Essays on International Economics

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Abstract

In my PhD dissertation, I have four essays on research topics related to international economics. These essays utilize some of the different economic tools and skill sets (that I have acquired during my graduate-school study experience in Australia, United States, and United Kingdom), to tackle the economic problems that I am interested in, so as to better understand the economics and economic policies in the real world.

Chapter 2:
Monetary Policy and Fiscal Rules for a Two-State Open Economy.

Chapter 3:
Business Cycle Synchronization between the Great Britain and the United States.

Chapter 4:
Oil Supply Shocks in a Non-Scale Economic Growth Model.

Chapter 5:
Canada's Loss of External Competitiveness: The Role of Commodity Prices and the Emergence of China

Table of Contents

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I would like to express my gratitude to my supervisor, Prof. Mike Wickens, for his guidance and advice during my PhD study at the University of York in England.

Declaration

This is to certify that

(i) the thesis comprises only my original work towards the degree Doctor of Philosophy in Economics, except for Chapter 5 which is co-authored with P. Medas (IMF).

(ii) due acknowledgement has been made in the text to all other material used.

(iii) This work has not previously been presented for an award at this, or any other, university.

Yuwen Dai

Chapter 1

Introduction

In this introductory chapter, I outline the abstract of each essay in order to give an overview of my dissertation.

Ch2: Monetary Policy & Fiscal Rules for a Two-State Open Economy
The seemingly never-ending economic woes of Euro Zone economies have been in the media coverage for a while, which makes us to cast doubt again on the optimality of the Euro Zone. The problem lies in the very different behavior of member economies, to which a contributing factor is the "one-size-fits-all" monetary policy that is imposed by a currency union. In order to understand better what is going on, this research project develops a two-state open economy DSGE model, in which the two state economies conduct the same monetary policy but retain their fiscal independence. In this constructed DSGE model, the central monetary authority has the policy instrument of the nominal interest rate. In each economy, the fiscal authority can levy non-distorting lump-sum tax transfer, and issue nominal risk-free debt, so as to finance a given process of public spending in their own economy. This DSGE model is calibrated to distinguish one state from the other. The (international) linkages between the two state economies are inter-state trade in goods, and inter-state bond borrowing. We investigate the macroeconomic effects of monetary policy where there is a single currency by considering two economic scenarios: a supply-side shock (such as a productivity increase), and a demand-side shock (such as a government expenditure increase) in the Home economy. The policy implications from this project are that monetary policy is wrong if inflation differentials persist. The wrong monetary policy would lead to too low real interest rates for high inflation countries, which would be indebted with too much borrowing. Markets are taking note of this by charging much higher risk premium.

Ch3: Business Cycle Synchronization between the UK and the US
In this chapter, I take up the research topic of business cycle synchronization between the Great Britain and the United States. In order to explore this topic, I examine the relationship between the UK and US economies for the past five decades. The measurement of the synchronization between these two economies is done by dating their business cycles and finding the turning points in their cycles. A GMM analytical framework is adopted, due to the need for a model that is free in measuring synchronization, and also the need for estimates that are robust to serial correlation and heteroscedasticity. Under this GMM framework, I conduct single and joint synchronization tests between the UK and US for each type of business cycles, and I find the US leads the UK for all three types of business cycles.

Ch4: Oil Supply Shocks in a Non-Scale Economic Growth Model
This research project extends Schubert and Turnovsky (2007) paper by introducing endogenous labor supply into their model economy, and investigates the dynamic effects of a one-time unanticipated permanent change in the price of the imported intermediate good on economic growth. The growth model considered here is a "non-scale" growth model, in which the economy has access to a perfect capital market. With the extension of endogenous labor supply, we find that output, capital, and input usage are all permanently much lower than the case for fixed labor supply. The short-run impacts on input usage and output are also larger than those in the fixed labor supply case.

Ch5: Canada's Loss of External Competitiveness, (joint with P. Medas[1])
After booming in the 1990s, Canadian exports have waned over the past decade. The loss of external competitiveness has been quite evident in the US market (the main destination of Canadian exports), where Canada has been overtaken by China as the leading exporter to the US. Using a dynamic panel data of Canadian merchandise exports across different sectors in the US market, we identify the main driving forces behind the weakening export performance in Canada, which include the slowdown in the US demand and the emergence of China as a major exporter. The appreciation of the Canadian loonie over the past decade, in part reflecting the rise in commodity prices, has also played a key role in explaining the loss of Canadian exporters share in the US market.

[1] Paulo Medas is a Senior Economist at the International Monetary Fund (IMF).

Chapter 2

Monetary Policy and Fiscal Rules for a Two-State Open Economy

Abstract

The seemingly never-ending economic woes of Euro Zone economies have been in the media coverage for a while, which makes us to cast doubt again on the optimality of the Euro Zone. The problem lies in the very different behavior of member economies, to which a contributing factor is the "one-size-fits-all" monetary policy that is imposed by a currency union. In order to understand better what is going on, this research project develops a two-state open economy DSGE model, in which the two state economies conduct the same monetary policy but retain their fiscal independence. In this constructed DSGE model, the central monetary authority has the policy instrument of the nominal interest rate. In each economy, the fiscal authority can levy non-distorting lump-sum tax transfer, and issue nominal risk-free debt, so as to finance a given process of public spending in their own economy. This DSGE model is calibrated to distinguish one state from the other. The (international) linkages between the two state economies are inter-state trade in goods, and inter-state bond borrowing. We investigate the macroeconomic effects of monetary policy where there is a single currency by considering two economic scenarios: a supply-side shock (such as a productivity increase), and a demand-side shock (such as a government expenditure increase) in the Home economy. The policy implications from this project are that monetary policy is wrong if inflation differentials persist. The wrong monetary policy would lead to real interest rates too low for high inflation countries, which would be indebted with too much borrowing. Markets are taking note of this by charging much higher risk premium.

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Key Words: Currency Union, Monetary Policy, Inflation, Interest Rate, Exchange Rate, Current Account, Fiscal Policy, DSGE (Dynamic Stochastic General Equilibrium) Modeling, Euro Zone.

2.1 Introduction

2.1.1 Motivation

The economic woes of Euro Zone economies seem never-ending. Would things get even worse before they get better? Or would they ever get better? All the media coverage on the Euro Zone debt crisis makes us to cast doubt again on the optimality of the Euro Zone. Similar concerns are also raised in Wyplosz (2006), and Goodhart (2007a, 2007b). Wickens (2010) points out that at the inception of the Euro Zone, the member countries were not suited to be an *optimal currency area*. Reasons include difference in productivity levels, real wage rates, the degree of stickiness in wages and prices, fiscal deficits, inflation, and government debt. These would cause countries to have very different economic performances.

The theory of the *optimal currency area* (OCA) was pioneered by economist Robert Mundell (1961), who is also often credited as the father of the Euro. The benefits of forming an OCA include lower and more stable inflation rate, lower interest rates, and increased trade. On the other hand, the costs of forming an OCA include no independence of monetary policy, and less ability to respond to shocks. To economists, an "optimal currency area" is one in which the gains from sharing a single currency outweigh the costs. There is some debate as to whether the Euro area qualifies though.

In the case of the Euro zone, Germany already had lower inflation rate and lower interest rate before the formation of the Euro Zone; so Germany is in the currency union for political reason, but not for economic reason. In the Euro area, the size and the correlation of shocks are very different across different economies in the same common currency union. Take Germany and Greece as an example. They have a very different size of their economies. Their difference is based upon different exports and imports, different industrial structures, different specializations, different wages, different union structures, etc. As a result, it would not at all unexpected for Germany to be booming at a time when Greece was in recession, or vice versa. Given that the shocks would be very different in nature and have different impact on each economy in the currency area, how much control over the common monetary policy will individual countries have? In the case of the Euro, there is very little control by the periphery countries. Germany is likely to dominate, which is in part by "design", as Germany has the best monetary policy to begin with, and it is also dominant in terms of its GDP size. Because of that, the European Central Bank (ECB) was built from the very beginning to respond less to shocks in periphery countries. In order to adjust to shocks, there should be ways other than internal devaluation to respond to shocks, which include labor mobility and fiscal equalization. Labor mobility is already controversial in the Europe; and fiscal equalization works in the US, but not in Europe. So the creation of the Euro was about politics and ideology, not a response to careful economic analysis.

2.1.2 Model Overview

Against this backdrop, we develop a Dynamic Stochastic General Equilibrium (DSGE) model for a two-state open economy, in which the two economies have the same monetary policy but independent fiscal policy. Real world examples include two countries in the Euro Zone, two states in the US economy, core and periphery countries, etc. In such economies, the two states have a fixed exchange rate and a common nominal interest rate as set by the central monetary authority, but they have different inflation rates in each individual economy. On the fiscal side, each state economy retains its fiscal independence by having its own fiscal authority which can levy non-distorting lump-sum tax transfer, and issue nominal risk-free debt in order to finance a given process of public spending in their state economy. The fiscal authority can also respond to the excessive change in the debt level, with a fiscal policy rule. On top of that, the linkages between these two state economies are: inter-state trade in goods, and inter-state borrowing in bonds.

2.1.3 Research Questions

The research questions addressed in this project derive from the interaction of monetary policy and fiscal rules in a currency union when each economy within the union has different economic situations, such as the Euro Zone economies nowadays. This research project seeks to answer the following two major research questions, which are outlined as follows:

Research Question (1): Is it appropriate for two economies to be in a currency union, if the two economies are different in economic size and have different inflation rates? What are the macroeconomic consequences of this inappropriate monetary policy under a currency union?

Research Question (2): Will both of the two economies be better off, if they are allowed to have their own monetary authority and conduct their own independent monetary policy, instead of having a common nominal interest rate as set by a central monetary authority?

2.1.4 Roadmap

Looking ahead, the rest of this chapter is structured as follows: In *Section 2.2*, we develop a two-state open economy DSGE model as our analytical framework, in which we model the consumers, firms, monetary authority, fiscal authority, and a trade sector in our currency union. In *Section 2.3*, we apply our DSGE model and conduct two economic experiments: a supply-side shock (for example, a productivity shock, or a migration shock), and a demand-side shock (for example, a government expenditure shock) in the Home economy. Based on our model estimates, we start from the impulse-response functions for the different types of shocks, and review the major implications from the domestic and open-economy variables for the supply and demand shocks that are present in the model. The joint behavior of the domestic variables and the open-economy variables (those linking the two economies) should be informative to identify the major impact of the shocks over this period. We conduct our two economic experiments under two monetary policy regime: a single currency regime, and a two-currency regime. *Section 2.4* offers our conclusion for this chapter.

2.2 Analytical Framework

Calibrated Dynamic Stochastic General Equilibrium (DSGE) models offer rigorous theoretical microeconomic foundations to tackle macroeconomic questions and policy issues. A number of two-country and multi-country DSGE models have emerged, for example, de Walque *et al.* (2005), Cristadoro *et al.* (2006), and IMF's GEM[1], GFM[2], GPM[3] models,[4] and GIMF[5] models. Large scale multi-country DSGE models are rare, which is due to the complexity of the economic modeling required to deliver the rich microeconomic foundations that are considered as the principal advantage of DSGE models. Global DSGE models with broad coverage are still some way off.

Following the footsteps of Smets and Wouters (2003, 2007), there has been an extensive literature which uses DSGE models to study the policy impacts and the stabilizing role played by monetary rules. Given the rising attention paid to fiscal policy, especially during the Euro Zone crisis, this research project contributes to the literature exploring the roles played by the joint conduct of monetary policy and fiscal policy in a DSGE framework for a currency union.

In the existing literature, Gali and Monacelli (2008) develop a tractable/analytical model for the analysis of optimal monetary and fiscal policy in a currency union, where monetary policy is in the form of a common interest rate at the union level, and fiscal policy is in the form of government spending rule at the country level. Ratto, Roeger, and Veld (2008) develop a DSGE open-economy model featured by nominal and real frictions, as well as financial frictions with liquidity constraints, and then they study the stabilization of monetary and fiscal policy rules, and estimate the model on Euro area data using Bayesian estimation techniques. On the other hand, Valli and Carvalho (2010) model a fiscal rule with primary balance targets to stabilize the debt-to-GDP ratio in a two-country open-economy New Keynesian DSGE model, with a forward-looking Taylor rule consistent with an inflation targeting regime.

Starting from first principles, we build our DSGE model from an economist's point of view, in the form of a macroeconomic model with microeconomic foundation, and with as few restrictive assumptions as possible. Our DSGE model and the various assumptions that it is built on are used to arrive at a simplified, but

[1]GEM = Global Economy Model.
[2]GFM = Global Fiscal Model.
[3]GPM = Global Projection Model.
[4]See Bayoumi (2004), Botman *et al.* (2007), Carabenciov *et al.* (2008a, 2008b, 2008c, 2013).
[5]GIMF = Global Integrated Monetary and Fiscal Model.

internally consistent view of the linkages in our "two-state" open economy. But our model is not meant to fully capture the real world. Instead, it is an abstraction of the reality, and it is to be used for cross checks in conjunction with our judgment and experience.

In our analytical framework for a currency union, we model consumers, firms, a central monetary authority, two separate fiscal authorities, and a trade sector. Building from that, we derive their behavioral equations explicitly from the inter-temporal optimization of our agents under budget constraint, technological constraint, and institutional constraint. Given these constraints, our representative agents "rationally" solve their constrained optimization problems, and "optimally" respond to the supply and demand shocks occurring in different markets, which are the cause of macroeconomic fluctuations in the economy. The behavioral equations describing aggregate variables are thus replaced by first-order conditions of our inter-temporal problems facing consumers, firms, and government. As a response, monetary policy and fiscal policy can play a role in alleviating the impact of these shocks, the extent of which can be studied by DSGE modeling and simulation, as a central tool for our macroeconomic analysis.

2.2.1 Household

To model the household, we adopt a "representative agent" approach, by which we assume that there are a large number of identical/homogeneous agents in our model economy.

2.2.1.1 Inter-temporal Utility

We assume that our representative agent has an inter-temporal utility function, in which he derives positive utility from the goods that he consumes[6].

$$U(C_{i,t}) \;=\; E_t\Big[\sum_{t=0}^{\infty}\beta^t\frac{(C_{i,t})^{1-\sigma}-1}{1-\sigma}\Big] \qquad (2.1)$$

where: 1) The utility function $U(.)$ is bounded, continuously differentiable, strictly increasing, and strictly concave. 2) The subscript index $i = 1, 2$, are State 1 (Home) and State 2 (Foreign), respectively. 3) $C_{i,t}$ is the level of consumption, to

[6]The consumption part of the utility function is a power utility, which is also known as the iso-elastic utility. With this utility form, the agent has constant relative risk aversion (CRRA).

be defined below. 4) The parameter $\sigma > 0$, is a non-negative constant.

2.2.1.2 Imperfect Substitutability in Consumption

Our representative agent has a consumption set, which includes both the tradeables from Home and the tradeables from Foreign. Following Wickens (2008)[7], we allow imperfect substitutability among the tradeables from the Home and Foreign economies, and we assume that the aggregation for the tradeables $C_{i,t}$ takes a Cobb-Douglas form,[8] which is a technological relation between total consumption in Home-produced goods and Foreign-produced goods.

$$C_{i,t} = \frac{(C_{i,t}^{H})^{\kappa}(C_{i,t}^{F})^{1-\kappa}}{\kappa^{\kappa}(1-\kappa)^{1-\kappa}} \tag{2.2}$$

where: 1) $C_{i,t}$ is the level of total consumption in each state economy. 2) $C_{i,t}^{H}$ and $C_{i,t}^{F}$ are levels of consumption in the tradeables produced at Home and Foreign, respectively.

The consumption expenditures in nominal terms are:

$$P_{i,t}C_{i,t} = P_{i,t}^{H}C_{i,t}^{H} + P_{i,t}^{F}C_{i,t}^{F} \tag{2.3}$$

[7] See Wickens (2008): Section 7.5.2.

[8] A Cobb-Douglas function is a special form of the CES (constant elasticity of substitution) function. A CES function can be written as: $Y = A\big[\alpha K^{\gamma} + (1-\alpha)L^{\gamma}\big]^{\frac{1}{\gamma}}$, in which the limiting case $\gamma = 0$ corresponds to a Cobb-Douglas function $Y = AK^{\alpha}L^{1-\alpha}$, with constant returns to scale.

In the literature, the consumption index C can also be assumed to take a more general CES functional form, as an aggregate of Home and Foreign goods. $C = \big[\nu^{\frac{1}{\theta}} C_H^{\frac{\theta-1}{\theta}} + (1-\nu)^{\frac{1}{\theta}} C_F^{\frac{\theta-1}{\theta}}\big]^{\frac{\theta}{\theta-1}}$, where $\theta > 0$ is the *intra-temporal* elasticity of substitution between Home and Foreign-produced goods C_H and C_F. See Sutherland (2005) for an example. The parameter ν determines Home consumers' preferences for Home-produced goods, and the parameter $(1-\nu)$ determines Home consumers' preferences for Foreign-produced goods. The size of the parameter ν could be determined by the relative size of the Home economy (n) and of the Foreign economy $(1-n)$, and also of the degree of openness λ. More specifically, $(1-\nu) = (1-n)\lambda$. Similarly, the Foreign economy has a consumption index as: $C^{*} = \big[\nu^{*\frac{1}{\theta}} C_H^{*\frac{\theta-1}{\theta}} + (1-\nu^{*})^{\frac{1}{\theta}} C_F^{*\frac{\theta-1}{\theta}}\big]^{\frac{\theta}{\theta-1}}$, with $\nu^{*} = n\lambda$. That is, Foreign consumers' preferences for Home-produced goods depend on the relative size of the Home economy and the degree of openness. The specification of ν and ν^{*} generates "home bias" in consumption. This bias only disappears when $\lambda = 1$.

In our model, we assume *inter-temporal* elasticity of substitution and imperfect substitutability among the Home and Foreign goods.

where: 1) $P_{i,t}$ are the general price levels in each state economy. 2) $P_{i,t}^{H}$ and $P_{i,t}^{F}$ are the nominal price levels of the tradeables produced at Home and Foreign, respectively.

2.2.1.3 Household Budget Constraint

The household flow budget constraint in nominal terms is:

$$P_{i,t}C_{i,t} + M_{i,t} + B_{i,t} + T_{i,t} \quad = \quad W_{i,t}L_{i,t} + D_{i,t} + M_{i,t-1} + (1 + R_t)B_{i,t-1}$$

where: The left-hand side represents the total money outflows from the household, and the right-hand side represents the total money inflows to the household. 1) $W_{i,t}L_{i,t}$ is the nominal wage income from labor supply. 2) $D_{i,t}$ is the dividend payment from firms. 3) $M_{i,t}$ is the nominal money holding of the household. 4) $B_{i,t}$ is the nominal domestic bond holding of the household. 5) $T_{i,t}$ is the lump-sum tax.

2.2.2 Firms

Having specified the demand side from the household's perspective in the previous part, let us now turn to the supply side of production by the firms in this two-state model economy.

2.2.2.1 Production Function

We assume that each economy produces one type of good, and they trade with each other because their consumers prefer to consume a variety of consumption goods, which is determined by their consumption preference of imperfect substitutability as stated in equation (2.2). In each economy, firms hire labor $L_{i,t}$ as their input during the process of output production. Their production function is assumed to take an "AL" form:

$$Y_{i,t} = A_i\mathcal{F}(L_{i,t}) = A_i(L_{i,t}) \tag{2.4}$$

This supply of goods is consumed by domestic residents, foreign residents, and domestic government.

$$Y_{i,t} \quad = \quad C_{i,t}^{H} + C_{j,t}^{F} + G_{i,t} \tag{2.5}$$

2.2.2.2 Firms' Budget Constraint

The firms' budget constraint in nominal terms is:

$$P_{i,t}Y_{i,t} = W_{i,t}L_{i,t} + D_{i,t} \tag{2.6}$$

where: 1) $W_{i,t}$ is the nominal wage rate. 2) $D_{i,t}$ is the dividend.

2.2.2.3 Price Determination

The price level is determined via the firms' wage-setting process, in which the real wage rate $\frac{W_{i,t}}{P_{i,t}}$ is equivalent to the marginal productivity of labor $\frac{\partial Y_{i,t}}{\partial L_{i,t}} = A_i$ in firms' production process.

$$P_{i,t} = \frac{W_{i,t}}{A_i} \tag{2.7}$$

2.2.3 Fiscal Authority

Each economy has its independent fiscal authority and sets its own fiscal policy at the individual country level. The fiscal authorities also purchase goods from domestic producers as part of public expenditure. In order to finance this government spending, the fiscal authority levies a lump-sum tax on domestic residents. But if there is not enough revenue to back their spending, the fiscal authority also issues government bonds to domestic residents and foreign residents so as to have a balanced government budget.

Under a monetary union, the absence of national currencies means that fiscal policy is the main policy instrument available to manage domestic macroeconomic cycles, and as a consequence of that, the importance of fiscal stability is even greater for members of a currency union.

2.2.3.1 Government Budget Constraint

The government's budget constraint in nominal terms is:

$$P^H_{i,t}G_{i,t} + M_{i,t-1} + (1 + R_t)B^G_{i,t-1} = T_{i,t} + M_{i,t} + B^G_{i,t} \tag{2.8}$$

where: The left hand side is money outflows from the government, whilst the right hand side is money inflows into the government. 1) $G_{i,t}$ is the level of government spending. 2) $(1+R_t)B^G_{i,t-1}$ is the re-payment of government bond borrowing from the previous period, plus the interest to the private sector. 3) $B_{i,t}$ is the revenue from the issuance of government bond in the current period. 4) $T_{i,t}$ is the size of tax revenue.

Money M is included in each government's budget constraint. This implies that any seigniorage revenue will go to each state's fiscal authority, but not to the central monetary authority.

2.2.3.2 Inter-State Bond Market

In the nominal household budget constraint, $B_{i,t}$ is the total bond holding by the household, the composition of which includes holdings of both domestic government issued bonds and foreign government issued bonds.

$$B_{i,t} = B^G_{i,t} + F_t \tag{2.9}$$

$$B_{j,t} = B^G_{j,t} - F_t \tag{2.10}$$

where: 1) $B_{i,t}$ is the total bond holding by the household. 2) $B^G_{i,t}$ is the household's holding of domestic government issued bonds. 3) F_t is the household's net holding position of foreign government issued bonds. For the two economies in a currency union, their net holding of the other government's issued bonds is of the same size, but with different signs.

2.2.4 Monetary Authority

An independent monetary authority has (at least) three conventional monetary policy instruments: the (nominal) exchange rate, the (nominal) interest rate, and the (nominal) money supply. In a monetary union, there is a central monetary authority which conducts monetary policy for its member economies. Under this same monetary policy regime, our two member economies have the same fixed nominal exchange rate, and a common nominal interest rate.

2.2.4.1 Interest Rate

The monetary authority has its monetary policy instrument: the nominal interest rate R_t.[9] We assume that the monetary authority adopts a Taylor (1993) style monetary rule and sets the nominal interest rate as follows:[10]

$$R_t = \pi_t + r + \tau(\pi_t - \pi) \tag{2.11}$$

where variables without a time subscript are their respective values at the steady state. That is, π is the steady-state value of the average inflation rate in the currency union; and r is the steady-state value of the real interest rate for the currency union. The parameter τ is a measure of how aggressive the monetary authority reacts to the deviation of the actual inflation ratio relative to the target inflation ratio. The intuition behind this monetary rule is that the nominal interest rate should be set in response to the deviation of the actual inflation rate from the target inflation rate.

The inflation rate is calculated from the weighted-average price level P_t between Home and Foreign.

$$1 + \pi_t = \frac{P_t}{P_{t-1}} \tag{2.12}$$

The weighted-average price level P_t between Home and Foreign depends on the relative size of the real GDP level in each economy.

$$P_t = P_{1,t}^{\mu} P_{2,t}^{1-\mu} \tag{2.13}$$

where μ is the weight, which is a proxy for the relative size of the Home economy in the currency union.

Alternatively, we can also calculate the average inflation rate for the currency

[9]The 2008 financial crisis has taught us that conventional monetary policy (which is conducted through setting the nominal interest rate or the nominal money supply) may not always be adequate, due to the fact that it is being constrained by a zero lower bound for the nominal interest rate. It is the unboundedness that complicates life; but in economics, the boundedness also complicates the work of economists.

[10]In the literature, an alternative approach is to use a Calvo-style inflation-forecast-based (IFB) interest rate rule, which depends on a discounted sum of current and future rates of inflation. See, for example, Levine, McAdam, and Pearlman (2007).

union, by taking the weighted-average of the inflation rates in each economy.

$$\pi_t = \mu\pi_{1,t} + (1-\mu)\pi_{2,t} \tag{2.14}$$

2.2.4.2 Money Supply

In order to facilitate monetary transactions in purchasing the consumption goods, we introduce the "cash-in-advance" (CIA) condition, originally due to Clower (1967), which exclusively focuses on the transaction demand for money, and assumes that all goods and services should be paid in full with cash at the time of purchase. Early examples include the work by Lucas (1982) and Svensson (1985), in which the demand for money is generated by postulating a "cash-in-advance" constraint.

As described by Clower (1967), goods buy money, and money buy goods, but goods do not buy goods, and because goods do not buy goods, a medium of exchange that serves to aid the process of transaction has its value. As a medium of exchange, money facilitates transactions and yields utility indirectly by allowing certain purchases to be made or by reducing the costs that are associated with transactions. The nature of the transaction technology in the economy then determines the demand for money.

With the "cash-in-advance" (CIA) condition, we impose a rigid restriction on the nature of transactions, and require that money balances be held to finance certain type of purchases, otherwise these purchases can not be made without money. So these CIA models capture the role of money as a medium of exchange by requiring explicitly that money (should) be used to purchase consumption goods.

To model the CIA condition, household money balance M is a pre-determined variable. That is, it is not exogenous, as it is endogenously determined by the CIA constraint.

This CIA specification can be represented by assuming that our representative agent faces a standard budget constraint, and a CIA constraint. This CIA constraint must be considered in solving the optimization problem of the household. The first-order conditions then imply a money demand equation.

In CIA models, the assumption on the timing is important. Following the timing convention as in Svensson (1985), Cooley and Hansen (1989, 1991), we assume that the goods market opens before the asset market.[11] Our representative agent enters the period with money holdings M_{t-1} and receives a lump-sum transfer T_t (in nominal terms). The goods market is assumed to open first, so our CIA constraint takes the form:

$$P_{i,t}C_{i,t} \leq M_{i,t-1}$$

where: 1) $C_{i,t}$ is the real consumption. 2) $P_{i,t}$ is the aggregate price level.

Expressed in real terms, the CIA constraint becomes:

$$\begin{aligned} C_{i,t} &\leq \frac{M_{i,t-1}}{P_{i,t}} \\ &= \frac{1}{1+\pi_{i,t}} \frac{M_{i,t-1}}{P_{i,t-1}} \end{aligned}$$

where: $1+\pi_{i,t} = \frac{P_{i,t}}{P_{i,t-1}}$ is 1 plus the inflation rate.

The timing of this CIA constraint is as follows: In time period $t-1$, the representative agent chooses the nominal money balance $M_{i,t-1}$, and carry this amount of money into time period t. The real value of these nominal money balance $M_{i,t-1}$ is determined by the aggregate price level $P_{i,t}$ in time period t. By this specification, we assume that income from production during time period t will not become available for consumption purchases during period t.

In the case of the Euro Zone, it is the individual central bank that supplies money supply $M_{i,t}$ to their own economy, in response to their national demand which is endogenously determined by consumption $M_{i,t} = P_{i,t}C_{i,t}$. That is why each government's budget constraint includes its own money supply $M_{i,t}$.

[11]By contrast, Lucas (1982) has the timing assumption that the asset market opens before the goods market. Under this timing assumption, the CIA constraint would then take the form:

$$C_{i,t} \leq \frac{1}{1+\pi_{i,t}} \frac{M_{i,t-1}}{P_{i,t-1}} - b_{i,t}$$

2.2.5 Inter-State Trade

Another linkage between these two economies is that they are allowed to trade with one another, in exchange for their consumption goods produced at each economy.

2.2.5.1 Terms of Trade

An economy's *terms of trade* is measured as the price of its exports relative to the price of its imports after adjusting for the exchange rate. But the nominal exchange rate is fixed at one in a currency union, so the terms of trade is just the price ratio between exports and imports for an economy.

$$P_{i,t}^{ToT} = \frac{P_{i,t}^{H}}{P_{i,t}^{F}} \tag{2.15}$$

where: 1) $P_{i,t}^{H}$ is the price of Home exports to Foreign. 2) $P_{i,t}^{F}$ is the price of Home imports from Foreign.

An increase in Home's terms of trade ratio means that Home exports to Foreign become relatively more expensive than Home imports from Foreign, and hence Home exports become less competitive, and this is a deterioration in Home's terms of trade.

2.2.5.2 Balance of Payments

The nominal *balance of payments* condition consists of current account and capital account here, as households are allowed to hold both domestic debt and foreign-issued debt, and therefore there is international (financial) capital movement.

$$BoP_i \; : \; P_{i,t}^{H}C_{j,t}^{F} - P_{i,t}^{F}C_{i,t}^{F} + R_t F_{t-1} = \Delta F_t \tag{2.16}$$

$$BoP_j \; : \; P_{j,t}^{H}C_{i,t}^{F} - P_{j,t}^{F}C_{j,t}^{F} - R_t F_{t-1} = -\Delta F_t \tag{2.17}$$

where: 1) $C_{j,t}^{F}$ are Home exports to Foreign. 2) $P_{i,t}^{H}C_{j,t}^{F}$ is the value of Home exports to Foreign, denominated in the common currency. 3) $C_{i,t}^{F}$ are Home imports from Foreign. 4) $P_{i,t}^{F}C_{i,t}^{F}$ is the value of Home imports from Foreign, denominated in the common currency. 5) $P_{i,t}^{H}C_{j,t}^{F} - P_{i,t}^{F}C_{i,t}^{F}$ is also the balance of trade. 6) F_t is Home's net position of Foreign bond holdings. 7) Home's net position of Foreign bond holding is the opposite of Foreign, in the sense that they are of the same size but with different signs.

The two *balance of payments* conditions can also be written as:

$$BoP_i \quad : \quad P^H_{i,t} C^F_{j,t} + (1 + R_t) F_{t-1} = P^F_{i,t} C^F_{i,t} + F_t \tag{2.18}$$

$$BoP_j \quad : \quad P^H_{j,t} C^F_{i,t} - (1 + R_t) F_{t-1} = P^F_{j,t} C^F_{j,t} - F_t \tag{2.19}$$

where: 1) The left-hand side is the value of exports this period, plus interest payments received this period from Home's net position of Foreign bond holding in the previous period. The sum of the left-hand side represents the total money inflows to the Home economy. 2) The right-hand side is the value of imports this period, plus payments for Home's net position of Foreign bond holding this period. The sum of the right-hand side represents the total money outflows from the Home economy.

The above approach is due to household's *intra-temporal* decisions, where the allocation of demand depends on the relative price of domestic and foreign-produced goods. [12]

In our model world, we only have a currency union, and we only have Home and Foreign in this currency union. For the currency union as a whole, the *balance of payments* condition has to hold. But within the currency union, each economy can have a current account surplus or a current account deficit. [13]

[12] An alternative approach is that the current account can also be explained by the *inter-temporal* decisions, which is represented by the Saving-Investment identity.

$$\begin{aligned} Y &= C + S^P + T \\ Y &= C + I + G + NX \\ C + S^P + T &= C + I + G + NX \\ S^P - I + (T - G) &= NX \\ (S^P + S^G) - I &= NX \end{aligned}$$

[13] In the real world, there are other countries outside this currency union. We can model our system to include a currency union, and the rest of the world. For this model system as a whole, the *balance of payments* condition has to hold. But within this model system, the currency union could run a current account surplus or a current account deficit, whilst the rest of the world would run a current account deficit or a current account surplus, which is of the same size but with a different sign as that of the currency union.

2.2.6 Macro Dynamics

For each economy in our currency union, the economy-wide resource constraint is derived from the household budget constraint, government budget constraint, balance of payments, and the national account identity.[14]

The economy-wide resource constraint (in nominal terms) can be written as:

$$\begin{aligned} P_{i,t}Y_{i,t} &= P^H_{i,t}C^H_{i,t} + P^F_{i,t}C^F_{i,t} + T_{i,t} + B^G_{i,t} - (1+R_t)B^G_{i,t-1} \\ &\quad + F_{i,t} - (1+R_t)F_{i,t-1} + M_{i,t} - M_{i,t-1} \end{aligned}$$

We deflate this nominal equation by the general price level to obtain the economy-wide resource constraint, in real terms:

$$\begin{aligned} Y_{i,t} &= \frac{P^H_{i,t}}{P_{i,t}}C^H_{i,t} + \frac{P^F_{i,t}}{P_{i,t}}C^F_{i,t} + \frac{T_{i,t}}{P_{i,t}} + \frac{B^G_{i,t}}{P_{i,t}} - (1+R_t)\frac{B^G_{i,t-1}}{P_{i,t-1}}\frac{P_{i,t-1}}{P_{i,t}} + \frac{F_{i,t}}{P_{i,t}} - (1+R_t)\frac{F_{i,t-1}}{P_{i,t-1}}\frac{P_{i,t-1}}{P_{i,t}} \\ &\quad + \frac{M_{i,t}}{P_{i,t}} - \frac{M_{i,t-1}}{P_{i,t-1}}\frac{P_{i,t-1}}{P_{i,t}} \end{aligned}$$

$$\begin{aligned} Y_{i,t} &= \frac{P^H_{i,t}}{P_{i,t}}C^H_{i,t} + \frac{P^F_{i,t}}{P_{i,t}}C^F_{i,t} + t_{i,t} + b^G_{i,t} - (1+R_t)\frac{b^G_{i,t-1}}{1+\pi_{i,t}} + f_{i,t} - (1+R_t)\frac{f_{i,t-1}}{1+\pi_{i,t}} \\ &\quad + m_{i,t} - \frac{m_{i,t-1}}{1+\pi_{i,t}} \end{aligned}$$

where: 1) $b^G_{i,t}$ is the holding of domestic government bonds in real terms. 2) $m_{i,t}$ is the real money balance.

Dynamic Optimization:
A representative agent's optimization problem is to maximize his utility:

$$U(C_{i,t}) = E_t\Big[\sum_{t=1}^{\infty}\beta^t\frac{(C_{i,t})^{1-\sigma}-1}{1-\sigma}\Big]$$

subject to the economy-wide resource constraint, and the CIA constraint[15]:

$$C_{i,t} \leq \frac{m_{i,t-1}}{1+\pi_{i,t}}$$

[14]See Appendix A1 for the consistency check of these 5 accounting identities.
[15]If money is held, the CIA constraint must be binding in some states of nature. The nominal interest rate will equal the discounted expected value of money.

Set up his Lagrangian problem (with $s > 0$):

$$\begin{aligned}\mathcal{L} &= \sum_{s=0}^{\infty}\Big\{\beta^s U(C_{i,t+s}) + \lambda_{i,t+s}\Big[Y_{i,t+s} - \frac{P^H_{i,t+s}}{P_{i,t+s}}C^H_{i,t+s} - \frac{P^F_{i,t+s}}{P_{i,t+s}}C^F_{i,t+s} - t_{i,t+s} \\ &\quad -b^G_{i,t+s} + (1+R_{t+s})\frac{b^G_{i,t+s-1}}{1+\pi_{i,t+s}} - f_{i,t+s} + (1+R_{t+s})\frac{f_{i,t+s-1}}{1+\pi_{i,t+s}} - m_{i,t+s} + \frac{m_{i,t+s-1}}{1+\pi_{i,t+s}}\Big] \\ &\quad +\lambda 2_{i,t+s}\Big(\frac{m_{i,t+s-1}}{1+\pi_{i,t+s}} - C_{i,t+s}\Big)\Big\}\end{aligned}$$

where: 1) The choice variables are: $C^H_{i,t+s}$, $C^F_{i,t+s}$, $b^G_{i,t+s}$, $f_{i,t+s}$, $m_{i,t+s}$. $b^G_{i,t+s}$, $f_{i,t+s}$, and $m_{i,t+s}$ are pre-determined variables. The time $(t-1)$ values of the predetermined variables are given at time t. 2) The parameter β is the discount factor. $\beta = \frac{1}{1+\rho}$. The parameter ρ is the time preference by our representative agent.

The first-order conditions with respect to the choice variables are:

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial C^H_{i,t+s}} &= (\beta^s C^{-\sigma}_{i,t+s})\Big(\kappa\frac{C_{i,t+s}}{C^H_{i,t+s}}\Big) - \lambda_{i,t+s}\frac{P^H_{i,t+s}}{P_{i,t+s}} - \lambda 2_{i,t+s}\kappa\frac{C_{i,t+s}}{C^H_{i,t+s}} = 0 \\ \frac{\partial\mathcal{L}}{\partial C^F_{i,t+s}} &= (\beta^s C^{-\sigma}_{i,t+s})\Big[(1-\kappa)\frac{C_{i,t+s}}{C^F_{i,t+s}}\Big] - \lambda_{i,t+s}\frac{P^F_{i,t+s}}{P_{i,t+s}} - \lambda 2_{i,t+s}(1-\kappa)\frac{C_{i,t+s}}{C^F_{i,t+s}} = 0 \\ \frac{\partial\mathcal{L}}{\partial b^G_{i,t+s}} &= \lambda_{i,t+s}(1+\pi_{i,t+s+1}) - \lambda_{i,t+s+1}(1+R_{t+s+1}) = 0 \\ \frac{\partial\mathcal{L}}{\partial f_{i,t+s}} &= \lambda_{i,t+s}(1+\pi_{i,t+s+1}) - \lambda_{i,t+s+1}(1+R_{t+s+1}) = 0 \\ \frac{\partial\mathcal{L}}{\partial m_{i,t+s}} &= \lambda_{i,t+s}(1+\pi_{i,t+s+1}) - \lambda_{i,t+s+1} - \lambda 2_{i,t+s+1} = 0\end{aligned}$$

The Euler equation[16] is:

$$\beta\Big(\frac{C_{i,t+s+1}}{C_{i,t+s}}\Big)^{-\sigma}\frac{1+R_{t+s}}{1+\pi_{i,t+s+1}} = 1 \tag{2.20}$$

In the long run, the Euler equation solves for the price level. This is because at the steady state, consumption disappears, the nominal interest rate is determined by the Taylor rule, the inflation is defined in terms of the price level, and the difference in the real interest rates is set equal to the risk premium.

[16] See Appendix A2 for the derivation of this Euler equation.

The model equations are:

$$
\begin{aligned}
C_{i,t} &= \frac{(C^H_{i,t})^{\kappa}(C^F_{i,t})^{1-\kappa}}{\kappa^{\kappa}(1-\kappa)^{1-\kappa}} \\
P_{i,t}C_{i,t} &= P^H_{i,t}C^H_{i,t} + P^F_{i,t}C^F_{i,t} \\
P_{i,t}C_{i,t} + M_{i,t} + B_{i,t} + T_{i,t} &= P_{i,t}Y_{i,t} + M_{i,t-1} + (1+R_t)B_{i,t-1} \\
Y_{i,t} &= A_i \times L_{i,t} \\
Y_{i,t} &= C^H_{i,t} + C^F_{j,t} + G_{i,t} \\
P_{i,t}Y_{i,t} &= P^H_{i,t}C^H_{i,t} + P^H_{i,t}G_{i,t} + P^H_{i,t}C^F_{j,t} \\
P^H_{i,t}G_{i,t} + M_{i,t-1} + (1+R_t)B^G_{i,t-1} &= T_{i,t} + M_{i,t} + B^G_{i,t} \\
B_{i,t} &= B^G_{i,t} + F_t \\
B_{j,t} &= B^G_{j,t} - F_t \\
P^H_{i,t}C^F_{j,t} + (1+R_t)F_{t-1} &= P^F_{i,t}C^F_{i,t} + F_t \\
P^H_{j,t}C^F_{i,t} - (1+R_t)F_{t-1} &= P^F_{j,t}C^F_{j,t} - F_t \\
\beta\Big(\frac{C_{i,t+1}}{C_{i,t}}\Big)^{-\sigma}\frac{1+R_t}{1+\pi_{i,t+1}} &= 1 \\
M_{i,t-1} &= P_{i,t}C_{i,t} \\
M_t &= \sum_{i=1}^{2} M_{i,t} = M_{1,t} + M_{2,t} \\
P_t &= P^{\mu}_{1,t}P^{1-\mu}_{2,t} \\
1+\pi_t &= \frac{P_t}{P_{t-1}} \\
R_t &= \pi + r + (1+\tau)(\pi_t - \pi)
\end{aligned}
$$

2.2.7 Programing & Calibration

We program our log-linearized model into Dynare[17] implemented in Matlab. The number of endogenous variables should equal to the number of log-linearized equations, in order for the model system to be viable. As a first step, we calibrate the model to be symmetric between the two state economies as our benchmark, so that we can check the stability of our model. Building from that, we calibrate the model to make the two state economies distinct from each other, for the purpose of conducting our economic experiments and investigating our research questions. In both the symmetric and asymmetric setup, we find that the two economies have

[17] See Adjemain *et al.* (2011) and other useful documentation available on their website www.dynare.org

a steady state.

The table below lists our calibration of the structural parameters that we use with the same values across all the experiments and extensions in this chapter.

We calibrate our representative agent's rate of time preference, and the size of the discount factor, in consistency with quarterly data empirically. So when we do simulation in Dynare, each time period of simulation represents a quarter, and the interpretation of impulse-response functions follow accordingly.

Clarida, Gali, and Gertler (1998) run regressions of the US federal funds rates on inflation and output, and find that inflation coefficients are below one prior to 1980 and above one since then.[18] So we calibrate our inflation coefficient $(1 + \tau)$ to be > 1.

Table 2.1. Baseline DSGE Model - Structural Parameter Calibration

Parameter	Value	Definition
κ	0.5	substitutability between consumption goods
μ	0.7	weight in calculating average price level
ρ	0.05	representative agent's rate of time preference
σ	4	a non-negative constant in the power utility function
τ	1.5	monetary authority's aggressiveness in the Taylor rule

2.2.8 Extension: Two-Currency Regime

In the main part of this chapter, we consider the case of a currency union for the two economies. As an extension (and as a comparison), we also allow each economy to have their own monetary authority in order to conduct their independent monetary policy. In the next section, these two cases (single currency regime and two-currency regime) will both be examined in our economic experiments.

Under a two-currency monetary regime, the independence in monetary policy includes a flexible exchange rate between the two economies, and a separate nominal interest rate for each economy. Money supply still satisfies the "cash-in-advance"

[18]See Cochrane (2011) for some theoretical discussion of the determinacy and identification with the Taylor rule.

condition in each economy.

In the CAPM (capital asset pricing model)[19], when there are two risky assets, we take into account the variance-covariance structure of returns to decide the proportions of each asset to hold in our portfolio. But in DSGE models, second moments are typically ignored, which means that the CAPM results can not be applied here.

Hence in our DSGE model, we use the existing *balance of payments* (BoP) condition, plus a no-arbitrage condition to link the two returns. This no-arbitrage condition is in essence an interest arbitrage equation, in which the returns are the same after risk adjustment.

Economic Modeling: Independent Monetary Policy

In this model extension, the main change is due to the fact that we have an independent monetary authority in each economy, so we discuss in detail its possible monetary policy instruments and responsibility as follows.

Monetary Authority:

(a) Nominal Exchange Rate

Without a common currency, each monetary authority also have the monetary policy instrument: the nominal exchange rate. In a currency union, the nominal exchange rate is exogenous, to be exact, it is given and fixed at one. Now we examine the case where the nominal exchange rate is allowed to float freely, and therefore the nominal exchange rate is endogenous in our new case without a currency union.

The nominal exchange rate measures the relative price of a currency with respect to another currency, that is, the rate at which one currency will be exchanged for another.

$$S_t = \frac{F\$}{H\$} \tag{2.21}$$

Take Home and Foreign for an example. The nominal exchange rate between Home and Foreign measures the corresponding units of Home currency for one

[19] In finance, the CAPM is used to determine a theoretically appropriate required rate of return of an asset, if we want to add that asset into an already well-diversified portfolio.

unit of Foreign currency. For each unit of Home currency, if there is an increase in the equivalent amount of Foreign currency, then this means that the Home currency becomes more valuable in the sense that it can exchange for more Foreign currency, and hence there is an appreciation in the Home currency and a depreciation in the Foreign currency. That is, a S_t increase is associated with an appreciation in the Home currency.

Terms of Trade:

As defined in *Section 2.2.4*, an economy's terms of trade is measured in terms of the price of its exports relative to the price of its imports after adjusting for the nominal exchange rate. Unlike the case with a currency union, the nominal exchange rate is not fixed at one now, and hence the terms of trade becomes:

$$P_{i,t}^{ToT} = S_t \frac{P_{j,t}^F}{P_{i,t}^F} \tag{2.22}$$

Real Exchange Rate:

In our model, each economy only produces a single good. So the terms of trade is equivalent to the real exchange rate.

$$Q_t = S_t \frac{P_{j,t}^F}{P_{i,t}^F} \tag{2.23}$$

Law of One Price:

In comparison, there is also the notion of *Law of One Price* (LOOP). LOOP is an economic law, which can be stated as: In an efficient market, all identical goods must have only one price. That is to say, for the same good, it should be purchased at the same price after adjusting for exchange rate, transportation cost, etc. LOOP applies to the price of a single good.

Purchasing Power Parity:

Purchasing Power Parity (PPP) is a measure of how much money would be needed to purchase the same amount of goods and services in two different countries. At the PPP rate, that size of money has the same purchasing power across the two countries. PPP applies to the general price level of all goods. But in our model, each economy only produces one good. So LOOP is equivalent to PPP here. If PPP is held exactly, then the real exchange rate would always equal one, $Q_t = 1$.

(b) Interest Rate:
With independent monetary policy, the monetary authority in each economy determines its own nominal interest rate $R_{i,t}$, and has the Taylor (1993) style interest rate rule as:

$$R_{i,t} = \pi_i + r_i + (1+\tau)(\pi_{i,t} - \pi_i) \tag{2.24}$$

Uncovered Interest Parity:
We assume that the "uncovered interest parity" (UIP) condition holds, in order to avoid international arbitrage opportunities in the bond market. This UIP condition involves expectation formation of the future exchange rate, and can be written as:

$$R_{i,t} = R_{j,t} + E_t \Delta s_{t+1} \tag{2.25}$$

where: 1) $R_{i,t}$ = the nominal interest rate at Home. 2) $R_{j,t}$ = the nominal interest rate at Foreign. 3) s_t = the logarithm of the nominal exchange rate between Home and Foreign.

We also assume that Home and Foreign bonds are perfect substitutes to each other, and as such, Home and Foreign have no incentive to hold the other's bonds. Therefore, the UIP condition is also the *balance of payments* condition.

Solve this UIP equation forward, we get:

$$s_t = \sum_{t=0}^{\infty} E_t(R_{j,t+s} - R_{i,t+s}) \tag{2.26}$$

The intuition behind this equation is that the nominal exchange rate will respond to any new information on the current and expected future nominal interest rate differentials.

(c) Money Supply:
On the money supply side, we still assume the "cash-in-advance" (CIA) condition, where money holdings are equal to the total expenditure on the purchase of consumption goods in each economy. Money market is assumed to clear in each economy. In a non-currency union case, each monetary authority is only responsible for money supply in its own economy.

New Model System

On the consumer side, the introduction of a flexible exchange rate changes the equation of their consumption expenditure:

$$P_{1,t}C_{1,t} = P^H_{1,t}C^H_{1,t} + \frac{1}{S_t}P^F_{1,t}C^F_{1,t} \tag{2.27}$$

$$P_{2,t}C_{2,t} = P^H_{2,t}C^H_{2,t} + S_t P^F_{2,t}C^F_{2,t} \tag{2.28}$$

The new model equations are:

$$\begin{aligned}
C_{i,t} &= \frac{(C^H_{i,t})^\kappa (C^F_{i,t})^{1-\kappa}}{\kappa^\kappa (1-\kappa)^{1-\kappa}} \\
P_{1,t}C_{1,t} &= P^H_{1,t}C^H_{1,t} + \frac{1}{S_t}P^F_{1,t}C^F_{1,t} \\
P_{2,t}C_{2,t} &= P^H_{2,t}C^H_{2,t} + S_t P^F_{2,t}C^F_{2,t} \\
P_{i,t}C_{i,t} + M_{i,t} + B_{i,t} + T_{i,t} &= P_{i,t}Y_{i,t} + M_{i,t-1} + (1 - R_{i,t-1})B_{i,t-1} \\
Y_{i,t} &= A_{i,t} \times L_{i,t} \\
Y_{i,t} &= C^H_{i,t} + C^F_{j,t} + G_{i,t} \\
P_{i,t}Y_{i,t} &= P^H_{i,t}C^H_{i,t} + P^H_{i,t}G_{i,t} + P^H_{i,t}C^F_{j,t} \\
P^H_{i,t}G_{i,t} + M_{i,t-1} + (1 + R_{i,t-1})B^G_{i,t-1} &= T_{i,t} + M_{i,t} + B^G_{i,t} \\
B_{i,t} &= F_t + B^G_{i,t} \\
B_{j,t} &= -F_t + B^G_{j,t} \\
\beta\Big(\frac{C_{i,t+1}}{C_{i,t}}\Big)^{-\sigma}\frac{1+R_t}{1+\pi_{i,t+1}} &= 1 \\
M_{i,t-1} &= P_{i,t}C_{i,t} \\
Q_t &= S_t\frac{P^F_{j,t}}{P^F_{i,t}} = 1 \\
R_{i,t} &= \pi_i + r_i + (1+\tau)(\pi_{i,t} - \pi_i) \\
R_{i,t} &= R_{j,t} + E_t\Delta s_{t+1}
\end{aligned}$$

2.3 Economic Experiments

In our analytical framework, there is a currency union between the two economies. On the fiscal side, each economy has its own independent fiscal authority. On the monetary side, there is only one central monetary authority, and the two economies are operated under the same monetary policy regime. Conventionally, there are three possible monetary policy instruments by a monetary authority: the nominal exchange rate, the nominal money supply, and the nominal interest rate. In our currency union, our two economies have a fixed exchange rate, so the nominal exchange rate is fixed at one in our model economy. The money supply passively satisfies the "cash-in-advance" (CIA) constraint in our model. Therefore, the only effective monetary policy instrument in our currency union is the nominal interest rate by the central monetary authority. It is the difference in inflation rates that matters for the conduct of joint monetary policy in a currency union with a common nominal interest rate.

The practical point of our exercise is to examine the (policy) implications of the European Central Bank (ECB) setting interest rates based on the average inflation in the Euro Zone. As the first part of our economic experiment, we simulate a productivity shock/gain to our Home/big economy, as productivity is the key to economic growth over the long run. This economic experiment is a positive supply shock. In our second economic experiment, we consider the implementation of *fiscal austerity* in the Home economy. This experiment is a negative demand shock. For both of these two economic experiments, we first discuss the case of a single currency monetary policy regime, and then the case of a two-currency policy regime for the same shock.

(Another possible economic experiment with this DSGE model is a migration shock to any of the economies. According to Robert Mundell (who is the father of the Euro), an important factor to make the shared common currency work is a mobile labor force. The intuition behind this key factor is that economies in a currency union lose their "degree of freedom" by giving up the right to adjust their (nominal) interest rates or re/devalue their currency when faced with (external or internal) economic shocks. But in Europe, only a small percentage of Europeans migrate across borders every year. By contrast, there are many more Americans who have inter-state migration each year. So migration shock would not be crucial for the Euro Zone.)

2.3.1 Productivity Shock

2.3.1.1 Single Currency Regime

First, we study the case of a single-currency monetary policy regime, for our Home economy and Foreign economy. As part of our economic experiment, we introduce a productivity shock into the Home production sector. This can be, for example, due to a technology advancement in the Home economy. A productivity shock is a shock from the supply side. For production, the output function in the Home economy is represented by: $Y_{1,t} = A_1 L_{1,t}$, where A_1 is the total factor productivity (TFP) in the production of Home output. It can also be thought of as some labor-augmenting technology, which makes labor more productive and hence there will be an increase in productivity and production. In our Matlab code, the increase in Home's productivity is proxied by a rise of e_1 to their TFP parameter A_1. In Figure 2.1, we present the impulse response functions of some endogenous variables of our interest (the price levels and interest rates) from our stochastic simulation in Dynare, for this productivity shock in the Home economy under a single currency policy regime.

For the productivity shock in the Home economy, the improvement in Home's productivity level A_1 will lead to an initial increase in Home's production level Y_1. Part of this Home production increase will be absorbed by domestic consumers. We assume[20] that the goods market is in equilibrium initially. There are two effects on the level/change of prices at play here. On the one hand, there is the *supply-side effect*, in the sense that an increase in productivity will increase potential output, which in turn will put downward pressure on the price level. In other words, the increase on the supply side will lead to a surplus and will cause a fall in the Home price of Home-produced goods P_1^H, which will contribute to a decrease in the general price level P_1 at Home.[21] On the other hand[22], there is also the *demand-side effect*, which is like the wealth effect, in the sense that our representative agent's income stream will increase and they will become/feel wealthier, which will encourage them to increase their consumption C_1, putting upward pressure on the price level and potentially leading to inflation. These

[20]Economists make assumptions all the time.

[21]The reasoning for the decrease in the Home's price level P_1 can also be shown via the relation of price determination: $P_{1,t} = \frac{W_{1,t}}{A_1}$. Take the partial derivative of $P_{1,t}$ with respect to A_1: $\frac{\partial P_{1,t}}{\partial A_1} = -\frac{W_{1,t}}{A_1^2} < 0$.

[22]Economists have two hands!

two effects from the supply side and the demand side counteract each other. In our DSGE modeling, we focus more on the short run or the medium run, with each time period representing a quarter. So the *supply-side effect* dominates the *demand-side effect* in our case. Then an increase in the factor productivity A_1 will cause an initial decrease in the price level P_1 before climbing back to its steady-state level, as shown in our simulation results. This initial decrease in the general price level would put deflationary pressure to the Home economy.

Given the larger share of the Home economy in the currency union, the central monetary authority will take this deflationary pressure from the Home economy into consideration when making the joint monetary policy for the currency union. A fall in the inflation level will leave room for the central monetary authority to ease its monetary policy. This is done by cutting the nominal interest rate for the currency union as a whole. The decrease in the nominal interest rate, along with the change in the inflation rate, will lead the real interest rates to behave differently for the two economies. Our impulse response functions show that the Home economy will still have a positive real interest rate first, as the initial change in the nominal interest rate is probably smaller than the change in Home's price level. This initial hike will be followed by a fall for Home's real interest rate, whilst Foreign's real interest rate will drop first before a rise. This is due to the relative magnitude of the change in the nominal interest rate and inflation rates, and the pace at which they are channeled through in each economy.

2.3.1.2 Two-Currency Regime

Next, we conduct the same economic experiment of a productivity shock in the Home economy, for the case of a two-currency monetary policy regime. The impulse response functions (for the price levels and interest rates) are presented in Figure 2.2. With a productivity shock from the supply side in the Home economy, there will be an expansion in the output of Home-produced goods. Assuming that nothing has changed on the supply side in the Foreign economy, the increase in Home production indicates that there will be a relative abundance of Home-made goods in the market. The relative surplus of Home production will initially put downward pressure on Home's price level. This will leave room for Home's central bank to ease its monetary policy stance, by cutting its nominal interest rate R_1.

The corresponding nominal interest rates for Home and Foreign are linked via the UIP condition, which in turn is reflected in the relative movement in their bi-lateral nominal exchange rate. This is the so-called "external revaluation", as compared to the "internal revaluation" in the case of a single currency regime.

Our impulse response function shows that the magnitude of the drop in inflation rate is smaller than the drop in nominal interest rate for both Home and Foreign, and therefore their real interest rates will also decrease, which in turn will reduce the borrowing cost for the private sector in order to finance the expansion in Home production. In general, the decrease in the real interest rate will encourage borrowing and spending, which will boost consumer and business confidence.

Comparing our simulation results from both the single currency and two-currency regimes, we find that in the case of a single currency, the pace and timing of the change in the inflation rates and the real interest rates occur differently, which is because of the counter-balancing forces from the sources of the shocks, the relative size of the two economies, and some "compromise" or adjustment made in between. But in the two-currency case, the real interest rates adjust quickly for both Home and Foreign.

2.3.2 Fiscal Austerity

The famous British economist John Maynard Keynes once said that "The boom, not the slump, is the right time for austerity". In an economy, such as the Euro Zone, where monetary policy is ineffective - both because of enfeebled financial system and (close-to) zero interest rate (at which the central bank could "intervene"), fiscal policy needs to be used. This is particularly true when the private sector has a huge structural excess of income over spending, and as a result of this lacklustre demand, economic recovery is weak, and therefore consideration should be given to greater near-term flexibility in the fiscal adjustment path. The double aims of fiscal policy must be to maintain aggregate demand, and at the same time, to improve aggregate supply for the whole economy.

But in reality, Europe rejects this view of Keynes and insists on implementing *fiscal austerity* during the economic slump after the sovereign debt crisis in the Euro area, which was triggered by the Global Financial Crisis (GFC) after the collapse of Lehman Brothers in September 2008. The economic rationale behind

Euro Zone's *fiscal austerity* policy is that following the introduction of the Euro in 1999, many Euro Zone countries, among them the PIIGS countries (Portugal, Ireland, Italy, Greece, and Spain), experienced a dramatic decline in their borrowing costs for both of their public and private sectors. As a consequence of that, cheap credit, which was often fed by capital from banks in the "core" economies of the Euro Zone, fueled a credit boom that led to high growth rates in those "periphery" economies of the Euro Zone. This in turn attracted more capital and investment to those economies. But at the same time, it also increased the indebtedness of households and firms. Current account balances had a sharp deterioration , and countries accumulated large foreign debt. Governments expanded their balance sheet, due to their mis-perception that the high growth rates would lead to permanent increase in their income.

So in our second economic experiment, we introduce *fiscal austerity* into our Home economy. In this case, the Home government conducts a contractionary fiscal policy, which is achieved by decreasing its government expenditure level. The contraction on government expenditure is a shock from the demand side. In our Matlab code, Home's contractionary expenditure policy is proxied by a decrease of size e_2 to their government spending variable g_1.

2.3.2.1 Single Currency Regime

In a single currency monetary policy regime, Home's expenditure shock is a contractionary shock on the demand side. Real GDP will fall due to the decreased demand for goods and services by the Home government, which in turn will contribute to a decline in the aggregate demand for the whole economy. In the short run, there is an excess capacity in production, and as a result of that, a contractionary shock from the demand side will push down costs and the general price level in the Home economy, putting deflationary pressure on the currency union. Faced with this deflationary pressure, the central bank will cut the nominal interest rate for the currency union, which will encourage the private sector to invest/innovate more so as to compensate for the lack of demand due to the government budget cut by the Home government. From our simulation results, we could see that Home and Foreign's real interest rates adjust at a different pace. This is again due to the pace at which the shock is channeled through in both the Home and Foreign economies via their financial and trade linkages. The corresponding impulse response functions (for the price levels and interest rates) are presented in Figure 2.3.

2.3.2.2 Two-Currency Regime

By the same reasoning, the economic analysis is similar in a two-currency monetary policy regime. Same as in the experiment of the productivity shock, the Home and Foreign economies gain "one degree of freedom" under a two-currency policy regime. The UIP condition links the relative movement of their (individual) nominal interest rates and their (bi-lateral) nominal exchange rates. The price level, nominal interest rates, and real interest rates will adjust more quickly to better reflect what is going on and what is needed in the Home and Foreign economies. In Figure 2.4, we present the corresponding impulse response functions (for the price levels and interest rates) from this expenditure shock in the Home economy under the two-currency policy regime.

2.4 Conclusion

2.4.1 Chapter Recap

This research project contributes to the literature on the appropriateness of a currency union. To that end, we develop a two-state open economy DSGE model, in which the two state economies are linked by their trade in goods with each other and bond borrowing with one another. Both economies operate under the same monetary policy regime by a central monetary authority, which fixes the nominal exchange rate at one, sets the common nominal interest rate for the currency union, and prints money to meet the "cash-in-advance" requirement in each economy. On the other hand, each economy retains its own fiscal independence, by having its own separate fiscal authority, which has the authority to levy non-distorting lump sum tax transfers, and issue nominal risk-free debt in order to finance a given process of public expenditure. Applying our DSGE model, we conduct economic experiments by imposing a productivity shock, and a government expenditure shock in the Home economy. With the possible setting of a two-currency monetary policy regime as our comparison, we find that the two economies would have different nominal interest rates, which are linked via the UIP (uncovered interest parity) condition. The difference in the magnitude of the adjustment in their interest rates better reflect what (interest rate) is needed in their economy.

From these two economic experiments, we reach the conclusion that asymmetric shocks to country inflation rates can better be dealt with by monetary policy in a multi floating currency system. A currency union is not always appropriate for its member economies. The problem lies with the lack of nominal exchange rate in a currency union to reflect the relative change of economic situations in both economies when faced with shocks. In addition to that, the "one-size-fits-all" nominal interest rate will not be appropriate for both economies, because the source of shocks originates from one economy which might be a supply-side shock or a demand-side shock. On top of that, the inflation rates in individual economies are not the same, which are mainly due to the different price dynamics going on in each market. Consequently, the real interest rates differ in each economy, and adjust more slowly in a single-currency monetary system as compared to a two-currency system, which in turn will further cause divergence in their competitiveness over time.

2.4.2 Policy Implications

What is the main takeaway from this research project? As Wickens (2010) points out, at the inception of the Euro Zone, the member countries do not satisfy an optimal currency area, due to the fact that individual Euro Zone economies have different inflation rates, which are unlikely to converge. The problem lies in the very different behavior of member economies, to which a contributing factor is the "one-size-fits-all" monetary policy that is imposed by the setting of a monetary union. The common nominal interest rate is set by the central monetary authority. However, inflation rates are unlikely to converge in individual Euro Zone economies, and their price levels are more likely to diverge. This difference in inflation rates will therefore cause each Euro Zone economy to have a different real interest rate, leading to difference in competitiveness over time.[23] Therefore, we have a "two-state" economy in the Euro Zone, with difference in real interest rates and in the degree of competitiveness among them, which is caused by the inappropriate monetary policy under a currency union. But as a currency union,

[23] The real interest rate is an important measurement of an economy's competitiveness. *First*, the real interest rate measures the opportunity cost of consumption. For each dollar in your pocket, you can either deposit it in your saving account with your bank, earning nominal interest rate while taking into account inflation; or you can use it for consumption. *Second*, the real interest rate measures the borrowing cost. The higher the real interest rate, the more expensive it is to finance new investment projects. *Third*, the real interest rate also measures the interest cost for debt. Higher real interest rate means that it costs more to pay the interest on your debt.

the European Central Bank (ECB) has the mandate of maintaining price stability around their inflation target for the Euro Zone as a whole. The difference in inflation rates will therefore cause each economy to have a different real interest rate, which in turn will lead to large difference in the degree of competitiveness among them over time. The problem is rooted from the inappropriate monetary policy under a currency union. Hence, the Euro Zone is not an optimal currency area.

Inflation difference is the underlying problem for the Euro Zone project. But which one is more important here? - [1] the difference in inflation expectations, [2] the short-term inflation differential, or [3] the long-term inflation differential among different Euro Zone economies?

Does [1] the difference in inflation expectations among each individual economy matter for a currency union as a whole? Simon, Matheson, and Sandri (2013) find that inflation expectations are strongly anchored to the central bank's inflation targets rather than being particularly altered by the current inflation levels; and on top of that, the anchoring of expected inflation has increased over time, whilst the impact of current inflation on expected inflation has diminished. Hence, there is some "stickiness" in inflation nowadays. In the case of a monetary union, inflation is stabilized at the union level, however, inflation is "destabilized" at the country level, and adjustment needs to come from each country, which is backed by fiscal policy, in order to be viable. At the country level in a currency union, there is no monetary authority that conducts independent monetary policy, and hence there is no inflation target to anchor the inflation expectation.

So [1] the difference in inflation expectations among individual economies does not matter for a currency union. Then between [2] the short-term inflation differential, and [3] the long-term inflation differential, which one is more important or more relevant here?

By the nature of DSGE modeling in Matlab-Dynare, our model simulation shows forecasting over the next few periods, on a quarterly basis. So by "design", we imply that it is the short-term inflation differential that matters. (To check whether the long-term inflation differential also matters, some empirical work with real data could be conducted as an econometric extension of this research project.)

Appendix: Mathematics Derivation

A1. 5 accounting identities - consistency check

In *Section 2.2*, we have 5 accounting identities. We rewrite these 5 equations here, in order to check their internal consistency.

(1) The household budget constraint is:

$$P_{i,t}C_{i,t} + M_{i,t} + B_{i,t} + T_{i,t} \quad = \quad W_{i,t}L_{i,t} + D_{i,t} + M_{i,t-1} + (1+R_t)B_{i,t-1}$$

where: $B_{i,t} = F_t + B^G_{i,t}$

(2) The firms' budget constraint is:

$$P_{i,t}Y_{i,t} \quad = \quad W_{i,t}L_{i,t} + D_{i,t}$$

(3) The government's budget constraint is:

$$P^H_{i,t}G_{i,t} + M_{i,t-1} + (1+R_t)B^G_{i,t-1} \quad = \quad T_{i,t} + M_{i,t} + B^G_{i,t}$$

(4) The balance of payments is:

$$P^H_{i,t}C^F_{j,t} - P^F_{j,t}C^F_{i,t} + R_tF_{t-1} \quad = \quad \Delta F_t = F_t - F_{t-1}$$

(5) The national income identity is:

$$P_{i,t}Y_{i,t} \quad = \quad P^H_{i,t}C^H_{i,t} + P^H_{i,t}G_{i,t} + P^H_{i,t}C^F_{j,t}$$

Substitute the firms' budget constraint into the household budget constraint:

$$P_{i,t}C_{i,t} + M_{i,t} + B_{i,t} + T_{i,t} \quad = \quad P_{i,t}Y_{i,t} + M_{i,t-1} + (1+R_t)B_{i,t-1}$$

Rewrite the household budget constraint as:

$$P_{i,t}C_{i,t} + M_{i,t} - M_{i,t-1} + T_{i,t} + B_{i,t} - (1+R_t)B_{i,t-1} \quad = \quad P_{i,t}Y_{i,t}$$

where: $B_{i,t} = F_t + B^G_{i,t}$

Rewrite the government budget constraint as:

$$P^H_{i,t}G_{i,t} \quad = \quad T_{i,t} + M_{i,t} - M_{i,t-1} + B^G_{i,t} - (1 + R_t)B^G_{i,t-1}$$

Substitute the rewritten government budget constraint into the household budget constraint, we get:

$$P_{i,t}C_{i,t} + P^H_{i,t}G_{i,t} + F_t - (1 + R_t)F_{t-1} \quad = \quad P_{i,t}Y_{i,t}$$

Write the consumption expenditure equation here:

$$P_{i,t}C_{i,t} \quad = \quad P^H_{i,t}C^H_{i,t} + P^F_{i,t}C^F_{i,t}$$

Substitute the consumption expenditure equation into the rewritten household budget constraint, we get:

$$P^H_{i,t}C^H_{i,t} + P^F_{i,t}C^F_{i,t} + P^H_{i,t}G_{i,t} + F_t - (1 + R_t)F_{t-1} \quad = \quad P_{i,t}Y_{i,t}$$

Write the current account balance equation here:

$$P^H_{i,t}C^F_{j,t} - P^F_{i,t}C^F_{i,t} + R_tF_{t-1} \quad = \quad F_t - F_{t-1}$$

Substitute the current account balance equation into the equation above, we get:

$$P^H_{i,t}C^H_{i,t} + P^H_{i,t}C^F_{j,t} + P^H_{i,t}G_{i,t} \quad = \quad P_{i,t}Y_{i,t}$$

It is our national income identity in this model economy.

So we have shown that these 5 accounting identities are internally consistent.

A2. Euler equation - derivation

In Section 2.2.6, we get the Euler equation from the dynamic optimization problem. In this appendix section, we show the derivation of this Euler equation.

The first-order conditions with respect to the choice variables are:

$$\frac{\partial \mathcal{L}}{\partial C^H_{i,t+s}} \quad = \quad (\beta^s C^{-\sigma}_{i,t+s})\Big(\kappa \frac{C_{i,t+s}}{C^H_{i,t+s}}\Big) - \lambda_{i,t+s}\frac{P^H_{i,t+s}}{P_{i,t+s}} - \lambda 2_{i,t+s}\kappa \frac{C_{i,t+s}}{C^H_{i,t+s}} = 0$$

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial C^F_{i,t+s}} &= (\beta^s C^{-\sigma}_{i,t+s})\Big[(1-\kappa)\frac{C_{i,t+s}}{C^F_{i,t+s}}\Big] - \lambda_{i,t+s}\frac{P^F_{i,t+s}}{P_{i,t+s}} - \lambda 2_{i,t+s}(1-\kappa)\frac{C_{i,t+s}}{C^F_{i,t+s}} = 0 \\
\frac{\partial \mathcal{L}}{\partial b^G_{i,t+s}} &= \lambda_{i,t+s}(1+\pi_{i,t+s+1}) - \lambda_{i,t+s+1}(1+R_{t+s+1}) = 0 \\
\frac{\partial \mathcal{L}}{\partial f_{i,t+s}} &= \lambda_{i,t+s}(1+\pi_{i,t+s+1}) - \lambda_{i,t+s+1}(1+R_{t+s+1}) = 0 \\
\frac{\partial \mathcal{L}}{\partial m_{i,t+s}} &= \lambda_{i,t+s}(1+\pi_{i,t+s+1}) - \lambda_{i,t+s+1} - \lambda 2_{i,t+s+1} = 0
\end{aligned}$$

Subtract the fifth F.O.C from the third F.O.C:

$$\begin{aligned}
-\lambda_{i,t+s+1}(1+R_{t+s+1}) + \lambda_{i,t+s+1} + \lambda 2_{i,t+s+1} &= 0 \\
-\lambda_{i,t+s+1} - \lambda_{i,t+s+1}R_{t+s+1} + \lambda_{i,t+s+1} + \lambda 2_{i,t+s+1} &= 0 \\
-\lambda_{i,t+s+1}R_{t+s+1} + \lambda 2_{i,t+s+1} &= 0 \\
-\lambda_{i,t+s+1}R_{t+s+1} &= -\lambda 2_{i,t+s+1} \\
\lambda_{i,t+s+1}R_{t+s+1} &= \lambda 2_{i,t+s+1}
\end{aligned}$$

The first F.O.C of the optimization problem gives us:

$$\begin{aligned}
\beta^s (C_{i,t+s})^{-\sigma}\Big(\kappa\frac{C_{i,t+s}}{C^H_{i,t+s}}\Big) - \lambda_{i,t+s}\frac{P^H_{i,t+s}}{P_{i,t+s}} - \lambda_{i,t+s}R_{t+s}\kappa\frac{C_{i,t+s}}{C^H_{i,t+s}} &= 0 \\
\beta^s (C_{i,t+s})^{-\sigma}\Big(\kappa\frac{C_{i,t+s}}{C^H_{i,t+s}}\Big) - \lambda_{i,t+s}\Big(\frac{P^H_{i,t+s}}{P_{i,t+s}} + R_{t+s}\kappa\frac{C_{i,t+s}}{C^H_{i,t+s}}\Big) &= 0 \\
\beta^s (C_{i,t+s})^{-\sigma}\Big(\kappa\frac{C_{i,t+s}}{C^H_{i,t+s}}\Big) - \lambda_{i,t+s}\frac{P^H_{i,t+s}C^H_{i,t+s} + R_{t+s}\kappa P_{i,t+s}C_{i,t+s}}{P_{i,t+s}C^H_{i,t+s}} &= 0
\end{aligned}$$

$$\beta^s (C_{i,t+s})^{-\sigma}\Big(\kappa\frac{C_{i,t+s}}{C^H_{i,t+s}}\Big) = \lambda_{i,t+s}\frac{P^H_{i,t+s}C^H_{i,t+s} + R_{t+s}\kappa P_{i,t+s}C_{i,t+s}}{P_{i,t+s}C^H_{i,t+s}}$$

Move it forward by 1 period:

$$\beta^{s+1} (C_{i,t+s+1})^{-\sigma}\Big(\kappa\frac{C_{i,t+s+1}}{C^H_{i,t+s+1}}\Big) = \lambda_{i,t+s+1}\frac{P^H_{i,t+s+1}C^H_{i,t+s+1} + R_{t+s+1}\kappa P_{i,t+s+1}C_{i,t+s+1}}{P_{i,t+s+1}C^H_{i,t+s+1}}$$

Take their ratio:

$$\beta\Big(\frac{C_{i,t+s+1}}{C_{i,t+s}}\Big)^{-\sigma}\frac{C_{i,t+s+1}}{C_{i,t+s}}\frac{C^H_{i,t+s}}{C^H_{i,t+s+1}} = \frac{\lambda_{i,t+s+1}}{\lambda_{i,t+s}}\frac{P_{i,t+s}C^H_{i,t+s}}{P_{i,t+s+1}C^H_{i,t+s+1}}$$

$$\beta\Big(\frac{C_{i,t+s+1}}{C_{i,t+s}}\Big)^{-\sigma}\frac{C_{i,t+s+1}}{C_{i,t+s}} = \frac{P^H_{i,t+s+1}C^H_{i,t+s+1}+R_{t+s+1}\kappa P_{i,t+s+1}C_{i,t+s+1}}{P^H_{i,t+s}C^H_{i,t+s}+R_{t+s}\kappa P_{i,t+s}C_{i,t+s}} \frac{\lambda_{i,t+s+1}}{\lambda_{i,t+s}}\frac{P_{i,t+s}}{P_{i,t+s+1}}\frac{(1+R_{t+s+1})P^H_{i,t+s+1}C^H_{i,t+s+1}}{(1+R_{t+s})P^H_{i,t+s}C^H_{i,t+s}}$$

$$\begin{aligned}
\beta\Big(\frac{C_{i,t+s+1}}{C_{i,t+s}}\Big)^{-\sigma} &= \frac{\lambda_{i,t+s+1}}{\lambda_{i,t+s}}\frac{1+R_{t+s+1}}{1+R_{t+s}}\frac{P^H_{i,t+s+1}C^H_{i,t+s+1}}{P_{i,t+s+1}C_{i,t+s+1}}\frac{P_{i,t+s}C_{i,t+s}}{P^H_{i,t+s}C^H_{i,t+s}}\\
\beta\Big(\frac{C_{i,t+s+1}}{C_{i,t+s}}\Big)^{-\sigma} &= \frac{\lambda_{i,t+s+1}}{\lambda_{i,t+s}}\frac{1+R_{t+s+1}}{1+R_{t+s}}\frac{1}{\kappa}\kappa\\
\beta\Big(\frac{C_{i,t+s+1}}{C_{i,t+s}}\Big)^{-\sigma} &= \frac{1+\pi_{i,t+s+1}}{1+R_{t+s+1}}\frac{1+R_{t+s+1}}{1+R_{t+s}}\\
\beta\Big(\frac{C_{i,t+s+1}}{C_{i,t+s}}\Big)^{-\sigma} &= \frac{1+\pi_{i,t+s+1}}{1+R_{t+s}}
\end{aligned}$$

So we get our Euler equation:

$$\beta\Big(\frac{C_{i,t+s+1}}{C_{i,t+s}}\Big)^{-\sigma}\frac{1+R_{t+s}}{1+\pi_{i,t+s+1}} = 1$$

Figure 2.1. Single Currency Regime - Home productivity shock - impulse response functions of price levels and interest rates

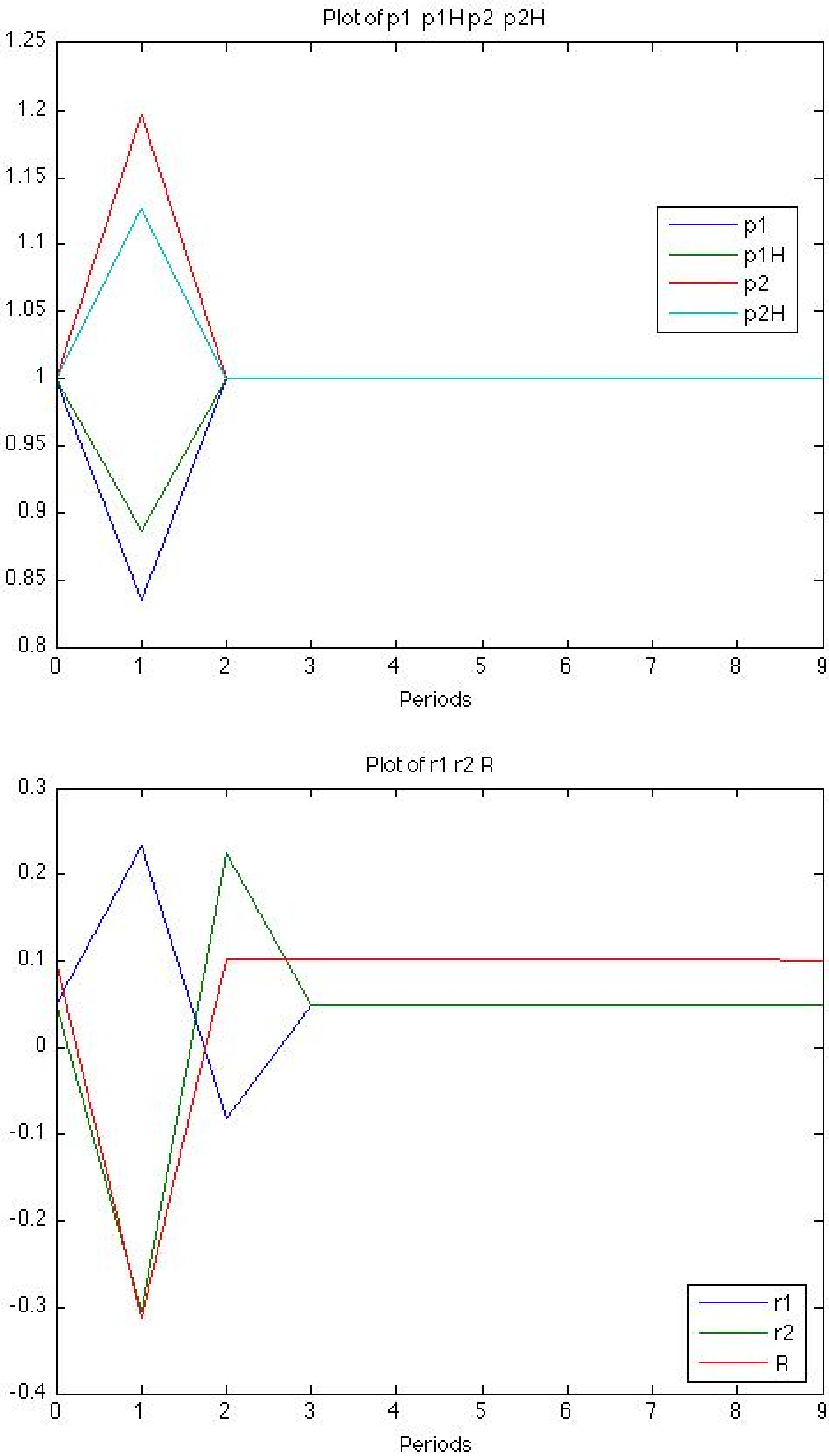


Figure 2.2. Two-Currency Regime - Home productivity shock - impulse response functions of price levels and interest rates

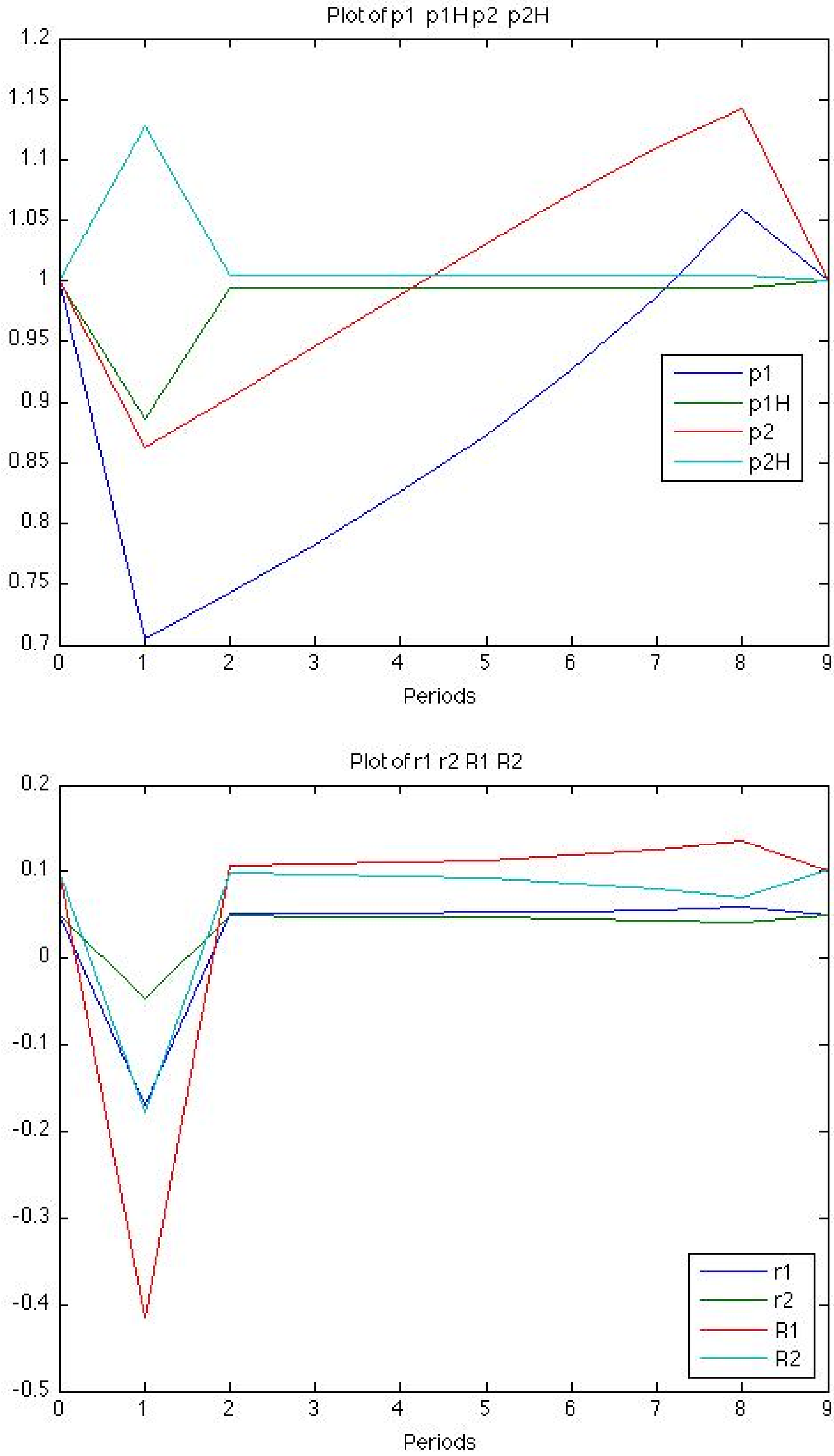


Figure 2.3. Single Currency Regime - Home expenditure shock - impulse response functions of price levels and interest rates

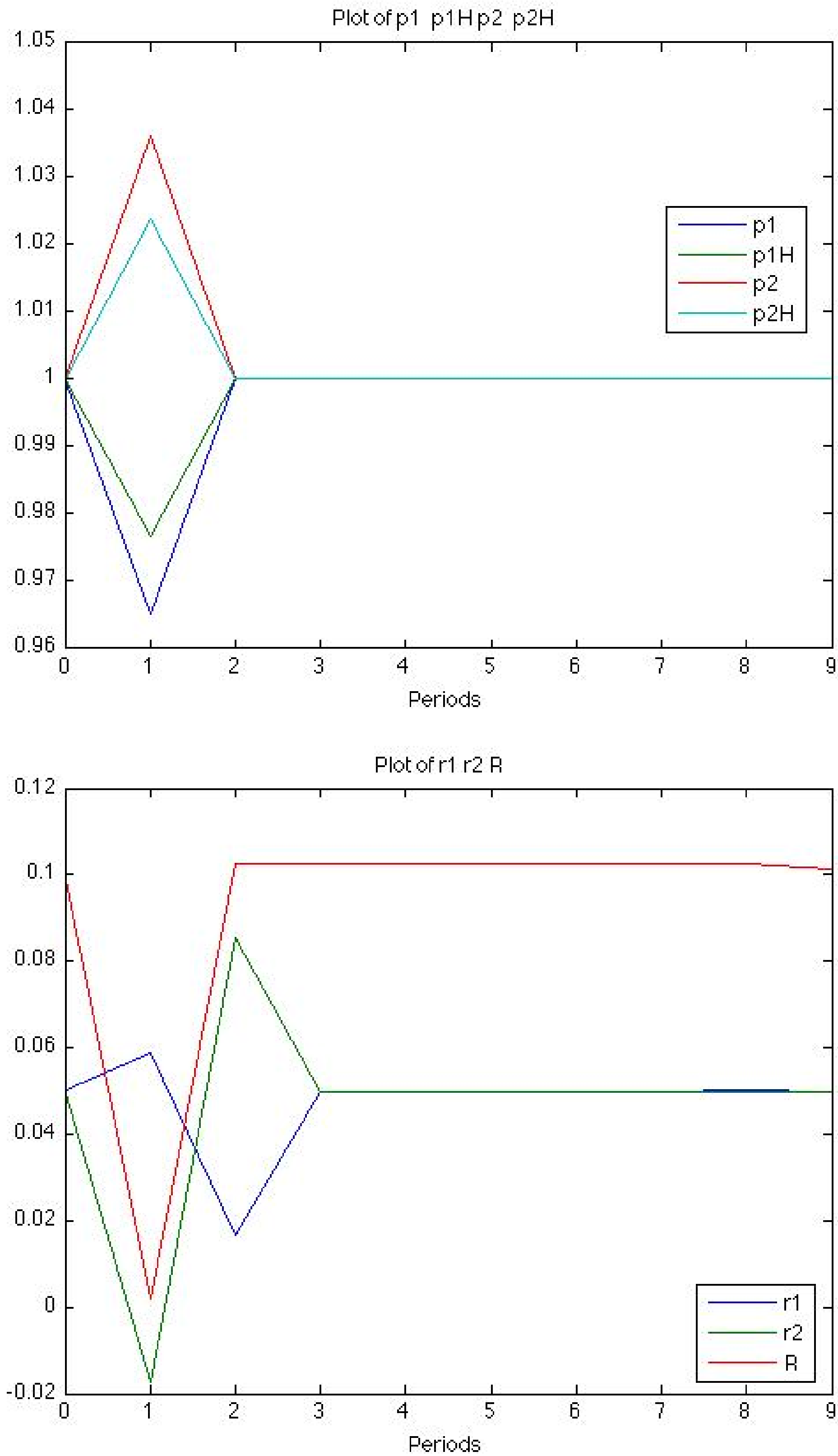
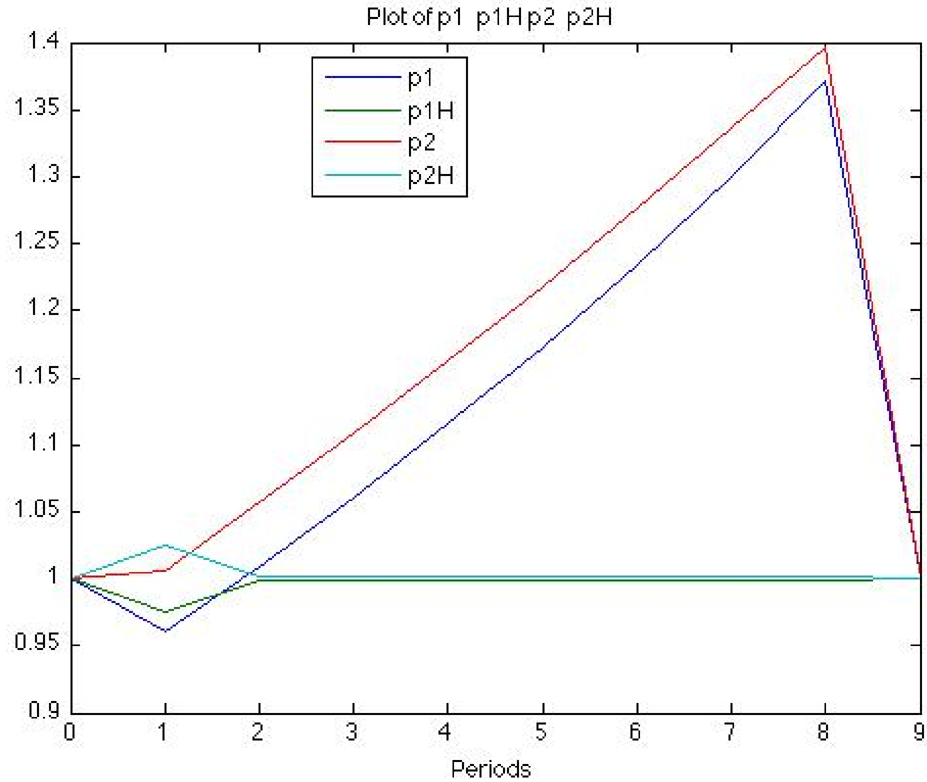
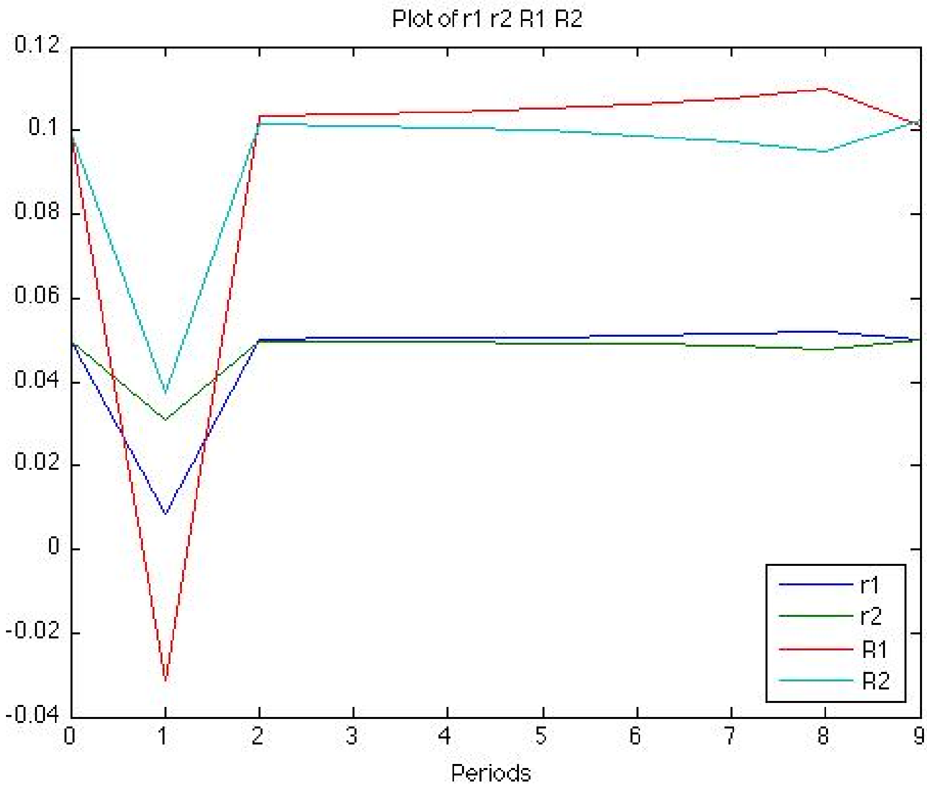


Figure 2.4. Two-Currency Regime - Home expenditure shock
- impulse response functions of price levels and interest rates





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Chapter 3

Business Cycle Synchronization between the Great Britain and the United States

Abstract

In this project, we take up the topic of business cycle synchronization between the Great Britain and the United States. In order to explore this research topic, we examine the relationship between the UK and the US economies for the past five decades. In particular, we measure the synchronization between these two economies, by dating their business cycles and finding the turning points in their cycles. We adopt a GMM analytical framework, as we want to seek a model that is free in measuring synchronization, and we also want to seek estimates that are robust to serial correlation and heteroscedasticity. Under our GMM framework, we conduct single and joint synchronization tests between the UK and the US for each type of business cycles. From these synchronization tests, we find evidence of correlation between the UK and the US business cycles, and in particular, the US leads the UK for all three types of business cycles.

JEL Classification: C18, C22, C32, E32.

Key Words: Business Cycle, Turning Point, GMM (Generalized Methods of Moments).

3.1 Introduction

3.1.1 Motivation

A common saying is that: "when the US sneezes, the rest of the world catches a cold."[1] Commonly used sayings sometimes represent distilled wisdom whilst at other times they represent mistaken inferences.

This research project investigates the broad question of whether the saying just cited represents distilled wisdom or mistaken inference. In *Section 4.1.3*, we refine this broad question down into a set of research questions that can be answered in a research paper.

Why is this question interesting? From the perspective of a policy maker, it is critical to understand the relationship between the UK and US economies. From the point view of an economist, it is also relevant and interesting to investigate whether models adequately capture the linkages in the business cycles between the UK and the US.

3.1.2 Project Topic

To address the broad questions outlined in *Section 4.1.1*, we study the business cycles in the UK and the US, with a focus on the topic of business cycle synchronization between these two countries. In particular, we attempt to identify and describe the main statistical characteristics of the UK and US business cycles during the past few decades, investigate whether expansionary (or recessionary) periods in one country are independent of the timing of expansions (or recessions) in the other country, measure the degree of synchronization between the UK and US economies, and consider among other things, the reasons for the connection between the business cycles in the UK and the US during the periods of time under study.

3.1.3 Research Questions

There are a few research questions which are addressed in this project. How to measure the synchronization between the business cycles in the UK and the US?

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[1]By a Google search, we are able to find more than 100 such quotes in the media over the past years.

How synchronized (or non-synchronized) are the UK and the US business cycles? What role does the correlation of GDP play in the synchronization of business cycles? How unusual is it that there was a classical cycle recession in the US in the Year 2001 but not one in the UK? Is this evidence of non-synchronization or is it just down to randomness?

3.1.4 Roadmap

In this paper, we attempt to answer our research questions by adopting the synchronization measures that are proposed in Harding and Pagan (2006a). Building from that, we extend their work by proceeding with the UK and US time-series data, with which we date their business cycles and find the turning points in their cycles. Under a GMM framework, we apply both single and joint synchronization tests, in order to answer the questions on the business cycle synchronization between the UK and the US.

The rest of this paper is structured as follows. In *Section 4.2*, the concepts of cycles are explored and comparisons among them are made with the various definitions that exist in the literature. In *Section 4.3*, we date the business cycles and find the turning points in the cycles. Next, we introduce in *Section 4.4* the moment conditions for the estimation of parameters which are related to the measurement of synchronization between the UK and the US business cycles. Then, we apply single and joint non-synchronization tests to the two economies in *Section 4.5*. As an extension, we also conduct correlation tests between the UK binary states and the US lagged or led binary states, and correlation tests between the growth rates of the UK GDP and the lagged or led growth rates of the US GDP. *Section 4.6* concludes.

3.2 Measures of Cycles

Cycles are defined in terms of turning points[2]. In the classical work on "*Measuring Business Cycles*" by Burns and Mitchell (1946), they define *specific cycles* in a series y_t in terms of *turning points* along its sample path. This tradition has been following and been central to the work at the National Bureau of Economic Research (NBER), and other institutions, such as the International Monetary

[2]See *Section 4.3* for a discussion on turning points.

Fund (IMF)[3] and the Organization for Economic Co-operation and Development (OECD).[4] These institutions measure the business cycle through locating the turning points in the series which are taken to represent the aggregate level of economic activity.

There are several different notions of a business cycle[5], among which the classical cycle and the growth cycle are the most common focus in the business cycle literature. The *classical cycle* refers to the recurrence of expansions and contractions in the absolute level of aggregate economic activity. On the other hand, the *growth cycle* reflects the fluctuations in the rate of economic growth, which takes the long-run trend rate of growth of the economy into account (Boehm and Moore, 1984). Harding and Pagan (2005) locate three traditions in this branch of literature, where the *classical cycle* is identified by the turning points in the level of the variable, the *growth cycle* is identified by the turning points in the level of the variable less a permanent component, and the *acceleration cycle* is identified by the turning points in the growth rate in the variable, which could be either a quarterly, or annual growth rate.

From a historical perspective, the *classical cycle* has been a major concern of the National Bureau of Economic Research (NBER), and its modified concept - the *growth cycle* has been the concentration of the International Economic Indicator (IEI) project at the NBER. Although the *growth cycle* has received much more justifiable attention since the 1960s, both of these cycle concepts are very useful for the development of business cycle theory among academic economists and policy-making advisors, which is due to the fact that the knowledge of these two cycles deepens our understanding of the essence of the recurring fluctuations in the economy. See Boehm (1982, 1983); Boehm and Moore (1984); and Moore and Zarnowitz (1984). On top of that, Harding and Pagan (2006a) point out that the choice of activity variable in the *acceleration cycle* means that we would be studying a cycle in the growth rates as distinct from the *growth cycle*.

In our synchronization measurement, we are interested in several measures of the business cycle, and we employ three types of cycles. They are classical cycle, growth cycle and acceleration cycle. Binary random variables are used to sum-

[3]See IMF(2002).
[4]See http://www.oecd.org/std/cli.
[5]See Harding and Pagan (2006b) for a description of more ways to measure the business cycle.

marize the expansion and contraction phases of business cycles. Each of them is defined in turn.

3.2.1 Classical Cycle

For the classical cycle, we use the turning points in the logarithm of the real GDP level, $\ln(GDP_t)$.

Let $S^C_{UK,t}$ be the binary variable that represents the classical cycle for the UK, and let $S^C_{US,t}$ be the binary variable that represents the classical cycle for the US.

$S^C_{UK,t}$ and $S^C_{US,t}$ take the value of one if the country is in a classical expansion at date t, and take the value of zero if it is in a classical recession at date t.

In the classical cycle, the peaks measure the dates from which economic activity suffers a sustained decline. On the other hand, the classical cycle troughs measure the dates from which economic activity ends its decline and begins a sustained increase.

3.2.2 Growth Cycle

For the growth cycle, we use the turning points in $\ln(GDP_t - a - b_t)$, where $(a + b_t)$ is the permanent component in the GDP data. (a is the time-invariant part, whilst b_t is the time-varying part in the trend component of the GDP data.)

Let $S^G_{UK,t}$ be the binary variable that represents the growth cycle for the UK, and let $S^G_{US,t}$ be the binary variable that represents the growth cycle for the US.

$S^G_{UK,t}$ and $S^G_{US,t}$ take the value of one if the country is in a growth expansion at date t, and take the value of zero if it is in a growth recession at date t.

The growth cycle peaks measure the points at which growth moves from being above trend rate to below trend rate, while the growth cycle troughs measure the points at which growth moves from below trend rate to above trend rate for a sustained period of time.

3.2.3 Acceleration Cycle

For the acceleration cycle, we use the turning points in $\ln(GDP_t) - \ln(GDP_{t-4})$, where 4 means 4 quarters, which is one year.

Let $S^A_{UK,t}$ be the binary variable that represents the acceleration cycle for the UK, and let $S^A_{US,t}$ be the binary variable that represents the acceleration cycle for the US.

$S^A_{UK,t}$ and $S^A_{US,t}$ take the value of one if the country is in an acceleration expansion at date t, and take the value of zero if it is in an acceleration recession at date t.

The peaks in the acceleration cycle measure the dates at which growth begins to slow down, whilst the troughs in the acceleration cycle measure the dates at which the rate of economic growth begins to increase.

3.3 Turning Points

The key idea behind business cycles is that a turning point occurs when one phase of expansion (or contraction) ends and another phase of contraction (or expansion) starts (Harding and Pagan, 2007). In order to detect the business cycle with a single series, we can therefore think of cycles in terms of turning points in the time series on the aggregate activity in the economy. However, this is a visual exercise, that is, it is done by eye and judgement. In some sense, this is like "eye econometrics". Examples include the charts by Burns and Mitchell (1946), Boehm and Moore (1984), and the business cycle dating committee at the National Bureau of Economic Research (NBER).

As modern economists, we want to go beyond this eye inspection and subjective judgement. To avoid this subjectivity, we have to check the time-series data with closer inspection. There are several reasons why a potential turning point might not be a real one. For instance, seasonal patterns[6], phase persistence[7], phase length[8], turning point alternation[9], etc. Hence, we need some considerable

[6] There are some examples of seasonal patterns, such as war and strikes.

[7] Phase persistence refers to the fact that a phase has to persist for some time, and if it lasts for only a short period of time, then some turning points will have to eliminated.

[8] Phase length follows two quarter rule, proposed by Harding and Pagan.

[9] A peak should be followed by a trough, and a trough should be followed by a peak, etc.

amount of scientific judgement and have to resort to computational methods on the final selection of the turning points.

3.3.1 Theoretical Development

From the perspective of theoretical development, Bry and Boschan (1971) first attempt to automate turning point selection. They developed a computer program which incorporated various smoothing and outlier detection rules, enforced judgements about the lengths of phase and cycle, and ensured that the turning points alternated. Their work involved the same kind of steps as in Burns and Mitchell (1946), and also reproduced Burns and Mitchell's decisions on turning points in a given set of time series. In their computer program, they adopted the rule that the phases must be at least 5 months in duration, whilst a complete cycle must last at least 15 months in length. But their procedure was implemented in FORTRAN on a digital computer, so numerical values had to be assigned specifically to test phase length.[10] In 1993, Mark Watson and Edwin Denson[11] developed their GAUSS program which implemented the Bry-Boschan (BB) business cycle dating algorithm.[12]

Building from that, Harding and Pagan (2002) develop the quarterly adaption of the Bry and Boschan (1971). Their dating algorithm is called BBQ, which stands for "Bry-Boschan Quarter". This computer program identifies the turning points in each of the time series via peak and trough dating. The BBQ program omits the smoothing rules in the BB algorithm but retains the key principles of the BB program. The modified BBQ (MBBQ) program is developed by James Engel.[13]

3.3.2 Analytical Framework

When we discuss business cycles, we have the tendency to present graphs of time series and recognize a cycle from the turning points in the time series under our study. These turning points are the peaks and troughs in the cycles, and the expansions and contractions are the periods between these cycle peaks and cycle troughs.

[10] For more information on BB's business cycle dating algorithm, see Chapter 2 of "Cyclical Analysis of Time Series: Selected Programs and Computer Program" in Bry and Boschan (1971).

[11] They did the work with the help of Robert King.

[12] An early version of this program was used in Watson (1994).

[13] James Engel's business cycle dating programs can be found on the website of National Centre for Econometric Research (NCER) in Australia. http://www.ncer.edu.au/data/

Harding and Pagan (2007) set some rules for locating the turning points in the cycle. When we refer to a turning point, it usually involves the location of local maxima and minima in the series, and this involves a change in the sign of the first derivative in discrete time, but is taken to be the first difference of the series under investigation. Hence, the rule involves studying quantities, such as Δy_t, where the series being studied is y_t. Same methods apply for other definitions of economic variables.

To illustrate this turning point rule, we can focus on a change in the variable y_t, Δy_t, and estimate the derivative by using an average of the y_t over some window around the potential turning point.

A peak in y_t at time t is being detected by examining whether the following sequence holds:

$$\{\Delta_2 y_t > 0, \Delta y_t > 0, \Delta y_{t+1} < 0, \Delta_2 y_{t+2} < 0\}$$

Likewise, a trough in y_t at time t is being detected by examining whether the following sequence holds:

$$\{\Delta_2 y_t < 0, \Delta y_t < 0, \Delta y_{t+1} > 0, \Delta_2 y_{t+2} > 0\}$$

This is the BBQ dating rule in our analytical framework for finding the turning points in the cycle.

3.3.3 Cycle Dating

We apply the BBQ algorithm to the time-series GDP data for the UK and the US, date the three types of their business cycles, and find the turning points in each type of the cycles for these two economies. Table 4.1 and Table 4.2 summarize the dating results of the turning points in the classical cycle, growth cycle and acceleration cycle for the UK and the US. From these two tables, we find that there are some features about the business cycles in these two economies. Among the three types of business cycles, the classical cycle has the fewest turning points, while the acceleration cycle has the most turning points for both economies. On the other hand, for all three business cycles, the first turning points start in the

Table 3.1. Business Cycle Turning Points - UK

Classical		Growth		Acceleration	
Trough	Peak	Trough	Peak	Trough	Peak
	1955q3		1955q3	1956q3	1957q2
1956q3	1961q2	1958q2	1960q1	1958q1	1960q1
1961q4	1973q2	1963q1	1964q4	1962q1	1964q1
1974q1	1974q3	1966q4	1968q1	1966q4	1968q1
1975q3	1979q2	1970q1	1970q4	1970q1	1970q4
1981q1	1990q2	1972q1	1973q1	1971q2	1973q1
1991q3	2008q1	1974q1	1974q3	1974q1	1975q1
2009q2		1975q3	1976q4	1975q3	1976q4
		1977q2	1979q2	1977q4	1979q2
		1981q2	1982q2	1980q2	1983q4
		1982q4	1984q1	1984q3	1985q2
		1984q3	1988q4	1986q2	1988q1
		1993q2	1994q4	1991q2	1994q3
		1995q2	2001q1	1999q2	2000q2
		2001q4	2004q1	2002q1	2004q1
		2005q1	2006q1	2005q1	2006q1
		2006q3	2007q4	2006q4	2007q3
		2010q1	2010q3	2009q1	2010q3

middle 1950s, and the latest turning point occurred during the Global Financial Crisis (GFC).

3.4 GMM Framework

In the previous section, we date the business cycles and find the turning points in the cycles for the UK and the US, which includes the turning points in the three types of business cycles, that is, classical cycle, growth cycle and acceleration cycle. As a by-product of the cycle dating, we also find the binary states of business cycles in these two economies.

In this section, we estimate the parameters which are related to the measures of synchronization between the UK and the US, by making use of the binary states from the cycle dating in the previous section. The notations and computer codes are based on and adapted from Hamilton (1994), and Harding and Pagan (2006a).

Table 3.2. Business Cycle Turning Points - US

Classical		Growth		Acceleration	
Trough	Peak	Trough	Peak	Trough	Peak
	1957q3		1955q3	1956q3	1957q3
1958q1	1960q1	1958q2	1960q1	1958q1	1959q2
1960q4	1969q3	1961q1	1966q1	1961q1	1962q1
1970q4	1973q4	1967q4	1969q1	1963q1	1964q1
1975q1	1980q1	1970q4	1973q2	1964q4	1965q4
1980q3	1981q3	1975q2	1976q1	1967q2	1968q2
1982q1	1990q3	1976q4	1977q3	1970q4	1973q1
1991q1	2007q4	1978q1	1978q4	1975q1	1976q2
2009q2		1980q3	1981q1	1977q1	1978q4
		1982q4	1986q1	1980q3	1981q3
		1987q1	1989q1	1982q3	1984q1
		1991q4	1992q4	1987q1	1988q2
		1993q3	1994q4	1991q1	1992q4
		1996q1	2000q2	1993q3	1994q3
		2003q1	2003q4	1995q4	1997q3
		2004q3	2005q1	1998q3	2000q2
		2009q3	2010q2	2001q4	2002q3
				2003q1	2004q1
				2007q1	2007q3
				2009q2	2010q3

In what follows, we use generalized methods of moments (GMM) as the analytical framework. There are three reasons for adopting the GMM framework. *First*, the main reason why we use GMM is that we want to seek a model which is free in measuring synchronization. *Second*, economic theory only provides us with information about the moments, but it does not provide information about the distribution from which the shocks are drawn. Unless one is willing to go beyond the information provided by economic theory, it is not possible to use maximum likelihood to estimate these models. *Third*, we also want to seek estimates that are robust to serial correlation and heteroscedasticity. That is why we adopt this methodology over a conventional bi-variate time-series analysis on GDP growth, as our concerns go beyond information on the size of the transmission of shocks and on any lead-lag relations.

The correlation between the two binary series is:

$$\rho^{j}_{UK,US} = Corr(S^{j}_{UK,t}, S^{j}_{US,t})$$

$$= \frac{Cov(S^j_{UK,t}, S^j_{US,t})}{\sqrt{Var(S^j_{UK,t})Var(S^j_{US,t})}}$$

The covariance[14] is

$$\begin{aligned} Cov(S^j_{UK,t}, S^j_{US,t}) &= E(S^j_{UK,t} - \mu^j_{UK})(S^j_{US,t} - \mu^j_{US}) \\ &= E(S^j_{UK,t}S^j_{US,t}) - \mu^j_{UK}\mu^j_{US} \end{aligned}$$

where j = classical cycle, growth cycle, acceleration cycle.

The variance[15] in each binary series is:

$$Var(S^j_{i,t}) = \mu^j_i(1 - \mu^j_i)$$

where i = UK, US.

3.4.1 Moment Conditions

We have the following moment conditions in the system.

$$ES^j_{i,t} - \mu^j_i = 0 \tag{3.1}$$

where i = UK, US; j = classical cycle, growth cycle, acceleration cycle.

$$E\Big[\frac{(S^j_{UK,t} - \mu^j_{UK})(S^j_{US,t} - \mu^j_{US})}{\sqrt{\mu^j_{UK}(1 - \mu^j_{UK})\mu^j_{US}(1 - \mu^j_{US})}} - \rho^j_{UK,US}\Big] = 0 \tag{3.2}$$

where j = classical cycle, growth cycle, acceleration cycle.[16]

Let $\theta' = (\mu^j_{UK}, \mu^j_{US}, \rho^j_{UK,US})$ be a vector of parameters for the population means (μ^j_{UK}, μ^j_{US}) in the UK and the US binary states, and the population correlation $\rho^j_{UK,US}$ between these binary states.

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[14] See the Mathematics/Statistics Appendix for the derivation of this covariance between the binary series.

[15] See the Mathematics/Statistics Appendix for the derivation of this variance in each binary series.

[16] Compare this equation with Harding and Pagan (2006a: p69), equation (26) to see the difference. Here we have two binary states for the UK and the US, respectively.

Let S be a $T \times 2$ matrix with typical elements S^j_{UK} and S^j_{US} in the two columns, respectively.

Then we can write the moment conditions as follows:

$$m_t(\theta, S_t) = \begin{pmatrix} S^j_{UK,t} - \mu^j_{UK} \\ S^j_{US,t} - \mu^j_{US} \\ (S^j_{UK,t}S^j_{US,t} - \mu^j_{UK}\mu^j_{US})/\sqrt{\mu^j_{UK}(1-\mu^j_{UK})\mu^j_{US}(1-\mu^j_{US})} - \rho^j_{UK,US} \end{pmatrix}'$$

where m_t is a $T \times 3$ matrix[17], and

$$g(\theta, \{S\}^T_{t=1}) = \frac{1}{T}\sum_{t=1}^{T} m_t(\theta, S_t) \tag{3.3}$$

In the m_t matrix, we have 3 moment conditions, and we also have 3 parameters in the system, $\theta' = (\mu^j_{UK}, \mu^j_{US}, \rho^j_{UK,US})$, so this is a just identified system.

3.4.2 Moment Estimation

Since this is a just identified model, we can use the method of moments. The estimators can be solved analytically via the following equations:

$$\mu^j_i = \frac{1}{T}\sum_{t=1}^{T} S^j_{i,t} \tag{3.4}$$

$$\rho^j_{UK,US} = \frac{\frac{1}{T}\sum_{t=1}^{T}(S^j_{UK,t}S^j_{US,t} - \mu^j_{UK}\mu^j_{US})}{\sqrt{\mu^j_{UK}(1-\mu^j_{UK})\mu^j_{US}(1-\mu^j_{US})}} \tag{3.5}$$

The next step is to obtain the S matrix.[18]

Let $\hat{\theta}' = (\hat{\mu}^j_{UK}, \hat{\mu}^j_{US}, \hat{\rho}^j_{UK,US})$ be the vector of parameters for the sample means in the UK and the US binary states and the sample correlation between these binary states.

[17] Compare this matrix with the one in Harding and Pagan (2006a: p69), equation (27) to see the difference. There they have a $T \times 1$ matrix.

[18] See the Mathematics/Statistics Appendix for a discussion on the consistent estimation of the S matrix.

The Newey-West (1987) estimate[19] of S is[20]:

$$\hat{S}_T = \hat{\Gamma}_{0,T} + \sum_{v=1}^{q}(1 - \frac{v}{q+1})(\hat{\Gamma}_{v,T} + \hat{\Gamma}'_{v,T}) \tag{3.6}$$

where q is the window width.

$$\hat{\Gamma}_{v,T} = \frac{1}{T}\sum_{t=v+1}^{T}[m_t(\hat{\theta}, S_t)][m_t(\hat{\theta}, S_{t-v})]' \tag{3.7}$$

where $\hat{\theta}$ is an initial consistent estimator of θ_0.

The window width q can be chosen either by using a "plug in rule" as in Harding and Pagan (2006a), or via an optimization procedure developed by Newey and West (1994). Following Harding and Pagan (2006a), we apply the "plug in rule" and set q equal to the integer part of $[T - n(n-1)/2]^{1/3}$, where T is the number of rows in the m_t matrix,[21] and n is the number of columns in the m_t matrix.[22] By this method, we get $q = 5$. On the other hand, we can also apply $[T - p]^{1/3}$, where p is the number of parameters.[23] By this optimization procedure, we also get $q = 5$.[24]

3.5 Synchronization Tests

In order to test for synchronization between the UK and the US economies, there are two possible tests. One test is to test for non-synchronization, while the other test is to test for perfect positive synchronization. We apply the non-synchronization test in this section.[25] By applying this test, we would like to investigate the research question: How synchronized are the UK and the US business cycles?

[19] A Newey-West estimator is used to provide an estimate of the covariance matrix of the parameters of a regression-type model when this model is applied in situations where the standard assumptions of regression analysis do not apply. The estimator is used to try to overcome autocorrelation, or correlation, and heteroskedasticity in the error terms in the models. This is often used to correct the effects of correlation in the error terms in regressions applied to time series data.

[20] See Hamilton (1994: p414), equations [14.1.19] and [14.1.20].

[21] T=191.

[22] n=3.

[23] p=3.

[24] $[T - n(n-1)/2]^{1/3} = [191 - 3(3-1)/2]^{1/3} = 5.7$.

[25] See the Mathematics/Statistics Appendix for the discussion on perfect synchronization tests.

3.5.1 Single Non-Synchronization Tests

It turns out that a good measure of the degree of business cycle synchronization[26] is provided by:

$$\rho^j_{UK,US} = Corr(S^j_{UK,t}, S^j_{US,t})$$

Analytical Framework

To test for non-synchronization, we have the null hypothesis and the alternative hypothesis as follows:

$$\begin{aligned} H_0 : \rho^j_{UK,US} &= 0 \\ H_1 : \rho^j_{UK,US} &\neq 0 \end{aligned}$$

where j = classical cycle, growth cycle, acceleration cycle.

Following *Section 4.4*, we let $\hat{\theta}' = (\mu^j_{UK}, \mu^j_{US}, 0)$ be the restricted parameter vector for the (strict) non-synchronization (SNS) case, where $\rho^j_{UK,US} = 0$.

Under the null hypothesis H_0, the test statistic is:

$$W_{SNS} = \sqrt{T}g(\theta_0, \{S\}^T_{t=1})'\hat{S}^{-1}_T\sqrt{T}g(\theta_0, \{S\}^T_{t=1}) \tag{3.8}$$

Under the alternative hypothesis H_1, the model is exactly identified, and hence $\bar{m}(\hat{\theta}) = 0$. The test statistic is a J test, the form of which is:

$$J = \bar{m}(\theta^j_0)S^{-1}_T\bar{m}(\theta^j_0)' \tag{3.9}$$

The J test is distributed χ^2_1 asymptotically.

For all three types of business cycles, the test statistic is for the null hypothesis of non-synchronization and has a χ^2 distribution at 1 degree of freedom, because there is only one restriction here.

The Results

In this part, we analyze the relationship between the UK and the US business cy-

[26]See, for example, Harding and Pagan (2006a).

Table 3.3. Single Non-Synchronization Tests - UK and US

1955-2011	Classical	Growth	Acceleration
μ_{UK}	0.881579	0.486842	0.442982
μ_{US}	0.868421	0.52193	0.482456
$\rho_{UK,US}$	0.339221	0.247092	0.322864
Test Statistic	7.530743	5.837898	10.38813
p-value	0.006065	0.015685	0.001268

cles. By inspection of the results from the synchronization tests, we characterize both the changing nature of the UK output growth and its evolving relationship with the US output growth since the late 1950s. Table 4.3 provides the results from the single non-synchronization tests, in each type of the business cycles for the entire sample period. For all three types of the cycles, the test statistics and the p-values are statistically significant. Hence, we reject the null, and conclude that the classical cycle, the growth cycle, and the accelerate cycle are all synchronized between the UK and the US economies. Overall, this table provides us with some comparative evidence on the synchronization between the UK cycles with those of the US. It suggests that the UK economy is highly synchronized with the US.

Extension: Leading Indicator Approach

In the previous part, we conduct synchronization tests in the contemporaneous binary states of the business cycles and find that all three types of business cycles are synchronized between the UK and the US for the entire sample period. In this part, we conduct some correlation tests in the leads and lags of the binary states for the US in this part, through which we want to investigate further the relationship between the UK and US business cycles. Previously, we apply the non-synchronization tests to the binary states[27] in the three types of business cycles for the UK and the US. In this extension, we fix the binary states for the UK, but lag and lead the binary states for the US by 1,2,3,4,5,6,7,8 periods[28], respectively. For example, if we lag the US binary states by 8 periods, then it means that the US business cycle leads the UK cycle by 2 years; while if we lead the US binary states by 8 periods, then it means that the US cycle lags that of the UK by 2 years.

[27] These binary states are the "by-products" from cycle dating in *Section 4.3.3*.
[28] 8 periods = 8 quarters = 2 years.

Table 3.4. Correlation between Binary States - UK and US

Periods	ρ^C	$pvalue^C$	ρ^G	$pvalue^G$	ρ^A	$pvalue^A$
-8	-0.13693	1.18×10^{-9}	0.137069	0.186253	-0.06502	0.509423
-7	-0.13622	1.09×10^{-9}	0.077245	0.471963	-0.13241	0.160761
-6	-0.05413	0.294544	0.018036	0.860676	-0.13671	0.085632
-5	0.066786	0.454561	0.022621	0.818801	-0.1229	0.125779
-4	0.143133	0.189027	0.062861	0.505698	-0.08252	0.370379
-3	0.257496	**0.024485**	0.138272	0.1558	0.028999	0.754461
-2	0.338387	0.011309	0.204326	**0.040585**	0.157418	0.058494
-1	0.338806	0.010186	0.244595	0.01910	0.267077	**0.00165**
0	0.339221	0.006065	0.247092	0.015685	0.322864	0.001268
1	0.298624	0.008026	0.217713	0.022217	0.284815	0.002558
2	0.217763	0.049286	0.170368	0.054449	0.139612	0.072155
3	0.177028	0.091663	0.176079	0.055485	0.011123	0.89242
4	0.136236	0.187614	0.136515	0.159638	-0.10047	0.283016
5	0.0551	0.524345	0.060574	0.556906	-0.17696	0.03996
6	0.054477	0.526891	-0.00732	0.940729	-0.19162	0.028637
7	0.013507	0.874798	-0.05761	0.526818	-0.18821	0.031655
8	0.012844	0.883393	-0.09906	0.276333	-0.12121	0.222114

The correlation between the binary states in GDP is:

$$\rho^{S,j}_{UK,US} = Corr(S^j_{UK,t}, S^j_{US,t+l})$$

where $l = -8, -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8$.

Then we proceed to find the correlation between the binary states for the UK and the lagged or led binary states for the US, in each type of business cycles. This gives us a good picture of the correlation between the business cycles in these two economies. Table 4.4 presents the correlation between the binary states for each type of business cycles in the UK and the US. From these results in the table, we can see that the US classical cycle leads the UK cycle by 3 periods; the US growth cycle leads that of the UK by 2 periods; and the US acceleration cycle leads the UK by 1 period. Hence, the US leads the UK for all three types of business cycles.

3.5.2 Joint Non-Synchronization Tests

Single non-synchronization tests are conducted in *Section 4.5.1* to test the hypothesis that for each type of the business cycles, none of them are synchronized.

An extension is also discussed in that part. In this subsection, we go on to conduct our joint non-synchronization tests which test the null hypothesis that all three types of the business cycles are jointly non-synchronized.

Analytical Framework

To test for joint non-synchronization in all three types of business cycles, we have the null hypothesis and the alternative hypothesis as follows:

$$H_0 : \rho^C_{UK,US} = \rho^G_{UK,US} = \rho^A_{UK,US} = 0$$

The alternative hypothesis is that at least one type of the business cycle is synchronized.

In this joint non-synchronization test, the moment conditions become:

$$m_t(\theta, S_t) = \begin{pmatrix} S^C_{UK,t} - \mu^C_{UK} \\ S^C_{US,t} - \mu^C_{US} \\ S^G_{UK,t} - \mu^G_{UK} \\ S^G_{US,t} - \mu^G_{US} \\ S^A_{UK,t} - \mu^A_{UK} \\ S^A_{US,t} - \mu^A_{US} \\ (S^C_{UK,t}S^C_{US,t} - \mu^C_{UK}\mu^C_{US})/\sqrt{\mu^C_{UK}(1-\mu^C_{UK})\mu^C_{US}(1-\mu^C_{US})} - \rho^C_{UK,US} \\ (S^G_{UK,t}S^G_{US,t} - \mu^G_{UK}\mu^G_{US})/\sqrt{\mu^G_{UK}(1-\mu^G_{UK})\mu^G_{US}(1-\mu^G_{US})} - \rho^G_{UK,US} \\ (S^A_{UK,t}S^A_{US,t} - \mu^A_{UK}\mu^A_{US})/\sqrt{\mu^A_{UK}(1-\mu^A_{UK})\mu^A_{US}(1-\mu^A_{US})} - \rho^A_{UK,US} \end{pmatrix}'$$

where $\theta' = (\mu^C_{UK}, \mu^C_{US}, \mu^G_{UK}, \mu^G_{US}, \mu^A_{UK}, \mu^A_{US}, \rho^C_{UK,US}, \rho^G_{UK,US}, \rho^A_{UK,US})$, and $S = (S^C_{UK} \sim S^C_{US} \sim S^G_{UK} \sim S^G_{US} \sim S^A_{UK} \sim S^A_{US})$.

The Results

From the results for the joint non-synchronization tests, the test statistic = 14.54598, and the p-value = 0.002249. So we reject the null, and conclude that at least one type of the business cycles is synchronized between the two economies. This result is consistent with our finding in the single non-synchronization tests.

3.6 Conclusion

In this paper, we focus on the real GDP as the time-series data in which the business cycles are to be located; and we employ three types of business cycles, which include classical cycle, growth cycle and acceleration cycle. Then we date these three business cycles and find the turning points in each type of cycles. Building from that, we use the GMM framework and estimate the parameters related to the measures of synchronization between the UK and the US for each type of business cycles. Under the GMM framework, we apply tests for single cycle synchronization, and find that the business cycles are synchronized between the UK and the US for the entire sample period. As an extension, we conduct tests in testing the correlation between the binary states in the UK and those lagged and led binary states in the US, for each type of business cycles. From these correlation tests between the binary states, we find that the US leads the UK for all three types of business cycles. In parallel to the single synchronization tests, we also conduct joint non-synchronization tests in the last part of this paper, where we find that there is evidence of joint synchronization, consistent with those found in the single non-synchronization tests.

Before closing this paper, let us return to the research questions that are asked in *Section 4.1.2*.

First of all, how to measure the synchronization between the business cycles in the UK and the US? One way to answer this, as is done in this paper, is to examine the real GDP data in these two economies, to employ three types of business cycles, and to test whether the business cycles are singly or jointly synchronized. There still remain some problems. For example, it usually requires many more years of data to determine with any degree of confidence whether the two business cycles are indeed synchronized over the different courses of their economic growth and development in history.

Second, how synchronized (or non-synchronized) are the UK and the US business cycles? The single synchronization tests show that the two economies are highly synchronized in their business cycles. The joint synchronization test confirms our finding from the single synchronization tests.

Third, how unusual is it that there was a classical cycle recession in the US in 2001 but not one in the UK? Is this evidence of non-synchronization or is it just down to randomness? The classical cycle recession in the US in 2001 was identified by the National Bureau of Economic Research (NBER), which chooses their turning points with reference to not only the GDP data, but also other economic fundamental variables, with a special focus on the unemployment rate. In the measurement here, we only focus on the real GDP data. So we get slightly different results from theirs. This difference is due to the choice of variables and methodology.

Appendix

A1. Mathematical derivation

Binary Covariance Derivation

The derivation of the covariance between the two binary series is:

$$\begin{aligned} Cov(S^j_{UK,t}, S^j_{US,t}) &= E[(S^j_{UK,t} - \mu^j_{UK})(S^j_{US,t} - \mu^j_{US})] \\ &= E(S^j_{UK,t}S^j_{US,t} - S^j_{UK,t}\mu^j_{US} - \mu^j_{UK}S^j_{US,t} + \mu^j_{UK}\mu^j_{US}) \\ &= E(S^j_{UK,t}S^j_{US,t}) - E(S^j_{UK,t})\mu^j_{US} - \mu^j_{UK}E(S^j_{US,t}) + \mu^j_{UK}\mu^j_{US} \\ &= E(S^j_{UK,t}S^j_{US,t}) - \mu^j_{UK}\mu^j_{US} - \mu^j_{UK}\mu^j_{US} + \mu^j_{UK}\mu^j_{US} \\ &= E(S^j_{UK,t}S^j_{US,t}) - \mu^j_{UK}\mu^j_{US} \end{aligned}$$

where j = classical cycle, growth cycle, acceleration cycle.

Binary Variance Derivation

The derivation of the variance in each binary series is:

$$\begin{aligned} Var(S^j_{i,t}) &= E(S^j_{i,t} - \mu^j_i)^2 \\ &= ES^j_{i,t} - (\mu^j_i)^2 \\ &= \mu^j_i - (\mu^j_i)^2 \\ &= \mu^j_i(1 - \mu^j_i) \end{aligned}$$

where i = UK, US.

Covariance-Variance Matrix

The covariance-variance matrix without adjustment for heteroskedasticity and autocorrelation is:

$$\begin{aligned}\Gamma_0 &= \frac{1}{T} m_t' m_t \\ \Gamma_v &= \frac{1}{T} \overline{m_v}' \underline{m_v}\end{aligned}$$

where $v = 1, 2, ...,$ and is just an index that runs from 1 to window width; and the overline means that we trim v rows from the top of the matrix m_t, while the underline means the we trim v rows from the bottom of the matrix m_t. This is the right way to trim the vectors, such that there are no missing values.

Consistent Estimation

In *Section 4.4.2*, we cover the moment conditions, where we show the covariance-variance matrix without and with adjustment for heteroskedasticity and autocorrelation. We introduce this statistical background more informally in this appendix.

The asymptotic variance of the sample mean $m(\bar{\theta}_0)$ is defined as

$$S = \lim_{T \to \infty} T \times E[m(\bar{\theta}_0)' m(\bar{\theta}_0)]$$

If $\{m_t(\theta_0)\}_{t=-\infty}^{\infty}$ is **serially uncorrelated**, then S can be consistently estimated by

$$S_T^* = \frac{1}{T} \sum_{t=1}^{T} m_t(\theta_0)' m_t(\theta_0)$$

This estimator is infeasible since θ_0 is unknown, and hence it is usual to use the feasible estimator

$$\hat{S}_T = \frac{1}{T} \sum_{t=1}^{T} m_t(\hat{\theta})' m_t(\hat{\theta})$$

where $\hat{\theta}$ is a feasible estimator of θ.

If $\{m_t(\theta_0)\}_{t=-\infty}^{\infty}$ is **serially correlated**, then some allowance must be made for that feature. These are several methods for doing this. A popular one due to Newey and West involves using the following Bartlett kernel with window width

q.

$$\tilde{S}_T = \Gamma_{0,T} + \sum_{v=1}^{q}(1 - \frac{v}{q+1})(\Gamma_{v,T} + \Gamma'_{v,T})$$

where

$$\Gamma_{v,T} = \frac{1}{T}\sum_{t=1}^{T} m_t(\hat{\theta})' m_{t-v}(\hat{\theta})$$

and the Bartlett kernel is $1 - \frac{v}{q+1}$.

Perfect Positive Synchronization Test

Following *Section 4.5*, it also turns out that for binary series, a necessary condition for perfect (positive) synchronization between the UK and the US is:

$$ES^j_{UK,t} = \mu^j_{UK} = \mu^j_{US} = ES^j_{US,t}$$

To test for perfect positive synchronization, we have the null hypothesis and the alternative hypothesis as follows:

$$\begin{aligned} H_0 &: \mu^j_{UK} = \mu^j_{US} \quad , \quad \rho^j_{UK,US} = 1 \\ H_1 &: \mu^j_{UK} \neq \mu^j_{US} \quad , \quad \rho^j_{UK,US} \neq 1 \end{aligned}$$

For this perfect positive synchronization test, we are testing at the boundary of the parameter space. It is not differentiable. We can not apply Taylor series approximation. It is a half normal distribution. We are required to make adjustment to the distribution of the test statistics. We need to use distribution theory that takes this feature of the hypothesis into account.

But given our results in the non-synchronization tests, we already know that the growth cycle is non-synchronized between the two economies, while the classical and acceleration cycles are synchronized, the evidence of which is not strong though. Hence, there is no need to proceed further to test for perfect positive synchronization.

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Chapter 4

Oil Supply Shocks in a Non-Scale Economic Growth Model

Abstract
This project extends Schubert and Turnovsky (2007) paper by introducing endogenous labor supply into their model economy. Building from that, we investigate the dynamic effects of a one-time unanticipated permanent change in the price of imported intermediate good on economic growth. The model considered here is a "non-scale" economic growth model, in which the economy has access to a perfect capital market. With the extension of endogenous labor supply, we find that output, capital, and input usage are all permanently much lower than the case for fixed labor supply. The short-run impacts on input usage and output are also larger than those in the fixed labor supply case.

JEL Classification: O41.
Key Words: Economic Growth, Non-Scale Model, Oil Shock.

4.1 Chapter Introduction

Schubert and Turnovsky (2007) build upon the class of model that is developed by Eicher and Turnovsky (1999) and study the impacts of an increase in the price of an intermediate input on economic growth of a small open economy. In that paper, Schubert and Turnovsky assume that the economy is populated with identical agents and that each individual's labor supply is fixed at unity.

In this project, we relax that assumption on fixed labor supply as in Schubert and Turnovsky (2007) paper, by introducing endogenous labor supply into their model economy, in order to see how their analytical framework evolves and what implications this endogenous labor supply has on the dynamic effects of a one-time unanticipated permanent increase in the price of the imported intermediate good.

The remainder of this paper proceeds as follows. Section 2 introduces the model setup with endogenous labor supply. Section 3 derives the macroeconomic equilibrium for this model economy. In section 4, we analyze the effect due to a change in the price of the intermediate imported input. Section 5 concludes.

4.2 Analytical Framework

Eicher and Turnovsky (1999) develop a one-sector "non-scale" open economy growth model.[1] Building from that, Schubert and Turnovsky (2007) include a foreign import, which is used as an intermediate input in domestic production. This economy is small, in the sense that it has no influence on its output price and input price in the world market. The domestic production includes one traded good, Y, which can be consumed or invested or exported. There is an imported intermediate good, which is used solely as an input in domestic production. Its relative price, expressed in terms of the traded final good, is p. It is assumed that this relative price of the imported good p remains constant over time.

The departure from Schubert and Turnovsky (2007) model is that we introduce endogenous labor supply and leisure choice, by relaxing the assumption on fixed labor supply in this analytical framework.

In our model, a representative agent obtains utility from his/her consumption and leisure choice, which is represented by an inter-temporal iso-elastic utility function over an infinite time horizon.

$$U_i = \int_0^\infty \frac{1}{\gamma} \left(C_i l^\theta\right)^\gamma e^{-\beta t} dt \tag{4.1}$$

where $-\infty < \gamma < 1$, $\theta > 0$.

[1] The "non-scale" growth model is a generalization of the neo-classical growth model to allow for non-constant returns to scale.

C_i is each individual's consumption choice. Each agent is endowed with a unit of time that is divided between leisure l and labor $(1-l)$.[2] θ measures the substitutability between consumption and leisure in utility by each agent. $\frac{1}{1-\gamma}$ measures the inter-temporal elasticity of substitution. β denotes the agent's rate of time preference, which is assumed to be constant over time.

The production function is:

$$Y_i \;=\; \alpha L_i^{1-\sigma-\xi} K_i^{\sigma} Z_i^{\xi} K^{\eta} = \alpha(1-l)^{1-\sigma-\xi} K_i^{\sigma} Z_i^{\xi} K^{\eta}$$

where $0 < \sigma < 1$, $0 < \xi < 1$, $0 < \sigma + \xi < 1$, $\eta < or = or > 0$.

α is like a proxy for total factor productivity (TFP) in the domestic production of the traded good Y_i. There are three private factors in producing the traded good Y_i. One factor is each individual's labor supply $L_i (= 1 - l)$. The second factor is private capital K_i. The third factor is imported intermediate good Z_i. On top of that, K is the economy-wide capital stock, with $K \equiv NK_i$.

In addition, η captures the spill-over effect from the economy-wide capital stock (for example, due to its production network), as in Romer (1986). We also assume that there are *constant returns to scale* in the three private factors - L_i, K_i, Z_i; but *total returns to scale* of degree $(1+\eta)$ in all factors. Depending on the qualitative nature of the spill-over effect, *aggregate returns to scale* can be increasing, constant, or decreasing.

The aggregate production function is obtained by summing individual production functions over the N agents.

$$Y \;=\; \alpha(1-l)^{\sigma_L} N^{\sigma_N} K^{\sigma_K} Z^{\sigma_Z} \tag{4.2}$$

where $\sigma_L = \sigma_N = 1-\sigma-\xi$, $\sigma_K = \sigma+\eta$, and $\sigma_Z = \xi$ are the shares of labor, capital and imported input in aggregate output, respectively. Thus, $\sigma_L + \sigma_K + \sigma_Z = 1+\eta$ measures total returns to scale of the aggregate production function.

A representative agent accumulates physical capital K_i. The investment in the accumulation of physical capital is associated with installation costs. We adopt a

[2] See Turnovsky (2009): pp13.

quadratic (convex) investment adjust cost function as in Hayashi (1982). Aggregating over the N individuals, this leads to:

$$\Phi(I,K) \quad = \quad I + h\frac{I^2}{2K} = I\left(1 + \frac{h}{2}\frac{I}{K}\right) \tag{4.3}$$

where the adjustment costs are proportional to the *rate* (rather than its level) of investment per unit of installed capital, $\frac{I}{K}$. The linear homogeneity of this function is necessary if a steady-state equilibrium showing ongoing growth is to be sustained.

For simplicity, it is further assumed that the capital stock does not depreciate, such that the net rate of capital accumulation per agent, taking population growth into account, is given by:

$$\dot{K}_i \quad = \quad I_i - nK_i$$

Here $n = \frac{\dot{N}}{N}$ is the growth rate of population. N is the population size, which equals total labor supply implied by full employment condition.

At the aggregate level, the economy faces the physical capital accumulation constraint:

$$\dot{K} \quad = \quad I \tag{4.4}$$

Domestic agents have access to a perfect world capital market, which allows them to accumulate bonds from the rest of the world. These world bonds are denominated in terms of the traded good and pay a fixed interest rate r in the world market, yielding a net rate of return of $(r-n)$ to individual agents. A representative agent's budget constraint in the short run, expressed in terms of the traded good, is:

$$\dot{B}_i \quad = \quad Y_i - C_i - pZ_i - \Phi(I_i, K_i) + (r-n)B_i$$

To the extent that the representative agent's income from production Y_i plus net interest income $(r-n)B_i$ exceed his consumption C_i, raw material costs pZ_i, investment costs $\Phi(I_i, K_i)$, he will accumulate his holdings of foreign (traded) bonds, that is, $B_i > 0$. Otherwise, $B_i < 0$, in which case the agent is a debtor.

The aggregate economy accumulates net foreign bonds, B, that pay an exogenously given world interest rate, r, subject to the accumulation equation:

$$\dot{B} = Y - C - pZ - I\left(1 + \frac{h}{2}\frac{I}{K}\right) + rB \tag{4.5}$$

For illustrational purpose, taxes are abstracted in this analytical framework.

Dynamic Optimization

We consider the general equilibrium that is generated in a centrally planned economy, in which the social planner chooses C, l, Z and I, together with K and B, so as to maximize the utility level of a representative agent as represented by equation (4.1), subject to the aggregate production function (4.2), the aggregate capital accumulation equation (4.3), and the aggregate resource constraint (4.5).

$$\begin{aligned} H &= \int_0^\infty \frac{1}{\gamma}\left(\frac{C}{N}l^\theta\right)^\gamma e^{-\beta t}dt - q^*\left(\dot{K} - I\right)e^{-\beta t} \\ &\quad -\lambda\left[\dot{B} - Y + C + pZ + I\left(1 + \frac{h}{2}\frac{I}{K}\right) - rB\right]e^{-\beta t} \end{aligned}$$

where

$$Y = \alpha(1-l)^{\sigma_L}N^{\sigma_N}K^{\sigma_K}Z^{\sigma_Z}$$

Optimality conditions

The first-order optimality condition with respect to consumption, C is:

$$N^{-\gamma}C^{\gamma-1}l^{\theta\gamma} = \lambda \tag{4.6}$$

The first-order optimality condition with respect to leisure, l is:

$$N^{-\gamma}\theta C^\gamma l^{\theta\gamma-1} = \lambda\sigma_L\frac{Y}{1-l} \tag{4.7}$$

The first two optimality conditions give us the consumption-output ratio:

$$\frac{C}{Y} = \frac{l}{1-l}\frac{\sigma_L}{\theta} \tag{4.8}$$

The first-order optimality condition with respect to the imported input, Z, is:

$$Z = \sigma_Z \frac{Y}{p} \tag{4.9}$$

The first-order optimality condition with respect to investment, I, is:

$$1 + h\frac{I}{K} = q \tag{4.10}$$

In these first-order conditions, λ is the shadow value (marginal utility) of wealth in the form of international traded bonds, q^* is the shadow value of capital, and $q = \frac{q^*}{\lambda}$ is the value of capital in terms of the (unitary) price of foreign bonds.

Equation (4.6) is the usual static optimality condition on consumption, which requires that along an optimal path, the marginal utility of consumption has to equal the shadow value of wealth. Equation (4.7) determines the optimal leisure choice. Equation (4.9) is the determination of optimally choosing the level of the imported input Z_i, which is equal to the marginal product of the imported input to its relative price in terms of the domestic good p. The production function belongs to the Cobb-Douglas type, which implies that the demand for the imported input is directly proportional to output and inversely proportional to its relative price. Equation (4.10) asserts that the marginal cost of investment is equated to the marginal value of installed capital q. This gives us a Tobin's Q theory of investment.

Euler equations

Next, the Euler condition with respect to foreign bond, B, is:

$$\beta - \frac{\dot{\lambda}}{\lambda} = r \tag{4.11}$$

The Euler condition with respect to capital, K, is:

$$\frac{\sigma_K}{q}\frac{Y}{K} + \frac{\dot{q}}{q} + \frac{h}{2q}\left(\frac{q-1}{h}\right)^2 = r \tag{4.12}$$

The Euler conditions are the dynamic efficiency conditions. Equation (4.11) is the dynamic efficiency condition on foreign bonds, which requires that the rate of return on consumption, denoted in terms of the traded good, is equal to the net growth interest rate. This implies a constant growth rate of the agent's marginal utility, with constant rate of time preference β, constant world interest rate r and constant population growth rate n. Equation (4.12) is the dynamic efficiency condition on capital stocks, which equates the rate of return on domestic capital to the rate of return on foreign (traded) bonds. The rate of return on domestic capital comprises three components. The first component is the marginal output per unit of installed capital, which is valued at the (shadow) price q. The second component is the capital gain, and the third component reflects the benefits arising from the fact that higher capital stock reduces installation costs.

Transversality conditions

Finally, the transversality conditions are:

$$\begin{aligned} lim_{t\to\infty}\lambda_i B_i e^{-\beta t} &= 0 \\ lim_{t\to\infty}\lambda_i q_i K_i e^{-\beta t} &= 0 \end{aligned}$$

These two transversality conditions are imposed in order to ensure that the agent's inter-temporal budget constraint is met.

4.3 Macroeconomic Equilibrium

4.3.1 Balanced Growth

Our objective is to analyze the dynamics of the economy in response to an oil shock along a stationary growth path, and hence we must determine the long-run growth rate along a balanced growth path for this economy. At the steady states, $\dot{q} = 0$. Equation (4.12) implies that along a balanced growth path, $\frac{Y_i}{K_i}$ remains constant. $\frac{Y_i}{Z_i}$ also remains constant at all points of time, which is implied by the optimality condition for the imported input - equation (4.9). So along the balanced growth path, Y_i, K_i and Z_i all grow at a common constant rate. With identical agents, the aggregate quantities are defined by $Y \equiv NY_i$, $K \equiv NK_i$, $Z \equiv NZ_i$, from which it follows that Y, K and Z also grow at a common rate, that is, $\hat{Y} = \hat{K} = \hat{Z}$.[3]

Taking percentage change of the aggregate production function and imposing the long-run equilibrium relationship in the growth rate $\hat{Y} = \hat{K} = \hat{Z}$ gives us the

[3]The "hat" denotes the growth rate in the variable.

steady-state growth rate:

$$\hat{Y} \equiv g = \frac{\sigma_N}{1-\sigma_K-\sigma_Z} n = \frac{1-\sigma-\xi}{1-\sigma-\eta-\xi} n \tag{4.13}$$

Therefore, the equilibrium growth rate depends on the production technology which is summarized by the shares σ_j in the aggregate production function. It also depends on the population growth rate n which is assumed to be exogenous in this model.

It is assumed that $\sigma_K + \sigma_Z < 1$, in order to have a positive balanced growth. Under constant aggregate returns to scale, the equilibrium growth rate equals the population growth rate $g = n$. In the presence of spill-over effect (from the economy-wide capital stock) as we have in this model, g is bigger or smaller than n, which depends on the qualitative form of this spill-over effect, $\eta > or < 0$, that is, whether we have increasing or decreasing aggregate returns to scale. This in turn also decides whether the introduction of an imported raw material aids or impedes the growth rate in the long run.

4.3.2 Consumption Dynamics

Agent's growth rate of consumption is determined by taking the time derivative of the optimality condition on consumption combined with the dynamic efficiency condition on foreign bonds.

$$\frac{\dot{C}}{C} = \frac{1}{1-\gamma}\left(r-\beta-\gamma n+\theta\gamma\frac{\dot{l}}{l}\right) \tag{4.14}$$

Knife-Edge Condition

When the assumption of inelastic labor supply is relaxed and labor is endogenously supplied, a much stronger knife-edge condition is required. (See Turnovsky (2002).) This is because the optimality condition on the consumption-output ratio $\frac{C}{Y} = \frac{l}{1-l}\frac{\sigma_L}{\theta}$ must now be taken into account. The fraction of time that is allocated to work is constant in steady state. This relationship thus implies that the steady-state consumption/output ratio must be constant, and as a result it imposes the equality of the long-run growth rates of consumption and output.

At the steady, $\dot{l} = 0$, so the consumption dynamics equation (4.14) becomes:

$$\frac{\dot{C}}{C} = \frac{r - \beta - \gamma n}{1 - \gamma} \equiv \psi \tag{4.15}$$

Thus, equating (4.13) and (4.14), we must have:

$$\psi \equiv \frac{r - \beta - \gamma n}{1 - \gamma} = \frac{\sigma_N}{1 - \sigma_K - \sigma_Z} n \equiv g \tag{4.16}$$

The condition (4.16) is the knife-edge condition for this growth model. That is, the return on foreign bonds, given the taste parameters and population growth, must be such that the implied growth rate of consumption is driven to equal the growth rate of production, which is determined by the population growth rate in conjunction with the productive elasticities, in accordance with the "non-scale" growth model.

4.3.3 Capital Dynamics

The dynamics of capital converge to a long-run steady state growth rate along a transitional growth path. In order to analyze this path, we first follow Eicher and Turnovsky (1999) to define the stationary "scale-adjusted" capital stock:

$$k \equiv \frac{K}{N^{\frac{\sigma_N}{1-\sigma_K-\sigma_Z}}}$$

In equilibrium, the growth rate of the aggregate capital stock is:

$$\frac{\dot{K}}{K} = \frac{q - 1}{h} \tag{4.17}$$

Then the growth rate of the stationary "scale-adjusted" capital stock is:

$$\frac{\dot{k}}{k} = \frac{q - 1}{h} - \frac{\sigma_N}{1 - \sigma_K - \sigma_Z} n \tag{4.18}$$

4.3.4 Dynamic System

The aggregate macroeconomic equilibrium is derived from the optimality conditions for individual agents. We begin by taking the time derivatives of: (i) the optimality condition for consumption (4.6), (ii) the equilibrium consumption-output ratio (4.8), and (iii) the production function (4.2). This leads to the relationships:

$$-\gamma n + (\gamma - 1)\frac{\dot{C}}{C} + \theta\gamma\frac{\dot{l}}{l} = \beta - r \tag{4.19}$$

$$\frac{\dot{C}}{C} - \frac{\dot{Y}}{Y} = \frac{\dot{l}}{l} + \frac{\dot{l}}{1-l} \tag{4.20}$$

$$\frac{\dot{Y}}{Y} = -\frac{\sigma_L}{1-\sigma_Z}\frac{\dot{l}}{1-l} + \frac{\sigma_N}{1-\sigma_Z}n + \frac{\sigma_K}{1-\sigma_Z}\frac{\dot{K}}{K} \tag{4.21}$$

The macroeconomic equilibrium can be expressed by the pair of differential equations in q and l:

$$\dot{q} = rq - \frac{(q-1)^2}{2h} - \sigma_K\frac{Y}{K} \tag{4.22}$$

$$\dot{l} = \frac{1}{F(l)}\left[r - \beta - \gamma n - \frac{1-\gamma}{1-\sigma_Z}\left(\sigma_K\frac{q-1}{h} - \sigma_N n\right)\right] \tag{4.23}$$

where

$$F(l) = \frac{1-\gamma(\theta+1)}{l} + \frac{1-\gamma}{1-l}\left(1 - \frac{\sigma_L}{1-\sigma_Z}\right) > 0$$

The dynamic system is (4.22) + (4.23) + (4.18):

$$\begin{aligned}
\dot{q} &= rq - \frac{(q-1)^2}{2h} - \sigma_K\alpha^{\frac{1}{1-\sigma_Z}}(1-l)^{\frac{\sigma_Z}{1-\sigma_Z}}(p^{-1}\sigma_Z)^{\frac{\sigma_Z}{1-\sigma_Z}}k^{\frac{\sigma_K+\sigma_Z-1}{1-\sigma_Z}} \\
\dot{l} &= \frac{1-\gamma}{F(l)}\left[\frac{r-\beta-\gamma n}{1-\gamma} - \frac{1}{1-\sigma_Z}\left(\sigma_K\frac{q-1}{h} - \sigma_N n\right)\right] \\
\dot{k} &= \left(\frac{q-1}{h} - \frac{\sigma_N}{1-\sigma_K-\sigma_Z}n\right)k
\end{aligned}$$

where q and l are jump variables, and k is a sluggish variable.

Here, we apply the general knife-edge condition (4.16). The macro dynamic equilibrium equations become:

$$\begin{aligned}
\dot{q} &= rq - \frac{(q-1)^2}{2h} - \sigma_K\alpha^{\frac{1}{1-\sigma_Z}}(1-l)^{\frac{\sigma_Z}{1-\sigma_Z}}(p^{-1}\sigma_Z)^{\frac{\sigma_Z}{1-\sigma_Z}}k^{\frac{\sigma_K+\sigma_Z-1}{1-\sigma_Z}} \\
\dot{l} &= \frac{1-\gamma}{F(l)}\left[g - \frac{1}{1-\sigma_Z}\left(\sigma_K\frac{q-1}{h} - \sigma_N n\right)\right] \\
\dot{k} &= \left(\frac{q-1}{h} - g\right)k
\end{aligned}$$

In order for the domestic capital stock, labor supply and output to converge to a balanced growth path with a constant rate of growth, the stationary solution to the dynamic system above must have at least one real solution, when $\dot{q} = \dot{k} = \dot{l} = 0$.

$$\begin{aligned}
\dot{q} &= r\tilde{q} - \frac{(\tilde{q}-1)^2}{2h} - \sigma_K \alpha^{\frac{1}{1-\sigma_Z}} (1-\tilde{l})^{\frac{\sigma_Z}{1-\sigma_Z}} (p^{-1}\sigma_Z)^{\frac{\sigma_Z}{1-\sigma_Z}} \tilde{k}^{\frac{\sigma_K+\sigma_Z-1}{1-\sigma_Z}} = 0 \\
\dot{l} &= \frac{1-\gamma}{F(\tilde{l})} \left[g - \frac{1}{1-\sigma_Z} \left(\sigma_K \frac{\tilde{q}-1}{h} - \sigma_N n \right) \right] = 0 \\
\dot{k} &= \left(\frac{\tilde{q}-1}{h} - g \right) \tilde{k} = 0
\end{aligned}$$

From the $\dot{k} = 0$ equation, we get:

$$\tilde{q} = 1 + gh \tag{4.24}$$

so $\tilde{q}$ is determined by the growth rate g.

Substitute this back into the $\dot{q} = 0$ and $\dot{l} = 0$ equations, we find that k and l are jointly determined. So we can take $\tilde{l}$ as given, and solve for $\tilde{k}$.

$$\tilde{k} = \left[r(1+gh) - \frac{g^2 h}{2} \right]^{\frac{1-\sigma_Z}{\sigma_K+\sigma_Z-1}} \sigma_K^{-\frac{1-\sigma_Z}{\sigma_K+\sigma_Z-1}} \alpha^{-\frac{1}{\sigma_K+\sigma_Z-1}} (1-\tilde{l})^{-\frac{\sigma_Z}{\sigma_K+\sigma_Z-1}} (p^{-1}\sigma_Z)^{-\frac{\sigma_Z}{\sigma_K+\sigma_Z-1}} \tag{4.25}$$

Linearize $\dot{q}$, $\dot{k}$ and $\dot{l}$ around their steady states,

$$\begin{pmatrix} \dot{q} \\ \dot{k} \\ \dot{l} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} q - \tilde{q} \\ k - \tilde{k} \\ l - \tilde{l} \end{pmatrix}$$

where:

$$\begin{aligned}
a_{11} &= r - g \\
a_{12} &= -\frac{\sigma_K(\sigma_K+\sigma_Z-1)}{1-\sigma_Z} \alpha^{\frac{1}{1-\sigma_Z}} (1-\tilde{l})^{\frac{\sigma_Z}{1-\sigma_Z}} (p^{-1}\sigma_Z)^{\frac{\sigma_Z}{1-\sigma_Z}} \tilde{k}^{\frac{\sigma_K}{1-\sigma_Z}-2} \\
a_{13} &= \frac{\sigma_K \sigma_Z \sigma_L}{1-\sigma_Z} \alpha^{\frac{1}{1-\sigma_Z}} (1-\tilde{l})^{\frac{2\sigma_Z-1}{1-\sigma_Z}} (p^{-1}\sigma_Z)^{\frac{\sigma_Z}{1-\sigma_Z}} \tilde{k}^{\frac{\sigma_K+\sigma_Z-1}{1-\sigma_Z}} \\
a_{21} &= \frac{\tilde{k}}{h} \\
a_{22} &= 0
\end{aligned}$$

$$
\begin{aligned}
a_{23} &= 0 \\
a_{31} &= -\frac{1-\gamma}{F(\tilde{l})}\frac{1}{1-\sigma_Z}\sigma_K\frac{1}{h} \\
a_{32} &= 0 \\
a_{33} &= \frac{1-\gamma}{F(\tilde{l})^2}[g-\frac{1}{1-\sigma_Z}(\sigma_K\frac{q-1}{h}-\sigma_N n)][\frac{1-\gamma(\theta+1)}{l^2}+\frac{1-\gamma}{(1-l)^2}(1-\frac{\sigma_L}{1-\sigma_Z})]
\end{aligned}
$$

So

$$
\begin{pmatrix} \dot{q} \\ \dot{k} \\ \dot{l} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ \frac{\tilde{k}}{h} & 0 & 0 \\ a_{31} & 0 & a_{33} \end{pmatrix} \begin{pmatrix} q-\tilde{q} \\ k-\tilde{k} \\ l-\tilde{l} \end{pmatrix}
$$

4.3.5 Non-Scale Growth

Recall from section 4.3.3 on the capital dynamics, the dynamics of capital converge to a long-run steady state growth rate along a transitional growth path. In this section, we also define the stationary "scale-adjusted" imported input and aggregate output.

The imported input is:

$$
Z = (p^{-1}\alpha\sigma_Z)^{\frac{1}{1-\sigma_Z}}(1-l)^{\frac{\sigma_L}{1-\sigma_Z}}K^{\frac{\sigma_K}{1-\sigma_Z}}N^{\frac{\sigma_N}{1-\sigma_Z}} \tag{4.26}
$$

Expressed in terms of its corresponding "scale-adjusted" per capita quantities,

$$
z = \frac{Z}{N^{\frac{\sigma_N}{1-\sigma_K-\sigma_Z}}}
$$

we obtain the direct relationship between the time path of capital and imported input.

$$
z = (p^{-1}\alpha\sigma_Z)^{\frac{1}{1-\sigma_Z}}(1-l)^{\frac{\sigma_L}{1-\sigma_Z}}k^{\frac{\sigma_K}{1-\sigma_Z}} \tag{4.27}
$$

The aggregate output is:

$$
Y = \alpha^{\frac{1}{1-\sigma_Z}}(p^{-1}\sigma_Z)^{\frac{\sigma_Z}{1-\sigma_Z}}K^{\frac{\sigma_K}{1-\sigma_Z}}N^{\frac{\sigma_N}{1-\sigma_Z}} \tag{4.28}
$$

Expressed in terms of its corresponding "scale-adjusted" per capita quantities,

$$y = \frac{Y}{N^{\frac{\sigma_N}{1-\sigma_K-\sigma_Z}}}$$

we obtain the direct relationship between the time path of capital and aggregate output.

$$y = \alpha^{\frac{1}{1-\sigma_Z}}(1-l)^{\frac{\sigma_L}{1-\sigma_Z}}(p^{-1}\sigma_Z)^{\frac{\sigma_Z}{1-\sigma_Z}}k^{\frac{\sigma_K}{1-\sigma_Z}} \tag{4.29}$$

4.3.6 Accumulation of Foreign Assets

For this open economy, we consider it to be small in the world financial market. Due to this, differential growth rates of consumption and domestic output can be sustained in the equilibrium. This also has implications on its net asset position. Rewrite the individual consumer's flow budget constraint here:

$$\dot{B}_i = Y_i - C_i - pZ_i - \Phi(I_i, K_i) + (r-n)B_i$$

Aggregating all the individual consumers' flow budget constraints, we get the aggregate net rate of accumulation of traded bonds by the private sector, which is also the current account balance in this economy.

$$\dot{B} = rB + Y - C - pZ - I\left(1 + \frac{h}{2}\frac{I}{K}\right)$$

Convert this to "scale-adjusted" form:

$$\dot{b} = (r-g)b + (1-\sigma_Z)\alpha^{\frac{1}{1-\sigma_Z}}(p^{-1}\sigma_Z)^{\frac{\sigma_Z}{1-\sigma_Z}}k^{\frac{\sigma_K}{1-\sigma_Z}} - \frac{q^2-1}{2h}k - c$$

In the Appendix of Schubert and Turnovsky (2007), they show that if the transversality conditions hold, then the linearized solution to the "scale-adjusted" per capita stock of bonds, starting from the initial stock of bonds, b_0, is given by:

$$b(t) = -\frac{M}{r-g} - \frac{L}{r-g-\mu_1}e^{\mu_1 t} + \frac{c(0)}{r-\psi}e^{(\psi-g)t}$$

where M and L are constants that are defined in their Appendix, and

$$c(0) = (r-\psi)\left[b_0 + \frac{M}{r-g} + \frac{L}{r-g-\mu_1}\right]$$

As noted in their Appendix, the above equation is the inter-temporal budget constraint in the economy, which shows that the present value of the resources available for initial consumption after the investment needs have been met along the transition path.

4.4 Shock Analysis

In this section, we analyze the impact of an oil shock, which is represented by an increase in the price of the intermediate input p.

4.4.1 Steady-State Changes

First, we consider the effects of an unanticipated permanent increase in the price of the imported input ($dp > 0$) at time $t = 0$. With perfect foresight, agents' expectations of the steady-state response determines the dynamic evolution of the economy. So we start our analysis with the study of the long-run steady state effects of an increase in the price of the intermediate input.

At the steady state,

$$\tilde{q} = 1 + gh$$

This implies that the long-run market price of installed capital $\tilde{q}$ does not change, because the steady-state growth rate g is independent of the intermediate input price in the economy.

Rewrite the "scale-adjusted" stationary quantities here:

$$\begin{aligned}
\tilde{k} &= [r(1+gh) - \frac{g^2 h}{2}]^{\frac{1-\sigma_Z}{\sigma_K+\sigma_Z-1}} \sigma_K^{-\frac{1-\sigma_Z}{\sigma_K+\sigma_Z-1}} \alpha^{-\frac{1}{\sigma_K+\sigma_Z-1}} (1-\tilde{l})^{-\frac{\sigma_Z}{\sigma_K+\sigma_Z-1}} (p^{-1}\sigma_Z)^{-\frac{\sigma_Z}{\sigma_K+\sigma_Z-1}} \\
\tilde{z} &= (p^{-1}\alpha\sigma_Z)^{\frac{1}{1-\sigma_Z}} (1-\tilde{l})^{\frac{\sigma_L}{1-\sigma_Z}} \tilde{k}^{\frac{\sigma_K}{1-\sigma_Z}} \\
\tilde{y} &= \alpha^{\frac{1}{1-\sigma_Z}} (1-\tilde{l})^{\frac{\sigma_L}{1-\sigma_Z}} (p^{-1}\sigma_Z)^{\frac{\sigma_Z}{1-\sigma_Z}} \tilde{k}^{\frac{\sigma_K}{1-\sigma_Z}}
\end{aligned}$$

From these, we see that the price of the imported input has the following long-run effects on the "scale-adjusted" capital stock, the demand for the imported input, and the aggregate output.

$$\frac{d\tilde{k}}{\tilde{k}} = -\frac{\sigma_Z}{1-\sigma_K-\sigma_Z}\left(\frac{dp}{p}+\frac{d\tilde{l}}{1-\tilde{l}}\right) < 0 \tag{4.30}$$

$$\frac{d\tilde{z}}{\tilde{z}} = -\frac{1}{1-\sigma_Z}\left(1+\frac{\sigma_K\sigma_Z}{1-\sigma_K-\sigma_Z}\right)\frac{dp}{p} - \frac{1}{1-\sigma_Z}\left(\sigma_L+\frac{\sigma_K\sigma_Z}{1-\sigma_K-\sigma_Z}\right)\frac{d\tilde{l}}{1-\tilde{l}} \tag{4.31}$$

$$\frac{d\tilde{y}}{\tilde{y}} = -\frac{\sigma_Z}{1-\sigma_Z}\left(1+\frac{\sigma_K}{1-\sigma_K-\sigma_Z}\right)\frac{dp}{p} - \frac{1}{1-\sigma_Z}\left(\sigma_L+\frac{\sigma_K\sigma_Z}{1-\sigma_K-\sigma_Z}\right)\frac{d\tilde{l}}{1-\tilde{l}} \tag{4.32}$$

They are all < 0. Therefore, an increase in the price of the imported intermediate input causes a decrease in the use of the input, which is intuitive. It also reduces the "scale-adjusted" capital stock and output levels (by different proportionate amounts) in the economy. Despite the fact that the growth rate of output does not change in the long run, we will have lower capital, lower output and reduced usage of the imported input following an oil shock along the new balanced growth path.

4.4.2 Impact Effects

Recall the "scale-adjusted" stationary imported input and aggregate output are:

$$z = (p^{-1}\alpha\sigma_Z)^{\frac{1}{1-\sigma_Z}}(1-l)^{\frac{\sigma_L}{1-\sigma_Z}}k^{\frac{\sigma_K}{1-\sigma_Z}}$$

$$y = \alpha^{\frac{1}{1-\sigma_Z}}(p^{-1}\sigma_Z)^{\frac{\sigma_Z}{1-\sigma_Z}}(1-l)^{\frac{\sigma_L}{1-\sigma_Z}}k^{\frac{\sigma_K}{1-\sigma_Z}}$$

The z equation shows that if there is an increase in the price of the imported input, then this will lead to an immediate reduction in its usage. In the short run, capital stock is fixed. So output production will decrease too.

$$\frac{dz(0)}{\tilde{z}} = -\frac{1}{1-\sigma_Z}\frac{dp}{p} - \frac{\sigma_L}{1-\sigma_Z}\frac{d\tilde{l}}{1-\tilde{l}} < 0$$

$$\frac{dy(0)}{\tilde{y}} = -\frac{\sigma_Z}{1-\sigma_Z}\frac{dp}{p} - \frac{\sigma_L}{1-\sigma_Z}\frac{d\tilde{l}}{1-\tilde{l}} < 0$$

Comparing the short-run effects and the long-run effects, we can see that the short-run decline in both the input usage and in output is less than the long-run decline. This implies that following the initial decrease, both $z(t)$ and $y(t)$ will continue to decline towards their steady state as the economy evolves over time.

For the "scale-adjusted" consumption, the initial response results from an increase in the price of the imported intermediate input can be derived as:

$$\frac{dc(0)}{dp} = (r-\psi)\left[\frac{1}{r-g}\frac{dM}{dp} + \frac{1}{r-g-\mu}\frac{dL}{dp}\right]$$

It can be shown that the eigenvalue μ_1 is independent of p, which implies that the asymptotic speed of convergence is independent of p. As argued by Schubert and Turnovsky (2007), $\frac{dc(0)}{dp} > 0$.

Last but not least, the effect of this import price change on the current account of this economy is:

$$\frac{d\dot{b}(0)}{dp} = -(\sigma_Z p^{-1}\alpha)^{\frac{1}{1-\sigma_Z}} k^{\frac{\sigma_K}{1-\sigma_Z}} - \frac{qk}{h}\frac{dq(0)}{dp} - \frac{dc(0)}{dp} < 0$$

This equation shows that there are two offsetting effects in operation here. The first effect is the direct effect of the increase in the price of the imported intermediate input, which decreases its usage, therefore reducing productivity and domestic output. This has an adverse effect on the trade balance. On the other hand, the second effect is represented by a reduction in $q(0)$ and a reduction in $c(0)$, which reduce domestic demand and therefore improve the current account balance.

4.5 Conclusion

In this project, we introduce endogenous labor supply into the analytical framework of Schubert and Turnovsky (2007) paper. By so doing, we further investigate the impacts of an increase in the price of an imported intermediate input (say oil) on the dynamics of a small open economy. This macro model also assumes that the economy has access to a perfect world capital market. Then we apply an oil shock to this model economy, which is represented by an increase in the price of the imported intermediate good. Our shock analysis shows that with the extension of endogenous labor supply, output, capital, and input usage are all permanently much lower than the case for fixed labor supply. The short-run impacts on input usage and output are also larger than the fixed labor supply case. By introducing endogenous labor supply, the changes in the variables (such as output, input usage, capital) are all in the same direction as in the case for fixed labor supply, but with a different (larger) magnitude. Same economics intuition applies here as in the Schubert and Turnovsky (2007) paper. The contribution of this project is that it generalizes the Schubert and Turnovsky (2007) paper in the dimension of labor supply, building from which we investigate the dynamic effects of a change in the price of imported intermediate good on economic growth.

Appendix

A1. Derivations

Derivation of Equation (4.2):

$$\begin{aligned} Y = NY_i &= \alpha[(1-l)N]^{1-\sigma-\xi} K^{\sigma} Z^{\xi} K^{\eta} \\ &= \alpha(1-l)^{1-\sigma-\xi} N^{1-\sigma-\xi} K^{\sigma+\eta} Z^{\xi} \\ &= \alpha(1-l)^{\sigma_L} N^{\sigma_N} K^{\sigma_K} Z^{\sigma_Z} \end{aligned}$$

Derivation of Equation (4.4):

$$\begin{aligned} \dot{K}_i &= I_i - nK_i \\ \left(\frac{\dot{K}}{N}\right) &= \frac{I}{N} - n\frac{K}{N} \\ \frac{\dot{K}N - K\dot{N}}{N^2} &= \frac{I}{N} - K\frac{n}{N} \\ \frac{\dot{K}}{N} - K\frac{\dot{N}}{N^2} &= \frac{I}{N} - K\frac{n}{N} \\ \dot{K} - K\frac{\dot{N}}{N} &= I - Kn \\ \dot{K} - Kn &= I - Kn \\ \dot{K} &= I \end{aligned}$$

Derivation of Equation (4.5):

$$\begin{aligned} \dot{B}_i &= Y_i - C_i - pZ_i - \Phi(I_i, K_i) + (r-n)B_i \\ &= Y_i - C_i - pZ_i - I_i\left(1 + \frac{h}{2}\frac{I_i}{K_i}\right) + (r-n)B_i \\ \left(\frac{\dot{B}}{N}\right) &= \frac{Y}{N} - \frac{C}{N} - p\frac{Z}{N} - \frac{I}{N}\left(1 + \frac{h}{2}\frac{I/N}{K/N}\right) + (r-n)\frac{B}{N} \end{aligned}$$

$$\begin{aligned}
\frac{\dot{B}N - B\dot{N}}{N^2} &= ... \\
\frac{\dot{B}}{N} - B\frac{\dot{N}}{N^2} &= ... \\
\dot{B} - B\frac{\dot{N}}{N} &= Y - C - pZ - I\left(1 + \frac{h}{2}\frac{I}{K}\right) + (r-n)B \\
\dot{B} - Bn &= ... - Bn \\
\dot{B} &= Y - C - pZ - I\left(1 + \frac{h}{2}\frac{I}{K}\right) + rB
\end{aligned}$$

Derivation of Equation (4.6):

$$\begin{aligned}
\frac{\partial H}{\partial C} = \frac{1}{\gamma}\gamma\left(\frac{C}{N}l^{\theta}\right)^{\gamma-1}\frac{1}{N}l^{\theta} - \lambda &= 0 \\
C^{\gamma-1}N^{-\gamma+1}l^{\theta(\gamma-1)}N^{-1}l^{\theta} &= \lambda \\
N^{-\gamma}C^{\gamma-1}l^{\theta\gamma-\theta+\theta} &= \lambda \\
N^{-\gamma}C^{\gamma-1}l^{\theta\gamma} &= \lambda
\end{aligned}$$

Derivation of Equation (4.7):

$$\begin{aligned}
\frac{\partial H}{\partial l} = \frac{1}{\gamma}\gamma\left(\frac{C}{N}l^{\theta}\right)^{\gamma-1}\frac{C}{N}\theta l^{\theta-1} + \lambda\left[\alpha\sigma_L(1-l)^{\sigma_L-1}(-1)N^{\sigma_N}K^{\sigma_K}Z^{\sigma_Z}\right] &= 0 \\
\theta C^{\gamma-1+1}N^{-\gamma+1-1}l^{\theta(\gamma-1)+\theta-1} - \lambda\sigma_L\frac{Y}{1-l} &= 0 \\
\theta C^{\gamma}N^{-\gamma}l^{\theta\gamma-\theta+\theta-1} - \lambda\sigma_L\frac{Y}{1-l} &= 0 \\
N^{-\gamma}\theta C^{\gamma}l^{\theta\gamma-1} &= \lambda\sigma_L\frac{Y}{1-l}
\end{aligned}$$

Derivation of Equation (4.8):

$$\begin{aligned}
\frac{N^{-\gamma}\theta C^{\gamma}l^{\theta\gamma-1}}{N^{-\gamma}C^{\gamma-1}l^{\theta\gamma}} &= \frac{\lambda\sigma_L\frac{Y}{1-l}}{\lambda} \\
\frac{\theta C}{l} &= \frac{\sigma_L Y}{1-l} \\
\frac{C}{Y} &= \frac{l}{1-l}\frac{\sigma_L}{\theta}
\end{aligned}$$

Derivation of Equation (4.9):

$$\begin{aligned}
\frac{\partial H}{\partial Z} = -\lambda\left[-\alpha(1-l)^{\sigma_L} N^{\sigma_N} K^{\sigma_K} \sigma_Z Z^{\sigma_Z - 1} + p\right] &= 0 \\
-\alpha(1-l)^{\sigma_L} N^{\sigma_N} K^{\sigma_K} \sigma_Z Z^{\sigma_Z - 1} + p &= 0 \\
-\sigma_Z \frac{Y}{Z} &= -p \\
\sigma_Z \frac{Y}{Z} &= p \\
\frac{Y}{Z} &= \frac{p}{\sigma_Z} \\
\frac{Z}{Y} &= \frac{\sigma_Z}{p} \\
Z &= \sigma_Z \frac{Y}{p}
\end{aligned}$$

Derivation of Equation (4.10):

$$\begin{aligned}
\frac{\partial H}{\partial I} = -q^*(-1)e^{-\beta t} - \lambda\left(1 + h\frac{I}{K}\right) e^{-\beta t} &= 0 \\
q^* - \lambda\left(1 + h\frac{I}{K}\right) &= 0 \\
q^* &= \lambda\left(1 + h\frac{I}{K}\right) \\
\frac{q^*}{\lambda} &= \left(1 + h\frac{I}{K}\right) \\
q &= 1 + h\frac{I}{K} \\
1 + h\frac{I}{K} &= q
\end{aligned}$$

Derivation of Equation (4.11):

$$\begin{aligned}
\frac{\partial H}{\partial B} &= \frac{d}{dt}\left(\frac{\partial H}{\partial \dot{B}}\right) \\
-\lambda(-r)e^{-\beta t} &= \frac{d}{dt}\left(-\lambda e^{-\beta t}\right) \\
r\lambda e^{-\beta t} &= -\dot{\lambda} e^{-\beta t} - \lambda e^{-\beta t}(-\beta) \\
\ldots &= -\dot{\lambda} e^{-\beta t} + \beta\lambda e^{-\beta t} \\
r\lambda &= -\dot{\lambda} + \beta\lambda
\end{aligned}$$

$$
\begin{aligned}
r &= -\frac{\dot{\lambda}}{\lambda} + \beta \\
r &= \beta - \frac{\dot{\lambda}}{\lambda} \\
\beta - \frac{\dot{\lambda}}{\lambda} &= r
\end{aligned}
$$

Derivation of Equation (4.12):

$$
\begin{aligned}
\frac{\partial H}{\partial K} &= \frac{d}{dt}\left(\frac{\partial H}{\partial \dot{K}}\right) \\
\left[-\lambda\left((-1)\alpha(1-l)^{\sigma_L}N^{\sigma_N}\sigma_K K^{\sigma_K-1}Z^{\sigma_Z} + \frac{h}{2}I^2(-1)\frac{1}{K^2}\right)\right]e^{-\beta t} &= ... \\
\left[\lambda\sigma_K\frac{Y}{K} + \lambda\frac{h}{2}\left(\frac{q-1}{h}\right)^2\right]e^{-\beta t} &= \frac{d}{dt}\left(-q^* e^{-\beta t}\right) \\
... &= -\dot{q}^* e^{-\beta t} - q^* e^{-\beta t}(-\beta) \\
... &= -\dot{q}^* e^{-\beta t} + \beta q^* e^{-\beta t} \\
\lambda\sigma_K\frac{Y}{K} + \lambda\frac{h}{2}\left(\frac{q-1}{h}\right)^2 &= -\dot{q}^* + \beta q^* \\
\sigma_K\frac{Y}{K} + \frac{h}{2}\left(\frac{q-1}{h}\right)^2 &= -\frac{\dot{q}^*}{\lambda} + \beta\frac{q^*}{\lambda} \\
... &= -\frac{(\dot{q\lambda})}{\lambda} + \beta q \\
... &= -\frac{\dot{q}\lambda + q\dot{\lambda}}{\lambda} + \beta q \\
... &= -\dot{q} - q\left(\frac{\dot{\lambda}}{\lambda}\right) + \beta q \\
\frac{\sigma_K}{q}\frac{Y}{K} + \frac{h}{2q}\left(\frac{q-1}{h}\right)^2 &= -\frac{\dot{q}}{q} - \frac{\dot{\lambda}}{\lambda} + \beta \\
\frac{\sigma_K}{q}\frac{Y}{K} + \frac{\dot{q}}{q} + \frac{h}{2q}\left(\frac{q-1}{h}\right)^2 &= \beta - \frac{\dot{\lambda}}{\lambda} = r
\end{aligned}
$$

Derivation of Equation (4.14):

$$
\begin{aligned}
N^{-\gamma}C^{\gamma-1}l^{\theta\gamma} &= \lambda \\
log\left(N^{-\gamma}C^{\gamma-1}l^{\theta\gamma}\right) &= log\lambda \\
logN^{-\gamma} + logC^{\gamma-1} + logl^{\theta\gamma} &= log\lambda
\end{aligned}
$$

$$
\begin{aligned}
-\gamma logN + (\gamma - 1)logC + \theta\gamma logl &= log\lambda \\
-\gamma dlogN + (\gamma - 1)dlogC + \theta\gamma dlogl &= dlog\lambda \\
-\gamma\frac{\dot{N}}{N} + (\gamma - 1)\frac{\dot{C}}{C} + \theta\gamma\frac{\dot{l}}{l} &= \frac{\dot{\lambda}}{\lambda} = \beta - r \\
-\gamma n + (\gamma - 1)\frac{\dot{C}}{C} + \theta\gamma\frac{\dot{l}}{l} &= \beta - r \\
(\gamma - 1)\frac{\dot{C}}{C} &= \beta - \gamma + \gamma n - \theta\gamma\frac{\dot{l}}{l} \\
\frac{\dot{C}}{C} &= \frac{1}{\gamma - 1}(\beta - \gamma + \gamma n - \theta\gamma\frac{\dot{l}}{l}) \\
&= \frac{1}{1 - \gamma}(r - \beta - \gamma n + \theta\gamma\frac{\dot{l}}{l})
\end{aligned}
$$

Derivation of Equation (4.21):

$$
\begin{aligned}
Y &= \alpha(1 - l)^{\sigma_L} N^{\sigma_N} K^{\sigma_K} Z^{\sigma_Z} \\
\frac{\dot{Y}}{Y} &= -\sigma_L\frac{\dot{l}}{1 - l} + \sigma_N\frac{\dot{N}}{N} + \sigma_K\frac{\dot{K}}{K} + \sigma_Z\frac{\dot{Z}}{Z} \\
\frac{\dot{Y}}{Y} &= -\sigma_L\frac{\dot{l}}{1 - l} + \sigma_N n + \sigma_K\frac{\dot{K}}{K} + \sigma_Z\frac{\dot{Z}}{Z} \\
\frac{\dot{Y}}{Y} &= -\sigma_L\frac{\dot{l}}{1 - l} + \sigma_N n + \sigma_K\frac{\dot{K}}{K} + \sigma_Z\frac{\dot{Y}}{Y} \\
\frac{\dot{Y}}{Y} - \sigma_Z\frac{\dot{Y}}{Y} &= -\sigma_L\frac{\dot{l}}{1 - l} + \sigma_N n + \sigma_K\frac{\dot{K}}{K} \\
(1 - \sigma_Z)\frac{\dot{Y}}{Y} &= -\sigma_L\frac{\dot{l}}{1 - l} + \sigma_N n + \sigma_K\frac{\dot{K}}{K} \\
\frac{\dot{Y}}{Y} &= -\frac{\sigma_L}{1 - \sigma_Z}\frac{\dot{l}}{1 - l} + \frac{\sigma_N}{1 - \sigma_Z}n + \frac{\sigma_K}{1 - \sigma_Z}\frac{\dot{K}}{K}
\end{aligned}
$$

Derivation of Equation (4.22): From equation (4.12),

$$
\frac{\sigma_K}{q}\frac{Y}{K} + \frac{\dot{q}}{q} + \frac{h}{2q}\Big(\frac{q - 1}{h}\Big)^2 = r
$$

$$
\begin{aligned}
\frac{\dot{q}}{q} &= r - \frac{\sigma_K}{q}\frac{Y}{K} - \frac{h}{2q}\left(\frac{q - 1}{h}\right)^2 \\
\dot{q} &= rq - \frac{h}{2}\frac{(q - 1)^2}{h^2} - \sigma_K\frac{Y}{K}
\end{aligned}
$$

$$\dot{q} = rq - \frac{(q-1)^2}{2h} - \sigma_K \frac{Y}{K}$$

where

$$Y = \alpha(1-l)^{\sigma_L} N^{\sigma_N} K^{\sigma_K} Z^{\sigma_Z}$$

in which

$$Z = \sigma_Z \frac{Y}{p}$$

Substitute Z expression into Y equation:

$$\begin{aligned}
Y &= \alpha(1-l)^{\sigma_L} N^{\sigma_N} K^{\sigma_K} \left(\sigma_Z \frac{Y}{p}\right)^{\sigma_Z} \\
&= \alpha(1-l)^{\sigma_L} N^{\sigma_N} K^{\sigma_K} \sigma_Z^{\sigma_Z} Y^{\sigma_Z} p^{-\sigma_Z} \\
\frac{Y}{Y^{\sigma_Z}} &= \alpha(1-l)^{\sigma_L} N^{\sigma_N} K^{\sigma_K} \sigma_Z^{\sigma_Z} p^{-\sigma_Z} \\
Y^{1-\sigma_Z} &= ... \\
Y &= \alpha^{\frac{1}{1-\sigma_Z}} (1-l)^{\frac{\sigma_L}{1-\sigma_Z}} N^{\frac{\sigma_N}{1-\sigma_Z}} K^{\frac{\sigma_K}{1-\sigma_Z}} \sigma_Z^{\frac{\sigma_Z}{1-\sigma_Z}} p^{-\frac{\sigma_Z}{1-\sigma_Z}}
\end{aligned}$$

Substitute Y equation above into the $\dot{q}$ equation:

$$\begin{aligned}
\dot{q} &= rq - \frac{(q-1)^2}{2h} - \sigma_K \frac{\alpha^{\frac{1}{1-\sigma_Z}} (1-l)^{\frac{\sigma_L}{1-\sigma_Z}} N^{\frac{\sigma_N}{1-\sigma_Z}} K^{\frac{\sigma_K}{1-\sigma_Z}} \sigma_Z^{\frac{\sigma_Z}{1-\sigma_Z}} p^{-\frac{\sigma_Z}{1-\sigma_Z}}}{K} \\
&= ... - \sigma_K \alpha^{\frac{1}{1-\sigma_Z}} (1-l)^{\frac{\sigma_L}{1-\sigma_Z}} N^{\frac{\sigma_N}{1-\sigma_Z}} K^{\frac{\sigma_K}{1-\sigma_Z}-1} \sigma_Z^{\frac{\sigma_Z}{1-\sigma_Z}} p^{-\frac{\sigma_Z}{1-\sigma_Z}} \\
&= ... - ...N^{\frac{\sigma_N}{1-\sigma_Z}} K^{\frac{\sigma_K - 1 + \sigma_Z}{1-\sigma_Z}} ... \\
&= ... - ... \left(\frac{K}{N^{\frac{\sigma_N}{1-\sigma_K-\sigma_Z}}}\right)^{\frac{\sigma_K - 1 + \sigma_Z}{1-\sigma_Z}} ... \\
\dot{q} &= rq - \frac{(q-1)^2}{2h} - \sigma_K \alpha^{\frac{1}{1-\sigma_Z}} (1-l)^{\frac{\sigma_L}{1-\sigma_Z}} k^{\frac{\sigma_K+\sigma_Z-1}{1-\sigma_Z}} \sigma_Z^{\frac{\sigma_Z}{1-\sigma_Z}} p^{-\frac{\sigma_Z}{1-\sigma_Z}} \\
&= rq - \frac{(q-1)^2}{2h} - \sigma_K \alpha^{\frac{1}{1-\sigma_Z}} (1-l)^{\frac{\sigma_Z}{1-\sigma_Z}} (p^{-1}\sigma_Z)^{\frac{\sigma_Z}{1-\sigma_Z}} k^{\frac{\sigma_K+\sigma_Z-1}{1-\sigma_Z}}
\end{aligned}$$

Derivation of Equation (4.23): From equation (4.19),

$$\theta\gamma\frac{\dot{l}}{l} = \beta - r + \gamma n - (\gamma - 1)\frac{\dot{C}}{C}$$

Plug in equation (4.20),

$$\begin{aligned}
\theta\gamma\frac{\dot{l}}{l} &= \beta - r + \gamma n - (\gamma-1)\left(\frac{\dot{Y}}{Y} + \frac{\dot{l}}{l} + \frac{\dot{l}}{1-l}\right) \\
&= \beta - r + \gamma n - (\gamma-1)\frac{\dot{Y}}{Y} - (\gamma-1)\frac{\dot{l}}{l} - (\gamma-1)\frac{\dot{l}}{1-l} \\
\theta\gamma\dot{l} &= (\beta - r + \gamma n)l - (\gamma-1)l\frac{\dot{Y}}{Y} - (\gamma-1)\dot{l} - (\gamma-1)\frac{l}{1-l}\dot{l} \\
\left[\theta\gamma + (\gamma-1) + (\gamma-1)\frac{l}{1-l}\right]\dot{l} &= (\beta - r + \gamma n)l - (\gamma-1)l\frac{\dot{Y}}{Y}
\end{aligned}$$

Plug in equation (4.21),

$$\begin{aligned}
... &= (\beta - r + \gamma n)l - (\gamma-1)l\left(-\frac{\sigma_L}{1-\sigma_Z}\frac{\dot{l}}{1-l} + \frac{\sigma_N}{1-\sigma_Z}n + \frac{\sigma_K}{1-\sigma_Z}\frac{q-1}{h}\right) \\
... &= ... - (\gamma-1)l\frac{\sigma_K}{1-\sigma_Z}\frac{q-1}{h} + (\gamma-1)\frac{\sigma_L}{1-\sigma_Z}\frac{l}{1-l}\dot{l} - (\gamma-1)l\frac{\sigma_N}{1-\sigma_Z}n
\end{aligned}$$

$$\begin{aligned}
&\left[\theta\gamma + (\gamma-1) + (\gamma-1)\frac{l}{1-l} - (\gamma-1)\frac{\sigma_L}{1-\sigma_Z}\frac{l}{1-l}\right]\dot{l} \\
&= (\beta - r + \gamma n)l - (\gamma-1)l\frac{\sigma_K}{1-\sigma_Z}\frac{q-1}{h} - (\gamma-1)l\frac{\sigma_N}{1-\sigma_Z}n
\end{aligned}$$

$$\begin{aligned}
&\left[\frac{\theta\gamma + \gamma - 1}{l} + (\gamma-1)\frac{1}{1-l} - (\gamma-1)\frac{\sigma_L}{1-\sigma_Z}\frac{1}{1-l}\right]\dot{l} \\
&= (\beta - r + \gamma n) - (\gamma-1)\frac{\sigma_K}{1-\sigma_Z}\frac{q-1}{h} - (\gamma-1)\frac{\sigma_N}{1-\sigma_Z}n
\end{aligned}$$

$$\begin{aligned}
\left[\frac{\gamma(\theta+1)-1}{l} + \frac{\gamma-1}{1-l}\left(1 - \frac{\sigma_L}{1-\sigma_Z}\right)\right]\dot{l} &= \beta - r + \gamma n - \frac{\gamma-1}{1-\sigma_Z}\left(\sigma_K\frac{q-1}{h} - \sigma_N n\right) \\
\left[\frac{1-\gamma(\theta+1)}{l} + \frac{1-\gamma}{1-l}\left(1 - \frac{\sigma_L}{1-\sigma_Z}\right)\right]\dot{l} &= r - \beta - \gamma n - \frac{1-\gamma}{1-\sigma_Z}\left(\sigma_K\frac{q-1}{h} - \sigma_N n\right) \\
F(l)\dot{l} &= r - \beta - \gamma n - \frac{1-\gamma}{1-\sigma_Z}\left(\sigma_K\frac{q-1}{h} - \sigma_N n\right) \\
\dot{l} &= \frac{1}{F(l)}\left[r - \beta - \gamma n - \frac{1-\gamma}{1-\sigma_Z}\left(\sigma_K\frac{q-1}{h} - \sigma_N n\right)\right]
\end{aligned}$$

Derivation of Equation (4.24):

$$
\begin{aligned}
\dot{k} &= \left(\frac{\tilde{q}-1}{h} - g\right)\tilde{k} = 0 \\
\frac{\tilde{q}-1}{h} - g &= 0 \\
\frac{\tilde{q}-1}{h} &= g \\
\tilde{q} - 1 &= gh \\
\tilde{q} &= 1 + gh
\end{aligned}
$$

Derivation of Equation (4.25):

$$
\begin{aligned}
\dot{q} &= r\tilde{q} - \frac{(\tilde{q}-1)^2}{2h} - \sigma_K \alpha^{\frac{1}{1-\sigma_Z}} (1-\tilde{l})^{\frac{\sigma_Z}{1-\sigma_Z}} (p^{-1}\sigma_Z)^{\frac{\sigma_Z}{1-\sigma_Z}} \tilde{k}^{\frac{\sigma_K+\sigma_Z-1}{1-\sigma_Z}} = 0 \\
&= r(1+gh) - \frac{(1+gh-1)^2}{2h} - \sigma_K \alpha^{\frac{1}{1-\sigma_Z}} (1-\tilde{l})^{\frac{\sigma_Z}{1-\sigma_Z}} (p^{-1}\sigma_Z)^{\frac{\sigma_Z}{1-\sigma_Z}} \tilde{k}^{\frac{\sigma_K+\sigma_Z-1}{1-\sigma_Z}} = 0 \\
&= r(1+gh) - \frac{g^2h^2}{2h} - ... \\
&= r(1+gh) - \frac{g^2h}{2} - \sigma_K \alpha^{\frac{1}{1-\sigma_Z}} (1-\tilde{l})^{\frac{\sigma_Z}{1-\sigma_Z}} (p^{-1}\sigma_Z)^{\frac{\sigma_Z}{1-\sigma_Z}} \tilde{k}^{\frac{\sigma_K+\sigma_Z-1}{1-\sigma_Z}} = 0
\end{aligned}
$$

$\tilde{k}$ and $\tilde{l}$ are jointly determined by the equation above.

$$
r(1+gh) - \frac{g^2h}{2} = \sigma_K \alpha^{\frac{1}{1-\sigma_Z}} (1-\tilde{l})^{\frac{\sigma_Z}{1-\sigma_Z}} (p^{-1}\sigma_Z)^{\frac{\sigma_Z}{1-\sigma_Z}} \tilde{k}^{\frac{\sigma_K+\sigma_Z-1}{1-\sigma_Z}}
$$

$$
\left[r(1+gh) - \frac{g^2h}{2}\right] \sigma_K^{-1} \alpha^{-\frac{1}{1-\sigma_Z}} (1-\tilde{l})^{-\frac{\sigma_Z}{1-\sigma_Z}} (p^{-1}\sigma_Z)^{-\frac{\sigma_Z}{1-\sigma_Z}} = \tilde{k}^{\frac{\sigma_K+\sigma_Z-1}{1-\sigma_Z}}
$$

$$
\left[r(1+gh) - \frac{g^2h}{2}\right]^{\frac{1-\sigma_Z}{\sigma_K+\sigma_Z-1}} \sigma_K^{-\frac{1-\sigma_Z}{\sigma_K+\sigma_Z-1}} \alpha^{-\frac{1}{\sigma_K+\sigma_Z-1}} (1-\tilde{l})^{-\frac{\sigma_Z}{\sigma_K+\sigma_Z-1}} (p^{-1}\sigma_Z)^{-\frac{\sigma_Z}{\sigma_K+\sigma_Z-1}} = \tilde{k}
$$

$$
\tilde{k} = \left[r(1+gh) - \frac{g^2h}{2}\right]^{\frac{1-\sigma_Z}{\sigma_K+\sigma_Z-1}} \sigma_K^{-\frac{1-\sigma_Z}{\sigma_K+\sigma_Z-1}} \alpha^{-\frac{1}{\sigma_K+\sigma_Z-1}} (1-\tilde{l})^{-\frac{\sigma_Z}{\sigma_K+\sigma_Z-1}} (p^{-1}\sigma_Z)^{-\frac{\sigma_Z}{\sigma_K+\sigma_Z-1}}
$$

Derivation of a_{11}, a_{12}, a_{13}, a_{33}:

$$
a_{11} = r - \frac{2(q-1)}{2h}
$$

$$
\begin{aligned}
&= r - \frac{q-1}{h} \\
&= r - g
\end{aligned}
$$

$$
\begin{aligned}
a_{12} &= -\sigma_K \alpha^{\frac{1}{1-\sigma_Z}} (1-\tilde{l})^{\frac{\sigma_L}{1-\sigma_Z}} (p^{-1}\sigma_Z)^{\frac{\sigma_Z}{1-\sigma_Z}} \frac{\sigma_K + \sigma_Z - 1}{1-\sigma_Z} \tilde{k}^{\frac{\sigma_K+\sigma_Z-1}{1-\sigma_Z}-1} \\
&= -\frac{\sigma_K(\sigma_K + \sigma_Z - 1)}{1-\sigma_Z} \alpha^{\frac{1}{1-\sigma_Z}} (1-\tilde{l})^{\frac{\sigma_Z}{1-\sigma_Z}} (p^{-1}\sigma_Z)^{\frac{\sigma_Z}{1-\sigma_Z}} \tilde{k}^{\frac{\sigma_K+\sigma_Z-1-1+\sigma_Z}{1-\sigma_Z}} \\
&= ...\tilde{k}^{\frac{2\sigma_Z-2+\sigma_K}{1-\sigma_Z}} \\
&= ...\tilde{k}^{\frac{\sigma_K}{1-\sigma_Z}-2} \\
a_{12} &= -\frac{\sigma_K(\sigma_K + \sigma_Z - 1)}{1-\sigma_Z} \alpha^{\frac{1}{1-\sigma_Z}} (1-\tilde{l})^{\frac{\sigma_Z}{1-\sigma_Z}} (p^{-1}\sigma_Z)^{\frac{\sigma_Z}{1-\sigma_Z}} \tilde{k}^{\frac{\sigma_K}{1-\sigma_Z}-2}
\end{aligned}
$$

$$
\begin{aligned}
a_{13} &= -\sigma_K \alpha^{\frac{1}{1-\sigma_Z}} \frac{\sigma_L}{1-\sigma_Z} (1-\tilde{l})^{\frac{\sigma_Z}{1-\sigma_Z}-1} (-1)(p^{-1}\sigma_Z)^{\frac{\sigma_Z}{1-\sigma_Z}} \tilde{k}^{\frac{\sigma_K+\sigma_Z-1}{1-\sigma_Z}} \\
&= \frac{\sigma_K\sigma_Z\sigma_L}{1-\sigma_Z} \alpha^{\frac{1}{1-\sigma_Z}} (1-\tilde{l})^{\frac{2\sigma_Z-1}{1-\sigma_Z}} (p^{-1}\sigma_Z)^{\frac{\sigma_Z}{1-\sigma_Z}} \tilde{k}^{\frac{\sigma_K+\sigma_Z-1}{1-\sigma_Z}}
\end{aligned}
$$

$$
a_{33} = -\frac{1-\gamma}{F(\tilde{l})^2} \left[g - \frac{1}{1-\sigma_Z} (\sigma_K \frac{q-1}{h} - \sigma_N n) \right] F'(\tilde{l})
$$

where

$$
F(\tilde{l}) = \frac{1-\gamma(\theta+1)}{l} + \frac{1-\gamma}{1-l}(1 - \frac{\sigma_L}{1-\sigma_Z})
$$

So

$$
F'(\tilde{l}) = -\frac{1-\gamma(\theta+1)}{l^2} + \frac{1-\gamma}{(1-l)^2}(1 - \frac{\sigma_L}{1-\sigma_Z})
$$

Thus

$$
a_{33} = \frac{1-\gamma}{F(\tilde{l})^2} [g - \frac{1}{1-\sigma_Z} (\sigma_K \frac{q-1}{h} - \sigma_N n)][\frac{1-\gamma(\theta+1)}{l^2} - \frac{1-\gamma}{(1-l)^2}(1 - \frac{\sigma_L}{1-\sigma_Z})]
$$

Derivation of Equation (4.26):

$$
Z = \sigma_Z \frac{Y}{p}
$$

$$
\begin{aligned}
&= \sigma_Z \frac{\alpha(1-l)^{\sigma_L} N^{\sigma_N} K^{\sigma_K} Z^{\sigma_Z}}{p} \\
\frac{Z}{Z^{\sigma_Z}} &= \sigma_Z \alpha (1-l)^{\sigma_L} N^{\sigma_N} K^{\sigma_K} p^{-1} \\
Z^{1-\sigma_Z} &= \ldots \\
Z &= \sigma_Z^{\frac{1}{1-\sigma_Z}} \alpha^{\frac{1}{1-\sigma_Z}} (1-l)^{\frac{\sigma_L}{1-\sigma_Z}} N^{\frac{\sigma_N}{1-\sigma_Z}} K^{\frac{\sigma_K}{1-\sigma_Z}} p^{-\frac{1}{1-\sigma_Z}} \\
Z &= (p^{-1}\alpha\sigma_Z)^{\frac{1}{1-\sigma_Z}} (1-l)^{\frac{\sigma_L}{1-\sigma_Z}} N^{\frac{\sigma_N}{1-\sigma_Z}} K^{\frac{\sigma_K}{1-\sigma_Z}} \\
Z &= (p^{-1}\alpha\sigma_Z)^{\frac{1}{1-\sigma_Z}} (1-l)^{\frac{\sigma_L}{1-\sigma_Z}} K^{\frac{\sigma_K}{1-\sigma_Z}} N^{\frac{\sigma_N}{1-\sigma_Z}}
\end{aligned}
$$

Derivation of Equation (4.27):

$$
\begin{aligned}
z &= \frac{Z}{N^{\frac{\sigma_N}{1-\sigma_K-\sigma_Z}}} \\
&= \frac{(p^{-1}\alpha\sigma_Z)^{\frac{1}{1-\sigma_Z}} (1-l)^{\frac{\sigma_L}{1-\sigma_Z}} K^{\frac{\sigma_K}{1-\sigma_Z}} N^{\frac{\sigma_N}{1-\sigma_Z}}}{N^{\frac{\sigma_N}{1-\sigma_K-\sigma_Z}}} \\
&= \ldots N^{\frac{\sigma_N}{1-\sigma_Z} - \frac{\sigma_N}{1-\sigma_K-\sigma_Z}} \\
&= \ldots N^{\frac{\sigma_N(1-\sigma_K-\sigma_Z) - \sigma_N(1-\sigma_Z)}{(1-\sigma_Z)(1-\sigma_K-\sigma_Z)}} \\
&= \ldots N^{\frac{\sigma_N - \sigma_N\sigma_K - \sigma_N\sigma_Z - \sigma_N + \sigma_N\sigma_Z}{\ldots}} \\
&= \ldots N^{\frac{-\sigma_N\sigma_K}{\ldots}} \\
&= (p^{-1}\alpha\sigma_Z)^{\frac{1}{1-\sigma_Z}} (1-l)^{\frac{\sigma_L}{1-\sigma_Z}} K^{\frac{\sigma_K}{1-\sigma_Z}} N^{\frac{-\sigma_N\sigma_K}{(1-\sigma_Z)(1-\sigma_K-\sigma_Z)}} \\
&= \ldots \left(\frac{K}{N^{\frac{\sigma_N}{1-\sigma_K-\sigma_Z}}} \right)^{\frac{\sigma_K}{1-\sigma_Z}} \\
&= \ldots k^{\frac{\sigma_K}{1-\sigma_Z}} \\
z &= (p^{-1}\alpha\sigma_Z)^{\frac{1}{1-\sigma_Z}} (1-l)^{\frac{\sigma_L}{1-\sigma_Z}} k^{\frac{\sigma_K}{1-\sigma_Z}}
\end{aligned}
$$

Derivation of Equation (4.28):

$$
\begin{aligned}
Y &= \alpha(1-l)^{\sigma_L} N^{\sigma_N} K^{\sigma_K} Z^{\sigma_Z} \\
&= \alpha(1-l)^{\sigma_L} N^{\sigma_N} K^{\sigma_K} (\sigma_Z \frac{Y}{p})^{\sigma_Z} \\
&= \ldots \sigma_Z^{\sigma_Z} Y^{\sigma_Z} p^{-\sigma_Z} \\
\frac{Y}{Y^{\sigma_Z}} &= \alpha(1-l)^{\sigma_L} N^{\sigma_N} K^{\sigma_K} \sigma_Z^{\sigma_Z} p^{-\sigma_Z} \\
Y^{1-\sigma_Z} &= \ldots \\
Y &= \alpha^{\frac{1}{1-\sigma_Z}} (1-l)^{\frac{\sigma_L}{1-\sigma_Z}} N^{\frac{\sigma_N}{1-\sigma_Z}} K^{\frac{\sigma_K}{1-\sigma_Z}} (\sigma_Z p^{-1})^{\frac{\sigma_Z}{1-\sigma_Z}}
\end{aligned}
$$

$$
\begin{aligned}
&= \ ...(p^{-1}\sigma_Z)^{\frac{\sigma_Z}{1-\sigma_Z}} \\
Y &= \alpha^{\frac{1}{1-\sigma_Z}}(1-l)^{\frac{\sigma_L}{1-\sigma_Z}}(p^{-1}\sigma_Z)^{\frac{\sigma_Z}{1-\sigma_Z}}K^{\frac{\sigma_K}{1-\sigma_Z}}N^{\frac{\sigma_N}{1-\sigma_Z}}
\end{aligned}
$$

Derivation of Equation (4.29):

$$
\begin{aligned}
y &= \frac{Y}{N^{\frac{\sigma_N}{1-\sigma_K-\sigma_Z}}} \\
&= \frac{\alpha^{\frac{1}{1-\sigma_Z}}(1-l)^{\frac{\sigma_L}{1-\sigma_Z}}(p^{-1}\sigma_Z)^{\frac{\sigma_Z}{1-\sigma_Z}}K^{\frac{\sigma_K}{1-\sigma_Z}}N^{\frac{\sigma_N}{1-\sigma_Z}}}{N^{\frac{\sigma_N}{1-\sigma_K-\sigma_Z}}} \\
&= ...N^{\frac{\sigma_N}{1-\sigma_Z}-\frac{\sigma_N}{1-\sigma_K-\sigma_Z}} \\
&= ... \\
y &= \alpha^{\frac{1}{1-\sigma_Z}}(1-l)^{\frac{\sigma_L}{1-\sigma_Z}}(p^{-1}\sigma_Z)^{\frac{\sigma_Z}{1-\sigma_Z}}k^{\frac{\sigma_K}{1-\sigma_Z}}
\end{aligned}
$$

Derivation of Equation (4.30):

$$
\begin{aligned}
\frac{d\tilde{k}}{\tilde{k}} &= \frac{\sigma_Z}{\sigma_K+\sigma_Z-1}\frac{dp}{p} - \frac{\sigma_Z}{\sigma_K+\sigma_Z-1}\frac{d(1-\tilde{l})}{1-\tilde{l}} \\
&= \frac{\sigma_Z}{\sigma_K+\sigma_Z-1}\frac{dp}{p} + \frac{\sigma_Z}{\sigma_K+\sigma_Z-1}\frac{d\tilde{l}}{1-\tilde{l}} \\
&= \frac{\sigma_Z}{\sigma_K+\sigma_Z-1}\left(\frac{dp}{p} + \frac{d\tilde{l}}{1-\tilde{l}}\right) \\
&= -\frac{\sigma_Z}{1-\sigma_K-\sigma_Z}\left(\frac{dp}{p} + \frac{d\tilde{l}}{1-\tilde{l}}\right)
\end{aligned}
$$

Derivation of Equation (4.31):

$$
\begin{aligned}
\frac{d\tilde{z}}{\tilde{z}} &= -\frac{1}{1-\sigma_Z}\frac{dp}{p} + \frac{\sigma_L}{1-\sigma_Z}\frac{d(1-\tilde{l})}{1-\tilde{l}} + \frac{\sigma_K}{1-\sigma_Z}\frac{d\tilde{k}}{\tilde{k}} \\
&= -\frac{1}{1-\sigma_Z}\frac{dp}{p} - \frac{\sigma_L}{1-\sigma_Z}\frac{d\tilde{l}}{1-\tilde{l}} + \frac{\sigma_K}{1-\sigma_Z}\frac{\sigma_Z}{\sigma_K+\sigma_Z-1}(\frac{dp}{p} + \frac{d\tilde{l}}{1-\tilde{l}}) \\
&= -\frac{1}{1-\sigma_Z}\frac{dp}{p} - \frac{\sigma_L}{1-\sigma_Z}\frac{d\tilde{l}}{1-\tilde{l}} + \frac{\sigma_K}{1-\sigma_Z}\frac{\sigma_Z}{\sigma_K+\sigma_Z-1}\frac{dp}{p} + \frac{\sigma_K}{1-\sigma_Z}\frac{\sigma_Z}{\sigma_K+\sigma_Z-1}\frac{d\tilde{l}}{1-\tilde{l}} \\
&= (-\frac{1}{1-\sigma_Z} + \frac{\sigma_K}{1-\sigma_Z}\frac{\sigma_Z}{\sigma_K+\sigma_Z-1})\frac{dp}{p} + (-\frac{\sigma_L}{1-\sigma_Z} + \frac{\sigma_K}{1-\sigma_Z}\frac{\sigma_Z}{\sigma_K+\sigma_Z-1})\frac{d\tilde{l}}{1-\tilde{l}}
\end{aligned}
$$

$$= -\frac{1}{1-\sigma_Z}(1-\frac{\sigma_K\sigma_Z}{\sigma_K+\sigma_Z-1})\frac{dp}{p} - \frac{1}{1-\sigma_Z}(\sigma_L - \frac{\sigma_K\sigma_Z}{\sigma_K+\sigma_Z-1})\frac{d\tilde{l}}{1-\tilde{l}}$$
$$= -\frac{1}{1-\sigma_Z}(1+\frac{\sigma_K\sigma_Z}{1-\sigma_K-\sigma_Z})\frac{dp}{p} - \frac{1}{1-\sigma_Z}(\sigma_L + \frac{\sigma_K\sigma_Z}{1-\sigma_K-\sigma_Z})\frac{d\tilde{l}}{1-\tilde{l}}$$

Derivation of Equation (4.32):

$$\frac{d\tilde{y}}{\tilde{y}} = -\frac{\sigma_Z}{1-\sigma_Z}\frac{dp}{p} + \frac{\sigma_L}{1-\sigma_Z}\frac{d(1-\tilde{l})}{1-\tilde{l}} + \frac{\sigma_K}{1-\sigma_Z}\frac{d\tilde{k}}{\tilde{k}}$$
$$= -\frac{\sigma_Z}{1-\sigma_Z}\frac{dp}{p} - \frac{\sigma_L}{1-\sigma_Z}\frac{d\tilde{l}}{1-\tilde{l}} + \frac{\sigma_K}{1-\sigma_Z}\frac{\sigma_Z}{\sigma_K+\sigma_Z-1}(\frac{dp}{p} + \frac{d\tilde{l}}{1-\tilde{l}})$$
$$= -\frac{\sigma_Z}{1-\sigma_Z}\frac{dp}{p} - \frac{\sigma_L}{1-\sigma_Z}\frac{d\tilde{l}}{1-\tilde{l}} + \frac{\sigma_K}{1-\sigma_Z}\frac{\sigma_Z}{\sigma_K+\sigma_Z-1}\frac{dp}{p} + \frac{\sigma_K}{1-\sigma_Z}\frac{\sigma_Z}{\sigma_K+\sigma_Z-1}\frac{d\tilde{l}}{1-\tilde{l}}$$
$$= (-\frac{\sigma_Z}{1-\sigma_Z} + \frac{\sigma_K}{1-\sigma_Z}\frac{\sigma_Z}{\sigma_K+\sigma_Z-1})\frac{dp}{p} + (-\frac{\sigma_L}{1-\sigma_Z} + \frac{\sigma_K}{1-\sigma_Z}\frac{\sigma_Z}{\sigma_K+\sigma_Z-1})\frac{d\tilde{l}}{1-\tilde{l}}$$
$$= -\frac{\sigma_Z}{1-\sigma_Z}(1-\frac{\sigma_K}{\sigma_K+\sigma_Z-1})\frac{dp}{p} - \frac{1}{1-\sigma_Z}(\sigma_L - \frac{\sigma_K\sigma_Z}{\sigma_K+\sigma_Z-1})\frac{d\tilde{l}}{1-\tilde{l}}$$
$$= -\frac{\sigma_Z}{1-\sigma_Z}(1+\frac{\sigma_K}{1-\sigma_K-\sigma_Z})\frac{dp}{p} - \frac{1}{1-\sigma_Z}(\sigma_L + \frac{\sigma_K\sigma_Z}{1-\sigma_K-\sigma_Z})\frac{d\tilde{l}}{1-\tilde{l}}$$

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Chapter 5

Canada's Loss of External Competitiveness: The Role of Commodity Prices and the Emergence of China

Abstract[1]
After booming in the 1990s, Canadian exports have waned over the past decade. The loss of external competitiveness has been quite evident in the US market (the main destination of Canadian exports), where Canada has been overtaken by China as the leading exporter to the US. Using a dynamic panel data of Canadian merchandise exports across different sectors in the US market, we identify the main driving forces behind the weakening export performance in Canada, which include the slowdown in the US demand and the emergence of China as a major exporter. The appreciation of the Canadian loonie over the past decade, in part reflecting the rise in commodity prices, has also played a key role in explaining the loss of Canadian exporters' share in the US market.

JEL Classification: F14, F31.
Key Words: commodity currency, Dutch disease, real effective exchange rate.

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5.1 Introduction

Canadian exports have been on a roller coast over the last two decades, surging from 22 percent of its GDP in 1990 to 40 percent by end-2000, and falling to 24 percent of GDP in 2010. Importantly, the composition of exports changed significantly as not all sectors were affected equally. The rise in exports in the 1990s was widespread, but the expansion in manufacturing was particularly impressive. After 2000, the fall in exports as a share of GDP was predominantly concentrated in manufacturing, while energy exports continued to expand and now represent about 25 percent of all merchandise exports (versus less than 10 percent in the 1990s). While exporters benefited from a depreciation of the Canadian real effective exchange rate (REER) in the 1990s, commodity prices surged in the 2000s and were accompanied by a large appreciation of the REER. This has led to a debate on whether Canada's loss of external competitiveness reflects some degree of "Dutch disease" phenomenon - in the sense that the rise in commodity exports, partly due to a temporary surge in commodity prices, explains the large appreciation of the currency and the subpar performance of manufacturing exports - or whether other factors, namely the increasing productivity gap vis-a-vis its trade partners and 'globalization forces' (e.g. the emergence of China and other economies), have played a greater role.

The striking deterioration in the Canadian export performance is likely due to several factors, as will be argued in this paper. It is nevertheless important to assess to what extent the recent commodity boom affected non-commodity firms' ability to compete. This is particularly relevant given that energy production and exports could potentially be more than doubled over the next decade or so, bringing with it new policy challenges. The development of the large reserves of non-renewable resources can generate important benefits for the country by raising exports, domestic income, and improving the standard of living. At the same time, commodity-rich countries face unique challenges that in some cases demand a re-thinking of the policy framework. To some degree, the domestic economy will have to continue to undertake a structural adjustment as Canada becomes more specialized in commodities - e.g. non-commodity sectors will have to adjust to persistently stronger currency and will likely represent a smaller share of exports and value-added in the economy. Another common challenge is how to buffer the economy from the boom-bust cycles in commodity prices, which could lead to persistent negative impacts on the non-commodity economy and result in lower welfare - in this regard economic policy can play a role in reducing the negative spillovers (see Medas and Zakharova, 2009). It is in this context that this paper looks at the key drivers of Canadian exports, with a special focus on commodities.

In recent years, there has been a renewed interest in studying the links between commodity prices, the exchange rate, and manufacturing production in Canada. A recent OECD (2012) report argues that the decline in central Canada's manufacturing base is correlated to the appreciation in the Canadian dollar (and the large increase in commodity prices). Beine, Bos, and Coulombe (2009) adopt a new approach to extract the commodity-price components from observed exchange rates and prices, and estimate that 42 percent of the manufacturing employment loss in Canada was due to exchange rate developments between 2002 and 2007. Shakeri, Gray, and Leonard (2012) make the argument that although there is evidence of "Dutch disease" effects, they seem to be relatively limited and not likely to be the only explanation for the loss of external competitiveness. The Bank of Canada has also argued that while the rise in commodity prices has had a substantial impact on the exchange rate over the past decade, the decline of manufacturing (as a share of GDP) is largely a process that started well before the recent boom in commodity prices and is common to other advanced economies. (See Carney (2012).)

This paper takes a new look at the causes of Canada's subpar export growth over the last decade by focusing on the performance of Canadian exporters in the US market. Since 2001, non-commodity exports to its neighbor have stagnated and, as a consequence, Canada's share of US imports has steadily declined. By looking at the US market, the destination for the vast majority of Canadian exports, the analysis can be focused on how different factors have affected the ability of Canadian firms to compete. As such, the paper is in a better position to ascertain how much of the decline in manufacturing was due to loss in exports competitiveness, and in particular, how much is related to the appreciation of the exchange rate and the boom in commodity prices. The empirical analysis starts by discussing the sensitivity of the Canadian exchange rate to commodity prices (energy and metals). We then investigate the extent to which the weaker Canadian export reflects the stronger Canadian dollar or other factors, using panel data for US imports by sectors. The detailed panel data allows us to better identify the impact of the exchange rate (and commodity prices) per sector and control for the emergence of China as a major exporter to the US market, while avoiding the potential bias associated with export data at an aggregate level.

The remainder of this paper is organized as follows. The next section provides an overview of the trade dynamics over the last decades, with a special focus on Canadian exports to its southern neighbor. In Section 6.3, we examine the linkage between commodity prices and the exchange rate. Section 6.4 identifies the main reasons behind Canadian exporters' loss of market share in the US market (especially the manufacturing sector) by using panel data analysis. In Section 6.5, we present key conclusions and discuss some policy options.

5.2 Canadian Exports Dynamics

5.2.1 Boom in the 1990s and stagnation in the 2000s

After booming in the 1990s, as firms benefited from the strong US demand, Canadian exports weakened considerably over the last decade. During the 1990s, export volumes expanded at a robust pace of an annual average of 8.5 percent, boosted by strong US imports growth (at an average of 10 percent on an annual basis). The free trade agreements with the US (CUFTA in 1989, replaced with NAFTA in 1994), have also likely helped to preserve Canada's position as the leading exporter to its southern neighbor throughout the 1990s (Romalis, 2005). However, the export performance deteriorated materially after 2000, with export growth stagnating up to 2007 and contracting sharply during the time of the financial crisis in 2008-09. Non-energy exports have been affected the most, remaining below their 2000 volume levels at the end of 2011; whilst energy exports have continued to expand albeit at a slower pace. (See Figures 1 and 2).

The recent loss of external competitiveness has been felt across many sectors in Canada, with the manufacturing sector being the most severely hit. After reaching a high in 2000, the two largest Canadian export sectors - automotive and machinery & equipment, have never fully recovered from the US recession in 2001. In particular, they stagnated until 2006-07, and suffered another adverse shock during the 2008-09 global crisis. (See Figure 3). The forestry industry has also experienced a collapse, reflecting the crisis in the US housing sector, and at the end of 2011 its export volumes remained 35 percent below 2000 levels. On the other hand, Canadian energy exports have continued to expand and were up 25 percent during the same period, benefiting from new discoveries and the opening up of market by its southern neighbor.

5.2.2 The importance of US markets to Canadian exporters

The development in the US markets has played a key role in the change of fortune for Canadian exporters. The US has by far been the largest destination for Canadian products, absorbing more than 75 percent of Canadian exports in the past decades. As such, it is natural that a weakening US demand is part of the explanation for the challenges faced by Canadian exporters in recent years. In particular, the growth of US import volumes decelerated from around 10 percent a year in the 1990s to 4.5 percent annually during the period of 2000-07. Over the same period, Canada's annual non-energy export growth also fell to a meager 1.5 percent. During more recent years, Canadian exporters have been affected by the international crisis in 2008-09 as well, further exacerbating the loss in external markets, for example, non-energy export volumes in 2011 remained 12 percent below 2000 levels.

Canadian firms have also faced a significant loss of market share in US markets. Canada used to be the uncontested leading exporter to the US until 2005. Between 1999 and 2011, Canada's market share declined by 5 percentage points to 14.33 percent of total US imports. This loss is equivalent to 6.5 percent of Canada's GDP, during which time China took over as the main exporter to the US. (See Figure 5). The case of manufacturing exports (e.g. machinery and transport equipment or manufactured products) is particularly striking and relevant, given their relative importance. (See Figures A1-A9). Namely, while Canadian exports represented 19 percent of the total machinery and transport equipment imports by the US in 1999, their share fell to 10.5 percent by 2011. As a consequence, while machinery and transport equipment accounted for close to half of Canadian exports to the US in 1999, by 2011 they represented just slightly more than a third. In comparison, Chinese market share surged from 5.5 percent to 25.5 percent in the US manufacturing market during the same period.

A key question is what factors explain Canada's declining external competitiveness. Even after taking into account the slowdown in US demand, the substantial loss of market share, in addition to the limited success in diversifying to other markets, suggests that there are other underlying causes for the loss of competitiveness. A potential explanation, being raised frequently in recent years, is the impact of the sharp rise in commodity prices, which likely fuelled the substantial appreciation of the Canadian exchange rate hurting manufacturing exports. This effect is likely to be of more importance after 2000 given the rising volumes of energy exports and their prices (in addition to metals). Other factors have also likely played important roles, including the increasing productivity gap relative to key competitors (Figure 6) and tighter competition from emerging economies (e.g. the emergence of China as a world exporter). In the next two sections, we test formally some of these hypotheses.

5.3 The Exchange Rate and Commodity Prices

This section discusses the sensitivity of the Canadian exchange rate to commodity prices. In particular, we examine the long-term relationship between the Canadian exchange rate and commodity prices. Figure 7 does indeed suggest that the movement of the Canadian real effective exchange rate (REER), especially during more recent periods, seems to be highly correlated with the movements of metal and energy prices in real terms. If the relationship is statistically significant and robust, it would be further evidence in favor of potential "Dutch disease" effects in light of the weak manufacturing export performance.

There are several studies discussing to what degree the Canadian dollar has become a commodity currency. A seminal paper by Amano and van Norden (1995) suggests that there was a negative relationship between energy prices and the relative strength of the Canadian dollar over the period of 1973-1993. By "negative", we mean that a rise in energy prices leads to a depreciation of the Canadian dollar versus the US dollar. Note that most studies focused on the bilateral exchange rate between Canada and the US, while we focus on the real effective exchange rate; nevertheless the conclusions should not differ substantially, given that the US remains the major trading partner for Canada.

However, more recent papers argue that this negative relationship has reversed over the past decade or so. For instance, Issa, Lafrance, and Murray (2006) find a positive relationship between the price of energy and the exchange rate from the

1990s onwards. The Bank of Canada argued that the rise in commodity prices accounted for about one half of the appreciation over the past decade. Moreover, other studies have also shown a positive relationship between energy and metal prices, and the exchange rate once we take into account the weight of energy and metals on total exports (one would expect greater correlation between the two when energy export volumes are relatively large). For example, Bayoumi and Muhleisen (2006) find that both energy and non-energy commodity prices have put some upward pressure on the Canadian exchange rate.

Our empirical analysis is in line with previous studies; however, we differ by focusing on the Canadian REER, which tends to be more relevant when analyzing trade competitiveness (the main goal of this paper).

Given that some variables of our interest may not be exogenous, we use a system VECM. Other papers tend to use a single error-correction equation, as they assume that the causality only goes one direction - from commodity prices to the exchange rate.

In order to test for a long-run equilibrium relationship between the real effective exchange rate and the commodity prices, we specify a vector error-correction model (ECM) as follows:

$$
\begin{aligned}
\Delta REER_t \;=\; & \alpha(REER_{t-1} - \beta_1 P_{energyt-1} - \beta_2 P_{metalst-1} - \beta_3 Productivity_{t-1}) \\
& + \gamma_1 \Delta REER_{t-1} + \gamma_2 \Delta P_{energyt-1} + \gamma_3 \Delta P_{metalst-1} \\
& + \gamma_4 \Delta Productivity_{t-1} + \gamma_5 spread_{t-4}
\end{aligned}
$$

where: $\Delta REER$, ΔP_{energy}, ΔP_{metals}, and $\Delta Productivity$ are the first differences of the logged Canadian real effective exchange rate, real energy prices, real metal prices, and an index of Canada-US productivity differential, respectively.

The terms in the bracket are the co-integrating equation, measuring the deviation of the system from its long-run equilibrium relationship. The ADF unit root tests confirm that the REER, commodity prices, and the Canada-US productivity differential index have unit roots at levels. In comparison, the Canada-US interest rate spread is found to be a stationary process. We test for co-integration using the system-approach developed by Johansen (1988), which confirms that there is a co-integration relationship at the 5 percent significance level.

The coefficient α is the error-correction parameter, which measures the adjustment speed of the exchange rate and other endogenous variables towards their long-run equilibrium. The spread variable is the Canada-US interest rate spread. (See Appendix for a detailed definition of the variables and data used.)

The regression also includes a measure of labor productivity in Canada relative to that in the US, which serves as a proxy for Canada's productivity gap with the trading partners of the US. According to the Harrod-Balassa-Samuelson hypothesis, it is expected that a persistent rise in productivity growth relative to the rest of the world will lead to an appreciation of the real exchange rate. Corsetti, Dedola, and Leduc (2005) report that in the face of a productivity shock, the real exchange rate did appreciate in the US and Japan, but depreciated in Italy and the UK, with mixed evidence on Germany.

Our VECM results (see Table 1) confirm that there is a positive long-run relationship between the Canadian REER, and both the energy and metal prices. The estimated long-run impacts (based on the quarterly sample) suggest that a 1 percent increase in the price of energy will lead to around 0.11-0.16 percent appreciation of the Canadian REER, whilst a 1 percent rise in the price of metals will result in a 0.38-0.49 percent appreciation of the Canadian REER. For example, the real energy prices grew by 60 percent between 2000 and 2007, and as such, the REER would be expected to appreciate by almost 10 percent. While significant, the estimates are somewhat lower than other recent estimates in the literature. Shakeri, Gray, and Leonard (2012) argue that for the post-2004 period, the exchange rate become more sensitive to commodity prices. They find that a 1 percent increase in energy prices would lead to an appreciation of 0.5 percent (and 0.7 percent for non-energy commodities). However, their finding is based on a limited sample (1992Q1-2007Q4) and focuses on Canada's bilateral exchange rate with the US.

Building from that, we also test the relationship between commodity prices and exchange rate by using a composite commodity price index (as a weighted average of metals and energy prices). The results (column IV in Table) confirm that there is a strong relationship between commodity prices and the exchange rate. In particular, a 1 percent increase in commodity prices would result in a 0.4 percent appreciation of the REER. This impact would be somewhat lower, if we include more recent period of the financial crisis into our sample under study. (See column V in Table 1.) Given that the composite commodity price index rose by 62 percent between 2000 and 2007, the expected appreciation in the Canadian exchange rate would be close to 25 percent.

On top of that, we find some weak evidence of a long-run impact of productivity gap (between Canada and US) as suggested by the Harrod-Balassa-Samuelson hypothesis. However, the estimated impact is relatively small and is not robust across samples. In the case of Canada, the fall in the productivity gap between 2000 and 2007 would imply a depreciation of 0.5 percent. In reality, the REER did not depreciate to compensate for the loss of productivity, as the effect of the commodity prices dominated and led to a substantial appreciation in the Canadian dollar. The relatively weak productivity growth in both advanced economies (e.g. US and Germany) and emerging markets (e.g. China and Mexico) have posted stronger productivity gains over the last decade, which has made Canadian firms more vulnerable to the appreciation of its currency and adverse economic shocks.

The regressions using higher frequency data after controlling for market volatility (as measured by the VIX index) are broadly in line with the results for the quarterly data. Periods of high market volatility may also be associated with large fluctuations in commodity prices, which potentially affect the estimates of the REER's sensitivity to commodity prices. The results (Table 1, columns VI and VII) confirm the effect of commodities prices on the REER. However, the regressions provide limited clues regarding the impact of the changes in the VIX on the Canadian REER. A positive relationship would indicate to us that investors perceive Canada as a safe haven during periods of market turmoil. There was some evidence of such an effect, but the results were only statistically significant at 10 percent and only for some specifications. (Note that a statistically significant negative estimate in the co-integrating equation indicates that over time the Canadian dollar would appreciate in response to a higher VIX). The lack of robustness may be related to the fact that the status as a safe haven is thought to be a relatively recent phenomenon.

5.4 Canadian Exports in the US Market

5.4.1 Methodology and Data

In this section, we conduct a dynamic panel data analysis to quantify the factors that have been the main driving forces behind Canada's loss of market share in the US. Following the discussion above, the focus will be on the movements of the exchange rate, driven by commodities prices, and the increased competition from the emergence of China in international trade.

A challenge, faced by other studies trying to identify the impact of movements in the exchange rate on manufacturing, is the need to differentiate those effects from the factor(s) driving the structural declining weight of the sector in advanced economies. The relatively weaker performance of manufacturing in Canada can be partly linked to a loss of external competitiveness vis-a-vis its trade partners (e.g. appreciation of the currency, weaker productivity growth). However, it may also reflect other forces, especially the common factors that have also affected other advanced economies (possibly linked to the effects of globalization). By looking at a panel data of imports to the US markets, we are able to better identify the factors that have hurt Canadian firms' competitiveness in their main destination market, and quantify which sectors have been the most affected. The focus is on market shares (instead of actual trade volumes), as it already adjusts for potential effects due to changes in the US demand that affect all exporters.

In addition, the methodology used allows us to better isolate the different factors, such as the increasing competition from China, since it has emerged as a large exporter in the US market. The sector-level trade data allow us to identify not only the overall effect on Canadian exports, but also which specific sectors (e.g. manufacturing) are more exposed to the movements in the exchange rate (or commodity prices) and/or competition from China.

Our empirical specification is as follows:

$$CAN_{i,t} \quad = \quad \beta_1 REER_t + \beta_2 \sum_t CHN_{i,t} + \beta_3 \sum_t X_t \qquad (5.1)$$

where: $CAN_{i,t}$ represents the Canadian share of US imports of good i at time t, $REER_t$ is the Canadian real effective exchange rate, $CHN_{i,t}$ is the Chinese share of US imports of good i at time t; X_t is a vector of control variables, including Canadian domestic demand, US GDP growth, and the dummies for the introduction of CUFTA/NAFTA.[2]

Statistically significant coefficients for the exchange rate would provide a measure of the exchange rate impact on the Canadian market share in the US market. A significant negative relationship for sector i indicates that an appreciation would result in a loss of market share in that sector and would be evidence in support of "Dutch disease" effects, given that the results in the previous section suggest commodity prices have been the key driver of the movements in the Canadian exchange rate. We focus in particular on the manufacturing sector. In addition, for some regressions we will include commodity prices as an instrument for the exchange rate, which would reinforce the evidence in favor if (or against) "Dutch disease"-type effects.

The inclusion of China's share as a dependent variable will also help to identify or control for the effect of China's emergence as a large player in international trade over the last decades, which was a significant exogenous shock to Canada (and other countries). As discussed by Cerra and Saxena (2002), Lardy (2004), and Rodrik (2006), China's emergence likely reflects Chinese government's own economic reform policies, which are exogenous to others.

The regressions use imports data at 4-digit levels of the SITC (Standard International Trade Classification) over the period from 1975 to 2010; however, the analysis is mainly centered on the pre-2008 period to exclude the effects of the international financial crisis; nevertheless we also present the results for the larger sample. The main regressions use data on manufacturing imports, which is constructed by including manufactured goods (SITC6), machinery and transport equipment (SITC7), and miscellaneous manufactured articles (SITC8). See data appendix for more details.

[2] CUFTA is a free-trade agreement (FTA) between Canada and the United States, entered in 1989. NAFTA, replaced CUFTA since 1994, is a free-trade agreement (FTA) among Canada, the United States, and Mexico. Romalis (2005) shows that CUFTA and NAFTA had a substantial impact on international trade volumes. Holding constant the total import volumes from the rest of the world, this implies that the Canada's share in the US market is expected to increase due to the implementation of CUFTA and NAFTA.

5.4.2 The Results

The dynamic panel analysis is based on GMM estimators suggested by Arellano and Bond (1991), as they ensure efficiency and consistency provided that the models are not subject to serial correlation of order two, and that the set of instrument variables used are valid (tested using Sargent-Hansen tests).[3] The use of GMM will help to address potential issues related to autocorrelation, especially as we have a small T (time series) relative to a large N (cross section).

In addition, to avoid potential problems from excessive number of instruments, we keep the number of lag instruments for the endogenous variables less than the number of groups. A potential problem with the GMM estimation method developed by Arellano and Bond (but also common to others) is that large number of instruments can lead to over-fitting bias of instrumented variables. The Arellano-Bond estimator potentially uses as instruments all the lagged information contained in the sample and increases the risk of over-fitting biases especially in cases where T (period units) is large. To reduce that risk, the regressions keep the lag periods instruments less than the number of groups. See Rodman (2009) and Arellano (2003) for further details.

Furthermore, we also test for the robustness of the results using additional and/or alternative instrument variables as will be discussed.

Non-Energy Exports

We first present a brief summary of the results for all non-energy exports, but the main discussion will center on the results for the manufacturing sector. The econometric results indicate that an appreciation of the REER puts downward pressure on the Canadian non-energy share in the US market. In particular, for the small sample (1975-2007), a 10 percent appreciation of the REER drives down the Canadian non-energy imports share by about 0.6 percentage points on average (Table 2). The estimated impact is somewhat larger when the regressions include commodity (energy and metals) or energy prices as instrument variables for the REER. This suggests that the rise in commodity prices was key in driving the loss of market share associated with the exchange rate appreciation. The results for the larger sample (1975-2010) show an even stronger impact of movements in the

[3] All regressions are tested for the presence of autocorrelation of order two.

exchange rate (and commodity prices) on Canada's market share (Table 2b). A 10 percent appreciation would lead to 0.8-0.9 percentage points fall in Canada's market share.

As conjectured, the competition effect from China is also significant. The Canadian non-energy import share falls by an estimated 13 basis points for every 1 percentage point increase in the Chinese share (Table 2). It is worthwhile noting that the estimates are based on a sample of disaggregated 4-digit import data, which provides a more robust measure of the impact of China than if we had used aggregate data on Canadian imports to the US. In reality, regressions for the different import groups do show that the impact of China varies considerably - in some cases being statistically significant (where both countries compete) while in others the impact is not significant. The results confirm the premise that the emergence of China in international trade had a significant impact (in some sectors) that is not being captured by movements in the exchange rate. Namely, the regressions captured the effect of changes in the REER (or relative prices), but do not capture the effect from a new entrant in the market that has a significant relatively lower price level (as China). By including Chinas share, we control for that effect.

In addition, other control variables have the expected effects. A stronger Canadian domestic demand drives down its exports, whilst Canadian firms benefited from the introduction of CUFTA.

Manufacturing Exports

We now turn to the manufacturing sector, where the evidence points to an even stronger impact of the exchange rate on Canada's market share (Table 3). In particular, a 10 percent increase in the REER results in a decline in the Canadian share in the US market by almost 0.9 percentage points (columns III and IV). To provide a sense of the magnitude involved, this elasticity suggests that the appreciation of the REER between 1999 and 2007 (close to 32 percent) would result in a fall of Canada's market share by 2.9 percentage points - helping explain a large proportion of the 4.5 percentage points in the actual fall during the period.

Additionally, to better identify the impact on the market share due to changes in the REER associated with movements in the commodity prices, we added commodity prices as an instrument variable. The estimated elasticities point to a stronger impact of exchange rate movements when linked to movements in commodity prices on the market share (columns V and VI). The results reinforce the argument that the rise in commodity prices, via REER appreciation, have played an important role in constraining the ability of Canadian firms to compete abroad. As was the case for the non-energy exports, the larger sample (1975-2010) shows an even larger impact of the exchange rate and commodity prices on Canada's market share - a 10 percent appreciation would lead to a 1.5 percentage points of fall in Canada's market share.

Overall, the REER appreciation did have a material negative effect on Canadian manufacturing exports. A simple counter-factual simulation shows that if the REER had stayed constant between 2000 and 2007, the Canadian manufacturing share in the US market would have been about 22.5 percent higher than the actual share in 2007. This would imply that export growth would have been higher by about 2.25 percentage points every year between 1999 and 2007, and manufacturing exports would have been higher by about 2.5 percent of GDP in 2007. While the appreciation of the exchange rate did not reflect only the commodity-price boom, the evidence does suggest that the rise in energy and metal prices did play a key role in putting upward pressure on the exchange rate, and dampening growth in the manufacturing sector.

The rise of China as a major trade player also had an impact on Canadian manufacturing exports. The results (Table 3, columns III to VI) indicate that a 1 percentage point increase in China's market share led to a decline of about 13 basis points in Canada's market share. While the elasticity may appear relatively small, the impact on Canada was significant given the large rise in China's share over the last decades. In particular, China's share in manufacturing rose by 14 percentage points between 1999 and 2007, which would imply, based on the estimated elasticity, a drop of close to 1.9 percentage points in Canada's market share. As there could be a possibility that Canada and China shares are reacting to a common shock, we tested the robustness of the results by introducing instrument variables for the China share: the productivity lag between Canada and China (column VII) and China productivity growth (column VIII). In both cases, the estimated impact of China remains statistically significant and is even larger (1 percentage point increase in China's share would lead to a fall in Canada's share by 18-26 basis points). The larger sample (1975-2010) shows a somewhat larger impact of China - a 1 percentage point increase in China's share would lead to a fall in Canada's share by 17 basis points. This estimated elasticity would imply, the China effect would explain about 40 percent of Canadas loss of market share in the 2000-2011 period.

The other control variables tend to have the expected sign. The introduction of CUFTA and NAFTA had a statistically significant positive (although small) impact on Canada's market share in most regressions. However, for regressions where the China share was not included (columns I and II), the impact of the trade agreements was negative. This may reflect the fact that when CUFTA/NAFTA was implemented, it was also a period of robust rise in Chinese exports to the US. By controlling for China's exports, it is possible to better identify the effect of the free-trade agreements. The domestic demand in Canada also tends to affect exports negatively to the US, possibly due to a substitution effect (when domestic market is strong, there is less incentive to export). Furthermore, we controlled for potential effects of changes in the US economy as, despite the fact that we use market share, it could be possible that changes in the US economy affect Canada more than other countries. We did not find robust evidence of a statistically significant impact although some specifications did suggest Canadian export share was sensitive to changes in the US economy.

5.5 Conclusion and Policy Options

Canada's waning export performance over the last decade reflects several factors, including its high dependence on the US markets and exposure to commodity prices. Exports as a share of GDP fell by 16 percentage points over the last decade. In part, this reflects the very large share of exports destined to the US market, making Canada highly exposed to developments in its southern neighbor. As domestic demand slowed down in the US in the 2000s, so did Canadian exports. At the same time, Canada also lost significant market share in the US, with material impact on exports and GDP.

Our findings indicate the loss of market share was due mainly to the large exchange rate appreciation, associated with rising commodity prices, and the emergence of China in the world market. The exchange rate appreciated by 32 percent between 1999 and 2007, mostly driven by the large rise in commodity prices. In turn, the analysis suggests that the appreciation of the currency explains close to 2/3 of the loss in manufacturing market share in the same period.[4] The integration of emerging economies to the international economy (especially China) has also played an important role. Our estimates indicate that a 1 percentage point increase in China's market share led to a decline of about 13 basis points in Canada's market share - as China's share in manufacturing rose by 14 percentage points between 1999 and 2007, this helped explain a drop of more than 1.75 percentage points in Canada's manufacturing market share.

The rising productivity gap relative to key trade competitors further hinders the ability of Canadian exporters to adjust to the strengthening of its currency and the stiffer competition from emerging markets. While Canadian firms would in any case be affected by the appreciation of the currency and the emergence of China, the weak productivity growth over the last decade has made it more difficult to adjust. Further efforts to address the weak productivity growth and increase competitiveness would help the structural adjustment by the non-commodity sectors - that will need to continue as the energy sector is expected to expand substantially and will likely put further upward pressure on the exchange rate.

The developments over the last decade also suggest that macro-fiscal policy will need to take into account the growing importance of the commodity exports. The share of the energy sector on the trade balance and GDP has grown, in part reflecting that Canada is becoming richer with the development of the vast natural resources, but also implying that the economy is more exposed to large swings in commodity prices. The non-energy trade balance is now significantly negative (deficit of 4 percent of GDP, while in the 1990s was in surplus) and is dependent on a large surplus on the energy trade balance. In reality, the increase in energy exports reflects much more about the rise in oil prices than volume increases, and as such, the positive impact (in incomes, export and fiscal revenues) could be quickly reversed. As energy production and associated fiscal revenues are expected to expand substantially over the next decade, Canada will increasingly face the same challenges faced by other commodity rich countries, especially regarding fiscal policy and management. It will be increasingly relevant for governments,

[4] For a longer sample including the crisis, 1999-2011, the 37 percent appreciation explained somewhat less than 60 percent of the loss in market share.

especially for resource-rich provinces, to better integrate in the policy framework an assessment of the effects of the greater dependence on commodities for the economy and public accounts, including quantifying and managing key risks.

Appendix: Data Resources

A1. Time-Series Data (annual, quarterly, and monthly)

- *Real effective exchange rate (REER) based on CPI*: computed by the IMF.
- *Commodity price index*: The energy price index and metals price index are from the Bank of Canada, with weights for price index from Canadian trade data.
  http://www.bankofcanada.ca/rates/price-indexes/bcpi/
  We deflate the commodity price index by the US GDP deflator (from Haver Analytics), to get the real commodity price index.
- *Canada-US 3-month interest rate spread*: based on the difference between the 3-month Canadian Prime Corporate Paper and the US 3-month non-financial commercial paper (both from Haver Analytics).
- *Canada-US labor productivity differential*: measured by GDP per person employed in 2011 EKS dollar, computed by the Conference Board.
- *Canada domestic demand*: based on national accounts data. (Source: Haver)

A2. Panel Data (annual)

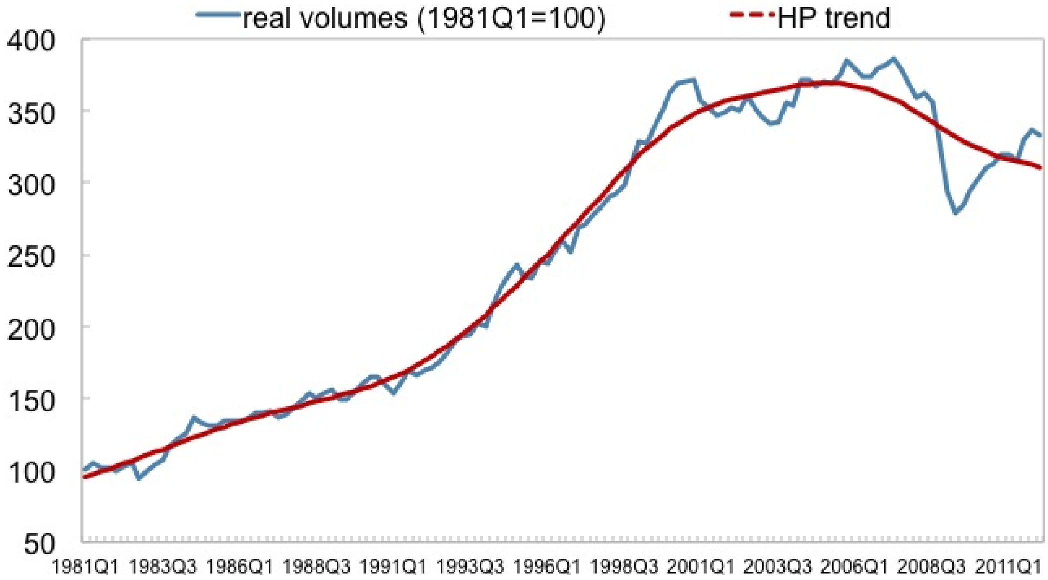
Standard International Trade Classification(SITC) data are from UN-Comtrade, which are complied and documented in Feenstra *et al.* (2005).

- SITC0 = food and live animals.
- SITC1 = beverages and tobacco.
- SITC2 = crude materials and inedible except fuels.
- SITC3 = mineral fuels, lubricants and related materials.
- SITC4 = animal and vegetable oils, fats and waxes.
- SITC5 = chemicals and related products.
- SITC6 = manufactured goods.
- SITC7 = machinery and transport equipment.

- SITC8 = miscellaneous manufactured articles.
- SITC9 = commodities and transactions not classified elsewhere in the SITC.

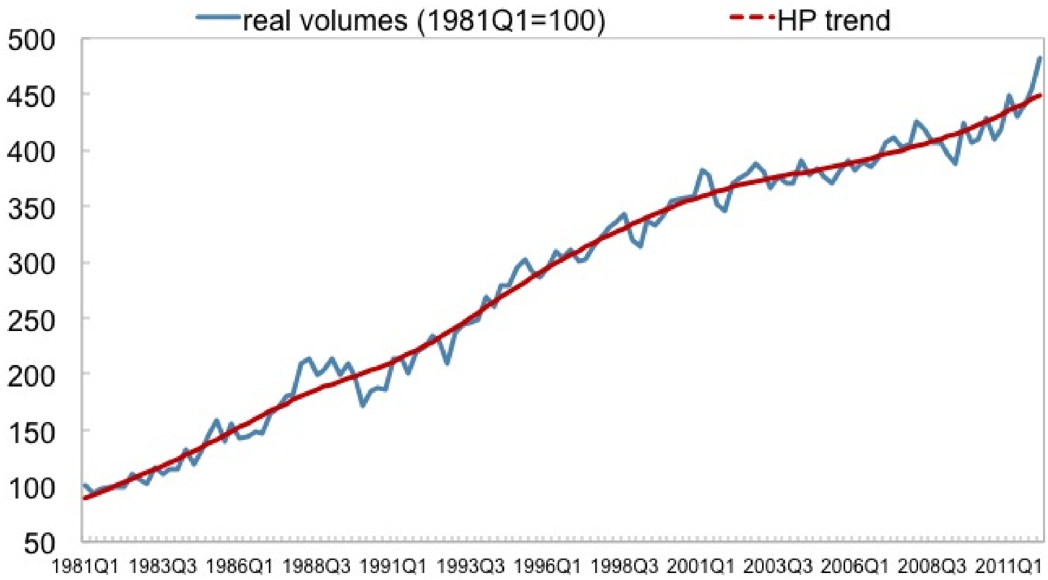
We define the manufacturing sector to be SITC6, SITC7 and SITC8. The non-energy sector is computed by excluding SITC3 from the SITC sectors.

Figure 1. Canadian non-energy exports



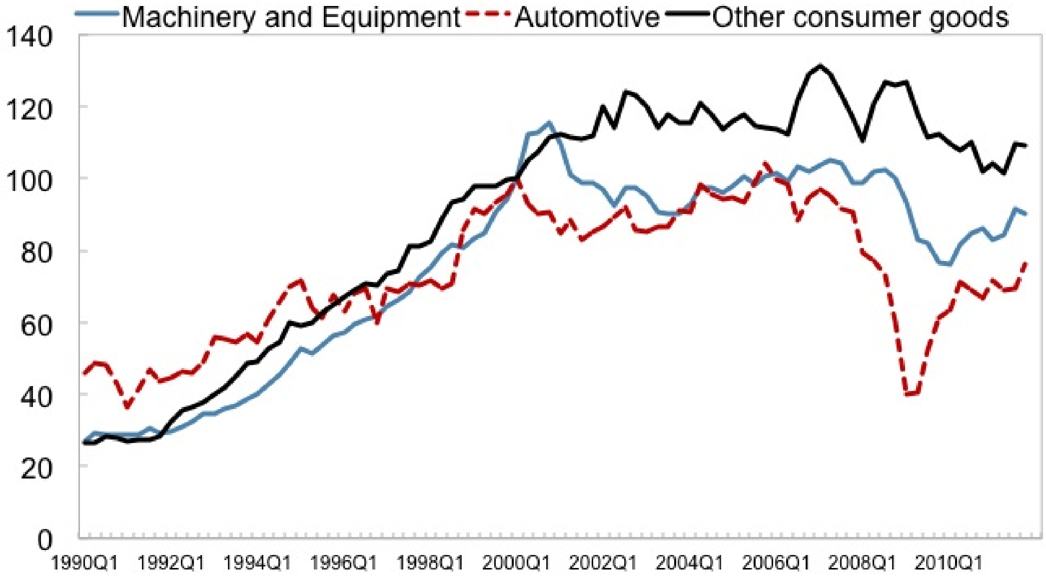
Source: Haver Analytics and authors' estimates

Figure 2. Canadian energy exports



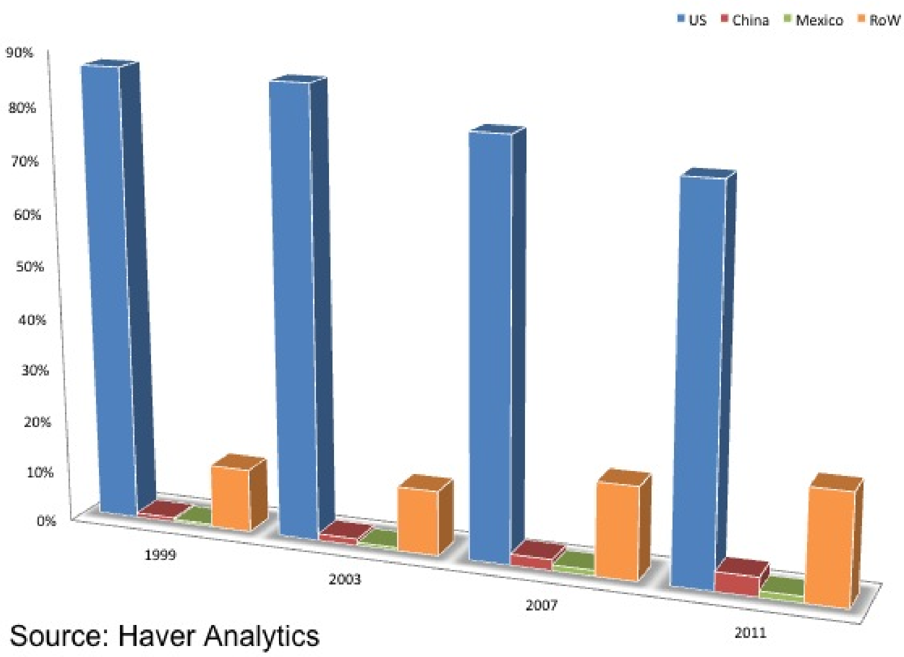
Source: Haver Analytics and authors' estimates

Figure 3. Canadian main exports



Source: Haver Analytics

Figure 4. Canadian exports share - by destination



Source: Haver Analytics

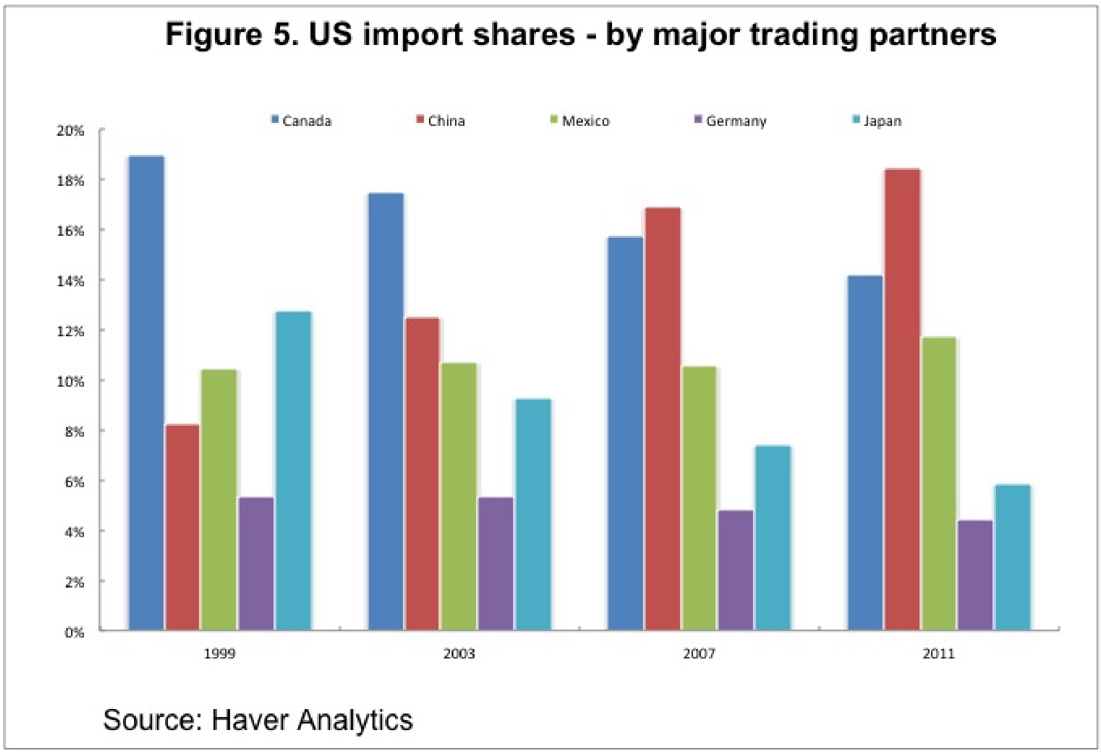


Figure 5. US import shares - by major trading partners
Canada
China
Mexico
Germany
Japan
20%
18%
16%
14%
12%
10%
8%
6%
4%
2%
0%
1999
2003
2007
2011
Source: Haver Analytics

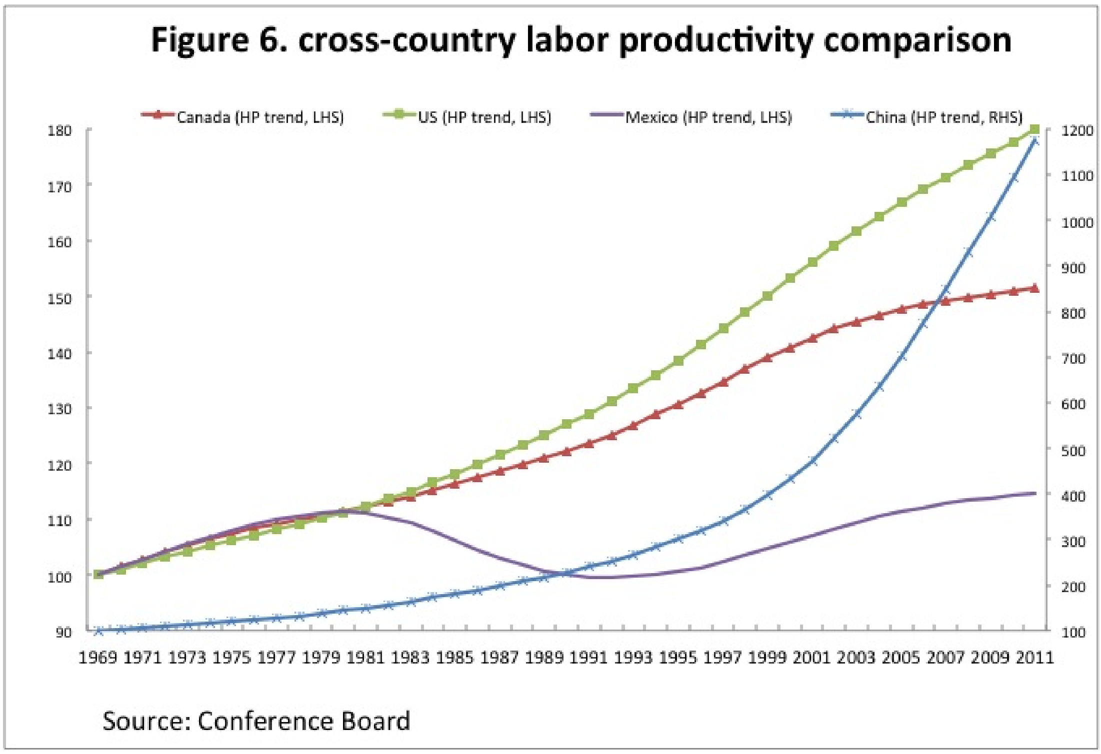
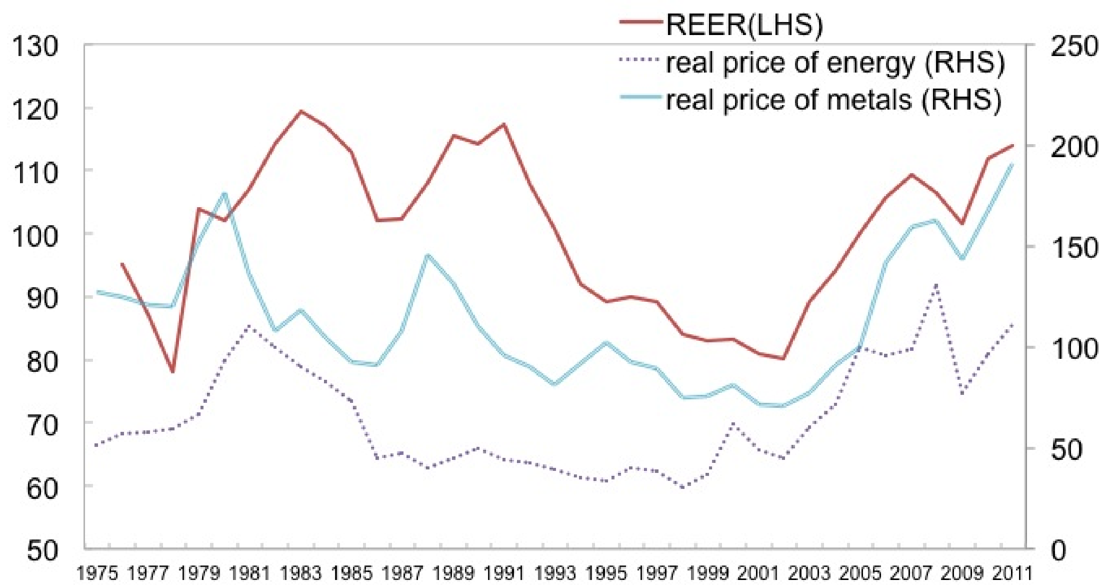


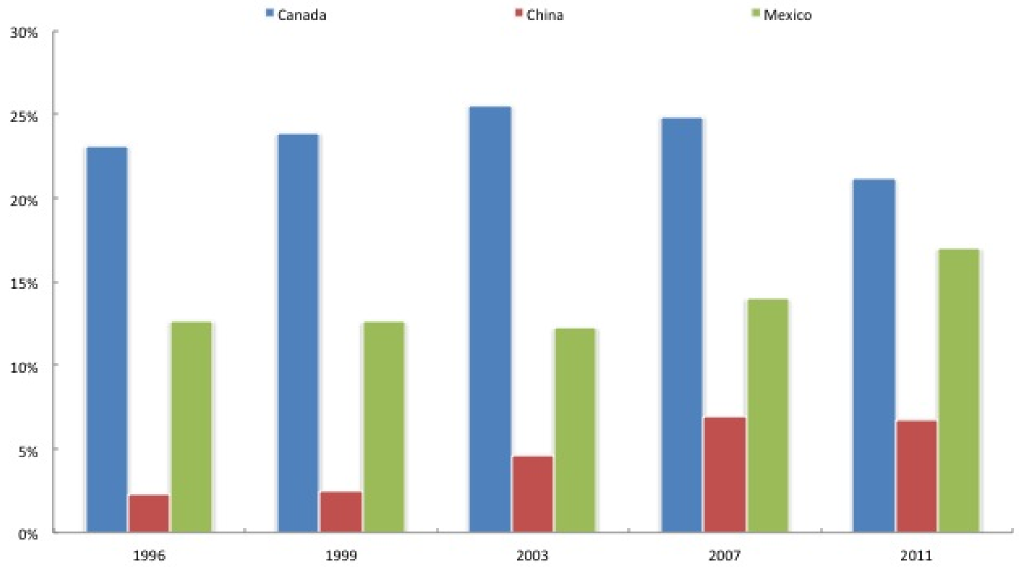
Figure 6. cross-country labor productivity comparison
Canada (HP trend, LHS)
US (HP trend, LHS)
Mexico (HP trend, LHS)
China (HP trend, RHS)
180
170
160
150
140
130
120
110
100
90
1200
1100
1000
900
800
700
600
500
400
300
200
100
1969 1971 1973 1975 1977 1979 1981 1983 1985 1987 1989 1991 1993 1995 1997 1999 2001 2003 2005 2007 2009 2011
Source: Conference Board

Figure 7. Canadian exchange rate vs. commodity prices



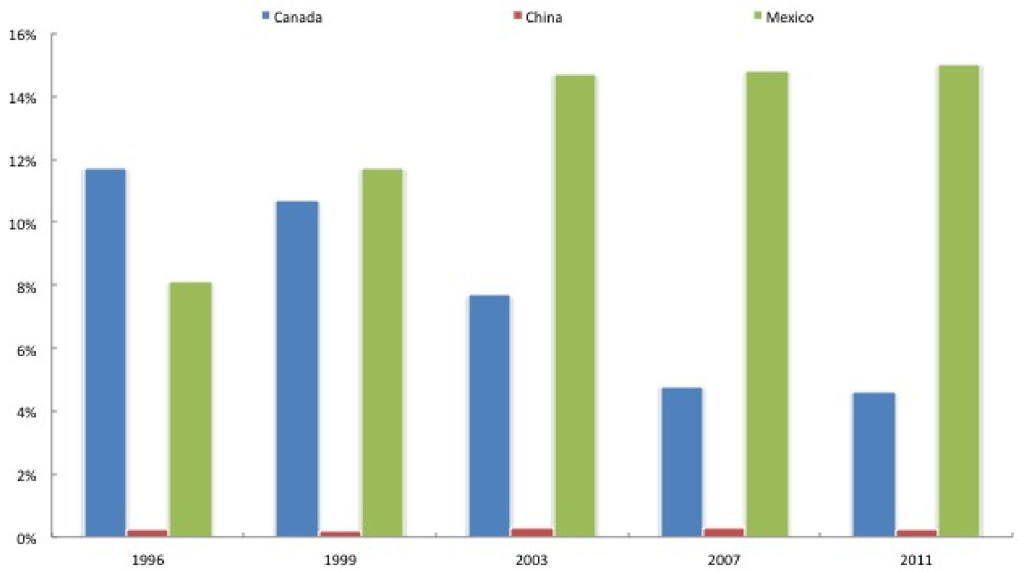
Sources: IMF, Bank of Canada, Haver Analytics

Figure A1. Shares in US import market - Food & Live Animals



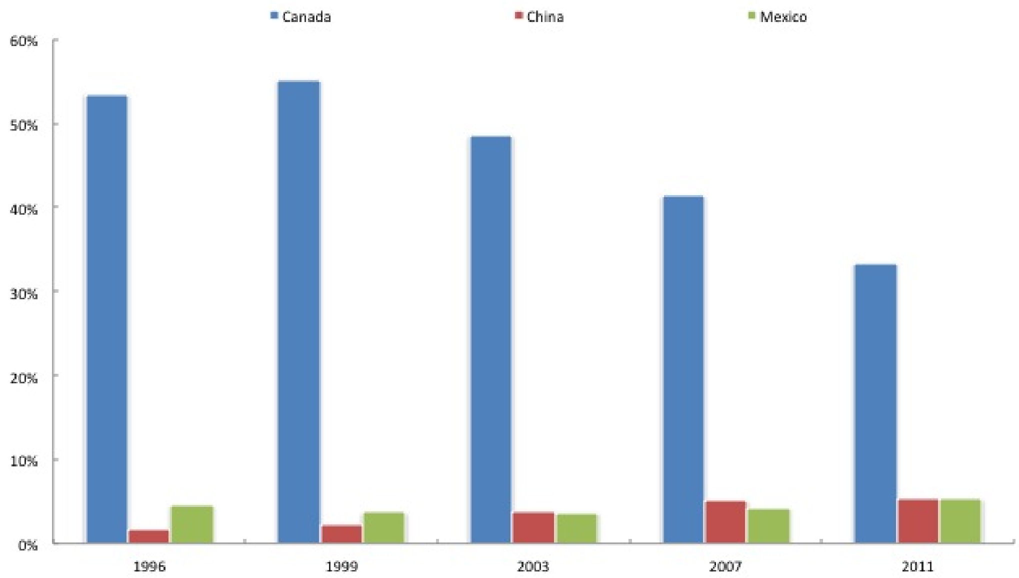
Sources: Haver Analytics, US Census Bureau

Figure A2. Shares in US import market - Beverages & Tobacco



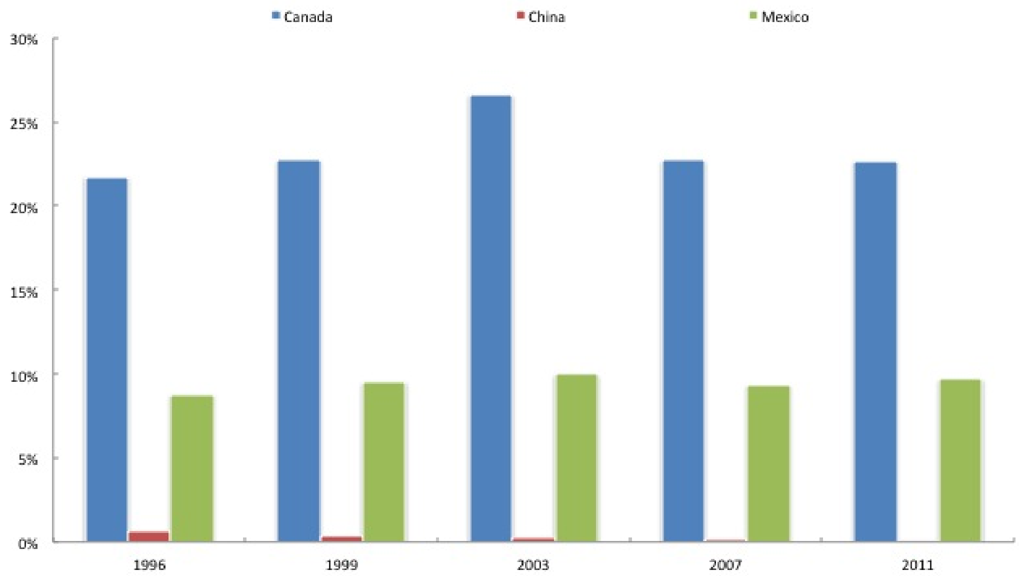
Sources: Haver Analytics, US Census Bureau

Figure A3. Shares in US import market - Crude Materials, Inedible, ex Fuels



Sources: Haver Analytics, US Census Bureau

Figure A4. Shares in US import market - Mineral Fuels, Lubricants & Related Materials



Sources: Haver Analytics, US Census Bureau

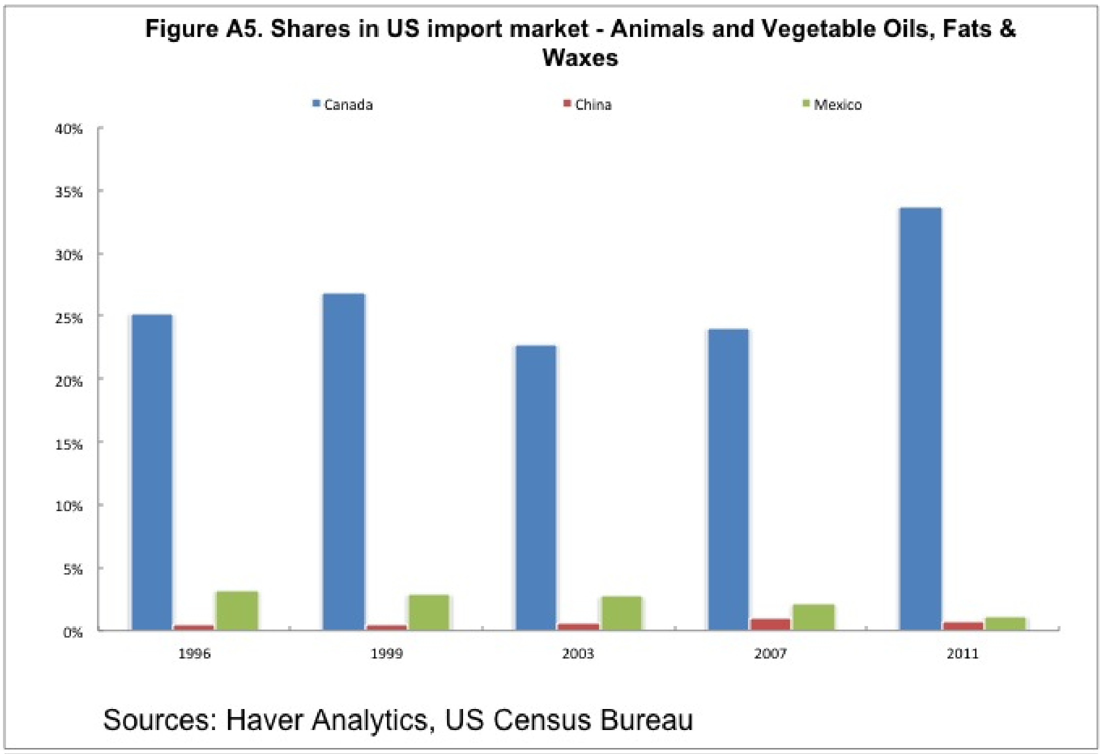


Figure A5. Shares in US import market - Animals and Vegetable Oils, Fats & Waxes
Canada
China
Mexico
40%
35%
30%
25%
20%
15%
10%
5%
0%
1996
1999
2003
2007
2011
Sources: Haver Analytics, US Census Bureau

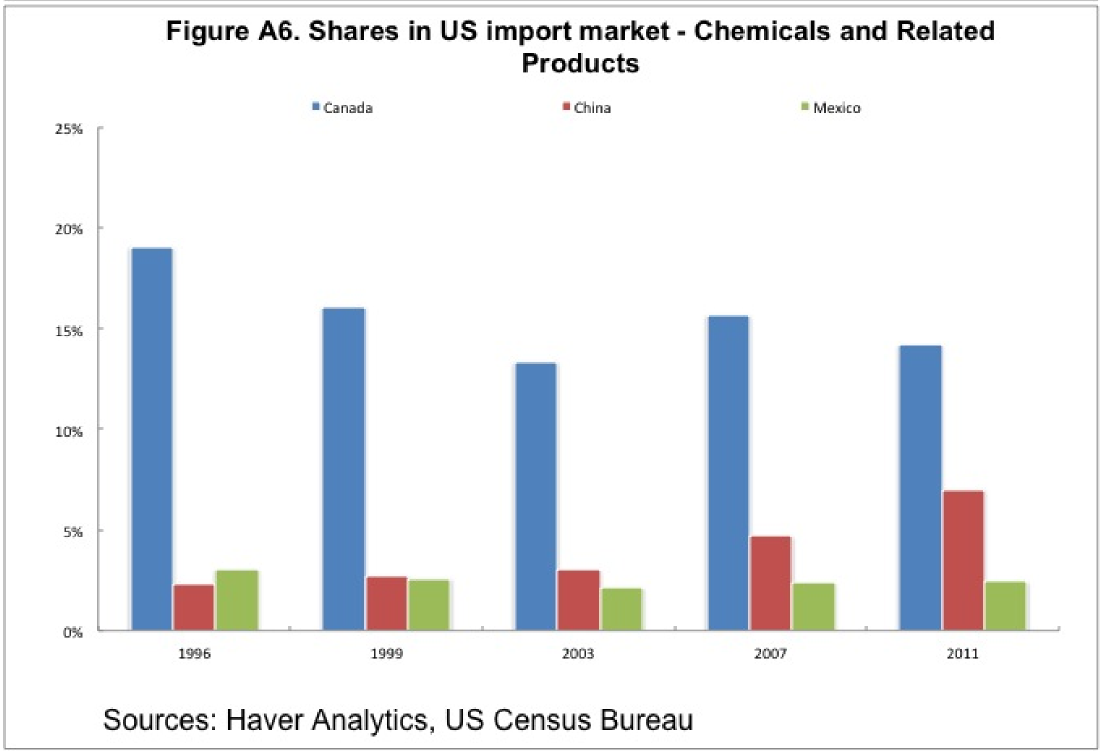
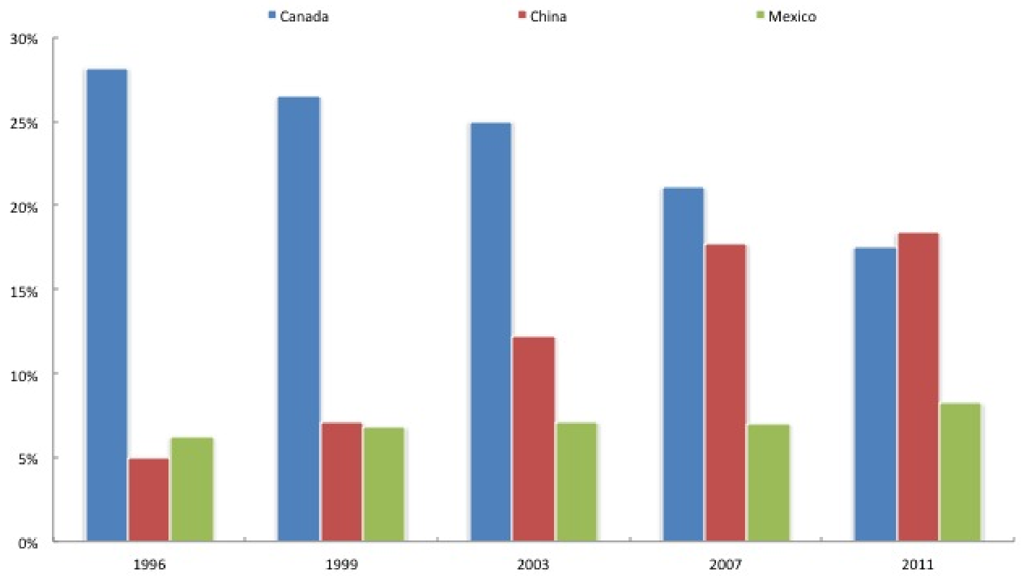


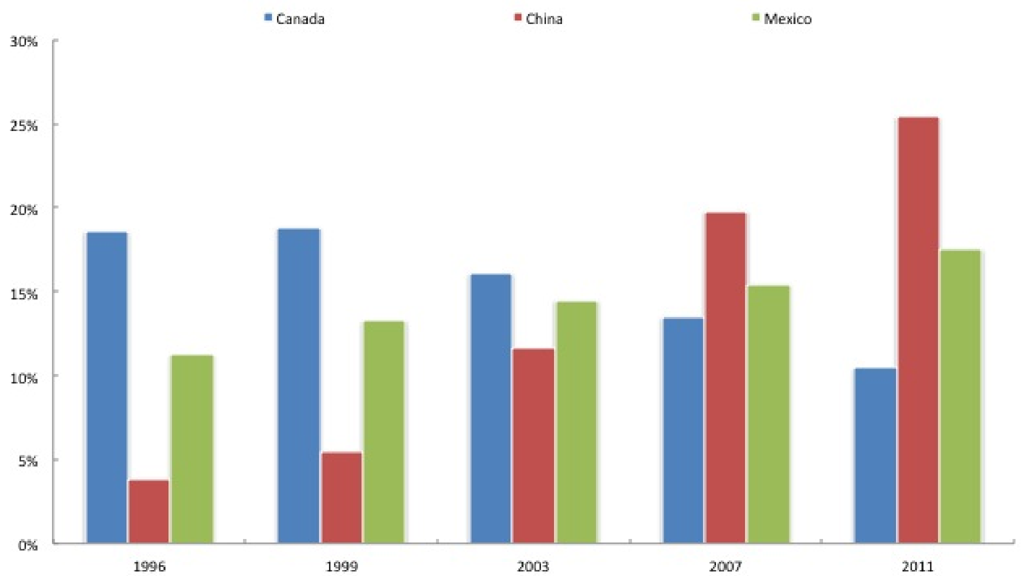
Figure A6. Shares in US import market - Chemicals and Related Products
Canada
China
Mexico
25%
20%
15%
10%
5%
0%
1996
1999
2003
2007
2011
Sources: Haver Analytics, US Census Bureau

Figure A7. Shares in US import market - Manufactured Goods



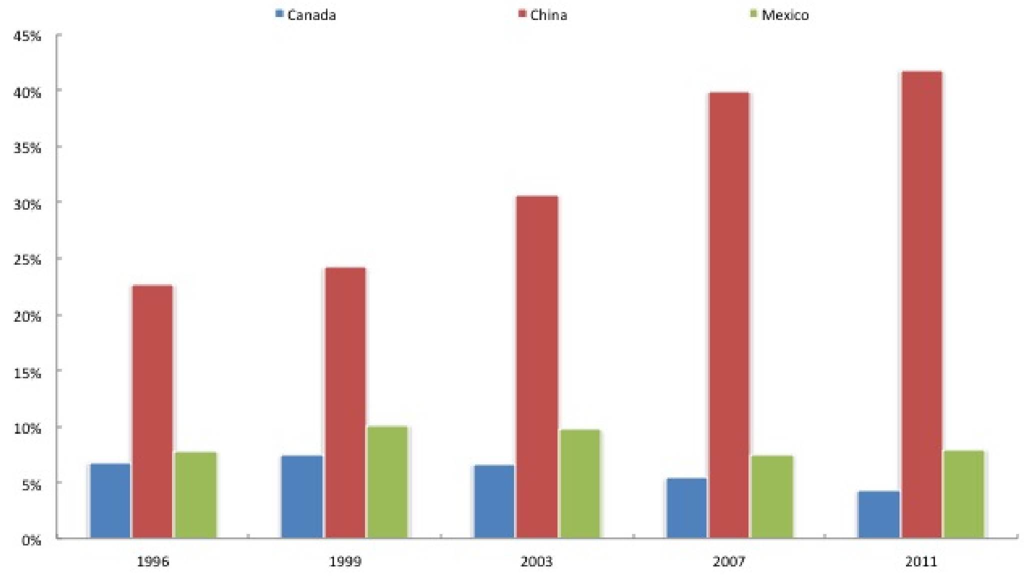
Sources: Haver Analytics, US Census Bureau

Figure A8. Shares in US import market - Machinery and Transport Equipment



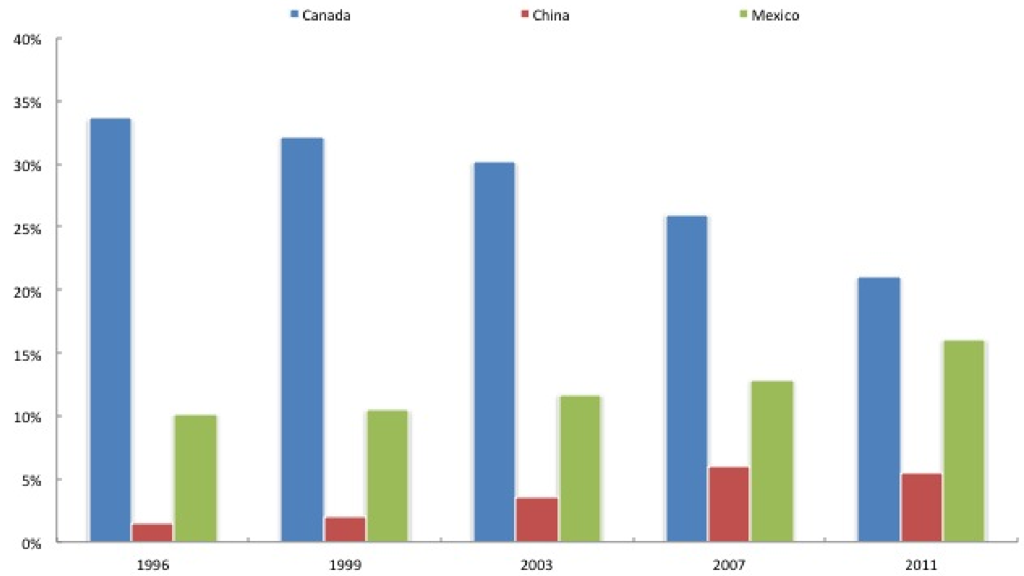
Sources: Haver Analytics, US Census Bureau

Figure A9. Shares in US import market - Misc Manufacture Articles



Sources: Haver Analytics, US Census Bureau

Figure A10. Shares in US import market - Commodities & Transactions



Sources: Haver Analytics, US Census Bureau

Table 1. Exchange Rate and Commodity Prices (Vector error-correction model)

Dependent variable: Real effective exchange rate (CPI based)

	I	II	III	IV	V	VI	VII
Cointegrating equation							
price of energy	-0.110 ***	-0.141 ***	-0.167 ***				
t-statistic	[-2.37]	[-4.93]	[-5.37]				
price of metals	-0.370 ***	-0.490 ***	-0.378 ***				
t-statistic	[-4.77]	[-9.27]	[-7.46621]				
composite commodity price				-0.388 ***	-0.329 ***	-0.369 ***	-0.278 ***
t-statistic				[-4.65]	[-4.62]	[-4.06]	[-4.09]
Canada-US productivity gap		-0.390	-0.985 ***				
t-statistic		[-1.24]	[-3.69]				
VIX index						-0.180 *	-0.157 *
t-statistic						[-1.74]	[-1.77]
error-correction parameter	-0.116 ***	-0.178 ***	-0.141 ***	-0.064 ***	-0.067 ***	-0.030 ***	-0.030 ***
t-statistic	[-4.70]	[-5.92]	[-3.82]	[-3.56]	[-3.18]	[-3.48]	[-3.28]
interest rate spread (1 year)	0.004 ***	0.004 ***	0.003 *	0.003 **	0.004 **	0.001 **	0.001 **
t-statistic	[2.92]	[3.00]	[1.72]	[2.05]	[2.24]	[2.10]	[1.99]
number of observations	112	105	122	112	129	210	263
sample period	1980Q1-2007Q4	1981Q4-2007Q4	1981Q4-2012Q1	1980Q2-2007Q4	1980Q1-2012Q1	1990M7-2007M12	1990M7-2012M5

*,**,*** indicate respectively statistical significance at the 10, 5, and 1% level.

Note: All variables are expressed in logarithms except for interest rate spread, which is the Canada-US 3-month interest rate spread (4 lags).

Table 2. Canada shares in the US non-energy import market (1975-2007)

Dependent variable: Canada share of US non-energy imports

	I	II	III
real effective exchange rate	-0.064 ***	-0.074 ***	-0.075 ***
p-value	0.000	0.000	0.000
China share (contemporaneous and lag)	-0.135 ***	-0.132 ***	-0.133 ***
p-value	0.000	0.000	0.000
Canada domestic demand growth (lagged)	-0.002 ***	-0.002 ***	-0.002 ***
p-value	0.000	0.000	0.000
dum_CUFTA/NAFTA	0.015 **	0.015 **	0.015 **
p-value	0.020	0.025	0.025
A-B test in AR(1) in 1st difference	0.000	0.000	0.000
A-B test in AR(2) in 1st difference	0.272	0.279	0.280
Hansen test of overid. restrictions	0.234	0.305	0.303
Number of observations	11374	11374	11374
Number of instruments	477	477	477
Number of groups	525	525	525

*,**,*** indicate respectively statistical significance at the 10, 5, and 1 percent level.

The dependent variable is the Canadian share of U.S. imports in the non-energy sector (as defined in the appendix). The independent variables are the log of the contemporaneous Canadian real effective exchange rate (REER), the China share of US imports of manufacturing (comtemporaneous and lag), the lagged Candian domestic demand growth, the lagged US GDP growth and a dummy for CUFTA/NAFTA (takes value 1 for years under CUFTA or NAFTA and zero otherwise). In regression II, the log of commodities prices is used as an instrument for the REER; in regression III, the log of energy price is used as an instrument for the REER.

The table presents the long-term elasticities. The standard errors used to test level of significance of the long-term coefficients are based on the delta method.

Table 2b. Canada shares in the US non-energy import market (1975-2010)

Dependent variable: Canada share of US non-energy imports

	I		II		III	
real effective exchange rate	-0.077	***	-0.089	***	-0.090	***
p-value	0.000		0.000		0.000	
China share (contemporaneous and lag)	-0.176	***	-0.170	***	-0.170	***
p-value	0.000		0.000		0.000	
Canada domestic demand growth (lagged)	0.000		0.000		0.000	
p-value	0.522		0.413		0.383	
dum_CUFTA/NAFTA	0.019	***	0.018	***	0.018	***
p-value	0.007		0.009		0.010	
A-B test in AR(1) in 1st difference	0.000		0.000		0.000	
A-B test in AR(2) in 1st difference	0.110		0.113		0.113	
Hansen test of overid. restrictions	0.455		0.474		0.470	
Number of observations	12831		12831		12831	
Number of instruments	522		522		522	
Number of groups	529		529		529	

*,**,*** indicate respectively statistical significance at the 10, 5, and 1 percent level.

The dependent variable is the Canadian share of U.S. imports in the non-energy sector. The independent variables are the log of the contemporaneous Canadian real effective exchange rate (REER), the China share of US imports of manufacturing (comtemporaneous and lag), the lagged Candian domestic demand growth, the lagged US GDP growth and a dummy for CUFTA/NAFTA (takes value 1 for years under CUFTA or NAFTA and zero otherwise). In regression II, the log of commodities prices is used as an instrument for the REER; in regression III, the log of energy price is used as an instrument for the REER.

The table presents the long-term elasticities. The standard errors used to test level of significance of the long-term coefficients are based on the delta method.

Table 3. Canada shares in the US manufacturing import market - Panel GMM (1975-2007)

Dependent variable: Canada share of US manufacturing imports

	I	II	III	IV	V	VI	VII	VIII
real effective exchange rate	-0.062 **	-0.086 ***	-0.092 ***	-0.093 ***	-0.106 ***	-0.108 ***	-0.073 ***	-0.084 ***
p-value	0.014	0.005	0.000	0.000	0.000	0.000	0.001	0.000
China share (contemporaneous and lag)			-0.134 ***	-0.138 ***	-0.132 ***	-0.132 ***	-0.264 ***	-0.178 **
p-value			0.001	0.001	0.001	0.001	0.001	0.011
Canada domestic demand growth (lagged)	-0.004 ***	-0.004 ***	-0.003 ***	-0.003 ***	-0.003 ***	-0.003 ***	-0.002 **	-0.002 ***
p-value	0.000	0.000	0.000	0.004	0.000	0.000	0.012	0.007
US GDP growth (lagged)				-0.045				
p-value				0.59				
CUFTA/NAFTA	-0.046 **	-0.047 **	0.027 ***	0.027 ***	0.026 ***	0.026 ***	0.044 ***	0.045 ***
p-value	0.027	0.022	0.002	0.002	0.002	0.002	0.000	0.0003
A-B test in AR(1) in 1st difference	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
A-B test in AR(2) in 1st difference	0.353	0.348	0.448	0.444	0.456	0.457	0.445	0.353
Hansen test of overid. restrictions	0.434	0.432	0.372	0.386	0.392	0.409	0.465	0.261
Number of observations	9300	9300	7010	7010	7010	7010	7010	7010
Number of instruments	308	308	291	292	291	291	282	282
Number of groups	309	309	299	299	299	299	299	299

*,**,*** indicate respectively statistical significance at the 10, 5, and 1 percent level.

The dependent variable is the Canadian share of U.S. imports in the manufacturing sector (as defined in the appendix). The independent variables are the log of the contemporaneous Canadian real effective exchange rate (REER), the China share of US imports of manufacturing (comtemporaneous and lag), the lagged Candian domestic demand growth, the lagged US GDP growth and a dummy for CUFTA/NAFTA (takes value 1 for years under CUFTA or NAFTA and zero otherwise). In regression II and V, the log of commodities prices is used as an instrument for the REER; in regression VI, the log of energy price is used as an instrument for the REER; in regression VII, the log of the productivity differential between Canada and China is used as an instrument for China's impact; in regression VIII, the log of Chinese productivity is used as an instrument variable for China's impact.

The table presents the long-term elasticities. The standard errors used to test level of significance of the long-term coefficients are based on the delta method.

Table 3b. Canada shares in the US manufacturing import market - Panel GMM (1975-2010)

Dependent variable: Canada share of US manufacturing imports

	I	II	III	IV	V	VI	VII	VIII
real effective exchange rate	-0.099 ***	-0.131 ***	-0.114 ***	-0.114 ***	-0.132 ***	-0.135 ***	-0.101 ***	-0.115 ***
p-value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
China share (contemporaneous and lag)			-0.178 ***	-0.171 ***	-0.170 ***	-0.168 ***	-0.354 ***	-0.261 ***
p-value			0.000	0.000	0.000	0.000	0.000	0.000
Canada domestic demand growth (lagged)	-0.002 **	-0.003 ***	-0.001 *	-0.002	-0.001 *	-0.002 *	-0.002 **	-0.002 *
p-value	0.021	0.009	0.101	0.111	0.069	0.053	0.018	0.052
US GDP growth (lagged)				0.063				
p-value				0.532				
CUFTA/NAFTA	-0.051 **	-0.052 **	0.039 ***	0.039 ***	0.037 ***	0.037 ***	0.050 ***	0.054 ***
p-value	0.016	0.012	0.000	0.001	0.001	0.001	0.000	0.000
A-B test in AR(1) in 1st difference	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
A-B test in AR(2) in 1st difference	0.405	0.400	0.414	0.418	0.419	0.419	0.371	0.372
Hansen test of overid. restrictions	0.447	0.421	0.108	0.116	0.207	0.219	0.195	0.196
Number of observations	10189	10189	7878	7878	7878	7878	7878	7878
Number of instruments	308	308	248	249	248	248	278	278
Number of groups	309	309	299	299	299	299	299	299

*,**,*** indicate respectively statistical significance at the 10, 5, and 1 percent level.

The dependent variable is the Canadian share of U.S. imports in the manufacturing sector (as defined in the appendix). The independent variables are the log of the contemporaneous Canadian real effective exchange rate (REER), the China share of US imports of manufacturing (comtemporaneous and lag), the lagged Candian domestic demand growth, the lagged US GDP growth and a dummy for CUFTA/NAFTA (takes value 1 for years under CUFTA or NAFTA and zero otherwise). In regression II and V, the log of commodities prices is used as an instrument for the REER; in regression VI, the log of energy price is used as an instrument for the REER; in regression VII, the log of the productivity differential between Canada and China is used as an instrument for China's impact; in regression VIII, the log of Chinese productivity is used as an instrument variable for China's impact.

The table presents the long-term elasticities. The standard errors used to test level of significance of the long-term coefficients are based on the delta method.

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