# Three Essays on a Financial Crisis: A New Keynesian DSGE Approach with Financial Frictions

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#### Abstract

This thesis aims at enhancing our understanding of a financial crisis by using New Keynesian frameworks with financial frictions and applying Bayesian methods to the dynamic stochastic general equilibrium (DSGE) models.

First, we use a Gertler and Karadi (2011) type closed economy DSGE model to investigate a source and the transmission mechanism of a financial crisis. We show that a collapse in borrowers' net worth could lead to a real contraction by limiting the the bankers' credit supply to non-financial firms. In addition, our simulation indicates that the central bank's credit market intervention could be an effective tool in alleviating the financial crisis by restoring the private financial intermediation.

Second, we simulate a sudden stop crisis in an emerging market economy by using a small open economy DSGE model with financial frictions. We show that foreign lenders' negative perception on an emerging market economy could actually lead to a recession via sudden stops in foreign capital inflow and the rise in cost of foreign borrowing. In addition, we establish that a sudden stop crisis could be aggravated by (i) the substantial degree of financial frictions in the economy, (ii) the heavy reliance on foreign resources in capital production, (iii) the choice of a fixed exchange rate regime, and so on.

Finally, we estimate the above small open economy DSGE model by using the data from South Korea and the US. We obtain sizable and significant estimates for key parameters in the model, which support the theoretical arguments above empirically.

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## Declaration

I hereby declare that the thesis is my own work and that it has not been submitted for any academic award at this, or any other, University. The work contained in the thesis is original and all sources are acknowledged as References.

## Chapter 1

## Introduction

The recent global financial crisis episode posed a number of challenges for macroeconomics as a discipline. First, the fact that the collapse of the US housing
market bubble resulted in such a sharp contraction in real activity requires the
macroeconomic theory to be able to deal with the linkages between the financial
markets and the real sector of the economy in a more systematic way. Second,
taking into account that emerging market countries which were not directly linked
to the event in the US housing market were significantly affected in the process of
the global financial crisis, more investigation into the international dimension of
financial crises is called for. Third, from a more practical perspective, there exists
a growing need for policy measures towards preventing or at least alleviating the
costs of financial crises, other than the monetary and fiscal policies as conventional
stabilising tools.

Indeed, there has been a number of developments in modern macroeconomics addressing these issues. First, there are a number of theoretical frameworks, which incorporate the linkages between the financial markets and the real economy. For example, Bernanke, Gertler and Gilchrist (1999), Kiyotaki and Moore (1997),

and Christiano, Motto and Rostagno (2010) have introduced financial frictions into otherwise conventional New Keynesian dynamic stochastic general equilibrium (DSGE) models, as have Yun (1996), Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2003). These studies typically utilise an agency problem in a loan contract as a source of financial frictions, in which the cost of external finance could be related to the borrowers' balance sheet conditions. Thus, the external shocks deteriorating the balance sheet could discourage the capital demand in the real sector through the increased cost of borrowing. However, as Woodford (2010) argues, the recent crisis originated from an abrupt contraction in credit supply rather than a reduction in credit demand owing to the problems of borrowers. Thus, in order for the analysis of the recent crisis event to be more relevant, one needs to allow for an abrupt contraction in credit supply and the active role for financial intermediaries in the process of the crisis. To this end, a new generation of New Keynesian DSGE models with financial frictions, such as Curdia and Woodford (2009) and Gertler and Kiyotaki (2010), explicitly incorporate financial frictions in the banking sector which can impede an efficient supply of credit.

Second, the global nature of the recent financial crisis makes the open economy framework increasingly important. Traditionally, the impact of the financial crisis on emerging market economies was analysed within models of currency crises that were particularly common in emerging market countries in the 1990s.<sup>2</sup> Existing literature on currency crisis covers a large range of issues, ranging from

<sup>&</sup>lt;sup>1</sup>Standard New Keynesian DSGE models incorporate imperfect competition and price stickiness *a la* Calvo (1983) in the goods market, into the real business cycle (RBC) framework, which features perfect competition and fully flexible prices.

<sup>&</sup>lt;sup>2</sup>Examples of the currency crisis episodes in the 1990s include the crises in Mexico (1994-95), a group of East Asian countries (1997), Russia (1998) and Brazil (1998-99).

'sudden stops' in capital inflows as in Calvo (1998) and 'speculative attacks' and 'self-fulfilling pessimism' as in Krugman (1999) to 'fear of floating' as in Calvo and Reinhart (2002), among others. All these arguments enrich and deepen our understanding of the mechanism and impact of financial crises in emerging market economies. However, these studies focus on events in the emerging markets as a trigger of crisis, but lack a role for a global shock in contrast to the recent global financial crisis experience. Moreover, implications of the pre-crisis conditions of an individual economy on the severity of the crisis are generally overlooked. In contrast, the recent analyses of DSGE models for open economy settings have provided more effective and systematic framework to deal with such issues. For instance, Gertler et al. (2007), Curdia (2007), and many others have proposed good benchmarks for small open economy DSGE models.

Third, the fact that the financial crisis broke out following a long period of low and stable interest rate raises question marks over the effectiveness of the conventional monetary policy in response to a financial crisis. That is, as Joyce, et al. (2012) argue, while the conventional monetary policy has been effective in achieving low and stable inflation, it has been unable to prevent asset market bubbles from forming, which might pave the way for financial crises. Hence, there has been substantial interest in alternative policy measures against the financial crisis, such as quantitative easing (QE), macroprudential policy, and expansionary fiscal policy.

Motivated by the above observations, this thesis attempts to enhance our understanding of financial crises by analysing the source and transmission of crises and evaluating the role of the pre-crisis economic conditions and the effectiveness of the stabilising policy tools. Main questions we attempt to answer are: (i) what kind of shock would trigger a financial crisis; (ii) how the shock would be transmitted to the economy; (iii) how some pre-crisis conditions affect the severity of the financial crisis; and (iv) how effective the conventional and unconventional policy measures would be in fighting a financial crisis.

To these ends, we construct a DSGE model with financial frictions for a closed economy in Chapter 2. Following Gertler and Karadi (2011), we propose a DSGE model where financial frictions result from the moral hazard or 'costly enforcement' problem in the banking sector to consider the role of banking sector explicitly, in addition to nominal rigidity in the final goods market and capital adjustment frictions in capital production. In addition, we allow for the important features of the recent financial crisis more explicitly. For example, we consider a negative shock to the banker's net worth as a trigger of the financial crisis, rather than the conventional capital quality shock in producing firms. We argue that a net worth shock presents a more realistic representation of the recent financial crisis, since the shock is directly involved in the events in the financial market rather than those in the non-financial firms' technology. In addition, in contrast to Gertler and Karadi (2011), we derive the policy rule for unconventional monetary policy or credit market intervention in a microfounded way. The resulting policy rule involves a clear and realistic policy structure, where the central bank tries to stabilise the contractions in private credit supply by enhancing the private bankers' balance sheet and restoring the private financial intermediation.

Main findings in Chapter 2 are as follows. First, we establish that a collapse in the bankers' net worth in the financial market could lead to a real recession in the economy, as the fall in the quality of capital in the non-financial firms' technology in the existing literature could. Both shocks reduce the quantity

of financial intermediation and increase the non-financial firms' cost of external finance by deteriorating the bankers' balance sheet, which lead to a reduction in output. Based on those observations, we argue that a fall in capital quality of non-financial firms may be one source of the decrease in banker's net worth. We also find that an economy with a high degree of financial frictions is more likely to be vulnerable to an unfavourable change in bankers' financial condition. That is, if the bankers have a tendency to conduct moral hazard in normal times, then the depositors might be doubtful about the bankers' behaviour. Hence, they would be likely to reduce the credit supply and require a higher risk premium to bankers in response to the deteriorations in the bankers' balance sheet, even if it turns out to be temporary and marginal. As a result, an economy with a high degree of financial frictions would face greater fluctuations in economic activities even when a small and temporary negative shock hits bankers' net worth. In addition, policy experiments in Chapter 2 indicate that conventional expansionary policy measures could alleviate the impact of a financial crisis so long as they are available to the authorities. However, there seems to be possibility that an expansionary conventional monetary policy is unavailable to the policy authority, such as zero lower bound (ZLB) of the nominal interest rate, especially in crisis periods. In addition, we find that an expansionary fiscal policy could be less effective than a monetary policy counterpart in stabilising the economy in the aftermath of a financial crisis. Not only could the former discourage the capital demand through the so-called 'crowding-out effect', but the former could also do so by limiting the bankers' credit supply to non-financial firms via the reduced profitability of financial intermediation, as compared to the latter.<sup>3</sup> In contrast,

<sup>&</sup>lt;sup>3</sup>Clearly, this argument is based on the assumption that the nominal interest rate in the

the direct credit market intervention by the central bank could be an effective tool to combat the financial crisis by moderating the credit contraction and the rise in capital cost. Such a stabilising effect of credit market intervention could be achieved either by restoring the bankers' balance sheet or by alleviating the non-financial firms' capital cost. This result is supported by the working of financial accelerator mechanism where the external finance premium is positively related with the bankers' leverage ratio.

Next, Chapter 3 extends the DSGE model with financial frictions in Chapter 2 to the model for a small open economy setting to analyse sudden stop crises in emerging market economies. Following Bernanke et al. (1999) and Gertler et al. (2007), we postulate the conventional Townsend (1979) type 'costly state verification' (CSV) problem between foreign lenders and domestic producing firms, i.e., entrepreneurs to consider the nature and effect of an abrubt rise in cost of foreign borrowings. Moreover, to investigate how foreigners' pessimism could be 'self-fulfilled' as an actual crisis in emerging market economies, we consider a negative shock to foreigners' evaluation of domestic entrepreneurs' net worth rather than an exogenous foreign interest rate shock, following Curdia (2007) and Ozkan and Unsal (2010). In addition, we conduct a set of experiments exploring the effects of the pre-crisis economic conditions in an emerging market economy on the severity of sudden stop crises. These include examining the role of the degree of foreigners' trust in the emerging market economy, the exchange rate regime in place, and the economy's reliance on foreign resources in capital production technology.

Our findings in Chapter 3 are as follows. First, we establish that the pre-crisis period is high enough for the central bank to implement an expansionary monetary policy.

working of financial accelerator mechanism in a small open economy setting is similar to that in a closed economy. That is, foreign lenders' negative perception regarding the financial soundness of the borrowers in an emerging market economy leads to a recession via sudden stops in foreign capital inflow and the resulting rise in cost of foreign borrowing. In addition, Chapter 3 identifies a number of an economy's environmental conditions that could aggravate the impact of sudden stops, which include: (i) the presence of substantial degree of financial frictions in the economy; (ii) the co-occurrence of a global recession and sudden stops in capital inflows into an emerging market economy; (iii) an economy's heavy reliance on foreign resources in production technology; and (iv) an economy's choice of the fixed exchange rate regime. That is, if an emerging market economy fails to gain the foreigners' trust in normal times, it could suffer a sudden stop crisis more severely, since the external finance premium imposed on the economy would increase highly sensitively in response to a distortion in entrepreneurs' balance sheet (perceived by foreign lenders). Our results also show that when a global recession overlaps with a sudden stop, the recovery from the crisis via an increase in the export is unlikely to be realised due to a contraction in the aggregate demand in foreign countries. In addition, when an emerging market economy relies heavily on the foreign resources for capital production, the shrinking in capital demand could be magnified due to the increased capital price as well as a rise in the cost of foreign borrowing. In addition, our results indicate that the response of an economy to a financial crisis initiated by an unfavourable shift in foreign lenders' perception regarding an emerging market economy is also shaped by the exchange rate regime that the economy adopts. That is, an emerging market economy with a high degree of foreign currency denominated debt is likely to choose a fixed

exchange rate regime to prevent the rise in cost of foreign borrowing, as the 'fear of floating' argument a la Calvo and Reinhart (2002) suggests. However, if a currency depreciation is limited under a fixed exchange rate regime, an improvement in the price competitiveness for domestic goods in foreign retail markets could be also restricted in a sudden stop crisis. Our simulation results indicate that a negative effect of a fixed exchange rate regime on the export demand for domestic goods could offset the benefit from stabilising the cost of foreign borrowings.

The above analyses in Chapter 2 and Chapter 3 are based on calibrated DSGE models, and hence, the validity of the arguments depends on the parameter values imposed in the model. In constrast, Chapter 4 estimates the small open economy DSGE model in Chapter 3, to evaluate the empirical validity of the arguments on sudden stop crises in emerging markets in Chapter 3. Following the recent development in estimation methodology for DSGE models, we apply Bayesian Markov Chain Monte Carlo (MCMC) methods to the model in Chapter 3, by using the observed data from the US and South Korea in 1995:Q1-2013:Q1.

Our findings in Chapter 4 are summarised as follows. First, we obtain a sizable and significant estimate for the sensitivity parameter of external finance premium to entrepreneurs' leverage ratio, which suggests that there exists a substantial degree of financial frictions in a loan contract between foreign lenders and domestic entrepreneurs. Accordingly, the emerging market economy could suffer a severe sudden stop crisis, since the foreign lenders are likely to increase the risk premium sensitively when they perceive a distortion in entrepreneurs' financial situation. Second, the parameter for the steady state share of domestic inputs in investment good composite is estimated to be much smaller than that for the steady state share of domestic goods in consumption bundle, which indicates that

capital producers in the economy relies heavily on foreign resources in capital production in normal times. In this environment, a currency depreciation due to a crunch in capital inflows could result in a rise in the capital good price which would decrease the production additionally by the aggravated cost condition, on top of the rise in the cost of foreign borrowing. Accordingly, the contraction in capital demand in the economy could be much more severe than that in an economy with a low degree of foreign resource reliance. Third, the Taylor rule coefficient attached on the nominal exchange rate is estimated to be positive but small, which suggests that there does not exist a high degree of 'fear of floating' in the economy. That is, the central bank in the economy does not adjust the nominal interest rate as sensitively to stabilise the nominal exchange rate, as in a free floating exchange rate regime. Fourth, the result from variance decomposition based on our Bayesian estimates indicates that foreign financial shocks might be a prime source of business cycle fluctuations in the emerging market economy. In contrast, the impacts of foreign aggregate demand turn out to be less important, which would undermine the plausibility of the theoretical hypothesis that a sudden stop crisis in the emerging market economy could be aggravated by the coincidence with the global recession, to some degree.

## Chapter 2

# Financial Crisis and Credit Market Intervention

## 2.1 Introduction

The recent financial crisis has revived interest in the linkage between the financial and real sectors of an economy, as a disruption in the financial market propagated to a sharp contraction in the economy. Indeed, researchers have attempted to develop theoretical frameworks to properly allow for the role of financial factors in the business cycle; Bernanke, Gertler, and Gilchrist (1999), Christiano, Motto, and Rostagno (2010), and Kiyotaki and Moore (1995). They tried to incorporate agency problem between borrowers and lenders in otherwise conventional New Keynesian DSGE models as in Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2003). However, as Woodford (2010) argues, to analyse the recent crisis episodes one needs to allow for an abrupt contraction in credit supply and the active role of financial intermediaries, as well as the discouraged capital demand by non-financial firms. Moreover, from a practical standpoint, more effective policy measures have been required to fight the financial crisis, since a conventional

expansionary monetary policy measure is not available to the authority in certain environments, such as zero lower bound (ZLB) of the nominal interest rate, which tends to take place in financial crisis periods. In addition, even though a conventional monetary policy could achieve the low and stable inflation for a long time, it was unable to prevent asset market bubbles from forming, as Joyce et al. (2012) argue, which has been widely accepted as a source of the recent crisis.

Motivated by the above observations, the objective of this chapter is twofold: investigating the role of financial frictions in a financial crisis; and evaluating the effectiveness of policy measures to fight a financial crisis. To these ends, we develop a dynamic stochastic general equilibrium (DSGE) model with financial frictions, following Gertler and Kiyotaki (2010) and Gertler and Karadi (2011). There are two advantages in using the model with banking sector as a benchmark model. First, it explicitly allows for financial intermediation where the banking sector could play an active role in the process of a financial crisis. In addition, we adopt the 'costly enforcement' problem rather than 'costly state verification' (CSV) approach a la Townsend (1979) and Bernanke et al. (1999), as a source of financial frictions, which provides more realistic underpinnings for the current moral hazard issue in the banking sector. In addition, while the previous literature considers a negative shock to quality of capital in non-financial firms' technology as a trigger of a financial crisis, we allow for a negative shock to bankers' net worth in their balance sheet. A negative net worth shock may result from a wide range of factors which deteriorate bankers' financial conditions, one of which would be an exogenous reduction in capital quality. We believe that our consideration as to

<sup>&</sup>lt;sup>1</sup>On the contrary, in the model *a la* Bernanke *et al.* (1999), the financial friction comes from an agency problem between households (i.e., depositors) and non-financial firms (i.e., final capital demanders), and the banking sector is dealt with as just a veil.

a trigger of crises better represents the notion that a financial crisis is triggered by events in the financial market. For example, a collapse in the stock market bubble or bad news about the bankers could be more relevant for describing a financial crisis, rather than events in the real sector such as a fall in the quality of capital. Third, we evaluate the effectiveness of credit market intervention by the central bank, which could be implemented when the function of the private financial intermediation is damaged. For this, the existing literature just assumes that the central bank tries to stabilise the risk premium to prevent the capital demand by non-financial firms from being reduced. In contrast, we design the alternative credit market intervention rule in a microfounded manner, where the central bank monitors the bankers' financial conditions and seeks to restore the private financial intermediation to stabilise the total credit supply to non-financial firms by enhancing the private bankers' balance sheet. This approach is based on the perspective that many of the negative shocks in the financial market deteriorate the bankers' financial conditions, which results in a financial crisis. Accordingly, the central bank seems to monitor the private bankers' financial conditions rather than the non-financial firms' borrowing conditions to prevent a financial crisis. We also provide a comparative analysis of the credit market intervention based on this credit market intervention rule and the rule in the existing literature, under which the central bank is assumed to aim at stabilising the risk premium for non-financial firms.

Our main findings in this chapter are summarised as follows. First, we show that a collapse in the bankers' net worth could lead to a real recession in the economy, as a decrease in the quality of capital in the non-financial firms' technology could. Both shocks reduce the credit supply and raises the cost of capital

finance for non-financial firms by deteriorating the bankers' balance sheet, which results in the production contractions. Second, we find that the degree of financial frictions plays a significant role in determining the severity of the financial crisis, in a way that a high degree of financial frictions magnifies the fluctuations of key economic variables. Thus, an economy with a high degree of moral hazard in the banking sector in normal times is likely to be more vulnerable to a negative shock in the financial market and to undergo a severer financial crisis, as the depositors in the economy could react more sensitively to a deterioration of bankers' financial state, even if the financial shock turns out to be temporary and small in the end. Third, we find that conventional monetary and fiscal policies could be effective in relieving the business cycle fluctuations in a financial crisis. However, they seem to be unavailable sometimes, especially in financial crisis periods, as mentioned above. In addition, we find the possibility that an expansionary fiscal policy could be contractionary to capital demand in the presence of financial frictions. This follows from the fact that not only an expansionary fiscal policy is limited in encouraging the production and capital investment due to the so-called 'crowding-out effect', but it also induces non-financial firms to shift the factor demand from capital to labour due to the discouraged credit supply for capital acquisition which is triggered by the fall in the profitability from financial intermediation. Fourth, our experiment uncovers that the credit market intervention could be an effective tool in fighting a financial crisis by directly moderating the contraction in total credit supply and the rise in cost of capital. In addition, we find that the credit market intervention rule to seek to restore the private financial intermediation by enhancing the bankers' balance sheet produces the qualitatively similar result to the rule to aim to stabilise the risk premium by directly supplying the credit to non-financial firms, in spite of the difference in how to operate the policy. This result is supported by the financial accelerator mechanism where the bankers' leverage ratio is positively related to the risk premium.

From our study, we contribute to the existing literature as follows. First, in order to simulate a financial crisis more realistically and systematically, we consider a shock arising from a financial market such as a collapse of bankers' net worth, rather than a non-financial shock in the conventional study such as a reduction in capital quality. Second, we derive the central bank's credit market intervention rule as an optimal behaviour, rather than just assuming it as in the existing literature. Third, while the traditional studies focus mainly on the impact of expansionary policies on the aggregate demand, we analyse the effect on the aggregate supply as well.

The remainder of this chapter is structured as follows. Section 2.2 sets up a New Keynesian DSGE model with financial frictions. Section 2.3 presents the solution to the model and parameter calibration for simulation. In section 2.4, we conduct a set of experiments about the financial crisis and the alternative stabilising policies. Section 2.5 concludes.

## 2.2 The Model

The model consists of households, bankers, non-financial firms, and government, which participate in markets for (wholesale and retail) goods, labour, capital, and credit. Households consume the retail goods and supply the labour to non-financial firms. They also deposit their savings by purchasing the private and public bonds and pay the lump-sum taxes to the government. Bankers engage in

the financial intermediation between households and non-financial firms. They are assumed to be able to divert the capital, which makes financial frictions in the deposit contract. Non-financial firms comprise wholesale firms, capital producers, and retail firms. Wholesale firms produce wholesale goods by using labour and capital, which are acquired from households and capital producers, respectively, and sell the wholesale goods to retail firms in a competitive manner. Capital producers combine final goods and the existing capital to update the capital goods into brand new ones, which are sold to wholesale firms for the use of producing wholesale goods. Retail firms differentiate the wholesale goods into their own varieties to gain a certain degree of monopolistic power, set the retail price for each of them under Calvo (1983) type price rigidity, and sell them to households, capital producers and government. Government conducts monetary and fiscal policy: it sets the nominal interest rate and implements public spending which is financed by taxes and public borrowing. Moreover, it may directly intervene in the credit market, if necessary. Each economic agent's behaviour is described in more detail below.<sup>2</sup>

### 2.2.1 Households

The economy is populated by a continuum of infinitely lived households of length unity, who consume, work and save. A representative household derives the lifetime utility from consumption,  $C_t$ , and labour,  $L_t$ , according to

$$E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, L_t\right) \tag{2.1}$$

<sup>&</sup>lt;sup>2</sup>Appendix A2 presents the derivation and log-linearisation process of equilibrium conditions of the model.

where  $\beta \in (0,1)$  is her subjective discount factor. Moreover, her utility function is assumed to belong to the constant relative risk aversion (CRRA) class, such as

$$U(C_t, L_t) = \frac{(C_t)^{1-\sigma}}{1-\sigma} - \frac{(L_t)^{1+\varphi}}{1+\varphi}$$
 (2.2)

where  $\sigma > 0$  is the inverse of the intertemporal elasticity of substitution in consumption and  $\varphi > 0$  is the inverse elasticity of labour supply. The representative household enters period t with one period (real) private and public bonds,  $B_{t-1}$  and  $D_{t-1}$ , respectively, both of which yield the gross (real) non-stochastic return,  $R_{t-1}$  over the period t. In addition, during period t, she supplies her labour,  $L_t$ , to non-financial firms at the real wage rate,  $W_t$ , per labour unit, and receives real dividends arising from the ownership of the firms,  $\Pi_t^o$ . Her budget is spent on the consumption,  $C_t$ , the payment of (real) lump-sum taxes,  $T_t$ , and the purchase of one period riskless bonds for the subsequent period,  $B_t$  and  $D_t$ . Thus her period budget constraint is given in real terms by

$$C_t + B_t + D_t \le W_t L_t + R_{t-1} B_{t-1} + R_{t-1} D_{t-1} + \Pi_t^o - T_t$$
(2.3)

for all t = 0, 1, 2, ...

The representative household seeks to maximise the lifetime utility in (2.1) subject to the period budget constraint in (2.3). The resulting first order conditions yield the following Euler equation for consumption and labour supply function:

$$1 = \beta E_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} R_t \right\} \tag{2.4}$$

and

$$W_t = (C_t)^{\sigma} (L_t)^{\varphi}, \qquad (2.5)$$

respectively. Euler equation in (2.4) establishes the negative relationship between the ratio of the current consumption to the future one,  $\frac{C_t}{C_{t+1}}$ , and the real interest rate,  $R_t$ , everything else being equal. Labour supply function in (2.5) implies that a fall in real wage,  $W_t$ , leads to reductions in labour supply,  $L_t$ , and consumption,  $C_t$ .

### 2.2.2 Bankers

Bankers engage in the financial intermediation between households and wholesale firms. At the end of period t, a representative private banker, j, is assumed to have available her own (real) net worth,  $N_t^j$ , which is the accumulation of her past profits from the financial intermediation. We also assume that she supplies the credit,  $Q_t S_t^j$ , to wholesale firms up to the end of period t, where  $S_t^j$  is the amount of financial claims on wholesale firms and  $Q_t$  is the real price of each claim. Then, in order to finance the credit supply, she needs to borrow from households,  $B_t^j$ , which is the difference between the values of the credit supplied,  $Q_t S_t^j$ , and her own net worth,  $N_t^j$ . Hence, the banker's balance sheet at the end of t is given by

$$Q_t S_t^j = N_t^j + B_t^j, (2.6)$$

which shows that the size of credit supplied,  $Q_t S_t^j$ , increases with the borrowing from households,  $B_t^j$ , and the banker's net worth,  $N_t^j$ . In addition, over the period t+1, the banker is required to pay the (gross) real riskless rate,  $R_t$ , on the borrowing

from households,  $B_t^j$ , and expects to earn the (gross) real capital returns,  $R_{t+1}^k$ , from the financial claims on wholesale firms,  $S_t^j$ . Then, the banker's net worth evolves over the period t+1, according to the difference between earnings on assets and borrowing costs, as:

$$N_{t+1}^{j} = R_{t+1}^{k} Q_{t} S_{t}^{j} - R_{t} B_{t}^{j}$$

$$= (R_{t+1}^{k} - R_{t}) Q_{t} S_{t}^{j} + R_{t} N_{t}^{j}, \qquad (2.7)$$

where we use the banker's balance sheet relation in (2.6) in the second equality. With perfect capital markets, capital returns,  $R_{t+1}^k$ , should be equal to riskless rate,  $R_t$ , since the positive risk spread,  $R_{t+1}^k - R_t$ , would induce bankers to expand her assets by borrowing additional funds from households. In contrast, with imperfect capital markets, the risk spread,  $R_{t+1}^k - R_t$ , could be positive due to restrictions on the bankers' ability to obtain borrowings from households.

Now we discuss the loan contracting problem between borrowers (i.e., bankers) and lenders (i.e., households) under capital market imperfections. First of all, we introduce the following moral hazard or capital enforcement problem a la Gertler and Karadi (2011). We suppose that at the end of t+1, a banker may decide to divert a fraction  $\omega$  of the gross return to capital project,  $R_{t+1}^k Q_t S_t^j$ , to transfer it to her family, say, in the form of large bonuses or dividends, and declare bankruptcy.<sup>3</sup> If the banker diverts the capital, depositors try to reclaim the funds,

<sup>&</sup>lt;sup>3</sup>Gertler and Karadi (2011) suppose that the banker may divert a fraction of fund,  $\omega Q_t S_t^j$ , at the beginning of the period. However, for analytical simplicity, we postulate the situation where the banker decides to divert a fraction from a total revenue,  $R_{t+1}^k Q_t S_t^j$ , at the end of period, which does not make a critical difference.

but it is assumed that it is too costly for the lenders to fully recover the funds. In the end, the bankers could still keep the diverted fraction  $\omega$  and households could only collect the remaining fraction  $1 - \omega$ . In this setup, the lenders are willing to supply funds to the banker, when it is expected that the bankers do not venture the moral hazard, which requires the expected returns to the banker from diverting the funds to be smaller than those from not doing so. Accordingly, in order for the lenders to participate in the loan contract, the following incentive constraint should be satisfied:

$$(R_{t+1}^k - R_t) Q_t S_t^j + R_t N_t^j \ge \omega R_{t+1}^k Q_t S_t^j.$$
 (2.8)

If the constraint in (2.8) is binding, the assets that the banker can supply to non-financial firms is determined by the following financial accelerator:

$$Q_t S_t^j = \left[1 - (1 - \omega) \frac{R_{t+1}^k}{R_t}\right]^{-1} N_t^j,$$
  
=  $\Psi_t N_t^j$ , (2.9)

where  $\Psi_t$  is the private leverage ratio. We obtain the private leverage ratio,  $\Psi_t \equiv \frac{Q_t S_t^j}{N_t^j}$ , in the form of increasing function in the risk premium,  $\frac{R_{t+1}^k}{R_t}$ :

$$\Psi_t = \Psi\left(\frac{R_{t+1}^k}{R_t}\right) = \left[1 - (1 - \omega)\frac{R_{t+1}^k}{R_t}\right]^{-1},\tag{2.10}$$

and, as shown in Appendix A2.2, it may be approximated around the steady state as:

$$\frac{\Psi_t}{\Psi} = \left(\frac{R_{t+1}^k/R_t}{R^k/R}\right)^{\Psi-1},\tag{2.11}$$

where  $\Psi$  and  $\frac{R^k}{R}$  are the steady state values for the private leverage ratio,  $\Psi_t$ , and the risk premium,  $\frac{R^k_{t+1}}{R_t}$ , respectively, and  $\Psi - 1$  is the sensitivity of the bankers' leverage ratio,  $\Psi_t$ , to the risk premium,  $\frac{R^k_{t+1}}{R_t}$ .

Equation (2.9) describes how the financial accelerator mechanism works in the model. First of all, the asset available for the banker,  $Q_t S_t^j$ , depends positively on her net worth,  $N_t^j$ , so that a decrease in  $N_t^j$  would directly reduce credit supplu to non-financial firms, and so, capital investment in non-financial firms. Second, holding  $N_t^j$  constant, the banker's credit supply,  $Q_t S_t^j$ , is determined by the private leverage ratio,  $\Psi_t$ . Thus, if the profitability from financial intermediation, i.e., the risk premium,  $\frac{R_{t+1}^{\kappa}}{R_t}$ , increases, the banker would be willing to supply more credit to non-financial firms. Moreover, the sensitivity of the banker's leverage ratio,  $\Psi_t$ , to the risk premium,  $\frac{R_{t+1}^k}{R_t}$ , is captured by the parameter,  $\Psi-1$ , which is inversely related to the capital diversion rate,  $\omega$ .<sup>5</sup> That is, the low degree of moral hazard,  $\omega$ , would result in the large value of  $\Psi - 1$ , so that the banker would expand credit supply,  $Q_t S_t^j$ , more sensitively in response to the given rise in the risk premium,  $\frac{R_{t+1}^k}{R_t}$ . In other words, in the economy with a lower degree of moral hazard, the bankers could be easier to obtain funds from households, so that they could expand credit supply,  $Q_t S_t^j$ , sensitively in response to the improved profitability from financial intermediation, i.e., the rise in risk premium,  $\frac{R_{t+1}^k}{R_t}$ .

<sup>&</sup>lt;sup>4</sup>Note that the private leverage ratio,  $\Psi_t$ , in (2.10) does not depend on firm specific factors, so that the financial accelerator relationship in (2.9) holds in the aggregate level as well as in the firm level. That is,  $Q_t S_t^j$  and  $N_t^j$  also imply the economy-wide financial assets privately intermediated,  $Q_t S_t^p$ , and the net worth for bankers in operation as a whole,  $N_t$ , respectively.

<sup>&</sup>lt;sup>5</sup>It follows from the steady state relationship,  $\Psi = \left[1 - (1 - \omega) \frac{R^k}{R}\right]^{-1}$ .

Next, we consider the bankers' survival time and the credit market conditions to derive the motion of the aggregate net worth,  $N_t$ . First of all, we assume the finite horizon for the individual bankers with the survival rate of  $\phi$  each period, which ensures that they never accumulate their own net worth enough to fully self-finance the capital investment. In addition, new bankers enter the banking sector in place of failed bankers, so that the aggregate net worth in the economy at the end of period t,  $N_t$ , consists of net worth of the successful bankers,  $N_t^e$ , and that of the newly entering ones,  $N_t^n$ . We assume that the existing bankers' net worth,  $N_t^e$ , is accumulation of profits from the financial intermediation, i.e.,  $N_t^e = \phi\left(R_t^kQ_{t-1}S_{t-1}^p - R_{t-1}B_{t-1}\right)$ , and that the newly entering bankers commence the business with the fixed amount of fund, F, which is transferred from the failed bankers', as a start up fund, i.e.,  $N_t^n = (1 - \phi) F$ . In addition, the overall value of bankers' net worth is subject to an exogenous shock,  $V_t$ , which is supposed to follow a first-order autoregressive (AR(1)) process given by

$$V_t = (V_{t-1})^{\rho_v} \exp\{\varepsilon_{v,t}\},$$
 (2.12)

where  $|\rho_v| < 1$ , and  $\varepsilon_{v,t}$  is a Gaussian white noise with mean zero and standard deviation  $\sigma_v$ . Then, the motion of the aggregate net worth may be expressed as:

<sup>&</sup>lt;sup>6</sup>The latter assumption ensures that the new bankers never operate their business solely by external finance. However, as discussed by Bernanke *et al.* (1999) and Gertler and Karadi (2011), the contribution of newly entering bankers' start up funds to the net worth evolution is quite small. Thus, for analytical simplicity and without loss of generality, we assume the amount transferred to the newly entering bankers is constant over time.

 $<sup>^{7}</sup>V_{t}$  includes all possible exogenous shock to affect the bankers' net worth. For example, we may take a collapse of stock market bubble, adverse rumour about an individual banker, and so on and so forth.

$$N_{t} = [N_{t}^{e} + N_{t}^{n}] \cdot V_{t}$$

$$= [\phi \{R_{t}^{k}Q_{t-1}S_{t-1}^{p} - R_{t-1}B_{t-1}\} + (1 - \phi)F] \cdot V_{t}$$

$$= [\phi \{(R_{t}^{k} - R_{t-1})\Psi_{t-1} + R_{t-1}\} N_{t-1} + (1 - \phi)F] \cdot V_{t}.$$
 (2.13)

Equation (2.13) shows that the aggregate net worth,  $N_t$ , increases with the capital returns,  $R_t^k$ , the amount of financial intermediation,  $\Psi_{t-1}$ , and the initial size of the net worth,  $N_{t-1}$ , while it decreases with the bankers' financing cost, i.e. the riskless rate,  $R_{t-1}$ .

#### 2.2.3 Wholesale Firms

Wholesale firms produce wholesale goods and sell them to retail firms in a competitive wholesale goods market. By the beginning of period t, they are assumed to acquire capital,  $K_{t-1}$ , from capital producers, which is combined with labour hired from households to produce wholesale goods,  $Y_{w,t}$ , over the period t, by the following Cobb-Douglas function<sup>8</sup>

$$Y_{w,t} = A_t (K_{t-1})^{\alpha} (L_t)^{1-\alpha},$$
 (2.14)

where  $\alpha$  is the share of capital in the production function.  $A_t$  denotes a level of total factor productivity (TFP), which obeys a first order autoregressive (AR(1))

<sup>&</sup>lt;sup>8</sup>Note that wholesale firms are assumed to be perfectly competitive and employ constant returns to scale (CRS) technology. These assumptions allow us to treat wholesale firms as a whole, so that we may write the production function as an aggregate relationship without firm specific superscripts.

process given by

$$A_t = (A_{t-1})^{\rho_a} \exp\left\{\varepsilon_{a,t}\right\},\tag{2.15}$$

where  $|\rho_a| < 1$ , and  $\varepsilon_{a,t}$  is a Gaussian white noise with mean zero and standard deviation  $\sigma_a$ . In order to finance the capital acquisition, wholesale firms issue the same amount of claims,  $S_{t-1}^p$ , as the desired capital,  $K_{t-1}$ , to bankers, which incurs the gross capital returns,  $R_t^k$ . For wholesale firms,  $R_t^k$  is the cost of capital finance. Following Gertler and Karadi (2011), we assume that there are no frictions in transactions between wholesale firms and bankers. That is, bankers have perfect information about the wholesale firms and there is no problem enforcing payoffs. Accordingly, asset market equilibrium implies

$$Q_{t-1}K_{t-1} = Q_{t-1}S_{t-1}^p, (2.16)$$

at the end of period t-1.<sup>10</sup> In addition, after finishing the production in period t, wholesale firms are assumed to resell the undepreciated capital goods,  $(1 - \delta) K_{t-1}$ , to capital producers at the price of  $\overline{Q}_t$ , in order to update them into the brand new capital goods. Then, wholesale firms' (real) total cost function is given by:

$$TC_{w,t} = W_t L_t + \left[ R_t^k Q_{t-1} K_{t-1} - (1 - \delta) \overline{Q}_t K_{t-1} \right],$$
 (2.17)

<sup>&</sup>lt;sup>9</sup>Within the model, only the bankers face the constraints on obtaining household funds. However, the constraints affect the supply of funds available to wholesale firms,  $Q_{t-1}S_{t-1}^p$ , and the associated capital returns,  $R_t^k$ , in the end. However, as long as wholesale firms pay the capital returns, the financing process is frictionless.

 $<sup>^{10}</sup>$ As discussed below, in the presence of credit market intervention by the central bank, the asset market equilibrium in (2.16) would be  $Q_{t-1}K_{t-1} = Q_{t-1}S_{t-1}^p = Q_{t-1}S_{t-1}^p + Q_{t-1}S_{t-1}^g$ , where  $S_{t-1}$  and  $S_{t-1}^g$  denote the total credit supply and the public credit supply, respectively.

where  $\delta$  is the depreciation rate for capital goods.<sup>11</sup>

Given that wholesale firms operate as price takers both in the wholesale goods market and in the factor markets, cost minimisation subject to the production technology implies the following demands for labour and capital goods, as:

$$W_t = (1 - \alpha) \left(\frac{Y_{w,t}}{L_t}\right) P_{w,t}, \qquad (2.18)$$

and

$$E_t \left\{ R_{t+1}^k Q_t \right\} = E_t \left\{ \alpha \left( \frac{Y_{w,t+1}}{K_t} \right) P_{w,t+1} + (1 - \delta) Q_{t+1} \right\}, \tag{2.19}$$

respectively, where  $P_{w,t}$  is the (real) wholesale good price.<sup>12</sup> Labour demand function in (2.18) implies that labour demand,  $L_t$ , increases with a production expansion,  $Y_{w,t}$ , and a rise in the real wholesale good price,  $P_{w,t}$ , but decreases with a rise in the real wage,  $W_t$ . Capital demand function in (2.19) suggests that capital demand,  $K_t$ , increases with a plan for production expansion,  $Y_{w,t+1}$ , and an expected rise in the wholesale good price,  $P_{w,t+1}$ , while it decreases with an expected rise in the required capital returns,  $R_{t+1}^k$ , other things being fixed. In addition, capital demand depends negatively on the current capital price,  $Q_t$ , but positively on the future capital price,  $Q_{t+1}$ .

<sup>&</sup>lt;sup>11</sup>Strictly speaking, the selling price of undepreciated capital,  $\overline{Q}_t$ , could differ from the market price of capital,  $Q_t$ . However, as discussed in Appendix A2.4, zero profit condition for capital producers implies  $\overline{Q}_t \simeq Q_t$  around the steady state. Hence, we use  $Q_t$  for both the selling price of undepreciated capital and the market price of capital, for notational simplicity.

<sup>&</sup>lt;sup>12</sup>Note that, in equations (2.18) and (2.19), the assumption of competitive wholesale firms requires the profit maximisation condition to be  $P_{w,t} = MC_{w,t}$ , with the real marginal cost of producing wholesale goods,  $MC_{w,t} = \left(\frac{1}{A_t}\right) \left[\frac{R_t^k Q_{t-1} - (1-\delta)Q_t}{\alpha}\right]^{\alpha} \left[\frac{W_t}{1-\alpha}\right]^{1-\alpha}$ . Appendix A2.3 provides more detailed derivation.

## 2.2.4 Capital Producers

Capital producers supply capital goods to wholesale firms, which are used to produce wholesale goods by wholesale firms. In order to do so, they engage in repair of existing capital goods and construction of new capital goods. In period t, competitive capital producers purchase the undepreciated capital goods,  $(1 - \delta)K_{t-1}$  from wholesale firms at the price of  $\overline{Q}_t$ , and combine them with investment goods,  $I_t$ , which are a fraction of final goods, to produce new capital goods,  $K_t^n$ . Following Ozkan and Unsal (2010), we specify the production function for new capital goods,  $K_t^n$ , as the one with capital adjustment costs,  $I_t^n$  given by

$$K_t^n = \left[ \frac{I_t}{K_{t-1}} - \frac{\kappa}{2} \left( \frac{I_t}{K_{t-1}} - \delta \right)^2 \right] K_{t-1}, \tag{2.20}$$

where  $\kappa > 0$  is the capital adjustment coefficient. Then, together with existing capital,  $(1 - \delta)K_{t-1}$ , new capital goods,  $K_t^n$ , are sold back to wholesale firms at the price of  $Q_t$  in period t, which are used for wholesale good production in period t + 1.

In this setup, the resulting economy-wide capital stock at the end of t accumulates according to

$$K_t = K_t^n + (1 - \delta) K_{t-1},$$
 (2.21)

and capital producers' (real) profit function is given by 14

<sup>&</sup>lt;sup>13</sup>In the presence of capital adjustment costs, the capital production function exhibits the nature of constant return to scale (CRS) and diminishing returns to investment good,  $I_t$ , which allows for variability in capital price,  $Q_t$ .

<sup>&</sup>lt;sup>14</sup>Note that the investment goods are just a fraction of final goods so that we assume that price index for investment goods,  $P_{I,t}$ , is equal to the consumer price index,  $P_t$ , without loss of generality. Accordingly, the real price of investment goods,  $\frac{P_{I,t}}{P_t}$ , remains unity at all times.

$$\Pi_{c,t} = Q_t K_t - \left[ I_t + \overline{Q}_t (1 - \delta) K_{t-1} \right].$$
 (2.22)

Then, the optimality condition for capital producers' problem with respect to the choice of  $I_t^{15}$  yields the following capital supply function:

$$Q_t = \left[1 - \kappa \left(\frac{I_t}{K_{t-1}} - \delta\right)\right]^{-1},\tag{2.23}$$

which is referred to as a Tobin's (1969) Q relation, modified to allow for the capital adjustment costs. The capital supply function in (2.23) implies that, given the existing capital stock,  $K_{t-1}$ , the capital investment,  $I_t$ , increases with the capital price,  $Q_t$ .

## 2.2.5 Retail Firms and Resource Constraint

In order to introduce price rigidity, which is one of the New Keynesians' main concepts, the model allows for monopolistically competitive retail firms, indexed by  $j \in [0,1]$ . They purchase wholesale goods,  $Y_{w,t}$ , from wholesale firms in a competitive wholesale market; costlessly diversify them into their own varieties,  $Y_t(j)$ , to gain a certain degree of price-setting power in the retail market; set the (nominal) retail price,  $\overline{P}_t(j)$ , on each variety in a monopolistically competitive manner under the price stickiness a la Calvo (1983); and sell them to consumers, i.e., households, capital producers and the government.

<sup>&</sup>lt;sup>15</sup>Note that the assumptions of CRS technology for capital production and perfect competitive capital market require capital producers to earn zero profit in equilibrium. In addition, in this environment, it can be shown that the effect of the existing capital stock on the capital producers' profit is negligible around the steady state, so that we may ignore the optimality condition with respect to the existing capital stock,  $K_{t-1}$ . See Appendix 2.4 for the details.

Suppose that consumers' preference over varieties belongs to a constant elasticity of substitution (CES) class. Then, the retail good composite,  $Y_t$ , and the corresponding consumer price index (CPI),  $P_t$ , are represented by the following Dixit and Stiglitz (1977) aggregator:

$$Y_{t} = \left[ \int_{0}^{1} Y_{t}(j)^{\frac{\epsilon - 1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon - 1}}, \qquad (2.24)$$

and

$$P_{t} = \left[ \int_{0}^{1} \overline{P}_{t} \left( j \right)^{1-\epsilon} dj \right]^{\frac{1}{1-\epsilon}}, \tag{2.25}$$

where  $\overline{P}_t(j)$  is the price for variety j, and  $\epsilon > 1$  is the elasticity of substitution among varieties. By construction, the retail good composite equals the wholesale goods as a whole in equilibrium, given by:

$$Y_t = Y_{w,t}. (2.26)$$

Consumers' expenditure minimisation suggests that each retail firm faces the downward sloping demand, given by

$$Y_{t}(j) = \left(\frac{\overline{P}_{t}(j)}{P_{t}}\right)^{-\epsilon} Y_{t}.$$
(2.27)

In this setting, they may set the price,  $\overline{P}_t(j)$ , to maximise their profit subject to the downward sloping demand curve for the variety j in (2.27). On the other hand, in order to introduce the nominal rigidity, we assume that retailers face the price stickiness a la Calvo (1983); that is, each retailer is able to reset its price,  $\overline{P}_t(j)$ , with a probability of  $(1-\theta)$  independently of the time elapsed since the

last adjustment, while with a probability of  $\theta$  it is not able to do so such that they keep the previous price,  $\overline{P}_{t-1}(j)$ . Then, the consumer price index in (2.25) can be expressed as the weighted average of two sets of price index, such as the previous price level,  $P_{t-1}$ , and the newly set price,  $\overline{P}_t$ , given by:<sup>16</sup>

$$P_{t} = \left[\theta \left(P_{t-1}\right)^{1-\epsilon} + \left(1 - \theta\right) \left(\overline{P}_{t}\right)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}, \tag{2.28}$$

which provides the dynamics for the aggregate price in the economy. 17

Now, we discuss the retail firm's price setting behaviour to determine  $\overline{P}_t(j)$  in (2.28). Suppose that an individual retailer, who is able to adjust the price at t, chooses  $\overline{P}_t(j)$  to maximise the current value of expected future profits while  $\overline{P}_t(j)$  remains effective. Then, her (real) profit maximisation problem in period t, when she is able to change her price, is given by:

$$\max_{\left\{\overline{P}_{t}(j)\right\}} \sum_{k=0}^{\infty} \theta^{k} E_{t} \left\{ \Lambda_{t,t+k} \left[ \left( \frac{\overline{P}_{t}(j)}{P_{t+k}} - P_{w,t+k} \right) Y_{t+k}(j) \right] \right\}, \tag{2.29}$$

subject to the sequence of demands for her variety

$$Y_{t+k}(j) = \left(\frac{\overline{P}_t(j)}{P_{t+k}}\right)^{-\epsilon} Y_{t+k}, \tag{2.30}$$

for  $k = 0, 1, 2, \dots$ , where  $\theta^k$  is the probability of keeping the retail price set at t,  $\overline{P}_t(j)$ , unchanged until t + k,  $\Lambda_{t,t+k} = \beta^k \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma}$  is her subjective intertemporal

<sup>&</sup>lt;sup>16</sup>It follows from the facts that all resetting firms will choose the same price,  $\overline{P}_t(j) = \overline{P}_t$ , since the cost and demand conditions which they face are assumed to be identical, and that firms keeping their prices unchanged have the same price distribution as the previous price index so that  $\left[\int_0^1 \overline{P}_{t-1}(j)^{1-\epsilon} dj\right]^{\frac{1}{1-\epsilon}} = P_{t-1}$ .

that  $\left[\int_0^1 \overline{P}_{t-1}(j)^{1-\epsilon} dj\right]^{\frac{1}{1-\epsilon}} = P_{t-1}$ .

17 In addition, it can be shown that, in the neighbourhood of the steady state, equation (2.28) can be written as  $P_t = (P_{t-1})^{\theta} (\overline{P}_t)^{1-\theta}$ .

substitution rate between t and t+k, and  $P_{w,t+k}$  is the retail firm's (real) marginal cost of purchasing the wholesale goods at period t+k.<sup>18</sup> The first order condition with respect to  $\overline{P}_t(j)$  implies the following optimal price setting rule for the retail firm:

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ \Lambda_{t,t+k} Y_{t+k} \left[ \frac{\overline{P}_t(j)}{P_{t-1}} - \mu P_{w,t+k} \left( \frac{P_{t+k}}{P_{t-1}} \right) \right] \left( \frac{1}{P_{t+k}} \right)^{1-\epsilon} \right\} = 0, \qquad (2.31)$$

where  $\mu = \frac{\epsilon}{\epsilon - 1}$  is the retail firm's desired (gross) mark-up, which is attached due to imperfections in the retail market.<sup>19</sup> Combining the aggregate price dynamics in (2.28) with the optimal price setting rule in (2.31) yields the following short-run dynamics for the consumer price index (CPI) (within the neighbourhood of the steady state):

$$\pi_t = (\mu P_{wt})^{\lambda} E_t \{ \pi_{t+1} \}^{\beta}, \qquad (2.32)$$

with  $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$  and the CPI inflation,  $\pi_t \equiv \frac{P_t}{P_{t-1}}$ , which is referred to as the New Keynesian Phillips curve (NKPC) in the literature. Equation (2.32) shows that the CPI inflation,  $\pi_t$ , rises with the inflation expectation,  $E_t \{\pi_{t+1}\}$ , and the wholesale good price,  $P_{w,t}$ , i.e., the marginal cost of wholesale good production.

Finally, note that the retail goods supplied by the retail firms,  $Y_t$ , are

<sup>&</sup>lt;sup>18</sup>Note that, since both wholesale and retail firms act as price takers in the perfectly competitive wholesale goods market, the retail firm j's marginal cost of purchasing the wholesale goods,  $MC_t(j)$ , is equal to wholesale firms' marginal cost of producing wholesale goods,  $MC_{w,t}$ , such that  $MC_t(j) = P_{w,t} = MC_{w,t}$ .

<sup>&</sup>lt;sup>19</sup>It can be shown that, in the neignhourhood of the steady state, equation (2.31) can be approximated as  $\frac{\overline{P}_{t}(j)}{P_{t-1}} = \mu \prod_{k=0}^{\infty} \left[ P_{w,t+k} \left( \frac{P_{t+k}}{P_{t-1}} \right) \right]^{(1-\beta\theta)(\beta\theta)^k}$ .

consumed by households, capital producers and government, so that the economywide resource constraint for retail goods is given by:

$$Y_t = C_t + I_t + G_t, (2.33)$$

where  $G_t$  denotes the government spending, which is discussed below.

## 2.2.6 Government Policy

Now, we turn to government policies. Government consists of monetary and fiscal authorities. The government attempts to stabilise the economy by using conventional and unconventional policy measures.

Monetary authority sets the nominal interest rate,  $R_t^n$ , conventionally, and it may directly intervene in the credit market when it is necessary. First of all, the central bank is assumed to adjust the (gross) short-term nominal interest rate,  $R_t^n$ , to stabilise the business cycle, by using the Taylor-type (1993) feedback rule with interest rate smoothing, given by:

$$\left(\frac{R_t^n}{R^n}\right) = \left(\frac{R_{t-1}^n}{R^n}\right)^{\alpha_r} \left(\frac{P_t}{P_{t-1}}\right)^{(1-\alpha_r)\alpha_\pi} \left(\frac{Y_t}{Y}\right)^{(1-\alpha_r)\alpha_y} \exp\left\{\varepsilon_{r,t}\right\},$$
(2.34)

where  $R^n$  and Y denote steady state values for nominal interest rate,  $R^n_t$ , and output,  $Y_t$ , respectively. We assume that a monetary policy shock,  $\varepsilon_{r,t}$ , is a Gaussian white noise process with mean zero and standard deviation  $\sigma_r$ , and that the values for Taylor rule coefficients,  $\alpha_r \in (0,1)$ ,  $\alpha_{\pi} > 1$  and  $\alpha_y > 0$ , are chosen by the central bank. Accordingly, it positively adjusts the nominal interest rate,  $R^n_t$ , in

response to inflation of consumer price index (CPI),  $\pi_t \equiv \frac{P_t}{P_{t-1}}$ , and output gap,  $\frac{Y_t}{Y}$ , to stabilise the economy's business cycle. In addition, short-term nominal interest rate,  $R_t^n$ , is linked to the real riskless rate,  $R_t$ , by the following Fisher equation:

$$R_t^n \equiv R_t E_t \{ \pi_{t+1} \} \,. \tag{2.35}$$

Moreover, the central bank is allowed to directly inject public funds,  $Q_t S_t^g$ , into the asset market, especially in a crisis period when the credit privately supplied,  $Q_t S_t^p = \Psi_t N_t$ , is shrinking. Accordingly, in the presence of credit market intervention, the private credit supply,  $Q_t S_t^p$ , is supplemented by the public credit supply,  $Q_t S_t^g$ , so that total amount of credit supply in the economy,  $Q_t S_t$ , is given by

$$Q_t S_t = Q_t S_t^p + Q_t S_t^g. (2.36)$$

Following Gertler and Karadi (2011), we assume that public credit supply,  $Q_t S_t^g$ , is a fraction,  $\Phi_t$ , of the total credit supplied,  $Q_t S_t$ , given by:

$$Q_t S_t^g = \Phi_t Q_t S_t, \tag{2.37}$$

with  $\Phi_t = \frac{S_t^g}{S_t} \in [0, 1)$ . Then, the total credit supply in (2.36) can be expressed as:

$$Q_t S_t = Q_t S_t^p + Q_t S_t^g$$

$$= \Psi_t N_t + \Phi_t Q_t S_t$$

$$= \frac{\Psi_t}{1 - \Phi_t} N_t.$$
(2.38)

In addition, the public credit supply,  $Q_tS_t^g$ , is assumed to be financed by issuing the special government bond,  $B_t^g$ , to households, which pays the riskless rate,  $R_t$ , to households.

Now, we determine the central bank's credit market intervention rule,  $\Phi_t$ . First of all, Gertler and Karadi (2011) suppose that the central bank supply the public fund,  $Q_t S_t^g = \Phi_t Q_t S_t$ , to non-financial firms at the capital returns,  $R_{t+1}^k$ , when the risk premium,  $\frac{R_{t+1}^k}{R_t}$ , rises rapidly, since the risk premium tends to soar in the crisis period. Accordingly, they propose the following credit market intervention rule

$$\frac{\Phi_t}{\Phi} = \left(\frac{R_{t+1}^k / R_t}{R^k / R}\right)^{\upsilon},\tag{2.39}$$

where  $\Phi$  is the steady state fraction of publicly intermediated assets and the feed-back parameter, v, is positive. Then, the central bank's credit market intervention could limit the rise in the cost of capital for wholesale firms, so that the reduction in capital demand,  $K_t$ , could be relieved. In this set up, the central bank finances the public credit supply to non-financial firms by issuing the public bond,  $D_t$ . Thus, the central bank earns the profit of  $(R_{t+1}^k - R_t) \Phi_t Q_t S_t$  from the public credit supply, which provides another source of government revenue.

Alternatively, we suppose that the central bank seeks to alleviate the fluctuation in the total credit supply,  $Q_tS_t$ , by increasing the degree of public fund injection,  $\Phi_t$ , since the financial crisis results from the contraction in private credit supply,  $Q_tS_t^p$ . In addition, we assume that the central bank is allowed to choose the degree of the intensity to which to intervene in the credit market and what variable to use as a control variable. Then, we follow the two step approach to ob-

tain the practical credit market intervention rule: first, we get the optimal credit market intervention rule by solving the central bank's problem to minimise the fluctuations of total credit supply; and then modify the rule to reflect the central bank's practice to intervene in the credit market.

First of all, we establish the central bank's problem to minimise the deviation of the total credit supply,  $Q_tS_t$ , from its steady state value, QS, with respect to the degree of public fund injection,  $\Phi_t$ , given by:

$$\min_{\Phi_t} (Q_t S_t - QS)^2 = \left(\frac{\Psi_t N_t}{1 - \Phi_t} - \frac{\Psi N}{1 - \Phi}\right)^2.$$
 (2.40)

Then, solving the above problem yields the optimal credit market intervention rule for the central bank as

$$\Phi_t = 1 - \left(\frac{1}{S}\right) \Psi_t N_t, \tag{2.41}$$

which implies that the central bank could effectively eliminate the fluctuation in the total credit supply,  $Q_tS_t$ , by the central bank's counteracting policy intervention, so that the total credit supply,  $Q_tS_t$ , remains at its steady state value, QS, at all times in spite of the fluctuation in the private credit supply,  $Q_tS_t^p$ . However, it is noticeable that the central bank does not necessarily react completely and instantly to the motion of private credit supply, as once the policy intervention by the central bank alleviates the initial impact of a financial shock, the economy could return to the steady state following the more stable path by the endogenous interaction of the economic variables. Considering this point, we modify the optimal credit market intervention rule in (2.41) by introducing the intensity coefficient,  $\nu > 0$ , to respond to the motion in private credit supply,  $Q_tS_t^p = \Psi_t N_t$ , as:

$$\Phi_t = 1 - \left(\frac{1}{S}\right) \left(\Psi_t N_t\right)^{\nu}, \qquad (2.42)$$

which may be interpreted as the myopic form of the optimal credit market intervention rule above. Moreover, noting that the crunch in the private credit supply,  $Q_tS_t^p$ , is triggered by the collapse of bankers' net worth,  $N_t$ , it is sufficient for the central bank to monitor and enhance the motion of bankers' net worth,  $N_t$ , to alleviate the financial crisis. Thus, as a policy control variable, we use the banker's net worth,  $N_t$ , rather than the private credit supply,  $\Psi_t N_t$ , and set the practical credit market intervention rule for the central bank as:

$$\Phi_t = 1 - (1 - \Phi) \left(\frac{N_t}{N}\right)^{\nu}. \tag{2.43}$$

where  $\nu > 0$  is the intensity coefficient of credit market intervention,  $\Phi_t$ , in response to the motion in bankers' net worth,  $N_t$ .<sup>20</sup> The derived credit market intervention rule in (2.43) suggests that the central bank injects the public fund,  $Q_t S_t^g = \Phi_t Q_t S_t$ , to bankers' balance sheet when the bankers' net worth,  $N_t$ , collapses so that the private credit supply,  $Q_t S_t^p = \Psi_t N_t$ , is expected to shrink. Thus, if the central bank tries to enhance the private bankers' balance sheet, the private bankers' difficulty in acquiring the households' deposit could be relieved and the private financial intermediation could be restored. In this setup, the central bank is assumed to acquire the households' deposit,  $D_t$ , at the riskless rate,  $R_t$ , and inject the public funds to private bankers at the same rate,  $R_t$ . Accordingly, the central bank could earn zero profit from the public fund injection to private bankers, but

<sup>&</sup>lt;sup>20</sup>The coefficient,  $(1-\Phi)$ , is attached for an equality in the steady state, i.e.,  $\Phi=1-(1-\Phi)\left(\frac{N}{N}\right)^{\nu}$ .

the private bankers could earn the profit of the risk spread,  $(R_{t+1}^k - R_t) \Phi_t Q_t S_t$ , by conducting the credit market intervention for the central bank.

In addition, note that in case the central bank obeys the rule in equation (2.43), the total credit supply in equation (2.38) may be reduced as:

$$Q_t S_t = \frac{\Psi_t N_t}{(1 - \Phi) \left(\frac{N_t}{N}\right)^{\nu}}$$

$$= S\left(\frac{\Psi_t}{\Psi}\right) \left(\frac{N_t}{N}\right)^{1 - \nu}. \tag{2.44}$$

Equation (2.44) implies that the central bank could relieve the fluctuations in total credit supply,  $Q_tS_t$ , by counteracting the private bankers' net worth,  $N_t$ , to a degree of credit market intervention parameter,  $\nu$ , which would lead to the alleviation of the recession in a financial crisis.

Lastly, fiscal authority implements government spending,  $G_t$ , which comprises current public spending,  $G_t^c$ , and expenditures on public credit supply,  $Q_t S_t^g = \Phi_t Q_t S_t$ , in the presence of credit market intervention, given by

$$G_t = G_t^c + \Phi_t Q_t S_t, \tag{2.45}$$

where current government expenditure,  $G_t^c$ , is assumed to be exogenously given by the following process:

$$G_t^c = \left(G_{t-1}^c\right)^{\rho_g} \exp\left\{\varepsilon_{q,t}\right\} \tag{2.46}$$

with  $|\rho_g| < 1$ , and  $\varepsilon_{g,t}$  being a Gaussian white noise with mean zero and standard deviation  $\sigma_g$ . The total government spending,  $G_t$ , is financed by lump-sum taxes,

 $T_t$ , the (net) issue of the public bond,  $(D_t - R_{t-1}D_{t-1})$ . In addition, if the central bank obeys the credit market intervention rule as in Gertler and Karadi (2011), the fiscal authority has an additional source of revenue from the public financial intermediation,  $(R_{t+1}^k - R_t) \Phi_t Q_t S_t$ . Thus, in this case, the government budget constraint is given (in real terms) by

$$G_t = T_t + (D_t - R_{t-1}D_{t-1}) + (R_{t+1}^k - R_t)\Phi_t Q_t S_t.$$
(2.47)

In contrast, either when the central bank does not intervene in the credit market or when it does so by following the credit market intervention rule in (2.43), there does not exist profit from the public financial intermediation for the authority. Thus, in such cases the government budget constraint is given by

$$G_t = T_t + (D_t - R_{t-1}D_{t-1}). (2.48)$$

# 2.3 Model Solution and Calibration

In this section, we discuss the solution method for the dynamic stochastic general equilibrium (DSGE) model, and deal with simulation strategy and parameter calibration for policy experiments.

### 2.3.1 Solution and Simulation Strategy

In the general equilibrium for our model, the infinite sequence of 21 endogenous variables,  $\{C_t, W_t, L_t, Q_t, S_t, K_t, N_t, V_t, Y_t, Y_{w,t}, A_t, P_{w,t}, I_t, G_t, G_t^c, \Psi_t, \Phi_t, \pi_t, R_t, Q_t, Q_t^c, Q_t^$  $R_t^k, R_t^n$  is determined to satisfy the 21 equilibrium conditions, which are listed in Appendix A1.1, given 4 temporary shocks,  $\{\varepsilon_{r,t}, \varepsilon_{v,t}, \varepsilon_{a,t}, \varepsilon_{g,t}\}$ . Technically speaking, our DSGE model belongs to a first order non-linear rational expectations (RE) system class, whose solution consists of a set of first order difference equilibrium equations relating the current variables to the past state of the system and current shocks, which is referred to as the policy function. As shown in Uhlig (1999), the analysis for the non-linear system may be conducted by the following procedure: (i) identifying the equilibrium conditions to construct a non-linear rational expectations (RE) system; (ii) transforming the non-linear rational expectations (RE) system into the linear one by using a first order Taylor expansion approximation around the steady state; (iii) choosing the parameter values by calibration; (iv) solving the first order linear rational expectations (RE) system by applying the numerical methods as in Blanchard and Kahn (1980), Klein (2000) and others; and then (v) investigating the properties of equilibrium path by analysing the impulse responses of the model economy to a certain shock.

Having transformed our non-linear model into the linear rational expectations (RE) system as in Appendix A1.2, by applying the log-linearisation technique presented in Appendix A2, we may write the model in the following linear first order difference equations system:

$$AE_t \{X_{t+1}\} = BX_t + CZ_{t+1}, \tag{2.49}$$

where  $X_t$  is a 21×1 vector of (log-deviated) endogenous variables,  $Z_t$  is a 4 dimensional vector of (log-deviated) exogenous stochastic shocks, A and B are 21×21 coefficient matrices, and C is a 21 × 4 coefficient matrix. Then, after parameter calibration discussed in the next section, we may solve the model numerically, by using, say, Michael Julliard's software DYNARE, given that Blanchard and Kahn (1980) conditions are satisfied.<sup>21</sup> Our numerical computation confirms that our model has a unique solution given some reasonable calibration of parameters, including a set of parameter values discussed in the next section.<sup>22</sup>

Having solved our DSGE model, we investigate the impulse responses of the model economy to diverse shocks under alternative economic environments to study the shock propagation process and the impact of the economic environmental change. First of all, in order to figure out how a disruption in the financial market propagates to the real economy, we investigate the impulse responses to a negative net worth shock (FA(NW) model). They are compared with the impulse responses to a negative capital quality shock (FA(CQ) model) to study sources of financial crisis. In addition, we compare the responses to a negative net worth shock in an economy with standard financial frictions (FA(NW) model) with those in the economy with a low degree of financial frictions (LFA(NW) model), to explore the role of financial accelerators in a financial crisis. Under the LFA(NW) model, the

<sup>&</sup>lt;sup>21</sup>Blanchard and Kahn (1980) show that the existence and uniqueness of the solution to the (stationary) first order linear system is determined by the relationship between the number of forward-looking (i.e., non-predetermined) variables in the system,  $n_x$ , and the number of unstable eigenvalues (i.e., eigenvalues outside the unit circle) of the coefficient matrix  $W = A^{-1}B$ ,  $n_u$ . That is, there exists a unique solution (determinacy), if  $n_x = n_u$ ; no stable solution, if  $n_x > n_u$ ; and an infinity of solution (indeterminacy), if  $n_x < n_u$ .

 $<sup>^{22}</sup>$ As shown in Appendix A1.2, our model has 6 forward-looking variables, such as  $\widehat{C}_{t+1}$ ,  $\widehat{R}_{t+1}^k$ ,  $\widehat{Y}_{w,t+1}$ ,  $\widehat{P}_{w,t+1}$ ,  $\widehat{Q}_{t+1}$ , and  $\widehat{\pi}_{t+1}$  in equations (A1.22), (A1.25), (A1.29), (A1.32) and (A1.35) in Appendix A1.2. Thus, Blanchard-Kahn condition requires the model to have 6 unstable eigenvalues for the transformed matrix,  $W = A^{-1}B$ , which turns out to be satisfied under some reasonable parameterisation, including our set of parameters.

degree of moral hazard in the banking sector, which is captured by the bankers' capital diversion rate,  $\omega$ , is low and the inverse sensitivity of the risk premium,  $\frac{R_{t+1}^{\kappa}}{R_t}$ , to the bankers' leverage ratio,  $\Psi_t$ , which is captured by the steady state value of financial accelerator,  $\Psi - 1$ , is taken to be high. Next, we study the effects of conventional monetary and fiscal policies in a financial crisis, by imposing both a negative net worth shock and expansionary policy shocks at the same time on the FA model (FA(NW+M) and FA(NW+F) models).<sup>23</sup> Lastly, we investigate the impact of the credit market intervention by the government. To this end, we introduce the public credit supply,  $\Phi_t Q_t S_t$ , to the economy with the standard degree of financial frictions (FA+CI model), where the central bank expands the public fund injection in response to a contraction in bankers' net worth,  $N_t$ , and a rise in the risk premium,  $\frac{R_{t+1}^k}{R_t}$ . Specifically, we assign non-zero value to the intensity parameter of the credit market intervention,  $\nu$ , and compare the impulse responses to a negative net worth shock from the FA(NW)+CI(CS) model with those from the model without credit market intervention (FA(NW) model). In addition, the effect of credit market intervention aiming to restore the private financial intermediation by enhancing the bankers' balance sheet, (FA(NW)+CI(CS) model) is compared with that of credit market intervention aiming at stabilising the nonfinancial firms' cost of external finance,  $\frac{R_{t+1}^k}{R_t}$ , by supplementing the private financial intermediation (FA(NW)+CI(RP) model) to study the operating mechanism of the central bank's credit market intervention. Parameter values used for each model are discussed in the subsequent part.

<sup>&</sup>lt;sup>23</sup>This configuration is to facilitate policy experiments. Hence, even though the assumption that the shock process in the policies,  $R_t^n$  or  $G_t^c$ , has the same structure as a shock to the bankers' net worth,  $V_t$ , is somewhat lacking in reality, it does not impede the aim of the policy experiments here

Table 2.1: Parameter Calibration (Common across Models)

Parameter	Value	Description
$\beta$	0.99	discount factor
$\sigma$	1.2	inverse of intertemporal elasticity of consumptions
arphi	3.0	inverse of elasticity of labour supply
$\phi$	0.975	bankers' survival rate
$\alpha$	0.3	capital share in production function
$\delta$	0.025	capital depreciation rate
$\kappa$	1.2	capital adjustment cost coefficient
heta	0.75	probability of not adjusting prices
$\epsilon$	6	elasticity of substitution among retail goods
$\alpha_r$	0.7	Taylor rule coefficient on interest rate smoothing
$lpha_\pi$	1.7	Taylor rule coefficient on inflation
$\alpha_y$	0.3	Taylor rule coefficient on output gap
$ ho_g$	0.8	persistence in government spending shock
$ ho_a$	0.8	persistence in technology shock
$ ho_v$	0.8	persistence in net worth shock
R	1.0101	rate of return to risk free asset in the steady state
$R^k$	1.0201	rate of return to capital in the steady state
C/Y	0.5614	consumption-to-output ratio in the steady state
I/Y	0.1386	investment-to-output ratio in the steady state
G/Y	0.3	government spending-to-output ratio in the steady state

## 2.3.2 Parameter Calibration

We calibrate the parameters,  $\{\beta, \sigma, \varphi, \phi, \omega, \alpha, \delta, \kappa, \epsilon, \theta, \alpha_r, \alpha_{\pi}, \alpha_y, \nu, \rho_a, \rho_g, \rho_v\}$ , and the steady state values for some endogenous variables,  $\{R, R^k, \Psi, \Phi, \frac{K}{N}, \frac{C}{Y}, \frac{I}{Y}, \frac{G}{Y}, \frac{G^c}{G}\}$ , which characterise the model economy. We assign to them the standard values in the literature, including Bernanke *et al.* (1999), Smets and Wouters (2003), Christensen and Dib (2007), Gali (2008) and Gertler and Karadi (2011). Table 2.1 presents the calibration result which is common across alternative models, (i.e., FA, LFA, and FA+CI models), and Tables 2.2 shows the parameter values which are different by model. Table 2.3 compares the parameter values calibrated for the FA model with those in the previous literature.

Table 2.2: Parameter Calibration (By Model)

Parameter	Description	LFA	FA	FA+CI
$\omega$	capital diversion rate	0.1905	0.381	0.381
$\Psi$	private financial accelerator in the	5.4798	2.6676	2.6676
	steady state			
$\Phi$	portion of credit market interven-	0	0	0.15
	tion in the steady state			
S/N	total credit-to-net worth ratio in	5.4798	2.6676	3.1384
	the steady state			
$G^c/G$	share of current expenditures in	0	0	0.8
	government spending			
$\nu$	credit market intervention coeffi-	0	0	0.5
	cient			

<sup>\*</sup>LFA: model with low financial frictions but without credit market intervention

First of all, we discuss the parameter values common across models. We set the quarterly discount factor,  $\beta$ , at 0.99, which also pins down the steady state quarterly riskless rate, R, at  $R = \frac{1}{\beta} = 1.0101$  (annually 4.1%). We fix the inverse of intertemporal elasticity of consumption,  $\sigma$ , and the inverse of labour supply elasticity,  $\varphi$ , at 1.2 and 3.0, respectively, in keeping with much of the literature. We take a quarterly risk spread,  $R^k - R$ , to be one hundred basis point, so that the steady state risk premium is pinned down at  $\frac{R^k}{R} = \frac{1.0201}{1.0101} = 1.0099$ . The bankers' quarterly survival rate,  $\phi$ , is set to be 0.975, so that the average duration of bankers is 10 years (i.e.,  $\frac{1}{1-\phi} = 40$ ). As is also within convention, we take the share of capital in production,  $\alpha$ , to be 0.3. In addition, we assign the conventional value of 0.025 to the quarterly capital depreciation rate,  $\delta$ , implying that capital stock is depreciated about 10 percent annually. The coefficient for capital adjustment cost,  $\kappa$ , is assumed to be 1.2, so that the inverse of elasticity of investment to the capital price,  $\kappa\delta$ , is calculated as 0.03. The elasticity of substitution among varieties,  $\epsilon$ , is

<sup>\*</sup>FA: model with standard financial frictions but without credit market intervention

<sup>\*</sup>FA+CI: model with standard financial frictions and credit market intervention

set to be 6, so that retail firms' desired mark-up is pinned down at  $\mu = \frac{\epsilon}{\epsilon - 1} = 1.2$ . In addition, we let the probability of retail firms keeping prices unchanged within a quarter,  $\theta$ , be equal to 0.75, implying that the average duration of retail price for a certain variety is a year (i.e.,  $\frac{1}{1-\theta} = 4$ ). Accordingly, the coefficient attached to the retail firms' marginal cost in New Keynesian Phillips curve in (2.32) is calculated as  $\lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta} = 0.0858$ . In addition, the steady state value of the investment-to-output ratio,  $\frac{I}{Y}$ , is calculated as 0.1386.<sup>24</sup> Then, by setting the steady state share of government expenditure in output,  $\frac{G}{Y}$ , as  $\frac{G}{Y} = 1 - \frac{I}{Y} - \frac{G}{Y} = 0.5614$ . In addition, we assume that the central bank sets Taylor rule coefficients,  $\alpha_r$ ,  $\alpha_\pi$ , and  $\alpha_y$ , to be 0.7, 1.7, and 0.3, respectively, which are in the range of conventional values in the literature. The persistence parameters for shocks from technology, government expenditure, and value of net worth,  $\rho_a$ ,  $\rho_g$ , and  $\rho_v$ , respectively, are all assumed to be 0.8, which also follows the conventional business cycle literature.

Next, the parameters which are different by model are calibrated as follows. First of all, following Gertler and Karadi (2011), we set the fraction of capital diverted by bankers,  $\omega$ , at 0.381, for the FA model, so that the corresponding values for private financial accelerator,  $\Psi$ , and bankers' asset-to-net worth ratio,  $\frac{S^p}{N}$ , in the steady state are calculated as  $\frac{S^p}{N} = \Psi = \left[1 - (1 - \omega) \frac{R^k}{R}\right]^{-1} = 2.6676$ , implying that bankers supply to non-financial firms approximately 2.7 times of funds as much as their own net worth in the steady state, by borrowing from households the difference between the credit supply and their own net worth. In contrast, for the LFA model representing an economy with a low degree of financial

 $<sup>\</sup>frac{1}{2^{4}}$ It follows from the fact that  $\frac{I}{Y} = \left(\frac{I}{K}\right) \left(\frac{K}{Y_{w}}\right) = \delta \left[\left(\frac{\alpha}{R^{k} - (1 - \delta)}\right) \left(\frac{\epsilon - 1}{\epsilon}\right)\right], where we use the steady state relations that <math>R^{k} = \alpha \left(\frac{Y_{w}}{K}\right) P_{w} + (1 - \delta)$  and  $P_{w} = \frac{\epsilon - 1}{\epsilon}$ .

Table 2.3: Parameter Calibration (Comparison by Author)

Parameter	BGG	SW	Gali	CD	GK	FA
$\beta$	0.99	0.99	0.99	0.99	0.99	0.99
$\sigma$		1	1.0			1.2
arphi	3.0	2	1.0			3.0
$\phi$	0.9728			0.9728	0.972	0.975
$\alpha$	0.35	0.30	0.33	0.3384	0.330	0.3
$\delta$	0.025	0.025		0.025	0.025	0.025
$\kappa$	10	0.692		0.59	69.12	1.2
heta	0.75	0.905	0.667	0.7418	0.779	0.75
$\epsilon$	6		6	6	4.7	6
$lpha_r$	0.9	0.8			0.8	0.7
$lpha_\pi$	1.1	1.7	1.5	1.4059	2.043	1.7
$\alpha_y$	0	0.5	0.125	0.2947	0.125	0.3
$\omega$					0.381	0.381
$ ho_g$	0.95				0.85	0.8
$ ho_a$	1.0	0.85	0.5	0.7625	0.85	0.8
$R^k - R$	0.02			0.0075	0.01	0.01
K/N	2	2.2		2	4	2.6676
G/Y	0.2	0.18			0.2	0.3

<sup>\*</sup>BGG (Bernanke, Gertler and Gilchrist, 1999), SW (Smets and Wouters, 2003), Gali (Gali, 2008, chapter 3), CD (Christensen and Dib, 2007), GK (Gertler and Karadi, 2011), FA (the benchmark model)

frictions, the bankers' capital diversion rate,  $\omega$ , is set at half the value in the FA model, i.e.,  $\omega=0.1905$ , which is linked to higher values of private financial accelerator parameter and steady state asset-to-net worth ratio, i.e.,  $\Psi=\frac{S^p}{N}=5.4798$ , than those in the FA model. On the other hand, for the FA+CI model, representing an economy in the presence of credit market intervention by the central bank, we assume that the size of credit market intervention is  $\Phi=\frac{S^g}{S}=0.15$  as compared to the size of asset market, and  $1-\frac{G^c}{G}=0.2$  as compared to total government spendings. Accordingly, the total leverage ratio is calculated as  $\frac{S}{N}=\frac{\Psi}{1-\Phi}=\frac{2.6676}{0.85}=3.1384$ , in the presence of credit market intervention. The credit market intervention parameter,  $\nu$ , is set to be 0.5, implying that the central bank increases the portion of public credit out of total credit supply,  $\Phi_t$ , by half as much as the contraction in bankers' net worth,  $N_t$ , in terms of percentage. On the contrary, the parameter,  $\nu$ , is set to be zero for the FA and LFA models, implying that the central bank does not react to the contraction in bankers' net worth.

# 2.4 Model Dynamics

#### 2.4.1 Transmission of Financial Crisis

This part explores the transmission mechanism of a financial crisis, implying the real recession in the economy triggered by an adverse shock in the financial market. To this end, we suppose the situation where the bankers' net worth collapses unex-

<sup>&</sup>lt;sup>25</sup>The steady state values of credit market intervention is tricky to calibrate, since credit market intervention is an exceptional event. However, in order to facilitate analysis, we calibrate the size of credit market intervention by considering the event in the United Kingdom in 2009. At that time, the Bank of England purchased 200 billion worth of private assets while the sizes of the UK Gilt market and the UK's government spendings are roughly 1,300 billion pounds and 700 billion pounds, respectively.

pectedly in the financial market. The solid lines in Figure 2.1 and 2.2 show how the model economy with the standard degree of financial frictions (FA(NW) model) reacts to a negative net worth shock. To study the role of financial accelerator in a financial crisis, they are compared to the impulse responses to the same shock under the environment with a lower degree of financial frictions (LFA(NW) model), which are represented by the dotted lines in Figure 2.1. In addition, to investigate sources of a financial crisis, the responses to a negative net worth shock are compared to those to a negative capital quality shock in the model with standard financial frictions (FA(CQ) model), which are shown in the dotted lines in Figure 2.2. However, one needs to note that our investigation on fluctuations from alternative models aims at understanding how the economy responds to the respective shock. Thus, the comparison here does not have quantitative implication.

First of all, we examine the motions in the solid lines in Figure 2.1 to investigate the transmission process of a financial crisis. A decline in bankers' net worth,  $N_t$ , in (2.12) and (2.13) immediately reduces the credit supply to non-financial firms,  $Q_tS_t^p$ , in an amplified manner, due to the balance sheet constraint in (2.9). That is, credit supply to non-financial firms,  $Q_tS_t^p$ , is reduced due to the deficiency in funds, which is due to not only the initial decline in the bankers' own net worth,  $N_t$ , but also the resulting deterioration in the bankers' leverage ratio,  $\Psi_t = \frac{Q_tS_t^p}{N_t}$ , where the latter would make it more difficult for them to obtain funds from households. In addition, the bankers who face deficiency in funds would impose the higher price on credit supply,  $Q_tS_t^p$ , i.e., the risk premium,  $\frac{R_{t+1}^k}{R_t}$ . Both the contraction in credit supply,  $Q_tS_t^p$ , and the increased cost of capital,  $\frac{R_{t+1}^k}{R_t}$ , would lead to contractions in capital demand,  $K_t$ , output,  $Y_{w,t}$  (=  $Y_t$ ), and investment,  $I_t$ , by (2.16), (2.19), (2.14) and (2.23). All in all, a disruption in the

financial market could lead to a production contraction in the real sector of an economy.

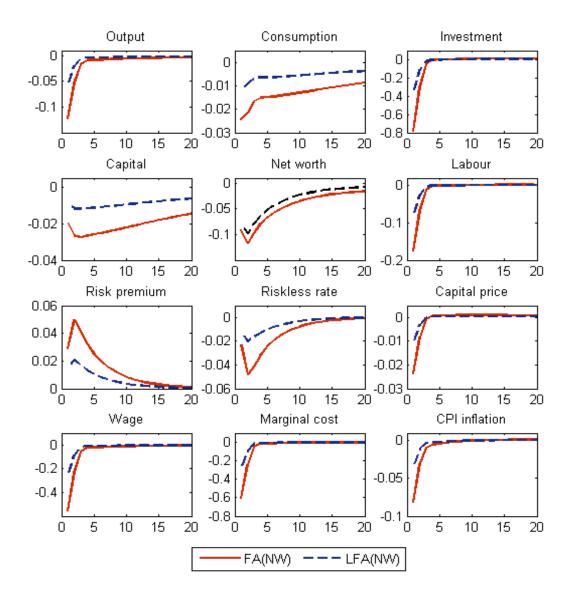
Now, we turn to demand side of the economy. As shown in the solid lines in Figure 2.1, the production contraction,  $Y_t$ , reduces the factor demands,  $K_t$  and  $L_t$ , by (2.14) and depresses the corresponding factor prices,  $Q_t$  and  $W_t$ , which decrease the real marginal cost, i.e., the real wholesale good price,  $P_{w,t}$ , and the inflation of retail price,  $\pi_t$ , by (2.32). In this event, the monetary authority eases the nominal interest rate,  $R_t^n$ , in response to the fall in CPI inflation,  $\pi_t$ , and production contraction,  $Y_t$ , by the Taylor-type feedback rule in (2.34), which, in turn, decreases the real riskless rate,  $R_t$ , by the Fisher equation in (2.35). In addition, the fall in wage,  $W_t$ , reduces households' consumption,  $C_t$ , by labour supply function in (2.5), which is limited to some degree by the fall in riskless rate,  $R_t$ , by Euler equation in (2.4). Overall, the production contraction,  $Y_t$ , corresponds to the reductions in consumption,  $C_t$ , and investment,  $I_t$ , by the resource constraint in (2.33).

The above transmission mechanism of a negative net worth shock to the model economy reflects the way how the recent financial crisis developed. That is, it is widely accepted that the recent crisis originated from the collapse of the bubble in the U.S. housing market, which distorted the financial intermediaries' balance sheet and discouraged the economic agents' activities by the reduction in credit supply to non-financial firms. That is what we have shown by our financial crisis simulation.<sup>26</sup>

Second, the role of financial accelerator mechanism in a financial crisis is

<sup>&</sup>lt;sup>26</sup>However, note that our model does not capture the economic agents' speculative behaviours in the recent financial crisis. Rather, it assumes that the economic agents try to minimise the hazardous impact of a negative shock on their welfare.

Figure 2.1: Responses to a Negative Net Worth Shock under Different Degree of Financial Frictions



<sup>\*</sup> The solid and dotted lines represent the impulse responses to negative shocks from net worth (NW) in the models with standard (FA) and low degree of financial frictions (LFA), respectively.

investigated by comparison of impulse responses of the economies with different degree of financial frictions. The dotted lines in Figure 2.1 represent the impulse responses to a negative net worth shock in the economy with lower degree of financial frictions, where the lower degree of moral hazard,  $\omega$ , exists in the steady state and the sensitivity of bankers' leverage ratio to the risk premium,  $\Psi - 1$ , is relatively high. That is, in the economy with a low degree of moral hazard, households react less sensitively even if the banker's balance sheet is deteriorated, so that bankers would have less difficulty in obtaining funds from households even in the financial crisis period. Accordingly, the bankers in the economy with a low degree of financial frictions feel it less necessary to reduce the credit supply to nonfinancial firms,  $Q_t S_t^p$ , or to raise the risk premium,  $\frac{R_{t+1}^k}{R_t}$ , than those in the economy with a high degree of financial frictions. This is confirmed in Figure 2.1, where the fluctuations in the economy with a lower degree of financial frictions (LFA) model) could be relieved, as compared to those in the economy with a standard degree of financial frictions (FA model). Based on the result, we may argue that the economy with a higher degree of moral hazard in the financial sector could be more vulnerable to the shocks in the financial market, and could experience the financial crisis more severely.

Third, we compare the impulse responses to a negative net worth shock (NW), with those to a negative capital quality shock (CQ) a la Gertler and Karadi (2011) in the FA model, to study the sources of financial crises. The capital quality shock,  $\xi_t$ , may be attached to capital,  $K_{t-1}$ , which provides a source of an exogenous variation in the effective amount of capital,  $\xi_t K_{t-1}$ . The presence of capital quality shock,  $\xi_t$ , affects equations associated with capital,  $K_{t-1}$ , such as asset market equilibrium condition in (2.16), production function in (2.14),

demand and supply for capital in (2.19) and (2.23), and capital accumulation in (2.21), as follows:

$$Q_{t-1}(\xi_t K_{t-1}) = Q_{t-1} S_{t-1}^p, \tag{2.50}$$

$$Y_{w,t} = A_t \left( \xi_t K_{t-1} \right)^{\alpha} \left( L_t \right)^{1-\alpha}, \tag{2.51}$$

$$E_t \left\{ R_{t+1}^k Q_t \right\} = E_t \left\{ \alpha \left( \frac{Y_{w,t+1}}{\xi_{t+1} K_t} \right) P_{w,t+1} + (1 - \delta) Q_{t+1} \right\}, \tag{2.52}$$

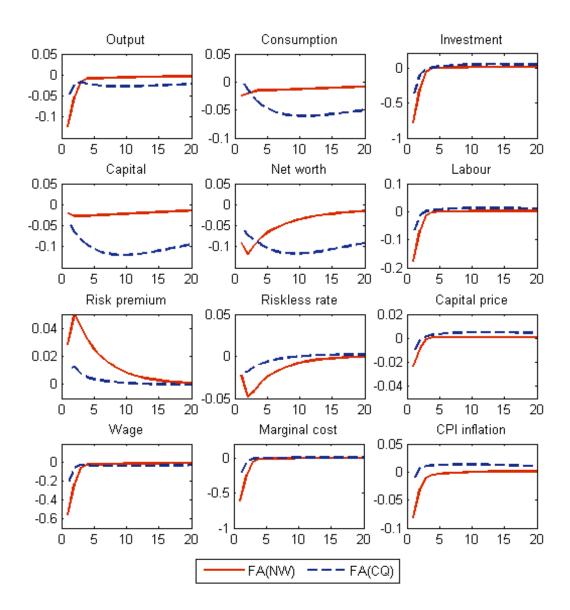
$$Q_t = \left[1 - \kappa \left(\frac{I_t}{\xi_t K_{t-1}} - \delta\right)\right]^{-1}, \qquad (2.53)$$

and

$$K_t = \xi_t \left[ K_t^n + (1 - \delta) K_{t-1} \right]. \tag{2.54}$$

The model with this configuration is denoted as the FA(CQ) model, and the impulse responses to a negative capital quality shock are displayed in the dotted lines in Figure 2.2, which are compared to the solid lines representing the responses to a negative net worth shock (which are the same as the solid lines in Figure 2.1). Comparison of pairs of lines in Figure 2.2 reveals that these two types of shocks produce the fluctuations with the same direction, regardless of the differences in the size of fluctuation and paths to the steady state. In fact, an unexpected fall in capital quality implies that the capital returns that bankers could expect is lowered, which leads to an endogenous fall in bankers' net worth. The process

Figure 2.2: Responses to a Negative Net Worth Shock and a Negative Capital Quality Shock



<sup>\*</sup> The solid and dotted lines represent the impulse responses to negative shocks from net worth (NW) and capital quailty (CQ) in the FA model with standard degree of financial frictions, respectively.

afterwards are the same as the scenario with a negative net worth shock. That is, the deteriorated bankers' balance sheet makes it difficult for bankers to obtain deposits from households, leading to either the shrinking in credit supply to non-financial firms or the rise in the cost of capital. Thus, we may argue that these two shocks are similar in the effect on the economy, in spite of the difference in the source of the shocks (i.e., shocks in the financial market versus those in production technology). Furthermore, the interpretation of the result might be that the capital quality shock could be one of the sources to reduce the bankers' net worth, like many other sources in the financial market, such as bad news for the banker, a collapse of bubble in asset value, and so on.

#### 2.4.2 Effect of an Expansionary Monetary Policy

Next, we investigate the effect of an expansionary monetary policy in a financial crisis. We suppose that the central bank decreases the nominal interest rate,  $R_t^n$ , when the value of bankers' net worth,  $N_t$ , collapses so that a financial crisis as discussed above is expected to occur. To simulate this, we impose negative shocks to both the central banks' feedback rule in (2.34) and bankers' net worth,  $N_t$ , in (2.12) at the same time. The dotted lines in Figure 2.3 represent the impulse responses to a negative net worth shock in the presence of an expansionary monetary policy (FA(NW+M) model), while the solid lines display those in the absence of the monetary policy (FA(NW) model), which are the same as those in the previous financial crisis experiment. Accordingly, the comparison between the two impulse responses reveals the role of an expansionary monetary policy under

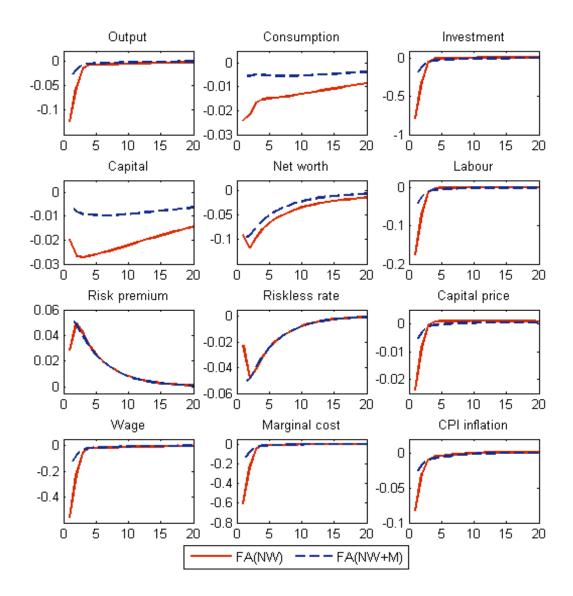
## a financial crisis.<sup>27</sup>

Given that the central bank reduces the nominal interest rate,  $R_t^n$ , to stabilise the economy in the financial crisis, household consumption,  $C_t$ , increases via the reduced real riskless rate,  $R_t$ , by the Euler equation in (2.4). The expansion in the aggregate demand would induce the increase in output production,  $Y_t$ , by the resource constraint in (2.33), which, in turn, encourages factor demands,  $K_t$  and  $L_t$ , by the production function in (2.14). In addition, the fall in real interest rate,  $R_t$ , would lead to a rise in risk premium,  $\frac{R_{t+1}^k}{R_t}$ , i.e., the profitability from financial intermediation for bankers in the FA(NW+M) model. Accordingly, it encourages bankers to increase the credit supply to non-financial firms,  $Q_t S_t^p = \Psi_t N_t$ , which increases capital demand,  $K_t$ , and the bankers' net worth,  $N_t$ , over time. Then, the increase in capital investment,  $I_t$ , induced by the increase in capital,  $K_t$ , by the capital supply in (2.23) makes another source of the aggregate demand growth by (2.33). All in all, an expansionary monetary policy could be an effective tool to stabilise the economy in a financial crisis, by stimulating the aggregate demand.<sup>28</sup> However, the rises in the factor prices,  $Q_t$  and  $W_t$ , induced by the expansions in factor demands,  $K_t$  and  $L_t$ , would raise the marginal production cost,  $P_{w,t}$ , and then, CPI inflation rate,  $\pi_t$ , by the New Keynesian Phillips curve in (2.32), which limits the central bank's expansionary position by the Taylor rule in (2.34).

<sup>&</sup>lt;sup>27</sup>One needs to note that the comparison of the impulse responses in each model does not have quantitative implication. It just show how an economy responds to the respective shocks.

<sup>&</sup>lt;sup>28</sup>However, this argument holds so long as the central bank is able to adjust the nominal interest rate,  $R_t^n$ , to stabilise the economy. In contrast, for example, under zero lower bound (ZLB) of the nominal interest rate, the central bank is unable to decrease the nominal interest rate, so that the validity of the argument might be undermined.

Figure 2.3: Effect of an Expansionary Monetary Policy under a Negative Net Worth Shock



<sup>\*</sup> The solid and dotted lines represent the impulse responses to a negative net worth shock in the absence (NW) and presence (NW+M) of an expansionary monetary policy in the FA model with standard degree of financial frictions, respectively.

### 2.4.3 Effect of an Expansionary Fiscal Policy

We turn to the effect of an expansionary fiscal policy in a financial crisis. We suppose that the fiscal authority expands the current public spending,  $G_t^c$ , when the value of bankers' net worth,  $N_t$ , collapses so that a financial crisis is anticipated. To simulate this, when the bankers' net worth,  $N_t$ , is hit by a negative shock in (2.12), we impose the positive shock on the current public spending process in (2.46) at the same time. The dotted lines in Figure 2.4 represents the impulse responses to a negative net worth shock in the presence of an expansionary fiscal policy (FA(NW+F) model), which are compared to the solid lines in Figure 2.4 representing those in the absence of the fiscal policy (FA(NW)), in order to study the role of an expansionary fiscal policy in the financial crisis.<sup>29</sup>

An exogenous increase in the current public spending,  $G_t^c$ , in the FA(NW+F) model, immediately increases aggregate demand,  $Y_t$ , by the resource constraint in (2.33), as compared to that in the FA(NW) model, as shown in Figure 2.4. However, the expansion in the current public spending,  $G_t^c$ , also raises the real riskless rate,  $R_t$ , which decreases household consumption,  $C_t$ , by the Euler equation in (2.4). It could dampen the initial increase in aggregate demand,  $Y_t$ , driven by the expansion in government spending. Accordingly, the size of the increase in factor demands,  $K_t$  and  $L_t$ , under an expansionary fiscal policy could be smaller than that under a monetary policy counterpart. Furthermore, the raised riskless rate,  $R_t$ , reduces the risk premium,  $\frac{R_{t+1}^k}{R_t}$ , so that the bankers are less willing to increase credit supply to non-financial firms,  $Q_t S_t^p = \Psi_t N_t$ , due to the reduced improvement in profitability from financial intermediation. It restricts the recovery in capital

<sup>&</sup>lt;sup>29</sup>One needs to note that the comparison for this part does not have quantitative implication.

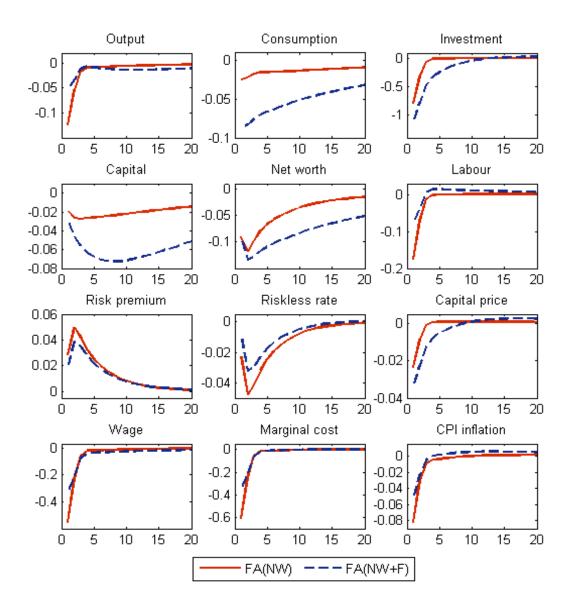
demand,  $K_t$ , and investment,  $I_t$ , by non-financial firms. Limitation of increases in consumption,  $C_t$ , and investment,  $I_t$ , under an expansionary fiscal policy makes it a less effective stabilising measure in a financial crisis than an expansionary monetary policy, as the conventional 'crowding-out effect' argument suggests.<sup>30</sup> Thus, we confirm the difference between the effects of conventional policies on the aggregate demand in the movement of the riskless rate,  $R_t$ . That is, an expansionary monetary policy lowers the riskless rate, while an expansionary fiscal policy raises it. Accordingly, the contraction in consumption is relieved under the former, while that in consumption is further aggravated under the latter. All of these points are consistent with the theoretical arguments in the existing literature.

Furthermore, it is noteworthy that the financial accelerator mechanism applies only to capital, while it does not to labour. That is, the reduced risk premium discourages only capital demand, by construction. Thus, facing the reduced increase in the credit supply and the raised cost of borrowing for capital acquisition, wholesale firms could become more reliant on labour than capital to correspond to the increased aggregate demand driven by an expansionary fiscal policy. The dotted lines in Figure 2.4 shows that a negative effect on the capital demand by the factor substitution is more than offsetting a positive effect by the increase in the aggregate demand,  $^{31}$  so that the capital investments,  $K_t$  and  $I_t$ , shrinks despite the expansionary fiscal policy. That is, an expansionary fiscal policy in a financial crisis could be contractionary to capital demand. In addition, the marginal cost

<sup>&</sup>lt;sup>30</sup>In contrast, more recent studies argue that fiscal policy could be quite effective under some environment. That is, it could be so in case where Ricardian equivalence does not hold (Gali, Lopez-Salido, and Valles, 2007) or where the economy reaches at the zero lower bound (ZLB) (Christiano, Eichenbaum, and Rebelo, 2009). However, the analysis in such an environment is beyond the scope of the thesis.

<sup>&</sup>lt;sup>31</sup>This result is partly due to the fact that the size of the increase in the capital has been already reduced significantly by the 'crowding-out effect.'

Figure 2.4: Effect of an Expansionary Fiscal Policy under a Negative Net Worth Shock



<sup>\*</sup> The solid and dotted lines represent the impulse responses to a negative net worth shock in the absence (NW) and presence (NW+F) of an expansionary fiscal policy in the FA model with standard degree of financial frictions, respectively.

of production,  $P_{w,t}$ , and the CPI inflation,  $\pi_t$ , are determined by the relative size of the rise in wage,  $W_t$ , and the fall in capital price,  $Q_t$ . Figure 2.4 shows that the former is greater than the latter, so that CPI inflation rises slightly in our experiment in the presence of the expansionary fiscal policy (FA(NW+F) model), as compared to those in the absence of the fiscal policy (FA(NW)). This point has not been highlighted in the previous literature.

#### 2.4.4 Effect of Credit Market Intervention

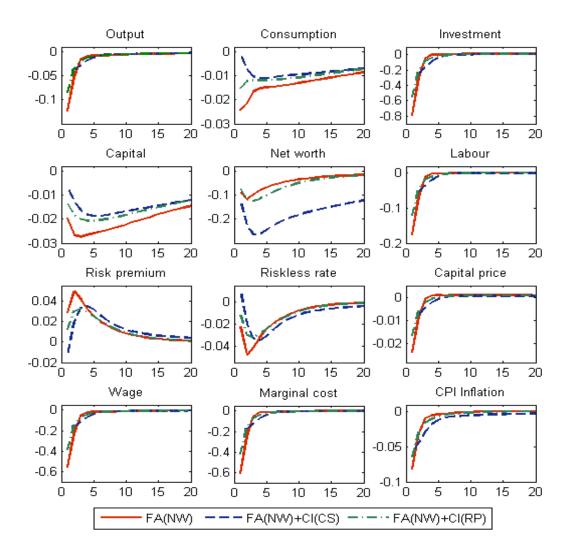
Now, we analyse the impact of credit market intervention by the central bank in a financial crisis. We suppose that bankers' net worth,  $N_t$ , collapses so that it is expected that the private credit supply to non-financial firms,  $Q_t S_t^p = \Psi_t N_t$ , shrinks in an amplified manner, and the risk premium imposed on non-financial firms by bankers,  $\frac{R_{t+1}^k}{R_t}$ , soars. In recognition of the collapse in bankers' net worth, the central bank may inject the public funds,  $\Phi_t$ , into private bankers' balance sheet, by the credit market intervention rule,  $\Phi_t = 1 - (1 - \Phi) \left(\frac{N_t}{N}\right)^{\nu}$  with  $\nu > 0$ , in (2.43), to restore the private financial intermediation. Alternatively, the monetary authority may use the risk premium,  $\frac{R_{t+1}^k}{R_t}$ , as a policy target, and supply the public credit directly to non-financial firms, following the credit market intervention rule,  $\frac{\Phi_t}{\Phi} = \left(\frac{R_{t+1}^k/R_t}{R^k/R}\right)^v$  with v > 0, in (2.39), to supplement the private credit supply. The impulse responses from the setup where the central bank follows the credit market intervention rule,  $\Phi_t = 1 - (1 - \Phi) \left(\frac{N_t}{N}\right)^{\nu}$  with  $\nu > 0$ , denoted by the FA(NW)+CI(CS) model, are represented by the dotted lines in Figure 2.5. On the other hand, those from the setup where the central bank follows the credit market intervention rule,  $\frac{\Phi_t}{\Phi} = \left(\frac{R_{t+1}^k/R_t}{R^k/R}\right)^v$  with v > 0, denoted by the FA(NW)+CI(RP) model, are represented by the dash-dot lines in Figure  $2.5.^{32}$ 

Figure 2.5 shows how the credit market intervention moderates the contractions in a financial crisis. First of all, to investigate the working of the credit market intervention in the FA(NW)+CI(CS) model, we examine the dotted lines in Figure 2.5. Since the central bank injects the public funds,  $Q_tS_t^g = \Phi_tQ_tS_t$ , to bankers' balance sheet at the lending rate,  $R_t$ , the initial collapse in bankers' net worth,  $N_t$ , is relieved. Then, based on the enhanced balance sheet,  $N_t$ , the bankers could increase the credit supply to non-financial firms,  $Q_tS_t^p = \Psi_tN_t$ , in an amplified manner, because bankers find it easier to acquire funds from households. In addition, the bankers' enhanced balance sheet makes it less necessary for them to raise the risk premium,  $\frac{R_{t+1}^k}{R_t}$ , because they suffer the deficiency in fund deposit less. The increased credit supply and lowered cost of capital allow non-financial firms to increase capital demand and production, so that the financial crisis triggered by the collapse of bankers' net worth could be effectively relieved.

Next, the dash-dot lines in Figure 2.5 show that the credit market intervention following the alternative rule,  $\frac{\Phi_t}{\Phi} = \left(\frac{R_{t+1}^k/R_t}{R^k/R}\right)^v$ , could also be effective to relieve a financial crisis. The FA(NW)+CI(RP) model implies that, when the risk premium,  $\frac{R_{t+1}^k}{R_t}$ , soars rapidly, the central bank injects the public fund directly to non-financial firms at the market rate,  $R_{t+1}^k$ . Then, non-financial firms could avoid the rise in cost of capital finance and deficiency in fund for capital so that the economic behaviours in the real sector could be effectively isolated from the disruption in the financial market originated from the distortion in the bankers' balance sheet.

<sup>&</sup>lt;sup>32</sup>One needs to note that our investigation on fluctuations from alternative models does not have quantitative implication.

Figure 2.5: Effect of Credit Market Intervention under a Negative Net Worth Shock



<sup>\*</sup> The dotted and dash-dot lines represent the impulse responses to a negative net worth shock under the credit market intervention with benchmark and alternative rules (FA(NW)+CI(CS) and FA(NW)+CI(RP) models), respectively, while the solid lines are those in the absence of the credit market intervention.

It is noteworthy that these two rules are different in a way the central bank operates. Under the first rule, the central bank monitors the financial conditions of bankers, and tries to restore the private financial intermediation by enhancing the private bankers' balance sheet. From the credit market intervention, government would earn zero profit, but the private bankers would earn the risk premium,  $\frac{R_{t+1}^k}{R_t}$ , regardless of the source of funds. In contrast, the monetary authority following the second rule checks the risk premium in the financial market which is related to the non-financial firms' cost condition for capital acquisition and seeks to stabilise the risk premium by supplying the credit for capital directly to non-financial firms. From the credit market intervention, the government would earn the market prevalent profit of  $\frac{R_{t+1}^k}{R_t}$ . However, these two rules are common in that both could be good ways to fight the financial crisis. In fact, Figure 2.5 clearly shows that the contractions triggered by a financial disruption are relieved in the presence of credit market intervention, no matter which rule the central bank follows. This is because these two seemingly different rules are closely related to each other, by the financial accelerator,  $\Psi_t = \left[1 - (1 - \omega) \frac{R_{t+1}^k}{R_t}\right]^{-1}$ , in (2.10). That is, the risk premium,  $\frac{R_{t+1}^k}{R_t}$ , and the bankers' leverage ratio,  $\Psi_t = \frac{Q_t S_t^p}{N_t}$ , are positively related, which implies that an improved profitability from financial intermediation,  $\frac{R_{t+1}^k}{R_t}$ , would encourage the bankers' financial intermediation activities,  $\Psi_t = \frac{Q_t S_t^p}{N_t}$ .

## 2.5 Conclusion

We have constructed a New Keynesian DSGE model with financial frictions to investigate how a financial disruption propagates to a real economy and how the financial intermediation of the banking sector plays a role in the process. We also explore how effective the conventional monetary and fiscal policies are to stabilise the economy when it is hit by a financial crisis. In addition to the conventional policy measures, we analyse the effect of the credit market intervention by the central bank, widely referred to as 'quantitative easing (QE)'. To these ends, we examine the impulse responses of the model economy to diverse shocks under the alternative environments: the shocks from bankers' net worth, wholesale firms' capital quality, the monetary and fiscal policies in the economies with a standard and low degree of financial frictions in the absence of credit market intervention by the central bank and the economy with a standard degree of financial frictions in its presence.

Our findings in this chapter can be summarised as follows. First, we show that an unexpected collapse in bankers' net worth in the financial market could lead to a real downturn in the business cycle by either reducing the credit supply to non-financial firms or increasing the cost of capital imposed on the firms. In addition, a negative shock to the capital quality in the non-financial firms' production technology could also result in the financial crisis via the deterioration in bankers' balance sheet. Based on the results, we argue that one of the sources of the collapse in bankers' net worth, which triggers a financial crisis, would be a fall in the capital profitability. Moreover, we find that a higher degree of moral hazard in the banking sector could make the economy more vulnerable to a financial crisis, since households would reduce the deposit more sensitively to a deterioration in bankers' balance sheet. Second, we find that the conventional monetary and fiscal policies are effective in relieving the business cycle fluctuations in a financial crisis, so long as they are available to the authorities. However, we also establish that an expansionary fiscal policy could be less effective in relieving the contraction in

capital demand than the monetary policy counterpart. This is because the former is not only involved in so-called 'crowding-out effect' by raising the real interest rate, but it could also discourage the bankers' credit supply for capital acquisition via the aggravated profitability from financial intermediation in a financial crisis. Third, we show that the credit market intervention by the central bank could dampen the contractions in capital investment effectively by either enhancing the bankers' balance sheet or stabilising the cost of capital for non-financial firms. In addition, we argue that the credit market intervention rule seeking to enhance the bankers' balance sheet produces the qualitatively similar result to the alternative rule aiming at stabilising the risk premium, since the bankers' leverage ratio and the risk premium are positively related to each other by the financial accelerator mechanism.

This chapter contributes to the existing literature as follows. First, we highlight the important role of a sudden collapse in borrowers' net worth as a trigger of a financial crisis, while the existing research usually considers non-financial shocks such as the capital quality shock. We argue that our consideration may provide the more relevant and realistic description on a financial crisis in the sense that a shock in the financial market leads to a real recession in the economy via the financial frictions. Second, we derive an optimal credit market intervention rule for the central bank, while the previous research just assumes the rule by the economic intuition. Moreover, under the derived credit market intervention rule, the central bank injects the public funds into the private financial intermediaries to enhance their balance sheet, and hence, to restore the private financial intermediation. Thus, the rule we derive seems to reflect the central bank's practical behaviour more realistically. Third, we point out the effect of conventional policies

on the production side via the impact on the bankers' profitability from the financial intermediation, while the traditional study focused mainly on the demand side. That is, on the one hand, we confirm that expansionary policies could encourage the aggregate demand as the traditional one suggests; on the other hand, we show that an expansionary fiscal policy could discourage the aggregate supply due to the aggravated profitability from financial intermediation, while an expansionary monetary policy could encourage it due to the improved one.

However, our research has some limitations on addressing the long-run growth effect of the shocks and capturing the economy's structural change. For example, in addition to triggering a financial crisis, a negative shock to bankers' net worth could cause some structural changes in economic agents' behaviours, which implies that the steady state itself could be changed. Indeed, the economies which are affected by the recent global financial crisis have become increasingly vulnerable to even a trivial shock to the financial markets as compared to the one prior to the crisis, which suggests that the economies' degree of financial frictions has increased in the wake of the crisis. Nevertheless, our research relying on a DSGE approach assumes the coefficients in the model are constant and are not affected by the shocks. It just considers the marginal effect of the shocks on the economy around the steady state due to the mean reverting property of the model. Therefore, the research could go further to the one that considers structural or long-run effects of the shocks in the future.

# Appendix A

# Appendix A1 The Model Solution

The model is a system consisting of 21 behavioural equations with 21 endogenous variables such as  $\{C_t, W_t, L_t, Q_t, S_t, K_t, N_t, V_t, Y_t, Y_{w,t}, A_t, P_{w,t}, I_t, G_t, G_t^c, \Psi_t, \Phi_t, \pi_t, R_t, R_t^k, R_t^n\}$  and 4 exogenous shocks such as  $\{\varepsilon_{r,t}, \varepsilon_{v,t}, \varepsilon_{a,t}, \varepsilon_{g,t}\}$ . A1.1 identifies the nonlinear equations characterising equilibrium in the model. These equations may be approximated around the steady state to be transformed into the linear equations, which is presented in A1.2. Derivation and log-linearisation process for equations is presented in Appendix A2.

### A1.1 Equilibrium Conditions

1. Consumption Euler equation:

$$1 = \beta E_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} R_t \right\} \tag{A1.1}$$

2. Labour supply:

$$W_t = (C_t)^{\sigma} (L_t)^{\varphi} \tag{A1.2}$$

3. Total credit supply:

$$Q_t S_t = \frac{\Psi_t}{1 - \Phi_t} N_t$$

$$= S\left(\frac{\Psi_t}{\Psi}\right) \left(\frac{N_t}{N}\right)^{1 - \nu}$$
(A1.3)

4. Financial accelerator:

$$\Psi_t = \left[1 - (1 - \omega) \frac{R_{t+1}^k}{R_t}\right]^{-1} \tag{A1.4}$$

5. Net worth evolution:

$$N_{t} = \left[\phi\left\{\left(R_{t}^{k} - R_{t-1}\right)\Psi_{t-1} + R_{t-1}\right\}N_{t-1} + (1 - \phi)F\right] \cdot V_{t}$$
(A1.5)

6. Production function:

$$Y_{w,t} = A_t (K_{t-1})^{\alpha} (L_t)^{1-\alpha}$$
 (A1.6)

7. Labour demand:

$$W_t = (1 - \alpha) \left(\frac{Y_{w,t}}{L_t}\right) P_{w,t} \tag{A1.7}$$

8. Capital demand:

$$E_{t}\left\{R_{t+1}^{k}Q_{t}\right\} = E_{t}\left\{\alpha\left(\frac{Y_{w,t+1}}{K_{t}}\right)P_{w,t+1} + (1-\delta)Q_{t+1}\right\}$$
(A1.8)

9. Capital supply:

$$Q_t = \left[1 - \kappa \left(\frac{I_t}{K_{t-1}} - \delta\right)\right]^{-1} \tag{A1.9}$$

10. Capital accumulation:

$$K_{t} = \left[ \frac{I_{t}}{K_{t-1}} - \frac{\kappa}{2} \left( \frac{I_{t}}{K_{t-1}} - \delta \right)^{2} \right] K_{t-1} + (1 - \delta) K_{t-1}$$
(A1.10)

11. New Keynesian Phillips curve:

$$\pi_t = (\mu P_{w,t})^{\lambda} E_t \{ \pi_{t+1} \}^{\beta}$$
(A1.11)

where 
$$\lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta}$$

12. Resource constraint:

$$Y_t = C_t + I_t + G_t \tag{A1.12}$$

13. Taylor rule:

$$\left(\frac{R_t^n}{R^n}\right) = \left(\frac{R_{t-1}^n}{R^n}\right)^{\alpha_r} (\pi_t)^{(1-\alpha_r)\alpha_\pi} \left(\frac{Y_t}{Y}\right)^{(1-\alpha_r)\alpha_y} \exp\left\{\varepsilon_{r,t}\right\}$$
(A1.13)

14. Fisher equation:

$$R_t^n \equiv R_t E_t \left\{ \pi_{t+1} \right\} \tag{A1.14}$$

15. Credit market intervention rule:

$$\Phi_t = 1 - (1 - \Phi) \left(\frac{N_t}{N}\right)^{\nu} \tag{A1.15}$$

16. Asset market equilibrium:

$$K_t = S_t \tag{A1.16}$$

17. Wholesale goods market equilibrium:

$$Y_{w,t} = Y_t \tag{A1.17}$$

18. Total government spending:

$$G_t = G_t^c + \Phi_t Q_t S_t \tag{A1.18}$$

19. Current government expenditure process:

$$G_t^c = \left(G_{t-1}^c\right)^{\rho_g} \exp\left\{\varepsilon_{g,t}\right\} \tag{A1.19}$$

20. Technology shock process:

$$A_t = (A_{t-1})^{\rho_a} \exp\{\varepsilon_{a,t}\}$$
 (A1.20)

21. Net worth valuation shock process:

$$V_t = (V_{t-1})^{\rho_v} \exp\left\{\varepsilon_{v,t}\right\} \tag{A1.21}$$

## A1.2 The Log-linearised Model

1. Consumption Euler equation:

$$\widehat{C}_t = E_t \left\{ \widehat{C}_{t+1} - \frac{1}{\sigma} \widehat{R}_t \right\} \tag{A1.22}$$

2. Labour supply:

$$\widehat{W}_t = \sigma \widehat{C}_t + \varphi \widehat{L}_t \tag{A1.23}$$

3. Credit supply:

$$\widehat{Q}_{t} + \widehat{S}_{t} = \widehat{\Psi}_{t} + \widehat{N}_{t} + \left(\frac{\Phi}{1 - \Phi}\right) \widehat{\Phi}_{t}$$

$$= \widehat{\Psi}_{t} + (1 - \nu) \widehat{N}_{t}$$
(A1.24)

4. Financial accelerator:

$$\widehat{\Psi}_t = (\Psi - 1) \left( \widehat{R}_{t+1}^k - \widehat{R}_t \right) \tag{A1.25}$$

5. Net worth evolution:

$$\widehat{N}_{t} = \phi \begin{bmatrix} \left( R^{k} \Psi \right) \widehat{R}_{t}^{k} - R \left( \Psi - 1 \right) \widehat{R}_{t-1} + \left( \Psi \left( R^{k} - R \right) \widehat{\Psi}_{t-1} + \left\{ \Psi \left( R^{k} - R \right) + R \right\} \widehat{N}_{t-1} \end{bmatrix} + \widehat{V}_{t}$$
(A1.26)

6. Production function:

$$\widehat{Y}_{w,t} = \widehat{A}_t + \alpha \widehat{K}_{t-1} + (1 - \alpha) \widehat{L}_t$$
(A1.27)

7. Labour demand:

$$\widehat{W}_t = \widehat{Y}_{w,t} - \widehat{L}_t + \widehat{P}_{w,t} \tag{A1.28}$$

8. Capital demand:

$$\widehat{R}_{t+1}^{k} = \left[1 - \frac{(1-\delta)}{R^{k}}\right] \left(\widehat{Y}_{w,t+1} - \widehat{K}_{t} + \widehat{P}_{w,t+1}\right) + \frac{(1-\delta)}{R^{k}} \widehat{Q}_{t+1} - \widehat{Q}_{t}$$
 (A1.29)

9. Capital supply:

$$\widehat{Q}_t = \kappa \delta \left( \widehat{I}_t - \widehat{K}_{t-1} \right) \tag{A1.30}$$

10. Capital accumulation:

$$\widehat{K}_t = \delta \widehat{I}_t + (1 - \delta) \, \widehat{K}_{t-1} \tag{A1.31}$$

11. New Keynesian Phillips curve:

$$\widehat{\pi}_{t} = \beta E_{t} \left\{ \widehat{\pi}_{t+1} \right\} + \left( \frac{(1-\theta)(1-\beta\theta)}{\theta} \right) \widehat{P}_{w,t}$$
(A1.32)

12. Resource constraint:

$$\widehat{Y}_t = \left(\frac{C}{Y}\right)\widehat{C}_t + \left(\frac{I}{Y}\right)\widehat{I}_t + \left(\frac{G}{Y}\right)\widehat{G}_t \tag{A1.33}$$

13. Taylor rule:

$$\widehat{R}_t^n = \alpha_r \widehat{R}_{t-1}^n + (1 - \alpha_r) \alpha_\pi \widehat{\pi}_t + (1 - \alpha_r) \alpha_y \widehat{Y}_t + \varepsilon_{r,t}$$
(A1.34)

14. Fisher equation:

$$\widehat{R}_t^n = \widehat{R}_t + E_t \left\{ \widehat{\pi}_{t+1} \right\} \tag{A1.35}$$

15. Credit policy rule:

$$\widehat{\Phi}_t = -\nu \left(\frac{1-\Phi}{\Phi}\right) \widehat{N}_t \tag{A1.36}$$

16. Asset market equilibrium:

$$\widehat{K}_t = \widehat{S}_t \tag{A1.37}$$

17. Wholesale goods market equilibrium:

$$\widehat{Y}_{w,t} = \widehat{Y}_t \tag{A1.38}$$

18. Total government spending:

$$\widehat{G}_t = \left(\frac{G^c}{G}\right)\widehat{G}_t^c + \left(1 - \frac{G^c}{G}\right)\left(\widehat{\Phi}_t + \widehat{Q}_t + \widehat{S}_t\right) \tag{A1.39}$$

19. Current government expenditure process:

$$\widehat{G}_t^c = \rho_q \widehat{G}_{t-1}^c + \varepsilon_{g,t} \tag{A1.40}$$

20. Technology shock process:

$$\widehat{A}_t = \rho_a \widehat{A}_{t-1} + \varepsilon_{a,t} \tag{A1.41}$$

21. Net worth valuation shock process:

$$\widehat{V}_t = \rho_v \widehat{V}_{t-1} + \varepsilon_{v,t} \tag{A1.42}$$

# Appendix A2 Derivation and Log-linearisation of Equilibrium Conditions

#### A2.1 Households' Behaviour

Appendix A2.1 provides derivations and linearising process for the equilibrium conditions related to the households' behaviours: the Euler equation in consumption and labour supply function.

Solution to the Households' Utility Maximisation Problem To solve the households' utility maximisation problem described in the text, we establish the associated Lagrangian as follows:

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{bmatrix} \frac{(C_t)^{1-\sigma}}{1-\sigma} - \frac{(L_t)^{1+\varphi}}{1+\varphi} \end{bmatrix} \\ -\lambda_t \left[ C_t + B_t + D_t - W_t L_t - R_{t-1} B_{t-1} - R_{t-1} D_{t-1} - \Pi_t^o + T_t \right] \right\}$$

where  $\lambda_t$  is the shadow price for the budget constraint in period t, i.e., the value in terms of utility of relaxing the budget constraint at the margin. Differencing the above Lagrangian with respect to  $C_t$ ,  $D_t$ , and,  $L_t$ , yields the following first order conditions (FOC):

$$[C_t] : (C_t)^{-\sigma} - \lambda_t = 0$$

$$[C_t]$$
:  $(C_t)^{-\sigma} - \lambda_t = 0$   
 $[B_t]$ :  $-\lambda_t + \beta \lambda_{t+1} R_t = 0$ 

$$[D_t] : -\lambda_t + \beta \lambda_{t+1} R_t = 0$$

$$[L_t] : -(L_t)^{\varphi} + \lambda_t W_t = 0.$$

The second and third conditions imply the evolution of the shadow price, i.e.,  $\frac{\lambda_{t+1}}{\lambda_t} = \frac{1}{\beta R_t}$ , which, in combination with the first condition, yields the equation for the intertemporal choice of consumption in (2.4) of the text:

$$1 = \beta E_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} R_t \right\}$$

which is called the Euler equation for the consumption.

In addition, substituting  $\lambda_t = (C_t)^{-\sigma}$  in the first condition into the third one, we obtain the labour supply:

$$W_t = \left(C_t\right)^{\sigma} \left(L_t\right)^{\varphi}$$

which corresponds to an equation (2.5) of the text.

Linearisation of Euler equation and Labour supply Now, we linearise the Euler equation in (2.4) and labour supply in (2.5) of the text by using a first order Taylor expansion.<sup>33</sup> First of all, we consider the Euler equation,  $\frac{1}{R_t} = \beta E_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \right\}$ . In the steady state where  $C_t = C_{t+1} = C$ , we obtain the steady state relation  $\beta = \frac{1}{R}$ . Then, using the fact that  $x_t = \exp \{ \log (x_t) \}$  and

The Taylor series expansion of a function f(x) of order n around  $x_0$  is established as  $f(x) \simeq \frac{f(x_0)}{0!} + \frac{f'(x_0)}{1!}(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 + \dots + \frac{f^{(n)}(x_0)}{n!}(x-x_0)^n$ , as discussed in Chiang (1974).

 $x_t \simeq x(1+\widehat{x}_t)^{34}$  the left hand side (LHS) and the right hand side (RHS) of the above equation can be approximated around the steady state as:

$$\frac{1}{R} \left[ 1 + \log \left( \frac{1/R_t}{1/R} \right) \right] \simeq \frac{1}{R} \left( 1 - \widehat{R}_t \right)$$

and

$$\beta E_{t} \left\{ \left( \frac{C}{C} \right)^{-\sigma} \left[ 1 + \log \left\{ \frac{\left( C_{t+1}/C_{t} \right)^{-\sigma}}{\left( C/C \right)^{-\sigma}} \right\} \right] \right\}$$

$$\simeq \beta E_{t} \left\{ \left[ 1 + \log \left( \frac{C_{t+1}}{C} \right)^{-\sigma} - \log \left( \frac{C_{t}}{C} \right)^{-\sigma} \right] \right\}$$

$$\simeq \beta E_{t} \left\{ 1 - \sigma \widehat{C}_{t+1} + \sigma \widehat{C}_{t} \right\}.$$

Combining the LHS and RHS yields the approximation for the Euler equation around the steady state as:

$$\widehat{C}_t = E_t \left\{ \widehat{C}_{t+1} - \frac{1}{\sigma} \widehat{R}_t \right\},\,$$

which is the equation (A1.22) in Appendix A1.2.

Next, we consider the labour supply,  $W_t = (C_t)^{\sigma} (L_t)^{\varphi}$ . It can be approximated around the steady state where  $W = C^{\sigma} L^{\varphi}$  as:

 $<sup>\</sup>overline{ ^{34}\text{We denote the steady state value of an arbitrary variable } x_t \text{ as } x, \text{ and a log-deviation from the steady state as } \widehat{x}_t \equiv \log \frac{x_t}{x}, \text{ which can be interpreted as a percentage change of } x_t, \text{ since } x_t \equiv x + \Delta x_t \text{ so that } \log \frac{x_t}{x} = \log \left(1 + \frac{\Delta x_t}{x}\right) \simeq \frac{\Delta x_t}{x}, \text{ where we use the relation, } \log(1+y) \simeq y \text{ for the small value of } y \text{ in the last equality. Then, by using a first order Taylor expansion, we may approximate } x_t \text{ around its steady state value, } x, \text{ as } x_t \simeq x(1+\widehat{x}_t). \text{ It follows from the facts that } x_t = x\left(\frac{x_t}{x}\right) = x \exp\left\{\log\left(\frac{x_t}{x}\right)\right\} \text{ and that } \exp\left\{\log\left(\frac{x_t}{x}\right)\right\} \simeq \frac{\exp\left\{\log\left(\frac{x}{x}\right)\right\}}{0!} \left[\log\left(\frac{x_t}{x}\right) - \log\left(\frac{x}{x}\right)\right] \simeq 1 + \widehat{x}_t.$ 

$$W\left(1+\widehat{W}_t\right) = C^{\sigma}L^{\varphi}\left(1+\sigma\widehat{C}_t+\varphi\widehat{L}_t\right),\,$$

so that

$$\widehat{W}_t = \sigma \widehat{C}_t + \varphi \widehat{L}_t,$$

which is the equation (A1.23) in Appendix A1.2.

#### A2.2 Bankers' Behaviours

Appendix A2.2 provides log-linearising process for the equations related to the private bankers' bahaviours. We deal with the bankers' balance sheet, (private) financial accelerator, and net worth evolution.

Linearisation of Balance Sheet and Financial Accelerator First of all, consider the bankers' balance sheet,  $Q_t S_t^j = N_t^j + B_t^j$ , in (2.6) of the text. Note that in the steady state  $Q = \left[1 - \kappa \left(\frac{I}{K} - \delta\right)\right]^{-1} = 1$ , since  $\frac{I}{K} = \delta$  as discussed in Appendix A2.4, and hence,  $S^j = N^j + B^j$ . By using  $x_t \simeq x(1 + \hat{x}_t)$ , the LHS and RHS of the equation are approximated around the steady state as:

$$(LHS) \simeq QS^{j} \left[ 1 + \widehat{Q}_{t} + \widehat{S}_{t}^{j} \right]$$

$$(RHS) \simeq N^{j} \left[ 1 + \widehat{N}_{t}^{j} \right] + B^{j} \left[ 1 + \widehat{B}_{t}^{j} \right].$$

Combining the LHS and RHS approximated, we obtain

$$\widehat{Q}_t + \widehat{S}_t^j = \left(\frac{N^j}{S^j}\right) \widehat{N}_t^j + \left(1 - \frac{N^j}{S^j}\right) \widehat{B}_t^j,$$

where we use  $1 = \frac{N^j + B^j}{S^j}$ .

Next, we consider the expression for private supply of credit,  $Q_t S_t^j = \Psi_t N_t^j$ , in (2.10) of the text. In the steady state, we obtain  $S^j = \Psi N^j$ , so that the above equation can be approximated around the steady state as:

$$\widehat{Q}_t + \widehat{S}_t^j = \widehat{\Psi}_t + \widehat{N}_t^j.$$

Now, consider the financial accelerator,  $\Psi_t = \left[1 - (1 - \omega) \frac{R_{t+1}^k}{R_t}\right]^{-1}$ , in (2.10) of the text, which may be rewritten as:

$$\frac{1}{\Psi_t} = 1 - (1 - \omega) \, \frac{R_{t+1}^k}{R_t}.$$

Note that the corresponding steady state relation is given by  $\frac{1}{\Psi} = 1 - (1 - \omega) \frac{R^k}{R}$ . By using  $x_t \simeq x(1 + \hat{x}_t)$ , the LHS and RHS of the equation are approximated around the steady state as:

$$(LHS) \simeq \frac{1}{\Psi} \left( 1 - \widehat{\Psi}_t \right)$$

$$(RHS) \simeq 1 - (1 - \omega) \frac{R^k}{R} \left( 1 + \widehat{R}_{t+1}^k - \widehat{R}_t \right).$$

Combining the LHS and RHS approximated, we obtain

$$\frac{\widehat{\Psi}_t}{\Psi} = \left(1 - \frac{1}{\Psi}\right) \left(\widehat{R}_{t+1}^k - \widehat{R}_t\right),\,$$

where we use the steady state relation,  $-(1-\omega)\frac{R^k}{R} = \frac{1}{\Psi} - 1$ . Thus, the above equation can be rewritten as:

$$\widehat{\Psi}_t = (\Psi - 1) \left( \widehat{R}_{t+1}^k - \widehat{R}_t \right),\,$$

which is the equation (A1.25) in Appendix A1.2. In addition, it implies the following relation:

$$\log\left(\frac{\Psi_t}{\Psi}\right) = \log\left(\frac{R_{t+1}^k/R_t}{R^k/R}\right)^{\Psi-1},$$

so that

$$\frac{\Psi_t}{\Psi} = \left(\frac{R_{t+1}^k / R_t}{R^k / R}\right)^{\Psi - 1}$$

which is the equation in (2.11) of the text.

**Linearisation of Net Worth Evolution** We linearise the evolution of net worth,  $N_t = [\phi\{(R_t^k - R_{t-1})\Psi_{t-1} + R_{t-1}\}N_{t-1} + (1-\phi)F] \cdot V_t$ , in (2.13) in the text, which can be rewritten as:

$$\frac{N_t}{V_t} = \phi \left\{ \left( R_t^k - R_{t-1} \right) \Psi_{t-1} + R_{t-1} \right\} N_{t-1} + (1 - \phi) F.$$

Note that in the steady state  $\frac{N}{V} = \phi \left[ \left( R^k - R \right) \Psi + R \right] N + (1 - \phi) F$  and we assume that V = 1, without loss of generality. Then, by using  $x_t \simeq x(1 + \widehat{x}_t)$ , the LHS is approximated as  $\frac{N}{V} \left( 1 + \widehat{N}_t - \widehat{V}_t \right)$  around the steady state. Each term of the RHS may approximated around the steady state as:

$$\phi R_t^k \Psi_{t-1} N_{t-1} \simeq \phi R^k \Psi N \left[ 1 + \widehat{R}_t^k + \widehat{\Psi}_{t-1} + \widehat{N}_{t-1} \right],$$

$$-\phi R_{t-1}\Psi_{t-1}N_{t-1} \simeq -\phi R\Psi N \left[ 1 + \widehat{R}_{t-1} + \widehat{\Psi}_{t-1} + \widehat{N}_{t-1} \right],$$

and

$$\phi R_{t-1} N_{t-1} \simeq \phi R N \left[ 1 + \widehat{R}_{t-1} + \widehat{N}_{t-1} \right].$$

Combining all these terms yields

$$\begin{split} \frac{N}{V} \left( 1 + \widehat{N}_t - \widehat{V}_t \right) &= \phi R^k \Psi N \left[ 1 + \widehat{R}_t^k + \widehat{\Psi}_{t-1} + \widehat{N}_{t-1} \right] \\ &- \phi R \Psi N \left[ 1 + \widehat{R}_{t-1} + \widehat{\Psi}_{t-1} + \widehat{N}_{t-1} \right] \\ &+ \phi R N \left[ 1 + \widehat{R}_{t-1} + \widehat{N}_{t-1} \right] + (1 - \phi) F. \end{split}$$

so that

$$\widehat{N}_{t} = \phi \left[ \begin{array}{c} \left( R^{k} \Psi \right) \widehat{R}_{t}^{k} - R \left( \Psi - 1 \right) \widehat{R}_{t-1} + \\ \Psi \left( R^{k} - R \right) \widehat{\Psi}_{t-1} + \left\{ \Psi \left( R^{k} - R \right) + R \right\} \widehat{N}_{t-1} \end{array} \right] + \widehat{V}_{t},$$

which is the equation (A1.26) in Appendix A1.2.

#### A2.3 Wholesale Firms' Behaviours

Appendix A2.3 provides a solution to wholesale firms' optimal production problem and log-linearising process for their behavioural equations: production function and demands on labour and capital.

Solution to Optimal Production Problem Wholesale firms' optimisation problem is solved through two steps: determining the optimal allocation among production factors by solving the cost minimisation problem; and then, determining the wholesale good price by solving the profit maximisation problem. First of all, we consider their cost minimisation problem. Given that wholesale firms operate in the perfectly competitive factor market, they take the associated factor price,  $W_t$  and  $Q_t$  given. The cost minimisation problem may be written (in real terms) as in (2.17) and (2.14) of the text:

$$\min_{\{K_t, L_t\}} TC_{w,t} = W_t L_t + \left[ R_t^k Q_{t-1} K_{t-1} - (1 - \delta) \overline{Q}_t K_{t-1} \right]$$

subject to

$$Y_{w,t} = A_t \left( K_{t-1} \right)^{\alpha} \left( L_t \right)^{1-\alpha}.$$

Let  $\lambda_t$  be the Lagrangian multiplier associated with the production function. Then the first order conditions are given by

$$[L_t] : W_t = \lambda_t (1 - \alpha) \left( \frac{Y_{w,t}}{L_t} \right)$$

$$[K_t] : R_{t+1}^k Q_t = \lambda_{t+1} \alpha \left( \frac{Y_{w,t+1}}{K_t} \right) + (1 - \delta) Q_{t+1},$$

where we use the steady state relation,  $\overline{Q}_t \simeq Q_t$ , which is discussed in Appendix A2.4. Substituting the above conditions into the production function in (2.14), we obtain

$$Y_{w,t} = A_t \left[ \frac{\lambda_t \alpha Y_{w,t}}{R_t^k Q_{t-1} - (1-\delta) Q_t} \right]^{\alpha} \left[ \frac{\lambda_t (1-\alpha) Y_{w,t}}{W_t} \right]^{1-\alpha}$$
$$= A_t \lambda_t Y_{w,t} \left[ \frac{\alpha}{R_t^k Q_{t-1} - (1-\delta) Q_t} \right]^{\alpha} \left[ \frac{1-\alpha}{W_t} \right]^{1-\alpha}$$

so that

$$\lambda_t = \left(\frac{1}{A_t}\right) \left[\frac{R_t^k Q_{t-1} - (1-\delta) Q_t}{\alpha}\right]^{\alpha} \left[\frac{W_t}{1-\alpha}\right]^{1-\alpha}.$$

In addition, substituting the optimality conditions into the total cost function in (2.17) yields

$$TC_{w,t} = W_{t} \left[ \frac{\lambda_{t} (1 - \alpha) Y_{w,t}}{W_{t}} \right] + \left[ R_{t}^{k} Q_{t-1} - (1 - \delta) Q_{t} \right] \left[ \frac{\lambda_{t} \alpha Y_{w,t}}{R_{t}^{k} Q_{t-1} - (1 - \delta) Q_{t}} \right]$$
$$= \lambda_{t} Y_{w,t}$$

so that

$$MC_{w,t} = \frac{\partial TC_{w,t}}{\partial Y_{w,t}} = \lambda_t.$$

Next, we consider the profit maximisation problem. Noting that wholesale firms sell the wholesale goods to retail firms in a perfectly competitive way, they act as price takers in the wholesale goods market, so that the (real) marginal revenue from selling the wholesale goods is equal to the (real) wholesale good price, i.e.,  $MR_{w,t} = P_{w,t}$ . In addition, production maximisation requires the (real) marginal revenue to be equal to the (real) marginal cost, i.e.,  $MR_{w,t} = MC_{w,t}$ , and hence, the wholesale good price to be equal to the marginal cost of production, i.e.,  $P_{w,t} = MC_{w,t} = \lambda_t$ . Then, all in all, we obtain the wholesale firms' demands for household labour and capital, respectively, as:

$$W_t = (1 - \alpha) \left(\frac{Y_{w,t}}{L_t}\right) P_{w,t}$$

$$R_{t+1}^{k}Q_{t} = \alpha \left(\frac{Y_{w,t+1}}{K_{t}}\right) P_{w,t+1} + (1 - \delta) Q_{t+1},$$

which are equations (2.18) and (2.19) in the text.

**Linearisation of Production function** We consider the production function,  $Y_{w,t} = A_t (K_{t-1})^{\alpha} (L_t)^{1-\alpha}$ , in (2.14) of the text. It can be approximated around the steady state where  $Y_w = A(K)^{\alpha} (L)^{1-\alpha}$  as:

$$Y_w \left( 1 + \widehat{Y}_{w,t} \right) = A \left( K \right)^{\alpha} \left( L \right)^{1-\alpha} \left[ 1 + \widehat{A}_t + \alpha \widehat{K}_{t-1} + \left( 1 - \alpha \right) \widehat{L}_t \right],$$

so that

$$\widehat{Y}_{w,t} = \widehat{A}_t + \alpha \widehat{K}_{t-1} + (1 - \alpha) \widehat{L}_t,$$

which is the equation (A1.27) in Appendix A1.2.

Linearisation of Demands for Labour and Capital Consider the labour demand,  $W_t = (1 - \alpha) \left(\frac{Y_{w,t}}{L_t}\right) P_{w,t}$ , in (2.18) of the text. In the steady state, we have  $W = (1 - \alpha) \left(\frac{Y_w}{L}\right) P_w$ . Using the relation  $x_t \simeq x(1 + \hat{x}_t)$ , the above equation is approximated around the steady state as:

$$W\left(1+\widehat{W}_{t}\right)\simeq\left(1-\alpha\right)\left(\frac{Y_{w}}{L}\right)P_{w}\left[1+\widehat{Y}_{w,t}-\widehat{L}_{t}+\widehat{P}_{w,t}\right]$$

so that

$$\widehat{W}_t = \widehat{Y}_{w,t} - \widehat{L}_t + \widehat{P}_{w,t},$$

which is the equation (A1.28) in Appendix A1.2.

Next, we linearise the capital demand function,  $R_{t+1}^k Q_t = \alpha \left( \frac{Y_{w,t+1}}{K_t} \right) P_{w,t+1} + (1-\delta) Q_{t+1}$ , in (2.19) of the text. The steady state relation for capital demand establishes that  $R^k Q = \alpha \left( \frac{Y_w}{K} \right) P_w + (1-\delta) Q$  and Q = 1, so that  $\frac{Y_w}{K} = \left[ \frac{\alpha P_w}{R^k - (1-\delta)} \right]^{-1}$ . Then, using the relation  $x_t \simeq x(1+\widehat{x}_t)$ , we obtain the approximated relations for the LHS and RHS of the above equation around the steady state as:

$$(LHS) \simeq R^{k} \left[ 1 + \widehat{R}_{t+1}^{k} + \widehat{Q}_{t} \right]$$

$$(RHS) \simeq \alpha \left( \frac{Y_{w}}{K} \right) P_{w} \left[ 1 + \widehat{Y}_{w,t+1} - \widehat{K}_{t} + \widehat{P}_{w,t+1} \right] + (1 - \delta) \left[ 1 + \widehat{Q}_{t+1} \right].$$

Combining the LHS and RHS yields the approximation for the capital demand function:

$$\widehat{R}_{t+1}^{k} = \left[1 - \frac{(1-\delta)}{R^{k}}\right] \left(\widehat{Y}_{w,t+1} - \widehat{K}_{t} + \widehat{P}_{w,t+1}\right) + \frac{(1-\delta)}{R^{k}} \widehat{Q}_{t+1} - \widehat{Q}_{t},$$

which is the equation (A1.29) in Appendix A1.2.

## A2.4 Capital Producers' Behaviours

In Appendix A2.4, we discuss capital producers' behaviours: capital supply and capital accumulation.

**Solution to Capital Producers' Problem** We consider the capital producers' profit maximisation problem described in (2.20), (2.22) and (2.21) in the text, which may be written as:

$$\Pi_{c,t} = Q_t K_t - \left[ I_t + \overline{Q}_t (1 - \delta) K_{t-1} \right] 
= Q_t \left[ K_t^n + (1 - \delta) K_{t-1} \right] - \left[ I_t + \overline{Q}_t (1 - \delta) K_{t-1} \right] 
= Q_t \left[ \frac{I_t}{K_{t-1}} - \frac{\kappa}{2} \left( \frac{I_t}{K_{t-1}} - \delta \right)^2 \right] K_{t-1} - \left[ I_t + \left( \overline{Q}_t - Q_t \right) (1 - \delta) K_{t-1} \right],$$

where  $(\overline{Q}_t - Q_t)$  is interpreted as the capital rental rate, charged by the whole-sale firms. Note that the assumption of the constant return to scale (CRS) in the capital producing technology,  $K_t^n = \left[\frac{I_t}{K_{t-1}} - \frac{\kappa}{2} \left(\frac{I_t}{K_{t-1}} - \delta\right)^2\right] K_{t-1}$ , and perfect

competition in the capital market, implies zero profit in equilibrium, i.e.,  $\Pi_{c,t} = 0$ , so that the equilibrium rental rate of the existing capital is specified as:

$$(1 - \delta)\left(\overline{Q}_t - Q_t\right) = Q_t \left[\frac{I_t}{K_{t-1}} - \frac{\kappa}{2} \left(\frac{I_t}{K_{t-1}} - \delta\right)^2\right] - \frac{I_t}{K_{t-1}}.$$

Moreover, since  $\frac{I}{K} = \delta$  (and hence,  $Q = \left[1 - \kappa \left(\frac{I}{K} - \delta\right)\right]^{-1} = 1$ ) at the steady state, the rental rate of the existing capital stock is of second order around the steady state, i.e.,

$$(1 - \delta) (\overline{Q} - Q) = Q \left[ \frac{I}{K} - \frac{\kappa}{2} \left( \frac{I}{K} - \delta \right)^{2} \right] - \frac{I}{K}$$
$$= \left[ \delta - \frac{\kappa}{2} (\delta - \delta)^{2} \right] - \delta = 0,$$

so that  $\overline{Q} = Q$ . Thus, in the steady state, the above capital producers' problem can be reduced to:

$$\Pi_{c,t} = Q_t \left[ \frac{I_t}{K_{t-1}} - \frac{\kappa}{2} \left( \frac{I_t}{K_{t-1}} - \delta \right)^2 \right] K_{t-1} - I_t.$$

The optimality conditions for the above profit maximisation problem with respect to  $I_t$  and  $K_t$  are:

$$\begin{split} \frac{\partial \Pi_{c,t}}{\partial I_t} &= Q_t \left[ 1 - \kappa \left( \frac{I_t}{K_{t-1}} - \delta \right) \right] - 1 = 0 \\ \frac{\partial \Pi_{c,t+1}}{\partial K_t} &= \frac{\kappa Q_{t+1}}{2} \left[ \left( \frac{I_{t+1}}{K_t} \right)^2 - \delta^2 \right] = 0. \end{split}$$

The first condition implies the capital supply function,  $Q_t = \left[1 - \kappa \left(\frac{I_t}{K_{t-1}} - \delta\right)\right]^{-1}$ , in (2.23) of the text, and the second one suggests that  $\frac{I_{t+1}}{K_t} = \delta$ , which is satisfied in the steady state.

**Linearisation of Capital Accumulation** Consider the capital accumulation equation,  $K_t = \left[\frac{I_t}{K_{t-1}} - \frac{\kappa}{2} \left(\frac{I_t}{K_{t-1}} - \delta\right)^2\right] K_{t-1} + (1 - \delta) K_{t-1}$ , in (2.20) and (2.21) of the text, which can be rewritten as:

$$\frac{K_t}{K_{t-1}} - (1 - \delta) = \frac{I_t}{K_{t-1}} - \frac{\kappa}{2} \left( \frac{I_t}{K_{t-1}} - \delta \right)^2.$$

Note that, in the steady state where  $K_{t-1} = K_t = K$ , the above equation is written as the following steady state relation:

$$\frac{K}{K} - (1 - \delta) = \frac{I}{K} - \frac{\kappa}{2} \left( \frac{I}{K} - \delta \right)^2,$$

so that

$$\left(\frac{I}{K} - \delta\right) \left[1 - \frac{\kappa}{2} \left(\frac{I}{K} - \delta\right)\right] = 0.$$

Accordingly, we establish the steady state relation,  $\frac{I}{K} = \delta$ . Then, by using the relation  $x_t \simeq x(1+\hat{x}_t)$ , the LHS and RHS of the capital accumulation equation above are approximated around the steady state as:

$$(LHS) \simeq \frac{K}{K} \left( 1 + \widehat{K}_t - \widehat{K}_{t-1} \right) - (1 - \delta) \simeq \widehat{K}_t - \widehat{K}_{t-1} + \delta,$$

and

$$(RHS) \simeq \frac{I}{K} \left( 1 + \widehat{I}_{t} - \widehat{K}_{t-1} \right) - \frac{\kappa}{2} \left[ \frac{I}{K} \left( 1 + \widehat{I}_{t} - \widehat{K}_{t-1} \right) - \delta \right]^{2}$$

$$\simeq \delta \left( 1 + \widehat{I}_{t} - \widehat{K}_{t-1} \right) - \frac{\kappa}{2} \left[ \delta \left( \widehat{I}_{t} - \widehat{K}_{t-1} \right) \right]^{2}$$

$$\simeq \delta + \delta \left( \widehat{I}_{t} - \widehat{K}_{t-1} \right),$$

where the last equality for the RHS follows from the fact that  $\frac{\kappa}{2} \left[ \delta \left( \widehat{I}_t - \widehat{K}_{t-1} \right) \right]^2$  can be ignored around the steady state since the squared value of a very small value (such as  $\widehat{I}_t - \widehat{K}_{t-1}$ ) is infinitesimal. Combining the LHS and RHS yields the approximation for the capital accumulation:

$$\widehat{K}_{t} = \widehat{K}_{t-1} + \delta \left( \widehat{I}_{t} - \widehat{K}_{t-1} \right)$$
$$= \delta \widehat{I}_{t} + (1 - \delta) \widehat{K}_{t-1},$$

which is the equation (A1.31) in Appendix A1.2.

**Linearisation of Capital Supply** Now, we linearise the capital supply function,  $Q_t = \left[1 - \kappa \left(\frac{I_t}{K_{t-1}} - \delta\right)\right]^{-1}$ , in (2.23) of the text, which can be rewritten as:

$$1 - \frac{1}{Q_t} = \kappa \left( \frac{I_t}{K_{t-1}} - \delta \right).$$

Note that in the steady state  $\frac{I}{K} = \delta$ , and hence,  $Q = \left[1 - \kappa \left(\frac{I}{K} - \delta\right)\right]^{-1} = 1$ . Then, using  $x_t \simeq x(1+\hat{x}_t)$ , the LHS and RHS of the above equation are approximated

around the steady state as:

$$(LHS) \simeq 1 - \frac{1}{Q} \left( 1 - \widehat{Q}_t \right) \simeq \widehat{Q}_t$$

$$(RHS) \simeq \kappa \left[ \frac{I}{K} \left( 1 + \widehat{I}_t - \widehat{K}_{t-1} \right) - \delta \right]$$

$$\simeq \kappa \delta \left( \widehat{I}_t - \widehat{K}_{t-1} \right).$$

Combining the LHS and the RHS yields the approximation for the capital supply function:

$$\widehat{Q}_t = \kappa \delta \left( \widehat{I}_t - \widehat{K}_{t-1} \right),\,$$

which is an equation (A1.30) in Appendix A1.2.

# A2.5 Retail Firms' Behaviours

Appendix A2.5 provides derivation and linearisation of retail firms' behaviour: aggregate price dynamics and New Keynesian Phillips curve (NKPC).<sup>35</sup>

**Linearisation of Aggregate Price Dynamics** Consider the price index,  $P_t = \left[\int_0^1 \overline{P}_t(j)^{1-\epsilon} dj\right]^{\frac{1}{1-\epsilon}} = \left[\theta\left(P_{t-1}\right)^{1-\epsilon} + (1-\theta)\left(\overline{P}_t\right)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}$ , in (2.25) and (2.28) of the text, which may be rewritten as:

$$\left(\frac{P_t}{P_{t-1}}\right)^{1-\epsilon} = \theta + (1-\theta) \left(\frac{\overline{P}_t}{P_{t-1}}\right)^{1-\epsilon}.$$

<sup>&</sup>lt;sup>35</sup>Discussion in this part is broadly based on Gali (2008, Chapter 3).

Note that in the zero inflation steady state,  $P_{t-1} = P_t = P$ , so that  $\overline{P} = P$  since  $P = \left[\theta P^{1-\epsilon} + (1-\theta)\overline{P}^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}$ . Then, by using the relation  $x_t \simeq x(1+\widehat{x}_t)$ , the LHS and RHS of the above equation are approximated around the steady state as:

$$(LHS) \simeq \left(\frac{P}{P}\right)^{1-\epsilon} \left[1 + (1-\epsilon)\left(\widehat{P}_t - \widehat{P}_{t-1}\right)\right]$$
  
$$\simeq 1 + (1-\epsilon)\left(\widehat{P}_t - \widehat{P}_{t-1}\right),$$

and

$$(RHS) \simeq \theta + (1 - \theta) \left(\frac{\overline{P}}{P}\right)^{1 - \epsilon} \left[1 + (1 - \epsilon) \left(\widehat{\overline{P}}_t - \widehat{P}_{t-1}\right)\right]$$
$$\simeq 1 + (1 - \theta) (1 - \epsilon) \left(\widehat{\overline{P}}_t - \widehat{P}_{t-1}\right).$$

Combining the LHS and RHS yields the linearised form of aggregate price dynamics:

$$\widehat{\pi}_t \equiv \widehat{P}_t - \widehat{P}_{t-1} = (1 - \theta) \left( \widehat{\overline{P}}_t - \widehat{P}_{t-1} \right).$$

**Derivation of Demand for the Individual Variety** Consider the composite of retail goods,  $Y_t = \left[\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj\right]^{\frac{\epsilon}{\epsilon-1}}$ , in (2.24), and the corresponding price index,  $P_t = \left[\int_0^1 \overline{P}_t(j)^{1-\epsilon} dj\right]^{\frac{1}{1-\epsilon}}$ , in (2.25) of the text. Now, note that consumers'

expenditure minimisation problem implies choosing  $Y_t(j)$  for any given (nominal) expenditure level,  $Z_t \equiv \int_0^1 \overline{P}_t(j) Y_t(j) dj$ , which can be written as the following Lagrangian:

$$\mathcal{L} = \left[ \int_{0}^{1} Y_{t}\left(j\right)^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}} - \lambda_{t} \left[ \int_{0}^{1} \overline{P}_{t}\left(j\right) Y_{t}\left(j\right) dj - Z_{t} \right].$$

Then the associated first order condition is:

$$[Y_t(j)]: \quad \left(\frac{\epsilon}{\epsilon-1}\right)(Y_t)^{\frac{1}{\epsilon}}\left(\frac{\epsilon-1}{\epsilon}\right)(Y_t(j))^{-\frac{1}{\epsilon}} - \lambda_t \overline{P}_t(j) = 0$$

so that

$$(Y_t)^{\frac{1}{\epsilon}} (Y_t(j))^{-\frac{1}{\epsilon}} = \lambda_t \overline{P}_t(j).$$

The above conditions hold for any varieties (i, j) so that:

$$\frac{Y_{t}\left(i\right)}{Y_{t}\left(j\right)} = \left[\frac{\overline{P}_{t}\left(i\right)}{\overline{P}_{t}\left(j\right)}\right]^{-\epsilon},$$

which can be substituted into the expression for expenditure,  $Z_t$ , to yield:

$$Z_{t} \equiv \int_{0}^{1} \overline{P}_{t}(i) Y_{t}(i) di = \int_{0}^{1} \overline{P}_{t}(i) \left[ \frac{\overline{P}_{t}(i)}{\overline{P}_{t}(j)} \right]^{-\epsilon} Y_{t}(j) di$$
$$= \frac{Y_{t}(j)}{(\overline{P}_{t}(j))^{-\epsilon}} \int_{0}^{1} \overline{P}_{t}(i)^{1-\epsilon} di = \frac{Y_{t}(j)}{(\overline{P}_{t}(j))^{-\epsilon}} (P_{t})^{1-\epsilon},$$

where the last equality follows from the price index,  $P_t = \left[ \int_0^1 \overline{P}_t \left( i \right)^{1-\epsilon} dj \right]^{\frac{1}{1-\epsilon}}$ . The above equation can be rewritten as:

$$Y_{t}\left(j\right) = \left[\frac{\overline{P}_{t}\left(j\right)}{P_{t}}\right]^{-\epsilon} \left(\frac{Z_{t}}{P_{t}}\right),$$

which can be substituted into the retail good composite,  $Y_t = \left[ \int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}}$ , to obtain:

$$Y_{t} = \left\{ \int_{0}^{1} \left[ \left( \frac{\overline{P}_{t}(j)}{P_{t}} \right)^{-\epsilon} \left( \frac{Z_{t}}{P_{t}} \right) \right]^{\frac{\epsilon-1}{\epsilon}} dj \right\}^{\frac{\epsilon}{\epsilon-1}}$$

$$= \left( \frac{Z_{t}}{P_{t}} \right) \left( \frac{1}{P_{t}} \right)^{-\epsilon} \left[ \int_{0}^{1} \overline{P}_{t}(j)^{1-\epsilon} dj \right]^{\frac{\epsilon}{\epsilon-1}}$$

$$= \left( \frac{Z_{t}}{P_{t}} \right) \left( \frac{1}{P_{t}} \right)^{-\epsilon} (P_{t})^{-\epsilon} = \frac{Z_{t}}{P_{t}},$$

so that  $Z_t = P_t Y_t$ . Combining the above two equations yields the demand schedule for the variety that the retail firm faces:

$$Y_{t}(j) = \left\lceil \frac{\overline{P}_{t}(j)}{P_{t}} \right\rceil^{-\epsilon} Y_{t},$$

which is the equation (2.27) in the text.

**Derivation and Linearisation of Optimal Price Setting Rule** Now, we consider the retailers' profit maximisation problem described in (2.29) and (2.30) of the text, which can be written as:

$$\max_{\left\{\overline{P}_{t}(j)\right\}} \quad \sum_{k=0}^{\infty} \theta^{k} E_{t} \left\{ \Lambda_{t,t+k} Y_{t+k} \left[ \left( \frac{\overline{P}_{t}(j)}{P_{t+k}} \right)^{1-\epsilon} - P_{w,t+k} \left( \frac{\overline{P}_{t}(j)}{P_{t+k}} \right)^{-\epsilon} \right] \right\}.$$

The first order condition for the problem with respect to  $\overline{P}_t(j)$  is obtained as:

$$0 = \sum_{k=0}^{\infty} \theta^{k} E_{t} \left\{ \Lambda_{t,t+k} Y_{t+k} \left[ (1-\epsilon) \left( \frac{\overline{P}_{t}(j)}{P_{t+k}} \right)^{-\epsilon} \left( \frac{1}{P_{t+k}} \right) - (-\epsilon) \left( \frac{P_{w,t+k}}{\overline{P}_{t}(j)} \right) \left( \frac{\overline{P}_{t}(j)}{P_{t+k}} \right)^{-\epsilon} \right] \right\}$$

$$= \sum_{k=0}^{\infty} \theta^{k} E_{t} \left\{ \Lambda_{t,t+k} Y_{t+k} \left[ \frac{\overline{P}_{t}(j)}{P_{t-1}} - \left( \frac{\epsilon}{\epsilon - 1} \right) P_{w,t+k} \left( \frac{P_{t+k}}{P_{t-1}} \right) \right] \left( \frac{1}{P_{t+k}} \right)^{1-\epsilon} \right\},$$

where  $\frac{\epsilon}{\epsilon-1} \equiv \mu$ , is interpreted as the desired markup, in that the optimal price setting rule is reduced to  $\frac{\overline{P}_t(j)}{P_t} = \left(\frac{\epsilon}{\epsilon-1}\right) P_{w,t}$  in the absence of price rigidity.<sup>36</sup> Moreover, note that in the zero inflation steady state all the agents in the economy choose the same quantities. Then, we obtain  $P_t = P_{t-1} = P$ ,  $\overline{P}(j) = P$  and  $P_{w,t+k} = P_w = \left(\frac{\epsilon-1}{\epsilon}\right)\left(\frac{\overline{P}(j)}{P}\right) = \frac{1}{\mu}$ . In addition, we establish that in the zero inflation steady state  $Y_{t+k} = Y$ ,  $C_t = C_{t+k} = C$ , and hence,  $\Lambda_{t,t+k} = \beta^k \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma} = \beta^k$ .

Imposing the steady state conditions upon the discount factor,  $\Lambda_{t,t+k}$ , and the levels of output,  $Y_{t+k}$ , and price,  $P_{t+k}$ , we obtain the approximation for the above optimal price setting rule as:

$$Y \cdot E_t \sum_{k=0}^{\infty} \left\{ \left(\beta \theta\right)^k \left( \frac{\overline{P}_t(j)}{P_{t-1}} \right) \left( \frac{1}{P_{t+k}} \right)^{1-\epsilon} \right\} = Y \cdot E_t \sum_{k=0}^{\infty} \left\{ \left(\beta \theta\right)^k \mu P_{w,t+k} \left( \frac{P_{t+k}}{P_{t-1}} \right) \left( \frac{1}{P_{t+k}} \right)^{1-\epsilon} \right\}.$$

Then, by using the relation  $x_t \simeq x(1+\hat{x}_t)$ , the LHS and RHS of the above equation are approximated around the steady state as:

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$$(LHS) \simeq \left(\frac{Y}{P^{1-\epsilon}}\right) \cdot E_t \sum_{k=0}^{\infty} (\beta \theta)^k \left(\frac{\overline{P}(j)}{P}\right) \left[1 + \widehat{\overline{P}}_t(j) - \widehat{P}_{t-1} - (1-\epsilon) \widehat{P}_{t+k}\right]$$

$$\simeq \left(\frac{Y}{P^{1-\epsilon}}\right) \cdot E_t \left[\frac{1}{1-\beta \theta} + \left(\frac{1}{1-\beta \theta}\right) \left(\widehat{\overline{P}}_t(j) - \widehat{P}_{t-1}\right) - (1-\epsilon) \sum_{k=0}^{\infty} (\beta \theta)^k \widehat{P}_{t+k}\right],$$

and

$$(RHS) \simeq \left(\frac{Y}{P^{1-\epsilon}}\right) \cdot E_t \sum_{k=0}^{\infty} (\beta \theta)^k \mu P_w \left(\frac{P}{P}\right) \left[1 + \widehat{P}_{w,t+k} + \widehat{P}_{t+k} - \widehat{P}_{t-1} - (1-\epsilon) \widehat{P}_{t+k}\right]$$

$$\simeq \left(\frac{Y}{P^{1-\epsilon}}\right) \cdot E_t \left[\frac{1}{1-\beta \theta} + \sum_{k=0}^{\infty} (\beta \theta)^k \left(\widehat{P}_{w,t+k} + \widehat{P}_{t+k} - \widehat{P}_{t-1}\right) - (1-\epsilon) \sum_{k=0}^{\infty} (\beta \theta)^k \widehat{P}_{t+k}\right].$$

Combining the LHS and RHS yields the linear approximation for the optimal price setting rule of the retailers around the steady state as:

$$\widehat{\overline{P}}_t(j) - \widehat{P}_{t-1} = (1 - \beta \theta) E_t \sum_{k=0}^{\infty} (\beta \theta)^k \left( \widehat{P}_{w,t+k} + \widehat{P}_{t+k} - \widehat{P}_{t-1} \right).$$

Now, we may transform the above equation into a first order difference equation. First of all, note that the second and third terms of the RHS can be written as:

$$(1 - \beta\theta) E_t \sum_{k=0}^{\infty} (\beta\theta)^k \left(\widehat{P}_{t+k} - \widehat{P}_{t-1}\right)$$

$$= (1 - \beta\theta) E_t \left\{ \left(\widehat{P}_t - \widehat{P}_{t-1}\right) + (\beta\theta) \left(\widehat{P}_{t+1} - \widehat{P}_{t-1}\right) + (\beta\theta)^2 \left(\widehat{P}_{t+2} - \widehat{P}_{t-1}\right) + \cdots \right\}$$

$$= (1 - \beta\theta) E_t \left\{ \widehat{\pi}_t + (\beta\theta) \left(\widehat{\pi}_{t+1} + \widehat{\pi}_t\right) + (\beta\theta)^2 \left(\widehat{\pi}_{t+2} + \widehat{\pi}_{t+1} + \widehat{\pi}_t\right) + \cdots \right\}$$

$$= (1 - \beta \theta) E_t \left\{ \left( 1 + \beta \theta + (\beta \theta)^2 + \cdots \right) \left[ \widehat{\pi}_t + (\beta \theta) \widehat{\pi}_{t+1} + (\beta \theta)^2 \widehat{\pi}_{t+2} + \cdots \right] \right\}$$

$$= E_t \sum_{k=0}^{\infty} (\beta \theta)^k \widehat{\pi}_{t+k}.$$

Substitution of this into the previous price setting rule yields the following first order difference equation:

$$\widehat{\overline{P}}_{t}(j) - \widehat{P}_{t-1} = (1 - \beta\theta) E_{t} \sum_{k=0}^{\infty} \left\{ (\beta\theta)^{k} \widehat{P}_{w,t+k} \right\} + \sum_{k=0}^{\infty} (\beta\theta)^{k} E_{t} \left\{ \widehat{\pi}_{t+k} \right\} 
= (1 - \beta\theta) E_{t} \left\{ \widehat{P}_{w,t} + (\beta\theta) \widehat{P}_{w,t+1} + (\beta\theta)^{2} \widehat{P}_{w,t+2} + \cdots \right\} 
+ E_{t} \left\{ \widehat{\pi}_{t} + (\beta\theta) \widehat{\pi}_{t+1} + (\beta\theta)^{2} \widehat{\pi}_{t+2} + \cdots \right\} 
= (1 - \beta\theta) \widehat{P}_{w,t} + \widehat{\pi}_{t} 
+ (\beta\theta) \left[ (1 - \beta\theta) E_{t} \sum_{k=0}^{\infty} \left\{ (\beta\theta)^{k} \widehat{P}_{w,t+1+k} \right\} + \sum_{k=0}^{\infty} (\beta\theta)^{k} E_{t} \left\{ \widehat{\pi}_{t+1+k} \right\} \right] 
= (1 - \beta\theta) \widehat{P}_{w,t} + \widehat{\pi}_{t} + (\beta\theta) E_{t} \left\{ \widehat{\overline{P}}_{t+1}(j) - \widehat{P}_{t} \right\}.$$

Furthermore, noting that  $\widehat{\overline{P}}_t(j) = \widehat{\overline{P}}_t$  by the assumption of homogeneity, the above equation can be written as

$$\widehat{\overline{P}}_{t} - \widehat{P}_{t-1} = (1 - \beta \theta) \, \widehat{P}_{w,t} + \widehat{\pi}_{t} + (\beta \theta) \, E_{t} \left\{ \widehat{\overline{P}}_{t+1} - \widehat{P}_{t} \right\}.$$

**Derivation of New Keynesian Phillips Curve** Now, by combining the aggregate price dynamics,  $\widehat{\pi}_t = (1 - \theta) \left( \widehat{\overline{P}}_t - \widehat{P}_{t-1} \right)$ , with the retail firms' optimal price setting rule,  $\widehat{\overline{P}}_t - \widehat{P}_{t-1} = (1 - \beta \theta) \widehat{P}_{w,t} + \widehat{\pi}_t + (\beta \theta) E_t \left\{ \widehat{\overline{P}}_{t+1} - \widehat{P}_t \right\}$ , we may derive the New Keynesian Phillips curve (NKPC), as:

$$\frac{\widehat{\pi}_t}{1-\theta} = (1-\beta\theta)\,\widehat{P}_{w,t} + \widehat{\pi}_t + (\beta\theta)\,E_t\left\{\frac{\widehat{\pi}_{t+1}}{1-\theta}\right\},\,$$

so that

$$\widehat{\pi}_t = \beta E_t \left\{ \widehat{\pi}_{t+1} \right\} + \lambda \widehat{P}_{w,t},$$

with  $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$ , which is the equation (A1.32) of Appendix 1.2. It shows that the current inflation is determined by the expectation on the future inflation and the current marginal costs.

Furthermore, by solving for  $\widehat{\pi}_t$  forward by the repeated substitution, we obtain the NKPC as the following forward-looking solution:

$$\widehat{\pi}_{t} = \lim_{k \to \infty} \beta^{k} E_{t} \left\{ \widehat{\pi}_{t+k} \right\} + \lambda \sum_{k=0}^{\infty} \beta^{k} E_{t} \left\{ \widehat{P}_{w,t+k} \right\}$$
$$= \lambda \sum_{k=0}^{\infty} \beta^{k} E_{t} \left\{ \widehat{P}_{w,t+k} \right\},$$

for  $\beta \in (0,1)$ , implying that the current inflation is determined by the discounted sum of the future expected marginal costs,  $\widehat{P}_{w,t+k} = \widehat{MC}_{t+k}$ .

**Resource Constraint** Consider the resource constraint,  $Y_t = C_t + I_t + G_t$ , in (2.33). Note that in the steady state, Y = C + I + G. Then, using  $x_t \simeq x(1 + \hat{x}_t)$ , the above equation is approximated around the steady state as:

$$Y\left(1+\widehat{Y}_{t}\right)=C\left(1+\widehat{C}_{t}\right)+I\left(1+\widehat{I}_{t}\right)+G\left(1+\widehat{G}_{t}\right),$$

so that we obtain

$$\widehat{Y}_t = \left(\frac{C}{Y}\right)\widehat{C}_t + \left(\frac{I}{Y}\right)\widehat{I}_t + \left(\frac{G}{Y}\right)\widehat{G}_t,$$

which is the equation (A1.33) in Appendix A1.2.

# A2.6 Government Policy

In Appendix A2.6, we derive and log-linearise the equations for government policies.

Linearisation of Conventional Monetary Policy Rule Consider the Taylortype feedback rule,  $\left(\frac{R_t^n}{R^n}\right) = \left(\frac{R_{t-1}^n}{R^n}\right)^{\alpha_r} (\pi_t)^{(1-\alpha_r)\alpha_\pi} \left(\frac{Y_t}{Y}\right)^{(1-\alpha_r)\alpha_y} \exp\left\{\varepsilon_{r,t}\right\}$ , in (2.34). Taking logarithms on both sides yields the approximation of the Taylor rule around the steady state as:

$$\widehat{R}_{t}^{n} = \alpha_{r} \widehat{R}_{t-1}^{n} + (1 - \alpha_{r}) \alpha_{\pi} \widehat{\pi}_{t} + (1 - \alpha_{r}) \alpha_{y} \widehat{Y}_{t} + \varepsilon_{r,t}$$

which is an equation (A1.34) in Appendix A1.2.

Derivation and Linearisation of Credit Market Intervention Rule We consider the central bank's problem to minimise the fluctuation in total credit supply,  $Q_tS_t$ , given by

$$\min_{\Phi_t} \quad \left(\frac{\Psi_t N_t}{1 - \Phi_t} - \frac{\Psi N}{1 - \Phi}\right)^2,$$

which is an equation (2.40) in the text. The first order condition with respect to the public credit supply,  $\Phi_t$ , is:

$$0 = 2 (\Psi_t N_t)^2 (1 - \Phi_t)^{-3} - 2 \left(\frac{\Psi N}{1 - \Phi}\right) (\Psi_t N_t) (1 - \Phi_t)^{-2}$$
$$= 2 (\Psi_t N_t) (1 - \Phi_t)^{-3} \left[\Psi_t N_t - \left(\frac{\Psi N}{1 - \Phi}\right) (1 - \Phi_t)\right],$$

so that in the presence of credit market intervention, i.e.,  $\Phi_t \in (0,1)$ , the optimal credit market intervention rule is:

$$\Phi_t = 1 - \left(\frac{1}{S}\right) \Psi_t N_t,$$

where we use the steady state relation,  $\frac{\Psi N}{1-\Phi} = QS = S$ . By introducing the intensity coefficient of credit market intervention,  $\nu > 0$ , as discussed in the text, we specify the credit market intervention rule as:

$$\Phi_t = 1 - (1 - \Phi) \left(\frac{N_t}{N}\right)^{\nu},$$

which is an equation in (2.43) in the text.

Next, we linearise the above credit market intervention rule. By using

 $x_t \simeq x(1+\widehat{x}_t)$ , the LHS and RHS of the above equation are approximated around the steady state as:

$$(LHS) \simeq \Phi(1+\widehat{\Phi}_t)$$
 
$$(RHS) \simeq 1 - (1-\Phi)\left(\frac{N}{N}\right)^{\nu} \left[1+\nu\left(\widehat{N}_t-0\right)\right].$$

Combining both sides yields

$$\widehat{\Phi}_t = -\nu \left(\frac{1-\Phi}{\Phi}\right) \widehat{N}_t,$$

which is the equation (A1.36) in Appendix A1.2.

**Linearisation of Total Credit Supply** We consider the total credit supply,  $Q_t S_t = \frac{\Psi_t}{1-\Phi_t} N_t$ , in (2.38) of the text, which can be rewritten as:

$$1 - \Phi_t = \frac{\Psi_t N_t}{Q_t S_t}.$$

Note that in the steady state,  $\Phi = 1 - \frac{\Psi N}{S}$ . By using  $x_t \simeq x(1 + \hat{x}_t)$ , the LHS and RHS of the above equation are approximated around the steady state as:

$$(LHS) \simeq 1 - \Phi \left( 1 + \widehat{\Phi}_t \right)$$

$$(RHS) \simeq \frac{\Psi N}{QS} \left( 1 + \widehat{\Psi}_t + \widehat{N}_t - \widehat{Q}_t - \widehat{S}_t \right)$$

$$\simeq (1 - \Phi) \left( 1 + \widehat{\Psi}_t + \widehat{N}_t - \widehat{Q}_t - \widehat{S}_t \right).$$

Combining the LHS and RHS yields

$$\widehat{\Phi}_t = \left(\frac{1 - \Phi}{\Phi}\right) \left(\widehat{Q}_t + \widehat{S}_t - \widehat{\Psi}_t - \widehat{N}_t\right),\,$$

so that

$$\widehat{Q}_t + \widehat{S}_t = \widehat{\Psi}_t + \widehat{N}_t + \left(rac{\Phi}{1-\Phi}
ight)\widehat{\Phi}_t$$

which is the first equation in (A1.24) in Appendix A1.2.

Alternatively, we may simplify the approximated total credit supply,  $\widehat{Q}_t + \widehat{S}_t = \widehat{\Psi}_t + \widehat{N}_t + \left(\frac{\Phi}{1-\Phi}\right) \widehat{\Phi}_t$ , by substituting the approximated the public credit supply,  $\widehat{\Phi}_t = -\nu \left(\frac{1-\Phi}{\Phi}\right) \widehat{N}_t$  as:

$$\widehat{Q}_t + \widehat{S}_t = \widehat{\Psi}_t + \widehat{N}_t - \left(\frac{\Phi}{1 - \Phi}\right) \nu \left(\frac{1 - \Phi}{\Phi}\right) \widehat{N}_t$$
$$= \widehat{\Psi}_t + (1 - \nu) \widehat{N}_t,$$

which is the second equation in (A1.24) in Appendix A1.2.

Linearisation of Total Government Spending Consider the government spending,  $G_t = G_t^c + \Phi_t Q_t S_t$ , in (2.45) of the text. Note that in the steady state,  $G = G^c + \Phi Q S$ , so that  $\Phi S = G - G^c$ . By using  $x_t \simeq x(1+\hat{x}_t)$ , the LHS and RHS of the above equation are approximated around the steady state as:

$$(LHS) \simeq G\left(1+\widehat{G}_t\right)$$

$$(RHS) \simeq G^c\left(1+\widehat{G}_t^c\right)+\Phi S\left(1+\widehat{\Phi}_t+\widehat{Q}_t+\widehat{S}_t\right).$$

Combining the LHS and RHS yields

$$\widehat{G}_t = \left(\frac{G^c}{G}\right)\widehat{G}_t^c + \left(1 - \frac{G^c}{G}\right)\left(\widehat{\Phi}_t + \widehat{Q}_t + \widehat{S}_t\right),\,$$

which is the equation in (A1.39) in Appendix A1.2.

# Chapter 3

# Sudden Stop Crisis in Emerging

# Markets

# 3.1 Introduction

Emerging market countries, which are characterised by a substantial degree of trade and financial openness, have usually been considered to be vulnerable to the events which occur in foreign countries. For example, a negative change in foreign investors' perception about an emerging market economy could cause the capital inflow to come to a standstill, a situation labeled as a 'sudden stop' (Calvo, 1998), leading to a drastic fall in economic activity. Indeed, emerging market economies experienced a number of such episodes throughout the 1990s, such as Mexico (1994-95), a group of East Asian countries (1997), Russia (1998) and Brazil (1998-99). In addition, the recent global financial crisis episode, triggered by the collapse of the housing market bubble in the United States in 2007-2008, shows how emerging market countries are affected by the crisis which breaks out in the global financial

hub.

As regards these events, many authors have provided stylised facts about sudden stop crises in emerging market countries through the theoretical and empirical works. First of all, Calvo and Reinhart (2000), Bleaney (2005) and Curdia (2007) found that when an emerging market economy is hit by a sudden stop, there tend to be a great currency depreciation, substantial contractions in investment and production, and a temporary growth in exports but a significant reduction in imports. In addition, Calvo, Izquierdo and Mejia (2004), Uribe and Yue (2006) and Chang and Fernandez (2010) showed that the foreign debt and 'balance-sheet effect' played an important role in business cycle fluctuations in emerging market countries. Furthermore, as Calvo and Reinhart (2000) point out, this kind of crisis tends to take place against a background of soft-pegged exchange rate regime, which supports the 'fear of floating' argument a la Calvo and Reinhart (2002). However, in spited of those common features as to sudden stop crises, it is also noteworthy that the specific pattern of crisis that an individual emerging market economy exhibited varied with the country's pre-crisis conditions, as Lane and Milesi-Ferretti (2010) pointed out.

Motivated by the above observations, this chapter aims at enhancing our understanding of sudden stop crises in emerging market countries. Our interest covers: (i) what triggers a sudden stop of the capital inflows into an emerging

<sup>&</sup>lt;sup>1</sup>'Fear of floating' refers to the phenomenon that an economy is reluctant to adjust exchange rates (de facto), even though it announces to adopt a free floating exchange rate regime (de jure). This phenomenon is prevalent particularly in emerging markets, where there exists a large amount of foreign currency denominated debt (domestic liability dollarisation, DLD). This is because of the inability of these countries to borrow abroad in their own currency, which is referred to as 'original sin' by Eichengreen and Hausmann (1999). In these circumstances, the 'fear of floating' could occur due to the concern about the negative impact of the currency depreciation on the economic activities, which is referred to as 'balance-sheet effect' in the literature.

market economy; (ii) how the economy is affected by the sudden stop; (iii) and what kind of pre-crisis conditions affect the sudden stop crisis. To these ends, this chapter extends a closed economy New Keynesian DSGE model with financial frictions as in Chapter 2 to the one allowing for the nature of a small open economy<sup>2</sup> to simulate a sudden stop crisis for an emerging market economy more realistically. We construct a New Keynesian DSGE model for a small open economy with financial frictions, which is based on Gertler, Gilchrist, and Natalucci (2007) and Curdia (2007, 2009). These kinds of models link an event in the financial sector to responses in real economic activities under a small open economy setting, so as to make them effective in analysing sudden stop crises in emerging market economies. Having constructed the DSGE model, we investigate the impulse responses of the model economy to a negative change in foreign lenders' evaluation on the domestic entrepreneurs' financial condition, to analyse the transmission of sudden stops to emerging market economies. In addition, we conduct a set of experiments to study what kind of roles the environmental conditions play in the development of the sudden stop crisis in the emerging market countries. In particular, we investigate the roles of exchange rate regime choice and an economy's degree of foreign input reliance in a sudden stop crisis under an open economy setting to enhance the understanding of the impact of foreign shocks to an emerging market economy's business cycle.

Our main findings are summarised as follows. First, we establish that foreign lenders' negative perception regarding an emerging market economy could

<sup>&</sup>lt;sup>2</sup>An emerging market economy described in the thesis implies a 'small open economy', unless otherwise mentioned. Accordingly, the economic variables in foreign countries are assume to be unaffected by the domestic agents' behaviours, except for the variations by the change in the exchange rate.

actually lead to a recession there via sudden stops in foreign fund inflow and the rise in cost of foreign borrowing. That is, foreigners' pessimistic outlook could be 'self-fulfilled', in the sense that the crisis in the emerging market economy is triggered by none other than the foreigners' own actions, as Calvo (1998) and Krugman (1999) suggest. Second, we identify some environmental conditions in emerging market economies that could make the sudden stop crisis aggravated. The conditions include: (i) the presence of substantial degree of financial frictions in the economy; (ii) the coincidence of sudden stops and a global recession; (iii) the choice of a fixed exchange rate regime; and (iv) an economy's heavy reliance on the foreign resources for capital production. That is, (i) a high degree of financial frictions could make a sudden stop crisis aggravated, since in this circumstances foreigners react susceptibly to even the temporary and slight distortions in entrepreneurs' financial condition; (ii) when a global recession coincides with the sudden stops, the recovery channel via the currency depreciation and the expansion in export in a sudden stop crisis could be interrupted due to the lack of global demand; (iii) when an emerging market economy relies heavily on foreign resources for capital production, it could suffer a sudden stop crisis more severely, as the currency depreciation in a sudden stop crisis would raise the capital production cost additionally so that the capital demand is further depressed; and (iv) a fixed exchange rate regime could be inferior to a floating exchange rate system in the face of sudden stop crises, in case a negative effect of a fixed exchange rate regime by limiting the improvement in price competitiveness for home goods offsets a positive effect by stabilising a rise in cost of foreign borrowing.

From our study, we contribute to the existing literature as follows. First, we establish an important role of a negative change in foreign lenders' perception

on an emerging market economy as a more primitive source of a financial crisis in the economy. In addition, we confirm the so-called 'self-fulfilling pessimism' argument a la Calvo (1998) and Krugman (1999) in a general equilibrium framework. Second, we point out the potential weakness of the 'fear of floating' argument a la Calvo and Reinhart (2002). That is, even if the argument holds in case a financial crisis in an emerging market economy is caused by a currency depreciation via a balance sheet effect, it may not in a different environment, e.g., in case their balance sheet is directly distorted by a collapse of borrowers' net worth. Third, we simulate the effect of 'processing trade' in a general equilibrium framework. It suggests that if an emerging market economy relies highly on the foreign inputs, it is not likely to recover through the currency depreciation and the resulting expansion in export, due to the aggravated production cost.

The remainder of this chapter is organised as follows. Section 3.2 sets up a New Keynesian DSGE model for a small open economy with financial frictions. Section 3.3 presents the simulation strategy and calibration. Section 3.4 discuss the transmission of the crisis and the role of pre-crisis conditions in the process of crisis through a set of crisis experiments. Section 3.5 concludes.

# 3.2 The Model

The small open economy model consists of the domestic and foreign blocks. Since we are assuming the small open economy, the behaviours in the foreign block is considered exogenously given, and the transactions across border are governed by

the law of one price (LOOP) and the uncovered interest parity condition (UIPC).<sup>3</sup> The domestic economy is populated by households, entrepreneurs, capital producers, retailers, and government. Households consume domestic and foreign goods, purchase domestic and foreign riskless bonds from domestic government and foreign financial intermediaries, supply labour to domestic entrepreneurs, and pay the taxes to the domestic government. Entrepreneurs produce wholesale goods by combining labour hired from domestic households and capital acquired from domestic capital producers, and sell them to domestic retail firms in a competitive way. In order to acquire capital, entrepreneurs use their own net worth and the foreign borrowings. Capital producers combine the existing capital goods purchased from entrepreneurs and domestic and foreign final goods, to construct the brand new capital goods. Retail firms purchase the wholesale goods from entrepreneurs, differentiate them to their own varieties, set the retail prices for the individual varieties in the environment of monopolistic competition and price stickiness a la Calvo (1983), and sell them to domestic and foreign consumers, i.e., domestic and foreign households, domestic capital producers and the domestic government. Government sets the nominal interest rate according to Taylor-type (1993) feedback rule and implement the government spending, which is financed by taxes and public bond issueing. Each agent's behaviours are described in more detail below.<sup>4</sup>

<sup>&</sup>lt;sup>3</sup>Note that it is assumed that there does not exist any frictions in the transactions across border such as trade costs and capital immobility. This is a configuration for the analysis to be focused on the financial frictions arising from the information asymmetry between foreign lenders and domestic borrowers.

<sup>&</sup>lt;sup>4</sup>Appendix B2 provides derivation and linearisation process for equilibrium conditions for our small open economy model.

#### 3.2.1 Households

Households consume home and foreign goods, work for entrepreneurs' firms, and deposit their savings at domestic and foreign bonds. They derive the lifetime utility from consumption,  $C_t$ , and labour,  $L_t$ , according to:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, L_t) \tag{3.1}$$

where  $\beta \in (0,1)$  is their subjective discount factor. Moreover, their utility function is assumed to be specified as:

$$U(C_t, L_t) = \frac{(C_t)^{1-\sigma}}{1-\sigma} - \frac{(L_t)^{1+\varphi}}{1+\varphi}$$
 (3.2)

where  $\sigma > 0$  is the inverse of the intertemporal elasticity of substitution in consumption and  $\varphi > 0$  is the inverse elasticity of labour supply. Suppose that they are allowed to access the international credit market (ICM) without any frictions, and hence, they can use both the domestic and foreign bonds,  $D_t$  and  $B_t^*$ , for consumption smoothing. Households are assumed to enter period t with domestic and foreign (real) bonds,  $D_{t-1}$  and  $B_{t-1}^*$ , which yield the gross (real) returns,  $R_{t-1}$  and  $R_{t-1}^*$ , respectively, over the period t. In addition, during the period t, they collect the nominal wage,  $W_t$ , from supplying the labour,  $L_t$ , and receive the real dividend arising from the ownership of firms,  $\Pi_t^o$ . Their budget is spent on

<sup>&</sup>lt;sup>5</sup>Henceforth, foreign variables or variables denominated in foreign currency are denoted by a superscript asterisk (\*).

<sup>&</sup>lt;sup>6</sup>Note that it is assumed that there does not exist domestic private financial intermediaries in the home country in order to focus our analysis on the implication of financial frictions between foreign lenders and domestic borrowers. This assumption reflects that many of emerging market economies do not have the sufficiently developed domestic financial system. Accordingly, in our model economy domestic households save their deposits either on the domestic public bonds,  $D_t$ , or the foreign private bonds,  $B_t^*$ , and domestic entrepreneurs borrow funds from foreign intermediaries only, as discussed later.

consumption,  $C_t$ , payment of the (real) lump-sum taxes,  $T_t$ , and purchase of the domestic and foreign bonds,  $D_t$  and  $B_t^*$ . Thus, their period budget constraint is given in real terms by

$$C_t + D_t + S_t B_t^* \le \left(\frac{W_t}{P_t}\right) L_t + R_{t-1} D_{t-1} + R_{t-1}^* B_{t-1}^* S_t + \Pi_t^o - T_t \tag{3.3}$$

for all t = 0, 1, 2, ..., where  $P_t$  is the domestic consumer price index (CPI) and  $S_t$  is the nominal exchange rate, which is defined as the price of foreign curreny in terms of domestic currency.

The households seek to maximise the lifetime utility in (3.1) subject to the period budget constraint in (3.3). The resulting first order conditions yield the following Euler equation for consumption, labour supply function, and uncovered interest rate parity condition (UIPC):

$$1 = \beta E_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} R_t \right\}, \tag{3.4}$$

$$\frac{W_t}{P_t} = \left(C_t\right)^{\sigma} \left(L_t\right)^{\varphi},\tag{3.5}$$

and

$$R_t = R_t^* \left( \frac{S_{t+1}}{S_t} \right). \tag{3.6}$$

Euler equation in (3.4) establishes the negative relationship between the ratio of the current consumption to the future one,  $\frac{C_t}{C_{t+1}}$ , and the real interest rate,  $R_t$ , everything else being equal. Labour supply function in (3.5) implies that an increase in the real wage,  $\frac{W_t}{P_t}$ , induces an increase in the labour supply,  $L_t$ , ceteris

paribus. Uncovered interest rate parity condition (UIPC) suggests that no matter where they invest, asset holders should earn the same returns in terms of domestic currency by the arbitrage. In addition, the uncovered interest rate parity condition in (3.6) points to the nature of capital mobility. That is, it implies that a fall in the domestic riskless rate,  $R_t$ , relative to the foreign one,  $R_t^*$ , would lead to the capital outflow and the domestic currency depreciation without any frictions.

Next, we turn to the domestic households' consumption allocation between the domestic and foreign goods. Their consumption bundle,  $C_t$ , is composed of domestic and foreign goods (in terms of domestic currency), denoted by  $C_{H,t}$  and  $C_{F,t}$ , respectively, which is represented by the following Dixit and Stiglitz (1977) aggregator:

$$C_t \equiv \left[ \eta^{\frac{1}{\gamma}} \left( C_{H,t} \right)^{\frac{\gamma - 1}{\gamma}} + \left( 1 - \eta \right)^{\frac{1}{\gamma}} \left( C_{F,t} \right)^{\frac{\gamma - 1}{\gamma}} \right]^{\frac{\gamma}{\gamma - 1}}, \tag{3.7}$$

where  $\eta \in (0,1)$  is the share of domestic goods in consumption bundle in the steady state,<sup>8</sup> and  $\gamma > 1$  is the elasticity of substitution between  $C_{H,t}$  and  $C_{F,t}$ . The corresponding consumer price index (CPI),  $P_t$ , is given by the following constant elasticity of substitution (CES) form,

$$P_{t} \equiv \left[ \eta \left( P_{H,t} \right)^{1-\gamma} + (1-\eta) \left( P_{F,t} \right)^{1-\gamma} \right]^{\frac{1}{1-\gamma}}$$
 (3.8)

<sup>&</sup>lt;sup>7</sup>Composites for domestic and foreign goods,  $C_{H,t}$  and  $C_{F,t}$ , are defined as  $C_{H,t} = \left[\int_0^1 C_{H,t}(j)^{\frac{\epsilon-1}{\epsilon}} dj\right]^{\frac{\epsilon}{\epsilon-1}}$  and  $C_{F,t} = \left[\int_0^1 C_{F,t}(j)^{\frac{\epsilon-1}{\epsilon}} dj\right]^{\frac{\epsilon}{\epsilon-1}}$ , respectively, where  $j \in (0,1)$  denotes varieties, and  $\epsilon > 1$  is the elasticity of substitution among varieties produced within a country. In addition, the corresponding price indices for domestic and foreign goods,  $P_{H,t}$  and  $P_{F,t}$ , in domestic currency, are given by  $P_{H,t} = \left[\int_0^1 P_{H,t}(j)^{1-\epsilon} dj\right]^{\frac{1}{1-\epsilon}}$  and  $P_{F,t} = \left[\int_0^1 P_{F,t}(j)^{1-\epsilon} dj\right]^{\frac{1}{1-\epsilon}}$ , respectively.

<sup>&</sup>lt;sup>8</sup>Accordingly,  $(1 - \eta)$  represents the degree of openness of the economy.

where  $P_{H,t}$  and  $P_{F,t}$  denote the nominal retail price indices in domestic currency for domestic and imported goods, respectively. Then, domestic households' expenditure minimisation implies the following demands for domestic and foreign goods in the domestic market:

$$C_{H,t} = \eta \left(\frac{P_{H,t}}{P_t}\right)^{-\gamma} C_t \tag{3.9}$$

and

$$C_{F,t} = (1 - \eta) \left(\frac{P_{F,t}}{P_t}\right)^{-\gamma} C_t. \tag{3.10}$$

Equations (3.9) and (3.10) suggest that they are positively related with the size of the total consumption bundle,  $C_t$ , and negatively related with the real prices of home and foreign goods,  $\frac{P_{H,t}}{P_t}$ , and  $\frac{P_{F,t}}{P_t}$ , respectively.

#### 3.2.2 Rest of the World

Economic agents in foreign countries import domestic retail goods, export foreign goods to home country, and engage in financial transaction of the foreign currency denominated bond. We have discussed the domestic consumers' demand for imported goods in the previous part, and will deal with the domestic entrepreneurs' foreign borrowing in the subsequent part. This part explores the export demand of home goods in the foreign countries.

First of all, we discuss some nature of domestic and foreign economies to characterise the transactions across border. First, we assume that there does not exist the local pricing, so that the law of one price (LOOP) holds for the home and foreign goods. Thus, the nominal prices of home and foreign goods in foreign currency,  $P_{H,t}^*$  and  $P_{F,t}^*$ , respectively, are linked to those in domestic currency,  $P_{H,t}$  and  $P_{F,t}$ , respectively, via the nominal exchange rate,  $S_t$ , given by<sup>9</sup>:

$$P_{H,t}^* = \frac{P_{H,t}}{S_t} \tag{3.11}$$

and

$$P_{F,t} = S_t P_{F,t}^*. (3.12)$$

In addition, note that, in the small open economy (SOE) setting, the rest of the world is so large that the portion of transaction with an individual emerging market economy is infinitesimal and the effects of the home country's behaviours are negligible. Then, we may assume that  $P_{F,t}^* = P_t^*$ , and  $C_t^* = Y_t^*$ , where  $C_t^*$ ,  $Y_t^*$ , and  $P_t^*$  denote the foreign households' consumption bundle, foreign output level, and foreign consumer price index, respectively. Moreover, the consideration of the small open economy suggests that the foreign variables, such as the foreign output level,  $Y_t^*$ , the inflation rate of the foreign consumer price index,  $\pi_t^* \equiv \frac{P_t^*}{P_{t-1}^*}$ , and the international riskless rate,  $R_t^*$ , are given exogenously to the domestic behaviours, according to the following first order autoregressive (AR(1)) process:

$$Y_t^* = (Y_{t-1}^*)^{\rho_{y^*}} \exp\{\varepsilon_{y^*,t}\},$$
 (3.13)

$$\pi_t^* = (\pi_{t-1}^*)^{\rho_{\pi^*}} \exp\left\{\varepsilon_{\pi^*,t}\right\},\tag{3.14}$$

and

$$R_t^* = (R_{t-1}^*)^{\rho_{r^*}} \exp\{\varepsilon_{r^*,t}\}, \qquad (3.15)$$

where  $|\rho_{y^*}| < 1$ ,  $|\rho_{\pi^*}| < 1$  and  $|\rho_{r^*}| < 1$ , and  $\varepsilon_{y^*,t}$ ,  $\varepsilon_{\pi^*,t}$ , and  $\varepsilon_{r^*,t}$  are Gaussian white noises with means all zeroes and standard deviations  $\sigma_{y^*}$ ,  $\sigma_{\pi^*}$ , and  $\sigma_{r^*}$ , respectively.

Next, we discuss the export demand for the home goods in the foreign countries,  $C_{H,t}^*$ . We assume that the foreign consumers' preference over home and foreign goods has an analogous structure to that of the domestic consumers in (3.7) in terms of functional form, the elasticity of substitution among varieties, and so on. Then, from the expenditure minimisation for foreign consumers, export demand for home goods,  $C_{H,t}^*$ , is given in foreign currency by:

$$C_{H,t}^* = \eta^* \left(\frac{P_{H,t}^*}{P_t^*}\right)^{-\gamma^*} C_t^*,$$
 (3.16)

where  $\eta^* \in (0,1)$  is the steady state share of home goods in the foreign consumers' consumption bundle,  $C_t^*$ , and  $\gamma^* > 1$  is the elasticity of substitution between home and foreign goods for the foreign consumers. For analytical simplicity, we suppose that the price elasticity of export demand for home goods in foreign countries is the same as that for home goods in domestic country, i.e.,  $\gamma^* = \gamma$ . In addition, we

define the real exchange rate,  $S_t$ , as

$$S_t \equiv \frac{S_t P_t^*}{P_t}. (3.17)$$

Then, under the assumptions of the small open economy (SOE) and the law of one price (LOOP) for home goods, the export demand for home goods in (3.16) may be expressed in domestic currency as

$$C_{H,t}^* = \eta^* \left(\frac{P_{H,t}}{S_t P_t^*}\right)^{-\gamma} Y_t^* = \eta^* \left(\frac{P_{H,t}}{S_t P_t}\right)^{-\gamma} Y_t^*.$$
 (3.18)

Equation (3.18) suggests that the export demand for home goods,  $C_{H,t}^*$ , in foreign countries increases with the foreign output level,  $Y_t^*$ , and the real exchange rate,  $S_t \equiv \frac{S_t P_t^*}{P_t}$ , but decreases with the real price of the home goods,  $\frac{P_{H,t}}{P_t}$ . 10

# 3.2.3 Entrepreneurs

Entrepreneurs are key players in the setup. They combine capital and labour to produce wholesale goods and sell them to domestic retailers in a perfectly competitive manner. They borrow from foreign lenders to finance capital acquisition. Entrepreneurs accumulate the profits in the form of net worth. We discuss each of entrepreneurs' activities in more detail below.

Thus, the import demand for foreign goods in (3.10) and the consumer price index in (3.8) may be expressed as  $C_{F,t} = (1-\eta)(S_t)^{-\gamma} C_t$  and  $1 \equiv \eta \left(\frac{P_{H,t}}{P_t}\right)^{1-\gamma} + (1-\eta)(S_t)^{1-\gamma}$ , respectively. Accordingly, we establish that the import demand is decreasing in the real exchange rate, while the CPI is increasing in it.

#### 3.2.3.1 Production of Wholesale Goods

Entrepreneurs purchase capital from capital producers in each period t-1, for the use in the subsequent period t. Capital,  $K_{t-1}$ , is used to produce wholesale goods,  $Y_{w,t}$ , in combination with labour hired from households,  $L_t$ , in period t, by the following Cobb-Douglas technology:

$$Y_{w,t} = A_t (K_{t-1})^{\alpha} (L_t)^{1-\alpha}, \qquad (3.19)$$

with  $\alpha$  denoting a share of capital in the production function.<sup>11</sup>  $A_t$  is a shock to total factor productivity (TFP), which is governed by the following first order autoregressive (AR(1)) process:

$$A_t = (A_{t-1})^{\rho_a} \exp\{\varepsilon_{a,t}\}, \tag{3.20}$$

where  $|\rho_a| < 1$ , and  $\varepsilon_{a,t}$  is a Gaussian white noise with mean zero and standard deviation  $\sigma_a$ . In addition, following Gertler *et al.* (2007), entrepreneurs are assumed to borrow from foreign lenders at the rate of  $R_t^k$  to finance the capital acquisition, and to resell the undepreciated capital goods,  $(1 - \delta) K_{t-1}$ , to capital producers immediately after finishing the wholesale good production. Then, their (real) cost function is given by:

$$TC_{w,t} = \left(\frac{W_t}{P_{H,t}}\right) L_t + \left[R_t^k Q_{t-1} K_{t-1} - (1-\delta) Q_t K_{t-1}\right], \tag{3.21}$$

where  $R_t^k$  is the required capital returns,  $Q_t$  is the real capital price, and  $\delta$  is the depreciation rate for the capital goods. We suppose that the entrepreneurs op-

<sup>&</sup>lt;sup>11</sup>The assumption of constant returns to scale allows us to write the production function as an aggregate relationship. Thus, we drop the firm specific subscript, for notational simplicity.

erate as price takers both in the wholesale goods market and in the production factor market. Then, cost minimisation in (3.21) subject to the production technology in (3.19) implies the following demands for labour and capital investment, respectively, as:

$$\frac{W_t}{P_{H,t}} = (1 - \alpha) \left(\frac{Y_{w,t}}{L_t}\right) P_{w,t},\tag{3.22}$$

and

$$E_t \left\{ R_{t+1}^k Q_t \right\} = E_t \left\{ \alpha \left( \frac{Y_{w,t+1}}{K_t} \right) P_{w,t+1} + (1 - \delta) Q_{t+1} \right\}, \tag{3.23}$$

where  $P_{w,t}$  is the real wholesale good price.<sup>12</sup> Labour demand function in (3.22) implies that labour demand,  $L_t$ , increases with a production expansion,  $Y_{w,t}$ , and a rise in the real wholesale good price,  $P_{w,t}$ , but decreases with a rise in the real wage,  $\frac{W_t}{P_{H,t}}$ . Capital demand function in (3.23) suggests that, given other things fixed, capital demand,  $K_t$ , increases with a plan for production expansion,  $Y_{w,t+1}$ , and an expected rise in the wholesale good price,  $P_{w,t+1}$ , while it decreases with rises in the required capital returns,  $R_{t+1}^k$ , and the (current) capital price,  $Q_t$ .

#### 3.2.3.2 Optimal Contracting Problem

Next, we explore the entrepreneurs' decision making to finance the capital acquisition. At the end of period t, the entrepreneurs are assumed to have available their own net worth,  $N_t$ , which is the accumulation of their past profit. Then, they borrow from abroad the difference between the capital demand,  $Q_tK_t$ , and the net

<sup>&</sup>lt;sup>12</sup>Note that since entrepreneurs sell the wholesale goods in a perfectly competitive market, the real wholesale good price,  $P_{w,t}$ , is required to equal the real marginal cost of producing the wholesale goods,  $MC_{w,t}$ , i.e.,  $P_{w,t} = MC_{w,t}$ , in the equilibrium.

worth,  $N_t$ , <sup>13</sup> so that the (real) entrepreneurs' balance sheet is given in domestic currency by:

$$Q_t K_t = N_t + S_t B_t^*, (3.24)$$

where  $B_t^*$  is the entrepreneurs' (real) borrowing from foreign lenders in foreign currency. Foreign lenders are assumed to pay the riskless rate,  $R_t = R_t^* \left( \frac{S_{t+1}}{S_t} \right)$ , to households and impose the gross capital returns,  $R_{t+1}^k$  on entrepreneurs.

Now, we discuss the loan contracting problem between foreign lenders and domestic entrepreneurs.<sup>14</sup> First of all, note that in the absence of financial frictions, credit market arbitrage implies that there would not exist any wedge between capital returns,  $R_{t+1}^k$ , and riskless rate,  $R_t = R_t^* \left( \frac{S_{t+1}}{S_t} \right)$ , such that

$$R_{t+1}^k = R_t = R_t^* \left( \frac{S_{t+1}}{S_t} \right). {(3.25)}$$

However, the presence of financial frictions would break the equality between  $R_{t+1}^k$  and  $R_t$ . To model the financial frictions, we postulate the 'costly state verification' (CSV) problem a la Townsend (1979) and Bernanke et al. (1999). We suppose that an individual entrepreneur suffers from an idiosyncratic shock,  $\omega_{t+1} \in (0, \infty)$ , to the capital returns,  $R_{t+1}^k$ . In addition, it is assumed that  $\omega_{t+1}$  is independently, identically and log-normally distributed across time and firms, with  $E\{\omega_{t+1}\} = 1$ , and with  $F(\omega_{t+1})$  and  $f(\omega_{t+1})$  denoting the cumulative distribution function (c.d.f.) and probability density function (p.d.f.) of  $\omega_{t+1}$ , respectively. Hence,

<sup>&</sup>lt;sup>13</sup>It is assumed that there does not exist domestic private financial intermediation in the economy to be consistent with the assumption for households' budget constraint in equation (3.3). Accordingly, domestic entrepreneurs are allowed to borrow from foreign financial intermediaries only.

<sup>&</sup>lt;sup>14</sup>The formal representation of the problem and the solution is presented in Appendix B2.2.

the  $ex\ post$  gross capital returns would be  $\omega_{t+1}R_{t+1}^k$ , where  $R_{t+1}^k$  is the  $ex\ post$  aggregate return to capital, i.e., the gross return averaged across firms. Then, given  $R_{t+1}^kQ_tK_t$  and  $S_tB_t^*$ , the debt contract is characterised by the contractual rate,  $R_t^b$ , and the entrepreneurs' default threshold,  $\overline{\omega}_{t+1}$ , as

$$R_t^b S_t B_t^* = \overline{\omega}_{t+1} R_{t+1}^k Q_t K_t. \tag{3.26}$$

In this setup, the 'costly state verification' (CSV) problem implies that borrowers can observe the realised capital returns,  $\omega_{t+1}R_{t+1}^kQ_tK_t$ , while lenders cannot do so without paying an auditing cost (or bankruptcy cost), which is assumed to be a fixed portion,  $\mu_b$ , of the capital returns,  $\omega_{t+1}R_{t+1}^kQ_tK_t$ , i.e.,  $\mu_b\omega_{t+1}R_{t+1}^kQ_tK_t$ . Then, if  $\omega_{t+1} \geq \overline{\omega}_{t+1}$ , the entrepreneur would repay the loan to foreign lenders at the contractual rate,  $R_t^b$ , and collect the remainder of the profit, i.e.,  $\omega_{t+1}R_t^kQ_tK_t - R_t^bS_tB_t^*$ ; but if  $\omega_{t+1} < \overline{\omega}_{t+1}$ , she would declare to default (and hence, receives nothing), while foreign lenders keep whatever they find after paying the auditing cost, i.e.,  $(1-\mu_b)\omega_{t+1}R_{t+1}^kQ_tK_t$ . Then, foreign lenders are willing to participate in the debt contract, when it is expected that the gross returns from the loan is larger than the opportunity cost of funds. Accordingly, foreign lenders' incentive constraint is given (in domestic currency) by

$$[1 - F(\overline{\omega}_{t+1})] R_t^b S_t B_t^* + (1 - \mu_b) R_{t+1}^k Q_t K_t \int_0^{\overline{\omega}_{t+1}} \omega_{t+1} dF(\omega_{t+1}) \ge R_t^* B_t^* S_{t+1}.$$
(3.27)

In addition, considering the entrepreneur's default threshold in (3.26) and balance sheet in (3.24), we may write the foreign lenders' participation constraint in (3.27)

$$\left[\Gamma\left(\overline{\omega}_{t+1}\right) - \mu_b G\left(\overline{\omega}_{t+1}\right)\right] R_{t+1}^k Q_t K_t = R_t^* \left(Q_t K_t - N_t\right) \left(\frac{S_{t+1}}{S_t}\right), \tag{3.28}$$

where  $\Gamma(\overline{\omega}_t)$  and  $\mu_b G(\overline{\omega}_t)$  denote the rates of total (expected) payment going to foreign lenders and their auditing cost, respectively, given by:

$$\Gamma\left(\overline{\omega}_{t+1}\right) \equiv \left[1 - F(\overline{\omega}_{t+1})\right] \overline{\omega}_{t+1} + \int_0^{\overline{\omega}_{t+1}} \omega_{t+1} dF\left(\omega_{t+1}\right), \tag{3.29}$$

and

$$\mu_b G\left(\overline{\omega}_{t+1}\right) \equiv \mu_b \int_0^{\overline{\omega}_{t+1}} \omega_{t+1} dF\left(\omega_{t+1}\right). \tag{3.30}$$

Then, the optimal contracting problem implies that the entrepreneur chooses the level of borrowings to maximise her expected profit,  $(1 - \Gamma(\overline{\omega}_{t+1})) R_{t+1}^k Q_t K_t$ , subject to the foreign lenders' incentive constraint in (3.28), which is given by

$$\left(1 - \frac{N_t}{Q_t K_t}\right) = \left[\Gamma\left(\overline{\omega}_{t+1}\right) - \mu_b G\left(\overline{\omega}_{t+1}\right)\right] \left(\frac{R_{t+1}^k}{R_t^*}\right) \left(\frac{S_t}{S_{t+1}}\right). \tag{3.31}$$

From the optimal borrowing equation in (3.31), we establish the external finance premium,  $\Psi_t$ , as the increasing function of entrepreneur's leverage ratio,  $\frac{Q_t K_t}{N_t}$ , given by

$$\Psi_{t} = \left[\Gamma\left(\overline{\omega}_{t+1}\right) - \mu_{b}G\left(\overline{\omega}_{t+1}\right)\right]^{-1} \left[1 - \frac{N_{t}}{Q_{t}K_{t}}\right]$$

$$= \Psi\left(\frac{Q_{t}K_{t}}{N_{t}}\right) \tag{3.32}$$

with  $\Psi' > 0$ . In addition, for analytical facilitation, we specify the external finance premium,  $\Psi_t$ , around the steady state, as:

$$\Psi_t = \left[\frac{Q_t K_t}{N_t}\right]^{\psi},\tag{3.33}$$

where  $\psi > 0$  denotes the elasticity of the external finance premium,  $\Psi_t$ , with respect to the entrepreneur's leverage ratio,  $\frac{Q_t K_t}{N_t}$ .<sup>15</sup> Equation (3.33) implies that if the foreign lenders perceive the increase in the entrepreneur's leverage ratio,  $\frac{Q_t K_t}{N_t}$ , they would charge the higher risk premium,  $\Psi_t$ , to entrepreneurs.

Now, we write the cost of foreign borrowing for entrepreneurs as:

$$R_{t+1}^k = R_t^* \Psi_t \left( \frac{S_{t+1}}{S_t} \right) = R_t \Psi_t,$$
 (3.34)

from equations (3.31) and (3.32). Equation (3.34) suggests that in the presence of information asymmetry in the loan contract, the uncovered interest rate parity in (3.6) is required to be augmented by the external finance premium factor,  $\Psi_t$ , which is increasing in the entrepreneurs' leverage ratio,  $\frac{Q_t K_t}{N_t}$ , by (3.33). Therefore, if the foreign lenders evaluate an entrepreneur's balance sheet as being distorted, the cost of foreign borrowing that the entrepreneur faces would be raised.

The shown in Gertler et al. (2007),  $\psi$  can be calculated from the steady state values of the risk premium,  $\frac{R^k}{R}$ , and the leverage ratio,  $\frac{K}{N}$ , given by,  $\psi = \frac{\ln(R^k/R)}{\ln(K/N)}$ .

#### 3.2.3.3 Net Worth Evolution

Lastly, we discuss the evolution of entrepreneurs' aggregate net worth,  $N_t$ . First of all, following Kiyotaki and Moore (1997) and Carlstrom and Fuerst (1997), we assume that entrepreneurs survive with the rate of  $\phi$  each period, and then, newly entering entrepreneurs fill the places of failed ones with a fixed amount of start up funds, F. In addition, note that the successful entrepreneurs' net worth is the accumulation of their past profits,  $R_t^k Q_{t-1} K_{t-1} - R_{t-1}^* \Psi_{t-1} B_{t-1}^* S_t$ . Accordingly, net worth in the economy at the end of period t is governed by the following law of motion:

$$N_{t} = \left[ \phi \left\{ R_{t}^{k} Q_{t-1} K_{t-1} - R_{t-1}^{*} \Psi_{t-1} B_{t-1}^{*} S_{t} \right\} + (1 - \phi) F \right] \cdot V_{t}, \tag{3.35}$$

where  $V_t$  is the foreign lenders' evaluation factor on the entrepreneurs' net worth,  $N_t$ , which includes foreign lenders' outlook on entrepreneurs' profitability, the economic situation in the emerging market economy, the availability of foreign currency, and so on and so forth. Under normal circumstances, foreigners trust the book value of the entrepreneurs' net worth,  $N_t$ , so that  $V_t = 1$ . However, in some periods, which is characterised by a sudden stop crisis, foreign lenders might have pessimism about the entrepreneurs, so that they would devalue the entrepreneurs' net worth, i.e.,  $V_t < 1$ . We assume here that foreigners' evaluation factor,  $V_t$ , is formulated exogenously to economic variables in the model, and follows a first-order autoregressive (AR(1)) process, as:

<sup>&</sup>lt;sup>16</sup>The assumption that entrepreneurs have the finite horizon, ensures that they never accumulate their own net worth enough to fully self-finance the capital investment. In addition, the assumption that they have their own net worth available from the start of the business guarantees that entrepreneurs never operate solely by external finance. Under these assumptions, the lenders' participation constraint in (3.28) is binding.

$$V_t = (V_{t-1})^{\rho_v} \exp\{\varepsilon_{v,t}\}, \tag{3.36}$$

where  $|\rho_v| < 1$ , and  $\varepsilon_{v,t}$  is a Gaussian white noise with mean zero and standard deviation  $\sigma_v$ .

# 3.2.4 Capital Producers

Capital producers supply the capital goods,  $K_t$ , to entrepreneurs, which will be used to produce the wholesale goods,  $Y_{w,t+1}$ , in the subsequent period. They combine investment goods,  $I_t$ , with the existing capital goods,  $K_{t-1}$ , to construct the new capital goods,  $K_t^n$ . To be consistent with the assumption for entrepreneurs, capital producers are assumed to acquire  $K_{t-1}$  from entrepreneurs after finishing the wholesale good production. In addition, the investment good composite,  $I_t$ , is composed of domestic and foreign final goods, denoted by  $I_{H,t}$  and  $I_{F,t}$ , respectively, which is given by:<sup>17</sup>

$$I_{t} \equiv \left[ (\eta_{i})^{\frac{1}{\gamma_{i}}} (I_{H,t})^{\frac{\gamma_{i}-1}{\gamma_{i}}} + (1-\eta_{i})^{\frac{1}{\gamma_{i}}} (I_{F,t})^{\frac{\gamma_{i}-1}{\gamma_{i}}} \right]^{\frac{\gamma_{i}}{\gamma_{i}-1}}, \tag{3.37}$$

where  $\eta_i \in (0,1)$  measures the steady state share of domestic inputs,  $I_{H,t}$ , in the investment good composite,  $I_t$ , and  $\gamma_i > 1$  is the substitutability between  $I_{H,t}$  and  $I_{F,t}$ . The corresponding investment price index,  $P_{I,t}$ , is given by:

$$P_{I,t} \equiv \left[ \eta_i \left( P_{H,t} \right)^{1-\gamma_i} + (1-\eta_i) \left( P_{F,t} \right)^{1-\gamma_i} \right]^{\frac{1}{1-\gamma_i}}.$$
 (3.38)

That is,  $I_{H,t} = \left[\int_0^1 I_{H,t}\left(j\right)^{\frac{\epsilon-1}{\epsilon}} dj\right]^{\frac{\epsilon}{\epsilon-1}}$  and  $I_{F,t} = \left[\int_0^1 I_{F,t}\left(j\right)^{\frac{\epsilon-1}{\epsilon}} dj\right]^{\frac{\epsilon}{\epsilon-1}}$ , where  $j \in [0,1]$  indicates the goods variety and  $\epsilon > 1$  is the elasticity of substitution among varieties.

Then, capital producers choose the optimal mix of domestic and foreign inputs according to:

$$I_{H,t} = \eta_i \left(\frac{P_{H,t}}{P_t}\right)^{-\gamma_i} I_t \tag{3.39}$$

and

$$I_{F,t} = (1 - \eta_i) \left(\frac{P_{F,t}}{P_t}\right)^{-\gamma_i} I_t.$$
 (3.40)

Next, we discuss the supply of the aggregate investment,  $I_t$ . Following Ozkan and Unsal (2010), we assume that the new capital goods,  $K_t^n$ , are produced by the capital production technology with capital adjustment costs, given by:

$$K_t^n = \left[ \frac{I_t}{K_{t-1}} - \frac{\kappa}{2} \left( \frac{I_t}{K_{t-1}} - \delta \right)^2 \right] K_{t-1}, \tag{3.41}$$

where  $\kappa > 0$  is the capital adjustment coefficient. In this setup, the economy-wide capital stock accumulates according to

$$K_t = K_t^n + (1 - \delta) K_{t-1},$$
 (3.42)

and capital producers' (real) profit function is given by 18

$$\Pi_{c,t} = Q_t \left(\frac{P_{H,t}}{P_t}\right) K_t - \left[\frac{P_{I,t}}{P_t} I_t + Q_t \left(\frac{P_{H,t}}{P_t}\right) (1 - \delta) K_{t-1}\right]. \tag{3.43}$$

Then, the optimality condition for capital producers with respect to the choice of

<sup>&</sup>lt;sup>18</sup>Note that the real capital price,  $Q_t$ , is measured by domestic price,  $P_{H,t}$ , since capital,  $K_t$ , is assumed to be non-tradable; while capital producers' real profit function,  $\Pi_{c,t}$ , is measured by the CPI,  $P_t$ , since capital producers are allowed to adquire the investment goods,  $I_t$ , from bothe domestic and foreign retail markets.

 $I_t$  yields the following capital supply function

$$Q_t \left(\frac{P_{H,t}}{P_t}\right) = \frac{P_{I,t}}{P_t} \left[ 1 - \kappa \left( \frac{I_t}{K_{t-1}} - \delta \right) \right]^{-1}. \tag{3.44}$$

Equation (3.44) suggests that the real capital price,  $Q_t$ , increases with the real investment good price,  $\frac{P_{I,t}}{P_t}$ , and investment good demand,  $I_t$ . In addition, note that the real investment good price,  $\frac{P_{I,t}}{P_t}$ , is affected by the relative size of  $\eta_i$  and  $\eta$  and that of  $\gamma_i$  and  $\gamma$  from equations (3.8) and (3.38). Suppose that the capital production relies heavily on foreign inputs as compared to consumption bundle, i.e.,  $\eta_i < \eta$ , with the sensitivities,  $\gamma_i$  and  $\gamma$ , being equal. Then, as compared to  $P_t$ ,  $P_{I,t}$  is more sensitively affected by the motion in foreign good price,  $P_{F,t} = S_t P_{F,t}^*$ . Accordingly, a rise in the nominal exchange rate,  $S_t$ , would raise  $P_{I,t}$  more than  $P_t$ , which leads to rises in  $\frac{P_{I,t}}{P_t}$  and  $Q_t$ . In contrast, under the assumption that  $\gamma_i = \gamma$  and  $\gamma_i = \eta$ ,  $\gamma_i = \eta_i$  and  $\gamma_i = \eta_i$  would show exactly the same changes, so that  $\gamma_i = \eta_i$  and  $\gamma_i = \eta_i$  and  $\gamma_i = \eta_i$  would show exactly the same change rate,  $\gamma_i = \eta_i$  and the real capital price,  $\gamma_i = \eta_i$  would not be affected by the nominal exchange rate,  $\gamma_i = \eta_i$  and  $\gamma_i = \eta_i$  would not be affected by the nominal exchange rate,  $\gamma_i = \eta_i$  and  $\gamma_i = \eta_i$  would not be affected by the nominal exchange rate,  $\gamma_i = \eta_i$ 

## 3.2.5 Retail Firms

Now, in order to introduce rigidity for domestic price,  $P_{H,t}$ , the model allows for monopolistically competitive retail firms, indexed by  $j \in [0,1]$ . They purchase the domestic wholesale goods,  $Y_{w,t}$ , from entrepreneurs; costlessly diversify them into their own varieties,  $Y_{H,t}(j)$ , to gain a certain degree of price-setting power in the domestic final goods market; set the monopolistically competitive price,  $\overline{P}_{H,t}(j)$ , for variety j, under the price stickiness a la Calvo (1983); and sell them at the

prices,  $\overline{P}_{H,t}(j)$  and  $\frac{\overline{P}_{H,t}(j)}{S_t}$ , in domestic and foreign retail markets.<sup>19</sup>

Suppose that the domestic final good composite,  $Y_{H,t}$ , and the corresponding domestic price index,  $P_{H,t}$ , are expressed as the following Dixit and Stiglitz (1977) aggregator:

$$Y_{H,t} = \left[ \int_0^1 Y_{H,t} \left( j \right)^{\frac{\epsilon - 1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon - 1}}, \tag{3.45}$$

and

$$P_{H,t} = \left[ \int_0^1 \overline{P}_{H,t} \left( j \right)^{1-\epsilon} dj \right]^{\frac{1}{1-\epsilon}}, \tag{3.46}$$

where  $\overline{P}_{H,t}(j)$  is the price for variety j, and  $\epsilon > 1$  is the elasticity of substitution among varieties. In addition, by construction, the wholesale goods as a whole,  $Y_{w,t}$ , are equal to retail good composite,  $Y_{H,t}$ , in the equilibrium, given by

$$Y_{H,t} = Y_{w,t}. (3.47)$$

Consumers' expenditure minimisation implies that each retail firm faces the downward sloping demand for variety j, given by

$$Y_{H,t}(j) = \left(\frac{\overline{P}_{H,t}(j)}{P_{H,t}}\right)^{-\epsilon} Y_{H,t}, \tag{3.48}$$

so that given the demand curve, the retailers may set the price,  $\overline{P}_{H,t}(j)$  on their own variety j, to maximise their profit.

<sup>&</sup>lt;sup>19</sup>Note that we exclude the possibility of local pricing, so that the foreign retail price is determined by the law of one price (LOOP), given by  $\overline{P}_{H,t}^*(j) = \frac{\overline{P}_{H,t}(j)}{S_t}$  in foreign currency. Hence, it is sufficient to discuss the determination of the domestic retail price,  $\overline{P}_{H,t}(j)$ , in order to establish its foreign price,  $\overline{P}_{H,t}^*(j)$ .

However, each retailer is also assumed to be confronted by Calvo (1983) type price stickiness, i.e., it is able to reset its price at  $\overline{P}_{H,t}(j)$ , with a probability of  $(1-\theta)$  independently of the time elapsed since the last adjustment, and with a probability of  $\theta$  it is not able to adjust the price so as to keep the previous price,  $\overline{P}_{H,t-1}(j)$  unchanged. Then, the aggregate price index in (3.46) can be expressed as two sets of price indices, the previous price level,  $P_{H,t-1}$ , and the newly set price,  $\overline{P}_{H,t}$ :

$$P_{H,t} = \left[\theta \left(P_{H,t-1}\right)^{1-\epsilon} + \left(1-\theta\right) \left(\overline{P}_{H,t}\right)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}},\tag{3.49}$$

so that

$$\left(\frac{P_{H,t}}{P_{H,t-1}}\right) = \left[\theta + (1-\theta)\left(\frac{\overline{P}_{H,t}}{P_{H,t-1}}\right)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}},$$
(3.50)

which provides the dynamics for the domestic price index in the economy. In this setting, a representative retailer's (real) profit maximisation problem at t, when she can adjust the price, is written as:

$$\max_{\left\{\overline{P}_{H,t}(j)\right\}} \quad \sum_{k=0}^{\infty} \theta^{k} E_{t} \left\{ \Lambda_{t,t+k} \left[ \left( \frac{\overline{P}_{H,t}(j)}{P_{H,t+k}} - P_{w,t+k} \right) Y_{H,t+k}(j) \right] \right\}, \tag{3.51}$$

subject to the sequence of demands for her variety

$$Y_{H,t+k}(j) = \left(\frac{\overline{P}_{H,t}(j)}{P_{H,t+k}}\right)^{-\epsilon} Y_{H,t+k}, \tag{3.52}$$

for  $k = 0, 1, 2, \dots$ , where  $\theta^k$  is the probability of keeping  $\overline{P}_{H,t}(j)$  unchanged from

t to t + k,  $\Lambda_{t,t+k} = \beta^k \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma}$  is the subjective rate of the intertemporal substitution between t and t + k, and  $P_{w,t+k}$  is the retail firm's (real) marginal cost of purchasing the wholesale goods at period t + k.<sup>20</sup> The first order condition with respect to  $\overline{P}_{H,t}(j)$  implies the following optimal price setting rule for the retail firm:

$$0 = \sum_{k=0}^{\infty} \theta^{k} E_{t} \left\{ \Lambda_{t,t+k} Y_{H,t+k} \left[ (1 - \epsilon) \frac{\left(\overline{P}_{H,t}(j)\right)^{-\epsilon}}{\left(P_{H,t+k}\right)^{1-\epsilon}} - (-\epsilon) P_{w,t+k} \frac{\left(\overline{P}_{H,t}(j)\right)^{-\epsilon-1}}{\left(P_{H,t+k}\right)^{-\epsilon}} \right] \right\}$$

$$= \sum_{k=0}^{\infty} \theta^{k} E_{t} \left\{ \Lambda_{t,t+k} Y_{H,t+k} \left[ \frac{\overline{P}_{H,t}(j)}{P_{H,t-1}} - \mu P_{w,t+k} \left( \frac{P_{H,t+k}}{P_{H,t-1}} \right) \right] \left( \frac{1}{P_{H,t+k}} \right)^{1-\epsilon} \right\} (3.53)$$

where  $\mu = \frac{\epsilon}{\epsilon - 1}$  is the retail firm's desired (gross) mark-up, which is attached due to imperfections in the retail market. Combining the aggregate domestic price dynamics in (3.50) with the optimal price setting rule in (3.53) yields the following short-run dynamics for the domestic price index (within the neighbourhood of the steady state):

$$\pi_{H,t} = (\mu P_{w,t})^{\lambda} E_t \{ \pi_{H,t+1} \}^{\beta},$$
(3.54)

with  $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$  and  $\pi_{H,t} \equiv \frac{P_{H,t}}{P_{H,t-1}}$ , which is called as the New Keynesian Phillips curve (NKPC) for the domestic retail goods. Equation (3.54) shows that the inflation of the domestic retail good price,  $\pi_{H,t}$ , rises with its inflation expectation,  $E_t \{\pi_{H,t+1}\}$ , and the (real) wholesale good price,  $P_{w,t}$ , i.e., the (real) marginal cost of the wholesale good production.

<sup>&</sup>lt;sup>20</sup>Note that the marginal cost of producing the wholesale good is equal to that of purchasing it, since the wholesale goods market is assumed to be perfectly competitive.

Finally, the home goods supplied by the domestic retailers are consumed by domestic and foreign households, domestic capital producers, and the government, so that the economy-wide resource constraint for the domestic final goods is given by:

$$Y_{H,t} = C_{H,t} + C_{H,t}^* + I_{H,t} + G_t, (3.55)$$

where  $G_t$  is government spending.

## 3.2.6 Government and Balance of Payment

Now, we turn to government policies. The central bank is assumed to adjust the (gross) short-term nominal interest rate,  $R_t^n$ , in response to inflation of consumer price index (CPI),<sup>21</sup> and deviations of output and nominal exchange rate<sup>22</sup> from their respective steady state values, to stabilise the business cycle fluctuations. Thus, it follows the Taylor-type feedback rule as in Taylor (1993) with interest rate smoothing, given by:

<sup>&</sup>lt;sup>21</sup>As Gali (2005) argues, in order for Taylor rule to be optimal in the small open economy, it should be based on the domestic price index (DPI),  $P_{H,t}$ , rather than the consumer price index (CPI),  $P_t$ . However, in practice, the majority of monetary authorities do not seem to make a policy decision strictly by DPI, so that we assume that it seeks to stabilise the CPI.

<sup>&</sup>lt;sup>22</sup>The assumption that the central bank seeks to stabilise the nominal exchange rate is controversial, since a large number of countries announce that they adopt the free floating exchange rate system, i.e.,  $\alpha_s = 0$ . However, as Reinhart and Rogoff (2004) and Levy-Yeyati and Struzenegger (2005) point out, these (de jure) floating systems tend to be heavily managed by the central banks (de facto), especially in emerging market countries where much of the country's debt is denominated in foreign currency. According to the 'fear of floating' argument a la Calvo and Reinhart (2002) suggest, such an environment would induce the emerging market countries to seek to stabilise the nominal exchange rate to prevent the value of foreign currency denominated debt from being deteriorated. The 'fear of floating' is supported by many authors, including Hausmann, Panizza, and Stein (2001), Calvo (2004), Lubik and Schorfheide (2007), and Bleaney and Ozkan (2008). In this sense, we include the nominal exchange rate stabilisation term,  $\alpha_s$ , in the central bank's feedback rule, and investigate the impact of the different degree of grip of the nominal exchange rate.

$$\left(\frac{R_t^n}{R^n}\right) = \left(\frac{R_{t-1}^n}{R^n}\right)^{\alpha_r} \left(\frac{P_t}{P_{t-1}}\right)^{(1-\alpha_r)\alpha_\pi} \left(\frac{Y_{H,t}}{Y_H}\right)^{(1-\alpha_r)\alpha_y} \left(\frac{S_t}{S}\right)^{(1-\alpha_r)\alpha_s} \exp\left\{\varepsilon_{r,t}\right\},$$
(3.56)

where  $R^n$ ,  $Y_H$  and S denote the steady state values for nominal interest rate,  $R_t^n$ , domestic output,  $Y_{H,t}$ , and nominal exchange rate,  $S_t$ , respectively, and  $\varepsilon_{r,t}$  is a monetary policy shock, which is a Gaussian white noise process with mean zero and standard deviation  $\sigma_r$ . The values for Taylor rule coefficients on CPI inflation, output gap, and nominal exchange rate,  $\alpha_\pi > 1$ ,  $\alpha_y > 0$ , and  $\alpha_s \ge 0$ , are assumed to be chosen by the central bank, which characterise the degree of the central bank's commitment to each of policy target. In particular, the values of  $\alpha_s$  represent the economy's choice of exchange rate regime. That is,  $\alpha_s = 0$  implies the free floating system as the central bank does not intervene in the foreign exchange market regardless of the motion in nominal exchange rate, while  $\alpha_s = \infty$  suggests the fixed exchange rate regime, since the monetary authority does not allow the fluctuation in nominal exchange rate. The nominal interest rate,  $R_t^n$ , set by the central bank is linked to the real riskless rate,  $R_t$ , by the following Fisher equation

$$R_t^n \equiv R_t E_t \left\{ \pi_{t+1} \right\}. \tag{3.57}$$

In addition, the fiscal authority implements the government spending,  $G_t$ , which is financed by revenues from lump-sum taxes,  $T_t$ , and (net) government bond issueing,  $D_t - R_{t-1}D_{t-1}$ . Accordingly, government budget constraint is given by

$$G_t = T_t + (D_t - R_{t-1}D_{t-1}). (3.58)$$

Furthermore, we assume that government spending,  $G_t$ , is exogenously given by the following process:

$$G_t = (G_{t-1})^{\rho_g} \exp\left\{\varepsilon_{g,t}\right\} \tag{3.59}$$

where  $|\rho_g| < 1$ , and  $\varepsilon_{g,t}$  is a Gaussian white noise with mean zero and standard deviation  $\sigma_g$ .

Lastly, the resources of the economy are determined by the households' budget constrain in (3.3). The substitution of the profits from entrepreneurs, capital producers and retailers into the households' dividend income,  $\Pi_t^o$ , yields the expression for the balance of payment for the economy,<sup>23</sup> given by:

$$0 = \left(P_{H,t}C_{H,t}^* - P_{F,t}C_{F,t} - P_{F,t}I_{F,t}\right) + P_tS_t\left(R_{t-1}^*B_{t-1}^* - B_t^*\right) + P_t\left(Q_tK_t - R_{t-1}^*\Psi_{t-1}\left(\frac{S_t}{S_{t-1}}\right)Q_{t-1}K_{t-1}\right).$$
(3.60)

In equation (3.60), the terms in the first bracket are the items of the current account, and the terms in the second and third brackets are those of the financial account which arise from households' financial transaction as depositors to foreign

The system of the profit from entrepreneurs is  $\Pi_t^E = P_{w,t}Y_{H,t} - W_tL_t - R_t^kP_tQ_{t-1}K_{t-1} + (1-\delta)P_tQ_tK_{t-1}$ ; that from capital producers is  $\Pi_t^C = P_tQ_tK_t - P_{I,t}I_t - (1-\delta)P_tQ_tK_{t-1}$ ; that from retailers is  $\Pi_t^R = P_{H,t}Y_{H,t} - P_{w,t}Y_{H,t}$ ; and the government budget constraint is  $G_t = T_t + (D_t - R_{t-1}D_{t-1})$ . In addition, we use the facts that  $P_tQ_t = P_{H,t}C_{H,t} + P_{F,t}C_{F,t}$ ;  $P_{I,t}I_t = P_{H,t}I_{H,t} + P_{F,t}I_{F,t}$ ;  $Y_{H,t} = C_{H,t} + C_{H,t}^* + I_{H,t} + G_t$ ; and  $R_{t+1}^k = R_t^*\Psi_t\left(\frac{S_{t+1}}{S_t}\right)$ .

financial intermediaries and that of the domestic entrepreneurs as borrowers from foreign financial intermediaries, respectively. It is noteworthy that a domestic currency depreciation improves the current account via the increased export and the decreased import. It also improves the financial account relating to households' transactions via the improved rate of returns from foreign investment for the domestic depositors. In contrast, it aggravates the financial account relating to entrepreneurs' transactions via the increased cost of foreign borrowings. By construction, the improvement in current account corresponds to the aggravation of financial account through the households' budget constraint, and vice versa.

# 3.3 Model Solution and Calibration

In this section, we discuss the solution method, strategy for experiments, and parameter calibration of the model.

# 3.3.1 Solution and Experiment Strategy

In the general equilibrium for our small open economy DSGE model, given 7 temporary shocks,  $\{\varepsilon_{r,t}, \varepsilon_{g,t}, \varepsilon_{a,t}, \varepsilon_{v,t}, \varepsilon_{y^*,t}, \varepsilon_{\pi^*,t}, \varepsilon_{r^*,t}\}$ , the infinite sequence of 34 endogenous variables,  $\{C_t, C_{H,t}, C_{F,t}, C_{H,t}^*, Y_t^*, Y_{H,t}, Y_{w,t}, A_t, L_t, K_t, N_t, B_t^*, I_t, I_{H,t}, I_{F,t}, P_t, P_{H,t}, P_{F,t}, P_{I,t}, P_t^*, W_t, Q_t, \pi_t, \pi_{H,t}, \pi_t^*, R_t^k, \Psi_t, R_t, R_t^*, R_t^n, S_t, S_t, V_t, G_t\}_{t=0}^{\infty}$ , is determined to satisfy 34 equilibrium conditions, which are listed in Appendices B1.1 and B1.2. Following Uhlig's (1995) procedure, we transform the non-linear rational expectations system to the linear one, and solve the linear rational expectations system by the numerical method after parameter calibration.

For calibration, as discussed shortly, we assign the conventional values to the model parameters,  $\{\beta, \sigma, \varphi, \gamma, \eta, \alpha, \psi, \nu, \phi, \theta, \alpha_r, \alpha_\pi, \alpha_y, \alpha_s, \rho_a, \rho_g, \rho_v, \rho_{y^*}, \rho_{\pi^*}, \rho_{r^*}\}$  and the steady state values for some endogenous variables,  $\{R, R^k, \Psi, \frac{K}{N}, \frac{C_H}{Y_H}, \frac{C_H^*}{Y_H}, \frac{G_H^*}{Y_H}, \frac{G_H^*}{Y_H}\}$ , following the literature on the small open economy DSGE model, including Gertler, Gilchrist, and Natalucci (2007), Curdia (2007 and 2009), Ozkan and Unsal (2010), Elekdag, Alp, and Lall (2012), and Yie and Yoo (2011). Table 3.3 presents the comparison of calibration in our model and those in the literature.

Having solved the small open economy DSGE model, we conduct various experiments using the impulse responses of the model economy to the diverse shocks under the alternative environments. First of all, we examine how an economy responds when foreign lenders' evaluation of the entrepreneurs' net worth turns negative abruptly, to investigate the transmission of a sudden stop crisis. To study the impact of a sudden stop crisis more clearly, the impulse responses to a negative net worth evaluation shock in the model with benchmark financial frictions are compared with those to an abrupt rise in the foreign interest rate, which are denoted as FA(NW) and FA(FI) models, respectively. In addition, we study the effect of a global financial crisis by analysing the model economy's reactions when it is hit by an unexpected contraction in foreign output as well as a sudden stop in foreign funds, which is labled the FA(NW+FO) model.

Next, we explore how the fluctuations in an emerging market economy in a sudden stop crisis are related to the economy's pre-crisis conditions. First of all, to investigate the role of financial frictions in a sudden stop crisis, we compare the responses in the FA(NW) model with those in an economy with a low degree of financial frictions, which are denoted as the LFA(NW) model. In the LFA(NW) model, foreigners keep trust in an emerging economy even if the entrepreneurs'

balance sheet is temporarily distorted, so that the sensitivity parameter of external finance premium to entrepreneurs' leverage ratio,  $\psi$ , is low. Second, the relative performance according to the alternative exchange rate regime is evaluated when an economy is hit by a sudden stop. For this, we compare the impulse responses in the FA(NW) model with those in the FA(NW)+FR model, which represent economies adopting a free floating exchange rate regime and a fixed exchange rate one, respectively. For the latter, we assign a very large value to the Taylor rule coefficient on the nominal exchange rate,  $\alpha_s$ , suggesting that the central bank adjusts the nominal interest rate to fix the nominal exchange rate at a certain level. Lastly, we analyse the effect of an economy's heavy reliance on the foreign resources for capital production, by exploring the impulse responses in the FA(NW)+RR model, where the steady state share of domestic inputs in the investment good composite,  $\eta_i$ , is much smaller than that of domestic goods in the consumption bundle,  $\eta$ . In contrast, in the FA(NW) model, we impose the same values on  $\eta_i$ and  $\eta$ , so that the capital price would not be affected by the nominal exchange rate, by construction.

### 3.3.2 Parameter Calibration

Now, we discuss parameter calibration for each of the models more specifically, which is shown in Tables 3.1 and 3.2. We start with the parameters for the benchmark FA model. For domestic and foreign households, we set the quarterly discount factors in home and foreign countries,  $\beta$  and  $\beta^*$ , respectively, to be all 0.99, which pin down the steady state quarterly riskless rates in home and foreign countries, R and  $R^*$ , at  $R = \frac{1}{\beta} = 1.0101$  and  $R^* = \frac{1}{\beta^*} = 1.0101$  (i.e., annually

4.1%). We fix the inverse of intertemporal elasticity of consumption,  $\sigma$ , and the inverse of labour supply elasticity,  $\varphi$ , at 1.5 and 3.0, respectively, in keeping with much of the literature. The elasticity of substitution between home and foreign goods,  $\gamma$ , and the share of home goods in the domestic households' consumption bundle,  $\eta$ , are set at 1.5 and 0.6, respectively. For entrepreneurs, we take the share of capital out of the output production,  $\alpha$ , and the quarterly capital depreciation rate,  $\delta$ , to be 0.3 and 0.025, respectively. We take a quarterly risk spread,  $R^k - R$ , to be 300 basis point in the steady state, so that the steady state risk premium is pinned down at  $\frac{R^k}{R} = \frac{1.0401}{1.0101} = 1.0198$ . In addition, we set the capital-to-net worth ratio in the steady state,  $\frac{K}{N}$ , at 2, so that the sensitivity of external finance premium to the capital-to-net worth ratio,  $\psi$ , is calculated as 0.0422, from the steady state relation,  $\frac{R^k}{R} = \left(\frac{K}{N}\right)^{\psi}$ . Entrepreneurs' survival rate,  $\phi$ , is set to be 0.975, which is conventional in the literature. For capital producers, we set capital adjustment cost coefficient,  $\kappa$ , at 1.2, so that the inverse of elasticity of investment to the capital price,  $\kappa\delta$ , is calculated as 0.03. In addition, the share of domestic inputs out of the investment good composite,  $\eta_i$ , is set to be 0.6, which is the same as the share of home goods in the consumption bundle,  $\eta$ . For retail market behaviours, the elasticity of substitution among varieties,  $\epsilon$ , is set to be 6, so that the retail firms' desired mark-up is pinned down at  $\mu = \frac{\epsilon}{\epsilon - 1} = 1.2$ . The probability of retail firms keeping prices fixed during a given period,  $\theta$ , is equal to 0.75, so that the coefficient attached to the retail firms' marginal cost,  $P_{w,t}$ , is set to be  $\lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta} = 0.0858$ . In addition, the steady state value of investment-to-output ratio,  $\frac{I_H}{Y_H}$ , is calculated as 0.0576.<sup>24</sup> The steady state values of export-to-output ratio and government spending-to-output ratio,  $\frac{C_H^*}{Y_H}$  and  $\frac{G}{Y_H}$ ,

 $<sup>^{24}\</sup>text{It follows from the steady state relation, } \frac{I_H}{Y_H} = \left(\frac{I_H}{I}\right)\left(\frac{I}{K}\right)\left(\frac{K}{Y_w}\right) = \eta_i \delta\left[\left(\frac{\alpha}{R^k - (1 - \delta)}\right)\left(\frac{\epsilon - 1}{\epsilon}\right)\right].$ 

Table 3.1: Parameter Calibration (Benchmark FA Model)

Parameter	Value	Description					
$\beta, \beta^*$	0.99	home and foreign discount factors					
$\sigma$	1.5	intertemporal elasticity of consumptions					
arphi	3.0	inverse elasticity of labour supply					
$\gamma$	1.5	substitutability between home and foreign goods					
$\eta$	0.6	share of home good in consumption					
$\alpha$	0.3	capital share in production function					
$\delta$	0.025	capital depreciation rate					
$\phi$	0.975	entrepreneurs' survival rate					
$\kappa$	1.2	capital adjustment cost coefficient					
${\gamma}_i$	1.5	substitutability between home and foreign inputs					
heta	0.75	probability of not adjusting domestic retail prices					
$\epsilon$	6	elasticity of substitution among retail goods					
$\alpha_r$	0.7	Taylor rule coefficient on interest rate smoothing					
$lpha_\pi$	1.7	Taylor rule coefficient on inflation					
$lpha_y$	0.2	Taylor rule coefficient on output gap					
$ ho_v$	0.8	persistence in net worth evaluation shock					
K/N	2	capital-to-net worth ratio in the steady state					
$C_H^*/Y_H$	0.2	export-to-output ratio in the steady state					
$G/Y_H$	0.2	government spending-to-output ratio in the					
		steady state					

are all assumed to be 0.2, so that the steady state value of consumption-to-output ratio,  $\frac{C_H}{Y_H}$ , is calculated as 0.5424. For the monetary authority, we assume that the central bank sets Taylor rule coefficients,  $\alpha_r$ ,  $\alpha_\pi$ , and  $\alpha_y$ , to be 0.7, 1.5, and 0.2, respectively. We assume the weak stabilisation of the nominal exchange rate in the FA model, such that  $\alpha_s$  is set to be 0.2. The persistence parameters for shocks, such as  $\rho_a$ ,  $\rho_v$ ,  $\rho_g$ ,  $\rho_{y^*}$ ,  $\rho_{\pi^*}$ , and  $\rho_{r^*}$ , are all set to be 0.8, as in the business cycle literature.

Next, we discuss the parameter calibrations for the alternative models, which are presented in Table 3.2. For the LFA model where the degree of financial frictions is low, we set the steady state value of the risk spread,  $R^k - R$ , at 100 basis

Table 3.2: Parameter Calibration (By Model)

	Description	FA	LFA	FA+FR	FA+RR
$R^k-R$	risk premium in the steady state	0.03	0.01	0.03	0.03
$\psi$	sensitivity of risk premium to lever-	0.0422	0.0142	0.0422	0.0422
	age ratio				
$\alpha_s$	Taylor rule coefficient on exchange	0.2	0.2	30	0.2
	rate				
$\eta_i$	share of domestic input in invest-	0.6	0.6	0.6	0.3
	ment good				
$I_H/Y_H$	investment-to-output ratio in the	0.0576	0.0576	0.0576	0.0288
	steady state				
$C_H/Y_H$	consumption-to-output ratio in the	0.5424	0.5424	0.5424	0.5712
	steady state				

<sup>\*</sup> LFA: the model with low degree of financial frictions

point, so that the sensitivity of the external finance premium to the entrepreneurs' leverage ratio,  $\psi$ , is calculated as 0.0142. For the FA+FR model where the economy adopts the fixed exchange rate regime, we assume that the Taylor rule coefficient on the nominal exchange rate,  $\alpha_s$ , is set at 30, where the nominal exchange rate,  $S_t$ , is almost fixed as in the fixed exchange rate regime, as is shown later. For the FA+RR model where capital producers rely heavily on the foreign inputs,  $I_{F,t}$ , to construct the new capital, we assume that the steady state share of domestic inputs out of investment good composite,  $\eta_i$ , is set at 0.3, suggesting that 70 percent of investment goods are imported from abroad. However, the steady state share of domestic goods in the consumption bundle,  $\eta$ , for the FA+FR model is set at the same level of 0.6 as in the FA model. Accordingly, in the FA+RR model, the steady state values of investment-to-output ratio and consumption-to-output ratio,  $\frac{I_H}{Y_H}$  and  $\frac{C_H}{Y_H}$ , are calculated as 0.0288 and 0.5712, respectively. Note that the parameters for the alternative models which are not otherwise mentioned above

<sup>\*</sup> FA+FR: the model with exchange rate stabilisation

<sup>\*</sup> FA+RR: the model with high degree of foreign input reliance

Table 3.3: Parameter Calibration (Comparison by Author)

Parameter	GGN	Curdia	OU	EAL	YY	FA
$\beta$	0.99	0.9840	0.99	0.9963	0.988	0.99
$\sigma$	5	1	1	1	1.4906	1.5
arphi	2	2	2	1	3	3
$\gamma$	1	1	1	1	1.5	1.5
$\eta$	0.5	0.75	0.75	0.575	0.7	0.6
$\alpha$	0.5	0.45	0.35	0.625	0.3	0.3
$\delta$	0.025	0.025	0.025	0.025	0.025	0.025
$\psi$				0.001	0.0038	0.0422
$\phi$	0.9728		0.9873	0.9728	0.98	0.975
$\theta$	0.75	0.75		0.419	0.7	0.75
$\epsilon$	6	6		7.6667	6	6
$lpha_r$				0.892	0.8058	0.7
$lpha_\pi$	2	2	1.5	1.760	1.3633	1.7
$\alpha_y$		0.75		0.019	0.6612	0.2
$\alpha_s$		$\{0.5, 2\}$		0.099	0.1475	0.2
$ ho_v$			0.5	0.576		0.8
$ ho_{y^*}$				0.956	0.9083	0.8
$R^k - R$	0.015	0.0063	0.0185	0.0301	0.0073	0.030
K/N	2.1	2	1.11		10	2

<sup>\*</sup> GGN (Gertler, Gilchrist, and Natalucci, 2007), Curdia (Curdia, 2007), OU (Ozkan and Unsal, 2010), EAL (Elekdag, Alp and Lall, 2012), YY (Yie and Yoo, 2011, mimeo), FA (the baseline model)

have the same values as the corresponding parameters in the benchmark FA model.

# 3.4 Model Dynamics

In this section, we explore the transmission process of foreign shocks to an emerging market economy under alternative economic environments. We consider the shocks arising from foreign lenders' net worth evaluation,  $\varepsilon_{v,t}$ , the foreign interest rate,  $\varepsilon_{r^*,t}$ , and foreign output,  $\varepsilon_{y^*,t}$ . In addition, we study the role of financial frictions and resource reliance to foreign country in the capital production, and the effects of exchange rate stabilisation by the central bank.

## 3.4.1 Transmission of Sudden Stop Crisis

We explore a transmission of a sudden stop crisis originating from foreign lenders' pessimism about the entrepreneurs in an emerging market economy. We suppose that foreigners devalue the entrepreneurs' net worth,  $V_t$ , so that they curtail lending to an emerging market country or raise the external finance premium to reflect their pessimism. The situation is represented by the FA(NW) model and the impulse responses from the model are shown in the solid lines in Figure 3.1. To study the transmission of a sudden stop crisis, we first trace the solid lines in Figure 3.1, and then compare them to the dotted lines in Figure 3.1. The dotted lines represent the impulse responses from the FA(FI) model, which supposes that

<sup>&</sup>lt;sup>25</sup>The motivation of foreign lenders' pessimism includes a doubt on the entrepreneurs' productivity, the availability of foreign currency, the outlook of the financial market situation, and so on. In addition, note that the foreigners' pessimism is not necessarily proven reasonable, *ex post*.

the foreign interest rate rises unexpectedly.<sup>26</sup> However, one needs to note that our investigation on fluctuations from alternative models aims at understanding how an economy responds to the respective shock. Thus, the comparison here does not have quantitative implication.

First of all, we examine the solid lines in Figure 3.1. A downturn in foreign lenders' outlook on entrepreneurs,  $V_t$ , in (3.36), decreases their evaluation on the entrepreneurs' net worth,  $N_t$ , in (3.35). This, then, leads to a rise in the entrepreneur's leverage ratio perceived by the foreigners,  $\frac{Q_t K_t}{N_t}$ , so that they impose the higher external finance premium,  $\Psi_t$ , on the entrepreneurs by (3.33), which, in turn, raises the entrepreneurs' cost of foreign borrowing,  $R_{t+1}^k$ , (i.e., required capital returns) in (3.34). Then, confronted with the raised cost of capital investment,  $R_{t+1}^k$ , the entrepreneur reduces the capital demand,  $K_t$ , in (3.23), which results in a fall in production,  $Y_{H,t}$ , and demand for domestic investment goods,  $I_{H,t}$ , by (3.19), (3.44) and (3.39).

Now, we turn to the demand side of the economy. The decline in factor demands,  $K_t$  and  $L_t$ , resulting from production contraction,  $Y_{H,t}$ , depresses the factor prices,  $Q_t$  and  $W_t$ , by (3.23) and (3.22), which, in turn, decreases the marginal cost of wholesale good production,  $P_{w,t}$ . Then, the New Keynesian Phillips curve in (3.54) implies a fall in the inflation of the home good price,  $\pi_{H,t}$ , in a staggered manner. On the other hand, the shrinkage in supply of foreign funds and the rise in external finance premium,  $\Psi_t$ , depreciate the domestic currency,  $S_t$ , by (3.34), in the foreign currency market. Then, under the stickiness in the home good price,  $P_{H,t}$ , the currency depreciation raises the real foreign good price in home country,

<sup>&</sup>lt;sup>26</sup>We obtain the results from the benchmark model similar to those in the existing literature such as Gertler *et al.* (2007) and Curdia (2007 and 2009).

 $\frac{P_{F,t}}{P_t} = \mathcal{S}_t$ , and reduces the real home good price in foreign countries,  $\frac{P_{H,t}^*}{P_t^*} = \frac{P_{H,t}}{S_t P_t^*}$ , by (3.12) and (3.11).<sup>27</sup> This, in turn, leads to a fall in the import demand in home country,  $C_{F,t}$ , and an increase in the export demand in foreign country,  $C_{H,t}^*$ , by (3.18) and (3.10), respectively. In addition, note that in spite of the fall in domestic good price,  $P_{H,t}$ , the currency depreciation would raise the domestic CPI inflation,  $\pi_t$ , due to the stickiness in the home good price,  $P_{H,t}$ , by (3.8). Then, the central bank would raise the domestic (nominal) riskless rate,  $R_t^n$ , to stabilise the CPI inflation, by the feedback rule in (3.56). The resulting increase in the real riskless rate,  $R_t$ , by the Fisher equation in (3.57), decreases consumption bundle,  $C_t$ , together with the reduced labour wage,  $\frac{W_t}{P_t}$ , by (3.4) and (3.5), which depresses the consumption demand for the home goods,  $C_{H,t}$ , in spite of the fall in the (real) home good price,  $\frac{P_{H,t}}{P_t}$ , by (3.9). All in all, despite the expansion in the export demand,  $C_{H,t}^*$ , the contractions in consumption,  $C_{H,t}$ , and investment,  $I_{H,t}$ , decrease the aggregate demand, which corresponds to the production contraction,  $Y_{H,t}$ , by the economy-wide resource constraint in (3.55).

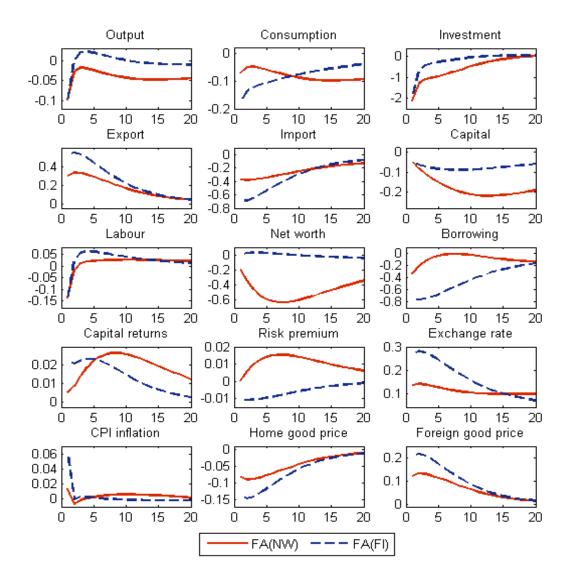
Next, we compare the impulse responses in the FA(NW) model with those in the FA(FI) models.<sup>28</sup> The dotted lines in Figure 3.1 show that an unexpected rise in the foreign interest rate produces the qualitatively similar real effects to a negative shock to foreigners' evaluation on the entrepreneurs' net worth. That is,

<sup>&</sup>lt;sup>27</sup>Note that CPI,  $P_t$ , is affected immediately by the foreign good price,  $P_{F,t}$ , and the nominal exchange rate under the law of one price (LOOP), while it reflects the change in the home good price,  $P_{H,t}$ , in a staggered manner, as the New Keynesian Phillips curve suggests. Accordingly, a currency depreciation,  $S_t$ , would lead to the rises in foreign good price,  $P_{F,t}$ , CPI,  $P_t$ , and real exchange rate,  $S_t = \frac{P_{F,t}}{P_t}$ , but the falls in real home good prices in home and foreign countries,  $\frac{P_{H,t}}{P_t}$  and  $\frac{P_{H,t}^*}{P_t^*} = \frac{P_{H,t}}{S_t P_t^*}$ .

28 Cespedes, Chang, and Velasco (2004) and Gertler *et al.* (2007) take the shocks from foreign

interest rate and country risk premium, respectively, while Curdia (2007) and Ozkan and Unsal (2010) consider the misperception shock on the entrepreneurs' productivity.

Figure 3.1: Responses to Shocks to Net Worth Evaluation and Foreign Interest Rate



<sup>\*</sup> The solid and dotted lines represent the impulse responses to a negative shock in foreigners' evaluation on the entrepreneurs' net worth (NW), and a positive shock in the foreign interest rate (FI), respectively.

an unexpected rise in the foreign interest rate may lead to the production contraction via the rise in the cost of foreign borrowing, and the overall contraction in aggregate demand and the demand shift from domestic one to foreign one. However, the difference between the two models lies in the channel from the financial shocks to the entrepreneurs' cost of foreign borrowings and real exchange rate. That is, while a shock to foreigners' evaluation on the entrepreneurs' net worth is transmitted to the real sector via the entrepreneurs' leverage ratio (perceived by foreigners) in the FA(NW) model, a shock to foreign interest rate propagates to the real sector by directly influencing the condition for foreign borrowing and the nominal exchange rate in the FA(FI) model. Accordingly, the risk premium and entrepreneurs' net worth are affected indirectly in the FA(FI) model, while they play a key role for shock propagation process in the FA(NW) model. Overall, in spite of the similar outcomes in the two alternative models, the FA(NW) model may be assessed as providing a more systematic explanation on a sudden stop crisis, as it relates the real fluctuations in emerging market economies to a more primitive shock, such as a change in foreigners' perception and the agency problem between foreign lenders and domestic entrepreneurs as a transmission channel.

## 3.4.2 Role of Financial Frictions under Sudden Stop Crisis

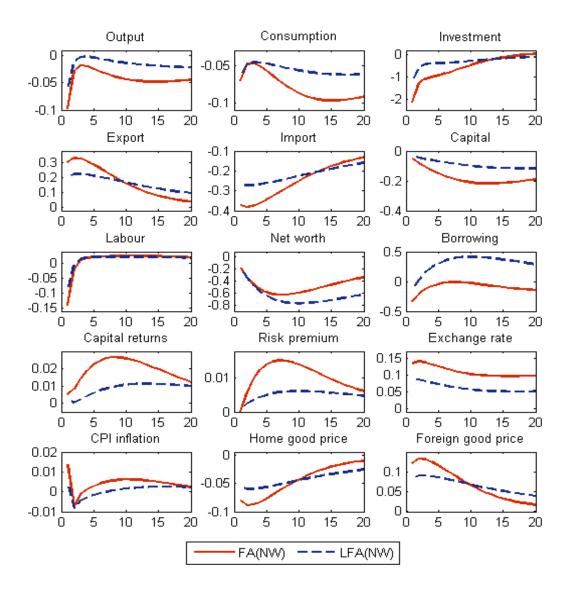
Next, we investigate the role of financial frictions in the transmission of a sudden stop, by comparing the impulse responses to an unfavourable change in foreign lenders' perception on entrepreneurs' net worth in economies with the different degree of financial frictions (FA(NW) abd LFA(NW) models). As discussed above, the LFA(NW) model represents an economy with a low degree of financial frictions,

so that it assumes a lower risk premium in the steady state,  $\frac{R^k}{R}$ , and hence, a lower sensitivity of risk premium to the entrepreneurs' leverage ratio,  $\psi$ , in (3.33) than the FA(NW) model. Such a representation suggests that foreign lenders in the LFA(NW) model would raise the risk premium,  $\frac{R^k_{t+1}}{R_t}$ , less sensitively to a rise in entrepreneurs' leverage ratio,  $\frac{Q_t K_t}{N_t}$ . This means that foreigners in the LFA(NW) model have still maintained trust in the entrepreneurs in the emerging market economy, even if entrepreneurs' balance sheet is evaluated being distorted temporarily. As a result, the rise in the cost of foreign borrowing that entrepreneurs face is dampened in the LFA(NW) model, so that the contraction in production and capital demand could be reduced. In addition, the dampened size of 'sudden stops' limits the magnitude of the real currency depreciation, which relieves contractions in aggregate demand.

Figure 3.2 shows the role of financial frictions in a sudden stop crisis. The dotted lines in Figure 3.2 represent the impulse responses to the same size of a negative shock to foreigners' evaluation on the entrepreneurs' net worth in the LFA(NW) model. In Figure 3.2, the dotted lines display dampened fluctuations, as compared to the solid lines which represent the impulse responses in the FA(NW) model. It suggests that an economy that fails to gain foreigners' trust in normal times, i.e., an economy with a high degree of financial frictions, could suffer a sudden stop crisis more severely than an economy with foreigners' confidence.<sup>29</sup>

<sup>&</sup>lt;sup>29</sup>We may assess the degree of confidence that an emerging market economy gains in the international financial market, by the sensitivity of risk premium to entrepreneurs' leverage ratio,  $\psi = \frac{\ln(R^k/R)}{\ln(K/N)}$ . By this criterion, foreigners may decide where to withdraw their fund from, when they are confronted with financial stress.

Figure 3.2: Role of Financial Frictions under a Negative Net Worth Evaluation Shock



<sup>\*</sup> The solid and dotted lines represent the impulse responses to a negative shock in foreigners' evaluation on entrepreneurs' net worth in the economies with standard (FA(NW)) and lower financial frictions (LFA(NW)), respectively.

## 3.4.3 Sudden Stops and Global Recession

In subsection 3.4.1, we confirm that sudden stops of capital inflow could lead to production contraction in an emerging market economy. However, as shown in the solid lines in Figure 3.1, which represent the impulse responses in the benchmark FA(NW) model, the production contraction,  $Y_{H,t}$ , under sudden stops is short-lived, as compared to the shrinkage in capital,  $K_t$ , foreign borrowing,  $B_t^*$ , and net worth,  $N_t$ . The fast recovery of the output relative to the credit lines, which is labeled 'Phoenix miracle' by Calvo, Izquierdo, and Talvi (2006), implies that entrepreneurs, who face the increase of cost of foreign borrowing for capital acquisition, try to reorganise production and finance, in such a way to hire more labour instead of capital and to use more internal financing rather than the external one. Furthermore, as shown in the solid lines in Figure 3.1, even though a sudden stop in foreign funds depresses production and domestic demand, it tends to encourage export demands for home goods in foreign countries via a real currency depreciation, which provides a source of the fast recovery from a sudden stop crisis.<sup>30</sup> However, when aggregate demand contraction in the world economy overlaps with sudden stops in foreign fund supply, this recovery process through an expansion in exports could be interrupted, so that the recession in the emerging market economy could be magnified and prolonged.

We explore the transmission of this kind of global financial crisis through Figure 3.3. The dotted lines in Figure 3.3 represent the impulse responses when negative shocks are imposed on both foreign lenders' evaluation on entrepreneurs'

<sup>&</sup>lt;sup>30</sup>Chari, Kehoe, and McGrattan (2005) even argue that a sudden stop could be expansionary in business cycle fluctuations. Curdia (2007) shows that a sudden stop could be expansionary when the sensitivity of export demand,  $\gamma^*$ , is sufficiently large.

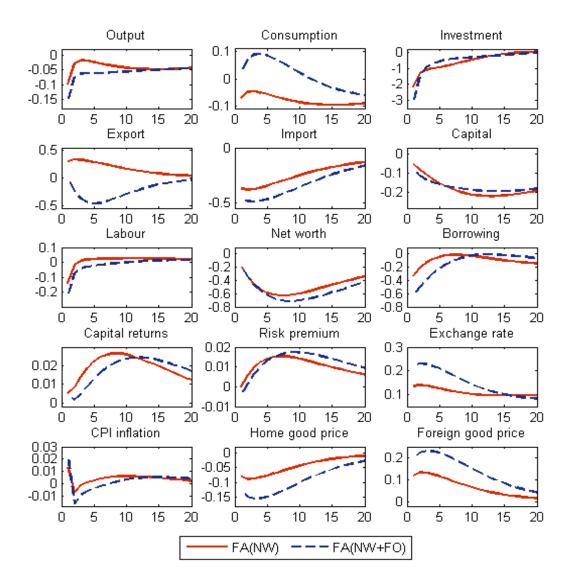
net worth,  $V_t$ , in (3.35) and foreign output,  $Y_t^*$ , in (3.13) at the same time, which we label the FA(NW+FO) model.<sup>31</sup> In contrast, the solid lines in Figure 3.3 represent the impulse responses to a negative shock in foreign lenders' evaluation on the entrepreneurs' net worth in the absence of contraction in world aggregate demand, which is the benchmark FA(NW) model. In the FA (NW+FO) model, the sudden stops are driven by both the foreigners' financial stress due to recession in the world economy and the foreigners' negative outlook on the emerging market economy due to, possibly, the vulnerability of an emerging market economy to the uncertainty in the global economy.

Comparison between the dotted and solid lines in Figure 3.3 reveals that a coincidence of global recession and sudden stops makes it difficult for an emerging market economy to recover from a sudden stop crisis through the export expansion channel, so that the recession in the emerging market economy could be intensified.<sup>32</sup> That is, in the FA(NW+FO) model, in spite of the currency depreciation,  $S_t$ , due to sudden stops, the export demand for home good,  $C_{H,t}^*$ , is significantly reduced due to a contraction in the aggregate demand in the world economy,  $Y_t^*$ , by (3.18), which magnifies the production contraction,  $Y_{H,t}$ , by the economy-wide resource constraint in (3.55). The additional production contraction,  $Y_{H,t}$ , amplifies the contractions in capital,  $K_t$ , and investment,  $I_{H,t}$ , by (3.19), (3.44) and (3.39). However, the contraction in export,  $C_{H,t}^*$ , raises the nominal exchange rate,  $S_t$ , in the foreign currency market, by (3.18). Then, the amplified domestic

<sup>&</sup>lt;sup>31</sup>This configuration assumes that the shock processes in foreigners' evaluation,  $V_t$ , and foreign output  $Y_t^*$ , has the same autoregressive structure, which may be lacking in reality. However it does not impede the aim of the experiment to simulate a global financial crisis, where sudden stops and global recession may coincide from a perspective of an emerging market economy.

<sup>&</sup>lt;sup>32</sup>However, one needs to note that our investigation on fluctuations from alternative models aims at understanding how the economy responds to the respective shock. Thus, the comparison here does not have quantitative implication.

Figure 3.3: Responses to Negative Shocks to Foreign Output and Net Worth Evaluation



<sup>\*</sup> The solid and dotted lines represent the impulse responses to a negative net worth evaluation shock without and with the global recession (FA(NW) and FA(NW+FO) models), respectively.

currency depreciation magnifies the rise in import price,  $\frac{P_{F,t}}{P_t}$ , and a fall in home good price,  $\frac{P_{H,t}}{P_t}$ , due to the price stickiness in home goods, by (3.8). Accordingly, the import demand for foreign good,  $C_{F,t}$ , is further depressed, but the consumption demand for home goods,  $C_{H,t}$ , increases, in the FA(NW+FO) model, by (3.10) and (3.9). In sum, in case of the coincidence of sudden stops and global recession, an emerging market economy could suffer the prolonged and amplified recession and experience the further demand shift from export to domestic consumption.

#### 3.4.4Exchange Rate Regime and Sudden Stops

We now turn to the impact of sudden stops of capital inflows under alternative exchange rate regimes: a free floating system and a fixed exchange rate regime. So far, we assume that the central bank adopts a free floating exchange rate regime. However, as Calvo (2002) argues, given that an emerging market economy is characterised by the high degree of 'domestic liability dollarisation' (DLD), it may have an incentive to fix the nominal exchange rate,  $S_t$ , at a certain level, to prevent a negative effect of the currency depreciation on real activity.<sup>33</sup> We now examine the case where an emerging market economy adopts a fixed exchange rate regime so that the central bank seeks to stabilise the fluctuations of the nominal exchange rate,  $S_t$ , completely and instantly, by counteracting the motion in foreign fund supply. That is, the central bank is assumed to adjust the nominal interest rate,  $R_t^n$ , in the feedback rule in (3.56), highly sensitively to the motion in the nominal <sup>33</sup>Note that the entrepreneurs' leverage ratio,  $\frac{Q_t K_t}{N_t} = \frac{Q_t K_t}{Q_t K_t - S_t B_t^*}$  increases when the nominal

exchange rate,  $S_t$ , rises, so that the risk premium that the entrepreneurs face,  $\frac{R_{t+1}^k}{R_t} = \Psi_t$ , would rise by the working of financial accelerator mechanism,  $\Psi_t = \left(\frac{Q_t K_t}{N_t}\right)^{\psi}$ .

exchange rate,  $S_t$ . To simulate the situation where an emerging market economy adopting a fixed exchange rate regime is hit by a 'sudden stop', we set the Taylor rule coefficient attached on nominal exchange rate,  $\alpha_s$ , in (3.56) to be 30, and impose a negative shock in foreign lenders' evaluation on the entrepreneurs' net worth, which is denoted as the FA(NW)+FR model. To investigate the role of a fixed exchange rate regime under a sudden stop crisis, the impulse responses in the FA(NW)+FR are compared with those in the benchmark FA(NW) model, which assigns an insignificant values to Taylor rule coefficient on nominal exchange rate, i.e.,  $\alpha_s = 0.2$ . Figure 3.4 displays the results of the experiment where the solid and dotted lines represent the impulse responses in the FA(NW) and FA(NW)+FR models, respectively.

As shown in Figure 3.4, a negative shock in foreign lenders' evaluation on the entrepreneurs' net worth,  $V_t$ , results in contractions in capital demand and output production in the emerging market economy, under both exchange rate regimes. That is, the distortion of the entrepreneurs' leverage ratio,  $\frac{Q_t K_t}{N_t}$ , raises the external finance premium,  $\Psi_t = \frac{R_{t+1}^k}{R_t}$ , due to the negative net worth evaluation shock, which leads to the contractions in capital demand,  $K_t$ , investment,  $I_{H,t}$ , and production,  $Y_{H,t}$ . It is noteworthy that in our simulation there does not exist significant differences in motions of risk premium,  $\Psi_t$ , capital returns,  $R_{t+1}^k$ , capital,  $K_t$ , and net worth,  $N_t$ . It implies that a fixed exchange rate regime may not have a significant stabilising effect on capital demand and production in the sudden stop crisis triggered by foreigners' pessimism.

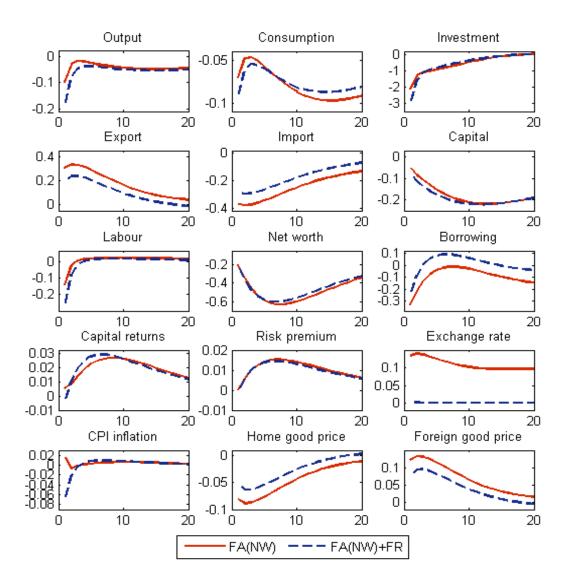
The result seems to be contradictory to the 'fear of floating' argument  $a \, la$  Calvo and Reinhart (2002) and Calvo (2004), where emerging market economies adopt a fixed exchange rate regime,  $de \, facto$ , to prevent an unfavourable balance

sheet effect of currency depreciation. However, note that a currency depreciation is just one source to distort the balance sheet. That is, a fixed exchange rate regime could be effective as a stabilisation tool in case where the entrepreneurs' balance sheet is distorted by an exogenous currency depreciation in an emerging market economy with a high degree of foreign debt. In contrast, for example, in case where a distortion in entrepreneurs' balance sheet (perceived by foreign lenders) is directly driven by foreigners' pessimism, the central bank is not able to stabilise the cost of foreign borrowing effectively, even though it could stabilise the nominal exchange rate under a fixed exchange rate regime.

However, the choice of exchange rate regime could affect the demand side significantly in an emerging market economy. While the real currency depreciation,  $S_t \equiv \frac{S_t P_t^*}{P_t}$ , resulting from a sudden stop, results in the rise in nominal exchange rate,  $S_t$ , under a free floating system, the deflation in the CPI,  $P_t$ , takes this adjusting role under the peg, instead of the nominal exchange rate,  $S_t$ .<sup>34</sup> Then, under the peg, the stickiness of home good price,  $P_{H,t}$ , implies the rises in real home good prices in home and foreign countries,  $\frac{P_{H,t}}{P_t}$  and  $\frac{P_{H,t}}{S_t P_t^*} = \frac{P_{H,t}}{S_t P_t}$ , and the fall in the real foreign good price,  $\frac{P_{F,t}}{P_t}$ , by (3.8). Changes in the real price structure decreases the demands for home goods in home and foreign countries,  $C_{H,t}$ ,  $I_{H,t}$ , and  $C_{H,t}^*$ , and increases the import demand for foreign goods,  $C_{F,t}$ , by (3.9), (3.39), (3.18) and (3.10), respectively. The additional contractions in demands for home goods,  $C_{H,t}$ ,  $I_{H,t}$ , and  $C_{H,t}^*$ , leads to the corresponding contraction in home good production,  $Y_{H,t}$ , by (3.55) and factor demands,  $K_t$  and  $L_t$ , by (3.23) and (3.22), respectively. In such an indirect way, a sudden stop in foreign fund supply could

 $<sup>^{-34}</sup>$ A real currency depreciation eventually leads to the rises in nominal exchange rate,  $S_t$ , and CPI inflation,  $\pi_t$ , in a floating system, while it results in the deflation of CPI,  $\pi_t$ , with the nominal exchange rate,  $S_t$ , being fixed under the peg.

Figure 3.4: Responses to a Negative Net Worth Evaluation Shock under a Fixed Exchange Rate Regime



<sup>\*</sup> The solid and dotted lines represent the impulse responses to a negative shock to foreigners' evaluation on entrepreneurs' net worth under floating (FA(NW)) and fixed exchange rate regimes (FA(NW)+FR), respectively.

aggravate the recession in the emerging market economy with the fixed exchange rate regime, as compared to that under a free floating counterpart. These observations point to the importance of the exchange rate regime choice as regards the economic performance in a sudden stop crisis.

### 3.4.5 Heavy Foreign Input Reliance and Sudden Stops

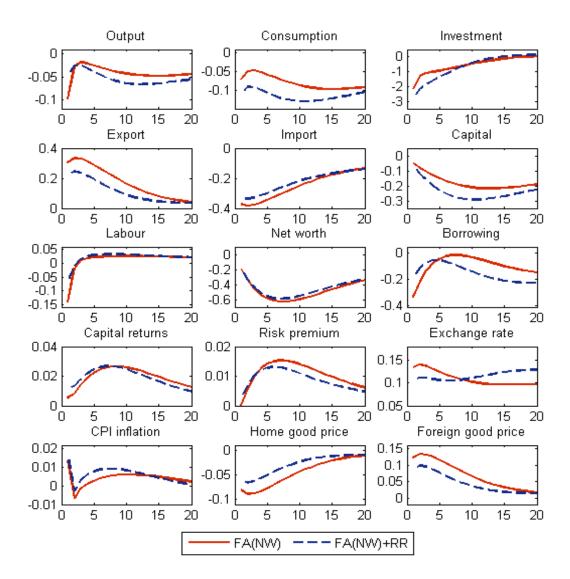
Lastly, we investigate the effect of a sudden stop when an emerging market economy relies on foreign economies in terms of resources for capital production, as well as funds for capital acquisition, which is referred to as 'processing trade' in the literature.<sup>35</sup> We suppose that the degree of capital producers' reliance on foreign input,  $I_{F,t}$ , in investment good composite,  $I_t$ , is greater than households' preference for imported good,  $C_{F,t}$ , out of their consumption bundle,  $C_t$ , in an emerging market country. To simulate this, we set the steady state share of domestic input in the investment good composite,  $\eta_i$ , in (3.37) to be 0.3, while that of domestic good in the consumption bundle,  $\eta$ , in (3.7) remains at 0.6, which is denoted by the FA(NW)+RR model. Thus, in the FA(NW)+RR model, capital producers acquire 70 % of their investment goods,  $I_t$ , from abroad, whilst households consume the imported goods by 40 % of their total consumption bundle in the steady state. Then, in the FA(NW)+RR model, the investment good price,  $P_{I,t}$ , in (3.38) is more affected than CPI,  $P_t$ , in (3.8), by the motion in foreign good price,  $P_{F,t} = S_t P_{F,t}^*$ , so that the real investment good price,  $\frac{P_{I,t}}{P_t}$ , and the real capital price,  $Q_t$ , in (3.44) moves in the same direction of nominal exchange rate,  $S_t$ , or foreign good price,

<sup>&</sup>lt;sup>35</sup>Braggion *et al.* (2007) and Curdia (2007) report that the import of capital and intermediate good for production of final goods takes a much larger portion of the total import as compared to that of consumption good.

 $P_{F,t}$ . In contrast, in the FA(NW) model with  $\gamma_i = \gamma$  and  $\eta_i = \eta$ ,  $P_{I,t}$  and  $P_t$  show the same behaviours, so that  $\frac{P_{I,t}}{P_t}$  remains at unity and the real capital price,  $Q_t$ , would not be affected by the nominal exchange rate,  $S_t$ . Figure 3.5 shows the effect of an emerging market economy's heavy reliance on foreign input when the economy is hit by a sudden stop in foreign fund supply, where the dotted and solid lines represent the impulse responses in the FA(NW)+RR and FA(NW) models, respectively.

As shown in dotted lines in Figure 3.5, a negative shock to foreign lenders' evaluation of entrepreneurs' net worth,  $V_t$ , results in declines in capital demand and output and a currency depreciation in both the FA(NW) and FA(NW)+RR models. However, in the FA(NW)+RR model, the currency depreciation has an additional effect on the cost conditions in capital production, which, in turn, affects the production cost conditions for entrepreneurs. That is, the currency depreciation,  $S_t$ , raises the foreign good price,  $P_{F,t} = S_t P_{F,t}^*$ , by (3.12), which increases the investment good price,  $P_{I,t}$ , and CPI,  $P_t$ , by (3.38) and (3.8). However, under the configurations for  $\eta_i$  and  $\eta$  in the FA(NW)+RR model,  $P_{I,t}$  rises more than  $P_t$ , so that the real investment good price,  $\frac{P_{I,t}}{P_t}$ , increases. Then, capital producers would impute the aggravated cost for capital production,  $\frac{P_{I,t}}{P_t}$ , to entrepreneurs, by raising the real capital price,  $Q_t$ , by (3.44), in the capital market. It implies the rise in cost of purchasing capital for entrepreneurs, which makes them decrease capital demand,  $K_t$ , by (3.23), additionally to the contraction in capital demand due to the rise in cost of foreign borrowing. The contraction in capital demand,  $K_t$ , leads to additional contractions in investment good composite,  $I_t$ , domestic investment goods,  $I_{H,t}$ , and output production,  $Y_{H,t}$ , by (3.44), (3.39), and (3.19),

Figure 3.5: Responses to a Negative Net Worth Evaluation Shock under a Heavy Foreign Input Reliance



<sup>\*</sup> The solid and dotted lines represent the impulse responses to a negative shock in foreigners' evaluation on entrepreneurs' net worth in the economy with standard (FA(NW)) and high degree of foreign resource reliance (FA(NW)+RR), respectively.

respectively.<sup>36</sup> Furthermore, in the FA(NW)+RR model, the rise in the real investment good price,  $\frac{P_{I,t}}{P_t}$ , triggered by the currency depreciation, could raise the real home good price,  $\frac{P_{H,t}}{P_t}$ , via the aggravated cost conditions, which leads to contractions in demands for domestic goods in home and foreign countries,  $C_{H,t}$  and  $C_{H,t}^*$ . It follows that an economy that relies heavily on foreign resources are likely to suffer sudden stop crises more severely via the impaired price competitiveness for the domestic product as well as the distorted cost conditions.

## 3.5 Conclusion

We have explored the implication of a sudden stop crisis in an emerging market economy: (i) what triggers a sudden stop of the international fund inflows to an emerging market economy; (ii) how an economy is affected by a sudden stop; and (iii) what pre-crisis conditions in an economy affect a sudden stop crisis. To these ends, we have used a small open economy DSGE model with financial frictions a la Gertler, Gilchrist, and Natalucci (2007), which emphasises the information asymmetry between foreign lenders and domestic entrepreneurs, and the positive relation between the entrepreneurs' financial condition and the cost of foreign borrowing that they are confronted with. In addition, following Curdia (2007) and Ozkan and Unsal (2010), we consider that foreign lenders' perception or evaluation of the domestic entrepreneurs' financial condition in an emerging market economy could play an important role as a trigger of a sudden stop crisis.

Our main findings are as follows. First of all, we confirm that foreigners'

<sup>&</sup>lt;sup>36</sup>However, the size of the production contraction,  $Y_{H,t}$ , is less than that of the domestic input demand for investment,  $I_{H,t}$ , due to the reduced steady state share of the domestic input for the investment,  $\frac{I_H}{Y_H}$ , by the decrease in  $\eta_i$ .

pessimism as to an emerging market economy's financial condition may result in a sudden stop of foreign fund inflow and real recession in the economy via the raised cost of foreign borrowing, as an exogenous rise in foreign interest rate does. Thus, we argue that the foreigners' pessimism could be one of the sources of the sudden stops, as the 'self-fulfilling pessimism' argument in Calvo (1998) and Krugman (1999) suggest. Second, we uncover that while a sudden stop crisis could be short-lived as the 'Phoenix miracle' in Calvo, Izquierdo, and Talvi (2004) suggest, certain characteristics that are typical of emerging market economies could make a sudden stop crisis further aggravated. We identify such environmental conditions as follows: (i) the presence of a high degree of financial frictions; (ii) a coincidence of sudden stops in capital inflow and global recession; and (iii) a heavy reliance on foreign input for capital production in an emerging market economy. That is, in case an emerging market economy fails to gain a high degree of trust from foreign lenders in normal times, foreign lenders could become highly sensitive to even temporary changes in the financial conditions in the economy, which magnifies the unfavourable consequences of the sudden stops. Next, when a global recession coincides with the sudden stops, the fast recovery from the sudden stop crisis via the currency depreciation and the increased export could be interrupted due to the lack of global demand. Lastly, if an emerging market economy relies heavily on foreign resources for the use of producing the intermediate goods, the currency depreciation in the process of a sudden stop crisis could even aggravate the crisis by increasing the imported input price and distorting the production cost conditions. Third, we show that the exchange rate regime choice of an emerging market economy has an important implication as regards the economic performance in a sudden stop crisis. In addition, our simulation results suggest that a fixed exchange rate regime could produce an inferior outcome to the floating counterpart when faced with sudden stops, in contrast to the 'fear of floating' argument a la Calvo and Reinhart (2002). This is due to the fact that if a rise in cost of foreign borrowing is driven by the factors other than the currency depreciation, say, foreign lenders' pessimism on an emerging market economy, then a fixed exchange rate regime is not an effective tool to prevent a sudden stop crisis, even if it could stabilise the nominal exchange rate. Rather, in this circumstances, the stabilised nominal exchange rate under a fixed exchange rate regime could reduce the improvement in price competitiveness for home good, which limits the increase in demands for home good in a sudden stop crisis. In our simulation, this negative impact of a fixed exchange rate regime in a sudden stop crisis turns out to offset its positive effect of stabilising the cost of foreign borrowing.

This chapter contributes to the existing literature as follows. First, we highlight the important roles of an unfavourable change in foreign lenders' perception on an emerging market economy as a trigger of a financial crisis in the economy and endogenous transmission mechanism via financial frictions in foreign borrowing contract. This is compared to the existing research where the exogenous shocks such as ones arising from the foreign interest rate or the sovereign risk premium. We argue that our consideration may provide the more primitive and relevant source of a financial shock in an emerging market economy and that it is the more coherent way to transmit the foreign shocks to the business cycle in an emerging market economy. In addition, our simulation confirms the so-called 'self-fulfilling pessimism' argument a la Calvo (1998) and Krugman (1999). Second, we point out the potential weakness of 'fear of floating' argument in Calvo and Reinhart (2002). That is, a fixed exchange rate regime motivated by the concern that

the entrepreneurs' balance sheet is negatively affected by a currency depreciation resultin in a sudden stop crisis is not likely to be an effective measure in the environment where the balance sheet is distorted directly by an unfavourable change in the foreign lenders' perspective, since they would require the higher external finance premium on non-financial firms in an emerging market economy even in the absence of a currency depreciation. Furthermore, we point out that under a fixed exchange rate regime, an emerging market economy would lose the opportunity to recover from a financial crisis via the improved price competitiveness for domestic goods and the resulting export expansion. Third, we show the so-called 'process trading' argument in a general equilibrium framework which is argued in a partial equilibrium model in the existing literature. In addition, while the previous research relying on a small open economy DSGE approach mainly point out the positive effect of a domestic currency on the export demand for an emerging market economy, we argue that in case the emerging market economy relies heavily on foreign inputs, the economy could fail to enjoy the recovery due to the further distortion in the production cost conditions and the resulting aggravation of price competitiveness for domestic goods.

## Appendix B

## Appendix B1 The Model Solution

The model consists of 34 behavioural equations with 34 endogenous variables, such as  $\{C_t, C_{H,t}, C_{F,t}, C_{H,t}^*, Y_t^*, Y_{H,t}, Y_{w,t}, A_t, L_t, K_t, N_t, B_t^*, I_t, I_{H,t}, I_{F,t}, P_t, P_{H,t}, P_{F,t}, P_{I,t}, P_t^*, W_t, Q_t, \pi_t, \pi_{H,t}, \pi_t^*, R_t^k, \Psi_t, R_t, R_t^n, R_t^*, S_t, S_t, V_t, G_t\}$  and 7 temporary exogenous shocks, such as  $\{\varepsilon_{r,t}, \varepsilon_{a,t}, \varepsilon_{v,t}, \varepsilon_{g,t}, \varepsilon_{y^*,t}, \varepsilon_{\pi^*,t}, \varepsilon_{r^*,t}\}$ . Appendix B1.1 identifies the nonlinear equations characterising equilibrium in the model. These equations may be approximated around the steady state to be transformed into the linear equations, which are presented in Appendix B1.2. Derivation and log-linearisation process for equations are exposed in Appendix B2.

## **B1.1 Equilibrium Conditions**

1. Consumption Euler equation:

$$1 = \beta E_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} R_t \right\}$$
 (B1.1)

2. Labour supply:

$$\frac{W_t}{P_t} = (C_t)^{\sigma} (L_t)^{\varphi} \tag{B1.2}$$

3. Uncovered interest rate parity condition (UIPC):

$$R_t = R_t^* \left( \frac{S_{t+1}}{S_t} \right) \tag{B1.3}$$

4. Consumption demand for home good:

$$C_{H,t} = \eta \left(\frac{P_{H,t}}{P_t}\right)^{-\gamma} C_t \tag{B1.4}$$

5. Import demand for foreign good:

$$C_{F,t} = (1 - \eta) \left(\frac{P_{F,t}}{P_t}\right)^{-\gamma} C_t \tag{B1.5}$$

6. Consumer price index (CPI):

$$P_{t} \equiv \left[ \eta \left( P_{H,t} \right)^{1-\gamma} + (1-\eta) \left( P_{F,t} \right)^{1-\gamma} \right]^{\frac{1}{1-\gamma}}$$
 (B1.6)

7. Law of one price (LOOP) for imported good:

$$P_{F,t} = S_t P_t^* \tag{B1.7}$$

8. Real exchange rate:

$$S_t \equiv \frac{S_t P_t^*}{P_t} \tag{B1.8}$$

9. Export demand for home good:

$$C_{H,t}^* = \eta^* \left(\frac{P_{H,t}}{\mathcal{S}_t P_t}\right)^{-\gamma} Y_t^* \tag{B1.9}$$

10. Production function:

$$Y_{w,t} = A_t (K_{t-1})^{\alpha} (L_t)^{1-\alpha}$$
 (B1.10)

11. Labour demand:

$$\frac{W_t}{P_{H,t}} = (1 - \alpha) \left(\frac{Y_{w,t}}{L_t}\right) P_{w,t} \tag{B1.11}$$

12. Capital demand:

$$E_{t}\left\{R_{t+1}^{k}Q_{t} - (1-\delta)Q_{t+1}\right\} = E_{t}\left\{\alpha\left(\frac{Y_{w,t+1}}{K_{t}}\right)P_{w,t+1}\right\}$$
(B1.12)

13. Balance sheet:

$$Q_t K_t = N_t + S_t B_t^* \tag{B1.13}$$

14. External finance premium:

$$\Psi_t = \left(\frac{Q_t K_t}{N_t}\right)^{\psi} \tag{B1.14}$$

15. Cost of foreign borrowing:

$$R_{t+1}^k = R_t^* \Psi_t \left( \frac{S_{t+1}}{S_t} \right)$$
 (B1.15)

16. Net worth evolution:

$$N_{t} = \left[ \phi \left\{ R_{t}^{k} Q_{t-1} K_{t-1} - R_{t-1}^{*} \Psi_{t-1} B_{t-1}^{*} S_{t} \right\} + (1 - \phi) F \right] \cdot V_{t}$$
 (B1.16)

17. Capital supply:

$$Q_t \left(\frac{P_{H,t}}{P_t}\right) = \frac{P_{I,t}}{P_t} \left[ 1 - \kappa \left( \frac{I_t}{K_{t-1}} - \delta \right) \right]^{-1}$$
 (B1.17)

18. Capital accumulation:

$$K_{t} = \left[\frac{I_{t}}{K_{t-1}} - \frac{\kappa}{2} \left(\frac{I_{t}}{K_{t-1}} - \delta\right)^{2}\right] K_{t-1} + (1 - \delta) K_{t-1}$$
(B1.18)

19. Investment demand for domestic input:

$$I_{H,t} = \eta_i \left(\frac{P_{H,t}}{P_t}\right)^{-\gamma_i} I_t \tag{B1.19}$$

20. Investment demand for foreign input:

$$I_{F,t} = (1 - \eta_i) \left(\frac{P_{F,t}}{P_t}\right)^{-\gamma_i} I_t$$
 (B1.20)

21. Investment good price index:

$$P_{I,t} \equiv \left[ \eta_i \left( P_{H,t} \right)^{1-\gamma_i} + (1 - \eta_i) \left( P_{F,t} \right)^{1-\gamma_i} \right]^{\frac{1}{1-\gamma_i}}$$
 (B1.21)

22. Wholesale goods market equilibrium:

$$Y_{H,t} = Y_{w,t} \tag{B1.22}$$

23. New Keynesian Phillips curve (NKPC) for home good:

$$\pi_{H,t} = (\mu P_{w,t})^{\lambda} E_t \{\pi_{H,t+1}\}^{\beta}$$
 (B1.23)

24. Resource constraint:

$$Y_{H,t} = C_{H,t} + C_{H,t}^* + I_{H,t} + G_t$$
 (B1.24)

25. Taylor rule:

$$\left(\frac{R_t^n}{R^n}\right) = \left(\frac{R_{t-1}^n}{R^n}\right)^{\alpha_r} \left(\frac{P_t}{P_{t-1}}\right)^{(1-\alpha_r)\alpha_\pi} \left(\frac{Y_{H,t}}{Y_H}\right)^{(1-\alpha_r)\alpha_y} \left(\frac{S_t}{S}\right)^{(1-\alpha_r)\alpha_s} \exp\left\{\varepsilon_{r,t}\right\}$$
(B1.25)

26. Fisher equation:

$$R_t^n \equiv R_t E_t \left\{ \pi_{t+1} \right\} \tag{B1.26}$$

27. Inflation of consumer price index:

$$\pi_t \equiv \frac{P_t}{P_{t-1}} \tag{B1.27}$$

28. Inflation of domestic price index:

$$\pi_{H,t} \equiv \frac{P_{H,t}}{P_{H,t-1}}$$
(B1.28)

29. Technology shock process:

$$A_t = (A_{t-1})^{\rho_a} \exp\left\{\varepsilon_{a,t}\right\} \tag{B1.29}$$

30. Net worth evaluation shock process:

$$V_t = (V_{t-1})^{\rho_v} \exp\left\{\varepsilon_{v,t}\right\} \tag{B1.30}$$

31. Government spending shock process:

$$G_t = (G_{t-1})^{\rho_g} \exp\left\{\varepsilon_{g,t}\right\} \tag{B1.31}$$

32. Foreign output shock process:

$$Y_t^* = (Y_{t-1}^*)^{\rho_{y^*}} \exp\{\varepsilon_{y^*,t}\}$$
 (B1.32)

33. Foreign CPI inflation shock process:

$$\pi_t^* = \left(\pi_{t-1}^*\right)^{\rho_{\pi^*}} \exp\left\{\varepsilon_{\pi^*,t}\right\} \tag{B1.33}$$

34. Foreign interest rate shock process:

$$R_t^* = (R_{t-1}^*)^{\rho_{r^*}} \exp\{\varepsilon_{r^*,t}\}$$
 (B1.34)

## **B1.2** The Log-linearised Model

1. Consumption Euler equation:

$$\widehat{C}_t = E_t \left\{ \widehat{C}_{t+1} - \frac{1}{\sigma} \widehat{R}_t \right\}$$
 (B1.35)

2. Labour supply:

$$\widehat{W}_t - \widehat{P}_t = \sigma \widehat{C}_t + \varphi \widehat{L}_t \tag{B1.36}$$

3. Uncovered interest rate parity condition (UIPC):

$$\widehat{R}_t = \widehat{R}_t^* + \widehat{S}_{t+1} - \widehat{S}_t \tag{B1.37}$$

4. Consumption demand for home good:

$$\widehat{C}_{H,t} = -\gamma \left(\widehat{P}_{H,t} - \widehat{P}_t\right) + \widehat{C}_t \tag{B1.38}$$

5. Import demand for foreign good:

$$\widehat{C}_{F,t} = -\gamma \left(\widehat{P}_{F,t} - \widehat{P}_t\right) + \widehat{C}_t \tag{B1.39}$$

6. Consumer price index (CPI):

$$\widehat{P}_t = \eta \widehat{P}_{H,t} + (1 - \eta) \widehat{P}_{F,t} \tag{B1.40}$$

7. Law of one price (LOOP) for imported good

$$\widehat{P}_{F,t} = \widehat{S}_t + \widehat{P}_t^* \tag{B1.41}$$

8. Real exchange rate:

$$\widehat{\mathcal{S}}_t \equiv \widehat{S}_t + \widehat{P}_t^* - \widehat{P}_t \tag{B1.42}$$

9. Export demand for home good:

$$\widehat{C}_{H,t}^* = -\gamma \left( \widehat{P}_{H,t} - \widehat{P}_t - \widehat{\mathcal{S}}_t \right) + \widehat{Y}_t^*$$
(B1.43)

10. Production function:

$$\widehat{Y}_{w,t} = \widehat{A}_t + \alpha \widehat{K}_{t-1} + (1 - \alpha) \widehat{L}_t$$
(B1.44)

11. Labour demand:

$$\widehat{W}_t - \widehat{P}_{H,t} = \widehat{Y}_{w,t} - \widehat{L}_t + \widehat{P}_{w,t} \tag{B1.45}$$

12. Capital demand:

$$\widehat{R}_{t+1}^{k} + \widehat{Q}_{t} = \left(1 - \frac{1 - \delta}{R^{k}}\right) \left(\widehat{Y}_{w,t+1} - \widehat{K}_{t} + \widehat{P}_{w,t+1}\right) + \left(\frac{1 - \delta}{R^{k}}\right) \widehat{Q}_{t+1}$$
 (B1.46)

13. Balance sheet:

$$\widehat{Q}_t + \widehat{K}_t = \left(\frac{N}{K}\right)\widehat{N}_t + \left(1 - \frac{N}{K}\right)\left[\widehat{S}_t + \widehat{B}_t^*\right]$$
(B1.47)

14. External finance premium:

$$\widehat{\Psi}_t = \psi \left( \widehat{Q}_t + \widehat{K}_t - \widehat{N}_t \right) \tag{B1.48}$$

15. Cost of foreign borrowing:

$$\widehat{R}_{t+1}^{k} = \widehat{R}_{t}^{*} + \widehat{\Psi}_{t} + \widehat{S}_{t+1} - \widehat{S}_{t}$$
(B1.49)

16. Net worth evolution:

$$\frac{\widehat{N}_t}{\phi R^k} = \left(\frac{K}{N}\right) \widehat{R}_t^k - \left(\frac{K}{N} - 1\right) \left(\widehat{R}_{t-1}^* + \widehat{\Psi}_{t-1} + \widehat{S}_t - \widehat{S}_{t-1}\right) + \widehat{N}_{t-1} + \left(\frac{1}{\phi R^k}\right) \widehat{V}_t$$
(B1.50)

17. Capital supply:

$$\widehat{Q}_t - \left(\widehat{P}_{I,t} - \widehat{P}_t\right) + \left(\widehat{P}_{H,t} - \widehat{P}_t\right) = \kappa \delta \left(\widehat{I}_t - \widehat{K}_{t-1}\right)$$
(B1.51)

18. Capital accumulation:

$$\widehat{K}_t = \delta \widehat{I}_t - (1 - \delta) \, \widehat{K}_{t-1} \tag{B1.52}$$

19. Investment demand for domestic input:

$$\widehat{I}_{H,t} = -\gamma_i \left( \widehat{P}_{H,t} - \widehat{P}_t \right) + \widehat{I}_t \tag{B1.53}$$

20. Investment demand for foreign input:

$$\widehat{I}_{F,t} = -\gamma_i \left( \widehat{P}_{F,t} - \widehat{P}_t \right) + \widehat{I}_t \tag{B1.54}$$

21. Investment good price index:

$$\hat{P}_{I,t} = \eta_i \hat{P}_{H,t} + (1 - \eta_i) \hat{P}_{F,t}$$
 (B1.55)

22. Wholesale goods market equilibrium:

$$\widehat{Y}_{H,t} = \widehat{Y}_{w,t} \tag{B1.56}$$

23. New Keynesian Phillips curve (NKPC) for home good:

$$\widehat{\pi}_{H,t} = \beta E_t \left\{ \widehat{\pi}_{H,t+1} \right\} + \left[ \frac{(1-\theta)(1-\beta\theta)}{\theta} \right] \widehat{P}_{w,t}$$
 (B1.57)

24. Resource constraint:

$$\widehat{Y}_{H,t} = \left(\frac{C_H}{Y_H}\right)\widehat{C}_{H,t} + \left(\frac{C_H^*}{Y_H}\right)\widehat{C}_{H,t}^* + \left(\frac{I_H}{Y_H}\right)\widehat{I}_{H,t} + \left(\frac{G}{Y_H}\right)\widehat{G}_t$$
(B1.58)

25. Taylor rule:

$$\widehat{R}_{t}^{n} = \alpha_{r} \widehat{R}_{t-1}^{n} + (1 - \alpha_{r}) \left[ \alpha_{\pi} \widehat{\pi}_{t} + \alpha_{y} \widehat{Y}_{H,t} + \alpha_{s} \widehat{S}_{t} \right] + \varepsilon_{r,t}$$
(B1.59)

26. Fisher equation:

$$\widehat{R}_t^n \equiv \widehat{R}_t + E_t \{ \widehat{\pi}_{t+1} \} \tag{B1.60}$$

27. Inflation of domestic consumer price index:

$$\widehat{\pi}_t \equiv \widehat{P}_t - \widehat{P}_{t-1} \tag{B1.61}$$

28. Inflation of domestic price index:

$$\widehat{\pi}_{H,t} \equiv \widehat{P}_{H,t} - \widehat{P}_{H,t-1} \tag{B1.62}$$

29. Technology shock process:

$$\widehat{A}_t = \rho_a \left( \widehat{A}_{t-1} \right) + \varepsilon_{a,t} \tag{B1.63}$$

30. Net worth evaluation shock process:

$$\widehat{V}_t = \rho_v \left( \widehat{V}_{t-1} \right) + \varepsilon_{v,t} \tag{B1.64}$$

31. Government spending shock process:

$$\widehat{G}_t = \rho_g \left( \widehat{G}_{t-1} \right) + \varepsilon_{g,t} \tag{B1.65}$$

32. Foreign output shock process:

$$\widehat{Y}_t^* = \rho_{y^*} \left( \widehat{Y}_{t-1}^* \right) + \varepsilon_{y^*,t} \tag{B1.66}$$

33. Foreign CPI inflation shock process:

$$\widehat{\pi}_t^* = \rho_{\pi^*} \left( \widehat{\pi}_{t-1}^* \right) + \varepsilon_{\pi^*,t} \tag{B1.67}$$

34. Foreign interest rate shock process:

$$\widehat{R}_t^* = \rho_{y^*} \left( \widehat{R}_{t-1}^* \right) + \varepsilon_{r^*,t} \tag{B1.68}$$

# Appendix B2. Derivation and Log-linearisation of Equilibrium Conditions

#### **B2.1** Households' Behaviours

Appendix B2.1 discusses the derivation and linearising process for the (domestic and foreign) households' behaviours: Euler equation in consumption, labour supply, uncovered interest rate parity condition (UIPC), domestic consumptions for home and foreign goods, domestic consumer price index (CPI), and export demand for home good in foreign countries.

Solution to Households' Utility Maximisation Problem To solve the households' utility maximisation problem described in the text, we write the problem as the following Lagrangian:

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{cases} \left[ \frac{(C_t)^{1-\sigma}}{1-\sigma} - \frac{(L_t)^{1+\varphi}}{1+\varphi} \right] \\ -\lambda_t \left[ C_t + D_t + S_t B_t^* - \left( \frac{W_t}{P_t} \right) L_t - R_{t-1} D_{t-1} - S_t R_{t-1}^* B_t^* - \Pi_t^o + T_t \right] \end{cases} \right\},$$

where  $\lambda_t$  is the shadow price for the budget constraint at the period t, i.e., the value in terms of utility of relaxing the budget constraint at the margin. Differentiating the Lagrangian with respect to  $C_t$ ,  $L_t$ ,  $D_t$  and  $B_t^*$  yields the following first order conditions (FOC):

$$[C_t] : (C_t)^{-\sigma} = \lambda_t$$

$$[L_t] : \frac{(L_t)^{\varphi}}{W_t/P_t} = \lambda_t$$

$$[D_t] : \frac{\beta \lambda_{t+1}}{\lambda_t} = \frac{1}{R_t}$$

$$[B_t^*] : \frac{\beta \lambda_{t+1}}{\lambda_t} = \frac{1}{R_t^*} \left(\frac{S_t}{S_{t+1}}\right).$$

The third and fourth conditions imply the evolution of the shadow price evaluated in domestic and foreign interest rate, respectively. Combining the first and second conditions yield the labour supply as:

$$\frac{W_t}{P_t} = (C_t)^{\sigma} (L_t)^{\varphi},$$

which is an equation (3.5) in the text. Substitution of the first condition to the third and fourth condition makes the Euler equation in consumption as:

$$1 = \beta E_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} R_t \right\},\,$$

which is an equation (3.4) in the text. In addition, combining the third and fourth conditions, we obtain the uncovered interest rate parity (UIPC) as:

$$R_t = R_t^* \left( \frac{S_{t+1}}{S_t} \right),$$

which is an equation (3.6) in the text.

#### Derivation of Domestic Consumptions for Home and Foreign Goods

Consider the domestic households' consumption bundle in (3.7) of the text,  $C_t \equiv \left[\eta^{\frac{1}{\gamma}} \left(C_{H,t}\right)^{\frac{\gamma-1}{\gamma}} + \left(1-\eta\right)^{\frac{1}{\gamma}} \left(C_{F,t}\right)^{\frac{\gamma-1}{\gamma}}\right]^{\frac{\gamma}{\gamma-1}}, \text{ and the corresponding price index in (3.8), } P_t \equiv \left[\eta \left(P_{H,t}\right)^{1-\gamma} + \left(1-\eta\right) \left(P_{F,t}\right)^{1-\gamma}\right]^{\frac{1}{1-\gamma}}. \text{ Households' allocation problem between } C_{H,t} \text{ and } C_{F,t} \text{ for } any \text{ given expenditure level, } Z_t \equiv P_{H,t}C_{H,t} + P_{F,t}C_{F,t}, \text{ can be formalised by the following Lagrangian:}$ 

$$\mathcal{L} = \left[ \eta^{\frac{1}{\gamma}} \left( C_{H,t} \right)^{\frac{\gamma-1}{\gamma}} + \left( 1 - \eta \right)^{\frac{1}{\gamma}} \left( C_{F,t} \right)^{\frac{\gamma-1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}} - \lambda_t \left[ P_{H,t} C_{H,t} + P_{F,t} C_{F,t} - Z_t \right].$$

Then, the associated first order conditions are:

$$[C_{H,t}] : \left(\frac{\gamma}{\gamma - 1}\right) (C_t)^{\frac{1}{\gamma}} (\eta)^{\frac{1}{\gamma}} \left(\frac{\gamma - 1}{\gamma}\right) (C_{H,t})^{-\frac{1}{\gamma}} - \lambda_t P_{H,t} = 0$$

$$[C_{F,t}] : \left(\frac{\gamma}{\gamma - 1}\right) (C_t)^{\frac{1}{\gamma}} (1 - \eta)^{\frac{1}{\gamma}} \left(\frac{\gamma - 1}{\gamma}\right) (C_{F,t})^{-\frac{1}{\gamma}} - \lambda_t P_{F,t} = 0$$

so that

$$\left(\frac{C_{H,t}}{C_{F,t}}\right) = \left(\frac{\eta}{1-\eta}\right) \left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\gamma}.$$

The above equation may be substituted into the total expenditure,  $Z_t \equiv P_{H,t}C_{H,t} + P_{F,t}C_{F,t}$ , to yield:

$$Z_{t} \equiv P_{H,t} \left[ \left( \frac{\eta}{1 - \eta} \right) \left( \frac{P_{H,t}}{P_{F,t}} \right)^{-\gamma} C_{F,t} \right] + P_{F,t} C_{F,t}$$

$$= \left[ \frac{C_{F,t}}{(P_{F,t})^{-\gamma}} \right] \left( \frac{1}{1 - \eta} \right) \left[ \eta \left( P_{H,t} \right)^{1 - \gamma} + (1 - \eta) \left( P_{F,t} \right)^{1 - \gamma} \right]$$

$$= \left[ \frac{C_{F,t}}{(P_{F,t})^{-\gamma}} \right] \left( \frac{1}{1 - \eta} \right) (P_{t})^{1 - \gamma},$$

where the last equality follows from the price index,  $P_t \equiv \left[\eta \left(P_{H,t}\right)^{1-\gamma} + \left(1-\eta\right) \left(P_{F,t}\right)^{1-\gamma}\right]^{\frac{1}{1-\gamma}}$ . It may be rewritten as:

$$C_{F,t} = (1 - \eta) \left(\frac{P_{F,t}}{P_t}\right)^{-\gamma} \left(\frac{Z_t}{P_t}\right),$$

and

$$C_{H,t} = \eta \left(\frac{P_{H,t}}{P_t}\right)^{-\gamma} \left(\frac{Z_t}{P_t}\right).$$

Substitution of the above equations for demands for foreign goods and home goods into the consumption composite,  $C_t \equiv \left[\eta^{\frac{1}{\gamma}} \left(C_{H,t}\right)^{\frac{\gamma-1}{\gamma}} + \left(1-\eta\right)^{\frac{1}{\gamma}} \left(C_{F,t}\right)^{\frac{\gamma-1}{\gamma}}\right]^{\frac{\gamma}{\gamma-1}}$ , yields:

$$C_{t} = \left[ \eta^{\frac{1}{\gamma}} \left\{ \eta \left( \frac{P_{H,t}}{P_{t}} \right)^{-\gamma} \left( \frac{Z_{t}}{P_{t}} \right) \right\}^{\frac{\gamma-1}{\gamma}} + (1-\eta)^{\frac{1}{\gamma}} \left\{ (1-\eta) \left( \frac{P_{F,t}}{P_{t}} \right)^{-\gamma} \left( \frac{Z_{t}}{P_{t}} \right) \right\}^{\frac{\gamma-1}{\gamma}} \right]$$

$$= \left( \frac{Z_{t}}{P_{t}} \right) \left( \frac{1}{P_{t}} \right)^{-\gamma} \left[ \eta \left( P_{H,t} \right)^{1-\gamma} + (1-\eta) \left( P_{F,t} \right)^{1-\gamma} \right]^{\frac{\gamma}{\gamma-1}} = \frac{Z_{t}}{P_{t}},$$

so that  $Z_t = P_t C_t$ . Substituting it into the above two optimality conditions, the following domestic demands for home and foreign goods are obtained as:

$$C_{H,t} = \eta \left(\frac{P_{H,t}}{P_t}\right)^{-\gamma} C_t$$

and

$$C_{F,t} = (1 - \eta) \left(\frac{P_{F,t}}{P_t}\right)^{-\gamma} C_t,$$

which are equations (3.9) and (3.10) in the text.

#### Linearisation of Domestic Consumptions for Home and Foreign Good

We linearise the domestic households' demands for home and foreign goods,  $C_{H,t} = \eta \left(\frac{P_{H,t}}{P_t}\right)^{-\gamma} C_t$ , and  $C_{F,t} = (1-\eta) \left(\frac{P_{F,t}}{P_t}\right)^{-\gamma} C_t$ , respectively. Note that, in the symmetric zero inflation steady state where  $P_H = P_F = P$ ,  $C_H = \eta C$  and  $C_F = (1-\eta) C$ . Then, by using  $x_t \simeq x(1+\hat{x}_t)$ , the consumption for home good may be approximated around the steady state as:

$$C_H \left( 1 + \widehat{C}_{H,t} \right) = \eta \left( \frac{P_H}{P} \right)^{-\gamma} C \left[ 1 - \gamma \left( \widehat{P}_{H,t} - \widehat{P}_t \right) + \widehat{C}_t \right],$$

and hence,

$$\widehat{C}_{H,t} = -\gamma \left(\widehat{P}_{H,t} - \widehat{P}_t\right) + \widehat{C}_t,$$

which is an equation (B1.38) in Appendix B1.2.

In addition, the domestic demand for foreign good is approximated around the steady state as:

$$C_F \left( 1 + \widehat{C}_{F,t} \right) = (1 - \eta) \left( \frac{P_F}{P} \right)^{-\gamma} C \left[ 1 - \gamma \left( \widehat{P}_{F,t} - \widehat{P}_t \right) + \widehat{C}_t \right],$$

and hence,

$$\widehat{C}_{F,t} = -\gamma \left(\widehat{P}_{F,t} - \widehat{P}_t\right) + \widehat{C}_t,$$

which is an equation (B1.39) in Appendix B1.2.

**Linearisation of Consumer Price Index (CPI)** Consider the consumer price index (CPI),  $P_t \equiv \left[\eta \left(P_{H,t}\right)^{1-\gamma} + \left(1-\eta\right) \left(P_{F,t}\right)^{1-\gamma}\right]^{\frac{1}{1-\gamma}}$  in (3.8) of the text, which may be rewritten as:

$$(P_t)^{1-\gamma} = \eta (P_{H,t})^{1-\gamma} + (1-\eta) (P_{F,t})^{1-\gamma}.$$

Note that, in the symmetric zero inflation steady state,  $\frac{P_H}{P} = \frac{P_F}{P} = 1$  and  $(P)^{1-\gamma} = \eta (P_H)^{1-\gamma} + (1-\eta) (P_F)^{1-\gamma}$ . Then, using  $x_t \simeq x(1+\widehat{x}_t)$ , the left hand side (LHS) and the right hand side (RHS) of the above equations may be approximated around the steady state as:

$$(LHS) \simeq (P)^{1-\gamma} \left[ 1 + (1-\gamma) \, \widehat{P}_t \right]$$

$$(RHS) \simeq \eta \, (P_H)^{1-\gamma} \left[ 1 + (1-\gamma) \, \widehat{P}_{H,t} \right] + (1-\eta) \, (P_F)^{1-\gamma} \left[ 1 + (1-\gamma) \, \widehat{P}_{F,t} \right].$$

Combining the LHS and RHS yields

$$\widehat{P}_t = \eta \widehat{P}_{H,t} + (1 - \eta) \, \widehat{P}_{F,t},$$

which is an equation (B1.40) in Appendix B1.2.

Linearisation of Foreign Demand for Home Good Consider the foreign demand for home good,  $C_{H,t}^* = \eta^* \left(\frac{P_{H,t}}{S_t P_t}\right)^{-\gamma} Y_t^*$ , in (3.18) of the text. Note that, in the steady state,  $C_H^* = \eta^* \left(\frac{P_H}{SP}\right)^{-\gamma} Y^*$ . Then, using  $x_t \simeq x(1+\hat{x}_t)$ , the foreign demand for home good may be approximated around the steady state as:

$$C_H^* \left( 1 + \widehat{C}_{H,t}^* \right) = \eta^* \left( \frac{P_H}{\mathcal{S}P} \right)^{-\gamma} Y^* \left[ 1 - \gamma \left( \widehat{P}_{H,t} - \widehat{P}_t - \widehat{\mathcal{S}}_t \right) + \widehat{Y}_t^* \right],$$

so that

$$\widehat{C}_{H,t}^* = -\gamma \left( \widehat{P}_{H,t} - \widehat{P}_t - \widehat{\mathcal{S}}_t \right) + \widehat{Y}_t^*$$

which is an equation (B1.43) in Appendix B1.2.

#### B2.2 Entrepreneurs' Behaviours of Entrepreneurs

Appendix B2.2 discusses a solution to the optimal contracting problem between entrepreneurs and foreign lenders, and log-linearising process for their net worth evolution.

Solution to Optimal contracting Problem<sup>37</sup> We consider the optimal contracting problem between the borrowers (i.e., domestic entrepreneurs) and the lenders (i.e., foreign financial intermediaries) a la Bernanke et al. (1999) and Gertler et al. (2007).<sup>38</sup> As discussed in the text, in the debt contract which is characterised as  $R^bSB^* = \overline{\omega}R^kQK$ , the foreign lenders' incentive constraint to participate in this debt contract is given by

<sup>&</sup>lt;sup>37</sup>This part is largely based on Bernanke et al. (1999) and Gertler et al. (2007).

 $<sup>^{38}</sup>$ Note that, in this section, we consider the steady state relation for analytical simplicity, so that we drop the time subscript t off from the equations.

$$[\Gamma(\overline{\omega}) - \mu_b G(\overline{\omega})] R^k QK \ge R^* (QK - N) \left(\frac{S}{S}\right)$$

$$= R(QK - N)$$

and the domestic entrepreneurs' expected profit is given by

$$[1 - \Gamma(\overline{\omega})] R^k Q K,$$

where the rate of total payment going to the foreign lenders,  $\Gamma(\overline{\omega}) \in [0,1]$ , and that of their auditing cost,  $\mu_b G(\overline{\omega})$ , are given, respectively, by:

$$\Gamma\left(\overline{\omega}\right) \equiv \left(1 - F(\overline{\omega})\right) \overline{\omega} + \int_{0}^{\overline{\omega}} \omega dF\left(\omega\right),$$

and

$$\mu_{b}G\left(\overline{\omega}\right) \equiv \mu_{b} \int_{0}^{\overline{\omega}} \omega dF\left(\omega\right).$$

Note that  $\Gamma(\overline{\omega})$  is strictly increasing and concave in  $\overline{\omega}$ , and that  $\mu_b G(\overline{\omega})$  is increasing in  $\overline{\omega}$ .<sup>39</sup> In addition, note that  $\Gamma(\overline{\omega}) - \mu_b G(\overline{\omega}) > 0$  for  $\overline{\omega} \in (0, \infty)$ , and that  $\lim_{\overline{\omega} \to 0} {\Gamma(\overline{\omega}) - \mu_b G(\overline{\omega})} = 0$ , and  $\lim_{\overline{\omega} \to \infty} {\Gamma(\overline{\omega}) - \mu_b G(\overline{\omega})} = 1 - \mu_b$ . The optimisation problem implies that the borrowers maximise their expected profit subject to the lender participation condition, which is given by:

Then, by the integration by parts,  $uv = \int v du + \int u dv$ , we obtain  $F(\overline{\omega})\overline{\omega} = \int_0^{\overline{\omega}} F(\omega) d\omega + \int_0^{\overline{\omega}} \omega dF(\omega)$ , so that  $\Gamma(\overline{\omega}) = [\overline{\omega} - F(\overline{\omega})\overline{\omega}] + \int_0^{\overline{\omega}} \omega dF(\omega) = \overline{\omega} - \int_0^{\overline{\omega}} F(\omega) d\omega$ . Accordingly, we establish that  $\Gamma'(\overline{\omega}) = 1 - F(\overline{\omega}) > 0$  and  $\Gamma''(\overline{\omega}) = -f(\overline{\omega}) < 0$ . In addition, it is obvious that  $\mu_b G'(\overline{\omega}) = \mu_b \overline{\omega} f(\overline{\omega}) > 0$ .

$$\begin{split} \max_{\{K,\overline{\omega}\}} \quad & \left[1-\Gamma\left(\overline{\omega}\right)\right]R^{k}QK \\ s.t. \quad & \left[\Gamma\left(\overline{\omega}\right)-\mu_{b}G\left(\overline{\omega}\right)\right]R^{k}QK \geq R\left(QK-N\right). \end{split}$$

Divide both sides of the above equations by RN, which is irrelevant to the problem, and denote the external finance by  $p(\overline{\omega}) \equiv \frac{R^k}{R}$ , and the steady state leverage ratio by  $k(\overline{\omega}) \equiv \frac{QK}{N} = \frac{K}{N}$ . Then, the Lagrangian is formulated as:

$$\mathcal{L} = \left[1 - \Gamma\left(\overline{\omega}\right)\right] pk + \lambda \left\{ \left[\Gamma\left(\overline{\omega}\right) - \mu_b G\left(\overline{\omega}\right)\right] pk - (k-1) \right\}.$$

The associated first order conditions are

$$[\overline{\omega}] : \qquad \lambda(\overline{\omega}) = \frac{\Gamma'(\overline{\omega})}{\Gamma'(\overline{\omega}) - \mu_b G'(\overline{\omega})}$$

$$[k] : \qquad p(\overline{\omega}) = \frac{\lambda(\overline{\omega})}{[1 - \Gamma(\overline{\omega})] + \lambda(\overline{\omega}) [\Gamma(\overline{\omega}) - \mu_b G(\overline{\omega})]}$$

$$[\lambda] : \qquad k(\overline{\omega}) = 1 + \lambda(\overline{\omega}) \frac{\Gamma(\overline{\omega}) - \mu_b G(\overline{\omega})}{1 - \Gamma(\overline{\omega})},$$

where we use  $k(\overline{\omega}) = \frac{1}{1 - [\Gamma(\overline{\omega}) - \mu_b G(\overline{\omega})] p(\overline{\omega})}$  and  $p(\overline{\omega}) = \frac{\lambda(\overline{\omega})}{[1 - \Gamma(\overline{\omega})] + \lambda(\overline{\omega})[\Gamma(\overline{\omega}) - \mu_b G(\overline{\omega})]}$  in the third condition.

Now, we investigate the properties of the above optimality conditions, under some reasonable restrictions. Suppose that  $\frac{\overline{\omega}f(\overline{\omega})}{1-F(\overline{\omega})} = \frac{G'(\overline{\omega})}{\Gamma'(\overline{\omega})}$  is increasing in  $\overline{\omega}$ , following Bernanke *et al.* (1999).<sup>40</sup> Then,  $\Gamma'(\overline{\omega}) - \mu_b G'(\overline{\omega}) = [1 - F(\overline{\omega})] \left[1 - \frac{\mu_b \overline{\omega} f(\overline{\omega})}{1-F(\overline{\omega})}\right]$  is decreasing in  $\overline{\omega}$ , implying that there exists an  $\overline{\omega}^*$  such that  $\Gamma'(\overline{\omega}) - \mu_b G'(\overline{\omega}) \stackrel{\geq}{\leq} 0$ 

<sup>&</sup>lt;sup>40</sup>Bernanke *et al.* (1999) show that this condition is satisfied if  $\overline{\omega}$  follows any monostically increasing transformation of the normal distribution.

for  $\overline{\omega} \leq \overline{\omega}^*$ .<sup>41</sup> Accordingly, for  $\overline{\omega} < \overline{\omega}^*$ , we establish that  $\lambda\left(\overline{\omega}\right) = \frac{\Gamma'(\overline{\omega})}{\Gamma'(\overline{\omega}) - \mu_b G'(\overline{\omega})} > 0$ . In addition, the assumption that  $\frac{\overline{\omega}f(\overline{\omega})}{1 - F(\overline{\omega})} = \frac{G'(\overline{\omega})}{\Gamma'(\overline{\omega})}$  is increasing in  $\overline{\omega}$ , implies that  $\left(\frac{G'(\overline{\omega})}{\Gamma'(\overline{\omega})}\right)' = \frac{G''(\overline{\omega})\Gamma'(\overline{\omega}) - G'(\overline{\omega})\Gamma''(\overline{\omega})}{\Gamma'(\overline{\omega})^2} > 0$ , so that  $G''(\overline{\omega})\Gamma'(\overline{\omega}) - G'(\overline{\omega})\Gamma''(\overline{\omega}) > 0$  for any  $\overline{\omega}$ . Thus, we see that  $\lambda'(\overline{\omega}) = \frac{\mu_b[G''(\overline{\omega})\Gamma'(\overline{\omega}) - G'(\overline{\omega})\Gamma''(\overline{\omega})]}{[\Gamma'(\overline{\omega}) - \mu_b G'(\overline{\omega})]^2} > 0$ . In addition, it can be shown that  $k'(\overline{\omega}) > 0$ , for  $\overline{\omega} < \overline{\omega}^*$  and that  $p'(\overline{\omega}) > 0$ , for  $\overline{\omega} < \overline{\omega}^*$ .

Next, we derive the financial accelerator from the above opitmality conditions, and establish its property. First of all, we invert the leverage ratio,  $k = k(\overline{\omega})$ , into  $\overline{\omega} = \overline{\omega}(k)$ , where  $\overline{\omega}'(k) > 0$  for k > 1. Then, substituting it into risk premium on the external finance,  $p = p(\overline{\omega})$ , yields the following expression for the financial accelerator

$$p = p \{ \overline{\omega}(k) \} = \Psi(k)$$

with  $\Psi'(k) > 0$  for k > 1. Now, given the equilibrium value of  $\overline{\omega}$ , it is straightforward to compute the implied external finance premium,  $p(\overline{\omega})$ , and the implied leverage ratio,  $k(\overline{\omega})$ .

Furthermore, without loss of generality, the above expression for the ex-

<sup>&</sup>lt;sup>41</sup>It implies that there exists an  $\overline{\omega}^*$  such that the net payoff to the lender,  $\Gamma(\overline{\omega}) - \mu_b G(\overline{\omega})$ , reaches a global maximum at  $\overline{\omega}^*$ . We may call the area where  $\overline{\omega} < \overline{\omega}^*$  so that  $\Gamma'(\overline{\omega}) - \mu_b G'(\overline{\omega}) > 0$ , 'non-rationing area'; it may be called 'rationing area', otherwise.

 $<sup>^{42}\</sup>text{Note that }\lim_{\overline{\omega}\to 0}\lambda\left(\overline{\omega}\right) = \frac{\Gamma'(0)}{\Gamma'(0)-\mu_bG'(0)} = 1, \text{ and that }\lim_{\overline{\omega}\to\overline{\omega}^*}\lambda\left(\overline{\omega}\right) = \frac{\Gamma'(\overline{\omega}^*)}{\Gamma'(\overline{\omega}^*)-\mu_bG'(\overline{\omega}^*)} = +\infty.$   $^{43}\text{It follows from the fact that } k'\left(\overline{\omega}\right) = \lambda'\left(\overline{\omega}\right)\left[\frac{\Gamma(\overline{\omega})-\mu_bG(\overline{\omega})}{1-\Gamma(\overline{\omega})}\right] + \lambda\left(\overline{\omega}\right)\left[\frac{\Gamma'(\overline{\omega})-\mu_bG'(\overline{\omega})}{1-\Gamma(\overline{\omega})}\right] + \Gamma'\left(\overline{\omega}\right)\lambda\left(\overline{\omega}\right)\left[\frac{\Gamma(\overline{\omega})-\mu_bG(\overline{\omega})}{(1-\Gamma(\overline{\omega}))^2}\right] = \frac{\lambda'(\overline{\omega})}{\lambda(\overline{\omega})}\left[k\left(\overline{\omega}\right)-1\right] + \frac{\Gamma'(\overline{\omega})}{1-\Gamma(\overline{\omega})}k\left(\overline{\omega}\right) > 0, \text{ for } \overline{\omega} \in (0,\overline{\omega}^*) \text{ and that } p'\left(\overline{\omega}\right) = \frac{\lambda'(\overline{\omega})(1-\Gamma(\overline{\omega}))}{[(1-\Gamma(\overline{\omega}))+\lambda(\overline{\omega})(\Gamma(\overline{\omega})-\mu_bG(\overline{\omega}))]^2} = \left(\frac{p(\overline{\omega})}{k(\overline{\omega})}\right)\left(\frac{\lambda'(\overline{\omega})}{\lambda(\overline{\omega})}\right) > 0, \text{ for } \overline{\omega} \in (0,\overline{\omega}^*). \text{ In addition, } we establish that <math display="block">\lim_{\overline{\omega}\to 0} p\left(\overline{\omega}\right) = \frac{\Gamma'(0)/[\Gamma'(0)-\mu_bG'(0)]}{[1-\Gamma(0)]+\lambda(0)[\Gamma(0)-\mu_bG(0)]} = 1, \text{ and that } \lim_{\overline{\omega}\to\overline{\omega}^*} p\left(\overline{\omega}\right) = \lim_{\overline{\omega}\to\overline{\omega}^*} \frac{1}{\lambda(\overline{\omega})} + \Gamma(\overline{\omega})-\mu_bG(\overline{\omega})}{\frac{1-\Gamma(\overline{\omega})}{\lambda(\overline{\omega})}} = \frac{p^*}{1-\Gamma(0)} + \lambda(0)[\Gamma(0)-\mu_bG(0)]} = 1, \text{ and that } \lim_{\overline{\omega}\to\overline{\omega}^*} k\left(\overline{\omega}\right) = \lim_{\overline{\omega}\to\overline{\omega}^*} \left\{\frac{1-\Gamma(\overline{\omega})}{\lambda(\overline{\omega})} + \Gamma(\overline{\omega})-\mu_bG(\overline{\omega})}{\frac{1-\Gamma(\overline{\omega})}{\lambda(\overline{\omega})}}\right\} = +\infty.$ 

ternal finance premium can be specified as the following form:

$$\frac{R^k}{R} = \left(\frac{QK}{N}\right)^{\psi},\,$$

where  $\psi$  is the steady state elasticity of  $p(\overline{\omega})$  with respect to  $k(\overline{\omega})$ , which is  $\frac{\partial p(\overline{\omega})/p(\overline{\omega})}{\partial k(\overline{\omega})/k(\overline{\omega})} = \frac{\lambda'(\overline{\omega})}{\lambda'(\overline{\omega})[k(\overline{\omega})-1]+\lambda(\overline{\omega})\frac{\Gamma'(\overline{\omega})}{1-\Gamma(\overline{\omega})}k(\overline{\omega})} > 0 \text{ for } \overline{\omega} \in (0, \overline{\omega}^*) \text{ (i.e., } p \in (1, p^*)).$ 

**Linearisation of Net Worth Evolution** First of all, note that the cost of foreign borrowing,  $R_{t+1}^k = R_t \Psi_t = R_t^* \Psi_t \left(\frac{S_{t+1}}{S_t}\right)$ , in (3.34) and the entrepreneurs' balance sheet,  $Q_t K_t = N_t + S_t B_t^*$ , in (3.24) can be approximated around the zero inflation symmetric steady state where  $R^k = R\Psi$ , and  $K = N + B^*$  as:

$$\widehat{R}_{t+1}^k = \widehat{R}_t^* + \widehat{\Psi}_t + \widehat{S}_{t+1} - \widehat{S}_t$$

and

$$\widehat{Q}_t + \widehat{K}_t = \left(\frac{N}{K}\right)\widehat{N}_t + \left(1 - \frac{N}{K}\right)\left(\widehat{S}_t + \widehat{B}_t^*\right),$$

respectively. Next, consider the entrepreneurs' net worth evolution,  $\frac{N_t}{V_t} = [\phi\{R_t^k Q_{t-1} K_{t-1} - R_{t-1}^* \Psi_{t-1} B_{t-1}^* S_t\} + (1-\phi) F]$ , in (3.35) in the text. We establish that the steady state relation for the net worth evolution is that  $\frac{N}{V} = [\phi\{R^k QK - R^* \Psi B^* S\} + (1-\phi) F]$ , where Q = 1 and V = 1. Then, by using the relation,  $x_t \simeq x(1+\widehat{x}_t)$ , the left hand side (LHS) around the steady state can be approximated as

$$\frac{N_t}{V_t} \simeq N \left[ 1 + \widehat{N}_t - \widehat{V}_t \right].$$

Each term of the right hand side (RHS) can be approximated around the steady state as:

$$\phi R_t^k Q_{t-1} K_{t-1} \simeq \phi R^k K \left[ 1 + \widehat{R}_t^k + \widehat{Q}_{t-1} + \widehat{K}_{t-1} \right]$$

and

$$-\phi R_{t-1}^* \Psi_{t-1} S_{t-1} B_{t-1}^* \simeq -\phi R^* \Psi S B^* \left[ 1 + \widehat{R}_{t-1}^* + \widehat{\Psi}_{t-1} + \widehat{B}_{t-1}^* + \widehat{S}_t \right]$$
$$\simeq -\phi R^k \left( K - N \right) \left[ 1 + \widehat{R}_{t-1}^* + \widehat{\Psi}_{t-1} + \widehat{B}_{t-1}^* + \widehat{S}_t \right],$$

respectively. By combining all these therms and dividing both sides by  $\phi R^k N$ , we obtain:

$$\frac{\widehat{N}_{t} - \widehat{V}_{t}}{\phi R^{k}} = \left(\frac{K}{N}\right) \left(\widehat{R}_{t}^{k} + \widehat{Q}_{t-1} + \widehat{K}_{t-1}\right) - \left(\frac{K}{N} - 1\right) \left(\widehat{R}_{t-1}^{*} + \widehat{\Psi}_{t-1} + \widehat{B}_{t-1}^{*} + \widehat{S}_{t}\right) 
= \left[\left(\frac{K}{N}\right) \widehat{R}_{t}^{k} + \widehat{N}_{t-1} + \left(\frac{K}{N} - 1\right) \left(\widehat{S}_{t-1} + \widehat{B}_{t-1}^{*}\right)\right] 
- \left(\frac{K}{N} - 1\right) \left(\widehat{R}_{t-1}^{*} + \widehat{\Psi}_{t-1} + \widehat{B}_{t-1}^{*} + \widehat{S}_{t}\right) 
= \left(\frac{K}{N}\right) \widehat{R}_{t}^{k} - \left(\frac{K}{N} - 1\right) \left(\widehat{R}_{t-1}^{*} + \widehat{\Psi}_{t-1} + \widehat{S}_{t} - \widehat{S}_{t-1}\right) + \widehat{N}_{t-1},$$

where we use  $\widehat{Q}_{t-1} + \widehat{K}_{t-1} = \left(\frac{N}{K}\right) \widehat{N}_{t-1} + \left(1 - \frac{N}{K}\right) \left(\widehat{S}_{t-1} + \widehat{B}_{t-1}^*\right)$  in the second equality. It can be written as

$$\frac{\widehat{N}_t}{\phi R^k} = \left(\frac{K}{N}\right) \widehat{R}_t^k - \left(\frac{K}{N} - 1\right) \left(\widehat{R}_{t-1}^* + \widehat{\Psi}_{t-1} + \widehat{S}_t - \widehat{S}_{t-1}\right) + \widehat{N}_{t-1} + \left(\frac{1}{\phi R^k}\right) \widehat{V}_t,$$

$$179$$

which is an equation (B1.50) in Appendix B1.2.

# Chapter 4

# A Bayesian Look at Small Open Economy DSGE Model with Financial Frictions

#### 4.1 Introduction

In the previous chapter, we have developed a small open economy (SOE) dynamic stochastic general equilibrium (DSGE) model with financial frictions to analyse the impact of a sudden stop of capital inflows on an emerging market country. We have argued that: (i) a high degree of financial frictions could make a sudden stop crisis aggravated, since in this circumstances foreigners react susceptibly to even the temporary and slight distortions in entrepreneurs' financial condition; (ii) when an emerging market economy relies heavily on foreign resources for capital production, it could suffer a sudden stop crisis more severely, as the currency depreciation in a sudden stop crisis would raise the capital production cost additionally so that the

capital demand is further depressed; and (iii) a fixed exchange rate regime could be inferior to a floating exchange rate system in the face of sudden stop crises, in case a negative effect of a fixed exchange rate regime by limiting the improvement in price competitiveness for home goods offsets the positive effect by stabilising a rise in cost of foreign borrowing. Our analysis in the previous chapter relied on a calibrated DSGE model, where the parameter values follow from those in the previous literature, such as Gertler, Gilchrist and Natalucci (2007), Curdia (2007), and Ozkan and Unsal (2010). We have confirmed the above arguments through the simulation and experiments based on the calibrated DSGE model.

However, in order for the above theoretical arguments to be empirically relevant, one needs to confirm that corresponding parameters in the calibrated model, which reflect our assumption on the economy's environmental conditions, reflect the real world well. More specifically, it is important to empirically establish that (i) the parameter for the sensitivity of external finance premium to entrepreneurs' leverage ratio,  $\psi$ , is positive and sufficiently large to confirm the presence of substantial degree of financial frictions in the economy; (ii) the steady state share of domestic input in investment good composite,  $\eta_i$ , is smaller than that of domestic goods in consumption bundle,  $\eta$ , to establish that the economy relies heavily on the foreign resources for capital production; and (iii) the Taylor rule coefficient attached on nominal exchange rate,  $\alpha_s$ , is positive and sufficiently large to verify that the central bank in the emerging market country seeks to stabilise the nominal exchange rate.

However, as Beltran and Draper (2008) point out, a calibrated DSGE model approach is not very obvious in how to calibrate the parameters, in particular for the newly emerged parameters, nor is always robust to alternative calibra-

tion. In contrast, an estimated DSGE model would quantify the average values of parameters based on the observed data, so that it could capture the features of business cycle more realistically. In this sense, we estimate the small open economy DSGE model in the previous chapter to assess how valid and plausible the above arguments are. To estimate our DSGE model, Bayesian methods and Markov Chain Monte Carlo (MCMC) algorithm are applied. One of the virtues of Bayesian method is that it provides a coherent way of combining prior information about parameters with the data as viewed through the DSGE model. In addition, as Elekdag *et al.* (2006) point out, it allows for a complete characterisation of uncertainty around the parameter values by simulating the posterior distributions.

Using the data series from the United States and South Korea and the DSGE model in the previous chapter, we find some empirical evidence supporting the above arguments. First of all, we obtain the sizable estimate for the sensitivity parameter of external finance premium to entrepreneurs' leverage ratio,  $\psi$ , which suggests that there exists a substantial degree of financial frictions in the economy so that the economy could be vulnerable to a sudden stop in foreign fund inflow. Second, the steady state share of domestic inputs in investment good composite,  $\eta_i$ , is estimated to be much smaller than that of domestic goods in consumption bundle,  $\eta$ , which indicates that the capital production in the emerging market country relies significantly on the foreign resources. Accordingly, it is plausible that the currency depreciation in a sudden stop crisis could result in a large volume of capital contraction due to an increase in capital production cost as well as a rise in cost of foreign borrowing. Third, our estimation result shows that Taylor rule coefficient on nominal exchange rate,  $\alpha_s$ , has a positive but small value, which

suggests that the central bank implements the monetary policy in a way to allow the nominal exchange rate to float freely. Fourth, our variance decomposition analysis indicates that the main sources of business cycle in the emerging market economy could be foreign financial shocks.

The remainder of this chapter is structured as follows. Section 4.2 outlines the small open economy DSGE model developed in the previous chapter. In section 4.3, we briefly sketch Bayesian estimation methods and discuss the data and priors used in the estimation. In section 4.4, we discuss the results from Bayesian estimation and variance decompositions and check the robustness of the estimates. Section 4.5 concludes the chapter.

#### 4.2 The Model

The model estimated in this chapter is the small open economy DSGE model with financial frictions, which is constructed in the previous chapter. Having derived all the equilibrium conditions in the previous chapter, this section briefly reviews them, paying attention to the parameters in the model which will be estimated.

First of all, households seek to make an optimal decisions between labour and consumption, between consumption and savings, and between domestic and foreign deposit, which yield the labour supply function, the Euler equation in consumption and the uncovered interest rate parity condition (UIPC), respectively, as:

$$\frac{W_t}{P_t} = (C_t)^{\sigma} (L_t)^{\varphi}, \qquad (4.1)$$

$$1 = \beta E_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} R_t \right\}, \tag{4.2}$$

and

$$R_t = R_t^* \left( \frac{S_{t+1}}{S_t} \right), \tag{4.3}$$

where  $\sigma > 0$  is the inverse of the intertemporal elasticity of substitution in consumption,  $\varphi > 0$  is the inverse elasticity of labour supply, and  $\beta \in (0,1)$  is the discount factor. By construction, we have  $\beta = \frac{1}{R}$  in the steady state. In addition, households allocate their consumption bundle,  $C_t$ , between home and foreign goods, by the following demands for home and foreign goods

$$C_{H,t} = \eta \left(\frac{P_{H,t}}{P_t}\right)^{-\gamma} C_t, \tag{4.4}$$

and

$$C_{F,t} = (1 - \eta) \left(\frac{P_{F,t}}{P_t}\right)^{-\gamma} C_t, \tag{4.5}$$

where  $\eta \in (0,1)$  is the share of domestic goods in the consumption bundle in the steady state, i.e.,  $\eta = \frac{C_H}{C}$  and  $\gamma > 1$  is the elasticity of substitution between  $C_{H,t}$  and  $C_{F,t}$ .  $P_t$  is the consumer price index (CPI), given by

$$P_{t} \equiv \left[ \eta \left( P_{H,t} \right)^{1-\gamma} + (1-\eta) \left( P_{F,t} \right)^{1-\gamma} \right]^{\frac{1}{1-\gamma}}, \tag{4.6}$$

where the import price (in domestic currency) is governed by the assumptions of the law of one price (LOOP) and the small open economy (SOE), so that

$$P_{F,t} = S_t P_t^*. (4.7)$$

Some fraction of the domestically produced goods are consumed by the agents in foreign countries. Export demand for domestic goods in foreign countries is given by

$$C_{H,t}^* = \eta^* \left(\frac{P_{H,t}}{\mathcal{S}_t P_t}\right)^{-\gamma^*} Y_t^*, \tag{4.8}$$

with the real exchange rate defined as

$$S_t \equiv \frac{S_t P_t^*}{P_t}. (4.9)$$

In equation (4.8),  $\eta^* \in (0,1)$  and  $\gamma^* > 1$  are the share of domestic goods in foreign households' consumption bundle and the price sensitivity of export demand, respectively, which are assumed to be identical to the corresponding values in domestic consumers, i.e.,  $\eta^* = \eta$  and  $\gamma^* = \gamma$ .

Second, domestic entrepreneurs produce the wholesale goods, by combining labour and capital by the production function,

$$Y_{w,t} = A_t (K_{t-1})^{\alpha} (L_t)^{1-\alpha}.$$
(4.10)

Then, the entrepreneurs' cost minimisation subject to the above production technology yields the following demands for labour and capital,

$$\frac{W_t}{P_{H,t}} = (1 - \alpha) \left(\frac{Y_{w,t}}{L_t}\right) P_{w,t},\tag{4.11}$$

and

$$E_{t}\left\{R_{t+1}^{k}Q_{t}-(1-\delta)Q_{t+1}\right\}=E_{t}\left\{\alpha\left(\frac{Y_{w,t+1}}{K_{t}}\right)P_{w,t+1}\right\},$$
(4.12)

where  $\alpha \in (0,1)$  and  $\delta \in (0,1)$  denote a steady state share of capital in the production function and the quarterly capital depreciation rate, respectively. In addition, entrepreneurs are assumed to finance the capital acquisition partly by foreign borrowing, so that the entrepreneurs' financial condition is expressed as the following balance sheet

$$Q_t K_t = N_t + S_t B_t^*. (4.13)$$

In addition, the foreign lenders facing the agency problem would impose the risk premium on the entrepreneurs according to the entrepreneurs' leverage ratio, so that the entrepreneurs are confronted with the following external finance premium:

$$\Psi_t = \left(\frac{Q_t K_t}{N_t}\right)^{\psi}. \tag{4.14}$$

In equation (4.14),  $\psi > 0$  denote the sensitivity of the risk premium to the entrepreneurs' financial condition, so that the bigger value of  $\psi$  relates the higher external finance premium to the given rise in leverage ratio. Then, the cost of foreign borrowing for the entrepreneurs is given by

$$R_{t+1}^k = R_t^* \Psi_t \left( \frac{S_{t+1}}{S_t} \right). \tag{4.15}$$

In addition, the economy-wide net worth is determined by

$$N_{t} = \left[ \phi \left\{ R_{t}^{k} Q_{t-1} K_{t-1} - R_{t-1}^{*} \Psi_{t-1} B_{t-1}^{*} S_{t} \right\} + (1 - \phi) F \right] \cdot V_{t}, \tag{4.16}$$

where  $\phi \in (0,1)$  is the entrepreneurs' survival rate.

Third, capital producers supply capital goods to entrepreneurs according to

$$Q_t\left(\frac{P_{H,t}}{P_t}\right) = \frac{P_{I,t}}{P_t} \left[1 - \kappa \left(\frac{I_t}{K_{t-1}} - \delta\right)\right]^{-1},\tag{4.17}$$

and the economy-wide capital stock at the end of period t, is given by

$$K_{t} = \left[ \frac{I_{t}}{K_{t-1}} - \frac{\kappa}{2} \left( \frac{I_{t}}{K_{t-1}} - \delta \right)^{2} \right] K_{t-1} + (1 - \delta) K_{t-1}, \tag{4.18}$$

where  $\kappa > 0$  is the capital adjustment cost coefficient. In addition, investment good,  $I_t$ , used for capital production is composed of home and foreign goods, so that capital producers' demands for home and foreign inputs are given by:

$$I_{H,t} = \eta_i \left(\frac{P_{H,t}}{P_t}\right)^{-\gamma_i} I_t, \tag{4.19}$$

and

$$I_{F,t} = (1 - \eta_i) \left(\frac{P_{F,t}}{P_{H,t}}\right)^{-\gamma_i} I_t,$$
 (4.20)

where  $\eta_i \in (0,1)$  and  $\gamma_i > 1$  are the share of domestic inputs in the investment good composite and the elasticity of substitution between home and foreign inputs, respectively. The corresponding price index for the investment goods,  $P_{I,t}$ , is

$$P_{I,t} \equiv \left[ \eta_i \left( P_{H,t} \right)^{1-\gamma_i} + \left( 1 - \eta_i \right) \left( P_{F,t} \right)^{1-\gamma_i} \right]^{\frac{1}{1-\gamma_i}}, \tag{4.21}$$

so that the motion of the real input price,  $\frac{P_{I,t}}{P_t}$ , i.e., the cost of capital production, is affected by the values of  $(\eta, \gamma)$  and  $(\eta_i, \gamma_i)$ . For instance, under the circumstances of the heavier reliance on the foreign input in the investment good composite as compared to the preference over imported goods in consumption bundle, i.e.,  $\eta_i < \eta$  and  $\gamma_i = \gamma$ , a currency depreciation would result in an increase in the real investment good price,  $\frac{P_{I,t}}{P_t}$ , as discussed in the previous chapter.

Fourth, retailers purchase the wholesale goods from entrepreneurs in a perfectly competitive manner; differentiate them into their own varieties; and then set the retail price on them under Calvo-type nominal rigidity. Retailers' optimal price setting behaviour yields the New Keynesian Phillips curve (NKPC) for domestic final goods:

$$\pi_{H,t} = (\mu P_{w,t})^{\lambda} E_t \{\pi_{H,t+1}\}^{\beta},$$
(4.22)

with  $\mu = \frac{\epsilon}{\epsilon - 1}$  and  $\lambda = \frac{(1 - \theta)(1 - \beta \theta)}{\theta}$ , where  $\epsilon$  captures the substitutability among varieties, and  $\theta$  denotes the possibility of keeping the previous retail price unchanged, i.e., price stickiness. The inflation of domestic price index is given by

$$\pi_{H,t} \equiv \frac{P_{H,t}}{P_{H,t-1}} \tag{4.23}$$

and the equilibrium condition in the wholesale goods market implies

$$Y_{H,t} = Y_{w,t}. (4.24)$$

Domestic final goods are sold to domestic and foreign households, capital producers and government, so that the resource constraint for domestic final goods is given by

$$Y_{H,t} = C_{H,t} + C_{H,t}^* + I_{H,t} + G_t. (4.25)$$

Fifth, the government conducts monetary and fiscal policies. The central bank adjusts the nominal interest rate according to the following Taylor-type feedback rule with interest rate smoothing,

$$\left(\frac{R_t^n}{R^n}\right) = \left(\frac{R_{t-1}^n}{R^n}\right)^{\alpha_r} \left(\pi_t\right)^{(1-\alpha_r)\alpha_\pi} \left(\frac{Y_{H,t}}{Y_H}\right)^{(1-\alpha_r)\alpha_y} \left(\frac{S_t}{S}\right)^{(1-\alpha_r)\alpha_s} \exp\left\{\varepsilon_{r,t}\right\}, \quad (4.26)$$

where the nominal interest rate,  $R_t^n$ , is linked to the real riskless rate,  $R_t$ , by the following Fisher equation

$$R_t^n \equiv R_t E_t \left\{ \pi_{t+1} \right\}, \tag{4.27}$$

and the CPI inflation is given by

$$\pi_t \equiv \frac{P_t}{P_{t-1}}.\tag{4.28}$$

In equation (4.26),  $\alpha_r \in (0,1)$  is the weight on the nominal interest rate in the previous period, and  $\alpha_{\pi} > 1$ ,  $\alpha_{y} > 0$ , and  $\alpha_{s} \geq 0$  are the policy parameters attached to the CPI inflation, output gap, and the nominal exchange rate. In the free floating exchange rate regime,  $\alpha_{s}$  is equal to zero, but, in practice, many emerging market countries, which announce to adopt a free floating system, are

thought of actually seeking to stabilise the value of domestic currency to some degree, so that  $\alpha_s > 0$ .

Lastly, there are seven exogenous shocks in the model, such as  $\varepsilon_{r,t}$ ,  $\varepsilon_{a,t}$ ,  $\varepsilon_{v,t}$ ,  $\varepsilon_{g,t}$ ,  $\varepsilon_{y^*,t}$ ,  $\varepsilon_{\pi^*,t}$ , and  $\varepsilon_{r^*,t}$ , which are assumed to be all Gaussian white noises. The shocks arising from the technology,  $A_t$ , net worth evaluation,  $V_t$ , government spending,  $G_t$ , foreign output,  $Y_t^*$ , foreign interest rate,  $R_t^*$ , and foreign CPI inflation,  $\pi_t^*$ , are assumed to obey the stationary first-order autoregressive process, given by

$$A_t = (A_{t-1})^{\rho_a} \exp\left\{\varepsilon_{a,t}\right\},\tag{4.29}$$

$$V_t = (V_{t-1})^{\rho_v} \exp\{\varepsilon_{v,t}\},$$
 (4.30)

$$G_t = (G_{t-1})^{\rho_g} \exp\left\{\varepsilon_{q,t}\right\},\tag{4.31}$$

$$Y_t^* = (Y_{t-1}^*)^{\rho_{y^*}} \exp\{\varepsilon_{y^*,t}\}, \qquad (4.32)$$

$$R_t^* = (R_{t-1}^*)^{\rho_{r^*}} \exp\{\varepsilon_{r^*,t}\},$$
 (4.33)

and

$$\pi_t^* = (\pi_{t-1}^*)^{\rho_{\pi^*}} \exp\{\varepsilon_{\pi^*,t}\}, \qquad (4.34)$$

where  $|\rho_a| < 1, \; |\rho_v| < 1, \; |\rho_z| < 1, \; \left|\rho_{y^*}\right| < 1, \; \left|\rho_{\pi^*}\right| < 1, \; \text{and} \; \left|\rho_{r^*}\right| < 1 \; \text{are the}$ 

persistence parameters in each shock process.

## 4.3 Estimation Methodology

A DSGE model can be estimated by using Bayesian methods, as described in An and Schorfheide (2007) and Canova (2011). In this section, we outline Bayesian methods for estimating the DSGE model and discuss prior densities and data to be used for estimation.<sup>1</sup>

#### 4.3.1 State-space Representation

In order to be estimated, a DSGE model should be firstly solved. A DSGE model can be solved following the procedure described in Uhlig (1999): (i) identifying equilibrium conditions to construct a non-linear rational expectations (RE) system; (ii) approximating the non-linear equations around the steady state to transform the non-linear rational expectations (RE) system into the first order linear one; (iii) solving the first order linear rational expectations (RE) system by using the numerical method, as shown in Blanchard and Kahn (1980) and Klein (2000); and (iv) writing the rational expectations (RE) solution in the state-space representation, discussed below.

Technically speaking, our DSGE model belongs to a non-linear rational expectations system with 34 endogenous variables and 7 exogenous shocks, which can be approximated around the steady state to obtain the linear rational expec-

<sup>&</sup>lt;sup>1</sup>More detailed discussion about Bayesian MCMC estimation methods for a DSGE model are presented in Appendix C2.

tations system as listed in Appendix B1.2. Then the rational expectations (RE) solution to the DSGE model can be represented by the following state-space form:

$$E\{x_{t+1}\} = Fx_t + Gz_{t+1}$$
  
 $y_t = H'x_t + v_t$  (4.35)

where  $x_t$  is  $34 \times 1$  vector of the state variables,  $z_t$  is 7 dimensional structural shocks,  $y_t$  is observed variables, and  $v_t$  is the measurement errors. As shown in (4.35), the state-space representation is made up of two equation blocks: state transition equations and observation equations. The transition equations govern the evolution of the state vector,  $x_t$  by F and G, which are  $34 \times 34$  and  $34 \times 7$  matrices of functions of structural parameters in the model, respectively. The observation equations relate the observables,  $y_t$ , to the state variables,  $x_t$ , through the  $7 \times 34$  matrix H'. In addition, structural shocks,  $z_t$ , and measurement errors,  $v_t$ , are assumed to be two independent Gaussian white noise series, i.e.,  $z_t \sim \mathcal{N}(0,Q)$  and  $v_t \sim \mathcal{N}(0,R)$ . The intuitive description on the estimation process by using the state-space representation in (4.35) is as follows. When the economy represented by the 34 dynamic equilibrium conditions listed in Appendix B1.2 is hit by some of the 7 structural shocks, 34 endogenous variables in the economy, i.e., the state variables, yield the general equilibrium path over time by the transition equations. Then, the generated movements of state variables are evaluated by the

<sup>&</sup>lt;sup>2</sup>We have 7 observables, as discussed below. Now that we have the same number of observables as the structural shocks, we can evaluate the likelihood function of the observed data. However, if we included more observables than structural shocks in the measurement equations, the model would be stochastically singular, as Ingram *et al.* (1994) and Ireland (2004) point out. In this case, the model predicts that certain combinations of the structural variables would be deterministic and be at odds with the data.

measurement errors at each time. Then, the prior beliefs about the parameter values are rectified by the. Bayesian MCMC algorithm, which will be discussed in more detail in the subsequent part.

Having represented the rational expectations (RE) solution to the DSGE model, we may apply Bayesian method and Markov Chain Monte Carlo (MCMC) procedure to estimate the DSGE model.

#### 4.3.2 Bayesian Estimation

The aim of implementing Bayesian method is to characterise the posterior density of the parameters. By Bayes' theorem, the posterior density of parameters,  $p(\theta|y)$ , is obtained by combining the likelihood for the data,  $\mathcal{L}(y|\theta)$ , and the prior density of parameters,  $p(\theta)$ , given by:

$$p(\theta|y) = \frac{\mathcal{L}(y|\theta) p(\theta)}{\int \mathcal{L}(y|\theta) p(\theta) d\theta},$$
(4.36)

where  $\theta$  is the parameter vector and y is the observed data. Given that the DSGE model is linear and the shocks are all independently and normally distributed, the likelihood of the data,  $\mathcal{L}(y|\theta)$ , can be calculated by applying the Kalman filter to the state-space representation of the model in (4.35), given by:

$$\log \mathcal{L}(y|\theta) = -\sum_{t=1}^{T} \left[ \frac{N_y}{2} \log(2\pi) + \frac{1}{2} \log \left| \Omega_{t|t-1} \right| + \frac{1}{2} v_t' \Omega_{t|t-1}^{-1} v_t \right], \tag{4.37}$$

where T is the number of periods,  $N_y$  is the number of observables,  $v_t = y_t - H'x_{t|t-1}$  is the prediction error for the observables, and  $\Omega_{t|t-1} = H'\Sigma_{t|t-1}H + R$  is the

associated covariance matrix.<sup>3</sup>

Having obtained the likelihood function,  $\mathcal{L}(y|\theta)$ , the posterior density of parameters,  $p(\theta|y)$ , can be estimated by a simulation method such as Markov Chain Monte Carlo (MCMC) algorithm. As discussed by Schorfheide (2000), the procedure follows a two-step approach. In the first step, the initial guess of the posterior mode,  $\theta_m$ , and the associated covariance matrix,  $\Sigma_m$ , are found by a numerical optimisation routine which maximises the posterior kernel,  $\mathcal{L}(y|\theta) p(\theta)$ . In the second step, a sequence of the parameters,  $\theta_j$ , is generated by an MCMC method, such as Random Walk Metropolis-Hastings (RWMH) algorithm, which is used to build the shape of the posterior density,  $p(\theta|y)$ . That is, (typically) starting from the posterior mode,  $\theta_m$ , for each step of the random draw,  $j = 1, \ldots, N$ , a candidate sample,  $\theta_i^*$ , is drawn from a proposal density,  $\theta_i^* \sim \mathcal{N}(\theta_{j-1}, c^2 \Sigma_m)$ , with c denoting a scale factor, and then, the jump from  $\theta_{j-1}$  to  $\theta_j^*$  is accepted with the acceptance rate, r, with

$$r = \min \left\{ 1, \frac{\mathcal{L}\left(y|\theta_{j}^{*}\right)p\left(\theta_{j}^{*}\right)}{\mathcal{L}\left(y|\theta_{j-1}\right)p\left(\theta_{j-1}\right)} \right\}, \tag{4.38}$$

and rejected with 1-r. In this fashion, the algorithm constructs the empirical posterior density,  $\widetilde{p}(\theta|y)$ , which converges to the true posterior density,  $p(\theta|y)$ , as the number of the chains approaches to the infinity.<sup>5</sup> Finally, based on the em-

<sup>&</sup>lt;sup>3</sup>The value of each term is obtained from the Kalman filter recursion for the given initial values  $x_{1|0}$  and  $\Sigma_{1|0}$ . As discussed in Appendix C2.1, they are calculated as:  $x_{t+1|t} = Fx_{t|t}$  and  $\Sigma_{t+1|t} = F\Sigma_{t|t}F' + GQG'$ , where  $x_{t|t} = x_{t|t-1} + \Sigma_{t|t-1}H(H'\Sigma_{t|t-1}H + R)^{-1}(y_t - H'x_{t|-1})$  and  $\Sigma_{t|t} = \Sigma_{t|t-1} - \Sigma_{t|t-1}H(H'\Sigma_{t|t-1}H + R)^{-1}H'\Sigma_{t|t-1}$ .

The covariance matrix,  $\Sigma_m$ , is the inverse of the negative Hessian matrix computed at the posterior mode,  $\theta_m$ , i.e.,  $\Sigma_m = \left[ -\left(\frac{\partial^2 \log(\mathcal{L}(y|\theta)p(\theta))}{\partial \theta \partial \theta'}\right)_{\theta=\theta_m} \right]^{-1}$ .

 $<sup>^5</sup>$ As shown in Johannes and Polson (2004), the sequence generated by the MCMC algorithm is the Markov chain, by construction, so that the empirical posterior density from any starting point,  $\theta_0$ , converges to the true posterior density, by the ergodic theory for Markov chains.

pirical posterior density,  $\widetilde{p}(\theta|y)$ , the estimates for mean, variance, and confidence interval are calculated as

$$E\left\{h\left(\theta\right)|y\right\} = \frac{1}{N_{sim}} \sum_{j=1}^{N_{sim}} h\left(\theta_{j}\right) \widetilde{p}\left(\theta_{j}|y\right), \tag{4.39}$$

where  $h(\theta)$  is a function of the posterior estimator of the parameters, and  $N_{sim}$  is the number of iterations net of those in the 'burn-in' period, which are discarded to avoid the potential dependency of the chains on the starting points.

In our estimation, we generate multiple chains of 100,000 replications, discarding the first 20 percent of the iterations,  $N_{sim} = 100,000-20,000 = 80,000$ . In addition, following Brooks and Gelman (1998), we generate 3 parallel sequences to check the convergence of the generated draws. We adjust the scale factor, c, attached on covariance matrix in the jumping distribution,  $\Sigma_m$ , to attain the acceptance rate, r = 0.25, following Roberts, Gelman and Gilks' (1997) suggestion.

#### 4.3.3 Data

The model is estimated using seven quarterly data series from the United States and South Korea ( $N_y = 7$ ). South Korea is chosen because it is an emerging market economy that experienced a sudden stop crisis in 1997-98 and was affected by the global financial crisis in 2007-2008, and the United States is adopted because the economic relationship between the Unite States and South Korea is very close and the size of the US economy is so large that it could be considered the world economy from a perspective of South Korea. For the United States, the data consist of the real gross domestic product (GDP), the consumer price index (CPI), and the rate of return for 3-month treasury bond (TB), which correspond to foreign output,

 $Y_t^*$ , foreign CPI inflation,  $\pi_t^*$ , and foreign interest rate,  $R_t^*$ , respectively. For South Korea, the observed variables are the real GDP, the CPI, the rate of return for 3-year corporate bond with rating of AA-, and the won/dollar nominal exchange rate, which are linked to domestic output,  $Y_{H,t}$ , domestic CPI inflation,  $\pi_t$ , capital returns,  $R_t^k$ , and the nominal exchange rate,  $S_t$ .<sup>6</sup> The annual rates of the US TB and South Korea's corporate bond are converted into the corresponding quarterly rates. All the data are detrended by the Hodrick-Prescott (HP) filter to obtain the stationary series,<sup>7</sup> and measured in terms of the percent deviation from the steady state (i.e., the corresponding Hodrick-Prescott trends) to be conformable to our log-linearised DSGE model. The sample runs from 1995:Q1 to 2013:Q1 (T = 73).

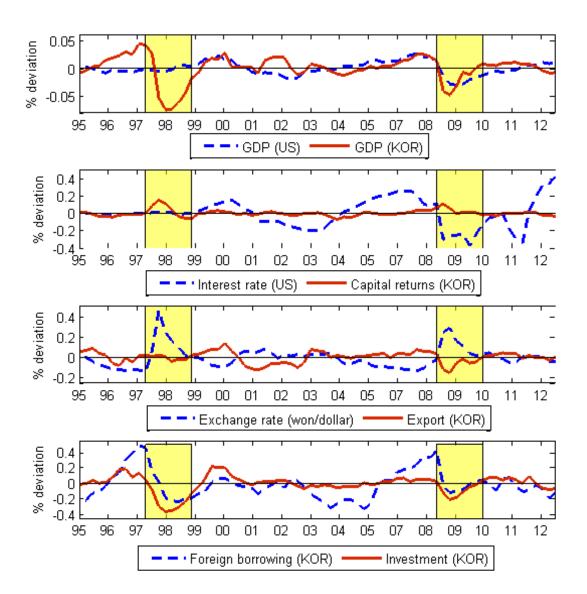
Figure 4.1 shows the movements in some main macroeconomic variables during the sample period, which include the real GDP and TB rate in the US, the won/dollar exchange rate, and GDP, corporate bond rate, investment and export in South Korea.<sup>8</sup> During the sample period, South Korea experienced two financial crisis episodes: the currency crisis in the late 1990s (1997:Q3-1999:Q2) and the global financial crisis in the late 2000s (2008:Q3-2010:Q2). The crisis periods are represented by the shaded areas in Figure 4.1. The prominent features

<sup>&</sup>lt;sup>6</sup>US data on GDP, CPI, and TB rate are extracted from the Bureau of Economic Analysis in the Department of Commerce (www.bea.gov), the Bureau of Labour Statistics in the Department of Labour (www.bls.gov), and the Board of Governors of the Federal Reserve System (www.federalreserve.gov), respectively. South Korean data are collected from the Bank of Korea (ecos.bok.or.kr).

<sup>&</sup>lt;sup>7</sup>The Hodrick-Prescott (HP) filter is used to isolate a 'cycle' (or deviation from the trend),  $c_t$ , from the original time series,  $z_t$ , which is assumed to be I(1). Thus,  $c_t = z_t - \mu_t$ , where the HP trend,  $\mu_t$ , is I(1) and  $c_t$  is I(0). The HP trend,  $\mu_t$ , is obtained by solving the problem,  $\min_{\mu_t} \sum_{t=1}^{T} \left[ (z_t - \mu_t)^2 + \lambda \left( \triangle^2 \mu_{t+1} \right)^2 \right]$ , where  $\triangle^j$  is the j-th order difference operator and  $\lambda$  is a weight on the trend. The choices for  $\lambda$  are conventionally recommended to be 100 for the annual data, 1600 for the quarterly data, and 14400 for the monthly data.

<sup>&</sup>lt;sup>8</sup>Figure 4.1 displays how economic variables move in crisis period, which are represented by the shaded area. Note that not all the variables in Figure 4.1 are the same as the data series used in estimation.

Figure 4.1: Actual Time Series in the US and South Korea



<sup>\*</sup> Vertical axes represent the percent deviation of the variables from the corresponding HP trends, and horizontal axes represent years.

Table 4.1: Parameters calibrated

	Description	Value
$\beta, \beta^*$	home and foreign discount factors	0.99
$R, R^*$	home and foreign interest rates in the steady state	1.0101
$\gamma$	substitutability between home and foreign goods	1.5
${\gamma}_i$	substitutability between home and foreign inputs	1.5
$\alpha$	capital share in production function	0.3
$\delta$	capital depreciation rate	0.025
$\phi$	entrepreneurs' survival rate	0.975
$\kappa$	capital adjustment cost coefficient	1.2
$\epsilon$	elasticity of substitution among retail goods	6
$C_H^*/Y_H$	export-to-output ratio in the steady state	0.2
$G/Y_H$	government spending-to-output ratio in the steady state	0.2

of the crisis periods are widened risk spreads<sup>9</sup>, a dramatic depreciation of the Korean won, and sharp and significant contractions in foreign borrowing, investment and GDP in South Korea.

#### 4.3.4 Priors for the Parameters

In this part, we discuss our prior beliefs on the parameters. First of all, following the practice of Bayesian estimation, we fix some parameters throughout the estimation procedure, by calibrating them in line with the existing literature. Calibrated parameters are listed in Table 4.1, where the values are identical to those in the previous chapter. The discount factors in home and foreign countries,  $\beta$  and  $\beta^*$ , are all set at 0.99, so that the corresponding steady state value of quarterly riskless rates are calculated as  $R(=\frac{1}{\beta}) = 1.0101$  and  $R^*(=\frac{1}{\beta^*}) = 1.0101$ . The substitutability between home and foreign goods in consumption bundle and in-

<sup>&</sup>lt;sup>9</sup>The risk spread is measured by the gap between the Korean capital returns (represented by the rate of returns for 3-year corporate bond with rating of AA-) and the US interest rate (represented by the 3-month US TB rate).

Table 4.2: Priors for the Parameters

	Description	Domain	Shape	Mean	S.E.
$\sigma$	intertemporal elasticity of consumption	$(0,+\infty)$	gamma	1.5	0.2
$\varphi$	elasticity of labour supply	$(0,+\infty)$	$\operatorname{gamma}$	3	0.5
$\psi$	sensitivity of risk premium	$(0,+\infty)$	$\operatorname{gamma}$	0.05	0.04
K/N	capital-to-net worth in the steady state	$(1,+\infty)$	gamma	2	0.3
$\theta$	probability of not adjusting retail price	(0, 1)	beta	0.75	0.2
$\eta$	share of home good in consumption	(0, 1)	beta	0.6	0.2
$\eta_i$	share of domestic input in investment	(0, 1)	beta	0.6	0.2
$\alpha_r$	persistence of interest rate in Taylor rule	(0, 1)	beta	0.7	0.2
$\alpha_{\pi}$	Taylor rule coefficient on inflation	$(1,+\infty)$	gamma	1.7	0.3
$\alpha_y$	Taylor rule coefficient on output gap	$(0,+\infty)$	gamma	0.2	0.1
$\alpha_s$	Taylor rule coefficient on exchange rate	$(0,+\infty)$	gamma	0.2	0.1
$ ho_{ u}$	persistence of net worth evaluation shock	(0, 1)	beta	0.7	0.1
$ ho_g$	persistence of government spending shock	(0, 1)	beta	0.7	0.1
$ ho_{r^*}$	persistence of foreign interest rate shock	(0, 1)	beta	0.7	0.1
$ ho_{y^*}$	persistence of foreign output shock	(0, 1)	beta	0.7	0.1
$ ho_{\pi^*}$	persistence of foreign inflation shock	(0, 1)	beta	0.7	0.1
$ ho_a$	persistence of technology shock	(0, 1)	beta	0.7	0.1
$\varepsilon_v$	S.E. of net worth evaluation shock	$(0,+\infty)$	inv. gam.	0.03	inf.
$\varepsilon_{m{g}}$	S.E. of government spending shock	$(0,+\infty)$	inv. gam.	0.03	inf.
$\varepsilon_{r^*}$	S.E. of foreign interest rate shock	$(0,+\infty)$	inv. gam.	0.03	inf.
$\varepsilon_{y^*}$	S.E. of foreign output shock	$(0,+\infty)$	inv. gam.	0.03	inf.
$\varepsilon_{\pi^*}$	S.E. of foreign inflation shock	$(0,+\infty)$	inv. gam.	0.03	inf.
$\varepsilon_a$	S.E. of technology shock	$(0,+\infty)$	inv. gam.	0.03	inf.
$\varepsilon_r$	S.E. of monetary policy shock	$(0,+\infty)$	inv. gam.	0.03	inf.

vestment inputs in investment good composite,  $\gamma$  and  $\gamma_i$ , respectively, are all set to be 1.5. We fix the capital share in the production technology,  $\alpha$ , quarterly capital depreciation rate,  $\delta$ , and entrepreneurs' survival rate,  $\phi$ , at 0.3, 0.025, and 0.975, respectively. Capital adjustment cost coefficient,  $\kappa$ , and elasticity of substitution among varieties,  $\epsilon$ , are pinned down at 1.2 and 0.6, respectively. The steady state shares of export and government spending out of the gross domestic product,  $\frac{C_H^*}{Y_H}$  and  $\frac{G}{Y_H}$ , are assumed to be all 0.2.

Next, we choose the prior densities for the estimated parameters by con-

sidering the theoretical restrictions for the parameters. Table 4.2 shows the priors for the estimated parameters, and Table C2 in Appendix C1 compares the priors in the existing literature. Following the conventions, gamma distributions are used for the parameters bounded to be positive, such as  $\sigma$ ,  $\varphi$ ,  $\psi$ ,  $\frac{K}{N}$ ,  $\alpha_{\pi}$ ,  $\alpha_{y}$ , and  $\alpha_s$ , and beta distributions are adopted for parameters for fractions or probabilities, such as  $\theta$ ,  $\eta$ ,  $\eta_i$ ,  $\alpha_r$ ,  $\rho_v$ ,  $\rho_a$ ,  $\rho_g$ ,  $\rho_{r^*}$ ,  $\rho_{y^*}$ , and  $\rho_{\pi^*}$ , since they are bounded between 0 and 1. Prior means for the estimated parameters are taken from the calibrated model in the previous chapter, which are standard in the business cycle literature, such as Elekdag et al. (2006), Adolfson et al. (2007), Adjemian et al. (2007), Yie and Yoo (2011) and Lee and Rhee (2013), as shown in Tables C2 in Appendix C1. The inverse of intertemporal elasticity of consumption,  $\sigma$ , and the inverse elasticity of labour supply,  $\varphi$ , are assumed to be centered at 1.5 and 3, respectively. The steady state value of entrepreneurs' leverage ratio,  $\frac{K}{N}$ , and the sensitivity of external finance premium to the leverage ratio,  $\psi$ , are assumed to be distributed around 2 and 0.05, respectively. The prior means for Taylor rule coefficients,  $\alpha_{\pi}$ ,  $\alpha_{y}$ , and  $\alpha_{s}$ , are set at 1.7, 0.2, and 0.2, respectively, where the strong grip for inflation and weak grips for output and exchange rate reflect the fact that the Bank of Korea has adopted inflation targeting and free floating exchange rate regime since 1998. We assume that the interest rate smoothing factor in Taylor rule,  $\alpha_r$ , and domestic retail price stickiness parameter,  $\theta$ , are centred at 0.7 and 0.75, respectively. The share of home goods in consumption bundle and that of domestic input in investment good composite,  $\eta$  and  $\eta_i$ , are assumed to be all distributed around 0.6, so that the real investment good price,  $\frac{P_{I,t}}{P_t}$ , and capital price,  $Q_t$ , would not be affected by the currency depreciation,  $S_t$ . The prior means for persistence coefficients, such as  $\rho_v$ ,  $\rho_g$ ,  $\rho_a$ ,  $\rho_{r^*}$ ,  $\rho_{y^*}$ , and  $\rho_{\pi^*}$ , are all set to be 0.7. In addition, Gaussian shocks, such as  $\varepsilon_{\nu}$ ,  $\varepsilon_{r}$ ,  $\varepsilon_{g}$ ,  $\varepsilon_{a}$ ,  $\varepsilon_{r^{*}}$ ,  $\varepsilon_{y^{*}}$ , and  $\varepsilon_{\pi^{*}}$ , are all assumed to follow the inverse gamma distributions with prior means of 0.03. Lastly, we choose relatively large values for the prior standard error of each parameter, to allow for the uncertainty about the prior belief on parameters.<sup>10</sup>

## 4.4 Estimation Results

Given the prior densities and the actual data as discussed above, our DSGE model are estimated by using Bayesian MCMC methods. In this section, we discuss the estimation results, which include the Bayesian estimates for the parameters, variance decomposition based on the estimates, and the robustness of Bayesian estimates.<sup>11</sup>

#### 4.4.1 Bayesian Estimates

Bayesian estimates for the parameters are summarised in Table 4.3, along with the 95 percent posterior confidence intervals (C.I.), which serve to measure the uncertainty surrounding these estimates. In addition, Figure 4.2 displays the empirical posterior densities constructed by the MCMC methods (which are shown in red solid lines), together with the corresponding priors (shown in gray solid lines) and posterior modes (shown in blue dotted lines).

<sup>&</sup>lt;sup>10</sup>The large values of prior standard errors suggest that the priors have fairly flat shapes. As Adjemian *et al.* (2007) argue, in case the data are very informative about the parameter, the loose priors could be well suited for the estimation of DSGE model.

<sup>&</sup>lt;sup>11</sup>In addition, Appendix C3 presents a discussion of convergence of MCMC sequence, and the associated test result for our model.

Table 4.3: Parameter Estimates

	Prior			P	osterior	
	Mean	S.E.	Mode	t-stat.	Mean	95% C.I.
$\sigma$	1.5	0.2	1.3575	6.0141	1.4283	[1.0493, 1.7893]
$\varphi$	3	0.5	2.2296	5.2108	2.2298	[1.5011, 2.8983]
$\psi$	0.05	0.04	0.0775	2.2972	0.0839	[0.0242, 0.1368]
K/N	2	0.3	1.4083	10.8268	1.4601	[1.2194, 1.6866]
$\theta$	0.75	0.2	0.8567	45.7865	0.8565	[0.8353, 0.8979]
$\eta$	0.5	0.2	0.9960	291.8509	0.9926	[0.9860, 0.9996]
$\eta_i$	0.5	0.2	0.0959	9.3891	0.0941	[0.0745, 0.1120]
$\alpha_r$	0.7	0.2	0.8710	24.0719	0.8543	[0.7955, 0.9204]
$\alpha_{\pi}$	1.7	0.3	1.6516	6.8351	1.7052	[1.3430, 2.0830]
$\alpha_y$	0.2	0.1	0.2943	3.3169	0.3747	[0.2124, 0.5422]
$\alpha_s$	0.2	0.1	0.0308	2.6724	0.0410	[0.0189, 0.0637]
$ ho_{ u}$	0.7	0.1	0.88884	30.1717	0.8715	[0.8118, 0.9390]
$ ho_g$	0.7	0.1	0.8751	3.9698	0.7052	[0.4155, 0.9959]
$ ho_{r^*}$	0.7	0.1	0.2451	4.8527	0.2523	[0.1624, 0.3405]
$ ho_{y^*}$	0.7	0.1	0.8457	20.4507	0.8422	[0.7752, 0.9080]
$ ho_{\pi^*}$	0.7	0.1	0.8632	14.8380	0.8103	[0.7094, 0.9150]
$ ho_a$	0.7	0.1	0.8751	22.2254	0.8629	[0.7992, 0.9282]
$arepsilon_{oldsymbol{v}}$	0.03	inf.	0.0291	3.8630	0.0343	[0.0172, 0.0496]
$\varepsilon_{m{g}}$	0.03	inf.	0.0138	2.4660	0.0292	[0.0062, 0.0659]
$\varepsilon_{r^*}$	0.03	inf.	0.1291	10.7472	0.1331	[0.1128, 0.1533]
$\varepsilon_{y^*}$	0.03	inf.	0.0055	12.2611	0.0056	[0.0048, 0.0064]
$\varepsilon_{\pi^*}$	0.03	inf.	0.0060	12.0405	0.0060	[0.0052, 0.0068]
$\varepsilon_a$	0.03	inf.	0.0269	4.5157	0.0295	[0.0194, 0.0402]
$\varepsilon_r$	0.03	inf.	0.0047	3.6309	0.0060	[0.0033, 0.0087]

First of all, our estimation results indicate the presence of substantial degree of financial frictions in the small open economy. That is, the sensitivity of the external finance premium to the entrepreneurs' leverage ratio,  $\psi$ , is estimated as 0.0839, and its 95 percent confidence interval ranges between 0.0242 and 0.1368, which is away from zero and statistically significant. These estimates suggest that foreign lenders might raise the risk premium by over 0.08 percent when they perceive one percent of the rise in entrepreneurs' leverage ratio. As discussed in the previous chapter, the raised risk premium due to the perceived distortion of entrepreneurs' leverage ratio would lead to contractions in capital demand and output production, which portrays a typical sudden stop crisis episode in emerging market countries. In addition, a posterior mean value for the steady state leverage ratio,  $\frac{K}{N}$ , is estimated as 1.4601 with a 95 percent confidence interval covering from 1.2194 to 1.6866, implying that the quarterly risk premium in the steady state is approximated as  $\frac{R^k}{R} = \left(\frac{K}{N}\right)^{\psi} = 1.0323$ . The estimated annual risk premium,  $13.6 = 100 * (1.0323)^4$ ) percent seems to be quite high as compared to the corresponding historical average, 6.3 percent in our sample period. However, it may reflect the impact of a financial crisis when the risk premium between the associated rates exceeded 18.9 percent in 1998:Q1, as Elekdag et al. (2006) argue. In short, the large values of the Bayesian estimates for the sensitivity of external finance premium to entrepreneurs' leverage ratio and the steady state risk premium suggest that there may exist a substantial degree of financial frictions in the economy, and that the economy would be vulnerable to foreign financial shocks.

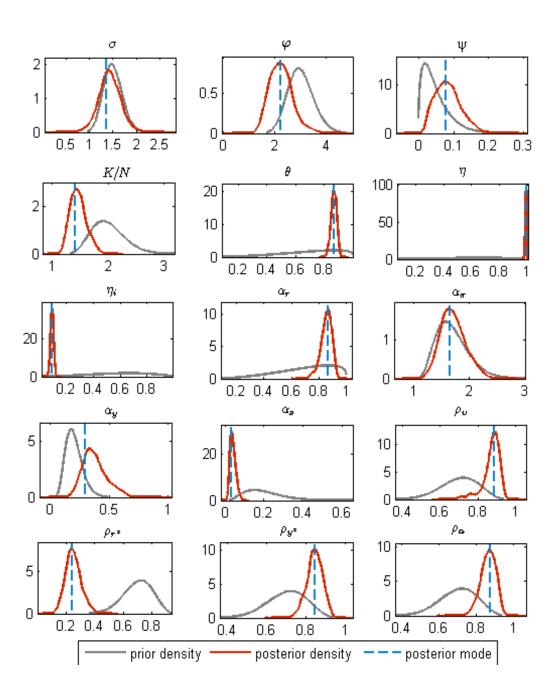
Second, on top of the channel through the cost of foreign borrowing, our estimation results suggest that a sudden stop crisis could be aggravated by the additional channel through the capital production cost. We have argued that the

heavier reliance on the foreign inputs in investment good composite relative to foreign goods in consumption bundle could raise the real capital production cost,  $\frac{P_{I,t}}{P_t}$ , 12 and the real capital price,  $Q_t$ , when the domestic currency is depreciated in a sudden stop crisis, so that the capital demand could be further discouraged by the increased capital price. Our estimation results reveal that this scenario could be actually realised in the economy. That is, the posterior means for the share of domestic goods in consumption bundle and investment good composite,  $\eta$  and  $\eta_i$ , are inferred as 0.9926 and 0.0941, respectively, implying that capital producers rely heavily on foreign input to produce the capital good, while households consume domestic goods much more than imported foreign goods. This environment may support our argument on a 'processing trade' in a sudden stop crisis empirically.

Third, we obtain an empirical characterisation of the central bank's monetary policy rule in the economy. The mean values of coefficients on CPI inflation, output gap and nominal exchange rate in Taylor rule,  $\alpha_{\pi}$ ,  $\alpha_{y}$  and  $\alpha_{s}$ , are estimated as 1.7052, 0.3747, and 0.0410, with 95 percent confidence intervals of (1.3430, 2.0830), (0.2124, 0.5422) and (0.0189, 0.0637), respectively. In addition, the posterior mean and 95 percent confidence interval for the inflation smoothing factor,  $\alpha_{r}$ , are inferred as 0.8543 and (0.7955, 0.9204), respectively. The large estimates for policy coefficients on CPI inflation but small value for that for coefficient on nominal exchange rate indicate that the central bank adjusts the nominal interest rate sensitively in response to CPI inflation, but it reacts less sensitively to the motion in nominal exchange rate. This implies that the central bank may implement the monetary policy in a way to respect the inflation targeting and the

 $<sup>^{12}</sup>$ Recall that this is because, under this condition,  $P_{I,t}$  is more strongly affected than  $P_t$  by the rises in nominal exchange rate and foreign good price, due to the large portion of foreign good price and the stickiness in domestic price.

Figure 4.2: Priors and Posteriors



free floating exchange rate regime, so that the degree of 'fear of float' as in Calvo and Reinhart (2002) is not substantial in the economy.

Lastly, we turn to the estimates for the remaining parameters. Our estimated mean values for the inverse of intertemporal sensitivity,  $\sigma$ , and the inverse of labour supply,  $\varphi$ , are 1.4283 and 2.2298, respectively, which are in the range of values commonly used in calibration-based studies. The posterior mean for Calvotype price stickiness parameter,  $\theta$ , is estimated to be 0.8656, with a 95 percent confidence interval covering the range between 0.8353 and 0.8979. Accordingly, the average duration of retail price lasts  $\frac{1}{1-\theta} = 7.4405$  quarters, i.e., almost two years, which is longer than that in the calibration-based model, where  $\theta = 0.75$  and  $\frac{1}{1-\theta} = 4$  quarters, i.e. one year, conventionally.

Overall, we obtain reasonable estimates in the sense that all of them are statistically significant and most of them are in the range of estimates in the existing studies relying on Bayesian methods to estimate the DSGE model, such as Elekdag et al. (2006), Adolfson et al. (2007), Adjemian et al. (2007), Yie and Yoo (2011) and Lee and Rhee (2013), as shown in Table C3 in Appendix C1. In addition, some of our estimates turn out to be away from the prior means, such as  $\varphi$ ,  $\psi$ ,  $\frac{K}{N}$ ,  $\theta$ ,  $\eta$ ,  $\eta_i$ ,  $\alpha_y$ , and  $\alpha_s$ , which suggests that the data are quite informative. That is, the observed data rectify the prior beliefs about parameter values by the Bayesian MCMC algorithm yielding the posterior estimates which are quite different from the priors.

### 4.4.2 Variance Decomposition

In this part, we discuss the results of variance decomposition, which explains how important a shock is in business cycle dynamics. Variance decomposition results are calculated from the impulse responses to each shock based on the parameter estimates in subsection 4.4.1. Table 4.4 presents the contribution of shocks on the fluctuations in main economic variables in percent.

As shown in Table 4.4, the variation in domestic output,  $Y_{H,t}$ , is mainly explained by foreign lenders' evaluation shock on entrepreneurs' net worth,  $\varepsilon_{v,t}$ , (88.14 percent of the overall variance), and foreign interest rate shock,  $\varepsilon_{r^*,t}$ , (5.28 percent), while the roles of shocks from foreign output,  $\varepsilon_{y^*,t}$ , domestic monetary policy,  $\varepsilon_{r,t}$ , and domestic fiscal policy,  $\varepsilon_{g,t}$ , are relatively small, which are estimated as 0.90, 0.85, and 0.01 percents, respectively. The main drivers for fluctuations of production factors, such as capital,  $K_t$ , and labour,  $L_t$ , are also the shocks from foreigners' evaluation,  $\varepsilon_{v,t}$ , and foreign interest,  $\varepsilon_{r^*,t}$ , which account for 87.23 and 6.91 percents for the former and 69.63 and 12.08 percents for the latter, respectively. This suggests that one of the main sources of the business cycle in the emerging market economy could be foreign financial shocks.

In addition, the non-negligible roles of  $\varepsilon_{v,t}$  and  $\varepsilon_{r^*,t}$ , are confirmed for the external finance premium,  $\Psi_t$ , and capital returns,  $R_t^k$ , capturing 25.84 and 72.03 percents of the overall variance for the former, and 60.09 and 35.57 percents for the latter. The shocks from foreigners' net worth evaluation,  $\varepsilon_{v,t}$ , and foreign interest rate,  $\varepsilon_{r^*,t}$ , also account for 77.65 and 7.41 percents for the variance of the foreign borrowing,  $B_t^*$ , respectively, and 86.93 and 5.19 percents for the variance of the nominal exchange rate,  $S_t$ , respectively. These indicate that it could be a main

Table 4.4: Variance Decomposition (in percent)

	$\varepsilon_{v,t}$	$\varepsilon_{r^*,t}$	$\varepsilon_{y^*,t}$	$\varepsilon_{\pi^*,t}$	$\varepsilon_{r,t}$	$\varepsilon_{g,t}$	$\varepsilon_{a,t}$
$Y_{H,t}$	88.14	5.28	0.90	3.77	0.85	0.01	1.06
$K_t$	87.23	6.91	1.45	3.77	0.29	0.00	0.35
$L_t$	69.63	12.08	4.11	3.23	2.47	0.01	8.48
$\Psi_t$	25.84	72.03	1.33	5.32	0.36	0.00	0.20
$R_t^k$	60.09	35.57	3.25	0.15	0.21	0.00	0.73
$S_t$	86.93	5.19	0.60	6.33	0.50	0.00	0.44
$B_t^*$	77.65	7.41	7.33	6.73	0.43	0.00	0.45
$N_t$	88.27	5.36	4.11	1.58	0.22	0.00	0.46
$C_{H,t}$	89.25	4.68	0.66	4.70	0.36	0.00	0.36
$C_{F,t}$	85.22	7.97	0.92	5.23	0.73	0.00	0.24
$C_{H,t}^*$	80.40	1.32	11.95	5.32	0.63	0.00	0.38
$I_{H,t}$	52.17	30.27	15.45	1.62	0.07	0.00	0.42
$I_{F,t}$	48.82	35.18	13.53	1.93	0.12	0.00	0.42
$\pi_t$	74.00	12.54	3.75	0.73	1.93	0.00	7.05
$\pi_{H,t}$	74.04	12.41	3.81	0.75	1.89	0.00	7.10

channel of foreign financial shock to propagates to the domestic production.

Furthermore, we find that the demand side of the emerging market economy is affected by the foreign output shock,  $\varepsilon_{y^*,t}$ , and foreign inflation shock,  $\varepsilon_{\pi^*,t}$ , as well as the foreign financial shocks,  $\varepsilon_{v,t}$  and  $\varepsilon_{r^*,t}$ . The foreign output shock,  $\varepsilon_{y^*,t}$ , and foreign inflation shock,  $\varepsilon_{\pi^*,t}$ , explain 11.95 and 5.32 percents of variations in the export demand,  $C_{H,t}^*$ , respectively, and 15.45 and 1.62 percents of variations in the demand for domestic investment good,  $I_{H,t}$ , respectively. However, the impacts on the emerging market economy of the foreign real shocks, such as  $\varepsilon_{y^*,t}$ , and  $\varepsilon_{\pi^*,t}$ , and the domestic shocks such as  $\varepsilon_{r,t}$ ,  $\varepsilon_{g,t}$  and  $\varepsilon_{a,t}$ , are found to be limited, as compared to the foreign financial shocks, such as  $\varepsilon_{v,t}$ , and  $\varepsilon_{r^*,t}$ .

#### 4.4.3 Robustness of the Result

We evaluate the robustness of our estimation result by reestimating the model with alternative and less informative priors. In the alternative model, the uniform distribution is assigned to the parameters bounded between 0 and 1, such as  $\theta$ ,  $\alpha_r$ ,  $\rho_v$ ,  $\rho_g$ ,  $\rho_{r^*}$ ,  $\rho_{y^*}$ ,  $\rho_{\pi^*}$ , and  $\rho_a$ , instead of beta distribution in the baseline model.<sup>13</sup> In addition, the normal distribution with mean of zero and standard deviation of 0.5 is assumed for the Taylor rule coefficient on the nominal exchange rate,  $\alpha_s$ , instead of gamma distribution in the baseline model, and the uniform distribution bounded 0 and 0.1 is taken for the sensitivity parameter of the external finance premium to entrepreneurs' leverage ratio,  $\psi$ , instead of gamma distribution. We implement otherwise the same estimation procedure as that in the baseline model. Table 4.4 compares the priors and the estimation results in the alternative model with those in the baseline model.

First of all, the posterior means of  $\psi$  and  $\frac{K}{N}$  are estimated as 0.0499 and 1.9381, respectively, in the alternative model, as compared to 0.0839 and 1.4601, respectively, in the baseline. Despite a fall in the posterior mean for  $\psi$ , and a rise in that for  $\frac{K}{N}$ , we obtain a similar steady state value of external finance premium, i.e.,  $\frac{R^k}{R} = \left(\frac{K}{N}\right)^{\psi} = 1.0348$ , to that in the baseline model, so that we may maintain the argument that there exists a substantial degree of financial frictions in the economy. Second, the alterative model estimates the posterior means for the steady state share of home goods in consumption bundle,  $\eta$ , and that of domestic input in investment good composite,  $\eta_i$ , as 0.7775 and 0.4568, respectively, so that

<sup>&</sup>lt;sup>13</sup>Note that, under the uniform distribution bounded between 0 and 1, the prior information on the mean value of the parameter cannot be considered other than 0.5, unlike under the beta distribution.

Table 4.5: Robustness of Result

	Alternative Model				Baseline Model		
	Prior		Posterior		Prior		Posterior
	Shape	Mean	Mean	95% C.I.	Shape	Mean	Mean
$\sigma$	gamma	1.5	1.4620	[1.4455, 1.4871]	gamma	1.5	1.4283
$\varphi$	gamma	3	2.6842	[2.6122, 2.8017]	gamma	3	2.2298
$\psi$	uniform	0.05	0.0499	[0.0429, 0.0554]	gamma	0.05	0.0839
K/N	gamma	2	1.9381	[1.8739, 2.0081]	gamma	2	1.4601
$\theta$	uniform	0.5	0.5418	[0.4959, 0.5784]	beta	0.75	0.8656
$\eta$	uniform	0.5	0.7775	[0.7431, 0.8104]	beta	0.6	0.9926
$\eta_i$	uniform	0.5	0.4568	[0.4376, 0.4759]	beta	0.6	0.0941
$\alpha_r$	uniform	0.5	0.5868	[0.5628, 0.6268]	beta	0.7	0.8543
$\alpha_{\pi}$	gamma	1.7	1.7155	[1.6732, 1.7529]	gamma	1.7	1.7052
$\alpha_y$	gamma	0.2	0.1881	[0.1770, 0.1973]	gamma	0.2	0.3747
$\alpha_s$	normal	0.0	0.0155	[0.0146, 0.0161]	gamma	0.2	0.0410
$ ho_{ u}$	uniform	0.5	0.5401	[0.4140, 0.6467]	beta	0.7	0.8715
$ ho_g$	uniform	0.5	0.5634	[0.5314, 0.6022]	beta	0.7	0.7052
$ ho_{r^*}^{\ \ \ \ \ }$	uniform	0.5	0.2682	[0.2184, 0.3428]	beta	0.7	0.2523
$ ho_{y^*}$	uniform	0.5	0.5040	[0.4744, 0.5240]	beta	0.7	0.8422
$ ho_{\pi^*}$	uniform	0.5	0.5746	[0.5034, 0.6215]	beta	0.7	0.8103
$ ho_a$	uniform	0.5	0.6124	[0.5754, 0.6389]	beta	0.7	0.8629
$\varepsilon_{\pmb{v}}$	inv. gam.	0.03	0.0813	[0.0626, 0.0986]	inv. gam.	0.03	0.0343
$\varepsilon_{m{g}}$	inv. gam.	0.03	0.0581	[0.0487, 0.0683]	inv. gam.	0.03	0.0292
$arepsilon_{r^*}$	inv. gam.	0.03	0.1299	[0.1104, 0.1498]	inv. gam.	0.03	0.1331
$\varepsilon_{y^*}$	inv. gam.	0.03	0.0078	[0.0067, 0.0089]	inv. gam.	0.03	0.0056
$\varepsilon_{\pi^*}$	inv. gam.	0.03	0.0062	[0.0054, 0.0070]	inv. gam.	0.03	0.0060
$\varepsilon_a$	inv. gam.	0.03	0.0190	[0.0160, 0.0220]	inv. gam.	0.03	0.0295
$\varepsilon_r$	inv. gam.	0.03	0.0368	[0.0314, 0.0423]	inv. gam.	0.03	0.0060

the latter is still much smaller than the former. Thus, our second argument that the economy relies heavily on the foreign input to produce the capital goods, so that the sudden stop crisis could be amplified by the currency depreciation via the distortion in capital price, is also robust to the change in priors. Third, Taylor rule coefficient on the nominal exchange rate,  $\alpha_s$ , is estimated as 0.0155 under the normal prior, which is still positive but very small. Thus, our third argument that the central bank implements the monetary policy in a way to respect a free floating exchange rate regime well, could be maintained even under the looser prior. Overall, even though there are some quantitative differences for some parameter estimates between the two alternative models, Bayesian estimates are broadly similar across models, as shown in Table 4.4. It suggests that our arguments are robust to priors taken for Bayesian estimation, and strongly backed up by the data.

## 4.5 Conclusion

We use Bayesian methods to estimate the small open economy DSGE model with financial frictions and to evaluate the empirical validity of arguments in the previous chapter. Combining data from the US and South Korea and the model proposed in the previous chapter by Bayesian methods, we obtain the significant Bayesian estimates with the right signs for the key parameters, which support our arguments empirically.

First of all, we obtain the empirical evidence for the presence of a substantial degree of financial frictions in the economy, which implies that the economy could be vulnerable to the foreign financial shocks. Second, the data uncover that capital producers rely heavily on the foreign inputs in the emerging market country, as compared to the households' consumption on foreign goods, which suggests that a sudden stop crisis could be amplified by the currency depreciation via the deterioration of capital price. Third, the positive but very small value of the estimate for Taylor rule coefficient on nominal exchange rate indicates that the central bank in the economy implements the monetary policy in a way to allow the nominal exchange rate to float freely. Fourth, the result from variance decomposition implies that the main source of business cycle in the emerging market economy comes from foreign financial shocks. Lastly, comparison of the estimation results from the alternative models with different priors suggests that the estimation result that we obtain are robust to the change in the prior belief about the parameters.

The above empirical findings result in the following practical implications. First, the policy authorities are advised to try to reduce the degree of financial frictions in the economy to make the economy more robust to foreign financial shocks. Second, since the economy's heavy reliance on the foreign inputs could be an obstacle to a rapid recovery from a sudden stop crisis, the authorities should try to reduce the degree of reliance on the foreign inpults by making the intermediate goods on its own or diversifying the sources of foreign inputs. Third, even though the estimation does not find the strong evidence that the economy adopts a fixed exchange rate regime, the authorities could be advised to conduct the exchange rate policy in more market friendly way because a fixed exchange rate regime could provide an inferior performance in a sudden stop crisis.

# Appendix C

## Appendix C1 Priors and Posteriors in the Literature

Table C1 Model and Data in the Literature

	Data source	Model type
EJT	South Korea	SOE NK with FF
ALLV	Euro area	SOE NK without FF
APM	Euro area	Closed NK without FF
YY	US & South Korea	SOE NK with FF
LR	US & South Korea	SOE NK with FF

<sup>\*</sup> EJT: Elekdag *et al.* (2006); ALLV: Adolfson *et al.* (2007); APM: Adjemian *et al.* (2007); YY: Yie and Yoo (2011, mimeo); LR: Lee and Rhee (2013).

<sup>\*\*</sup> SOE NK with FF: small open economy New Keynesian DSGE model with financial frictions; SOE NK without FF: small open economy New Keynesian DSGE model without financial frictions; Closed NK without FF: closed economy New Keynesian DSGE model without financial frictions.

Table C2 Priors in the Literature

	EJT	ALLV	APM	YY	LR
$\beta$	0.99	0.999	0.99	0.988	
$\eta$	B(0.6, 0.1)	0.69		0.7	
$\gamma$		5		N(1.5, 0.2)	
$\sigma$	G(3,1)		N(1, 0.38)	G(1.5, 0.2)	
$\varphi$	G(3,1)	1		$G\left(3,0.5\right)$	
$\alpha$	0.37	0.29	0.3	0.3	
$\psi$	B(0.07, 0.03)			$G\left(1,0.5\right)$	N(0.07, 0.2)
$\kappa$		N(7.7, 1.5)	$N\left( 4,2\right)$		$IG\left(0.5,0.5\right)$
$\epsilon$	G(8,3)		4.3	6	
heta	B(0.6, 0.1)	B(0.68, 0.05)	B(0.75, 0.05)	B(0.7, 0.1)	B(0.65, 0.1)
$\alpha_r$	B(0.8, 0.2)	B(0.8, 0.05)	N(0.75, 0.1)	B(0.7, 0.1)	B(0.8, 0.1)
$\alpha_{\pi}$	G(3,0.5)	N(1.7, 0.1)	N(1.5, 0.05)	G(1.5, 0.2)	N(1.5, 0.05)
$\alpha_y$	G(1,0.8)	N(0.13, 0.05)	N(0.13, 0.1)	G(0.4, 0.2)	N(0.13, 0.05)
$\alpha_s$		N(0, 0.05)		N(0.05, 1)	
K/N	G(2,0.3)				$G\left(2,0.1\right)$
$C_H^*/Y_H$		0.25		0.3	
$ ho_{}$	B(0.5, 0.25)	B(0.85, 0.1)	B(0.85, 0.1)	B(0.7, 0.2)	B(0.8, 0.1)
ε	IW(1, 0.75)	B(0.85, 0.1)	U(2, 1.2)	$IG(0.01,\inf)$	IG(0.005, 0.1)

<sup>\*</sup> The density functions are represented as follows: B= beta, G= gamma, N= normal, U= uniform, IW= inverse wishart, and IG= inverse gamma. The first number in the parenthesis is the mean and the second one is the standard error.

Table C3 Posterior Means in the Literature

	EJT	ALLV	APM	YY	LR	BL
$\sigma$	0.741		1.9591	1.4906		1.4283
$\varphi$	1.889		1.8004	3.3139		2.2298
$\psi$	0.048			0.0038	0.0099	0.0839
K/N	1.759				1.3141	1.4601
$\theta$	0.387	0.891	0.9098	0.5246	0.461	0.8656
$\eta$	0.689					0.9926
$\alpha_r$	0.054	0.881		0.8058	0.903	0.8543
$\alpha_{\pi}$	2.307	1.730	1.5762	1.3633	1.494	1.7052
$\alpha_y$	0.054	0.104	0.125	0.6612	0.118	0.3747
$\alpha_s$	0.040	-0.009		0.1475		0.0410
$ ho_a$	0.987	0.886		0.3236	0.774	0.8629
$\rho_{r^*}$	0.825			0.8471	0.791	0.2523
$\varepsilon_a$	0.765			0.0754	0.021	0.0295
$arepsilon_{oldsymbol{v}}$					0.08	0.0343
$arepsilon_{r^*}$	1.176			0.0010	0.002	0.1311
$\varepsilon_r$	0.579	0.135		0.0029	0.002	0.0060

<sup>\*</sup> BL: posterior means in the baseline model

### Appendix C2 Bayesian MCMC Methods

In this part, we present the procedure of Bayesian MCMC methods for estimating a DSGE model. It includes the state-space representation of a DSGE model, the Kalman filter for the likelihood for data, numerical methods for the posterior mode, the Markov Chain Monte Carlo (MCMC) Algorithm for the posterior density, and the diagnostic for convergence of MCMC.

#### C2.1 State-space Representation and Kalman Filter<sup>14</sup>

The log-linearised DSGE model can be solved by the solution method in Blanchard and Kahn (1980), Klein (2000), and others. Then, the rational expectations solution to a linear system can be represented by the following state transition equation:

$$E\{x_{t+1}\} = Fx_t + Gz_{t+1},$$

where  $x_t$  is a vector of endogenous variables,  $z_t$  is a vector of structural shocks, and the matrices F and G are functions of the model's parameters. The state transition equation governs the time evolution of the state vector,  $x_t$ . In addition, to estimate the model, we allow for the following observation equation:

$$y_t = H'x_t + v_t,$$

where  $y_t$  is a vector of observed variables and  $v_t$  are measurement errors. The observation equation links the observables,  $y_t$ , to the state variables,  $x_t$ , through

 $<sup>^{14}</sup>$ This part is based on Hamilton (1994), Tsay (2005), Beltran and Draper (2008), and Canova (2011).

the matrix H'. We assume here that  $z_t$  and  $v_t$  are two independent Gaussian white noise series, i.e.,  $z_t \sim \mathcal{N}(0, Q)$  and  $v_t \sim \mathcal{N}(0, R)$ , respectively.

Then, the Kalman filter, proposed by Kalman (1960), updates the state variable from  $x_{t|t-1}$  to  $x_{t|t}$ , recursively, by using the newly available data,  $y_t$ . The joint distribution of  $x_t$  and  $y_t$  conditional on the observations,  $y^{t-1} \equiv \{y_{t-1}, \dots, y_1\}$ , is given by

$$\begin{bmatrix} x_t \\ y_t \end{bmatrix}_{y^{t-1}} \sim \mathcal{N} \left( \begin{bmatrix} x_{t|t-1} \\ y_{t|t-1} \end{bmatrix}, \begin{bmatrix} \Sigma_{t|t-1} & \Sigma_{t|t-1}H \\ H'\Sigma_{t|t-1} & \Omega_{t|t-1} \end{bmatrix} \right),$$

where  $x_{t|t-1} \equiv E\{x_t|y^{t-1}\}, y_{t|t-1} \equiv E\{y_t|y^{t-1}\}, \Sigma_{t|t-1} \equiv E\{(x_t - x_{t|t-1})(x_t - x_{t|t-1})'|y^{t-1}\}, \Sigma_{t|t-1}H \equiv E\{(x_t - x_{t|t-1})(y_t - y_{t|t-1})'|y^{t-1}\}, H'\Sigma_{t|t-1} \equiv E\{(y_t - y_{t|t-1})(x_t - x_{t|t-1})'|y^{t-1}\}$  and  $\Omega_{t|t-1} \equiv E\{(y_t - y_{t|t-1})(y_t - y_{t|t-1})'|y^{t-1}\} = H'\Sigma_{t|t-1}H + R$ . In addition, the property of multivariate normal distribution<sup>15</sup> allows the above distribution to be reduced to the distribution of  $x_t$  conditional on the observation of  $y_t$ , and  $y^{t-1}$ , given by

$$x_t|y_t, y^{t-1} \sim \mathcal{N}(x_{t|t}, \Sigma_{t|t}),$$

where  $x_{t|t} = x_{t|t-1} + \Sigma_{t|t-1}H(H'\Sigma_{t|t-1}H + R)^{-1}(y_t - H'x_{t|t-1})$  and  $\Sigma_{t|t} = \Sigma_{t|t-1} - \Sigma_{t|t-1}H(H'\Sigma_{t|t-1}H + R)^{-1}H'\Sigma_{t|t-1}$ . Then, the knowledge of  $x_t$  given  $y^t$  can be used

<sup>15</sup> It can be shown that the random vectors x and y, whose joint distribution is multivariate normal, has the following properties: (i)  $E(x|y) = \mu_x + \Sigma_{xy}\Sigma_{yy}^{-1}(y - \mu_y)$ , and (ii)  $Var(x|y) = \Sigma_{xx} - \Sigma_{xy}\Sigma_{yy}^{-1}\Sigma_{yx}$ .

to predict  $x_{t+1}$  via the state transition equation. That is,

$$x_{t+1|t} = E\left\{x_{t+1}|y^t\right\} = E\left\{Fx_t + Gz_{t+1}|y^t\right\}$$
$$= Fx_{t|t},$$

and

$$\Sigma_{t+1|t} = E\left\{ \left( x_{t+1} - x_{t+1|t} \right) \left( x_{t+1} - x_{t+1|t} \right)' | y^t \right\}$$

$$= E\left\{ F\left( x_t - x_{t|t} \right) \left( x_t - x_{t|t} \right)' F' | y^t \right\} + E\left\{ G\left( z_{t+1} \right) \left( z_{t+1} \right)' G' | y^t \right\}$$

$$= F\Sigma_{t|t} F' + GQG'.$$

In addition, we obtain

$$y_{t+1|t} = E\{y_{t+1}|y^t\} = E\{H'x_{t+1} + v_{t+1}|y^t\}$$
  
=  $H'x_{t+1|t}$ ,

and

$$\Omega_{t+1|t} = E\left\{ \left( y_{t+1} - y_{t+1|t} \right) \left( y_{t+1} - y_{t+1|t} \right)' | y^t \right\} 
= E\left\{ H'\left( x_{t+1} - x_{t+1|t} \right) \left( x_{t+1} - x_{t+1|t} \right)' H | y^t \right\} + E\left\{ \left( v_{t+1} \right) \left( v_{t+1} \right)' | y^t \right\} 
= H' \Sigma_{t+1|t} H + R.$$

Consequently, the Kalman filter is summarised as the prediction equations for the observables and the updating equations for the state variables, as:

$$v_t = y_t - H' x_{t|t-1},$$

$$\Omega_{t|t-1} = H' \Sigma_{t|t-1} H + R,$$

$$x_{t+1|t} = Fx_{t|t},$$

$$\Sigma_{t+1|t} = F\Sigma_{t|t}F' + GQG',$$

where  $x_{t|t} = x_{t|t-1} + \Sigma_{t|t-1}H(H'\Sigma_{t|t-1}H + R)^{-1}(y_t - H'x_{t|t-1})$  and  $\Sigma_{t|t} = \Sigma_{t|t-1} - \Sigma_{t|t-1}H(H'\Sigma_{t|t-1}H + R)^{-1}H'\Sigma_{t|t-1}$ .

In addition, the Kalman filter algorithm can be used to evaluate the likelihood function of the observables. Given a DSGE model and data, the likelihood function under the assumption of the independent and identical normal distribution is

$$p(y_1, \dots, y_T | F, G, H', Q, R) = p(y_1 | F, G, H', Q, R) \prod_{t=2}^{T} p(y_t | F, G, H', Q, R)$$
$$= p(y_1 | F, G, H', Q, R) \prod_{t=2}^{T} p(v_t | F, G, H', Q, R)$$

where  $y_1 \sim \mathcal{N}(H'x_{1|0}, \Omega_{1|0})$  and  $v_t = y_t - H'x_{t|t-1} \sim \mathcal{N}(0, \Omega_{t|t-1})$  with  $\Omega_{t|t-1} = 0$ 

 $H'\Sigma_{t|t-1}H + R$ . Consequently, assuming that  $x_{1|0}$  and  $\Sigma_{1|0}$  are given, and taking the logarithms, we have the log likelihood function of data, as:

$$\log \mathcal{L}\left(y^{T}|F,G,H',Q,R\right) = \log p\left(y_{1},\cdots,y_{T}|F,G,H',Q,R\right)$$

$$= \log \prod_{t=1}^{T} (2\pi)^{-N/2} \left|\Omega_{t|t-1}\right|^{-1/2} \exp\left(-\frac{1}{2}v_{t}'\Omega_{t|t-1}^{-1}v_{t}\right)$$

$$= -\sum_{t=1}^{T} \left[\frac{N}{2}\log(2\pi) + \frac{1}{2}\log\left|\Omega_{t|t-1}\right| + \frac{1}{2}v_{t}'\Omega_{t|t-1}^{-1}v_{t}\right],$$

where  $N = \dim(y_t)$ .

#### C2.2 Posterior Density and Posterior Mode

Having obtained the likelihood for data,  $\mathcal{L}(y|\theta)$ , the Bayes theorem relates the prior density for the parameters,  $p(\theta)$  and the likelihood function,  $\mathcal{L}(y|\theta)$ , to the posterior density of the parameters,  $p(\theta|y)$ , according to:

$$p(\theta|y) = \frac{\mathcal{L}(y|\theta) p(\theta)}{\int \mathcal{L}(y|\theta) p(\theta) d\theta}$$

where  $\theta$  is the vector of unknown parameters and y is the observed data. In addition, since the data, y, are fixed, the marginal distribution,  $p(y) = \int \mathcal{L}(y|\theta) p(\theta) d\theta$ , does not depend on  $\theta$ , so that, instead of the posterior density,  $p(\theta|y)$ , posterior kernel,  $\mathcal{K}(\theta|Y)$ , can be used for estimation, given by

$$\mathcal{K}(\theta|y) \equiv \mathcal{L}(y|\theta) p(\theta) \propto p(\theta|y),$$

where  $\propto$  implies the proportionality. In addition, the log posterior kernel,  $\log \mathcal{K}(\theta|y)$ , can be expressed as the sum of the log likelihood,  $\log \mathcal{L}(y|\theta)$ , and the log prior,  $\log p(\theta)$ :

$$\log \mathcal{K}(\theta|y) = \log \mathcal{L}(y|\theta) + \log p(\theta).$$

Next, we maximise the log posterior kernel to estimate the posterior mode,  $\theta_m$ . That is,

$$\arg\max_{\theta}\log\mathcal{K}\left(\theta|y\right) = \arg\max_{\theta}\left[\log\mathcal{L}\left(y|\theta\right) + \log p\left(\theta\right)\right],$$

to get  $\left[\frac{\partial \log \mathcal{L}(y|\theta)}{\partial \theta} + \frac{\partial \log p(\theta)}{\partial \theta}\right]_{\theta=\theta_m} = 0.^{16}$  Furthermore, by using a first order Taylor expansion, the optimality condition can be approximated around the initial guess of parameter values,  $\theta_0$ , given by:

$$\begin{split} & \frac{\partial \log \mathcal{L}\left(y|\theta\right)}{\partial \theta} + \frac{\partial \log p\left(\theta\right)}{\partial \theta} \\ & \simeq & \left[\frac{\partial \log \mathcal{L}\left(y|\theta_{0}\right)}{\partial \theta} + \frac{\partial \log p\left(\theta_{0}\right)}{\partial \theta}\right] + \left[\frac{\partial^{2} \log \mathcal{L}\left(y|\theta_{0}\right)}{\partial \theta \partial \theta'} + \frac{\partial^{2} \log p\left(\theta_{0}\right)}{\partial \theta \partial \theta'}\right] \left(\theta - \theta_{0}\right) = 0 \end{split}$$

so that the posterior mode,  $\theta_m$ , is obtained as:.

$$\theta_{m} - \theta_{0} = \left[ \frac{\partial^{2} \log \mathcal{L} (y|\theta_{0})}{\partial \theta \partial \theta'} + \frac{\partial^{2} \log p (\theta_{0})}{\partial \theta \partial \theta'} \right]^{-1} \left[ \frac{\partial \log \mathcal{L} (y|\theta_{0})}{\partial \theta} + \frac{\partial \log p (\theta_{0})}{\partial \theta} \right].$$

In addition, we can calculate the covariance matrix,  $\Sigma_m$ , by the inverse of the

The optimality condition,  $\left[\frac{\partial \log \mathcal{L}(y|\theta)}{\partial \theta} + \frac{\partial \log p(\theta)}{\partial \theta}\right]_{\theta=\theta_m} = 0$ , suggests that if  $\log \mathcal{L}(y|\theta)$  is flat so that  $\frac{\partial \ln \mathcal{L}(y|\theta)}{\partial \theta}$  is close to zero, then  $\theta_m$  is dominated by the prior,  $p(\theta)$ ; while if  $\log p(\theta)$  is flat so that  $\frac{\partial \log p(\theta)}{\partial \theta} = 0$ , then  $\theta_m$  is dominated by the data.

negative Hessian matrix evaluated at the posterior mode,  $\theta_m$ , given by:

$$\Sigma_{m} = \left[ -\left( \frac{\partial^{2} \log \mathcal{K} (\theta|y)}{\partial \theta \partial \theta'} \right)_{\theta = \theta_{m}} \right]^{-1}.$$

### C2.3 Markov Chain Monte Carlo (MCMC) methods<sup>17</sup>

In practice, a posterior density is estimated by using the simulation methods,<sup>18</sup> such as Monte Carlo integration, Importance sampling (IS), and Markov Chain Monte Carlo (MCMC) algorithm. The strategy of the MCMC method is to generate the random draws for the parameters,  $\theta$ , from the proposal posterior density, which are accepted or rejected according to the relative value of the target density at the candidate point,  $\theta_j^*$ , to that at the current point,  $\theta_{j-1}$ . Then, the algorithm constructs an empirical histogram, which converges to the true posterior density as the iteration approaches to the infinity, by the ergodic property of the Markov chain.<sup>19</sup> The well known examples of the MCMC method are the Gibbs sampler, the Metropolis-Hastings (MH) algorithm, and the random walk Metropolis-Hastings (RWMH) algorithm, among which we outline the RWMH procedure.

 $<sup>^{17}</sup>$ This part is based on Johannes and Polson (2002), Tsay (2005), Gamerman and Lopes (2006), and Greenberg (2012).

<sup>&</sup>lt;sup>18</sup>Difficulty in applying the analytical approach arises from the fact that the integrals for the posterior density do not generally have a closed-form solution.

<sup>&</sup>lt;sup>19</sup>A Markov chain of the sequence  $\{x_i\}$  has the property that  $p(x_{i+1}|x_i, x_{i-1}, \ldots) = p(x_{i+1}|x_i)$ , where i refers to an index of Monte Carlo interation. That is, the next state depends only on the current state and not on the sequence of events that precedes it. A Markov chain is known to be 'irreducible' and 'aperiodic'. In other words, the chain can enter any state from any state but the visits to state i can occur only at irregular times. It can be shown that if an irreducible and aperiodic chain has a proper invariant distribution,  $p(\theta|y)$ , then it is unique and stationary distribution of the chain. That is,  $\lim_{j\to\infty} prob\left[\theta_j|\theta_0,y\right] = p(\theta|y)$ . See Meyn and Tweedie (2009) for details.

 $\langle$ Step 1> The algorithm is initialised by setting the number of iteration,  $j=1,\ldots M$ , and specifying a starting value,  $\theta_0$ , which can be drawn from the proposal distribution, (or jumping distribution,)  $\mathcal{N}(\theta_m, c^2\Sigma_m)$ , where  $\theta_m$  is the posterior mode,  $\Sigma_m$  is the inverse of negative Hessian matrix computed at  $\theta_m$ , and c is the scale factor.

Step 2> Then, we evaluate the log likelihood for data,  $\mathcal{L}(y|\theta)$ , and prior for parameter,  $p(\theta)$ , at the starting point,  $\theta_0$ , which involves: (i) evaluating  $p(\theta_0)$  for given  $\theta_0$ ; (ii) using the numerical methods such as Klein's (2000) method to solve the model for given  $\theta_0$ ; and using the Kalman filter to evaluate  $\mathcal{L}(y|\theta_0)$ .

Step 3> Next, a candidate sample,  $\theta_j^*$ , is drawn from a proposal density,  $\theta_j^* = \theta_{j-1} + \varepsilon \sim \mathcal{N}\left(\theta_{j-1}, c^2 \Sigma_m\right)$ , and the log likelihood for data,  $\mathcal{L}\left(y \middle| \theta_j^*\right)$ , and prior for parameter,  $p\left(\theta_j^*\right)$ , are evaluated by the above procedure. Then, kernel values,  $\mathcal{K}\left(\theta \middle| y\right) = \mathcal{L}\left(y \middle| \theta\right) p\left(\theta\right)$ , evaluated at the current point,  $\theta_{j-1}$ , and the candidate,  $\theta_j^*$ , are compared, so that the algorithm returns  $\theta_j = \theta_j^*$  with the acceptance rate,  $r = \min\left\{1, \frac{\mathcal{L}\left(y \middle| \theta_j^*\right) p\left(\theta_j^*\right)}{\mathcal{L}\left(y \middle| \theta_{j-1}\right) p\left(\theta_{j-1}\right)}\right\}$ ; and it returns  $\theta_j = \theta_{j-1}$ , with 1 - r.

Step 4> If j < M, then the algorithm proceeds from j to j+1 and repeats the procedure from <Step 3>. If j=M, then the algorithm stops, and, based on the constructed posterior density,  $\widetilde{p}(\theta|y)$ , the posterior estimates are calculated as  $E\{h(\theta)|y\} = \frac{1}{M} \sum_{j=1}^{M} h(\theta_j) \, \widetilde{p}(\theta_j|y)$ , where  $h(\theta)$  is a function of the posterior estimates.

#### C2.4 Convergence of MCMC

In theory, the empirical posterior density,  $\widetilde{p}(\theta|y)$ , generated by the MCMC algorithm, converges to the true posterior density,  $p(\theta|y)$ , as the iteration approaches

to the infinity. This is because the ergodic theory for Markov chains implies that the sequence generated by the MCMC algorithm converges to the true density wherever it starts, as shown in Johannes and Polson (2004). However, in practice, the problem is how fast the convergence occurs. If the sequence lacks convergence, it is still affected by the starting value,  $\theta_0$ , so that the true posterior density,  $p(\theta|y)$ , may not be considered to be well represented by the simulated draws,  $\tilde{p}(\theta|y)$ . In contrast, as argued by Brooks and Gelman (1998), if the sequence converges, at least two things should occur: (i) the empirical posterior density should remain the same within a sequence; and (ii) it should be the same across sequence. Based on the idea, Brooks and Gelman (1998) propose the diagnostic to check the convergence, which consists of 'between variance' and 'within variance' 'Within variance' and 'between variance' are specified by

$$\widehat{W} = \frac{1}{J} \sum_{i=1}^{J} \frac{1}{I} \sum_{i=1}^{I} (\Psi_{ij} - \overline{\Psi}_{\cdot j})^{2}$$

and

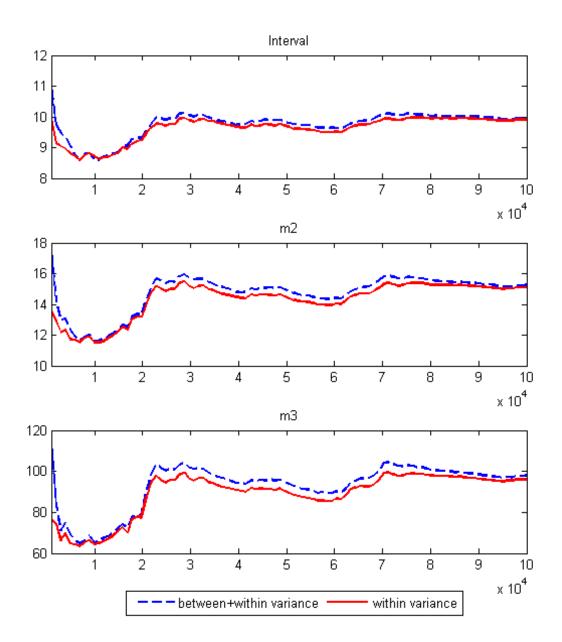
$$\widehat{B} = \frac{1}{J-1} \sum_{i=1}^{J} \left( \overline{\Psi}_{\cdot j} - \overline{\Psi}_{\cdot .} \right)^{2},$$

respectively, where  $\Psi_{ij}$  is the  $i^{th}$  draw out of I in the  $j^{th}$  sequence out of J,  $\overline{\Psi}_{\cdot j}$  is the mean of  $j^{th}$  sequence, and  $\overline{\Psi}_{\cdot \cdot}$  is the mean across all available data. In this setup, the convergent sequence requires 'between variance' to go to zero, i.e.,  $\lim_{I\to\infty} \widehat{B} = 0$  and 'within variance' to settle down at a constant, i.e.,  $\lim_{I\to\infty} \widehat{W} = 0$ . DYNARE reports the convergence test result using the red lines and blue lines, where the former represents the 'within variance',  $\widehat{W}$ , and the letter depicts the

variance as a whole,  $\widehat{W} + \widehat{B}$ . Thus, the convergence of the Markov chain requires (i) red and blue lines to get close; and (ii) red lines to settle down at a constant.

Figure C1 displays multivariate diagnostic for convergence in our baseline model. Each panel in Figure C1 represents 'within' and 'within+between' variances, which is constructed based on an 80 percent confidence interval around the parameter mean; based on a variance; and based on third moments, respectively. Figure C1 shows the red (solid) lines and blue (dotted) lines take almost the same path, implying that there does not exist a significant difference among parallel sequences from the early stage of iterations. However, the two lines settle down at a constant after 30,000 iterations pass, which suggests that the random draws within sequence converges to the true posterior density,  $p(\theta_j|y)$ , after 30,000 iterations pass. In other words, the empirical posterior density,  $\tilde{p}(\theta_j|y)$ , constructed through over 30,000 iterations of random draws could be considered the true posterior density,  $p(\theta_j|y)$ , in our simulation.

Figure C1 Multivariate Diagnostic for Convergence



<sup>\*</sup> Horizontal axis in each panel represents the iterations and verial axis represents the size of variance.

# Chapter 5

# Conclusion

This thesis aims at enhancing our understanding of the financial crises by using New Keynesian DSGE frameworks with financial frictions. Our main interests lie in the following issues: (i) the source and the transmission mechanism of a financial crisis; (ii) the role of pre-crisis conditions on the impact of a financial crisis; and (iii) the effectiveness of policy measures to fight a financial crisis. To these ends, we have constructed New Keynesian DSGE models with financial frictions for closed and small open economies, conducted a set of simulations and experiments, and estimated parameters in the model by using Bayesian MCMC methods and data from the US and South Korea.

Our main findings can be summarised as follows. In Chapter 2, we find that a collapse in borrowers' net worth and a distortion in their balance sheet could trigger a financial crisis, by reducing the bankers' credit supply and raising the cost of external finance. Such an effect of a negative shock to borrowers' net worth on credit supply and cost of external finance turns out to produce a similar outcome to the conventional negative shock to capital quality in non-financial firms' technology. This is because a negative shock to capital quality decreases the borrowers' net worth by the reduced capital returns, which results in a distortion in bankers' balance sheet and a production contraction in the real sector of the economy. Based on the observation, we argue that a fall in the efficiency of the capital in non-financial firms' technology is one of the events causing a distortion in bankers' balance sheet, which would lead to a financial crisis in the end.

In addition, we have evaluated the effectiveness of diverse policy measures in a financial crisis in Chapter 2. First of all, our simulation results indicate that conventional expansionary monetary and fiscal policies could relieve the business cycle fluctuations in a financial crisis, as long as they are properly working. However, under certain circumstances, such as zero lower bound (ZLB) of the nominal interest rate, the central bank is unable to adjust the nominal interest rate properly, so that a conventional monetary policy would not be an effective tool to fight a financial crisis. In addition, an expansionary fiscal policy could be less effective in stabilising the economy in a financial crisis. This is because an expansionary fiscal policy tends to increase the interest rate, which decreases the bankers' credit supply by reducing the profitability from financial intermediation as well as discouraging the capital demand by the 'crowding-out effect'. In contrast, the credit market intervention by the central bank could effectively attenuate a financial crisis by either restoring the private bankers' financial intermediation or relieving the cost of external finance for non-financial firms. We have proposed two alternative operating rules that the central bank could follow when it implements the credit market intervention. That is, the central bank could either inject the public fund into the private bankers' balance sheet to encourage the private financial intermediation or supply the public fund directly to non-financial firms to relieve the contraction in capital demand. Comparison of the impulse responses under these two rules indicate that the credit market intervention by any of these two rules produce the similar outcomes in a financial crisis. The similarity between the two rules is supported by the financial accelerator mechanism which positively links the external finance premium to the bankers' leverage ratio.

Under the setting of a small open economy, Chapter 3 points out that foreign lenders' negative perception on entrepreneurs' financial conditions in an emerging market economy could lead to an actual financial crisis, even if their pessimism turns out to be groundless. That is, when foreign lenders have a negative evaluation of the domestic entrepreneurs' net worth, it could cause a sudden stop crisis in an emerging market economy with a high degree of foreign currency denominated debt via the reversal of capital out of the economy and the spike in the cost of foreign borrowing, as the 'self-fulfilling pessimism' argument in Calvo (1998) and Krugman (1999) suggests. A foreigners' negative perception of the entrepreneurs' net worth has a similar effect on production to an exogenous rise in the foreign interest rate, in that both shocks increase the cost of foreign borrowing and reduce the foreign fund supply for capital investment. The difference between the two lies in that the latter directly increases the cost of foreign borrowing, while the former does so via the distortion of the entrepreneurs' balance sheet. Hence, a change in foreigners' pessimism may reflect the more primitive source of a sudden stop crisis than an exogenous shock to the foreign interest rate.

In addition, Chapter 3 has explored the role of a number of pre-crisis conditions in the transmission of the financial crisis, such as (i) the degree of financial frictions, (ii) the coincidence of global recession and a sudden stop, (iii) the degree of the economy's reliance on the foreign resources in capital production,

and (iv) the choice of exchange rate regime. First, we have uncovered that the presence of a high degree of financial frictions in an emerging market economy could lead to large business cycle fluctuations when the economy is hit by a sudden stop. As discussed in Chapter 3, the degree of financial frictions is negatively related to foreign lenders' trust in an emerging market economy in normal times. Thus, if foreign lenders do not have a sufficient trust in an emerging market economy, they would react susceptibly to even a temporary and slight distortion in entrepreneurs' balance sheet (perceived by foreign lenders), so that they would reduce the credit supply or impose the high risk premium on entrepreneurs. This implies that an emerging market economy that fails to gain the trust from foreign lenders in normal times would experience a severer sudden stop crisis. Second, we have shown that if an emerging market economy faces a global recession at the same time when it is hit by a sudden stop, the financial crisis in the economy could be amplified and prolonged. The standard economics has predicted that while a sudden stop raises the cost of foreign borrowing and contractions in production and capital investment, it also could encourage the export demand in foreign countries via the domestic currency depreciation and the improved price competitiveness of domestic goods. However, our simulation results imply that when a sudden stop coincides with a global recession, the increase in export demand is restricted by a contraction in the overall aggregate demand in foreign economies, so that the sudden stop crisis in the emerging market economy could be aggravated. Third, we have found that the degree of an emerging market economy's reliance on foreign input in capital production could also affect the severity of a sudden stop crisis. That is, if an emerging market economy relies heavily on foreign input in capital production as compared to the households' preference over foreign goods, a currency depreciation

in a sudden stop crisis could lead to a rise in the real capital price, which could aggravate the cost condition for producing domestic goods. In such a case, an emerging market economy could suffer a severer production contraction in a sudden stop crisis by the rise in factor price as well as the rise in cost of finance. Fourth, our simulation results point to the importance of the choice of the exchange rate regime for the business cycle fluctuations in a sudden stop crisis. The conventional 'fear of floating' argument a la Calvo and Reinhart (2002) suggests that emerging market economies with large foreign currency denominated debts actually seek to stabilise the nominal exchange rate even though they announce that they adopt a free floating exchange rate regime. However, we have shown that a fixed exchange rate regime could produce the inferior performance in a sudden stop crisis to a free floating exchange rate regime. That is, the stabilised nominal exchange rate under a fixed exchange rate regime could reduce a possible increase in export demand for domestic goods by limiting the improvement of price competitiveness of domestic goods in a sudden stop crisis, while it could relieve a rise in cost of foreign borrowing. Our simulation result shows that the negative effect of a fixed exchange rate regime on the business cycle fluctuation could offset the positive effect.

In Chapter 4, we have evaluated the empirical validity of the theoretical arguments in Chapter 3, by estimating the small open economy DSGE model in Chapter 3 by the Bayesian MCMC methods and data from the US and South Korea. First, we have obtained the sizable estimates for sensitivity parameter of external finance premium to entrepreneurs' leverage ratio, implying that there exists a high degree of financial frictions in the emerging market economy. This suggests that the economy could be highly vulnerable to foreign lenders' evaluation

on the economy's financial condition. Second, the steady state share of domestic input in capital producers' investment good composite is estimated to be much smaller than that of domestic goods in households' consumption bundle, which suggests that capital producers in the economy rely heavily on foreign resources. In this circumstances, it is plausible that the economy suffers a severe and prolonged contraction by the aggravated capital price as well as the spike of the cost of foreign borrowing if it is hit by a sudden stop. Third, the Taylor rule coefficient attached on the nominal exchange rate is estimated to be negligible, which indicates that the central bank in the economy does not adjust the nominal interest rate that much to stabilise the nominal exchange rate. This suggests that the 'fear of floating' argument could be irrelevant to analyse financial crises in the South Korean economy. Fourth, our results from variance decomposition indicate that the main sources of business cycle fluctuations in the economy come from foreign financial shocks, such as shocks to foreign lenders' perception on the economy's financial condition and foreign interest rate. However, the domestic and foreign factors such as shocks to foreign output and domestic interest rate are less important in explaining the fluctuations in the economy.

Overall, the thesis establishes that the economy's business cycle could stem from distortions in the microeconomic conditions in the financial market such as borrowers' financial soundness (perceived by the lenders), and that it could be affected by the economy's environmental conditions, such as the degree of financial frictions, the degree of reliance on foreign inputs, and the choice of exchange rate regime. Accordingly, from a theoretical perspective, these findings point to the importance of the microeconomic conditions in the economy's macroeconomic performance. They also suggest that researchers, who want to enrich DSGE models

in terms of the reality, need to consider the problems that the economic agents face in reality and try to equip the models with the optimal behaviours as solutions to the problems. In addition, from a practical perspective, the policy authorities should consider the microeconomic problems agents face for the policy to be more effective. That is, they need to try to design the policy rule in a microfounded way since the economic agents are rational. In addition, in order to relieve or prevent a financial crisis, the economy needs to improve the environmental conditions. Specifically, the economy is required to gain the credibility in financial markets in normal times to reduce the degree of financial frictions; it should reduce the degree of reliance on foreign resources in terms of production factors as well as finance; and it would rather conduct a monetary policy in a market friendly way in order to avoid a potential negative effect of a fixed exchange rate regime on the business cycle in an economy hit by a sudden stop.

As summarised above, this thesis has analysed important issues as to financial crises, but it has also some limitations. For example, our analyses are mainly based on the investigation and comparison of the impulse responses of diverse shocks under alternative environments. Even though this approach offers many interesting insights about the transmission of a fianancial crisis, comparison of fluctuations to alternative shocks does not provide the quantitative implication in a strict sense. In addition, our study could be improved by reflecting the more realistic aspects of the recent financial crisis episodes, such as zero lower bound (ZLB) of the nominal interest rate. Moreover, taking the recent increase in the size or importance of South Korean economy into account, we could conduct the crisis experiments by a two country DSGE model rather than our small open economy DSGE model. In addition, our Bayesian estimation based on the US and South

Korean data could be lacking in providing the general evidence for the arguments put forward in this thesis. Clearly, it is of great importance to check the general validity by enlarging the sample and accumulating the episodes. These are the issues for future research.

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