

**Splitting the Atom: *An Interpretation and Defence of Hume's Lead
Argument against Infinite Divisibility in the Treatise***

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ABSTRACT

This thesis explains and defends Hume's lead argument against the infinite divisibility of a finite extension (T 1.2.1). The structure of the thesis is as follows: First, I construct a comprehensive version of Hume's lead argument, which I will divide into four parts, each with its own chapter.¹ The first chapter provides a thorough textually-based defence of Hume's first principle that "...the capacity of the mind is limited, and can never attain a full and adequate conception of infinity." (T 1.2.1.2 SBN 26)

Chapter two explains Hume's argument from the principle that the mind is limited to the conclusion that the only adequate idea the mind can use to form ideas of extended objects is an indivisible coloured or tangible point, i.e. Hume's "least idea." I will argue that Hume's "least idea," contrary to widespread opinion,¹ is decidedly non-empiricist and follows from a priori and transcendental arguments. I will also reply to the criticisms that it is ridiculous to colour an extensionless point and that extensionless points cannot form an extension.

Chapter three unpacks Hume's second principle that "...whatever is capable of being divided *in infinitum*, must consist of an infinite number of parts, and that 'tis impossible to set any bounds to the number of parts, without setting bounds at the same time to division." (T 1.2.1.2 SBN 26-7) Following Thomas Holden, I endeavour to show that Hume's divisibility principle is not a mathematical, but a metaphysical principle, which depends upon the actual parts doctrine.

Chapter four interprets Hume's *adequacy* principle, which states "WHEREVER ideas are adequate representations of objects, the relations, contradictions and agreements of the ideas are all applicable to the objects." (T 1.2.2.1 SBN 29) Commentators generally interpret Hume as claiming that we can infer from our clear and adequate ideas of space the nature of space itself. However, I will argue that Hume simply means that when two ideas contradict, we infer that the objects of these two ideas do not both exist—a far more innocuous move. In this final chapter I will lay out Hume's argument for why the idea of an infinitely divisible extended object is contradictory, despite our intuitive belief to the contrary. Under this reading, the case for Hume's atomism and relational theory of extension becomes more potent, and is seen to be consistent with the philosophical elements found in his *Treatise*.

¹ In particular, Baxter, Donald. "Hume's Theory of Space and Time in Its Skeptical Context." *The Cambridge Companion to Hume: Second Edition*. Ed. David Fate Norton and Jacqueline Taylor. Cambridge: Cambridge University Press, 2009. 105-45 and Jacquette, Dale. "Hume on Infinite Divisibility and Sensible Extensionless Indivisibles." *Journal of the History of Philosophy* 34.1 (1996): 61-78. Print.

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DECLARATION

I declare that all of the work found in this thesis is completely my own, and that under no circumstances did I violate any concept of the ethical code for academic conduct. This thesis is an exemplar of academic integrity. I also never presented this work for an award at any other University. All of the sources utilized for the construction of this thesis are plainly recognized as References.

INTRODUCTION

“Nothing is more dangerous to reason than the flights of the imagination, and nothing has been the occasion of more mistakes among philosophers.” T 267

This thesis explains and defends Hume’s lead argument against the infinite divisibility of a finite extension (T 1.2.1). His conclusion—that extended objects are composed of indivisible extensionless points—has received much criticism.² Norman Kemp Smith writes “Hume’s own positive teaching, that space and time consist of physical points is, I think we must agree, one of the least satisfactory parts of his philosophy,”³ and C.D. Broad writes “[T]here seems to me to be nothing whatever in Hume’s doctrine of space except a great deal of ingenuity wasted in recommending and defending palpable nonsense.”⁴ Antony Flew specifically writes Hume’s lead argument has two fundamental principles, and “[b]oth he mistakes to be obvious: although one is true, surely, only in his own highly artificial interpretation, the other is without qualification false. Certainly both are fundamental.”⁵ It is simply untrue that Hume was wrong that a finite extension is ultimately composed of indivisible points or ‘atoms’. In the recently published *The Structure of Objects* (2008) Kathrin Koslicki writes “Whether there *in fact* are any atoms is an open question.”⁶ I will not argue that Hume closes this question. I will argue that any faults in Hume’s argument are not due to Hume’s ‘misunderstanding’ or being ‘confused’ about mathematical issues.⁷ In fact, one of Hume’s overarching goals (and a goal I hope to make explicit through the course of this thesis) is to prove that the nature of extended objects and their parts is a metaphysical, and *not* a mathematical question.

² Smith, Norman Kemp. *The Philosophy of David Hume: A Critical Study of its Origins and Central Doctrines*. 2nd. New York, NY: Palgrave Macmillan, 2005. Print., p 300; Flew, Antony. "Infinite Divisibility in Hume's Treatise." *Hume: A Re-Evaluation*. Ed. Donald W. Livingston and James T. King. New York, NY: Fordham University Press, 1976. 257-70. Print.; C.D. Broad, ‘Hume’s Doctrine of Space’, *Proceedings of the British Academy*, vol. xlvii (1961), 176.

³ Smith, Norman Kemp., p 287.

⁴ C.D. Broad, p 176.

⁵ Flew, p 258.

⁶ Koslicki, Kathrin. *The Structure of Objects*. N.p.: Oxford Scholarship Online, 2008. *University Press Scholarship Online*. Web. 18 May 2013., p. 14.

⁷ Alexander Rosenberg writes “Both [of Hume’s] arguments trade on misunderstandings of limits and of the possibility of infinite series summing to finite magnitudes,” and “Because of its confusions about the infinite, its admixture of psychological issues with physical and mathematical ones, and its fundamental circular character, Hume’s treatment of space and time has been of little influence in subsequent discussions of these concepts in the philosophy of science.” Rosenberg, Alexander. "Hume and the Philosophy of Science." *The Cambridge Companion to Hume: Second Edition*. Ed. David Fate Norton. Cambridge: Cambridge University Press, 1993. 80-89. Print., p 83-4.

I will first construct what I take to be a comprehensive version of Hume's lead argument, which I will divide into four parts, each with its own chapter.ⁱⁱ The first chapter analyses Hume's principle that "...the capacity of the mind is limited, and can never attain a full and adequate conception of infinity." (T 1.2.1.2 SBN 26) I will consider all the ways the Humean mind is limited, whether or not the mind is limited to mental images, and what it means to form a full and adequate conception of infinity. I will conclude that the Humean mind is not strictly limited to mental images, and through a special use of the imagination and abstraction can form a notion of infinity. However, that the mind cannot attain a *full and adequate* conception of infinity follows from Hume's understanding of *full and adequate* as being an idea that is determinate, admits of no further addition, and resembles an impression in certain respects.

Chapter two explains Hume's argument from the principle that the mind is limited to the conclusion that the only adequate idea the mind can use to form ideas of extended objects is an indivisible coloured or tangible point, i.e. Hume's "least idea." I will argue that Hume's "least idea," contrary to widespread opinion,⁸ is decidedly non-empiricist and follows from a priori and transcendental arguments. I will also reply to the criticisms that it is ridiculous to colour an extensionless point and that extensionless points cannot form extension by their conjunction or combination.

Chapter three unpacks Hume's divisibility principle that "...whatever is capable of being divided *in infinitum*, must consist of an infinite number of parts, and that 'tis impossible to set any bounds to the number of parts, without setting bounds at the same time to division." (T 1.2.1.2 SBN 26-7) I endeavour to show that Hume's divisibility principle is not a mathematical, but a metaphysical principle, which depends upon the actual parts doctrine, which maintains that divisibility logically presupposes the *actual* pre-existence of separate and distinct parts. The actual parts doctrine was the majority position in Hume's day, and remains so amongst modern mereologists. In fact, the divisibility principle and Hume's 'atomism' are readily received by mereologists as a possible model for extended objects. This proves that Hume's principle is not a mathematical blunder that "...trade[s] on misunderstandings of limits and of the possibility of infinite series summing to finite

⁸ In particular, Baxter, Donald. "Hume's Theory of Space and Time in Its Skeptical Context." *The Cambridge Companion to Hume: Second Edition*. Ed. David Fate Norton and Jacqueline Taylor. Cambridge: Cambridge University Press, 2009. 105-45 and Jacquette, Dale. "Hume on Infinite Divisibility and Sensible Extensionless Indivisibles." *Journal of the History of Philosophy* 34.1 (1996): 61-78. Print.

magnitudes,”⁹ but the strong metaphysical principle Hume envisioned it to be. In fact, Hume is vindicated for his insistence on the possibility of extended objects being composed of points, despite the mathematicians’ (and their purported demonstration for infinite divisibility) ardently informing him otherwise.

Chapter four interprets Hume’s *adequacy* principle, which states “WHEREVER ideas are adequate representations of objects, the relations, contradictions and agreements of the ideas are all applicable to the objects.” (T 1.2.2.1 SBN 29) Commentators generally interpret Hume as claiming that we can infer from our clear and adequate ideas of space the nature of space itself. For example, Robert Fogelin writes “...the transition from claims about our *ideas* of space and time to assertions about space and time *themselves*...is a match for anything found in the writings of the rationalists.”¹⁰ One could not imagine a more damning criticism.

However, this makes too heavy weather over what I take to be an innocuous claim. I think Hume simply means that when two ideas contradict, we infer that the objects of these two ideas do not both exist. That this reading is faithful to Hume is evinced by his argument that our idea of an infinitely divisible finite extension is contradictory. If extended objects (whether physical or mental) exist, and if Hume is right that his atomism and relational theory of extension are *possible* (chapter three), then because the only alternative to his theory—the theory of infinite divisibility—is “...utterly impossible and contradictory,” (T 1.2.3.1 SBN 39) it is “...certain that [extended objects] actually do exist conformable to” (T 1.2.3.1 SBN 39) Hume’s theory. In this chapter I will lay out Hume’s argument for why the idea of an infinitely divisible extended object is contradictory, despite our intuitive belief to the contrary. Under this reading, the case for Hume’s atomism and relational theory of extension becomes more potent.

⁹ Rosenberg, 83.

¹⁰ Fogelin, Robert. "Hume and Berkeley on the Proofs of Infinite Divisibility." *The Philosophical Review* 97.1 (1988): 47-69. Print., 53.

CHAPTER ONE: Imagination, Abstraction, and Infinite Ideas

Principle one in Hume's lead argument is that 'Tis universally allow'd, that the capacity of the mind is limited, and can never attain a full and adequate conception of infinity." Prima facie Hume means the mind's capacity is not infinite. Donald Baxter concurs: "Hume means simply that the mind cannot have an infinite number of ideas."¹¹ However, I propose to analyse this premise in the context of Book 1 and discover what jewels we may unearth. I will break Hume's principle into two parts. The first part considers what Hume means by "the capacity of the mind is limited," and each section will consider different limits. The second part explains why Hume does not think our limited minds can "attain a full and adequate conception of infinity."

Part One: The *Limited Mind*

The *mind* for Hume "...is nothing but a heap or collection of different perceptions, united together by certain relations" (T 1.4.2.39 SBN 207). I can draw three limits from this passage. First, the mind is limited to perceptions. Second, these perceptions must be *different*. Third, these perceptions are united by certain relations.

I. The *Mind's* Perceptions: Impressions, Memory Ideas, and Three Types of Imaginary Ideas

There are five types of perceptions discussed in Book One: Impressions, memory ideas, and three types of imaginary ideas. Hume's most general distinction is between impressions and ideas. Impressions are more vivid and forceful and include "...all our sensations, passions and emotions." (T 1.1.1.1 SBN 1) Memory ideas are the less vivid replicas, or "...faint images of these [impressions] in thinking and reasoning." (T 1.1.1.1 SBN 1)

By my count there are three kinds of imaginary *Ideas*: reassembled ideas (e.g. winged horses, fiery dragons, and monstrous giants [SBN 10]), and two productive ideas, one legitimate and other illegitimate. Legitimate imaginary ideas include the missing shade of blue (SBN 6), the idea of succession in a rapidly wheeling burning coal (SBN 35), ideas such as the six billion inhabitants of the world (SBN 108), external objects that resemble perceptions (T 1.2.6), the anticipated *idea of an effect* following a particular cause, (T 1.3.6) and most importantly for Hume's lead argument, insensibly small ideas of extension (SBN

¹¹ Baxter, Donald, "Skeptical Context," 106.

28, 48). Critically, legitimate imaginary ideas resemble perceptions and are formed using relations derived from experience, while illegitimate imaginary ideas fail in these respects. Examples of illegitimate ideas include perfect equality (SBN 48), perfect identity (SBN 205), non-particularized abstract idea (SBN 18), material or immaterial substance (T 1.4.5), necessary connexion (SBN 160), and most importantly for Hume's lead argument, the idea of infinite divisibility (SBN 26) and the idea of a vacuum (SBN 53-65). Thus the "heap" of perceptions that makes up the *mind* includes impressions, memory ideas, reassembled imaginary ideas, and productive imaginary ideas that resemble perceptions. Arrogantly, the philosophers feign that they have a fifth kind of idea such as infinite divisibility. However, such imaginary ideas are "unjust," (SBN 157) "inadequate," (SBN 160), "unclear," (SBN 164, 168), "deceiving," (SBN 162) "unintelligible" (SBN 169, 220) "falsely suppos'd" (SBN 207) "fictional," (SBN 209) and "unknown and invisible" (SBN 220). For Hume the mind is truly limited to the first four kinds of perceptions, and only by intellectual hubris peculiar to the philosopher does it think it has an adequate idea of the last kind.

II. Memory and Reassembling Imagination

The memory reproduces original complex impressions in the form of complex ideas.¹ The memory is a faculty with which "we repeat our impressions" (T 1.1.3.8 SBN 8) and which "...preserves the original form, in which its objects were presented." (T 1.1.3.3 SBN 9) Memory ideas are less vivid replicas of original impressions and their composition in sensation.

Simple impressions do not necessarily remain connected in the "original form" of complex impressions. They can be separated and recombined to generate reassembled imaginary ideas. Hume writes the imagination "...is not restrain'd to the same order and form with the original impressions" and has "...*the liberty...to transpose and change its ideas.*" (T 1.1.4.10 SBN 10) Hume notes wing horses, fiery dragons, and monstrous giants as evidence. Such reassembling is possible because the imagination can separate ideas wherever it perceives a *difference*. Hume writes "Where-ever the imagination perceives a difference among ideas, it can easily produce a separation." (T 1.1.4.10 SBN 10) For example, the imagination can separate the idea of a horse, a bird's wings, and the colour pink, and reassemble these impression-based ideas to form a complex imaginary idea that lacks (as of now) an empirical object referent. This makes possible Hume's separability principle, which states "Every thing, that is different is distinguishable; and every thing, that is

distinguishable, may be separated... if on the contrary they be not different, they are not distinguishable; and if they be not distinguishable, they cannot be separated.” (T 1.2.3.10 SBN 36) What is critical about reassembled imaginary ideas is their *simple* components *are* derived one-to-one from preceding impressions.

III. Determinacy Criterion

Another limit is that “different” perceptions must be distinctly and particularly conceived as having fixed quantities and qualities. I will call this Hume’s determinacy criterion. The determinacy criterion is a consequence of all ideas being directly or indirectly derived from original impressions. Ideas of “qualities” include any colour, smell, taste, feeling, or sound and vary by degrees of vivacity. Quantities are in terms of number (SBN 14-5). Hume writes that “...no object can appear to the senses; or in other words, that no impression can become present to the mind, without being determin’d in its degrees of quantity and quality...as a strong impression must necessarily have a determinate quantity and quality, the case must be the same with its copy or representative.” (T 1.1.7.4-5 SBN 19) All simple impressions have a determinate quantity or quality. Thus all simple copied ideas have determinate quantities and qualities as well.

The mind must also encounter or invoke a determinate quantity or quality in order to find or make an idea conceivable. This is because the qualities and quantities that the mind utilises must be traceable to preceding impressions. Hume writes, “Now as ‘tis impossible to form an idea of an object, that is possest of quantity and quality, and yet is possest of no precise degree of either; it follows, that there is an equal impossibility of *forming* an idea, that is not *limited* and confin’d in both these particulars.” (T 1.1.7.6 SBN 20, emphasis added) The mind’s conceptions (i.e. formed ideas) must have determinate qualities derived from the senses and must have determinate quantities. Even theoretical or conceptual ideas abide by this limit. For example, in order to conceive his indivisible points Hume bestows a “colour and tangibility” upon them, which are qualities derived from impressions.¹² Hume writes “There is nothing but the idea of their colour and tangibility making them conceivable...Upon the removal of the idea of these sensible qualities, they are utterly annihilated to the thought or imagination.” (T 1.2.4.15 SBN 38-9) By contrast, the mind

¹² Rosemary Newman concurs that Hume’s indivisible minimum is a conceptual or theoretical idea: “A mathematical point, on the other hand, represents the idea at the limit of the imagination’s divisional capacities, and to *bestow a colour or solidity on these points* furnishes them with some sort of reality, albeit it seems only an ideal one. (T 40)” Newman, Rosemary. “Hume on Space and Geometry.” *Hume Studies* 7.1 (1981): 1-31. Print., 18.

cannot conceive an idea that lacks a determinate quality and quantity. For example, the “inference is erroneous” to suppose that an abstract idea “...represent no particular degree either of quantity or quality” because “...’tis utterly impossible to conceive any quantity or quality, without forming a precise notion of its degrees.” (T 1.1.7.2 SBN 18) This limit is extended to the imagination, as any conceived fiery dragon must be of a particular colour and have a particular number of horns. Neither I, nor Hume is claiming that these horns must be countable; we are simply claiming that the negation is by definition absurd, namely, that a fiery dragon has an indeterminate or imprecise number of horns. A dragon can gain or lose a horn, but at any particular time she must have a particular number.

IV. Associative Relations

The mind is also limited to certain relations to associate and unite perceptions. These relations are resemblance, identity, spatial or temporal contiguity, quantity or number, quality in terms of degree, contrariety, and cause and effect. (SBN 14-5) This associating and uniting is an act of the imagination. Hume writes to “...understand the full extent of these relations, we must consider, that two objects are connected together in the imagination” (T 1.1.4.3 SBN 10) and “...two ideas are connected together in the imagination.” (T 1.1.5.1 SBN 13) The imagination transitions easily between and along so-related objects. Hume writes “...qualities of objects are united together by a very close relation, the mind, in looking along the succession, must be carry’d from one part of it to another by an easy transition...This easy transition is the effect, or rather the essence of relation.” (T 1.4.3.3 SBN 220) I will often call this “easy transition” the inertia of the mind.

These seven relations are *derived from experience*. The mind has received original impressions that resemble each other, are contiguous with each other, or differ in quantity or number. Memory-ideas retain the relations of complex impressions. Furthermore, when the imagination separates and reassembles simple ideas it can only unite them using these relations. The fiery dragon’s wings are contiguous with its body, and the number of horns on its back is *greater than* the number of horns on its crown, etc.

V. Legitimate and Illegitimate *Productive Imaginary Ideas*

The distinction between *productive* legitimate and illegitimate imaginary ideas is important to Hume’s lead argument for two reasons. First, infinite extension is a legitimate imaginary idea, and is therefore a *construction*. The mind can only construct ideas of

extension using the determinate ideas of coloured or tangible points. Infinite divisibility requires an infinite number of parts. Therefore, the only idea of infinite divisibility the mind can form is of an extension of infinite parts. The number of parts of an extension cannot at the same time be both infinite and finite in number. Therefore a finite extension is not infinitely divisible.

Furthermore, the productive imagination employs reason; it reasons that smaller parts exist beyond the threshold of sensation and in its illegitimate application, produces the imaginary idea of infinite divisibility. Second, Hume's idea of extension or space as a *relation* of a finite number of coexistent coloured or tangible points contiguously arranged is a *legitimate* imaginary idea, formed through the comparison of resembling complex ideas and a further distinction of reason. By contrast, the abstract general idea of extension void of content, or the Newtonian conception of a vacuum, is illegitimate. Hume's relational theory of extension is the backdrop for his lead argument, and a failure to appreciate his relational theory, and to contrast it with the Newtonian vacuum, is a theme of this thesis.

Legitimate productive imaginary ideas abide by the limits of the mind. They are conceived with particular quantities and qualities and involve the seven experiential relations. The critical difference between the productive imagination and the reassembling imagination is that the former can produce *simple* ideas not derived directly from a preceding impression. Legitimate ideas of the productive imagination are the consequence of "sound reason," (SBN 48) "just and natural" (SBN 225) reasoning, "judgment" (SBN 108), and are often described as "realities," (SBN 108) or "conclusions of judgment" (SBN 123). Examples include the missing shade of blue (SBN 6), idea of successive moments in a rapidly wheeling burning coal (SBN 35), and most importantly, extremely small parts of extension (SBN 28, 48). Productive imaginary ideas that are constructed using simple ideas derived directly from preceding impressions include ideas such as the 6 billion inhabitants of the world (SBN 108), an external object that resembles perceptions, (T 1.2.6) the anticipated *idea of an effect* following a particular cause (T 1.3.6), and most importantly, Hume's relational idea of extension (SBN 34). Legitimate uses of the imagination often seek the truth of how things really are. The imagination fills in the missing shade of blue because it reasons such a shade exists; or generates the idea of six billion people because it reasons (under the right circumstances) that six billion people exist; or creates ideas of insensibly small parts of extension because it reasons that they exist; or makes a distinction of reason from two resembling complex impressions of contiguous and coexistent simples because it reasons that

there is a shared aspect or “manner of appearance” (SBN 34) between complex impressions of sight or touch.

By contrast, illegitimate imaginary ideas are usually the ill-advised fictions of philosophers who feign they have an idea or notion such as a perfect equality (SBN 48), perfect identity (SBN 205), full and adequate idea of infinity (SBN 26), abstract idea without a particular quantity or quality (SBN 18), necessary connexion between two external objects (SBN 160), or empty space or vacuum (T 1.2.5). These ideas are illegitimate because they are not conceived with a particular quantity or quality and do not involve experiential relations. Again, Hume often describes these ideas as “unjust,” (SBN 157) “inadequate,” (SBN 160), “unclear,” (SBN 164, 168), “deceiving,” (SBN 162) “unintelligible” (SBN 169, 220) “falsely suppos’d” (SBN 207) “fictional,” (SBN 209) and “unknown and invisible” (SBN 220). Hume references the contrast between legitimate and illegitimate imaginary ideas in the opening paragraph of Part IV section IV. Hume writes:

...I must distinguish in the imagination betwixt the principles which are permanent, irresistible, and universal; such as the customary transition from causes to effects, and from effects to causes: And the principles, which are changeable, weak, and irregular; such as those I have just now taken notice of. The former are the foundation of all our thoughts and actions, so that upon their removal human nature must immediately perish and go to ruin. The latter are neither unavoidable to mankind, nor necessary, or so much as useful in the conduct of life; (T 1.4.4.1 SBN 225)

The “changeable, weak, and irregular” principles produce notions of material or immaterial substances, substantial forms, accidents, and occult qualities espoused by ancient philosophers (SBN 219). These ideas are avoidable and unnecessary. The “principles which are permanent, irresistible, and universal” produce imaginary ideas that are “the foundation of all our thoughts and actions” by employing the seven relations I enumerated earlier and the inertia of the mind.

VI. Legitimate Imaginary Ideas

The relations of resemblance and contiguity and the inertia of the mind are the “certain principles” that produce legitimate imaginary ideas. The productive imagination can fill gaps in a causal series. Hume writes: “...two objects are connected together in the imagination, not only when the one is immediately resembling, contiguous to, or the cause of the other, but also when there is interposed betwixt them a third object, which bears to both of them any of these relations. This may be carried on to a great length;” (T 1.1.4.3 SBN 11)

Hume explains that if there exists a spatial, temporal, or qualitative gap or gaps between two resembling impressions or ideas, the imagination is capable of filling in the gap or gaps with a resembling idea or ideas. For example, when Hume receives a letter from a continental friend he interposes resembling and contiguous objects such as ships, horses, and delivery people to complete the causal series. Again, this use of the imagination moves to seek the truth of how things really are. The imagination completes the causal series and furnishes the mind with ideas of ships and delivery people. These ships and delivery people can be said to exist, even though these objects are not given.

The imagination's ability to form ideas of not given objects from the association of resembling and contiguous given objects is not only limited to *causal* series. In fact, a paradigm example is the missing shade of blue. In this case, the imagination conceives a *simple* idea that is not immediately derived from a prior impression. Hume considers whether a person presented with a gradation of contiguous colours of blue, "...descending gradually from the deepest to the lightest," (T 6) except a particular shade, can imagine the colour of this missing shade. Because the person has never been acquainted with this "one particular shade," this perception is neither an impression nor a memory-idea. Hume asks "...whether 'tis possible for him, from his *own imagination*, to supply this deficiency, and raise up to himself the idea of that particular shade, tho' it had never been conveyed to him by his senses?" (T 1.1.1.10 SBN 6, emphasis added) Hume's answer is an unequivocal yes; but how can the mind do this? Because the missing shade is a *simple* idea that is not derived from a corresponding, and exactly resembling, prior simple impression, the merely reassembling imagination cannot account for this idea. To *produce* the missing shade of blue, the imagination must proceed in a different way. The mind perceives either impressions or ideas of closely resembling and contiguous objects with a determinate quantity and quality. In this case the relation and quality are similar but *lighter-than* or *darker-than*. Using these relations, the mind is led by a kind of inertia to complete the series or fill in the gaps, and posit the missing shade of blue. What makes this "sound reason" that is "natural" and "just" is that it uses experienced relations that hold between particular experienced objects, and extrapolates that these relations continue to hold for unobserved objects in that same series. This same operation of the imagination will be how the mind forms legitimate ideas of insensibly minute parts of extension. A similar—but critically different—act of the imagination also produces the illegitimate idea of infinite divisibility.

VII. Illegitimate Imaginary Ideas

The idea of infinite divisibility is an illegitimate imaginary idea, for Hume. Illegitimate imaginary ideas proceed from a similar process as legitimate ones, but the limits of the mind's reasoning powers are overstepped when objects or entities without a determinate quantity or quality, or that have relations not derivable from experience, are posited.¹³ Illegitimate fictions include an abstract idea that has "no particular degree either of quantity or quality," (SBN 18) a complete and full idea of infinity (SBN 26), the perfect standard of equality (SBN 48), the idea of a vacuum (SBN 65), the idea of a necessary connection between two external objects (SBN 157-60), the perfect numerical identity of objects (SBN 202) including soul or self (SBN 252), and a material or immaterial substance that is the cause of our perceptions (SBN 220). For example, so long as the mind limits its understanding of a causal relationship to the rules found in Section XV of Part III: *Rules by which to judge of causes and effects*, it reasons justly and naturally. However, the philosopher violates the limits of the mind by going beyond objects of experience, and positing an invisible, unobservable, and consequently unintelligible idea or relation of a necessary connection, power, force, energy, efficacy, or productive quality (SBN 157) that obtains *in* the external world (and not just an internal impression of the mind) to explain why "effects" follow from "causes." The mind cannot genuinely conceive of such a force or energy because it does not resemble an impression or an idea copied from an impression.

VIII. The Legitimate Idea of a Particular Extension and the Illegitimate Abstract Idea of Extension

For Hume, the idea of a particular extension and the abstract idea of extension are different. The idea of a particular extension is a relational one and has content *and* form and is a legitimate imaginary idea, while the abstract or general idea of extension is form *without* content, and is an illegitimate imaginary idea. Hume's lead argument focuses specifically on the legitimate idea of a particular extension, which is the idea of points of a particular colour or of particular colours disposed in a certain manner. The idea is formed through association

¹³ Frasca Spada has a nice discussion how natural "mental inertia" leads to the formation of illegitimate imaginary ideas. She writes "This feigning of a perfect standard of tone, light, velocity, equality and uniformity is actually an expression of a very general tendency of the mind... This sort of mental inertia... is also very easily applied to explain why it is so difficult for us to give up the idea of an infinite divisibility." Frasca Spada, Marina. "Part 2: Reality and Coloured Points in Hume's Treatise." *British Journal for the History of Philosophy* 6.1 (1998): 25-46. Print., p 36.

of contiguous and coexistent simple ideas that are derived from a preceding complex impression. Hume writes:

The table before me is alone sufficient by its view to give me the idea of extension. This idea, then, is borrow'd from, and represents some impression, which this moment appears to the senses. But my senses convey to me only the impression of colour'd points, dispos'd in a certain manner. If the eye is sensible of any thing farther, I desire it may be pointed out to me. But if it be impossible to shew anything farther, we may conclude with certainty, that the idea of extension is nothing but a copy of these colour'd points, and of the manner of their appearance. (T 1.2.3.4 SBN 34)

The idea of a particular extension (in Hume's example, the table before him) is the *idea* of contiguous and coexistent coloured simple "points." It is content in a form. The idea of extension Hume describes is not a table. The table *gives* the idea of extension by providing enough materials for the mind to *form* the idea of extension. The memory-idea of the table is not one and the same with the associated idea of the table's extension. The idea of the table's extension is the idea of a *relation* that requires a comparison and association by the imagination of two or more ideas. It is a *conception* formed by the imagination. Hume writes, "Now 'tis certain we have an idea of extension; for otherwise why do we talk and reason concerning it? 'Tis like-wise certain, that this idea, as *conceiv'd by the imagination*, tho' divisible into parts or inferior ideas, is not infinitely divisible, nor consists of an infinite number of parts." (T 1.2.2.8 SBN 32, emphasis added) Extension is an *idea of relation* formed by the imagination.¹⁴ Hume writes: "After identity the most universal and comprehensive *relations* are those of *Space* and *Time*, which are the sources of an infinite number of comparisons, such as *distant, contiguous, above, below, before, after, &c.*" (T 1.1.5.5 SBN 14, emphasis added) So how does the imagination form the idea of the table's extension from the complex impression of the table?

The imagination forms the relational idea of extension by associating, comparing, and binding resembling and contiguous ideas derived from preceding impressions. Hume writes: "That we may understand the full extent of these relations, we must consider, that two objects are connected together in the imagination...when the one is immediately resembling, contiguous to, or the cause of the other." (T 1.1.4.5 SBN 11) The table is a complex

¹⁴ Dale Jaquette argues that extension is a copied memory-idea derived immediately from a complex sense impression. He writes "Hume indicates that immediate sense impressions of spatial entities without intervention by other faculties of mind are sufficient to produce the idea of extension." Jaquette, pg. 418. This is wrong because Hume calls the idea of extension an imaginary idea as the quote above indicates. Hume also says that space is an abstract idea (SBN 34). Therefore, the formation of extension or space follows the same logic as set forth in T.1.1.7, which involves "intervention by other faculties of mind."

impression with contiguous and coexistent “colour’d” simple “points.” After the mind receives such an impression, it compares the simple ideas and associates them based on their resemblance and contiguity. After such a comparison the mind forms the idea of a relation of coloured points being contiguous and coexistent. Hume writes “...the idea of extension, which is compos’d of the ideas of these points.” (T 1.2.3.16 SBN 39) This idea of extension is not a memory-idea. Memory-ideas will always be of particular tables, chairs, bovines, etc. The *idea* of extension is formed posterior to complex sense impressions, and requires a comparison by the imagination of simple ideas. The idea of extension is always content in a form. The idea of coexistent and contiguous coloured points is a legitimate imaginary idea because it has a certain quantity and quality and resembles past or present impressions.

The abstract or *general* idea of extension—form without content—requires a bit more work. To generate the idea of form devoid of content requires a *distinction of reason*. The mind has varied ideas of extended objects of all shapes, sizes, and colours. The mind compares these different complex ideas and, despite the contrariety, can identify what is *common*. Through a *distinction of reason* the mind can identify a common ‘spatial’ aspect, even though the imagination cannot separate it from complex ideas. Hume writes, “...we begin to distinguish the figure from the colour by a *distinction of reason*; that is, we consider the figure and colour together, since they are in effect the same and undistinguishable; but still view them in different aspects, according to the resemblances, of which they are susceptible.” (T 1.1.7.18 SBN 25) The mind strains to conceive the form void of content. This strained idea becomes the abstract idea of an extension. Hume writes “...we omit the peculiarities of colour, as far as possible, and found an abstract idea merely on that disposition of points, or manner of appearance, in which they agree.” (T 1.2.3.5 SBN 34) This is Hume’s psychological account of how we come think we have an abstract idea of an extension void of content. Obviously, such an idea is formed by the imagination. However, is it a legitimate or illegitimate one?

Hume’s definitive answer is that the idea of an extension void of content is illegitimate. Hume calls such an idea a vacuum. It is an illegitimate imaginary idea because it lacks a determinate quantity or quality and does not resemble an impression. Any legitimate idea of extension is always conjoined in its conception with the idea of contiguous and coexistent coloured points. Abstracting from content or a negation of content does not produce an *actual* idea. As Wayne Waxman explains, the mind lacks an “aspect-seeing power.” Waxman writes: “The question for Hume was whether abstractions like these

deserve to be counted as perceptions. His answer was negative. To accord them this status requires that we suppose ourselves endowed with a special abstractive mental capability, a power of “aspect-seeing” as it were, whereby intuitively and immediately to apprehend (perceive) abstracta (distinctions of reason).”¹⁵

The idea of form without content is still an idea. However, such an idea does not correspond to or resemble any impression. The idea of extension or space is not something separate and distinct from contiguous and coexistent coloured points. Extension is merely a *relation* that holds between particular contents, i.e., tangible points. If you destroy the idea of individual coloured (or otherwise tangible) points you destroy the idea of extension. Let us borrow an example from Locke, who explains how the idea of a flock is the idea of individual sheep arranged in a certain manner. It follows that if you destroy every sheep you destroy the flock. Additionally, the idea of a single sheep is not the idea of a flock. The idea of a sheep is *categorically* different from the idea of a flock. Analogously, the idea of extension involves the relation of coloured and simple points contiguously arranged. If you destroy the coloured and simple points you destroy the relation, and thereby destroy extension. Furthermore, the idea of one simple and coloured point is categorically different from the idea of extension. One simple and coloured point is not extended. It is extensionless. Hume’s lead argument is designed to eliminate the possibility that the idea of a finite extension contains an infinite number of these coloured points, or that each of these points is infinitely divisible.

IX. Humean Understanding of Contradictory Ideas

The final limit of the mind I want to discuss is the impossibility of having a clear idea of a contradiction. This is critical for the lead argument because Hume believes the idea of a finite extension contradicts the idea of infinite divisibility. Hume writes a “contradiction...’tis impossible...that it can be conceiv’d.” (T 1.3.9.10 SBN 111) For example, Hume writes “We can form no idea of a mountain without a valley.” (T 1.2.2.7 SBN 32) To understand what makes a contradictory idea *inconceivable*, let us first consider what makes an idea conceivable.

I will argue that for Hume, an idea is conceivable if it resembles an impression or impressions. Hume often employs the maxim “*That whatever the mind clearly conceives includes the idea of possible existence*, or in other words, that *nothing we imagine is*

¹⁵ Waxman, Wayne. "The Psychologistic Foundations of Hume's Critique of Mathematical Philosophy." *Hume Studies* 22.1 (1996): 123-67. Print., p 132.

absolutely impossible” (T 1.2.2.7 SBN 32) to argue things like a coloured and tangible point is possible (SBN 38), or that a change in the course of nature is possible (SBN 89). Hume often uses “conceives” interchangeably with “imagines.” “Conception” for Hume is always an *act* of the mind (and usually the imagination). Hume writes, “...the *imagination*...run[s] along the parts of space and time in *conceiving* its objects,” (T 1.1.4.2 SBN 11) and “...the *imagination* *conceive*[s] it with all its particular circumstances and proportions.” (T 1.1.7.7 SBN 20) “Clear” ideas are usually impressions or ideas derived from impressions. Hume writes “...impressions are clear and precise, the ideas, which are copy’d from them, must be of the same nature” (T 1.3.1.7 SBN 72-3) Thus “*whatever the mind clearly conceives includes the idea of possible existence*” intimates that imagined ideas that resemble impressions are possible. Ideas resemble impressions by satisfying the determinacy criterion and having relations derived from experience.

Importantly, Hume believes that whenever two ideas contradict each other we can infer that the object or objects of these ideas cannot exist. He writes that if there is a “...formal contradiction...’tis impossible not only that it can exist, but also that it can be conceiv’d.” (T 1.3.9.10 SBN 111) For example, Hume writes “We can form no idea of a mountain without a valley, and therefore regard it as impossible.” (T 1.2.2.7 SBN 32) Because we cannot form an idea of a valley-less mountain, we therefore regard it as impossible. Hume expresses this inference from the contradiction of ideas to the impossibility of their objects when he writes “WHEREVER ideas are adequate representations of objects, the relations, *contradictions* and agreements of the ideas are all applicable to the objects.” (T 1.2.2.1 SBN 29, emphasis added) Supposedly, if our idea of a mountain and valley are “adequate representations” of mountains and valleys the “relations, contradictions and agreements” of our *ideas* of a mountain are applicable to mountains. Our idea of a mountain necessarily implies a valley, and therefore we regard it as impossible that an actual mountain as an object exists without a valley.

The mind cannot conceive a contradiction by combining contradictory ideas into a complex idea and therefore, supposes the object of such a complex idea to be impossible. The mind cannot form a complex idea XY from two ideas that contradict each other. Because there can be no object without an idea of it, there is no object XY, e.g., no square triangle or valley-less mountain. This is an exhaustive account of the first half of principle one which states “the capacity of the mind is limited.” I will now consider a criticism levelled against this principle by Antony Flew.

X. Equating Conceiving with Image Formation

Antony Flew argues that Hume erroneously equates conceiving with image formation, and reads him as rejecting the idea of infinity because the mind cannot form an infinite image. Contra Flew, there are a multitude of legitimate perceptions in Hume's *Treatise* that lack a one-to-one corresponding image. Consequently, Hume could and *does* have an image to represent a notion of infinity, even though we cannot form an infinite image. Ideas without a one-to-one image correspondence involve a legitimate use of the imagination to complete or continue a resembling series. Distinct ideas, such as the 10,000th part of a grain of sand, are comprehended through our adequate ideas of proportions and decimals (i.e. mathematical expressions of the relation of quantity). Much like we use a particular idea abstractly to represent an unlimited number of tokens (T 1.1.7), the mind employs an individual image to represent an unlimited number of distinct ideas beyond the threshold of sensation.

Flew writes that Hume "...equates conceiving with imagining, and mistakes it that to imagine—or at any rate to be able to imagine—is always to form—or at any rate to be able to form—the appropriate mental image or images."¹⁶ This is a common interpretation. Baxter writes "Ideas for Hume are like images."¹⁷ Mark Pressman writes "Hume maintains that ideas "are images" (T 6),"¹⁸ Arthur Pap says, "'Idea' is in Hume's usage synonymous with 'mental image'"¹⁹ and D.C.G MacNabb notes "Hume thought of ideas as images, and primarily as visual images."²⁰ I am not denying that Hume associates ideas with mental images. Mental images can be representative of a vast number of different and distinct legitimate ideas. This includes an 'image' of unlimited aggregation or removal of simple ideas, which is Hume's image or notion of infinite extension or infinite division. Equating conceiving with image formation is contrasted with Descartes' position that it is impossible to form a mental image of a chiliagon, but it is easy to conceive of one. Hume would say we have an adequate idea of decimals and proportions, and thus can understand what is meant by a chiliagon. However, we would employ the same inadequate image to represent either a chiliagon (1,000 sides) or myriagon (10,000 sides). In this way, Hume does not strictly

¹⁶ Flew, Antony. 257-70.

¹⁷ Baxter, Donald. "Hume's Theory of Space and Time in Its Skeptical Context." pg. 106.

¹⁸ Pressman, H. Mark. "Hume on Geometry and Infinite Divisibility in the *Treatise*." *Hume Studies* 23.2 (1997): 227-44. Print., pg. 227.

¹⁹ Arthur Pap, *Semantics and Necessary Truth* (New Haven: Yale University Press, 1958), 75.

²⁰ D.C.G. MacNabb, "Hume," in *The Encyclopedia of Philosophy*, edited by Paul Edwards (New York: MacMillan and The Free Press, 1967) Volume IV, 79.

equate conceiving with image formation, as legitimate ideas of vastly large, small, or complex objects can have a 2-1 or 100-1 idea-to-image correspondence.

The mind conceives insensibly small parts by extrapolating that a series of impressions decreasing in size continue beyond the threshold of sensation. These ideas are legitimate because the *smaller-than* relation is derived from experience, and the imagined parts resemble impressions and are determinate, that is, particular, though insensible, contents are attributed to them. The limited nature of sight and touch determines that our *impressions* consist of sensible minima. Hume writes:

Put a spot of ink upon paper, fix your eye upon that spot, and retire to such a distance, that at last you lose sight of it; 'tis plain, that the moment before it vanish'd the image or impression was perfectly indivisible. 'Tis not for want of rays of light striking on our eyes, that the most minute parts of distant bodies convey not any sensible impression; but because they are remov'd beyond that distance, at which their impressions were reduc'd to a *minimum*, and were incapable of any farther diminution. (T 1.2.1.3 SBN 27)

The mind receives diminishing ink spot impressions and associates them through resemblance and contiguity. The resemblance and contiguity of these numerically distinct impressions generates the idea that these are different impressions of *one single* ink spot. In addition, the mind is led by a kind of inertia to continue the *smaller-than* relation beyond the threshold of sensation and posit smaller parts of the sensibly indivisible impression. This act of the imagination supplements, and to an extent rectifies, the inadequacy of our sense faculties and seeks the truth of how things 'really' are. Hume writes:

Nothing can be more minute, than some ideas, which we form in the fancy; and images, which appear to the senses; since there are ideas and images perfectly simple and indivisible. The only defect of our senses is, that they give us disproportion'd images of things, and represent as minute and uncompounded ***what is really great*** and compos'd of a vast number of parts. This mistake we are not sensible of; but taking the impressions of those minute objects, which appear to the senses, to be equal or nearly equal to the objects, and ***finding by reason***, that there are other objects ***vastly more minute***, we too hastily conclude, that these are inferior to any idea of our imagination or impression of our senses. (T 1.2.1.5 SBN 28, emphasis added)

In fact, microscopes confirm this just 'reasoning.'²¹ However, our ability to reason that sensibly indivisible impressions (the ink spot at its vanishing point) are composed of *smaller* parts does not mean that infinitely small ideas exist. Because the mind is finite it cannot have

²¹ Hume writes "A microscope...renders them visible, produces not any new rays of light, but only spreads those, which always flow'd from them; and by that means both gives parts to impressions, which to the naked eye appear simple and uncompounded, and advances to a minimum, what was formerly imperceptible."(T 1.2.1.4 SBN 28)

an *idea* of a part of extension that is *infinitely* small. The mind can, however, form distinct ideas of parts smaller than any that are sensible. Hume writes “This however is certain, that we can form ideas, which shall be no greater than the smallest atom of the animal spirits of an insect a thousand times less than a mite.”(T 1.2.1.5 SBN 28) How is it that the mind can form an idea of something minute as to be insensible?

These ideas of the minute involve the application of our adequate ideas of decimals and proportions. In Section VII Hume explains that the mind does not have an adequate idea of one thousand, but *does* have an adequate idea of proportions, ratios, or decimals. Using our idea of proportion, the mind can produce ideas of the very large and of the minutely small. Hume writes “...when we mention any great number, such as a thousand, the mind has generally no adequate idea of it, but only a power of producing such an idea, by its adequate idea of the decimals, under which the number is comprehended.” (T 1.1.7.12 SBN 23) Mathematical expressions of proportion in the form of decimals and ratios—though never perfectly observed in experience—call to mind the inexact proportions of *greater-than* and *less-than* that are experienced. We know that $\frac{1}{2}$ is *greater-than* $\frac{1}{4}$, or that 1,000 is *less-than* 10,000. In this indirect way, the mind can understand what is meant by “the smallest atom of the animal spirits of an insect a thousand times less than a mite.”

The mental image of these vast quantities will be inadequate and cannot faithfully represent every part or member of these complex ideas. Yet, because we have an adequate ideas of proportions derived from experience, and understand their mathematical expressions, we know that the thousandth and ten thousandth parts of a grain of sand are distinct and different ideas, even though we cannot form a mental image that adequately represents either. Hume writes:

...the imagination reaches a minimum, and may raise itself an idea, of which it cannot conceive any sub-division, and which cannot be diminished without a total annihilation. When you tell me of the thousandth and ten thousandth part of a grain of sand, I have a distinct idea of these numbers and of their different proportions; but the images, which I form in my mind to represent the things themselves, are nothing different from each other, nor inferior to that image, by which I represent the grain of sand itself, which is suppos'd so vastly to exceed them. (T 1.2.1.3 SBN 27)

The mental images of the grain of sand and a piece ten thousand times inferior are indistinguishable. Still, the thousandth and ten thousandth parts are “distinct” and adequate ideas, for Hume. As Frasca Spada puts it, there exists “...a relation, expressed in terms of numbers and their proportions, between those levels,” but there is “[a]n imaginal apparatus which always employs exactly the same materials to represent to the mind each of the

different levels.”²² Having a particular image represent a vast number of individuals is analogous to Hume’s account of abstract ideas. Hume writes that the vast use of decimals and proportions “...seems to be an instance parallel to the present one of universal ideas.” (T 1.1.7.12 SBN 23) How does this parallel instance work?

Ideas of the vastly minute are formed similarly to abstract ideas; in the case of abstract ideas, a particular idea becomes general in its representation; in the case of the vastly minute, the minimal image becomes general in its representation as well. The minimal image of a complete grain of sand represents the thousandth or the ten-thousandth part of that grain. An abstract idea is nothing but a collection of particular ideas annexed to a general term, but through reason “Abstract ideas...may become general in their representation.” (T 1.2.7.6 SBN 20) The mind performs a similar operation with respect to the insensibly minute. The mental image is identical regardless of whether it represents the thousandth or ten-thousandth part. It is through decimals and proportions that the image can become general in its representation. Consequently, the lack of a mental image of these parts “...is never felt in our reasonings.” The minimal image can stand for either the thousandth or ten-thousandth parts, each of which is a distinct idea. However, these distinct ideas are not direct copies of distinct impressions, and thus do not have one-to-one corresponding images. As Frasca Spada explains, Hume “...is talking precisely of a distinct idea existing despite the absence of a corresponding image.”²³ And whatever is distinct is different. These different ideas are represented by the same image. There could be a two-to-one or a hundred-to-one idea to image correspondence.

The position that Hume strictly equates conceiving with image formation ignores the vast world of perceptions acquired by this imaginative reasoning “...which peoples the world, and brings us acquainted with such existences, as by their removal in time and place, lie beyond the reach of the senses and memory. By means of [this reasoning] I paint the universe in my imagination, and fix my attention on any part of it I please,” (T 1.3.9.3 SBN 108). This universe of perceptions includes the insensibly small and the vastly large. In Hume’s world, there is an abundance of distinctly conceived and adequate ideas that do not have one-to-one corresponding images. Though the “imaginal apparatus which always employs exactly the same materials to represent to the mind each of the different levels” is a form of conceiving

²² Frasca Spada, “Reality and Coloured Points in Hume’s Treatise,” pg. 41.

²³ Frasca Spada, Marina. “Hume’s Conception of Space.” *Studies in the History of Philosophy and Science* 21.3 (1990): 371-411. Print.

associated with image formation, the utility of these images is nearly unlimited. They can be used to represent vast parts of the real world of which the mind cannot form distinct images.

Flew argues that Hume denies a full and adequate conception of infinity because the mind cannot form a mental image of an infinite collection of objects. Flew writes: “Since conceiving is thus identified with imagining, and since both are always thought of in terms of forming mental images, to “attain a full and adequate conception of infinity” would be to have a clear and distinct idea, or image, of infinity.”²⁴ But distinct ideas for Hume are not limited to mental images. Furthermore, Hume does have a mental image of an unlimited aggregate of simple ideas that, while not picturing infinity, *represents* it. He also has a “quicksilver” image-idea that represents infinite divisibility. So the question remains: if it is not because Hume equates conceiving with mental image formation, why does he think the mind cannot have a full and adequate conception of infinity?

Part Two: A Full and Adequate Conception of Infinity

Principle one states that “the capacity of the mind is limited, and can never attain a full and adequate conception of infinity.” Part one delineated the “limits” of the mind, and part two considers what is meant by a “full and adequate conception of infinity.” Frasca Spada ties this premise to Port Royal’s axiom “it is the nature of the finite mind to be incapable of comprehending infinity” and the Cartesian proposition “Our mind, because it is finite, can know nothing certain of the infinite.” Frasca Spada concludes, “In fact, in Hume’s text, “comprehending” and “knowing something for certain” are transformed into “having a full and adequate conception.””²⁵ This is not quite right. Pace Frasca Spada, I will define “full and adequate conception” according to Hume’s uses of these terms in Book One.

This part will be broken into three sections. The first defines the relevant terms and explain why a full and adequate conception of infinity, besides being oxymoronic, could not be constructed by a legitimate use of the imagination. The second explains what the legitimate idea of infinity is, within Hume’s system. This involves the employment of a mental image to represent infinity. The third section will respond to Flew’s criticism that “...there is no insuperable difficulty about learning the ordinary uses of the words “infinite” and “infinity.” We can perfectly well understand what is meant by talk of a series’ being

²⁴ Flew, 259.

²⁵ Frasca Spada, “Hume’s Conception of Space,” 397.

infinite or going on to infinity”²⁶ and show that this criticism misses the mark because Hume does endorse this ordinary language theory of meaning for the term ‘infinity.’ Hume’s premise is that the mind cannot have a *full and adequate* conception of infinity, which is quite different from a *notion* or language functioning idea of infinity.

I. Relevant Terms

Conception: Conceiving is an *act* of the mind involving the imagination. A “conception” is an imaginary idea. There is no impression of infinity, and thus there is no memory-idea, either. Infinity is an imaginary conception of a particular kind.

Full and Adequate: An adequate idea has a determinate quantity or quality, has sensibly-derived content (and therefore resembles an impression), and faithfully represents an object. I do not think “full” is a technical term for Hume. I would guess he means *can admit no further addition*. Therefore, a *full and adequate* idea has a determinate quantity or quality, resembles an impression, faithfully represents its object and admits of no further addition.

It follows that the limited mind can never attain a *full and adequate* conception of infinity. A *full and adequate* conception of infinity, as defined by Hume, is almost an oxymoron. A full and adequate conception is an imaginary idea that resembles an impression, is determinate with fixed quantity and quality, and admits no further addition. Infinity is commonly defined as “an indefinitely great amount or number,”²⁷ and is therefore never a fixed or definite quantity. Because impressions have a determinate quantity and quality, an indeterminate idea like infinity can never resemble an impression. Also, an idea of infinity can never be complete or full, as it is that which always admits of addition or division. Considering infinite division, Hume writes “The number of fractions bring it no nearer the last division, than the first idea it form’d. Every particle eludes its grasp by a new fraction; like quicksilver, when we endeavour to seize it.” (T 1.2.4.14 SBN 44) Infinity cannot be a full and adequate idea because it is never a determinate quantity or quality. By contrast, a fraction such as the ten-thousandth part of a grain of sand is a full and adequate idea because it resembles an impression (i.e. a grain of sand), has a fixed quantity and quality, and admits of no further addition or division without becoming another idea.

²⁶ Flew, 259.

²⁷ <http://dictionary.reference.com/browse/infinity?s=t>

II. A Legitimate Conception of Infinity: Infinite Extension

Importantly, Hume believes there is a conception of infinity the mind can have. It is reminiscent of Locke's idea of infinity as "a growing and fugitive idea, still in a boundless Progression that can stop no where."²⁸ Hume begins with his conceptual "least idea." Insofar as he bestows a colour and solidity upon this idea, it is conceivable by the mind. He then augments this idea and realises it would become infinitely extended. Hume writes:

I first take the least idea I can form of a part of extension...I then repeat this idea once, twice, thrice, &c. and find the compound idea of extension, arising from its repetition, always to augment, and become double, triple, quadruple, &c. till at last it swells up to a considerable bulk, greater or smaller, in proportion as I repeat more or less the same idea. When I stop in the addition of parts, the idea of extension ceases to augment; and were I to carry on the addition *in infinitum*, I *clearly perceive*, that the idea of extension must also become infinite. (T 1.2.2.2 SBN 29-30, emphasis added)

Hume's phrase "clearly perceive" indicates this idea's legitimacy. We understand that *if* the human mind had an infinite capacity to augment extension it could augment extension to infinity. The mind clearly perceives what infinity would be like, even though it cannot form a *full and adequate* conception of it. The imagination can form this notion of an infinite extension because it infers that the relation of *greater-than* (quantity) in a series of resembling impressions or ideas can be repeated again and again. The mind can conceive this notion of infinity insofar as the augmenting always involves the addition of determinate parts that resemble impressions. This "image" of an unlimited aggregate can then represent our idea of infinity even though no mental image is adequate.

III. Flew's Criticisms

The following is Flew's criticism of Hume's first principle:

In Hume's formulation, straightforwardly interpreted, it is surely untrue. For there is no insuperable difficulty about learning the ordinary uses of the words "infinite" and "infinity," and we can perfectly well understand what is meant by talk of a series being infinite or going on to infinity. But, of course, Hume is not construing the expression "attain a full and adequate conception" in any such studiously simple-minded way. For he equates conceiving with imagining, and mistakes it that to imagine—or at any rate to be able to imagine---is always to form—or at any rate to be able to form—the appropriate mental image or images.

I do not think Hume denies a full and adequate conception of infinity because he mistakenly construes conceiving with image formation. A plethora of ideas in Hume's philosophy do not

²⁸ An Essay Concerning Human Understanding, Book II chap. XVII, sect. 12, P.H. Niddich (ed.) (The Clarendon Press: Oxford, 1975). P. 216.

have a one-to-one image correspondence. Following from his account of abstraction, a single image, such as a grain of sand, can represent an unlimited number of distinct ideas, such as the thousandth or ten-thousandth parts thereof. These ideas become *distinct* through the attribution of content found in an impression and form determined by our adequate ideas of proportions and decimals.

The mind can employ an idea of infinity in a similar way. Flew is right that Hume does not believe we can form an adequate mental image of infinity. However, we can form an image of augmenting simple ideas, and have the notion that that procedure could continue forever. This idea, or “image” if you will, of infinity, or its inverse (i.e. repeated division), can *represent* our ideas of infinity. In this way Hume is comfortable using the term “infinite” in the *Treatise*. For example, when discussing the conjunction of impressions and resembling ideas, Hume writes “Such a constant conjunction, in such an *infinite number* of instances, can never arise from chance” (T 1.1.1.8 SBN 4, emphasis added). He means that from the multitude of resembling impressions and ideas, we extrapolate that the constant conjunction will continue forever. Or, when discussing abstract ideas, Hume writes “If ideas be particular in their nature, and at the same time finite in their number, ‘tis only by custom they become general in their representation, and contain an *infinite number* of other ideas under them.” (T 1.1.7.16 SBN 24) For example, the mind has had a finite number of main impressions. However, the number of instances continues to grow, and the imagination extrapolates that this could continue forever. To summarise, we have a legitimate imaginary idea of an infinite number as being the repeated addition or removal of parts forever. And we can surmise from Hume’s writing that he believes this idea or notion of infinity is a meaningful term for use in language.

By my reading, Flew’s criticism that “there is no insuperable difficulty about learning the ordinary uses of the words “infinite” and “infinity,” and we can perfectly well understand what is meant by talk of a series’ being infinite or going on to infinity” misses the mark. Hume *does* hold that we have an ordinary understanding of infinite, and this ordinary understanding is grounded on the notion of the unceasing addition or removal of parts. The fact is, Hume writes that the mind can never attain a *full and adequate* conception of infinity. If Hume had written that ‘the mind is limited and never attains a notion of infinity’, then Flew’s criticism would be accurate. However, Hume understands a *full and adequate* conception as an idea that has a determinate quantity or quality, resembles an impression, and

admits of no further addition without becoming another idea. That the limited mind cannot have this idea of infinity is manifestly true.

IV. Closing Remarks

I explained how the “clearly perceived” idea of infinity is the repeated aggregation of simple ideas. I believe the inverse reasoning applies as well, and that Hume would have to admit that we can conceive of infinite division. I think Hume’s description of infinite division as “Every particle eludes its grasp by a new fraction; like quicksilver, when we endeavour to seize it” illustrates that he believes he does have this notion, or can conceive of infinite division. This mental image of quicksilver can represent the idea of infinite division, and because “...*whatever the mind clearly conceives includes the idea of possible existence*” infinite division would seem to be a legitimate conception. Moreover, the infinite division of a finite extension would seem possible unless Hume can demonstrate that the infinite divisibility of a finite extension would require for its formation, the combination of contradictory ideas. Just as that sort of contradiction rules out square triangles and valley-less mountains, it might rule out infinitely divisible finite extensions. As it turns out, the incomplete “quicksilver” *notion* of infinite divisibility is inconsonant with the idea of a finite extension. This is because the idea of a finite extension must be composed of indivisible coloured points. The key is that Hume believes that having the idea of a finite extension is possible *only if* the idea is found through the association of unextended and therefore, indivisible, coloured points.

It may seem Hume ‘cooked the books’ by demanding that our idea of infinity be full and adequate. However, Hume’s principle is not designed to eliminate every notion of infinity. It allows for the notion of infinite divisibility (such as the “quicksilver” idea). His principle is intended to secure the premise that the mind could only attain a full and adequate idea of a finite extension, and the premise that “...the idea, which we form of any finite quality, is not infinitely divisible.” (T 1.2.1.2 SBN 27) The consequence is that any idea the mind forms of extension logically reduces “...to inferior runs, which will be perfectly simple and indivisible.” (T 1.2.1.2 SBN 27) Hume’s aim is to show that any idea the mind forms of extension must in theory be formed through the association of these indivisible coloured points; and because infinite divisibility requires an infinite number of parts (chapter 3, actual parts doctrine), the only idea of infinitely divisible extension the mind can form is an infinite extension, i.e. an infinite number of indivisible coloured points. Just as one can (and indeed,

must) combine the idea of a mountain with that of a valley, and cannot combine the idea of a square with the idea of a triangle, one can combine the idea of a finite extension with the idea of finite divisibility, and cannot combine the idea of a finite extension with the idea of infinite divisibility.

CHAPTER TWO: Hume's "Least Idea"

This chapter explains how we get from principle one (that "The capacity of the mind is limited, and can never attain a full and adequate conception of infinity") to the conclusion that an extensionless, indivisible, coloured point is the only adequate idea we can use to construct *conceptions* (i.e. imaginary ideas) of extension. Hume aggregates this "least idea" to conceive infinite extension. Infinite divisibility would require an infinite number of parts (Hume's actual parts doctrine, chapter three). The only idea of an infinite number of parts that the mind can construct is an infinite number of least ideas form an infinite extension. To Hume, the idea of infinite divisibility is identical with an infinite extension. Our idea of a finite extension is necessarily composed of a finite number of parts. Therefore, the idea of infinite divisibility contradicts the idea of a finite extension. The contradiction makes the inference to the finite divisibility of objects themselves more potent (chapter four). Therefore, Hume's "least ideas" are the only parts the mind can use to construct ideas of extension through association is critical because Hume's argument requires this contradiction.

I will argue that Hume's "least idea" is the product of an a priori argument. Hume cites indivisible sensible and imaginable minimums, not to ground his "least idea," but as empirical evidence to reinforce his conclusions. Baxter believes Hume's "least idea" is a copy of a sensible minimum, while Flew and Newman argue Hume's "least idea" is the minimal mental image of the imagination. I will explain why these readings are problematic and give an exposition of what I take to be Hume's a priori argument for his "least idea." Afterwards, I will consider standard mathematical complaints against Hume's "least idea" and the question of how extensionless points can produce extension by their conjunction. Hume is primarily concerned with the nature and limits of our ideas of extended objects and extension, which are discovered through logic and introspection. Standard mathematical objections, in the form of geometrical demonstrations or references to mathematical models that require infinite divisibility, inform us more about the powers of the imagination, and less about our ideas of extension derived from sense experience.

Part One: Hume's Perceptual Minimums

I. Perceptual Minimums

Baxter argues Hume's "least idea" is a memory-idea copy of a sensible minimum such as the ink spot at its vanishing point. Hume tells us to "[p]ut a spot of ink upon paper, fix your eye upon that spot, and retire to such a distance, that at last you lose sight of it; 'tis plain, that the moment before it vanish'd the image or impression was perfectly indivisible." Baxter writes "This last image is indivisible; it is a minimal impression. Minimal ideas are simply less vivid copies of such minimal impressions,"²⁹ and Baxter goes on to argue that "Armed with [these] minimal ideas Hume proceeds in *Treatise* 1.2 to argue confidently that space, (or as he calls it) "extension" is not infinitely divisible."³⁰ Is this memory-idea "...the least idea" the mind "...can form of a part of extension," and are we "certain that there is nothing more minute than this idea"?³¹

We know the senses have the defect of "...giving us disproportion'd images of things, and represent[ing] as minute and uncompounded what is really great and compos'd of a vast number of parts." The imagination reasons that a minimum impression, such as the ink spot at the brink of vanishing, is actually composed of smaller parts. Hume references our "...taking the impressions of those minute objects, which appear to the senses, to be equal or nearly equal to the objects, and finding by reason, that there are other objects vastly more minute." We are very much in the habit of finding, upon approaching a distant object, a number of parts that were not apparent at first. Conversely, from a gradation of diminishing ink spot impressions sharing the relation *smaller-than* the imagination's inertia would naturally continue the series beyond the threshold of sensation and posit even smaller resembling parts. So which idea is the smallest? Which is Hume's "least idea?" Is it the mountain-dot that first appears on the horizon, the speck of dirt that is barely visible on the mountain's surface, or both? Is it the copied idea of an indivisible impression at the brink of its vanishing point or or the imaginary idea of an insensibly smaller part of that minimum indivisible impression?

Newman argues Hume's "least idea" is the minimum imaginary idea of a part beyond the threshold of sensation. She writes:

...that which reveals reason to be false in advancing to infinity cannot be minima sensibilia, for these are powerless to prevent such advance, but must be those minimal

²⁹ Baxter, 108.

³⁰ Baxter, 108.

³¹ (T 1.2.2.2 SBN 29)

images which belong to the imagination...it is the minimal image reached by the imagination when it seeks to repeatedly divide a finite idea of extension (including an idea of the threshold of sight) which must be the idea corresponding to a minimal part of extension.³²

Flew also interprets this minimal image as Hume's "least idea," but complains it follows from Hume's error of equating conceiving with image formation. I think Flew's criticism might reveal a flaw in Newman's thinking. Hume says we have a "distinct idea" of the thousandth and ten thousandth parts of a grain of sand, which is represented by the same mental image. However, Flew might ask, why are these distinct ideas not the "least idea" the mind can form? Why is it the minimal mental image which arbitrarily represents either the 1,000th or 10,000th part, instead of the distinct idea of the 100,000th part? There is clearly a flaw with equating conceiving with image formation. Furthermore, and most importantly, is it rational to talk of ideas having size, images or not? Can the distinct idea of the 10,000th part be smaller than the idea of the 1,000th part? I think we can make better sense of these puzzling questions if we consider Hume's relational theory of extension.

II. Hume's Relational Theory of Extension

Thinking of an idea as having *size* or *extension* is bizarre. Thus asking which idea is smaller, the minimal imaginary image or the copied memory idea of an indivisible impression is equally bizarre. Allison thinks so and writes of extended ideas having size "...seems deeply paradoxical and is often dismissed as manifest nonsense."³³ However, it is only "manifest nonsense" if you ignore Hume's relational theory of extension. For Hume, extension is a relation between coexistent and contiguous coloured points. Hume writes that extension is "...merely those of the manner or order, in which objects exist." (T 1.2.4.2 SBN 40) "Space" is a general term that does not have any reality save its reference to particular collections of contiguous and coexistent objects, impression, or ideas. In this way, our ideas can be "extended" or "spatial" by having the content of coexistent and contiguous parts. For example, my idea of a boat is extended because the idea is composed of masts, hulls, and rudders that are contiguous and coexistent.

Ideas can be extended by being complex ideas of visible or tangible objects. In this way ideas can be of different sizes based upon the number of simple parts they are composed of. Thus, one object, impression, or idea has no extension because extension is a relation that requires at least two objects, impressions, or ideas. Hume writes:

³² Newman, 11.

³³ Allison, Henry. *Custom and Reason in Hume*. Oxford: Clarendon Press, 2008. Print. 51.

...let us take one of those simple indivisible ideas, of which the compound one of *extension* is form'd, and separating it from all others, and considering it apart, let us form a judgment of its nature and qualities. 'Tis plain it is not the idea of extension. For the idea of extension consists of parts; and this idea...is perfectly simple and indivisible. (T 1.2.3.13-4 SBN 38)

Hume's relational theory of extension follows the same logic as mathematical points and lines. A mathematical point has no length, breadth or depth (Hume's indivisible simple). A line is defined as two or more mathematical points. Thus length or extension is only generated if you have two or more mathematical points.³⁴ Therefore, if the question is posed, which is smaller, the single indivisible imaginary mental image or the single indivisible memory idea, then the answer is that they are the *same non-size*. This is because one impression or one minimal image has no extension. It therefore has no size, either. They are *sizeless* perceptions. Thus the *minimum* perception at any threshold of annihilation, whether it is sensational, memorial, or imaginary, is not *the* "least idea," but can *represent* the "least idea." This is why Hume systematically enumerates all of the *minimums* (with italicised emphasis) that perception provides, because any of these minimums can generate a "least idea." He writes "...the imagination reaches a *minimum*, and may raise up to itself an idea, of which it cannot conceive any sub-division, and which cannot be diminished without a total annihilation" (T 1.2.1.3 SBN 27) and "...their impressions were reduc'd to a *minimum*, and were incapable of any farther diminution," (T 1.2.1.4 SBN 27) and "A microscope or telescope...advances to a *minimum*, what was formerly imperceptible." (T 1.2.1.4 SBN 28) Any one of these *minimums*, because they are indivisible and singular, and therefore extensionless and sizeless, can represent the "least idea."

Importantly, the "least idea" is not a minimum reached by a process of reduction or division. The mind does not get tired, or reach old age, and stop the divisional process. The imagination has the power to carry on division and generate a notion of infinite divisibility (i.e. Hume's quicksilver idea) that always introduces a new threshold with a new minimum. What prevents this reasoning from proceeding *infinitum*? Consequently, it is best to read Hume's "least idea" as the consequence of an a priori argument. The minimum sensation (via eyes or microscope) and the minimum image merely represent this indivisible "least idea" that is a product of reasoning. Hume simply reinforces his a priori argument with experiments

³⁴ "A surface is defin'd to be length and breadth without depth: A line to be length without breadth or depth: A point to be what has neither length, breadth nor depth. 'Tis evident that all this is perfectly unintelligible upon any other supposition than that of the composition of extension by indivisible points or atoms. How else cou'd anything exist without length, without breadth, or without depth?" (T 1.2.4.9 SBN 42)

of the mind. The discovery of minimum impressions and ideas of the mind confirms his reasoning. Furthermore, if Hume's argument only relies upon his relational account of extension then it would be question begging. Thomas Holden writes "...if Hume's argument did indeed rest on his prior commitment to the thesis that whatever is extended is made up of an array of simple first elements, then it would be entirely question-begging...he can scarcely assume that extended things must be collections of such simples in order to refute infinite divisibility: this would be a manifest *petitio*."³⁵

Part Two: Hume's A Priori Arguments for his "Least Idea"

There is textual evidence that Hume's argument is a priori. First, Hume writes "In rejecting the infinite capacity of the mind, we suppose it may arrive at an end in the division of its ideas; nor are there any possible means of evading the evidence of this conclusion." (T 1.2.1.2 SBN 27) "The evidence of this conclusion" indicates to me that Hume believes he has provided a demonstration. Hume also writes that "...the division would be stopt short by the indivisible parts, which we should *immediately* arrive at." (T 1.2.2.2 SBN 29, emphasis added) The phrase "immediately arrive at," to me, indicates a logical implication. Hume also writes, "'Tis therefore certain, that the imagination reaches a *minimum*," and the language "therefore certain" indicates to me the necessary consequence of a logical argument.

I believe I can attribute the following argument to Hume for his "least idea." It follows from his principle that because "...the capacity of the mind is limited, and can never attain a full and adequate conception of infinity," then...

1. *Any idea of a finite extension, had by a finite mind, must contain a finite number of parts. Because the parts are finite in number, it immediately follows that the parts resolve themselves into indivisible parts or 'points.'*
2. *The idea of extension is a conception formed by the imagination, but it is still an idea. Therefore its parts must resolve into indivisible points as well.*
3. *The imagination can bestow a colour or solidity upon these indivisible points to make them adequate for the purposes of constructing conceptions of extension.*

Corollary: Hume's "least idea" is one of these indivisible coloured or tangible points. These points are the only "parts" that the limited mind can use to form a particular conception of extension.

Section one describes the transcendental arguments Hume employs to establish that our ideas of finite objects are composed of indivisible points (1). Section two revisits Hume's idea of

³⁵ Holden, Thomas. "Infinite Divisibility and Actual Parts in Hume's Treatise." *Hume Studies* 28.1 (2002): 3-26. Print., pg. 17.

extension as a *conception* formed by the imagination and discusses how a full and adequate idea is an imaginary conception as well. The conception of a full and adequate idea enables Hume to propose a dilemma between choosing whether or not our imaginary conceptions must be composed of ideas that are indivisible or infinitely divisible (3&4). We discover that our imaginary ideas of extension can only be constructed using indivisible simple coloured or tangible ideal points (4). This is important because the idea of infinity is an imaginary conception. Therefore, the only legitimate conception of infinity entails an infinite number of indivisible coloured or tangible points, which generates an infinite extension.

I. Transcendental Arguments for Ideas of Finite Parts

Any idea of a finite extension, had by a finite mind, must contain a finite number of parts. Because the parts are finite in number, it immediately follows that the parts resolve themselves into indivisible parts or 'points.'

Hume's empiricism demands that all ideas be traceable to preceding impressions. Hume's "least idea" is hardly given in sense experience, outside of contrived instances such as the ink spot experiment, which is not without its controversy.³⁶ For example, C.D. Broad writes "...so long as I am sure that I am seeing the spot at all, I am fairly sure that the sense-datum which is its visual appearance is *extended*, and not literally punctiform. So I very much doubt whether there are punctiform visual sense-data. The case for punctiform tactual sense-data would seem to be still weaker."³⁷ If I can demonstrate that Hume's "least idea" is a product of a priori arguments—and not a copy of a minimal impression (Baxter; Jacquette)—then the existence of an unextended "punctiform" sense impression is inconsequential to Hume's argument. Plus these perceptual 'points' are "...so minute and so confounded with each other, that 'tis utterly impossible for the mind to compute their number." (T 1.2.4.19 SBN 45) Thus Hume's "least idea" is not given in casual sense experience. The same is true of the minute parts of memory-ideas, which are copies of impressions. I cannot discern the minute parts that compose my memory-idea of the Eiffel tower. Furthermore, I could never enumerate all of the parts and conclude that they are finite in number. Thus Hume needs

³⁶ Peter Simons writes "...no one has ever perceived a point, or will ever do so, whereas people have perceived individuals of finite extent." Simons, Peter. *Parts: A Study in Ontology*. Oxford: Oxford University Press, 1987. Print., p 42. Fortunately, my "a priori" reading of Hume's "least idea" does not rest on the reality of an indivisible impression.

³⁷ Broad, p 166.

transcendental arguments for the claim that our impressions and memory-ideas of extended objects consist of simple indivisible points.

A transcendental argument begins with an idea and considers the conditions that make it possible. One such argument starts with the idea of an extended object, such as a line, surface, or three-dimensional object as having a terminus point or terminus points (edges and sides). An example is the idea of the Eiffel Tower having terminus points where the building ceases and the air begins. Hume argues that if the idea of a surface terminates, then the *terminus* points must be indivisible. Otherwise our ideas of a line or surface would never terminate, as the endpoint or edge would “...elude the grasp by a new fraction; like quicksilver, when we endeavour to seize it.” Therefore, insofar as we conceive a line or surface or object with terminus points, we must necessarily have the idea of indivisible points. Hume writes “...if we have no such idea, ‘tis impossible we can ever conceive the termination of any figure.” (T 1.2.4.16 SBN 44) Thus the ideas we form of a line, a surface composed of lines, or an object composed of lines and surfaces (Eiffel tower), must be composed of indivisible points. Hume writes that the “...schoolmen were so sensible of the force of this argument, that some of them maintain’d, that nature has mix’d among those particles of matter, which are divisible *in infinitum*, a number of mathematical points, in order to give a termination to bodies.” (T 1.2.4.15 SBN 44) Hume, however, rejects this admixture of indivisible and infinitely divisible particles as absurd. The sentiment remains the same. Peter Simons in his recent book *Parts* writes “It is fair to say that the hybrid position resulting from adding [atoms and infinitely divisible quantities] to a mereology has rarely been seriously entertained. Since the issue of atomism has been one which has been aired predominantly in metaphysics and the philosophy of nature, the compulsive tendency to a uniform picture of the world has left it out in the cold, where we too shall leave it.”³⁸ The termination of our ideas of extended objects requires the idea of an indivisible point. If our ideas are meant to uniformly represent nature, then it follows transcendently that our complex impression or ideas of *terminated* extended objects must be composed of points, even though these points are indiscernible through sensation or introspection.

Another one of Hume’s transcendental arguments explains that indivisible points condition the possibility of conceiving objects as equal, smaller-than, or greater-than one another (T 1.2.4.18 SBN 45). Equality, presumably, is true of two objects if they have the same number of parts. But if our ideas of extended objects are composed of an infinite

³⁸ Simons, Peter. *Parts: A Study in Ontology*. Oxford: Oxford University Press, 1987. Print., p 42.

number of parts, then the book and the ocean liner would be the same ‘size’ because they would both contain the same number of parts—infinity! Now, if the idea of a book and an ocean liner are composed of simple same-sized individual parts, then I can justly say the ocean liner has a greater dimension because it has more coexistent and contiguous simple parts. However, Hume explains that such a standard is “just,” but entirely “useless.” When we endeavour to count all of these parts they “...are so minute and so confounded with each other, that ‘tis utterly impossible for the mind to compute their number.” (T 1.2.4.19 SBN 45) Two objects are only theoretically or conceptually composed of an equal or different number of indivisible parts. However, this conception conditions the possibility of conceiving different sized objects. And because we *do have* conceptions of different sized objects, the indivisibility of simple parts is transcendently true.

Hume’s use of transcendental reasoning is not an aberration in Part II. Hume employs a similar transcendental argument for indivisible moments of time. Hume writes “‘Tis evident, that time or duration consists of different parts: For otherwise we cou’d not conceive a longer or shorter duration.” (T 1.2.3.8 SBN 35) Hume’s point is that the possibility of our conceptions of longer and shorter durations is conditioned by the reality of indivisible non-coexistent moments, even though these moments are insensibly brief. By analogy, our ideas of longer and shorter extensions are conditioned by the possibility of them being composed of a different number of fundamental parts. Because we *do* have ideas of different sized extended objects, it must be true that our ideas are composed of simple fundamental parts.

My a priori interpretation of Hume’s argument attributes to Hume a richer system than copy-principle empiricism. If *all* our ideas are directly copied from preceding impressions, then Hume’s “least idea” would have to be copied from a preceding impression (Baxter, Jacquette). However, all one needs to do is assert that “...no one has ever perceived a point, or will ever do so, whereas people have perceived individuals of finite extent,”³⁹ and Hume’s “least idea” would be denied, and his lead argument would fall to ruins. I agree with Graciela De Pierris that “In the original whole of extension, however, the minima out of which it is composed appear always confounded with one another and not as discrete parts of a larger whole. It is in precisely this way that Hume acknowledges the phenomenological appearance of continuity, and, at the same time, can nevertheless claim [through

³⁹ See FN 10.

transcendental argument] that extension is composed of simple and indivisible minima.”⁴⁰ My reading involves transcendental arguments from memory-ideas of extended objects and the imaginative synthesis with the idea of a full and adequate conception. However, because all of these ideas are *indirectly* traceable to preceding impressions I do not violate Hume’s empiricism, but bolster it by diverting emphasis away from his copy principle and towards the claim that our empirical reality and judgements therein, is inextricably bound with the operations of the imagination.

I will now consider mathematical objections to Hume’s position that our ideas of finite extensions must be composed of indivisible simples. This objection is that infinite divisibility implies the division and sub-division of *proportionally* smaller parts, which can be expressed by decreasing fractions adding up to but never exceeding one (i.e. $1/2 + 1/4 + 1/8$, etc.). I will then consider an objection to Hume’s adequacy claim, which maintains that it is nonsensical to colour something that has no extension and that the contiguity of unextended simples can never comprise extension (or in other words, how can objects with zero extension when added together generate a positive extension).

II. Mathematical Objections

It is often objected that infinite divisibility involves proportionally smaller parts, i.e. $1/2$, $1/4$, $1/8$, of a whole etc. An impression or object would need to be composed of points (i.e. $1/2$) that are both indivisible and divisible. However, I agree with Hume that marking this real number as an indivisible point or “unit,” while holding that it is also infinitely divisible, though perhaps mathematically viable, is an arbitrary denomination. (This is especially the case if we are considering our ideas of extended objects, and not necessarily mathematical constructions.) Hume writes:

‘Tis in vain to reply, that any determinate quantity of extension is an unite; but such-a-one as admits of an infinite number of fractions, and is inexhaustible in its sub-divisions. For by the same rule... twenty men may be consider’d as an unite. The whole globe of the earth, nay the whole universe *may be consider’d as an unite*. That term of unity is merely a fictitious denomination, which the mind may apply to any quantity of objects it collects together. (T 1.2.2.3 SBN 30)

A mathematical model such as Cantor’s real number line may be constructed with indivisible real numbers (units) that are themselves infinitely divisible (fractions/rational numbers), but

⁴⁰ Pierris, Graciela De. "Hume on Space, Geometry, and Diagrammatic Reasoning." *Synthese* 186 (2012): 169-89. Print., p. 172.

our idea of extension need not be expressed in terms of mathematical constructions. Hume would say the real number as an indivisible “unity,” if taken as an idea of extension, is “a fictitious denomination,” which the mind applies to the collection of rational numbers. If the mind can divide and sub-divide a unit of extension, such as a real number, then this unit is divisible, and can never be a simple element in an idea of a finite quantity. Hume recognised such a mathematical model but rejected it as “frivolous” because nestled intervals of infinitely smaller proportional parts could never logically be the composition of a finite idea of extension. Hume writes:

It has been objected to me, that infinite divisibility supposes only an infinite number of *proportional* not *aliquot* parts, and that an infinite number of proportional parts does not form an infinite extension. But this distinction is entirely frivolous. Whether these parts be call'd aliquot or proportional, they cannot be inferior to those minute parts we conceive; and therefore cannot form a less extension by their conjunction. (T 1.2.2.fn 1 SBN 30)

Why is this distinction entirely frivolous? First, it is frivolous because Hume’s relational theory of extension entails that a “perfectly simple and indivisible” point has no extension or size, and consequently there is no idea that exists smaller than what is sizeless (thus two proportionally smaller parts “cannot *form* a less extension by their conjunction”). Second, I think Hume would grant the mathematician his idea of infinite divisibility (Hume’s ‘quicksilver idea’ of infinite divisibility). However, the legitimacy of this idea is moot for Hume’s argument. Because infinite divisibility involves an infinite number of parts (chapter three), and any idea the mind *constructs* of a finite extension must be composed of a finite number of indivisible coloured or tangible points, no idea the mind constructs of a finite extension contains an infinite number of parts. The “quicksilver” idea, and its mathematical expression, is a different idea, resulting from a different operation of the imagination.

Many commentators dismiss Hume’s footnote and ignore how his relational theory provides a response to the mathematical objection. Flew uses this mathematical objection writing: “...absolutely crucially, to say that something may be divided in infinitum is not to say that it can be divided into an infinite number of parts. It is rather to say that it can be divided, and sub-divided, and sub-sub-sub divided as often as anyone wishes: infinitely, without limit.”⁴¹ James Franklin also challenges Hume on mathematical grounds writing: “...Hume was wrong about the supposed impossibility of infinite divisibility... In any case, we are dealing with mathematics... The infinite divisibility of space and time is possible. (This is

⁴¹ Flew, 260.

because there exists a consistent model which incorporates infinite divisibility, namely the set of infinite decimals.)”⁴² And Robert Fogelin levels the same criticism: “This argument isn’t any good...It is true that if we take a finite extension (however small) and repeat it ad infinitum, we will get an infinite extension. That, however, is quite beside the point, because the proof of infinite divisibility depends upon the possibility of constructing ever smaller finite extensions, as in the sequence [1/2, 1/4, 1/8, etc.] whose sum approaches, but does not exceed, 1.”⁴³ Other commentators who have similarly criticised Hume include John Laird, Mark Pressman, and J.M.M.H. Thijssen.⁴⁴

First, given Hume’s relational theory of extension, the mathematical distinction between *aliquot* (same-sized) parts and *proportional* parts is entirely frivolous. The idea of a simple and indivisible ‘point’ (which we established through transcendental argument) entails that the point is extensionless and sizeless. This means one: the idea of this point having proportionally smaller parts is impossible, and two: if this indivisible point *did have* proportionally smaller parts (which it does not), then those parts’ “conjunction” would form an extension and a length, and would therefore have a size and be ‘larger’ than the indivisible conceptual minimum they compose, which is absurd. Hume dismisses the distinction between aliquot and proportional parts using the same reasoning that delivered him the conclusion that the minute parts of an indivisible impression cannot be smaller than the impression (The parts form a greater extension “by their conjunction” than the indivisible impression). The difference is that the imagination can conceive that the indivisible impression *is* composed of smaller parts, while the imagination, for obvious reasons, cannot conceive that the *conceptual* minimum (i.e. “least idea”) is composed of smaller parts. Consequently, it is absurd to suppose that the indivisible minimum that the mind conceives has proportionally smaller parts *ad infinitum*. The mathematical objection concerns mathematical constructions, and need not necessarily concern our ideas of extension.

Second, I do not think the mathematician’s conception of a continuum of proportionally smaller parts is really an objection to Hume. As he says, it is “entirely frivolous.” The continuum of proportionally smaller parts seems like a mathematical

⁴² Franklin, James. “Achievements and Fallacies in Hume’s Account of Infinite Divisibility.” *Hume Studies* 20.1 (1994): 85-101. Print

⁴³ Fogelin, Robert. “Hume and Berkeley on the Proofs of Infinite Divisibility.” *The Philosophical Review* 97.1 (1988): 47-69. Print., pg. 51.

⁴⁴ John Laird, *Hume’s Philosophy of Human Nature* (Archon Books, 1967), 65-7; Pressman, *Hume on Geometry and Infinite Divisibility in the Treatise*, *Hume Studies* 23. (1997): 227-44, 241.; Thijssen, “David Hume and John Keill and the Structure of Continua,” *Journal of the History of Ideas* 53 (1992): 271-86, 280.

expression of the repeated removal of parts *ad infinitum*. But Hume has no gripe with this idea of infinity, with its expression being the “quicksilver idea.” Furthermore, as we have already seen, Hume has an explanation for how the mind forms and employs adequate ideas of decimals and proportions. That we can never have a *full and adequate* conception of infinite divisibility, and the infinite number of parts it entails, is a different matter, and must be rejected because of the finite mind.

Hume also has the resources to provide a *psychological* explanation for how the mathematician forms the concept of infinite divisibility. The mind receives a series of decreasing impressions bearing the *smaller-than* relation. Using the imagination the mind can posit ever smaller portions of extension by continuing the *smaller-than* relation to insensible levels. Through the mind’s adequate idea of proportions and decimals these ideas become distinct, despite the fact that these ideas are represented with the same mental image. Expressions such as the 10,000th part of a grain of sand, are legitimate products of the imagination because they are determinate, and can be conceived as having colour and solidity, which enables the parts to resemble the whole. Removing those properties from any member of the series is not contemplated by the associative rule (“resembling, but smaller than”) that created the series. And while the relation *smaller-than* is observed, the mathematicians’ relation “infinitely smaller-than” (like the mathematicians’ equality or point of intersection) is not. The parts are determined by the imagination’s power of associating with an impression, a series of ideas, each member of which series bears a relation to the impression (e.g. the *smaller-than* relation) of a type that has been observed. Infinite divisibility is a relation not derivable from experience, and is an idea that can never be determinately given in accordance within the limits of the imagination. Infinitely small can never be conceived with a particular quantity or quality, can never resemble an impression, and is therefore indeterminate. Consequently, the mind can never attain an adequate conception of an infinite number of proportionally smaller parts. The notion that proportionally smaller parts continues ‘ad infinitum’ does not prove the infinite divisibility of a *finite extension*, so much as it proves the power of the imagination to deceive us when we are not careful.

III. Solidifying and Colouring the Surfaceless and Extending the Extensionless

A criticism levelled against Hume by the physicalist is how we can colour or solidify something extensionless. As Harry Bracken writes, “Raynor and Ayers appear to interpret

Hume as on the one hand insisting that primary qualities cannot be conceived apart from the secondary, and on the other that in case of *minima* they can be...I agree that Hume sometimes holds both points and is hence caught in a contradiction.”⁴⁵ Without the primary quality of extension it seems as though the secondary quality of colour cannot be ‘painted on.’ This is related to, but critically different from, the objection that it impossible for an aggregate of extensionless points to form an extension. This physicalist objection was expressed by Bayle who writes, “...that several nonentities of extension joined together will never make up an extension,”⁴⁶ and Kemp Smith writes: “In other words, two *unextended* sensibles, *if contiguous*, will generate what is genuinely extended!”⁴⁷ I will reply to both objections in tandem. My response is that Hume does not need “genuine” extension, but only an *apparent* extension. Importantly, I will explain it is a *categorical mistake* to view a fundamental part (or what immediately whittles to a point through a priori reasoning) of a particular extension as itself extended. The idea or impression of a particular extension is a compound idea or impression, or Lockean simple mode⁴⁸; the formation of which necessarily requires two or more simple parts. Therefore, categorically speaking a simple part of extension cannot be extended. I will introduce a bit of set theory to help elucidate this point. Finally, it is not *contradictory* to colour or solidify extensionless points. The colour or solidity of these points cannot be rejected a priori. It is a priori truth that an impression of extension is a compound of parts and that a compound of parts is coloured (extension being a ‘primary quality’). It follows that a part cannot be extended, but it does not follow that a part cannot be coloured. The reason being that extension is a relation and *not* a part and not an attribute of any part, while colour is an attribute of each part that comprises the whole. I will utilise some nifty work by Pressman to elucidate this claim. The colour or solidity of these parts is a positive fact that follows transcendently from the *appearance* of extension as being colored or solidified, in accordance with Hume’s principle that the parts must resemble the whole, even in the case of the insensibly minute and sensibly (but not intellectually) confounded points that, when associated and bound by the imagination, comprise an impression of extension.

⁴⁵ Bracken, Harry. "On Some Points in Bayle, Berkeley, and Hume." *History of Philosophy Quarterly* 4.4 (1987): 437. Print.

⁴⁶ Bayle, Pierre, ‘Zeno of Elea’, *Historical and Critical Dictionary: Selections* ed. and trans. Richard H. Popkin (Indianapolis, Ind.: Bobbs-Merrill Company, Inc. 1965), p 359-60.

⁴⁷ Kemp Smith, pg 300.

⁴⁸ [Essay II xii 4-5](#)

Kemp-Smith criticises the logical possibility of how a composition of extensionless points could, when strung together, be extended. Fortunately there is a strong rational argument from set theory that demonstrates how this is possible.

Steenburgh's argument for durationless parts of time can be turned into an argument for unextended parts of an extension. Steenburgh argues for the intelligibility of durationless parts of time by identifying the essence of time as being an order of "concreta," where "concreta" is impressions, ideas, or objects. Essential to time is change from the non-existence of a moment, to existence, and back to non-existence. Steenburgh also distinguishes between "proper duration," i.e. succeeding impressions, and "fictive duration," i.e. applying duration to an unchanging impression. Steenburgh argues that it is logically necessary that proper duration is composed of durationless moments. He writes:

Given that the essence of time is the order of concreta... and given the distinction between proper and fictive duration, we have at hand the premises sufficient to prove that there must be durationless moments. The argument is as follows: the T!'s in a proper duration cannot themselves have proper duration. The sense 'cannot' is logical. No element in an order is identical with that order. It follows that all T!'s—all moments or parts of time—are devoid of real, or proper, duration.⁴⁹

Steenburgh symbolises an unchanging object, or moment of time, as "T!" A real or proper duration is comprised of a 'changing existent,' which Steenburgh notates as {~T!, T!, ~T!}, for example, a musical note does not exist, exist, and then not exist. Each T! is an element or moment of the notation set of proper duration, and because "no element in an order is identical with that order," it is logically the case that a moment or part of time cannot itself be or have proper duration.

The same principles of set theory apply to extension. E! represents the indivisible extensionless minimum of sensation (the minimum ink spot) or the least idea (which follows from the finite nature of the mind). A composition of E!'s forms the set {E!, E!}, which becomes extension. {E!, E!} is not E!, and because "no element in an order is identical with that order," it follows that a part of extension cannot itself be or have extension. The following is essentially Steenburgh's set theory about time, applied by Hume to extension:

Every idea, that is distinguishable, being also separate, let us take one of those simple indivisible ideas, of which the compound one of extension is form'd, and separating it from all others, and considering it apart, let us form a judgment of its nature and qualities. 'Tis plain it is not the idea of extension. For the idea of

⁴⁹ Steenburgh, E.W. Van. "Durationless Moments in Hume's Treatise." *Bicentary Papers*. Ed. G.P. Morice. Edinburgh: The University Press Edinburgh, 1977. 181-86. Print., p 184. In the article Steenburgh notates an unchanging impression as E!, but I have changed it to T! for clarity.

extension consists of parts; and this idea, according to the supposition, is perfectly simple and indivisible. (T 1.2.3.13-4 SBN 38)

It is logically the case that the idea of a part is not identical with our idea of a whole (which is composed of parts). In the case of extension, our idea of a part cannot be itself the whole, i.e. extended. However, this does not violate Hume's principle that the parts must resemble the whole because extension, as opposed to colour or solidity, is not a resembling quality inherent in the members of a set, but a relation that holds between the members—a relation that none of the individual members (by itself) inherently possess. With the possible exception of the identity relation ($A=A$), relations require, for their fulfilment, more than one object. That is the case with the "smaller-than" relation. All of the soldiers in an army can have resembling uniforms, but none of the soldiers is an army.

Most importantly, Hume's lead argument only needs our fundamentally indivisible *ideas* to form an *appearance* of extension. I take it as obvious that our ideas of extended objects only have *apparent* extension. I have the Eiffel Tower idea, and this idea is extended inasmuch as it has coexistent and contiguous parts. My Eiffel Tower idea cannot be *physically* divided. No one can open up my brain and cut the Eiffel Tower in half. The idea and its parts are not genuinely extended but have the *appearance* of extension. Our dreams are another example of apparent extension or dimension. When I dream that I am on a boat in the Caribbean, this idea of the boat, and all of its parts, have no *genuine* extension. The only genuinely divisible "boat" would be the synapses firing in my brain that correspond to my dream-boat, but we are discussing ideas and not synapses. The fact that the my dream-boat does not divide into an infinite number of parts follows from the fact that it is a finite mind having this idea, and that the mind can only gather and associate a finite number of parts.

According to his relational theory Hume would need to maintain that sensational extension, like dream extension, is merely *apparent*, but differs in terms of force, liveliness and vivacity (FLV). Impressions of extension are associated contiguous and coexistent coloured or tangible—but not individually—extended—points. The increased FLV of impressions of extension leads to belief and the claim that it is *genuine* or *real* extension, in contrast with apparent ideas of extension. But this distinction is fictive and can be eliminated with sound reason. For Hume, both sensations and ideas occur in the limited mind, thus Hume's lead argument could equally be applied to impressions. Whether external objects, which are presumably the cause of our impressions and ideas have physical extension is an entirely different question, which I will consider in chapter four.

The lead argument can be equally applied to impressions, as well as ideas, to generate the same conclusion that our impressions are composed of indivisible minima whose aggregation forms the *appearance* of physical extension. It would be as follows: The faculties of sensation are limited. We cannot have a full and adequate impression of infinitely smaller or larger. This limit is confirmed by the ink spot experiment. The minutely small limit *is* the empirical threshold before annihilation. If this threshold did not exist, then the faculties of sense would be infinite in their power. This threshold, or most minute impression, is indivisible to sight or touch, and cannot itself be extended. Thus it cannot be ‘physical’ either, if physical implies divisible parts. Therefore, the ‘physical reality’ of impressions of sight and touch is merely an appearance. My reading is similar to De Pierris’ who, responding to Holden, writes:

I agree, in particular that all of the indivisible parts of extension with which Hume is concerned—that is, all sensible minima—necessarily have the same size. Yet I disagree with Holden that Hume’s conception of extension and sensible minima derives from contemporaneous early modern theories of matter. Hume’s sensible minima are not physical atoms, and, more importantly, the sense in which such minima all have the same “size” is entirely different...than any notion applicable to physical atoms.⁵⁰

Extended sensible objects are composed of innumerable coloured or tangible points that are indivisible and not physical. However, the composition and conjunction forms the vivid and forceful appearance of ‘physical’ extended objects, just as ideas and dreams have the appearance of being physical. The difference between a dream world and the world of sensation differs merely in belief,⁵¹ which is grounded in FLV. Thus Hume writes “...the *belief*, or *assent*, which always attends the...senses, is nothing but the vivacity of those perceptions they present; and that this alone distinguishes them from the imagination.” (T 1.3.5.6 SBN 86)

Whether the *conceptual* threshold that constitutes Hume’s “least idea” is somehow coloured or tangible is a different question. The colour or tangibility of these fundamental parts follows transcendently from the fact that *appearances*--the whole made up by the parts—is coloured or tangible, and because the parts must *resemble* the whole, the parts are themselves coloured or tangible. Hume writes:

Now such as the parts are, such is the whole. If a point be not consider’d as colour’d or tangible, it can convey to us no idea; consequently the idea of extension,

⁵⁰ De Pierris, fn 12.

⁵¹ “An opinion, therefore, or belief may be most accurately defin’d, A LIVELY IDEA RELATED TO OR ASSOCIATED WITH A PRESENT IMPRESSION.” (T 1.3.7.5 SBN 96)

which is compos'd of the ideas of these points, can never possibly exist. But if the idea of extension really can exist, as we are conscious it does, its parts must also exist; and in order to that, must be consider'd as colour'd or tangible. (T 1.2.3.16 SBN 39)

An idea of a particular extension is a legitimate imaginary idea if it resembles prior impressions, i.e. is coloured or tangible. Because the whole is coloured or tangible, we know that each of its parts is coloured or tangible as well.

Yet, we just learned a part of a set must be distinct from the set. Therefore if the set is coloured, then the part would supposedly need to be uncoloured in order to be distinct from the set. However, this is not necessarily the case because a part can still *resemble* the set without being identical with it. For example, **E!** is not identical with **{E!, E!}**, but an aspect of **E!**, viz. its colour, can resemble an aspect of the set **{E!, E!}**. Pressman explains how coloured, indivisible, surfaceless points are logically possible. He writes that we can state Hume's position as "...if x is extended, x is coloured, but not as: if x is coloured, x is extended. To claim all conceptions of extension are coloured is to assert that if x is a (conception of) extension, x is coloured; it isn't to assert that if x is coloured, x is extended. Hume, then, is only committed to the following *consistent* set of claims: x is extended if and only if x is divisible [i.e. has parts]; if x is extended, x is coloured; and a visible point [or its conceptual counterpart, the "least idea"] is coloured but not extended."⁵² It is true a priori that extension is composed of parts and that extension, following from Hume's position on primary qualities, must be coloured or tangible. However, it is not blocked a priori that a point could be coloured or tangible. In fact, their colour or tangibility is demanded by the limits Hume places on the mind which dictates that our legitimately conceived ideas must be determinate in quality and origin and resemble the impressions from which they are directly or indirectly derived. Furthermore, because extension is a relation, it 'kicks in' only when the imagination discovers and associates coloured or tangible points as being contiguous and coexistent. The mind would be incapable of discovering a *relation* held by a single point in virtue of its singularity, and therefore no point itself can be 'extended.' Yet, a single point, in order to be associated and bound by the imagination, must resemble other members in its set, and therefore must be coloured or tangible.

Conceptually, it might be odd to colour or solidify a surfaceless entity. We certainly cannot literally *picture* such an object. This might be further evidence that Hume does not

⁵² Pressman, 235.

equate conceiving with image formation. However, there can be no formal logical complaint against colouring an extensionless point.

And our conclusion has been established and defended that...

Hume's "least idea" is one of these indivisible coloured or tangible points. These points are the only "part" that the limited mind can use to form a conception of a finite and particular extension.

Chapter 3: Hume's Divisibility Principle and Actual Parts Doctrine

Divisibility presupposes the existence of actual parts that will be divided into. E.g. if you slice a piece of paper in half, the halves have to exist in order to make possible the slicing.

Consequently, infinite divisibility presupposes the existence of an infinite number of actual parts that will be divided into.

This chapter emphasises the *metaphysical* character of Hume's argument. Hume did not intend to provide an empirically derived geometry. If Hume anticipated contemporary discrete geometries, it was inadvertent.⁵³ Instead, I read Hume as a forerunner for 20th century atomic mereologies that attempt to describe extended objects. Hume argues that problems relating to extension are resolved through the metaphysical employment of reason and introspective analysis of our ideas. One of Hume's most important premises is derived introspectively, to wit: "...nor does any thing ever appear extended, that is not either visible or tangible." (T 1.2.3.15 SBN 38) Because mathematical objects are neither visible nor tangible they cannot be extended, and we learn nothing about extension by studying or analysing them. If infinite divisibility and being composed of an infinite number of parts are true only of mathematical objects, we need not concern ourselves with them. We only need to concern ourselves with ideas like Humean points that resemble impressions and conform to rules based on relations derived there from. Mathematical models are simply models involving the relations of abstract entities. If a mathematical model conflicts with our ideas—e.g. a mathematical model indicating our ideas can always be smaller *infinitum*—then such a mathematical model is rejected in favour of our *ideas* of extension, which are, presumably, directly or indirectly derived from *impressions*. Thomas Holden summarises this nicely: "If there is a coherent mathematical model of infinite divisibility, this merely shows that there can be no purely *formal* or *mathematical* complaint against it. It certainly does not show that there could be no *metaphysical* complaint against that model being translated into an actually existing physical structure [which is presumably coloured or tangible]."⁵⁴ Similarly, there is no guarantee that a consistent mathematical model will be descriptive of impressions.

⁵³ Donald Baxter, Marina Frasca-Spada, Dale Jacquette, and especially James Franklin praise Hume on this score.

⁵⁴ Holden, 12.

The metaphysical character is apparent from Hume's divisibility principle: "...whatever is capable of being divided *in infinitum*, must consist of an infinite number of parts." (T 1.2.1.2 SBN 26) This is not a mathematical principle that "trade[s] on a misunderstanding...of the possibility of infinite series summing to finite magnitudes,"⁵⁵ but the metaphysical principle that division is really decomposition; that it assumes the actual pre-existence of separate and distinct parts. This principle is important because it entails that an infinitely divisible extension would have to be the composition of an infinite number of "least ideas." An infinite number of least ideas generate an infinite extension. The idea of an infinite extension contradicts with the idea of a finite extension, and therefore the idea of a finite extension is not infinitely divisible.

This chapter first explains and defends the actual parts doctrine held by Hume, which maintains that divisibility, if it were possible, presupposes the *actual* pre-existence of parts, and consequently, that infinite divisibility presupposes the existence of an infinite number of actual parts. I distinguish the actual parts doctrine from the Aristotelean potential parts doctrine, to shed light on why Hume adopts the former and not the latter. Besides being the majority view in Hume's day, the actual parts doctrine continues to have its adherents; it is one of the fundamental presuppositions of modern mereology. I will conclude this chapter by explaining how Hume's 'atomism,' with respect to impressions of extension, is an acceptable mereological system.

I. Hume's Actual Parts Doctrine

Hume's actual parts doctrine is based on his atomistic assumption that "...everything in nature is individual." (T 19) This means that parts (simple impressions) are not wholes (complex impressions) occur naturally, which means that if we encounter a composite, it must have been constructed (by the imagination) from pre-existing parts, and not given as a composite. Because the imagination lacks the capacity to assemble an infinite number of parts, we can assume that any composite (complex idea or impression) is composed of a finite number of parts. By contrast the Aristotelian *potential* parts doctrine that holds a part does not exist until it is created through division. Hume's opponents often adopt the potential parts doctrine, arguing for the infinite divisibility of extensions, and against Hume's theory that impressions are composed of points prior to their decomposition in thought. For example, Flew argues that a part does not exist until *after* division, the 'part' prior to division existing

⁵⁵ Rosenberg, 83.

only *potentially*. The actual parts doctrine maintains “The parts are already embedded in the architecture of the whole: division merely separates or unveils them, it does not create them anew,”⁵⁶ while the potential parts doctrine maintains “...the parts of a given continuant (such as a body) are not distinct existents prior to their being actualized by a positive operation of division. Rather division creates these parts anew—it does not simply separate pre-existing parts.”⁵⁷ Consider the statue of David. The actual parts doctrine might be committed to the assertion that the figure of ‘David’ exists prior to Michelangelo’s chiselling. The ‘David’ part, though hidden within the marble, exists prior to separation from the block. By contrast, the potential parts doctrine claims that the David part and his parts have no ontological reality, besides their potentiality, prior to Michelangelo’s chiselling. According to the potential parts doctrine Michelangelo brings the ‘David’ part into existence, while the actual parts doctrine maintains that the ‘David’ part is only revealed by Michelangelo’s work. On the surface, the former is the more plausible of the two positions. Check you are being consistent with the quotation marks

The potential parts doctrine is grounded on the claim that the basic unit of the ‘material’ world is an Aristotelian substance, or a quantity of matter arranged in a particular form. The actual parts doctrine became the reigning metaphysical position by the 17th century and was held by Descartes and the Cartesians, Walter Charleton, Isaac Barrow, Ralph Cudworth, Pierre Bayle, Leibniz, and Newton and the Newtonians.⁵⁸ Evidence of the doctrine can be found in many known works of the period. According to the actual parts doctrine, the possibility of division into n pieces is conditioned by the reality of n actually existing separate and distinct parts—the two halves of the paper make possible the slicing. Therefore, if something is infinitely divisible, it must consist of an infinite number of pre-existing “*actually* separate and distinct parts.” The wide acceptance of this doctrine is why Hume claims principle two--“...that whatever is capable of being divided in infinitum, must consist of an infinite number of parts”—is “obvious.” For example, In *A Letter to the Learned Mr. Henry Dodwell*, Anthony Collins asserts, “Matter, being a divisible substance, consist[s] always of separable, nay of *actually* separate and distinct parts.”⁵⁹ Physical matter, because it occupies space, “always” has actually separate and distinct parts, and thus always divisible *ad*

⁵⁶ Ibid, 7.

⁵⁷ Ibid, 9.

⁵⁸ Ibid, 9.

⁵⁹ Anthony Collins, *A Letter to the Learned Mr Henry Dodwell*, 2nd Edn (1709). From Eighteenth Century Collections online, accessible via the University library catalogue., emphasis added.

infinitum. The actual parts doctrine is famously employed by Bayle in his attack on Epicurean atoms. Bayle writes:

Nor is it less impossible or inconceivable that [extended objects] should be composed of Epicurean atoms, that is, of extended and indivisible corpuscles; for every extension, how small forever, hath a right and left side, an upper and lower side: therefore it is a conjunction of distinct bodies; and I may deny of the right side what I affirm of the left, for there are two sides not in the same place: a body cannot be in two places at once; and consequently every extension which fills several parts of space contains several bodies.⁶⁰

The epicurean atom is a purportedly *indivisible* physical atom. But regardless of how small such an atom is, *if it occupies space*, it will always have a left and right side. Even a quark would have a left and right side, the left side *actually* distinct and separate from the right because they occupy different “parts of space.” What can be separated by thought is actually distinct and separate. A space-occupying ‘physical’ atom will always be divisible *ad infinitum* because it will always have a left and a right side. Hume writes, “The system of physical points...is too absurd to need a refutation. A real extension, such as a physical point is supposed to be, can never exist without parts, different from each other; and whatever objects are different, they are distinguishable and separable by the imagination.” (T 1.2.4.3 SBN 40) This seems to imply that physical points would be infinitely divisible; only by making his points non-physical and unextended, does Hume avoid Bayle’s criticism.

The actual parts doctrine is not an antiquated metaphysical position. In fact, the majority of contemporary mereologists adhere to it. For example, Theodore Sider takes it as given in *Four-Dimensionalism* that the actual parts doctrine is true of extended objects. He writes “Talk of cutting and slicing must be taken with a grain of salt: the parts are there whether or not they are physically separated from the whole.”⁶¹ Sider presents an argument similar to Bayle’s. Sider writes, “Suppose the desk is divided into halves. Since the halves are spatially separated, they are distinct objects. These objects were presumably not created by division, so they must have existed before division. But surely it does not matter whether the division ever *actually* occurs, so anything *potentially* divisible must actually have parts.”⁶² Sider’s arguments are evidence that the actual parts doctrine, which is the lifeline of Hume’s divisibility premise, is still maintained as a robust and defensible position.

⁶⁰ Pierre Bayle, *Historical and Critical Dictionary: Selections*, ed. And trans. Richard H. Popkin (Indianapolis, Ind.: Bobbs-Merrill Company, Inc., 1965). See esp. the article ‘Zeno of Elea’, *ibid.*, 350-88.

⁶¹ Sider, Theodore. *Four Dimensionalism: An Ontology of Persistence and Time*. Oxford: Oxford University Press, 2001. Web., pg. 2.

⁶² *Ibid.*, 90.

II. Hume's Rejection of the Potential Parts Doctrine

Hume likely rejects the potential parts doctrine because of its affinities with the ancient philosophical notions of substance and substantial form, which for Hume are fictive, i.e. illegitimate imaginary ideas. According to the potential parts doctrine, David has potential existence as a substance and takes form only when actualised by Michelangelo. According to Hume, the imagination in its improper application forms the illicit idea of substance. The imagination naturally groups resembling impressions and associates them. The imagination makes distinct, yet related visible or tangible impressions: impressions of one enduring 'object.' For example, the mind groups temporally contiguous and resembling house impressions and, when receiving an impression of the front of the house, might supplement it with the idea that there is a resembling back of the house. Insofar as any idea shares certain qualities with my impressions and resembles them, the idea is a natural and legitimate product of the imagination. However, positing an enduring Aristotelian "substance" to account for the resemblance and coherence between impressions and their copies is an illegitimate use of the imagination. Hume writes, "...the imagination is apt to feign something unknown and invisible, which it supposes to continue the same under all these variations; and this unintelligible something it calls a *substance, or original and first matter.*" (T 1.4.3.4 SBN 220) A "*substance*" is illegitimate because it does not resemble impressions and has qualities that are not derived from experience. The mind cannot form a determinate idea of a pure *substance*, and therefore deludes itself if it believes it has an adequate idea of it. Because Hume finds the notions of substance and substantial forms to be unintelligible, the potential parts doctrine is unintelligible as well.

The actual parts doctrine is more consonant with Hume's philosophy. The uncreated parts of an extended object are "hidden" and are neither impressions nor memory-ideas, and can only be imaginary. How can these ideas be formed? During the division of an object there are decreasingly smaller impressions. The imagination is led by a kind of inertia to continue the series of "smaller-than, but otherwise resembling," and the viewer conceives of smaller parts that would be revealed by prolonged division. When imagining these smaller parts, the mind reasons that these smaller parts exist prior to division. They are determined in thought before they are separated in practice.

With the contrast between the actual and the potential parts doctrine, and why Hume favours the former, established, (?) we will return to Flew's criticism. Flew writes: "...to say that something is divisible into so many parts is not to say that it consists of—that it is, so to

speak, already divided into—that number of parts. A cake *may be* divisible into many different numbers of equal slices without it thereby consisting in, through already having been divided into, any particular number of such slices.”⁶³ Flew thereby advocates the potential parts doctrine. Flew is claiming that pieces of cake do not actually exist until they are made actual division. Hume and his contemporaries would say that Flew is mistakenly confusing the identity of a slice of cake, as defined as a ‘slice of cake,’ with the ‘matter’ or composite parts⁶⁴ that compose the “slice.” It follows, from my reading of Hume, that Hume would say parts are “actual” prior to division, not because of their pre-existing matter or substance, but because ideas of parts are determinate prior to division, that is, determined to have real qualities that resemble impressions, before we have impressions of them. The idea of a part that resembles a whole, before any impression of that part, is not fictive. The idea that the whole and the part are composed of the same substance, of which we have no separate and distinguishable impression, *is* fictive.

III. Mereology and the Actual Parts Doctrine

This section proves Hume was right that conceiving atomic extension is not impossible. Hume’s contemporary opponents were geometers who claimed they had proof that extension *must* be infinitely divisible.⁶⁵ Surprisingly, some recent commentators still counter Hume’s argument with archaic geometrical demonstrations.⁶⁶ Geometry was considered the reigning science of ‘physical’ objects—a science Hume supposedly ‘misunderstood’-- until the 1920’s when Stanislaw Lesniewski developed mereology, which has evolved into the formal theory of part--whole metaphysics. In mereology an atomic system is admissible. Its presuppositions include a finite domain and finite divisibility of its parts. By looking at how Hume’s philosophy of extension could fit into the “one-element trivial model” of mereology, we can see that there is nothing illogical about conceiving that

⁶³ Flew, 260. (emphasis added)

⁶⁴ For Hume, the composite parts would be colour’d and tangible indivisible simples, which are not technically ‘material.’

⁶⁵ Hume’s contemporary opponents include Issac Borrow and the Port Royal Society. For an excellent discussion on this topic See Fogelin, Robert, 47-69. Print.

⁶⁶ Most notably, see Garrett, Don. *Cognition and Comittment in Hume's Philosophy*. New York, NY: Oxford University Press, 1997. Print., 74-5. Garrett writes that Hume’s “spatial minima” “...resolve some geometrical paradoxes, however, only at the price of engendering many others,” and goes on to discuss how the Pythagorean theorem demands that a right triangle with two equal legs of 10 spatial minima would require that the hypotenuse be composed of 14.14 minima, which is impossible according to Hume’s theory. Garrett writes “For Hume, some line segments forming a right angle cannot be (and cannot be conceived to be) closed by a third to form a triangle.” Mark Pressman also argues against Hume using geometrical demonstrations. He writes “The problem [with Hume’s position] is that Euclid demonstrates that every geometric segment is bisectable into equal segments (book 1, Proposition 1).” Pg 233.

which is extended as being composed of indivisible unextended atoms. This reveals that mathematical complaints against Hume's simply miss the mark.

Mereology is the logical study of parts and wholes. The actual parts doctrine is a fundamental principle of mereology. Mereology takes it as given that wholes are composed of *actual* parts. Mereology constructs logical systems that describe the different ways parts can relate to wholes. The standard, formal mereological system is *classical extensional mereology* famously known in two forms: the Calculus of Individuals of Leonard and Goodman, and the Mereology of Lesniewski.⁶⁷ Mereology is a way to understand extended objects. Kathrin Koslicki writes "...the vast majority would protest that insofar as we have any understanding of the notions of parthood and composition at all, this understanding derives from standard mereology."⁶⁸ David Lewis, a proponent of mereology and the actual parts doctrine it presupposes, writes: "What is the *general* notion of composition, of which the mereological form is supposed to be only a special case? I would have thought that mereology already describes composition in full generality."⁶⁹ We can surmise that Lewis would deny that Flew's cake is a "special case." Flew's cake is a composite object, and the only logical way we can understand composite objects is through extensional mereology, which presupposes the actual existence of parts of any given whole. As we have already seen, mereologist Ted Sider is also a proponent of the actual parts doctrine.

Hume 'atomism' and relational theory of extension can be accommodated by mereology. Hume's "least idea," i.e. his conceptual coloured point, is similar to the "atom" of 20th century mereology (save its colour or tangibility, which is a mark of Hume's empiricism). Hume's "least idea" is "...perfectly simple and indivisible" and has no parts, or in mereological terms, has no *proper parts*. A proper part is a part completely enclosed in another part. For example, a circle enclosed in a square is a proper part of the square. A mereological atom is defined as "...an individual with no proper parts; it is accordingly indivisible either in fact or in theory...Atoms in this strict mereological sense are not to be confused with atoms in the sense of physics and chemistry, which may have numerous proper parts and are far from indivisible, even in fact."⁷⁰ The mereological atom can be used to construct a complete and coherent logical system, such as the system SC delineated by Peter

⁶⁷ Simons, pg 1.

⁶⁸ Koslicki, pg 21.

⁶⁹ "Against Structural Universals", *Australasian Journal of Philosophy*, 64, 25–46, reprinted in Lewis (1999), pp. 78–107., pg 97. In this passage, Lewis is attacking the Uniqueness of Composition and Armstrong's work on *structural universals* which fall outside the domain of CEM

⁷⁰ Simons, pg 16.

Simons in *Parts: A Study in Ontology*. Such a system has a closed or finite domain. Simons writes “...any finite model of SC is perforce atomistic, and has the cardinality of $2^n - 1$, where n is the number of atoms.”⁷¹ For example, if there are three atoms then there are seven objects in the finite domain. The objects include the first second and third atom, plus the remaining four combinations of the atoms. The diagram below should be helpful. A, B, and C are atoms; D, E, F and G are the combinations of atoms that generate the four additional parts:

A	B	C
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A+B= D; B+C= E; A+C= F; A+B+C= G

As we would suspect, no object in an atomic mereology is infinitely divisible. Koslicki writes, “An *atomic* mereology requires that every object either is itself an atom or is composed of atoms. An *atomless* mereology requires that every object is infinitely divisible into further proper parts.”⁷²

What would Hume’s atomic mereology look like? First, Hume’s conceptual atoms are *disjoint*, meaning, none of the atoms *overlap*. Simons writes “Individuals are *disjoint* iff (if and only if) they do not overlap, i.e. they are disjoint iff they have no part in common.”⁷³ Hume’s atoms, because they have no parts, are non-overlapping parts. We can take from Hume’s adoption of Maliezeu’s argument that only the disjoint atoms exist as fundamental parts. The composite parts such as D, E, and G in the diagram above are “fictitious unities,” for Hume.⁷⁴

Two or more atoms in an aggregate are associated and bound by the imagination according to the relations contiguity and resemblance because the imagination fixes the unity, it is “fictitious.” Hume would endorse an atomic mereological system as long as it distinguishes between legitimate parts (units or atoms; A, B, and C above) and fictitious parts (collections of atoms or units; D, E, F, and G above). Furthermore Hume’s mereology would not allow atoms to have *improper parts* (which some systems do allow). An improper part is a part-of-or-equal—in other words, improper parts include identical parts. Simons writes “If x and y

⁷¹ *Ibid*, 41.

⁷² Koslicki, pg 15.

⁷³ Simons, 13.

⁷⁴ That term of unity is merely a fictitious denomination, which the mind may apply to any quantity of objects it collects together; nor can any such a unity any more exist alone than number can, as being in reality a true number. But the unity, which can exist alone, and whose existence is necessary to that of all number, is of another kind, and must be perfectly indivisible, and incapable of being resolved into any lesser unity. (T 1.2.2.3 SBN 30-1

are identical, then we say x is an *improper* part of y (and vice versa).”⁷⁵ However, Hume argues the idea of identical atoms is conceptually impossible. He writes “’Tis impossible for the mind to preserve any notion of difference betwixt two bodies of the same nature existing in the same place at the same time.” Though identical atoms may be logically consistent within a given mereological system, such a conception surpasses the limits of the mind. Hume’s mereological construction is inextricably constrained by conceptual limits. And finally, in order for these ‘atoms’ to be conceivable, they must be conceived as coloured or tangible. His mereology, or idea of a finite extension, would involve a finite domain of *disjoint, non-identical, non-overlapping*, coloured or tangible atoms that are contiguously related and bound by the imagination.

However, mereology does not assert whether atoms actually exist. Simons writes “The system SC, like other classical mereologies, does not pronounce one way or the other on the question whether or not there are atoms, and whether everything is composed of atoms.”⁷⁶ Well, which system is the preferred one when applying it to actual objects of sense experience? Simons writes: “The issues involved in the question whether the atomistic or the atomless mereology is in some sense the ‘correct’ one in application to the physical world are among the deepest that mereology has to face, and, despite some work which has been done on the alternative axiomatic bases of atomistic and atomless mereology...the question has not yet been squarely faced among mereologists.”⁷⁷ It seems that recourse to mereology—the formative metaphysic of the part/whole relationship—does not prove Hume correct. However, it does provide evidence that Hume was not wrong or confused about ideas of extended objects. It also closes the question whether there is an *a priori* complaint against Hume’s argument. Hume is correct that his conceptual atomism is logically possible. He writes “Here then is an idea of extension, which consists of parts or inferior ideas, that are perfectly divisible: consequently this idea implies no contradiction: consequently ‘tis possible for extension really to exist conformable to it: and consequently all the arguments employ’d against the possibility of mathematical points are mere scholastic quibbles, and unworthy of our attention.” (T 1.2.2.8 SBN 32) Thus, in one fell swoop Hume has correctly refuted his geometrical opponents who maintain that geometrical demonstrations prove the impossibility of finitely divisible atoms.

⁷⁵ Simons, pg 11.

⁷⁶ Simons, pg 41.

⁷⁷ Simons, pg 42.

CHAPTER FOUR: Hume's *Adequacy Principle*

I believe Hume's *adequacy* principle that ““WHEREVER ideas are adequate representations of objects, the relations, *contradictions* and agreements are all applicable to the objects” is his way of stating that when component ideas of a complex idea contradict, we infer the existence of such an entity is impossible. The contradictory ideas in Hume's lead argument are the idea of finite extension and the idea of infinite divisibility. I will call an emphasis on the contradiction the ‘light reading.’ Commentators often give a ‘strong’ Cartesian reading that Hume means we can infer the nature of space and time ‘themselves’ from our clear ideas of space and time. I will conclude that interpreting the adequacy principle as limiting inferences about reality to contradictory complex ideas is less problematic, textually loyal, and consistent with Hume's scepticism. In this chapter I will first complete my version of Hume's lead argument and then defend the light reading of the adequacy principle.

I. **Contradictory Ideas: Infinite Divisibility and Finite Extension**

Hume's “least idea” is an indivisible coloured or tangible point. These points are the only “part” that the limited mind can use to form a conception of a finite and particular extension.

Chapter two explained the move from “...the capacity of the mind is limited and can never attain a full and adequate conception of infinity” to the mind must *construct* or *form*⁷⁸ ideas of extension using Hume's indivisible coloured or tangible points. Infinity is an imaginary conception because it is neither an impression nor memory-idea. Conceptions are mental constructions. We know through transcendental reflection that conceptions of a particular extension resolve themselves into minima. It follows conceptions of particular extensions are constructed using coloured or tangible points. If one of these points were infinitely divisible, then such an idea would never be adequate because its parts “...elude the grasp by a new fraction; like quicksilver when we endeavour to seize it.” Conceptions of extension must begin with a coloured or tangible point. Hume writes, “I first take the least idea I can *form* of a part of extension, and being certain that there is nothing more minute than this idea.” The conception of a particular extension is a compound idea that requires two or more coloured or tangible points, and a relation between them (e.g. contiguity without

⁷⁸ “...the idea, which we *form* of any finite quality” (SBN 27); “And we ought rather to conclude, that the difficulty lies in enlarging our *conceptions* so much as to *form* a just notion of a mite” (SBN 28);

overlap), and therefore a coloured or tangible point is unextended and indivisible.⁷⁹ According to Hume, the conception of a particular extension is formed by the imagination of coexistent, contiguous, and disjoint non-overlapping coloured or tangible points.⁸⁰

Divisibility presupposes the existence of actual parts that will be divided into. For example, if you are going to slice a piece of paper in half, the halves have to exist in order to make possible the slicing.

Consequently, infinite divisibility presupposes the existence of an infinite number of actual parts that will be divided into.

Chapter three discussed the actual parts doctrine behind Hume's divisibility principle which maintains that actually existing parts condition the possibility of division. The actual parts doctrine usurped the Aristotelian potential parts doctrine and became the reigning metaphysical position in the 17th and 18th centuries, and remains so among modern mereologists. This principle couches Hume's argument in its appropriate metaphysical context, as opposed to the mathematical one with which it is often confused. If one cut is possible, then two parts must actually exist. If an infinite number of cuts are possible, then an infinite number of parts must actually exist prior to division.

When the mind conceives infinite divisibility, it must conceive an infinite number of actual "parts." (7)

Conceptions of particular extensions must be composed of indivisible coloured or tangible points. (5)

Therefore, the only conception of infinitely dividing a finite extension would necessarily involve an infinite number of indivisible coloured or tangible points.

*The conception of an infinite number of indivisible coloured or tangible points is the conception of an infinite extension, i.e. ←-*****-→*

*Therefore, the only conception of infinite divisibility the mind could form would necessarily involve the divisibility of an infinite extension, i.e. ←-*****-→. Infinite extension is not a finite extension, i.e. ←-*****-→ is not ******

*Therefore, the mind contradicts itself if it believes it has a conception of a finite extension being infinitely divisible, i.e. if it thinks ***** is ←-*****-→. The*

⁷⁹ "...let us take one of those simple indivisible ideas, of which the compound one of extension is form'd, and separating it from all others, and considering it apart, let us form a judgment of its nature and qualities. 'Tis plain it is not the idea of extension. For the idea of extension consists of parts; and this idea, according to the supposition, is perfectly simple and indivisible." (T 1.2.3.13-4 SBN 38)

⁸⁰ "Now 'tis certain we have an idea of extension; for otherwise why do we talk and reason concerning it? 'Tis likewise certain, that this idea, as *conceiv'd by the imagination*, tho' divisible into parts or inferior ideas, is not infinitely divisible, nor consists of an infinite number of parts: For that exceeds the comprehension of our limited capacities." (T 1.2.2.8 SBN 32, emphasis added); "... 'tis also necessary we shou'd preserve the idea of their colour or tangibility in order to *comprehend them by our imagination*. There is nothing but the idea of their colour or tangibility, which can render them *conceivable* by the mind." (T 1.2.3.15 SBN 38, emphasis added); "Now such as the parts are, such is the whole. If a point be not consider'd as colour'd or tangible, it can convey to us no idea; consequently the idea of extension, which is compos'd of the ideas of these points, can never possibly exist. But if the idea of extension really can exist, as we are conscious it does, its parts must also exist; and in order to that, must be consider'd as colour'd or tangible." (T 1.2.3.16 SBN 39)

conception that a finite extension is infinitely divisible is contradictory, confused, muddled, bizarre, stupid, and unholy.

That our conceptions of particular extensions must be constructed using coloured or tangible indivisible points with the finite divisibility principle generates the conclusion that the conception of an infinitely divisible extension is identical with the conception of an infinite extension. Hume writes “Upon the whole, I conclude, that the idea of an infinite number of parts is individually the same idea with that of an infinite extension.” (T 1.2.2.2 SBN 30) The idea of an infinite extension contradicts the idea of a finite extension therefore, the idea of a finite extension is not infinitely divisible.

Hume’s idea of infinite extension as the relentless aggregate of “least ideas”⁸¹ is reminiscent of Locke’s idea of infinity as “...a growing and fugitive idea, still in a boundless Progression that can stop no where.”⁸² The formation of an adequate idea of an infinite extension, presupposes the mental ability to assemblage an infinite number of coloured or tangible points, an ability we lack. Cognising an infinite assemblage may be possible where forming an infinite assemblage is not. However, why is our idea of infinite divisibility not akin to this legitimate conception of an infinite extension, but the inverse? I think Hume does believe we can form this idea of infinite divisibility and expresses it as his ‘quick silver’ idea. However, both the relentless aggregate and quicksilver ideas are never determinate. Regardless of whether our idea of infinity involves same-sized parts (infinite extension) or proportionally smaller parts (quick silver idea), either idea is inconsonant with any idea *WE* form of a finite extension, whose finite nature necessarily makes it determinate. The consequence of rejecting the infinite power of the mind is that any idea we *construct* of a finite extension is composed of indivisible points. The notion of infinite divisibility (quicksilver idea) entails that each part has a proportionally smaller part, which is made distinct by its expression as a fraction. The 1,000th part of the grain of sand is distinct from the 1,001th part. Whatever is distinct is distinguishable, and whatever is distinguishable is different. Therefore the notion of infinite divisibility (quicksilver idea) logically pre-supposes the prior existence of an infinite number of different and distinct parts. Infinitely dividing a part into proportionally smaller parts presupposes an infinite number of proportionally smaller parts. But if a finite extension, to be cognised at all, must be cognised as being

⁸¹ Hume writes “...were I to carry on the addition *in infinitum*, I clearly perceive, that the idea of extension must also become infinite.” (T 1.2.2.2 SBN 30)

⁸² An Essay Concerning Human Understanding, Book II chap. XVII, sect. 12, P.H. Niddich (ed.) (The Clarendon Press: Oxford, 1975). P. 216.

composed of a fixed number of indivisible points, it cannot be cognised as being composed of an infinite number of points or parts. Therefore, the quicksilver idea of infinite divisibility contradicts any idea of a finite extension we can have.

*“WHEREVER ideas are adequate representations of objects, the relations, **contradictions** and agreements are all applicable to the objects.” (T 1.2.2.1 SBN 29, emphasis added)*

When we can form no clear and adequate complex idea from contradicting component parts, we regard the object in question as impossible. Hume writes “...if it be a contradiction to suppose, that a finite extension contains an infinite number of parts, no finite extension can be infinitely divisible.” Our idea of a finite extension entails its composition of a finite number of parts which, by the actual parts doctrine, precludes its infinite divisibility. Therefore, an actual finite extension cannot be thought as being infinitely divisible.

II. Hume’s Adequacy Principle

Hume’s ‘adequacy principle’ has made considerable noise in the literature. I read it as an innocuous inference from contradictory ideas. Oddly enough most commentators interpret Hume as claiming that we can infer from our clear ideas of space and time the nature of a mind-independent space and time. Newman sympathetically writes “Hume’s assertion that there must be conformity between the nature of reality and our ‘adequate ideas’⁸³...if it is not merely a trivial analytic truth must surely be regarded as a metaphysical postulate,”⁸⁴ and Fogelin remarks “...the transition from claims about our *ideas* of space and time to assertions about space and time *themselves*... is a match for anything found in the writings of the rationalists,” and concludes “Hume certainly owes us... a defense of the general rationalist principle that adequate ideas of objects are *eo ipso* true of them.”⁸⁵ Baxter uncomfortably acknowledges, “Hume’s proof that finite extension is not infinitely divisible depends crucially, even excruciatingly, on the assumption that our minimal ideas are adequate. On this assumption depends the inference from features of minimal ideas, and features of compounds of them, to features of space or extension itself. It is this assumption, however, that seems the least defensible.”⁸⁶ In the argument Dale Jaquette attributes to Hume, one moves from “complex sense impressions” to “extended things” through the adequacy principle without

⁸³ If we read Hume as a perceptual-idealist, Hume simply means is that reality consists of our adequate ideas, so there can be no non-conformity.

⁸⁴ Newman. pg 4.

⁸⁵ Fogelin, pg 54.

⁸⁶ Baxter, Donald. "Hume's Theory of Space and Time in Its Skeptical Context."

reference to a contradiction.⁸⁷ George S. Pappas even argues it is a second copy thesis.⁸⁸ To James Franklin the *adequacy principle* is a fallacy of Berkelian proportions insofar as it denies a certain kind of real existence (that which is infinitely divisible) from the nature of perceptions (that they are finitely divisible). Franklin writes, “Now if Berkeley is susceptible to the fallacy, “It is not conceivable, so it cannot be,” then we are not surprised; it is precisely the grossness of his fallacies that makes Berkeley so useful as target practice for undergrads...But what are we to make of it when Hume, the paragon of rationality in the century of “reason” does the same? *We make nothing of it, because we are too flabbergasted.*”⁸⁹

I believe all of this over-reads the text. Firstly, Franklin is wrong that neither Berkeley nor Hume commits this fallacy. Wayne Waxman sums it up nicely writing, “I know of no thinker prior to Hume so keenly cognizant as he of the dangers of inferring from how our natures oblige or prevent us from thinking about reality, how reality “in itself” is or must be.”⁹⁰ Setting aside Franklin’s fallacy charge, I will first provide textual evidence for the light reading. I will then provide the (minimal) textual evidence for the stronger Cartesian reading expressed by these commentators. I will explore the difficulties with the Cartesian reading, and conclude that my lighter reading is preferable.

III. Textual Evidence for the Light Reading

Hume writes “...whatever *appears* impossible and contradictory upon the comparison of two ideas, must be *really* impossible and contradictory, without any farther excuse or evasion.” (T 1.2.2.1 SBN 29) The two ideas being compared are infinite divisibility and a finite extension. If these ideas contradict each other, their combination is impossible. Hume writes “If therefore any finite extension be infinitely divisible, it can be no contradiction to suppose, that a finite extension contains an infinite number of parts: And *vice versa*, if it be a contradiction to suppose, that a finite extension contains an infinite number of parts, no finite extension can be infinitely divisible.” (T 1.2.2.2 SBN 29) Additionally, on the following page Hume does not make a claim about extension itself until *after* generating the contradiction. Furthermore when Hume recapitulates his argument at T 1.2.4 he writes, “’Tis therefore

⁸⁷ Jacqueline, Dale. "Hume on Infinite Divisibility and Sensible Extensionless Indivisibles."

⁸⁸ Pappas, George S. "A Second Copy Thesis in Hume?" *Hume Studies* 27.1 (1991): 51-59. Print.

⁸⁹ Franklin, James. "Achievements and Fallacies in Hume's Account of Infinite Divisibility." *Hume Studies* 20.1 (1994): 85-101. Print., pg. 93.

⁹⁰ *Ibid*, 147.

possible for space and time to exist conformable to this idea: and if it be possible, 'tis certain they actually do exist conformable to it; since their infinite divisibility is utterly impossible and contradictory.” (T 1.2.4.1 SBN 39)

I think Hume was sceptical regarding claims about ‘objects’ as distinct from perceptions. He writes “As long as we confine our speculations to the *appearances* of objects to our senses, without entering into disquisitions concerning their real nature and operations, we are safe from all difficulties, and can never be embarrass’d by any question” (T 1.2.5.26 n 12), and “...since we may suppose, but never can conceive a specific difference betwixt an object and impression; any conclusion we form concerning the connexion and repugnance of impressions, will not be known certainly applicable to objects.” (T 1.4.5.21 SBN 241) Hume is consistently sceptical about our ability to discern objects themselves. Hume believes that, at best, our imagination can form “relative ideas” of them from impressions, but that we can never fully understand the related objects. He writes “the farthest we can go towards a conception of external objects, when suppos’d *specifically* different from our perceptions, is to form a relative idea of them, without pretending to comprehend the related objects.” (T 1.2.6.9 SBN 68) Consequently, I have trouble attributing to Hume a principle that “is a match for anything found in the writings of the rationalists” inferring the nature of objects themselves from our clear and ‘adequate’ ideas. Instead, I view Hume as confining his speculation to ideas and impressions, with the caveat that he *is* willing to make claims about objects when his ideas contradict. For example, Hume writes “We can form no idea of a mountain without a valley, and therefore regard it as impossible.”

IV. Strong Cartesian Reading

There is some textual evidence that supports the stronger Cartesian reading. Hume writes “I first take the least idea I can form of a part of extension, and being certain that there is nothing more minute than this idea, I conclude, that whatever I discover by its means must be a real quality of extension.” (T 1.2.2.2 SBN 29) Robert Fogelin and George Pappas cite this passage when articulating the stronger reading.⁹¹ However, if the adequacy of his minute idea was *eo ipso* true of extended objects themselves, then Hume would not need to undergo a demonstration at all. Talk of contradictions would be entirely superfluous. If we take the strong Cartesian reading, Hume could immediately infer from the adequacy of his “least

⁹¹ Fogelin, 53; Pappas, 52.

idea” that extended objects are composed of indivisible minima. If we take the strong Cartesian reading it becomes befuddling why Hume provides a demonstration at all.

By my reading of Hume X is real if 1) X is an impression; 2) if X is a copy of an impression; or 3) X is formed (by the imagination) from impressions and copies according to associative rules. Space is not an impression (1). Space is not a copy of an impression (2). Space is, however, formed by the imagination from impressions and their copies according to associative rules (3), so space is real. An extension is a determinate space because it is formed by the imagination from least ideas, i.e. the idea of a coloured or tangible point, which are formed by the imagination from impressions according to associative rules of contiguity and resemblance (3), it is real. Giving extension “real qualities” does not necessarily make it mind-independent; it can just mean that it has qualities formed by the mind (3) that are not fictive.

Frasca Spada supports the adequacy principle by seeing Hume as an indirect realist. She explains that because any external objects we suppose must necessarily resemble our impressions and ideas, external objects are constrained by perception. She writes “The fact that no process of division is infinite says something about external reality. Since whatever we suppose about reality must be applicable to perception, this feature of perception—its always presenting indivisibles, sooner or later—places a limit on reality itself. Reality cannot here go against perception.”⁹² However, this attributes the “It is not conceivable, so it cannot be” fallacy to Hume that Franklin falsely levels, which claims that “...the world is restricted by the powers the mind has.”⁹³ I agree with Franklin that this is a fallacy. It is erroneous to limit external reality by the limits of perception. For example, just because we can only perceive four dimensions does not mean twelve could not exist. In Frasca Spada’s discussion, just because each level of *our* perception has a minimum does not mean that reality itself has a minimum.

The second troubling passage for my lighter reading is when Hume writes “But our ideas are adequate representations of the most minute parts of extension” (T 1.2.2.1 SBN 29) without qualification or argument. Jaquette puzzles over this passage as well. He writes, “From a Kantian perspective, this solution seems hopelessly naive. It may even be inconsistent with Hume’s philosophical scepticism about the existence and nature of the external world. What Hume proposes is that adequate ideas are those that agree with their

⁹² Frasca Spada, Marina. "Part 2: Reality and Coloured Points in Hume's Treatise." pg 45.

⁹³ Franklin, 91.

objects. But what access can we possibly have to the objects themselves independently of our impressions and ideas?”⁹⁴ Jacquette’s criticism is well taken and targets what is wrong with the stronger reading. By what means can Hume test the adequacy of his ideas? How does Hume know if his idea is adequate or not? Hume admits that “...nothing is ever present to the mind but perceptions,” (T 1.2.6.8 SBN 67) and all perceptions are either impressions or ideas. We never have the impression of a distinct and mind-independent external body. We are only ever acquainted with impressions, never mind-independent objects allegedly causing or responsible for them. Therefore, we can never compare mind-dependent ideas with mind-independent objects to confirm the relation of resemblance or of adequate representation.

Such considerations motivate Jacquette to maintain that ‘adequacy’ is determined by an ideas’ comparison—not with an object—but with a present impression. Jacquette writes:

One answer is in immediate sense impressions. It has been so long since I have seen the Tower of London, that my idea of the White Tower now is of a round building with three copulas. Is this an adequate idea or not? The best answer is to visit the site again and compare the idea with my immediate sense impressions. That is as close as I can get to the object itself, and the problem no doubt admits of no other kind of resolution...If, on the contrary, my impressions of the Tower reveal it to be a square structure with four copulas, then the first idea must be judged inadequate... When we check the idea of extension as heir to the finite divisibility limitations of its originating sense impressions by comparing it with those impressions, we naturally find it adequate by Hume’s [empirical] criterion.

Jacquette argues that an idea is ‘adequate’ when it resembles an immediate impression: he calls this Hume’s “empirical criterion.” For example, Jacquette’s idea of a minimum indivisible is confirmed to be ‘adequate’ by conducting the ink spot experiment. However, Jacquette’s troubling phrase is that immediate impressions are “as close as [we] can get to the object itself.” But Hume is intensely critical of the purported resemblance between impressions and external objects. Again, Hume writes that, “The farthest we can go towards a conception of external objects, when suppos’d *specifically* different from our perceptions, is to form a relative idea of them, without pretending to comprehend the related objects.” How are sense impressions the ‘closest’ we can get to objects themselves? When has he compared his impression of an external object with the external object itself to note the resemblance? If the only conception of an external object is a “relative idea,” presumably formed from impressions by the imagination, then can Jacquette’s claim be reduced to the thesis that ideas are adequate when they resemble our imaginary ideas? Jacquette retreats to Hume’s ‘naturalism’ to complete the interpretive picture. Jacquette writes, “...if by empirical criteria

⁹⁴ Jacquette, 75.

[Hume's "least idea"] is an adequate idea of extension, then imagination according to Hume's naturalism compels belief that extension in reality must also be finitely divisible." It is natural to trust our imagination and believe external objects exist and resemble our impressions. As I have noted before, insofar as our ideas of external objects resemble impressions (as opposed to an unintelligible substance, like matter for instance) our conceptions are legitimate. However, careful reflection reveals that this "...universal and primary opinion of all men is soon destroyed by the slightest philosophy, which teaches us, that nothing can ever be present to the mind but an image or perception, and that the senses are only inlets, through which these images are conveyed, without being able to produce any immediate intercourse between the mind and the object."⁹⁵ Interpreting Hume as sceptic or naturalist, realist or indirect realist (or neither), is tricky, and falls beyond the scope of this thesis. Suffice it to say that if we can avoid this murky swamp by adopting the light reading, and properly constraining the imagination, so much the better. Jaquette is close, but I do not think Hume's "least idea" is a copy of a minimal impression. Hume says nothing of the sort in Part II. Instead, I read Hume's least idea as being the consequence of a priori arguments, which becomes conceivable through the use of the imagination to continue a series of impressions or ideas according to associative rules.

Baxter notoriously interprets Hume as a Pyrrhonian Sceptic.⁹⁶ Baxter says the trouble with "criticism of Hume's adequacy assumption is that it overlooks the sceptical context of Hume's treatment of space and time."⁹⁷ Baxter argues that Hume's scepticism limits his analysis to only ideas and impressions, i.e. ideas and "the appearances of [purported] objects to our senses." (T 1.2.5.26 n. 12) Baxter thinks adequate ideas are applicable to purported objects as they appear, i.e. impressions. Ideas are adequate when they are copies of antecedent impressions, and therefore our 'clear ideas' can be reversely known to be true of their prior impressions. Baxter brushes aside Hume's uses of the words *real* and *really* as "incautious uses," and argues that "...once he takes it to be understood that he is confining his attention to these appearances, he sometimes feels free to reapply the contrast between appearance and reality. In effect he is then distinguishing how an object really appears from how it apparently appears".⁹⁸ Baxter believes the familiar reader will know

⁹⁵ Hume. *An Enquiry concerning Human Understanding*, sec. XII, pt I.

⁹⁶ Baxter, Donald L.M. *Hume's Difficulty: Time and Identity in the Treatise*. New York, NY: Routledge, 2008. Print., chapter one: "Interpreting Hume as a Metaphysician and Skeptic," pg. 6-16.

⁹⁷ Baxter, "Hume's Theory of Space and Time in Its Skeptical Context." pg. 112.

⁹⁸ Baxter, "Hume's Theory of Space and Time in Its Skeptical Context." Pg. 116.

when he is speaking carelessly and remain cognisant of Hume's scepticism. Thus Baxter concludes: "Talk by Hume's critics of infinitely divisible space as a set of points ordered as a continuum would for him amount merely to empty words. Given the copy principle, since there could be no impression of such a set with its elements ordered in such a way, there could be no idea of it. Beyond that, this conceit of a mathematical continuum is not forced on us by appearances, and there is no way to know if it is true of reality."⁹⁹ According to Baxter, Hume's scepticism keeps him agnostic about any mind-independent space or time.

I agree with Baxter that Hume 'defines' adequacy in terms of impressions. Or at least, an idea is adequate when it has a determinate quantity or quality and resembles an impression.¹⁰⁰ I am also sympathetic with reading Hume as a sceptic. However, I disagree with Baxter that Hume limits his inferences in Part II to only impressions. This reading goes off script too much: if this is what Hume meant, then he would have said 'WHEREVER ideas are adequate representations of *impressions*, the relations, contradictions and agreements of the ideas are all applicable to the *impressions*.' But he does not use the term "impression," he uses the term "object," which means something different. Furthermore, Hume begins Section III with a direct reference to impressions and the copy principle, and writes "Let us apply this principle, in order to discover farther the nature of our ideas of space and time," (T 1.2.3.1 SBN 33) which leads his discussion in a new direction. If the adequacy principle is merely a reversal of the copy principle, why would Hume redundantly begin two sections with the same principle? It also renders the titles of section I and II baffling as Hume shifts from "*Of the infinite divisibility of our ideas of space and time*"¹⁰¹ to "*Of the infinite divisibility of space and time*." Assenting to Baxter's reading leaves us in the uncomfortable position of acknowledging Hume desperately required a more scrutinising editor.

Like Baxter, Waxman argues similarly that Hume is averse to making inferences about reality itself from the nature of our ideas. Waxman argues that our ideas of space and time are post-imaginative constructions, and therefore, are meaninglessly applied to the original 'objects' from which they were derived. Waxman says:

I know of no thinker prior to Hume so keenly cognizant as he of the dangers of inferring from how our natures oblige or prevent us from thinking about reality, how reality "in itself" is or must be. It was not reality he was concerned to demarcate, but our ability to conceive it. He operated not as a metaphysician but as a

⁹⁹ Ibid, 119.

¹⁰⁰ That this resemblance could be bestowed upon an idea, and not directly copied, by the mind is evinced by Hume's conceptual minimum.

¹⁰¹ Emphasis added

philosophical psychologist; his interest lay not in the ideas we lack, but in those we have, or pretend to have, and what human nature determines us to believe or disbelieve. To the objection, is it not at least *possible* that, by sheer serendipity, things unknown and unconceived by us answer perfectly to our notions of infinite divisibility, his answer would therefore be an unflinching no. For the ability even to pose the question presumes that we have in our possession an idea of space and time that leaves open the possibility of their infinite divisibility; and this, for him, is precisely the point at issue. What Hume's examination of the origin of the ideas of space and time reveals is that they are bound up by *content* with the activity and affects of the imagination; and since this implies that these ideas cannot possibly hold of anything (perceptions included) prior to and independently of imagination, it follows that any attempt so to apply them—the only ideas of space and time we have—would violate their sense-conditions and so yield only nonsense.¹⁰²

Waxman makes a bold claim. He staunchly maintains that inferring the nature of objects themselves from our ideas of them is “utterly impossible.” The contradiction Waxman finds is between pre-imaginative objects themselves and post-imaginative abstract ideas of relation, e.g. space and time. According to Waxman, the stronger reading is ‘utterly impossible’ because our ideas of space and time are inextricably linked to the operations of the imagination. Complex ideas such as space and time have already been filtered through the imagination, which associates and binds these simple “manifolds of minima which the imagination is free to divide and recombine to the full extent of its powers.”¹⁰³ To reapply these ideas to pre-conceptualised material seems impossible and absurd. It would be like asking if the Homer Simpson on my movie screen is infinitely divisible *in reality*. First, there is no Homer Simpson in reality. To posit one would be wrong. And further, technically, the Homer Simpson on the movie screen is a visual product of a reel of data being projected. Asking if space itself is divisible would be like asking if the reel is Homer Simpson, and if the reel is infinitely divisible. It is just a stupid question. According to Waxman the pre-imaginative raw manifolds are incommensurate with the post-imaginative perceptual products (i.e. space and time).

Waxman is right that the imagination plays a vital role in determining our idea of extension as a relation. Unfortunately, Waxman backs himself into a dilemma. We can all agree that Hume maintains that the idea of extension is formed a posteriori. Hume believes that, to be adequate, a conception of any extension must be that of coexisting coloured or tangible points arranged contiguously, with lineage to past or present impressions, but not copying, any particular impression (“The table before me is alone sufficient by its view to

¹⁰² Waxman, 147.

¹⁰³ *Ibid*, 147.

give me the idea of extension”¹⁰⁴). If Waxman maintains that our ideas of space “cannot possibly hold of anything (perceptions included) prior to and independently of the imagination,” then either (A) our idea of extension cannot apply to impressions of objects such as tables, chairs, bovines, etc. (let alone objects themselves), and is applicably inert or (B) impressions are *also* post-imaginative conceptualizations. The consequence of (A) is that impressions of tables, chairs, bovines, etc. are not composed of contiguous coexistent coloured or tangible minima, which is absurd. Hume obviously denies (A). He writes, “...since the idea of duration cannot be deriv’d from such an object, it can never in any propriety or exactness be apply’d to it, nor can any thing unchangeable be ever said to have duration. Ideas always represent their objects or impressions, from which they are deriv’d, and can never without a fiction represent or be apply’d to any other.” (T 1.2.3.11 SBN 37) The idea of time can rightfully be applied to succeeding impressions or objects. Similarly, the idea of extension can rightfully be applied to complex ideas consisting of impressions related to each other by the imagination according to rules of association, viz. contiguity and resemblance, e.g. tables, chairs, and bovines, etc. If Hume held (A) this statement would be utter nonsense.

Bizarrely, Waxman and his Kantian affinities attributes (B) to Hume. Waxman argues that Hume kept his readers in the dark, and did not want to overwhelm them with the insight that our impressions are imaginative products. Waxman writes “...the re-reader: since, in T 1ii, Hume tended to withhold those implications of the T I i doctrines he was not prepared to develop and explore until T I iii...and T I iv...For, by the end of the first book of the *Treatise*, he expected us to have grasped that the ultimate implication of the seeds sown in T I i is that “The memory, senses, and understanding are, therefore, all of them founded on the imagination, or the vivacity of our ideas.” (T 1.4.7.3 SBN 265)”

From an interpretive standpoint, Waxman should have heeded Hume’s dictum that “Nothing is more dangerous to reason than the flights of the imagination, and nothing has been the occasion of more mistakes among philosophers.” (T 1.4.7.3 SBN 265) Waxman’s reading leans heavily on the quote at SBN 265. But this passage only asserts that the imagination produces the idea of a distinct continued and external existence, when it seems like the senses should. Hume writes “Without this quality [i.e. the proclivity of the imagination], by which the mind enlivens some ideas beyond others ...we cou’d never assent to any argument, nor carry our view beyond those few objects, which are present to the

¹⁰⁴ T 1.2.3.4 SBN 34

senses.” (T 1.4.7.3 SBN 265) Hume is simply saying that it is the imagination that *produces* this particular idea. To read Hume here as claiming that impressions are products of the imagination is an interpretive leap.

I do not think Hume believes that coloured or solid impressions are post-imaginative conceptualisations. Impressions “enter with the most force and violence.” Impressions are forced upon us. The imagination is different. The imagination is a faculty that can separate or reassemble complex impressions or memory-ideas, or can create new complex ideas according to associative rules. Furthermore, imaginary ideas always have less vivacity than memory ideas, and memory ideas always have less vivacity than impressions; ipso facto imaginary ideas have less vivacity than impressions. Coloured or solid impressions (unlike taste, smell, or sound impressions) when thought as being extended in space—an act of the imagination—become extended objects, prior to any thought of their possible permanence. The first work of the imagination, to wit, imagining the experience to be extended in space, is only performed on the most forceful, lively, and vivid impressions; dream-like experiences, faint images in fog, and mirages are exempt. The second work of the imagination—the attribution of permanent substance—is so far removed from even vivid impressions, as to be “fantastic.” By definition, substance is neither contiguous with, nor does it resemble particular impressions. Extension, by contrast, is the relation between impressions that, being coloured or solid, resemble each other. Even if you did a Waxmanian flip, and claim that vivacity or *belief* determined the type of perception, there would still be the different faculties distinguishable by the different levels of vivacity. Imaginary ideas would still be produced by the faculty of imagination; impressions would still be received by senses or be passions, and would be distinguishable by vivacity. Even if one were dogmatically sceptical of the existence of external objects, and refused them entry into the explanatory story, the mind would be what fixes and determines perceptions according to vivacity—not the imagination. The Humean mothership that fixes perceptions is not the imagination because the imagination is the faculty that constructs the complex ideas (“extended table in space”) that are less FLV than complex impressions (“table”). Instead, under Waxman’s hypothesis, the Humean mothership that determines the nature of perceptions would still be the *mind*, which, independently of the perceptions it produces (which I carefully demarcated in chapter one), is *completely unknowable*. Waxman’s claim is philosophically intriguing. However, if his intention is to provide an interpretation of Hume’s adequacy principle in the context of Hume’s lead argument against infinite divisibility, then it becomes too far-fetched. The re-re-

reader Waxman refers to apparently read Kant's *Critique* in between his first and third readings of Hume's *Treatise*.

My light reading avoids all of these difficulties. First, it explains why Hume does not immediately infer from his clear "least idea" to the 'atom' itself, but instead provides a demonstration leading to a contradiction. Second, my reading does not attribute careless uses of the words 'real,' or 'really' to Hume, and explains why he unqualifiedly uses the term 'extension,' and 'space and time' instead of impressions of extension or space and time or space and time *as they appear*. My reading also avoids the quagmire of interpreting Hume's scepticism with regards to external objects. Only a dogmatic and delusional sceptic would maintain that complex ideas with contradictory component parts could possibly exist, e.g. that a square circle or a valley-less mountain could exist. Furthermore, it maintains a form of Hume's scepticism without confused recourse to bizarre Kantian imaginative syntheses. However, a dash of Kant, I think, is the most defensible reading. Allowing that the imagination systematises our experience (constrains complex ideas, according to rules, from impressions) explains Hume's emphasis on the associative and relational aspects of space.

V. Concluding Remarks

The only task left is to grapple with the problematic passage "But our ideas are adequate representations of the most minute parts of extension; and thro' whatever divisions and subdivisions we may suppose these parts to be arriv'd at, they can never become inferior to some ideas, which we form"—a statement that has sent many a commentator on an ill-advised Cartesian or Kantian interpretive journey. I believe, once again, that the interpretive error is that commentators fail to appreciate the relational character of Hume's theory of extension. There is nothing "inferior" in the universe to what is indivisible and simple. What is indivisible and simple is extensionless and therefore sizeless. This is a barebones ontological claim. As early as section I of Part II Hume explicates how each perceptual level has its own threshold and minimum perception. This confirms his a priori reasoning that our complex impressions and ideas of extended objects resolve themselves into unextended points. *Regardless* of the imagination's proclivity to carry on division beyond these thresholds, insofar as it conceives (and legitimately so) smaller parts *plural*, these parts will form a 'superior' size by their conjunction. Therefore, through the a priori arguments Hume employs in part I, and the empirical analysis of perceptual thresholds, coupled with introspective analysis of the mind's contents, Hume already secures the conclusion that our

simple and indivisible ideas are adequate representations of simple and invisible objects (or simple and indivisible anything for that matter). This is a brute ontological fact resulting from extension being a relation in T 1.1.5 and Hume's doctrine of abstraction at T 1.1.7. The idea of extension without content (the idea of extension as having meaning beyond the particular solid or coloured impressions from which it is derived) is meaningless according to Hume's doctrine of abstraction. Therefore, extension is merely a relation that must be thought as holding amongst coexistent and contiguous 'parts.' Consequently, fundamental parts must be unextended points without ontological qualification. Hume writes "... 'tis a principle generally receiv'd in philosophy, that every thing in nature is individual, and that 'tis utterly absurd to suppose a triangle really existent, which has no precise proportion of sides and angles. If this therefore be absurd in *fact and reality*, it must also be absurd *in idea*." (T 1.1.7.6 SBN 19)

When Hume says "But our ideas are adequate representations of the most minute parts of extension" he is not making a bold Cartesian claim, but is arguing that extension has no existence independent of its reference to particular coexistent and contiguous things. A claim he already establishes in part I and part II section I of the *Treatise*. If an entity is one, then it is simple, indivisible and extensionless. Therefore, our idea of an indivisible minimum *is* or *would be* an adequate representation of an indivisible minimum object. They would be the same non-size. The only question is whether there are such indivisible atomic elements of extension itself, which is what Hume proceeds to demonstrate by revealing that our idea of a finite extension contradicts our idea of infinite divisibility.

APPENDIX

ii Chapter One: The Imagination, Abstraction, and Infinite Ideas

1. *"The capacity of the mind is limited, and can never attain a full and adequate conception of infinity."*

Chapter Two: Hume's Least Idea

1. *Any idea of a finite extension, had by a finite mind, must contain a finite number of parts. Because the parts are finite in number, it immediately follows that the parts resolve themselves into indivisible parts or 'points.'*
2. *The idea of extension is a conception formed by the imagination, but it is still an idea. Therefore its parts must resolve into indivisible points as well.*
3. *The imagination can bestow a colour or solidity upon these indivisible points to make them adequate for the purposes of constructing conceptions of extension.*

Corollary: Hume's "least idea" is one of these indivisible coloured or tangible points. These points are the only "parts" that the limited mind can use to form a particular conception of extension.

Chapter Three: Hume's Divisibility Principle and the Actual Parts Doctrine

4. *Divisibility presupposes the existence of actual parts that will be divided into. For example, if you are going to slice a piece of paper in half, the halves have to exist in order to make possible the slicing.*
5. *Consequently, infinite divisibility presupposes the existence of an infinite number of actual parts that it will be divided into.*

Chapter Four: An Infinity for Contradictions and Hume's Adequacy Principle

6. *When the mind conceives infinite divisibility, it must conceive an infinite number of actual "parts." (7)*
7. *The idea of an extension having an infinite number of parts is not a memory-idea but a conception formed by the imagination.*
8. *Conceptions of particular extensions must be composed of indivisible coloured or tangible points. (4)*
9. *Therefore, the only conception of infinite divisibility the mind can conceive involves an infinite number of indivisible coloured or tangible points.*
10. *The conception of an infinite number of indivisible coloured or tangible points generates the conception of an infinite extension, i.e. ←-*****-→*
11. *Therefore, the only conception of infinite divisibility the mind can form involves the divisibility of an infinite extension, i.e. ←-*****-→. Infinite extension is not a finite extension, i.e. ←-*****-→ is not ******

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12. *Therefore, the mind contradicts itself if it believes it has a conception of a finite extension being infinitely divisible, i.e. if it thinks ***** is \leftarrow ***** \rightarrow . The conception that a finite extension is infinitely divisible is contradictory, confused, muddled, bizarre, stupid, and unholy.*

(Inference from ideas to objects) When two ideas contradict, we can infer that the objects of these ideas do not both exist. For example, we cannot conceive of a square triangle, and therefore regard it as impossible. Moreover, Hume writes “WHEREVER ideas are adequate representations of objects, the relations, *contradictions* and agreements are all applicable to the objects.” (T 1.2.2.1 SBN 29)

13. *The idea of the infinite divisibility of a finite extension contradicts the only idea of infinite divisibility the mind can form.*
14. *Therefore, a finite extension is not infinitely divisible but is finitely divisible. Finite divisibility presupposes a finite number of parts (6).*
15. *Therefore, a finite extension is composed of unextended indivisible parts or points.*

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