TOLL COMPETITION IN HIGHWAY TRANSPORTATION NETWORKS

by

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The candidate confirms that the work submitted is his own, except where work which has formed part of jointly authored publications has been included. The contribution of the candidate and the other authors to this work has been explicitly indicated below. The candidate confirms that appropriate credit has been given within the thesis where reference has been made to the work of others.

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To Constance with Love: Thank you

To Oscar: Sleep well in feline heaven

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Abstract

Within a highway transportation network, the social welfare implications of two different groups of agents setting tolls in competition for revenues are studied. The first group comprises private sector toll road operators aiming to maximise revenues. The second group comprises local governments or jurisdictions who may engage in tax exporting. Extending insights from the public economics literature, jurisdictions tax export because when setting tolls to maximise welfare for their electorate, they simultaneously benefit from revenues from extra-jurisdictional users. Hence the tolls levied by both groups will be higher than those intended solely to internalise congestion, which then results in welfare losses. Therefore the overarching question investigated is the extent of welfare losses stemming from such competition for toll revenues.

While these groups of agents are separately studied, the interactions between agents in each group in competition can be modelled within the common framework of Equilibrium Problems with Equilibrium Constraints. Several solution algorithms, adapting methodologies from microeconomics as well as evolutionary computation, are proposed to identify Nash Equilibrium toll levels. These are demonstrated on realistic transportation networks. As an alternative paradigm to competition, the possibilities for co-operation between agents in each group are also explored.

In the case of toll road operators, the welfare consequences of competition could be positive or adverse depending on the interrelationships between the toll roads in competition. The results therefore generalise those previously obtained to a more realistic setting investigated here.

In the case of competition between jurisdictions, it is shown that the fiscal exter-

nality of tax exporting resulting from their toll setting decisions can substantially reduce the welfare gains from internalising congestion. The ability of regulation, co-operation and bilateral bargaining to reduce the welfare losses are assessed. The research thus contributes to informing debates regarding the appropriate level of institutional governance for toll pricing policies.

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Chapter 1

Introduction

1.1 Motivation

For nearly a century, economists in the tradition of Pigou (1920) have advocated the use of tolls as a means of addressing inefficiencies associated with the use of congested roads. The implementation of tolls also potentially generates a large amount of revenue which could be used to compensate users inconvenienced by the implementation of a pricing scheme (Hau, 2005). Here, the government is treated as "a black box, through which revenue flows in and out, without diversion or impediment" (Manville and King, 2013, p. 230).

In reality, public economics literature posits, and numerous examples attest to the fact that governments compete for a variety of target objectives such as the opportunity to host hall mark events (Westerbeek *et al.*, 2002), to attract investment capital (Douglass, 2002; Parkinson *et al.*, 2004), for grants from higher level government (Lever, 1999), tourism (Hong, 2008) as well as crucially, for funds (Ferreira *et al.*, 2005).

The politically motivated desire to maximise social well-being of their electorate on the one hand, while utilising toll revenues to reduce local taxation and finance public goods desired by their residents on the other, point to the possibility of strategic jurisdictional competition for toll revenues. Levinson (2001) provides econometric evidence substantiating the use of tolls precisely as such a strategic instrument, mirroring that of excise tax competition discussed in mainstream public economics literature (e.g. Jacobs *et al.*, 2010). Again applying econometric techniques, Rork reached these same conclusions and notes that "toll increases in other states will be met by increases in the home state"(Rork, 2009, p. 137). The literature has thus

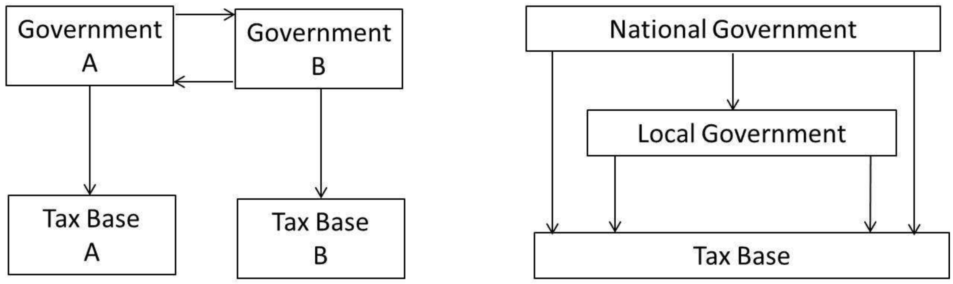

Figure 1.1: Left: Horizontal Tax Competition. Right: Vertical Tax Competition

recognised the potential for such policy interactions in the context of transportation networks controlled by different jurisdictions (De Borger and Proost, 2012).

When jurisdictions engage in competition for toll revenues in the setting of a multi-level governance structures, they also generate distortions into the economy as a result of their tax and expenditure policies i.e. fiscal decisions (Kenyon, 1997; Boadway *et al.*, 1998; Devereux *et al.*, 2007). The relevance of such fiscal induced distortions, also known as fiscal externalities, has gained increasing attention in the transportation systems management literature in recent years (De Borger *et al.*, 2005, 2007; Zhang *et al.*, 2011).

Drawing directly from distinctions made in the public economics literature (Ferreira *et al.*, 2005; Devereux *et al.*, 2007; Jacobs *et al.*, 2010), two governmental organisational structures and the competition that arises in each instance are illustrated in Fig. 1.1. The left panel of this figure illustrates horizontal or *inter-jurisdictional* competition, i.e. competition between governments at the same level or with similar responsibilities (e.g. between two states in the US or two local authorities in the UK (Kenyon, 1997; Boadway *et al.*, 1998; Shah, 2001; Devereux *et al.*, 2007) or between different countries that constitute the EU (De Borger and Proost, 2012)).

On the other hand, the right panel illustrates vertical or inter-governmental competition (Kotsogiannis, 2010) which refers to competition between different levels and types of responsibilities (e.g. between federal and state governments in the context of federalist nations like the US, or central versus local governments in the context of the unitary nations like the UK (De Borger and Proost, 2012)).

As in all other sectors of the economy, the state of institutional governance of transport matters because it is critical to the successful delivery of policy (Pemberton, 2000; van Zuylen and Taale, 2004; Taylor and Schweitzer, 2005; Marsden and May, 2006). Pemberton (2000) illustrates the interaction of decision makers in local government and points out that when developing integrated transport strategies in Tyne and Wear (North East England), officials in other cities comprising the region such as Sunderland, North Tyneside, South Tyneside and Gateshead perceived an "over dominance in terms of policy determination by Newcastle City Council" (Pemberton, 2000, p. 300). Marsden and May (2006) recognise that the effectiveness of integrated transportation strategies are diluted when there are splits in institutional responsibilities. They noted that multiple tiers of government "appears to create extra transactional barriers and impedes the implementation of the most effective measures for cutting congestion" (Marsden and May, 2006). Studying the interactions between a municipal road authority and a motorway authority, van Zuylen and Taale (2004) demonstrated that independent decision making could result in sub-optimal outcomes if each authority acted independently.

Thus one of the key issues examined in this thesis is therefore whether the corrective effect on efficiency (intended with the use of tolls) is dwarfed by the distortions induced through inter-jurisdictional competition. In particular, it examines the question of what the appropriate level of governance of toll pricing should be. Following Banzhaf and Chupp (2012), the key consideration could be presented as follows. While local jurisdictions are more likely to pay attention to unique local conditions, they will also ignore inter-jurisdictional spillovers. In transportation networks, the main spillover is congestion since "congestion in metropolitan areas often crosses local government and even state lines" (TRB, 1994, p. 58). On the other hand, a higher echelon of government (e.g. central or federal level) may take into account such spillovers but are also likely to impose uniform standards and policies which may not cater well to local needs and circumstances without significant adaptation or deviation.

Two particularly distinct approaches have been advocated and these have, in fact, been adopted. In the UK, particularly to encourage local authorities to investigate toll pricing, provisions in the Transport Act 2000 allow for revenues from toll pricing schemes to be specifically hypothecated (or earmarked) for funding *local* schemes. On the other hand, in Sweden, while a government at the local level initially introduced a toll pricing scheme, responsibility for managing the scheme presently rests with an agency instituted at the national level (National Transport Agency *Transportstyrelsen*) (Börjesson *et al.*, 2012).

The policy relevance of this research goes beyond national governments. In particular, it also applies to increasingly important supranational bodies such as the European Union (EU) and the Eurasian Economic Union (EEU). For example, the EU has adopted a commitment to *the principle of subsidiarity* which states that "that if reasonably possible, goods and services should be provided by the level of government that is closest to the people" (Voorhees, 2005, p. 94). In practice, subsidiarity means that the implementation of policies, such as toll pricing, should be decentralised to the member states. However, it has been shown that allowing individual member states themselves to decide toll charges on portions of the TransEuropean Network (TEN-T) corridors that inevitably pass through their jurisdiction could, in fact, reduce welfare versus a cooperative federally led policy (De Borger *et al.*, 2007).

Regarding the competition for funds that motivates inter-jurisdictional competition, it was noted that these funds could be used to finance infrastructure provision. This is because in most countries, the highway infrastructure of roads have traditionally been provided by governments (Zhang, 2005). However, several commentators (e.g. Mills, 1995; Nijkamp and Rienstra, 1995; Walker and Smith, 1995; Dunn, 1999; de Rus and Romero, 2004) have highlighted the trend towards a renaissance of private sector involvement in the provision of roads through a Public Private Partnership (PPP). A PPP can be regarded as "a contractual agreement between a public agency (federal, state or local) and a private sector entity" (Gordon *et al.*, 2013, p. 73). In this thesis, the private sector is assumed to be synonymous with profit driven enterprises. It should be mentioned that PPPs are not unique to transportation and have been used as a financing mechanism in other sectors of the economy as such as education (Ball *et al.*, 2003) and healthcare (McKee *et al.*, 2006). In the UK, PPPs are often referred to as Public Finance Initiatives (PFI) (Allen, 2003).

Privately operated roads are not entirely novel ideas (Viton, 1995; Bain, 2009a). Bain (2009a) reports that in the UK, from the mid-1600s to 1878 when they were abolished, roads were largely constructed and maintained by private companies (known as "turnpike trusts") through the private financing mechanism of "turnpike bonds". Parallel historical developments are also found across the Atlantic in the US as documented in Viton (1995).

In modern times, there are two ways through which private sector participants are financially compensated for funding the construction and maintenance of highway facilities in a PPP (Lockwood *et al.*, 2000; Debande, 2002; Lockwood, 2007; Hensher and Chung, 2011). The first is through the mechanism of "shadow tolls" where the government pays a private sector financing consortium based on some agreed measure in relation to the volume of traffic using the road (Grimsey and Lewis, 2004). The second is through tolls levied on the *users* of facilities. In this case, tolls are paid by the road users *themselves* who use the facility. It is assumed throughout this thesis, that when "tolls" are referred to, it implies that the users themselves pay for the use of the road.

Examples of such private toll road projects include the Guangzhou-Shenzhen superhighway in China, SR91 in California, USA and the Birmingham North Relief Road (M6 Toll) in the UK (Fishbein and Babbar, 1996; Dunn, 1999; Engel *et al.*, 2002; Pugh and Fairburn, 2008). In Europe, about a third of the motorways are privately owned (Verhoef, 2007) and private roads in the USA and other countries are becoming more common (Winston and Yan, 2011; van den Berg and Verhoef, 2012). When private operators are engaged in toll road construction, it is usually the case that the government awards a private sector participant, selected through competitive tenders, a concession to construct and maintain a road for a period of time.

It is in the public interest to ensure that concession rights awarded to private operators, which could be in place for a lengthy period, are not abused. The private sector demands high returns for taking on the risks associated with PPPs (Beesley and Hensher, 1989; Nijkamp and Rienstra, 1995). These risks stem primarily from both the demand uncertainty (Nagae and Akamatsu, 2006; Chen and Subprasom,

2007) associated with the lengthy period before the original investment is recovered and empirical evidence suggesting that traffic forecasts tend to overestimate traffic levels and consequently, the revenue potential of toll roads, resulting in "optimism bias" (Bain, 2009b). At the same time, the construction of a profitable road does not necessarily translate into an increase in social surplus and could result in "cherry picking" of projects (Yang and Meng, 2000; Ortiz *et al.*, 2008). Similarly, a road that enhances social welfare may not be provided by the private sector because it is not profitable. This reflects the divergence between welfare and revenue objectives (Mills, 1995; Ortiz *et al.*, 2008).

In the absence of non-compete clauses in concession agreements, an operator with a concession could potentially face competition with other operators doing the same on other roads in the transportation network. This is not an altogether unrealistic prospect. In fact, private operators compete with one another, as well as with existing alternative toll free roads for patronage and toll revenues, on road networks in the Republic of Ireland (Roughan & O'Donovan *et al.*, 2011) and Sydney (Li and Hensher, 2010). Such practical examples motivate the specific focus of this thesis, which is not on the much broader subject of PPP where extensive literature already exists (Gómez-Ibáñez and Meyer, 1993; Walker and Smith, 1995; Grimsey and Lewis, 2004) but instead on the consequences on social welfare as a result of *competition* between these private sector toll road operators in realistic networks.

In setting a toll that maximises revenue, the literature (e.g. Edelson, 1971; Verhoef *et al.*, 1996) has highlighted that the operator would be incentivised to take into account congestion of the road under his control. In this way, the toll should induce a more efficient usage of the road, an observation entirely in accordance with the Pigouvian ideal of improving efficiency. However, as noted above, because the private sector levies tolls so as to maximise revenue (rather than social welfare), the operation of the private toll road will inevitably result in another distortion. Thus in the face of competition between road operators, the issue investigated is whether the efficiency improvements achieved with the use of tolls is offset by the distortion due to the commercial (revenue maximising) interests of the private operators.

As will be discussed in subsequent chapters, there is extensive literature on competition between private operators. However, the insights have been predominantly obtained using traffic networks that abstract significantly from reality. This thesis contributes by investigating the transferability of the insights to a more complex network featuring multiple Origin Destination (OD) pairs. In addition, an implicit assumption of this literature is that the toll road operators do not engage in anti-competitive practices when they are faced with competition. As the number of such private toll road operators in a network is likely to be small, this assumption is unrealistic as they will recognise their interdependencies with rivals. In this case, it would be of interest to regulators to understand the existence (and policy implications) of incentives for toll road operators to engage in unfair trade practices such as collusion in toll setting.

When studying the effects of toll road operator and inter-jurisdictional competition in highway transportation networks, it should be emphasised that the objectives of the various agents are interrelated through an implicit relationship characterising the equilibrium route choices of the users. This implicit relationship arises because tolls influence the generalised travel costs incurred by users which then has a bearing on their choice of routes through the network in turn affecting the objectives of these agents.

In studies that have investigated competition in simple networks with a single OD pair, so long as restrictive assumptions are imposed, these toll levels could be analytically determined. In the more realistic setting with multiple links and multiple OD pairs such as those investigated in this thesis, the analytical approach would then no longer be practical. In such a setting, determination of these toll levels would require the application of solution algorithms.

1.2 Objectives and Scope

From the above discussion, two distinct groups able to utilise tolls to *compete* for toll revenues from highway users were identified. The first group are toll road operators that compete for toll revenues to maximise revenues. The second group comprises jurisdictions that compete for toll revenues in order to finance public goods desired by their residents while looking after the welfare of their residents.

On the one hand, toll pricing has been proposed to address inefficiency in the highway transportation system. On the other hand, efficiency losses result from the toll setting decisions made by each of these groups. Thus the aim of this thesis is to investigate the extent of welfare losses stemming from competition for toll revenues *a*) amongst toll road concessionaires, and separately, *b*) amongst jurisdictions. In order to answer this question, this thesis applies transportation network analysis tools within a modelling framework underpinned by principles of industrial organisation and public sector economics with a view to:

1. assess the transferability of findings regarding private sector toll road competition from simple network models to a more realistic network setting;
2. investigate the incentives for, and consequences on social welfare of, collusion between toll road operators;
3. study the welfare impacts of inter-jurisdictional competition for toll revenues;
4. assess the welfare implications when jurisdictions share toll pricing revenues while setting tolls non-cooperatively;
5. develop solution algorithms taking into account route choices of the users in support of the above;
6. test the algorithms developed with realistic networks and demonstrate the applicability of the methods to realistic problems; and
7. draw policy conclusions and develop policy recommendations informed by the modelling results.

The model developed will assume a single user class, a single time period and will focus exclusively on highway transportation.

While toll pricing in traffic networks is recognised as a powerful transportation demand management measure as it encourages travellers to adjust all aspects of their travel behaviour (de Palma and Lindsey, 2011), this thesis will restrict attention to the impacts of toll pricing on the route choice (and demand) impacts. In this case, all other responses to changes in the generalised cost of trips as a result of tolls and congestion is modelled by means of an elastic demand function which captures trip suppression.

In this regard, the traffic model used is based on equilibrium assignment where two different route choice principles are separately considered. The first is based on Wardrop's user equilibrium principle (Wardrop, 1952) which leads to the Deterministic User Equilibrium (DUE) model. The second is the more general Stochastic User Equilibrium (SUE) model (Daganzo and Sheffi, 1977; Sheffi, 1985). As will be demonstrated through this thesis, the applicability of the solution algorithms developed will depend on the routing paradigm assumed.

Technology of toll pricing and the costs of toll collection are not considered. As noted in de Palma and Lindsey (2011), exploitation of innovations in electronic toll collection have resulted in substantial reductions in the cost of toll collection which can be done without impeding traffic flow.

As this thesis will be concerned with measuring welfare effects of different toll pricing strategies, welfare changes are reported relative to that obtained in the no toll base equilibrium.

A word on the mathematical notation used is in order. This thesis adopts the convention that, unless otherwise stated, all vectors, distinguished by **bold** font, are column vectors. In this connection, the superscript T is used to indicates the transpose of a matrix. Notation and abbreviations are defined when introduced. A full set of the abbreviations used is provided in Appendix F.

1.3 Organization of the Thesis

The thesis is structured following the outline in Fig. 1.2.

Chapters 2 to 5 develop the theoretical background. Following a literature review, it will be highlighted in Chapter 2 that the common underlying theme in a large

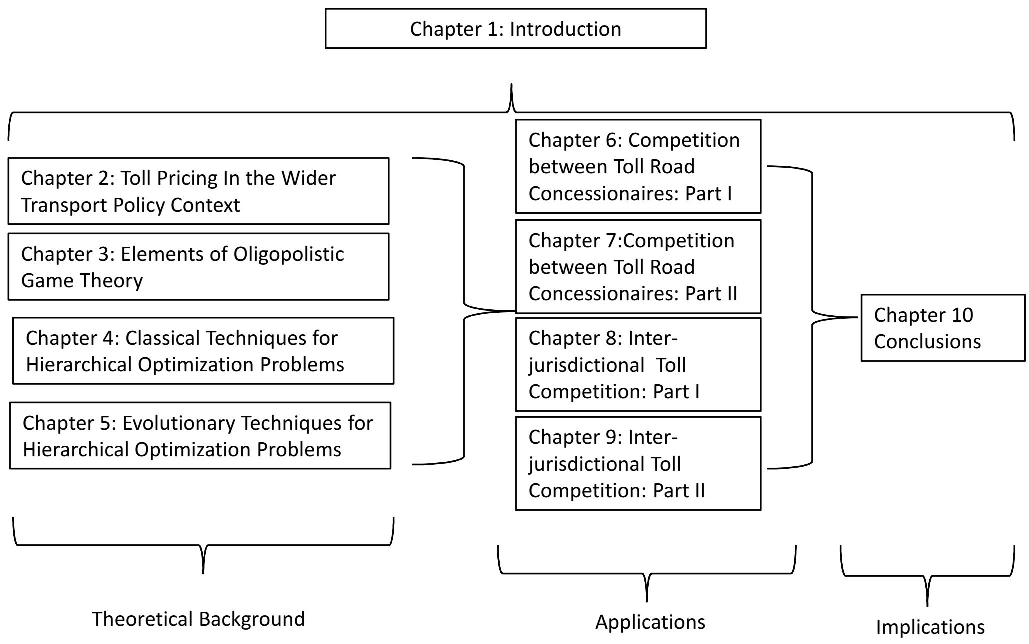

Figure 1.2: Outline of the Thesis

portion of the existing literature on toll pricing has implicitly assumed that toll pricing decisions are made by a single regulator. However, in order to study the social welfare consequences of toll revenue competition (whether by private sector toll road operators or by jurisdictions), this thesis argues that this basic paradigm must be augmented by a framework capable of capturing the *interactions* of multiple agents, each implementing toll pricing in pursuit of some individual objective. These interactions can be analysed by drawing on principles from game theory which is the subject of Chapter 3.

Chapter 4 subsequently integrates the game theoretic model describing interactions between the agents responsible for toll setting with the underlying route choice model. Here, drawing on analogy with economic theory of the Stackelberg model, it will be emphasised that these agents can be modelled as "leaders" who set tolls anticipating the responses of users to the toll pricing implementations, in pursuit of their objectives. Thus, this integrated model fits within the framework of a class of mathematical problems known as Equilibrium Problem with Equilibrium Constraints (EPECs). The notion of the equilibrium constraint emphasises that the objectives of leaders are implicitly interrelated through, and constrained, by an underlying route choice model specifying the equilibrium route choice (and demand) condition in a highway transportation network.

The purpose of the EPEC framework is two fold. Firstly, it allows for two alternate suppositions as to how these leaders would behave depending on whether they were assumed to either *a*) *compete* amongst themselves, resulting in a Non-Cooperative EPEC (NCEPEC) or, *b*) *cooperate*, resulting in a Multiobjective Optimisation Problem with Equilibrium Constraints (MOPEC). As the objectives of these leaders could be improved if there was a possibility that they could act cooperatively rather than competitively, both assumptions of the EPEC framework are relevant for this study. Secondly, the EPEC framework allows for the development of algorithmic approaches to determine the toll levels, predicated on the assumption of the leaders' behaviour. In this regard, it should be emphasised that research into solution methods for EPECs has only begun recently and continues to be a daunting research challenge. Assuming that these leaders were to compete amongst themselves, Chapter 4 outlines two solution algorithms that can be used to solve the NCEPEC.

As a result of the intrinsic nonsmooth nature of the EPEC formulation, Chapter 5 exploits recent developments in the field of evolutionary computation to develop two further solution algorithms that capture both of the aforementioned behavioural assumptions of the leaders. The crucial difference between the approaches developed here and those in Chapter 4 is that the approaches here are not predicated on the intrinsic assumption that the equilibrium link flows and demands obtained as solutions of the underlying traffic models are differentiable. In addition, Chapter 5 exploits the concept of bilateral bargaining from Axiomatic Bargaining Theory to bridge the gap between the two extreme assumptions of non-cooperative behaviour and full cooperation amongst the leaders.

Chapters 6 to 9 focus on applications of the theoretical and algorithmic principles discussed in the theoretical background to the two topics of competition for toll revenues amongst revenue maximising toll road operators (Chapters 6 and 7) and inter-jurisdictional toll revenue competition (Chapters 8 and 9). Each topic studied is divided into two chapters each, distinguishing in this way between the underlying route choice model assumed i.e. DUE and SUE. This distinction is introduced with the intent of highlighting that the applicable solution algorithm, developed in the earlier chapters, is dependent on the underlying route choice model adopted.

Chapter 10 is the concluding chapter of the thesis, in which the work through the thesis is summarised, and the main conclusions are drawn. Possible avenues for future research are discussed.

Chapter 2

Toll Pricing In the Wider Transport Policy Context

2.1 Introduction

As summarised in Chapter 1, this thesis focuses on assessing the welfare implications induced by toll pricing policies initiated by various agents seeking to influence the travel behaviour of road users in order to achieve some goals. This chapter has two aims. Firstly, it sets the context of this research by both outlining the theoretical justification for toll pricing and by means of an overview of relevant research in this area. Secondly, this chapter identifies literature on the twin topics of private sector involvement in highway transportation networks and competition between (governmental) jurisdictions for toll revenues. In doing so, research gaps are identified with a view to consolidating the direction of research undertaken in this work.

To economists, the costs a road user takes into account, when deciding to use an already crowded road, excludes the costs his presence imposes on others. Similarly, the use of vehicles produces a variety of emissions that contribute to smog, global warming and acid rain (Joireman *et al.*, 2004). In both these aforementioned instances, the additional costs (congestion delays in the former, environmental impacts in the latter) are not taken into account by the user, resulting in a market failure, which economists argue, could be rectified through the imposition of a marginal cost toll.

This marginal cost pricing principle can be straightforwardly generalised from the often used pedagogical single road link case to network of roads where in the latter case, a toll should be levied for the use of each and every congested road in the network. While the pursuit of this principle serves to maximise social welfare, it also dictates that all congested roads in the network are subjected to tolls. This requirement is particularly onerous being both impractical and costly (Verhoef *et al.*, 1996). Thus an extensive literature has developed, focusing on practical toll pricing implementations, relaxing the stringent requirement that all road links in a network are designated as tollable (i.e. available for toll pricing). Nevertheless, as will be stressed in this chapter, whilst providing many useful insights, the underlying premise of this literature is still restrictive in so far as it implicitly presumes that toll pricing strategies are implemented by a *single benevolent* regulator seeking to maximise social welfare for *all* road users.

Against this backdrop, it is recognised that the growth in demand for highway capacity has begun to outstrip the funds available to satisfy the growing need for continued expansion and maintenance of the highway transportation system (e.g. Forkenbrock, 2005; Zhang *et al.*, 2009). As a result, there has been a global trend towards supplanting the available resources by involving private sector participants in the development of toll roads. In return for financing the construction of additional highway capacity, these private sector entities levy a toll charge on users who utilise the facility when the toll road is open to traffic. However, the tolls introduced by these private sector participants are intended to maximise revenue in order for the original investment to be recouped and to obtain a return on their investments. Thus the toll introduced with this objective in mind will inevitably differ from one introduced for the purpose of maximising social welfare. Furthermore, with increasing private sector participation, a toll road operator exercising control over the toll on a particular road could face competition from others doing likewise elsewhere on the network. This is not an altogether unrealistic possibility since such competition is an existing occurrence on Sydney's orbital motorway network (Li and Hensher, 2010). Thus with toll road competition, *multiple* agents are able to simultaneously exercise control over toll levels that influence the route choice of users.

Furthermore, arising naturally as a consequence of transportation networks transcending multiple jurisdictions, the introduction of a toll pricing policy in one jurisdiction has an impact on the welfare of another. As will be reviewed in this chapter, an emerging literature suggests that when implementing a toll pricing policy, a jurisdiction is politically motivated to view users from outside its jurisdiction, as a source of congestion impeding local residents and as a source of toll revenue, whilst paying less heed to their welfare.

The remainder of this chapter is structured as follows. Following this introduction, the traditional justification for toll pricing, based on the argument that highway traffic imposes costs on society beyond that borne by users themselves, is outlined in the next section. As noted above, reasons of practicality constrain the regulatory authority's ability to impose tolls on all congested roads in the network. Hence, several practical toll pricing designs have been suggested and these are briefly discussed herein. Subsequently, Section 2.3 challenges the usual implicit premise that toll pricing policies are implemented by a single and benevolent regulator. Instead, in order to evaluate the wider welfare implications, the need for a framework that is able to fully take into account the actions of *multiple interacting agents*, each exercising control over a subset of all toll pricing variables, is emphasised. Two specific instances of multiple agent involvement in toll setting decisions lie at the heart of this thesis. The first discussed in Section 2.4 is toll (revenue) competition amongst independent revenue maximising private sector toll road operators. The second, the focus of Section 2.5, is toll (revenue) competition amongst different jurisdictions. Section 2.4 starts of with the necessary background on private sector involvement in highway transportation systems before focusing on the specific issue of competition between private toll road operators. The literature review presented in this section shows that the welfare implications of allowing competition between toll road operators pivots on the intrinsic interrelationships between the toll roads in competition. The assumption of benevolence of governments implementing toll pricing strategies is questioned in Section 2.5 where it is argued, extending insights from the public economics literature, that governments generate various externalities in pursuit of their fiscal policies. One of these externalities, relevant to a discussion on the governance of transportation networks, is *tax exporting*, based on the argument that while jurisdictions are concerned with the welfare of their own residents, they treat users from outside their jurisdictions traversing the transportation networks within their jurisdiction as a source of revenue and as a source of congestion impeding local residents. The literature review presented in this section suggests that when jurisdictions engage in tax exporting, the toll pricing policies introduced by welfare maximising jurisdictions, in fact, become *indistinguishable* from that of revenue maximising private toll road operators. The possibility of tax exporting further raises the policy relevant question of whether toll pricing should be viewed primarily as a local traffic management issue devolved to local governments or whether it should be instituted within a national framework managed by a higher level (e.g. central) government. Section 2.6 summarises the key issues discussed in this chapter, focusing on the research gaps addressed in this thesis.

2.2 The Classical Justification for Toll Pricing

2.2.1 Congestion Externality on a Single Link

Congestion is a "condition that affects transportation networks when the demand for a facility temporarily exceeds capacity" (Iacono and Levinson, 2011, p. 69). In practice, this phenomenon manifests itself in the familiar "stop-start" conditions experienced by many drivers during the morning and evening rush hours in many cities around the world "when vehicles interact to impede each others' progress"(DfT, 2013*a*). Ignoring tailbacks and queues resulting from accidents and roadworks, economists, in the tradition of Pigou (1920), have long contended that congestion arises because the costs that a road user takes into account when making his decision to use the road *excludes* the costs his presence in the traffic stream imposes on others. Thus "[d]riving on a congested road leads to a relatively long travel time for the driver involved, but also increases the travel times for all other drivers on the same road" (Emmerink *et al.*, 1995, p. 582). In the parlance of economists, the increase in travel time experienced by others due to the road user's action is termed a "(negative) externality" (Knight, 1924; Walters, 1961; Button, 1993; Small and Verhoef, 2007; Parry *et al.*, 2007). The presence of negative externalities in the highway transportation market is symptomatic of an underlying market failure (MasCollel *et al.*, 1995; Lipsey and Chrystal, 1999) which can be remedied by an imposition of a toll.

Although the term "externality" is widely used in economics, there is, in fact, "no consensus on its exact definition and interpretation" (Verhoef, 1994, p. 273). However, a convenient working definition relevant for the ensuing discussion is that an externality is the "indirect effect of a consumption activity or a production activity on the consumption set of a consumer, the utility function of a consumer, or the production function of a producer" (Laffont, 2008, p. 192). The word "indirect" emphasises that "the effect concerns *an agent other than the one exerting this economic activity* and that this effect does *not* work through the price system" (Laffont, 2008, p. 192, italics added). Echoing this, Mishan (1971) stresses that the essential feature of an externality is that the "effect produced is not a deliberate creation but an *unintended* or *incidental* by-product of some otherwise legitimate activity" (Mishan, 1971, p. 2, italics as per original). Thus applied in the highway transportation context, the act (of using the congested road) has the unintended consequence of increased costs being imposed (on *other* road users).

Congestion is unfortunately not the only negative externality associated with the use of roads. Other externalities, such as noise annoyance, visual intrusion, pollution, and accidents have been noted (Santos *et al.*, 2010). However, studies estimating the external costs of transport have shown that congestion costs are one of the most significant component of externalities in the UK (Sansom *et al.*, 2001), Europe (Friedrich and Quinet, 2011) and the US (Delucchi and McCubbin, 2011). For this reason, and because the congestion externality is the most tangible one since "a large share of the population faces congestion on a daily basis" (Emmerink *et al.*, 1995, p. 582), the exposition here will develop the traditional justification for toll pricing advanced by economists, assuming that congestion is the only relevant externality distorting the market.

Consider uninterrupted traffic flow on a single road of a given distance on portions of road away from conflicts such as on or off-ramps and friction from pedestrians and intersections (Morrison, 1986). The traffic flow on the road is described by an engineering speed-flow curve (Haight, 1963; Gerlough and Huber, 1975; Li, 2008) which can be converted with the known distance to produce a travel time-flow relationship.

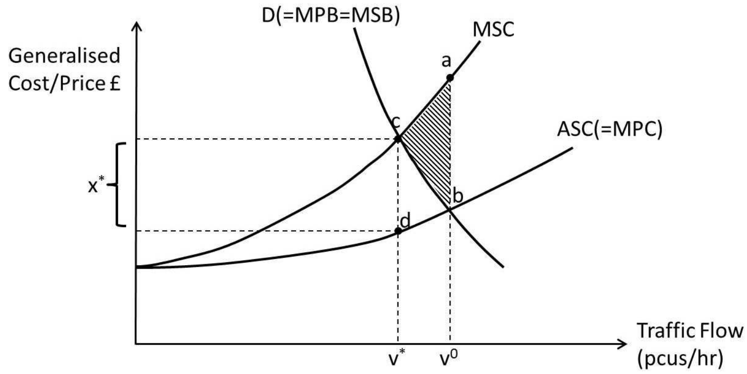

FIGURE 2.1: Depicting the Economist's Argument for Toll Pricing

Traffic flow is measured in passenger car equivalent units (pcus) per hour (TRB, 2010), a unit designed to capture the impacts of different vehicle compositions on capacity by translating them into an equivalent stream of passenger cars.

Adopting a partial equilibrium approach, the economic rationale for toll pricing can be illustrated with reference to Fig. 2.1. Assuming homogeneous users, the travel time-flow relationship can be multiplied by a common value of time (Abrantes and Wardman, 2011) and when other elements of costs independent of traffic flow (Hau, 2005) are added to it, the Average Social Cost curve is obtained. This is the curve labelled on Fig. 2.1 as ASC which is equal to the Marginal Private Cost (MPC) curve since it reflects the costs each user individually or "privately" incurs in deciding to undertake the trip (Hau, 2005; Steinmetz, 2011). However, each user also generates additional costs, such as congestion delay to other users by his presence, that he ignores when making the decision to use the road. Both the private costs and these additional costs are reflected in the MSC (Marginal Social Cost) curve which explains why the MSC curve lies above the ASC for each unit of traffic flow and rises much faster than the latter.

On the demand side, the inverse demand curve for trips, labelled on Fig. 2.1 as D, is assumed to fully reflect Marginal Private Benefits (MPB) which is in turn equivalent to Marginal Social Benefits (MSB) when there are no positive externalities of road use (Small and Verhoef, 2007). The equilibrium in the highway market thus occurs at the intersection of D and ASC at point $\mathtt{b}$ with traffic flow of $\mathtt{v}^0$ pcus/hr.

However, economists view this free-market equilibrium as inefficient. This is because for all drivers between $\mathtt{v}^*$ and $\mathtt{v}^0$, the Marginal Social Costs *exceeds* the Marginal Social Benefits. This results in a welfare loss given by the shaded area abc. The socially optimal traffic flow should, instead, be given by the intersection of D and MSC at point c of $\mathtt{v}^*$ pcus/hr. However, in the absence of any incentive to do so, this optimum would not be realised, and there is a discrepancy between the costs users privately incur (ASC) and costs borne by society (MSC), resulting in a market failure as the guiding first best dictum that prices should reflect marginal costs is violated (Lipsey and Chrystal, 1999).

To correct the market failure, a charge, referred to synonymously as the "marginal cost toll" or "Pigouvian toll" should be levied on each road user to rectify the discrepancy (Button, 1993; Emmerink *et al.*, 1995; Small and Verhoef, 2007). On Fig. 2.1, this toll is given by the line segment marked cd, corresponding to a toll of $\mathtt{x}^*$. It can be seen that this toll is equal to the difference between MSC and ASC at the optimal traffic flow $\mathtt{v}^*$ pcus/hr. This difference between MSC and ASC is also known as the "marginal external congestion costs" (Small and Verhoef, 2007). Charging every user of the congested link a toll equivalent to the marginal external congestion cost is then said to *internalise* the externality so that road users would be incentivised to take into account the congestion costs imposed on others when making their decisions to use the road (Emmerink *et al.*, 1995; Hau, 2005).

The internalisation of the congestion externality, through the imposition of the Pigouvian marginal cost toll, thus corrects the market failure and in doing so, maximises social welfare. With the first best toll, the welfare loss (area abc) is eliminated. In the absence of any other distortions in the economy (Button, 1993; Small and Verhoef, 2007), the removal of this welfare loss results in a (Pareto) efficient allocation of resources (MasCollel *et al.*, 1995; Lipsey and Chrystal, 1999). It is thus the pursuit of the efficiency objective in transportation systems management that advocates the use of tolls to correct the market failure (Rouwendal and Verhoef, 2006).

2.2.2 Congestion Externalities on a Network

The model presented so far has outlined the rationale for toll pricing on a single road. Nevertheless, the marginal cost pricing principle can, and has been, generalised to a network of roads (Beckmann *et al.*, 1956; Yang and Huang, 1998, *inter alia*). In the literature (e.g Yang and Huang, 1998, 2005), this policy is referred to as either "first best" or "system optimal" pricing and the resulting tolls, equal to the marginal external congestion costs on each and every congested link, termed the first best or system optimal toll vector (Verhoef, 2002*a*).

Though this thesis is only concerned with the route choice and demand impacts of a toll pricing policy, it should be emphasised that in the wider transport policy context, toll pricing is viewed as a powerful transportation demand management measure stimulating users to adjust all aspects of their behaviour (de Palma and Lindsey, 2011).

2.2.3 Practical Toll Pricing Schemes

While first best pricing maximises efficiency, it also imposes the unrealistic requirement that all congested links are designated as tollable. There are several reasons why this cannot be achieved in practice. Firstly, despite technological innovations (see e.g. de Palma and Lindsey, 2011, for a recent review), the costs associated with collecting tolls on every congested road link in a highway network would still be potentially prohibitive. Secondly, the public support for tolls is very low (May, 1986; Jones and Hervik, 1992; Small, 1992; Emmerink *et al.*, 1995) and a toll free route is usually provided, to enhance acceptability of any implementation. Furthermore, legislation in some countries, such as France, specifically require that toll free alternatives are provided (Raux *et al.*, 2007).

Second Best Toll Pricing

Abstracting from the theoretical first best model enunciated above, the practical constraints to implementing first best pricing in the real (or "second best") world, immediately implies that for maximum social welfare, the tolls, levied on road users on the available subset of road links designated tollable, should then no longer be set equal to the marginal external congestion costs. Following Lipsey and Lancaster (1956), this is simply an application of the more general theory of the second best.

The main repercussion of this theorem on transportation research has been to engender a stream of research on "second best toll pricing" (SBTP) problems (e.g. Marchand, 1968; Lévy-Lambert, 1968; Verhoef *et al.*, 1996; Liu and McDonald, 1998, 1999). In these references, the situation considered has been that of a tolled road alongside an untolled road in a two link highway network connecting a single origin destination (OD) pair. As second best problems are "more difficult to solve than first best problems" (Rouwendal and Verhoef, 2006, p. 109), research on SBTP has, in tandem, been extended from the two link, single OD pair case to general networks (i.e. with an arbitrary number of links and with multiple OD pairs) where only a subset of the links were tollable (e.g. Yang and Lam, 1996; Verhoef, 2002*a*,*b*) or where the tollable links were constrained to form a closed cordon around a congested city centre (e.g. Sumalee, 2004*a*) and, relaxing the assumption of user homogeneity, to take into account users distinguished by socio-economic status through differences in their perceptions in the value of time (e.g. Yang and Zhang, 2002). Optimal tolls in the presence of other distortions in the economy, such as those arising due to labour taxes, have also been considered (e.g Parry and Bento, 2001). Nevertheless, the common implicit underlying assumption in this literature is that the entire network is managed by a single welfare maximising regulator.

The literature has been further expanded in several directions. Two directions are closely related to the issues investigated in this thesis. Firstly, as will be discussed in Section 2.4.2, the literature has further investigated private toll road operator(s) playing a role in setting toll(s). Secondly, in the light of recent literature discussed in Section 2.5 below, a second best situation also arises naturally because local jurisdictions implementing toll pricing would usually only be limited to doing so within its spatial boundaries.

First Best Pricing as a Benchmark

Though first best pricing cannot be practically implemented, it nonetheless represents a theoretical benchmark as it gives the upper bound of social welfare gains achievable with marginal cost pricing, against which practical toll pricing schemes can be assessed. Faced with constraints which prevent absolute adherence to the marginal cost pricing principle, the literature on toll pricing has thus centred on attempts to maximise such a metric.

Types of Toll Pricing Schemes

Given the difficulties associated with the implementation of first best pricing, practical methods for toll pricing are second best schemes that adopt a combination of one of the following five forms and where the level of charge could possibly vary by time of day (TRB, 1994; Hau, 2006; de Palma and Lindsey, 2011):

- Facility-based schemes or point tolls: where a highway user is charged a fee for using the road. Examples are tolls on bridges and tunnels in Hong Kong (Loo, 2003) as well as the Birmingham North Relief Road (M6 Toll) in the UK (Pugh and Fairburn, 2008). Facility-based schemes dominate in North America (de Palma and Lindsey, 2011). A variant of the facility-based toll pricing scheme, known as the High Occupancy/Toll (HOT) lanes, is particularly prevalent in the US. Examples include the SR-91 in California (Sullivan, 2006) or the I-394 in Minnesota (Buckeye and Munich, 2004). On an otherwise untolled road, HOT lanes (one or more lanes of a multi-lane highway) give drivers the option of either paying a toll for its use (with the toll waived if the vehicle is carrying a stipulated minimum number of passengers, usually 4) or travelling in the other lanes without the need to pay a toll.
- Cordons: a pricing scheme whereby vehicles intending to enter a particular area (e.g. the city centre) “pay a toll to cross a cordon in the inbound direction, in the outbound direction, or possibly in both directions” (de Palma and Lindsey, 2011, p. 1381). Furthermore, a scheme may have single or multiple

cordons. The Stockholm congestion charging scheme (Börjesson *et al.*, 2012) is an example of a cordon scheme.

- Zonal Schemes: where a fee has to be paid to enter or exit, or to travel within a predefined area. An example of this is London's Congestion Charging Scheme (Leape, 2006; Santos and Fraser, 2006; Givoni, 2012). This is sometimes known as an "Area License Scheme".

- Distance Based Schemes: where the charges vary with the distance travelled (Forkenbrock, 2005). A national distance-based heavy goods vehicle charge is applicable in Austria, Czech Republic, Germany, Slovakia and Switzerland (Suter and Walter, 2001; Borgnolo *et al.*, 2005; Broaddus and Gertz, 2008; Poliak, 2009). In recent years, national distance based charging schemes have been proposed in both the UK and the Netherlands but plans have been put on hold due to a lack of political support (de Palma and Lindsey, 2011).

- Hybrid schemes: a combination of one or more of the aforementioned methods. For example, the toll pricing scheme currently in operation in Singapore comprises both facility based schemes with tolls on several arterials as well as cordon toll pricing within the Central Business District (CBD) (Menon, 2006).

2.2.4 Comparison of Alternative Toll Pricing Schemes

Several commentators have reported, using model based studies, that distance based charging is an effective instrument for reducing congestion and meeting the efficiency objective (e.g. May and Milne, 2000; Verhoef and Rouwendal, 2004; Mitchell *et al.*, 2005; Balwani and Singh, 2009). Verhoef and Rouwendal (2004) showed that in some cases, a flat distance charge could result in first best efficiency gains. On the other hand, May and Milne (2000) found that for the cordon scheme they tested with a highway assignment model, though the most operationally feasible, was not as effective in efficiency terms, as other pricing schemes tested when assessed across a number of metrics. This finding was contradicted by both Mun *et al.* (2003) and Santos (2004). Mun *et al.* (2003) showed that the cordon scheme they tested, based on Japanese data, attained an economic welfare level very close to the system optimum levels. Santos (2004) simulated cordon pricing implementations for eight English towns and noted that while the distributional impacts varied (Santos and Rojey, 2004), the cordons were effective in the meeting the twin objectives of increasing social welfare and reducing environmental impacts of traffic. This inconsistency can be reconciled as May and Milne recognised that alternative cordon designs could have been more effective as they only tested one specific scheme design.

Subsequent research has demonstrated that the extent of attainable efficiency gains with cordon pricing is highly sensitive to the design of the toll pricing scheme itself, specifically the location of toll points (May *et al.*, 2002; Sumalee *et al.*, 2005). In view of this, Sumalee (2004*a*,*b*, 2008) and Zhang and Yang (2004) developed heuristics, based on graph theory, to design toll pricing cordons that sought to maximise social welfare but at the expense of heavy computational burden. Shepherd *et al.* (2008) subsequently developed a faster, albeit heuristic, approach to accelerate the cordon scheme design process. Much less knowledge has been acquired about the design of zonal schemes though it is of practical relevance (e.g. in London's Congestion Charging Scheme). Nevertheless, there has been some research innovations in this direction (e.g. Maruyama and Sumalee, 2007).

2.2.5 Environmental Externalities

As noted above, congestion is not the only externality in highway transportation. Another externality, which has received increasing attention in recent years, is the unintended negative consequences of road traffic on the environment. Johansson-Stenman (2006) has shown that tolls should internalise both congestion and environmental externalities while Yin and Lawphongpanich (2006) investigate modifications to the first best pricing rule to highlight the trade-off encountered in attempting to internalise both congestion and environmental externalities.

In an era when environmental concerns have been raised over ozone layer depletion, green house gas production and global warming (ECMT, 1998; Joireman *et al.*, 2004), some toll pricing schemes have been implemented specifically with the aim of internalising environmental externalities. Both Suter and Walter (2001) and Broaddus and Gertz (2008) point out that the objective of heavy vehicle toll pricing in Switzerland and Germany respectively was the internalisation of the environmental impacts of heavy goods vehicles. Similarly, the objective of Milan's ECOPASS scheme (also known as "Area C" (Percoco, 2014)), implemented in January 2008, was introduced primarily for the purpose of improving the air quality in Milan through traffic restraint (Rotaris *et al.*, 2010). While the analysis in Invernizzi *et al.* (2011) has attributed the observed reduction in carcinogenic Particulate Matter to the implementation of the scheme, Percoco (2014) applied econometric techniques to show that the environmental benefits of the scheme may have been offset as a result of significant modal switch to powered two-wheelers (i.e. motorbikes and scooters).

2.3 Multiple Agents in the Governance of Toll Pricing Policy

Thus far, consistent with large portions of the literature on toll pricing (e.g. Yang and Lam, 1996; Yin, 2000; Zhang and Yang, 2004), the discussion has implicitly assumed that a *single* regulator exercises control over all aspects of toll pricing policy variables (e.g. the location of toll points and the toll levels) with the aim of maximising a measure of social welfare for all travellers regardless of origin or destination over the *entire* network. Such a governance structure can be termed a "centric pricing scheme" (Zhang *et al.*, 2011, p. 298). This assumption points toward "an implicitly benevolent vision of government" (Manville and King, 2013, p. 230). However, the adequacy of the conventional assumption of decision making by a sole regulatory agency in transportation networks is questionable since, in reality, "the situation where one single government controls an isolated network is probably the exception rather than the rule" (Verhoef, 2008, p. 362).

The primary reason for motivating a multiple agent framework in the governance of toll pricing is intimately related to the fact that transportation networks, generally spanning multiple jurisdictions, are usually managed by different governments (Taylor and Schweitzer, 2005; Rodrigue *et al.*, 2013) where a jurisdiction is "the road authority responsible for maintaining the road" (Levinson, 2000, p. 72). Within a nation, the (spatial and statutory) boundaries of a jurisdiction (e.g. local government) are usually defined by the national government, possibly enshrined in legislation, mainly for the purposes of clarifying public service responsibilities and for administrative reasons. For example, as stipulated in the provisions of the Greater London Authority Act of 1999, Transport for London (TfL) exercises jurisdiction over many strategic roads within Greater London. Likewise, the Highways Agency (HA) in England is tasked with operating, maintaining and improving England's Strategic Road Network, and thus exercises jurisdiction over motorways and trunk roads (HA, 2014). As transportation networks are meant to connect users between disparate geographical locations, it is inevitable that the routes users take to reach their destinations would involve travel over networks that are controlled/regulated by a jurisdiction different from where they commenced their trip.

When introducing a toll pricing scheme, a jurisdiction will usually be limited to introducing toll points on roads within its jurisdictional boundaries. Furthermore, a local regulatory authority would be primarily concerned with traffic conditions in its own jurisdiction and would, arguably, place less weight on conditions outside its sub-network. Taken together, this results in the so-called "distributed congestion pricing scheme" (Zhang *et al.*, 2011, p. 300). Since transportation networks transcend artificial jurisdictional boundaries, the introduction of toll pricing (or the application of any transport policy instrument, for that matter) in one jurisdiction would inevitably have impacts on another (van Zuylen and Taale, 2004). The outcome in transportation networks is thus "affected by the policy decision of several governments, and the implications of these policies strongly interact" (De Borger and Proost, 2012, p. 35). In such instances, robust assessments of the overall welfare implications of implementing toll pricing policies can only be conducted by considering the outcomes on multiple jurisdictions *simultaneously*. Thus, while serving as a useful benchmark, the paradigm of a single regulator is insufficient in evaluating policy outcomes when different agents interact, by each introducing a toll pricing policy, to maximise an individual local welfare measure. Furthermore, as will be discussed in Section 2.5, jurisdictions cannot be presumed to act benevolently towards *all* users when doing so which has significant implications for overall societal welfare.

A second motivation for a multiple agent framework relates to the observed global trend towards deregulation which has taken place in many, including transportation, industries over the last three to four decades. Within transportation, this has seen an increased number of private sector profit maximising entities participating in the provision of transportation services. For example, in the public transport industry in the UK and elsewhere (de Rus, 2006; Johnson and Nash, 2012), it is evident that, in an era of deregulation and privatisation, different agents, either acting individually or as a group, could exercise leverage over key public transport policy instruments such as fares and service frequencies (e.g Harker, 1988; Williams and Abdulaal, 1993; Zubeita, 1998; Li *et al.*, 2012).

Alongside the observed trend towards deregulation, there has been increased interests shown by governments around the world in engaging private sector operators to build and manage portions of the highway network (Roth, 1996; Yang and Huang, 2005). While deregulation and private sector engagement has led to broader participation into the management of highway transportation systems, broader participation has engendered more conflict in the decision-making process (Stough and Rietveld, 1997; Dunn and Sussman, 2011). The conflicts arise partly because the objectives of the private sector entities (profit or revenue maximisation) naturally differ from those of the public sector (welfare maximisation) (Mills, 1995; Gordon *et al.*, 2013). Thus a multiple agent framework is necessary in order to fully take into account the consequences of tolls, introduced by multiple revenue maximising private operators, on social welfare.

2.4 Private Sector Involvement in Infrastructure Provision

The internalisation of traffic externalities, discussed in Section 2.2 above, has been a key motivation leading to the introduction of the few toll pricing schemes currently in operation across the globe. However, as a result of both historical and political reasons, Larsen (1995) points out that the objective for introducing urban toll cordons in Norway was “purely financial” (Larsen, 1995, p. 188) with road tolls constituting 25% of road construction funding in Norway (Brathen and Odeck, 2009, p. 377). Besides toll roads, cordons or “toll rings” have been implemented in several Norwegian cities such as Bergen, Stavanger and Oslo. Though recent literature (e.g. Ieromonachou *et al.*, 2006) indicates that these schemes have begun to turn their focus towards the internalisation of the congestion externalities of traffic, these toll facilities were originally intended to raise revenues to fund highway infrastructure investments.

Nevertheless, the Norwegian example still remains an exception to the norm. In the majority of other countries around the world, tax revenues (usually receipts from fuel taxes) are used by governments to fund construction and maintenance of road networks such as the Interstate Highway System in the US (Winston, 2010; Geddes, 2011) or the network in the UK (Mackie and Smith, 2005). At the same time, with the declining purchasing power of fuel tax revenues due to inflation (Parry *et al.*, 2007), improved fuel efficiency of new vehicles (US-CBO, 2011) and the diversion of fuel tax revenues to fund general non-transportation related budgetary commitments (Goel and Nelson, 2003), it has been recognised, in the literature examining the finances of the US Highway Trust Fund, that the future stream of such revenues could be insufficient to satisfy the growing demand for increased highway capacity and its continued maintenance (Forkenbrock, 2005; Zhang *et al.*, 2009; Geddes, 2011; Robitaille *et al.*, 2011; Schank and Rudnick-Thorpe, 2011). These reasons have played a pivotal role in the observed global trend in leveraging private sector resources, through the use of Public Private Partnerships (PPPs), to augment highway transportation infrastructure provision (Gómez-Ibáñez *et al.*, 1991; Fishbein and Babbar, 1996; Poole, 1998; Ortiz *et al.*, 2008).

2.4.1 Private Toll Roads

Private sector participation in toll road construction is neither new nor theoretical but is accelerating in pace across the globe (Fishbein and Babbar, 1996; Yang and Huang, 2005). The Build-Operate-Transfer (BOT) concession model (see e.g. Yang and Meng, 2000; Tsai and Chu, 2003) is an example of a PPP in highway transportation where the government awards a private sector participant (known as the "concessionaire"), selected through a competitive tendering process, with concessions to collect tolls from users of these roads (Engel *et al.*, 2002) for a pre-specified duration ("concession period") when the road is opened to traffic. In return, the concessionaire secures private capital to fund both the construction of the new road and its continued maintenance over the life of the concession. After the concession expires, the road is then transferred back to the government (and tolls may be abolished). Thus such roads are referred to as "private toll roads".

Advocates of a market based approach to infrastructure provision (e.g. Geltner and Moavenzadeh, 1987; Nijkamp and Rienstra, 1995; Roth, 1996; Engel *et al.*, 2002) point to four main reasons in support of private sector participation in the provision of highway infrastructure. Firstly, by drawing on private sector resources such as pension funds (Della Croce, 2012; Armistead, 2013), the total budget available to meet infrastructure requirements increases. Secondly, they argue that the private sector would be at least as efficient as a government run entity. While there is evidence suggesting that "PPPs demonstrate clearly superior cost efficiency over traditional procurement" (Infrastructure Partnerships Australia, 2007, p. 1), others highlight that the argument of increased cost efficiency of the private vis-á-vis the public sector is "more sociological or cultural than economic" (Geltner and Moavenzadeh, 1987, p. 15). Thirdly, it is highlighted that tolls are a more equitable means of financing road provision (Gittings, 1987; US-CBO, 2011) as they will only be paid by those who directly derive a benefit from using the facility whereas traditional financing sources are derived from fuel taxes which are paid by *all* motorists including those who may not use the facility. Fourthly, it has been noted that economic efficiency of highway usage could be improved. The argument advanced in support of this last point is that as fuel taxes are levied across all motorists who purchase fuel, irrespective of their individual contribution to congestion, they are unable to *directly* target the congestion externality (Parry and Small, 2005). Echoing this view, a report by the US Congressional Budget Office (CBO) has recognised that "fuel taxes cannot provide a strong incentive for people to avoid overusing highways-that is, to forgo trips for which the costs to themselves and others exceed the benefits."(US-

CBO, 2011, p. vi). On the other hand, revenue maximising concessionaires, as will be discussed below, are motivated to take the congestion externality into account in setting the toll(s).

As a road uniquely occupies space (Zhang and Levinson, 2005), the award of a toll road concession inevitably confers a degree of monopoly power to the concessionaire. One way to counter this monopoly power is to introduce competition. Beyond competitive auctions between bidders for the award of the concession itself (Ubbels and Verhoef, 2008*a*) (resulting in so-called "competition *for* the market" (Etro, 2007)), there could also be direct competition between toll road concessionaires (also termed "competition *in* the market" (Etro, 2007)). Thus in the absence of non-compete clauses in the concession agreement, a concessionaire could potentially face direct competition, for toll revenues, from toll roads controlled by other consortia. Such competition can be observed, for example, between concessionaires exercising control over tolls on the different segments constituting Sydney's urban orbital motorway network (Li and Hensher, 2010; Roads and Maritime Services, 2013) as well as between several PPP financed toll roads around Dublin, Republic of Ireland (Roughan & O'Donovan *et al.*, 2011). Thus an understanding of the welfare implications of the latter case of toll road competition *in* the market is crucial to inform future regulatory policy.

2.4.2 Competition between Private Toll Roads

The SBTP problem was previously introduced in Section 2.2.3 (see p. 20) in relation to the practical difficulties associated with implementing Pigouvian welfare maximising first best tolls. This section discusses the insights from another class of SBTP problems that naturally arise in connection with private toll roads since each concessionaire would be limited to deciding the toll level(s) only on the road(s) that they have been awarded concessions for.

Table 2.1 lists a selection of the extensive literature contributing insights relevant to understanding toll competition amongst toll road providers utilising networks connecting **a single OD pair**. These works are differentiated by the network structure considered (i.e. either parallel or serial links) and whether link capacity was endogenously determined. Alongside the first best pricing benchmark and the non-intervention equilibrium, the papers listed in this table have considered one or more of the following ownership regimes:

- a public toll free road in competition with a privately operated toll road,
- a monopoly, where a single private concessionaire maximising revenue exercises control over both roads and
- a duopoly, where two independent privately operated toll roads compete for toll revenues.

The nature of the network used (i.e. two parallel and/or two serial links) is shown by tick symbol, ✔, in the appropriate column if it was considered in the reference cited or cross symbol, ✗, otherwise. Furthermore, a tick symbol, ✔, in the column labelled "Endogenous Capacity Determination" indicates that the reference also considered the determination of link capacity alongside the choice of the toll level.

Table 2.1: Selection of Literature Discussing Private Concessionaire Toll Road Operation in highway networks connecting a **single OD pair**, differentiated by network structure (i.e parallel or serial links) and whether capacity choice, in addition to tolls, was endogenously modelled.

Reference	Network Structure Parallel	Serial	Endogenous Capacity Determination
de Palma and Leruth (1989)	✔	✗	✔
de Palma (1992)	✔	✗	✗
Mills (1995)	✔	✔	✗
Viton (1995)	✔	✗	✗
Verhoef *et al.* (1996)	✔	✗	✗
de Palma and Lindsey (2000)	✔	✗	✔
Engel *et al.* (2004)	✔	✗	✗
Acemoglu and Ozdaglar (2007*a*)	✔	✗	✗
Acemoglu and Ozdaglar (2007*b*)	✔	✔	✗
de Palma *et al.* (2007)	✔	✗	✔
Small and Verhoef (2007)	✔	✔	✔
Mun and Ahn (2008)	✗	✔	✗
van den Berg and Verhoef (2012)	✔	✗	✗
van den Berg (2013)	✗	✔	✔

As noted in Table 2.1, some authors exclusively considered the determination of the toll level (e.g. Mills, 1995; Verhoef *et al.*, 1996) treating capacity as fixed while others study how private operators determine both tolls and capacity of the road(s) they control. In this latter case, some e.g. de Palma and Leruth (1989), assume that tolls and capacity are determined in two stages. In these two stage games, concessionaires decide capacity levels on the link each exercises control over at the first stage, and tolls are chosen at a second stage, with capacities determined from the first stage held fixed. Alternatively, tolls and capacities could be optimised simultaneously (e.g. van den Berg, 2013). In this case, Verhoef *et al.* (2010) have shown in a general network context that the link capacity is determined by its flow in the second best equilibrium which implies that a closed form analytical expression for the link capacities is available under several widely held technical assumptions (see also extensions in Wu *et al.* (2011) and Wang *et al.* (2013)).

In economic theory, the "short run" is the time period over which the inputs of, at least, one factor of production cannot be varied (Lipsey and Chrystal, 1999). Thus models that assume fixed capacity while endogenously determining toll levels can be regarded as "short run" models. This may be "more realistic, as capacity is a long run decision while fees can be changed more easily" (van den Berg, 2013, p. 186). Focusing exclusively on the use of tolls as the strategic variable, assuming that the road capacities remain unchanged, this thesis thus aims at developing short run models of toll revenue competition.

In terms of modelling of congestion effects, the majority of the literature cited in Table 2.1 have tended to emphasise the static congestion model where (generalised) travel time on a link is described by its time-flow relationship and where the build up of queues is not explicitly considered. Others (e.g. de Palma and Leruth, 1989; de Palma and Lindsey, 2000; de Palma *et al.*, 2007) take into account dynamics utilising the "bottleneck" model of Vickrey (1969) (see e.g. Arnott *et al.*, 1990, 1993, for a review of this model). In concert with the majority of the literature, and for numerical tractability, this thesis will limit focus to the case of static congestion.

A common strand of the literature listed in Table 2.1 is the use of a *game theoretic framework* to describe the interactions between competing concessionaires. In tandem with the multiple agent framework underlying this thesis, this framework lends weight to the argument of the necessity to take into account decision variables of *all* concessionaires who simultaneously exercise control over tolls on their respective links so as to maximise revenues. The concept of Nash Equilibrium (Nash, 1950*a*, 1951) is subsequently applied to determine the decision variables offered by each toll road operator in the resulting non-cooperative game. In concert with this literature, Chapter 3 of this thesis reviews the relevant game theoretic concepts underpinning the strategic interactions of these toll road concessionaires.

Toll Setting By Private Concessionaires

Before turning attention to toll setting by concessionaires in networks, Fig. 2.2 illustrates how a concessionaire would decide the toll level on a **single link** in order to maximise revenues. Adapted from Winston (2010), this figure reproduces Fig. 2.1 on p. 18, where the curves labelled D, ASC and MSC remain as previously defined. To recapitulate, these are the inverse demand, Average Social Cost and Marginal Social Cost curves respectively. However, this figure also includes an additional "Marginal Revenue" curve (labelled MR). Applying microeconomic theory (Lipsey and Chrystal, 1999), each point on the MR curve gives the additional revenue earned by the concessionaire as a result of the toll levied on an additional unit of traffic. The MR curve lies below the inverse demand curve, D, because the reduction in toll required to earn the additional revenue will require lowering the toll not only for the additional unit of traffic but also for all other units of traffic before it (Lipsey and Chrystal, 1999).

The industrial economics literature (e.g Belleflamme and Peitz, 2010) emphasises that a revenue maximiser will set prices from their knowledge of the inverse demand function at the point where the MR curve intersects the MSC curve. This occurs at point $\mathtt{f}$ on Fig. 2.2 resulting in traffic flow of $\mathtt{v}^{\mathtt{p}}$ pcus/hr. From their knowledge of the inverse demand curve, D, reflecting the users' willingness to pay, the concessionaire would set the price of the road at $\mathtt{p}^1$. At the level of traffic flow $\mathtt{v}^{\mathtt{p}}$ pcus/hr, the ASC, reflecting the cost borne by the users themselves is $\mathtt{p}^0$. Thus, the toll that the revenue maximising concessionaire would levy is given as the difference between $\mathtt{p}^1$

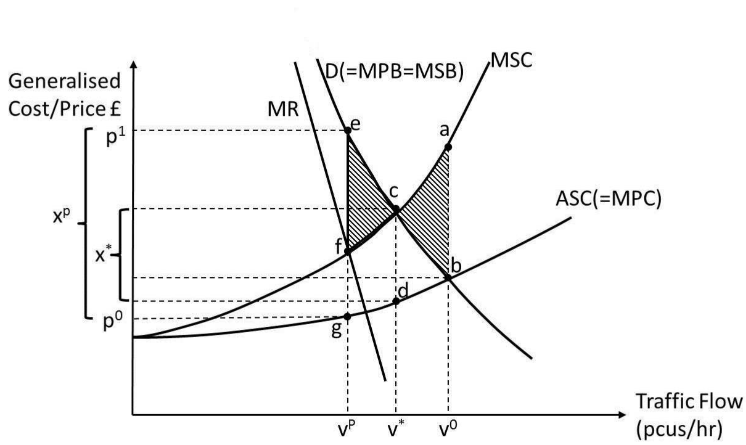

Figure 2.2: Illustrating Toll Setting by A Revenue Maximising Toll Road Concessionaire on a Single Link (Winston, 2010, p. 127)

and $\mathtt{p}^0$. On Fig. 2.2, this is given by the line segment marked `eg` corresponding to a toll of $\mathtt{x}^\mathtt{p}$.

It is immediately evident that $\mathtt{x}^\mathtt{p}$ is *greater* than $\mathtt{x}^*$, the toll required to fully internalise traffic congestion. Thus traffic flow $\mathtt{v}^\mathtt{p}$ pcus/hr is *lower* than the socially optimal level $\mathtt{v}^*$ pcus/hr. The resulting implication is that for all traffic flow between $\mathtt{v}^\mathtt{p}$ and $\mathtt{v}^*$, the Marginal Social Benefits (MSB) is *greater* than the Marginal Social Cost of the trips. This results in a welfare loss from revenue maximising pricing vis-á-vis first best pricing as shown by the area `cef`.

As noted earlier (see Section 2.2.1) and reinforced in Fig. 2.2, the equilibrium in the absence of any tolls, implies the traffic flow of $\mathtt{v}^0$ pcus per hour and a welfare loss given by the shaded area `abc`. From this discussion, it is *ambiguous* whether privatisation increases or reduces welfare vis-á-vis an unpriced public road (Winston, 2010). Hence when toll setting responsibilities are assigned to private revenue maximising concessionaires, the fundamental question of interest is whether the corrective effect of internalising the congestion externality intended with toll pricing (area `abc`) is *larger or smaller* than the welfare loss generated by a toll aimed at maximising revenue (shaded area `cef`).

Both Verhoef *et al.* (1996) and de Palma and Lindsey (2000) considered, amongst other scenarios, a private toll road concessionaire in competition with a toll-free road. These authors have shown analytically that the revenue maximising toll can be decomposed into:

1. the marginal external congestion cost of the link, and
2. a demand related markup.

These two components can be separately identified on Fig. 2.2. Recall that the toll levied by the private operator is $\mathtt{x^p}$ which is the line segment marked `eg`. The component of the toll reflecting marginal external congestion cost (item 1 above) is given by the difference between ASC and MSC at traffic flow of $\mathtt{v^p}$ pcus/hr. In Fig. 2.2, this corresponds to the line segment marked `fg`. The demand related markup component, (item 2 above), is given by the difference between the MR and D curves at traffic flow of $\mathtt{v^p}$ pcus/hr. Graphically, this component is represented on Fig. 2.2 by the line segment marked `ef`.

Since the revenue maximising toll includes an element reflecting the marginal external congestion cost, this implies that a private toll road concessionaire has an incentive to internalise the congestion externality on the toll road, in turn supporting a conclusion reached earlier by Edelson (1971). Intuitively, congestion is just as detrimental for the toll road concessionaire as it is for users since any increase in the travel time costs that users face due to congestion, *ceteris paribus*, reduces the willingness to pay for the use of the toll road. On the other hand, the demand related mark up is a direct consequence of the revenue maximisation objective of the concessionaires.

The crucial issue then is what happens in the presence of competition. Does the welfare loss associated with toll setting by private concessionaires (area `cef`) *grow* or *shrink* with competition? The literature listed in Table 2.1 has emphasised that the answer to this question pivots on whether the competition takes place between parallel links or between serial links. These two cases are considered in turn below.

Toll Competition Between *Parallel* Links

Two links are said to be parallel "if they connect the same pair of nodes in the same direction" (Bell and Iida, 1997, p. 22). This graph theoretic definition is convenient and clearly applicable in networks that comprise a single OD pair, but as will be noted later, becomes restrictive in more general networks. It can be seen from Table 2.1 that these networks of parallel links appear to have been studied most frequently "for no apparent empirical reason" (Ubbels and Verhoef, 2008*b*, p. 176).

As noted above, the concessionaire's revenue maximising toll consists of both a component related to the marginal external congestion cost and a demand related markup. In these parallel networks, as the number of competing concessionaires (i.e. the *intensity* of competition) increases, the component related to the marginal congestion cost increases while the demand related mark up component diminishes. Thus in the limit, the toll set by each competing concessionaire will equal that required to fully internalise congestion. In the case of a single OD pair, this implies that the competitive toll approaches the (Pigouvian) first best levels. (For a proof, see Small and Verhoef, 2007, p. 200).

This finding has led Engel *et al.* (2004) and Acemoglu and Ozdaglar (2007*a*), *inter alia*, to further hypothesise that when several concessionaires are engaged in toll revenue competition on *parallel* links connecting a single OD pair, toll competition could substitute for toll regulation since in the limit, toll levels would be equal to the welfare maximising Pigouvian first-best levels. It should be understood that this limiting result, while theoretically interesting, would not be realised in reality as it is "impossible to have an infinite number of (parallel) competing roads (van den Berg and Verhoef, 2012, p. 971, parenthesis as per original).

Toll Competition Between *Serial* Links

However, the conclusion that toll levels tend toward the welfare maximising levels as competing operators increase no longer holds true in the case of toll competition between *serial* links. Serial interdependencies arise when the journey requires "travel over a sequence of road links" (Mills, 1995, p. 137). Therefore, travel between that OD pair necessitates the use of two or more links *together* highlighting the complementary nature of such links.

Consider a serial link (such as a corridor) connecting a single OD pair divided into

2 segments where each segment is controlled by an independent revenue maximising concessionaire. Then in the competitive equilibrium, each concessionaire would set a toll that not only internalises the congestion externality of the segment under his control as in the parallel link case above, but also the congestion externality of the other segment as well. On top of this, each would add a demand related markup (Mills, 1995; Small and Verhoef, 2007; Acemoglu and Ozdaglar, 2007*b*; Mun and Ahn, 2008; van den Berg, 2013). The end result is that the toll that has to be paid to travel between the OD pair is higher than what a concessionaire in control of both segments together (i.e. a "monopolist") would charge. This is a result of the over-internalisation of the congestion externality and the markups being charged for each individual segment of the serial link.

Originating from the industrial economics literature, such an outcome is known as "double marginalisation" (Spengler, 1950; Tirole, 1988; Economides and Salop, 1992). Double marginalisation occurs when one concessionaire ignores the effect of the toll he sets on the revenue of the other concessionaire in the chain leading to the result that competition between complements results in higher prices. Thus toll revenue competition between concessionaires on *serial* links results in higher tolls for users, lower revenues for each concessionaire and a reduction in social welfare (Acemoglu and Ozdaglar, 2007*b*; De Borger and Proost, 2012; van den Berg, 2013).

In stark contrast to the case of competition between concessionaires controlling parallel links, these negative impacts are *magnified* as the number of serial competitors (i.e. *intensity* of competition) increase (for a proof, see Small and Verhoef, 2007, p. 200). Thus it is evident that whether competition between toll operators in the road network would improve efficiency is intrinsically related to the interdependencies of the particular links in competition.

Toll Competition In General Networks

In addition to the literature listed in Table 2.1, some authors (Yang and Woo, 2000; Yang *et al.*, 2009; Koh and Shepherd, 2010), have studied competition between toll road concessionaires in more general networks (i.e. those comprising both parallel and serial road links with multiple OD pairs). When more than two operators are possible, this is the setting of oligopolistic competition (Lipsey and Chrystal, 1999) as will be discussed in Chapter 3.

The discussion above has stressed that the network structure (more specifically, the relationship of the links in competition) plays a crucial role in determining the welfare implications of competition between toll road concessionaires. Extension of the analysis to networks raises two key issues, both of which are investigated in this thesis.

The first issue is the extent of transferability of policy relevant insights gained from these studies using single OD pair networks to the latter, more general but more realistic, setting. In a general network context, the graph theoretic definition for parallel links of Bell and Iida (1997) as given above, while correct, would be limited, since it neglects the message that "parallel" links in a general network setting should be interpreted, more generally, to mean that travellers have *alternative* route options as "links are *substitutes* for each other" (Mun and Ahn, 2008, p. 368, italics added). This more general definition is adopted in the work reported here. Similarly, serial links in the context of general networks should emphasise that "links are *complements* to each other" (Mun and Ahn, 2008, p. 368, italics added) which stresses, e.g. with two serial links, that "a user has to use *both* facilities in order to consume" (van den Berg, 2013, p. 186, italics added).

In practice, it is recognised that these definitions are not entirely unambiguous because of the complication that, in reality, a road link in a general network serves multiple OD pairs so that "the same link can be a substitute for some users and a complement for others" (De Borger and Proost, 2012, p. 38). Furthermore, when toll levels change, traffic usually has the opportunity to reroute, and any categorisation of links based on the untolled base equilibrium reference point could potentially break down.

The second issue is that an analytical approach (applied in the references cited in Table 2.1) to determine the toll levels in the resulting competitive equilibrium would not be feasible as there is no tractable analytical solution in this more general setting. To address this, this thesis contributes to the literature by proposing solution algorithms for the determination of the toll levels in an oligopolistic competitive equilibrium, explicitly taking into account the route choice of users.

Collusion

Another issue of policy relevance stemming from toll road competition is ensuring that users of the highway network, who ultimately incur the burden of toll payments, are not exploited by revenue maximising toll road concessionaires possessing varying degrees of spatial monopoly power. This motivates a further line of inquiry investigated in this thesis, which is to investigate the possibilities for *collusion* between toll road concessionaires. While Engel *et al.* (2004) have suggested that toll competition on *parallel* links could substitute for toll regulation, this conclusion is predicated on the assumption that concessionaires do not engage in anti-competitive practices such as collusion. In his study of private toll roads, Levinson (2006) writes that "(m)arkets function best when no producer . . . , can *collude*, to affect the price" (Levinson, 2006, p. 89, italics added). Thus the open question would be the existence or otherwise, of incentives for concessionaires to engage in collusion and the resulting welfare impacts of such actions.

The origin of the study of collusion is very old. Discussions of this issue can be traced as far back as the founding father of economics, Adam Smith, who warned that "people of the same trade seldom meet together, even for merriment and diversion, but the conversation ends in a conspiracy against the public, or in some contrivance to raise prices" (Smith, 1776, p. 145, para. c27). Similarly, Cournot (see Chapter 3) writes that if competitors "should come to an agreement so as to obtain the greatest possible income, the results . . . would not differ, so far as consumers are concerned, from those obtained in treating of a monopoly" (Cournot, 1838, p. 80). The modern study of collusion between firms can be found in the industrial economics literature (e.g. Romp, 1997; Porter, 2005; Peters, 2008; Belleflamme and Peitz, 2010).

While overt formal coordination using agreements has been made illegal under existing anti-trust legislation (Cooper, 1986; Rhodes, 2008; Davis *et al.*, 2010), *tacit* collusion or "conscious parallelism" (Macleod, 1985; Normann, 2000) is still possible (Davis *et al.*, 2010) because "collusion does not have to be enshrined in formal agreement if discipline can be maintained without it" (Beesley, 1990, p. 299). As a result, tacit collusion can be very difficult for regulators to detect (Porter, 2005). In tacit collusion, oligopolistic firms choose higher prices due to mutual recognition of their interdependence (Henderson and Quandt, 1980; Cooper, 1986). As there are likely to be only a few toll road concessionaires operating in the road network at any one time, it is postulated that they would recognise their mutual interdependence and thus have an incentive to manipulate the market to increase toll revenues.

Though tacit collusion has been investigated other network industries such as deregulated electricity markets (Bolle, 1992), telecommunications (Maillé *et al.*, 2009) and even between public transport operators (Beesley, 1990; Williams and Martin, 1993), discussion or analysis of this policy question has hardly surfaced in the literature on private sector toll road concessionaire competition. Tacit collusion can be characterised by indirect communication where "firms *infer* rivals' intentions from their actions or from market outcomes" (Porter, 2005, p. 148, italics added). Extending this idea, this thesis addresses this gap in the literature by means of an intuitive modelling approach to enhance understanding of the possibilities and social welfare consequences of collusion between toll road concessionaires.

2.5 Governmental Competition for Toll Revenues

Recognising that transportation networks span jurisdictional boundaries, an emerging literature has argued that jurisdictions are politically motivated to employ tolls to facilitate "tax exporting". Tax exporting, a concept originating from the public economics literature, is "the shifting of tax burdens by a locality to non-residents" (Wildasin, 1987, p. 591). When applied to transportation systems analysis, this strand of literature does not suggest that residents are exempted from the tolls but rather, has emphasised that jurisdictions implementing toll pricing only view extra-jurisdictional users (i.e. users from outside the local jurisdiction such as non residents traversing the network) as a source of toll revenue without regard for their welfare in the toll setting decisions. As will discussed in this section, this view, found in a small but growing literature (Levinson, 2001; De Borger *et al.*, 2007, 2008; De Borger and Proost, 2012), further challenges the conventional implicit assumption of *benevolence* of jurisdictions introducing toll pricing.

Tax exporting was never raised as an issue in Singapore where a toll pricing scheme was introduced in 1975 because being a city state, "transport planning in Singapore is facilitated by the fact that there is only one level of government for the island. This avoids the problems of overlapping jurisdictions between urban and suburban areas that are common in, for example, the United States. It also avoids conflicts over priorities among municipal, state (or county) and national decision makers" (Watson and Holland, 1978, p. 21). Similarly, conflicts between jurisdictions did not emerge as an issue during the implementation of London's Congestion Charging Scheme introduced in 2003. While "there was opposition London boroughs, this did not ultimately represent a major hurdle" (Ison and Enoch, 2005, p. 133). This is due, in part, to the "vertically integrated transportation system" (Manville and King, 2013, p. 236) where, TfL, the sole agency responsible for the implementation of the scheme, exercises considerable influence over the road network in Greater London (Richards, 2005). The structure accorded by the system was "critical to the introduction of congestion charging" (Livingstone, 2004, p. 491). Furthermore, by law, the revenues, net of operating costs, from the London scheme must be spent on measures to further the Mayor's Transport Strategy applicable across *all* London boroughs (House of Commons, 2009).

2.5.1 Multiple Level Governments and Externalities

Yet, the governance of toll pricing in Singapore and London can be regarded as atypical. More usually, toll pricing schemes are designed and implemented within a political and institutional setting with multi-level governance structures (Taylor and Schweitzer, 2005). As a result, there is a division of responsibilities between sub-national (i.e. state or local) governments and a national (e.g. in UK context) or federal (e.g. in the US context) level government. For example, in the US, the responsibility for transportation policy "is shared vertically by local, state and federal governments, and horizontally across a wide range of local and regional governments and agencies" (Manville and King, 2013, p. 230). The mainstream public economics literature highlights that in the process of making their individual tax decisions, each level of government generates fiscal externalities (Dahlby, 1996; Hoyt, 2001; Dahlby and Wilson, 2003; Spahn, 2007) i.e. unintended fiscal impacts on a different government as a result of a decision a government makes.

Both vertical and horizontal fiscal externalities have been identified in the literature. A *vertical* fiscal externality "occurs in a federation when the taxes or expenditures of one level of government affect the budget constraint of another level of government" (Dahlby and Wilson, 2003, p. 917). It is important to note that vertical fiscal externalities act in both directions (Hoyt, 2001). In other words, just as federal taxes affect state revenues, it is also the case that state taxes affect federal revenues. For example, in the US, the fuel tax consists of a portion payable to the federal and another to the state government (FHWA, 2012). This could result in a tax level that is too high when a state government increases its tax levels ignoring the reduction in federal tax revenues with the unintended consequence that the shared tax base is reduced (Flowers, 1988; Besley and Rosen, 1998). In this regard, Devereux *et al.* (2007) found evidence of significant interaction in US fuel tax rates between governments at the state and federal levels.

A *horizontal* fiscal externality occurs "when two or more jurisdictions *at the same level of government* encounter costs, or draw benefits from, some policy or action" (Spahn, 2007, p. 167). An example is the competition for funds between local governments for the same tax base (Ferreira *et al.*, 2005). The Tiebout model (Tiebout, 1956) is a "monopolistically competitive model of many similar communities differentiated by the offerings of the public sector as well as other amenities that influence people's locational choices" (Ulbrich, 2011, p. 41). Assuming perfect mobility of households, the Tiebout model hypothesises that competition between jurisdictions is welfare improving because it allows for better matching of diverse household preferences (De Borger and Proost, 2012), others (Oates and Schwab, 1988; Wilson, 1999) suggests that such competition "induces a race to the bottom in the relevant tax rates, potentially resulting in inefficiently low provision of public goods" (Brülhart and Jametti, 2006, p. 2028). In this way, the fiscal externality arising from the competition for mobile capital could result in tax rates being too low. Applying this in the transportation context, De Borger and Proost (2012) note that faced with elastic demand from traffic passing through its borders (e.g. hauliers who have the option to refuel elsewhere), having a low fuel tax rate in Luxembourg increases its tax revenues (Rietveld and van Woudenberg, 2005), potentially at the expense of its neighbours (Swedish Environment Protection Agency, 2000).

Tax exporting is another instance of a horizontal fiscal externality. There is a political motivation, because of vote maximisation, for local politicians to tax export by acting in the interests of their residents (Besley and Case, 1995; Wassmer, 2005) but disregard (positive or negative) spillovers of local policies on other jurisdictions (Spahn, 2007). Knight (2004) studied US congressional votes in matters of transportation funding and found empirical evidence that a politician's probability of supporting a transportation project *increases* in own-district spending but *decreases* in the tax burden associated with aggregate spending.

As will be discussed below, tax exporting is intimately related to issues of equity and in particular, the utilisation of the toll revenues collected from a toll pricing scheme.

2.5.2 Equity and Revenue Use

As discussed in Section 2.2, the basic justification for the use of tolls as a policy measure to internalise externalities and correct market failure, is rooted in the pursuit of efficiency. More precisely, internalising the congestion externality is viewed as a Pareto efficient policy under the Kaldor-Hicks criteria (Hicks, 1939; Kaldor, 1939) as long as those who are made better off could *potentially* compensate those that made worse off, such that a Pareto improving outcome results (Scitovsky, 1941; Johansson, 1991). However, there is yet another objective of transport policy that has been overlooked in this discussion thus far: the issue of distributional fairness or equity (Rosen and Gayer, 2008; Levinson, 2010).

A review of the literature supports the view expressed in May (1986) that equity issues, whether real or perceived, have been used to dismiss restraint proposals such as toll pricing. Thus equity concerns could partially explain why in many cities toll pricing may not be on the political agenda, and why schemes have been considered, but not implemented in cities such as Edinburgh (Rajé *et al.*, 2004; Saunders, 2005; Gaunt *et al.*, 2006, 2007), Manchester (Ahmed, 2011), New York (Schaller, 2010) and Hong Kong (Borins, 1988). There is an increasing consensus that equity arguments are only a partial explanation for the more general issue of acceptability of toll pricing as a policy measure (Schade and Schlag, 2000, 2003; Jaensirisak *et al.*, 2005). Giuliano summarises the issue succinctly in noting that "distributional equity may present an apparently legitimate basis for opposition that is actually motivated by other reasons" (Giuliano, 1992, p. 349).

It has been noted that "political acceptability depends very much on the distribution (and perception of the distribution) of gains and losses to a proposed change" (Levinson, 2010, p. 33). In particular, three groups of road users will be made worse off as a result of tolls (Hau, 2005, p. 92): *a*) users who remain on the tolled road because they incur the toll charge, *b*) users who are inconvenienced because they suppress/retime their trips, divert to alternative routes or modes and *c*) existing users of alternative routes/modes who might experience increased congestion (due to diversions). However, it is *the government*, in collecting toll revenues, that is made better off (Hau, 2005). With respect to the aforementioned Kaldor-Hicks criteria for Pareto improving outcomes, it is the use of this revenue that allows for the translation of the *potential* compensation into *actual* compensation. Thus it has been consistently emphasised that equity effects, real or perceived, will depend on how the revenue collected is used (Small, 1992; Armelius and Hultkrantz, 2006; King *et al.*, 2007; Schuitema and Steg, 2007; May *et al.*, 2010; Levinson, 2010).

Consequently, models of how the toll revenues collected from pricing could be used have been proposed (Goodwin, 1989; Jones, 1991; Small, 1992). Goodwin (1989) suggested a three way split sharing the revenues between roads, public transport and tax reductions. Similarly, Small (1992) proposed a revenue distribution model to make toll pricing practical and politically viable by monetary reimbursements to all travellers as a group, substitution of toll pricing revenues for general taxes and the provision of new transportation services. In this way, “earmarking” or “revenue hypothecation” could be used as a means to increase support for toll pricing proposals. However, there are two often neglected caveats to this argument.

Firstly, it is crucial to bear in mind that the categorisation of users who potentially lose out from implementation of toll pricing is conducted at the *aggregate* level. This does not translate directly into impacts at the *individual* level (Small, 1992). For example, in order to encourage local governments in the UK to consider the use of toll pricing to reduce congestion and to increase the public acceptability of scheme proposals, provisions in the Transport Act 2000 explicitly permitted local governments to retain the revenues collected from toll pricing for the purposes of meeting *local* transportation needs. However, since transportation networks naturally transcend jurisdictional boundaries, it is inevitable that individuals originating from outside the jurisdiction implementing the scheme would utilise the road network in the jurisdiction. Therefore “the people who benefit from congestion relief and revenue use do not neccessarily coincide with those who pay the fees or who suffer the inconvenience in order to avoid them” (Small, 1992, p. 362).

Secondly, the revenue distribution argument implicitly assumes that jurisdictions are benevolent and regard the welfare of all users *equally*. However, it should be recognised that “[t]he viability of any redistribution plan, in other words, pivots on the credibility of the institution doing the collecting” (Manville and King, 2013, pp. 230-231). While Manville and King (2013) point out that it is difficult to promote toll pricing to the public because they do not perceive governments who promise to recycle the revenues as credible, this thesis looks towards an alternative literature, discussed below, which argues that if jurisdictions tax export by treating extra-jurisdictional users traversing their road networks solely as a revenue source but ignoring their welfare, the implementation of distributed/decentralised toll pricing schemes can significantly reduce social welfare compared to a cooperative pricing policy, as will be analysed next.

Welfare Effects of Tax Exporting

Assuming that the government is "a black box, through which revenue flows in and out, without diversion or impediment" (Manville and King, 2013, p. 230), the judicious use of toll revenues can alleviate equity concerns associated with toll pricing policies. In reality, with tax exporting and the consequent desire to raise revenue from extra-jurisdictional users, jurisdictions would display behaviour similar to "that of a profit-maximizer with respect to its tolling of foreign travellers" (Ubbels and Verhoef, 2008*b*, p. 177). As a result, the studies, to be discussed below, have concluded that the fundamental insights from private toll road concessionaire competition (as discussed in Section 2.4.2, p. 35), similarly apply when the "operators" in this context are governments.

Fig. 2.3 reproduces Fig. 2.1 and as before, the MSC and ASC curves refer to the Marginal Social Cost and Average Social Cost respectively (see p. 18). However, two inverse demand curves are shown on Fig. 2.3. The first, labelled D(Local), refers to the inverse demand function pertaining to local (i.e. jurisdiction) traffic and is equal to the Marginal Social Benefit (MSB) of local users only. The other curve, D(Total) (equal to MSB(Total)), refers to the inverse demand function (equal to Marginal Social Benefits) for both local *and* extra-jurisdictional traffic.

As the Pigouvian rule does *not* distinguish between local and extra-jurisdictional users, the welfare maximising traffic flow level should be given at point $\mathtt{c}$ where D(Total) intersects the MSC resulting in a traffic flow of $\mathtt{v}^*$ pcus/hr. As previously emphasised, traffic should be charged the first best toll, $\mathtt{x}^*$, in order to eliminate the welfare loss due to the congestion externality as given by the area $\mathtt{abc}$.

Consider next the toll setting decision from the perspective of a local jurisdiction. The literature, elaborated further below, assumes that the jurisdiction is only concerned with the welfare of local users only and the toll revenues collected from extra-jurisdictional users. Since the jurisdiction is assumed to be interested in the MSB experienced by *local* users only, in setting the toll, they will equate MSC with MSB(Local).

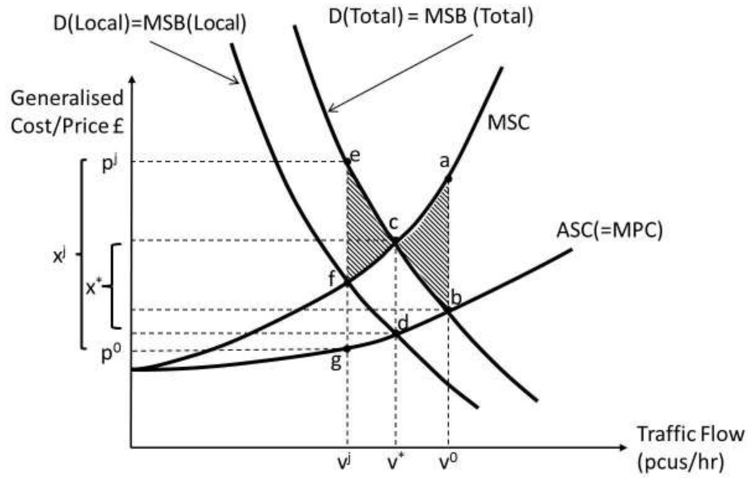

Figure 2.3: Illustrating Toll Setting with Tax Exporting Intentions by a Jurisdiction (Adapted from De Borger and Proost, 2008, p. 331)

This intersection occurs at point f with traffic flow of v^j pcus/hr. However, they are also interested in the toll revenues that can be earned from extra-jurisdictional users (e.g. through traffic). Assuming that the jurisdiction is unable to discriminate between local and extra-jurisdictional users, the price the jurisdiction will charge to *all* users for the use of the road, will be p^j. At the traffic flow of v^j, p^0 is the costs users themselves incur in making the trip (as given by the ASC at point g). Thus the toll, being the difference between p^j and p^0, is x^j (shown on Fig. 2.3 by the line segment marked eg).

Hence, exactly similar to the situation of toll setting by a revenue maximising toll road concessionaire (see the discussion on p. 35), a tax exporting jurisdiction levies a toll that consists of **two** components. The first is related to the objective of internalising congestion, stemming from the desire to maximise welfare for local users only (shown on Fig. 2.3 by the line segment marked fg). This rectifies the discrepancy between MSC and ASC, i.e. the marginal external congestion costs, at the traffic flow of v^j pcus/hr. The second component, the demand related markup, similar to that toll road concessionaires levy, is because of the desire to maximise revenue from extra-jurisdictional users (shown on Fig. 2.3 by the line segment marked ef). Therefore the toll a tax exporting jurisdiction sets, x^j, *exceeds* x^*, the first best toll intended solely to internalise congestion. As a consequence of the "profit-maximising behaviour of governments" (Ubbels and Verhoef, 2008*b*, p. 177), this means that MSB *exceeds* MSC, resulting in a welfare loss vis-á-vis first best pricing, given by the area `cef`.

Since "jurisdictions care for the well being of their citizens, but they are also interested in potential tax and toll revenues" (De Borger and Proost, 2012, p. 35), the fundamental question of interest is whether the welfare improvments achieved by internalising the congestion externality through tolls (area `abc`), is outweighed by the welfare decreasing fiscal externality of tax exporting (area `cef`). This is the underlying theme of the literature reviewed in the following section.

2.5.3 Evidence of Inter-jurisdictional Tax Exporting

The propensity for jurisdictions to engage in tax exporting can be substantiated by a combination of insights obtained from econometric analysis, reviews of literature documenting attempts to introduce toll pricing schemes as well as conclusions drawn from model based studies. These are summarised in this section.

Econometric Analysis

Econometric analysis by Levinson (2001) and Rork (2009) utilising data from the US, provides empirical evidence supporting the existence of tax exporting behaviour. Levinson (2001) was interested in the decision of whether a state would introduce toll pricing or otherwise. He found that the more non-resident workers a state has, the greater the likelihood of it introducing toll pricing. Furthermore, Levinson (2001) found evidence of a beggar-thy-neighbour policy effect: for a state with a large number of residents commuting out of state and if those neighbouring states toll, it would also likely retaliate by imposing its own tolls. This beggar-thy-neighbour effect arising as a consequence of predatory tax competition is a phenomenon documented in the public economics literature (Ambrosanio and Bordignon, 2006, p. 313) and will be observed in the case studies in this thesis.

Rork (2009), building on Levinson (2001), applied econometric techniques to study the intensity of the inter-jurisdcitional competition for toll revenues. Rork (2009) concluded that a 10% increase in per capita toll revenues of neighbouring states was met with a 4% increase in per capita toll revenues in the home state (Rork, 2009, p. 137).

Analysis of Proposed Toll Pricing Schemes

Further evidence of tax exporting behaviour can be found in the literature documenting attempts to implement various toll pricing schemes around the world. While the literature on these post mortems have tended to focus on the acceptability of toll pricing as an instrument of demand restraint (Schade and Schlag, 2000, 2003; Jaensirisak *et al.*, 2005), this literature also supports the view that perceptions of tax exporting intentions by the scheme proponents could have exacerbated public opposition to the proposals.

Tax exporting was one of the issues underlying discussions surrounding Edinburgh's congestion charging proposal. Commentators studying the collapse of Edinburgh's proposal (Rajé *et al.*, 2004; McQuaid and Grieco, 2005; Saunders, 2005; Gaunt *et al.*, 2006), which was rejected by over 70% of voters at the referendum (Saunders, 2005), have highlighted that the scheme would strongly affect drivers of those vehicles originating *outside* the jurisdiction of City of Edinburgh Council (CEC) who proposed the scheme. For example, "half of the working population of mid-Lothian [sic] travel into Edinburgh every day for work and would be directly affected by the charge" (Laird *et al.*, 2007, p. 181). This was further compounded by CEC explicitly providing an exemption for its own residents from having to pay the toll to cross the outer cordon (Rajé *et al.*, 2004; Gaunt *et al.*, 2006; Laird *et al.*, 2007). This exemption was opposed by three neighbouring authorities (Fife, Midlothian and West Lothian) who subsequently launched a legal challenge to the scheme (BBC, 2005).

While Scottish legislation facilitates toll pricing proposals, it also requires that toll revenues are earmarked for transport projects (Laird *et al.*, 2007). Thus simultaneous with the toll pricing proposals, CEC had also proposed a revenue sharing scheme where the toll revenues, net of collection costs, would be used for transport projects that would benefit residents of adjacent local authorities "in proportion to the trip origins of those paying the congestion charge" (Laird *et al.*, 2007, p. 163). With the revenue sharing proposals, it was forecast that slightly more than half of the charge payers would originate from within CEC's jurisdiction, so CEC would retain slightly more than half of the revenues with the rest being distributed to these adjacent local authorities for their disbursements on transport schemes (Gaunt *et al.*, 2006, p. 89).

Similarly, Schaller (2010) highlights that local opposition to the charging proposal for Manhattan (New York City) came from elected officials and civic groups in the four New York City boroughs *outside* Manhattan such as eastern Queens and southern Brooklyn. Also it was proposed that the revenue collected from drivers would be used to fund transit improvements benefiting the residents of New York City (TRB, 2011, p. 118). This proposal ultimately stalled in the New York senate and was subsequently abolished. The potential for tax exporting was also raised regarding Manchester's proposed toll pricing scheme as evidenced in parliamentary debates (Hansard, 2008) where members of the UK House of Commons emphasised that residents from jurisdictions *outside* the jurisdiction of the scheme's proponents would incur the charges but not necessarily benefit from public transport improvements associated with the use of toll revenues.

The toll pricing scheme in Stockholm has been in operation since 2007, after a six month trial culminating in a referendum (Stockholmförsöket, 2006). Studies of the chronology of events leading to its permanent implementation (Eliasson *et al.*, 2008; Börjesson *et al.*, 2012) provide further evidence suggesting that tolls are used (or perceived as being used) as an instrument of tax exporting. In ascending order of jurisdiction, the Swedish hierarchy of government comprises the municipality, the county and the national government, with responsibility for local transportation devolved to the municipality (Börjesson *et al.*, 2012). Thus the city of Stockholm, one of the 26 municipalities constituting Stockholm County, stressed that it was entirely within its authority to propose such a scheme and decide on the scheme characteristics as well as restrict voting in the referendum to residents of its municipality only. On the other hand, neighbouring municipalities within the county opposed the scheme, highlighting that as the tolls would negatively affect their residents, they should also be entitled to vote in the referendum (Eliasson *et al.*, 2008; Isaksson and

Richardson, 2009).

Thus even though "there is no such thing as a regional referendum in Swedish legislation" (Isaksson and Richardson, 2009, p. 255), several (but not all) surrounding municipalities also organised simultaneous local referenda on the scheme, some of which were politically motivated. While the official referendum held in the city of Stockholm returned a majority in favour of the scheme, the overall result of the parallel referenda, held in the surrounding municipalities, returned a majority against (Gullberg and Isaksson, 2009). Under Swedish law, the toll charge is legally interpreted as a tax. As Swedish legislation explicitly stipulates that "Swedish municipalities cannot levy taxes on other than their own citizens" (Börjesson *et al.*, 2012, p. 10), the decision was referred to the national government. Consequently, when the toll pricing scheme was reintroduced on a permanent basis, responsibility for management of the scheme was transferred from the municipality to the national level through a newly created National Transport Agency (*Transportstyrelsen*) (Börjesson *et al.*, 2012, p. 10).

The evolution of the Stockholm scheme also raises an underlying question of whether responsibility for the implementation and management of toll pricing schemes should rest at the national/federal level or whether toll pricing should be viewed primarily as a traffic management policy tool with responsibilities devolved to local jurisdictions.

Model Based Studies

From a survey of the literature, Table 2.2 presents several model based studies that have provided insights on toll competition between governments. This table lists the references and, apart from Laird *et al.* (2007) (see later), the nature of the governmental competition investigated i.e. whether horizontal or vertical competition or both. As mentioned previously (see Fig. 1.1 on p. 2), "horizontal" or inter-jurisdictional competition refers to competition between governments at the same level while "vertical" or inter-governmental competition refers to competition between governments at different levels.

In addition, for each reference listed, the network representation used in the studies as well as the policy instrument(s) available to the parties are identified. As Table 2.2 shows, the entire spectrum of network representation have been considered. Some studies (e.g. Proost and Sen, 2006) do not utilise any network representation while others (e.g. De Borger *et al.*, 2005) base their findings on networks with two links. In addition, toll revenue competition in more general networks i.e. those with multiple links and OD pairs have also been investigated (e.g Zhang *et al.*, 2011).

The common assumption in the studies shown in Table 2.2 is that governments are benevolent welfare maximising agents acting to further the interests of their *own* residents but they regard extra-jurisdictional users both as a source of revenue and contributing to congestion in the local network neglecting their welfare.

There are, at least, two similarities between the literature investigating competition between governments and those that have investigated competition between private toll road concessionaires (see Table 2.1, p. 31).

Firstly, in both cases there have been a mix of studies that have focused on the use of tolls alone as the policy instrument and those that have allowed for the endogenous determination of link capacities as well. In the latter case, in the case of competition between governments as shown in Table 2.2, the works listed all utilised so-called "two stage" games i.e. strategic encounters between parties where tolls are chosen in the second stage after capacity had been determined in the first stage. In the case of competition between governments, this thesis will focus exclusively on the development of short run models where tolls are used as the sole instrument of competition.

Secondly, a common theme in the both the studies shown in Table 2.2, with the exception of Laird *et al.* (2007), and the literature on private sector toll road competition, discussed in Table 2.1), is that a *game theoretic framework* is employed to describe the interactions between the agents in each case in setting tolls to maximise some objective function. Thus this observation lends justification to the approach of modelling the strategic interactions of these two groups of *prima facie* unrelated agents (i.e. toll road concessionaires and jurisdictions), exercising control over toll levels in transportation networks, within a common unifying framework that is adopted within this thesis.

Table 2.2: Overview of Literature Relevant to Competition between Governments Distinguished by Nature of Competition, Network Representation and Policy Instruments Alllowed

Reference	Nature of Competition	Network Representation	Policy Instrument(s)
Laird *et al.* (2007)	-	No Network	Tolls only
De Borger *et al.* (2005)	Horizontal	Two Links (Parallel)	Tolls only
De Borger *et al.* (2007, 2008)	Horizontal	Two Links (Serial)	Tolls and Capacity
Ubbels and Verhoef (2008*b*)	Horizontal	Two Links (Serial)	Tolls and Capacity
Proost and Sen (2006)	Vertical	No Network	Tolls and Parking Fees
Gühnemann *et al.* (2011)	Horizontal	General	Tolls only
Zhang *et al.* (2011)	Horizontal	General	Tolls only
Gühnemann *et al.* (2014)	Horizontal and Vertical	General	Tolls only

Though Laird *et al.* (2007) did not specifically study competition between governments *per se*, these authors provided insights to enhance understanding of toll setting behaviour by a jurisdiction implementing toll pricing within a distributed pricing context. For this reason, it is included in the literature review in Table 2.2. Using a strategic land-use and transport interaction model, Laird *et al.* (2007) investigated the welfare implications stemming from the use of toll revenues if the toll pricing scheme proposed for Edinburgh, as discussed above, had been implemented. Laird *et al.* (2007) showed that if CEC (the scheme proponents) could exercise full authority over the disbursements of revenues collected, they would be incentivised to levy a toll up to 4 times higher than if there was additional oversight from a higher level government such as the Scottish Executive. In addition, CEC would be inclined to redistribute the revenues collected to their own citizens (e.g. by reducing local taxation) rather than to invest in transportation that would benefit extra-jurisdictional users as well. Thus while non-Edinburgh resident users contributed to the toll revenues, the benefits would be reaped by the residents in CEC's jurisdiction, emphasising the possibility of CEC exhibiting tax exporting behaviour.

While De Borger *et al.* (2005) considered toll revenue competition in a network consisting of two links in *parallel*, De Borger *et al.* (2007, 2008) studied the same situation but with two links in *series*. In all cases, each link was subject to tolls by a different government. Local traffic circulating within each government's jurisdiction was limited to using the link within each jurisdiction. In addition, these works also allowed for "transit traffic", i.e. traffic passing through the network but with origins and destinations *outside* either jurisdiction. This transit traffic had the option of using either link in the parallel case but had to use both links in the serial case. Thus when traversing the network, transit traffic would interact with local traffic i.e. contribute to and suffer from local congestion. Each government was assumed to be interested in the welfare of their own jurisdiction as well as revenues from tolls (if any) on transit. The authors considered three toll pricing regimes: *a*) differentiated tolls on transit and local traffic, *b*) uniform tolls for transit and local traffic and *c*) no tolls on transit but tolls on local traffic.

In the case of *parallel* links, De Borger *et al.* (2005) concluded that independent (or non-cooperative) setting of tolls on transit by each government was not detrimental to welfare. In particular, the scenario with no tolls on transit but tolls being limited to local traffic fared the worst in welfare terms. In contrast to this, in the *serial* case, De Borger *et al.* (2007, 2008) reported that preventing jurisdictions from individually deciding toll levels on transit would increase welfare. De Borger *et al.* (2007, 2008) attribute this to the double marginalisation problem, discussed previously when private toll road concessionaires compete for toll revenues on serial links (see p. 37). In the instances here, each government would set its toll, ignoring the reduction in revenues to the other government controlling the other link in the series. This would result in higher tolls with a lower network wide welfare vis-á-vis a cooperative toll pricing scheme. This should be contrasted against the substitutability of links in the parallel case, where the opportunity for rerouting available to transit traffic, would have cushioned the ability of individual governments to charge very high tolls.

The negative welfare implications of tax exporting were again highlighted in Ubbels and Verhoef (2008*b*) who studied competition between a regional government and a city government. Each government was responsible for setting the toll (and capacity) of one road each in a two link *serial* network. In doing so, each jurisdiction was assumed to maximise the welfare of its own inhabitants only and the toll revenues collected. The residents of the region had to use both roads in their commute while city residents only needed to use the city's road in their commute. As Ubbels and Verhoef (2008*b*) did not allow for reverse commuting (i.e. residing in the city and working in the region), the set up of the problem they considered was implicitly asymmetric as the region's residents would be subject to tolls by the city but the region was unable to tax export to the city's residents.

Ubbels and Verhoef (2008*b*) simulated exhaustive combinations of possible toll pricing strategies by each jurisdiction including the non-cooperative game as well as situations in which either jurisdiction could act first. In all scenarios, it was shown that the city was incentivised to set tolls higher than the first best level in order to extract revenues from residents of the region using the city's road. While it has been noted that the toll chosen by the jurisdiction is a "complex expression involving the relative share of local and transit traffic, the slope of the cost function and the elasticities of demand" (p. 330, footnote 3, De Borger and Proost, 2008), an analytical expression for the two link case in Ubbels and Verhoef (2008*b*) confirms that beyond internalising the congestion externality (which is expected behaviour of a welfare maximising government), the city further adds a "demand related monopolistic mark-up" (Ubbels and Verhoef, 2008*b*, p. 189). This mark up (shown previously on Fig. 2.3 on p. 47 as the line segment marked `ef`), is due to the city government's desire to maximise the revenues collected from the extra-jurisdictional commuters. In one instance, Ubbels and Verhoef (2008*b*) showed that that the toll for use of the city road was up to 12 times higher compared to the first best benchmark as a result of the demand-related mark up. Note that the cooperative pricing between jurisdictions is equivalent to the Pigouvian first best as there are only two roads in their model but this is no longer true in more general networks.

Ubbels and Verhoef (2008*b*) further concluded that a non-cooperative game between jurisdictions had particularly negative consequences for overall social welfare in this serial setting and could, in fact, be worse than not implementing any toll pricing policies to internalise congestion, resulting in a "Prisoner's Dilemma" (Chapter 3 discusses this game theoretic topic further). This was the consistent message reported in Zhang *et al.* (2011) and Gühnemann *et al.* (2011, 2014) who studied inter-jurisdictional competition in more general networks with multiple OD pairs where each government managed a subset of the network.

The "Stackelberg" game (von Stackelberg, 1934, see also further discussion in Chapter 4) is a model of industrial organisation where a dominant firm (Varian, 2010, p. 499) (henceforth, the Stackelberg leader) is able to gain a profit advantage over its competitors, known as "followers", by being able to act first as a result of its ability to anticipate the followers' reactions to its strategies (Church and Ware, 2000). Ubbels and Verhoef (2008*b*) noted that the overall welfare consequences were relatively similar regardless of whether the region or the city assumed the role of the Stackelberg leader and acted first (i.e. *before* its rival) when setting the toll level or whether they acted simultaneously (i.e. *at the same time* as its rival) when doing so, as conventionally assumed in a non-cooperative game (see Chapter 3).

In contrast to the previous cases, Proost and Sen (2006) investigated the effects of vertical competition i.e. revenue competition between different levels of government. These authors considered an urban government in charge of the city and a regional government *where the city is a part of the region.* Thus while the urban government was interested in the welfare of the urban residents exclusively, the regional government was concerned with the welfare of *both* the commuters (i.e. residents of the region outside the city) and the city residents. This is in contrast to the set up used in Ubbels and Verhoef (2008*b*) who had assumed that the city and region were two distinct, non-overlapping jurisdictions.

In the first set of tests, Proost and Sen assumed that the city exercised control over parking fees which its own inhabitants and commuters from the region had to pay while the regional government controlled a cordon toll payable only by commuters accessing the city. The revenue from the cordon tolls was redistributed to the commuters while the city redistributed revenues from parking charges to its own residents only.

In the non-cooperative game that Proost and Sen tested in a strategic transport model devoid of network representation, the outcome was only marginally worse than the fully cooperative solution. This was because of the jurisdictional overlap as the region took into account the welfare of the city inhabitants as well. Nevertheless, because the city exercised full control of the parking charges which it redistributed to its own residents only, the city continued to have an incentive to tax export by levying high parking fees on commuters.

Proost and Sen (2006) also considered revenue sharing where the region gave the city a share of the cordon toll revenues. In this case, the revenue sharing strategy was shown to reduce the parking charges the city set and thus counteracted the appetite of the city to tax export.

Gühnemann *et al.* (2011) studied competition in a large network setting between two neighbouring jurisdictions (Peak Distict and Sheffield), both located within the environmentally sensitive Transpennine corridor in northern England using a traffic assignment model. In the non-cooperative game between these two jurisdictions, Gühnemann *et al.* (2011) found that the Peak District would act as a tax exporter and was incentivised to levy a high cordon toll to extract revenues from extra-jurisdictional traffic. The ability of the Peak district to do so was due to its strategic location in the centre of the Transpennine corridor such that alternative routes avoiding the toll cordon around the jurisdiction were far more costly in terms of generalised travel time. In a general network context, if extra-jurisdictional users were able to avoid traversing through toll points controlled by a jurisdiction, this would reduce the opportunity for tax exporting. This also explains why De Borger *et al.* (2005) who studied toll competition between jurisdictions in a *parallel* two link setting where possibility for traffic to reroute exists, had concluded that non-cooperative toll pricing was not detrimental to welfare.

Similar conclusions were reached in Gühnemann *et al.* (2014) who, extending the horizontal toll revenue competition model between Peak District and Sheffield developed in Gühnemann *et al.* (2011), added a vertical competition scenario. In Gühnemann *et al.* (2014), the HA, applying distance-based tolls on the strategic motorway and trunk roads, took on the role of the "higher level" government agent taking into account the welfare of the entire network and sharing toll revenues with the Peak District and Sheffield. The tests showed that a coordinated/cooperative pricing strategy would deliver the largest global (i.e. from a network wide perspective) efficiency gains. The authors also reported that preventing Peak District (the jurisdiction that stood to gain most by tax exporting) from introducing tolls would also improve welfare but this would be infeasible because tolls introduced by both Sheffield and the HA would adversely impact the welfare of Peak District.

Similar to the previous authors, Zhang *et al.* (2011) studied distributed toll pricing schemes where toll pricing was implemented within each jurisdictions independently and schemes where jurisdictions cooperatively decided the toll levels. Zhang *et al.* (2011) showed that toll pricing policies introduced by a jurisdiction could have both beneficial as well as detrimental welfare impacts on another jurisdiction. In other words, the welfare impact of toll pricing on other jurisdictions is not necessarily adverse.

Nevertheless, Zhang *et al.* (2011) highlighted that it was difficult to generalise the outcome of non-cooperative policies (i.e. whether they would be beneficial or detrimental) which is dependent on both the network structure and composition of trip origins and destinations. While cooperation among regions in the setting of tolls would improve network wide social welfare, the authors noted that this could mean some jurisdictions being made better off at the expense of others.

Furthermore, Zhang *et al.* (2011) also modelled the situation in which one jurisdiction acted as a Stackelberg leader taking into account the reactions of other jurisdictions when setting their tolls. However, similar to Ubbels and Verhoef (2008*b*) discussed above, they showed that the presence of a Stackelberg leader did not have any material effect on the overall welfare outcome of competition between jurisdictions.

Discussion

It is clear from the review presented thus far that the implementation of toll pricing in one jurisdiction could have negative impacts on the welfare of another. In this way, the jurisdiction negatively affected could "retaliate" by introducing a similar policy in an attempt to alleviate the impacts. However, it is important to note that impacts may not always be adverse and a cooperative pricing scheme could equally result in jurisdiction(s) benefitting at the expense of others.

This literature has consistently highlighted that the incentive for jurisdictions to tax export. In this way, when setting tolls, **jurisdictions pursue two simultaneous objectives**: "improving transport conditions as far as their own residents are concerned and generating profit or tax revenue from through traffic" (De Borger and Proost, 2012, p. 39). To reiterate, the key issue, addressed in this literature and further investigated in this thesis through the case studies in Chapter 8 and 9, revolves around the question whether the welfare improvements achieved by using tolls to (imperfectly) internalise the congestion externality, is outweighed by the welfare decreasing fiscal externality of tax exporting, brought about by the desire to maximise toll revenues from extra-jurisdictional users.

De Borger *et al.* (2008) argue that the serial network structure is more realistic than the parallel network. For example, the Trans-European Networks for Transport (TEN-T) in the EU (EC, 2014) and the Interstate Highway System in the US (FHWA, 2014) are networks comprising mainly *serial* transport corridors. This has important practical implications. As a result of the double marginalisation problem stemming from the desire to maximise revenue from extra-jurisdictional users, overall welfare could be improved if jurisdictions were prevented from introducing tolls (De Borger and Proost, 2012). In other words, any toll pricing policy should be coordinated at a federal/national level. However, this would conflict with supranational bodies such as the EU, with its commitment to subsidiarity (Voorhees, 2005).

It can be seen that these issues are inseparable from the underlying question as to what the appropriate level of government for the governance of toll pricing should be. In other words, whether toll pricing should be instituted at a national level or whether it should be viewed as a local traffic management tool with responsibility devolved to jurisdictions. While local officials are better able to identify transportation needs in their jurisdictions than federal officials (Dilger, 2012), they would neglect inter-jurisdictional spillovers which the federal government would be best placed to adequately account for while ensuring that national interests are met

(Banzhaf and Chupp, 2012). From the literature reviewed above, two opposite ends of the spectrum have been advocated thus far. On the one hand, in Sweden, following the permanent reintroduction of toll pricing in Stockholm, responsibility for the management of the scheme has been transferred to the national level. On the other hand, in the UK, as facilitated by legislation, the approach adopted is one whereby toll revenues are hypothecated or earmarked for meeting *local* needs.

It is well documented in the public economics literature (e.g. Ulbrich, 2011) that central governments are able to both correct inter-jurisdictional spillovers and influence the activities of lower level governments using revenue sharing agreements. For example, in Canada, a country with a federal system of government, the lower level (provincial) governments receive a share of the federal gas tax revenues (Department of Finance, Canada, 2005). In the toll pricing context, revenue sharing is of practical relevance since, in the case of Edinburgh (Gaunt *et al.*, 2006; Laird *et al.*, 2007), CEC had explicitly proposed sharing toll revenues collected from users originating in neighbouring jurisdictions with these authorities.

Though the public economics literature recognises that revenue sharing can be used to minimise inter-jurisdictional spillovers (Ladd, 2005; Brillantes and Tiu Sonco, 2007), there is, however, only limited literature regarding the effects of revenue sharing within a transportation context. While Proost and Sen (2006) showed that revenue sharing did reduce the incentives for tax exporting, it should be noted that these authors focused entirely on vertical competition between two levels of governments with overlapping tax bases. Thus the consequences of revenue sharing, in the case of horizontal competition investigated in this thesis within the more, realistic serial network structure remain to be investigated.

Despite the aforementioned similarities between private revenue maximising concessionaires and tax exporting jurisdictions in their toll setting decisions, there is one crucial difference. While private sector concessionaires intent on engaging in restrictive practices would be legally prohibited from making binding agreements to collude, (national and lower level) governments have ample opportunity to partake in, and conclude binding bilateral or multilateral agreements (e.g. Keohane, 1990). For example, the ten local governments constituting Greater Manchester, UK form the so called Greater Manchester Combined Authority with responsibilities for transportation planning and policy in the region (GMCA, 2014). Thus a limitation of the majority of the literature above is the assumption that governments engaging in toll competition play non-cooperative games (see Chapter 3 for further discussion on Game Theory).

Besides the non-cooperative case, De Borger and Pauwels (2010) have also allowed for jurisdictions to cooperate, through bilateral bargaining, in the setting of tolls (and capacities). These authors show that cooperation could reduce the welfare losses associated with independent uncoordinated toll setting decisions. In this thesis, the welfare impacts of bilateral cooperative agreements between jurisdictions with regard to both revenue sharing and toll pricing will be investigated.

Where competition between governments in general networks have been studied previously (Gühnemann *et al.*, 2011; Zhang *et al.*, 2011; Gühnemann *et al.*, 2014), the resulting toll levels were obtained using grid search methods and heuristic approaches. This suggests the need for the development of solution algorithms that will take into account the route choices of the users of the network as the jurisdictions react to the toll levels chosen by their rivals. This thesis will thus contribute through the development of solution methods that allow for the determination of toll levels when these different governments compete for toll revenues from extra-jurisdictional users while simultaneously maximising welfare of local users.

Several studies reviewed above (Ubbels and Verhoef, 2008*a*; Zhang *et al.*, 2011) have shown that allowing a jurisdiction to play the role of the Stackleberg leader which could act first and decide toll levels before other follower jurisdictions, taking into account the reactions of these followers, does not have a material effect on the overall welfare consequences. For this reason, this thesis will not consider the case of any government acting as a Stackleberg leader and thus assume that the jurisdictions engage on an "equal footing".

2.6 Summary

This chapter began by outlining the argument, advanced by economists (Pigou, 1920; Knight, 1924), for the implementation of toll pricing in traffic networks which is intended to correct a market failure arising from the discrepancy between private and social costs of highway use. This strand of literature has emphasised that tolls are required to ensure that road users take into account the externalities they impose on others when using the road network. However, the first best pricing system that this principle dictates, when generalised to a network context, requires that users are charged a toll for use of all congested roads in a network. Due to the impracticalities associated with this requirement, large portions of literature has focused on the second best toll pricing problem where, amongst other second best issues, only a subset of links are subject to tolls or where links are constrained to form a cordon around a congested city centre.

The classical argument for toll pricing is predicated on the assumption of a single benevolent regulator exercising control over all aspects of toll pricing seeking to maximise welfare for all network users. As a result of both deregulation and the recognition that in reality, transportation networks transcend jurisdictional boundaries, this assumption has been challenged. These observations point to the need to augment the single regulator paradigm with a framework capable of encompassing multiple agents in the execution of toll pricing policies.

As traditional fuel tax based financing of infrastructure are found to be insufficient to satisfy needs for system expansion, private sector participation in the highway transportation sector has increased. This has led to increased interests in private toll roads. The multiple agent framework discussed in this chapter allows for a discussion of competition between toll road concessionaires in highway networks which is substantiated by practical examples of private toll road competition. As stressed in this chapter, the answer to the question of whether social welfare is improved or otherwise as a result of competition between concessionaires pivots on with the interrelationships of the toll roads in competition i.e. whether they serve as substitutes or complements in a journey (Engel *et al.*, 2004; Small and Verhoef, 2007; Mun and Ahn, 2008; van den Berg and Verhoef, 2012; van den Berg, 2013).

Additionally, the multiple agent framework is essential in evaluating outcomes when jurisdictions engage in competition for toll revenues in which case the assumption of benevolence of governments is called into doubt. The literature reviewed in this chapter has argued that in implementing toll pricing strategies, jurisdictions might be concerned only with the welfare of their residents but simply regard extra-jurisdictional users as a source of revenue (e.g De Borger *et al.*, 2005, 2007, 2008; Ubbels and Verhoef, 2008*b*; Laird *et al.*, 2007; Zhang *et al.*, 2011; De Borger and Proost, 2012). The general theme of this strand of literature is that this is because of the incentive of jurisdictions independently implementing a decentralised pricing scheme to tax export. In seeking to maximise revenues from extra-jurisidictional users, it was shown that the behaviour of jurisdictions become indistinguishable from revenue maximising toll road concessionaires when, in setting toll charges, they also levy a mark up taking the toll level beyond that required to (imperfectly) internalise congestion. In a decentralised pricing scheme, it is further argued that strategies for the use of toll revenues which are intended to alleviate equity concerns of toll pricing strategies may in fact, worsen the tax exporting implications. This is because while tolls are paid by all users, local users would most likely be the beneficiaries of use of the toll revenues collected.

With regard to private toll road competition, the first question to be investigated in this thesis is the extent of transferability of policy relevant insights established in the case of networks comprising exclusively of a single OD pair to more realistic network settings with multiple OD pairs. Another policy relevant question is whether toll road operators would engage in collusion and if so, what the policy implications of such collusive behaviour would be.

In the setting of inter-jurisdictional competition for toll revenues, the policy relevant questions investigated in this thesis stem from whether responsibility for toll pricing should be instituted at the national level or whether it should be viewed as a local demand management measure with responsibilities for toll setting devolved to local jurisdictions. While revenue sharing has been used in the public economics literature to allay inter-jurisdictional spillovers, its effects have not been investigated in the transportation context. Hence this research will investigate the welfare impacts of revenue sharing strategies when jurisdictions compete for toll revenues.

Furthermore, while private sector toll road concessionaires are legally prevented with existing anti-trust legislation from making binding collusive agreements, it is often the case that jurisdictions can cooperate in a multitude of ways. Yet this issue has not been investigated extensively within a toll pricing context. Thus, this thesis will study the welfare consequences as a result of jurisdictions engaging in a bilateral cooperative agreements in both toll setting and revenue sharing.

In order to investigate the welfare implications of competition between toll road concessionaires and between jurisdictions, solution methodologies that are able to simultaneously take into account both the decision making behaviour of the multiple agents exercising control over tolls and the route choice decisions of users in a general traffic network, are needed. This thesis thus seeks to develop solution algorithms applicable in such a context.

In concert with the literature reviewed herein and the research gaps identified, the next chapter thus proceeds to draw on elements of game theory to develop a sound behavioural model applicable to the study of interactions between these multiple agents when each is confronted with others implementing toll pricing policies simultaneously.

Chapter 3

Elements of Non-Cooperative Oligopolistic Game Theory

3.1 Introduction

This thesis investigates competition, which requires an understanding of the *interactions*, amongst multiple agents exercising control over toll pricing policies in highway transportation networks. A tool most applicable to the study of such interactions is game theory, which is the "systematic study of how rational agents behave in strategic situations" (Jehle and Reny, 2011, p. 305), and a "game" is any situation in which individuals "interact in a setting of strategic interdependence" (MasCollel *et al.*, 1995, p. 219). The setting of competition between toll road concessionaires and governmental jurisdictions, discussed in this thesis, is characterised by rivalry between a *small* number of these agents. The microeconomic literature terms such settings of "competition among the few" (Intriligator, 2002, p. 205) as "oligopolistic competition". Thus this chapter will also draw extensively from concepts articulated in the economics of industrial organisation.

The aim of this chapter is to outline elements of game theory that support the development of a robust model characterising behaviour of toll road concessionaires and government jurisdictions, when they interact with their rivals, when exercising control over toll setting decisions in a highway network.

While the route choice model describing behaviour of users in a highway network is intrinsically intertwined with the interactions amongst these agents when each independently decides toll levels, this chapter focuses *solely* on the behaviour of these agents. In this way, the agents' behavioural model is, at least temporarily, artificially disentangled from the underlying route choice model. Subsequently, a model integrating both the behaviour of agents using the game theoretic concepts outlined in this chapter and the route choice decisions of users in highway networks, is presented in Chapters 4 and 5 within a unified modelling framework.

3.1.1 Scope of Coverage

As the scope of Game Theory is relatively broad, it is not the intention to utilise every aspect of this rich field in formulating a framework to study interactions between decision makers. The boundaries of the game theoretic models developed in this thesis are demarcated with reference to the classification of games based on the following generic characteristics as summarised in Zagare and Slantchev (2010):

- Rules that govern play: Two broad categories are distinguished here namely Non-Cooperative Games and Cooperative Games. The former explicitly assumes that each participant in a game, referred to as a "player" in game theory parlance, "acts independently, without collaboration or communication with any of the others" (Nash, 1951, p. 286). Cooperative games are games where players are allowed to form coalitions and make binding agreements (Friedman, 1983). This thesis focuses primarily on non-cooperative games. However as will be seen later in this chapter, the non-cooperative outcome may be worse for players than if they could cooperate. Thus the possibilities for cooperation to improve the position of *all* players simultaneously will also be discussed.

- Rewards to players: The rewards to players (known in Game Theory parlance as "payoffs") are either "constant sum" or "variable sum". When the players have diametrically opposed interests, i.e. where a gain to one player immediately implies a loss to the other, a constant sum game is being played (Raghavan, 1994). In such a game, the payoffs of the player sum to a constant. The "zero-sum game" is a special case of a constant sum game which has been normalised such that the payoffs sum to zero (Fudenberg and Tirole, 1991). On the other hand, in a variable sum game, the players have *both* competitive and complementary motives. It has been noted that games "most characteristic of economic life are n-person variable sum games. Two person constant sum games, though much studied by mathematicians working in game theory, have little or no applicability to economics" (Friedman, 1983, p. 210). Thus this thesis will be concerned with variable sum games alone.

- Information available: When the payoffs are "common knowledge", the game is said to be a one of complete information. Adapting from Aumann (1976), information is common knowledge among the players if every player knows it, every player knows that every player knows it, and so on. This thesis assumes throughout that payoffs are common knowledge and players possess all necessary information in order to make rational decisions. In addition, the acquisition of such information is assumed to be free of transaction costs.

- Representation and timing: In strategic games, players make their moves simultaneously while players make moves sequentially in so-called "Extensive Form" games (Hart, 1992). A further distinction is made between "single shot" games and the class of "repeated games". In a single shot game, the strategic encounter is a static "snapshot" of interaction that takes place as a one off while repeated games are those where "a one shot game is played a number of times" (Carmichael, 2005, p. 198) and "players interact more than once" (Carmichael, 2005, p. 198). This thesis will restrict focus to the case of the various decisions makers, in control of highway transportation networks, engaged in single shot strategic games.

3.1.2 Definitions

A strategic game, $\mathcal{G}$, is defined by the triplet $\{\mathcal{N}, X, \phi\}$ where $\mathcal{N}$ is the set of n players $\{1, 2, \ldots, n\}$. $X = X_1 \times X_2 \times \cdots \times X_n$ is the strategy or action space of these n players. If X is a finite set, then $\mathcal{G}$ is a finite game. On the other hand, if X is a subset of $\mathbb{R}^n$ i.e. Euclidean (real) space, then $\mathcal{G}$ is referred to as a continuous game. In general, games employed in economic modelling are continuous games (Fudenberg and Tirole, 1991; Alós-Ferrer and Ania, 2001; Reny, 2008). Thus, except when necessary to emphasise certain key concepts, finite games are not discussed at length in this thesis. The action $x_i \in X_i$ each player takes will be referred to as her strategy and the collective actions of all players constitutes a strategy profile, $\mathbf{x} = (x_1, \ldots, x_i, \ldots, x_n)^\intercal$.

The game theory literature (Osborne and Rubinstein, 1994) distinguishes between strategies and actions. This distinction becomes critical in Extensive Form games which are not discussed in this thesis. Thus the terms "actions" and "strategies" will be used synonymously.

$\boldsymbol{\phi}$ is the set of payoff functions for the players where each element is the individual player's payoff function, ϕ_i, and measures what player i,$i \in \mathcal{N}$, obtains by taking a particular strategy which is dependent on the actions which all other players take. As games where payoffs might depend on the outcomes of exogenous events occurring with some probability (with consequences not known at the time of taking action) are not considered in this thesis, the strategy profiles together with the consequent payoffs are therefore sufficient to determine the outcome of the game.

To focus attention specifically on player i, it will be convenient to use the notation $\mathbf{x}_{-i}$ to emphasise the actions of every player *excluding* player i i.e. $\mathbf{x}_{-i} \triangleq (x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n)^\intercal$. With a slight abuse of notation, the collective action can then be written as $\mathbf{x} \triangleq (x_i, \mathbf{x}_{-i})$. Writing $(x_i, \mathbf{x}_{-i})$ must not be taken to mean that the components of $\mathbf{x}$ are reordered so that actions of player i becomes the first block.

Each player in $\mathcal{G}$ acts obeying the "rationality postulate of non-cooperative behaviour" (Gabay and Moulin, 1980, p. 272) which means that each player seeks to maximise individual payoffs and a larger payoff is always preferred to a smaller one.

The rest of this chapter is organised as follows. To solve a game is to predict the outcome of the game. Thus following this introduction, Nash Equilibrium (NE), the most commonly used solution concept in game theory (Fudenberg and Tirole, 1991; Sethi, 2008) is discussed. Employing the concept of best responses, the determination of NE in a finite game is demonstrated. As noted, this thesis focuses on games where players have infinitely many strategies rather than a finite action set. Thus two models of oligopolistic competition from economics are discussed in

Section 3.3. The first is that of Cournot competition where players (firms) engage in non-cooperative determination of production levels. The second model discussed is Bertrand competition which, by construction, differs from the Cournot model in that players are assumed to compete in setting prices, rather than prices. While the economic models of competition serve the primary purpose of emphasising the interdependencies between decision makers in determining outcomes within a continuous game setting, it will be highlighted that these economic models of competition are not directly applicable in the highway transportation context as both congestion costs and toll prices constitute generalised costs which is the key determinant of users' route choices in highway networks. By influencing the users' routing options, the payoffs of multiple agents are, in turn, affected. However, the application of the NE solution concept can result in paradoxical outcomes even though the players are rational utility maximising economic agents. Games may also have more than one Nash Equilibria and unless players are allowed to randomise over their actions, there may be no Nash Equilibria. These properties of the Nash non-cooperative outcome are discussed in Section 3.4. Section 3.5 summarises.

3.2 Nash Equilibria

In this section, the definition of Nash Equilibrium is introduced. It draws attention to the notion engendered by the term "equilibrium" reflecting a steady state of play in the game where "each player holds the correct expectation about the other players' behaviour and acts rationally" (Osborne and Rubinstein, 1994, p. 14).

Whilst other solution concepts for games are also available, such as iterative elimination of dominated strategies (MasCollel *et al.*, 1995), the NE concept is particularly appealing as it emphasises both to the simultaneity of decisions of players and draws attention to the individual pursuit of self-interest, encapsulating within this single principle, the key characteristics of non-cooperative games (Fudenberg and Tirole, 1991).

Definition 3.1. Nash Equilibrium (NE) (Nash, 1950*a*, 1951):

A combined strategy profile $\mathbf{x}^* = (x_1^*, x_2^*, ..., x_n^*)^\intercal \in X$ is a Nash Equilibrium for the game $\mathcal{G} \triangleq \{\mathcal{N}, X, \boldsymbol{\phi}\}$ if for each player i,$i \in \mathcal{N}$, x_i^* is a best response to the best response actions of all other players, $\mathbf{x}_{-i}^*$ such that Eq. 3–1 is satisfied.

$$\phi_i(x_i^*, \mathbf{x}_{-i}^*) \geq \phi_i(x_i, \mathbf{x}_{-i}^*) \ \forall x_i \in X_i \ , \forall i \in \mathcal{N} \qquad \text{(Eq. 3–1)}$$

Thus the NE is a point of simultaneous best responses where each player is doing her best given what other players are doing.

Corollary 3.1. At an NE, no player can benefit (increase individual payoffs) by unilaterally deviating.

Based on Corollary 3.1, as all players do not have an incentive to change strategies i.e. deviate from their current position, such a point is indeed an equilibrium. Exploiting the fixed point theorem (Kakutani, 1941), Nash (1950*a*) showed that an NE is a fixed point of the so-called "best response functions". These best response functions are defined next. Subsequently, Example 3.1 demonstrates how the NE can be obtained by identifying best responses.

3.2.1 Best Response Function

Definition 3.2. Best Response Function (Shy, 1995, p. 21):

In the game $\mathcal{G} \triangleq \{\mathcal{N}, X, \boldsymbol{\phi}\}$, the best response function of player i is the function $R_i(\mathbf{x}_{-i})$ that, for given actions $\mathbf{x}_{-i}$ of all other players i.e. $\{1, 2, \ldots, i{-}1, i{+}1, \ldots, n\}$, assigns an action $x_i = R_i(\mathbf{x}_{-i}), x_i \in X_i$, that maximises player i's payoff, $\phi_i(x_i, \mathbf{x}_{-i})$.

To be mathematically precise, a distinction should be made between functions and correspondences. By definition, a "correspondence from a domain set X to a range set Y associates each element of the domain set x in X, a non-empty subset of Y,$Q(x)$" (Ali Khan, 2008, p. 270). On the other hand, "a function is a correspondence such that $Q(x)$ is a *singleton* for each x in X" (Ali Khan, 2008, p. 270, italics added). In other words, the function mapping is unique but a correspondence does not assume uniqueness. Thus, the phrase "best response function", implicitly assumes that the best response is unique. While this phrase is retained in this thesis since it is frequently encountered in the literature, it should be noted that this should be better referred to as the "best response correspondence" since, as will be discussed (see Section 3.4.3 below), uniqueness is, by no means, guaranteed.

Example 3.1. Applying Best Response to identify NE in a Finite Game

In the game shown in Game 3.1 adapted from Gibbons (1992), both Players I and II have three actions each. This will be indicated as $X_i, i \in \{\mathrm{I}, \mathrm{II}\}$ where strategy set X_{I} are the actions {"Top", "Middle", "Bottom"} abbreviated T, M, B respectively. Similarly, X_{II} is the strategy set comprising actions {"Left", "Centre", "Right"}, abbreviated L,C,R respectively. This is an example of a finite game with reference to the finite or countable action space of each player. Following established convention in game theory literature (Fudenberg and Tirole, 1991; Gibbons, 1992), the first value in each cell shows the payoff that Player I (the "row" player) would get if he plays a strategy in the corresponding row. The second value in each cells shows the payoff that Player II (the "column" player) would get if he plays a strategy in the corresponding column. For example, if Player I plays T and II plays strategy L then I gets is 0 and II gets 4. Similarly, if I plays strategy B and II plays C, then I's payoff is 3 and II gets 5.

		Player II		
		L	C	R
	T	$0,4$	$4,0$	$5,3$
Player I	M	$4,0$	$0,4$	$5,3$
	B	$3,5$	$3,5$	$6,6$

GAME 3.1: Example of Matrix Game (Gibbons, 1992, p. 7)

This process of identifying the NE of Game 3.1 proceeds as follows. Firstly, for each strategy of Player I, determine the optimal action that II should take to maximise her payoff. Such an action would be II's best response satisfying Definition 3.2. Secondly, determine the best response that I should take for each strategy of II. In more general games, these steps need to be repeated until all best response actions to all actions of competitors have been identified. The intersection of these best responses is the NE as this occurs when the action of a player is the best response to the action of the other following Definition 3.1.

To illustrate the first step, suppose Player I plays T, then the best strategy for II is to play L as evident from the first line of Game 3.1. This is because Player II can get a payoff of 4 with action L but a lower payoff with any other. For concreteness, consider all other alternatives open to II exhaustively: if II plays C, the payoff to II would be 0 and playing R leads to a payoff of 3. Based on this line of reasoning, the best response of II to each and every strategy i.e. {T,M,B} of player I, is summarised in the left hand panel of Table 3.1. Similarly, the right hand panel of Table 3.1 shows the best response of I to a given strategy of II.

TABLE 3.1: (Left)Best Response of Player II to given strategy of Player I in Game 3.1 (Right) Best Response of Player I to given strategy of Player II in Game 3.1

Strategy of Player I	Best Response of Player II
T	L
M	C
B	R

Strategy of Player II	Best Response of Player I
L	M
C	T
R	B

In the payoff matrix in Game 3.2 (which reproduces Game 3.1), these various best responses are underlined. The only cell where both players' payoffs are simultaneously underlined is cell {B,R} with payoff to each of 6. This is the NE of the game since both players' actions are simultaneously best responses to each other.

		Player II		
		L	C	R
	T	$0, \underline{4}$	$\underline{4}, 0$	$5, 3$
Player I	M	$\underline{4}, 0$	$0, \underline{4}$	$5, 3$
	B	$3, 5$	$3, 5$	$\underline{6}, \underline{6}$

GAME 3.2: Payoff Matrix with Best Response of each Player to opponent's strategy underlined (Gibbons, 1992, p: 10)

This section has demonstrated that the process of determining best response actions can be used to identify NE and thus solve a finite game. In the next section, game theory applications from economics are introduced. The primary difference is that in the economic applications, the strategy spaces of the players are more usually subsets of the real line (Alós-Ferrer and Ania, 2001; Reny, 2008) rather than discrete action spaces that was used in Example 3.1. Nevertheless, the principle that the NE is the simultaneous best response of all players remains and that at an NE, no player should have an incentive to unilaterally deviate from their chosen strategies.

3.3 Economic Models of Competition

This section presents two basic models of oligopolistic competition from microeconomic theories of industrial organisation. The first model is that of Cournot (Cournot, 1838) competition where firms engage in competition to determine the quantity of production (i.e. output levels) each should make in order to maximise profits. The second model is that of Bertrand competition (Bertrand, 1883) in which profit maximising firms compete in prices instead.

3.3.1 Cournot Quantity Competition Model

The Cournot (Cournot, 1838, Ch. 7) model described in this section is a model of competition between oligopolistic firms (the players in the game) where each individually decides on the quantity offered for sale for a homogeneous good to maximise profits. Each firm is assumed to be unaware of the output levels of the good that its competitors are producing and any communication between firms is prohibited.

For consistency, throughout this thesis, x_i is used to denote the strategic variable of player i,$i \in \mathcal{N}$. Since the Cournot game is a game in quantities, let x_i be the quantity of the good produced by firm i. The payoff in this case is given by a profit function, which for firm i,$i \in \mathcal{N}$ gives the difference between the revenue earned from the sale of the good in question and the costs incurred in its production. The interdependency between the firms arises since the revenue each earns will depend, via the inverse demand function, on the output of the other firms. In this way the profit/payoff function for this firm can be written as shown in Eq. 3–2,

$$\phi_i(x_i, \mathbf{x}_{-i}) = \underbrace{P(Q)x_i}_{\text{Revenue}} - \underbrace{C_i(x_i)}_{\text{Costs}}, \tag{Eq. 3–2}$$

where $Q = x_i + \sum_{j \in \mathcal{N}, j \neq i} x_j$ is total industry output, $P(Q)$ is the inverse demand function giving price as a function of total industry output, Q, and $C_i(x_i)$ is the production cost function.

Since other firms in the industry will also be choosing output levels simultaneously, a Cournot game ensues and the objective is to determine the profit maximising output levels for each firm. The resulting vector of production levels is known as a Cournot-Nash Equilibrium. A critical assumption of the Cournot model is the underlying "conjectural variation" which is how a firm believes its rival(s) will react to its strategy choice. This is discussed next.

3.3.2 Conjectural Variation

In Chapter 4 (see Section 4.5), more general first order conditions explicitly allowing for the case when a firm's production level can be zero ($x_i = 0$), resulting in a complementarity problem formulation (Karamardian, 1971), will be discussed. However, for simplicity, the current exposition restricts attention to the case when the production of firm i,$i \in \mathcal{N}$ is strictly positive (i.e. $x_i > 0$) in the resulting NE. With this assumption, the first order condition for firm i's profit maximisation problem in Eq. 3–2 can be written as Eq. 3–3,

$$\frac{\partial \phi_i}{\partial x_i} = P(Q) + \frac{\partial P(Q)}{\partial Q} \boxed{\frac{\partial Q}{\partial x_i}} x_i - \frac{\partial C_i(x_i)}{\partial x_i}. \qquad \text{(Eq. 3–3)}$$

From the definition of Q, the boxed term on the RHS of Eq. 3–3 can be decomposed into Eq. 3–4,

$$\frac{\partial Q}{\partial x_i} = 1 + \frac{\partial \left(\sum_{j \in N, j \neq i} x_j \right)}{\partial x_i}. \qquad \text{(Eq. 3–4)}$$

The partial derivative term on the RHS of Eq. 3–4 is the "conjectural variation" (Intriligator, 2002; Estrin *et al.*, 2008). In economic models of competition, firm i has to make an assumption or "conjecture" about how its competitors will react to its change in output. Specifically, the Cournot model assumes that the firm's competitors will *not* react to its changes in output and thus the conjectural variation is 0. Though this assumption has been criticised as being naive (Kamien and Schwartz, 1983), it is consistent with basic economic models of competition and also underlies much modelling applications in the highway transportation context (e.g. Williams and Abdulaal, 1993; Yang *et al.*, 2009). Thus the models of competition between interacting decision makers formulated in this thesis will likewise assume that the conjectural variation is zero. Example 3.2 demonstrates how the Cournot-Nash Equilibrium can be determined in such a game setting.

Example 3.2. Cournot Duopoly (Estrin *et al.*, 2008)

This example assumes that the oligopolistic market under consideration is a duopoly i.e. one where two firms exclusively supply the entire market. Thus $\mathcal{N} = \{1, 2\}$. In this market, the inverse demand function adopts a linear form explicitly given by $P(Q) = 100 - Q = 100 - (x_1 + x_2)$. Each firm has cost function written as $C_i(x_i) = 10x_i, i \in \{1, 2\}$ i.e. fixed costs are zero. Following Eq. 3–2 above, an explicit expression for Firm 1's profit function is shown in Eq. 3–5.

$$\begin{aligned} \phi_1(x_1, x_2) &= P(Q)x_1 - C_1(x_1) \\ &= (100 - x_1 - x_2)\, x_1 - 10x_1 \\ &= 90x_1 - x_1^2 - x_1x_2 \end{aligned} \tag{Eq. 3–5}$$

Firm 2's profit function, obtained in a similar fashion, is shown in Eq. 3–6.

$$\phi_2(x_1, x_2) = 90x_2 - x_2^2 - x_1x_2 \tag{Eq. 3–6}$$

Since it is assumed that the conjectural variation is zero, then by applying the first order conditions for a profit maxima for each producer in Eq. 3–3 and Eq. 3–4, the system of equations is given by each sub-equation in Eq. 3–7

$$\frac{\partial \phi_1}{\partial x_1} = 90 - 2x_1 - x_2 = 0 \tag{Eq. 3–7a}$$

$$\frac{\partial \phi_2}{\partial x_2} = 90 - 2x_2 - x_1 = 0 \tag{Eq. 3–7b}$$

By rearranging each sub-equation in Eq. 3–7, the best response functions, $R_i(\mathbf{x}_{-i})$, for each player can be obtained as shown in Eq. 3–8. As noted in Definition 3.2, these best response correspondences, also known as reaction functions, identify the optimal action for each player to take to maximise individual profit, given the action of its rival.

$$R_1(x_2) \to x_1 = 45 - \frac{x_2}{2} \tag{Eq. 3–8a}$$

$$R_2(x_1) \to x_2 = 45 - \frac{x_1}{2} \tag{Eq. 3–8b}$$

There are two ways to identify the NE (more specifically, a Cournot-Nash Equilibrium) in this game. The first is analytical and proceeds by solving the system of first

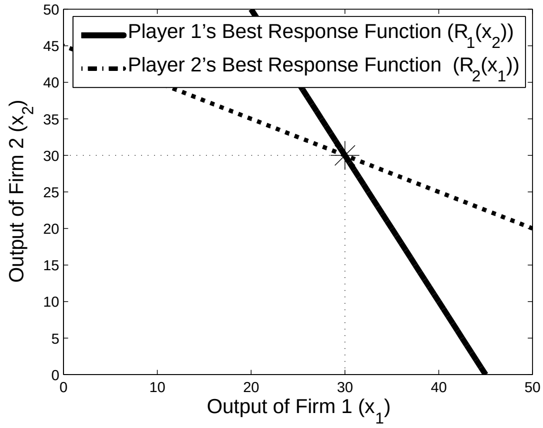

FIGURE 3.1: Producer 1's Best Response Function to strategies of Producer 2, $R_1(x_2)$ shown by continuous line. Producer 2's Best Response Function to strategies of Producer 1, $R_2(x_1)$ shown by broken line. $*$ indicates intersection at Cournot-Nash Equilibrium quantities of 30 units each.

order conditions in Eq. 3–7 simultaneously. Note that the second order conditions also have to be verified to ascertain that the solution vector thus determined indeed corresponds to a profit maximum for each producer. An alternative approach to determine the NE is to graph the best response functions. In Fig. 3.1, the continuous line is the best response function for firm 1 i.e. $R_1(x_2)$. On the other hand, the broken line is the equivalent function for firm 2 i.e. $R_2(x_1)$. The intersection of these best response functions is the NE of the game as this intersection represents the point where the best responses of the players coincide.

Applying either method, the NE of $x_1^* = x_2^* = 30$ (Estrin *et al.*, 2008, p. 320) is obtained. This is highlighted on Fig. 3.1 with an asterisk, $*$. Substitution of the NE quantities into Eq. 3–7 gives profit to each firm of £900.

3.3.3 Bertrand Price Competition Model

The next economic model of competition discussed is the Bertrand model (Bertrand, 1883). In contrast to Cournot model of interdependent profit maximising firms engaging in quantity competition, the Bertrand model is characterised by oligopolists competing in *prices* instead. Such a model will be demonstrated through two examples involving duopolists presented next. For consistency, throughout this thesis, x_i

is used to denote the strategic variable of player i, $i \in \mathcal{N}$. Thus in these examples, note that $x_i, i \in \{1, 2\}$ now represents the *price* chosen by firm i.

Example 3.3. Bertrand Duopoly with Perfect Substitutes (Church and Ware, 2000)

Consider the case of price competition in a setting of firms supplying perfect substitutes. There are two firms in this competitive price setting duopoly and assume that neither firm faces a capacity constraint. There are no fixed costs and the per unit costs of production are equal and written as $C_i = C_j = C, i \in 1, 2, i \neq j$.

As the products are perfect substitutes, it is postulated that buyers will purchase from the firm offering the lower of the two prices. If firm 1 charges a higher price than firm 2 (i.e. $x_i > x_j$), then no consumer buys from firm 1 and firm 2 captures the entire market. Similarly, if firm 2 charges a higher price than firm 1 (i.e. $x_j > x_i$), firm 2 makes no sales and firm 1 captures the entire market. If both firms charge the same price (i.e. $x_i = x_j$), then the demand is split equally between the two firms. The foregoing discussion can be summarised in Eq. 3–9 with $d(\cdot)$ being the *market* demand function and the demand faced by firm i is $d_i(\cdot)$.

$$d_i(x_i, x_j) = \begin{cases} 0, \text{if } x_i > x_j, i \neq j \\ d(x)/2, \text{if } x_i = x_j = x \\ d(x_i), \text{if } x_i < x_j, i \neq j \end{cases} \tag{Eq. 3–9}$$

However, Eq. 3–9 also means that the best reply correspondences are discontinuous and standard calculus techniques cannot be used to identify the NE (Serrano and Feldman, 2013, p. 235). Instead of applying calculus, the NE (more precisely the Bertrand-Nash Equilibrium) can be deduced by exhaustively considering the following six possible cases and identifying whether there exists an incentive, in each case, to profitably deviate or otherwise (Church and Ware, 2000, p. 257):

1. $x_1 > x_2 > C$: This outcome cannot be a NE because firm 1 could profit by lowering its price, x_1, infinitesimally below x_2 to capture the entire market.

2. $x_2 > x_1 > C$: Similar to Case 1 above, this outcome cannot be a NE because firm 2 could profit by lowering its price, x_2, infinitesimally below x_1 to capture the entire market.

3. $x_1 > x_2 = C$: This outcome cannot be a NE because Firm 2 makes zero profit.

Firm 2 could profitably deviate by increasing its price x_2 infinitesimally above C but still remaining below x_1 and thus capturing the entire market.

4. $x_2 > x_1 = C$: Similar to Case 3 above, this outcome cannot be a NE because Firm 1 makes zero profit. Firm 1 could profitably deviate by increasing its price, x_1 infinitesimally above C but still remaining below x_2 and thus capturing the entire market.

5. $x_1 = x_2 > C$: This outcome cannot be a NE because either firm could profit by lowering its price infinitesimally below the other to capture the entire market.

6. $x_1 = x_2 = C$: This outcome is the only NE as neither firm can profitably deviate (cf. Corollary 3.1). If either attempts a lower price, each would capture the entire market but incur a loss. On the other hand, by increasing its price above per unit production costs, they would earn zero since all consumers would buy from its competitor.

Therefore, in the Bertrand model, it can be seen that there is a tendency of each firm to undercut the other (as noted in Case 5) until price is equal to the per unit cost of production (i.e. marginal cost) and the two firms share the market equally. Note that price cannot be lower than marginal costs for in that case, both firms would be better off shutting down than making a loss. Thus Bertrand competition between perfect substitutes results in prices that are equal to the marginal cost of production, obtaining the exact same outcome as in the case of a perfectly competitive market. This effect has been termed the "Bertrand Paradox" as two firms competing in prices are sufficient for the perfectly competitive outcome and profits above the cost of capital are entirely eroded (Shy, 1995; Church and Ware, 2000).

The next example considers the more realistic setting of differentiated products which are imperfect substitutes i.e. products are similar but not identical. In this case, even with equal per unit production costs, the Bertrand paradox does not hold.

Example 3.4. Bertrand Duopoly with Differentiated Products (Serrano and Feldman, 2013)

As the products are differentiated, they are imperfect substitutes. Unlike the case with homogeneous products assumed in Example 3.3, the demand facing a firm does not fall to zero when a firm charges a different price from its rival(s). The demand facing firm 1 can be expressed as Eq. 3–10.

$$Q_1 = 50 - x_1 + \frac{x_2}{2} \tag{Eq. 3–10}$$

Analogously, the demand facing firm 2 can be expressed as Eq. 3–11.

$$Q_2 = 50 - x_2 + \frac{x_1}{2} \tag{Eq. 3–11}$$

On the cost side, the per unit production costs are assumed to be equal for both firms. Thus the total cost function for firm i is $C_i(Q_i) = 25Q_i$, $i \in \{1, 2\}$.

The profit function for firm 1 is the difference between revenues and costs. By substituting into the profit function, the expression for Q_1 from Eq. 3–10, the profit function is analytically obtained as shown in Eq. 3–12.

$$\begin{aligned} \phi_1(x_1, x_2) &= x_1(Q_1) - 25(Q_1) \\ &= x_1(50 - x_1 + \frac{x_2}{2}) - 25(50 - x_1 + \frac{x_2}{2}) \\ &= 75x_1 - x_1^2 + \frac{x_1 x_2}{2} - \frac{25x_2}{2} - 1250 \end{aligned} \tag{Eq. 3–12}$$

Analogously, using the expression for Q_2 from Eq. 3–11, the profit function for firm 2 can be written as Eq. 3–13.

$$\begin{aligned} \phi_2(x_1, x_2) &= x_2(Q_2) - 25(Q_2) \\ &= 75x_2 - x_2^2 + \frac{x_2 x_1}{2} - \frac{25x_1}{2} - 1250 \end{aligned} \tag{Eq. 3–13}$$

The assumption of zero conjectural variation is likewise applicable in the Bertrand model. In this case, each firm takes the price its rival sets as fixed when setting its price. In this way, the first order conditions for a profit maximum for each firm are given by Eq. 3–14.

$$\frac{\partial \phi_1}{\partial x_1} = 75 - 2x_1 + \frac{x_2}{2} = 0 \tag{Eq. 3–14a}$$

$$\frac{\partial \phi_2}{\partial x_2} = 75 - 2x_1 + \frac{x_1}{2} = 0 \tag{Eq. 3–14b}$$

By rearranging these first order conditions, the best response functions of each firm to its competitor's prices is shown in the sub-equations in Eq. 3–15.

$$R_1(x_2) \rightarrow x_1 = \frac{75}{2} + \frac{x_2}{4} \tag{Eq. 3–15a}$$

$$R_2(x_1) \rightarrow x_2 = \frac{75}{2} + \frac{x_1}{4} \tag{Eq. 3–15b}$$

The Bertrand-Nash Equilibrium can then be obtained by the simultaneous solution of the system of equations in Eq. 3–15 giving prices of £50 each (Serrano and Feldman, 2013, p. 236). Note that the second order conditions also have to be verified to ascertain that the solution vector thus determined corresponds to a profit maximum for each producer.

However, it should be pointed out that neither the Cournot nor the Bertrand models are directly applicable to the study competition in general transportation networks due to the unique features of transportation networks. While tolls are equivalent to prices, the difference is that tolls are only one component of overall OD generalised costs that ultimately determine route choices of the users. This is further elaborated in Chapter 4 (see Section 4.2). Furthermore, an assumption of the economic models is that the good in question is sold in a single market. The closest analogue of the "market" in the highway transportation context would, arguably, be an OD pair (Verhoef *et al.*, 2010), and in general transportation networks, the traffic on a road is composed of the flow between multiple OD pairs (Ortuzar and Willumsen, 1994). Thus when different toll road concessionaires engage in competition for revenues, they would be implicitly serving multiple markets simultaneously. Nevertheless, the models articulated here, rooted in the industrial economics literature, serve to underscore the *interdependencies between players* since the payoff attainable by a player is inherently dependent on the strategies chosen by its rivals.

3.4 Properties of Nash Equilibria

The game theory literature (e.g. Myerson, 1991; Fudenberg and Tirole, 1991) notes that a game may possess one or more of the following properties:

1. There may be no NE when players are limited to "pure" strategies, or
2. the NE may be (Pareto) inefficient, or
3. there may be multiple NE.

These three properties are discussed in turn in this section.

3.4.1 Absence of NE in Pure Strategies

In some games, players may choose probability distributions over the set of actions available to them. When they do so, they are said to be playing "mixed strategies". At the opposite end, if players do not engage in any randomisation, they are playing "pure strategies". It is thus convenient to view mixed strategies "as random variables whose values are pure strategies" (Kurz, 1994, p. 1170) since any pure strategy can be represented as "the mixed strategy that puts zero probability on all of the player's other pure strategies" (Gibbons, 1992, p. 33). While Nash (1950*a*, 1951) proved that NE will always exist in a finite game when certain technical conditions are satisfied, this does not preclude the possibility that the NE may exist in mixed strategies and that they may be no NE when only pure strategies are allowed.

However, it is "hard to motivate games with mixed actions in economic modelling" (Shy, 1995, p. 34). Some commentators go as far as to suggest that mixed strategies "have been given much more attention by game theorists than is justified by any resulting illumination of human behaviour"(Bowles, 2004, p. 34). In any case, consistent with assumptions made in several microeconomic applications of game theory (e.g. MasCollel *et al.*, 1995, p. 389), this thesis will also restrict consideration to cases where decision makers *do not* randomise over the (continuous) strategy spaces and only play pure strategies. It should be highlighted that this restriction did not prevent the identification of NE in the games to be discussed in the case studies presented in Chapters 6 to 9.

3.4.2 Pareto Inefficiency of NE

Being the outcome of a non-cooperative game, the NE outcome maximises each individual player's payoff. However, this can lead to outcomes that are inefficient "in the sense that there exist alternative outcomes that are both feasible and preferred by *all* the players" (Sethi, 2008, p. 375, italics added). Such an alternative is known as a Pareto Optimal outcome as formally stated in Definition 3.3.

Definition 3.3. Pareto Efficiency (Shy, 1995, p. 22):

In a game $\mathcal{G} \triangleq \{\mathcal{N}, X, \boldsymbol{\phi}\}$, let $\mathbf{a}$,$\mathbf{b} \in X$ be two feasible strategy profiles.

1. Strategy profile **a** Pareto Dominates strategy **b** if the following conditions are satisfied:

   (a) For every player i, $\phi_i(\mathbf{a}) \geqslant \phi_i(\mathbf{b})$, $i \in \mathcal{N}$ **and**

   (b) there exists at least one player i such that condition 1a holds with strict inequality i.e. $\phi_i(\mathbf{a}) > \phi_i(\mathbf{b})$, $i \in \mathcal{N}$

2. Strategy profile **a** is Pareto Efficient (also called Pareto Optimal) if there does **not** exist any strategy which Pareto Dominates **a**.

3. Strategy profiles **a** and **b** are called Pareto noncomparable if for some player i, $\phi_i(\mathbf{a}) > \phi_i(\mathbf{b})$ but for some other player j, $j \neq i$, $\phi_j(\mathbf{a}) < \phi_j(\mathbf{b})$

An outcome of a game is Pareto Optimal if it satisfies Definition 3.3. This means that the payoffs of all players are simultaneously maximised such that no player can be made better off without making another worse off. These solutions serve as a benchmark against the NE outcome. As will be shown later (see Example 3.6), the Pareto Optimal outcome may not be unique.

It should be highlighted that the zero-sum game is an example of a non-cooperative game in which the NE is also Pareto Optimal. This is because the zero-sum game is a pure conflict game where the interests of the players are diametrically opposed. On the other hand, as noted in the introduction to this chapter, in most strategic situations encountered in practice, elements of common interests as well as conflict aspects are simultaneously present and are not mutually exclusive (Schelling, 1980; Bowles, 2004; Zagare and Slantchev, 2010).

As a result of players focusing solely on the pursuit of self-interests/personal gain in non-cooperative games, common interests could be sacrificed, thereby resulting in an outcome that is not Pareto Optimal. This section uses two example games to discuss such an outcome. Example 3.5 demonstrates this in the finite game setting of the Prisoner's Dilemma, whilst Example 3.6 shows this in the case of the Cournot game based on the parameters introduced previously in Example 3.2.

Example 3.5. Prisoner's Dilemma (Axelrod, 1984)

In this game, two suspects are interviewed by the police in relation to a major crime. They are interviewed separately (e.g. kept in separate cells throughout) and each is assumed not to know how the other is responding. Each suspect can either confess to the crime or deny their involvement. If neither confess, the police are unable to convict them on the major crime but can still charge them for a lesser offence carrying a lighter penalty. If just one of them confesses to the major crime, they can both be convicted. The dilemma arises because if one of the suspects confesses, but the other one does not, the one confessing will be freed as a reward for acting as an informer. The payoff matrix (see Game 3.3) shows the years in prison each has to serve as a result of their joint decisions and thus a lower number of years in prison served corresponds to a higher payoff.

		Player II	
		Deny	Confess
Player I	Deny	$1, 1$	$10, \underline{0}$
	Confess	$\underline{0}, 10$	$\underline{5}, \underline{5}$

GAME 3.3: Payoff Matrix for Prisoner's Dilemma (Carmichael, 2005, p. 59)

Applying the principle of best response to Game 3.3, several deductions can be made. If Player II denies, the best response for I is to confess. If II confesses, the best response for I is to confess. Similarly, the best response for I is to confess. As was done in Example 3.1, these best responses are underlined in Game 3.3. The only cell that is underlined twice is the cell {Confess,Confess} where each player's strategy is the best response to that of the other and thus is a NE satisfying Definition 3.1. However, it is obvious that this outcome is not Pareto Efficient according to Definition 3.3 since they both could have lower sentences (or higher payoffs) if they had both denied the crime.

Example 3.6. Profits from Collusion (Friedman, 1983; Peters, 2008)

Suppose the Cournot producers of Example 3.2 were given the opportunity to communicate and decide on output levels so that they control the entire market. Thus instead of acting independently/non-cooperatively, it is now assumed that they formed a cartel to maximise joint profits. The profit function for this cartel is given by Eq. 3–16,

$$\begin{aligned} \phi_C(x_1, x_2) &= \phi_1(x_1, x_2) + \phi_2(x_1, x_2), \\ &= 90x_1 - x_1^2 - 2x_1x_2 + 90x_2 - x_2^2, \end{aligned} \qquad \text{(Eq. 3–16)}$$

where the profit functions, $\phi_1(x_1, x_2)$ and $\phi_2(x_1, x_2)$, are exactly as used previously in Eq. 3–5 and Eq. 3–6. The first order conditions for a profit maximum of this cartel are shown in Eq. 3–17,

$$\frac{\partial \phi_{\mathrm{C}}}{\partial x_1} = 90 - 2x_1 - 2x_2 = 0, \qquad \text{(Eq. 3–17a)}$$

$$\frac{\partial \phi_{\mathrm{C}}}{\partial x_2} = 90 - 2x_1 - 2x_2 = 0. \qquad \text{(Eq. 3–17b)}$$

Taken together, these first order conditions imply that as long as the total output of the cartel is 45 units (vis-á-vis 60 in total in competition), joint profit would be maximised. At the total output level of 45, the price will be £55 and *total* costs are £450 (since individual production costs are equal), the joint profit would be £2025. However, the *distribution* of this joint profit between the two producers cannot be uniquely determined but should satisfy Eq. 3–18.

$$\phi_1(x_1, x_2) + \phi_2(x_1, x_2) = 2025 \qquad \text{(Eq. 3–18)}$$

Eq. 3–18 defines the Profit Possibility Frontier, also known as the "Pareto Front" which identifies, for a given market, "the profit possibilities technically attainable" (Friedman, 1983, p. 24). This front is shown by the continuous line defined by Eq. 3–18 on each panel of Fig. 3.2. A more detailed discussion on general methods to identify Pareto Fronts is presented in Chapter 5 (see Section 5.4).

If the two firms were allowed to negotiate, a possibility explicitly prohibited under the assumptions of non-cooperative game theory, then the share of individual profits would lie on this Pareto Front. For concreteness, suppose each producer agreed to produce half of the total joint profit maximising output of 45 units or 22.5 units each, (which is not unreasonable since each has equal per unit production costs), then the profits each would obtain would be £1012.5. This point is marked with an asterisk $*$ and lies on the Pareto Front. Recall from Example 3.2, the profits obtained with the Cournot-Nash solution were £900 each (this is indicated on each panel of Fig. 3.2 with a $+$).

It is clear from both panels of Fig. 3.2 that the Cournot-Nash solution is not Pareto Optimal. This is because the Cournot-Nash solution lies in the interior of the Pareto Front. Another way to see this as follows. Holding the profit from firm 1 fixed at the Cournot-Nash profit of £900, one can move upwards (in the direction of the arrow towards the Pareto Front) and hence increase the profit of firm 2, *without* reducing the profit accruing to firm 1. This example once again shows the Pareto inefficiency of the Cournot-Nash outcome as the joint profit outcome Pareto Dominates it following Definition 3.3.

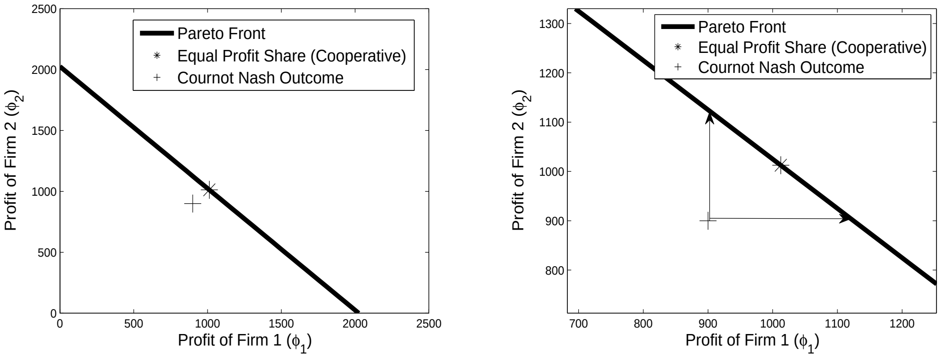

FIGURE 3.2: (Left): The Pareto Front (continuous line), Profits with Cournot-Nash Solution (+), Joint Profit Maximisation (∗) (Right): The Cournot-Nash Solution is *Not* Pareto Optimal

3.4.3 Multiple NE

The proofs of existence of NE (for finite games (Nash, 1950*a*, 1951) and for continuous games (Glicksberg, 1952)) do not rule out the possibility of non-uniqueness. In this section, an example demonstrates the existence of multiple NE in a finite game. Multiple NE in continuous games are a distinct possibility when the best response functions are no longer singletons but are instead, multivalued maps or correspondences. This could arise as a result of a player's payoff function exhibiting multiple maxima (Son and Baldick, 2004). Such non-uniqueness will be shown in the case studies presented in Chapters 6 to 9.

Example 3.7. Multiple NE in Finite Game

In Game 3.4, applying the definition of best responses results in two best responses depending on the strategy of the other. I's best response is to play strategy A if II chooses strategy A and to play strategy B if II chooses strategy B. The same can be seen from the perspective of II. There are two intersections of the best responses and therefore two pure strategy NE.

		Player II A	Player II B
Player I	A	$1, 2$	$0, 0$
Player I	B	$0, 0$	$2, 1$

GAME 3.4: A Game with Multiple NE (Fudenberg and Tirole, 1991, p. 19)

As noted above, the definition of the best response function given in Definition 3.2 implicitly assumes that players are either able to locate the global best response or that the payoff functions are unimodal (i.e. possess a single global maximum). However, in practical applications, it cannot be assumed that these payoff functions are indeed unimodal for given strategies of rivals. In such cases, each player could instead potentially locate a "*local maximum* of his payoff function given the strategies of other players" (Alós-Ferrer and Ania, 2001, p. 167, italics added). Following Son and Baldick (2004), when payoff functions exhibit multiple maxima, a distinction has to be made between *Local* Nash Equilibria (abbreviated LNE) and Nash Equilibria.

Definition 3.4. Local Nash Equilibrium (LNE) (Son and Baldick, 2004, Definition 2, p. 306):

In a game $\mathcal{G} \triangleq \{\mathcal{N}, X, \boldsymbol{\phi}\}$, a combined strategy profile $\mathbf{x}^* = (x_1^*, x_2^*, \ldots, x_n^*)^\intercal \in X$ is a local NE if there exists some $\epsilon > 0$ such that Eq. 3–19 is satisfied,

$$\phi_i(x_i^*, \mathbf{x}_{-i}^*) \geq \phi_i(x_i, \mathbf{x}_{-i}^*) \ \forall x_i \in B_i^\epsilon(x_i^*) \ , \forall i \in \mathcal{N} \qquad \text{(Eq. 3–19)}$$

where $B_i^\epsilon(\hat{x}_i) = \{x_i \in X_i \mid \|x_i - \hat{x}_i\| < \epsilon\}$.

Clearly, an NE that satisfies Definition 3.1 is also a local NE satisfying Definition 3.4. However, the reverse is not true. While LNE points satisfy Eq. 3–19 within a *local neighbourhood* (defined by $B_i^\epsilon(\hat{x}_i)$) of a given solution, they do not satisfy Definition 3.1 when viewed over the *entire strategy space* X. Thus, a distinction is made between LNE and NE solutions with the LNE being a "weaker" condition. The possibility of such LNE solutions in continuous games will be shown in the case studies. To give emphasis to the fact that the outcome satisfying Definition 3.1 *over the entire strategy space*, this outcome is often referred to in this thesis as the "global NE".

3.5 Summary

This chapter sought to outline a behavioural model, developed from game theoretic principles, of how a rational agent would act to maximise individual payoff, in the face of rivals doing the same simultaneously. Abstracting elements from the vast framework of non-cooperative game theory, the Nash Equilibrium (NE) principle is highlighted as the relevant solution concept of such strategic interactions. At the NE, players have no incentive to select an alternative strategy as they cannot improve their payoff unilaterally, and thus this point is an equilibrium.

An example of determination of NE in a finite game was used to emphasise the concept of best responses that underlie the NE. In extending this concept to continuous games, where the strategy spaces of players are subsets of the real number line, which this thesis focuses on, two examples of competition between oligopolistic producers were highlighted. The first example of Cournot competition focused on competition when players choose output levels whilst in the second example of Bertrand competition, price was the strategic variable. In both cases, it was shown that the NE is the intersection of the best response functions as at that point, every player is doing the best given the strategies chosen by their rivals. In these simple models, the intersection could be obtained by the solution of a set of simultaneous equations representing the best response functions.

At the same time, it was emphasised that economic models of competition are not directly applicable to transportation networks as route choices of users which influence the payoffs of decision makers are influenced not by tolls alone but by the generalised costs of travel where tolls are one component and congestion costs are yet another. However, the economic models serve to highlight the strategic interdependencies between players which must be taken into account in establishing a sound behavioural model studying the interaction of decision makers pursuing disparate objectives in the transportation network context. The next chapter will subsequently integrate the game theoretic model introduced here with models of users' route choice behaviour thereby setting out a unified modelling framework applied in the case studies.

However, an NE may not exist when players are restricted to playing pure strategies. Though a proof of the existence of NE is beyond the scope of the present research, the existence of NE in the games discussed in this thesis will be demonstrated through numerical tests reported in the case studies. As pointed out in this chapter, the NE could be inefficient as individual players focus only on maximising personal gain and in so doing, neglect aspects of decision making reflecting mutual interests. In this context, the Prisoner's Dilemma was used to show that players could be better off if they cooperated. Similarly, the Cournot-Nash solution was shown not be Pareto Optimal as one player could be made better off without making another worse off. Furthermore, if the payoff function of one or more players exhibits multiple maxima, multiple NE would exist and a distinction was made between Local Nash Equilibria (LNE) and NE. The equilibrium point in a LNE is a best response only within a *local neighbourhood* while the NE is the best response globally i.e over the entire strategy space. These characteristics of NE outcomes will be seen in the case studies presented in this thesis found in Chapters 6 to 9.

Chapter 4

Deterministic Approaches to Hierarchical Optimisation Problems[4.1]

4.1 Introduction

While this thesis separately discusses both private sector toll road competition and inter-jurisdictional competition for toll revenues, these two seemingly different agents can be studied within a common modelling framework. Before any assessment of the welfare implications of such competition can be investigated, robust algorithms need to be developed and applied in order for the equilibrium outcomes to be determined and evaluated. Furthermore, the appropriate algorithm will be dependent on the underlying model describing travellers' route choices. In order to address these issues, this chapter firstly outlines a unified mathematical framework of Equilibrium Problem with Equilibrium Constraints (EPEC) (Mordukhovich, 2004, 2005) that permits both private operator and inter-jurisdictional competition/collaboration in highway networks to be investigated within a common framework before secondly, proposing robust algorithms for their resolution in this and the subsequent chapter. The proposed methods will then be subsequently applied in the case studies reported in Chapters 6 to 9.

The EPEC framework has already been applied to study competition in a variety of disciplines such as investigating competitive practices in European electricity markets (Pang and Fukushima, 2005; Leyffer and Munson, 2010) as well as in highway transportation (Yang *et al.*, 2009). As will be discussed in this chapter, this

[4.1] This chapter draws extensively on Koh (2013).

EPEC model is the appropriate mathematical model for two reasons. Firstly, it is applicable in the context of non-cooperative behaviour between the two groups of decision makers discussed in this thesis (private toll road concessionaires or local governments) with each seeking to maximise individual objectives, contingent on the decisions made by others. Secondly, the model is also applicable in the case when these players are assumed to cooperate, a relatively neglected area where there is limited literature and are used as benchmarks in this thesis. At the same time, while the EPEC represents the most appropriate mathematical formulation for the problems discussed in this thesis, it should be recognised that the research on solution algorithms for EPECs has only just begun Steffensen and Bittner (2014).

The rest of this chapter is organised as follows. Following this introduction, the Stackelberg model of competition also known as a "Leader-follower" game as proposed in von Stackelberg (1934) is introduced to motivate the mathematical model of a generic bilevel programming problem (BLPP). While conventionally serving as the gateway to studying transportation systems management by a single regulatory agency, this BLPP also simultaneously serves as the building block of the EPEC model applicable when interacting multiple agents are considered. The EPEC model is discussed in Section 4.3. As will be highlighted, both BLPPs and EPECs are mathematical programs characterised by a "hierarchical" structure and thus this chapter will refer to them collectively as hierarchical optimisation problems. Furthermore, the solution algorithms developed in the context of the single regulator paradigm can be straight forwardly extended to encompass that of multiple agents. This extension is discussed in Section 4.4. As shown in Chapter 3, oligopolistic competition has been actively studied in the industrial organisation literature. Thus an algorithm from that domain can be straightforwardly extended to study competition in highway transportation systems. This discussion is the subject of Section 4.5. Section 4.6 summarises and sets out the relation of this chapter to the overall thesis.

4.2 The Bi-Level Programming Problem

In transportation systems management, it is usually assumed that it is not possible to dictate or somehow coerce users to select a particular path through a traffic network to achieve the facility manager's desired outcome. For example, it is evident that a toll road operator cannot force traffic to use the road he controls. Instead, assuming that users make rational decisions, the manager is limited to using the toll level to *influence* the travellers' route choices. Thus the "transmission mechanism" at work here is as follows: a change in the toll levels would affect the generalised costs of travel which would in turn influence the equilibrium route choices. This would then result in a new routing pattern (with implied demand levels) across the network, manifesting in changes in traffic volumes which would impact on the system manager's objectives. For brevity, such as "transmission mechanism" is referred to as the reaction or response of the highway users to the system manager's policies.

Therefore, by their choice of appropriate toll levels, system managers can exert an influence on the route choices of users. This has led the literature (e.g. Fisk, 1984; Friesz and Harker, 1985) to notice the parallels in this regard with the classic Stackelberg game (von Stackelberg, 1934) first cursorily introduced in Chapter 2. As noted therein, the Stackelberg model of industrial organisation assumes that there is a single dominant firm (known as the "leader") in the market alongside several smaller firms (referred to collectively as the "followers"). The leader obtains a first mover advantage (Lieberman and Montgomery, 1988), enabling it to earn profits above the cost of capital by virtue of the fact that it knows how the followers will react to its strategy. In addition, there is an element of sequential decision making in this model as the leader is assumed to act first, confident that the followers will obey their reaction functions, after which the followers react, taking the leader's strategy as given. von Stackelberg's model of behaviour of the leader and follower firms will be referred to as the "leader-follower" paradigm throughout this thesis. These behavioural assumptions stand in contrast to the assumptions behind the Cournot-Nash game discussed in Chapter 3 where each player in the Nash non-cooperative game acts simultaneously and do so by only reacting to the strategies of its rivals.

To complete the analogy, the transportation systems manager should therefore act as the Stackelberg leader while users of the system are modelled collectively as the followers (Fisk, 1984). The most important lesson from the Stackelberg analogy though is that the systems manager has to take the reactions of the users into account when applying any policy instrument such as tolls (Dickson, 1981; Fisk, 1984; Harker and Friesz, 1984). Just as in the Stackelberg model, the road users (followers) treat the leader's policy/decision variables as an exogenous input in making their route choice decisions. This requirement of the "leader-follower" paradigm has important implications for the appropriate mathematical model as well as the solution algorithms developed in this chapter.

4.2.1 A General BLPP

The Stackelberg model discussed above can be formulated as shown in Eq. 4–1. This is a specific case of the more general class of multi-level programming/hierarchical optimisation problems restricted to two levels viz. "upper level" and "lower level". The distinguishing feature of BLPPs is that the constraint region of the upper level problem (Eq. 4–1a) is defined by yet another optimisation problem (a lower level problem) in Eq. 4–1b. Furthermore, this lower level constraint is always active. For example, in the case of the Stackelberg model with one follower, the upper level problem is the optimisation problem faced by the market leader who is taking into account the reaction of the follower whose optimisation problem appears as a constraint into the leader's problem.

$$
\text{BLPP}\begin{cases}
\quad \max_{x\in X} \; U(\mathbf{x},\mathbf{y}(\mathbf{x})) & \text{(Eq. 4–1a)}\\
\text{subject to} & \\
\quad \mathbf{y}(\mathbf{x}) = \arg\max_{y\in Y} L(\mathbf{x},\mathbf{y}) & \text{(Eq. 4–1b)}
\end{cases}
$$

with $U, L\colon \mathbb{R}^{n_1} \times \mathbb{R}^{n_2} \mapsto \mathbb{R}$. X represents the decision space of the upper level problem (Eq. 4–1a). $X = \{(x_1, x_2, \ldots, x_{n_1})^\intercal \in \mathbb{R}^{n_1} \,|\underline{\mathrm{x}}_i \leq x_i \leq \bar{x}_i \;, i = \{1, 2, \ldots, n_1\}\}$ Arising from the "leader-follower" analogy of BLPPs, the terms "leader's variables"

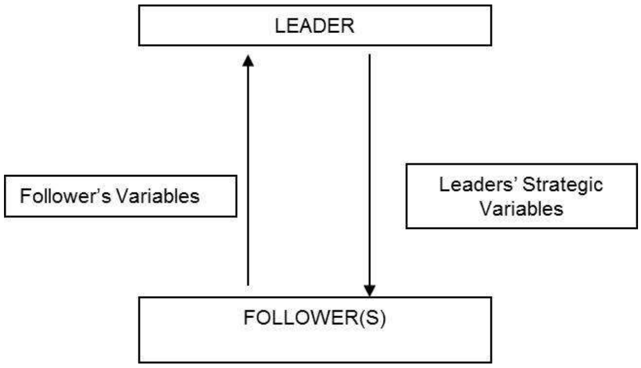

FIGURE 4.1: Pictoral Representation of a BLPP adapted from Sumalee (2004*a*)

and "upper level variables" are interchangeably used when referring to $\mathbf{x}$. Similarly Y represents the decision space for the lower level problem (Eq. 4–1b) with $Y = \left\{ (y_1, y_2, \ldots, y_{n_2})^\intercal \in \mathbb{R}^{n_2} \left| \underline{y}_j \leq y_j \leq \bar{y}_j \right. , j = \{1, 2, \ldots, n_2\} \right\}$.

The BLPP has been a subject of extensive research and several monographs on this subject have been published to date (Luo *et al.*, 1996; Shimizu *et al.*, 1997; Bard, 1998; Outrata *et al.*, 1998; Dempe, 2002). At the same time, applications of BLPP can be found in diverse fields ranging from chemical engineering (Gümüş and Floudas, 2001) to robotics (Luo *et al.*, 1996) and transportation systems management (Migdalas, 1995). In tandem, there has been much work on the development of solution methodologies (see Dempe, 2002; Colson *et al.*, 2007, for a review).

Fig. 4.1, adapted from Sumalee (2004*a*), captures three main characteristics of BLPP (Wen and Hsu, 1991; Oduguwa and Roy, 2002):

a) the decision-making units are interactive and exist within a hierarchical structure,

b) decision making is sequential from higher to lower level. The lower level decision maker executes its policies after decisions are made at the upper level, and

c) each unit independently optimises its own objective functions but is influenced by actions taken by other units.

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4.2.2 Mathematical Program with Equilibrium Constraints

In this section, a Mathematical Program with Equilibrium Constraints (MPEC) is formally defined emphasising the fact that the MPEC is a special case of the BLPP with the difference that the lower level program is defined as a Variational Inequality (VI) Problem.

Variational Inequality

Adapting the definition from Nagurney (1999) to the context of BLPPs, a Variational Inequality Problem (VIP) is the problem of finding a n_2 dimensional vector $\mathbf{y}^*$, for given and fixed $\mathbf{x}$, such that the condition Eq. 4–2 is satisfied with $A\colon \mathbb{R}^{n_1} \times \mathbb{R}^{n_2} \mapsto \mathbb{R}^{n_2}$ and where $\mathcal{F}$ defines a feasible region.

$$\text{VIP} \left\{ \; A(\mathbf{x}, \mathbf{y}^*)(\mathbf{y} - \mathbf{y}^*) \geqslant 0, \mathbf{y} \in \mathcal{F} \right. \qquad \text{(Eq. 4–2)}$$

With the definition of the VIP from Eq. 4–2, the MPEC is formulated as the BLPP in Eq. 4–3.

$$\text{MPEC} \begin{cases} \displaystyle\max_{x \in X} \quad U(\mathbf{x}, \mathbf{y}(\mathbf{x})) & \text{(Eq. 4–3a)} \\ \text{subject to} & \\ \mathbf{y}(\mathbf{x}) \quad \leftarrow \text{SOL}\{\mathcal{V}(\mathbf{x})\} & \text{(Eq. 4–3b)} \end{cases}$$

In Eq. 4–3, the notation $\mathbf{y}(\mathbf{x}) \leftarrow \text{SOL}\{\mathcal{V}(\mathbf{x})\}$ is used as shorthand to emphasise that the lower level variable, $\mathbf{y}$, is obtained as the solution[4.2] for a VIP parametrised in the upper level decision vector, $\mathbf{x}$. As VIPs encapsulate equilibrium conditions (Nagurney, 1999), the MPEC can be thought of as an optimization problem where the constraint region specifies an equilibrium in a given parametric system. Note that the closely related class of Mathematical Programs with *Complementarity Constraints* (Leyffer, 2005) is a special case of the MPEC where the constraint region

[4.2] In general, the solution of Eq. 4–2 may not be unique. However for the case study applications considered in this thesis (in Chapters to 6 to 9), uniqueness of the demands and link flows solution to the VIP encapsulating traffic equilibrium, for each vector of the leader's variables $\mathbf{x}$ or tolls, is theoretically assured under several assumptions which will be made explicit in those chapters (Smith, 1979; Cantarella, 1997).

is defined by a Complementarity Problem (CP) parametrised in $\mathbf{x}$ with a CP being a specific case of the more general VIP (Karamardian, 1971; Nagurney, 1999). The CP is briefly discussed in Section 4.5.

From BLPP to MPEC and vice-versa

Consider the BLPP in Eq. 4–1. If the lower level problem is replaced by its Karush Kuhn Tucker (KKT) (Karush, 1939; Kuhn and Tucker, 1951) conditions, then the MPEC shown in Eq. 4–3 is obtained (Nagurney, 1999, Proposition 1.2, p. 6). When the lower level problem (Eq. 4–1b) is convex, then the KKT conditions characterise global optimality (Luo *et al.*, 1996). Conversely, the lower level VIP (Eq. 4–3b) in the MPEC can be formulated as an optimization problem *only if certain symmetry conditions are met* (Nagurney, 1999). Thus the BLPP is obtained.

In transportation systems management, a convex mathematical optimisation problem resulting in link flows (and demands) over a highway transportation network that satisfy Wardrop's DUE condition (Wardrop, 1952) was formulated in Beckmann *et al.* (1956). Similarly, a mathematical optimisation problem for link flows that satisfy Stochastic User Equilibrium (SUE) is formulated in Fisk (1980). In these optimisation approaches to the determination of equilibrium link flows and demands, the bilevel program for the system manager adopts the form of Eq. 4–1 where the $\mathbf{y}(\mathbf{x})$ is the vector of link flows satisfying the equilibrium conditions predicated on the system manager's control vector $\mathbf{x}$.

Subsequently, it has been shown (Smith, 1979) that Wardrop's equilibrium condition can be formulated as a VIP. This VIP formulation is more general and allows situations where the separability assumption[4.3] does not hold such as the *asymmetric* traffic assignment problem (Dafermos, 1980). When this separability assumption is violated, the VIP can no longer be formulated as an optimisation problem. Though this case is not considered in this thesis, the bilevel program of the system manager must be modelled as an MPEC.

[4.3]This assumption means that the link's generalised travel time is dependent on its own link flow alone and independent of the link flow(s) of all other links.

Furthermore, under the assumption of travellers choosing routes according to the SUE routing model, as will be discussed in Chapter 7, the equilibrium link flows and demands can be obtained as the solution of a non linear system of equations (Cantarella, 1997) (see Section 7.2). Such a system of equations, parametrised in the leader's variables, is another instance of a VIP (Nagurney, 1999, Proposition 1.1, p. 5). Once again, because the leader's problem is constrained by a VIP, her optimisation problem can also be expressed in the form of the MPEC in Eq. 4–3.

In view of the above, the MPEC formulation is more general than the BLPP formulation. Another reason to focus on the MPEC representation is related to the core theme of this thesis. The MPEC formulation extends naturally to the case of *multiple* leaders, and emphasises that the actions of these leaders are constrained by a condition specifying equilibrium in the transportation network under study. This extension is discussed later in this chapter (see Section 4.3).

4.2.3 Computational Difficulties of MPECs

When all functions (both objectives and constraints) in both upper and lower level problems of the BLPP are linear, the resulting BLPP is known as a linear BLPP. However, even in this deceptively "simple" case, this BLPP is still Nondeterministic Polynomial time hard (NP hard) (Ben-Ayed and Blair, 1990). This means that an exact solution cannot be found in polynomial time. Even if both the upper level and the lower level are convex programs, the resulting BLPP itself can be non-convex (Ben-Ayed, 1993). Non convexity suggests the possibility of multiple optima. Ben-Ayed and Blair (1990) demonstrated the failure of both the Parametric Complementarity Pivot Algorithm (Bialas and Karwan, 1984) and the Grid Search Algorithm (Bard, 1983) to locate the optimal solution of the linear-BLPP. Since then, progress has been made in solving the linear-BLPP and techniques including implicit enumeration (Candler and Townsley, 1982), penalty based methods (Aiyoshi and Shimizu, 1981) and methods based on KKT conditions (Fortuny-Amat and McCarl, 1981) have been developed. See Wen and Hsu (1991) for a review of the algorithms available for the linear-BLPP.

In transportation systems management, with the exception of relatively restrictive cases, no explicit formulation is generally available for responses of the users at the lower level (Fisk, 1984, p. 304). The lower level model thus incorporates the leader's strategies but only *implicitly.* This contributes to the difficulty with solving the BLPP and thus requiring algorithmic approaches for its resolution.

One proposal to solve the general BLPP was the Iterative Optimisation Algorithm (IOA) (Allsop, 1974; Steenbrink, 1974) which some authors (e.g. Migdalas, 1995, p. 395) term "Block Coordinate Descent". This method involves solving the upper level problem for fixed $\mathbf{y}$ and using the solution thus obtained to solve the lower level problem, and repeatedly iterating between the upper and lower level programs until some convergence criterion is met. However, the IOA was later shown to be an exact method for solving a Cournot-Nash game (Fisk, 1984; Friesz and Harker, 1985) rather than the Stackelberg game that the BLPP is meant to reflect. This is because the design of the IOA implicitly assumes that the leader is myopic as he does not take into account the follower's reaction to her policy (Dickson, 1981; Fisk, 1984; Friesz and Harker, 1985) and thus violates the "leader-follower" paradigm of the Stackelberg model where as emphasised, the leader must be modelled as being endowed with knowledge of the follower's reaction function which the leader knows the follower will obey.

Another difficulty with solving MPECs stems from the fact that they fail to satisfy certain technical conditions (known as constraint qualifications) at any feasible point (Chen and Florian, 1995; Scheel and Scholtes, 2000). The penalty interior point algorithm was proposed for MPECs in Luo *et al.* (1996). However, a counter example in Leyffer (2005) showed that this algorithm could in fact converge to a non-stationary point. Subsequent research has led to the development of many other techniques to solve the MPEC such as the piecewise sequential quadratic programming (Luo *et al.*, 1996), branch-and-bound (Bard, 1988), nonsmooth approaches (Dempe, 2002; Outrata *et al.*, 1998) and smoothing methods (Facchinei *et al.*, 1999).

Migdalas (1995) outlines several transportation systems management problems that can be posed as MPECs, formulated on the assumption of a single regulator at the upper level applying policy instruments to influence the route choices of users. These encompass the network design problem (Friesz *et al.*, 1992; Chiou, 2005), the traffic signal setting problem (Fisk, 1984; Teklu *et al.*, 2007), the OD matrix estimation problem (Maher *et al.*, 2001) and also most relevant to this thesis, the optimal toll pricing and design problem (Shepherd and Sumalee, 2004; Sumalee, 2004*a*,*b*; Koh *et al.*, 2009).

The Cutting Constraint Algorithm (CCA) (Lawphongpanich and Hearn, 2004) will be used to solve the MPECs that are formulated in the case studies described in Chapters 6 and 8. In the CCA, the MPEC is first reformulated as a single level optimisation problem with additional constraint(s). At the first step, a first additional constraint reflects the VI representation of the Wardrop's DUE condition (Smith, 1979) as a convex combinations of extreme points. These extreme points are generated by solving an auxiliary shortest path problem and additional constraints are added iteratively until the entire system satisfies Wardrop's DUE condition at termination. A more detailed description of the CCA can be found in Chapter 6 (see Section 6.4).

4.3 Equilibrium Problem with Equilibrium Constraints

This equilibrium constraint and hierarchical structure is also present in the class of Equilibrium Problems with Equilibrium Constraints (EPECs) depicted in Fig. 4.2. The EPEC may be thought of as a multi-leader generalization of the Stackelberg game (Sherali, 1984; Mordukhovich, 2005; Leyffer and Munson, 2010; Hu and Fukushima, 2012). Compared to an MPEC, the EPEC aims at "finding some equilibrium (rather than minimum) points subject to constraints described by the parametric variational systems" (Mordukhovich, 2005, p. 379).

In this regard, researchers have conjectured that there could be two possible behaviours of the leaders at the upper level (Outrata, 2004; Mordukhovich, 2005).

At one extreme, the leaders could act cooperatively which can be modelled as a Multiobjective Optimisation Problem with Equilibrium Constraints (MOPEC) (Ye and Zhu, 2003; Mordukhovich, 2004). Equilibrium in the upper level in this latter case can then be characterised by the principle of Pareto Optimality (see Definition 3.3). The rest of this chapter focuses on the NCEPEC while a discussion of the MOPEC is found in Chapter 5.

At the other extreme, the leaders could act non-cooperatively and engage in a Nash non-cooperative game amongst themselves resulting in a Non-Cooperative EPEC (NCEPEC) with the equilibrium of the upper level leaders characterised by the NE condition (Definition 3.1). Furthermore, as the Nash non-cooperative game between the leaders can be formulated as a VIP (Facchinei and Pang, 2003, p. 24), and the lower level problem is another VIP parametrised in the vector of leaders' variables, some authors have coined the term "Bilevel Variational Inequality"(Yang and Huang, 2005; Yang *et al.*, 2009) for the NCEPEC. It must be emphasised that in the NCEPEC, each leader anticipates the reactions of the followers only when making its decisions. However, in the relationship between the leaders themselves, no leader is assumed to have any dominant or leadership position over the others. Therefore, leaders engage each other in the Nash non-cooperative game on an *equal footing*. As noted previously, the situation where one leader is able to act as a Stackleberg leader with respect to the others, by anticipating their reactions to his actions, is beyond the scope of this thesis. Taking into account both the Nash non-cooperative game between the leaders and the "leader-follower" relationship between the followers, the NCEPEC can be viewed as a game which aims to "find an equilibrium point where no leader can improve his objective given the strategies chosen by the other leaders *and* those chosen by the other followers" (Zhang, 2010, p. 119, italics added).

4.3.1 The Non-Cooperative EPEC

In the discussion of games in Chapter 3, it was assumed that each player's actions were chosen from some discrete (as in the Prisoner's Dilemma, see p. 83) or continuous (as in the Cournot game, see p. 73) action space. In these cases, the action a player chooses does not affect the actions available to all other players. Thus each player acts in an "unconstrained action space" i.e. the actions open to each player are not affected by strategies chosen by their competitors. However, the NCEPEC

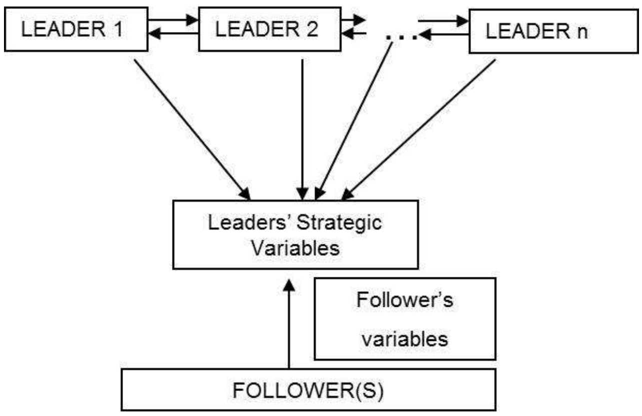


FIGURE 4.2: Equilibrium Problem with Equilibrium Constraints

is a special case of these Nash non-cooperative games discussed in Chapter 3. In the NCEPEC, each player's action is specifically constrained by a condition that dictates an equilibrium, formulated as a VIP, in the parametric system under study (Mordukhovich, 2004, 2005; Hu and Ralph, 2007; Soon, 2011; Hu and Fukushima, 2012).

Given both the Stackelberg relationship between leaders and followers as well as the Nash non-cooperative game between the leaders themselves, the terms "player" and "leader" are used synonymously in this thesis. Recall, from Chapter 3 that $\mathbf{x}_{-i}$ denotes the strategies of all other players in the game *excluding* that of player i i.e. $-i \triangleq i \in \mathcal{N} \backslash i$. Then the optimisation problem facing player/leader i selecting her optimal strategy to maximise her payoff, given the strategies chosen by all other leaders and taking into account responses of the followers may be represented by the MPEC in Eq. 4–4.

$$\forall i \in \mathcal{N}, \text{ Player } i \text{ solves: } \begin{cases} \max\limits_{x_i \in X_i} \phi_i(x_i, \mathbf{x}_{-i}, \mathbf{y}) \\ \text{subject to} \\ \mathbf{y} \leftarrow \mathrm{SOL}\{\mathcal{V}(x_i, x_{-i})\} \end{cases} \tag{Eq. 4–4}$$

The constraint in Eq. 4–4 emphasises the VIP specifying an equilibrium problem parametrised in the strategies of *all* players in the game i.e. the strategy of leader i as well as all other leaders. Furthermore, as player i takes into account the responses of the followers, player i's payoff is *dependent* on the solution of the lower level VIP.

Hence the NCEPEC is in essence a series of interrelated MPECs with the coupling constraint, that all players jointly face, being defined by a VIP. A solution of Eq. 4–4, if it exists, should be an NE satisfying Definition 3.1 (Mordukhovich, 2005).

It is beyond the scope of this thesis to prove the *existence* of an NE (or even the weaker LNE condition as stated in Definition 3.4) in a general NCEPEC, an issue which continues to require further research (Pang and Fukushima, 2005; Yang and Huang, 2005; Facchinei and Kanzow, 2010). Thus, following the approach in Ehrenmann (2004), its existence will be numerically shown and verified (employing numerically estimated best response correspondences as discussed in Chapter 3) in the case studies reported in Chapters 6 to 9 when applied to several games formulated as NCEPECs. To find the LNE solutions of these games, two possible algorithms are described in the rest of this chapter.

4.4 Fixed Point Iteration

The first class of algorithms for solving NCEPECs discussed is based on Fixed Point Iteration (FPI). This category covers numerical methods that have already been successfully applied to solve Equilibrium Programming problems (Ortega and Rheinboldt, 1970; Zangwill and Garcia, 1981) in the absence of a binding equilibrium constraint.

While the algorithm was initially suggested to solve a Cournot-Nash game in Harker (1984), it has been adapted to identify LNE in the NCEPEC (Su, 2005; Leyffer and Munson, 2010). It should be remarked that one of the earliest NCEPECs model was that of intercity carrier competition where the upper level players are competing shipping firms who optimise profits engaged in a Nash non-cooperative game subject to a combined mode choice and highway assignment model represented as a VIP at the lower level (Fisk, 1984, 1986). The FPI algorithm was suggested for the resolution of the problem but no numerical results were presented.

The pseudocode of the FPI algorithm, which solves the NCEPEC as a series of interrelated MPECs of $|\mathcal{N}|$ leaders, is given in Algorithm 4.1. Starting with an initial strategy vector, each leader's MPEC is solved in turn, with all other players' strategies held fixed. This means that in applications of the FPI algorithm, a separate algorithm to solve each player's MPEC (termed the "inner MPEC") at each iteration is required. The strategy vector is then updated. If a convergence criteria e.g. one based on a proximity measure such as the euclidean norm of difference in strategy vectors between successive iterations being less than a pre-specified tolerance, ϵ, the algorithm is deemed to have converged and the algorithm terminates. Otherwise, the iteration counter is updated and the process is repeated. In the updating process

Algorithm 4.1 Fixed Point Iteration (FPI) for NCEPECs

Input: Maximum Iterations, G
Input: Termination Tolerance, $\epsilon(> 0)$
$g \leftarrow 0$
Choose initial $\mathbf{x}^g = \{x_1^g, \dots, x_i^g, \dots, x_n^g\}^\intercal$
while $g < G$ **do**
  for $i = 1$ to $|\mathcal{N}|$ **do**
    Solve MPEC for player i in Eq. 4–4 for x_i^{g+1}
  end for
  if $\|x_i^{g+1} - x_i^g\| \quad \forall i \in \mathcal{N} \leqslant \epsilon$ **then**
    Terminate
  else
    $g \leftarrow g + 1$
  end if
end while

of the vector of strategies at iteration g, two slightly different variants have been proposed viz. Gauss-Seidel and Gauss-Jacobi (Ehrenmann, 2004; Su, 2005). With the Gauss-Seidel variant, updated values are used as soon as they are obtained while in the latter, updated values are used only after all $|\mathcal{N}|$ MPECs have been solved.

By way of illustration of the differences between each variant, suppose that the algorithm reaches a point where the next step would be to solve player i's problem. To get to this point, the algorithm would have solved all preceding players' (i.e. $1, 2, \dots, i-1$) MPECs. In the Gauss-Seidel variant, the strategy vector treated as fixed when solving player i's problem, written as $\mathbf{x}_{-i}$ is the vector comprising the elements $\mathbf{x}_{-i}^g = \{x_1^{g+1}, \dots, x_{i-1}^{g+1}, x_{i+1}^g, \dots, x_n^g\}^\intercal$. On the other hand, with the Gauss Jacobi variant, updating of the strategy vector only occurs when *all* $|\mathcal{N}|$ individual MPECs have been solved. Therefore in solving player i's problem, the strategy vector treated as fixed is $\mathbf{x}^g_{-i} = \{x^g_1, \ldots, x^g_{i-1}, x^g_{i+1}, \ldots, x^g_n\}^\intercal$.

Research into the application of FPI to NCEPECs has suggested that the Gauss-Seidel variant outperforms the Gauss-Jacobi variant (Su, 2005, p. 33). Thus in this thesis, the Gauss-Seidel FPI variant will be applied to the numerical examples reported in the case studies.

Interestingly, the FPI algorithm closely resembles that of the IOA, wrongly suggested as the way to tackle BLPPs and MPECs. The subtle but important difference though is that instead of iterating between the upper level and lower level problems which the IOA does, the iteration in the FPI is over each leader's MPECs, with the strategies of all others held fixed. It has been pointed out that the FPI algorithm directly relates to the intuitive reasoning behind the concept of NE. In each iteration of the algorithm, each leader treats as fixed all the other leaders' strategies whilst maximising her individual objective. When all leaders have computed their own strategies, the game then moves onto a new vector of strategies to which each leader must again react. The game ends when no firm has an incentive to change strategy which is precisely the definition of NE as given in Definition 3.1 (Harker, 1984; Ehrenmann, 2004).

In fact, this algorithm has been used by several authors to solve similarly structure NCEPECs encountered in the modelling of deregulated electricity markets (Hu and Ralph, 2007; Ehrenmann and Neuhoff, 2009; Zhang, 2010). While convergence remains to be proved for general cases, the literature suggests that the FPI algorithm converge to an LNE when each player's payoff function satisfies a technical condition known as "diagonal strict concavity" (Cardell *et al.*, 1997; Contreras *et al.*, 2004). Intuitively interpreted, this condition requires that "each player has more control over his payoff than other players have over it" (Contreras *et al.*, 2004, p. 197). Since it is beyond the scope of this thesis to prove that payoff functions for the players in the NCEPECs considered in this thesis do in fact satisfy this condition, the FPI algorithm can only be regarded as a heuristic procedure for solving the NCEPEC.

4.5 Sequential Linear Complementarity Problem Algorithm

Based on the KKT conditions that characterise the optimality conditions for each player in a Cournot game introduced in Chapter 3, an alternative algorithm for solving the NCEPEC is developed. This section outlines the theoretical basis of the proposed method concluding with an outline of the solution procedure.

4.5.1 Oligopolistic Competition as a Complementarity Problem

Consider the optimisation problem facing player i,$i \in \mathcal{N}$ engaged in a Cournot game discussed in Chapter 3 where the payoff function is given by Eq. 3–2. Thus the player faces the optimisation problem in Eq. 4–5.

$$\max_{x_i \in X_i} \phi_i(x_i, \mathbf{x}_{-i}) = \phi_i(\mathbf{x}) \qquad \text{(Eq. 4–5)}$$

It is assumed throughout that player i's payoff/objective function, $\phi_i(x_i, \mathbf{x}_{-i}), i \in \mathcal{N}$, is concave if opponents' strategies, $\mathbf{x}_{-i}$, are held fixed. This assumption was also made in Yang *et al.* (2009)(p. 41) and has indeed been verified to hold in the games that will be formulated in Chapters 6 to 9.

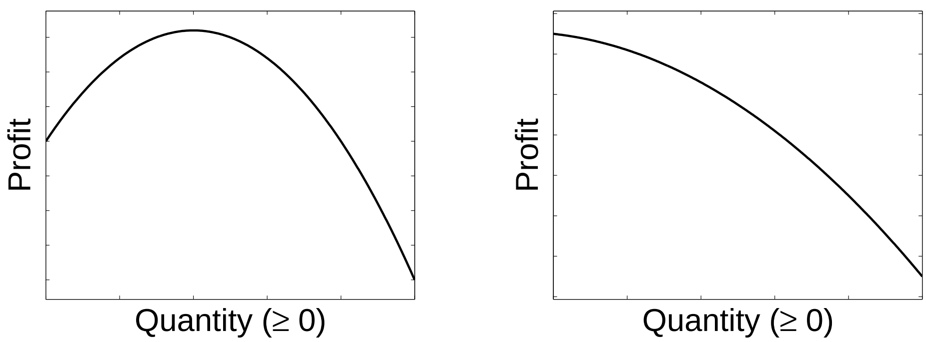


FIGURE 4.3: Complementarity Case 1 (left): Optima of the payoff function in the positive orthant. Complementarity Case 2 (right): The payoff function, being concave, must have decreasing slope in the positive orthant.

Focusing on player i, the two (and only) possibilities for a payoff maxima for this player's payoff function that can arise are illustrated in the left and right panels of Fig. 4.3. In the left panel, the maxima of the profit function occurs where output is *strictly* positive i.e. $\partial\phi_i(x_i, \mathbf{x}_{-i})/\partial x_i = 0$ when $x_i > 0$. The notation $\mathbb{R}_+$ to emphasise that x_i is strictly positive. On the other hand, it could be that the maximum is attained in the negative space. Recall that the Cournot-Nash game is a game in quantities (see Section 3.3.1), thus production levels cannot be negative. Hence with values of $x < 0$ being ruled out, the only possibility, given concavity of the profit function, is that the the profit curve must be falling or equivalently that derivative of the profit curve with respect to player i's output must be negative i.e. $\partial\phi_i(x_i, \mathbf{x}_{-i})/\partial x_i < 0$ when output is 0. The so called "complementarity conditions" allow for these two possibilities to be simultaneously taken into account. These conditions are summarised in Eq. 4–6.

$$\text{CP} \left\{ \begin{array}{l} F_i(x_i) = -\frac{\partial\phi_i(\mathbf{x})}{\partial x_i} \geqslant 0 \\ x_i \frac{\partial\phi_i(\mathbf{x})}{\partial x_i} = 0 \\ x_i \geqslant 0 \end{array} \right\} \forall i \in \mathcal{N} \qquad \text{(Eq. 4–6)}$$

Eq. 4–6 defines a Complementarity Problem (CP) (Karamardian, 1971; Kolstad and Mathiesen, 1991; Facchinei and Pang, 2003; Konnov, 2007) which can be viewed as a special case of the more general VIP (Nagurney, 1999, Proposition 1.4, p. 9). Written in vector form, the CP is to find $\mathbf{x} \in \mathbb{R}^n_+$ where $\mathbf{F} : \mathbb{R}^n \rightarrow \mathbb{R}^n$ such that the three conditions, in Eq. 4–7 are satisfied.

$$\begin{aligned} &\mathbf{F}(\mathbf{x}) \geqslant 0 \\ &\mathbf{x}\mathbf{F}(\mathbf{x}) = 0 \\ &\mathbf{x} \geqslant 0 \end{aligned} \qquad \text{(Eq. 4–7)}$$

4.5.2 Solution Algorithm

The first order Taylor expansion of $\mathbf{F}(\mathbf{x})$ at $\mathbf{x}^0$ (some arbitrary starting vector) can be written as $LF(\mathbf{x}|\mathbf{x}^0) = \mathbf{F}(\mathbf{x}^0) + \nabla_{\mathbf{x}}\mathbf{F}(\mathbf{x}^0)(\mathbf{x} - \mathbf{x}^0)$. Then, following Kolstad and Mathiesen (1991), the resulting Linear Complementarity Program (LCP($\mathbf{M}, \mathbf{z}$)) is to find $x \in \mathbb{R}^n_+$ such that the system in Eq. 4–8 is satisfied.

$$\mathrm{LCP}(\mathbf{M}, \mathbf{z}) \begin{cases} LF(\mathbf{x}|\mathbf{x}^0) = \mathbf{z} + \mathbf{M}\mathbf{x} \geqslant 0 \\ \mathbf{x}^{\intercal}(\mathbf{z} + \mathbf{M}\mathbf{x}) = 0 \\ \mathbf{x} \geqslant 0, \end{cases} \tag{Eq. 4–8}$$

where

$$\mathbf{z} = \mathbf{F}(\mathbf{x}^0) - \mathbf{x}^0 \nabla_{\mathbf{x}} \mathbf{F}(\mathbf{x}^0), \tag{Eq. 4–9}$$

and

$$\mathbf{M} = \nabla_{\mathbf{x}} \mathbf{F}(\mathbf{x}^0), \tag{Eq. 4–10}$$

where $\nabla_{\mathbf{x}} \mathbf{F}(\mathbf{x}^0)$ is the Jacobian of $\mathbf{F}(\mathbf{x})$ evaluated at some arbitrary vector $\mathbf{x}^0$ i.e. a matrix of the partial derivatives of $\mathbf{F}(\mathbf{x})$ with respect to $\mathbf{x}$. The diagonals of $\nabla_{\mathbf{x}} \mathbf{F}(\mathbf{x}^0)$ are $\partial F_i(x_i)/\partial x_i$ and the off diagonals are $\partial F_i(x_i)/\partial x_j, j \neq i$.

Thus Kolstad and Mathiesen (1991) proposed to solve the Cournot game by iteratively solving a sequence of linear complementarity problems taking the form of Eq. 4–8. Hence this method is known as the Sequential Linear Complementarity Problem (SLCP) Algorithm. However, it is important to emphasise that the algorithm was originally proposed for the classical Nash non-cooperative game where the players' strategies are not coupled by an equilibrium constraint. Nevertheless, with little modification, SLCP can be applied to compute LNE strategies for NCEPECs.

The pseudocode in Algorithm 4.2 outlines the SLCP algorithm to solve NCEPECs with an emphasis on the required modifications to the original proposal of Kolstad and Mathiesen (1991). Starting with an initial vector of strategy variables, the lower level VIP is solved to determine the first and second order derivatives (i.e. $\mathbf{F}(\mathbf{x}^g)$ and $\nabla \mathbf{F}(\mathbf{x}^g)$ at iteration g respectively) of each leader's objective/payoff function (see lines 5 to 6 of Algorithm 4.2). These derivatives can be obtained by either numerical finite differencing techniques (Morton and Mayers, 2005) or sensitivity analysis (Yang and Huang, 2005) and thus allow the computation of $\mathbf{z}$ and $\mathbf{M}$ using Eq. 4–9 and Eq. 4–10 respectively. These serve as inputs to the LCP to be solved to obtain a revised strategy vector as a starting point input into the next iteration.

The resulting LCP can be solved by application of Lemke's algorithm (Cottle *et al.*, 2009). If the absolute maximum of the elements in the vector of first order derivatives of each leader's payoff function is less than some pre-specified (small) termination tolerance (ϵ), the algorithm is deemed to have converged and SLCP terminates. Otherwise, the iteration counter is incremented and the process repeated.

It is evident that by explicitly incorporating the resolution of the lower level VIP for a given vector of the leader's strategies (Line 5 in Algorithm 4.2) to enable computation by $\mathbf{z}$ and $\mathbf{M}$ (Lines 7 and 8) at each iteration, the SLCP algorithm continues to respect the leader-follower paradigm critical in the NCEPEC. Compared to the FPI algorithm, instead of cyclically solving each leader's MPEC until the system converges, the SLCP algorithm solves all leaders' strategies simultaneously. This suggests that SLCP should be more computationally efficient vis-á-vis any of the FPI variants. Though the SLCP algorithm will be extensively applied in the case studies reported in this thesis, it should be emphasised that it is beyond the scope of this thesis to theoretically establish convergence of the algorithm to a LNE point of the NCEPEC in Eq. 4–4.

Algorithm 4.2 Sequential Linear Complementarity Problem (SLCP)

1: Input: Termination Tolerance, $\epsilon(> 0)$
2: $g \leftarrow 0$
3: Choose initial $\mathbf{x}^g = [x_1^g, \ldots, x_i^g, \ldots, x_n^g]^\intercal$
4: **while** Not Converged **do**
5:    Solve Lower Level VI with $\mathbf{x}^g$
6:    Obtain $\mathbf{F}(\mathbf{x}^g)$ and $\nabla\mathbf{F}(\mathbf{x}^g)$
7:    Compute $\mathbf{M}$ using Eq. 4–10
8:    Compute $\mathbf{z}$ using Eq. 4–9
9:    Solve LCP($\mathbf{M}, \mathbf{z}$) in Eq. 4–8 for $\mathbf{x}^{g+1}$
10:    **if** $\max \left|\mathbf{F}(\mathbf{x}^{g+1})\right| \leqslant \epsilon$ **then**
11:       Terminate
12:    **else**
13:       $g \leftarrow g + 1$
14:    **end if**
15: **end while**

4.6 Summary

This chapter has discussed the bilevel programming problem (BLPP) based on the Stackelberg game from economics. The Stackelberg game is characterised by the leader-follower relationship where the leader implements her strategy, taking into account the reactions of the follower(s). The emphasis has been on the requirement in modelling of toll pricing problems that the system manager is modelled as the Stackelberg leader with the road users collectively playing the role of the followers.

The single leader Stackelberg game can be extended to that of multiple leaders in the form of an Equilibrium Problem with Equilibrium Constraints (EPEC). It is emphasised that this is the common modelling framework, appropriate for separately studying the problems of both *a*) private toll road concessionaires in competition and *b*) jurisdictions in competition. constituting the principle topics investigated in this thesis. Furthermore, the EPEC framework allows for the case where either these leaders act cooperatively or engage in Nash non-cooperative game amongst themselves.

In the case when the leaders' are engaged in a Nash non-cooperative game, a simple Fixed Point Iteration (FPI) algorithm was outlined as a solution methodology to determine the LNE strategies. In addition, an algorithm based on solving a sequence of Linear Complementarity Problems (SLCP) was proposed. The SLCP algorithm was originally proposed for solving oligopolistic games from economics. However, by ensuring that the Stackelberg leader-follower paradigm is adhered to, the original proposal can be easily adapted to solve EPECs. In contrast to the FPI algorithm solving each leader's MPEC iteratively until the entire system converges, the SLCP attempts to solve the entire NCEPEC simultaneously. Both the FPI and SLCP algorithms are implemented to determine LNE in NCEPECs in the case studies in Chapters 6 to 9.

In the next chapter, the situation when leaders are assumed to cooperate, which can be modelled as a Multiobjective Optimisation Problem with Equilibrium Constraints (MOPEC) is discussed. In addition, an alternative algorithm for identifying LNE in games where leaders act non-cooperatively in the NCEPEC is also outlined.

Chapter 5

Evolutionary Approaches to Hierarchical Optimization Problems[5.1]

5.1 Introduction

It has been recognised that the various hierarchical optimisation problems formulated in Chapter 4 “are intrinsically nonsmooth and require the use of generalized differentiation for their analysis and applications” (Mordukhovich, 2004, p. 479). In this thesis, an alternative approach, based on Evolutionary Algorithms (EAs), is suggested.

In recent years, several EAs have been proposed and applied to optimisation problems in a multitude of disciplines. These EA variants include Genetic Algorithms (GA) (Goldberg, 1989), Evolution Strategies (Schwefel, 1995), Particle Swarm Optimization (Kennedy *et al.*, 2001) and Differential Evolution (DE) (Storn and Price, 1997).

The differences between conventional/classical optimisation methods and EAs when applied to optimisation problems are outlined to support the application of EAs to EPECs. Classical methods generally operate on a *single* trial point, transforming it using search directions computed based on first (and possibly, second) order conditions until a criteria indicating convergence to a stationary point is met (Bazaraa *et al.*, 2006; Luenberger and Ye, 2008; Nocedal and Wright, 1999). The ability to

[5.1]Large portions of this chapter are based on Koh (2013), Section 5.5 draws on Koh (2012) and Section 5.6 is new.

exploit derivative information is a double edged sword. On the one hand, derivative information allows for verification of the algorithm's convergence to a stationary point thereby theoretically guaranteeing that a (local) minimum has been found when the algorithm converges. On the other hand, in the case of many multimodal problems encountered in practice, these derivative based algorithms are unable to distinguish between local and global minima. EAs differ from these classical methods in three key aspects (Michalewicz, 1999; Deb, 2001; Yu and Gen, 2010).

Firstly, to generate search directions, canonical EAs rely only on function evaluations and obviate the use of derivatives. Thus EAs are applicable to nonsmooth and non-differentiable optimisation problems such as MPECs and EPECs. There have been a large number of successful applications of EA to problems of these nature (e.g Wang *et al.*, 2007; Amjady and Nasiri-Rad, 2010; Deb *et al.*, 2013). In this respect alone, EAs are thus similar to other "direct search methods" (see e.g. Lewis *et al.*, 2000, for a review) so named as they essentially rely only on objective function evaluations to generate search directions. Techniques such as the pattern search algorithm of Hooke and Jeeves (1961) or the method of dividing rectangles proposed in Jones *et al.* (1993) are examples of direct search techniques that are not classed as EAs.

Secondly, EAs generally operate with a *population* of trial points instead. The principle behind the EA is that of improving each member of the population throughout the operation of the algorithm by way of an analogy with Darwin's theory of evolution. Details of an EA's population based structure are discussed in this Chapter.

Thirdly, classical methods are "deterministic" in the sense that they do not utilise any randomisation to assist in the generation of the next trial point. These algorithms tend to converge to an optimum closest to the initial trial point specified. By using a population of trial points with random search directions based on function evaluations, EAs are generally more suited to exploring the entire problem surface and are intended therefore to identify the global optimum rather than just an arbitrary local optimum. However for an EA to do this, extensive function evaluations are required which implies an increased computational burden vis-á-vis classical methods. It should be noted that there may be no convergence proof that EAs are indeed able to locate the global (or even a local optimum) except under certain restrictive assumptions (Rudolph, 1994). This is why EAs are usually regarded as heuristics. At the same time, the proposed EA for solving NCEPECs introduced in this chapter is, in fact, uniquely endowed with a theoretical convergence proof that an NE has been found, vindicating to some extent its computational burden.

The rest of this chapter is organised as follows. In the next section, the template of a generic EA is outlined to provide an overview of its operation and its key operators. Section 5.3 shows how the EA variant used in this thesis, DE, can be tailored to solve MPECs. By modifying its operating mechanism, DE can then be extended to solve MOPECs. These modifications are discussed in Section 5.4. In a similar way, and a novel contribution of this thesis, a DE based approach to identify NE in NCEPECs is proposed as discussed in Section 5.5. On the one hand, the NCEPEC assumes that players focus on the relentless pursuit of self-interest in a Nash non-cooperative game and in so doing sacrifice elements of mutual interests and, as discussed in Chapter 3 (see Section 3.4.2), could result in Pareto sub-optimal outcomes. On the other hand, as this chapter will highlight, the solution of the MOPEC, reflecting the outcomes available to players should they co-operate instead, is not generally unique. While there are potentially gains for all parties from moving from the NCEPEC outcome to the cooperative outcomes obtained as solutions to the MOPEC, the remaining question is how these gains should be shared between the players. To this end, Section 5.6 provides an overview of axiomatic bargaining theory that provides a framework for answering this question. Section 5.7 summarises.

5.2 Generic Evolutionary Algorithm

In an EA, denote the trial population at iteration or generation g by $\mathcal{P}^g$ which comprises $\pi \equiv |\mathcal{P}|$ members. An illustration of such a population is given in Eq. 5–1. Each individual or member of $\mathcal{P}^g$, is a single trial point, also known as a "chromosome" in the EA literature (Goldberg, 1989; Deb, 2001) and denoted by $\mathbf{x}_k^g = (x_{k,1}^g, \ldots, x_{k,n_1}^g), k = \{1, \ldots, \pi\}$, of n_1 dimensions that represents the leader's variables in an MPEC.

$$\mathcal{P}^g = \begin{pmatrix} \mathbf{x}_1^g \\ \vdots \\ \mathbf{x}_k^g \\ \vdots \\ \mathbf{x}_\pi^g \end{pmatrix} = \begin{pmatrix} x_{1,1}^g & x_{1,2}^g & \cdots & x_{1,n_1}^g \\ \vdots & \vdots & \ddots & \vdots \\ x_{k,1}^g & x_{k,2}^g & \cdots & x_{k,n_1}^g \\ \vdots & \vdots & \ddots & \vdots \\ x_{\pi,1}^g & x_{\pi,2}^g & \cdots & x_{\pi,n_1}^g \end{pmatrix} \qquad \text{(Eq. 5–1)}$$

Algorithm 5.1, adapted from Michalewicz (1999), gives the template of a generic EA for optimisation. The user specifies a termination criteria such as the maximum number of iterations allowed, G, the number of members of the population of trial points, π, as well as control parameters specific to the EA variant utilised. These control parameters are discussed later.

At the start of the algorithm, an initial parent population is randomly generated (Line 3 of Algorithm 5.1). These population members are evaluated and the "fittest"[5.2] population member is identified. The fittest population member is the chromosome in the population corresponding to the lowest (highest) value of the objective function for a minimisation (maximisation) problem.

Subsequently, the parent population is used to produce a child population, $\mathcal{C}^g$, by application of *recombination* operators (Line 6 of Algorithm 5.1). Each member of this child population is evaluated and subsequently, a *selection* operator (Line 8) picks members of $\mathcal{C}^g$ to becomes the parents for the next generation $\mathcal{P}^{g+1}$. Line 9 ensures that the fittest population member found at each iteration is retained to be a parent for the next generation. The process is repeated until some criteria, such as the maximum number of iterations allowed, is reached.

Algorithm 5.1 reflects the typical operation of an EA and highlights that the two key procedures in an EA are the recombination and selection operators. These specific operators in the DE variant are detailed in the next section. To adapt the canonical DE to solve MOPECs and NCEPECs, additional modifications to the selection operator are required as will be highlighted in Sections 5.4 and 5.5 respectively.

[5.2]The term "fittest" used in EAs is derived from its analogy with evolution where Darwin's concept of survival of the fittest is a cornerstone.

Algorithm 5.1 Outline of a Generic Evolutionary Algorithm for Optimisation (Michalewicz, 1999)

```
Input: Maximum Iterations, G,Population Size, π, Control Parameters
g ← 0
Randomly generate Parent Population P^0
Evaluate fitness of P^0
while g < G do
    C^g ←(Recombination)— P^g
    Evaluate fitness of C^g
    P^{g+1} ←(Selection)— C^g
    Retain the fittest population member found at iteration g
    g = g + 1
end while
```

5.3 Differential Evolution

Differential Evolution for Bi-Level Programming (DEBLP) was suggested in Koh (2007) to solve MPECs in transportation systems management. It extends the GA Based Approach (Yin, 2000; Sumalee, 2004*a*) by replacing the use of binary coded GA strings with real coded DE chromosomes (Price *et al.*, 2005) as the search mechanism. DE is a simple algorithm that utilises perturbation and recombination to optimise multi-modal functions and has already demonstrated remarkable success when applied to the optimisation of numerous real-world problems (Storn and Price, 1997; Price, 1999; Price *et al.*, 2005).

A description of the operation of DEBLP to solve the MPEC outlined in Chapter 4 (see Eq. 4–3) is given in the rest of this section. Recall that the lower level problem of the MPEC in transportation systems management (Eq. 4–3b) is a VIP stipulating the followers' route choice equilibrium condition in a highway transportation system. Thus for a fixed vector of the leader's strategies $\mathbf{x}$, this VI can be solved by executing a (DUE/SUE) traffic assignment where the lower level vector, $\mathbf{y}$, in this context, encapsulates equilibrium link flows and demands.

This consideration motivated the development of the DEBLP heuristic which sought to integrate DE's well-documented global search capability to optimise the upper level problem with existing (DUE/SUE) traffic assignment algorithms. Most crucially, DEBLP continues to respect the crucial "leader-follower" paradigm of the

MPEC.

5.3.1 Generate Parent Population

When the algorithm begins, real parameters in each dimension i of each member k of $\mathcal{P}$, constituting the parent population, are randomly generated within the lower and upper bounds of the domain of the upper level variables of the BLPP as given by Eq. 5–2 where $rand(0,1)$ is an operator returning a pseudo random number generated from an uniform distribution between 0 and 1.

$$x_{k,i} = rand(0,1)(x_i^u - x_i^l) + x_i^l, k \in \{1, ..., \pi\}, i \in \{1, ..., n_1\}. \qquad \text{(Eq. 5–2)}$$

Thus each chromosome, $\mathbf{x}_k$, represents a possible vector of the leader's decision variables in the MPEC.

5.3.2 Evaluation

The evaluation process used to determine the fitness of each trial point in $\mathcal{P}^g$ has to be developed bearing in mind the "leader-follower" paradigm of the Stackelberg model. As stressed in Chapter 4, this implies that the leaders have to be modelled as taking into account the response (reaction) of the followers to his strategy $\mathbf{x}$. This can be accomplished by a "two stage" or hierarchical evaluation procedure shown in Algorithm 5.2.

Algorithm 5.2 Hierarchical Evaluation of Chromosome $\mathbf{x}_k$,$k \in \mathcal{P}$

1: Input $\mathbf{x}_k$
2: Execute (DUE/SUE) Traffic Assignment to obtain $\mathbf{y}(\mathbf{x_k})$
3: Evaluate $U_k = U(\mathbf{x}_k, \mathbf{y}(\mathbf{x_k}))$

In the first stage, each individual k vector of the leader's decision variables $\mathbf{x}_k$, a (DUE/SUE) traffic assignment is executed resulting in equilibrium link flows and demands represented here by $\mathbf{y}$. With $\mathbf{y}$ so obtained, the upper level objective U can be evaluated to obtain the value of the upper level objective function in the MPEC, U_k, measuring the fitness of chromosome $\mathbf{x}_k$. Thus in relation to the description of the operation of the algorithms discussed in this thesis, it is implicitly assumed that all references to evaluation implies that evaluation is carried out following the manner of Algorithm 5.2.

It is emphasised that this procedure *differs* from the IOA described in Chapter 4 (see Section 4.2) as the evaluation procedure obviates any iteration between the upper and lower level problems. Instead, entirely consistent with the "leader-follower" paradigm, the leader's vector $\mathbf{x}_k$ being manipulated by DE is an input to the lower level traffic assignment. One obvious drawback of doing this is the resulting increase in computational burden which has been significantly reduced by advances in computing power.

5.3.3 Recombination Operator in DE: Mutation and Crossover

The recombination process in DE is achieved through the mutation and crossover process which aims to produce a child vector $\mathbf{w}_k$ from the parent. This is accomplished by stochastically adding to the parent vector, the factored difference of two other randomly chosen vectors from the population as shown in Eq. 5–3 (Storn and Price, 1997).

$$w_{k,i} = \begin{cases} x_{s_1,i} + \lambda(x_{s_2,i} - x_{s_3,i}) & \text{if } rand(0,1) < \chi \text{ or } i = intr(1, n_1) \\ x_{k,i} & \text{otherwise} \end{cases} \qquad \text{(Eq. 5–3)}$$

In Eq. 5–3, s_1,s_2 and $s_3 \in \{1, 2, \ldots, \pi\}$ are randomly chosen population indices distinct from each other and also distinct from the current population member index k. $rand(0, 1)$ is an operator that returns a pseudo random real number between 0 and 1. Similarly, $intr(1, n_1)$ is an operator that returns a pseudo random *integer* between 1 and n_1, i.e. the number of upper level decision variables in the MPEC. The mutation factor, λ, between 0 and 2 (Storn and Price, 1997, p. 344), is a parameter controlling the magnitude of the perturbation and, χ, is the probability (between 0 and 1) that controls the ratio of new components in the offspring. Both λ and χ are user-defined control parameters. The or condition in Eq. 5–3 ensures that the child vector $\mathbf{w}_k$ will differ from its parent $\mathbf{x}_k$ in at least one dimension. Note that the mutation and crossover strategy shown in Eq. 5–3 is not the only possible methods available though this is the one used in this work. Other techniques are found in the literature (Storn and Price, 1997; Price, 1999; Price *et al.*, 2005). Thus, the aim of the recombination operator in DE is to create the child vector $\mathbf{w}_k$ "by adding the weighted difference between two population vectors to a third vector" (Storn and Price, 1997, p. 343).

5.3.4 Enforce Bound Constraints

Application of the recombination process described in Section 5.3.3 above, however, can generate child vectors that lie outside the bound constraints of the leader's decision variables (i.e. $w_{k,i} < \underline{x}_i$ or $w_{k,i} > \bar{x}_i$). There are several ways to ensure satisfaction of these constraints. One could set each dimension violated equal to the bound of the variable violated or regenerate it within the bounds. However, applying these methods could lead to a loss of diversity which could lead to premature convergence. Instead, a compromise proposed in Price (1999) is used where out of bound values in each dimension i are reset to half way between its pre-mutation value and the bound violated as shown in Eq. 5–4.

$$w_{k,i} = \begin{cases} (x_{k,i} + \underline{x}_i)/2 \text{ if } w_{k,i} < \underline{x}_i \\ (x_{k,i} + \bar{x}_i)/2 \text{ if } w_{k,i} > \bar{x}_i \\ \quad w_{k,i} \text{ otherwise} \end{cases} \tag{Eq. 5–4}$$

Thus in DE, an individual child vector is created by application of the recombination operators (mutation and crossover) as discussed in Section 5.3.3 and the enforcement of bound constraints in Section 5.3.4. This is summarised in Algorithm 5.3.

Algorithm 5.3 Creation of Child Chromosome with Differential Evolution

1: Input: Current Population $\mathcal{P}$, Index k of current population member, Mutation Factor λ, Probability of Crossover χ
2: Randomly choose 3 integer indices s_1, s_2, s_3 between 1 and π such that $s_1 \neq s_2 \neq s_3 \neq k$.
3: Apply Recombination (Mutation and Crossover) using Eq. 5–3.
4: Enforce Bound Constraints using Eq. 5–4.
5: Output: child vector $\mathbf{w}_k$

5.3.5 Selection

Each child chromosome $\mathbf{w}_k$ is then evaluated using Algorithm 5.2 to obtain its fitness. This fitness can then be compared with that of its parent $\mathbf{x}_k$. In other words, comparison takes place between the same k^{th} member of the child population, $\mathcal{C}^g$, and the same k^{th} member of the parent population, $\mathcal{P}^g$. The chromosome that returns a larger value of the upper level objective between the two (assuming maximisation) is then selected to become a member of the parent population in the next generation. This procedure is summarised in Eq. 5–5.

$$\mathbf{x}_k^{g+1} = \begin{cases} \mathbf{w}_k^g & \text{if } U(\mathbf{w}_k^g, L(\bullet)) \geq U(\mathbf{x}_k^g, L(\bullet)) \\ \mathbf{x}_k^g & \text{otherwise} \end{cases} \qquad \text{(Eq. 5–5)}$$

The DE-specific techniques of child/trial vector creation as summarised in Algorithm 5.3 serves as the building block of two further algorithms designed to address EPECs which are then applied to the case studies. These two algorithms will be discussed in the rest of this chapter.

5.4 Multiobjective Optimisation Problems with Equilibrium Constraints

As pointed out in Chapter 4 (see Section 4.3), the multiple leader-follower EPEC model in which the upper level leaders are assumed to cooperate can be formulated as a Multiobjective Optimisation Problem with Equilibrium constraints (MOPEC) (Mordukhovich, 2004). Such a program is shown in Eq. 5–6.

$$\text{MOPEC} \begin{cases} \underset{\mathbf{x} \in X}{\text{Maximise}}\ \mathbf{\Phi}(\mathbf{x}, \mathbf{y}(\mathbf{x})) = (\phi_1(\mathbf{x}, \mathbf{y}(\mathbf{x})), \ldots, \phi_n(\mathbf{x}, \mathbf{y}(\mathbf{x})))^\intercal & \text{(Eq. 5–6a)} \\ \qquad \text{subject to} & \\ \qquad \mathbf{y}(\mathbf{x}) \leftarrow \text{SOL}\{\mathcal{V}(\mathbf{x})\} & \text{(Eq. 5–6b)} \end{cases}$$

In contrast to usual optimisation programs which optimise an objective function returning a scalar value (a "scalar objective function"), the objective function in the MOPEC, $\mathbf{\Phi}(\mathbf{x})$, is a *vector*, and can be viewed as the "concatenation" of the set of $\mathcal{N}$ individual objective functions i.e. $\mathbf{\Phi} : \mathbb{R}^{n_1} \times \mathbb{R}^{n_2} \mapsto \mathbb{R}^n$.

The upper level problem (Eq. 5–6a) is a standard Multiple Objective Optimisation Problem (MOP) that has been extensively discussed in the literature (Cohon, 1978; Chankong and Haimes, 1983; Sawaragi *et al.*, 1985; Osyczka, 1998; Deb, 2001; Miettinen, 2001; Marler and Arora, 2004; Ehrgott, 2005). The specific difference in the MOPEC is that this upper level MOP is constrained by a VIP (Eq. 5–6b) which, as emphasised in Chapter 4, specifies an equilibrium condition in the parametric system predicated on the upper level decision vector $\mathbf{x}$. Therefore, any solution methodology for MOPECs must continue to obey the leader-follower paradigm of Stackelberg games and this can be achieved by applying the hierarchical evaluation procedure outlined in Algorithm 5.2.

In single objective optimisation problems e.g. Bazaraa *et al.* (2006), the goal is to find a single optimal solution. However, in MOPs including MOPECs, due to the potential contradiction of objectives, there is usually no single "best" solution (Miettinen, 2001). Instead, the solution concept for MOPs is based on the principle of Pareto Optimality stated in Definition 3.3. Thus optimality in the context of MOPs means that solutions are optimal if each component of the objective function *cannot be improved without deterioration to at least one other component* (Miettinen, 2001; Marler and Arora, 2004; Ehrgott, 2005). In general, it is expected that there are a number of these Pareto Optimal solutions known as the "Pareto Front" (when viewed in objective function space) or "Pareto Set" (when viewed in solution space). Thus, the twin goals of solving MOPs are firstly, to identify the Pareto Front pertaining to the problem at hand and secondly, to find solutions as diverse as possible on this Pareto Front (Deb, 1999, 2001; Robič and Filipič, 2005; Tan *et al.*, 2005).

5.4.1 Interpretation of Multiobjective Formulation

While MOPECs in the form of Eq. 5–6 have been previously formulated in the transportation systems management literature (Friesz, 1981; Friesz *et al.*, 1993; Anderson *et al.*, 1998; Taber *et al.*, 1999; Yin, 2002; Balling *et al.*, 2003; Cantarella and Vitetta, 2006; Sumalee *et al.*, 2009), it is crucial to stress that these works have all focused their investigations on the choices facing a *single* regulator confronted with multiple, conflicting objectives. Friesz (1981) formulated and Friesz *et al.* (1993) simulated a model of the tradeoffs, faced by a regulator, taking into account conflicting objectives of user costs, travel distance, land take required in the network design problem and proposed two potential solution algorithms. Taber *et al.* (1999) applied a GA to a model of Provo, Utah, USA to consider three conflicting objectives of minimising travel time, per capita cost and land use changes in the network design problem while Sumalee *et al.* (2009) constructed a GA to design toll cordons that maximise both efficiency and toll revenues but minimizing distributional equity impacts within the context of a sole regulator.

In the aforementioned works, each element of $\mathbf{\Phi(x, y)}$ reflects a different objective of the single regulator. However, in concert with the principle theme of this thesis, each element of $\mathbf{\Phi(x, y)}$ reflects the objective of a *different* agents exercising control over toll pricing strategies in the highway network. Therefore while the model formulation of the MOPEC in Eq. 5–6 is the same as in antecedent literature, the underlying paradigm of this thesis that motivates the formulation itself represents a departure from the previous studies. Here, the focus is instead on the Pareto Optimal tradeoffs among multiple decision makers with conflicting objectives.

5.4.2 Classical Techniques for MultiObjective Problems

In this section, two classical methods for solving MOPs are reviewed. Both methods share in common the feature that the objective function in the MOPEC in Eq. 5–6 is transformed such that it returns a scalar instead of a vector. The first approach is the weighting method, proposed in Zadeh (1963). This method aggregates all n objectives into a single objective function that could potentially be optimised by classical techniques. The second approach is the ϵ constraint method, proposed in Haimes *et al.* (1971), which also operates by operates by selecting in turn, one component objective from $\mathbf{\Phi(x, y)}$ to serve as the objective function while the other objectives enter as constraints to the reformulated optimisation problem.

Note that these two methods are not the only two available for MOPs in the literature. However, the literature (Miettinen, 1999; Deb, 2001; Miettinen, 2001; Marler and Arora, 2004; Ehrgott, 2005) indicates that these methods are the most suitable for the application of classical optimisation techniques which were designed to handle scalar objective functions.

Weighting Method

When applied to MOPECs, the weighting method first associates each objective function with a weighting coefficient, $\varpi_i, i \in \{1, \ldots, n\}$ and then subsequently maximises the resulting weighted sum of objectives (Miettinen, 1999). This procedure thus transforms the original MOPEC into the Weighted MOPEC in Eq. 5–7 with a scalar objective function.

$$\text{Weighted MOPEC} \begin{cases} \underset{\mathbf{x} \in X}{\text{Maximise}} \quad \sum_{i=1}^{n} \varpi_i \phi_i(\mathbf{x}, \mathbf{y}(\mathbf{x})) & \text{(Eq. 5–7a)} \\ \text{subject to} & \\ \quad \mathbf{y}(\mathbf{x}) \quad \leftarrow \text{SOL}\{\mathcal{V}(\mathbf{x})\} & \text{(Eq. 5–7b)} \end{cases}$$

where $\varpi_i \geqslant 0$ and $\sum_{i=1}^{n} \varpi_i = 1$. It can be proven that the optimal and unique solution of Eq. 5–7, for a given weight combination, is Pareto Optimal (Miettinen, 1999, p. 79, Theorem 3.1.1). This implies that an MPEC solver can be used to solve the MOPEC to determine a single Pareto Optimal solution. It follows from this idea that repeated solution of Eq. 5–7 with a different set of weights each time could be used to generate the entire Pareto Front.

Though this idea seems simple and promising, at least three caveats are necessary. Firstly, it is difficult, if not impossible, to verify that the solution of Eq. 5–7 is unique, which is a required condition for a solution discovered to be a Pareto Optimal point. Secondly, this approach succeeds in getting points from all parts of the Pareto Front *only when the Pareto Front is convex.* (Das and Dennis, 1997). However, as stressed in Chapter 4, the general MPEC is non-convex (Luo *et al.*, 1996). Therefore, there is no *a priori* reason to expect that the Weighted MPEC formulated would be convex and thus not all Pareto Optimal solutions can be identified by repeated application of this method. Thirdly, the use of an evenly distributed set of weights *does not* result in an even spread of points on the Pareto Front (Das and Dennis, 1997; Miettinen, 1999; Marler and Arora, 2004).

Despite these drawbacks, the weighting method has been applied to the solution of MOPECs in the transportation systems management literature and elsewhere (Marler and Arora, 2004). For example, Friesz *et al.* (1993) applied the weighting method to identify Pareto Optimal points in a multiobjective network design problem.

ϵ-Constraint Method

In the ϵ-Constrained Method, one of the objective functions in $\mathbf{\Phi(x, y)}$ is selected to be optimised and all other objective functions are converted into constraints by setting a lower bound (in the case of simultaneous maximisation of all objectives) for them (Chankong and Haimes, 1983; Miettinen, 1999; Deb, 2001). Thus the MOPEC may be formulated as shown in Eq. 5–8 with the other objectives constrained to achieve some minimum "target" value, $T_i, i \in \{1, \ldots, n\}, i \neq \nu$.

$$\epsilon \text{ Constraint} \begin{cases} \underset{\mathbf{x} \in X}{\text{Maximise}} \quad \phi_\nu(\mathbf{x}, \mathbf{y}(\mathbf{x})) & \text{(Eq. 5–8a)} \\ \text{subject to} & \\ \phi_i(\mathbf{x}, \mathbf{y}(\mathbf{x})) \geqslant T_i, i \in \mathcal{N}, i \neq \nu & \text{(Eq. 5–8b)} \\ \qquad \mathbf{y}(\mathbf{x}) \quad \leftarrow \text{SOL}\{\mathcal{V}(\mathbf{x})\} & \text{(Eq. 5–8c)} \end{cases}$$

While it can be proven that the ϵ constraint method is indeed able to identify Pareto Optimal solutions (Miettinen, 1999, p. 86, Theorem 3.2.3), the main difficulty is that the above proof holds with the qualification that the solution of Eq. 5–8 is unique. As is the case with the weighting method, in practical applications, verification that this uniqueness condition is satisfied is likely to be difficult. In practice, deciding the appropriate value of the target values $T_i, i \in \mathcal{N}, i \neq \nu$ for some objectives would be difficult and problem dependent. It is inevitable that the choice of some target values $T_i, i \in \mathcal{N}$ could result in an infeasible optimisation problem (Deb, 2001).

Other methods to solve MOPs, that can be adapted to solve MOPECs, can be found in the literature (Stadler, 1988; Osyczka, 1998; Miettinen, 1999; Deb, 2001; Marler and Arora, 2004; Ruzika and Wiecek, 2005; Ehrgott, 2005). Techniques include goal programming (Charnes *et al.*, 1955; Schniederjans, 1995) and the weighted Tchebycheff method (Steuer and Choo, 1983). In addition, a nonsmooth method for solving MOPs has also been proposed (Haarala *et al.*, 2007) and applied to solve a MOPEC formulated in Mordukhovich *et al.* (2007). In general, in common with both the weighting and ϵ-constraint methods, these techniques operate by transforming the MOP into a mathematical optimisation problem amenable to application of classical optimisation methods.

5.4.3 Multiobjective Evolutionary Algorithms (MOEAs)

Another particular area where extensive research has been undertaken is in the application of EAs to MOPs. As highlighted earlier, the aim in solving MOPs is not to identify just a single solution but as many diverse Pareto Optimal solutions as possible (Deb, 2001; Yin, 2002; Coello Coello *et al.*, 2007). In this respect, as EAs operate with populations of trial solutions (Eq. 5–1 refers), EAs offer the potential for conducting simultaneous parallel search. This is an important feature giving EAs an advantage over the classical methods. Though a MOEA is a heuristic, lacking any theoretical proof that an EA can indeed identify one Pareto Optimal solution, it has been highlighted that the "ability of an EA to find multiple optimal solutions ... makes EAs unique in solving multiobjective problems" (Deb, 2001, p. 8). This accounts for the rapid expansion of research into Multiobjective Evolutionary Algorithms (MOEA).

One of the earliest MOEAs was the Vector Evaluated Genetic Algorithm (Schaffer, 1985). Since then, a multitude of MOEAs have been proposed. These have included the Non Dominated Sorting Genetic Algorithm (Srinivas and Deb, 1994; Deb *et al.*, 2002), Niched Pareto Genetic Algorithm (Horn *et al.*, 1994), Multiobjective Genetic Algorithm (Fonseca and Fleming, 1998), Strength Pareto Archived Evolution Strategy (Zitzler and Thiele, 1999) and Pareto Archived Evolution Strategy (Knowles and Corne, 2000), amongst others. At the same time, literature documenting successful applications of MOEA in a multitude of disciplines abound. These have ranged from the medical sciences (Yu, 1997), truss optimization in civil engineering (Narayanan and Azarm, 1999), hydrology (Kollat and Reed, 2006) as well as MOPECs in transportation systems management (Yin, 2002; Sumalee *et al.*, 2009).

As a result of the rapid developments in MOEA, it is recognised that "evolutionary techniques constitute probably the most successful approach for solving MOPs in practice" (Ehrgott and Wiecek, 2005, p. 707). In view of this, the MultiObjective Self Adaptive Differential Evolution (MOSADE) Algorithm, a DE based MOEA proposed by Huang *et al.* (2007), will be used to generate the Pareto Fronts for the MOPECs formulated in this thesis.

MOSADE is a MOEA that operates in a similar way to a generic EA (Algorithm 5.1) but with two key differences. The first difference is in the selection operator. In solving MOPs, it is no longer possible to compare fitness of two chromosomes using Eq. 5–5 as there are multiple objectives. Instead, the selection process that determines which chromosome is "fitter" is based on the principle of Pareto Optimality (as given in Definition 3.3).

The second difference is the use of an "archive" as a repository to store potentially Pareto nondominated solutions discovered as the algorithm proceeds. It is envisaged that solutions contained in this archive would approximate the Pareto Front at termination. However, because of the stochastic nature of EAs, it is possible that the application of recombination operators could eliminate such solutions if they are not set aside during the intermediate stages (Knowles and Corne, 2000; Marler and Arora, 2004). As the algorithm proceeds, this archive is updated so that a store of "good" solutions discovered is retained. Such an archiving strategy reflects the "elitism" principle of the EA literature (Goldberg, 1989; Michalewicz, 1999) which recommends retention of high quality solutions discovered during the operation of the algorithm.

The operation of MOSADE is described with reference to Algorithm 5.5 as follows. When MOSADE begins, a parent population, $\mathcal{P}^0$, is randomly generated and evaluated. At the same time, the external archive, $\mathcal{A}$, is initialised with $\mathcal{P}^0$. Subsequently, a child population, $\mathcal{C}^g$, is generated using the DE recombination operators of mutation and crossover (Algorithm 5.3) and these are evaluated.

In the selection phase, a Pareto Domination (Definition 3.3) comparison between parent($\mathbf{x}$) and child ($\mathbf{w}$) chromosomes takes place. If the parent dominates the child ($\mathbf{x} \prec \mathbf{w}$), the child is discarded and the algorithm moves on to perform Pareto Domination comparison on the next member in the population (Lines 18 to 20 in Algorithm 5.5). However, in the cases when the child either Pareto Dominates the parent ($\mathbf{w} \prec \mathbf{x}$) or is Pareto Incomparable with respect to the parent ($\mathbf{w} \parallel \mathbf{x}$), the child becomes the parent for the next generation (Lines 22 and 25). The next step in this case is to decide if the child enters the archive. This decision process for this step is shown in Algorithm 5.4.

As can be seen, a candidate, $\mathbf{w}$, enters the archive if it either *a*) Pareto Dominates any existing member of the archive or *b*) is Pareto Incomparable with respect to any member of the archive. In the former instance, the existing members in the archive that are dominated by $\mathbf{w}$ will be removed from the archive.

As noted above, an additional aim in solving MOPs is to identify as many diverse solutions of the Pareto Front as possible. Thus returning to Line 30 of Algorithm 5.5, should the size of the archive ($|\mathcal{A}|$) exceed the user specified size, $\bar{\mathcal{A}}$, the k^{th} Nearest Neigbour technique (Friedman *et al.*, 1977) is applied to identify the closest (in terms of euclidean distance) neighbours to a chromosome in function space. These neighbours identified are then removed from the archive.

Additionally, MOSADE dynamically updates the mutation factor, λ, and probability of crossover, χ, so that fewer control parameters are required from the user. Details of this process can be found in Huang *et al.* (2007).

5.5 Evolutionary Algorithms for NCEPECS

This section describes the development of an EA to find solutions of NCEPECs. As noted in Chapter 4, such a solution should be a LNE satisfying Definition 3.4. Thus as a first step, EA methods for identifying LNE are discussed. As EAs rely only on function evaluations, the methodologies proposed can be adapted to determine

Algorithm 5.4 Update Archive $\mathcal{A}$ (Huang *et al.*, 2007)

Input: vector entering into the archive $\mathbf{w}$
if any archive member Pareto Dominates $\mathbf{w}$ **then**
    discard $\mathbf{w}$
else if $\mathbf{w}$ Pareto Dominates a subset of existing archive members **then**
    accept $\mathbf{w}$ into Archive.
    delete the dominated members.
else if $\mathbf{w}$ is Pareto Incomparable with existing archive members **then**
    accept $\mathbf{w}$ into Archive.
end if
Output: Updated Archive $\mathcal{A}$

LNE in the NCEPEC (Eq. 4–4) as long as the evaluation of chromosomes is carried out following the hierarchical evaluation method discussed in Algorithm 5.2.

5.5.1 Literature Review

The ability of EAs to optimise nonsmooth and non-differentiable functions has resulted in the development of an EA version of the FPI Algorithm discussed in Chapter 4 (see Section 4.4). The algorithm operates as outlined in the Algorithm 4.1 with the only difference being that an EA, rather than a conventional gradient-based optimisation method, is used to solve each player's optimisation problem (Son and Baldick, 2004; Chen *et al.*, 2006; Razi *et al.*, 2007; Rajabioun *et al.*, 2008; Hajimirsadeghi *et al.*, 2009; Ladjici and Boudour, 2010).

Another stream of research has exploited the principle of co-evolution which abstracts from host-parasite or predator-prey co-evolution observed in nature. The idea is summarised thus: "There are many examples in nature of organisms that evolve defense to parasites that attack them only to have the parasites evolve in ways to circumvent the defenses, which results in the hosts' evolving new defenses, and so on in an ever-rising spiral-a "biological arms race""(Mitchell, 1996, p. 20).

In applying the co-evolutionary concept to the detection of NE, each sub-population, embodying the strategies of a single player, player $i, i \in \{1, \ldots, n\}$, is evolved separately. However, the fitness of each member of a single sub-population is determined by evaluation against *all other sub-populations*. By analogy with co-evolution observed in nature, the aim is to evolve fitter individuals/strategies in each sub-

Algorithm 5.5 MultiObjective Self Adaptive Differential Evolution (Huang *et al.*, 2007)

Input: π, G, payoff functions
Input: Maximum Archive Size $\bar{\mathcal{A}}$
$g \leftarrow 0$
$\mathcal{A} \leftarrow \emptyset$
Randomly initialize a population of π parent strategy profiles $\mathcal{P}$
Evaluate payoffs to players with $\mathcal{P}$
$\mathcal{A} \leftarrow \mathcal{P}^0$
while $g < G$ **do**
for $j = 1$ to π **do**
use Algorithm 5.3 to create child strategy profiles vector w_j
$\mathcal{C}_j^g \leftarrow \mathbf{w}_j$
end for
Evaluate payoffs to players with $\mathcal{C}$
for $j = 1$ to π **do**
$\mathbf{x} \leftarrow \mathcal{P}_j^g$
$\mathbf{w} \leftarrow \mathcal{C}_j^g$
Apply Definition 3.3 to determine if $\mathbf{x} \prec \mathbf{w}$
if $\mathbf{x} \prec \mathbf{w}$ **then**
discard $\mathbf{w}$
$\mathcal{P}_j^{g+1} \leftarrow \mathbf{x}$
else if $\mathbf{w} \prec \mathbf{x}$ **then**
$\mathcal{P}_j^{g+1} \leftarrow \mathbf{w}$
Use Algorithm 5.4 to decide if $\mathbf{w}$ enters $\mathcal{A}$
else if $\mathbf{x} \parallel \mathbf{w}$ **then**
$\mathcal{P}_j^{g+1} \leftarrow \mathbf{w}$
If $\mathbf{x} \notin \mathcal{A}$, use Algorithm 5.4 to decide if $\mathbf{x}$ enters $\mathcal{A}$
Use Algorithm 5.4 to decide if $\mathbf{w}$ enters $\mathcal{A}$
end if
end for
if $|\mathcal{A}| > \bar{\mathcal{A}}$ **then**
Use k^{th} Nearest Neighbour algorithm (Friedman *et al.*, 1977) to delete nearest neighbours in function space from $\mathcal{A}$
end if
Update DE Control Parameters: χ and λ
$g \leftarrow g + 1$
end while
Output: Pareto Optimal Solutions

population to counter the fitter strategies of the opponent populations as they too evolve. Algorithm 5.6 outlines the search algorithm for locating the NE that incorporates the concept of co-evolution.

Algorithm 5.6 Coevolutionary Algorithm for Identifying NE

1: Input: Maximum Iterations, G
2: $g \leftarrow 0$
3: Initialize n sub-populations of size π, $\mathcal{P}_i, \{i = 1, 2, \ldots, n\}$
4: Randomly select one member from each $\mathcal{P}_j$ as the initial strategy: $\mathbf{x}^0 = (x_1^0, x_2^0, \ldots, x_n^0)$
5: **while** $g < G$ **do**
6:    **for** $i = 1$ to n **do**
7:       Apply EA to evolve $\mathcal{P}_i$
8:       Update strategy vector $\mathbf{x}^g$ with the fittest member of $\mathcal{P}_i$
9:    **end for**
10:    $g \leftarrow g + 1$
11: **end while**

In Algorithm 5.6, each sub-population $\mathcal{P}_i, \{i = 1, 2, \ldots, n\}$ comprising π members each, encodes the strategies of each player in the game. Thus there are as many sub-populations as players. An EA is subsequently used to evolve each separately for a number of specified iterations (Line 7 of Algorithm 5.6). However, as pointed out in Pedroso (1996), the best solution of each population $\mathcal{P}_i, \{i = 1, 2, \ldots, n\}$ found at iteration g may not necessarily be the best when the strategy vector is updated (Line 8). Hence at the start of next iteration, it is essential to evaluate the entire population again to determine the fittest member following the revelation of all player's strategies. One of the first applications of this method to several examples from economics such as the Cournot game as discussed in Chapter 3 can be found in Curzon Price (1997). It should be pointed out that a Particle Swarm based co-evolutionary algorithm for the NCEPEC was proposed in Koh (2010).

In practice, there have been opposing views with regards to the performance of co-evolutionary methods as outlined in Algorithm 5.6. Son and Baldick found that simple co-evolutionary algorithms failed to converge to the LNE in one instance (Son and Baldick, 2004, p. 310) while produced misleading results in another (Son and Baldick, 2004, p. 314). On the other hand, both Razi *et al.* (2007) and Koh (2010) did not report such difficulties in their applications of the co-evolutionary concept.

In this thesis, another proposal for the identification of NE, known as Nash Domination, is discussed. The attractiveness of Nash Domination is its close theoretical relationship to the definition of NE itself and in particular, Corollary 3.1. Nash Domination can be further integrated into an EA, enabling NE strategies to be identified. In addition, the application of Nash Domination provides theoretical proof of convergence of the algorithm to an NE and is a unique feature of the proposed EA.

5.5.2 Nash Domination Evolutionary Multiplayer Optimisation

As noted in Corollary 3.1 in Chapter 3, at an NE, players would not benefit from unilaterally deviating from their current strategies. This observation has led Lung and Dumitrescu (2008) to define Nash Domination, a concept analogous to Pareto Domination employed as the selection criteria in MOPECs as discussed in Section 5.4. This section defines Nash Domination before outlining an EA to identify the NE in an NCEPEC which explicitly embodies this concept.

In comparing two strategy profiles, the Nash Domination principle operates by counting the number of players that could potentially benefit if each player deviates from one profile to the other *unilaterally.* Then the strategy profile resulting in *fewer* players being incentivised to unilaterally deviate *away* from it is deemed to be closer to an NE following Corollary 3.1.

Let $\mathbf{a}$ and $\mathbf{b}$ be two strategy profiles, $\mathbf{a}, \mathbf{b} \in X$ where $\mathbf{a} \equiv \{a_1, a_2, \ldots, a_i, \ldots, a_n\}^\intercal$ and $\mathbf{b} \equiv \{b_1, b_2, \ldots, b_i, \ldots, b_n\}^\intercal$. Then the strategy profile written as $(b_i, \mathbf{a}_{-i})$ has the interpretation that player i uses strategy b_i while every other player uses strategies from $\mathbf{a}$ i.e. $(b_i, \mathbf{a}_{-i}) \equiv \{a_1, \ldots, a_{i-1}, b_i, a_{i+1} \ldots a_n\}^\intercal$. Similarly, $(a_i, \mathbf{b}_{-i}) \equiv \{b_1, \ldots, b_{i-1}, a_i, b_{i+1} \ldots b_n\}^\intercal$.

Let Ψ_a be the number of players that could benefit by unilaterally switching to b_i when everyone else plays $\mathbf{a}_{-i}$. Similarly, let Ψ_b be the number of players that could benefit by unilaterally switching to a_i when everyone else plays $\mathbf{b}_{-i}$.

It should be obvious that $0 \leqslant \Psi_a, \Psi_b \leqslant n$, since there may be either none or anywhere up to a maximum of all n players that could benefit from unilateral deviation. The procedure to determine Ψ_a and Ψ_b is given in Algorithm 5.7.

Algorithm 5.7 Nash Domination Comparison

Initialise $\Psi_a = 0$, $\Psi_b = 0$
for $i = 1$ to n **do**
    if $\phi_i((b_i, \mathbf{a}_{-i}), \mathbf{y}) > \phi_i(\mathbf{a}, \mathbf{y})$ **then**
        $\Psi_a = \Psi_a + 1$
    else if $\phi_i((a_i, \mathbf{b}_{-i}), \mathbf{y}) > \phi_i(\mathbf{b}, \mathbf{y})$ **then**
        $\Psi_b = \Psi_b + 1$
    end if
end for

Applying Algorithm 5.7, one, and only one, of the following outcomes must be true (Lung and Dumitrescu, 2008, Remark 4, p. 365):

1. $\Psi_a < \Psi_b \implies$ **a** Nash Dominates **b**, written as $\mathbf{a} \prec_{\mathrm{N}} \mathbf{b}$, or

2. $\Psi_b < \Psi_a \implies$ **b** Nash Dominates **a**, written as $\mathbf{b} \prec_{\mathrm{N}} \mathbf{a}$, or

3. $\Psi_a = \Psi_b \implies$ **a** and **b** are Nash Non Dominated with respect to each other, written as $\mathbf{a} \parallel_{\mathrm{N}} \mathbf{b}$.

Proposition 5.1. All Nash Non Dominated chromosomes are NE.

Proof. See Lung and Dumitrescu (2008), Proposition 9, p. 366. □

Proposition 5.1, therefore, theoretically assures that the solution found by application of the Nash Domination principle will not just be a LNE but also the NE when viewed in the full strategy space. Thus based on Proposition 5.1, an EA for determination of NE in NCEPECs known as Nash Domination Evolutionary Multiplayer Optimisation (NDEMO), was proposed in Koh (2012). The crucial change required, compared to the Generic EA given in Algorithm 5.1, is in the selection operator. Each chromosome represents a strategy profile i.e. strategies of all players in the game. A chromosome is now judged to be "fitter" compared to another if it Nash Dominates its competitor.

NDEMO operates as shown in Algorithm 5.8. The user specifies the maximum number of iterations, G, the population size, π, the termination tolerance, $\epsilon(> 0)$, the control parameters of DE i.e. Mutation Factor, λ, and Probability of Crossover, χ, and a procedure to evaluate payoffs. An initial population at iteration $g = 0$ of parent strategy profiles, $\mathcal{P}^g$, is generated randomly and then evaluated. Subsequently, a child population of strategy profiles, $\mathcal{C}^g$, are created by applying the DE operators given in Algorithm 5.3 and these are evaluated.

At each iteration, parent and child strategy profiles are compared one by one, following the Nash Domination Comparison procedure of Algorithm 5.7. Again, as in canonical DE, the Nash Domination comparison takes place between the same k^{th} member of the child population, $\mathcal{C}^g$, and the same k^{th} member of the parent population, $\mathcal{P}^g$. It should be noted that this procedure can be computationally demanding when applied to the NCEPECs discussed in this thesis since this necessitates a separate traffic assignment for each unilateral deviation so as to compute Ψ_a and Ψ_b to determine the Nash Domination status.

From this procedure, chromosomes identified as Nash Non Dominated are placed in a temporary population $\mathcal{T}$. However, this also means that the size of $\mathcal{T}$ (denoted $|\mathcal{T}|$) could potentially exceed the user defined population size, π. If this happens, $\mathcal{T}$ is randomly trimmed so that there will always be only π parents for the next generation (lines 28 to 30 of Algorithm 5.8). Convergence is checked by computing the standard deviation of the population (σ). If σ is less than ϵ, the population is judged to have converged and the algorithm terminates. Otherwise, the counter is increased and the process is repeated.

5.6 Bargaining

While Pareto Optimality (cf. Definition 3.3) means that the outcomes to the players acting cooperatively would lie on the Pareto Front, this still "leaves open a large number of possibilities since the Pareto Frontier usually contains many points" (Bergin, 2005, p. 281) and is not, in general, unique. Thus even after some computational effort had been expended to identify the Pareto Front, it would still be necessary to " specify how colluding firms select a point" (Schmalensee, 1987, p. 357) on this front. In general, the key question of interest in the study of bargaining

Algorithm 5.8 Nash Domination Evolutionary Multiplayer Optimization (NDEMO) Koh (2012)

Input: π, G, Termination Tolerance $\epsilon(> 0)$, DE Control Parameters, payoff functions
$g \leftarrow 0$
Randomly initialise a population of π parent strategy profiles $\mathcal{P}^g$
Evaluate payoffs to players with $\mathcal{P}^g$
Compute standard deviation (σ) of $\mathcal{P}^g$
while $g < G$ **or** $\sigma \geqslant \epsilon$ **do**
  for $j = 1$ to π **do**
    Apply Algorithm 5.3 to create child strategy profiles vector $\mathbf{b}$
    $\mathcal{C}_j^g \leftarrow \mathbf{w}_j$
  end for
  Evaluate payoffs to players with $\mathcal{C}$
  $\mathcal{T} \leftarrow \emptyset$
  for $j = 1$ to π **do**
    $\mathbf{x} \leftarrow \mathcal{P}_j^g$
    $\mathbf{w} \leftarrow \mathcal{C}_j^g$
    Apply Algorithm 5.7 to determine if $\mathbf{x} \prec_{\mathrm{N}} \mathbf{w}$
    if $\mathbf{x} \prec_{\mathrm{N}} \mathbf{w}$ **then**
      discard $\mathbf{w}$
      $\mathcal{T} \leftarrow \mathbf{x}$
    else if $\mathbf{w} \prec_{\mathrm{N}} \mathbf{x}$ **then**
      discard $\mathbf{a}$
      $\mathcal{T} \leftarrow \mathbf{w}$
    else
      $\mathcal{T} \leftarrow \mathbf{x}$
      $\mathcal{T} \leftarrow \mathbf{w}$
    end if
  end for
  if $|\mathcal{T}| > \pi$ **then**
    Randomly trim $\mathcal{T}$ until $|\mathcal{T}| = \pi$
  end if
  Compute standard deviation (σ) of $\mathcal{P}_j$
  if $\sigma \leqslant \epsilon$ **then**
    Terminate
  else
    $\mathcal{P}^{(g+1)} \leftarrow \mathcal{T}$
    $g \leftarrow g + 1$
  end if
end while
Output: Nash Non Dominated Solutions

is how the "surplus" or gains, obtained by moving from the non-cooperative outcome to the cooperative outcome on the Pareto Front, should be divided between the participants (Serrano, 2008).

There are two approaches to the problem of surplus division between parties (Nash, 1953; Serrano, 2008). In the first, or strategic approach, "the cooperative game is reduced to a non-cooperative game" (Nash, 1953, p. 129). This approach is concerned with the evolution of the bargaining process; namely, the "exact specification of the details of negotiation ... and the identification of behaviour that would occur in those protocols" (Serrano, 2008, p. 371). This approach is not investigated in this thesis. Instead, the focus is on the second, or normative approach, through Axiomatic Bargaining Theory. This approach operates by outlining "properties that it would seem natural for the solution to have" (Nash, 1953, p. 129) and then proceeds to identify a solution that agrees with those principles. In the literature, these principles are known as "axioms".

In this thesis, the focus is on the bargaining problem restricted to only two parties "who have the opportunity to collaborate for mutual benefit in more than one way" (Nash, 1950*b*, p. 155). Clearly, the negotiations between the parties may breakdown but so long as "there are feasible outcomes which all the participants prefer to the disagreement outcome, then there is an incentive to reach an agreement" (Roth, 1979, p. 5). The disagreement outcome is synonymously referred to as the "Status Quo Point" (Schmalensee, 1987; Napel, 2002; Serrano, 2008) which, following Fisher *et al.* (1991), may also be referred to as the "Best Alternative To a Negotiated Agreement" (abbreviated BATNA). The BATNA corresponds to the players' payoff in the (fully competitive) non-cooperative NE outcome and thus can be interpreted as the "backstop payoffs" (Dixit and Skeath, 2004, p. 569) to the parties should negotiations fail and an agreement not be reached. Outcomes of bargaining are therefore either "cooperation agreements specifiying a surplus distribution or final disagreement" (Napel, 2002, p. 10).

To select an outcome, amongst the many on the Pareto Front, two axiomatic bargaining paradigms are explored in this thesis. These are:

1. Utilitarian Solution: Under the classical utilitarian principle, which dates back to the mid-19$^{\text{th}}$ century (Thomson, 1994), this solution aims at "maximizing the sum of the individual agents' utilities" (Moulin, 1988, p. 13). Extending this notion, the utilitarian view argues that "a society is just if it maximizes the total or average utility of the society" (Cudd, 1996). A philosophical discussion of notions of fairness, a hotly debated and subjective topic, is outside the scope of this thesis. An example of the utilitarian approach had, in fact, been previously illustrated in Example 3.6 in Chapter 3 (see p. 83). In that example, the discussion had explicitly assumed that the colluding firms, forming a cartel, would maximise *joint* profits (see Eq. 3–16). Furthermore, as the colluding parties were entirely symmetric (i.e. with equal production costs), it was assumed that they would also share these joint profits equally. Two drawbacks can be identified with this solution. Firstly, this solution might not be unique (Thomson, 1994). Secondly, "the utilitarian solution may require a very unequal distribution" (Bergin, 2005, p. 284) of outcomes.

2. The Nash Bargaining Solution (NBS) (Nash, 1950*a*) is obtained by "maximising the product of utility gains from the disagreement point"(Thomson, 1994, p. 1243). The NBS is further elaborated in the next section.

It should be emphasised that the term "Utilitarian Solution" should not be interpreted as referring to utility as a measure of social welfare. If the intention is to indeed to refer to social welfare, this will be made clear in any discussions.

Though not examined in this thesis, there are a number of other paradigms, which postulate how the gains from cooperation may be allocated between the players (see e.g. the discussions in Kalai and Smorodinsky, 1975; Roth, 1979; Perles and Maschler, 1981; Schmalensee, 1987; Thomson, 1994; Binmore, 1998; Muthoo, 1999; Napel, 2002; Binmore, 2007). For example, the Egalitarian Solution argues that the best way is to ensure that gains are shared equally between parties. However, applying this solution concept could potentially require a very large reduction in welfare of one party for a small gain in welfare of the other (Bergin, 2005).

5.6.1 Nash Bargaining and Nash Bargaining Problem with Equilibrium Constraints (NBPEC)

In this section, the Nash Bargaining Problem (Nash, 1950*a*), a normative approach to surplus division based on Axiomatic Bargaining Theory, is discussed. Several definitions and assumptions are in order. Formally, a bargaining problem is defined by the tuple $(\mathcal{B}, \boldsymbol{\phi}^{\mathrm{N}})$ where $\mathcal{B}$ is a set of feasible payoffs (the Bargaining Set) and $\boldsymbol{\phi}^{\mathrm{N}} \triangleq \left(\phi_1^{\mathrm{N}}, \phi_2^{\mathrm{N}}\right)^{\intercal}$ is the vector of BATNAs i.e. payoffs each would obtain in the non-cooperative outcome. Following Thomson (1994), it is assumed that the set $\mathcal{B}$ is bounded and closed. The boundary of $\mathcal{B}$ constitutes the Pareto Front. For a non-trivial solution, it is assumed that there is at least one point in $\mathcal{B}$ that strictly Pareto dominates $\boldsymbol{\phi}^{\mathrm{N}}$.

The axioms that the NBS (often called "Nash Axioms") satisfy are as follows:

1. Individual Rationality: This axiom requires that "all agents should strictly gain from the compromise" (Thomson, 1994, p. 1248).

2. Pareto Optimality: This axiom requires that all feasible gains from cooperation should be exhausted and thus "no available mutual gain should go unexploited" (Dixit and Skeath, 2004, p. 572).

3. Scale Invariance: This axiom requires that linear rescaling should have no effect on payoff calculations and no effect on outcomes (Thomson, 1994).

4. Independence of Irrelevant Alternatives: This axiom requires that eliminating feasible alternatives, other than the BATNA, that would *not* have been chosen, should not affect the bargaining solution (Myerson, 1991).

Adapting from Nash (1950*b*) and Theorem 7.4.1 (p. 144) in van Damme (1991), the 2-person Nash Bargaining Problem with Equilibrium Constraints (NBPECs) can be formulated in Eq. 5–9. Therefore, this NBPEC is also another instance of the class of MPECs as discussed in Chapter 4 since it incorporates a VIP predicated on the upper level decision vector.

$$
\begin{aligned}
\underset{x \in X}{\text{Maximise}} \quad & \mathcal{Z}(\mathbf{x}, \mathbf{y}(\mathbf{x})) = (\phi_1(\mathbf{x}, \mathbf{y}(\mathbf{x})) - \phi_1^{\mathrm{N}})(\phi_2(\mathbf{x}, \mathbf{y}(\mathbf{x})) - \phi_2^{\mathrm{N}}) \\
\text{subject to} \quad & \mathbf{y}(\mathbf{x}) \leftarrow \mathrm{SOL}\{\mathcal{V}(\mathbf{x})\} \\
& \phi_1(\mathbf{x}, \mathbf{y}(\mathbf{x})) \geqslant \phi_1^{\mathrm{N}} \\
& \phi_2(\mathbf{x}, \mathbf{y}(\mathbf{x})) \geqslant \phi_2^{\mathrm{N}}
\end{aligned}
\qquad \text{(Eq. 5–9)}
$$

As noted previously, the objective function in the NBPEC maximises the product of the gains to both parties vis-á-vis the BATNA. Aside from the VIP constraint, the other constraints stipulate that in the solution, both parties obtain, at least, their BATNAs. This thus ensures that Axiom 1 of Individual Rationality, as defined above, is satisfied. In passing, it should be pointed out that in contrast to the NBS, the Utilitarian Solution does not satisfy this axiom (Thomson, 1994, p. 1248) since it is quite possible that one party may obtain lower payoff vis-á-vis the BATNA.

In bargaining problems *without* the VIP constraint that have been previously studied, Nash (1950*a*) proved that the solution of Eq. 5–9 is the only solution satisfying Axioms 1 to 4 above. It is beyond the scope of this thesis to prove that these axioms are met when the bargaining problem under consideration includes an equilibrium constraint. Despite this, the formulation will be applied heuristically in this work and it will be demonstrated, through the numerical examples, that in the solution to the NBPEC, that at the very least, Axioms 1 and 2 can be seen to be satisfied. In other words, it will be shown that the solution of the NBPEC will illustrate that not only will the agents improve their outcomes vis-á-the BATNA (Axiom 1) but also that it is Pareto Optimal (Axiom 2).

Thus, this formulation, whilst applied heuristically in this work, serves to bridge the gap between the outcome obtained from the non-cooperative pursuit of self-interest underlying the NCEPEC formulation in Eq. 4–4 and the cooperative paradigm of mutual interest underscoring the MOPEC formulation in Eq. 5–6.

5.7 Summary

This chapter proposed the use of Evolutionary Algorithms (EAs) for the resolution of the hierarchical optimisation problems formulated in Chapter 4 which are recognised to be difficult optimisation problems characterised by nonsmoothness, non-differentiability and multimodality. By design, EAs do not utilise derivative information to generate search directions when solving optimisation problems. Thus EAs are applicable to a variety of nonsmooth problems including EPECs.

A generic template of an EA for optimisation was outlined focusing on both its population based structure and its mechanism for generation of search directions by the iterative application of recombination and selection operators. Subsequently, Differential Evolution (DE), a member of the class of EAs, was detailed. Two crucial aspects of DE when applied to VIP constrained hierarchical optimisation problems were emphasised. Firstly, by ensuring that the fitness of a trial vector is evaluated using a hierarchical evaluation strategy, DE could be applied to solve MPECs whilst maintaining the crucial leader-follower relationship characterising Stackelberg games. Secondly, DE's recombination and selection mechanisms were described. DE's recombination operator subsequently formed the basis of two further algorithms ,introduced in this chapter, designed to solve MOPECs and NCEPECs.

By tailoring the selection mechanism to ensure that the fitness of trial vectors, or chromosomes, were compared on the basis of Pareto Domination criteria, DE could be easily extended to identify the Pareto Fronts in MOPECs. Following a review of several techniques to solve Multiple Objective Problems, the DE based MOSADE algorithm proposed by Huang *et al.* (2007) was outlined. MOSADE will be applied to generate Pareto Fronts in the case studies discussed in subsequent chapters of this thesis.

Subsequently, as a novel contribution in this thesis, a DE based approach to identify NE in NCEPECs was described. The idea is based on the principle of Nash Domination proposed in Lung and Dumitrescu (2008). When comparing "fitness" of two strategy profiles, this principle means that the strategy profile resulting in *fewer* players having an incentive to unilaterally deviate away from it is judged to be closer to an NE.

By making a change to the selection operator of EAs to ensure that fitness of strategy profiles was assessed utilising the Nash Domination principle, a DE based algorithm (NDEMO) was proposed for the identification of NE in NCEPECs where the leaders were assumed to act non-cooperatively. In particular, it was highlighted that convergence to the NE and not merely an arbitrary LNE, is theoretically assured by the application of the Nash Domination principle and this is a unique feature of the proposed EA.

When moving from the Nash non-cooperative outcome of the NCEPEC to the Pareto Front generated as a solution of the MOPEC, surplus to the players will be created. How should the surplus be shared between the players? This is the question of interest in (two person) Axiomatic Bargaining Theory. The aim of this normative approach to bargaining is not to explain the evolution of the bargaining process *per se* but to understand how bargaining should be resolved between rational parties according to some desirable criteria or axioms. Two of the many possible answers to this question as elucidated in the literature were introduced. In the Utilitarian view, the total gains should be maximised. On the other hand, the Nash Bargaining Solution (NBS) (Nash, 1950*b*) seeks to maximise the product of gains to the two parties. In this chapter, adapting the methodology from the literature, this chapter proposed the Nash Bargaining Problem with Equilibrium Constraints (NBPEC) to bridge the gap between the solution of the NCEPEC and the Pareto Front of the MOPEC.

The effectiveness of MOSADE and NDEMO will be subsequently assessed when applied to the case studies formulated in Chapters 6 to 9.

Chapter 6

Competition between Toll Road Concessionaires: Part I[6.1]

6.1 Introduction

This chapter is the first of two chapters that studies toll revenue competition between toll road concessionaires in a highway network with multiple OD pairs. In line with the research objectives of this thesis, the purpose of this chapter is three-fold.

Firstly, the welfare and revenue impacts of competition between revenue maximising toll road concessionaires are assessed against two alternative toll pricing policies: *a)* a second best welfare maximising toll pricing policy, providing an indication of the upper bound of the welfare gains attainable, with tolls charged on the set of pre-specified tollable links and *b)* a revenue maximising concessionaire, termed a “monopolist”, deciding toll levels on all tollable links in the network, giving an indication of the upper bound of the revenues attainable. As discussed in Chapter 2, whether competition is socially beneficial, or otherwise, vis-á-vis monopoly control, depends critically on the intrinsic interrelationships between the links in competition i.e. whether competition takes place between concessionaires controlling either *parallel* or *serial* links and this distinction will be maintained through the numerical tests reported in this chapter.

Secondly, several algorithms for the identification of the LNE toll vector, when concessionaires are in competition, are applied and their performance evaluated. As described in Chapter 4, these algorithms included the Fixed Point Iteration (FPI) and the Sequential Linear Complementarity Problem (SLCP) approach. In

[6.1] This chapter draws extensively on Koh and Shepherd (2010).

addition, the Synchronous Iterative (SI) algorithm introduced by Yang and Huang (2005) will be described and tested in this chapter. It is shown that the SLCP algorithm, a novel contribution of this thesis, is effective and and computationally efficient in locating LNE. In addition, the ability of the proposed Nash Domination Evolutionary Multiplayer Optimisation (NDEMO) algorithm, to locate LNE points in NCEPECs, as described in Chapter 5, is assessed.

Thirdly, to address a gap identified in the literature review from Chapter 2, the possibility for tacit collusion between toll road concessionaires is investigated. To do so, this chapter outlines an intuitive modelling approach that allows a toll road concessionaire to infer its rival's intentions from its observation of market outcomes.

The rest of this chapter is organised as follows. The next section introduces the additional notation required to mathematically formulate the various toll pricing policies tested in this chapter. In Section 6.3, the MPEC models of second best welfare maximisation as well as revenue maximisation by a monopolist are developed. Subsequently, the mathematical model of toll revenue competition between concessionaires is outlined and this is shown to be an instance of a NCEPEC. As highlighted in Chapter 4, the MPECs and the NCEPEC are hierarchical optimisation problems characterised by the presence of a binding equilibrium constraint surfacing in the form of a VIP. More specifically, this chapter assumes that the lower level VIP reflects users' route choices obeying Wardrop's DUE principle. While this principle is extensively employed in transportation network analysis, it suffers from the drawback that DUE link flows are not necessarily differentiable everywhere (Patriksson, 2004). Thus the MPEC and NCEPEC models formulated in this chapter are characterised by a nonsmooth active DUE constraint. Consequently, specialised algorithms have to be applied to solve the resulting formulations in order to determine the optimal toll levels in each case. Thus Section 6.4 describes the Cutting Constraint Algorithm (CCA) which will be used to solve the MPEC formulations. Since the NCEPEC model of competition between concessionaires can be decomposed into a series of inter-related MPECs, CCA can also be embedded within the FPI algorithm to solve the problem to determine (L)NE tolls. Section 6.5 describes the network that used for the numerical tests in this chapter before presenting the results of the algorithms and assessing the welfare implications of the alternative toll pricing policies. In order to address a research gap highlighted in Chapter 2, Section 6.6 investigates the possibilities for collusion between toll road concessionaires. It is shown that when moving from no collusion through partial collusion to full collusion, a path is drawn from the NCEPEC (Nash non-cooperative) to the Utilitarian Solution/monopoly outcome. Interestingly, in the presence of LNE, it is found that collusion could be welfare enhancing. In addition, tests of the Nash Bargaining Problem with Equilibrium Constraints (NBPEC) introduced in Chapter 5, directly extending the approach proposed in Nash (1950*b*), is tested as an alternative paradigm as to how the surplus created by moving from to solution under competition to the monopoly solution could potentially be allocated amongst the concessionaires. Section 6.7 summarises.

6.2 Notation

This section outlines the notation used in this chapter where they have not been previously defined.

In the Nash non-cooperative game discussed in Chapter 3, $\mathcal{N}$ denoted the set of players in a game. Since the specific game considered in this chapter is between toll road concessionaires, this set of concessionaires will similarly be denoted by $\mathcal{N}$.

With regards to the highway network, let $\mathcal{L}$ represent the set of all links. The subset of these links upon which a toll can be levied (referred to as "tollable" links) is denoted by the set $\mathcal{J}$, $\mathcal{J} \subseteq \mathcal{L}$. The location of these tollable links are assumed to have been pre-defined and summarised in a $|\mathcal{L}| \times |\mathcal{J}|$ incidence matrix $\boldsymbol{\Xi}$,with elements Ξ_{lj} equal to 1 if and only if link l,$l \in \mathcal{L}$ is tollable link j,$j \in \mathcal{J}$, 0 otherwise.

With one exception to be discussed in the next chapter (see Section 7.5), it will be assumed in the numerical tests when competition is discussed, that each toll road concessionaire can only decide the toll level on a *single* link in the highway network. Thus the number of concessionaires, in the competitive scenarios to be examined, is equal to the number of tollable links in the network, i.e. $|\mathcal{N}| = |\mathcal{J}|$.

Let $\mathbf{x}$ be the vector of link tolls, $\mathbf{x} = [x_i], i \in \mathcal{N}$. The decision space of such tolls is denoted by $X_i = \{x_i : 0 \leqslant x_i \leqslant \bar{x}_i\}, i \in \mathcal{N}$ with $\bar{x}_i$ being a pre-specified upper bound of tolls on the link. It is assumed that tolls are measured in time units of seconds.

Let $\mathbf{v}$ be the vector of link flows of all links in the network where each element is $v_j,\ j \in \mathcal{L}$. The travel time will depend, through congestion, on the flow on the link. For each link j, $j \in \mathcal{L}$, this can be represented as a monotonically increasing, continuous function $t_j(v_j)$ of the flow v_j on link j only. Thus the generalised travel time function $c_j(v_j, x_j)$, given link flow, v_j, and toll level, x_j, is given by Eq. 6–1.

$$c_j(v_j, x_j) = \begin{cases} t_j(v_j) + x_j, & j \in \mathcal{J} \\ t_j(v_j), & \text{otherwise} \end{cases} \qquad \text{(Eq. 6–1)}$$

This generalised travel time function can be collected in vector form as $\mathbf{c}(\mathbf{v}, \mathbf{x})$.

The network also contains OD movements in a set $\mathcal{K}$, with $q_k,\ k \in \mathcal{K}$ denoting the travel demand for OD movement k and $\mathbf{q}$ collects all demands in a vector. As only a single user class is considered, it is assumed that all travellers in the network perceive travel times in the same way, regardless of socio-economic status. Following Yang and Huang (2005), it is assumed that the OD demand is a function of the equilibrium OD generalised travel times between that OD pair only and that this function is strictly monotone, invertible and decreasing function of generalised travel times, μ_k. The exposition will be concerned primarily with its inverse which returns OD generalised travel times as a function of the demand for OD pair k which can be written as $d_k^{-1}(q_k),\ k \in \mathcal{K}$. In this way, $\mathbf{d^{-1}(q)}$ represents the vector of inverse demand functions.

Let $\mathcal{R} = \{1, 2, \ldots, |\mathcal{R}|\}$ denote the index set of all acyclic routes with $\mathcal{R}_k \subseteq \mathcal{R}$ being the subset of such routes that serve OD pair k, $k \in \mathcal{K}$. The relationship between routes and links is specified through the $|\mathcal{L}| \times |\mathcal{R}|$ link-route incidence matrix $\boldsymbol{\Delta}$, with elements Δ_{jr} equal to 1 only if link j is part of route r, and equal to 0 otherwise $(j \in \mathcal{L}; r \in \mathcal{R})$. Similarly, the relationship between routes and OD movements is specified by the $|\mathcal{K}| \times |\mathcal{R}|$ OD-route incidence matrix $\boldsymbol{\Gamma}$ with elements Γ_{kr} equal to 1 if $r \in \mathcal{R}_k$ and equal to 0 otherwise $(k \in \mathcal{K}; r \in \mathcal{R})$. Finally, let $\mathbf{f}$ be a $|\mathcal{R}|$ vector of route flows.

6.3 Problem Formulation

In this section, the mathematical formulation of the various toll pricing policies described in the introduction, namely second best welfare maximisation, monopoly revenue maximisation and competition between toll road concessionaires, are outlined. As has been extensively discussed in Chapter 4, these toll pricing models are examples of the class of hierarchical optimisation problems which are characterised by a constraint expressed as a VIP, stipulating an equilibrium condition in a given parametric system.

In this chapter, the VIP takes the form of Wardrop's DUE route choice principle (Wardrop, 1952). This principle stipulates that in equilibrium, the generalised travel times of all used routes connecting an OD pair are equal and routes with higher generalised travel times are not used. It will be further assumed that the demand for travel for each OD pair is responsive to the generalised travel times experienced by that OD pair. Thus the DUE route choice model with elastic demand can be represented by the VI in Eq. 6–2,

$$\mathbf{c}\left(\mathbf{v}^{*},\mathbf{x}\right)^{\intercal}\left(\mathbf{v}-\mathbf{v}^{*}\right)-\mathbf{d}^{-1}\left(\mathbf{q}^{*}\right)^{\intercal}\left(\mathbf{q}-\mathbf{q}^{*}\right)\geqslant 0,\ \ \forall\left(\mathbf{v},\mathbf{q}\right)\in\mathcal{D} \qquad \text{(Eq. 6–2)}$$

where $\mathcal{D}$ is the set of feasible link flows and demands defined by a linear equation system of flow conservation constraints of the traffic assignment program (Beckmann *et al.*, 1956; Sheffi, 1985) as defined by Eq. 6–3.

$$\mathcal{D}=\left\{(\mathbf{v},\mathbf{q}):\mathbf{v}=\mathbf{\Delta f}\text{ and }\mathbf{q}=\mathbf{\Gamma f}\text{ where }\mathbf{f}\geqslant 0,\mathbf{f}\in\mathbb{R}^{|\mathcal{R}|}\right\} \qquad \text{(Eq. 6–3)}$$

It has been established (Smith, 1979; Dafermos, 1980) that the solution to the VI in Eq. 6–2, given toll vector, $\mathbf{x}$, results in *unique* vectors of link flows, $\mathbf{v}^{*}(\mathbf{x})$ and demands, $\mathbf{q}^{*}(\mathbf{x})$ satisfying Wardrop's DUE route choice principle. In what follows, the notation in Eq. 6–4 is used to specifically denote such a vector of link flows and demands that solves the VIP stipulated in Eq. 6–2, given some given toll vector, $\mathbf{x}$.

$$\{\mathbf{v}^*(\mathbf{x}), \mathbf{q}^*(\mathbf{x})\} \leftarrow \text{SOL}\{\mathcal{V}(\mathbf{x})\} \qquad \text{(Eq. 6–4)}$$

Following the literature (e.g. Yang *et al.*, 2009), the costs associated with toll collection are ignored.

6.3.1 Welfare Maximising Second Best Toll Pricing

The social surplus or welfare, measures the difference between what users are willing to pay, as given by the area under the inverse demand functions, and the total generalised travel time (excluding tolls) experienced by users in their journeys through the network (Yang and Huang, 2005) as shown in Eq. 6–5.

$$\mathcal{W}_{\text{DUE}}(\mathbf{x}) = \sum_{k \in \mathcal{K}} \int_0^{q_k} d_k^{-1}(w)dw - \sum_{j \in \mathcal{L}} v_j t_j(v_j) \qquad \text{(Eq. 6–5)}$$

Therefore, in the case of welfare maximising second best tolls, toll levels are chosen, for each link in the set of tollable links, $\mathcal{J}$, to maximise such a welfare measure, subject to link flows and demand satisfying the DUE route choice principle. The resulting MPEC is formulated as shown in Eq. 6–6.

$$\begin{aligned} \underset{x \in X}{\text{Maximise}} \quad & \mathcal{W}_{\text{DUE}}(\mathbf{x}) = \sum_{k \in \mathcal{K}} \int_0^{q_k} d_k^{-1}(w)dw - \sum_{j \in \mathcal{L}} v_j(\mathbf{x}) t_j(v_j(\mathbf{x})) \\ \text{subject to} \quad & \{\mathbf{v}^*(\mathbf{x}), \mathbf{q}^*(\mathbf{x})\} \leftarrow \text{SOL}\{\mathcal{V}(\mathbf{x})\} \end{aligned} \qquad \text{(Eq. 6–6)}$$

As the optimisation problem in Eq. 6–6 is constrained by a VIP, this problem is thus an MPEC as discussed in Chapter 4. Eq. 6–6 can be solved by a variety of algorithms as discussed in Chapters 4 and 5. For the purposes of the numerical tests reported in Section 6.5, the Cutting Constraint Algorithm (CCA), to be further elaborated in Section 6.4, was applied to solve the problem.

6.3.2 Monopoly

If a single revenue maximising monopolist was permitted to decide toll levels on all tollable links in the network, then the set of concessionaires, $\mathcal{N}$ would collapse to a singleton. The mathematical optimisation problem facing this monopolist with revenue function $\phi_{\mathrm{M}} : \mathbb{R}^{|\mathcal{J}|} \times \mathbb{R}^{|\mathcal{J}|} \mapsto \mathbb{R}$, can be represented by Eq. 6–7. This problem is also an MPEC and in the numerical tests to be discussed in Section 6.5, CCA was applied to solve this problem.

$$\begin{aligned} \underset{x \in X}{\text{Maximise}} \quad & \phi_{\mathrm{M}}(\mathbf{x}, \mathbf{v}(\mathbf{x})) = \sum_{j \in \mathcal{J}} x_j v_j = (\boldsymbol{\Xi}\mathbf{x})^{\intercal} \mathbf{v}(\mathbf{x}) \\ \text{subject to} \quad & \{\mathbf{v}^*(\mathbf{x}), \mathbf{q}^*(\mathbf{x})\} \leftarrow \mathrm{SOL}\{\mathcal{V}(\mathbf{x})\} \end{aligned} \qquad \text{(Eq. 6–7)}$$

6.3.3 Oligopolistic Competition Between Toll Road Concessionaires

Having laid out the models of second best welfare maximisation and of monopolistic ownership of toll roads, the model of competition between concessionaires is developed in this section. Each concessionaire's optimisation problem can be framed as the MPEC in Eq. 6–8.

$$\forall i \in \mathcal{N}, \text{ concessionaire } i \text{ solves: } \begin{cases} \underset{x_i \in X_i}{\text{Maximise}} \ \phi_i(\mathbf{x}, \mathbf{v}(\mathbf{x})) = v_i(\mathbf{x}) x_i \\ \text{subject to} \\ \{\mathbf{v}^*(\mathbf{x}), \mathbf{q}^*(\mathbf{x})\} \leftarrow \mathrm{SOL}\{\mathcal{V}(\mathbf{x})\} \end{cases} \qquad \text{(Eq. 6–8)}$$

Each concessionaire seeks to maximise the toll revenue from levying a toll on the link under his control. The revenue attainable will depend on the equilibrium link flows and demands as reflected in the DUE constraint. Furthermore, these link flows and demands will also be affected by the toll levels chosen by other concessionaires exercising control over other tollable links elsewhere in the network. In this way, the formulation in Eq. 6–8 points to the interdependencies amongst these concessionaires, who are players engaged in a toll setting game. This inter-play of the concessionaires in each aiming to maximise individual revenues, reacting to the other concessionaires' toll levels, while anticipating the route choices of network users, results in a NCEPEC as discussed in Chapter 4. Thus while there is a Nash non-cooperative game amongst the concessionaires, there is a Stackelberg game between each concessionaire and the travellers. As noted in Chapter 4, solving the NCEPEC entails identifying an NE toll vector satisfying Definition 3.1.

6.4 Solution Algorithms

Since the lower level VIP constraint to the MPECs formulated in Eq. 6–6 and Eq. 6–7 both describe equilibrium route choices obeying the DUE principle, the equilibrium link flows may not be differentiable everywhere (Patriksson, 2004). It was pointed out in Chapter 4 (see Section 4.2.3) that MPECs do not satisfy certain "constraint qualifications" (Chen and Florian, 1995; Scheel and Scholtes, 2000) that are necessary regularity conditions for solving non-linear optimisation problems. For these reasons, these MPECs cannot be simply solved as single-level non-linear optimisation problems by embedding the VIP directly as constraints. Thus, a specialised algorithm has to be applied to solve these MPECs. In this section, the Cutting Constraint Algorithm (CCA) (Lawphongpanich and Hearn, 2004), used to solve the MPECs (Eq. 6–6 and Eq. 6–7) formulated in Section 6.3 above, is described. Furthermore, when applied to the identification of (L)NE points in the NCEPEC describing toll revenue competition, the FPI algorithm operates by iteratively solving each concessionaire's revenue maximisation problem (Eq. 6–8) for fixed vector of tolls of all other concessionaires, in turn, until the entire sequence converges. Since each concessionaire's optimisation problem is an MPEC, CCA can also be used in this case. Following discussion of the CCA, the Synchronous Iterative (SI) Algorithm (Yang and Huang, 2005) specifically designed to solve the NCEPEC formulation only is described.

6.4.1 Cutting Constraint Algorithm

From convex set theory (Bazaraa *et al.*, 2006, Theorem 2.1.6, p. 43), the feasible set of link and demand flows, $(\mathbf{v}, \mathbf{q}) \in \mathcal{D}$, can be defined as a convex combination of the set of extreme points. Exploiting this idea, Lawphongpanich and Hearn (2004) show that the VI in Eq. 6–2 can be reformulated as Eq. 6–9.

$$\mathbf{c}\left(\mathbf{v}^{*}, \mathbf{x}\right)^{\intercal}\left(\mathbf{u}^{e}-\mathbf{v}^{*}\right)-\mathbf{d}^{\mathbf{-1}}\left(\mathbf{q}^{*}\right)^{\intercal}\left(\mathbf{h}^{e}-\mathbf{q}^{*}\right) \geqslant 0 \quad \forall e \in \boldsymbol{E} \qquad \text{(Eq. 6–9)}$$

where $(\mathbf{u}^e, \mathbf{h}^e)$ is the vector of extreme link and demand flows indexed by the superscript e, and $\boldsymbol{E}$ is the set of all extreme points of $\mathcal{D}$.

Together with an initial extreme point, generated by solving an initial shortest path problem (e.g. all or nothing assignment), and the constraints defining feasible flows, the single level problem (labelled P0) in Eq. 6–10 is solved to find the optimal tolls at iteration g.

$$\text{P0}\begin{cases} \underset{(\mathbf{x},\mathbf{v},\mathbf{q})}{\text{Maximise}} \;\; U\left(\mathbf{x},\mathbf{v},\mathbf{q}\right) \\ \text{subject to} \\ (\mathbf{v},\mathbf{q}) \in \mathcal{D} \\ \mathbf{c}\left(\mathbf{v},\mathbf{x}\right)^{\intercal}\left(\mathbf{u}^{e}-\mathbf{v}\right)- \;\; \mathbf{d}^{-1}\left(\mathbf{q}\right)^{\intercal}\left(\mathbf{h}^{e}-\mathbf{q}\right) \geqslant 0 \quad \forall e \in \boldsymbol{E} \end{cases} \qquad \text{(Eq. 6–10)}$$

Subsequently, new extreme points are found by solving Problem P1 in Eq. 6–11 given solutions of Problem P0. Problem P1 is in effect a shortest path problem. If the gap function is positive at the end of this step (indicating that the Wardrop's DUE condition is satisfied), the algorithm terminates. Otherwise, the iteration counter is incremented and the process repeated.

$$\text{P1}\begin{cases} \underset{(\mathbf{u},\mathbf{h})}{\text{Minimise}} \quad \mathbf{c}\left(\mathbf{v},\mathbf{x}\right)^{\intercal}\mathbf{u}-\mathbf{d}^{-1}\left(\mathbf{q}\right)^{\intercal}\mathbf{h} \\ \text{subject to} \quad (\mathbf{u},\mathbf{h}) \in \mathcal{D} \end{cases} \qquad \text{(Eq. 6–11)}$$

The CCA is summarised in Algorithm 6.1.

6.4.2 Synchronous Iterative Algorithm

Chapter 4 described two deterministic (i.e. gradient-based) methods that can be used to identify (L)NE points of the NCEPEC model of toll revenue competition between concessionaires. The first was the Fixed Point Iteration (FPI) algorithm (see Section 4.4) and the second was the Sequential Linear Complementarity Problem

Algorithm 6.1 Cutting Constraint Algorithm (Lawphongpanich and Hearn, 2004)

$g \leftarrow 0$
Solve P1 to obtain $(\mathbf{u}^g, \mathbf{h}^g)$
Include $(\mathbf{u}^g, \mathbf{h}^g)$ into $\boldsymbol{E}$
while Not Converged **do**
    Solve P0 with all Extreme Points in $\boldsymbol{E}$ to obtain $(\mathbf{x}^g, \mathbf{v}^g, \mathbf{q}^g)$
    Solve P1 with $(\mathbf{x}^g, \mathbf{v}^g, \mathbf{q}^g)$ to obtain $(\mathbf{u}^{g+1}, \mathbf{h}^{g+1})$
    if $\mathbf{c}\left(\mathbf{v}^g, \mathbf{x}^g\right)^\intercal \left(\mathbf{u}^{g+1} - \mathbf{v}^g\right) - \mathbf{d}^{-1}\left(\mathbf{q}^g\right)^\intercal \left(\mathbf{h}^{g+1} - \mathbf{q}^g\right) \geqslant 0$ **then**
        Terminate
    else
        Include $(\mathbf{u}^{g+1}, \mathbf{h}^{g+1})$ into $\boldsymbol{E}$
        $g \leftarrow g + 1$
    end if
end while

(SLCP) approach (see Section 4.5).

In addition to these two algorithms, this section details the Synchronous Iterative (SI) algorithm proposed by Yang and Huang (2005). The SI algorithm begins by recognising that the first order conditions of the revenue maximisation problem for player i as stated in Eq. 6–8, for given toll level strategies of all other players, $\mathbf{x}_{-i}$, can be written as Eq. 6–12.

$$\frac{\partial \phi_i(x_i, \mathbf{x}_{-i}, \mathbf{v})}{\partial x_i} = \frac{\partial v_i(x_i, \mathbf{x}_{-i})}{\partial x_i} x_i + v_i(x_i, \mathbf{x}_{-i}) = 0 \qquad \text{(Eq. 6–12)}$$

Note that this assumption is only valid if the toll level chosen by concessionaire i, x_i, is strictly positive (interior optimum) (Yang *et al.*, 2009, p. 27). By rearranging Eq. 6–12, an expression for the concessionaire i's toll as shown in Eq. 6–13 is obtained.

$$x_i = -v_i(x_i, \mathbf{x}_{-i}) \left(\frac{\partial v_i(x_i, \mathbf{x}_{-i})}{\partial x_i} \right)^{-1} \qquad \text{(Eq. 6–13)}$$

The toll level, x_i, in Eq. 6–13 is only optimal for fixed $\mathbf{x}_{-i}$. Thus as other competitors update their tolls, the toll satisfying Eq. 6–13 from the perspective of concessionaire i would also change. In order to take this into account, Algorithm 6.2 was suggested in Yang and Huang (2005).

The proposed method works as follows. Starting from an arbitrary initial toll vector at iteration g, a traffic assignment is carried out in order to obtain the derivative term i.e. $\left(\partial v_i(x_i, \mathbf{x}_{-i}) / \partial x_i\right), i \in \mathcal{N}$. The derivative then facilitates the computation of an "auxiliary" toll vector, $\tilde{x}_i$. using Eq. 6–13.

Subsequently, the Method of Successive Averages (MSA) (Line 9 of Algorithm 6.2) is employed to combine the current toll vector and auxiliary toll vector to form the toll vector for the next iteration and the process is repeated until convergence is met. The convergence criteria was not explicitly specified in Yang and Huang (2005). Thus, for consistency with the criteria utilised in the FPI algorithm (see Algorithm 4.1), in the numerical results reported in the next section, convergence was deemed to have been achieved when the change in the toll vector obtained between successive iterations was less than some pre-specified tolerance, ϵ.

Algorithm 6.2 Synchronous Iterative Algorithm (Yang and Huang, 2005)

1: Input: Termination Tolerance, $\varepsilon(> 0)$
2: $g \leftarrow 0$
3: Choose initial $\mathbf{x}^g = \{x_1^g, \ldots, x_i^g, \ldots, x_n^g\}^\intercal$
4: **while** Not Converged **do**
5:   Execute (DUE) Traffic Assignment given $\mathbf{x}^g$ to obtain link flows $\mathbf{v}$
6:   **for** $i = 1$ to $\mathcal{N}$ **do**
7:     Compute $\partial v_i(x_i^g, \mathbf{x}_{-i}^g)/\partial x_i$
8:     Compute $\tilde{x}_i$ using Eq. 6–13
9:     Compute $x_i^{g+1} = x_i^g + \frac{1}{g+1}(\tilde{x}_i - x_i^g)$
10:   **end for**
11:   **if** $\|x_i^{g+1} - x_i^g\| \leqslant \varepsilon \quad \forall i \in \mathcal{N}$ **then**
12:     Terminate
13:   **else**
14:     $g \leftarrow g + 1$
15:   **end if**
16: **end while**

6.5 Numerical Tests

This section first describes the highway network and its associated demand parameters. Subsequently, the scenarios tested are detailed. Finally, the results of tests conducted are presented.

6.5.1 Highway Network and Demand Parameters used in Tests

The highway network, upon which the numerical tests were conducted, is shown in Fig. 6.1. Each link in this network has a travel time function of the form $t_j = t_j^0 + \beta_j (v_j/\kappa_j)^{\rho_j}$ where t_j^0, β_j, κ_j, ρ_j refer to the free flow travel time, coefficient, capacity and the power respectively associated with link j. These parameters are given in Table 6.1.

TABLE 6.1: Link travel time parameters for network in Fig. 6.1 used in numerical tests

Link j	A node	B node	t_j^0 (secs)	β_j	κ_j (pcus/hour)	ρ_j
1	1	2	45	9.55	1800	4.5
2	2	1	45	9.55	1800	4.5
3	2	3	108	108	1100	3
4	2	4	120	120	1100	3.1
5	3	6	270	57.27	1100	3.5
6	3	2	108	108	1100	3
7	3	5	90	90	1100	3.2
8	4	6	274.5	58.23	1100	3
9	4	2	120	120	1100	3.1
10	4	5	96	96	1100	3.1
11	5	3	90	90	1100	3.2
12	5	4	96	96	1100	3.1
13	5	6	72	72	1100	3.1
14	6	3	270	57.27	1100	3.5
15	6	4	274.5	58.23	1100	3
16	6	7	45	9.55	1800	4.5
17	6	5	72	72	1100	3.1
18	7	6	45	9.55	1800	4.5

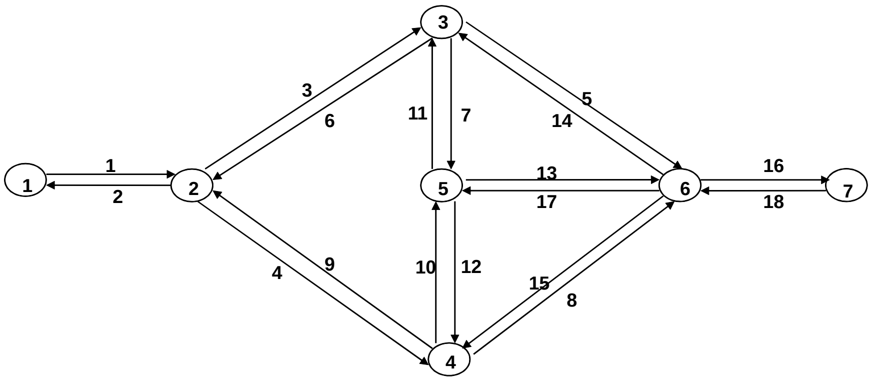


FIGURE 6.1: Highway Network Used for Studying Competition between Toll Road Concessionaires (DUE) Koh and Shepherd (2010)(Link numbers are indicated on arcs.)

On the demand side, the demand function, giving trips as a function of the generalised costs, adopts the "power law" form with a constant elasticity of demand given by Eq. 6–14,

$$q_k = q_k^0 \left(\mu_k/\mu_k^0\right)^{\eta_k}, k \in \mathcal{K}, \tag{Eq. 6–14}$$

where q_k^0, μ_k^0 are respectively, the demand and generalised travel times for each OD pair in the base (i.e. no toll equilibrium) respectively and η_k is an exogenous parameter. The details of the base demands and base generalised travel times are given in Table 6.2. While more generally, η_k could adopt different values for different OD pairs, it is assumed here that η_k is -0.58 for all OD pairs. If origin nodes 1 and 7 are viewed as residential suburbs and destination node 5 is viewed as the CBD, then the base demand matrix can be interpreted as capturing the typical morning commuting patterns within a monocentric city (Mun *et al.*, 2003).

TABLE 6.2: Base OD Demands and Generalised Travel Times for Network shown in Fig. 6.1 used in numerical tests

Origin Node	Destination Node	Base Demands (q_k^0) (pcus/hr)	Base Generalised Travel Times (μ_k^0) (secs)
1	5	637	1125
1	7	1027	1050
5	1	522	675
5	7	391	600
7	1	964	1050
7	5	442	850

The power law demand function in Eq. 6–14 finds support in transport planning guidance issued by the Department for Transport (DfT) in the UK (HA, 1997). This functional form has a "a well-behaved formulation that is simple to apply" (DfT, 2013*b*, p. 58). However, this demand function is endowed with a property that can potentially be a drawback when applying it to the study of revenue maximising toll road ownership. In economic theory (e.g. Lipsey and Chrystal, 1999), the elasticity of demand is defined as the percentage increase/decrease in quantity of a good (equivalently trips in this instance, q_k) demanded as a result of a percentage decrease/increase in price (equivalently generalised travel times in this context, μ_k)[6.2].

[6.2]Under the standard assumption of downward sloping demand curves, generalised

For the power law demand function in Eq. 6–14, the elasticity of trip demand with respect to generalised travel times is constant throughout the entire range of the demand function and given by η_k. For this reason, the power law demand function is synonymously referred to as the constant elasticity demand function.

The elasticity η_k =-0.58, $\forall k \in \mathcal{K}$ used is consistent with real world estimates (see e.g. de Jong and Gunn, 2001, Table 3, p. 146, where short run elasticities of car kilometres with respect to car travel times in the range of -0.52 to -0.62 are reported). With η_k =-0.58, this implies inelastic demand and so a 1% increase in the toll level results in a 0.58% *reduction* in travel demand. Since the increase in the toll outweighs the decrease in travel demand, total revenue rises. In general then, with inelastic demand, the revenue always increases, as the toll increases, *ceteris paribus* and thus the revenue curve, from the perspective of the concessionaire, does not have a maximum point. In this way, the upper bound of the toll level becomes active and constrains the (revenue maximising) toll solution. This analysis of the demand formulation is vastly simplified because it has thus far ignored the interactions with the travel time flow curves (reflecting the "supply side" component of the system) in the demand supply equilibrium and it is the interactions of these two components that ultimately determine the resulting demand levels. These interactions will be investigated in the numerical tests to follow.

6.5.2 Description of Scenarios

In the numerical tests, two parallel and two serial link scenarios were considered. In each scenario, the links mentioned are the only links in the network designated as tollable.

In Scenario 1, links 3 and 4 in Fig. 6.1 are the only tollable links and no toll free alternative route is available for some trips, such as those from origin node 1. In Scenario 2, links 7 and 10 are the only tollable links with a toll-free link (link 17) available for trips to destination node 5. Competition involving serial links are studied in Scenario 3, which involves tolls on both links 3 and 7 where there is the

travel times and the volume of travel (demand level) are inversely related.

possibility for users to avoid both links and in Scenario 4, tolls links 1 and 3 where there is no alternative route for trips from origin node 1 to avoid the toll on link 1.

It should be pointed out that in the numerical tests, the maximum allowable toll for each tollable link $\bar{x}_i$, $i \in \mathcal{N}$ is 5000 seconds. The bound of 5000 was chosen as this translates into a practical toll level of approximately £6[6.3] which is considered to be reasonable maximum on the basis of acceptability for a toll on one link.

6.5.3 Results of Tests

In this section, the results obtained by application of the three algorithms used to identify the NE toll vector when the concessionaires engage in toll revenue competition are first discussed. Subsequently, the welfare impacts of competition are assessed against outcomes obtained under both a second best welfare maximising toll pricing policy and a monopolistic revenue maximising policy.

Competitive Tolls obtained by Different Algorithms

Table 6.3 shows the resulting tolls, number of iterations and CPU times required for each algorithm tested to converge to the NE solution. The FPI algorithm was judged to have converged when the change between tolls between successive iterations, defined by the tolerance parameter, ϵ, was less than 0.0001 (see Algorithm 4.1). In the SLCP algorithm, the termination criteria is based on the absolute maximum of the elements in the vector of numerically estimated first order derivatives of each player's payoff function (see Algorithm 4.2) and this was set to 0.0001 as well. Figures in the column labelled "Iterations" refer to the number of iterations required to converge to the above defined criteria. As shown the resulting tolls are almost identical when comparing across algorithms. However, the toll, obtained by FPI, on link 7 in scenario 3, is somewhat different. This could likely be a result of the imprecision in CCA when applied to solve each player's inner MPEC within FPI.

[6.3] The parameters of the model used here, originated from Sumalee (2004*a*) where a value of time of 7.63 pence per minute was assumed. This implies that the bound of 5000 seconds is equal to £6.35.

The difference in computational times obtained by FPI and SLCP are relatively pronounced for all scenarios tested. SLCP used much fewer iterations and is considerably faster than FPI, requiring less than 1% of the CPU time taken by the former. The SI algorithm also performed fairly efficiently but it is slowed down considerably by the use of MSA embedded within the formulation of the algorithm.

Welfare Impacts of Competition

Table 6.4 reports the tolls, revenues and the change in social welfare for each scenario under *a*) competition, *b*) monopoly and *c*) (second best) welfare maximisation. Throughout, the column labelled "Welfare Change" reports the difference in social welfare with tolls under the relevant toll pricing policy, computed using Eq. 6–5, and the welfare in the base untolled equilibrium and this is measured in generalised seconds.

This table shows that when there are no alternative routes available (as in the case of Scenario 1 where links 3 and 4 are tolled), the monopolist can charge the maximum toll allowable for link 3. In fact, the upper bound of the toll here is a binding constraint on the toll in the monopoly case. The toll on link 4 is lower due to the slightly longer free-flow travel time. A check with both tolls set at 5000 seconds showed that the total revenue was indeed lower than shown in Table 6.4 (being only 3,555,289 seconds). As may be expected with the monopolistic case, the impact on welfare is negative. As noted above previously, the monopolist's toll attaining the bound, in the case of link 3, could be attributable to the inherent characteristic of the power law demand function form assumed as discussed previously. However, in the case of two competing concessionaires, each player has no alternative but to succumb to the strategy charged by the other and hence ultimately both are only able to charge a much lower toll (around 10% of what the monopolist would charge). The overall welfare change for Scenario 1 under competition is close to that of second best social welfare maximisation, but is as expected marginally lower.

The more interesting case emerges in Scenario 2 when there is a toll free alternative (link 17 in Fig. 6.1) available for travel into destination Zone 5. In this situation,

Table 6.3: Results of NE tolls in competition game between concessionaires obtained by 3 alternative algorithms: FPI, SLCP and SI

		FPI			SLCP			SI		
	Links	Toll (secs)	Iterations	CPU Time (secs)	Toll (secs)	Iterations	CPU Time (secs)	Toll (secs)	Iterations	CPU Time (secs)
Scenario 1	3	530.63	25	1213.7	530.55	6	9.8	529.99	78	29.3
	4	505.65			505.62			505.04		
Scenario 2	7	141.37	25	1200.8	141.36	6	9.5	143.36	89	33.5
	10	138.29			138.29			139.41		
Scenario 3	3	248.62	23	1211.2	248.65	5	8.3	248.63	54	20.3
	7	97.84			92.54			95.50		
Scenario 4	1	5000	19	914.9	5000	5	8.8	5000	40	15.0
	3	35.30			35.22			35.29		

Table 6.4: Tolls, Revenues and Social Welfare Change under Alternative Toll Pricing Policies

		Competition			Monopoly			Second Best Welfare Maximisation		
	Link	Toll (secs)	Revenue (secs)	Welfare Change (secs)	Toll (secs)	Revenue (secs)	Welfare Change (secs)	Toll (secs)	Revenue (secs)	Welfare Change (secs)
Scenario 1	3	530.63	461,882	87,633	5000	2,543,530	-1,581,256	510.93	449,583	87,818
Parallel	4	505.65	420,293		4986.7	1,013,577		488.13	407,301	
Total Revenue			882,175			3,557,108			856,883	
Scenario 2	7	141.37	105,295	187,422	713.19	280,255	150,587	181.83	116,209	202,311
Parallel	10	138.29	100,848		709.53	266,465		179.30	110,580	
Total Revenue			206,143			546,720			226,783	
Scenario 3	3	248.65	146,756	-88,020	242.01	147,209	-76,956	0	0	95,795
Serial	7	98.52	54,309		92.54	54,275		141.89	74,027	
Total Revenue			201,065			201,482			74,027	
Scenario 4	1	5000	3,552,057	-1,590,050	5000	3,553,670	-1,585,945	488.21	836,935	87,818
Serial	3	35.2	11,122.00		26.73	10,513		22.74	20,008	
Total Revenue			3,563,179			3,564,184			856,943	

even a monopolist controlling both links 7 and 10 together cannot charge the maximum allowed toll of 5000 seconds on each link to maximise his revenue. Here the tolls are limited to around 700 seconds (though the impact on welfare is positive even under monopoly control). In the case of competition, Table 6.4 shows that the tolls charged and the total revenue earned are even lower than under the second best welfare maximising toll level. Thus it could be concluded that where there is an untolled alternative, competition could have the effect of driving tolls down even below the socially optimal level which was not the case for Scenario 1 where there was no untolled alternative. This possibility of tolls being driven down below the second best welfare maximising level does not seem to have been highlighted previously in the literature.

The first two scenarios have focused on parallel competing links. Since parallel links are the equivalent of substitutes in the route choice decisions of users, the findings thus supports insights from the literature discussed in Chapter 2 (See Section 2.4.2) that competition between substitutes could be welfare enhancing and result in lower tolls vis-á-vis monopoly ownership.

Serial competition is considered in both Scenarios 3 and 4. In these cases, tolls are higher under competition than if a monopolist were able to exercise control over both links. In this case, competition results in lower welfare compared to monopolistic control. This observation is in line with the literature discussed in Chapter 2 and is a manifestation of the double marginalisation problem, as serial concessionaires are incentivised to internalise the congestion externality not only of the segment under his control but also that of other segments in the series before each adding a demand related markup.

It is also worth noting that where there are alternative free routes, as in Scenario 3, then the tolls are relatively low even under monopoly, whereas in scenario 4, where link 1, exclusively used by trips originating from origin node 1, has no toll free alternative, then the upper bound constrains the solution. In this way, the concessionaire on link 1 exerts some degree of “power” over the concessionaire on link 3. The toll on link 1 in scenario 4, in both the competitive and monopolistic cases, attaining the upper bound of 5000 seconds, is again most likely to be a result of the power law demand functional form assumed. In the serial cases, both the competitive and monopolistic solution results in a negative welfare change compared to the second best welfare maximising tolls implying that with competition, society is worse off which has crucial implications for policy makers. Thus the results suggest that regulators should not allow direct competition in the serial link cases as it could potentially be worse for social welfare than doing nothing.

The welfare impacts of each scenario under competition, monopoly and welfare maximisation may be summarised using the unitless index of relative welfare improvement, ω (Verhoef *et al.*, 1996). This index can be computed according to Eq. 6–15 and gives an indication of the welfare gains attainable under each toll pricing policy relative to the theoretical first best benchmark. With first best pricing (i.e marginal cost tolls on every link (see Chapter 2)), the social welfare change relative to the untolled base equilibrium is 460,853 secs.

$$\begin{aligned}\omega &= \frac{\text{Welfare in scenario under study} - \text{Base Welfare}}{\text{Welfare from First Best Pricing} - \text{Base Welfare}} \\ &= \frac{\text{Welfare change in scenario under study}}{\text{Welfare change from First Best Pricing}}\end{aligned} \tag{Eq. 6–15}$$

TABLE 6.5: Index of Relative Welfare Improvement ω under Competition, Monopoly and Second Best Welfare Maximisation in Each Scenario

	Competition	Monopoly	Second Best Welfare Maximisation
Scenario 1 (Parallel)	0.19	-3.43	0.19
Scenario 2 (Parallel)	0.41	0.33	0.44
Scenario 3 (Serial)	-0.19	-0.11	0.21
Scenario 4 (Serial)	-3.45	-3.44	0.19

For each scenario tested in this chapter, Table 6.5 reports the index of relative welfare improvement under competition, monopoly and second best welfare maximisation. Emphasising the previous discussion, this table shows that the welfare obtained under competition between the parallel link pairs tested (Scenarios 1 and 2) is always higher than that obtained under monopoly and thus shows that competition in this case is welfare enhancing vis-á-vis monopolistic control. This confirms the insights from the literature discussed in Chapter 2. On the other hand, in the case of competition between serial link pairs (i.e. Scenarios 3 and 4), the index of relative welfare improvement is always lower under competition compared to monopolistic control, supporting insights from the literature that competition in this case is would deteriorate social welfare.

6.5.4 Tests with NDEMO

While both SLCP and SI produced results broadly similar to that obtained with FPI, it must be borne in mind that the former methods require first (and in the case of SLCP, second) order derivatives. In the implementation of the numerical tests, finite differencing, used in much numerical work in the engineering sciences, was applied to determine these derivatives. However, the very existence of such derivatives has been challenged when traffic routing obeys Wardrop's DUE principle (Patriksson, 2004). Thus despite their superior computational efficiency vis-á-vis FPI as demonstrated in these examples, both SLCP and SI should only be regarded as heuristics for the NCEPEC formulation. The heuristic nature of the SI algorithm is also acknowledged in Yang and Huang (2005).

This observation motivated the application of the alternative evolutionary algorithm (Nash Domination Evolutionary Multiplayer Optimisation or NDEMO), as described in Chapter 5 (see Section 5.5) to identify NE points in the NCEPEC but obviating the need for derivative information. While it was highlighted the NDEMO is theoretically guaranteed to identify an NE (see Proposition 5.1 in Chapter 5), NDEMO is envisaged to be computationally demanding. In this section, tests of NDEMO for finding NE in the case of Competition in Scenarios 1 and 2 as defined above are reported. NDEMO requires several user specified parameters as outlined previously and these are summarised in Table 6.6. As mentioned in Chapter 5 (see Algorithm 5.8), NDEMO terminates when the standard deviation of the population of chromosomes, encoding tolls strategies of all players, is at least equal to some termination tolerance, ϵ.

As NDEMO is a population based stochastic algorithm, the population means (i.e. toll levels) and standard deviations are tracked at each iteration. The population

Table 6.6: Parameter Settings for NDEMO

Parameter	Value
Maximum Number of Iterations	400
Population size(π)	20
Mutation amplification factor (λ)	0.45
Crossover factor (χ)	0.35
Termination tolerance (ϵ)	1.00E-06

means are displayed in the left hand panels of Fig. 6.2 and Fig. 6.3 for Scenarios 1 and 2 respectively. At the same time, the right hand panels show, for both scenarios, the standard deviation of the population, σ, at each iteration until the termination tolerance, ϵ, is met. The results reported in Table 6.7 are from the last run of NDEMO. 120 iterations were required to achieve this tolerance in the case of Scenario 1 while 84 iterations were required for Scenario 2. However, comparing the CPU times taken in Table 6.7 with those reported in Table 6.3, it is noticeable that NDEMO takes much longer than either SLCP or SI to converge to an NE, reflecting the heavier computational burden expended applying NDEMO.

Table 6.7: Results obtained by application of NDEMO to Scenarios 1 and 2

Scenario	Link	NDEMO	Solution of SLCP Algorithm in Table 6.3	CPU time (secs)
1	3	530.63	530.63	1273
	4	505.61	505.65	
2	7	141.36	141.37	728
	10	138.28	138.29	

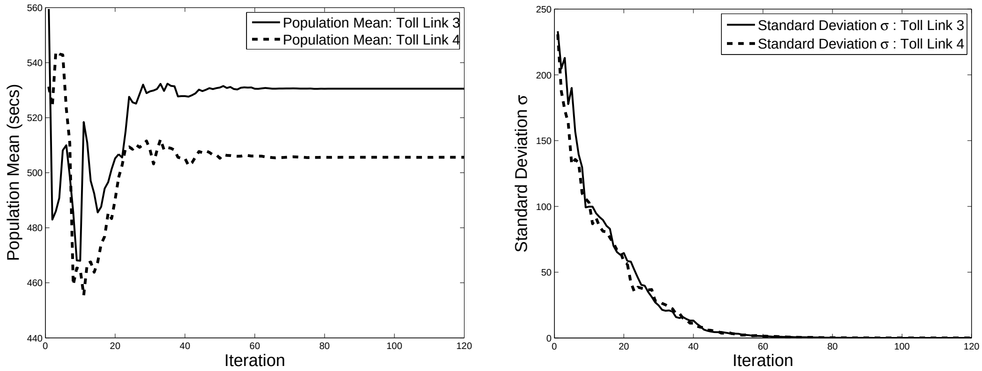


FIGURE 6.2: Population Mean (Left Panel) and Standard Deviation (Right Panel) at each Iteration of NDEMO for Scenario 1: Competition on Links 3 and 4

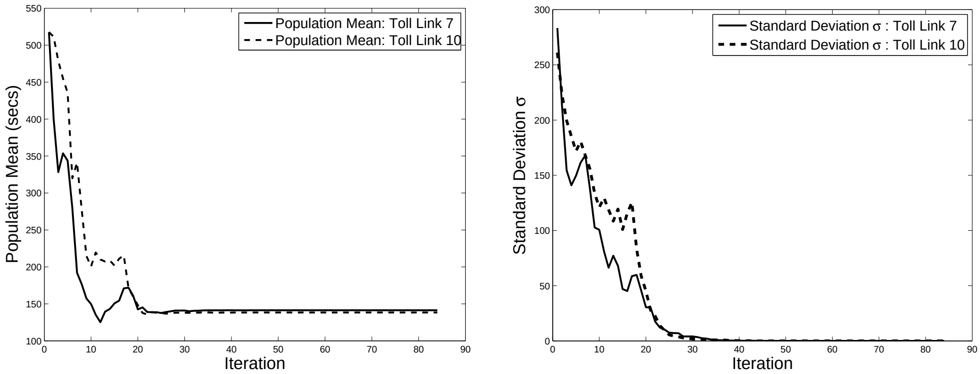


FIGURE 6.3: Population Mean (Left Panel) and Standard Deviation (Right Panel) at each Iteration of NDEMO for Scenario 2: Competition on Links 7 and 10

6.6 Collusion

The numerical tests presented in Section 6.5 have shown that competition between parallel links results in lower tolls and revenues accruing to the concessionaires and higher welfare is obtained by society. Based on this finding, the policy conclusion endorses fundamental insights from the literature reviewed in Chapter 2 (see Section 2.4.2). However, this conclusion rests on the crucial assumption that toll road concessionaires do not engage in anti-competitive practices in order to increase their revenues.

Arguably, since there would usually be a small number of toll road concessionaires operating in the road network, it is inevitable that they would recognise their mutual interdependence and engage in some form of collusion. From the perspective of the concessionaires, the competitive outcome reported in each scenario above is not Pareto Optimal since one concessionaire would be able to increase his revenues without making the other worse off. However, if the concessionaires were assumed to *cooperate* instead, then a different set of outcomes (i.e. tolls/revenues/welfare impacts) would result. Thus the potential for and consequences of collusion are investigated in this section.

6.6.1 Multiobjective Optimisation Problem with Equilibrium Constraints

As discussed in Chapter 5 (Section 5.4 refers), cooperation between concessionaires facing the DUE route choice constraint can be modelled as a Multiobjective Optimisation Problem with Equilibrium Constraints (MOPEC). As noted therein, the solution of the MOPEC is the identification of the Pareto Front. This front shows the Pareto Optimal (see Definition 3.3) revenue combinations such that any point on this front, viewed in revenue space, satisfies the condition that one concessionaire cannot increase his revenues without reducing the revenues to the other.

By the definition that the revenues attained under monopoly as reported in Table 6.4 maximises the total revenues attainable, it is hypothesised that will lie on this Pareto Front. In the context of collusion, this solution can also be interpreted as a "merger" between the concessionaires of the erstwhile independent concessionaires who aligned their interests totally.

In this and the next chapter, because the scenarios studied will involve two concessionaires exclusively, the MOPEC considered can be written as Eq. 6–16.

$$\begin{aligned} &\underset{\mathbf{x}\in X}{\text{Maximise}} && \boldsymbol{\Phi}(\mathbf{x}) = (\phi_1(\mathbf{x},\mathbf{v}(\mathbf{x})),\phi_2(\mathbf{x},\mathbf{v}(\mathbf{x})))^\intercal \\ &\text{subject to} && \{\mathbf{v}^*(\mathbf{x}),\mathbf{q}^*(\mathbf{x})\} \leftarrow \text{SOL}\{\mathcal{V}(\mathbf{x})\} \end{aligned} \qquad \text{(Eq. 6–16)}$$

As noted in Chapter 5, the MOSADE algorithm (Algorithm 5.5) will be used to solve Eq. 6–16 so as to identify the Pareto Front. However, non uniqueness of this front implies that there is, in fact, an entire range of possibilities for bargaining and negotiation between the two parties (assuming that it was not illegal to do so). Faced with so many possibilities, how would two concessionaires collude in practice in order to settle on a point on this Pareto Front? A closely related question is whether they would be willing to do so. In order to "specify how colluding firms select a point" (Schmalensee, 1987, p. 357) on this front, two approaches based on Axiomatic Bargaining Theory, as discussed Section 5.6, are investigated. The first, in the next section, is based on "signalling" to achieve the outcome predicted by the Utilitarian Solution i.e. a solution that maximises the total revenues, through an intuitive approach to modelling tacit collusion. The second, discussed in Section 6.6.3, is to identify the Nash Bargaining Solution (NBS).

To aid discussion, Table 6.8 identifies the correspondence between key terms encountered in the Axiomatic Bargaining Theory literature (e.g. Roth, 1979; Moulin, 1988; Bergin, 2005) and their equivalent terminology encountered in the Industrial Organisation literature. Firstly, the Pareto Front, obtained from solving the MOPEC in Eq. 6–16, can be viewed as the upper boundary of the "Bargaining Set" (Bergin, 2005, p. 283). Secondly, the non-cooperative NE solution of the NCEPEC characterising competition between the concessionaires is synonymous with the BATNA since in the absence of any cooperative agreement, each party would obtain this payoff. Finally, the monopoly solution obtained using Eq. 6–7 is analogous to the Utilitarian Solution[6.4] which, by definition, seeks to maximise the *total* gains to concessionaires. The last column in this table, relating to the collusion parameter α, will be discussed in Section 6.6.2.

6.6.2 Utilitarian Approach to Collusion

As noted in Chapter 2 (see Section 2.4.2), one possibility of modelling tacit collusion is to extend the idea that "firms infer rivals' intentions from their actions or from market outcomes" (Porter, 2005, p. 148). This section describes an intuitive

[6.4] As noted previously in Section 5.6, an inevitable confusion arises from the terminology used in the Axiomatic Bargaining Theory and the work reported here. To reiterate the point made previously, it should be emphasised that the term "Utilitarian Solution" does not refer to utility as a measure of social welfare. If the intention is to indeed to refer to social welfare, this will be made clear in the discussions.

Table 6.8: Equivalence of Terminology in Axiomatic Bargaining Theory and Industrial Organisation Literature

Notation	Axiomatic Bargaining Literature	Industrial Organisation Literature	Collusion Parameter α
Boundary of $\mathcal{B}$	Boundary of Bargaining Set	Pareto Front	NA
ϕ_i^{N}, $i \in \mathcal{N}$	Status Quo Point or Best Alternative to A Negotiated Agreement (BATNA)	(Fully) Competitive NE Outcome	0
ϕ^{M}	Utilitarian Solution	Monopoly Solution (Fully) Collusive Outcome	1

modelling approach to achieve this goal. Specifically, the objective function of each highway concessionaire in Eq. 6–8 is modified by introducing a (unitless) parameter, α, $0 \leqslant \alpha \leqslant 1$, in order to represent the degree of cooperation between the concessionaires. Therefore when setting tolls, they do not only maximise their own revenues but also α proportion of the revenues of their rivals, taking their rival's tolls as fixed [6.5]. It is implicitly assumed that each concessionaire reciprocate the actions of their rival and would do likewise.

Since the tests reported below are conducted on the same Scenarios 1 to 4 as used previously involving two concessionaires, the revised NCEPEC in this case can be explicitly written as Eq. 6–17.

$$\begin{aligned} &\underset{x_1 \in X_1}{\text{Maximise}} \quad \phi_1(x_1, v_{j_1}) = v_{j_1}(\mathbf{x})x_1 + \alpha v_{j_2}(\mathbf{x})x_2,\ j_1, j_2 \in \mathcal{J},\ j_1 \neq j_2 \\ &\text{subject to} \quad \{\mathbf{v}^*(\mathbf{x}), \mathbf{q}^*(\mathbf{x})\} \leftarrow \mathrm{SOL}\{\mathcal{V}(\mathbf{x})\} \end{aligned} \qquad \text{(Eq. 6–17a)}$$

$$\begin{aligned} &\underset{x_2 \in X_2}{\text{Maximise}} \quad \phi_2(x_2, v_{j_2}) = v_{j_2}(\mathbf{x})x_2 + \alpha v_{j_1}(\mathbf{x})x_1,\ j_1, j_2 \in \mathcal{J},\ j_2 \neq j_1 \\ &\text{subject to} \quad \{\mathbf{v}^*(\mathbf{x}), \mathbf{q}^*(\mathbf{x})\} \leftarrow \mathrm{SOL}\{\mathcal{V}(\mathbf{x})\} \end{aligned} \qquad \text{(Eq. 6–17b)}$$

It can be seen that when $\alpha = 0$, each equation collapses to the form of Eq. 6–8. In

[6.5] This is not unrealistic possibility since toll rates are openly available information.

the case when $\alpha = 1$, the objective of each player becomes one of maximising the *total* toll revenue of both players i.e. the Utilitarian Solution. Therefore, it is expected that when $\alpha = 1$, the solution of this system, for which the SLCP algorithm can be applied, should result in the same monopoly tolls and revenues as that reported in Table 6.4. Furthermore, the link revenues accruing to each concessionaire should lie on the Pareto Front. When $\alpha \neq 0$, each is assumed to take a proportion (as specified by α) of the other concessionaire's revenue into consideration in individual toll setting decisions. Thus by their charging of different set of tolls, obtained from solving the system in Eq. 6–17, each concessionaire can be interpreted as sending a "signal" to the other that there is an intent to collude for mutual gain.

In the following exposition, it is necessary to distinguish between "full" and "partial" collusion. Partial collusion can be thought of as the situation when α is strictly less than 1 and full collusion refers to the case where $\alpha = 1$. In this same vein, *full* competition corresponds to the case when $\alpha = 0$ and this is the same as "no collusion".

6.6.3 Nash Bargaining

The Nash Bargaining Problem with Equilibrium Constraints (NBPEC) was introduced in Chapter 5 (Section 5.6.1 refers). Bearing in mind the nonsmooth nature of the NBPEC in Eq. 5–9 due to the binding DUE constraint, this NBPEC can be solved by adapting the Cutting Constraint Algorithm (CCA) (Lawphongpanich and Hearn, 2004) (see Algorithm 6.1). In the results reported below, Nash Bargaining will be investigated in Scenarios 1 and 2.

6.6.4 Collusion and Nash Bargaining in Scenario 1: Links 3 and 4

In this section, collusion between concessionaires in Scenario 1 between links 3 and 4 is investigated. The left panel of Fig. 6.4 depicts the Pareto Front and confirms that there is a spectrum of possible outcomes that could be reached through some form of collusion depending on the bargaining possibilities between the two players.

Superimposed on the Pareto Front shown on the left panel of Fig. 6.4 are the revenues to each concessionaire in the fully competitive case (which is the situation when $\alpha = 0$, marked by +) and in the the monopoly solution ($\alpha = 1$, marked by *, from Table 6.4). This figure confirms two predictions. Firstly, the fully competitive solution is not Pareto Optimal as it lies in the interior of the Pareto Front. Secondly, the monopoly solution does indeed lie on the Pareto Front. It is clear that there is a relatively large difference between these two solutions in terms of individual revenues to players. Thus there seems to be some scope whereby concessionaires can collude to increase their revenues.

Furthermore, from the left panel of Fig. 6.4, it can be seen that at the fully competitive solution with $\alpha = 0$, both concessionaires earn relatively similar amounts of around 400,000 secs each. However, when $\alpha = 1$, the revenues attainable from link 3 is much higher compared to that from link 4. Examination of the network parameters in Table 6.1, shows that both the free flow travel time and power parameter for link 3 are marginally lower than that of link 4. This means that, *ceteris paribus*, comparing between the two, link 3 is the "faster" link. Therefore, in this sense, the concessionaire on link 3 can be considered the "stronger" player in this game. This accounts for the higher revenues under full collusion attainable by the concessionaire on link 3.

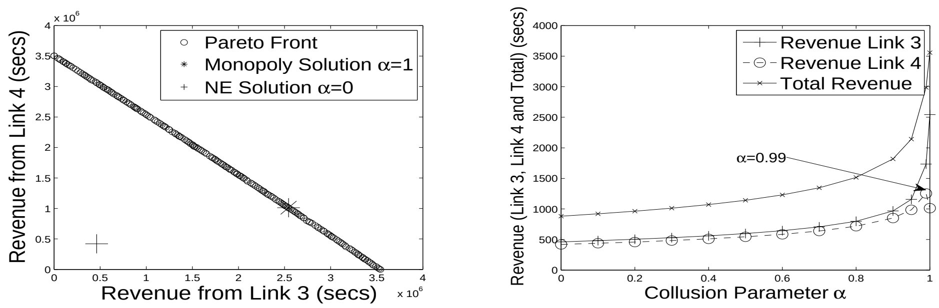


FIGURE 6.4: Left: Scenario 1-Collusion on link 3 and link 4: Pareto Front, NE ($\alpha = 0$) and Monopoly ($\alpha = 1$) solutions. Right: Revenues to concessionaire on link 3 and link 4 as α varies between 0 and 1

To investigate this issue further, the revenues for link 3 and 4 as well as the total revenues, when the collusion parameter, α, is varied between 0 and 1 are plotted on the right panel of Fig. 6.4. Two facts are evident from this figure. Firstly, as

α increases, both the revenue from tolls on link 3 and total revenues increase in accordance with intuition. Secondly, the revenue from link 4 rises up till $\alpha = 0.99$ but then drops off as indicated. The reason is that, when moving from a collusion level (α) of 0.99 to 1.0, the maximum allowable toll (of 5000 secs) becomes active on link 3 (see Table 6.9). While this bound is theoretical and possibly attributable to the characteristics of the power law demand functional form assumed, it is often the case that for acceptability reasons, concessionaires would cap the maximum toll levels concessionaires can charge (Gómez-Ibáñez *et al.*, 1991; Tsai and Chu, 2003). In such this case, to generate more total revenues, the concessionaire on link 4 is compelled to accept a reduction in revenues. This is because, as noted above, link 4is the weaker of the two concessionaires due to the link characteristics. It is possible to conjecture then that this weaker concessionaire might not be incentivised to collude fully without an explicit revenue sharing arrangement.

As noted previously, full competition between parallel links results in higher welfare compared to monopoly. Thus even if only partial collusion is reached, the welfare attainable with any $\alpha \neq 0$ is lower than under the fully competitive solution. As seen in Table 6.9, the index of relative welfare improvement ω computed according to Eq. 6–15 deteriorates compared to the fully competitive outcome whenever $\alpha \neq 0$. This implies that any collusion (partial or full) for this link pair results in a deterioration of welfare.

Table 6.9: Tolls on Links 3 and 4 and Index of Relative Welfare Improvement as α varies

α	Link 3 Toll (secs)	Link 4 Toll (secs)	Index of Relative Welfare Improvement ω
0	530.63	505.65	0.19
0.2	591.52	567.77	0.18
0.4	677.67	655.00	0.16
0.6	814.69	793.00	0.11
0.9	1455.48	1434.98	-0.35
0.95	1910.25	1889.84	-0.75
0.99	3523.17	3502.64	-2.21
1	5000.00	4986.73	-3.43

Nash Bargaining

As noted above, collusion between concessionaires to achieve the Monopoly outcome is unlikely to take place as with a cap of the maximum toll allowed, then in order to generate more revenues, the weaker player (link 4) would have to accept a reduction in revenues.

Table 6.10: Scenario 1: Revenues under Competition, Monopoly and Nash Bargaining

	Revenue (secs)				
Link	Competitive (BATNA)	Monopoly (Utilitarian) (see Table 6.4)	%Share Monopoly	NBS	%Share NBS
3	461,882	2,543,530	72%	1,804,294	51%
4	420,293	1,013,557	28%	1,749,223	49%
Total	882,175	3,557,108		3,553,517	

Table 6.10 confirms this. It is clear that both players can gain by making a move from the competitive outcome to the Utilitarian outcome through colluding since there is substantial revenue to be earned by doing this. But in collusive agreements is the share or proportion of the total each would get. These shares are shown in the column labelled "% Share Monopoly" and each cell shows the percentage of the total monopoly profits each concessionaire receives. In terms of the total monopoly profit, the concessionaire on link 3 gets a 72% share but the concessionaire on link 4 gets only a 28% share. Therefore, this supports the finding above that collusion towards achieving the Utilitarian Solution would likely be difficult as the concessionaire on link 4 is unlikely to agree to this much smaller revenue share.

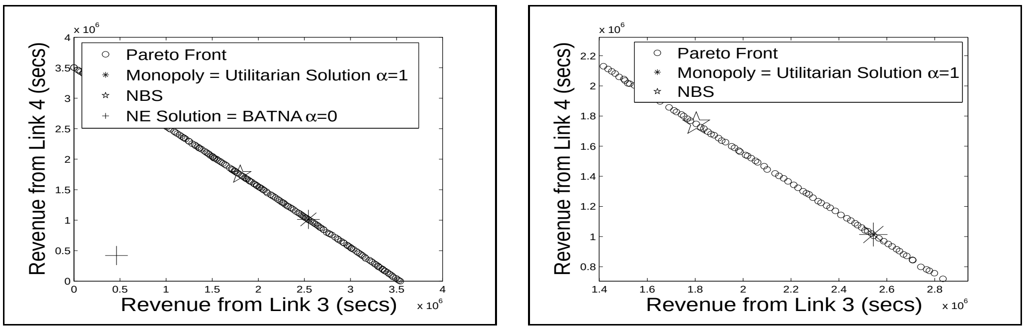


Figure 6.5: Left: NBS for Scenario 1 Right: Zoom in of NBS for Scenario 1

Next consider the Nash Bargaining Solution (NBS) which was determined by using CCA to solving the NBPEC in Eq. 5–9. With the NBS, total profits are not maximised but rather the product of the two parties gains vis-á-vis their individual BATNAs. The interesting finding with NBS is that the revenue share each gets as a proportion of the total is almost equal. Thus the 28% share of the total monopoly revenues becomes a 49% share of the Nash Bargaining Revenues for the weaker player. However, the concessionaire on link 3 would prefer the Utilitarian Solution as it obtains nearly twice as much compared to the NBS. In addition, both the left and right panels of Fig. 6.5 confirm that the NBS solution, indicated on each panel of these charts by ☆, is Pareto Optimal as it lies on the Pareto Front.

Note that the NBS, like the Utilitarian/Monopoly Solution (see Table 6.5) also results in a social welfare loss. In both cases, the index of relative welfare improvement ω (Verhoef *et al.*, 1996) is -3.43.

6.6.5 Collusion and Nash Bargaining in Scenario 2: Links 7 and 10

In this section, tests involving collusion between concessionaires in Scenario 2 (Links 7 and 10) is discussed. The Pareto Front obtained as the solution to Eq. 6–16 as generated by MOSADE, is shown on the left panel of Fig. 6.6. This plot also shows the implied link revenues to each concessionaire at both the monopoly solution and the fully competitive solution (cf. Table 6.4).

However, in contrast to that which had been hypothesised, as illustrated on the right panel of Fig. 6.6, the optimal toll revenues implied by the solution of Eq. 6–17 with $\alpha = 1$ does not in fact lie on this Pareto Front. The reasons for this is investigated in this section.

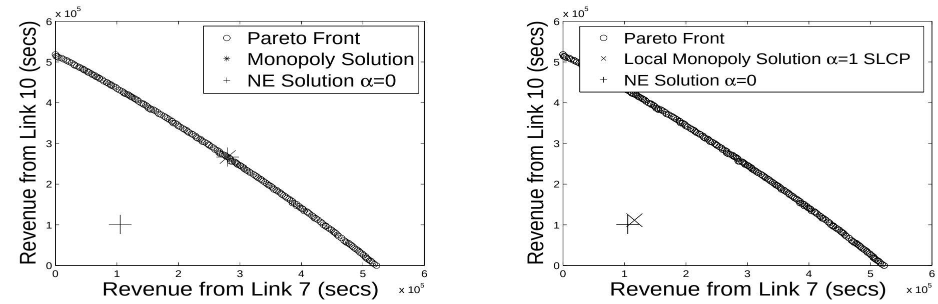


FIGURE 6.6: Left: Scenario 2: Collusion between Concessionaires on link 7 and link 10: Illustration of Pareto Front alongside solutions for fully competitive NE (indicated by +) and Monopoly (indicated by *), Right: Solutions obtained with SLCP when $\alpha = 1$ **does not** lie on the Pareto Front.

The left and right panels of Fig. 6.7 respectively show the tolls on links 7 and 10 as the collusion parameter varies from $\alpha = 0$ to $\alpha = 1$ in steps of 0.1. It can be seen from these diagrams that the toll level obtained by SLCP and NDEMO are similar up until 0.6. However, beyond $\alpha = 0.7$, the tolls obtained by these algorithms differ.

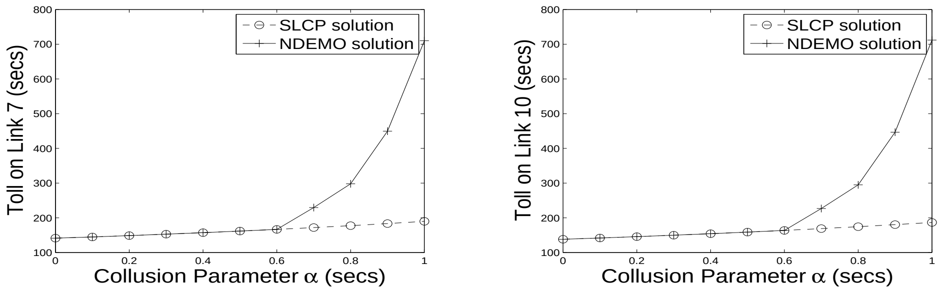


FIGURE 6.7: Toll levels as α varies for link 7 (Left) and link 10 (Right)

Restricting attention to the situation when the tolls obtained by the two algorithms differ (i.e. for values of $\alpha \geqslant 0.7$), Table 6.11 shows the tolls and revenues obtained by SLCP compared to that obtained by NDEMO. It is clear that the revenues obtained by NDEMO are higher and this suggests that the SLCP algorithm located a LNE instead of the global NE. To understand this further, best response functions, following Definition 3.2, were numerically estimated as the derivative of each concessionaires' objective in Eq. 6–17 with respect to each concessionaire's toll level for each value of α considered. As previously highlighted in Chapter 3, the intersection of these best response functions represent LNE points.

TABLE 6.11: Toll Revenues with tolls (in parenthesis) found by SLCP and NDEMO for $\alpha \geqslant 0.7$

	SLCP		NDEMO	
α	Link 7 (secs)	Link 10 (secs)	Link 7 (secs)	Link 10 (secs)
0.7	114,544	109,659	117,637	110,212
	(171.96)	(168.99)	(229.54)	(226.74)
0.8	115,404	110,461	146,556	136,936
	(177.56)	(174.54)	(297.84)	(295.05)
0.9	115,986	111,002	203,663	188,776
	(183.49)	(180.40)	(449.62)	(446.78)
1	116,198	111,206	280,255	266,465
	(189.76)	(186.57)	(713.19)	(709.53)

These plots are shown in Figs. 6.8 to 6.9 for values of α from 0.7 to 1 in steps of 0.1. In each plot, the continuous line is the best response function for the concessionaire on link 7 while the broken line is the best response function for the concessionaire on link 10. It is clear from these figures that two LNE exist.

Fig. 6.10 which plots the total revenue surface as tolls vary on links 7 and 10 in turn. It can be seen that the solution obtained by SLCP with $\alpha = 1$ turns out to be the *local* optimum of this total revenue function. Thus the solutions found by SLCP is an LNE that satisfies Definition 3.4 while NDEMO found the global NE and this can be verified in the surface plot in Fig. 6.10.

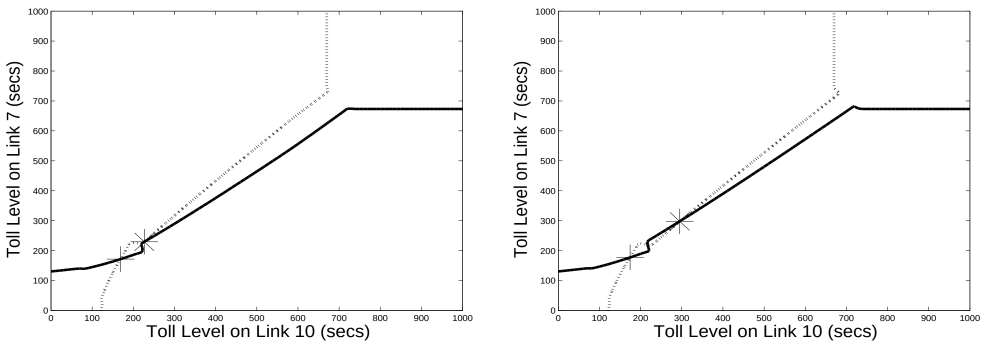


FIGURE 6.8: Best response function for Concessionaire on Link 7 (continuous line) and for Concessionaire on Link 10 (broken line) for two different α. Solutions found by SLCP indicated by + while solution found by NDEMO indicated by *. (Left): $\alpha = 0.7$ (Right): $\alpha = 0.8$

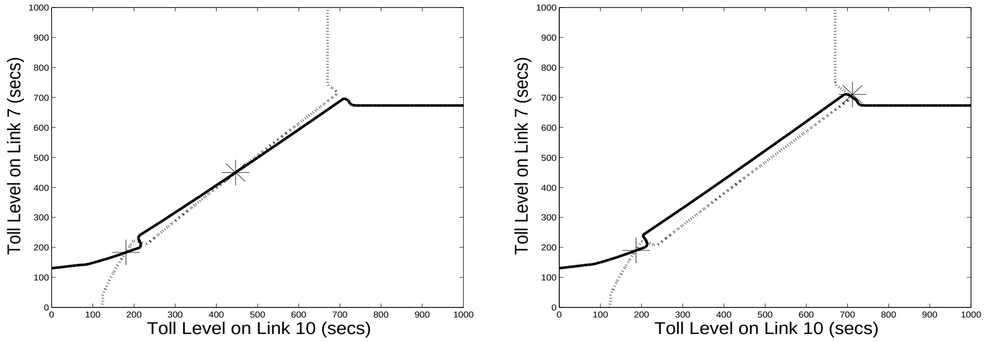


FIGURE 6.9: Best response function for Concessionaire on Link 7 (continuous line) and for Concessionaire on Link 10 (broken line) for two different α. Solutions found by SLCP indicated by + while solution found by NDEMO indicated by *. (Left): $\alpha = 0.9$ (Right): $\alpha = 1.0$

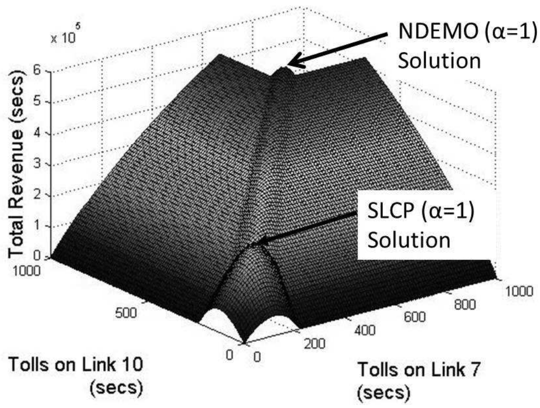


FIGURE 6.10: Surface Plot of Total Revenues from Tolls on both links 7 and 10 when $\alpha = 1$. Solution found by SLCP with $\alpha = 1$ is a local optimum. Solution found by NDEMO with $\alpha = 1$ is the global optimum.

A policy implication arising from this analysis is that if concessionaires move away from the competitive outcome toward the local monopoly outcome, the increase in the toll could be relatively small. If this is the case, these revisions might not be enough to attract the attention of regulators which would make the task of detecting collusive behaviour even more difficult. At the same time, even if the concessionaires fail to identify the global solution, they would still be able to increase revenues compared to being in full competition.

Next, the welfare impacts of the two LNE arising from collusion in this scenario are considered. Recall that the toll levels in the fully competitive outcome were *lower* than that obtained under second best social welfare maximisation (cf. Table 6.4).

As shown in Fig. 6.7, the tolls obtained by SLCP and NDEMO are similar until $\alpha > 0.6$. Thus the index of relative welfare improvement ω for all $\alpha \leqslant 0.6$, computed according to Eq. 6–15, will be identical as shown in the first four rows of Table 6.12.

TABLE 6.12: Index of Relative Welfare Improvement ω under Two Different LNE as collusion parameter α varies

α	ω (SLCP Solution)	ω (NDEMO Solution)
0	0.41	
0.2	0.42	
0.4	0.43	
0.6	0.43	
0.7	0.44	0.41
0.8	0.44	0.41
0.9	0.44	0.40
1	0.44	0.33

Interestingly, in this scenario, it is found that the tolls obtained by SLCP under partial collusion could be welfare improving. Under full competition i.e. $\alpha = 0$, the index of relative welfare improvement ω is 0.41. In contrast, it is found that all LNE solutions found by SLCP results in a higher ω (second column of Table 6.12). Recall that in this scenario, the tolls under competition were *lower* than the second best welfare maximising tolls (see Table 6.4). The higher tolls obtained with some degree of collusion here results in a welfare increase. However, it is important to bear in mind that the welfare increase is not due to collusion *per se* but only because collusion resulted, in the LNE case, in toll levels that were closer to those of a (second best) welfare maximising level. Had the concessionaires managed to locate the global NE, it is also clear from Table 6.12 that in that case, welfare would be lower ($\omega = 0.33$) compared to the fully competitive outcome ($\omega = 0.41$).

Nash Bargaining

Table 6.13 reports the revenues accruing to the players with full competition (i.e. the BATNA), the Monopoly outcome (i.e. Utilitarian Solution) and the NBS. Because of the local optima obtained in full collusion, as discussed above, the link based revenues to concessionaires under the Monopoly is taken directly from Table 6.4

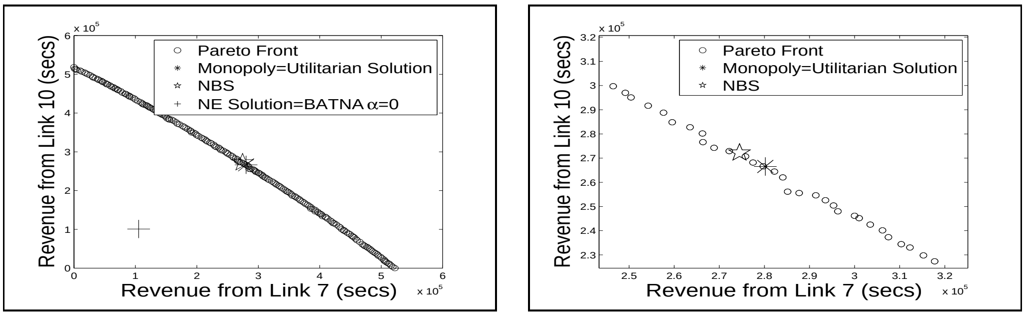


Figure 6.11: Left: NBS for Scenario 2 Right: Zoom in of NBS for Scenario 2

(p. 155). The key question in Axiomatic Bargaining is how the surplus should be distributed between the parties. As mentioned above, the columns labelled "% Share Monopoly" and "% Share NBS" show the percentage share each concessionaire obtains as a percentage of the total revenue achieved under the Monopoly Solution and the NBS respectively.

Comparing the NBS with the Utilitarian Solution/Monopoly in Table 6.13, it can be seen that total revenue is not maximised under the NBS. This means that the Monopoly solution is not only a Pareto Optimal outcome (see Fig. 6.11 where this solution is indicated by *) but it also ensures that the total surplus is maximised. Next, as verified in Figure 6.11, the NBS lies on the Pareto Front as indicated by the ☆ marker. This ascertains that that Axiom 2 of Pareto Optimality holds in the NBS.

Table 6.13: Scenario 2: Revenues under Competition, Monopoly and Nash Bargaining

	Revenue (secs)				
Link	Competitive (BATNA) (see Table 6.4)	Monopoly (Utilitarian)	%Share Monopoly	NBS	%Share NBS
7	105,295	280,255	51.3%	274,484	50.2%
10	100,848	266,465	48.7%	272,214	49.8%
Total	206,143	546,720		546,698	

From an inspection of the network parameters shown in Table 6.1 (p. 149), the free flow travel time of link 7 is 90 secs and its capacity is 1100 pcus/hr. With the same capacity, the free flow travel time on link 10 is 96 secs. Thus, *ceteris paribus*, link 7 is the faster of the two links, and should be more attractive to users. In this way, link 7 is the "stronger" player and receives 51.3% slice of the surplus under the Utilitarian Solution. Thus this approach to the allocation of surplus suggests that the stronger concessionaire in control of the "faster" link, at least on the basis of free flow time alone, should receive a higher share.

On the other hand, with the NBS, the percentage share of the surplus obtained by link 7 decreases, albeit marginally to 48.7%. The network parameters in Table 6.1 show that the power of the link travel time function for link 10 is lower at 3.1 and thus, the congestion effect of link 10 is lower than for link 7 at 3.2. This could be used to justify the higher percentage share of the surplus allocated to link 10 under the NBS. However, it can be seen that the Utilitarian Solution and the NBS are not significantly different which makes this example less interesting. More significant differences will be encountered in examples to follow in this and the subsequent chapter. The more interesting question is whether the Utilitarian or the NBS division of surplus, if any, are viewed as fair or equitable. The answer to this is subjective and beyond the scope of the current research. Nevertheless, the social welfare change under the NBS, as measured by the index of relative welfare improvement, ω is 0.33 which is the same as that under the monopoly solution. This is the same as that under monopoly, discussed above and represents a decrease over that obtained in competition (0.41) (see Table 6.12). The adverse impact on welfare as a result of Nash Bargaining as demonstrated in Scenarios 1 and 2 thus underscores the need for regulators to prevent bilateral bargaining between toll road concessionaires controlling parallel toll roads.

6.6.6 Collusion in Scenario 3: Links 3 and 7

In this test, collusion is considered between the concessionaires controlling the serially interdependent links 3 and 7. The Pareto Front, produced by MOSADE, on the assumption that players cooperate is shown in Fig. 6.12. The revenues obtained with full collusion between the concessionaires (i.e. $\alpha = 1$) is also indicated on this figure indicated by a *. In contrast to Scenario 2 above, this is exactly the same as the monopoly solution reported in Table 6.4 as there is no evidence of a local optimum.

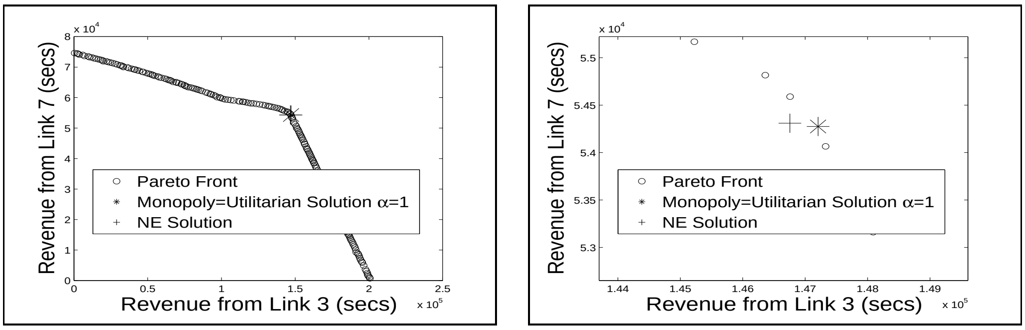


FIGURE 6.12: Left: Pareto Front, Monopoly and (fully) Competitive NE Revenues in Scenario 3 Right: zoom of region of interest on left panel.

It can be seen that the fully competitive NE outcome, although not Pareto Optimal, lies very close to the monopoly solution. For this reason, the right panel of Fig. 6.12 zooms in on the region of interest. The proximity of these two solutions suggests that in this scenario, the benefits of collusion and bargaining are, in fact, small[6.6].

Examining the network topology, it is evident that traffic has ample opportunity to avoid the toll on link 7. In comparison, link 4 (untolled in this scenario) is the only opportunity to avoid the toll on link 3. As discussed above, in any case, link 3 has both a lower free flow time and a lower power parameter vis-á-vis link 4. Taken together, this suggests that link 3 is the stronger player in this scenario.

Regardless of whether *total* revenues increase, the existence of otherwise or incentives to collude depend on whether the concessionaire individually are better off than their status quo position. As the collusion parameter α varies from 0 to 1, the revenues accruing to each concessionaire from collusion is plotted. This is termed the "collusion path" and this is shown in Fig. 6.13. It can be seen that while link 3 is a strong player as noted above, the collusion path also suggests that the concessionaire on Link 7 might not be incentivised to collude *fully.* This is because the revenue to this concessionaire in full collusion is *lower* compared to that attainable under partial collusion. This arises, since as noted in Chapter 5, there is no guarantee that Axiom 1 of Individual Rationality would be satisfied in the monopoly/Utilitarian Solution.

[6.6]For this reason, Nash Bargaining is not discussed in this scenario and the next. More interesting cases are presented in the next chapter.

Consider now the impacts on welfare if there was in fact collusion. Table 6.14 presents the index of relative efficiency ω. This confirms, as expected, that welfare losses are reduced as α increases.

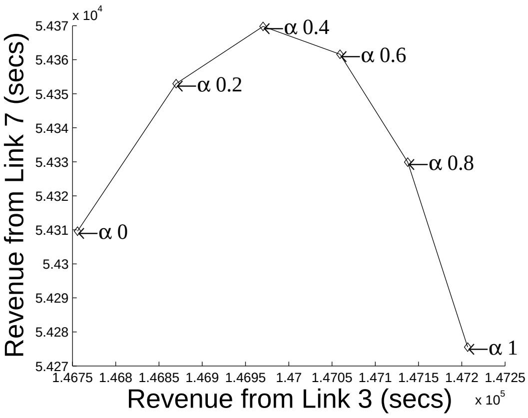


Figure 6.13: Collusion Path showing Revenues to Concessionaires on Link 3 (x-axis) and 7 (y-axis) as α varies from 0 (Competitive Solution) to 1 (Monopoly Solution).

Table 6.14: Scenario 3: Index of Relative Welfare Improvement ω as degree of collusion increases

Collusion Parameter α	Index of Relative Welfare Improvement ω
0	-0.19
0.2	-0.18
0.4	-0.18
0.6	-0.17
0.8	-0.17
1	-0.17

Thus in this serial setting, while it is better for social welfare if concessionaires did collude (compare social welfare under competition against monopoly for this link pair in Table 6.4), full collusion to aim for the Utilitarian Solution of maximising *total* revenues is unlikely to take place. This once again emphasises the divergence between private (profit maximising incentives) and social (welfare maximising incentives).

6.6.7 Collusion in Scenario 4: Links 1 and 3

In this section, collusion between the concessionaires on links 1 and 3 is described. In this scenario, as the collusion parameter α varies from 0 to 1, the toll on link 1 remains constant at 5000 seconds (the upper bound). This is because trips from Origin Zone 1 has no other alternative but to use this link for travel to the rest of the network. Thus route choice is not an option for these trips. However, the toll for link 3 decreases smoothly towards the monopoly toll as shown in Fig. 6.14.

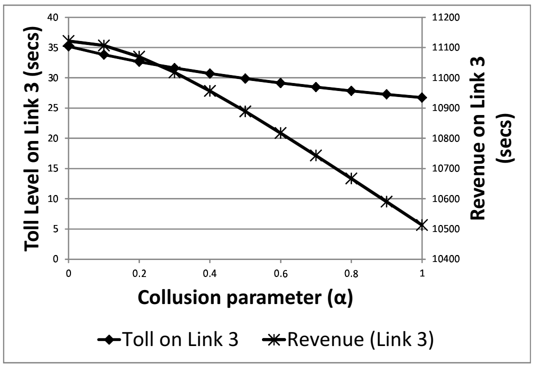


FIGURE 6.14: Scenario 4: Toll and revenue to concessionaire on link 3 as collusion parameter (α) varies between 0 and 1

This figure also shows the revenue for link 3, which decreases as the level of collusion increases. This is the same effect as was seen for Scenario 1 with the stronger player (link 1 in this case) being constrained by the maximum allowable toll. Again, it can be seen that in order to generate more revenue *in total*, the weaker player must accept a lower revenue. Thus, there is no incentive for the concessionaire on link 3 to collude here without an explicit revenue sharing agreement.

Looking at the outcomes in boths Scenario 1 and Scenario 4 (where the upper bound on the toll becomes active), the results suggest that if a concessionaire is limited by some upper bound on the toll level, or more generally, if there is a cap on the maximum possible toll, then there might be no incentive for the other concessionaire to collude.

6.7 Summary and Policy Implications

Comparing competitive, monopolistic and second best welfare maximising solutions for both parallel and serial link toll operation, these tests confirmed that in the case of competition between parallel links, competitive tolls are lower than monopoly tolls and are close to second best welfare tolls. In the serial link case, the experiments confirmed that the competitive tolls were greater than (or equal to) the monopoly tolls and that the welfare level is lower under competition than under monopoly. This suggests that regulators should not allow direct competition in the serial link case. Such findings are fully in line with the literature as discussed in Chapter 2.

However, in general networks, with both the serial and parallel cases, it was also shown that the presence of untolled alternatives would reduce the toll levels and so reduces opportunities for monopolistic behaviour.

Furthermore, this chapter investigated the performance of several alternative algorithms for locating the NE of the NCEPEC describing competition between revenue maximising toll road concessionaires. The first was a Fixed Point Iteration (FPI) algorithm which solves the NCEPEC by decomposing it into a series of inter-related MPECs. These MPECs are solved using the Cutting Constraint Algorithm (CCA) for toll pricing. The next algorithm, a novel contribution of this thesis, is the extension of an existing Sequential Linear Complementarity Problem (SLCP) approach to find NE in games where players are bound by an equilibrium constraint. Finally, the Synchronous Iterative (SI) algorithm (Yang and Huang, 2005; Yang *et al.*, 2009) was also tested and it was found that the reliance on the Method of Successive Averages (MSA) inherent in the algorithm slowed down the algorithm considerably. Both SLCP and SI were found to give similar solutions as the FPI approach but with significantly lower computation time. This is attributable to the fact noted by Zhang (2010) that the FPI is a single solution approach as it solves each MPEC *individually* until the entire system converges, while both SLCP and SI aim to solve each problem in the NCEPEC *simultaneously*. At the same time, both SLCP and SI rely on derivative information whose existence has been questioned in the literature. In order to counter this drawback, an alternative Evolutionary Algorithm NDEMO was developed and tested on 2 scenarios. Furthermore, it was shown that SLCP terminated at a LNE when applied to one instance in the study of collusion between toll road concessionaires while NDEMO could locate the NE. While there is theoretical assurance regarding NDEMO's ability to locate the NE (see Proposition 5.1), the algorithm was found to be computationally demanding from experiences with the numerical tests.

This chapter also applied a MOEA to demonstrate that the solution obtained under monopoly is one solution amongst a number of solutions that maximise the objectives of two concessionaires simultaneously. This set of solutions constitute the Pareto Front when viewed in revenue space (combinations of tolls that maximise both concessionaire's revenues *simultaneously*) satisfying the condition that one concessionaire could not increase revenue without the other incurring a reduction in revenue. It was highlighted that this example of simultaneous maximisation of revenues subject to the DUE condition was an instance of a MOPEC when the toll concessionaires were assumed to cooperate as discussed in Chapter 4.

The Pareto Front was subsequently used as a benchmark to study collusive behaviour amongst toll road concessionaires, an issue that has not been discussed in the literature. It was shown that there is an entire spectrum of Pareto Optimal revenue combinations that could be attained through collusion. In order to achieve one of these solutions in practice, an intuitive formulation was developed using a collusion parameter, α, that was used to reflect the degree of cooperation between two concessionaires. It is recognised that this approach is the equivalent of the Utilitarian Solution in Axiomatic Bargaining Theory. Implicit in the assumption was that concessionaire would be willing to reciprocate the action of the other through signalling behaviour. Even for the simple examples presented, there is potential for multiple equilibria to be obtained in collusion.

In the scenarios tested, it was found that as collusion increases from none ($\alpha = 0$) through to full collusion ($\alpha = 1$) then the tolls map from the fully competitive outcome to the monopoly solution. Where a local monopoly solution exists then the collusive behaviour can also map toward this local monopoly rather than the global one, which could be more acceptable to the public in terms of toll levels and welfare changes. It is quite possible that the global monopoly solution may not be stable and could explain why it may not be located by SLCP. At the same time, it could also make collusion more difficult for regulators to detect because the resulting toll changes could be relatively small.

In general, collusion between parallel links in competition was found to be welfare deteriorating while collusion between serial links was found to be welfare increasing. However, if competition resulted in toll levels being below the second best welfare maximising levels (as shown in Scenario 2, see p. 172), it is possible that collusion could be welfare enhancing even in the case of competition between parallel links. It is emphasised that this is not because of collusion *per se* but because collusion resulted in tolls that were closer to the (second best) welfare maximising level. In cases where the one concessionaire is limited by an upper bound on the toll level (which could occur in practice as regulators, for acceptability reasons, cap the toll charges), then to increase total revenue the other concessionaire might have to accept a reduction in revenue. Thus in such an instance, there would be limited incentive to collude without some form of agreement to share revenues set up in advance. Such an agreement might be facilitated through knowledge of the Pareto Front.

The Utilitarian Solution was shown to be both Pareto Optimal and maximises the total possible surplus available to be allocated between the bargaining parties. On the other hand, it suffers from the drawback that it can result in one party being made worse off i.e. attaining lower revenues vis-á-vis their individual BATNAs. This motivated the application of the Nash Bargaining Problem with Equilibrium Constraints (NBPEC) which can also be solved using the CCA. While the Nash Bargaining Solution (NBS) does not aim to maximise *total surplus*, in the examples where it was tested, it was shown to lead to Pareto Optimal outcomes as well as ensuring that the parties are not made worse off than under full competition.

Chapter 7

Competition between Toll Road Concessionaires: Part II

7.1 Introduction

The previous chapter applied several algorithms to study competition between toll road concessionaires where assuming that travellers routed in the network following Wardrop's DUE principle principally with a view to examining the transferability of insights from the literature regarding competition between toll road concessionaires to networks with multiple OD pairs. Consistent with the insights from economic theory discussed in Chapter 2, it was demonstrated that the welfare impacts of competition were intrinsically dependent on the relationships between the tolled links in competition. In the case of competition between concessionaires controlling links parallel to each other (i.e. parallel competition), it was shown that competition could result in tolls that were lower vis-à-vis a monopolist controlling both links together. Furthermore, the tolls under parallel competition approached the level determined by a benchmark second-best welfare maximising policy but, interestingly, could also be lower than the latter due to concessionaires undercutting each other. On the other hand, in the case of serial competition (i.e. competition between links in series), the results of the numerical tests showed that competitive tolls were either greater than or equal to the tolls set by a revenue maximising monopolist. Serial competition thus resulted in welfare even lower than that if a monopolist could exercise control over the links. These findings resonate well with economic theory which attributes the decrease in social welfare to the double marginalisation problem arising from the serial concessionaires who in their toll setting decisions, ignore the reduction in revenues to other links in the series, thereby setting tolls that are too high. The main policy conclusion demonstrated was that serial competition could be welfare deteriorating while parallel competition could be welfare improving.

However, due to the size of the network used in the last chapter, there was insufficient opportunity to study the effects of increases in the *intensity* of competition. The scenarios investigated therein only focused on two concessionaires. In this chapter, with a much larger network being used, it would be possible to investigate the effects of increased competition on welfare.

As noted in the last chapter, the problem of determining NE tolls in the case of competitive toll concessionaires is an instance of a NCEPEC which could be solved by adapting the Sequential Linear Complementarity Problem (SLCP) approach following that discussed in Chapter 4. The numerical comparisons against other algorithms presented in the previous chapter suggest that this algorithm was the most computationally efficient when compared against the Fixed Point Iteration (FPI) approach and the Synchronous Iterative (SI) algorithm. The NDEMO algorithm, whilst able to successfully identify the NE, did so at the expense of imposing a heavy computational burden.

These policy findings were all developed under the assumption that the binding equilibrium constraint governing the routing (and demands) in the transportation network was specified by an elastic demand Deterministic User Equilibrium (DUE) model. Fundamental to the DUE model is Wardrop's principle (Wardrop, 1952) which stipulates that, in equilibrium, all used paths between an O-D pair have equal and minimum generalised travel times while all unused paths have greater or equal generalised travel times. Wardrop's DUE principle implicitly assumes that users are endowed with perfect knowledge (Fisk, 1980; Bell and Iida, 1997) and are thus fully aware of the generalised travel times of all alternative routes connecting each OD pair. However, for various reasons such as habit and hysteresis (Goodwin, 1977; Blase, 1980) or familiarity with the network (Lotan, 1997), this assumption is clearly restrictive. Furthermore, empirical evidence suggests that both habit and knowledge of the network are equally significant determinants of route choice behaviour (Li and Wong, 1994; Prato *et al.*, 2012). In any case, it is unrealistic to expect that road users possess perfect information when shaping their route choice decisions.

Thus, in this chapter, the assumption that only the lowest generalised travel time routes connecting each OD pair are used in an equilibrium, is relaxed. In particular, travellers are assumed to vary in their perception of generalised travel times. Rather than generalised route travel times assumed to be "deterministic" reflecting perfect information across users, there is, instead, a stochastic variation in the perception of these generalised travel times across users. The analogous concept of equilibrium relevant in this context, that of Stochastic User Equilibrium (SUE), is subsequently formulated as the binding constraint when the interactions and strategies of competitive toll road concessionaires are studied.

The adoption of an SUE framework allows for the exploitation of two inter-related advantages. Firstly, the mathematical construct of the SUE problem, as discussed below, assures smoothness and differentiability. This allows for further exploitation of the derivative based algorithms such as SLCP, explored in the previous chapter, to compute NE toll levels when toll concessionaires compete for toll revenues on the highway network. Secondly, this allows for an investigation of whether the policy recommendations developed under the strict Wardropian DUE principle are valid when routing in the network is characterised by variations in travel time costs perceptions of users.

The rest of this chapter is organised as follows. In the next section, the SUE traffic assignment model is detailed. This will serve, in place of the Wardropian DUE formulation as the binding equilibrium constraint, facing various decision makers exercising control over toll pricing policies to maximise their respective objectives. As in the previous chapter, these objectives are namely (second best) welfare maximisation and revenue maximisation where a monopolist decides toll levels on each link in the set of predefined tollable links. These scenarios will also serve as welfare and revenue benchmarks when oligopolistic competition, featuring a number of concessionaires, each controlling a single link in the network engage in a game amongst themselves to maximise individual toll revenue. These toll pricing models are formulated in Section 7.3. As demonstrated in the last chapter, the SI algorithm was slowed down by the MSA procedure inherent in the algorithm. Thus in this chapter, the SI algorithm is excluded from the tests. Instead, FPI, SLCP and NDEMO will be applied to the resolution of the game between competitive concessionaires, applied to a network of Edinburgh[7.1], UK as reported in Section 7.4. While 12 scenarios are studied, NDEMO was only applied to a selection of these due to the lengthy run time requirements.

In addition, as described in Section 7.5, a test was conducted which allowed for toll road concessionaires to manage multiple links in the network, thereby emphasising both the interdependencies between links and the policy repercussions on the assignment of concessions to concessionaires. Collusion between toll road concessionaires is investigated in this network setting in Section 7.6 with a (cooperative) EPEC solution as a benchmark by means of the Pareto Front generated by application of the MOSADE Multiobjective Evolutionary Algorithm. In addition, the NBPEC formulation describing the Nash Bargaining approach to the division of surplus from colluding concessionaires is investigated. Section 7.7 summarises the insights obtained and draws conclusions.

7.2 Stochastic User Equilibrium Traffic Assignment

Recall, as defined in Chapter 6 (see Section 6.2), $\mathcal{R}$ denotes the index set of all acyclic routes in the network. Let $\mathcal{R}_k \subseteq \mathcal{R}$ be the subset of such routes serving OD pair k, $k \in \mathcal{K}$. Similarly, the $|\mathcal{L}| \times |\mathcal{R}|$ link-route incidence matrix is $\mathbf{\Delta}$, with elements Δ_{jr} equal to 1 only if link j is part of route r, and equal to 0 otherwise ($j \in \mathcal{L}; r \in \mathcal{R}$). Lastly, the OD-route incidence matrix $\mathbf{\Gamma}$ has dimensions $|\mathcal{K}| \times |\mathcal{R}|$, again each element of $\mathbf{\Gamma}$, Γ_{kr} equal to 1 if $r \in \mathcal{R}_k$ and equal to 0 otherwise ($k \in \mathcal{K}; r \in \mathcal{R}$). All other notation, having been previously defined, will not be repeated again.

[7.1] This network model, as used in Sumalee (2004*a*), comprises 550 OD pairs and 344 one way links. The tests conducted on such a network serve a two-fold purpose. Firstly, utilising a larger network compared to that used in Chapter 6, allows for an investigation of the policy implications of increasing the *intensity* of competition (i.e. beyond scenarios involving two competitors). Secondly, the performance of the proposed algorithms can be evaluated within a more realistic setting.

7.2.1 Mathematical Formulation

It is assumed that rational road users are utility maximisers or more appropriately in this context, (generalised travel) time minimizers. Thus, individuals choose routes amongst those available in proportions specified by a random utility model. Following the literature (Sheffi, 1985; Yang and Huang, 2005), the utility $\mathfrak{U}_{r,k}$ of a representative traveller on route r connecting OD pair k, is given by Eq. 7–1.

$$\mathfrak{U}_{r,k} = -\theta \hat{c}_{r,k} + \xi_{r,k} \qquad \text{(Eq. 7–1)}$$

Eq. 7–1 implies that travellers choose routes on the basis of its observable characteristics (namely its generalised route travel times, $\hat{c}_{r,k}$ with a scaling parameter θ) as well as unobservable route specific characteristics, the latter being represented by $\xi_{r,k}$. Consistent with the previous chapters, it is assumed that these (generalised) link travel times are continuous, differentiable and monotonically increasing function of link flows $\mathbf{v}$. The generalised travel times (including tolls x_j if any) on link j, $j \in \mathcal{L}$ is, as defined in the previous chapter (see Eq. 6–1), given as $c_j(v_j, x_j)$. It is assumed that generalised *link* travel times are additive such that the generalised travel time of route r,$r \in \mathcal{R}_k$ is simply the sum of the generalised travel times of each link that comprise the route. Then the generalised route travel times for OD pair k, $k \in \mathcal{K}$ can be mapped to these link travel times by means of Eq. 7–2.

$$\hat{\mathbf{c}}_k = \mathbf{\Delta}^\intercal \mathbf{c}(\mathbf{v}, \mathbf{x}), k \in \mathcal{K} \qquad \text{(Eq. 7–2)}$$

If it is assumed that the unobservable route specific characteristics, $\xi_{r,k}$, in Eq. 7–1 are normally distributed variables, then one would obtain the probit-based route choice model (Connors *et al.*, 2007). On the other hand, the focus in this thesis is on the more tractable multinomial logit-based route choice model. Such a model arises if $\xi_{r,k}$ are assumed to be independently and identically distributed Gumbel variates (see e.g. McFadden, 1974). Then, for each OD movement $k \in \mathcal{K}$, let $\mathbf{p}_k(\hat{\mathbf{c}}_k; \theta)$ denote the vector function with individual elements $p_r(\hat{c}_k; \theta), r \in \mathcal{R}_k$; these elements denoting the proportion of travellers who select route r on OD movement k given the scaling/dispersion parameter, θ. In the multinomial logit model, these elements are given by Eq. 7–3.

$$p_r(\hat{\mathbf{c}}_k;\theta) = \frac{\exp(-\theta\hat{c}_r)}{\sum_{z\in\mathcal{R}_k}\exp(-\theta\hat{c}_z)} \quad r\in\mathcal{R}_k\ k\in\mathcal{K} \qquad \text{(Eq. 7–3)}$$

Then $\mathbf{p}(\hat{\mathbf{c}};\theta)$ is the vector representation of $\mathbf{p}_k(\hat{\mathbf{c}}_k;\theta)$ i.e. with elements $\mathbf{p}(\hat{\mathbf{c}};\theta) = (\mathbf{p}_1(\hat{\mathbf{c}}_1;\theta), \mathbf{p}_2(\hat{\mathbf{c}}_2;\theta), \ldots, \mathbf{p}_{|\mathcal{K}|}(\hat{\mathbf{c}}_{|\mathcal{K}|};\theta))^\intercal$, $k\in\mathcal{K}$. Associated with the choice probability, $s_k(\mathbf{c}_k;\theta)$ gives the expected maximum utility for travellers on OD movement k (Sheffi, 1985; Bell and Iida, 1997). This is sometimes referred to synonymously as either "composite costs" (e.g. Leurent, 1997) or "satisfactions" (e.g. Maher, 1998). In this thesis, $s_k(\mathbf{c}_k;\theta)$ is referred to as "composite generalised travel time" for OD pair k,$k\in\mathcal{K}$. In the logit model, this composite generalised travel time function, has a closed form expression[7.2] as given by Eq. 7–4. This can be collected in vector form as Eq. 7–5 where ln(.) and exp(.) denote *element-by-element* application of these functions on the input argument.

$$s_k(\hat{\mathbf{c}}_k;\theta) = -\frac{1}{\theta}\left(\ln(\sum_{r=1}^{|\mathcal{R}_k|}\Gamma_{kr}(\exp(-\theta\hat{c}_k)))\right), \quad k\in\mathcal{K} \qquad \text{(Eq. 7–4)}$$

$$\mathbf{s}(\mathbf{c};\theta) = -\theta^{-1}\ln(\mathbf{\Gamma}\exp(-\theta\hat{\mathbf{c}})) \qquad \text{(Eq. 7–5)}$$

On the demand side, a demand function $d_k(s_k)$ for each OD pair k,$k\in\mathcal{K}$, gives the *level* of demand, q_k, as a function of the composite generalised travel time, s_k. It is assumed that this demand function is monotonically decreasing in s_k, separable and a function solely of the composite generalised travel time of OD pair k. Such assumptions mean that its inverse, $d_k^{-1}(.)$ exists. Hence $d_k(s_k) = q_k$ and $d_k^{-1}(q_k) = s_k,\ k\in\mathcal{K}$.

7.2.2 SUE route choice model

A stochastic user equilibrium (SUE) is attained in the network when no user can reduce *perceived* generalised travel times by unilaterally changing routes (Daganzo and Sheffi, 1977; Sheffi, 1985). Additionally, since the demand for travel for each OD

[7.2] Amongst others, Connors *et al.* (2014) note that, in contrast to the logit model assumed here, an analytical expression for the composite generalised travel time function does not exist in a probit-based SUE assignment.

pair is responsive to the composite generalised travel time experienced by that OD pair, the resulting route choice model is a SUE model with elastic demand (Maher, 2001). It can be shown that the Wardrop's DUE principle is a special, limiting case of SUE (Sheffi, 1985; Yang and Huang, 2005)[7.3].

The convex set of feasible demands and link flows to a traffic assignment program was defined in Chapter 6 (see Equation Eq. 6–3). Note that the definition of this feasible set is independent of whether routing satisfies a Wardrop's DUE (as discussed in Chapter 6 or the (logit) SUE principle as used here. For completeness, the definition of this feasible set is restated in Eq. 7–6.

$$\mathcal{D} = \left\{ (\mathbf{v}, \mathbf{q}) : \mathbf{v} = \boldsymbol{\Delta}\mathbf{f} \text{ and } \mathbf{q} = \boldsymbol{\Gamma}\mathbf{f} \text{ where } \mathbf{f} \geqslant 0, \mathbf{f} \in \mathbb{R}^{|\mathcal{R}|} \right\} \qquad \text{(Eq. 7–6)}$$

Cantarella (1997) shows that for any given vector of tolls $\mathbf{x}$, there is a corresponding unique SUE vector of link flows (and demands), $(\mathbf{v}^*, \mathbf{q}^*) \in \mathcal{D}$, that solves the system of equations $\mathcal{S}(\mathbf{x})$ in Eq. 7–7, parametrised in the toll vector $\mathbf{x}$.

$$\mathcal{S}(\mathbf{x}) \begin{cases} \mathbf{v}^*(\mathbf{x}) = \boldsymbol{\Delta}\mathbf{q}\mathbf{p}(\boldsymbol{\Delta}^\intercal(\mathbf{t}(\mathbf{v}) + (\boldsymbol{\Xi}\mathbf{x})); \theta) \\ \mathbf{q}^*(\mathbf{x}) = \mathbf{d}(-\theta^{-1} \ln (\boldsymbol{\Gamma} \exp (-\theta \boldsymbol{\Delta}^\intercal(\mathbf{t}(\mathbf{v}) + \boldsymbol{\Xi}\mathbf{x})))) \\ \{\mathbf{v}, \mathbf{q}\} \in \mathcal{D} \end{cases} \qquad \text{(Eq. 7–7)}$$

For brevity, the shorthand notation in Eq. 7–8 will be used to denote specifically such a unique vector of link flows (and demands) that solves Eq. 7–7 given toll vector $\mathbf{x}$.

$$\{\mathbf{v}^*(\mathbf{x}), \mathbf{q}^*(\mathbf{x})\} \leftarrow \text{SOL}\{\mathcal{S}(\mathbf{x})\} \qquad \text{(Eq. 7–8)}$$

As $\mathcal{S}(\mathbf{x})$ represents a system of equations, this is a special case of the more general VI (Nagurney, 1999, Proposition 1.1, p. 5).

In contrast to the SUE principle described here, Wardrop's DUE principle implies that some paths may not be used at equilibrium for some demand level. Nevertheless

[7.3] A larger θ, *ceteris paribus*, implies a lower variance in perception of generalised route travel times across users. In the limiting case, the effects of the random component $\xi_{r,k}$ in Eq. 7–1 is dominated by the measured route travel time component $\hat{c}_{r,k}$ implying that users choose routes based only on $\hat{c}_{r,k}$, thereby recovering Wardrop's DUE principle.

it is possible that as demand increases, these hitherto unused paths get used. This implies that there is a discontinuity in the path flows and thus the response surface of the path flows to generalised travel time differences is not necessarily continuous or differentiable (Bell and Iida, 1997, p. 85). On the other hand, in a logit model typified by Eq. 7–3, there is a non-zero probability[7.4] of path utilisation. As a result, the SUE map in Eq. 7–7 is smooth and differentiable (Davis, 1994; Bell and Iida, 1997; Connors *et al.*, 2007). This is the key characteristic distinguishing the MPECs and EPECs examined in this chapter from those constrained by Wardrop's DUE presented in Chapter 6. The smoothness of the logit assignment model is a feature that can be exploited when applying derivative based algorithms for the determination of tolls under the alternative toll pricing models.

Note that the logit model satisfies the axiom known as Independence of Irrelevant Alternatives (McFadden, 1974, Axiom 1, p.109). which is also assumed in the Nash (Nash, 1950*b*) Bargaining Model (see Axiom 4 in Section 5.6.1). This axiom means that the ratio of the probabilities of choosing one alternative to that of choosing another is *independent* of the probability of choosing other available alternatives (Florian and Fox, 1976; Ortuzar and Willumsen, 1994). As shown in Florian and Fox (1976), this is a disadvantage of applying the multinomial logit model to the traffic assignment problem, and can bias the assignment results. This drawback is highlighted in situations featuring overlapping routes (i.e. where a link is part of more than a single route). Proposals to remedy this undesirable property include the use of the nested and cross-nested logit models (Vovsha, 1997; Vovsha and Bekhor, 1998; Prashker and Bekhor, 1999, 2004). At a more general level, the probit model (Connors *et al.*, 2007; Uchida *et al.*, 2007) could be used as an alternative SUE model as it does not suffer from this drawback of the logit model but still retains the smoothness of the SUE formulation. On the other hand, van der Weijde *et al.* (2013) demonstrate that the differences between the logit variants and the probit models in realistic applications are, in fact, relatively small, particularly in relation to assessing the welfare impacts critical to guiding policy analysis. In view of this observation and the advantage that the logit model has over the probit in that the

[7.4] van der Weijde *et al.* (2013) highlight that although that the probability that a route is used can approach zero, it never reaches it.

former is endowed with a closed form expression for the composite generalised travel time function (see Eq. 7–4), this thesis will restrict the focus of SUE assignment to that specified by the simple multinomial logit model.

7.3 Problem Formulation

Following the brief overview of the SUE traffic assignment model, this section mathematically formulates the three models of toll pricing decision making investigated in this chapter. These models serve as benchmarks of the NCEPEC model of competition between toll road concessionaires. The first two models are MPECs which feature, in each case, a single regulator deciding tolls on the set of tollable links, $\mathcal{J}$, aiming to maximise their respective objective. These are the model of welfare maximising second best toll pricing which assumes that a regulator aims to maximise social welfare, giving a benchmark on the upper bound of efficiency gains achievable and the model of revenue maximisation by a single concessionaire i.e. the monopolist, giving a benchmark on the upper bound of revenues achievable. Subsequently, the third model formulated is that of competition between revenue maximising concessionaires. Note that these are all hierarchical optimisation problems constrained by the VIP in Eq. 7–7, parameterised in the leader's toll variables, stipulating that users' route choice adhere to the SUE principle.

7.3.1 Welfare Maximising Second Best Toll Pricing

With a SUE assignment, social welfare can be measured using Eq. 7–9.

$$\mathcal{W}_{\text{SUE}}(\mathbf{x};\theta) = \sum_{k\in\mathcal{K}} \int_{0}^{q_k} d_k^{-1}(w)dw - \mathbf{q}^{\intercal}\mathbf{s}(\mathbf{c};\theta) + (\boldsymbol{\Xi}\mathbf{x})^{\intercal}\mathbf{v}(\mathbf{x}) \qquad \text{(Eq. 7–9)}$$

Eq. 7–9 can be interpreted as follows. The inverse demand function pertaining to OD pair k, $k \in \mathcal{K}$ gives the marginal benefit of an additional unit of travel between that OD pair and so its integral (the first term in Eq. 7–9) is the total measure of travellers' benefits from travel between the OD pairs. The second term gives the total composite generalised travel times (inclusive of tolls) incurred in doing so.

Finally, the third term reflects the fact that toll revenues are a transfer payment (from the users of the tolled links to the collecting agency) and therefore, do not represent real resources and so do not affect social surplus. Therefore these toll revenues have to be added back in the computation of social welfare measured at the aggregate level.

Therefore in the situation, of a (second-best) welfare maximising toll pricing policy, toll levels are chosen, for each link in the set of pre-defined tollable links, to maximise such a welfare measure. Mathematically, the resulting optimisation problem facing the regulatory authority implementing such a policy can be formulated as the MPEC shown in Eq. 7–10 with the SUE conditions in Eq. 7–7 constituting the (always) binding equilibrium constraint.

$$\begin{aligned} \underset{x \in X}{\text{Maximise}} \quad & \mathcal{W}_{\text{SUE}}(\mathbf{x};\theta) = \sum_{k \in \mathcal{K}} \int_0^{q_k} d_k^{-1}(w)dw - \mathbf{q}^{\intercal}\mathbf{s}(\mathbf{c};\theta) + (\boldsymbol{\Xi}\mathbf{x})^{\intercal}\mathbf{v}(\mathbf{x}) \\ \text{subject to} \quad & \{\mathbf{v}^*(\mathbf{x}), \mathbf{q}^*(\mathbf{x})\} \leftarrow \text{SOL}\{\mathcal{S}(\mathbf{x})\} \end{aligned} \qquad \text{(Eq. 7–10)}$$

7.3.2 Monopoly

Aside from the equilibrium constraint reflecting that traveller's route choices follow a SUE elastic demand model instead of DUE, the MPEC of a single toll concessionaire, the "monopolist", deciding toll levels on each pre-defined tollable link in the network does not differ from that given in Chapter 6 (see Eq. 6–7 in Section 6.3.2). The revenue maximisation problem facing such a monopolist is set forth in Eq. 7–11.

$$\begin{aligned} \underset{x \in X}{\text{Maximise}} \quad & \phi_{\text{M}}(\mathbf{x}, \mathbf{v}(\mathbf{x})) = (\boldsymbol{\Xi}\mathbf{x})^{\intercal}\mathbf{v}(\mathbf{x}) \\ \text{subject to} \quad & \{\mathbf{v}^*(\mathbf{x}), \mathbf{q}^*(\mathbf{x})\} \leftarrow \text{SOL}\{\mathcal{S}(\mathbf{x})\} \end{aligned} \qquad \text{(Eq. 7–11)}$$

7.3.3 Oligopolistic Competition Between Toll Road Concessionaires

As stated in Chapter 6 (see Section 6.3.3), in oligopolistic competition, there are, a minimum of two, competing concessionaires, defined by an index set $\mathcal{N}$. Each concessionaire is assumed to choose toll levels on the link which he exercises control over in order to maximise revenue by solving Eq. 7–12, responding to the toll levels set by all other concessionaires but taking into account the reaction of the users.

$$\forall i \in \mathcal{N}, \text{ Concessionaire } i \text{ solves: } \begin{cases} \underset{x_i \in X_i}{\text{Maximise}} \;\; \phi_i(\mathbf{x}, \mathbf{v}(\mathbf{x})) = v_i(\mathbf{x})x_i \\ \text{subject to} \\ \{\mathbf{v}^*(\mathbf{x}), \mathbf{q}^*(\mathbf{x})\} \leftarrow \text{SOL}\{\mathcal{S}(\mathbf{x})\} \end{cases} \qquad \text{(Eq. 7–12)}$$

As all other toll concessionaires are envisaged to be doing the same simultaneously, a game thus ensues. The objective then is to determine the NE toll vector satisfying Definition 3.1 such that no concessionaire can improve his payoff by unilaterally deviating from his chosen toll level. As stressed throughout this thesis, this problem is an NCEPEC since each concessionaire's payoff are jointly coupled through the active SUE constraint.

7.3.4 Solution Methods

In the numerical tests to be reported in the next section, because of the smoothness and differentiability of the SUE assignment model mentioned in Section 7.2, and in contrast to the DUE formulation of these equivalent problems described in Chapter 6, each of the MPECs in Eq. 7–10 and Eq. 7–11 can be solved as standard non-linear programming problems by embedding the SUE conditions in Eq. 7–7 directly as constraints without the need for specialised algorithms such CCA. The ability to embed gradients directly in the optimisation problem is something distinctive about adopting a SUE as opposed to DUE approach.

In the case of competition between the toll road concessionaires, when the FPI algorithm is applied, the "inner MPEC" (line 7 of Algorithm 4.1) i.e. the revenue maximisation MPEC for concessionaire $i, i \in \mathcal{N}$ with all other concessionaires' tolls held fixed, can also be solved as a standard non-linear optimisation problem with the SUE conditions as constraints. In the application of the SLCP algorithm, the smoothness of the SUE constraints allows for first and second order derivatives i.e. $\mathbf{F}(\mathbf{x})$ and $\nabla\mathbf{F}(\mathbf{x})$ (see Section 4.5) to be computed without difficulties.

In this thesis, the IPOPT interior point solver (Wächter and Biegler, 2006) available as an option in the freely available OPTI toolbox for MATLAB (Currie and Wilson, 2012) is used as the non-linear programming solver.

7.4 Numerical Tests

The network of Edinburgh with 550 OD pairs, as used in Sumalee (2004*a*), was used for the numerical tests reported in this chapter. Each of the 344 links in this network has a travel time function specified by $t_j = t_j^0 + \beta_j(v_j/\kappa_j)^{\rho_j}$. Details of the link parameters are available in Sumalee (2004*a*). The logit dispersion parameter θ was set at 0.01. Two demand functions were considered. For consistency with the previous chapter, the first demand function used in the numerical tests was based on the power law functional form shown in Eq. 7–13, with q_k^0 and s_k^0 being the demand and composite generalised travel time for each OD pair in the base (i.e. no toll) equilibrium respectively. Following Sumalee (2004*a*), η_k was set to -0.58 for all 550 OD pairs.

$$q_k = q_k^0 \left(s_k/s_k^0\right)^{\eta_k}, \quad k \in \mathcal{K} \tag{Eq. 7–13}$$

To overcome any possible limitations of the power law demand function as mentioned in Chapter 6, an additional set of tests were conducted with the exponential demand function (Eq. 7–14). In this function, γ_k was set to 0.0005 for all 550 OD pairs to both replicate similar welfare changes attainable with first best pricing relative to the base under both demand functions and similar elasticities in the base case.

$$q_k = q_k^0 \exp(-\gamma_k(s_k - s_k^0)) \tag{Eq. 7–14}$$

This alternative functional form was chosen as it has been used by several authors in the literature (e.g. Mills, 1995; Yang *et al.*, 2009, *inter alios*). Furthermore, besides the power law form in Eq. 7–13, it is an alternative functional form finding support in UK transport planning guidance (HA, 1997; DfT, 2013*b*). In addition, the exponential demand function “fits conveniently with the logit choice model” (Bell and Iida, 1997, p.139).

7.4.1 Base Network Characteristics

While the base demands, q_k^0, were the same as that used in Sumalee (2004*a*), the composite generalised travel times in the base, s_k^0, were obtained from a fixed demand SUE assignment (using the Optimal Step Length algorithm of Maher (1998)) of the base demands. In all other assignments, the Elastic Demand SUE model was solved using the algorithm described in Maher (2001) with the base composite generalised travel times obtained. During the network building stage, 20 paths were generated for each of the 550 OD pairs by application of Yen's K-shortest path algorithm (Yen, 1971). Due to the structure of the network, 2 of these OD pairs had 3 and 4 paths each. In total, 10967 paths were generated which were used consistently in all tests reported here.

7.4.2 Description of Test Scenarios

In total, 12 test scenarios were developed. Fig. 7.1 gives an overview of the locations of these links within the Edinburgh network while Table 7.1 shows the link flows and volume/capacity ratios for the first set of 6 primary link pairs from the base (no toll) SUE assignment. These ranked amongst the highest in the network in the untolled base equilibrium.

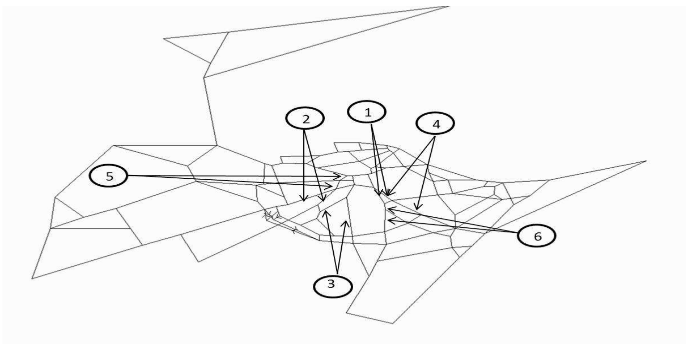


FIGURE 7.1: Highway Network of Edinburgh from Sumalee (2004*a*) showing location of first 6 link pairs tested.

In order to investigate the effects of increasing the intensity of competition, the remaining 6 scenarios were formed through combination and extensions of these first 6 link pairs. The tolled links tested in Scenarios 7 and 8 were formed through incremental combination of Scenarios 1 and 2 as shown in the left and right panes of Fig. 7.5. Likewise, Scenarios 9 and 10 were created as incremental combinations of Scenarios 1, 2 and 3 as shown in Fig. 7.6. These scenarios all involve links that are parallel to each other, topologically providing alternative routes for traffic from the south of the network to access the city centre of Edinburgh. Scenario 11 is obtained by adding link 143 to Scenario 4 while Scenario 12 is obtained by a southward extension of Scenario 6 which involve toll competition between links in series (see Fig. 7.7).

As reported in Sumalee (2004a) and consistent with the assumption used in Chapter 6, the value of travel time assumed was 7.63 pence per minute and in the numerical tests, the upper bound on the toll level $\bar{x}_j$, $j \in \mathcal{J}$ was 5000 secs (approximately £6).

TABLE 7.1: Link flows and volume capacity ratios in the no toll base equilibrium of links that constitute the first 6 scenarios in numerical tests

Scenario	Link	Link Flows (pcus)	Volume Capacity Ratio
1	284	4,268.35	2.85
	285	4,990.69	3.33
2	258	3,986.60	2.66
	259	2,606.43	1.74
3	229	1,503.17	1.50
	230	2,473.09	1.65
4	284	4,268.35	2.85
	286	2,942.73	1.96
5	243	3,601.43	2.40
	247	4,805.46	3.20
6	291	2,731.47	1.82
	296	4,095.47	2.73

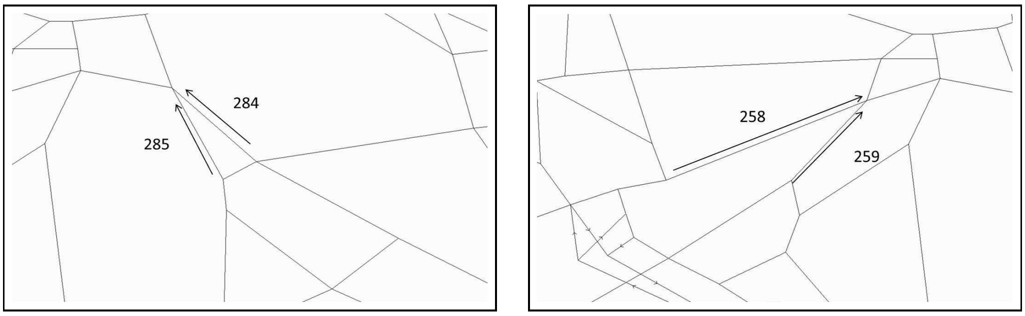


Figure 7.2: Links Constituting Scenarios 1 (left) and 2 (right) (Arrows show direction of travel upon which a toll is levied with link numbers indicated alongside.)

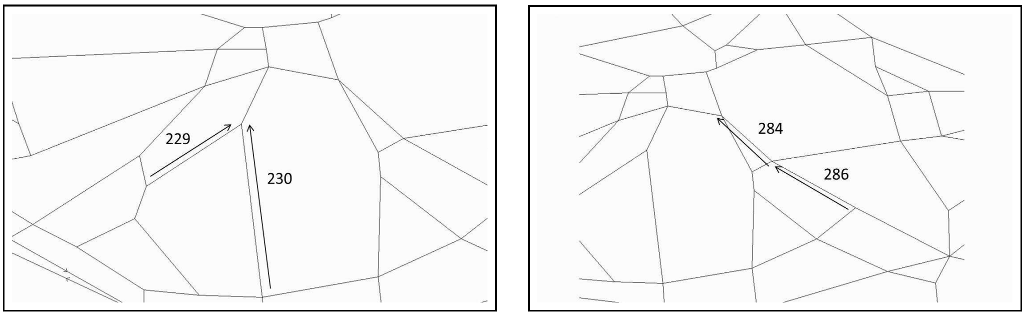


Figure 7.3: Links Constituting Scenarios 3 (left) and 4 (right) (Arrows show direction of travel upon which a toll is levied with link numbers indicated alongside.)

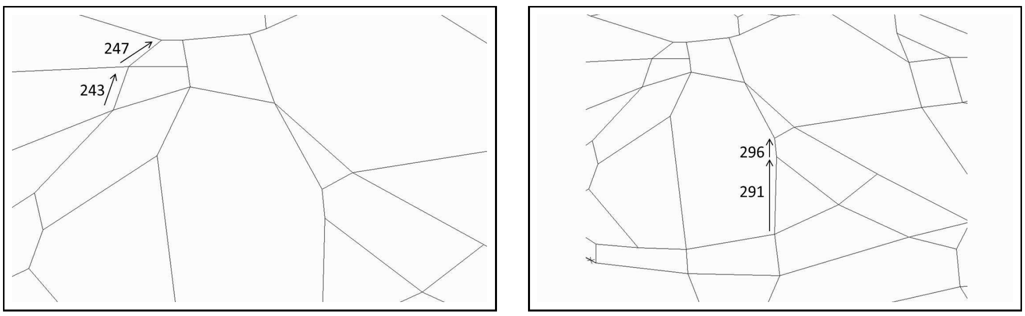


Figure 7.4: Links Constituting Scenarios 5 (left) and 6 (right) (Arrows show direction of travel upon which a toll is levied with link numbers indicated alongside.)

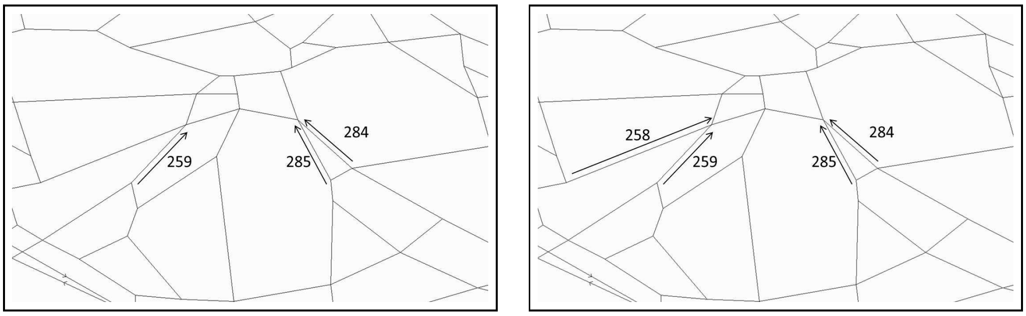


FIGURE 7.5: Links Constituting Scenarios 7 (left) and 8 (right) (Arrows show direction of travel upon which a toll is levied with link numbers indicated alongside.)

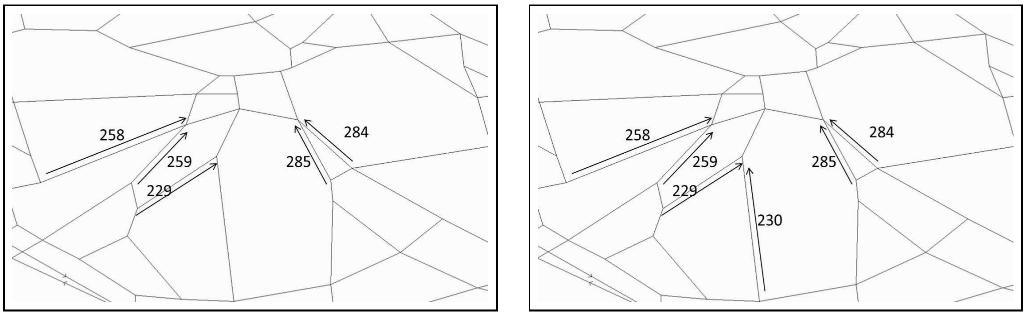


FIGURE 7.6: Links Constituting Scenarios 9 (left) and 10 (right) (Arrows show direction of travel upon which a toll is levied with link numbers indicated alongside.)

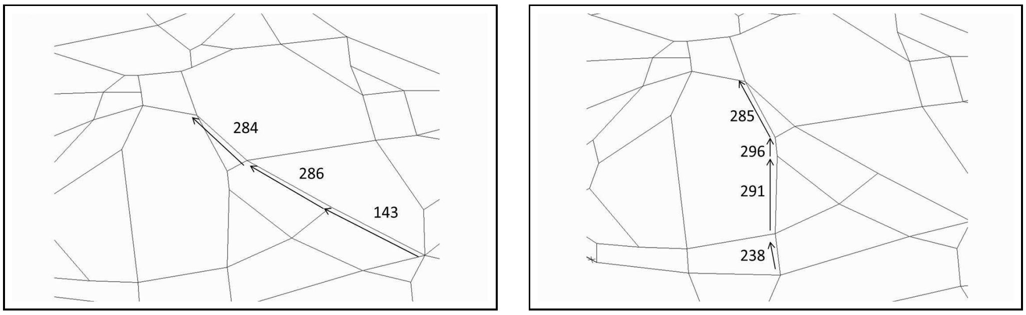


FIGURE 7.7: Links Constituting Scenarios 11 (left) and 12 (right) (Arrows show direction of travel upon which a toll is levied with link numbers indicated alongside.)

7.4.3 Results of Alternative Algorithms to determine Nash Equilibrium Tolls

For the case of the power law demand function, the FPI and SLCP algorithms were applied to all scenarios tested to determine the NE tolls. The results are shown in Tables 7.2, 7.3 and 7.4. The FPI algorithm was terminated when the change between successive iterations was less than 0.0001 (see Algorithm 4.1). In

Table 7.2: Equilibrium tolls in competition game between concessionaires for Scenarios 1 to 6 with Power Law Demand Function obtained by 3 algorithms: FPI, SLCP and NDEMO

		FPI			SLCP			NDEMO		
Scenario	Link	Toll (secs)	Iterations	CPU time (secs)	Toll (secs)	Iterations	CPU time (secs)	Toll (secs)	Iterations	CPU time (secs)
1	284	1,088.48	11	680	1,088.48	6	304	1,088.48	61	35,112
	285	1,132.19			1,132.19			1,132.19		
2	258	1,432.40	5	817	1,432.40	6	353		NOT RUN	
	259	855.16			855.16					
3	229	611.21	8	702	611.21	4	187	611.21	58	30,437
	230	912.02			912.02			912.02		
4	284	732.37	6	868	732.37	4	187	732.37	59	31,707
	286	691.06			691.06			691.06		
5	243	467.17	7	1,566	467.17	4	246		NOT RUN	
	247	497.79			497.79					
6	291	501.60	8	1,125	501.60	5	255	501.59	52	30,206
	296	740.21			740.21			740.21		

Table 7.3: Equilibrium tolls in competition game between concessionaires in Scenarios 7 to 9 obtained by 3 algorithms: FPI, SLCP and NDEMO

		FPI			SLCP			NDEMO		
Scenario	Link	Toll (secs)	Iterations	CPU time (secs)	Toll (secs)	Iterations	CPU time (secs)	Toll (secs)	Iterations	CPU time (secs)
7	284	1,114.27	11	1,487	1,114.52	5	554	1,114.52	92	113,672
	285	1,161.91			1,162.20			1,162.20		
	259	735.19			742.47			742.47		
8	284	1,124.72	18	3,482	1,124.72	17	2,627	1,124.75	189	180,437
	285	1,173.83			1,173.83			1,173.84		
	258	1,429.66			1,429.66			1,432.95		
	259	905.36			905.36			904.91		
9	284	1,127.28	12	3,590	1,127.28	5	1,684	1,127.28	124	269,336
	285	1,175.54			1,175.54			1,175.54		
	258	1,449.73			1,449.73			1,449.73		
	259	989.70			989.70			989.70		
	229	789.69			789.69			789.69		

Table 7.4: Equilibrium tolls in competition game between concessionaires in Scenarios 10 to 12 obtained by 3 algorithms: FPI, SLCP and NDEMO

		FPI			SLCP			NDEMO		
Scenario	Link	Toll	Iterations	CPU time	Toll	Iterations	CPU time	Toll	Iterations	CPU time
10	284	1,220.04	13	4,226	1,224.12	7	2,339	1,224.12	162	347,420
	285	1,276.03			1,280.21			1,280.21		
	258	1,474.98			1,482.61			1,482.61		
	259	1,044.94			1,063.70			1,063.70		
	229	805.44			941.61			941.61		
	230	1,198.81			1,231.31			1,231.31		
11	284	730.14	10	2,134	730.14	6	603		NOT RUN	
	286	697.41			697.41					
	143	243.43			243.43					
12	238	221.67	16	3,753	221.67	20	1,628		NOT RUN	
	285	761.25			761.25					
	291	894.73			894.73					
	296	639.65			639.65					

the SLCP algorithm, the termination criteria is based on the absolute maximum of the elements in the vector of first order derivatives of each player's payoff function (see Algorithm 4.2) and this was set at 0.0001 as well. These were exactly the same convergence criterion used in each algorithm to that used in Chapter 6. In addition, NDEMO (using control parameters as given in Table 6.6) was also applied to a selection of these scenarios. However, it was not applied in all scenarios due to the lengthy CPU times as shown in the tables. As an illustration of the computational burden, in Scenario 10, (the scenario with the largest number of parallel competing concessionaires), NDEMO took over 340,000 seconds (approximately 4 days) to satisfy the convergence criterion based on the standard deviation of the population, σ was less than the termination tolerance, ϵ, of 1.00E-06.

Compared to FPI and SLCP, the NDEMO algorithm as discussed in Chapter 5 does not rely on derivative information but uses instead a pairwise comparison procedure (Algorithm 5.7 refers) to determine Nash Dominance status of the chromosomes (which encode the strategic variables of the competing concessionaires) that requires extensive function evaluations in each iteration for each member of the population. Since there is a need to solve a SUE traffic assignment each time such a Nash Dominance check is performed, this added to the computational burden intrinsic in the design of NDEMO. Though there is theoretical assurance in NDEMO's ability to locate a Nash Equilibrium in the EPEC, should one exist, the evidence presented in these tables overwhelmingly suggest that NDEMO is the most computationally demanding of the three tested. This finding supports the insight of Fletcher (1987) who notes that "it must be appreciated that the existence of convergence and order of convergence results is *not guarantee of good performance in practice*" (Fletcher, 1987, p. 20, italics added). Nevertheless, in the scenarios where it was applied, the results of NDEMO agrees very well with those obtained by the alternative algorithms.

While the results in all 12 scenarios obtained by the different algorithms were similar, the SLCP algorithm was found to be the most computationally efficient in all 12 scenarios tested. The results obtained by the SLCP algorithm are subsequently verified using numerical estimates of the best response functions. A selection of these best response functions are included in Appendix A. These figures are used to demonstrate that the solution found by the SLCP algorithm reported in Tables 7.2 to 7.4 coincide with the intersections of these functions. It should be noted that there is no evidence to suggest the existence of multiple NE in any of these scenarios tested.

In the case of competition between *parallel* links, the best response function for each concessionaire is positively related (i.e. upward sloping best response functions). This arises since any increase in the toll by a competitor operating on a parallel link, results in traffic rerouting to the alternative i.e. substitute link. *Ceteris paribus*, this increases the traffic flow and thereby, the congestion externality and hence increases the revenue maximising toll level (De Borger and Van Dender, 2006). This is brought out in the tests (see e.g. Fig. A.1 in Appendix A).

On the other hand, in the case of *serial* links, the best response functions are negatively sloped (van den Berg, 2013). The economic insight for this observation is as follows: an increase in the toll set by a serial competitor would increase generalised travel times which decreases overall demand for travel. Reduction in demand also implies, *ceteris paribus*, that the congestion externality is reduced. Since a revenue maximising concessionaire internalises the congestion externality in setting the toll, the optimal revenue maximising toll would also be consequently reduced as a result. This implies a negative relationship between each concessionaires' tolls in the case of serial links (see e.g. Fig. A.3 in Appendix A).

In the scenarios which involve competition between more than 2 concessionaires (i.e. Scenarios 7 to 12), the procedure to numerically estimate the best response functions have to be adapted. In these scenarios, the toll levels for all other players (except for the two being considered) were held fixed at the NE levels obtained by SLCP shown in Tables 7.3 and 7.4), during the grid search for the 2 players shown on the respective axes. In all scenarios, it is noted that the solution obtained by SLCP reported in Tables 7.2, 7.3 and 7.4 do indeed coincide with the intersection of the best response functions thus verifying that an NE, being mutual best responses, has been found. Note that in this case, only a subset of the best response functions are shown due to space constraints as the number of graphs required to illustrate all possible combinations exhaustively would increase exponentially with the number of links in competition. For example, in Scenario 1 with 2 concessionaires in competition, a single figure would suffice but in Scenario 10, with 6 concessionaires in competition, 15 figures would be required.

TABLE 7.5: Equilibrium tolls in competition game between concessionaires for Scenarios 1 to 6 with Exponential Demand Function obtained by SLCP

Scenario	Link	Toll (secs)	Iterations	CPU time (secs)
1	284	924.98	7	355
	285	976.71		
2	258	1,155.65	8	470
	259	728.46		
3	229	535.66	4	187
	230	868.48		
4	284	656.12	4	187
	286	625.87		
5	243	374.05	5	308
	247	417.75		
6	291	396.44	9	458
	296	613.57		

As the numerical tests with the power law demand function suggest that the SLCP approach was the most computationally efficient, SLCP was the only algorithm used when these same scenarios were rerun with the Exponential Demand function (Eq. 7–14). The results of these tests are shown in Tables 7.5 and 7.6. Numerical best response functions were subsequently constructed to verify the solutions obtained by SLCP as shown in Appendix B. These diagrams verify that the competitive tolls obtained by SLCP as reported in Tables 7.5 and 7.6 occur at the intersections of the best response functions and are thus judged to be NE. As was the case with the power law demand function, it should be again noted that there is no evidence to suggest the existence of multiple NE for any of these scenarios tested.

Table 7.6: Equilibrium tolls in competition game between concessionaires for Scenarios 7 to 12 with Exponential Demand Function obtained by SLCP

Scenario	Link	Toll (secs)	Iterations	CPU time (secs)
7	284	947.79	8	887
	285	1,008.68		
	259	748.74		
8	284	949.08	10	1545
	285	1,009.39		
	258	1,154.47		
	259	723.98		
9	284	939.85	9	3031
	285	998.64		
	258	1,174.66		
	259	782.59		
	229	637.41		
10	284	1,039.56	15	5012
	285	1,099.51		
	258	1,183.96		
	259	776.49		
	229	720.50		
	230	1,071.05		
11	284	655.49	8	803
	286	624.99		
	143	192.34		
12	238	574.26	7	570
	285	677.97		
	291	1,281.69		
	296	501.86		

7.4.4 Comparison of Results against Benchmarks

Tables 7.7 to 7.11 show the outcomes (tolls/revenues/social welfare) under the competitive, monopolistic and the (second best) social welfare maximisation toll pricing policies for the 12 scenarios where the power law demand function was applied. Throughout, the column labelled "Welfare Change" reports the difference in social welfare with tolls under the relevant toll pricing policy, computed using Eq. 7–9,

Table 7.7: Tolls, Revenues and Social Welfare Change under Alternative Toll Pricing Policies (Scenarios 1 to 3) with Power Law Demand Function. Figures in parentheses show percentage change vis-á-vis the scenario's (second best) welfare maximising policy.

			Competition			Monopoly			Second Best Welfare Maximisation	
Scenario	Links	Toll (secs)	Revenue (secs)	Welfare Change (secs)	Toll (secs)	Revenue (secs)	Welfare Change (secs)	Toll (secs)	Revenue (secs)	Welfare Change (secs)
1	284	1,088.48	3,232,453	240,924	5,000.00	6,364,178	-8,308,684	571.75	2,047,543	1,014,578
Parallel	285	1,132.19	3,766,563	(24%)	5,000.00	7,089,031	(-819%)	592.72	2,446,491	
Total			6,999,015			13,453,209			4,494,035	
2	258	1,432.40	3,234,617	-355,956	1,568.36	3,328,708	-1,089,205	722.59	2,255,224	896,010
Parallel	259	855.16	1,319,808	(-40%)	1,070.87	1,300,102	(-122%)	298.58	689,137	
Total			4,554,425			4,628,810			2,944,361	
3	229	611.21	621,424	-381,503	833.92	630,120	-918,079	236.23	307,948	185,217
Parallel	230	912.02	1,316,558	(-206%)	1,104.27	1,400,895	(-496%)	329.11	698,975	
Total			1,937,982			2,031,015			1,006,922	

TABLE 7.8: Tolls, Revenues and Social Welfare Change under Alternative Toll Pricing Policies (Scenarios 4 to 6) with Power Law Demand Function. Figures in parentheses show percentage change vis-á-vis the scenario's (second best) welfare maximising policy.

Scenario	Links	Competition Toll (secs)	Competition Revenue (secs)	Competition Welfare Change (secs)	Monopoly Toll (secs)	Monopoly Revenue (secs)	Monopoly Welfare Change (secs)	Second Best Welfare Maximisation Toll (secs)	Second Best Welfare Maximisation Revenue (secs)	Second Best Welfare Maximisation Welfare Change (secs)
4	284	732.37	1,513,602	-914,658	696.74	1,525,342	-667,162	222.48	799,352	412,225
Serial	286	691.06	946,824	(-222%)	630.86	947,772	(-162%)	308.87	709,976	
Total			2,460,425			2,473,115			1,509,327	
5	243	467.17	709,773	-754,928	401.52	718,251	-414,301	184.05	519,499	313,508
Serial	247	497.79	1,067,251	(-241%)	448.31	1,089,051	(-132%)	158.79	624,211	
Total			1,777,024			1,807,303			1,143,710	
6	291	501.60	494,533	-1,413,203	287.32	417,584	-801,316	171.42	389,562	213,934
Serial	296	740.21	1,318,943	(-661%)	720.67	1,496,607	(-375%)	174.20	611,981	
Total			1,813,476			1,914,191			1,001,543	

Table 7.9: Tolls, Revenues and Social Welfare Change under Alternative Toll Pricing Policies (Scenarios 7 and 8) with Power Law Demand Function. Figures in parentheses show percentage change vis-á-vis the scenario's (second best) welfare maximising policy.

		Competition			Monopoly			Second Best Welfare Maximisation		
Scenario	Links	Toll (secs)	Revenue (secs)	Welfare Change (secs)	Toll (secs)	Revenue (secs)	Welfare Change (secs)	Toll (secs)	Revenue (secs)	Welfare Change (secs)
7	284	1,114.52	3,351,923	-202,169	5,000.00	6,365,683	-9,076,178	584.67	2,091,616	1,089,981
Parallel	285	1,162.20	3,909,451	(-19%)	5,000.00	7,089,249	(-833%)	606.29	2,499,420	
	259	742.47	1,020,120		834.45	1,230,429		189.64	441,094	
Total			8,281,494			14,685,360			5,032,131	
8	284	1,124.72	3,401,150	229,470	5,000.00	6,372,641	-9,552,256	602.23	2,150,752	1,991,997
Parallel	285	1,173.83	3,965,619	(12%)	5,000.00	7,094,206	(-480%)	624.18	2,569,037	
	258	1,429.66	3,293,525		1,629.44	3,531,484		744.08	2,322,012	
	259	905.36	1,475,641		1,244.99	1,754,459		353.94	810,736	
Total			12,135,935			18,752,790			7,852,537	

Table 7.10: Tolls, Revenues and Social Welfare Change under Alternative Toll Pricing Policies (Scenarios 9 and 10) with Power Law Demand Function. Figures in parentheses show percentage change vis-á-vis the scenario's (second best) welfare maximising policy.

		Competition			Monopoly			Second Best Welfare Maximisation		
Scenario	Links	Toll (secs)	Revenue (secs)	Welfare Change (secs)	Toll (secs)	Revenue (secs)	Welfare Change (secs)	Toll (secs)	Revenue (secs)	Welfare Change (secs)
9	284	1,127.28	3,412,232	719,020	5,000.00	6,377,526	-10,628,481	613.33	2,188,670	2,196,977
Parallel	285	1,175.54	3,983,479	(33%)	5000.00	7,098,548	(-484%)	636.04	2,615,396	
	258	1,449.73	3,465,418		1,918.35	4,075,024		805.97	2,513,122	
	259	989.70	1,891,792		1,716.96	2,467,479		485.60	1,111,322	
	229	789.69	868,713		1,465.87	975,834		400.81	509,565	
Total			13,621,635			20,994,412			8,938,075	
10	284	1,224.12	3,893,576	2,123,552	5,000.00	9,041,630	-12,539,583	803.31	2,809,258	2,958,360
Parallel	285	1,280.21	4,543,821	(72%)	5000.00	11,008,641	(-424%)	830.69	3,348,242	
	258	1,482.61	3,610,747		2,344.31	6,170,362		901.21	2,806,585	
	259	1,063.70	2,121,671		5,000.00	801,944		665.75	1,500,231	
	229	941.61	1,367,956		5,000.00	1,556,683		722.87	920,413	
	230	1,231.31	2,228,962		5,000.00	6,363,112		820.01	1,676,656	
Total			17,766,732			34,942,373			13,061,385	

Table 7.11: Tolls, Revenues and Social Welfare Change under Alternative Toll Pricing Policies (Scenarios 11 and 12) with Power Law Demand Function. Figures in parentheses show percentage change vis-á-vis the scenario's (second best) welfare maximising policy.

		Competition			Monopoly			Second Best Welfare Maximisation		
Scenario	Links	Toll (secs)	Revenue (secs)	Welfare Change (secs)	Toll (secs)	Revenue (secs)	Welfare Change (secs)	Toll (secs)	Revenue (secs)	Welfare Change (secs)
11	284	730.14	1,503,877	-1,092,202	695.61	1,518,038	-797,020	222.50	799,403	412,229
Serial	286	697.41	913,784		628.87	915,419		308.87	709,969	
	143	243.43	120,913		208.41	121,090		0.06	71	
Total			2,538,574			2,554,547			1,509,443	
12	238	221.67	145,153	-3,265,676	136.59	190,439	-1,258,087	38.33	89,690	437,866
Serial	285	761.25	1,689,647		709.37	1,825,144		269.01	1,116,212	
	291	894.73	390,144		215.04	312,663		145.40	332,242	
	296	639.65	821,238		526.77	1,102,102		105.5173	372,704	
Total			3,046,182			3,430,348			1,910,848	

and the welfare in the base untolled equilibrium and this is measured in generalised seconds.

These tables show that despite pricing being able to raise a substantial revenues from tolls, the possibility of large social welfare losses cannot be ignored. This reflects the potential divergence between commercial revenue interests and social welfare interests highlighted in Mills (1995).

Comparing across Tables 7.7, 7.9 and 7.10, it is inferred that as the number of parallel links are added, the social welfare increases. In Scenario 1, when concessionaires on Links 284 and 285 are in competition, the change social welfare relative to the untolled base equilibrium, whilst positive (240,924 secs) is only 24% of that achievable by the second best welfare maximising toll pricing policy. As the number of competitors, reflecting the increased *intensity* of parallel competition, increase (from 2 competitors in Scenario 1 to 6 in Scenario 10), the social welfare gains achievable from competition tends to that achievable with a second best welfare maximising toll pricing policy. For example, in Scenario 10 with 6 competing concessionaires, the resulting social welfare under oligopolistic competition is approximately 2.1 million seconds or 72% of the second best social welfare (of 2.9 million seconds). This suggests that insights discussed in Chapter 2 (see Section 2.4.2) are supported in a more realistic network with multiple OD pairs. The policy implication is that competition should be encouraged between links that could potentially serve as (imperfect) substitutes.

Recall that in Chapter 6 (in particular, Scenario 2 in Section 6.5.3), the results indicated the possibility of tolls under competition being *lower* than those of a second best welfare maximising policy when there was an untolled alternative. In this case, while there are a multitude of untolled alternatives in the network, the results show that all the competitive tolls, (regardless of the demand function assumed), are always higher than under a second best welfare maximising policy. From the numerical tests, there is no indication that competition reduced toll prices. Instead, the opposite was found and that increased intensity of competition could lead to an increase in toll prices. This can be observed in the NE toll levels for Links 284 and 285. These two links competed against themselves in Scenario 1 and each was involved in Scenarios 7 to 10 where the number of links in parallel competition was increased from 3 to 6. As Fig. 7.8 illustrate, for both case of power law demand function(left panel) and exponential demand function (right panel), the tolls on links 284 and 285, in fact, *increase*, rather than decrease, as competition increases.

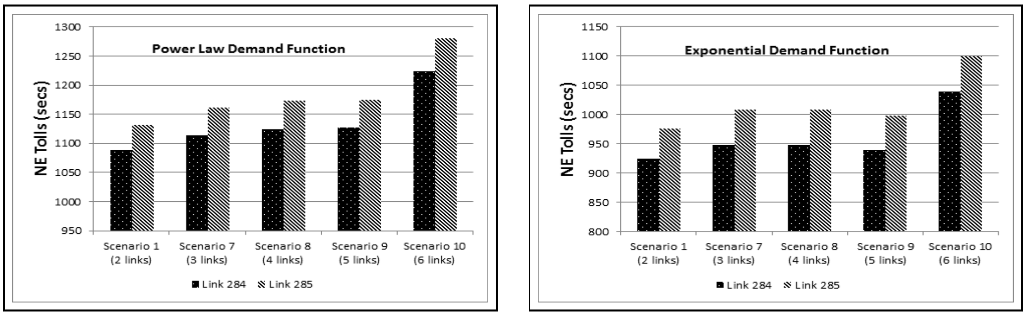


FIGURE 7.8: NE tolls may rise as the number of links in competition increases (Left): Power Law Demand Function (Right): Exponential Demand Function

A plausible explanation for the rise in toll levels as the intensity of competition increases is that when more links are in competition, there is less opportunity for users, travelling in the general direction facilitated by these links, to avoid the tolls and this means that more congestion is internalised. As noted in Chapter 2 (see Section 2.4.2), because concessionaires take congestion into account in setting revenue maximising tolls, the congestion internalisation component of the toll also increases and therefore the toll rises. At the same time, more congestion is internalised and therefore the welfare gain increases as a result of the increased intensity of competition.

Now consider the scenarios involving competition between serially interdependent links (see Scenarios 4 to 6 in Table 7.8 and Scenarios 11 and 12 in Table 7.11), the social welfare under competition is lower than that obtained under monopolistic control where a single concessionaire decides toll levels over all tollable links in a series. Again, this endorses findings from the economics literature within a SUE framework. The reason for the lower social welfare vis-á-vis monopoly is due to the phenomenon of double marginalisation (Economides and Salop, 1992; Small and Verhoef, 2007) as discussed in the Chapter 2. The double marginalisation problem arises when a toll concessionaire sets the toll level on a link without taking into account the reduction in revenues to other links in the series. This results in tolls that are too high.

When the intensity of serial competition is increased, the consequences of double marginalisation are clearly magnified. This can be inferred by comparing the social welfare change under competition in Scenarios 4,5 and 6 with similar from Scenarios 11 or 12. For example, in Scenario 4 (Table 7.8) with links 284 and 286 in serial competition , the welfare change under competition was slightly over -900,000 secs. On the other hand, in Scenario 11 (Table 7.11), when an additional link (143) participated in the serial competition in this set, the welfare change under competition exceeded -1,000,000 secs.

In these scenarios, the resulting tolls from the game between competitive concessionaires *exceeds* the toll levels a revenue maximising monopolist would set. Even after taking into account the mathematical characteristics of the power law demand function as discussed in Chapter 6, none of the tolls in these scenarios ever attain the upper bounds for the monopolist pursuing a revenue maximising policy. It is also clear that the welfare changes associated with a monopolist operating these serial links is higher (despite being negative) when compared to the competitive outcomes. The policy implication is thus that, regardless of the routing paradigm assumed, toll competition between links exhibiting strong serial interdependence should be discouraged.

However, it is not possible to assess the implications of the competitive outcomes vis-á-vis the monopoly outcomes in the case of Scenarios 7 to 10 as the tolls are constrained by the pre-specified upper bound, a possibility envisaged with the use of the power law demand function. Thus as mentioned, the tests were additionally carried out with the exponential demand function (see Eq. 7–14). The results (alongside the monopolistic outcomes and the social welfare maximising outcomes) are presented in Tables 7.12 to 7.16. As predicted, with the exponential demand function, it can be observed that, in these cases, the upper bound (on tolls) of 5000 secs is never active.

Firstly, the insights obtained with the power law demand function are confirmed by the exponential demand function. This suggests that the policy insights are independent of the functional form of the demand and welcome news as "the demand curve is usually unknown and difficult to estimate in practice" (Yang *et al.*, 2004, p. 478).

Secondly, for the case of parallel links, the social welfare change is consistently higher in the case of competition than that attainable by allowing a revenue maximising monopolist the opportunity to decide the tolls on all tollable links. This stands in stark contrast to that of the situations involving competition between serial links. In the latter case, the social welfare change is higher comparing monopolistic control versus competition. In such cases, a monopolist will be incentivised to take into account the effects of the toll on a given link on other links in the series in his revenue maximising decisions and in so doing, obviate the double marginalisation problem associated with competition.

The welfare impacts of each scenario under competition, monopoly and welfare maximisation are summarised using the index of relative welfare improvement (Verhoef *et al.*, 1996), ω, as used in Chapter 6 (see Section 6.5.3). These are shown in Tables 7.17 and 7.18 for the power law and exponential demand functions respectively. Recall that ω is computed using Eq. 6–15 and thus measures the efficiency improvement in each scenario relative to the first best theoretical benchmark. As mentioned previously (see Section 7.4), both demand functions were calibrated to produce similar elasticities in the base to obtain similar welfare changes attainable with first best pricing of approximately 35,189,300 seconds.

Not surprisingly the numbers are relatively small because there are few tollable links relative to the 344 links in the network. Nevertheless, Tables 7.17 and 7.18 emphasise the key messages discussed above. Firstly, in the scenarios of competition between parallel links, relative efficiency is always higher (or less negative) compared to that attainable with monopoly control. In the case of serial competition, monopoly control is better for efficiency in that ω always higher (or less negative) than under competition. Secondly, as intensity of competition (i.e. more links are involved) increases in the parallel case, welfare improves and tends towards the second best welfare maximising case. At the same time, increased intensity of serial competition magnifies the negative impacts of serial competition.

Table 7.12: Tolls, Revenues and Social Welfare Change under Alternative Toll Pricing Policies (Scenarios 1 to 3) with Exponential Demand Function. Figures in parentheses show percentage change vis-á-vis the scenario's (second best) welfare maximising policy.

			Competition			Monopoly			Second Best Welfare Maximisation	
Scenario	Links	Toll (secs)	Revenue (secs)	Welfare Change (secs)	Toll (secs)	Revenue (secs)	Welfare Change (secs)	Toll (secs)	Revenue (secs)	Welfare Change (secs)
1	284	924.98	2,672,041	2,053,008	1,496.62	2,987,978	1,103,258	757.09	2,380,040	2,173,221
Parallel	285	976.71	3,204,289	(94%)	1,463.68	3,804,961	(51%)	787.79	2,889,708	
Total			5,876,330			6,792,940			5,269,748	
2	258	1,155.65	2,665,336	1,846,973	1,286.47	2,752,474	1,625,080	961.26	2,475,564	1,947,271
Parallel	259	728.46	1,084,157	(95%)	889.19	1,095,758	(83%)	539.90	958,286	
Total			3,749,493			3,848,232			3,433,850	
3	229	535.66	543,514	600,858	757.75	551,687	317,598	436.18	444,458	724,644
Parallel	230	868.48	1,395,379	(83%)	1,066.57	1,496,714	(44%)	602.74	1,214,319	
Total			1,938,893			2,048,401			1,658,777	

Table 7.13: Tolls, Revenues and Social Welfare Change under Alternative Toll Pricing Policies (Scenarios 4 to 6) with Exponential Demand Function. Figures in parentheses show percentage change vis-á-vis the scenario's (second best) welfare maximising policy.

		Competition			Monopoly			Second Best Welfare Maximisation		
Scenario	Links	Toll (secs)	Revenue (secs)	Welfare Change (secs)	Toll (secs)	Revenue (secs)	Welfare Change (secs)	Toll (secs)	Revenue (secs)	Welfare Change (secs)
4	284	656.12	1,386,411	223,260	625.74	1,397,016	370,871	343.27	1,084,686	798,586
Serial	286	625.87	863,314	(28%)	575.27	864,518	(46%)	386.55	774,946	
Total			2,249,725			2,261,534			1,859,632	
5	243	374.05	568,909.93	390,246	310.88	566,495	570,677	220.01	516,934	754,842
Serial	247	417.75	974,626.00	(52%)	387.16	1,007,640	(76%)	259.86	887,195	
Total			1,543,536			1,574,135			1,404,129	
6	291	396.44	521,404	12,111	294.01	492,338	262,231	273.66	583,547	614,230
Serial	296	613.57	1,206,198	2%	587.31	1,291,667	(43%)	272.91	851,065	
Total			1,727,602			1,784,005			1,434,611	

Table 7.14: Tolls, Revenues and Social Welfare Change under Alternative Toll Pricing Policies (Scenarios 7 and 8) with Exponential Demand Function. Figures in parentheses show percentage change vis-á-vis the scenario's (second best) welfare maximising policy.

		Competition			Monopoly			Second Best Second Best Welfare Maximisation		
Scenario	Links	Toll (secs)	Revenue (secs)	Welfare Change (secs)	Toll (secs)	Revenue (secs)	Welfare Change (secs)	Toll (secs)	Revenue (secs)	Welfare Change (secs)
7	284	947.79	2,758,783	2,081,963	1,493.79	3,061,110	1,252,529	623.10	2,173,280	2,357,105
Parallel	285	1,008.68	3,302,079	(88%)	1,461.63	3,921,258	(53%)	732.64	2,694,546	
	259	748.74	823,671		778.26	850,120		271.27	530,452	
Total			6,884,533			7,832,488				
8	284	949.08	2,762,647	4,013,608	1,534.30	3,092,086	2,750,127	795.47	2,475,811	4,185,122
Parallel	285	1,009.39	3,303,718	(96%)	1,502.66	3,959,607	(66%)	829.50	3,002,706	
	258	1,154.47	2,663,498		1,306.25	2,882,043		964.87	2,485,149	
	259	723.98	1,100,078		1,166.66	1,091,278		553.36	979,012	
Total			9,829,942			11,025,014				

Table 7.15: Tolls, Revenues and Social Welfare Change under Alternative Toll Pricing Policies (Scenarios 9 and 10) with Exponential Demand Function. Figures in parentheses show percentage change vis-á-vis the scenario's (second best) welfare maximising policy.

		Competition			Monopoly			Second Best Second Best Welfare Maximisation		
Scenario	Links	Toll (secs)	Revenue (secs)	Welfare Change (secs)	Toll (secs)	Revenue (secs)	Welfare Change (secs)	Toll (secs)	Revenue (secs)	Welfare Change (secs)
9	284	939.85	2,724,320	4,681,989	2,007.11	3,086,386	1,787,564	785.75	2,443,001	4,799,102
Parallel	285	998.64	3,256,881	(98%)	1,987.84	3,851,851	(37%)	820.14	2,960,083	
	258	1,174.66	2,785,456		1,441.28	3,069,301		1,072.43	2,756,427	
	259	782.59	1,427,347		1,273.00	1,640,883		776.04	1,387,964	
	229	637.41	676,967		1,097.52	743,653		664.17	671,822	
Total			10,870,971			12,392,074				
10	284	1,039.56	3,052,075	6,243,958	2,360.73	3,827,390	3,462,055	1,194.71	3,216,445	6,467,290
Parallel	285	1,099.51	3,655,649	(97%)	2,358.19	4,860,276	(54%)	1,202.80	3,972,027	
	258	1,183.96	2,820,894		1,586.85	3,624,678		1,156.56	2,972,889	
	259	776.49	1,448,421		1,842.30	1,503,827		937.90	1,563,296	
	229	720.50	976,487		1,979.76	1,154,237		1,012.75	986,377	
	230	1,071.05	1,878,779		2,202.47	2,775,705		1,193.99	2,149,903	
Total			13,832,305			17,746,113				

Table 7.16: Tolls, Revenues and Social Welfare Change under Alternative Toll Pricing Policies (Scenarios 11 and 12) with Exponential Demand Function. Figures in parentheses show percentage change vis-á-vis the scenario's (second best) welfare maximising policy.

		Competition			Monopoly			Second Best Welfare Maximisation		
11	284	655.49	1,384,083	124,918	624.51	1,394,793	280,329	342.14	1,081,998	799,485
Serial	286	624.99	861,748	(16%)	573.68	862,933	(35%)	387.89	776,654	
	143	192.34	124,345		188.27	124,790		23.24	23,349	
Total			2,370,176			2,382,516				
12	238	574.26	86,827	-1,252,895	575.71	86,951	-1,010,856	650.99	100,870	421,779
Serial	285	677.97	1,585,513	(-297%)	656.37	1,623,793	(-240%)	373.11	1,343,668	
	291	1,281.69	281,940		1,346.85	300,845		298.06	437,673	
	296	501.86	595,093		397.78	568,837		195.66	519,542	
Total			2,549,374			2,580,426				

Table 7.17: Index of Relative Welfare Improvement ω under Competition, Monopoly and Second Best Welfare Maximisation in Each Scenario with *Power Law Demand Function*

Scenario	Number of Competitors	Competition	Monopoly	Welfare Maximisation
Parallel Competition				
Scenario 1	2	0.01	-0.24	0.03
Scenario 2	2	-0.01	-0.03	0.03
Scenario 3	2	-0.01	-0.03	0.01
Scenario 7	3	-0.01	-0.26	0.03
Scenario 8	4	0.01	-0.27	0.06
Scenario 9	5	0.02	-0.30	0.06
Scenario 10	6	0.06	-0.36	0.08
Serial Competition				
Scenario 4	2	-0.03	-0.02	0.01
Scenario 5	2	-0.02	-0.01	0.01
Scenario 6	2	-0.04	-0.02	0.01
Scenario 11	3	-0.03	-0.02	0.01
Scenario 12	4	-0.09	-0.04	-0.04

Table 7.18: Index of Relative Welfare Improvement ω under Competition, Monopoly and Second Best Welfare Maximisation in Each Scenario with *Exponential Demand Function*

Scenario	Number of Competitors	Competition	Monopoly	Welfare Maximisation
Parallel Competition				
Scenario 1	2	0.06	0.03	0.06
Scenario 2	2	0.05	0.05	0.06
Scenario 3	2	0.02	0.01	0.02
Scenario 7	3	0.06	0.04	0.07
Scenario 8	4	0.11	0.08	0.12
Scenario 9	5	0.13	0.05	0.14
Scenario 10	6	0.18	0.10	0.18
Serial Competition				
Scenario 4	2	0.01	0.01	0.02
Scenario 5	2	0.01	0.02	0.02
Scenario 6	2	0.00	0.01	0.02
Scenario 11	3	0.00	0.01	0.02
Scenario 12	4	-0.04	-0.03	0.01

7.5 Multiple Links Per Concessionaire

Thus far, it has been assumed that under competition, each concessionaire exercises control only on a *single* link in the network. This may have resulted in an impression that the SLCP approach is only applicable in this particular special case. In this section, a modified SLCP approach is applied to study the case when a concessionaire exercises control over two links in the network. For brevity, the results reported below were only conducted with the exponential demand function (Eq. 7–14).

7.5.1 Nash Game Variants Considered

Allowing multiple links per concessionaire facilitates the development of three interrelated Nash non-cooperative game variants which enable a study of the policy implications arising from the assignment of links to toll road concessionaires.

This specific test is applied to four tollable links shown in the Fig. 7.9 numbered as 281, 286, 291 and 296. In the case of competition, the four exhaustive possibilities for assignment of control (i.e. toll setting responsibilities) over these links to concessionaires are as follows:

1. "Disaggregated (non-cooperative) Nash Game": there are 4 concessionaires controlling one link each

2. "Integrated Parallel Competition": there are 2 concessionaires (labelled A and B) such that A controls links 284 and 286 and B controls links 291 and 296

3. "Extended Serial Competition - Variant I": there are 2 concessionaires (labelled X and Y) such that X controls links 284 and 291 and Y controls links 286 and 296

4. "Extended Serial Competition - Variant II": there are 2 concessionaires (labelled X and Y) such that X controls links 284 and 296 and Y controls links 286 and 291

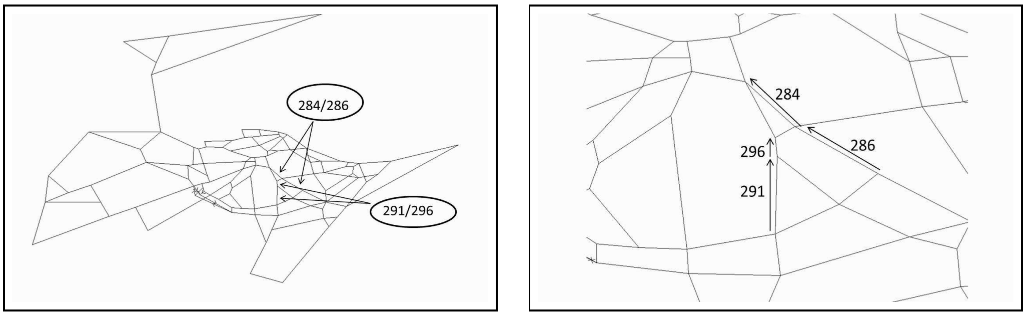


Figure 7.9: Left: Location of Links in Network Right: Location of Links close up

7.5.2 Results, Verification and Policy Insights

The outcomes (tolls/revenues/welfare changes) of each variant of the non-cooperative Nash game as described above are shown in Tables 7.19 and 7.20. In addition, by applying numerically estimated best response functions, the NE tolls in these games obtained by SLCP are verified in Appendix C (see Figs. C.1 - C.4). In these figures, the dashed line in each diagram shows the numerical estimates of the best response function for the player indicated on the x-axis while the continuous line shows the numerical estimates of the best response function for the player indicated on the y-axis. It can be observed from these diagrams that the solution of the SLCP approach coincides with the intersection of these best response functions in all the Nash non-cooperative game variants investigated.

In addition, Table 7.21 shows the outcomes associated with a monopolist (i.e. controlling all four links) as well as under a (second best) social welfare maximising policy which, as usual, form benchmarks in terms of the revenue and welfare possibilities to support an assessment of the various game variants. The main insights from this test are summarised thus. While the Disaggregated Nash (non-cooperative) Game is welfare enhancing, obtaining 41% of the maximum second best welfare gain, the Integrated Parallel Competition scenario seems to be the most socially beneficial way of link assignment to the concessionaires. It is also seen that the tolls are lower when either player controls both serial links than when there are serial links engaged in competition. As a recurring theme indicated in this and the previous chapter, this is once again a manifestation of the double marginalisation problem.

While competition seems to be welfare enhancing in both the Integrated Parallel

Table 7.19: Tolls, Link Revenues and Social Welfare Change: Disaggregated (non-cooperative) Nash Game and Integrated Parallel Competition. Figures in parentheses gives percentage changes in welfare for the scenario relative to the second best welfare maximisation scenario shown in last section of Table 7.21.

Link	Toll (secs)	Link Revenue (secs)	Welfare Change (secs)
Disaggregated Nash Game			
284	567.97	1,104,746	724,351
286	882.78	1,438,825	(41%)
291	416.95	533,670	
296	660.90	1,487,939	
Total Revenue		4,565,180	
Link	**Toll (secs)**	**Link Revenue (secs)**	**Welfare Change (secs)**
Integrated Parallel Competition			
Concessionaire A: 284 and 286			
284	537.22	1,139,301	1,144,043
286	846.14	1,408,684	(65%)
Revenue A		2,547,985	
Concessionaire B: 291 and 296			
291	295.24	492,331	
296	632.81	1,597,377	
Revenue B		2,089,708	
Total Revenue		4,637,693	

Table 7.20: Tolls, Revenues and Social Welfare Change: Extended Serial Competition variants. Figures in parentheses gives percentage changes in welfare for the scenario relative to the second best welfare maximisation scenario shown in last section of Table 7.21.

Link	Toll (secs)	Link Revenue (secs)	Welfare Change (secs)
Extended Serial Competition - Variant I			
Concessionaire X: 284 and 291			
284	533.01	939,388	410,243
291	352.33	323,747	(23%)
Revenue X		1,263,135	
Concessionaire Y: 286 and 296			
286	1354.78	1,775,663	
296	1197.04	1,589,772	
Revenue Y		3,365,435	
Total Revenue		4,628,570	

Link	Toll (secs)	Link Revenue (secs)	Welfare Change (secs)
Extended Serial Competition - Variant II			
Concessionaire X: 284 and 296			
284	532.56	1,103,952	-824,265
296	612.27	1,438,007	(-47%)
Revenue X		2,541,959	
Concessionaire Y: 286 and 291			
286	879.56	1,426,383	
291	464.86	569,848	
Revenue Y		1,996,231	
Total Revenue		4,538,189	

Table 7.21: Tolls, Link Revenues and Social Welfare Change: Monopoly and Second Best Welfare Maximisation. The figure in parentheses gives the percentage change in welfare for the Monopoly scenario relative to the second best welfare maximisation scenario.

Link	Toll (secs)	Link Revenue (secs)	Welfare Change(secs)	
Monopoly				
284	414.99	934,756	557,101	(32%)
286	1334.99	1,773,822		
291	0.00	0		
296	1140.54	2,112,278		
Total		4,820,856		
Link	**Toll (secs)**	**Link Revenue (secs)**	**Welfare (secs)**	
Second Best Welfare Maximisation				
284	250.53	784,566	1,763,486	
286	719.79	1,322,903		
291	216.13	464,574		
296	459.81	1,465,097		
Total		4,037,139		

Competition and Disaggregated (non-cooperative) Nash Games scenarios (both obtain higher percentages of the second best welfare maximising outcome compared to monopoly), it is clear that the structure of concessionaire control in terms of the assignment of links to concessionaires also has significant social welfare implications.

As can be observed, the results indicate that Extended Serial Competition could be the worst strategy amongst the control possibilities tested for policy makers to pursue. Both variants of Extended Serial Competition involve concessionaires each controlling multiple but geographically separate portions of road links exhibiting a high degree of serial interdependence/complementarity. In this case, because each controls two of such serial links, the problem of double marginalisation is further exacerbated with a reduction in social welfare social welfare. The problem of double marginalisation not only affects users and society at large with welfare losses but also adversely impacts the concessionaires since revenues are lower in both of these game variants vis-á-vis Integrated Parallel Competition.

In the worst case of Variant II, the Extended Serial Competition game also results in a negative social welfare change. This implies that pursuit of this particular variant is worse than taking no action. Thus, it might be even better, choosing between the two, to hand control over to the revenue maximising monopolist, since there is the possibility of a 32% social welfare gain, than to allow such competition.

For each scenario tested, Table 7.22 summarises the index of relative welfare improvement ω which, as noted previously, measures the welfare change from each scenario relative to the theoretical first best welfare benchmark. By definition, the second best welfare maximisation scenario would attain the highest measure of relative efficiency. Aside from this, the Integrated Parallel Competition scenario results in the next highest ω. Furthermore, the monopoly scenario results in a similar magnitude of efficiency changes as the Disaggregated (non-cooperative) Nash game variant. It is also clear that either variants of Extended Serial Competition fare the worst in welfare terms compared to the monopoly solution as discussed above.

TABLE 7.22: Index of Relative Welfare Improvement ω for Each Multiple Link Scenario

Scenario	ω
Second Best Welfare Maximisation	0.05
Integrated Parallel Competition	0.03
Monopoly	0.02
Disaggregated Nash	0.02
Extended Serial Competition - Variant I	0.01
Extended Serial Competition - Variant II	-0.02

In summary, the policy implications arising from the numerical tests conducted in this section serves to affirm the premise that "private operators, if allowed on a network, should serve full-length corridors but should face competition when doing so" (Small and Verhoef, 2007, p. 201) in a realistic network situation with route choices described by an elastic demand SUE model.

7.6 Collusion and Nash Bargaining

The previous chapter examined both the potential for collusion between two concessionaires and the normative approach to Nash Bargaining in a DUE setting. The correspondence between the terminology utilised in the Axiomatic Bargaining literature and equivalent terminology in the Industrial Organisation literature were given in Table 6.8 and these terms will also be utilised in this section.

It was noted that the relevant benchmark to study the possibilities for collusion between concessionaires is based on the principle of Pareto Optimality (cf. Definition 3.3) and it was emphasised that, as reflected in the Pareto Front, there is an entire range of potential outcomes attainable by each through engaging in some form of collusion and bargaining. It was shown that the NE revenues attainable under the (fully) competitive regime would not be Pareto Optimal as one concessionaire could increase his revenues without reducing that accruing to the other. In this chapter, with route choice following the SUE principle, the benchmark MOPEC under consideration can be expressed as Eq. 7–15.

$$\begin{aligned} &\underset{\mathbf{x}\in X}{\text{Maximise}} \quad \boldsymbol{\Phi}(\mathbf{x}) = (\phi_1(\mathbf{x}, \mathbf{v}(\mathbf{x}))), \phi_2(\mathbf{x}, \mathbf{v}(\mathbf{x})))^{\intercal} \\ &\text{subject to} \quad \{\mathbf{v}^*(\mathbf{x}), \mathbf{q}^*(\mathbf{x})\} \leftarrow \text{SOL}\{\mathcal{S}(\mathbf{x})\}. \end{aligned} \tag{Eq. 7–15}$$

In addition, an intuitive approach was developed, adhering to the Utilitarian approach to the division of gains with *total* revenues being maximised, in order to model the situation in which erstwhile independent concessionaires could move from the non-Pareto Optimal point to the monopoly solution on the Pareto Front through a form of "signalling" behaviour, allowing them to infer rivals' intentions from observation of market outcomes. Such signalling could take the form of setting tolls in such a way that would signal, to their rivals, their intention to collude and mutual reciprocation was assumed. This was achieved by the use of a unitless scalar parameter, α, $0 \leqslant \alpha \leqslant 1$, to represent the degree of cooperation between the concessionaires. With the two concessionaire scenarios, the modified NCEPEC, taking into account SUE route choices, can be explicitly written as the system in Eq. 7–16.

$$\begin{aligned} &\underset{x_1\in X_1}{\text{Maximise}} \quad \phi_1(x_1, v_{j_1}) = v_{j_1}(\mathbf{x})x_1 + \alpha v_{j_2}(\mathbf{x})x_2,\ j_1, j_2 \in \mathcal{J},\ j_1 \neq j_2 \\ &\text{subject to} \quad \{\mathbf{v}^*(\mathbf{x}), \mathbf{q}^*(\mathbf{x})\} \leftarrow \text{SOL}\{\mathcal{S}(\mathbf{x})\} \end{aligned} \tag{Eq. 7–16a}$$

$$\begin{aligned} &\underset{x_2\in X_2}{\text{Maximise}} \quad \phi_2(x_2, v_{j_2}) = v_{j_2}(\mathbf{x})x_2 + \alpha v_{j_1}(\mathbf{x})x_1,\ j_1, j_2 \in \mathcal{J},\ j_2 \neq j_1 \\ &\text{subject to} \quad \{\mathbf{v}^*(\mathbf{x}), \mathbf{q}^*(\mathbf{x})\} \leftarrow \text{SOL}\{\mathcal{S}(\mathbf{x})\} \end{aligned} \tag{Eq. 7–16b}$$

This framework would allow policy makers to study the possibilities of such tacit collusion tactics by examining the existence or otherwise of incentives for concessionaires to engage in "concious parallelism"(Macleod, 1985).

However, a drawback of the Utilitarian Solution to the division of gains is that it could result in one concessionaire being made worse off, thereby violating the Axiom 1 of Individual Rationality (see Section 5.6.1). In such a situation, the party that could lose out would not be willing to collude since they could, individually, do better by playing the non-cooperative game. As a result, an alternative Nash Bargaining Solution (NBS) (Nash, 1950*b*) was introduced that would satisfy this axiom. For completeness, the Nash Bargaining Problem with Equilibrium Constraints (NBPEC) considered in this chapter, specifically taking into account users' route choices obeying the SUE principle, may be cast as Eq. 7–17,

$$\begin{aligned} \underset{x \in X}{\text{Maximise}} \quad & \mathcal{Z}(\mathbf{x}, \mathbf{v}(\mathbf{x})) = (\phi_1(\mathbf{x}, \mathbf{v}(\mathbf{x})) - \phi_1^{\mathrm{N}})(\phi_2(\mathbf{x}, \mathbf{v}(\mathbf{x})) - \phi_2^{\mathrm{N}}) \\ \text{subject to} \quad & \{\mathbf{v}^*(\mathbf{x}), \mathbf{q}^*(\mathbf{x})\} \leftarrow \mathrm{SOL}\{\mathcal{S}(\mathbf{x})\}. \\ & \phi_1(\mathbf{x}, \mathbf{y}(\mathbf{x})) \geqslant \phi_1^{\mathrm{N}} \\ & \phi_2(\mathbf{x}, \mathbf{y}(\mathbf{x})) \geqslant \phi_2^{\mathrm{N}}. \end{aligned} \qquad \text{(Eq. 7–17)}$$

In this section, collusion and Nash Bargaining between toll road concessionaires, again concentrating exclusively on scenarios with two concessionaires (i.e. Scenarios 1 to 6), are investigated. Firstly, taking the perspective of the concessionaires, the revenue impacts of the Utilitarian Solution achieved by the signalling behaviour, as represented by Eq. 7–16, is used to identify the existence of incentives, or otherwise, for parties to move through to fully collusive outcome. Subsequently, tests are conducted to identify the NBS. Finally, the the welfare consequences of collusion and Nash Bargaining are summarised.

As was done in the previous chapter, the Pareto Fronts are generated by application of the MOSADE algorithm (Algorithm 5.5 refers) to heuristically solve Eq. 7–15. The tests will be carried out for the Exponential Demand function only. Furthermore, because of the smooth and differentiable properties of the SUE map discussed in Section 7.3.4, the NBPEC in Eq. 7–17 can be solved by embedding the SUE constraints directly within the IPOPT interior point solver (Wächter and Biegler, 2006), a standard non-linear programming solver.

7.6.1 Revenue Impacts of the Utilitarian Approach to Collusion

The Pareto Fronts serving as benchmarks for the 6 scenarios tested are shown in the left panes of Figs. 7.10 to 7.15. These figures confirm two predictions. Firstly, the monopolist solution (equivalent to the situation when the collusion parameter, α, is equal to 1) reported in Table 7.12 all lie on the Pareto Front as indicated by the * on these diagrams. Unlike in Scenario 2 in Chapter 6 (see p. 169), there is no evidence to suggest the existence of multiple optimum in this case. Secondly, the fully competitive NE outcome is not Pareto Optimal (in terms of individual revenues attainable by each the players) as that revenue tuple lies in the interior of the Pareto Front.

It is important, however, to distinguish between total revenue and individual revenue in investigating the possibilities for collusion. While the *total* revenue increases as α increases from 0 at the (fully competitive) NE solution to 1 in the fully collusive solution, the revenue accruing to each concessionaire does not increase linearly. Plotted on the right pane in Figs. 7.10 to 7.15 is the corresponding "collusion path". This plots the implied revenues to each concessionaire, obtained by using the SLCP algorithm to solve Eq. 7–16, varying α between 0 and 1 in steps of 0.2. Such a path shows the resulting distribution of revenues between the players if they were to collude so as to move from the fully competitive outcome to the fully collusive Monopoly outcome.

In the case of parallel competition, the numerical tests suggest that players are incentivised to collude. In all three scenarios as shown in the right hand panes of Figs. 7.10 to 7.12, the revenue accruing to either player increases as α increases. However, in exactly such a case as discussed above, collusion would be detrimental for societal welfare. At the same time, they may not collude fully since the collusion path is an inverted U shaped as shown in the right hand panes of Figures 7.10 to 7.15. This implies that the maximum revenue accruing to each concessionaire individually could occur at a point *before* full collusion (i.e. $\alpha = 1$) is reached since one party could in fact lose out in terms of obtaining lower revenues one could get if they were to collude fully. In such cases, there is little incentive to collude fully unless there is an explicit revenue sharing agreement to enforce collusion. However, if a comparison is made by only comparing the revenues attainable by each in the fully competitive outcome ($\alpha = 0$) vis-á-vis the fully collusive outcome ($\alpha = 1$), it is clear that in the scenarios involving parallel competition, both concessionaires would be made better off.

On the other hand, collusion in serial links would be potentially positive for social welfare since it has been highlighted that the monopolistic outcome in these cases was shown to lead result in lower tolls and, correspondingly, more positive (or at the very least, less detrimental) welfare impacts as the double marginalisation problem is avoided. However, the results show that there may not always be an incentive for concessionaires to collude fully in this case. While Fig. 7.13 (Scenario 4) suggests that *both* players could increase their revenues by moving towards the fully collusive outcome, Figs. 7.14 and 7.15 (Scenarios 5 and 6 respectively) show that the link revenue under full collusion obtained by one player (indicated on the x-axis) is in fact less than that obtained in the fully competitive NE. Thus, there may be no incentive for players to move to full collusion even if it would improve social welfare (as discussed in Section 7.6.3 below).

Therefore the evidence suggests that while it would be, in fact, welfare improving for concessionaires to collude in the serial link case, the numerical examples highlight the fact that they are may not be incentivised to do so. On the other hand, concessionaires might potentially be incentivised to collude in the parallel link case which results in negative social welfare consequences. Thus regulators need to be aware of such possibilities particularly in the case of links that serve as alternative connections between OD pairs.

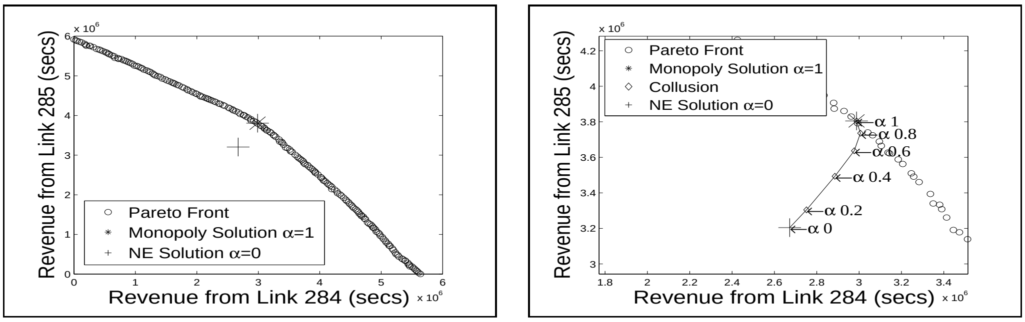


FIGURE 7.10: (Left) Scenario 1: The fully competitive NE (+) is not Pareto Optimal while the monopoly solution (*) lies on the Pareto Front. (Right): The "Collusion Path" as competitors move from the fully competitive NE ($\alpha = 0$) to the monopoly solution ($\alpha = 1$).

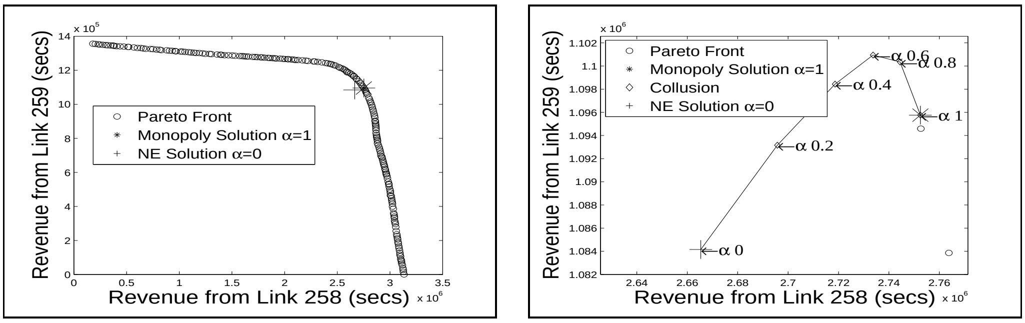


FIGURE 7.11: (Left) Scenario 2: The fully competitive NE (+) is not Pareto Optimal while the monopoly solution (*) lies on the Pareto Front. (Right): The "Collusion Path" as competitors move from the fully competitive NE ($\alpha = 0$) to the monopoly solution ($\alpha = 1$).

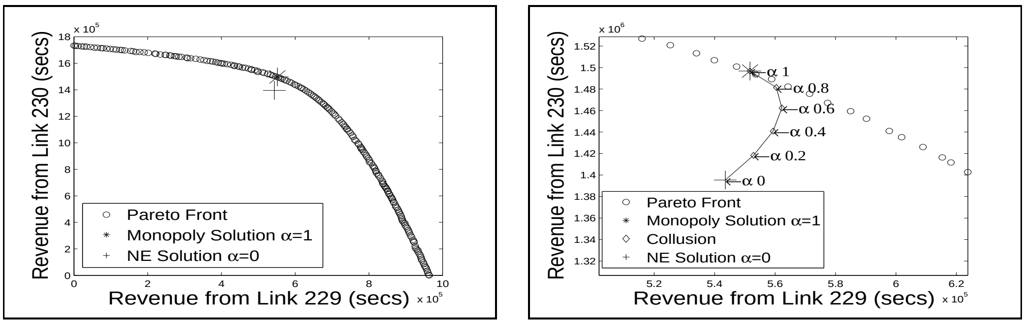


FIGURE 7.12: (Left) Scenario 3: The fully competitive NE (+) is not Pareto Optimal while the monopoly solution (*) lies on the Pareto Front. (Right): The "Collusion Path" as competitors move from the fully competitive NE ($\alpha = 0$) to the monopoly solution ($\alpha = 1$).

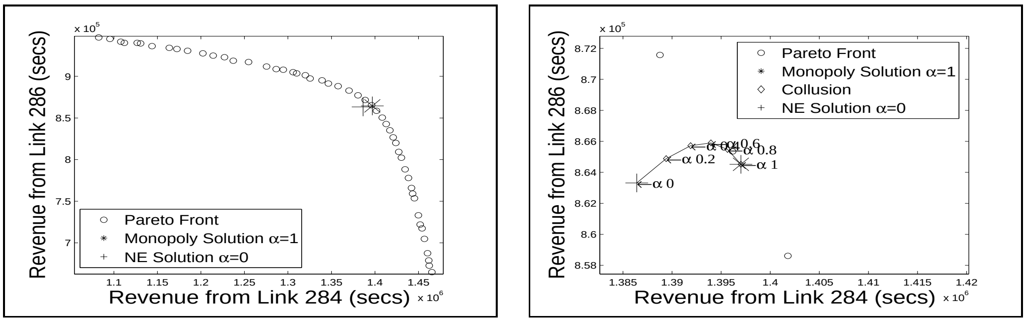


FIGURE 7.13: (Left) Scenario 4: The fully competitive NE (+) is not Pareto Optimal while the monopoly solution (*) lies on the Pareto Front. (Right): The "Collusion Path" as competitors move from the fully competitive NE ($\alpha = 0$) to the monopoly solution ($\alpha = 1$).

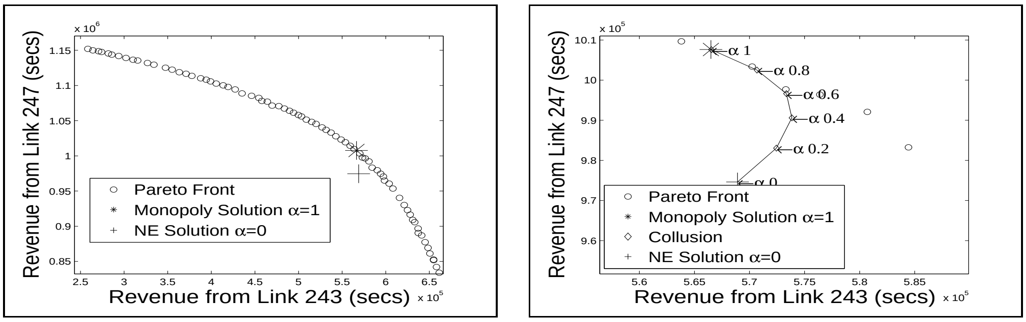


FIGURE 7.14: (Left) Scenario 5: The fully competitive NE (+) is not Pareto Optimal while the monopoly solution (*) lies on the Pareto Front. (Right): The "Collusion Path" as competitors move from the fully competitive NE ($\alpha = 0$) to the monopoly solution ($\alpha = 1$).

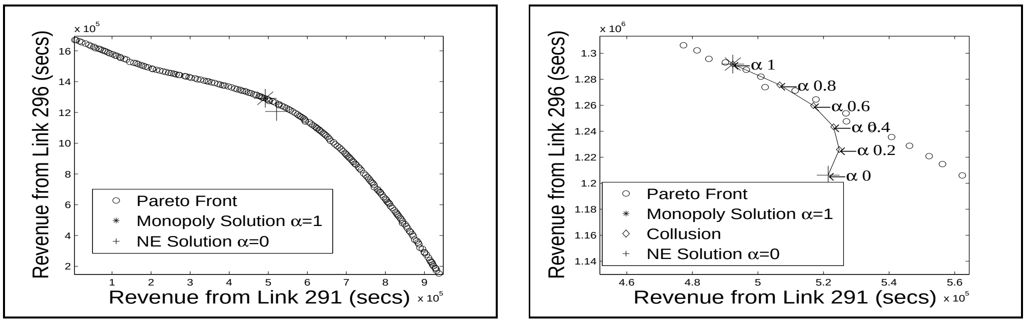


FIGURE 7.15: (Left) Scenario 6: The fully competitive NE (+) is not Pareto Optimal while the monopoly solution (*) lies on the Pareto Front. (Right): The "Collusion Path" as competitors move from the fully competitive NE ($\alpha = 0$) to the monopoly solution ($\alpha = 1$).

7.6.2 Nash Bargaining Solution

In the scenarios of collusion involving parallel links (i.e Scenarios 1 to 3), comparing the fully competitive outcome with the fully collusive outcome *alone* (ignoring the intermediate points of α), it was noted that the concessionaires could, arguably, agree to the Utilitarian Solution to division of the surplus obtained from cooperation. This deduction is based on recognising that moving from competition to monopoly, both gain and neither would lose out compared to the BATNA.

However, for collusion involving the serial link pairs (i.e Scenarios 4 to 6), it is not possible to draw the same broad brush conclusion. The revenues accruing to each concessionaire controlling each link in the serial scenarios in the case of competition and monopoly are reproduced in the third and fourth columns of Table 7.23. The column labelled "% Change A" measures the percentage change in revenue between monopoly and competition relative to the competitive outcome.

In Scenario 4, both concessionaires gain when they move from the fully competitive outcome ($\alpha = 0$) to the fully collusive/monopoly outcome ($\alpha = 1$) and so in this scenario the Utilitarian Solution satisfies the axiom of Individual Rationality (Axiom 1 in Section 5.6.1). However, this is still not sufficiently convincing since as shown in Table 7.23, with full collusion, the concessionaire controlling link 284 finds that the revenue earned increases by around 10,000 secs (i.e. 1,397,016 secs in monopoly versus 1,386,411 secs in competition) or 0.8% vis-á-vis the fully competitive outcome but the concessionaire controlling link 286 finds that with full collusion, the increase in revenue amounts to only 1,000 secs (864,518 secs under monopoly versus 863,314 secs under competition) or 0.1% compared to that attainable under full competition. Thus in this case, it is highly unlikely, in the absence of any side payment, that the concessionaire controlling link 286, obtaining roughly *10 times lower* than the increase obtained by the concessionaire on link 284 will be willing to accept this outcome based on the Utilitarian approach to the division of revenue.

However, in Scenario 5 and 6, it is clear that in this case, Axiom 1 is not satisfied under the Utilitarian Solution to the division of gains. This is because one concessionaire (link 243 in Scenario 5 and link 291 in Scenario 6) *loses out* with full collusion. As shown in the column labelled "% Change A" in Table 7.23, these concessionaires face a 0.4% and 5.6% *reduction* in revenue respectively vis-á-vis the fully competitive outcome. Thus even though *total* revenues have increased with collusion, the gains accrue to the other concessionaire in the series. Thus a rational concessionaire that obtains lower revenues than the BATNA would not be willing to agree to the Utilitarian Solution. This sets the scene for the alternative Nash Bargaining approach.

TABLE 7.23: Revenues under Competition, Monopoly and Nash Bargaining for Scenarios 4, 5 and 6

		Revenues (secs)				
Scenario	Link	Competitive ($\alpha = 0$)	Monopoly ($\alpha = 1$)	% Change A	NBS	% Change B
4	284	1,386,411	1,397,016	0.8%	1,392,591	0.4%
Serial	286	863,314	864,518	0.1%	868,617	0.6%
Total		2,249,725	2,261,534		2,261,208	
5	243	568,910	566,495	-0.4%	583,044	2%
Serial	247	974,626	1,007,640	3.4%	989,466	2%
Total		1,543,536	1,574,135		1,572,511	
6	291	521,404	492,338	-5.6%	536,235	3%
Serial	296	1,206,198	1,291,667	7.1%	1,239,092	3%
Total		1,727,602	1,784,005		1,775,328	

With Nash Bargaining, total revenues are not maximised. Instead, the objective is to maximise the product of the gains relative to the BATNA and to ensure that the parties do not attain lower revenues than the BATNA. Figs. 7.16 to 7.17 superimpose on the Pareto Fronts, the revenues accruing to each concessionaire in the NBS, obtained by solving Eq. 7–17, reported in Table 7.23. The results show that the NBS, indicated on Figs. 7.16 to 7.17 with a ☆ does indeed satisfy the Axiom 2 of Pareto Optimality in that all the gains from cooperation are exhausted and no concessionaire can be made better off without making another worse off.

With the Utilitarian Solution (indicated by a * marker), none of the concessionaires in Scenario 4 lose out, relative to the competitive outcome, by moving to this solution. However, in Scenarios 5 and 6, at least one concessionaire would lose out. In Table 7.23, the column labelled "% Change B" reports the percentage change in revenue between NBS and competition relative to the competitive outcome. This emphasises that with the NBS that none of the concessionaires lose out. For example, in Scenario 6, instead of suffering a 5.6% decrease relative the BATNA with the Utilitarian Solution, the concessionaire on link 291 gains by 3%. It is interesting that in the three scenarios tested, all concessionaires benefit by approximately the same proportion relative to their individual BATNAs. This suggest another advantage of Nash Bargaining in that it seems to incentivise the player who could be worse off with the Utilitarian Solution as the surplus seems to be shared more equally. Table 7.24 shows that the percentage share of total revenues or “slice of the revenue pie” each gets is similar under either of these axiomatic bargaining paradigms.

While the question of whether the NBS to sharing gains is viewed as fair or just is a subjective issue outside the scope of this research, it is evident that attaining the NBS in practice would be possible in Scenarios 5 and 6 given that none lose out. Thus the concessionaires would have an incentive to work towards that outcome. Furthemore, as will be seen in Section 7.6.3, the NBS is also positive for social welfare.

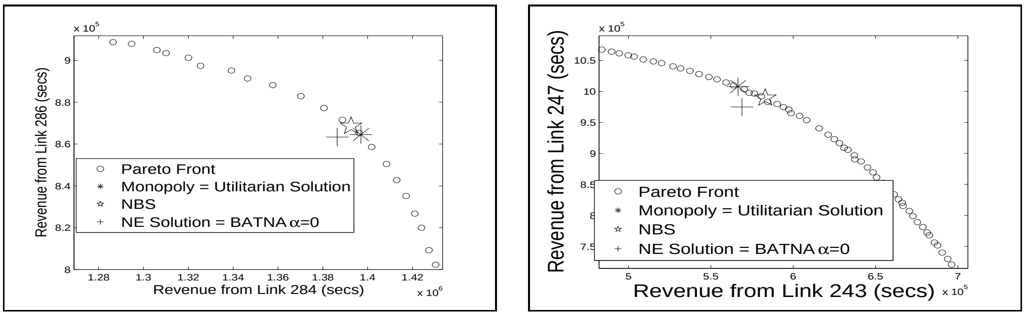


Figure 7.16: Left: Pareto Front, NBS, Monopoly and Competitive Solutions (Scenario 4)
Right: Pareto Front, NBS, Monopoly and Competitive Solutions (Scenario 5)

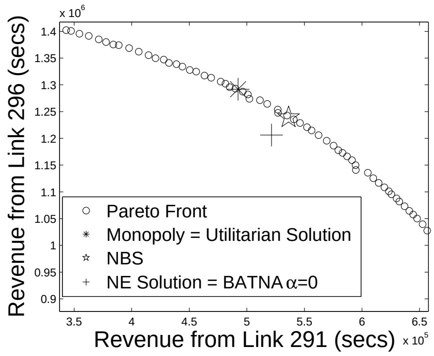


Figure 7.17: Pareto Front, NBS, Monopoly and Competitive Solutions (Scenario 6)

Table 7.24: Percentage Share in Revenues under Monopoly and NBS

Scenario	Link	%Share Monopoly	%Share NBS
4	284	62%	62%
	286	38%	38%
5	243	36%	37%
	247	64%	63%
6	291	28%	30%
	296	72%	70%

7.6.3 Welfare Impacts of Utilitarian Collusion and Nash Bargaining

In this section, the welfare impacts of collusion are summarised. This may again be assessed using the index of relative welfare improvement ω (Verhoef *et al.*, 1996) as the collusion parameter, α, varies from 0 to 1. The left panel of Fig. 7.18 focuses on collusion in the case of parallel competition (Scenarios 1 to 3) while the right panel focuses on collusion in the case of serial competition (Scenarios 4 to 6). Each of these figures shows how ω varies as α varies.

Welfare Impacts of Utilitarian Collusion

In the case of parallel competition, it can be seen that ω *decreases* as the degree of cooperation/collusion increases (i.e α increases from 0 to 1) where the concessionaires moving towards the Utilitarian Solution. It is clear that even if there is partial collusion, welfare will deteriorate compared to the fully competitive outcome. On the other hand, in the case of serial competition, ω *increases* as the degree of collusion increases. Thus welfare improves even with partial collusion. However, as noted above for Scenarios 5 and 6, concessionaires in the case of serial competition may not be incentivised to collude fully if that resulted in one of these concessionaires receiving lower revenues from collusion in the absence of a revenue-sharing agreement.

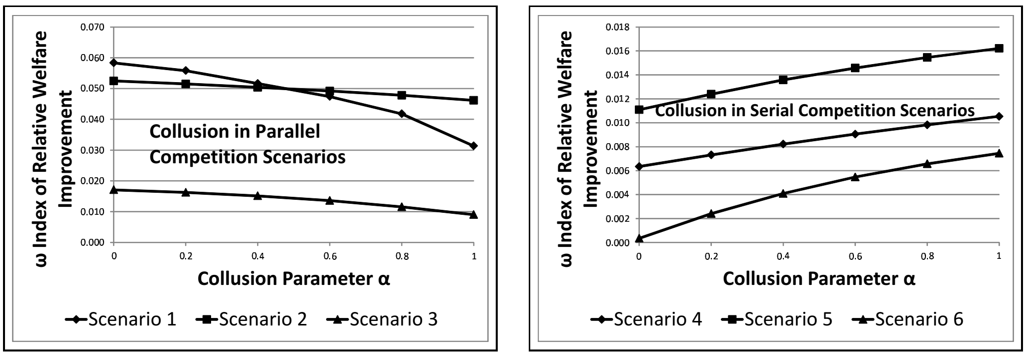


FIGURE 7.18: Index of Relative Welfare Improvement as Collusion Parameter, α, varies for Parallel Competition Scenarios (Left) and Serial Competition Scenarios (Right)

Welfare Impacts of Nash Bargaining

As discussed above, in Scenarios 4 to 6 with serially interdependent links, welfare is enhanced by ensuring that a monopolist controls the entire both links together, obviating the double marginalisation problem, rather than allowing competition. Table 7.25 compares the welfare change (relative to the untolled base equilibrium) achieved under the Utilitarian (Monopoly) solution and NBS and also reports the index of relative welfare improvement in each case. As this index is identical in both scenarios, it is evident that both of these solutions are better for social welfare than allowing competition in this case.

TABLE 7.25: Tolls and Welfare Change under Monopoly (Utilitarian) and Nash Bargaining

Scenario	Link	Monopoly Toll (secs)	Monopoly Welfare (secs)	ω	NBS Toll (secs)	NBS Welfare (secs)	ω
4	284	625.74	370,871	0.011	618.51	375,504	0.011
	286	575.27			584.36		
5	243	310.88	570,677	0.016	327.76	574,797	0.016
	247	387.16			372.22		
6	291	294.01	262,231	0.007	347.38	233,970	0.007
	296	587.31			566.24		

Combining this result with the possibility of the NBS being an acceptable solution to the parties as discussed previously, collusion and bargaining in this case could turn out to be welfare enhancing. This illustrates a possible rare instance where the commercial (revenue maximising) objectives and social (welfare maximising) objective are not at odds with each other.

7.7 Summary and Policy Implications

In the previous chapter, with routing in a congested network is characterised by Wardrop's Deterministic User Equilibrium (DUE) principle, it was demonstrated that concessionaires engaged in competition between links offering alternatives for travel between OD pairs would be positive for social welfare. On the other hand, competition between links that exhibit serial interdependencies i.e. where the travel journey could not be completed without using the entire route, would lead to deterioration of social welfare. The impacts of such serial competition could be worse than assigning responsibility for toll setting to a monopolist.

In this chapter, link flows and demands described by an elastic Stochastic User Equilibrium (SUE) model served instead as the active VI constraint in the NCEPEC formulation of the game between competing concessionaires. In addition, tests of a monopolist granted the right to collect tolls on all predefined tollable links in the network as well as the paradigm of a (second best) welfare maximising toll pricing policy were used as benchmarks. These benchmarks allowed for the determination of the extent of both the revenue as well as social welfare impacts of competition in a network with users routing according to the SUE principle, allowing for stochastic variation in perceptions of generalised route times across users. As part of the numerical tests, both the power law and exponential demand function were also used to study 12 scenarios. The primary differentiation between the scenarios was whether the tollable links served as alternatives for travel (i.e. parallel links) or whether they exhibited strong serial interdependencies (e.g. links that comprise a corridor). Furthermore, with the larger network used in this chapter, the scenarios investigated competition scenarios involving up to 6 concessionaires. This allows for an investigation into the effects of increasing the intensity of the competition in this way extending the insights from the previous chapter.

The overwhelming policy implication from the results of numerical tests conducted in both chapters is that, regardless of the demand function (power law or exponential) and routing paradigm assumed (DUE or SUE), competition between parallel links should be encouraged. It was also observed that that tolls could rise as a result of increased intensity of competition and this is attributable to the congestion internalisation component of the toll which increases welfare. When two links in a realistic network are the only links where tolls are charged, there is ample opportunity for traffic to reroute away from these links. When more links serving the same general direction of traffic movement are tolled, this reduces the opportunity for rerouting. When more traffic is captive to the tolls as intensity of competition increases, the congestion internalisation component of each concessionaire's toll would also increase. At the same time, as a direct result of increased competition, the demand related markup would fall. This explains why although tolls increase with increased intensity of competition, welfare also increases.

At the same time, competition between links that exhibit serial interdependencies should be restricted or curtailed. It was shown numerically that a monopolist in control of these links together would charge a lower toll resulting in better welfare gain. Although the toll increases with increased intensity of serial competition (as was the case with increased intensity of parallel competition), this is not due to more congestion being internalised but due to the the double marginalisation problem with the demand related markup being charged for each segment of a serial corridor. This explains why welfare decreases in the case of increasing competition between serial links.

These results were further emphasised in an additional test which allowed concessionaires to control a corridor comprising multiple links. It was shown that the allowing concessionaires the right to manage a corridor comprising serial links but facing competition from others doing the same on other corridors was a far more socially beneficial policy than allowing unbridled competition between serial links or monopolistic operation. Yet the worst strategy in terms of social welfare change vis-á-vis the no toll base, amongst the cases examined, was the assignment of such rights to concessionaires to set tolls on multiple but disparate portions of links that showed a high degree of serial interdependence. A clear message stemming from the numerical examples was the potential deterioration in social welfare with competition between serial links due to the double marginalisation problem as a concessionaire operating independently does not take into account, in his toll setting decisions, the reduction in revenues he would have on other links in the series.

Similar to the previous chapter, this chapter applied MOSADE, a DE-based Multi-Objective Evolutionary Algorithm to benchmark the revenue possibilities of cooperation between toll road concessionaires. It demonstrate that the link revenues obtained under monopoly is one solution amongst a number that maximise the objectives of two concessionaires simultaneously. Such solutions constituted the Pareto Front. It was shown that the Nash non-cooperative game solution was not Pareto

Optimal. This is because at the fully competitive outcome, one player could increase his revenues without his opponent suffering a loss.

The formulation, introduced in Chapter 6, was used to reflect the degree of cooperation between two concessionaires with $\alpha = 0$ representing the fully competitive outcome and $\alpha = 1$ representing the fully cooperative outcome. By repeatedly applying the SLCP algorithm each time for different α values, it was shown that the collusion path characterising the move across the spectrum could be traced. It was previously established in the tests that monopoly formation in the case of parallel links could be detrimental to social welfare but such monopoly formation in the case of serial links, on the other hand, could be positive. In all cases, it was shown that as collusion increases from none ($\alpha = 0$) through to full collusion ($\alpha = 1$) then the revenues accruing to each player maps from the NE solution to the monopoly solution. In the case of parallel links, it was shown that there was an incentive for concessionaires to collude. Unfortunately this would be detrimental to social welfare. Thus regulators should be aware of the potential for concessionaires to engage in collusion through such signalling behaviour.

In the case of serial links, it was demonstrated that with full collusion, while *total* revenues increase, the revenue earned by one concessionaire could be below that obtained at the fully competitive NE. This would act as a disincentive to the player who would then be worse off to collude unless a revenue sharing agreement was set up in advance. This motivated the study of bilateral Nash Bargaining as an alternative to Utilitarian Collusion. It was demonstrated that the Nash Bargaining Solution (NBS) could be obtained as the solution of the proposed Nash Bargaining Problem with Equilibrium Constraints (NBPEC). The numerical results verified that the NBS was not only Pareto Optimal but both parties to the NBS bargaining agreement would obtain higher returns than under the non-cooperative solution. In this way, the NBS would counter the disincentive (to one concessionaire) of the monopoly (equivalently Utilitarian) solution. The numerical tests point to the advantage of the NBS in that it seemed to result in more equal allocation of the surplus created. In addition, welfare improvements could be obtained with the NBS because it alleviates the double marginalisation problem associated in competition between serial links.

In this case, the NBS is beneficial from both the concessionaires' perspective as well as society's perspective.

In terms of the algorithmic implementations, it was demonstrated that the approach that solves the NCEPEC as a Sequential Linear Complementarity Problem (SLCP) seemed to be the most computationally efficient when compared against both the Fixed Point Iteration (FPI) and the NDEMO algorithm. As noted previously, the SLCP approach solves the NCEPEC simultaneously while the FPI solves each concessionaire's revenue maximisation problem in turn until the entire system converges. This accounts for why the SLCP approach outperforms the FPI algorithm in computational efficiency. It is vital to reiterate that the ability to apply the SLCP approach is primarily due to the smoothness of the SUE equilibrium constraints.

Chapter 8

Inter-jurisdictional Toll Competition: Part I[8.1]

8.1 Introduction

This chapter studies the problem of toll revenue competition between jurisdictions. As mentioned in Chapter 2, individual jurisdictions have an incentive to engage in tax exporting as the toll revenues it can raise from extra-jurisdictional users traversing its network would contribute to its local welfare. It is shown that this problem of inter-jurisdictional toll competition can be formulated as a NCEPEC. The toll revenue competition game is studied using a small network model with travellers' route choices described by the elastic demand Wardropian DUE principle (Wardrop, 1952). Subsequently, applying simple grid search techniques and making use of tools of game theory discussed in Chapter 3, the existence of LNE in this game is investigated numerically. The existence of multiple LNE is shown to exist under a range of assumptions.

The rest of this chapter is structured as follows. The additional notation required to describe toll competition between jurisdictions is developed in the next section to model the competition in the simplest possible setting of two adjacent regulatory authorities. The network representation described in Section 8.3 allows for an examination of the response surfaces i.e. how one authority would react to the other authority's choice of toll levels. Two cases are considered. The first, discussed in Section 8.4, assumes that the cities are identical with respect to both trip demands and network topology. The second case, discussed in Section 8.5 subsequently introduces asymmetry by considering the impacts of competition should one of the

[8.1] Large portions of this chapter is based on Koh *et al.* (2013).

cities be more attractive as a destination vis-á-vis the other. In all cases, it is demonstrated that incentivised by revenues from extra-jurisdictional users, authorities have an incentive to set high tolls to tax export. As a consequence, the welfare enhancing corrective effect of tolls aimed at internalising the congestion externality could be offset by the welfare decreasing fiscal externality of tax exporting. In the extreme case, social welfare would be higher had the authorities not introduced tolls in the first instance.

As discussed in Chapter 2, revenue sharing between jurisdictions has been suggested in the Edinburgh toll pricing scheme proposals (Gaunt *et al.*, 2006; Laird *et al.*, 2007). However, within a horizontal toll revenue competition context, the welfare implications of revenue sharing have not been investigated. Therefore, Section 8.6 investigates this issue which has both theoretical and practical relevance. One of the aims of the analysis is to understand the extent to which revenue sharing can alleviate the impacts of the fiscal externality of tax exporting.

Aside from the special case discussed, revenue sharing is shown not to result in Pareto Optimal outcomes. The reason for this is because while revenue sharing reduces the welfare loss from the fiscal externality of tax exporting, it cannot address the welfare loss stemming from the fact that jurisdictions only regard extra-jurisdictional users as adding to congestion in the network but not paying attention to their welfare. Finally, relaxing the assumption of non-cooperative behaviour underlying the NCEPEC, jurisdictions are allowed to engage in bilateral bargaining. It is shown that that the Nash Bargaining Solution may not be unique but unlike revenue sharing, Nash Bargaining can result in Pareto Optimal outcomes. Section 8.7 summarises.

8.2 Notation and Problem Formulation

In this section, the second best toll pricing problem (SBTP) is formulated in two forms. Firstly, to benchmark the welfare level attainable with a practical scheme, it is assumed a high level regulator/independent arbitrator sets tolls to maximise welfare for the entire network. Subsequently, the situation of two jurisdictions engaged in toll revenue competition is formulated. Unless new notation are introduced, all remain defined as previously used.

8.2.1 Global Regulator Problem

For a set of links in a traffic network $\mathcal{L}$, a subset of these $\mathcal{J}$, $\mathcal{J} \subseteq \mathcal{L}$ have been predefined to be tollable. Thus in this case, toll levels are chosen for each tollable link to maximise welfare subject to travellers routing according to Wardrop's DUE condition. The objective function reflects the difference between the benefits users receive from travel between the OD pairs and the generalised travel times costs expended in doing so.

This MPEC is termed a "Global Regulator Problem" (abbreviated as GRP) to distinguish it from the local variant to be discussed in the next section. This SBTP problem therefore corresponds to a "centric" pricing scheme (Zhang *et al.*, 2011, p. 298) as it models the setting of a single regulator deciding the toll levels on each link in $\mathcal{J}$ in order to maximise the benefit for all users i.e. regardless of origin or destination. This model is formulated in Eq. 8–1.

$$\text{GRP}\begin{cases} \underset{x \in X}{\text{Maximise}} \quad \mathcal{W}_{\text{DUE}}(\mathbf{x}) = \sum_{k \in \mathcal{K}} \int_0^{q_k} d_k^{-1}(w)dw - \sum_{j \in \mathcal{L}} v_j t_j(v_j) \\ \text{subject to} \quad \{\mathbf{v}^*, \mathbf{q}^*\} \leftarrow \text{SOL}\{\mathcal{V}(\mathbf{x})\} \end{cases} \qquad \text{(Eq. 8–1)}$$

As an MPEC, the (always) active constraint to the GRP is a VI. This is written as $\{\mathbf{v}^*, \mathbf{q}^*\} \leftarrow \text{SOL}\{\mathcal{V}(\mathbf{x})\}$. As introduce in Chapter 6 (see Section 6.3, p. 143), this expression is read as meaning that the unique vector of link flows $\mathbf{v}^*$, and demands, $\mathbf{q}^*$, satisfying Wardrop's DUE route choice principle (Wardrop, 1952), are obtained as the solution of the VIP formulation for the static traffic assignment problem, parametrised in the regulator's toll vector $\mathbf{x}$.

8.2.2 Inter-jurisdictional Toll Revenue Competition

In this section, the model is extended to allow for toll revenue competition between jurisdictions which are termed "authorities". To this end, it will be assumed that there are two regulatory authorities (labelled A and B), each authority having their own pre-defined subset of network links over which they may charge a toll. For simplicity, it is assumed that each authority i has a *single* toll level $x_i \geqslant 0$ that it chooses to levy on a predefined set of links in the network. Together, the tolls to be determined can be collected in the vector $\mathbf{x} = (x_A, x_B)^\intercal$. The decision space of the single toll level decided by each authority is denoted by $X_i = \{x_i : 0 \leqslant x_i \leqslant \bar{x}_i\}, i \in \{\mathrm{A}, \mathrm{B}\}$. It follows that $\mathbf{x} \in X_\mathrm{A} \times X_\mathrm{B}$.

It is assumed that each authority can only decide the toll level on the links in its subnetwork over which it exercises jurisdiction. The definition of the subnetwork is assumed to be predefined. Thus the highway network containing the set of links $\mathcal{L}$ can be further partitioned such that

1. $\mathcal{L}_i$ refers to the set of links within the subnetwork of Authority $i, i \in \{\mathrm{A}, \mathrm{B}\}$ such that $\mathcal{L}_\mathrm{A} \cap \mathcal{L}_\mathrm{B} = \emptyset$ and $\mathcal{L} = \mathcal{L}_\mathrm{A} \cup \mathcal{L}_\mathrm{B}$ and

2. $\mathcal{J}_i$, $\mathcal{J}_i \subseteq \mathcal{L}_i$ refers to the set of tollable links in Authority $i, i \in \{\mathrm{A}, \mathrm{B}\}$.

As noted in Chapter 2 (see Section 2.5.3, p. 48), each authority is interested in the welfare of its jurisdiction's residents. Following Zhang *et al.* (2011), it is assumed that these residents correspond to trips that originate from zones located within its spatial jurisdictional boundaries. On this assumption, the OD movements are partitioned into two mutually exclusive and exhaustive sets, such that $\mathcal{K}_i$ is the index of OD movements *originating* in Authority $i, i \in \{\mathrm{A}, \mathrm{B}\}$ with $\mathcal{K} = \mathcal{K}_\mathrm{A} \cup \mathcal{K}_\mathrm{B}$ and $\mathcal{K}_\mathrm{A} \cap \mathcal{K}_\mathrm{B} = \emptyset$.

In addition, the link flow vector $\mathbf{v}$ is partitioned such that $\tilde{v}_{ji}$ denotes the flow on link j of demand originating in Authority i (which will be referred to as "authority link flows"), clearly with $v_j = \tilde{v}_{j\,\mathrm{A}} + \tilde{v}_{j\,\mathrm{B}},\ j \in \mathcal{L}$. In vector notation, if the authority link flows are collected in a $|\mathcal{L}| \times 2$ matrix $\tilde{\mathbf{V}}$ then they are related to the aggregate link flow vector by $\mathbf{v} = \tilde{\mathbf{V}}\,\mathbf{1}$ where $\mathbf{1} = (1,1)^\intercal$.

Authority A's objective function is given by Eq. 8–2.

$$\begin{aligned}\phi_{\mathrm{A}}(\mathbf{x}) = \phi_{\mathrm{A}}(x_{\mathrm{A}}, x_{\mathrm{B}}) = & \sum_{k \in \mathcal{K}_{\mathrm{A}}} \int_0^{q_k} d_k^{-1}(z)dz - \sum_{j \in \mathcal{L}} \tilde{v}_{j\,\mathrm{A}} t_j(\tilde{v}_{j\,\mathrm{A}} + \tilde{v}_{j\,\mathrm{B}}) \\ & + \delta \sum_{j \in J_{\mathrm{A}}} x_{\mathrm{A}} \tilde{v}_{j\,\mathrm{B}} - \delta \sum_{j \in J_{\mathrm{B}}} x_{\mathrm{B}} \tilde{v}_{j\,\mathrm{A}}\end{aligned} \qquad \text{(Eq. 8–2)}$$

This measure of social welfare for Authority A in Eq. 8–2 is analogous to the objective function of the GRP in Eq. 8–1 *but* restricted to the subnetwork of Authority A. This function is equivalent to objective functions found elsewhere in the literature (e.g. Ubbels and Verhoef, 2008*b*; Zhang *et al.*, 2011) and is intended to encapsulate the idea introduced in Chapter 2 (see, p. 59), that in setting the tolls, jurisdictions pursue two simultaneous objectives. To recapitulate, these are as follows: "improving transport conditions as far as their own residents are concerned and generating profit or tax revenue from through traffic" (De Borger and Proost, 2012, p. 39).

The first term in Eq. 8–2 is the Marshallian measure of trips made from origins zones located within Authority A's subnetwork. The second term represents the generalised travel times (excluding tolls) for traffic with an origin in Authority A's jurisdiction. Thus the net effect of the first and second terms is a measure of the social surplus for users originating from zones located within A's subnetwork only. Therefore this accounts for the jurisdiction's desire to use tolls to internalise congestion and improve the transport conditions for its own residents. As can be seen, the social surplus of extra-jurisdictional users (i.e. those traversing its network) *does not* enter into the social surplus calculations.

The third and fourth terms together reflect the jurisdiction's desire to raise revenues from extra-jurisdictional users. The third term represents the toll revenue spent by residents from Authority B within Authority A; this is a transfer payment that increases the welfare of Authority A at the expense of Authority B. Similarly, the fourth term represents the toll revenue spent by residents from Authority A on links controlled by Authority B, i.e. those with origins in Authority A and travelling on tolled links in Authority B. Again, this is a transfer payment and increases the welfare of Authority B at the expense of Authority A. The parameter $\delta, \delta \in (0, 1)$ is a tax exporting parameter, for which shall be assumed to be common value for both authorities. Therefore, the net effect of the third and fourth term is the revenue gained from (or lost to, if negative) Authority B.

For the moment, the tests assume that $\delta = 1$ which means that both Authority A and B are concerned with the net effect of the revenues gained from the counterpart jurisdiction. Variations in δ are explored in Section 8.6.

Authority B's objective function is exactly analogous, *mutatis mutandis*, and the details are discussed below. As toll revenues are transfer payments, they cancel at the aggregate (i.e. network wide) level (Ubbels and Verhoef, 2008*a*, p. 179). Therefore, Eq. 8–3 holds at the aggregate network level for given toll vector $\mathbf{x}$ and with δ being equal for both jurisdictions.

$$\phi_{\mathrm{A}}(\mathbf{x}) + \phi_{\mathrm{B}}(\mathbf{x}) = \mathcal{W}_{\mathrm{DUE}}(\mathbf{x}) \tag{Eq. 8–3}$$

Given the objective function in Eq. 8–2, the optimisation problem facing Authority A can now be stated. Authority A is assumed to maximise social welfare of its own residents by adjusting the toll level of links over which it has control, anticipating the impact of the toll on travellers' route and demand decisions, but reacting to the toll level levied by Authority B. Thus it is assumed that Authority A does not anticipate the effect that its toll will have on Authority B's response, but it simply reacts to the toll set by Authority B. This means that neither of them possess superior information to enable them to act as a Stackelberg leader taking into account the reactions of the other authority in its toll setting. Therefore, A and B are assumed to play a Nash non-cooperative game. Assume, for the moment, that Authority B has already decided its toll level $\tau_{\mathrm{B}} \in X_{\mathrm{B}}$, and that this is known to Authority A. Authority A is then supposed to determine its own toll level x_{A} by solving the MPEC formulated as shown in Eq. 8–4.

$$\begin{gathered} \underset{x_{\mathrm{A}} \in \mathbf{X}_{\mathrm{A}}}{\text{Maximise}}\ \phi_{\mathrm{A}}(\mathbf{x}) = \phi_{\mathrm{A}}(x_{\mathrm{A}}, \tau_{\mathrm{B}}) \\ \text{Subject to} \end{gathered} \tag{Eq. 8–4a}$$

$$\mathbf{c}\left(\tilde{\mathbf{V}}\,\mathbf{1}, \mathbf{x}\right)^{\intercal} \cdot \left(\mathbf{v} - \tilde{\mathbf{V}}\,\mathbf{1}\right) - \mathbf{d}^{-1}\left(\mathbf{q}^{*}\right)^{\intercal}\left(\mathbf{q} - \mathbf{q}^{*}\right) \geqslant 0,\ \ \forall\left(\mathbf{v}, \mathbf{q}\right) \in \mathcal{D} \tag{Eq. 8–4b}$$

While the optimisation problem in Eq. 8–4 has a similar bilevel structure to the GRP, the key difference is that Eq. 8–4 is defined in terms of link flows *disaggregated by authority.* In general networks, for any given toll vector, uniqueness of the authority link flows cannot be guaranteed, even though the assumptions guarantee uniqueness of the *total* link flows. Therefore, if applied in a general network, Eq. 8–4 maximises social welfare in two ways: partly through the toll, but additionally by assuming that the authority link flows can be controlled over-and-above the toll effect. In this chapter, attention will be restricted in the numerical tests reported below to special network structures in which the uniqueness of the total link flows automatically assures uniqueness of the authority link flows. On this restrictive assumption, it will be assumed that network structure ensures uniqueness of the authority link flows and that link flows and demands $(\tilde{\mathbf{V}}, \mathbf{d})$ are uniquely determined by the VI constraint for a given toll vector $\mathbf{x}$.

In an analogous way to the behaviour of Authority A, Authority B determines its toll level, x_{B}, conditional on the toll level of Authority A by considering its own counterpart to objective function Eq. 8–2. This is shown in Eq. 8–5.

$$\begin{aligned}\phi_{\mathrm{B}}(\mathbf{x}) = \phi_{\mathrm{B}}(x_{\mathrm{A}}, x_{\mathrm{B}}) = \sum_{k \in \mathcal{K}_{\mathrm{B}}} \int_0^{q_k} d_k^{-1}(w)dw - \sum_{j \in \mathcal{L}} \tilde{v}_{j\,\mathrm{B}} t_j(\tilde{v}_{j\,\mathrm{A}} + \tilde{v}_{j\,\mathrm{B}}) \\ + \delta \sum_{j \in \mathcal{J}_{\mathrm{B}}} x_{\mathrm{B}} \tilde{v}_{j\,\mathrm{A}} - \delta \sum_{j \in \mathcal{J}_{\mathrm{A}}} x_{\mathrm{A}} \tilde{v}_{j\,\mathrm{B}}\end{aligned} \qquad \text{(Eq. 8–5)}$$

This objective function has a similar interpretation to the welfare objective of Authority A (see Eq. 8–2) as discussed previously. The first and second terms measure the social surplus of trips with origins in B's subnetwork. The third term measures the revenue "gained" from Authority A's users who incur the charge it levies while the fourth term measures the revenue "lost" to Authority A by its users.

Directly analogous to the competition between highway concessionaires discussed previously, the inter-play of these two authorities in each aiming to maximise its jurisdictional welfare measure by setting a toll, conditional on the other authority's toll, while anticipating the impact on the travellers, leads to another example of a NCEPEC. In this game, a vector of tolls, $\mathbf{x}^*$, is an NE when neither authority is able to increase its jurisdictional welfare by unilaterally deviating from its chosen toll level such that the condition in Eq. 8–6 is satisfied.

$$\phi_i(\mathbf{x}^*) = \phi_i(x_i^*, \mathbf{x}_{-i}^*) \geq \phi_i(x_i, \mathbf{x}_{-i}^*) \ \forall x_i \in X_i \ , i \in \{\mathrm{A}, \mathrm{B}\} \qquad \text{(Eq. 8–6)}$$

Note that each authority's optimisation problem assumes that the authorities can only determine their own toll, within each authority's decision space, conditional on the toll set by its counterpart authority, but places no further restriction on the admissible tolls. That is to say, the conditions require that, as far as one authority is concerned, their toll gives a global optimum solution to their individual MPEC, conditional on the other authority's toll setting.

However, if rather than each authority determining a global optimum toll conditional on the other authority's toll choice, there is the possibility that each authority only determines a *local* optimum to their individual MPEC. In this case, the condition in Eq. 8–6 need only to hold *within a local neighbourhood* of a given toll vector leading to the condition in Eq. 8–7. Following Definition 3.4 (see p. 86), points that satisfy Eq. 8–7 are Local Nash Equilibria (LNE). Thus for an LNE, each authority only needs to establish optimality within a neighbourhood of the given solution, $\mathcal{B}_i^{\epsilon}(\hat{x}_i)$.

$$\phi_i(x_i^*, \mathbf{x}_{-i}^*) \geq \phi_i(x_i, \mathbf{x}_{-i}^*) \ \forall i, \forall x_i \in \mathcal{B}_i^{\epsilon}(x_i^*) \ , i \in \{\mathrm{A}, \mathrm{B}\} \qquad \text{(Eq. 8–7)}$$

where $\mathcal{B}_i^{\epsilon}(\hat{x}_i) = \{x_i \in X_i \mid \|x_i - \hat{x}_i\| < \epsilon\}$.

Since the LNE conditions are weaker, the solution set to the NE problem is contained within the solution set to the LNE. Both kinds of solution are relevant for investigation, since it is not clear which is a more realistic representation of the behaviour of authorities in setting their tolls. This is an issue highlighted in the numerical tests reported in the next section.

8.3 Numerical Tests

Thus jurisdictions could set tolls non-cooperatively with the goal of individually maximising an individual jurisdiction welfare measure (Eq. 8–2 and Eq. 8–5). This section reports the results of tests conducted on a traffic network to determine the LNE in this game.

8.3.1 Highway Network and Demand Parameters used in Tests

Fig. 8.1 shows the highway network used for the numerical tests. Each link in this network takes the form $t_j(v_j) = t_0(1 + 0.15(v_j/\kappa_j)^4), j \in \mathcal{L}$ where t_j^0 and κ_j refer to the free flow travel time and capacity associated with link j. The free flow travel time t_j^0 is 450 seconds for all links except of links 2,5,8 and 11 which is 1000 secs. The capacity parameter κ_j is 1500 pcus/hr for all links except for 2,5,8 and 11 which is 3000 pcus/hr. Links 2,5,8 and 11 therefore represent a high capacity bypass that avoids travel through the town centre.

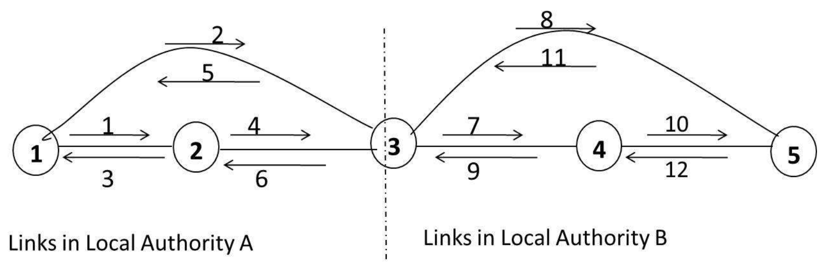


FIGURE 8.1: Highway Network for Numerical Tests (Link Numbers indicated on arcs)

There are 12 OD pairs and all nodes excluding Node 3 are origin or destination zones. There are two Central Business Districts (CBD) (zones 2 and 4) located within each authority respectively. The dotted line through Node 3 in Fig. 8.1 demarcates the boundary of jurisdiction of the two authorities. The base demand represents a typical morning peak with dominant flows to the CBDs from the suburb of each local authority (zones 1 and 5). However, demand to and from other zones is also present to represent the associated problems of through traffic. Travellers are assumed to respond to generalised prices using a demand function based on the power law form (Eq. 8–8), where, as before, q_k^0 and μ_k^0 are the base (i.e. untolled equilibrium) demands and generalised travel times for OD pair k respectively and $\eta_k, k \in \mathcal{K}$ is the elasticity parameter of -0.58, assumed to be equal for all OD pairs.

$$q_k = q_k^0 \left(\mu_k/\mu_k^0\right)^{\eta_k}, k \in \mathcal{K} \qquad \text{(Eq. 8–8)}$$

Authority A is assumed to set a uniform common toll[8.2] on links 1 and 6 to simulate a cordon into its CBD zone 2 while Authority B sets a separate uniform common toll on links 7 and 12 to simulate a cordon for travel into its CBD zone 4. In this way, a situation which may arise in reality is represented, namely that of cities who both wish to set up a cordon charge around their CBD with the aim of maximizing the welfare of their residents. The notation $\{x_{\mathrm{A}}, x_{\mathrm{B}}\}$ is used to indicate the toll levied on links 1 and 6 and the other single toll on links 7 and 12 respectively.

As noted in Section 8.2, a key property required of the problem formulation is uniqueness of link flows disaggregated by authority, at any given toll vector. This is established for the particular network under consideration in Appendix D, requiring some mild additional assumptions that are readily verifiable during the numerical tests, and indeed have been verified to hold. Considering Authority A's network (by symmetry, analogous implications can be drawn for Authority B's network), uniqueness is established by a combination of *a*) identifying routes that would never be efficient under Wardrop's DUE conditions; *b*) applying conservation-of-flow at the authority level and *c*) noting where authority flows do and do not mix. In the case of Authority A's network (analogous properties hold for Authority B's network, by symmetry), there is mixing of the flows between authorities on links 1, 3 and 6 only, whereas links 2 and 4 carry flows from origins in Authority A exclusively and link 5 carries flow from origins in Authority B exclusively.

In the tests, two different cases are considered. While the network as shown in Fig. 8.1 was used in both cases so as to preserve symmetry between the sub-network within Authority A and Authority B, the difference is the trip matrix used in each

[8.2] The uniform common toll means that the tolls on either tollable entry (inbound) link into each CBD are equal.

case. In Case 1, the base demand is symmetric and so represents the case where cities are equal in both production and attraction and network supply. In Case 2, the base demand is adjusted so that the city in Authority A is seen as stronger in terms of its ability to attract users. Details of the matrix used in each case are given in the relevant sections.

8.4 Tests with Symmetric Demands: Case 1

This section reports on numerical test carried out on the assumption that the base demands (and base generalised travel times) are symmetric in both cities. Table 8.1 shows the matrix used. With these demand parameters, the welfare change with theoretical first best pricing benchmark (i.e. marginal cost pricing on all links) relative to the untolled base is approximately 21,468 seconds.

TABLE 8.1: Base Trips and Base Costs used for Case 1

Origin	Destination	Base Trips d_k^0 (pcus/hr)	Base Generalised Travel Times μ_k^0 (secs)
1	2	1000	488.08
1	4	200	1,389.75
1	5	100	1,839.86
2	1	100	450.11
2	4	100	901.67
2	5	100	1,351.78
4	1	100	1,351.78
4	2	100	901.67
4	5	100	450.11
5	1	100	1,839.86
5	2	200	1,389.75
5	4	1000	488.08

8.4.1 Case 1: Solution to the GRP

As a benchmark against other solutions to be discussed, it is assumed that a single regulator is in place to determine the uniform toll on both Links 1 and 6 and another uniform toll on Links 7 and 12 resulting in the GRP defined by Eq. 8–1. The GRP

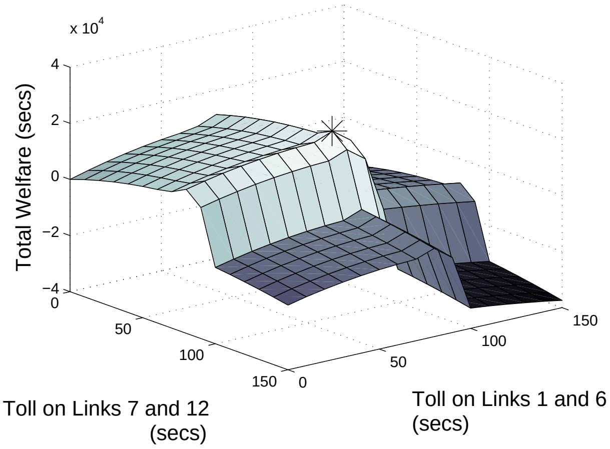


FIGURE 8.2: Case 1: Surface Plot of Total Welfare (secs) (GRP Toll and Total Welfare as shown in Table 8.2 indicated by $*$).

can be solved with the Cutting Constraint Algorithm (CCA) described in Chapter 6 (see Algorithm 6.1, p. 147).

The welfare surface of the GRP for Case 1 is shown in Fig. 8.2. Notice that the surface plots points to the existence of a single global optimum around a toll combination of $\{79.88, 79.88\}$ secs. Beyond toll levels of around 90 seconds from either authority, there is a reduction in benefits which continues to be the case as toll levels are increased to 1000 seconds.

Table 8.2 shows the tolls and welfare obtained in the GRP. As expected, due to symmetry, both authorities' welfare increases by the same amount vis-á-vis the base no toll equilibrium and that tolls are equal on links in both jurisdictions. The last column of Table 8.2 shows the relative welfare improvement ω, which as noted in Chapter 6 measures the welfare change attained by the GRP relative to first best pricing computed according to Eq. 6–15. As the second best GRP tolls are almost able to fully internalise congestion in this network, the index of relative welfare improvement is very close to 1.

8.4.2 Case 1: Inter-jurisdictional Toll Revenue Competition

For the NE problem defined by Eq. 8–6 and assuming that $\delta = 1$ (i.e. no revenue sharing), a grid search method was used to allow for a detailed exploration of the

Table 8.2: Tolls and Welfare under GRP for Case 1 obtained by CCA

Toll A (secs)	Toll B (secs)	Total Welfare Change (secs)	Welfare Change A (secs)	Welfare Change B (secs)	ω
79.88	79.88	20,296	10,148	10,148	0.95

response surfaces. To explore the potential solutions, the welfare for each authority was evaluated for a given toll pair with tolls ranging between 0-1000 seconds. Subsequently, the best response functions for each authority to the toll set by the other can be numerically estimated. As noted in Chapter 3, the intersections of these best response functions are possible LNE. These are shown in Fig. 8.3 where the continuous lines are the best response functions for Player A for any given toll of player B i.e. $R_{\mathrm{A}}(\mathbf{x}_{\mathrm{B}})$ and the broken line are the best response functions for Player B for any given toll of player A i.e. $R_{\mathrm{B}}(\mathbf{x}_{\mathrm{A}})$ following Definition 3.2. It can be seen that the best response functions intersect at 4 distinct points.

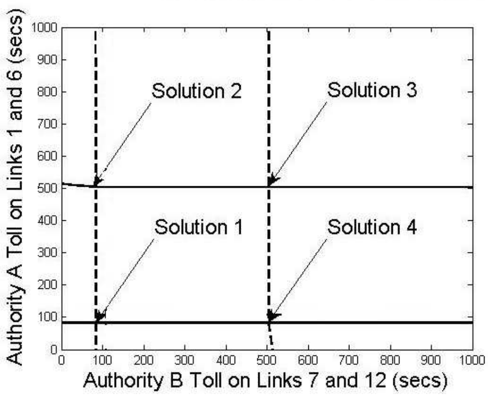


Figure 8.3: Case 1: Numerical Estimates of Best Response Functions. The continuous lines are best response functions for Authority A, ($R_{\mathrm{A}}(\mathbf{x}_{\mathrm{B}})$ while broken lines are best response functions for Authority B ($R_{\mathrm{B}}(\mathbf{x}_{\mathrm{A}})$)

Based on the welfare surfaces provided by the grid search and best response function plots, Table 8.3 gives the local welfare for each authority associated with each of these solutions. In this table, the figure in parentheses next to the local welfare measure gives the preference ranking, from the perspective of the authority, for a given outcome in the game. The last column of Table 8.3 summarises the relative welfare index, ω computed according to Eq. 6–15 which measures the network wide welfare change relative to first best pricing.

Table 8.3: LNE Tolls, Welfare and Index of Relative Welfare Improvement for LNE Solutions in Case 1, Figures in parenthesis indicate Authority's preference ranking ($\delta = 1$)

Solution Number	Toll A (secs)	Toll B (secs)	Welfare Change A (secs)	Welfare Change B (secs)	Total Welfare Change (secs)	ω
1	85	85	9,096 (2)	9,096 (2)	18,192	0.85
2	505	85	24,076 (1)	-101,839 (4)	-77,763	-3.67
3	505	505	-86,872 (3)	-86,872 (3)	-173,744	-8.18
4	85	505	-101,839 (4)	24,076 (1)	-77,763	-3.67

It is clear that the tolls obtained in each and every LNE solution in Table 8.3 is higher than the GRP reported in Table 8.2. This is, as noted in Chapter 2 (see p. 47), the toll a jurisdiction sets, when attempting to raise revenues from extra jurisdictional users, as implied by its payoff functions (Eq. 8–4 and Eq. 8–5), contains a demand related markup (Ubbels and Verhoef, 2008*b*). It is this markup that takes the toll above the level required to imperfectly internalise congestion. The consequence of this higher toll, as a result of the fiscal externality of tax exporting, is the lower welfare for all LNE solutions vis-á-vis the GRP outcome, as evidenced by a reduction in the index of relative welfare improvement ω. Thus in general, the consequence of both jurisdictions simultaneously attempting to extract revenues from extra-jurisdictional can have a adverse impact on welfare.

As both players begin to toll then, from Table 8.3, Authority A would clearly prefer Solution 2 while Authority B would prefer the diametrically opposed, Solution 4. If it is assumed that the authorities have full information about the change in welfare over the full range of tolls, then for a given toll of their opponent, they would move towards a toll of around 505 secs. In response, the second mover would also set a toll of around 505 secs and the authorities would end up at Solution 3 which results in a classic Prisoner's Dilemma (see Example 3.5, p. 82), where both authorities end up worse off than in the no toll case or if they had cooperated. In other words, the fiscal externality has such a negative impact that it *outweighs* the welfare improving corrective effect of tolls intended to imperfectly internalise congestion. As noted in Chapter 2 (see Section 2.5.3, p. 48), similar conclusions have been reported by others (e.g. Ubbels and Verhoef, 2008*b*; Zhang *et al.*, 2011; Gühnemann *et al.*, 2011).

As the authorities are entirely symmetric, discussion can be focused entirely on Authority A. Fig. 8.4 shows how welfare for Authority A varies with its own toll, for given values of tolls set by Authority B (i.e. 85 secs and 505 secs as identified in Table 8.3). Notice that there is a local maximum around a toll of 85 secs followed by a minimum then another maximum around a toll of 505 secs. It is worth noting here that the best response of player A and B do not appear to be affected by the toll set by the other authority above a toll of 85 secs. This suggests that there is little or no interaction between the players in the high toll regime. Section 8.4.3 will further explore the reasons behind this.

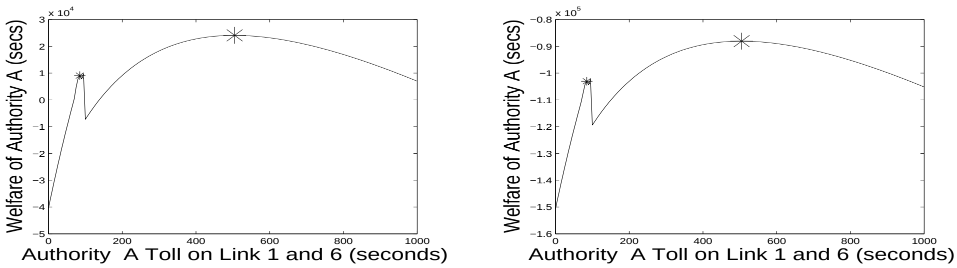


FIGURE 8.4: Welfare of Authority A showing 1) Local Optimum at 85 secs 2) Global Optimum (Larger Asterisk) at 505 secs when Authority B levies a toll of 85 secs. Right: Welfare of Authority A showing 1) Local Optimum at 85 secs 2) Global Optimum (Larger Asterisk) at 505 secs when Authority B levies a toll of 505 secs

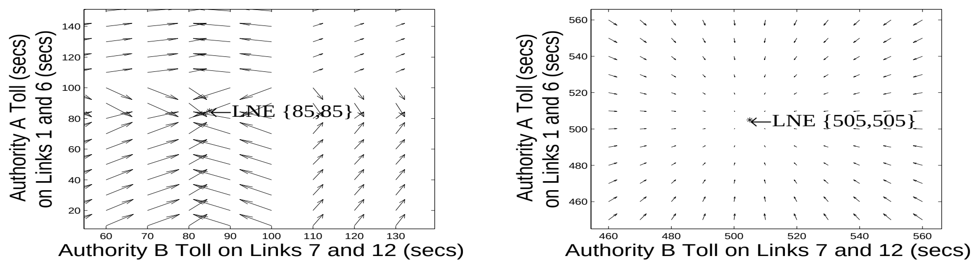


FIGURE 8.5: Left: Vector fields around toll vector of {85,85} secs for Case 1 ($\delta = 1$) where some arrows point *away* from this solution when tolls exceed 100 seconds. Right: Vector fields around toll vector of {505,505} secs for Case 1 ($\delta = 1$) where arrows point *towards* this solution.

To examine these solutions, the vector fields around the reaction functions at each point on the grid are plotted in Fig. 8.5. The arrows in each plot plot show the finite differenced approximations to the gradients of the welfare surfaces for each player with respect to their own toll and point towards the direction each player should move when selecting their toll levels given the current tolls to maximise its objective (Eq. 8–2 or Eq. 8–5). The left panel of Fig. 8.5 shows the vector fields centred on the LNE labelled Solution 1 with the toll tuple of $\{85, 85\}$ secs, while the right panel shows the vector fields centred on the LNE labelled Solution 3 $\{505, 505\}$ secs. From inspection of these vector fields it can be seen that these are LNE satisfying Eq. 8–7. Furthermore, it is evident from these figures that the basin of attraction is smaller around Solution 1 (the left panel of Fig. 8.5) and as a toll set by the other player moves beyond 100 seconds the players may well be attracted to Solution 3.

Similar plots show that the basin of attraction around Solutions 2 and 4 are also relatively small and that Solution 3 is the only solution which satisfies Eq. 8–6. Solutions 1, 2 and 4 are therefore only NE in a local neighbourhood i.e. LNE satisfying Definition 3.4 and Eq. 8–7.

An alternative way to view the outcome of each authorities' decision making and whether or not they act in a local neighbourhood or otherwise when setting tolls is to use a pay-off matrix approach as introduced in Chapter 3. Game 8.1 shows the pay-off matrix in terms of welfare changes for Authorities A and B given the tolls can only be set at values of 0, 85 or 505 (taken from the knowledge of where the possible LNE occur as identified above). Based on established convention in game theory literature (Fudenberg and Tirole, 1991; Gibbons, 1992), the first value in each cell shows the payoff that Authority A (the "row" player) would get if it uses a strategy in the corresponding row and Authority B uses the strategy in the corresponding column.

		Authority B		
		0	85	505
	0	$0, 0$	$-40.4, 50.7$	$-150, 66.8$
Authority A	85	$50.7, -40.4$	$9.1, 9.1$	$-101.8, 24.1$
	505	$66.8, -150$	$24.1, -101.8$	$-86.8, -86.8$

Game 8.1: Case 1: Pay-off matrix (thousand seconds) near each LNE solution (Welfare A, Welfare B)

It is noticeable that both players have an incentive to move away from the no toll situation, assuming that the other player does not charge. That is, both have a first mover incentive. If player A moves first, then player A has an incentive to move through to toll of 85 and then to a toll of 505. Player B would then respond accordingly and with these limited decisions available to the players the outcome is always the NE solution which satisfies the condition in Eq. 8–6i.e. it supports the view that solution 3 is the NE rather than just an LNE.

Next, the payoff matrix is widened to include some more local decisions around the solution at {85,85} secs as shown in Game 8.2. If either authority only considers *local* moves around tolls of 85 secs, then it is possible to remain in solutions 1,2 and 4, i.e. the {85,85} secs solution or one of the other {85,505} secs solutions. This can be seen for example by examining the local decision around the {85,85} secs pay-off cell. From this cell, there is no benefit for either player to increase or decrease the toll and so this is an LNE. However, it is noticeable that as soon as one authority charges above 90 seconds then they are incentivised to move towards solution 3 i.e. the NE solution.

		B				
		0	80	85	90	505
	0	0,0	-38.0,49.4	-40.4,50.7	-42.7,49.5	-150,66.8
	80	49.4,-38.0	10.1,10.1	7.7, 11.5	5.4,10.2	-104.4,26.5
A	85	50.7,-40.4	11.5,7.7	9.1, 9.1	6.7, 7.8	-103.0,24.1
	90	49.5,-42.7	10.2,5.4	7.8, 6.7	5.4, 5.4	-104.4,21.7
	505	66.8,-150	26.5,-104.4	24.1, -103.0	21.7,-104.4	**-86.8,-86.8**

Game 8.2: Case 1: Pay-off matrix (thousand seconds) with local moves around {85, 85} secs (Welfare A, Welfare B)

From a policy perspective, it is interesting that Solution 1 with tolls set at {85,85} secs is in the vicinity of the GRP solution (see Table 8.2, p. 252) such that both authorities experience an increase in welfare. It could be argued that such a solution may be found if the upper bounds of the toll sets considered were somehow restricted to within the range 0-90 seconds. As this is only a theoretical network example, it is difficult to conclude more generally about the scale issue but it is argued that in reality, there is usually an upper bound on the toll determined by, amongst other factors, public and political acceptability. Otherwise as solutions 1, 2 and 4 are only NE in a local neighbourhood (i.e. LNE) then these are unlikely to be obtained in a game with full information. Since acquiring information to support decision making may be costly, these LNE solutions could also be relevant as others studying NCEPECs elsewhere have noted that "limits to rationality or knowledge of players may lead to meaningful local Nash equilibria" (Hu and Ralph, 2007, p. 818).

Application of NDEMO

The section reports the results of obtained by NDEMO when it was applied to determine the LNE of this inter-jurisdictional game. The control parameters of NDEMO used for this test were the same as those used in tests in Chapters 6 and 7 (see Table 6.6). As NDEMO is a population based algorithm, the population mean (i.e. toll levels corresponding to each player) are tracked at each iteration. These are displayed in the left hand panel of Fig. 8.6. As mentioned in Chapter 5 (Algorithm 5.8 refers), NDEMO terminates when the standard deviation of the population (encoding the strategic variables of each player in the game) satisfies the termination tolerance criteria. Thus the right hand panel of Fig. 8.6 shows the evolution of the standard deviation of the population at each iteration. In this test, NDEMO required 80 iterations (and 1284 cpu secs) to converge to the pre-specified tolerance criteria. Note that the results reported in Table 8.4 is the mean of each population (encoding each player's strategies) at the final iteration.

TABLE 8.4: Case 1: Tolls and Welfare obtained by application of NDEMO

Toll A (secs)	Toll B (secs)	Welfare Change A (secs)	Welfare Change B (secs)	Total Welfare Change (secs)
504.85	504.97	-88,006	-88,005	176,011

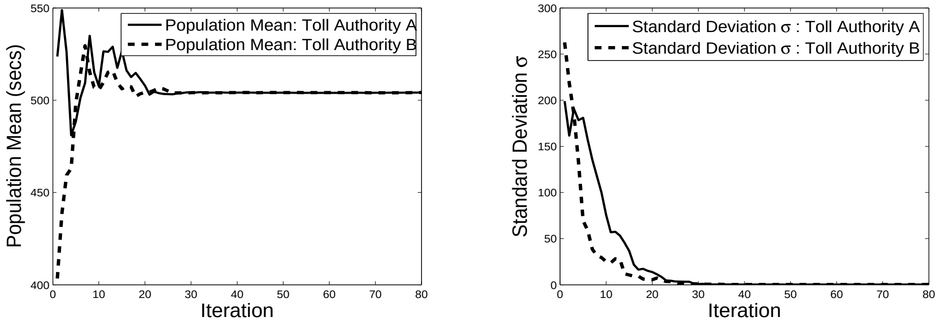


FIGURE 8.6: Case 1-Population Mean (Left Panel) and Standard Deviation (Right Panel) at each Iteration of NDEMO

The results in Table 8.4 lend further justification to the view that the tuple of approximately {505,505} secs as determined by the grid search is the NE satisfying Eq. 8–6 and not just an LNE satisfying Eq. 8–7. Nevertheless, it must be emphasised that the ability of any algorithm to detect the NE does not necessarily reflect realistic decision making behaviour, for which further research is required. Furthermore, the development of an algorithm to identify all possible equilibria in a game with continuous strategy spaces continues to be a subject for further research.

8.4.3 Case 1: Exploring the potential for multiple LNE

As noted earlier the optimal toll for Authority A does not appear to be affected by the toll set by Authority B in the high toll regime. This section first of all explains how this comes about by focussing on flow regimes and then explores which other factors can influence whether or not multiple LNE may exist.

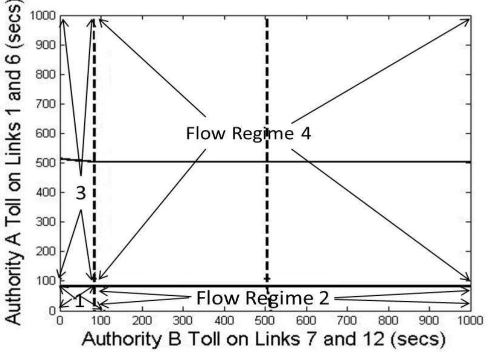


FIGURE 8.7: Flow regimes with alternative toll tuple $\{x_{\mathrm{A}}, x_{\mathrm{B}}\}$ combinations superimposed on numerical estimates of each player's best response functions

Figure 8.7 shows where the flows on the network can be decomposed into 4 “regimes” depending on the toll tuple $\{x_{\mathrm{A}}, x_{\mathrm{B}}\}$ and that these flow regimes correspond to the contours from Figure 8.3. The following insights can be drawn regarding traffic flows in these 4 regimes.

- When there are no tolls, the bypass links are not used at all. Hence all traffic regardless of destination utilise links through the town centre. This is due to the difference in free-flow travel times for using the bypass compared with the town centre route. Within regime 1, as the tolls are increased then eventually some users begin to use the bypass links 2 and 11 and hence obtain a “mixed traffic regime” i.e. flows on both the town centre route and flows on the bypass. Regime 1 is characterized by the set of tolls below 100 secs.

- In Regime 2, once the tolls set by Authority B (on links 7 and 12) increase beyond 100 secs, all through traffic in Authority B’s subnetwork uses the bypass links. That is, a toll greater than 100 seconds invokes the use of links 8 and 11 (the links that comprise the bypass in Authority B) but not links 2 or 5 which is still a function of tolls set by Authority A. The only traffic using the tolled links 7 and 12 are effectively captive (as in equilibrium they have no competitive alternative route across the range of feasible toll levels) to those links and there is a separated regime in B’s subnetwork. This means that subnetworks such as link 8 versus links 7 and 10 do not have the same cost at equilibrium and this is obtained by segregation of OD demands. By symmetry, Regime 3 is similar, *mutatis mutandis*, to flow regime 2 but responds to (Authority A’s) tolls exceeding 100 secs on links 1 and 6.

- In Regime 4, all bypass links are used and the traffic using the tolled routes is only “captive traffic” i.e. traffic that have destinations within the tolled area i.e. zone 2 or zone 4 which do not have any competitive alternative route across the range of feasible toll levels. All other traffic uses the bypass links. Each sub-network is in equilibrium but with higher costs for through traffic.

These regimes all come about because of the extremely low delays experienced on the bypass links relative to those on the through links. With the base demands it seems that the delays which result on the bypass links are negligible compared to the free flow travel times of 1000 seconds and that the assignment becomes an all-or-nothing assignment in regimes 2 to 4.

Understanding these regimes helps explain why the optimal toll set by A does not appear to be affected by the toll set by B in the high toll regime. Solution 3 lies in the separated flow regime so that the toll is in effect only affecting captive users and no more re-routing in response to a toll is possible. This separated regime implies that the optimal toll for player A is dependent only on the demand towards the central zone (node 2) and that the welfare function can only be increased by affecting the consumer surplus of own residents heading towards node 2 and the congestion experienced on link 1 plus the amount of revenue collected on link 6 from those non-residents travelling to node 2. All other flows and link costs are fixed once the tolls exceed 100 seconds. This sub-problem faced by player A is not influenced by the toll set by player B as all those who enter A's network from authority B have not been charged a toll in B's network by definition. They have either come from zone 4 via link 9 without charge or have come from zone 5 via the bypass link 11 again with no charge. This explains why there is no interaction effect between players once we are in this separated regime. The next section investigates how the number of LNE solutions varies as the elasticity of travel demand varies.

8.4.4 Case 1: Number of Potential LNE with changes in Elasticity of Demand

As mentioned in Chapter 6 (see p. 150), the power law demand function (Eq. 8–8) used for this network implies a constant elasticity of demand specified by the parameter $\eta_k, k \in \mathcal{K}$. This parameter represents the percentage change in demand as a result of a percentage increase in generalised travel times (inclusive of tolls) for each OD pair k. Thus η_k can be varied to assess the impact of an (absolute) increase in elasticity on the number of potential LNE in the network.

In this test, from the base elasticity of demand of -0.58, this was varied in steps of 0.25 between -0.75 to -1.75 (applied to all OD pairs equally). Figures 8.8 to 8.10

show the resulting numerically estimated best response functions. In these figures, the continuous lines show best response functions for Authority A, $R_{\mathrm{A}}(\mathbf{x}_{\mathrm{B}})$ and broken lines show the best response functions for Authority B, $R_{\mathrm{B}}(\mathbf{x}_{\mathrm{A}})$.

It is evident from Figures 8.8 to 8.10 that for elasticities up to -1.5, there are four LNE solutions, and with elasticity of -2, there is a single LNE solution (in the mixed flow regime). This demonstrates that there can exist combinations of network and elasticity values which exhibit only one LNE (and hence one NE) solution and that in this case, there would not be a "Prisoner's Dilemma".

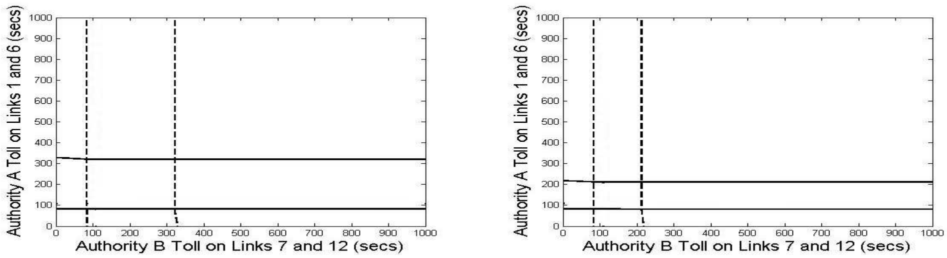


FIGURE 8.8: Left Panel: Best Response Functions for Case 1 with $\eta_k = -0.75, \forall k \in \mathcal{K}$ Right Panel: Best Response Functions for Case 1 with $\eta_k = -1, \forall k \in \mathcal{K}$

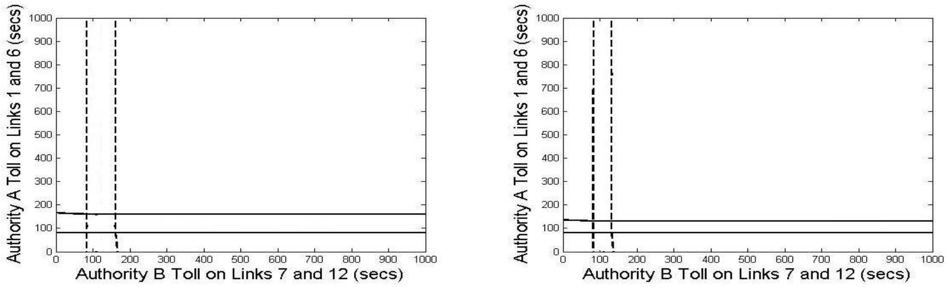


FIGURE 8.9: Left Panel: Best Response Functions for Case 1 with $\eta_k = -1.25, \forall k \in \mathcal{K}$ Right Panel: Best Response Functions for Case 1 with $\eta_k = -1.5, \forall k \in \mathcal{K}$

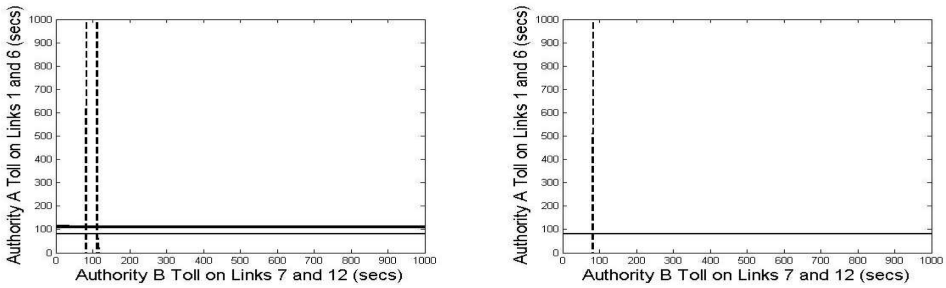


FIGURE 8.10: Left Panel: Best Response Functions for Case 1 with $\eta_k = -1.75, \forall k \in \mathcal{K}$ Right Panel: Best Response Functions for Case 1 with $\eta_k = -2, \forall k \in \mathcal{K}$

The figures also show that as demand becomes more elastic, the NE solution tends towards the low toll regime rather than the high toll regime. The left panel of Fig. 8.11 shows the graph of welfare for Authority A as the toll it sets varies (with Authority B's toll held fixed) in the case when the elasticity is kept at the base value of -0.58. It is noted that that the global optimum of welfare (as its own toll varies) in this case occurs to the *right* of the local optimum and is in the high toll regime. In contrast, the right panel of Fig. 8.11 shows the same graph with elasticity of -1.25. In this case, it is observed that the global optimum occurs to the *left* of the local optimum in the low toll regime. This demonstrates that as demand becomes more elastic, the NE solution moves from a high toll regime to a low toll one. This has the policy implication in that if (absolute) elasticity is higher then the authorities are less likely to end up in a Prisoner's Dilemma, the users will face lower tolls and residents could see an increase in total welfare. A similar conclusion was also obtained by Ubbels and Verhoef (2008*b*) who also concluded that the negative welfare impacts of inter-jurisdictional competition would be reduced with a higher (absolute) elasticity of travel demand.

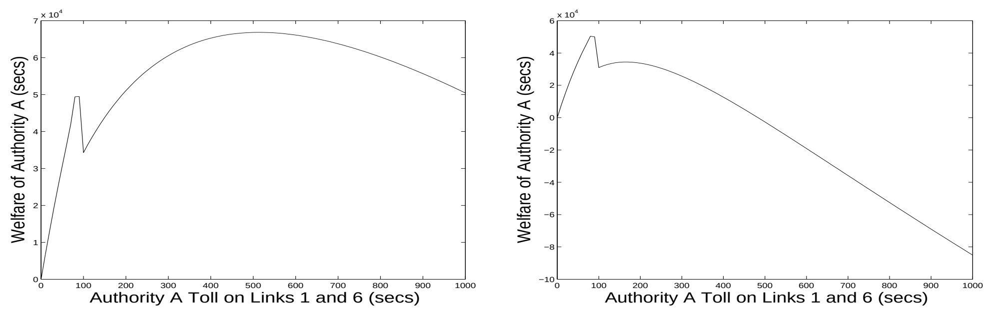


FIGURE 8.11: (Left) Global optimum of own authority welfare is in the high toll regime and to the *right* of the local optimum at elasticity of -0.58 (base case) as own authority toll varies. (Right) Global optimum of own authority welfare is in the low toll regime and to the *left* of the local optimum at elasticity of -1.25 as own authority toll varies.

It is also noticeable that the low toll LNE does not change as elasticity increases. This is again down to the specific parameters in the network and in particular it is related to the very small impact on delay on the bypass links as a small proportion of the flow is diverted from link 1 to link 2 for example. With low levels of through traffic, the congestion impact on the bypass links is only a fraction of a second so that the optimal toll is always in the same integer range. This is network specific and is not expected to be generalised.

8.5 Tests with Asymmetric Demands: Case 2

In constructing the asymmetric case, the number of trips originating from each zone is held constant as in Case 1, but these are re-distributed these so that the CBD in Authority A is now seen as more attractive relative to the CBD in Authority B. Specifically, as shown in Table 8.5, the total number of trips from A to B is reduced from 500 to 200 while the number from B to A increases from 500 to 800. The results reported in this section thus utilise this matrix with the network parameters (and topology) as in Case 1. Given these demand parameters, the first best welfare change relative to the untolled base is approximately 22,571 seconds.

TABLE 8.5: Base Trips and Base Costs used for Case 2

Origin	Destination	Base Trips d_k^0 (pcus/hr)	Base Generalised Travel Times μ_k^0 (secs)
1	2	1300	488.08
2	1	100	450.34
2	4	100	900.04
2	5	100	1,350.06
4	1	100	1,361.26
4	2	100	910.92
4	5	100	450.02
5	1	200	1,849.35
5	2	400	1,399.00
5	4	700	488.08

8.5.1 Case 2: Solution to the GRP

The welfare surface of the GRP for Case 2 is shown in Fig. 8.12. In addition, searching over the entire surface confirms that similar to Case 1, there exists only a single maximum at a toll of {90,80} secs. The results are summarised in Table 8.6. It is interesting that Authority B suffers from a welfare reduction (vis-á-vis the untolled base equilibrium) even in the GRP. The last column of Table 8.6 shows

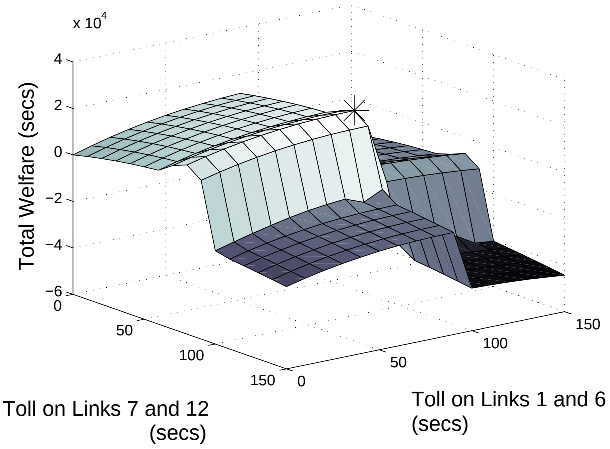


FIGURE 8.12: Case 2: Surface Plot of Total Welfare (secs) (optimum toll and welfare as shown in Table 8.6 indicated by $*$).

TABLE 8.6: Tolls and Welfare under the GRP for Case 2 found by CCA

Toll A (secs)	Toll B (secs)	Total Welfare Change (secs)	Welfare Change A (secs)	Welfare Change B (secs)	ω
90.36	79.99	21,418	62,012	-40,593	0.95

the relative welfare improvement index ω which as noted in Chapter 6 measures the welfare change attained by the GRP relative to first best pricing computed according to Eq. 6–15. As with Case 1, the high ω, close to 1, is due to tolls being charged on the most congested links in the network.

8.5.2 Case 2: Inter-jurisdictional Toll Revenue Competition

Following the same procedure described in Section 8.4.2, numerically estimate best response functions were obtained in order to identify the LNE solutions. These numerically estimated best response functions are shown in Fig. 8.13 where the continuous lines are the best response function for Player A for any given toll of player B i.e. $R_{\mathrm{A}}(\mathbf{x}_{\mathrm{B}})$ and the broken lines are the best response function for Player B for any given toll of player A i.e. $R_{\mathrm{B}}(\mathbf{x}_{\mathrm{A}})$ following Definition 3.2. It is clear that once again there are 4 LNE solutions that satisfy the condition in Eq. 8–7.

It is the case that in all 4 LNE, the impact on B's welfare is adverse, recall that

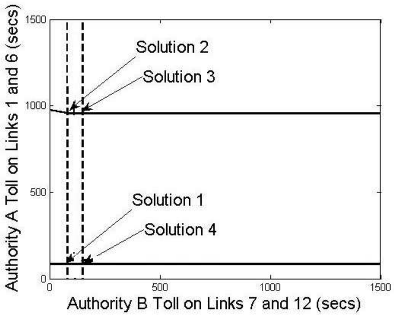


FIGURE 8.13: Case 2: Numerical Estimates of Best Response Functions. Continuous lines show best response functions for player A. $R_{\mathrm{A}}(\mathbf{x}_{\mathrm{B}})$ while Broken lines show best response functions for player B $R_{\mathrm{B}}(\mathbf{x}_{\mathrm{A}})$

TABLE 8.7: LNE Tolls, Welfare and Index of Relative Welfare Improvement for LNE Solutions in Case 2, Figures in parenthesis indicate Authority's preference ranking ($\delta = 1$)

Solution Number	Toll A (secs)	Toll B (secs)	Welfare Change A (secs)	Welfare Change B (secs)	Total Welfare Change (secs)	ω
1	85	81	58,025 (3)	-36,666 (1)	21,359	0.95
2	955	81	150,671 (1)	-392,798 (3)	-242,127	-10.73
3	955	150	142,737 (2)	-422,454 (4)	-279,717	-12.39
4	85	150	49,777 (4)	-65,860 (2)	-16,083	-0.71

B suffers from negative welfare even under the GRP benchmark (cf. Table 8.6). Compared to Case 1, it seems that the outcome will favour the stronger player. Solution 2 and Solution 3 are both highly favoured outcomes for Player A with the same toll level of 955 set by Authority A which demonstrates the strength of Authority A. Comparing the preference ranking in Case 2 with that from Case 1 and with reference to Table 8.7, now Authority A gives Solution 1 {85,81} secs nearer to the global regulator outcome {90,80} secs (cf. Table 8.6) a lower ranking while Authority B actually prefers this.

Whilst Authority B appears to be worse off in all cases, the response surface around the no-toll situation shows that both players have an incentive to move from the no-toll case so long as the other city is assumed not to toll. As expected, this incentive is stronger for Authority A. However, once the game begins, Authority B always ends up in a Prisoner's Dilemma and is worse off as a result in all 4 LNE solutions. Similarly, Authority A is always better off. This is in stark contrast to Case 1 where both authorities ended up worse off. It is, however, possible to show that solution

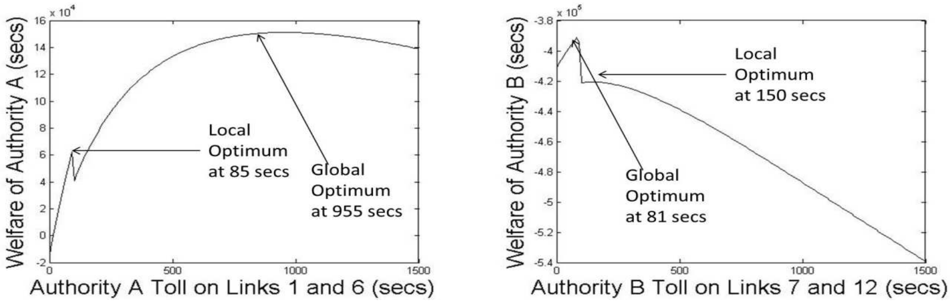


FIGURE 8.14: (Left) Case 2: Authority A's objective as its toll varies when B tolls at 81 secs (Right) Case 2: Authority B's objective as its toll varies when A tolls at 955 secs

2 is the NE which favours the stronger player, A, and is its preferred outcome. This can be inferred from Table 8.7 since if A moves first, it sets a toll of 955 secs and B responds (assuming full information) with a toll of 81 secs, conversely if B moves first, it sets a toll of 81 secs and A would respond with a toll of 955 secs. In other words, assuming perfect information, regardless of who moves first, the same solution results. This suggests that Solution 2 is not only an LNE but also the NE.

To ascertain this observation, the left panel of Fig. 8.14 plots the welfare of Authority A as its own toll x_{A} varies assuming Authority B sets a toll of 81 secs. It can be seen that A's toll of 955 secs in response of B's toll of 81 secs is the global optimum of its welfare function. Similarly, the right panel shows the welfare of Authority B as its own toll x_{B} varies assuming Authority A sets a toll of 955 secs and thus confirms that the toll of 81 secs is a global optimum of its welfare function, in response to A's toll of 955 secs. As both are simultaneously best responses within the entire strategy space and in line with Definition 3.1, the tuple {955,81} secs is the NE.

The findings support the findings of an econometric study by Levinson (2001) mentioned in Chapter 2 who found that that the more non-resident workers a state (in the US context in which the research was done) has, the greater the likelihood of toll pricing. Since Authority A has a larger number of non-resident workers (compared to Authority B as more commute to work in its jurisdiction) and so A has a stronger incentive to apply tolls to raise revenue from these extra-jurisdictional users and this is evident from the results shown in Table 8.7. The negative welfare experienced by Authority B offer a reason why strong objections were raised by neighbouring jurisdictions surrounding both Edinburgh and Stockholm when these authorities proposed to implement pricing schemes. In addition, a policy implication is that if there is extensive commuting by non-urban residents towards a city for employment as in many cities in Europe (De Borger and Pauwels, 2010), the city importing a large number of extra-jurisdictional users could be incentivised to tax export by setting tolls to extract revenues from extra-jurisdictional users.

The analysis also sheds some light on why large cities such as London or Stockholm can start the game and gain an advantage while smaller authorities (when including set up and operating costs that are ignored here) decide that in fact the benefits of going alone are not even there so this explains why there is a no-move case for the smaller regions especially if they think that the other larger town will retaliate and they may end up being even worse off.

Application of NDEMO

The NDEMO algorithm was also applied to determine the NE tolls and exactly the same control parameters used in Case 1 are used here. The population mean and standard deviations are plotted in the left and right hand panes of Fig. 8.15.

In this test, NDEMO required 120 iterations (and 2115 cpu secs) to converge to the pre-specified tolerance criteria. The results, reported in Table 8.8, is the mean of each population (encoding each authority's tolls) at the final iteration. These results augment the view that Solution 2 in Table 8.7 is the NE.

TABLE 8.8: Case 2: Tolls and Welfare obtained by application of NDEMO

Toll A (secs)	Toll B (secs)	Welfare Change A (secs)	Welfare Change B (secs)	Total Welfare Change (secs)
954.51	80.59	150,672	-392,798	-242,126

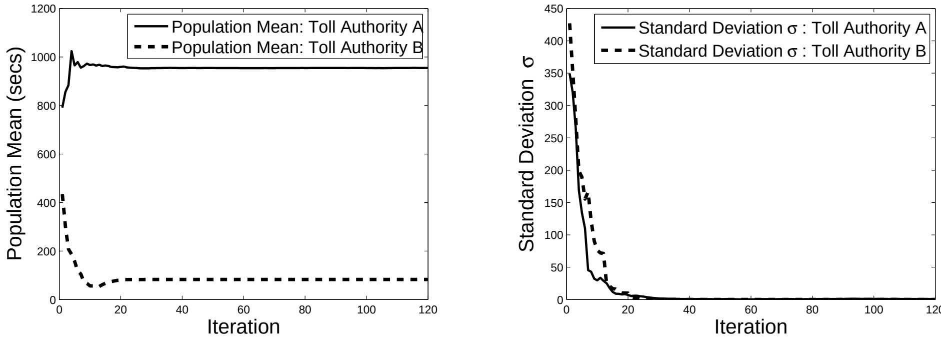


FIGURE 8.15: Case 2-Population Mean (Left Panel) and Standard Deviation (Right Panel) at each Iteration of NDEMO

8.6 Revenue Sharing

Thus far, the analysis has assumed that $\delta = 1$ in each jurisdiction's objective function (Eq. 8–2 and Eq. 8–5). This means that beyond improving transport conditions as far as their residents are concerned, each jurisdiction is also interested in maximising the net revenues raised from extra-jurisdictional users.

As highlighted in Chapter 2 (see p. 60), the public economics literature (e.g. Ladd, 2005; Ulbrich, 2011) views revenue sharing as a tool that central governments can use to correct inter-jurisdictional spillovers and to influence the activities of lower level governments. In the toll pricing context, revenue sharing has practical relevance as the City of Edinburgh Council (CEC), the proponents of the Edinburgh scheme, had suggested a revenue sharing arrangement with local authorities surrounding it so that revenues paid by users would be returned to these authorities in proportion to the trip origins (Gaunt *et al.*, 2006; Laird *et al.*, 2007). Furthermore, while Proost and Sen (2006) studied revenue sharing in the case of vertical competition, there is no literature on the effects of revenue sharing when jurisdictions are engaged in horizontal toll revenue competition. A revenue sharing agreement can be modelled using the parameter δ taking values between 0 and 1 (inclusive).

When $\delta = 1$, there is no revenue sharing and so the toll revenues are retained by the authority levying the toll and redistributed *only* within that authority. The case with $\delta = 1$ is also known as full tax-exporting since the authority levying the tolls retain the revenues and it contributes to its welfare measure.

When $\delta = 0$, there is full revenue sharing which means that there is full recycling of revenues back to those, regardless of origin, who paid the toll. Crucially $\delta = 0$ implies that the authority is no longer interested in the net revenues from extra-jurisdictional users but will only be concerned with the welfare of its residents. Likewise, this is the case with no tax exporting since the toll revenues from extra-jurisdictional users do not add to local welfare. For intermediate values of δ, there is degree of sharing of revenues collected i.e. some proportion of revenues are returned to the relevant authority.

For the case of $\delta = 1$ used so far, the numerical results above confirm the existence of multiple LNE. Therefore, in order to identify LNE for the range of δ values tested, a grid search of each authority's welfare as each toll varies between 0 and 1000 second was conducted before contour plots obtained by finite differenced approximation of the gradients were produced. These contour plots are best response functions. As noted in Chapter 3, the intersection points of best response functions are LNE. To further reduce the set of LNE to focus on the NE, the FPI algorithm was then applied to locate the NE based on the intersections.

However, attempts to apply the SLCP Algorithm (Algorithm 4.2) to this problem failed due to changes in the used paths. Since SLCP relies on the derivatives, it encountered numerical difficulties due to discontinuities in each jurisdiction's objective function. The failure is not entirely surprising given that it has been documented, as previously noted, that the equilibrium link flows (and demands) satisfying the Wardropian DUE route choice principle are not necessarily differentiable everywhere (Patriksson, 2004). Attempting to change the starting point in the SLCP algorithm did not circumvent this issue. As the SI algorithm (Algorithm 6.2, p. 148) is only designed for identifying LNE when players optimise revenue/profit, it cannot be tailored to the problem of players maximising welfare as considered in this chapter.

In order to benchmark the tradeoffs when jurisdictions are assumed to cooperate, the MOPEC in Eq. 8–9 is formulated. In this case, the Pareto Front will show points which maximise both jurisdictions' objectives simultaneously such that no one jurisdiction can be made better off without making another worse off. It is hypothesised that the GRP solution lies on this Pareto Front.

$$\begin{aligned} \underset{\mathbf{x}\in X}{\text{Maximise}} \quad & \mathbf{\Phi}(\mathbf{x}) = (\phi_{\mathrm{A}}(\mathbf{x}), \phi_{\mathrm{B}}(\mathbf{x}))^{\intercal} \\ \text{subject to} \quad & \{\mathbf{v}^*(\mathbf{x}), \mathbf{q}^*(\mathbf{x})\} \leftarrow \mathrm{SOL}\{\mathcal{V}(\mathbf{x})\}. \end{aligned} \qquad \text{(Eq. 8–9)}$$

8.6.1 Revenue Sharing in Case 1: Symmetric Demands

Table 8.9: Case 1:Tolls and Welfare Change as Revenue Sharing Parameter δ varies

δ	Toll A (secs)	Toll B (secs)	Welfare Change A (secs)	Welfare Change B (secs)	Total Welfare Change (secs)
0	80.00	80.00	10,146	10,146	20,292
0.2	81.30	81.39	10,020	10,104	20,125
0.4	159.27	159.32	-19,798	-19,777	-39,575
0.6	240.24	242.72	-32,337	-31,442	-63,779
0.8	356.08	356.08	-54,030	-54,030	-108,059
1	504.67	504.67	-88,006	-88,005	-176,011

Table 8.9 shows the tolls and welfare obtained by applying the FPI algorithm for values of δ between 0 and 1 inclusive in steps of 0.2. This table shows that as δ is reduced i.e. when there is more revenue sharing, then there is a tendency for the solution to move towards the lower toll regime. In fact, in the extreme case when $\delta = 0$, the welfare obtained with revenue sharing is very close to that obtained in the solution of Problem GRP (see Table 8.2). Any remaining difference is a result of numerical imprecision. In this case, the tolls with full revenue sharing are equal to the GRP solution. However, this is not generally true when the demand is asymmetric as will be shown later or when there is more extensive mixing of link flows in an example discussed in Chapter 9.

The Pareto Front, obtained as the solution of Eq. 8–9, by the use of MOSADE (cf. Algorithm 5.5) is shown in Fig. 8.16. In addition, the jurisdictional welfare of Authority A and B (last two columns of Table 8.9 for the range of δ) are also plotted on this figure. Given the scale of the diagram, the solutions with $\delta = 0$ and $\delta = 0.2$ are very close. Thus for clarity, they are separately plotted on the right panel.

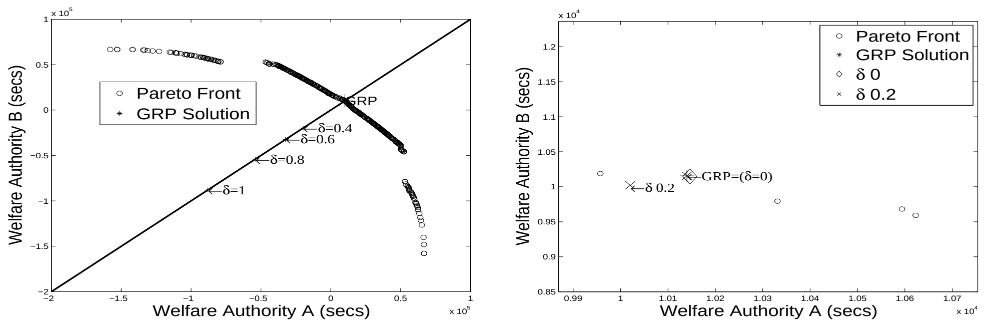


FIGURE 8.16: Left: Case 1- Cooperation and Revenue Sharing between Authorities: Pareto Front and jurisdiction welfare as revenue sharing parameter δ varies. Right: Zoom in of plot in left panel showing welfare for GRP (indicated by $*$) and for $\delta = 0$ and $\delta = 0.2$

It is observed that the NE outcome *without* revenue sharing (i.e. $\delta = 1$) on the left panel of Fig. 8.16 is not Pareto Optimal since the welfare of both players lie in the interior of the Pareto Front. However, as hypothesised, the solution of the GRP (see Table 8.2, p. 252), which aims to maximise the social welfare of *all* users lies on this front as indicated on both panels of Fig. 8.16 by a $*$.

As δ decreases from 1 to 0, an increasing proportion of the revenue is returned to the counterpart authority. Conversely, this means that the emphasis given to the revenue term in each authority's objective function (net effect of the third and fourth terms in (Eq. 8–2) and (Eq. 8–5)) is reduced. This implies that the impact of the demand related markup component in each authority's toll (see discussion on p. 47) also diminishes and this reduces the equilibrium toll level. For this reason, each authority's welfare tends toward to the Pareto Front as δ decreases. When $\delta = 0$, the welfare outcome lies on the Pareto Front. Furthermore, with fully symmetric demands, the path joining the welfare outcome with $\delta = 1$ to the welfare outcome obtained as the solution to the GRP lies on the 45° line.

Recall that in the absence of revenue sharing, in the low toll LNE, the index of relative welfare improvement ω is 0.85 (see line 1 in Table 8.3, p. 253). This means that even without revenue sharing, the congestion externality is adequately internalised. Thus it is the fiscal externality, arising from the desire to raise revenues that explains why tolls are above the GRP solution. Full revenue sharing then eradicates this fiscal externality and this is sufficient to restore welfare levels to that attainable by the GRP. However, this is only because of the limited interactions in this network, as highlighted above. The ability of revenue sharing to do this should be contrasted with the situation where there are extensive interactions over the network as will be discussed in Chapter 9 (see Section 9.5, p. 304) as well when demands are asymmetric as in Case 2 discussed next.

8.6.2 Revenue Sharing in Case 2: Asymmetric Demands

For Case 2, Table 8.10 shows the results obtained by the FPI algorithm for selected values of δ between 0 and 1. As reported in Table 8.6 (see p. 264), the solution of the GRP, modelling the situation where a single regulator sets tolls to maximise welfare for the entire network would set a toll of approximately {90, 80} secs. On the other hand, it can be seen that the NE tolls with full revenue sharing ($\delta = 0$) (first line of Table 8.10) is *not* the same as that obtained by solving the GRP.

TABLE 8.10: Case 1:Tolls and Welfare Change as Revenue Sharing Parameter δ varies

δ	Toll A (secs)	Toll B (secs)	Welfare A Change A (secs)	Welfare B Change B (secs)	Total Welfare Change (secs)
0	102.73	79.99	9,064	-13,286	-4,222
0.2	171.61	80.60	18,584	-27,629	-9,045
0.4	263.59	81.20	34,843	-58,788	-23,945
0.6	395.04	81.78	59,781	-115,609	-55,828
0.8	609.77	82.03	96,634	-218,552	-121,918
1	953.17	83.05	150,585	-392,325	-241,740

In Case 2, the stronger authority is still able to charge more than in the GRP (102 secs compared to 90 in the latter). The question then becomes why Authority A would do this even though net revenue collected from extra-jurisdictional users do not add to welfare since $\delta = 0$. The answer is that Authority A does not include in its objective/welfare function the social surplus experienced by the extra-jurisdictional users[8.3]. In this way, Authority A merely regard extra-jurisdictional users as adding to congestion in its network and hindering its users when in fact, *all* users contribute

[8.3]The first two terms of its objective function Eq. 8–2 only relate to benefits experienced by Authority A's residents i.e. trips originating within zones in A.

to and suffer from congestion by hindering each other. This explains why, despite the absence of any revenue incentive, Authority A still charges a toll higher than the GRP. This serves to stress that the elimination of the fiscal externality does not result in the welfare level achievable by a single regulator concerned with the welfare of all users equally in attempting to imperfectly internalise the congestion externality.

The results presented in Table 8.10 show that while Authority B would prefer revenue sharing, Authority A would prefer not to do this. The implication, when comparing the asymmetric case with the symmetric case, is that there may be a greater need for regulation when there exists a stronger player (as is the case with in other sectors of the economy). For society to be better off as a whole in Case 2, there needs to be a regulator in place which could also offset any disbenefits to those residents from Authority B by re-distribution of the revenues collected.

As in Case 1, MOSADE was applied to generate the Pareto Front showing the (Pareto Optimal) welfare tradeoff possibilities between the two authorities i.e. solution to Eq. 8–9. The jurisdictional welfare to Authority A and B (last two columns of Table 8.10 for different δ) are also indicated on this diagram. It is evident that the NE solution without revenue sharing (i.e. at $\delta = 1$) on Fig. 8.17 is not Pareto Optimal as both players are inside the Pareto Front. The solution of GRP, which maximises the social welfare of all users, lies on this Pareto Front.

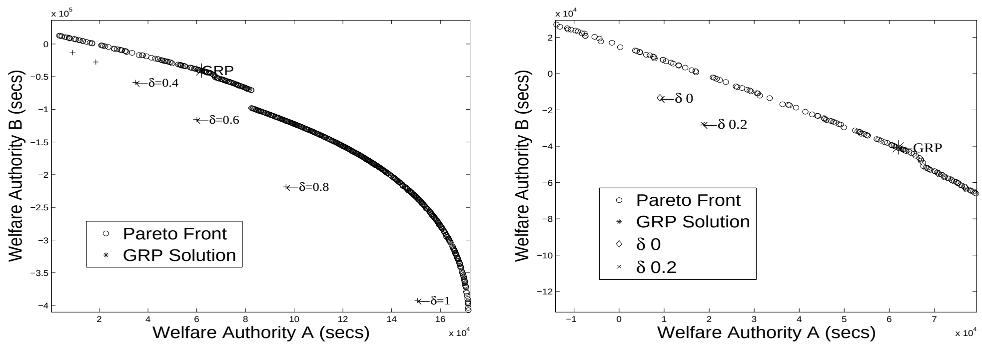


FIGURE 8.17: Left: Case 2- Cooperation and Revenue Sharing between Authorities: Pareto Front and jurisdiction welfare as revenue sharing parameter δ varies. Right: Zoom in of plot in left panel showing welfare for GRP (indicated by $*$) and for $\delta = 0$ and $\delta = 0.2$

It can be seen from Fig. 8.17 that Authority A is the stronger player. Through the range of revenue sharing parameters tested, Authority A consistently obtains higher welfare. The welfare of Authority B, given the parameters used, is consistently negative and worse than doing nothing (i.e. not imposing a toll).

Most crucially, even though full revenue sharing results in lower welfare losses for Authority B compared to no revenue sharing, it can be seen that in contrast to Case 1 with symmetric demands, full revenue sharing, does *not* result in a Pareto Optimal outcome. Thus while revenue sharing may address the welfare loss resulting from the fiscal externality, it is unable to correct the welfare loss stemming from the fact that in setting its toll, Authority A does not account for the full congestion externality caused by the interactions of *both* local and extra-jurisdictional users.

8.6.3 Nash Bargaining in Case 2: Asymmetric Demands

Up to this juncture, the research has assumed, following large portions of the literature, that jurisdictions act non-cooperatively in setting tolls. However, as noted in Chapter 2 (see p. 60), unlike toll road concessionaires, jurisdictions do not face legal restrictions preventing them from making binding agreements or collaborating with each other. Thus, following De Borger and Pauwels (2010), the assumption of non-cooperative behaviour is relaxed. Specifically, Authority A and B can cooperate when setting tolls through bilateral bargaining (Nash, 1950*b*). In this way, the tolls will be obtained by solving the Nash Bargaining Problem with Equilibrium Constraints (NBPEC) in Eq. 5–9 (see p. 135). In the numerical example to follow, the CCA (see p. 147 in Chapter 6) was applied to solve the NBPEC.

The Best Alternative to A Negotiated Agreement (BATNA) (Fisher *et al.*, 1991) is the backstop payoffs each player obtains should no bargaining agreement be reached. In bilateral bargaining between toll road concessionaires discussed in Chapters 6 and 7, it is assumed, following Schmalensee (1987) that should negotiations fail, toll road concessionaires would act non-cooperatively and obtain the payoffs at the NE outcome. A complication with bilateral bargaining between jurisdictions is that the BATNA is not necessarily unique as jurisdictions could continue to share toll pricing even if there is no bargaining agreement. In this way, the NBS is not unique but dependent on the BATNA assumed.

As the test only considers Case 2, the BATNA used in the first variant (labelled NBS (I)) is the outcome shown in the ultimate line of Table 8.10 i.e. welfares without revenue sharing ($\delta = 1$) with tolls of $\{953.17, 83.05\}$ secs. This means that if negotiations fail, both jurisdictions fall back to acting non-cooperatively *without* revenue sharing. The BATNA used in the second variant (NBS (II)) is the outcome shown in the first line of this table i.e. welfares with tolls of $\{102.73, 79.99\}$ secs when $\delta = 0$. This means that if negotiations fail, both jurisdictions act non-cooperatively but share toll revenues when doing so.

While only two variants are considered here, it is obvious that a number of BATNAs are possible depending on the extent of revenue sharing δ possible should negotiations collapse.

From Authority A's perspective, full revenue sharing means that it returns to Authority B the toll revenues it collects from users originating from zones within Authority B's subnetwork. As previously discussed, Table 8.10 had shown that there is a welfare loss if these jurisdictions act non-cooperatively regardless of whether they share revenues or not. Even with full revenue sharing, the index of relative welfare improvement, ω (Verhoef *et al.*, 1996) is -0.19 (see Table 8.11). Furthermore, as shown previously in Fig. 8.17, full revenue sharing does not result in a Pareto Optimal outcome.

TABLE 8.11: Jurisdiction Toll and Welfare with Revenue Sharing, GRP, NBS(I) and NBS(II)

Authority	Full Revenue Sharing		GRP		NBS (I)		NBS (II)	
	Toll (secs)	Welfare Change (secs)	Toll (secs)	Welfare Change (secs)	Toll (secs)	Welfare Change (secs)	Toll (secs)	Welfare Change (secs)
A	102.73	9064	90.36	62,012	84.76	54,532	42.09	19,626
B	79.99	-13,286	79.99	-40,593	79.93	-36,485	80.21	-3,923
Total		-4222		21,419		18,047		15,703
ω		-0.19		0.95		0.80		0.70

Next, consider the GRP. The values in these columns in Table 8.11 are taken directly

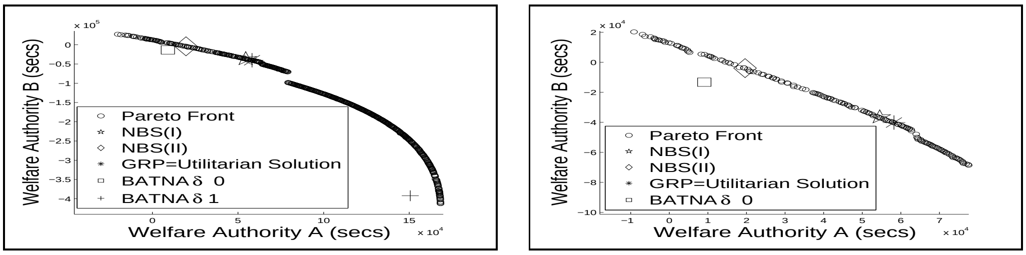


FIGURE 8.18: Left: Pareto Front, GRP, NBS(I), NBS(II)and BATNA with and without revenue sharing. Right: Zoom in confirming Pareto Optimality of NBS(I) and (II).

from Table 8.2. The GRP is the analogue of the Utilitarian Solution[8.4] as discussed in Chapter 5 (see p. 133) since it aims to maximise the welfare of the entire network. Though the Authority B is worse off compared to doing nothing, the GRP yields positive welfare gain to society.

Finally, the two NBS variants are reported. As noted, for NBS(I), the BATNA, acting as a constraint to the NBPEC, is the individual welfare without revenue sharing and in NBS(II), the BATNA is the individual welfare with full revenue sharing. While the NBS does not result in maximum network wide social welfare gain, the constraints to the NBPEC ensure that the welfare of each authority will be greater than the relevant BATNA. This is verified by comparing each jurisdiction's welfare under NBS(I) in Table 8.11 against the same for $\delta = 1$ in Table 8.10. Similarly, each jurisdiction's welfare under NBS(II) is also greater than (or less negative than) that attainable with $\delta = 0$ in Table 8.10. Therefore, this verifies that the NBS satisfies Nash Axiom 1 (p. 134) of Individual Rationality.

Table 8.11 shows that Authority B suffers a welfare loss under both NBS variants, although a lower one with NBS(II). Nevertheless, unlike full revenue sharing, both NBS variants are Pareto Optimal since they lie on the Pareto Front. This is indicated on Fig. 8.18 by the ☆ and ⋄ markers for NBS(I) and NBS(II) respectively. However, for either NBS outcome to be realised, it also requires Authority A to agree to one of these solutions. This seems unlikely as A has more to gain by not cooperating. A discussion of this issue would require a strategic approach to bargaining problems (Nash, 1953) beyond the Axiomatic approach that this thesis focuses on.

[8.4] In contrast to bargaining between toll road concessionaires (see footnote 6.4), the term "utilitarian" here does indeed have a utility or welfare interpretation.

The more interesting result is NBS (II). In contrast to all other solutions in Table 8.11, NBS (II) requires a lower toll for Authority A but a higher toll for Authority B. While Authority B suffers the lowest welfare loss in NBS (II), the network wide index of relative welfare improvement also falls ($\omega = 0.70$ compared to $\omega = 0.80$ in NBS (I)). Authority B would prefer NBS (II) but as noted, Authority A might object to this as it could obtain higher jurisdictional welfare with either GRP or NBS (I). The results show that the NBS is non unique and dependent on the BATNA assumed. However in both cases tested, the NBS satisfies Axiom 1 of Individual Rationality so that both parties would obtain at least their BATNA.

While revenue sharing does not necessarily result in a Pareto Optimal outcome, the NBS does. This verifies that Axiom 2 of Pareto Optimality is satisfied. This has a policy implication for the governance of toll pricing. One way to achieve Pareto Optimality with toll pricing policies in a multi-level governance setting is to assign the task to a higher level national regulator or arbitrator. This is precisely the GRP, which as shown, results in a Pareto Optimal outcome. However, because of ideological reasons or concerns that federal officials may not take into account unique local conditions (Banzhaf and Chupp, 2012), this might not be the desired approach to the governance of toll pricing schemes. In that case, an alternative possibility would be to allow jurisdictions to bargain bilaterally to attain Pareto Optimality.

8.7 Summary and Policy Implications

This chapter has focused on competition for toll revenues between jurisdictions. The simplest possible setting of two jurisdictions competing for toll revenues through their choice of toll levels on a cordon surrounding their CBDs with the aim of maximising a jurisdictional measure of social welfare was constructed. Travellers were assumed to route in the network obeying Wardrop's DUE principle.

To benchmark the welfare impacts of such horizontal competition for toll revenues, a Global Regulator Problem (GRP) was introduced. The inter-jurisdictional toll revenue competition problem was subsequently formulated and shown to be an instance of a NCEPEC. Employing grid search techniques (by evaluating welfare to each jurisdiction in the range of tolls tested in combination with the application of finite difference numerical approximations to obtain the best response functions), basic tools of game theory were subsequently applied to enumerate the LNE. It was shown that multiple LNE could exist in the toll revenue competition game and this was due to multiple maxima in the welfare/payoff function for each jurisdiction.

The sensitivity of the number of LNE to the travel demand elasticity was investigated. It was shown that multiple NE existed over a large range of elasticity values tested. For the network examined, it was only if travel demand was highly elastic would there be a single NE solution.

Focusing only on maximising jurisdictional welfare which incorporates the twin objectives of improving transport conditions for their residents and maximising revenues from extra-jurisdictional users, it was shown that there is an incentive for each jurisdiction to introduce tolls. Regardless of the trip matrix assumed, it was shown numerically that in all LNE, the tolls were always higher (and network wide social welfare always lower) than the GRP benchmark. This is because of the demand related markup component in each jurisdiction's toll associated with their desire to maximise toll revenues from extra-jurisdictional users. Thus the higher toll (vis-á-vis the GRP) is because of the *fiscal externality* of tax exporting. This finding is entirely consistent with the literature (e.g. De Borger *et al.*, 2007, 2008; Ubbels and Verhoef, 2008*b*) reviewed in Chapter 2 (see Section 2.5.3, p. 48).

Furthermore, depending on the tolls authorities levy, it was possible that the NE outcome could result in Prisoner's Dilemma outcome. This is because of the retaliatory response from an authority adversely impacted by the policies of another jurisdiction. In this case, competition for toll revenues not only reduced welfare below the GRP but also made each city worse off than if they had not been incentivised to toll in the first instance. In this extreme case, the fiscal externality has such an adverse impact on social welfare that it, in fact, outweighs any welfare gains from imperfectly internalising the congestion externality. Again, this finding resonates with previous findings in the literature (Ubbels and Verhoef, 2008*b*; Zhang *et al.*, 2010).

By allowing for asymmetries in demand, the analysis showed that stronger cities could be incentivised to introduce toll pricing with the aim of increasing jurisdictional welfare. However, a weaker region could suffer disbenefits as a result. This is likely to occur, for example, if there were a lot of residents commuting out of jurisdiction which is the usual case in many regions in Europe (De Borger and Pauwels, 2010). This concurs with findings reported elsewhere in the literature (e.g. Levinson, 2001). The result contributes to explaining the reason behind objections raised by jurisdictions when their neighbours propose introducing tolls as was the case observed in both Edinburgh and Stockholm (Gaunt *et al.*, 2006; Laird *et al.*, 2007; Eliasson *et al.*, 2008; Börjesson *et al.*, 2012)

The objective function for jurisdictional welfare used allowed for the study of revenue sharing/tax exporting which can be modelled by introducing the unitless parameter δ that can take a value between 0 and 1 inclusive. Revenue sharing could take the form of δ proportion of toll revenues returned to the jurisdiction from where the tolls were collected. When $\delta = 0$, all toll revenues are returned to the authority from which charge payers originate. When $\delta = 1$, the revenue is retained by the collecting authority for that authority's own disposal. In this sense, as toll revenues collected from extra-jurisdictional users was counted as a jurisdiction's social welfare and so this allows for the modelling of tax exporting.

In the case when cities were symmetric and if the interactions were weak, it was possible for revenue sharing to attain Pareto Optimal outcomes. This is because with limited interactions, the GRP was sufficiently effective in correcting the congestion externality. Therefore, with symmetric demand, revenue sharing was able to correct the remaining fiscal externality.

However, when asymmetries in demand were allowed for, with the network topology held fixed, revenue sharing did not reduce toll levels to the level of the GRP and the stronger authority was able to charge a higher toll. The end result is that the outcome with revenue sharing was not Pareto Optimal. The reason for this is that even though full revenue sharing eliminates the tax exporting incentive from each jurisdiction's objective function, each jurisdiction is only concerned with maximising the welfare of its own users (residents) and view extra-jurisdictional users as a source of congestion impeding their users. Further asymmetries, involving extensive interactions between users from both jurisdictions, will be discussed in the subsequent chapter.

Furthermore, relaxing the assumption of non-cooperative behaviour, a sensitivity test considered the NBPEC which models the situation of the jurisdictions engaged in bilateral bargaining when setting tolls. This could be modelled by solving the Nash Bargaining Problem with Equilibrium Constraints (NBPEC) employing CCA to obtain the Nash Bargaining Solution (NBS). It was emphasised that the NBS was not unique but dependent on the assumption regarding the BATNA vector which differed depending on whether full revenue sharing was possible. Two cases out of the various possible BATNA were considered depending on whether in the absence of an agreement, jurisdictions would share revenues or otherwise.

In both cases, it was shown that the Nash Axiom of Individual Rationality was satisfied in that both parties obtain higher welfare than their relevant BATNAs by engaging in bilateral bargaining. Most crucially, it was shown that unlike revenue sharing, the NBS is Pareto Optimal. The policy implication of this result is that should a national regulator (solving the GRP problem) not be the preferred approach to the institutional governance of toll pricing, bilateral bargaining between jurisdictions could be an alternative strategy to encourage jurisdictions to work towards Pareto Optimal outcomes when implementing toll pricing.

Although the welfare function introduced in this chapter allows for revenue sharing, it suffers from the disadvantage that it requires link flows disaggregated by authority. While it is shown in Appendix D, with additional mild assumptions, that these link flows are indeed unique for the example network used in this chapter, this is not true in general if route choices follow Wardrop's DUE principle. Furthermore, the SLCP approach failed to locate NE toll levels in this network as it encountered numerical difficulties when attempts were made to obtain first and second order derivatives. The failure of SLCP encountered stems from the recognition that the DUE link flows are not necessarily differentiable everywhere(Patriksson, 2004).

The inter-related issues of non-uniqueness and nonsmoothness therefore pave the way in Chapter 9 to consider the situation in which travellers route according to the more general SUE principle which, as noted in Chapter 7, is both smooth and differentiable. This also guarantees uniqueness of authority based link flows without requiring further restrictive assumptions, thereby allowing for the application of SLCP on a larger network, building on the analysis presented in this chapter.

While SLCP could not be applied to the numerical examples in this chapter because of the nonsmoothness of the DUE constraints, the NDEMO algorithm, on the other hand, was able to identify the NE. However, extensive function evaluations were required. Nevertheless, it should be emphasised that the ability of NDEMO to detect the NE is not indicative of realistic decision making behaviour, an issue for which further research is required. Regarding future developments, the next step would be to develop algorithms capable of identifying all possible game equilibria.

Chapter 9

Inter-jurisdictional Toll Competition: Part II[9.1]

9.1 Introduction

In this chapter, competition for toll revenues between two jurisdictions is discussed under the assumption that the users' route choices follow the elastic demand (logit) SUE principle. The smoothness (i.e. differentiability) of the SUE constraints is exploited to enable application of the gradient-based SLCP algorithm to determine LNE toll levels in the game between jurisdictions. This serves to contrast against the previous chapter where toll revenue competition between jurisdictions was investigated on the assumptions where routing obeyed the Wardrop's DUE principle. In that setting, attempts to apply SLCP to identify LNE failed as a result of numerical difficulties encountered due to the documented non-differentiability of DUE link flows.

The rest of this chapter is organised as follows. Section 9.2 formulates the mathematical structure of the problem in the same two instances as discussed in the previous chapter. In the first, it is assumed that a global regulator decides the toll levels for all tollable links in the network. In the second, it is assumed that jurisdictions engage in an inter-jurisdictional game with the objective of maximising a jurisdictional social welfare measure that explicitly incorporates the revenues raised from extra-jurisdictional users. This mathematical formulation is applied to Section 9.3 to a numerical example discussed in Chapter 8 (Case 1 on p. 250) where the SLCP algorithm intended to identify LNE failed due to changes in the active

[9.1]Large portions of this chapter are based on Watling *et al.* (2014).

path sets because of non-differentiability of link flows under the DUE assumptions. Exploiting smoothness of the SUE equilibrium constraints, this chapter shows that SLCP is indeed able to successfully identify LNE when applied to this same network. However, it is also shown that, depending on the logit dispersion parameter assumed, the smoothness may also result in convergence to a LNE rather than the global NE. One key characteristic of this network, as discussed in Chapter 8 was that there was limited interactions between users originating from the two different jurisdictions resulting in a separated traffic flow regime at high toll levels.

Thus in a second numerical example, Section 9.4 adapts a 62 link network from Zhang *et al.* (2011) that features extensive interactions in each subnetwork between users from both jurisdictions. In general, the numerical examples show that inter-jurisdictional toll competition has an adverse impact on social welfare. Section 9.5 investigates the implications of revenue sharing and Nash Bargaining between jurisdictions in this network. Revenue sharing can reduce and even eliminate the incentive to extract revenues from extra-jurisdictional users. In doing so, it remedies the welfare losses stemming from tax exporting. However, revenue sharing cannot address the failure of jurisdictions to take into account welfare of these extra-jurisdictional users when non-cooperatively setting tolls. For this reason, it performs poorly compared to a toll pricing policy that considers the welfare of all users equally. On the other hand, it is shown that Nash Bargaining can yield Pareto Optimal outcomes. Section 9.6 summarises.

9.2 Problem Formulation

9.2.1 Preliminaries

In discussing the elastic demand SUE model, this section will make use of the notation used in Chapter 7 (see Section 7.2). As previously done in Chapter 8, the study focuses on the case of horizontal competition between two jurisdictions/authorities, identified as Authority A and Authority B.

As defined previously, the set of all links in the traffic network is denoted by $\mathcal{L}$.

Suppose each authority has their own pre-defined subset of network links over which they may charge a toll. These tollable links are identified by means of a link-authority incidence matrix $|\mathcal{L}| \times 2$ matrix, $\mathbf{\Lambda}$ with elements Λ_{ji} equal to 1 only if link j may be tolled by authority i and equal to 0 otherwise ($j \in \mathcal{L}; i \in \{\mathrm{A}, \mathrm{B}\}$).

It is assumed that each authority i has a single, non-negative toll $x_i, i \in \{\mathrm{A}, \mathrm{B}\}$ that they may determine and levy on their tollable links. Thus the toll vector is a two element vector representing the toll for each authority and can be written as $\mathbf{x} = (x_\mathrm{A}, x_\mathrm{B})^\intercal$.

Based on the assumptions and subsequent discussion in Chapter 7 (see Section 7.2), when the route choices of users are described by a elastic demand logit SUE model, there exists an unique SUE vector of link flows (and demands), $(\mathbf{v}^*, \mathbf{q}^*) \in \mathcal{D}$, that solves the system of equations $\mathcal{S}(\mathbf{x})$ in Eq. 9–1 for given toll vector $\mathbf{x}$ (Cantarella, 1997).

$$\mathcal{S}(\mathbf{x}) \begin{cases} \mathbf{v}^*(\mathbf{x}) = \mathbf{\Delta} \mathbf{q} \mathbf{p}(\Delta^\intercal(\mathbf{t}(\mathbf{v}) + (\mathbf{\Lambda}\mathbf{x})); \theta) \\ \mathbf{q}^*(\mathbf{x}) = \mathbf{d}(-\theta^{-1} \ln(\mathbf{\Gamma} \exp(-\theta \mathbf{\Delta}^\intercal(\mathbf{t}(\mathbf{v}) + \mathbf{\Lambda}\mathbf{x})))) \\ \{\mathbf{v}, \mathbf{q}\} \in \mathcal{D} \end{cases} \tag{Eq. 9–1}$$

where $\mathcal{D}$ is a convex feasible set of demands and link flows satisfying Eq. 9–2.

$$\mathcal{D} = \left\{ (\mathbf{v}, \mathbf{q}) : \mathbf{v} = \mathbf{\Delta}\mathbf{f} \text{ and } \mathbf{q} = \mathbf{\Gamma}\mathbf{f} \text{ where } \mathbf{f} \geqslant 0, \mathbf{f} \in \mathbb{R}^{|\mathcal{R}|} \right\} \tag{Eq. 9–2}$$

Thus once again for brevity, the shorthand notation in Eq. 9–3 will be used to denote specifically such a vector of link flows (and demands) that solves Eq. 9–1 given toll vector $\mathbf{x}$.

$$\{\mathbf{v}^*(\mathbf{x}), \mathbf{q}^*(\mathbf{x})\} \leftarrow \mathrm{SOL}\{\mathcal{S}(\mathbf{x})\} \tag{Eq. 9–3}$$

9.2.2 Global Regulator Problem

Consider the situation of a regulator deciding tolls on all tollable links $\mathcal{J}$ in the network in order to maximise social welfare. The problem can be framed as the MPEC in Eq. 9–4. This is the SUE analogue of the Global Regulator Problem

(GRP) introduced in Chapter 8 (see Eq. 8–1).

$$\text{GRP}\begin{cases}\underset{x\in X}{\text{Maximise}} & \mathcal{W}_{\text{SUE}}(\mathbf{x};\theta)=\sum_{k\in\mathcal{K}}\int_{0}^{q_k} d_k^{-1}(w)dw-\mathbf{q}^{\intercal}\mathbf{s}(\mathbf{c};\theta)+(\mathbf{\Lambda x})^{\intercal}\mathbf{v}\\ \text{subject to} & \{\mathbf{v}^{*}(\mathbf{x}),\mathbf{q}^{*}(\mathbf{x})\}\leftarrow\text{SOL}\{\mathcal{S}(\mathbf{x})\}\end{cases}\qquad\text{(Eq. 9–4)}$$

where $\mathbf{s}(\mathbf{c};\theta)$ is the composite generalised travel times experienced by each user as given by Eq. 9–5.

$$\begin{aligned}\mathbf{s}(\mathbf{c};\theta)&=-\theta^{-1}\ln\left\{\mathbf{\Gamma}\exp\left(-\theta\mathbf{\Delta}^{\intercal}\left(\mathbf{c}(\mathbf{v})\right)\right)\right\}\\&=-\theta^{-1}\ln\left\{\mathbf{\Gamma}\exp\left(-\theta\mathbf{\Delta}^{\intercal}\left(\mathbf{t}(\mathbf{v})+\mathbf{\Lambda x}\right)\right)\right\}\end{aligned}\qquad\text{(Eq. 9–5)}$$

An equivalent MPEC was used to benchmark the welfare gains attainable in the case of competition between toll road concessionaires as described in Chapter 7 (see Eq. 7–10, p. 190). As explained therein, the objective function gives the social surplus experienced by all users. The first and second terms together measure the difference between the benefit these users receive from highway travel between all OD pairs (i.e. the Marshallian measure given by the integral under the inverse demand or marginal benefit functions) and the generalised composite costs (inclusive of tolls) incurred in doing so. The revenue collected from tolls are added as they represent a transfer payment from users to the collecting agency.

Due to the smoothness of the SUE constraints as discussed in Chapter 7, this GRP can be solved by directly embedding the SUE constraints directly into an optimisation algorithm. In the tests to be discussed later, similar to that done in Chapter 7, the interior point solver IPOPT (Wächter and Biegler, 2006) available as an option in the OPTI toolbox for MATLAB (Currie and Wilson, 2012) was used to solve this problem.

9.2.3 Inter-Jurisdictional Game

In the case of jurisdictions competing for toll revenues, it is assumed that each regulatory authority has jurisdiction over setting tolls in its own local area only and that its responsibility is only to residents, which are assumed to be trips that originate in its area.

Authority Specific Objective Function

As was previously done in Chapter 8, the OD matrix is partitioned into two mutually exclusive and exhaustive sets, such that $\mathcal{K}_i$ is the index of OD movements *originating* in Authority $i, i \in \{\mathrm{A}, \mathrm{B}\}$ with $\mathcal{K} = \mathcal{K}_\mathrm{A} \cup \mathcal{K}_\mathrm{B}$ and $\mathcal{K}_\mathrm{A} \cap \mathcal{K}_\mathrm{B} = \emptyset$. In this way, the $|\mathcal{K}|$-dimensional column vectors $\mathbf{q}_\mathrm{A}$ and $\mathbf{q}_\mathrm{B}$ are defined according to Eq. 9–6.

$$\mathbf{q}_\mathrm{A} = \begin{cases} q_k, \text{if } k \in \mathcal{K}_\mathrm{A} \\ 0, \text{if } k \in \mathcal{K}_\mathrm{B} \end{cases} \quad \mathbf{q}_\mathrm{B} = \begin{cases} 0, \text{if } k \in \mathcal{K}_\mathrm{A} \\ q_k, \text{if } k \in \mathcal{K}_\mathrm{B} \end{cases} \tag{Eq. 9–6}$$

Thus the vector of link flows, $\mathbf{v}$, and demands, $\mathbf{q}$, can be partitioned such that the conditions in Eq. 9–7 is satisfied.

$$\mathbf{v}_\mathrm{A} + \mathbf{v}_\mathrm{B} = \mathbf{v} \tag{Eq. 9–7a}$$

$$\mathbf{q}_\mathrm{A} + \mathbf{q}_\mathrm{B} = \mathbf{q} \tag{Eq. 9–7b}$$

where $\mathbf{v}_\mathrm{A}$ and $\mathbf{v}_\mathrm{B}$ are $|\mathcal{L}|$-dimensional column vectors denoting the authority based link flows i.e. link flows from origins in Authorities A and B respectively.

In the inter-jurisdictional competition for toll revenues, Eq. 9–8 is the SUE analog of Authority A's objective function introduced in the last chapter (see Eq. 8–2, p. 244). As mentioned in Chapter 8, δ,$0 \leqslant \delta \leqslant 1$ is a (unitless) revenue sharing/tax exporting parameter. assumed throughout to be common for both jurisdictions.

$$\mathcal{W}_\mathrm{A}^{\mathrm{SUE}}(x_\mathrm{A}, \mathbf{v}_\mathrm{A}, \mathbf{v}_\mathrm{B}, \mathbf{q}_\mathrm{A}, \mathbf{q}_\mathrm{B} | x_\mathrm{B}) = \sum_{k \in \mathcal{K}_\mathrm{A}} \int_0^{q_k} d_k^{-1}(w)dw - \mathbf{q}_\mathrm{A}{}^\intercal \mathbf{s}(\mathbf{c};\theta) + (\mathbf{\Lambda x})^\intercal \mathbf{v}_\mathrm{A} + \boxed{\delta \left(\mathbf{\Lambda} \begin{pmatrix} x^\mathrm{A} \\ 0 \end{pmatrix} \right)^\intercal \mathbf{v}_\mathrm{B} - \delta \left(\mathbf{\Lambda} \begin{pmatrix} 0 \\ x^\mathrm{B} \end{pmatrix} \right)^\intercal \mathbf{v}_\mathrm{A}} \tag{Eq. 9–8}$$

To understand Eq. 9–8, consider the extreme case where $\delta = 0$. This means that there is full revenue sharing where Authority A returns in full, all revenues collected from users originating in Authority B and that Authority B does the same.

The first term in Eq. 9–8 is the summation of the Marshallian measure of user benefits but only for trips made from origins located in Authority A i.e. $\mathcal{K}_{\mathrm{A}}$. The second term is the composite generalised travel time experienced by trips with origins in Authority A. As defined in Eq. 9–6, $\mathbf{q}_{\mathrm{A}}$ is 0 if the origin is in Authority B. Thus trips with an origin in Authority B, do not play any role in the first two terms. The third term reflects the fact that A recycles the revenues collected from its own users *only.* Since $\delta = 0$, the last 2 boxed terms in Eq. 9–8 do not play a role in the computation of jurisdictional welfare.

Next consider the other extreme case where $\delta = 1$. As previously, this means that Authority A retains all the revenues collected from users from Authority B traversing its subnetwork and paying its tolls and Authority B also does the same. However, as revenues are retained by the jurisdiction imposing the toll, the two boxed terms affect jurisdictional welfare. The first boxed term states that B's users paying a toll to A will increase the welfare of A by the amount of toll revenue paid to A. Conversely, in the second additional term, A's users paying a toll to B will increase the social welfare of Authority B and therefore equally reduce the social welfare of A by the amount of toll revenue paid.

In this way, Eq. 9–8 allows for the modelling of jurisdictions pursuing two objectives simultaneously: "improving transport conditions as far as their own residents are concerned and generating profit or tax revenue from through traffic" (De Borger and Proost, 2012, p. 39). In Sections 9.3 and 9.4, the numerical examples will assume the absence of revenue sharing and thus $\delta = 1$. Variations in δ are discussed in Section 9.5.

Formulation of NCEPEC

In the game between jurisdictions under DUE routing constraints as discussed in Chapter 8 (see Section 8.2.2), it was necessary to impose restrictive assumptions to ensure uniqueness of such variables disaggregated by authority as the standard assumptions of the DUE model can only assure uniqueness of *total* link flows. In contrast, with a SUE assignment model with elastic demand applied here, uniqueness of path flows is assured (Bell and Iida, 1997). Consequently the formulation is valid, in general networks, without requiring any further restrictive assumptions.

Let $\mathbf{\Omega}^{\mathrm{A}}$ be the $\mathcal{K}\times\mathcal{K}$ diagonal matrix with diagonal entries $\Omega^{\mathrm{A}}_{kk} = 1$ if OD movement k has an origin in Authority A and equal to zero otherwise. This matrix can be defined *a priori* from the network structure. Similarly the counterpart matrix for Authority B, $\mathbf{\Omega}^{\mathrm{B}}$, can be analogously defined.

Suppose that $\{\mathbf{v}^*(\mathbf{x}), \mathbf{q}^*(\mathbf{x})\}$ is an elastic SUE solution given by Eq. 9–1 given toll vector $\mathbf{x}$. Then the link flows and demands disaggregated by Authority to be uniquely determined as shown in Eq. 9–9.

$$\mathbf{q}^*_{\mathrm{A}}(\mathbf{x}) = \mathbf{\Omega}^{\mathrm{A}}\mathbf{q}^* \qquad \mathbf{v}^*_{\mathrm{A}}(\mathbf{x}) = \mathbf{\Delta}\mathbf{q}^*_{\mathrm{A}}\mathbf{p}(\mathbf{\Delta}^{\intercal}(\mathbf{t}(\mathbf{v}) + (\mathbf{\Xi}\mathbf{x})); \theta) \qquad \text{(Eq. 9–9a)}$$

$$\mathbf{q}^*_{\mathrm{B}}(\mathbf{x}) = \mathbf{\Omega}^{\mathrm{B}}\mathbf{q}^* \qquad \mathbf{v}^*_{\mathrm{B}}(\mathbf{x}) = \mathbf{\Delta}\mathbf{q}^*_{\mathrm{B}}\mathbf{p}(\mathbf{\Delta}^{\intercal}(\mathbf{t}(\mathbf{v}) + (\mathbf{\Xi}\mathbf{x})); \theta) \qquad \text{(Eq. 9–9b)}$$

Thus, for a given revenue sharing parameter δ, Authority A, in setting a toll level to maximise its objective in Eq. 9–8, solves the MPEC in Eq. 9–10 that is a variant of the GRP in Eq. 9–4.

$$\begin{aligned} \underset{x_{\mathrm{A}}\in X_{\mathrm{A}}}{\text{Maximise}} \quad & \mathcal{W}^{\mathrm{SUE}}_{\mathrm{A}}(x_{\mathrm{A}}, \mathbf{v}_{\mathrm{A}}, \mathbf{v}_{\mathrm{B}}, \mathbf{q}_{\mathrm{A}}, \mathbf{q}_{\mathrm{B}} | x_{\mathrm{B}}) \\ \text{subject to} \quad & \{\mathbf{v}^*(\mathbf{x}), \mathbf{q}^*(\mathbf{x})\} \leftarrow \mathrm{SOL}\{\mathcal{S}(\mathbf{x})\} \\ & \mathbf{v}_{\mathrm{A}} + \mathbf{v}_{\mathrm{B}} = \mathbf{v} \\ & \mathbf{q}_{\mathrm{A}} + \mathbf{q}_{\mathrm{B}} = \mathbf{q} \end{aligned} \qquad \text{(Eq. 9–10)}$$

Similar to the GRP (Eq. 9–4), the constraints to Eq. 9–10 define a unique allocation of link flows and demands disaggregated by authority. From Eq. 9–9, the disaggregated link flows are differentiable functions of the total link flows and demands $(\mathbf{v}^*(\mathbf{x}), \mathbf{q}^*(\mathbf{x}))$ which are in turn differentiable functions of the toll vector $\mathbf{x}$. Therefore, a smooth single-level problem equivalent to Eq. 9–10 can be formulated as Eq. 9–11.

$$\begin{aligned}\text{Maximise } \phi_{\mathrm{A}}(x_{\mathrm{A}}|x_{\mathrm{B}}) &= \mathcal{W}_{\mathrm{A}}^{\mathrm{SUE}}(x_{\mathrm{A}}, \mathbf{v}_{\mathrm{A}}^{*}(\mathbf{x}), \mathbf{v}_{\mathrm{B}}^{*}(\mathbf{x}), \mathbf{q}_{\mathrm{A}}^{*}(\mathbf{x}), \mathbf{q}_{\mathrm{B}}^{*}(\mathbf{x})|x_{\mathrm{B}}) \\ \text{subject to} \quad x_{\mathrm{A}} &\geqslant 0\end{aligned} \qquad \text{(Eq. 9–11)}$$

At the same time, noting that Authority B will be doing likewise, the optimisation problem it faces can be analogously stated as a single level optimisation problem given by as Eq. 9–12.

$$\begin{aligned}\text{Maximise } \phi_{\mathrm{B}}(x_{\mathrm{B}}|x_{\mathrm{A}}) &= \mathcal{W}_{\mathrm{B}}^{\mathrm{SUE}}(x_{\mathrm{B}}, \mathbf{v}_{\mathrm{A}}^{*}(\mathbf{x}), \mathbf{v}_{\mathrm{B}}^{*}(\mathbf{x}), \mathbf{q}_{\mathrm{A}}^{*}(\mathbf{x}), \mathbf{q}_{\mathrm{B}}^{*}(\mathbf{x})|x_{\mathrm{A}}) \\ \text{subject to} \quad x_{\mathrm{B}} &\geqslant 0\end{aligned} \qquad \text{(Eq. 9–12)}$$

Since both Authority A and Authority B each non-cooperatively aim to maximise an individual measure of jurisdictional welfare, each reacting to the toll set by the other but taking into account the route choices of the users, this results in a NCEPEC. In this game, a vector of tolls, $\mathbf{x}^{*}$, is a NE when neither authority is able to increase its jurisdictional welfare by unilaterally deviating from its chosen toll level. Thus the condition in Eq. 9–13 is satisfied.

$$\phi_i(\mathbf{x}^*) = \phi_i(x_i^*, \mathbf{x}_{-i}^*) \geq \phi_i(x_i, \mathbf{x}_{-i}^*) \;\; \forall x_i \in X_i \;, i \in \{\mathrm{A}, \mathrm{B}\} \qquad \text{(Eq. 9–13)}$$

As shown in Chapter 4 (see Section 4.5.1, p. 105), the NE problem can be formulated as a CP (Complementarity Problem), based on the KKT conditions for the simultaneous maximum of each jurisdiction's optimisation problem (Eq. 9–11 and Eq. 9–12).

Based on the discussion in Chapter 3 and the experience in the previous chapter, because of possible local maxima in each jurisdiction's welfare or payoff functions, solutions may only satisfy Eq. 9–13 locally i.e. LNE solutions may be obtained. In this case, when jurisdictions are only able to verify optimality within a neighbourhood of a given solution, the sets of solutions satisfying the weaker LNE condition in Eq. 9–14 would also be relevant (see Definition 3.4).

$$\phi_i(x_i^*, \mathbf{x}_{-i}^*) \geq \phi_i(x_i, \mathbf{x}_{-i}^*) \;\; \forall i, \forall x_i \in \mathcal{B}_i^{\epsilon}(x_i^*) \;, i \in \{\mathrm{A}, \mathrm{B}\} \qquad \text{(Eq. 9–14)}$$

where $\mathcal{B}_i^{\epsilon}(\hat{x}_i) = \{x_i \in X_i \mid \|x_i - \hat{x}_i\| < \epsilon\}$.

9.3 Example 1: Tests on Network from Chapter 8

In this section, the first network used is that discussed in the previous Chapter (see Fig. 8.1). The network parameters remain unchanged and the focus will be restricted to symmetric demands in the Base Matrix (i.e. Case 1 as described in Section 8.4). In this chapter, the only difference is that the routing of the users is assumed to follow a SUE elastic demand model.

9.3.1 Exploration of Nash Equilibria as logit dispersion parameter (θ) is varied

Chapter 8 demonstrated the existence of 4 LNE in this network when the traveller routing follows a DUE model. In this section, it is shown that multiple LNE continue to exist under SUE for large θ (small dispersion), but as θ decreases and there is a departure from DUE, then only one LNE solution exists, which also satisfies the NE conditions over the entire strategy space (and thus is also the unique NE solution following Definition 3.1).

As was done in the Chapter 8, a grid search was conducted over the entire tollable space of 1000 secs for each authority. This enabled numerical estimation of the best response functions. As noted previously, intersections of these best response functions identify toll combinations that could be potentially LNE.

Examples of such best response function plots for the cases $\theta = 0.005$ and $\theta = 0.5$ are shown in the left and right panes of Fig. 9.1. Where the best response functions are near horizontal or vertical, this is indicative of a low level of interaction between the two problems solved by the authorities. This is an artefact of the network and demand scenario selected in this example; this will not always be the case, as will be seen in Example 2 (see Section 9.4). With $\theta = 0.005$ there is only one point of intersection, which demonstrates there is only one LNE solution, and hence only one NE solution, at a toll of around 300 secs set by each authority. With $\theta = 0.5$ there

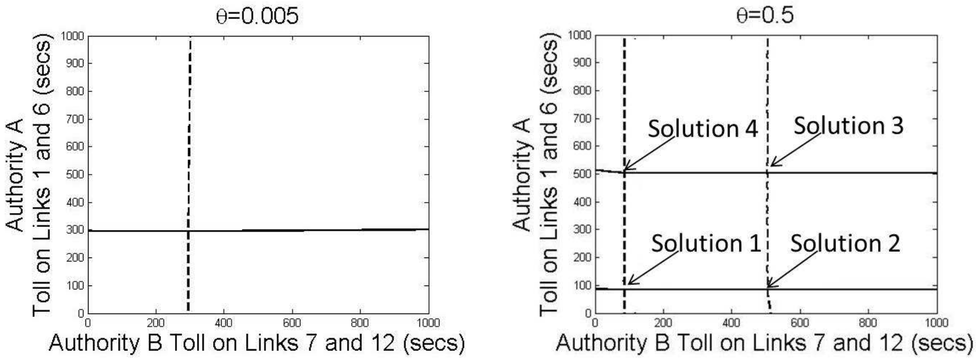


FIGURE 9.1: Left: Numerically estimated best response functions for both Authority A (continuous line) and Authority B (dashed line) $\theta = 0.005$ with a single intersection (1 NE). Right: Numerically estimated best response functions for both Authority A (continuous line) and Authority B (dashed line) $\theta = 0.5$ with 4 intersections (4 LNE)

are four points of intersection, therefore four LNE solutions, and it can be verified (by checking the welfare levels at each LNE) that there is one NE solution, which is the one marked "Solution 3" in the right panel of Fig. 9.1

9.3.2 Solution by SLCP Algorithm

Firstly, a grid search was carried out for a large range of θ to estimate the best response functions. This allows an identification of the number of potential LNE. Next, by starting SLCP relatively close to each of the LNE identified through the grid search mentioned above, SLCP was applied to determine the LNE solutions corresponding to the nearest starting point. Tables 9.1 present the LNE solutions (either a single LNE or all 4 where they exist) and the impacts on jurisdictional welfare relative to the no toll base equilibrium.

With competition between jurisdictions, it can be seen that welfare improves as θ increases since more users are assigned to the cheaper town centre route, avoiding the more costly bypass. Table 9.1 also shows that for $\theta \geqslant 0.02$, the NE solution (Solution 3) did not vary with θ, and this coincides (as may be anticipated) with the solution found under DUE as reported in Chapter 8.

From Table 9.1, there are two types of LNE solutions, symmetric (i.e. equal toll for both jurisdictions) and asymmetric. In the asymmetric solutions, the highest welfare improvement (or in some cases, least bad deterioration in welfare) is obtained for the city charging the higher toll, for all θ. In all cases where multiple LNE exist, the total welfare change (given by the sum of welfare change in A and welfare change in B) for both Solutions 2 and 4 is lower than that attainable at Solution 1, while all three of these solutions are preferable to the NE (outcome shaded in grey in Table 9.1) on total welfare grounds.

In terms of numerical implementation, a question of interest is how SLCP performs in locating the NE (Solution 3 if there are multiple NE) rather than each of the arbitrary LNE solutions. In order to understand this, a grid of initial conditions between 0 and 1000 secs in steps of 10 was generated for each jurisdiction, thereby generating 10201 distinct start points for application of the SLCP algorithm. The performance of SLCP in locating each of these LNE solution is reported in Table 9.2.

Table 9.2 shows that when $\theta = 0.02$, the solutions found by SLCP were distributed evenly among each of the four LNE. This suggests that for this case, if the initial starting point was chosen randomly, there is almost equal probability of locating each of the LNE solutions. In contrast, for the case of $\theta = 1$, 98% of the solutions converged to the Solution 3, the NE solution while Solution 1 was found in 0.02% of these cases. This suggests that the performance of SLCP algorithm in this respect is therefore sensitive to the value of θ.

To understand why this is the case, the welfare changes for Authority A as the toll it sets varies, assuming that Authority B sets the NE (Solution 3) toll, are plotted in Figs. 9.2 and 9.3. These figures show how changes in the logit dispersion parameter θ change the shape of the welfare function and highlights the transition from multiple optima to one with a single optimum.

It can be seen that when $\theta = 0.5$ (left panel of Fig. 9.2), the welfare function exhibits an obvious peak at the NE solution with the LNE solution being quite distinctly a local optima. On the other hand, the global optimum of the welfare function corresponding to the NE solution is less obvious when $\theta = 0.02$ (right panel of Fig. 9.2). It is clear then that while SUE smooths the problem, it also makes the local optima of the welfare function less distinct and not easily distinguishable from the

Table 9.1: Tolls (secs) and Welfare Change by Authority (secs) corresponding to LNE Solutions, NE solutions indicated by shaded cells

	LNE Solution 1			**LNE Solution 2**				**LNE Solution 3**			**LNE Solution 4**			
θ	x_{A} $(=x_{\mathrm{B}})$ (secs)	Welfare Change A (secs)	Welfare Change B (secs)	x_{A} (secs)	x_{B} (secs)	Welfare Change A (secs)	Welfare Change B (secs)	x_{A} $(=x_{\mathrm{B}})$ (secs)	Welfare Change A (secs)	Welfare Change B (secs)	x_{A} (secs)	x_{B} (secs)	Welfare Change A (secs)	Welfare Change B (secs)
0.005	295.4	-41,845	-41,845											
0.007	257.0	-33,459	-33,458					SINGLE LNE=NE						
0.009	231.6	-28,190	-28,190											
0.01	221.7	-26,157	-26,156											
0.02	159.3	-12,691	-12,690	498.8	165.2	-10,880	-103,358	497.6	-98,661	-98,660	165.2	498.8	-103,358	-10,880
0.03	123.8	-3,560	-3,560	505.2	127.1	5,680	-105,498	504.0	-93,868	-93,868	127.1	505.2	-105,498	5,680
0.04	107.9	913	913	504.0	107.9	14,265	-105,168	504.0	-91,416	-91,416	107.9	504.0	-105,168	14,265
0.5	85.6	9,646	9,645	504.0	85.6	23,822	-102,092	504.0	-87,843	-87,843	85.6	504.0	-102,092	23,822
1	85.4	9,372	9,372	504.2	85.3	23,960	-102,475	504.0	-87,843	-87,843	85.3	504.2	-102,475	23,960

TABLE 9.2: Percentage of runs with 10201 alternative starting points converging to each LNE solution

θ	Solution 1	Solution 2	Solution 3	Solution 4
0.02	27.36%	24.86%	23.08%	24.70%
0.05	0.54%	6.54%	86.31%	6.62%
0.5	0.08%	2.98%	94.61%	2.33%
1	0.02%	0.99%	98.30%	0.69%

global optima. Thus the basin of attraction around this local optimum is larger with a lower θ which implies that SLCP is not easily able to distinguish between these two solutions. Furthermore, the left and right panels of Fig. 9.3 show that as θ is further reduced, the welfare plots also display a single optimum which corresponds to the case where there is only one NE.

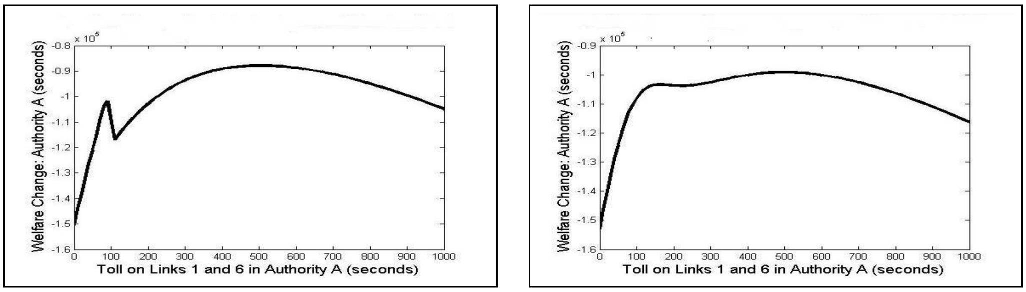


FIGURE 9.2: Left: Welfare Change for Authority A as toll it sets varies (θ=0.5) assuming Authority B sets the NE toll of 504.0 Right: Welfare Change for Authority A as toll it sets varies (θ=0.02) assuming Authority B sets the NE toll of 497.6.

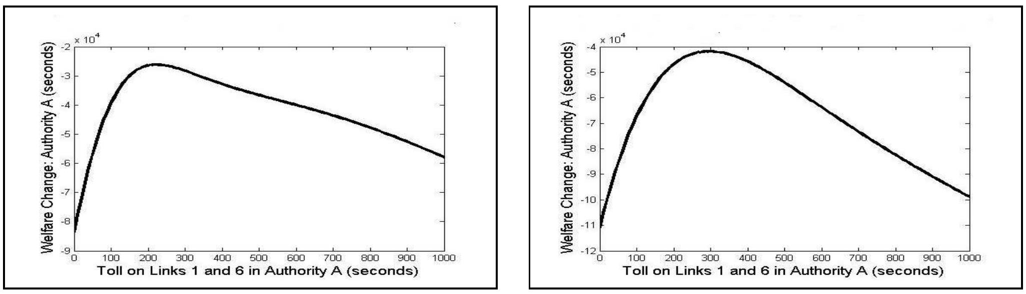


FIGURE 9.3: Left: Welfare Change for Authority A as toll it sets varies (θ=0.01) assuming Authority B sets the NE toll of 221.7(secs) Right: Welfare Change for Authority A as toll it sets varies (θ=0.005) assuming Authority B sets the NE toll of 295.4 (secs)

9.3.3 Comparison of LNE solutions with Solution to GRP

As noted above, four LNE were found to exist for $\theta \geqslant 0.02$. In such cases, some LNE solutions result in positive changes in welfare for both jurisdictions relative to the no toll base equilibrium. However, as noted above, there is a NE solution that results in the highest overall negative welfare change because when jurisdictions aim to extract revenues from extra-jurisdictional users, they are both simultaneously incentivised to set a high toll. Thus, to avoid this situation, it may be necessary to regulate the tolls whereby a regulator institutes tolls with the aim of maximising total welfare. In this setting, a single regulator rather than the individual jurisdictions determines both cordon tolls with the aim of maximising total welfare and this leads to the GRP (see Eq. 9–4). Due to the symmetry of this example, the tolls obtained as a solution to the GRP are identical and each city's welfare change is equal to half the total (i.e. network wide) welfare change.

TABLE 9.3: Tolls (secs) and Total Welfare Change (secs) obtained as solutions to GRP

θ	Toll A (secs)	Toll B (secs)	Total Welfare Change (secs)
0.005	61.19	61.19	9,835
0.007	60.12	60.11	10,556
0.009	59.27	59.27	11,333
0.01	59.19	59.19	11,740
0.02	58.79	58.79	15,522
0.03	60.07	60.08	18,441
0.04	61.87	61.88	20,404
0.5	79.00	78.99	21,114
1	79.38	79.38	20,705

Table 9.3 shows the solution of the GRP. Comparing Table 9.1 with Table 9.3, it is evident that the total welfare change (i.e. sum of Welfare Change A and Welfare Change B) at the Solution 1 LNE moves further away from the GRP solution as θ is decreased (users becoming less cost sensitive). As θ decreases, the tolls charged in the Solution 1 LNE become higher (and are always higher than the tolls obtained as the solution of the GRP) and total welfare decreases. For the cases where there is only one LNE/NE (θ <0.02), this suggests that in this network, there is no solution for the Nash game which is "close" to the GRP solution. Thus as

dispersion increases (i.e. a smaller θ) then under competition, there is no possibility of achieving a positive welfare change, and a regulator would be required to obtain this. Furthermore, with decreasing θ, as users are assumed to be less cost sensitive in terms of their route choices and decisions to travel, competition between cities would result in greater dis-benefits for both cities, with higher tolls imposed on the users.

For cases with higher θ, the Solution 1 LNE is seen to be closer to the regulator solution, and so there may be a greater chance that authorities in such a case would accept regulation. However, as the gap between the GRP solution and the Solution 1 LNE increases with a decrease in θ, then there is a greater need for regulation of the authorities. It is also of interest to note that as θ decreases even further, that the welfare to be gained even under the GRP will also decrease. This may suggest that if users are less cost-sensitive, then there is less benefit to be gained from toll pricing in general, as would be expected.

9.4 Example 2: Grid Network

Fig. 9.4 shows the network with 20 nodes and 62 links from Zhang *et al.* (2011) that is used as the second example. Each link j, $j \in \mathcal{L}$ in this network has a travel time function of the form given by Eq. 9–15.

$$t_j = t_j^0 + \beta_j (v_j/\kappa_j)^4 \qquad \text{(Eq. 9–15)}$$

where t_j^0, β_j and κ_j are the free flow travel time, coefficient and capacity for link j,$j \in \mathcal{L}$. The travel time parameters of this network are uniform throughout but differ depending on whether they are vertical or horizontal links (Zhang *et al.*, 2011). Each horizontal link, such as link 1, connecting nodes 1 and 2, has a free flow time t_j^0 of 2.5 mins, β_j of 0.375 and a capacity, κ_j, of 700 pcus per hour. On the other hand, vertical links such as link 34 connecting nodes 1 and 6, has t_j^0 of 5 mins, β_j of 0.75 and κ_j of 1000 vehicles per hour. Note that tolls will be reported in the same units as travel time i.e. in minutes. The broken line down the centre of

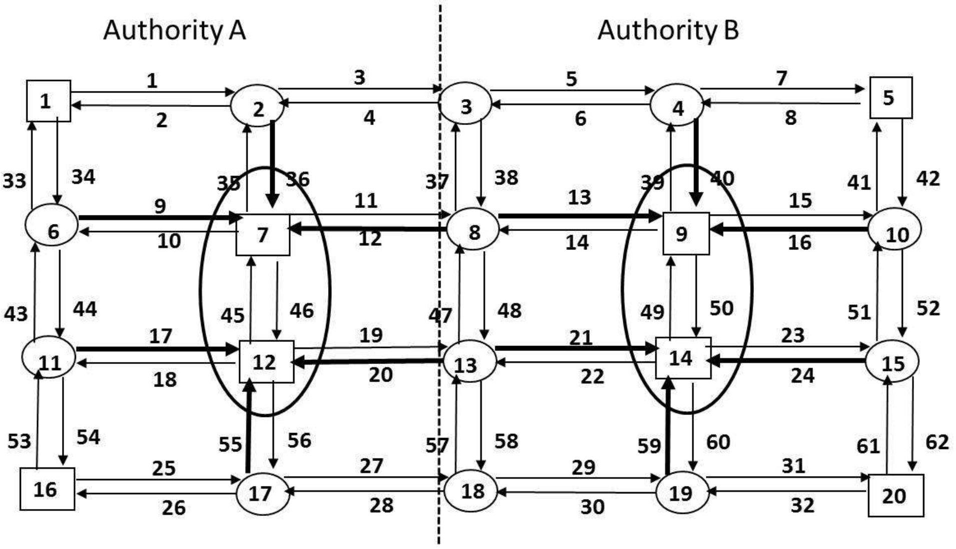


FIGURE 9.4: Grid Network from Zhang *et al.* (2011). Rectangular nodes represent origins and destinations while circular nodes only serve to connect links in the network. The dotted line demarcates the separation of the authority. The thicker lines in each authority represent the tolled links that form a closed cordon.

the network demarcates the boundaries of the authorities such that the subnetwork located to left of this line contains the jurisdiction of Authority A. The subnetwork of Authority B is rightward of this line.

While the network parameters follow that as reported in Zhang *et al.* (2011), the demand characteristics differ. The demand function adopted is based on the power law demand function in Eq. 9–16 with the elasticity of demand η_k set to -0.58 for all OD pairs, consistent with parameters used previously.

$$q_k = q_k^0 \left(s_k / s_k^0\right)^{\eta_k}, \quad k \in \mathcal{K} \qquad \text{(Eq. 9–16)}$$

The centroids are shown in Fig. 9.4 as rectangular boxes. Two zones constitute the Central Business District (CBD) in each authority (Zones 7 and 12 in A and Zones 9 and 14 in B). There are also two residential suburbs in each authority (Zones 1 and 16 in A and Zones 5 and 20 in B). Furthermore, there are trips between the suburbs as well as within the CBD zones representing through traffic. In total, there are 60 OD pairs. Details of the base matrix, intended to reflect a typical AM peak commuting flow pattern, and the associated base composite generalised travel times for $\theta = 10$ are shown in Appendix E in Tables E.1 and E.2, respectively. The base matrix is entirely symmetric for both authorities. The dominant movement is from the suburbs of each authority to the its own CBD comprising 1000 pcus/hour. Furthermore, there is demand from all suburbs in either jurisdiction to the other jurisdiction's CBD of 500 pcus/hr.

The toll cordon for Authority A is around its two CBD zones (7 and 12). In this way, links 9, 12, 17, 20, 36 and 55, distinguished by the thick lines in Fig. 9.4, form the set of tollable links in A that will be subject to a single toll. Similarly, Authority B is assumed to set a single toll on links 13, 16, 21, 24, 40 and 59 (also distinguished by the thick lines) so as to form a closed cordon around its CBD (Zones 9 and 14).

9.4.1 Solution of Global Regulator Problem

Table 9.4: GRP Tolls, Total and Jurisdictional Welfare

	Jurisdiction Level			Network Wide		
θ	Toll A (=Toll B) (mins)	Welfare Change A (mins)	Welfare Change B (mins)	Total Welfare Change (mins)	First Best Welfare Gain (mins)	ω
0.2	16.81	51,431	51,417	102,849	175,265	0.59
0.4	28.33	37,534	37,546	75,080	137,454	0.55
0.6	28.05	36,857	36,863	73,720	130,093	0.57
0.8	28.25	36,688	36,688	73,376	127,266	0.58
1	28.18	36,619	36,619	73,237	125,722	0.58
10	28.26	36,099	36,099	72,197	120,557	0.60

The penultimate column of Table 9.4 shows the welfare gain obtainable with marginal cost tolls on all links for the range of θ considered. This gives the maximum possible welfare gain forming the theoretical first best pricing benchmark.

For each θ, Table 9.4 reports the solution of the GRP. Due to symmetry, the GRP tolls for each cordon in either jurisdiction are equal. The last column of this table gives the index of relative welfare improvement, ω, computed according to Eq. 6–15 which reports the extent of welfare improvement of each pricing scenario studied relative to the theoretical first best benchmark.

It is noticeable that the optimal tolls (in mins) in the GRP are much higher in this example than in Example 1 of the previous section, despite using the same demand elasticity, similar free flow travel times and the same functional form for link travel time. To understand this, it is helpful to distinguish between captive trips and non-captive trips. Captive trips are those that have destinations within the tolled zones (i.e. CBDs of either authority) and are therefore captive to any cordon toll introduced. On the other hand, an example of a non-captive trip is the movement from Zone 16 to Zone 5; such trips have a choice of avoiding the tolled links. For $\theta = 10$, in the untolled base equilibrium, approximately 120 (out of 200) trips from this particular OD pair utilise a route that includes one or more of the tollable links. With the GRP toll, captive trips are suppressed. While trip suppression is also present in Example 1, the difference is that Example 2, the captive and non-captive trips share untolled entry links. The effect of this is that in this example, as the captive trips are suppressed, the non-captive trips benefit substantially from congestion relief.

Because of these shared entry links, as captive trips are suppressed, non-captive trips experience a reduction in costs because of the freeing up of untolled entry links. Since elastic demand is used, the number of these non-captive trips *increases* compared to the untolled base equilibrium for each θ. This increase in the non-captive trips, along with the decongestion benefits, then counter the suppression of the captive trips and so a greater welfare gain is possible with a much higher toll. Note that even with lower demands than the modified matrix used here and with different tollable links, Zhang *et al.* (2011) also reported tolls of a similar order of magnitude (between 9 and 30 minutes) in their equivalent of the GRP.

Note that this effect was not observed in Example 1 because the introduction of a toll only results in non-captive trips rerouting around the more costly bypass, and as the toll increases, more trips are routed onto the bypass so that the costs faced by non-captive trips is always increasing with tolls.

A second noticeable feature of Table 9.4 is that the toll level is much lower when $\theta = 0.2$ being 16 mins versus a value around 28 mins for higher θ. In general, as θ decreases users are less sensitive to costs, and so for any given toll level, the suppression achieved is lower with a lower θ. At the same time, this also means that the "indirect generation" effect caused by the freeing up of untolled entry links

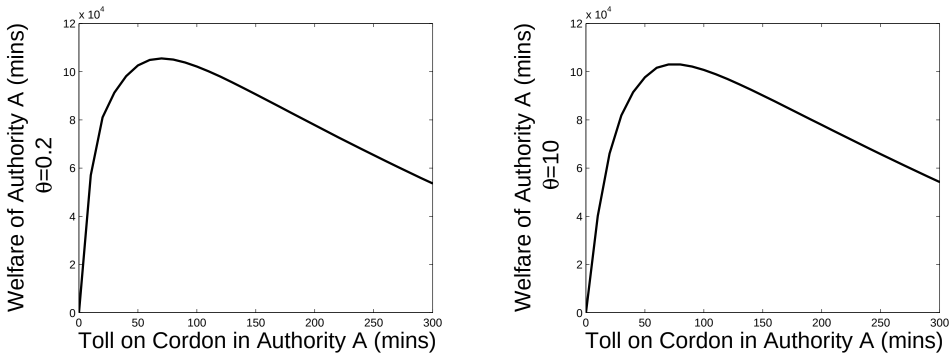


FIGURE 9.5: Welfare Change for Authority A, as own cordon toll varies, assuming Authority B sets no toll on cordon in B), for two different logit dispersion parameter, θ. Left: $\theta = 0.2$. Right: $\theta = 10$

will also be lower. In addition to the generation effect, there is a noticeable change in use of paths for certain OD pairs as users become less cost sensitive, and are more dispersed across the available routes. This dispersion of traffic across paths combined with a lower demand response affects different elements of each authority's welfare function, so that a relatively lower toll becomes optimal with a lower θ. In contrast, with a higher θ, because users become more sensitive to costs, more captive OD pairs are suppressed for a given toll level. This also implies that there is more freeing up of shared entry links. Essentially, the trade-off between benefits arising to different OD pairs within the welfare function varies as the demands and route choices vary with changes in users' sensitivity to cost.

9.4.2 Incentives to Engage in Inter-jurisdictional Game

Consider now the situation when the jurisdictions retain the full revenue from tolls or the case of no revenue sharing. Thus the analysis in this section will assume that $\delta = 1$, equally that revenues raised from extra-jurisdictional users add to local welfare. The left panel of Fig. 9.5 show the welfare of Authority A as the toll it sets varies for the case of $\theta = 0.2$, assuming that Authority B sets a toll of 0. Thus, assuming Authority B does not toll, Authority A finds that it can obtain higher jurisdictional welfare by introducing a toll. The right panel gives the same illustration for the case of $\theta = 10$. Due to symmetry, the same can be said for Authority B (assuming that Authority A does not toll). Thus, both panels of Fig.

TABLE 9.5: NE Tolls and Welfare Change with Inter-jurisdictional Competition

θ	Toll A (=Toll B) (mins)	Jurisdiction Level Welfare Change A (mins)	Welfare Change B (mins)	Aggregate Level Welfare Change (mins)	ω
0.2	65.52	10,590	10,561	21,151	0.12
0.4	66.27	4,678	4,677	9,355	0.07
0.6	67.31	3,306	3,306	6,613	0.05
0.8	68.03	2,641	2,641	5,281	0.04
1	68.47	2,270	2,270	4,540	0.04
10	69.89	1,125	1,125	2,250	0.02

9.5 confirm that based on the potential for increased jurisdictional welfare, there is an incentive for either authority to introduce a toll and begin the game.

9.4.3 Identification of LNE with SLCP

As the incentives to engage in inter-jurisdictional competition exists, the SLCP algorithm was applied to this problem to determine the NE toll levels. Table 9.5 reports the results.

Figs. 9.6 to 9.8 shows the numerically estimated best response functions for both authorities. In these figures, the best response function of Authority A to any toll level of Authority B is indicated by the continuous line. Similarly the best response function for Authority B to any toll level of Authority A is indicated by the broken lines. As noted in Chapter 3, the NE is the intersection of these best response functions. These figures serve to numerically verify that the solution reported in Table 9.5 obtained by SLCP coincides with the intersection of these best response functions. These figures show that these best response functions are negatively sloped which is similar to the case of toll road competition between *serial* links as discussed in Chapter 7 (see Section 7.4.3, p. 201). This is indicative of complementary linkages between the two jurisdictions.

The NE tolls in Table 9.5 are nearly three times higher compared those obtained as solutions to the GRP in Table 9.4. Thus the welfare change for each jurisdiction while positive, is far lower than under the GRP. The last column of Table 9.5 reports

ω, i.e. the index of relative welfare improvement measuring the welfare change under competition relative to first best pricing. In the case of $\theta = 10$, ω reduces from close to 0.6 under GRP to around 0.02 under competition. While the welfare change under competition is still positive (indicating that positive welfare gain can still be obtained), there is a Prisoners Dilemma outcome since both jurisdictions are worse off than if they had *cooperated* e.g. allowing an independent regulator to maximise welfare for both jurisdiction as modelled by the GRP. Compared to Example 1 and the example in Chapter 8, it can be seen in Example 2 that while welfare is reduced as a result of the fiscal externality of tax exporting, it does not have as adverse an impact as to offset the positive welfare impacts brought about by the use of tolls to internalise congestion.

The higher tolls obtained is due to a combination of both the over-internalisation of externalities as well as revenue maximising/tax exporting incentive of each jurisdiction. On the former (over-internalisation) issue, the double marginalisation problem, arises because in a serial setting, each authority does not take into account the reduction in revenues to the other authority when setting its toll. This reflects research presented in Chapter 7 on competition between toll road concessionaires. On the latter (revenue maximising) issue, this arises since extra-jurisdictional users who travel on a jurisdiction's road network contribute only to the toll revenue component in each jurisdiction's objective function. Such a finding thus extends a conclusion from the literature, notably De Borger *et al.* (2007) and Ubbels and Verhoef (2008*b*) to a more general network setting under SUE, and is line with findings in Zhang *et al.* (2011) and the previous results presented in Chapter 8.

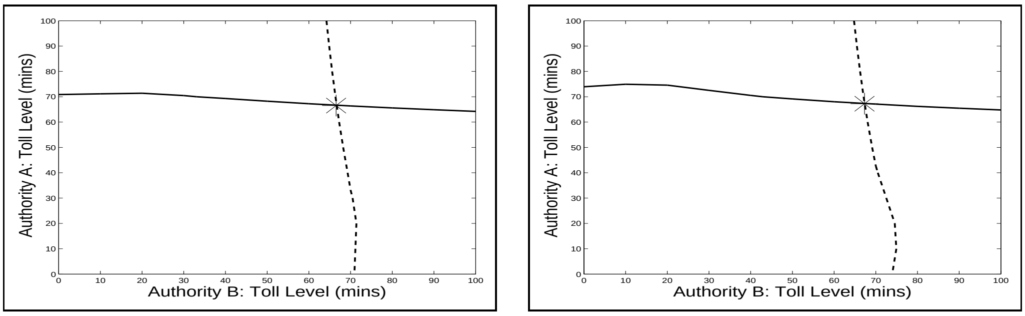


FIGURE 9.6: Left: Intersection of numerically estimated best response functions for $\theta = 0.2$ coincides with the cordon toll obtained by SLCP, of 65.52 mins for both authorities A and B. Right: Intersection of numerically estimated best response functions for $\theta = 0.4$ coincides with the cordon toll obtained by SLCP, of 66.27 mins for both authorities A and B.

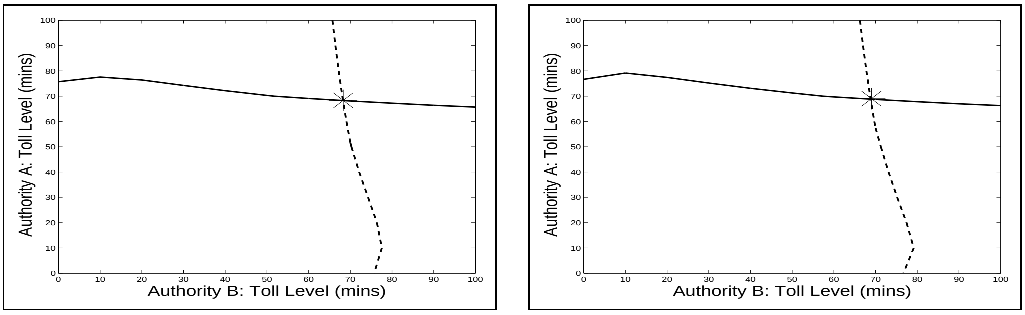


FIGURE 9.7: Left: Intersection of numerically estimated best response functions for $\theta = 0.6$ coincides with the cordon toll obtained by SLCP, of 67.31 mins for both authorities A and B. Right: Intersection of numerically estimated best response functions for $\theta = 0.8$ coincides with the cordon toll obtained by SLCP, of 68.03 mins for both authorities A and B.

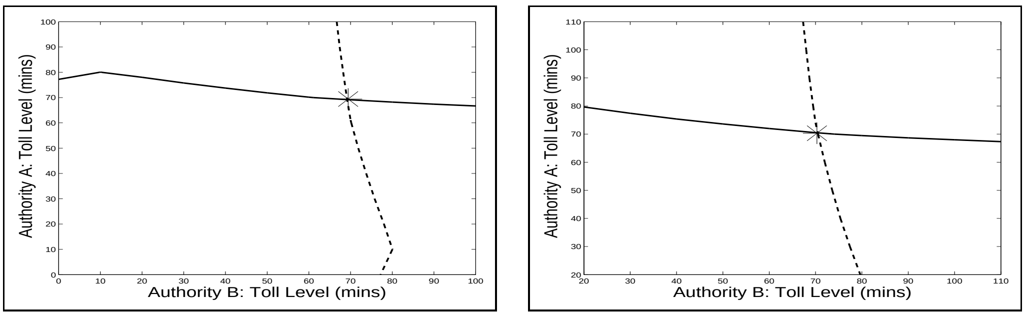


FIGURE 9.8: Left: Intersection of numerically estimated best response functions for $\theta = 1$ coincides with the cordon toll obtained by SLCP, of 68.47 mins for both authorities A and B. Right: Intersection of numerically estimated best response functions for $\theta = 10$ coincides with the cordon toll obtained by SLCP, of 69.89 mins for for both authorities A and B.

9.5 Revenue Sharing

In the results presented so far, attention has been restricted to the case of no revenue sharing (i.e. $\delta = 1$). In this section, the analysis is focused on how changes in δ affect the NE toll levels and the resulting welfare implications. Focusing on the network used in Example 2 as shown in Fig. 9.4, extensive grid searches confirm that a single LNE exists for all combinations of δ and θ reported here, regardless of the starting point of the SLCP algorithm. Thus it is appropriate to refer to this solution as "the NE solution".

Table 9.6 shows the NE tolls as the revenue sharing parameter δ is varied for three representative values of dispersion parameter θ alongside the welfare change over the network. Note that the results for $\delta = 1$ corresponds to the results reported, for equivalent θ, in Table 9.5. The left panel of Fig. 9.9 shows the variation in the NE toll graphically. The right panel shows the index of relative welfare improvement ω as the revenue sharing parameter δ varies. This is elaborated further below.

Table 9.6: Tolls and Welfare Change as Revenue Sharing Parameter, δ varies

Revenue Sharing Parameter δ		Dispersion Parameter θ=0.2	θ=0.4	θ=0.8
0	Toll A=Toll B (mins)	11.12	7.86	6.53
	Total Welfare Change (mins)	93,512	58,276	45,621
0.2	Toll A=Toll B (mins)	14.57	11.09	21.11
	Total Welfare Change (mins)	101,574	65,525	69,109
0.4	Toll A=Toll B	18.47	30.04	30.03
	Total Welfare Change	102,315	74,704	73,075
0.6	Toll A=Toll B (mins)	37.48	39.52	39.85
	Total Welfare Change (mins)	79,649	66,325	64,691
0.8	Toll A=Toll B (mins)	50.57	50.88	51.77
	Total Welfare Change (mins)	56,524	46,033	43,668
1	Toll A=Toll B (mins)	65.52	66.27	68.03
	Total Welfare Change (mins)	21,151	9,355	5,281

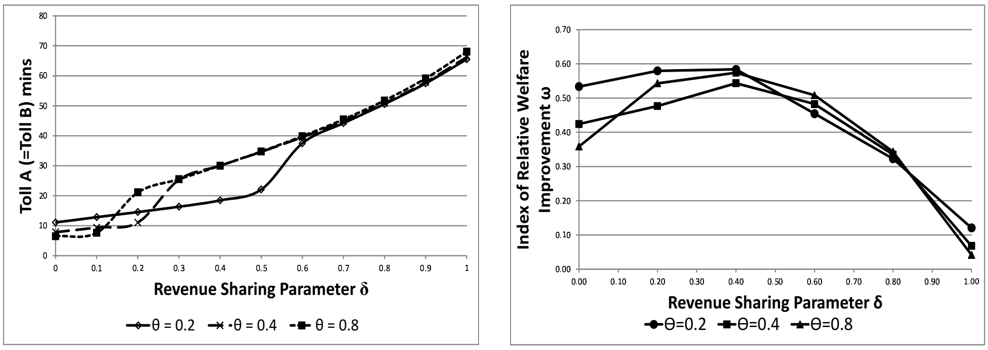


Figure 9.9: (Left): NE Toll Levels (mins) and (Right): Index of Relative Welfare Improvement ω as Revenue Sharing Parameter (δ) varies

To understand the results, consider the two extreme points of $\delta = 0$ (full revenue sharing) and $\delta = 1$ (no revenue sharing). When $\delta = 0$, the NE tolls are lower than that obtained under as the solution of the GRP. This can be seen by comparing each θ for $\delta = 0$ against the corresponding solutions to the GRP reported in Table 9.4. This is in contrast to the results presented in Chapter 8 and the reason for this will be discussed further below (see Section 9.5.1).

Next, consider the case of $\delta = 1$. In this case, the highest tolls should be expected since the authority retains *all* revenues from tolls including those collected from extra-jurisdictional users. Given these two points, it would then be expected that as δ increases from 0 towards 1, the NE tolls should increase. An increase in δ increases the revenue retained by the authority, and in the extreme case of no revenue sharing, the tolls are highest. The results in Table 9.6 confirm this expected phenomenon, and this continues to hold for each of the θ values tested.

An additional effect is that as the toll levels rise with an increase in δ, the number of paths effectively utilised by an OD pair decreases; that is to say, although all paths are used at least a small (non zero) amount at any value of δ due to the nature of the underlying SUE assignment model, the number that are used by a significant amount of traffic decreases. As the tolls increase, analysis of the path flows shows that with an increase in δ, non-captive OD pairs begin to avoid using the tolled links by rerouting away from these less attractive tolled links. This explains the apparent change in toll regime in Fig. 9.9, which notably appears at different levels of δ for different dispersion parameters θ. That is to say, sensitivity to costs (as controlled by θ) has a complex interaction effect with the assumption on revenue sharing/tax exporting (as controlled by δ), at the NE solution.

Where the phenomenon of a change in the toll regime precisely occurs depends on both θ and δ, and this is difficult to predict *a priori*. But in general, as θ increases, because users become more sensitive to costs, this would be expected to take place at lower values of δ. For example, for $\theta = 0.2$, there is a noticeable "break" at $\delta = 0.6$ and an NE toll of 37.48 mins compared to the NE toll of 18.47 mins at $\delta = 0.4$. However, when users become more cost sensitive this "break" occurs earlier. A noticeable change can be observed in the left panel of Fig. 9.9 in the case of $\theta = 0.8$ of between $\delta = 0$ and $\delta = 0.2$.

If in the competitive case, some higher-level regulatory authority were able to influence the revenue sharing agreement through δ (given knowledge of θ), so as to achieve the highest total welfare gains, then the "optimum" value occurs around $\delta = 0.4$ for all cases of θ. This is shown in the right panel of Fig. 9.9 which shows the trend in the index of relative welfare improvement ω (Verhoef *et al.*, 1996) measuring the overall welfare gains as the revenue sharing parameter δ varies (relative to the theoretical first best benchmark for given θ reported in the penultimate column of Table 9.4). The welfare gains achieved in such a way seems to be close to the benefits achieved by the GRP (for equivalent θ) reported in Table 9.4.

It should be recognised that the welfare that is obtained close to that with GRP tolls when $\delta = 0.4$ is most likely network-specific and only reflects the fact that there is a trade-off being made by the local decision makers with respect to congestion impacts on their own residents (the first three terms in Eq. 9–8) and revenues from neighbours (the last two terms in Eq. 9–8). This implies that for this network at least, if indeed a revenue sharing policy were to be pursued, some degree of revenue sharing resulting in tolls close to the GRP solution could be better for welfare than full revenue sharing. Therefore, in any given network, any overall regulator of jurisdictions would need to evaluate NE solutions for varying δ, in order to determine an optimal level of revenue sharing/cooperation for the region as a whole.

9.5.1 Pareto Inefficiency of Full Revenue Sharing

Consider the situation in which the authorities could cooperate. In this case, the problem under consideration can be represented as the MOPEC in Eq. 9–17.

$$\begin{aligned} &\underset{\mathbf{x}\in X}{\text{Maximise}} && \mathbf{\Phi(x)} = (\phi_{\mathrm{A}}(\mathbf{x}), \phi_{\mathrm{B}}(\mathbf{x}))^{\intercal} \\ &\text{subject to} && \{\mathbf{v}^*(\mathbf{x}), \mathbf{q}^*(\mathbf{x})\} \leftarrow \mathrm{SOL}\{\mathcal{S}(\mathbf{x})\}. \end{aligned} \qquad \text{(Eq. 9–17)}$$

The solutions of this MOPEC should satisfy the principle of Pareto Optimality given in Definition 3.3. In this context, Pareto Optimality implies that that no jurisdiction should be able to be improve its welfare without reducing the welfare of the other.

As was done in previous chapters, the MOSADE algorithm is applied to obtain the Pareto Fronts as the solution to the MOPEC in Eq. 9–17. These are shown in Fig. 9.10 for the case of $\theta = 0.4$ (left panel) and $\theta = 0.8$ (right panel). The jurisdiction level welfare obtained as a solution to the GRP problem (see Table 9.4) is also plotted on each of these figures. It can be seen in each case that this individual jurisdictional welfare lies on the Pareto Front.

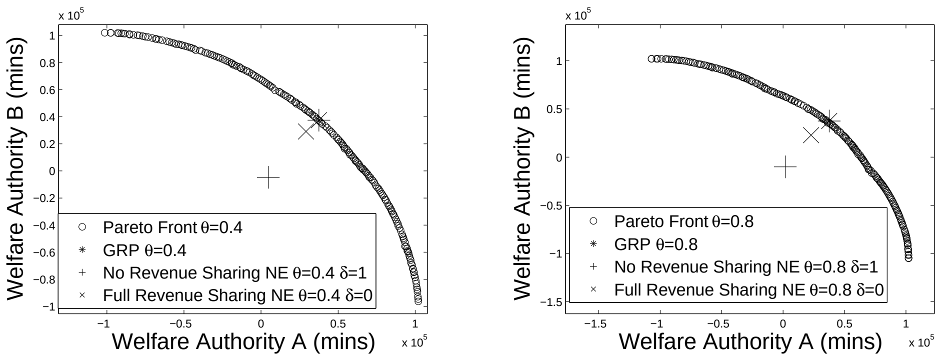


FIGURE 9.10: Left: Cooperation between Authorities in Example 2: Illustration of Pareto Front alongside authority welfare for $\delta = 1$ and $\delta = 0$ with welfare under GRP. Left: Case of $\theta = 0.4$. Right: Case of $\theta = 0.8$

Next, the welfare to each jurisdiction assuming no revenue sharing i.e. $\delta = 1$ is also shown on these figures (indicated on each panel of Fig. 9.10 with $+$). It is clear that this point lies in the *interior* of these Pareto Fronts. This means that at least one jurisdiction could be made better off without making the other worse off. The reason this solution lies inside the Pareto Front is, as discussed above, due to the welfare decreasing effect of the revenue objective in each authority's welfare function i.e. tax exporting.

Finally, the welfare to each jurisdiction assuming full revenue sharing or equivalently, no tax exporting is also plotted on these figures (i.e $\delta = 0$, indicated on each panel of Fig. 9.10 with $\times$). It is evident that jurisdictional welfare in this case, also lies in the *interior* of these Pareto Fronts. This is contrasted with the equivalent problem investigated in the network discussed in the last chapter (see Fig. 8.16 in Section 8.6.1) where the revenue sharing outcome in that case was on the Pareto Front. However, that particular example was a special case due to the *limited* interactions between users from either jurisdiction particularly in the high toll regime.

Returning to the example at hand, while demand is symmetric between the jurisdictions, the difference is that, in each jurisdiction, there are more extensive interactions between local and extra-jurisdictional users. The extensive interactions is brought out by analysing the path flows in the competitive tolled equilibrium with $\delta = 0$. In the case of $\theta = 0.4$, with tolls at the GRP toll level, only 12 of the 62 links (approximately 20%) are exclusively used by traffic originating from one authority. In other words, 80% of links are used by traffic from both authorities.

However, even when tax exporting is eliminated with $\delta = 0$, each jurisdiction is only concerned with the welfare of its own users (see Eq. 9–8). As a result, they fail to take into consideration the fact that on the 80% of links that carry traffic from both authorities, extra-jurisdictional users also suffer from congestion. Since this spillover is disregarded in the jurisdiction's toll setting decision, the toll is lower than the GRP toll.

As pointed out in the previous chapter, it is possible to allow jurisdictions to engage in bilateral bargaining in order to correct the inter-jurisdictional spillovers. The NBS solution of the bilateral bargaining problem can be obtained by solving the NBPEC for which again, due to the smootheness of the SUE constraints, can be solved by directly embedding the SUE conditions as constraints into the non-linear programming solver.

Superimposed on the Pareto Fronts for $\theta = 0.4$ and $\theta = 0.8$ in the left and right panel of Fig. 9.11 respectively, are the jurisdictional welfares obtained by the NBS, plotted with the ☆ markers as well as the GRP (Utilitarian) solutions. However, in this example, because of the symmetry of the network and the base demands, the tolls obtained by bilateral Nash bargaining are not materially different (to 3 decimal places) from that obtained under the GRP solution as reported in Table 9.4. This conclusion is invariant to the choice of BATNA assumed (i.e. whether there is revenue sharing should negotiations fail). Nevertheless, this example verifies once again that Pareto Optimal outcomes are attainable with the NBS, unlike full revenue sharing.

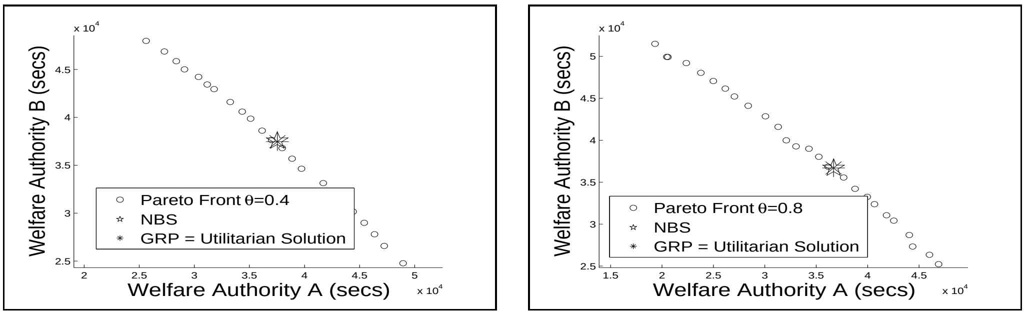


Figure 9.11: Pareto Front, GRP, NBS(I), NBS(II)and BATNA with and without revenue sharing Left: $\theta = 0.4$. Right: $\theta = 0.8$

9.6 Summary and Policy Implications

This chapter has investigated toll competition between jurisdictions on the assumption that users route according to the SUE route choice principle. This toll competition problem can be formulated as a NCEPEC where at the upper level, each authority aims non-cooperatively to maximise social welfare of their residents by introducing tolls in its subnetwork, anticipating the reactions of travellers while reacting to the tolls set by the other authority.

In the case where there was no revenue sharing, jurisdictions were assumed to focus on the twin objectives of maximising welfare for their residents and maximising the toll revenues from extra-jurisdictional users. In competition with others for toll revenues, they would set tolls higher than the GRP level. The resulting effect in physically adjacent regions (modelled as networks with serial dependencies) is exacerbated by the double marginalisation (Spengler, 1950; Economides and Salop, 1992) problem. The results have shown that horizontal competition between jurisdictions for toll revenue could result in welfare losses that significantly offset the welfare gains from employing tolls to internalise congestion. In the worst case, as illustrated in Example 1 (and in the previous chapter), toll competition could result in lower total welfare than if there were no tolls at all.

As was done in the previous chapter, revenue sharing was modelled by assuming that each authority returned a proportion of the revenues collected from extra-jurisdictional users to the counterpart authority. The results presented in Example 2 show that even with full revenue sharing, the solution obtained was not Pareto Optimal when there are extensive interactions between users from both jurisdictions. Revenue sharing was shown to reduce tax-exporting thereby alleviating welfare losses from this fiscal externality. However, revenue sharing is unable to address the failure of jurisdictions acting non-cooperatively to take into account the welfare of extra-jurisdictional users. Specifically, they ignore, in their toll setting decisions the transport specific congestion externality in that extra-jurisdictional users both contribute to as well as suffer from congestion when circulating in their subnetwork. Thus a potential policy implication of this research is that when pricing is decentralised to the local level, individual jurisdictions need to take into account the interactions of all users rather than solely that of local users.

As highlighted in Chapter 2 (see p. 59), a key question of interest is the appropriate level of governance of toll pricing policies i.e. whether toll pricing policies should be regulated at the national level or whether it should be treated as a local traffic management issue with responsibilities devolved to local governments/jurisdictions. In view of the above findings, the results of the numerical tests suggest that a regulated environment for toll pricing, as represented by the GRP, would be most favourable in terms of social welfare compared to all revenue sharing arrangements considered.

However, this may not be the most preferred approach to the institutional governance of toll pricing such as concerns that a federal/national regulator might not sufficiently cater to local circumstances. This observation motivated the study of bilateral bargaining between jurisdictions modelled using the NBPEC encapsulating Nash Bargaining (Nash, 1950*b*). Due to the symmetry of the network and demands tested in this chapter, the Nash Bargaining Solution (NBS) yielded welfare levels close to that obtained by the solution of the GRP. Nevertheless, the numerical tests in reported in this (and the previous) chapter show that bilateral bargaining can yield Pareto Optimal outcomes. The policy implication here is that bilateral bargaining could facilitate a decentralised approach which would be more closely aligned to local interests without overlooking inter-jurisdictional spillovers.

The smoothness of the SUE formulation used in this chapter is exploited in two ways. Firstly, it facilitated the mathematical formulation of the toll competition problem between jurisdictions as a single level Complementarity Problem (CP). Secondly, it enabled application of the SLCP algorithm which was successfully applied to identify LNE points in the inter-jurisdictional game in the numerical examples. However, the numerical tests show that while SUE smooths the problem and assures differentiability, there is still the potential for the SLCP algorithm to miss the global peak and identify LNE rather than the NE.

Chapter 10

Conclusions and Further Research

10.1 Summary

From the point of view of economists, the theoretical argument for introducing pricing for road use in the form of tolls is that an unpriced highway network results in a welfare loss to society because highway users do not take into account the costs their use of the highway network imposes on others. At the same time, the literature has implicitly assumed that the transportation network is managed by a sole benevolent regulator keen on maximising welfare for all users of the network.

As highlighted in Chapter 2, in reality, transportation networks transcend jurisdictional boundaries and are managed by multiple governments. In implementing fiscal policies, governments generate fiscal distortions due to the competition for funds to finance public goods desired by their residents. In recognition of the desire to leverage private sector resources to overcome budgetary constraints faced by governments in their attempts to meet highway infrastructure requirements, the involvement of the private sector (which was assumed in this thesis to be synonymous with revenue maximising entities) in the operation of toll roads has increased around the world.

When these private sector participants have successfully obtained concessions for the development of toll roads, these concessionaires set tolls to maximise revenue. This objective then implies that the toll a concessionaire levies will be different from one introduced so as to maximise social welfare. In the face of competition between toll road concessionaires for toll revenues, the specific question of interest is the extent to which welfare losses stemming from the use of tolls aimed at maximising revenue outweigh the welfare gain from using tolls used to internalise the congestion externality.

As a consequence of the desire for revenues to finance public goods that benefit their residents, a growing literature reviewed in Chapter 2 has alluded to the possibility that local governments/jurisdictions may introduce toll pricing in order to compete with other jurisdictions for toll revenues. Furthermore, jurisdictions may not behave benevolently towards all users who traverse the transportation networks within their jurisdiction when a toll pricing policy is in place. While they are politically motivated to look after the welfare of their own residents, they may regard users from outside a jurisdiction, (referred to as extra-jurisdictional users in this thesis) as a source of revenue and as a source of congestion impeding local residents. In doing so, jurisdictions introduce a fiscal externality of tax exporting which reduces welfare compared to toll pricing solely aimed at imperfectly internalising congestion.

Therefore the aim of this thesis has been to investigate the extent of welfare losses stemming from competition for toll revenues *a*) amongst toll road concessionaires, *b*) and separately, amongst jurisdictions.

Since the research question revolves around the central theme of competition, regardless of who the specific players were, or the precise details of the game, Chapter 3 focused on the principles of oligopolistic non-cooperative game theory which is used as the tool to study how these players would interact when they are engaged in a strategic encounter with each other where each is pursuing his own objective. In this context, the Nash Equilibrium principle was identified as the appropriate solution concept for games but as recognised therein, there could be multiple Nash equilibria. Furthermore, it was also highlighted that if these players could act cooperatively rather than unilaterally pursuing self-interests, they could, in fact, obtain better outcomes for themselves.

Chapters 4 developed the mathematical modelling framework applied in the numerical examples. Following the literature, the model of a sole regulator was framed as a Stackelberg leader-follower game or Mathematical Program with Equilibrium Constraints (MPEC) where the actions of the regulator was constrained by the equilibrium condition in the lower level route choice model expressed as a variational inequality.

In tandem with the policy requirement to take into account the interactions amongst multiple agents involved in toll pricing decisions in transportation networks, Chapter 4 showed that the problem could be posed as an Equilibrium Problem with Equilibrium Constraints (EPEC) which directly extends the MPEC. In this formulation, the actions of all leaders implementing toll pricing decisions were inextricably linked as users respond to tolls set by all players in their DUE/SUE route choices. While the problem formulation is theoretically attractive for studying the problems posed in this thesis, the study into EPECs itself had only just begun in earnest recently and theoretically proven solution algorithms for such problems are not widely available. Thus in order to identify solutions that reflect the Nash Equilibria of the games (as well as solutions that reflect the Pareto Optimal benchmark), along with the recognition that analytically obtaining these were infeasible in larger networks, the research sought to identify a number of heuristic methodologies with this objective in mind. By adapting existing solution methods from microeconomics and the field of evolutionary computation, several algorithms for solving the EPEC formulation were suggested in Chapters 4 and 5. Subsequently, these algorithms were applied extensively to the case studies involving competition amongst toll road concessionaires and separately, amongst jurisdictions in Chapters 6 to Chapters 9.

The rest of this chapter is structured as follows. Following this overview, Sections 10.2 and 10.3 summarise the findings from the case studies involving competition amongst toll road concessionaires and amongst jurisdictions respectively. In these sections, policy implications are summarised. An evaluation of the performance of the algorithms that were proposed and applied in this thesis is presented in Section 10.4. Section 10.5 summarises the key policy messages and novel contributions of this thesis. Potential areas identified for further research are outlined in Section 10.6.

10.2 Competition between Toll Road Concessionaires

Using network models comprising exclusively either parallel or serial links connecting a single OD pair, the literature, as summarised in Chapter 2, has emphasised that whether competition enhances or reduces welfare crucially depends on whether the competition took place between concessionaires exercising control (i.e. toll setting decisions) over **parallel** or **serial** links. The key messages from the literature are as follows:

- competition between parallel links is welfare enhancing with welfare increasing with the intensity of competition;
- competition between serial links deteriorates welfare with welfare decreasing with the intensity of competition, and
- in the case of serial links, tolls would be higher and welfare lower with competition vis-á-vis a monopolist in control of the entire serial corridor.

It was highlighted in Chapter 2 that the issue of collusion between toll road concessionaires has not been discussed in the literature. As the number of toll road operators in a network would likely be small, it was postulated that concessionaires would recognise their mutual inter-dependence and thus would be incentivised to engage in collusion to increase their revenues.

Building on and extending the literature, the objectives of the research[10.1] specifically in relation to competition amongst toll road concessionaires were to:

1. assess the transferability of findings regarding private sector toll road competition from simple network models to a more realistic network setting and
2. investigate the incentives for, and consequences on social welfare of, collusion between toll road operators.

Competition between toll road concessionaires was studied in Chapters 6 and 7. In Chapter 6, tests were conducted on a small network model where users chose routes

[10.1] See p. 8 with objectives numbered accordingly.

in accordance with Wardrop's DUE principle. Though the SLCP algorithm was able to successfully identify the NE in this case, it was recognised that this should only be viewed as a heuristic since the DUE is not necessarily differentiable. Further tests were subsequently conducted on a larger network of Edinburgh in Chapter 7 where the underlying route choice model was based on the (logit) SUE principle which, as outlined in Chapter 7, is smooth and differentiable.

As noted in the literature review, in a realistic network with multiple OD pairs, links cannot be unambiguously categorised into being substitutes (in the case of parallel links) or complements (in the case of serial links). This is because a link might serve as a complement for some OD pairs while simultaneously serving as a substitute for other OD pairs. While it might be difficult to identify the degree of substitutability or complementarity between links, a pragmatic approach was adopted in this thesis to categorise links in order to assess the transferability of findings.

With the smaller network tested in Chapter 6, it was still relatively easy to identify these parallel and serial links through inspection of both the network topology and the OD matrix. With the larger network of Edinburgh used in Chapter 7, the first set of 6 scenarios involving two links in competition were identified based on their volume capacity ratios in the untolled base equilibrium. Subsequently, 6 more scenarios were formed based on incremental combinations of the initial 6 scenarios with reference to the network topology.

The welfare consequences of competition, as summarised by the index of relative welfare improvement, were assessed against two benchmark models. Firstly, to benchmark the welfare levels attainable where the same links in competition would be the only links in the network subject to a second best welfare maximising tolls. Secondly, to benchmark the revenues attainable, where these links were tolled by a single revenue maximising concessionaire termed the "monopolist". In the case of competition, the tests assumed that each concessionaire was able to exercise control over one single link in the network. In this case, there were up to 6 concessionaires (and hence 6 links) in competition in the network. Subsequently, the analysis was extended in Chapter 7 where a concessionaire could exercise control over two links.

Objective 1: Transferability of findings to general networks

Results of the numerical tests carried out in Chapters 6 and 7 extend the insights of the literature to a more general network context.

In all cases of competition between *parallel* links, the toll levels were always lower and welfare changes were always higher compared to when a monopolist exercised control over the links [10.2].

However, it is more difficult to predict, *a priori*, in the case of parallel competition, whether toll levels would be higher or lower compared to *the second best welfare maximising levels*. In Chapter 6, the possibility was highlighted that competition between concessionaires controlling parallel links, in the presence of an untolled alternative, could result in toll levels lower than those obtained by a policy of second best welfare maximisation. On the other hand, with the larger network of Edinburgh, used in Chapter 7, despite the presence of a large number of toll-free alternatives, the competitive toll levels were always higher than those obtained under a second best welfare maximisation policy.

The larger network employed in Chapter 7 allowed for the investigation into the effects of increasing the *intensity* of competition. It was shown therein that increasing the number of concessionaires in parallel competition could, in fact, increase, rather than reduce, the competitive toll levels. An increase in the number of concessionaires in parallel competition in the network reduces the opportunity for users to avoid travelling through one of these tolled links. However, this also increases the marginal external congestion cost of the link controlled by the concessionaire. As highlighted in the literature review (see p. 35), the toll that a revenue maximising concessionaire charges incorporates the link's marginal external congestion cost (as well as a demand related markup). Thus as more congestion is internalised due to higher volumes, this accounts for the higher toll levels with increased intensity of competition.

On the other hand, in all cases of serial competition, it was found that competitive

[10.2] This holds true independent of whether routing follows a DUE or SUE model and independent of the functional form of demand assumed.

tolls were always higher and welfare always lower vis-á-vis a monopolist in control of all links in the series. As noted in Chapter 2 (see p. 37), this arises because of both the double marginalisation problem and that the concessionaire will also internalise the congestion externalities associated with other links in the series. The double marginalisation problem occurs because a concessionaire does not take into account the reduced revenues to concessionaires controlling other segments of the serial link in his toll setting decision.

A further test allowing for concessionaires to decide tolls on two links demonstrated that the extent of welfare gains would also depend on the assignment of links to concessionaires. It was demonstrated that welfare could be further enhanced by organising concessionaires such that a concessionaire controlled the entire corridor (comprising two links in a series) but faced competition from a concessionaire controlling another parallel corridor.

Objective 2: Incentives for, and welfare impacts of, collusion

In this research, the study of collusion was focused on scenarios involving two concessionaires. In order to benchmark the revenue possibilities to concessionaires if they were to collude, the Pareto Front, identifying revenue tuples satisfying the criteria that one concessionaire could not increase his toll revenue without making another worse off, was generated using the MOSADE algorithm. It was shown that besides the monopoly solution, there were a number of solutions on the Pareto Front.

In all tests, it was shown that the revenues attained by each concessionaire in competition were not Pareto Optimal. Therefore, a surplus would be created by the move from the NE solution to the Pareto Front. Since the incentives for collusion ultimately depend on the distribution of the surplus amongst the concessionaires, two possibilities for surplus division were investigated in the research as follows:

- The first is the Utilitarian approach to collusion based on reciprocal signalling behaviour where each concessionaire took into account a proportion, representing the degree of collusion, of the toll revenues earned by his rival while each was independently setting the toll level for the link under his control. In this regard, the collusion path, as a function of the degree of collusion/cooperation amongst the two concessionaires, reflecting the move from none through to full collusion would be traced.

- The second is based on Nash Bargaining. This aims to maximise the product of gains to the parties relative to the fully competitive outcome. As a novel contribution, the research proposed to model this as a Nash Bargaining Problem with Equilibrium Constraints (NBPEC) in order to determine the Nash Bargaining Solution (NBS).

In all cases tested, the collusion path was an inverted U shaped. This means that the revenues to one concessionaire reaches a maximum before full collusion or the Utilitarian/monopoly revenues are attained. If that is the case, there might not be an incentive to collude fully unless again some form of compensation takes place. However, this does not rule out partial collusion.

In cases where one concessionaire could be constrained by an upper bound on the toll level (see e.g. Scenario 1 Fig. 6.4, p. 165), then to increase total revenue, the other concessionaire might have to accept a reduction in revenue. Thus in such an instance, there would be limited incentive to collude without some form of agreement to share revenues set up in advance.

It was shown that with the Utilitarian approach to collusion on *parallel* links, any move away from the fully competitive outcome toward the monopolistic outcome always resulted in higher tolls. These higher tolls implied higher revenues to both concessionaires and lower welfare compared to the fully competitive outcome. Thus even if there is concessionaires only collude partially (since the collusion path is an inverted U shape as noted above), welfare (as measured by the index of relative welfare improvement) would deteriorate (see Tables 6.9, p. 166 and the left panel of Fig. 7.18, p. 234).

However, an exception to this general observation was highlighted in Chapter 6 (see Table 6.12, p. 172). In this case some degree of collusion could, in fact, be welfare enhancing. It was emphasised that this was not because of collusion *per se* that improved welfare but rather because full competition resulted in tolls below the second best welfare maximising level.

Next, consider the Utilitarian approach to collusion in the case of *serial* links. As highlighted above, in the case of serial links, the welfare change obtained in the monopolistic outcome was always higher than under full competition. However, whether the concessionaires would be incentivised to collude fully in this case cannot be answered *unambiguously.* In Example 4 in Section 7.6.1, it was shown that the revenues to each concessionaire was higher with full collusion compared to full competition and thus the incentive to collude fully exists. In this instance, collusion would therefore be welfare improving. However, there are counter examples to this. It was shown (see Scenarios 5 and 6 in Section 7.6.1) that in both cases, the revenue accruing to one concessionaire from full collusion could be *lower* vis-á-vis the fully competitive outcome. In this case, then there is no incentive to collude unless there was a compensation by the concessionaire made better off to the concessionaire who would be made worse off with collusion e.g. through an explicit revenue sharing agreement. This emphasises a disadvantage of the Utilitarian approach to collusion where one party could obtain lower revenue vis-á-vis the fully competitive outcome even though is total revenue increased, thereby violating the Axiom of Individual Rationality.

On the other hand, the Nash Bargaining Solution was shown to satisfy both the Nash Axioms (see p. 134) of Individual Rationality (no player obtains lower revenues than at the fully competitive outcome) and Pareto Optimality (the NBS lies on the Pareto Front). Furthermore, the results suggest that the NBS could result in a more equal allocation of the surplus gained from moving from the NE to the Pareto Front (see e.g Table 7.23, p. 232).

Nash Bargaining was applied to the case of collusion involving concessionaires controlling parallel links in Chapter 6 and to the case of serial links in Chapter 7. In the case of parallel links, the NBS resulted in similar levels of welfare loss as the monopoly solution (as measured by the index of relative welfare improvement).

In the case of serial links, it was shown that because the NBS satisfies the Axiom of Individual Rationality, they would be incentivised to collude. This then resolves the double marginalisation problem associated with serial competition. Therefore, welfare was found to improve in this case. It was noted that this is a rare instance where the commercial (revenue maximising) objective and social (welfare maximising) objective were not in conflict with each other.

To summarise, the following are the conclusions from the study of collusion between concessionaires:

- **There may not always be an incentive for concessionaires to collude fully towards the Utilitarian Solution.**
- **With few exceptions, partial collusion can worsen welfare in the case of parallel links. Similarly, in the case of serial links, partial collusion can improve welfare.**
- **The Nash Bargaining Solution was shown to be both Pareto Optimal and ensures that no concessionaire obtains lower revenues than in the fully competitive outcome.**

10.2.1 Policy Implications

Endorsing existing policy insights from the literature, it is emphasised that on the basis of welfare alone, interrelationships between the links should be taken into account when concessions are awarded. In particular, competition between parallel links should be encouraged while competition between links in a series (such as links that comprise a corridor) should be curtailed.

It was emphasised that allowing concessionaires the right to manage a corridor comprising serial links but facing competition from others doing the same on other corridors would be the most welfare enhancing policy to pursue. Similarly, concessionaires should be prevented from including non-compete clauses in the concession agreements. This would keep open the possibility of developing future welfare enhancing parallel toll roads.

As mergers, acquisitions and corporate takeovers accelerate in pace across the corporate world (Deloitte, 2014), regulators should scrutinise planned mergers between toll road concessionaires. The findings implies that, on the basis of efficiency, mergers between concessionaires that control parallel links should not be prevented since in such an instance, there is a move from a situation of welfare enhancing parallel competition to that of a welfare deteriorating monopoly control. On the other hand, if such a merger involves concessionaires controlling serial links, it could potentially be welfare improving as the double marginalisation problem is resolved.

In the case of parallel links, it was noted that tolls may rise as a result of increased competition. The possibility of an increase in tolls as a result of increased competition would make it harder to justify increased private sector involvement in toll roads to the public. Even though this enhances welfare by increasing the internalisation of the congestion externality, the notion of welfare itself is arguably an abstract concept to the users who have to bear the very real cost of increased toll payments.

Related to the point above, there is a potential regulatory issue in that it makes it difficult to discern whether an observed rise in the toll level is indeed a genuine outcome of competition, justified on the grounds of increased need to internalise congestion or whether it is an outcome of tacit collusion where concessionaires are setting higher tolls to exercise their spatial monopoly power to extract more revenue from users.

Collusion, through signalling behaviour, achieved by means of regular revision of tolls, could result in toll levels tending towards the monopolistic outcome. The regulatory framework could be designed to curtail the frequency of revisions to toll levels.

It was highlighted that concessionaires might not have an incentive to collude if tolls were capped (see e.g. Scenario 1, Fig. 6.4, p. 165). In this case, placing a cap on the maximum toll levels chargeable would serve two goals. It would make tolls more acceptable to the public and also dis-incentivise concessionaires deciding to collude.

It was shown that there could be local NE stemming from signalling behaviour which might be achieved through relatively modest increases in tolls. Thus if the resulting toll revisions are indeed small, the colluding concessionaires could escape detection unless revisions to toll levels are scrutinised carefully.

10.3 Inter-jurisdictional Competition

A common theme of the literature discussed in Chapter 2 is that in its implementation of a toll pricing policy, a jurisdiction is motivated to maximise the welfare of local users but would regard extra-jurisdictional users utilising its network as a source of congestion impeding local users and as a source of revenues.

It was highlighted in Chapter 2 that the discussions of revenue sharing can be found in the public economics literature. As described in Chapter 2 (see p. 60), this was suggested by the proponents of the toll pricing scheme in Edinburgh but the effects of revenue sharing have not been discussed in the context of horizontal inter-jurisdictional toll revenue competition. In addition, it was recognised also that a majority of the literature had conventionally assumed that governments play non-cooperative games when engaged in toll competition. In reality, jurisdictions are not prevented from making binding agreements with one another. With few exceptions, the possibility of jurisdictions engaging in bargaining had also not been studied extensively.

Building on and extending the literature, the objectives of the research[10.3] specifically in relation to inter-juridictional competition were to:

3. study the welfare impacts of inter-jurisdictional competition for toll revenues and

4. assess the welfare implications when jurisdictions share toll pricing revenues while setting tolls non-cooperatively;

Inter-jurisdictional competition was studied in Chapters 8 and 9 under two alternative assumptions of users' route choice behaviour: in Chapter 8, users chose routes following the DUE principle and the SUE principle was applied in Chapter 9.

The numerical tests were all conducted in a setting with two jurisdictions. In its toll setting decision, each jurisdiction was modelled as deciding a single toll level on a set of pre-defined links constituting a closed cordon around its central business

[10.3] See p. 8 with objectives numbered accordingly.

district.

The welfare implications of inter-jurisdictional competition for toll revenues were assessed against the benchmark of the Global Regulator Problem (GRP). This models the situation where a single regulator sets tolls, on the set of tollable links within each jurisdiction to maximise the welfare of all users. It was emphasised that the GRP could be viewed as a regulated environment for toll pricing such as that directed by a higher level government agent. Implicitly, the GRP tolls recognises that within a jurisdiction, both local users and extra-jurisdictional users contribute to and experience congestion.

Objective 3: Welfare impacts of inter-jurisdictional competition

Results of the numerical tests carried out in Chapters 8 and 9 show that inter-jurisdictional competition for toll revenues can substantially reduce welfare compared to the regulated outcome.

On the assumption that each jurisdiction retained full control over the revenue it collects from all users, the literature review emphasised that there were two simultaneous objectives of the toll pricing policy the local jurisdictions pursued in their toll setting decisions (see p. 59). These were: firstly, to internalise congestion externalities experienced by local users *only* and secondly, to raise revenues from extra-jurisdictional users. The second objective is based on the desire to tax export (a fiscal externality) resulting in a welfare loss. Therefore, the extent of welfare losses rests on whether the welfare improvements achieved by employing tolls to internalise the congestion externality, would be dwarfed by the welfare decreasing fiscal externality of tax exporting. In order to examine these issues, two networks were considered.

The first network discussed in Chapter 8 had limited interactions (at least in the high toll regime) between extra-jurisdictional users and local users, resulting in a separated traffic flow regime. In this network, the tolls were able to internalise congestion significantly (thereby meeting the first objective of the jurisdiction's toll setting decision). Thus, tests with this network allowed for a focus on the welfare losses resulting primarily from the fiscal externality of tax exporting. In the second network tested in Chapter 9, the interactions between these two groups of users were more extensive. This network highlights an additional drawback of non-cooperative toll setting: it only accounts for congestion experienced by local users only but not that experienced by extra-jurisdictional traffic. This point is further emphasised in the discussion on revenue sharing possibilities below.

Regardless of the network structure or the underlying routing paradigm assumed, the results showed that non-cooperative tolls were always higher than the regulated outcome. This stems from a combination of the desire by the jurisdictions to tax export by adding a demand-related markup as well as the double marginalisation problem associated with serially interdependent networks. The overall outcome is the observed reduction in welfare.

However, the extent of welfare losses varies. This is complicated by the existence of multiple Local Nash Equilibria (LNE) in the inter-jurisdictional competition game in Chapter 8 (Table 8.3, p. 253). In the extreme case (see Solution 3 in Example 1, Table 8.3, p. 253), the results show that the welfare decreasing fiscal externality of tax exporting could *dominate* the welfare enhancing effect of internalising the congestion externality. In other words, welfare would be higher had the jurisdictions not been incentivised to introduce tolls in the first instance, resulting in a Prisoner's Dilemma.

However, with other LNE solutions (e.g. Solution 1 in Example 1, Table 8.3, p. 253), or in the case of the second network tested where there was only a single NE (see Table 9.5, p. 301), it was shown that the welfare gain, even though better than not implementing any tolls, was always lower than attainable with the GRP/regulated tolls. In this case, the fiscal externality while significantly reducing the welfare gains (compared to the regulated outcome) did not outweigh the welfare gains achieved from internalising congestion.

When asymmetries in demand was allowed for, the jurisdiction that had more extra-jurisdiction users commuting into it was shown to have an incentive to charge a toll higher than the regulated toll (see Table 8.7, p. 265). There are two reasons for this observation. Firstly, these extra-jurisdictional users were regarded by the importing jurisdiction as contributing to congestion in its subnetwork. Secondly, the presence of additional extra-jurisdictional users enhances the ability of the jurisdiction importing these additional users to extract revenue. The combined effect is that the welfare of the jurisdiction exporting its residents is reduced significantly and the welfare of the importing jurisdiction improves. Such a finding provides an explanation for the opposition encountered by authorities surrounding Edinburgh and Stockholm, when these latter jurisdictions proposed the implementation of toll pricing.

Objective 4: Welfare implications of sharing toll revenues while setting tolls non-cooperatively

Full (partial) revenue sharing was modelled by assuming that a jurisdiction returns in full (a proportion of) the toll revenues collected from extra-jurisdictional users to the authority from where these extra-jurisdictional users originate.

Allowing for some degree of revenue sharing reduces the toll levels and improves welfare achievable with toll pricing compared to the situation where jurisdictions retained the revenues raised from extra-jurisdictional users. In this way, revenue sharing can correct and even eliminate (in the case of full revenue sharing) the welfare loss resulting from the *fiscal* externality of tax exporting associated with the desire to raise revenue from extra-jurisdictional users.

In the case of symmetric demands and weak interactions (see Example 1 in Chapter 8, p. 271), eliminating the fiscal externality was sufficient to result the same Pareto Optimal outcome as under the regulated (i.e. GRP) solution. However, in more general cases, when there were asymmetries in demand (see Example 2 in Chapter 8, p. 273), or when there were extensive interactions between users from both jurisdictions (see Example 2 in Chapter 9, p. 307), revenue sharing was shown to be no longer Pareto Optimal with tolls and welfare being vastly different from the GRP/regulated solution. The reason is because revenue sharing does not address the failure of jurisdictions acting non-cooperatively to take into account the welfare of extra-jurisdictional users. Specifically, they ignore, in their toll setting decisions, the transport specific congestion externality in that extra-jurisdictional users *both contribute to and suffer from congestion* when circulating in their subnetwork and this results in a welfare loss. This should be contrasted against the GRP toll which takes into account the welfare of all users.

It should be noted that with partial revenue sharing, it was possible for welfare gain to be similar to the level obtained with the GRP toll. However, it should be emphasised that this only occurs because, with partial revenue sharing, there was still an incentive to extract revenues from extra-jurisdictional users and it was this effect that brought the tolls close to the GRP levels.

Recognising the limitations of revenue sharing, the assumptions of non-cooperative behaviour was relaxed such that jurisdictions could act cooperatively and engage in bilateral bargaining. In this way, Nash Bargaining was also applied and tested in 2 examples (see p. 274 and p. 309). It was shown that both parties would obtain higher welfare gains than acting non-cooperatively, in some cases close to the regulated outcome, through by bilateral bargaining. Furthermore, the bilateral bargaining outcome, unlike full revenue sharing, was Pareto Optimal.

To summarise, the following are the conclusions from the study of revenue sharing between jurisdictions:

- **vis-á-vis no revenue sharing, allowing for some degree of revenue sharing reduces toll levels and improves the extent of welfare gains achievable with toll pricing,**

- **in general, revenue sharing does not result in the regulated outcome and is not Pareto Optimal, with the exception of a special case of limited interactions and symmetric demands and**

- **bilateral Nash bargaining was shown to result in Pareto Optimal outcomes and ensured that jurisdictions would obtain higher welfare gains than acting non-cooperatively.**

10.3.1 Policy Implications

The underlying issue motivating this research into inter-jurisdictional competition has been on the appropriate level of governance for toll pricing policies (see p. 59). Specifically, the question of interest is whether toll pricing should be designed and instituted at the federal/central level or whether it should be viewed as a demand management tool with responsibilities devolved/decentralised to local jurisdictions. The results suggest that on the grounds of social welfare alone, a regulated environment for toll pricing would be the most favourable compared to the implementation of non-cooperative toll pricing policies. This would require a central/higher level regulator to take into account inter-jurisdictional spillovers. However, for various reasons such as concerns that federal/national approach implementing a "one size fits all" policy may not take into account local conditions, this may not be the preferred approach.

If responsibility for toll pricing is to be decentralised to the local level, the research has emphasised the potential for tolls to be levied in order to extract revenues from extra-jurisdictional users which has an adverse impact on welfare. Therefore there is a need for a regulatory authority to ensure that individual jurisdictions *a*) do not set tolls in order to tax export and *b*) take into account the interactions of all users rather than solely that of local users, recognising that both groups of users contribute to and suffer from congestion. The policy advice is particularly relevant in the case of cities that attract high inward commuting flows. The numerical results suggest that should such a city implement toll pricing policy, it would have a strong incentive to set tolls in order to extract revenues from extra-jurisdictional users. Regulators should reduce the incentive to tax export by e.g. requiring that toll revenues are also invested in transportation projects that benefit all users rather than just local users and preventing toll revenues being diverted to reduce local taxation.

As a good "middle ground" to facilitate a decentralised approach to toll pricing, the research has alluded to, and demonstrated the potential, of bilateral bargaining. This has the advantage of being more closely aligned to local interests than a national level approach without overlooking inter-jurisdictional spillovers, in the process resulting in Pareto Optimal outcomes.

Only in the absence of these possibilities, should some form of revenue sharing be pursued. While revenue sharing may not equate to a fully cooperative outcome, revenue sharing could, as a minimum, address the tax exporting incentive which was shown to constitute a significant component of welfare loss.

10.4 Algorithms

The focus of primary interest in this thesis has been on leaders engaged in a Nash non-cooperative game amongst themselves which, as emphasised throughout, could be modelled as a Non-Cooperative Equilibrium Problem with Equilibrium Constraints (NCEPEC). The objective of solving the NCEPEC was to identify the NE in the game. In this regard, it should be recognised that the computational complexity of determining pure strategy NE in Nash non-cooperative games where the players were not bound by a binding VI constraint is known to be high (Daskalakis *et al.*, 2009). At the same time, it is evident that the identification of LNE toll strategies in general networks would require the development and application of solution algorithms.

Therefore, with regards to solution algorithms, this thesis set forth the objectives of[10.4]:

5. developing solution algorithms taking into account route choices of the users in support of the above; and
6. testing the algorithms developed with realistic networks and demonstrating the applicability of the methods to realistic problems;

In order to achieve these goals, several solution algorithms for the NCEPEC were proposed and applied in this thesis in the case studies. Broadly speaking, these algorithms fall into two categories: those that rely on derivative information and

[10.4] See p. 8 with objectives numbered accordingly.

those that do not. In the former category, the research discussed an adaptation of the Fixed Point Iteration (FPI) algorithm, adaptation of the Sequential Linear Complementarity Problem (SLCP) algorithm and in the case of competition between toll road concessionaires, the Synchronous Iterative (SI) algorithm. The algorithm in the latter category was the Nash Domination Evolutionary Multiplayer Optimisation (NDEMO) algorithm.

This distinction is necessary because as highlighted throughout this thesis (see e.g. Chapter 7) the smoothness of the SUE model compared to the non-differentiability of the DUE link flows plays an important role in determining the most appropriate solution algorithm. In this context, it should be emphasised that while both SLCP and SI could be successfully applied in a DUE setting in Chapter 6 where derivatives were obtained by means of finite differencing, these procedures should only be viewed as heuristics.

The FPI algorithm had been proposed in earlier work for solving Nash non-cooperative games (Harker, 1984) and has been subsequently extended to solving NCEPECs (Su, 2005). This is a "workhorse" algorithm which has been applied in the study of NCEPECs formulated in other disciplines e.g. deregulated electricity markets (Hu and Ralph, 2007).

FPI is intended to locate LNE points of the NCEPEC by decomposing and solving a series of inter-related MPECs until the system converges. The primary advantage of the FPI algorithm is that it is intuitively related to the concept of Nash Equilibria since it models each player's search for the best response to the strategies of all other players. At the same time, FPI relies on the use of an MPEC solver to solve each player's MPEC, given the toll strategies of all other players in the game. In this thesis, this was solved using CCA.

To identify NE tolls in the case of competition between toll road concessionaires, the SI algorithm proposed by Yang and Huang (2005) was also tested. The primary advantage of the SI algorithm over SLCP is that SI does not rely on second order derivatives. However, three drawbacks of the SI algorithm can be identified. Firstly, it requires the computation of first order derivatives of the link flows with respect to tolls, which as noted above, may not exist under DUE. Secondly, it relies on the method of successive averaging which was shown to reduce the efficiency of the algorithm. Thirdly, the SI algorithm is specifically designed to identify LNE in the case of competition between toll road concessionaires only, (where players optimise revenues) and cannot be extended to identifying NE in the case of inter-jurisdictional competition (where players optimise welfare and revenues).

The first novel algorithmic contribution of this thesis is the extension of the SLCP algorithm to identify LNE points in NCEPECs. SLCP was originally proposed to identify NE a Cournot Game in Kolstad and Mathiesen (1991). By posing the NCEPEC as a complementarity problem, SLCP solves the NCEPEC by iteratively solving a sequence of linear complementarity problems and thus aims to solve the system of inter-related MPECs *simultaneously*. This explains why it was concluded that SLCP was more efficient than FPI in all applications. It was noted that SLCP encountered numerical difficulties when it was applied to determine LNE in the case of inter-jurisdictions competition when routing following the DUE model in Chapter 8. However, the ability of SLCP to exploit derivative information also presents with it a drawback. Although heuristically applied in the case of DUE link flows, SLCP converged to a LNE in the study of collusion in Chapter 6 (see Table 6.11, p. 170) Furthermore, it was found to be dependent on the starting point used. It was shown in Chapter 9 (see Table 9.2, p. 294) that while the differentiability of the SUE link flows allows for the application of SLCP, the smoothing effect achieved by SUE could make the local optima less distinct and not easily distinguishable from the global optima and this resulted in SLCP identifying a LNE solution rather than the NE solution.

The second novel algorithmic contribution of this thesis is the population based NDEMO algorithm to identify NE points in NCEPECs. This in turn extends earlier work by Lung and Dumitrescu (2008) who developed the Nash Domination principle to solve general Nash non-cooperative games. The premise of this principle is that at an NE, players should not have an incentive to unilaterally deviate from their chosen strategies. When used to compare two strategy profiles (each embodying possible strategies of all players), this principle operates by counting the number of players that would be incentivised to switch from one profile to the other. In this way, the strategy profile resulting in a lower number of players deviating away from it was deemed to be closer to a NE compared to the other.

In NDEMO, the parent profiles are generated randomly while a population of child strategies are created by application of Differential Evolution and in comparisons between each parent strategy profile and each child strategy profile, the Nash Dominance principle was applied.

In contrast to FPI, SLCP and SI, NDEMO is distinguished by two characteristics: *a*) it operates with a population of trial points and *b*) it does not require differentiability of link flows. These two distinguishing features of NDEMO are at once both its advantages as well as its weaknesses. Since it obviates the use of derivatives, there is theoretically no restriction as to whether the underlying route choice model follows the DUE or SUE principle. Indeed, NDEMO was able to identify the NE tolls in Chapter 8 when the SLCP approach encountered numerical difficulties due to non-differentiability of DUE link flows. However, application of the Nash Domination principle requires extensive function evaluations which necessitates carrying out a (DUE/SUE) traffic assignment each time for each unilateral deviation of a player. This points to the intrinsic computational complexity in solving EPECs using evolutionary algorithms in that the lower level traffic assignment problem must be solved each time in keeping with the Stackelberg leader-follower principle upon which the NCEPEC is founded.

From a theoretical perspective (see Proposition 5.1, p. 129), there is theoretical assurance that NDEMO can, in fact, identify the NE of the NCEPEC and not just a LNE and this was seen in the numerical tests. *The unique feature of NDEMO is that despite being based on Evolutionary Algorithms, it is endowed with a convergence proof that an NE has been located.* However, despite this, NDEMO was also found to be the most computationally demanding amongst all algorithms for solving NCEPECs tested. As an example of the computation burden, NDEMO took close to 4 days (see p. 200) to identify the NE. Most importantly, it should be emphasised that the ability of any algorithm to identify NE does not necessarily reflect realistic decision making behaviour.

If the leaders were to cooperate, this could be modelled as a Multiobjective Optimisation Problem with Equilibrium Constraints (MOPEC). The objective of solving the MOPEC was the identification of Pareto Optimal points. The algorithm applied to do this was the Multiobjective Self Adaptive Differential Evolution (MOSADE) algorithm. The primary advantage of MOSADE is that the algorithm does not rely on any user input control parameters beyond the population size and the size of the archive of solutions retained. However, it should be recognised that this algorithm is a heuristic. There is much scope for further research into theoretically convergent algorithms for solving MOPECs and as noted, this has only begun in earnest recently.

Another novel contribution in this thesis is the formulation of the Nash Bargaining Problem with Equilibrium Constraints. This directly extends proposals in Nash (1950*b*) to incorporate the equilibrium route choice condition of the traffic assignment model. This model bridges the gap between the fully cooperative MOPEC and the non-cooperative paradigm of the NCEPEC. Though it is recognised that this is a heuristic formulation of the bilateral Nash bargaining problem, it has been demonstrated, through the numerical tests, that the solutions obtained satisfied the Nash Axioms of Individual Rationality and Pareto Optimality.

10.5 Summary of Contributions

Returning to the overarching question of the extent of welfare losses as a result of competition in the two groups of agents with toll setting responsibilities, viz. toll road concessionaires and (governmental) jurisdictions, these are summarised as follows:

- Tests with network models with multiple links and multiple Origin Destination pairs showed that the welfare effects of competition amongst toll road concessionaires for toll revenues depend crucially on the interrelationships between the toll roads in competition. Thus, the findings obtained generalise results obtained previously in the literature to more realistic settings.

- In the case of inter-jurisdictional competition for toll revenues, the welfare gains from imperfectly internalising congestion are significantly reduced by the fiscal externality of tax exporting resulting from their toll setting decisions. In general while it is possible to conclude that overall welfare would be lower compared to that achievable by a regulated outcome, the numerical tests also suggest that the extent of welfare losses could be network specific. In addition, this is complicated by the possible existence of multiple local Nash equilibria.

To summarise, the following are the novel contributions of the research:

- investigate the effects of collusion and the incentives for collusion between toll road concessionaires,
- proposed and demonstrated two effective algorithms for identification of Nash Equilibria in Non-Cooperative Equilibrium Problems with Equilibrium Constraints (NCEPECs): SLCP and NDEMO,
- propose and formulate the Nash Bargaining Problem with Equilibrium Constraints (NBPEC) to bridge the gap between the fully competitive NCEPEC and the fully cooperative MOPEC,
- demonstrate the possibility of multiple Local Nash Equilibria (LNE) solutions in inter-jurisdictional competition for toll revenues,
- demonstrate Pareto suboptimality of revenue sharing between jurisdictions and show that bilateral Nash bargaining can overcome this.

The primary policy implications are as follows:

- Awards of toll road concessions should take into account intrinsic interrelationships between the toll roads,
- regulators should be aware of the potential for concessionaires to collude,
- judged solely on the basis of welfare, a nationally/federally led cooperative toll pricing policy would be desirable,

- but by being able to incorporate both local needs and take into inter-jurisdictional spillovers, bilateral bargaining could be exploited as a means of facilitating a devolved approach to toll pricing.

It is worth highlighting that while bilateral bargaining between toll road concessionaires would be viewed by anti-trust regulators as anti-competitive, bargaining between jurisdictions should be encouraged.

Limitations

With regard to private sector toll road concessionaire competition, this study assumed that the underlying PPP model was free of transaction costs. In reality, PPPs involve a high degree of risks and transaction costs (e.g. Becker and Patterson, 2005).

With regard to inter-jurisdictional competition, the research assumed that jurisdictions implementing toll pricing would focus only on the twin objectives of maximising welfare for its own users and extracting toll revenues from extra-jurisdictional users. In reality, in the process of developing their transportation strategies, jurisdictions would face a number of conflicting objectives which would have repercussions on the policy implications obtained in this thesis.

10.6 Further Research

This thesis has focused on competition solely within the context of highway transportation networks alone. Competition between public transport providers has been investigated previously (e.g. Harker, 1988; Li *et al.*, 2012) but these have been considered in isolation from the highway mode. There is much scope for combining these two modes within a multimodal model that could be applied to study e.g. competition between private roads simultaneously with privately operated public transport.

In a similar vein, vertical competition between governments (inter-governmental competition) was not investigated in this thesis. Conceivably a high level regulator (e.g. central government agent) would indeed influence the policies of either concessionaires, jurisdictions or even both together within this framework. This would lead directly to a vertical extension of the bilevel programming problem presented in Chapter 4 and result in the study of an even more complex class of hierarchical optimisation problems known as the tri-level programming problem. It is speculated that this genre of optimisation problems would be extremely complex to formulate and solve. While there exist some preliminary work in this area (Zhang *et al.*, 2010), this is still relatively uncharted territory where significant theoretical and methodological advances could be made.

In the NCEPEC, the toll road concessionaires/jurisdictions are modelled as Stackleberg leaders taking into account route choice of users, but non-cooperative Nash players in the game amongst themselves. Consider an alternative situation in which a player e.g. a toll concessionaire was able to take into account *both* the reactions of the users and the reaction of the other toll concessionaires when setting his tolls. This player would then become the Stackelberg leader amongst the leaders in the toll pricing game. In microeconomic theory, a Stackelberg firm desires to be the leader as it is able to earn higher profits by taking into account the behaviour of the follower firms (Varian, 2010). In the setting of a network with two parallel links connecting a single OD pair, Shepherd and Sumalee (2008) showed the paradoxical result that, in fact, the Stackelberg toll concessionaire could be made worse off by being the leader. It would be of definite interest to understand whether such paradoxical results hold in general networks. In considering this question in general networks, another area for research would be the development of efficient solution algorithms to solve the above resulting formulation whilst continuing to take into account the route choice of users.

This thesis has studied the Bargaining Problem restricted to bilateral settings (i.e. 2 bargaining parties). There is extensive scope to extend the proposed NBPEC to multilateral bargaining situations. In addition, further work could extend alternative paradigms of surplus distribution between bargaining parties (e.g. proposals in Kalai and Smorodinsky, 1975; Perles and Maschler, 1981) to bargaining problems incorporating route choice equilibrium constraints.

While the algorithms proposed in this thesis have been able to identify a single LNE in the game, it is recognised that this is only the beginning of fruitful research in locating NE in NCEPECs. At least three further research issues can be identified in this regard. Firstly, since the ability of any algorithm to detect NE does not necessarily reflect realistic decision making behaviour, it would be useful to be able to identify *all* LNE solutions. While some progress has been in this direction in the context of general Nash non-cooperative games (Judd *et al.*, 2012), whether techniques employed therein would be transferable to detecting multiple equilibria in NCEPECs is at present an open question. Secondly, this thesis has only focused on single shot games i.e. players meet once in a single encounter. In reality, rational agents repeatedly interact and it is known that the (finite and infinite) repetition of play affects the outcomes of games (Osborne and Rubinstein, 1994). It would be of interest to investigate how the dynamics of interactions affect a rational agents' decision making behaviour in transportation networks and how this impacts on the policy implications. Thirdly, by exploiting the fact that a Nash game can be posed as a variational inequality problem, it is hypothesised that projection methods (see e.g. Tinti, 2005, for a review) are also applicable in this context. It should be remarked that a number of alternative approaches based on minimizing the Nikaido-Isoda function (Nikaidô and Isoda, 1955) have also been proposed for single level Nash non-cooperative games, and applied in Krawczyk and Uryas'ev (2000). Development of algorithms based on these principles could be an useful area of research given the underlying practical relevance of the EPEC framework both within and outside transportation systems management.

Appendix A

Verification of NE tolls obtained by SLCP: Power Law Demand Function

This appendix provides numerical estimates of the best response functions (cf. Definition 3.2) in the case of competition between toll road concessionaires with the power law demand function.

These are used to numerically verify that the solutions reported in main text (see Tables 7.2 to 7.4) where the SLCP algorithm was applied, to locate the NE. These figures show that the solutions reported therein is the intersection of these best response functions for the tests with the power law demand function (Eq. 7–13). In each graph, the dashed line shows the best response function for the player indicated on the x-axis. Similarly, the continuous line shows the best response function for the player indicated on the y-axis.

In the scenarios which involve competition between more than 2 concessionaires (i.e. Scenarios 7 to 12), the procedure to numerically estimate the best response functions was modified. In these cases, the toll levels for all other players (except for those being considered) were fixed at the NE levels obtained by SLCP shown in Tables 7.3 and 7.4, during the grid search for the 2 players shown on the respective axes. Due to space constraints, only a selection of these are shown.

A.1 Scenarios 1 to 6 with Two Links

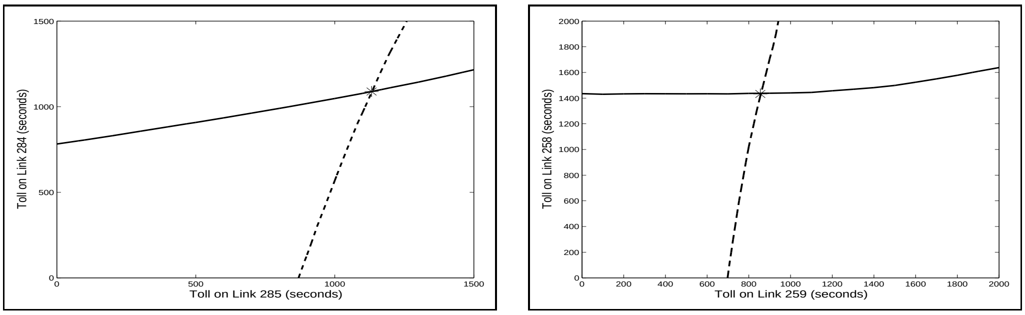


FIGURE A.1: (Left) Scenario 1: Intersection of best response functions coincides with SLCP tolls for Links {284, 285} of {1,088.48, 1,132.19} secs. (Right) Scenario 2: Intersection of best response functions coincides with SLCP tolls for Links {258, 259} of {1,432.40, 855.16} secs.

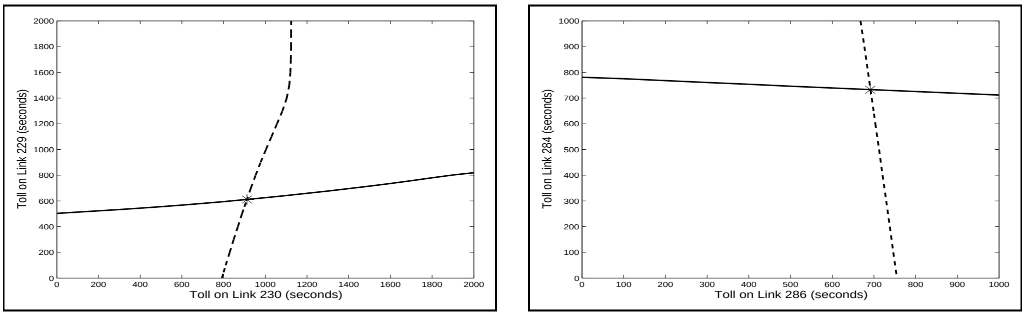


FIGURE A.2: (Left) Scenario 3: Intersection of best response functions coincides with SLCP tolls for Links {229, 230} of {611.21, 912.02} secs. (Right) Scenario 4: Intersection of best response functions coincides with SLCP tolls for Links {284, 286} of {732.37, 691.06} secs

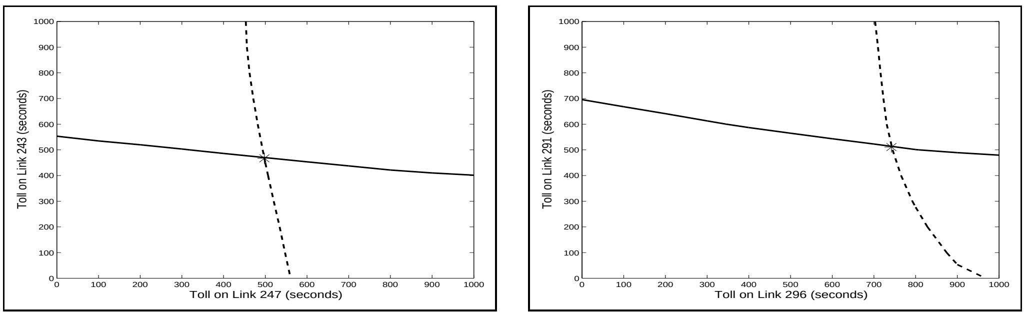


FIGURE A.3: (Left) Scenario 5: Intersection of best response functions coincides with SLCP tolls for Links {243, 247} of {467.17, 497.79 } secs. (Right) Scenario 6: Intersection of best response functions coincides with SLCP tolls for Links {291, 296} of {501.60, 740.21} secs.

A.2 Scenarios 7 and 11 with Three Links

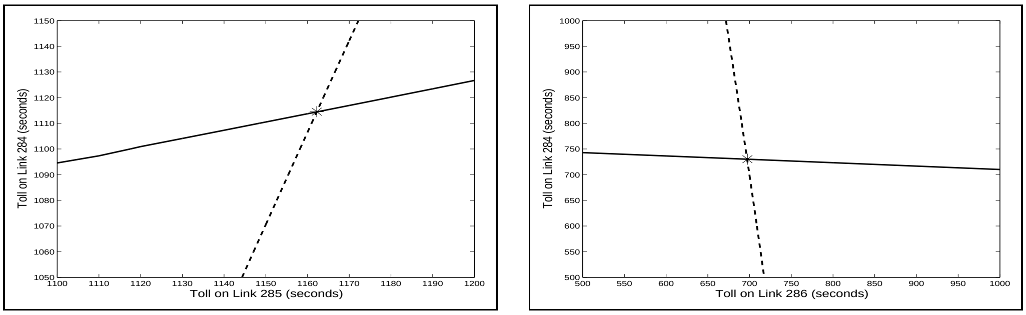


Figure A.4: (Left) Scenario 7: Intersection of best response functions coincides with SLCP tolls, *all other link tolls fixed at NE*, for Links {284, 285} of {1,114.52, 1,162.20} secs. (Right) Scenario 11: Intersection of best response functions coincides with SLCP tolls, *all other link tolls fixed at NE*, for Links {284, 286} of {730.14, 697.41} secs

A.3 Scenarios 8 and 12 with Four Links

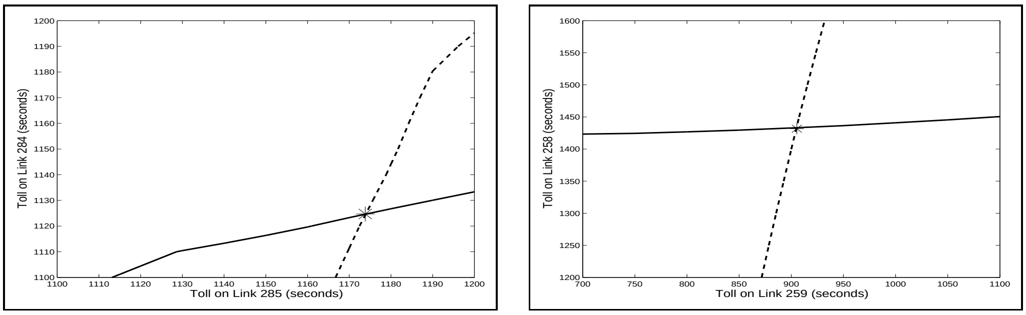


Figure A.5: (Left) Scenario 8: Intersection of best response functions coincides with SLCP tolls, *all other link tolls fixed at NE*, for Links {284, 285} of {1,124.72, 1,173.83} secs. (Right) Scenario 8: Intersection of best response functions coincides with SLCP tolls, *all other link tolls fixed at NE*, for Links {258, 259} of {1,429.66, 905.36} secs.

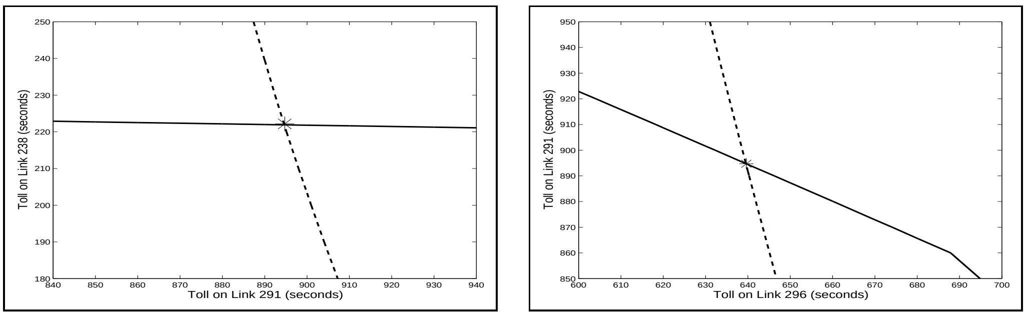


Figure A.6: (Left) Scenario 12: Intersection of best response functions coincides with SLCP tolls, *all other link tolls fixed at NE*, for Links {238, 291} of {221.67, 894.73} secs. (Right) Scenario 12: Intersection of best response functions coincides with SLCP tolls, *all other link tolls fixed at NE*, for Links {291, 296} of {894.73, 639.65} secs.

A.4 Scenarios 9 and 10

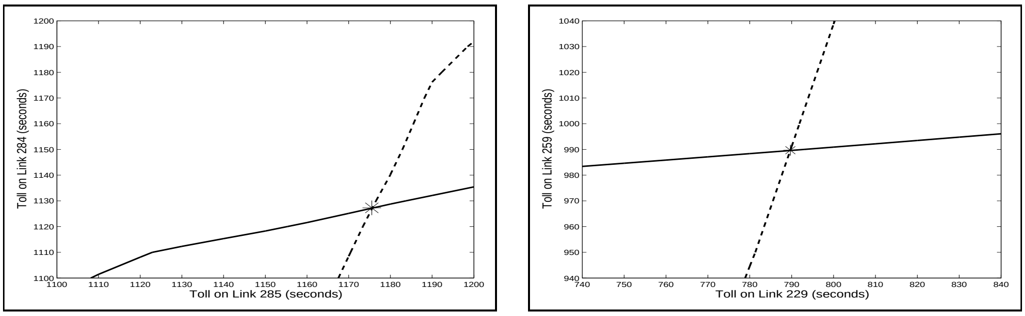


Figure A.7: (Left) Scenario 9: Intersection of best response functions coincides with SLCP tolls, *all other link tolls fixed at NE*, for Links {284, 285} of {1,127.28, 1,175.54} secs. (Right) Scenario 9: Intersection of best response functions coincides with SLCP tolls,*all other link tolls fixed at NE*, for Links {259, 229} of {989.70, 789.69} secs.

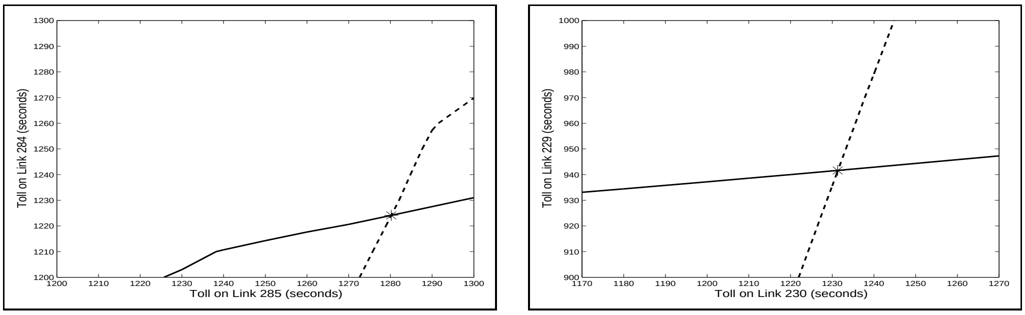


Figure A.8: (Left) Scenario 10: Intersection of best response functions coincides with SLCP tolls,*all other link tolls fixed at NE*, for Links {284, 285} of {1,224.12, 1,280.21} secs. (Right) Scenario 10: Intersection of best response functions coincides with SLCP tolls,*all other link tolls fixed at NE*, for Links {229, 230} of {941.61, 1,231.31} secs.

Appendix B

Verification of NE tolls obtained by SLCP: Exponential Demand Function

This appendix provides numerical estimates of the best response functions (cf. Definition 3.2) in the case of competition between toll road concessionaires with the exponential demand function. These are used to numerically verify that the solutions reported in main text (see Tables 7.5 and 7.6) where the SLCP algorithm is applied, to locate the NE. These figures show that the solutions reported therein is the intersection of these best response functions for the tests with the exponential demand function (Eq. 7–14). In each graph, the dashed line shows the best response function for link indicated on the x-axis. Similarly, the continuous line shows the best response function for the player indicated on the y-axis.

B.1 Scenarios 1 to 6 with Two Links

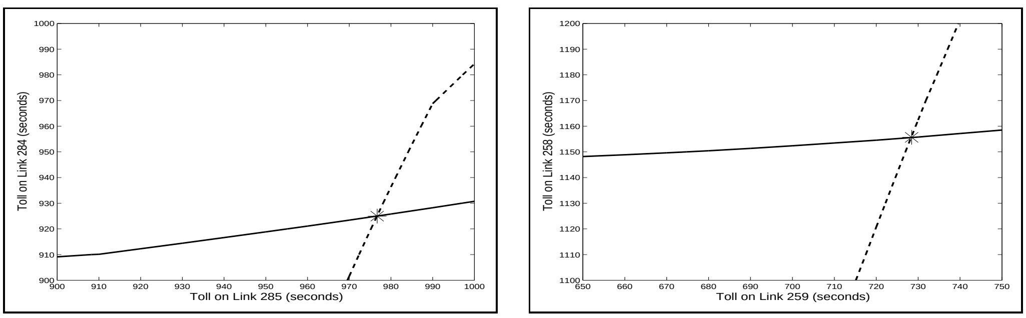


Figure B.1: (Left) Scenario 1: Intersection of best response functions coincides with SLCP tolls for Links {284, 285} of {924.98, 976.71} secs. (Right) Scenario 2: Intersection of best response functions coincides with SLCP tolls for Links {258, 259} of {1155.65, 728.46} secs.

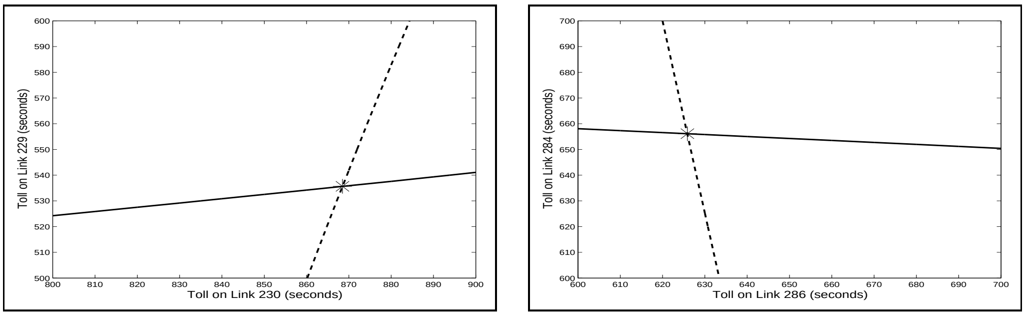


FIGURE B.2: (Left) Scenario 3: Intersection of best response functions coincides with SLCP tolls for Links {229, 230} of {535.66, 868.48} secs. (Right) Scenario 4: Intersection of best response functions coincides with SLCP tolls for Links {284, 286} of {656.12, 625.87} secs.

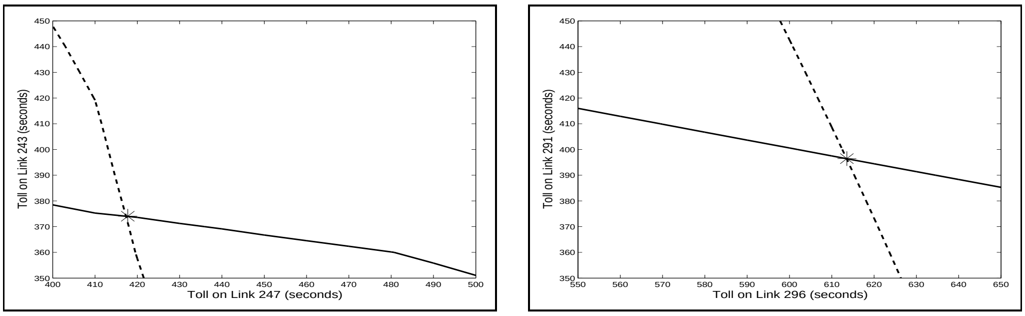


FIGURE B.3: (Left) Scenario 5: Intersection of best response functions coincides with SLCP tolls for Links {243, 247} of {374.05, 417.75} secs. (Right) Scenario 6: Intersection of best response functions coincides with SLCP tolls for Links {291, 296} of {396.44, 613.57} secs.

B.2 Scenarios 7 and 11 with Three Links

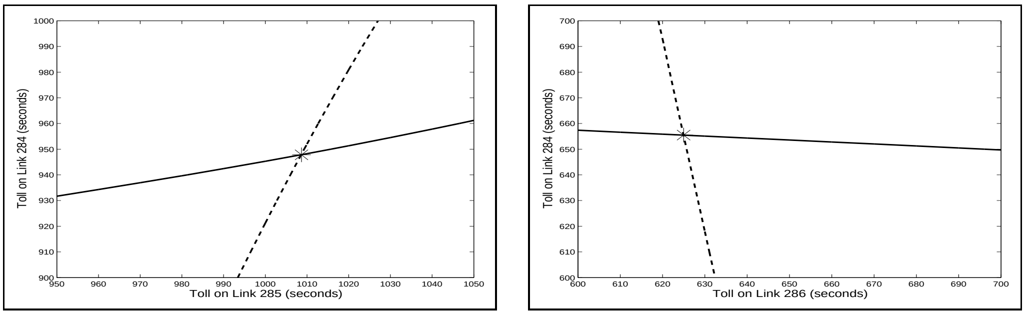


FIGURE B.4: (Left) Scenario 7: Intersection of best response functions coincides with SLCP tolls, *all other link tolls fixed at NE*, for Links {284, 285} of {947.79, 1,008.68} secs. (Right) Scenario 11: Intersection of best response functions coincides with SLCP tolls, *all other link tolls fixed at NE*, for Links {284, 286} of {655.49, 624.99} secs.

B.3 Scenarios 8 and 12 with Four Links

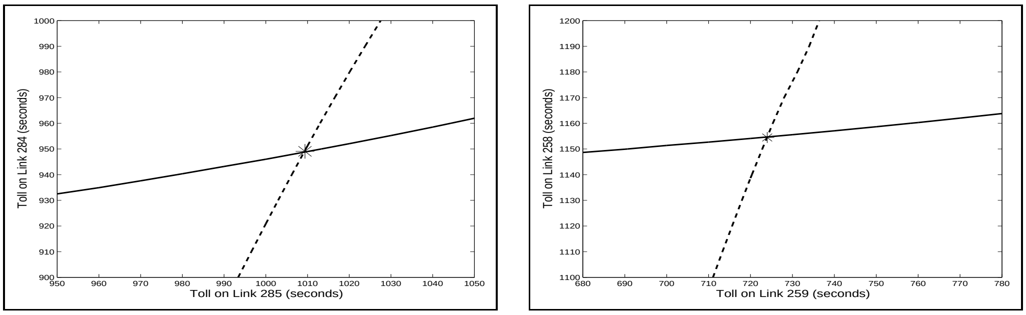


FIGURE B.5: (Left) Scenario 8: Intersection of best response functions coincides with SLCP tolls,*all other link tolls fixed at NE*, for Links {284, 285} of {949.08, 1,009.39} secs. (Right) Scenario 8: Intersection of best response functions coincides with SLCP tolls,*all other link tolls fixed at NE*, for Links {258, 259} of {1,154.47, 723.98 } secs.

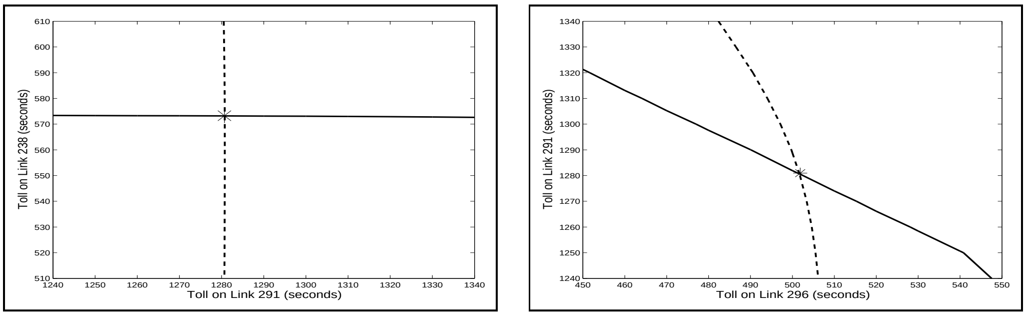


FIGURE B.6: (Left) Scenario 12: Intersection of best response functions coincides with SLCP tolls,*all other link tolls fixed at NE*, for Links {238, 291} of {574.26, 1,281.69} secs. (Right) Scenario 12: Intersection of best response functions coincides with SLCP tolls,*all other link tolls fixed at NE*, for Links {291, 296} of {1,281.69, 501.86}secs

B.4 Scenarios 9 and 10

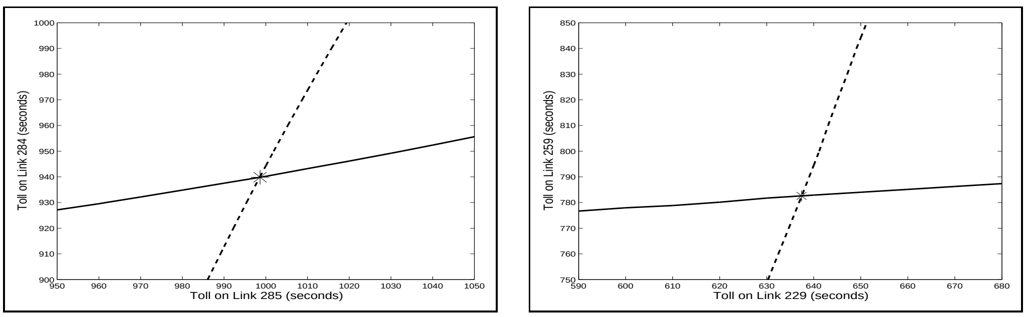
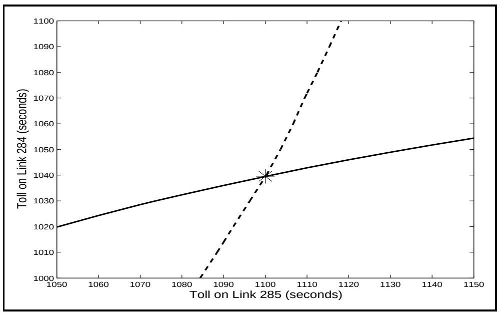


FIGURE B.7: (Left) Scenario 9: Intersection of best response functions coincides with SLCP tolls, *all other link tolls fixed at NE*, for Links {284, 285} of {939.85, 998.64} secs. (Right) Scenario 9: Intersection of best response functions coincides with SLCP tolls,*all other link tolls fixed at NE*, for Links {259, 229} of {782.59, 637.41} secs



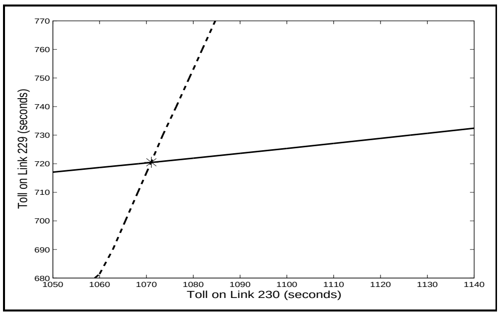
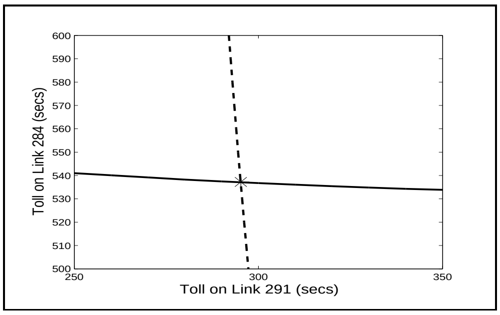


FIGURE B.8: (Left) Scenario 10: Intersection of best response functions coincides with SLCP tolls,*all other link tolls fixed at NE*, for Links {284, 285} of {1,039.56, 1,099.51} secs (Right) Scenario 10: intersection of best response functions coincides with SLCP tolls,*all other link tolls fixed at NE*, for Links {229, 230} of {720.50, 1,071.05} secs

Appendix C

Verification of NE tolls obtained by SLCP reported in Section 7.5

This appendix provides numerical estimates of the best response functions (cf. Definition 3.2) in the case of competition between toll road concessionaires as described in Section 7.5. These are used to numerically verify that the solutions reported in main text (see Tables 7.19 and 7.20) where the SLCP algorithm is applied to locate the NE. The following figures show that the solutions reported therein is indeed the intersection of these best response functions. Note that in these tests, only the exponential demand function (Eq. 7–14) was used. In each graph, the dashed line shows the best response function for the player indicated on the x-axis. Similarly, the continuous line shows the numerical estimates of the best response function for the player indicated on the y-axis.



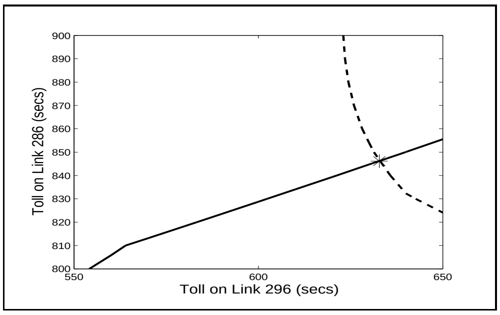


FIGURE C.1: Integrated Parallel Competition: (Left) Intersection of best response functions coincides with SLCP tolls reported in first section of Table 7.19, *all other link tolls fixed at NE*, for Links {284, 291}. (Right) Intersection of best response functions coincides with SLCP tolls reported in first section of Table 7.19, *all other link tolls fixed at NE*, for Links {286, 296}

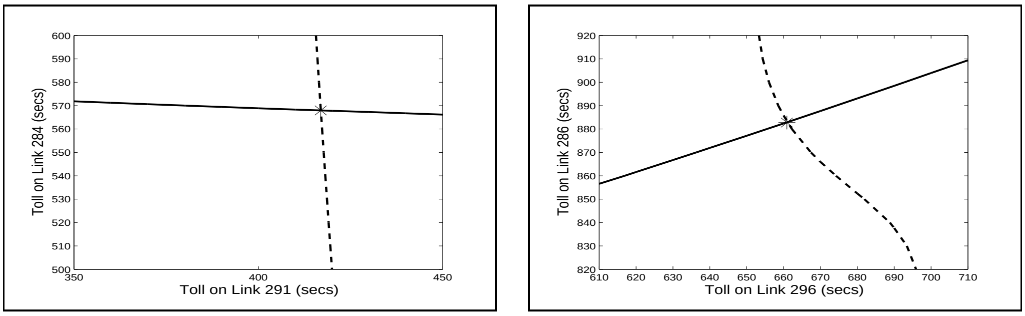


Figure C.2: Disaggregated Nash Game: (Left) Intersection of best response functions coincides with SLCP tolls reported in second section of Table 7.19, *all other link tolls fixed at NE*, for Links {284, 291}. (Right) Intersection of best response functions coincides with toll tuple obtained by SLCP tolls reported in second section of Table 7.19, *all other link tolls fixed at NE*, for Links {286, 296}

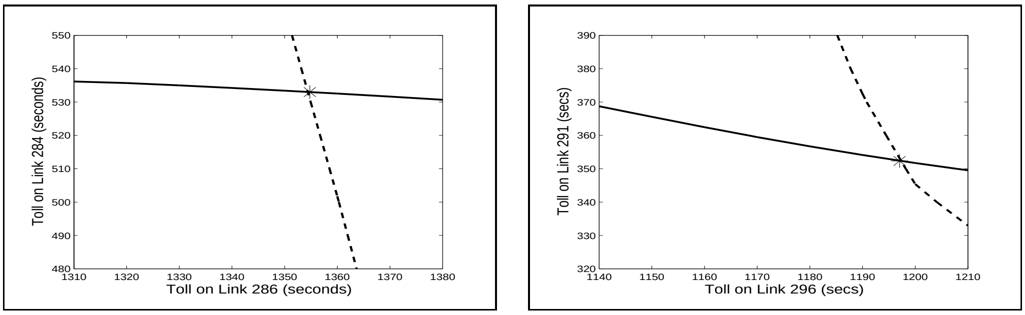


Figure C.3: Extended Serial Competition with Exponential Demand Function Variant I: (Left) Intersection of best response functions coincides with SLCP tolls reported in third section of Table 7.19, *all other link tolls fixed at NE*, for Links {284, 286}. (Right) Intersection of best response functions coincides with SLCP tolls reported in third section of Table 7.19, *all other link tolls fixed at NE*, for Links {291, 296}

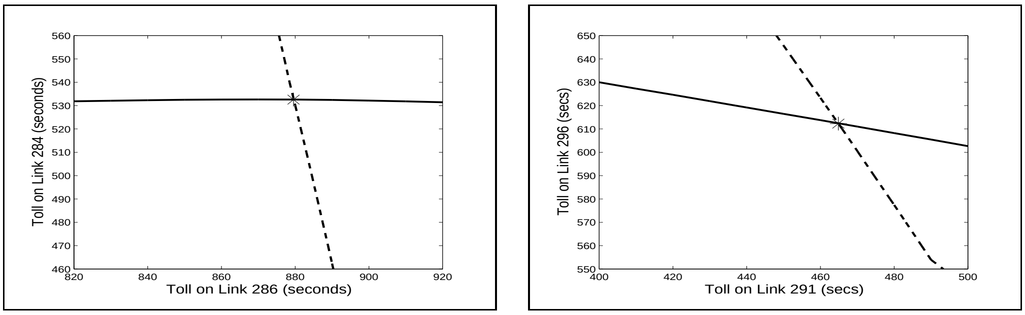


Figure C.4: Extended Serial Competition with Exponential Demand Function Variant II: (Left) Intersection of best response functions coincides with toll tuple obtained by SLCP reported in third section of Table 7.19, *all other link tolls fixed at NE*, for Links {284, 286}. (Right) Intersection of best response functions coincides with SLCP tolls reported in third section of Table 7.19, *all other link tolls fixed at NE*, for Links {296, 291}

Appendix D

Uniqueness of Equilibrium Link Flows Disaggregated by Authority

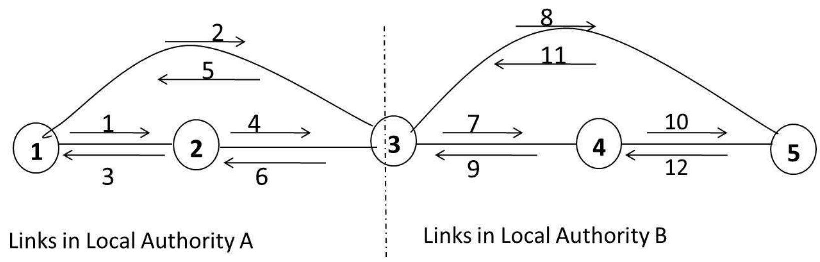


FIGURE D.1: Highway Network Used in Chapter 8

In this section, uniqueness of the equilibrium link flows disaggregated by authority (i.e. "authority link flows"), at any given toll vector, for the network previously shown in Fig. 8.1 and reproduced in Fig. D.1 is established with the aim of justifying the approach set out in Chapter 8. In order to do so, some mild additional assumptions are made. Let c_j^* denote the equilibrium generalized cost on link $j, j \in \mathcal{L}$ corresponding to a given solution to Eq. 8–6. Formally, for any given toll vector solution $\mathbf{x}^*$ to Eq. 8–6 these are given uniquely by the elements of vector $\mathbf{c}^*$ i.e. $\mathbf{c}^* = \mathbf{c}\left(\tilde{\mathbf{V}}^*(\mathbf{x})\ \mathbf{1},\ \mathbf{x}\right)$

Specifically the following assumptions are made:

$$c_3^* + c_2^* > c_4^* \qquad \text{(Eq. D–1)}$$

$$c_4^* + c_5^* > c_3^* \qquad \text{(Eq. D–2)}$$

$$c_{10}^* + c_{11}^* > c_9^* \qquad \text{(Eq. D–3)}$$

$$c_9^* + c_8^* > c_{10}^* \qquad \text{(Eq. D–4)}$$

With these assumptions uniqueness of authority flows can be established through the following steps, and applying where relevant, conservation of flow principles:

1. The assumptions on the cost functions and demand functions are sufficient to guarantee uniqueness of the equilibrium *total* link flows and OD demands (Sheffi, 1985), so the issue can be equivalently posed as follows: in the given network structure, is this uniqueness sufficient to also guarantee uniqueness of the link flows disaggregated by authority?

2. At equilibrium, intra-authority OD movements will never use the links of the other authority. For example, one possible route from node 1 to node 2 is to follow the route given by the link sequence {2,7,10,11,6}, but since link costs are strictly positive it follows that such a route will always have higher cost than the route following links {2,6}, and so this earlier route can never appear in an equilibrium solution at any toll vector. An analogous argument may be made for all intra-authority OD movements, so for such movements we need only consider the routes that use links strictly within that authority's jurisdiction.

3. The network structure is entirely equivalent to one in which an additional bi-directional, dummy link is added by dividing node 3 in two and inserting the link between the two nodes resulting from the divided node 3. The only flow on the left-pointing direction of this link will be (all of) that demand travelling from Authority B (node 4 or 5) to Authority A (node 1 or 2), there will be no intra-authority demand using it given remark 2. Returning to the original network definition, it is possible to represent the demand from Authority B as if it were from an origin at node 3 with OD flow to nodes 1 and 2 equal to the relevant OD flows from the sum of nodes 4 and 5 (noting that such sums are unique since the individual demands are unique by remark 1). By symmetry, the same argument may be made regarding demand from Authority A to B, if thinking from the perspective of Authority B's network.

4. Considering Authority A's network, links 2 and 4 take traffic into node 3. In view remark 2, such links could never be part of an equilibrium route for traffic from Authority B. Therefore links 2 and 4 only carry Authority A's demand, and these flows are unique since the total link flows are unique by remark 1.

5. Eq. D–2 means that for demand travelling from node 2 to nodes 4 or 5, it is more costly (at equilibrium) to travel on the indirect route to node 3 (via links 3 and 2) than via the direct route via link 4, and so such demand will never use the indirect route. This implies that the only Authority A flow on link 2 is that demand from node 1 (destined for nodes 2, 4 or 5). All the remaining demand from node 1 to these other nodes must use link 1. Since at equilibrium we uniquely determine the total demand from node 1 (as the sum of demands to nodes 2, 4 and 5), and since in remark 4, Authority A's flow on link 2 can be uniquely determined and since by the argument just made this flow on link 2 can only be from node 1, then by subtracting the (unique) link 2 flow from the (unique) total demand from node 1, then we have uniquely determined the flow on link 1 that is due to demand from node 1. Now since no demand from node 2 would ever use link 1, so that the only Authority A demand on link 1 is that from node 1, and this has been uniquely determined. Thus the Authority A flow on link 1 is unique, and by subtraction from the total link 1 flow (which is unique by remark 1) then the Authority B flow on link 1 is also unique.

6. Eq. D–2 implies that it is never efficient for demand from node 2 to travel to node 1 via the indirect route of links 4 and 5, in preference to the direct route via link 3. In particular, it means that link 5 is not used by demand from node 2; neither is this link on a route from node 1. Therefore no Authority A flow uses link 5, only Authority B flow and so this must equal the total flow on link 5, which is unique by remark 1.

7. Since by remark 3, the total Authority B demand arriving at node 3 (and destined for nodes 1 and 2) is uniquely determined, and since links 5 and 6 are the only exit nodes from node 3, and since by remark 6. the Authority B flow on link 5 is unique, then it follows that the Authority B flow on link

6 can be uniquely determined by conservation of Authority B flow at node 3. By subtraction from the total link 6 flow, the Authority A flow on link 6 is then also unique.

8. Consider node 2. By remarks 4, 5 and 7, the Authority B flow on links 1, 4 and 6 is uniquely determined. By remark 1, the total Authority B OD flow that is destined for node 2 is uniquely determined, and by definition of the OD matrices in Tables 8.1 and 8.5, there is no Authority B OD flow originating at node 2. Therefore, applying conservation-of-flow at node 2 to the Authority B flow, then the Authority B flow on link 3 may be uniquely determined, as it is then the only unknown in the conservation equation. By subtraction from the total link 3 flow, the Authority A flow on link 3 is then also unique.

9. Remarks 4 to 8 establish uniqueness of the authority flows on links 1-?6, i.e. those under Authority A's jurisdiction. By symmetry, equivalent arguments can be made about links 7-12 (i.e those in Authority B's jurisdiction), exploiting assumptions Eq. D–3 and Eq. D–4 in place of Eq. D–1 and Eq. D–2.

Appendix E

Base Matrices Used in Example 2 in Chapter 9

The base trip matrix (in pcus/hr) for the network shown in Fig. 9.4 used in Example 2 of Chapter 9 (see Section 9.4, p. 296) is given in Table E.1. The base composite generalised travel time matrix (in minutes) for $\theta = 10$ is given in Table E.2.

TABLE E.1: Base Trip Matrix (pcus/hr) used in Example 2, Chapter 9 (see Section 9.4).

		Destinations							
		1	**5**	**7**	**9**	**12**	**14**	**16**	**20**
Origin	**1**	0	200	1000	500	1000	500	200	200
	5	200	0	500	1000	500	1000	200	200
	7	200	200	0	200	100	200	200	200
	9	200	200	200	0	200	100	200	200
	12	200	200	100	200	0	200	200	200
	14	200	200	200	100	200	0	200	200
	16	200	200	1000	500	1000	500	0	200
	20	200	200	500	1000	500	1000	200	0

TABLE E.2: Base Composite Generalised Travel Time Matrix (in minutes) corresponding to Base Matrix in Table E.1 used in Example 2 of Chapter 9 (see Section 9.4) for $\theta = 10$.

		Destinations							
		1	**5**	**7**	**9**	**12**	**14**	**16**	**20**
Origins	**1**	0	31.51	25.29	34.80	31.14	40.64	23.79	48.28
	5	31.51	0	34.80	25.29	40.64	31.14	48.28	23.79
	7	7.74	17.18	0	9.72	5.93	15.49	13.57	22.98
	9	17.18	7.74	9.72	0	15.49	5.93	22.98	13.57
	12	13.57	22.98	5.93	15.49	0	9.72	7.74	17.18
	14	22.98	13.57	15.49	5.93	9.72	0	17.18	7.74
	16	23.79	48.28	31.14	40.64	25.29	34.80	0	31.51
	20	48.28	23.79	40.64	31.14	34.80	25.29	31.51	0

Appendix F

Abbreviations

ASC Average Social Cost.

BATNA Best Alternative To a Negotiated Agreement.

BBC British Broadcasting Corporation.

BLPP Bilevel Programming Problem.

BOT Build-Operate-Transfer.

CBD Central Business District.

CBO Congressional Budget Office.

CCA Cutting Constraint Algorithm.

CEC City of Edinburgh Council.

CP Complementarity Problem.

CPU Central Processing Unit.

DE Differential Evolution.

DEBLP Differential Evolution for BiLevel Programming.

DfT Department for Transport.

DUE Deterministic User Equilibrium.

EA Evolutionary Algorithm.

EC European Commission.

EEU Eurasian Economic Union.

EPEC Equilibrium Problem with Equilibrium Constraints.

EU European Union.

FHWA US Federal Highway Administration.

FPI Fixed Point Iteration.

GA Genetic Algorithm.

HA Highways Agency.

HOT High Occupancy/Toll.

hr hour.

IOA Iterative Optimisation Algorithm.

LCP Linear Complementarity Problem.

LNE Local Nash Equilibrium.

mins minutes.

MOEA Multiobjective Evolutionary Algorithm.

MOP Multiobjective Problem.

MOPEC Multiobjective Optimisation Problem with Equilibrium Constraints.

MOSADE MultiObjective Self Adaptive Differential Evolution.

MPB Marginal Private Benefit.

MPC Marginal Private Cost.

MPEC Mathematical Program with Equilibrium Constraints.

MR Marginal Revenue.

MSA Method of Successive Averages.

MSB Marginal Social Benefit.

MSC Marginal Social Cost.

NBPEC Nash Bargaining Problem with Equilibrium Constraints.

NBS Nash Bargaining Solution.

NCEPEC Non-Cooperative Equilibrium Problem with Equilibrium Constraints.

NDEMO Nash Domination Evolutionary Multiplayer Optimisation.

NE Nash Equilibrium.

OD Origin Destination.

pcu Passenger Car Units.

PFI Public Finance Initiative.

PPP Public Private Partnership.

RHS Right Hand Side.

SBTP Second Best Toll Pricing.

secs seconds.

SI Synchronous Iterative.

SLCP Sequential Linear Complementarity Problem.

SOL solution of.

SUE Stochastic User Equilibrium.

TEN-T TransEuropean Network for Transport.

TfL Transport for London.

TRB Transportation Research Board of the US National Academy of Science.

UK United Kingdom.

US United States.

VI Variational Inequality.

VIP Variational Inequality Problem.

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