Uncertainty and Interpretability Studies in Soft Computing with an application to Complex Manufacturing Systems

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This Thesis is dedicated to my parents, my brothers and my sister.

Because family represents the very first source of strength for anybody.

To the memory of Ines Carbajal Rodriguez.

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Abstract

In systems modelling and control theory, the benefits of applying neural networks have been extensively studied. Particularly in manufacturing processes, such as the prediction of mechanical properties of heat treated steels. However, modern industrial processes usually involve large amounts of data and a range of non-linear effects and interactions that might hinder their model interpretation. For example, in steel manufacturing the understanding of complex mechanisms that lead to the mechanical properties which are generated by the heat treatment process is vital. This knowledge is not available via numerical models, therefore an experienced metallurgist estimates the model parameters to obtain the required properties. This human knowledge and perception sometimes can be imprecise leading to a kind of cognitive uncertainty such as vagueness and ambiguity when making decisions. In system classification, this may be translated into a system deficiency - for example, small input changes in system attributes may result in a sudden and inappropriate change for class assignation.

In order to address this issue, practitioners and researches have developed systems that are functional equivalent to fuzzy systems and neural networks. Such systems provide a morphology that mimics the human ability of reasoning via the qualitative aspects of fuzzy information rather by its quantitative analysis. Furthermore, these models are able to learn from data sets and to describe the associated interactions and non-linearities in the data. However, in a like-manner to neural networks, a neural fuzzy system may suffer from a lost of interpretability and transparency when making decisions. This is mainly due to the application of adaptive approaches for its parameter identification.

Since the RBF-NN can be treated as a fuzzy inference engine, this thesis presents several methodologies that quantify different types of uncertainty and its influence on the model interpretability and transparency of the RBF-NN during its parameter identification. Particularly, three kind of un-

certainty sources in relation to the RBF-NN are studied, namely: entropy, fuzziness and ambiguity.

First, a methodology based on Granular Computing (GrC), neutrosophic sets and the RBF-NN is presented. The objective of this methodology is to quantify the hesitation produced during the granular compression at the low level of interpretability of the RBF-NN via the use of neutrosophic sets. This study also aims to enhance the disitnguishability and hence the transparency of the initial fuzzy partition. The effectiveness of the proposed methodology is tested against a real case study for the prediction of the properties of heat-treated steels.

Secondly, a new Interval Type-2 Radial Basis Function Neural Network (IT2-RBF-NN) is introduced as a new modelling framework. The IT2-RBF-NN takes advantage of the functional equivalence between FLSs of type-1 and the RBF-NN so as to construct an Interval Type-2 Fuzzy Logic System (IT2-FLS) that is able to deal with linguistic uncertainty and perceptions in the RBF-NN rule base. This gave raise to different combinations when optimising the IT2-RBF-NN parameters.

Finally, a twofold study for uncertainty assessment at the high-level of interpretability of the RBF-NN is provided. On the one hand, the first study proposes a new methodology to quantify the a) fuzziness and the b) ambiguity at each RU, and during the formation of the rule base via the use of neutrosophic sets theory. The aim of this methodology is to calculate the associated fuzziness of each rule and then the ambiguity related to each normalised consequence of the fuzzy rules that result from the overlapping and to the choice with one-to-many decisions respectively. On the other hand, a second study proposes a new methodology to quantify the entropy and the fuzziness that come out from the redundancy phenomenon during the parameter identification.

To conclude this work, the experimental results obtained through the application of the proposed methodologies for modelling two well-known benchmark data sets and for the prediction of mechanical properties of heat-treated steels conducted to publication of three articles in two peer-reviewed journals and one international conference.

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Nomenclature

Roman Symbols

$\hat{a}_i$	Ambiguity produced by the ith fuzzy rule
$\hat{S}$	Similarity matrix
$\\| \cdot \\|$	Euclidean norm
$\tilde{A}$	Embedded Type-2 Fuzzy Set
^{-}a	Lower boundary of the non-standard set a
A	Embedded type-1 Fuzzy Set
b^{+}	Upper boundary of the non-standard set b
$C(m)$	Confusion
C_A	Centroid of a Type-1 Fuzzy Set
$C_{\tilde{A}}$	Centroid of a Type-2 Fuzzy Set
$compat(\cdot, \cdot)$	Compatibility between any two Fuzzy sets
$E(m)$	Dissonance
e_H	Entropy
E_i^p	Overall Fuzziness of the Receptive Unit ith
E_{RMS}	RMS Error
F	Falsity
$f(\cdot)$	Activation function
fe_k^i	dimensional kth Fuzziness of ith fuzzy set
G^i	Fuzzy consequence
$G_{1,2}$	Global uncertainty of a system
h_i^1	Upper height of the ith fuzzy rule
h_i^2	Lower height of the ith fuzzy rule
I	Indeterminacy
$i_{i\cup j}$	Indeterminacy produced by the fusion of the granules i and j
J_x	Primary membership
$L_{A,B}$	Multidimensional length of a resulting granule from the union between A and B

m_k^i	kth mean of the i fuzzy rule
N	Clusters's matrix
N	Total number of input-output data pairs
$N_i(\cdot)$	Neutrosophic index for measuring the disorder throughout the granulation process
T	Truth
$T(m)$	Total uncertainty
u_x	Secondary variable-$u \in j_x$
$V(m)$	Innate contradiction
w_i	Output fuzzy weight
w_l^i	ith Left output weight
w_r^i	ith right output weight
x_p	Input vector
x_{nor}	Normalised input vector
y_p	Model output
Greek Symbols	
α	Learning rate
γ	Momentum
μ_i	Membership Function of type-1 of the set A
$\mu_{\tilde{A}}(x,u)$	Secondary membership
μ_i	mean vector of the ith fuzzy rule
P_p	Performance index
μ_{Ov}	Overlapping coefficient
σ_i^2	variance of the ith fuzzy rule

Acronyms

ANN	Artifitial Neural Network
RBF-NN	Radial Basis Function Neural Network.
BBA	Basic Belief Assignments.
IT2-RBF-NN	Interval Type-2 Radial Basis Function Neural Network.
FCM	Fuzzy C-Means.
GrC	Granular Computing.
IT2-FS	Interval Type-2 Fuzzy Set.
MF	Membership Function.
FS	Fuzzy Set.
FLS	Fuzzy Logic System.
MISO	Multiple-Input-Single-Output.
MIMO	Multiple-Input-Multiple-Output.
T2-FS	Type-2 Fuzzy Set.
IFL	Intuitionistic Fuzzy Logic.
T1-FS	Type-1 Fuzzy Set.
FOU	Footprint Of Uncertainty.
NL	Neutrosophic Logic.
MCP	McCulloch-Pitts Model
MLP	Multilayer Perceptron
RU	Receptive Unit

1 INTRODUCTION

UNCERTAINTY is frequently found in real situations and it usually represents a deficiency in the information [Klir and Wierman, 1999]. That means, uncertainty is part of the world and in systems is inevitable as it appears in almost every measurement; either as a consequence of the theoretical framework used for quantifying it or due to the combination of measurement errors and resolution limits of the measuring instruments [Klir and Wierman, 1999, Liu, 2004]. For this reason, the correct processing and quantification of information becomes vital when its understanding involves the knowledge of uncertain events.

For more than two hundred years, the study and understanding of uncertainty has been a pivotal issue in order to make decisions and create models that imitate the human reasoning when dealing with real complex systems. Furthermore, there has been a controversy about which is the best mathematical framework that is capable of capturing and then faithfully characterising situations under uncertainty. It is generally believed that the very first studies associated to uncertainty began in the seventeenth century with Fermat and Pascal who laid the fundamental groundwork of probability theory by deriving the exact probabilities for problem gambling. Subsequently, in 1931 a significant contribution from Von Mises who proposed the concept of sample space initialised the field of applied mathematics by unifying the fields of probability and measure theory. Next, this idea was overtaken by Kolmogorov's axiomatisation which laid the foundations of modern probability theory. In addition to this, different points of view for capturing the uncertainty were proposed in the twentieth century. On the one hand, L. Zadeh [Zadeh, 1965, 1968] proposed the concept of fuzzy set denoted by a class of objects where each element in the set is characterised by a degree of membership in the closed interval $[0, 1]$. This type of theory was proposed to deal with uncertainty that comes out from approximate reasoning. On the other hand, Dempster, 1967 and Shafer, 1976 [Shafer, 1976] introduced the theory of evidence which allows to make decisions based on the available evidence collected from different sources. As an extension to fuzzy set theory, L. Zadeh introduced in 1978 the possibility theory [Negoita et al., 1978] which is devoted to handling of incomplete information [Dubois and Prade, 1988] and it is considered within fuzzy set theory an alternative to probability. In 2002, Liu developed a new branch of mathematics devoted to unify the concept of uncertainty under a generic framework used in order to study the behaviour of random, fuzzy and rough events [Liu, 2004]. The question of which is the best theoretical framework to quantify and describe uncertainty within these theories is highly difficult to answer. Because it is clear that several types of uncertainties exist and hence it is also clear that uncertainty is multidimensional. That means that usually the quantification of uncertainty is problem-dependent and if this quantification is just conceived in terms of only one theory, its multidimensional nature is obscured [Klir and Wierman, 1999]. As it is pointed out by the theorem of *Gödel*, mathematics is not immune to uncertainty.

In systems engineering especially in systems modelling and making decision, the understanding about the nature of uncertainty has drawn a lot of attention from some practitioners and researches in the last three decades. This is mainly due to the increasing interest for understanding the influence that each model component and the associated parameters have for contributing with an uncertain and indeterminate system behaviour in the output model. Usually, uncertainty can be catalogued into Aleatory uncertainty and epistemic uncertainty. The former is originated by the system variability which reflects the inherent randomness of the nature. This type of uncertainty never disappears by collecting more information and sometimes can be also referred as random uncertainty, stochastic uncertainty, real-world uncertainty or natural variability. The latter arises as a lack of knowledge of the physical of world and a lack of measuring and modelling the physical world [Li et al., 2013]. Therefore, uncertainty in system engineering can be attributed to different sources, i.e.

- Parameter uncertainty. This type of deficiency comes from the model parameters which sometimes are estimated in advanced representing an input to the the mathematical model. For example when simulating the dynamic of a car during a crash, an important parameter is the initial car speed.

- Parametric variability. This type of uncertainty is produced by the variability of input variables of model. An example is when monitoring and estimating the final flow acceleration of a liquid in a pipeline; this prediction can be inaccurate since the stochastic behaviour of the liquid contained the pipeline and the environmental conditions add some uncertainty.

- Structural uncertainty. This source frequently comes out when we are uncertain about the functional form of the model, and hence it produces a deficiency in order to reflect properly reality producing an uncertainty about a adequate data processing.

- Algorithmic uncertainty. This is a numerical uncertainty that results from numerical errors and numerical approximations by the implementation of a computer model.

In this context, two major quantification problems are usually found in literature, i.e. 1) forward uncertainty propagation and 2) inverse uncertainty propagation. The former aims to quantify the uncertainty propagated from uncertain inputs in the system outputs. For example the evaluation of low-order moments of the outputs such as mean and the standard deviation, the evaluation of the output reliability based on the system performance, and the assessment of the probability distribution of the output model. The latter quantification basically aims to evaluate the discrepancy (called bias correction) between the results obtain from a mathematical model and experimental results.

As can be seen, the understanding of uncertainty in system engineering can be accredited to several factors. For this reason transparency and interpretability play an important role for a good system knowledge. In other words, the more interpretable the information of a system under study, the better its understanding. Particularly in system modelling, extracting information and converting it to 'easy to interpret' knowledge is a crucial but not a trivial task, especially in the case of modelling very complex systems and non-linear processes [Zhou and Gan, 2006, 2008]. Conventional approaches that are usually based on differential equations to system modelling offer a poor performance when modelling complex and uncertain systems.

In order to gain insights of the system being modelled (to a certain degree), fuzzy modelling has shown to be an effective and a popular tool since it can formulate the system behaviour by qualitatively expressing the system knowledge with linguistic rules in a transparent and interpretable way rather by a quantitative analysis [Kandola, 2001]. That means, a fuzzy model is fully transparent if it is possible to identify, understand and analyse the influence of each system parameter in the model output. Particularly, transparency is a measure used to validate how reliable and accurate are the linguistic rules and hence the associated fuzzy sets necessary to make a fuzzy system an interpretable model. In this regard, in literature efforts for creating fuzzy systems with a good balance between interpretability and accuracy have been proposed. As pointed out in [Casillas, 2003], one of the main objectives in fuzzy modelling is to construct models that have a good balance between accuracy and interpretability. However, this is a contradictory purpose as not always this balance can be achieved. Basically, the reasons of having fuzzy models with a high degree of accuracy and low degree of interpretability or viceversa depends mainly on what requirements are pursued.

In the specialized literature, some researchers have created and studied systematic rule-based systems that are functionally equivalent to fuzzy logic systems and neural networks citarrr. Particularly, the Radial Basis Function Neural Network (RBF-NN) has shown to be a prominent architecture to modelling complex systems in system identification and control. The merits of the RBF-NN is that inherits some significant properties from fuzzy systems such as the ability to model systems via the use of linguistic rules which can be generated based on some prior human expert knowledge or heuristics. However, opposite to fuzzy systems, the RBF-NNs suffer from some loss of interpretability and hence transparency as a consequence of the learning process which is usually carried out through the use of gradient descent-based approaches. The analysis of this deficiency in transparency and hence in interpretability might aid to improve the RBF-NN performance and then reduce its black-box properties.

Relevance contributions by using the RBF-NN as a fuzzy system can be found in literature [Chen and Linkens, 2001b, Cho and Wang, 1996, Jang and Sun, 1993, Nelles, 2001]. Specifically, in manufacturing processes the reputation of RBF-NNs for system identification have been extensively exploited [Raviram et al., 2009, Wu et al., 2010]. For instance, in the aerospace industry neural fuzzy systems have been applied to acquire a relationship between the mechanical properties of a titanium alloy and the processing parameters involved for its heat treatment [Yu et al., 2010]. These type of processes represent in the manufacturing industry a highly difficult challenge since expert knowledge is often of very high importance to fulfil the production requirements dictated by the customers. Therefore, models constructed from data such as the RBF-NN falls into the interpretability scrutiny of experts in order to confirm the system's validity [Panoutsos and Mahfouf, 2010a]. Furthermore, the black-box properties of the RBF-NN hinders its interpretability due to a lack of transparency. For this reason, some authors have developed methodologies whose main objective is to achieve a good level of interpretability without losing accuracy . In literature, the existing research work in improving the interpretability in neural fuzzy systems have been focused on creating systematic data-driven structures that usually includes the initial model self-generation, input selection process, partition validation, parameter optimisation and rule-base simplification . Compared to neural networks and fuzzy systems, a neural fuzzy model (for example the RBF-NN) posses the ability to approximate any real nonlinear function by explicit knowledge representation in the form of if-then rules, the ability to mimic cognitive reasoning in human understandable terms, the facility for processing linguistic information from humans and then combine it with numerical data. Even so, neural fuzzy systems inherit the shortcoming of being black-box models and therefore the criticism of not providing any information of how they work [Benítez et al., 1997].

Although in literature a large number of publications in relation to transparency and interpretability in fuzzy systems can be found, an small number of articles address the problem of transparency and hence the associated uncertainty created by this lack of interpretability in neural structures, particularly in RBF networks. A significant amount of methodologies dealing with approximate and uncertain reasoning can be listed in soft computing theory. This means that neural fuzzy properties can be studied not only from the existing theory in neural networks and fuzzy systems, but also from the new developments in computational intelligence ranging from evolutionary computing, fuzzy uncertainty, possibility theory, intuitionistic sets theory, interval type-2 fuzzy sets, computing with perceptions, etc. For instance, in [Pal and Bezdek, 1994] a review of the existing uncertainty measures is provided. In that article, all the merits and drawbacks for applications are discussed. Basically the type of uncertainty treated in fuzzy sets theory deals with situations where the set boundaries are not sharply defined. Moreover, in [Pal and Bezdek, 1994] probabilistic uncertainty is sometimes related to fuzziness in the sense of the belongingness of elements or events to crisp sets giving a higher dimensional meaning to probability theory in fuzzy sets theory.

Among the latest and general proposals to deal with uncertainty in fuzzy logic is intuitionistic sets logic [Atanassov, 1986]. This theory was proposed by Atanassov as a new branch of fuzzy logic that represent the uncertainty of rules and facts through the association of falsity and truth to two different values. In other words, this type of analysis can be translated into a problem for quantifying the uncertainty propagation through the inference engine which is employed in fuzzy logic systems to make decisions.

Another good example, is neutrosophy which is a generalisation of fuzzy logic that deals with the "*origin and scope of neutralities as well as their* *interactions with other spectra*" [Wang et al., 2005]. This new logic is based on the infinitesimal calculus in order to use tuples that associate the truth, indeterminacy/uncertainty and falsity to an event. Different successful applications of neutrosophy can be found in literature. Particularly these applications proposed the analysis and quantification of uncertainty in neural networks by the exploration and exploitation of soft computing techniques. For instance, in [Kraipeerapun et al., 2007] P. Kraipeerapun introduced a new framework based on ensemble neural networks and interval neutrosophic sets for binary classification. The purpose of that study was to quantify the associated error and vagueness (uncertainty) during the process of classification. In [Kharal, 2014] the author introduced a new neutrosophic multicriteria decision making method (MCDM) in which the mathematical foundations of neutrosophy sets theory was successfully applied for classification purposes.

Quite recently, some researches have explored the advantages of neural fuzzy systems of type-2 with the view of quantifying the linguistic uncertainty that is not handled by the fuzzy sets of type-1. A good example was provided in [Castro et al., 2011], where a novel integration of an interval type-2 fuzzy inference system based the Takagi-Sugeno-Kang reasoning and an adaptive network was introduced. In that work, the authors created a hybrid methodology capable of dealing with uncertainty that resulted from the imprecision during the parameter identification.

In system modelling, the understanding and then the quantification of uncertainty can be carried out by the use of existing methodologies in soft computing. Particularly the uncertainty that result from improper data, bad modelling as a consequence of wrong interpretations or human mistakes, imprecision originated by language granularity, vagueness and inconsistency which result by redundant linguistic rules producing conflict and hence contradictions.

1.1 PROBLEM STATEMENT

In soft computing several theories have been proposed in order to deal with various types of incomplete and uncertain information. Particularly, fuzzy logic and probability theory might be seen as the main mathematical frameworks dealing with uncertainty [Li et al., 2013]. Moreover, the unification of two or more different methodologies to quantify uncertainty has become a popular tool in soft computing literature. For instance, in [Kocadağlı and Aşıkgil, 2014] a new evolutionary Monte Carlo algorithm was introduced in order to train a Bayesian neural network for the time series forecasting of weekly sales of a finance magazine.

In [Denoeux, 2000] a new classifier based on a multilayer neural network and on the Dempster-Shafer theory of evidence was introduced. On the one hand, the authors proposed an specific architecture based on an input layer, two hidden layers and one output layer to evaluating the patterns as evidence and then presenting them as Basic Belief Assignments (BBA) which are pooled using the Dempster's rule combination. On the other hand, the methodology performance was compared to different statistical and neural network techniques.

The authors in [Kraipeerapun et al., 2006] proposed a systematic procedure based on two different frameworks to quantify the uncertainty in mineral prospectivity. The main purpose of that study is to construct a methodology based on three neural networks in order to estimate the associated truth, uncertainty and falsity when predicting the degrees of favourability for gold deposits. Furthermore, researches in the area of statistics have paid a lot of attention in constructing simple and more transparent systems from the perspective of complexity reduction. Particularly, in achieving a trade-off between complexity reduction and how well the system prediction is during the training process. Methodologies such as support vector machines [Smola and Schölkopf, 2004, Suykens and Vandewalle, 1999, Vapnik, 2000], orthogonal least squares [Chen et al., 1991] and input selection [Zhang et al., 2004] have shown to be an excellent tool for complexity reduction while preserving transparency and interpretability in system modelling.

More importantly, such methodologies have proved to enhance fuzzy interpretability when applied in fuzzy modelling.

More examples can be found in literature, especially in the sense of improving trade-off between accuracy and interpretability of fuzzy rule-based systems by using adaptive learning methodologies from neural networks theory and single and multi-objective evolutionary approaches [Ishibuchi and Nojima, 2007, Ishibuchi and Yamamoto, 2004, Pulkkinen and Koivisto, 2008]. On the one hand, in the 1990s efforts were focused on improving the accuracy more precisely in system modelling and control theory. Particularly, an emphasis on accuracy maximisation [Wang, 1992] was placed by the application of evolutionary techniques whose cost was a lack in transparency and hence the complexity of such systems increased importantly [Cordón et al., 2001]. On the other hand, in the last decade; various methodologies for designing interpretable fuzzy models which are constructed from data were conducted. First, it was suggested to consider the structure of a fuzzy model as a twofold taxonomy in order to discriminate the role of each component associated to the fuzzy model interpretability.

In this context in [Zhou and Gan, 2008], a deep insight of the different components involved in achieving an interpretable fuzzy model were classified into two different levels: a) low-level interpretability and b) high-level interpretability. The former refers to the optimization of the membership functions in terms of semantic criteria related to a fuzzy set level and the latter involves the interpretability associated to coverage, completeness and consistency of the rules in terms of the criteria on fuzzy rule level leading the complexity reduction to a moderate number of rules and their associated consistency.

In spite of the large number of research works that have been proposed for evaluating the interpretability in fuzzy systems, this issue is still an open field in neural fuzzy systems theory. Moreover, a reduced number of attempts can be found in relation to the importance of evaluating the uncertainty and its association with fuzzy interpretability in neural fuzzy systems. This limitation can be translated into an appealing field to be explored because having an interpretable model allows us to incorporate to it prior or expert knowledge.

Particularly, in RBF-NN modelling there is a lost of transparency and hence of interpretability that results from the application of adaptive algorithms used for the associated parameter identification. In this sense, this deficiency may produce a grade of uncertainty that might be expressed into several mathematical frameworks. Such an uncertainty can affect the interpretability of the RBF-NN and therefore its transparency and performance.

Since an RBF-NN can be seen as a type of fuzzy system, this research work addresses the issue of uncertainty quantification and its relationship with system interpretability during the parameter identification of the RBF-NN. Especially, the functional equivalence between the RBF-NN and fuzzy systems allows us to explore and exploit a significant number of existing soft computing tools for uncertainty quantification and the evaluation of system interpretability. For this reason, a group of different soft computing tools will be studied and then used for the uncertainty evaluation, including Neutrosophic sets theory, GrC, IT2-FSs and uncertainty theory. Due to the nature of the system considered in this research work, the following types of cognitive uncertainty are suggested to be studied:

- Linguistic uncertainty
- Fuzziness
- Entropy
- Ambiguity
- Uncertainty produced as a consequence of the redundancy among the fuzzy sets.

Finally, such uncertainty studies are tried against a real case study and well known benchmark data sets for manufacturing processes with particular application in the prediction of mechanical properties of heat-treated steels.

1.2 RESEARCH AIMS

The aim of this research work is to quantify the uncertainty produced during the parameter identification of the RBF-NN for modelling purposes - and to study the relationship between this uncertainty and the interpretability of the RBF-NN. This research work also suggests to take advantage of the functional equivalence between the RBF-NN and fuzzy systems of type-1 for exploiting and exploring alternative tools from soft computing in order to quantify the network uncertainty and extract information from the associated interpretability.

Basically this study consists of the identification and analysis of different sources of uncertainty in the RBF-NN at two different levels of interpretability, i.e. a) at low-level of interpretability and b)at high-level of interpretability. Therefore, the major aims of this research work can be listed as follows:

- The first study aims to identify and quantify the uncertainty due to a ravenous behaviour that results from a granular inclusion throughout the granulation process which is employed for the initial parameter identification of the RBF-NN. Hence, an index is suggested to handle and minimise this type of uncertainty having an impact in the creation of a more parsimonious fuzzy rule base.

- Secondly, an interval type-2 RBF neural network (IT2-RBF-NN) and the corresponding parameter identification process are suggested in order to deal with the linguistic uncertainty that is associated to the interpretation of words and linguistic propositions contained in the fuzzy rule base. The intention of this study is to execute a group of simulations for evaluating the performance of the proposed IT2-RBF-NN with two different types of clustering approaches, i.e. a) Fuzzy C-Mean (FCM) and b) Granulation on the one hand. On the other hand, the objective of this architecture is to explore the benefits of computing with words by dealing with the uncertainty that results from the semantic framework.

- Finally this research work aims to provide a twofold study that consists in the interpretation of two types of fuzzy uncertainty measures based on the fuzzy entropy and the ambiguity produced during the parameter identification of the RBF-NN and the proposed IT2-RBF-NN architecture. The first study evaluates the information contained at each receptive unit and hence suggests the use of neutrosophic sets theory to develop a methodology capable of enhance the RBF-NN interpretability. The last study, suggests a similarity measure that quantifies two types of fuzzy uncertainty in relation to the redundancy between the fuzzy rules; i.e. a) Fuzziness and b) ambiguity.

1.3 CONTRIBUTIONS

The main contribution of this research work is to provide a number of different methodologies for uncertainty quantification based on the interpretability of the RBF-NN during its associated parameter identification. Such methodologies allow us to create a more transparent neural fuzzy model based on the RBF-NN. Under these circumstances, it is possible to evaluate the distinguishability and then the interpretability of the RBF-NN. Such methodologies also allow us to investigate the RBF-NN performance based on fuzzy uncertainty theory and its association to a good trade-off between accuracy and interpretability during the parameter identification of the network. Basically, a number of uncertainty studies will be presented according to two main levels of interpretability i.e. a) high-level of interpretability and b) low-level of interpretability. Therefore, the main contribution of this research work can be listed as follows:

- In chapter 3, a systematic neural fuzzy modelling based on the Fuzzy C-Means (FCM)and neural networks is used in manufacturing processes with an special application for impact energy prediction on heat-treated steels using a data set collected at six different labs.

- In Chapter 4 a twofold contribution is provided; firstly, it is proposed the application of a systematic modelling framework based on the RBF-NN and Granular Computing (GrC) for modelling a real case study in manufacturing processes. The modelling framework was initially developed in [Panoutsos and Mahfouf, 2010a] and then was successfully applied in this research work to exploit the advantages of granulation enhancing the transparency of the initial rule base at the low-level of interpretability of the RBF-NN [Zhou and Gan, 2008]. Secondly, a new clustering approach based on granulation and neutrosophic sets was introduced. This study investigates the significance of each input by evaluating the distinguishability of the fuzzy rules during the initial clustering stage. Moreover, a new compatibility criteria is developed in order to measure the uncertainty produced by a ravenous behaviour that results from the overlapping between the fuzzy rules. Finally, experimental results were run in order to compared the performance of the granulation with and without the application of neutrosophic sets.

- In chapter 5, an Interval Type-2 Radial Basis Function Neural Network (IT2-RBF-NN) is proposed. Such a framework is functionally equivalent to Interval Type-2 Fuzzy Systems and the RBF-NN. The major contribution of this network is twofold - first the IT2-RBF-NN not only provides a new methodology for dealing with linguistic uncertainty and then with perceptions, but also in a like-manner to its type-1 counterpart, the IT2-RBF-NN interpretability can be treated at two different levels of linguistic information. Secondly, this chapter also provides the corresponding parameter identification of the new IT2-RBF-NN which is different to that used for training its type-1 counterpart.

  A further experimentation was carried out in order to verify the model performance of the IT2-RBF-NN and then compared to its counterpart the RBF-NN or as it is called here the RBF-NN. Therefore, some results for modelling some popular benchmark data sets and the real case study employed in chapter 4 are provided respectively. With conclusive evidence, the simulation results showed the RBF-NN might be a prominent tool to cope with linguistic uncertainties and then perceptions.

- Finally in Chapter 6, two studies about fuzzy uncertainty quantification during the parameter identification of the RBF-NN and the proposed IT2-RBF-NN are provided. Firstly, the proposed methodology exploits and explores the functional equivalence between the RBF-NN and a number fuzzy logic systems of type-1 [Hunt et al., 1996]. Thus, two new uncertainty measures based on neutrosophic sets and used to evaluate the fuzziness and ambiguity in the rule base of the RBF-NN are introduced. Such measures allows the RBF-NN to evaluate on the one hand the distinguishability in the rule base, and on the other hand the ambiguity that comes out from selecting one choice among different options in the RBF-NN rule base. The second part of this chapter contributes with an study about the relationship between the similarity of fuzzy sets and the uncertainty associated to the fuzzy rules redundancy in both the RBF-NN and the proposed IT2-RBF-NN. That means, in the time this study estimates the similarity between the shape and distance of the fuzzy sets involved in the rule base, a similarity matrix is being constructed in order to evaluate the uncertainty associated to the redundancy of each of those fuzzy sets.

The work in this thesis has contributed in part or full to the following publications and revisions:

- Rubio Solis, A. and Panoutsos, G. "Interval Type-2 Radial Basis Function Neural Network: A modelling framework". IEEE Transactions on Fuzzy Systems, 11(99), 2014.
- AR Solis, G. Panoutsos, "Granular computing neural-fuzzy modelling: A neutrosophic approach", Applied Soft Computing, Elsevier, 13(9), 4010-4021, 2013.
- Rubio Solis, Adrian, Panoutsos, G., "Fuzzy Uncertainty Assessment in RBF Neural Networks using Neutrosophic Sets for Multiclass Classification", 2014 IEEE International Conference on Fuzzy Systsems, Beijing China, 2014.

Journal Papers in Preparation:
(To be submitted to Materials Science and Technology and Soft Computing respectively.)

- Performance of the Interval Type-2 Radial Basis Function Neural Network in Materials Science.
- Interpretability aspects when computing with words: An Especial Application for the Prediction of Mechanical Properties of Heat-treated Steels

Other activities

- Invited reviewer on the topic: Rough sets Theory for the International Journal of Machine Learning and Cybernetics.
- Invited reviewer on the topic: Granular computing and Rough sets Theory for the International Journal of Machine Learning and Cybernetics, Springer.

1.4 OUTLINE OF THE THESIS

The structure of this thesis is organised in 7 chapters and one appendix. In this chapter the basic notions necessary to understand the contributions of this research work are described. The next 6 chapters describe the current contributions and the conclusion of this thesis. Therefore, the document is organised as follows:

Chapter 2, covers the main soft computing techniques that may be useful to deal with uncertainty in systems modelling. These include a general review of Fuzzy Sets (FS) theory, including theory related to Fuzzy Systems (FSs), Granular Computing and the modus ponens or inference mechanism which is crucial to understand how a Fuzzy system handles the information. Secondly, an uncertainty-based information theory for crisp and fuzzy sets is reviewed as it may play an important role in the development of this research. Finally, this chapter briefly reviews the theory of artificial neural networks, particularly that information related to RBF neural networks making reference to its functional equivalence to fuzzy systems.

Chapter 3 includes on the one hand a detailed description of heat treatment process from a metallurgical point of view and manufacturing processes. Details on the mechanical, physical and chemical properties of ferrous and non-ferrous materials were included. Consequently, an overview of steel making and of mechanical testing for materials is reported. On the other hand, a data-driven modelling framework based on the RBF-NN theory and Fuzzy C-Means (FCM) was applied for the prediction of mechanical properties of heat-treated steels in manufacturing processes. The realisation of a systematic model based on neural fuzzy systems aims to mimic the human reasoning ability to express complex system with simple linguistic rules. Finally, experimental results were accounted graphically and numerically.

Chapter 4 is concerned with enabling the RBF-NN for extracting information in a more distinguishable form by the use of granular computing (GrC) and the quantification of uncertainty through the application of neutrosophic sets. An initial experimentation was carried out to investigate the RBF-NN performance with the aid of granulation [Panoutsos and Mahfouf, 2010a]. The aim of this experimentation was to predict transparently the initial rule base of the RBF-NN and for the prediction of the mechanical properties of heat-treated steels. The associated parameter identification process of the RBF-NN model was firstly estimated by the granulation (GrC) of input raw data and consequently optimised by the application of a gradient-descent based approach. The main role of the granulation process was to generate the initial fuzzy rule base of the RBF-NN according to the compability of the input data. A new compatibility criteria that quantifies the uncertainty during the granulation process and that is a consequence of an excessive overlapping between the fuzzy sets during the formation of the rule base was proposed. Particularly, this new compatibility measure was used as a granular constraint for evaluating the interpretability throughout the granulation process.

In Chapter 5 concentrates in the development of a systematic data-driven modelling based on the RBF-NN and Interval Type-2 Fuzzy Sets for systems modelling purposes; such a methodology was named: Interval Type-2

Radial Basis Function Neural Network (IT2-RBF-NN). The aim of the proposed network structure is for dealing with the linguistic uncertainty that is not quantified by its type-1 counterpart the RBF-NN. More importantly, this new structure was developed to inherit not only the properties of the RBF-NN and fuzzy systems of type-2, but also to be susceptible to the parameter estimation employed in the RBF-NN. Therefore, the associated parameter identification process that is able to deal with interval fuzzy sets is also developed. Experimental results by using the proposed IT2-RBF-NN are conducted through the modelling of two well-known benchmark data sets and the real case study for mechanical prediction of heat-treated steels proposed in this thesis.

Chapter 6 considers on the one hand a fuzzy uncertainty assessment methodology by using RBF neural networks and neutrosophic sets for multi-class classification. The idea of this methodology is to create a more transparent and interpretable training process that can explore and exploit the information contained at each receptive unit (RU) of the RBF-NN. On the one hand, a neutrosophic measure for quantifying the fuzziness among the fuzzy sets (RUs) is proposed. On the other hand, an assessment of ambiguity associated to the nonspecificity and representing a cognitive uncertainty based on neutrosophic sets is conducted. On the other hand, is provided an uncertainty assessment of ambiguity and entropy based on the similarity among fuzzy rules in the rule base either a) the RBF-NN, or b) the proposed IT2-RBF-NN architecture. And a comparison analysis of the uncertainty assessment suggested for impact energy prediction is provided. The similarity measure suggested in this chapter is based on the distance and shape of the receptive units functions.

Finally, chapter 7 includes a detailed conclusion of this research project and the future directions will also be discussed.

2 A BACKGROUND TO SOFT COMPUTING TECHNIQUES

THE main objective of this chapter is to provide an insight about the existing techniques found in soft computing. A particular emphasis will be put on Fuzzy Logic, Fuzzy modelling and Neural Networks. As far as Fuzzy Logic is concerned, a review of the different types of uncertainty is included. This is mainly due to the type of topic considered in this research work.

2.1 INTRODUCTION

The term soft computing was coined by Zadeh, the inventor of fuzzy set theory, to be an extension to fuzzy logic. Basically, soft computing is a partnership of several problem-solving paradigms such as fuzzy logic (FL), Probabilistic Reasoning (PR), Neural Networks (NNs) and Evolutionary Computation (EC) [Bonissone, 1997]. Moreover, this collection of different methodologies exploits the advantages of human tolerance for imprecision and uncertainty to achieve tractability, robustness, and low solution cost.

In this partnership, fuzzy logic is mainly concerned with imprecision and approximate reasoning; neural networks with learning and curve-fitting; and probabilistic reasoning with uncertainty and belief propagation [Seising, 2010]. In this regard, uncertainty plays an important role behind fuzzy logic and neural networks in dealing with information obtained from sources which are non-linear behaviour, time-varying behaviour and the interaction with uncertain/indeterminate environments.

2.2 FUZZY SETS

The concept of fuzzy sets was introduced and formalised by [Zadeh, 1965] as an extension of conventional set theory. The aim of fuzzy sets lies in modelling the impreciseness of human reasoning by representing uncertainty for the variables that are used by assignment of a set of values to the variable.

A crisp set usually represents a dichotomisation of individuals to be members or not into two groups in a given universe of discourse (which it is known as the domain of a function). However, many classification concepts suffer from the lack of this property, for example the group of tall people, sunny days or cheap cars. From a mathematical standpoint, the definition of a classical set of objects X is called the universe where its generic elements are denoted by x. Therefore a crisp set can be represented by the notation $X = \{x_1, x_2, \ldots x_n\}$ and defined by a property that is satisfied by its members: $X = \{x|P(x)\}$ where $P(x)$ is a proposition of the form "x *has the property* P". The membership in a classical subset A of X is usually viewed as the characteristic function μ_A from x to $\{0, 1\}$ Such that

$$\mu_A(x) = \begin{cases} 1, & \text{iff } x \in A \\ 0, & \text{iff } x \notin A \end{cases} \tag{2.1}$$

where $\{0, 1\}$ is the valuation set and the characteristic function $\mu_A \to \{0, 1\}$. A fuzzy set can be defined mathematically by assigning to each x over the universe of discourse a value representing its grade of membership in the fuzzy set. For example, a fuzzy set might represent the set of cloudy days with the maximum and minimum value of 1 and 0 to those days that are sunny and completely cloudy respectively. This means that values of 20% can be designated to those days that are partially cloudy. If the valuation set is allowed to be a real interval $[0, 1]$, A is called a fuzzy set, and $\mu_A(x)$ is the grade of membership of x in A. The closer the value of $\mu_A(x)$ to 1, the more x belongs to A, and where A is a subset of X that clearly has no sharp boundaries. From this notation, A is completely characterised by the set of pairs

$$A = \{(x, \mu_A(x)), x \in A\} \tag{2.2}$$

Zadeh proposed a convenient notation, where a fuzzy set of X is defined as

$$A = \sum_{i=1}^{n} \mu_A(x_i)/x_i \tag{2.3}$$

If X is not finite it is said to be:

$$A = \int_x \mu_A(x)/x \tag{2.4}$$

In fuzzy set theory, containment, union, intersection and complement are defined in terms of their MFs. Therefore, such definitions lead to the following expressions [Mendel, 1995].

Containment

$$A \subseteq B \Longleftrightarrow \mu_A(x) \leq \mu_B(x), \forall x \in X \tag{2.5}$$

Union

$$\mu_{A \cup B}(x) = max[\mu_A(x), \mu_B(x))], \forall x \in X \tag{2.6}$$

Intersection

$$\mu_{A \cap B}(x) = min[\mu_A(x), \mu_B(x))], \forall x \in X \tag{2.7}$$

Complement

$$\mu_{\overline{B}}(x) = 1 - \mu_B(x), \forall x \in X \tag{2.8}$$

However, the "max" and "min" are not the only operators which can describe union and intersection of fuzzy sets. Zadeh proposed two operators for union and intersection [Zadeh, 1965], namely: union based on the maximum and algebraic sum represented by $\mu_{A \cup B}(x) = \mu(A)(x) + \mu_B(x) - \mu_A(x)\mu_B(x)$ and intersection which is based on minimum and algebraic product and expressed by $\mu_{A \cap B} = \mu_A \mu_B$. Basically, the authors [Höhle, 1978] and [Alsina et al., 1983] were the pioneers that introduced the t-norm and the t-conorm into fuzzy set theory be the operations for the intersection and union of fuzzy sets. Since that, many other researches have proposed various types of t-operators. Particularly, in [Gupta and Qi, 1991] a review of the most prominent examples about t-norms is provided. Further contributions of t-norms and t-conorms which have axiomatic basis [Mendel, 1995] have been proposed and represented by the symbols $\star$ and $\oplus$ respectively. Examples of t-conorms (also known as s-norm) are *bounded sum*:$x \oplus y = min(1, x + y)$, *drastic sum* $x \oplus y = x$ if $y = 0$, y if $x = 0$, 1 if $x, y > 0$. And examples for the t-norm are *bounded product*: $x \star y = max[0, x + y - 1]$ and *drastic product*: $x \star y = x$ if $y = 1$, y if $x = 1$, and 0 if $x, y < 1$.

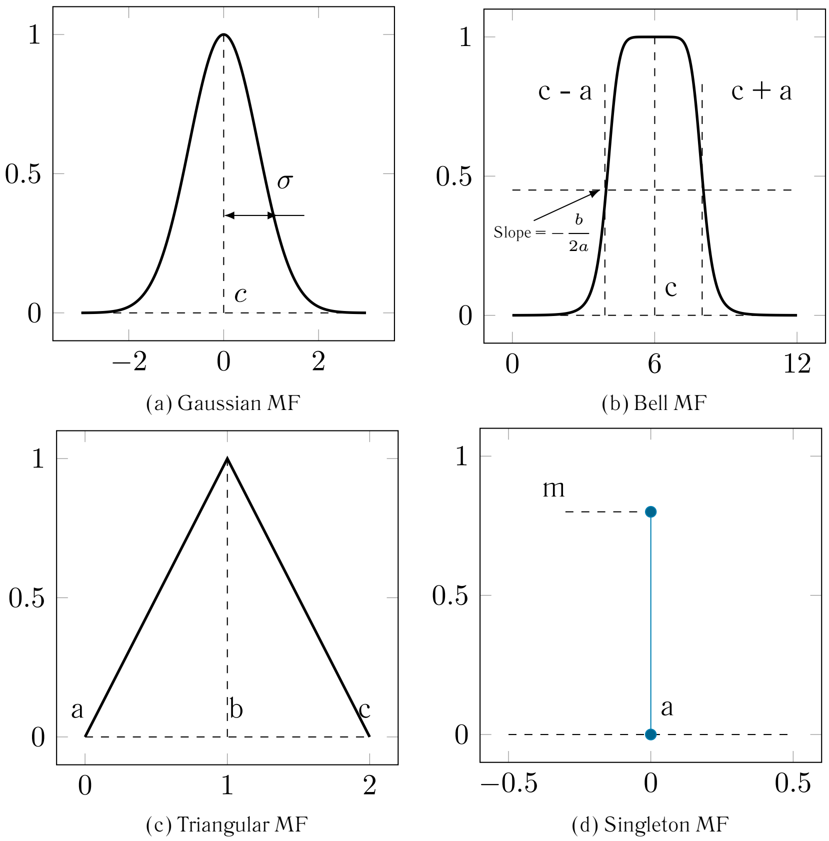


Fig. 2.1 Shapes for Membership Functions (MFs)

By using the extension principle it is possible to define some other basic operations from set theory into fuzzy set theory. A fuzzy set is completely characterised by its membership function (MF). For this reason is more convenient to express the MF through a mathematical formula. In Fig. 2.1 the most popular MFs are depicted whose expressions are given below.

Triangular MF

$$F_{\Delta}(x; a, b, c) = \begin{cases} 0, & x \leq a \\ \frac{x-a}{b-a}, & a \leq x \leq b \\ \frac{c-x}{c-b}, & b \leq x \leq c \\ 0, & x \geq c \end{cases} \tag{2.9}$$

Trapezoidal MF

$$F_{T}(x; a, b, c, d) = \begin{cases} 0, & x \leq a \\ \frac{x-a}{b-a}, & a \leq x \leq b \\ 1, & b \leq x \leq c \\ \frac{d-x}{d-c}, & c \leq x \leq d \\ 0, & x \geq d \end{cases} \tag{2.10}$$

Gaussian MF

$$f(x; c, \sigma) = exp\left(-\left[\frac{x-c}{\sigma}\right]^2\right) \tag{2.11}$$

Generalised Bell MF

$$f(x; a, b, c) = \frac{1}{1 + |\frac{x-c}{a}|^{2b}} \tag{2.12}$$

$$f(x; a) = m \tag{2.13}$$

Where the trapezoidal MF is an special case of the triangular MF.

2.2.1 FUZZY LOGIC SYSTEMS

A Fuzzy Logic System (FLS) is a nonlinear mapping of a crisp input vector (feature) X_p into an scalar output y_p where the pth output vector case can be decomposed into a collection of multi-input/single-output systems.

As it is pointed out in [Mendel, 1995], an FLS is able to simultaneously process numerical data and linguistic knowledge. Furthermore, it has been proved Mendel [1995], Wang [1992] any FLS can be considered as a linear combination of fuzzy basis functions and hence as a nonlinear universal aproximator. As it is described in [Mendel, 1995], an FLS contains four elements, namely: a) fuzzifier, b) an inference engine, c) a fuzzy rule base and a defuzzifier; in Fig. 2.2 the general structure of an FLS is illustrated.

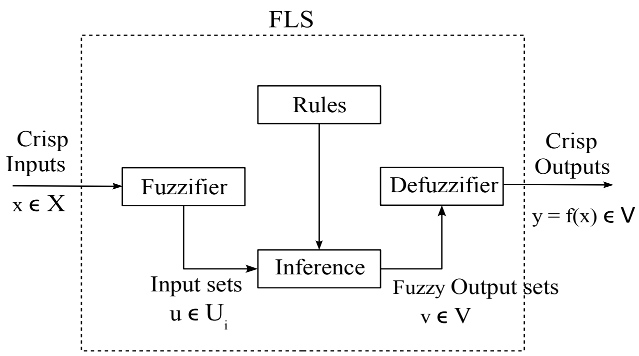


Fig. 2.2 Fuzzy Logic System (FLS) [Mendel, 1995].

- The *fuzzifier* plays a twofold role in an FLS; on the one hand the fuzzifier maps crisp numbers into fuzzy sets, and on the other hand the fuzzifier is needed to activate the fuzzy rules expressed through the use of linguistic variables associated to fuzzy sets. Basically, the most popular mathematical expressions used for converting the crisp input numbers into the corresponding fuzzy sets defined in the linguistic rule base are the equations (2.9)-(2.12).

- *Rules* (fuzzy rule base). A fuzzy rule base is a collection of predefined linguistic IF-THEN rules set up either by expert knowledge or by experts in the area. Since fuzzy rules are the vehicle of knowledge representation, the flexibility of the rule base structure is determined by the form of the rules. Particularly in fuzzy modelling, the rules of a multiple-antecedent and multiple-consequent FLS can be expressed as follows [Mendel, 1995]. The basic form of a rule is

$$R^i : IF\, x_1\, is\, F_1^i\, and\, x_2\, is\, F_2^i\, and\, \ldots\, x_n\, is\, F_n^i\, THEN\, v\, is\, G^i \quad (2.14)$$

where $i = 1, \ldots, M$, $k = 1, \ldots, n$ inputs, F_k^i and G_i are fuzzy sets in $U_i \subset R$ and $V \subset R$ respectively (R denotes the real line), $\vec{x} = (x_1, x_2, \ldots, x_n) \in X_1 \times X_2 \ldots \times X_n$ and $v \in V$. According to Mendel [1995], different adaptations of the fuzzy rule based can be obtained if the rules are:

1. *Incomplete IF rules.* A rule base may contain a set of rules whose antecedents are only a subset of the n inputs, e.g.

$$IF\, x_1\, is\, F_1^i\, and\, x_2\, is\, F_2^i\, and\, \ldots\, x_m\, is\, F_m^i\, THEN\, v\, is\, G^i$$

Such rules are *incomplete IF rules*, and apply regardless the $x_{m+1} \ldots x_n$ antecedents. However, these rules can be treated as complete IF rules if the antecedents $x_{m+1} \ldots x_n$ are considered as elements of a fuzzy set called INCOMPLETE (IN for short), where by definition $\mu_{IN}(x_k) = 1, \forall x \in R$, i.e.

$$IF\, x_1\, is\, F_1^i\, and\, x_2\, is\, F_2^i\, and\, \ldots\, x_m\, is\, F_m^i\, THEN\, v\, is\, G^i$$

If and only if:

$$IF\, x_1\, is\, F_1^i\, and\, x_2\, is\, F_2^i\, and\, \ldots\, x_m\, is\, F_m^i\, and\, x_{m+1}\, is\, IN \\ \ldots and\, x_n\, is\, IN\, THEN\, v\, is\, G^i$$

2. *Mixed rules.* Suppose a rule uses two different connective oper-

ators such as "*and*" and "*or*" in the following way:

$$IF\ x_1\ is\ F_1^i\ and\ x_2\ is\ F_2^i\ and\ \ldots\ x_m\ is\ F_m^i\ or\ x_{m+1}\ is\ F_{m+1}^i \\ \ldots and\ x_n\ is\ F_n^i\ THEN\ v\ is\ G^i$$

Hence, such a rule can be expressed as the following two rules:

$$R^1 : IF\ x_1\ is\ F_1^i\ and\ x_2\ is\ F_2^i\ and\ \ldots\ x_m\ is\ F_m^i\ THEN\ v\ is\ G^i$$

and

$$R^2 : IF\ x_{m+1}\ is\ F_{m+1}^i\ and \ldots and\ x_n\ is\ F_n^i\ THEN\ v\ is\ G^i$$

Where both rules can be seen as two *incomplete if rules* (see [Mendel, 1995]).

3. *Comparative rules*. Some rules are comparative, e.g. "The largest the u", "the smaller the v". However, according to [Mendel, 1995] this type of rules must be first formulated as IF-THEN rules, for example: "IF u is L" THEN "v is S", where L is a fuzzy set representing Large and S small.
4. *Unless rules*. This type of fuzzy rules employ the connective "unless" and can be put into the format of 2.2.1 if the De Morgan's Law is used. For example, the rule

$$v\ is\ G^i \text{ unless } x_1\ is\ F_1^i\ and\ x_2\ is\ F_2^i\ and\ \ldots\ x_n\ is\ F_n^i$$

which can be first expressed as

$$IF\ (x_1\ is\ not\ F_1^i\ or\ x_2\ is\ not\ F_2^i\ or\ \ldots\ x_n\ is\ not\ F_n^i)\ THEN \\ v\ is\ G^i, \text{where } not\ F_k^i \text{ is a fuzzy set}$$

5. *Quantifier rules*. The last case includes a quantifier "some" or "all". The former quantifier is mostly applied by the operator union to the number of antecedents which include "some" and

the intersection to the elements that employ the latter quantifier. By using De Morgan's Law, $\overline{A \cap B} = \bar{A} \cup \bar{B}$, therefore the rule can be expressed as

$$IF\ x_1\ is\ F_1^i\ and\ x_2\ is\ F_2^i\ and\ \ldots\ x_n\ is\ F_n^i$$

- The *inference engine* of an FLS is used for mapping fuzzy sets into fuzzy sets, that means that the inference engine handles the way the rules are combined. There is a vast number of inference engines, however just an small number of them are used. The aim of an inference engine is to mimic the way the human beings make decision based on a linguistic representation.

- The *defuzzifier* maps output sets into crisp numbers. This conversion is context dependent which means that for example whether the problem is about control theory, the output is an action.

2.2.2 FUZZY MODUS PONENS

The modus ponens in crisp sets is a well known deduction rule in logic (as described in 2.2.2). Basically, from the fact $x\ is\ A$ and the rule IF$x\ is\ A$ THEN $y\ is\ B$, a new fact B can be deduced. However, if there is not certainty that $x\ is\ A$, hence it is difficult to make any deduction about y.

$$\begin{array}{l} x\ is\ A \\ \text{IF}x\ is\ A\ \text{THEN}\ y\ is\ B \\ \hline y\ is\ B \end{array}$$

In this context, the extension of the classical modus ponens into fuzzy set logic facilitates to reason with gradual truth, vague knowledge and imprecise information. That means, a generalised version of the modus ponens can be written as:

$$\frac{\begin{array}{l}\text{Premise 1(Fact) } x\ is\ A^* \\ \text{Premise 2(rule) IF} x\ is\ A \text{ THEN } y\ is\ B\end{array}}{\text{Conclusion } y\ is\ B*}$$

Where A^* and A are usually fuzzy sets on the universe of discourse X and B^* and B represented by fuzzy sets on the universe Y. The generalised modus ponens holds that the higher the degree of the premise, the higher the degree of truth in the conclusion. A system's interpretation for the generalised modus ponens in fuzzy systems is illustrated in Fig. 2.3 The diagram is a fuzzy composition where the first relation is merely a fuzzy set A^*. Consequently the term $\mu_{B^*}(y)$ is obtained from a sup-star composition.

$$\mu_{B^*} = sup\left[\mu_{A^*}(x) \star \mu_{A\to B}(x,y)\right]. \tag{2.15}$$

Different implications have been proposed since fuzzy logic was applied into the area of control theory and modelling. Below the three most popular inference engines are listed.

- The Minimum implication was proposed by Mamdani [Mamdani, 1974] for simplicity computation reasons and expressed as

$$\mu_{A\to B}(x,y) \triangleq min\left[\mu_A(x), \mu_B(x)\right]. \tag{2.16}$$

- Larsen [Martin Larsen, 1980] proposed a product implication which was again introduced for computation purposes rather than cause and effect.

$$\mu_{A\to B}(x,y) \triangleq \mu_A(x)\mu_B(x). \tag{2.17}$$

- The minimum and product inference engine is the most widely mechanism in engineering applications due to it preserves the cause and effect, i.e. $\mu_{p\to q}(x,y)$ is fired only when the antecedent and consequent part of the rules are true.

$$\mu_{A\to B}(x,y) \triangleq min\left[\mu_A(x)\mu_B(x)\right] \tag{2.18}$$

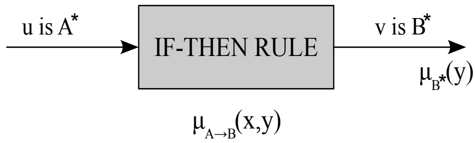


Fig. 2.3 System's interpretation for generalised modus ponens.

FUZZY MODELLING

The application of fuzzy sets into modelling aims to express complex systems in the form of fuzzy implications. In fuzzy modelling of a process, a fuzzy implication is particularly called a fuzzy process law. In general, any data-driven fuzzy model is frequently constructed by employing the physical properties of the system, the observational data and empirical knowledge [Sugeno and Kang, 1986].

The use of expert knowledge in fuzzy modelling aims to compile all this information in linguistic (control and modelling) rules. From this view, a fuzzy reasoning model is a set of rules in the IF-THEN form to describe input-output relations of a system. Consider a collection of P data points $X_1, X_2, \ldots, X_P$ in an n dimensional space that combines input and output dimensions that can give rise to any type of generic representation (MIMO, MISO, etc). Thus, a single input-output data pair can be written as

$$X_p = \{x_1, x_2, \ldots, x_n, y_p\}; X_p \in R^{n+1}, p = 1, \ldots, P \tag{2.19}$$

Hence, let $x = (x_1, x_2, \ldots, x_n) \in R^n$ be inputs and $y \in R$ the output. The target of modelling is to identify the non-linear function $y = f(x) : R^n \to R$ with P given input-output data pairs. A fuzzy model based on the Takagi-Sugeno-Kang (TSK) implication can be represented as a partnership of fuzzy rules:

$$R_i : IF\ x_1\, is\, A_{1i}\, and\, x_2\, is\, A_{2i}\ \ldots\ and\, x_n\, is\, A_{ni}\ THEN\ y = f_i(x) \quad (2.20)$$

Where $x = (x_1, x_2, \ldots, x_n) \in U_1 \times U_2 \times U_n$ and $y \in V$ are the linguistic variables, A_{ji} are fuzzy sets of the universes of discourse $U_i \in R(i = 1, 2, \ldots, n)$, and R_i represents the ith rule, $i = 1, 2, \ldots, p$ and finally, $fi(x)$ can take three main values: (1) singleton, (2) fuzzy sets, and (3) linear function. Note that if $f_i = k$(constant) the fuzzy model may be seen as a fuzzy Mamdani model. A general architecture of a fuzzy model is illustrated in Fig. 2.4 which is composed of three principal modules.

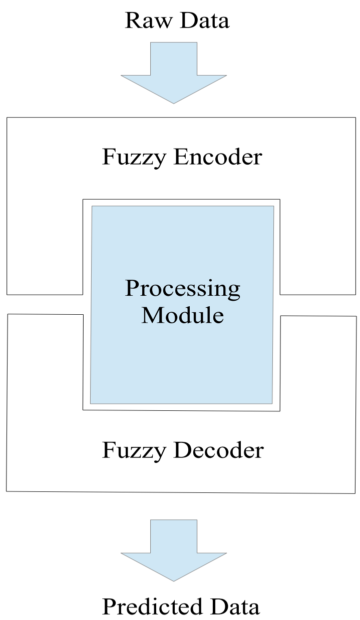


Fig. 2.4 General topology of a fuzzy model.

The essential role of the fuzzy encoder and fuzzy decoder is to encode/decode information (the input vector x) coming from the environment in which the modelling takes place. Such information might be heterogeneous in nature involving numerical quantities, intervals as well as fuzzy sets. The transformation of the external information into a compatible set during the encoding level with the one being used in the processing stage is carried out by distinct matching procedures. In fact, such methodologies are considered the primary mechanisms of the fuzzy encoding. Quite frequently these procedures depend on the extensive usage of necessity and compatibility measures. For instance, (a) fuzzy C-means in which an objective function leads the search of the clustering process, and (b) The granular compression approach where a certain number of granules are formed from raw data into fuzzy sets. Fuzzy sets (linguistic labels) forming the interface to the computational part of the architecture shown in figure 2.4 should satisfy a few general requirements to assure a proper functionality and flexibility of the entire system:

- **Interpretability** . It refers to the capability of the fuzzy model to express the behaviour of the system in an understandable way. This is a subjective property that depends on a number of several factors such as: the input variables, fuzzy rules, linguistic terms, the shape of the fuzzy sets and the most important the model structure. The term of interpretability encloses different criteria such as compactness, completeness, consistency, or transparency.

- **Accuracy** this concept refers to the capability of the fuzzy model to faithfully represent the modelled system. The closer the fuzzy model to the system, the higher its accuracy. Due to the similarity between the response of the real system and the fuzzy model is why the fuzzy models are considered a function approximation model.

As Zadeh stated in its principle of Incompatibility [Zadeh, 1973], 'as the complexity of a system increases, the ability to make precise and yet significant statements about its behaviour diminishes until a threshold is reached beyond which precision and significance (or relevance) become almost mutually exclusive characteristics'. As a consequence, the fuzzy modelling can be divided into two main groups:

- Linguistic fuzzy modelling. The goal is to obtained fuzzy models with a good interpretability.

- Precise fuzzy modelling. The main objective is to obtain fuzzy models with a good accuracy.

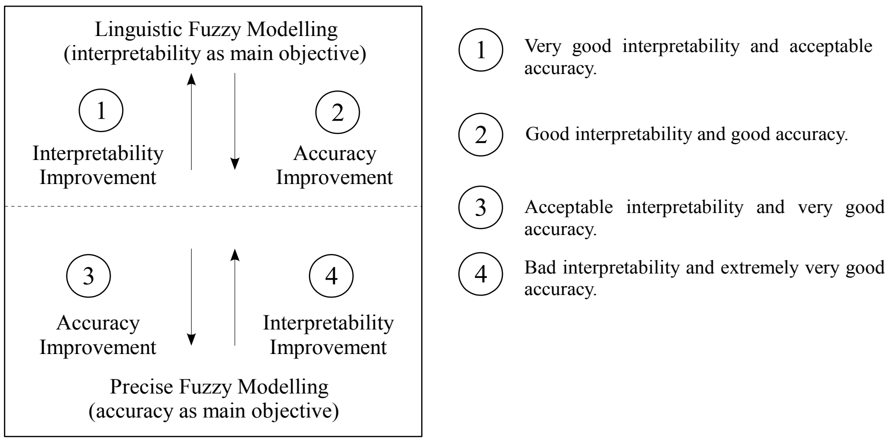


Fig. 2.5 Improvements of interpretability and accuracy [Casillas, 2003].

The computational module shown in Fig. 2.5 can vary significantly depending upon the problem at hand. Moreover, this stage is the so called 'inference engine' in fuzzy systems. One out of the most popular approaches is the neural networks in which the collection of rules is encapsulated. The following list mentions the most commonly fuzzy models used in engineering:

1. Tabular representations
2. Fuzzy grammars
3. Fuzzy relational equations
4. Fuzzy neural networks
5. Rule- based models
6. Fuzzy regression models

2.3 TYPE-2 FUZZY SETS AND SYSTEMS

Frequently, the main reason for using Type-2 Fuzzy Sets (T2-FS) among the community of fuzzy practitioners is due to their ability to model and minimise the effects of linguistic uncertainty [Mendel and John, 2002]. Moreover, Zadeh presented a more powerful argument for the use of fuzzy sets for manipulating perceptions [Zadeh, 2001a]. That is, the human cognition for grouping and describing objects mostly is done by performing a variety of physical and mental tasks without any underlying assumption, for example the perception of what is the size, height, colour, volume, weight of an object, where the object can be any physical or abstract entity. Indeed, the idea of perceptions goes more at hand with the human ability to represent objects by means words and propositions drawn from a natural language.

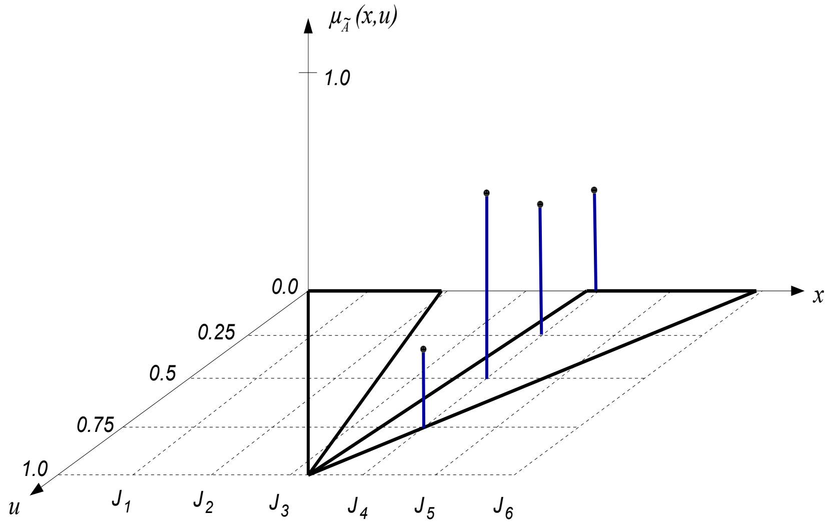


Fig. 2.6 T2 Fuzzy Membership [Mendel and John, 2002]

In this regards, as it is mentioned in [John and Coupland, 2007], T2-FS is a framework capable of computing with words since they do not have crisp membership functions (or just Type-1 fuzzy membership functions). In other words, computing with words leads to computing with perceptions as a result of manipulating fuzzy quantities. The term type-1 fuzzy sets has gained more popularity among practitioners since the introduction of the concept of 'T2-FS' which was proposed by Zadeh in 1975 [Zadeh, 1975]. The research area of T2-FS is now well established in academia activity. A more detailed of an historical review about T2-FSs is given in [John and Coupland, 2007]. Since the inception of T2-FS, the number of research works and publications has grown importantly due to the vast existing theory that fully define type-1 fuzzy sets (T1 FS) on the one hand, and the consolidation of the mathematical basis necessary for defining uncertain rule-base fuzzy logic systems on the other hand. Particularly works done by John and [John, 1996, 1998], Mendel [Mendel, 2001, 2003], John and Mendel [Mendel and John, 2002], and Karnik and Mendel [Karnik and Mendel, 1998a] opened this field to a wider audience that has used it into areas such as robotics, medicine, complex systems modelling, etc.

TYPE-2 FUZZY SETS

Before going directly to the review of theory of interval type-2 fuzzy systems, it would be worth to provide some foundations of type-2 fuzzy sets theory (T2-FS). Therefore, as it was proposed in [Mendel, 2001, 2007b, Mendel and John, 2002] type-2 fuzzy $\tilde{A}$ is characterised by a type-2 membership function T2-MF $\mu_{\tilde{A}}(x,u)$ and defined as

$$\tilde{A} = \{(x,u), \mu_{\tilde{A}}(x,u) | \forall u \in J_x \subseteq [0,1]\} \tag{2.21}$$

Where $\mu_{\tilde{A}}(u,x)$ is a type-2 membership function that characterizes $\tilde{A}$, $x \in X$ and $u \in J_x \subseteq [0,1]$. In which $0 \leq \mu_{\tilde{A}}(u,x) \leq 1$ and can also be stated as:

$$\tilde{A} = \int_{x\in X} \int_{u\in J_x} \mu_{\tilde{A}}(u,x)/(x,u) \quad J_x \in [0,1] \tag{2.22}$$

According to Fig. 2.6 $\tilde{A} = \{\mu_{\tilde{A}}(u,x) | \forall x \in X\}$ or defined as

$$\tilde{A} = \int_{x\in X} \mu(\tilde{A})(x)/x = \int_{x\in X} \left[\int_{u\in J_x} f_x(u)/u\right] \tag{2.23}$$

For discrete universes of discourse $\tilde{A}$ can be defined as

$$\tilde{A} = \sum_{x \in X} \left[\sum_{u \in J_x} f_x(u)/u \right] /x \tag{2.24}$$

The bounded triangular area represented in 2.6 was called by John and Mendel [Mendel and John, 2002] the Footprint of Uncertainty which means is the union of all the primary membership functions.

$$FOU(\tilde{A}) = \bigcup_{x \in X} J_x \tag{2.25}$$

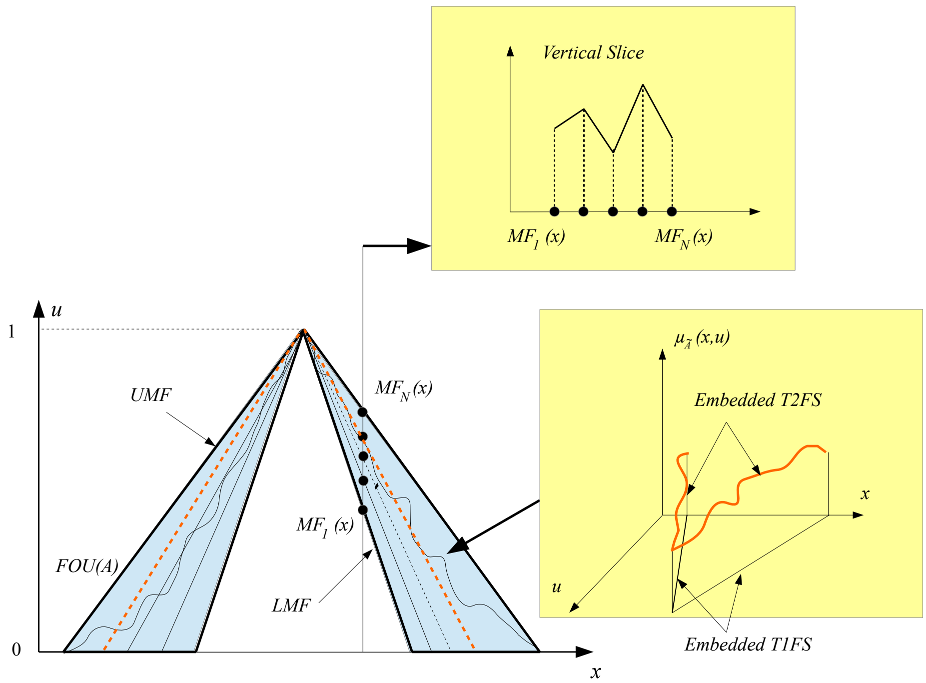


Fig. 2.7 Vertical slice and embedded type-2 fuzzy sets theorem representations [Mendel and John, 2002].

In [Mendel and John, 2002] two representation theorems for T2-FS are proposed, namely: a) Vertical-slice representation and b) wavy-slice representation. While the former representation is based on the mathematical expression for the slice of membership functions illustrated in Fig. 2.6, the latter uses the concept of embedded type-2 fuzzy set $\tilde{A}_e$ which is defined as follows, see e.g Fig. 2.7

$$\tilde{A}_e = \sum_{i=1}^{N} [f_{x_i}(u_i)/u_i]/x_i \quad u_i \in J_{x_i} \subseteq U = [0,1] \tag{2.26}$$

where a type-1 embedded set is

$$A_e = \sum_{i=1}^{N} u_i/x_i \quad u_i \in J_{x_i} \subseteq U = [0,1] \tag{2.27}$$

And A_e has N elements, one each from $J_{x_1}, \ldots, J_{x_N}$ namely $u_1, \ldots, u_N$. That means there is a total of $\amalg_{i=1}^{N} M_i A_e$ type-1 sets. Therefore, the representation theorems for T2-FS are stated as [Mendel and John, 2002]:

- Vertical-slice representation

$$\tilde{A} = \bigcup_{\forall x \in X} vertical\ slices(x) \tag{2.28}$$

- Wavy-slice representation

$$\tilde{A} = \bigcup_{\forall j} Embedded\ T2 - FS(j) \tag{2.29}$$

In Fig. 2.7 The representation theorems mentioned above are illustrated. Such representation theorems are considered as covering theorems since the union of all the vertical slices and the union of all the embedded type-1 fuzzy sets T1-FS cover the whole FOU.

TYPE-2 FUZZY LOGIC SYSTEMS

Basically a T2 Fuzzy Logic System (T2-FLS) consists of the same number of components than its T1 counterpart, namely (a) a fuzzifier, (b) an Inference engine, and (c) a defuzzifier which uses a type-reducer component that combines in a similar way than T1 defuzzifier the fired-rule output sets from the inference engine obtaining a type-reduced set.

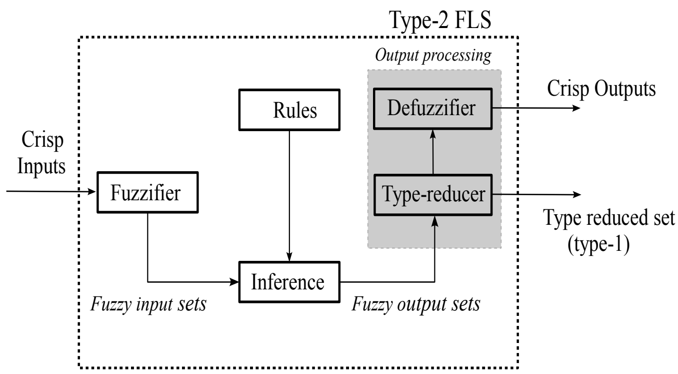


Fig. 2.8 T2 Fuzzy Logic System taxonomy

As illustrated in Fig. 2.8 the general taxonomy of a T2 Fuzzy Logic System (T2 FLS) can be seen as a system having k inputs $x_1 \in X_1, \ldots, x_n \in X_n$ and one output y_p where $p = 1, \ldots, P$ is the number of vector data presented at the input [Mendel, 2007a]. Therefore, one T2 fuzzy rule rule can be stated as follows:

$$R^i : IF\, x_1\, is\, \tilde{A}_1^i\, and\, x_2\, is\, \tilde{A}_2^i\, and\, \ldots\, x_n\, is\, \tilde{A}_n^i\, THEN\, y_p\, is\, \tilde{G}^i,\; i = 1, \ldots, M \tag{2.30}$$

The R^i represents the input-output relationship where the input space is $X1 \times \ldots \times X_n$ and the output space Y, and the T2 Fuzzy Set $\tilde{A}^i = \tilde{A}_1^i \times \ldots \times \tilde{A}_1^n$. Hence the ith rule can be rewritten as:

$$R^i : \tilde{A}_1^i \times \ldots \tilde{A}_1^n \rightarrow G^i,\; i = 1, \ldots, M \tag{2.31}$$

Similar to type-1 fuzzy systems, the inference engine combines rules and give a mapping from T2-FS to output T2-FS. Usually, the antecedents are connected through the $t-norm$ (intersection of fuzzy sets) and hence combined by the sub-star composition. This means that the rules can be either combined by using a $t-conorm$ (union of fuzzy sets) or during the defuzzification process.

As the centroid of T2-FS is concerned, usually is calculated into discrete domains because if its practicality. In a similar way, the centroid of a type-2 fuzzy set can be calculated from the following equation:

$$C_A = \frac{\sum_{k=1}^{n} x_k \mu_A(x_i)}{\sum_{k=1}^{n} \mu_A(x_i)} \tag{2.32}$$

A discretized $x-domain$ into n points, that is $\tilde{A} = \sum_{i=1}^{n}[\int_{u \in J_x} f_{x_k}(u)/u]/x_k$ can be defined by using the Extension Principle as is described below:

$$C_{\tilde{A}=\int_{\theta_1 \in J_{x_1}} \cdots \int_{\theta_n \in J_{x_n}}} [f_{x_1} \star \ldots \star f_{x_n}] \Big/ \frac{\sum_{k=1}^{n} x_k \theta_k}{\sum_{k=1}^{n} \theta_k} \tag{2.33}$$

where $C_{\tilde{A}}$ is a type-1 fuzzy set. In this sense, the computation of $C_{\tilde{A}}$ involves the computation of:

$$a(\theta) \triangleq \frac{\sum_{k=1}^{n} x_k \theta_k}{\sum_{k=1}^{n} \theta_k} \tag{2.34}$$

$$b(\theta) \triangleq [f_{x_1} \star \ldots \star f_{x_n}] \tag{2.35}$$

In order to compute the tuples (a, b), an intensive process of all the combinations $\theta = [\theta_1, \ldots, \theta_n]$ is perfomed in order to obtained α tuples (a_1, a_α), where $\theta \in J_{x_k}$.

Despite the attractive advantages of T2-FS for dealing with linguistic uncertainties, its implementation results to be expensive in terms of computation as a consequence of the use of a type-reduction process which is quite intensive. In this context, different types of representations have been proposed in order to decrease such a computational burden. Particularly, this load decreases significantly when the secondary membership function is defined as an interval renaming the T2-FS as interval type-2 fuzzy sets (IT2-FS) [liang2000interval, mendel2006interval]. New developments such as zslices representation for type-2 fuzzy sets, $\alpha-$plane representation [Mendel et al., 2009], geometric type-2 [Coupland and John, 2007] and *quasi*-type-2

Fuzzy Logic Systems [Mendel and Liu, 2008] have contributed to the computational simplicity for the application of Type-2 Fuzzy Sets in real world problems.

The use of IT2-FSs whose secondary membership function could be either zero or one simplify importantly the number of computations required to obtain the type-reduced set. For the sake of completeness appendix **??** provides a brief review of IT2-FS including meet and join operations for interval sets.

INTERPRETABILITY IN FUZZY LOGIC SYSTEMS

Due to the properties of transparency and interpretability, fuzzy models have led some researches to create generic models for the prediction of nonlinear systems properties [Casillas et al., 2003, Chen and Mahfouf, 2010, Juang and Chen, 2013, Paiva and Dourado, 2004, Setnes et al., 1998b]. The richness of fuzzy set theory has been exploited into different areas such as medicine, robotics, control theory, systems modelling, and mathematics.

Particularly, one out of the major purposes of complex systems modelling is to developing reliable and transparent models that provide an interpretable insight into real-world systems. To cast system behaviour in historic perspective, several data-driven modelling techniques have been developed as a fundamental mechanism to understand natural phenomena via the use of linguistic terms.

Three main categories have been frequently used for system modelling, namely: a) white-box models, in which the mathematical characterisation has easy-to-interpret parameters and all the necessary information is available, b) black-box modelling, where there is not prior information about the system establishing opaque relationships between the input and the output based on observational data, and c) Gray-box modelling, which represents a combination and exploitation of the capabilities of the two previous modelling techniques.

In general, complexity modelling involves a trade-off between simplicity and accuracy of the model. Particularly, data-driven models based on fuzzy systems offers an interesting expression of dynamic systems trough fuzzy implications (inference engine) based on observational data and empirical/expert knowledge.

Within this context, a number of fuzzy systems have been constructed from data by using adaptive learning methodologies and evolutionary computation in order to increase the interpretability and hence the transparency (e.g. Chen model, [Chen and Linkens, 2001a], Leng model [Leng et al., 2005] with an on-line extraction of fuzzy rules and Talamantes-Silva model, [Zhu et al., 2003].

2.4 NEUTROSOPHIC LOGIC

Neutrosophy was born as a branch of philosophy employed to explain the origin, nature and scope of neutralities as well their interaction with ideational spectra [Smarandache, 1999]. Basically, neutrosophy studies a proposition, event, theory, concept or entity as $'A'$ in relation to its opposite denoted as $anti - A$ or $not\ A$ and the neutralities $neu - A$ which is not A, $< not - A >$ and that which is neither A nor $Anti - A$ are referred as to $non - A$ ideas. This new type of logic was developed to mathematically model uncertainty, vagueness, ambiguity, inconsistency, contradictions, paradoxes, incomplete language/systems and This new logic can be fitted into the category of paraconsistent logics. However, this new framework needs to be specified from a technical point of view. From a fuzzy perspective, this new logic not only may consider the associated truth-membership $'T'$ and falsity-membership $'F'$ supported by evidence, but also the associated indeterminacy/uncertainty-membership $'I'$.

According to Gershenson [Gershenson, 2001], neutrosophy is a logic structure based on axioms that makes the study of any system incomplete, in other words just believed. Moreover, Gershenson commented that neutrosophy is a concept that involves the study of many systems because it contains them. That means that the study of a system does not finished and it can always be improved. Smarandache proposed to define a set based on the tuple $< T, I, F >$, where T, F, and I are the true, falsity and indeterminacy associated to an event or a set respectively. Compared to fuzzy set theory where a set is defined to measure the associated true in the closed interval $[0, 1]$, a neutrosophic set can be defined through the use of infinitesimal numbers which means that a number T can be evaluated in the interval $]^{-}0, 1^{+}[$. The mathematical framework of neutrosophy argues with the idea of Gershenson that as less-incomplete the ideas of a sytem, the more are useful, since the human being can not perceive the associated true, falsity and indeterminacy of a system. Therefore, a neutrosophic set still needs to be defined from a technical point of view.

Even though, the notion of fuzzy entropy (sometimes referred as uncertainty) encloses various theories, such a measure just deals with disorder quantification among fuzzy sets. The concept of Neutrosophy was introduced by Smarandache as an extension/combination of the fuzzy logic, intuitionistic logics, paraconsistent logic and the three-valued logics that uses an indeterminate value [Ashbacher, 2002]. Moreover, a neutrosophic set employs the non-standard analysis, a formalization of analysis and a branch of mathematical logic, which rigorously defines the infinitesimals [Wang et al., 2005]. The informal idea behind an infinitesimal value is an infinitively small number, i.e. x is said to be infinitesimal if and only if for all positive integers n, the ratio $|x| < 1/n$. Furthermore, let $\cdot > 0$, be a such infinitesimal, and $1^{+} = 1 + \cdot$ a non-standard number, where $'1'$ it is the standard part and $'\cdot'$ its non-standard part, and $^{-}0 = 0 - \cdot$ in which the same logic works. Smarandache defines $]^{-}a, b^{+}[$ a non-standard interval, where ^{-}a and b^{+} can be viewed as the lower and upper boundary within a closed interval.

$$(^{-}a) = \{a - x : x \in R+, x \text{ is infinitesimal}\} \tag{2.36}$$

$$(b^{+}) = \{b + x : x \in R+, x \text{ is infinitesimal}\} \tag{2.37}$$

In neutrosophic terms, the elements of a neutrosophic interval [Smarandache, 2001] can be defined as $^{-}a = a - x$ and $b^{+} = b + x$. The definition of the neutrosophic components based on the previous concepts can be represented by T, F, and I within a standard or non-standard real subsets of $]^{-}0, 1^{+}[$ [Smarandache, 2001]. Where T, F, and I are the truth, falsehood, and the indeterminacy related to a mathematical event respectively. Following the definition of T, F, and I, $^{-}0$ and 1^{+} are numbers infinitively small but less than 0 or infinitively small but greater than 1, and hence belong to the non-standard unit interval. By extension the lowest value of $]-a, b+[$ might be introduced by the inf $]^{-}a, b^{+}[\ = \ -a$, and the highest sup $]^{-}a, b^{+}[\ = \ b+$. These numbers can related to T, F, and I percentages as follows:

sup T = t_{sup}, inf T = t_{inf}
sup I = i_{sup}, Inf I = i_{inf}
sup F = f_{sup}, inf F = f_{inf}

A generalisation of T, F, and I are real standard and non-standard subsets, included in the non-standard unit interval]-0, 1+[where:

$$^{-}0 \leq inf(T) + inf(I) + inf(F) \leq sup(T) + sup(I) + sup(F) \leq 3^{+} \quad (2.38)$$

The superior (sup) and inferior (inf) sum is:

$nsup = \text{sup(T)+ sup(I) + sup(F)}]^{-}0, 3^{+}[$

May be as high as 3 or 3+, while inf(T)+inf(I)+inf(F)$\in]^{-}0, 3^{+}[$, may be as low as 0 or $^{-}0$. This non-restriction allows paraconsistent, and incomplete information to be characterised in neutrosophic set logic, i.e. the sum of all these three components if they are defined as intervals, single points, and superior limits can be > 1 (for paraconsistent information coming from different sources) or < 1 (for incomplete information). According to [Smarandache, 2001], this new representation is closer to the human mind reasoning and characterises the imprecision of knowledge or linguistic inexactitude. While intuitionistic fuzzy logic (IFL) can not describe this representation because in IFL the components T (truth), I (Indeterminacy), F (falsehood) are restricted either to t+i+f=1 or $t + f \leq 1$, if T, I, F are all reduced to the points t, i, f respectively, or to sup T + sup I + sup F = 1 if T, I, F are subsets of $[0, 1]$. Opposite to this, in neutrosophic logic (NL) the components T, I, F can be represented by standard or non-standard subsets included in the unitary non-standard interval $]0^-, 1^+[$ [Smarandache, 2001]. Due to this reasoning, a linguistic representation of the elements T, I, F can be interpreted as intervals, standard or non-standard real sets, discrete, continuous, single-finite sets, operations under intersection or union, fuzzy numbers, normal distribution, etc. For this reason the tuple <t, i, f> represents the truth value, indeterminacy value and falsehood value. One can use all this information in order to define a punctual view of neutrosophic sets from a fuzzy perspective. The definition of fuzzy sets just deals with the truth of an event, while IFL and NL cope with a broader scheme considering the uncertainty-based information.

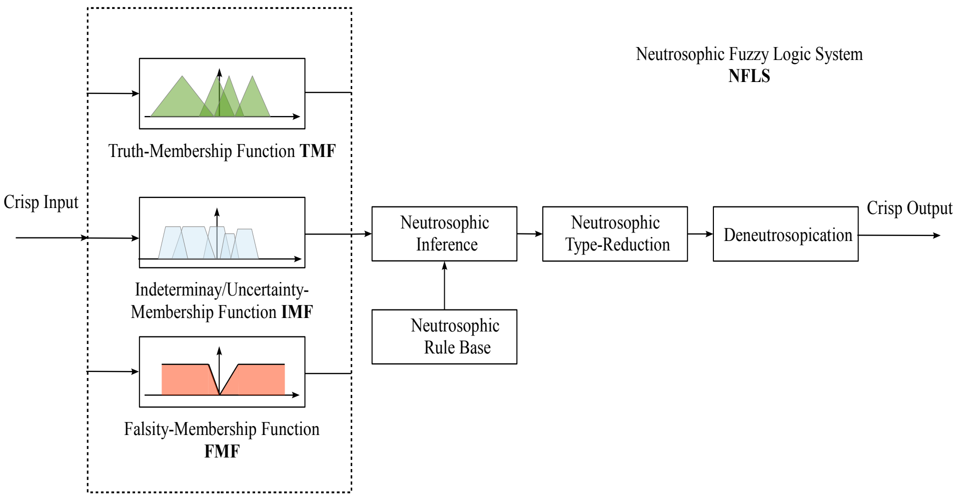


Fig. 2.9 Neutrosophic Fuzzy Logic System Structure, [Wang et al., 2005]

Although the elements of a Neutrosophic Set (NS) have an associated degree of truth, falsity and uncertainty/indeterminacy that lie in the non-standard set $]0^-, 1^+[$, in this research work such sets will be defined from the point of view of standard analysis and hence each element of the tuple

$< T, I, F >$ is defined in the closed interval $[0, 1]$. Therefore, a Neutrosophic Fuzzy Logic System (NFLS) may be seen as illustrated in Fig. 2.9.

In [Smarandache, 2010a] it was introduced a set of extensions of the fuzzy T-norm and T-conorm. In that article the authors covered both the N-norm and N-conorm for non-standard and standard sets. However, for technical applications the domain of definition will be considered in the interval $[0, 1]$. Therefore, the N-norm (N_n) and N-conorm N_c can stated as:

$$N_n : ([0, 1] \times [0, 1] \times [0, 1])^2 \to [0, 1] \times [0, 1] \times [0, 1] \tag{2.39}$$

and

$$N_c : ([0, 1] \times [0, 1] \times [0, 1])^2 \to [0, 1] \times [0, 1] \times [0, 1] \tag{2.40}$$

If any two given neutrosophic sets x and y, the corresponding N-norms are $N_n(x(T_1, I_1, F_1), y(T_1, I_1, F_1)) = (N_nT(x, y), N_nI(x, y), N_nF(x, y))$. Where N_n must satisfy the following axioms:

1. Boundary conditions: $N_n(x, 0) = 0, N_n(x, 1) = x$
2. Commutativity: $N_n(x, y) = N_n(y, x)$
3. Monotonicity: If $x \leq y$ then $N_n(x, z) \leq N_n(y, z)$
4. Associativity: $N_n(N_n(x, y), z) = N_n(x, N_n(y, z))$

According to [Smarandache, 2010a], there are cases where not all the axioms are satisfied. This is due to some type of operations, for example neutrosophic normalisation. In that case, such operation are called N-pseudo-norms. In a like manner to Fuzzy Sets theory, the operator N_n may represent the *and* operator and the *intersection* operator in neutrosophic logic and Neutrosophic Sets theory respectively. For instance the " $\wedge$ " can be the algebraic product if any two sets $x(T_1, I_1, F_1)$ and $y(T_2, I_2, F_2)$, hence $N_n = (T_1 \wedge T_2, I_1 \vee I_2, F_1 \vee F_2)$. Thus if any $J \in \{T, I, F\}$, the most known $N - norms$ as in fuzzy logic and fuzzy sets theory the $T - norms$ are

- The algebraic product $N-norm$: $N_{n-algebraic}J(x,y) = x \cdot y$
- The bounded $N-norm$: $N_{n-bounded}J(x,y) = max\{0, x+y-1\}$
- The default min $N-norm$: $N_{n-min}J(x,y) = min\{x,y\}$

In relation to the $N-conorms$, N_c may represents the *or* operator and the *union* operator in neutrosophic logic and neutrosophic sets theory respectively. Therefore if any $J \in \{T, I, F\}$

- The algebraic product $N-conorm$: $N_{c-algebraic}J(x,y) = x+y-xy$
- The bounded $N-conorm$: $N_{c-bounded}J(x,y) = max\{1, x+y-1\}$
- The default max $N-conorm$: $N_{c-max}J(x,y) = max\{x,y\}$

2.5 GRANULAR COMPUTING

Before going directly to the concept of Granular Computing (GrC), it would be worth to mention the roots of granulation. The concept of granulation was firstly proposed in [Zadeh, 1997] as a computational paradigm based on the human cognition where three basic concepts underlie this ability, namely: a) granulation, b) organisation and c) causation. The first concept refers to the decomposition of a whole into parts; the second concept involves the ability of humans for integrating parts into a whole; and causation involves the association between effects and causes.

the concept of granulation is inspired by the abstract way the human beings granulate information and reason with it [Zadeh, 1997]. This mechanism represents the point of departure for information granulation (IG) where the granules can be a) crisp (c-granules) or b) fuzzy (f-granules). Although the former types of granules have been applied successfully in conjunction with other methodologies such as Demspter-Shafer theory [Butenkov, 2004], probabilistic reasoning [Zadeh, 2002], decision trees [Pedrycz and Sosnowski, 2001], etc. it suffers from the ability to reason with entities/objects as can be done by using f-granules. For example, the anatomy of a human is mostly represented by fuzzy granules rather than crisps. That is the size

and features of the ears, eyes, legs, hair, etc. are not sharply defined. This example is clearly related to the association of a clump of fuzzy granules instead of a single fuzzy granule. In this environment of partial knowledge, attributes such as similarity, compatibility, distance, functionality, etc. may result from the association between two or more granules (intergranularity). Formally speaking, the fuzziness of granules may represent the human ability to make decisions under an uncertain environment.
Furthermore, the concept of information granulation can be see as a generalisation which may be applied to different concepts [Zadeh, 1997]. Zadeh proposed five types of generalisation modes which can be defined as [Zadeh, 1997]:

- Fuzzification (f-generalisation). In this type, a fuzzy granule is replace by a fuzzy granule (See Fig.).
- Granulation (g-generalization). This type is about the partition of a set into a group of granules.
- Randomization (r-generalization). In this type, a variable is replaced by random variable.
- Usualization (u-generalization). In this type, a proposition expressed as $X\ is\ A$ is replaced with usually (X is A).
- Fuzzy granulation (f.g-generalisation). This process involves a progression from fuzzy sets to granulated fuzzy sets (see Fig. 2.11)

Some combinations between two or more of the cases mentioned above can be done. In the context of information granulation, emerging frameworks such as Granular Computing (GrC) are proposed as processing mechanisms of complex information entities [Bargiela and Pedrycz, 2003a]. In other words, GrC aims to represent information in the form of some aggregates and their corresponding processing. Granular Computing extracts information from numerical data to mimic the ability of the human beings to develop a granular view of the "world" and "objects" according to their similarities such as proximity, functionality, size, orientation, shape, etc. This means that GrC serves a way of achieving data compression through the use of words and information granulation for representation when the information is so imprecise and the environment involves uncertainty an partial truth. Perhaps some of the most practical reasons of its emerging popularity are the necessity of information granulation and its simplicity derived from granulation in solving problems. For instance, in performing some tasks like driving in city traffic, where the human kind (driver) employs the perception for estimating some variables such as distance, speed, direction, shape, intent, likelihood, truth, and other attributes of physical and mental objects.

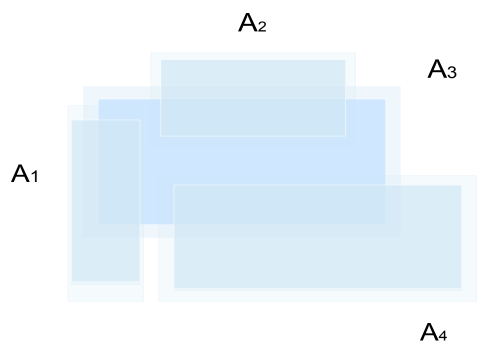
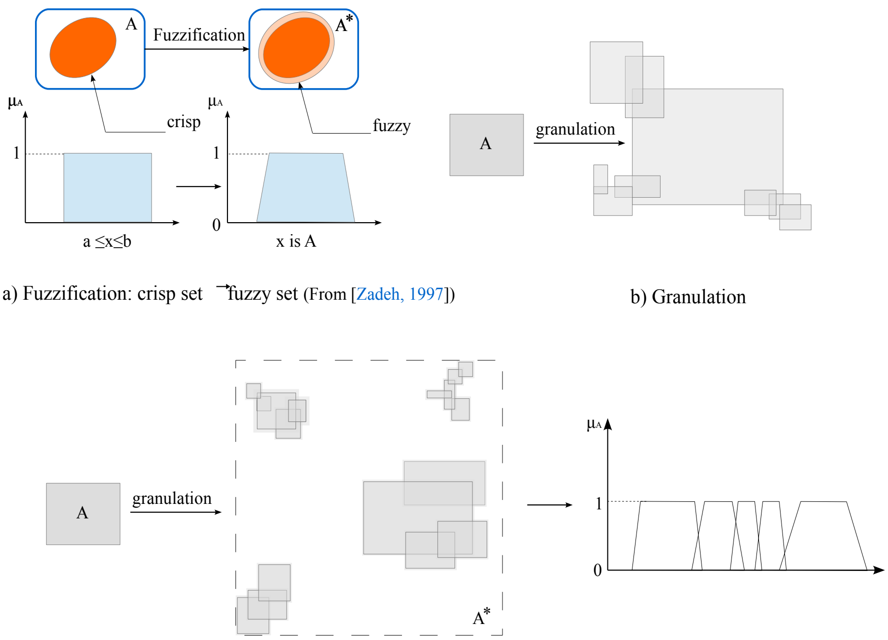


Fig. 2.10 f-granularity

More specifically, perceptions are, for the most part, fuzzy granules in the sense that: (a) the boundaries of perceived classes are fuzzy, and (b) the values of the perceived attributes have a granular structure. In Fig. 2.10, a general granular structure (f-granule) is illustrated. where $A = A_1 + A_2 + A_3 + A_4 = \sum_j A_j; A \in U_i$ is the set of the fuzzy sets. Even though the term of GrC is relatively recent, this concept has been already used in different areas such as granularity in artificial intelligence, fuzzy and rough set theory, cluster analysis, etc.



a) Fuzzification: crisp set → fuzzy set (From [Zadeh, 1997])

b) Granulation

c) Fuzzy granulation

Fig. 2.11 Granulation

2.6 UNCERTAINTY BASED-INFORMATION

Uncertainty usually emerges as a consequence of a type of deficiency when dealing with information. Measurement errors and resolution limits are two of the major reasons of uncertainty which is an inseparable companion of almost any type of measurement. In Fig. 2.12 the different types of uncertainty in fuzzy set theory are listed [Pal et al., 1992, 1993]. The information obtained from a system is frequently not fully reliable because of the incomplete, fragmented, vague and contradictory measurements [Klir and Wierman, 1999]. In machine learning an effective way of dealing with uncertain information is through the use of probabilistic inference mechanisms and some other theories that have have been demonstrated to be capable of characterising situations under uncertainty.

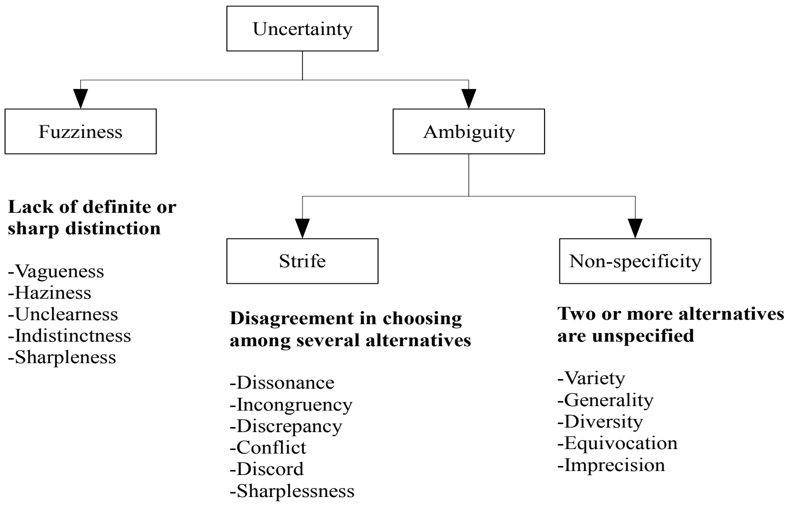


Fig. 2.12 Uncertainty measures

The most visible of such theories are fuzzy sets [Zadeh, 1965], evidence theory [Shafer, 1976], possibility theory [Dubois, 2006, Zadeh, 1999b] and the theory of fuzzy measures [Ishii and Sugeno, 1985].

The nature of uncertainty-based information depends on the mathematical theory within which uncertainty pertaining to various problem-solving situations is formalised [Klir and Wierman, 1999]. To make this clear, different concepts have been suggested by various authors. In [Shafer, 1976] an uncertainty measure based on the evidence was introduced. This type of uncertainty usually emerges due to limitations of evidence gathering, interpretation system, and as a difficulty for specifying the exact solution (non-specificity) or just due to randomness in the system (probabilistic). Tu put it more simply, these types of uncertainty are confined to describe situations where there is no ambiguity about set- boundaries but rather to the belongingness of events or elements to crisp sets [Pal et al., 1992]. For instance, in [Yager, 1983], Yager introduced the concept of entropy and specificity in the framework of Shafer's theory. On the one hand, The concept of entropy was generalised from the probabilistic framework and specificity on the other hand was defined from a possibilistic point of view. Such uncertainty measurements proved to be complementary measures of quality of a piece of evidence. Hohle proposed in [Hohle, 1981, Höhle, 1982] a measure to quantify the level of confusion present in a body of evidence. Smets [Kaufmann and Swanson, 1975] developed a distinct type of measure for the information content of an evidence.

In table 2.1, [Pal et al., 1992, 1993], a list of non-fuzzy uncertainty measures is presented. Particularly, the authors emphasized the uncertainty of a system as a composite measure of two different types [LAMATA and MORAL, 1988]. The point of departure lies in the fact to consider the non-specific and probabilistic aspects of uncertainty in a system.

Even though in table 2.1 three different measures (G_1, G_2, T) that quantify the uncertainty in a system are provided, there is still an ignorance to account the complete uncertainty that results from randomness. Under these circumstances the authors in [Pal et al., 1992, 1993] discussed the properties of G_1, G_2 and T. The term probabilistic in the third column in table 2.1 represents more the uncertainty due to randomness or chance. Consequently, according to [Yager, 1983] the first measure of uncertainty $E(m)$ indicates the degree of dissonance (conflict) in a body of evidence. where $m(A)$ is the degree of evidence or belief of an element x that belongs to the set A but not to any B such that $B \subset A$. And (F, m) is the body of evidence with F as the set of all the subsets of A. In fact, Yager suggested that specificity is associated to a possibility distribution. Moreover, Yager generalised this idea introducing the concept of non-specificity $J(m)$. In this sense, Hohle in [Hohle, 1981, Höhle, 1982] proposed a measure to represent conflict $C(m)$ when two evidential claims $m(A)$ and $m(B)$ conflict within the same body of evidence. The term $U(r)$ was introduced by Higashi and Klir in [Higashi and Klir, 1982] in order to measure non-specificity which satisfies the axioms of the Shannon's entropy. A different measure was proposed by Smet [Smets, 1983], such an expression does not represent a generalisation of the Shannon's entropy which makes it interpretable in terms of randomness.

Table 2.1 A catalog of uncertainty measures [Pal et al., 1993].

Author	Sum	Probabilistic	Non-specific
Yager [Yager, 1983]		$E(m) = \sum_{A\in F} m(A) log Pl(A)$ (Dissonance)	$J(m) = 1 - \sum_{A\in F} (m(A)/\lvert A\rvert)$ (Non-specificity)
Hohle [Hohle, 1981, Höhle, 1982]		$C(m) = -\sum_{A\in F} m(A) log Bel(A)$ (Confusion)	
Hirashi & Klir [Higashi and Klir, 1982]			$U(r) = \sum_{i=1}^{n} m(A_i log\lvert A_i\rvert)$ (Non-specificity)
Smets [Smets, 1983]		$L(m) = -\sum_{A\subseteq F} log Cm(A)$	
Dubois & Prade [Dubois and Prade, 1985]			$I(m) = \sum_{A\in F} m(A) log\lvert A\rvert$
Klir & Ramer [KLIR and Ramer, 1990]	$D(m) = -\sum_{A\in F} m(A) log \left[\sum_{B\in F} M(B)\lvert A\cap B\rvert/\lvert B\rvert\right] (discord)$		
Lamata & Moral [LAMATA and MORAL, 1988]		$V(m) = EV_{Bel}(-log(Pl(x))))$ (innate contradiction)	$L(m) = log\left(\sum_{A\subset X} m(A)\lvert A\rvert\right)$ (imprecision)
Pal, Bezdek & Hemashina		$G(m) = -\sum_{A\in F} m(A) log_2 m(A)$	$I(m) = \sum_{A\in F} m(A) Log\lvert A\rvert$ (Non-specificity)
Lamata & Moral [LAMATA and MORAL, 1988]	$G_1(m) = E(m) + I(m)$ (Global uncertainty)		
Lamata & Moral [LAMATA and MORAL, 1988]	$G_2(m) = V(m) + W(m)$ (Global uncertainty)		
Klir & Ramer	$T(m) = D(m) + I(m)$ (Total uncertainty)		
Pal, Bezdek & Hemashina	$T(m) = D(m) + I(m)$ (Average total uncertainty)		

In response to the uncertainty index $E(m)$ proposed by Yager, Klir and Ramer point out that the measure of dissonance in that expression is unsatisfactory, and $m(A)$ and $m(B)$ conflict. For this reason, the authors suggested in [KLIR and Ramer, 1990] a measure of conflict that solves the problem. In this context, the first seven rows of table 2.1 list a number of basic measures of uncertainty including probabilistic and non-specific events. Therefore, the terms G_1, G_2 and T can be seen as the global $G_{1,2}$ and T total uncertainty in a system. The first two terms G_1 and G_2 are composite measures that exhibit a trade-off of the assessment of their factors. For example, the global uncertainty G_1 balances dissonance against non-specificity. This means that G_1 accounts for only one element of uncertainty, i.e. conflict.

As far the term G_2 is concerned, Lamata & Moral pointed out that some terms such as $I(m)$ can not be extend to a generalised class of fuzzy measures because this measure is only expressed in terms of a Basic Probabilistic Assigment (BPA). For this reason, in [LAMATA and MORAL, 1988] the authors proposed G_2 to circumvent this problem, however according to [Pal et al., 1993] no motivation is provided by Lamata & Moral to define an expression that considers the imprecision W and V the degree of surprise as a measure for global uncertainty. Similar to the uncertainty G_1, T was defined by Klir & Ramer to represent the total uncertainty based on conflict [Pal et al., 1993]. Nevertheless, the term $D(m) = \sum_{A \in F} m(A) Log \left[\sum_{B \in F} m(B)|A - B|/B\right]$ is difficult to interpret because it only captures the uncertainty due to randomness in a partial way.

Finally, the average total uncertainty T defined by [Pal et al., 1993] consider the deficiencies mentioned above and introduces a new term for conflict $D(m) = \sum_{A \in F} m(A) Log \left[\sum_{B \in F} m(B)|A \cap B|/B\right]$ to overcome such problems and group of axioms that any measure of global and total uncertainty must satisfy.

2.7 FUZZY UNCERTAINTY BASED-INFORMATION

The concept of information is too broad to be captured completely by a single definition. According to Table 2.1, the entropy of a variable is defined in terms of its subjective probability distribution and can be a good measure of randomness or uncertainty. In the areas of pattern recognition, machine learning, image processing, speech recognition, etc., it is often required to get some idea about the degree of ambiguity (fuzziness) present in a fuzzy set.

A measure of fuzziness is a kind of cognitive uncertainty and it is expected to give the average amount of information caused by the uncertainty area from one linguistic term to other [Wang et al., 2012]. This notion has been extended to fuzzy set theory by the concept of Shannon's entropy sometimes referred as a measure of uncertainty. Zadeh defined the entropy of a fuzzy subset A for a finite set $x_1, x_2, \ldots, x_n$ with respect to the probability distribution $p_1, p_2, \ldots, p_n$ as:

$$H^P = -\sum_{i=1}^{n} \mu_A(x_i) p_i log(p_i) \tag{2.41}$$

p is defined on an event x_i is a function $p(x_i)$, which can have values only in the interval $[0, 1]$. A set of these functions assigns the degree of possessing some property p by the event x_i constitutes what is called a property set. In other words, p_i is the probability of occurrence of x_i, and H^p can be viewed as a weighted version of Shannon entropy measure where the memberships μ_A are used as weights. Kaufman in [Kaufmann and Swanson, 1975] defined the entropy of a fuzzy set as:

$$H^k = \{-1/log(n)\} \sum_{i=1}^{n} \Phi_i log(\Phi_i) \tag{2.42}$$

Where $\Phi_i = \mu_i / \sum_(i = 1)^n \sum_i, i = 1, 2, \ldots, n$. However, the drawback of this measure is that it does not depend on the absolute values of μ_i, but on their relative ones. Deluca and Termini [De Luca and Termini, 1972] used a different expression, based on Shannon's entropy to define the entropy of a fuzzy set as follows.

$$H = -k \sum_{i=1}^{n} \mu_i log(\mu_i) + (1 - \mu_i) log(1 - \mu_i) \tag{2.43}$$

Where k is a normalising constant, and equation (2.43) is used to express an average amount of fuzziness, ambiguity in a fuzzy set A. Pal and Pal in [Pal and Pal, 1989] also defined a fuzziness measure based on exponential entropy as:

$$H = -k \sum_{i=1}^{n} \mu_i e^{(\mu_i)} + (1 - \mu_i) e^{(1-\mu_i)} \tag{2.44}$$

Any measure of fuzziness including the entropy in a system should satisfy the following properties:

(a) H is minimum iff $\mu_i = 0 \; or \; 1 \, \forall \, i$

(b) H is maximum iff $\mu_i = 0.5 \, \forall \, i$

(c) $H \geq H^*$ is the entropy of a fuzzy set A^*, a sharpened version of A. (A^* is a sharpened version of A if $\mu^* \leq \mu$ for μ in $[0, 0.5]$ and $\mu* \geq$ for μ in $[0, 0.5]$).

(d) $H = H'$, where H' is the entropy of the complement set.

Referring back to equations (2.41)-(2.44), the definition of fuzziness is conceptually different from the probabilistic information. Their arithmetic sum may not yield any meaningful quantity. In other words, if $p_i = \mu_i$ such description infers that the average fuzzy information yielded by a fuzzy set with 'n' elements is 'equivalent' to the average amount of Shannon information yielded by n independent binary Shannon information sources. Based on this, fuzzy information can be transferred to Shannon information and inversely [Pal and Pal, 1992].

2.8 NEURAL NETWORKS

Artificial Neural Networks (ANN) are computational models inspired by the structure and functions of biological neural networks. In a broad sense, an ANN mimics a massively parallel distributed processor made up of simple processing units or simply neurons; having a natural propensity for storing experiential knowledge and making it available for use. An artificial neuron is a mathematical model that executes the basic operation of an ANN, and whose basic structure is composed of three main elements:

- *Synapses or connecting links.* A connecting link is characterised by a weight or strength ${}_{kj}$ which multiples an input x_j connected to a neuron k where in a different manner to biological neurons, the artificial neuron range may lie between negative and positive values.

- *Adder.* This element aims to sum all the input signals which are weighted by the corresponding synapses. The operations at this stage represent a linear combiner or model.

- *Activation function.* The role of this element is to limit the output of a neuron or just simply squash the permissible neuron output to some finite value.

- *The Bias* is used to increase or lower the network input of the activation functions depending whether it is positive or negative, respectively.

The model for a neuron as represented in Fig. 2.13 can be expressed by the following two equations:

$$u_k = \sum_{j=1}^{m} w_{kj} x_j \tag{2.45}$$

$$y_k = \varphi(u_k + b_k) \tag{2.46}$$

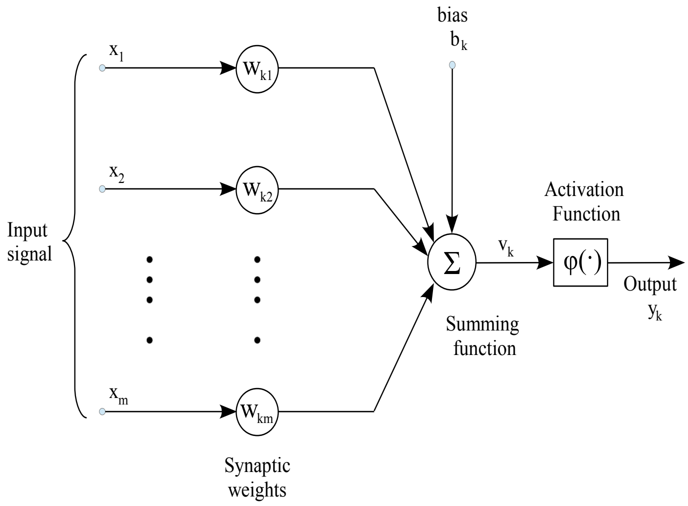


Fig. 2.13 Artificial Neural Network model

where $x_1, x_2, \ldots, x_m$ are the network inputs, $w_{k1}, w_{k2}, \ldots, w_{km}$ are the synaptic weights, b_k the bias, and $\varphi(\cdot)$ the activation function. The role of the bias b_k is an affine transformation to the output u_k of the linear combiner which can be stated as:

$$v_k = u_k + b_k \tag{2.47}$$

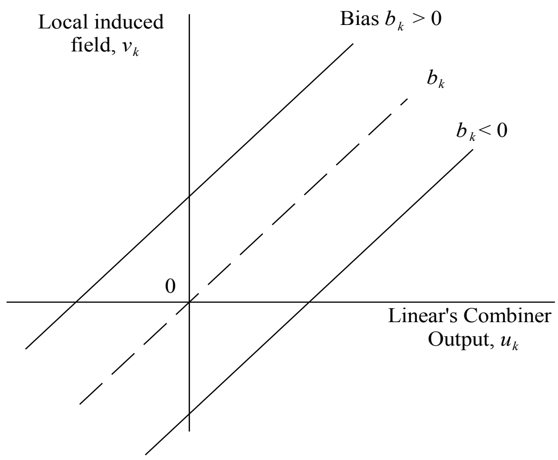


Fig. 2.14 Affine transformation as a consequence of $v_k = b_k$ at $u_k = 0$.

The bias is an external signal which can be added to (1.1) and finally the network output written as (See Fig. 2.14):

$$v_k = \sum_{j=0}^{m} w_{kj} x_j \tag{2.48}$$

$$y_k = \varphi(v_k) \tag{2.49}$$

where the value of x_0 is usually equal to $+1$ and its synaptic weight $w_{k0} = b_k$. The neural structure illustrated in Fig. 2.13 is considered a one-layer network whose parameters can be calculated by using Least Square approximations if the input-output relationship is linear. However, if approximation of non-linear functions is done by using linear neurons, no benefit in terms of computational burden compared to other traditional algorithms such as regression techniques is shown [Haykin and Network, 2004].

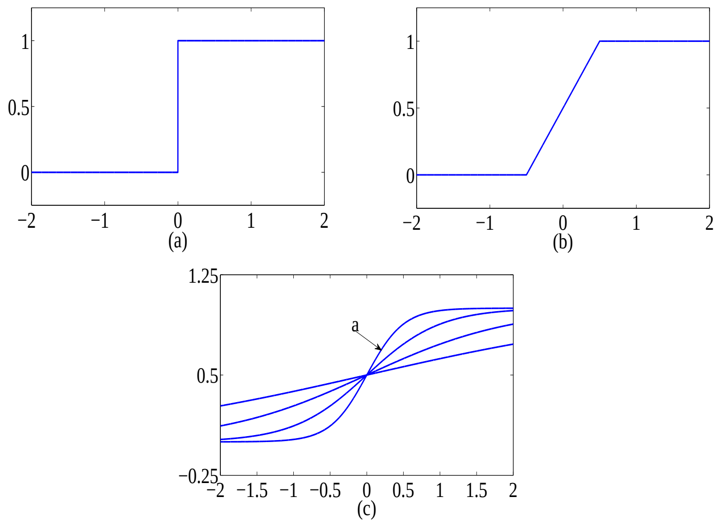


Fig. 2.15 Activation functions: (a) Threshold function, (b) Piecewise linear function and (c) Sigmoid function.

Although non-linear relationships can be approximated by using non-linear activation functions, the accuracy depends mainly on the value of the weights or synaptic values when a neural network is trained. The model presented in Fig. 2.13 is known as the McCulloch-Pitts model (MCP) [McCulloch and Pitts, 1943], and various types of activation functions can be found frequently ranging from 0 to 1, or if it is desirable from -1 to +1; in which case the shape of the activation function is antisymmetric with respect to the origin. As it is illustrated in Fig. 2.15, the following expressions show the most popular activation function used in neural networks

- Piecewise-linear Function is an activation function whose amplitude is 1, and can be seen as an approximation to the model of a linear amplifier.

$$\varphi(v) = \begin{cases} 1, & v \geq \frac{1}{2} \\ v, & +\frac{1}{2} > v > -\frac{1}{2} \\ 0, & v \leq -\frac{1}{2} \end{cases} \tag{2.50}$$

  - A linear combiner arises if the region of operation is maintained with no saturation.
  - The piecewise-linear function reduces to a threshold function if the operation factor is made infinitely large.

- Threshold Function is usually known as Heaviside function, where the mathematical expression of the output neuron is

$$\varphi(v) = \begin{cases} 1, & if\ v \geq 0 \\ 0, & if\ v \leq 0 \end{cases} \tag{2.51}$$

  for the output y_k the threshold function can be stated as

$$y_k = \begin{cases} 1, & if\ v_k \geq 0 \\ 0, & if\ v_k \leq 0 \end{cases} \tag{2.52}$$

A multilayer network whose connections between the units do not form a directed cycle are called feed-forward networks or just multilayer perceptron (MLP) whose functional architecture is different to that based recurrent connections. From a mathematical view, any feed-forward network with a single hidden layer can approximate almost any continuous function or compact subset in R^n under some minor constraints with respect to the type of activation function employed. As mentioned in [Hornik, 1991], multilayer feed-forward networks under general conditions are universal approximators emphasising that not all the available activation functions perform equally under the same conditions. Particularly when using sigmoid functions [Cybenko, 1989], a multilayer network behaves as a universal approximator.

A Multilayer network can employ a variety of parameter identification methodologies (learning technique), the most popular is the back propagation technique which pretends to adjust the weight of each connection in order to reduce the output error that is compared to the correct answer (learning pattern) to compute the value of a predefined cost function. This error is then fed back in order to estimate the negative gradient of the cost function at the current learning step. This kind of learning methodology or non-linear optimization technique is used for finding the local minimum and usually is known as well as the steepest descent or the method of the steepest descent. The gradient descent approach calculates the derivative of the cost function with respect to each free parameter of the network and then such variables are adjusted such that the neural error decreases after a number of computational steps known as training, this means that the gradient descent approach can be only applied on networks with differentiable activation functions.

2.8.1 RBF NEURAL NETWORKS

Although the Radial Basis Function neural network (RBF-NN) and the Multilayer Perceptron model (MLP) are non-linear feedforward networks, some remarkable differences can be listed [Haykin and Network, 2004].

- The RBF network usually has only a single hidden layer in its basic form while the MLP may have more than one.

- Typically the computation of the neurons in the hidden and output layers of the MLP network share the same model, whereas the computation of the neurons in the hidden layer of the RBF network obeys a different purpose to those in the output layer.

- Opposite to its counterpart, the model of the neuron in the hidden and output layer of the RBF network are non-linear and linear respectively (as a classifier). In other words, for classification purposes, the hidden and output layers are nonlinear. This may be different when the MLP is used for solving nonlinear regression problems and hence the output layer should be linear.

- While in the RBF network, the argument of the activation function in the hidden layer neurons compute the Euclidean norm (distance), the activation function of each neuron in the hidden layer of the MLP computes the inner product of the input vector and the synaptic weight vector of that unit.

- RBF networks use exponential decaying nonlinearities to construct local approximations to nonlinear input-output mappins. In contrast, the MLP carries out a global approximation to nonlinear mappings.

In the most essential respects, the taxonomy of the RBF network is illustrated in Fig. 2.16. The input layer consists of n nodes where $'n'$ represents the dimensionality of the input vector. Usually the number of nodes in the hidden layer is equal to the number of training data, however, problems of over fitting may arise. The point of departure for the construction of the RBF network lies on the basic methodology of radial basis functions which involves the selection of a number of functions or Receptive fields Units (RUs) [Broomhead and Lowe, 1988] with the following form:

$$F(x) = \sum_{i=1}^{M} w_i \Phi\left(\| x - x_i \|\right) \tag{2.53}$$

where $\{f_i\left(\| x - x_i \|\right) | i = i, 2, \ldots M\}$ is the number of functions generally nonlinear which are also known as radial basis functions, and $\| \cdot \|$ is the

Euclidean norm. The points x_i are taken to be the centers of the of the radial basis functions or receptive units (RUs).

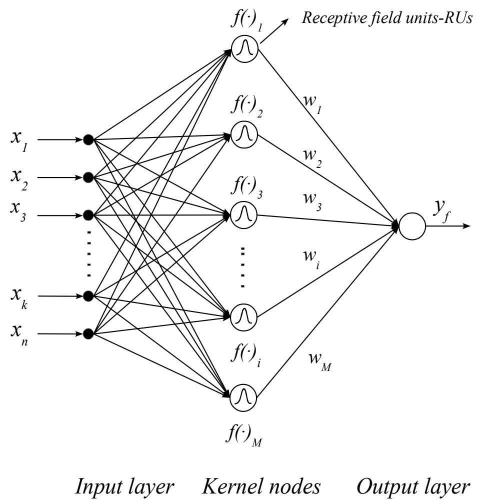


Fig. 2.16 Radial Basis Function Neural Network architecture-RBF-NN.

Each RU in the RBF-NN computes a radially symmetric function, where usually the strongest firing strength or neuron output is obtained when the current input data is at the centre of the that RU or the associated norm is zero. As mentioned in [Bishop, 1995] the roots of the RBF-NNs derive from exact interpolation of real multidimensional spaces, which means that mutidimensional vectors are mapped onto the corresponding target vector. As in MLP architectures, the addition of a bias in the linear sum of the output layer includes a compensation for the difference between the value over the data set of the RUs and the corresponding average value of the target outputs.

According to the theory of multivariable interpolation in highdimensional spaces, the interpolation problem can be stated as [Haykin and Network, 2004]:

$$F(x_i) = d_i, i = 1, 2, \dots M \tag{2.54}$$

Eq. 2.54 indicates that the interpolation surface is constrained to pass through all the training points. In this sense, the following representation with uncertain weights $w_i's$ can be obtained:

$$\begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & \dots & \Phi_{1n} \\ \Phi_{21} & \Phi_{22} & \Phi_{23} & \dots & \Phi_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \Phi_{M1} & \Phi_{M2} & \Phi_{M3} & \dots & \Phi_{PM} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_M \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_M \end{bmatrix}$$

where the vector $d = [d_1, d_2, \dots, d_M]^T$ and $w = [w_1, w_2, \dots, w_M]^T$ and $M$and$P$ are the number of hidden neurons and the total amount of training points respectively.
A matrix representation including the term $\{\Phi = f_{ij}|(i,j) = 1, 2, \dots M\}$ can be written as:

$$\Phi w = x \tag{2.55}$$

According to the Michelli's theorem, the $ij - th$ element $(\Phi_{ij} (\| x_i - x_j \|))$ of the interpolation matriz Φ is nonsingular. That means, the vector w can be represented as

$$w = \Phi^{-1} x \tag{2.56}$$

Although there is a large number of radial basis functions that are covered by the Midhelli's theorem, the following functions are the most popular [Haykin and Network, 2004]:

- Multiquadratics:

$$\Phi(r) = (r^2 + c^2)^{1/2} \text{ for some c} > 0 \text{ and } r \in R \tag{2.57}$$

- Inverse Multiquadratics:

$$\Phi(r) = \frac{1}{(r^2 + c^2)^(1/2)} \text{ for some c} > 0 \text{ and } r \in R \tag{2.58}$$

- Gaussian functions:

$$\Phi(r) = exp\left(-\frac{r^2}{2\sigma^2}\right) \text{ for some } \sigma > 0 \text{ and } r \in R \tag{2.59}$$

The Gaussian function is of particular interest in practice because such a function only depends on the Euclidean distance of the vectors $x - x_i$. Especially the multivariate Gaussian function is a Green function $G(x, \xi)$ in which x and ξ are the paremeter and the argument respectively. A Green function plays a role for a linear differential operator that is similar to that for the inverse matrix for a matrix equation (for instance 2.56, for a deeper explanation see [Haykin and Network, 2004]). The most popular function in modelling and function approximation when using RBF Networks is usually the multivariate Gaussian function.

$$G(x, x_i) = exp\left(-\frac{1}{2\sigma^2} \parallel x - x_i \parallel^2\right) \tag{2.60}$$

The activation functions of the hidden layer are now defined by the Green's functions that we call here f_i which are connected to the output layer that consists of a single linear unit, being fully connected to the hidden layer. The output layer is a weighted sum of the output of each hidden unit. The RBF network architecture presented in Fig. 2.16 assumes that the Green's function $G(x, x_i)$, here asf_i is positive definite for all i [Girosi et al., 1995].

It has been shown that significant benefits from neural networks are inherited to the RBF networks, particularly those benefits that derive from their computational power that is based on their parallel distributed architecture and their ability for learning and generalising tasks. Therefore, some important properties and capabilities of the RBF networks can be listed.

- Non-linearity. According to the type of activation functions, a neural network can be defined as a linear or non-linear systems. That means if the hidden layer contains non-linear nodes the network is non-linear itself.

- No prior assumptions. A neural network is an input-output mapping whose parameters can be estimated after a teaching process of a desired data set. Therefore at each iteration, one target pattern is presented to the neural network and the connection (weights) and activation function parameters are calculated in a predefined order. Thus the neural network is learns from examples constructing a mapping for the problem at hand. This **'non-parametric'** estimation is usually employed when no prior assumptions are made on a statistical model for the input data [Haykin, 1994]. That means that a probabilistic distribution model is not needed as an arbitrary decision boundary is found for an pattern-classification task by using a set of patterns or examples. Frequently the term of non-parametric estimation is done into the study of statistical inference in which is carried out a model-free estimation.

- Adaptivity. Neural networks posse the capability for adaptation in terms of its connection weights according to the environment. This capability has brought to the study and application of adaptive control, adaptive signal processing and adaptive classification. Moreover, the capability of adaptation makes the neural network more robust in its performance when the network is working under a non-stationary environment. Nevertheless, it does not mean that a more robust performance leads to robustness, since there are example where constants in the systems produce rapid and sudden responses of the neural network. In contrast, it is more beneficial to have values that allow the system to ignore spurious disturbances and then just respond to meaningful changes.

- Contextual information retrieval. The information contained at each neuron is fully affected by all the other neurons in the network.

- Uniformity of analysis and design. The neural network has an enviable position among classification techniques as the type of the different available neurons can be used indistinctly at different neural models. This makes it possible to share theories and learning approaches in a wide spectrum of applications. And finally a neural networks mimics closely the biological nervous systems which means that neural network performance degrades gracefully under adverse operation conditions.

2.9 SUMMARY

In this chapter some background knowledge related to this research work is provided. Particularly, relevant information to fuzzy sets theory and uncertainty based information has been viewed in more detailed. In addition, the basics of granular computing and neutrosophic sets theory are reviewed since it is of great importance for the development of this research work. Finally some information related to Neural Networks with special emphasis in Radial Basis Functions Neural networks is included.

Next chapter will provide on the one hand a background on manufacturing processes including the importance of the different types of tests that are helpful for understanding the behaviour of some heat treated steels under certain operation conditions. Consequently, on the other hand a neural fuzzy framework based on the Radial Basis Function Neural Networks (RBF-NNs) and Fuzzy C-Means (FCM) is applied for modelling a data set of 1661 Charpy test measurements and their associated test parameters which were collected at 6 different labs and provided by the TATA Steel Company, Yorkshire, UK.

3 SOFT COMPUTING FOR COMPLEX MANUFACTURING PROCESSES

A review of manufacturing processes for steel industry and some preliminary results for the mechanical properties prediction of heat-treated steels by using the RBF-NN and Fuzzy C-Means (FCM) are provided. Particularly, in this chapter an emphasis about the functional equivalence between the RBF-NN and Fuzzy Systems of Type-1 is put on. This equivalence is mainly employed for constructing a Fuzzy System of Type-1 based on the RBF-NN.

3.1 INTRODUCTION

In modern manufacturing systems, the processing and then the representation of the information has played a crucial factor for massive production, mainly to respond effectively to the severe competitiveness and the increasing demand of quality product in the market. Since manufacturing facilities are more complex and highly sophisticated, modern manufacturing systems represent a great opportunity to exploit ideas with great potential which can enhance their performance and then make them more flexible. That means flexibility may bring benefits such as increased production and product customisation. However, if this new property is not properly controlled, it may lead to ineffective decision-making, customer dissatisfaction and higher costs.

Complexity in manufacturing systems are heavily accredited to the following components:

- *Product structure*, that is the amount of different end user products, number and type of sub-assemblies, cycle times and type and sequence of resources required to produce such a variety of products.

- *the structure of the plant*, the number of resources, layout, maintenance tasks idle time and performance measures.
- *the planing and scheduling functions*, that is basically based on three main elements
  1. The planning and scheduling strategies.
  2. The information processing for planning and scheduling.
  3. The decision-making process.
- *Information flow* which on the one hand is largely based on internal decision-making and team working, and on the other hand on external information processing that includes interaction with other plants, suppliers and customers.
- *The dynamism, variability and uncertainty of the environment*, this includes customer changes, breakdowns, absenteeism, data inaccuracy and unreliability.
- *Other elements* such as training, technology upgrade and political information.

Particularly the understanding of manufacturing processes that transform raw material from its raw form to the final product is vital to increase competitiveness in industry and to achieve a good trade-off between flexibility and complexity. Furthermore, this understanding involves large amount of data and non-linear effects and interactions throughout the entire process. For instance, in steel making the heat treatment process is used to develop the required mechanical properties in a range of alloy steels. Therefore, an adequate estimation of the heat treatment regimens is crucial to obtain the required steel grade accuracy at a reduced cost. Nevertheless, the prediction of appropriate heat treatment regimens depends largely on the both the chemical composition of the steels and the related process conditions of the treatment. Thus, by predicting properly such optimal conditions is not an easy task since it may involve a deep understanding of the influence that each component of the process has to contribute with uncertain predictions or behaviours.

The nature of uncertainty and variability in manufacturing systems, specially those related to determine the most appropriate process conditions for steel making may result due to the following reasons:

- Highly non-linear interaction and non-linear behaviour of the individual processes such as casting, forming, machining, joining, heat treatments and finishing.

- Measurement uncertainty that results from the parametric variability. This type of uncertainty is usually produced by the variability of the inputs of the process, that includes raw material, the chemical composition, manufacturing precision, planing and scheduling.

- Parameter uncertainty. This source of uncertainty is due to a wrong estimation of the initial parameters that will be used in a process. For example, machining speed, viscosity, initial temperature, cooling temperature, etc.

For gaining a thorough description of manufacturing systems and awareness of the extent of the problems that entails the associated complexity and of the causes and effects of each action during the entire process, soft computing has proved that is a promising research field that can help in the development of new intelligent manufacturing systems which provide a deeper understanding of each of its components. An intelligent manufacturing systems will be able to continuously improve the productivity through the effective use of all the resources, especially the insights and the gained experience from the front-line operators and experts. In particular, there is a growing concern in the manufacturing of materials such as heat treated steels and iron alloys which are massively used in the construction of different products such as aircraft, automobiles, appliances and medical equipment. For this reason, knowledge and understanding of the uses, limitations and strengths of the mechanical properties of heat treated steel in different types of manufacturing is of primary concern to properly design, construct and maintain equipment and tools. Principally the prediction of mechanical properties of materials such as ductility, toughness, elasticity, fusibility and hardness on the basis of their composition and preceding treatment defines the final product manufacture properties of a given size and form.

Since the importance of the understanding about the mechanisms and limitations behind the different tests used to obtain the mechanical testing results is crucial in manufacturing industry. Firstly this chapter provides a background of manufacturing processes and its relationship to heat treated steel and, secondly it describes the application of various concepts of different disciplines from soft computing such as fuzzy logic and neural networks to properly predict mechanical properties of heat treated steel. Thus, the content of this chapter consists of:

- An overview of manufacturing processes and the mechanical tests used to obtain the different mechanical properties of heat treated steels in manufacturing including the limitations and sources of errors of such tests.

- A description of the application of an RBF Neural Network (RBF-NN) in a real case study for the prediction of impact test energy of heat treated steel data set which was provided by TATA Steel Company, Yorkshire, UK.

  Particularly, impact testing becomes an interesting study case as it produces complex results due to the multitude of standards that exist, the low repeatability of the experimental results under the same input test conditions and the highly non-linear behaviour of the test represent a good opportunity to using Neural Networks (NN) for impact energy test prediction. By applying an RBF-NN, the proposed modelling framework is capable of exploiting and exploring its functional equivalence with fuzzy systems of type-1 and new advances of fuzzy set theory in order to model in a transparent and interpretable form the data set given in the case study which helps to understand the importance of each element in the final chemical composition and the so-called steel purity as well the influence of the heat treatment process.

3.2 MANUFACTURING PROCESSES

In modern manufacturing, productivity is related to important factors that define the quality and cost of the production at any organisation. Because of manufacturing entails a large number of independent activities of converting raw materials into a usable form of products or goods for human being needs, the preservation of the physical and mechanical properties of the material product is crucial. The different stages of a process of manufacturing should be aimed at achieving certain well-accepted goals in terms of: a) meeting the design specifications and b) service requirements of the product including efforts of finding the most economical methods of manufacturing. Particularly, manufacturing processes used for transforming metals into some usable products require to have specific properties such as fusibility (melting point), malleability, ductility and divisibility which is known as the capability of materials to be machined. The properties of ferrous and non-ferrous materials in manufacturing processes play an important role in the fabrication of new products. This is mainly due to:

- **Mechanical properties.** include hardness, fatigue, creep, elasticity and strength.

- **Physical properties.** include melting point, electric and magnetic properties, density, specific heat and thermal conductivity.

- **Chemical properties.** This property represents an important factor in the design of materials since it helps to define the material composition to be resistant in both normal and hostile environment conditions. For instance, the most important factors are: toxicity, flammability, general degradation of the material as a consequence of the environment including oxidation, corrosion which can lead the material under fracture conditions.

- **Manufacturing properties or fabrication properties of materials** that determine the ease of their welding, shearing, machining, etc.

Furthermore, in steel manufacturing industry the selection of the correct quality of steel for a particular application, and the optimum heat treatment frequently involves all the operating conditions of the steel. Typically the operating conditions which must be considered are summarised in the following list:

1. Service conditions
   - The operating environment which can have either corrosive or oxidising effects on steel.
   - The final operating temperature, for example a temperature fluctuating between low and high values.
2. Mechanical requirements.
   - Magnitude of stress.
   - Type of the possible shock loading.
   - Degree of rigidity of flexibility required.
   - Weight limitations.
   - Type of stress, for example: tensile, bending, compressive, etc.
   - The nature of the stress during the operating which can be constant, periodically or alternating.
3. Ease of manufacture
   - Weld-ability
   - Forgeability
   - Heat treatment response
   - Machinability

Where the basic steel manufacturing processes encompass various categories which are:

- Casting. is the process where a liquid material is poured into a mold that has a hollow cavity of a specific desired shape and then allowed to solidity.

- Molding. This process comprises two different stages in order to cast a product. The first stage forces granular or powdered material (plastic) into a heated mold cavity under using a great pressure which together with the application of heat turns out in the fill of the mold cavity with the raw material.

- Shearing or cutting. is the process of shaping materials using different cutting operations such as: a) punching, b) piercing, c) shearing, c) blanking, parting and trimming.

- Forming operations (hot forming). is the process of changing the shape of hot metals by applying high pressure and then the metal is brought to the viscous or plastic state by subjecting it to elevated temperatures flowing without rupture by the effect of the high pressure. The main hot-forming operations are: forging, rolling, extruding and upsetting.

Another important group of manufacturing processes are the **machining processes** which are used to remove excess metal from a work-piece to bring the work-piece to the desired shape and size of a product. The major machining categories are:

- **Hole making operations** are drilling, reaming, boring and taping. Drilling is the process of making holes, reaming enlarges the drilled hole to a precise size, boring enlarges the already made hole considerably with a boring tool and tapping is used for thread cutting in the drilled hole.

- **Shape changing processes** are turning, facing, shaping, planning, milling, threading, parting and broaching.

- **Sawing process** which is a process used for cutting pieces from raw stock.
- **Grinding** is a finishing operation frequently carried out after milling, turning.
- **Unconventional methods of machining** which include electric, discharge machining, electrochemical machining, ultrasonic machining, laser machining, etc.

Finishing processes are a type of processes used to improve characteristics, appearance, or durability of a surface. Examples of finishing processes cover deburring, cleaning, painting and coating. Assembly or jointing processes are employed for connecting or attaching individual components to finally assembly a product. For instance, bolts, nuts, screws, rivets and wire stitches. Finally the heat-treatment process is used for modifying the mechanical properties of metals to prepare them for applications that require properties different from those inherent in the base metal. Such processes cover different categories of heat-treatment processes such as **hardening** used for increasing the **hardness** of a work-piece, **case-hardening** used for the surface hardness of a material, **tempering** to make the metal composition tougher and harder, and **annealing** employed to remove hidden stress and improve grains.

Physical, chemical, mechanical and fabricating properties play an important role in the behaviour and performance of any material in manufacturing. During the past decades new manufacturing technologies have been developed in order to enhance the material properties. Particularly such improvements have been focused on factors governing the mechanical properties of metals which are

- Crystal structure of metal defines the ease of formability of a metal piece when loads are applied on. As a consequence of such loads, deformations of the metal take place due to slipping of atomic structure along the slip planes of the metal piece. The formability depends mainly on the available number and directions of the slip. Metals with face-centred cubic (fcc) metals crystal lattice like cooper, silver, etc are easy to form.

- Alloying elements play a significance role in the determination of the mechanical properties of metals. Common alloying elements are Nickel, Chromium, Carbon, manganese, tungsten. Principally, the incorporation of carbon helps in increasing properties such as hardness, and tensile strength, and impact strength, Chromium increases strength to suit in high temperature applications, and nickel increases toughness.

- Working temperatures affect significantly the properties of metals as follows: a) the tensile strength, elastic limit falls when the temperature of the material increases, b) the modulus of elasticity decreases steadily and the elongation falls with an increase in temperature.

- Effect of heat treatment involves heating and cooling of metals in specific ways to obtain certain desired properties. On the one hand, heat-treatment relieves internal stress in a metal that got developed in the course of passing through various manufacturing processes. On the other hand, heat-treatment refines grains and their size ensuring improved mechanical properties and heat-treatment helps altering the microstructure of metals and changes the surface chemistry of the final product by deleting or adding elements such as carbon, thus increasing the hardness of the metals.

- Cold- and hot-working. While cold-working usually increases the tensile strength and hardness but decreases the ductility, in the hot-working treatment, the heated metal undergoes to a plastic deformation while temperature usually goes above 800° degrees.

- Geometry of product has an important role in increasing the strength of a metal as a consequence of a unevenly distributed stress.

- Rate and type of loading is applied very slowly and not continuously but with pauses during the treatment where the metal has opportunity to strain-harden. Finally, smaller average strains (deformations) are observed in the metal piece if a load is applied quickly but continuously.

3.3 AN OVERVIEW OF STEEL PROPERTIES AND STEEL MAKING PROCESSES

Due to its wide variety and range of application in industry, heat treated steel has proved to be a popular material in manufacturing. Such variety depends mainly on its carbon content being the most widely used those steels that have a carbon content ranging between 0.1-0.25. The different types of steel that are produced can be found into four main categories and according to their chemical composition as follows:

- Carbon steels.
- Alloy steels.
- Stainless steels.
- Tool steels.

The popularity of steel use in manufacturing industry is mainly due to 1) its abundance in the earth's crust in the form of the element Fe_2O^3 where a not difficult process is required to convert it into Fe and 2) the great variety of microstructures and thus a wide range of mechanical properties that can exhibit after a heat treatment process. Moreover, the importance of its popularity often is a consequence of the type of mechanical properties that can be obtained from steel such as ductility, brittleness, yield strength, tensile strength, etc. For this reason before describing the heat treatment process, it would be worth to briefly examine some basic properties of steel and the main stages that comprise the steel production in manufacturing processes, however a further and a detailed examination can be gained in [Tenner et al., 2001] and some other books [Leslie, 1981, Thelning and Black, 1984]. Basically, steel is an alloy based on iron with carbon that contributes up to 2.1% out of the total weight of the metal piece. Even though steel and cast iron are alloys made of carbon and iron, the main difference between steel and cast iron lies on the amount of carbon that both metals contain. While steel contains less than 2.0% percent of carbon, the cast iron contains more than 2.0% of carbon with or without other alloying elements. Steels are usually classified into two main groups (a) carbon steels and (b) alloys. While the former type of steels are mainly made of carbon and iron that frequently are known as straight or plain alloys, the later group of steels are those to which one or more alloying elements that are added to modify certain properties. Even iron is the main component in different types and forms of steel, other elements are commonly contained in its chemical composition, some of them unwanted or even intentionally added. Carbon steels are by far the most used and produced type of steels worldwide accounting for about 92% out of the total production in the world. The different categories of carbon steel are classified as

- High-carbon steel, with a carbon above 50%.
- medium-carbon steels, with a percentage (%) ranging from 0.2-0.49.
- low-carbon steels, with a percentage (%) ranging from 0.05-0.19.
- extra-low-carbon steels, with a percentage (%) ranging from 0.015-0.05.
- ultra-low-carbon steels, with a percentage (%) less than 0.015.

Where the most common alloying components are:

- Nickel (Ni): This element is usually added to steel alloys in order to increase the resistance of the material to heat and corrosion as well the ductility of steel working as refining action. The amount of nickel in steels can be up to 5%.

- Manganese (Mn) is a brittle and metallic element that works as an additive to protect the metal surface against corrosion.

- Phosphorus (P) is a non-metallic element that increases the protection of metals to corrosion.

- Chromium (Cr) is used in the steel production mainly to protect the material to corrosion and oxidation.

- Silicon (Si) is frequently used as a deoxidizer in steel production.
- Sulphur (S) is a non-metallic element that can cause steel to be porous and and prone to cracking.
- Carbon (C) is the most popular element employed in the steel production as the main strengthening component in carbon steels.

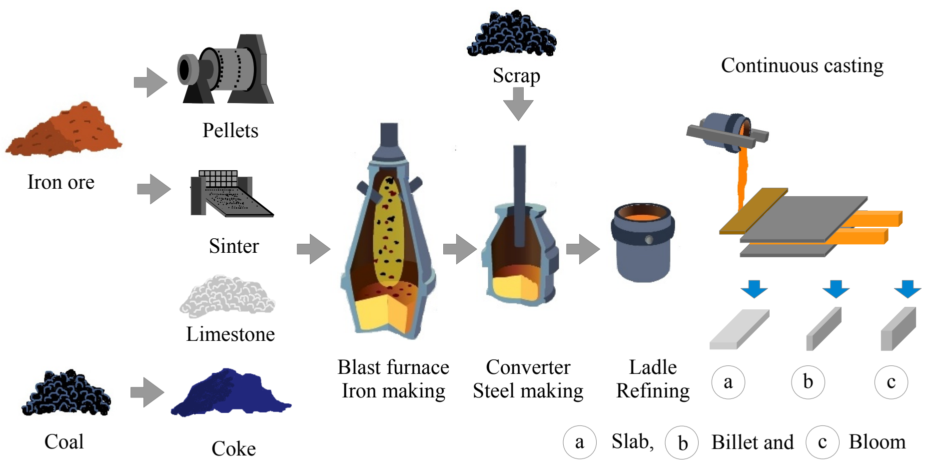


Fig. 3.1 Steel making process.

The basic procedure for steel making is composed of the following steps (see Fig. 3.1):

- The initial stage of the steel-making process consists in mixing the iron ore with limestone and coke in a blast furnace where are melted. The purpose of the blast furnace is to chemically reduce and physically transform the iron ore into liquid removing sulphur and other impurities by using limestone, and coke as an enriching agent in order to obtain clinker usually called sinter.
- Once inside the furnace, the materials require some time to descend to the bottom where a liquid iron and a liquid slag are obtained. However, the liquid produced at the bottom still contains a high percent-

age of carbon which is removed by reprocessing the melted iron several times up to a desired carbon grade is achieved. Finally this liquid is continuously cast into ingots according to the specifications of the product.

- Alternatively, the liquid iron can be obtained by using an Electric Arc Furnace (EAF). This procedure involves the melting of the scrap charge by electric arcs. Finally, after ingots a process of rolling, forging and heat treatment are necessary to produce the final geometrical and mechanical properties of the product.

3.3.1 THE CRYSTAL STRUCTURE OF STEEL

Since steel is an alloy made of iron and carbon (including or not some other alloying elements) it is a prerequisite to describe the structure of the iron and thus of metals. The basic atomic structure in metals is arranged in a regular three-dimensional pattern which is known as crystal structure. This structure can be visualised as a series of cubes piled up side by side and one on the top of another. The corners of the cube are atoms and each corner is shared by eight or even more adjoining cubes or cells.

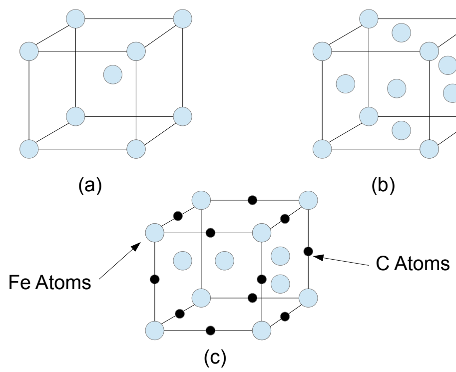


Fig. 3.2 (a)BCC, (b) FCC and (c) FCC crystal structure of austenite.

As it is illustrated in Fig. 3.2 the configuration of the atomic arrangement can be classified into a) one atom at the centre of the cell called as bodycentred-cubic (BCC) and b) with atoms at the centre of each wall of the cell called as faced centre cubic (FCC). The former structure is obtained at low temperatures up to 911° termed as ALPHA-iron (α) structure or simply ferrite, and the latter structure exists up to 1400° termed as GAMMA-iron (γ) structure or austenite, at which temperature crystals turn back into the BCC arrangement usually termed δ crystals (the iron is known as well as Delta-ferrite).

3.3.2 HEAT TREATMENT PROCESS

Heat treatment is usually carried out to develop the required mechanical properties such as ductility and toughness in a range of alloy steels [Tenner et al., 2001]. Indeed, the main effect produced by the heat treatment on most metals and iron-alloys is to increase their properties. Among alloys, the most significant increase is produced on the metallurgical structure and thus in the mechanical properties of steels. Basically the heat treatment can be catalogued according two main needs: 1) as an intermediate process in the manufacture of an specific product, e.g. annealing for cold forming in order to improve machinability and 2) as an application dependent process (usually as a finishing process) to cause specific properties such as hardening. Even the study of heat treatment covers a large amount of phenomena and properties, in this section only the essential information related to steels, the processes involved throughout the heat treatment and the structural modifications suffer the carbon alloys as well as the effects of alloying elements on the heat treatment of the steel are examined. Changes in the metallurgical structure of the steel and hence on its mechanical properties. Heat treatment has been used in most of the ferrous metals aand alloys in order to modify their properties, however steels suffer the most dramatic increase on its. In manufacturing the heat treatment process is usually a group of different industrial and metalworking activities employed to alter the chemical and physical composition of a material [Totten and Howes, 1997]. Frequently materials such as steels and including suffer the most dramatic changes as a consequence of the application of a heat treatment.

As soon as heat is applied

3.4 MECHANICAL TESTING

During manufacture and assembly of products, mechanical testing is crucial to ensure that any ferrous or non-ferrous materials and particularly steels complies the mechanical property requirements, applicable standards and specifications of the final components. This process of routine testing is usually carried out in-house for interpretation purposes of the final product quality. Furthermore, this valuable testing knowledge is needed when interpreting and assessing test results from other material suppliers. Usually mechanical testing can be classified according the type of mechanical property to be studied, namely: a) static or b) dynamic. This is due to mechanical properties which can be classified according to two main properties, namely: (a) static and (b) dynamic. While the former is a property independent of the loading rate at which a force is applied to a test piece, the latter is a property that depends on it. The main types of mechanical property tests that are usually employed for heat-treated steel are:

- Tensile testing: This test results in the determination of values such as Tensile Strength (TS), the Proof Stress (PS), the Yield Stress of the material (YS), and the elongation and reduction of area of the specimen.

- Impact testing: This test is used to measure the resistance to failure of a material to a suddenly applied force.

- Hardness testing. This is a test method dependent that measures the resistance of a material to permanent indentation.

Not all the mechanical static (strength, elasticity, plasticity, ductility, hardens and malleability) and dynamic(creep, fatigue, toughness and brittleness) properties can be directly measure by using the above tests. However, such tests are important for designing engineering steels mainly in order to inferred properties of the material.

3.4.1 TENSILE TESTING

The tensile test is the most commonly procedure employed for determining mechanical properties such as strength, toughness, ductility and strain-hardening. Particularly, the tensile strength is one of the most significant mechanical properties in material engineering that mainly corresponds to the maximum amount of stress that any material can resist before failure. Typically there are three different types of definitions of tensile strength which are:

- **Ultimate strength or tensile strength.** This type of mechanical property refers to the maximum stress that any material can withstand during a tensile test.
- **Yield strength**. Defines which is the maximum stress a material can withstand without deformation. This measure is useful to determine the maximum elongation of a material under the application of an specific load.
- **Breaking strength**. Is the ultimate stress where the material fails.

From the tensile test it is possible to obtain three direct measures which are the ultimate tensile, reduction in area and maximum elongation. Moreover, some other values such as the Young's modulus, Possion's radio, yield strength and the strain hardening can be estimated from the direct results mentioned above. The tensile test basically consists in the preparation of a test piece (specimen) which usually can be found in three different forms, namely: a) solid and round, b) tubular or c) flat shape. The specimen usually is stipulated to have the form as illustrated in Fig. 3.3 with a uniform central gauge length and shape both affecting the final test results. The international specifications for the test specimen dimensions are usually regulated by the ASTM standards, however the British standards cover a wide range of forms and dimensions [Tenner, Tenner et al., 2001]. In research's Tenner, a deeply summary related to the different British test standards can be found. In practice the tensile test consists in gripping in the jaws of a tensile machine a predefined cross section specimen which is subjected to a tensile force which is gradually increased by suitable increments of load. At each load increase the length of the specimen is measured by a device up to the test piece fails. Throughout the application of a tensile force, a strain-stress diagram can be plotted (See Fig. 3.3). This diagram depicts the mechanical behaviour of the test piece including the plastic and elastic zones. At first (a) a uniform static deformation is exhibited by the test piece with no proportion to the applied load. This means that after the application of a load the specimen dimensions will return to its original size (elastic zone) obeying the Hooke's law which states that the strain produced is proportional to the stress applied. At the slope 0-(a) the value stress/strain is constant which is know as the Young's Modulus of elasticity. If the specimen is stressed beyond the point (a), the curve form deviates from its straight shape to a

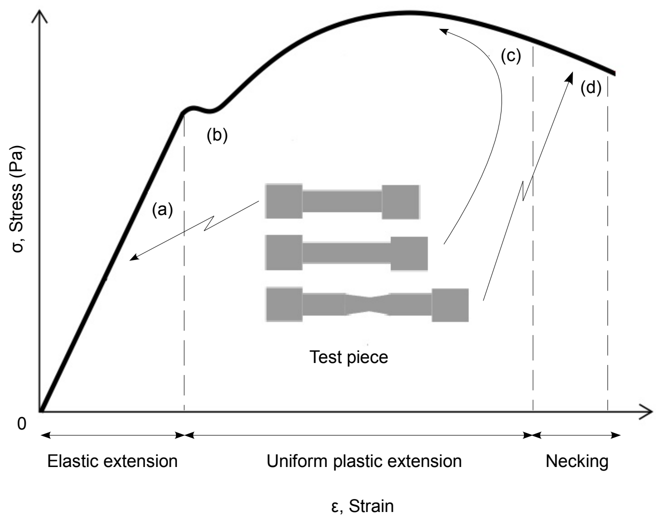


Fig. 3.3 Tensile strength Curve.

3.4.2 HARDENING TESTING

Basically, hardness is the material's resistance to deformation - in materials engineering, three different types of hardness measurements can be found, namely: scratch, indentation and rebound. The first type aims to measure how resistant a material is to plastic deformation due to friction produced by a sharp object. The second measurement refers to the resistance offered by a material to material deformation when a compression load is applied constantly by using a sharp object. The last type aims to indicate the dynamic hardness level in relation to elasticity. Since indentation hardness is of an enormous importance in engineering a brief introduction to the hardness test in order to measure indentation will be provided in this section. In this sense, a variety of this type of hardness tests exist which include Brinell, Knoop, Vickers and Rockwell. For example, the standard Rockwell basically consists in the application of a constant load over the surface area of indentation in a piece where one is the penetrator and the other is the specimen to be tested. This test usually employs a single diamond cone penetrator of a 120° with a rounded off peak of 0.2 mm. Such a penetrator can be replaced by a ball made from a hard metal whose diameter is test dependent. Usually the specimen must be 8 times as thick as the indentation made.

3.5 IMPACT ENERGY IN HEAT TREATED STEELS

Heat treatments are usually carried out to develop the required mechanical properties such as ductility and toughness in a range of alloy steels [Tenner et al., 2001]. In fact, many parts of a machine need to be designed to stand impact loads and absorb the energy of the impact through an elastic action. Materials that must resist an impact usually range from areas such medicine and food packaging and storage up to areas such as industrial products and aerospace and defence. Particularly aerospace and defence need materials engineered for structural applications that must be highly capable of absorbing rapidly applied forces [Louden et al., 1988]. For example, during the operation, military or commercial aircrafts can be hit by runway debris, hail or maintenance tools producing an important internal damage to an structural component and lead to performance failure.

Impact energy test is frequently employed to ascertain the fracture characteristics of materials; it basically estimates the impact energy of a standard size/shape bar of square cross section during its fracture by another standard type of cantilever equipment. As it is illustrated in Fig. 3.4, where a typical impact energy procedure is depicted; a load is applied as an impact blow from a weighted pendulum hammer, which is released from a specific height. the specimen is placed on a base and suddenly hit by the pendulum that fractures it.

The fracture often propagates from an initial fatigue crack which is produced artificially prior to the test. The energy produced due to the impact of the pendulum is absorbed by the specimen during the fracture and then measured by the angle of displacement of the pendulum. There are two main types of impact energy procedures, namely: (a) Izod test and (b) and Charpy test. While Charpy impact test usually uses a V-notch specimen that opposes to hammer (see Fig. 3.5 (a)), the Izod test is often used for nonmetallic materials and the test specimen may be either notch or unnotched.

The necessary energy to fracture the specimen usually is measured in Joules and from a modelling point of view both types of impact energy procedures are not compatible as there is not conversion from one type to the other. Moreover, according to what materials are being tested, specimen of metals are usually squared, and polymers are usually rectangular being struck perpendicular to long axis of the rectangle.

The standard Charpy impact test specimen consists of a bar of metal, or other material whose dimensions are usually $55 \times 10 \times 10$ having a notched machined across one of the larger dimensions. The Izod test like the Charpy test is also used to test materials at low temperature to emulate conditions that may occur in real conditions of use of the material. Opposite to the Izod test, Charpy is one of the most popular and standardised impact techniques used as an economical quality control method to determine the notch sensitivity and impact toughness of engineering materials.

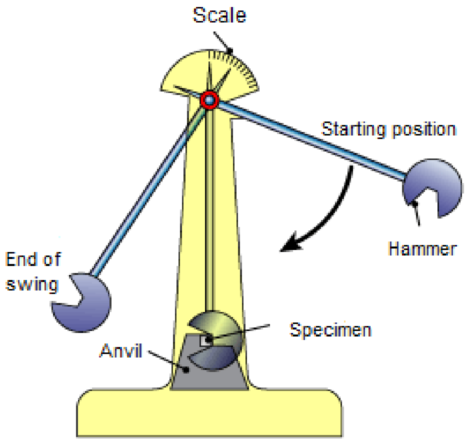


Fig. 3.4 Charpy impact test.

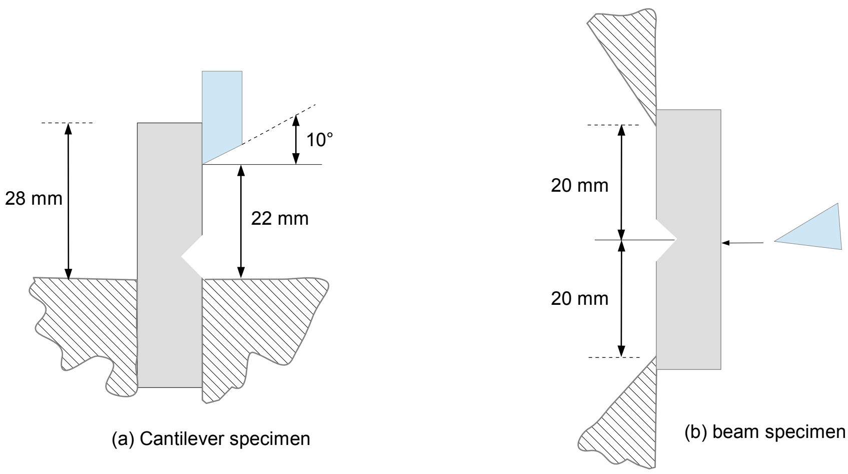


Fig. 3.5 (a) Cantilever arrangement and (b) beam arrangement of the Izod and Charpy specimen.

The Charpy test is frequently applied to composites, ceramics and polymers. By applying the Charpy test to identical specimens at different temperatures and then plotting the impact energy as a function of temperature, the ductile-to-brittle transition becomes an important property, including some factors such as low temperatures, high strain rates and stress concentrators (notch) that involve the toughness of the specimen and then the material.

The cantilever arrangement of the Izod specimen and the 3-point beam arrangement of the Charpy impact test are illustrated in Fig. 3.5. Usually the test conditions depends on the customer preferences and needs which can include tests at sub zero temperatures or the chemical composition of the specimen. The modelling of impact energy test is usually quite complicated mainly due to the following reasons:

- The non-linear behaviour of the process.
- High-interaction between the multiple-variable input spaces.
- Measurement uncertainty of the industrial data.
- High-complexity of the optimisation space.
- Low repeatability in impact test results with similar statistical properties.
- Sparse data space.

3.5.1 NEURAL-FUZZY MODELLING ON IMPACT ENERGY TEST

Neural-fuzzy modelling is a framework that uses on the one hand the capabilities of fuzzy systems such as fuzzification, linguistic rules, fuzzy sets based-inference engine and defuzzification in order to create transparent and interpretable models. On the other hand a neural fuzzy model preserves the functional approximation and learning capabilities as well as generalisation properties of neural networks to approximate highly non-linear and complex real systems.

Furthermore, a neural-fuzzy model is able to represent real systems by the construction of linguistic rules and quantifying the uncertainty in a simple way which can be translated into fuzzy numbers or fuzzy sets associated with linguistic labels.

3.6 DATA-DRIVEN MODELLING OF IMPACT ENERGY TEST APPLIED ON HEAT TREATED STEELS

Charpy test has been used for more than a hundred of years for the toughness assessment of metallic materials including steels. Moreover, the Charpy test has also been used for characterising the ductile-to-brittle transition temperature (DBTT) of materials [Rossoll et al., 2002]. Basically the impact test provides the information necessary to understand the behaviour of a material under dynamic loads compared to the information provided by just analysing the data obtained from tensile strength tests where the load is slowly applied and sometimes known as static load. The knowledge and representation of the impact test properties is of engineering importance as it can be estimated the amount of energy absorbed by a material before fracturing. Therefore, this information can be used to estimate which mechanical properties of the material (steel) are the most appropriate in order to withstand a load without fracturing.

Laboratory experiments usually are performed in order to replicate as nearly as possible the service conditions to which the materials undergo. Hence the impact test conditions must be correctly chosen in order to represent the most severe conditions to which the material fractures, for example a) the deformation of the material at relatively low temperatures, b) the triaxial stress state which is caused by the presence of a notch (a notch reproduces the same effect of a crack in the presence of a blow), and c) a high strain rate (i.e. the rate of deformation). The last decade, a larger community of researches have embraced the construction of data-driven models through the application of soft computing techniques for predicting the mechanical steel properties.

Indeed, there is much evidence of successful applications, for example at the university of Sheffield, Professor D. A. linkens was a pioneer in the construction of dynamical system identification with the help of soft computing techniques, specifically neural fuzzy systems. Linkens proposed different types of models for mechanical property prediction of hot rolled steels and C-Mn steels [Chen and Linkens, 2001a,b]. The main purpose of such models were to construct from numerical data a linguistic representation of mechanical tests that includes an initial fuzzy model self-generation based on neural neural networks, partition validation, parameter optimization and rule-base simplification.

Due to the functional equivalence established in [Jang and Sun, 1993] between a type of fuzzy systems and neural networks, some researches have exploited and explored the theory and new advances found in fuzzy logic to create models that have a good balance between accuracy(precision) and transparency(interpretability). For instance, in [Zhang and Mahfouf, 2011] a new methodology to accurately represent in an interpretable form complex high-dimensional datasets concerned to the prediction of mechanical properties of alloy steels by correlating them to the conditions of the heat treatment and the associated chemical composition of the steel.

The new methodology consists of an initial Mamdani fuzzy model based on a hierarchical clustering approach and its corresponding improvement by using a high-performance particle optimisation (PSO) based multi-objective optimisation mechanism.

Based on the experiments presented in [Panoutsos and Mahfouf, 2010a] this section describes the application of a neural fuzzy model that is functionally equivalent to a type of fuzzy systems (deeply examined in Chapter 6 as functionally equivalent to a group of type-1 fuzzy systems) for modelling the Charpy impact test. The data-driven modelling of the impact energy test usually includes the combination of two or more techniques from soft computing, for example: fuzzy logic, neural networks, genetic algorithms and evolutionary strategies.

The real case study proposed in this research work is a collection of different experiments carried out at six different test sites (provided by the TATA Steel Company, Yorkshire UK). where the data set consists of 1661 measurements on heat-treated steel. In order to be familiar with the process and its data, it would be worth to provide an insight of the collected data.

Table 3.1 Statistics of Impact Energy Test dataset

Variable	Type	Min.	Max.	Mean	SD
Test depth, mm	Input	5.5	146.0	20.8	14.5032
Specimen size, mm	Input	11.0	381.0	172.488	80.8380
Test site	Input	1	6	3	0.4984
C (wt-%)	Input	0.13	0.52	0.3942	0.0575
Si (wt-%)	Input	0.11	0.38	0.2548	0.0318
Mn (wt-%)	Input	0.41	1.75	0.8409	0.2172
S (wt-%)	Input	0.0008	0.052	0.0167	0.0089
Cr (wt-%)	Input	0.11	3.25	1.0752	0.2447
Mo (wt-%)	Input	0.02	0.98	0.2394	0.0860
Ni (wt-%)	Input	0.03	4.21	0.3683	0.5190
Al (wt-%)	Input	0.003	0.047	0.0270	0.0048
V (wt-%)	Input	0.0010	0.26	0.0077	0.0223
Hardening temperature °C	Input	810.0	980.0	864.0157	15.4689
Cooling temperature °C	Input	1	3	1.5	0.3830
Tempering temperature °C	Input	190.0	730.0	647.1927	49.9249
Test temperature °C	Input	-59.0	23.0	-5.7869	26.4486
Impact Energy	Output	3.4667	245.33	89.6419	32.9701

The Charpy data set consists of 1661 measurements on heat-treated steel represented in a matrix format whose rows represent a different heat treatment batch and where each column of data is describing the variables process (inputs) and its corresponding results (output). A basic initial processing stage is done by providing some information related to the max-min variable values and the associated correlation measures as illustrated in Table 3.1. Since the input variable values (See Table 3.1) are defined over different ranges, a normalisation process is necessary to produce a data set whose importance among the variables is similar. Due to the reasons mentioned above and the complexity of the data space and its sparsity, there are areas of high density (popular steel grades). Fig. 3.6 illustrates such areas, in which a number of various samples of Carbon(%), Mn (%), test depth (mm), and the size of the specimen are shown. In Fig. 3.8, the basic neural fuzzy modelling framework used throughout this research work is described.

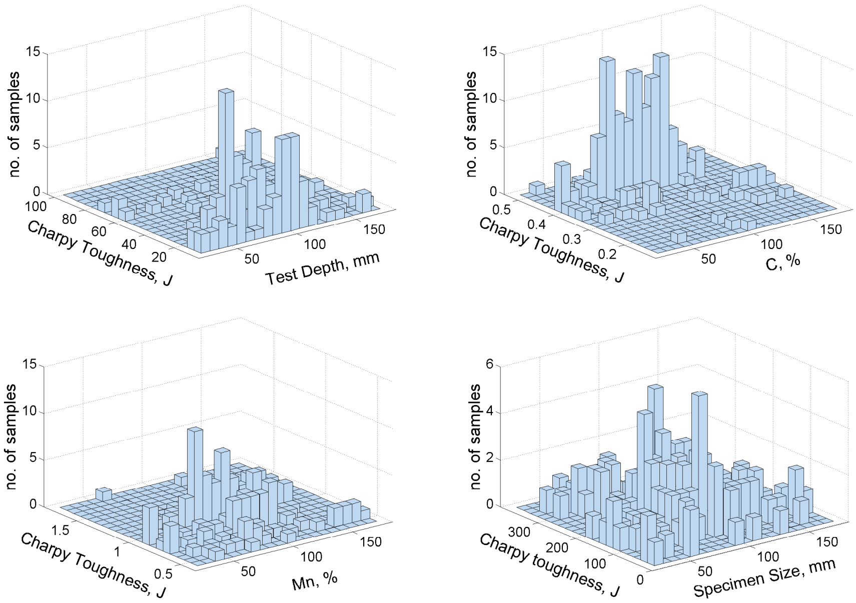


Fig. 3.6 Data density example

Neural-fuzzy modelling is a framework that uses on the one hand the capabilities of fuzzy systems such as fuzzification, linguistic rules, fuzzy sets based-inference engine and defuzzification in order to create transparent and interpretable models. On the other hand a neural fuzzy model preserves the functional approximation and learning capabilities as well as generalisation properties of neural networks to approximate highly non-linear and complex real systems. Furthermore, a neural fuzzy model is able to represent real systems through linguistic rules and quantify the uncertainty in a simple way which can be translated into fuzzy numbers or fuzzy sets associated with linguistic labels. Considering the functional equivalence between the RBF-NN and the Tagaki Sugeno type-0 FS (or type-1 Mamdani inference engine), an RBF-NN combines the input-output n+1 dimensional space $(x_1, \ldots, x_k, \ldots, x_n, y_{n+1})$ where x_k represents the input partition and the corresponding output y_{n+1} as is illustrated in Fig. 3.7.

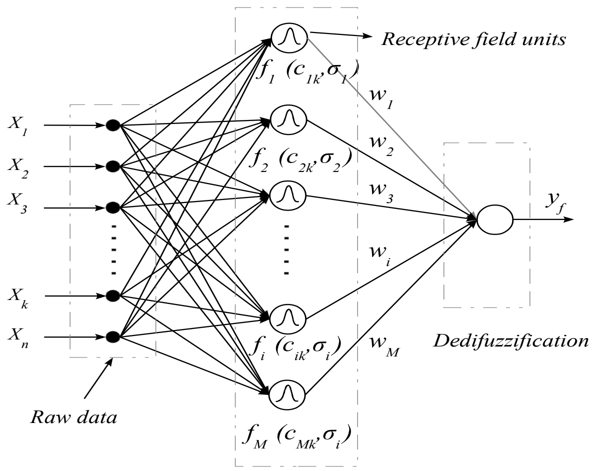


Fig. 3.7 RBF-NN structure.

According to the flow diagram illustrated in Fig. 3.8, an initial information extraction is done by normalising the Charpy data set (Raw Data) and computing some correlation measures. In addition to this information processing, some other researches such as [Tenner et al., 2001] suggests an additional processing stage for data cleaning that aims to remove faulty outlying points. Tenner proposed several sources for outlier points due to the following reasons:

(i) Data handling errors (faulty data)
(ii) Measurements/process faults (faulty data)
(iii) Typographical errors
(iv) Incorrect treatment prescription (valid data)

four different methodologies can be used in order to find the sources for faulty points:

- basic (max-min and correlation)
- structured (analysis of similar input vectors)
- multivariate (principal component analysis PCA)
- learn detection (model based analysis)

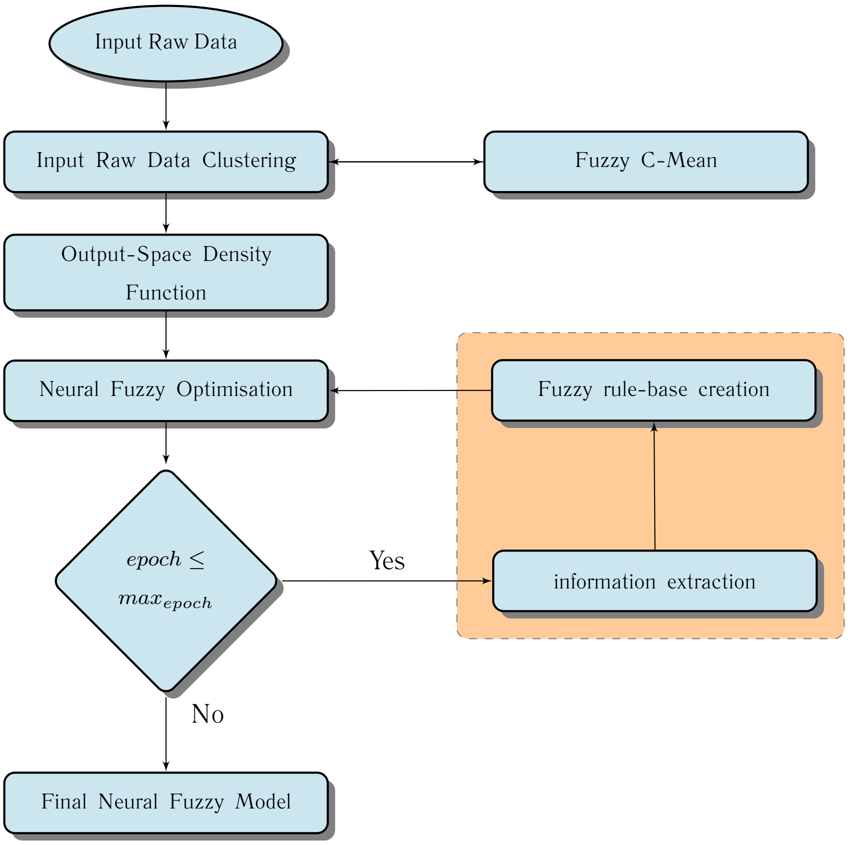


Fig. 3.8 Data-driven model based on RBF Neural Networks and Fuzzy Clustering.

The process of normalisation for the input raw data can be done in different ways and it is mainly problem-dependent, however some of the most popular methodologies scale the input data into the closed interval $[-1, 1]$ or between $[0, 1]$.

The purpose of the application of a normalisation process is to scale data from a problem and reducing it into an specific range while preserving the data integrity and eliminating the redundancy in the data. That means that all the data (input data) are consistent and hence satisfy all the constraints (limits) of a predefined range. Moreover, the normalisation process must ensure that even properties such as direct redundancy which means that the data set is found in two different locations or if the data can be expressed/calculated from other data items (indirect redundancy) are preserved.

Usually a normalisation process for scaling the input data x_i between the limits $[0, 1]$ is recommended when training a neural network whose activation function is a sigmoid. Therefore, the following expression can be used:

$$x_{nor} = \frac{x_i}{max(x_k)_{k=1,\dots,N}}, x_i \in R^n \tag{3.1}$$

where N is the number of inputs and x_k is the kth element of the original data set. A better normalisation equation can be used to normalise the minimum value from the data set to zero, and to adjust its maximum value to one stated as:

$$x_{nor} = \frac{x_i - min(x_k)_{k=1,\dots,N}}{max(x_k)_{k=1,\dots,N} - min(x_k)_{k=1,\dots,N}} \tag{3.2}$$

A process for normalising the input data between $[-1, 1]$ is usually employed when a tangent activation function is used in the hidden layer of a neural network. Hence, the following equation can be computed:

$$x_{nor} = 2 * \frac{x_i - min(x_k)_{k=1,\dots,N}}{max(x_k)_{k=1,\dots,N} - min(x_k)_{k=1,\dots,N}} - 1 \tag{3.3}$$

In Table 3.2, the max-min values and some correlation measures of the normalised Charpy data used during the training stage and obtained by using 3.3 are illustrated. For cross validation purposes, the data set was split into training, checking and testing sets in order to avoid over-fitting which enables the model to improve its generalisation properties. The data set used to train the RBF Neural Network (RBF-NN) consists of 1084(65%), which are composed of just normalised raw data. The checking and testing data are 277 (17%) and 300 (18%) respectively. Following the flow from Fig. 3.8, the clustering procedure employed for the initial parameter identification process is the Fuzzy C-Means which allows each data point to belong to one or several clusters to a degree specified by a membership grade.

Table 3.2 Statistics of the normalised Impact Energy Test dataset

Variable	Type	Min.	Max.	Mean	SD
Test depth, mm	Input	-1	0.9851	0.1918	0.1322
Specimen size, mm	Input	-1	1	0.4389	0.3705
Test site	Input	-1	1	0.5721	0.4685
C (wt-%)	Input	-1	1	0.3077	0.2306
Si (wt-%)	Input	-1	0.7778	0.2386	0.1811
Mn (wt-%)	Input	-1	1	0.3304	0.2422
S (wt-%)	Input	-1	1	0.3485	0.2941
Cr (wt-%)	Input	-1	0.9745	0.1534	0.0715
Mo (wt-%)	Input	-1	1	0.1959	0.1045
Ni (wt-%)	Input	-1	1	0.2485	0.1455
Al (wt-%)	Input	-1	1	0.2130	0.1681
V (wt-%)	Input	-1	1	0.1959	0.0557
Hardening temperature °C	Input	-1	1	0.1920	0.1154
Cooling temperature °C	Input	-1	1	0.4150	0.2402
Tempering temperature °C	Input	-1	1	0.1846	0.1431
Test temperature °C	Input	-1	1	0.6375	0.6198
Impact Energy	Output	3.4667	245.33	89.6419	32.9701

The mechanism behind the Fuzzy C-Means algorithm (FCM) is to partition n-dimensional P data points into M fuzzy clusters. By minimising an objective function J_m based on each cluster centre location v_i the FCM algorithm creates a fuzzy partition space where each data point x_p can belong to several clusters with a membership grade u_{pi}. The FCM algorithm constructs a matrix $\hat{U}$ whose elements have a range defined in the interval $[0, 1]$. The objective function is defined as follows:

$$J_m(\hat{U}, v) = \sum_{p=1}^{P} \sum_{i=1}^{M} u_{pi}^m d_{pi}^2 \tag{3.4}$$

where u_{pi} is the membership between 0 and 1 of the element x_p, m the fuzziness exponent, the variable $d_{pi} = \parallel x_p - v_i \parallel_A$ is the Euclidean distance between the element x_p and i_{th} cluster center, and v_i is the vector of centers $v_i = (v_1, v_2, \ldots, v_M)$. The computation of u_{pi} and v_i can be stated as:

$$v_i = \frac{\sum_{p=1}^{P} u_{pi}^m x_p}{\sum_{p=1}^{P} u_{pi}^m} \tag{3.5}$$

and

$$u_{pi} = \frac{1}{\sum_{i=1}^{M} (\frac{d_{pi}}{d_{pk}})^{2/(m-1)}} \tag{3.6}$$

Where the input vector $x_p = [x_1, \ldots, x_n]$ and k is the kth iteration used for the clustering process. Basically, the FCM algorithm is an iterative process that in a batch mode operation the clustering procedure determines the cluster center v_i and the corresponding matrix $\hat{U}$ as follows [Cannon et al., 1986]

Step 1 fix the number of clusters M, $2 \leq M \leq P$. Fix m between $1 < m \leq \infty$. Choose any inner product induced norm metric $\parallel \cdot \parallel$, e.g.

$$\parallel x - v \parallel^2 = \parallel x - v \parallel^T A \parallel x - v \parallel \tag{3.7}$$

Step 2 Initialise $\hat{U}$ matrix, $\hat{U}^{(0)}$.

Step 3 at p_{th} step calculate the centers vectors $v_i = \{v_1, v_2, \ldots, v_M\}$ by using 3.5.

Step 4 update $\hat{U}_p, \hat{U}_{p+1}$ by using 3.6.

Step 5 If $\parallel \hat{U}_{k+1} - \hat{U}_k \parallel < \epsilon$ then stop, otherwise go to step 3.

The output-space density obtained from the application of the FCM is then used for establishing the initial parameters for the hidden units of the RBF-NN. The width of the Gaussian function in the RBF-layer is calculated via the following expression [Pedrycz, 1998]

$$\sigma_i = \frac{1}{r} \left(\sum_{l=1}^{r} \|v_j - v_i\| \right)^{1/2} \tag{3.8}$$

in which v_l is the nearest neighbour to the centroid v_i and $r \geq 1$, usually the value of r is 2, however it may be depend on the type of problem. Once the initial parameter values are estimated, the information extraction can be obtained through the exploitation and exploration of an initial fuzzy rule-base which can be created by M fuzzy rules that corresponds to the final number of receptive units (hidden layer neurons) at the RBF-NN, thus one fuzzy rule can be stated as:

$$R_i : \textbf{IF } x_1 \ is\ A_1^i \textbf{ AND } x_2 \ is\ A_2^i \textbf{ AND } x_N \ is\ A_M^i \textbf{ THEN } y \ is\ Y_p \qquad (3.9)$$

where $i = 1, \ldots, M$, M is the total number of rules or receptive units, A_1^i is the fuzzy antecedent at the i_{th} fuzzy rule, y is the output linguistic variable and Y_p is the consequent fuzzy set.

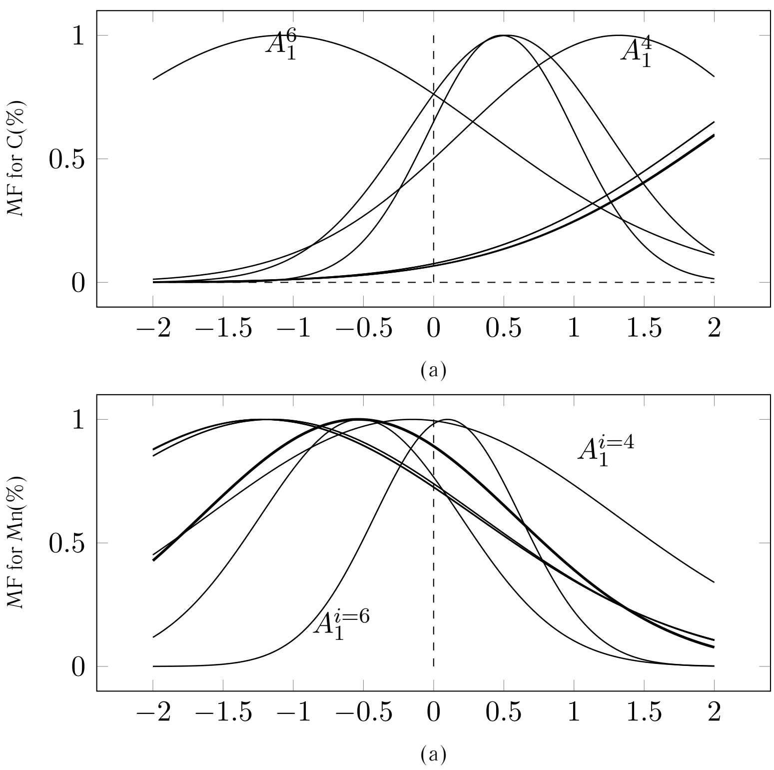


Fig. 3.9 Final distribution in the Universe of discourse of the C(%) and Mn(%) after Fuzzy C-Means (FCM).

To enable a discussion about the results obtained from the FCM process and hence in relation to the initial fuzzy rule base, it would be worth to provide an illustrative example of the final shape of the MFs after FCM. Therefore, in Fig. 3.9 the initial universe of discourse after the application of FCM for the dimension that linguistically describes the Carbon (C-%) and Manganese (Mn-%) is presented. One fuzzy rule that linguistically represents one neuron of the proposed case study can be stated as:

$$\widetilde{R}^1\text{: IF } Testdepth \text{ is } A_1^1 \text{ and } Test\ site \text{ is } A_2^1 \text{ and } C \text{ is } 3^1 \text{ and } Si \text{ is } A_4^1 \text{ and } Mn \text{ is } \widetilde{A}_5^1 \text{ and } S \text{ is } A_6^1 \text{ and } Cr \text{ is } A_7^1 \text{ and } Mo \text{ is } A_8^1 \text{ and } Ni \text{ is } A_9^1 \ldots$$

$$\ldots\text{THEN the Impact Energy is } \widetilde{B}^1 \tag{3.10}$$

Where the multidimensional i_{th} fuzzy set is $A_i = [A_1^i, \ldots A_P^i]$, and P is the total number of inputs. After Fuzzy C-Means, the final rule base is not yet finally constructed. As can be seen from Fig. 3.9(a,b), a high degree of redundancy and a lack of distinguishability in terms of overlapping is still exhibited by the membership functions (MFs). In this context, according to [Zhou and Gan, 2008], in interpretability-oriented fuzzy modelling, each MF of a variable is expected to represent a linguistic label with a clear semantic meaning and thus at least one point in the universe of discourse should have a value equal to one, it means a MF should be normal. Moreover, the normality in fuzzy sets seems to be self-evident and hence the traditional term sets in the universe of discourse should contain not only the intermediate sets, but also left and right-shoulders sets [Zhou and Gan, 2008]. For example, the linguistic variable *age* whose term sets are *young*, *adult* and *old*. It seems that the terms *old* and *young* may reach normality, however, when it comes to the term *adult*, it is difficult to achieve a conclusion. In this sense, the universe of discourse presented in Fig. 3.9 does not employ subnormal MFs which may be debatable due to the type of problem. Particularly, the generation of fuzzy models for mechanical property prediction has demonstrated a satisfactory performance without the use of left and right shoulders and subnormal MFs. In order to better discriminate the role of each multidimensional fuzzy set in the universe of discourse, a supervised parameter identification process is used. In other words, a parameter learning based on a gradient descent methodology is employed.

An example of the initial fuzzy rule-base extracted from the FCM results and that contains only 3 out of the 16 inputs featured by 5 fuzzy sets that compose the input data space can be depicted as illustrated in Fig. 3.10.

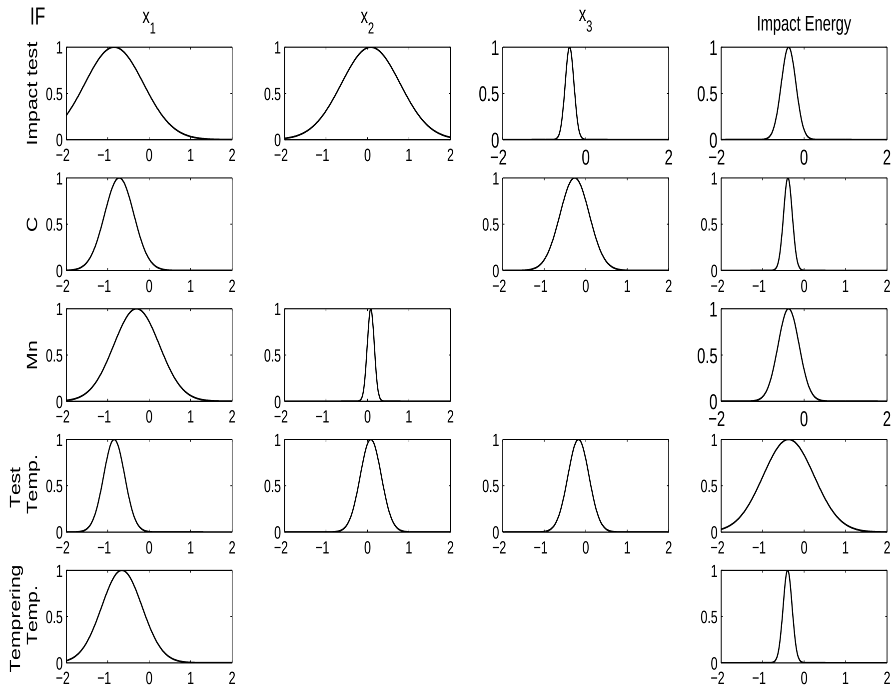


Fig. 3.10 Fuzzy rule-base example

To verify the physical interpretation of the initial model obtained after FCM, in Fig. 3.11 is illustrated the 3-D surface responses and the data density along the surface of 2 out of the 16 input variables versus the measured impact energy (Joules).

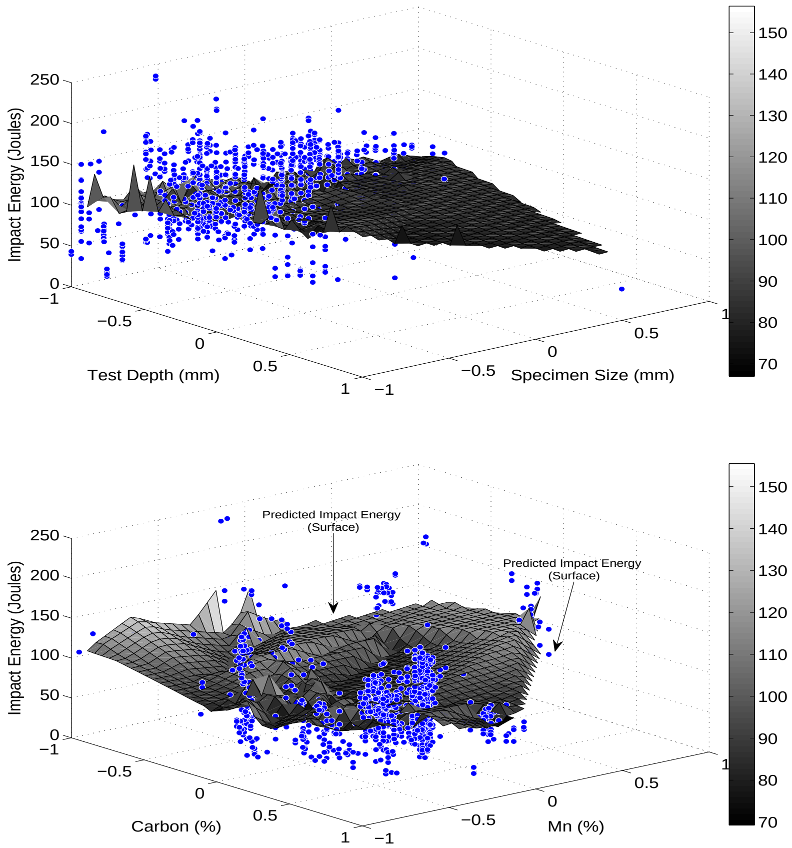


Fig. 3.11 Response surface for six out of 16 parameters of the Charpy test.

As can be seen from Fig. 3.11, two different types of data are shown, namely: (a) the measured impact energy (blue points) and (b) the predicted impact energy (3-D surface). Both data, the measured and the predicted impact energy are represented in terms of four different variables, i.e. 1) test site and the size of the specimen and 2) Carbon and Mn (%) respectively. Such results are obtained just after the application of the Fuzzy C-Mean algorithm in order to cluster the raw data. It is evident from the figures that the surface created by the fuzzy model/initial rule base (RBF network) is not able to cover most of the data. However, the initial location of the centers offers a good approximation of the rule base parameters which will be further optimised by the application of a learning approach based on the gradient descent.

3.6.1 FUZZY RULE OPTIMISATION

Over-training represents that a neural fuzzy system learns to represent noise in data instead of the true underlying process. The cross-validation process consists of randomly choosing data for training and then periodically the prediction accuracy of the model is investigated. The process of validation (checking) on network accuracy for the prediction of impact test results indicates that the process of training must be finalised when the error of the validation increases meaning that the generalisation properties of the model have begun to deteriorate. The fuzzy rule-base optimisation consists in the application of an adaptive Back Error Propagation approach (adaptive-BEP) which has been proven in the past to be very efficient in the proposed type of system [Chen and Linkens, 2001b]. This is due that a conventional BEP usually leads the objective function to a good local minimum by using a small learning rate, but often it does not represent the optimal performance of the system due to the algorithm 'getting stuck' in local minima. In order to overcome this issue a momentum and a continuously adaptive version of BEP is used. Hence, a performance index can be defined as:

$$P_p = \frac{1}{P}\sum_{p=1}^{P} e_p^2 \tag{3.11}$$

in which P is the number of training points. The update rule for the output weight is:

$$w_i(p+1) = \gamma w_i(p) - \beta e_p g_i \tag{3.12}$$

where $g_i = \frac{A_i}{\sum_i A_i}$, $A_i = exp(- \parallel x - c_i \parallel^2 / \sigma_i^2)$, and the update rule for the width is:

$$\sigma_i(p+1) = \gamma\sigma_i(p) - \beta e_p g_i (w_i(p) - y_p)\frac{(x_k(p) - C_{ik})^2}{\sigma_i^3} \tag{3.13}$$

And the update rule for the ith centre is:

$$C_{ik}(p+1) = \gamma C_{ik}(p) - \beta e_p g_i (w_i(p) - y_p)\frac{(x_k(p) - C_{ik})}{\sigma_i^2} \tag{3.14}$$

Where:
β learning rate;
γ momentum;
t iteration number;
d_p pth output from the data;
y_p pth output from the model and $e_p = (y_p - d_p)$;

The energy index is used to update the adaptation algorithm as follows:

- if $Pi_p(t+1) \geq Pi_p(t)$ then

$$\alpha(t+1) = h_d\alpha(t), \; \gamma(t+1) = 0$$

- if $Pi_p(t+1) < Pi_p(t)$ and $\left|\dfrac{\Delta Pi}{Pi(t)}\right| < \delta$ then

$$\alpha(t+1) = h_i\alpha(t), \; \gamma(t+1) = \gamma_0 \tag{3.15}$$

- if $Pi_p(t+1) < Pi_p(t)$ and $\left|\dfrac{\Delta Pi}{Pi(t)}\right| \geq \delta$ then

$$\alpha(t+1) = \alpha(t), \; \gamma(t+1) = \gamma(t)$$

Where h_d and h_i are the decreasing and increasing factors, respectively. And δ is the threshold for the rate of the relative index. That means the performance index follows the behaviour of the RMSE whose constrains are:

$$\begin{aligned} 0 < h_d < 1 \\ h_i > 1 \end{aligned} \tag{3.16}$$

Once the parameter optimisation process have been completed, the final fuzzy model is obtained. Therefore, the modelling results will be discussed in the next section.

3.6.2 PRELIMINARY MODELLING RESULTS

Due to the variability produced by the FCM, several trials were performed with a different number of clusters and hence through cross-validation experimentation it was found that the optimum number of fuzzy rules (hidden layer neurons) is 9 for the prediction of the Charpy Impact test. Fig. 3.12 and table 3.3 show the effects of hidden layers on mean squared prediction accuracy for impact energy prediction. That means prediction accuracy of the RBF network was compared by evaluating the Root Mean Square Error (RMSE), where Eq. 3.17 the terms y_p and d_p are the current model output and the desired pattern respectively. In Table 3.3, it can be seen the results obtained from different trials ranging between 6 and 100 clusters. The various experimental results shown in Table 3.3 do not represent all the information that can be extracted from the neural fuzzy modelling framework used in this section. However, it contains the information required to decide which model could have a good balance between accuracy and interpretability. Moreover, for future comparisons in Fig. 3.12 the results were obtained by rearranging the data for training, checking and testing are provided.

$$E_{RMS} = \left(\frac{1}{P} \sum_{p=1}^{p} (y_p - d_p)^2 \right)^{1/2} \tag{3.17}$$

Table 3.3 RMSE of the neural fuzzy framework

Number of clusters	Training	Checking	Testing
6	20.10	20.95	22.78
9	18.78	19.48	21.78
15	15.46	19.65	21.90
30	14.8	19.85	20.80
50	13.74	22.01	22.45
100	15.30	22.20	24.12

According to [Gacto et al., 2010, 2011, Zhou and Gan, 2008], the number of MFs should not be arbitrary, but it should be according to the number of individual entities the human beings can handle and store efficiently at the short-term memory [Pedrycz et al., 1998, Valente de Oliveira, 1995]. From this idea and according to the cross-validation experimentation results obtained by using a different number of rules, in this research work it is more convenient to select a model that contains only 9 rules. Furthermore, a model with a moderate number of rules is easier to be interpreted and hence studied enhancing the consistency of the fuzzy rule base. From table 3.3, it is evident that a fuzzy model with only 9 rules shows a better performance in terms of generalisation (checking and testing) than models with a smaller or larger number of rules. This can be confirmed with those results obtainedin Fig. 3.12. In fact, a model that contains more rules not necessarily behaves more accurately than models with a moderate number of MFs. This also implies a lack of interpretability and hence of distinguishability. Fig. 3.12 illustrates the data fit by using 9 rules for the prediction of impact energy for two different simulations of 1300 epochs each one. It is important to note that the final clusters used for testing the model are those obtained when the checking performance trend has stopped growing as illustrated below.

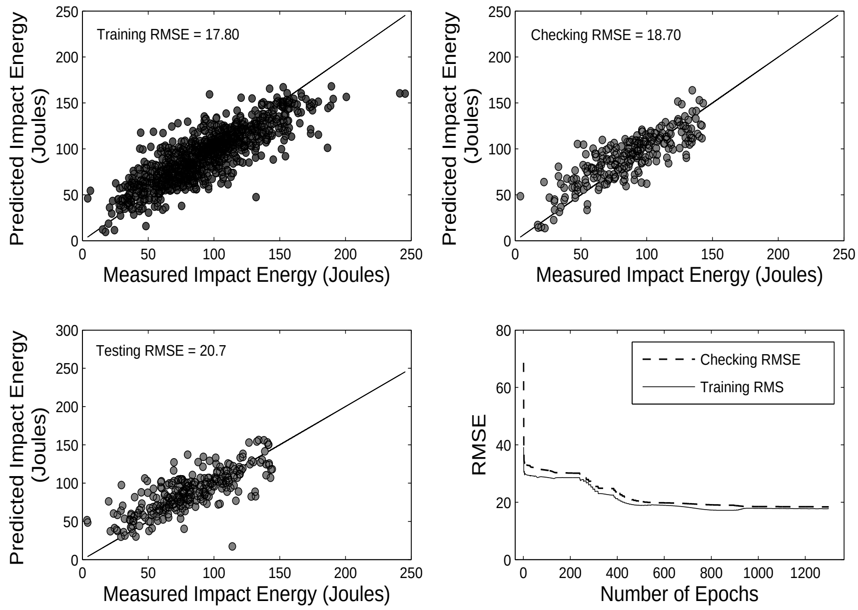


Fig. 3.12 Data fit, Charpy Impact Test Prediction by using Fuzzy C-Means as the clustering approach for the construction of the initial fuzzy rule base.

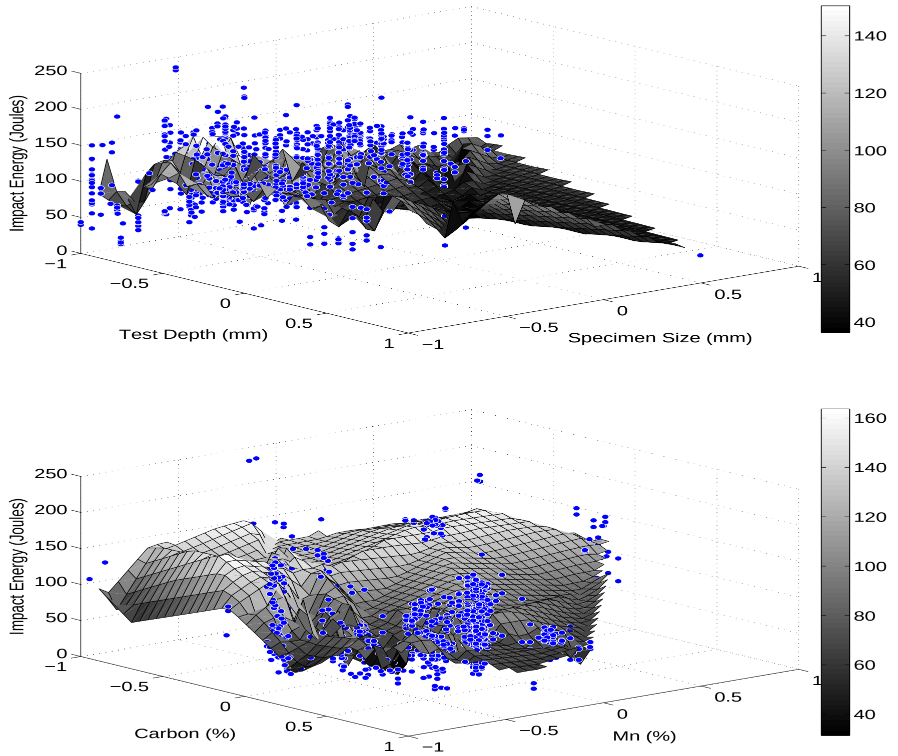


Fig. 3.13 Response surface of the RBF NN after the training process.

To provide more information about the RBF NN performance, in Fig. 3.13, 3.14 and 3.15 are shown the response surfaces after the process of cross-validation. Particularly, Fig. 3.13 and 3.14 show 4 out of the 16 variables from the input space. As can be seen from the results, the impact energy values are defined in the interval $[0, 250]$ whose units are Joules. It is also evident from Fig. 3.13, Fig. 3.14 and Fig. 3.15 the response surfaces covers the majority of the measured impact energy (pattern). This reflects good generalisation properties (testing) whose response surface keeps a similar shape to that one obtained after training. Moreover, more information may be extracted from Fig. 3.14 and 3.15. For example, in Fig. 3.14, the size of the specimen appears to affect slightly the predicted impact energy in relation to the test depth which are measured in millimetres.

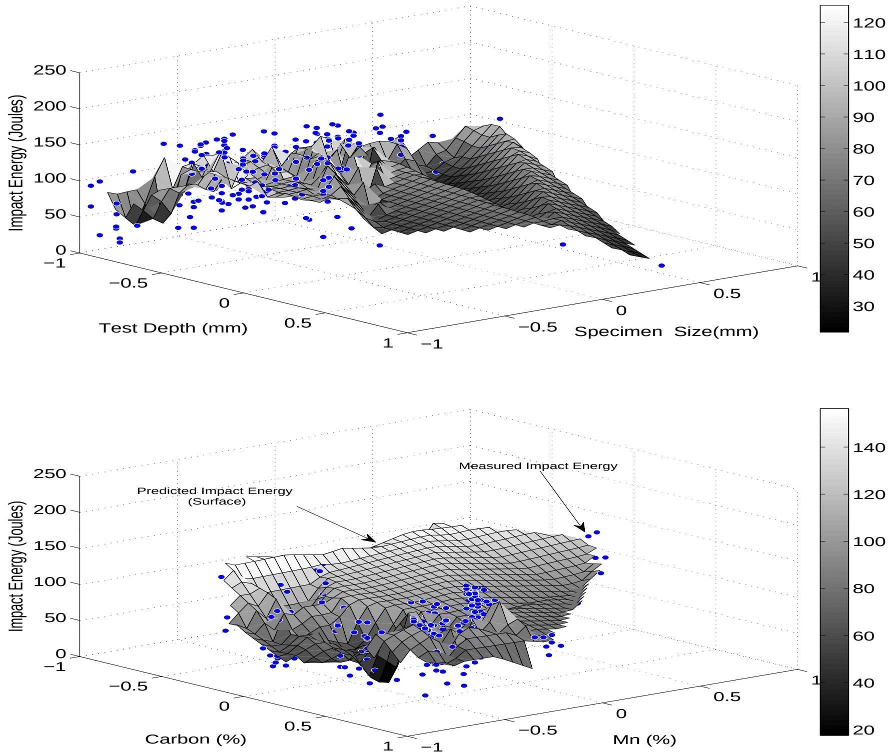


Fig. 3.14 Response surface of the RBF NN after the checking process.

In this sense, in Fig. 3.15, it can be observed that there are areas where a small increase/reduction in the amount of Carbon and Mn affects importantly the prediction of the impact energy. Indeed, the analysis of the response surface may help to understand the sensitivity of the RBF network which can be calculated by summing the effects of small changes to each input variable across the given data set [Tenner et al., 2001]. As it is mentioned in [Zhang and Mahfouf, 2011], while a fuzzy model can provide information from the surface response based on limited inferences mechanisms for the unseen part of the data due to the process of fuzzification, a neural network is purely a fitting function. In this regards, the RBF NN according to [Hunt et al., 1996, Jin and Sendhoff, 2003] can be interpreted as a type of fuzzy systems of type-1 inheriting properties such as transparency (information extraction), interpretability (rule-base creation) and distinguishability.

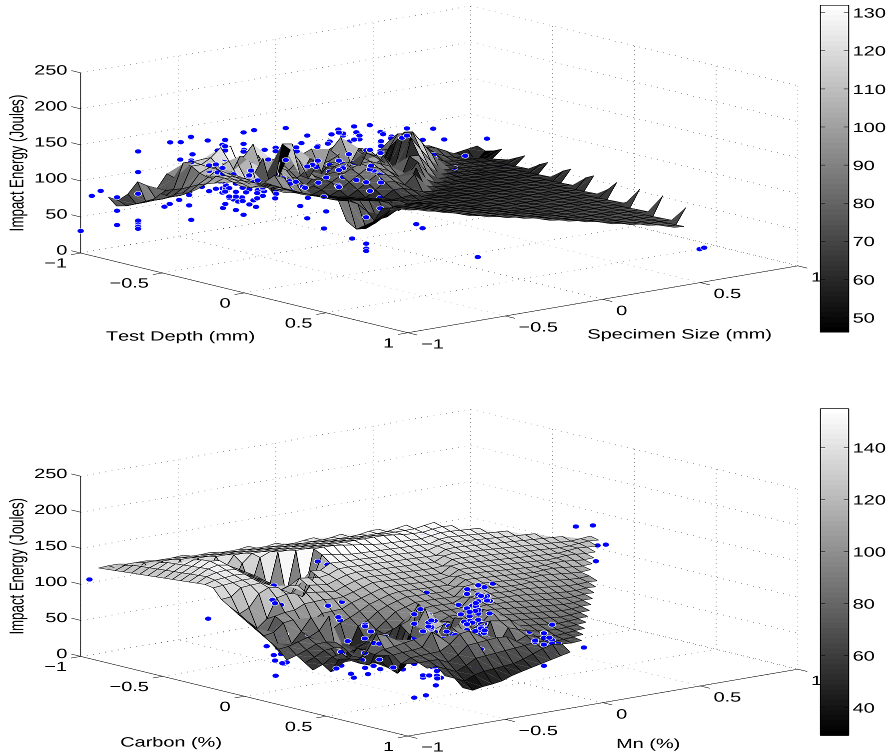


Fig. 3.15 Response surface of the RBF NN after the testing process.

The rest of this Thesis work will be focused in exploiting and exploring various concepts developed into fuzzy set theory and neural networks for function approximation purposes. Finally, in Appendix **??** is illustrated the final shape of response surfaces after the process of clustering by applying 1) granulations and 2) the well-known FCM approach.

3.7 SUMMARY

In this chapter a background on mechanical tests of heat treated steels and its importance for manufacturing process as well as a modelling of a real case study for impact energy prediction were provided. The presented modelling framework combines the ability of fuzzy sets and RBF neural networks for function approximation through the exploration and exploration of information extraction.

A detailed hybrid methodology for the parameter identification of the

RBF neural network was described, including the initial FCM-based clustering approach and the application of an adaptive gradient descent approach. Finally some results were discussed including the creation of a multidimensional rule-base.

In the next chapter, a modelling framework based on RBF neural networks, Granular Computing (GrC) with an application of Neutrosophic Sets (NS) for the analysis and evaluation of uncertainty will be introduced.

4

LOW-LEVEL INTERPRETABILITY IN THE RBF-NN USING GRANULAR COMPUTING AND NEUTROSOPHIC SETS.

This chapter provides a new methodology based on Granular Computing (GrC) and neutrosophic sets in order to evaluate the associated uncertainty that results from a ravenous behaviour during the merging operation at the granulation stage. First, the construction of neutrosophic sets is based on a Shannon criterion in order to extract information in relation to the distinguishability at the granulation process. Secondly, such an information is used to quantify the uncertainty/fuzziness when forming new granules and finally such an information is used in conjunction with the compatibility criterion employed at the granulation process for making decisions and creating a more transparent fuzzy rule base.

The main motivation for creating a framework that is able to quantify the uncertainty during the granulation process lies on the idea that when applying an adaptive learning algorithm, a lost of interpretability is produced during the parameter identification of the RBF-NN. For this reason, a more transparent and distinguishable initial fuzzy rule base might aid to create a more parsimonious inference engine. In order to compare the proposed methodology, some preliminary simulation results based only on granulation and the RBF-NN are provided.

4.1 INTRODUCTION

The objective of fuzzy modelling in system engineering is the development of reliable and understandable models which can describe the system behaviour through the construction of a linguistic rule base. That means, in order to gain a deeper insight into the system being modelled, fuzzy systems formulate the system knowledge based on transparent and interpretable linguistic rules. Accordingly it is possible to associate a semantic meaning to each term of the linguistic rules in order to characterise the system behaviour.

In spite of an RBF-NN is a black-box methodology, it can be seen as a fuzzy inference model of type-1 [Hunt et al., 1996]. That means, a parameter identification procedure in the RBF-NN can be employed in a similar way to that used in fuzzy systems [Chen and Linkens, 2001a]. In other words, the RBF-NN parameters can be estimated systematically from observational data, i.e a procedure that includes an initial fuzzy model self-generation methodology, the corresponding parameter optimisation and the rule-base simplification. Usually in fuzzy systems theory, a parsimony model is associated to its interpretability as a consequence of a good distinguishable rule base that defines the level of transparency in the fuzzy inference engine. Compared to fuzzy systems, the RBF-NN frequently suffers from a loss of interpretability during the optimisation parameter which is usually carried out by the application of a gradient descent-based approach [Chen and Linkens, 2001b]. In fuzzy logic systems, transparency plays an important role as it evaluates the level of interpretability in the rule base. In this regard, a collection of different constraints must be considered when constructing interpretable fuzzy systems [Hefny, 2007, Mencar et al., 2007a]. For instance, distinguishability is a metric usually employed for evaluating how much is affected the interpretability of a fuzzy system as a consequence of the overlapping between two or more fuzzy sets. In [Zhou and Gan, 2008] it was categorised the role of each component and each procedure employed during the parameter identification of systematic fuzzy logic systems. In a deeper context, the authors described a fuzzy model based on two different levels of interpretability, namely: a) low-level interpretability and b) high-level interpretability. While the low-level of interpretability consists in the optimisation of the MF's based on a fuzzy semantic criteria, the high-level of interpretability refers to the evaluation of a criteria that contemplates the coverage, completeness and consistency of the rules in order to achieve a good model interpretability. The criteria that can be employed to evaluate the degree of transparency at the low-level of interpretability is the evaluation of the distinguishability among the fuzzy sets (overlapping in the MF's), a moderate number of MF's, the coverage and completeness of the input space and the type of normalisation used in the input space. And the criteria that can be considered at the high-level of interpretability are the transparency, consistency and readability of the rule structure as well as a criterion that evaluates the parsimony and simplicity of the rule base. A common procedure to train the RBF-NN is to first choose the centres in the hidden layer by using an unsupervised methodology to reflect in somehow the initial distribution of the input training data [Girosi et al., 1995]. In particular, clustering algorithms have been widely used to partition the input space - for instance the *k-means algorithm* [Huang, 1998], the Fuzzy C-means (FCM) method Bezdek [1981] and recently Granular Computing (GrC) [Panoutsos and Mahfouf, 2010a].

Particularly computational paradigms such as Granular Computing (GrC) have been exploited for processing information in a transparent and interpretable way in order to estimate the initial RBF-NN parameters at the low-level interpretability. Unlike popular clustering approaches such as *Fuzzy C-Means (FCM) - granulation* is a technique in the field of GrC that mimics the human cognition in terms of grouping information together according to predefined similarity measures [Panoutsos and Mahfouf, 2010a]. Compatibility operators such as cardinality, orientation, density and multi-dimensional length represent an important element into granulation acting on both in raw data and information granules formed from raw data that finally provide a framework for human-like information processing where information granulation is intrinsic. Therefore, such individual entities are merged into dense information granules whose similarity [Panoutsos and Mahfouf, 2010a] can be evaluated in a variety of different ways depending mainly on the application at hand. Transparency plays an important role as a measure of interpretability and distinguishability, i.e. the more interpretable the information of a system under study, the better its understanding. Even though granulation as an explanatory data analysis represents a useful clustering approach, and has demonstrated its powerful as a tool for estimating the initial parameters of the RBF-NN, there is not a measure which leads how much a granule must grow. This phenomenon produces a grade of inclusion uncertainty among the new granules as a consequence of a ravenous behaviour. And a loss of transparency and then of interpretability might be loss. This lack of interpretability raises an important question concerning the use of new logics that posses the fuzzy capabilities of an expert system able for making decisions based on uncertainty. To exemplify a case study of this phenomenon, in this chapter the use of a new logics that is able to handling the uncertainty is proposed. In this context, Neutrosophy [Neutrosophy, 2002] is a three-valued logic that is the generalisation of fuzzy logic, intuitionistic logic [Atanassov, 1986], paraconsistent logic [Priest and Tanaka, 2009] and paradoxic logic [Elkan et al., 1994]. Neutrosophic sets theory is devoted to the description of events that are true and false at the same time. Moreover, it studies the scope of neutralities of events based on the idea of a tripartition (true, falsehood, indeterminacy/uncertainty) which was initially proposed by J. H. Lambert as a new logic capable of investigating the credibility of one witness by the contrary testimony of another [Smarandache, 2010b]. The application of neutrosophic provides an extra dimension which makes the compatibility criterion able to measure the overlapping behaviour through the evaluation of the fuzzy entropy (uncertainty) produced during the granulation. This measure persuades the compatibility search in eliminating potential granules that increase the granular overlapping producing a reduction in model transparency and affecting the consistency of the rules. In other words, as it is pointed out in [Pal and Bezdek, 1994] fuzzy uncertainty arises when boundaries are not sharply defined resulting in vagueness or linguistic imprecision. In this sense, several measures have been proposed to evaluate the fuzzy uncertainties [Pal and Bezdek, 1994, Wang et al., 2012]. Particularly in this work is used that presented in [De Luca and Termini, 1972] in order to evaluate the overlapping as a cognitive uncertainty (fuzziness) that can be interpreted as the imprecision in the transition area from one linguistic term to another. Therefore, in this chapter, a twofold study is presented - on the one hand, a process of granulation is carried out at the low-level interpretability in order to estimate the initial location of the centres in the hidden layer of the RBF-NN. On the other hand, it is proposed a new methodology based on the granulation process developed in [Panoutsos and Mahfouf, 2010a] and neutrosophic sets (Gr-NS) in order to quantify the uncertainty/fuzziness associated to the overlapping among the granules during the clustering stage. Hence, the main contributions in this chapter can be listed as follows:

- A description of the RBF-NN components in terms of low-level interpretability and high-level interpretability.
- A low-level interpretability process of granulation for an initial RBF-NN parameter identification.
- A methodology based on GrC and neutrosophic sets for quantifying the uncertainty that comes out from the overlapping phenomenon produced during the granulation process is presented. Such a methodology evaluates the distinguishability of the granules that are being formed at each iteration of the granulation process with the objective to construct a more transparent and interpretable initial fuzzy rule base. Such an uncertainty evaluation is carried out by the use of a proposed index that is based on a Shannon criterion. This study also suggests that the final optimisation of the RBF-NN depends heavily on the initial cluster positions which are used to define the initial fuzzy rules.

4.2 INTERPRETABILITY IN THE RBF-NN STRUCTURE

According to [Jang and Sun, 1993], RBF-NNs and Fuzzy Logic Systems (FLSs) of type-1 are functionally equivalent under some mild conditions. Thereby, properties from neural networks and fuzzy logic systems can be exploited and explored from a unified framework. That implies the RBF-NN may be interpreted in the language of Fuzzy Logic and viceversa.

However, a major criticism arises when the associated parameter identification is carried out by adaptive learning techniques that overshadow the interpretability and hence the transparency of the unified methodology [Jin and Sendhoff, 2003]. In [Jin and Sendhoff, 2003], the authors proposed a number of interpretability conditions for neural networks based on the RBF-NN structure and fuzzy systems of type-1. Such conditions can be listed into three headings, which are:

- The fuzzy partitioning of all the variables in the fuzzy system should be complete and distinguishable. That means the physical meaning of the fuzzy partitioning is clear and easy-to-interpret leading to a reduced universe of discourse with and only the necessary rules to describe a system.
- The fuzzy rules must be consistent. For example, if two any antecedents in a fuzzy rule are the same but produce a completely different consequent, therefore, there is an inconsistency.
- The number of rules in the premise part should be as small as possible avoiding over-fitting. Because a large number of training rules may come out in learning perfectly the training data.

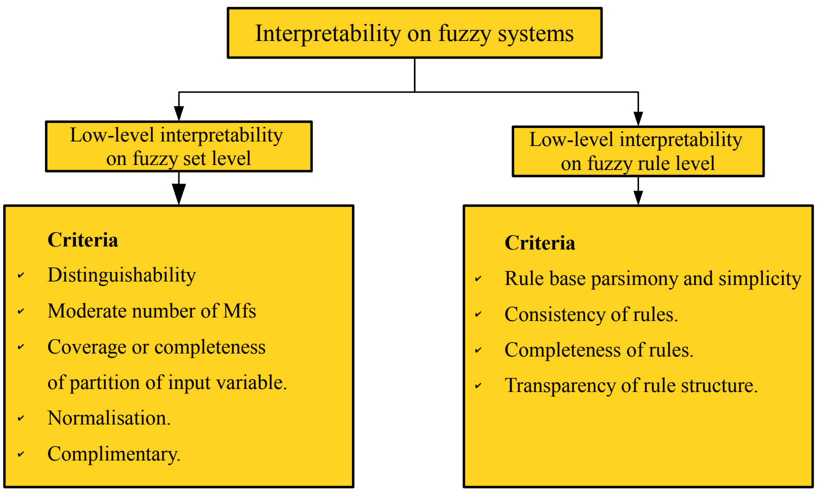


Fig. 4.1 Interpretability levels for Fuzzy Logic Systems.

For instance in [Zhou and Gan, 2008] a categorisation of interpretability for fuzzy modelling is proposed - Fig. 4.1 shows such a categorisation which is mainly divided into two levels of interpretability, i.e. a) low level of interpretability and b) high level of interpretability. On the one hand, the authors in [Zhou and Gan, 2008] suggested several criteria to achieve a low-level of interpretability by optimising the MFs on fuzzy set level. Basically the improvement lies on the modification of the MFs by defining some semantic constraints which are based on the distinguishability of the universe of discourse, a moderate number of MFs, the coverage and completeness of the partition of the input space, normalisation and the complimentary. On the other hand, operations on the fuzzy rule base are performed to achieve a high-level of interpretability whose main purpose is to create a compact and consistent fuzzy rule base. Such operations may cover the creation of a parsimony rule base and its associated level of simplicity, consistency of rules, completeness of rules and transparency of rules structure. However, in fuzzy modelling the categorisation presented in Fig. 4.1 may only be applied on linguistic fuzzy modelling. According to [Gacto et al., 2011], when dealing with the trade-off of accuracy-interpretability two fields of study may be considered:

1. Luinguistic Fuzzy Modelling. This field is mainly devoted to construct interpretable models through the use of linguistic Fuzzy rule-based systems (FRBSs). Such systems are heavily based on linguistic rules (or Mamdani) whose interpretability is associated to the preservation of the semantic of the MFs.

2. Precise Fuzzy modelling (PFM). This field is focused on the construction of accurate fuzzy models by means Takagi-Sugeno FRBSs. In contrast to Mamdani-based FRBSs, these models employ fuzzy systems without an associated meaning.

Since the RBF-NN can be regarded as a FRBS of type-1 - the interpretability taxonomy of the network can also be categorised at two different levels. This classification must be defined in relation to the parameter identification process of the RBF-NN. This means, the interpretability categorisation in the RBF-NN consists on identifying the elements for a low-level and high-level of interpretability at two different stages, i.e a) during the clustering of the initial raw data, which is used to identify the initial fuzzy rule-base, and b) the optimisation of the MFs location by using an adaptive procedure that is usually based on gradient descent approaches. In Fig. 4.2, a proposed structure for categorising the interpretability at the RBF-NN is presented.

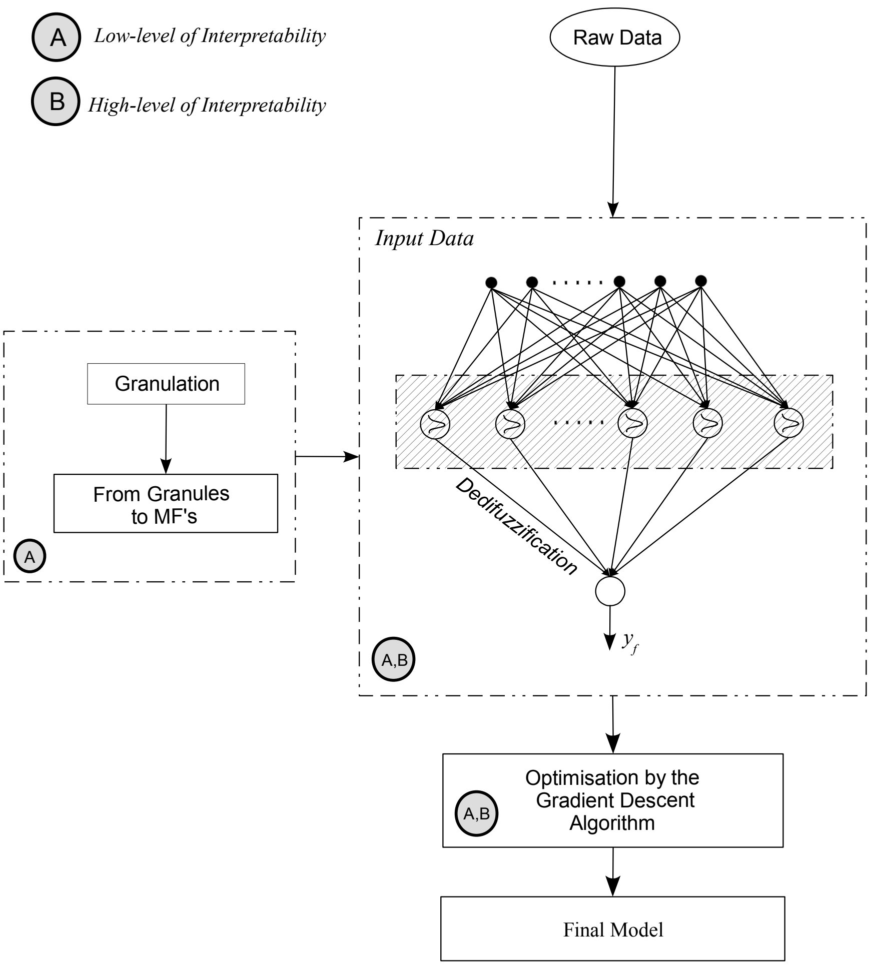


Fig. 4.2 Interpretability levels at the RBF-NN taxonomy

According to [Zhou and Gan, 2008], the elements that may be involved at the low-level of interpretability of the RBF-NN includes:

(a) The distinguishability of the MFs and the associated semantic - this includes the process of granulation *(during the construction of the initial rule base)*, and the optimisation of the MFs parameters (location). As it is pointed out in [Park and Sandberg, 1993], an initial clustering approach is required to position the centres of the radial basis function which are eventually moved toward the majority of the data by the application of a gradient descent approach. For this reason, the initial location and therefore the associated distinguishability play an important role for the final construction of the fuzzy rule.

(b) A moderate number of MFs. In other words, the number of fuzzy rules should be as small as possible while preserving a satisfactory system's performance. An smaller number of rules allows us to better understand the associated meaning of a MF. However, the evolution in computation makes possible the analysis of high-dimensional problems and the extraction of features which allow the readability of the associated fuzzy sets.

(c) Coverage and completeness of the partition space at two different stages, i.e. at the end of the granulation process, and at the end of the optimisation process of the location of the MFs. This implies that every data should be represented linguistically by a fuzzy set over its universe of discourse. Incompleteness can be interpreted as the over-fitting phenomenon in the RBF-NN and hence in the proposed model. In [Zhou and Gan, 2008] the authors described incompleteness as a deficiency in the correct partition of the fuzzy space during the parameter optimization process.

(d) Normalisation. In the RBF-NN, the highest value is determined by distance between the centre of a Radial Basis Function and every input vector.

(e) Complimentary. For each element in the universe of discourse, the sum of all its associated MFs should be close to one. This assures a uniform distribution of the meanings in all the elements.

According to [Zhou and Gan, 2008], several techniques have been used to achieve a low-level of interpretability for fuzzy modelling, such techniques include:

1. Regularization approaches for parameter estimation.

2. Multi-objective optimization for antecedent parameter estimation.

3. Fuzzy set merging techniques.

4. User-oriented interactive technique.

In a like-manner, the high-level of interpretability at the RBF-NN and the proposed IT2-RBF-NN should be mainly defined in relation to the interpretability of the fuzzy rule base of both modes. Therefore, the elements that may be taken into account to achieve a high-level of interpretability are:

(a) Rule base parsimony and simplicity. According to [Zhou and Gan, 2008], The best model is the simplest one that fittest the system behaviours well - this includes a fuzzy rule base with the smallest number of rules that preserves a satisfied level of performance leading to a better global understanding of the system.

(b) Transparency of rule structure. The proposed IT2-RBF-NN and the RBF-NN can be seen as a generalised framework for fuzzy modelling - this implies that both fuzzy rule structures are either Mamdani type or Takagi-Sugeno (TS) type. The former is the most widely used structure, this is because the consequent part of a Mamdani rule structure are fuzzy sets and therefore transparency is supposed to be a default property. However, the transparency and properties such as distinguishability and interpretability are enormously affected by the learning process.

(c) Consistency. The degree of consistency for the proposed IT2-RBF-NN is fully determined by the absence of contradictory fuzzy rules, i.e. two similar rules with a similar premise should have a similar consequent.

(d) Completeness. For any input vector to the RBF-NN models, at least one fuzzy rule must be fired, however, due to the nature of the network, usually one or more rules in the fuzzy rule base are activated.

(e) Readability of fuzzy rules. According to [de Oliveira, 1999], a good degree of readability may be achieved if the number of different conditions for each premise part should not exceed 7 ± 2. The main reason comes out from a study in Cognitive Psychology that states that the maximum number of different entities that a human can handle efficiently should not exceed such an amount. This ability may be translated into the structure of a fuzzy system as the number

    A fuzzy set usually associates the meaning of a linguistic variable to a semantic rule, i.e. every value of the linguistic variable over the universe of discourse may be represented by a linguistic term with a clear semantic meaning.

Therefore, the interpretability levels in the RBF-NN are considered in order to study the benefits of the application and advances in fuzzy set theory.

4.3 GRANULATION OF DATA

Before going directly with the details of the granulation technique employed in this chapter, it would be worth to review the underlying principle of granulation and how this methodology into the emerging paradigm of Granular Computing (GrC) concentrates to extract information from numeric data. The point of departure lies on the existing clustering algorithms that are usually divided into two main categories, namely; a) hierarchical clustering [Johnson, 1967] and b) partitioning clustering [Linhui, 2001]. The former algorithms are frequently used for partitioning objects into optimally homogeneous groups on the basis of empirical measures or similarity measures classifying objects to different groups according to their similarity. The latter groups data in predefined clusters or finding areas with higher data density. In this context, the granulation process aims to cluster data with similar features. To achieve the information grouping, granulation usually employs a compatibility measure that calculates a 'compatibility index' based on the granular similarity.

The term granule was initially defined by Zadeh [Zadeh, 1996a] into the field of fuzzy logic as a set of points having the form of a clump of elements drawn together by similarity. Moreover, in that work Zadeh denotes a word as a label of a granule which is seen as a fuzzy set playing the role of a fuzzy constraint on a variable. Zadeh highlighted the importance of granulation as a process that mimics the human cognition with the ability of information compression. Thereafter, the term Granular Computing (GrC) was first introduced by T. Y. Lin as a new multidisciplinary study [Lin, 1997]. This conceptual paradigm of GrC is related to the processing of complex information entities - information granules that are formed by abstracting numeric data and of the derivation of knowledge from information [Bargiela and Pedrycz, 2003a]. The rationale behind information granulation in this research work lies on the representation of information granules as hyperboxes positioned in a highly dimensional data space [Pedrycz and Bargiela, 2002, Yao et al., 2013]. The mathematical formalism is based on interval analysis that according to [Pedrycz and Bargiela, 2002] provides a more roust framework for the analysis of information density of the granular structures that arise as a consequence of a process of granulation. Pedrycz proposed the first clustering approach that granulate the information from raw data that are usually in the form of numeric [Pedrycz and Bargiela, 2002]. The aim of that methodology is to capture the information through the process of data organisation in the form of granules which are finally compressed based on some similarities. According to Pedrycz [Pedrycz and Bargiela, 2002], a clustering methodology based on granulation obeys a level of abstraction which is achieved through a process of condensation of the original data (which may be numeric or granules) into granules. Furthermore, Pedrycz pointed out that the more condensation, the larger the sizes of the information granules that realises this aggregation. However, under ravenous situations this is always not happening, since the nature of data does not follow an order all the time. Therefore, the basic idea of the clustering approach proposed in [Pedrycz and Bargiela, 2002] is carried out by the following iterative process:

- Find the two closest information granules according to some predefined compatibility criteria, and on this basis build a new granule embracing them. The purpose behind this idea is to reduce the size of the data set while the clustering process condenses data.

- Repeat the first step until enough data condensation has been accomplished or a predefined criterion is met.

Where a granule A is a hyperbox (or box) in R^n that is fully described by its lower (l) and upper corner (u). Therefore, a granule can be expressed as $A(l,u) \in R^n$, if $l = u$ the granule reduces to a single point. Moreover, the box may be defined over a family of relations defined in R^n such that $A \in \wp(R^n)$, where $\wp(\cdot)$ is a class of sets. As stated in [Pedrycz and Bargiela, 2002], the volume of $V(A)$ can be used to calculate the compatibility of two similar granules A and B and it is advantageous to consider the expression

$$exp(-V) \tag{4.1}$$

Note that similarity is usually a measure used to quantify the compatibility of two or more individual entities and it is frequently calculated from the distance $\| \cdot \|$ between such objects, where $(\cdot)$ may be any metric. In line with the compatibility measure, it attains its maximum value 1 when the volume hyperbox reduces and 0 otherwise. Therefore, the granulation process can make sure only dense and compact granules are being obtained. In Fig. 4.3 some geometric properties of a resulting granule $'C'$ by merging two compatible granules $'A'$ and $'B'$ is illustrated. In order to finally calculate the compatibility between two granules A and B, the volume of a resulting granule D can be calculated as follows [Pedrycz and Bargiela, 2002].

$$V\left(D\right) = \prod_{i=1}^{n} length_i\left(D\right) \tag{4.2}$$

where

$$length_i\left(D\right) = max\left(u_B\left(i\right), u_A\left(i\right)\right) - min\left(l_B\left(i\right), l_A\left(i\right)\right) \tag{4.3}$$

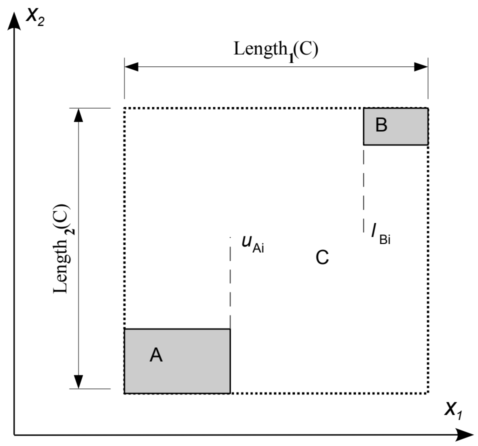


Fig. 4.3 Resulting information granule $'C'$ by merging granules A and B [Pedrycz and Bargiela, 2002].

Therefore, the compatibility can be calculating in the form

$$compat\,(A,B) = 1 - d\,(A,B)\,e^{-\alpha V(D)} \tag{4.4}$$

where

$$d\,(A,B) = (\| \, l_A, l_B \, \| + \| \, u_A, u_B \, \|) \tag{4.5}$$

In agreement with the research work in [Pedrycz and Bargiela, 2002], Panoutsos extended this idea where the compatibility measure includes the volume of granules, the associated density, cardinality and the length of the resulting granule and of the entire data space [Panoutsos and Mahfouf, 2010a]. In essence, the extended version of the granulation approach maintains the iterative procedure divided into two main steps as follows:

- Find the two most compatible information granules by using the Eq. 4.6 and then merge them together as a new information granule containing both original granules.
- Repeat the process of finding the two most compatible granules until a satisfactory data abstraction level is achieved.

In Fig. 4.4, a flow chart of the granulation process used in this chapter is described. Even in this work the input raw data is normalised between [-1, 1], usually this option is problem-dependent. The granular process basically is divided into three main steps: (a) raw data; at this stage each datum is viewed as a granule in the input space and hence compressed into compact and dense granules, (b) input-space data granulation; during this iterative process the initial number of granules is reduced according to their compatibility in which various similarity measures can be considered, such as: the size of the granules, the cardinality, overlapping among granules, orientation, etc. And finally (c) output space-density function represents the linguistic interpretation of the final group of dense granules that preserve the original features of the raw data.

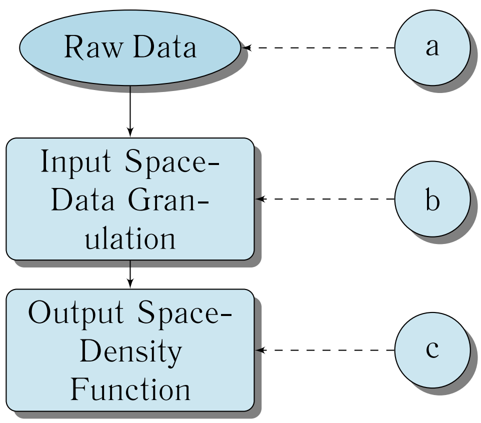


Fig. 4.4 Data granulation process.

For agreement reasons with [Pedrycz and Bargiela, 2002], in this chapter some of the terms employed in [Panoutsos and Mahfouf, 2010a] will be written exactly with the same notation used in [Pedrycz and Bargiela, 2002]. Therefore, in a similar way $compat(A, B)$ defines the merging operation of two different granules A and B. However, the compatibility measure extended in [Panoutsos and Mahfouf, 2010a] is not based on the volume of the resulting granule, but it uses the multidimensional length and the cardinality of each granule including a weighting term w_k which is viewed as a dimensional importance factor. The compatibility defines the most important concept during the granulation process.

$$compat(A, B) = D_{MAX} - d_{A,B}e^{(-\alpha R)} \tag{4.6}$$

Where:

$$R = \frac{card_{A,B}/Cardinality_{MAX}}{L_{A,B}/Length_{MAX}} \tag{4.7}$$

And D_{MAX} is the maximum possible distance in the data set, and $d_{A,B}$ is the weighted multidimensional average distance between two granules A and B.

$$d_{A,B} = \frac{\sum_{k=1}^{n} w_k(max(u_{Ak}, u_{Bk}) - min(l_{Ak}, l_{Bk}))}{n} \tag{4.8}$$

with w_k playing the importance weight for the dimension k, and n the total number of dimensions. In Eq. 4.6, α weights the requirements between distance and cardinality/length, the term $Cardinality_{MAX}$ is the total number of granules in the data set. $Length_{MAX}$ is the maximum possible length of a granule in the data set which may sometimes be as large as the dimensions of the data set boundaries. In Eq. 4.8, l_{Ak} and u_{Ak} are the lower and upper limits (corners) of the granule 'A' respectively and in Eq. 4.9 L_{AB} is the multidimensional length of the resulting granule.

$$L_{A,B} = \sum_{k=1}^{n}(max_{xk} - min_{xk}) \tag{4.9}$$

To illustrate the meaning of the terms in Eq. 4.9, in Fig. 4.5 is provided a graphic representation of the terms max_{xk} and min_{xk}. Moreover, to exemplify the compatibility calculation, in Fig. 4.6 is depicted a 2-dimensional granular space where the granules A and B are merged (Figure taken from [Solis and Panoutsos, 2013]).

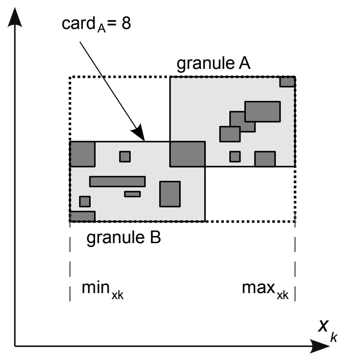


Fig. 4.5 Terms associated to Eq. 4.9.

The term α is employed as a threshold in the interval $[0, 1]$ in order to balance the terms of 'distance' and 'density' (Cardinality/size) and w_i weights each dimension according to the problem at hand [Bargiela and Pedrycz, 2003a]. According to the dimensions provided in Fig. 4.6, granules A and B produce the following values if the values of $w_k = 1$ for $n = 2$:

$$D_{MAX} = \sum_{k=1}^{n=2} (1 - (1)) = 4 \tag{4.10}$$

$$d_{A,B} = \frac{(max\,(0.9, 0.55) - min\,(0.4, -0.1))}{2} + \frac{(max\,(0.2, -0.1) - min\,(-0.8, -0.2))}{2} \tag{4.11}$$

Unlike set theory, here the union of two granules is obtained as the merging operation of two granules A and B. Fig. 4.7 shows the union of employed in granulation which is the resulting granule C.

$$card_{A,B} = card_A + card_B = 15 granules \tag{4.12}$$

where $card_A$ is the associated cardinality of the granule A. As can be seen from Fig. 4.7, the number of granules in A is eight.

$$\begin{aligned} Cardinality_{MAX} &= card_A + card_B + \cdots + card_H \\ &= 8 + 7 + 10 + 2 + 11 + 3 + 2 = 43 \end{aligned}$$

where $L_{A,B} = 2$, $Length = 3.93$ and the proposed value of $\alpha = 0.35$. Therefore, the compatibility between the granules A and B is:

$$compat(A, B) = 4 - e^{(-0.35\times0.682)} = 3.123 \tag{4.13}$$

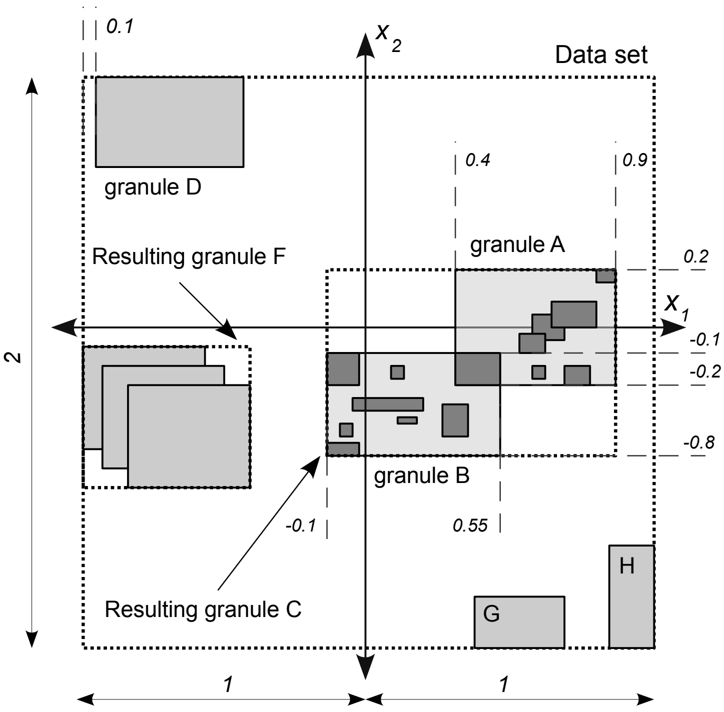


Fig. 4.6 Compuation of the resulting granule 'C'

As it is pointed out in [Bargiela and Pedrycz, 2003a], the exponential form of the compatibility is associated with the normalisation of all the values in the interval $[0, 1]$. In particular, the extended version of the compatibility criterion proposed by Panoutsos in [Panoutsos and Mahfouf, 2010a] favours the formation of compact granules with a high cardinality. Moreover, Eq. 4.6 does not need to normalise the original data set, since the reference distance (D_{MAX}) to measure the compatibility may be the size of the data set boundaries. The compatibility criterion now includes those entities/granules with a high density and indirectly it preserves the properties shown by the volume V term used in Eq. 4.4.

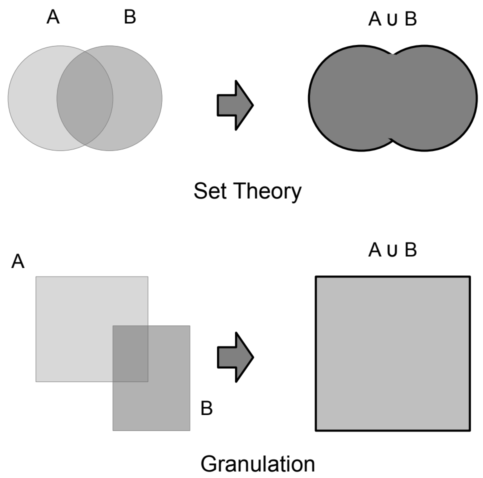


Fig. 4.7 Union of two granules 'A' and 'B'.

As it is suggested in chapter 3, the geometrical boundaries of each final information granule are used to estimate the initial values of the RBF parameters C_i and σ_i which are illustrated in Figure 4.8. The average hyperbox boundaries of each granule are utilised to calculate the initial C_i as follows:

$$C_i = [C_{i=1,k=1}, \ldots, C_{Mn}] \tag{4.14}$$

where M is the number of centers and n the total number of input data points.

$$C_{ik} = \frac{1}{2}(max_{xk} - min_{xk}) \tag{4.15}$$

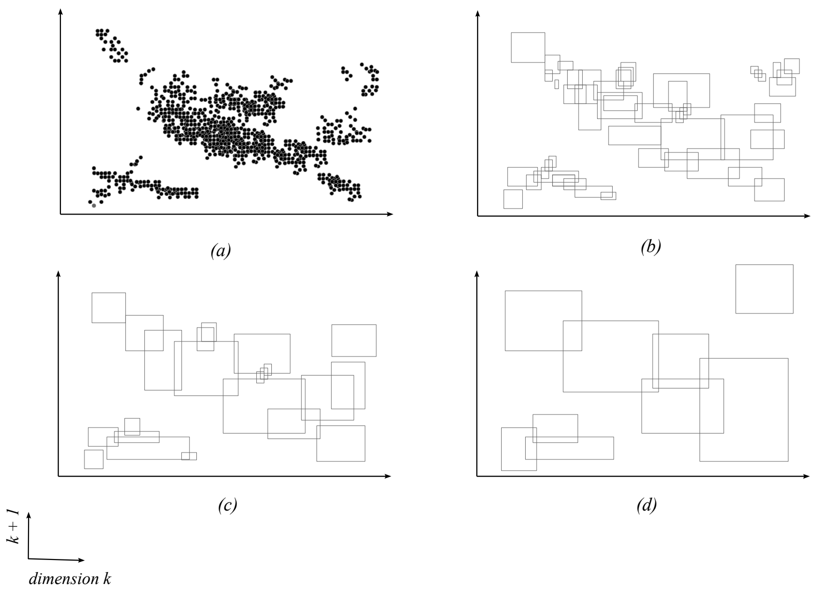


Fig. 4.8 (a) Raw data, (b) 60 information granules, (c) 20 information granules and (d) the final granules.

Here, the width of the Gaussian function in the RBF-layer is calculated via the following expression

$$\sigma_i = \frac{1}{r}\left(\sum_{j=1}^{r} \|C_j - C_i\|\right)^{1/2} \tag{4.16}$$

in which C_j is the nearest neighbour to the centroid C_i and r is usually 2.

4.3.1 MODELLING RESULTS BY USING GRANULATION

This section describes those results obtained by using a process of granulation for the initial clustering of the input raw data in order to create the initial fuzzy rule base which is then optimised by applying a self-adaptive Back Error Propagation approach that is described in section 3. As mentioned above, granulation is an iterative process that finds the two most compatible data at each iteration (iter); merging them geometrically into a new granule up to a predefined number of granules are formed/achieved. According to

Fig. 4.9 such final granules are used to create the initial multidimensional fuzzy rule base that represents a clump of abstract objects drawn together through the extraction of information about their distinguishability, similarity, proximity or functionality [Zadeh, 1997].

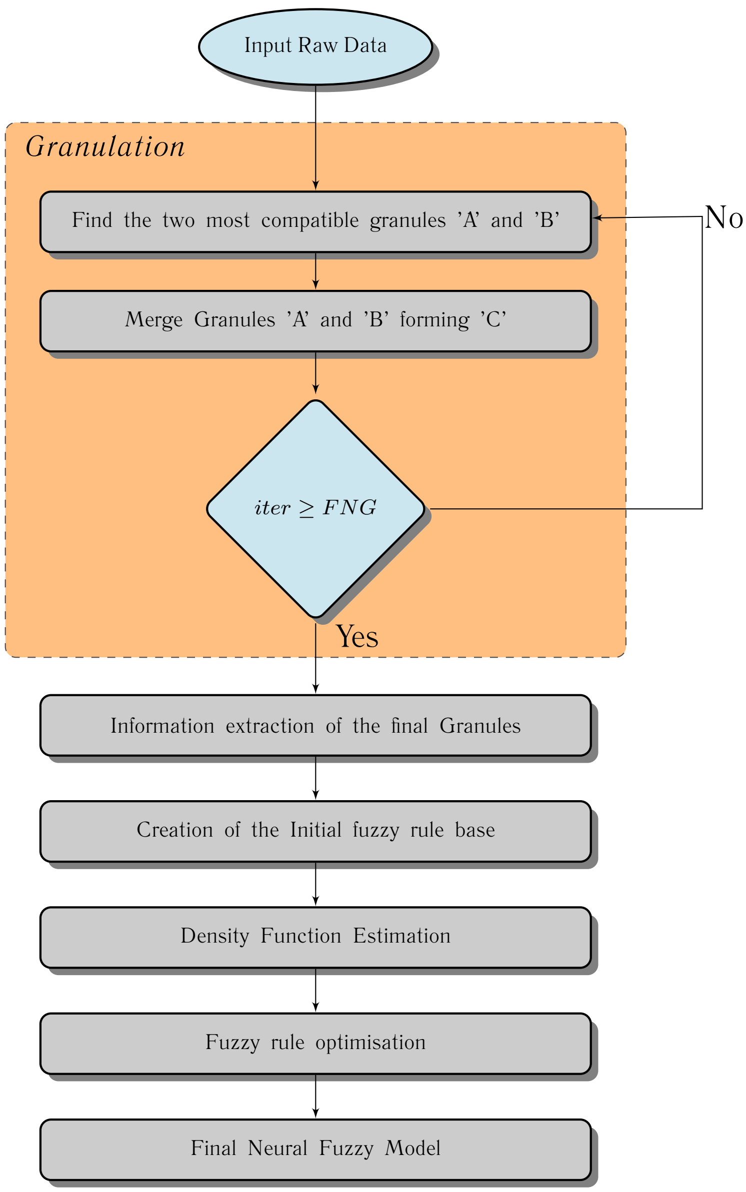


Fig. 4.9 Data-driven model based on RBF Neural Networks and Fuzzy Clustering.

For comparison reasons and hence for cross-validation purposes, the input raw data set used in this chapter was identically divided to that presented in chapter 3 into three main data sets, namely: training data 1084 (65%), validation data 277 (17%) and test data 300 (18%). Similar to those results obtained by using FCM as the initial clustering approach in chapter 3, in this section a group of experiments with a different number of granules (Fuzzy sets) is proposed. Therefore, it was found that a simulation with less than 6 or more than 18 granules is not considered in order to avoid over-fitting or under-representation of the raw data that may occurs during the training stage. The geometrical properties of the final granules are used to construct the initial multidimensional fuzzy rule base, for example the rule 1 is constructed from the geometrical properties of the granule 1 as follows:

$$\begin{aligned}\widetilde{R}^1\text{: IF } x_1 \text{ is } A^1_1 \text{ and } x_2 \text{ is } A^1_2 \text{ and } x_3 \text{ is } A^1_3 \text{ and } x_4 \text{ is } A^1_4 \text{ and } x_5 \text{ is } \widetilde{A}^1_5 \text{ and } x_6 \text{ is }\\ A^1_6 \text{ and } x_7 \text{ is } A^1_7 \text{ and } x_8 \text{ is } A^1_8 \text{ and } x_9 \text{ is } A^1_9 \ldots \text{ and } x_{16} \text{ is } A^1_{16}\\ \ldots\text{THEN the Impact Energy is } \widetilde{B}^1 \qquad (4.17)\end{aligned}$$

where $\vec{x_p} = [x_{k=1}, \ldots x_n]$ is the normalised input raw data whose limits are defined in the ith fuzzy granule $A^i_k = [max_{Ak}, min_{Ak}]$ at dimension k which is employed for identifying the initial values of σ_i and centres C_{ik}.

Different trials were performed in order to investigate at which value of α the final granules offer a good level of compactness and distiguishability. In Fig. 4.10, the final compatibility index behaviour is presented using a weighting factor '$\alpha = 0.35$'. Such a figure also depicts a typical evolution of the compatibility measure, as expected the index reduces dramatically (falls-off) which represents less compatible (dissimilar information) is merged towards the end of the granulation process. This may be also used as a criterion to terminate the iterative process - the optimal number of granules can be estimated from graphically finding the point of intersection of the two tangent lines to the curve of compatibility as is illustrated in Fig. 4.10. Due to the variability of neural network training, 3 different trials were carried out in order to determine the optimal model. This includes to rearrange randomly the original data for training, checking and testing. That means the data used per each run will be different, but the proportion in data for training, checking and testing will be kept, i.e. 65 % (1084 data points) for training, 17 % for checking (277 data points) and 18 % for testing (300 data points).

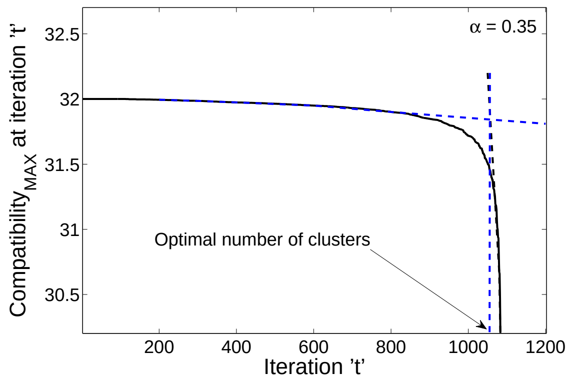


Fig. 4.10 Compatibility behaviour throughout the training stage.

A comparison of the RMSE from 3 runs and with a different number of fuzzy rules (granules/centers) using the RBF-NN with granulation and the well-known Fuzzy C-Means (FCM) clustering approach are shown in Table 4.1. An Index based on the Root-Mean-Square Error (RMSE) in order to measure the training, checking and testing performance and an initial partition space of 9 granules is suggested. In Fig. 4.11, a plot of experimental results by using granulation and the adaptive-BEP are illustrated. This is mainly due that impact energy is a highly non-linear property in relation to the steel composition, and then the impact energy dataset comes out difficult to be modelled as a consequence of the multitude of standards that exists, and the variety of results. It is as well evident from Fig. 4.11 that some scatter data represent the lack of ability of the RBF-NN by using granulation to correctly classify all the points, particularly those at the checking and testing stage. Since the compatibility criterion (4.6) is based on the multidimensional length of each granule and its cardinality, the granular index decreases while the numbers of iterations increases as less compatible granules are merged.

Statistically speaking, the fuzzy model that employed a granulation approach to create the initial fuzzy rule base outperformed the model that employed FCM (See table 4.1). Furthermore, the application of granulation to construct the initial fuzzy rule of the RBF model results more transparent than just using FCM. This is because, the readability of a single fuzzy rule is through the analysis of the elements of the compatibility criterion.

Similarly to those results illustrated in Fig. 4.11, the experimental results obtained when using FCM (See Fig. 4.12) still preserve the misclassification of some points which clearly confirm that the scatter data are statistically similar but represent a different point. In other words, some training data fed into the T1-RBF-NN will describe a similar input space but a scatter output space. Unlike the FCM clustering algorithm, the process of granulation encompasses a transparent and distinguishable process at the low level of interpretability.

Table 4.1 RMSE using Granulation and FCM

No. of rules	6	9	15	20	25	6	9	15	20	25
RMSE	Granulation					FCM				
First arrangement										
Training	20.16	18.71	16.01	15.79	15.50	21.05	18.95	16.23	16.12	15.91
Checking	20.56	19.75	19.27	19.87	21.15	22.17	20.45	19.12	20.31	21.62
Testing	21.23	21.32	21.68	21.94	22.13	22.78	21.78	21.90	21.92	23.88
Second arrangement										
Training	19.70	16.91	15.83	15.75	15.33	20.50	19.18	15.86	15.79	15.55
Checking	19.82	19.65	21.30	21.42	22.40	21.36	20.01	20.15	19.42	21.02
Testing	20.26	21.38	22.41	22.14	22.22	22.97	22.30	22.19	22.60	22.47
Third arrangement										
Training	19.47	16.76	15.69	15.42	15.10	20.10	18.78	15.46	15.35	15.05
Checking	19.45	19.20	20.80	20.90	22.10	20.95	19.48	19.65	19.42	21.02
Testing	19.83	20.91	21.75	21.77	21.74	22.78	21.78	21.90	21.92	21.85

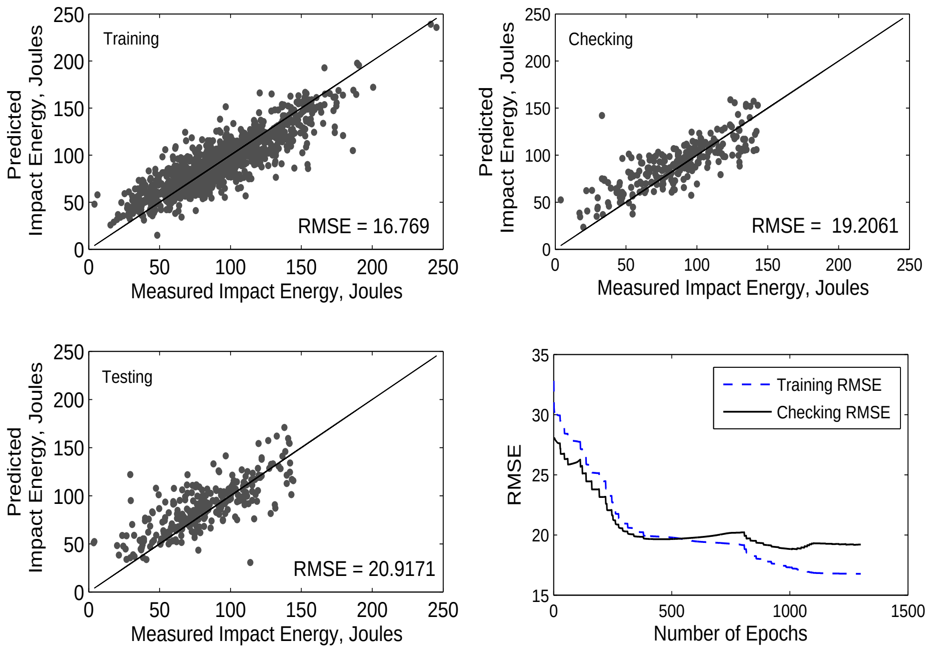


Fig. 4.11 *Data fit-impact energy by using granulation*

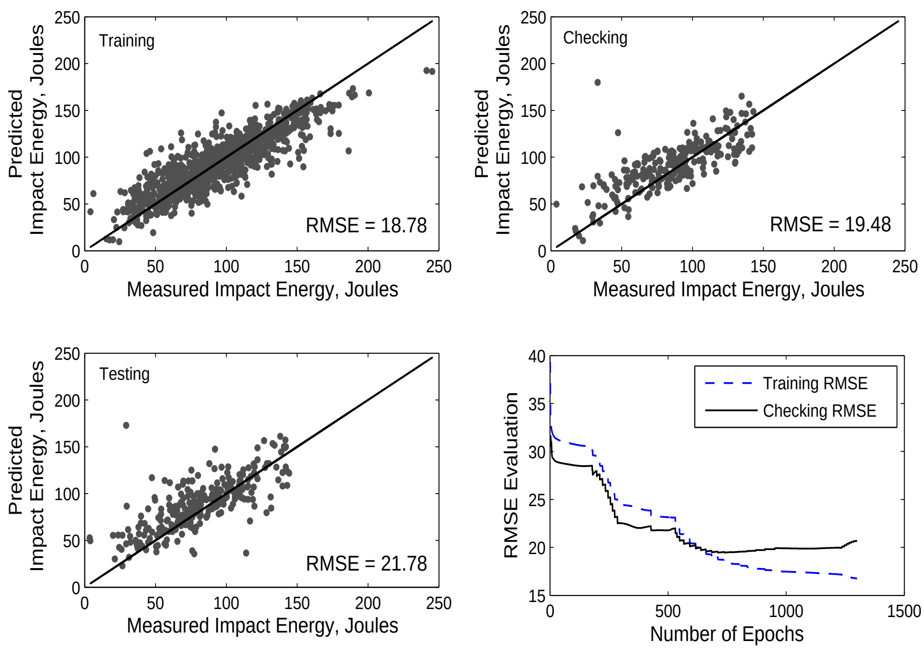


Fig. 4.12 *Data fit-impact energy by using the FCM algorithm*

4.4 MODELLING OF CHARPY IMPACT TEST BY USING NEUTROSOPHIC SETS

The granulation process as described above, aims to compress the initial data into compact and dense granules based in the resulting cardinality and the multidimensional length of any two merged granules. Furthermore, this methodology exploits as much as possible the density ($'richness'$ of information) of the granules. To exemplify the evolution of the granulation process, in Fig. 4.13 a typical granular compression over five hierarchical levels is illustrated. The "level (1)" 80 data points taken from the Charpy data set used in Chapter 3 are used as the initial set to be compressed into 32 granules. These granules are presented as input data to "level (2)" of granulation where are compressed into 24 and 14 granules to be used as the input data to "level (3)" and "level (4)" respectively. Finally, at "level (5)" the predefined granules map conveniently onto the linguistic entities (MFs) that are used as the initial parameters of for the rule base of the RBF Network. It is self evident from Fig. 4.13 that the different hierarchical levels of granulation preserve the essential features of the raw data. However, it is also evident from Fig. 4.13, no assumption about the maximum size of the granules is made. This means that the granules keep increasing and then overlapping other granules. Although it is supposed that the formation of closely separated granules is avoided by the very nature of maximisation information density [Pedrycz and Bargiela, 2002], a grade of inclusion uncertainty may be produced. This ravenous behaviour can be translated as a lack of distinguishability due to the overlapping which is not considered into the compatibility measure expressed in Eq. 4.6.

In Fig. 4.14, the evolution of an iterative process of granulation per dimension is depicted in detailed. As shown in Fig. 4.14, the phase (1) of granulation results in a mixture of granules in "level (4)". The output space-density per dimension is employed to construct the initial MFs as it is illustrated in Fig. 4.14(b). Consequently, a multidimensional fuzzy rule is obtained as follows:

$\widetilde{R}^1$: IF $Test\ depth$ is A_1^1 and $Specimen\ size$ is A_2^1 and $Test\ site$ is A_3^1 and

C is A_4^1 and Si is $\widetilde{A}_5^1$ and Mn is A_6^1 and S is A_7^1 and Cr is A_8^1 and Mo is A_9^1 . . . and $Test\ temperature$ is A_{16}^1

$$\ldots \text{THEN the Impact Energy is } \widetilde{B}^1 \tag{4.18}$$

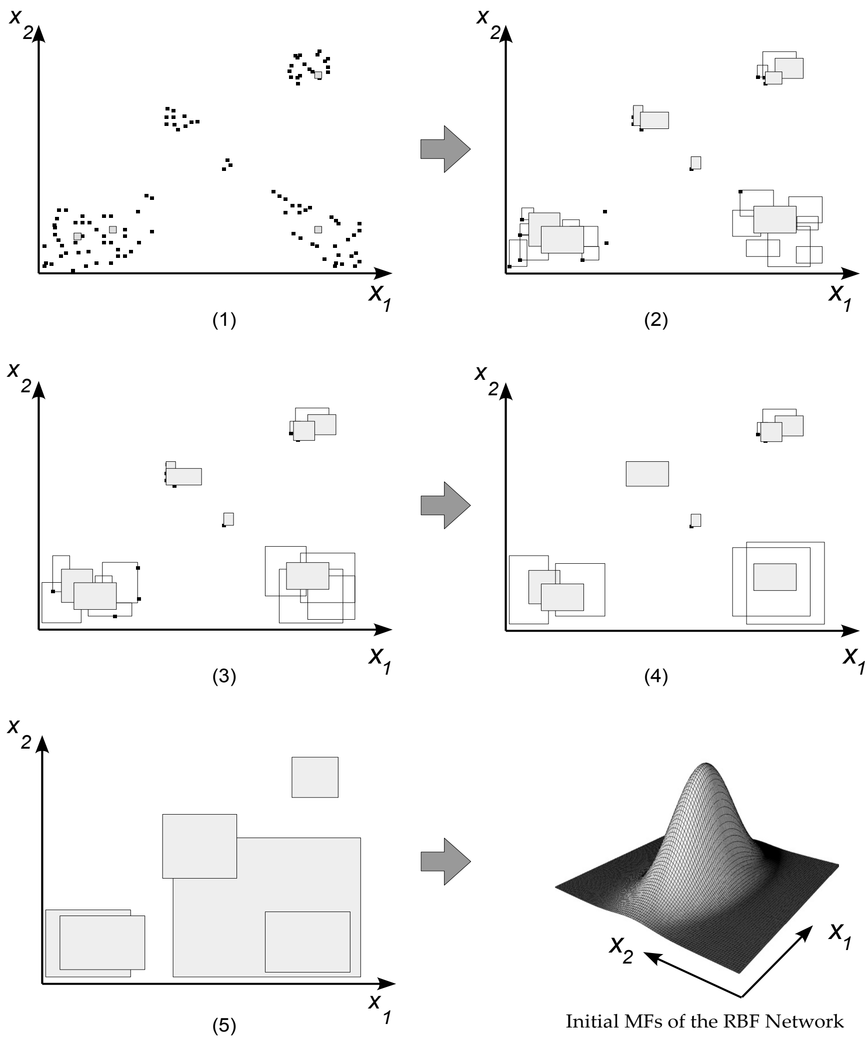


Fig. 4.13 Granulation evolution for 80 data input points extracted from the Charpy Impact test data set

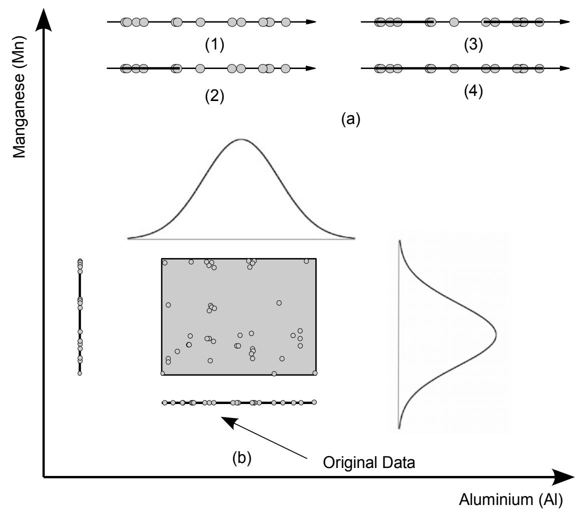


Fig. 4.14 Dimenisonal granulation evolution and final density function extraction

To illustrate the final shape of the MFs after granulating the 80 data points extracted from the Charpy data set, in Fig. 4.15, the discourse of universe of two out of the sixteen dimensions is presented. It is instructive to point out the high degree of overlapping created after granulation. It is believed in this research work that the overlapping caused by the merging stage is significant and this may produce a lack of sharpness in the distinction of the rules.

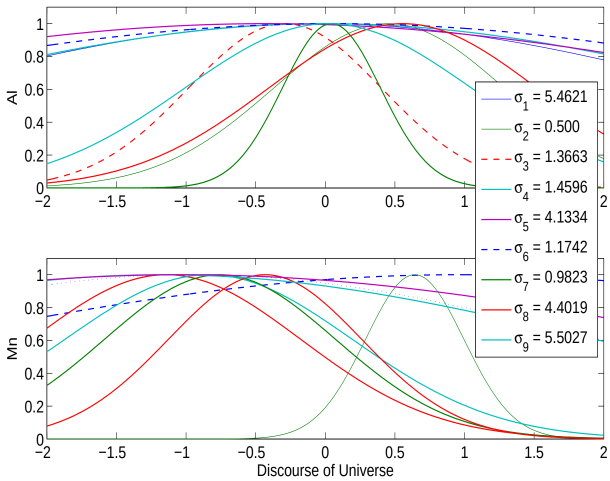


Fig. 4.15 *Data fit-impact energy by using the FCM algorithm*

As illustrated in Fig. 4.15, a high degree of overlapping may lead to the creation of fuzzy rules whose MFs are not distinct enough from each other so as to represent a linguistic term with a clear semantic meaning [Zhou and Gan, 2008]. This behaviour may result on the one hand in the creation of inconsistent rules that contribute to make uncertain/indeterminate decisions. This inconsistency may be translated in the construction of a fuzzy rule base with contradictory rules. In other words, the presence of rules with a similar premise should have a similar consequence (See Fig. 4.16). And on the on other hand, this level of overlapping hinders the creation of a transparent and hence interpretable fuzzy rule. For this reason, in this chapter a methodology based on granulation and neutrosophic sets that is capable to quantify the overlapping as a source of uncertainty when making decisions is proposed. The aim is to attenuate such a behaviour and enhance the transparency and hence the interpretability of the final granular space (initial fuzzy rule base for the RBF model). The point of departure lies on the hypothesis that if the granulation compatibility index in Eq. 4.6 favours the merging of two granules that will lead to less accumulated uncertainty when forming new granules. Therefore, the resulting multidimensional granules, and hence the "fuzzy rules" will be more distinguishable and interpretable.

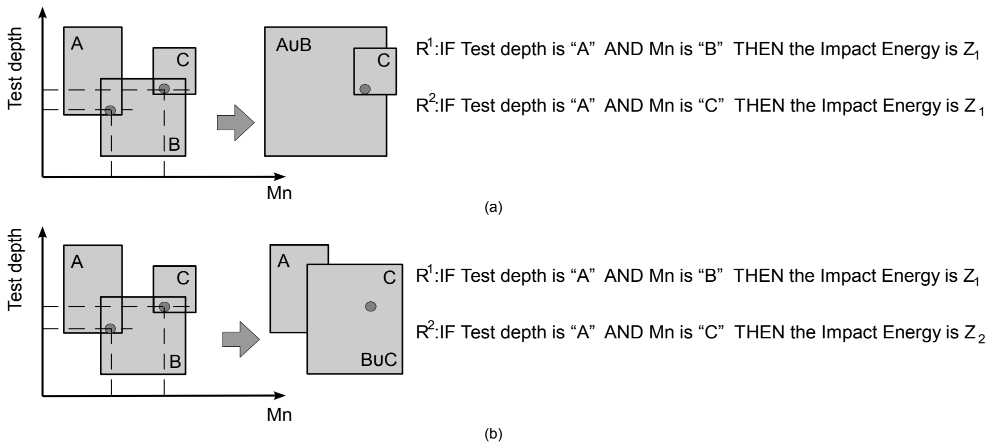


Fig. 4.16 Consistency of fuzzy rules after granulation

In order to quantify and then attenuate an excessive level of overlapping, the idea behind the proposed methodology is to applied the concept of neutrosophy. This new field aims to study the origin, nature and scope of neutralities as well as their interaction with different ideational spectra [Maji, 2013]. Neutrosophy considers every proposition, event or entity $< A >$ in relation to its opposite $Anti - A$ and the neutralities $neu - A$ which is not A, $< not - A >$ and that which is neither A nor $Anti - A$ are referred as to $non - A$ ideas. To put it more simply, this new type of logic deals with contradictions, paradoxes, incomplete language/systems and it can be fitted into the category of para-consistent logics. However, this new framework needs to be specified from a technical point of view. From a fuzzy perspective, this new logic not only may consider the associated truth-membership and falsity-membership supported by evidence, but also the associated indeterminacy/uncertainty-membership.

Under these circumstances, the proposed methodology aims to define a neutrosophic set in order to measure how much two granules ”A” and "B" overlap each other (Truth-membership whose short name is "T"), and then use the associated falsity-membership ("F") and an exponential version of the Shannon's entropy (uncertainty/indeterminacy-membership, "I") to quantify the level of distinguishability between two or more granules. Therefore, the pseudo-code of the proposed methodology which will be called here as granulation with neutrosophic sets (Gr-NS) can be stated as illustrated in Algorithm 1:

In what follows, a deeper explanation of each line of the pseudo-code will be provided. The **input** $e_j nor$ represents the normalised input data in the interval $[0, 1]$ for training the RBF-N and the corresponding **Output** of the methodology is the desired number of granules M whose geometrical properties are used to calculate the initial fuzzy rule base of the RBF model. Each linguistic variable is represented by a crisp granule as

$$g_i = ([l_{i1}, u_{i1}], \ldots, [l_{ik}, u_{ik}], \ldots, [l_{in}, u_{in}]) \tag{4.19}$$

where $i = 1, \ldots, M$ fuzzy rules.

At line #2, a lower triangular matrix *compat* is initialise to zero. In order to discriminate correctly the compatibility between two different granules, the elements in the diagonal of the matrix *compat* will be kept to zero throughout the granulation. At line #3, the variable *granule* is used to update the size of the matrix *compat* since at each iteration two different granules are merged. The compatibility between two any different granules i and j is calculated through the lines $\#8-13$.

Algorithm 1 Granulation with Neutrosophic Sets (Gr-NS)

Input: e_{jnor}
Output: $g_i, i = 1, \ldots, M$
1: $iter \leftarrow 1$
2: $compat \leftarrow 0$
3: $granule \leftarrow 0$
4: $iter_{MAX} \leftarrow (cardinality_{ejnor} - M)$
5: **while** $iter \leq iter_{MAX}$ **do**
6: $\quad j \leftarrow 1$
7: $\quad m \leftarrow iter_{MAX} - 1$
8: $\quad$ **while** $j \leq (iter_{MAX} - 1) - granule$ **do**
9: $\quad\quad i \leftarrow j + 1$
10: $\quad\quad$ **while** $i \leq iter_{MAX} - granule$ **do**
11: $\quad\quad\quad compat(i, j) = D_{MAX} - \left\{ i_{i \cup j} - d_{i,j} e^{(-\alpha R)} \right\}, compat \in R^{m \times m}.$
12: $\quad\quad$ **end while**
13: $\quad$ **end while**
14: $\quad$ Find the two most compatible elements i and j of the matrix *compat*.
15: $\quad$ Merge the two most compatible granules i and j.
16: $\quad iter_{MAX} \leftarrow iter_{MAX} - 1$
17: $\quad m \leftarrow iter_{MAX}$
18: $\quad granule \leftarrow granule + 1$
19: **end while**
20: Calculate $Ci = [C_{i=1,k=1}, \ldots, C_{Mn}]$, where $C_{ik} = \frac{1}{2} (max_{xk} - max_{xk})$
21: Calculate $\sigma_i = \frac{1}{r} \left(\sum_{j=1}^{r} \parallel C_j - C_i \parallel \right)^{1/2}$

The equation proposed in [Panoutsos and Mahfouf, 2010a] is used to compute the compatibility, however a new term $i_{i \cup j}$ is introduced. Such a term quantifies the uncertainty that results from a lack of distinguishability during the process of granulation. As pointed out above, a lack of distinguishability when merging two granules may arise due to a high level of overlapping. For example, in Fig. 4.17 is illustrated the resulting overlapping over other granules after merging the granules i and j.

In fuzzy set theory, fuzziness is a type cognitive uncertainty that is caused by the uncertainty transition area from one linguistic term to another [Wang et al., 2012]. In other words, fuzziness measures the distinction between one set and its complement. Since the granules are crisp sets, the overlapping level may be used as the degree of fuzziness between two or more granules. Therefore the uncertainty $i_{i\cup j}$ based on fuzziness can be through the following function

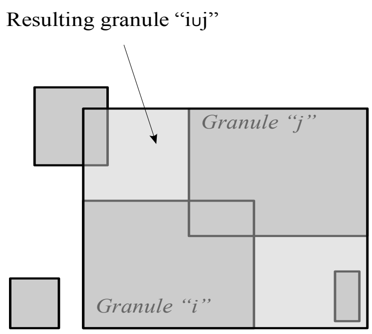


Fig. 4.17 Resulting overlapping when merging two granules i and j.

$$i_{i\cup j} = \frac{1}{n^2}\sum_{k=1}^{n} i_k \tag{4.20}$$

where n is the number of dimensions of the input data, and i_k is calculated as the dimensional fuzziness when merging two granules [Pal and Pal, 1993].

$$i_k = C + \frac{1}{s_k}\sum_{i,j=1,i\neq j}^{M}\left(\mu_{ij}e^{(1-\mu_{ij})} + (1-\mu_{ij})e^{\mu_{ij}}\right), C \in [0,1] \tag{4.21}$$

where M is the maximum number of intervals $[l_{ik}, u_{ik}]$ (See Eq. 4.4) at the dimension "k" and $s_k = 1/\sum_{j=1,i\neq j}^{n}\mu_{ji}$.

Note that M is continuously updated throughout the granulation process. That means at iteration 1, M is equal to the cardinality of the original data set, and at the end of the compression process M is equal to the number of final granules. Eq. 4.21 is an exponential version of the Shannon's entropy whose functional form to measure Fuzzy Uncertainty (FU) without reference to probabilities was firstly defined by Deluca and Termini in [De Luca and Termini, 1972]. Such a measure can be stated as:

$$H(A) = -K \sum_k \mu_k log \mu_k + (1 - \mu_k)\, log\, (1 - \mu_k) \tag{4.22}$$

where K is a normalising constant, A is a fuzzy set in the universe of discourse X. The term μ_{ij} which is usually denoted as p_j, in Eq. 4.21 such a term usually represents the probability of an event j and where $0 \leq p_j \leq 1$ and $\sum_k p_j = 1$. Here μ_{ij} is computed as the membership that indicates the degree of overlapping of the interval $[l_{ik}, u_{ik}]$ upon the interval $[l_{jk}, u_{jk}]$.

$$\mu_{ij} = \frac{[l_{ik}, u_{ik}] \cap [l_{jk}, u_{jk}]}{L_j}, \; L_i = |u_{ik} - l_{ik}|, i \neq j \tag{4.23}$$

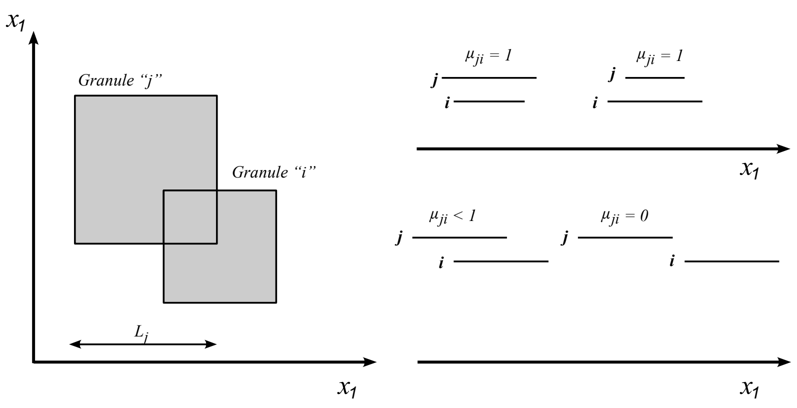


Fig. 4.18 Overlapping membership representation using intervals.

To get a better insight of Eq. 4.23, in Fig. 4.18 the interpretation of a granular membership μji function is depicted, where the granule g_j overlaps the granule g_i. According to [De Luca and Termini, 1972], the maximum value of the fuzziness is when the term $\mu_{ij} = 0.5$ as illustrated in Fig. 4.4.

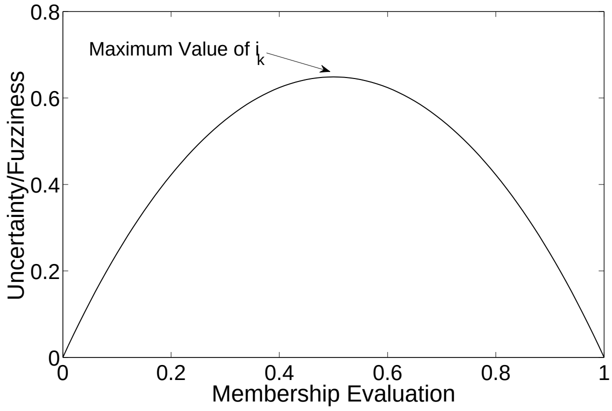


Fig. 4.19 Uncertainty/fuzziness evaluation

The construction of neutrosophic set can be defined as:

$$t_i = \mu_i;\ i_i = i_{i\cup j};\ f_i = 1 - \mu_i \tag{4.24}$$

where μ_i is the degree of overlapping of the granule i upon the granule j. The compatibility criterion is a minimisation cost function; hence the granulation will follow the 'path' of the minimum uncertainty. The disorder 'produced' during the granulation process in terms of uncertainty/indeterminacy could be evaluated by using the tuple $< t, i, f >$ as a histogram of such components as follows:

$$N_i(iter) = \frac{1}{n \times card_{i,j}} e^{-f(iter)} \times i(iter) \tag{4.25}$$

where n is the number of dimensions, $card_{i,j}$ the cardinality of the new merged granule and $iter$ represents the current iteration. Finally, the flow diagram in Fig. 4.20 illustrates the sequence for clustering those granules whose overlapping is that diminishes as much as possible the entropy-based uncertainty.

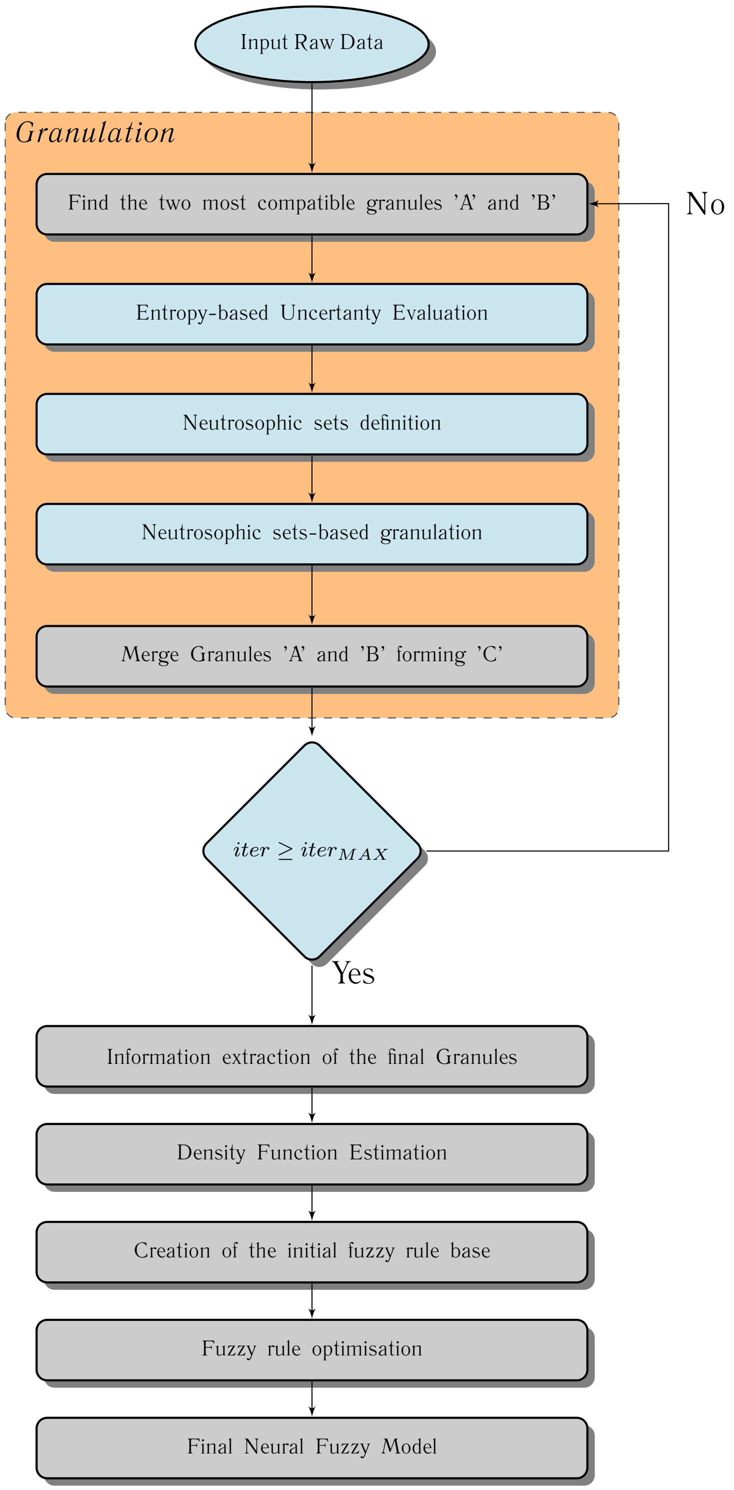


Fig. 4.20 Data-driven model based on RBF-NNs and Gr-NS.

4.4.1 MODELLING RESULTS BY ESTIMATING THE UNCERTAINTY IN THE LINGUISTIC SCENARIO AND GRANULATION INFORMATION 'COVERAGE'

Taken in its broad sense, granulation iterative methodology described by [Panoutsos and Mahfouf, 2010a] considers the proximity between any two entities and its cardinality and length as a compatibility measure. However, as it was described above there are some situations in which distance measures do not produce the best orientation and distribution of the new merged granules. More specifically, this can represent a loss of transparency in the final linguistic rules, and their characterisation. For example, in Fig. 4.16(a) the two final granules produce a misinterpretation of the consequence of the linguistic scenario, and hence this composition bears a lack of parsimonious modelling.

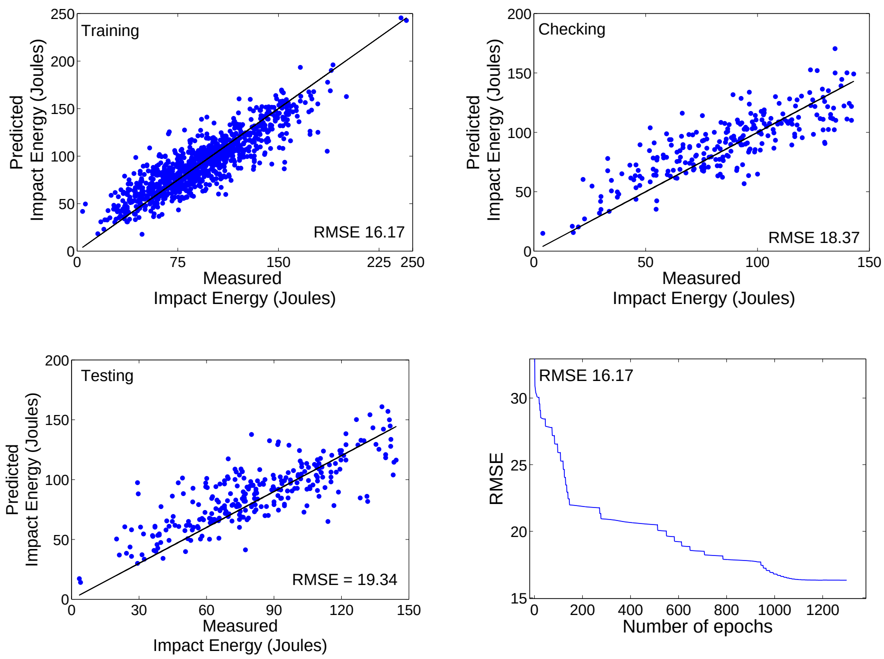


Fig. 4.21 Data fit-Impact energy by using Gr-NS.

The $i \cup j$ resulting granule in Fig. 4.16(a) covers an area (lower left of the granule) where raw data - information - simply does not exist despite following the compatibility objective. As a further example of the application of neutrosophic sets, Fig. 4.21 illustrates the final modelling experiments by using granulation and neutrosophic sets. One of the major motivations to include the uncertainty under this merging process is to eliminate as much as possible this undesirable granulation behaviour, and promote a better granular coverage under a neutrosophic scheme, where the granules are strongly linked with the raw data/information. Furthermore, the term $i \cup j$ is introduced to estimate the indeterminacy produced by the overlapping created in each dimension considering just intervals or simply the corresponding face of a granule. Once the final compression is obtained this information is captured by the proposed neutrosophic scheme based on the T1-RBF-NN. As a comparison study, the simulations were carried out using the same initial parameters were identical to those used by just using granular computing (Section 4.3). In table 4.2, it is shown a comparison of two previously obtained results via FCM, granulation and those obtained by means of the use of neutrosophic sets. Therefore, the second and third arrangement presented in table 4.1 and used for running two different experiments by using Gr-NS.

Table 4.2 RMSE performance by using FCM, GrC and Gr-NS.

9 rules	Training	Checking	Testing
GrC [Panoutsos and Mahfouf, 2010a]	14.66	21.24	20.42
	Second arrangement		
FCM	18.78	19.48	21.78
Granulation	16.91	19.65	20.91
Gr-NS	16.48	19.10	19.73
	Third arrangement		
FCM	19.18	20.01	22.30
Granulation	16.76	19.20	20.91
Gr-NS	16.10	18.37	19.34

Even though, in [Panoutsos and Mahfouf, 2010a] the training performance is better, the proposed neutrosophic scheme proved to be efficient and more robust bearing an enhanced generalisation (testing) reducing the errors of the predicted results which is very significant to this type of industrial data. The final granular scenario after granulation can be seen as a fuzzy model representation due to its own characteristics and hence it may be assumed that the interpretability of the final granular discourse is automatically given due to the formation of the granules and their corresponding interpretation as linguistic fuzzy rules. However, from the experimental results obtained by just using granulation and shown in Fig. 4.22 in the 'C' and 'test depth' dimensions demonstrate that the compatibility index suffers from a lack of distinguishability among the granules.

In addition, Fig. 4.22 confirms that the process of granulation tends to group data according to similar properties, but it never takes into account the orientation and overlapping during the granule formation. Particularly overlapping affects negatively the transparency and then the distinguisability of the final granules. Fig. 4.23 shows the final shape of the MFs after the application of the proposed neutrosophic scheme based on granulation.

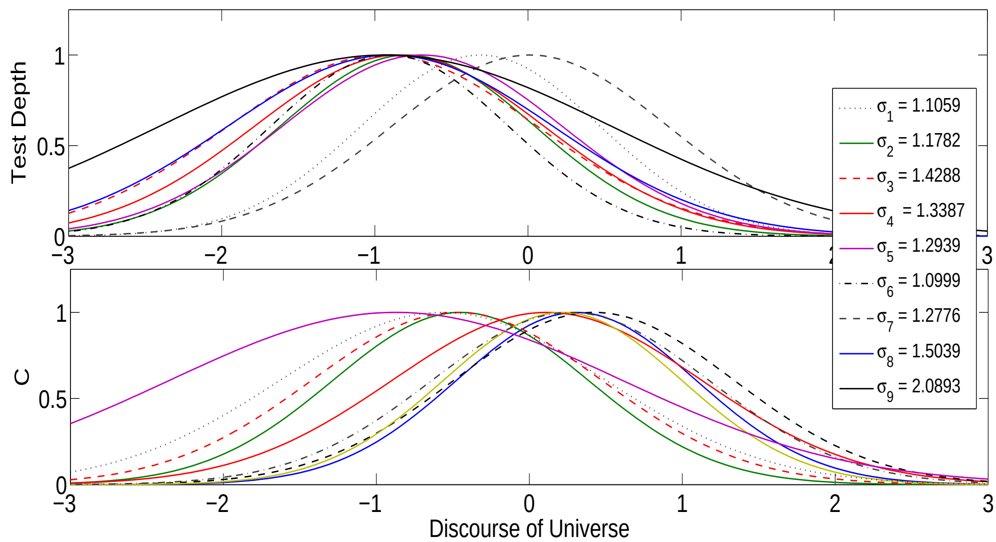


Fig. 4.22 Final shape of the MFs after granulation.

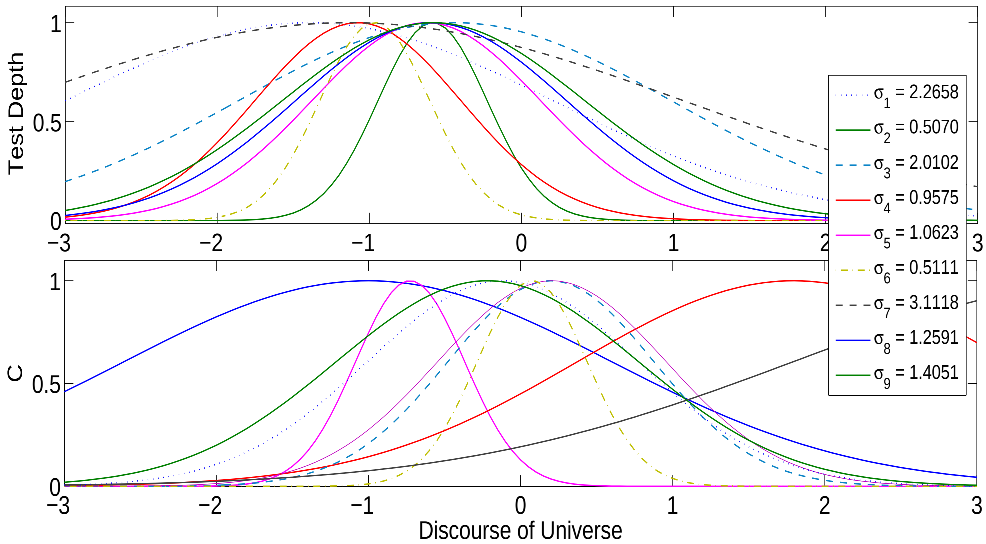


Fig. 4.23 Final shape of the MFs after granulation and neutrosophic sets.

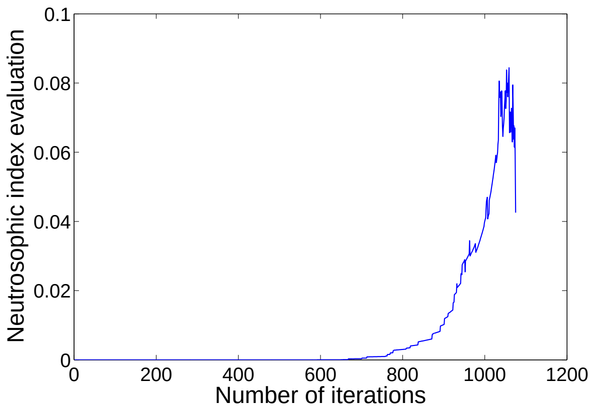


Fig. 4.24 Neutrosophic index behaviour throughout granulation.

As can also be observed from Fig. 4.23, the compatibility criterion now guides the process of granulation to form granules whose overlapping is more moderate. This means that even the beauty of fuzzy models is the construction of more transparent models, when non-separable data are under study, some overlapping is necessary. Therefore, the creation of an hyperplane that completely separates the input vectors is not always possible. From Fig. 4.24, the proposed neutrosophic algorithm pretends to efficiently diminish this overlapping without affecting the powerful of granulation in grouping data according to similar features. Such an index reflects the behaviour of the compatibility expression in terms of the tuple $< t, i, f >$ and the final distribution of the resulting granules.

4.5 SUMMARY

In this chapter a systematic modelling framework based on Granular Computing (GrC), the RBF-NN and neutrosophic sets is proposed. The suggested approach uses a neutrosophic logic concept to estimate inherent information uncertainty/indeterminacy due to the merging operation during the information granulation process. The uncertainty index, calculated via a Shanon entropy criterion, is iteratively calculated throughout granulation and this results in a final GrC-T1-RBF-NN inference system with a more robust rule-base with better representation of the given raw data information. This approach was applied to a real industry data set, based on the measurement of Charpy toughness of heat treated steel, a process that is particularly know for the production of sparse and uncertain data. The proposed methodology is successfully applied to the industrial dataset and the results show an improved generalisation and model interpretability performance compared with similar modelling attempts. Moreover, such results obtained by the proposed methodology led to the publication of an article in the peer reviewed journal "Soft Computing" with the title: *Granular Computing neural-fuzzy modelling: A neutrosophic approach.*

In the next chapter, an uncertainty assessment methodology is proposed in order to explore and exploit the information contained and processed during the training process.

5 IT2-RBF-NN: INTERVAL TYPE-2 RADIAL BASIS FUNCTION NEURAL NETWORK

An Interval Type-2 Radial Basis Function Neural Network (IT2-RBF-NN) that is functionally equivalent to Interval Type-2 Fuzzy Systems and the well-known RBF-NN is introduced in this chapter.

The main contribution of this chapter is twofold, on the one hand the creation of a new network that is able to deal with linguistic uncertainty is introduced. And on the other hand, an adaptive parameter identification procedure based on the gradient-descent approach is provided.

The motivation for the development of an IT2-RBF-NN is to deal with linguistic uncertainty at two different levels of interpretability. This opens up a new area of research study for systems modelling by means , perceptions and the creation of clustering approaches based on words.

5.1 INTRODUCTION

As it was pointed out in [Mendel, 1995], fuzzy logic systems are able to handle numerical data and linguistic information. That means that fuzzy logic systems tend to perform an inference procedure based on two types of information knowledge. One the one hand, numerical knowledge refers to *objective knowledge* frequently found in engineering problems. On the other hand, the linguistic representation of information through *subjective knowledge* that is usually abstract and it is impossible to quantify in mathematics [Mendel, 1995]. In this regard, the application of fuzzy sets in data-driven models both types of knowledge can be coordinated. For instance, in literature a large number of fuzzy logic systems of type-1 applied on real and complex systems can be found [Coza and Macnab, 2006, Feng, 2006, Kosko, 1992, Lee, 1990, Mamdani, 1977, Tong, 1977].

The fuzzy inference engine plays an important role in fuzzy logic systems since it represents the mechanism to combine the IF-THEN rules from the rule base into a mapping from the input data to fuzzy output sets. Each rule is seen as an individual inference activated by an antecedent (input data, MF, singleton, etc) and then mapped into another output space (consequence) that usually is fuzzy, crisp or interval sets. Although, fuzzy logic systems are able to represent real problems by using linguistic rules, there is a problem when it comes to process/compute with words. This type of problems arise when the MFs in the rule base are difficult to be determined as a consequence of a controversy between two or more experts [Mendel, 2001, 2007b]. For example, when a group of people are asked to specify which length dimensions should be to classify a car like an small vehicle. Therefore, it would be worth using an interval that capture the opinion of people with similar answers. In other words, the use of crisp MFs to inference the opinion of the people would not be enough. Moreover, the type of MFs, i.e. triangular, Gaussian, trapezoidal, etc, is crucial as it is problem-dependent. This raises questions about uncertain linguistic information when processing data with fuzzy systems, especially neural fuzzy systems either in control theory or systems modelling.

Zadeh not only introduced the concept of Fuzzy sets (FSs) [Zadeh, 1965], but also proposed the idea of Fuzzy Sets of Type-2 (T2-FSs, 1975). Therefore, it became common to call FSs of Type-1 as T1-FSs - and T2-FSs to those FSs that have a MF of type-2, which mean that a T2-FS is a fuzzy-fuzzy-set. However, it was not until 1998 that Mendel and Karnik [Karnik and Mendel, 1998a] defined the basis for type-2 fuzzy systems. In that article, Mendel and Karnik introduced all the components that a fuzzy system of type-2 should have, i.e. a) a fuzzifier, b) a rule base, c) type-reducer and a defuzzifier. In a like-manner to fuzzy systems of type-1, the input data is fuzzify into a MF of type-2 and then processed by an inference engine for T2-FSs. Consequently, in order to get a crisp number, a type-reducer was proposed in order to obtain fuzzy sets of type-1 from T2-FSs. Finally, the defuzzifier produces a crisp number from the FSs that is the output of the type-reducer. Type-2 Fuzzy Set theory is a growing research field [John and

Coupland, 2007]. The reason behind is its ability to deal with uncertainty in four different ways: 1) The words that are used in the antecedent and the consequent part could mean different to different people, 2) The information obtained from a group of experts in relation to one rule can have a different meaning, 3) noisy training data and 4) the noisy measurements that can activate the inference engine. However, the application of type-2 fuzzy set theory in engineering can result expensive in computational terms. Principally, this computational load results from the large number of calculations required to obtain the MFs of grade 2 of each input, and the number of iterations that are needed to execute the type-reducer [Karnik and Mendel, 1998b, 2001, Wu and Mendel, 2009]. In this sense, interval type-2 fuzzy sets have become a popular tool among researchers and practitioners due to its easy understanding and low computational burden compared to fuzzy systems of type-2 [Liang and Mendel, 2000]. Furthermore, the concept of interval offers a great chance to understand real complex systems from a linguistic perspective handling better with knowledge and rule uncertainty. Such properties are still described by the classical elements through the use of a fuzzifier, rule-base and defuzzifier that constitutes the basic taxonomy in fuzzy systems of type-1.

This Chapter details the development of an Interval type-2 Radial Basis Function Fuzzy Neural Network (T2-RBF-FNN) and the corresponding learning methodology for its parameter identification. The advantage of the functional equivalence of radial basis function neural networks (RBF-NN) to a class of type-1 fuzzy logic systems (T1-FLS) is exploited in order to propose a new interval type-2 equivalent system; it is systematically shown that the type equivalence (between RBF and FLS) of the new modelling structure is maintained in the case of the IT2 system. The new IT2-RBF-NN incorporates interval type-2 fuzzy sets within the radial basis function layer of the neural network in order to account for linguistic uncertainty in the system's variables. The antecedent and consequent part in each rule in the IT2-RBF-NN is an interval type-2 fuzzy set and the consequent part is of Mamdani type with interval weights, which are used for the Karnik-Mendel type-reduction process in the output layer of the network.

The structural and parametric optimisation of the IT2-RBF-NN parameters is carried out by a hybrid approach that is based on estimating the initial rule base and footprint of uncertainty (FOU) directly via the granulation approach used in chapter 4, and an adaptive Back Error Propagation approach (adaptive-BEP) proposed in this chapter. The effectiveness of the new modelling framework is assessed in two parts. Firstly the IT2-RBF-NN is tested against a number of popular benchmark datasets, and secondly it is demonstrated the good performance and the very good computational efficiency of the proposed framework in modelling the Charpy impact dataset.

5.2 T1-RBF-NN STRUCTURE AND FUZZY LOGIC SYSTEMS OF TYPE-1

As it is deeply described in appendix A, and fully explained in [Jang and Sun, 1993], an RBF-NN can be seen as a Fuzzy System of type-1 if the following conditions are met:

- The number of receptive fields in the hidden layer (see Fig. 5.1) is equal to the number of fuzzy rules.
- The MF's within each rule are chosen as Gaussian functions.
- The T-norm operator used to compute each rule's firing strength is multiplication.
- Both the T1-RBF-NN and the FIS under consideration use the same defuzzification method, that is: either the centre of gravity or weighted sum to estimate their overall outputs.

Generally stated, the Jang-Sung result showed that the standard RBF-NN is functionally equivalent to a type of Takagi-Sugeno fuzzy systems if the value of the output weights w_i (Fig. 5.1) are used as linear functions of the input vector $\vec{x}_p$. That means that the canonical form of each local inference

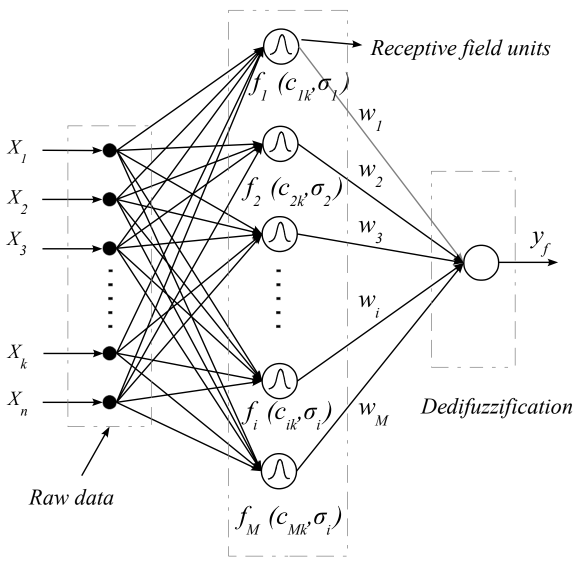


Fig. 5.1 RBF Neural Network Structure

engine (Receptive Units, RUs) in the RBF-NN can be expressed as:

$$R_i: \; if\, x_1\, is\, F_1^i \wedge x_2\, is\, F_2^i \wedge \ldots \wedge x_n\, is\, F_n^i \;\; then\, w_i(\vec{x}_p) = a_1 x_1 + \ldots + a_n x_n + b_1 \tag{5.1}$$

where each fuzzy rule is premised on its own input vector $\vec{x}_p$, i.e. $\vec{x}_p \in R^n$, F_n^i are the linguistic labels of the fuzzy sets describing the qualitative state of the input vector, and the conjunction operator $\wedge$ is the T-norm in the RBF-NN. In [Hunt et al., 1996], the authors generalised the result obtained by Jang-Sung, by using ellipsoidal basis functions which means no restriction on the width of the basis functions, the output of each rule is given by a linear combination (removing the restriction of just using a constant) and the removal of Gaussian functions as the only type of MFs to be used.

In terms of fuzzy logic applications, this type of networks are now functionally equivalent to a wider number of fuzzy sets of type-1. Particularly, the work of Hunt, Hant & Smith creates a framework where the basis functions are more independent.

5.3 IT2-RBF-NN STRUCTURE

Before delving into the description of the constituents of the proposed IT2-RBF-NN, it would be worth mentioning some important features that make this network a generalised fuzzy framework for modelling purposes. This implies that according to the appendix A.1, the proposed IT2-RBF-NN can not only be seen as fuzzy model based on the Mamdani inference but also as a:

1. Takagi-Sugeno Fuzzy model (TS-FM) [Hunt et al., 1996]. In contrast to the Mamdani FM, a TS-FM defines (A.7) as follows:

$$y_f = \frac{\sum_{i=1}^{M} \mu_{B^i}(y) w_i}{\sum_{i=1}^{M} \mu_{B^i}(y)} \tag{5.2}$$

   where $w_i = a_1 x_1 + \ldots + a_n x_n + b_i$, such that $i = 1, \ldots, M$ represents M linear local models as the consequent part of each IF-THEN rule.

2. Local model network. Since the proposed IT2-RBF-NN represents a type of extension of fuzzy logic systems and inherits some properties from neural networks such as universal approximation, adaptation and generalisation properties, practical advantages from one paradigm may be used to the other under appropriate interpretations. This includes, learning algorithms, the use of a priori expert knowledge to pre-construct a fuzzy model, and the ability of the IT2-RBF-NN to express a system by the use of local models. To put it more simply, an IT2-RBF-NN may be seen as a non-linear system that is decomposed into sub-models which are integrated by smooth interpolation functions over an operating space [Foss and Johansen, 1993].

Besides, each interval Gaussian function that is premised by the input vector $\vec{x}_p$ may have a different width (spheroidal) or not (ellipsoidal). In the former case, each MF can be expressed as

$$\mu_{B^i} = exp[-(\vec{x}_p - \vec{c}_i)' \Delta_i (\vec{x}_p - \vec{c}_i)] \tag{5.3}$$

with

$$\Delta_i = \begin{bmatrix} \frac{1}{\sigma_{i1}^2} & 0 & \cdots & 0 \\ 0 & \frac{1}{\sigma_{i2}^2} & \cdots & 0 \\ \vdots & & \ddots & \\ 0 & & & \frac{1}{\sigma_{in}^2} \end{bmatrix}$$

Therefore, the components of the IT2-RBF-NN can be listed as follows: the fuzzifier is that of singleton type whose T-norm is the multiplication and the type-reducer is that proposed by Karnik and Mendel [Liang and Mendel, 2000, Liu et al., 2012]. The IT2-RBF-NN configuration is illustrated in Fig. 5.2 - from a structural point of view the IT2-RBF-NN has a total of 4 layers which are described below.

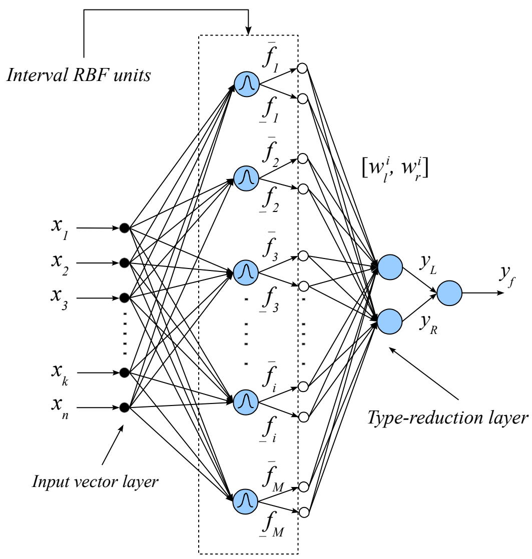


Fig. 5.2 Structure of the proposed IT2-RBF-NN

Input Layer. The input data are multidimensional crisp data represented by $\vec{x}_p = [x_1, ..., x_n] \in R^n$. Only the current states are fed into the layer as the input data $\vec{x}_p$ and then forwarded to the next layer.

Hidden RBF Layer. This Layer is a twofold layer that performs the fuzzification process of the input data $\vec{x}_p$ and produces the upper and lower interval MF $[\bar{f}_i, \underline{f}_i]$ as it is illustrated in Fig. 5.3. Similar to T1-RBF-NN, a process of clustering based on data granulation [Panoutsos and Mahfouf, 2010a] is used in order to estimate the initial parameters of the RBF receptive units. In agreement with the existing terminology used in IT2-FS theory [Liang and Mendel, 2000, Mendel, 2004, Wu and Mendel, 2007], here five different types of MFs are proposed.

1. First, an n-dimensional Gaussian MF having a fixed standard deviation σ_i and an uncertain mean m_k^i is considered as follows

$$f_i(\vec{x}_p) = exp\left[-\frac{\|\vec{x}_p - m_k^i\|^2}{(\sigma_i)^2}\right], \; m_k^i \in [m_{k1}^i, m_{k2}^i] \tag{5.4}$$

in which $\vec{x}_P = (x_1, \ldots, x_n)^T$, and where
$M:$ number of rules;
$i:$ $1, \ldots, M$;
$n:$ number of antecedents at iteration p, and $k:$ $1, \ldots, n$.

For example, the n-dimensional upper MF $\bar{f}_i$ can be stated as:

$$exp\left[-\frac{\sum_{k=1}^{n} \overline{\phi}_k(x_k)}{(\sigma_i)^2}\right] \triangleq \bar{f}_i(\overline{\phi}_{kl}(x_k), \sigma_i) \tag{5.5}$$

in which

$$\overline{\phi}_k(x_k) \begin{cases} (x_k - m_{k1}^i)^2, & x_k \leq m_{k1}^i \\ 0, & m_{k1}^i \leq x_k \leq m_{k2}^i \\ (x_k - m_{k2}^i)^2, & x_k > m_{k2}^i \end{cases} \tag{5.6}$$

and the n-dimensional lower MF $\underline{f}_i$ is

$$exp\left[-\frac{\sum_{k=1}^{n} \underline{\phi}_k(x_k)}{(\sigma_i)^2}\right] \triangleq \underline{f}_i(\underline{\phi}_k(x_k), \sigma_i) \tag{5.7}$$

where

$$\underline{\phi}_k(x_k) \begin{cases} (x_k - m_{k2}^i)^2, & x_k \leq \dfrac{m_{k1}^i + m_{k2}^i}{2} \\ (x_k - m_{k1}^i)^2, & x_k > \dfrac{m_{k1}^i + m_{k2}^i}{2} \end{cases} \tag{5.8}$$

Note that from Eq. 5.5 the value of $\bar{f}_i(\vec{x}_p) \approx 1$ when $\sum_{k=1}^{n} \underline{\phi}_k(x_k) \approx 0$, either if $(x_k - m_{kl})^2 \to 0$ or $x_k \in [m_{k1}^i, m_{k2}^i]$.

2. In like manner for an n-dimensional Gaussian primary MF having a fixed mean m_k^i and an uncertain standard deviation σ_i

$$f_i(\vec{x}_p) = exp\left[-\frac{\|\vec{x}_p - m_k^i\|^2}{(\sigma_i)^2}\right], \ \sigma_i \in [\sigma_i^1, \sigma_i^2] \tag{5.9}$$

in which $\vec{x}_P = (x_1, \ldots, x_n)^T$, $m_k^i = (m_1^i, \ldots, m_n^i)^T$ and where $M :$ number of rules, $i :$ $1, \ldots, M$, $n :$ number of antecedents at iteration p, and $k :$ $1, \ldots, n$.
Correspondingly, the n-dimensional upper MF $\bar{f}_i$ is

$$exp\left[-\frac{\sum_{k=1}^{n} (x_k - m_k^i)^2}{(\sigma_i^1)^2}\right] \triangleq \bar{f}_i(m_k^i, \sigma_i^1; \vec{x}_p) \tag{5.10}$$

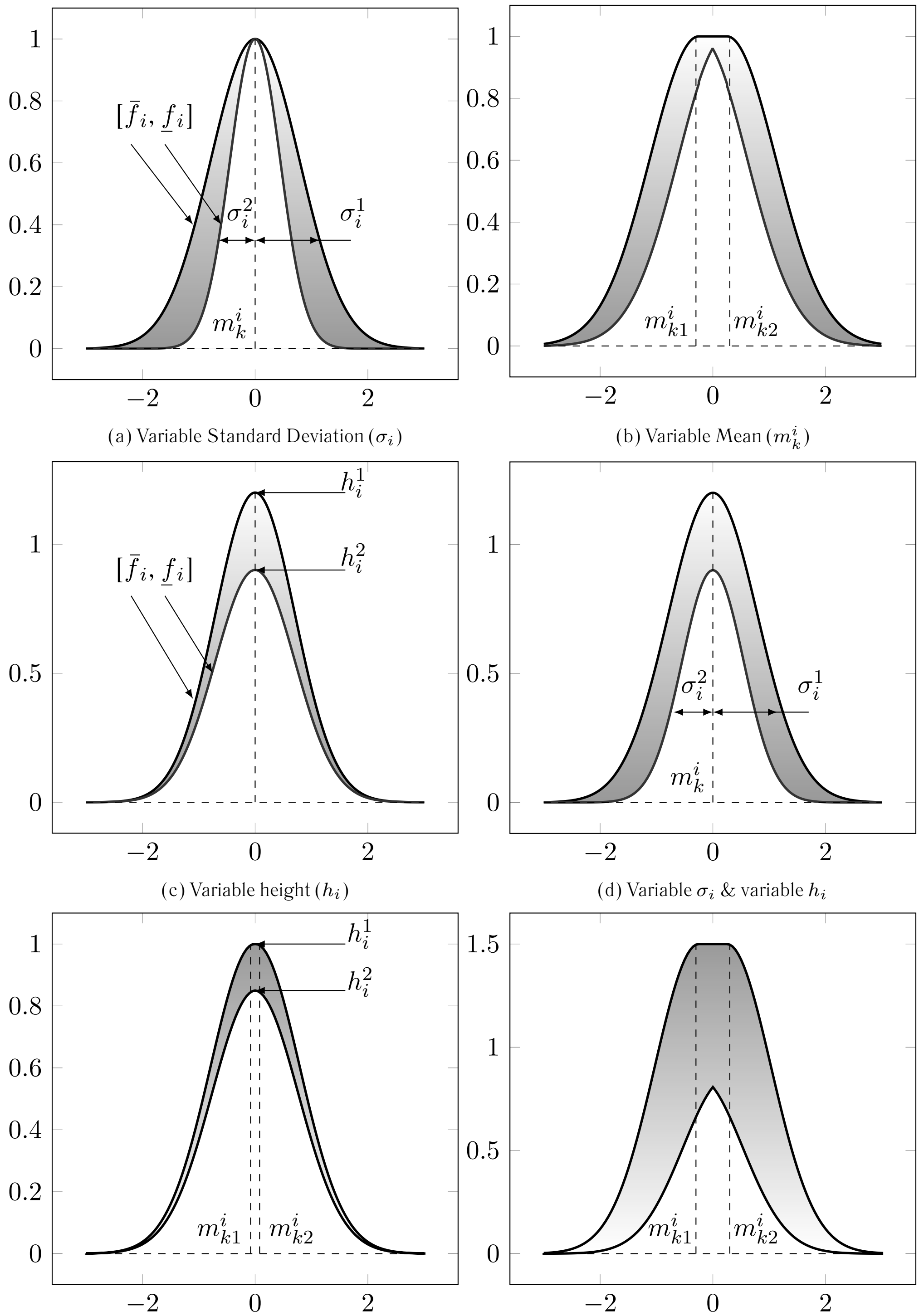
and the n-dimensional lower MF $\underline{f}_i$ is

$$exp\left[-\frac{\sum_{k=1}^{n} (x_k - m_k^i)^2}{(\sigma_i^2)^2}\right] \triangleq \underline{f}_i(m_k^i, \sigma_i^2; \vec{x}_p) \tag{5.11}$$

3. For an n-dimensional Gaussian primary MF having a fixed mean m_k^i, a fixed standard deviation σ_i, and an uncertain height h_i defined as

$$f_i(\vec{x}_p) = h_i * exp\left[-\frac{\|\vec{x}_p - m_k^i\|^2}{(\sigma_i)^2}\right], \ h_i \in [h_i^1, h_i^2] \tag{5.12}$$

where the n-dimensional upper MF $\bar{f}_i$ is (See Fig. 5.3)



(a) Variable Standard Deviation (σ_i)

(b) Variable Mean (m_k^i)

(c) Variable height (h_i)

(d) Variable σ_i & variable h_i

(e) Variable h_i and variable m_k^i

(f) Variable h_i and variable m_k^i

Fig. 5.3 Interval type-2 Membership Functions for the receptive units in the IT2-RBF-NN

$$h_i^1 * exp\left[-\frac{\sum_{k=1}^{n}(x_k - m_k^i)^2}{(\sigma_i)^2}\right] \triangleq \bar{f}_i(m_k^i, \sigma_i, h_i^1; \vec{x}_p) \tag{5.13}$$

and the n-dimensional lower MF $\underline{f}_i$ is

$$h_i^2 * exp\left[-\frac{\sum_{k=1}^{n}(x_k - m_k^i)^2}{(\sigma_i)^2}\right] \triangleq \underline{f}_i(m_k^i, \sigma_i, h_i^2; \vec{x}_p) \tag{5.14}$$

4. An n-dimensional Gaussian primary MF having a fixed mean m_k^i, an uncertain standard deviation σ_i, and an uncertain height h_i that can be stated as

$$f_i(\vec{x}_p) = h_i * exp\left[-\frac{\|\vec{x}_p - m_k^i\|^2}{(\sigma_i)^2}\right],\ h_i \in [h_i^1, h_i^2],\ \sigma_i \in [\sigma_i^1, \sigma_i^2] \tag{5.15}$$

hence the n-dimensional upper MF $\bar{f}_i$ is (see Fig. 5.3(d))

$$h_i^1 * exp\left[-\frac{\sum_{k=1}^{n}(x_k - m_k^i)^2}{(\sigma_i^1)^2}\right] \triangleq \bar{f}_i(m_k^i, \sigma_i^1, h_i^1; \vec{x}_p) \tag{5.16}$$

and the n-dimensional lower MF $\underline{f}_i$ is

$$h_i^2 * exp\left[-\frac{\sum_{k=1}^{n}(x_k - m_k^i)^2}{(\sigma_i^2)^2}\right] \triangleq \underline{f}_i(m_k^i, \sigma_i^2, h_i^2; \vec{x}_p) \tag{5.17}$$

5. Finally, in order to calculate an n-dimensional Gaussian primary MF having an uncertain mean m_k^i, a fixed standard deviation σ_i, and an uncertain height h_i; a combination of those equations used for the case 2, 3 and 4 can be used. Fig. 5.3(e) and 5.3(f) particularly illustrate two cases that reflects the properties of adjusting the the heigh and the mean of the MF. For example, when the difference $|m_k^1 - m_k^2| \to 0$, the shape of the MF is almost identical to that MF obtained by just adjusting the height as illustrated in Fig. 5.3(e).

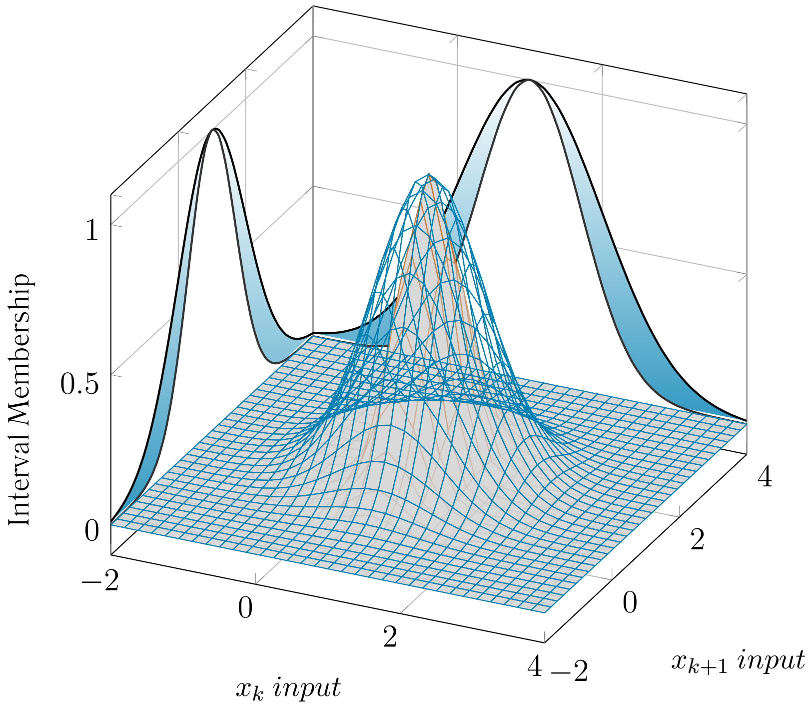


Fig. 5.4 Two-dimensional interval RBF MF

In particular, 5.4 and 5.9 expressed in one dimension leads to a piecewise-linear interpolating function which represents the simplest form of exact interpolation [Mendoza et al., 2009]. Thus, the generalisation to several dimensions is straightforward insomuch as basis functions represent a mapping from n-dimensional input space $\vec{x}_p$ to one-dimensional target space.

Moreover, the RBF approach introduces a set of M basis functions, one for each data point which takes the values $\|\vec{x}_p - \vec{m}\|$ to be Euclidean, between $\vec{x}_p$ and $\vec{m}$. For illustrative purposes, in Fig. 5.4 the shape of the MF in two dimensions with uncertain standard deviation is illustrated.

Type-Reduction Layer (TRL) . Regardless of singleton or non singleton fuzzification and the type of minimum or product t-norm, the firing strength in the hidden layer is an interval type-1 set that can be calculated by its left-most and right-most points $\underline{f}_i$ and $\bar{f}_i$. The TRL is the type-reduction method proposed by Karnik and Mendel [Wu and Mendel, 2009] which is the extension of the type-1 defuzzification process, and hence the functional equivalence of the weighted average sum in the T1-RBF-NN. We propose a type-reduction layer based upon the Karnik-Mendel center of sets type-reducer in order to combine the output consequent set which is shown in 5.18 and 5.19.

$$y_l = \frac{\sum_{i=1}^{L} \bar{f}_i w_l^i + \sum_{i=L+1}^{M} \underline{f}_i w_l^i}{\sum_{i=1}^{L} \bar{f}_i + \sum_{i=L+1}^{M} \underline{f}_i} \tag{5.18}$$

$$y_r = \frac{\sum_{i=1}^{R} \underline{f}_i w_r^i + \sum_{i=R+1}^{M} \bar{f}_i w_r^i}{\sum_{i=1}^{R} \underline{f}_i + \sum_{i=R+1}^{M} \bar{f}_i} \tag{5.19}$$

Where $[w_l^i, w_r^i]$ represent the centroid interval set of the consequent type-2 fuzzy set of the ith rule.

4) Output Layer. The output layer finally computes the average of y_l and y_r.

$$y_f = \frac{y_l + y_r}{2} \tag{5.20}$$

5.4 PARAMETER IDENTIFICATION OF THE IT2-RBF-NN

In this section the proposed IT2-RBF-NN is a system having a center-of-sets type reduction, product inference rule and a singleton fuzzy output space. Since the proposed model is a type of network that falls within the general class of non-linear layer feed-forward networks, the adaptive-BEP approach can be applied on the estimation of the antecedent parameters σ_k^i and m_k^i, and the consequent parameters $[w_l^i, w_r^i]$ of the MFs. The derivatives that are needed to implement the steepest-descendent parameter-tuning algorithm are derived in [Mendel, 2004]; it is explained in detail what are the challenges in the calculation of the IT2-FS derivatives as compared to the simpler type-1 FS ones. This section, provides a hybrid algorithm based on granular computing (data granulation) for identifying the initial parameters of the hidden RBF layer and a learning method that uses a momentum term γ with an adaptive learning rate α for the optimisation of the IT2-RBF-NN parameters. In a like manner to those experimental results carried out in chapter 4, the aim of the granulation stage is to group similar data (given raw data) whose effectiveness lies on a compatibility-best designed measure mentioned in chapter 4. The proposed adaptive learning algorithm is used to optimise the RBF parameters and the output weights; these are now intervals and represent interval fuzzy sets in the premise and consequent part of the fuzzy rules. The IT2-RBF-NN structure includes a type-reducer stage based on the Karnik-Mendel approach that is an ascending sort process. This iterative procedure results in a number of permutations which must be considered when training the IT2-RBF network [Hagras, 2006]. In this research work it is used the same assignation when naming the active branch that was employed in [Hagras, 2006] in order to calculate the switching points L and R.

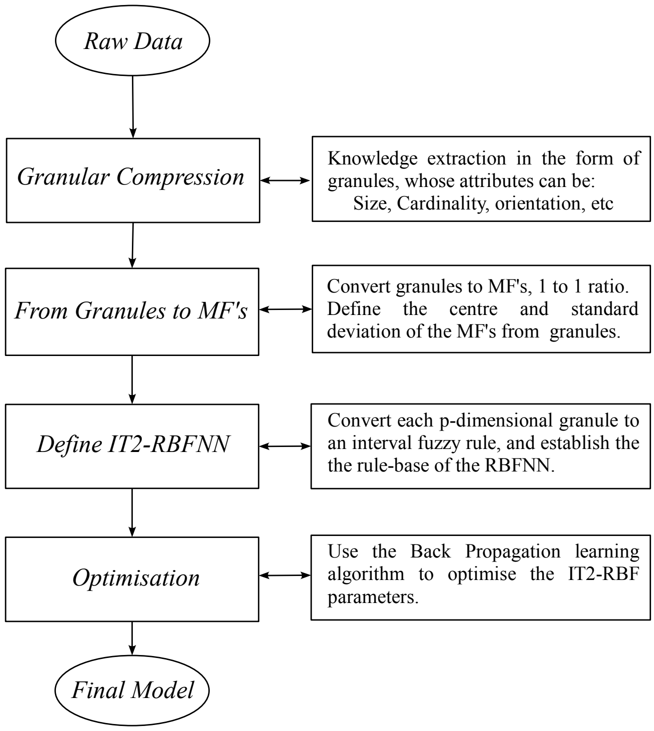


Fig. 5.5 Overview of the GrC-based IT2-RBF-NN framework

An overview of the overall framework is depicted in Fig. 5.5 which comprises the rule base formation, and parametric optimisation of the IT2-RBF-

NN system. Starting from the raw data, a GrC-based algorithm is used to extract the information granules that subsequently will form the rule base of the system. Each n-dimensional granule corresponds to one fuzzy rule. In this step the FOU for each MF is also estimated. Finally, following the definition of the IT2-RBF-NN system (as in Section 5.3) a parametric optimisation is performed via the adaptive BEP algorithm. The data granulation procedure fully described in chapter 4, where a compatibility measure was employed for grouping data according to pre-defined similarities, and the parametric optimisation of the system is deeply described in the following section.

FROM GRANULES TO MEMBERSHIP FUNCTIONS

The final geometrical boundaries of each information granule after compression are used to estimate the initial value of m_k and σ_i. The average hyper-box boundaries of each granule are utilised to calculate the initial m_k no matter if it is a fixed mean or not. Indeed, it is initially let free m^i_{k1} and m^i_{k2} by using $m^i_{k2} = |\Delta m^i_k| + m^i_{k1}$ and $\sigma^2_i = |\Delta\sigma_i| + \sigma^1_i$ when the IT2-RBF parameters are optimised.

$$m^i_{k1} = [m^i_{11}, m^i_{21}, ..., m^i_{p1}] \tag{5.21}$$

in which

$$m^i_{k1} = \frac{max_{X_k} - min_{X_k}}{2} \tag{5.22}$$

And for the estimation of σ^2_i [34].

$$\sigma^2_i = \frac{1}{r}\left(\sum_{j=1}^{r} \|m^j_{k1} - m^i_{k1}\|\right)^{1/2} \tag{5.23}$$

where $j \neq i$, j is the nearest neighbour to the neuron i, and $r \geq 2$. Once, the initial IT2-RBF parameters are estimated we obtain the very first interval

MFs with uncertain mean and uncertain standard deviation.

5.4.1 LEARNING METHODOLOGY

The goal of the proposed adaptive-BEP approach is the estimation of the parameters σ_k^i and m_k^i and $[w_l^i, w_r^i]$ that characterise the antecedent and consequent of the MFs respectively. Our start point is the derivation of the equations necessary for the cases when a) the standard deviation is fixed and the mean (M) is uncertain and when b) the mean is fixed and the uncertain deviation (SD) is uncertain. Therefore, the adaptive learning methodology is used to overcome the drawback in leading the objective function (performance index) to a local minimum by just using the gradient descent. The adaptive-BEP approach for training the IT2-RBF-NN [Hagras, 2006, Panoutsos and Mahfouf, 2010a] must track the corresponding parameters σ_k^i and m_k^i in the corresponding antecedent active branch which may be different at each iteration t as a consequence of the different values of L and R during the type-reduction process that sorts the consequent weights w^i's in increasing order, and hence the dependency of y_L and y_R on the output layer parameters may also be changed. By using a learning methodology based on a BEP algorithm for P input-output training data $(\vec{x}_p : d_p)$; $p = 1, ..., P$, the following cost error function should be minimised:

$$e_p = \frac{1}{2}(y(\vec{x}_p) - d_p)^2 \tag{5.24}$$

The performance index utilised during the optimisation stage is as follows:

$$P_{iter} = \frac{1}{P}\sum_{p=1}^{P} e_p^2 \tag{5.25}$$

where p is the total number of training points. Since the proposed IT2-RBF-NN model falls within the family of feed-forward networks, the proposed learning methodology first processes the information in only one direction from the input layer through the hidden neurons and finally compute the network output. Consequently, an adaptive Back Error Propagation approach (adaptive-BEP) based on an the gradient descent is applied to update the IT2-RBF-NN parameters. This is done by firstly comparing the current output network with the desired pattern through the computation of the Root-Mean-Square-Error (RMSE). Therefore, the error is then fed back through the IT2-RBF-NN by computing the associated derivatives. In Appendix B a complete procedure for the computation of the corresponding derivatives is provided. The final adaptive-BEP equations for the IT2-RBF-NN optimisation in the corresponding active branch areas follows:

1. *For uncertain mean*

The update rule for the centre of each MF:

$$\Delta m_{k1}^{i}(t+1) = -\alpha \frac{\partial e_p}{\partial m_{k1}^{i}} + \gamma \Delta m_{k1}^{i}(t) \tag{5.26}$$

$$\Delta m_{k2}^{i}(t+1) = -\alpha \frac{\partial e_p}{\partial m_{k2}^{i}} + \gamma \Delta m_{k2}^{i}(t) \tag{5.27}$$

The update rule for the width of each MF:

$$\Delta \sigma_i(t+1) = -\alpha \frac{\partial e_p}{\partial \sigma_i} + \gamma \Delta \sigma_i(t) \tag{5.28}$$

The update rule for the output weight:

$$\Delta w_{l}^{i}(t+1) = -\alpha \frac{\partial e_p}{\partial w_{l}^{i}} + \gamma \Delta w_{l}^{i}(t) \tag{5.29}$$

$$\Delta w_{r}^{i}(t+1) = -\alpha \frac{\partial e_p}{\partial w_{r}^{i}} + \gamma \Delta w_{r}^{i}(t) \tag{5.30}$$

2. *For uncertain standard deviation*

The update rule for the centre of each MF:

$$\Delta m_{k}^{i}(t+1) = -\alpha \frac{\partial e_p}{\partial m_{k}^{i}} + \gamma \Delta m_{k}^{i}(t) \tag{5.31}$$

The update rule for the width of each MF:

$$\Delta\sigma_i^1(t+1) = -\alpha\frac{\partial e_p}{\partial\sigma_i^1} + \gamma\Delta\sigma_i^1(t) \tag{5.32}$$

$$\Delta\sigma_i^2(t+1) = -\alpha\frac{\partial e_p}{\partial\sigma_i^2} + \gamma\Delta\sigma_i^2(t) \tag{5.33}$$

3. *For uncertain height*

The update rule for the centre of each MF:

$$\Delta m_k^i(t+1) = -\alpha\frac{\partial e_p}{\partial m_k^i} + \gamma\Delta m_k^i(t) \tag{5.34}$$

The update rule for the width of each MF:

$$\Delta\sigma_i(t+1) = -\alpha\frac{\partial e_p}{\partial\sigma_i} + \gamma\Delta\sigma_i^1(t) \tag{5.35}$$

The update rule for the height of each MF:

$$\Delta h_i^1(t+1) = -\alpha\frac{\partial e_p}{\partial h_i^1} + \gamma\Delta h_i^1(t) \tag{5.36}$$

$$\Delta h_i^2(t+1) = -\alpha\frac{\partial e_p}{\partial h_i^2} + \gamma\Delta h_i^2(t) \tag{5.37}$$

4. *For uncertain height and uncertain standard deviation.* In order to tune a variable term h_i and σ_i, it is only necessary to include in the adaptive-BEP Eq. (5.79) and (5.80) for the height and Eq. (5.75) and (5.76) for the standard deviation.

5. *For uncertain height and uncertain mean.* In a like manner to the arrangement established in the case 4, the equations that can be used for parameter identification are (5.69) and (5.70) for m_k^i, (5.71) for σ_i and (5.79) and (5.80) for h_i. Where 't' is the iteration number and the performance index 'Pi' is monitored by the adaptation algorithm which is defined as follows:

- if $Pi(t+1) \geq Pi(t)$ Then

$$\alpha(t+1) = h_d \alpha(t),\ \gamma(t+1) = 0$$

- if $Pi(t+1) < Pi(t)$ and $\left|\dfrac{\Delta Pi}{Pi(t)}\right| < \delta$ Then

$$\alpha(t+1) = h_i \alpha(t),\ \gamma(t+1) = \gamma_0 \tag{5.38}$$

- if $Pi(t+1) < Pi(t)$ and $\left|\dfrac{\Delta Pi}{Pi(t)}\right| \geq \delta$ Then

$$\alpha(t+1) = \alpha(t),\ \gamma(t+1) = \gamma(t)$$

where h_d and h_i are the decreasing and increasing factor, respectively, and δ is the threshold for the rate of the relative index based on the Root-Mean-Square Error (RMSE). Hence, the following conditions must be involved:

$$0 < h_d < 1, \quad h_i > 1 \tag{5.39}$$

5.5 SIMULATION RESULTS

To illustrate the benefits of Type-2 FS in processing linguistic uncertainty, this section is devoted to compare the performance of the proposed IT2-RBF-FNN and the T1-RBF-NN for three different example simulations. The first data set is the Iris plant database [Fisher, 1936], which is perhaps one of the most popular benchmarking datasets in pattern recognition. The second simulation uses the E.coli data set, which has been used with expert systems for the prediction of Cellular Localisation sites [Horton and Nakai, 1996, Nakai and Kanehisa, 1991]. And finally, the last case study under simulation is the predictive modelling of the Charpy Toughness of heat-treated steel; a manufacturing process that exhibits very high uncertainty in the measurements due to the thermomechanical complexity of the Charpy test itself [Panoutsos and Mahfouf, 2010b, Solis and Panoutsos, 2013]. The way the IT2-RBF-FNN is implemented in this chapter will be established according to the problem. The rest of this section is divided depending on the variable to be tuned, that is 1) the first two experimental simulations for classifying the Iris data set and for the cellular localisation sites prediction, the variables proposed to be tuned are the uncertain mean and the uncertain standard deviation, while the last study case; the five possible configurations proposed in this work will be test and whose acronyms for representing them are:

- Uncertain mean, IT2-RBF-FNN-(M).
- Uncertain standard deviation, IT2-RBFNN-(SD).
- Uncertain height, IT2-RBF-FNN-(H).
- Uncertain height and uncertain standard deviation, IT2-RBF-FNN-(H-SD).
- Uncertain height and uncertain, mean IT2-RBF-FNN-(H-M).

5.5.1 EXAMPLE 1: IRIS PLANT CLASSIFICATION

This example employs the proposed IT2-RBF-FNN and its type-1 counterpart in order to model the Iris plant database which was created by R.A. Fisher [Fisher, 1936]. The data set contains three main categories, namely; a) Iris Setosa, b) Iris Versicolour and c) Iris Virginica of 50 instances each, where each category refers to a type of an iris plant and whose main classification feature is that one category is linearly separable from the two others and the latter are non linearly separable each other. The parameter identification of the IT2-RBF-FNN-(M), IT2-RBF-FNN-(SD) and T1-RBF-NN comprised a training process by means the proposed adaptive-BEP described in the appendix and its corresponding validation by means of a testing stage. Five different simulations were carried out whose initial data used for training both models consist of 105 (70%) and 45 (30%) for testing which were selected randomly. In training the IT2-RBF-FNN and its type-1 counterpart, 1300 epochs were used each of which has 105 time steps where there is no repetition in these 105 training data. It was also considered using the same number of parameters and rules for all the models in order to fairly evaluate their performance under the same simulation conditions.

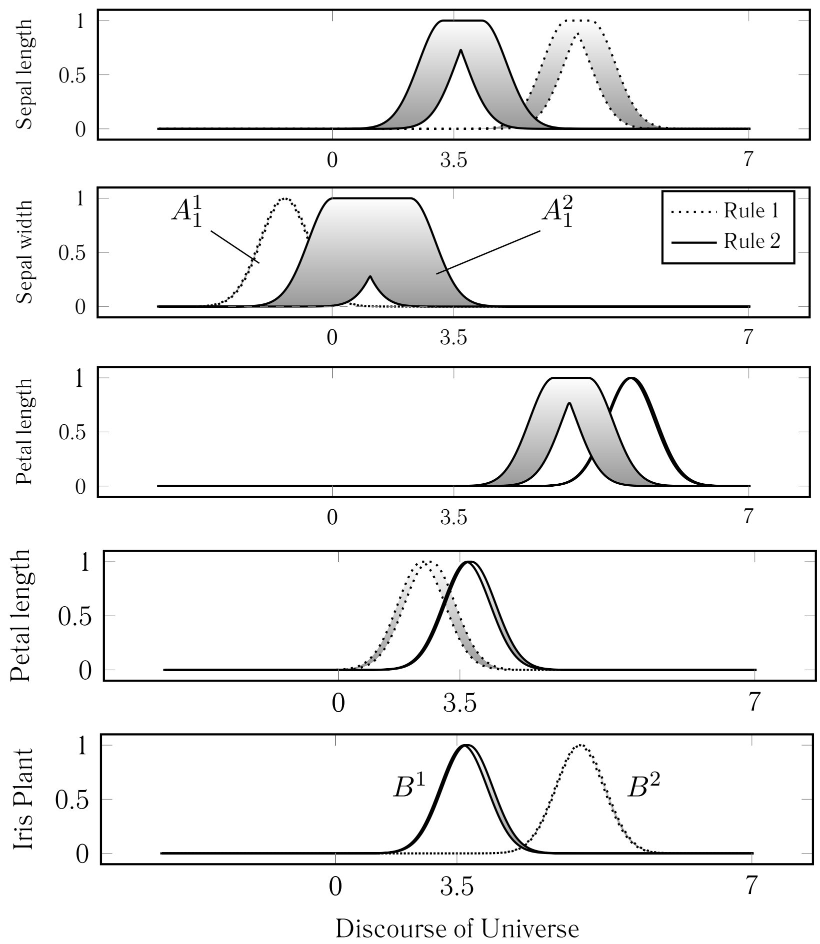


Fig. 5.6 Interval fuzzy rule example (Iris Classification with IT2-RBF-FNN-(M))

An example of the fuzzy rule base is illustrated in Fig. 5.6 by using an IT2-RBF-NN with an uncertain mean. Hence one rule for the IT2-RBF-FNN with uncertain mean (M) and uncertain standard deviation (SD) can be stated as:

$$\widetilde{R}^i : \text{IF } x_1 \; is \; \widetilde{A}_1^i \; and \; x_2 \; is \; \widetilde{A}_2^i \ldots \; and \; x_n \; is \; \widetilde{A}_n^i \text{THEN } y \; is \; \widetilde{B}^i \tag{5.40}$$

$$\widetilde{R}^1 : \text{IF Sepal length is } \widetilde{A}_1^1 \text{ and Sepal width is } \widetilde{A}_2^1 \text{ and Petal length is } \widetilde{A}_3^1 \ldots$$
$$\ldots\text{and Petal width is } \widetilde{A}_4^1 \text{ THEN the Iris Plant is } \widetilde{B}^1 \quad (5.41)$$

In Fig. 5.7(a), the initial distribution of the universe of discourse for neuron 1 (from top to bottom, See IT2-RBF-FNN structure in Fig. 5.2, section 5.3) is shown, as obtained via the data granulation algorithm.

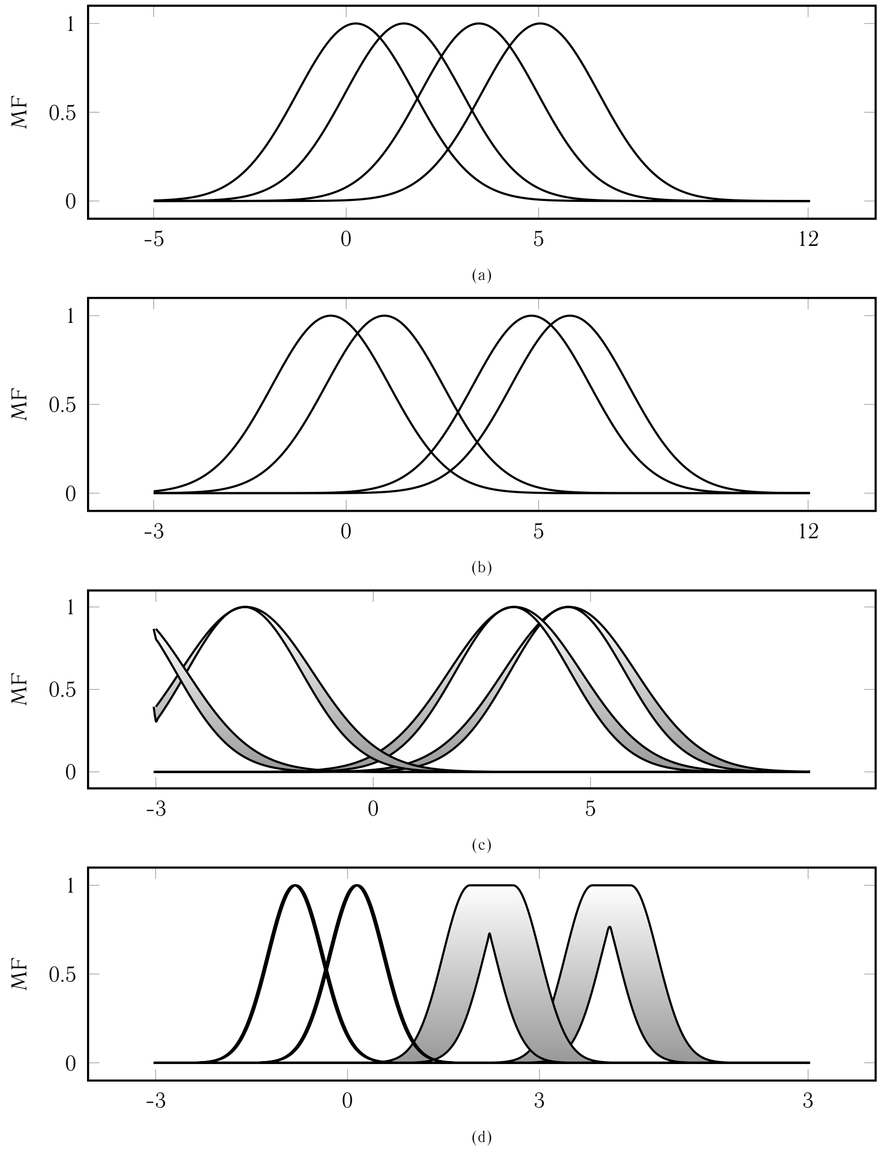


Fig. 5.7 (a) Initial and final distribution of the (b) T1-RBF-NN, (c) IT2-RBF-FNN-(SD) and (d) IT2-RBF-FNN-(M) for the simulation 1.

This distribution is used as the starting point for training the T1-RBF-NN and both the IT2-RBF-FNN-(M) and (SD) modelling structures. Even though, in Fig. 5.7(b), (c) and (d) the final shape of the distributions for the T1-RBF-NN and IT2-RBF-FNN are similar, the ability of T2-FS for dealing with linguistic uncertainty improved the performance of the neural network as shown in Fig. 5.8. Each result shown in table 5.1 (RMSE performance) for the IT2-RBF-FNN-(M) and IT2-RBF-FNN-(SD) is the average value of 5 different runs. The results show that the RMSE performance of the IT2-RBF-FNN is better than that of the T1-RBF-NN. The results are presented on different size models, two different cases have been considered, one with just three rules, and one with five rules.

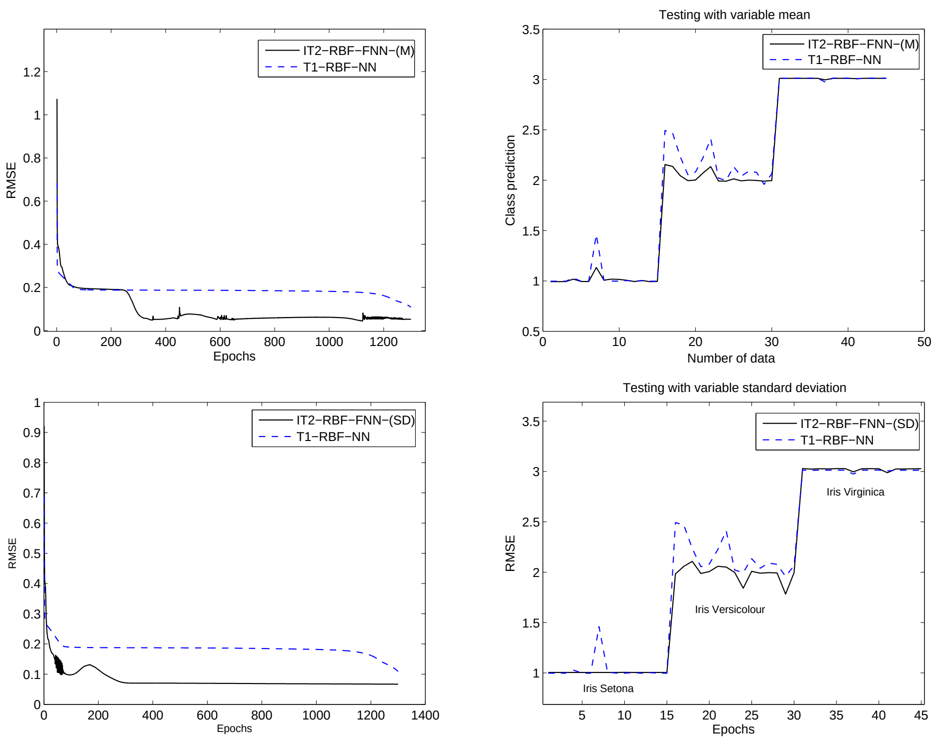


Fig. 5.8 Training RMSE and Testing performance for the simulation 1 with 3 rules and uncertain standard deviation

Table 5.1 Performance of the T1-RBF-NN and IT2-RBF-FNN models for the Iris Database prediction in example 1

Models	T1-RBF-NN	IT2-RBF-NN-(M)	IT2-RBF-FNN-(SD)	T1-RBF-NN	IT2-RBF-F NN-(M)	IT2-RBF-FNN-(SD)
number of rules	3	3	3	5	5	5
number of parameters	18	18	18	30	30	30
Training RMSE	0.1277	0.0962	0.0891	0.1127	0.067	0.07200
Testing RMSE	0.1910	0.1092	0.1209	0.1410	0.082	0.094

Table 5.2 Performance of the T1-RBF-NN and IT2-RBF-NN models for the Cellular localisation sites prediction in example 2

Models	T1-RBF-NN	IT2-RBF-NN-(M)	IT2-RBF-FNN-(SD)	T1-RBF-NN	IT2-RBF-NN-(M)	IT2-RBF-NN-(SD)
Clustering methodology	Fuzzy C-Means			Data granulation		
number of rules	5	5	5	5	5	5
number of parameters	30			30		
Training RMSE	0.1489	0.1210	0.1191	0.1219	0.051	0.0920
Testing RMSE	0.1612	0.1591	0.1430	0.1400	0.087	0.1002

Table 5.1 and 5.2 show the experimental results obtained in relation to Iris data classification and Ecoli Data set approximation respectively. The latter are presented in the next section - both experimental simulation applied the proposed IT2-RBF-FNN with an uncertain standard deviation (IT2-RBF-FNN-SD) and an uncertain means (IT2-RBF-FNN-M). On the one hand, in table 5.1 the IT2-RBF-FNN utilised granulation with 3 and 5 fuzzy granules as the initial parameter identification approach. On the other hand, in table 5.2 the initial parameter identification was carried out with two different clustering approaches, namely: a) Fuzzy C-Means (FCM) and b) granulation.

5.5.2 EXAMPLE 2: ECOLI DATA SET CLASSIFICATION

The objective of this simulation is the prediction of the cellular localisation sites of the E.coli proteins [Nakai and Kanehisa, 1991]. Proteins from E.coli data set are classified into 8 classes with 8 attributes each. The attribute information of the cellular sites are signal sequence recognition methods (particularly those of McGeoch and von Heijne) [Nakai and Kanehisa, 1991], the presence of charge of N-terminus of predicted lipoproteins and 3 different scoring functions on the amino acid contents used for predicting if such information is inner or outer membrane, cleavable or uncleavable and sequence signal. According to [Nakai and Kanehisa, 1991] and for statistical purposes, 336 observations were obtained of which we carried out 5 different simulations (different data arrangements) with 202 (70%) data for training and 134 (30%) for testing which were selected randomly for each simulation. This example compares the performance of the proposed hybrid learning methodology by using the FCM and GrC with the same fixed learning rate and without the adaptive momentum term. For comparison purposes, table 5.2 provides the average RMSE of five different runs of the data set for IT2-RBF-FNN-(SD), (M) and the T1-RBF-NN with 5 rules for training and testing. Fig. 5.9 shows the actual predicted output of the IT2-RBF-FNN-(M) and the IT2-RBF-FNN-(SD) of the first simulation for class identification of the localisation site of the proteins.

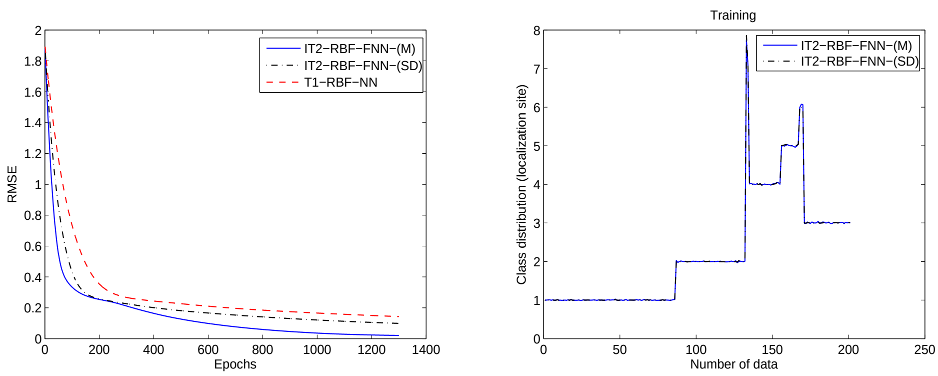


Fig. 5.9 Performance of the IT2-RBF-FNN using data granulation with 5 rules for the simulation number 1.

It can be concluded from the results shown in Table 5.2 that in general the IT2-RBF-FNN outperforms its type-I equivalent system, while the data granulation algorithm provides better quality granules/rules that are easier to optimise as compared to the FCM algorithm for setting the initial rule base of the system.

5.5.3 EXAMPLE 3: MECHANICAL PROPERTY PREDICTION OF HEAT TREATED STEEL

This example is used to verify the effectiveness of the proposed IT2-RBF-FNN over a real industrial case study. The example consists of a data set related to the Impact Energy Test of Heat treated grade steel described deeply in chapter 4. Particularly, impact energy is a highly non-linear property in relation to the steel composition, and difficult to be modelled. The Charpy toughness data set used in this section in a like manner to those experimental results presented throughout this research work consists of 1661 measurements on heat-treated steel (TATA Steel, Yorkshire, UK). The data set has 16 input dimensions, and 1 output (Impact Energy, Joules) and the chemical composition, test parameters and heat treatment conditions are described in table 4.1, chapter 4. For cross-validation, the data have been split into training, checking and testing data sets, in order to avoid overfitting and hence enhancing the generalisation properties when modelling the Charpy test. The data used to train the IT2-RBF model consists of 1084 (65%), which are composed of just raw data. The checking and testing data are 277 (17%) and 300 (18%) respectively. The selection of Data was set to identically match the data set used in [Solis and Panoutsos, 2013] and [Panoutsos and Mahfouf, 2010b] for comparison purposes. However, the granular approach employed in this chapter does not consider the uncertainty used for improving the distinguishability of the universe of discourse. The proposed architecture is capable of extracting knowledge from data and providing an interval linguistic representation which can lead to a computing with words (CWW) framework.

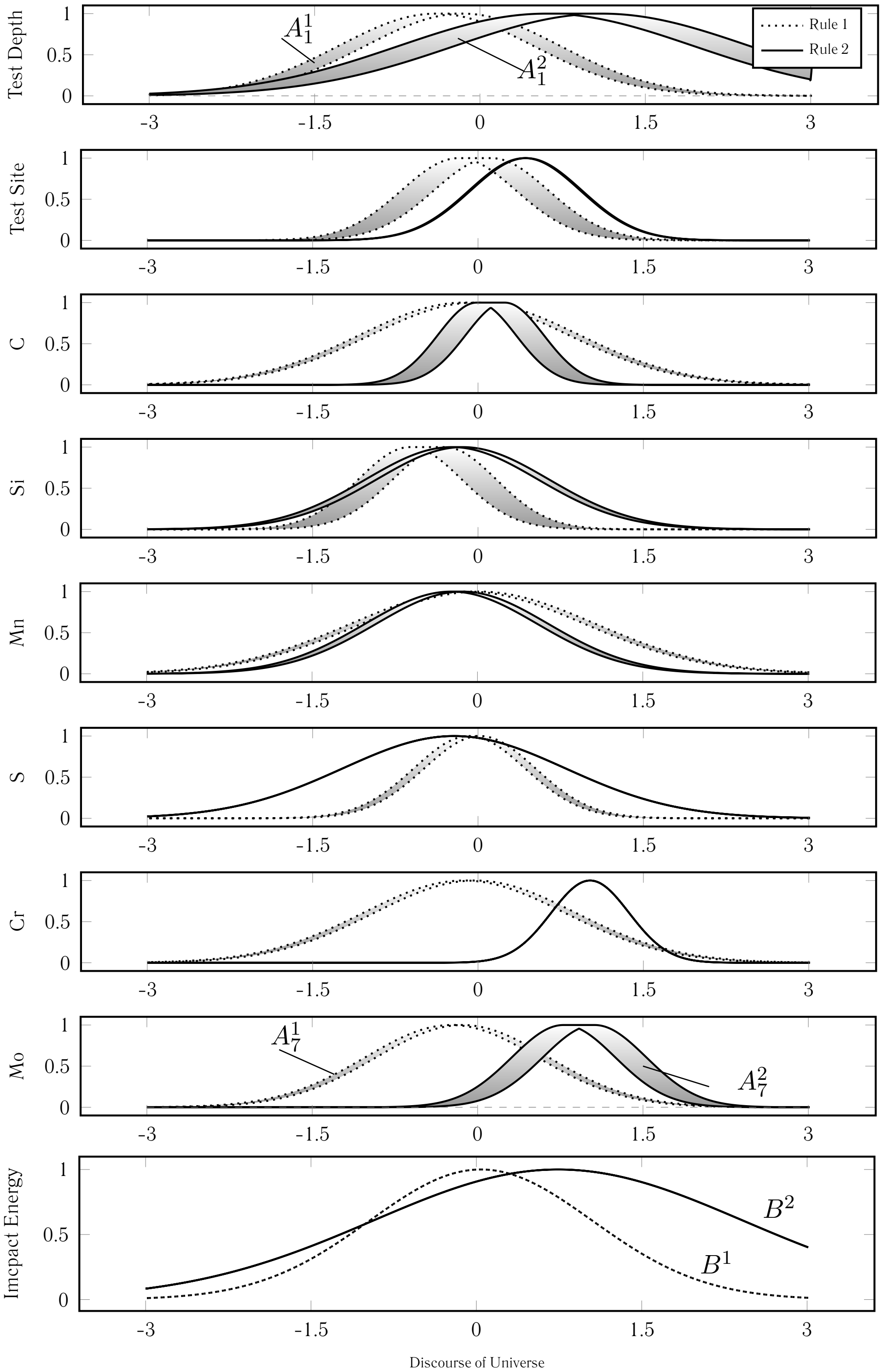


Fig. 5.10 Interval fuzzy rule example (Impact Energy modelling-IT2-RBF-FNN).

The proposed network also offers a good level of interpretability and transparency by using expert knowledge of the physical process while the preservation of a good level of generalisation is assured. Furthermore, the learning technique used here shows a faster convergence to a better solution as a consequence of an enhanced construction of the interval fuzzy rules in comparison to its type-1 counterpart. The application of the IT2-RBF-NN let us to model uncertainties that are not possible in type-1 fuzzy systems. Part of the linguistic rule base is shown as an example in Eq. 5.43 and Fig. 5.10 which illustrates 8 out of the 16 input variables with a 2-rules comparison and an uncertain mean after the optimisation. It is also worth noting that the rule base is represented not only by type-2 fuzzy sets but also by type-1 sets which are classified as $\underline{f} = \bar{f}$. An interval type-2 singleton rule can be stated as:

$$\widetilde{R}^i : \text{IF } x_1 \; is \; \widetilde{A}_1^i \; and \; x_2 \; is \; \widetilde{A}_2^i \ldots \; and \; x_n \; is \; \widetilde{A}_n^i \quad \text{THEN } y \; is \; \widetilde{B}^i \tag{5.42}$$

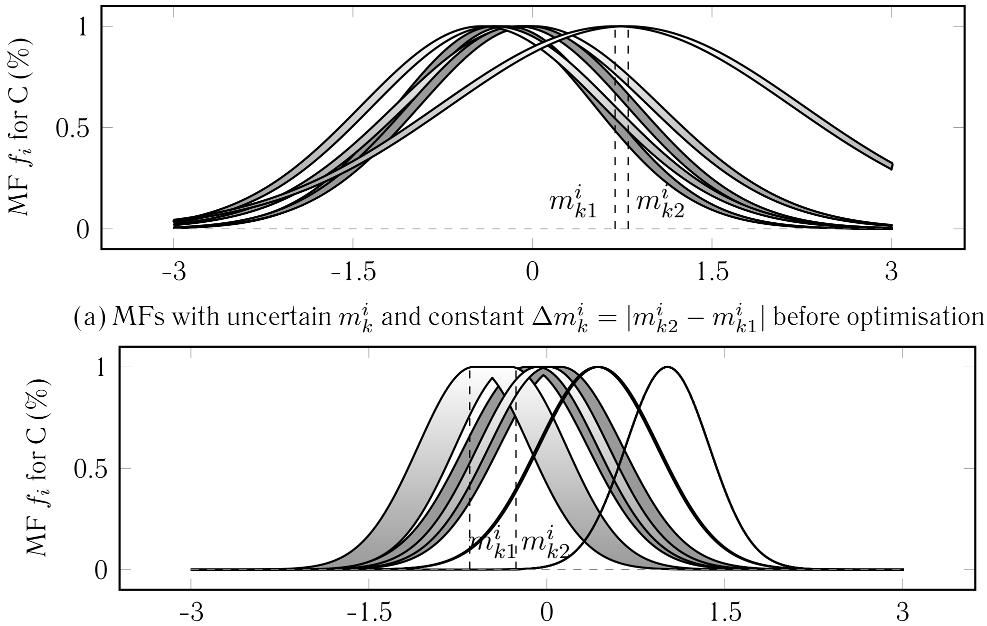
Where a rule for the IT2-RBF-NN just taking into account 8 out of the 16 input variables can be stated as:

$$\widetilde{R}^1\text{: IF } Testdepth \text{ is } \widetilde{A}_1^1 \text{ and } Test\ site \text{ is } \widetilde{A}_2^1 \text{ and } C \text{ is } \widetilde{A}_3^1 \text{ and } Si \text{ is } \widetilde{A}_4^1 \text{ and } Mn \text{ is } \widetilde{A}_5^1 \text{ and } S \text{ is } \widetilde{A}_6^1 \text{ and } Cr \text{ is } \widetilde{A}_7^1 \text{ and } Mo \text{ is } \widetilde{A}_8^1 \text{ and } Ni \text{ is } \widetilde{A}_9^1 \ldots$$

$$\ldots\text{THEN the Impact Energy is } \widetilde{B}^1 \tag{5.43}$$

5.5.4 SIMULATION RESULTS BY USING UNCERTAIN MEAN, IT2-RBF-FNN-M

This sections presents the simulation results obtained by using the IT2-RBF-NN whose MFs are with a) uncertain mean $[m_{k1}^i, m_{k2}^i]$ and a fixed standard deviation σ_i. On the one hand, as illustrated in the Fig. 5.11(a) the initial difference $\Delta m_k^i = |m_{k2}^i - m_{k1}^i|$ was set to be constant for all the interval MFs whose location is obtained from the data granulation.



(a) MFs with uncertain m^i_k and constant $\Delta m^i_k = |m^i_{k2} - m^i_{k1}|$ before optimisation.

(b) Optimised MFs with uncertain m^i_k.

Fig. 5.11 (a) Initial and (b) final distribution of MFs with 'uncertain mean' - for simplicity showing 5 out of 9 IT2 fuzzy MFs

On the other hand, Fig. 5.11 shows the optimised shape (after optimisation) of the MFs along 'C' dimension. Particularly, the experimental results illustrated in Fig. 5.12 and obtained by using the IT2-RBF-FNN-(M) show the benefits of the application of IT2-FSs since the linguistic representation leads to a faster parameter identification of the proposed architecture reducing the number of training steps (See RMSE). It is evident as well that the modelling performance was enhanced. However, the IT2-RBF-FNN-(M) model is not able to predict correctly some scatter data due to the nature of the data (statistically similar), certain degree of redundancy among the fuzzy rules and the low repeatability of the Charpy test. Such results confirm that the proposed IT2-RBF-FNN-(M) provides more degrees of freedom resulting in a more robust classifier both in training and generalisation properties.

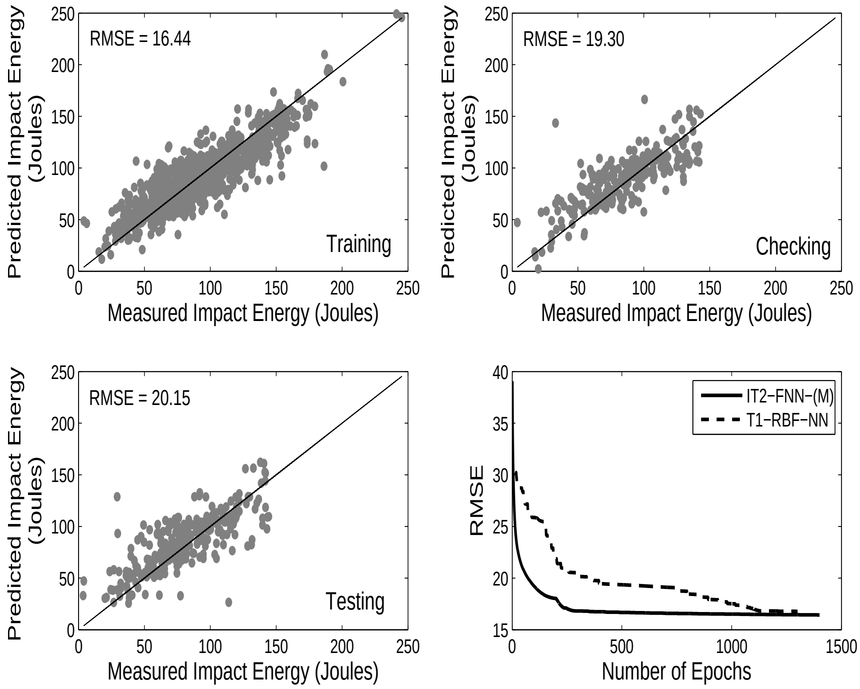
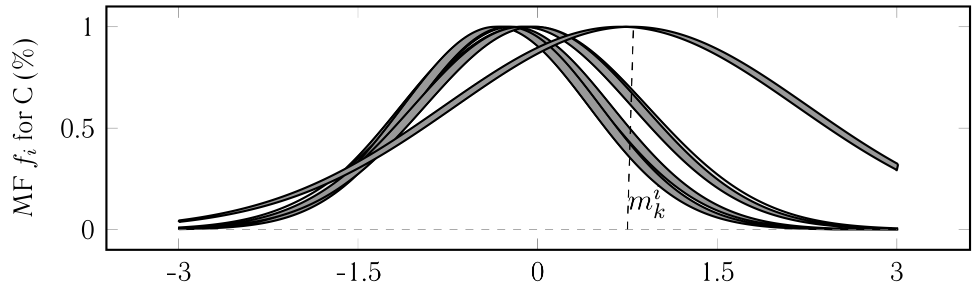


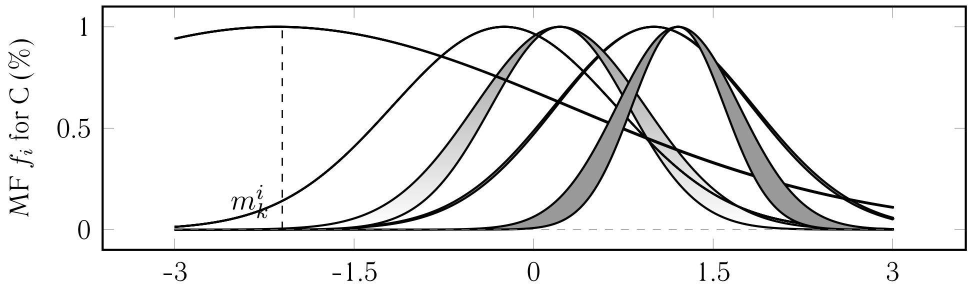
Fig. 5.12 Data fit-Impact Energy by using uncertain mean.

5.5.5 SIMULATION RESULTS BY USING UNCERTAIN STANDARD DEVIATION, IT2-RBF-FNN-(SD)

In a like manner to those experimental results obtained with the IT2-RBF-NN-(M), in this section the results obtained by using the configuration of the IT2-RBF-NN with a variable SD are displayed in Fig. 5.13. From Fig. 5.11(a) and Fig. 5.13(a) it is possible to observe that the initial MFs share an identical distribution with different parameters. This is because the initial MFs parameters are similarly obtained by using the data granulation, however the posterior optimisation of the a) variable standard deviation $[\sigma_1^i, \sigma_1^i]$, b) the mean m_k^i and c) the output weights defined in the interval $[w_l^i, w_r^i]$ by using an uncertain standard deviation leads the MFs to a more parsimonious universe of discourse as illustrated in Fig. 5.13(b).



(a) Optimised membership functions with uncertain s_i.



(b) Optimised membership functions with uncertain s_i.

Fig. 5.13 (a) Initial and (b) final distribution of MFs with 'uncertain standard deviation' - for simplicity showing 5 out of 9 IT2 fuzzy MFs

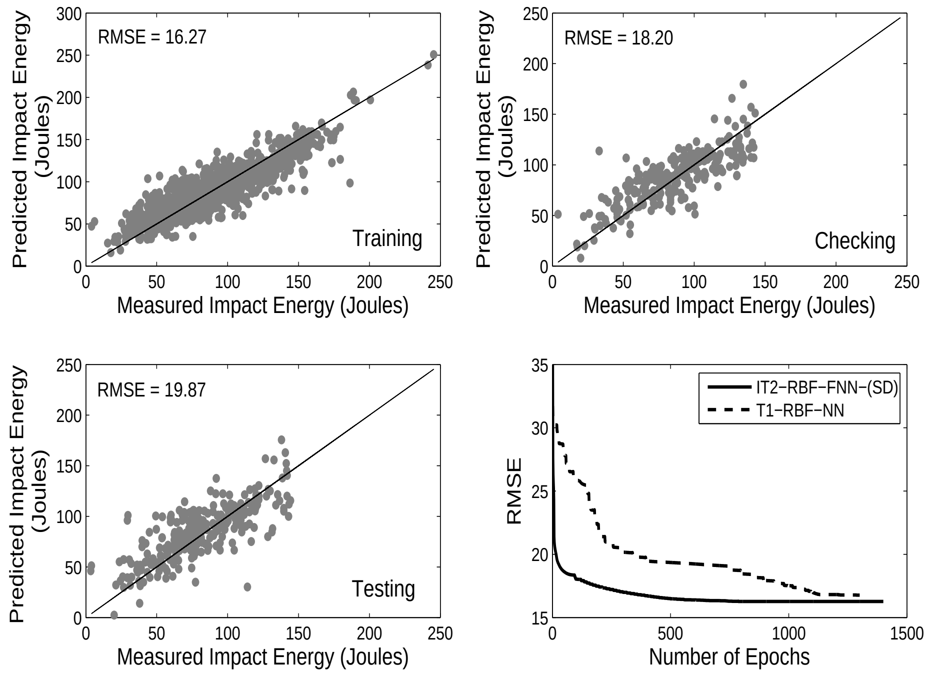


Fig. 5.14 Data fit-Impact Energy by using uncertain standard deviation.

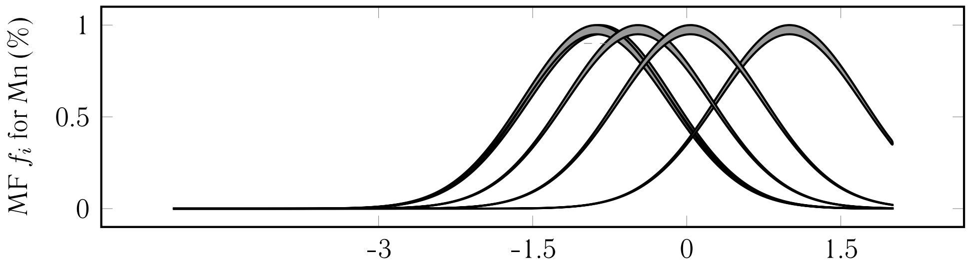
From Fig. 5.13(b), it can be also noticed that the newly optimised rules are more distinguishable than those initially provided by the data granulation process illustrated in Fig. 5.13(a), and it is clear from Fig. 5.14 that the results obtained by the proposed IT2-RBF-FNN-(SD) outperformed the T1-RBF-NN and IT2-RBF-FNN-(M). Nevertheless, as it is also illustrated in Fig. 5.14 the proposed model suffers from the same lack of ability to predict scatter data.

5.5.6 SIMULATION RESULTS BY USING UNCERTAIN HEIGHT, IT2-RBF-FNN-(H)

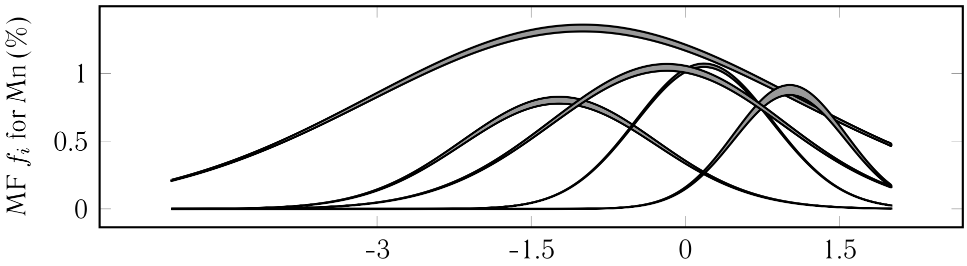
This section is devoted to examine those experimental results obtained by using the IT2-RBF-NN model with an uncertain height $h_i \in [h_i^1, h_i^2]$, a fixed mean m_i and a fixed standard deviation σ_i. The initial parameters of the IT2-RBF-NN structure were the final geometrical properties of the final granules with a coefficient $\alpha = 0.35$. Some constraints are necessary for the optimization of the IT2-RBFNN parameters in order to avoid σ_i and h_i having negative values.

$$0.4 < h_i^1, h_i^2 < 5.0 \tag{5.44}$$

Firstly, in Fig. 5.15 (a) and 5.15(b) the initial and final shape of 5 out of 9 fuzzy rules at dimension 'Mn' is illustrated, where the initial difference $h_i^1 - h_i^2 = 0.05$ and the initial values of the free parameters h_i, m_k^i and σ_i are obtained from the granulation process. Secondly, Fig. 5.16 shows the experimental results obtained for nonlinear identification of the given impact energy data set. Even the process of training is performed for 1400 iterations, the final parameters used for testing the proposed architecture are those found when the checking evaluation stops decreasing. Especially those results shown in Fig. 5.16 also demonstrate the ability of the proposed IT2-RBF-NN-(H) for quickly defining the fuzzy linguistic rules - as it is depicted by the RMSE plot.



(a) Initial shape of the MFs with uncertain height (h_i).



(b) Optimised membership functions with uncertain height (h_i).

Fig. 5.15 (a) Initial and (b) final distribution of MFs with 'uncertain height' - for simplicity showing 5 out of 9 IT2 fuzzy MFs

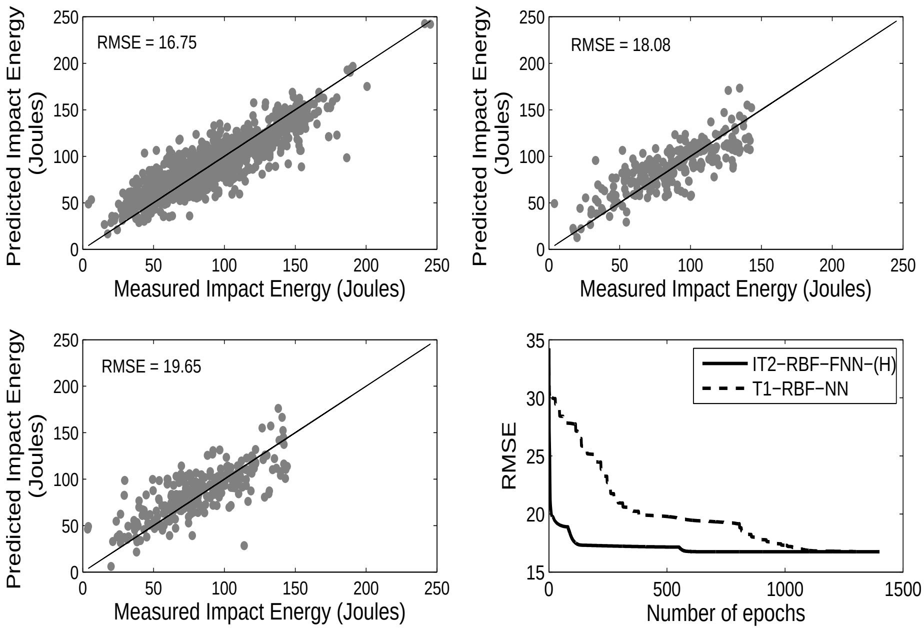
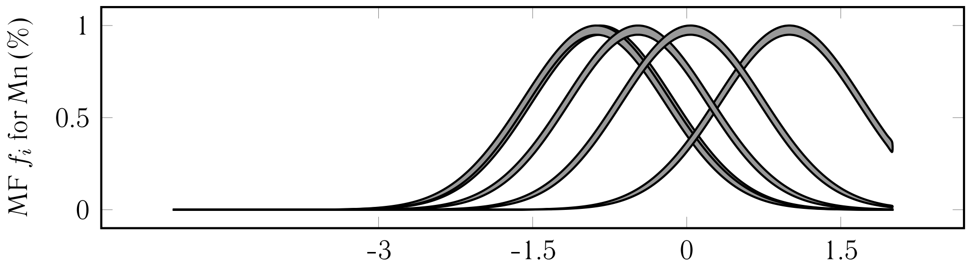


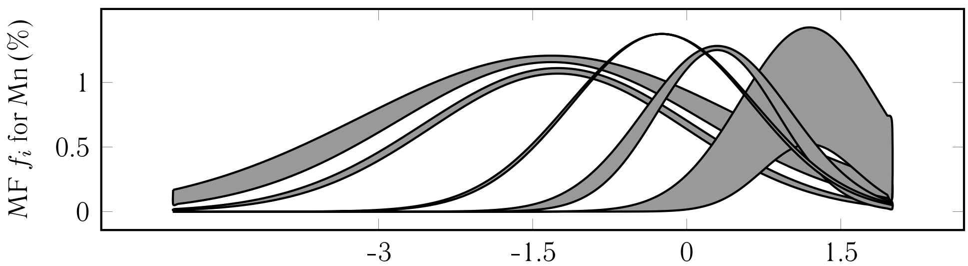
Fig. 5.16 Data fit-Impact Energy by using uncertain height h_i.

5.5.7 SIMULATION RESULTS BY USING UNCERTAIN HEIGHT AND UNCERTAIN STANDARD DEVIATION, IT2-RBF-FNN-(H-SD)

This section presents those results obtained by varying the height and the deviation of the MFs. The initial parameters were identically set up to those initial values used in the sections 5.5.5 and 5.5.6. For example, Fig. 5.17(a) illustrates the initial shape of the MFs which are quite similar to those initial MFs over the dimension 'Mn' described in 5.5.6, and here the 'linguistic dimension' Mn is used as well as an illustrative example. It is clear from Fig. 5.17, the results in somehow are similar to the final distribution described by the results obtained in Fig. 5.11 and Fig. 5.15. However, the difference between the lower and the higher MF is bigger and this combination of having an uncertain height and an uncertain standard deviatios has enhanced the performance of the proposed IT2-RBF-FNN-(H-SD) architecture. From Fig. 5.18 it is depicted the performance of the proposed IT2-RBF-FNN-(H) which outperformed the results obtained by using the T1-RBF-NN.



(a) Initial shape of the MFs with uncertain h_i and uncertain σ_i.



(b) Optimised MFs with uncertain h_i and uncertain σ_i.

Fig. 5.17 (a) Initial and (b) final distribution of MFs with uncertain h_i and uncertain σ_i - for simplicity showing 5 out of 9 IT2 fuzzy MFs

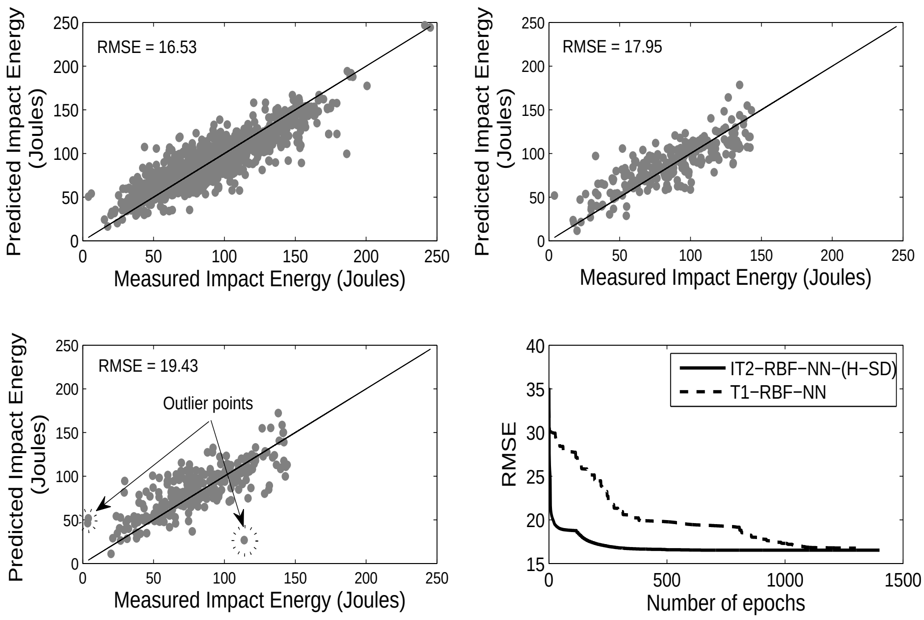


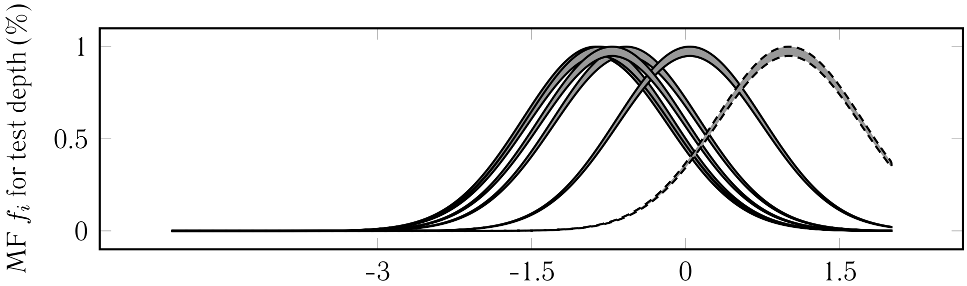
Fig. 5.18 Data fit-Impact Energy by using uncertain height h_i and uncertain mean σ_i.

Particularly a significant improvement in generalisation properties and a faster identification of the linguistic rule base parameters was achieved. In other words, the procedure of non-linear identification carried out by means the IT2-RBF-FNN-(H-SD) structure favours a better classification of most of the outlier points produced when using the IT2-RBF-FNN-(M) and IT2-RBF-FNN-(SD) as is illustrated in Fig. 5.18. Following the order for training, checking and testing figures, the three outlier points at the testing stage in Fig. 5.18 are supposed to be classified with an impact energy value 4.07, 5.07 and 112.10 Joules, but their corresponding statistical properties are more similar to those points categorised within the impact energy range between 30-50 Joules. In general, this network is able of achieving a good balance between training and checking while preserving a good level of generalisation. However, it would be worth proposing as a further study for the development of a clustering approach which can provide interval fuzzy sets as the initial parameters for the cross-validation procedure.

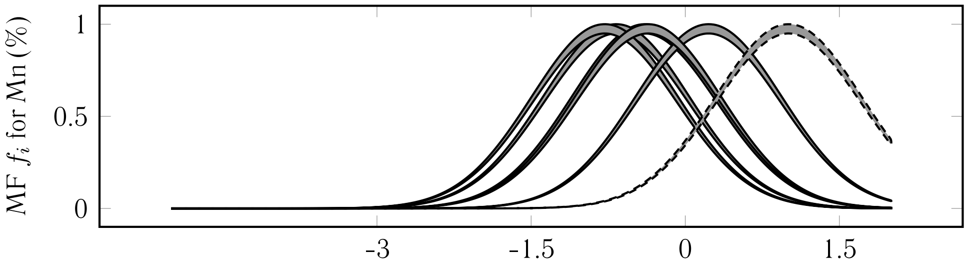
5.5.8 SIMULATION RESULTS BY USING UNCERTAIN HEIGHT AND UNCERTAIN MEAN, IT2-RBF-FNN-(H-M)

This chapter has been concerned on the development of transparent models by the use of interval type-2 fuzzy sets. Moreover, the associated parameter identification procedure for the IT2-RBF model including the different configurations was developed. The main idea behind the use of neural-fuzzy modelling lies on the concept for the quest of more accurate, user-friendly and intelligent models. Such models must be designed under the idea of transparency as a consequence of elements that are meaningful to the user. In other words, a well-defined semantic of the information is essential when designing computing with words systems and user-centric models. For example, in Fig. 5.19 is offered a representation of two out of the sixteen inputs, namely: test depth and Mn ($\%Mn$) dimensions where interestingly varying the height and the value of the interval centre of the Gaussian MFs, the IT2-RBFNN captures the capacity of the fuzzy systems to characterise the domain of knowledge and the relationship among fuzzy rules in terms of the language of logic dependencies. This means it is possible to reflect the ability of fuzzy systems to create rule-based systems that imply a certain level of accuracy and rules of higher generality when modelling high-dimensional systems (e.g. manufacturing systems).

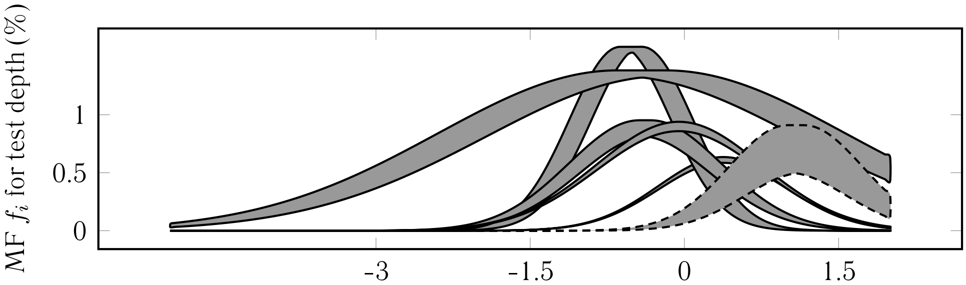
It is clear form Fig. 5.19 (c) and (d) that when $|m_k^1 - m_k^2| \rightarrow 0$ the MF behaves as a word expressed just in terms of its associated height and hence the properties contained in a MF defined by using an uncertain mean disappear. The purpose of the IT2-RBF-NN model is achieved since the information contained in the receptive units reveals associations between fuzzy sets that defined the linguistic input-output space. In Fig. 5.20, the experimental simulations by using the proposed IT2-RBF-FNN-(H=M) for predicting the impact energy in terms of words are illustrated. Even the training performance is not as good as that obtained by just using the T1-RBF-NN and the previous configurations of the IT2-RBF-FNN, the checking and testing results are comparable to such models, even better than some of them.



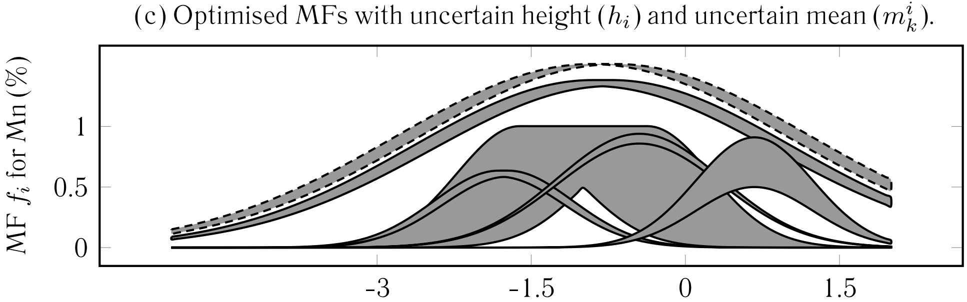
(a) Initial shape of the MFs with uncertain height (h_i) and uncertain mean (m_k^i).



(b) Initial shape of the MFs with uncertain height (h_i) and uncertain mean (m_k^i).



(c) Optimised MFs with uncertain height (h_i) and uncertain mean (m_k^i).



(d) Optimised MFs with uncertain height (h_i) and uncertain mean (m_k^i).

Fig. 5.19 (a) Initial and (b) final distribution of MFs with 'uncertain height h_i and uncertain mean m_k^i' - for simplicity showing 5 out of 9 IT2 fuzzy MFs

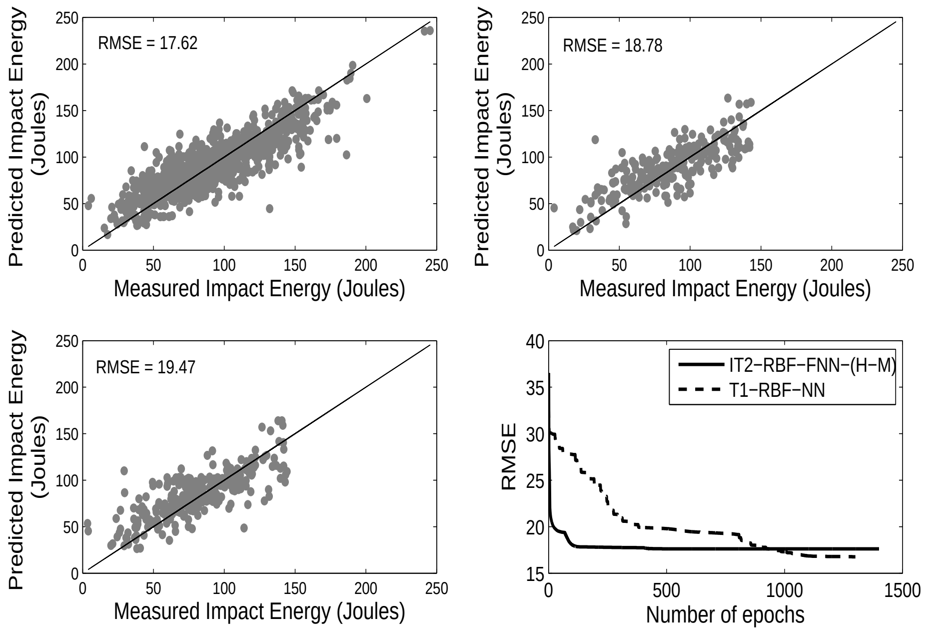


Fig. 5.20 Data fit-Impact Energy by using uncertain height (h_i) and uncertain mean (m_k^i).

5.5.9 COMPARISON ANALYSIS FOR IMPACT ENERGY MODELLING RESULTS

From the experimental results presented in the previous sections, it is clear that the proposed methodology outperformed its type-1 counterpart mainly in generalisation terms. Also the IT2-RBF-NN proved its efficiency and high accuracy for fitting data particularly by adapting the value of σ_i and the corresponding h_i.

In practical terms, the results obtained by using an uncertain height demonstrated that fuzzy logic can be evaluated over an interval $[0, 1^+[$. A further study in this direction can be done within the theory of neutrosophic sets. Table 5.3 shows 5 different types of modelling results obtained by using the proposed structure and the type-1 RBF-NN.

Table 5.3 Modelling performance

Model	*Number of parameters*	*training*	*checking*	*testing*
T1-RBF-NN	162	16.76	19.25	20.91
IT2-RBF-FNN-(M)	162	16.44	19.30	20.15
IT2-RBF-FNN-(SD)	162	16.27	18.20	19.87
IT2-RBF-FNN-(H)	162	16.75	18.08	19.65
IT2-RBF-FNN-(H-SD)	162	16.53	17.95	19.43
IT2-RBF-FNN-(H-M)	162	17.62	18.78	19.47

All the experimental simulation were carried out by employing an adaptive BEP approach and its corresponding version developed for identifying the IT2-RBF-FNN parameters. Even such results depicted similar behaviours, the initial value of the learning rate α and the output layer weights $[w_l^i, w_r^i]$ per experiment was different.

5.6 SUMMARY

In this chapter a new data-driven IT2 Fuzzy Logic modelling framework, which is based for the first time on a Radial Basis Function - Neural Network is presented. The good performance of IT2-FLS as opposed to their T1 equivalent is known, as well as the vast array of T1-RBF-NN-based implementations, which offer functional equivalence to T1-FLS, universal approximation capability and a plethora of clustering and parametric optimisation methodologies that help optimise the linguistic rule base. The presented IT2-RBF-FNN outperforms its T1 equivalent T1-RBF-NN counterpart and also maintains its functional equivalence to a T2-FLS. Furthermore, a systematic approach for capturing knowledge out of raw data sets via a GrC-based framework and use this information to define an equivalent footprint of uncertainty is used, and then it is optimised as a whole IT2-FLS via an adaptive-BP approach.

The proposed methodology is tested against three case studies, which include two benchmark problems and one real industrial case study that poses particular challenges in terms of uncertainty and data scarcity. In each case study we demonstrate the results of the proposed IT2-RBF-FNN with two different implementations, one with a variable mean and one with a variable standard deviation. In all three cases the IT2-FLS outperforms its T1 equivalent, which is in line with previous results from other authors in non-T1-RBF-NN fuzzy logic structures. Furthermore, the uncertain standard deviation implementation seems to outperform the uncertain mean in every case. Absolute raw performance however, on this occasion, was not the main goal of the proposed structure; it is expected that the use of alternative optimisation techniques (parametric and/or structural) may provide an even better overall result. The main contribution of this work is the creation of the RBF-NN-based implementation of an IT2-FLS, and its direct comparison with a T1-FLS equivalent structure. This new implementation also opens up the potential for other researchers in the field, who already work with the popular RBF implementations of T1-FLS to try the proposed IT2 structure. As a further conclusion to the presented methodology, the granular computing framework provides an almost intuitive way of automatically setting the footprint of uncertainty of IT2-FLS. Therefore a systematic and automatic methodology that can be used (even beyond T1-RBF-NN) to capture knowledge from raw data and use this knowledge to establish the FOU of IT2-FLS was created.

6 STUDIES FOR UNCERTAINTY ASSESSMENT IN THE RBF-NN AND THE IT2-RBF-NN.

A twofold study at the low level of interpretability and high-level of interpretability of the RBF-NN in order to quantify fuzzy uncertainty is provided. The first part of this study consists in the development of a methodology based on neutrosophic sets for the evaluation of vagueness among the fuzzy rules by using an overlapping coefficient throughout the parameter optimisation stage. Consequently, an index is proposed to evaluate the ambiguity associated with one-many-relations when making decisions during the parameter identification process. Secondly, the last part of the study provides a methodology for quantifying ambiguity, fuzziness and entropy that is produced due to the resulting redundancy in the fuzzy rule base at each iteration of the parameter identification process of the RBF-NN and the IT2-RBFNN. This information analysis might be employed for enhancing both the low and high-level of interpretability of the RBF-NN and the IT2-RBF-NN.

6.1 INTRODUCTION

In fuzzy rule-based systems, interpretability is assumed to be a natural property [Alcalá et al., 2006, Casillas, 2003, Jin, 2000, Johansen and Babuska, 2003, Mencar et al., 2007b, Mikut et al., 2005]- interpretable intelligent systems are always desired for applications in a wide range of areas such as medicine, robotics, control, economics, etc. Moreover the readability and comprehensibility are crucial for the construction of fuzzy systems capable of explaining humanistic systems (i.e. systems whose behaviour is strongly influenced by human judgement, perception or emotions[Zadeh, 1975]).

A vast number of different efforts have been made for the development of linguistically interpretable neural fuzzy models from data, i.e. neural systems capable of representing fuzzy systems that preserve meaningful features such as interpretability, transparency and then distinguishability [Alcalá et al., 2007, Cpałka et al., 2014, Łapa et al., 2014, Lughofer, 2013, Mencar et al., 2011]. That means the extraction of information in a transparent way is a cornerstone for parameter identification of neural fuzzy systems for representing input-output data samples.

As mentioned by [Paiva and Dourado, 2004], transparency is a measure of linguistic interpretability of the rules issued from the training of a neural-fuzzy system. A lack of knowledge representation and interpretability is a common issue among neural-fuzzy systems mainly as a consequence of the training process utilised for parameter identification. Indeed, the interpretability is born as a natural property in the birth of fuzzy systems. An adequate balance between accuracy and interpretability is not an easy task as both abilities are affected when the complexity of the system increases. Zadeh pointed out in its principle of incompatibility [Zadeh, 1975], "As the complexity of a system increases, our ability to make precise and yet significant statements about its behaviour diminishes until a threshold is reached beyond which precision and significance (or relevance) become almost mutually exclusive characteristics".

Even transparency and interpretability are two properties closely related, it does not mean that both concepts match. In other words, the transparency of a fuzzy system can be considered as a measure to validate how interpretable is the fuzzy rule base [Riid and Rüstern, 2003]. Moreover, an improvement in the readability of fuzzy systems through the use of a moderate number of system variables, fuzzy sets, and the avoidance of constructing an inconsistent rule base has not a lot of in common with transparency.

Particularly, efforts on fuzzy modelling have been focused on increasing the interpretanbility and distinguishability of the rule base while maintaining a good modelling performance in systems design [Zhou and Gan, 2008]. For instance, in [Juang and Chen, 2012] a data-driven interval-type-2 neural fuzzy system with high learning accuracy and improved model interpretability is proposed. Juang and Chen built a type-2 fuzzy model whose design is twofold, i.e. (1) an initial clustering approach was used to generate accurate fuzzy rules with good accuracy and (2) a gradient descent and ruled-ordered recursive least square algorithms for learning the antecedent and consequent parameters of the proposed network. In [Rhee and Choi, 2007], Rhee and Choi proposed an off-line methodology based on interval type-2 fuzzy set theory for estimating the initial parameters of the RBF-NN. This work is shown to improve the classification performance and to control the linguistic uncertainty produced throughout the construction of the inference mechanism.

As it is described above, interpretability and accuracy is a pivotal element that must be considered when designing data-driven fuzzy models [Nauck et al., 1997, Paiva and Dourado, 2004]. The smallest number of aspects that must be considered throughout the construction of fuzzy models and especially neural fuzzy systems are [Guillaume, 2001]:

- The amount of fuzzy rules might be small enough to be understandable - according to [Bodenhofer and Bauer, 2003], it is advisable to exclude any rule weight or degrees of plausibility.

- Each rule represents an input-output model relationship (locally) and therefore the rules are consistent. That means, two or more similar rules lead to similar conclusions.

- The structural representation of the rule base is easy-to-interpret containing an small number of features (model inputs).

- The shape, parameters and mathematical expression of the MFs should be intuitively comprehensible.

- The inference engine should produced mathematically and linguistically correct consequences (model outputs).

In order to enhance the trade-off between interpretability and accuracy, some researchers have employed fuzzy uncertainty theory to quantify the behaviour of each component in a fuzzy model. For example, in [Wang et al., 2012], it was introduced a mechanism to quantify ambiguity associated to the construction of a fuzzy tree for modelling purposes. Such a methodology was able to measuring the fuzzy decision as the averaged classification ambiguity of the tree's root. Usually in fuzzy trees this kind of uncertainty is evaluated recursively from the leaf nodes to its root which means a higher consuming time. Alternatively, the authors proposed a novel mechanism based on ambiguity quantification to select from a large data set a reduced number of representative samples so as to minimise the adjustment of the fuzzy decision when adding samples to the training set. Because of this, the construction of the fuzzy tree was faster on the one hand, and it was just needed an small number of rules on the other hand.

However much of the work related to the RBF-NN concerns with function approximation [González et al., 2003, Oh et al., 2011, Park and Sandberg, 1991], fuzzy rule extraction [Sarimveis et al., 2002], and granular computing [Panoutsos and Mahfouf, 2010c] and so as not to achieved a good level of transparency and accuracy. The RBF-NN posses the characteristic of fuzzy sets that the RUs values can be defined in the interval $[0, 1]$ as the correlated truth of an event. In a like manner, the learning capabilities of the RBF-NN has some parametric flexibility that can be studied into other fields of fuzzy logic. In that case, for parameter identification purposes recent theories such as intuitionistic sets logic, interval type-2 fuzzy sets and neutrosophy might aid not only to quantify the associated uncertainty to the RBF-NN, but also to enhance its interpretability while preserving a good level of accuracy. Particularly, neutrosophy is a generalisation of fuzzy logic based on the fact that a proposition can be true (T), indeterminate (I) and false (F) - a tuple $< T, F, I >$ can be defined over the real domain with no restrictions. Besides, Neutrosophy is a branch of philosophy capable of dealing with prepositions which are true and false at the same time. This implies that during the parameter identification process (cross-validation) of the RBF-NN and the proposed IT2-RBF-NN, the associated uncertainty may be studied from different fuzzy perspectives. Under these circumstances, three major uncertainty frameworks can be exploited and hence applied to improve the understanding of the network. On the one hand, entropy and fuzziness (cognitive uncertainty) quantify the impurity of a crisp (real) set and the uncertainty transition area from one linguistic rule to another respectively [Wang et al., 2012]. On the other hand, ambiguity is another type of cognitive uncertainty that is produced as a result of choosing one from two or more alternatives [Hartley, 1928].

The scope of uncertainty theory in fuzzy logic is not limited just to entropy, fuzziness and ambiguity [Pal and Bezdek, 1994, Xiaoshu and Fanlun, 2000, Yager, 2002], but also to fuzzy relations [Yu et al., 2007] and approximate reasoning [Dubois and Prade, 1991] have been proposed. In the design of fuzzy systems, uncertainty appears due to the lack of information, and it mainly comes into three different disguises that covers the Probabilistic Uncertainty (PU), Resolutional uncertainty (Ru) and Fuzzy Uncertainty (FU) [Pal and Bezdek, 1994]. The first two types of uncertainty are closely related to belongingness of elements or events to crisp sets and the ambiguity of specifying the exact solution respectively.

In this chapter, the development of several experimental studies which are divided in two main sections is proposed, i.e. (1) the first section exploits and explores the functional equivalence established between the RBF-NN and Fuzzy Logic Systems of type-1 (FLS) so as to quantify the uncertainty. (2) the second section proposes the calculation of three measures of uncertainty based on their relationship to the redundancy in the fuzzy rule base. To begin, the first study is mainly concerned to the development of a neutrosophic mechanism which is firstly used to measure the fuzziness I_k produced as a consequence of the dimensional overlapping area among RUs via defining the neutrosophic set $< T, F, I_k >$. T and F are used to measure the overlapping area between two RUs and its complement respectively. Secondly, an index I_{ji}^k is suggested in order to measure the non-specificity (ambiguity) by the RUs throughout the training stage of the RBF-NN. The performance of the uncertainty evaluation carried out by the application of neutrosophic sets will be compared to the experimental results provided in the second section of this chapter.

To conclude this chapter, It follows the same idea of estimating the ambiguity and the fuzziness in relation to entropy, but the proposed evaluation quantifies the uncertainty that results from the redundancy created during the cross-validation process in the RBF-NN and in the IT2-RBF-NN architecture. The redundancy is measured by applying a similarity measure that compares the shape and proximity of two fuzzy sets. For this, experimental results show that under some considerations, a similarity matrix can be constructed from the hidden layer neurons in the RBF-NN in order to evaluate the redundancy and hence the similarity during the construction of the RBF-NN rule base.

6.2 UNCERTAINTY ASSESSMENT IN THE RBF NEURAL NETWORK USING NEUTROSOPHIC SETS

As it was mentioned in chapter 5 and deeply explained in appendix A, a functional equivalence between type-1 fuzzy systems and the RBF-NN can be established under some restrictions. Besides, in chapter 5 it was possible to demonstrate as well that the RBF-NN can be extended to a specific type of IT2-FSs based on distance (kernel functions). Under these circumstances, the existing tools developed so far so as to measure fuzzy uncertainty may be applied on the RBF network under some restrictions. For that reason, in this section is introduced a new methodology that includes two types of uncertainty assessment based on neutrosophic sets, namely: on the one hand, the vagueness among fuzzy rules which is estimated calculating the fuzziness [Xiaoshu and Fanlun, 2000] between two fuzzy sets $\widetilde{A}_k^i$ and $\widetilde{A}_k^l$ with respect to the kth input using an overlapping coefficient [Inman and Bradley Jr, 1989]. And on the other hand, the ambiguity during the fuzzy rule construction is estimated - such an uncertainty is associated with one-to-many relations, i.e. situations with two or more alternatives influence in making decisions during the learning process of the RBF-NN.

The first step of the proposed methodology is to define the tuple $< T_i, F_i, I_i >$ in the RBF-NN taxonomy and then use this information to calculate the associated type of uncertainty. Secondly, a process of identification must be carried out in order to calculate the RBF parameters. In Fig. 6.1 the proposed structure with neutrosophic RUs is illustrated indicating the role of the tuple $< T, F, I >$.

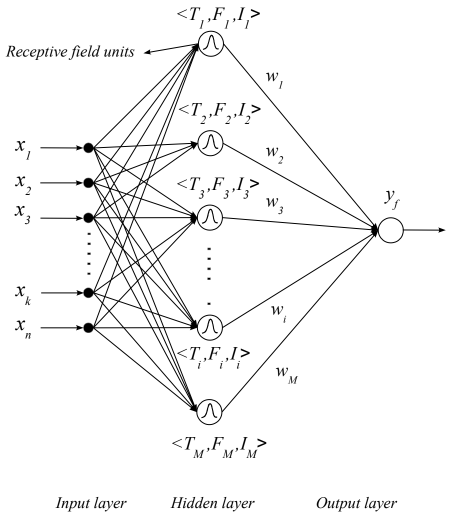


Fig. 6.1 RBF-NN structure based on NS

The hidden layer of the RBF-NN can be treated as a fuzzy inference engine that maps an input observed universe of discourse $U \subset R^n, k = 1, ..., n$ characterized by a MF $\mu_A(x) : U \rightarrow [0, 1]$ into a nonfuzzy $Y \in R$ set. From this layer a rule based system can be described as follows:

$$\widetilde{R}^i : \text{IF } x_1 \; is \; \widetilde{A}_1^i \; and \; x_2 \; is \; \widetilde{A}_2^i \; \ldots \; and \; x_n \; is \; \widetilde{A}_n^i$$
$$\text{THEN } w_i = a_1 x_1 + \ldots a_n x_n + b_i \quad (6.1)$$

If w_i is c, hence the RBF model may be seen as a Mamdani inference model where the output of each RU is

$$\mu_{A^i}(\vec{x}_p) = f_i \left(exp \left[- \frac{\|\vec{x}_p - \vec{x}\|^2}{\sigma_i^2} \right] \right) \quad (6.2)$$

In other words, the network output which is computed by Eq. 6.3 may be seen as the weighted sum of each normalised truth $\mu_{A^i} = T_i$ of the event p, where each event is the pth input vector during the parameter identification stage.

$$y_f = \frac{\sum_{i=1}^{M} w_i \mu_{A^i}}{\sum_{i=1}^{M} \mu_{A^i}} \quad (6.3)$$

From this perspective each neutrosophic RBF unit can be represented by the tuple $< T_i, F_i, I_i >$ where T_i can be defined as the firing strength or its normalised value. Usually, F_i and I_i are defined as the complement of a given fuzzy set $\widetilde{A}_k^i$ and its associated uncertainty respectively. Therefore, the proposed elements T_i, F_i and I_i of the neutrosophic tuples are calculated in this paper according to fuzziness and ambiguity.

6.2.1 FUZZINESS

Fuzziness or vagueness [Pal and Pal, 1989, Wang et al., 2012] has been a measure widely used in the development of fuzzy set theory and as an alternative measure of randomness for describing uncertainty. As mentioned in [Kosko, 1990] there are some theoretical differences between fuzziness and uncertainty which can be explained with examples and with theorems. To put it more simply, while fuzziness is conceived by the treatment of fuzzy sets, uncertainty theory gets more information by considering both aspects of possibility of truth (belief in) and the possibility of falsehood. The latter is mainly studied into the field of possibility theory. Furthermore, the semantic difference between both theories concerns by the fuzzy side on expressing "*blurry*" situations and by the "*uncertainty*" side on the expression of *not-exactly-known* reality. However, there are similarities that make both measures share a common point of view. For example, both theories handle with such similarities in terms of their individual capabilities to represent uncertainty numerically in the unit interval $[0, 1]$ and that both measures - fuzziness and randomness (uncertainty) can combine sets and propositions associatively, commutatively and distributively. Fuzziness is mainly associated with respect to the linguistic uncertainty of fuzzy terms. In [Pal and Bezdek, 1994] a review of a number of well known measures of fuzziness for discrete fuzzy sets is presented. The proposed fuzziness measure to be used in this work and defined in [Xiaoshu and Fanlun, 2000, Yager, 2002] can be written as follows:

$$fe_k^i(\mu_{Ov}) = \begin{cases} (1-\mu_{Ov})^{\alpha} e^{\mu_{Ov}} + \mu_{Ov}^{\alpha} e^{(1-\mu_{Ov})}, & i \neq j \\ \\ 0, & i = j. \end{cases} \tag{6.4}$$

Where $\alpha \in [0,1]$ and μ_{Ov} represents the area that the fuzzy set $\widetilde{A}_k^l$ overlaps the fuzzy set $\widetilde{A}_k^i$ $(i = 1, \ldots, M)$ and can be obtained as:

$$\mu_{Ov} = \frac{Ov_{\widetilde{A}_k^i \widetilde{A}_k^l}}{\widetilde{A}_k^i}, \; \mu_{Ov} \in [0,1] \tag{6.5}$$

Note that the value of fe_k^i is zero if $i = j$, that means the overlapping area is just computed for two different MFs. In the case of $i \neq j$, an exponential version of the Shannon's entropy is used. the value of fe_k^i is 1 if the MF $'i'$ is fully overlapping the MF $'j'$. In Fig. 6.2, the proposed fuzziness measure is depicted for different values of α. Such measure is related to the truth or MF in each N-RBF unit. Nevertheless, measures based on a combination between the truth and falsity of an event can be calculated as well. The overlapping coefficient $Ov_{\widetilde{A}_k^i \widetilde{A}_k^l}$ is used to calculate the area under the smaller of the fuzzy distributions $\widetilde{A}_k^i$ and $\widetilde{A}_k^l$ as is illustrated in Fig. 6.3. Therefore, $Ov_{\widetilde{A}_k^i \widetilde{A}_k^l}$ can be calculated as follows [Inman and Bradley Jr, 1989]:

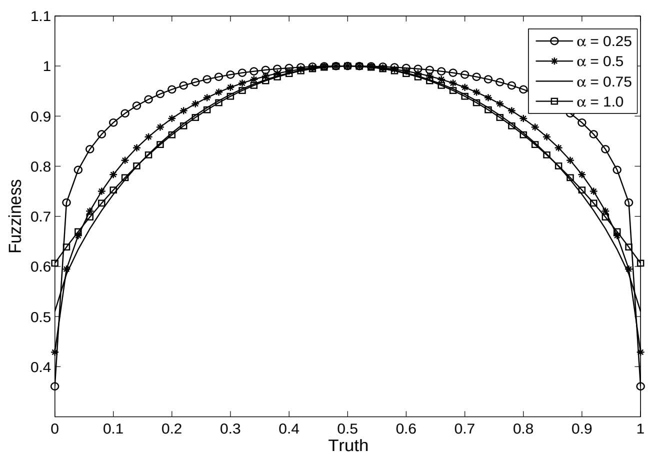


Fig. 6.2 Fuzziness (fe_k^i)

$$Ov_{\widetilde{A}_k^i \widetilde{A}_k^l} = \int_a^b min\left[\widetilde{A}_k^i(x), \widetilde{A}_k^l(x)\right] dx \tag{6.6}$$

The expression(5.4) represents the fuzziness per dimension in the ith rule between the fuzzy sets $\widetilde{A}_k^i$ and $\widetilde{A}_k^l$. However, the fuzziness must be an average dimensional measure per neuron at pattern p which can be obtained as follows:

$$E_i^p(fe_k^i) = \frac{1}{M \times n} \sum_{k=1}^{n} \sum_{i=1, i \neq j}^{M} fe_k^i(\mu_{Ov})) \tag{6.7}$$

Where M and n are the number of rules and dimensions respectively. In order to define the neutrosophic sets based on the evaluation of the fuzziness in the fuzzy rules construction, the value of the local uncertainty/indeterminacy I_k between two fuzzy sets $\widetilde{A}_k^i$ and $\widetilde{A}_k^i$ is obtained as follows:

$$\hat{U}_{ik}^p = \begin{cases} \frac{1}{(1+e^{g \times fe_k^i})}, & \mu_{Ov} < \hat{t}; \\ \\ \frac{(e^{g \times fe_k^j}) - e^{g \times fe_k^i})}{(e^{g \times fe_k^i}) + e^{g \times fe_k^i})}, & \mu_{Ov} > \hat{t}. \end{cases} \tag{6.8}$$

When $i = j$ the value of $\hat{U}_{ik}^p$ is zero. Where $\hat{t} \in [0, 1]$ and $g \in R$.

Therefore the local uncertainty per RU can be defined as

$$I_i = \frac{1}{M \times n} \sum_{k=1}^{n} \sum_{i=1, i \neq j}^{M} \hat{U}_{ik}^p \tag{6.9}$$

And the overall network uncertainty at pattern p is defined as:

$$I_p = \frac{1}{M \times n} \sum_{p=1}^{P} \sum_{k=1}^{n} \sum_{i=1, i \neq j}^{M} \hat{U}_{ik}^p \tag{6.10}$$

Where P is the number of training patterns, T_i is defined as the truth $\mu_{\widetilde{A}_k^i}$ associated to a N-RBF unit, and $F_i = 1 - \mu Ov$ is the falsity.

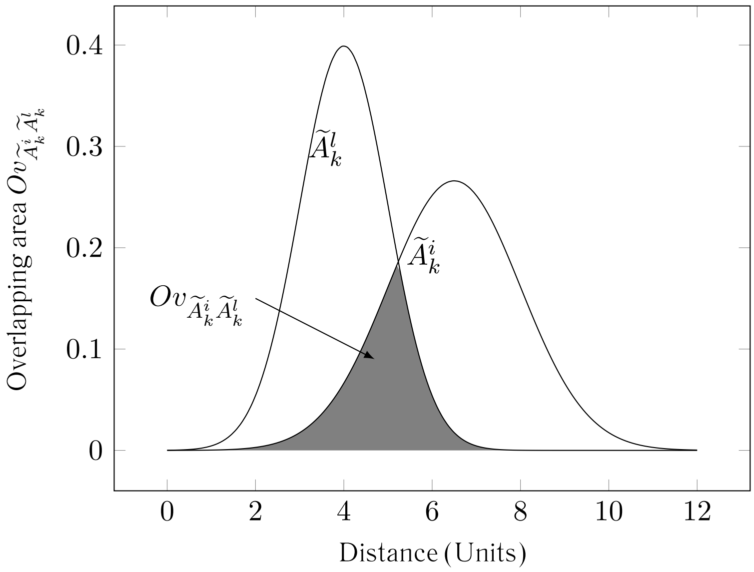


Fig. 6.3 Overlapping Area between the fuzzy sets A_j and A_i

6.2.2 AMBIGUITY

Usually in fuzzy set theory ambiguity [Wang et al., 2012] includes three main types of uncertainty measures, namely: a) nonspecificity, b) dissonance and c) confusion.

The proposed measure of ambiguity is associated with nonspecificity based on neutrosophic sets which represents a cognitive uncertainty. In the RBF-NN, the ambiguity is caused by the uncertainty of choosing one from all the normalized outputs (normalized firing strengths) in the hidden layer when classifying the input data. Therefore, the larger the number of alternatives, the higher the ambiguity is [Pal and Pal, 1989].

In this paper, the ambiguity is defined as the indeterminacy in choosing which fuzzy rule (receptive field unit) defines correctly the input data according to its normalized output. Thus, the tuple $< T_i, F_i, I_{ik}^p >$ is defined as follows:

The truth is calculated by:

$$T_i = \frac{\mu_{A^i}(\vec{x}_p)}{\sum_{i=1}^{M} \mu_{A^i}(\vec{x}_p)} \tag{6.11}$$

The falsity is calculated by:

$$F_i = max\,[T_i]_{i \neq j} \tag{6.12}$$

The ambiguity/indeterminacy is obtained by using the equation defined in [Wang et al., 2012] and is depicted in Fig. 6.2.2;

$$I_{ik}^{p} = Ambiguity_i = 1 - |T_i - F_i| \tag{6.13}$$

Therefore, the total neural ambiguity can be calculated by the following expression

$$I_A = \frac{1}{M \times n} \sum_{p=1}^{P} \sum_{k=1}^{n} \sum_{i=1}^{M} I_{ik}^{p} \tag{6.14}$$

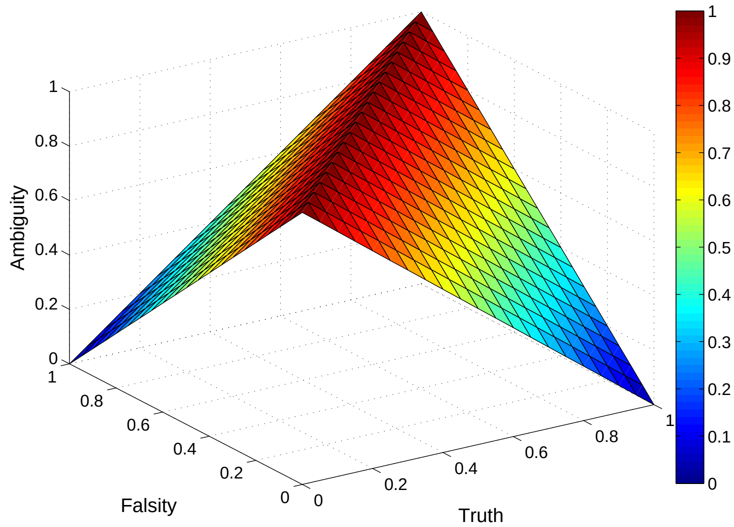


Fig. 6.4 Ambiguity (I_{ik}^{p})

6.2.3 PARAMETER IDENTIFICATION METHODOLOGY

The parameter identification consists of two main stages: a) a process of granulation [Panoutsos and Mahfouf, 2010c] where are calculated the initial parameters of the RBF-NN and b) their corresponding optimization by using an adaptive gradient descent approach including the uncertainty from two different perspectives based on *fuzziness* and *ambiguity*. The flow diagram of the fuzzy uncertainty assessment by using RBF-NN's and NS for classification is depicted in Fig. 6.5.

The energy expression and the objective function is obtained respectively as follows:

$$Pi = \sum_{p=1}^{P} \sum_{i=1}^{M} E_i^p e_p^2 \tag{6.15}$$

where $E_i^p e_p^2$ represents the neutrosophic inference mechanism throughout the learning process. And the fuzzy inference can be established as the weighted normalised average expressed in (11). Therefore, the update rule for the output weight is:

$$w_i(p+1) = \gamma w_i(p) - f e_k^i \beta e_p g_i; \tag{6.16}$$

Where $g_i = \frac{\mu_{A^i}(\vec{x}_p)}{\sum_j \mu_{A^i}(\vec{x}_p)}$ and the update rule for the width is:

$$\sigma_i(p+1) = \gamma \sigma_i(p) - f e_k^i \beta e_p g_i (w_i(p) - y_p) \frac{(x_i(k) - m_k^i)^2}{\sigma_i^3}; \tag{6.17}$$

And the update rule for the ith centre is:

$$m_k^i(p+1) = \gamma m_k^i(p) - f e_k^i \beta e_p g_i (w_i(p) - y_k) \frac{(x_i(k) - m_k^i)}{\sigma_i^2}; \tag{6.18}$$

Where β is the learning rate and γ is the momentum. The energy index is used to update the adaptation algorithm as follows:

- if $Pi(t+1) \geq Pi(t)$ Then

$$\alpha(t+1) = h_d \alpha(t), \; \gamma(t+1) = 0$$

- if $Pi(t+1) < Pi(t)$ and $\left| \frac{\Delta Pi}{Pi(t)} \right| < \delta$ Then

$$\alpha(t+1) = h_i \alpha(t), \; \gamma(t+1) = \gamma_0 \tag{6.19}$$

- if $Pi(t+1) \; < \; Pi(t)$ and $\left|\dfrac{\Delta Pi}{Pi(t)}\right| \geq \delta$ Then

$$\alpha(t+1) = \alpha(t), \; \gamma(t+1) = \gamma(t)$$

Where h_d and h_i are the decreasing and increasing factors, respectively. As it is mentioned in [Panoutsos and Mahfouf, 2010c], the value of the constrains are:

$$\begin{aligned} 0 < h_d < 1 \\ h_i > 1 \end{aligned} \tag{6.20}$$

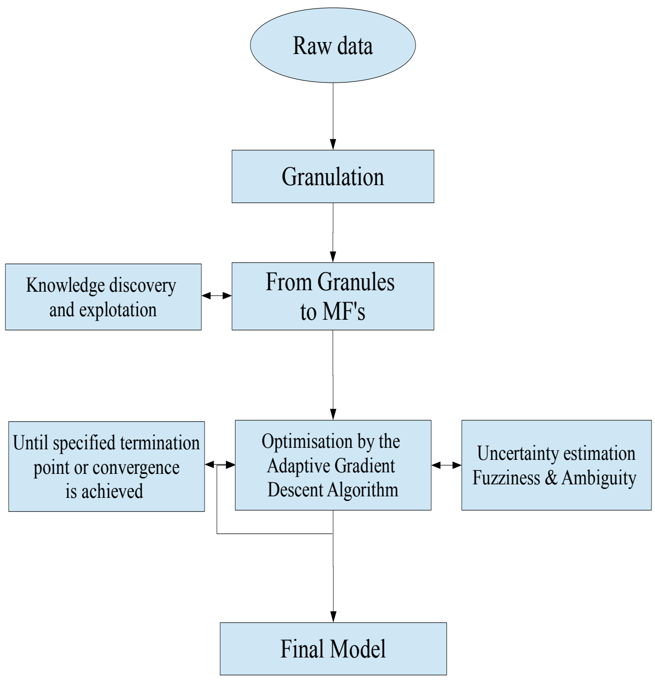


Fig. 6.5 Neutrosophic parameter identification process

6.3 EXPERIMENTS AND ANALYSIS

To investigate fully the effectiveness and efficiency of the proposed methodology, two different problems of 4 and 16 dimensional space are reported in this part. Firstly an assessment of uncertainty due to the fuzziness by using the Iris plant database is modelled. As it is mentioned in [Tenner et al., 2001], when a linear model is developed, the determination of the importance for the model inputs is directly related to the coefficients of the model. Nevertheless, in neural fuzzy systems the interpretation and then the estimation process of the weights of the network (which can be regarded as the linear model coefficients) is much more complicated to some extent. For this reason, Iris data set has represented a popular benchmark data set which combines three different classes, two linearly related each other and both non-linearly with the third one. Secondly, the real case study presented in chapter 5 for the predictive modelling of the Charpy Toughness of the Heat treated steel is used. Because of impact energy test exhibits very high uncertainty in the measurements as a consequence of its thermomechanical complexity, the developed methodology is intended to reflect such uncertainty through the assessment of the local and global fuzziness and ambiguity of the RBF-NN. It is worth mentioning that the two different study cases carried out in this chapter use the same training methodology and its corresponding cross-validation process. However such methodology is viewed from a neutrosophic point of view, this means that the proposed structure can be treated as an RBF-NN architecture capable of evaluating the tuple $< T_i, F_i, I_i >$ where its elements $T, F, I \in [0, 1]$. For example if the value of $F = I = 0$; the usual RBF-NN is being employed, otherwise an RBF-NN is taking into account the associated falsity and indeterminancy/uncertainty produced by the network.

6.3.1 EXAMPLE 1: IRIS PLANT CLASSIFICATION

In this part, the application of the developed structure based on the RBF network is intended to carry out the prediction of the iris dataset. Two different experimental studies were carried out, to be specific: 1) an experimental study for modelling the Iris data set by using the tuple $< T_i, 0, 0 >$, and 2) an experimental study for evaluating the tuple $< T_i, F_i, I_i >$. The former model used the cross validation methodology employed in chapters 4 and 5, meanwhile the latter model, a cross-validation process with the proposed training methodology was used. Since the previous results in chapter 5 demonstrated that less than 3 or no more than 5 N-RBF units are needed to accurately classify the data, in this section only 3 units are proposed to be used. In table 6.1 the statistics properties and attribute information of the a) Iris Setosa, b) Iris Versicolour and c) Iris Virginica and the correct percentage (%) of the average classification accuracy for the class 1, 2, and 3 by using the tuple $< T_i, F_i, I_i >$ are shown. This experiment also investigates the performance of the proposed neutrosophic frameworks. Such methodologies demonstrated the ability for creating a more distinguishable discourse of universe where the RBF-NN when classifying the IRIS data set. The training process employs 100% of the data set and estimates at the same time the network uncertainty caused by the overall and individual RU fuzziness.

Fig. 6.6 illustrates respectively the final distribution of the universe of discourse in the dimension 4 of the Iris data set by using the tuple $< T_i, 0, 0 >$ and $< T_i, F_i, I_i >$, the local uncertainty E_i^p and the overall network uncertainty I_p behaviours due to the fuzziness. Specially, in Fig. 6.6(c), the assessment of uncertainty clearly indicates the relationship of the fuzziness and the classification of the different Iris categories. While the term RU_a represents a neutrosophic RBF unit by using the tuple $< T_i, 0, 0 >$, the term R_b is used for representing the corresponding neutrosophic RBF unit by using the tuple $< T_i, F_i, I_i >$.

Table 6.1 Iris Database statistics, attributes and average classification accuracy

Summary Statistics	*Min*	*Max*	*Mean*	*SD*
Sepal Length (cm)	*4.3*	*7.9*	*0.83*	*5.84*
Sepal Width (cm)	*2.0*	*4.4*	*0.43*	*5.84*
Sepal Length (cm)	*1.0*	*6.9*	*1.76*	*5.84*
Sepal Width (cm)	*0.1*	*2.5*	*0.76*	*5.84*
Name	class 1 %	class 2 %	class 3 %	
Iris	100	97.66	99	

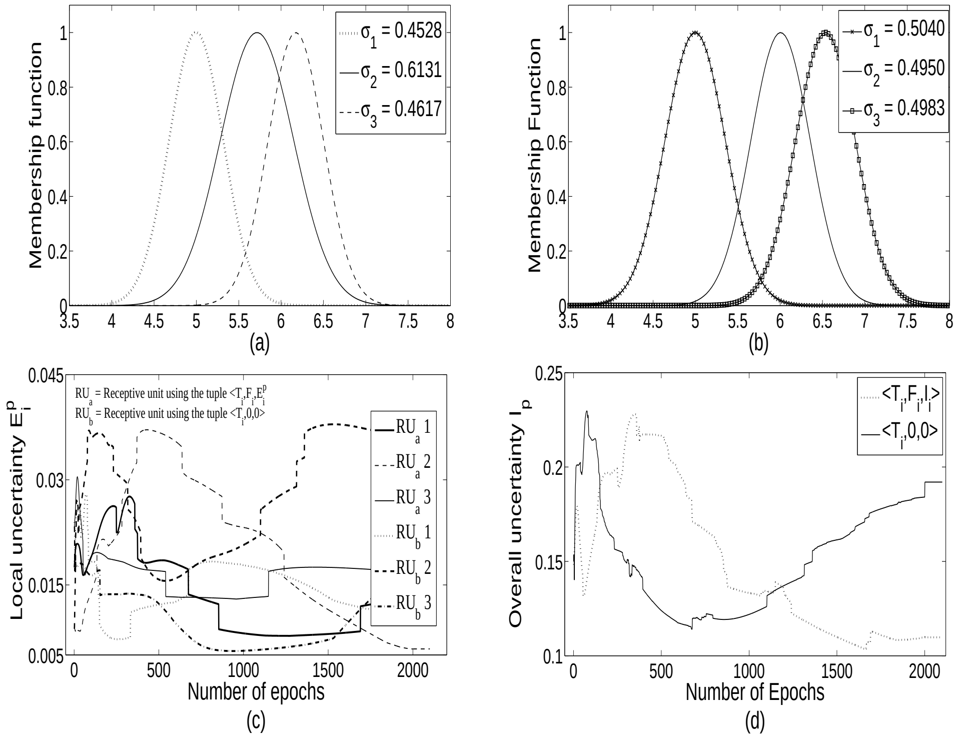


Fig. 6.6 (a) Final distribution using the tuple $< T, 0, 0 >$, (b) Final distribution using the tuple $< T, F, I_k >$, (c) local uncertainty E_j^k performance and (d) the overall uncertainty I_k produced by the overlapping among the RUs throughout the training process.

From Fig. 6.6, it is also obvious that for this case in particular, the neural network uncertainty I_p diminished importantly when using the tuple $< T_i, F_i, I_i >$ during the training. This means that it is possible to exploit the information contained in the RUs, and then manipulate the transparency and interpretability of the information per RU. The inclusion of fe_k^i in this study aims to unify the concept of uncertainty and the evaluation of truth under a neutrosophic framework.

6.3.2 EXAMPLE 2: IMPACT ENERGY TEST

In this example, the experiments are established into three different simulations, namely:

1. An experimental simulation applying the cross-validation methodology by using the truth associated to each N-RBF unit.

2. An experimental simulation by using the proposed fuzziness measure for uncertainty assessment.

3. An experimental simulation by using the proposed ambiguity measure for uncertainty assessment.

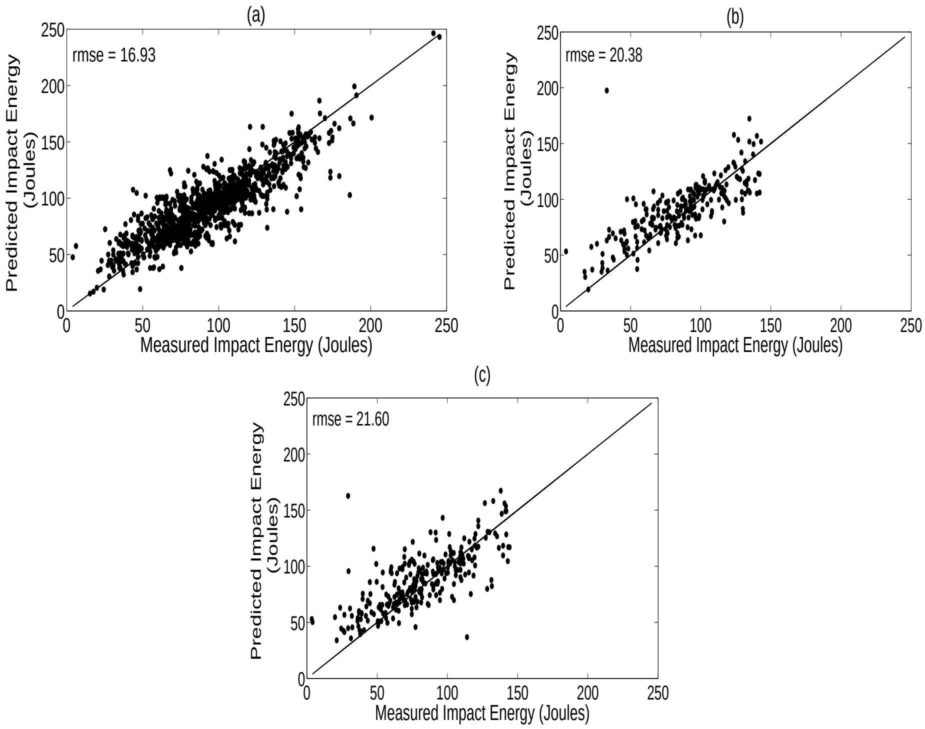


Fig. 6.7 Performance of (a) Training, (b) Checking and (c) Testing using the tuple $< T, F, I_k >$

Basically, the two experiments performed in this example assess the uncertainty caused by the fuzziness and ambiguity during the training process of the RBF-NN for the prediction of the impact energy. The example consists of a data set related to the Impact Energy Test of Heat treated grade steel. For comparison reasons, The selection of Data was set to identically match the data set used in chapter 5. The chemical composition, test parameters and heat treatment conditions are shown in table 6.3.1. The input space is defined by 16 input dimensions, and 1 output (Impact Energy, Joules), and the data set employed to train the RBF network consists of 1084 (65%), which are composed of just raw data. The checking and testing data are 277 (17%) and 300 (18%) respectively.

In Fig. 6.7, a plot of the modelling results evaluating the fuzziness are illustrated. Such results are obtained by using the proposed gradient descent algorithm and the tuple $< T_i, F_i, I_i >$ where the term I_p is the overall fuzziness which is computed using the Eq. (6.9). In Fig. 6.8, the final distribution by assessing the fuzziness of the fuzzy sets at dimension 3 (Test site test parameter) and the local uncertainty E_i^p are illustrated. Fig. 6.8(b) illustrates the behaviour of the overlapping of the entire RBF-NN throughout the training process.
As it is illustrated in Fig. 6.8(a), the higher the overlapping per dimension, the larger the local uncertainty per receptive unit (see Fig. 6.8(b)). In this sense an RBF network shares the capability of fuzzy systems for dealing with situations where set-boundaries are not sharply defined [Smarandache, 2005] and the proposed fuzziness measure of the final distribution per RU contributes to the interpretability of the RBF-NN. To investigate the RBF-NN performance based on the ambiguity assessment, the proposed adaptive gradient descent algorithm [Panoutsos and Mahfouf, 2010c] using the term I_{ik}^p in the energy equation (13) instead of the term fe_k^i is employed. In Fig. 6.7, a plot of the simulation results is presented, the results are comparable to those obtained by evaluating the overall fuzziness and to the RBF-NN of Mamdani type presented in chapter 4 in and [Panoutsos and Mahfouf, 2010c].

The overall ambiguity index I_A is the average ambiguity of the M normalised output of the RUs. Even though, Fig. 6.7(d) shows that the overall ambiguity behaviour over the span of the training process posses a decreasing trend, and the use of a measure based on ambiguity enhanced the training performance as presented in table 6.2, the final ambiguity value is never zero. This is mainly due to high non-linear property of the steel composition and heat treatment regime. Moreover, some outliers points are equally misclassified in either by evaluating the overall fuzziness or by evaluating the overall ambiguity.

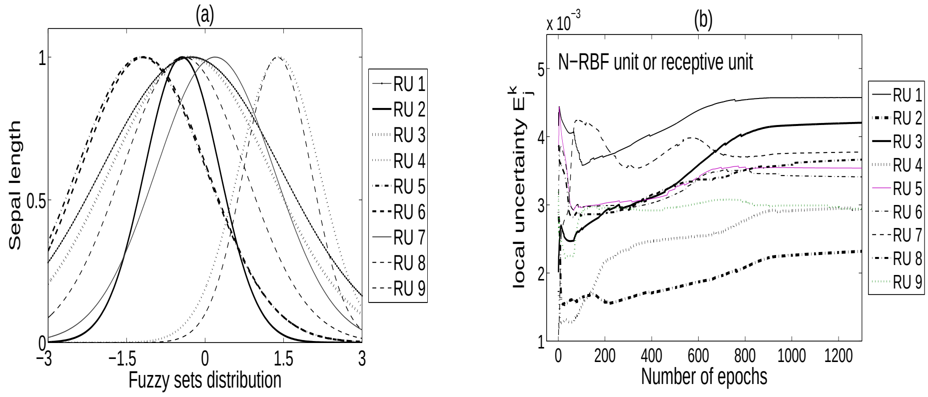


Fig. 6.8 (a) Final distribution using the tuple $< T, 0, 0 >$, (b) Final distribution using the tuple $< T_i, F_i, I^p_{ik} >$

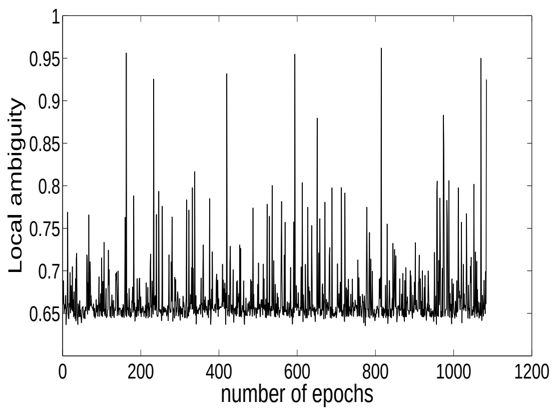


Fig. 6.9 (a) Ambiguity behaviour of the N-RBF unit number 7 at dimension C(%)

Fig. 6.9 illustrates a typical behaviour of the ambiguity generated by the RU unit number 7 during the process of training of the RBF-NN. It is evident from Fig. 6.9 the ambiguity is accordance the variability of data. In other words, such ambiguity evaluation demonstrates that the ambiguity is high when the neuron is not capable of representing the information contained in he fuzzy rule, otherwise the value is low.

Table 6.2 Performance of the optimised RBF-NN for modelling the Charpy test.

Model	Number of rules	Training	Checking	Testing
$< T, 0, 0 >$	*9*	*16.76*	*19.25*	*20.91*
$< T, F, I_k >$	*9*	*16.93*	*20.38*	*21.60*
$< T, F, A_t >$	*9*	*16.66*	*20.25*	*21.39*

Finally, in order to compare the RBF-NN performance by evaluating the ambiguity, fuzziness and the associated truth-membership at each RU, in Table 6.2 shows a comparison between three different types of uncertainty assessment, namely: using a) the tuple $< T_i, 0, 0 >$, b) the tuple $< T_i, F_i, I_i >$ and c) the tuple $< T_i, F_i, I^p_{ik} >$ which is the RBF-NN of Mamdani type.

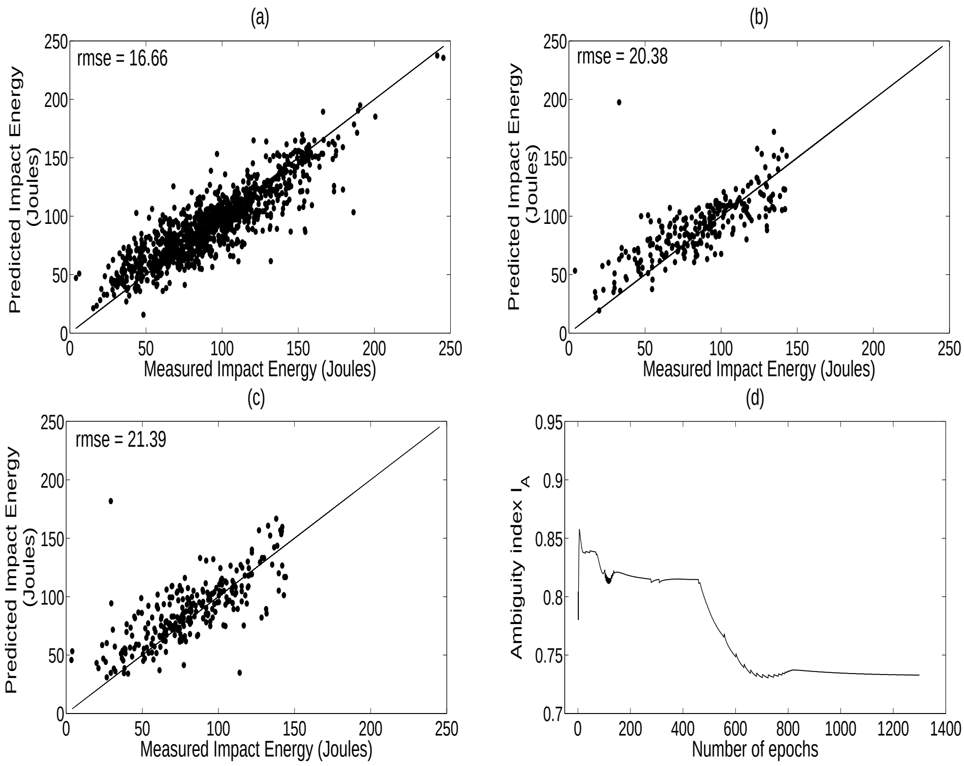


Fig. 6.10 Performance of (a) Training, (b) Checking and (c) Testing using the tuple $< T, F, I_A >$ and (d) the behaviour of the overall ambiguity I_A

As it is described in [Panoutsos and Mahfouf, 2010c], in certain cases where some data were wrongly predicted mainly at checking and testing stages; it can be concluded that such misclassification is a consequence of process repeatability of the data set (Charpy test experiments) which turns out in noisy data (or wrong data and outliers).

Particularly, the nature of the Charpy test produces very high data scatter and due to its low repeatability in obtaining the same results under the same input conditions, the performance of the RBF-NN is affected. In the view of the former results, the use of neutrosophic sets is not only the generalisation of fuzzy sets but also such sets can be exploited in order to increase the transparency and interpretability of systems functionally equivalence to fuzzy and then neutrosophic frameworks.

6.4 SIMILARITY-BASED UNCERTAINTY MEASURES IN THE RBF-NN AND THE IT2-RBF-NN

The easiest way of introducing interpretability into a learning algorithm is to employ a parameter identification procedure that includes parameters and the associated hypermarameters that have a clear interpretation of their meaning [Gibbs and MacKay, 1997]. Furthermore, a clear understanding of the effects from each model input, how their interact and the importance of each input can aid in helping to enhance the model distinguishability, transparency and hence model validation and selection and indirectly model performance.

This section includes the study of various similarity-based uncertainty measures for the RBF-NN (type-1 RBF-NN) and the proposed IT2-RBF-NN architecture. Such similarity is used for understanding the importance of each hidden neuron and hence the associated uncertainty due to fuzzy rule base redundancy. In [Wu and Mendel, 2008] is mentioned the relationship between compatibility, similarity and proximity. Basically such a relationship is based on the properties shared by a mapping $s : X \times X \rightarrow$ where two fuzzy sets A_i and A_j are defined on the domain X. Such properties are the *1. Symmetry* $s(A, B) = s(B, A)$, *2. reflexivity* $s(A, 1) = 1$, and *3)* *transitivity* $s(A, B) \geq s(A, C) \wedge s(C, B)$ where C is any another fuzzy set. The evaluation of compatibility usually encompasses similarity and proximity but not the opposite, since most of similarity measures are based on distance and hence compatibility measures how similar two entities are in relation to attributes such as proximity, geometrical shape, density, etc.

Uncertainty measures such as the Shannon entropy has been extensively studied [Pal and Bezdek, 1994] and used for constructing fuzzy models as entropy represents a measure that expresses conflict among evidential claims within a probabilistic body of evidence. This type of uncertainty quantifies the outcome attributed to randomness or in other words uncertainty that results from probabilistic events. In [Pal et al., 1992], Yager stated that another type of uncertainty is produced as a consequence of deficiencies from the system that is quantifying it. This second uncertainty measure exhibits the lack of ability to accurately specify the solution. However, both types of uncertainty do not deal with the linguistic imprecision or vagueness in fuzzy systems. For example, a die is thrown and you are asked to guess the outcome, frequently this kind of assumption is based on the evaluation of probabilistic events, and moreover if it is required that a machine quantifies the outcomes from a group of experiments (throws), probably the results vary in each try producing a deficiency during the information processing (ambiguity). The computation of the experiments by using words can be carried out by means a fuzzy machine which labels the outcome of each throw as *high, low, small, etc*, building a perceptual computer that deals with linguistic assumptions. Nevertheless, the fuzzy machine will probably produce uncertain predictions as a consequence of several factors such as 1. a poor definition of the vocabulary used for describing each throw, since "*words mean different to different people*" [Mendel, 2003] and 2. an incorrect election of the elements that must be used by the fuzzy machine, for instance: a) the type of fuzzy set employed in the fuzzy machine, b) the associated T-norm used in the inference engine, and c) the defuzzifier. Furthermore, the similarity, proximity and compatibility employed for discriminating the importance of each input when modelling real problems is crucial, and it is directly related to the uncertainty produced during the construction of fuzzy machines that have a good balance of interpretability and accuracy. Usually this misinterpretation is due to the redundancy in the fuzzy rule base when two or more fuzzy sets are mathematically processing the same input. Some authors have extended the concept of some uncertainty measures developed for type-1 fuzzy sets into interval type-2 fuzzy systems [Wu and Mendel, 2007], interval valued fuzzy sets [Türkşen, 1996], and intuitionistic fuzzy sets [Szmidt and Kacprzyk, 2001].

This section provides a group of experimental simulations that shows how a similarity measure that is often used to measure redundancy when constructing fuzzy models can be employed for enhancing the transparency of the RBF-NN and the proposed IT2-RBF-NN. Moreover, two entropy measures and one ambiguity measure will be defined based on its relationship to redundancy during the fuzzy rule construction. In this context, a methodology is suggested to first calculate the similarity in the rule base and then construct a matrix which meet the three properties that any similarity possesses , i.e. a) symmetry, b) reflectivity and c) transitivity.

In [Wu and Mendel, 2008] is suggested that IT2-FSs can be employed for computing with words (CWW) and hence for making judgements. In that article, Wu and Mendel consider three different ideas that can be translated into any type of fuzzy system either a fuzzy model for automatic control or hybrid/neural fuzzy systems for modelling under the corresponding assumptions. Zadeh coined the phrase "Computing with words"-CWW that states that *the objects can be treated as abstract words and propositions drawn from natural language*. Secondly, in [Nikravesh, 2005] was pointed out CWW is *fundamentally different from the traditional expert systems which are simply tools to 'realise' an intelligent system but are not able to process natural language which is imprecise, uncertain and partially true*.

Finally, Wu and Mendel [Wu and Mendel, 2008] stated that *words mean different to different people*. In this work, it is suggested that all these statements can not only be defined by any fuzzy model but also captured by the RBF-NN and the IT2-RBF-NN that might be considered as an special case of a general type-2 RBF-NN.

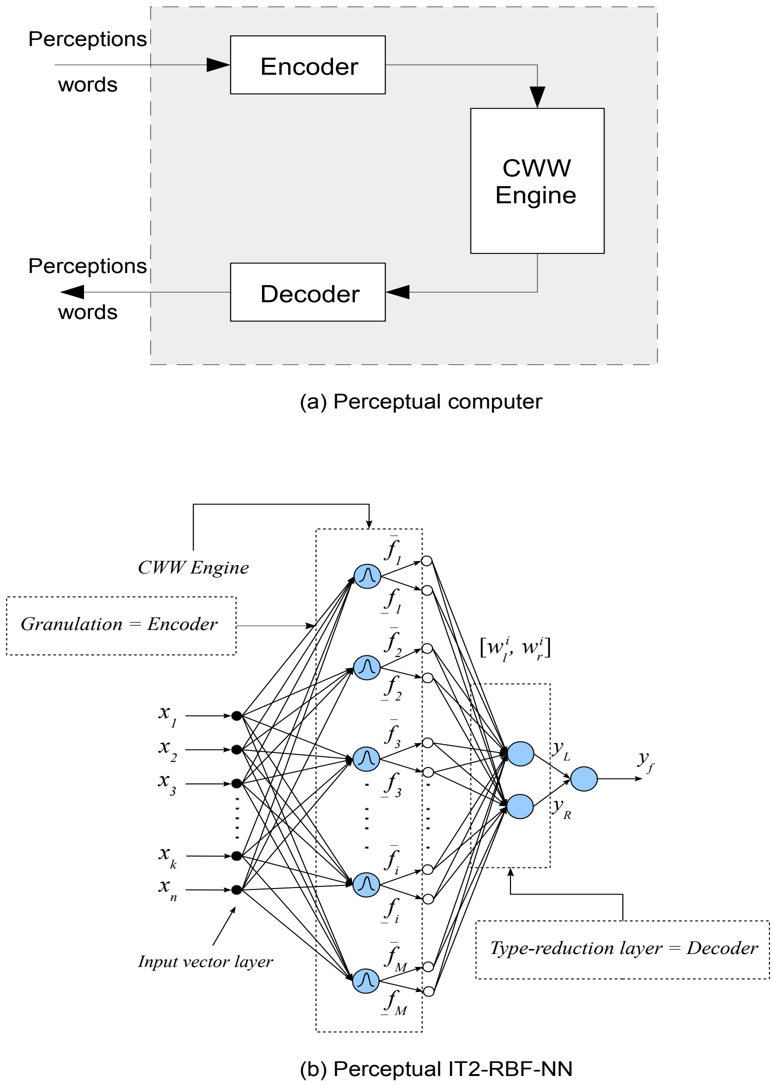


Fig. 6.11 Representation of the elements considered to estimate the similarity between two interval type-2 MFs based on their shape their distance.

For this reason, the knowledge extraction due to the information contained at each receptive units (RU) in both the RBF-NN and the IT2-RBF-NN can be studied from a fuzzy set theory perspective on the one hand. On the other hand, a deeper understanding in the construction of the RBF-NN rule base can be achieved by enhancing the transparency and interpretability of the fuzzy rules.In addition, the RBF-NN can be seen not only as a neural fuzzy system but also as an engine for computing with perceptions, thus an fuzzy inference engine capable of processing words drawn from a natural language and an expert intelligent system can be established by using the RBF-NN. In Fig. 6.11 (a) a typical perceptual computer and (b) an inference engine for CWW and hence with perceptions based on the IT2-RBF-NN are depicted respectively.

The main target of the study provided in this section is to understand better how the fuzzy sets (RUs) interact throughout the cross-validation process. In a similar way to the perceptual computer the lack of a parsimonious fuzzy model represents the redundancy created by a high level of overlapping between two or more fuzzy sets firing the same input data space. It is crucial to interpret and hence distinguish clearly the role of each fuzzy set during the training and checking process. A further study about perceptual computers can be found in [Zadeh, 1999a, 2001b, 2002].

6.5 SIMILARITY-BASED UNCERTAINTY MEASURES IN THE RBF-NN and IT2-RBF-NN.

Since Zadeh introduced the concept of fuzzy sets, researches have developed similarity measures for type-1 fuzzy sets. In [Bustince et al., 2007, Lee-Kwang et al., 1994, Wu and Mendel, 2008] is presented a summary of more than 50 existing similarity measures for type-1 fuzzy sets including some measures for IT2-FSs. In [Wu and Mendel, 2008] an overview of the number considerations that must be meet any similarity measure is provided. Basically, a similarity measure s_{ij} between two fuzzy sets A_i and A_j has the following properties:

- Reflexivity: $S(A_i, A_j) = 1,\ when\, i = j.$
- Symmetry: $S(A_i, A_) = S(A_j, A_i).$
- Transitivity: $S(A, A) \geq s(A, C) \wedge S(C, B)$ where C is any another fuzzy set.

In this section the similarity measure s_{ij} described and used for estimating the uncertainty u_{ij} produced throughout the optimisation process of the inference engine for a) the RBF-NN and b) the IT2-RBF-NN is based on that presented in [Jaccard, 1908] and generalised for interval type-2 fuzzy sets in [Wu and Mendel, 2008].

6.5.1 SIMILARITY FOR THE RBF-NN AND THE IT2-RBF-NN

Basically, in [Wu and Mendel, 2008] s_{ij} is calculated by using two different measures of similarity; i.e. a measure based on the shape of the IT2-MFs comparing the upper and the lower MFs of two IT2-FSs $\tilde{A}$ and $\tilde{B}$, and a similarity measure based on the distance between them, thus a twofold expression was suggested as follows:

$$s_{ij}(\tilde{A}, \tilde{B}) = (s_1(\tilde{A}, \tilde{B}), s_2(\tilde{A}, \tilde{B})) \tag{6.21}$$

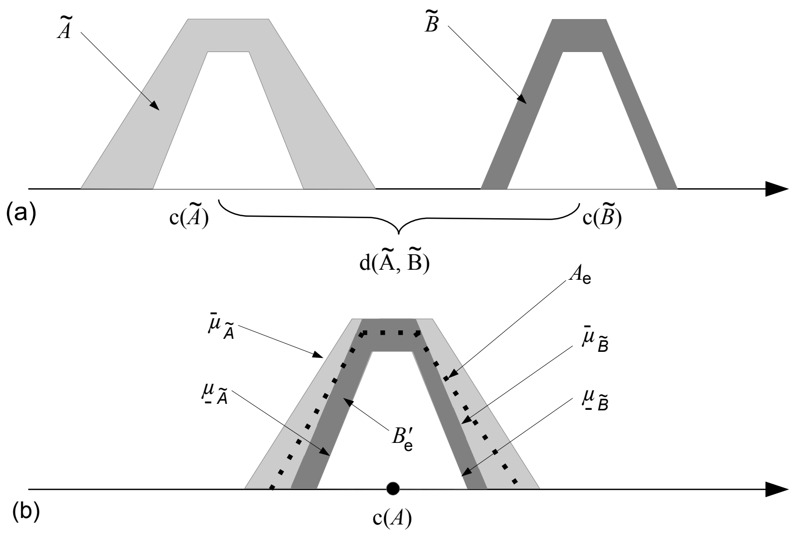


Fig. 6.12 Representation of the elements considered to estimate the similarity between two interval type-2 MFs based on their shape their distance.

The distance metric used to obtain the term s_2 may be problem-dependent, it means that the geometrical properties may be used to estimate the distance between two different fuzzy sets according to the nature of the problem and the user needs. In [Johanyák and Kovács, 2005], a summary of existing distance-based similarity measures between two fuzzy sets is presented comparing their performance and geometrical properties. Fig. 6.12 illustrates the elements employed for calculating the similarity measure s_1 by using the centre of each fuzzy set $\tilde{A}$ and $\tilde{B}$ and the distance between them. Fig. 6.12(a) shows that both fuzzy sets $\tilde{A}$ and $\tilde{B}$ must be moved in order to make coincide their centroids as illustrated in Fig. 6.12(b).

Therefore, the embedded T1 FSs A_e and B_e' of $\tilde{A}$ and $\tilde{B}$ respectively represent the shape of the IT2-FSs as illustrated in Fig. 6.12 where two measures can be obtained:

$$s_{1,l} \equiv min_{\forall A_e, B_e'} \frac{\text{card}(A_e \cap B_e')}{\text{card}(A_e \cup B_e')} \tag{6.22}$$

$$s_{1,r} \equiv max_{\forall A_e, B_e'} \frac{\text{card}(A_e \cap B_e')}{\text{card}(A_e \cup B_e')} \tag{6.23}$$

The cardinality used in 6.22 is obtained by the expression defined in [De Luca and Termini, 1972] as the power set. Moreover, the measure s_1 can be seen as mentioned in [Wu and Mendel, 2008]:

$$s_{1,interval}(\tilde{A}, \tilde{B}) = \bigcup_{\forall A_e, B_e'} \frac{\text{card}(A_e \cap B_e')}{\text{card}(A_e \cup B_e')} = [s_{i,l}, s_{i,r}] \tag{6.24}$$

Since there are not closed-form equations for calculating the centroid of $[s_{1,l}, s_{1,r}]$, similar to [Wu and Mendel, 2008], here s_1 is defined for interval type-2 FSs as the ratio of the average cardinalities of the $FOU(A_e \cap B_e')$

and $FOU(A_e \cup B'_e)$, i.e.

$$\begin{aligned} s_1 &= \frac{AC[FOU(\tilde{A} \cup \tilde{B})]}{AC[FOU(\tilde{A} \cup \tilde{B})]} \\ &= \frac{\text{card}(\overline{\mu}_{\tilde{A}}(x) \cap \underline{\mu}_{\tilde{B}'}(x)) + \text{card}(\underline{\mu}_{\tilde{A}}(x) \cap \overline{\mu}_{\tilde{B}'}(x))}{\text{card}(\overline{\mu}_{\tilde{A}}(x) \cup \underline{\mu}_{\tilde{B}'}(x)) + \text{card}(\underline{\mu}_{\tilde{A}}(x) \cup \overline{\mu}_{\tilde{B}'}(x))} \\ &= \frac{\int_X \min(\overline{\mu}_{\tilde{A}}(x), \underline{\mu}_{\tilde{B}'}(x)) + \int_X \min(\underline{\mu}_{\tilde{A}}(x), \overline{\mu}_{\tilde{B}'}(x))}{\int_X \max(\overline{\mu}_{\tilde{A}}(x), \underline{\mu}_{\tilde{B}'}(x)) + \int_X \max(\underline{\mu}_{\tilde{A}}(x), \overline{\mu}_{\tilde{B}'}(x))} \end{aligned} \tag{6.25}$$

$C_{\tilde{A}}$ and $C_{\tilde{B}}$ denote the centroids of $\tilde{A}$ and $\tilde{B}$ which are computed by using the closed-form equations $C_{\tilde{A}} = [c_l(\tilde{A}), c_r(\tilde{A})]$ and $C_{\tilde{B}} = [c_l(\tilde{B}), c_r(\tilde{B})]$ and their corresponding centres can be obtain as:

$$c(\tilde{A}) = \frac{[c_l(\tilde{A}), c_r(\tilde{A})]}{2} \tag{6.26}$$

$$c(\tilde{B}) = \frac{[c_l(\tilde{B}), c_r(\tilde{B})]}{2} \tag{6.27}$$

When all the uncertainties disappear the sets $s_{1,l}$ and $s_{1,r}$ become T1-FSs, and hence the following expression is use [Jaccard, 1908].

$$s_1(A, B) = \frac{\text{card}(A \cap B')}{\text{card}(A \cup B)} = \frac{\int_X min(\mu_A(x), \mu_{B'}(x))dx}{\int_X max(\mu_A(x), \mu_{B'}(x))dx} \tag{6.28}$$

In order to estimate the similarity between two fuzzy sets A_i and A_j either IT2-FS or T1-FSs at the hidden layer of the RBF-NN (IT2-RBF-NN) during the cross-validation process and considering their shape and distance, this research work proposes a process that consists of the following steps:

- Train the IT2-RBF-NN by applying either the self-adaptive learning process suggested . During the training process, instead of using an embedded T1-FS A_e use the output of each receptive unit per input datum, and then use the following expression if it is an IT2-RBFNN:

$$s_{ij} = \frac{\sum_{p=1}^{P} min(\overline{A}_i \cap \overline{A}_j) + \sum_{p=1}^{P} min(\underline{A}_i \cap \underline{A}_j)}{\sum_{p=1}^{P} max(\overline{A}_i \cup \overline{A}_j) + \sum_{p=1}^{P} max(\overline{A}_i \cup \overline{A}_j)} \tag{6.29}$$

Otherwise, use the expression given below:

$$s_{ij} = \frac{\sum_{p=1}^{P} min(\overline{A}_i \cap \overline{A}_j)}{\sum_{p=1}^{P} max(\overline{A}_i \cup \overline{A}_j)} \tag{6.30}$$

where s_{ij} represents the similarity between the fuzzy set A_i and A_j, $p = 1, \ldots, P$ is the whole training data, $i = 1, \ldots, M$ is the number of rules, and A_i is the MF at each receptive unit obtained for IT2-FSs as described in Chapter 5.

- In a like manner calculate the similarity at checking and testing stage.

Note that the calculation of s_2 is not necessary as the MF for both the RBF-NN and the IT2-RBF-NN is based on the distance between the centre of the MF and the corresponding p input. Thus, it means the larger the number of input data closest to two fuzzy sets A_i and A_j, the more similar such fuzzy sets are.

In other words, the behaviour of two fuzzy sets A_i and A_j will be too similar such that their firing strengths will hold similar values throughout the training process due to the proximity to the input data. Therefore, only the value of the firing strength of each receptive unit/hidden neuron/interval neuron/fuzzy set is needed. Moreover, no mathematical proof is necessary since 6.25 and 6.22 calculate the similarity between two fuzzy sets A_i and A_j based on distance through their shape similarity.

6.5.2 UNCERTAINTY MEASURES ASSESSMENT

As described above, the more similar two fuzzy sets, the higher their overlapping. Therefore, an uncertainty measure that results from redundancy among the fuzzy sets in the hidden layer of the RBF-NN can be proposed. Several authors [Chen and Linkens, 2001b, Jin, 2000] have employed distance-based measures for assessing how redundant two fuzzy sets are, i.e. similar fuzzy rules that result in unnecessary structure leading to the construction of a low-interpretable model.

For example in [Chen and Linkens, 2001a], for the prediction of hot-rolled steels properties a fuzzy model was constructed by using a similarity index that was employed to increase the interpretability while preserving accuracy modelling. Such a similarity index aids to remove redundant fuzzy rules merging similar fuzzy sets in order to create a common fuzzy set during the process of rule base simplification.

This redundancy representation might results in a lack of transparency and interpretability during the optimization of the rule base in the RBF-NN/IT2-RBF-NN. This deficiency can be translated into a source of uncertainty due to areas in the rule base where the redundancy or simply an overlapping between two or more fuzzy rules is very high affecting the trade-off between simplicity, interpretability and accuracy. For this reason, in this section an uncertainty measure based on fuzzy similarity is proposed, first a matrix representation can be constructed for the RBF-NN as follows:

$$\hat{S} = \begin{bmatrix} 1 & s_{12} & \cdots & s_{1j} & \cdots & s_{1M} \\ s_{21} & 1 & & \cdots & \cdots & s_{2M} \\ \vdots & \vdots & \ddots & & & \vdots \\ s_{i1} & s_{i2} & & s_{ij} & \cdots & s_{iM} \\ \vdots & \vdots & \cdots & \cdots & \ddots & \vdots \\ s_{M1} & s_{2M} & \cdots & s_{ij} & \cdots & 1 \end{bmatrix} \tag{6.31}$$

Here it is used s_{ij} in order to denote the similarity between the fuzzy set i and j. Therefore, the uncertainty produced per RU might be calculated by means two different ways:

- Firstly, the ambiguity associated to each RU is related to one-to-many relations and can be estimated as follows:

$$\hat{a}_i = \frac{1}{M-1}\sum_{j=1}^{M} s_{A_i,A_j}, i \neq j \tag{6.32}$$

- Secondly, the network entropy produced by all the input data due to their similarity can be calculated as [Pal and Bezdek, 1994]:

$$u_p = \frac{1}{P*(M-1)} \sum_{p=1}^{P} \sum_{i=1}^{M} s_{ij}^p * (1 - s_{ij}^p) \tag{6.33}$$

$$\hat{u}_p = \frac{1}{P*(M-1)} \sum_{p=1}^{P} \sum_{i=1}^{M} s_{ij}^p * log(s_{ij}^p) \tag{6.34}$$

6.6 EXPERIMENTAL SIMULATIONS

Experimental simulations are carried out in this part in order to test the suggested methodology used for evaluating the ambiguity and uncertainty generated throughout the cross-validation process. In this section just those results that involve the RBF-NN and the IT2-RBF-NN-(SD) that here is being called IT2-RBF-NN are considered. In this sense, the experimental studies for evaluating the similarity among the *receptive units (RUs)* in the RBF-NN and the proposed IT2-RBF-NN are illustrated and hence analysed. Therefore, this section presents the experimental results in the following order:

- First a summary of the matrix representation of the proposed similarity measure for the training and checking process in the RBF-NN is provided.

- Secondly, results related to the similarity evaluation in the IT2-RBF-NN architecture are illustrated.

- Finally, a comparison of the uncertainty behaviour based on that similarity used for evaluating the redundancy in the fuzzy rule base of the RBF-NN and IT2-RBF-NN are illustrated.

6.6.1 EXPERIMENTAL RESULTS FOR EVALUATING THE SIMILARITY IN THE RBF-NN RULE BASE

This section provides those results obtained by using the proposed similarity measure for the RBF-NN at three different stages of the cross-validation process, i.e. training, checking and testing. In table 6.3 the matrix representation of the similarity among the fuzzy sets throughout the training process and contained in the RBF-NN are presented. As can be seen, it is not difficult to realise that the elements in the main diagonal must be one. Such elements, are not considered when calculating the RU uncertainty and the overall uncertainty at each epoch of the training.

Table 6.4 and 6.5 shows the similarity matrix for the checking and testing. It is difficult to know exactly what to do with so many values, in [Wu and Mendel, 2008] it was suggested to measure the correlation between any two out of all the measures included there. Here, it is suggested to average the uncertainty produced by this similarity either per RU and the overall network uncertainty.

Table 6.3 Similarity matrix representation during the training process for the RBF-NN.

FS	1	2	3	4	5	6	7	8	9
1	1.0000	0.0000	0.5883	0.0911	0.2853	0.0103	0.1179	0.2798	0.2618
2	0.0000	1.0000	0.0000	0.0001	0.0000	0.0002	0.0001	0.0000	0.0000
3	0.5883	0.0000	1.0000	0.1043	0.2332	0.0070	0.1455	0.3038	0.2929
4	0.0911	0.0001	0.1043	1.0000	0.0511	0.0200	0.2971	0.0731	0.1377
5	0.2853	0.0000	0.2332	0.0511	1.0000	0.0137	0.1287	0.2467	0.1826
6	0.0103	0.0002	0.0070	0.0200	0.0137	1.0000	0.0246	0.0040	0.0065
7	0.1179	0.0001	0.1455	0.2971	0.1287	0.0246	1.0000	0.1562	0.2226
8	0.2798	0.0000	0.3038	0.0731	0.2467	0.0040	0.1562	1.0000	0.4442
9	0.2618	0.0000	0.2929	0.1377	0.1826	0.0065	0.2226	0.4442	1.0000

Table 6.4 Similarity matrix representation during the checking process for the RBF-NN.

FS	1	2	3	4	5	6	7	8	9
1	1.0000	0.0000	0.6044	0.0915	0.3475	0.0323	0.1109	0.2811	0.2479
2	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.6044	0.0000	1.0000	0.1148	0.2567	0.0152	0.1484	0.2956	0.2942
4	0.0915	0.0000	0.1148	1.0000	0.0541	0.0274	0.3715	0.0673	0.1482
5	0.3475	0.0000	0.2567	0.0541	1.0000	0.0262	0.1177	0.2518	0.1805
6	0.0323	0.0000	0.0152	0.0274	0.0262	1.0000	0.0646	0.0082	0.0144
7	0.1109	0.0000	0.1484	0.3715	0.1177	0.0646	1.0000	0.1594	0.2574
8	0.2811	0.0000	0.2956	0.0673	0.2518	0.0082	0.1594	1.0000	0.4511
9	0.2479	0.0000	0.2942	0.1482	0.1805	0.0144	0.2574	0.4511	1.0000

As can be seen from table 6.3 and 6.4, there are areas where the uncertainty evaluation is zero - this can be induced due to the non-existent overlapping when defining the location of the RUs. Particularly, the redundancy in the column and row number two is zero. However, the values provided in the tables 6.3 and 6.4 represent the last iteration of the training and tchecking process. This means, the uncertainty evaluation can be completely different indicating the behaviour of the croos-validation process.

Table 6.5 Similarity matrix representation during the testing process for the RBF-NN.

FS	1	2	3	4	5	6	7	8	9
1	1.0000	0.0000	0.5767	0.0729	0.3541	0.0102	0.1174	0.3082	0.2803
2	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.5767	0.0000	1.0000	0.0933	0.2670	0.0091	0.1569	0.3001	0.2766
4	0.0729	0.0000	0.0933	1.0000	0.0433	0.0207	0.2830	0.0656	0.1148
5	0.3541	0.0000	0.2670	0.0433	1.0000	0.0180	0.1315	0.2503	0.1748
6	0.0102	0.0000	0.0091	0.0207	0.0180	1.0000	0.0241	0.0083	0.0089
7	0.1174	0.0000	0.1569	0.2830	0.1315	0.0241	1.0000	0.1489	0.1986
8	0.3082	0.0000	0.3001	0.0656	0.2503	0.0083	0.1489	1.0000	0.3896
9	0.2803	0.0000	0.2766	0.1148	0.1748	0.0089	0.1986	0.3896	1.0000

Table 6.6 Similarity matrix representation during the training process for the IT2-RBF-NN.

FS	1	2	3	4	5	6	7	8	9
1	1.0000	0.1069	0.1174	0.0815	0.5004	0.5171	0.4421	0.0076	0.0001
2	0.1069	1.0000	0.1273	0.2602	0.0629	0.1552	0.1009	0.0218	0.0002
3	0.1174	0.1273	1.0000	0.1751	0.0831	0.1507	0.0753	0.0138	0.0000
4	0.0815	0.2602	0.1751	1.0000	0.0553	0.0981	0.0629	0.0255	0.0000
5	0.5004	0.0629	0.0831	0.0553	1.0000	0.3616	0.2906	0.0050	0.0000
6	0.5171	0.1552	0.1507	0.0981	0.3616	1.0000	0.3992	0.0063	0.0000
7	0.4421	0.1009	0.0753	0.0629	0.2906	0.3992	1.0000	0.0092	0.0000
8	0.0076	0.0218	0.0138	0.0255	0.0050	0.0063	0.0092	1.0000	0.0001
9	0.0001	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	1.0000

In other words, the parameter identification process is based on gradient-descent approaches which heavily depends on the initial search point. Finally, in table 6.5, the similarity behaviour of each RU is very much alike to that presented in table 6.3 and 6.4. This behaviour is depicted by the RUs in the proposed IT2-RBF-NN and shown in table 6.6 and 6.7. This is due to both models employed the same initial output weights.

Table 6.7 Similarity matrix representation during the checking process for the IT2-RBF-NN.

FS	1	2	3	4	5	6	7	8	9
1	1.0000	0.0643	0.1447	0.0907	0.5150	0.4584	0.5213	0.0109	0.0000
2	0.0643	1.0000	0.0863	0.2971	0.0397	0.1163	0.0419	0.0143	0.0000
3	0.1447	0.0863	1.0000	0.1970	0.1174	0.2167	0.0739	0.0075	0.0000
4	0.0907	0.2971	0.1970	1.0000	0.0679	0.1475	0.0616	0.0156	0.0000
5	0.5150	0.0397	0.1174	0.0679	1.0000	0.3387	0.3222	0.0071	0.0000
6	0.4584	0.1163	0.2167	0.1475	0.3387	1.0000	0.3603	0.0039	0.0000
7	0.5213	0.0419	0.0739	0.0616	0.3222	0.3603	1.0000	0.0052	0.0000
8	0.0109	0.0143	0.0075	0.0156	0.0071	0.0039	0.0052	1.0000	0.0000
9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000

6.6.2 UNCERTAINTY BEHAVIOUR

The uncertainty behaviour produced as a result of the redundancy in the fuzzy rule base and in relation to those results obtained during the training and checking process for modelling the charpy data set are presented in Fig. 6.13 and Fig. 6.14 respectively. The data set employed for estimating the redundancy-based uncertainty is the same to that used in chapters 3 and 4.

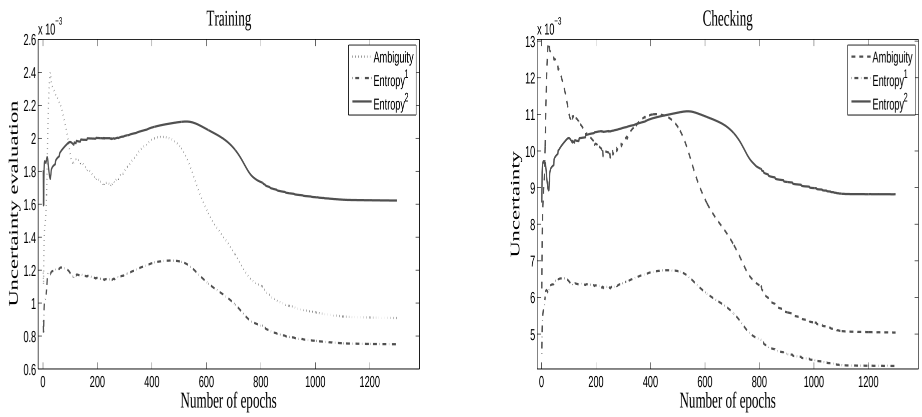


Fig. 6.13 Uncertainty behaviour for the RBF-NN.

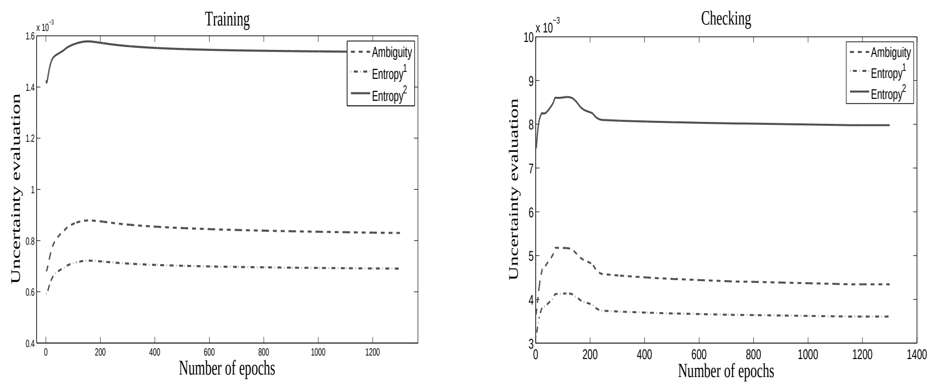


Fig. 6.14 Uncertainty behaviour for the IT2-RBF-NN.

Particularly Fig. 6.14 illustrates the ambiguity and entropy evaluated by the equations 6.33 and 6.34 with respect to the training stage.The uncertainty behaviour related to the checking stage is shown in Fig. 6.14. As can be seen, the uncertainty behaviour of both cross-validation stages describe similar trends either on modelling by using the RBF-NN or the IT2-RBF-NN leveling off approximately after 1000 epochs of training. In this context, those results shown in Fig. 6.14, the ability of the proposed IT2 network architecture for dealing with linguistic uncertainty aids for creating a more parsimonious universe of discourse. This can be translated in a lower level of ambiguity and entropy as is depicted in Fig. 6.14.

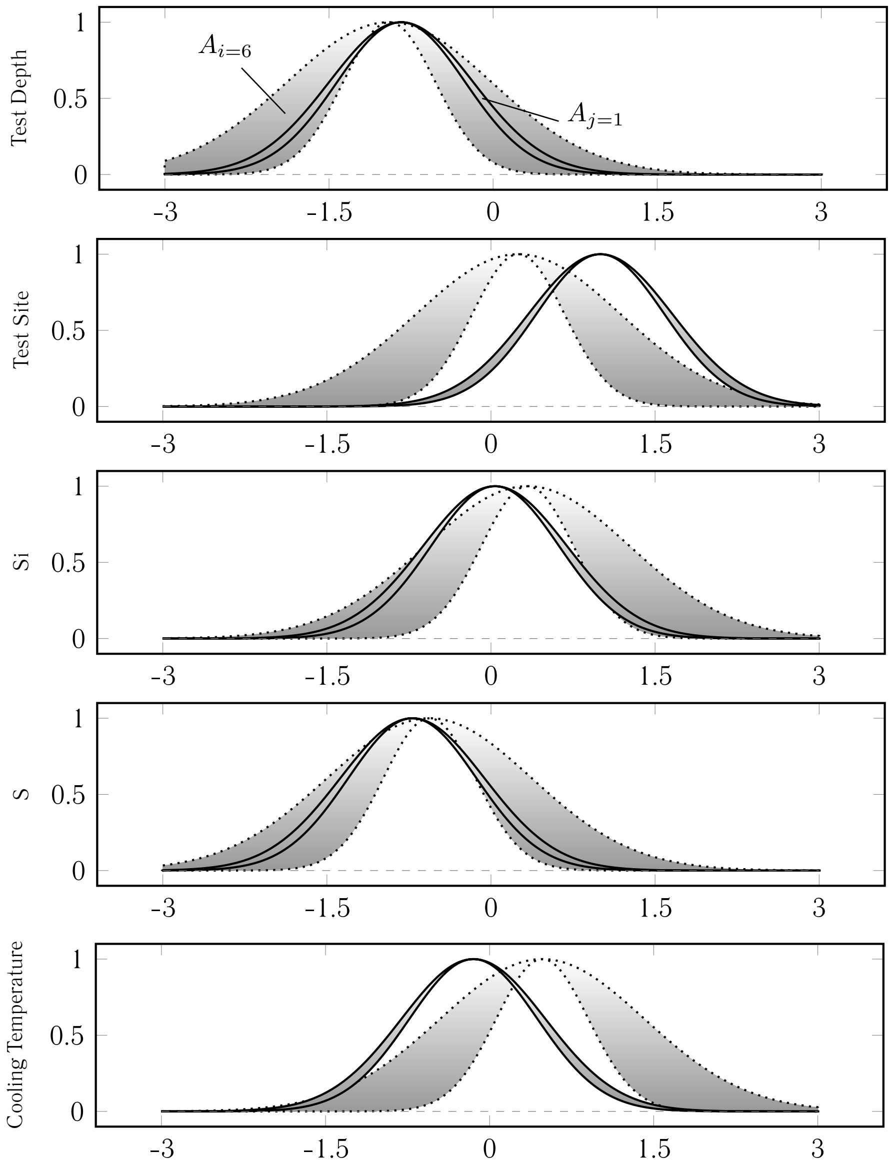


Fig. 6.15 Interval type-2 Fuzzy sets 3 and 4 used for graphically exemplify the similarity measure for the training process.

As the entropy calculation is concerned, such evaluation was made at all stages of the croos-validation procedure - i.e. the training, checking and testing. Where entropy [1] and entropy [2] are defined by the right terms of (6.33) and (6.34) respectively.

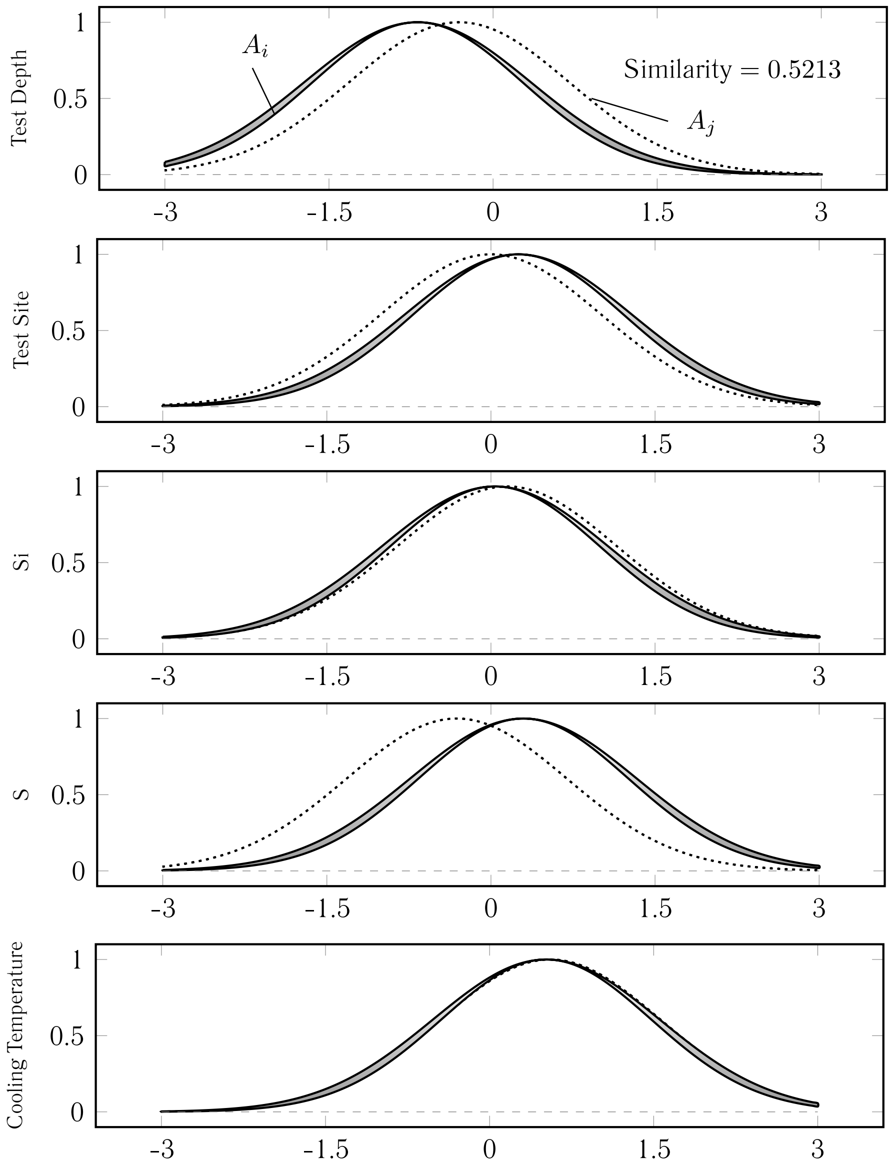


Fig. 6.16 Interval type-2 Fuzzy sets 3 and 4 used for graphically exemplify the similarity measure for the training process.

[1] $s^p(1 - s^p_{ij})$
[2] $s^p_{ij} log(s^p_{ij})$

Where $entropy^1$ and $entropy^2$ is the entropy obtained by using the expressions 6.33 and 6.34 respectively. In order to show the effectiveness of the proposed uncertainty assessment due to fuzzy rule redundancy, on the one hand in Fig. 6.14 illustrates the similarity between the fuzzy sets 3 and 4 (s_{34} or $s_{43} = 0.1970$) in 5 out of 16 dimensions that compose the input space.

On the other hand, Fig. 6.16 shows the similarity between the fuzzy sets 1 and 7 (or $s_{16} - s_{61} = 0.5171$). From Fig. 6.15 and 6.16 it can be concluded, the more similar two fuzzy sets, the higher their firing strength throughout the cross-validation process.

Nevertheless, it is also clear according to the results presented above that no similarity value is higher than 0.6. This is because the similarity value is being weighted more on shape than on distance (Euclidean distance). No proof is provided in this section, since it would required a further study how to weight individually both elements, i.e. the distance and the shape which are intrinsic in the Gaussian function employed in the RBF model. A further example can be seen in Fig. 6.16 - there the value of similarity is about 0.5271. This means that even the MFs are so close, the role of the form of the MFs play a crucial role when evaluating the similarity.

6.7 SUMMARY

The study included in this chapter is twofold, on the one hand a methodology for exploiting the functional equivalence between RBF-NNs and fuzzy systems of type-1, and the application of neutrosophic sets theory was presented. On the other hand, an study for uncertainty assessment based on the relationship between similarity and the redundancy in the fuzzy rule base was provided. The first methodology could managed to exploit and explore the information contained in each receptive unit of the RBF-NN. Notwithstanding, the black-box properties of the RBF-NN, two measures were obtained, namely: a) fuzziness and b) ambiguity. Firstly, a fuzziness measure to examine the agreement between two fuzzy rules (Gaussian fuzzy rules) by using an overlapping coefficient was defined. Secondly, an ambiguity index was constructed based on the associated true and falsity of each fuzzy rule which is contained in each N-RBF unit (neuron). An adaptive Back Error Propagation approach by using the neutrosophic sets based on fuzziness and ambiguity was employed for parameter identification. Hence, such methodology was tested against a benchmark data set and real industrial data of high dimensionality and complex nature. The resulting models produced comparable performance to that obtained by just using fuzzy sets of (RBF-NN), and due to the transparency of the process, expert knowledge can be used for improving the the interpretability and distinguishability during the fuzzy modelling.

The second methodology explores and uses the information obtained by measuring the redundancy created in the fuzzy rule base during the cross-validation process of the RBF-NN and the IT2-RBF-NN. A representation matrix for the similarity between fuzzy sets was proposed and then a relationship between similarity and entropy/ambiguity was established. Experimental results show that the uncertainty behaviour is quite similar to that behaviour exhibited by the ambiguity and fuzziness obtained by the application of neutrosophic sets.

The results obtained in the first part of this chapter led to the writing of an article that was presented at the *IEEE International Conference on Fuzzy Systems (FUZZ-IEEE) in Beijing, China.*

Next chapter will draw the conclusions of the presented thesis, and the future work related to this project will be discussed as well.

7 CONCLUSIONS AND FUTURE WORK

In this research work, we have elaborated a number of fuzzy methodologies for quantification uncertainty based on two different levels of interpretability of the RBF Neural Network (RBF-NN). The development of these methodologies aims to improve the interpretability of the RBF-NN. We believe this improvement may aid to better understand the influence that each model component and the associated parameters have for contributing with an uncertain and indeterminate system behaviour in the RBF-NN model. Therefore, the RBF-NN is used as the core mechanism to construct neural-fuzzy inference models with a special application for modelling manufacturing systems. Such methodologies follow two main directions:

1 *At the low level of interpretability of the RBF-NN.* In order to achieve this level of interpretability some criteria such as distinguishability and consistency during the granulation compression and throughout the optimisation of the initial fuzzy rule were used.

2 *At the high-level of interpretability of the RBF-NN.* At this level the criteria such as consistency, readibility and transparency of the final fuzzy rule were employed.

It was also considered the development of an Interval Type-2 RBF network which is able not only to deal with knowledge representation but also to deal with uncertainty. In this sense, the categorisation of the RBF-NN interpretability allows us to discrimate the role of each of its components as well as their contribution to produce uncertain behaviours in the RBF-NN output.

In what follows, conclusions of this thesis and suggestions about future work directions are presented.

7.1 CONCLUSIONS

The design of logic-driven and interpretable neural-fuzzy models has been an ongoing challenge in the area of data analysis and systems modelling. For this reason, this research work takes advantage from the functional equivalence between the RBF-NN and fuzzy sets of type-1 in order to describe the RBF-NN as a neural fuzzy system with adaptation capabilities to extract IF-THEN fuzzy rules from input and output sample benchmark data sets and from real experimental results obtained from steel-making industry.

In chapter 4, it was discussed the methodological and algorithmic issues of the granulation compression (low-level of interpretability of the RBF-NN) which was initially proposed in [Pedrycz and Bargiela, 2002] and finally extended in [Panoutsos and Mahfouf, 2010a]. Consequently a systematic modelling framework based on the RBF-NN, Granular Computing (GrC) and Neutrosophic Sets (NSs) was proposed. The aim of such a methodology is to mimic the ability of human cognition in order to group similar information (granules) together based on a number of similarity measures - In the computational case: proximity, cardinality and length. Moreover, the proposed methodology employs the Neutrosophic Logic concept (NL) to estimate the inherent information uncertainty/indeterminacy due to the merging operation during the information granulation process. The uncertainty/indeterminacy is calculated via a Shannon's entropy measure and then used to enhance the distinguishability at the low-level of interpretability of the RBF-NN. A Neutrosophic index was proposed to measure the disorder during the process of granulation in terms of the uncertainty that resulted from a high level of overlapping. It was observed that the final position and the level of distinguishability among the granules have a significant influence in the final interpretability and hence transparency of the initial fuzzy rule base.

As mentioned in [Pedrycz, 2005], information granulation in the fuzzy rules implies a certain level of accuracy and transparency or user friendliness. However, sometimes having fewer number of granules (more general rules) implies a reduced accuracy by the readability and the associated degree of transparency of the resulting granular universe. In other words, the higher the granularity, the better the specificity of the fuzzy rules obtained from the final granules. Compared to traditional clustering approaches such as FCM, granulation is more transparent since its components are more meaningful to the user. This means there is a well-defined semantic of the information granules. The simplified rule base after granulation is then more efficient in computational terms and linguistically tractable. From our perspective a useful qualitative and linguistic description of the low-level of interpretability in the RBF-NN may contribute importantly to establish more solid basis for the final construction of the fuzzy model. From the experimental results it was proven that the compatibility criterion not only favours a transparent and distinguishable fuzzy rule but also to contribute to eliminate redundant rules and hence to improve their consistency.

The second modelling framework proposed in chapter 5 consists in the functional extension of the RBF-NN (viewed as a fuzzy Logic System of type-1) into a generalised Interval Type-2 Logic System. Such a new framework is called "Interval Type-2 Radial Basis Function Neural Network (IT2-RBF-NN). In a like manner to interval FLSs and its counterpart the RBF-NN, the suggested structure includes a fuzzifier, rule base, fuzzy inference engine, type-reducer and defuzzifier. On the on hand, the hidden layer plays the role of fuzzifier and inference engine, and on the other hand, the type reducer and the defuzzifier are performed by the output layer of the IT2-RBF-NN. The IT2-RBF-NN may be seen as a generalised inference engine since under some mild conditions the consequent part can be used either as a) Mamdani inference or b) TSK inference [Hunt et al., 1996]. The structural and parametric optimisation of the IT2-RBF-NN is carried out by a hybrid approach that is based on estimating the initial rule base and footprint of uncertainty (FOU) directly via the granulation algorithm employed in chapter 4. Consequently, an adaptive Back Error Propagation approach (adaptive-BEP) was developed in order to optimise the rule base parameters. The reduced set in the output layer is obtained by a Karnik and Mendel type-reduction process which is considered during the application of the adaptive-BEP. Although important advances and closed-form equations for computing the type-reduced set have been proposed, in this research work the point of departure is based on the Karnik-Mendel algorithm. This is due to the weighted average approach used by the RBF-NN. Finally, the effectiveness of the proposed framework is tested against a number of popular benchmark data sets, and used to model a real manufacturing process. A further number of advantages offered by the proposed IT2-RBF-NN can be listed as follows:

- A good computational performance compared to its type-1 counterpart, the RBF-NN.
- The ability to deal with linguistic uncertainty.
- Advances in type-2 and interval type-2 fuzzy sets theory may be applied under the corresponding conditions.
- Similarly to the RBF-NN and FLSs of type-1, the interpretability in the IT2-RBF-NN can be categorised into two different levels.
- Since the proposed framework uses GrC as the initial process for extracting information (encoder), the IT2-RBF-NN may be seen as a Computing With Words (CWW) Engine whose output are crisp data.
- The IT2-RBF-NN may be used not only for modelling purposes, but also into control theory.

Finally, in chapter 6 a twofold study demonstrated that various types of uncertainty can be evaluated from the linguistic information obtained during the cross-validation process for the RBF-NN and the proposed IT2-RBF-NN architecture. The first study was focused on the application of neutrosophy in order to exploit the information contained in each receptive unit (neuron/fuzzy rule) at the two levels of interpretability of the RBF-NN. Two measures on fuzzy uncertainty were calculated, i.e. a) fuzziness and b) ambiguity. Due to the proposed uncertainty evaluation it was possible on the one hand to measure the agreement between fuzzy rules by using an overlapping index (fuzziness) and to evaluate the ambiguity created as a result of the associated truth and falsity of each fuzzy rule on the other hand. The two proposed methodologies based on the associated fuzziness and ambiguity showed a comparable performance to that obtained by just using fuzzy sets of type-1. Moreover, the simplicity of the proposed methodology in this first study also added to the computational efficiency of the model which resulted in a more interpretable structure. In respect to the second uncertainty study, a methodology for measuring the uncertainty produced as a consequence of a redundancy phenomenon in the rule base of the RBF-NN and the IT2-RBF-NN was suggested. Similar to fuzzy rule reduction, this second study took advantage of existing similarity indices to measure the uncertainty produced during the cross validation process for both neural models. In other words, the shape of the MFs, their proximity and the overall cardinality were used to estimate among the fuzzy sets and hence the related uncertainty in the hidden layer of both a) the RBF-NN and b) the IT2-RBF-NN. From this study, a symmetric matrix was constructed in order to prove that it is possible to evaluate the rule base of both models as is done in fuzzy logic systems.

7.2 FUTURE WORK

As part of the future work, we are interested in designing a highly transparent and interpretable mechanism based on the RBF-NN and fuzzy logic for making multi-objective decisions with a good trade-off between accuracy and generalisation, e.g. [Alcalá et al., 2007, Obajemu et al., 2014, Wang and Mahfouf, 2012]. This also includes the granulation process at the low-level of interpretbility which should be extended to deal with IT2-FSs. At the high-level of interpretability, the vast number of similarity and uncertainty measures available in literature may aid to understand the role of each component at the RBF-NN. The application of new techniques such as Multi-objective Evolutionary Algorithms (MOEAs), has demonstrated its power in a wide range of engineering problems. A hybridisation strategy between the RBF-NN and MOEAs could be a powerful combination opening a host of opportunities for solving complex and combinatorial problems. Particularly, the nature of MOEAs allows an optimisation search based on the decomposition of a Multiple Objective Problem (MOP) into several single-objective optimisation problems.

Furthermore, we believe the development of the IT2-RBF-NN may open up a new field of action from the point of view of kernel methods to compute with perceptions. This can be translated into a number of research works that involve interpretable models with kernels and fuzzy logic of type-2. The necessity to solve problems under an uncertainty environment is a corner stone in decision making theory. This means that the IT2-RBF-NN could be combined with existing frameworks from machine learning, e.g. Gaussian processes and Bayesian theory in order to account different types of uncertainty when making decisions. This also consider real time applications for extracting information and hence modelling real complex manufacturing systems.

Even though, the computational burden to identify the parameters of the IT2-RBF-NN was low, compared to its type-1 counterpart was higher. This is mainly due to the kind of type-reducer employed for combining the consequences in the fuzzy rule base. In this context, in the specialised literature a wide range of type-reducers [Wu, 2012] can be explored into the IT2-RBF-NN structure with the premise of reducing the computation load.

Finally, the application scope of the proposed methodologies can not only be used for pattern classification, but also for other areas such as control theory and evolutionary robotics.

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A FUZZY LOGIC SYSTEMS OF TYPE-1 AND TYPE-2

In this appendix, on the one hand a detailed description about the functional equivalence between the well-known RBF-NN and FLSs of Type-1 is provided. And on the other hand a review about theory of Fuzzy Sets of Type-2 is provided.

A.1 FUNCTIONAL EQUIVALENCE BETWEEN THE RBF-NN AND FUZZY LOGIC SYSTEMS OF TYPE-1

In [Jang and Sun, 1993], Jan and Sun established a functional equivalence between the RBF-NN and Fuzzy Logic Systems of type-1 under some mild conditions. Consequently, in [Hunt et al., 1996], the authors extended such an equivalence which was finally revised in [Andersen et al., 1998]. Particularly, this functional equivalence demonstrates that the RBF-NN can be considered as a Fuzzy Inference System (FIS) sharing properties such as function approximation, IF-THEN rules classification, low and high level interpretability, etc. Therefore advances in fuzzy set theory may be applied on RBF-NNs under some restrictions [Andersen et al., 1998]. Of this the RBF-NN can be seen as a FLS if [Hunt et al., 1996, Jang and Sun, 1993]

1. The number of receptive fields in the hidden layer (see Fig. A.1) is equal to the number of fuzzy rules.

2. The *MF*'s within each rule are chosen as Gaussian functions.

3. The T-norm operator used to compute each rule's firing strength is multiplication.

4. Both the T1-RBF-NN and the FIS under consideration use the same

defuzzification method, that is: either the centre of gravity or weighted sum to estimate their overall outputs.

In general, an FLS can be treated as an inference engine (see Fig. A.2) that maps an input observed universe of discourse ($U \subset R^n$, where $k = 1, \ldots, n$) characterized by an MF $\mu_A(x) : U \rightarrow [0, 1]$ into the nonfuzzy $Y \in R$ set. In this research work, a multi-input-single-output (MISO) fuzzy system $f : U \subset R^n \rightarrow R$ is considered having n inputs $x_k \in [x_1, ..., x_n]^T \in U_1 \times U_2 \times .. \times U_k .. \times U_n \triangleq U$ where the ith rule has the form [Wu and Er, 2000]:

$$R^i : IF\ x_1\ is\ F_1^i\ and \ldots\ x_k\ is\ F_k^i\ and \ldots \\ and\ x_n\ is\ F_n^i\ THEN\ y\ is\ G^i;\ i = 1, \ldots, M \quad (A.1)$$

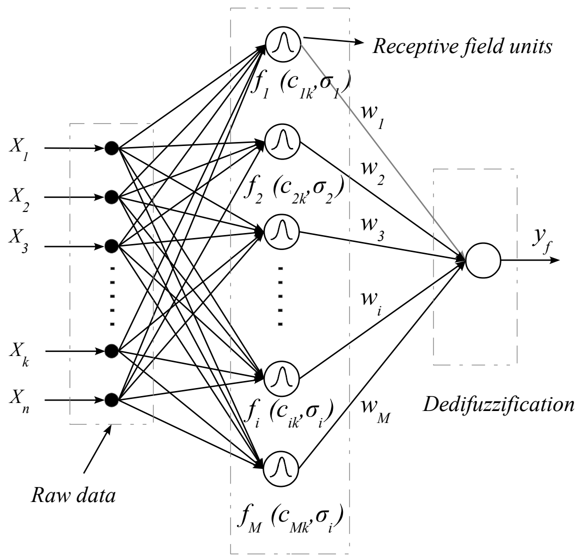

Fig. A.1 RBF Neural Network Structure

And $F_1^i \times ... \times F_n^i = A^i$, hence Eq. A.1 can be expressed as:

$$R^+ : F_1^i \times ... \times F_n^i \rightarrow G^i = A^i \rightarrow G^i; i = 1, ..., M \quad (A.2)$$

A rule R^i is described by the MF $\mu_{R^i}(\vec{x}_p, y) = \mu_{R^i}[x_1, ..., x_n, y]$, where $\vec{x}_p = [x_1, ..., x_n] \in X_1, ..., X_p = R^p$ and the following implication (Mamdani) can be used:

$$\mu_{R^i}(\vec{x}_p, y) = \mu_{A^i \to G^i}(\vec{x}_p, y) = \left[T_{k=1}^{n} \mu_{F_k^i}(x_k) \star \mu_{G^i}(y)\right] \tag{A.3}$$

Consequently, the functional equivalence established in [Jang and Sun, 1993] can be expressed from a fuzzy perspective if each firing strength f_i of each hidden receptive unit of the RBF-NN is defined as

$$\mu_{R^i}(\vec{x}_p, y) = \mu_{A^i \to G^i}(\vec{x}_p, y) = f_i \left(exp\left[-\frac{\|\vec{x}_p - \vec{\overline{x}}\|^2}{\sigma_i^2} \right] \right) \tag{A.4}$$

where the vector $\vec{\overline{x}} = [\overline{x}_1, ..., \overline{x}_n] \in X_1, ..., X_p$ constitutes the centre of the Gaussian MFs, while σ_i is a parameter defining the width of the MFs. In other words, for $k = 1, ..., n$ input, the Cartesian product of the fuzzy sets $F_1^i, ..., F_n^i$ in the universe of discourse $X_1, ..., X_p$ defined in R^n is a fuzzy set with the following membership function [Rutkowska, 2002]

$$\begin{aligned} \mu_{F_1^i \times ... \times F_n^i \to G^i} &= \prod_{k=1}^{n} \mu_{F_k^i}(x_k) \\ &= exp\left[-\left(\frac{\sum_{k=1}^{n}(x_k - \overline{x}_k)}{\sigma_i} \right)^2 \right] \\ &= exp\left[-\frac{(x - \overline{x})^T (x - \overline{x})}{\sigma_i^2} \right] \end{aligned} \tag{A.5}$$

Hence the combination of M firing strengths of the RBF-NN can be represented through the rule combiner shown in Fig. A.2 and mathematically as

$$B = A^i \circ \left[R^1, R^2, ..., R^M\right] \tag{A.6}$$

Under these conditions, the adaptive filter layer in Fig. A.2 can represent the weighting layer in the T1-RBF-NN shown in Fig. A.1 as:

$$y_f = \frac{\sum_{i=1}^{M} \mu_{B^i}(y) w_i}{\sum_{i=1}^{M} \mu_{B^i}(y)}; \ \mu_{B^i} = \mu_{A^i \to G^i}(\vec{x}_p, y_f) \tag{A.7}$$

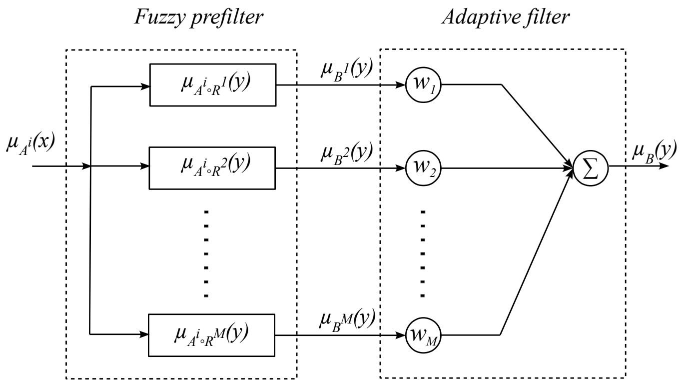

Fig. A.2 Fuzzy Inference Engine used by an FLS

A.2 TYPE-2 FUZZY SETS

This section provides a review of some of the most important definitions necessary to understand in more detailed the model proposed in chapter 5, and those studies provided in chapter 6.

A further description of IT2-FS theory can be found in [Liang and Mendel, 2000, Mendel et al., 2006]. Without loss of generality, when all the secondary MFs of a T2-FS are defined as intervals, such that $\mu_{\tilde{A}}(x, u) = 1$ they are called interval type-2 fuzzy sets IT2-FSs [Mendel et al., 2006]. Hence an IT2-FS can be defined as:

$$\tilde{A} = \int_{x \in X} \int_{u \in J_x} 1/(u, x), \qquad J_x \subseteq [0, 1]. \tag{A.8}$$

As is illustrated in the Fig. A.3, a vertical slice or a T2-MF, for example at $x = x_1$ can be expressed by the following equation.

$$\mu_{\tilde{A}}(x = x_1) = \mu_{\tilde{A}}(x_1) = \int_{u \in J_{x_1}} 1/u, \qquad J_{x_1} \subseteq [0, 1]. \tag{A.9}$$

Therefore, $\tilde{A}$ can be re-express in a vertical slice manner as

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) | \forall x \in X\}. \tag{A.10}$$

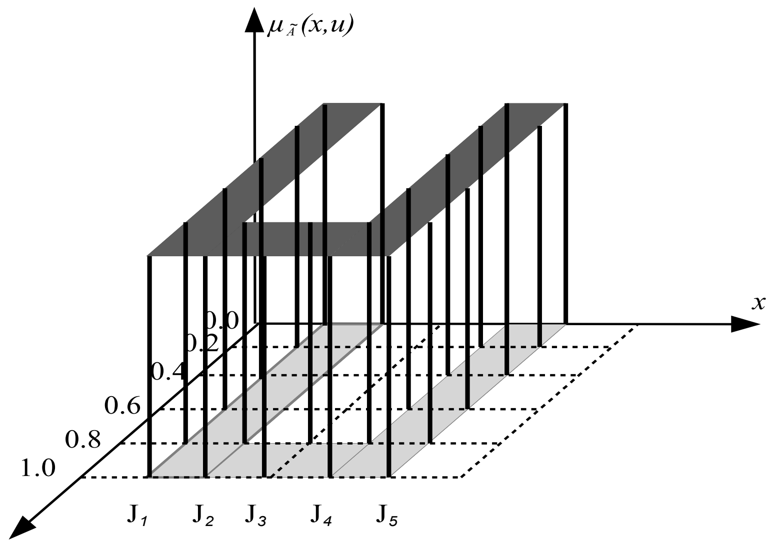

Fig. A.3 Interval Type-2 Membership Function for discrete universe of discourse

If an IT2-FS $\tilde{A}$ is discrete, hence it can be expressed as:

$$\tilde{A} = \sum_{k=1}^{n} \left[\sum_{u \in J_{x_i}} 1/u \right] \Big/ x_k = \left[\sum_{l=1}^{M_1} 1/u_{1l} \right] \Big/ x_1 + \ldots + \left[\sum_{l=1}^{M_n} 1/u_{nl} \right] \Big/ x_n. \tag{A.11}$$

Where $+$ denotes union, the discourse of universe $U \in X$ is defined by the vector $x_p = [x_1, \ldots, x_n]$ and if the discretization of each u_{kl} contains the same number of elements, hence $M_1 = M_2 = \ldots = M_n \equiv M$. Similarly to T2-FS, the FOU for IT2-FSs is defined as mentioned in chapter 2, thus the upper and lower bound of the FOU can be expressed as:

$$\overline{\mu}_{\tilde{A}} \equiv \overline{FOU(\tilde{A})} \qquad \forall x \in X \tag{A.12}$$

$$\underline{\mu}_{\tilde{A}} \equiv \underline{FOU(\tilde{A})} \qquad \forall x \in X \tag{A.13}$$

From the the equations A.12 and A.13, $\tilde{A}$ can be expressed as:

$$\tilde{A} = 1/FOU(\tilde{A}) \tag{A.14}$$

Note that $J_x = [\underline{\mu}_{\tilde{A}(x)}, \overline{\mu}_{\tilde{A}(x)}]$. Therefore, an embedded IT2-FS $\tilde{A}_e$ has

n-elements containing one element from $J_{x1}, J_{x2}, \ldots,\ J_{x_n}$ and one from $u_1, u_2, \ldots,\ u_n$, each element with a secondary MF equal to 1. i.e.

$$\tilde{A}_e = \sum_{k=1}^{n} [1/u_k] \Big/ x_k \qquad u_k \in J_{x_k} \subseteq U = [0,1] \tag{A.15}$$

Hence, from the equation represented above $\tilde{A}$ can be represented through the union of all its embedded whose total number is $\text{II}_{k=1}^{n} = M_k A_e$ and whose representation can be as follows:

$$\tilde{A} = \sum_{j=1}^{n_A} \tilde{A}_e^j \tag{A.16}$$

where $(j = 1, \ldots, n_A)$, and

$$\tilde{A}_e^j = \sum_{k=1}^{n} \left[1/u_k^j \qquad u_k^j \in J_{x_k} \subseteq U = [0,1]\right] \tag{A.17}$$

and

$$n_A = \text{II}_{k=1}^{n} M_k \tag{A.18}$$

where M_k is the discretization levels of secondary variable u_k^j at each of the $n\ x_k$.

B DERIVATIVES FOR THE LEARNING PROCEDURE

For simplicity, the IT2-RBF-NN under consideration has 'n' inputs and one output. Hence, according to the description provided in section 5.3, the first three optimisation cases that must be considered are: a) having a fixed standard deviation σ_i with a variable mean m_k^i defined on the values $[m_{k1}^i, m_{k2}^i]$, b) having a fixed mean m_k^i with a variable standard deviation σ_i defined on the values $[\sigma_i^1, \sigma_i^2]$ and that case with a fixed deviation σ_i, fixed mean m_k^i and variable height h_i.

a) Fixed Standard deviation with a variable mean. To tune the mean m_k^i of Gaussian MF with a fixed standard deviation σ_i in the ith rule [Hagras, 2006] and for the k input, we have the following equations

$$m_{k1}^i(p+1) = m_{k1}^i(p) - \alpha \left. \frac{\partial e_p}{\partial m_{k1}^i} \right|_p \tag{B.1}$$

$$m_{k2}^i(p+1) = m_{k2}^i(p) - \alpha \left. \frac{\partial e_p}{\partial m_{k2}^i} \right|_p \tag{B.2}$$

where:

$$\frac{\partial e_p}{\partial m_{k1}^i} = \left[\frac{\partial e_p}{\partial y(\vec{x}_p)} \frac{\partial y(\vec{x}_p)}{\partial y_l} \frac{\partial y_l}{\partial m_{k1}^i} + \frac{\partial e_p}{\partial y(\vec{x}_p)} \frac{\partial y(\vec{x}_p)}{\partial y_r} \frac{\partial y_r}{\partial m_{k1}^i} \right]$$
$$\frac{\partial e_p}{\partial m_{k2}^i} = \left[\frac{\partial e_p}{\partial y(\vec{x}_p)} \frac{\partial y(\vec{x}_p)}{\partial y_l} \frac{\partial y_l}{\partial m_{k2}^i} + \frac{\partial e_p}{\partial y(\vec{x}_p)} \frac{\partial y(\vec{x}_p)}{\partial y_r} \frac{\partial y_r}{\partial m_{k2}^i} \right]$$

for the standard deviation σ_i

$$\sigma_i(p+1) = \sigma_i(p) - \frac{1}{2}\alpha(y(\vec{x}_p) - d_p)\left[\frac{\partial y_l}{\partial \sigma_i} + \frac{\partial y_r}{\partial \sigma_i}\right] \quad \text{(B.3)}$$

and for the interval consequence weight $[w_l^i, w_r^i]$ we have two expressions.

$$w_l^i(p+1) = w_l^i(p) - \frac{1}{2}\alpha(y(\vec{x}_p) - d_p)\left[\frac{\partial y_l}{\partial w_l^i} + \frac{\partial y_r}{\partial w_l^i}\right] \quad \text{(B.4)}$$

$$w_r^i(p+1) = w_r^i(p) - \frac{1}{2}\alpha(y(\vec{x}_p) - d_p)\left[\frac{\partial y_l}{\partial w_r^i} + \frac{\partial y_r}{\partial w_r^i}\right] \quad \text{(B.5)}$$

Hence, by using the chain rule the corresponding derivatives are:

$$\left.\frac{\partial e_p}{y(\vec{x}_p)}\right|_p = y(\vec{x}_p) - d_p \quad \text{(B.6)}$$

$$\left.\frac{\partial y(\vec{x}_p)}{\partial y_l}\right|_p = \left.\frac{\partial y(\vec{x}_p)}{\partial y_r}\right|_p = \frac{1}{2} \quad \text{(B.7)}$$

$$\left.\frac{\partial y_l}{\partial m_{k1}^i}\right|_p = \left[\frac{\partial y_l}{\partial \bar{f}_i}\frac{\partial \bar{f}_i}{\partial m_{k1}^i} + \frac{\partial y_l}{\partial \underline{f}_i}\frac{\partial \underline{f}_i}{\partial m_{k1}^i}\right] \quad \text{(B.8)}$$

$$\left.\frac{\partial y_r}{\partial m_{k1}^i}\right|_p = \left[\frac{\partial y_r}{\partial \bar{f}_i}\frac{\partial \bar{f}_i}{\partial m_{k1}^i} + \frac{\partial y_r}{\partial \underline{f}_i}\frac{\partial \underline{f}_i}{\partial m_{k1}^i}\right] \quad \text{(B.9)}$$

For σ_i, the partial derivatives are

$$\left.\frac{\partial y_l}{\partial \sigma_i}\right|_p = \left[\frac{\partial y_l}{\partial \bar{f}_i}\frac{\partial \bar{f}_i}{\partial \sigma_i} + \frac{\partial y_l}{\partial \underline{f}_i}\frac{\partial \underline{f}_i}{\partial \sigma_i}\right] \quad \text{(B.10)}$$

$$\left.\frac{\partial y_r}{\partial \sigma_i}\right|_p = \left[\frac{\partial y_r}{\partial \bar{f}_i}\frac{\partial \bar{f}_i}{\partial \sigma_i} + \frac{\partial y_r}{\partial \underline{f}_i}\frac{\partial \underline{f}_i}{\partial \sigma_i}\right] \quad \text{(B.11)}$$

where the partial derivatives of the upper and lower MFs with respect to m_{k1}^i and m_{k2}^i are:

$$\frac{\partial \bar{f}_i}{\partial m_{k1}^i} = \begin{cases} 2\frac{(x_k - m_{k1}^i)\bar{f}_i(\bar{\phi}_k(x_k), \sigma_i)}{(\sigma_i)^2}, & x_k \leq m_{k1}^i \\ 0, & m_{k1}^i \leq x_k \leq m_{k2}^i \\ 0, & x_k > m_{k2}^i \end{cases} \tag{B.12}$$

$$\frac{\partial \underline{f}_i}{\partial m_{k1}^i} = \begin{cases} 0, & x_k \leq \frac{m_{k1}^i + m_{k2}^i}{2} \\ 2\frac{(x_k - m_{k1}^i)\underline{f}_i(\underline{\phi}_k(x_k), \sigma_i)}{(\sigma_i)^2}, & x_k > \frac{m_{k1}^i + m_{k2}^i}{2} \end{cases} \tag{B.13}$$

$$\frac{\partial \bar{f}_i}{\partial m_{k2}^i} = \begin{cases} 0, & x_k \leq m_{k1}^i \\ 0, & m_{k1}^i \leq x_k \leq m_{k2}^i \\ 2\frac{(x_k - m_{k2}^i)\bar{f}_i(\bar{\phi}_k(x_k), \sigma_i)}{(\sigma_i)^2}, & x_k > m_{k2}^i \end{cases} \tag{B.14}$$

$$\frac{\partial \underline{f}_i}{\partial m_{k2}^i} = \begin{cases} 2\frac{(x_k - m_{k2}^i)\underline{f}_i(\underline{\phi}_k(x_k), \sigma_i)}{(\sigma_i)^2}, & x_k \leq \frac{m_{k1}^i + m_{k2}^i}{2} \\ 0, & x_k > \frac{m_{k1}^i + m_{k2}^i}{2} \end{cases} \tag{B.15}$$

In order to compute the related derivatives to y_r and y_l expressed in (B.12), (B.13), (B.14) and (B.15) with respect to the MF parameters [Panoutsos and Mahfouf, 2010a], hence it is necessary to know where exactly the antecedent and consequent parameters are located. This means that the different possible permutations produced during the type-reduction process must be considered. In other words, the computational burden increases as the number of iterations increase at each type reduction of the interval type-2 fuzzy sets. In section 5.3 the procedure required to process the permutations is described in detail. Therefore, the corresponding derivatives can be categorised into four different expressions as follows:

$$\frac{\partial y_l}{\partial \bar{f}_i} = \begin{cases} \dfrac{(w_l^i - y_l)}{\left(\sum\limits_{i=1}^{L} \bar{f}_i + \sum\limits_{i=L+1}^{M} \underline{f}_i\right)}, & i \leq L \\ 0, & i > L \end{cases} \tag{B.16}$$

$$\frac{\partial y_l}{\partial \underline{f}_i} = \begin{cases} \dfrac{(w_l^i - y_l)}{\left(\sum\limits_{i=1}^{L} \bar{f}_i + \sum\limits_{i=L+1}^{M} \underline{f}_i\right)}, & i > L \\ 0, & i \leq L \end{cases} \tag{B.17}$$

$$\frac{\partial y_r}{\partial \bar{f}_i} = \begin{cases} \dfrac{(w_r^i - y_r)}{\left(\sum\limits_{i=1}^{R} \underline{f}_i + \sum\limits_{i=R+1}^{M} \bar{f}_i\right)}, & i > R \\ 0, & i \leq R \end{cases} \tag{B.18}$$

$$\frac{\partial y_r}{\partial \underline{f}_i} = \begin{cases} \dfrac{(w_r^i - y_r)}{\left(\sum\limits_{i=1}^{R} \underline{f}_i + \sum\limits_{i=R+1}^{M} \bar{f}_i\right)}, & i \leq R \\ 0, & i > R \end{cases} \tag{B.19}$$

and with respect to the standard deviation σ_i.

$$\frac{\partial \bar{f}_i}{\partial \sigma_i} = 2\frac{\sum_{k=1}^{n} \overline{\phi}_k(x_k)\bar{f}_i(\overline{\phi}_k(x_k), \sigma_i)}{(\sigma_i)^3} \tag{B.20}$$

$$\frac{\partial \underline{f}_i}{\partial \sigma_i} = 2\frac{\sum_{k=1}^{n} \underline{\phi}_k(x_k)\underline{f}_i(\underline{\phi}_k(x_k), \sigma_i)}{(\sigma_i)^3} \tag{B.21}$$

Following the same procedure given above, the derivatives of $\partial y_l/\partial w_l^i$ and $\partial y_r/\partial w_r^i$ are as follows:

$$\frac{\partial y_l}{\partial w_l^i} = \begin{cases} \dfrac{\bar{f}_i}{\left(\sum\limits_{i=1}^{L} \bar{f}_i + \sum\limits_{i=L+1}^{M} \underline{f}_i\right)}, & i \leq L \\ \dfrac{\underline{f}_i}{\left(\sum\limits_{i=1}^{L} \bar{f}_i + \sum\limits_{i=L+1}^{M} \underline{f}_i\right)}, & i > L \end{cases} \tag{B.22}$$

$$\frac{\partial y_r}{\partial w_r^i} = \begin{cases} \dfrac{\underline{f}_i}{\left(\sum\limits_{i=1}^{R} \underline{f}_i + \sum\limits_{i=R+1}^{M} \bar{f}_i\right)}, & i \leq R \\ \dfrac{\bar{f}_i}{\left(\sum\limits_{i=1}^{R} \underline{f}_i + \sum\limits_{i=R+1}^{M} \bar{f}_i\right)}, & i > R \end{cases} \tag{B.23}$$

According to the analysis given above, a number of different permutations are produced in the antecedent and consequence rules respectively - for example if $i \leq L$, $i \leq R$ and $x_k > \frac{m_{k1}^i + m_{k2}^i}{2}$ (also $m_{k1}^i \leq x_k \leq m_{k2}^i$) and then substituting the corresponding equations into (B.1) and (B.3) we have the expressions in (B.24) and (B.25) for σ_i and m_{k1}^i. A similar procedure can be followed to compute the different permutations of m_{k2}^i.

$$m_{k1}^i(p+1) = m_{k1}^i(p) - \alpha(y(\vec{x}_p) - d_p)\frac{(x_k - m_{k1}^i)\underline{f}_i(\underline{\phi}_k(x_k), \sigma_i)}{(\sigma_i)^2}\left(\frac{w_r^i - y_r}{\sum_{i=1}^{R}\underline{f}_i + \sum_{i=R+1}^{M}\bar{f}_i}\right) \tag{B.24}$$

$$\sigma_i(p+1) = \sigma_i(p) - \frac{\alpha(y(\vec{x}_p) - d_p)}{(\sigma_i)^3}\times$$
$$\times\left(\frac{\sum_{k=1}^{n}\bar{\phi}_k(x_k)\bar{f}_i(\bar{\phi}_k(x_k), \sigma_i)(w_l^i - y_l)}{\sum_{i=1}^{L}\bar{f}_i + \sum_{i=L+1}^{M}\underline{f}_i} + \frac{\sum_{k=1}^{n}\underline{\phi}_k(x_k)\underline{f}_i(\underline{\phi}_k(x_k), \sigma_i)(w_r^i - y_r)}{\sum_{i=L+1}^{M}\underline{f}_i + \sum_{i=R+1}^{M}\bar{f}_i}\right) \tag{B.25}$$

From (B.4) and (B.5), we now define the two possible permutations for the consequence weights $[w_l^i, w_r^i]$ respectively in the output layer of the IT2-RBF-NN by substituting the related derivatives from (B.22) and (B.23). For example if $i \leq L$ and renaming the denominator from (B.19) and (B.17) as follows:

$$y_{lden} = \sum_{i=1}^{L}\bar{f}_i + \sum_{i=L+1}^{M}\underline{f}_i \tag{B.26}$$

And

$$y_{rden} = \sum_{i=1}^{R}\underline{f}_i + \sum_{i=R+1}^{M}\bar{f}_i \tag{B.27}$$

Therefore if $i \leq L$

$$w_l^i(p+1) = w_l^i(p) - \frac{1}{2}\alpha(y(\vec{x}_p) - d_p)\frac{\bar{f}_i}{y_{lden}} \tag{B.28}$$

Otherwise

$$w_l^i(p+1) = w_l^i(p) - \frac{1}{2}\alpha(y(\vec{x}_p) - d_p)\frac{\underline{f}_i}{y_{lden}} \tag{B.29}$$

For $i \leq R$

$$w_r^i(p+1) = w_r^i(p) - \frac{1}{2}\alpha(y(\vec{x}_p) - d_p)\frac{\bar{f}_i}{y_{rden}} \tag{B.30}$$

and $i > R$

$$w_r^i(p+1) = w_r^i(p) - \frac{1}{2}\alpha(y(\vec{x}_p) - d_p)\frac{\underline{f}_i}{y_{rden}} \tag{B.31}$$

b) Fixed mean with a variable standard deviation. As described previously, a similar procedure can be used to optimise the standard deviation $\sigma_i \in [\sigma_i^1, \sigma_i^2]$ with a fixed mean m_k^i. The methodology is then carried out by using the adaptive-BEP approach for learning the premise parameters as

$$\sigma_i^1(p+1) = \sigma_i^1(p) - \frac{1}{2}\alpha(y(\vec{x}_p) - d_p)\left[\frac{\partial y_l}{\partial \sigma_i^1} + \frac{\partial y_r}{\partial \sigma_i^1}\right] \tag{B.32}$$

$$\sigma_i^2(p+1) = \sigma_i^2(p) - \frac{1}{2}\alpha(y(\vec{x}_p) - d_p)\left[\frac{\partial y_l}{\partial \sigma_i^2} + \frac{\partial y_r}{\partial \sigma_i^2}\right] \tag{B.33}$$

Where

$$\frac{\partial \bar{f}_i}{\partial \sigma_i^1} = 2\frac{\sum_{k=1}^{n}(x_k - m_k^i)^2 \bar{f}_i(m_k^i, \sigma_i^1; \vec{x}_P)}{(\sigma_i^1)^3} \tag{B.34}$$

$$\frac{\partial \underline{f}_i}{\partial \sigma_i^2} = 2\frac{\sum_{k=1}^{n}(x_k - m_k^i)^2 \underline{f}_i(m_k^i, \sigma_i^2; \vec{x}_P)}{(\sigma_i^2)^3} \tag{B.35}$$

And in order to compute m_k^i

$$m_k^i(p+1) = m_k^i(p) - \frac{1}{2}\alpha(y(\vec{x}_p) - d_p)\left[\frac{\partial y_l}{\partial m_k^i} + \frac{\partial y_r}{\partial m_k^i}\right] \tag{B.36}$$

c) Fixed mean with variable height. The procedure to optimise the height $h_i \in [h_i^1, h_i^2]$ with a fixed mean m_k^i and a fixed standard deviation σ_i. The adaptive learning methodology is

$$h_i^1(p+1) = h_i^1(p) - \frac{1}{2}\alpha(y(\vec{x}_p) - d_p)\left[\frac{\partial y_l}{\partial h_i^1} + \frac{\partial y_r}{\partial h_i^1}\right] \tag{B.37}$$

$$h_i^2(p+1) = h_i^2(p) - \frac{1}{2}\alpha(y(\vec{x}_p) - d_p)\left[\frac{\partial y_l}{\partial h_i^2} + \frac{\partial y_r}{\partial h_i^2}\right] \tag{B.38}$$

Where

$$\frac{\partial \underline{f}_i}{\partial h_i^1} = 0 \tag{B.39}$$

$$\frac{\partial \bar{f}_i}{\partial h_i^2} = 0 \tag{B.40}$$

$$\frac{\partial \bar{f}_i}{\partial h_i^1} = \bar{f}_i(m_k^i, \sigma_i^1, h_i^1; \vec{x}_p) \tag{B.41}$$

$$\frac{\partial \underline{f}_i}{\partial h_i^2} = \underline{f}_i(m_k^i, \sigma_i^2, h_i^2; \vec{x}_p) \tag{B.42}$$

And in order to compute m_k^i

$$m_k^i(p+1) = m_k^i(p) - \frac{1}{2}\alpha(y(\vec{x}_p) - d_p)\left[\frac{\partial y_l}{\partial m_k^i} + \frac{\partial y_r}{\partial m_k^i}\right] \tag{B.43}$$

d) Finally the last two configurations proposed in this chapter is a procedure to optimise the height $h_i \in [h_i^1, h_i^2]$ with a fixed mean m_k^i and a uncertain standard deviation σ_i and a procedure to optimise the height $h_i \in [h_i^1, h_i^2]$ with an uncertain mean m_k^i and a fixed standard deviation σ_i. These two configurations can be conducted by combining the equations defined in the sections (a), (b) and (c). For example, to identify the partial derivatives of the former configuration, the expressions (B.32), (B.33) for computing σ_i, B.37, B.38 for a variable h_i and B.43 for m_k^i must be used respectively. Therefore, the parameter identification for the latter configuration can be done by utilising the equations B.37, B.38 for the height h_i, B.1 and B.2 for tuning an uncertain mean m_k^i and B.3 for optimising a fixed deviation. That means, the number of combinations for tuning the IT2-RBF-NN is 2^3, however in this research work it is only presented six out of the total.