# AN EXPLORATORY STUDY OF MATHEMATICS TEACHERS' BELIEFS AND CLASSROOM PRACTICES IN STATE SCHOOLS AND PRIVATE PREPARATORY COURSES: AN INSTITUTIONAL PERSPECTIVE

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April 2006

The candidate confirms that the work submitted is his own and that appropriate credit has been given where reference has been made to the work of others.

# DEDICATION

I dedicate this thesis to my dear parents

Macide KARAAGAC

and

Ramazan KARAAGAC

# ACKNOWLEDGMENT

This thesis has been not just a piece of academic writing but reified expression of my professional and personal development. There are, therefore, so many people who directly and indirectly contributed to it and supported me to get through rough and bumpy road of PhD. I would like to express my sincere gratitude for them. Please do accept my heartfelt apologies if I omitted someone mistakenly.

First of all, I am deeply grateful to my supervisors Dr John Threlfall and Dr John Monaghan for their invaluable guidance, continuous support and encouragement. They painstakingly read and commented on the drafts of the study and offered invaluable advice and stimulating discussions. Dr. Monaghan's enthusiasm for his work and passion for research in mathematics education has been inspirational for me. I felt privileged to have been working with him. Dr. Threlfall always provided exceptionally insightful comments and more importantly he has always been there for me.

My sincere gratitude also goes to my close friend Dr Erhan Bingolbali for his support both intellectually and emotionally. My thanks also go to Dr. M. Fatih Ozmantar and Dr. Ali Delice for their encouragement and academic support.

Special thanks go to my sponsor, the Turkish Ministry of Education, for funding this research and the teachers who took part in this study for their cooperation. Without them I would not be able to complete this thesis.

Last, but by no means the least, I am grateful to all my family members provided constant love, support and encouragement which gave me strength in completing this seemingly endless journey. I, therefore, would like to dedicate this thesis to my parents for always being there for me when I needed most.

# ABSTRACT

This study explores mathematics teachers' classroom practices in Turkey and is centrally informed by socio-cultural theories. The research examines mathematics teachers' instructional practices in relation to the wider institutional context in which teaching practices are situated.

The study takes a naturalistic approach, with minimum prior assumptions on the way in which teachers' classroom practices are examined. The structure of the examination of practices is grounded in the data itself. A multiple-case study methodology was used for this purpose. The main data included observation of four mathematics teachers' lessons from different institutional backgrounds (two state school and two private preparatory courses). I video recorded teachers' lessons while they were teaching the topic 'functions' over a period of one month, a total of 52 lessons. Other sources of data included semi-structured interviews and a questionnaire administered to 87 teachers.

The findings from the analysis of interviews suggested that all of the teachers described their lessons in the same manner and I conclude that all four teachers' instructional practices contain two main elements: 'content', where the theory of the mathematical knowledge to be taught is presented; and 'example solving', where the theoretical knowledge presented was essentially put into practice. Analysis of the video data suggests different patterns of practices in the teachers of different institutions.

My attempt to make sense of these differences revealed an emergent theme that I pursued: that the institutional context influences teachers' practices more than I expected and more than is reported in the mathematics education literature.

The findings reveal associations between specific instructional materials, teaching practices and institutions. The analysis of data also shows that the institutional context influences teachers' practices to an extent that teachers subordinate their own views regarding their teaching practices, i.e. teachers adopt teaching practices the institution they are working in promotes, even though they believe that these are not the most appropriate teaching practices to facilitate student understanding of mathematics. On the basis of these findings: I argue that individual differences in teachers' practices may be reduced by the institutions, depending on the degree of influence of the institution concerned; I argue that institutions influence mathematics teachers' professional development; I introduce the construct 'contextual density' to describe the varying degrees of influence of institutions on teachers.

# ACKNOWLEDGMENT OF AUTHORSHIP

I hereby would like to acknowledge the authorship of the thesis and declare that this PhD. thesis is my own work. It has led to several conference papers which have already been published. One of the papers is a joint publication and prepared in cooperation with Dr. John Threlfall. Content of the papers made use of the relevant chapters of the thesis with some modifications. The list of the papers published based on this study is presented below.


Karaagac, M. K., & Threlfall, J. (2004). The tension between teacher beliefs and teacher practice: the impact of the work setting. In M. J. Høines & A. B. Fuglestad (Eds.), *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 137-144). Bergen, Norway, 14-18 July 2004.

Karaagac, M. K. (2004). An aspect of teacher practice in Turkish classrooms: differences in time given to students to do mathematics in two different types of schools. In O. McNamara, (Ed.), *Proceedings of the British Society for Research into Learning Mathematics*, 24(2), 39-44. University of Leeds, UK, June 2004.

Karaagac, M. K. (2005a). Differences in teachers' selection and use of examples in classrooms: an institutional perspective on teacher practice. In D. Hewitt (Ed.), *Proceedings of the British Society for Research into Learning Mathematics*, 25(2), 43-48. The Open University, UK, June 2005.

Karaagac, M. K. (2005b). The tension between teacher beliefs and teacher practice: the impact of the institutional context. In D. Hewitt (Ed.), *Proceedings of the British Society for Research into Learning Mathematics*, 25(3), 69-74. Lancaster University, UK, November 2005.

# TABLE OF CONTENTS

# LIST OF TABLES

# LIST OF FIGURES

# LIST OF ABBREVIATIONS

The following abbreviations have been used in the thesis.

| | |
|---|---|
| **SS** | State School |
| **PC** | Private Preparatory Course |
| **EUU** | University Entrance Examination |
| **OSS** | Student Selection Examination |
| **OYS** | Student Placement Examination |
| **PEQ** | Past Examination Questions |
| **AE** | Active Example |
| **PE** | Passive Example |

# CHAPTER 1- INTRODUCTION

Mathematics education in Turkey is a fast growing research area, and yet, it is in its early stages of development. There is more and more interest in this area as time passes. Yet, there are still several fundamental questions left unanswered. One of them is regarding mathematics teachers' classroom practices. My review of literature on mathematics teachers' classroom practices yielded very little. This is not an unexpected result considering the fact that educational research in general does not have a long history after the establishment of the Turkish Republic. This situation has led me to focus on high school mathematics teachers' classroom practices in Turkey. Because of the great importance of university entrance examination, certain private educational institutions have emerged. These institutions teach the same mathematical topics to their students, but they focus on students' preparation of examination in terms of mathematical knowledge and examination psychology. Therefore I have included teachers from these institutions into my sample to have comparable and more meaningful results.

With regard to the research on mathematics teachers and mathematics education as a whole, there has been a considerable change in the way researchers carry out their studies since the 1980s. The focus merely on the students' and teachers' cognition has begun to include recognition of the broader context within which the teaching and learning took place. Thus, a major focus in research in mathematics education has become the influence of socio-cultural contexts on the students' and teachers' cognitions and mathematical practices.

This led me to explore mathematics teachers' classroom practices at the high school level, with recognition of the importance of the broader context in which teaching and learning took place. As many researchers (see for example Fang 1996; Nespor, 1987) have pointed out, teachers' practices need to be understood in relation to their beliefs. In mathematics education this especially includes teachers' beliefs about teaching and learning mathematics. Therefore, teachers' practices are examined against the background of their beliefs about teaching and learning mathematics. I also include teachers' beliefs about their work setting, which is in agreement with my consideration of the socio-cultural context of teaching as significant element in understanding teachers' practices. In the following section I will provide brief background information regarding the Turkish educational system and the institutions in which the present study took place.

## 1.1 Research Context

In the Turkish education system, curriculum subject-matter, syllabuses, textbooks and teachers' guidelines for all schools are subject to national regulations, prescribed by central government. The norms are followed by individual schools and teachers, so that the system operates in the same mode and at the same rate in every corner of the country. The norm in educational management seems to be that 'central government knows best'. Head teachers, then, regard themselves primarily as executors of regulations and norms issued from above. Local directors of education are intermediate links in the chain of command with limited innovative powers. In Turkey the administrative structure of Turkish education is centred on the National Ministry of Education, which may indeed have one of the largest bureaucratic hierarchies in the world (Karakaya, 2004, p. 197). The Minister takes the final decision, affecting the administration of all the schools in the country. His signature must appear on all orders, even in the case of relatively minor matters, in schools. All educational activities for each school operate within a framework of regulations set up by the central ministry. Therefore, Turkish education is highly centralized where policy-making and administration of schools are conducted and regulated at ministerial level. This is even more apparent in the case of secondary education. Karakaya (1994) states:

> It must be pointed out that it is difficult to imagine a system in which less opportunity is given to individual schools and teachers to exercise initiative, and in which all changes and adjustments must come from a place remote from the real school situation (Karakaya, 1994, p. 197).

The basic structure of the Turkish national education system is outlined in the Basic Law on National Education (Law no. 1739). This system has four main levels: pre-school, basic education, Secondary education and Higher education.

Pre-school level involves initial education of children before the age of 6. Pre-school education, which is optional, is carried out in independent kindergartens, nursery classes in primary schools and preparation classes. Basic education comprises the education of children in the 6-14 year age group. It provides children with basic knowledge and ensures their physical, mental and moral development in accordance with national objectives. Eight years of basic education is compulsory for all Turkish citizens who have reached the age of six. This level of education is free of charge in public schools. There are also private schools under state control. Secondary education comprises education of students in the 15-17 year age group. It encompasses two categories of educational institutions, namely general high schools (majority) and vocational and technical high schools where a minimum of three years of schooling is implemented after the basic education. General high schools are educational institutions that prepare students for institutions of higher learning. Higher Education involves universities and accepts students with sufficient scores at the university entrance examination (UEE).

In Turkey, the UEE is of great importance for students and their families as it plays a crucial role in the students' future lives and career choices. It is essentially a gateway for most students for their future career. A satisfactory score in the UEE is a pre-condition for all students who want to continue their education at the university level. However, this examination is extremely competitive. For example, among 1 728 076 students who sit the examination only 192 632 were successfully placed in a 4 year program in 2004[1]. The capacity of the good departments in universities, those from which one can easily get a job after graduating, is roughly 200,000. This situation puts a great pressure on students. As a matter of fact, a considerable number of students take the UEE two or three times as they do not get a score high enough to get a place in a good university for the first time. A common metaphor used to express the competitive nature of preparation for the examination is "Horse racing".

In the UEE, there are two main parts: social sciences and physical sciences. Table 1.1 shows subjects examined in each part.

| **Social sciences** | **Physical sciences** |
|---|---|
| Turkish Language | Mathematics |
| History | Physics |
| Geography | Chemistry |
| Psychology & Sociology | Biology |

**Table 1.1** The subjects examined in UEE.

High school students choose their modules, or take classes, according to the part they have chosen after the first year of high school. Despite this separation, every student

[1] See http://www.osym.gv.tr.

needs to answer questions from all subjects to do well in the actual examination[2]. 45 out of 180 questions in UEE are mathematical and this makes mathematics 'a bridge through which everyone have to pass' whether they like it or not. Even those who have chosen social sciences also need to answer mathematics questions correctly to get a good score, as it constitutes 25% of the examination. The examination system was changed a number of times in the past. For example, unlike the current system[3], which is single examination, it used to have two legs (OSS-student selection examination, and OYS-student placement examination) and the second leg was significantly harder to pass than the first leg.

In the last 25 years, parallel to the growing number of students and changes in the UEE regulations, 'Private Educational Institutions' or 'Private Preparatory Courses' (PC) have gradually appeared[4]. These institutions are at the forefront of the Turkish education agenda and an issue of debate. Until 1979 there were few of these courses and they existed only in the big cities. In 1983, opening PCs was legally banned (Law No: 2843) as they were claimed to 'break the law of equal right to have education' and their preparatory mission was proposed to be undertaken by the state schools (SS) (Öztürk, 1994, p. 4; Tunay, 1992, p. 17). However, by 1984 they were allowed to continue their existence (Law No: 3035, Öztürk, 1994, p. 4).

Since these institutions play an important role at a crucial stage of students' life they are an integral part of the Turkish education system. The number of these institutions, which was 174 in 1984, is not less than 4000 throughout Turkey today.

There may be several reasons why they exist. However, the major reason seems to be the discrepancy between what is taught in state schools and what is required for a good score in the UEE. This gap, whose existence was confirmed by one of the Ministry of Education reports (see Ergün, 1990, p. 24-25), may be one major reason for the existence of these private courses. Another reason why the number of PEI increased rapidly may be the nature of the test. It is a multiple-choice test and existing literature suggests that coaching for testing helps students to raise their scores. Bunting & Mooney (2001) examined the effects of practice and coaching on test results for

[2] The information presented here is describes the regulations at the time of the data collection and may have changed in the later years.
[3] The information based on single leg examination (OSS) in 2004, which was modified later on.
[4] Despite the fact that some of the regulations date back to 1915 (Tunay, 1992).

educational selection at 11 years of age and claimed that 'The effect of sustained coaching over a period of 9 months is shown to be substantial' (ibid, p. 243). These institutions are regarded as an important helping hand for the student in their test preparation.

Despite the widespread existence of these private institutions in Turkey and the magnitude of the debates of academics about testing as it relates to classroom teaching and that these institutions seem to be a significant part of students' experiences of mathematics, there is lack of research on these institutions. The extant research does not seem to go much beyond a survey of student demographics, student attitudes towards PCs (Şirin, 1998; Tunay 1992; Öztürk, 1994) or the correlation between mock examinations in PCs and UEE (Dogan, 1999). My review of literature resulted in no research on instructional practices of the mathematics teachers working in these institutions. Similarly, and surprisingly, the same could be said for mathematics teachers working in the SS at the high school level. This is, perhaps, because mathematics education is a new area of research in Turkey. Therefore, there is a need for research on mathematics teachers' instructional practices in SS and PCs.

### 1.2 Research Focus and Research Questions

The present research aimed to explore mathematics teachers' classroom practices from two types of institutions (SS and PC). In order to achieve a better understanding of teachers' instructional practices, I also examined the broader context in which teaching takes place. This is in line with Lerman's (2001) suggestion that educational research should focus on some particular issues taking into account the social context in which the research is conducted, thus, integration of these two dimensions into the research. This approach provided a basis for my research foci and research questions, which are as follows.

**Research Focus 1 (RF1):** The first research focus is, essentially, on mathematics teachers' classroom practices. It deals with the ways in which the mathematical activities have been framed by the teacher. Using Lerman's (2001) metaphor, this focus aims to 'magnify' teachers' instructional practices to explore essential facets of teaching activities taking place in the two types of institutions. Therefore the present research focus has the following research questions:

**RQ1a:** What are the important features of practice from the point of view of the teachers in the two institutions?

**RQ1b:** How can teachers' practices be described in terms of these features?

**Research Focus 2 (RF2):** My second research focus is mainly on how the broader context in which teaching takes place influences the teachers' practices. This 'zoom out' is aimed to complement the 'zoom in' of RF1 as it is anticipated to provide a broader picture of the context of the present research. This is addressed through two research questions:

**RQ2a:** What are teachers' beliefs and perceptions about teaching and learning mathematics and how do they relate to their practices?

**RQ2b:** What are teachers' perceptions of the broader institutional context in which their practices are situated and how do they relate to their practices?

RF2 and related research questions was intentionally left relatively broad. This stemmed from my realisation of the importance of being open to possible themes that may emerge.

## 1.3 Methodology

I used case study methodology for my study as I needed authentic in-depth data rather than broad but surface data. Four mathematics teachers made up the 'cases' of this multiple-case study. Two teachers were selected from each of two institutions (PEIs and SSs) on the basis of their willingness to participate and their timetable. The feasibility of the research was a major determinant of such a selection process. These teachers' lessons were video recorded when they were teaching the topic of functions. These video recordings were the main source of information on teachers' practices. Case study methodology also allowed me to carefully explore teachers' beliefs and perceptions about teaching and learning mathematics as well as their ideas about the institutions in which they work. Two separate interviews for each teacher were designed and conducted. One set of interview questions probed the teachers own explanation of their teaching and another probed the teachers' beliefs to complement the first one.

In addition to the main data source (four teachers), many other mathematics teachers from both types of institutions were interviewed. I also collected various teaching materials used in both types of institutions such as mock examination booklets, examination papers and various tests. Small amounts of data were also collected from students of these teachers to see their perceptions of the case teachers' practices. I therefore interviewed a small number of students. One aim of this multi-faceted approach is to enhance interpretability (Robson, 1993, p. 291) of the findings. Additionally, such an approach is regarded as a highly useful particularly in case studies (Cohen and Manion, 1994). This multi-faceted approach provided extra support for the inferences made on the basis of main data source.

### 1.4 Summary of the Chapters

This section summarises the contents of the following chapters.

In chapter 2, I provide a literature review in order to situate my research within the extant literature, to provide the rationale behind the research foci and to inform the discussion of the findings. In the first section, I review the research on teachers' beliefs in relation to their practices. The second section provides a brief review of research on teachers' goals and decisions. In the third section, I attend to the existing research in teacher socialisation. The last section presents the current trends in research on teachers, which strives to establish the lack and the importance of examining teachers' practices from an institutional perspective.

In chapter 3, I provide a detailed description of the methodology used in the study. In the first section, I present the aims of the present study, its nature, piloting and methodological issues surrounding the case study as a method. This is followed by the presentation of the issues regarding research tools and data collection and the sources of information. In the fourth section, I present the analysis of the data. The last section concludes with considerations of reliability and validity.

Chapter 4 presents the results of the present study. The results chapter is made up of four main sections. I provide background information and findings for each case (teachers). The first two sections are allocated to PC teachers, while the third and fourth sections were allocated to SS teachers. The section allocated for each teacher is divided into 4 main sub-sections. In the first, I present the teacher's personal information, his/her weekly schedule and my contact with him/her as background information. Secondly, I present the organisation of his/her teaching of the topic of functions. This is followed by analysis of a specific lesson in detail in the third section. The fourth section presents the teachers' beliefs and goals.

In Chapter 5, I bring the findings presented in the results chapter together. This is to synthesise the results of the study in order to make sense of the discussion in the Chapter 6. In the first section of this chapter, I present comparison of the SS and PC teachers' overall lesson structures. The second and the third section present a synthesis of the findings from the teachers' classroom practices in 'content' and 'example solving' segments of their lessons, respectively.

Chapter 6 is the discussion chapter. I discuss findings of the study in relation to the literature presented in Chapter 2. In the first section, I will attend to the meaning of the term 'institution' in this study. In the second section, I argue that there are associations between certain materials and practices and certain institutions. I argue in the third section that the institutions may be so influential on teachers' practices that teachers may subordinate their own personal views in relation to their practices. In this section, I maintain that teachers may prefer to teach in the way that the institution they are working in suggests even though they clearly think otherwise. That may mean that individual differences in teachers' practices may even be reduced by the institutions depending on the 'density of the institutional context', which I explain in the fourth section. In the final section of this chapter, I discuss the influence of institutions on teachers' professional development.

Chapter 7 is the conclusion chapter. It presents the conclusions of the study, which clarifies the contribution of the current study to the extant literature, and presents my suggestions for further research. These are followed by the limitations of the current study.

# CHAPTER -2 LITERATURE REVIEW

In this chapter I will present the pertinent literature in order to situate the present research into the existing body of knowledge on teachers. In the first section, I review the research on teachers' beliefs in relation to their practices. In the second section, I present research on teachers' goals and decisions. In the following section, I attend briefly to the existing research in teacher socialisation. The last section briefly presents the trends in research on teachers.

## 2.1 Research on Teachers' Beliefs in Relation to Practices

Understanding teachers is one of the major goals of educational research. There has been considerable theoretical and empirical research devoted to the understanding of teachers and their instructional practices (Shavelson & Stern,1981; Clark & Petersen, 1986; Day, Pope & Denicolo,1990; Fang 1996). Findings from various branches of educational research indicate that a large number of factors may influence teachers' practices (Clark & Petersen, 1986; Borko & Putnam, 1996). For example, different types of teachers' knowledge (e.g., subject matter knowledge, pedagogical content knowledge), curriculum, and textbooks used may all influence teachers' practices. However, research on teachers suggests that beliefs are one of the major factors affecting teaching (Thompson, 1992; Calderhead, 1996; Pajares, 1992; Kagan, 1992 Richardson, 1996). In fact, "there has been a considerable amount of research on teachers' beliefs based on the assumption that what teachers believe is a significant determiner of what gets taught, how it gets taught, and what gets learned in the classroom" (Wilson & Cooney, 2002, p. 128). Moreover, in her recent article, Speer (2005) argues that research on teachers' beliefs is abundant because other factors fail to fully explain the nature of teachers' instruction. As a result, it is argued (Pehkonen & Törner, 1999, p.5) that the link between teachers' beliefs and their practice is considered to be well-established. Pehkonen & Törner (1996, 1999) say, for example, that beliefs act as a 'regulating system' that drives actions. Others contend that "beliefs are instrumental in defining tasks and selecting the cognitive tools with which to interpret, plan, and make decisions regarding such tasks; hence, they play a critical role in defining behaviour and organizing knowledge and information" (Pajares, 1992, p. 325). Standen (2002) stresses that :

> The concept of beliefs has a significant and primary role in understanding personal meaning with which teachers imbue their practice...a more appropriate way of investigating why teachers choose one course of action in preference to others is through the construct of teachers' beliefs...teachers' beliefs provides a suitable framework for explaining how teachers negotiate the different dilemmas that are presented in the daily routines of the classroom. This is particularly evident in the way in which beliefs filter the knowledge considered appropriate to use in the classroom and in providing a frame for interpreting the challenges that emerge (p. 26-27).

In this vein, Speer (2005) states that:

> Beliefs appear to be, in essence, factors shaping teachers' decisions about what knowledge is relevant, what teaching routines are appropriate, what goals should be accomplished, and what the important features are of the social context of the classroom (p. 365).

Despite the fact that beliefs may be regarded as "the best indicators of the decisions individuals make throughout their lives" (Pajares, 1992, p. 307), there has not been a consensus on what is to be taken as 'belief' (Hekimoglu, 2004). The need for a commonly agreed definition of belief is acknowledged and a number of definitions have been put forward. Hart (1989), for example, uses "the word belief to reflect certain types of judgments about a set of objects" (p. 44). Lester et al. (1989) state that "beliefs constitute the individual's subjective knowledge about self, mathematics, problem solving, and the topics dealt with in problem statements" (p. 77). Nespor (1987) talks about 'belief systems' rather than beliefs:

> Belief systems often include affective feelings and evaluations, vivid memories of personal experiences, and assumptions about the existence of entities and alternative worlds, all of which are simply not open to outside evaluation or critical examination in the same sense that the components of knowledge systems are (p. 321).

Ponte (1994) (as cited in Pehkonen & Törner, 1999) puts beliefs and conceptions in one basket and states that: "beliefs and conceptions are regarded as part of knowledge. Beliefs are the incontrovertible personal 'truths' held by everyone, deriving from experience or from fantasy, with a strong affective and evaluative component" (p. 169). Some researchers define the term from more of a mathematics education point of view. Schoenfeld (1992), for example, interprets beliefs "as individual understandings and feelings that shape the ways that the individual conceptualizes and engages in mathematical behavior" (p. 358). Another example is Thompson (1992), who states that "a teacher's conceptions of the nature of mathematics may be viewed as that teacher's conscious or subconscious beliefs, concepts, meanings, rules, mental images, and preferences concerning the discipline of mathematics."

Despite the existence of a large number of definitions used by researchers, it is also accepted as a 'messy construct' (Pajares, 1992) to define. Nespor (1987) uses the term 'entangled domain' to refer to the messiness of the issues surrounding research on beliefs. This 'messiness' may be because belief does not lend itself easily to empirical investigation. Others suggest that searching for a common ground in the definition of beliefs is not a useful endeavour (Pehkonen & Furinghetti, 2001, p.52). Moreover, it is argued that the search for a common definition is less important than understanding teacher practice (Wilson & Cooney, 2002). Searching for a common definition of belief and making it at the forefront of the agenda of one's research endeavour may be a 'misrouting' of the effort on studies on teachers since such an approach may result in mixing means with a desired goal. Leder Pehkonen & Törner (2002) collected a number of definitions suggested and used by leading educational researchers and devised a questionnaire in which they listed these definitions. The questionnaire required the readers to indicate to what extent they agreed with the given definition of 'belief'. The questionnaire was completed by leading researchers in mathematics education who are interested in belief related research. Having reviewed the research on teachers beliefs, knowledge and other similar concepts used in the area (such as conceptions, views) and having analysed the results of this questionnaire, Pehkonen & Törner (1999) "admit that the different assumptions of researchers make it impossible to reach a complete agreement on the mutual relationships in the triad 'beliefs - conceptions - knowledge'" (p. 27). This conclusion is shared by other researchers who surveyed the area (Mcleod & Mcleod, 2002). Mcleod & Mcleod (2002) proposed that "there is no single definition of the term 'belief' that is correct and true, but several types of definitions that are illuminative in different situations" (p. 120). Therefore what seems to be possible and in fact desirable for a researcher is to present the definition of belief used in the study (Pajares, 1992).

It seems that messiness is inevitable in research on beliefs. I, therefore, will not make a new definition of belief for this study but I will settle for an operational definition, since I do not directly focus on teachers' beliefs per se. I will use the term 'professed belief' which is "defined as those stated by teachers" (Speer, 2005, p. 361). Speer (2005) very recently focuses on the distinction between 'professed beliefs' and 'attributed' beliefs and states that this distinction has become "a fixture of research on teachers" (p. 361). The current study is not the first to adopt such an approach. In fact, most research studies focusing directly on beliefs avoid giving an explicit definition and settle for an operational definition (Hekimoglu, 2004). I will present the interview extract or refer to the relevant episode of video recording in order for the reader to be able to judge the validity of my conclusions on teachers' beliefs.

Despite the arguments for teachers' beliefs as being one major determinant of teachers' behaviours, empirical investigations also show that teachers' beliefs and their instructional practices may not be entirely consistent all of the time (Thompson, 1984, 1985, 1992; Kesler, 1985; Raymond, 1997; Karaagac, 2004). For example, Kesler (1985) (as cited in Thompson 1992) reports some variation among senior high school teachers in the degree of consistency between their conceptions and their instructional practices. He classified teachers as having dualistic/absolutist and multiplistic/relativist views. He reports that the teachers with dualistic views show consistency, but not those with a multiplistic view. Thompson (1984) used case study methodology to examine the relationship between beliefs and instructional practices of three elementary mathematics teachers. Thompson examined both belief-belief and belief-practice inconsistencies. That is to say she was examining not only the discrepancy between beliefs and observed practices of teachers but any possible inconsistencies between two beliefs. Although two of the teachers show relatively more consistency between their beliefs and practices, one teacher with 3.5 years of teaching experience, Lynn, had inconsistencies between her separate beliefs and between her beliefs and practices. For instance, Thompson reports Lynn's inconsistent beliefs:

> Lynn did not have an integrated conceptual system with regard to mathematics. Her views of mathematics as 'cut and dried' and essentially prescriptive in nature, allowing little opportunity for creativity, appeared to be in sharp contrast with her remarks about the mental disciplinary effects of

> its study…it was clear from Lynn's comments that she was referring to mathematical activities that call into play creativity and inventiveness as well as activities involving formal, logical reasoning (p. 122).

To Thompson, having 'an integrated conceptual system' is important to achieve having a consistency between different beliefs and also to help teachers to have beliefs about teaching mathematics consistent with their instructional practices. Thompson said:

> The fact that so many of her views in this regard were discrepant with her instructional practice suggests the absence of an integrated conceptual system operating to modify her actions. Had she an integrated conceptual system, one would expect to have seen some evidence that she experienced conflict between, say, her attempt to individualize instruction and the fact that she consistently gave every student identical worksheets (p. 122).

Raymond (1997) also reports a belief-instructional practice inconsistency in an elementary school mathematics teacher. In the vein of Thompson (1984), she also used case study methodology on six teachers within the first two years of their professional lives. She focused on Joanna who was in her second year as a teacher. Raymond states that among all the teachers 'Joanna's beliefs about learning and teaching mathematics were the most inconsistent with her practice' (p. 553). Raymond asked Joanna to illustrate the influence of her beliefs on her practice on a model. Despite the fact that in the procedure of this research there was a clear indication of the researchers' interest in the consistency-inconsistency aspect of beliefs and practices of teachers, Joanna, a mathematics teacher, had inconsistencies between her beliefs and her practice. One of these, Raymond points out, was in her use of manipulatives:

> In another example, Joanna made a connection between a pedagogical belief and a related practice. She claimed that her belief in teaching mathematics through manipulatives encouraged her to provide base-ten blocks for her students. Recall that the only time Joanna actually used the manipulatives during this 10-month period, she described the class period as "chaotic" and decided that she needed to change things before she tried to implement them a second tine. Thus, one cannot conclude that she actually teaches mathematics through manipulatives to any degree. I do not believe that Joanna saw the distinction between intentions and actual practice. She seemed to view her mathematics teaching practice in terms of what she

> wanted to do, or thought should do, rather than by what she actually accomplished...It was as if she thought that believing in good mathematics teaching practices was a way of practicing good mathematics teaching (p. 569).

It is curious to observe such discrepancies between teachers' beliefs and practices considering the fact that teacher's beliefs and practices are expected to be in harmony with one another (Festinger, 1957). In fact an observed inconsistency between teachers' beliefs and practices may be a valuable source of information in our understanding of teachers' practices. For example, one question that comes to mind in such a situation is 'what is it that is more influential on teachers' practices than their own beliefs?' The answer to this question could be very illuminating in research on teachers since it could reveal other sources of powerful influence on teachers' practices. However, answering such a question satisfactorily and coming to sound conclusions depends on the extent to which the teacher is aware of such a discrepancy. Thompson (1992) states that "in the case of observed discrepancies between professed mathematical beliefs and practice, one must question the extent to which teachers are aware of such discrepancies and if so how they explain them" (p. 135) The review of research in this area shows the studies which report such cases where teachers are found to be unaware of the situation (see for example Raymond, 1997; Thompson, 1984). Sadker and Sadker (1994) (cited in Ghosh, 2004) report teachers who were shocked when they saw themselves in the videotapes of their actual classroom interactions with the students when these did not seem to be in agreement with their expressed beliefs. This may lead to a change in teachers' beliefs about themselves as teachers and thus their identities as teachers. This may be another reason why examining inconsistency between teachers' beliefs and practices may be a productive source of information in our understanding of teachers. Smagorinsky et al. (2004) report such a case, where a beginning teacher experienced tensions between her constructivist beliefs that she picked up from her university education and her classroom practices in the high school where she began to teach. This led her to experience tensions which affected her identity as a teacher. Therefore, reconciling beliefs and practices or reconciling different beliefs may profoundly affect how teachers see themselves as mathematics teachers and thus their identities as mathematics teachers.

## 2.2 Research on Teachers' Goals and Decisions in Relation to Their Practice

Although some teachers (Pinnegar & Carter, 1990) may report that teaching is very intuitive and 'common sense', there is ample evidence to suggest that it is in most part based on teachers' decisions, judgements and pedagogical thoughts (Shavelson & Stern, 1981; Shavelson, 1983; Clark & Peterson, 1986). The National Institute of Education (1975) (as cited in Shavelson & Stern, 1981, p. 457) states that:

> Though it is possible, and even popular, to talk about teacher behavior, it is obvious that what teachers do is directed in no small measure by what they think....To the extent that observed or intended behavior is 'thoughtless,' it makes no use of the human teacher's most unique attributes. In so doing, it becomes mechanical and might well be done by a machine (p. 7).

Therefore, any teaching act is the result of a decision, whether conscious or unconscious, that the teacher makes after the complex cognitive processing of the information available. This reasoning leads to the hypothesis that the basic teaching skill is decision making (Shavelson, 1973, p.18). Yet, what is not clear is the nature of the relationship between teachers' thinking, their decision-making processes and their instructional practices. Nisbett & Ross (1980) acknowledged this over a quarter of a century ago:

> We also say little about precisely how people's judgements affect their behaviour. This is neither an oversight nor a deliberate choice. We simply acknowledge that we share...psychology's inability to bridge the gap between cognition and behaviour, a gap that in our opinion is the most serious failing of modern cognitive psychology (p. 11).

Despite the fact that there are valuable developments in our understanding of teachers' thought processes since the time of this quote, it seems to me that our understanding of the relationship between teachers' thought processes and teachers' instructional behaviour is still in its infancy. This is perhaps because of the fact that the more we begin to uncover human psychology, the more we realise that it is more complex than we previously thought. However, central to my argument here is the significance of teachers' thought processes in understanding teachers' practices. One of the most significant aspects I would like to turn my attention to is teachers' goals.

Shavelson & Stern (1981)[1] argue for the need for research on teaching to examine teacher' goals and the way they are linked with their instructional practices, and not just 'behaviour' alone, which cannot account for the variations in teachers' behaviours arising from teachers' goals, judgements and decisions in the classroom. They also stress that linking teachers' goals to their practice provides a better basis for educational reforms. Since the review of research on teachers by Shavelson & Stern in 1981, there seems to be relatively little interest in the study of teachers' goals in educational research. This is acknowledged by Boyer & Tiberghien (1989) and relatively more recently by Donnelly (1999), who states:

> Teachers' aims are not a popular subject for research....Few studies can be found in the literature, and within those that exist, teachers' goals are often seen as the foil for other issues, rather than having any particular interest in their own right. Examination of two handbooks of research on teaching reveals almost nothing in this area. The chapter on the goals of science teaching in the Handbook of Research on Science Teaching and Learning (Gabel 1994) is concerned with the views of almost anyone other than teachers themselves. In the parallel volume on teachers and teaching (Wittrock 1986) the subjects again figure barely at all. It appears, then, that the aims of teachers are not judged to be particularly significant, except perhaps as the targets of attempts to shift them (p. 18).

It is possible that Donnelly (1999) may not have seen all the research on this area, and having less number of studies on this area does not mean they are not significant in our understanding of teachers' practices. In fact, as the discussion chapter of this thesis will suggest, this area seems to be a relatively less developed and yet an essential aspect of research on mathematics teachers.

Focusing on the relationship between mathematics teachers' goals and beliefs, Schoenfeld and his colleagues (who called themselves the Teacher Model Group) has published a series of articles (Aguirre & Speer, 2000; Schoenfeld, 2000; Schoenfeld, et al., 2000; Zimmerlin & Nelson, 2000) in a special issue of Journal of Mathematical Behavior in 2000. In order to make very detailed analyses (minute by minute) of

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[1] The word 'intention' is used interchangeably with 'goal'. They also use the word 'behavior' frequently. This is possibly because these were the most commonly used terms by process-product researchers in the time of the publication.

lessons of established and novice mathematics teachers, they categorised teachers' goals as 'overarching goal(s)', 'local goal(s)' and they added Saxe's (1991) notion of 'emergent goal(s)'. They define 'goals' as "the things you want to accomplish" (Schoenfeld, 2000, p. 250) or "the cognitive constructs that describe (at various levels of detail) what the teacher wants to accomplish" (Aguirre & Speer, 2000, p. 332). They also define "overarching goals" as teachers' "goals for students over the course of weeks, months, or the year; unit goals; lesson goals; goals for particular parts of a lesson" (Schoenfeld, 2000, p. 250) and "local goals" are the ones which are specific to certain moments in the lesson.

Goals may be epistemologically oriented ('I want the students to understand and experience physics/ mathematics as a sense-making discipline'); they may be content-oriented ('students should know the three measures of central tendency and their properties'); they may be socially oriented, at various levels of grain size ('I want the class to function as a community of inquiry', or 'I want this student to feel rewarded for having ventured a question'). Goals may be pre-determined (e.g., as part of the lesson image) or they may be emergent (e.g., when the class seems restless, or an interesting issue arises in dialogue with the students). And, of course, multiple goals can be (and usually are) operative at the same time.

Rich & Almozlino (1999) and Donnelly (1999) report differences in goals of the teachers of different subjects. History teachers are reported to have a more uniform set of goals. In fact, the majority of history teachers' goals are found to be mainly around three goal statements. By contrast, science teachers' statements of goals were ranged more widely. In explaining this finding Donnelly (1999) uses the expression 'shared sense of purpose' and suggests that there was less sense of a clear, shared goal among science teachers. He later suggests that the uniformity of the history teachers' vision may be explained by a curriculum development project on history. One could infer, then, that history teachers would 'normally' have a wide range of goals like science teachers if the project mentioned was not implemented. Boyer & Tiberghien (1989) found that teachers' as well as students' representations of goals are strongly influenced by the constraints of the educational system. Donnelly (1999) asked English, science and history teachers about their goals. He found little systematic difference between teachers of the same types of institutions (state schools). Despite that he acknowledges the lack of teachers' explicit reference to the influence of National Curriculum, he associates having common goals among state school teachers with the existence of 'a heavily centralised school system...as a result of the statutory authority of the National Curriculum' (p. 30).

### 2.3 Research on Teacher Socialisation

In this section I will specifically focus on teacher socialisation, even though it may be considered as a part of larger body of knowledge as occupational socialisation, or more generally, socialisation of individuals. Despite the increasing recognition of the significance of the process of becoming a teacher (Brown & Borko, 1992, p. 209) it seems that there is relatively little done on professional socialisation of mathematics teachers. Much of the early work on professional socialisation took into account other professions, mostly medical students, rather than teachers (Lacey, 1985, p. 4076). Scarcity of research on this area appears to be at two levels. Firstly, there are plenty of studies on how beginning teachers react when they face the daily realities of teaching, in terms of possible change in their beliefs, knowledge and their practices, but there is little done on how existing teachers are influenced by the institutional context in which they work. Secondly, the extant literature regarding, in particular, mathematics teachers seems to be weak in terms of professional socialisation of mathematics teachers and how they become mathematics teachers in the context of institutional settings. Research on teacher socialization views the teacher as a member of a professional culture and therefore the process of becoming a teacher is viewed as becoming member of that culture (Brown & Borko, 1992). However, as Lacey (1985), possibly the most quoted study on this area, points out, 'it is important to notice that the process of professional socialization does not end at the point of entry into the profession or at any arbitrary point during the early career of the teacher' (p. 4073). Therefore, it should be taken as a continuing process through which teachers' beliefs, values, their perceptions of self and their socio-cultural context, and their identities as teachers evolve.

Two prominent reviews on teacher socialisation, Brown & Borko (1992) and Zeichner & Gore (1990), identify three major traditions, or paradigms as Zeichner & Gore (1990, p. 329) prefer, in this area. This is despite the fact that most of the research reviewed rarely presented a clear articulation of these paradigms (ibid, p. 329). These paradigms are: functionalist, interpretive and critical approaches.

**Functionalist Paradigm**

The functionalist paradigm, as the historically earliest perspective on teacher socialisation, emphasises the conception of teaching as the passive acquisition of the professional culture promoted by the socio-cultural context. This paradigm stresses the idea that individuals fit into society, stressing a lack of autonomy of the individuals. Merton et al. (1957), for example, defines socialisation as:

> ...the process by which people selectively acquire the values and attitudes, the interests, skills and knowledge – in short the culture – current in groups to which they are, or seek to become, a member (p. 287).

Much of the work by functionalist researchers assumes a simple 'filling of empty vessels' mechanism by which the individuals passively acquire those values and skills that are necessary to become a member of the profession (Lacey, 1985, p. 4074). One of the major problems of this perspective is the basic acceptance of teachers' professional development as a process with an end product, 'a person fully matured and capable of taking his or her place in society' (Lacey, 1977, p. 18). Further criticisms of the basic assumptions of the functionalist paradigm can be found in Lacey (1985).

**Interpretive Paradigm**

The interpretive paradigm can be traced back to its roots in the German idealist school of thought (Zeichner & Gore, 1990, p. 330). It challenges the ontological assumptions of the functionalist approach (Burrell & Morgan, 1979, p. 32). Many of the recent studies with the interpretive approach to teacher socialisation have been inspired by the work of Lacey (1977), who attempted to understand student teachers from their perspectives. Lacey aimed at 'developing a model of the socialization process that would encompass the possibility of autonomous action by individuals and therefore the possibility of social change emanating from the choices and strategies adopted by individuals' (Lacey, 1985. p. 4076). The model he created is established around the notion of 'social strategy'. To him,

> A social strategy is reducible to actions and ideas but it is only interpretable in the context of a specific situation. A social strategy involves the actor in the selection of ideas and actions and working out their complex interrelationships (action-idea systems) in a given situation. The selection of these action-idea systems as a student [teacher] moves from situation to situation, need not be consistent (p.67-68).

Lacey (1977) defines three main action-idea systems depending on the individual's specific goals and the constraints of the institutional context in which they work. These are: internalized adjustment, strategic compliance, and strategic redefinition. Internalized adjustment refers to a strategy where the individual fully complies with the institution by producing the promoted behaviours, valued practices and takes on the expected role as a teacher. The individual believes that the constraints of the situation are for the best. Strategic compliance refers to a strategy where the teacher complies with the institutional context and the constraints of the situation but retains 'private reservations' (ibid, p. 72) about them. The individuals act in accordance with the institutional requirements even though it may be inconsistent with their own beliefs and values, as an adaptation to the institutional constraints. Finally, strategic redefinition refers to a strategy where the individual successfully changes the situation. They manage to do so by causing or enabling those with formal power to change their interpretation of what is appropriate in the situation.

**Critical Paradigm**

The central concern of the research within critical paradigm is justice, equality and freedom. The issues related to class, gender and race are key foci in critical paradigm research. For example, teaching might be seen as mostly work of one gender (Apple, 1987). Zeichner & Gore (1990) observe that the functionalist and interpretive paradigms have clear links between the teacher and occupational socialisation literature and that this is not the case for the critical paradigm. The critical paradigm within the teacher socialisation research literature seems to exist very much as a theoretical and philosophical perspective and there are very few empirical studies of socialisation from this point of view (Brown & Borko, 1992, p. 225).

In this thesis the interpretive paradigm has been adopted as it emphasises the influence of the socio-cultural context of the teachers, while keeping the teachers' autonomy. I believe that teachers can keep their autonomy while they socialise and take roles in line with the 'community of practice' (Wenger, 1998) in which they operate.

## 2.4 Trends in Research on Teachers

One basic assumption of teacher research is that teacher practice is significantly influenced by teacher thinking processes and even determined by it (Clark & Petersen 1986). And yet, most of the research on teacher thinking was carried out after the mid 1970s (Clark & Petersen, 1986). In an attempt to organise the research on teachers, Clark & Petersen (1986) report a shift in the trend in teacher research after the mid 1970s and divides the area into two main research paradigms until the mid 1980s. Along with the trends in educational research in general, research on teachers experienced another shift in the early 1990s from focusing on teachers' cognitions as a major influence on teachers' practices towards recognising the influence of the socio-cultural context on teachers' practices.

The early trend in teacher research is what is commonly called 'process-product research', 'teacher effectiveness' or 'teacher behaviour' (Brophy & Good, 1986). Influenced by behaviourist research especially in psychology (Calderhead, 1990), this approach gained prominence among researchers especially up until the mid 1970s. In an attempt to improve students' mathematical achievement it was thought that changing certain behavioural parameters in teachers' behaviours (for example clarity, enthusiasm, variability, and task orientation) will make a dramatic impact on students' achievement. Process-product researchers generally assumed the unidirectional causality between teacher behaviour and student behaviour, with teachers' behaviour directly affecting students' achievement. Almost mechanical relationships were offered in this research and the relationships between teachers' behaviour and student outcomes were commonly expressed with if-then sentences or similar to mathematical formulas. For example Doyle (1977) states:

> Within the process-product paradigm the effectiveness question is formulated in terms of relationships between measures of teacher classroom behaviors

> (processes) and measures of student learning outcomes (products). This approach is based on a two-factor criterion-of-effectiveness structure that relates teacher variables directly to effectiveness indicators. The structure of the paradigm corresponds in essence, therefore, to a prediction formula: Define the criterion and find its predictors...Any number of process and criterion variables can be inserted into the formula, and the empirical associations can then be calculated. As a result, the paradigm can be used by investigators who differ markedly in their definitions of appropriate variables (p. 165).

Similar explanations are offered in Brophy & Good (1986), who reviewed process-product research extensively.

Another trend in teacher research began to emerge when the focus of studies turned to teachers' thinking processes and teachers' cognition rather than merely teachers' behaviour. The change of focus was, as one would expect, influenced by the advances in cognitive psychology and trends in qualitative methodology and conceptions of teaching as being a thoughtful profession (Fang, 1996). Thompson (1992) situates this shift in its historical context:

> Research in teaching began a shift in the 1970s from a process-product paradigm, in which the object of study was teachers' behaviours, to a focus on teachers' thinking and decision making processes... The shift of focus to teachers' cognition, in turn led to an interest in identifying and understanding the composition and structure of belief systems (p. 129).

The change took place because several limitations had been pointed out in the behavioural basis of process-product research. Shavelson & Stern (1981), who reviewed research on this area, point out two reasons for the shift of trend. First, the behavioural model is 'conceptually incomplete' (p. 455) and cannot account for the variations in teachers' behaviours arising from teachers' judgements and decisions. Second, linking teachers' intentions to their behaviour would provide a better basis for educating teachers and for reforms. According to Clark & Petersen (1986), the process of teaching involves two basic domains: (1) teachers' thought processes (or teacher cognition) and (2) teachers' actions and their observable effects. Teachers' thought processes occur "inside teachers' heads and are unobservable" (ibid, p. 257).

In the last 15 years there seems to have been another shift in teacher research including research in mathematics education. The research trend is generally called socio-cultural research. The underlying assumptions of the socio-cultural approach are based on the works of Russian researcher Lev Vygotsky. Particularly influenced by Marxist ideas, especially the works of Marx himself (Wertsch, 1991, p. 21), Vygotsky regarded human thinking as dependent on the socio-cultural context within which the individuals are situated. Although the empirical studies in socio-cultural research have grown in the last 15 years, the recognition of the influence of context dates back long before the 1990s. For example Dewey (1916) states:

> The environment consists of the sum total of conditions which are concerned in the execution of the activity characteristic of a living being. The social environment...is truly educative in its effect in the degree in which an individual shares or participates in some conjoint activity. By doing his share in the associated activity, the individual appropriates the purpose which actuates it, becomes familiar with its methods and subject matters, acquires needed skill, and is saturated with its emotional spirit...The deeper and more intimate educative formation of disposition comes, without conscious intent, as the young gradually partake of the activities of the various groups to which they may belong (p.26).

However, educational research has only begun to recognise the significance of the context on understanding teachers' and students' beliefs and practices in mathematics education. The empirical evidence is beginning to emerge in mathematics education regarding various contexts as influential on teachers' and students' beliefs, norms values, practices and cognitions. The movement which recognises the socio-cultural context as an integral part of human functioning has a starting point in the 'genetic law of cultural development', put forward by Vygotsky in the 1930's. Vygotsky (1978) argued that:

> every function in the child's cultural development appears twice: first, on the social level, and later, on the individual level; first between people (interpsychological), and then inside the child (intrapsychological). This applies equally to voluntary attention, to logical memory, and to the formation of concepts. All the higher functions originate as actual relations between human individuals (p. 57, emphasis in the original).

Vygotsky argued for mediational processes and that all mental functions are mediated. Although Vygotsky has argued for socio-cultural influences on human mental functioning and worked on semiotic mediation, he is criticised for not directly addressing many of the critical issues he gave rise to. One of his strong contemporary interpreters Wertsch (1991) points out that 'in certain essential aspects he did not succeed in providing a genuinely sociocultural approach to mind' (p.46). In particular, he failed to take broader context into account on human mental functioning. In fact, 'he did little to spell out how specific historical, cultural, and institutional settings are tied to various forms of mediated action' (ibid, p. 46). Accordingly, individuals cannot be considered in isolation from their socio-cultural context and therefore it is necessary to look at the broader context in which individuals are living to have a more comprehensive understanding of their practices. Brown, Stein & Forman (1996) acknowledge this trend in mathematics education, moving from cognitive psychological frameworks towards recognition of socio-cultural processes and regarding the cognitivist frameworks they state:

> These frameworks locate learning within the individual with little or no attention to the social and cultural processes that influence the development of thinking and understanding. Until very recently, research in mathematics education in the United States followed the assumption that learning consists of the development of increasingly sophisticated knowledge structures and cognitive skills devoid of context...however, mathematics education researchers have begun to recognize that these frameworks leave large portions of classroom activity unexamined and major aspects of learning unaccounted for (p. 64).

They go on to say that:

> ...More recently, perspectives that take into account the role of social, institutional, and cultural factors in the processes of teaching and learning have begun to appear within the mathematics education research literature...One of the perspectives that has been gaining attention within the mathematics education research community is sociocultural theory which emphasizes that the construction or appropriation of knowledge occurs in a social, institutional, and cultural context (p. 65).

Daniels (2001) draws attention to the "insufficient empirical study of socio-institutional effects" and also a "tendency to under-theorise differences between schools in terms of institutional effects on the social formation of mind" (p. 135). He argues for the significance of institutional approach and states that it "may be regarded as points for development in contemporary post-Vygotskian theory and research" (ibid. p.135). Zeichner & Gore (1990) also point out the "lack of attention to institutional and cultural contexts" (p. 341) from the perspective of teachers' professional socialisation. Wertsch (1991) approaches the issue in relation to the production of 'utterances', 'speech genres' and states that "meaning is inextricably linked with historical, cultural, and institutional setting" (p. 66). Bingolbali (2005) recognises that "the effects of institutions can be manifested in the way that lecturers, in the case of this study, teach and assess students studying in different departments" (p. 26). Despite such recognition, there is still a lack of empirical research in mathematics education that has a particular focus of teachers practices in relation to the institutional context in which they work.

Very recently Bingolbali (2005) and Castela (2004) studied the influence of institutions on students' understanding of certain mathematical concepts. Bingolbali & Monaghan (2004, 2005) have focused on aspects of learning and teaching the derivative in mechanical engineering and mathematics departments from an institutional perspective at the undergraduate level. In their 2004 paper, they discuss mechanical engineering students' tendency towards rate of change aspects of the derivative and mathematics students' tendency towards tangent-oriented aspects. They partially attribute these different tendencies to the calculus practices to which students are exposed in each department. That is to say, as the students are taught in a way that emphasises different aspects of derivative, they develop a tendency towards different aspects of derivative concept, or simply, they learn differently. The researchers note that lecturers 'privilege' different aspects of the derivative when teaching in different departments, that is, mechanical engineering calculus lecturers privilege the rate of change aspect whilst mathematics calculus lecturers privilege the tangent aspects of the derivative concept. In understanding and interpreting why mechanical engineering and mathematics students develop different tendencies towards different forms of the derivative concept and why the lecturers privileged different aspects of the derivative, in their subsequent paper (Bingolbali & Monaghan, 2005) they argue that it is indispensable to take into account the institutional settings in which teaching and learning occurs. In this connection, they state that each department's characteristics and goals explicitly or implicitly impart particular value judgements with regard to mathematics. Interpretations of these value judgements shape both lecturers' and students' perception of, and practices in, teaching and learning mathematics in different departments. They conclude their paper by stating that cognitive functioning is influenced by others, by the setting and by the way individuals position themselves in settings and hence suggesting that the researchers should look at the issues under investigation from an institutional perspective.

While existing studies often focus on students, there seems to be a gap in the literature specifically on how institutions influence teachers' classroom practices. Despite the fact that teaching is beginning to be recognised as inherently 'cultural activity' (Stigler & Hiebert, 1998, 1999; LeTendre et al., 2001), and 'that teachers' instructional practices are profoundly influenced by the institutional constraints that they attempt to satisfy, the formal and informal sources of assistance on which they draw, and the materials and resources that they use in their classroom practice' (Cobb et al., 2003, p. 13), the current attempts to situate mathematics teachers' classroom practices within the institutional context in which teachers are working does not seem to go beyond the theoretical level (ibid, 2003). This is where the current study is situated within mathematics education literature; an attempt to reveal the influence of institutions on mathematics teachers' classroom practices.

While this study is informed by socio-cultural theories, differing theoretical perspectives (cognitive dissonance, constraints and affordances etc.) are also visible throughout the discussion. The initial path the study takes is a naturalistic approach, with minimum prior assumptions on the way in which teachers' classroom practices are examined, so that the structure of the examination of practices is grounded in the data itself. However, as the data is analysed, emerging findings proved to be hard to make sense of without the help of differing theoretical positions. The use of theory, therefore, follows the emergence of findings from the data analysis rather than preceding them. Since different theoretical positions are used to explain different aspects of the findings, these differing positions did not seem to lead to a potential danger of 'poisoning' the study with use of theories with incompatible assumptions to explain the same finding.

# CHAPTER –3 METHODOLOGY

The main objective of this chapter is not a linear presentation of the research processes that took place but more considerations of the way in which the research was carried out. Undertaking educational research is a messy process at the level of PhD, and yet, it is very personal journey. I am of the view that a PhD is a very personal product and it reflects the background and the personality of the researcher. Many research techniques and tools have been developed parallel to my own personal development. In a sense, this could be perceived as mixture of 'trial and error' and 'learning by doing'. Thus this chapter contains not only clues regarding the messiness of the educational research process and the chain of events that took place, but it also contains clues to my personal journey from being 'peripheral participant', as Lave & Wenger (1991) would say, in the educational research community to being an 'active participant' of it.

In the first section, I present the aims of the present study, its nature, piloting and methodological issues surrounding the case study as a method. This is followed by the presentation of the issues regarding research tools and data collection and the sources of information. In the fourth section, I discuss the analysis of the data. The chapter concludes with considerations of reliability and validity.

## 3.1 Research Design

### 3.1.1 Research Foci and Questions

The present research aims to explore mathematics teachers' classroom practices from two types of institutions (SS and PC). In order to achieve a better understanding of teachers' instructional practices, I examine the broader context in which teaching takes place. This approach is in line with Lerman's (2001) metaphor of camera focus. He states that:

> From a sociocultural perspective an object of research on mathematics teaching and learning can be seen as a particular moment in the zoom of a lens. Researchers focus on a specific part of a complex process whilst taking account of the other views that would be obtained by pulling back or zooming in. Researching teaching and learning mathematics must be seen in the same way. Thus in zooming out researchers address the practices and meanings within which students become school-mathematical actors, whilst zooming in enables a study of mediation and of individual trajectories within the classroom. In each

choice of object of research the range of other settings have to be incorporated into the analysis (p. 87).

Lerman argues that researchers should focus on some particular issues (zoom in) taking into account the importance of the social context (zoom out) in which the research is conducted. From this perspective, an aim of educational research should, therefore, be integrating these two dimensions. As to this study, my approach incorporates 'zoom in' along with 'zoom out'. The former constitutes my first research focus and the latter constitutes the second.

**Research Focus 1 (RF1):** The first research focus is, essentially, on mathematics teachers' classroom practices. It deals with the ways in which the mathematical activities have been framed by the teacher. Using Lerman's (2001) metaphor, this focus aims to 'magnify' teachers' instructional practices to explore essential facets of teaching activities taking place in the two types of institutions. Therefore the present research focus has the following research questions:

**RQ1a:** What are the important features of practice from the point of view of the teachers in the two institutions?
**RQ1b:** How can teachers' practices be described in terms of these features?

**Research Focus 2 (RF2):** My second research focus is mainly on how the broader context in which teaching takes place influences the teachers' practices. This 'zoom out' is aimed to complement the 'zoom in' as it is anticipated to provide a broader picture of the context of the present research. This is addressed through two research questions:

**RQ2a:** What are teachers' beliefs and perceptions about teaching and learning mathematics and how do they relate to their practices?
**RQ2b:** What are teachers' perceptions of the broader institutional context in which their practices are situated and how do they relate to their practices?

RF2 and related research questions was intentionally left relatively broad. This stemmed from my realisation of the importance of being open to possible themes that may emerge.

### 3.1.2 Research approach

The distinction between 'qualitative' and 'quantitative' approaches has been an area of debate for some time in social sciences. It has been rather commonly held that these terms refer to traditions each containing a set of ontological and epistemological assumptions regarding the researcher's perception of social reality, and associated notions and labels. The quantitative label is often associated with a view which conceives the social world like the natural world as if it were a hard, external and objective reality and that the research needs to be directed towards analysing the causal relationships and regularities between selected factors in that world (Cohen & Manion, 1994, p. 7). Such an approach is often said to be linked with a family of labels such as: positivistic, natural science based, hypothetico-deductive, and scientific (Robson, 1993, p. 18). Qualitative approaches, on the other hand, are said to favour an alternative view of social reality which stresses the importance of the subjective experience of individuals in the creation of the social world, so that research of a qualitative nature is seen as focusing upon understanding of the way in which individuals interpret the world in which they live (Cohen & Manion, 1994, p. 8). Denzin & Lincoln (2000) put it this way:

> Qualitative research involves an interpretive, naturalistic approach to the world. This means that qualitative researchers study things in their natural settings, attempting to make sense of, or to interpret, phenomena in terms of the meanings people bring to them (Denzin, 2000, p. 3).

'Qualitative' research is often linked with the term 'interpretive' (Robson, 1993, p.18). 'Quantitative' approaches generally frequently thought of as involving deduction of a hypothesis from a theory and testing it and modifying it with regard to the outcomes. In 'qualitative' approaches, on the other hand, the theory is generally seen arising after the data collection rather than having it at the beginning.

In spite of the influence of this distinction, some researchers have pointed out the dangers of making such a dichotomy and assuming that studies have to subscribe to a single approach (Hammersley, 1995, p. 2-3). Bryman (2004) points out that the distinction between quantitative and qualitative research is frequently exaggerated (p. 449). Moreover, it is acknowledged that a quantitative research approach can be employed for the analysis of qualitative studies and a qualitative research approach can be employed to examine the rhetoric of quantitative researchers (ibid, p. 450). From this perspective, it will suffice to say that the present study can be regarded as a mixture of 'qualitative' and 'quantitative' approaches, in the sense that it builds upon both qualitative and quantitative forms of data analysis (for example, emergence of themes in the analysis of interview, and emergence of lesson structure in the analysis of video recordings).

Robson (1993, p. 42) distinguishes between three types of social research on the basis of the purpose of the enquiry: descriptive, explanatory and exploratory. He acknowledges that one study may have more than one purpose but one predominates. Descriptive studies aim to portray an accurate profile of a person, an event or a situation. It could be qualitative as well as quantitative and requires researcher to have a great deal of background information regarding the phenomenon. Explanatory studies, as their name suggests, seek an explanation to an event or a problematic situation in either a qualitative or a quantitative manner. Exploratory studies essentially aim to find out what is happening or to seek insights into a situation or to assess a phenomenon under a new light. Robson (1993, p. 42) also states that these kind of studies are, usually but not necessarily, 'qualitative'.

The present research sets out to understand, essentially, what is happening in mathematics lessons and to seek insights into mathematics teachers' instructional practices. From this perspective, the present study has the characteristics of a descriptive and exploratory research which contains both qualitative and quantitative elements.

### 3.1.3 Case Study Method

Case study has been one of the most widely used research strategies in social sciences such as education, law and sociology. A case study is ideally a detailed portrayal and intensive description of a 'case'. In other words, case study is 'an umbrella term for a family of research methods having in common the decision to focus on enquiry around an instance [or case]' (Adelman et. al., 1984). This definition implies the idea that doing a case study is employing a number of pre-specified research instruments. However, case study should not be regarded as a research method package coming with the title 'case study' (Adelman et al., 1984, p. 94). Robson[1] (1993) defines it in the following way:

[1] It seems to me that Robson is overlooking historical cases by this definition.

> Case study is a strategy for doing research which involves an empirical investigation of a particular contemporary phenomenon within its real life context using multiple sources of evidence (Robson, 1993, p. 5).

A 'case' is something whose boundaries can be defined. Lou Smith, one of the first educational ethnographers, used the term 'bounded system' (cited in Stake, 1997, p. 406) to define 'case'. It is interesting that the two authorities may have different views of the term 'case'. Robson (1993, p. 146) states that 'contemporary phenomenon' or 'case' can be virtually everything. Stake (1995, p. 2), on the other hand, states that 'custom has it that not everything is a case'. He goes on to say that 'we cannot make precise definitions of cases or case studies because practices already exist for case study in many disciplines. Having reviewed common flaws in the literature concerning the definition of case study and its distinctive features, Yin (1989) gives his definition, which is similar to the one suggested by Robson (1993):

> A case study is an empirical enquiry that:
>
> - Investigates a contemporary phenomenon within its real life context: when
> - the boundaries between phenomenon and context are not clearly evident;
> - and multiple sources of evidence are used (Yin, 1989, p. 23).

Finally, according to Hammersley (1992), 'case' is 'the phenomenon (located in space/time) about which data are collected and/or analysed, and that corresponds to the type of phenomena to which the main claims of a study relate' (p. 184). In making sense of case study research, two points should be considered: firstly, any person with teaching experience knows the importance of giving a specific example in order to understand certain features of a general problem. Therefore, it casts light on prominent facets of the problem. Secondly, unlike qualitative studies, many quantitative studies are criticised because of their neglect of context. On the other hand, case study method is useful to examine the phenomenon at hand in its natural context. Additionally, sometimes there is no way to gain a thorough understanding of the problem other than using a case study approach, which is generally performed qualitatively. The importance of the case study as a research strategy lies on these points. Case studies are quite strong in examining the phenomenon in its real life context. This is probably the reason why many PhD studies are carried out employing this approach, in spite of its critics on the generalisability issue, which will be examined later on.

According to Stake's (1995) categorisation, there are three basic types of case study: 'intrinsic case study', 'instrumental case study' and 'collective case study'. It is their purposes that differentiate one from another. If the particular case is investigated to gain a better understanding of salient features of the phenomenon and its intricate relationship with its context, then it is 'an intrinsic case study'. The main interest here is not the typicality of the case, but its particularity in showing a specific trait. If the case is examined due to its typicality or representativeness of other cases and therefore the emphasis of the study is not the case's particularity, then it is 'an instrumental case study'. It is important to note that it is not necessarily that the case should be a typical example of possible cases. A typical example of a phenomenon can be studied as a case to understand the common features of certain phenomena. We might also choose to study more than one case to get at 'healthier' generalisations from the case. In this case, the study is 'a collective case study'. Collective case studies sacrifice time and depth of the information gathered from the 'case' in return for better grounds for generalisations.

Case study proponents generally emphasise that although case study strategies are not safe grounds on which to make generalisations, their depth and contextualisations means that analytical generalisations based on examination of cases are of value. It seems that the problem arises from the common perception that the only way of making generalisations is through statistical inference. Yin (1989) views this as 'a fatal flaw'. He perceives generalising from cases to 'theory' as analogous to scientists' generalisation from experimental results to theory, and multiple cases as analogous to multiple experiments using 'replication logic'. It seems to me that such a perception of generalisation is based on the quantitative research tradition. Mitchell (1983) draws attention to the distinction between 'statistical inference' and 'logical inference':

> A good deal of the confusion has arisen because of a failure to appreciate that the rationale of extrapolation from a statistical sample to a parent universe involves two very different and even unconnected inferential processes - that of statistical inference which makes a statement about the confidence we may have that the surface relationships observed in our sample will in fact occur in the parent population, and that of logical or scientific inference which makes a statement about the confidence we may have that the theoretically necessary or logical connection among the features observed in the sample pertain also to the parent population (Mitchell, 1983, p. 207).

In 'statistical inference', generalisations are made depending mainly on the degree to which it can be 'safely' claimed that the sample is chosen from the populations and that it has typical characteristics of the population. Researchers employing this logic can be confident in their claims or generalisations, to a degree determined by statistical confidence limits in their studies. 'Logical inference', on the other hand, is viewed in the following way: 'the inference about the logical relationship between two characteristics is not based upon the representatives (cis) of the sample and therefore upon its typicality, but rather upon the plausibility or upon the logicality of the nexus between two characteristics' (ibid, p. 198). Hence, logical inference is based on the strength of the logical relationship between the 'case' and the population.

Case study researchers share commonalities in the way they think about the issue of generalisations based on case(s). Mitchell's (1983) notion of logical inference can be seen in Stake (1995) under the label of 'direct interpretation'. A similar term can be observed in Yin (1989, 2003) with the label 'analytical generalization'. It seems to me the meaning of the terms they use for making generalisations based on case studies are closely related. Similarly, Stake (1995) makes distinction between 'direct interpretation' and what he calls 'categorical aggregation'. It seems that the way Stake defines 'categorical aggregation' is quite similar to 'statistical inference' as defined by Mitchell (1983). Although the meanings of the terms they use are similar, Mitchell and Stake have different aims in their approaches. While Mitchell's aim in using the terms is to develop a theory, Stake takes direct interpretation and categorical aggregation as different ways of interpreting case study data. Stake (1995) establishes a connection between interpretations and generalisations: 'this is all important to case researchers because we have choices to make in terms of how much we should organise our analyses or interpretations to produce the researcher's propositional generalisations (which I have been calling assertions) or to provide input into the researcher's naturalistic generalisations' (p. 86). What is implicitly claimed here is that interpretations are pillars of the generalisations in the case studies.

These two forms of interpretations, direct interpretation and categorical aggregation are explained, by Stake (1995), in the following way:

> Two strategic ways that researchers reach new meanings about cases are through direct interpretation of the individual instance and through aggregation of instances until something can be said about them as a class. Case study relies

on both of these methods. Even with intrinsic case study, the caseworkers sequences the action, categorizes properties, and makes tallies in some intuitive aggregation. Even with instrumental case study, some important features appear only once... (Stake, 1995, p. 74).

Categorical aggregation is basically a way of interpretation in which the researchers look for a pattern of behaviour of the case or repeatedly occurring events around the case. For instance, suppose a researcher has chosen a mathematics teacher as a case and he is making classroom observations on a regular basis. The researcher observes that in the introduction of a new topic, the teacher prefers to solve a problem on the blackboard and explain the topic using this particular example. When the observer realises that this teaching style is embedded in his 'case', he interprets these observational data as a basis for his interpretation, of course not without supporting this finding with other sources of information. As can be seen in this example, the case study researcher looks for consistency with regard to certain conditions. These aggregations can be made within a case as well as over a number of cases. Scott & Usher (1999) state that:

> Theory development is either cumulative, in that as more and more cases are studied, the database becomes more extensive and rich and findings more reliable, enabling the researcher to generalize to larger populations, or theory developed from one or more cases can then be tested as to its validity and reliability by examining further cases (p. 88).

Direct interpretation is another way of getting at new interpretations or generalisations from the case or cases. Sometimes a single event might be a crucial part of the case in the development of the events in its natural setting. The kind of interpretation the researcher makes of such cases is direct interpretation. Probably the most important point to make about such an interpretation is to emphasise the validity of the data and the subjectivity of the interpretation. However, it is not unusual for the caseworkers 'to make assertions on a relatively small database, invoking the privilege and responsibility of interpretation' (Stake 1995, p. 12). Therefore, the researcher takes a greater responsibility in direct interpretations.

Bassey (1984) has a different perspective in making generalisations in case studies. He points out the distinction between what he calls 'open' and 'closed' generalisations:

> An open generalization is a statement in which there is a confidence that it can be extrapolated, beyond the observed results of the sets of events studied, to similar events, with the expectation that it will be similarly applicable. A closed ganeralization is a statement which refers to specified set of events and without

> extrapolation to similar events. A closed generalization is descriptive; an open generalization is both descriptive and predictive (Bassey, 1984, p. 111).

The approach in Bassey's paper can be regarded to have a utilitarian point of view because he constantly asks the question 'what (kind of) generalisations are useful to improve practice?' Because open generalisations are relatively more likely to provide reliable predictions, they are useful to help teachers to make decisions in the classroom. A closed generalisation, however, can be employed by a teacher in trying to 'relate what has happened in other classrooms to what is happening in his [own classroom]...perhaps the case study is potentially more useful to teachers than the closed generalization' (ibid, p. 117-118). From Bassey's point of view, an important criterion in judging the merit of case study is the extent to which the details are sufficient and appropriate for teachers working in a similar situation to relate their decision making to that discerned in the case research and 'the relatability of a case study is more important than its generalisability' (ibid. p. 119). This sounds like logical inference. It is intriguing that he claims that we should eschew the pursuit of generalisations, unless their usefulness is apparent; instead we should encourage the descriptive and evaluative study of single pedagogic events.

It seems reasonable to suggest that we need to consider the case study approach not only in itself, but also in terms of its place relative to other approaches. In selecting any research strategies there are trade-offs we face with and that we can not get everything we want. We sacrifice one thing at the cost of getting some more from another (Hammersley, 1992). Nisbet & Watt (1984) make a similar point. They believe that case study data is 'strong in reality' but difficult to organise. The data obtained from other research strategies, on the other hand, is generally 'weak in reality' but susceptible to ready organisation. Case study is regarded as strong on reality because it studies the phenomenon in its real life context, and this strength in reality provides a 'natural' (rather than statistical) basis for generalisation. Moreover, saying that case study offers a weaker basis for generalisation to large, finite populations as opposed to surveys does not mean that it provides no basis for generalisations (Hammersley, 1992).

### 3.1.4 Piloting

The piloting took place in March 2001 for a month. The aim of piloting was to get initial data, which was expected to help to frame the structure of the main study and to examine the feasibility of the research project. Four mathematics teachers (two from each type of school) were the subjects of the piloting. They were willing to participate in this piloting. It involved interviewing mathematics teachers of SSs and PCs as well as classroom observations. 32 classes were observed (15 SS classes and 17 PC classes). Three classes (two PC and one SS) were also video recorded. Three groups of students were interviewed. The piloting helped me to refine my thinking about the research and my methodology. For example, the data collected have been used in constructing interview questions and questionnaire items.

Having an open approach to teachers' classroom practices helped me to advance my ideas about the project without the imposition of any prior framework. Therefore piloting was also a nervous, and yet, exciting experience towards recognising the feasibility of the project. I was not sure if the data I intended to collect could be obtained. It was also a training opportunity for me as a researcher to learn how to observe teachers from the point of view of someone other than a colleague or a student. I also developed my interviewing skills along the way. My prior thinking 'what could be hard about asking a few simple questions?' has disappeared and admittedly the results of interviews were not as good as I supposed they would be. However, such a first-hand experience helped me towards becoming a skilled interviewer much more than I first thought it would.

After the piloting and a fair amount of literature review I have developed two research foci for the study in line with Lerman's (2001) metaphor of camera focus (see research foci and research questions). That is, 'zoom in' on teachers' classroom practices and 'zoom out' to explore teachers own beliefs and perspectives about teaching/learning mathematics and the socio-cultural environment in which teaching practice takes place.

Another result of the piloting was the selection of the mathematical topic for the study. Having reviewed the distribution of the topics throughout the entire year for both PC and SS classes, I decided to select the topic of functions. Functions were the topic that seemed to be ideal for data collection. It was not small on the syllabus but it was not too long for me to cover either. The teaching of the topic of functions in the SS and PC was close to the mid-point between the beginning and middle of the year, which seemed to be ideal to collect the data. The topic of functions is also relatively well elaborated in the literature, which could help future studies to combine the findings of current study with previous findings.

## 3.2 Main Data Collection

### 3.2.1 Sampling Strategy

The trade-off Hammersley (1992) suggests not only exists in the generalisability of the findings but also in the sampling strategy of the research. Unlike experimental or survey style enquiries, case studies rely less on sampling, partly because one cannot include every possible case into the study (Robson, 1993, p. 154) Another reason for this may well be that the case studies do not lead to a conclusion in the sense of statistical generalisation from the sample to every member of the class of the sample. Miles & Huberman (1994, p. 30) suggest that in multiple-case study design the number of cases to be studied cannot be decided on statistical grounds, but on conceptual grounds in which the selection strategy needs to be explicit. The trade-off in this study's sampling strategy is that it emphasises authenticity and examination of the phenomenon in its natural context and feasibility of the research, and deemphasises statistical generalisability of findings. The present research is a multiple-case study which involves two teachers from each type of institution (PC and SS), making a total of four cases. Multiple-case sampling is more robust than single case studies and adds confidence to the interpretations of the researchers on a conceptual basis rather than a statistical basis (Miles & Huberman, 1994, p. 29).

As the study is descriptive and exploratory in nature, the sampling choice was made mainly on the basis of the feasibility of the research. However, I also looked for teachers who were not unrepresentative of teachers in the Turkish context. I also looked for more experienced teachers on the basis that experienced teachers would also be more experienced in the institution in which they are working. Another reason for such a choice is my observation[2] that mathematics teachers in Turkey are not particularly expressive and thus experience would help them exemplify their points with regard to their past experiences.

In the SS leg of the field visit, I was given permission by local representatives of the Ministry of Education to carry out my study in 5 high schools. I visited each high school and managed to talk to most of the mathematics teachers and obtained their weekly timetables. Some of the teachers welcomed my intention and some of them

[2] This includes piloting and my 'apprenticeship of observation' (Lortie, 1975).

appeared to be intimidated by the prospect of being video recorded for a month. Once one teacher put himself forward as a possible case (Mahir) the task was to find a willing teacher who had a non-ovelapping timetable with him, which happened to be Ayten. It is my personal judgement that both teachers were willing to participate partly because they are very experienced, self confident teachers. As to the PC side of the data, I visited 4 different PCs and talked to mathematics teachers. Unlike SS, the head teachers of the PCs did not require formal permission and my relationship with PC teachers was less formal in comparison to SS teachers. Five teachers put themselves forward for the task and Saban and Nuri had non-overlapping timetables and, as they were in the same PC, it was easier for me to deal with technicalities of the video recording. It is of particular importance to acknowledge that all of the teachers who took part in this study gave the researcher permission to use their images in the thesis. Such a sampling strategy, a common sampling strategy in case studies, is called 'purposive sampling' (Robson, 1993, p. 141-142; Cohen & Manion, 1994, p. 89).

### 3.2.2 Interviews

This section is divided into a number of parts under two main headings. In the first, I will provide information about interviews as a research tool. The second will inform the reader about the rationale of the use of interviews for this study, its preparation and implementation in the field and analysis of interview data.

#### *3.2.2.1 Interview as a research tool*

*Types of interviews*

The interview is a flexible way of collecting personal information. The information that needs to be elicited may vary from competence and skill to act in a certain way to how individuals make sense of the phenomenon at hand and what they know or think about it. Cohen & Manion (1994) mention three purposes that an interview can be used for. First, it may be used as the principal means of gathering information having direct bearing on the research objectives: what a person knows, likes dislikes and what a person thinks. Second, it may be used to test hypotheses or to suggest new ones; or as an explanatory device to help identify variables and relationships. Third, the interview may be used in conjunction with other methods in a research undertaking: to follow up unexpected results or validate other methods, or go into respondents' motivations and their reasons for responding as they do.

There are various types of interviews that may be used for different purposes of research. The most common ground for division is at the level of structure. Robson (1993) makes a distinction between structured interviews, semi-structured interviews, and unstructured interviews (p. 231). *Structured interviews* involve asking questions that have been fully prepared beforehand and responses are sequenced in a certain way. Although there is an insurance of covering all the interest area and not going off the focus, there is less room for making modifications during the interview. Robson (1993) calls this 'effectively a questionnaire where the interviewer fills in the responses' (p. 231). Therefore, the major drawback is its insensitivity to emergent themes that may be of interest to interviewer (Fontana & Frey, 1998). In *semi-structured interviews*, as used by many naturalistic researchers, the interviewer has a list of topics that need to be addressed and thus they require preparation of a basic set of questions but have flexibility to follow up issues that emerge during the interview. Thus, the interviewer is able to modify the wording and sequence of the questions and may add some probing questions to follow interesting themes that come up during the interview. In *unstructured interviews*, or as Cohen & Manion (1994) call them ,non-directive interviews, there is a minimal structure in the interview and thus wording and sequence of the questions emerge from the discussion in the setting. For this type of interview, the interviewer has a general area of interest but lets the conversation develop within the area of interest through reflecting and rephrasing the respondents' statements.

*Use of interviews in belief and practice research*

Interviews have been used widely in research on teachers' beliefs and in studies which examine teachers' practices (Thompson, 1992) and suggested to be one of the most powerful tools of qualitative research (Fontana & Frey, 1998). I have used interviews to capture teachers' beliefs and perceptions in order to understand teachers' practices from their own perspective. Interview is widely used in mathematics education research to understand students' and teachers' beliefs, perceptions and their way of sense making. To be more precise, it seems that there is hardly any belief research that does not make use of interview as a method. Despite its common use in belief research, a regular criticism the interview receives is its indirect way of accessing interviewees' (teacher or student) minds. To alleviate the problem, as several researchers including Tomlinson (1989) and Woods (1997) have argued, the interview questions should be based on personal experience, rather than being merely at an abstract level. An interview that seeks to understand the phenomenon from the interviewee's perspective needs to take into account the contextual elements. This is likely (1) to reduce the possibility of interviewees' accounts being thought up during the interview and (2) to decrease the possibility of eliciting answers that the interviewees may think are expected from them. In line with this approach, Woods (1996) emphasises the need for eliciting teachers' beliefs through indirect questions, which are based on concrete contextual experiences.

#### *3.2.2.2 Interviews in this study*

*Rationale for choosing semi-structured interviews for this study*

The interviews had to be executed during empty slots in the teachers' schedule, and this limited the time allocated. Although it is essential to be open about the interesting issues that surfaced during the execution of interviews, this time limitation was one reason for not choosing the unstructured interview. Another reason was the effect of long interviews on the teachers' emotional state and willingness to give authentic answers. A structured interview was rejected on the basis of the exploratory nature of the study. The trade-off between the need for flexibility to pursue possible emergent issues during the interview and keeping up the interview agenda suggested semi-structured interviews as a choice for this study.

*The preparation of interviews*

Since the current research is exploratory in nature, I tried to keep my personal understanding of teaching in the two schools out of the way in the way in which the research was designed. It was mainly the piloting and relevant literature that affected the way the interview items were constructed. The general categories of questions come from belief research literature: teachers' beliefs about teaching mathematics, teachers' beliefs about learning mathematics and teacher beliefs about the nature of mathematics (Speer, 2005). However, the piloting suggested that it may be unproductive to try to explore teachers' beliefs about mathematics by direct questioning as the atmosphere of the interview with those teachers who have not been interviewed before can sometimes be negatively affected. However, teachers' beliefs can also be inferred from their answers to context specific answers. Thus two interviews were constructed. The first aimed at understanding teachers' beliefs and practices with reference to their practices as a way of accessing their beliefs indirectly, while the second aimed at understanding their beliefs and views about wider contextual issues through direct questioning.

Teacher interview-1 was designed for understanding teachers' views on their own classroom practices. In other words, I wanted to see their practice from their eyes. How do they think mathematical activities are structured in their classes? What are/should be the roles of teacher and students? Therefore, teacher interview-1 was expected to enable me to see the classroom practice from the teacher's perspective. (See Appendix A)

Teacher interview-2 was intended to get at teachers' beliefs and perceptions about mathematics and the institutions in which they work. This interview was guided by two questions: "What are the teachers' views about the way mathematics should be taught and learned?" and "What are the teachers' goals and how do they relate to classroom practice from the teachers' perspectives?" (See Appendix B)

*In the field*

The interviews were carried out with four teachers (cases) after the completion of the video recordings or near to the end of the period of video recordings. This was mainly intended to gain time for teachers to be comfortable talking to me regarding their beliefs and instructional practices. I am of the opinion that with the deliberate delay of the interviews, I reached a level of closeness that allowed teachers to be comfortable talking to me casually about their professional lives in and out of the classroom. In addition to the main interview data collected from the cases (four teachers) I also interviewed mathematics teachers from a variety of SSs and PCs. In total 24 teachers were interviewed, of which 15 were from SSs and 9 were from PCs.

### 3.2.3 Video Recordings

This section is divided into a number of parts under two main headings. In the first, I provide information about video recordings as a research tool. The second will inform the reader about the rationale of the use of video recordings for this study and its implementation in the field.

#### *3.2.3.1 Video recordings as a research tool*

Video data provide a means of incorporating 'quantitative' and 'qualitative' approaches in educational research, in particular research focusing on teachers' classroom practices. Since the availability of video recorders is expanding, teacher researchers are beginning to recognise it as a viable research method. It is beginning to emerge as a promising way to collect authentic data on teachers' classroom practices. It is particularly favoured by the researchers who are interested in teachers' classroom practices in different countries. For example, the 'Third International Mathematics and Science Study Video Study', which is also known among mathematics and science educators as 'TIMSS 1999 Video study' (Stigler et al., 1999; Hiebert et al., 2003a, 2003b), is based principally on video recordings. In brief, a number of internationally known researchers compared teaching mathematics in seven different countries based on large numbers of video recordings (See Hiebert 2003a, 2003b). This is, as has been claimed, the first time researchers have used video to collect a national sample of teaching (Stigler & Hiebert, 1999, p. 17). Another international comparative research study based on video recordings is the 'Learners Perspectives Study' (Clarke, 2003). Despite its growing popularity among teacher researchers, there seems to be very little written on the use of video recordings as a research method.

I will now present advantages and disadvantages of the use of video recordings as a data collection method and its relative merits over others where appropriate.

One of the major advantages of the use of video data to emerge is based on its compatibility with current trends among teacher researchers, which regards teaching as a very complex process in the context of the complex environment of the classroom. It involves a myriad of verbal as well as behavioural exchanges between teacher and students. A researcher observing a teacher can only attend to certain aspects of his teaching. Therefore, it is impossible even to notice, let alone capture, all of the significant aspects of interactions in real time. Through the use of video, it is possible to capture a number of aspects that are taking place simultaneously in the classroom. This exceeds by far important limitations of observation methods in which researcher simply ticks the boxes if certain teacher or student behaviour is observed.

Another advantage of the use of video is that it enables researchers to capture verbal as well as visual clues regarding what is taking place in the classroom. Recording lessons and capturing verbal expressions undoubtedly helps understanding the teaching and learning process in the classroom. However, it is not always the teachers' or students' words that matter. We know through experience that it is often the case that visual clues play significant functions in the way, say, students perceive and interpret the meaning of the behaviours of the teacher. Eye contact is a significant one of them. Video data enables researchers to be able to identify such subtleties, which certainly helps explain teaching processes.

The use of video is particularly useful in exploratory studies in that it allows novel research questions to emerge from data, and at the same time it provides researchers the means to test these questions in both a qualitative and a quantitative manner (Jacobs et al., 1999).

Video also increases the reliability of the inferences made based on the data. This is possible since the video data can be analysed as many times as the researcher requires or wishes. More reliable inferences can be made in two main ways. Firstly, video data enables what Jacobs et al. (1999) call a 'cyclical analytical process' in the analysis of the data. This is a significant aspect of video data, in that 'conventional data collection methods must be collected and analysed linearly, but video data allow for a unique iterative process' (p. 718). To them, this cyclical process involves repeated use of video data to generate themes, make discoveries and frame hypotheses. This then could help in establishing a coding system. The researchers could then test their coding system using the videos. This, in return, helps to validate emergent themes or hypotheses and come to clearer interpretations. Secondly, video data enables coding and interpreting data from multiple perspectives. Once the researchers have developed a coding system or made certain inferences based on the data, they can always ask their colleagues to test out if their coding system is working well or to see if their inferences match with others' inferences. This is a remarkable advantage of video data over many other observation methods as it allows external verification of conclusions.

Although video is a valuable and effective way of collecting data, it is not free from limitations. One major limitation is that the video recorder will only record what its user focuses it on. The video will only record what happens in a certain area of classroom, or certain individuals or groups of students. This selection of focus for the recorder means that certain activities in the classroom are excluded. Although this could be overcome through use of a number of video recorders, as used by Clarke (2003) in the 'Learners Perspectives Study', it may entail a higher possibility of distraction of the natural flow of social interactions. This brings another downside of video recording as a data collection instrument. As the video recorder is clearly situated in the classroom, there is always a possibility of distracting teachers and students from being 'natural' or 'normal'. However, relatively speaking, this is also problematic for other observation methods. It is also hard to predict which situation would more be likely to distract teacher and/or students from being natural; the recording of a machine or constant observation of a human. Since a video recorder is a machine, in the era of digital technology and its wealth of electronic machines, it may be less of a distraction. Conversely, as the teacher and students may be aware that their behaviours are to be archived for repeated watching, it may be more distracting than human observer methods. To overcome such limitations, Pirie (1996) and Lesh & Lehner (2000) (as cited in Powell et al., 2003) suggest combining video recordings with other data collection methods such as interviews and written documents in order to reach a more comprehensive examination of teachers' and students' classroom practices.

#### *3.2.3.2 Video recordings in this study*

I video recorded lessons of two teachers (Ayten and Mahir) from SS and two teachers (Nuri and Saban) from PC while they were teaching the functions. It takes approximately a month to complete teaching the topic of functions in both institutions. I have visited two schools at different times and thus video recordings took place at different times. While in SS, functions are taught in November, in PCs it is taught in March. It was a challenge to carry out the video recordings without missing any lessons and being able to socialise between and after the lesson to get teachers' impressions of the lessons and classroom activities. I used a tripod in order to make sure that the video recordings are of high quality with a minimum amount of shake. A single tape can hold one lesson. As the number of tapes was limited, during the evenings and nights, I converted the content of each video tape to digital format and kept it in the hard drive of my computer and burned then to CDs. The content of each tape was individually copied to the hard drive of my computer and then it was compressed in order to fit into CDs. This procedure took over 6 hours for each tape. The fact that some days I was recording four lessons made a hectic schedule for data collection. I managed to capture almost all of the lessons the four teachers taught with a small number of missed lessons. I established a weekly timetable to be able to cope with the tightness of time for interviews, video recording and storage of the data. I attended and observed 2 -3 lessons before the teachers started teaching functions in order for the teacher as well as the students to feel comfortable with my existence in the classroom.

### 3.2.4 Complementary Sources of Information

#### *3.2.4.1 The questionnaire*

Having realised the emergence of the significance of institutions as an influence on teachers practices, I designed a questionnaire (See Appendix C) to check if such an argument is supported by a larger number of teachers. As with other research methods, questionnaires have advantages and disadvantages. A major advantage of this method is that it is very efficient in terms of time and effort (Robson, 1993, p. 243). It allows data to be collected from large numbers of respondents within a short period of time. In fact, I managed to distribute the questionnaire to 10 PCs and 10 SSs and collect them within only a week and in total, 87 teachers in both institutions completed the questionnaire. However, questionnaires also have down sides. As Robson (1993) points out, there is always the possibility of the responses being superficial and that questionnaire items must be constructed extremely carefully to have meaningful responses. Several items in the questionnaire I designed suffered from such a crucial mistake and as a result I had to exclude some items in analysis.

#### *3.2.4.2 Socialisation with teachers (informal interview)*

In order to get at teachers' perceptions of the broader institutional context and to establish a trusting relationship with the teachers, I socialised with teachers out of the classroom. Such an approach could be called an 'informal interview' since no interview items were prepared beforehand and there is no video or tape recording of the talk. The the information was captured through notes I made after the conversation ended. My socialisation with teachers involved visiting teachers out of classes, talking to them between classes and discussing the teachers' instructional activities with them during the lessons.

### 3.3 Data Analysis

In this section, I will provide explanations of the process of data analysis. Before going into specifics, I would like to point out that one significant issue during the data analysis is to ensure the anonymity of teachers' identities. Having completed collection of the video data, I asked permission for the use of screen captures of their video recordings in the thesis. All four teachers (cases) gave me permission to do so. Therefore use of the screen captures that can be seen in the results chapter are not without the teachers' consent. In order to ensure the anonymity of the teachers, I have changed their real names in the thesis.

### 3.3.1 Analysis of Video Data

My starting point in analysing the video data is the teachers' own description of the classroom activities, therefore the data itself. The results from piloting, as well as early findings through repeated examination of teachers' classroom practices (interviews as well as video recordings from the main data), revealed that the teachers followed particular patterns in their practices. The lessons contained academic and non-academic elements[3]. The academic elements involve teaching and/or learning activities and the non-academic elements are generally class management activities or other activities, which do not involve formal teaching and/or learning. Academic parts of lessons involved what teachers from both type of institution called 'content' and 'example solving' (a direct translation of Turkish phrases that were used by teachers in the interviews). I have not used the phrase "problem solving" since the meaning attached to "problem solving" is different from "example solving" in its context[4]. I will now use the term 'segment' for each of these instructional activities in teacher practice and any subsection of these segments will be referred to as 'phases'. In the following part of this chapter I will provide detailed explanations of how to pinpoint these instructional activities in teachers' practices.

**Content:** This is the theoretical segment of the lesson where teachers provide definitions of concepts and explain the concepts and procedures in mostly abstract and/or out of context terms. During the explanations, teachers may give typical examples of concept(s) or abstract description of phases of mathematical processes involved. The starting point for this segment, as it is observed in video data, is mostly when teachers write the title or sub-subtitle or sub-section of the topic to be taught on the blackboard or when teachers begin to explain the topic verbally. The endpoint, despite slight variations, is when the teachers signal to students that he has completed his explanations. In most cases teachers wait for students to write down the text on the board to their notebooks or they carry on with the example solving segment.

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[3] This is my distinction, not from the data itself.

[4] In educational research the phrase "problem solving" generally refers to open ended questions, which mainly require students to investigate/speculate on the solution. "Example solving" is much more straightforward and generally requires procedural knowledge and calculation skills in this context.

**Example solving:** In this segment of the lesson, actual mathematical examples (or problems) with numerical context are processed. It is essentially the application part of the lesson. The start is commonly marked by the teachers distinctly when they state that they will start solving some examples or when they write the Turkish word for 'example' on the board. The endpoint is when the teacher or a student comes to the board to complete the solution and relevant explanations. In the analysis of the teachers' practices, I realised that example solving is an important feature of the mathematics teachers' practices in both institutions. Being a basic constituent of the teachers' practices I focused on this segment to further my understanding of what is happening in this segment of the lesson. Further attention to this segment revealed that not all teachers made use of examples in the same way. Selection and use of examples differed significantly. Repeated viewing of the video data suggested that the teachers' practices (for all four case teachers) in the example solving segment are made up of three phases: presentation, engagement and resolving.

***Presentation Phase:*** Teachers present the example that they would like to give in this phase. The video data indicated that it mainly involves teachers' writing the example(s) on the board and, when necessary, stating the problem verbally. This is the most straightforward of the three phases. As it is the first phase of the example solving, the starting point is defined by the teacher stating that he is giving an example or the teacher beginning to write an example on the board. The end point, however, is defined by the completion of the writing of the example or teachers stating what the question is verbally.

***Engagement Phase:*** In this phase of the example solving, teachers generally wait for students to engage in the solution of the example in presentation, i.e. they let them try to solve it. It begins directly after the end of presentation phase and ends with the beginning of the resolving.

***Resolving Phase:*** This phase of the example solving involves a demonstration of the solution of the example. This phase is not as clear-cut as the previous two. In some cases the solution is provided by students. However, it is mainly, as observed in the data, the teacher that demonstrates the solution and explains the processes involved. The resolving phase generally begins with teachers' utterances as an attempt to get the attention of the students which signals that the example on board is about to be solved.

The end of this phase is marked by the end of solution. If the resolving involves a student attempting to demonstrate the solution then it is marked by end of the student's demonstration of solution or the teacher's further explanation on the students' solution on the board.

I have examined (in terms of time allocated, and teachers' and students' activities) these three phases in every example for each video recorded lesson[5]. This gave me three time intervals (presentation, engagement, resolving) and teachers' instructional activities for each example solving session. In terms of the allocated time, I calculated the mean time spent for each phase of example solving. This gave me weighted mean of each phase of a lesson. I made these calculations for each teacher and this gave me the time spent by a teacher for each phase of example solving across a series of lessons.

Further elaboration of the examples used in the mathematics lessons revealed that there is a distinction between two different types of examples used in teachers' practices. I called them 'active examples' and 'passive examples'.

***Passive examples*** are the examples used by teachers when they want to exemplify typical cases of broader categories. They are intended to exemplify a concept or a procedure. Passive examples, in this respect, are part of teachers' explanations. For instance: the teacher shows that f(x)=3 is an example of the concept 'constant function'. In such a case teacher conveys the message that 'f(x)=3' is one example of the concept 'constant function. Similarly, when the teacher demonstrates how to find the inverse of a function after defining the concept, it is a demonstration of a procedure. Although such an explanation involves how to perform the inversion procedure, it also demonstrates how to act in other instances.

***Active examples*** are more like exercises in the sense that they require students or teacher to solve them. Active examples require making use of a variety of previously acquired mathematical knowledge. These examples can be regarded as exercises since their use constitutes practice of prior knowledge.

[5] These analysed lessons excludes the SS lessons allocated merely for checking students' homework and PC lessons which only involves solving questions of a quiz.

The important difference between these two types of examples is that active examples do not primarily provide understanding of the mathematical ideas involved. They are just the practice of certain mathematical procedures or a solution technique with the help of the previously explained mathematical knowledge. However, since passive examples are generally used at the beginning of a topic, they help students to understand the abstract mathematical information teachers set out to teach. Another distinction between the two types is that unlike active examples, passive examples do not generally end with a question mark and teachers do not generally expect students to solve them. In the case of active examples, the nature and the expression of such examples need solution and teachers encourage students to engage in the solution.

### 3.3.2 Analysis of Interview Data

With the purpose of understanding teachers' classroom practices in mind, I used the interviews to support (or reveal the peculiarity of) the video data and used the initial categories, as described in piloting, as a starting point for analysis. For the first interview, these categories were: teachers' planning; the theoretical part of classroom instructions (content); and the examples they use and the materials they employ during teaching. In the second interview, I included teachers' beliefs about how mathematics should be taught, how students learn mathematics and teachers' perceptions of the broader institutional context. However, the theme that emerged from the data (influence of institutional context) affected the way they were analysed. Therefore, while there were initial categories that were derived from piloting data and relevant literature. later on I examined the interviews in relation to two aspects:

(1) teachers' practices from their own perspective (content, example solving) and

(2) the impact of the wider institutional context.

Therefore I focused more on information that interviews contained on the following aspects:

1- Content.;

2- Example solving activity;

3- Teachers' perceptions of their work place.

It should be noted that in presenting the results of the interviews I do not make a clear distinction between beliefs and practices. Although theoretically speaking they are distinct concepts, in data analysis it is not that easy to make this distinction. There are some utterances that can be clearly categorised as an expression of belief or an expression of their practice. However, many expressions may be an indication of practice and belief at the same time. In fact, if one takes the approach of bringing teachers' classroom experiences into the process of interview as suggested by Tomlinson (1989) and Woods (1997) to elicit more authentic answers, then it seems that there is little point in making such a division in terms of analysing interviews. In reporting research they are treated as different concepts but when it comes to analysis it is not as clear cut as one expects. Although the distinction is used by many researchers, it seems it is not explicitly stated by most researchers studying teachers' beliefs and practices. Regarding this often unstated tricky issue, Cooper (2003), however, is an exception. Having a similar methodological issue in the data analysis stage, Cooper (2003) explicates the problem:

> Beyond the multifaceted characteristics of both beliefs and practices is the fact that, empirically speaking, they are often difficult to distinguish from each other. Attempts to do so sometimes isolate cause and effect, thereby distorting the data. It is not surprising, therefore, that initial efforts to organize the data under the two simple categories of beliefs and practices in the sample coding phase failed (p. 418).

In an attempt to overcome the issue, Cooper makes a distinction between the data coming from observations and those coming from the interviews. Speech, action, and events heard or observed are distinguished from ideas inferred from practice and interviews. She explains this as:

> Further data analysis suggested that a more fruitful path was to accept the interconnectedness of beliefs and practices but distinguish between those that were *manifest* in observable events, practice, or talk about practice and those that were discussed in the abstract or were *inferred* from practice. I regrouped the former as *operational beliefs and practices* and the latter as *conceptual beliefs and practices*. This resulted in a more holistic profile of the teachers (p. 418). (Italics in original)

In analysing the interviews, this issue is considered significant. Therefore in understanding the interview extracts throughout the thesis, the reader should be aware of this matter.

Another aspect that needs consideration is the issue of translation of the original transcript of the interviews. I paid careful attention to make sure that, on the one hand, the meaning of the translation is understandable to the English speaking audience and, on the other hand, the translation is faithful to the original Turkish text. Such a trade-off proved to be elusive at times. It was hard to keep the meaning unchanged while translating the text for certain parts of the transcript. In such cases I asked some Turkish doctoral students to help me translate the text. We compared my translation to theirs and discussed the best possible solution. It was tricky especially when the translation of the original text did not make much sense when left 'intact'. In such cases, in order to make it more understandable to the reader, I was forced to insert words in brackets -'[]'-. that were not actually uttered in the original interview transcription.

### 3.3.3 Analysis of Questionnaire Data

Based on the teachers' answers to certain items, which contained signs of misunderstandings of the questions asked, I realised that verbal expression of these items required a bit more clarity. Unfortunately this was after the data collection. Lack of piloting in construction of the questionnaire items seems to be the likely reason for this. This resulted in omission of from the analysis of some items (1a, 1d, 2a, 2b) that did not work as previously expected. Time limitation for the study also led me to analyse items that are easier to manage within the time limit (1b, 1c, 2c). In analysing the results of these items, I first entered the teachers' background information (past teaching experience, current institution). I then entered the data from the items (multiple choice section) into the statistical analysis program SPSS and obtained descriptive statistics for these items. This gave me overall picture of the teachers' answers. I also made use of the explanation part (the blank spaces for teachers to explain their answers) of the items to support the results obtained from the multiple choice section. In analysing the item 2c, which required participants to comment on whether teachers' views would be affected by the institution in which they work, I categorised the teachers' responses into 4 groups: Yes, No, It depends, Not Applicable. This was also entered into SPSS for analysis. The teachers' explanations were used in the thesis to exemplify their responses.

### 3.3.4 Presentation of the Results

Based on the data collected from four teachers, results chapter consists of four main sections. Each section is divided into four sub sections. First, I present the teachers' personal information and my contact with them as background information. Secondly, I present the organisation of their teaching of the topic of functions. This is followed by a detailed description of a specific lesson in the third section. The fourth section presents the teachers beliefs and goals. Here is the organisation of the results chapter for each teacher:

1-Background information

2-The teacher's organisation of the teaching of functions

3-Analysis of the teacher's teaching.

4-The teacher's beliefs

For each mathematics teacher, one lesson was selected to depict a detailed description of the teacher's instructional practices. The analysis of the data suggested that not all lessons contained content and example solving segments. Some lessons contained only example solving segments. To select comparable lessons from four teachers, I selected lessons which contained both of those segments. That meant that the lesson chosen needed to contain the teacher's presentation of some theoretical information and some examples. Considering the entire set of video recordings and the fact that some lessons were missed, the lessons in which inverse functions were introduced were chosen. For all four teachers this lesson contained both content and example solving segments. These lessons were also not at the beginning of the topic, and this may give some confidence in terms of teachers being 'themselves' in the lessons. In presenting the detailed description of the lessons, it was ensured that the amount of detail presented is also comparable. That is to say that I tried not only to describe the lessons as objectively as possible but also tried to describe the teachers' practices in those lessons in enough detail for the reader to get a feel for them.

### 3.4. Issues of Reliability and Validity

In this section I will briefly discuss, first, the issue of reliability and validity in case study methodology, and then, I will present validation or verification strategies (as Creswell, 1998, puts it) used in this study to ensure the validity and the accuracy of the interpretations (or conclusions).

One of the most frequently criticised aspects of the case study tradition is the reliability and validity of findings. LeCompte & Goetz (1982) note that 'qualitative' research can be criticised on the basis of 'its failure to adhere to canons of reliability and validity' (p. 31). However, the terms reliability and validity are criticised by researchers in the 'qualitative' tradition as they are based on assumptions of 'quantitative' research. For example, Wolcott (1990) argues that the term 'validity' has become a confusing term because it has been specified in one domain and yet transferred to another. Wolcott (1990) even suggests that 'validity neither guides nor informs' his work.

It seems to me that this is not different from case study researchers' distinctions regarding the use of term 'generalisation' between 'qualitative' and 'quantitative' senses (see Stake, 1995). Regarding 'generalisability' in case study methodology, proponents of 'qualitative' research offer alternative terms for demonstrating credibility of the findings (Janesick, 2000). For example, Lincoln & Guba (1985) put forward 'trustworthiness' and use other terms such as 'credibility', 'transferability', 'conformability' and 'dependability' and Creswell (1998) prefers the term 'verification'. Therefore, it seems to me that the way out of this issue, as in the case for generalisability issue, is not total removal of the terms 'reliability' and 'validity' from the dictionary of research but replacement of them with terms that are in line more with the case study methodology.

I make use of suggestions by Stake (1995, p. 87) to ensure the validity of my conclusions. However, before going into the strategies used to ensure rigour in the research process, it is important to be aware of the position I am taking here. The present study is a multiple-case study and interpretive in nature. There is no 'correct' interpretation of the (especially qualitative) data but there are many possible interpretations. What is presented in this thesis is this researcher's own interpretation of the raw data. I am of the idea that any human perception and understanding of events, cases, and situations is subjective; it is not possible for me to be purely objective towards the data at hand and the conclusions arrived at are inevitably the products of methodology and the researcher's background[6]. It is a matter of the degree of subjectivity that gives research an 'acceptable' level of rigour. Ironically, the term 'acceptable degree' is not standardised either, despite the fact that there are certain broad guidelines for increasing the rigor in research. Hence, the strategies presented here are those guidelines followed by the researcher during the investigation. They are research processes suggested by Stake (1995, p. 87) to ensure 'the validation of naturalistic generalisation[s]' (Stake, 1995, p. 87) for case study research and verification strategies for 'qualitative' research suggested by Creswell (1998, p. 201-203).

[6] It would be interesting to examine a juxtaposition of this idea with the Lortie's (1975) notion of 'apprenticeship of observation'.

The first strategy used in this study is that I provide raw data (see Chapter 4) prior to my discussion so that readers can externally judge the validity and the quality and accuracy of my interpretations. This should also enable readers to consider their own alternative interpretations. Although my narrative description of the teachers' classroom practices are subjective selections of teacher behaviours, they are still an important part of the validation process through which readers can judge themselves the quality of the interpretations. In a similar vein, Stake (1995) states that:

> The reader will take both our narrative descriptions and our assertions: narrative descriptions to form vicarious experience and naturalistic generalizations, assertions to work with existing propositional knowledge to modify existing generalizations. To assist the reader in making naturalistic generalizations, researchers need to provide opportunity for vicarious experience (ibid, p. 86).

Such a detailed narrative description of events, as Creswell (1998) argues, enables readers to transfer information to other settings and to determine whether the findings can be transferred.

The second strategy used in this study is the use of multiple sources of evidence to reinforce/support the truthfulness of findings and the accuracy of the interpretations. For example, having more than one case for each institution enabled me to arrive at interpretations that are supported by data from other cases. Similarly, different data sources from the same case also helped me to come to conclusions (For example belief-practice inconsistency). This strategy also involves negative case analysis, that is, refining interpretations or findings from one case in the light of evidence that refutes a previous interpretation. To Robson (1993), this is a significant part of building an explanation. One may expect that such a negative case would 'ruin' the researcher's claim. On the contrary, it helps the researchers to improve their interpretations. For example, this happened when I interpreted the data on the use of examples as uniformity of practice for teachers of the same institution (i.e. teachers of the same institution use examples in the same or similar ways). This interpretation is challenged by the findings from SS teachers where different teachers' emphasised different phases of the example solving segment. This allowed me to refine my interpretation and further my argument through saying that not all the institutions influence teachers' practices in the same standardised way. That is to say, some institutions may influence more than others on the basis of certain factors (see section 6.4). Therefore, using such an approach enabled me to confirm or sometimes dismiss the quality and accuracy of my interpretations. However, I must admit that it is a stressful process as it sometimes leaves the researcher in the middle of nowhere with little consistent interpretation to make of an enormous amount of data.

The third strategy used in this study is the review of research design by my peers and especially my supervisors. The research was designed under the close scrutiny of my supervisors. They not only suggested strategies to use to ensure rigour in the methodology but they also raised numerous issues regarding the design of the research tools. They challenged the usability and validity of the schedule for the interviews and items for the questionnaire and pointed out issues surrounding the analysis of video data. For example, in connection with checking my differentiation between different phases of example solving segment we watched the excerpts from video recordings of the mathematics teachers' lessons and discussed the use of such an approach.

The fourth strategy used in this study is prolonged engagement and close subjective, and yet professional, relationship with participants in the field. The case teachers were fully cooperative when I explained my research, and in fact two teachers put themselves forward to take part in my research before I had asked them to. Yet, there is always the possibility for the participants to utter the words they may think I would expect to hear. It seems to me that there is and will be such a possibility in any research with teachers. One way to minimize such a possibility is through a close relationship with the participants in and out of the classroom and building trust. Interviews were scheduled towards the end of the period of the video recordings to ensure accuracy of data collected. Fetterman (1989) asserts that 'working with people day in and day out, for long periods of time is what gives ethnographic research its validity and vitality' (p. 46).

The fifth strategy used in this study is the use of external audits. This strategy involves an external consultant, the auditor, to examine both the process and the product of the account, assessing their accuracy (Creswell, 1998). The term 'external' refers to the fact that the auditor should have no connection to the study. It allows research to be examined with regard to the degree to which the findings, interpretations and conclusions are supported by the data. In my study, I submitted several research reports to national (British) and international conferences where I discussed my research with experienced researchers. The conferences involved my presentation of findings and discussion around my interpretations of the data.

# CHAPTER -4 RESULTS

## 4.1 The Case of Nuri

### 4.1.1 Background Information

From the beginning of my contact with Nuri he treated me as a colleague rather than an 'outsider'. When I explained what I was trying to do he asked a few questions about the nature of my research and after that he was comfortable with me in and out of his classroom at any time. My relation with Nuri was at a very personal level. I invited him to my house for the interview as the environment of the PC was hectic. He also invited me to his house for the other interview. Thus I can comfortably say that he was not distracted by my presence in his classroom in a significant way.

Nuri was an experienced mathematics teacher in his mid thirties. He was graduated from a pure mathematics department and had never worked in a state school. At the time of the interview he had 12 years of experience as a PC teacher and during the interview he identified himself as 'PC teacher'. He was head of the mathematics department in the private course where he was working. He taught 24 lessons per week in both weekends and weekdays. He was responsible for teaching mathematics to five classes in total: 3 weekend classes and 2 weekday classrooms. Like other PC teachers, Nuri had a pretty hectic schedule during the weekends, teaching both morning and afternoon sessions. I observed him in 2 different classrooms. He covered the topic of functions in a month. As I was video recording other teachers at the time, I managed to video record 10 lessons on functions in total. I also observed him several times without video recording.

### 4.1.2 Organisation of Nuri's Teaching

Nuri completed the topic of functions in 7 lessons in one of the classes I observed. Most of the time Nuri's teaching was made up of cycles of content and active examples solved by him. However, he used passive examples few times just after the content segment of the lesson. However, he had strong tendency towards sequencing his teaching in cycles of content and active examples. Nuri's emerging practice pattern could be seen in Figure 4.1.

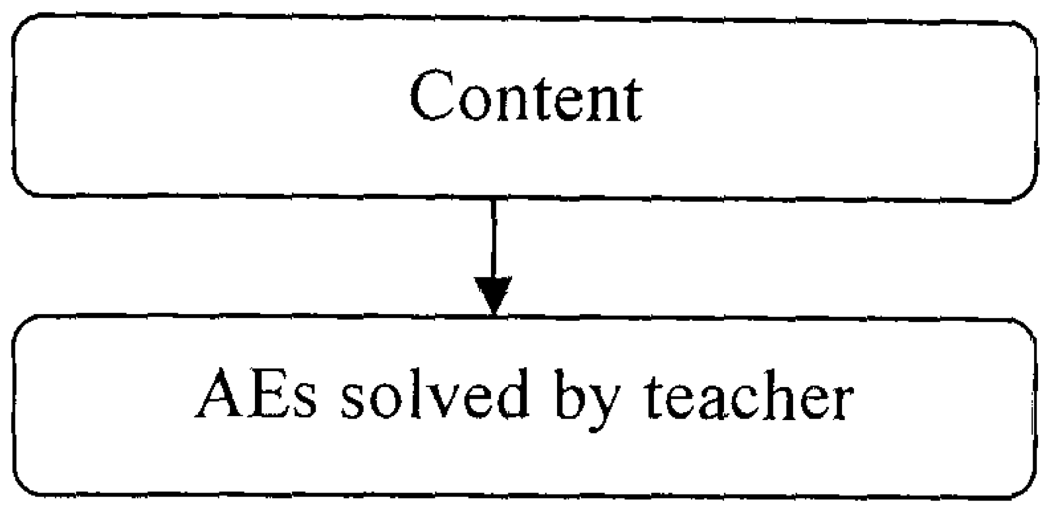



**Figure 4.1** Nuri's practice pattern.

Table 4.1 and 4.2 summarise his teaching of functions. The second column in Table 4.1 indicates the lesson code in my records. The third column indicates the 'content' part of the lesson and the other two columns show the number of passive examples (PE) and active examples (AE) used[1] and the total time spent on them. The number of past examination questions (PEQ) is also presented under active examples, within brackets.

| | **Lesson Code** | **Content** | **Passive Examples** | | **Active Examples (PEQ.)** | |
|---|---|---|---|---|---|---|
| | | | Number | Time | Number | Time |
| 1 | Nuri-1 | 10m 50s | 2 | 1m 20s | 8 | 17m 45s |
| 2 | Nuri-2 | 8m 48s | - | - | 12 | 20m 24s |
| 3 | Nuri-3 | - | - | - | 11 | 25m 18s |
| 4 | Nuri-4 | 4m 44s | - | - | 10(3) | 26m 7s |
| 5 | Nuri-5 | 7m 52s | 1 | 1m 38s | 11 | 28m 1s |
| 6 | Nuri-6 | - | - | - | 12 | 30m 40s |
| 7 | Nuri-7[2] | - | - | - | 13 | 35m 55s |

**Table 4.1** Summary of Nuri's practice.

Table 4.2 indicates the organisation and the sequence of the topics taught, in other words, how the content segments in his lessons are distributed throughout the topic. The table indicates that Nuri's organisation is based on spending very little time with content, which is the theoretical part of mathematics, and solving a lot of examples. He typically writes the rules, formulas or definitions on the board and makes brief explanations of the concept or the procedure involved. This is generally limited to a short period of time. As a result typical lesson of Nuri is dominated by examples.

[1] See Methodology chapter for further information on the terms used.
[2] This is a special lesson where Nuri used examples from one quiz for the entire lesson

| Lesson | Code | Content |
|---|---|---|
| 1 | Nuri-1 | • Definition of function (2m 55s)<br>• Explanation with mothers-kids metaphor (1m 39s)<br>• Being a function (1m 53s)<br>• Finding number of functions (1m 8s)<br>• Onto function (1m 16s)<br>• "Orten"[3] function (44s)<br>• 1-to-1 function (1m 15s) |
| 2 | Nuri-2 | • Checking if a function is 1-to-1 on its graph (2m 57s)<br>• Constant function (41s)<br>• Identity function (1m 9s)<br>• Linear function (32s)<br>• Writing function out of graph (45s)<br>• Basic calculations on functions (2m 44s) |
| 3 | Nuri-3 | - |
| 4 | Nuri-4 | • Condition for an inverse to be a function (46s)<br>• The procedure of $y = f(x) \Leftrightarrow x = f^{-1}(y)$ (43s)<br>• Emphasis of the same procedure (25s)<br>• Inversing the functions of the form $ax + b$ (35s)<br>• Inversing the functions of the form $\frac{ax+b}{cx+d}$ (2m 15s) |
| 5 | Nuri-5 | • Composite functions (1m 43s)<br>• That g° f(x) refers to g(f(x)) (52s)<br>• Properties of composite functions (2m 48s)<br>• Permutation Function (2m 29s) |
| 6 | Nuri-6 | - |
| 7 | Nuri-7 | - |

**Table 4.2** Summary of the content segments in Nuri's recorded functions lessons.

There is a tendency in Nuri's lessons to become more and more example oriented towards the end of the topic. While Nuri spent 10 minutes of the first lesson as content, this is decreased as the topic is developed. The last lesson is totally example oriented. In fact the last lesson consisted of no content at all. Nuri solved 13 examples without presenting any theoretical information at all. Lesson 4, which is analysed in detail in the next sub-section, falls midway between the first lesson and the last, balancing Nuri's preferences on content vs. examples. In this lesson, although Nuri allocated very limited time (less than 5 minutes), it still has content. However, like his other lessons, this lesson is dominated by examples.

| | Percentage | Number of examples |
|---|---|---|
| Passive | ~ 4 | 3 |
| Active | ~ 96 | 77 |
| **Total** | 100 | 80 |

**Table 4.3** Distribution of examples used by Nuri.

[3] I could not find the corresponding term in English and thus used the original term.

Considering Nuri's organisation of the whole topic, another interesting aspect that strikes me is Nuri's tendency towards using active examples rather than passive examples. This can be seen in Table 4.3. Out of the total of 80[4] examples he solved in 7 lessons, Nuri used only 3 passive examples. The remaining 77 examples were active examples. Another remarkable aspect of his teaching was that although he solved many active examples, he did not let his students to solve any of them on the board. Table 4.4 shows the number of active examples solved by students and Nuri.

| | Total | Solved by Nuri | Solved by students |
|---|---|---|---|
| Active Examples | 77 | 77 | 0 |
| **Percentage** | 100 | 100 | 0 |

**Table 4.4** Examples solved by Nuri and students.

A detailed description of one of his lessons is presented in the next section.

[4] This number includes examples from his last lesson (Nuri-7).

### 4.1.3 Nuri's Teaching

Lesson Code: Nuri-4

To describe the teaching/learning activities during the lesson, I would like to present the main phases of the lesson as a summary.

- Nuri presented and solved two examples as a follow up from the previous lesson.
- Nuri then explained the condition for an inverse of a function to indicate a function in very brief terms. He warned students about the questions that appeared in the past examinations regarding inverse functions.
- He then gave 1 example regarding the condition and he solved it himself. He continued warning students about common pitfalls. He then gave a past examination question and immediately solved it himself.
- Nuri explained a manipulation technique and he put enormous emphasis on the importance of it in the university entrance examination.
- He then gave 2 more examples which were immediately solved by him and continued his warnings.
- Nuri reminded students of the manipulation he previously presented.
- He then gave another past examination question and solved it.
- Nuri then explained the procedure of inversing the functions of the form $f(x)=ax+b$.
- Nuri gave an example of this content.
- He then explained the procedure of inversing the functions of the form $f(x)=\dfrac{ax+b}{cx+d}$.
- He then gave 3 examples and followed by another past examination question. He solved all of them himself and explained the use of the numerical value technique on such type of examples.

At the beginning of the lesson, Nuri said "*I will present you another example and this is asked [in one of the past examinations].*" Without looking at any source (textbook, quiz or any note) he gave an example.

For $f(x)=\dfrac{x+1}{x}$, what is the expression of *f(x-2)* in terms of *f(x)?*

When he finished presenting the example he added "*During the examination, it is possible to solve this type of question by using numerical value [technique] but it is risky. I mean, I need to warn you on that. We can solve this type of question by numerical values with the condition of being very careful.*" He then started solving the example. He obtained *f(x-2)* by putting x-2 in $f(x)=\dfrac{x+1}{x}$, where he wrote the expression $f(x-2)=\dfrac{x-1}{x-2}$ right away and thus skipped putting x-2 in the place of x in numerator and denominator.

$$f(x-2)=\frac{x-1}{x-2}$$

$$f(x)=\frac{x+1}{x}$$

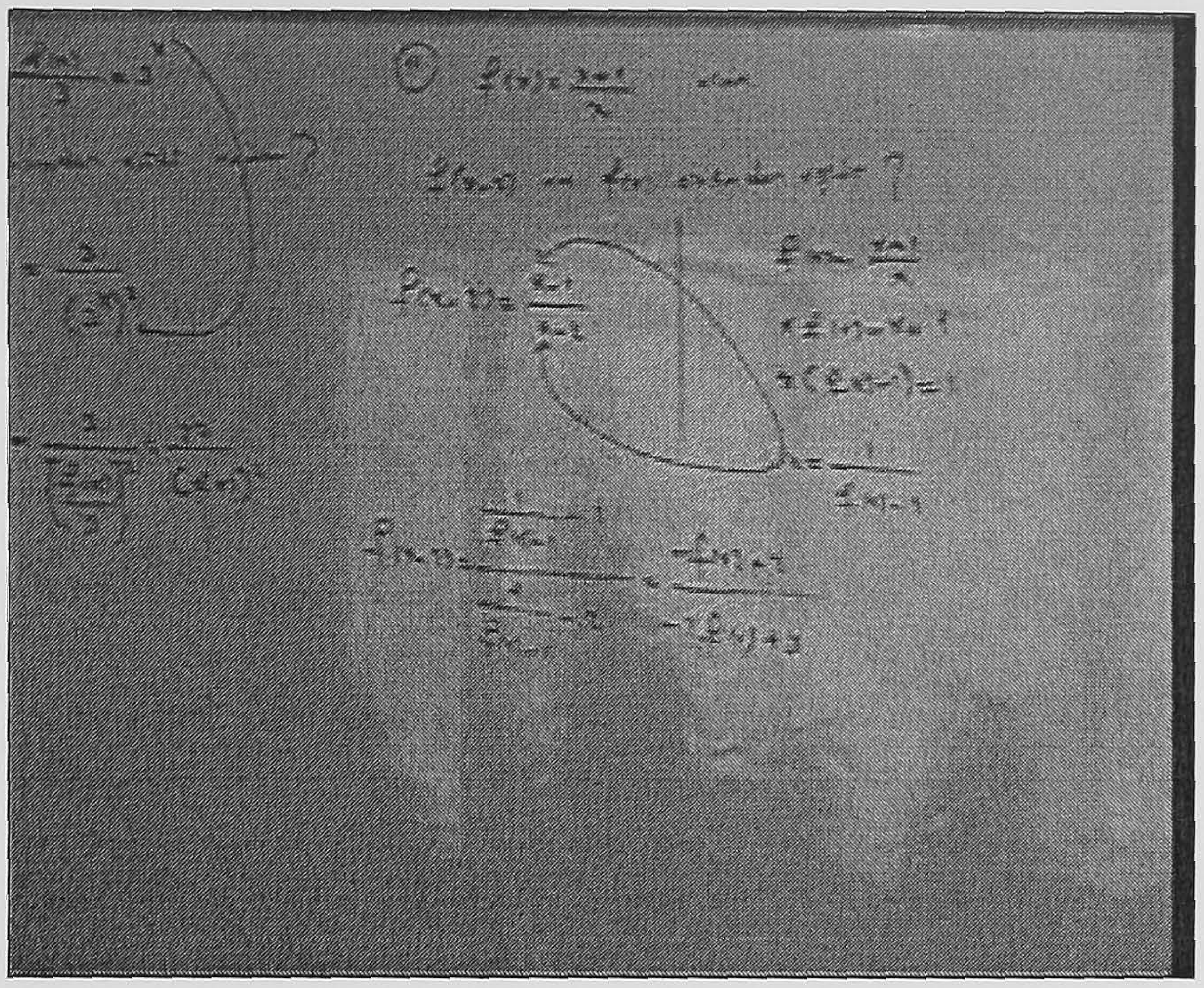
$$x f(x) - x = 1$$

$$x(f(x) - 1) = 1$$

$$x=\frac{1}{f(x)-1}$$

He then said "*Now my aim is to get rid of these 'x's (pointing x in the expression obtained) and to do anything to be able to write 'x's in terms of f(x). My friends, I am going to solve this in a classical way. Watch me (carefully).*" From the expression of *f(x)*, he obtained *x* in terms of *f(x)*. When he had finished he asked if anyone had a problem with understanding this. He then drew two arrows from the expression $x=\dfrac{1}{f(x)-1}$ to the *x*'s in the first expression $f(x-2)=\dfrac{x-1}{x-2}$ (See capture) to indicate that he put $\dfrac{1}{f(x)-1}$ wherever he found an *x* in *f(x-2)*. Similarly to his behaviour in the previous placement process (putting '*x-2*' in *f(x)*), it should be noticed that he skipped some of the middle steps in which he needs to make some symbolic calculations on the expression.

He added "*and this is the expression of f(x-2) in terms of f(x). Look, imagine that the*



*options [in the examination question] full of expressions like this and imagine you give numerical value to them. It would take a while [to calculate the result] and thus my advice for these kinds of questions is solving them...*(Pointing to two examples and solutions on the board) *Both of these types of questions have been asked [in the past examinations]*".

He cleared the board and while writing another example said "*I think you can solve this*

$$\boxed{f(x-2)=\frac{\dfrac{1}{f(x)-1}-1}{\dfrac{1}{f(x)-1}-2}=\frac{-f(x)+2}{-2f(x)+3}}$$

*one quickly...this is relatively easier example compared to others.*"

An interesting conversation took place after this example was put on the board. One of

For $f(x)=2^{x}+2^{-x}$ what is the expression of *f(2x)* in terms of *f(x)?*

the students whispered to the teacher "*after the camera came in you started asking hard questions*". The teacher smiled and said "*wait a minute, wait a minute, there is an assault here! I am asking hard questions after the camera came in, is it correct?*" There was a slight humming in the classroom. One student suggested "*the topic is hard*". Nuri asked "*is Functions a topic that you don't know much about?*" One student said

"*Exactly, it is hard, it is as complex as arabic*[5]" The teacher said "*is it?*" and laughed loudly. However, there was no humming or talk when the teacher turned back to the example like the cut of a knife. He said "*Now my friends, I put 2x wherever I see x.*" The board looked like this:

$$f(2x) = 2^{2x} + 2^{-2x} = \left(2^{x}\right)^{2} + \left(2^{-x}\right)^{2}$$
$$= (2^{x} + 2^{-x})^{2} - 2.2^{x}.2^{-x}$$
$$= [f(x)]^{2} - 2$$

When the teacher reached the end of first line $(2^{x})^{2} + (2^{-x})^{2}$ he said "*isn't it like $a^2+b^2$. How do we express $a^2+b^2$? Yes! $(a+b)^2$ ..Yes tell me*" He did not complete the formula $\{a^2+b^2 = (a+b)^2 - 2.a.b$, where $a=2^x$, $b=2^{-x}\}$ he was using. He immediately started writing the second line. When he finished the second line, he asked if everyone was OK with it. He then said "*I told you it is easy but you did not believe me you see $2^{x} + 2^{-x}$ is equal to f(x) and multiplication of these is 1 (pointing $2^x$ and $2^{-x}$) so the result is $[f(x)]^{2} - 2$. It's OK isn't it, my friends?*"

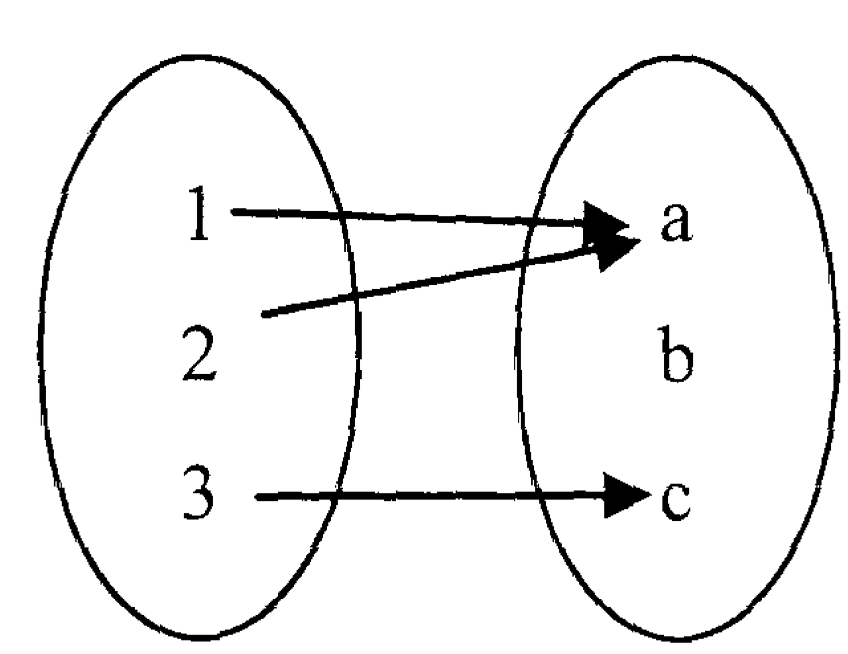
He began teaching 'inverse functions' by saying "*Well, we know some formulas like what? The inverse of the ax+b is x-b/a isn't it? There are some memorised routine formulas and templates. But it is very interesting, my friends, they [examination preparers] got out of the [well-known] formulas. There are very original questions as*

**Inversing a Function**

In order for inverse of f(x) to be able to indicate a function, it should be 1-to-1 and onto.

Ex: A={1,2,3}  B={a,b,c}

F: A→B  A  B




[5] Turkish people use this expression 'as complex as arabic' when something is very difficult to learn.

*you will see in a few moments. I mean, your knowledge of routine templates doesn't help you, my friends. There are really nice, really original questions [among past examination questions]. Now we will go through them together. First of all I say this, very important!*" He started writing while explaining, "*In order for inverse of f(x) to be able to indicate a function, it should be 1-to-1 and onto.*" He turned around and repeated it again and then said "*Now I will give you a very simple example.*" He wrote an example on the board. He explained that while this indicates a function its inverse does not make a function, because the element 'b' in set B is not mapped into A. "*because it is not 1-to1 and onto, thus its inverse is not a function. We agreed? And this [information] is asked in the past examinations. While students were learning how to inverse, they [examination preparers] they suddenly asked a question based on definition. I am sure my friends a lot of people who ignored [the definition of inverse function about] 1-to-1 and onto have stuck on that question because they didn't know what to do or what to think due to lack of this information*" He then wrote that because the function is not 1-to-1 and onto, thus, $f^{-1}(x)$ is not a function. He cleared the other part of the board and said "*I will bring this question into your perspective now. This is a UEE question*". When he finished writing the root of the question he said "*the question is exactly like this.*" When he finished the first function he asked "*do you think this function can be inversed?*" and then answered "*because both 2 and 3 went to 11 and so it is neither 1-to-1 nor onto, so it cannot be inversed,*" then added "*maybe we should not say that it cannot be inversed lets say its inverse is not a function*"

He then gave f2={(1,12) (2,10),(3,11)} and stated that "*yes my friends the function is*

> **OSS:** Defined from A={1,2,3} to B={10,11,12}, which of the functions given below can be inversed?
>
> f1={(1,10)(2,11)(3,11)}
> neither 1-to-1 nor onto, it cannot be inversed

*both 1-to-1 and onto so* $f^{-1}$ *can be obtained...Lets find its inverse. My friends, the most practical way of finding inverse of a function is changing the place of x's and y's, in order to find inverse of a function the place of x's and y's are changed*" then he wrote the inverse function of *f2*. He then reminded students that (pointing to the inverse of *f2*) "*This is not asked in that examination but I gave you this as an extra...As you can see a question like this is asked and you need to be careful about it. We cannot inverse any* *random function*". He then put emphasis on the information he was presenting "*This information is very, very important*" and wrote it with a red pen to emphasise its significance (see box).

| Let y=f(x) be a 1-to-1 and onto function. |
|---|
| $y = f(x) \Leftrightarrow x = f^{-1}(y)$ |

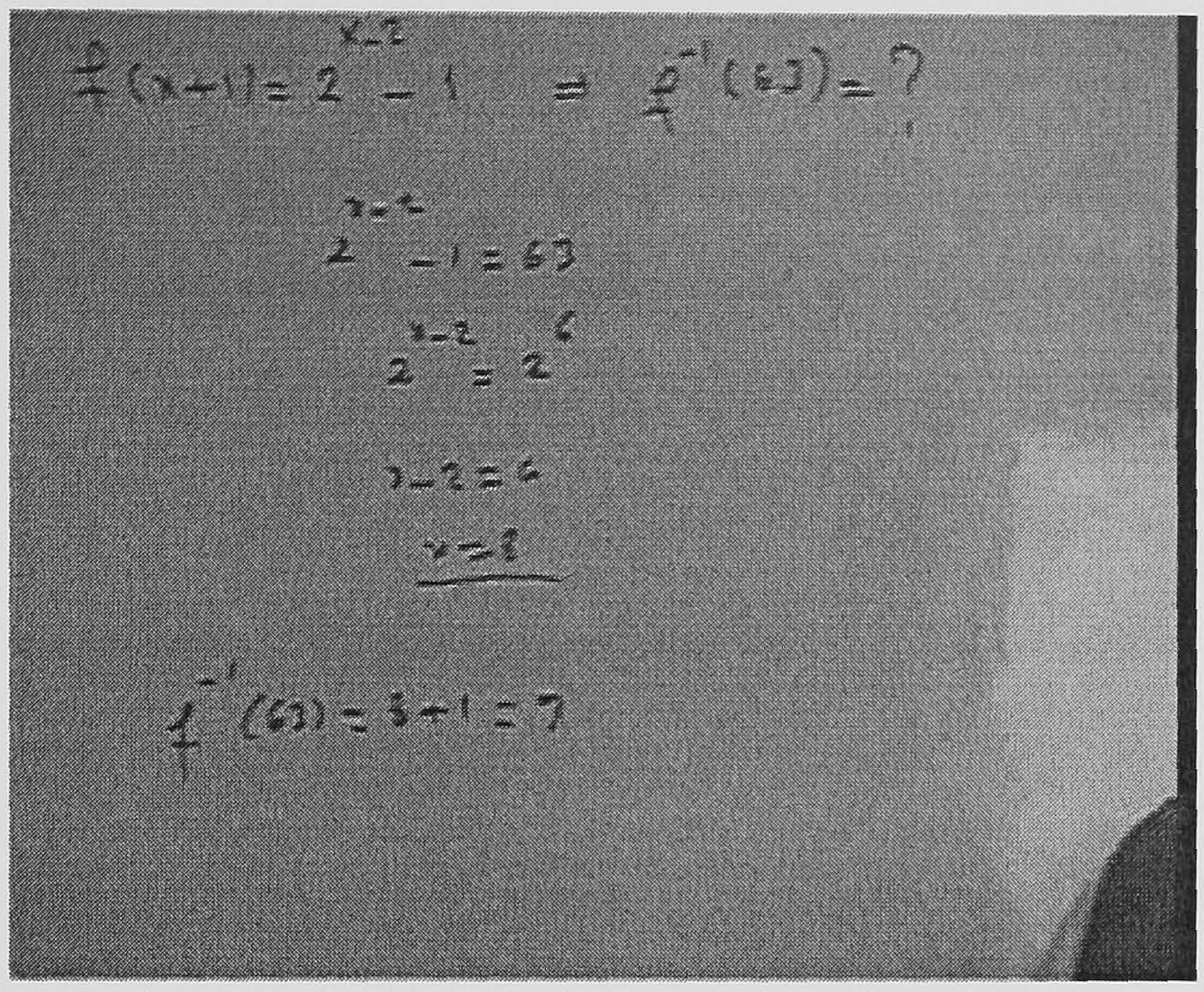
He said "*it has a lot of practical usages, this is very, very, very important! Especially in reading graphs many of you have trouble. Just a minute stop writing and listen to me carefully, pay some attention here* (pointing to y=f(x) on the board). A *function takes x's to y's. What is a function?...It is a tool to take x's to y's. But in the inverse function y's go to x's* (drawing arrows from x to *y* and *y* to *x*). *Exactly the opposite. While the function takes x's to y's, the inverse function takes y's to x's. Don't forget! When you see the inverse function* (pointing to '-1' in $x = f^{-1}(y)$) this value (pointing to '*y*' in $x = f^{-1}(y)$) *is the value of y. That's the way it is my friends...I have not taught you inversion techniques yet but I will start its examples! As yet we have not learned how to inverse but we will solve some examples!*" After this explanation he gave an example:

| $f(x+1)=2^{x-2}-1 \Rightarrow f^{-1}(63) = ?$ |
|---|

In solving the example Nuri pointed to 17 in $f^{-1}(17) = ?$ and said "*My friends, what did I tell you? I told you that this is the value of y, didn't I. 17. what is y here?* (pointing to *3x+5* on the example) *3x+5.*" He then underlined *f(2x-1)=3x+5* in the example (see capture) and said "*Look, be careful now! Is f going to other side [of he equation] as* $f^{-1}$ *according to this rule? So can I write it like this*[6]." Nuri wrote '$2x-1 = f^{-1}(3x+5)$' and said "*so I want inside* (pointing and underlining *3x+5*) *to be [equal to] 17.*" He then equalled 17 to *3x+5* and found *x*=4 and put this value in $2x-1 = f^{-1}(3x+5)$ which gave him $f^{-1}(17) = 7$. He then added "a*s you can see we can find the answer without inversing the function.*" He then gave another example: "*What are we going to do? We are going to equalise 2x-2-1 to 63.*" From 2x-2-1=63, he quickly found *x*=8 and then he

[6] Many times Nuri is asking questions like this but his voice and attitude strongly suggests that he is not expecting an answer instead he is expecting students to pay more attention to his words.

put 8 in $f(x+1)=2^{x-2}-1$ and found $f^{-1}(63)=9$. In solving the equation '$2^{x-2}-1=63$', he again skipped the middle steps that are expected, such as writing $2^{x-2}-1=63+1$ after $2^{x-2}-1=63$.



The board was cleared after that. He said "*We started 'Inversing a Function' and before giving any inversing formulas we started inversing. Now! Before presenting a very good past examination question, I would like to stress again that for y=f(x) be a 1-to-1 and*

**97-OSS**

$$f(x): R-\{-1\} \rightarrow R-\{3\}$$
$$x=\frac{f(x)+2}{3-f(x)} \Rightarrow f^{-1}(x)=?$$

*onto function,* $y=f(x) \Leftrightarrow x=f^{-1}(y)$ *and this is very important*" and wrote it on the board. Then he continued "*In 1997, there is was an original question, very good question. I am bringing this into the perspective.*" He then wrote the example and said "*this is a question from UEE in 1997*".

After a short pause he said "*My friends, there are two ways to solve this. The first is the naïve way. You manipulate the equality [and make necessary multiplications] so that you can find f(x) then you inverse it and this is one way. Isn't it a long job?...[as second way] This is where the originality of the question lies* (pointing to $x=f^{-1}(y)$ on the board) *What is x equal to? Inverse of f. I write* $f^{-1}(y)$ *wherever I see x. Well, what f(x)* *is equal to? y.*" He wrote $f^{-1}(y)=\frac{y+2}{3-y}$, and said "*my friends, this [question] is finished*[7]. *It already gave you the function itself. It was in fact the same as its inverse. What am I going to write whenever I see y? x.* (pointing to $f^{-1}(y)=\frac{y+2}{3-y}$) *It is already the inverse function whether I write x or not. My friends, that means* $f^{-1}(x)=\frac{x+2}{3-x}$. *As you can see this is a very good question. As you can see we have not seen inversion yet but we just gave you the logic of inversion. So this is a past examination question appeared like this.*" After a short pause Nuri wrote a title on the board: 'Inversion

| |
|---|
| $y=f(x)=ax+b$ |
| $y-b=ax$ |
| $x=\frac{y-b}{a}$ |
| $f^{-1}(x)=\frac{x-b}{a}$ |

methods' and said "*My friends, Very simple! If you want to inverse a function simply leave x alone. This is so simple. If you want to inverse a function, what ever you do, you leave x alone.*" Nuri then showed inversing $ax+b$ by leaving $x$ alone (see box):

He then gave an example and said "*be careful, I want you to tell me the inverse.*" A few students suggested some expressions. Without commenting on the students responses, Nuri said "*I have written this down in this way on purpose, why should I keep it that way. Let's write like this 2x-4.*" And he solved it (see box):

| |
|---|
| $f(x)=-4+2x \Rightarrow f^{-1}(x)=?$ |
| $f^{-1}(x)=\frac{x-(-4)}{2}=\frac{x+4}{2}$ |

Nuri said "*I will bring a second template into perspective, but when we are working on this template we need to be careful as there are some related original [past*

[7] Nuri meant that it is easy after that and thus the question is solved.

*examination] questions*". He wrote a function but left two values (in brackets) blank (see box).

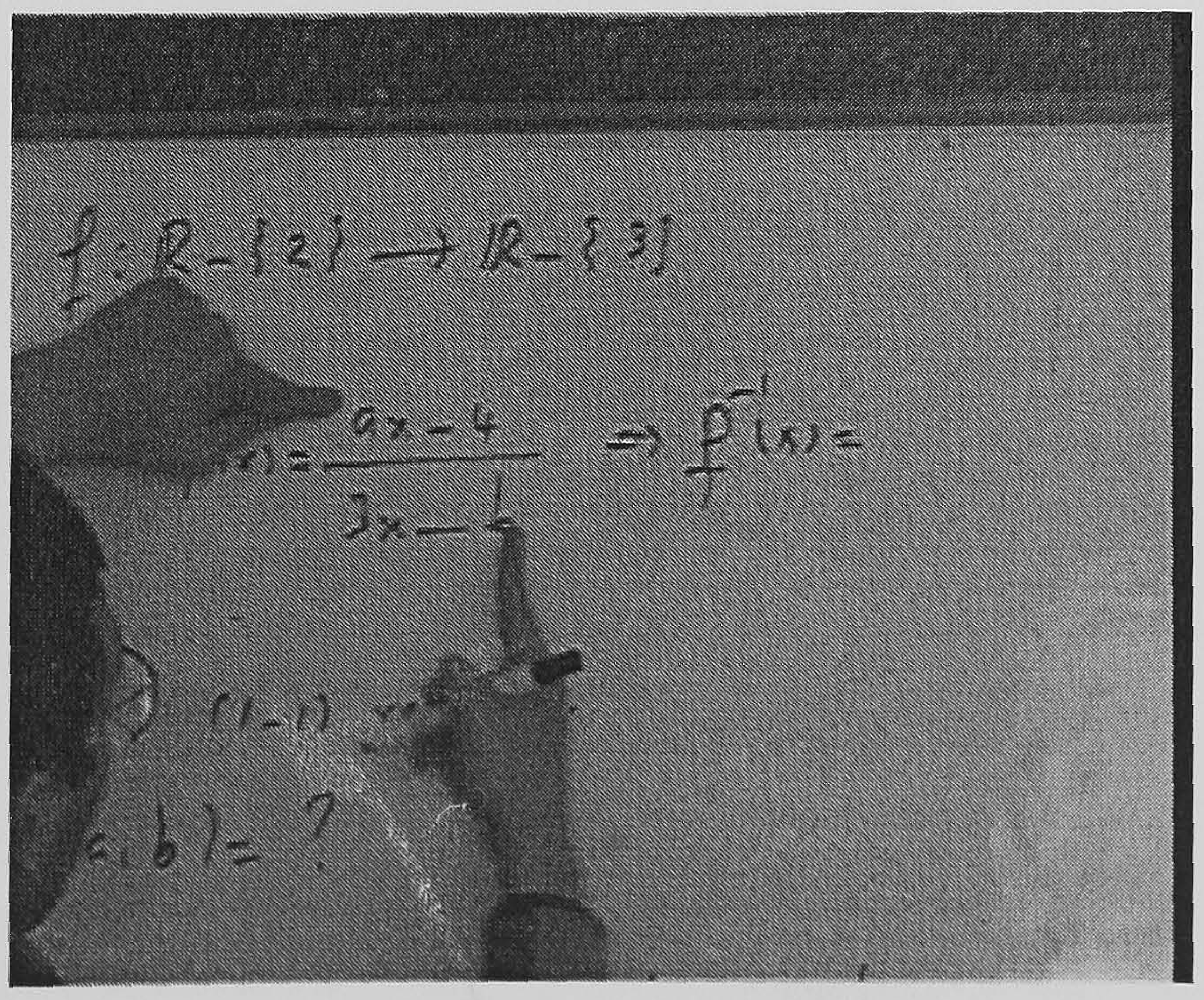
$$f(x): R-\{...\} \rightarrow R-\{...\}$$
$$f(x)=\frac{ax+b}{cx+d}$$
$$cx+d=0 \Rightarrow x=-\frac{d}{c} \text{ undefined}$$
$$f^{-1}(x)=\frac{-dx+b}{cx-a}$$
$$cx-a=0 \Rightarrow x=\frac{c}{a}, \text{ f}^{-1}\text{(x)undefined}$$

He drew students' attention to why there was a need to exclude some values from the domain and range sets and explained that it was because there were values that made the function undefined. He asked what this value was and then explained that it was the value that made the denominator zero.

He then he found $x=-\frac{d}{c}$ and wrote it in the brackets for the domain. Nuri then said "*these functions are 1-to-1 and onto within given sets. Very detailed takes time, I won't go into that. Previously, I told you that inverse of 1-to-1 and onto functions are also functions. Now we will inverse this* (pointing to the function, *f(x)*). *Did not I tell you that inversion is leaving x alone? But dear friends, if you do these calculations/manipulations [solving] the question takes a long time. Yes! And here is the practical way:* (drawing red coloured circles around a and d in $f(x)=\frac{ax+b}{cx+d}$) *Change the place and the signs of a and d*" and wrote $f^{-1}(x)=\frac{-dx+b}{cx-a}$. Then continued "*Does the inverse of this function make a function? Don't you think that should also be well defined? In that case the value making the denominator* (pointing to '*cx-a*' in $f^{-1}(x)$) *should also be excluded. Thus for* $x=\frac{c}{a}$, $f^{-1}(x)$ *is undefined* (pointing to the range set). *Then I am excluding* $\frac{c}{a}$ *from here.*" Nuri wrote it in the brackets for the range set. After this he interestingly said "*You will realise on the next few examples* *that I did not give you that much [theoretical] information for nothing.*" He immediately gave another example about which Nuri stated that it was from the examination in 1997.

| |
|---|
| **97-OSS** |
| $f(x): R-\{2\} \rightarrow R-\{3\}$ |
| $f(x)=\dfrac{ax-4}{3x-b}$ |
| and f(x) is 1-to-1 and onto. |
| (a,b)=? |

Unlike the previous examples, the example he gave this time was from a book. He said "*This is a very good question. Why? Many [students] just got used to inversing a function and don't know about definitions, and this will be a very hard question for them. Why is 2 excluded from the domain? Because it makes the denominator zero.*" He equalised *3x-b* to *0* and found that *b=6*. He then said "*because it is 1-to-1 and onto let's find the inverse of it.*" He then inversed the function by changing the place and signs of '*a*' and '*–b*' and found $f^{-1}(x)=\dfrac{bx-4}{3x-a}$.



He asked "*why is 3 excluded [from the range]?*" then answered himself "*because 3 makes the denominator zero*" He then equalised '*3x-a*' to *0* and found *a=9*. He then wrote *(a,b)=(6,9)*. One student said "*it is (9,6)*" and he realised the mistake he made and said "*yes, you are right*" and changed to *(a,b)=(9,6)*. He then said "*This was a past examination question. Now I will go back to this formula and will give examples. Be careful!*" Nuri wrote another example (see box) and immediately said "*come on [hurry up]*".

$$\boxed{f(x)=\frac{1-2x}{3x-5} \Rightarrow f^{-1}(x)=?}$$

Students started humming and guessing the answer but he started solving it. He jokingly said "*What a democratic classroom*[8]*, everybody has different answers*". He pointed to the formula $f(x)=\frac{ax+b}{cx+d}$ and said "*this is the template, let's organise this* (pointing the function in the example) *similarly*" and wrote $f(x)=\frac{1-2x}{3x-5}=\frac{-2x+1}{3x-5}$ then wrote the inverse function $f^{-1}(x)=\frac{5x+1}{3x+2}$. Having just completed the solution he gave another example (see box).

$$\boxed{f(x)=\frac{-2}{2-4x} \Rightarrow f^{-1}(x)=?}$$

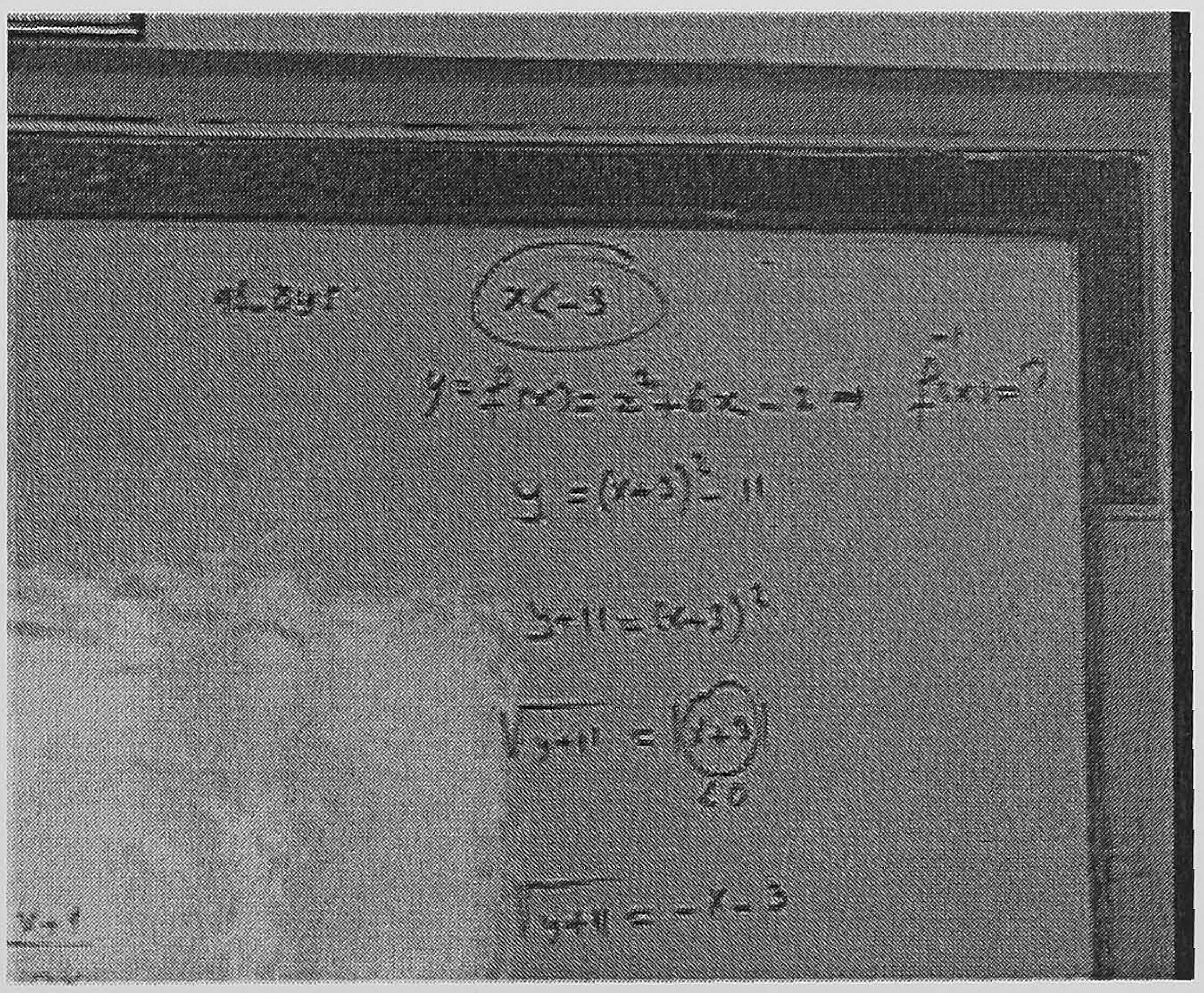
After a short time student started guessing. He did not comment on the correctness or incorrectness of their answers but started solving it himself. He said "*If you think of it like this [form], you may get confused, it can be written as* $\frac{0x-2}{-4x+2}$." He then changed the place of '*0*' from numerator and '*2*' from denominator to find the inverse function. Having waited for students to note the solution for a short time, Nuri pointed out "*this isn't the only thing asked in the past examinations. Although this is an OYS*[9] *question we have to solve it. As far as I know this is from 1998. Yes. 98 OYS*". He wrote the question without looking at any source.

[8] The teacher is being sarcastic here since they are expected to find 'the' answer not 'an' answer.
[9] The teacher referring to second stage of the examinations in the past. See section 1.1 for background information on examinations.

**98-OYS**

For $x<3$, $f(x)=x^2+6x-2$ then $f^{1}(x)=?$

After waiting for a short time he started his explanations "*What did I tell you about finding an inverse practically?...No matter what you do leave x alone somehow. My friends, as you can see there is $x^2$ and 6x here. This is actually $(x+3)^2$. Why?* (pointing the $x^2$) *the square of the first [term] and* (pointing the 6x) *multiplication of first and second [term] with 2. Hmm. What is the square of 3? 9. What should you take away? 11.*" and wrote '$y=(x+3)^2-11$'.He asked if everyone is OK with it. Although he got mixed responses, he continued the solution. When he square rooted both sides of the equation he emphasised that "*this is a dangerous question*" because the absolute value that comes with the manipulation requires extra attention to the condition '$x<3$' given in the question. He even made a joke of it by saying "*some students think this is there for decoration flower to look good* (everyone laughing). *Not at all, be careful! It is dangerous! The information is given [in the question has a use], it cannot be ignored.*"



He explained that because x<3 the expression '$x+3$' should be multiplied by *-1* to remove the absolute value signs. He then left x alone and said "*leaving x alone means inversing the function. Then I found the inverse if this function.*" Then he wrote $f^{-1}(x)=-3-\sqrt{x+11}$. He waited for a while for students to write and then stated "*It is possible to solve this example by numerical value technique. You may find it difficult to use here. I don't want to use that technique for every question but a regularly studying student would assign a value to x in the root to remove the root. Then calculates the result then checks the options.*" He considered the condition '*x<3*'assigned -*7* to *x* and found $f^{-1}(-7) = -5$ then he manipulated the equality and found that *f(-5)=* -*7*. He then showed the correctness of the result by putting these values (-*5* and -*7*) in the function.

| |
|---|
| $f^{-1}(-7) = -3-2 = -5$ |
| $f(-5) = -7$ |

He said "*As you can see you can solve it from the options but you need to be careful about it. Why? Because there will be many calculations to be made and you will have to check the options one by one and then you need to go back to the function itself [to compare]. This may take your time but it is possible. So this question can be solved by using the options*" and the lesson finished.

### 4.1.4. Nuri's Beliefs

At the beginning of the interview I asked him to introduce himself briefly. He identified himself in relation to his institution.

> **R** Could you shortly introduce yourself please?
> **T** I am a mathematics teacher, Nuri *****[10]. I started 1991. Since 91 I have actively been private courser. Course teacher. I never worked in state schools. I am a graduate of Istanbul University. I am a private courser in short.
> **R** Private courser?
> **T** Yes

During the interviews, many times he referred to himself as a 'private courser'. This is a direct translation from the Turkish expression 'dersaneci', which translates as 'the one whose job is situated in a private course' or 'the one who works for a private course'. This identification is important in that it signifies his perception of himself as a teacher and the strength of his affiliation to the institution where he is working. Another example of this is when I asked the aim of teaching mathematics.

> **R** What do you think is the aim of teaching mathematics?
> **T** As PC teacher, the aim of mathematics teaching is not teaching maths basically, but preparing them for the examination they will take. Make them able to solve the questions that they will face in the exam. Our aim is not teaching mathematics deeply and with its theory. As an educator in PC, our aim is to prepare them for the examination so that they can solve questions quickly in the shortest time. We have limited time...From our perspective, teaching practical mathematics[11] defines mathematics teaching.

In answering my question, he established himself as a PC teacher then he answered my question and further he used the word 'our aim' or 'our perspective' rather than 'my aim' or 'my perspective'. The reader should be able to see Nuri's affiliation to the private courses in his words in the following excerpts.

> **T** I always tell my students that mathematics is a lesson which you can learn by listening during the lesson. If a student listens to you very carefully he has done most of his job 50 even 60 percent of it. I also tell them this: You have to study these topics for a number of weeks out of the classroom on your own, but I could give them to you within two hours as a summary and with its important parts. On the one hand, you may study for a week to learn it but I could give you as a ready made summary. This means the teacher should

[10] He has given his surname here.
[11] Here 'practical mathematics' means mathematics which involves solving questions quickly and in the shortest time.

be listened to very carefully. Having listened to the teacher carefully, the lesson should be reviewed [at home] and studied within the same day or within a few days. I mean the knowledge presented [in the lesson] should be repeated and reviewed within a short time to learn.

**R** Then repetition/revision is important?

**T** Yes repetition/revision is very important. Besides, tests and materials are also important factors. In development of mathematics, [using] test [preparation] books are also an important factor. We encourage students to solve examples from question banks[12]. To review and to consolidate.

Nuri saw students' role as listener, passive 'receiver' rather than students as actively involved in the flow of the lesson. He considered that the important part of learning can be achieved by careful listening and this should be supported by reviews using quizzes and examination preparation books. Along with these he stressed the importance of question banks.

**R** What do you suggest your students to do?

**T** First of all I say to them: topics in mathematics are like rings of a chain. Primarily they should follow the lessons carefully. I mean if a student attends to one lesson and misses 2, what happens? A student with no understanding of square roots will not understand exponential numbers. The one with no understanding of square roots and exponential numbers will not understand factorisation. The one with no understanding of factorisation or ratios will not be able to solve equations. The one with no understanding of solving equations will not be able to solve word questions even if he manages to make an equation out of given information. Because mathematics is like a chain, regularity is very important. Secondly, during the lesson students should be very attentive all the time but this is partly up to the teacher [to keep their attention].

**R** Do you ask them to only listen?

**T** Yes. I ask them to listen carefully. I suggest them to solve examples the same day. He should review the examples and try to solve them without looking at solutions because the knowledge is still fresh in their minds. Because it is new it can be learned if they repeat within 1-2 days. Plus, students should solve examples from the topic we taught during that week and solve examples from tests, quizzes and question banks. Once they finish we ask them to review the past topics. And they should ask questions they could not solve. And trying to solve examples from mock examinations, this will make them better at mathematics. But this learning cannot be reflected in the mock examination results right away. The more you solve examples, the more experienced you become and experience of mathematics [and/or] experience of examples grows.

[12] The expression 'question bank' refers to textbooks which includes huge number of questions that are similar to the ones in the examinations. These books do not contain any theoretical information.

He stressed the importance of listening to the teacher a number of times; the extract above is another example of it. Along with listening to the teacher, he also highly valued repetition and revision. He viewed that students learn through revision at home. He expressed his perception of mathematics with the metaphor of a 'chain'. He viewed mathematical topics as the rings of chains and closely linked to each other. Nuri stressed the crucial importance of examples and that students should especially learn 'model examples' in the first place.

**R** What is the importance of giving examples? I mean how do they help students?

**T** Right. The meaning of giving examples is putting the theoretical information into practical terms. Examples are everything for us, I mean, in terms of private courses. Providing high number and different types of examples is everything for a teacher who works in a private course.

**R** Hmm...

**T** I mean I could say that for a private courser, the most important materials are examples and summarising topics with a variety of examples. So, I could say that examples are everything for us.

**R** What do you mean by summarising with examples?

**T** ...Even if we know the formulas we are unable to use them for all examples, aren't we? Even if we know the definitions we may not be able to solve the examples. But in private courses there is what is called model examples, yes model examples. Once students understand these model examples, they can solve some other examples too. As a result, what we do as a first step is to present these model examples to students.

**R** Hmm...

**T** Once students we give them these model examples, students are able to solve following examples on their own. But if you don't give them these model examples, students are unable to pass to the other examples.

The expression 'examples are everything for us' reveals the importance of examples quite clearly. In these extracts, Nuri also reveals his perception of the role of teachers as providing 'model examples'.When I socialised with Nuri, quite often I saw students asking him the questions they could not solve in the quizzes or question banks. I even see long queues of students for him. He also expressed the importance of these model examples when I asked about students bringing their questions to him to be solved. To him they were 'key' examples.

**R** Do the students learn from asking you the questions they could not solve?

**T** Yes they do learn. It came to my attention that students were stuck on certain types of examples and they stuck and they were stuck on a number of examples that are the same type. Once you solve one of them you open the lock and they are able to solve others as well.

During the interviews he also provided clues as to what 'model examples' are referring to. The following extract also outlines his views about teaching mathematics.

> **R** You also ask some specific types of examples, that have very particular solutions, are they really important?
>
> **T** I expressed this before many times. The past university examination questions are our guiding examples, our model examples. We shape our teaching around them. After that the [new types of] examples that students may not be able to guess [coming]. Because every year one or two new [previously unknown] types of question comes up. Our other mission is to present them some of the new types of examples in the lesson. Students should say this 'I never thought of these types of examples'. Yes, you present the content then give the model examples and model like examples but along with those, depending on the classes, you should also give examples that students see for the first time...This is one glamorous and enjoyable side of our job. Examination preparation books or examination preparation magazines or the examples we give are compared to the examples that show up that year and we say 'we taught these types of examples [to our students] and in fact these types showed up.

The last three interview extracts reveal a lot about his views on teaching mathematics, and his role as a teacher. Firstly, Nuri sees past examination questions (PEQ) as a source of 'model examples' and he derives the model examples from PEQ. Hence, it seems quite clear that PEQ are in the driving seat of his teaching. Secondly and possibly more importantly, he used 'we' and 'us' to position himself within the institution he works when he talked about the significance of 'model examples'. I commented "So this becomes something to be proud of?" He replied:

> **T** Yes. [We say to students] You see I taught you this example type. I solved it in the class and a similar question was asked in the actual examination. This is of course something that piles up with the years with a lot of [teaching] experience. I mean, because you [teachers] identify and examine past examination questions appeared by now, then you could come up with a question somewhere around them [questions]. And that is quite an advantage for you [teachers].

To him, PC teachers develop experience around past examination questions and they derive 'model examples' they teach in the lesson from them. Thus, guessing the questions in the coming examination becomes easier for them and this signifies how good a teacher they are in the eyes of the students. Along with model examples, Nuri also stressed the importance of helping students out of regular classes by solving the questions students could not. This was stressed in the following extract.

**T** Another factor in mathematics is teachers. I mean, a student is unable to learn a mathematical topic from one teacher but is able to learn from another teacher even though they are teaching the same topic.

**R** Even though they teach the same topic

**T** Yes, even though they teach same topic. I have witnessed this many times and it is very interesting: Students say 'my teacher taught me functions but I did not understand anything at all but when you taught I learned a lot of stuff. This is especially true for those who do not do well in mathematics. If you do not explain it and repeat it many times students are not able to absorb it. It is also important that teachers help their students. This is another factor. We deal with our students out of classes and we see this: students are learning the model examples that we taught in our classes. And then they use quizzes and question banks and they raise their level of mathematics. They ask us the questions they could not solve and this is another way of raising success.

**T** One day two students came up to me and asked if I could solve their questions. We sat together, and after I asked 3-4 examples one student said 'oh my god I understood this.' I mean the student was very cheerful and was sharing this with me. When they could not solve an example it feels like eating a hot pepper, but when they manage to solve its like eating baklava[13], and they got happy like this...they should listen very carefully, they should study regularly and they should solve a lot of examples. I mean they should solve tens of hundreds of examples of certain models so that these models will be established in their minds.

I inquired what makes him successful. His reply was again in connection with the institution he is working.

**R** What would make you think you are successful?

**T** Because we consider this from university examination perspective, a rise in the number of correct answers in students [the mock examination] results is a success.

**R** So the sign of success is...

**T** The sign [of success of a teacher] is the rise in the number of examples students are able to solve in the mock examination.

To be able to see his views on both institutions (SS and PC), I asked him to compare two institutions through an imaginary situation.

**R** Let's imagine two groups of students at identical level. I mean they have the same ability level. Let's say one group attends schools but not private courses and another group attends private course but

[13] It is a traditional Turkish dessert.

not schools for the whole 3 high schools years. After the graduation what difference do you see in these students?

**T** Majority of the ones with only school education will have limited practicality. In terms of example solving practice, in terms of example solving techniques they will not be developed.

**R** What aspect do they develop?

**T** They would develop more theoretical [mathematics] and be more inclined to definitions. They would understand definitions a bit better. For example if they go to university, I could say that they would be better at university mathematics. The ones which attend private courses would have clearly better number of correctly solved mathematics questions in the university entrance examination. I mean if both [groups of students] sit the university examination, the ones going to private course will solve most of the questions, but ones with only state school education will get stuck son ome of the questions. I mean, one [group] will know more theoretical side of mathematics based on definitions. Other [group] will know only practical side of mathematics. For example, if a student is exposed to only private course mathematics, this student will have questions during the university.

**R** Which ones do you think knows more mathematics?

**T** I think the one which goes to both state school and private course would know more mathematics. (Both laugh) It also depends on students' capacity.

**R** Lets say they have identical capacity

**T** If identical, state school students would know more mathematics

**R** Then, would that be correct to say that school students would be better in terms of definitions and concepts but private course students will be better in terms of solutions and procedures.

**T** Yes. We could say that.

He made a number of interesting points. He made a clear distinction between mathematics taught in SSs and the one taught in PCs. He coined them as 'state school mathematics' and 'private course mathematics'. To him while the former was more theory laden and based on definitions of mathematical concepts, the latter tend to be more practical and based on solving examples. In other words, the former was aiming at 'know-that', the latter was aiming at 'know-how'. He used this distinction not only in this part of the interview but also in many other instances in and out of classroom conversations. When I asked about institutional differences he made references to the goals of the institutions. To him different goals meant different institutions, or vice versa.

**R** Could you compare state schools with private courses?

**T** The aim of SS, the aim of high schools is to prepare students for life. To help him learn some job skills and to prepare him for higher educational institutions. They are a one step lower to the higher institutions. Private courses, however, prepare students directly for the university entrance examination.

**R** Do you mean it has nothing to do with university education?
**T** We prepare students only for the university examination.
**R** Does this difference in goals reflect the classroom life?
**T** I see that because students have the primary goal of going to a university, they primarily work on private course mathematics [rather than state school mathematics] and they pay attention to state school mathematics only secondarily. I mean, study only enough to get a passing grade. So the reflection is this: While students study state school mathematics just a day before the examination [at school], but for the private course mathematics, I mean the mathematics we teach, they study regularly during the week. Goals are different.

As can be observed in the extract, Nuri is making the distinction between 'goals' of SSs and 'the goal' of PCs. He perceived SSs goal as widely defined as preparing for life and, he mentioned, in particular, university education. PCs goal, on the other hand was singular and very particular: examination.

**R** What do you think is the aim of school mathematics?
**T** The aim of mathematics taught in state schools is mainly teaching mathematics with its theories and to prepare them for university mathematics and prepare them for the mathematics they will see there. In my opinion, what school mathematics differs from PC mathematics is that it is more theoretical and solving questions theoretically…and spending more time on them and emphasis on [learning the] process of solving [not the answer].

**R** Imagine a teacher teaching in one case in PC and in another case in SS. What difference would you see?
**T** SS teachers give definitions and proofs of theories. I mean teacher goes into details and makes more explanations. He gives a long solution or theoretical solution of an example. Private course teacher Nuri, however, tries to solve one example within one minute. For example, polynomials, say, a second degree polynomial, $ax^2+bx+c$ type. While we solve this example by numerical value technique, in state schools they take it as $ax^2+bx+c$ and solve it as a polynomial. On the one side [PC] there is a time limitation and the goal of getting the answer quickly, on the other side [SS] there are details, theories and definitions.
**R** Are they any differences in terms of the number and type of examples?
**T** Yes there are. But I observed that a few individual school teachers try to help students and give students some extra questions. That means, they realise the lack of practical side and to recover they prepare extra quizzes.
**R** Do you think state school teachers are also affected by the [university entrance] examination?
**T** I think some of them are affected and there are some who try to help students prepare. Because if you just suffice with school mathematics [and not study PC mathematics], it is extremely hard to be successful at university entrance examination.

Although he viewed teaching mathematics in SSs as teaching based on definitions theories and proofs, he mentioned a few individual SS teachers who may adopt some PC practices because these teachers 'realised the lack of practical side' in SS. He also stressed that 'SS mathematics' is insufficient for students in the university examination. Although he saw 'SS mathematics' as inadequate for students, he did not approve of 'PC mathematics' altogether. In fact he clearly disapproved of some of the techniques he used.

**R** What do you think about using numerical values to solve questions?

**T** Yes, this is a part our system. In terms of university preparation, preparation for university entrance examinations, this is part of our system...Using numerical values is of interest to them and they like it very much. 'Let's assign 1 to the value of 'a', and after that, lets give the options, lets put 1 for wherever you see 'a', what a simple thing, isn't it.' This is a part of our system, I mean, as a private course it is a part of us, we make use of it.

**R** Do you mean it is one of the indispensables of private courses?

**T** To me [Yes], look, sometimes you may not be able to remember the solution of a question. Because the students may become nervous during the examination they may not be able to do things they can do normally. But if you approach them like 'you can solve it using numerical values' they can make use of a second method and they can possibly solve the question in a practical manner with ease.

**R** Do you think it is a healthy method in terms of mathematics?

**T** In terms of mathematics teaching it is not a healthy method. Because it keeps students away from formulas, it keeps them away from definitions. I mean without understanding the definition, without understanding the formulas, they want to solve questions. That's not healthy in terms of mathematics education.

Here is another instance of his disapproval.

**R** Sometimes teachers are exposed to student question like 'What should we do for this kind of questions?' Do you think this mechanises mathematics? I mean, does what is taught in PC mechanise mathematics?

**T** Of course. I mean, whether we like it or not, we do it. There are some [question] forms these forms should be learned. The aim here is to bring the students to such a level that they can solve these [question] forms. Whether we like it or not, we have to mechanise a bit.

**R** Is it PC mathematics characteristic?

**T** Whether one likes it or not because the characteristic of university entrance examination is to deal with practical side of mathematics[14]. In the university entrance examination it is not important the way you solve the question. It is not important how students solve the question. Sometimes we solve factorisation

[14] Here the term 'practical' refers to solving questions in a shortest time and quickest way without dealing with any theoretical aspect of the question at all.

questions through using options. We say to students 'you may not know the topic at all; you may not understand this topic at all but let's solve this question together using options.' And students get surprised 'oh my god how easy it was'. I mean you have different approaches to the different questions but because there are some question forms in some topics that do not change, we mechanise it. We say 'for this kind of question you should do it this way, you should not think something else' for another question [we say] 'do not get into such interpretations'. I could not express well but we basically do not want them to go deep into it. Sometimes through doing too much interpretation, students make istakes. ...we expect them to produce more practical and superficial solutions.

**R** Lets ignore [for a moment] SS mathematics or PC mathematics. Considering using numerical value [technique] to solve the question, do you think it is an ideal way to teach mathematics?

**T** It is not healthy. In my opinion, solving a question using numerical value [technique] is only going for easy[15] ride but it perfectly fits with PC [mathematics teaching] approach. It attracts students' attention. Students like it because students aim is solving the question in any possible way and they like it and when you teach such techniques he mentions this to his fellow students [proudly] 'we have solved this question this way', 'Our teachers solved it with such an ease', 'Wow I never thought of this way' but it is not an ideal way to teach mathematics. From my perspective, it is going for an easy way a kind of escape to an easy way.

Here is another instance.

**R** How should it [mathematics] be taught? I mean, what methods should be used?

**T** In state schools theory must given but it should be consolidated by examples.

**R** Do you think proofs should be taught?

**T** I think proofs should be given. But proof should not be used [by teachers] as something to scare students...Because proofs are an important step towards understanding mathematics. I mean just giving practical mathematics and turning students into a mechanical thing is not right. In that case you would not provide students healthy mathematical thinking. You would not give students a mathematics that was required at the university level. For example, lecturers at universities are complaining and saying that because students are preparing only for the examination students do not know some topics like derivative or integration. But some departments like physics, mathematics and engineering require these topics. Because students lack these [mathematical topics], the level of education [at universities] drops. Because we only aim at university examination this is a big issue and at the schools students do not want to learn these [topics], universities suffer from it [as a result].

[15] The Turkish word 'kolay' is translated as 'easy' but it does not exactly correspond to the meaning of it. The reader should take into account that 'kolay' clearly connotes disapproval in this context.

Nuri also commented that they present the theoretical parts quite briefly. I prompted him for further explanations using a metaphor.

**R** You previously commented that 'we give the mathematics as summary. Whatever students need to know whatever necessary'. Can we say that it is like giving a pill?

**T** You know there are ready made [computer] programs. Yes. We give our students like ready made programs in relation to students' needs. I mean we don't want students to think in terms of [theoretical] mathematics. We don't want them to think deeply and we don't want them to think from a variety of perspectives. We have some practical methods and techniques. Their learning of these techniques and methods is important to us. We give the topic as a summary as an outline, only the parts that are necessary. For example, in the topic of relations. Relations is a very good example of this. When teaching relations in a state school, you have to teach the properties of a relation like reflectivity, symmetry, anti-symmetry, transitivity and explain them one by one. After that you should go on [teaching] equivalence relations and define this and then you should teach the properties of equivalence relations and then you should go on [teaching] modular mathematics. You should do some proofs in equivalence relations. But look, as I am teaching relations in a private course, I do not go into the properties of relations because there is no question about the properties of equivalence relations at all. But in order for me to teach modular mathematics, students should understand the concept of equivalence relation. So, I make an introduction and say this: 'My friends listen, before I go into giving practical side of modular mathematics you should know this. In fact, modular mathematics comes from equivalence relations and it is an equivalence relation.' And that's it and I go to the next item. They teach at school, but I only say it verbally. But at the school this should be given with its properties and its proofs. That's what I mean as program. I did what my job requires for me to do. I skipped something that would never be asked at the university entrance examination. For example, there is the property of closedness, union and distribution. We give this to our students very quickly. But when we come to neutral element, when we come to finding the inverse of an element, we teach them carefully and emphasise them because there are past university examination questions on them. Because the property of closedness is not asked before we just state its name and we go onto the next one. But school gives these [properties] in detail with their definitions.

**R** Does it mean that what you do is decided by past university entrance examination questions, as a guide?

**T** Yes. Past university entrance examination questions give us directions.

I also asked him "*What would be happen if there were no private courses?*" and he stated:

**T** If there were no private courses, considering the education state schools provide today, students' success rate in the entrance examination would drop seriously. Today we have low rate of entrance and that would be much lower. Unfortunately the mean number of mathematics questions students solve in the examination is around 7 or 8.

**R** Out of 45 questions?

**T** Yes, out of 45. So, these numbers would be much lower.

## 4.2 The Case of Saban

### 4.2.1 Background Information

Saban was one of the mathematics teachers working in a private course. He had been in this position for just over 8 years. In these 8 years he never worked in a state school. Despite being in his mi-thirties, he seemed to be a very experienced private course teacher. This was probably because his teaching experience has only been in private courses. When I asked if I could video record his lessons and do some interviews he was very comfortable with the idea. This is probably because I explained him clearly that my aim was not to make any value judgements about his competence as a teacher. He had signs of nervousness no more than at the beginning of first video recorded lesson. I interviewed with him on four occasions, on one of which I was invited to his home. In one occasion, in the first 10 minutes, I felt he was uneasy. He was not the Saban that I knew in and out of the classroom. Having realised this unusual situation, I tried to understand the cause of it by indirect questions. He revealed to me that his mother had some health problems and he was not feeling well but had promised to be interviewed. I gave my condolences and rescheduled the interview for another date. Other than on this occasion, I felt that the atmosphere of the interviews was pretty personal and from my perspective Saban was totally himself during my contact my contact with him. He was also surprisingly comfortable with recording our conversations. He was teaching in a number of different classrooms over 26 lessons per week. Most of his lessons were on weekends.

### 4.2.2 Organisation of Saban's Teaching

In organising his teaching, Saban seemed to be making use of his past teaching experience. According to him in his first few years he prepared many lesson notes and made himself ready for what to teach and how to present:

> **R** Can you talk about your teaching organisation?
>
> **T** The template and outline of topic is ready as I taught it for years. I made a lot of notes in the 2nd and 3rd years of my teaching. In the 4th and 5th years I organised myself. I mean, it is like which example to teach and how much time to spend on it. Which explanations to be made in which specific occasions and to what extent it will be expanded? I divide them [main title of the topic] into sub-sections. After I have done it I give sub-sections one by one. I give them and I solve their examples with them. I mean I don't give the [theory of] topic together and then solve mixed examples later on.

**R** Do you mean like sub-section and examples, sub-section and examples?

**T** I present a title shortly and then I give the examples that can be solved in that way. I am trying to give the application of that topic.

He divided the topics into a number of small parts or sub-sections. He organised his lessons in a manner that reflected this division. Further details of his teaching will be presented in the next section. I will now focus on how he specifically organised his lessons in the topic of functions. I video recorded him in a total of 12 lessons in 3 different classes. He completed the topic of functions in one of the classrooms I video recorded in over 5 lessons. Table 4.5 summarises his teaching in these lessons.

| **Lesson Code** | | **Content** | **Passive Examples** | | **Active Examples (PEQ. )** | |
|---|---|---|---|---|---|---|
| | | | Number | Time | Number | Time |
| 1 | Saban-1 | 4m 21s | - | - | 12 | 21m 36s |
| 2 | Saban-2 | 2m 31s | - | - | 13 | 39m 13s |
| 3 | Saban-3 | 5m 50s | - | - | 8 | 19m 20s |
| 4 | Saban-4 | 58s | - | - | 10 | 25m 50s |
| 5 | Saban-5 | 1m 21s | - | - | 8 | 27m 44s |

**Table 4.5** Summary of Saban's practice.

Table 4.5 shows that Saban spent most of his time solving active examples. This resulted in a cycles of content and active examples, which could be seen in Figure 4.2.

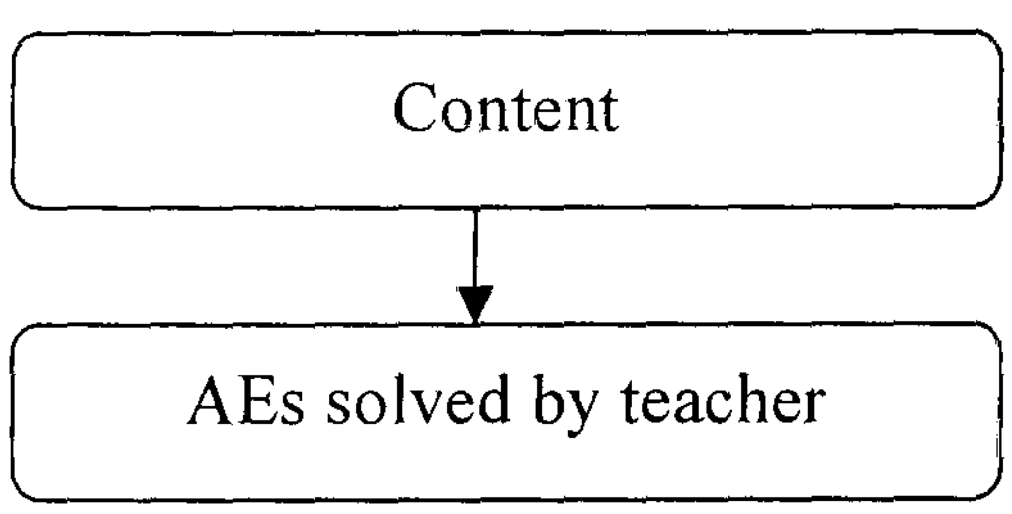



**Figure 4.2** Saban's practice pattern.

He solved a number of examples in a very short time. He also spent little time on the content segment of his lessons (the theoretical part). In these content segments he typically wrote the rules, formulas or definitions on the board and made brief explanations of the concept or the procedure involved. This was generally limited to a short period of time. In short, Sabans lessons were dominated by active examples. Table 4.6 presents a summary of content segments in Saban's lessons.

| Lesson | Code | Content |
|---|---|---|
| 1 | Saban-1 | • Conditions for being a function (3m 4s)<br>• Domain and Range sets (1m 17s) |
| 2 | Saban-2 | • Function types (2m 31s) |
| 3 | Saban-3 | • Constant functions (27s)<br>• Liner functions (20s)<br>• Inverse functions (3m 5s)<br>• $(f^{-1}(x))^{-1} = f(x)$ (5s)<br>• Inversing the functions of the form $ax+b$ (1m 25s)<br>• Inversing the functions of the form $\frac{ax+b}{cx+d}$ (33s) |
| 4 | Saban-4 | • The procedure of $y = f(x) \Leftrightarrow x = f^{-1}(y)$ (25s)<br>• Inversing the functions at the form of $ax^2+bx+c$ (23s) |
| 5 | Saban-5 | • Composite functions (47s)<br>• Properties of composite functions (34s) |

**Table 4.6** Summary of the content segments in Saban's recorded functions lessons.

The tendency in Saban's teaching is quick presentation of content and then linking this information to the questions that appeared in the past UEE.

| | Percentage | Number of examples |
|---|---|---|
| Passive | 0 | 0 |
| Active | 100 | 61 |
| **Total** | 100 | 61 |

**Table 4.7** Distribution of examples used by Saban.

Saban's lessons were dominated by active examples. Saban's strong tendency to use active examples can be observed in Table 4.7. Out of the total of 61 examples he solved in 5 lessons, Saban did not use any passive examples and all 61 examples were active examples. Table 4.8 shows the number of active examples solved by students and Saban.

| | Total | Solved by Saban | Solved by students |
|---|---|---|---|
| Active Examples | 61 | 61 | 0 |
| **Percentage** | 100 | 100 | 0 |

**Table 4.8** Examples solved by Saban and students.

Despite the fact that Saban solved many active examples, he did not let his students to solve any of them on the board. The next section will provide detailed description of his teaching in one of his lessons.

### 4.2.3 Saban's Teaching

Lesson Code: Saban-3

To describe teaching/learning activities during the lesson, I would like to present the main phases of the lesson as a summary.

- Saban explained what a constant function was in very brief terms.
- He then gave 2 examples regarding constant functions which he himself solved. He demonstrated 2 methods to solve each of these two examples. He explained their relation to the examination.
- Saban explained what a linear function is in very brief terms.
- He then gave an example of a linear function and gave information about linear functions in the examination.
- Saban made some explanations about inverse functions, which he named as the first focal point of the inverse function topic. This was followed by a simple example.
- He then gave an example with 3 sub-questions.
- Saban then explained more focal points, the second and third. For the third point he wrote a note that explains how the third point is used in solving examination questions.
- He gave 2 examples.
- Saban then explained a fourth focal point.
- He then gave 2 more examples. He solved the second example using two methods.

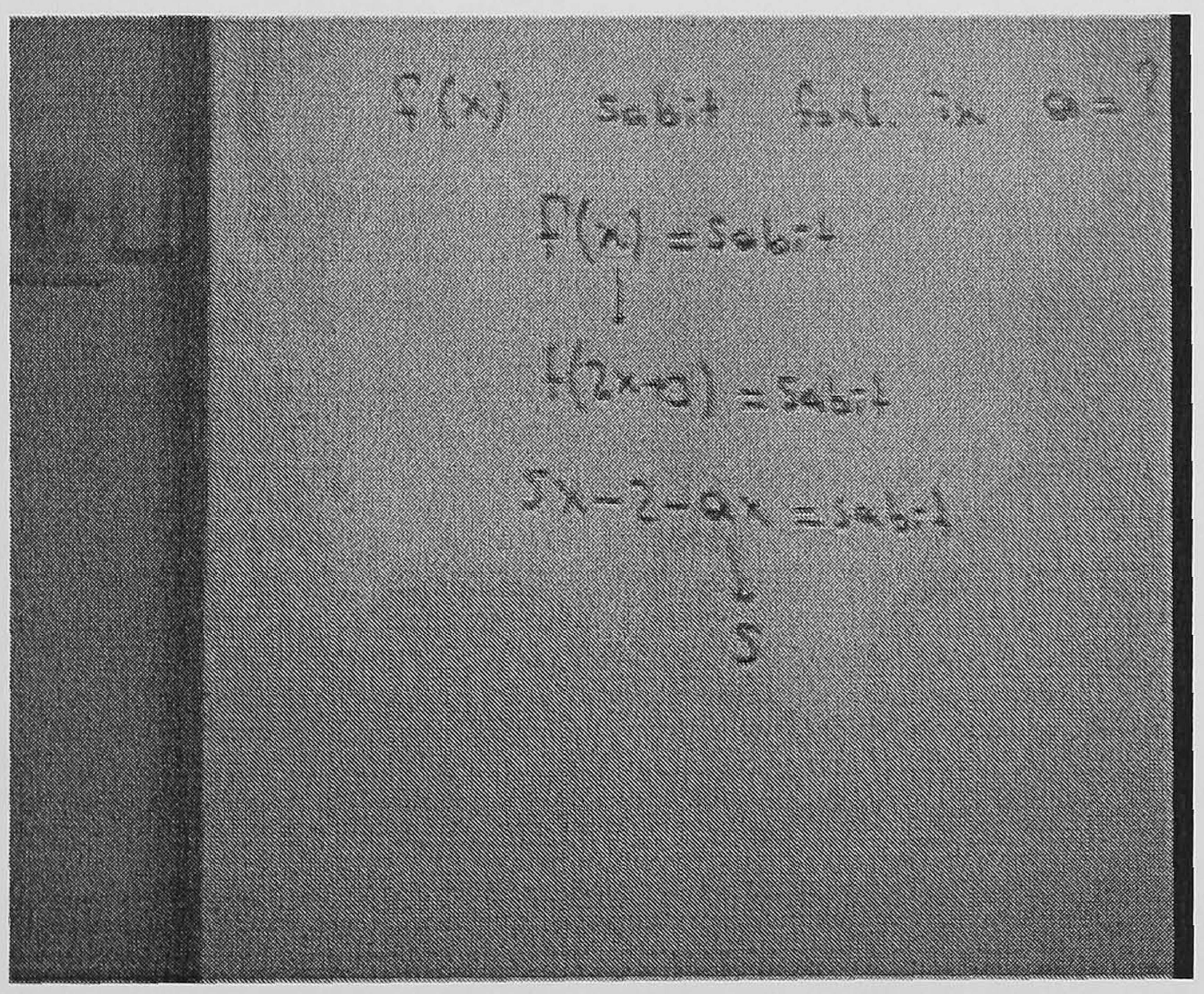
At the beginning of the lesson, Saban wrote a title on the board 'Constant Function'. He stated: "*f(x) equals to a numerical constant. No matter what you write inside (of parentheses, referring to argument of the function), write 1 or write 2 or –1, the result is the same constant. That means you can solve any question (involving constant function) with two methods: you can solve it by equalising the polynomials, or you can also solve it by numerical value like this*".

| |
|---|
| Constant Function |
| f(x)= constant or |
| f(1)= f(0)= f(2)= f(-1)=.....= constant |

And that was all his explanations on the topic of constant function. He immediately went onto writing an example on the board. Without using any source he wrote the function *f(2x+3)=5x-2+a-bx* then he had quick look at the expression and changed his mind and cleared '*+a-bx*', instead added '*-ax*'(see box).

| |
|---|
| Ex: f(2x+3)=5x-2-ax |
| and if f is a constant function a=? |

He immediately solved the example. He explained the reasoning of the solution he produced. He pointed out that if a function is constant then it does not matter what is in the parentheses of f (pointing to the argument of function), it (the image) will be the same constant value. If there is one value for 'a' that makes 5x-2-ax constant than it is 5 because the only other term containing x is +5x therefore they (+5x and-ax) could be cancelled together.




f(x)= constant
↓
f(2x+3)= constant
5x-2-ax= constant
↓
5

He called this '*first method*' and wrote '$1^{st}$ method' in the upper left corner of the solution and wrote '$2^{nd}$ method' then demonstrated another solution. He wrote f(0)=f(1) and after a second of thinking he changed his mind and stated, "*f(0) may not give you the results you wanted, that makes it a bit awkward*" (and erased f(0)=f(1)). "*Let's begin with giving 0 to x, and put this in the function, f(3)...let's give –1 to x, this gives you f(1)*"[1] and calculated f(3) and f(1) (see box).

[1] In other words let us assign 0 to the value of x and calculate all x's as 0.

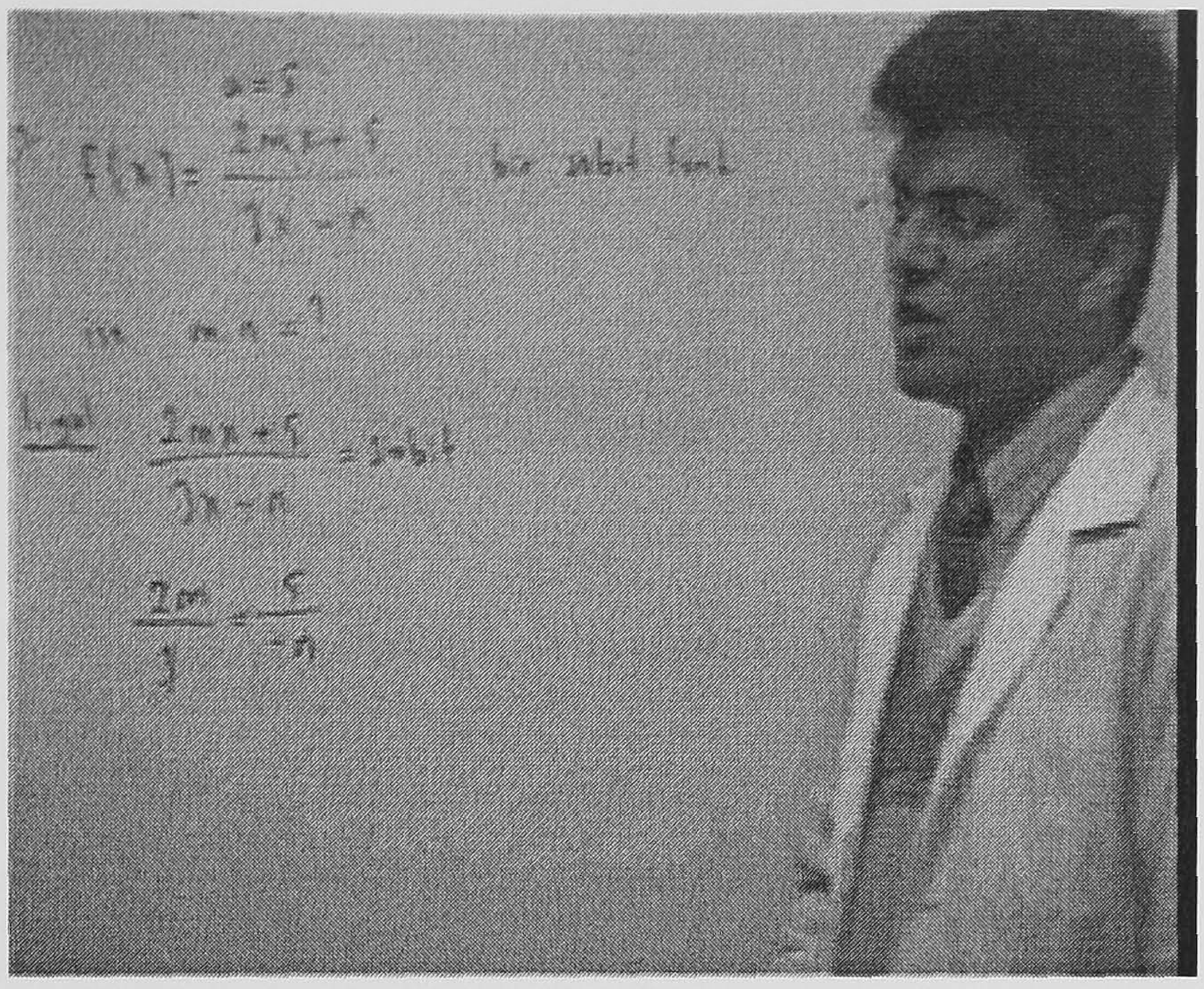
| |
|---|
| For x=0 ,    f(3)=-2 |
| For x=-1  , f(1)= -5-2+a |

At this point of the solution he said "*if it is a constant function they are equal, f(3) equal to f(1)*". He then equalised *–2* and *–5-2+a* and found *a=5*. He added "*There is no question of this type among the past university examination questions, there isn't. Among function types*[2]*, only inverse function and composite function is asked*". Then he emphasized this by repeating this sentence again. He then gave another example:

| |
|---|
| Ex: If $f(x)=\dfrac{2mx+5}{3x-n}$ is a constant function then m.n=? |

He indicated, "*f(x) is a function containing one variable (x) and in the structure of a/b*" He also solved this example by two methods and he made it clear by writing 1st method and 2nd method at the top left corner of the each solution as he did in the previous example. In the 1st method he explained that it was linked to sequences and constant sequence. He said "*Look, if* $f(x)=\dfrac{2mx+5}{3x-n}$ *is equal to a constant value then it requires that the polynomial in the numerator can be divided by the polynomial in the denominator. That means the ratio of the coefficients of x's is equal to the ratio of the numerical values.*" and wrote: $\dfrac{2m}{3}=\dfrac{5}{-n}$. He then solved it (see box).

[2] 'Function types' in this context refers to a subsection of the topic of functions. To him, this included constant, linear, inverse and composite functions.

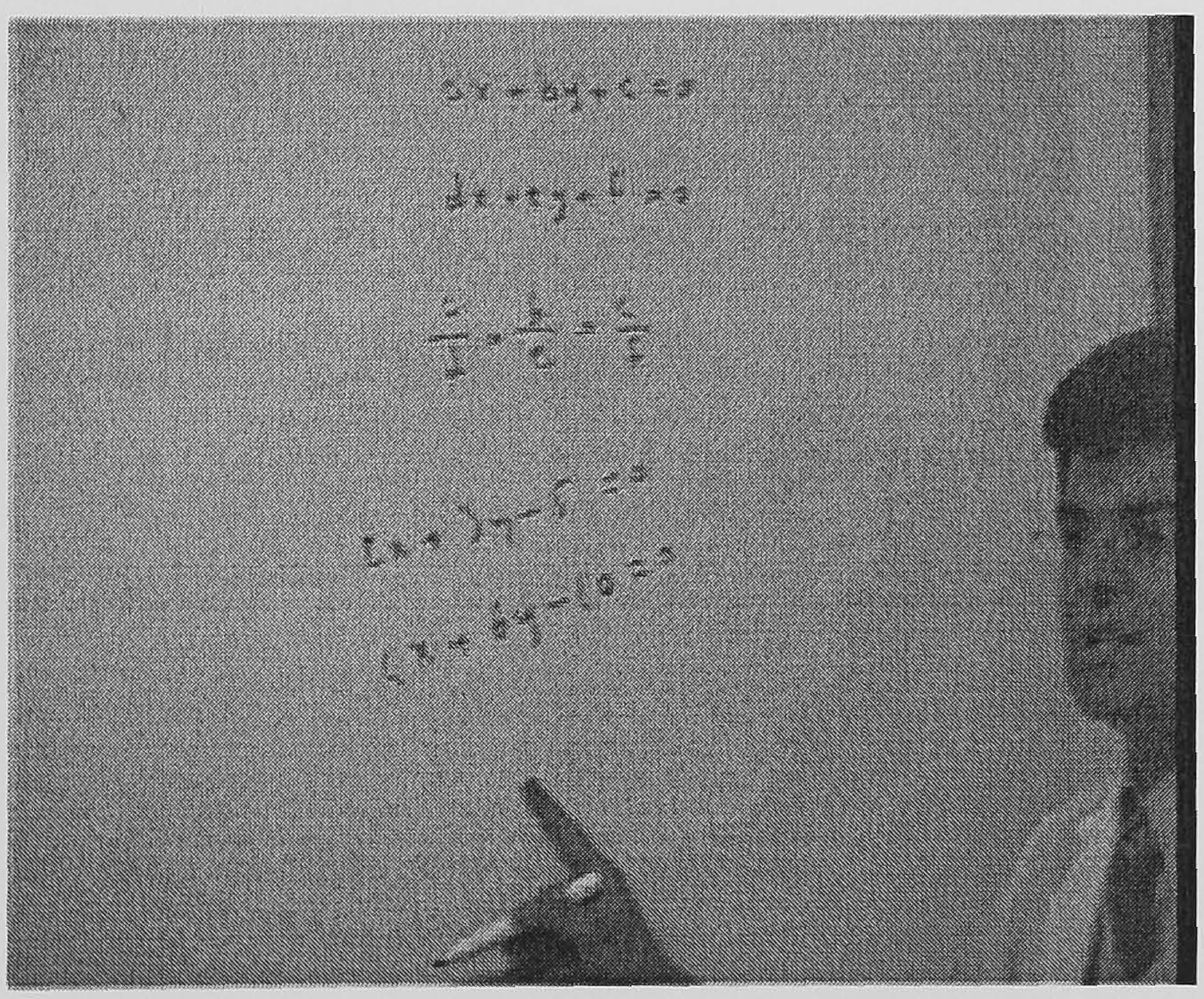


$$\frac{2m}{3} = \frac{5}{-n}$$

2mn = -15

mn = -15/2

After that Saban extended his explanation to analytical geometry. He said, "*If that equality is correct then expressions show the same line. Remember this expression*" and wrote: $ax+by+c=0$ and $dx+ey+f=0$, "*in order for these lines to indicate the same line, (in other words) to have infinite solution, it is required that ratio of the coefficients of x's is equal to ratio of the coefficients of y's and is equal to c over f*".



He then explained that 2x+3y+5=0 and $4x+6y+10=0$ indicates the same line as division of the corresponding coefficients of these two expressions is equal. He linked this to the topic of sequences that if they are constant sequences the ratios of the coefficients are equal. After that Saban started to explain the second solution method, as he put it, "*in case you could not make sense of the first method*". He said, "*You could assign a numerical value to x, let's say 0, we find f(0) and it is* $\frac{5}{-n}$, *we assign another numerical* *value, say 1, then you find f(1), it is* $\frac{2m+5}{3-n}$ *, if it is constant function f(0) equal to f(1)*".

And he performed the solution by equalising values of *f(1)* and *f(0)*.

| |
|---|
| For x=0, $f(0)= \frac{5}{-n}$ |
| For x=1, $f(1)= \frac{2m+5}{3-n}$ |
| $f(0) = f(1)$ |
| $\frac{5}{-n} = \frac{2m+5}{3-n}$ |

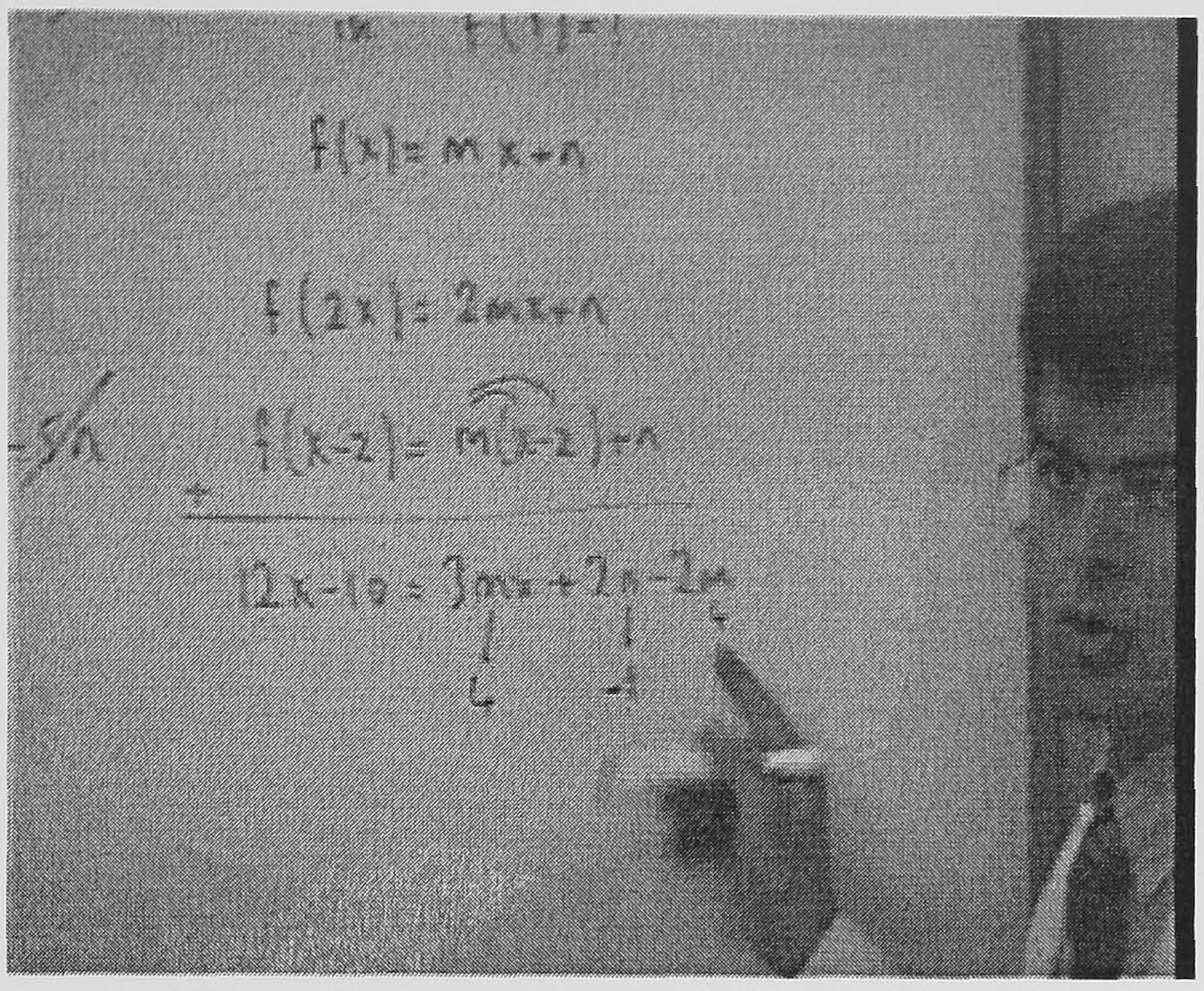
He said "*there is not much question in the past examinations until this part, not any... We learned constant function, identity function and we will have one more; linear function and then we will go onto inverse function. Our real focus is on inverse function*". He asked if there was anyone who had a problem with these examples. Up to this point in the lesson, Saban had spent only 9 minutes for the explanations of constant functions and providing two examples and solving each of them using 2 different methods. He wrote a title '*Linear Function*' and said "*If f(x) is given as linear then you can say it is mx+n*" and wrote *f(x)=mx+n*. After repeating this sentence exactly, he brought out his notes from his pocket and wrote an example.

| |
|---|
| f(x) is a linear function. |
| if f(2x)+f(x-2)=12x-10 then |
| f(1)=? |

Referring to '*f(x)* is a linear function' he said "*they*[3] *don't have to add this but no problem if they do. If they don't you could guess it (is in fact a linear function). In that case you could use the options and solve it by using options*". Saban solved the example by putting '*2x*' and '*x-2*' in the function '*mx+n*' then added them together and equalised the result of this addition to *12x-10* (given in the question). He found the equality *12x-10=3mx+2n-2m*. He used the property of the equal polynomials to find *m* and *n*. However, he skipped an expected step in the solution. That is writing the equalities (*12m=3* and *2n-2m=-10*) separately. He stated that '*because they are equal polynomials*' then he drew a downward arrow and wrote *4* under it and wrote *4* under

[3] Here 'they' refers to either examination questions or experts preparing the examination questions. Based on the linguistic characteristics of his sentence, it is not possible to identify which one.

the $m$ in the expression $2n$-$2m$. He said "*if you write 4 to m then n should be 1*" and drew a downward arrow and wrote 1 under n.




He wrote the function $f(x)=4x-1$ without reminding students on what basis he wrote that function[4]. Having found $f(1)=3$, he said "*They may not say that it is a linear function, but in that case they ask what f(x) is. In that case all of the options would be polynomials from the first degree like 4x-1, 4x+2 or 3x-4. By inspecting the options you could guess that f(x) is in fact a linear function. Once you decide that it is linear, you could do use the form mx+n and solve similarly. I wrote this* (referring to 'f(x) is linear function' in the example) *because the title is linear functions.*"

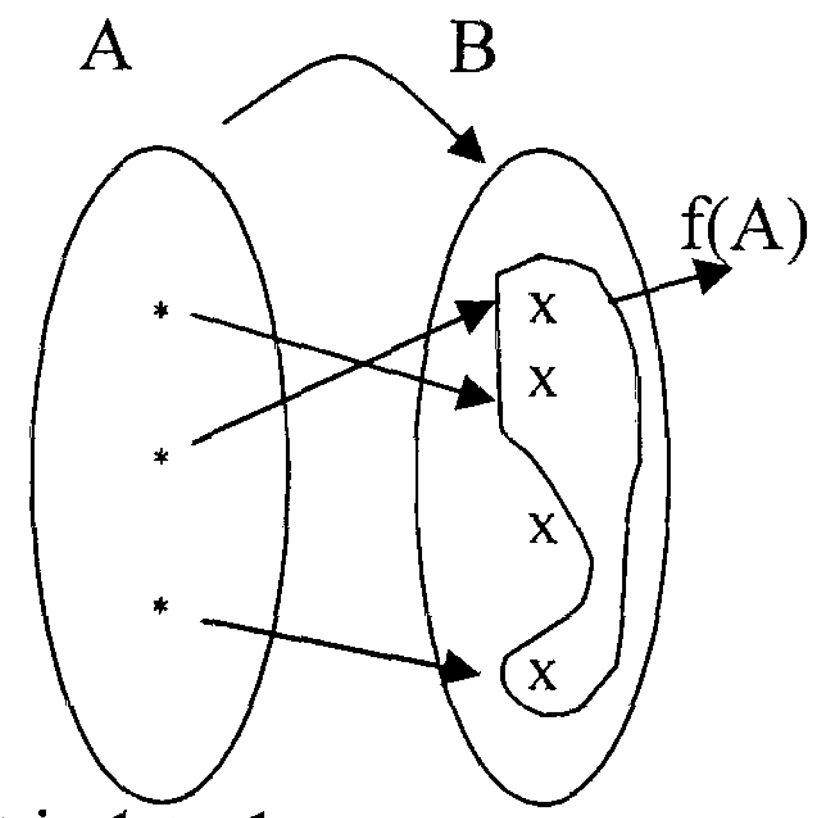
Saban then wrote the title '*Inverse Function*' and said, "*This is our main focus, I mean, this is frequently the part university examination questions come from…We have a number of subsections*" and added "*One…In order for a function to have an inverse it should be 1 to 1 and onto. If a function is not 1 to 1 and onto it does not have an inverse.*" And he wrote that on the board.

| |
|---|
| Inverse Function:<br>(1) In order for a function to have an inverse it should be 1 to 1 and onto |

[4] Because he assumed f(x)=mx+n at the beginning of the example, he just put the value of m and n.

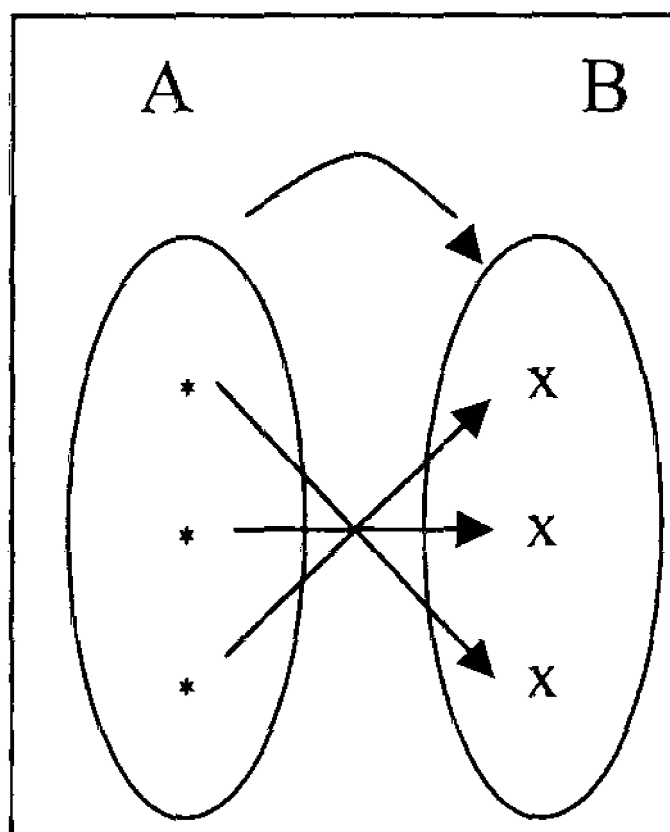
He explained what is meant by '1 to 1' and 'onto' on a diagram where the function was 1 to 1 but not onto.

Inverse Function
(1) In order for a function to have an inverse it should be 1 to 1 and onto.




It is 1 to 1
Not onto – $f(A) \subset B$

He pointed out, "*Think about it like this. Onto means that f(A), range set, I mean the set made up of the images should be equal to B, but (in this diagram) it is covered by B. Onto means f(A) is covering B. As you see there is one element left out in the range set. Thus it is 1 to 1, yes, each element corresponds to 1 element, but not onto as it (f(A)) does not cover set B, at least there is one element left out. The rule here is that, in the inverse function, all the arrows should be turned back. That means, if domain is A and range is B, then for the inverse function domain is B and range is A. When you turned the arrows back, there is one element left out. If there is an element in the domain, it (inverse function) does not make a function, even though it (f(x)) does (make a function). Therefore it must be 1 to 1 and onto to have an inverse*". Saban then drew another diagram and showed that it is 1 to 1 and onto.

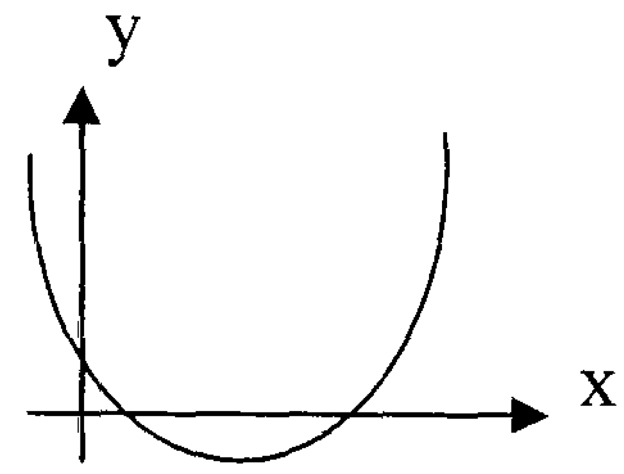



It is 1 to 1 and onto.

Saban said, "*It is 1 to 1 and onto and thus it has an inverse and you can turn the arrows back. Turning an arrow back means changing the place of x and y... changing the place* *of x and y. Lets come back to examples that are of our interest*" and then he wrote an example with three sub-questions.

Ex: Which of the following functions is/are 1 to 1 and onto.

**a)**




**b)** A={1,2,3}

B={a,b,c} f:A→B

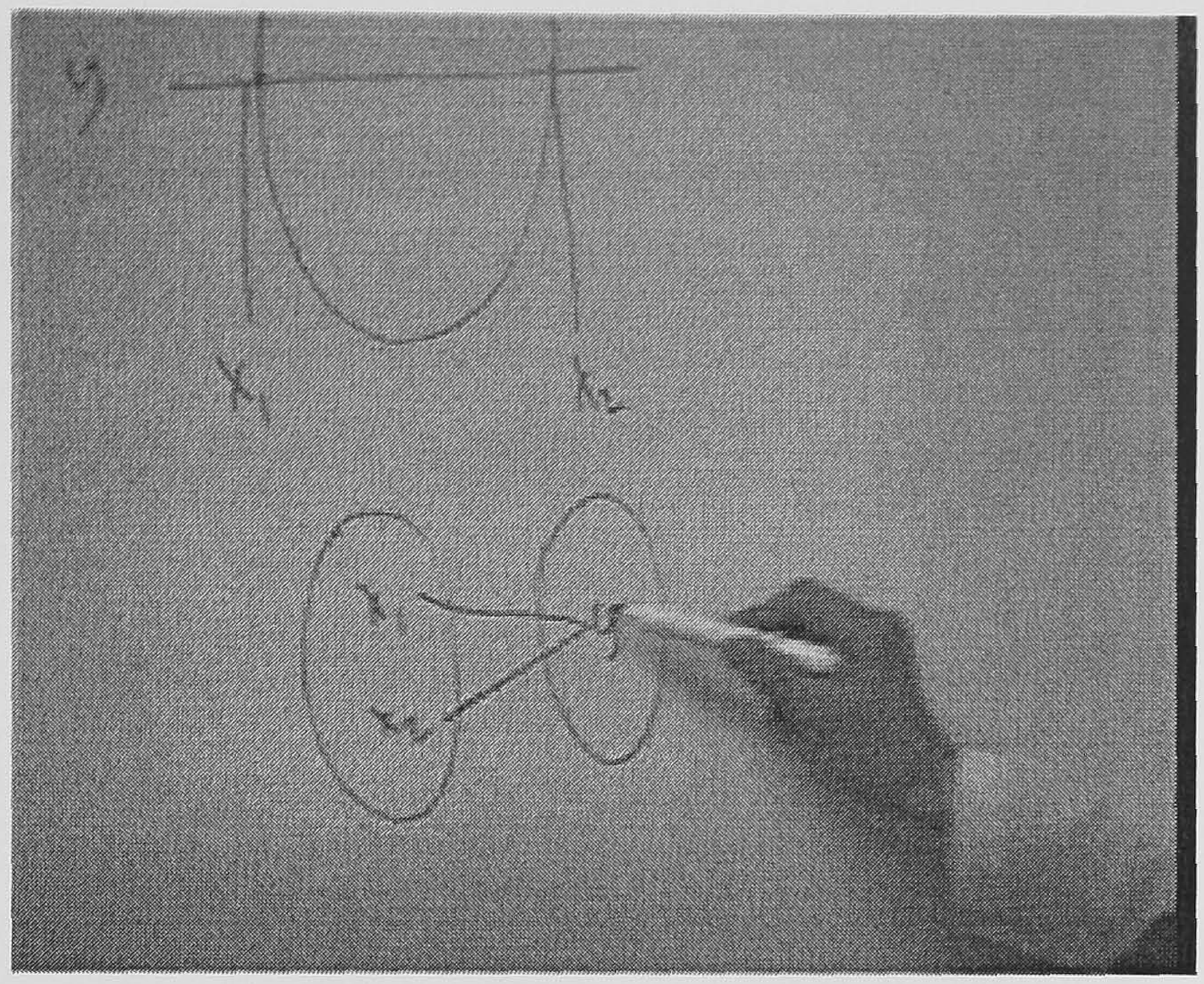
f ={ (1,b) (2,a) (3,a) }

**c)** A={a,b,c}

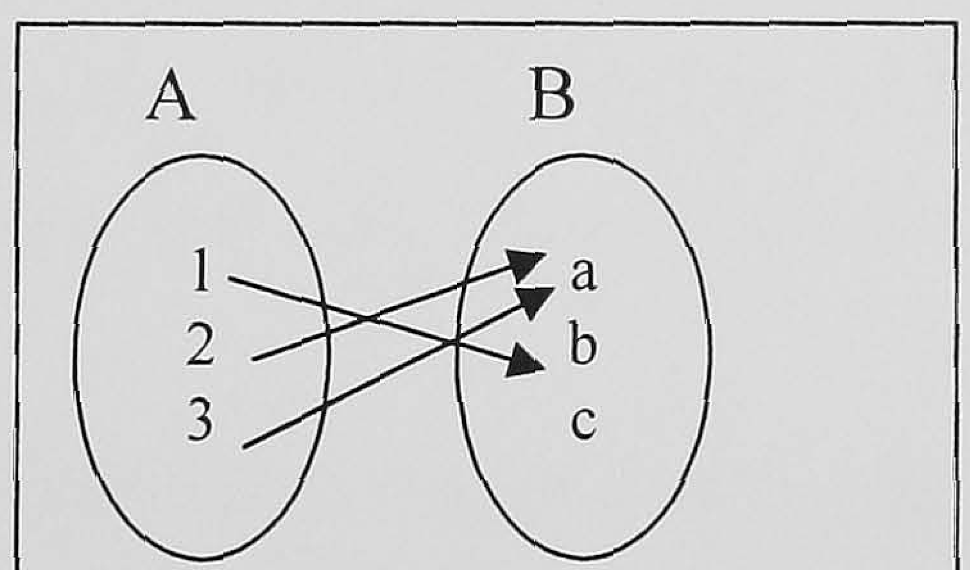
B={x,y,z} f:B→A

f ={ (x,b) (y,a) (z,c) }

Having finished writing the example, Saban said "*there was one question in 1988* (examination)" then he started solving it. He used a vertical line technique to show that the graph given, in option a, indicates a function. He said, "*draw lines parallel to x axis. It always intersects (with graph) at only one point. It means it is a function*", he wrote '*Function*' next to the graph. He also showed that the inverse of it does not indicate a function by using the same technique: "*In order for its inverse to be function you exchange x and y, so lines should be drawn parallel to y axis. If it intersects at more than one point the inverse does not indicate a function, but itself is*", he wrote '*inverse is not (a function)*' next to graph. He then explained the technique again with similar wording. He then drew another graph and a diagram and extended his explanations, regarding why, if the line drawn intersects at two points, it does not indicate a function.



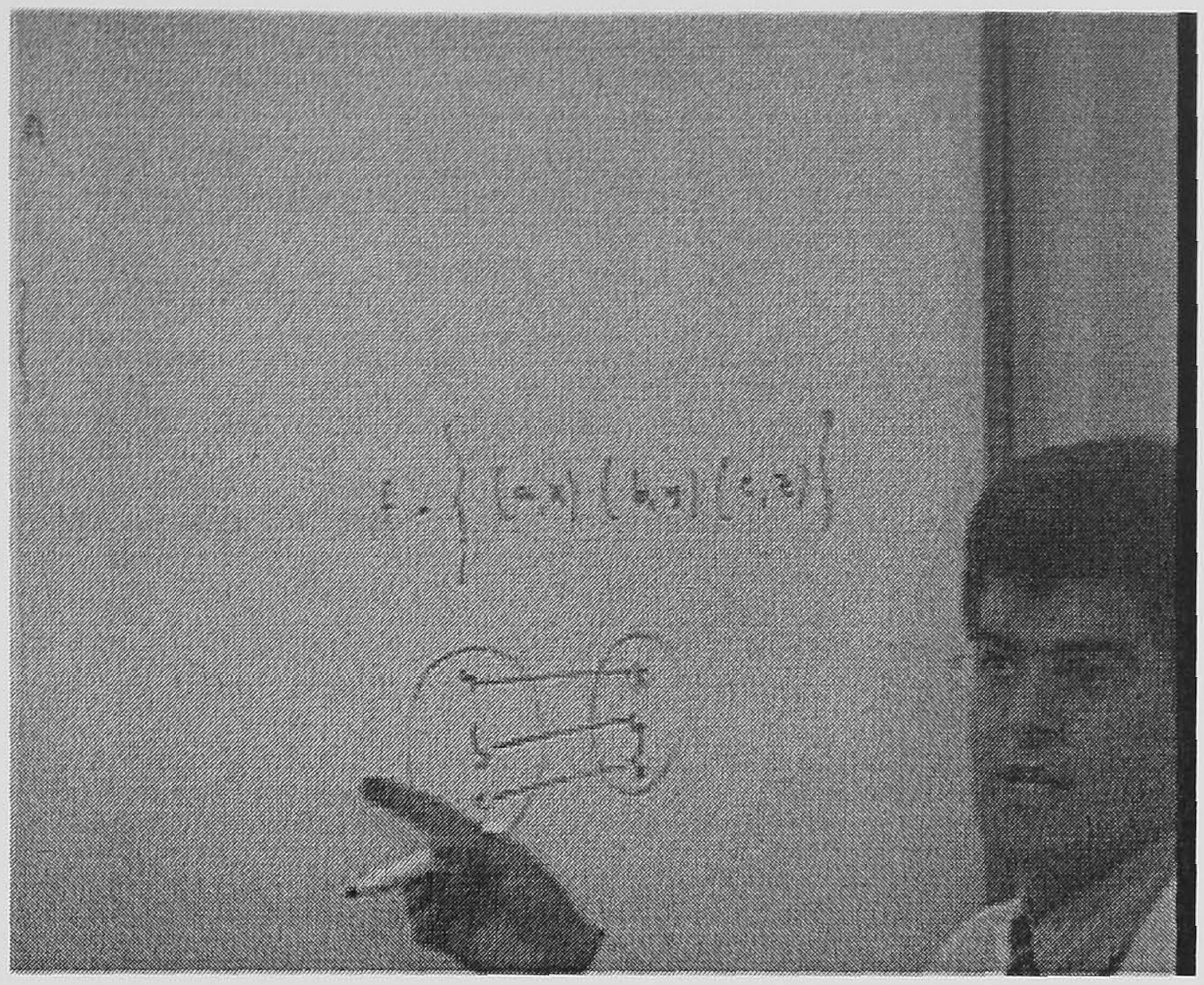
Saban then started solving the option b by a warning "*for this type of question the practical method is this. Firstly, the number of elements (in two sets) should be equal otherwise it can't be 1 to1 and onto. So the number of elements should be equal*" After this, as a second method, he pointed each elements of the set A and B on *f =*{(1,b)(2,a)(3,a)} and said "*the element 'a' is written twice, then it is possibly 1 to 2, we could see it by a drawing a diagram*" and he drew a diagram.




Function but inverse is not.

He explained that (by pointing to the element 'c' in the diagram) "*there is an element left out. When you turn the arrows back, it doesn't indicate a function because of this (element)*". He wrote '*it is a function but its inverse is not*'. He then went onto solving the option c. He called for students to pay attention to the order of domain and range sets of the function in the examination questions: "*be careful! It is from B to A. Sometimes they*[5] *give you (the function) from B to A but it (function) is given (with elements) from A to B. You may start thinking there are two answers*". Saban then solved the example by drawing a simple diagram for it.

[5] Again he is referring to examination questions or experts preparing the examination questions.



He then warned the students again: "*they may trick you with questions like this.*" And he wrote another function in the form of ordered pairs and drew a diagram for it (see capture). The function he gave seemed like a well-defined function, but in fact it was not because the domain and range sets were swapped. Therefore, the first elements of the pairs in the function were from range set rather than the domain. And thus it did not make a function and he asked them to pay attention to this detail in the examination and added, "*This is the focal point about the definition (of inverse function), there is only one examination question in 1988, you can check it out*".

He put second and third focal points on the board with short explanations for each. For the second one he said "*inverse of the inverse of a function is equal to itself*". For the third one he said, "*this is the rule to inverse a function*". He verbally explained while writing the third, which was step-by-step instructions to inverse a function. He called this '*general rule*'. He then repeated step-by-step instructions he made during the demonstration of the third point.

(2) $\left(f^{-1}\right)^{-1} = f$

(3) If f(x)= mx+n then $f^{-1}(x) = \dfrac{x-n}{m}$

| | | |
|---|---|---|
| Let's say f(x)=y | → | f(x)= mx+n |
| x is written | → | y-n=mx $x = \dfrac{y-n}{m}$ |
| in terms of y | | |
| change the place | → | $y = \dfrac{x-n}{m}$ |
| of x and y | | $f^{-1}(x) = \dfrac{x-n}{m}$ |

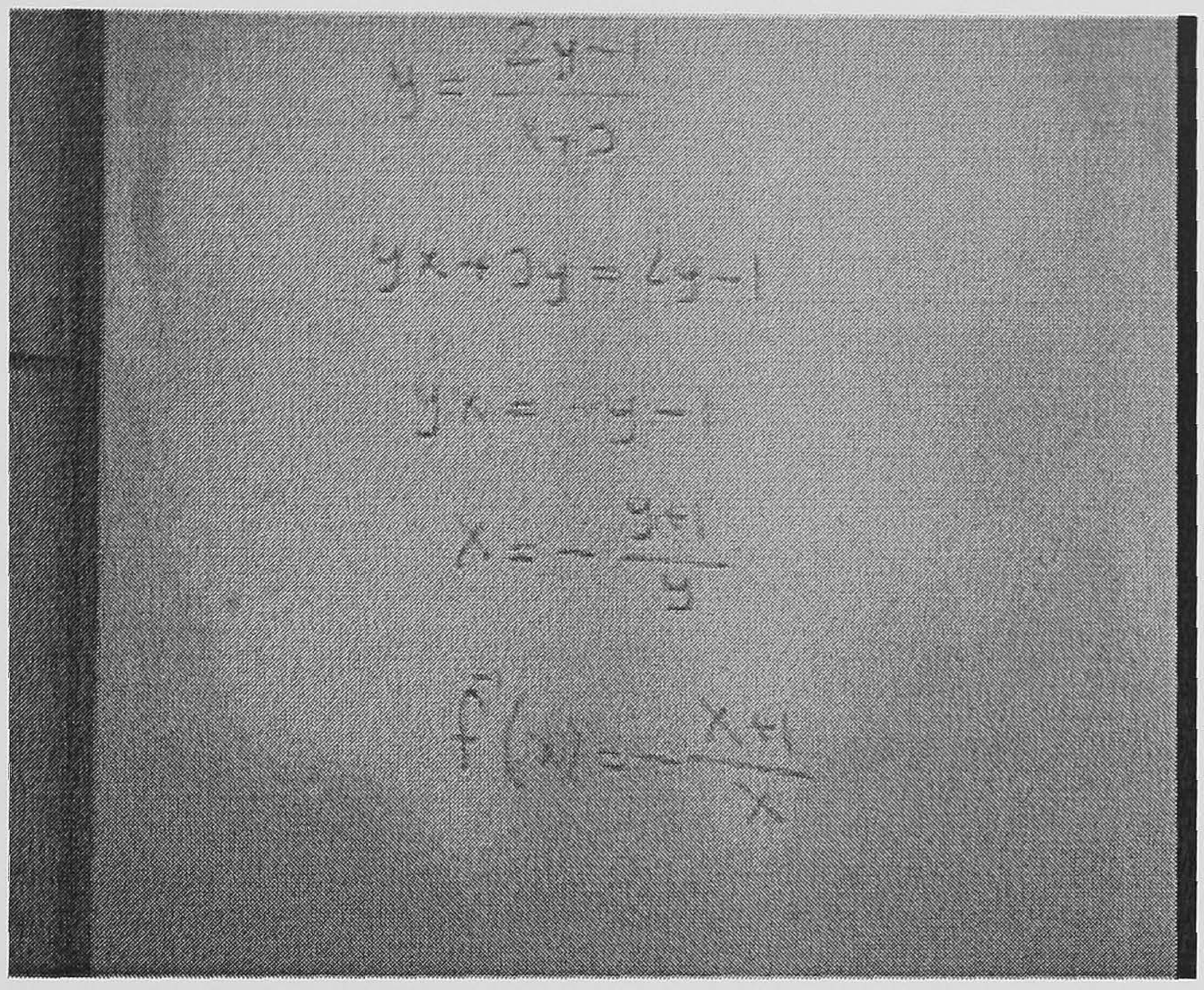
He stated "*In order to find the inverse of a function, f(x) is called y, then x is written in terms of y, then the place of x and y is changed*". Saban said, "*this note must be given here*" then wrote a note on the board with red pen (to have extra attention of students).

> Note: This method is generally used for the questions that contain only one x (in the rule of the function). Other questions are solved through specific methods.

He said "*It is generally used for the questions that include only one x, you can use it for some others, but for the ones that include only one x, it is necessary*". He went onto examples and added, "*I won't come up with the example 'f(x)=2x-1 and what is its inverse?' That would be too simple (in relation to examination questions). Because we are interested in the use of this rule* (referring to note), *let me give an example, which is from a past examination*" and he wrote the example (see box).

> Ex: f is defined as a 1to1 and onto function.
>
> If $f(x) = \dfrac{2f(x)-1}{x+3}$ then $f^{-1}(x) = ?$

Unlike other examples, this time he seemed to let the students have a bit more time to solve this example (25 seconds). Saban solved the example with a practical method that he suggested. In reply to one student's suggestion "*we can find f(x) first*", Saban said "*you can find f(x), but the calculations take a long time... Look* (pointing to the function), *check how many x are there...there is only one. For the questions that include only one x, you don't have to find f(x) by leaving f(x) alone on one side. Do this!*" then he solved it by the steps he gave in the third point. Rather than finding *f(x)* first and then inversing it, he wrote the expression in a way that leaves x alone on the one side of equality. Then what is remained on the other side was actually the on inverse function (See capture).



Saban said "*it is better if you use this for the questions that include only one x. At least you would be doing fewer manipulations*" Saban then wrote another example.

| |
|---|
| f is defined as a 1to1 and onto function. |
| If $x = \dfrac{3f(x)+2}{f(x)-5}$ then $f^{-1}(x)$=? |

When others were trying to solve this one student asked why he could not get the answer (showing his notebook), Saban commented that he was using such a long method. He answered similarly to another student whose notebook was shown to him. Like all other examples, Saban started solving the example himself. He did it by using a very practical method. Again, rather than finding *f(x)* and then $f^{-1}(x)$, he said "*look how many x is there?*" and answered right away "*only one*" and used the method he demonstrated. He put y in the place of *f(x)* and, as shown below, found that what is given as equal to x is actually the inverse function itself (see the solution).

$$\boxed{x = \frac{3y+2}{y-5} \rightarrow f^{-1}(x) = \frac{3x+2}{x-5}}$$

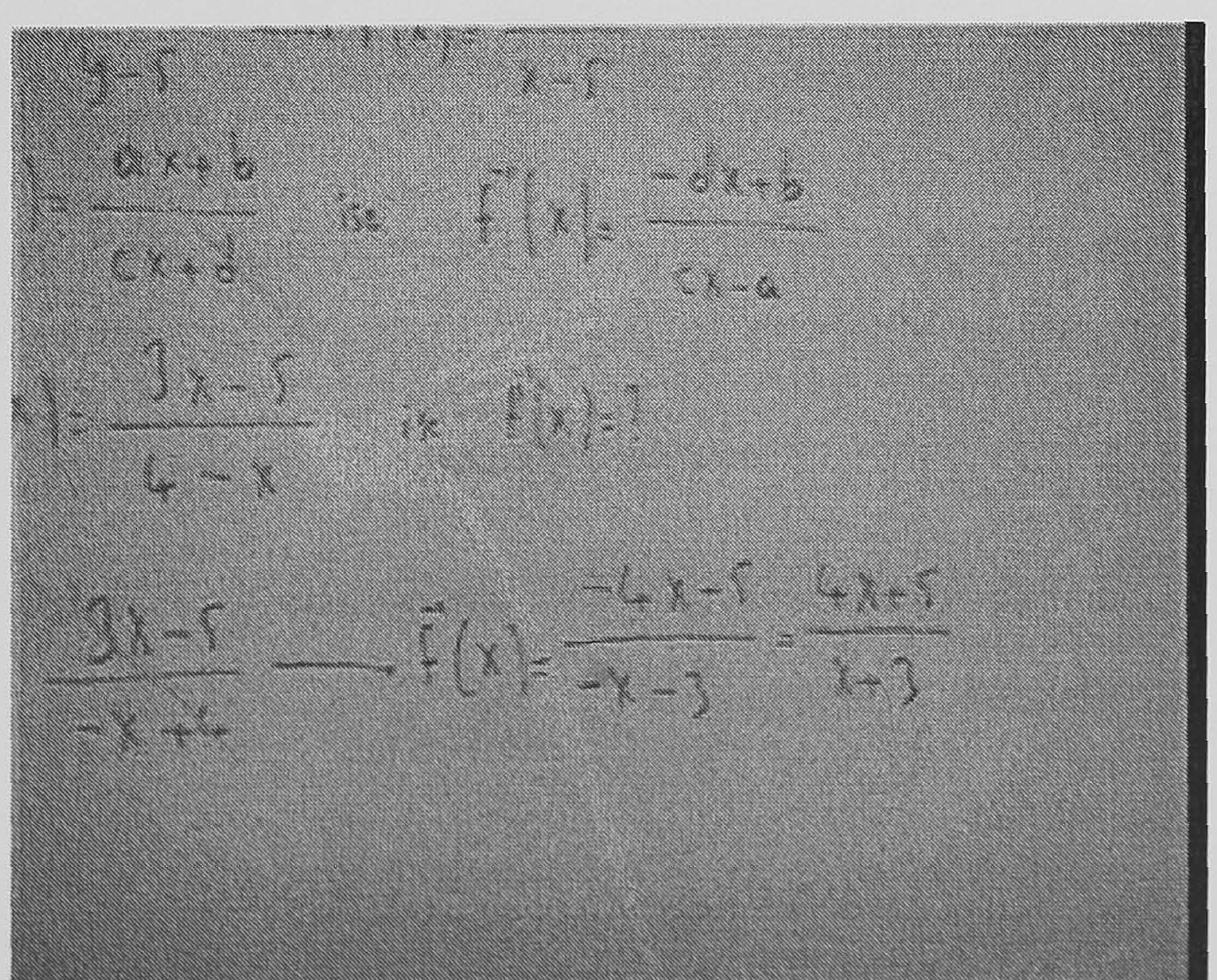
He added "*if you attempt to find f(x), your calculations get longer and longer. It is not wrong but longer. Don't forget this. I am not saying what you said is incorrect. It is correct, you can find f(x) then find its inverse but if you want to gain time you should try this (method)*" He asked if any students had questions about this and after receiving no question, Saban wrote the fourth focal point.

| |
|---|
| (4) $f(x)=\dfrac{ax+b}{cx+d}$ then $f^{-1}(x)=\dfrac{-dx+b}{cx-a}$ |

He said "*this is a special method if* $f(x)=\dfrac{ax+b}{cx+d}$, *the general rule is still correct* (pointing to the third point) *but this includes two x* (pointing to the expression), *the special method is that, without making any manipulation, the numbers on this diagonal* (drew a diagonal line from 'ax' to '+d' on the function) *should change place and their signs. So* $f^{-1}(x)=\dfrac{-dx+b}{cx-a}$. *The condition for this is that the terms containing x should be the first terms and one should be under another*" then he repeated his explanation (see box). He then gave an example (see box).

| |
|---|
| Ex: If $f(x)=\dfrac{3x-5}{4-x}$ then $f^{-1}(x)=?$ |

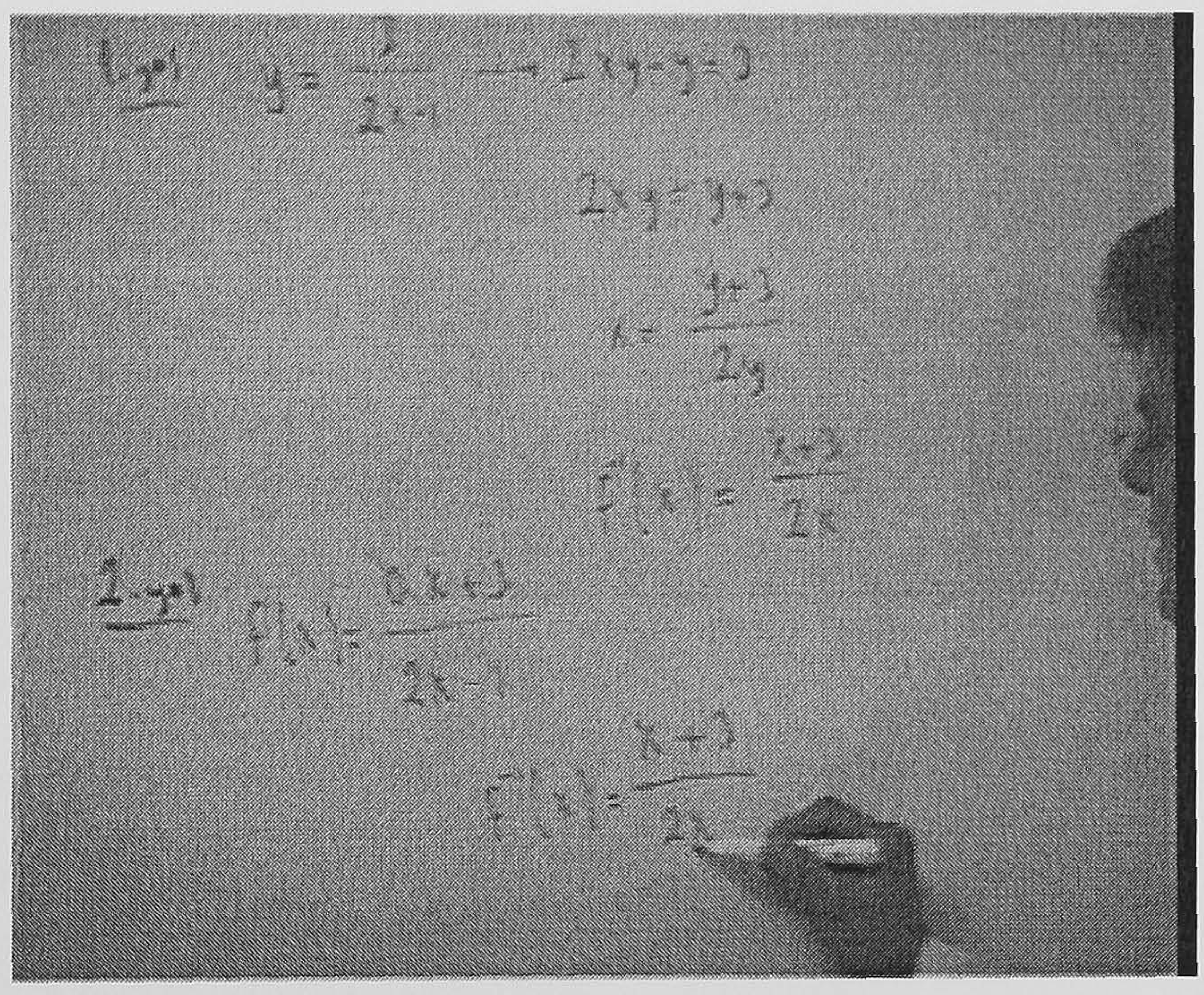
One student guessed the answer "$\dfrac{4x-5}{x-3}$". Saban said "*Look!* (pointing '4-x') *you should change the order of this*" and solved it by rewriting the expression in the denominator so that the term with x is the first term. Then he changed the number in the diagonal and found the answer.



He added, "*Don't forget that the coefficient of x in the numerator and the term without x in the denominator should change their places and signs…I will give you past examination questions on the next lesson.*" Saban then gave another example (see box)

| |
|---|
| Ex: If $f(x) = \dfrac{3}{2x-1}$ then |
| what is $f^{-1}(x) = ?$ |

He said, "*You could solve it by general method or you could solve it by the special method I just gave you*". Some of the students started shouting the answer out and he nodded. He also solved this example by two methods and he again made it clear by writing $1^{st}$ method and $2^{nd}$ method at the top left corner of each solution.



First, he used the general method and added, "*for this (kind of) example this (method) is troublesome, for this it is difficult. But if you use this all you have to do is because there is no term with x on the numerator you should write 0x.*" and he solved it by using the method he gave on the fourth point (see capture). He said "*xs should be the first and under each other. That's enough for today*". And the lesson was ended.

### 4.1.4 Saban's Beliefs

I have so far mentioned Saban's organisation of teaching. To explain how he organises his lessons in detail I will make use of an example he gave on factorisation during the interview. He said he teaches factorisation of $a^2$-$b^2$ as a sub-section under the title of factorisation. He solves many types of examples in that part and then goes onto the next sub-section, factorisation of $a^3$-$b^3$. It is his organisation of sub-sections based on his experience of teaching and the textbooks he examined. He evaluates this approach as having the disadvantage of letting students know what formulation to use for an example and therefore it being memorisation for the student. He aimed that at the end of the factorisation topic, students should know what factorisation approach should be used at any randomly given exercise. However, to him this approach has more advantages. He said "*I gave them different example types on factorisation of difference of squares, then students start realising to use difference of squares at some different example types if he has the potential to realise*". Interestingly when asked about the teacher's contribution, he stated "*to make solution methods memorised through solving many examples...when the student is able to decide which method to use for an example, it gets automated, became learned!*" The teacher did not think he is providing a theoretical base of mathematics but his sub-sections are built on the basis of different example types he has experienced in the past. Based on his comparison of his teaching with some other teachers, I made a table of his teaching style. If there are three sub-sections in a topic, he claims that he gives a number of examples and does not proceed to the next sub-section unless he feels it is appropriate to do so.

| Saban | Other teachers |
|---|---|
| Sub-section | Sub-section |
| Examples | Sub-section |
| Sub-section | Sub-section |
| Examples | Examples |
| Sub-section | Examples |
| Examples | Examples |

When he was asked how much time he spends on sub-sections he replied that he does not even make explanations in the sub-section but just writes. He stated,

> **T** From our point of view, they do not even need to know where the information on the sub-section derived from or its proof; he should only know what to do when he comes across such an example.
> **R** Isn't that mechanising?
> **T** It should do. To me it should
> **R** Is that the expectation?
> **T** The examination leads you towards this...

Saban seems to believe that what PC contributes to students is the "*use of mathematical information*" and that it is the schools that need to teach basic mathematical concepts. He also criticises the education system for making students have a tendency to put their efforts into getting better marks rather than learning mathematics and thus students are not coming to PC with basic knowledge of mathematics.

The written information on the board does not seem to be important for Saban. Its use for him is that it may make students remember afterwards since the mathematical knowledge will be written down as well as listened to. The ideal learning scenario for his students comprises students trying to learn the topic from the textbooks or university preparation magazines available on the market before it has actually been taught. This needs to be followed by careful listening to the teacher during the lesson and asking teachers whenever there is a problem with understanding the mathematics taught. It should be followed by solving quite a lot of examples. He believed that for the topic of functions it is useless if students do not solve 200-250 examples afterwards. He added, "*it becomes memorised if not practiced enough*". He believes that the students are different in their mathematical abilities and that some of the students are able to adjust the mathematical knowledge they learned to new type of problems and some others are stuck when the structural features of an example differ from their previously learned ones.

Saban used a water pipe metaphor to explain the role of the teacher. He believes that the teacher is someone who clears up the unwanted materials blocking water from flowing in a pipe. He states that there should be some water first, therefore students should have some mathematical abilities as a first condition and there is no use in clearing up the pipe if there is no water flowing. The teacher therefore has the job of dealing with the students' lack of understandings or misunderstandings, not "*pushing from behind*".

Saban also said that he provides the alternative solutions of an example. He presents the example and then presents the solution and then he talks about its variations by changing its structural features like replacing x with 2x or $x^2$. This will let students know what to do for differing types of examples and, to him, this is what they (PC) are aiming at. Saban is trying to make his students flexible about variations of examples and the school does not provide this. He gives the example of a constant function:

**T** All the books define constant functions as *f(x)=x* is a constant function. I also teach this but I add some more to that as a note. No matter what the term [or function], inside [of the brackets] will be equalised to the outside [of the brackets] and I provide an example and I explain through that [example]. There are two ways. You can assign a numerical value to x and [calculate] the inside [of the brackets] and then equalise this to the outside [of the brackets]. Did you get 2 then equalise to 1, did you get 2 then equalise to 2 or [the second way] equalise the polynomials. I give this two ways but books don't mention that and students suppose that a constant function is always equal to *x*. I add that if it [inside of the brackets] was *2x+1* then it [the image] should be *2x+1*.

Saban also provides another example on coefficients of polynomials to make his point. He claims that he provides students with "the opportunity to be able to think more flexibly" by providing them examples that will contribute to their understanding of mathematics. He believes that many students are unable to solve problems if they are not given different types of examples.

He also stated that what he provides his students with mathematical knowledge is directed as well as constrained by the past university entrance examinations. When prompted for why he was providing different representations of functions (for example with diagram and number pairs), Saban clearly stated that he provides this information since there are past university entrance examination (UEE) questions that require students to know different representations of a function.

**R** You gave two sets like A, B and then assign values in set A to set B. And also as pairs like *(1,a) (2,b)* etc. Also you present a function such as $f = \frac{(x+1)}{(1+x^2)}$ What is the aim of [teaching] it?

**T** Well firstly it is because there are examples of past university entrance examination questions. If it wasn't in the past examination then I would not teach these and that's my first aim. Secondly, to make it more visual because you can switch one another. I mean if a student is asked one representation (of a function) in the university entrance examination but given another representation, the student will be able to switch. There isn't much other reasoning behind that

**R** Hmm...

**T** To be more precise we depend on the university entrance examination. I mean, we try to provide our students how to solve questions in the examination at 80 percent. For the remaining 20 percent, students should make use of different textbooks.

When asked if their teaching is being determined by UEE, Saban utters "*Unfortunately!*" He answers the question of why as "that's the rule! If it wasn't I would teach differently".

Saban seems to have a perception of state school (SS) mathematics as being heavily dependent on theories and proofs. He stated that if he was in SS, he would not "*exhaust*" students by proofs but he would present the example at the level of students in the class. He added, regarding PC lessons:

> **T** Because the questions of university entrance examination do not change [from student to student], I have to teach the same things to everybody...I don't change what I teach from one classroom to another but maybe some examples...I don't give easy examples in [mathematically] good classrooms

He therefore presents the same mathematical information in different classes but varies the difficulty of the examples across classes, depending on the competence of the students in them.

I found out that some teachers are making use of metaphors in explaining especially the definition of the concept of function at a level that students can grasp. I asked if he uses any; he stated that he does not use metaphors in the classroom to explain mathematical content[6] to his students and does not find them useful.

With regard to the purpose of examples in mathematics, he comments:

> **T** It is the use of mathematical knowledge. I mean, to make use of the information they memorised should be used and it can be used with an example. To be more precise they can try their knowledge on an example. Without examples how will I know what they know? Without examples, the knowledge will remain as memorised. Isn't it that children need to practice the information they memorised?
> **R** Yes.
> **T** And that practicing is done with examples.
> **R** Then the example you give on the board
> **T** Application...application

Saban regards encouraging students to go through the examples he gave during his lessons as pointless. He expects his students to have some motivation that originates

[6] The term 'content teaching' or 'content' is translation of Turkish expression 'konu' or 'konu anlatimi' and it refers to presentation of theoretical information in mathematics lessons without exemplification.

from themselves, rather than encouraging them to study himself. To him. that motivation should naturally come if students are attending a PC. Otherwise, simply "*it is not much of a concern to me, if they study, it is good; if they don't they come to a PC again next year*".

He draws upon his teaching experience when he is asked about the arrangement of examples in his mind.

> **R** Do you have a sequence in your mind for the examples you give?
> **T** I have a sequence [in my mind] and I present with that sequence.
> **R** How did you decide that sequence?
> **T** Entirely up to me and I consult a number of books...I sequence examples in line with sub-topics I teach.
> **R** Hmmm...
> **T** In the first year of my teaching, we were given a seminar and there is an organisation [and order] of topics. That's already in mind...in the notebooks to be more precise. I also studied sequence of topics using a number of different magazines. In the second year I study the sequence again from different sources. In the fourth year I made myself teaching notes. In the fifth year another notes and that is enough for 6th, 7th and 8th years.

With regard to use of examples, he commented that he does not give time for students to solve them in the first few examples but maybe 1 or 2 afterwards. Moreover, Saban did not seem to be interested in the individual needs of his students. During the interview he stated that:

> **T** I have to make good use of time. I have to use the time given to me. If I deal with each student I could barely solve 10 questions in a 40 minute lesson, because you will present the example and then wait for them to solve, and this takes 1 minute then I will solve, which takes 2 minutes. Then I will wait till they copy it down in their notebooks. This makes 3 minutes.[---] If there are some special (e.g. difficult) examples, I may wait a little bit.

Saban disapproves of the possible teacher question 'did you understand it?' He would rather ask, 'do any of you have a question about this?' as he thinks that even if students do not understand, they would not dare to state it publicly.

I attempted to explore Saban's reasoning behind presenting different types of solutions for an example. He explained his reasoning in a substantial amount of time. In doing so he gave two real life examples of himself. His first reaction to my question was that "*in order students not to memorise*" He goes on to say that:

**T** I present different ways (of solving an example), so that he can solve in different ways. None of the examples have single solution. So that he can search for different ways. Isn't (our mission) opening their horizons...(What happens) if I solve (examples) in just one way and make them conditioned to one way. Some examples are solved much more easily if using a different (from usual) way. A student should adopt one of these ways so that when he comes across an example he won't think about which way to go but he will use that way straight away.

There needs to be made a distinction between memorising formula and memorising a solution method. Saban does not make this distinction himself clearly but when he talks about them he uses the same Turkish phrase "*memorise*" with two different meanings. Moreover, considering his overall approach these aspects of students' learning, one can observe that he has different positions towards these two. He disapproves of stating the mathematical knowledge and leaving it as it is and therefore left it *unpractised* (in his words "*memorised*"). On the other hand, he thinks that students should know different ways of solving an example, but they should *stick to* (in his words "*memorise*") one way when they attempt to solve an example.

**T** Unless a student solves 50, 60 may be 100 examples using the ways I taught, he would go for their way. (In that case) It does not matter even if I teach them 10 ways. If someone doesn't think that there are more ways to solve than a single one, then it is hard for him to find a second way during the examination. I mean my aim is to prevent memorising and opening them to different approaches.

Saban thinks that if a student has a potential and given opportunity, he can extend his thinking. He made this point by two examples. I will present one of them: he came across the expression 'virtual line' in a book. He tried to make sense of what 'virtual line' could mean in mathematics and someone else suggested that equator could be a 'virtual line'. Therefore he extends his thinking by suggesting that it does not necessarily have to be thought about in terms of mathematics, and 'virtual line' could be thought about in terms of science. Saban thinks of meridians as 'virtual lines' after this suggestion. Although nobody tells him the meridian is a virtual line he can infer from information available to him. Likewise Saban thinks that students with potential should be given the opportunity to extend their thinking and presenting different solution methods is a way of achieving this.

**T** If one has potential and when you open up a door to him, he can think of varieties like braches of a tree then he can generate different ways.[---] In considering 'virtual line', I always thought of something in mathematics. I never managed to get out of that *template*.

**R** So are you trying to get students out of that template?

**T** (Teacher nods to mean yes). It is not right to memorise one way and try to use the same template for all the examples. (Students should say to himself) There must be some other dimensions, other ways. Students will decide which (way to use) not me. I will only show him 1 or 2 examples.

What was interesting to hear in the interview is Saban's answer to my question of whether he is selecting examples to be used in a lesson beforehand. He used the word 'template' again to describe his selection process. He referred to his preparations for teaching in his first few years of teaching. He used to go through the exercise books on the market designed for UEE preparation in his first 3 or 4 years. He made groups of examples and during teaching he used to select from these groups to use in classrooms.

**T** After the fifth year example templates have been constructed (in my mind). There was no need to select any more. I generate examples that fit into these templates...I have my grouping in my mind. I only change the numbers and put the numbers into the template (and use it in the class)

He also said that he used to write the examples down among his notes in the past, but not anymore. Unless he saw an 'original' example in a source (book, quiz, mock examination), he commented that he does not make notes of examples prior to a lesson. He wrote down the examples that are 'original' to him and accumulated a collection of examples with time.

The question of how he knows that the number of examples he gives is enough for his students has been answered by Saban as "*intuition, depends on classroom*".

I asked him if he gives examples with specific form (like multiple-choice, open ended etc.) and if he presents examples with possible options. Saban answered, "*We don't give options generally except for some examples that options can be used to solve it*". When I inquired what makes an example that requires options to be presented, Saban gave me an example:

**T** A man climbs up a set of steps by moving 3 steps in each move and he climbs down by moving 2 steps in each move. We know that he made 6 more moves in climbing down than climbing up. Then what is the number of steps in this staircase?

He commented that for such an example, students can use the options and try calculating how many moves the man does in climbing up and down and then calculate the difference so that he can compare the result with the given in number in the root of the example. He said:

> **T** For instance let's say 30 is one of the options, if he moves 3 by 3 it makes 10 movements and climbs down 2 by 2 and it makes 15 moves. The difference is 5 then it is not right (option). For 36 if he moves 3 by 3 it makes 12 movements and climbs down 2 by 2 and it makes 18 moves. The difference is 6. If the student will use options like this I would present it. But if there is no need for options to solve the example I don't give the options.

When I asked if that is a healthy method, Saban interrupted me *"that saves time in the university entrance examination...if the students know in which questions they can use the options, they will save some time in the examination*". I tried to keep my initial question and rephrased it again and asked if he thinks it is appropriate in terms of mathematics education. He said "*I think it is quite appropriate*" after a short pause he referred to the meaning of the phrase 'mathematics education'[7] and commented:

> **T** Does mathematics education refers to training of the brain or is it just teaching mathematics? It isn't appropriate for (mathematics) teaching but it is a way. It is a way for the students to use their brain
>
> **R** How do you use numerical value technique, could you explain a bit?
>
> **T** Well. I present the options and say if the question has a variable and if it its value is not asked, that variable can be given any value. I make them write this [rule] down. After this I give 8 to 10 examples. For instance, this [numerical value technique] can be used [on questions] in [the topic of] exponential numbers, in [the topic of] functions, [in the topic of] equation construction problems, in [the topic of] divisibility and I present numerical value technique as a second way of solution, where necessary.
>
> **R** Well in order to use numerical values
>
> **T** The student should have a potential and the options should be presented
>
> **R** Hmm. Do you approve of it?
>
> **T** Let alone approving of it, we clearly advise it.
>
> **R** You *advise* it?
>
> **T** Because in the university entrance examination, more or less 10 to 15 questions are asked like this.
>
> **R** 10 to 15 questions out of 45 is one third or one quarter.

[7] The phrases 'mathematics education' and 'mathematics teaching' does not correspond to the meaning of the phrases that were used in original interview in Turkish.

**T** Yes, for those who have got potential there are questions that can be solved by numerical value.

I asked Saban about the tools he uses in his lessons. He commented that he uses "*only quizzes and rarely a book...a question bank*".

Saban explained that he presents the summary of the mathematical content and then uses quizzes. However, the use of quizzes changes depending on the student's level of mathematics. In the best classes he hands out the quizzes before the actual lesson begins and then starts presenting the mathematical content. To him this has an advantage. For those students who are familiar with the topic he teaches, it is time for dealing with the quiz rather than listening to the teacher while he explains something they are already familiar with. For those students who do not know the topic, they can listen to the teacher and deal with the quiz after the lesson. In mathematically average classes, he hands out the quizzes and then presents the mathematical content and starts solving examples from the quiz. When I asked Saban why he uses quizzes in this way, he remarked "*to save time*".

I then explored his ideas about the textbooks he uses. He commented that he uses question banks. He stated that he used to use the books with some summary of the theoretical information along with exercises targeted for the examination, he said, "*after the 4th or 5th year (of teaching mathematics) one would not need books with some content*". In Turkey there are also textbooks that have been approved by the Ministry of Education and used in state schools. These textbooks have a lot of theoretical information including proofs for some theories. When I prompted his approach to these books we had an interesting conversation:

**R** Then you use only question banks. How about textbooks approved by Ministry of Education? Do you use them?
**T** No. They aren't important for us.
**R** Why don't you use them?
**T** Why should we?
**R** Aren't they any good?
**T** I don't scrutinise them.
**R** Hmm..
**T** I never examined one.

He does not seem to have any intention of examining one. He confirms that he has some Ministry of Education approved textbooks but he said that he is not "*idealist*" and he used to go through all the mathematics textbooks in the past but he became fed up with it. Saban also did not like the fact that there are many examination preparation textbooks on the market, and many of which he thought are useless. He said:

> **T** It would be good if you manage to examine and follow one Ministry of Education approved but they probably have heavy theoretical information and proofs in them. But I never get into that.

Saban has different ideas about the benefit of using quizzes for the student and for the teacher. In his words "*ideally, they should be designed to help teachers*". To him teachers generally do not provide enough number of examples during their teaching if they do not have a strong memory. In this case, he thought, quizzes help. If properly designed, quizzes can go along with the mathematical content the teacher is teaching. For example, after explaining the definition of function, the teacher can refer to the quiz he hands out and can solve the first 3 questions related to the definition of function. And the teacher can consider the fourth question and realise that it is related to the constant function and can start teaching the constant function. This is how quizzes ideally can be used by a teacher from Saban's point of view.

> **T** You may or may not have examples in your (the teachers) mind. Rather than copying them down from another source and students also struggle (trying to write examples down). No need it is ready made examples. You already handed it out all you have to do is to say let's have a look at question number 2.

The help of quizzes to the students is that they experience different types of examples. With regard to the importance of the use of different types of examples, he stated that it is important for the student to know "*what to do for each type of question*"

When I prompted further explanations, Saban made comments on the significance of quizzes to PCs. To him "*this is a tradition of PCs. It so much a tradition that PCs even advertise themselves as giving 500 quizzes for their students*". He regarded these one paged question papers as something that is central to teaching mathematics in PCs. PC teachers even get some negative feedback from students if they do not hand out quizzes. If teachers did not give enough quizzes, students would react to that.

> **R** Do you get any reaction from your students if you do not hand quizzes out?
>
> **T** Yes we do get reactions. This became a convention here. All PCs give quizzes. Let me put it this way: I give 3 quizzes and some other teachers give 1 quiz and there is even a reaction to that. Why I did give 3 and others didn't?

**R** Is it that students react to that by talking to the PC manager?
**T** Of course.
**R** Can we then say that it is an inevitable part of PC mathematics?
**T** Hmm.yes. It became so. It became inevitable somehow...Students have a lot of textbooks. In each you there are 100, 200, 500 questions for each topic and students yet ask for a couple of quizzes. That's what I think. But they are still inevitable part of PCs that's right. We give ...hmm...We have to give.

Saban accepts quizzes as being a part of PC culture and yet Saban did not seem satisfied with the quizzes at hand. In explaining so he also gives some clues to the ideal teaching scenario from his perspective.

**T** ...from my point of view quizzes are not functioning properly. In PCs they are regarded as something to use no matter whether it is good or not. Again from my own perspective like this...How do I teach functions? First I give the definition of it. Secondly, I give (function) f within (function of) g. Thirdly, I give types of functions like constant function, identity function, inverse function and then composite function. I follow this sequence. It would be good if quizzes contain 5-6 examples for each sub-section I teach and if it goes alongside with my teaching sequence. What happens is rather than me giving all examples, I hand out the quizzes and then explain the definition of function and I give an example from me then if the quiz has 5 examples in it I solve 3 of them and let them do the other 2. Students immediately get used to it (solving examples on definition of functions)...then I don't get the heavy load. If the quiz is sequenced with the topics then you can give the load to the student and quiz...There are some good quizzes and some bad ones. (For instance) there are 5 questions of an unnecessary example type (that is not like UEE questions), but there is only 1 question for the type of example I teach.

As can be observed in the extract, Saban has his own specific sequence of teaching in his mind. His perspective on an ideally constructed quiz is that it follows his teaching sequence and that the quiz contains questions that exemplify his personally constructed types of examples. For each type of example there needs to be 5 questions and this will help him to teach and it will also save time for him as well as for students. To him this will take the load from the teacher to the students.

I asked if Saban makes any preparation prior to his lessons. He said that he did not make preparations in the last 3 years despite the fact that he acknowledged the need for it. He explained what he used to do in his first few years with some enthusiasm:

**T** I used to make preparations in the past. I would go through all the questions in the books, I used to make lesson notes for each topic again and again. I would think about my teaching sequence and I would reconsider my examples and I would select each example I will use in the classroom one by one. I am fed up with it now. You will ask me how did it happen in just 8 years. Well, it happened somehow

As we can see along with some previous extracts, Saban has a knowledge and experience that has accumulated throughout the years. He simplified the mathematical content he supposes to teach and by going through textbooks and question banks he prepared himself a knowledge base for teaching mathematics in a PC. This consisted of different example types along with a summary of the mathematical content he needs to teach. However, practising teaching the same topics to all classes seems to make his teaching a routine job. After 8 years of experience, he uses his experience to make teaching decisions in his lessons.

I asked Saban the purpose of PCs and SS. He stated that the only purpose of PCs is to prepare students for UEE and PCs organise everything to maximise student achievement at UEE including teacher selection. As to the purpose of SSs, he stated that they aim at educating citizens. My attempt to delve into this resulted in his criticism of the education system of Turkey in general. Although he was out of my scope, one of his comments is significant to my attempt to explore his perspectives. He thinks that the main problem is lack of having a specific target at the level of the teachers as well as at management level. This is another place where I felt challenged to bring him back to the context of classroom.

I prompted him to compare SSs and PCs in terms of what happens in the classrooms. He firstly stated that there used to be a difference between these two institutions in terms of number of the students in a class. He stated that this difference is not that wide now. Secondly, he commented,

> **T** unfortunately the quality of teachers in SSs are not as high as those in PCs. This is because old teachers are not renewing themselves and not adding new knowledge to their existing knowledge base. Because they don't have a target (like PC teachers do)[8]

What is interesting is when this extract is put side by side with one of the previous extracts where he said "*I am fed up with it now. You will ask me how did it happen in just 8 years. Well, it happened somehow*". He either did not realise this seemingly contradictory nature of these comments or he felt that he renewed himself by adding original examples into his old knowledge base of examples and felt that that is enough

[8] This is of course his value judgement and I just quote what he stated.

for a teacher to refresh his knowledge. As a researcher who conducted the interviews personally and spent quite significant time socialising in and out of his classrooms with this teacher, I tend to feel that he subscribes to the latter explanation.

He seems to believe that SS teachers have to teach whatever is in the textbook, but in PCs, teachers have the flexibility to organise the information differently. He also saw some difference in emphasis in PC and SS teachers.

> **T** There is also a difference in terms of content teaching and explanations. In SSs, mathematics is 5 lessons in a week and it goes pretty slowly. It does not matter if the teacher solves 3 examples or 5. The teacher teaches what is written in the textbook. However, it isn't like this in PCs. Summary of (mathematical) content should be presented as brief as possible. And then students should be directed to exercises. Therefore PCs puts more emphasis on practising.

Saban then reflected on what he saw as a difference between teachers of two different types of institutions and summarised his main points as: the number of students (in a classroom), teaching, quality of teachers and then he added "*psychology of students*". He explained that when students go to PCs they have the aim of passing UEE with a good score but they do not have the same perception when they go to SSs.

## 4.3 The Case of Ayten

### 4.3.1 Background Information

Ayten is a veteran state school mathematics teacher. She had over 30 years of experience. She worked 3 years in basic education (6-14 year olds) and 27 years in secondary education (15-17 year olds). She was thinking about retiring at the time the research was carried out. Similar to many of her colleagues in the same school she was teaching 20 hours a week. When I explained the research project she willing to let me video record her lessons for a month. At the same time, I observed that she had hesitations on being interviewed. Over the first three weeks I explained my intentions and the purpose of the data collection during the lesson breaks. I socialised with her during the lesson breaks and talked about the students and her teaching. Having realised the purpose of the research project she indicated her willingness to be interviewed and scheduled the time for the session.

### 4.3.2 Organisation of Ayten's Teaching

I video recorded Ayten in 16 lessons in which she completed teaching functions. Based on her teaching practice as well as her lesson notes that I have obtained from her, I observed that her organisation of teaching functions was divided into sub-sections such as; 'Definition of Function', 'Constant Function' and 'Composite Functions'. These sub-sections may last more than one lesson including examples. The organisation and the order of these sub-sections are in agreement with the syllabus. The order is:

1-Definition of Function;

2-Types of Functions (1-to-1, constant etc.);

3-Inverse Functions;

4-Composite Functions.

Throughout the topic of functions a pattern emerged from her practice. For the *Content* parts, she regularly wrote the theoretical information on the board and gave explanations. Her explanations were followed by, and many times overlapped with, passive examples. Having completed her explanations, she gave students time to write their notebooks. The content and subsequent passive examples were followed by active examples on a regular basis. Having solved the first few active examples, Ayten then let the students solve active examples she presented. Ayten's emerging practice pattern for each subsection is shown in the Figure 4.3.

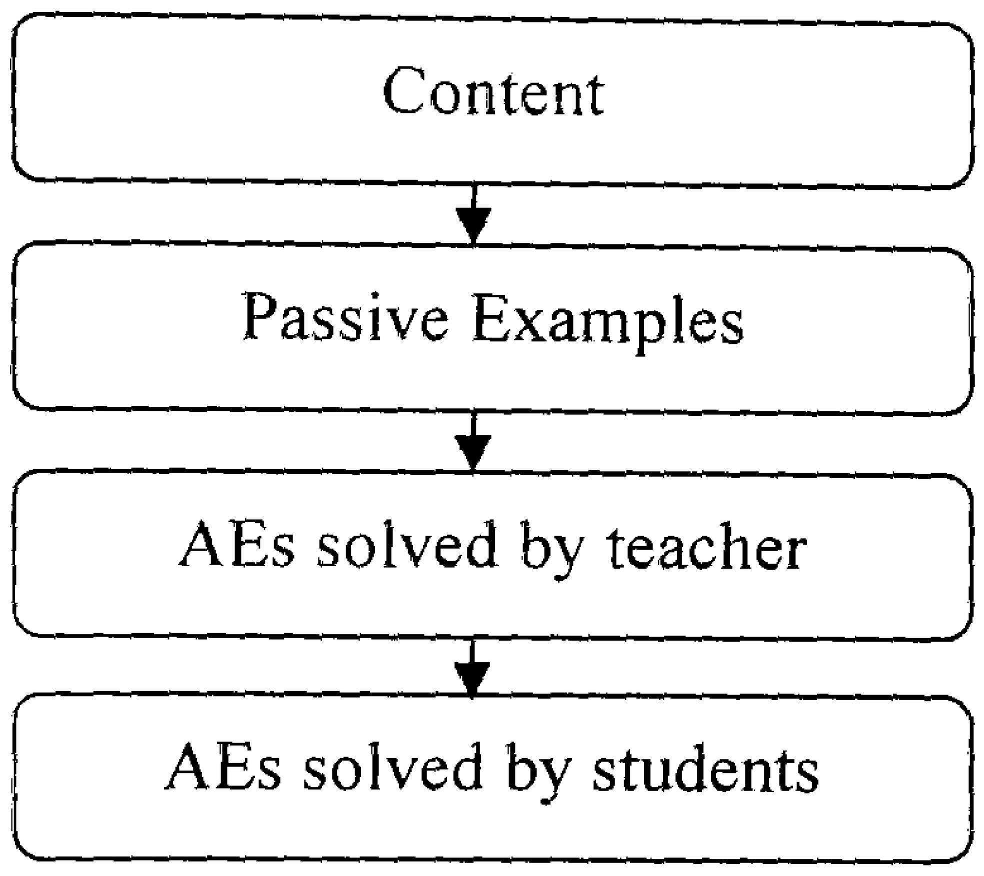



**Figure 4.3** Ayten's practice pattern.

This pattern starts with each new sub-section and makes a cycle throughout the topic a number of times. During content and use of passive examples, Ayten stays at the front of the classroom and makes eye contact with almost every student in the classroom. Ayten allocates most of her lessons to observing individual students' mathematical practices in their notebooks throughout the active examples. This pattern can be clearly seen in the following section, where I present a description of one of her lessons. The organisation of Aytens's practice in teaching functions is presented in Table 4.9.

| | **Lesson Code** | **Content** | **Passive Examples** | | **Active Examples (PEQ.)** | |
|---|---|---|---|---|---|---|
| | | | Number | Time | Number | Time |
| 1 | Ayten-1 | 4m 28s | 9 | 4m 9s | 3 | 27m 54s |
| 2 | Ayten-2 | 6m 15s | 9 | 4m 48s | 6 | 27m 7s |
| 3 | Ayten-3 | - | - | - | 9 | 39m 55s |
| 4 | Ayten-4 | - | - | - | 6 | 39m 16s |
| 5 | Ayten-5 | - | 1 | 3m 13s | 3 | 33m 59s |
| 6 | Ayten-6[1] | - | - | - | - | - |
| 7 | Ayten-7 | 12m 58s | 9 | 2m 55s | 4 | 18m 7s |
| 8 | Ayten-8 | 4m 49s | 7 | 2m 6s | 4 | 24m 35s |
| 9 | Ayten-9 | 4m 20s | 5 | 2m 54s | 8 | 32m 27s |
| 10 | Ayten-10 | 4m 48s | 5 | 2m 44s | 4 | 22m 39s |
| 11 | Ayten-11 | - | 3 | 2m 48s | 8 | 41m 50s |
| 12 | Ayten-12 | 4m 11s | 2 | 2m 48s | 5 | 23m 26s |
| 13 | Ayten-13 | - | 1 | 1m 13s | 9 | 42m 58s |
| 14 | Ayten-14 | 4m 19s | - | - | 10 | 34m 35s |
| 15 | Ayten-15 | - | - | - | 4 | 26m 46s |
| 16 | Ayten-16 | - | - | - | 7 | 38m 43s |

**Table 4.9** Summary of Ayten's practice.

The table suggests that Ayten uses passive examples extensively throughout the topic. She also solves a number of active examples. Ayten utilised the passive examples to help students understand the theory she presented in the content segment. Table 4.10 presents a summary of content segments in Ayten's lessons.

[1] In this entire lesson Ayten checked students' notebooks to see if they have done their homework.

| Lesson | Code | Content |
|---|---|---|
| 1 | Ayten-1 | • The definition of Function (2m 43s)<br>• Domain and range sets (1m 45s) |
| 2 | Ayten-2 | • Condition of being a function (3m 48s)<br>• Formula for calculating number of functions (2m 27s) |
| 3 | Ayten-3 | |
| 4 | Ayten-4 | |
| 5 | Ayten-5 | |
| 6 | Ayten-6 | |
| 7 | Ayten-7 | • 1-to-1, onto, permutation functions (6m 33s)<br>• Constant Function (3m 18s)<br>• Identity Function (2m 7s) |
| 8 | Ayten-8 | • Inverse function (2m 51s)<br>• The procedure of $y = f(x) \Leftrightarrow x = f^{-1}(y)$ (1m 58s) |
| 9 | Ayten-9 | • Finding Inverse of a function (1m 41s)<br>• Inversing the functions of the form ax+b (1m 4s)<br>• Conditions for $f(x) = f^{-1}(x)$ (1m 35s) |
| 10 | Ayten-10 | • Inversing the functions of the form $\frac{ax+b}{cx+d}$ (4m 48s) |
| 11 | Ayten-11 | |
| 12 | Ayten-12 | • Inverse of Permutation function (1m 11s)<br>• Composite functions (3m) |
| 13 | Ayten-13 | |
| 14 | Ayten-14 | • Properties of composite functions (4m 19s) |
| 15 | Ayten-15 | |
| 16 | Ayten-16 | |

**Table 4.10** Summary of the content segments in Ayten's recorded functions lessons.

As can be observed in the table, some of her lessons are devoted to solving active example and therefore contain no content segment. In such lessons, she solved some examples herself and she then asked students to solve the example. In fact, in order to encourage students to solve examples on the board, she gave high grades to students who came to the board to give a solution. Table 4.11 shows the distribution of the examples used by Ayten.

| | Percentage | Number of Examples |
|---|---|---|
| Passive | ~32 | 42 |
| Active | ~68 | 90 |
| **Total** | 100 | 132 |

**Table 4.11** Distribution of examples used by Ayten.

The distribution of the active examples solved by Ayten and the students can be seen in Table 4.12.

| | Total | Solved by Ayten | Solved by students |
|---|---|---|---|
| Active Examples | 90 | 45 | 45 |
| **Percentage** | 100 | 50 | 50 |

**Table 4.12** Examples solved by Ayten and students.

The table indicates that out of 90 examples presented, 45 were solved by students. That is to say, half of the active examples were solved by students and that shows a high level of participation of students in her lessons. The next section presents one of her lessons in detail.

### 4.3.3 Aytens's Teaching

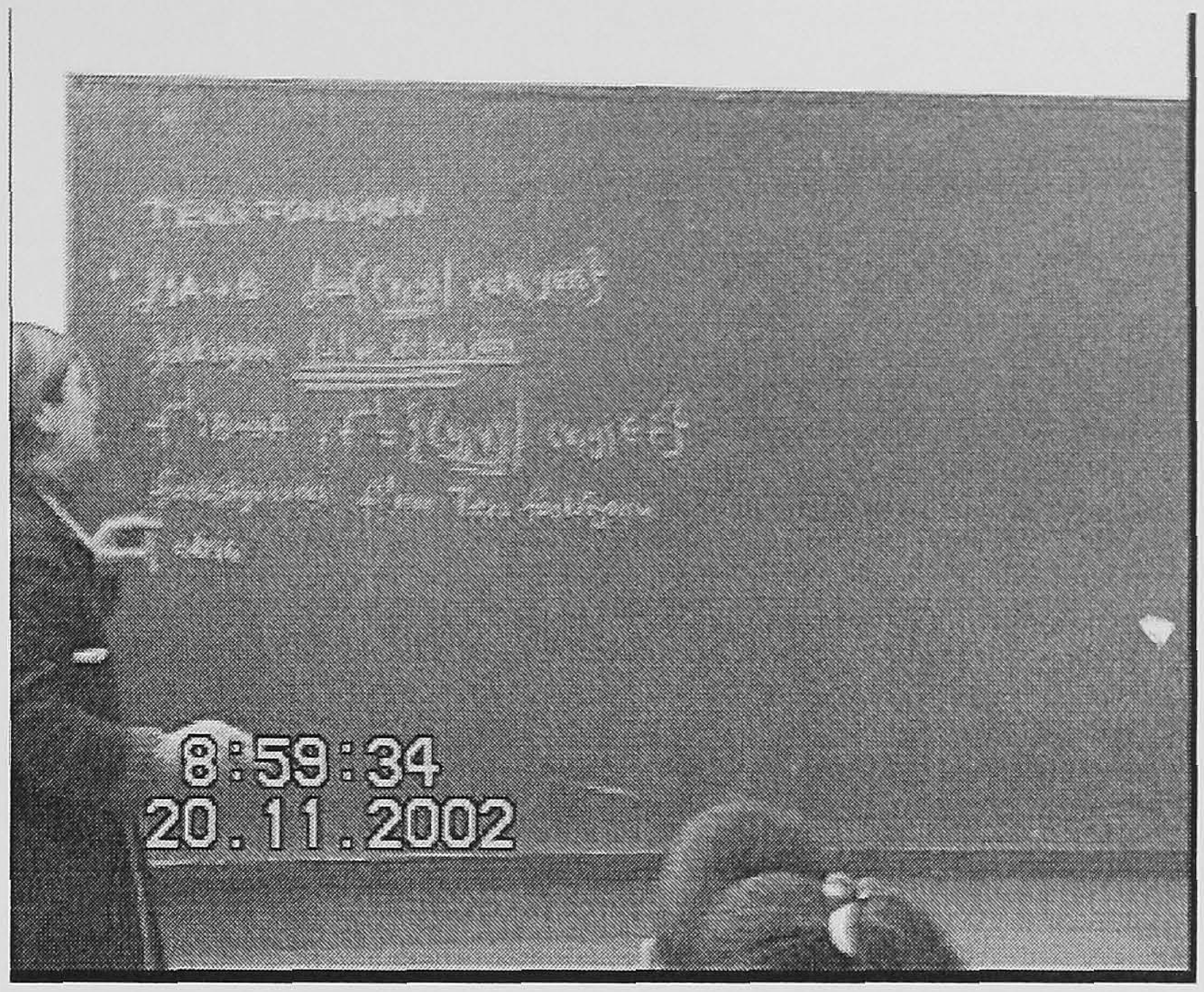
Lesson Code: Ayten-8

Before describing any teaching/learning activities during the lesson, I would like to present the main phases of the lesson as a summary. This is intended to help the reader to visualise the overall flow of the lesson.

- Ayten explained what an inverse function is. She wrote it on the board and made sure students copied it down into their notebooks. She explained the main points of inverse functions by two simple examples. She demonstrated expected student actions herself.
- Ayten explained a simple manipulation of [given function f(x), if $f(x)=y$, then $f^{-1}(y) = x$ ], the importance of which she emphasised greatly. Again she wrote it on the board and made sure students copy it down into their notebooks. She explained it further on five simple manipulation practices. She herself demonstrated expected actions.
- She then put two examples on the board which required the use of manipulation along with other processes and this time she waited for some time for students to solve them, then she solved them with the students.
- She then wrote two more examples, which again required manipulation of functions. She waited for students to solve them, but this time she waited quite a long time and she dealt with students' individual needs by walking between desks and checking their progress on their notebooks. She asked a student to solve the problem on the board 8 minutes after the example was presented on the board. She put a clue for solving the second example on the board. She asked a student to solve the problem on the board 16 minutes after the example she presented on the board.
- She then wrote three more examples in the last few minutes of the lesson and waited for students to solve them. The bell rang and they are left unsolved.

The lesson started with an issue from the previous lesson. Ayten was annoyed about one student's behaviour. She held her finger upwards and kept looking into his eyes for some time seriously. She said "*Don't reply me! When I say keep quiet you can't* (she made coughing sound). *Please, don't. This is a classroom, not a place you can behave however you like.*" After that intense moment she reminded the class that they had completed function types and now they will learn inverse functions. She wrote a formal definition of the inverse function on the board.

INVERSE FUNCTION
A function f: A→B, when
f:{ (x,y) | x∈A, y∈B}is 1 to 1 and onto,
The function $f^{-1}$:B→A
$f^{-1}$:{ (y,x) | (x,y)∈f} is called the inverse function of f.



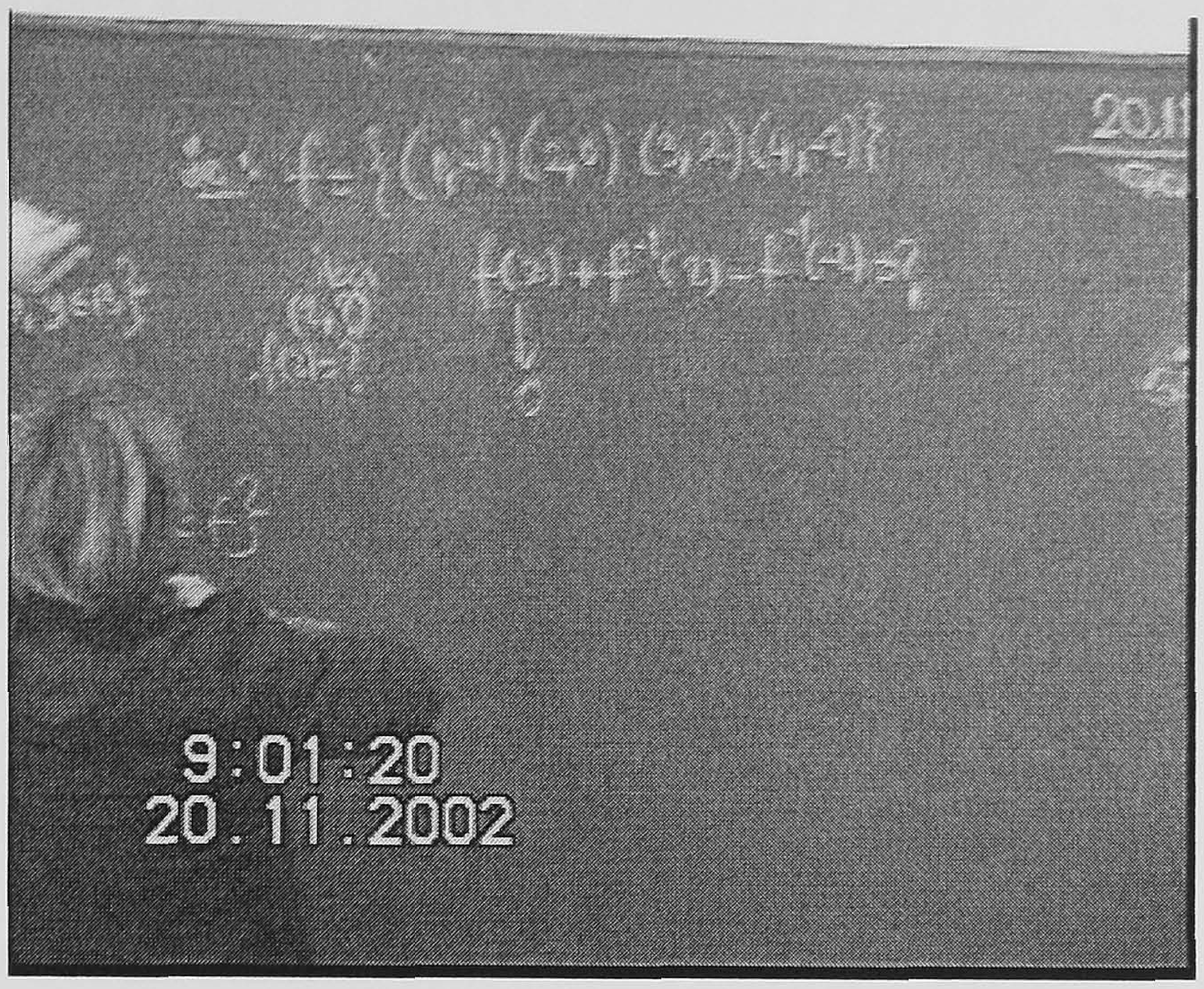

She then said, "*this is the story part, it is obvious that you did not understand this.*" She then explained two points: "*I want to emphasize two points. A function has to be 1 to 1 and onto in order to be inversed*" and "*second point, while f contains (x,y)'s the inverse function contains what?...*(she waited here for students to respond) *(y,x)'s where x and y have changed their place*". She underlined 1 to 1 and onto and *(x,y)* and *(y,x)* on the board. Ayten then wrote a function as an example:

f:{(1,a),(2,b),(3,c),(d,4)}

She expected students to tell her the elements of the inverse function. She said, "*would you tell me the elements of the inverse of f*" under that she wrote $f^{-1}$*:{(a,1)(b,2)(c,3)(d,4)}*. For each element, students told her the elements of the inverse. Just after finishing that example, Ayten wrote another:

f:{(1,-1),(2,0),(3,2),(4,-2)} what is
$f(2)+f^{-1}(2)+f^{-1}(-2)=?$

She saw some student were not paying attention to the board but writing down notes and said, "*look, please just listen, don't write!*". She said "*f is given, what is the meaning of this* (pointing f(2) )" After a moment she gave them a clue by asking "*when x is 2, what is y*". She pointed to *(2,0)* on the function and put downward arrow under *f(2),* wrote "*0*" and asked "*did you see? Did you understand?*"




Although students said "*yes*" altogether, she repeated her explanation one more time. This time she wrote *f(2)=?* means *(2,?)*.

| |
|---|
| f(2) = ? means (2,?) |

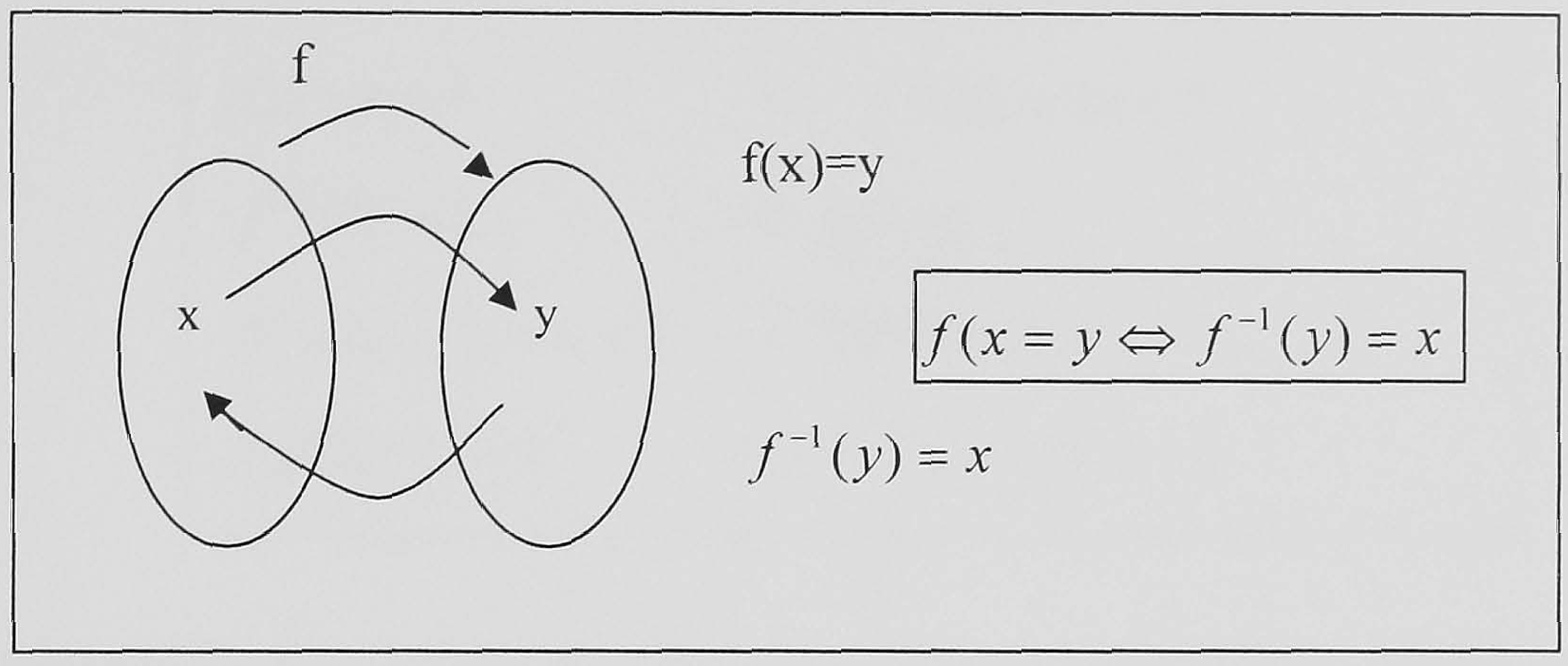
To obtain the value of $f^{-1}(2)$ she wrote the elements of the inverse function under *f*, and wrote *3* under $f^{-1}(2)$ and *4* under $f^{-1}(-2)$. She added, "*Let's make note of this*".

| | |
|---|---|
| f:{(1,-1),(2,0),(3,2),(4,-2)} what is | |
| $f^{-1}$:{(-1,1),(0,2),(2,3),(-2,4)} | |
| f(2)+ $f^{-1}$(2)- $f^{-1}$(-2)=? | f(2)=?means (2,?). |
| $\downarrow$ $\quad\downarrow$ $\quad\downarrow$ | |
| 0 $\quad$ 3 - $\quad$ 4 = -1 | |

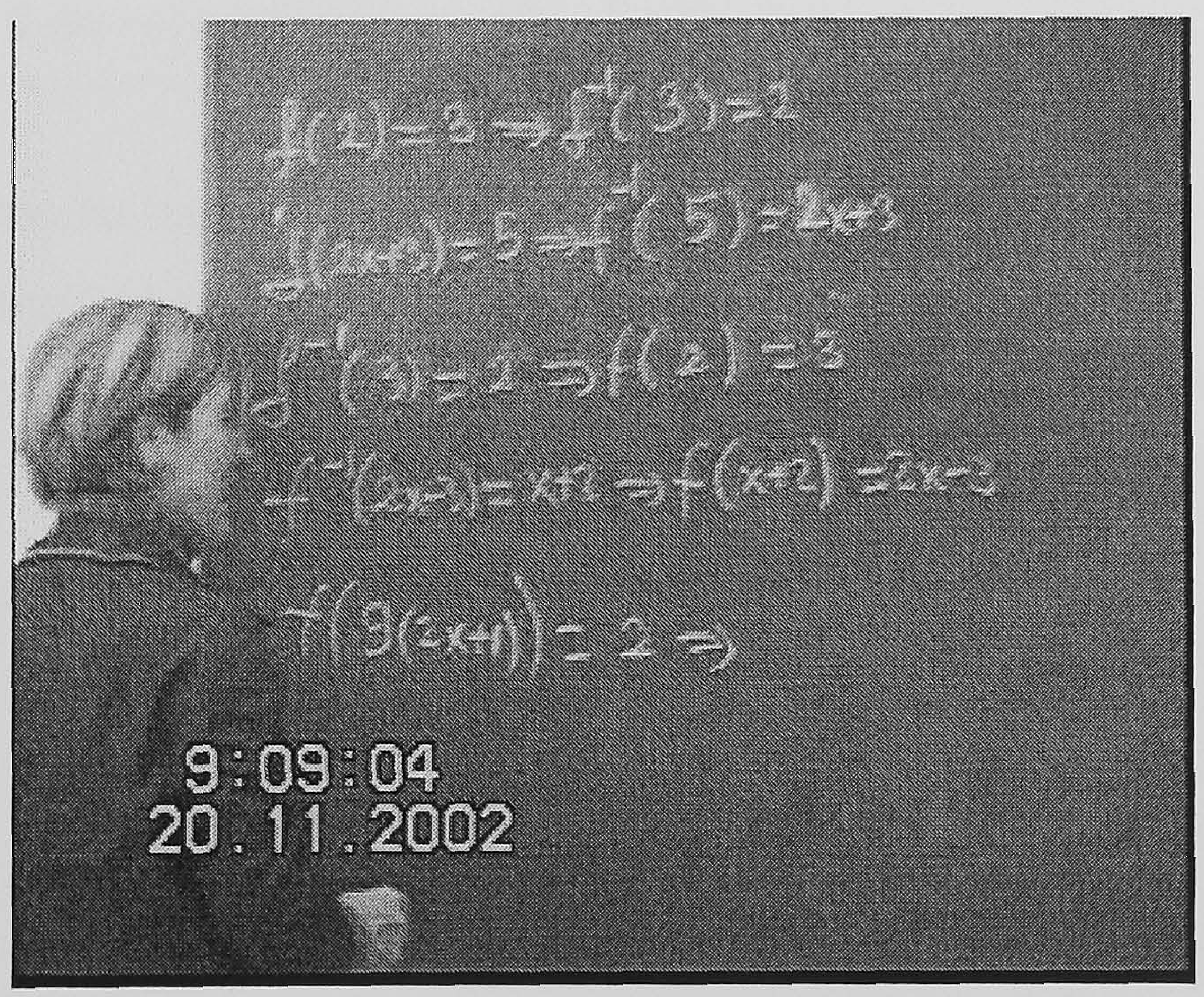
She explained "*when f(2) is asked we will think of it as what is y when x is 2, when* $f^{-1}(2)$ *is asked we will think of it as what is x when y is 2*" and wrote her explanation on the board as a note, which looked like the box below:

| | |
|---|---|
| Note: When f(x) is given | $\nearrow$ If asked f(2) then x=2, y=? |
| | $\searrow$ If asked $f^{-1}(2)$ then y=2, |

Ayten then gave a signal to students to copy the board into their notebooks. She walked around the classroom while they were writing. After a few minutes of silence she drew a diagram on the board and started making explanations.




By referring to the argument[2], she stated "*if f hugs x and takes it y and if* $f^{-1}$ *is inverse of f, then* $f^{-1}$ *hugs y and takes it to x.*" and repeated her explanations again in a similar manner. She highlighted "$f(x = y \Leftrightarrow f^{-1}(y) = x$" and said "*It is very important*". Ayten let students copy the board down to their notebooks. As the board was full, she asked the daily responsible student to clear part of the board. She wrote 5 examples with 2 parts for each.




She wrote the left hand side part and for each example she expected students to join her by telling her what to write on the second part (right hand side) where there is a need for manipulation of the first part. With these examples she tried to show the procedure she had just presented. That is, how $f(...) = ...$ type of equalities can be turned into

[2]The expression placed within the parentheses next to f, is called the argument of the function

$f^{-1}(...) = ... \; f^{-1}(y) = x$ and vice versa with simple manipulation. The board looked like the box below:

| | | |
|---|---|---|
| *f(2)=3* | => | $f^{-1}(3) = 2$ |
| *f(2x+3)=5* | => | $f^{-1}(5) = 2x + 3$ |
| $f^{-1}(3) = 2$ | => | *f(2)=3* |
| $f^{-1}(2x-3) = x + 2$ | => | *f(x+2)=2x-3* |
| *f( g(2x+1))=2* | => | $f^{-1}(2) = g(2x+1)$ |

After completing this, she said "*Copy them down immediately. You will see a lot of questions related to this.*" To a student's question "*Does it come in the university examination?*" she replied, "*Yes, I may write some...Copy them down fast and don't talk among yourselves.*" At that point, she looked at her notes for the first time in the lesson and started writing two examples on the board.

f: R→R  f(x-a)=x+2a and $f^{-1}(3) = 0$ => a=?

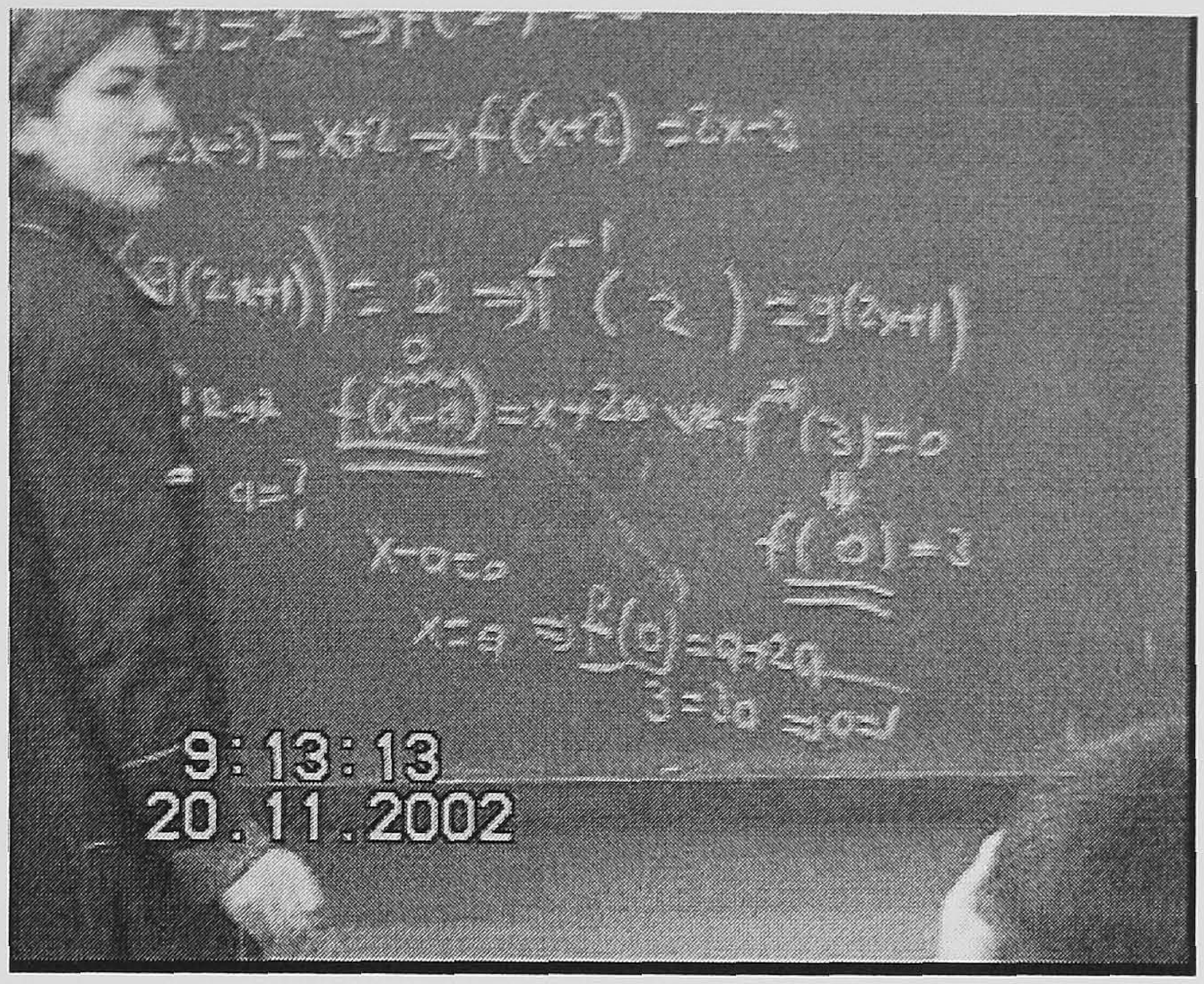
f(-3x+m)=x-5 and $f^{-1}(-1) = 3$ => m=?

The student who is responsible cleaning the board came to the board and cleared the remaining written part. After waiting around 2 minutes for students to copy the examples down and think about them, teacher began to solve the first example. She performed the solution by, first, obtaining *f(0)=3* from the equality $f^{-1}(3) = 0$, in which she used the manipulation demonstrated previously, then she showed the similarity between the two expressions.

f(x-a)=x+2a
f(0)=3.

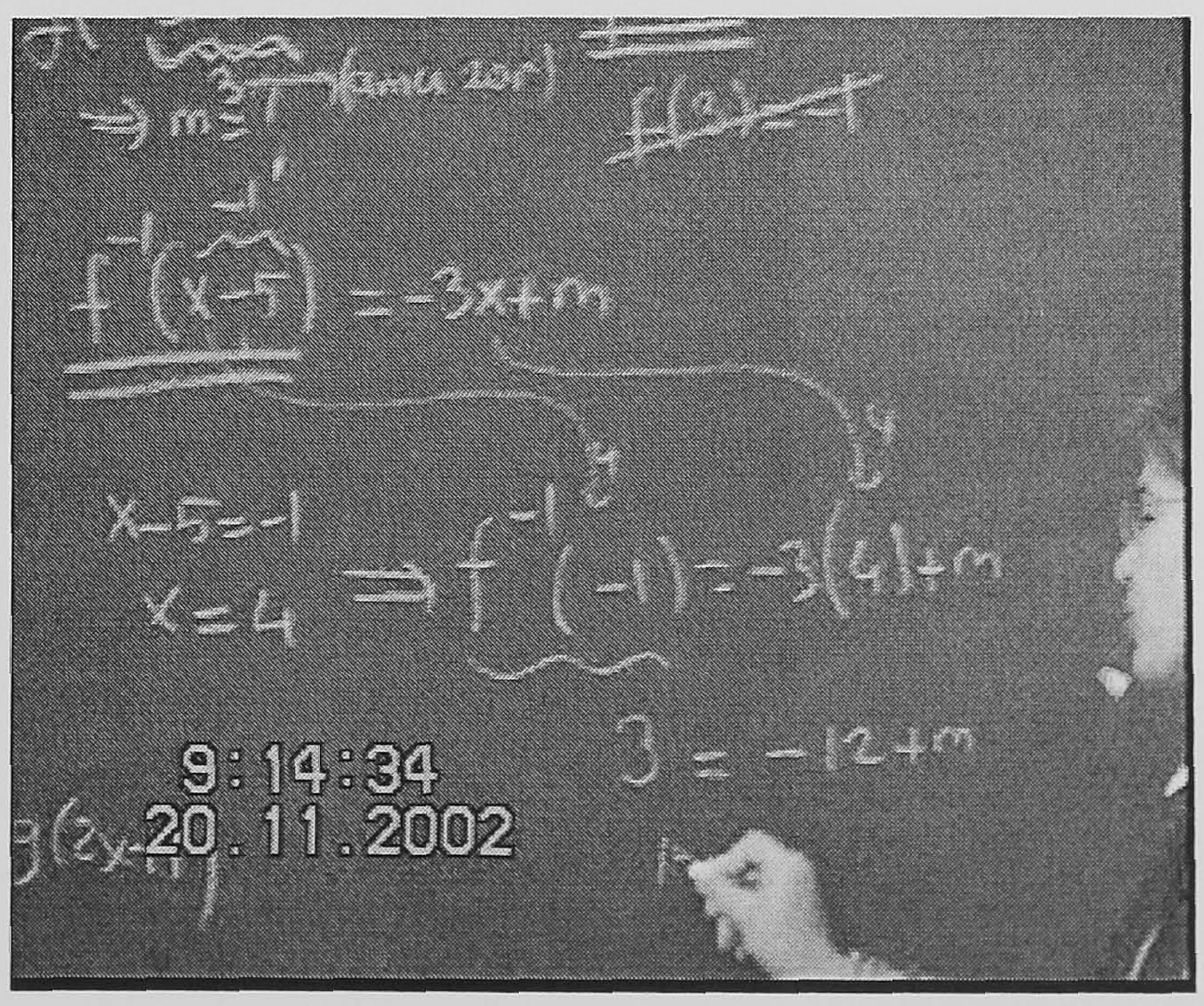
f(x-a)=x+2a
f(a-a)=a+2a
f(0)=3a
As f(0)=3, then 3a=3 => a=1.

She then equalised the corresponding expression in the two functions. This gave her two equalities: *x-a=0* and *x+2a=3*. From the first one she obtained *x=a* and then she put the x value in the second expression.




In order to solve the second example, Ayten obtained *f(3)=-1* from the given expression $f^{-1}(-1)=3$, with similar manipulation. However, she explained that it is hard to solve it this way since it is hard to use the similarity between expressions *f(3)=-1* and *f(-3x+m)=x-5*. She crossed out $f^{-1}(-1)=3$ and wrote '*hard to do*'.[3] She said, "*it is hard this way, then, I will change this one* (pointing *f(-3x+m)=x-5*)". She obtained $f^{-1}(x-5)=-3x+m$ by manipulation of *f(-3x+m)=x-5*. And similar to the previous example, she showed the similarity between the obtained expression and the given equality.

$$f^{-1}(x-5)=-3x+m$$
$$f^{-1}(-1)=3.$$




[3] She has done this as if she is a student trying to solve the problem and stuck with the first manipulation and now trying another way.

She equalised *x-5* to *–1* (which are actually the numbers in function brackets) and obtained $x=4$. She then put *4* as $x$ in the expression $[f^{-1}(x-5)=-3x+m]$.

$$f^{-1}(-1)=-3.4+m$$
$$3=-12+m$$
$$15=m$$

After completing the solutions she said, "*aren't these examples beautiful?...You will love them*", with some enjoyment. She let students copy the board into their notebooks. Ayten asked the president of the class about missing students and the president told her numbers of missing students. She signed the classroom book, making a note of the missing students. After waiting for a minute she started writing two more examples from her personal notes.

1) g: R→R g(3x-4)=6x-5 then $g^{-1}(-2)=?$

2) f(x) a linear function
$f^{-1}(1)=2$ , $f^{-1}(4)=-1$ => f(3)=?

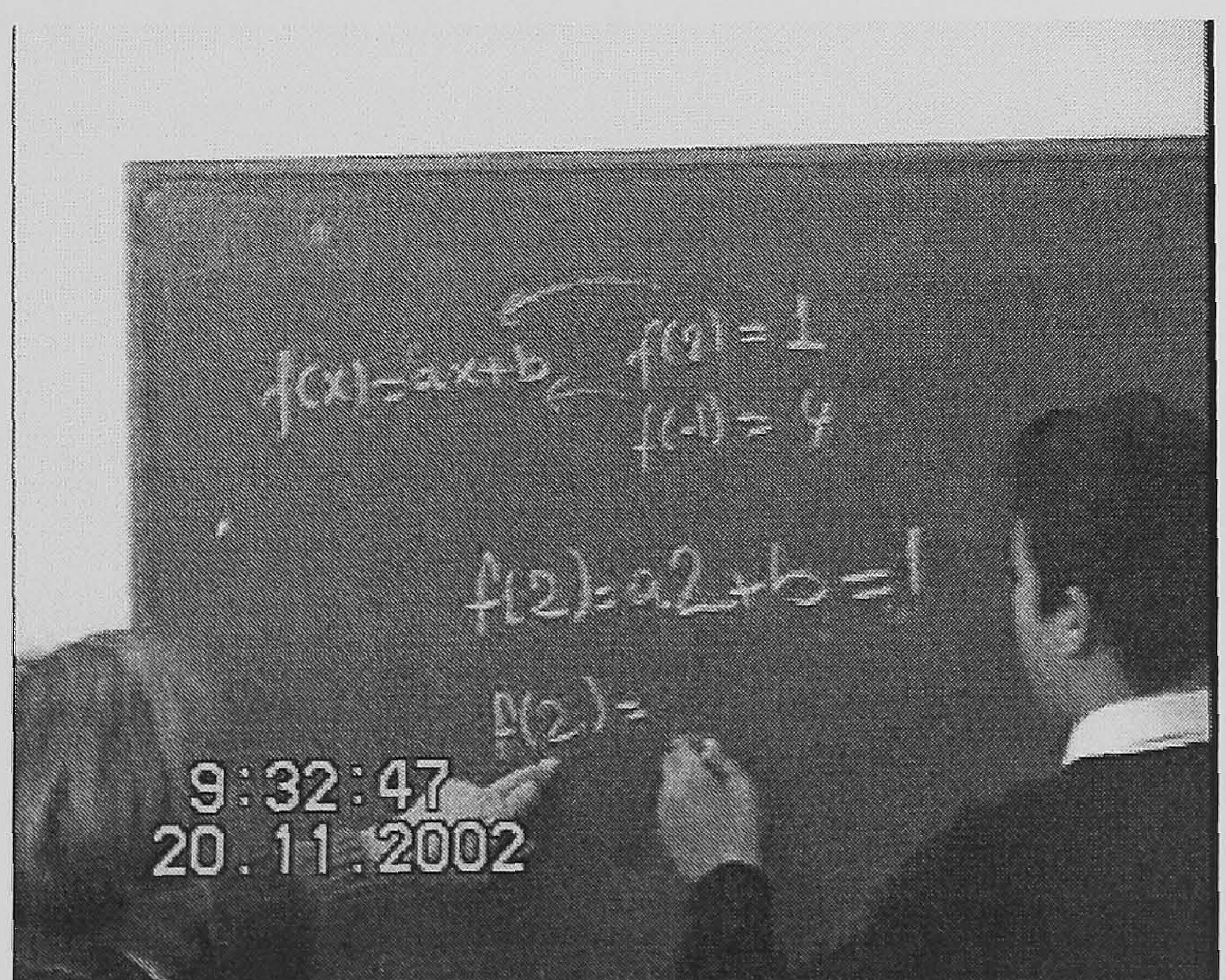
While writing she said "*those who finish [the solution of the example] please let me know I will come next to you. Don't say the answer out loud. Some of your friends complained that it is affecting them negatively [when some students shout out the answer]. Be sensitive about it. Just call me I will come next to you*". Having completed writing the examples she started walking between desks and dealing with students individually while students are trying to solve the examples. Meanwhile she said, "*[I observed that] You learned this thing [manipulation] well, if it is [function] so then the inverse [of it] is such and such. If the inverse [of the function] is so, then it [function] is such and such* (referring to manipulation without naming what it is)".

She dealt with a great deal of the students individually and checked their progress through inspecting the students' solutions in their notebooks. She encouraged some of them: "*yes very good*"," you are *doing well keep going*". She reminded some of the students of their mistakes. After more than 7 minutes of student engagement, she called for a student to volunteer to demonstrate the solution on the board. While he was solving the example on the board she commented on his solution to explain it to the other students. She commented on every line of his solution. Although she was happy with it in general she complained about the messiness of his solution. She asked him to

$$\boxed{\begin{array}{ll} & f(2) = 1 \\ f(x) = ax+b. & \\ & f(-1) = 4 \end{array}}$$

be more organised and tidy in terms of mathematical notations. Before going into the solution of the second example she reminded the students that 'what 'linear function' means is ax+b' and wrote not only general form of linear function but also manipulated versions of two equalities given in the example:

Ayten kept walking between desks and checking students' solutions. She was so involved into dealing with individual students that once she sat next to a student and started writing a solution into her notebook and discussed it with that student. Then she realised that the student had copied down a number incorrectly and that was why the student could not reach the correct answer she had in her notes. She commented that "*The answer I have in my notes is different but it is not important. What is important is the correctness of the solution method. You should still be careful when copying it down. If you don't copy it down correctly, you can't reach the correct answer.*" One student was not able to reach the answer and asked for help. Ayten wanted to see his progress but he had already erased his calculations from his notebook, as he could not get the correct answer! She invited him to the board to solve it. It had been 16 minutes




when this example had been written on the board for students to solve. The student seemed to have very little understanding of the linear function. This may partly be caused by his being nervous as he was in front of all his classmates. When Ayten realised that, she pointed to *f(2)=1* and said "*what does it mean?...it means write 2 wherever you see x, now write*" When he wrote: $f(2)=a.2+b$ Ayten asked him to equalise to 1.

She told him to use the second equality in the clue she gave '$f(-1)=4$'. Although the student had made some mistakes in basic calculations, she had supported him to carry out calculations at each step of the solution. The student found the value of $a$ and $b$ and then put values in the expression $ax+b$ and then found $f(3)$, which is the answer of the example. She was expecting him to solve it on his own as she said "*what is the point, if you are going to solve it with me*". She then turned to him, while he was at the board and said "*After putting these values in* (pointing the function $ax+b$), *is it hard for you to do...No!...*(looking at him) *please...you are not studying hard enough*". It took around 4 minutes for student to solve the example with the support of the teacher. After this Ayten asked the responsible student to clear the board. She then wrote 3 more questions on the board.

| | |
|---|---|
| 3) | $f^{-1}(x-1)=\dfrac{7x-12}{2}$ => f(8)=? |
| 4) | f(x)=3x+m and $f^{-1}(5)=2$ then m=? |
| 5) | $f(x-1)=\dfrac{x+1}{x-3}$ => $f^{-1}(2)=?$ |

While walking among the students again, she said, "*come on let's do these, I want to see how much you have progressed*". After a minute the bell rang as a sign for the end of the lesson.

### 4.3.4. Ayten's Beliefs

I asked Ayten to talk about the way she teaches.

> **T** When I am teaching I don't give content all together and then the examples. I put the topic into small sub-sections as you observed. I give content and then I give examples relevant to that part. Next, I go onto another part. I give the content of it then I give examples relevant to that. I mean, I do it [teaching] this way. I just don't just give all the formulas and definitions and theorems together and then go on to solve the examples after that. I present the content along with examples. When I finish the topic my examples are also finished. If there is still something that students did not understand I do some more examples in an extra lesson. That's how I teach.

That outlined her teaching organisation. She continued to talk about these subsections. She especially talked about use of examples as an important part of her teaching.

> **T** When I am solving examples I write 4-5 examples on the board, as you observed. I solve 1-2 of them and then I let the students deal with the rest. When they are solving I walk between desks. I am monitoring their notebooks to check who did what and to what extent they progressed. If they come to a certain stage in the solution [and stuck], I tell them 'after that, you may think of it like so and so' or 'come on try a bit more' and I try to motivate them and encourage them by doing so. Students are active in classes but in general it isn't very much. There is involvement but not as much as I wanted, which is 40 students raising their hands to solve the examples.

She stressed that she solves the first few and shows how to do it and then expects students to solve the following examples. To her, demonstration of solution steps is not the only function of the examples. She also used examples to make sure that students are able to solve examples similar to those she had solved previously. In a way, examples were used to inform her about students' progress. In line with this, she made use of students' notebooks to see individual students' progress. In this way she could manage to provide motivational support for those who needed it in their engagement with examples. She valued students' active involvement in the classroom. From her perspective, the ideal situation is when all students raise their hand to actively participate in the course of the lesson.

> **T** From my perspective, the [important] thing about mathematics is that no matter how much you give [theory and definitions] they don't comprehend it in the moment you are presenting. But, if you explain it by its examples, the knowledge fits to its place. For example, we simply say "In a right angled triangle, the median of the hypotenuse is equal to half of the hypotenuse" Even if they

write it down 10 times or even 100 times, if a student does not use it in 10 examples, it will not fit to its place. I am of this opinion.

Ayten views that theories and definitions are necessary to teach, but unless they were exemplified the knowledge presented 'will not fit to its place'. I asked what she meant by this expression.

**R** What do you mean by 'it will not fit to its place'?

**T** I mean they cannot comprehend completely. I mean, they have to consolidate it. In [my opinion of] learning there is a thing called consolidation. Students do the consolidation by examples. It [piece of knowledge] fits to students' mind when they use it in solving an example. Even if I write it on the board ten times, students memorise writing it, but they have to use it in solving examples. That's what I think. It is true for mathematics. I mean, if kids use the knowledge in solving examples, it fits into their mind. I never have the attitude 'write this and memorise it'. Never. Neither in analytical geometry, in geometry, nor in mathematics. I don't have the attitude 'memorise these formulas' I try not to prescribe formulas. If I have to give one I underline it many times by saying 'this is a formula' and I say that 'you have to memorise or have to know this'....As I said my lessons are mixture of content and examples. But if you ask me a percentage I suppose I only reserve 15-20 minutes of it for presenting the content I mean the essence of the lesson, the remaining is examples.

**R** So is it nearly half of it?

**T** It is not half exactly but more than half is on examples. But it can still be considered as half.

As can be observed in the extract, Ayten meant 'comprehension' or 'understanding' by the term 'it will not fit to its place'. This explains the previous extract where she said "if a student does not use it in 10 examples, it will not fit to its place". She therefore valued repetition by similar examples. She highly valued examples because, to her, students consolidate the mathematical knowledge presented through solving examples. She believed that students will not have to memorise the theoretical knowledge if they consolidate by examples. Another indication of her high regard of examples in her lessons was how much time she allocates for examples. She stated that nearly half of her lessons are reserved for examples and the other half is content. Although she briefly explained the use of examples when she talked about the organisation of teaching previously (in the second extract) in this section, I asked further questions to explore her views about examples.

**R** Do you solve 2-3 examples after each type [of functions] or is 1 enough?

**T** No. 3-4. I mean Let me give you one student's notebook [so that you see how many solve]. I solve so many examples. Kerem, you observed this [in my lessons].

**R** Why do you put emphasis on examples?

**T** As to the question 'Why do I put emphasis on examples?'...hmm ...I believe I teach better that way. That's my opinion. You know that there are verbal [theoretical] parts in mathematics. When I present that [part], kids write it down but they cannot understand on their own when they read it. If that was true [that they learn by writing down] then there are many mathematics textbooks on the market, many resources. They bring them to me and ask. They even got books with solved examples. Students bring me the solved examples from the books and ask me to explain. Why? Because they don't understand. I don't believe mathematics is such a subject that students read and understand or put into practice [right away]. Do I make myself clear? That's why I pay more attention to solving examples. I think the knowledge fits to its place when they solve examples. I mean, rather than knowledge written on the notebook with a certain template, [I prefer teaching] through solving examples... No matter how many times you give Menelaus' theorem to students [they don't understand]. When they solve 10 examples it fits to its place. They don't need to memorise it. I am trying to keep students away from memorising. Away from memorising formula or a rule.

She is making a number of points in this extract. Firstly, she perceives that she is solving many examples for each sub-section of the topic. Secondly, she refers to students' notebooks as an indicator of her teaching. Thirdly, she clearly states the value of examples in her mind. Fourthly, she believes that students do not understand the theoretical part, or content, unless it is exemplified many times. Fifthly, she seems to be opposed to memorisation in mathematics. Finally, as she stressed in the previous extract, she viewed that student learning cannot be achieved merely by reading or making notes. Students learn by being actively involving in the lesson and engaging in the solution of the examples and thats how 'it fits to its place'. The following extract also demonstrates that she values students' participation in her lessons.

**T** ...I have a student called Caner, with whom we got rid of the bridges

**R** What do you mean?

**T** I mean, our dialog has been damaged. I tried talking to him personally, I tried talking to his parents...but our dialog has gone. He could not handle and it is getting worse. Last term he got 5 [in mathematics in his term report]. I mean, it is very important [for students] to like mathematics but it is very hard to engage with 40 students individually. Honestly, I can not do that but I am doing as much as I can.

She values engaging students individually so that she could remedy their deficiencies. To follow up her views on her use of students notebooks, I asked "Do you expect students to write down everything you presented [on the board]?" She commented,

**R** I expect them to copy down everything and write down everything [on the board]

**T** Why do you expect them to write?

**R** Why do you expect them to write?...hmm...So that they could repeat at home. It is their puberty period, and they are not able to concentrate well. I observe that. Because of this some students are able to understand if they repeat what I wrote at home. [But] Some others are able to understand in the lesson. I believe that something can be learned better by seeing by writing and by visualising. If they only see [things on the board], it can be lost [in their mind]. If they read it again in their notebooks they could say 'ohh I remember my teacher had also taught this'. But if they don't write, the important point that I presented could be cleared up [in their mind].

**R** You mean...to keep it in mind?

**T** To keep it in mind and to consolidate and to be able to repeat in the future.

**R** Why is repetition so important?

**T** It is absolutely necessary for consolidation. Students cannot learn only by my explanations.

Based on this and previous extracts, it seems that Ayten was aiming students comprehension and putting high value on students' understanding of the mathematics she teaches. She stated how well she used the textbook in the classroom. I asked,

**R** You expressed your satisfaction with the book you are using. Did you choose it yourself?

**T** The school has made the selection. We [mathematics teachers] said [to headmaster] we want to give homework but we are unable to because of a lack of textbook. Let's select a textbook. And that book is really good. Using that textbook students really consolidate what they learned well.

The book she uses was selected by the permission of the school administration. When I asked the type of book she is using she gave an interesting answer,

**R** Is it one of the books approved by Ministry of Education?

**T** Yes, it approved by Ministry of Education...How dare we [use something else]...? Who are we [to make such a decision] (both laughing)

Ayten's comment was carrying a bit of exaggeration but in fact it was a clear indication of the fact that she was bound by a higher authority, the Ministry of Education. When I prompted her to explain the use of examples she made clear the link between the textbook and the way she teaches.

**R** Do you expect students to examine the examples you gave during the lesson?

**T** Of course I do.

**R** How do you expect them to do that?

**T** I always tell my students don't just look or read the example, copy it down to another sheet of paper and try to solve it. If you can't go back to your notebooks. In fact, I generally use examples that are very similar to the textbook [I use]. Although there may be some extra examples, I generally use the textbook. I have examples similar to the ones in the textbook and when I have solved the examples [in a lesson] students have to solve them. They start solving [at home] the examples of the textbook and when they are stuck they open their notebooks and learn how to solve them. If the students still cannot solve it, they bring it to me. When I see the example [they couldn't solve on the textbook] I immediately ask 'give me your notebook'. I do it because I cannot explain each and every time. I open their notebook and show the similar example [that I solved during the lesson] and I ask them to examine that [previously solved example] and I say 'if you still cannot understand this, come back to me'. They then understand it.

Ayten used examples that are similar to the textbook she is using and asked students to solve the examples from the textbook. When they could not solve an example she advised them to make use of their notebooks. She continued

**T** Students understand [mathematics] much better when they study on their own. I explain, I give examples but they understand when they consolidate.

It seems that she is of the opinion that students learn only when they study on their own. She also said that existence of a teacher is vital for students to be able to understand.

**T** Students cannot learn mathematics by reading it from the textbook. I always said that and I can argue that against anyone. Is it not possible to learn from the textbook? It is [possible]. I did learn [from books] to be able to teach but students do not do that.

I asked about the institution where she is working and private courses.

**R** Can you compare state schools and private courses in terms of their aims?

**T** The aim of state school is comprehension of the topic, to teach its essence. That's it. [nothing more] If I give the content I can get away with only 3-4 examples, and nobody can say anything. I am not responsible [for extra examples].

**R** You just mean explaining the content properly?

**T** Yes.

**R** So what is the aim of private courses?

**T** The aim…hmm..i never worked in a private course but the aim of a private course is to consolidate with examples.

Ayten seems to see the aim of state school as providing students with an understanding of the mathematics. In the follow up of her explanations on the difference between PCs and SSs, she gave her husband as an example.

**R** My husband is a geography teacher. He knows the content of the topics very well. I cannot describe how good his content knowledge [of geography] is amazing. But, now, he started to work in a private course. He *used to* say I am very good [at geography]. But he isn't. There is a mountainous difference between teaching content and solving examples.

**T** Do you mean, now, he does not feel as comfortable as he used to be in the state school [he was teaching]?

**R** No he doesn't. Let me explain. He has to see the relationships in every example that students bring to him. O.K. Knowing the content very well helps him a lot but if he had taught his lessons in the state school with examples and linked them with university examination questions he would have been much more comfortable now.

**T** Do you mean that mathematics teachers who work in a state school may not be comfortable teaching in a private course?

**R** Of course they don't. [4] Kerem, [are you aware of] what are you asking? Of course they don't. they have to be able to answer every question the students bring. [Are you aware of] what are you asking? [If the teacher cannot solve an example] students say 'I took this example to that teacher and he couldn't solve'. Would you like to be a teacher like that Kerem? You tell me. I personally don't like to be...In a state school teacher does not have to solve every example [student bring] but it is not like that in a private course. In a private course the teachers are responsible for solving every example students request you to solve. They have to.

**T** What is the teacher responsible for in a state school?

**R** What is the teacher is responsible for in state school…hmm...I don't know. I think this is up to the individual teacher's conscience. It really is.

This extract is a very interesting one and it says a lot about what she believes in relation to what it means to be a PC or a SS teacher. Ayten seems to believe that the difference between PC teachers and SS teachers is the type of knowledge required in teaching. While SS teachers need to have a good conceptual knowledge of the subject matter, PC teachers need to be have good procedural understanding of the subject in order to be prepared for every type of example they may be asked to solve. To her, this makes "*a mountainous difference*" between teaching in PC and teaching in a SS. She reported her husband's experience of changing the type of institution he worked in. According to her, his experience of such a change affected how confident he is about himself as a teacher.

[4] Based on her intonation and emphasis of the words I observe Ayten was very surprised with being asked a question that has an obvious answer.

She explained[5] that she gives examples immediately after each bit of content and goes onto another bit. When all the content is covered, then she gives examples that cover all the content she taught previously. I asked about this. Ayten revealed a distinction between different uses of examples.

> **R** Is there a difference between examples after the each content bit and examples you give afterwards?
>
> **T** Yes. The examples [after the content] only cover that bit of the content but the examples [I give] afterwards may cover previously given contents. A bit harder
>
> **R** How is it harder? Harder [for students ] to calculate?
>
> **T** No, no. In fact the calculation may not be that hard but these examples require more than a single bit of information. In one [type of] example they need to inverse a function but in the other [type] they have to inverse the function and then have to find the composite function with another [function] and then they have to find the image of the composite function at one given point. It brings together a few bits of content.

She indicated that she sometimes leaves some examples on the board unsolved and tells her students to solve them at home. She said

> **T** I leave some [examples] on the board. I ask them to solve them at home and I also ask this in the examination. They *have to* solve them. Because I did [like that] in the first examination, and in every examination I select one of these [examples left for students to solve] in the examinations.
>
> **R** So you prepare these [examples to left for home] beforehand.
>
> **T** No I don't. If I solve a similar example, in order not to give same example repeatedly, I tell them to solve at home, but I put a mark on these examples in my notes to indicate it was not solved during the lesson. In preparing the examination questions, I go through these marked examples I include 2 of them in the examination. Then some of them regretfully say 'Oh no. I did not solve that at home'. After that they feel the urge to solve them every time.

She commented that she prepares the content and examples she is going to give beforehand using mostly Ministry of Education approved books, and makes notes for her teaching and updates these notes every year. I asked her how much she uses these notes during the lesson.

> **T** Is there a 1 to 1 correspondence between your teaching and these notes?
>
> **R** Yes, of course. There is a 1 to 1 correspondence.

Regarding the examples she uses during her lessons she said

[5] She supported this explanation with a drawing on a piece of paper.

**T** I don't expect them to reach the answer for every example. For some examples I tell them 'this and that is needed to be done. OK. Can we do these calculations? After talking about it [the way to solve the example] I leave the solution. I don't have to solve it. If a student is able to understand how to solve it, and able to see the reasoning in the example, then it is OK. I can leave it.

It seems that she is putting emphasis on the process of solving the example, and students' understanding of the reasoning involved rather than finding the answer. I asked if she gives options for the examples.

**T** If there is an example in the university examination that I give during the lesson, then I do. But other than that I never give the options.
**R** Is there a specific reason for this?
**T** Yes, there is. If I write the options they start thinking about tricks to solve it or they try eliminating the options. No. Students have to learn the knowledge. The tricks and other stuff should be [used] later...[when students ask] if we solve it like that [using tricks] in the [school] examination do you accept? I tell them: No. You cannot use that in the examination. I do not [even] give mark for this.
**R** So the important aspect for you is...
**T** The important thing is that they have to know the main points...but in the out of school examinations they can use it. Let them be successful in those examinations too.

This confirms that Ayten is more interested in students' understanding of the topics and the reasoning behind the way in which examples were solved rather than finding the answer. She also indicates that such trick methods are only acceptable for examinations other than the ones in state school examinations.

## 4.4 The Case of Mahir

### 4.4.1 Background Information

Mahir was an experienced mathematics teacher and had been working in state schools for 26 years. He worked in secondary schools (ages 12-15) for 8 years and in the last 18 years he has worked in high schools (ages 16-18). His teaching experience was only in state schools.

My personal relationship with him goes back to my high school years. In fact, he was one of my mathematics teachers in high school. This was the most likely reason he was totally comfortable with me observing and interviewing him. Another was that I assured him I was not going to evaluate his mathematical competence. He was close to retirement, thus he had nothing to be uneasy about. Being the highest graded student in my high school classroom, I had a good interpersonal relation with him then. When I contacted him with the research project in mind, he suggested that I could observe him as a case, before I even requested for him to be involved. This situation has created a very comfortable environment for both interviews and observations for both of us.

### 4.4.2 Organisation of Mahir's Teaching

I video recorded him in 14 lessons in which he completed the teaching of functions (with 2 missed lessons). His organisation of teaching functions was divided into sub-sections in agreement with the syllabus. The order is:

1-Definition of functions;

2-Types of functions (1-to-1, constant etc.);

3-Inverse functions;

4-Composite functions.

As presented in the methodology chapter, I viewed his practice from the teacher's perspective, to be more precise a content-example perspective. For the content part he made sure that every student had a copy of the theoretical part in their notebooks. To do this he stated the definitions and theories out loud so that students could write them down. However, he spent most of his time solving examples of both active and passive types. Writing down the theoretical information was followed by passive examples almost all the time. Afterwards he then solved a few examples himself and then he let students solve them on the board. Mahir's teaching for these subsections can be seen as cycles of content, passive examples, and active examples (solved by teacher or student).

Mahir's emerging practice pattern for each subsection is shown in the Figure 4.4. This practice pattern starts with each new sub-section and the package cycles throughout the topic a number of times.

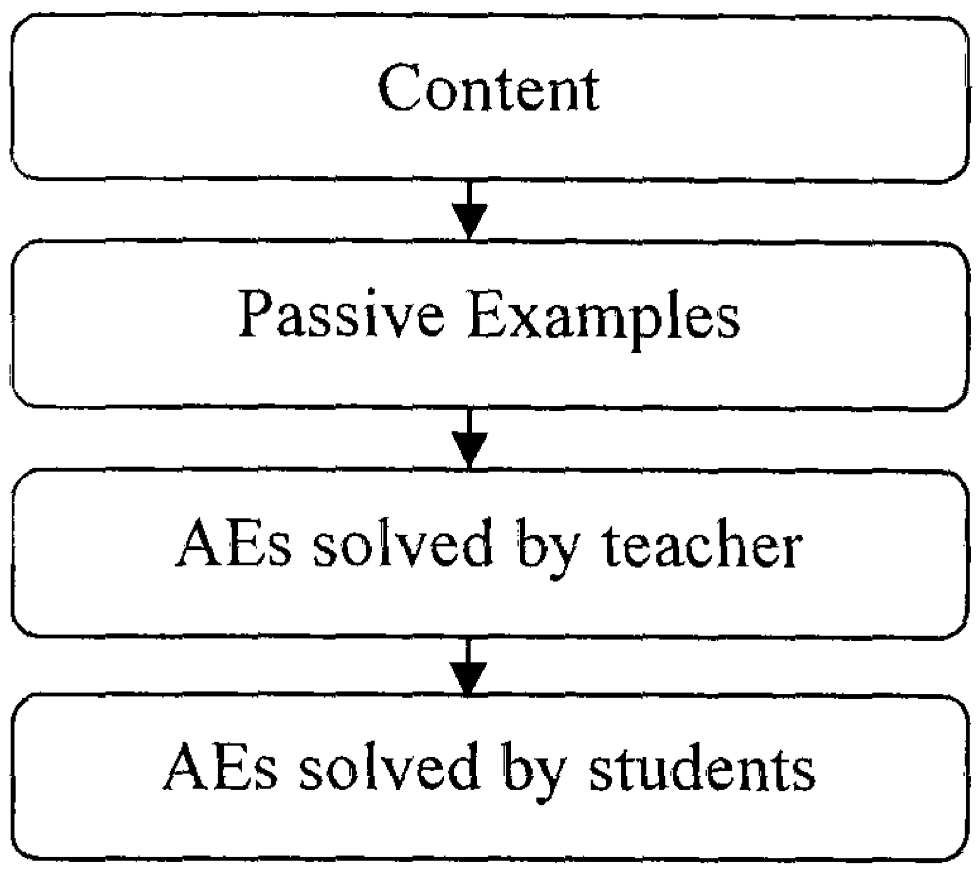



**Figure 4.4** Mahirs's practice pattern.

The organisation of Mahir's practice in teaching functions is presented in Table 4.11.

| | **Lesson Code** | **Content** | **Passive Examples** | | **Active Examples (PEQ. )** | |
|---|---|---|---|---|---|---|
| | | | Number | Time | Numb er | Time |
| 1 | Mahir-1 | 6m 47s | 6 | 8m 49s | 4 | 15m 23s |
| 2 | Mahir-2 | - | 1 | 4m 8s | 7 | 29m 14s |
| 3 | Mahir-3 | - | - | - | 5 | 33m 14s |
| 4 | Mahir-4 | 14m 22s | 3 | 7m 16s | 2 | 10m 3s |
| 5 | Mahir-5 | 14m 1s | 11 | 7m 28s | 3 | 7m 58s |
| 6 | Mahir-6 | - | - | - | 8 | 29m 33s |
| 7 | Mahir-7 | - | - | - | 11 | 37m 33s |
| 8 | Mahir-8 | - | - | - | 10 | 35m 36s |
| 9 | Mahir-9 | - | - | - | 5 | 37m 50s |
| 10 | Mahir-10 | - | - | - | 8 | 34m 40s |
| 11 | Mahir-11 | - | 7 | 8m 31s | 4 | 20m 50s |
| 12 | Mahir-12 | - | 1 | 4m 5s | 8 | 37m 25s |
| 13 | Mahir-13 | - | - | - | 4 | 32m 24s |
| 14 | Mahir-14 | - | - | - | 5 | 40m 37s |

**Table 4.13** Summary of Mahir's practice.

Table 4.13 and 4.14 suggests that Mahir spends some of his lessons on solving active examples. In these lessons Mahir solved many examples but he encouraged students to come to the board and solve the examples. This includes both him and students solving them. Even content, where theoretical information was presented, seemed to take less time in relation to examples. One can hardly miss that whenever Mahir presents theoretical information (Mahir-1, 4, 5 and 11) he utilises passive examples to support students' understanding of the concept or procedure involved. This shows a regularity of use of passive examples along with theoretical information to help students get the hang of the mathematics involved.

| Lesson | Code | Content |
|---|---|---|
| 1 | Mahir-1 | • Definition of function.(4.11)<br>• Definition of domain and range sets (2.36) |
| 2 | Mahir-2 | - |
| 3 | Mahir-3 | - |
| 4 | Mahir-4 | • Constant function definition (4.21)<br>• Identity function (1.57)<br>• 1-to-1, onto function (8.04) |
| 5 | Mahir-5 | • Possible number of functions formula (0.39)<br>• Possible number of 1-to-1 functions formula (2.05)<br>• Possible number of constant functions formula (1.45)<br>• Inverse Function (9.32) |
| 6 | Mahir-6 | - |
| 7 | Mahir-7 | - |
| 8 | Mahir-8[1] | - |
| 9 | Mahir-9 | - |
| 10 | Mahir-10 | - |
| 11 | Mahir-11 | • Permutation function definition (2.48) |
| 12 | Mahir-12 | - |
| 13 | Mahir-13 | - |
| 14 | Mahir-14 | - |

**Table 4.14** Summary of content segments in Mahir's recorded functions lessons.

Table 4.15 shows the number of examples solved by Mahir.

| | Percentage | Number of Examples |
|---|---|---|
| Passive | ~26 | 29 |
| Active | ~ 74 | 84 |
| **Total** | 100 | 113 |

**Table 4.15** Distribution of examples used by Mahir.

The table indicates that Mahir solves a lot of active examples. However, the number of passive examples used is not an insignificant number, 29 making 26 percent of the examples used throughout Mahir's practice. What does this mean in terms of examples per lesson? In fact this means that Mahir presented 6 active examples and 2.07 passive examples per lesson. Table 4.16 shows the number of active examples solved by students and Mahir.

| | Total | Solved by Mahir | Solved by students |
|---|---|---|---|
| Active Examples | 84 | 46 | 38 |
| **Percentage** | 100 | ~55 | ~45 |

**Table 4.16** Examples solved by Mahir and students.

[1] 2 lessons were missed between 8 and 9, where he taught composite functions (content).

Out of 84 active examples presented and solved throughout the topic of functions, Mahir solved 55 percent of them. Mahir encouraged students to solve 45 percent of the examples. This suggests the active participation of students in Mahir's lessons.

### 4.4.3 Mahir's Teaching

Lesson Code: Mahir-5

Before describing any teaching/learning activities during the lesson I would like to present the main phases of the lesson as a summary. This is intended to help the reader to visualise the overall flow of the lesson.

- Mahir gave the formula for calculating the number of possible functions for two given sets. He dictated the formula to students and he made sure that everybody wrote it down. He made gave explanations of an example.
- Mahir then dictated the formula for calculating the number of possible 1-to-1 functions for given two sets and all students wrote it down. He wrote the formula on the board and gave some explanations of the example.
- Before introducing the term 'inverse function', Mahir quickly reminded students of some of the terms he had introduced previously: constant function, identity function, 1-to-1 function, onto function. He used examples and diagrams to help students understand these terms and he made some explanations through a series of question-answer sequences.
- Mahir arranged his explanations in such a way that students realise 'in order for a function to have an inverse, the function should be 1-to-1 and onto'.
- Once it was clear then he dictated a formal definition of 'inverse function' and made further explanations on a diagram he drew. He also wrote and explained a simple manipulation [given function f(x), if $f(x)=y$, then $f^{-1}(y)=x$] and he emphasised the importance of this verbally, as well as through drawing a rectangle around it.
- He then wrote two more examples, which required manipulation of functions. He waited for students to solve them, but he waited longer than he did in the previous examples. He walked between desks and checked a few students' notebooks. Eventually he solved the examples himself.
- The example he gave in the last minute was interrupted by the bell ringing but he solved it quickly and the lesson ended.

Mahir started the lesson by dictating a series of formulas and giving examples related to each of them. During this formula dictation time he kept the textbook in his hand and carried it with him all the time. He used the textbook for the things he asked students to write down. For each item he read it from textbook. For the first one he read "*The number of functions that can possibly be defined from set A to set B is n(B) to the power* *n(A). That means the number of elements of A to the power elements of B."* He made sure that every student had written it down to their notebooks.

He immediately gave an example verbally, "*If the number of elements in set A is 3 and the number of elements in set B is 5....Then the number of functions that can be defined from A to B is 125*"

He demonstrated the solution verbally through interacting with students. He interacted with students asking simple factual questions like "*How many elements does set A have?*" and "*How many elements does set B have?*" The way he stated the questions and the way he used the tonality of his voice suggests that although he was asking questions he was not expecting students to answer. He seems to have done this to keep their attention alive and to make the theoretical information he is teaching as simple as possible for students to understand. The second formula he dictated to students is "*T:e number of 1-to-1 functions that can be defined from set A to set B is, for s(A)=m , s(B)=n and* $m \leq n$*, permutation of n with m. How do we write this?*" He wrote $P(n,m) = \dfrac{n!}{(n-m)!}$ on the board. He then gave an example of finding the number of 1-to-1 functions that can be defined from a set with 3 elements to a set with 5 elements. This time he wrote the solution on the board. He again solved it by asking factual questions as he did in the previous example.

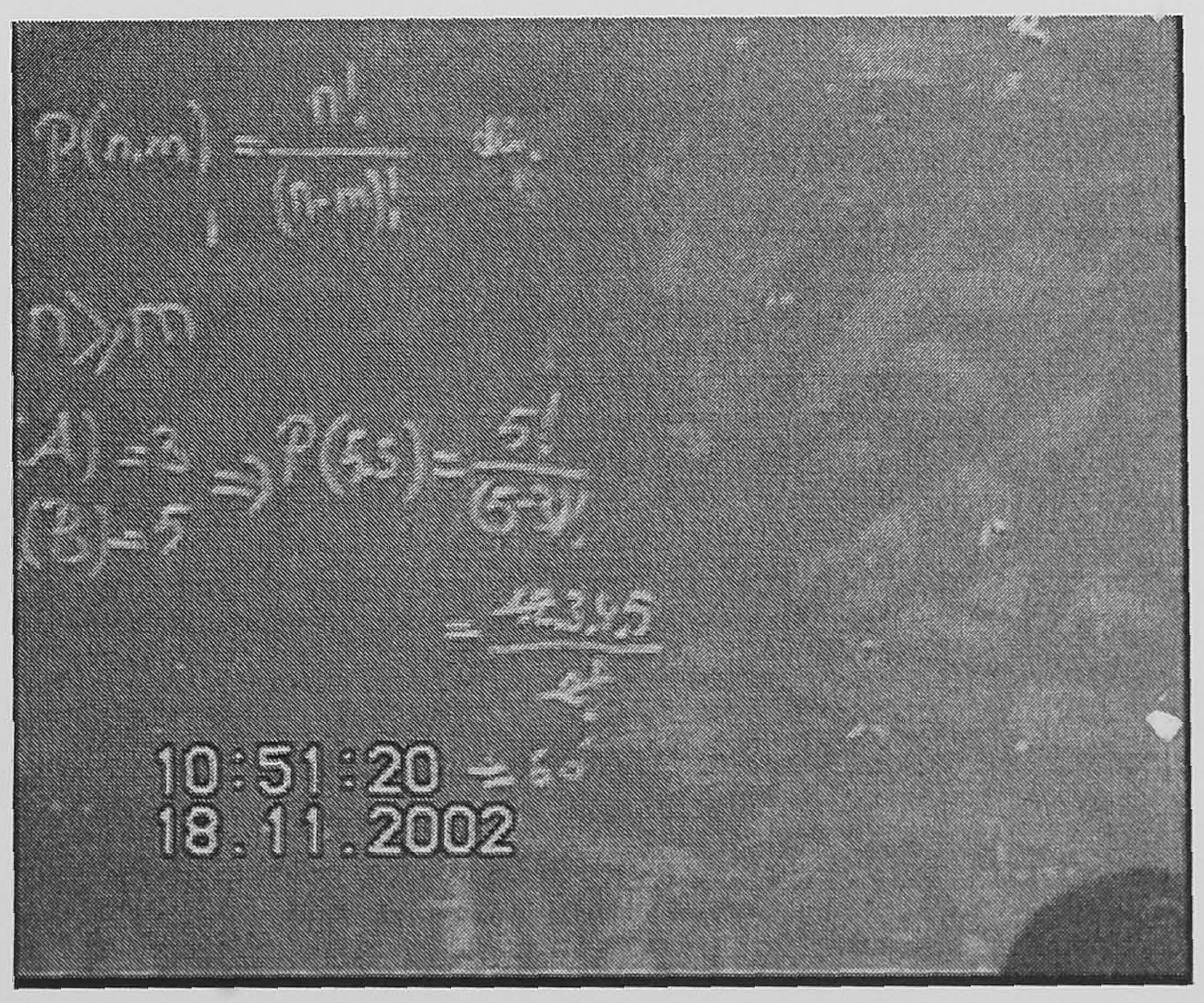
$$P(n,m) = \frac{n!}{(n-m)!}$$

n≥m

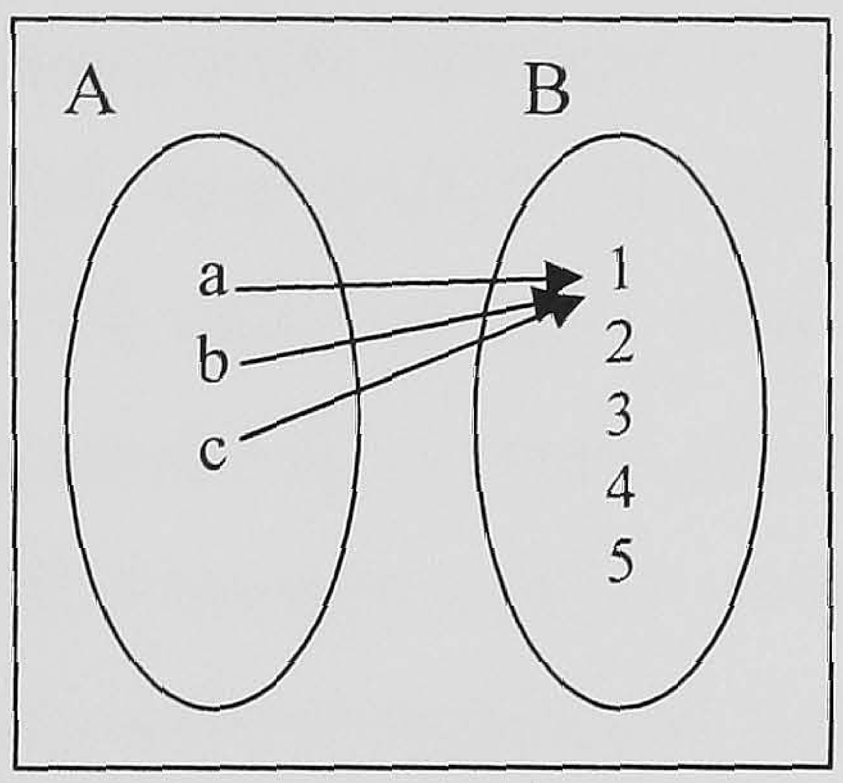
s(A)=3=> $P(5,3) = \dfrac{5!}{(5-3)!}$

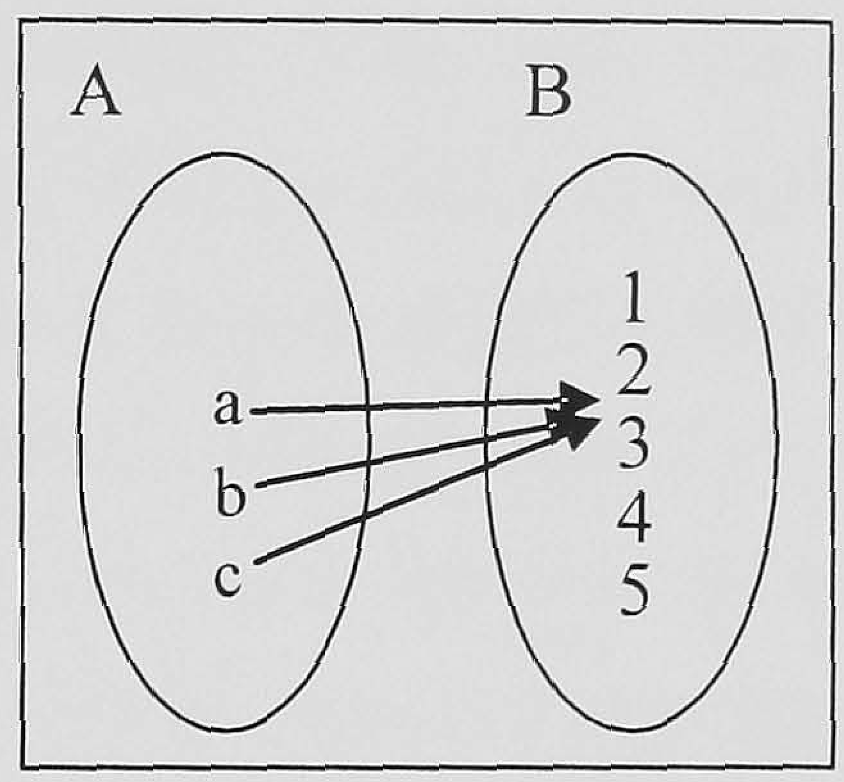
s(B)=5 $\quad = \dfrac{1.2.3.4.5}{1.2}$

$= 60$

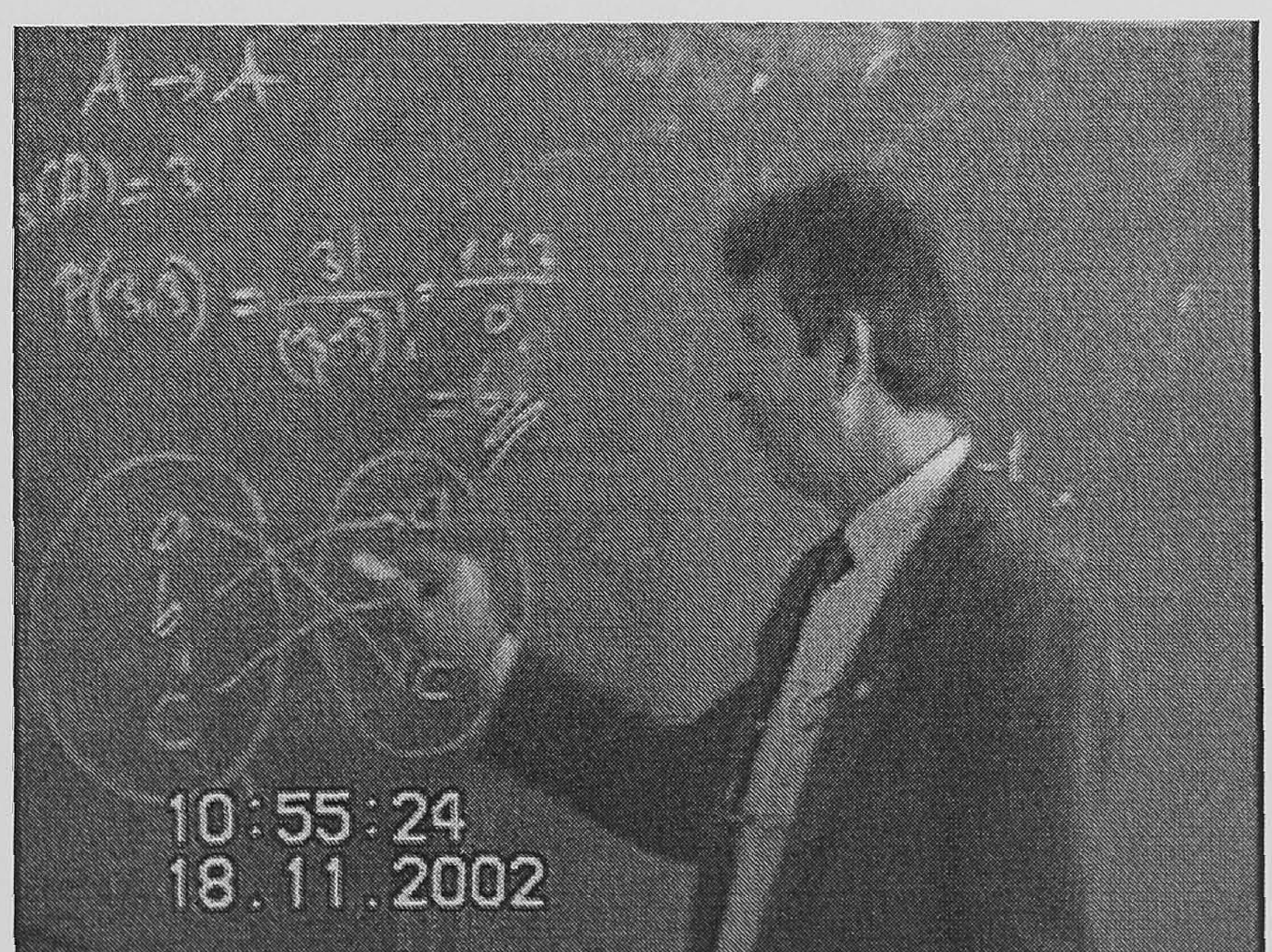



In presenting the third formula he dictated to students "*The number of constant functions that can be defined from set A to set B is equal to the number of elements of* *set B*". He gave an example of this on a diagram. After drawing it he said "*We said it is a constant function lets take all [elements] to 1* (drew arrows to 1 from each element of set A). *Can we match up all with 1? Yes we can and this is the first. Secondly,* (he drew another diagram and drew arrows to 2 from each element of set A) *we can match them up 2 and this is the second one. We can match up with 3, 4 and 5. So we obtain 5 new functions.*"



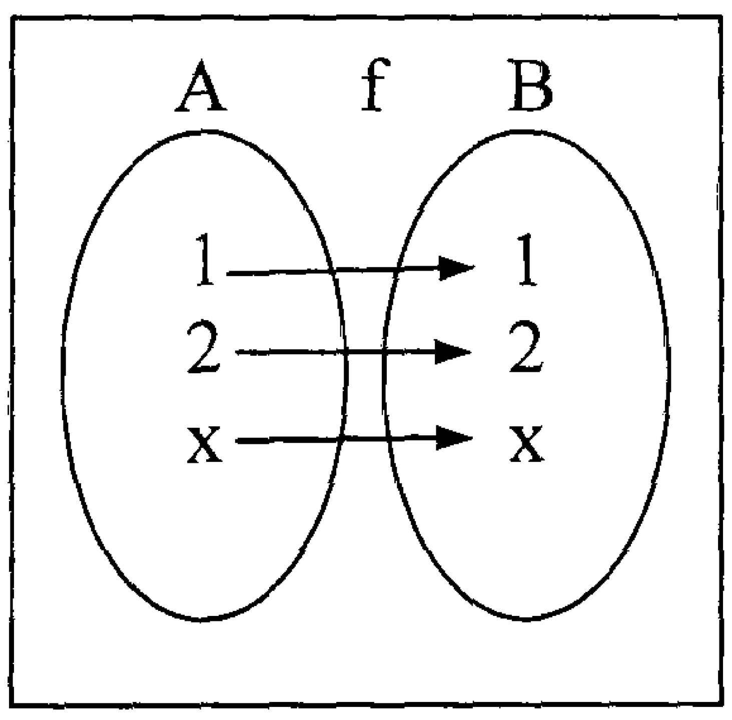

After the explanations, Mahir waited some time for students to finish copying down notes from the board to their notebooks. One student asked "*What should we write [title for] these?*"[2] He said "*[the title is] the number of functions…We just saw the number of a constant function. We asked how many functions can be defined from A to A*". Upon some noise from the class he warned them to listen and then he continued where he left off his explanation "*Let's have a look at the number of elements of A. How many?…3. Then we can define 3 constant functions. Well, how many 1-to-1 functions can we define?*" Mahir then demonstrated the solution on board. He used the formula above in the solution and found 6.




[2] Which seem to be intended to learn what should be the title for his notes.

He then said "*we can write 6 1-to-1 functions from A to A. OK. Well...What kind of functions are these 1-to-1 functions?*" He answered again by drawing a diagram and explaining it.

Some students suggested "*onto*". He said "*You see it is 1-to-1 and onto. So how many 1-to-1 and onto functions that are from A to A? 6*". He then signalled students that he will start teaching inverse functions by stating a title of the topic "Inverse function". One student asked "*is it a title?*" He answered "*Yes, the title*" He warned students to keep quiet again. He said "*OK. Let's write it down, inverse function. Well before this some of you missed the last few lessons, they don't know what I taught. Let's recap for them. We presented function types and the first one is constant function.* (pointing to the diagram he previously drew to explain the formula on the number of constant functions) *What is this an example of? This is (example of) a constant function. Isn't this domain?* (pointing to set A) *Yes set A is the domain and this is* (pointing to set B) the *range. All elements of the domain set (matches up) with one element of the range set. Aren't they all going to 1? So this kind of function we a call constant function. How many elements are there in the image set for this constant function? What is the number of elements for f(A)?* (pointing to 1 on the diagram) *1....OK. Secondly we said identity function. We defined it as function that matches up with itself.*" He drew a diagram that paired 1 with 1 and 2 with 2.

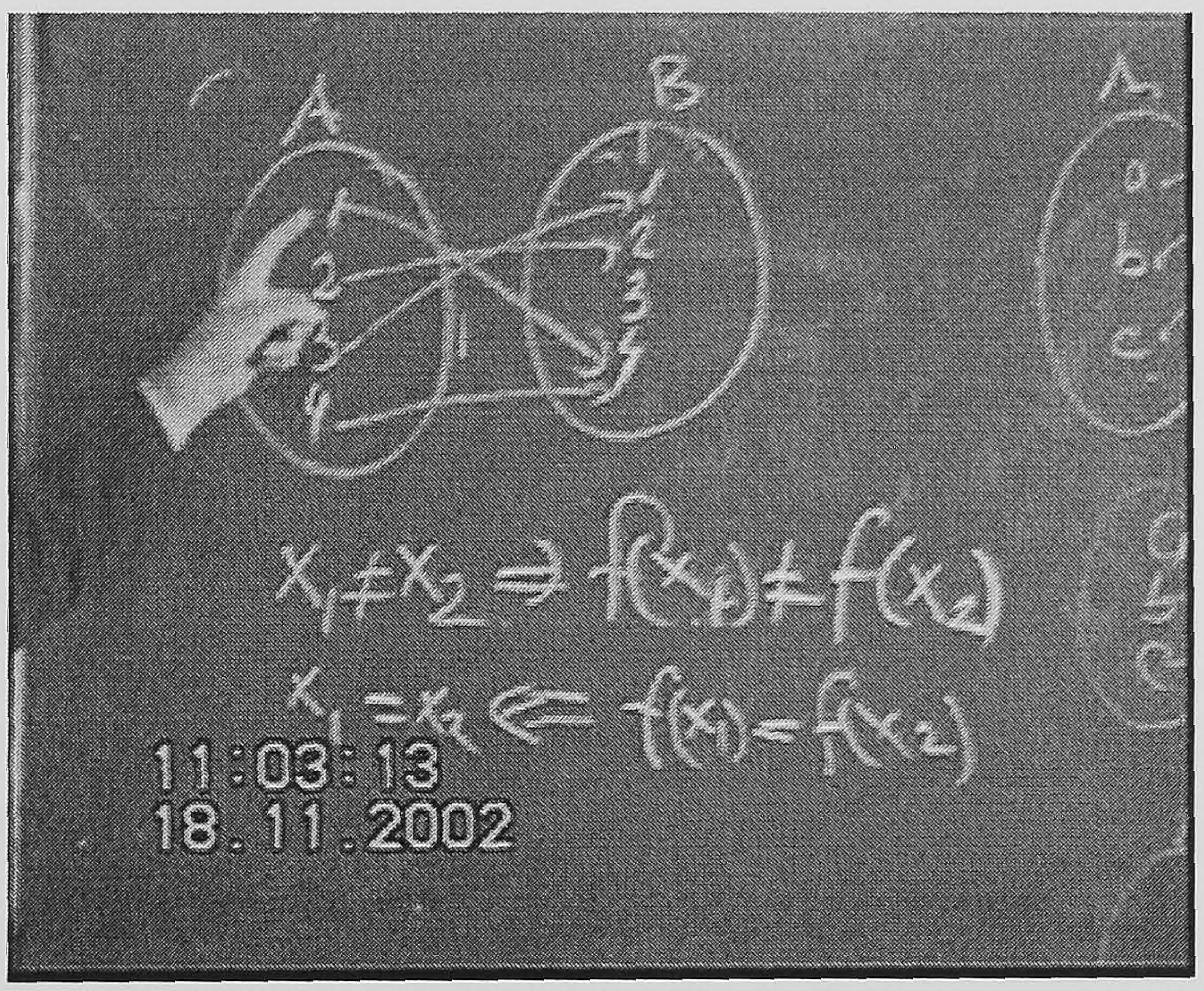



He said "*What do we call that matches 1 with 1 and 2 with 2? Identity function. So what is that going to match x with? x. So what is f(x) equal to? Equal to x. We said there is no term that includes x in the constant function. There isn't a constant term in an identity function and what is the coefficient of x? 1.*" He then wrote an example on the board.

| |
|---|
| In order for $f(x)=x+(m-3)$ to be a constant function what should be the value of m? |

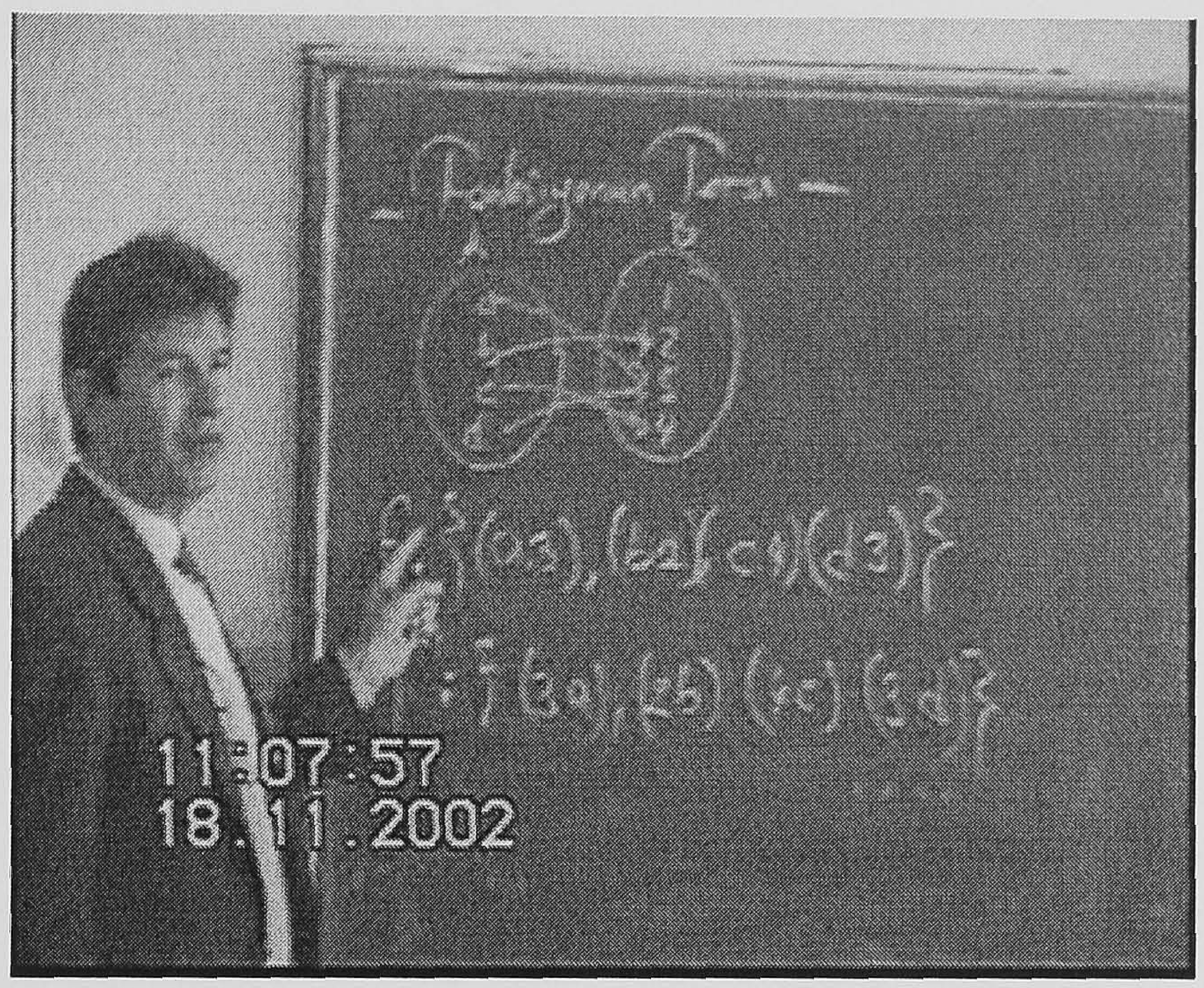
He then asked "*what is the function? It should be* $f(x)=x$ *so what is this?* (pointing to $(m-3)$ in the expression). *Redundant. We should remove it.*" He then equalised $m-3$ to 0 and found $m=3$. He added "*What if it was* $f(x)=(m-3)x+n-5$*?...We said the coefficient of* $x$ *should be 1 and the constant term should be 0.*" He equalised $m-3$ to $1$ and $n-5$ to $5$ and found $m=4$, $n=5$. He then said "*What kind of function is it when m is 4 and n is 5? An identity function.*"

He then explained another function type (the onto function) on a diagram. He said "*if there is a free element in the range set*" and wrote $f(A)$ $f(A) \subset B$. He then explained that "*if there isn't any element free then it is called an onto function. I mean* $f(A)=B$*. Lastly the 1-to-1 function. If the differing elements in the domain*". He drew a diagram while he was explaining. He then made his explanations on the diagram and wrote the condition for a function to be 1-to-1 in its formal structure.

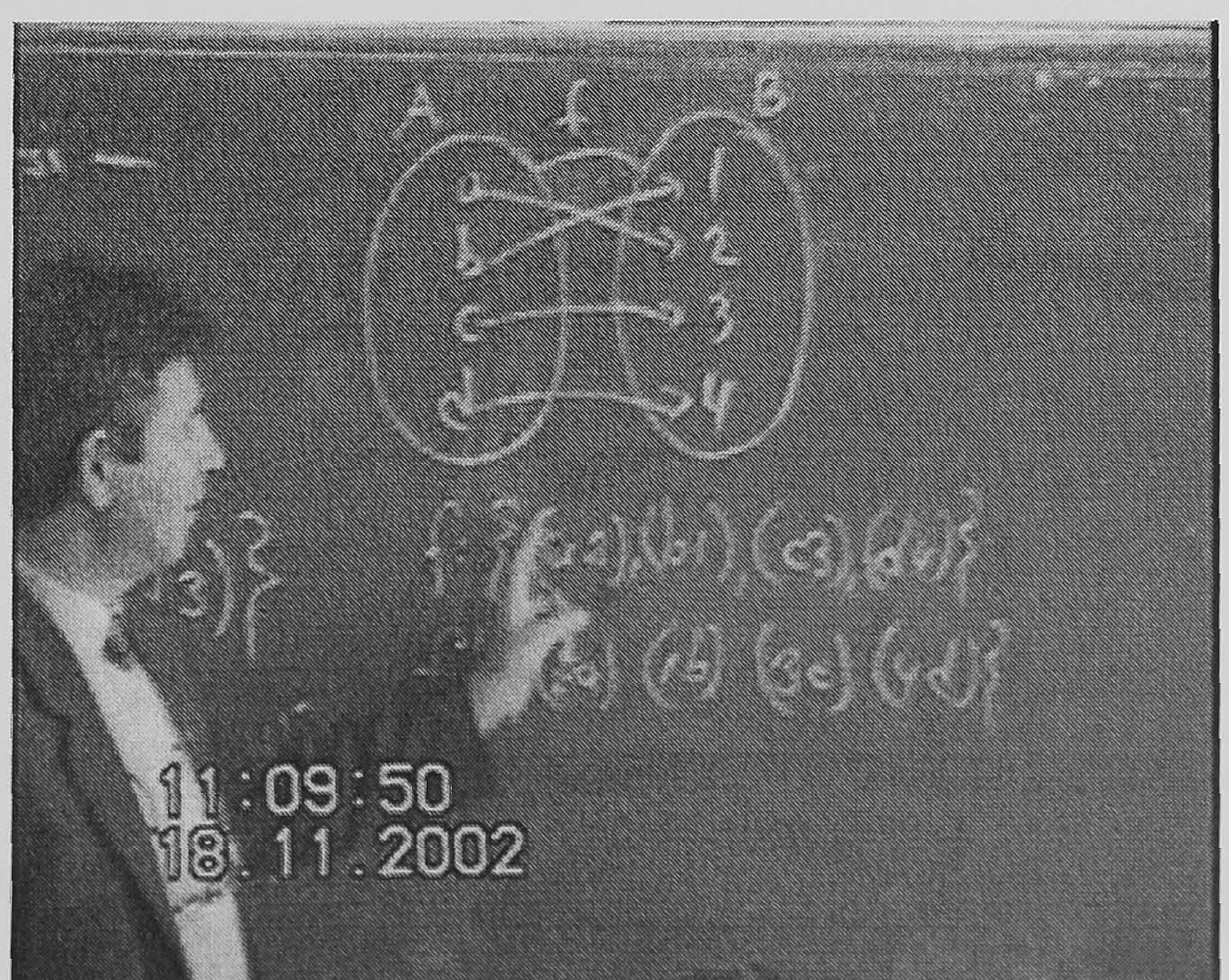



He then made changes to the diagram so that it was an example of the 1-to-1 and onto function. He then got back to the title he had them written before: inverse function. He asked the responsible student to clean the board. While students were talking to each other and/or making notes he went to his desk and found the inverse function section in the textbook he uses. When the blackboard was cleaned he asked them to listen.

He wrote a title on the board '*Inverse Functions*' and asked "*what is the condition for a function to have an inverse?*" Without answering he drew a diagram and wrote a relation between two sets in the form of listed pairs, $f:\{(a,3)(b,2)(c,4)(d,3)\}$, based on the diagram. He asked and answered 3 simple questions to show that the relation he wrote on the board indicates a function. "*Do all elements of the domain set have an image? Yes, they do.*" "*Is there an element without an image? No*" "*Is there an element with more than one image?... No...Is the relation f a function? Yes it is a function. If we get the inverse of it...how do we get the inverse relation? We change the places of the elements*" and wrote $f^{-1}:\{(3,a)(2,b)(4,c)(3,d)\}$.

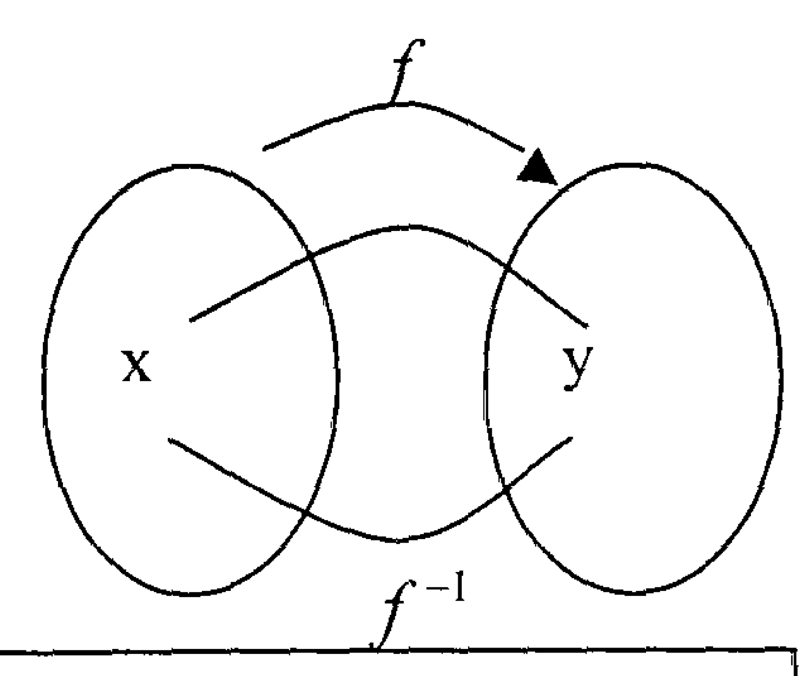



He used a series of question-answer sequences for students to realise a condition for the inverse function. He said "*Let's see...Is the inverse of f a function?*" Students answered '*No*' together as they did for many questions he asked. He said "*No, why... Because 3 goes to a, and 3 goes to d. How many images does 3 have? It has 2 images. So, the inverse is not a function. Well then tell me what is needed for the inverse of a function to be a function?*" He waited some time for students to answer his question. Some of them said "*onto*". He said "*Yes, it should be onto*" He then immediately drew another diagram and wrote a relation between two sets in the form of listed pairs, exactly as he did previously. Then he wrote the inverse of it.



He asked "*does it indicate a function? Do all elements have images? Yes they do. How many images do they have? Only 1. Then f relation is a function...Well let's consider the inverse of f. 2 goes to a, 1 to b, 3 to c and 4 to d. Then this is also a function. That means for the inverse of f to indicate a function f should be 1-to-1 and onto.*" He wrote 1-to-1 and onto below f and asked "*is it enough to be 1-to 1? It isn't enough to be 1-to 1, why? If there was 5 as an element* (he wrote a blurry 5 on the range set), *5 would be free in the inverse* (pointing to the domain set), *it would not have an image in the inverse. So what happens then? It wouldn't make a function. That means in order for a function to have an inverse, the function given to us should be a 1-to-1 and onto function.*" He wrote "*it should be a 1-to-1 and onto function*" on the board. He then wrote "*Definition:*" on the clean side of the board and underlined it for its importance. Some students said "*please let us write them*" and he answered "*write them, hurry this is the definition of inverse function...Let's write it down without talking, hurry up*". He allocated some time for students to write them down, then he looked at the textbook and started dictating "*If f is a 1-to-1 and onto function from set A to set B, the inverse of f is a function from B to A. This function of* $f^{-1}$ (wrote $f^{-1}$ on the board) *is called the 'inverse function of f'*" He dictated the definition to students in a very slow pace so that all students could write it down. Despite this, some students asked Mahir to repeat the last few words, and he did repeat them. He thus made sure that everyone had a copy of the formal definition of an inverse function. He then drew a diagram on which to make some explanations.

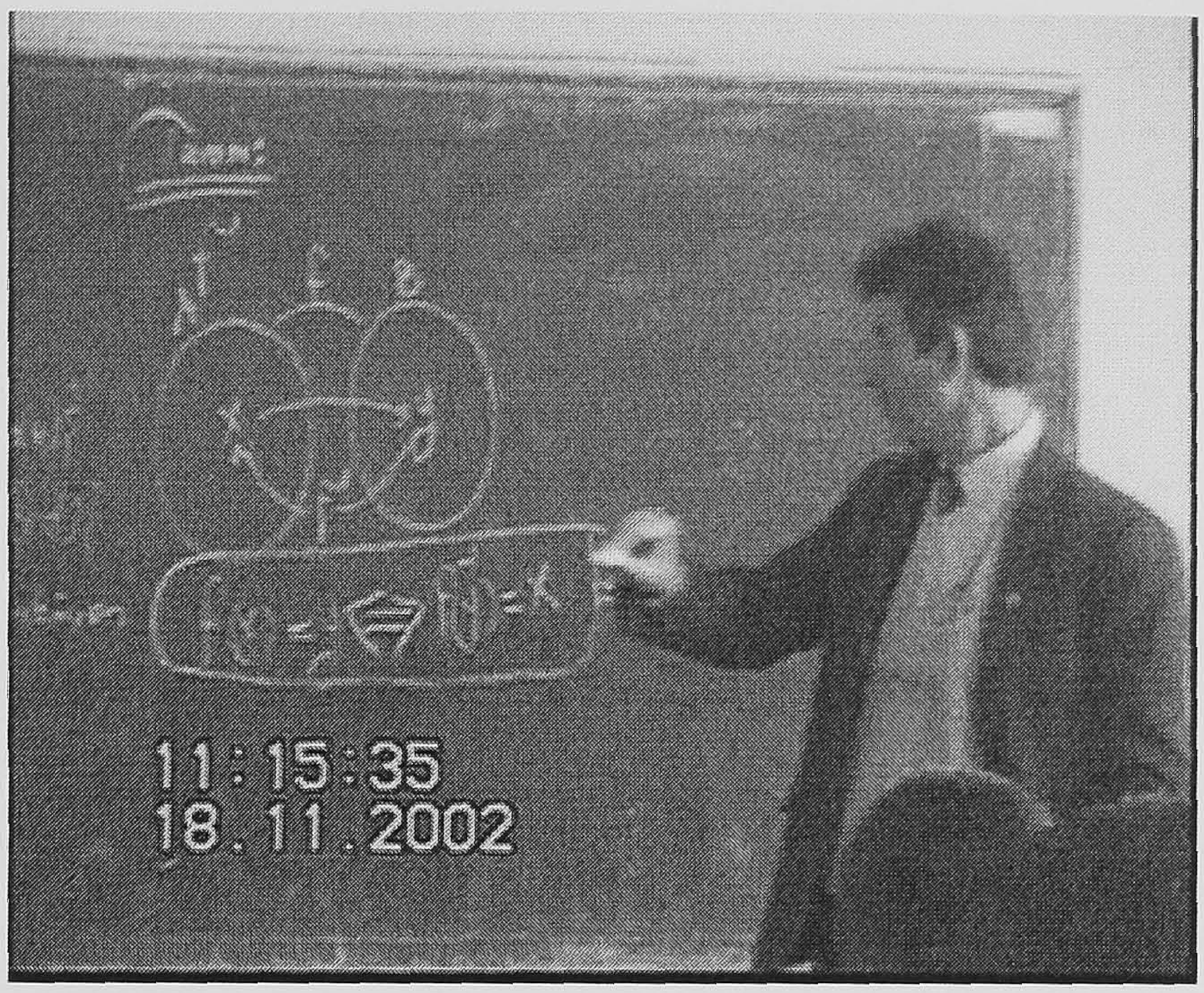
Definition:




$$\boxed{f(x) = y \Leftrightarrow f^{-1}(y) = x}$$

He asked "*through the f relation, where does x go? To y.* (pointing to x then y) *So we can write this down as f(x)=y. This is a 1-to-1 and onto function. Its inverse is also what? a function.* (drew an arrow from $y$ to $x$ and wrote $f^{-1}$ on it) *And that is the inverse of f isn't it.* (pointing to the inverse) *So how do we write this?*" and wrote '$f^{-1}(y) = x$'.

He added "*so if f(x) is equal to y then* $f^{-1}(y)$ *is equal to x. If* $f^{-1}(y)$ *is equal to x then f(x) is equal to y.*" He wrote '$f(x) = y \Leftrightarrow f^{-1}(y) = x$' and he highlighted the importance of it by drawing a rectangle around it.




He added "*We shouldn't forget this!*" He gave some time for students to copy down the blackboard. After looking at his textbook, he wrote an example

$$\boxed{f(x) = x^2 - 4x + 2 \Rightarrow f^{-1}(2) = ?}$$

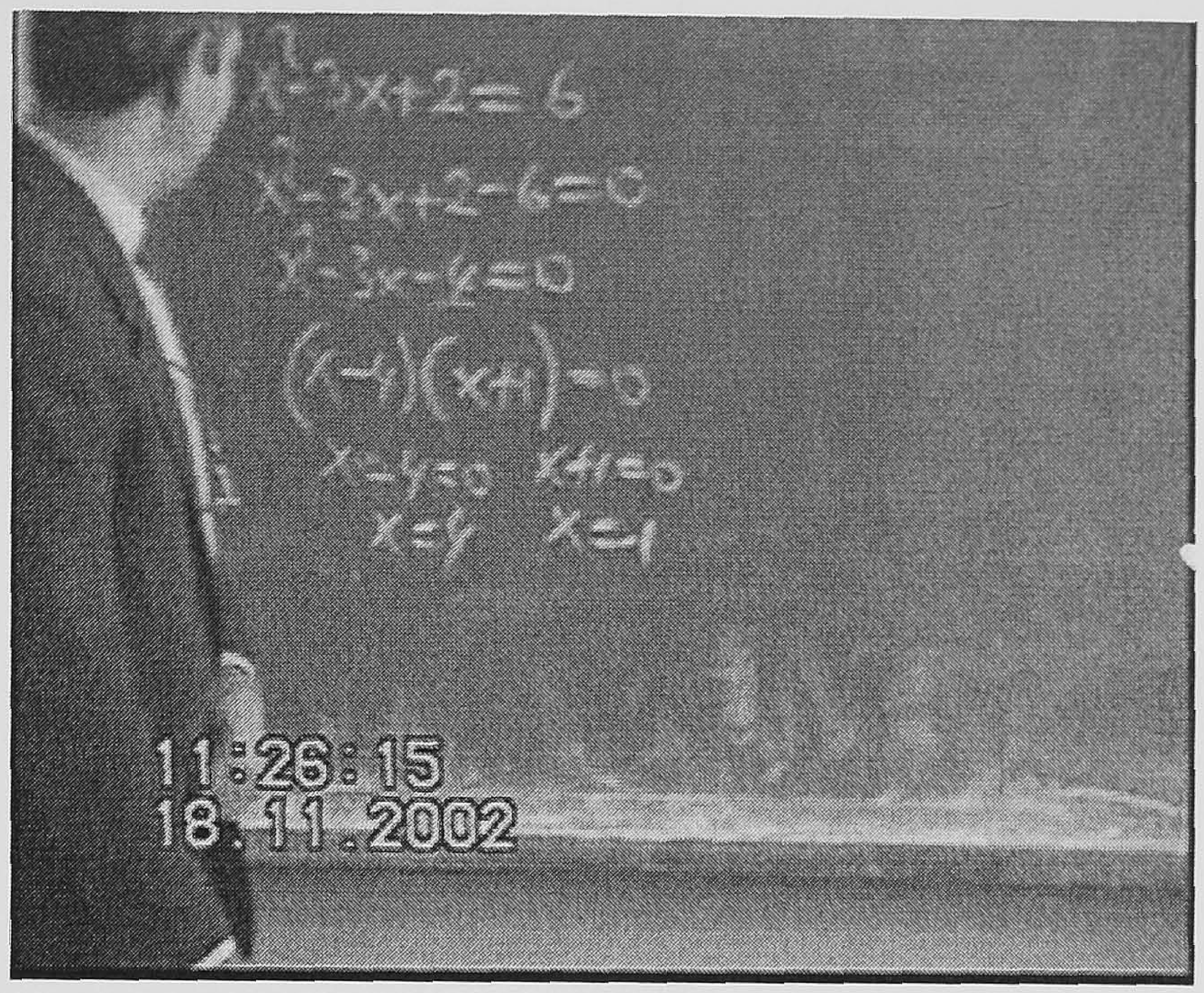
He read it out loud for all students to have an accurate copy of the example and then he said "*The one above is our definition and the one below is our example.*" While students are trying to solve the example, he walked between desks and inspected students' activities and commented on some students' work. At this point of the lesson the class was quite noisy because students were comparing their results with others and discussing their solutions. After this engagement time, Mahir turned to students and asked, "*look at this here, what is* $f^{-1}(y)$ *equal to? Equal to x. What is y here* (pointing '$f^{-1}(2) = ?$' in the example)*? What is the value of f(x)*" Some students suggested $x^2 - 4x + 2$ but he said "*What is f(x) equal to? y. What is the value of y? 2. Then (that means)* $x^2 - 4x + 2$ *is equal to 2.*" *He wrote* $x^2 - 4x + 2 = 2$. He then paused and looked at the students and he seemed like checking if there is a sign of confusion on the students' faces. He asked, "*Did we understand this?*" Many students answered altogether "*No we didn't*". He said, "*I will repeat it again. Look, look!.* (He pointed the formula '$f(x) = y \Leftrightarrow f^{-1}(y) = x$') *if f(x) is equal to y then* $f^{-1}(y)$ *is equal to what? x.*" Mahir pointed $f^{-1}(y) = ?$ and wrote $f^{-1}(2)$ exactly under $f^{-1}(y) = x$ on the right hand side of the formula in order for students to be able to understand why the value of $y$ is equal to $2$ in that example. He then asked "*what is the value in the place of y? 2*" He then wrote 2 under $y$ in $f(x) = y$ on the left hand side of the formula and said "*let's come to this place, what is f(x) equal to? 2. f(x) is given to us* (pointing $f(x)=x^2-4x+2$ in the example). *What is it?* $x^2-4x+2$." He then equalised $x^2-4x+2$ to $2$. He then cancelled out $2$ from both sides and wrote $x^2-4x=0$, then he obtained $x(x-4)=0$, then he divided it into two parts. $x=0$ and $x-4=0$ thus $x=4$. He said "*What is the solution set? 0 and 4*".

One student asked "*are you going to ask questions in this form (in the examination)?*". Mahir replied "*maybe*", he then asked for half of the blackboard to be cleared. While a student was doing the clearing, in the meantime he inspected some students' notes and commented on some of them. The class was noisy again. He then went close to his desk where the textbook is left open. He looked at the textbook, turned back and said "*you are making noise*" and gave an authoritarian frozen look that seem to have a meaning of 'keep quiet'. He said "*there is another example*" and wrote the example and said "*what is the value of* $f^{-1}(6)$ *?*

| |
|---|
| f: R-{-1}$\rightarrow$ R |
| $f(x)=x^2-3x+2 \Rightarrow f^{-1}(6) = ?$ |

He then walked to the back of the classroom and looked at the board from there. He explained the domain and range set in response to a student question. The student who asked the question behaved as if he could not read it in a funny way. The teacher realised this and put his hand on his neck and started slapping slowly in a playful manner. Some students were trying to solve the example and some were observing the incident. When the last playful slap made quite a noise the students all started laughing along with him. Mahir then walked to blackboard and said "*OK. That's enough...Listen...OK. Listen*" He hit to blackboard with his hand few times to get attention of all. When he realised that, Ali, the student next to the one he slapped, was still very active he said, "*Ali...behave! Or I will do the same to you. So behave!*" He then started to describe the solution. He made similar explanations to the previous one and solved it in exactly the same way by equalising $x^2-3x+2$ to $6$. As he did in the solution of the previous example, he showed each step of the solution explicitly. At the end he said "*what is the solution set?*" and he verbally stated the solution set.

$$
\begin{aligned}
&x^2-3x+2=6\\
&x^2-3x+2-6=0\\
&x^2-3x-4=0\\
&(x-4)\,(x+1)=0\\
&x-4=0 \quad x+1=0\\
&x=4 \quad x=-1
\end{aligned}
$$




One student asked "*why did not we do it the same way?*" He replied "*we did do it the same way!*" Students started humming. Another student pointed the previous example and asked, "*there...we did not factorise it*". Mahir and another student replied at the same time "*We did factorise it there!*" and Mahir alone "*We did it for both...We took the common multiplier there*" Mahir then went back to the previous example on the board and showed that he had solved it in the same way. The student seemed convinced about it and Mahir let them write it down and in the mean time he talked to another student individually and explained to her the solution he had presented. Although I could not capture the conversation between them very clearly, he seemed to be comparing the last two examples with each other as she was turning pages forward and backward. Another student raised her hand and Mahir saw it and nodded slightly. Having received permission to leave her desk (the nodding), she came close to him and showed her notebook. He was looking at her book and the classroom as very noisy. He suddenly put his head up and shouted with a very serious and authoritarian tone "*Behave!... You are messing it up! Please* " When Mahir finished looking at her solution, he disapproved of it and sent her back to her seat. He then looked at the textbook and said "*Look! We have another example.*" He wrote the example:

$$
\begin{gathered}
f{:}R \rightarrow R\\
f(x)=3^{2x+1}\\
f^{-1}(28)=?
\end{gathered}
$$

When he finished he read it aloud as he did for the previous examples. One student started guessing answers, when the bell rang as a sign of the end of the lesson. He immediately went back to blackboard and said "*Have a look at here! Again similarly what f(x) is going to be equal to? y.* (wrote *y* under 28 in the example) *Isn't it the value of . It is the value of y.*" He then equalised $3^{2x+1}$ to *28* and found $x=1$. He then put the chalk back in its place and said "*Have a nice lunch*" as it was lunch break for the school. The lesson ended.

### 4.4.4 Mahir's Beliefs

The interviews took place in two sessions. He arranged a free room in the school for the interviews. Looking at my interview questions, he stated that he could attempt to answer some of the questions without me asking them. Once he had finished with his explanations, I went through some second level questions, which were not covered by his explanations.

At the beginning of the interview, Mahir indicated his concern about students' psychological readiness for instruction. He believes that students come to high schools or primary schools, with some prejudice against mathematics. To him, the primary source of this is the impressions they get from their sisters or brothers that mathematics is a hard subject. He put some responsibility on himself and other mathematics teachers. He relates this to a general impression mathematics teacher have as looking "tough". Students regard mathematics as a subject that they cannot be successful in before actually attending to school. He believes that getting rid of this is the first condition to teach mathematics to students. He said:

> **T** In order to overcome that, mathematics teachers, I mean us, should be nicer towards them. When we enter the classroom, we should have a more smiling face...if a student likes his teacher, he likes the subject. This is how it is in Turkey, in fact, this is how it is in the world. If a student has a liking for his teacher, then he has a liking for the subject. If he has a liking for the subject, then he wouldn't have a fear of failure in the subject. If you consider mathematics teachers, there are generally tough looking teachers. We should get rid of this tough look. I am doing my best on this matter. We should not make them scared by our appearances. They already come with some prejudice and if they get scared by our appearance then they get rid of paper and pencil. If not from their hands they get rid of it from their minds, I mean psychologically. If students are like ours, then they start listening to the teachers and if they start listening carefully, they could learn [mathematics] with the slightest review at home.

In this extract there are also clues to his ideas about his perception of the students and how students learn. Mahir seems to have confidence in the students' potential to learn which is indicated in his expression '*students like ours*'. He also seems to believe that students learn greatly by listening carefully and asking questions. He, however, prioritised listening to any other student activity and, to him, failing this creates a chain of unfruitful activities. He believes that after listening to the teacher carefully and asking about the things they could not understand, students should review their lesson notes in notebooks at home. He seems to suggest a chain of student activities for students:

- Listening to the teacher
- Reflecting on it by writing down the lesson notes
- Questioning the teacher's explanations, asking for explanation
- Reviewing the lesson notes at home

He indicated that students should listen to him when he is explaining, and then when he prompts they should copy the blackboard down into their notebooks.

> **T** I tell students not to write when I am explaining. I tell them to listen first. They should listen to me and then write them down. If they try to write and listen at the same time they can't think about it. They just copy things down [automatically]. There is no opportunity to reflect on what I say. Then they don't ask questions and if they don't ask questions there is no learning. In this case, they will have unanswered questions in their mind during home reviews. That means they didn't learn it. To learn, there shouldn't be any question in mind...They must get answers to their questions in the classroom. Listen carefully and get the answers for the questions. I see no reason for them to be unsuccessful if they do so.

From the extract, Mahir seems to consider notebooks as an important material for student learning. To him students learn by listening to him and then ask any questions they have about his explanations and then write things down, which should be followed by reviews at home. He also points out that he is making extensive use of notebooks as learning material.

> **T** I am a kind of person who believes that enough examples should be solved in the classroom. The notebooks I suggest them to buy and use are running out of pages during a semester and they need another one. I solve more than the necessary number of examples.

He indicated that during the reviews at home, notebooks should be used for two things; for going through definitions and for re-solving the examples written. He also considered examples as a "*consolidation tool*" to make students more fluent with solutions.

> **R** Could you talk about examples? What is the use of them?
> **T** What is an example...examples are consolidation tools.
> **R** Consolidation tools?
> **T** They are a kind of tool to consolidate what is learned. For instance, we solve many examples. What's the use of this? It consolidates things that are learned. It makes them more practical, it makes the solution more practical, easier. This easiness is in our head and by

means of examples. We used two examples in composite functions [as part of syllabus]. The examples were alike and it was easier to solve the third and fourth examples [after first two]. These examples help solving more practically, and faster

Mahir perceives examples as developmental materials that make further solutions easier. To him, examples resemble one another and are linked to one another. The solution of one helps for the following examples. His beliefs regarding mathematics seem to follow a sequential logic where one piece links and/or leads to another. This is true for his example selection as well as the theoretical information he teaches. To him, students build their knowledge of examples which helps in solving further and more complex ones. He believes that mathematics should be taught step by step from easiest to hardest. In explaining his view he uses a ladder metaphor, where easiest examples are signified by the first few steps and the hardest ones are at the top of the ladder. In this metaphor, learning corresponds to climbing up. He stated

**T** No matter which topic I am teaching I always start with the easiest part.

**R** With the easiest.

**T** With the easiest yes. For instance, if I am teaching addition then I try to teach adding 1 with 1. For instance, in teaching functions, I try to explain what a function is. Once they have learned it with some examples. Then I try to teach basic calculations with functions, like additions of two functions, multiplication of two functions, multiplication of functions with 2, subtraction of and dividing functions. I mean students learn basic calculations in functions.

**R** Do you select these functions?

**T** Yes. I select them from the easiest ones. It is a common thing. Most of our colleagues begin with easiest and climb to the hardest. I mean we go up the stairs slowly. I the try to go from lowest step of the ladder to the highest step of the ladder. Students learn by stepping up higher. On the last step the functions are learned. In teaching composite functions, I explain how to get the composite function verbally and then I immediately give a very simple example. Let's say, x+1 and x+2, what's the composite function of these functions? They put x+2 in x+1 and they get the answer. Afterwards I make it harder. I give second function more complex.

**R** Do you mean squared or cubed?

**T** No. It may not be squared or cubed but you higher the level than x+1 and change it with 2x+1. You raise it to $\frac{3x+1}{5}$. You can raise it to a square like $\frac{x^2+1}{3}$. And then we try to let them inverse the function and then put it into the second function. I give the inverse function of a function and then ask them to inverse it first and then get the composite function. As I explained from easiest to hardest.

**R** So...you have a way of doing it then.

**T** I believe when there is a sequence I could teach better and students learn better

In this extract it can be inferred that Mahir has his own particular sequence in his mind as to what makes an example hard or easy. It mainly consists of raising the numbers involved in the functions. It could be:

(1) raising the number to be added to the function;

(2) raising the coefficient of the variable in the expression;

(3) making it division of two functions;

(4) squaring or cubing the variables in the expression;

which produce a harder example. Sticking to this sequence is significant not only for students to learn better but also for him to feel better about his teaching.

Mahir also believes that the students learn better by "*living it*" as he put it. He emphasises the significance of students' learning through experience.

**T** It is a good method and I used this sometimes. [In teaching function] I bring the students to the board and I define a function by pairing the students. I make the boys the domain set and the girls the range set. I tell boys to choose one girl. If they all choose different girls then I say this is a 1-to-1 function. If some boys choose the same girl then I say this isn't 1-to-1. If all choose the same girl then I say this is a constant function.

**R** Hmm...

**T** I mean teaching them by living/experiencing it. After that, they don't forget the constant function or 1-to-1 functions. They don't forget because they experienced it, they saw it.

This is also evident in his ideas regarding student participation. Although he suggests listening as a necessary behaviour in students' learning, they should be actively participating in the lesson, rather than passively listening.

**R** Do you invite them to come to board?

**T** Of course. I always tell them 'come to the board and don't wait for me to invite you [come without asking] and you should compete with each other as to who will come to the board fist and most. The more you come to board the better for you. You learn better on the board than in your seats.

**R** What's the difference?

**T** Let me put it this way. When the kids come to board, they gain self-confidence. Some kids are afraid of coming to board and they overcome it by doing so...It helps students to trust in themselves.

**R** So you encourage them then?

**T** Yes, I encourage them, I implement it. In some classrooms I even give candies to encourage them to come to board, but they still don't come.

It seems that Mahir is not only interested in students' cognitive development but also in trying to improve their educationally desired qualities. He is working on students' self-confidence, through inviting them to the board and making them more actively involved in the lesson.

Mahir is of the idea that the aim of examinations in the school is to inform teachers to what extent students have learnt and as a result of this he tries to cure the parts that are not well understood.

> **T** ...If the exam results are at a certain level we consider the topic has been learnt. If under, it means they did not learn it well and this is the aim of examinations. To inform us to what extent students have learnt. After the examinations we try to make up for the weak parts. For instance, if students are unable to solve a certain type of questions then it means they did not learn it and after the examination I try to teach them again. I mean I explain it and try to teach it again.

When I asked for the purpose of mathematics in schools, Mahir gave a broad answer.

> **R** What do you think is the purpose of mathematics in schools?
>
> **T** I believe that our lives are mathematics. I think our lives are all mathematics. For instance, when you were born you get a birth date and it is expressed by means of mathematics. You go to school and need books and pencils. There is mathematics again. How far is your home from school? It's mathematics again...Mathematics connects people to life. I can't think of life without mathematics. I believe that school mathematics is a lesson that prepares for life.

This was too general to focus, and I probed more specifically for the school mathematics. He answered the question by considering the type of school he is working in.

> **R** Hmm...to prepare for life. Lets talk about high school grade 1, 2 and 3 mathematics.
>
> **T** The students we have in this school are preparing for university. The graduates of our school aim at university...But unfortunately students' goal is getting good grades. This is an issue of the education system. It starts from the GPA. The students' aims is getting good marks rather than learning things. It should not be like that. All I am trying say is that they put all their effort on preparing for the university examination.
>
> **R** Well,...hmm...what's your aim then?
>
> **T** The aim of high school is to get the students into the universities and to prepare them for it [further education]...The aim of the mathematics we teach is to prepare them for the [education they will get at] university and secondarily for the examination. This is true for

other subjects in high school. They prepare students for university [education not the examination].

As can be seen, the teacher is not happy about the students' goals in the school is being just a high grade. He seems to expect that the students should come to school with the aim of learning rather than getting high grades. Mahir narrated an interesting conversation between him and his students when he was explaining the aim of mathematics in state schools. The conversation reflects an important point about his view of the psychology of students in SS and PC.

**T** ...one day I was talking to my students I said 'you are attending private courses and you are studying hard'. They said 'of course. When we are having some test there is no noise [in the private course] but here [in the school] some students make a lot of noise I mean they are unable to concentrate and therefore the atmosphere [to study and learn] is not created.' I asked them 'why the atmosphere isn't created?' they replied 'we don't know'. I asked 'is it because you are paying for it' they replied 'it probably is' and continued 'we are like in competition there [in a private course] , everyone has the goal of [entering the] university' I asked 'you are Ayse[3] here [in the school] and you are Ayse there [in a private course], what's the difference? Are you becoming Fatma there? Do you change your brain?' they said 'No. we are same but the atmosphere is not the same. There isn't a serious [studying] atmosphere here. We are unable to create it [a studying atmosphere]...The education here [in the school] is compulsory and the education in private courses is intentional. They go to private courses because they want to, but, they attend state schools because they have to. This is because they can't go to university without graduating from high school. They come to school because they are required to. Because they come with that [intention in mind] they don't take it seriously but they go to private courses with intention and they pay for it so they take it seriously there. That's my opinion.

From the conversation between him and his students, it can be said that the environment is not only different for the teachers but also for students. He attempted to close the gap between the '*atmosphere*' of the two types of institutions by asking a few question and tried to motivate his students to create a better studying environment. Failing this, Mahir turns his attention to the nature of the two institutions to explain why this being the case. I prompted him for further explanation.

**R** What do private courses do?
**T** There isn't much of a difference. Private courses prepare [students] for the university entrance examination. We prepare [students] for [education at] university.
**R** You primarily aim at university and secondarily university

[3] Ayse and Fatma are Turkish names.

examination?

**T** Of course.

**R** We have got a bit more of a general purpose. Preparing for life in general. Private courses aim only at the university entrance examination.

Mahir makes a distinction between the goals of the institutions as well. He perceived that while SS have a "*general purpose*" and prepare students "*for life in general*", PC have a singular goal "*only the university entrance examination*". Mahir quoted his students on talking about the teaching of mathematics in PCs[4].

**T** My students said 'we learn in state school in detail but private course isn't like that. They [teachers in private courses] give the main points and immediately go on to solving examples.' This is the difference between school and private course. We try to give topics in detail, try to teach in detail. In private course they give the main points and solve examples right away. My students told me that, in private course, they were given the main points of the functions topic and it was followed by solving past examination questions and examples that are likely to appear on the next examination.

Mahir made a distinction between SSs and PCs in terms of how much detail teachers provide in the theoretical part of the mathematics. To him, SS teachers give more detailed information while PC teachers only provide the main points. I asked him to delve into the word 'detail'. He used the ladder metaphor to explain his reasoning.

**R** You talked about details. What's that 'detail' then?

**T** We start from the smallest thing. From the easiest and go towards harder. As far as I know it isn't gradual in private courses. The definition of function can be given [in a private course] but the questions may be asked from the highest step of the ladder. Some from middle step. We don't do that way. What we do is we try to climb the ladder step by step. We give the hardest [example] at the end. But from what I heard it isn't like that in private courses. They go straight into solving examples, and these examples may be very hard [for students to solve], some may be easy. I mean it isn't from easy to hard.

He believed that SS teachers begin with the easiest examples and gradually make the examples harder[5]. In contrast, PC teachers did not do sequence the examples in this manner. To him SS teachers' instruction required students to climb the first step of the ladder first and then the second step and so on.

**R** What if there wasn't [a university entrance] examination?

**T** As I said it before, because we prepare our students for university [not

[4] Possibly because he never worked in a PC.

[5] How is he making the examples harder is explained previously.

the examination] there would not be any difference at all. We would give the same education to our students.

**R** Would that influence your classroom instruction, such as the type or number of examples you give?

**T** No.

**R** Then can we say that the university examination does not affect your classroom instruction?

**T** Of course [it doesn't].

Mahir clearly indicates that his instruction is not affected by the existence of UEE.

# CHAPTER -5 SYNTHESIS

This section is designed to bring the results presented in the previous chapter together. The aim is to synthesise the results of the study which could help making sense of the discussion in the following chapter. In the first section of this chapter, I present a comparison of the SS and PC teachers' overall lesson structures. In the second section, I present a synthesis of the findings from the teachers' classroom practices in the 'content' segment of their lessons. The final section consists of a synthesis of the findings from the 'example solving' segment of the teachers' classroom practices.

## 5.1 Organisation of the Overall Lesson Structure

The video recordings suggested that each mathematics teacher I observed had some differences in the structure of different lessons. That is to say, there was no standard lesson structure that they followed in each lesson. However, each teacher had a pattern of structuring their lessons. SS teachers Ayten and Mahir spent time on the content segment to explain theoretical information and then supported their explanations with passive examples. This was followed by a number of active examples for students to practice. They then go back to a content segment for the following topic. This cycle was the common overall structure for SS teachers' practices. In contrast, PC teachers' (Nuri and Saban) lessons involved quick presentation of the theoretical information (content segment) and this is followed by several active examples solved in a very short period of time. Thus, their lessons were dominated by active examples. To make the comparison easy and to help the reader to visualise the teachers' lesson structures I have made an illustration of the lesson structures of two teachers from different institutions (Saban as the PC teacher and Ayten as the SS teacher) in Appendix D. In order to have more comparable lessons, the lessons chosen are actually the lessons that are presented in the results section, where the teachers are introducing the concept of inverse function. Each box in the illustration represents a segment of lesson (content, example solving) and the size of each box also represents the time spent on each segment.

## 5.2 Content Segment of the Lesson

In this section, I will attend to similarities and differences between the four teachers' practices in the *Content* segments of a lesson. I will illustrate the teachers' practices of *Content* with exemplary instances from the cases. Before going into the exemplary instances, we need to recap the meaning of the *Content* segment. This segment is basically the theoretical segment of the lesson where teachers provide definitions of concepts and explain the concepts and procedures in mostly abstract terms. During the explanations, teachers may give typical examples of concept(s) or abstract description of the phases of the mathematical processes involved. Therefore, teachers give passive examples in this theoretical part of the lesson to help students comprehend facts, definitions and theory.

| |
|---|
| INVERSE FUNCTION<br>A function f: A→B, when<br>f:{ (x,y)! x∊A, y∊B}is 1 to 1 and onto,<br>The function $f^{-1}$:B→A<br>$f^{-1}$:{(y,x)! (x,y)∊f}is called inverse function of f. |

The teachers in this study have shown similarity in the way they order their content segments. First they introduced the definition of the term function, and then they explained types of functions. The types of functions involved are; constant functions, 1-to-1 functions, onto functions, and identity functions. Teaching these functions is followed by inverse functions. Composite functions are taught as the last content segment. This seems logical as it does not seem to make much sense trying to teach, for instance, inverse functions prior to 1-to-1 functions.

The main sign of the beginning of this segment is that all teachers wrote the title or sub-subtitle of the topic part to be presented. After this point, the PC and SS teachers showed remarkable differences between them. SS teachers, Ayten and Mahir, gave the theoretical part in abstract terms. When the content segment involved a concept like (function, constant function), they consistently provided formal definitions. They also used passive examples to support the students' comprehension of the topic. For example, in the lesson I presented in 4.3.3., Ayten wrote the definition of the inverse function on the board and then explained it verbally. Her explanations are based on passive examples she provided. She said:

> This is the story part, it is obvious that you did not understand this. {Ayten-5}

She made explanations on two points

> I want to emphasize two points. A function has to be 1 to 1 and onto in order to be inversed...second point, while f contains (x,y)'s the inverse function contains what?...(she waited here for students to respond) (y,x)'s where x and

y have changed their place (She underlined 1 to 1 and onto and (x,y) and (y,x) on the board). {Ayten-5}

There are three main points that arise from this exemplary instance: Firstly, Ayten acknowledges that students "*didn't understand*" it. The explanation she made based on a definition is focusing on conditions for a function to be inversed and the meaning of inversion as changing the place of values in each element -like (2,3)- of the function. Therefore, what Ayten emphasises, and thus expects students to learn, based on this definition is to *know that*[1] a function needs to be 1-to-1 and onto to be inversed and the process of inversion (as swapping the place of x and y).

Secondly, Ayten is aware that theoretical information on inverse functions in the form of a formal definition is not easy for students to make sense of. What she does in order to rectify this situation is to use passive examples[2]. Therefore, Ayten used passive examples as a complement to theoretical information in the content segment. That is to say, she used passive examples to support students' comprehension[3] of the abstract terms in the definition. In trying to understand teachers' practices, it does, therefore, seem rather helpful to think of passive examples as an integral ingredient of a content segment. (Yet they are still within the broad category of examples). Nuri's use of passive examples in connection with the content segment, which will be mentioned further in this section, also points in the same direction. In fact, it seems rather logical to use passive examples after the theoretical information presented. Used this way passive examples would, at least theoretically, support students' comprehension.

In contrast, PC teachers, first of all, established connections between the topic they are teaching and the UEE. They did this through explaining the type of questions that appeared in the past examinations and the type of questions that are expected in the next examination the students will sit. This not only puts the students in the context of an examination, but also makes students aware of expected question types. PC teachers made the connections prior to the topic and, in a sense, they used this information as 'advanced organizers' (Ausubel, 1960). In teaching inverse functions {Nuri-4}, Nuri

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[1] As opposed to know-how.

[2] Interview data supports this argument. She did acknowledge the use of examples with the aim of helping students to comprehend the theoretical information.

[3] Note that it is emphasising 'know-that' again.

exemplified this practice. Having written the title 'Inversing a Function' on the board. He said

> Well, we may know some formulas like what? The inverse of ax+b is (x-b)fa isn't it? There are some memorised routine formulas and templates. But it is very interesting, my friends, they [examination preparers] got out of the well-known formulas and templates in the topic of functions. There are very original past examination questions in functions and you will see them in a few moments. I mean, your knowledge of routine formulas or templates doesn't help you [in the examination], my friends. There are really good and quite original questions [among the past examination questions]. Now we will go through all of them together.{Nuri-4}

Nuri is warning students that the theoretical information they may have is not relevant to deal with examination questions in functions. He is informing students that the explanations he is about to make based on the example he is about to give is what is necessary for the examination. Nuri added:

> First of all I will explain this, it is very important! In order for the inverse of f(x) to be able to indicate a function, it should be 1-to-1 and onto. Now I will give you a very simple example.

He wrote an example[4] on the board. He explained that while this indicates a function, its inverse does not make a function, because one element 'b' in the range set is not mapped into A by inversing.

> Because it is not 1-to1 and onto, thus its inverse is not a function. Did we understand this? And this aspect [of inverse functions] is asked in the past examinations. It is really an intriguing aspect. While [before the examination] students were learning how to inverse [and the formulas], they [examination preparers] suddenly asked a question based on [this aspect of] the definition. I am sure my friends a lot of people who ignored [the definition of inverse function about] 1-to-1 and onto have stuck on that question. Because they wouldn't know what to do [in the examination] or what to think due to lack of this information...I will bring this question into your perspective now. This is an OSS[5] question. {Nuri-4}

Nuri then wrote a question that had appeared in a past examination and made further explanations on this aspect. As you can see Nuri used a passive example in such a way

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[4] The example he gave was rare instance of use of passive example in PC lessons.

[5] Please refer to university examination background information in chapter 3.

that it informs students' knowledge of examinations. Nuri also made sure that students are aware of the aspect of inverse functions that has been asked about in the UEE in the past. This example also demonstrates how Nuri made a smooth transition from content to an active example through putting the example in the context of examinations.

One important characteristic of PC teachers' practices during the content segment is that, unlike SS teachers, PC teachers focused extensively on the *know-how* aspect as exemplified above. This is despite the fact that they were teaching the same topic. An exemplary case (of a constant function) from Saban's lesson (See section 4.2.3.) further demonstrates this well. Saban wrote a title on the board "*Constant Function*". He said:

| |
|---|
| Constant Function<br>f(x)= constant or<br>f(1)= f(0)= f(2)= f(-1)=.....= constant |

> *f(x) is equal to a numerical constant. It doesn't matter what you write inside [of parentheses]* (referring to argument of the function). *Write 1 or write 2 or –1, it is up to you, the result is the same constant all the time. That means you can solve any question [involving constant function] with two methods: you can either solve it by equalising the polynomials, or you can also solve it by numerical value [technique] like this.* {Nuri-4}

Saban immediately went on to writing an active example on the board. He then demonstrated the 2 ways of solving the example[6]. Saban, like the other PC teacher Nuri, was quick to finish the content segment. And that was all his theoretical explanations on the topic of constant function. Although it requires one to define what is theoretical or not, which I am not going into, strictly speaking, it seemed to me that there was almost no theoretical aspect in this content segment. The information is presented to students in a way that privileges the know-how aspect over the know-that aspect. This is evident in the transition he made: "*Write 1 or write 2 or –1, it is up to you, the result is the same constant all the time. That means you can solve any question [involving constant function] with two methods*". It seems that the definition of constant function suggests to Saban the kind of solution method he can use.

It is important to be aware that similar to the other PC teacher, Nuri, Saban made a smooth transition from content to an active example through putting the example in the context of the examination. This is in contrast to the SS teacher Ayten's comment that

[6] Having observed Saban in many lessons and across different classrooms, I could say that this was a common mode of behaviour for him.

"*This is the story part, it is obvious that you did not understand this.*" Ayten used passive examples to make the transition from content to active examples, probably since the information she presented is relatively more theoretical.

Mahir's practice when he taught inverse functions also suggests the privileging of know-that rather than know-how. Having written the title, he asked:

> what is the condition for a function to have an inverse? {Mahir-5}

Mahir gave 2 passive examples after this question. Both involved a di agram of a relation. He then used a question-answer technique[7] and expected students to come to *know that* 'in order for a function to have an inverse, the function given to us should be an onto function' {Mahir-5}. The first passive example was a counter example, in which the relation given was a function but not the inverse of it since it was not onto. At the end of the first passive example he asked:

> Then tell me what is needed for the inverse of a function to be a function? {Mahir-5}

Some students answered "*onto*". The second passive example, however, was 1-to-1 and onto. Similar to the first PE, he used a question-answer technique. For instance, he asked:

> Is it enough to be 1-to-1[to have an inverse]? {Mahir-5}

By using this and some other questions to ensure students *know that* 'a function should be not only onto but also 1-to-1 to have an inverse' he then wrote "*Definition*:" and then dictated the definition from the textbook he used {Mahir-5}.

As we can see from the example above, similar to the other SS teacher Ayten, Mahir is privileging theoretical information or *know-that* during the content segment. He achieved this through a question-answer technique. He asked simple questions which students could easily answer and provided answers. He used the technique in a way that brings students to a conclusion: 'a function should be not only onto but also 1-to-1 to have an inverse'{Mahir-5}. This is in contrast to Nuri, where he put the same condition in the context of PEQs and supported his argument with an examination question that appeared in the past UEEs. It is also worth noting that in contrast to PC teachers, Mahir did not even mention the examination.

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[7] He asked a simple question and expected some answers to lead students towards a conclusion. Mahir used this technique in all of his lessons.

### 5.3 Teachers' Selection and Use of Examples

The examples teachers used and the way in which examples were utilised in the lesson constitute an important part of understanding teachers' classroom practices. This section therefore is focused on the teachers' use of examples in the course of their lessons, how teachers are similar to and/or different from each other. In doing so, I found the notion of 'privileging' useful. Borrowed from Wertsch (1991), Kendal & Stacey (2001, 2002) used 'privileging' as a key term for understanding the classroom practices of teachers who began to incorporate CAS calculators into their teaching. They define the term:

> Privileging is a construct to describe a teacher's individual way of teaching and includes decisions about what is taught and how it is taught (2001, p. 245). Privileging includes decisions about what is taught and how it is taught including: what is emphasized in the content (what is stressed and what is not stressed); what representations are preferred and ignored; the attention paid to procedures and concepts, rules, and meaning; and how much is explained or left to the students to work out for themselves (2002, p. 197).

They also point out that:

> Privileging reflects the teacher's underlying beliefs about the nature of mathematics and how it should be taught … it is moderated by institutional knowledge about students and school constraints and is manifested through teachers' practice and attitude (2002, p. 197).

With regard to my findings, I find the term useful in describing what is similar and different across these four case teachers. This section is divided into two sub-sections, where the first is mainly for quantitative indicators of teachers' practices and the second is mainly for qualitative indicators.

#### 5.3.1 Quantitative Indicators

All four cases have shown interesting privileging patterns in the way teachers made their selection and in their use of examples. The results also produced a number of questions. I will present an overall picture of these similarities and differences, and then I will go into the details of them.

#### *5.3.1.1 Overview of example solving segment across teachers*

To begin with, Table 5.1 shows the number of lessons analysed and the total number of active examples solved by each teacher. SS teachers, Ayten and Mahir solved a similar number of examples, 90 and 84 respectively, in a similar number of lessons, 15 and 14 respectively. Data collected from PC teachers' lessons also showed similarity between teachers' practices. Nuri and Saban solved 64 and 51 active examples in 6 and 5 lessons respectively.

| Teacher | Institution | Total number of active examples solved | Total number of lessons analysed |
|---|---|---|---|
| Nuri | PC | 64 | 6 |
| Saban | PC | 51 | 5 |
| Ayten | SS | 90 | 15 |
| Mahir | SS | 84 | 14 |

**Table 5.1** Number of active examples solved and the number of lessons analysed.

Although SS teachers seems to be solving more examples than PC teachers, the picture is in fact the other way round when we consider the number of AE solved per lesson. Table 5.2 shows the mean of the number of AE solved by each teacher.

| Teacher | Institution | The mean number of active examples solved per lesson | Standard Deviation | Institutional Mean |
|---|---|---|---|---|
| Nuri | PC | 10.7 | 1.37 | 10.5 |
| Saban | PC | 10.2 | 2.28 | |
| Ayten | SS | 6 | 2.36 | 6 |
| Mahir | SS | 6 | 2.69 | |

**Table 5.2** The mean number of active examples solved per lesson.

The table suggests two major points: a similarity between teachers from the same institution, and a difference between teachers from differing types of institutions. Firstly, the number of examples solved by PC teachers, Nuri and Saban, per lesson is very similar. Likewise, SS teachers, Ayten and Mahir, solved the same number of active examples. This means that there exists a similarity *within* institutions. Secondly, the table shows a distinctive pattern *between* institutions. PC teachers showed a tendency towards solving more examples than SS teachers per lesson. While the PC teachers solved 10.5 AE per lesson, SS teachers solved only 6 examples per lesson. This means that PC teachers, Nuri and Saban, solved 75 percent more number of AEs than SS teachers, Ayten and Mahir, did per lesson. Immediate questions that come to mind are "Can the similarity between teachers of the same institution be explained merely on the grounds of possible cognitive similarities?", "why do PC teachers have a propensity for solving more examples than their colleagues in SS?" and "To what extent is it possible to explain the tendency on the grounds of cognitive differences?" These questions will be addressed in the following chapter.

Now, I will address another question "How long does it take for teachers to solve the examples?" Figure 5.1 shows the mean time teachers spent on single AEs.

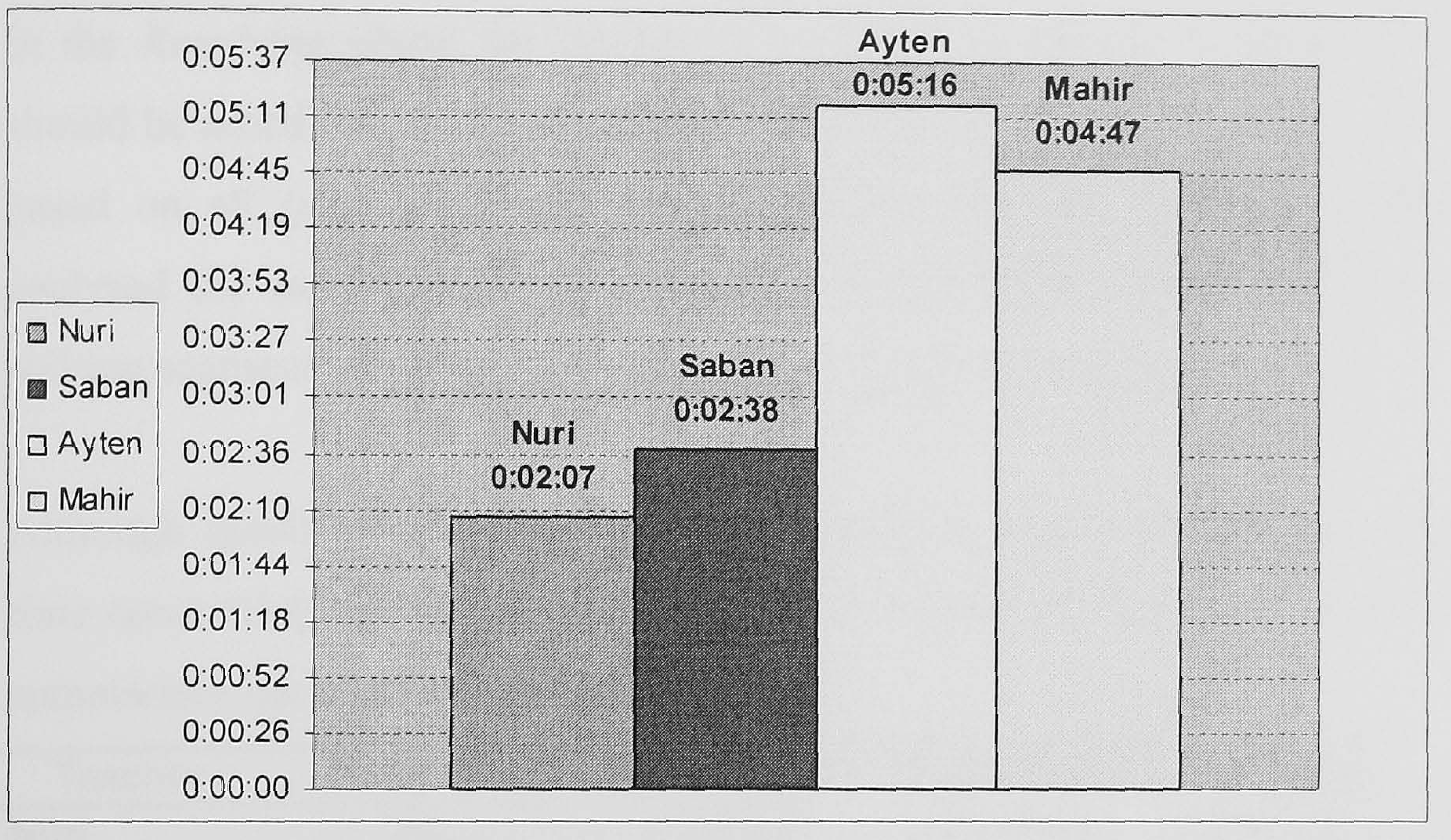



**Figure 5.1** The mean time teachers spent per active example.

Figure 5.1 suggests a *similarity within* institutions and a *difference between* the teachers of different institutions. It is clear that PC teachers Nuri and Saban have a tendency towards solving an example within much less time than SS teachers, Ayten and Mahir. While Ayten, at the highest end, allocate 5 minutes and 16 seconds for each example, Nuri, at the lowest end, allocate 2 minutes and 7 seconds, which is much less than half of Nuri's. That is to say, PC teachers spent 2 minutes 23 seconds and SS teachers spent 5 minutes and 2 seconds per example. The questions arising from this analysis in addition to the ones presented above are "What are the teachers doing in these example solving segments?", "What clues do the teachers' activities[8] within example solving segments provide us in order to explain the differences between Nuri-Saban and Ayten-Mahir?" and "Do PC teachers differ considerably in terms of what they do in an example solving segment of a lesson?" Analysis of the teachers' activities in the example solving segments will be addressed in the following section.

[8]Here and throughout this section, the use and meaning of word 'activities' is not as it is used by activity theory research.

### *5.3.1.2 Details of example solving segments*

Before going into the details of what is happening within example solving sessions, I remind the reader of the phases of an example solving segment. There are 3 main phases of an example solving segment: *Presentation*, *Engagement* and *Resolving*. *Presentation* is the phase in which the teacher presents the example, which will be solved later on. *Engagement* is the phase when students are engaged in solving the example presented. In the *Resolving* phase the teacher or a student solves the example on the board. It should be noted that the data presented here (e.g. presentation phase), is the mean value based on all (e.g. presentation phase of) the example segments of all the lessons analysed for each teacher. For instance, the presentation phase for Ayten's example solving segment is based on 90 examples in 15 lessons (See table 5.1).

Although quantifying qualitative example solving segments data was an arduous and time consuming process, it provided further insights into teachers' practices. Table 5.3 summarises the results for all four teachers.

| Teacher | Institution | Presentation | Engagement | Resolving |
|---|---|---|---|---|
| **Nuri** | PC | 00:00:23 | 00:00:32 | 00:01:12 |
| **Saban** | PC | 00:00:31 | 00:00:27 | 00:01:40 |
| **Ayten** | SS | 00:00:29 | 00:03:15 | 00:01:32 |
| **Mahir** | SS | 00:00:25 | 00:01:21 | 00:03:01 |

**Table 5.3** Mean time spent by four teachers in each phase of the example solving segment.

The production of a table for four teachers leads to a number of interesting and striking results. To make the results more visual I present the results of Table 5.3 in the form of a graph in Figure 5.2.

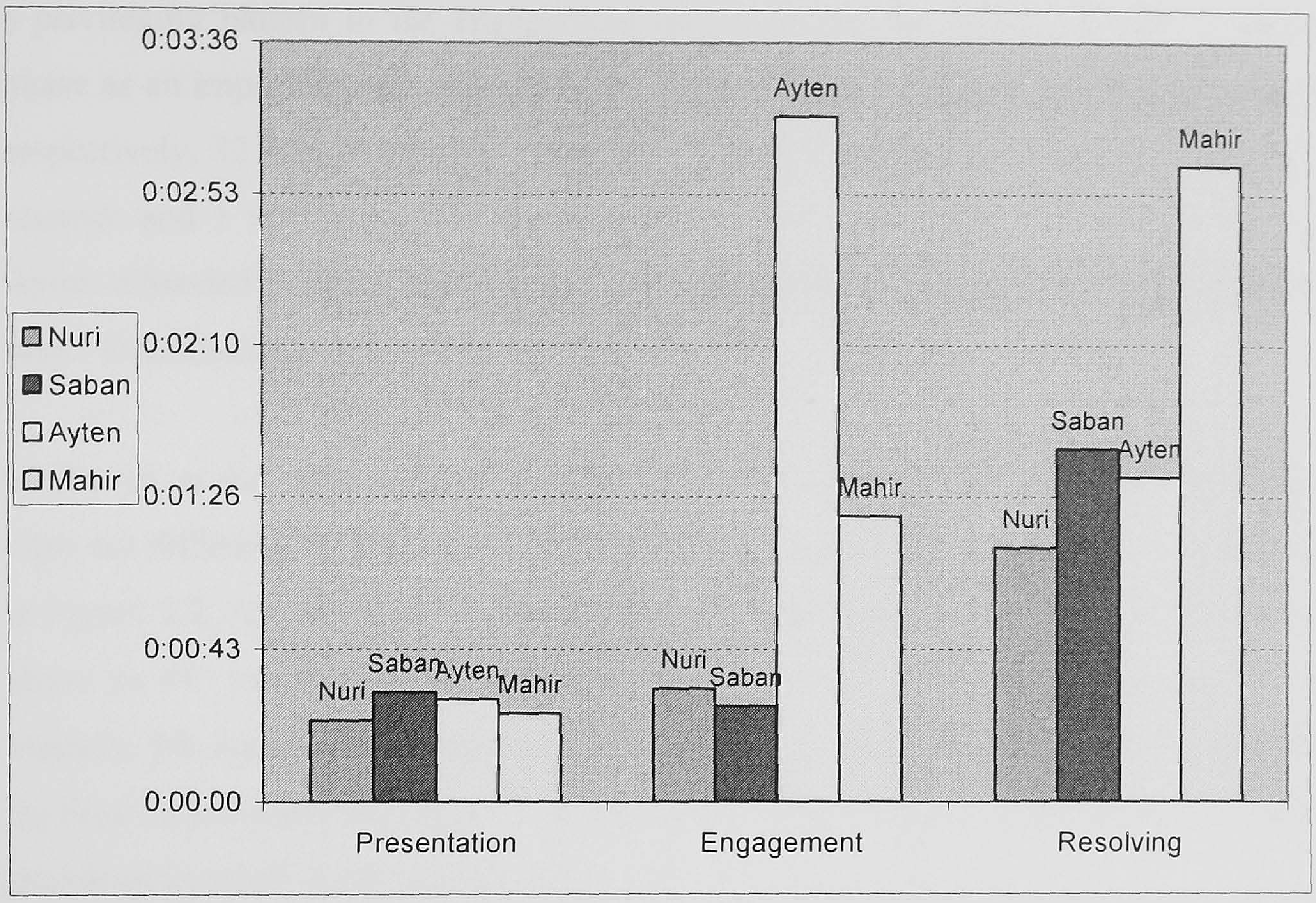



**Figure 5.2** Graph of the allocated times for each phase of the example solving.

First of all, all four teachers seem to have similar presentation times. This is hardly remarkable when we consider that teachers present the examples in a similar fashion (See next section).

Yet what is quite remarkable are the results of the engagement phase. This is further support for the similarity of teachers' practices *within* institutions and difference *between* teachers of differing institutions. PC teachers, Nuri and Saban, spent only 23 and 31 seconds for students to solve the example. It seems that PC teachers have a tendency towards providing much less opportunity for students to tackle the example. For example, the mean number of active examples Nuri solved per lesson is 10.7 (See table 5.2), and Nuri allocated only 32 seconds per example for students to work out the example. It is questionable whether students can first, understand the example, and then form a solution strategy and, last, obtain a solution within 32 seconds (engagement phase)[9]. A similar remark can be made for Saban. He allocated less time in his engagement phase (27 seconds) than Nuri did (32 seconds) and this figure is even less than his presentation time (31 seconds).

Another interesting aspect of this result is that PC teachers allocated significantly less time to the engagement phase than their colleagues in SS. Therefore there seems to exist

[9] I refrain from making any value judgements on the effectiveness of teachers' practices.

a privileging pattern in the engagement phase. Ayten and Mahir seemed to take this phase as an important part of the lesson. While PC teachers, Nuri and Saban, allocated, respectively, 32 and 27 seconds, SS teachers, Ayten and Mahir, allocated 3 minutes 15 seconds and 1 minute and 21 seconds, respectively, i.e. Mahir allocated 3 times and Ayten allocated 6 times more time on the engagement phase of the example solving than PC teachers.

Moreover, at the resolving phase where examples are solved no the board, the trend of clear cut differences between SS and PC in the results slightly changes. As can be seen in Figure 5.2, Ayten, an SS teacher, allocated a similar amount of time to the resolving phase as PC teachers. In the resolving phase Mahir is the highest among all four teachers. He spent a lot of time on the resolving phase. In fact, he spent 63 percent of his time on the resolving phase. The reason behind this slight change in the trend will be explained in detail in the next section.

Additionally, Figure 5.3 can be examined to compare institutional mean value.

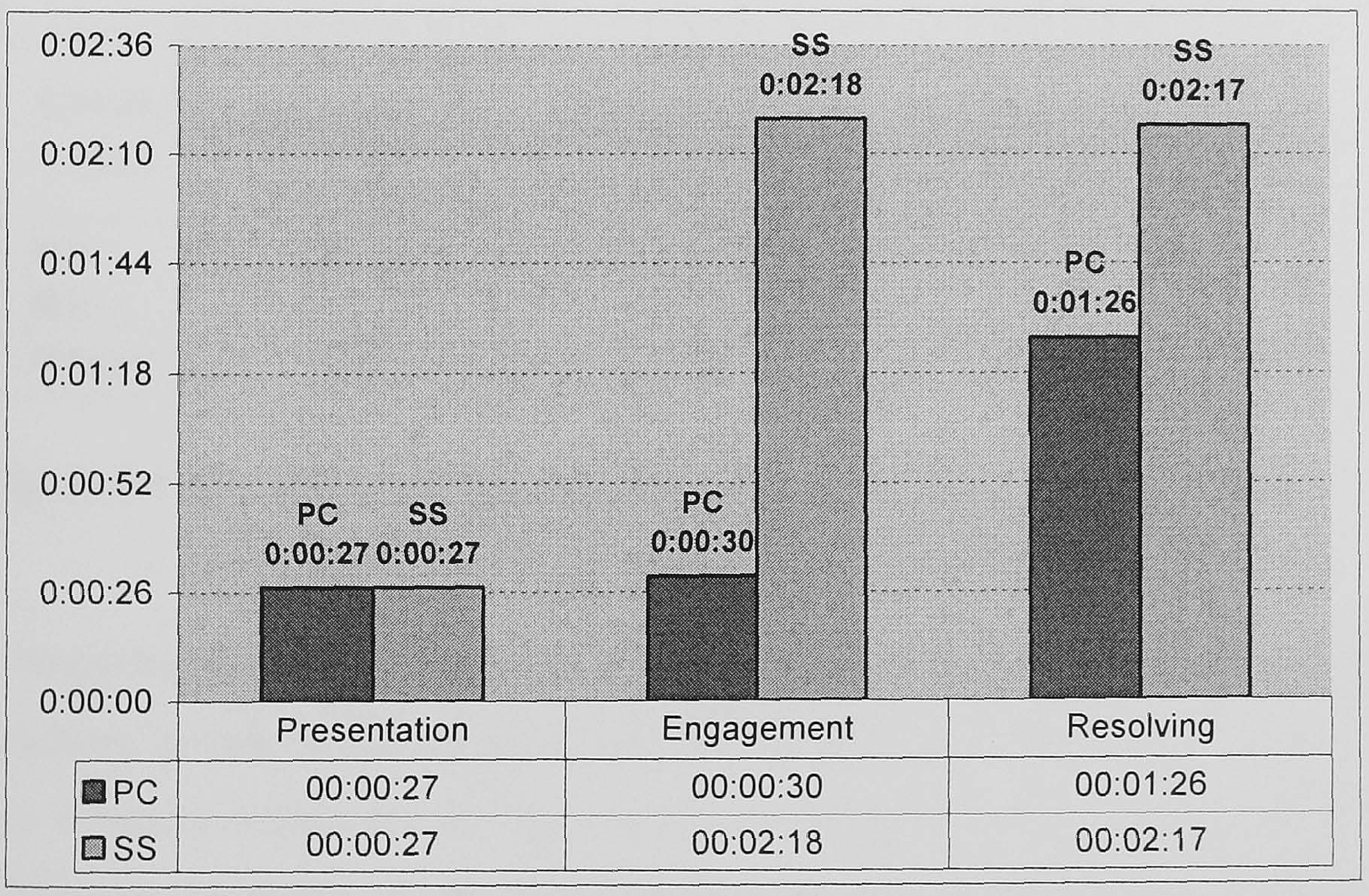
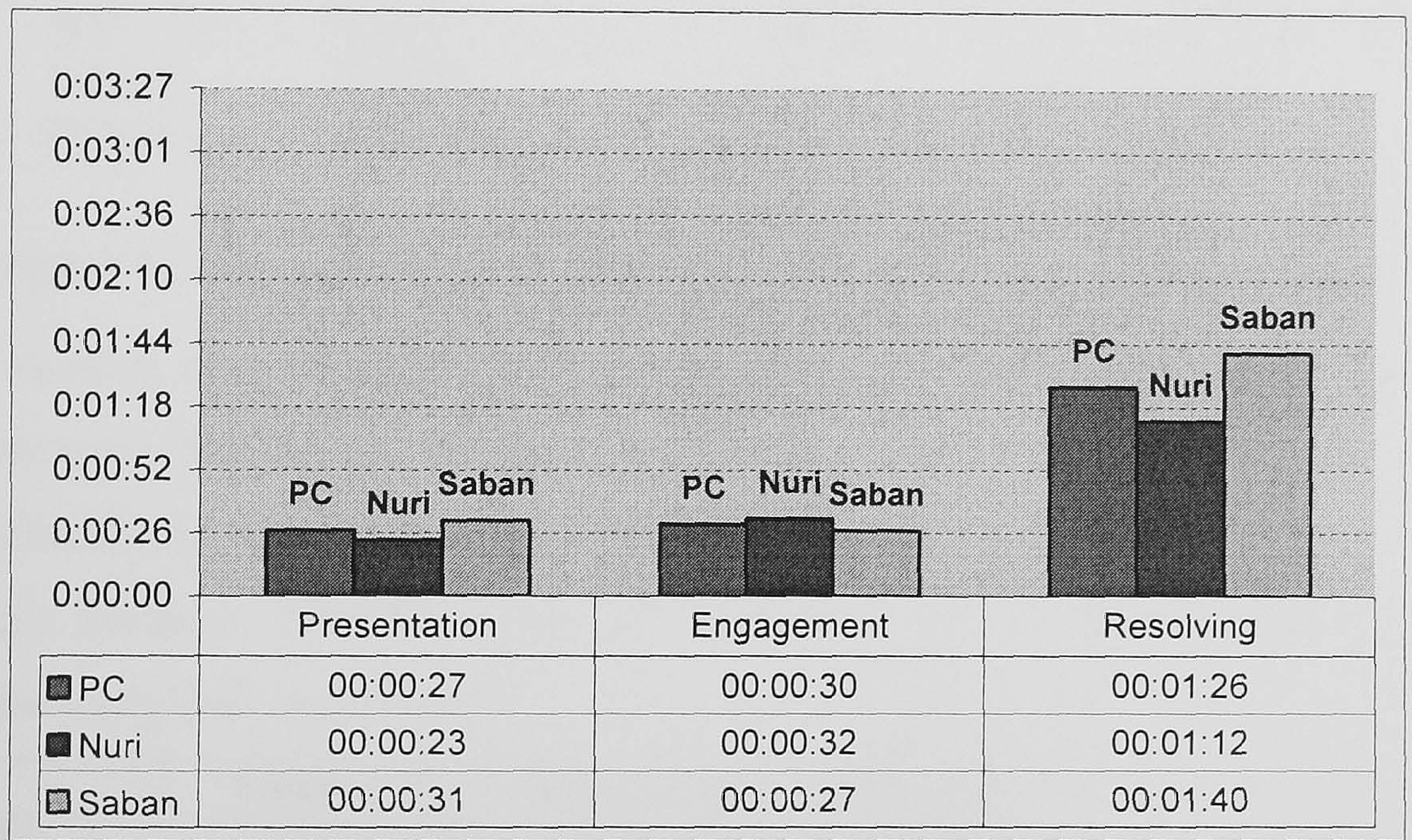



**Figure 5.3** Comparison of institutional means on each the example solving phases.

Figure 5.3 illustrates the privileging patterns between PC and SS teachers' practices in terms of the time they spent on each phase of an example solving session. Interestingly enough, in Nuri's and Saban's (PC) classroom practices the emphasis is on the resolving phase. Whereas Ayten's and Mahir's emphasis is on the engagement phase and the resolving phase equally, they still differ markedly from PC teachers. The difference between PC and SS teachers is in the resolving phase: 59[10] percent. The biggest difference between PC and SS teachers' example solving sessions is at the engagement phase: 360 percent.

By now, the difference *between* institutions has been presented. There exists another interesting result that comes out of this analysis: The diversity *within* institutions. While PC teachers' practices are almost identical, SS teachers' practices indicate some differences. To make the previous point more clear one should compare Figure 5.4 with Figure 5.5. Figure 5.4 shows the values for the two PC teachers and the institutional mean (PC teachers combined). Figure 5.5 shows the values for the two SS teachers and the institutional mean (SS teachers combined).



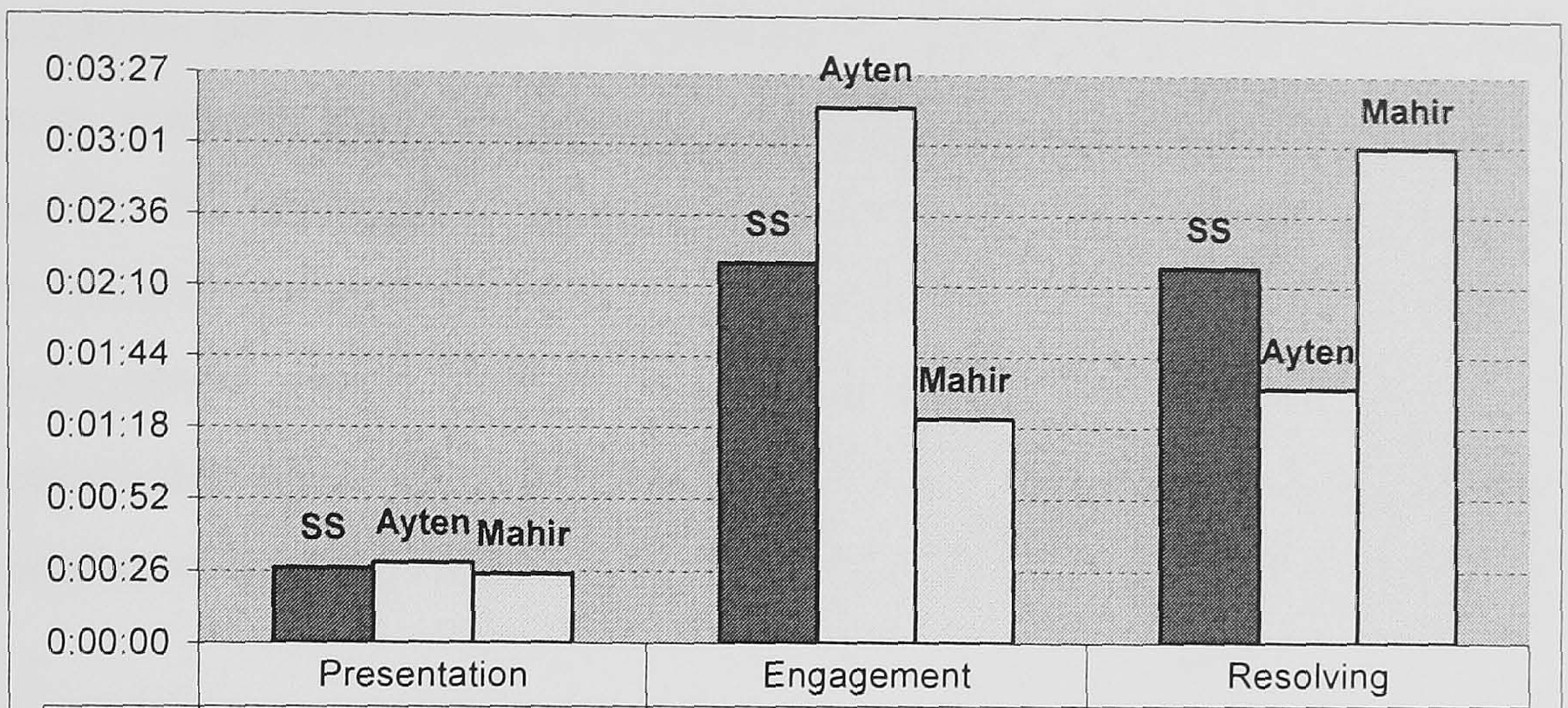

| | Presentation | Engagement | Resolving |
|---|---|---|---|
| PC | 00:00:27 | 00:00:30 | 00:01:26 |
| Nuri | 00:00:23 | 00:00:32 | 00:01:12 |
| Saban | 00:00:31 | 00:00:27 | 00:01:40 |

**Figure 5.4** PC's and the two PC teachers' means on example solving phases.

Comparing graphs given in the two figures enables one to notice the distribution of teachers' practices in each phase of the example solving session. Figure 5.4 shows that the diversity within PC is very small. That means the two teachers do not have much difference in terms of the time they allocated for each phase of example solving. However, SS data show just the opposite, a high degree of diversity. This is clear in both the engagement and the resolving phase. Mahir privileged the resolving phase, but Ayten privileged the engagement phase. While Mahir spent only 1 minute 21 seconds in the engagement phase, Ayten spent 3 minutes and 15 seconds, which is more than twice

[10] This and the following calculations are based on total seconds. Thus each value is converted to seconds before calculations made. For example, 00:02:17 (2minute 17seconds) is calculated as 137 seconds.

the time Mahir allocated. At the resolving phase Mahir allocates far more time than Ayten. Mahir, in fact, allocates 3 minutes and a second which almost doubles the time Ayten allocates 1 minute 32 seconds.




| | Presentation | Engagement | Resolving |
|---|---|---|---|
| SS | 00:00:27 | 00:02:18 | 00:02:17 |
| Ayten | 00:00:29 | 00:03:15 | 00:01:32 |
| Mahir | 00:00:25 | 00:01:21 | 00:03:01 |

**Figure 5.5** SS's and the two SS teachers' means on example solving phases

Another way of understanding this difference is to look at the point of differences between teachers of the same institution. Table 5.4 shows the absolute values of the differences between teachers of the same institution (See also Figure 5.2). That is to say, for instance, 0:00:08 in the presentation phase of the PC indicates the difference between Nuri and Saban.

| | Presentation | Engagement | Resolving |
|---|---|---|---|
| PC | 0:00:08 | 0:00:05 | 0:00:28 |
| SS | 0:00:04 | 0:01:54 | 0:01:29 |

**Table 5.4** Diversity within the institutions.

The biggest difference between PC teachers, Nuri and Saban, is 28 seconds in the resolving phase and is relatively marginal in comparison to the corresponding value in SS, e.g. 1 minute 29 seconds. This also supports the idea that there exists less diversity among PC teachers. In SS, there are two major points where SS teachers differ from each other. At the resolving phase the difference is 1 minute 29 seconds. The biggest difference lies at the engagement phase: 1 minute 54 seconds. That means at the engagement phase Ayten doubles Mahir, but at the resolving phase Mahir doubles Ayten.

This being the case one cannot help but ask "What is happening in Ayten's and Mahir's engagement and resolving phase that is different?" This is what will be addressed in the next section.

To put it briefly, results in the example solving segment shows remarkable quantitative differences *between* institutions. PC teachers evidently solved a greater number of examples per lesson than SS teachers. PC teachers also spent remarkably less time on each example than SS teachers. Comparisons *within* institutions suggest interesting results. PC teachers' practices show almost the same pattern. Both teachers privileged the resolving phase. However, there is a diversity within SS. While Ayten privileged the engagement phase, Mahir's practice privileged the resolving phase. (Mahir is still remarkably different from both PC teachers.) These similarities and results sparked a number of questions which will, at least partly, be resolved in the next section.

### 5.3.2 Qualitative Indicators

In this section I will point out the similarities and differences between four teachers in their use of examples and what activities they do during an example solving segment in qualitative terms.

#### *5.3.2.1 The connection between examples and the teachers' goal-in-context*

During the lessons I observed, PC teachers regularly put the example they are about to solve or the example they are solving in the context of the university entrance examination[11]. They made implicit as well as explicit connections between the past examination questions (PEQs) and the example at hand. These associations were made at various phases of the example solving segment. However, more often than not the connections were made just before the presentation phase. Section 5.2. indicated that this similar feature is also consistently the case for the content segment of the PC lessons but not SS lessons. Most of the time the introduction to a particular example type was before the example is presented. PC teachers consistently made explicit associations by explaining that a similar example appeared in the past examinations. To illustrate this, let's examine a PC teacher Nuri's lesson, which I presented in section 4.1.3. Before the first example of the lesson Nuri said:

> I will present you another example and this is asked [in one of the past examinations].{Nuri-4}

Although Nuri did not use the word 'examination', 'past examination' or 'UEE', it was almost impossible not to recognise that Nuri meant a past UEE question. Both sides,

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[11] Which PC teachers' stated during the interviews as their goal.

teacher and students, seem to be clear about what was meant but it was not, at least for that occasion, explicit. For the same item Nuri stated, during the engagement phase:

> During the examination, it is possible to solve this type of question by using *numerical value* [technique] but it is risky. I mean, I need to warn you of that. We can solve this type of questions by *numerical values* with the condition of being very careful. {Nuri-4}

Nuri is making an explicit link between the example he selected to solve and the past examination questions. Moreover, he is also suggesting what kind of solution methods would best fit the type of this example. He also said:

> Now my aim is to get rid of these 'x's (pointing to x in the expression obtained) and to do anything to be able to write 'x's in terms of f(x). My friends, I am going to solve this in a *classical way.*{Nuri-4}

As can be seen in the extract Nuri explicitly labelled the technique he used in solving the example '*in a classical way*'. After solving the same example he said:

> Look, *imagine* that the options [of the examination question] full of expressions like this and *imagine* you give numerical value to them. It would take a while [to calculate the results for each option] and thus *my advice* for these kinds of questions is to *solve* them. {Nuri-4}

In this extract, Nuri is making a link between the structure and particular features of the past examination questions and the solution techniques he is using to reach the answer. It is fascinating to see that the teacher is putting the students in an imaginary examination situation where a similar question is asked and then suggesting how students may reach the answer. The assumption in Nuri's expression is that he is suggesting not how to solve the example but, in fact, how to reach the answer in general. However, for this specific example he advises students to 'solve it'. Nuri labelled this '*the classical way*'. It is worth noting that the last four extracts are Nuri's comments on only one example, and, based on a number of video recordings of his lessons, it is safe to say that he frequently established connections between the examples he uses and his goal-in-context. Saban, the other PC teacher, also explicated the link between the examples he used and PEQs. For example he made a similar comment after solving the first example of the lesson that I described in 4.2.3. He said:

> There is no question of this type among the past university examination questions, there isn't[12]. Among function types[13], only inverse function and

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[12] Although that example that did not appear among PEQs, (1) it is within the scope of examination and may be asked in the future (2) it helps students understand other parts of the topic.

composite function are asked. Among function types, they asked only inverse function and composite function. {Saban-3}

Saban is directing the students' attention to the types that were asked in PEQs. He is making sure that students are focusing especially on the parts where PEQ appeared.

In contrast to PC teachers, SS teachers did not make connections between the example they are solving and PEQs during the example solving segments. Interestingly enough, even when students brought up the topic of examinations SS teachers did not delve into it. Although this occurred on very rare occasions, comments on an example that attempt to relate the example on the board to the PEQs came from students. However, they did not seem to be substantiated by the SS teachers. For example in the lesson of Ayter I presented in 4.3.3, one student asked "Does it come in the university examination?" She replied, "*Yes, I may write some…Copy them down fast and don't talk amongst you.*" She then continued her instructions. Generally, UEE related issues came from students rather than teachers in SS.

#### *5.3.2.2 Phases of the example solving segment*

The data presented in Table 5.2 and Figure 5.1 indicate that PC teachers solved more examples than SS teachers per lesson and SS teachers allocated more time per example. What is really happening in PC and SS classrooms then? What is it SS teachers are doing in the extra time in relation to PC teachers? To understand teachers' practices let us examine the teachers during three phases of the example solving segment.

*Presentation phase*

Nuri, as explained above, establishes associations between the examples he is using and the UEE. In the presentation phase (23 seconds) he often wrote "Ör." which is a short form of "Örnek", which means "example". He then wrote the example on the board. This completed the presentation phase. More often than not, he wrote the example without looking at any sources. However, sometimes Nuri selected the examples he is using from the 'Quizzes' produced specifically by the PC in which he is working. In fact, Nuri used a 'Quiz' for one entire lesson {Nuri-7}. Saban acted similarly to Nuri in

[13] 'function types' in this context refers to a subsection of the topic of functions. To him, this included constant, linear, inverse and composite functions.

his activities during the presentation phase. He did not use any source except 'Quizzes' either.

SS teachers, Ayten and Mahir, sometimes used the textbook they selected for their examples. In fact, this textbook is selected by the committee of mathematics teachers of that school to use for that year. Most of the time, as observed in her lessons, Ayten selected her examples from her notes that she prepared beforehand. Mahir sometimes wrote the examples with no apparent source and sometimes used different textbooks students brought to the classroom. Therefore, unlike PC teachers, there was less regularity among SS teachers' sources of examples. PC teachers used nothing except 'Quizzes'.

*Engagement phase*

The video data suggests that for the engagement phase the difference between teachers of different institutions were not only significant quantitatively (See Figure 5.3) but also qualitatively. PC teachers privileged the resolving phase over the engagement phase and the engagement phase did not seem to have any function in their practice. In comparison to PC teachers, the engagement phase was longer in SS teachers' example solving segments. Moreover, the engagement phase seemed to have an important function in SS teachers' example solving segments. In fact, this phase seemed to have no instructional function in the PC teachers' practices.

For PC teachers, this phase was a quiet time of the example solving segment. During the engagement (32 seconds) phase, Nuri generally walked from one side of the classroom to other and he mostly stayed in front of the board. Occasionally, he asked students to suggest a solution during this phase and sometimes Nuri talked about PEQs in relation to that example. Thus, Nuri's engagement phase was mostly quiet and there was minimum teacher-student interaction. Saban did not even attempt to ask students to suggest a solution. His engagement phase was also a quiet time as he mostly walked between desks with no apparent intention of interacting with students or checking their progress on the example he presented.

In sharp contrast to PC teachers, the engagement phase constituted an essential part of SS teachers' practice. It was, for example, privileged over others and it was, seemingly, the most significant phase of Ayten's practice. Table 5.3 and especially Figure 5.2 shows that Ayten distinctively privileged the engagement phase (3 minutes 15 seconds) over other phases. Ayten spent 62 percent of her example solving time on the engagement phase. It seemed to me that, for Ayten, the engagement phase was not just an important part of the example solving segment but also the highlight of the entire lesson. Once Ayten presented her examples (presentation phase), she consistently started 'walking between desks'[14] and dealing with students individually. In Ayten's lessons, both teacher and students seemed to be aware of an implicit contract between them that the engagement phase is a time for students to try to work out the example in their seats. Ayten's part of the contract involved checking individual students' progress. An excellent example of this 'walking between desks' activity is in the lesson described in section 4.3.3. In that part of the lesson, she wrote an example from her notes. She said:

> Those who finish [the solution of the example] please let me know I will come next to you. Don't say the answer loud. Some of your friends complained to me that it is affecting them negatively [when some students shout out the answer]. Be sensitive about it. Just call me I will come next to you. {Ayten-8}

Having examined a number of students while students were working on the example she said:

> [I observed that] You learned this thing [manipulation] well, if it is [function] so then the inverse [of it] is such and such. If the inverse [of the function] is so, then it [function] is such and such {Ayten-8}

She dealt with a great deal of the students individually and checked their progress through inspecting their solutions in their notebooks. In fact Ayten checked 14 students one by one, which is about one quarter of the whole student population in the classroom. She made a variety of comments to different students. Here are some examples:

> 'Yes, keep going, keep going',
>
> 'you are going very good',
>
> 'look what is the example asking you, it is asking you -2 in the inverse of g',
>
> 'Well, where did 7 come from? Where?',
>
> 'you are making a mistake here',

---

[14] Similar "walking between desks" activity has also been acknowledged as a significant and culture specific 'lesson event' in Australian and Japanese mathematics lessons (Clarke, 2003) by the learners perspective study group Clarke (2002)

'what do you find when you put 2 in the place of x. Why 2? It [example] is asking you [the image of] -2.',

'look look you are making a mistake here'

She even got slightly angry with one simple mistake one student making. She realised that one of the students whose notebook she examined copied the example down mistakenly and commented on it.

For Ayten, this 'walking between desks' activity, from my observations, appeared to have three instructional functions:
1-Monitoring students' progress through inspecting their notebooks and encouraging every individual to become involved in the example solving activity;
2-Actively supporting students' progress and making sure that every student is able to solve the example in their notebooks;
3-Monitoring the students notebooks' so that they will have a correct copy of the example and its solution.

On many occasions, Ayten knelt down or sat beside the student (or students) and engaged them in conversation about the example they were attempting. She was so keen on supporting the students' progress that a few times I saw her sitting next to a student and solving the example in the student's notebook herself. 'Walking between desks' was such a regular activity that when I watched the lesson in fast motion Ayten seemed like a bee collecting honey from different flowers. In a sense, Ayten acted during the engagement phase that utilises examples for student progress. I cannot help but note here that the notebook and examples are used by Ayten not only as a tool to contribute to their understanding but also as a tool to inform her about students' cognitive development. She, hence, used this tool to check their progress by identifying mistakes of her students.

Mahir also used the engagement phase for students to try to solve the example. The most distinctive feature of his practice during the example solving sessions is that he encouraged a whole class discussion of the example. Although Mahir mainly 'opened the discussion' in the resolving phase, Mahir encouraged students to try to work out the example during the engagement phase by discussing the example with individual students in a way that other students can also hear. Some other times Mahir inspected a number of students' notebooks and their solutions. Although Mahir also 'walked between desks' during the engagement phase, he did not inspect students' notebooks as much as Ayten did. The difference between Mahir's and Ayten's engagement phase was not only in the time they allocated (Mahir: 1minute 21 seconds, Ayten 3 minutes 15 seconds). They also seemed to use it as a different way to achieve the same goal. Ayten approached students individually and discussed their solution on a 1-to-1 basis, Mahir, however, created an environment in which the whole class was encouraged to discuss the example on the board. In short, Mahir was making sure that each and every student was involved in the lesson by whole class discussion. Ayten was making sure that each and every student was involved in the lesson by attending to each students' needs individually.

Unlike the PC teachers, the SS teachers, despite having different approaches;

1-Encouraged students to be actively involved in the lesson;

2-Encouraged students to work on examples in their notebooks so that they could see their progress;

3-Wanted to make sure that all students are improving.

Therefore SS teachers used this phase in a functional as well as a versatile way and, probably more importantly, seemed to be trying to achieve a different goal in comparison to PC teachers.

*Resolving phase*

The resolving phase is another the phase where the differences between institutions are highly noticeable. PC teachers privileged this phase over others (See Figure 5.3). Mahir also privileged this phase over others but as we can see that Mahir used the resolving phase in a different way in comparison to PC teachers. As noted above, Ayten privileged the engagement phase over this phase. She, however, seems to have used this phase similar to the way in which Mahir did.

For Mahir, an SS teacher, the resolving phase seemed to be a time for whole class discussion. His general conversational style was very much on a question-answer basis. It was so common that he even asked questions whose answer was known. Having this style of conversation he asked a series of questions about the information readily given in the example. He then started to ask questions about the way the example should be solved and he solved the example. During these question-answer sessions, Mahir expected students to become involved in the lesson by suggesting to him a way to solve the example. To demonstrate this point I will refer to an example he gave during the lesson I presented in 4.4.3.

$$\boxed{f(x) = x^2 - 4x + 2 \Rightarrow f^{-1}(2) = ?}$$

When he was demonstrating the solution of this example (the resolving phase) he asked 14 questions in total. Some of them are:

"look at here, what is $f^{-1}(y)$ equal to? Equal to x"

"What is y here?" (pointing to '$f^{-1}(2) = ?$' in the example)

"What is the value of *f(x)*?" (Some students suggested $x^2 - 4x + 2$).

"What is *f(x)* equal to? *y*".

"What is the value of *y*? *2*

"Did we understand this?"

"What is the solution set? 0 and 4"

During the resolving phase of this example Mahir made sure that all students managed to follow each of the solution steps of the example one by one without skipping any middle step between two. This is achieved by a series of question-answers.

Not question answer sessions, but whole-class discussion sessions got him angry as sometimes students got so loud in discussion among themselves, or in suggesting a solution to him, that the noise became unbearable. Ironically, Mahir got quite irritated a number of times because of the over-involvement of students.

Similar to Mahir, Ayten, the other SS teacher, also solved the example in a step-by-step manner. She did not ask as many questions as Mahir did during the solution but her resolution phase similarly involved a series of questions and answers. For instance, she asked 6 questions during the resolving phase of this example. When she was solving the examples, she asked students to become involved in the solution. In the lesson I presented in 4.3.3., she saw that some students were not paying attention to her solution by doing it themselves and she said[15]:

> "Now my friends, let's look here. Don't try to solve it on your own. Let's solve it together"

Although the explanations above identify how teachers acted when they were solving the examples, it is important for the reader to be aware that not all examples were solved by teachers. This is another aspect of the resolving phase where PC and SS teachers differ greatly. Table 5.5 shows the mean number of examples solved by students and teachers per lesson.

| | Institution | Total per lesson | Teacher Solved | Student solved |
|---|---|---|---|---|
| **Nuri** | PC | 10.7 | 10.7 | 0 |
| **Saban** | PC | 10.2 | 10.2 | 0 |
| **Ayten** | SS | 6 | 3 | 3 |
| **Mahir** | SS | 6 | 3.29 | 2.71 |

**Table 5.5** The mean number of examples solved by students and teachers per lesson.

Although, in comparison to SS teachers, PC teachers solved 75 percent more examples per lesson, none of the examples were solved by the students. SS teachers, however, let (and sometimes pushed) students to solve the examples; the number of examples solved was fairly evenly balanced between teachers and students. Students solved 50 percent of examples in Ayten's lessons and 45 percent of the examples in Mahir's lessons. This is another indication of student involvement in SS mathematics lessons, and lack of it in PC lessons. PC teachers always solved the example themselves.

One can easily observe that for Nuri's teaching, the resolving phase (1 minute 12 seconds) is evidently the privileged phase of the example solving segment. The same can also be said for Saban's teaching (1 minute 40 seconds). A number of perspectives can help the reader to visualise what PC teachers' practices look like during this phase. First of all, in the entire course of the teaching of functions, PC teachers never asked students to come to the board and solve the example. Neither was there any sign of an attempt (generally a hand in the air in SSs or a similar indication) by students to come to the board. It was as if there was an unseen agreement between teacher and students that 'students are not supposed to come to the board and actively demonstrate the solution'.

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[15] One could see that this is a comment where the implicit cultural contract between student and teacher is reminded to students by the teacher.

In fact one interesting incident, which took place in the first few minutes of Nuris's first lesson {Nuri-1}, nicely demonstrates this unseen agreement. When Nuri was writing on the board, one student asked in a way that everybody can hear: 'Excuse me, may I come to board to solve an example?' Nuri did not make any comment on it. However, many students laughed at the comment. It seems that everybody in the classroom knew that it was clearly a ridiculous request to make as it was against this unseen regulation. Moreover, this request was an attempt to ridicule the teacher but did not bring a negative action by the teacher in return[16]. Therefore Nuri did not show any attempt of involving students in the flow of the lesson.

Secondly, both PC teachers Saban and Nuri solved the examples in a way that skips some possible middle steps of the solution. For instance, during the lesson described in 4.1.3. Nuri solved an example where he had to put the expression $\frac{1}{f(x)-1}$ wherever the $x$ exists in the expression $f(x-2)=\frac{x-1}{x-2}$ (see capture in 4.1.3.). This gave him the expression $f(x-2)=\frac{\frac{1}{f(x)-1}-1}{\frac{1}{f(x)-1}-2}$. From that he directly wrote $\frac{-f(x)+2}{-2f(x)+3}$ without showing the middle steps that lead to the last one. My version of the solution of this example suggests the existence of at least 2 or 3 more steps that may be shown to the students, which were skipped by Nuri perhaps for the sake of solving the example quickly.

Thirdly, and possibly most importantly, in the resolving phase both Saban and Nuri often showed alternative techniques of reaching the answers. These special techniques are differentiated by common solution methods. They even have particular names. The name Nuri used for the common solution method is '*classical way*'. One technique he used in the resolving phase, as well as suggesting that students use during the examination, is called the *'numerical value technique'*. The last example of the lesson I presented in 4.1.3. illustrates the commonality of the use of this technique. He said:

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[16] Despite such incidents, the students I contacted showed a lot of respect to PC teachers as, to many of them, they were more knowledgeable than SS teachers which, again for them, was evident in their quickness of solving the examples.

> It is possible to solve this example by the *numerical value technique*. You may find it difficult to use here. I don't want to use that technique for every question but a regularly studying student would assign a value to x in root to remove the root. Then calculates the result then checks the options...(making some calculations)...As you can see you can solve it from the options but you need to be careful about it. Why? Because there will be many calculations to be made and you will have to check the options one by one and then you need to go back to the function itself [to compare]. This may take your time but it is possible. So this question can be solved by using the options. {Nuri-4}

Nuri not only suggested and demonstrated the use of the technique but also warned students that it may not be the quickest way to reach the answer for every question. It is worth noting that although Nuri uses this special technique to reach the answer to a question and teaches it, during the interviews he disapproved of this technique which creates a tension between his professed beliefs and his practice. This issue will be addressed in section 6.3 in the discussion chapter.

Upon the centrality of examples in the PC teachers practice (see quantitative indicators) and their privileging of the resolving phase, I would like to exemplify this technique as Nuri used in the lesson {Nuri-5} since this was the time where he explained it fully. The example in fact was a past examination question with the numbers changed. Nuri wrote the example first. The example basically gave a function of $f$ and its composite function with another function ($g$) and required the reader to find the unknown function. Nuri solved it by putting $g(x)$ in the expression of $(fog)(x)$. Then he completed the expression in the square and found the square root of both sides, which gave him 2 possible answers.

> $f(x)=x^2+4x$ and
> $(fog)(x)=x^2+6x+5$
> then what is g(x)?

$$[g(x)]^2+4g(x) = x^2+6x+5$$
$$[g(x)]^2+4g(x)+4 = x^2+6x+5+4$$
$$(g(x)+2)^2 = (x+3)^2$$
$$g(x)=x+1 \text{ or } g(x)=-x-5$$

During the resolving phase Nuri said:

> This is a very good question I have only changed the numbers involved...It is one of the original past examination questions. This is regarding composite functions and asked before.

One student immediately asked "We should find the one in the options shouldn't we..hmm..we should tick that one right?" Nuri replied:

If they put these in two options they will have to cancel it out [from the examination results later on]. Because one group of students will select one option another group will select another option.

Nuri then added:

What will the options be like? (wrote g(x)=x+1) x+1. Imagine you are in the examination, as a normal human being, you tried hard to reach the answer but you couldn't no matter what you did. My dear friends, I keep saying this all the time: give a numerical value. Look here now carefully. If it was me [in the examination] I would give 0 [to *x*] here (pointed $[g(x)]^2+4g(x) = x^2+6x+5$ and calculated $fog(0)$). What is fog(0)? It is 5. Well, does fog(0) means f(g(0)) ? yes and it is 5. Let me put some options randomly. One is x+1 another is x-3 another one 2x-4.

He wrote each option then calculated the value of $g(0)$ for each option and then added:

What is $g(0)$ my friends? (pointing to the option $g(x)=x+1$) it is 1. If this option is correct then it should confirm the equality $f(g(0))=5$ [as calculated before]. Lets write 1 in the place of $g(0)$ (he did and he calculated $f(1)$ using $f(x)=x^2+4x$ and found 5) Is it confirming? Yes.

Nuri did the same procedure of finding $g(0)$ and then putting this value in $f(g(0))$ and comparing the result with the one he finds using $f(x)=x^2+4x$ in other options. He then added:

You can easily solve the examination questions that are coming from the topic of factorisation, from the topic of ratios and proportions, and from the topic of functions. Did I not show you earlier? Yes I did. But I warn you on the questions of functions. If, during the examination, one of you decides to use the numerical value technique and want to use this technique, be very careful! It [using this technique] isn't as simple as factorisation. If you mix up what to assign and to which variable then you get more confused, and you cannot then get out of the trouble. But it can be solved.

Nuri then pointed to the first solution, as he put it the 'classical way', and stated:

Look at this solution. This isn't really a simple way, isn't it? Look my friends, it is during the examination. Imagine the [pressure of] examination day psychology. During the examination you are supposed to see completing the square! It isn't that easy. Now you sit here like drinking coffee in a café and you are comfortable. Easy here. In the examination it is not that easy. I will now give you an example. It is a really an original example on composite functions and you will see that. Now let's do another example... {Nuri-5}

Saban also often used the numerical value technique and taught his students with very clear instructions regarding how to get the answer by using this technique. He wrote down the steps involved in this technique on the board. Moreover, he seemed to be quite masterful in his use of this technique as he showed a lot of flexibility. Similar to Nuri, Saban also disapproved of this technique as a way of teaching mathematics.

# CHAPTER -6 DISCUSSION

The aim of this research was to explore the instructional practices of teachers in Turkish state schools and private courses. As a largely unexplored area in the Turkish educational context, I wanted to see mathematics teachers' instructional practices without missing the broader picture. In line with Lerman's (2001) suggestion that researchers should focus on some particular issues (zoom in) taking into account the importance of the social context (zoom out) in which the research is conducted. From this perspective, an aim of educational research should, therefore, be integrating these two dimensions. As to this study, my approach incorporates 'zoom in' along with 'zoom out'. Using Lerman's (2001) metaphor, I 'magnified' teachers' practices to explore main facets of teaching activities in Turkish state schools and private courses and I also wanted to make sense of the teachers' practices without ignoring the broader portrait. This approach let an important theme emerge that suggests the significance of teachers' work settings in understanding teachers' practices. In this section I will discuss this thoroughly.

In the previous section I put the results together to reveal differences in the practices of SS and PC teachers. I composed the findings from the results chapter and made explicit comparisons between PC and SS teachers' classroom practices.

Considering the differences between PC and SS teachers, the question 'how do we interpret and make sense of these differences?' gains importance. Understanding teachers' practices is one of the major goals of educational researchers. Since teachers play a significant role in students' classroom experiences and their learning, making sense of what teachers do and, more importantly, why they do what they do is of paramount importance. Here are some of the questions that may arise: Are the similarities observed between Nuri and Saban, and between Ayten and Mahir a matter of coincidence? Are the differences between Nuri/Saban and Ayten/Mahir stemming from individual differences? Alternatively, can these similarities or differences in teachers' practices be attributed to institutional differences (or institutional influences on teachers)? In the spirit of the sceptic educational researcher, I will not dismiss either possibility right away. However, I will argue that institutional context has a significant influence on teachers' practices and that the differences between teachers cannot be fully understood without considering the institutional context teachers are exposed to.

In the first section, I will explain what is meant by 'institution' in this study. It will be followed by arguments about the influence of institutions on teachers' practices. I will do so through discussing a small number of interview extracts in which teachers refer to the institutions in which they are working. In the second section, based on the analysis of the data, I will further argue that there are associations between certain materials and practices and certain institutions. In the following section, I will argue that the institutions may be so influential on teachers' practices that teachers subordinate their own personal views in the decision making process. In other words, teachers may prefer to teach in the way that the institution they are working in suggests, even though they clearly think otherwise. That may mean that individual differences in teachers' practices may even be reduced by the institutions depending on influential power of the institution, which I will discuss in the fourth section. In the last section, I will discuss some further influences of institutions on teachers' professional development.

### 6.1 Why an Institutional Perspective?

The term 'institution' is an interesting one. Despite being widely used in educational research, there seems to be a paucity of research on this area (see section 2.4) and lack of specificity in its definition. In educational research it generally refers to schools when the organisational aspect of the schools is put into perspective (see for example Olson, 1992, Olson 2003; Zeichner, Tabachnick, & Densmore, 1987; Zeichner & Gore, 1990; Cobb et al., 2003). Thus schools are regarded as organisations with educational goals in educational research. I will therefore use it to refer to educational organisations. I take a pragmatic approach to this issue and use the term as it is specified in the context that the research was carried out. The term 'institution' is used in preference to 'school' because some may not regard PCs as schools in a Turkish context. When the term 'school' is used, it evokes an image that is different from PCs in Turkey. The possible reasons for this perception may be due to the cultural and contextual differences I discuss throughout the thesis. It seems also likely that it stems from the formal name used for PCs. I translated the Turkish term 'Dersane' as Preparatory Private Course (PC). However, in official documents they are referred to as 'Private Educational Institutions' ('Özel Eğitim Kurumları' in Turkish). Since state schools can be seen as educational institutions, the term institution seemed to be relevant to cover both state schools and private courses for the purposes of this research and it fits nicely within the framework I adopted from Cole (1996) (see Figure 6.1) in which institutions are seen as one level of many embedded context levels.

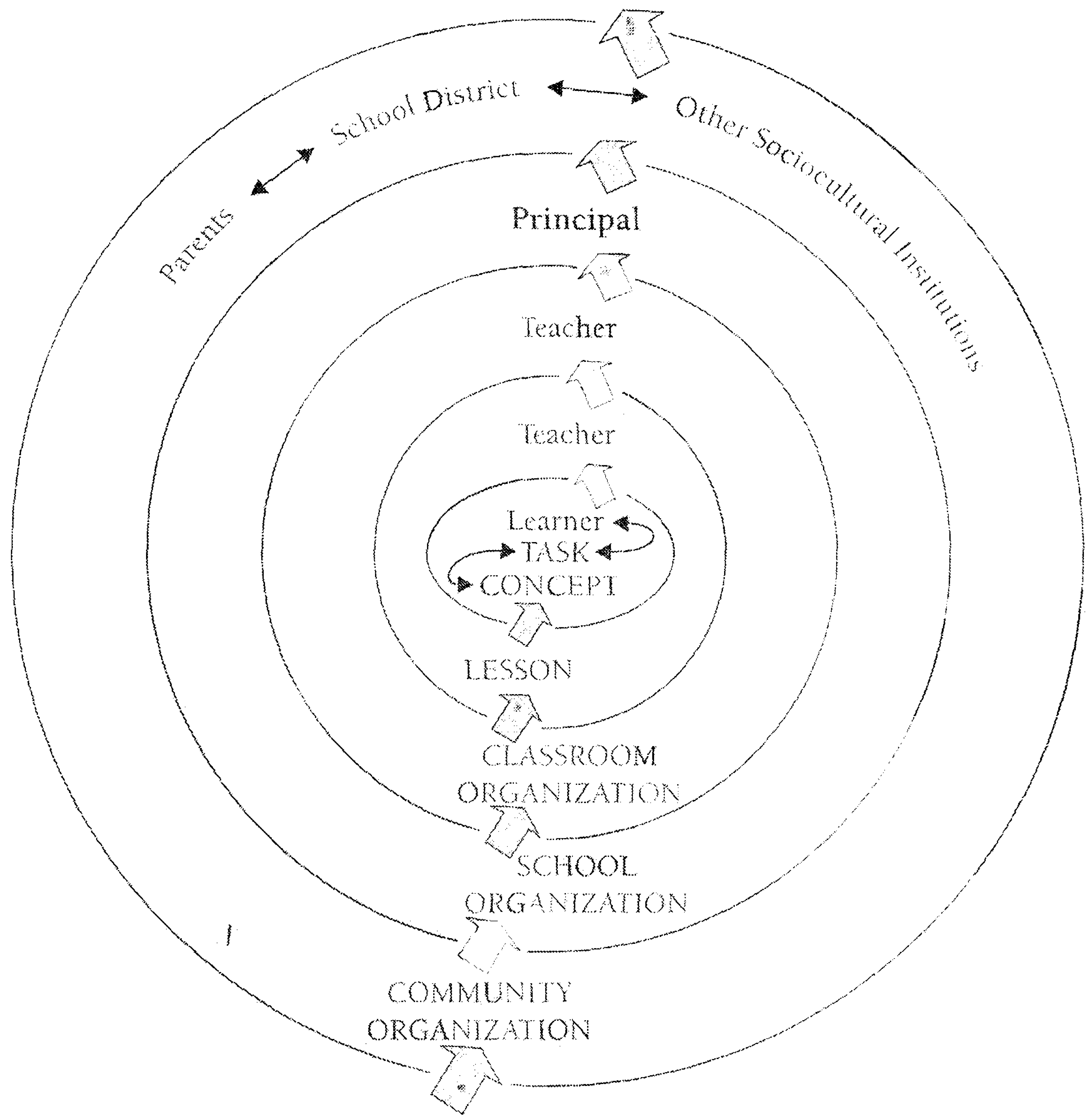


**Figure 6.1** Concentric circles representing the notion of context (Cole, 1996, p.133)

Talbert & McLaughlin (1993) also used a similar framework, in which they centrally have 'a view of teaching as permeated by multiple layers of context, each of which has the capacity to significantly shape educational practice' (Talbert & McLaughlin, 1993, p. 188). Lim & Hang (2003) made use of activity theory to study integration of ICT in Singaporean schools within a similar 'concentric' circles framework. They perceived each level as an activity system and argued that "the activity system in each circle influences the activity system in the innermost circle" (p. 54), nevertheless, "nothing is unidirectional in such an interactive system" (p. 54). Therefore, I see institutions as part of a wider educational context with close bidirectional relationships. In this connection, I regard the relationship between the teachers' beliefs, practices, perceptions and institutional context as by no means unidirectional. However, since the initial focus of the study does not include the influence of teachers on the broader context, I will mostly focus on the single direction of the relationship leaving the other direction of the relationship for future studies.

There are a number of reasons that point to institutional parameters in understanding why PC teachers' practices are different from SS teachers' practices. Many of the differences between the practices of SS teachers and PC teachers can be indicative of and traced back to the institutional context that they are 'exposed' to in their daily professional lives. Teachers referred to their overarching goal, as derived from the institutional goals, in both their daily classroom practices and in explaining their practice during the interviews. Moreover, they clearly referred to the goal of the institution as a central impact on their practice, for example:

**R** You gave two sets like A, B and then assign values in set A to set B. And also as pairs like *(1,a) (2,b)* etc. Also you present function such as $f = \dfrac{(x+1)}{(1+x^2)}$ What is the aim of [teaching] it?

**T** Well firstly it is because there are examples of past university entrance examination questions. If it wasn't in the past examination then I would not teach these and that's my first aim. Secondly, to make it more visual because you can switch one another. I mean if a student is asked one representation (of function) in the university entrance examination but given another representation, student will be able to switch. There isn't much other reasoning behind that

**R** Hmm...

**T** To be more precise we depend on the university entrance examination. I mean, we try to teach our students how to solve questions in the examination at 80 percent. For the remaining 20 percent, students should make use of different textbooks.

I believe that his expression "*To be more precise, we depend on the university entrance examination*" (line 12-13) is a quite convincingly clear way of revealing the influence of the institution. A second look at the way Saban expresses this idea discloses another important aspect: he prefers to use 'we' as a personal pronoun for his sentence rather than 'I', which may indicate his affiliation to a 'community of practice' (Lave & Wenger, 1991; Wenger, 1998).

Before going any further I would like emphasise two points. Firstly, the reader should view that the arguments made here (the following sections in this chapter) are within the limitations of the case study methodology (with the support of a questionnaire). Any claim put forward should be taken within these limitations. Secondly, I will try to avoid repetition of the data previously presented. However, I will exemplify my arguments with a small portion of the findings as a representative of more substantial data.

## 6.2 Associations between Practices and Institutions

Despite the fact that teachers in this study taught the same mathematical topic, they taught it differently. Some of the instructional materials and practices observed in one institutional context did not exist in the other. In fact, the data seem to suggest an association between certain instructional materials and practices, and certain institutions. For example, the textbook used by SS teachers in their classroom was specifically used in SS classrooms, but not in PC classrooms. Ayten used this textbook in her lessons and gave homework to her students regularly. Giving homework is also inherent to the SS context. PC teachers did not even mention homework in any of the video recorded lessons or during the interviews. However, giving homework was an integral part of SS teachers' practices. Ayten, for instance, allocated two separate lessons for checking students' homework. She evaluated their homework and decided to give grades to the owner of some of the homework. This was not the case in PC. In the context of PC, use of quizzes and use of past examination questions were very common. They also highly promoted the use of 'question banks', books with large number of questions that are similar to questions of the UEE. Question banks are exercise books that have a very specific target of UEE. The questions in these books are generated by the author of the book or a group of mathematics teachers. These teachers many times are those who are working in the same PC and come together to publish a question bank or generate quizzes. Another inherent practice was teaching short solution techniques such as the numerical value technique or use of options. Quizzes were also an integral part of PC teachers' lessons. A quiz is a one-page two-sided exercise sheet with questions on it. Many times quizzes have as many as 20 questions with options. The answers for those quizzes are also included for students to check their answers. Quizzes are divided into two by a vertical line on each side. This is exactly as the format of original UEE booklet structure. Both Nuri and Saban made use of quizzes when necessary. For instance, when I asked about quizzes Saban said:

**T** This is a tradition of private courses. It so much of a tradition that private courses even advertise themselves as giving 500 quizzes for their students[to attract students].

The term 'tradition' (line 1) is quite a powerful signal of quizzes being an inherent part of private courses. It indicates that the use of quizes is not a casual practice but very much a customary part of PC teaching/learning. It is so much so that Saban even stated that PC teachers even get some negative feedback from students if they do not hand out quizzes. I personally observed in my social interactions with PC teachers and with a number of PC managers that this 'negative feedback' from students could be very influential in the process of teachers' future employment. What's more, Saban stated that even giving a small number of quizzes would be an issue from the students' perspective:

1 **R** Do you get any reaction from your students if you do not hand quizzes
2 out?
3 **T** Yes, we do get reactions. This became a tradition here [in PC]. All
4 private courses give quizzes. Let me put it this way: I give 3 quizzes
5 and some other teachers give 1 quiz and there is even reaction to that.
6 Why do I give 3 and others don't.
7 **R** Is it that students react to that by talking to the manager of the private
8 course?
9 **T** Of course.
10 **R** Can we then say that it is an inevitable part of mathematics in private
course?
11 **T** Hmm Yes. It became so. It became inevitable somehow...Students
12 have a lot of textbooks. In each there are 100, 200, 500 questions for
13 each topic, and yet, students ask for a couple of quizzes. That's what I
14 think. But they are still an inevitable part of private courses. That's
15 right. We give ...hmm...We have to give.

It should be noted that Saban used the expression 'have to' and this, I believe, is a compelling indication of the inherency of use of quizzes in PCs. It can be seen in the teachers' expressions that it is very much associated with the expectations from the institutional context. The inevitability of the association can be also seen when he talked about the content segment of a lesson in the interview:

1 **T** From our point of view, they [students] do not even need to know
2 where the information in the sub-section derived from or its proof; he
3 should only know what to do when he comes across such a question.
4 **R** Isn't that mechanising?
5 **T** It should do. To me it should.
6 **R** Is that the expectation?
7 **T** The examination leads you towards this...

This was the case especially for PC teachers' practices rather than SS teachers' practices. I will explain its whys and wherefores in section 6.4. However, the results seem to suggest that finding an association between certain institutions and certain practices is not limited to the teachers I have examined as cases in this study. I believe that it is a common state of affairs for other teachers too. In fact, analysis of the questionnaire data supports this idea. The questionnaire was devised to see if other teachers also associate certain practices and solution methods with certain institutions and whether an institutional context is such an influential parameter for all the teachers. The questionnaire provided a mathematical problem and two possible solution methods with their mathematical explanations. Solution-1 was a common solution method for the problem as I observed in both institutions (SS and PC). However, solution-2, which employs NVT to reach the answer, was observed only in PC lessons. In the questionnaire, which was completed by 87 SS and PC teachers in total, I asked "which of the solution methods do you think **a typical state school teacher** would be more likely to use?" The results (see Table 6.1) showed a strong association of solution-1 with the practice of 'a typical state school teacher'. In fact, 86.2 percent of the teachers perceived that a typical state school teacher would **only** use solution-1.

| | Solution-1 | Solution-2(NVT) | Both | N/A | Total[1] |
|---|---|---|---|---|---|
| Percentage | 86.2 | 1.1 | 11.5 | 1.1 | 99.9 |
| Count | 75 | 1 | 10 | 1 | 87 |

**Table 6.1** Result of the questionnaire item regarding a typical SS teacher's practice.

This item seems to support my argument that certain solution methods are associated with SS. In fact, SS teachers were associated strongly (86.2 %) with **only** solution-1, where the solution requires some level of understanding of the theoretical side of the mathematics involved. This, in fact, is in line with part of the overarching goal of SS teachers. In another item in the questionnaire I asked "which of the solution methods do you think **a typical private course teacher** would be more likely to use?" The results (see Table 6.2) showed a strong association between solution-1 with the practice of 'a typical state school teacher'

| | Solution-1 | Solution-2(NVT) | Both | N/A | Total |
|---|---|---|---|---|---|
| Percentage | 3.4 | 63.2 | 32.2 | 1.1 | 99.9 |
| Count | 3 | 55 | 28 | 1 | 87 |

**Table 6.2** Result of the questionnaire item regarding a typical PC teacher's practice.

The answer to this question is divided between only the 'Solution-2' and 'Both' options with a relatively strong tendency towards the former. However, the majority (95.4 percent = 63.2 percent + 32.2 percent) of the teachers believed that a typical PC teacher would use NVT for solving the same example. In fact, solution -2 requires a minimum understanding of the theoretical mathematics involved and this is in line with the overarching goal of PCs, as Nuri put it (during the interview) "*Our aim is not teaching*

[1] In the tables the total figure may not add up to 100 since numbers are rounded.

*mathematics deeply and with its theory. As an educator in PC, our aim is to prepare them for the examination so that they can solve questions quickly in the shortest time.*"

The questionnaire results provide strong support for the validity of the argument (PC teachers use both of the methods and suggest NVT for students) based on results from other sources (interviews and video analysis) and therefore support the conclusion that there is an association between certain practices and certain institutions.

Although the content of mathematics taught in PC and SS were basically the same, the aspects of mathematical topics privileged in PC teachers' practices were different from the aspects of the mathematics privileged in SS teachers' practices. Moreover, mathematics taught in SS and mathematics taught in PC were labelled differently. Nuri for example made a clear distinction between 'PC mathematics' and 'SS mathematics'.

**R** Could you compare state schools with private courses?

**T** The aim of SS, the aim of high schools is to prepare students for life. To help him learn some job skills and to prepare him for higher educational institutions. They are one step lower to the higher institutions. Private courses, however, prepare students for directly for the university entrance examination.

**R** Do you mean it has nothing to do with university education?

**T** We prepare students only for the university examination.

**R** Does this difference in goals reflect the classroom life?

**T** I see that because students have the primary goal of going to a university, they primarily work on private course mathematics [rather than state school mathematics] and they pay attention to state school mathematics only secondarily. I mean, they study barely enough to get a passing grade. So the reflection is this: While students study state school mathematics just a day before the examination [at school], but for private course mathematics, I mean the mathematics we teach, they study regularly during the week. Goals are different.

First he makes the distinction between the overall goals of SS and PCs in order to differentiate between the two institutions. He then labels the mathematics taught in PC and in SS (lines 11-13, 15-16). This identification is important in that it signifies that his perception of mathematics is very much in relation to the institution in which he is working in. Another example of this is when I asked him about the aim of teaching mathematics.

**R** What do you think is the aim of teaching mathematics?

**T** As PC teacher, the aim of mathematics teaching is not teaching maths basically, but preparing them for the examination they will take. Make them able to solve the questions that they will face in the exam. Our aim is not teaching mathematics deeply and with its theory. As an educator in PC, our aim is to prepare them for the examination so that they can solve questions quickly in the shortest time. We have limited time...From our perspective, teaching practical mathematics[2] defines mathematics teaching.

In answering my question, firstly, he 'positions' (Davies & Harré, 1990) himself within the institution he is working (lines 2, 5, 6, 8) (also see following extract line 4, 6-8, 10-12, 15, 17-20, 24) and then he states that PC mathematics is defined as 'practical mathematics'. He explained this extensively when I asked him to comment on one of his previous remarks.

**R** You previously commented that 'we give the mathematics as summary. Whatever the student needs to know whatever necessary'. Can we say that it is like giving a pill?

**T** You know there are ready made [computer] programs. Yes. We give our students like ready made programs in relation to students' needs. I mean we don't want students to think in terms of [theoretical] mathematics. We don't want them to think deeply and we don't want them to think from a variety of perspectives. We have some practical methods and techniques. Their learning these techniques and methods is important to us. We give the topic as a summary as an outline, only the parts that are necessary...as I am teaching relations in a private course, I do not go into the properties of relations because there is no question about the properties of equivalence relations at all (in the university entrance examination)...They teach this at school, but I only say it verbally. But at the school this should be given with its properties and its proofs. That's what I mean as program. I did what my job requires me to do. I skipped something that would never be asked at the university entrance examination...But when we come to neutral element, when we come to finding the inverse of an element, we teach them carefully and emphasise them because there are past university examination questions on them. Because the property of closedness is not asked before we just state its name and we go onto the next one. But school gives these [properties] in detail with their definitions.

**R** Does it mean that what you do is decided by past university entrance examination questions, as a guide?

**T** Yes. Past university entrance examination questions gives us directions.

I have observed the same labelling in other teachers too. For example, during the interview when I asked Mahir; "OK. You present the definitions and students are writing them down. What do you expect them to do?" He replied "*In our mathematics,*

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[2] Here 'practical mathematics' means mathematics which involves solving questions quickly and in the shortest time.

*everything starts with definitions. Let's take complex numbers. What is a complex number? To know what a complex number is he has to learn the definition.*" The labelling meant that teachers are making a categorisation and thus relating certain features with a certain category. For example, when one labels some students as low achievers and some high achievers, he is inevitably establishing associations between, say, certain examination results with certain students. Therefore, labelling mathematics taught in SS and mathematics taught in PC means the teachers are associating certain features of mathematical practices with certain institutions. Central to the distinction between SS mathematics and PC mathematics, from the teachers' perspective, is the distinction between the goals of the two institutions. The teachers teach different mathematics simply because of the different goals of the different institutions.

The significance of these associations lies in the assumption that our perceptions and our practices are interrelated and perhaps it is a cyclical relationship. Our perceptions and expectations within a certain context will lead us to show certain behaviours within that specific context. In turn, this will affect how we perceive the context and what is an available course of action in that context. For example, eating in the context of a restaurant is generally associated with paying the bill (unless of course one owns it), but eating in the context of home is not. The result of these associations in practice is that, when we eat in a restaurant it will lead us to asking for the bill, whilst when we eat at home the same behaviour practice would be awkward and even rude. Similarly, if teachers perceive that certain institutional contexts are linked to certain practices, then when they are in a certain institution they will have, to say the least, a tendency towards having a practice in accordance with that institutional context.

I find the notions of 'constraints', 'affordances' and 'attunements' useful in understanding these associations. The term 'affordance' was coined by Gibson (1977), whose initial interest was in understanding what motivates behaviour (Reed, 1988). According to him before one can act, action must be perceived as possible, that is, the perception of an affordance motivates the doing of an action. He saw affordances as a precondition for activity defining allowable actions between the object or context and agent; however, the existence of an affordance does not necessarily imply that activity will occur (Brown, Stillman, & Herbert, 2004). For a person, for example, water affords drinking, not breathing but drowning (Gibson, 1977) but for a fish it affords breathing. Gibson (1979) explained that:

> the verb to afford is found in the dictionary, but the noun affordance is not. I have made it up. I mean by it something that refers to both the environment and the animal in a way that no existing term does. It implies the complementarity of the animal and the environment (p. 127)

Tanner & Jones (2000) adopted the term affordances in educational settings 'to describe a potential for action, the capacity of an environment or object to enable the intentions of the student within a particular problem situation' and 'constraints are provided by the properties of the environment, problem situation or social context which limit possible actions'. Therefore the term could be used for both an object or for an environment. For example, Drijvers (2003) examined the affordances of graphic calculators in relation to their use in mathematics classes and Boaler (1999, 2002) examined the constraints and affordances the school environment provided for students. To Greeno & MMAP (1998) the regularities of a person's practice, or routine parts of a teacher's practice, are not thought of in terms of narrow conceptions of ability but by their attunements to the different constraints and affordances provided by the context.

It seems that institutions have their affordances and constraints and this is closely related to the goals institutions have. Accordingly, those teachers and students who are attuned to these constraints and affordances will see particular practices as part of particular institutions. Therefore people will be attuned[3] to a variety of features of the context to a certain extent[4]. I view the teacher's perception of associations between institutions and certain practices as a result of teachers' attunements to institutional constraints and affordances. This may be illustrated by Figure 6.2.

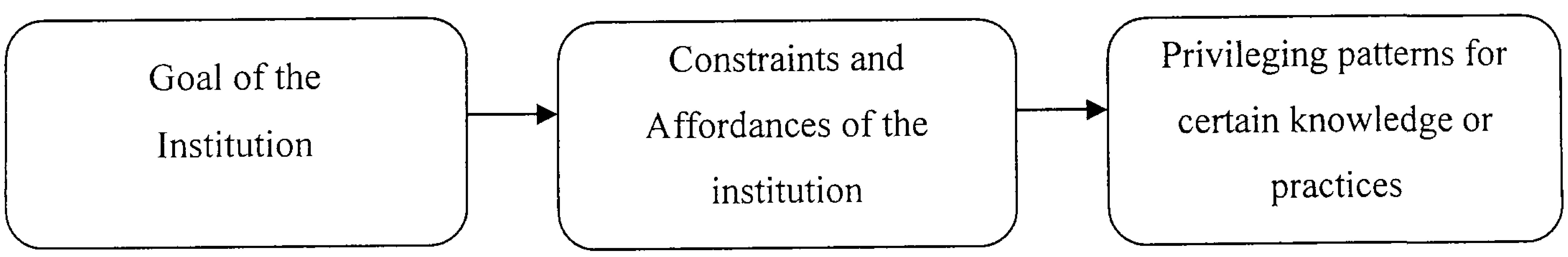


**Figure 6.2** Link between goals, constraint & affordances of an institution and privileging patterns.

For instance, PC teachers in this study associated NVT with a PC environment because NVT is an affordance in the context of PC. This is inevitably linked with the overarching goal of the institution, examination preparation. As stated by many

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[3] Wertsch (1998) would probably use the notion of 'appropriation' in such a context.

[4] I will discuss the relationship between teachers and their institutional context to a certain extent later in this chapter.

teachers, UEE requires solving a lot of questions within a time limit. This limitation in turn generates a constraint on PC teachers' practices. In other words, the teachers who work in PCs, who adopt the goals of the institution will need to teach some short way of reaching the answer. One common method utilised by the PC teachers for this purpose is teaching the use of NVT. It therefore explains why NVT is commonly practised among PC teachers. Therefore, the privileging patterns observed in the data (see section 5.1) indicate the possible mediation of constraints and affordances of the institution. This line of reasoning is also reflected in Grossman et al. (1999) when they, from the activity theoretical perspective, stated:

> An activity setting has a cultural history through which community members have established specific outcomes that guide action within the setting. The overriding motive for a setting, then, while not specifying the actions that take place, provides channels that encou rage and discourage particular ways of thinking and acting (p. 7).

It seems clear that they use the terms 'discourage' and 'encourage' referring to the similar meaning as my use of 'constraints' and 'affordances'. It is also important to see the significance of the overarching goal (or 'overriding motive' as they use it) that seems to regulate the constraints and affordances of the institutional context.

In a similar vein, Boaler (2002) has studied the development of over 300 students' mathematical knowledge over the period of 3 years from the age of 13 to the age of 16 in two schools. The schools were similar in the sense that at the beginning of the research, students from each school had similar attainment levels, but they were different in the sense that the schools showed different teaching methods. The students in one school (Amber Hill) were grouped by the teachers' perception of their ability and they were taught mathematics using a 'traditional textbook approach' (Boaler, 1999, p. 262); the teachers explained methods and procedures on the board at the start of lessons, and then students practised the procedures in textbook questions. The students of another school (Phoenix Park) were taught in mixed ability groups and they were taught using a more open ended approach; no textbooks or schemes were used and students were able to negotiate solutions with each other. The results showed that the traditional teaching based school (Amber Hill) was relatively less successful in the examinations at the end of three years. Boaler (2002) concluded that "knowledge and practices are intricately related and that studies of learning need to go beyond knowledge to consider the practices in which students engage and in which they need to engage in the future"

(p. 43) and thus "students did not learn *less* at Amber Hill: but they learned *different* mathematics." (p. 43) (italics in original). This is, as she interpreted, because of the different constraints and affordances students were attuned to in the different school contexts (Boaler, 1999). She mentioned several constraints and affordances students were attuned to in Amber Hill as being an impediment to their success, and she also listed several constraints and affordances students attuned to as having a role in their success in the examination at the end of three years. The differences in the students' experiences over the three years had a huge impact upon their mathematical perceptions and behaviours. She argued that "this was not to do with the clarity of their learning of mathematical concepts, but with the way they interacted with the broad activity systems of the classroom and the real world". These differences in affordances and constraints provided by different institutional contexts lead students to develop different identities as learners and the 'identities that were developed at the two schools that had an important impact upon the students' knowledge development and use' (Boaler, 1999, p. 264) and thus their mathematical practices.

This perspective is not different from one of Nuri's comments on what students learn in different institutions. Regarding SS students, he said:

**T** They will develop more theoretical [mathematics] and be more inclined to definitions. They would understand definitions a bit better. For example if they go to university, I could say that they would be better at university mathematics. The ones which attend private courses would have clearly a better number of correctly solved mathematics questions in the university entrance examination. I mean if both [group of students] sit the university examination, the ones going to private course will solve most of the questions, but the ones with only state school education will get stuck on some of the questions. I mean, one [group] will know the more theoretical side of mathematics based on definitions. The other [group] will know only the practical side of mathematics. For example, if a student is exposed to only private course mathematics, this student will have questions during the university.

**R** Which ones do you think knows more mathematics?

**T** If identical, state school students would know more mathematics

**R** Then, would that be correct to say that school students would be better in terms of definitions and concepts but private course students will be better in terms of solutions and procedures.

**T** Yes. We could say that.

Bingolbali (2005) studied undergraduate students' conceptions of the derivative from different departments (mathematics and mechanical engineering) within the same university. To him different departments have different goals and thus they offer specific courses for the needs of students in these departments. For example whilst the mathematics department offered abstract mathematics, the engineering department offered computer-based technical drawing. Bingolbali views these department specific courses as affordances of these departments. He argues that the affordances of the departments enable students to form particular conceptions with regard to the departments in general and their characteristics in particular.

### 6.3 Belief-Practice Inconsistency

Bandura (1986) argues that the beliefs that individuals hold are the best indicators of the decisions that they make during the course of everyday life. It is also the case in mathematics education. There are a large number of studies on beliefs related to mathematics and in particular to teachers' beliefs that emphasise the importance of the beliefs in influencing mathematical practices (see for example Thompson, 1984, 1992). In fact Llinares (2002) argues that it is the teachers' beliefs about their practice that need to be changed in order to improve teachers' practices. Ernest (1989) states that beliefs wholly 'regulate' a teacher's teaching practice in the classroom. Pehkonen & Torner (1996, 1999) go one step further and argue that beliefs act as a 'regulating system' that drive actions. However, the data at hand shows that teachers' beliefs may not be fully in line with their instructional practice. In fact, it may not always be teachers' beliefs that 'regulate' their instructional practices: the institutional context seems to have a major role in 'regulating' teachers practices. I will demonstrate this point on PC teachers, in particular on Nuri. Although I some found discrepancy between Nuri's beliefs and his practice in more than one aspect, I will focus on the discrepancy in the use of the numerical value technique (NVT) as the data on this issue is more readily available than other aspects. Having examined the case of Nuri, I will examine similar cases in previous studies and compare them with the current study. It will then be followed by an interpretation of the case through a major theory in social psychology.

I have observed Nuri, a PC teacher, a number of times teaching through NVT. Despite his frequent use of this technique, when I asked about his ideas on this technique he disapproved of the use of it as a way of teaching mathematics[5]:

[5] Rather than referring to the relevant section, I will present the interview extracts for the reader to see the belief-practice disparity more closely.

**R** What do you think about using numerical values to solve questions?

**T** Yes, this is a part of our system. In terms of university preparation, preparation for university entrance examinations, this is part of our system...Using numerical values is of interest to them and they like it very much. 'Let's assign 1 to the value of 'a', and after that, lets give the options, lets put 1 for wherever you see 'a', what a simple thing, isn't it.' This is a part of our system, I mean, as a private course it is a part of us, we make use of it.

**R** Do you mean it is one of the indispensables of private courses?

**T** To me [Yes], look, sometimes you may not be able to remember the solution of a question. Because the students may become nervous during the examination they may not be able to do things they can do normally. But if you approach them like 'you can solve it using numerical values' they can make use of a second method and they can possibly solve the question in a practical manner with ease.

**R** Do you think it is a healthy method in terms of mathematics?

**T** In terms of mathematics teaching it is not a healthy method. Because it keeps students away from formulas, it keeps them away from definitions. I mean without understanding the definition, without understanding the formulas, they want to solve questions. That's not healthy in terms of mathematics education.

It is also observable in the following extract.

**R** Let's ignore [for a moment] SS mathematics or PC mathematics. Considering using numerical value [technique] to solve the question, do you think it is an ideal way to teach mathematics?

**T** It is not healthy. In my opinion, solving a question using numerical value [technique] is only going for an easy[6] ride but it perfectly fits with PC [mathematics teaching] approach. It attracts students' attention. Students like it because students' aim is solving the question in any possible way and they like it and when you teach such techniques he mentions this to his fellow students [proudly] 'we have solved this question this way', 'Our teachers solved it with such ease', 'Wow I never thought of it this way' but it is not an ideal way to teach mathematics. From my perspective, it is going for the easy way a kind of escape to an easy way.

The extracts seem to have clear indications of a discrepancy between what Nuri believes and what Nuri practices. This was not the first case where researchers found a discrepancy between teachers' beliefs about teaching and learning mathematics and their instructional practices (see section 2.1 for further discussion of such a discrepancy). Raymond (1997), for example, reports a teacher named Joanna who

[6] The Turkish word 'kolay' is translated as 'easy' but it does not exactly correspond to the meaning of it. The reader should take into account that 'kolay' clearly connotes disapproval in this context.

seemed to have discrepancies between her beliefs and her practices. Although it is possible that Joanna could have been influenced by the researchers' interest in the consistency-inconsistency aspect and she was trying to be consistent in the eye of the researcher, both of which Raymond did not seem to acknowledge, the existence of inconsistencies shows the magnitude of the situation. Regarding this, Raymond (1997) said:

> Although Joanna provided two seemingly valid examples of how beliefs influenced her practice, she was unaware of the many inconsistencies between her professed beliefs and actual practice (p. 568)...It is likely that Joanna is not fully aware of conflicts in her classroom (p. 570).

It seems that Joanna was not aware of the inconsistencies. To me the teachers' awareness of inconsistencies is one of the most significant aspects of belief-practice research. This is because it could possibly lead to a change in the teachers' beliefs or the teachers' practices, which many reform movements failed to do. Although there is disagreement among researchers as to which (belief vs. practice) would (and therefore should be focused on) change first, most of the teacher researchers in mathematics education assume a cyclical relationship between changing beliefs and changing practice (Lerman, 2002, p. 235). Yet, it seems that research on belief-practice consistency has not focused enough on the awareness aspect of the issue. Neither Thompson's (1984) report on Lynn, nor Raymond's (1997) report on Joanna indicate that the teachers who are reported to have beliefs inconsistent with their instructional practice were fully aware of the situation. Raymond stated that Joanna 'managed' the contradictions rather than resolved them:

> Joanna may not have recognized the dilemmas...In not consciously acknowledging and reconciling either existence of opposing beliefs or the inconsistency between beliefs and practice, Joanna successfully managed the contradictions without completely resolving them (p. 570).

Thompson (1984) interpreted these discrepancies in relation to the extent to which teachers reflect upon their actions, their belief or the subject matter (p. 123). She called this the 'integratedness of the teacher's conceptual system'. She seems to view that consistency of beliefs meant a change in the teacher's instructional practice. To Thompson (1984), 'an integrated conceptual system' would be 'operating to modify her actions' (p. 122) and 'the failure to reflect on their actions in relation to their beliefs and in the needs and pressures, beliefs seem to have little, if any, effect on their teaching' (p.124). Therefore it seems that whether the teachers are aware of the inconsistency has a significant effect on their instructional practices. Awareness of such an inconsistency may influence teachers' to change their practices through reflection. It is also possible that inconsistency may lead to a change in the teachers' beliefs and this may eventually change in their practices. This approach is in line with Prawat (1992, p. 357), who argued that in order for teachers' to change their beliefs, the first step is that they must go through three steps:

1-They must be 'dissatisfied with their existing beliefs in some way';

2-They must find alternatives both intelligible and useful in extending their understanding to new situations;

3-They must find a new way to connect the new beliefs with their existing beliefs.

The awareness aspect is the major difference between the teachers I report and the ones other researchers report. One aspect of originality of the present study is that when the other researchers observe a teacher with belief-instructional practice inconsistency they also point out that the teacher is unaware of the situation. On the contrary, in the case of Nuri, the teacher is fully aware of the inconsistency between his beliefs about NVT and his practice (his usage of the technique).

In order to interpret the situation I will make use of the research in social psychology, where there are a number of theories in cognitive consistency. I will briefly explain the essence of a relevant major theoretical perspective in social psychology and then I will apply it to the current research. To social psychologists this awareness creates a 'tension' and this normally leads to a change (Eagly & Chaiken, 1993, p. 455). The popularity of theories of cognitive consistency dates back to 1950s. Although several theoretical approaches appeared, Festinger's theory of cognitive dissonance is arguably the most well known among them. Eagly & Chaiken (1993) explains the consistency principle as follows:

> The people's mental representations of their beliefs, attitudes, and attitudinally significant behaviours, decisions and commitments tend to exist in harmony with one another, and that disharmony motivates cognitive changes designed to restore the harmony (p. 469).

Festinger (1957) formulated dissonance theory in such a way that enabled him to examine any type of inconsistency between what he called 'cognitive elements'. This is because of his broad consideration of what constitutes the 'cognitive element'. To him, anything that the person recognises or 'knows' is a cognitive element. He explained it as "the things a person knows about himself, about his behaviour, and about his surroundings" (Festinger, 1957, p. 9). When two cognitive elements contradict with each other, dissonance arises. One of the most common examples of this is two cognitive elements 'I smoke cigarettes' and 'smoking is a health hazard'. This situation is illustrated in Figure 6.3, where dotted lines indicate inconsistency and therefore existence of a dissonance or tension.

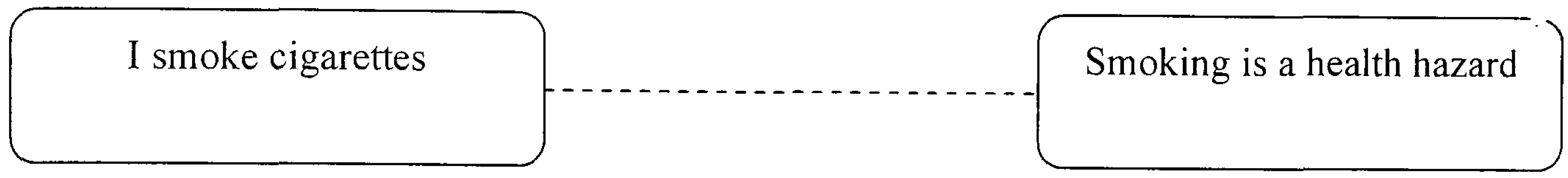


**Figure 6.3** Example of a cognitive dissonance.

According to Festinger (1957), when the person has these two cognitive elements he will experience dissonance and the magnitude of the dissonance experienced depends on the importance of these elements to the person. Central to Festinger's theory is the assumption that existence of dissonance results in tension and thus pressure to eliminate or reduce the dissonance. This requires the perceiver to avoid thinking about the elements or change one or more cognitive elements, which depends on the strength of the element. However, changing or eliminating a cognitive element may not be possible in some cases. For a heavy smoker it will be hard to change either the element 'I smoke cigarettes', or 'smoking is a health hazard'. Festinger argued that when two elements are hard to eliminate and not possible to avoid thinking about, a person can add 'consonant elements' such as 'I only smoke one pack a day' and 'more people die from car accidents than smoking'. Adding a consonant element may not completely eliminate the cognitive element(s) but it could reduce the significance of them to the person, allowing one to restore the harmony. The situation is illustrated in the Figure 6.4.

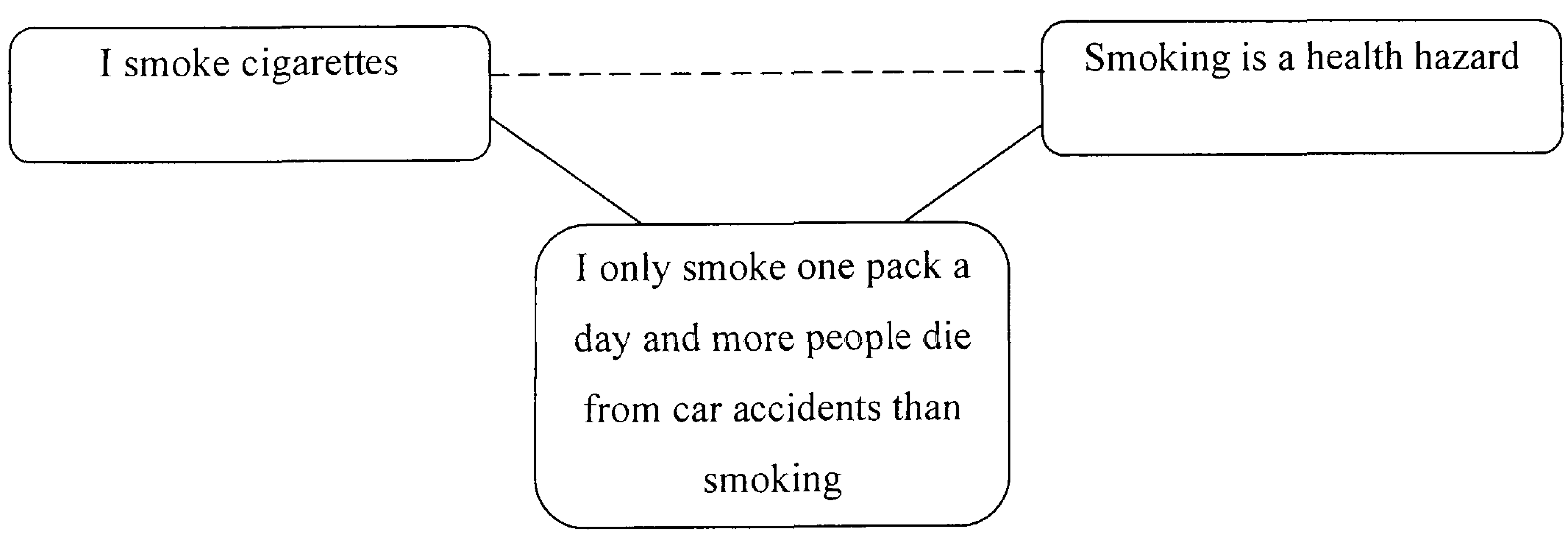


**Figure 6.4** Example of a cognitive dissonance with added consonant element.

In this case, adding a consonant element enables the person who smokes not to perceive 'smoking is a health hazard' as important, thus reducing the magnitude of the tension and restoring the harmony among cognitive elements. In the case of the PC teacher Nuri, he clearly disapproved of the NVT as a teaching method a number of times during the interviews, and yet, he is well aware of his practice of using NVT. Figure 6.5 illustrates the case of Nuri.

| The teacher's belief about the use of NVT | - - - - - - | The teachers' practice (use of NVT in the classroom) |
|---|---|---|

**Figure 6.5** Nuri's cognitive dissonance.

Festinger (1957) suggests four such changes, which are presented here in a form pertinent to the present case:

1. The teacher avoids thinking about the elements that conflict (specifically his belief that NVT is harmful), and so experiences disharmony only transiently;
2. The teacher tries to change his practice (e.g. stops using NVT);
3. The teacher changes his beliefs (e.g. comes to consider NVT as less harmful);
4. The teacher incorporates into his thinking additional "consonant elements" which effectively 'water down' the tension between his beliefs and his practice, and thereby reduce the motivation to change.

The first one is not possible because during the interview the case of inconsistency is very clear. However, during teaching it may be possible to avoid thinking about it. As for the second option, change in practice, Nuri has to stop using NVT as a technique. The teachers perceive the use of NVT as very much linked with PC and it is part of their job to teach certain short cut techniques (see section 6.2). There is a possibility that he may not be regarded as a good PC teacher if he does not teach these short cut techniques. At the end of the day this is why students are attending the PC, to learn these tricks and special techniques for the examination. As to the third option, the teacher may start considering NVT as less harmful. However, the extent of his disapproval is quite strong. He strictly rejects this as proper mathematics teaching. Nuri, however, goes for the fourth option e.g. he incorporates into his thinking additional consonant elements. Hence, based on the interview data, the fourth of these seems to fit the current case. Consider the following excerpt:

**R** What do you think about using numerical values to solve questions?

**T** Yes, this is a part of our system. In terms of university preparation, preparation for university entrance examinations, this is part of our system...Using numerical values is of interest to them and they like it very much. 'Let's assign 1 to the value of 'a', and after that, let's give the options, lets put 1 for wherever you see 'a', what a simple thing, isn't it.' This is a part of our system, I mean, as a private course it is a part of us, we make use of it.

**R** Do you mean it is one of the indispensables of private courses?

**T** To me [Yes], look, sometimes you may not be able to remember the solution of a question. Because the students may become nervous during the examination they may not be able to do the things they can do normally. But if you approach them like 'you can solve it using numerical values' they can make use of a second method and they can possibly solve the question in a practical manner with ease.

First of all Nuri is well aware of his practice; he shows no sign of unawareness. Secondly, it seems that Nuri perceives the use of techniques such as NVT as part of the overall goal of the institution he is working in. He clearly links his practice with the institution he is working in order to make sense of his practice (lines 2-4, 7, 8, 10, 12, 15, 16). This link was also examined in section 6.2. Similar observations can also be made in the following excerpt from an interview with Nuri.

**R** Sometimes teachers are exposed to students' questions like 'What should we do for these kind of questions?' Do you think this mechanises mathematics? I mean, does what is taught in PC mechanise mathematics?

**T** Of course. I mean, whether we like it or not, we do it. There are some [question] forms. These forms should be learned [and be taught]. The aim here is to bring the students to such a level that they can solve these [question] forms. Whether we like it or not, we have to mechanise a bit.

**R** Is it your, I mean is it PC mathematics characteristic?

**T** Whether one likes it or not because the characteristic of university entrance examination is to deal with practical side of mathematics[7]. In the university entrance examination it is not important the way you solve the question. It is not important how student solves the question. Sometimes we solve factorisation questions through using options. We say to students 'you may not know the topic at all; you may not understand this topic at all but let's solve this question together with using options.' And students get surprised 'oh my god how easy it was.' I mean you show different approaches to the different questions but because there are some question forms in some topics that do not change, we

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[7] Here the term 'practical' refers to solving questions in a shortest time and quickest way without dealing with any theoretical aspect of the question at all.

mechanise it. We say 'for this kind of question you should do it this way, you should not think something else' for another question [we say] 'do not get into such interpretations'. I could not express well but we basically do not want them to go deep into it [theoretical mathematics]. Sometimes through doing too much interpretation, students are making mistakes. ...we expect them to produce more practical and superficial solutions.

**R** Let's ignore [for a moment] SS mathematics or PC mathematics. Considering using numerical value [technique] to solve the question, do you think it is an ideal way to teach mathematics?

**T** It is not healthy. In my opinion, solving a question using numerical value [technique] is only going for an easy[8] ride but it perfectly fits with PC [mathematics teaching] approach. It attracts students' attention. Students like it because students' aim is solving the question in any possible way and they like it and when you teach such techniques he mentions this to his fellow students [proudly] 'we have solved this question this way', 'Our teachers solved it with such ease', 'Wow I never thought of it this way' but it is not an ideal way to teach mathematics. From my perspective, it is going for the easy way a kind of escape to an easy way.

As can be seen in the extract Nuri is referring to his overarching goal (derived from an institutional goal) to explain the discrepancy between his practice and his belief. The question forms he mentions here are the types of examples in the examination. He refers to them as 'model examples'. Because he believes that he should teach these model examples in order for students to be ready for the examination, it is not that significant even if it has negative consequences. He aims at students' being mechanically ready for certain forms of questions so that they will be able to solve them in the examination and because they are ready they will not lose time to think about how to solve it. His comment "*whether we like it or not, we do it*" is so powerful in that it clarifies the extent of the influence of institutions on teachers' practices. It indicates that the teacher is putting the priority of the institution he is working her in front of what he personally believes. I believe that the influence of an institutional context cannot be expressed made clearly than this comment which was repeatedly expressed by the teacher who has first hand experience of the institutional influences as an 'insider'.

Using Festinger's (1957) analysis, the use of NVT by a principled teacher might be seen as an example of 'forced compliance', which, as his practical studies show, can be lived with on the basis of financial reward. The teacher has chosen to work for the private

[8] The Turkish word 'kolay' is translated as 'easy' but it does not exactly correspond to the meaning of it. The reader should take into account that 'kolay' clearly connotes disapproval in this context.

school, where the pay is good and the status is high, and accepts the consequences of doing so. Despite the fact that many researchers argue that beliefs have a significant power in explaining teachers' instructional practices, they may not always be the major factor that 'regulate' teachers' actions. Although, it is generally felt that "beliefs have a strong shaping effect on behavior" (Schoenfeld, 1998, p. 19), teachers' beliefs about how mathematics should be taught can be overwhelmed by the goals in particular institutional contexts. In other words, institutional goals can be more powerful to drive actions than beliefs do within 'certain institutions'.

Like Nuri, data from Saban showed discrepancies between his beliefs and his instructional practice. In particular, he also disapproved of the NVT. Although Nuri refers to the institutional goal to explain the discrepancy between his practice and his belief, Saban's reaction was slightly different from Nuri's reaction to the situation.

In contrast to PC data, SS data did not show a discrepancy between teachers' beliefs and their instructional practice[9]. It seems that what they believe about how to teach mathematics and how students learn is reflected in their instructional practices. The question that arises from this is why this happens in the context of PC but not in SS. Hence, there is a need for an explanation of what is meant by 'certain institutions' (see previous paragraph). I will address this issue and explain further what 'certain institutions' refer to in the following section.

## 6.4 Teachers' Goals and Institutional Context

Previously I have shown differences between SS and PC teachers' practices and argued that it is mainly the institutional context that is likely to explain these differences. I then showed that teachers establish associations of certain practices with certain institutions and that in turn influences how they perceive the culture of the institution in relation to their classroom practices. In the previous section, I demonstrated that the influence of institutions could be so powerful that it could subordinate teachers' beliefs in driving the teachers' instructional practices. One major element I mentioned in the previous sections is, derived from the institution in which they work, the teachers' goals. In this section, I will focus on the teachers' goals in relation to the institutional context in

[9] It should not be taken that there is no discrepancy at all, there may be, but there was no data that confirms it.

which teachers operate. I will argue that the institutional context within which PC teachers work is more influential than the institutional context within which SS teachers work.

In any study focusing on teachers' practices it is important that the researcher takes the teaching goals into account. However, the goals in teaching are not researched extensively (Aguirre, 2000). The goals teachers hold could be overarching goals as well as goals that are more context specific or as Schoenfeld (1998) puts it, 'local goals'. Teachers' overarching goals possibly become a background for teachers' local goals. The same could be possibly be said for teachers' emergent goals (Saxe 1991), which are created in moment-to-moment interactions with students e.g. teachers' emergent goals are derived from their overarching goals. Cobb (1986) suggests considering 'more general contexts associated with overall goals and the specific contexts of on-going activity as being mutually interdependent' (p. 4). That is to say that the overarching goals that teachers have shapes their local goals and vice versa.

In this study, I observed teachers' actions in both institutions and their behaviour supports such a dialectical relationship. For example, use of one page quizzes is very common in PCs (not SSs) and the teachers' overarching goal of 'preparing students for the examination' can be localised to the classroom activity level by producing a local goal like 'helping students to be able to solve examples quickly'. One way of achieving this quickness, as PC teachers suggested, is by making them practise a variety of types of examples in and out of the classroom through quizzes. Similarly, SS teachers' overall goal of 'students should understand mathematical concepts' can be localised to the classroom activity level by producing a local goal like 'to make sure that all students write down the definitions of the mathematical concepts taught.' In fact both SS teachers suggested that the students should have a copy of the definitions as they need to see the definition of concepts in order to understand them.

However, the overall picture of the four teachers suggests that SS teachers seemed to be less influenced by their institutional context in comparison to PC teachers. This is based on an analysis of the interviews as well as the analysis of the classroom practices. I demonstrated this in section 6.3 where I showed that some of the PC teachers' instructional practices are in fact against their beliefs on how mathematics should be taught. This was not the case for SS teachers. Moreover, during the interviews SS teachers seemed to use relatively more personal expressions such as 'I think' or 'I believe' in comparison to PC teachers, who preferred 'we' and 'us'. In other words, in comparison to PC teachers' practices, SS teachers' practices are mediated more by their personal preferences. Data on SS teachers indicated relatively less influence of the institutional context. In contrast, PC teachers used expressions that suggest the institutional context is a factor in explaining and making sense of their practice. This was also the case for teachers' practices. Considering sections 5.2 and 5.3, the analysis of the teachers' instructional practices illustrated that there was little variation among PC teachers' instructional practices; in fact they were almost identical in many respects. SS teachers, however, varied in certain areas despite the fact that the related local goal inferred from the teachers' practices seems to be the same; 'student engagement in the lesson'. For example, Ayten's use of examples suggested that the emphasis was on the 'engagement' phase. Mahir's use of examples suggested that he emphasised the 'resolving phase'. Ayten walked among the desks of students and checked their work in the 'engagement' phase to insure that all students are trying their best to solve the example presented on the board. Mahir tried to involve students in the lesson by creating an environment in the 'resolving' phase where students could discuss their methods and make suggestions to the teacher. Lastly, while PC teachers' practices seemed to be directly influenced by the context, SS teachers' practices seemed to be indirectly influenced. For example, PC teacher Saban said, "*Because the questions of university entrance examination do not change [from student to student], I have to teach the same things to everybody*", "*To be more precise we depend on university entrance examination*" and Nuri said "*I did what my job requires for me to do. I skipped something that would never be asked at the university entrance examination.*" On the other hand, SS teachers did not comment on such powerful influence and yet it was available indirectly. For example, when I asked "*did you choose the textbook?*" Ayten stated "*The school has made the selection*" and that "*I generally use examples that are very similar to the textbook*". When I asked "*is it one of the books approved by Ministry of Education*? she commented that "*Yes, it approved by Ministry of Education...How dare we [use something else]...? Who are we [to make such a decision].*" This clearly indicated the indirect influence of institutions on their practice.

It is acknowledged that 'institutions can exert more or less strict regulation on the practices of people involved as subjects' (Castela 2004, p. 42). In this study, the influence of the institution appears to be stronger in PC, compared to SS. However, it is a matter of degree. That is to say, SS teachers are only influenced by institutional contexts to a limited extent, and PC teachers' personal preferences and beliefs only have a limited influence on their practice. Teachers could be influenced by the institution in which they work more or less, depending on a number of parameters. It seems to me that the way in which institutions influence individuals is unlikely to stem from a single aspect of the relationship between teachers and institutions. That would be underestimating the complexity involved in teachers' professional socialisation process and would show a lack of understanding of the historical development of the research on teaching.

Having said that, it is a challenge to determine the factors affecting teacher socialisation and in particular how the institutional context influences them. Since the significance of the institutional context was an emergent theme in the current study, the factors I present may not give the complete picture. However, data suggested these factors to be important. Before going into these factors, I would like to clarify how the thesis positions itself on the issue. I am of the opinion that institutions should not be thought of as just buildings, but as a collection of elements and interaction between them. These elements include the way in which the buildings are structured and the physical layout of the room and furniture. They also include the rules and regulations on paper and how individual positions and different roles are constructed by these rules and regulations. However, institutions are not complete without the 'actors' involved.[10] That is to say institutions would not be functional without the people, who take various positions (for example, the position of head teacher, head of mathematics department or mathematics teacher). Therefore, when the institution is mentioned one should not simply assume non-human elements. It is in fact the interaction between different elements that shapes the way institutions operate. Ignoring the human factor would be deterministic and as teacher socialisation literature suggests (Zeichner & Gore, 1990), functionalist. Such a perception lacks sufficient understanding of how individuals' beliefs, their perceptions of themselves, and the way actors play out their positions and roles, may vary. On the other hand, ignoring the physical layout of the building, the rules, the regulations and the positions as reflected in these regulations would be overestimating human autonomy and would assume human behaviour operates without the restrictions of the context.

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[10] Lave (1988, p. 151) uses the term 'arena' to refer to contexts without humans and exemplifies this with supermarkets without their customers during the night.

The power of an institution to influence its 'insiders' is not an inherent property. It is dynamic, as it depends very much on the actors involved in it. The influence of the context can only be put into practice through the agents involved in the institution. However, the influence of the institution can be sufficiently powerful that individual differences among the 'insiders' of an institution are reduced. It follows that, ironically, even though the influential power of an institution depends on the individuals in that institution, it is indicated by the extent that, within that institution, individual differences are reduced. In this study I conclude that the influential power of SS is relatively less than PC since in the context of PC the data indicated less variation among the teachers' instructional practices. There may be a huge number of factors that affect this. However, I would like to propose a few factors based on the data at hand.

A major factor that determines such an influence is the extent of specificity of the overarching goal of the institution. Wertsch (1985) maintains, from a socio-cultural perspective, that "the motive that is involved in a particular activity setting specifies what is to be maximized in that setting. By maximizing one goal, one set of behaviours and the like, over others, the motive also determines what will be given up if need be in order to accomplish something else" (p. 212). I propose that the more specific the overarching goals[11] are, the more likely it will be to produce context specific practices (again depending on the agents). Au and Carroll (1997) (as cited in Gallego et al., 2001) have documented teachers' dissatisfaction with generalities and teachers' requests for guidelines that are 'specific enough to guide practice'. From this perspective, SS teachers may be unable to answer questions on their goals of instruction directly because SS do not provide sufficiently specific guidelines to formulate a goal for their instruction. This may be a reason why PC teachers easily turned to their overarching goals in explaining their practice during the interviews while SS teachers seemed to be 'beating around the bush'. In an institution where the overarching goal is clearly expressed, institution specific practices are likely to be more easily produced because the benchmark for the agents involved is clear. Therefore, when making decisions, agents will have less space for personal interpretation. Less input of interpretation in the decision making process means more likelihood of production of stereotypical practices or practices that are specific to the context. Hargreaves (1986) argues that teachers are

[11] I would like to emphasise that the goal I refer to is the overarching goal, not 'local goals' (Schoenfeld, 1998) or 'emergent goals' (Saxe, 1991).

not controlled by or passively responding to the exigencies and constraints of classroom life, rather teachers respond on the basis of their interpretation of the situation, experienced problems and dilemmas. However, when the goals are specified very clearly, teachers have relatively little space to interpret. Returning back to my data, SS teachers have loosely defined overarching goal(s), which enables them to have some space for interpretations. However, the goal of a PC, as expressed by the teachers, is relatively more specific. Many PC teachers I interviewed, and socialised with (mathematics teachers as well as teachers of other subjects) in and out of classrooms have stated their goal in a simple way. Nuri for example puts it as:

R What do you think is the aim of teaching mathematics?
T As PC teacher, the aim of mathematics teaching is not teaching maths basically, but preparing them for the exam they will take. Make them able to solve the questions that they will face in the exam. Our aim is not teaching mathematics deeply and with its theory. As an educator in PC, our aim is to prepare them for the examination so that they can solve questions quickly in a shortest time. We have a limited time...From our perspective, teaching practical mathematics defines mathematics teaching.

In my opinion, the expressions 'As PC teacher' (line 2) and 'As an educator in PC' (line 6) suggest that the teacher is explicitly 'positioning' (Davies & Harré, 1990) himself within the institution he is working in in answering the question. He is doing so implicitly as well: 'Our aim' (line 5), 'We', 'From our perspective' (line 8). From his perspective he identifies himself not as a teacher but as a PC teacher. What this extract seems to suggest is that the teacher adopts the goal of the institution and that the teacher saw the goal of the institution as 'teaching for the examination'. It was not only expressed in specific terms and straightforwardly but it was also a singular entity. On the other hand SS teachers saw their goal as a multi-dimensional entity and they indicated broader issues. SS teachers perceived their job as preparing students for their future life, and primarily their education in university or colleges. PC teachers see it as only 'the' examination. This distinction between SSs and PCs can be illustrated in Figure 6.6.

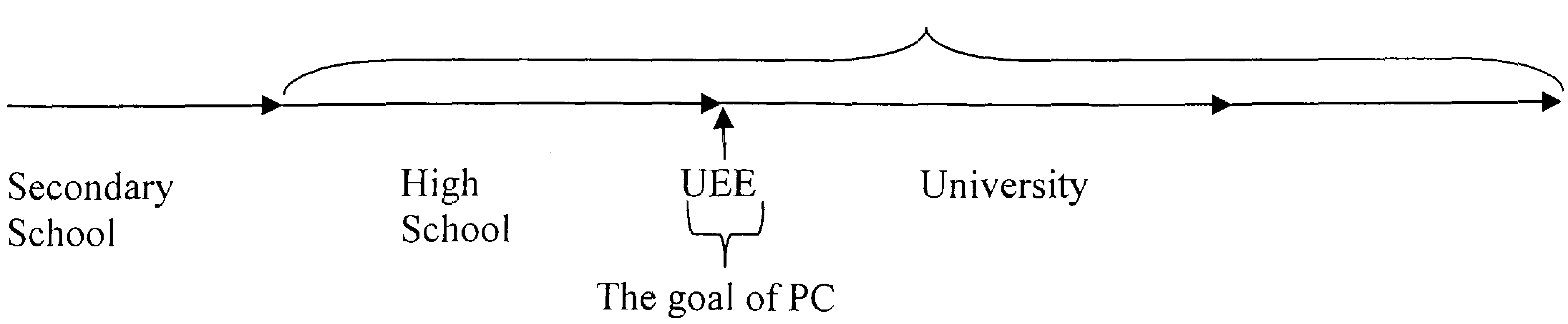


**Figure 6.6** The goal of PC and SS in terms of students' life span.

SS teachers' perspectives are wider than those of PC teachers. To elucidate this point further, I would like to refer to two concepts teachers used. They are 'eğitim' and 'öğretim' in Turkish. To me the closest translation of 'eğitim' is 'education' and the closest translation of 'öğretim' is 'instruction' or 'teaching'. The term 'eğitim' has clearly a broader meaning than 'öğretim'. A number of mathematics teachers made the distinction between 'eğitim' and 'öğretim'. It was perceived that while the goal of PCs is no more than 'öğretim' (instruction), the goal of SSs is 'eğitim' (education) along with 'öğretim' (instruction). That is to say, PC is regarded as an institution which focuses merely on students' acquisition of academic procedural knowledge. In contrast, SS is regarded as an institution where students not only learn academic subjects such as mathematics and science but it is also an institution to socialise and learn how to behave in society, or how to be a good citizen in general. In fact, during the interviews I found it quite challenging to pinpoint SS teachers' perceptions of their overarching goals, it proved to be quite elusive. From this perspectives Connell (1985) seems to have a point as he perceives teaching as

> a labour process without an object. At best, it has an object so intangible that it cannot be specified in anything but vague and metaphorical ways (p.70).

He may be criticised as perceiving teaching as simply 'a labour process' and ignoring all the complexities involved, nevertheless he has a point in arguing its vagueness. It is this vagueness of the SS context that seems to enable, or better require, SS teachers to interpret the situation in making decisions related to instructional practice. On the other hand, it seems that having a specific goal makes it easier for PC teachers to judge whether certain instructional practices are 'acceptable' or not. This is, of course, an interpretation of the teacher. However, it is minimal in PC as Nuri repeatedly puts it "*whether we like it or not, we do it*".

Another factor, not unrelated to the previous factor mentioned that affects the extent to which institutions influence the actors involved is the magnitude of the constraints and affordances of the institution. The greater the constraints and affordances, the higher the likelihood of the influence of context. Admittedly it is not quite straightforward to establish the magnitude of the constraints and affordances for an institution. Moreover, there is always the factor of human agency that mediates between what is potentially available and what is actually put into practice. In this study, I observed a relatively smaller degree of institutional constraints and affordances on SS teachers' practices in comparison to their colleagues working in PCs. One indication of this is that I did not observe belief-practice inconsistency (as a result of institutional constraints and affordances) in the data I collected from SS teachers. In contrast, the discrepancy between teachers' beliefs and practices in the data collected from PC teachers was clear (see section 6.3). I have also observed associations between certain practices and certain institutions, which I argued (see section 6.2) to be the result of institutional constraints and affordances. For example, the questionnaire showed that NVT was considered as a solution method typical of PC teachers' practices, but not SS teachers' practices. Hence, NVT was seen as an affordance of PCs. In the institutional context of PC, the teachers are expected to use this example solving technique. This may result in similarities in the PC teachers' instructional practices. NVT is one affordance that was identified in this study which was observed to be producing institutionally stereotypical practices. There may be some others, which may collectively produce a stronger case. Additionally, since PC has quite a specific goal, the constraints that PC have (on teachers) might be stronger. This might limit diversity in PC teachers' practices. The argument here is that the magnitude of constraints and affordances teachers experience in an institutional context seems to be a significant factor on the extent to which the teachers are influenced. It seems, therefore, that constraints limit certain practices that are institutionally 'unacceptable', and that affordances promote certain practices that are institutionally 'acceptable'.

So far I have discussed the influence of institutions on teachers' instructional practices. In doing so I argued that there are associations between certain materials and practices and types of institutions. I also argued that the institutions may be so influential on teachers' practices that teachers subordinate their beliefs, that is, teachers may prefer to teach in the way that the institution they are working in suggests, even though they believe otherwise. I also explored how institutions operate and what may affect the way institutions influence teachers' instructional practices. The phrase 'contextual density' can be used to refer to the influential power of institutions, since it metaphorically comprises the meaning discussed. I have attempted to develop the idea behind this concept. However, since the influence of the institutions on teachers' instructional practices was an emergent theme for the current study, therefore I could only develop the viewpoint within the constraints of the data at hand. This start will hopefully enable future studies to examine complexities of 'contextual density' in detail.

## 6.5 Institutions and Teachers' Professional Development

I hope I have demonstrated the effects of the institutions on teachers' practices, and have argued that it is important to take the institutional parameters into account in order to understand teachers' instructional practices. Furthermore, I claimed that the influential power of the institutions or 'contextual density' of an institution, on teachers may not be the same for all educational institutions. In this section, I will use an activity theoretical approach to demonstrate two possible implications of the current perspective on research on teachers.

With the rise of studies with a socio-cultural focus in mathematics education, activity theory has gained prominence. Lev Vygotsky can be considered as the father figure of socio-cultural research. Central to socio-cultural theories in general and activity theory in particular is the idea of human activities being mediated. Vygotsky, however, is also criticised for focusing solely on mediated action and ignoring the wider cultural and historical context in which the activity occurs. (Wells 1994 a,b as cited in Daniels 2001). Engeström (1993) developed activity theory to enable examination of mediated activity within the broader and macro level rather than solely on the individual and micro level. His addition to Vygotsky's original subject-object-tool model is three new elements which brings the context into perspective. They are: division of labour or roles, community, and rules. Through incorporation of these elements he puts the wider context into the account of human practice. Daniels (2001, p. 78-79) reports a difference between Wertsch and Engeström in terms of what is brought to the foreground and what remains in the background of the research in their use of Vygotsky's ideas: Wertsch (1998) is focusing on mediational means, while Engeström (1993) is focusing on the context. However, Wertsch also recognises (Wertsch & Rupert, 1993) the significance of some contextual elements such as authority and values in social settings in understanding human practice.

I would like to employ this approach to make sense of the teachers' practices within their institutional context. In understanding the concepts used in this study and whether they fit within the original formulation of them in the previous accounts of activity theory, one should take into account that activity theory is more an approach to data rather than a 'theory' per se in the strict sense of the word (Engeström, 1993). As Kuutti (1996) puts, it is 'a philosophical and cross-disciplinary framework for studying different forms of human practices as development processes, both individual and social levels' (p. 25). Thus, as Engeström (1993) argues:

> Activity theory is not a specific theory of a particular domain, offering ready-made techniques and procedures. It is a general, cross-disciplinary approach, offering conceptual tools and methodological principles, which have to be concretised according to the specific nature of the object under scrutiny. (p.97)

A detailed explanation on the underlying assumptions and concepts used by activity theorists can be seen in Cole & Engeström (1993).

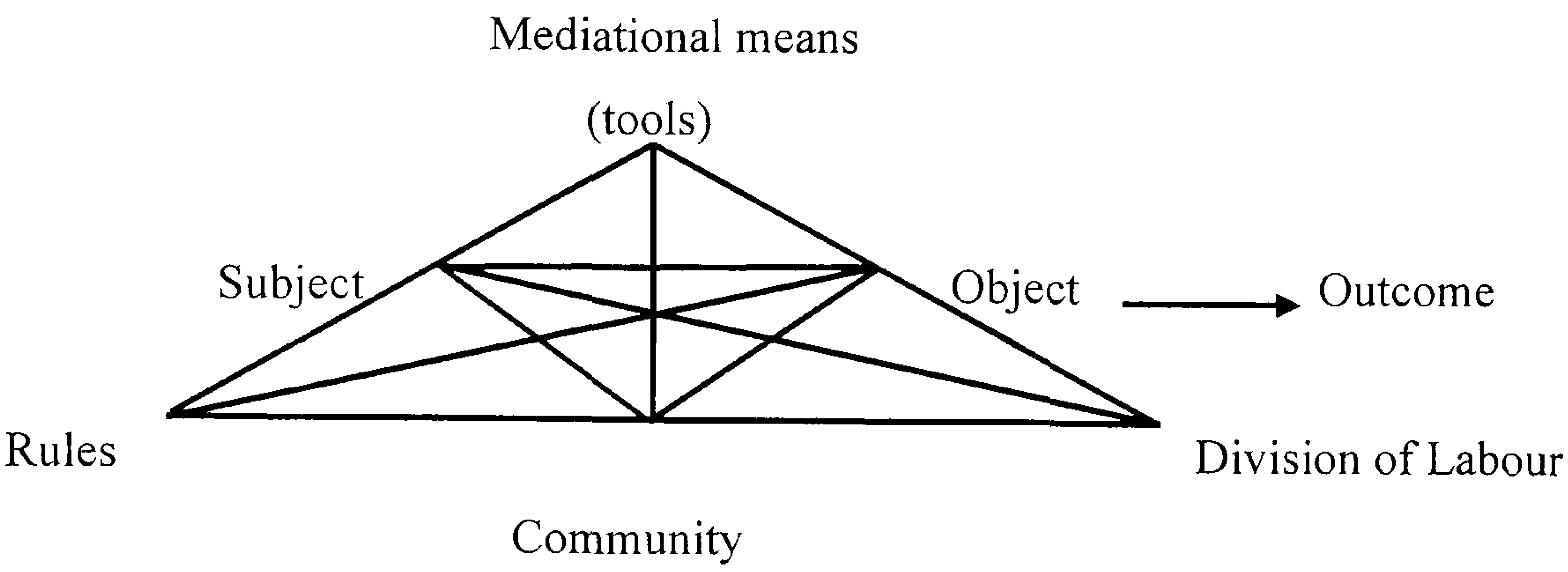


**Figure 6.7** Activity theory diagram.

I will now make use of the activity theory as a lens to view the current study, and then I will illustrate the possible implications of this study. I will do so through making use of some of the concepts involved in the activity theory (Cole & Engeström, 1993; Engeström, 1991) diagram. (See Figure 6.7)

Since tools transform "the entire flow and structure of mental functions" (Vygotsky, 1981), I would like to direct my attention to practices that involve the use of institution specific tools such as NVT (see section 6.2) in PC, an institution with greater contextual density. I have previously proposed (see section 6.3) that it caused Nuri to have a discrepancy between his beliefs and practices, and yet he managed to reconcile his beliefs and his practice through referring to the institutional context he is working in. The reaction of Saban is interestingly different from Nuri. When I prompted if NVT is a healthy method to teach mathematics, Saban interrupted me and referred to the goal of the institution in which he is working:

**1** **T** That saves time in the university entrance examination...if students
**2** know for which questions they can use the options, they will save
**3** some time in the examination.

When I asked again he said "*I think it is quite appropriate*". After a short pause he referred to the meaning of the phrase 'mathematics education' and commented

**T** Is mathematics education refers to training of brain or is it just teaching mathematics? It isn't appropriate for teaching [mathematics] but it is a way. It is a way for students to use their brain.

**R** How do you use numerical value technique, could you explain a bit?

**T** Well in order to use numerical value the student should have a potential and the options should be presented

**R** Hmm. Do you approve of it?

**T** Let alone approving of it, we clearly advise it.

**R** You *advise* it?

**T** Because in the university entrance examination, more or less 10 to 15 questions are asked like this.

He disapproves of it for teaching mathematics but he accepts it as a way to solve examples. It seems to me that he needed to adjust the meaning of 'mathematics education' from 'teaching mathematics' to 'training of brain' in order to have consistency between his beliefs and his practice of using NVT. He believed that it is not his job to 'teach mathematics', since he does not consider himself as a 'mathematics teacher'. He perceived himself as a 'brain trainer' rather than a mathematics teacher, which he seems to regard as someone who teaches mathematics using 'acceptable' methods. It was also observable when I asked "what do you think is the aim of teaching mathematics?" He said:

**T** ...I believe that one's ability to solve a problem and the method that he created to solve that problem shows its effects on social life. I mean the aim [of mathematics] is to exert the use of brain a little differently. If someone is able to solve a second degree equality question [and find the roots] using his own interpretation, if he is able to do that interpretation, when he faces other issues he will be open to different interpretations.

**R** You mean mathematics education changes one's perspective?

**T** I am saying it can change and he does not condition himself to a single solution while trying to solve a real problem of a social life. He can produce alternative solutions. It is open to debate whether one needs to have complete knowledge about something in order to produce it [solution]. If he can use his brain well he can calculate pros and cons.

**R** Are you saying mathematics gives interpretation ability?

**T** Yes. I think it does at. At least contributes. When solving question I show 3 different methods. Someone can solve a question with only a single way, that's possible. But when he looks at the question and tries to find different solutions and is able to find different solutions to the same problem that means he is able to use his brain in different areas. If someone is able to use his brain in mathematics, he has the ability to think about different perspectives and a variety of points.

It seems that Saban preferred to leave his identity as 'mathematics teacher' and stick to using the tool specific to that institutional context. In fact he rather considered himself as a 'brain trainer'[12]. It seems that with the extended exposure to institutional constraints and affordances and his 'appropriation' of the tool in that activity setting he formed an identity around the use of this context specific tool. This is in line with Hung & Chen's (2002) argument who state:

> From an activity theory perspective, these rules, tools and roles serve as mediators within the activity system. Through enculturation (participants performing their specific roles through the assistance of tools and according to the rules of the community), participants learn to be, forming an identity particular to that community (p. 247).

Similarly, since the institutional context and in particular 'communities of practices' (Lave & Wenger, 1991; Wenger, 1998) in which the teachers live partially defines what role they have and what they do and the way they practise their job, it is also part of who they are. Hence, teachers' identities seemed to be mediated by the context in which they live their professional lives. Therefore, I would like to argue that mathematics teachers learn to become a mathematics teacher within the context, in particular, the *institutional context* in which they teach. Studying beginning teachers' professional development Calderhead & Shorrock (1997) maintains that:

> ...becoming a teacher is not simply a matter of *doing* what teachers do, it is also a matter of *being* a teacher. The latter involves a personal investment, a feeling of being at ease in the role of teacher, an acceptance of teaching as being part of one's identity, being able to reconcile one's own values with those of the institution and the colleagues with whom one works. This does not mean that the new teacher has to accept those institutional values, but that he or she has to develop a way of working within them...Part of being a teacher is developing relationships with children, establishing a human rapport with others, and contributing through one's own personal qualities to the working environment of the classroom and school (p. 194) (italics in original)

In this connection, I would like to make use of the distinction between what Sfard (1998) calls the 'acquisition' metaphor and the 'participation' metaphor for learning. She accepts, notwithstanding, that they are not mutually exclusive. Although they are seemingly contradictory , as she accepts, she considers that they are offering different

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[12]In Festinger's (1957) terminology Saban incorporated into his thinking an additional 'consonant element' which removes a possible tension between his beliefs and his practice regarding NVT.

perspectives rather than competing opinions. Bruner (1996) makes a similar distinction between learning *about* and learning *to be* within the framework of 'communities of practices'. Hung & Chen (2002) make use of this approach and re-conceptualise activity theory within the framework of 'communities of practices' to examine the relationship between learning *about* and learning *to be* in teaching. They state:

> Learning to be and learning about are deeply intertwined. We learn 'how' through practice; and through practice, we learn to be. In other words, practice shapes our dispositions and belief systems – our identity in a particular profession. When we learn more about a particular domain or profession, we identify more with that profession – forming an identity of that profession...From an activity theory perspective, these rules, tools and roles serve as mediators within the activity system. Through enculturation (participants performing their specific roles through the assistance of tools and according to the rules of the community), participants learn to be, forming an identity particular to that community (p. 247).

Learning *about* in the context of teachers' professional development involves learning an accumulation of factual knowledge of information regarding what teachers do or 'knowing that' (Brown & Duguid, 2000, p. 128) However, as Lortie (1975) pointed out more than 30 years ago, this is not enough for professional development as it is not the same as learning *to be*. Learning to be not only requires knowledge about teaching but it comes through the process of enculturation (Hung, 1999). The process of enculturation takes place inevitably within certain institutions. Moreover, it is the contextual density of those institutions that seems to be affecting the enculturation process. This is because an institution with greater contextual density is more likely to have agents with little personal preference for action and this will result in more institution specific practices through usage of institution specific tools. This will inevitably produce 'tensions'[13] (Freeman, 1993, p. 488) or 'dilemmas' (Berlak & Berlak, 1981) between individual and institutional factors. In this study, for example, I have shown (see section 6.3) that NVT is a context specific tool and it caused such a clash between individual and institutional factors. Through the reconciliation of these factors and resolution of the 'tensions' or 'dilemmas', teachers reflect on their practices and this requires them to perceive their professional roles as teachers differently. It is, hence, these tensions that 'help' them to

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[13] Note that Freeman used the term slightly differently from the activity theoretical sense. My use here is more in line with his usage.

come to understanding themselves differently. From an activity theoretical point of view Wertsch & Rupert (1993) argue:

> A focus on the importance of mediational means in shaping human action does not imply a static body of knowledge or set of analytic practices. Instead, the tension involved in the interaction between mediational means and the individuals using those results in a continuous process of transformation and creativity. In this process, however, we again stress the creation of new ideas and practices occur through operating on existing mediational means (p. 230).

In a similar vein, Kennedy (1990) (as cited in Freeman, 1993) maintains that the notion of tension helps to reframe the 'improvement of practice problem'. In agreement with this line of reasoning, Freeman (1993) argues that:

> To develop their classroom practice, teachers need to recognize and redefine these tensions. In this process of renaming what they know through their experience, the teachers critically reflect on-and thus begin to renegotiate- their ideas about teaching and learning (p. 488).

It seems to me that it is not only the teachers' practices that are influenced by the institutional context but also their perception of self and therefore their professional identity. Studying the influence of institutions on students' understanding of certain mathematical concepts, Castela (2004) comes to a similar conclusion. He compares students from two different French higher education institutions and claims that the institutions 'regulate'[14] students' and teachers' practices:

> When persons 'enter' an institution, their life in this institution is submitted to collective *constraints* and *expectations* that regulate their actions. These constraints and expectations specify their position as *subjects* of the institution. Several subject positions exist in a given institution: for example, students, lecturers and assistants at a university (p. 41-42) (italics in original).

He then goes on to argue that institutions not only influence the way students and teachers act upon their 'positions' but they also influence how these actors come to perceive themselves:

> Institutions can exert more or less strict regulations on the practices of people involved as subjects because they have some stability in time and space. Thus

[14] It is interesting to see Castela's (2004) use of the word 'regulate' to indicate the influence of institutions on teachers' practice and compare it with the use of exactly the same term in Ernest's (1989) and Pehkonen & Torner's (1996, 1999) writings to indicate the influence of 'beliefs' o n teachers' practice.

> they give these persons opportunities to adapt themselves to the institutional customs—in other words to learn. From this comes a certain invariance of situations and practices inside the institution, whatever the individual specificities of actors may be. Institutional regulations are not only exerted through direct communication between subjects in different positions. They also come from a complex system of sub-institutions— institutional artefacts—that act on the practices of the community and thus have a mediating part between the institution and its subjects (p. 42).

It is important to note that, although Castela (2004) examined the students, rather than teachers, in their institutional context, he comes to similar conclusions. First of all, I observe the notion of 'contextual density' (see section 6.4), in his approach when he states "institutions can exert more or less strict regulations" (p. 42). Secondly, I observe another similarity between my argument that certain artefacts and instructional practices are associated with certain institutions (see section 6.2) when he states "a certain invariance of situations and practices inside the institution" (p. 42). Thirdly, I made use of the idea of 'constraints' (Tanner & Jones, 2000) and 'affordances' (Gibson, 1977) (see section 6.2) in order to understand institutions. He acknowledges this, implicitly, when he states "when persons 'enter' an institution, their life in this institution is submitted to collective *constraints* and *expectations* that regulate their actions."

# CHAPTER-7 CONCLUSION

In this chapter, I will first present the conclusions of the study in relation to the two research foci. In this first section I will present the conclusions and contributions of the current study to the extant literature, and this will also incorporate my suggestions for further research. This will be followed by the limitations of the current study in the second section.

## 7.1 Conclusion and Suggestions for Further Studies

The main aim of this study was to explore Turkish mathematics teachers' instructional practices in relation to the broader context in which the teaching takes place. This approach is informed by the current research trend in mathematics education: the socio-cultural approach to mind. Lerman (2001) uses the metaphor of a camera focus to suggest that educational research should focus on some local practices (zoom in) taking into consideration the wider social, cultural and institutional context (zoom out) in which the local practices are carried out. From this perspective, an aim of educational research should, therefore, be to integrate these two dimensions into the research. In this study I 'zoomed in' to mathematics teachers' classroom practices and 'zoomed out' to the wider socio-cultural context. The former comprised the first focus and the latter comprised the second.

Concerning the first research focus, I wanted to use teachers own beliefs/perceptions of their practices as a frame of reference for the study. In this connection, I made use of the ways in which the mathematical activities have been framed by the teacher in order to explore the teachers' practices. Using Lerman's (2001) metaphor, this was to 'magnify' (or 'zoom in' to) teachers' instructional practices to explore essential facets of teaching activities taking place in the classroom. Regarding the second research focus I used the concentric circles structure used by Cole (1996), Talbert & McLaughlin (1993) and Lim & Hang (2003) (See Figure 6.1 in section 6.1) as a frame of reference to be able to understand and operationalise the concept of context. Early data analysis suggested that the institutional context plays a bigger and more significant role than was anticipated. In connection with this, I realised that a fundamental gap in the literature of mathematics education, that the teachers' classroom practices were not empirically examined in relation to the institutional context in which the teaching takes place. The extant literature does not seem to go beyond the theoretical level when it comes to the issue of institutional influences in relation to mathematics teachers' classroom practices (See for example Popkewitz, 1988). I, therefore, focused my attention on the institutional context, which constituted a level in Cole's (1996) concentric circles model of context.

In order to explore mathematics teachers' practices, I interviewed a number of teachers in SSs and PCs. The findings from the interviews suggested that all of the teachers described their lessons in the same manner. In fact, they portrayed their lessons in the form of sequences of 'content' and 'example solving'. The finding was consistent across all the teachers with varying degrees of explicitness in formal as well as informal interviews. Such a description style was detected in piloting data and it was supported by the main data. Hence, as far as the mathematics teachers of both institutions were concerned, their instructional practices contained two main elements: content, where, according to them, the theory of the mathematical knowledge to be taught is presented; and 'example solving', where the theoretical knowledge presented was essentially put into practice.

Having identified this pattern, I wanted to examine teachers' practices from such a perspective. I therefore turned my attention to the actual events that took place in the classroom. This required careful examination of mathematics teachers' instructional practices in the video data. In total 52 lessons were observed for this purpose. I watched the mathematics lessons of the four teachers several times to ensure the accuracy of my interpretations and that they are based entirely on the data itself. Although it was apparent that the four teachers did not have exactly the same instructional patterns, the analysis of the video data suggested different patterns of practices in the teachers of different institutions. I have found a variety of differences between PC and SS teachers' actual practices:

- PC teachers have a different organisation of mathematics lessons. Specifically their lessons had more 'pace' in that one lesson contained several organisational segments in comparison to SS lessons.
- The 'content' segment of mathematics lessons in PC teachers' practices were more like summaries rather than long explanations. For PC teachers this segment was aiming at application of the theory and thus it was privileging the 'know-how' aspect. On the other hand, the same segment in SS teachers practices were more emphatic and more theoretical, privileging the 'know-that' aspect.

- PC teachers solved more examples in each lesson than SS teachers, and solved each example using significantly less time than SS teachers.
- PC teachers used example solving segments of their lessons primarily to demonstrate solution methods, including alternative solution methods, 'tricks' and 'short cuts'.
- The primary focus in SS teachers' example solving segments of their lessons was student engagement and their active participation.
- Unlike PC teachers, SS teachers problematised the examples by either creating a suitable atmosphere for discussion or demanding that students try to solve the examples by themselves.
- Mathematics lessons in PC were much less interactive than mathematics lessons in SS. SS teachers required students to be more active in the lesson and thus SS teachers were receiving much more feedback about students progress than PC teachers.

Although the analysis of video data revealed differences between the instructional practices of these teachers, it did not provide much evidence as to why that was the case. At the beginning of the discussion of these findings I asked 'how do we interpret and make sense of these differences?' Making sense of these differences required more than detailed examination of practices, in other word, 'zoom in'. In order to interpret these differences I looked for clues in the teachers' beliefs and perceptions regarding teaching and learning mathematics as well as in the broader socio-cultural context, or 'zoom out'. This is when the second research focus played a significant role.

The findings revealed associations between certain instructional materials, teaching practices and certain institutions. The data also revealed that the institutional context could be so influential on teachers' practices that it could even subordinate teachers' own personal views regarding their teaching practices. In other words, teachers may prefer to teach in the way that the institution they are working in suggests even though they clearly think otherwise. Based on such findings, I argued that this may entail the individual differences in teachers' practices being reduced by the institutions, depending on the 'contextual density of the institution'. 'Contextual density' is a term I coined to signify that not all institutions may influence teachers to the same degree. Based on these findings I argued that institutions may influence teachers' professional development.

In 2002 some researchers in the field of mathematics education edited a book entitled 'Beliefs: a hidden variable in mathematics education' in which a number of researchers discussed the teachers' and students' beliefs in relation to their practices (Leder, Pehkonen & Törner, 2002). One major issue that was agreed upon is the dialogical relationship between beliefs and practices. Since (1) teachers' practices are more immediately observable than the teachers' beliefs and perceptions, and (2) my starting point was to examine the teachers' practices and beliefs/perceptions about teaching and learning mathematics were not central, the present study indicated the influence of institutions on teachers' practices rather than the teachers' beliefs and perceptions. The questionnaire that was administered has provided interesting results in this connection. The overwhelming majority of teachers confirmed the findings from the case teachers that institutions influence their classroom practices as well as their beliefs to a certain extent. A minority of teachers stated that institutions influenced their practice but not their beliefs. I believe this line of inquiry has more to it than I managed to reveal in a questionnaire. It seems to me that it would be well worth studying the nature and direction of the influence of institutions on mathematics teachers' beliefs and perceptions about teaching and learning mathematics.

Moreover, the findings suggest different privileging patterns for the teachers of different institutions. This, I believe, may emerge as an important aspect of mathematics teachers' professional development. This finding is supported by Boaler (1999) and Bingolbali (2005) who maintain that development of students' mathematical knowledge is significantly affected by the institutional context. However, there seems to be a lack of research on how teachers' content knowledge of mathematics and their pedagogical content knowledge is developed in relation to the institutional context that teachers are teaching. A future study with such a focus would be a valuable contribution to our understanding of teachers' professional development

It seems that early choices teachers are making in terms of where to practice their profession, in other words their choice of institution, affects their professional development considerably. If that is the case, which it apparently is, then teachers' choices of workplace may have considerable consequences in terms of educational research on teachers' professional development.

I argued that institutions exert a certain level of pressure depending on the 'contextual density'. SS teachers do not seem to be affected by the institution as much as PC teachers, despite that there may be other explanations. In this connection, it is also important not to overlook the fact that individuals are not totally constrained by the context and it is possible to resist the constraints that an institutional context impose. Therefore, I do not ignore the possibility that there may be certain personal and/or cultural parameters that may have a role in such a difference. I believe there is enough reason to suggest that such parameters need to be explored and this thread is well worth following up in further research.

### 7.2 Limitations of the Study

Thus far, I have briefly outlined the answers to research questions and noted my main arguments. However, this study, as other educational research, is not without its shortcomings and biases and those need to be noted.

The first limitation of the present study is from a methodological point of view. The data, despite its richness, is collected from a relatively small number of cases (teachers). Although I avoided having a single case as the main source of information and secured a certain level of confidence in my findings through the verification strategies, I presented in section 3.4, it is still not to the extent that enables one to make confident generalisations[1]. Considering the time and effort required to carry out a larger scale study, I am of the opinion that the decision to examine four cases has given the optimal balance between feasibility of the research and confidence in the findings of the study. I was well aware of such a limitation throughout my thesis and I endeavoured to keep my writings within its limits and notified the reader when the argument has gone slightly beyond the evidence at hand. This is, as Hammersley (1992) and Schoenfeld (1985) observes, simply a trade-off in educational research and needs to be made explicit for the reader to be able to judge its merits.

A second limitation is related to the translation of the data. The data is collected from a number of Turkish teachers and translated to English by the researcher. Despite the fact that I paid considerable attention to the translation of the data, it is not without its

[1] It should be pointed out here, as it was in section 3.2, that the study was not intended to have findings that will be generalised in the statistical sense of 'sample to universe'.

shortcomings. Having experienced the issue of translation in my research, I believe that it is not possible to have perfect translation and that there will always be certain meanings that will be lost in translation. However, I could comfortably claim that I made a considerable effort to be faithful to the meaning in the original data for this matter.

Another limitation of this study is that the interpretations I make are based on observations of the products of the influence of institutions on teachers and I posit the existence of the process of this institutional influence. For example, my argument for the influence of institutions on teachers' practices does not capture the moment(s) when institutional constraints influence teachers' practices. If one could capture a conversation between the head teacher and a mathematics teacher, in which the head teacher makes certain suggestions to the teachers, then this would be an ideal moment/situation for a researcher to be able to see how the institutions influence becomes apparent in the context of the interaction between different agents. However, in this study I took a relatively open and flexible approach to teachers' practices and the significance of the influence of institutional context on teachers' practices emerged as the result of data analysis. The study, hence, was not designed to capture such moments and examine these interactions, which carries evidence regarding institutional constraints and affordances on teachers' practices, in depth. I believe that research designed to examine this issue would have the potential to provide significant insights into how educational institutions operate and how people with different roles adopt positions and how these positions shape the power of the agents in the setting. This may, in turn, help us gain further insight into how and why teachers' classroom practices are influenced. For example, I believe it would be revealing to see how teachers' interactions with different human agents in the institutional context take place and how they position themselves in relation to others. In fact, I presented signals on how PC teachers position themselves during the interviews but I did not follow up this issue as it is out of the scope of the present study.

Finally, another limitation of this study is the researcher's bias. This is especially an issue when the research is not highly structured. The fact that the present study is exploratory in nature and open to emergent issues makes it inevitably vulnerable to researcher bias. For example, in examining teachers' practices through video data it is unavoidable that what I have noticed as significant is a function of my background such as my attitudes towards the phenomenon under scrutiny. This was also the case for the present study. This is why I am of the opinion that the thesis at hand is not only the product of the data but also the product of the researcher's background to a certain extent and therefore further research is required to confirm its findings.

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**Appendix A:** Teacher Interview-1

A-Content of lesson

1-How much time (what percentage of your lesson) do you devote to presenting knowledge and procedures in a lesson?

2-How do you present knowledge and procedures? For example, do you write on the blackboard and ask them to write them down? And is this a common pattern for your teaching?

3-Do you use analogies/metaphors like mother-children analogy for teaching?

B-Examples provided

1-What do you think is the main purpose of giving examples? In what way do you think it helps students?

2-On what basis do you choose the examples you use in your lessons?

3-Do you prefer to give examples in a certain form (multiple-choice, open-ended)? Why?

C-Materials/Artefacts Used

1-What materials do you use in teaching?

2-How do you use the materials?

3-How do you think quizzes help students?

D-Planning Decisions

1-What kind of preparations do you do in terms of planning the lesson? For example prior to teaching functions, what preparations do you do?

**Appendix B:** Teacher interview-2

A-Teachers' views on mathematics taught

1-How do you think mathematics should be taught?

2-What do you think is/are the aim(s) of teaching mathematics in school? For example, what is the purpose of teaching functions?

3-What do you think students should do to learn mathematics (before, during and after the lessons)?

4-What, do you think, should be the role of mathematics teacher in classroom? What counts as a success at the end of the end of teaching (for example, functions)?

5-Do you think the mathematics taught in SSs different from the mathematics taught in PCs? Could you explain it further? (What are the differences?)

6-(If the answer is yes for the first question) What do you think is/are the reason(s) for this difference?

B-Teachers' views on institutions

1-Could you compare SSs and PCs in terms of aim(s) of teaching mathematics?

2-What do you think are the reasons for the difference between PCs and SSs? Is it only because of examination system?

3-Do you think that PCs can be regarded as complementary to SSs? Could you explain it further?

**Appendix C:** Questionnaire

**Information**

| |
|---|
| Please state your current workplace:<br>A State School ❑ , A Private Course ❑ |
| Please state your professional experience in terms of the number of years.<br>...years in state schools ...years in private courses. |

**Item-1**

In the following you will see an example and two possible ways of solving it (1. and 2.). They both results in the correct answer but uses different techniques. Please examine the example and two solution methods and answer the following questions.

**Example**

For given $x < 3$ and $f(x) = x^2 - 6x - 2$ what is $f^{-1}(x)$?

$a)3 - \sqrt{x+11}$ $b)3 + \sqrt{x+14}$ $c)2 - \sqrt{x+14}$ $d)2 - \sqrt{x+11}$

**Solution 1**

$f(x) = x^2 - 6x - 2$

To make the expression a square we add and subtract 9 from the function.

$y=x^2-6x-2+9-9$

$y=x^2-6x+9-11$

$y=(x^2-6x+9)-11$

$y=(x-3)^2-11$

$y+11=(x-3)^2$ we get the square root of both sides

$\sqrt{y+11} = |x-3|$ and as $x<3$, $|x-3| = -(x-3)$

$\sqrt{y+11} = 3 - x$ we multiply both sides with -1

$-\sqrt{y+11} = -3 + x$ and take 3 to other side

$x = 3 - \sqrt{y+11}$ now we must swap the place of $x$ and $y$ because it's the inverse function.

$y=f^{1}(x)=3 - \sqrt{x+11}$

Then the answer is *a)*.

**Solution 2**

Let's assign a numerical value to x. Because x<3 let's say 2 to x in the function.

$f(2)= (2)^2-6.2-2$

$f(2)= -10$

Then $f^{1}(-10)=2$

Lets try the options one by one. Let's put -10 and when we find 2 it is the correct answer.

$a)3 - \sqrt{-10+11} =3-1=2$

$b)3 - \sqrt{-10+14} =3-2=1$

$c)2 - \sqrt{-10+11} =2-1=1$

$d)2 - \sqrt{-10+14} =2-2=0$

Since *a)* is the only option that matches with the result of first calculation, the answer is *a)*.

(please tick the option)

| | |
|---|---|
| **a)** If you were to use this example in your classroom, which of the solution methods would **you** be more likely to use? Why? | Solution 1 ❑ Solution 2 ❑ Both ❑ None ❑ |
| | |
| **b)** Which of the solution methods do you think **a typical state school teacher** would be more likely to use? Why? | Solution 1 ❑ Solution 2 ❑ Both ❑ None ❑ |
| | |
| **c)** Which of the solution methods do you think **a typical private course teacher** would be more likely to use? Why? | Solution 1 ❑ Solution 2 ❑ Both ❑ None ❑ |
| | |
| **d)** Which of the solution methods would you like to see **your students** to use? Why? | Solution 1 ❑ Solution 2 ❑ Both ❑ None ❑ |
| | |

**Item-2**

Two teachers are discussing the solution methods given above. They are trying to explain in each other why they would choose the method they do.

**Ali** says that "I think students should learn solution1 because it always gives an accurate result. I also think it is more important for me how student solve the example rather than the answer. I want my students to be able to solve the example rather than get the answer quickly. To me in teaching mathematics the solution 2 is not acceptable since it sacrifices speed for the accuracy."

**Veli**, however, says that "I think students should learn both and use solution 2 whenever they can because it is much quicker. I don't mind how student get the answers as long as they get the correct answers. Many mathematics questions can be solved by solution 2 to get the answer quickly. To me the method the students use is not crucially important, it doesn't matter which way they choose as long as it is quick enough to get the answer."

1-What inferences can you make about these teachers? Please explain.

2-Which one is closer to the way of your thinking? Please explain.

3-Do you think the place teachers are working affects their views (on such matters)? Please explain.

**Appendix D:** Illustration of the lesson structures of PC and SS teachers.

**Structure of Saban's lesson**

**Content:**
The demonstration of constant function in brief.

↓

**Example solving:**
2 active examples solved by the teacher using two methods of solution for both.

↓

**Content:**
The demonstration of linear function in brief.

↓

**Example solving:**
1 active example solved by the teacher.

↓

**Content:**
The demonstration of first point of inverse function.

↓

**Example solving:**
A active example with three sub-sections is solved by the teacher.

↓

**Content:**
The demonstration of second and third point of inverse function.

↓

**Example solving:**
2 active examples solved by the teacher.

↓

**Content solving:**
The demonstration of fourth point of inverse function.

↓

**Example:**
2 active examples solved by the teacher. 2 methods used for the last one.

**Structure of Ayten's lesson**

**Content:**
The definition of Inverse function and explanations on 2 passive examples.

↓

**Content:**
Basic manipulation related to inverse functions and explanations on 5 passive examples.

↓

**Example solving:**
2 active examples solved by teacher.

↓

**Example solving:**
2 active examples solved by students.