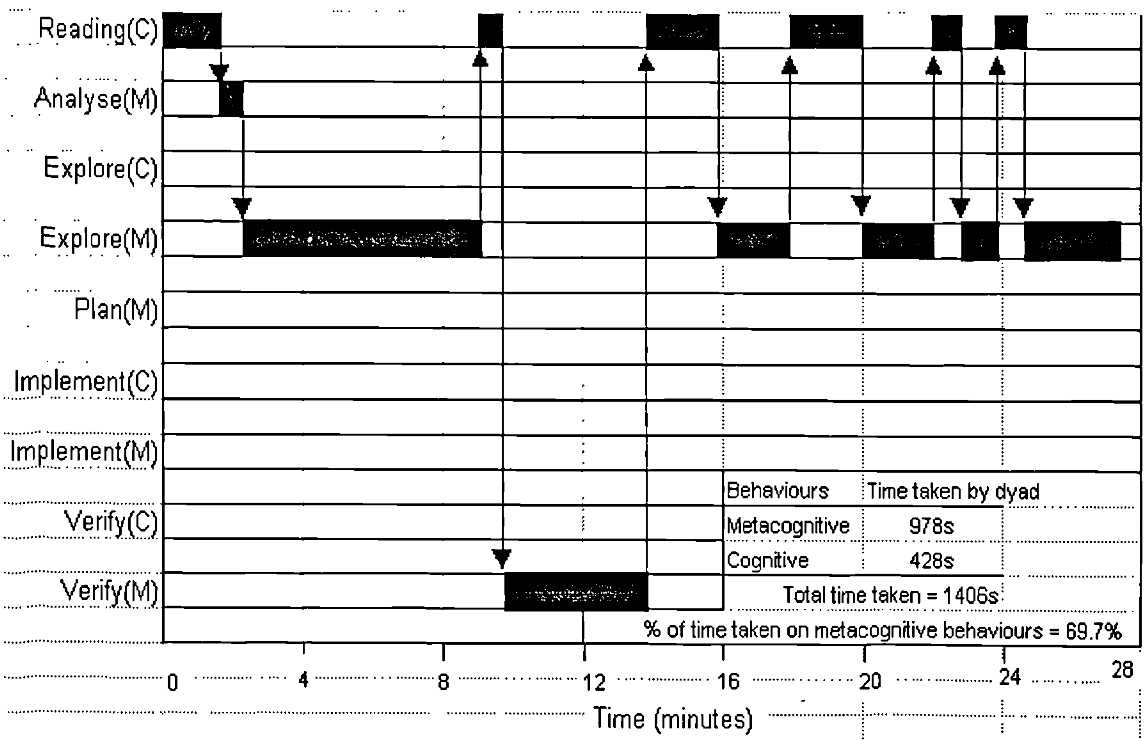


Figure 6.2: A timeline representation of ES and J (T/LA) solving 1N3

Episode 1: Reading (Items 1 - 16)

They began by reading the word problem aloud. J and ES returned to the word problem and read parts of the word problem by themselves. They appeared to have difficulties in understanding the meaning of the word problem. This episode took them 80 seconds.

Episode 2: Analysis (Items 17 - 20)

J persuaded ES not to draw a diagram because he did not know how to show the relationship between the givens and the unknown. J proceeded to inform ES the strategy to solve the problem in the next episode. The goal of the word problem was not clarified here.

Episode 3: Exploration (metacognitive) (Items 21 - 147)

They engaged in a series of calculations which were removed from the goal of the word problem. There appeared to be an order to their strategy but without regulating and monitoring the procedure, they did not recognise that they were moving further and further away from the goal of the problem. J would occasionally persuade ES to accept what he was doing by giving a brief explanation for his method.

J: you just take 400 minus 28 first / 400 minus 28
then you have the left / the left over
ES: the rest
J: then the Jing Hao and / Joe
then divide by 8
then they can each / each part
then they say how many more
so we just / times / 7
then 1
then 7 minus the answer
then you will get the answer (6)
ES: mm, yeah
so (3) received 28 marbles
J: I told him already

J was the leader of the two and appeared to dominate the word problem solving. There were occasions when E meekly gave in to J's suggestions though he might have considered J's suggestion to be leading them nowhere at that time. This was evidenced by his whispers only magnified by the micro-phone.

Towards the end of this episode when they derived at a 'solution', J discovered that something was amiss, evidenced by the following exchange of their protocol.

J:	wait huh / back space we don't need that actually / if I'm not wrong 400 minus	{calculator: press 400 - }
ES:	minus 28	
J:	28 times 7 7 more what! / then? (6)	{calculator: press 28*7}

Finally he decided to read the word problem again.

Episode 4: Reading (Items 148 - 151)

Both of them reread parts of the word problem focusing on '*7 times the total number of marbles Joe Ee and Mun Fai received*'.

Episode 5: Verification (metacognitive) (Items 152 - 169)

J suddenly announced to ES that what they had been doing was flawed. He tried to locate the source of the error but was unable to do so. He appeared to sense that the solution was incorrect but his limited resources were not able to help him identify what was flawed.

J:	we did wrong for this question / if I'm not wrong (3) you see huh (8) 4	{calculator: press 4}
ES:	28 / 196	
J:	196 plus 176 plus 28 equals 400 okay, our answer is correct. The answer is 20, right? (2) if I'm not wrong, we are wrong.	{calculator: press 196+176+28=}

Episode 6: Reading (Items 170 - 185)

ES decided to read the whole problem again. J followed his example. During a short period in this episode, J questioned ES if he knew how to do the word problem. ES declared that he was not sure. Then they continued reading parts of the word problem. This took 75 seconds.

Episode 7: Exploration (metacognitive) (Items 186 - 230)

J returned to the strategy used in Episode 3. ES immediately reminded him that they had used that strategy before and it did not work. J had a sudden flash of insight and announced that *'I think I know already'*. However, this insight was again flawed. He began by making a series of guesses to determine the number which is required to divide by 400. Fortunately, ES monitored on his strategy and pointed to him on a few occasions that that *'cannot be'*. At the end of this episode, J desperately declared, *'I don't know how to do'*.

Episode 8: Reading (Items 231 - 242)

J decided to read the word problem again. This time both of them read parts of the word problem slowly and deliberately, pausing occasionally. This reading episode took 120 seconds.

Episode 9: Exploration (metacognitive) (Items 243 - 268)

J began another series of 'wild goose chases' (Schoenfeld, 1985). ES was consistently monitoring J's strategy and telling him *'cannot lah'* which indicated that he did not think the strategy worked. Towards the end of this episode, J evaluated the status of their word problem solving.

J: got any ideas? (3)
ES: received
J: got any ideas? (3)
I'm asking you (4)
ES: I'm thinking / I'm thinking
I don't know (20)
wrong
I got nothing! (12)

Episode 10: Reading (Items 269 - 270)

ES read '*7 times the total number of marbles Joe Ee and Mun Fai received (7)*' again.

Episode 11: Exploration (metacognitive) (Items 271 - 277)

J referred to CRIME and pointed out the sections they had done. However, their reference to CRIME did not appear to help them. They appeared reluctant to try other heuristics, i.e. draw a diagram, to aid them in solving the word problem.

Episode 12: Reading (Items 278 - 283)

They read the word problem in unison and paused for some time.

Episode 13: Exploration (metacognitive) (Items 284 - 292)

When they were told that their time was up and they had to either give up or provide a solution for the problem, they chose to provide a solution which was calculated in episode 3 for the word problem.

The protocol of ES and J (T/LA) could be summarised as:

1. Reading and rereading parts of the word problem; and
2. Exploration (metacognitive), in which they failed to clarify the meaning of the goal of the word problem at the start of the word problem solving enterprise. As a result, their strategy was far removed from the goal of the word problem.

Initially, J appeared to be confident and worked with certainty on his flawed strategy. Though both of them monitored their progress, their limited metacognitive resource was not able to 'rescue' them from their flawed strategy. Eventually, J appeared to have given up hope and engaged in a series of 'wild goose chases'. ES constantly rejected J's suggestions. However, his limited resource was also not able to provide an appropriate alternative. These weaknesses contributed to their failure in solving the word problem.

2.3 **The Full Analysis of SM and B's (C/HA) Think Aloud Protocol**

The overall structure of the solution analysis for the N3 word problem protocol is shown in Figure 6.3. This involves 145 'items' by the interacting students.

Episode 1: Reading (Items 1 - 9)

They began by reading the word problem together and each of them would return to certain parts of the word problem and reread them again. They devoted 43 seconds to this episode.

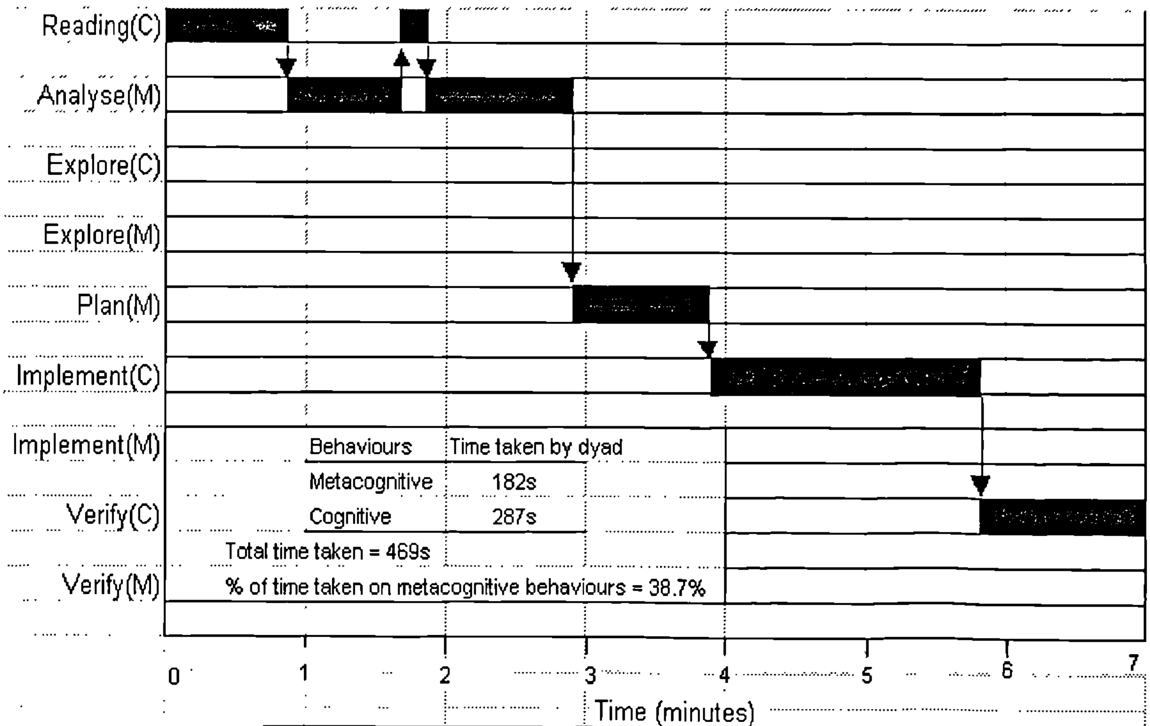


Figure 6.3: A timeline representation of SM and B (C/HA) solving 1N3

Episode 2: Analysis (Items 10 - 22)

They re-examined the implicit and explicit relations between the givens and the goals of the word problem.

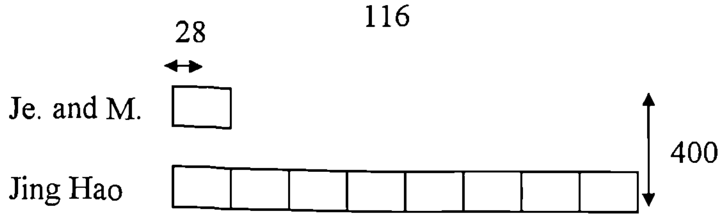
SM: 1 unit equals 28 / 28
Jing Hao received seven times the total number of marbles as Joe Ee =
B: = no / and Mun Fai
SM: that means /the total number of marbles Joe Ee and Mun Fai received 7 units huh?
B: Yeah / 7 units

Episode 3: Reading (Items 23 - 24)

SM returned to rereading the problem. It appeared that he wanted to ensure that he had understood the bit '*Jing Hao received 7 times the total number of marbles Joe Ee and Mun Fai received*' before he proceeded to represent the conditions of the word problem on a diagram.

Episode 4: Analysis (Items 25 - 50)

The students devoted 78 seconds to this. They appeared to be making an effort in ensuring that their diagram vividly expressed the relationship between the givens and the unknowns of the word problem.



Episode 5: Planning (Items 51 - 68)

They proceeded to make explicit plans for the approach they were going to take in finding the solution of the word problem. They clarified all doubts and stated explicit goals before they moved on to implementation.

SM: 8 units equals 400
B: 400 divide by 1,2,3,4,5,6,7,8 / 8 units
then you minus 28 =
SM: = 28
B: then you get [Jing Hao
SM: [1 unit minus 28 equals Mun Fai
B: Mun Fai's
then [later you get Jing Hao's
SM: [how many more?
Jing Hao's
B: amount of money
then later you minus / the amount

Episode 6: Implementation (cognitive) (Items 69 - 130)

They executed the strategy they made in episode 5. Their actions appeared systematic and there was a deliberateness in transforming the givens into the goals of the word problem. However, they did not make any assessment during this episode. Hence, it was appropriate that this episode was categorised as implementation (cognitive). It appeared that there was no need to make assessments since they had clarified what needed to be accomplished in episodes 4 and 5.

Episode 7: Verification (cognitive) (Items 131 - 145)

After calculating the solution of the word problem, they did not use a test to check the reasonableness of the solution. They typed the final statement and appeared satisfied that they had obtained the solution to the word problem.

The protocol could be summarised as an orderly progression of activity, Read → Analyse ⟷ Plan → Implement (cognitive) → Verify (cognitive) which led to a successful solution.

2.4 The Full Analysis of B and P's (C/LA) Think Aloud Protocol

The overall structure of the solution analysis for the N3 word problem protocol is shown in Figure 6.4. This involves 170 items by the interacting students.

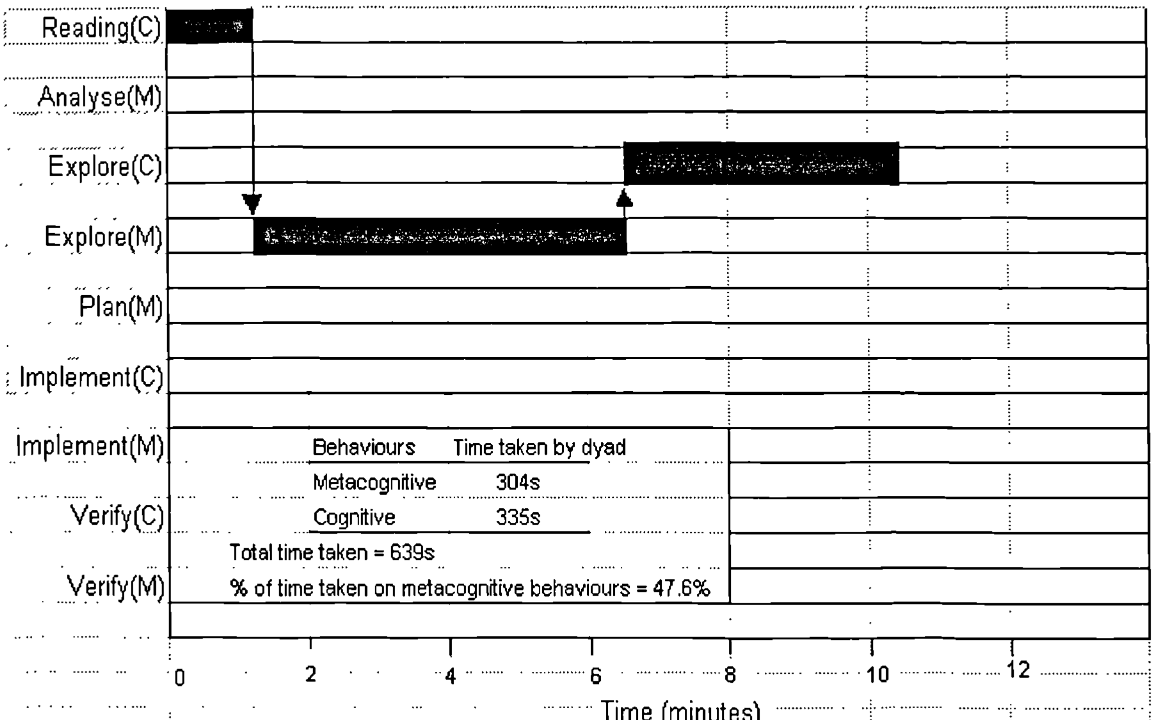


Figure 6.4: A timeline representation of B and P (C/LA) solving 1N3

Episode 1: Reading (Items 1 - 12)

The students began by reading the word problem in unison. Then, each of them took their time to reread parts of the word problem again. It appeared that for them, rereading the word problem would help them clarify their doubts. They devoted 79 seconds to this.

Episode 2: Exploration (metacognitive) (Items 13 - 99)

Throughout this episode, B was the major source of new ideas.and procedures, most of which were inappropriate and all of which were eventually rejected by P.

P: no / then Mun Fai received 7 times
so, each / Mun Fai and Joe Ee have 1 unit.

and

B: if Jing Hao / no, if Mun Fai is 28 marbles
28 plus 28
P: no, cannot be
[because 28 is not decided 1 unit yet

and

B: so it's 252
P: 252
B: so there's still missing 100 over marbles /
P: how many marbles did [Jing Hao receive?
B: [take 196
P: this is wrong
because [we haven't found Mun Fai
B: [no but then the total number of marbles is 400
the 56 is Joe Ee plus Mun Fai together /
so if Joe Ee has 28 marbles
P: but Mun Fai
we don't know how many marbles are there.

However, P was not able to dissuade B from pursuing an inappropriate strategy. On many occasions, P tried to direct B's attention to the need of finding the number of marbles Mun Fai received and not to assume that Mun Fai had 28 marbles. However, B rejected P's suggestion without informing him of the reason. Towards the end of the episode, B finally declared, '*Mun Fai and Joe Ee cannot be the same*'. This exclamation from B led them to the next episode.

Episode 3: Exploration (cognitive) (Items 100 - 170)

Even though B understood that Mun Fai and Joe Ee could not be the same, he continued to make assumptions with regard to the number of units each of them were represented in this episode.

B: it can only be half
if Joe Ee is half and Mun Fai is 1 unit,
28 times 2 equals 56
56 plus 28 is 8, 8, 4
P: see it is 7 times more
B: 7 times
ninety, 96 / 94 marbles / you get 94 marbles
Jing Hao must be the rest

During this episode, there was only one occasion when P challenged B's assumption.

P: but 28
they never tell us that it's half the unit =
B: = they never tell us how many, how much is 28 is not 1
unit
they never tell us =
P: = yeah
B: so, if 28 is half
28 times 2 equals 56
Mun Fai has 56, so Joe Ee has 28
so, 56 plus 28 / 56 plus [28 equals
P: [28 you get 94 =

Finally, P was persuaded by B to use his strategy, a strategy that appeared to be dependent on the assumptions B had made and not on the conditions of the word problem. The strategy employed involved using the numbers from the word problem statements and using a variety of calculations to reduce the discrepancies between the givens and the goals. This apparently flawed strategy led to their failure in their word problem solving.

The protocol of B and P (C/LA) could be summarised as:

1. Reading the word problem; and
2. Exploration (cognitive), where the pair was observed to take the numbers out of the word problem context and used different operations to manipulate these numbers. The students also appeared to be constantly making assumptions based on their own understanding and not relying on the conditions provided by the word problem.

It seemed that they had hoped exploration would allow them to obtain a solution. P often engaged B in local assessments, but they did not make appropriate metacognitive decisions which could direct them to the goal of the word problem. They also did not clarify each other's uncertainties at all stages of their word problem solving. In addition, in B and P's word problem solving, there were many occasions when P gave in to B's flawed strategies even though he did not agree with them. Initially, P would make an effort to argue and challenge B's suggestions. However, it appeared that B often refused to listen and accept P's arguments or challenge. This tension might have arose from B's perception of P as a 'weaker' word problem solver or it might be that B was genuinely unable to comprehend P's argument. It appeared that this tension and their weaknesses mentioned above contributed to their failure in most of their word problem solving..

2.5 The Full Analysis of L and JK's (T/HA) Think Aloud Protocol

The overall structure of the solution analysis for the N3 word problem protocol is shown in Figure 6.5. This involves 73 items by the interacting students (see Appendix H and I).

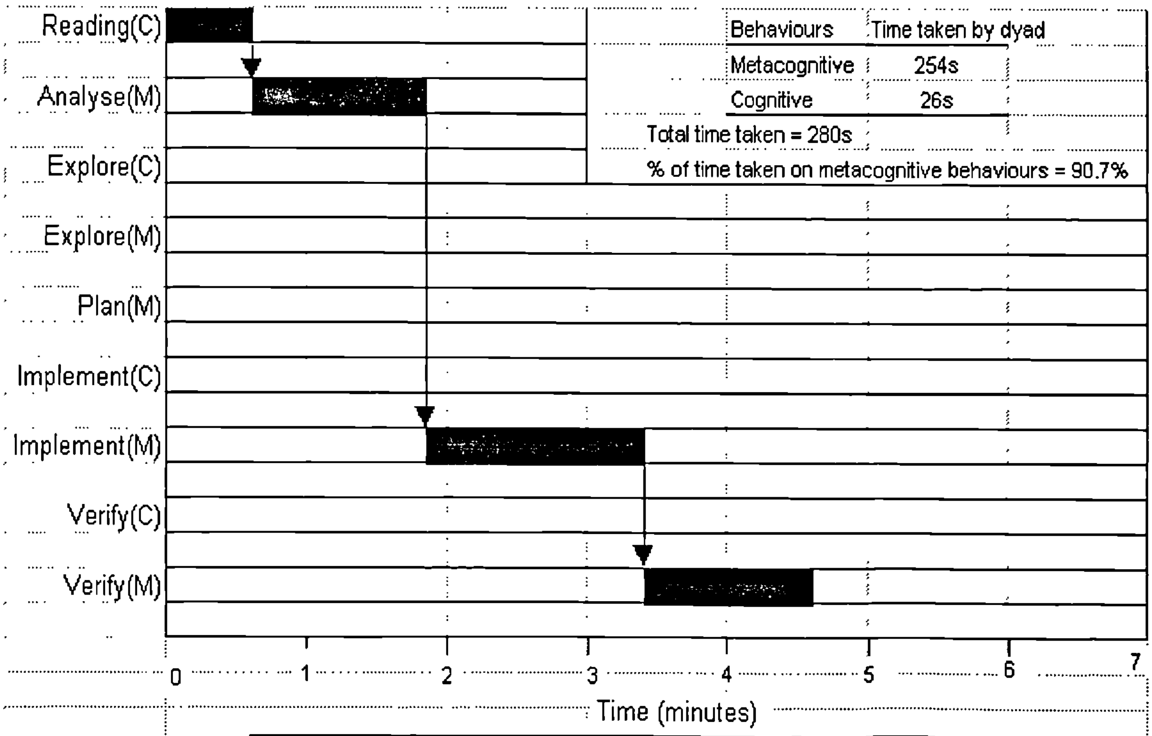


Figure 6.5: A timeline representation of L and JK (T/HA) solving 1N3

Episode 1: Reading (Item 1)

Both L and JK began by reading the word problem statements.

Episode 2: Analysis (Items 2 - 25)

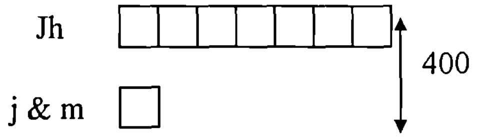
L started drawing the block diagram. He noted the conditions of the word problem and directed JK to label the diagram to demonstrate the relationship between the givens and the unknown. During this episode, both of them monitored each other's understanding and their perspective of the word problem.

JK: draw 28 marbles. The total unit is 8 units leh.
L: how do you know?/
Oh, okay.

and

L: And this should be / Jing Hao and this should be for / Joe and Mun
JK: no, this is 7.
1 unit 7

The diagram was drawn which included all the vital information.



Episode 3: Implementation (metacognitive) (Items 26 - 54)

In this episode, working statements for each immediate step during the word problem solving process appeared to be carefully thought through and confidently typed into the working space provided in WordMath. These working statements provided a logical and systematic sequence to the solution. First, they agreed to find the number of Mun Fai's marbles. Then, it was possible to find the number of Jing Hao's marbles. While calculating the value of Jing Hao's marbles, L reflected on the strategy they were about to use and decided to use a 'finer' strategy:

L: Jing Hao. 7 units which is 50 times 7 which is 350 /
no need lah.
Take 400 minus 50
faster / and easier.

Episode 4: Verification (metacognitive) (Items 55 - 73)

After L had successfully calculated the number of more marbles Jing Hao received than Mun Fai, he led JK to assess the reasonableness of the result by using a test to ascertain that the total number of Mun Fai and Jing Hao's marbles (calculated) added to Joe Ee's number of marbles (given) equalled 400 marbles (given).

L: Now let's check {calculator:
so if / if / Mun Fai is 22, then [Joe plus press 350-22=
and 22+28=}
JK: [Joe plus 28 equal 50
L: is 50 {calculator:
[50 times press 50*7=}
JK: [50 times 8
L: 7 is 350 {calculator:
so to find whether it's correct or not, plus 50 to get the press 350+50=}
total and we're correct.

The protocol for L and JK (T/HA) could be summarised as a well-regulated progression of activity, Read → Analyse → Implement (metacognitive) → Verify (metacognitive), which led to their success in solving the word problem. They also seemed in control of their cognitive actions throughout their word problem solving process, as illustrated by the exchanges in the above episodes 2, 3 and 4.

2.6 The Full Analysis of HM and XY's (T/LA) Think Aloud Protocol

The overall structure of the solution analysis for the N3 word problem protocol is shown in Figure 6.6. This involves 146 items by the interacting students (see Appendix I).

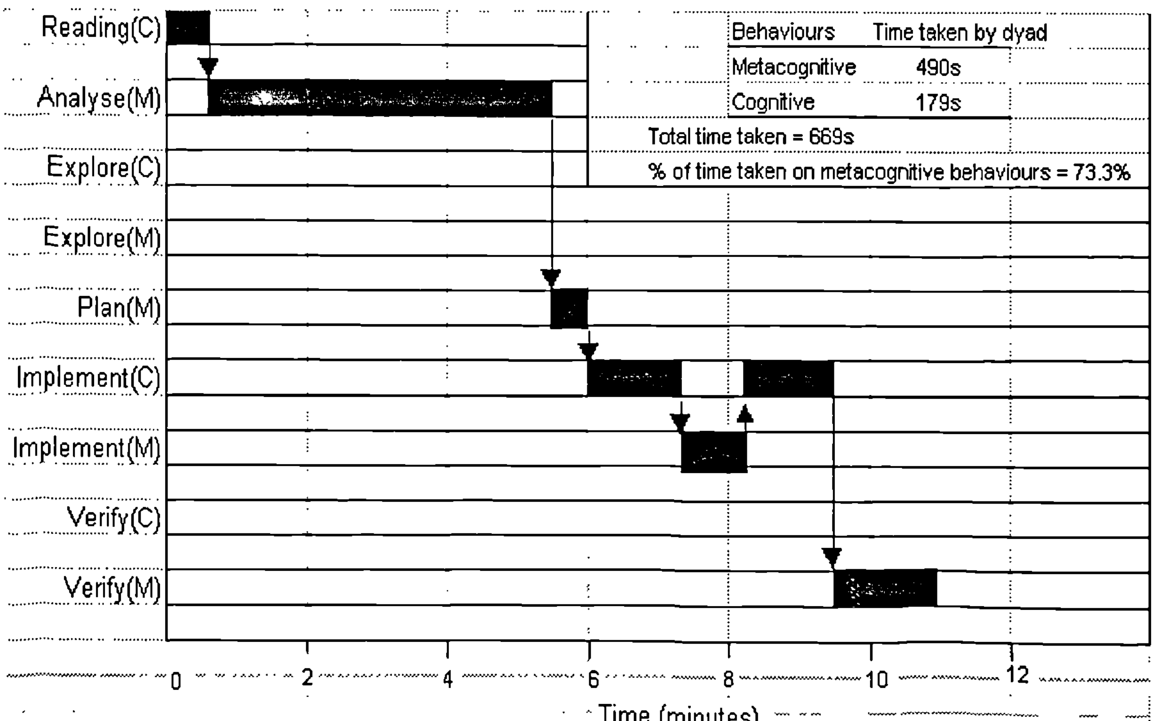


Figure 6.6: A timeline representation of HM and XY (T/LA) solving 1N3

Episode 1: Reading (Item 1)

They began by reading the word problem aloud.

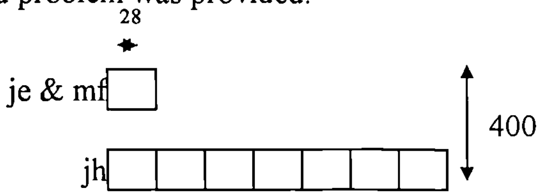
Episode 2: Analysis (Items 2 - 53)

HM started to draw the block diagram. XY intermittently monitored HM's diagram and suggestions by referring her to the conditions of the word problem situation.

XY: What's this? (referring to the block HM had drawn)
HM: 7 times. I'm drawing 7 times =

XY: = but it is 7 times of / Joe and Mun Fai you know

HM accepted XY's challenge and started making modifications to her previous diagram and re-labeling the blocks. XY continued to monitor HM's suggestion, seeking clarification on the diagram she was drawing. As a result, HM seemed to be forced to clarify the relationship between the givens and the unknown from the word problem statements which led to drawing a diagram where all the vital information for solving the word problem was provided.



Before moving to the next episode, HM reminded XY of the goal to the word problem, '*the question asked how many more marbles did Jing Hao receive than Mun Fai*' and directed XY, by referring to the drawn diagram, to how the solution could be calculated. XY appeared to be unsure of HM's strategy and asked for clarification.

XY: That one is what? {referring to the diagram}
[what's this?
HM: [This is unknown / unknown because Mun Fai's
so let's say this is one small unit /
XY: okay.

Episode 3: Planning (Items 54 - 74)

HM described the approach to solve the word problem. With the aid of the diagram drawn, she explained the procedure used to solve the word problem. The plan was explicit and XY approved of HM's suggestion.

Episode 4: Implementation (cognitive) (Items 75 - 93)

They executed the strategy that was developed from their plan. In this episode, they appeared to be engaged in a well-structured series of calculations which was orderly, evidenced by the following exchanges.

HM: so 8 units is equal to 400 minus
[28 times 8
XY: [28 times 8
28 times 8 is (5)
HM: do you need a calculator?

XY: 28 times 8 / 224 {calculator: press 28*8=}
HM: yes
XY: so / minus 400 {calculator: press 400-224=}
HM: the answer is [176
XY: [176, 176
HM: yes, [1unit
XY: [so / 1 small unit? =
HM: = yes
XY: 176 / divided by 8
HM: which is?
XY: 22 huh?
HM: {calculator: press 176/8=}
XY: 176 / divide / [22
HM: [22

Episode 5: Implementation (metacognitive) (Items 94 - 112)

This short episode lasted for 76 seconds whereby the students evaluated the results for the sub-goals, relating the calculated values to the conditions of the word problem and articulating the subsequent steps they needed to take.

HM: this is for =
XY: = 22 / Mun Fai has 22 marbles
HM: yes, so we have to [find Jing Hao
XY: [find 1 big unit
HM: yes

Episode 6: Implementation (cognitive) (Items 113 - 125)

They continued to calculate the '*1 big unit*' which enabled them to calculate the number of Jing Hao's marbles. Then, the difference between the number Jing Hao and Mun Fai's marbles was calculated, which led to the solution.

Episode 7: Verification (metacognitive) (Items 126 - 146)

Both of them checked the solution of the word problem by testing that the solution satisfied the conditions of the word problem.

HM: Let's check.
XY: okay, so
HM: this is / let's check whether this is 50 / 400
so 328 [plus 50
XY: [plus 50
328 plus 50?
HM: wait wait wait
350 / because 350 is Jing Hao's

XY: yeah
HM: It's 400, so the answer is correct.
XY: okay

The protocol could be summarised as an orderly progression of activity, Read → Analyse → Plan → Implement (cognitive) → Implement (metacognitive) → Implement (cognitive) → Verify (metacognitive) which led to a successful solution.

2.7 The Full Analysis of K and SJ's (C/HA) Think Aloud Protocol

The overall structure of the solution analysis for the N3 word problem protocol is shown in Figure 6.7. This involves 110 items by the interacting students (see Appendix I).

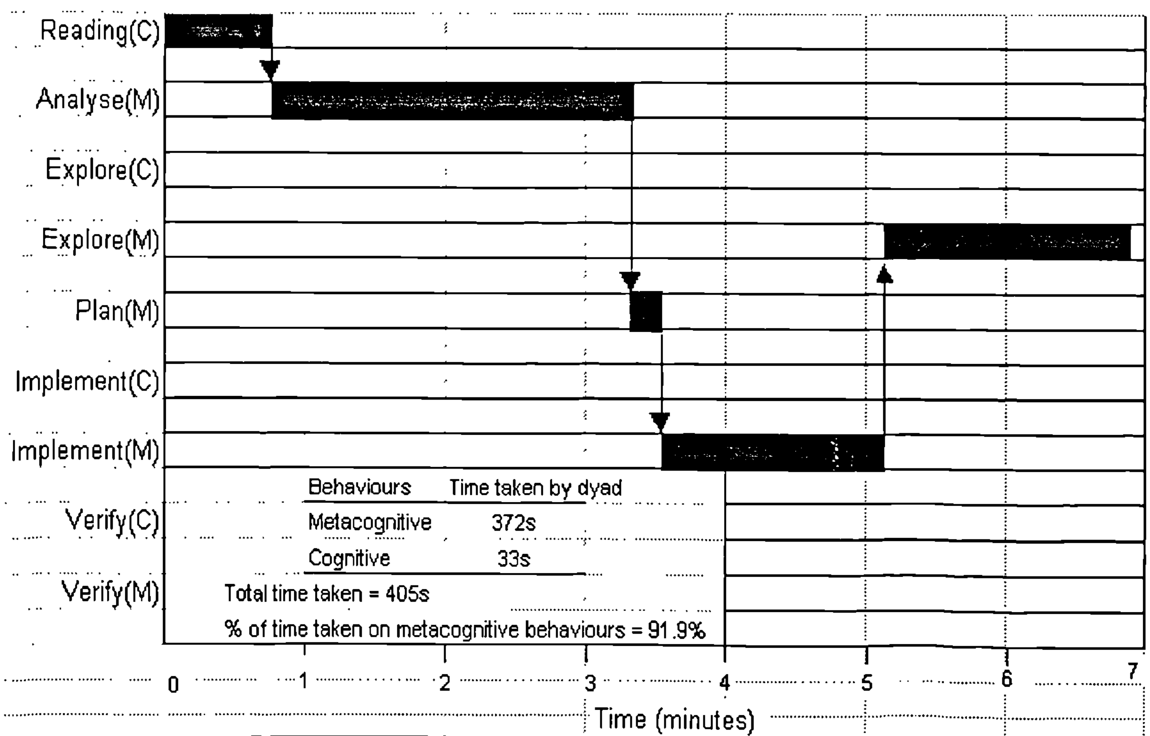


Figure 6.7: A timeline representation of K and SJ (C/HA) solving 1N3

Episode 1: Reading (Items 1 - 2)

They read the word problem aloud.

Episode 2: Analysis (Items 3 - 38)

Analysis was goal driven as they drew the diagram to show the relationship between the givens and the unknown. SJ constantly directed K to the conditions of the word problem as K labeled the diagram. He also reminded K of the goal of the word problem.

SJ: eh (6) wait wait / erm this should put Joe Ee / Joe Ee and Mun Fai {referring to the unit drawn}
K: no =
SJ: = because =
K: = Joe Ee received 28 marbles you see =
SJ: = no, because Jing Hao received seven times the total number / of the number of marbles Joe Ee and Mun Fai received
K: but they already stated Joe Ee received 28 marbles
SJ: I know. We can draw another [model
K: [orh

K understood SJ's rationale for labeling Joe Ee and Mun Fai as one unit and started making modifications to the previous diagram. Their final diagram demonstrated that they had clarified their doubts and had established exactly what they knew and needed to find.

Episode 3: Planning (Items 39 - 45)

Their plan was overt and they were able to define the strategy which appeared to lead them to the word problem solution if implemented.

Episode 4: Implementation (metacognitive) (Items 46 - 87)

They executed their plan and typed the working statements in an orderly manner which would lead them to the outcome. SJ continued to check on K's strategy to find the value of Mun Fai's marbles. SJ's insistence to seek clarification forced K to explain the rationale for his strategy.

SJ: why not 400 divide by 8?
By, this is 7
K: okay you see huh
SJ: yeah
K: to find this question mark, we don't have any method, so we can actually use 28
can actually fit in here / to all the units you see
so, we have eight 28s
SJ: yeah, I get it

Episode 5: Exploration (metacognitive) (Items 88 - 110)

Though they had calculated the exact values of Jing Hao and Mun Fai's marbles, which were vital information for success, K, at this stage, misinterpreted the goal of the word problem. SJ tried to reject K's subsequent procedure, directing him to see that there was a flaw in his procedure. Instead of convincing K, SJ was persuaded by K to accept his flawed procedure. This new procedure, a reallocation of resource, moved them away from the goal of the word problem and hence, they were not successful in this word problem solving.

The protocol of K and SJ (C/HA) could be summarised as:

1. Reading of the word problem;
2. Accurate analysis of the word problem. This was indicated by the diagram drawn which correctly represented the relationship between the givens and the unknown; and
3. Exploration (metacognitive), in which they failed to clarify the uncertainties they encountered with regard to the goal of the word problem. As a result, they reallocated their resource away from the goal of the problem.

They did not solve the word problem successfully.

2.8 The Full Analysis of E and XF's (C/LA) Think Aloud Protocol

The overall structure of the solution analysis for the N3 word problem protocol is shown in Figure 6.8. This involves 75 items by the interacting students (see Appendix I).

Episode 1: Reading (Item 1)

K and SJ began by reading the word problem aloud.

Episode 2: Analysis (Items 2 - 16)

They decided to draw a diagram. However, they did not establish the goal of the word problem. Therefore, they appeared to be confused to what needed to be drawn.

E: draw the 3 of them together (3)
XF: 3 / how? (7)
E: draw the 3 of them together
XF: 1 part?
E: yes (4)
XF: why must we draw? / Why must we draw 3 of them? (5)

Episode 3: Reading (Item 17)

After trying to seek clarification from E for the rationale of drawing '*3 of them together*' and not getting any appropriate response, XF returned to rereading the word problem silently, indicated by the movement of the cursor on the screen.

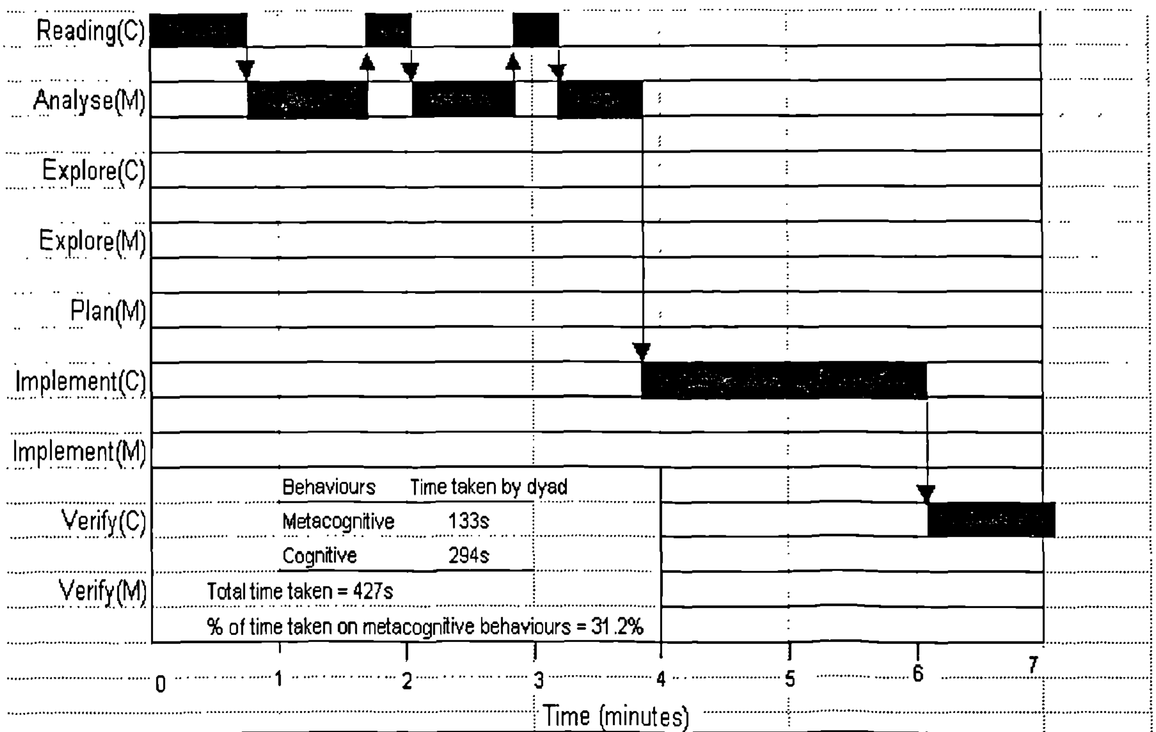


Figure 6.8: A timeline representation of E and XF (C/LA) solving 1N3

Episode 4: Analysis (Items 18 - 21)

Suddenly, XF had a sudden flash of insight to what needed to be drawn first and this decision was supported by a rationale:

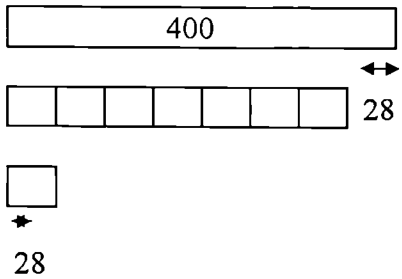
XF: we draw (2) eh (5) ah:: another model with 7 parts
E: why?
XF: eh because they say Jing Hao received 7 times

Episode 5: Reading (Items 22 - 23)

XF verbalised parts of the word problem statement to E, reminding him of the condition which would enable them to draw the number of units representing the number of Joe Ee's marbles.

Episode 6: Analysis (Items 24 - 35)

XF continued to direct E to draw the whole diagram and label all the vital information from the word problem. The final diagram is shown below.



Episode 7: Implementation (cognitive) (Items 36 - 64)

After establishing the relationship between the givens and the unknown from the word problem, both of them proceeded to implementation. They appeared to be engaged in a coherent and well-structured series of calculations. However, their focus appeared to be on the calculations and not on monitoring their word problem solving process.

Episode 8: Verification (cognitive) (Items 65 - 75)

They observed that the solution was appropriate and were satisfied with the outcome. They confidently typed the final statement to indicate that the word problem was solved and a solution was provided. Again, they did not check on the reasonableness of the solution or devise a test to check if the solution satisfied the conditions of the word problem.

The protocol for XF and E (C/LA) could be summarised as an orderly progression of activity, Read → Analyse → Read → Analyse → Read → Analyse → Implement (cognitive) → Verify (cognitive), which led to their success in solving the word problem.

3. **Numerical Presentation and Description of Dyads' Think Aloud Protocol Data**

The think aloud protocols were categorised into episodes (cognitive or metacognitive). The time taken for each episode (cognitive or metacognitive) coded for each dyad in posttest 2 and delayed posttest 2 of the N2, N3, N4 and F2 word problems was recorded. Then, a profile of the students' metacognitive contribution in the word problem solving process was calculated using the time taken (and %) devoted to metacognitive behaviours divided by the total time taken for the whole word problem solving process. The same was done for the students' cognitive behaviours. This was to examine the relationship between the length of time (and %) students devoted to cognitive and metacognitive behaviours and their mathematical word problem solving performance. These data are displayed in Tables 6.1 to 6.8.

3.1 **Length (and %) of Time Devoted to Metacognitive and Cognitive Behaviours**

The following display tables, Tables 6.1 and 6.2; 6.3 and 6.4; 6.5 and 6.6; and 6.7 and 6.8, show the time taken and the percentage of behaviours coded as metacognitive and cognitive for post-test 2 (1N2, 1N3, 1N4 and 1F2) and delayed post-test 2 (2N2, 2N3, 2N4 and 2F2) of the word problems N2, N3, N4 and F2 (see Appendix C) respectively.

	Dyads in School 1				Dyads in School 2				
Behaviour Category	A and CC* (T/HA)	ES and J (T/LA)	SM and B* (C/HA)	B and P (C/LA)	L and JK (T/HA)	HM and XY* (T/LA)	K and SJ* (C/HA)	E and XF* (C/LA)	Total
Metacognitive	436 (80.4)	310 (71.8)	197 (60.5)	539 (40.9)	398 (97.1)	451 (94.4)	663 (82.2)	314 (70.4)	3308 (69.6)
Cognitive	106 (19.6)	122 (28.2)	128 (39.5)	778 (59.1)	12 (2.9)	27 (5.6)	143 (17.8)	132 (29.6)	1448 (30.4)
Total	542 (100)	432 (100)	325 (100)	1317 (100)	410 (100)	478 (100)	806 (100)	446 (100)	4756 (100)

* correct solution

Table 6.1: Time in Seconds (and %) devoted to Cognitive and Metacognitive Behaviours Per Dyad for Word Problem N2 during Posttest 2 (1N2)

	Dyads in School 1				Dyads in School 2				
Behaviour Category	A and CC* (T/HA)	ES and J* (T/LA)	SM and B* (C/HA)	B and P (C/LA)	L and JK* (T/HA)	HM and XY* (T/LA)	K and SJ* (C/HA)	E and XF* (C/LA)	Total
Metacognitive	298 (96.1)	452 (76.1)	197 (60.5)	539 (40.9)	378 (95.7)	621 (96.9)	663 (82.2)	314 (70.4)	3462 (71.6)
Cognitive	12 (3.9)	142 (23.9)	128 (39.5)	778 (59.1)	17 (4.3)	20 (3.1)	143 (17.8)	132 (29.6)	1372 (28.4)
Total	310 (100)	594 (100)	325 (100)	1317 (100)	395 (100)	641 (100)	806 (100)	446 (100)	4834 (100)

* correct solution

Table 6.2: Time in Seconds (and %) devoted to Cognitive and Metacognitive Behaviours Per Dyad for Word Problem N2 during Delayed Posttest 2 (2N2)

	Dyads in School 1				Dyads in School 2				
Behaviour Category	A and CC* (T/HA)	ES and J (T/LA)	SM and B* (C/HA)	B and P (C/LA)	L and JK* (T/HA)	HM and XY* (T/LA)	K and SJ (C/HA)	E and XF* (C/LA)	Total
Metacognitive	466 (66.0)	978 (69.7)	182 (38.7)	304 (47.6)	254 (90.7)	490 (73.3)	372 (91.9)	133 (31.2)	3179 (63.6)
Cognitive	240 (34.0)	428 (30.3)	287 (61.3)	335 (52.4)	26 (9.3)	179 (26.8)	33 (8.1)	294 (68.8)	1822 (36.4)
Total	706 (100)	1406 (100)	469 (100)	639 (100)	280 (100)	669 (100)	405 (100)	427 (100)	5001 (100)

* correct solution

Table 6.3: Time in Seconds (and %) devoted to Cognitive and Metacognitive Behaviours Per Dyad for Word Problem N3 during Posttest 2 (1N3)

	Dyads in School 1				Dyads in School 2				
Behaviour Category	A and CC* (T/HA)	ES and J* (T/LA)	SM and B* (C/HA)	B and P (C/LA)	L and JK* (T/HA)	HM and XY* (T/LA)	K and SJ (C/HA)	E and XF* (C/LA)	Total
Metacognitive	262 (95.6)	740 (93.1)	182 (38.7)	304 (47.6)	381 (94.8)	612 (95.9)	372 (91.9)	133 (31.2)	2986 (73.7)
Cognitive	12 (4.4)	55 (6.9)	287 (61.3)	335 (52.4)	21 (5.2)	26 (4.1)	33 (8.1)	294 (68.8)	1063 (26.3)
Total	274 (100)	795 (100)	469 (100)	639 (100)	402 (100)	638 (100)	405 (100)	427 (100)	4049 (100)

* correct solution

Table 6.4: Time in Seconds (and %) devoted to Cognitive and Metacognitive Behaviours Per Dyad for Word Problem N3 during Delayed Posttest 2 (2N3)

	Dyads in School 1				Dyads in School 2				
Behaviour Category	A and CC* (T/HA)	ES and J (T/LA)	SM and B* (C/HA)	B and P (C/LA)	L and JK* (T/HA)	HM and XY* (T/LA)	K and SJ* (C/HA)	E and XF* (C/LA)	Total
Metacognitive	750 (74.3)	578 (47.7)	200 (37.9)	62 (8.3)	206 (91.2)	1306 (98.6)	255 (77.8)	121 (29.7)	3478 (60.0)
Cognitive	260 (25.7)	633 (52.3)	328 (62.1)	681 (91.7)	20 (8.8)	19 (1.4)	73 (22.2)	285 (70.3)	2299 (40.0)
Total	1010 (100)	1211 (100)	528 (100)	743 (100)	226 (100)	1325 (100)	328 (100)	406 (100)	5777 (100)

* correct solution

Table 6.5: Time in Seconds (and %) devoted to Cognitive and Metacognitive Behaviours Per Dyad for Word Problem N4 during Posttest 2 (1N4)

	Dyads in School 1				Dyads in School 2				
Behaviour Category	A and CC* (T/HA)	ES and J (T/LA)	SM and B* (C/HA)	B and P (C/LA)	L and JK* (T/HA)	HM and XY* (T/LA)	K and SJ* (C/HA)	E and XF* (C/LA)	Total
Metacognitive	694 (96)	350 (71.8)	200 (37.9)	62 (8.3)	301 (94.7)	715 (96.2)	255 (77.8)	121 (29.7)	2698 (63.1)
Cognitive	29 (4)	137 (28.2)	328 (62.1)	681 (91.7)	17 (5.4)	28 (3.8)	73 (22.2)	285 (70.3)	1578 (36.9)
Total	723 (100)	487 (100)	528 (100)	743 (100)	318 (100)	743 (100)	328 (100)	406 (100)	4276 (100)

* correct solution

Table 6.6: Time in Seconds (and %) devoted to Cognitive and Metacognitive Behaviours Per Dyad for Word Problem during Delayed Posttest 2 (2N4)

	Dyads in School 1				Dyads in School 2				
Behaviour Category	A and CC (T/HA)	ES and J (T/LA)	SM and B* (C/HA)	B and P* (C/LA)	L and JK* (T/HA)	HM and XY* (T/LA)	K and SJ* (C/HA)	E and XF (C/LA)	Total
Metacognitive	539 (78.7)	1390 (72.9)	256 (70.9)	466 (57.0)	133 (62.7)	334 (94.4)	210 (92.5)	947 (69.7)	4275 (72.2)
Cognitive	146 (21.3)	516 (27.1)	105 (29.1)	351 (43.0)	79 (37.3)	20 (5.6)	17 (8.5)	410 (30.3)	1644 (27.8)
Total	685 (100)	1906 (100)	361 (100)	817 (100)	212 (100)	354 (100)	227 (100)	1357 (100)	5919 (100)

* correct solution

Table 6.7: Time in Seconds (and %) devoted to Cognitive and Metacognitive Behaviours Per Dyad for Word Problem F2 during Posttest 2 (1F2)

	Dyads in School 1				Dyads in School 2				
Behaviour Category	A and CC (T/HA)	ES and J (T/LA)	SM and B* (C/HA)	B and P* (C/LA)	L and JK* (T/HA)	HM and XY* (T/LA)	K and SJ* (C/HA)	E and XF (C/LA)	Total
Metacognitive	417 (93.9)	799 (89.9)	256 (70.9)	466 (57.0)	203 (93.1)	1302 (91.8)	210 (92.5)	947 (69.7)	4600 (80.3)
Cognitive	27 (6.1)	90 (10.1)	105 (29.1)	351 (43.0)	15 (6.9)	116 (8.2)	17 (8.5)	410 (30.3)	1131 (19.7)
Total	444 (100)	889 (100)	361 (100)	817 (100)	218 (100)	1418 (100)	227 (100)	1357 (100)	5731 (100)

* correct solution

Table 6.8: Time in Seconds (and %) devoted to Cognitive and Metacognitive Behaviours Per Dyad for Word Problem F2 during Delayed Posttest 2 (2F2)

3.2 Comparison of Length (and %) of Time Devoted to Metacognitive and Cognitive Behaviours in Treatment and Control Groups

Tables 6.9 and 6.10 show the time taken (and %) of behaviours coded as metacognitive and cognitive in treatment groups (A and CC; ES and J; L and JK; and HM and XY) and control groups (SM and B; B and P; K and SJ; and E and XF) for post-test 2 (1N2, 1N3, 1N4 and 1F2) and delayed post-test 2 (2N2, 2N3, 2N4 and 2F2) of the N2, N3, N4 and F2 word problems respectively. This is to compare the length of time (and %) devoted to metacognitive and cognitive behaviours between treatment and control students, and examine if this relates to students' word problem solving.

	1N2		1N3		1N4		1F2	
Behaviour	Treatment	Control	Treatment	Control	Treatment	Control	Treatment	Control
Metacognitive	1595 (85.7)	1713 (59.2)	2188 (71.5)	991 (51.1)	2840 (75.3)	638 (31.8)	2396 (75.9)	1879 (68.0)
Cognitive	267 (14.3)	1181 (40.8)	873 (28.5)	949 (48.9)	932 (24.7)	1367 (68.2)	761 (24.1)	883 (32.0)
Total	1862 (100.0)	2894 (100.0)	3061 (100.0)	1940 (100.0)	3772 (100.0)	2005 (100.0)	3157 (100.0)	2762 (100.0)

Table 6.9: Time in Seconds (and %) devoted to Cognitive and Metacognitive Behaviours in Treatment and Control Dyads for Word Problems N2, N3, N4 and F2 during Posttest 2 (1N2, 1N3, 1N4 and 1F2)

The results in Table 6.9 appear to indicate that treatment students devote more time to metacognitive behaviours compared with control students. With regard to the 1N3 word problem, three out of the four pairs of students in the Treatment groups were successful in their word problem solving. However, the group which was not successful devoted more time to regulating their word problem solving compared to those who were not successful in their word problem solving in the control groups. For example, ES and J (T/LA) was not successful but they devoted 69.7% of their time to metacognitive actions compared to Group B and P (C/LA) who devoted 47.6% of their time to metacognitive actions (see Table 6.3).

	2N2		2N3		2N4		2F2	
Behaviour	Treatment	Control	Treatment	Control	Treatment	Control	Treatment	Control
Meta-cognitive	1749 (90.2)	1713 (59.2)	1995 (94.6)	991 (51.1)	2060 (90.7)	638 (31.8)	2721 (91.6)	1879 (68.0)
Cognitive	191 (9.8)	1181 (40.8)	114 (5.4)	949 (48.9)	211 (9.3)	1367 (68.2)	248 (8.4)	883 (32.0)
Total	1940 (100.0)	2894 (100.0)	1741 (100.0)	1940 (100.0)	2271 (100.0)	2005 (100.0)	2969 (100.0)	2762 (100.0)

Table 6.10: Time in Seconds (and %) devoted to Cognitive and Metacognitive Behaviours in Treatment and Control Dyads for Word Problems N2, N3, N4 and F2 during Delayed Posttest 2 (2N2, 2N3, 2N4 and 2F2)

The results in Table 6.10 also appear to indicate that treatment students devote more time to metacognitive behaviours compared with control students. With regard to the 2N3 word problem, the only dyad that was not successful in word problem solving during posttest 2 was successful in their delayed posttest 2. The time devoted to metacognitive activities had also risen. For example, ES and J (T/LA) now devoted 93.1% instead of 69.7% of their time to metacognitive behaviours for the 2N3 word problem (see Tables 6.3 and 6.4).

With regard to the 2N4 word problem, ES and J (T/LA), which was the only treatment dyad not successful in posttest 2, was still not successful in their word problem solving (see Tables 6.5 and 6.6) in delayed posttest 2. However, they devoted more time to metacognitive behaviours. For example, they devoted 47.7% of their time during post-test 2 compared to 71.8% of their time devoted to metacognitive behaviours during delayed post-test 2. Visual inspection suggests that their failure on both occasions was due to their inability to select the appropriate strategy even though they devoted more time to regulating their word problem solving on their second attempt. This weakness appeared to be consistent with ES and J's (T/LA) word problem solving. Their limited metacognitive resources (see section 2.2) often prevented them from selecting the appropriate strategy even though they monitored their word problem solving behaviours (see section 2.2 for the full analysis of ES and J's protocol). However, on the whole, it was observed that they made an improvement in their word problem solving performance in delayed posttest 2, being able to solve 50% of the word problems compared with 0% in posttest 2. This finding concurs with the results in Chapter Five, section 3.2, number 6 d, where it was observed that lower achievers appeared to demonstrate a delayed improvement compared with higher achievers and the greater improvement was evident between posttest and delayed posttest means. This phenomenon will be discussed in Chapter Seven, section 2.

Based on the above results, there is no evidence that metacognitive training inevitably leads to efficaciousness in mathematical word problem solving performance. However, there is evidence that lower achievers in the treatment group (refer to the analysis of ES and J's protocol in section 2.2 and HM and XY's protocol in section 2.6) are devoting more time to regulating and monitoring their word problem solving process even after a prolonged period of six weeks without metacognitive instruction (see Tables 6.1 to 6.8).

3.3 Length (and %) of Time Devoted to Episodes (Metacognitive and Cognitive) in modified Artzt and Armour-Thomas' Framework

Tables 6.11 and 6.12 show the percentage of cognitive and metacognitive behaviours coded by category for the students while solving the N3 word problem during posttest 2 and delayed posttest 2 respectively. The N3 word problem was chosen so that analysis on students' devotion to metacognitive behaviours can be described in conjunction with the analysis of their think aloud protocol data in section 2.

In posttest 2 (see Table 6.11), across all dyads, there were 3179 seconds (63.6% of both metacognitive and cognitive behaviours) devoted to behaviours coded as metacognitive. Of these 1262 seconds were in the category *explore (metacognitive)* (39.7% of all metacognitive behaviours) and 920 seconds in the category *analyse* (28.9% of all metacognitive behaviours). In other words, the greatest percentage of metacognitive behaviours was in explore (metacognitive), followed by analyse. In the category *explore (metacognitive)*, ES and J (T/LA) had the highest percentage of this behaviour – 61.1% for explore (metacognitive) and L and JK (T/HA), HM and XY (T/LA), E and XF (C/LA), A and CC (T/HA) and SM and B (C/HA) had the lowest percentage of this behaviour - 0% for explore (metacognitive). In the category *analyse*, B and P (C/LA) had the lowest percentage of this behaviour - 0% for analyse, and HM and XY (T/LA) had the highest percentage of this behaviour - 41.1% for analyse.

Across all dyads, 1822 seconds (36.4% of both metacognitive and cognitive behaviours) were devoted to behaviours coded as cognititve (see Table 6.11). Of these, 847 seconds were in the category *read* (46.5% of all cognitive behaviours) and 462 seconds were in the category *implement (cognitive)* (25.4% of all cognitive behaviours). In other words, the greatest percentage of cognitive behaviours was in read, followed by implement (cognitive).

	Dyads in School 1				Dyads in School 2			
Behaviour Category	A and CC* (T/HA)	ES and J (T/LA)	SM and B* (C/HA)	B and P (C/LA)	L and JK* (T/HA)	HM and XY* (T/LA)	K and SJ (C/HA)	E and XF* (C/LA)
Meta-cognitive								
Analyse	110 (15.6)	12 (0.9)	133 (28.3)	0 (0.0)	93 (33.2)	275 (41.1)	164 (40.5)	133 (31.2)
Explore	0 (0.0)	859 (61.1)	0 (0.0)	304 (47.6)	0 (0.0)	0 (0.0)	99 (24.4)	0 (0.0)
Plan	30 (4.3)	0 (0.0)	49 (10.4)	0 (0.0)	0 (0.0)	55 (8.2)	13 (3.2)	0 (0.0)
Implement	176 (24.9)	0 (0.0)	0 (0.0)	0 (0.0)	88 (31.4)	76 (11.4)	96 (23.7)	0 (0.0)
Verify	150 (21.2)	107 (7.6)	0 (0.0)	0 (0.0)	73 (26.1)	84 (12.6)	0 (0.0)	0 (0.0)
Cognitive								
Read	106 (14.9)	428 (30.3)	52 (11.1)	79 (12.4)	26 (9.3)	27 (4.0)	33 (8.1)	96 (22.5)
Explore	134 (19.0)	0 (0.0)	0 (0.0)	256 (40.1)	0 (0.0)	0 (0.0)	0 (0.0)	0 (0.0)
Implement	0 (0.0)	0 (0.0)	173 (36.9)	0 (0.0)	0 (0.0)	152 (22.8)	0 (0.0)	137 (32.1)
Verify	0 (0.0)	0 (0.0)	62 (13.2)	0 (0.0)	0 (0.0)	0 (0.0)	0 (0.0)	61 (14.3)

* correct solution

Table 6.11: Time in Seconds (and %) devoted to Cognitive and Metacognitive Behaviours by Word Problem Solving Dyads for Word Problem N3 during Posttest 2 (1N3)

In delayed posttest 2 (see Table 6.12), across all dyads, there were 2986 seconds (73.7% of both metacognitive and cognitive behaviours) devoted to behaviours coded as metacognitive. Of these 984 seconds were in the category *analyse* (33.0% of all metacognitive behaviours), followed by 761 seconds in the category *implement (metacognitive)* (25.5% of all metacognitive behaviours). The greatest percentage of metacognitive behaviours was in analyse and then in implement (metacognitive). In the category analyse, Group SM and B (C/LA) had the highest percentage of this behaviour – 40.5% for analyse. Group HM and XY (T/LA) now had the highest percentage of time devoted to implement (metacognitive) - 50.5% for implement (metacognitive).

	Dyads in School 1				Dyads in School 2			
Behaviour Category	A and CC* (T/HA)	ES and J* (T/LA)	SM and B* (C/HA)	B and P (C/LA)	L and JK* (T/HA)	HM and XY* (T/LA)	K and SJ (C/HA)	E and XF* (C/LA)
Meta-cognitive								
Analyse	70 (25.5)	178 (22.4)	133 (28.3)	0 (0.0)	148 (36.8)	158 (24.8)	164 (40.5)	133 (31.2)
Explore	0 (0.0)	182 (22.9)	0 (0.0)	304 (47.6)	0 (0.0)	0 (0.0)	99 (24.4)	0 (0.0)
Plan	30 (10.9)	141 (17.7)	49 (10.4)	0 (0.0)	15 (3.7)	42 (6.6)	13 (3.2)	0 (0.0)
Implement	60 (21.9)	132 (16.6)	0 (0.0)	0 (0.0)	151 (37.6)	322 (50.5)	96 (23.7)	0 (0.0)
Verify	102 (37.2)	107 (13.5)	0 (0.0)	0 (0.0)	67 (16.7)	90 (14.1)	0 (0.0)	0 (0.0)
Cognitive								
Read	12 (4.4)	55 (6.9)	52 (11.1)	79 (12.4)	21 (5.2)	26 (4.1)	33 (8.1)	96 (22.5)
Explore	0 (0.0)	0 (0.0)	0 (0.0)	256 (40.1)	0 (0.0)	0 (0.0)	0 (0.0)	0 (0.0)
Implement	0 (0.0)	0 (0.0)	173 (36.9)	0 (0.0)	0 (0.0)	0 (0.0)	0 (0.0)	137 (32.1)
Verify	0 (0.0)	0 (0.0)	62 (13.2)	0 (0.0)	0 (0.0)	0 (0.0)	0 (0.0)	61 (14.3)

* correct solution

Table 6.12: Time in seconds (and %) devoted to Cognitive and Metacognitive Behaviours by Word Problem Solving Dyads for Word Problem N3 during Delayed Posttest 2 (2N3)

Across all dyads, 1063 seconds (26.3% of both metacognitive and cognitive behaviours) were devoted to behaviours coded as cognitive (see Table 6.12). Of these, 374 seconds were in the category *read* (35.2% of all cognitive behaviours) and 310 seconds were in the category *implement (cognitive)* (29.2% of all cognitive behaviours). In other words, the greatest percentage of cognitive behaviours was still in read, followed by implement (cognitive).

4. **The Qualitative Component: A Summary**

This sub-section describes an analysis that is related to research question 4, which is to explore the role of metacognition in word problem solving and its influence on word problem solving performance. This summary was stimulated by the research literature (Goos & Galbraith, 1996) which presents the characteristic structure of two students' problem solving protocols (op cit p. 254) and relates it to their problem solving performance. In this study, I am interested in summarising the students' overall progression of word problem solving activity based on the four word problems (N2, N3, N4 and F2), and establish a unique progression of word problem solving activity for each dyad. Then I want to compare each dyad's unique progression of word problem solving activity and look for emergent patterns that might arise from this comparison. The next step is to cluster students with similar progression of word problem solving activity into categories. This section presents the summary findings of the above form of analysis. The findings from this analysis will help to relate the role of students' metacognition during word problem solving to their word problem solving performance.

The analyses in sections 2 and 3 based on students' think aloud protocols suggest that the progression of students' word problem solving activity are represented by five types of cognitive-metacognitive word problem solving models (see Appendix L). These models were generated from the modified Artzt & Armour-Thomas cognitive-metacognitive model described in Chapter Four, Figure 4.1. The five main types of cognitive-metacognitive word problem solving models will be labeled as Type P, Type Q, Type R, Type S and Type T cognitive-metacognitive word problem solving models. In the following sub-sections, the types of cognitive-metacognitive word problem solving models will be described and a brief summary given with regard to the progression of students' word problem solving activity. The arguments presented are based on the full set of data obtained from all of the students' four (N2, N3, N4 and F2) word problem solving think aloud protocols, although only one of them - the N3 word problem (see Appendix C and Chapter Four, section 5.2.2) will be used as an illustration. These summaries, together with the analyses carried out in sections 3 and 4, demonstrate that control students' work at the metacognitive control level for the higher/lower achievers stand in contrast to the metacognitive control behaviour of higher/lower achievers in the treatment groups after metacognitive training. For example, treatment lower achievers, students HM and XY, whose progression of word problem solving activity was Type P (see section 4.1) cognitive-metacognitive word problem solving, devoted 91.2% of their time to metacognitive behaviours (see Table 6.5) on the N4 word problem compared with control lower achievers, students B and P, whose progression of word problem solving activity was Type R (see section 4.3) cognitive-metacognitive word problem solving, devoted 8.3% of their time to metacognitive behaviours (see Table 6.5).

4.1 **Type P Cognitive-metacognitive Word Problem Solving Model**

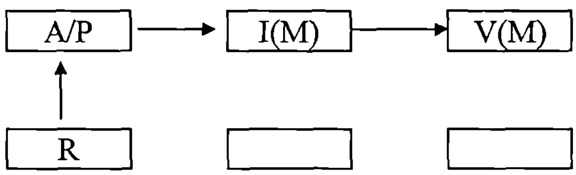


Figure 6.9: Type P Cognitive-metacognitive Word Problem Solving Model

A Type P cognitive-metacognitive word problem solving model represents progression of word problem solving activity that is well-regulated and controlled. In Type P, the outcome of the students' word problem solving appears to be influenced by their ability to be systematic in their progression of word problem solving activity. The progression of word problem solving activity of L and JK (able to solve 75% of the word problems), K and SJ (able to solve 75% of the word problems), and HM and XY (able to solve 100% of the word problems) is the Type P cognitive-metacognitive word problem solving (see Appendix L). Figure 6.9 illustrates Type P cognitive-metacognitive word problem solving.

With respect to the N3 word problem (see Appendix C), the dyads, whose progression of word problem solving activity is Type P cognitive-metacognitive word problem solving, are described as follows:

4.1.1 L and JK's (T/HA) Progression of Word Problem Solving Activity

L and JK's analysis during the word problem solving process was coherent and they appeared to have clarified all their doubts at the beginning of the session. Descriptions from section 2.5 showed them to be consistently focused, goal driven, well-regulated and explicit in their decision making. They also engaged in assessments (local and global) throughout their word problem solving. These strengths appeared to have contributed to their success in their word problem solving. Figure 6.9 illustrates the distinctive characteristic of the students' progression of word problem solving activity. They devoted the highest percentage of their time, 33.2% (see Table 6.11) to analysing the word problem situation, followed by 31.4% and 26.1% of their time to implement (metacognitive) and verify (metacognitive) respectively (see Table 6.11). They appeared to be satisfied only when they had checked that their solution satisfied the conditions of the word problem.

During delayed post-test 2, the protocol for L and JK (T/HA) could also be summarised as a well-regulated progression of activity, Read → Analyse → Implement (metacognitive) → Verify (metacognitive), which led to their success in solving the word problem. They also seemed to be in control of their cognitive actions throughout their word problem solving, as illustrated by the following exchanges.

L: so this is Sharon
JK: must plus another / eh Sharon part
L: how come?
JK: then can put 7 times lah
L: but / you don't know how much Cindy got what?
JK: you put here Sharon, Cindy.
then / wait wait you put here Sharon, then here Cindy what /
because Sandra is 7 times / of the total amount

The control of their word problem solving was evidenced by the percentage of time they devoted to metacognitive activities: analyse (36.8%); plan (3.7%); implement (metacognitive) (37.6%); and verify (metacognitive) (16.7%) (see Table 6.12). Their progression of word problem solving activity remained as Type P as illustrated in Figure 6.9.

This pair is one of the two T/HA pairs. Out of the four word problems, they only failed (1N2, see Appendix C) in one of their word problem solving attempts during posttest 2. The cause of this failure appeared to stem from L's careless misinterpretation of the concept in the word problem. In delayed post-test 2, they were successful in all their word problem solving.

4.1.2 **K and SJ's (C/HA) Progression of Word Problem Solving Activity**

Type P cognitive-metacognitive word problem solving is a typical description of K and SJ's progression of word problem solving activity. The students consistently demonstrated their ability to analysis the word problem coherently by drawing diagrams to show the relationship between the facts of the word problem situation. They also appeared to regulate their decisions throughout their word problem solving. However, with respect to the N3 word problem (see section 2.7), they appeared to be moving towards the goal of the problem, when suddenly, K misinterpreted the relationship between the givens and the unknown in the diagram they had drawn. They monitored their strategy but their assessment was weak, so the assessment did not help them move away from the inappropriate chosen path. They devoted the last 99 seconds (24.4%) of their word problem solving to explore (metacognitive) (see Table 6.11).

4.1.3 **HM and XY's (T/LA) Progression of Word Problem Solving Activity**

Type P cognitive-metacognitive word problem solving is also a typical description of HM and XY's progression of word problem solving activity. HM and XY also analysed their word problem coherently by drawing diagrams to show the relationship between the facts of the word problem situation. Next, they made explicit plans which they appeared to implement with consistent evaluation. They would end their word problem solving with verification to check if the solution met the conditions of the word problem. These strengths contributed to their success in their word problem solving. With respect to the N3 word problem (see section 2.6), they devoted 73.3% of their time to metacognitive activities of which 41.1% was devoted to analyse, 8.2% to plan; 11.4% to implement (metacognitive) and 12.6% to verify (metacognitive) (see Table 6.11).

During delayed post-test 2, the protocol for HM and XY (T/LA) could also be summarised as a well-regulated progression of activity, Read → Analyse → Plan → Implement (metacognitive) → Verify (metacognitive), which led to their success in solving all the word problems. The control of their word problem solving for N3 was again evidenced by the percentage of time they devoted to metacognitive activities: analyse (24.8%); plan (6.6%); implement (metacognitive) (50.5%); and verify (metacognitive) (14.1%) (see Table 6.12). Their progression of word problem solving activity remained as Type P during delayed post-test 2 as illustrated in Figure 6.9.

4.2 **Type Q Cognitive-metacognitive Word Problem Solving Model**

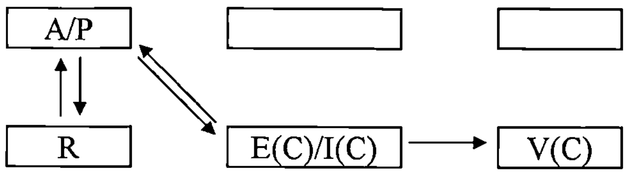


Figure 6.10: Type Q Cognitive-metacognitive Word Problem Solving Model

A Type Q cognitive-metacognitive word problem solving model represents progression of word problem solving activity that tends to focus on word problem solving activities such as reading, analysis and planning before proceeding to implementation. Students would also occasionally engage in explore (cognitive). During exploration, if the students 'hit' on the appropriate strategy, they would return to analysis and/or planning before implementing their strategy. The engaged implementation episode, like the exploration, would unlikely be controlled and regulated. The outcome of the students' word problem solving would appear to depend on the first stage whereby the students would have formulated clear goals to guide them solve the word problems. The progression of word problem solving activity of SM and B (able to solve 100% of the word problems), and E and XF (able to solve 75% of the word problems) is the Type Q cognitive-metacognitive word problem solving (see Appendix L) . Figure 6.10 illustrates Type Q cognitive-metacognitive word problem solving.

With respect to the N3 word problem, the dyads, whose progression of word problem solving activity is Type Q word problem solving, are described as follows:

4.2.1 SM and B's (C/HA) Progression of Word Problem Solving Activity

The progression of word problem solving activity of SM and B's word problem solving is the unique Type Q cognitive-metacognitive word problem solving. With respect to the N3 word problem (see section 2.3), they devoted the first phase of their word problem solving to analysing (28.3%) and planning (10.4%) activities (see Table 6.11). When they had clarified their goals, they proceeded to implement. Monitoring during implementation and verification episodes were apparently deemed unnecessary. For them, they appeared confident that the clear goals at the beginning of the word problem solving would lead them to the solution. This consistent progression of word problem solving activity worked well for them and they were successful in all their word problem solving.

4.2.2 E and XF's (C/LA) Progression of Word Problem Solving Activity

The progression of word problem solving activity of E and XF's word problem solving is also the unique Type Q cognitive-metacognitive word problem solving. Their analysis of the word problem was coherent and they clarified all their doubts. This enabled them to set all the goals at the beginning of the session which resulted in their success in their word problem solving. For this pair, detailed verifications deemed unnecessary for they had assumed that their solution was correct and did not bother to make an assessment of it. This consistent progression of word problem solving activity worked well for them for three of the four word problem solving. With respect to the N3 word problem (see section 2.8), they devoted a substantial amount of time to cognitive activities (68.8%, see Table 6.3): read (22.5%), implement (32.1%) and verify (14.3%) (see Table 6.11).

4.3 Type R Cognitive-metacognitive Word Problem Solving Model

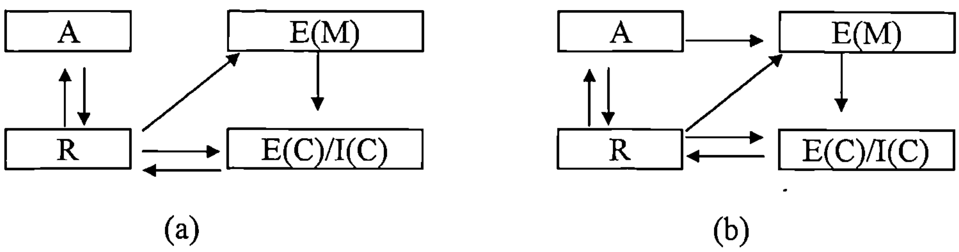


Figure 6.11: Type R Cognitive-metacognitive Word Problem Solving Models

A Type R cognitive-metacognitive word problem solving model represents progression of word problem solving activity that is dominated by exploration and the students tend to have difficulties in solving the word problems. The progression of word problem solving activity of B and P (able to solve 25% of the word problems), and ES and J (unable to solve the word problems) is the Type R cognitive-metacognitive word problem solving (see Appendix L). While B and P, and ES and J monitored their exploratory activities, their monitoring activities did not seem to have a positive effect on their word problem solving outcome. Figure 6.11 illustrates the two forms of Type R cognitive-metacognitive word problem solving.

With respect to the N3 word problem, the dyads, whose progression of word problem solving activity is Type R word problem solving, are described as follows:

4.3.1 B and P's (C/LA) Progression of Word Problem Solving Activity

Type R cognitive-metacognitive word problem solving is a typical description of B and P's progression of word problem solving activity. B and P appeared to have limited resources to aid them in their word problem solving. They engaged in exploration and tried making local assessments at the beginning of the word problem solving session, but their metacognitive decisions were weak and they did not help them. When the students appeared to have 'exhausted' all ideas, they decided to use the superficial coping strategy (Verschaffel & De Corte,1997, p. 87): *"look at the numbers; they will tell you what to do"* which moved them further away from the goal of their word problem solving. With respect to the N3 word problem (see section 2.4), they devoted 40.1% (see Table 6.3) of their time to cognitive activities: read (12.4%) and explore (cognitive) (40.1%) (see Table 6.11).

4.3.2 ES and J's (T/LA) Progression of Word Problem Solving Activity

Type R cognitive-metacognitive word problem solving is also a typical description of ES and J's progression of word problem solving activity during posttest 2. This dyad also appeared to have limited resources which might have contributed to their failure in their word problem solving. They tried making assessments throughout their word problem solving, but the decisions they made did not help them move away from inappropriate chosen solution paths. With respect to the N3 word problem (see section 2.2), ES and J devoted most of their word problem solving activities to exploration (metacognitive) : 61.1% (see Table 6.11). Not only did they fail in solving N3 word problem, but they also failed in their other word problem solving during posttest 2.

In delayed posttest 2, however, ES and J were successful in solving the N3 word problem. The time devoted to exploration (metacognitive) was reduced to 22.9%. They devoted more time to metacognitive activities such as analyse (22.4%), plan (17.7%), implement (16.6%) and verify (13.5%) (see Table 6.12). Figure 6.12 is a cognitive-metacognitive word problem solving model that represents ES and J's progression of word problem solving activity during delayed posttest 2. This model is a combination of the progression of word problem solving activity of Type P (see section 4.1) cognitive-metacognitive word problem solving and the metacognitive behaviour, *exploration*. This model is labeled as Type T cognitive-metacognitive word problem solving (see Figure 6.12).

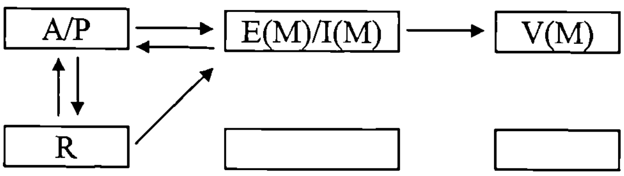


Figure 6.12: Type T Cognitive-Metacognitive Word Problem Solving Model

The following is a summary of ES and J's progression of word problem solving activity during delayed posttest 2 which is represented by Type T cognitive-metacognitive word problem solving. With respect to N3 for delayed posttest 2, the protocol for ES and J (T/LA) could be summarised as:

1. Reading of the word problem;
2. Exploration (metacognitive), where the 'flawed' strategy was evaluated and found to be ineffective. As a result, the 'flawed' strategy was abandoned. Then, new information was discovered and a new procedure was proposed; and
3. An orderly progression of activity, Analyse → Plan → Implement (metacognitive) → Verify (metacognitive), which led to their success in solving the word problem.

This dyad was successful in solving 50% of the word problems during delayed posttest 2.

4.4 Type S Cognitive-metacognitive Word Problem Solving Model

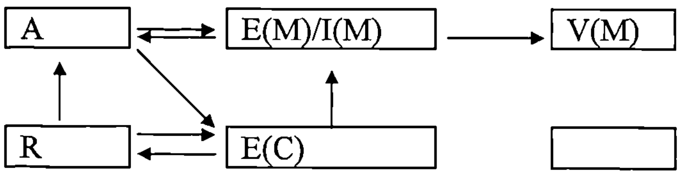


Figure 6.13: Type S Cognitive-metacognitive Word Problem Solving Model

A Type S cognitive-metacognitive word problem solving model represents progression of word problem solving activity that is dominated by exploration. Unlike the dyads, whose progression of word problem solving activity is Type R cognitive-metacognitive word problem solving (see Figure 6.11), the exploratory activities in Type S appear to be well-regulated and controlled. This usually has a positive influence on the students' word problem solving outcome. The progression of word problem solving activity of A and CC (able to solve 75% of the word problems) is the Type S cognitive-metacognitive word problem solving. Figure 6.13 illustrates Type S cognitive-metacognitive word problem solving.

With respect to the N3 word problem, the dyad, A and CC, whose progression of word problem solving activity is Type S cognitive-metacognitive word problem solving, is described as follows:

4.4.1 A and CC's (T/HA) Progression of Word Problem Solving Activity

The progression of word problem solving activity of A and CC's word problem solving is the unique Type S cognitive-metacognitive word problem solving. Most of the time, their word problem solving was dominated by exploratory activities. They analysed and planned at the beginning of their word problem solving session, but it was their regulation during exploration that appeared to work well for them. This behaviour, coupled with their verification (metacognitive), often enabled them to be successful in their word problem solving. With respect to the N3 word problem (see section 2.1), they devoted 66% (see Table 6.3) of their time to metacognitive behaviours: 15.6% on analyse; 4.3% on plan; 24.9% on implement; and 21.2% on verify (see Table 6.11).

During delayed posttest 2, the protocol for A and CC (T/HA) could be summarised as a well-regulated progression of activity Read → Analysis → Plan → Implement (metacognitive) → Verify (metacognitive). This progression of word problem solving activity is represented by Type P cognitive-metacognitive word problem solving as illustrated in Figure 6.9 (see section 4.1). This progression of word problem solving activity may have contributed to 75% success in their word problem solving. During delayed posttest 2, the control of their word problem solving process for N3 was evidenced by the percentage of time, 95.6% (see Table 4), they devoted to metacognitive activities: analyse (25.5%); plan (10.9%); Implement (21.9%); and verify (37.2%) (see Table 6.12).

5. Presentation and Description of Students' Metacognitive Knowledge

The analysis of the teacher interview (see Appendix D) and student questionnaire (see Appendix E) data provides accounts of students' metacognitive knowledge (see Chapter Two, section 2) before the intervention process. Summary descriptions of the students' metacognitive knowledge are based on the students' own opinion and their teachers' judgment as presented below.

5.1 A and CC's (T/HA) Metacognitive Knowledge

In the teacher's opinion, A is good in mathematics, especially in 'visualising' (interview data). A is aware (questionnaire data) that his common errors are making calculation errors and misreading the numbers (givens) in the word problems. He is also aware (questionnaire) that he tends to be fast, confirmed by his teacher (interview), and misinterprets the goals of the word problems by missing out some important conditions in the word problems. A's strategic awareness appears to be well-developed. He doesn't see the need for drawing models (interview) but relies on his own strategy, that is to '*look at the question's meaning to understand the question better; try to do the first step mentally to get ready for what I will have to write; try to do the first step then the second; and read the question again in case I left out something*'. He is very accurate in assessing the level of familiarity and the quality of his solutions. The common reason he gives for his difficulty in solving some of the word problems in the pretest is his inability to understand the question, '*The question was hard and I just couldn't understand.*' In summary, the person, task and strategy components of A's metacognitive knowledge (Garofalo & Lester, 1985) appear to be well-developed. Finally, A believes that he must '*try to solve more word problems in the near future and use new methods to solve the questions to improve.*'

According to the teacher, CC is also good in mathematics, better than A in 'thinking' (interview). CC thinks (questionnaire) his common errors stem from his calculation mistakes and drawing inaccurate models because he has not understood the word problems. CC is fairly accurate in assessing the level of familiarity and quality of his solutions. A major difficulty identified in not being able to solve some of the word problems in the pretest is '*I don't understand the question. I don't know where to start.*' In summary, the person and task components of CC's metacognitive awareness appear to be fairly well-developed. Finally, CC believes that he must '*practise more often and learn new methods to solve the problems.*'

5.2 ES and J's (T/LA) Metacognitive Knowledge

In ES teacher's judgment, ES is very weak in mathematics, especially his mathematical concept foundation (interview). ES is aware (questionnaire) that his major source of errors is computation, confirmed by his teacher (interview). ES is quite accurate in assessing the level of familiarity and the quality of his solutions. It is interesting to note that for ES, the main source of assessing the difficulty of word problems is dependent on the size of the numbers (givens) in the word problems. For example, he identifies (questionnaire) word problems 3 and 4 as word problems he has done well because '*the numbers are simple*', word problems 2 and 5 as word problems he has not done well because '*the number are harder and bigger*' and word problems 6, 7, 8, 9 and 10 as difficult because '*the fractions are more complicated*'. In summary, the person and task components of ES's metacognitive awareness appear to be well-developed. Finally, ES believes that solving more word problems daily would help him get better at solving word problems.

In the judgment of J's teacher, J is also very weak in mathematics, but he is quicker than ES in grasping concepts (interview). However, in the teacher's opinion, he is 'unmotivated' and has shown not to be interested in mathematics (interview). For J, he is aware (questionnaire) that he often draws inaccurate models and he is not good at solving word problems involving fractions. In order to overcome the former difficulty, he claims that he needs to read the word problem carefully. Most of the time, he is dependent (questionnaire) on 'guessing' to solve word problems. He also does not see (questionnaire) the necessity of checking and keeping track of what he is doing while solving word problems. He believes (questionnaire) that he should try and work hard on word problems in order to improve in solving word problems. Like ES, J's accurate assessment of the level of familiarity and quality of solutions are dependent on the size of the numbers (givens) in the word problems. For example, word problems 2, 3 and 4 are identified as word problems he has difficulties because '*it is hard in number*'. In summary, the person and task components of J's metacognitive awareness appear to be well-developed.

5.3 **SM and B's (C/HA) Metacognitive Knowledge**

According to SM's teacher, SM is good in mathematics (interview). SM is aware (questionnaire) that he is careless in his calculations. He is also aware (questionnaire) that he '*gets confused of the numbers*' in word problems because they are '*big numbers*'. In the judgment of his teacher, this difficulty (interview) is due to SM's language 'disability' which causes him to misinterpret the requirements of the word problem. SM's strategic awareness appears to be well-developed. For example, he says (questionnaire) that it is important to read and understand the word problem clearly in order to identify the goals of the word problem before planning all the steps required to solve the word problem. When this is done, he would use the steps planned to keep track of his solution. He is quite accurate in assessing the level of familiarity and quality of his solutions. One reason for identifying the difficulty in solving some of the word problems in the pretest is that '*I cannot keep track of what I am doing.*'. In summary, the person, task and strategy components of SM's metacognitive awareness appear to be well-developed. Finally, SM believes that in order to overcome difficulties in solving difficult word problems, he should '*draw models; do workings; read the story sums a few times; and understanding the story.*'

In the opinion of B's teacher, B is good in mathematics (interview). B is aware (questionnaire) that he makes too many calculation errors, confirmed by his teacher (interview), because he is too fast in his calculation. He is also aware (questionnaire) that he tends to read the word problem too fast and might misunderstand the goal of the word problem. As a result, he might provide an answer which is not the goal of the word problem. This weakness might be the result of what his teacher calls an 'impulsive nature' (interview) and his desire (questionnaire and interview) to reserve more time in thinking 'quicker' ways in solving difficult word problems. B keeps track (questionnaire) by solving the word problem mentally to get an estimate of the answer first or 'putting' the steps in his brain first and then reproducing the working step by step on paper. However, by using this strategy, he tends (questionnaire) to forget to provide a statement for each working step. B believes that in order to get better at solving difficult word problems, '*I have confident and I love the subject. I practise every day and learn the steps to do the sums and I learn how to do short-cuts.*' B is accurate in assessing the level of familiarity and quality of his solutions. In summary, the person, task and strategy components of B's metacognitive awareness appear to be well-developed.

5.4 **B and P's (C/LA) Metacognitive Knowledge**

In the opinion of B's teacher, B is a bright boy and will only perform when one showers him with attention (interview). In mathematics, according to his teacher's judgment, he is under-performing (interview). B is aware (questionnaire) that his major source of difficulty in solving word problems is his inability to understand the word problems and as a consequence, he might draw the models incorrectly. B's assessment of the level of familiarity and quality of his solutions are quite accurate. In summary, the person and task components of B's metacognitive awareness appear to be fairly well-developed. Finally, B believes that he '*should practice more and harder to get better.*'

According to the judgment of P's teacher, P is weak in mathematics and slower than B in grasping new topics (interview). His teacher observes that P has a short concentration span and would tend to give up easily when the word problems are too difficult (interview). For P, he is aware (questionnaire) that he has difficulties in expressing himself and does not often write a sentence after each working step. He knows that this is a weakness because without the sentence he tends to forget what he has done when he checks his word problem solving. He is also aware (questionnaire) that the other errors arise from careless calculations and drawing inaccurate models. It is interesting to note that for him, drawing models in solving word problems is optional. He feels (questionnaire) that it is more important to '*revise my work everyday doing my tuition book. I can also take notes down if I don't really know how to do the sum. The most important is pay attention in class.*' P is quite accurate in assessing the quality of his level of familiarity and quality of his solutions. In summary, the person and task components of P's metacognitive awareness appear to be well-developed.

5.5 L and JK's (T/HA) Metacognitive Knowledge

In the opinion of L's teacher, L is very good in mathematics and is one of the best in his class (interview). L perceives (questionnaire data) that his common errors are mechanical (i.e. carelessness), also confirmed by his teacher (interview). His strategic awareness is well-developed. For example, he is able to describe in detail (questionnaire) how he would plan and write down all the working steps and their statements systematically so that he can keep track of what he is doing. He explains that this method also enables the examiner to follow the logic of his solution. He also claims (questionnaire) that he enjoys thinking of the shortest way while solving word problems. He is able (questionnaire) to give an accurate assessment of the difficulty and level of familiarity of word problems and a sound estimate of the correctness of his solutions. In summary, the person, strategy and task components of L's metacognitive knowledge appear to be well developed, as he is able to appreciate his abilities and weaknesses and have some understanding of the reasons. Finally, L believes that he needs to '*do more exercises in other assessment books*' to become better at solving word problems.

In JK teacher's judgment, JK is also very good in mathematics, better than L when it comes to calculating and thinking about solutions (interview). According to his teacher, JK is not as 'impulsive' as L (interview) but he knows that a major source of his errors, like L, is mechanical (questionnaire). He has shown the same level of perception as L in judging the familiarity level of questions or in estimating the quality of his solution (questionnaire). However, it appears that due to what his teacher calls 'language disability' (interview), he has not been able to provide detailed descriptions with regard to the strategy components of his metacognitive knowledge. Finally, JK believes that in order to overcome difficulties in solving word problems is to practice solving word problems. In summary, the person and task components of JK's metacognitive knowledge appear to be fairly well developed.

5.6 **HM and XY's (T/LA) Metacognitive Knowledge**

According to HM teacher's opinion, HM is weak in mathematics compared to her classmates (interview). HM is a Taiwanese and only started her Singapore education when she was 10 years old. In her teacher's judgment, HM has made tremendous improvement (interview) since she first started her education in Singapore two years ago. HM declares (questionnaire) that the major source of her errors arises from her desire to use a 'quick' method or her forgetfulness in using '*some important numbers or clues in the word problems*'. She is also aware (questionnaire) that she makes many calculation mistakes. Her strategic awareness is quite well-developed. For example, when she is stuck, she tries to change all units or fractions to the ratio format because '*working in ratio is easier*'. In summary, the task and strategy components of her metacognitive knowledge appear to be well-developed. Finally, HM believes that in order to improve word problem solving, she needs to '*look out for ideas to solve unfamiliar word problems and do practice more sums*'.

In XY teacher's opinion, XY is also weak in mathematics compared to her classmates (interview). The teacher observes (interview) that XY possesses an unrealistic mathematics anxiety, which she declares stems from her own imagination. XY sees (questionnaire) the effectiveness of writing short notes beside her method in order to keep track of what she is doing; and not to rely on mental calculations because '*I want to prevent making calculation mistakes*'. She is accurate in assessing the level of familiarity and quality of her solution. In summary, the person and strategy components of XY's metacognitive awareness appear to be well-developed. Finally, XY believes that in order for her to be better at solving word problems is to '*do more exercise books and always revise my work*'.

5.7 K and SJ's (C/HA) Metacognitive Knowledge

According to K teacher's opinion, K is good in mathematics but his major setback is what she calls a 'language handicap' (interview). He is aware (questionnaire) that he has a language problem and what his teacher calls 'an impulsive nature', confirmed by the teacher (interview). Hence, most of the errors he makes are due to misinterpreting the word problem or overlooking important conditions in the word problem statements. He is also aware (questionnaire) that he is weak in mental calculation, so he would make all calculations on paper. He is accurate in assessing the familiarity and quality of his solution and has a reasonable understanding for them. In summary, the person and strategy components of K's metacognitive awareness appear to be well-developed. Finally, K believes that he needs to '*practice and do more maths sum and to draw simple models because they helped to solve the word problems without the need for workings*'.

According to SJ's teacher, SJ is also good in mathematics and is considered one of the best in his class (interview). SJ is aware (questionnaire) that the major source of his errors is mechanical. Hence, he usually checks all calculations when he thinks the 'math' is difficult for him or when he thinks he has made an error. His strategic awareness is quite well-developed. For example, in order to keep track of his word problem solving, he would '*read my solution again and again and keep in mind what the question wants*'. He is very accurate in assessing the familiarity and quality of his solutions. In summary, the person, strategy and task components of SJ's metacognitive awareness appear to be well-developed. Finally, SJ believes that in order to get better at solving word problems or those he has difficulties in is to read the word problems a few times before answering them, and to check for careless mistakes.

5.8 E and XF's (C/LA) Metacognitive Knowledge

In the opinion of the teacher, E is very weak in mathematics (interview). According to the teacher (interview), not only does E have what she calls 'poor language ability', but he also has a poor foundation in his mathematical concepts. In addition, though he needs guidance in solving multistep word problems, E's teacher observes that he has a good attitude and is willing to try (interview). For E, he is aware (questionnaire) that his major weakness in mathematics is his inability to solve word problems - *'I cannot understand the question even I draw model'*. He believes that one way of overcoming this difficulty is to *'listen to teacher for ways to solve the problems and to practice doing all those difficult questions'*. He is very accurate in assessing the level of familiarity and quality of his solutions on the word problems in the pretest, and is able to give reasons for them. In summary, the person and task components of E's metacognitive awareness appear to be well-developed.

According to XF's teacher, XF is also very weak in mathematics (interview). In the teacher's judgment, XF also has what she calls 'poor language ability' and a poor concept foundation. Unlike E, the teacher observes that XF is not willing to try (interview). For example, according to his teacher (interview), *'whatever is given to him, he gives up. He'll wait for help to be given to him. If you don't help him, he'll do something else. Definitely not your work!'* For XF, he is aware (questionnaire) that his weakness lies in his inability to understand how to draw models and how to label the models. He believes (questionnaire) that by reading the word problem carefully many times and then trying to draw the models can help him overcome this difficulty. He claims (questionnaire) that he is also careless in his calculation. XF is quite accurate in assessing the level of familiarity and the quality of his solutions. In summary, the person and task components of XF's metacognitive awareness appear to be fairly well-developed.

6. Students' Metacognitive Knowledge: Its Influence on Word Problem Solving

The purpose of the teacher interview schedule and student questionnaire is to provide accounts of the students' metacognitive knowledge in the case study design. However, the descriptions of the students' metacognitive knowledge from the teacher interview and student questionnaire data, and the analysis of the students' word problem solving in section 2 illuminated why students behaved in a certain manner during word problem solving. Specifically, it was interesting to observe that each student carried with him/her metacognitive awareness that appeared to influence their word problem solving.

It was observed that students, whose person, task and strategy components of their metacognitive knowledge, appeared to be well-developed seemed to outperform those students whose metacognitive knowledge appeared to be fairly well-developed. For example, students SM and B, L and JK, and HM and XY, all appeared to have well-developed person, task and strategy components and these dyads were able to solve all the word problems in posttest 2 and delayed posttest 2, except L and JK who were not able to solve the N2 word problem in posttest 2. In contrast, student B's person and task components appeared to be fairly well developed. Though P's, his partner, person and task components appeared to be well developed, he was unable to 'rescue' the dyad's word problem solving. It was observed that during word problem solving, P often assessed B's suggestion or provided ideas but they were often ignored by B. They were only able to solve one word problem successfully during posttest 2. This observation appears to agree with researchers like Garofalo and Lester (1985, p. 168) who claim that the interactions of person, task and strategy knowledge have an influence on the decision to regulate one's activity and this in turn affects mathematical performance.

In addition, based on my observation of the students' description of their metacognitive knowledge in section 5 and the analysis of the students' word problem solving in section 2, some form of the students' metacognitive awareness was reflected in their word problem solving. For example, both B and P confessed that they often drew inaccurate models because they did not understand most word problems (see section 5.2). Then, in section 2.4, the students drew flawed diagrams as a result of misinterpreting the word problem. The lack of metacognitive awareness seemed to be one of the factors that contributed to their failure in word problem solving. Another example involved L and JK who liked to write down all the working steps that led to the word problem solution (see section 5.5). In their presentation of the word problem solution in section 2.5, the students ensured that all the working steps and a statement for each of them was provided '*so that the answer is clear to the marker*' (questionnaire). This was one of the factors that led to their success in word problem solving. On the other hand, it is not known if there exists a change in the students' metacognitive awareness after the intervention, especially amongst the treatment students. The present study is not able to address this issue but it would be worthwhile to examine the difference between students' metacognitive knowledge before and after metacognitive training in future studies.

Chapter Seven
Discussion and Interpretation of Findings

1. Introduction

The presentation and description of the quasi-experimental data and think aloud protocol data in Chapters Five and Six provided insights about the effect of metacognitive training on students' word problem solving in a computer environment. The main findings based on the four research questions (see Chapter One, section 4) suggest that

1. metacognitive training results in a greater improvement in mathematical word problem solving performance (see Chapter Five, section 3.2, number 2 and number 4, and section 4.2, number 1);
2. higher achievers demonstrate an overall benefit from cognitive apprenticeship instruction compared with lower achievers (see Chapter Five, section 5) from pretest to delayed posttest;
3. there is no statistical significant evidence that the benefits from metacognitive training on mathematical word problem solving performance varies with the level of students' mathematical achievement (see Chapter Five, section 3.2, number 6c and section 4.2, number 3c); and
4. treatment dyads devote more time to regulating and monitoring (metacognitive behaviours) their word problem solving process (see Chapter Six, section 3.2); the progression of dyads' word problem solving activity can be represented by five distinct cognitive-metacognitive word problem solving models, and students' word problem solving relates to these five types of cognitive-metacognitive word problem solving (see Chapter Six, section 4); and there is a relationship between students' metacognitive knowledge and their word problem solving (Chapter Six, section 6).

This chapter seeks to discuss what influences metacognitive training may have on students' word problem solving in a computer environment and the role of metacognition on students' word problem solving with and without metacognitive training based on the research questions. The review of literature and past research in Chapter Two provide the theoretical framework for this discussion. In order to provide a more holistic and integrated interpretation of the effect of metacognitive training on the mathematical word problem solving of Singapore 11-12 year olds in a computer environment, I will seek to interpret the findings of this study in the light of all the possible influences that metacognitive training may have on students' word problem solving which were illuminated from this study. They will be discussed under the following three main broad headings, namely: learning environment and mathematical word problem solving performance; cognitive perspective and word problem solving; and affect and word problem solving.

In section 2, the discussion of the relationship between the learning environment and students' mathematical word problem solving performance is based on the summary of findings from quasi-experimental study (Chapter Five) and case study (Chapter Six). It was observed that the students were immersed in a learning environment that may have influenced the students' word problem solving performance. The learning environment includes training the students with a CRIME strategy; the pedagogical approach used to promote metacognitive awareness; and using WordMath in word problem solving.

In section 3, the discussion of the relationship between cognitive perspective and students' word problem solving is based on my exploration of the think aloud protocol data in the case study design. There are three subsections in section 3. The first two subsections focus on the role of metacognition and cognition in word problem solving. In the final subsection, there is a summary of the progressions of dyads' word problem solving activity which are represented by five different types of cognitive-metacognitive word problem solving models. This is followed by a discussion about the relationship between dyads' progression of word problem solving activity and their word problem solving outcome.

In section 4, the discussion of the relationship between the affective issues and students' word problem solving is based on the analysis of think aloud protocol, teacher interview and student questionnaire data. My observation reveals that affective factors appear to influence students' word problem solving performance. This observation will be discussed under two headings: students' beliefs and word problem solving performance; and effective dyad collaboration.

Finally, in section 5, there is a summary of the factors, from sections 2 to 4, that appear to have influenced students' word problem solving performance in a computer environment. These factors emerged from my exploration of the factors, namely explicit metacognitive training, cognitive apprenticeship instruction and collaborative interaction, from the theoretical framework presented in Chapter One, section 3, Figure 1.1 and mathematical word problem solving performance. It is hoped that the new emergent factors, see Figure 7.1, will contribute to existing studies in the arena of metacognition in word problem solving.

2. Learning Environment and Mathematical Word Problem Solving Performance

The findings in the present study extend the findings of previous studies showing a positive influence of metacognitive training on students' word problem solving in a computer environment (see Chapter Two, section 4). In particular, based on analysis of mathematical achievement test data from the quasi-experimental study (see Chapter Five, section 3.2) and analysis of word problem solving think aloud protocols from the case study (see Chapter Six, sections 2, 3 and 4), the findings are summarised as follows.

The findings from the quasi-experimental study (see Chapter Five) suggest that:

1. metacognitive training results in a greater improvement in mathematical word problem solving performance (Chapter Five, section 5); and
2. though higher achievers in the treatment classes benefit more from metacognitive training, as evidenced by the improvement in their pretest and posttest 1 means, and their pretest and delayed posttest 1 means, lower achievers demonstrate a greater delayed benefit from metacognitive training than do higher achievers as evidenced by their improvement from posttest 1 to delayed posttest 1 means (Chapter Five, section 3.2, number 6d).

The findings of the case study suggest that lower achievers in treatment dyads appear to demonstrate a delayed benefit compared with higher achievers as evidenced by their improvement from posttest 2 to delayed posttest 2 means (see Table 7.1 and Chapter Six, section 3.2).

In this section, the first subsection will provide suggestions about why metacognitive training results in a greater improvement in mathematical word problem solving performance The argument is that treatment students were immersed in a learning environment that differed from the control students. The differences lay in the use of the metacognitive strategy, CRIME, which enabled students to become aware of their thinking processes during word problem solving; on the pedagogical approach, which was supposed to lead students to this awareness in a computer environment with WordMath; and on the use of WordMath, which appeared to enhance students' word problem solving processes. The second subsection addresses possible reasons for the observed phenomena of treatment lower achievers in quasi-experimental and case study designs demonstrating a delayed benefit from metacognitive training.

2 1 Mathematical Word Problem Solving Performance of Treatment Students

A possible reason why metacognitive training may result in a greater improvement in mathematical word problem solving performance might be attributed to the 'explicitness' in the pedagogical approach and relative 'completeness' (see Chapter Two, section 6) of the computer environment with CRIME (see Appendix F) during word problem solving.

From a practical perspective, training in the use of the self-questioning process in CRIME (see Appendix F), which comprises metacognitive components (see Chapter Two, section 7), may teach students a specific strategy to approach a word problem instead of them giving up when they fail to understand it. From a metacognitive perspective, the self-questioning process was intended to induce students to explicitly self-regulate their learning. The questions in CRIME were formulated to encourage students to explicitly focus their attention on cognitive components. Specifically, CRIME enabled students to use questions to guide them to connect ideas to their prior knowledge, analyse information, reconceptualise the word problem by integrating the information into a coherent representation and to self-monitor their progress by assessing and correcting their mistakes. In this manner, they were engaged in complex knowledge making which, in turn, enhances learning (King, 1992). When the treatment students solved the word problems, they seemed to explicitly carry out the metacognitive components enhanced by the CRIME strategy: Careful reading, Recalling possible strategies, Implementing strategies, Monitoring and Evaluating. For example, HM and XY (T/LA) demonstrated the metacognitive components in CRIME when they solved the N3 word problem (see Chapter Six, section 2.6 and Appendix I). They read the word problem carefully, referring to the word problem situation again when needed; they recalled that the model approach (Appendix B) would help them solve this word problem and started drawing the diagram; they implemented their strategies once they had drawn the models that depicted the word problem situation; they monitored and regulated their word problem solving while drawing the models and implementing their strategies; and finally, they engaged in evaluation whereby they used a test to ascertain that the solution met all the conditions in the word problem. In contrast, none of the control students were so intense in their approach to solving the word problems with their own metacognitive strategies (if present) as illustrated in CRIME (see Chapter Six, sections 2.3, 2.4, 2.7 and 2.8). King (1992) concludes that explicit prompts in the form of checklists help students to be more strategic and systematic when solving problems. Cardelle-Elawar (1995) also agrees that by questioning themselves, students become more aware of what they already know and feel more challenged to critically and creatively acquire by themselves the information needed to solve the word problem. In this study, the self-inquiry process might have guided the treatment students to build understanding by reflecting on the processes required to solve the word problem and as a consequence they improved in their mathematical word problem solving performance. However, it is not known if the treatment students exhibited these 'CRIME' behaviours before the intervention process and how their initial behaviours might be modified with metacognitive training with CRIME. This issue was not explored in the present research but is worthwhile for it to be examined in future studies.

Both treatment and control students were instructed in a computer environment with WordMath which focused on collaborative word problem solving. The main difference was that treatment students were trained to use the self-questioning process in CRIME (see Chapter Four, section 4), which provided treatment students an opportunity to practice metacognitive behaviour. From this perspective, the classroom environment could be considered 'complete' (see Chapter Two, section 6). For example, the intervention approach (see Chapter Four, section 4), in both cognitive apprenticeship instruction with WordMath and metacognitive training with CRIME, was to engage treatment students in all aspects of word problem solving identified in the Singapore mathematics syllabus (see Appendix A), including the nature of the word problems, representation used for those word problems, strategy selection (see Appendix B) and cognitive monitoring. Furthermore, both the modeling and scaffolding teaching methods were designed to allow the full task to be perceived by the students. Modeling provided the students with a schema for the application of the CRIME processes (see Appendix F). This schema included information concerning how these processes were used to solve word problems. Scaffolding encouraged successive approximation of the entire range of skills involved in completing the task (see Chapter Four, section 4). Finally, the 'treatment' computer environment was characterised as 'complete' in that the social aspect of learning was considered. Children worked in pairs and were encouraged to solve word problems collaboratively, focusing on the CRIME processes.

The other reason why metacognitive training may result in a greater improvement in mathematical word problem solving performance might be attributed to the computer environment, WordMath, which was used as a mediator for metacognitive experiences (see Chapter Two, section 5). Based on my observation, WordMath appears to encourage students to focus on the meaning of the representation constructed in the computer environment (Noss & Hoyles, 1996, p. 228). For example, in the analysis episode of L and JK's (T/HA) word problem solving (see Appendices H and I), they devoted 33.2% (see Chapter Six, Table 6.1) of their metacognitive behaviours to deriving meanings from the representation they had drawn.

JK: draw 28 marbles.
The total unit is 8 units leh
L: How do you know?/
Oh okay, so we need to find
we make it 8 units because / we put Jing Hao over Joe and Mun
so, it would be / 8 over / 1
and this should be / Jing Hao
and this should be / Joe and Mun
JK: no, this is 7
1 unit 7
7 of the total unit
L: oh yeah

minus 1
this is for Joe and Mun
and the total will be /
JK: 400
L: 400 / 400 marbles
so / now you know that =
JK: = 8 / 8 units is 400
L: so the total will be / [8 units
JK: [8 units is 400

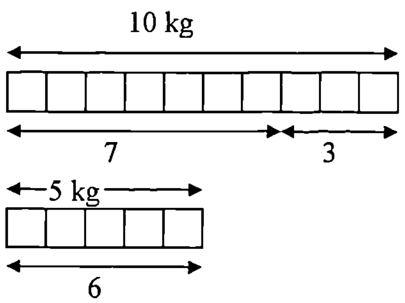
Dörfler (1993) argues that computers and computer software can be viewed from two perspectives. First, the hardware and software can be viewed as amplifiers of human capabilities. Viewed in this manner, the tool supports the actions on the meta-level by condensing and curtailing complex processes to easily manipulable unities or entities (Dörfler, 1993, p. 162). On the other hand, when the tool is viewed as a reorganiser of objects on which the activity, the actions and the work are carried out (Dörfler, 1993, p. 163), the learning process makes conscious and explicit this change of objects by changing the tools, to reflect on it and to exploit it. For example, WordMath can be compared with the traditional tools for drawing the models like pencil and ruler. With WordMath tools, blocks, partitioning of blocks, and systems of relationships amongst them can become the objects of the cognitive activity and are no longer just the products of the drawing activity. The diagrams drawn in WordMath as a whole can be edited, manipulated, separated into parts, labeled, coloured and so forth. In this study, the WordMath tool is viewed as having activity, actions and work carried out which bring about the reorganisation of cognitive activity. There is evidence to support this perspective. One example is from L and JK's (T/HA) word problem solving as mentioned above whereby the tool appears to enable the students to focus their attention on a 'higher' meta-level (e.g. monitoring the way they were drawing the diagram). Another example can be seen from E and XF's (C/LA) word problem solving (see Chapter Six, section 2.8 and Appendix I).

XF: So we draw (2) eh (5) ah:: another model with 7 parts
E: why?
XF: Eh because they say Jing Hao received 7 times
E: total number of =
XF: = number of marbles Joe Ee and {referring to the problem statement}
one / okay / so eh / 28
E: why?

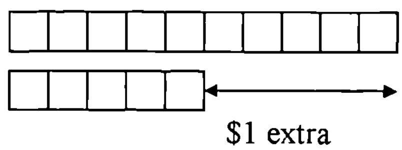
XF: Eh (2) this represents Joe Ee / 28
E: 8 units equal to 400 (2)
XF: yeah / eh
E: 8 units equal 400
XF: wait (2)
we type 28 first
E: 28? =
XF: = yeah / 28
okay so now / we do the /question

The above example appears to suggest that using WordMath enables students to reorganise their cognitive activity and shift their focus to a (higher) 'meta-level' (Dörfler, 1993). It must be emphasised that drawing the diagram can be done without the tool considered. However, these drawing actions have to be carried out in full and might entail considerable problems (e.g. erasing a diagram using a rubber if the relationships between the blocks are not accurate). The tool, WordMath, appears to free the students from that work load by condensing the unfolded action into a single click of the button.

On the other hand, there were occasions when WordMath tools had neither helped students reorganise their cognitive activity nor enabled them shift their focus to a higher 'meta-level'. With regard to the N4 word problem (see Appendix C) in posttest 2, dyads B and P (C/LA), and ES and J (T/LA) failed in their word problem solving. Both of them appeared to have difficulties in representing the word problem situation using WordMath tools. Both of them had made the assumption that the unit drawn with WordMath tools represented 1 kg of rice. ES and J's, and B and P's diagrams for the N4 word problem in posttest 2 are illustrated below.



ES and J's diagram for N4



B and P's diagram for N4

For ES and J, their final answer to the N4 word problem was $7. The following exchange illustrates J explaining to ES how the diagram was drawn and how he managed to get the final answer.

J: 10 kg, 7 dollars
then this one/
ES: 3 dollars short
J: 3 dollars
we need
you see/ 10 kg, 7 dollars not enough
you add 3, 10 dollars for 10 kg
then 5 dollars, 6
5 kg, 6 dollars
ES: 5 kg, 6 dollars
J: then got extra 1 dollar
so, it's 7 dollars
agree?
ES: Yeah yeah yeah

For B and P, their final answer was $30. The following exchange illustrates how the dyad came to this conclusion.

B: so if she buys two packs of 5 kg of rice, she has
P: two packs of [5 kg
B: [1 dollar/ she needs 1 dollar more
so the bag / the 10 kg of rice bag is 2 dollars more expensive (4)
2 times 5, 10
10 dollars
P: orh, see if she buys 3 dollars / 2 dollars more expensive than the / 5 kg rice
so is / 5 times 2 you get 10
B: uh huh
P: so is 10 dollars
if 5 / if 5 kg
5 kg is altogether is / 10 dollars (3)
then 1 unit is 2
how much does she have?
Then if / so is / so is 1 kg is 2 dollars
15 kg / she buys 15 kg / 2 [equals 30
B: [15 times 2 is 30 dollars
so she brought 30 dollars with her.

WordMath tools were not able to minimise the 'drawing' workload for the dyads. In fact, drawing models using WordMath tools appeared to have added an extra burden to the dyads' word problem solving. In most of ES and J's word problem solving during posttest 2, the dyad tried to avoid drawing diagrams using WordMath tools. However, during delayed posttest 2, ES and J used WordMath tools to draw diagrams in order to represent the word problem situation. Their representations were appropriate, and they were successful in 50% of their word problem solving (see Table 7.1). Dyad B and P was successful in one of their word problem solving in posttest 2, the F2 word problem. It was the only word problem which the diagram drawn with WordMath was appropriate. They were able to use the appropriate diagram to solve the word problem. Based on L and JK, E and XF, B and P, and ES and J's word problem solving with WordMath, it is appropriate to suggest that WordMath appears to enhance the treatment and control students' metacognitive behaviours during word problem solving, and it also appears to enhance the students' metacognitive and cognitive behaviours during word problem solving. These assertions merit further research.

2.2 Mathematical Word Problem Solving Performance of Treatment Lower Achievers

The findings from this study reveal that treatment lower achievers demonstrated a greater delayed benefit from metacognitive training than did higher achievers (see Chapter Five, section 3.2, number 6d and Chapter Six, section 3.2). These findings concur with Cardelle-Elawar's (1995) study (see Chapter Two, section 4). In Cardelle-Elawar's study, they found that low achievers with explicit metacognitive training outperformed the control group where students still relied more on the teacher for the right answer. The training in the use of self-questioning process in accordance with the set of questions derived from Mayer's model, led low achievers to progressively a) understand how to approach a problem, b) identify the appropriate schema, and c) verify their solutions (Cardelle-Elawar, 1995, p. 93). Cardelle-Elawar's (1995) study suggests that metacognitive training provides a classroom structure for low achievers to think for themselves and to recognise their limitations which in turn promotes problem solving success (op cit p. 93). However, the analysis in Cardelle-Elawar's study was based on the pretest and posttest results. In the present study, the evidence is based on posttest and delayed posttest results in the quasi-experimental study and case study. It might be that treatment lower achievers needed some time for internalisation (Díaz et al, 1990, p. 134) to occur before positive metacognitive influence could prevail in their mathematical word problem solving performance from posttest to delayed posttest. It is not known if this delayed improvement on the treatment lower achievers would also prevail in their word problem solving over an extended period of time (e.g. one year). It would be interesting to monitor the treatment students' word problem solving performance by planning for a longitudinal study (Cohen et al, 2000, p. 174) and examining if the phenomena just described would also prevail over an extended period of time.

The following subsection is an attempt to suggest other possible reasons why treatment lower achievers, particularly dyad ES and J (T/LA) from the case study, were not successful in their word problem solving during post-test 2 but were successful in delayed post-test 2.

2.2.1 Factors that Might Contribute to the Delayed Improvement in ES and J's (T/LA) Mathematical Word Problem Solving Performance

It was interesting to note that the progression of activity for treatment lower achievers, ES and J, was Type R cognitive-metacognitive word problem solving (see Chapter Six, Figure 6.11) during posttest 2 and Type T cognitive-metacognitive word problem solving during delayed posttest 2 (see Chapter Six, Figure 6.12). Moreover, this pair was more confident and was more successful in their word problem solving in delayed post-test 2. It appears that there is a delayed benefit of metacognitive training in ES and J's word problem solving. Not only were they engaging in more occasions of metacognitive activities (see Chapter Six, Tables 6.2, 6.4, 6.6 and 6.8), but they were also more efficacious in their mathematical word problem solving performance (solved 50% of the word problems) during delayed posttest 2. In this sub-section, I argue that ES and J did not show a positive effect in their think-aloud protocol immediately after the training not only because of their need for more time before internalisation of the influence of metacognitive training to occur, but also because of the following three issues which relate to ES and J's (T/LA) word problem solving: the issue of word problems used, the issue of dyad's social interaction, and the issue of motivational aspects of cognition.

Varschaffel and De Corte (1997) state that the process of skilful solution of a word problem starts with the construction of a network of representation of the basic semantic relationships between the main quantities in the problem. Throughout that constructive process of problem representation, different kinds of knowledge seem to play an important role. They highlight three types of knowledge - schemata of problem situations, linguistic knowledge and knowledge about the game of school word problems. It appears that ES and J were not able to trigger the appropriate schema and map the incoming information onto it by assigning the known and unknown quantities correctly to its slots. This is evidenced by their consistent declaration that '*I don't know how to draw*' or '*I don't know how to do*' or '*I've got nothing*' (see Chapter Six, section 2.2). According to Verschaffel and De Corte (1997), these students have failed to understand and represent these word problems because they did not possess the appropriate situational problem solving schema. It might be that the method to use part-whole models had not helped them in understanding. There is evidence that this might be the case for the dyad often drew inappropriate diagrams during posttest 2 (see section 2.1). Krutetskii (1976, p. 326) warns that, to some extent, diagrams may be a hindrance for particular students of low mathematical ability, '*for whom visual images really blind thinking, push it onto a concrete plane and hinder the interpretation of a problem in general form*'. ES and J might have also misinterpreted or miscomprehended the linguistic expression in the word problem statements. They appeared to have not only made an incorrect interpretation, but also were uncertain what the correct interpretation should be when they were confronted with errors (see Chapter Six, section 2.2). This weakness may have prevented them from building appropriate representations of the word problem situation (Varschaffel & De Corte, 1997).

According to Carr and Biddlecomb (1998, p. 85), social interaction can either hinder or facilitate the development of reflective thought and children's construction of mathematical knowledge. In collaborative work, students experience monitoring through others' critiques of their own reasoning and through hearing others voice the same reasoning (see Chapter Two, section 7). However, during ES and J's word problem solving, the dominating member, J, was constantly contributing ideas in order to make progress in their word problem solving while ES constantly accepted his judgment without much restraint. This pair work did not appear to be engaged in effective collaboration. However, in delayed posttest 2, ES appeared to show signs of being more 'courageous' and decided to speak up and give suggestions to alternative views when they were moving further away from the goal of the word problem (see section 4.2 for the exchanges between ES and J). Carr and Biddlecomb (1998, p. 87) claims that as children challenge the logic of other children and point out alternative views of the problem, perturbation may result. It is this socially induced cognitive conflict and subsequent perturbation that is believed to cause children to think about what they are doing and to bring to consciousness their understanding of what they are doing. This situation might have occurred during delayed posttest 2 when ES decided to play a more collaborative role in word problem solving. He challenged J about his suggestions and monitored their word problem solving. This issue will be revisited in section 4.2.

Finally, the factor which might have attributed to ES and J's posttest 2 failure is the motivational factor (Mayer, 1998). In Chapter Two, section 5.1, I have briefly discussed the importance of acknowledging the influence of affective issues in students' word problem solving. This issue was evident in ES and J, especially J's attitude towards the whole study. ES and J appeared to be inattentive and not motivated during the training sessions. This was especially true for J, the more dominating member of the dyad. There were occasions when I had to stand behind the dyad during the first four training sessions in order to ensure that they were engaging in the training activities. During the think aloud protocol training sessions, J was often late for the sessions. Once, he was one and a half hours late, apologising when he arrived and stating that he was having lunch with his friend and had forgotten about the time. On another occasion, J sought my permission to miss a video-recording session because he wanted to attend and support his friends in a badminton tournament. All these showed that he was not totally engaged in the metacognitive training. Gourgey (1998) argues that students who are not used to thinking metacognitively might resist having to do so, especially if they have been passive learners for many years. They do not understand why it is important to be more active in their learning and feel uncomfortable with the extra effort required. When I returned to video-record this dyad during delayed post-test 2, J appeared to be more attentive to what he was doing. There were many factors that might have contributed to this more positive attitude which enabled him to work more collaboratively with his partner in delayed posttest 2. However, these factors are not identified in this study.

The above discussion suggests that there is a need for treatment lower achievers to be given time before the influence of metacognitive training could be internalised. This internalisation of metacognitive awareness can be promoted by students' motivational aspects of cognition (Reeve, 1996). Based on the observation of ES and J's word problem solving, it also suggests that there is a need for students to be provided with a wide range of word problem solving strategies (i.e. tabulation techniques) besides the model approach; for students to be familiar with linguistic expressions found in word problems; for students to be motivated and see the importance and value of metacognitive training; and for students to be encouraged to work collaboratively during word problem solving. The educational and pedagogical implications for lower achievers will be explicated in Chapter Eight, section 2.

In section 2, a summary of the findings from the quasi-experimental design and case study design was provided. For each observation, some possible reasons for the phenomenon that occurred were also provided. In the following section, section 3, the discussion is based on my exploration of the think aloud protocol data in the case study design. Though the following observations are not conclusive, the findings from the case study provide important insights to the role of metacognition and its influence on students' (higher and lower achievers) word problem solving performance.

3. Cognitive Perspective and Word Problem Solving

The modified Artzt and Armour-Thomas framework (see Chapter Four, section 5.2.1 and Appendix G) provides useful information with respect to identifying when, where and in what frequency dyads used the word problem solving activities at the cognitive and metacognitive levels, and how these levels of thought might have influenced the word problem solution. It is possible that a certain balance of both cognitive and metacognitive processes within a dyad is necessary for word problem solving efforts to result in a solution. The framework is also used to identify the characteristic word problem solving behaviour in the students' think-aloud protocols and to classify them in terms of their levels of cognitive processing attributes. This classification was then used to generate cognitive-metacognitive word problem solving models (see Chapter Four, Figure 4.1) which represent distinct progression of the eight pairs of students' word problem solving activity based on the four word problems (N2, N3, N4 and F2). The modified Artzt and Armour-Thomas' framework, the modified Artzt and Armour-Thomas' cognitive-metacognitive word problem solving models, and the data display tables were then used to suggest the role of metacognition and the role of cognition in students' mathematical word problem solving in subsections 3.1 and 3.2 respectively, and finally in subsection 3.3, there is a discussion about the relationship between the unique characteristics of dyads' progression of word problem solving activity and their word problem solving performance.

3.1 **Role of Metacognition in Word Problem Solving**

Two distinct roles of metacognition emerged from the framework and cognitive-metacognitive word problem solving models of the eight dyads in the case study. They are the generation of metacognitive behaviours and the ability to know when and how to use metacognitive behaviours.

a) **Generation of Metacognitive Behaviours**

First, the generation of metacognitive behaviours seems important. The current literature supports the importance of metacognitive behaviours such as active monitoring and subsequent regulation of cognitive processes during problem solving (Garofalo & Lester; 1985; Schoenfeld, 1985). With respect to the N3 word problem (see Appendix C), the five dyads that solved the word problem devoted 90.7% (L and JK), 73.3% (HM and XY), 66% (A and CC), 31.2% (E and XF), and 38.7% (SM and B) (see Chapter Six, section 3.1, Table 6.3) to metacognitive activities. Of these metacognitive activities, the episode type 'analyse' was coded with the highest percentage amongst the groups. Only L and JK (T/HA), A and CC (T/HA) and HM and XY (T/LA) engaged in verification activities, 26.1%, 21.2% and 12.6% respectively. When examining the specific instances of these types of metacognitive statements, one gets a better understanding of the ways in which they serve to enhance and propel the word problem solving process.

For example, the statements made by K in K and SJ (C/HA), '1 *question mark is equal to 22. So, they are asking how many more marbles did Jing Hao receive more than Mun Fai. So, the difference is actually 6 question marks. Correct?*' served to help the dyad understand the status of the word problem solution and the direction in which to go to continue the solution process. Other statements made by different students were: '*1 unit is 50. You see, how much more did Jing Hao more/ than Mun Fai. After 50 minus 28, then you got Mun Fai*' and '*28 times 7 equals to 196. But then Joe Ee is 196 plus 28, so it is 200 over. But then if plus another 28, it's not even 300 but there are 400 marbles!*' Such statements often change the flow of conversation and appropriately redirect efforts of the dyads. For example, after B in B and P (C/LA) (see Chapter Six, section 2.3) had stated that '*it's not even 300 but there are 400 marbles*', the dyad decided that '*Mun Fai and Joe Ee cannot be the same*'. Then they redirected their efforts to look into the possibility that '*Joe Ee is half and Mun Fai is 1 unit.*'

In a different way, the more 'local' monitoring statements such as '*This is wrong. We haven't found Mun Fai,*' '*No, more than 28. It's Joe Ee and Mun Fai,*' and '*Wait, this should put Joe Ee and Mun Fai (referring to the diagram)*' served to control the dyad and to keep it from going off on wrong tangents by reminding each other of the conditions of the word problem that must be met and by suggesting the next small steps to take. For example, the following are exchanges made by L and JK (T/HA) when they were solving the N3 word problem during delayed posttest 2. They had just completed reading the word problem and were trying to draw an appropriate diagram. They had already drawn a block and had relabeled it '$28'. Then L reminded JK that '*Sharon is 7 times of the total amount*'. This reminder prevented them from going off on wrong tangents which some dyads had made for they had thought that 'Sharon is 7 times of the 28'.

L: so this is Sharon {referring to the diagram}
JK: must plus another / eh Sharon part
L: how come?
JK: then can put 7 times lah
L: but / you don't know how much Cindy got what
JK: you put here Sharon, Cindy.
then / wait wait you put here Sharon
then here Cindy what /
because Sharon is 7 times / of the total amount

The absence of consistent monitoring and regulating of the word problem solving process can be seen in the progress of B and P (C/LA) who did not solve the word problem. Both students engaged in a flawed strategy whereby they took numbers out of the word problem context and used different operations to manipulate the numbers. If they had been monitored by another metacognitive statement such as '*The answer doesn't make sense*' or '*Maybe we cannot assume that Joe Ee has 1 unit and Mun Fai has half unit*,' the dyad might have had a chance of getting back on track.

It was also observed that the generation of metacognition helped students when they used heuristic strategies in word problem solving. According to Schoenfeld, (1985, p. 23), heuristic strategies are rules of thumb for successful problem solving, general suggestions that help an individual to understand a problem better or to make progress toward its solution. Some strategies include working forward from the data, drawing figures, varying the problem and working backward. The modified Artzt and Armour-Thomas framework (see Appendix G) was a useful tool for investigating the occurrence of heuristics demonstrated in the episodes of the dyads' word problem solving. In addition, it also helped in examining how the presence of metacognitive behaviours during the use of heuristic strategies had led dyads into solutions. For example, in the present study, it was observed that all dyads returned several times to reading the word problem in order to make progress toward the word problem solution. Most often, they returned to the words of the word problem to gain a clearer understanding. They could often be heard reminding one another of the conditions that had to be met in the solution of the word problem. Most of the time, when the students returned to reading the word problem, they appeared to bring new insights with them. These students would most probably bring with them a higher level of understanding although they were just reading the word problem again. This was observed in the analysis of E and XF's (C/LA) word problem solving protocol (Chapter Six, section 2.8), where they devoted 22.5% (see Chapter Six, Table 6.11) of their cognitive activities to reading on the N3 word problem protocol. The first reading episode was followed by analysis which the dyad appeared confused with regard to drawing the appropriate diagram to represent the word problem situation. They resorted to rereading the question again. From the second reading episode, XF gained a new insight - '*another model with 7 parts ... because they say Jing Hao received 7 times*'. When challenged to provide evidence that Jing Hao was represented by seven parts, XF resorted to rereading the question again, verbalising parts of the word problem statement to E and pointing out to him the evidence. In contrast, there were some dyads who were not able to bring to a 'higher level of understanding' or have new insights when they returned to reading the word problem a number of times. For example, ES and J (T/LA) devoted 30.3% of their time to reading (see Chapter Six, Table 6.11). Every time they reached an impasse (see Chapter Six, section 2.2), they went back to reading the word problem statements. On some occasions, reading the word problem again appeared to bring new insights to their word problem solving. On other occasions, reading appeared to be a deliberate attempt to clarify their uncertainties. Nevertheless, this return to reading did not appear to have helped them move towards the goal of the word problem solution.

De Corte et al (1996) claims that one major way in which heuristics can be helpful in solving a problem is as tools or resources that the problem solver uses in transforming the original problem into a familiar routine tasks for which (s)he has a ready-made solution. They also claim that heuristics do not guarantee that one will find the solution of a given problem; however, because they induce a systematic and planned approach to the task- in contrast to a trial-and-error strategy- they substantially increase the probability of success. From the study, it appears that two higher achiever dyads from treatment and control groups and a treatment lower achiever dyad solved their word problems using a systematic and planned approach. Their progression of word problem solving activity was the Type P cognitive-metacognitive word problem solving (see Chapter Six, Figure 6.9) which led to their success in most of their word problem solving. In contrast, two lower achiever dyads from treatment and control groups and a treatment higher achiever dyad tend to use a trial-and-error strategy to solve their word problems. For the treatment and control lower achievers, their engaged trial-and-error strategy led them further and further away from the goal of the word problem (see Chapter Six, sections 2.2 and 2.4). On the other hand, for the treatment higher achievers, A and CC (T/HA), this trial-and-error approach worked well for them. Their progression of word problem solving activity was the Type S cognitive-metacognitive word problem solving (see Chapter Six, section 4.4). Their exploration usually sparked the analysis which then sparked further exploration and, then more analysis. This sequence of behaviours would usually lead them into solutions for their word problem solving.

b) When and How To Use Metacognitive Behaviours

The ability to know when and how to use metacognitive behaviours when they are needed are important determinants of word problem solving. Gourgey (1998) points out that a common pattern of poor metacognition is seen in the example of novice students in Schoenfeld's (1987) study who seized upon a solution strategy and failed to ask themselves if the strategy was leading to their goal. He claims that students frequently perform inappropriate operations because they have not clarified the relationships among the facts in the problem, fail to consider exactly what needs to be done and why. This lack of careful attention to sense-making and clarification often leads to illogical solution attempts. In addition, Mayer (1998) posits that one of the factors that discriminate successful problem solvers from unsuccessful problem solvers is their ability to know not only what to do, but also when to do it with regard to using their cognitive skills. He calls this the ability to control and monitor cognitive processes. For example, in the present study, the three dyads that did not solve the N3 word problem during posttest 2 devoted 91.9% (K and SJ), 69.7% (ES and J), and 47.6% (B and P) (see Chapter Six, Table 6.3) to metacognitive activities. Visual inspection of K and SJ's word problem solving protocol suggests that K and SJ (C/HA) failed in their word problem solving because they had misdirected their goal towards the end of their word problem solving (see Chapter Six, section 2.7). The dyad had found the number of marbles Jing Hao and Mun Fai had. When they were about to find the number of more marbles Jing Hao received than Mun Fai, K reallocated his resources. Though SJ recognised that K's procedure was flawed and tried to direct K to the correct path by reminding K of the condition of the word problem, he was not able to convince K. Instead he was persuaded by K to accept his flawed procedure. The new procedure, a reallocation of resource, led them away from the goal of the word problem and hence, they failed in their word problem solving. In another example, 61.1% of ES and J's time on metacognitive activities was devoted to explore (metacognitive) (see Chapter Six, Table 6.11). Though they monitored their activities, evidenced by the occasion '*It cannot be*', they failed to obtain the correct solution (see Chapter Six, section 2.2).

The above examples from the present study suggest that the occurrence of metacognitive behaviours on its own does little to ensure successful word problem solving. This concurs with Stillman and Galbraith's (1998) study which reveals that though the successful groups in their study displayed a high number of key points where metacognitive decisions would influence cognitive actions, this alone was not a guarantee of success. They argue that the opportunities for metacognitive decisions to be made does not ensure that they will be made nor if such decisions are made, that they will be appropriate. They emphasise that a rich store of knowledge of metacognitive strategies and their facility developed over an extended period of use is a likely prerequisite to productive decision making (Stillman & Galbraith, 1998, p. 182). This point was also argued for in the discussion of treatment lower achievers' delayed improvement in metacognitive training in sections 2.2 and 2.2.1.

3.1.1 Word Problem Solving Impasse

The analysis of the think aloud protocol data also led to an observation that was related to the students' responses when they were 'stuck' while solving the word problems. It was observed (see Chapter Six, section 2) that students from different conditions responded differently when they were 'stuck'. For example, for SM and B (C/HA), and A and CC (T/HA), their immediate response to being 'stuck' for all word problems was to return to reading and analysis; while B and P's (C/LA) immediate response was to return to explore (cognitive). This calls for a discussion on why students do what they do when they are faced with difficulties and how metacognitive behaviours appear to play a part in helping students overcome their impasse.

In solving the N4 word problem (see Appendix C) during posttest 2, HM and XY (T/LA) were exploring (metacognitive) for 179 seconds when they realised that they were 'stuck'.

XY: it doesn't match what! (3)
this plus is extra right?
HM: We're just doubling / doubling it (3)
so, this and this is 1 dollar extra
XY: hm mm
HM: but this doesn't match
this, the minus and plus sign doesn't match
so / what we have to do is to make this minus sign become add sign /
XY: how do to do that? (17)

HM: Is that true / that 4 kg is 4?
5 kg is 4 dollars?
XY: 5 kg is / how do you get the 4 dollars?
HM: 3 plus 1
XY: 5 kg is 4 dollars
then why do you add together?
HM: Let me try
if 5 kg is 4 dollars right so (4)
XY: 5 kg is 4 dollars, then 10 kg is 8 dollars (4)
HM: 5 kg is 4 dollars / yeah
10 kg is 8 dollars
so, 8 dollars minus 3 because of [the 4
XY: [but why do you add the 3 dollars plus 1 dollar?
HM: Because one is short and one is extra /

During the 17 seconds pause, HM was silently referring to the word problem. When she proposed that 5 kg was 4 dollars, HM and XY analysed this idea with reference to the diagram they had initially drawn. XY also checked HM's suggestion. This apparently good control and monitoring appeared to have helped in HM and XY's word problem solving success. During delayed posttest 2, their apparent good control and monitoring strategies when they were 'stuck' usually led them away from inappropriate paths into paths of solution.

In sharp contrast, B and P (C/LA) appeared to engage in explore (cognitive) when they realised they were 'stuck'. As described in Chapter Six, section 2.4, when B and P realised that their solution was incorrect in Episode 2, they did not proceed to reread the word problem nor analyse the word problem based on the diagram they had drawn. Instead, they continued making inappropriate assumptions with regard to the relationship between the givens and the unknowns in the word problem situation. Their inappropriate diagrams reflected this behaviour and this behaviour appeared to be consistent with B and P's (C/LA) word problem solving (see section 2.1).

English and Halford (1995) believe that a failure to make appropriate connections between the external representations of computational concepts and procedures is not the only cause of children's poor mental models. Their difficulties may also stem from their inability to see the links between related computational procedures as well as between these procedures and the underlying numeration concepts. Hiebert and

Carpenter (1992) also assert that some students have difficulties in using representations because the students do not bring with them the kind of knowledge of quantities that teachers expect and it is not easy for the students to relate their interactions with the representation system (i.e. part/whole schema) to existing networks. These explanations might be possible reasons why B and P might not be able to draw appropriate diagrams to represent the word problem situation. It might be that these students need instruction on how to solve word problems using other strategies besides the model approach so that connections can be made between the relationships in the word problem situation and the representation(s) they construct. This issue was also discussed in section 2.2.1 in this chapter.

From a metacognitive-cognitive perspective, Hegarty et al (1995) assert that successful problem solvers use metacognition to create an internal representation or mental map of the givens, the relations among the givens and the goals found in the problem. From Hegarty et al's study, it was found that successful problem solvers used an object-based representation for arithmetic word problems. These solvers tend to construct mental models of the situation depicted in a problem. On the other hand, according to Hegarty et al (1995), less successful problem solvers used a more impoverished propositional representation. They constructed a mental model based on the numbers and keywords found in a problem. This might be the case in B and P's (C/LA) word problem solving.

In addition, Kaplan and Davidson (1988) suggest three reasons why individuals may have difficulties in problem solving. The first reason involves stereotypy (Kaplan & Davidson, 1988). In this case, the word problem solver becomes fixed on one particular path to solution. They cite McAfee and Leong (1994, p. 144) who assert that '*Poor students may have the requisite knowledge and skills but fail to use them correctly or at the appropriate time. These students lack flexibility and may stick to one strategy even when it does not lead them to successful solutions*'. The second reason is to do with the inability to generate any plans or procedures for solving a problem (Kaplan & Davidson, 1988). For example, if a word problem is sufficiently novel or requires unavailable knowledge, the word problem solver may simply not know how or where to begin. According to Kaplan and Davidson (1988), when these students reach such points of impasse, instructional intervention offering problem solving strategies and encouraging self-reflection has been found to improve problem solving performance. The final reason why some have trouble solving problems is that the students do not monitor and evaluate their knowledge and solution procedures (Nickerson et al, 1985). For these students, 'metacognitive shortfall' occurs when the word problem solvers do not assess bias in their models, do not realise when a model can be extended or do not reconsider a conclusion after receiving additional information (Perkins, 1989). Treatment lower achievers, ES and J, initially demonstrated the same progression of word problem solving activity, Type R, as B and P during posttest 2 (see Chapter Six, section 4.3). They also engaged in exploring when they reached an impasse (see Chapter Six, Figure 6.11). However, their progression of word problem solving activity became Type T after a period of six weeks' absence of metacognitive training and they were successful in 50% of their word problem solving. Hence, there is cause to suggest that B and P (C/LA) need metacognitive training so that they will be able to discern when they have to move away from inappropriate solution paths, relocate their resources so that they might effectively solve the word problem by constantly monitoring their solution paths.

3.2 **Role of Cognition in Word Problem Solving**

In this study, cognitive activity was evident in all groups. From section 3.1, we have seen the important role of metacognition; however, without the presence of students who were able to follow or implement the cognitive statements, the word problem solving could not be moved on or completed. For example, in the protocol of A and CC (T/HA) solving the N3 word problem (see Chapter Six, section 2.1), the students proceeded with the plan proposed by A. In the midst of computation, CC noticed that their solution was not satisfactory. Through the combined cognitive efforts of performing the calculations and metacognitive efforts of evaluating their solutions, the students decided that they had to take a new approach, and hence they were able to move on towards the goal of their word problem solving. In another example, in SM and B's (C/HA) word problem solving (see Chapter Six, section 2.3), the intense metacognitive decisions made during their initial analysis and planning episodes helped them advance in their computation during the implementation episode. This sequence of activities led them to success in word problem solving. In Stillman and Galbraith's (1998, p. 183) study, they also observed that all groups who were successful in their problem solving displayed a high number of key points where metacognitive decisions could influence cognitive action. As a cautionary note, Artzt and Armour-Thomas (1992) highlight the complexity of the interrelationship between metacognitive and cognitive processes, and suggest that an appropriate interplay between the two is necessary for successful problem solving. In addition, Davidson and Sternberg (1998) asserts that the links between cognition and metacognition still need further examination for it is not fully understood how these metacognitive processes develop in relation to cognitive ones and vice versa. Though this is beyond the present study, this issue should be further studied in the light of students' word problem solving.

3.3 Cognitive-Metacognitive Word Problem Solving Models

Five distinct types of cognitive-metacognitive word problem solving models emerged from the analysis of think aloud protocol data that describe the progression of dyads' word problem solving activity. They are Type P, Type Q, Type R, Type S and Type T cognitive-metacognitive word problem solving models (see Appendix L and Chapter Six, section 4). For Type P cognitive-metacognitive word problem solving (see Chapter Six, section 4.1), the progression of word problem solving activity in the model is well-regulated and controlled and this usually leads to the dyads' success in their word problem solving. The students, whose progression of word problem solving activity is Type Q cognitive-metacognitive word problem solving (see Chapter Six, section 4.2), tend to focus their word problem solving on reading, analysis and planning. When they have clarified their doubts and developed their goals for their solution, they would proceed to implementation and verification. These last two episodes would unlikely be monitored. The success of their word problem solving is usually dependent on the first phase of their word problem solving process. In contrast, students, whose progression of word problem solving activity is Type R cognitive-metacognitive word problem solving (see Chapter Six, section 4.3), devote most of their time to exploratory activities which are usually not monitored. Their sequence of activity looks very.similar to the pattern of problem solving behaviours of the novice students that Schoenfeld (1985) described in his study, where they engaged in 'wild goose chases'. In Type S cognitive-metacognitive word problem solving (see Chapter Six, section 4.4), the students' progression of word problem activity is dominated by exploratory activities which are monitored. Unlike the students, whose progression of word problem solving activity is Type R, their word problem solving appears to be influenced by the effective regulation and control of their exploratory activities. Students, whose progression of word problem solving activity is Type T cognitive-metacognitive word problem solving (see Chapter Six, Figure 6.12), show some word problem solving features which are similar to Type P cognitive-metacognitive word problem solving, but the students would also devote time to explore (metacognitive). The following table, Table 7.1, illustrates the distinctive progression of word problem solving activity of the eight dyads' word problem solving during posttest 2 and delayed posttest 2. This is followed by a discussion of the relationship between the dyads' distinctive progression of word problem solving activity and their word problem solving performance.

Dyads	Condition	Post-test 2	% Successful in Word Problem Solving	Delayed Post-test 2	% Successful in Word Problem Solving
A and CC	T/HA	Type S	75%	Type P	75%
ES and J	T/LA	Type R	0%	Type T	50%
SM and B	C/HA	Type Q	100%		
B and P	C/LA	Type R	25%		
L and JK	T/HA	Type P	75%	Type P	100%
HM and XY	T/LA	Type P	100%	Type P	100%
K and SJ	C/HA	Type P	75%		
E and XF	C/LA	Type Q	75%		

Table 7.1: Summary Table of Dyad's Cognitive-metacognitive Word Problem Solving Behaviours during Posttest 2 and Delayed Posttest 2

3.3.1 Type P Cognitive-metacognitive Word Problem Solving

The dyads, whose progression of word problem solving activity is Type P cognitive-metacognitive word problem solving, comprise of a pair of treatment higher achievers (L and JK) and a pair of treatment lower achievers (HM and XY), and a pair of control higher achievers (K and SJ). The pair of lower achievers (HM and XY) was successful in all their word problem solving during posttest 2 and delayed posttest 2 (see Table 7.1). The progression of word problem solving activity for HM and XY, and L and JK, on both occasions, posttest 2 and delayed posttest 2, was the Type P cognitive-metacognitive word problem solving. In addition, the pair of higher achievers (A and

CC), who was successful in the N3 word problem during posttest 2 and delayed posttest 2, had progression of word problem solving activity represented by Type S in posttest 2 and Type P in delayed posttest 2 (see Chapter Six, section 4.4.1). These findings suggest that students, whose progression of word problem solving activity is Type P cognitive-metacognitive word problem solving, are more likely to be successful in their word problem solving compared with students, whose progression of word problem solving activity is represented by other cognitive-metacognitive word problem solving models.

The pair of control higher achievers (K and SJ), whose word problem solving activity was Type P cognitive-metacognitive word problem solving, might have attributed their success to their already developed metacognitive strategies in word problem solving. English and Halford (1995) claim that successful students possess powerful strategies for dealing with novel problems, can reflect on their problem solving activities and can monitor and regulate those strategies efficiently and effectively. Their ability to diagnose and monitor their understanding is a significant predictor of their mathematics achievement (English & Halford, 1995).

3.3.2 Type Q Cognitive-metacognitive Word Problem Solving

Students, whose progression of word problem solving activity was the Type Q cognitive-metacognitive word problem solving, include a pair of control higher achievers (SM and B) and a pair of control lower achievers (E and XF). The progression of word problem solving activity in the Type Q cognitive-metacognitive word problem solving appeared to work for these dyads and they were quite successful in their problem solving attempts (see Table 7.1). This success appeared to be attributed to the metacognitive activities (analysis and planning) during their word problem solving. Davidson and Sternberg (1998) believe that for these students, their word problem solving is dependent on effective planning which involves dividing a problem situation into parts and a sequence of actions is developed for how to accomplish the goal of each part. Furthermore, for these students, the process of problem decomposition (Holyoak, 1995) is preferable to devising and implementing a global plan in order to reach the overall goal for the entire problem. This is because completing a series of 'subgoals' often requires fewer steps and results in fewer errors. Davidson & Sternberg conclude by quoting Goldin and Hayes-

Roth (1980) who assert that good problem solvers tend to spend relatively more time on higher level of planning and exercise more deliberate control over the planning process than do poor planners.

3.3.3 Type R Cognitive-metacognitive Word Problem Solving

Two pairs of lower achievers had progression of word problem solving activity which was the Type R cognitive-metacognitive word problem solving. One dyad (ES and J) had metacognitive training and the other dyad (B and P) did not have metacognitive training. Both of them were not successful in most of their word problem solving (see Table 7.1). Like the novice students in Schoenfeld's (1985) study, this result appears to suggest that students, whose progression of word problem solving activity is Type R cognitive-metacognitive word problem solving, are less likely to be successful in their word problem solving. However, analysis of ES and J's word problem solving suggests that students who had explicit metacognitive training are capable of modifying their progression of word problem solving activity, from a Type R cognitive-metacognitive word problem solving to a Type T cognitive-metacognitive word problem solving (see Table 7.1). This modification of the progression of word problem solving activity appears to contribute to one of the factors that enables ES and J demonstrate improvement in word problem solving performance. This positive influence might not be demonstrated immediately after the training but there is evidence that the influence will be delayed (see sections 2.2 and 2.2.1).

3.3.4 Type S Cognitive-metacognitive Word Problem Solving

A pair of treatment higher achievers (A and CC) was the only dyad whose progression of word problem solving activity was the Type S cognitive-metacognitive word problem solving. The students in Goos & Galbraith's (1996) study also moved into the exploration episode in their problem solving when needed, and their success depended on the quality of the subject's control decisions. Likewise, A and CC continuously demonstrated effective control decisions in their exploration. This word problem solving behaviour contributed to their success in 75% of their word problem solving during posttest 2. During delayed posttest 2, the students' progression of word problem solving activity was the Type P cognitive-metacognitive word problem solving and they were also 75% successful in their word problem solving.

The five different cognitive-metacognitive word problem solving models have provided insights to the relationship between treatment and control higher and lower achievers' progression of word problem solving activity and their word problem solving performance. My observation suggests that students, whose progression of word problem solving activity is Type P cognitive-metacognitive word problem solving, are more likely to be successful in word problem solving. This manifestation of the progression of word problem solving activity represented by Type P can be accomplished with metacognitive training as evidenced by HM and XY, a pair of lower achievers who had metacognitive training and were successful in all their word problem solving (see Table 7.1) during posttest 2 and delayed posttest 2. If students are not able to learn Type P cognitive-metacognitive word problem solving, having the progression of word problem solving activity which is represented by Type T cognitive-metacognitive word problem solving via metacognitive training (see Chapter Six, Figure 6.12) may also lead to improvement in word problem solving performance, as evidenced by ES and J's word problem solving (see section 2.2.1). This is because in Type P and Type T cognitive-metacognitive word problem solving, students tend to devote more time to metacognitive activities which appear to have a positive influence on students' word problem solving performance (see Chapter Six, sections 4.1 and 4.3.2). It was also observed that some students (a pair of higher and lower achievers), who did not have metacognitive training, had progression of word problem solving activity which was Type Q cognitive-metacognitive word problem solving and they were quite successful in word problem solving (see Table 7.1). As mentioned in section 3.3.2, these students' success may be a result of their engaged metacognitive activities during word problem solving in the first phase of their word problem solving. Other factors might also attribute to their success, some of which will be discussed in the next section.

4. Affect and Word Problem Solving

Research has provided clues to the variables that are likely to impact positively or negatively on a group's performance. For example, Mayer (1998, p. 50) suggests that one of the prerequisites for successful problem solving is based on the motivational aspects of cognition, that is the problem solver's will. Others, like Sternburg (1998), highlight that metacognition interacts with many other aspects of the students (e.g. abilities, personalities and learning styles). As mentioned in Chapter Three, section 3.2.2, the purpose of the teacher interview schedule and student questionnaire is to provide descriptive accounts of students' metacognitive knowledge during mathematical word problem solving. These accounts provide background information of the cases in the case study design. As I began to describe these students' metacognitive knowledge and analyse their think aloud protocol data, a relationship between the students' metacognitive knowledge and their word problem solving began to emerge. My observation reveals that there are affective factors that relate to students' word problem solving. The affective factor, the motivational factor, was described in treatment lower achievers' (ES and J) word problem solving (see section 2.2.1). In this section, the affective issue will be discussed under two headings: students' beliefs and word problem solving performance; and effective dyad collaboration.

4.1 Students' Beliefs and Word Problem Solving Performance

Schoenfeld (1992, p. 358) states that beliefs are interpreted as an individual's understandings and feelings that shape the ways that the individual conceptualises and engages in mathematical behaviour. From his studies, he notes that the students' mathematical beliefs shape their behaviour in ways that have extraordinarily powerful (and often negative) consequences (op cit p. 359). For example, Schoenfeld (1985) observed that when students believed that all 'problems' could be solved in a certain amount of time, they would give up on a problem after a few minutes of unsuccessful attempts, even though they might have solved it had they persevered.

Based on my observations (see Chapter Six, section 5) in the present study, it appears that students' beliefs also have an influence on their word problem solving performance.

The different beliefs of the sixteen students (see Chapter Six, section 5) based on the questionnaire items are summarised as follows:

a) being able to solve a word problem is dependent on following an 'effective' heuristic (e.g. read the word problem, understand the story, draw models, do the working);
b) being able to solve a word problem is dependent on checking for careless mistakes;
c) being able to solve a word problem is dependent on one's confidence and love for the subject;
d) being able to solve a word problem is dependent on the size of the numbers in the word problem and the number of steps required to solve the word problem; and
e) being an effective word problem solver is to '*practice doing more exercises*'.

The students whose beliefs are indicated by a), b) and c) were quite successful in word problem solving (see the analysis of dyads A and CC, SM and B, L and JK, HM and XY, K and SJ, and E and XF's word problem solving in Chapter Six, section 2, and Table 7.1). In contrast, those students whose beliefs are indicated by d) were four lower achievers, ES, J, B and P (see Chapter Six, section 5). They were unsuccessful in most of their word problem solving (see Table 7.1). Nevertheless, it is a concern to me to hear that Singapore students have developed beliefs indicated by e). These beliefs came from twelve students, higher and lower achievers (see Chapter Six, section 5). Students whose beliefs are that '*doing more exercises*' would enable them to become an effective word problem solver might have stemmed from the Singapore mathematics curriculum which promotes word problem solving that focuses on mastery of relatively small chunks of subject matter and word problems that can be completed in a short amount of time (see Chapter Two, section 3.1). The teachers (I admit to being one of them) have also played a part in inculcating these beliefs. We have encouraged students to solve different types of word problems in order to expose them to different strategies. We have given them 'tons' of word problems for homework in the belief that 'practice makes perfect'. This present study has changed my beliefs. The complementary interplay of practice and being aware of one's own metacognitive processes, I believe, is one of the factors that determines success in word problem solving. This calls for raising awareness amongst educators to be aware of our students' mathematical beliefs in the mathematics classroom. Curriculum policy needs to look into how we, educators, can promote 'positive' mathematical beliefs amongst our students.

4.2 Effective Dyad Collaboration

As noted in Chapter Two, section 7, a growing body of literature suggests that children can prompt each other to reflect on their thinking and that this improves mathematics performance (Steffe, 1994; Artzt & Armour-Thomas, 1992). In Artzt and Armour-Thomas' (1992) study, they examined the role of metacognition in group problem solving and found that personalities and attitudes of the participants rather than the ability level predicted dictated whether children would share metacognitive insights. Participants in groups that worked well together were more likely to function metacognitively in that they attempted to understand, analyse, plan and verify the problem. In contrast, children in groups that did not function as well tended to engage in more cognitive activities in that they read the problem and explored but the exploration tend to be trial and error. Carr and Biddlecomb (1998) conclude that having children work together does not guarantee a better quality of interaction including metacognitive awareness. The following looks into a) the students' communication skills and the intentions of the dyads, and b) how students are paired according to their metacognitive knowledge which might be important considerations for effective pair collaboration. It is also interesting to note that effective collaborating pairs usually demonstrate success in word problem solving (e.g. L and JK; and HM and XY).

In the present study, the dyads had similar academic profiles (see Chapter Four, Table 4.3). Despite this similarity between students in a dyad, the higher/lower achievers from respective dyads functioned rather differently. Like the findings from Artzt and Armour-Thomas' (1992) study, some variables that may have contributed to these differences were the personalities and attitudes of the dominating member in each dyad. For example, ES and J (T/LA) hardly worked together and were unsuccessful in solving all the word problems during posttest 2, whereas in L and JK's (T/HA) word problem solving, the dyad was very interactive and managed to be successful in their word problem solving. In ES and J, the more assertive member, J, on a number of occasions, got fixed on his fault strategy and was not receptive to ES's feedback. In contrast, in L and JK, each member of the dyad challenged each other's strategies and this forced each of them to overtly express their ideas. However, during delayed post-test 2, ES in ES and J decided to oppose some of J's suggestion. This challenge managed to 'save' them from many inappropriate solution paths and they were successful in 50% of their word problem solving attempts. The following is an exchange between ES and J to demonstrate how ES managed to convince J to accept his strategy.

J: {drew a flawed diagram}
ES: no / find 1 part what / find 1 part
J: this is 2 units {referring to the diagram}
ES: 2 units
why draw one more / for what?
J: where got draw one more?
ES: here {pointing to the diagram}
J: 2/ you know
7 times more
this times 2 plus 1
ES: just now can just 400 divided by 8
J: {modified the diagram}
ES: 400 divided by 8
[50
J: [how about / 400 divided by 8?
Then how?
ES: 50 / then after that the 50 / 50 is Sharon and Cindy
then 50 minus 28 lor
J: what you say?/
say once more

It is also observed that success in word problem solving appears to be related to how students are paired according to similar metacognitive knowledge and not according to similar academic profiles. For example, HM and XY (T/LA) were successful in all their word problem solving. One can attribute this to the effect of the metacognitive training they had. However, I would like to suggest that it may also be the similar metacognitive knowledge of mathematical word problem solving each student possesses that relates to their success in all their word problem solving. HM believes (questionnaire) that while solving word problems, she needs to '*draw one or more models, refer to every step I do, read the problem twice; keep track of what I do; and avoid all careless mistakes*', and XY believes (questionnaire) that while solving word problems, she needs to '*draw a* *model or table for question which I'm not sure of; write short words beside the method so that I know what I am doing'*. XY also keeps track of her word problem solving by referring to the models, tables, or the word problem again or to look at the short notes she has written (questionnaire). In their word problem solving of the N3 word problem, they devoted 41.1% (see Chapter Six, Table 6.11) to analysis which was mainly focused on ensuring that all the vital information were represented on the diagram. They consistently kept track of their cognitive actions during word problem solving, as illustrated by the following exchange after the pair had drawn the diagram which represented the word problem.

HM: The question asked how many more marbles did Jing Hao receive than Mun Fai.

XY: So we have to find Mun Fai

HM: Let me see (pauses for 3 seconds).
This is the unknown {pointing to the diagram} / unknown because of Mun Fai.
So let say this is one small unit /

XY: Okay

Another example of how dyads with similar metacognitive knowledge relates to their word problem solving is demonstrated in E and XF's (C/LA) word problem solving (see Chapter Six, section 5.8). Both of them were considered the 'worst cases' in their class, judged by their teacher (interview). In fact, in XF's teacher's opinion, XF was considered a 'hopeless case' (interview). However, they were successful in 75% of their word problem solving. This might be attributed to XF's beliefs (questionnaire) to '*read the question many times and draw model to help you*' and E's belief (questionnaire) to '*draw model; to check every step for my math; and think carefully for every step*' that saved them from many flawed paths of solution. They demonstrated such metacognitive knowledge while they solved the N3 word problem. Both devoted 31.2% (see Chapter Six, Table 6.11) of their metacognitive behaviours to analysis where they ensured that the diagram drawn had all the conditions of the word problem (see Chapter Six, section 2.8), and 22.5% of their cognitive behaviours to reading. On the other hand, ES and J (T/LA) had undergone metacognitive training but both of them had very negative feelings towards mathematics, confirmed by the teacher (interview). J believes (questionnaire) in '*guessing*' and ES believes (questionnaire) in '*drawing part-whole* *models'*. In addition, J occasionally keeps track of his word problem solving and does not believe (questionnaire) in checking except *'when it is PSLE'*, and ES keeps track of his word problem solving so that it will prevent him from day dreaming and *'play with my things'* (questionnaire). They were not successful in all their posttest 2 word problem solving. Other factors that contributed to ES and J's failure in word problem solving during posttest 2 were discussed in section 2.2.1.

In brief, it is observed that there is a relationship between grouping students' according to their metacognitive knowledge and their success in word problem solving. Though the above observation is not conclusive, it is in line with Artzt and Armour-Thomas' (1992) findings, and this issue merits further development.

5. Conclusion

In Chapter One, I proposed Figure 1.1 as the theoretical framework for the research study. The factors that might contribute to increase learning in mathematical word problem solving performance are based on providing students with metacognitive training, providing a cognitive apprenticeship approach to instruction, and ensuring that the students are involved in collaborative interaction during word problem solving. It was hoped that these factors would influence students to become aware of their own thinking process and mathematical word problem solving knowledge structure, which in turn might increase their mathematical word problem solving performance. However, my exploration of the relationship between these factors and mathematical word problem solving performance has shown that what takes place in word problem solving is not just influenced by metacognitive training, collaborative interaction and cognitive apprenticeship instruction (see Chapter One, Figure 1.1). From my exploration in this study and the above discussion, I discover that there exist other factors or sub-factors which contribute to a complex interaction in influencing students' mathematical word problem solving performance. This interaction is illustrated in Figure 7.1

The framework shown in Figure 7.1 emerged from this exploration. It attempts to summarise and capture the interplay of various factors (as indicated by the two-way arrows) that influence students' mathematical word problem solving performance and provides initial directions for other emergent research designs of a similar nature as this study. The effect of metacognitive training on the mathematical word problem solving of Singapore 11-12 year olds in a computer environment, WordMath, was explored through mathematical achievement tests and observing students' think aloud protocols in word problem solving. In this chapter, I have first shown how the explicitness and completeness of the environment in metacognitive training with CRIME in WordMath environment led to efficacy in word problem solving performance. At the same time, what emerged from the findings of this discussion are underlying student personal factors which influence their word problem solving. For example, it appears that the levels of students' mathematical achievement play an important part in how metacognitive training can influence their word problem solving. Furthermore, the period of time provided for students to internalise metacognitive concepts in metacognitive training is an important consideration. Then, in my exploration to examine the role of metacognition and its influence in word problem solving performance from the think aloud protocol data and accounts of students' metacognitive knowledge from teacher interview and student questionnaire, it is observed that though generating metacognition is important, it is more crucial to know when and how to use metacognitive behaviours in order to be successful in word problem solving. This has an impact on how metacognitive training should be carried out in the classroom. The focus is on providing facilities for a rich store of knowledge of metacognitive strategies to be developed over an extended period of use (Stillman & Galbraith, 1998, p. 182). It is also observed that students' progression of word problem solving activity represented by five distinctive cognitive-metacognitive word problem solving models also influence how students become effective word problem solvers. Finally, it also appears that the affective factors such as the motivational aspect of students' cognition; students' personal characteristics (e.g. attitude); students' mathematical beliefs; and how students are paired according to their metacognitive knowledge have a part to play in effective pair collaboration, which in turn influence students' word problem solving. Hence, for this study to have an impact on students' mathematical word problem solving with metacognitive training in the context of the Singapore mathematics classroom, it entails an understanding of all these different factors.

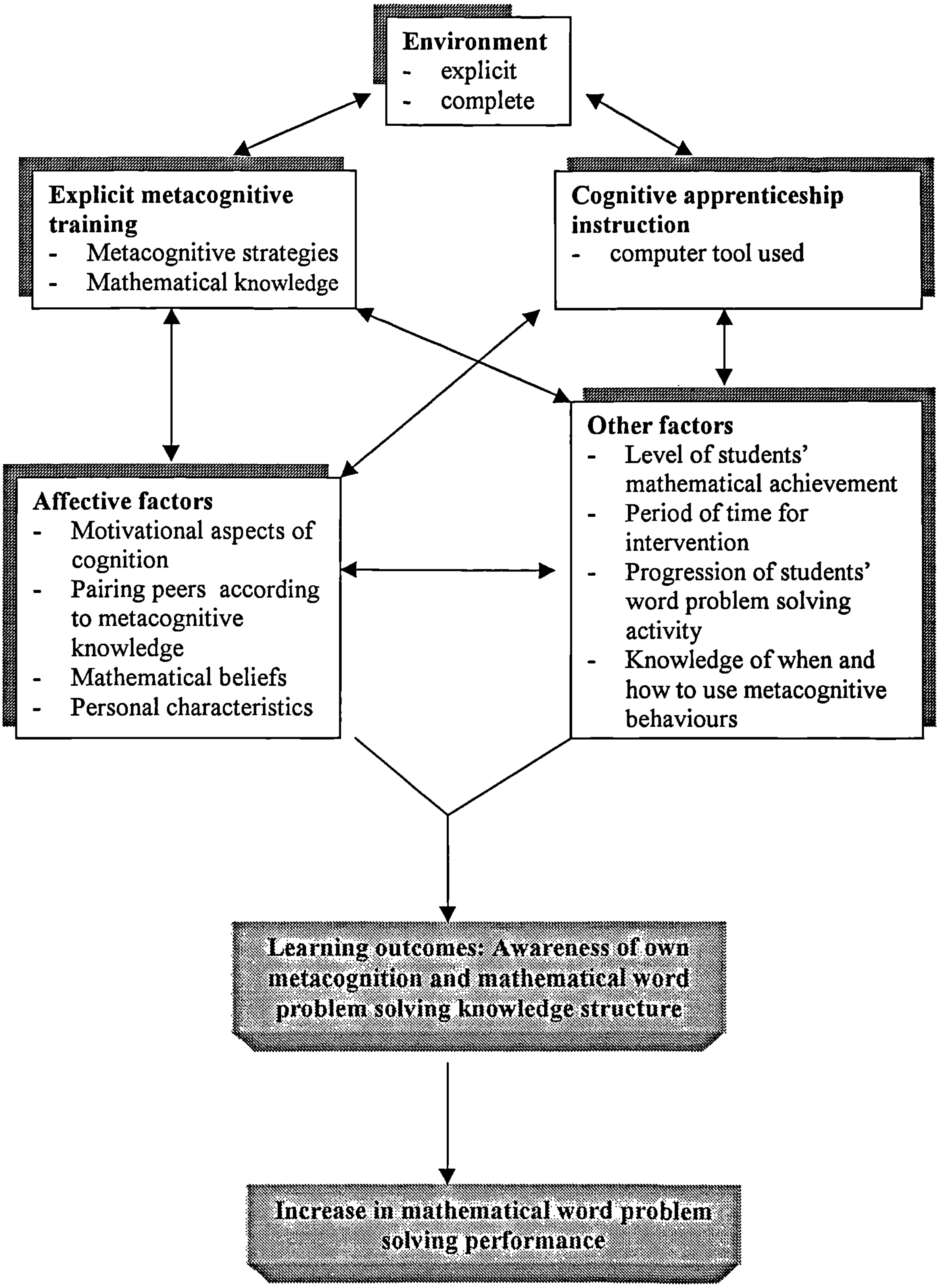


Figure 7.1: Framework of Factors influencing Mathematical Word Problem Solving Performance

Chapter Eight
Conclusion of Study

1. Introduction and Overview of Chapter

In the previous chapter, I discussed and interpreted some of the findings of this study in relation to both the mathematical achievement test data as well as the think aloud protocol, interview and questionnaire data that were presented in Chapters Five and Six respectively. I also provided a new theoretical framework (see Chapter Seven, section 5, Figure 7.1) that includes the possible factors, emerged from this study, which might influence word problem solving performance. This concluding chapter will begin by summarising findings related to this study and then discuss the educational and pedagogical implications of the findings. A critique, showing some of the limitations of the study, is made before identifying possible contributions that this study has made to the existing field of research. The chapter ends with suggestions for further research.

2. Overview of the Findings of this Research

The two main aims of this study are to investigate the effect of metacognitive training on students' word problem solving performance, and explore the role of metacognition in word problem solving in a computer environment (see Chapter One, sections 2.1 and 2.2), and four research questions (see Chapter One, section 4) were developed. The findings of this study can be summarised with respect to the four research questions in Chapter One, section 4, and the discussion from Chapter Seven.

With respect to research questions 1, 2 and 3, analysis of quasi-experimental data (see Chapter Five) reveals that

1. metacognitive training seems to promote efficaciousness in mathematical word problem solving performance. The factors may have led to this efficacy in mathematical word problem solving are the relative completeness in the use of the metacognitive strategy, CRIME, to promote metacognitive awareness; the explicitness in the pedagogical approach which led to students' metacognitive awareness in a computer environment with WordMath; and the use of WordMath, which appears to enhance students' word problem solving processes (see Chapter Seven, section 2);

2. higher achievers benefit more from cognitive apprenticeship instruction compared with lower achievers; and
3. there is not significant evidence that the benefits from metacognitive training on mathematical word problem solving performance varies with the level of students' mathematical achievement. However, visual examination of pretest, posttest 1 and delayed posttest 1 means from quasi-experimental data reveals that lower achievers appear to only show the full benefit from metacognitive training after a period of time. This result concurs with the findings from the analysis of case study data. Some factors that suggest how this phenomenon occurred are related to how treatment lower achievers needed more time to internalise the metacognitive concepts; their inadequate mathematical word problem solving knowledge; ineffective social interaction; and lack of motivation to learn metacognitive skills (see Chapter Seven, sections 2.2 and 2.2.1).

With respect to research question 4, analysis of case study data reveals that

1. Generating metacognitive behaviours, and knowing when and how to use them during word problem solving are important determinants for success in word problem solving. Furthermore, when students reached an impasse in word problem solving, the generation of metacognitive behaviours usually helped them overcome their difficulties. Finally, the interrelationship between metacognitive and cognitive processes also influences success in word problem solving;
2. Five distinct cognitive-metacognitive word problem solving models were constructed for students' progression of word problem solving activity. The progression of students' word problem solving activity, represented by the models, seem to relate to the students' success in word problem solving. Students, whose progression of word problem solving activity is Type P cognitive-metacognitive word problem solving, are more likely to be successful in their word problem solving. In contrast, students, whose word problem solving activity is Type R cognitive-metacognitive word problem solving, are more likely to fail in their word problem solving. The findings also appear to indicate that metacognitive training can modify students' progression of word problem solving activity from one model to one that is more likely to enable them to be successful in their word problem solving (see Chapter Seven, section 3.3);

3. There is a relationship between affective factors and students' word problem solving. Effectiveness of dyad collaboration is influenced by students' mathematical beliefs and how they are paired according to their metacognitive knowledge, which in turn influence their word problem solving performance. Pairs of students with similar metacognitive knowledge are more likely to be successful in word problem solving. In contrast, pairs of students with dissimilar metacognitive knowledge are more likely not to be involved in collaborative interaction. Instead the dominating peer appear to take control of the word problem solving. Other affective factors include the motivational aspects of cognitive which relates to students' motivation to engage in metacognitive training.

3. Educational and Pedagogical Implications

The quantitative and qualitative data showed the importance of metacognitive training in students' word problem solving in a computer environment. These data point to a need to formulate a specific curriculum policy to introduce explicit metacognitive training to primary school students to promote metacognitive awareness in the primary mathematics curriculum. Heeding the advice of Stillman and Galbraith (1998), the focus of metacognitive training is to provide facilities that enable students to develop a rich store of metacognitive strategies over an extended period of use (see Chapter Seven, section 3.1b). For example, Pressley and Associates (1990) provide concrete guidelines about teaching metacognitive skills. These are summarised as follows:

1. Select a few strategies to begin, and teach these strategies across the various content areas in the mathematics curriculum. Additional strategies are introduced only when the initial strategies have been fairly well established;
2. Describe the strategies and model their use. Comment aloud how the strategies are performed;
3. Model the strategies again, re-explain those aspects of using the strategies that are not well understood;
4. Explain why the strategies are used, what they can accomplish, and the specific situations in which they are used;

5. Provide guided practice by having students use the strategies for as many appropriate tasks as possible. Provide reinforcement and feedback on how the students can improve their execution of the strategies;
6. Encourage students to monitor their performance when using the strategies;
7. Encourage generalisation of the strategies by having students use them with different type of materials in various content areas as well as their continued use;
8. Increase students' motivation to use strategies by heightening student awareness that they are acquiring valuable skills that are at the heart of competent functioning; and
9. Emphasise reflective processing rather than speedy processing. Try to eliminate as much as possible high anxiety on the part of students.

At the heart of Pressley et al's (1990) practical advice is the notion that there is a need to recognise the affective issues in order to produce effective cognitive functioning. The findings from this study also indicate that affective issues relate to word problem solving performance. Law and Tan (2000) believes that students learn to be confident in their thinking and learning processes through the acquisition of metacognitive beliefs. Presumably, metacognitive beliefs guide decision making at critical junctures in learning, and with metacognitive beliefs, students can evaluate aspects of the learning situation (i.e. does personal resources match task requirement?). Paris and Winograd (1990) list four beliefs which students need to develop which influence their orientation to learning. First, students need to develop beliefs about themselves as self-directed learners. Second, students need to realise the utility of various cognitive strategies such as questioning and reflective thinking. Third, students need to develop control beliefs which enable them to develop their own power to control and direct their own thinking. Students need to believe that their actions are responsible for successful performances and that failure is neither inevitable or uncontrollable. Finally, students need to understand the purpose of their own learning, and have positive expectations for their performance and value success. The development of these beliefs has implications for their learning in a classroom that promotes metacognitive awareness.

There is also a need to train in-service and pre-service teachers in the cognitive apprenticeship approach to teaching mathematics. This involves immersing the teachers into a pedagogical approach that encourages novice-expert interactions in the form of modeling, coaching and scaffolding. It is believed that such an approach may enable the novice to internalise critical cognitive skills, such as metacognitive skills, demonstrated by the expert.

The findings from this study also suggest that effective peer collaboration relates to word problem solving performance. There is a growing awareness amongst Singapore educators about the virtue of group collaboration (Curriculum Planning Division, 1995; Yahaya, 1997). A caveat is that teachers, who encourage collaborative work in mathematics classrooms, need to be aware of the existing group dynamics, and consistently monitor and make necessary changes in groupings. It is hoped that this consistent monitoring will maximise students' collaborative interaction.

In brief, students' metacognitive awareness can be promoted by informing them about effective word problem solving strategies, and discussing cognitive and motivational characteristics of thinking. The effect of metacognitive training include transferring of responsibility for monitoring learning from teachers to students, and fostering positive self-perception, affect and motivation. This effect of metacognitive training, I believe, will enhance students' mathematical performance as evidenced by the findings in the present study.

4. A Critique of the Study

I realise that although this study has made significant contributions to the existing field of research in the arena of metacognition, it is not without its limitations. Some limitations with regard to validity and reliability issues were explicated in Chapter Three, section 2. Here, I would like to present two more limitations that are present in this research.

The first limitation has to do with the research paradigm adopted in this study. I am aware that attempting to find the influence of metacognitive training from an analysis of mathematics achievement tests from a small sample in the quasi-experimental design, and translation from one or two word problem solving sessions per dyad in the case study design, is not sufficient. Hence, studying a large sample in a true experimental design and a number of word problem solving sessions over a period of time would probably provide a deeper understanding to ascertain how students' mathematical word problem solving performance is influenced by metacognitive training. However, due to time and resource constraints of a doctorate programme, it was not possible to do a longitudinal study. I could only carry out condensed fieldwork and as a result was only able to obtain snapshots of the influence of metacognitive training within a limited period of time. Practical constraints also meant that I could not really spend as much time as may be necessary familiarising with students. Hence, I was unable to provide a more detailed account of the 8 dyads under investigation.

The second limitation has to do with the think aloud protocol analysis. My focus in the case study is on the role of metacognition in word problem solving and its influence on word problem solving outcomes. While the think aloud protocol analysis is useful as a data collection technique, the process itself also had unavoidable flaws. Think aloud protocol analysis is a time consuming procedure. As a result, the present study is limited to analysing eight pairs of students' progression of word problem solving activity. More definite results may emerge if there are more cognitive-metacognitive word problem solving models to be compared. Hence, the cognitive-metacognitive word problem solving models can only provide interesting tentative representatives of students' progression of word problem solving activity and show how they relate to students' word problem solving performance.

5. Contributions to Research

Although there has been some research carried out in exploring the role of metacognition in students' problem solving in Singapore mathematics classrooms, these studies have generally focused on secondary students' use of metacognition in problem solving in a non-computer environment. This study has investigated the role of metacognition in relation to metacognitive training in word problem solving in the primary school. Although this study represents an initial exploration into the relationship between metacognition and word problem solving in a computer environment amongst primary students, the findings of this study provide evidence that the role of primary students' metacognition, influenced by metacognitive training in a computer environment, can play in contributing to primary students' word problem solving performance. This represents a further contribution to research on primary students' metacognition influenced by metacognitive training as well as providing a way forward in understanding how this influence can help primary students in word problem solving.

In the exploration of the progression of students' word problem solving activity, five distinct types of models were constructed: Type P, Type Q, Type R, Type S and Type T cognitive-metacognitive word problem solving. Each dyad had progression of word problem solving activity which was represented by one of these cognitive-metacognitive word problem solving models. The progression of dyad's word problem solving activity appear to relate to their word problem solving performance. It was also observed that there was a possibility that the progression of dyad's word problem solving activity could be modified when the students became more aware of their cognitive processes via metacognitive training. The outcome of this initial exploration of students' progression of word problem solving activity would prove useful for researchers who wish to study students' word problem solving behaviour.

The findings from the teacher interview and student questionnaire revealed that effective pair collaboration was influenced by how students were paired according to similar metacognitive knowledge. In Artzt and Armour-Thomas' study (1992), they provide similar results but their focus is on small group settings. The findings from this study which focuses on pair collaboration represents a further contribution to research on effective pair collaboration and provide a way forward in understanding how pairing students according to similar metacognitive knowledge has an influence on their interaction and on their word problem solving.

6. Suggestions for Further Research

A number of studies have been carried out in America and Australia, but this study itself represents a preliminary study in this area of the effect of metacognitive training on Singapore 11-12 year old students' mathematical word problem solving in a computer environment and further research is needed in this area. The suggestions for further research presented in Chapters Six and Seven include examining students' metacognitive knowledge before and immediately after the intervention process; examining how WordMath or a computer tool can enhance students' metacognitive behaviours; monitoring the delayed benefit of metacognitive training on lower achievers using a longitudinal study; exploring the relationship between cognitive and metacognitive behaviours during word problem solving; and exploring the relationship between students' word problem solving performance and how they are paired according to their metacognitive knowledge. The following represents some other suggestions for further research.

First, further studies can be carried out in terms of exploring other variables which influence students' word problem solving performance with metacognitive training. This could involve investigating the relationship between word problem solving performance, metacognitive training and variables like task types, computer tools, students' age, group size, and gender. The outcome of these findings could provide further factors which might influence students' word problem solving with metacognitive training in Singapore mathematics classrooms.

Another major area of research is to explore the relationship between the effect of metacognitive training on younger and older students. The implications of this type of research would be significant in informing studies in metacognitive training among mathematics students in the primary and secondary levels. For example, if the findings of such studies indicate that the effect of metacognitive training is more significant with younger than older students, there may be a need to explore why this is so and explore the possibility of introducing metacognitive training sessions to younger students.

This study has constructed five distinct cognitive-metacognitive word problem solving models, representatives of the progression of students' word problem solving activity, which are related to word problem solving performance. Further exploration and investigation is needed into these progressions of word problem solving activity, and how these progressions of word problem solving activity relate to students' word problem solving performance.

Another suggestion is to explore the difference in influence of CRIME when it is used in computer and non-computer (i.e. using pencil and paper) environments. Such studies would illuminate the influence of metacognitive training on students' word problem solving performance in computer and non-computer environments. I noted in Teong et al (2000) that the type and role of metacognitive decisions in word problem solving differ amongst students in computer and non-computer environments and those who had metacognitive training in a computer environment. These were only indicative findings and more research is needed in this area. The results would allow teachers and educators to think about where and how metacognitive training should be undertaken with students.

Finally, this study has also shown that word problem solving performance and the willingness to engage in metacognitive training are influenced in some measure by affective factors: students' metacognitive knowledge; their mathematical beliefs; and motivation. According to McLeod (1992) and Vauras et al (1999), the role of affect in metacognitive training is still at its infancy. Further research can explore other affective factors that contribute to students' word problem solving. These findings will provide further dimensions to our understanding of the relationship of affect, metacognitive training and word problem solving performance.

References

Artzt, A & Armour-Thomas, E (1992) Development of a cognitive-metacognitive framework for protocol analysis of mathematical problem solving in small groups. *Cognition and Instruction, 9*, p. 137-175.

Bangert-Drowns, R (1993) The word processor as an instructional tool: a meta-analysis of word processing in writing instruction. *Review of Educational Research, 63*, p. 69-93.

Behrens, J & Smith, M (1996) Data and Data Analysis. *Handbook of Educational Psychology*. (Editors: Berliner, DC & Calfee, RC) New York: MacMillan Library Reference, p. 945-989.

Bruner, J (1985) Vygotsky: a historical and conceptual perspective. *Culture, Communication and Cognition: Vygotskian Perspectives* (Editor: Wertsch, J) Cambridge: Cambridge University Press.

Callanhan, L & Garofalo, J (1987) Metacognition and school mathematics. *Arithmetic Teacher, 34(9)*, p. 22-23.

Canney, G & Winograd, P (1979) *Schema for reading and reading comprehension performance* (Tech Rep No 120) Centre for the Study of Reading.

Cardelle-Elawar, M (1992) Effects of teaching metacognitive skills to students with low mathematics ability. *Teaching & Teacher Education, 8*, p. 109-121.

Cardelle-Elawar, M (1995) Effects of Metacognitive Instruction on Low Achievers in Mathematics Problems. *Teaching & Teacher Education, 11(1)*, p. 81-95.

Carr, M & Biddlecomb, B (1998) Metacognition in Mathematics from a Constructivist Perspective. *Metacognition in Educational Theory and Practice* (Editors: Hacker, D, Dunlosky, J & Graesser, A) Mahwah, New Jersey: Lawrence Erlbaum Associates, p. 69-91.

Chee, H (1995) *Teaching of problem sums.* (unpublished article, Ministry of Education, Singapore)

Chinnappan, M & Lawson, M (1996) The Effects of Training in the use of Executive Strategies in Geometry Problem Solving. *Learning and Instruction, 6(1)*, p. 1-17.

Clements, D (1990) Metacomponential Development in a Logo Programming Environment. *Journal of Educational Psychology, 82(1)*, p. 141-149.

Cohen, L & Manion, L (1994) *Research Methods in Education* (4th edition) London: Routledge.

Cohen, L, Manion, L & Morrison, K (2000) *Research Methods in Education* (5th edition) London: Routledge.

Collins, A & Brown, J (1988) The Computer as a Tool for Learning Through Reflection. *Learning Issues for Intelligent Tutoring Systems* (Editors: Mandl, H & Lesgold, A) New York: Springer-Verlag, p. 1-18.

Collins, A, Brown, J & Holum, A (1991) Cognitive Apprenticeship: Making Thinking Visible. *American Educator, Winter.*

Collins, A, Brown, J & Newman, S (1989) Cognitive apprenticeship: Teaching the craft of reading, writing and mathematics. *Knowing learning and instruction: Essays in honor of Robert Glaser* (Editor: Resnick, L) Hillsdale: Lawrence Erlbaum Associates, p. 453-494.

Cook, T & Campbell, D (1979) *Quasi-Experimentation. Design & Analysis Issues for Field Settings.* Boston: Houghton Mifflin Company.

Creswell, J (1998) *Qualitative Inquiry and Research Design. Choosing Among Five Traditions.* Thousand Oaks: Sage Publications.

Crook, C (1995) *Computers and the Collaborative Experience of Learning* (second edition) London: Routledge.

Curriculum Development Institute of Singapore (1996) *Primary Mathematics 6B* (second edition) Singapore: Federal Publications (S) Pte Ltd.

Curriculum Planning & Development Division (1998) *Syllabus. Mathematics, Primary. EM1, EM2 & EM3 Streams.* Ministry of Education, Singapore.

Curriculum Planning Division (1995, reprint) *Mathematics Syllabus. Primary One to Primary Four. Primary Five and Primary Six (EM1 & EM2 Streams).* Ministry of Education, Singapore.

Damon, W & Phelps, E (1989) Strategic uses of peer learning in children's education. *Peer relationships in child development* (Editors: Berndt, T & Ladd, G) New York: Wiley.

Davidson, J & Sternburg, R (1998) Smart Problem Solving: How Metacognition Helps. *Metacognition in Educational Theory and Practice* (Editors: Hacker, D, Dunlosky, J & Graesser, A) Mahwah: Lawrence Erlbaum Associates, p. 47-68.

De Corte, E, Greer, B & Verschaffel, L (1996) Mathematics teaching and learning. *Handbook of Educational Psychology* (Editors: Berliner, D & Calfee, R) New York: MacMillan Library Reference, p. 491-549.

Delclos, V & Harrington, C (1991) Effects of Strategy Monitoring and Proactive Instruction on Children's Problem Solving Performance. *Journal of Educational Psychology, 83(1),* p. 35-42.

Díaz, R, Neal, C & Williams, M (1990) Social origins of self-regulation. *Vygotsky and Education. Instructional Implications and Applications of Sociohistorical Psychology* (Editor: Moll, L) Cambridge: Cambridge University Press, p. 127-154.

Dillenbourg, P (1999) Introduction: What Do You Mean by "Collaborative Learning"? *Collaborative Learning. Cognitive and Computational Approaches* (Editor: Dillenbourg, P) Amsterdam: Pergamon, p. 1-19.

Dörfler, W (1993) Computer Use and Views of the Mind. Learning from Computers: *Mathematics Education and Technology* (Editors: Keitel, C & Ruthven, K) Berlin: Springer-Verlag, p. 159-186.

Ely, M, Anzul, M, Friedman, T, Garner, D & Steinmetz, A (1991) *Doing Qualitative Research: Circles within Circles.* London: Flamer Press.

Ely, M, Vinz, R, Downing, M & Anzul, M (1997) *On Writing Qualitative Research. Living by Words.* London: The Falmer Press.

English, L & Halford, G (1995) *Mathematics Education. Models and Processes.* Mahwah, New Jersey: Lawrence Erlbaum Associates.

Ericsson, KA & Simon, HA (1993) *Protocol Analysis. Verbal Reports as Data.* Cambridge, Massachusetts: The MIT Press.

Ferguson, G & Takane, Y (1989) *Statistical Analysis in Psychology and Education* (sixth edition) New York: McGraw-Hill Book Company.

Flavell, JH (1976) The Metacognitive aspects of problem solving. *The Nature of Intelligence* (Editor: Resnick, L) Hillsdale: Lawrence Erlbaum Associates, p. 231-235.

Flavell, JH (1981) Cognitive monitoring. *Children's oral communication skills* (Editor: Dickson, WP) New York: Academic, p. 35-60.

Fong, H K (1999) *Top Marks.* Singapore: Longman.

Garner, R (1988) Verbal Report Data on Cognitive and Metacognitive Strategies. *Learning and Study Strategies: Issues in Assessment, Instruction and Evaluation* (Editor: Weinstein, CE, Goetz, ET & Alexander, PA) San Diego: Academic Press, p. 63-76.

Garofalo, J & Lester, FK (1985) Metacognition, Cognitive Monitoring and Mathematical Performance. *Journal for Research in Mathematics Education, 16(3),* p. 163-176.

Goos, M & Galbraith, P (1996) Do It This Way! Metacognitive Strategies in Collaborative Mathematical Problem Solving. *Educational Studies in Mathematics, 30*, p. 229 - 260.

Gourgey, A (1998) Metacognition in basic skills instruction. *Instructional Science, 26(1-2)*, p. 81-96.

Green, A (1995) Verbal Protocol Analysis. *The Psychologist, 8(3)*, p. 126-129.

Green, C & Gilhooly, K (1996) Protocol analysis: practical implementation. *Handbook of Qualitative Research Methods for Psychology and the Social Sciences* (Editor: Richardson, J). Leicester: The British Psychology Society, p. 55-74.

Hegarty, M, Mayer, R & Monk, C (1995) Comprehension of arithmetic word problems: A comparison of successful and unsuccessful problem solvers. *Journal of Educational Psychology, 87*, p. 18-32.

Henkel, R (1976) *Tests of Significance* (Editor: Uslaner, E) Beverley Hills: Sage Publications.

Hiebert, J & Carpenter, TP (1992) Learning and Teaching with understanding. *Handbook of research on mathematics teaching and learning* (Editor: Grouws, D A) New York: Macmillan, p. 65-97.

Holyoak, KJ (1995) Problem solving. *Thinking* (Editors: Smith, E & Osherson, D) Cambridge: MIT Press.

Howitt, D & Cramer, D (2000) *An Introduction to Statistics in Psychology. A Complete Guide for Students* (second edition) Essex: Pearson Education Limited.

Hoyles, C, Sutherland, R & Healy, L (1991) Children talking in computer environment: New insights into the role of discussion in mathematics learning. *Language in Mathematical Education. Research and Practice* (Editors: Durkin, K & Shire, B) Milton Keynes: Open University Press, p. 162-176.

Huberman, A & Miles, M (1998) Data Management and Analysis Methods. *Collecting and Interpreting Qualitative Materials* (Editors: Denzin, N & Lincoln, Y) Thousand Oaks: Sage Publications, p. 179-210.

Iversen, G & Norpoth, H (1976) *Analysis of Variance*. (Editor: Uslaner, E) Beverly Hills: Sage Publications.

Jaeger, RM & Bond, L (1996) Quantitative Research Methods and Design. *Handbook of Educational Psychology*. (Editors: Berliner, DC & Calfee, RC) New York: MacMillan Library Reference, p. 877-898.

Jarvela, S (1998) Socioemotional aspects of students' learning in a cognitive-apprenticeship environment. *Instructional Science, 26*, p. 439-472.

Kail, RV Jr & Bisanz, J (1982) Cognitive Strategies. *Handbook of research methods in human memory and cognition* (Editor: Puff, CR) New York: Academic Press, p. 229-255.

Kaplan, CA & Davidson, JE (1988) *Hatching a theory of incubation effects.* Carnegie Mellon University, Department of Psychology.

King, A (1991) Effects of Training in Strategic Questioning on Children's Problem Solving Performance. *Journal of Educational Psychology, 83(3),* p. 307-317.

King, A (1992) Comparison of self-questioning, summarising and notetaking-review as strategies for learning from lectures. *American Educational Research Journal, 29(2),* p. 303-323.

King, A (1994) Guiding Knowledge Construction in the Classroom: Effects of Teaching Children How to Question and How to Explain. *American Educational Research Journal, 31(2),* p. 338-368.

Kramarski, B & Mevarech, Z (1997) Cognitive-metacognitive training within a problem solving based Logo environment. *British Journal of Educational Psychology, 67(4),* p. 425-446.

Kreutzer, M, Leonard, C & Flavell, J (1975) An interview study of children's knowledge about memory. *Monographs of the Society for Research in Child Development, 40* (1, Serial No. 159).

Kruteskii, V (1976) *The psychology of mathematical abilities in school children.* (Editors: Kilpatrick, J & Wirszup, I) Chicago: The University of Chicago Press.

Kubiszyn, T & Borich, G (1990) *Educational Testing and Measurement. Classroom Application and Practice* (3rd Edition) Brown Higher Education.

Lave, J (1988) *Cognition in practice.* Cambridge: Cambridge University Press.

Law, LC & Tan, AG (2000) Reading 1. Thinking, Metacognition, Critical Thinking. *Psychology of Cultivating Creativity. Conceptions and Competence.* Singapore: Lingzi Mass Media, p. 129-141.

Lee, T (1999) *Using Qualitative Methods in Organisational Research.* Thousand Oaks: Sage Publications.

Leik, R (1997) *Experimental Design and The Analysis of Variance.* California: Pine Forge Press.

Leo, E & Galloway, D (1996) Conceptual links between Cognitive Acceleration through Science Education and Motivational Style: a critique of Adey and Shayer. *International Journal of Science Education, 18(1),* p. 35-49.

Lesh, R (1982) Modeling students' modeling behaviours. *Proceedings of 4th Annual PME meeting, Athens, GA.*

Lester, FK & Garofalo, J (ed) (1982) *Mathematical Problem Solving Issues in Research.* Philadelphia: The Franklin Institute Press.

Lester, FK (1983) Trends and issues in mathematical problem solving research. *Acquisition of mathematics concepts and processes* (Editors: Lesh, R & Landau, M) New York: Academic Press, p. 229-261.

Lester, FK (1994) Musing about mathematical problem solving research: 1970-1994. *Journal for Research in Mathematics Education, 25(6),* p. 660-675.

Lester, FK, Garofalo, J & Kroll, DL (1989) *The role of metacognition in mathematical problem solving: A study of two grade seven classes.* Final report to the National Science Foundation of NSF project MDR 85-50346.

Lim, ST (1990) What is the overall goal in teaching? *Mathematics Newsletter. Curriculum Planning Division, 1(3),* p. 1-5.

Looi, C & Tan, B (1997) *A Cognitive-Apprenticeship-based Environment for Learning Word Problem Solving.* Unpublished article, Singapore.

Loper, A & Murphy, D (1985) Cognitive Self-Regulatory Training for Underachieving Children. *Metacognition, Cognition and Human Performance. Volume 2* (Editors: Forrest-Pressley, D, MacKinnon, G & Waller, T) Orlando: Academic Press, p. 223-266.

Maqsud, M (1998) Effects of metacognitive instruction on mathematics achievement and attitude towards mathematics of low mathematics achievers. *Educational Research, 40(2),* p. 237-243.

Markman, E (1979) Realising that you don't understand: Elementary school students' awareness of inconsistencies. *Child Development, 50,* p. 643-651.

Marzano, RJ, Brandt, RS, Hughes, CS, Jones, BF, Presseisen, BZ, Rankin, SC & Suhor, C (1988) *Dimensions of Thinking: A Framework for Curriculum and Instruction.* U.S.A., Virginia: Association for Supervision and Curriculum Development.

Mayer, R & Wittrock, M (1996) Problem Solving Transfer. *Handbook of Educational Psychology* (Editors: Berliner, D & Calfee, R) New York: MacMillan, p. 47-62.

Mayer, R (1998) Cognitive, Metacognitive, and Motivational Aspects of Problem Solving. *Instructional Science, 26(1-2),* p. 49-63.

McCrindle, A & Christensen, C (1995) The Impact of Learning Journals on Metacognitive and Cognitive Processes and Learning Performance. *Learning and Instruction, 5,* p. 167-185.

McLeod, D (1992) Research on Affect in Mathematics Education: A Reconceptualisation. *Handbook of Research on Mathematics Teaching and Learning* (Editor: Grouws, D) New York: Macmillan, p. 575-596.

Mevarech, Z & Kapa, E (1996) The effects of a problem-solving based Logo environment on children's information processing components. *British Journal of Educational Psychology, 66*, p. 181-195.

Mevarech, Z (1999) Effects of Metacognitive Training Embedded in Cooperative Settings on Mathematical Problem Solving. *The Journal of Educational Research, 92(4)*, p. 195-205.

Miles, MB & Huberman, AM (1998) Data Management and Analysis Methods. *Collecting and Interpreting Qualitative Materials* (Editors: Denzin, N & Lincoln, Y) Beverly Hills: Sage Publications, p. 179- 210.

Ministry of Education (1998) *Learning to Think. Thinking to Learn. Towards thinking schools, learning nation.* Singapore

Mok, CH (1997) *The Thinking Programme. Explicit Teaching of Thinking. Teacher's Manual.* Secondary One. Curriculum Planning Division. Ministry of Education, Singapore.

Montague, M & Bos, C (1990) Cognitive and metacognitive characteristics of eighth grade students' mathematical problem solving. *Learning Individual Differences, 2*, p. 371-388.

Nelissen, J (1987) *Children learning mathematics: A study on construction and reflection in elementary school children.* The Netherlands.

Newton, R & Rudestam, K (1999) *Your Statistical Consultant. Answers to your data analysis questions.* Thousand Oaks: Sage Publications.

Nickerson, RS, Perkins, DN & Smith EE (editors) (1985) *The Teaching of Thinking.* Hillsdale: Lawrence Erlbaum Associates.

Norušis, MJ (1999) *SPSS 9.0 Guide to Data Analysis.* New Jersey: Prentice Hall.

Noss, R & Hoyles, C (1996) *Windows on Mathematical Meanings. Learning Cultures and Computers.* Dordrecht: Kluwer Academic Publishers.

Oladunni, M (1998) An experimental study on the effectiveness f metacognitive and heuristic problem solving techniques on computational performance of students in mathematics. *International Journal of Mathematical Education and Science Technology, 29(6)*, p. 867-874.

Paris, SG & Winograd, P (1990) How metacognition can promote academic learning and instruction. *Dimensions of thinking and cognitive instruction* (Editors: Jones, BF & Idol, L) Hillsdale: Erlbaum, p. 15-52.

Paris, SG, Lipson, MY & Wixson, KK (1983) Becoming a strategic reader. *Contemporary Educational Psychology, 8,* p. 293-316.

Perkins, DN (1989) Reasoning as it is and could be: An empirical perspective. *Thinking across cultures: The Third International Conference on Thinking* (Editors: Topping, DM, Crowell, DC & Kobayashi, VN) Hillsdale: Lawrence Erlbaum Associates.

Pirie, S (1991) Peer discussion in the context of mathematical problem solving. *Language in Mathematical Education. Research and Practice* (Editors: Durkin, K & Shire, B) Milton Keynes: Open University Press, p. 143-161.

Polya, G (1973) *How to solve it: a new aspect of mathematical method.* Princeton: Princeton University Press (original work published 1945).

Pressley, M & Associates (1990) *Cognitive strategy instruction that really improves children's academic performance.* Cambridge: Brookline Books.

Reed, S (1999) *Word Problems. Research and Curriculum Reform.* Mahwah: Lawrence Erlbaum Associates.

Reeve, J (1996) *Motivating Others: Nurturing Inner Motivational Resources.* Allyn and Bacon.

Robson, C (1993) *Real World Research. A Resource for Social Scientists and Practitioner-Researchers.* Oxford: Blackwell.

Rogoff, B (1998) Cognition as a collaborative process. *Handbook of Child Psychology (5th edition) Vol. 2 Cognition, Perception and Language* (Editors: Damon, W, Kuhn, D & Siegler, RS) New York: John Wiley, p. 679-744.

Schoenfeld, A (1982) Some thoughts on problem solving research and mathematics education. *Mathematical Problem Solving: Issues in Research* (Editors: Lester, FK & Garofalo, J) Philadelphia: Franklin Institute Press, p. 27-37.

Schoenfeld, A (1983) 'Episodes and Executive Decisions' in Mathematical Problem Solving. *Acquisition of mathematics concepts and processes* (Editors: Lesh, R & Landau, M) New York: Academic Press, p. 345-395.

Schoenfeld, A (1985) *Mathematical Problem Solving.* San Diego: Academic Press.

Schoenfeld, A (1987) What's the fuss about metacognition?. *Cognitive Science and mathematics education* (Editor: Schoenfeld, A) Hillsdale: Lawrence Erlbaum Associates, p. 189-215.

Schoenfeld, A (1992) Learning to think mathematically: problem solving, metacognition and sense making in mathematics. *Handbook of research on mathematics thinking and learning* (Editor: Grouws, D) New York: MacMillan, p. 334-370.

Schroeder, TL & Lester, FK (1989) Developing Understanding in Mathematics via problem solving. *New Directions for Elementary School Mathematics.* Reston: National Council of Teachers of Mathematics, p. 39.

Silver, E (1982) Knowledge organisation and mathematical problem solving. *Mathematical Problem Solving: Issues in Research* (Editors: Lester, FK & Garofalo, J) Philadelphia: Franklin Institute Press.

Silver, E (editor) (1985) *Teaching and Learning Mathematical Problem Solving: Multiple Research Perspectives.* Hillsdale: Lawrence Erlbaum Associates.

Silverman, D (1993) *Interpreting Qualitative Data. Methods for Analysing Talk, Text and Interaction.* London: Sage Publications.

Silverman, D (2000) *Doing Qualitative Research. A Practical Handbook.* London: Sage Publications.

Stake, R (1995) *The Art of Case Study Research.* Thousand Oaks: Sage Publications.

Steffe, LP (1994) Introduction to this special issue: Mathematical learning in computer microworlds. *Journal Research in Mathematics Education, 8*, p. 85-86.

Sternburg, R (1998) Metacognition, abilities, and developing expertise: What makes an expert student?. *Instructional Science, 26(1-2),* p. 127-140.

Stillman, G & Galbraith, P (1998) Applying Mathematics with Real World Connections: Metacognitive Characteristics of Secondary Students. *Educational Studies in Mathematics, 36,* p. 157-195.

Tay-Koay, SL (1999) Thinking Processes of Adolescents in Classrooms. *Growing Up in Singapore. Research Perspectives on Adolescents* (Editors: Chang, A, Gopinathan, S & Ho, WK) Singapore: Prentice Hall, p. 71-85.

Teong, SK (1997) A Critical Review of the Revised Mathematics (Pri) Curriculum in Singapore Schools (unpublished MEd Essay, University of Leeds)

Teong, SK, Threlfall, J & Monaghan, J (2000) The Effects of Metacognitive Training in Mathematical Word Problem Solving in a Computer Environment. *Proceedings of the 24^{th} Conference of the International Group for the Psychology of Mathematics Education Volume 4* (Editors: Tadao, N & Masataka, K) p. 193-200.

The Straits Times (3 June 1997) *Vision for a Total Learning Environment: Thinking Schools, Learning Nation.*

Tilley, A (1996) *An Introduction to Psychological Research and Statistics* (third edition). Queensland: Pineapple Press.

Vauras, M, Rauhanummi, T, Kinnunen, R & Lepola, J (1999) Motivational vulnerability as a challenge for educational interventions. *International Journal of Educational Research, 31,* p. 515-531.

Verschaffel, L & De Corte, E (1997) Word Problems: A Vehicle for Promoting Authentic Mathematical Understanding and Problem Solving in the Primary School? *Learning and Teaching Mathematics. An International Perspective* (Editors: Nunes, T & Bryant, P) East Sussex: Psychology Press Ltd, p. 69 - 97.

Wertsch, J (1990) The voice of rationality in a sociocultural approach to mind. *Vygotsky and Education. Instructional Implications and Applications of Sociohistorical Psychology* (Editor: Moll, L) Cambridge: Cambridge University Press, p. 111-126.

Width, A & Ahotola, O (1978) Analysis of Covariance. Beverly Hills: Sage Publications.

Williams, M (1999) Introduction. *Integrating Technology into Teaching and Learning. Concepts and Applications* (Editor: Williams, M) Singapore: Division of Instructional Science, National Institute of Education, p. 1-8

Wilson, M (1996) Asking Questions. *Data Collection and Analysis* (Editors: Sapsford, R & Jupp, V) London: SAGE, p. 94-120.

Wong, P (1997) Metacognition in mathematical problem solving. *Education in Singapore. A Book of Readings* (Editors: Tan, J, Gopinathan, S & Ho WK) Singapore: Prentice Hall, p. 157-170.

Yahaya, Z (1997) Small Group Learning in the Mathematics Classroom. *The Mathematics Educator, 2,* p. 83-92.

Yeap, B & Menon, R (1996) Metacognition during mathematical problem solving. *Educational Research Association, Singapore and Australian Association for Research in Educational Joint Conference 1996.*

Yeap, BH & Kuar, B (1996) Problem Solving in Regular Classroom: The Use of Pattern Observation. *Education Research Association, Singapore and Australian Association for Research in Education Joint Conference 1996.*

Yeap, BH (1997) *Mathematical problem solving: A focus on metacognition.* (unpublished MEd dissertation, Nanyang Technological University, Singapore)

Yee, WH (1998) The Thinking Programme. *Mathematics Newsletter, 1(18),* Curriculum Planning & Development Division, Ministry of Education, Singapore, p. 7-8.

Yin, RK (1989) *Case Study Research: Design and Methods* (second edition) Newbury Park: Sage Publications.

Yip, J, Eng, SP & Yap, J (1997) 25 Years of Educational Reform. *Education in Singapore. A Book of Readings* (Editors: Tan, J, Gopinathan, S & Ho, WK) Singapore: Prentice Hall, p. 3-32.

Appendix A: Singapore Primary Education System and The Revised Mathematics Syllabus

1. Singapore Primary Education System

In 1991, the 'Improving Primary School Education (IPSE)' report recommended a number of changes to Singapore primary education system (Yip et al, 1997). As a result of this report, the Singapore primary education system was modified. In brief, primary education is structured to have a foundation stage of four years (7 years old to 10 years old) and an orientation stage of two years (11 years old and 12 years old). Streaming students takes place at the end of the foundation stage at Primary 4. These students are streamed into EM1, EM2 and EM3 classes, based on two scores from their English, Mathematics or Mother Tongue end-of-the-year examination results. Sometimes, some students remain in the EM2 stream even though they qualify for the EM1 stream. This phenomenon also occurs to students who are streamed into the EM3 stream but choose to remain in the EM2 stream. The school usually makes a recommendation for the stream the child qualifies for but the final say lies on the parents' decision. The table below shows the classification of the three streams:

	English	Mother Tongue	Mathematics
EM1	>=85%	>=85%	>=85%
EM2	49%<score<85%	49%<score<85%	49%<score<85%
EM3	<=49%	<=49%	<=49%

At the end the orientation stage at Primary Six, the students are required to sit for a national examination, the Primary School Leaving Examination (PSLE). The scores in this examination will determine the type of programme students qualify for when they enter Secondary Schools. If their scores are above a certain aggregate (taken from the four core subjects: English, Mother Tongue, Mathematics and Science), they will qualify for a four-year secondary education in the Special stream where English and Mother Tongue are taken as first languages or the Express stream where English is taken as a first language and Mother Tongue is taken as a second language. If the scores are below a certain aggregate, the students will qualify for a five-year secondary school education in the Normal (Academic) stream or Normal (Technical) stream.

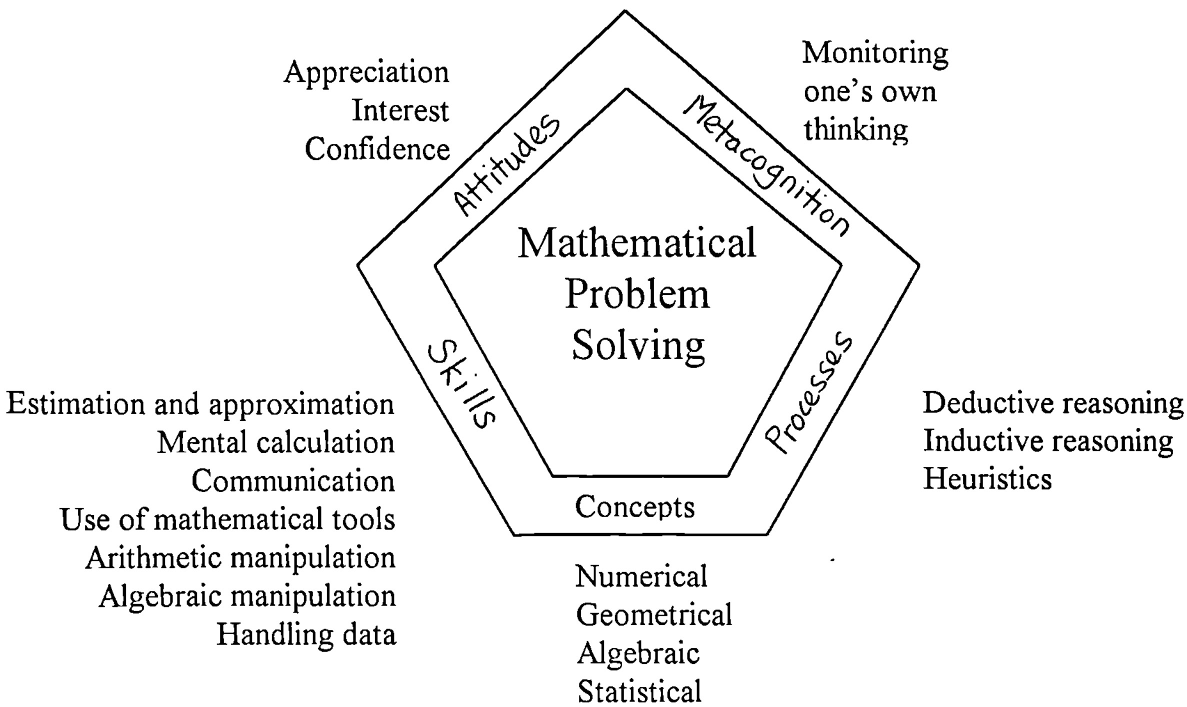
The students are given a grade for each subject in their PSLE result. The grades and corresponding scores in the Primary School Leaving Examination are defined as follows:

Grade	Score
A*	91% and above
A	75% to 90%
B	60% to 74%
C	50% to 59%
D	35% to 49%
E	20% to 34%
U	below 20%

2. **The Revised Mathematics Syllabus**

The Ministry of Education (MOE), Singapore, reviews the school syllabuses every 8 to 10 years. It is an ongoing process to ensure that they continue to meet the needs of the students and the nation. The last revision of the primary and secondary mathematics syllabus was in 1992.

The spirit and emphases of the revised syllabus are best encapsulated in the following framework for mathematics education in Singapore schools (Curriculum Planning Division, 1995).



The primary aim of the mathematics curriculum is to enable students develop their ability in mathematics problem solving. In this context, the word 'problem' covers a wide range of situations from routine mathematical problems to open-ended investigations that make use of relevant mathematics. The attainment of problem solving ability is dependent on five independent related components - Concepts, Skills, Processes, Attitudes and Metacognition. The above framework encompasses the whole Singapore mathematics curriculum from primary to secondary schools (Curriculum Planning Division, 1995).

Appendix B: The Model Approach

In 1992, the Revised Mathematics Syllabus was implemented in Singapore primary and secondary schools. In the primary school, word problem solving form a major part of the curriculum in upper (9 to 12 years old) mathematics. It requires students to solve mathematical word problems encoded in situations such as the following:

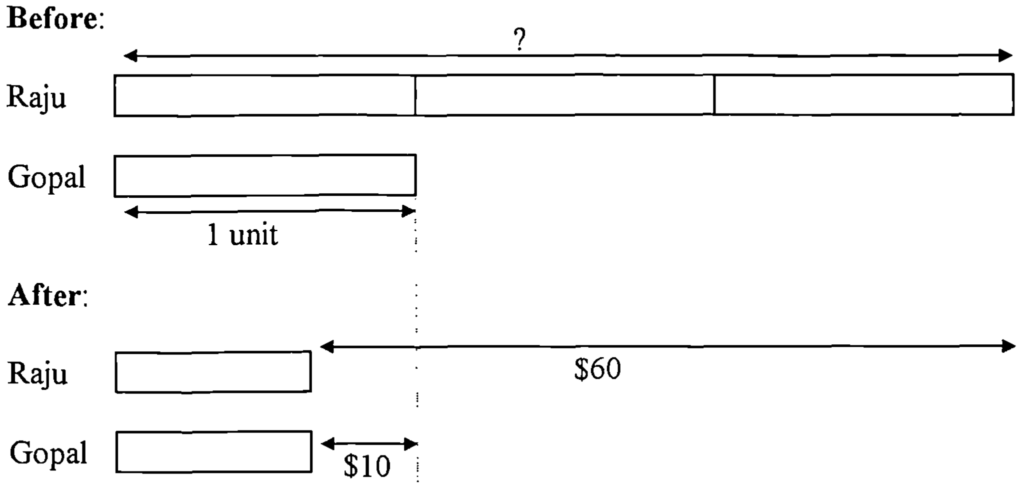
Raju had 3 times as much money as Gopal. After Raju spent $60 and Gopal spent $10, they each had an equal amount of money left. How much money did Raju have at first? (Curriculum Development Institute of Singapore, 1996, p. 69)

The word problems are designed to depict real-life situations such as daily expenditure and grocery shopping. In schools, students learn to use the 'model approach' (Fong, 1999) as a tool to solve word problems. The model approach refers to a method of solving mathematical word problems in which diagrams are drawn to represent the word problem situation. According to Fong (1999, p. 49-50), the general steps for using this approach are:

(1) Read the word problem. Then represent the word problem situation using a bar diagram;
(2) Fill in the diagram with all the given information;
(3) Divide the bar diagram into equal units;
(4) Form a proportional[1] statement by equating the number of units to a quantity. The value of the quantity may be obtained by computing some given figures in the word problem;
(5) Use the proportion method to obtain the answer.

[1] Proportion: a relation between 2 variables in which the ratio remains constant.
e.g. 2 pencils cost 80 cents. Ratio = 80 : 2 = 40 : 1
4 pencils cost $1.60. Ratio = 160 : 40 = 40 : 1 Ratio remains constant.

With regard to the above word problem, a student will probably draw the following two sets of models:



Then, using the above sets of models, the student will form a proportional statement by equating the number of units to a quantity (2 units represent $50) and work out the answer as shown below.

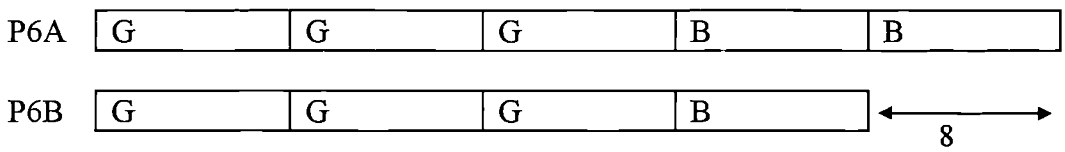
2 units --- $60 - $10 = $50
1 unit --- $50 ÷ 2 = $25
Raju had (3 units) = $25 x 3
= $75

This technique of model building to solve the word problem is an approach to making a word problem concrete. It is claimed that by drawing blocks, students can 'visualise' the word problems more clearly.

Model building is a powerful approach that can be best illustrated by word problems, often involving fractions, ratios and percentage, which appear difficult. When models are drawn to show the word problem situation, the solution becomes clearer or sometimes obvious. The following is another example.

⅗ of P6A and ¾ of P6B are girls. Both classes have the same number of girls and P6A has 8 more boys than P6B. How many pupils are there in P6A? (Looi & Tan, 1997)

The appropriate model is



Then, forming a proportional statement by equating the number of units to a quantity (i.e. 1 unit represents 8 pupils), the solution can be calculated as follows:

1 unit --- 8 pupils
P6A has (5 units) = 8 x 5
= 40 pupils

Word problems like the above can be solved using variables. However, it has been reported that children face difficulties with the concept of variables (Chee, 1995). Hence the model approach ('pictorial algebra') was introduced to Singapore primary schools. This model approach was developed locally more than ten years ago by Hector Chee, a mathematics teacher, and has since been used extensively in the Singapore primary mathematics curriculum.

Appendix C : Word Problems for Quasi-experimental Design and Case Study Design

1. Word Problems for Quasi-Experimental Design

a) Pretest Word Problems

1. A farmer bought a total of 85 ducks and geese for $445. When 15 ducks died, there was an equal number of ducks and geese left. If each goose cost $3 more than each duck, find the cost of each goose.
2. Mr Lim had a bag of sweets for his class. After giving each pupil 8 sweets, he had 3 sweets left. If he had given only 5 sweets to each pupil, he would have 108 sweets left. How many pupils were there in his class?
3. There are altogether 15 marbles in the three boxes A, B and C. If we move 3 marbles from A to B, move 2 marbles from B to C, and then move 1 marble from C to A, there will be the same number of marbles in the boxes. How many marbles are there in A, B and C at first?
4. A class of 43 pupils donated a total of $306 to the Community Chest. Each boy donated $2 more than each girl. If there were 5 more boys than girls, how much did each girl donate?
5. There is a group of children. A boy in the group says, "Not counting me, the number of boys is half the number of girls." A girl in the group says, "Not counting me, there is the same number of boys and girls." How many children are there in the group?
6. There are some red, yellow and blue beads in a box. $\frac{1}{3}$ of the beads is red and $\frac{1}{4}$ of the beads is yellow. If 35 blue beads are removed from the box, there is half as many blue beads as yellow beads now. How many beads are there altogether in the box at first?
7. Alice read $\frac{1}{4}$ of a book on Sunday. She read 6 more pages on Monday than on Sunday. If she still had 36 pages to read, how many pages did she read on Sunday?
8. Jim had a sum of money. He spent $\frac{3}{5}$ of it on a pair of shoes and 4 pairs of socks. Each pair of socks cost $\frac{1}{5}$ as much as the pair of shoes. He had $36 left. What was the cost of the pair of shoes?

9. David and Betty each had an equal amount of money at first. After David had spent $18 and Betty had spent $42, Betty had $\frac{2}{3}$ as much money as David. How much money did each of them have at first?
10. A tank was $\frac{1}{5}$ full. When 700 ml of water was poured into the tank, it became $\frac{2}{3}$ full. Find the capacity of the tank.

b) **Posttest 1 Word Problems**

1. A shopkeeper bought a total of 85 books and magazines for $445. When 15 magazines were sold, there was an equal number of books and magazines left. If each book cost $3 more than each magazine, find the cost of each book.
2. Nancy bought some balloons for the children in the Children's Home. After giving each child 8 balloons, she had 3 balloons left. If she had given only 5 balloons to each child, she would have 108 balloons left. How many children were there in the Children's Home?
3. There are altogether 15 sweets in the green, yellow and red containers. If we move 3 sweets from the green container to the yellow container, move 2 sweets from the yellow container to the red container, and then move 1 sweet from the red container to the green container, there will be the same number of sweets in all the containers. How many sweets are there in the green, yellow and red containers at first?
4. A company of 43 workers donated a total of $306 to the Ju Eng Home for the Aged. Each male worker donated $2 more than each female worker. If there were 5 more male worker than female workers, how much did each female worker donate?
5. There is a group of mothers and their children. A mother in the group says, "Not counting me, the number of mothers is half the number of children." A child in the group says, "Not counting me, there is the same number of mothers and children." How many mothers and children are there in the group?
6. Mrs Tan made some pineapple, apple and strawberry tarts. $\frac{1}{3}$ of the tarts was pineapple and $\frac{1}{4}$ of the tarts was apple. If 35 strawberry tarts were given away, there was half as many strawberry tarts as apple tarts. How many tarts did Mrs Tan make?

7. John completed $\frac{1}{4}$ of his school assignment on Sunday. He wrote 6 more pages on Sunday than on Saturday. If he still had 36 pages to write, how many pages did he write on Saturday?
8. Eunice was given a sum of money. She spent $\frac{3}{5}$ of it on a blouse and 4 pairs of shorts. Each pair of shorts cost $\frac{1}{5}$ as much as the blouse. She had \$36 left. What was the cost of the blouse?
9. Tim and Zoe each had an equal amount of money at first. After Tim had spent \$18 and Zoe had spent \$42, Zoe had $\frac{2}{3}$ as much money as Tim. How much money did each of them have at first?
10. A container was $\frac{1}{5}$ full. When 700 ml of orange syrup was poured into the container, it became $\frac{2}{3}$ full. Find the capacity of the container.

c) **Delayed Posttest 1 Word Problems**

1. Mrs Deva bought a total of 85 durians and mangoes for \$445. After giving away 15 mangoes, there was an equal number of durians and mangoes left. If each durian cost \$3 more than each mango, find the cost of each durian.
2. On Children's Day, Miss Teo brought some marbles for her class. After giving each student 8 marbles, she had 3 marbles left. If she had given only 5 marbles to each student, she would have 108 marbles left. How many students were there in her class?
3. Jane puts 15 beads into three cups A, B and C. If she moves 3 beads from A to B, moves 2 beads from B to C, and then moves 1 bead from C to A, there will be the same number of beads in the cups. How many beads are there in A, B and C at first?
4. Tom bought 43 fiction and non-fiction books for \$306. Each fiction book cost \$2 more than each non-fiction book. If there were 5 more fiction books than non-fiction books, how much did each non-fiction book cost?
5. There is a group of hawkers. A male hawker in the group says, "Not counting me, the number of male hawkers is half the number of female hawkers." A female hawker in the group says, "Not counting me, there is the same number of male and female hawkers." How many hawkers are there in the group?
6. There are some yellow, purple and white orchids in a flower shop. $\frac{1}{3}$ of the orchids is yellow and $\frac{1}{4}$ of the orchids is purple. If 35 white orchids are sold, there is half as

many white orchids as purple orchids now. How many orchids are there altogether in the flower shop at first?

7. Ali read $\frac{1}{4}$ of a Science magazine on Wednesday. He read 6 more pages on Thursday than on Wednesday. If he still had 36 pages to read, how many pages did he read on Wednesday?
8. Simon was given a sum of money to purchase new clothes for Chinese New Year. He spent $\frac{3}{5}$ of it on a pair of trousers and 4 T-shirts. Each T-shirt cost $\frac{1}{5}$ as much as the pair of trousers. He had \$36 left. What was the cost of the pair of trousers?
9. David and Betty each had an equal number of 20-cent coins at first. After David had given away 18 coins and Betty had spent 42 coins, Betty had $\frac{2}{3}$ as many coins as David. How many 20-cent coins did each of them have at first?
10. An oil tank was $\frac{1}{5}$ full. When 700 ml of oil was poured into the oil tank, it became $\frac{2}{3}$ full. Find the capacity of the oil tank.

2. **Word Problems for Case Study Design**

a) **Posttest 2 Word Problems**

N1 : John and Evan had an equal number of stamps. John lost 24 of his stamps. Then Evan had 5 times as many stamps as John. How many stamps did John have at first?

N2 : Now, Jenny is 11 years old and her mother is 35 years old. How many years ago was her mother 4 times as old as Jenny?

N3 : Joe Ee, Mun Fai and Jing Hao shared 400 marbles amongst themselves. Joe Ee received 28 marbles. Jing Hao received seven times the total number of marbles Joe Ee and Mun Fai received. How many more marbles did Jing Hao receive than Mun Fai?

N4 : Mrs Low goes to the provision shop with some money to buy rice. If she buys 10 kg of rice, she has \$3 short. If she buys 5 kg of rice, she has \$1 extra. How much does she have?

F1 : Now, Cindy is 12 years old and her uncle is 30 years old. In how many years' time will their total age be 50 years?

F2 : A bottle weighs 2.5 kg when it is $\frac{1}{3}$ filled with cooking oil. It weighs 3.3 kg when

Appendix D : Teacher Interview Schedule

1. In your opinion, what is the child's overall mathematics ability?
2. a) In which area (in word problem solving) is the child weak/strong? (Give some examples)
   b) Is (s)he aware of this weakness/strength?
3. a) From your own observation, what does the child do when (s)he is given a word problem?
   b) Is (s)he confident in solving the problem by him/herself?
   c) Does (s)he need help? In which area does (s)he need help?
   d) Does (s)he rush into the problem?
   e) Or does (s)he think for some time before trying to solve the word problem?
4. a) What are the common mistakes the child usually makes in solving word problems?
   b) Is (s)he aware of her/his mistakes?
   c) Does (s)he make an effort to remember and correct her/his mistake(s)?
5. Are there things the child tends to forget to do when (s)he is solving mathematics word problems?
6. a) In an assessment, when does the child usually complete her/his work?
   b) If (s)he completes early, what does (s)he do?

Appendix E: Student Questionnaire

Please refer to the booklet in the pocket at the end of the thesis.

Appendix F : CRIME

It's a **CRIME** to **DRAW SMALL PARTS BEFORE BACKWARDS !**

C	**AREFUL READING**	Have I read and understood what I am supposed to find?
R	**ECALL POSSIBLE STRATEGIES**	**Draw** (solve by **drawing** models/diagrams) **Small** (simplify problem using **small** numbers) **Parts** (solve **part(s)** of the problem first) **Before** (use **before**-after concept) **Backwards** (solve by working **backwards**)
I	**MPLEMENT STRATEGY/STRATEGIES**	• Have I labeled the models/diagrams to show the relationship between the knowns and unknowns? • Can I re-arrange the blocks to see if there is a clearer relationship? • Can I write Mathematics statement(s) to work out my answer?
M	**ONITOR**	• Am I on the right track? • Am I getting closer to the goal? • Am I still using the strategy I have chosen? • Do I need to reread the problem and use another strategy?
E	**VALUATION**	• Does the answer make sense? • Can I check the answer by using a test? • Can I solve the problem in a different way?

Appendix G : Original and Modified Artzt & Armour-Thomas' Cognitive-Metacognitive Framework

1. **Original Artzt & Armour-Thomas' Cognitive-Metacognitive Framework for Protocol Analysis of Problem Solving in Mathematics (1992, Appendix)**

Episode 1:	Reading the problem (cognitive)
Description:	The student reads the problem.
Indicators:	The student is observed as reading the problem or listening to someone else read the problem. The student may be reading the problem silently or aloud to the group.

Episode 2:	Understanding the problem (metacognitive)
Description:	The student considers domain-specific knowledge that is relevant to the problem. Domain-specific knowledge includes recognition of the linguistic, semantic, and schematic attributes of the problem in his or her own words and represents the problem in a different form.
Indicators:	The student may be exhibiting any of the following behaviours: (a) restating the problem in his/her own words; (b) asking for clarification of the meaning of the problem; (c) representing the problem by writing the key facts or by making a diagram or list; (d) reminding himself or herself or others of the requirements of the problem; (e) stating or asking himself or herself whether (s)he has done a similar problem in the past; and (f) discussing the presence or absence of important pieces of information.

Episode 3:	Analysing the problem (metacognitive)
Description:	The student decomposes the problem into its basic elements and examines the implicit and explicit relations between the givens and goals of the problem.
Indicators:	The student is engaging in an attempt to simplify or reformulate the problem. An attempt is made to select an appropriate perspective of the problem and to reformulate it in those terms.

Episode 4:	Planning (metacognitive)
Description:	The student selects steps for solving the problem and a strategy for combining them that might potentially lead to problem solution if implemented. The student may also select a representation for the information in the problem. In addition, the student may assess the status of the problem solution and make decisions for change if necessary.
Indicators:	The student describes an approach that (s)he intends to use to solve the problem. This may be in the form of steps to be taken or strategies to be used.

Episode 5a:	Exploring (cognitive)
Description:	The student executes a trial-and-error strategy in an attempt to reduce the discrepancy between the givens and the goals.
Indicators:	The student engages in a variety of calculations without any apparent structure to the work. There is no visible sequence to the operations performed by the student.

Episode 5b:	Exploring (metacognitive)
Description:	The student monitors the progress of his or her or others' attempted actions thus far and decides whether to terminate or continue working through the operations. This differs form analysis in that it is less well structures and it is further removed from the original problem. If one comes across new information during the exploration, (s)he may return to analysis in the hope of using that information to better understand the problem.
Indicators:	(a) The student draws away from the problem to ask himself/herself or someone else what has been done during the exploration; (b) The student gives suggestions to other students about what to try next in the exploration; and (c) The student evaluates the status of the exploration.

Episode 6a:	Implementing (cognitive)
Description:	The student executes a strategy that grows out of his/her understanding, analysis, and/or planning decisions and judgments. Unlike exploration, the student's actions are characterised by a quality of systematicity and deliberateness in transforming the givens into the goals of the problem.
Indicators:	The student appears to be engaging in a coherent and well structured series of calculations. There is evidence of an orderly procedure.

Episode 6b:	Implementing (metacognitive)
Description:	The student engages in the same kind of metacognitive process as in the exploring (metacognitive) phase of problem solving, monitoring the progress of his/her attempted actions. Unlike the exploratory phase, however, the metacognitive decisions build on, check or revise those previously considered decisions. Furthermore, the student may consider a relocation of his/her problem solving resources, given the time constraint within which the problem must be solved.
Indicators:	During the implementation phase, the student draws away from the work to see what has been done or where it is leading.

Episode 7a:	Verifying (cognitive)
Description:	The student evaluates the outcome of the work by checking computational operations.
Indicators:	The student redoes the computational operations (s)he did before to check that it was done correctly.

Episode 7b:	Verifying (metacognitive)
Description:	The student evaluates the solution of the problem by judging whether the outcome reflected adequate problem understanding, analysis, planning, and/or implementation. Should the student discover a discrepancy in this comparison search, (s)he engages in new decision making for correcting the faulty metacognitive and/or cognitive processing that led to the incorrect solution. The ability to adjust one's thinking on the basis of evaluative information is another indication of self-regulatory competence. Should the evaluation of the problem solution indicate an adequacy of or congruence with metacognitive and cognitive processing, the mental reiteration ends.
Indicators:	After the student has decided that the solution or part of the solution has been obtained, (s)he may review the work in several ways: (a) The student checks the solution process to see whether it makes sense. (b) The student checks to see if the solution satisfies the conditions of the problem. (c) The student explains to a groupmate how the solution was obtained.

Episode 8:	Watching and listening (uncategorised)
Description:	This category only pertains to students who are working with other people. The student is attending to the ideas and work of others.
Indicators:	The student appears to be listening to a group member who is talking or watching a group member who is writing.

2. **Modified Artzt & Armour-Thomas' Cognitive-Metacognitive Framework for Protocol Analysis of Word Problem Solving in Mathematics**

The examples will be taken from students solving the N4 word problem (see Appendix C)

Episode 1:	Reading the word problem (cognitive)
Description:	The student(s) read(s) the word problem.
Indicators:	The student reads the word problem statement silently or aloud. The student rereads the word problem or verbalises parts of the word problem statement.

Episode 2:	Analysing the word problem (metacognitive)
Description:	1. The student considers domain-specific knowledge that is relevant to the word problem. Domain-specific knowledge includes recognition of the linguistic, semantic, and schematic attributes of the problem in his or her own words and represents the problem in a different form; and 2. The student decomposes the word problem into its basic elements and examines the implicit and explicit relations between the givens and goals of the word problem.

Indicators:	1. The student exhibits any of the following behaviours: (a) rephrases the word problem in his/her own words; (b) asks for clarification of the meaning of the word problem; (c) represents the word problem by typing the key facts into the working space in WordMath or by making a diagram or list; (d) reminds himself/herself or peer of the requirements of the word problem, for example, '5 kg not 5 dollars'; (e) states or asks himself/herself whether (s)he has done a similar word problem in the past, for example, 'I know already. I did this in the University of Leeds one'; and (f) discusses the presence or absence of important pieces of information. 2. The student makes an attempt to simplify or reformulate the word problem. Examples of statements reflecting such analysis are: 'for example she has 100 dollars to buy 10 kg, she only has 97 dollars. Just supposing,' and 'You can see the connection, 10 kg and 5 kg. There's, you know, like a ratio 1 is to 2.'

Episode 3:	Planning (metacognitive)
Description:	1. The student selects steps for solving the word problem and a strategy for combining them that might potentially lead to word problem solution if implemented; and 2. The student assesses the status of the word problem solution and make decisions for change if necessary.
Indicators:	The student describes an approach that (s)he intends to use to solve the word problem. This may be in the form of steps to be taken or strategies to be used. An example of statements that reflect planning is: 'You first write 10 kg minus 3 dollars equal to 5 kg plus 1 dollar. Then I change the sign, both signs to minus signs or additional signs.'

Episode 4a:	Exploring (cognitive)
Description:	The student executes a trial-and-error strategy in an attempt to reduce the discrepancy between the givens and the goals.
Indicators:	The student engages in a variety of calculations without any apparent structure to the work. There is no visible sequence to the operations performed by the student.

Episode 4b:	Exploring (metacognitive)
Description:	The student monitors the progress of his/her or the peer's attempted actions and decides whether to terminate or continue working through the operations. This differs form analysis in that it is less well structured, and it is further removed from the original word problem. If (s)he comes across new information during the exploration, (s)he may return to analysis in the hope of using that information to better understand the word problem.

Indicators:	1. The student draws away from the word problem to ask himself/herself or someone else what has been done during the exploration. Examples of such statements are: 'What are you doing?' and 'What am I doing?'; 2. The student gives suggestions to peer about what to try next in the exploration. An example of such a comment is: 'Cannot, cannot, the number must be small'; and 3. The student evaluates the status of the exploration. Examples of such statements are: 'I don't think we're right,' and 'I think that's the answer!'

Episode 4a:	Implementing (cognitive)
Description:	The student executes a strategy that grows out of his/her understanding, analysis, and/or planning decisions and judgments. Unlike exploration, the student's actions are characterised by a quality of a systematic and deliberateness in transforming the givens into the *goals of the word* problem.
Indicators:	The student appears to be engaging in a coherent and well structured series of calculations. There is evidence of an orderly procedure.

Episode 4b:	Implementing (metacognitive)
Description:	The student engages in the same kind of metacognitive process as in the exploring (metacognitive) phase of word problem solving, monitoring the progress of his/her attempted actions. Unlike the exploratory phase, however, the metacognitive decisions build on, check or revise those previously considered decisions. Furthermore, the student may consider a relocation of his/her problem solving resources, given the time constraint within which the problem must be solved.
Indicators:	During the implementation phase, the student draws away from the work to see what has been done or where it is leading. The following example of statements reflects this: 'No, see, how much does she have? The money / so 4 plus 1 is 5.'

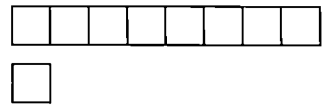
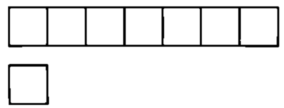
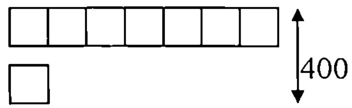
Episode 5a:	Verifying (cognitive)
Description:	The student evaluates the outcome of the work by checking computational operations.
Indicators:	The student redoes the computational operations (s)he did before to check that it was done correctly.

Episode 5b:	Verifying (metacognitive)
Description:	The student evaluates the solution of the word problem by judging whether the outcome reflected adequate word problem understanding, analysis, planning, and/or implementation. Should the student discover a discrepancy in this comparison search, (s)he engages in new decision making for correcting the faulty metacognitive and/or cognitive processing that led to the incorrect solution. The ability to adjust one's thinking on the basis of evaluative information is another indication of self-regulatory competence. Should the evaluation of the word problem solution indicate an adequacy of or congruence with metacognitive and cognitive processing, the mental reiteration ends.
Indicators:	After the student has decided that the solution or part of the solution has been obtained, (s)he may review the work in several ways: (a) The student checks the solution process to see whether it makes sense. For example, 'Yeah it matches you know.' (b) The student checks to see if the solution satisfies the conditions of the word problem. For example, 'So you are saying 5 kg equals 4 dollars. That means when she has 5 dollars and she buys this 4 dollars worth of rice, she has 1 dollar extra. So, it should be correct.' (c) The student explains to peer how the solution was obtained. For example, 'If every 5 kg is 4 dollars/ times 2 which will give me 10 kg which is 8 dollars.'

Appendix H : Dyads' Think Aloud Protocols

1. **L and JK's (T/HA) Full Think Aloud Protocol**

L was controlling the mouse and JK was using the keyboard

Code	Time	Item				Typed into WordMath
R	26 s	1	L & JK	Joe Ee, Mun Fai and Jing Hao shared 400 marbles amongst themselves. Joe Ee received 28 marbles. Jing Hao received seven times the total number of marbles Joe Ee and Mun Fai received. How many more marbles did Jing Hao receive than Mun Fai?		
		2	L	so /		
		3	JK	draw 28 marbles		
		4		The total unit is 8 units leh		
		5	L	How do you know?/		
		6		Oh Okay.	{drawing & labeling}	jh 
		7		So we need to find		
		8		We make it 8 units because / we put Jing Hao over Joe and Mun.		
		9		So it would be: 8 over: 1.		
		10		And this should be / Jing Hao		
		11		and this should be for / Joe and Mun		
		12	JK	no, this is 7.		
		13		1 unit 7		
		14		7 of the total unit.		
		15	L	oh yeah.	{making changes}	jh 
A	93 s	16		Minus 1		
		17		this is for Joe and Mun	{labeling}	j &
		18		and the total will be /		
		19	JK	400		
		20	L	400	{labeling}	jh 
		21		400 marbles		
		22		so / now you know that =		j &

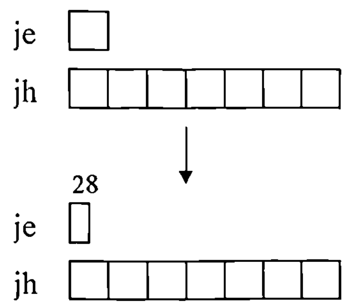
Code	Time	Item				Typed into WordMath
		23	JK	= 8		
		24		8 units is 400		
		25	L	so the total will be / [8 units		
		26	JK	[8 units is 400	{typing}	8 units --- 400
		27	L	you do leh		
		28		so 8 units 400		
		29		1 unit will be 400 divided by 8 which will give me 50		
		30	JK	1 unit	{typing}	1 unit --- 50
		31		50 /		
		32		6 =		
		33	L	= so to find Mun Fai / [take 50 minus 28		
		34	JK	[take 50 minus 28	{typing}	1 unit --- 50 - 28
		35	L	wait wait		
		36	JK		{typing}	1 unit --- 50 n
		37	L	wrong		
I (M)	88 s	38		not n		
		39		it's m		
		40	JK	50 minus / equals	{typing}	mf --- 50 - 28 = 22
		41	L	equals 22		
		42	JK	22		
		43	L	so now to find =		
		44	JK	= 7 units		7 units
		45	L	Jing Hao.		
		46		7 units which is 50 times 7 which is 350 /		
		47		no need lah.		
		48		Take 400 minus 50		
		49		faster / and easier		

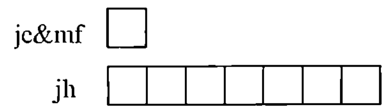
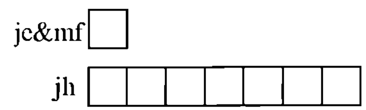
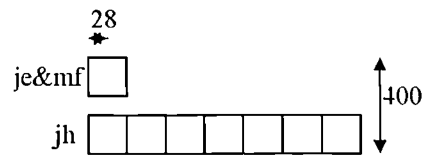
Code	Time	Item				Typed into WordMath
		50	JK	[350	{typing}	7 units --- 350 350 - 22 = 328
		51	L	[then find the difference between 350 minus 22.		
		52	JK	equal 328	{typing}	
		53	L	wait check	{calculator: press 350-22=}	
		54		350 minus 22 / 328		
		55		Now let's check.		
		56		So if / if / Mun Fai is 22 then [Joe plus	{press 22+28=}	
		57	JK	[Joe plus 28 equal 50		
		58	L	is 50	{press 50*7=}	
		59		[50 times		
		60	JK	[50 times		
		61		8		
		62	L	7	{using calculator: press 350+50=}	
		63		is 350		
V (M)	73 s	64		so to find whether it's correct or not		
		65		plus 50 to get the total and we're correct.		
		66		So		
		67	JK		{typing}	350 - 22 = 328
		68	L	hey wait		
		69	JK		{typing}	Jing Hao has 328 marbles more than Mun Fai.
		70	L	328 marbles		
		71		more than Mun Fai.		
		72		M U N F A I		
		73		328		
				{They entered the (correct) answer into the box at the end of the screen.}		

2. **HM and XY's (T/LA) Full Think Aloud Protocol**

HM was controlling the mouse and XY was using the keyboard

Code	Time	Item				Typed into WordMath
R	27 s	1	HM & XY	Joe Ee, Mun Fai and Jing Hao shared 400 marbles amongst themselves. Joe Ee received 28 marbles. Jing Hao received seven times the total number of marbles Joe Ee and Mun Fai received. How many more marbles did Jing Hao receive than Mun Fai?		
A	275 s	2	HM	So/ draw first		
		3	XY	This is?		
		4	HM	This is 28		28
		5	XY	28?		
		6	HM	28	{labeling}	
		7	XY	7 times (2)		
		8		so (2)	{HM: drawing}	
		9		what's this?		
		10	HM	7 times		
		11		I'm drawing 7 times =	{drawing}	
		12	XY	= but it is 7 times of / Joe and Mun Fai you know		
		13	HM	this is Joe Ee / j e	{modifying first unit}	
		14		and this is Jing Hao / j h		
		15		this should be (3) should be divided by 2 here	{XY: labeling}	
		16		to be like this		
		17		I think we'll erase this (3)	{HM: adding one block to the diagram and then erasing the added block}	
		18		We'll draw (3) another like that		
		19		and this one we have to make it one unit		



Code	Time	Item			Typed into WordMath
		20	XY	mm	
		21	HM	I think we'll erase these two (2)	
		22	XY	what / what are you trying to do?	
		23	HM	This is actually Joe Ee and Jing / and (2) Joe Ee and Mun Fai's	
		24	XY	mm mm (1) but we don't know Mun Fai's what.	
		25	HM	Yeah /	{XY: labeling} 
		26		we just leave it like this /	
		27		and here (3) this one we'll write Joe Ee and Mun Fai	
		28		plus (2) mun / m f	
		29		and this should be /	
		30	XY	28	{labeling} 
A		31		28	
		32		then?	
		33	HM	So altogether they have [400	
		34	XY	[400	
		35	HM		{labeling} 
		36	XY	and	
		37	HM	the question asked how many more [marbles did Jing Hao receive than Mun Fai.	
		38	XY	[marbles	
		39		so we have to find [Mun Fai	
		40	HM	[one	
		41	XY	what?	
		42	HM	let me see (3)	{adding an arrow to the diagram}
		43		this is the unknown {pointing to the first unit}	
		44		so we write here (3)	
		45		this is too small to write	

Code	Time	Item			Typed into WordMath
		46	XY	I know	
A		47		just there	
		48		that one is what?	
		49		[What's this?	
		50	HM	[This is unknown/ unknown because Mun Fai's	
		51		so let's say this is one small unit /	
		52	XY	okay	
		53	HM	so 1, 2, 3, 4, 5, 6, 7, 8	
		54		so there / [8 units plus	
		55	XY	[I know / 8 units plus	
		56	HM	plus 28 =	
		57	XY	= 28	
		58	HM	times	
		59	XY	no / why do you times?	
P	55 s	60	HM	plus plus	
		61	XY	8 units plus /	
		62	HM	8 units plus 28 times / [8	
		63	XY	[what's the times for?	
		64	HM	Because 28	
		65		this is 28	
		66		so here also should be 28 for 1 unit	
		67		28 for 1 unit	
		68		and / 1 unit is 28 plus 1 small unit	
		69		so each of them there is 28 / and 1 times/	
		70	XY	1 unknown =	
		71	HM	= yeah / so 8 units	
		72		altogether there are 8 small units plus =	
		73	XY	= yeah	
		74	HM	8 small units plus 28 times 8	

Code	Time	Item				Typed into WordMath
		75	XY	plus	{typing}	8 small units + 28
		76	HM	plus 28 times /		
		77	XY	8? =		8 small units + 28 x 8 = 400
		78	HM	= yes		
		79	XY	this is 400		
		80	HM	yes		
		81	XY		{typing}	
		82	HM	so 8 units is equal to 400 minus [28 times 8		
		83	XY	[28 times 8	{typing}	8 small units --- 400 - 28x8 = 176
I(C)	76 s	84		28 times 8 is (5)		
		85	HM	do you need a calculator?		
		86	XY	28 times 8	{using calculator: press 28*8=}	
		87		224		
		88	HM	yes		
		89	XY	so / minus 400	{press 400-224=}	
		90	HM	the answer is [176		
		91	XY	[176		
		92		176		
		93	HM	yes		
		94		[1 unit		
		95	XY	[so / 1 small unit? =		
		96	HM	= yes		
I(M)	76 s	97	XY	176 / divide by 8	{typing}	1 small unit --- 176/8 =
		98	HM	which is?		
		99	XY	22 huh?		

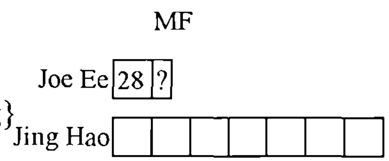
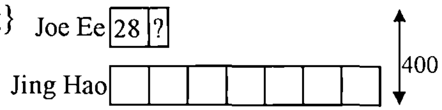
Code	Time	Item				Typed into WordMath
		100	HM		{using calculator: press 176/8=}	
		101	XY	176 / divide		
		102		[22		
		103	HM	[22		
I(M)		104		this is for =		
		105	XY	= 22 / Mun Fai has 22 marbles		
		106	HM	yes		
		107		so we have to [find Jing Hao		
		108	XY	[find 1 big unit		
		109	HM	yes		
		110		which is 28 plus /		
		111	XY	22 =		
		112	HM	= yes		
		113		1 unit		
		114	XY	equals to [50	{typing}	1 unit --- 28 + 22 = 50
		115	HM	[50		
		116		there is 7 units so is [350		
		117	XY	[350	{typing}	7 units --- 50 x 7
		118	HM	350		
I(C)	76's	119		7 times equals		
		120	XY	350		
		121		so		
		122	HM	350 minus 22		
		123		equals to		
		124	XY	equal 328	{typing}	350 - 22 = 328 marbles
		125		328		

Code	Time	Item				Typed into WordMath
		126	HM	yes / let's check		
		127		write the final statement first		
		128	XY		{typing final statement}	Jing Hao has 328 more marbles than Mun Fai
		129	HM	has / has 328 more marbles		
		130		Mun Fai		
		131		Let's check		
		132	XY	okay		
		133		so		
		134	HM	this is /		
V(M)	84 s	135		let's check whether this is 50		
		136		400		
		137		so 328 [plus 50		
		138	XY	[plus 50		
		139		328 plus 50		
		140	HM	wait wait wait		
		141		350		
		142		because 350 is Jing Hao's		
		143	XY	yeah		
		144	HM	It's 400		
		145		so the answer is correct		
		146	XY	okay		
				{They entered the (correct) answer into the box located at the end of the screen.}		

3. **K and SJ's (C/HA) Full Think Aloud Protocol**

K was controlling the mouse and SJ was using the keyboard

Code	Time	Item				Typed into WordMath
R	33 s	1	K & SJ	Joe Ee, Mun Fai and Jing Hao shared 400 marbles amongst themselves. Joe Ee received 28 marbles. Jing Hao received seven times the total number of marbles Joe Ee and Mun Fai received. How many more marbles did Jing Hao receive than Mun Fai?		
A		2	K	Erm (2) erm (2)		
		3		hummm / must draw 3 models first		
		4	SJ	yeah / this is [28	{drawing}	[28]
		5	K	[this is 28		
		6		Joe Ee's Joe Ee's marbles		
		7	SJ	eh (6) wait wait		
		8		erm this should put Joe Ee / Joe Ee and Mun Fai		
		9	K	no =		
		10	SJ	= because =		
	164 s	11	K	= Joe Ee received 28 marbles you see =		
		12	SJ	= no, because Jing Hao received seven times the total number / of		
		13		the number of marbles Joe Ee and Mun Fai received		
		14	K	but they already stated Joe Ee received 28 marbles		
		15	SJ	I know we can draw another [model		
		16	K	[orh		
		17	SJ		{drawing model}	Joe Ee [28][?]
		18		this should be	{reducing size of 28}	
		19		we should indicate this as / put this as question mark because we don't know / how many	{labeling question mark}	
		20		sorry		

Code	Time	Item				Typed into WordMath
		21	K	then you have to draw JE plus MF	{labeling}	
		22	SJ	this is MF huh?		
		23	K	Okay / bigger		
		24		okay		
		25	SJ	wait, wait /		
		26		these two units erm should be equal to 1 unit		
		27	K	7		
		28	SJ	this is Jing Hao's	{drawing & labeling}	
		29	K	erh Jing Hao		
A		30		then may be want to make /	{labeling}	
		31	SJ	yeah		
		32	K	slightly more equal		
		33	SJ	erm (1) [how many		
		34	K	[then you also have to indicate [everything equal to 400 marbles		
		35	SJ	[yeah, okay	{labeling}	
		36		erm 400		
		37	K	okay		
		38	SJ	okay		
		39		so every 1 unit they =		
		40	K	= so every 1 unit equals 28 plus something; 28 plus something; 28 plus something		
P	13 s	41	SJ	yeah =		
		42	K	= so actually 8 units of these 28		
		43		so we		
		44		okay / go down		
		45		okay =	{typing}	28 x 8 =

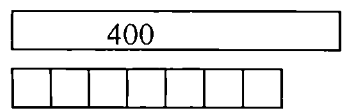
Code	Time	Item			Typed into WordMath
		46	SJ	= okay	
		47		28 times 8 /	
		48		208	
		49	K	224	
		50		use the calculation/ the calculator	
		51	SJ		{using calculator: press 28*8=}
		52	K	equal 224	
		53	SJ	hey wait	
		54		this [is	
		55	K	[because	
		56	SJ	this is	
		57		oh yeah okay	
		58	K	everything is sliced	
I(M)	96 s	59		sliced, sliced, sliced	
		60		actually the total there is this 8 question marks	
		61	SJ	yeah	
		62		no =	
		63	K	= correct, 8 question marks	
		64	SJ	why not 400 divide by 8?	
		65		By, this is 7	
		66	K	okay you see huh	
		67	SJ	yeah	
		68	K	to find this question mark, we don't have any method	
		69		so, we can actually use 28	
		70		can actually fit in here	
		71		to all the units you see	
		72		so we have eight 28s	

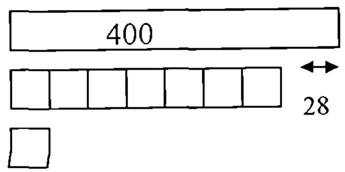
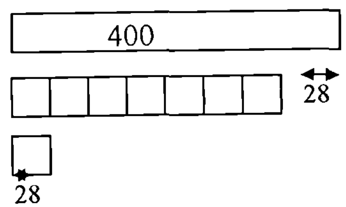
Code	Time	Item	Speaker	Utterance	Action	Typed into WordMath
I(M)		73	SJ	yeah I get it		
		74		so / total		
		75	K	224	{typing}	28 x 8 = 224
		76		so 400 minus =		400 -
		77	SJ	= 224		
		78	K	equals [to 176		
		79	SJ	[to 176		
		80	K	which is equal to		
		81	SJ	divide / then divide		
		82	K	which is 8 question marks =	{typing}	400 - 224 = 176 [8?]
		83	SJ	= 8 question marks		
		84	K	so 1 question mark is equal to =	{typing}	1? = 22
		85	SJ	= 176 divide by 8		
		86		equal to 22 =		
		87	K	= 22		
E(M)		88		so they are asking how many more marbles did Jing Hao receive more receive more than Mun Fai		
		89		so the difference right / actually is 6 question marks [correct?		
		90	SJ	[yeah correct		
		91		no no no		
		92		6 question marks plus [6		
	99 s	93	K	[because Mun Fai is 1 you see		
		94		Jing Hao has 7		
		95		so the difference is 6 question marks / correct?		
		96	SJ	But / if 1 unit have / 1 question mark		
		97		then how bout the question mark, 28 / 28?		
		98	K	Precisely / the 28, we deducted from the 400 already		
		99		that is why now we know that there is 1 question mark		
		100		that is why there is only 176		

Code	Time	Item				Typed into WordMath
		101	K	you know what I mean		
		102	SJ	orh / okay		
		103	K	6 question marks	{typing}	6? = 132
E(M)		104		22 times 6 /		
		105		132		
		106	SJ	132		
		107	K	better use the calculator just in case		
		108	SJ	okay	{using calculator: press 6*22=}	
		109		okay correct		
		110	K		{typing final statement}	Jing Hao received 132 marbles more than Mun Fai.

{They entered the (incorrect) answer into the box located at the end of the screen.}

4. **E and XF's (C/LA) Full Think Aloud Protocol**
XF was controlling the mouse and E was using the keyboard

Code	Time	Item				Typed into WordMath
R	38 s	1	E & XF	Joe Ee, Mun Fai and Jing Hao shared 400 marbles amongst themselves. Joe Ee received 28 marbles. Jing Hao received seven times the total number of marbles Joe Ee and Mun Fai received. How many more marbles did Jing Hao receive than Mun Fai?		
A	56 s	2	E	Draw model loh		
		3	XF	(7)	{about to start drawing}	
		4	E	draw the 3 of them together (3)		
		5	XF	3		
		6		how? (7)		
		7	E	draw the 3 of them together		
		8	XF	1 part?		
		9	E	yes (4)		
		10	XF	why must we draw? /		
		11		why must we draw 3 of them? (5)		
		12	E	what? (2)		
		13	XF	why must /		
		14	E	cannot hear	{labeling}	
		15		400 marbles		
		16	XF	can or not?	{drawing}	
R	28 s	17		{reading the question silently indicated by the movement of the		
		18		pointer} (5)		
A	40 s	19		so we draw (2) eh (5) ah:: another model with 7 parts		
		20	E	why?		
		21	XF	Eh because they say Jing Hao received 7 times		

Code	Time	Item				Typed into WordMath
R	30 s	22	E	total number of =		
		23	XF	= number of marbles Joe Ee and {referring to the word problem statement; indicated by the movement of the cursor on the word problem statements}	{drawing and labeling}	
A		24		one		
		25		okay / so eh / 28		
		26	E	why?		
		27	XF	Eh (2) this represents Joe Ee		
		28		28 (12)		
		29	E	8 units equal to 400 (2)		
	37 s	30	XF	yeah / eh	{drawing an arrow}	
		31	E	8 units equal 400		
		32	XF	wait (2)		
		33		we type 28 first		
		34	E	28? =		
		35	XF	= yeah / 28	{labeling}	
I(C)		36		okay so now / we do the / question		
		37	E	8 units equal to 400 =		
		38	XF	= yeah		
		39	E	50? =	{typing}	8 units --- 400 1 unit --- 50
		40	XF	= yeah		
		41	E	find 7 units /		
		42	XF	no		
		43		correct correct		

Code	Time	Item				Typed into WordMath
		44	E	7 units for Jing Hao	{typing}	7 units ---
		45		find 6 units (2)		
		46		no (2) 7 units		
		47	XF	7 units		
		48	E	50 times 7	{typing}	7 units --- 350
		49		350?		
		50		350?		
		51	XF	Yeah		
		52	E	50 minus 28? =		
		53	XF	= yeah		
		54	E		{typing}	mun fai --- 50 - 28
		55	XF	28		
I(C)	137 s	56		12 / eh no no [22		
		57	E	[22	{typing}	mun fai --- 50 - 28 = 22
		58		so 350 minus 22? (2)		
		59	XF	350 /		
		60		yeah correct		
		61	E	equals (2)	{typing}	350 - 22 =
		62	XF	328		
		63	E	328? =	{typing}	350 - 22 = 328
		64	XF	= yeah		
		65	E	go down	{typing final statement}	
		66	XF	Jing Hao		
		67		capital letter		
V(C)	61 s	68	E		{typing}	He received
		69	XF	received		
		70	E	328		

Code	Time	Item			Typed into WordMath
		71	XF	more marbles	
V(C)		72		marbles	
		73		than Mun Fai	
		74	E	full stop	He received 328 more
		75		328	marbles than Mun Fai.
				{They entered the (correct) answer into the box located at the end of the screen.}	

Appendix I: Video Illustration of Dyads' Word Problem Solving (Compact Disc)

Please refer to the compact disc in the pocket at the end of the thesis.

Appendix J: Student Profiles used in Analysis within Quasi-experimental Designs

1. Student profile used in quasi-experimental design 1

	School	Metacognitive Training	Mathematical Achievement	1998 Mathematical Score	Pretest Score	Posttest Score	Delayed Posttest Score
1	school2	Treatment	HA	86	5	4	7
2	school2	Treatment	HA	78	0	7	5
3	school2	Treatment	HA	81	3	7	10
4	school2	Treatment	HA	80	2	4	6
5	school2	Treatment	HA	81	0	4	5
6	school1	Treatment	HA	82	10	9	10
7	school1	Treatment	HA	90	8	10	10
8	school1	Treatment	HA	90	6	10	10
9	school1	Treatment	HA	84	9	9	9
10	school1	Treatment	HA	84	7	8	9
11	school2	Treatment	LA	54	0	0	0
12	school2	Treatment	LA	57	2	0	1
13	school2	Treatment	LA	62	2	1	5
14	school2	Treatment	LA	62	0	5	4
15	school2	Treatment	LA	65	2	2	2
16	school1	Treatment	LA	68	8	7	7
17	school1	Treatment	LA	74	7	9	7
18	school1	Treatment	LA	74	5	6	9
19	school1	Treatment	LA	72	6	6	8
20	school1	Treatment	LA	65	4	6	8
21	school2	Control	HA	85	3	4	7
22	school2	Control	HA	82	1	1	3
23	school2	Control	HA	87	5	6	6
24	school2	Control	HA	81	3	3	5
25	school2	Control	HA	81	1	4	4
26	school1	Control	HA	82	7	8	8
27	school1	Control	HA	75	8	10	8
28	school1	Control	HA	82	8	8	7
29	school1	Control	HA	76	8	6	6
30	school1	Control	HA	78	6	7	6
31	school2	Control	LA	66	0	1	0
32	school2	Control	LA	62	0	1	1
33	school2	Control	LA	65	1	0	0
34	school2	Control	LA	54	2	1	1
35	school2	Control	LA	62	0	1	1
36	school1	Control	LA	61	1	2	3
37	school1	Control	LA	55	2	3	2
38	school1	Control	LA	56	3	3	3
39	school1	Control	LA	56	1	2	2
40	school1	Control	LA	59	5	5	7

2. **Student profile used in quasi-experimental design 2**

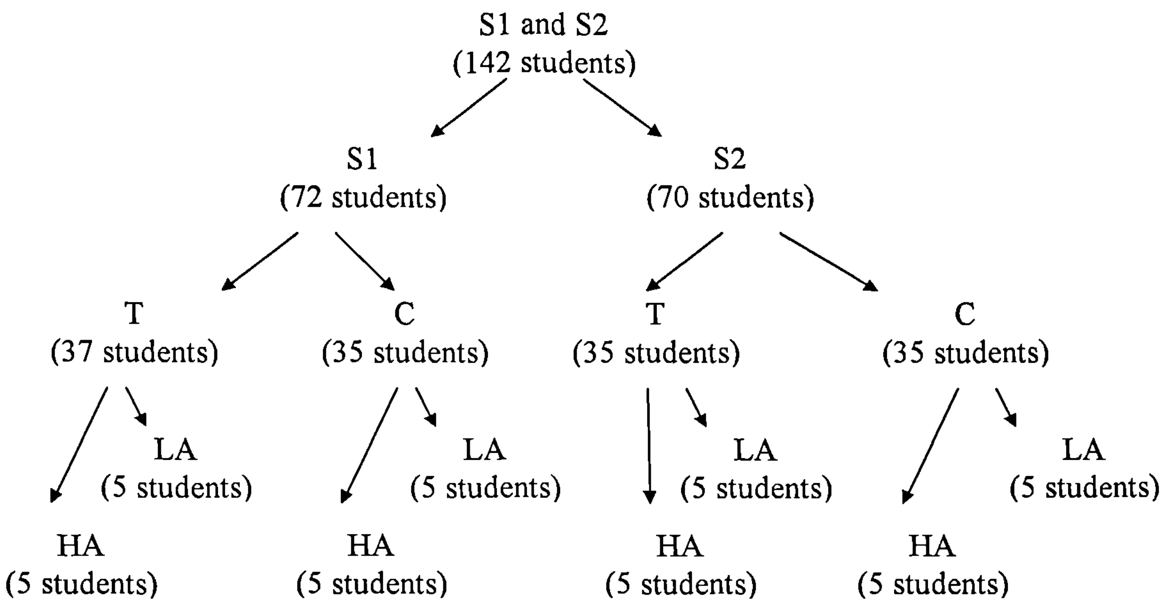
	Metacognitive Training	Mathematical Achievement	1998 Mathematical Score	Pretest Score	Posttest Score	Delayed Post-test Score
1	Treatment	HA	75	0	0	5
2	Treatment	HA	82	7	7	10
3	Treatment	HA	78	0	7	5
4	Treatment	HA	76	0	3	3
5	Treatment	HA	81	0	4	5
6	Treatment	HA	80	2	4	6
7	Treatment	HA	79	2	6	6
8	Treatment	HA	80	2	3	4
9	Treatment	HA	76	3	3	3
10	Treatment	HA	81	3	7	10
11	Treatment	HA	78	4	3	5
12	Treatment	HA	83	4	8	10
13	Treatment	HA	86	5	4	7
14	Treatment	HA	77	5	5	6
15	Treatment	HA	79	6	9	6
16	Treatment	HA	76	6	8	9
17	Treatment	HA	90	6	10	10
18	Treatment	HA	82	10	9	10
19	Treatment	HA	77	7	8	10
20	Treatment	HA	78	7	5	7
21	Treatment	HA	78	7	6	9
22	Treatment	HA	80	7	7	8
23	Treatment	HA	83	7	6	8
24	Treatment	HA	76	7	8	9
25	Treatment	HA	82	7	7	7
26	Treatment	LA	72	1	2	3
27	Treatment	LA	63	4	0	2
28	Treatment	LA	74	1	2	3
29	Treatment	LA	74	0	5	6
30	Treatment	LA	67	1	2	5
31	Treatment	LA	62	0	5	4
32	Treatment	LA	54	0	0	0
33	Treatment	LA	67	0	2	2
34	Treatment	LA	73	1	2	3
35	Treatment	LA	71	1	0	5
36	Treatment	LA	57	2	0	1
37	Treatment	LA	73	0	3	6
38	Treatment	LA	74	2	7	6
39	Treatment	LA	62	2	1	5
40	Treatment	LA	68	8	7	7
41	Treatment	LA	74	7	9	7

42	Treatment	LA	74	5	6	9
43	Treatment	LA	70	5	4	4
44	Treatment	LA	72	6	6	8
45	Treatment	LA	65	4	6	8
46	Treatment	LA	65	2	2	2
47	Treatment	LA	72	1	1	1
48	Treatment	LA	70	5	4	4
49	Treatment	LA	55	1	2	1
50	Treatment	LA	60	0	1	1
51	Control	HA	84	3	5	4
52	Control	HA	85	3	4	7
53	Control	HA	76	1	2	5
54	Control	HA	82	1	1	3
55	Control	HA	78	6	4	6
56	Control	HA	75	0	2	2
57	Control	HA	87	3	4	7
58	Control	HA	81	1	5	5
59	Control	HA	76	2	1	2
60	Control	HA	81	3	3	5
61	Control	HA	87	5	6	6
62	Control	HA	77	1	1	2
63	Control	HA	77	3	3	5
64	Control	HA	75	4	1	6
65	Control	HA	76	1	1	3
66	Control	HA	81	1	4	4
67	Control	HA	80	2	4	5
68	Control	HA	76	1	1	3
69	Control	HA	75	8	10	8
70	Control	HA	82	8	8	7
71	Control	HA	82	7	8	8
72	Control	HA	76	8	6	6
73	Control	HA	78	6	7	6
74	Control	HA	80	3	5	4
75	Control	HA	79	4	5	7
76	Control	LA	61	1	2	3
77	Control	LA	55	0	0	1
78	Control	LA	54	2	1	1
79	Control	LA	55	2	3	2
80	Control	LA	54	2	4	4
81	Control	LA	53	2	2	4
82	Control	LA	60	3	3	5
83	Control	LA	58	3	6	4
84	Control	LA	56	3	3	3
85	Control	LA	56	3	2	4
86	Control	LA	63	3	4	5
87	Control	LA	57	3	3	4

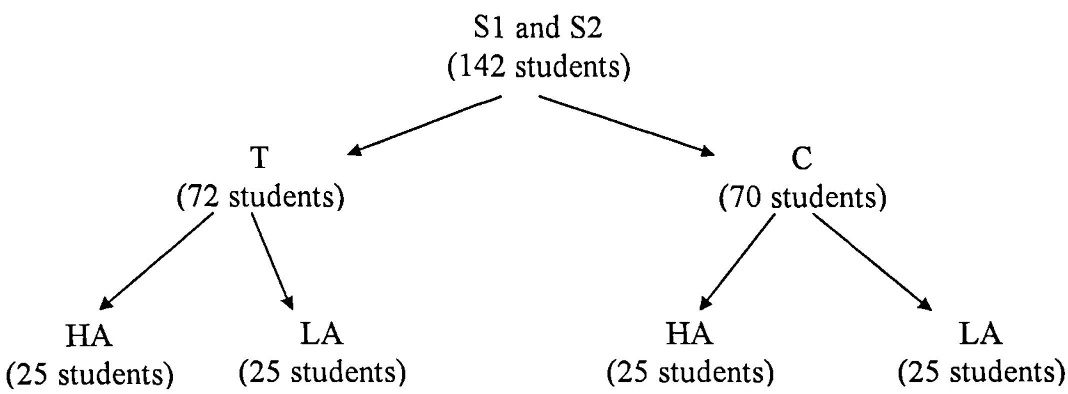
88	Control	LA	56	3	3	4
89	Control	LA	58	4	1	3
90	Control	LA	64	4	5	4
91	Control	LA	57	4	3	5
92	Control	LA	60	5	4	7
93	Control	LA	59	5	5	7
94	Control	LA	59	5	5	7
95	Control	LA	72	6	7	6
96	Control	LA	73	1	2	2
97	Control	LA	61	1	1	2
98	Control	LA	70	2	4	5
99	Control	LA	61	1	2	3
100	Control	LA	60	1	3	2

Appendix K : Assigning Students into Quasi-experimental Design 1 (Blocking) and Quasi-experimental Design 2

1. Quasi-experimental Design 1 (Blocking) (see Chapter Four, section 2.2.1 a)

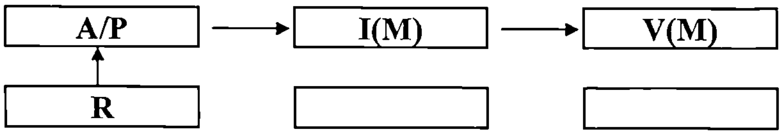


2. Quasi-experimental Design 2 (see Chapter Four, section 2.2.1 b)



Appendix L: Summary of Dyads' Cognitive-Metacognitive Word Problem Solving Models

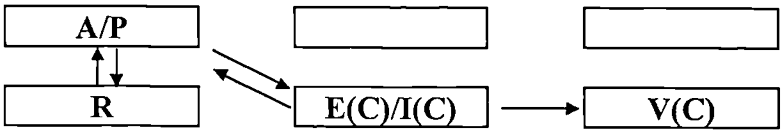
1. **Type P Cognitive-Metacognitive Word Problem Solving Model**



Posttest
L and JK (T/HA); HM and XY (T/LA); K and SJ (C/HA)

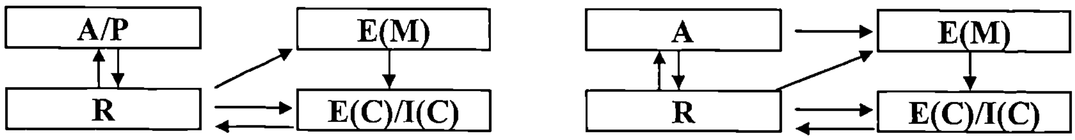
Delayed Posttest
L and JK (T/HA); HM and XY (T/LA); A and CC (T/HA)

2. **Type Q Cognitive-Metacognitive Word Problem Solving Model**



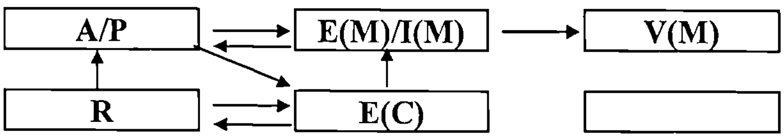
Posttest
SM and B (C/HA); E and XF (C/LA)

3. **Type R Cognitive-Metacognitive Word Problem Solving Models**



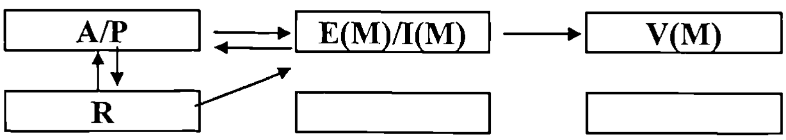
Posttest
B and P (C/LA); ES and J (T/LA)

4. **Type S Cognitive-Metacognitive Word Problem Solving Model**



Posttest
A and CC (T/HA)

5. **Type T Cognitive-Metacognitive Word Problem Solving Model**



Delayed Posttest
ES and J (T/LA)

Appendix E: Student Questionnaire

University of Leeds

Mathematical Word Problem Solving

Name ______________________________ (   )

School ______________________________

Class ____________

Date ____________

INSTRUCTIONS

Write your name, register number, school, class and date clearly in the spaces provided.

Answer ALL questions.

Think carefully before writing.

Write as much as possible for each question.

This Question Booklet consists of 7 printed pages.

[Turn over

1 a) You have taken the word problem solving exercise. Write down all the things you usually do when you solve these word problems.

b) Why do you do these things?

Things I do when I solve word problems	Reasons for doing them

Things I do when I solve word problems	Reasons for doing them

2 a) What kind of mistakes do you usually make when you solve mathematical word problems?
b) Why do you make these mistakes?
c) List the ways which you think will prevent yourself from making these mistakes?

Mistakes I make	Reasons for making them	Ways to prevent making these mistakes

Mistakes I make	Reasons for making them	Ways to prevent making these mistakes

3. a) Do you keep track of what you are doing when you solve a mathematical word problem?
   b) If you do, name at least two ways you keep track of what you are doing.

4 a) Do you check when you are solving mathematical word problems?
   b) When do you think it is useful to check?
   c) Why?

5. What do you usually forget to do when you solve mathematical word problems?

____

____

____

____

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____

6 a) From the exercise you have taken, write down the question number(s) of the word problem(s) which you think you have done very well.

b) Give reasons to why you think you are good at solving these word problems.

Examples	Reasons why I am good at solving these problems

Examples	Reasons why I am good at solving these problems

7 a) From the exercise you have taken, write down the question number(s) of the word problem(s) which you think you have done very badly.

b) Given reasons to why you think you are not god at solving these word problems.

Examples	Reasons why I am not good at solving these problems

Examples	Reasons why I am good at solving these problems

c) What do you think you can do to get better at solving these word problems?