Innovation by Asymmetric Firms

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September 2013

Abstract

One need only look at the list of the world's most valuable firms, including Apple Inc. and Microsoft, to understand that there is a link between innovation and success. However, little has been done to explore *why* some firms are more innovative. In this thesis we explore one possible reason that some firms are more innovative than others: innate ability.

The first essay explores the importance of abilities on the *innovative process*, defined as a firm's ability to spot *and* implement new technologies. We observe that if the *more* able firm possesses both an ability and timing advantage, it *always* becomes the dominant firm. However, if an *ex ante* low ability firm has an investment timing advantage it can become the *ex post* market leader if and only if the *a priori* ability gap is not too large.

The second essay analyses whether a firm's incentive to agglomerate, when research spillovers are location based, survives the existence of asymmetric abilities which may generate *heterogeneous* unit costs. First, we find that agglomeration is never optimal, not even when the firms are symmetric, due to the threat of rapidly escalating of price competition. Second, where a firm is better able to both reduce its own costs *and* assimilate a rival's economic knowledge, it becomes more aggressive in terms of both location and investment, leading to increasingly asymmetric outcomes.

The third essay examines the impact abilities have on venture capital funding. Specifically, we consider the impact of venture capital from the *firms'* perspectives. We find evidence of both a *direct* and *indirect* impact of venture capital. Furthermore, we find that the commonly held assertion that venture capital spurs success is too vague. Instead, venture capital *only* spurs innovation amongst the "lucky", chosen few, but unambiguously suppresses innovation of non-VC-backed firms.

Contents

Acknowledgements

I would like to express my sincerest gratitude to all those who have made the completion of this thesis possible.

First and foremost I would like to thank my wife, without whom I would have given up a thousand time. Her endless supply of tea, and her unconditional love and support have enabled me to keep my sanity throughout.

Second, and equally important, I would like to thank my supervisor Dr. Bipasa Datta for her encouragement, guidance and support. I would also like to extend my gratitude for her contributions towards this body of work. Without her help, completing this thesis would have been impossible.

Third, I would like to thank my Dad and my Brother for their hard work proofreading this thesis. Whilst I'll blame them for any remaining errors, without their efforts this work would have sounded like gibberish. I would also like to thank my Mother, otherwise she might feel left out.

Finally, I would like to offer an extended thank you to all my family and friends who have had to put up with me being miserable during this time. To them all, at the behest of Dr. Mark "Big Man" Eslick, I give a Haiku

I've been a sad git
Now this trial is over
I can go play now

Declaration

I hereby declare that this thesis is my own, original work and that none of the material contained within has previously been submitted for a degree in this, or any other, awarding institution.

The idea for Chapter 3 was reached jointly with Dr. Bipasa Datta, with Bipasa helping to form the model specification. Whilst the results presented in this thesis were derived and written by myself, Bipasa offered extensive help throughout and comments during editing.

Chapter 4 was co-authored with Dr. Bipasa Datta. The initial idea for the chapter was my own, but Bipasa helped form the underlying model. Whilst I derived the results, Bipasa contributed to the writing of the introduction and model sections of the paper, and with general comments on the rest.

Papers based on Chapters 2 and 4 have been published as University of York Discussion Papers:

Beacham, M. and Datta, B. (2012). The effect of Stackelberg cost reductions on spatial competition with heterogeneous firms. *University of York Discussion Paper*, No. 12/14. [Online]. Available at: http://www.york.ac.uk/media/economics/documents/discussionpapers/2012/1214.pdf [Accessed 1/5/2014]

Beacham, M. and Datta, B. (2013). Who Becomes the Winner? Effects of Venture Capital on Firms' Innovative Incentives - A Theoretical Investigation. *University of York Discussion Paper*, No. 13/33. [Online]. Available at: http://www.york.ac.uk/media/economics/documents/discussionpapers/2013/1333.pdf [Accessed 1/5/2014]

Introduction

In recent decades, and with society's growing reliance on technology, the innovative abilities of firms have become an increasingly important factor in driving and maintaining their success. In fact, one need only look at the list of the world's most valuable firms, which includes such notable - and innovative - companies as Apple Inc., Exxon Mobil, Google and IBM, to notice that a firm's innovative efforts are positively correlated with their success (Forbes, 2013b). However, this idea is not just one held by economists but also by those in charge of businesses themselves. In a survey of 216 worldwide executives, the Economist Intelligence Unit (2008, p.6) observed that "70% of ... respondents [believed] product innovation [was] critical or very important to maintaining corporate competitiveness", by reducing production costs, generating new customer opportunities, or both. However, despite the apparent belief that innovation was crucial for success, a significant (innovative) performance gap persisted between firms. For example, 90% of the top performing companies surveyed described themselves as successful or very successful at driving innovation in new product development compared to only 33% for all other respondents (Economist Intelligence Unit, p.6). A similar gap persisted across the firms' stated abilities to improve their manufacturing processes, falling from 53% for the most innovative firms to only 20% for all others.

For an example of this disparity, one need only consider the market for search engines in which, as of September 2013, Google controls approximately 89.7% of the global market (StatCounter, 2013). In contrast, its nearest rivals, Bing, Yahoo! and Baidu, control a paltry 4.1%, 2.9% and 1.1% respectively (StatCounter, 2013). Google's innovative success, and subsequent market dominance, has come from its active pursuit of an "innovative culture".[1] This approach to innovation has played a crucial role in determining Google's corporate culture, with two unique strategies standing out. First, their implementation of "20% time", which enables employees to spend 20% of their time working on individual side projects, allowing Google to reap the benefits of research pathways that would have otherwise been ignored (Forbes, 2013a). Whilst this has now, to some extent, been reigned in, this has harboured a creative and innovative atmosphere amongst its employees. In fact, the success of this strategy has seen it implemented by *other* firms. Second, Google has consistently promoted creative work spaces, once again hoping to attract more innovative employees and boost innovative output (BBC, 2008). This attention to detail and willingness to take risks has seen Google develop into a creative powerhouse.

Whilst many firms try to mimic Google's strategies, very few are able to turn these into sustainable success. The question is, then, why are firms that are employing similar innovative strategies almost destined to diverge, with some firms being consistently more successful. Obviously, *neither* a desire to be innovative *nor* simply imitating a rival's innovation strategies is to be enough to guarantee success. Therefore, in this thesis we explore *one* of the reasons that may explain the innovation disparity between firms: innate ability. In essence, this assertion is simple and assumes that some firms are more likely to be innovative than others. Of course, the difference in abilities could also be driven by a wide range of factors, including but not limited to: the research skills of employees; the quality of the equipment and raw materials; or luck. This thesis examines three distinct ways in which abilities influence the strategic innovation decisions made by and the composition of the final product market.

Chapter 2 examines how abilities are important in influencing the *innovative process* (defined as the ability to spot and implement new innovative ideas) and determining the composition of the final product market. To the author's knowledge, no attempt has been made to examine the innovative process in this way. To do this, we propose a model of endogenous cost selection in which

[1] For example, see Google's "eight pillars of innovation" which offers up advice including: "strive for continual innovation, not instant perfection"; and "look for ideas everywhere" (Google, 2013).

firms are heterogeneous with respect to both their *ex ante* abilities to implement a cost reducing technology and their investment timings. In modelling innovation in this way, it is possible to evaluate the importance of both investment timing and innovative ability in determining the composition of the final product market or, more interestingly, whether it is possible for an *ex ante* low ability firm to become an *ex post* market leader. In doing this, we extend the existing linear spatial literature in *two* ways. First, firms are *a priori* heterogeneous in respect of both their ability to implement cost reductions and the timing of their investments. Second, the *ex post* cost differential is endogenous and determined by the strategic interactions of the firms.

The results suggest that abilities do, indeed, play a crucial role in determining which firm will be most innovative. To summarise: (*i*) if a firm possesses both an investment timing and ability advantage it always becomes the *ex post* efficient firm; and (*ii*) if a firm possesses only an investment timing advantage, it can become the *ex post* efficient firm if and only if the *a priori* ability gap is not too large. Our results appear to both agree with and extend upon the existing literature. Whilst the results presented here agree that the *ex post* low cost firm would emerge as the market leader, they also suggest that this firm could be either the *ex ante* efficient or the inefficient firm. If, as here, the firms are *a priori* heterogeneous with respect to their effectiveness at reducing costs and the timing of their investments then a firm's unit cost is no longer a measure of its efficiency but, instead, a direct consequence of strategic interactions *given* the firms' relative efficiencies (investment timing and cost reduction effectiveness).

Chapter 3 takes a slightly different approach and examines whether firms of asymmetric abilities would ever find it optimal to agglomerate. In essence, we analyse whether a firm's incentive to agglomerate, when research spillovers are location based, survives the existence of asymmetric abilities which may generate *heterogeneous* costs. To address this question we employ a three-stage, innovation-location-price Hotelling (1929) model with quadratic transport costs and make two key assumptions: *i*) the closer the firms are to each other, the greater the benefit they receive from their rivals' efforts in R&D; and *ii*) firms are a priori heterogeneous with respect to their innovative abilities, defined as the ability to implement cost reductions and assimilate a rival's research (absorptive capacity). It is the assumptions, that the firms absorptive capacities may differ, that is new to the literature.

We find a couple of results that both complement and contrast with the existing literature. First, contrasting to the existing literature, we find that the firms never find it optimal to (partially) agglomerate, not even if the firms are *a priori* symmetric. Instead, the problem of rapidly escalating price competition drives the firms to maximal differentiation, the classic d'Aspremont *et al* (1979) result. Second, and complementing the existing literature, location based spillovers, combined with asymmetric firms, leads to increasingly asymmetric outcomes. This result is driven by the existence of *heterogeneous absorptive capacities.* As the firms locate closer to one another, the more able firm assimilates *more* knowledge than it leaks to its inefficient rival, and this allows it to become *more* predatory. In turn, this increases its ability to drive a less able firm away from the market, generating greater demand and profits, and this induces it to invest *more.* In contrast, a less able firm's investment strategy is related to the heterogeneity of the consumers: when consumers become increasingly homogenous, the weaker firm must act "soft" to mitigate price competition and minimise spillovers.

Finally, in Chapter 4 we examine the roles that abilities *and* venture capital play in spurring innovation and/or firm success. It has become established in the empirical literature that venture capital plays an important role in the promotion of innovation at industry level and the professionalisation of firms at micro-level (Da Rin et al (2013)). However, whilst the venture capital-to-success link has been well explored, the mechanisms behind *how* and *why* certain venture-backed firms are, apparently, more successful is important and has, to the authors' knowledge, been ignored within the majority of the literature. *i*) what impact does venture capital have on a firm's incentives to invest in innovation?; *ii*) how do rival, non-venture capital-backed firms respond?; and *iii*) does the prospect of receiving venture capital funding in the future, and its associated benefits, spur innovation *ex ante*? To address these questions, we consider a stylised two-period, multi-stage game in which innovation is uncertain and firms are of different innovative abilities. In order to simplify proceedings, we turn the tables on the existing literature and assume that venture capital is *exogenous* or, like the firms of the empirical literature, passive. Nonetheless, we try not to lose any of the key features that venture capital possesses. Therefore, we assume venture capital funding is a package consisting of three things: *i*) an equity stake in the firm; *ii*) pecuniary funds; and *iii*) value-adding services such as monitoring, implementing formal HR procedures or improved marketing.

We find evidence that venture capital plays an important role in the success of a firm. First, the addition of venture capital also has a profound impact on competition *directly after* it has been granted. In essence, venture capital tips the balance of competition in favour of the firm that receives it, regardless of the firm's relative ability level. It does this by inducing the venture capital-backed firm to invest more and the rival firm less, improving the relative probability of success for the portfolio firm. Therefore, we suggest that the commonly held belief that venture capital spurs innovation is too simplistic, as it clearly damages the prospect of the firms it *does not* support. We also find, somewhat weaker, evidence for venture capital's impact on innovation *indirectly*, prior to its provision. Whilst a number of these results suggest an *ambiguous* impact of venture capital, this ambiguity should *not* be misinterpreted as *no* effect. Rather, one should interpret our *indirect effect* results more broadly: given the specification, it is likely that *future* venture capital will have an impact on first period efforts, though it is not possible to say whether this impact is positive or negative. The reason for this is that venture capitalists alter *future* expected payoffs and, therefore, the firms' incentives to invest.

The remainder of the thesis is set out as follows. In Chapter 1 we examine the relevant literature. In Chapters 2 - 4 we present our essays on innovation and ability. Chapter 5 concludes.

Chapter 1

Literature Review

1.1 Introduction

The research presented in Chapters 2, 3 and 4 has a clear focus on investment, research and innovation by firms of asymmetric abilities. The theoretical and empirical literature relating to investment and innovation has spread to almost every important economic concept and, consequently, this leaves a broad spectrum of possible literature to explore. Our focus, however, spans only *two* broad topics: *i*) Hotelling's (1929) linear spatial model and horizontal product differentiation; and *ii*) venture capital and its links to innovation and success.

With regards to Hotelling's (1929) model of linear spatial competition and our research, two important observations have been clearly, and repeatedly, made: *i*) linear spatial models are sensitive to their specification; and *ii*) *ex post* cost asymmetries are supported in equilibrium if and only if they are not too large. It is these rigorously explored observations that have been crucial in allowing Hotelling's (1929) model to become an important part of the Industrial Economist's toolbox. However, whilst much has been done to examine the impact of innovation on a firm's location decision, there is an apparent need to incorporate the *innovative process* instead of maintaining the *status quo* of exogenous cost differentials. Only by explicitly examining the investment decisions of asymmetric firms, and the subsequent innovative outcomes, is it possible to understand how such *ex post* asymmetries arise and their impact on the final product market.

Second, we examine the literature examining the interactions, and their consequent outcomes, between venture capitalists and entrepreneurs. The majority of this subsection is focused purely on the empirical literature; a focus that persists within the venture capital literature itself.[1] Whilst we do examine *some* theoretical ideas, this literature is almost exclusively focused on the deriving an optimal (convertible security) contract to incentivise both the entrepreneur and venture capitalist to invest efficiently. From an empirical perspective, similar results have been repeatedly observed: venture capital spurs innovation at an industry level and success at firm level. However, regardless of whether one examines the theoretical or empirical literature, there always exists an unpalatable, implicit assumption: venture capitalists are the *sole* drivers behind their interactions with entrepreneurs. Yet, surely this suggestion must be incorrect because it tells only *half* of the story. Quite simply, the literature has frequently, and implicitly, assumed that firms are passive entities in these interactions and, consequently, overlooked an important question: how do *firms* respond to the prospect of receiving venture capital and how do they behave afterwards?

The rest of the chapter proceeds as follows. First, section 1.2 examines the literature pertinent to linear spatial competition, both broadly and related to asymmetric firms. Second, section 1.3 explores the venture capitalist literature. Finally, section 1.4 concludes.

1.2 Spatial competition: a review

In this section, we examine literature regarding linear spatial competition. More specifically, the focus of this review is on two specific elements. First, we examine the broad literature exploring linear spatial competition, à la Hotelling (1929). In doing so, we examine why this model has become so widely used within the industrial organisation literature but also uncovering some of the issues that have led to oft-contradictory results.[2] Second, we evaluate the impact of asymmetric costs and research spillovers within the spatial compe-

[1] See Da Rin *et al* (2013).

[2] In an excellent review of the more general aspects of spatial competition, Biscaia and Mota (2013, p.856) observe, "spatial competition models have seen regular growth in terms of the number of publications. Furthermore, most of those models have been published in journals with at least moderate impact."

tition literature. However, in this instance we do not simply focus on linear spatial competition but also examine the role of other specifications, most notably Salop's (1979) circular city model. This additional literature allows us to demonstrate that the issues created by cost asymmetries are not the preserve of linear spatial models but, instead, are present within other spatial settings.

The main conclusions can be summarised as follows. First, the spatial competition literature suggests that it is, perhaps, the most useful tool for the analysis of horizontal differentiation. However, it is obvious that the specification of a Hotelling (1929) style model is important because it is naturally susceptible to discontinuities and corner issues that one must take care to address (Biscaia and Mota (2013)). Yet, the sheer volume of research into these issues has considerably reduced the number of potential pitfalls that one might unwittingly encounter. Second, the work on asymmetric costs yields an equally interesting set of results, most notably that spatial competition can handle cost asymmetries and, by extension, asymmetric firms. However, these models are not without their issues. That is, even after accounting for the more specific issues of spatial competition, where cost differentials are sufficiently large, issues of equilibrium existence still arise.

Hotelling's (1929) model and extensions

Hotelling (1929) developed what has become one of the most influential models in industrial organisations, and the first to really evaluate horizontal differentiation. In the model, consumers were uniformly distributed along a linear "city" of unit length and purchased a single unit of a homogeneous good differentiated only by their location. Each consumer, upon purchasing the good, has to pay a linear transportation cost that represents cost of travelling to collect their preferred good. For the firms' part, they simultaneously select locations and then prices. The result suggested that the firms would be driven to agglomerate in the centre of the market because, by moving towards their rival in the location space, the could steal a rival firm's market share. However, such a conclusion is less than desirable because it leads to a situation in which both firms earn zero profits; the "Bertrand (1883) paradox".

It is obvious that this model and its results are based on a number of critical assumptions: *i*) linear transport costs; *ii*) firms locating within the city's boundaries; *iii*) homogeneous goods; *iv*) consumer densities; and *v*) simultaneous moves at each stage. We examine the consequence of each of these assumptions in turn.[3]

D'Aspremont *et al* (1979) (henceforth DGT), examined whether agglomeration was really the optimal outcome for firms by examining the role that transport costs play in equilibrium. By replicating Hotelling's (1929) model, but replacing linear transport costs with ones that were quadratic in distance, the results were completely reversed. The issue, they found, was that discontinuities existed in the original specification where an indifferent consumer was not located between the two firms and the application of quadratic transport costs solved this by eliminating discontinuities in the reaction functions. Therefore, whilst firms still had an incentive to locate towards the centre (a direct, market stealing effect), these were more than countered by rapidly intensifying price competition (an indirect, price effect). Thus, the firms prefer to locate at the opposite extremes of the market.

The importance of the transport cost specification have also been explored by Gabszewicz and Thisse (1986) and Anderson (1988). Analysing a Hotelling (1929) model where transport costs contain both linear and quadratic components, they concluded that no price equilibrium existed when the firms are located too close together and in most cases no location-price equilibrium existed at all. Again, discontinuities in the reaction functions - identical to those found by d'Aspremont *et al* (1979) - undermine equilibrium and can only be removed by eliminating the linear portion of the transport cost (Anderson, 1988). In related work, Economides (1986) examined Hotelling's (1929) model using "intermediate" transport costs; between linear and quadratic. Formally, the model assumes that for a given distance between the firms, d, the transport cost should be such that $f(d) = d^{\alpha}$ where $\alpha \in (1, 2)$.[4] The results suggest that whilst minimal differentiation never obtains, maximal differentiation is not the only outcome. That is, for a range of $\alpha \in (1, 2)$ it is concluded that firms may locate somewhere between the maximal and minimal locations. These results

[3] Other assumptions are made within this model, for example duopoly market and complete information. However, they are not crucial for Chapters 2 and 3 so, for the sake of brevity, we ignore them here. The more interested reader may with to see Biscaia and Mota (2012) for a full discussion of all the assumptions.

[4] Setting $\alpha = 1$ or 2 is consistent with Hotelling's or DGT's models respectively.

point to one important result: it is the degree of convexity of transport costs, not convexity *per se*, that ensures continuous reaction functions and once these are established minimal differentiation never obtains.

At this point, it is worth noting that the specification of the transport cost function says something about the *types* of cost consumers face. With linear transportation costs, it is realistic to assume that such costs are simply physical, shoe-leather costs. In contrast, quadratic transport costs better represent consumer tastes/disutility (Biscaia and Mota (2013)). Nonetheless, given the obvious importance of transport costs for the existence of equilibrium, it is likely be that Hotelling's (1929) model handles information about a product characteristics better than physical locations.

To this end, Hotelling's (1929) model has been expanded to incorporate multi-dimensional product characteristics. Irmen and Thisse (1998), extends the DGT model by allowing for an n-dimensional product space in which consumers may have a preferred product characteristic. They find that where consumers prefer one product characteristic sufficiently more than the others, both firms maximise differentiation in this one product space and minimise differentiation in all others. That is, maximising product differentiation across a *single* product characteristic may be sufficient to soften price competition enough to support minimal differentiation across all others. This prompts them to conclude that "Hotelling was almost right" because *both* minimal and maximal differentiation occur within the same model despite the use of quadratic transport costs.[5] A similar result is derived by Ben-Akiva *et al* (1989) but, in this model, each firm locates within a linear city but is also given a "brand" - essentially another product characteristic. Similar to Irmen and Thisse (1998), they find agglomeration only occurs if brand heterogeneity is sufficiently large or price competition is softened sufficiently to allow them to both locate in the centre of the market without exacerbating price competition.

Consumer densities also play a crucial role in determining the locations of the firms. Tabuchi and Thisse (1995) examine what occurs if the original uniform density were replaced by a triangular density with the peak at the centre point of the market. Under this assumption, "when the density is concave,

[5] Tabuchi (1994) had previosuly found an identical result but only within a 2-dimensional product space.

symmetric and not 'too much' different from the uniform density, there exists a unique Nash location equilibrium" in which firms locate symmetrically (Tabuchi and Thisse, 1995, p.223). However, when concavity is "excessive" only asymmetric equilibria exist, despite the model remaining symmetric. Anderson *et al* (1997) finds similar results using log-concave density functions observing that symmetric equilibria exist if and only if the distribution of consumers is neither too asymmetric nor too concave. When the density function is too concave, similar to Tabuchi and Thisse (1995), only asymmetric equilibria exist.

Tabuchi and Thisse (1995), and Lambertini (1994, 1997) also examine the possibility that firms can locate outside of the market.[6] Given a quadratic transport cost function to ensure an equilibrium, the models suggest that price competition is so fierce that the firms would prefer to locate outside of the market. Formally, for an unbounded city with consumers uniformly distributed over $[0, 1]$, the result is a symmetric equilibrium in which the firms locate at $-1/4$ and $5/4$ respectively. Intuitively, this result is driven by the contrasting forces discussed within the DGT framework: *i*) direct effects (market stealing); and *ii*) strategic effects (exacerbating price competition). Given that firms strictly prefer to locate outside of the city's boundaries, it is clear that the fear of exacerbating price competition is, indeed, incredibly strong.

The author has found this assumption to yield the most vocal complaints and divide opinion. Whilst such a scenario is easy to imagine for physical locations, for example out-of-town retailers, exploring this assumption within the framework of consumer tastes is less intuitive. As an example, imagine a city bounded by $[0, 1]$ where 0 represents "sweet" chocolate and 1 represent "bitter" chocolate. The existence of firms outside of the consumer distribution simply suggests that firms produce product types that were either unknown by consumers *ex ante* or that the firms produce goods that are not the exact preference of any consumer. Whilst this is not good from a social welfare perspective, there is no reason why firms cannot, or should not be able to, produce goods that are not meet any single consumer's specific preferences.

Finally, Lambertini (2002), examines the impact of sequential entry into the Hotelling model and evaluates the impact that time between moves plays

[6] Under this assumption the firms are free to locate anywhere along the real axis.

on the location choices of each firm. Within the paper he notes that within single-period Stackelberg games the equilibrium collapses to the simultaneous move (maximal differentiation) game due to the severity of price competition.[7] However, as the time between the firms' moves increases, the closer to the centre the Stackelberg leader locates with the late entrant locating at one extreme end of the market.

The main conclusion to be drawn regarding the basic, linear-city model as proposed by Hotelling (1929) is that it offers a malleable and useful tool for examining the impact of horizontal differentiation on product market competition. However, as can be seen from extension work, it is not a completely predictable tool and can be particularly sensitive to the specification of the model. Moreover, even a slight tweak to the original model's specification can have huge implications for equilibrium solutions and welfare implications. However, the linear spatial model - with complete information - has seen much work done to evaluate all the possible implications and is now well understood (Biscaia and Mota (2013)). Consequently, potential pitfalls have been well mapped and this has strengthened the importance of spatial competition as a tool for industrial economists to draw upon in evaluating horizontal differentiation.

Asymmetric costs

Schultz and Stahl (1985) were the first to propose an examination of the reasons for non-existent equilibrium within linear spatial models other than mill price undercutting or transport cost issues. After observing that payoffs are *not* determined by the firms' locations *per se* but the distance between them, they explore reasons that may push firms from established equilibria and note that asymmetric production technologies were a likely cause. Ziss (1993) formalised this idea, analysing exogenous cost asymmetries within a DGT model. The model implied that, despite price equilibria existing for all cost differentials, a location equilibrium can only be supported where the cost differential between the firms is not too large. Matsumura and Matsushima (2009) then proposed two extensions. In the first, they allow the firms to locate anywhere on the real

[7] See also Nevin (1987) and Bonnano (1987)

axis, finding the firms optimally choose to locate *asymmetrically* outside of the city with the low cost (efficient) firm locating more centrally.[8] Second, they questioned whether pure strategies are the only equilibria within this model, observing that for all "large" cost differentials a solution in mixed strategies always existed. Intuitively, for large cost differentials the equilibrium becomes an effective game of "cat and mouse" with the *ex ante* efficient firm attempting to prevent entry by locating next to the inefficient firm and the inefficient firm trying to get away.

It is for this reason that the Hotelling (1929) model "is [assumed] inconvenient for investigating endogenous production costs" as a full examination of location equilibrium conditions is difficult to obtain (Matsumura and Matsushima, 2009: p216). Nonetheless, from a theoretical standpoint, the existence of mixed strategy solutions may explain differences in market structure across seemingly similar markets; especially where firms face asymmetric costs. Bester *et al.* (1996), using a DGT model, find an infinity of mixed strategy equilibria and argues that, without some coordination mechanism, firms face a strictly positive probability that they would locate at the same point. Thus, where firms face asymmetric production costs, simple coordination failure can be the difference between a high cost firm remaining in the market or being driven out of business. Whilst this result is intuitive, Martin (2001) offers an alternative - and equally depressing argument for this result. If firms *do not* randomly select locations but make such decisions in private, this would create the same propensity for coordination failure and drives a similar result.

Unfortunately, within these models no firms select their own costs but, instead, are randomly assigned as either the high or low cost firm. Whilst some work has been done to examine endogenous cost selection within a linear spatial setting, more often than not they make assumptions that prevent cost differential undermining equilibrium outcomes. Matsumura and Matsushima (2004) and Kumar and Saha (2008) examine cost reduction decisions where one firm is publicly owned and the other privately. Where the level of public ownership is high, the private firm's costs will become lower than those of the public firm due to its excessive investment and the public firm's incentive to

[8] Given that the firms are also able to locate further from one another a larger cost disparity is supported than in Ziss' (1993) original specification.

maximise social welfare. However, when the level of public ownership is low, the standard DGT result obtains. Therefore, in this framework, it is possible that the incentives of the firms never clash and asymmetric firms can compete without issue. Similarly, Matsumura and Matsushima (2010a, 2010b) also acknowledge that any level of cost differential can be supported if the firms are able to license their innovations. Quite simply, the lower cost firm licenses its innovation to a high cost firm but extracts all of its rival's profits in the process, again preventing a clash of incentives.

One reason that might explain this dearth of cost reduction literature within the linear spatial field is that there has been a much greater focus on models that examine spillovers. Building on the seminal works of d'Aspremont and Jacquemin (1988) and Kamien *et al* (1992), who introduced the concept of exogenous spillovers, much headway has been made analysing the implications of endogenising spillovers within a spatial setting.[9] The addition of spillovers to spatial models is a good match because it is often assumed that spillovers should only act locally, or inversely proportional to the distance between the firms. Moreover, anecdotal evidence, such as agglomeration in Silicon Valley, are often cited as examples of firms agglomerating in order to benefit from positive spillover effects between *local* firms.

Mai and Peng (1999) were the first to use a linear spatial setting to examine the incentives for firms to invest in cost-reduction if spillover rates are inversely proportional to the distance between the firms. The equilibrium balances both competitive and cooperative forces, similar to the original DGT results. However, now the lure of additional cost reductions through spillovers offer firms an added incentive to locate closer to one another and, when the cooperative force becomes dominant, it is possible that minimal differentiation is restored. Piga and Poyago-Theotoky (2005) examine a similar model but assume R&D is quality-improving. Again, the spillover rate is linked to the distance between the firms but, in this case, is never sufficiently large to lead

[9]See Kamien and Zang (2000), Amir *et al* (2003) and Katsoulacos and Ulph (1998) for examples of endogenous spillovers outside of a location setting.

to agglomeration.[10, 11]

Zhang and Li (2013), similar to Matsumura and Matsushima (2004) and Kumar and Saha (2008), examine a public-private firm relationship with innovation spillovers. They find that the spillover rate is the key determinant of whether it is the public or private firm that invests more. For sufficiently small spillover rates the Matsumura and Matsushima (2004) result holds. However, above a certain level the public firm is induced to heavily invest in cost reduction in order to maximise social welfare. In addition, the private firm suppresses investment with high spillovers in order to maximise its profits. However, neither agglomeration nor maximal differentiation are supported equilibria.

Despite all of this work within the linear spatial field, much has been done in non-linear markets, especially using a Salop (1979) model. Salop, in an attempt to mitigate the corner difficulties associate with Hotelling's (1929) model, analysed a model in which consumers were located around a circular city. Whilst the paper does not analyse a two-stage location-price game, assuming locations to be exogenous, it has been influential in opening up new avenues of spatial research.

Within such a framework, both Aghion and Schankerman (2004) and Syverson (2004) analyse exogenous cost asymmetries and the impact of public policy aimed at intensifying competition. Both apply a Bayesian set-up, with firms knowing only their own (stochastic) production costs and assume prices are set prior to learning their rivals' locations and costs; though the distribution of costs are common knowledge. Similar to the linear spatial literature, an issue again arises due to the potential for price undercutting. Syverson (2004) assumes that an Eaton and Lipsey (1978) "no mill-price undercutting rule"

[10]Interestingly, Sun (2013) finds no difference between cost-reducing or quality-improving R&D. Using a three-stage (location-investment-price) model, with linear, endogenous spillovers dependent on distance, Sun (2013) evaluates the differences between the two R&D types. They conclude that there exists a duality between cost-reducing and quality-improving investments given symmetric conditions and knowing one outcome allows a simple derivation of the other.

[11]A number of papers also examine similar models but within a location-quantity setting. The results are similar, suggesting that the spillover rate plays a key role in determining the locations of the firms, but the models are suffciently divergent in their approach to not completely fit within our discussion. Consequently, they are ommited (see Long and Soubeyran (1998) and Mota and Brandão (2004)).

must be satisfied, similar to that derived by Ziss (1993). This assumption ensures that each firm always faces a positive market share in any state of the world. However, it is observed that heightened competition makes it harder for inefficient producers to operate profitably and high cost firms are forced out of the market.[12]

Aghion and Schankerman (2004) take a different approach and assume that decisions are based upon average costs which, as noted by Alderighi and Piga (2010), suggests firms only needs a positive market share *on average*. Under these conditions similar results obtain regarding the structure of the market and market entry. However, they add the benefits from enabling increased competition come in three forms: *i*) high cost firms face reduced market shares; *ii*) increased incentives to reduce costs; and *iii*) stimulating entry by new low cost firms. Nonetheless, Aghion and Schankerman's (2004) assumptions regarding the firms' decisions are not strong enough as the result imply that some firms may have negative market shares in equilibrium (Alderighi and Piga (2010)). This occurs because, by ignoring the possibility that one firm may be able to undercut another, "a highly efficient firm's reach could potentially extend beyond its nearest neighbour's position" (Alderighi and Piga, 2008, p.3).

It is also possible to extend these result by allowing firms to select their own locations. Vogel (2008) adds endogenous locations to this framework and observes that more efficient firms are more isolated in equilibrium. However, most intriguing is the result that the marginal cost of its rival does not matter *per se*, but does through the mechanism by which it alters the average marginal cost, whether or not they are direct competitors.

Research into endogenous cost reductions within a Salop framework have also been attempted. Scalera and Zazzaro (2005) examine the impact of changing market conditions on a firm's incentives to reduce their costs. They find that cost reducing investments are increasing in market size and entry costs. In related work, Ebina and Shimizu (2012) allow firms to invest in both cost-reducing and quality-improving R&D. In line with the literature on multiple dimensions, firms only differentiate in one aspect, cost reduction. Whilst,

[12] Syverson (2004) also tests his results empirically and observes persistent productivity differences. This is chalked up to "spatial substitution barriers" (Syverson, 2004, p.1218).

strictly speaking, minimum differentiation is not achieved in either dimension they suggest that the idea is, essentially, the same.

To summarise, the cost asymmetry literature is not without its issues. However, it is comforting to note that the problems that create difficulties in the linear spatial cases are also an issue in the circular city model. The most notable similarity is that, regardless of the spatial setting, one must make one crucial assumption: an Eaton and Lipsey (1978) no mill-price undercutting condition. However, what has been noted in the linear spatial models is that, even where a pure strategy location equilibrium cannot be guaranteed, a mixed strategy solution must exist. Such a result is not yet present within a Salop framework.

1.3 Venture capital and innovation: a review

In this section we consider literature linking venture capital (henceforth VC) backing to greater levels of innovation and success amongst VC-backed firms. Rather than examining all literature relevant to venture capital, we take an empirically focused approach to help us derive a theoretical model (see Chapter 4) that captures the reasons why such a link exists. Whilst we do examine some theoretical literature, we do this to highlight two points: *i*) the theoretical literature is focused on the derivation of optimal contracts; and *ii*) there is an implicit assumption that the venture capitalist drives this relationship. Obviously, this dearth of theoretical literature within our review may overlook some relevant theoretical ideas but, given the nature of empirical research, we should still capture relevant theoretical ideas that we have omitted.

After examining a cross-section of the theoretical literature, we examine the empirical observations which can be split into three broad areas. First, we examine the impact of VC at an industry level, observing a consistent link between VC-backing and innovation. Second, we examine the impact of venture capitalists on *individual* firms. The results here are also consistent: VC does not spur innovation amongst individual firms but does, in general, lead to stronger firm performance. Finally, we evaluate the empirical evidence that suggests *why* VC may help spur success; through a hands-on approach and their provision of value-adding services.

1.3.1 Theoretical literature

The bulk of the theoretical VC literature focuses heavily on how the design of optimal contracts can drive efficient investment by an entrepreneur and an optimal level of involvement by a venture capitalist. At the heart of this discussion has been the increasing realisation that the venture capitalists contract of choice involves *convertible securities*: a debt-like security which the venture capitalist has the *option* to convert into an equity-like security. However, such contracts are rarely used by more traditional financing methods or, as Schmidt (2003, p.1140) observes, "convertible securities are very rarely used by banks or other outside investors who finance the bulk of small (but more established and less risky) companies". Consequently, the established literature examines how convertible securities impact the venture capitalist-entrepreneur relationship in three ways: *i*) how such contracts act as an impetus for venture capitalists to exert their own, uncontractible effort; *ii*) how venture capitalists decide to refinance or liquidate existing deals; and *iii*) how such contracts affect control rights (the rights a venture capitalist may have over a portfolio firm's decisions). We examine *i*) and *iii*) in turn.[13]

First, Schmidt (2003) examines a double moral hazard model in which a venture capitalist only exerts effort after an entrepreneur has done so. With convertible contracts, a venture capitalist will only exert effort if they have exercised their conversion rights, and this will only happen if the entrepreneur has exerted enough effort initially. Therefore, conversion acts as a way of inducing *both* agents to invest efficiently.[14] Using a similar double hazard model, Hellman (2006) argues that convertible securities may take many forms - not simply debt-to-equity - and explores the case in which final outcomes may differ based on the exit decision of the venture capitalist (either by IPO or acquisition). The main finding is that such cash flow rights do, indeed, differ between the two exit cases. In the IPO case the entrepreneur's equity stake is automatically preserved (and this ensures efficient investment) but, for

[13]We ignore the question of refinancing as this is irrelavent to our model in Chapter 4. However, an interested reader may wish to read Repullo and Suarez (2004) and Dessí (2005), both of which find convertible securities to be optimal in the refinancing decision.

[14]See also Casamatta (2003) who examines the possibility of an outsider offering value-adding services and that the provision of convertible contracts is dependent on the level of "advice" given.

acquisitions, additional cash flow rights (additional payments from the firm's future profits) are allocated to the venture capitalist.

Second, within an incomplete contracting framework, Berglöf (1994) found an intuitively appealing result in which control rights are given to the entrepreneur in good states and the investor in bad states. Simply, when things are less risky, there is less need for a venture capitalist to "step in". However, more recently, Cestone (2002) examined a model in which cash flow rights and control rights are offered independently and find: *i*) these rights follow a joint pattern; and *ii*) risky claims should be be negatively correlated to control rights. The latter result, it is argued, is driven by the inclusion of contractual contingencies: by appropriately designing a contract, entrepreneurs can induce venture capitalist effort *and* limit interference. However, one big issue with this body of literature is that control rights are *not* discrete - to be offered only to a venture capitalist or entrepreneur - but are instead a specific set of voting rights that proportionally *divide* control. To this end, de Bettignies (2008) examines a model with an additional element of *joint* control and finds that convertible contracts work well in this situation, especially where the efforts of a venture capitalist and entrepreneur complement each other well.

Whilst a lot has been done regards contracting - including optimally inducing entrepreneur and VC effort - the literature still ignores many key questions. The most obvious is *how*, given the apparent usefulness of convertible contracts, would firms react *ex ante* and *ex post* to the potential opportunity to accept such deals. If such contracts induce optimal levels of effort, then surely this would also act as an incentive for firms to invest more efficiently even prior to being offered such a contract. However, as is also the case in the empirical literature, the firms are assumed to be passive.

1.3.2 Empirical Literature

Before reviewing the empirical literature linking VC to increased innovation, we observe that testing this hypothesis is fraught with endogeneity issues because VC and innovation are linked in two opposing ways (Hirukawa and Ueda (2011) and Da Rin *et al* (2013)).[15] On the one hand, VC may lead to a direct

[15] See also Sørensen (2007)

increase in innovation through the provision of value-adding services, such as monitoring the firm's performance or professionalising the firm (*VC-first*). On the other hand, with greater levels of innovation one would also expect venture capitalists to have more opportunities to fund innovative firms (*innovation-first*). This endogeneity issue has been dealt with particularly well at industry level, using changes to VC legislation to remove the issue of innovation-first VC. At firm level, this issue is harder to remove because it would require examining a mixture of firms that are randomly assigned VC (Dessí and Yin (2010)). This, then, may go some way to explaining why the VC-to-innovation link is observed regularly in the industry level data and not in firm level analysis, though this still demonstrates better performance post-VC funding.[16]

Industry level

At the industry level, Kortum and Lerner (2000) were the first to observe a significant link between VC funding and innovation. Examining data across 20 US manufacturing industries between 1965-92, they estimated the impact that VC funding had on patent counts. To remove the possibility that innovation may drive growth in VC funding, they split the data using a 1979 clarification to the Employee Retirement Income Security Act that allowed pension funds to invest in venture capital. This sudden expansion in VC funds was, therefore, unrelated to technological progress and offered a natural way of examining a link from VC to innovation. Their conclusion was clear, despite only accounting for less than 3% of corporate R&D, VC accounted for 8% of innovation in the US during the relevant period.

Hirukawa and Ueda (2008) replicated these results using data across 19 US industries but a longer time period (1968-2001). These results were more pronounced and survived a period of extremely high VC-growth in the late 1990s. However, using total factor productivity (TFP) as a measure of innovation they find VC has no impact. Hirukawa and Ueda (2011) then explicitly test which way Granger-causality runs: VC-first or innovation-first? Using both patent counts and TFP as measures of productivity, they observe mixed

[16]This interesting issue is also picked up by Gompers and Lerner (1999, 2001) when they observe that some of the most succesful high-tech innovators in the US, such as Microsoft and Apple Computers, have benefited from VC backing.

results. Within the patent data results, they find no evidence for either hypothesis. In contrast, using TFP offered evidence for both hypotheses. These final results are an important observation within the literature as they suggest a keen understanding of intertemporal effects is important within this analysis.

Similar results exist within Europe. Popov and Roosenboom (2009), examining data across 21 European economies between 1991-2004, estimating that whilst private equity investment accounts for 8% of aggregate industrial spending (R&D and private equity), it accounts for between 8-12% of innovation. Popov and Roosenboom (2012) then conduct a similar study but examine the VC-impact disparity between high- and low-VC economies. They find that, even within Europe, VC can be a potent stimulus for innovation - at similar levels to the US - but this only holds for high-VC economies. Specifically, they found that within economies with more VC-friendly structures (i.e. lower barriers to entrepreneurship or lower taxes on capital gains) private equity accounts for around 3.9% of corporate R&D spending but 10.2% of innovation. Work by both Faria and Barbosa (2013) and Geronikolau and Papachistou (2012) use European data to check which direction Granger-causality runs. Similar to the work of Hirukawa and Ueda (2011), both hypotheses receive some empirical support.

These empirical studies suggest that venture-capital does appear to play a crucial role in harbouring innovation *at industry level*. More importantly, despite VC's relatively small stake in the funding of R&D, it has become widely accepted that it has played a huge part in the success story of US innovation (Dessí and Yin (2010)). Worryingly, the US approach has been seen as crucial to the promotion of VC across the globe, without really understanding *how* it promotes such impressive levels of innovation. Therefore, at a macro-level, there is much to be done to fully understand the impact of numerous macroeconomic factors on VC's success before employing such sweeping policy changes.[17]

[17] It is important to note that Popov and Roosenboom (2012) do hint that venture capital may be impacted by public policy within an economy. However, they do not examine this specifically.

Firm level

At firm level, VC appears to have no link to innovation *per se* but does appear to have other, real impacts on a firm's potential for success. It has been argued that this lack of evidence with regards to innovation may be caused by firm-level endogeneity - i.e. "forward-looking selection effects" (Da Rin *et al*, 2013). That is, empirical work in this area compares similar VC- to non-VC-backed firms to minimise the impact of potential selection bias. However, in comparison to the industry-level analysis, it is much harder to completely, or even satisfactorily, eliminate this problem. In fact, as Dessí and Yin (2010) despair, the only way to truly eliminate such an issue would be to randomly assign a group of firms a variety of financing sources which, obviously, is not possible.

Hellman and Puri (2000), using a selection of survey and commercially available data for 173 hand-picked Silicon Valley start-ups, observe that firms pursuing an innovator strategy are more likely to obtain VC funding and see a reduction in time needed to bring a product to market. Most intriguing, however, is their assertion that, "firms are more likely to consider VC a milestone event than obtaining financing from some other kind of financier" (Hellman and Puri, 2002, p.962). Though a reason for this is not given, all three of these findings are consistent with a venture capitalist possessing at least one of two skills: *i*) a higher ability to seek out innovative firms *ex ante*; or *ii*) offering benefits beyond those of traditional finance methods through the use value-adding services *ex post*. Also using US firm level data, Puri and Zarutskie (2012) compare VC- and non-VC-backed firms' growth rates. Whilst the results suggest that VC may be irrelevant in the creation of new firms (accounting for only 0.11% of new firms), they note consistently faster growth, though this does not necessarily translate into larger profits. Peneder (2010), using data on 132 Austrian firms, found similarly impressive results: VC-backed firms grew 70% quicker than equivalent non-VC-backed firms.

Two studies offer potential explanations for these results. First, Chemmanur *et al* (2011) examine US census data and observe TFP is significantly higher both pre- and post-VC compared to non-funded firms. Therefore, it may be that VC selection effects are being captured and that venture capitalists simply back those firm predisposed to grow faster. However, Da Rin and Penas (2007), using Dutch data, find that TFP growth is significantly larger with VC-backing because venture capitalists push their portfolio firms into adopting more in-house R&D practices as well as investing in absorptive capacity. Consequently, even though pre-selection effects may be captured, these may not completely undermine the firm level growth results but suggest they are simply upwardly biased.

To examine whether *ex ante* or *ex post* effects are important, Kaplan *et al* (2009) and Baum and Silverman (2004) examine the characteristics that attract venture capitalists to certain firms. Kaplan *et al* (2009) examine whether venture capitalists are more likely to back "the horse" (the firm's business idea) or "the jockey" (the management team). They observe that whilst VC-backed firms do, indeed, grow much faster, it was the business idea - not management team - that was the most important factor.[18] Baum and Silverman (2004), using data on 204 Canadian biotechnology start-ups and 407 incumbents, examine whether venture capitalists "pick" (*ex ante* selection) or "build" (*ex post* mentoring) their chosen firms. They find a combination of both effects with venture capitalists more likely to invest in firms that have demonstrated some innovation (alliance participation or patents) and, thereafter, they perform better.[19]

Our biggest criticism of this literature is not that it is impossible to completely remove selection bias but, instead, that firms are treated in a rather unusual way. Whilst venture capitalists offer various levels of funding and value-adding services, the decisions of the firms are completely ignored. Da Rin *et al* (2013) are especially scathing about this issue with respect to its *ex ante* implications. They argue that given the potential benefits hinted at in the empirical literature it would make sense that firms were not passive pre-VC but would rather change their initial actions to improve the likelihood of ob-

[18] In a related result, Wasserman (2003) finds that manager turnover is more likely when managers have succesfully developed a product rather than when they have performed poorly. Simply, once a firm has developed, the skills that made the initial CEO so succesful in developing a product or idea may be less important once the firm faces a different scenario.

[19] In related literature, there is some evidence that venture capitalists attempt to eliminate asymmetric information before making such as choice. Dahiya and Ray (2011) and Hoenen *et al* (2012) suggest that a venture capitlaist may use a range of signals, for example patents, to screen weaker firms before choosing either the "horse" or the "jockey."

taining this competitive advantage. It is this underlying assumption that, from an industrial organisations point of view, we object to and one that may understate the benefits of VC in spurring innovation by ignoring pre-VC success driven by the lure of future benefits.

Value adding services

Given the overwhelming body of evidence that VC-backed firms are more successful and industries more innovative, the question should be *why* do we observe this? We begin by noting a consistently observed empirical fact: venture capitalists are far from passive investors and this is often assumed to be a key reason behind the VC-to-success link. As Bottazzi *et al* (2008, p.489) observe, "the VC literature identifies a broad role for the investor, which goes beyond the simple provision of finance. Venture capitalists may engage in a number of value-adding activities, including monitoring, support, and control. Those activities are largely non-contractible, yet may have real consequences". Quite simply, venture capitalists do not just invest money but also time and considerable amounts of effort which have significant benefits for a firm.[20]

The most obvious use of a venture capitalist's time, and in line with traditional financing methods, is monitoring. Monitoring is made easier by the large amount of time that venture capitalists spend with their portfolio firms, which has been estimated at around 80 hours per year on-site and a further 30 hours on the phone with senior management (Sahlman (1990) and Gorman and Sahlman (1989)). Similar to what one would expect, the empirical results suggest that monitoring increases as and when the need arises; as measured by management changes (Lerner, 1995) or agency costs (Gompers, 1995). However, monitoring is far from being the only value-added service that venture capitalists offer.

A number of studies have examined the scope of "professionalisation" of VC-backed firms. Hellman and Puri (2002), analysing data on 170 young high-tech Silicon Valley start-ups, examined the impact of VC on the development

[20] A large body of literature examines venture capital from an alternative angle; screening. Caselli *et al* (2009), using Italian data, found that firm is more likely to recieve VC funding if it has already demonstrated some innovative ability. Similar results have been found within the US (Hellman and Puri, 2000; Mann and Sager, 2007) and Germany (Engel and Keilbach, 2007).

new firms finding that venture capitalists alter a firm's inner workings. By far the biggest impact came from the professionalisation of the firm, with venture capitalists implementing improvements at every level of the firm; from replacing the original founders with external CEOs to simply formulating HR policies or improving a firm's marketing strategies. Both Wasserman (2003) and Kaplan *et al* (2012) also observe a "paradox of success" in which VC-backed firms appear to more frequently replace existing senior management with external CEOs in spite of their initial success.[21] This willingness to bring in new management, despite success, suggests that they are willing to make tough decisions to improve a firm's prospects. Finally, Hochberg (2012), examining 2,827 IPOs between 1983-94, found that even after controlling for endogeneity issues these improvements to corporate governance were not simply a consequence of *ex ante* selection biases. To put it another way, venture capitalists did not select "better" firms *per se*.[22]

Venture capitalists, though, do not simply impact on a firm's inner workings but also on their external interactions. Lindsey (2008) examined an uncommon, yet important, value-adding service in the form of strategic alliances between firms with a common venture capitalist. The results suggest that having a common venture capitalist blurs the lines between the firms' boundaries and increased the probability that two firms would work together. Hsu (2006) finds similar results, demonstrating a significant increase in cooperation between firms if they are VC-backed. In related literature, Ozmel *et al* (2013) argues that strategic alliances can alter the relationship between a firm and its backer. Using data on biotechnology firms, the study found that whilst accepting VC increased the chances of future VC and alliance activity, alliance activity only increased the chances of future alliance activity. The reason for this result is that alliance activity is likely to conflict with a venture capitalist's

[21] Though Kaplan *et al* (2012) do note that after accounting for the skills of managers, there is no difference between the performance of internal and external CEOs.

[22] These results regarding activity, monitoring and proffesionalisation are dependent on many, often overlooked, factors. First, the importance of legal systems is discussed by Botazzi *et al* (2009). Second, trust between the firm and its backer are examined in Botazzi *et al* (2012). Third, the impact on innovation may depend on whether the backer is private or government run and what the funds are invested for (Bertoni and Tykvová (2012); Cumming *et al* (2005)). Fourth, a venture capitalist's tolerance to failure (Tian and Wang (in press)). Finally, the scale of risk involved and contract types play a role too (Kaplan and Strömberg (2003, 2004)).

interests. Lerner *et al* (2003) find similar results but add that the likelihood of turning to a strategic partner is inversely related to the quality of public markets.

The main issue with this body of literature is that a firm's *ex ante* strategic decisions may be viewed as assuming that firms are passive. In fact, the empirical literature exclusively examines the impact of VC on firms whilst trying to *remove* any selection (firm) effects. However, this decision has created a large gap in the literature that, as yet, no one has attempted to fill. Specifically, from the *firm's perspective*, three non-trivial questions can be examined: *i*) how does the promise of VC change a firm's *ex ante* innovation decisions?; *ii*) how does VC improve a firm's probabilities of success; and *iii*) does VC alter the decisions of a firm *ex post*? As of yet, no work - empirical or theoretical - has been offered to answer these questions. Instead, it has been the case that such questions have been ignored in favour of implicitly assuming that VC was the impetus between the VC-to-success link. However, it is necessary to ask whether a combination of both firm and venture capitalist effort is required for such benefits to come to fruition.

1.3.3 Summary

There is ample evidence to suggest that venture capitalists do help to promote innovation at industry level and more general success at the firm level. More specifically, whilst industry level results suggest that VC may spur innovation, as measured by patents and TFP, at firm level evidence points to the professionalisation of firms. Empirically, the professionalisation of firms covers a wide range of activities and this provision of value-adding services (including hiring new CEOs, monitoring firm performance and even harbouring more collaborative efforts) is what adds to the success of firms. Whilst there is ample evidence to support all of these notions, it is unrealistic to ignore the actions of firms entirely. Rather, there is scope to explore the decisions of firms both pre- and post-VC to examine how firms react to both the potential for receiving VC funding and after they have actually received it.

1.4 Conclusion

Whilst this review is not exhaustive, it does offer a good insight into the conclusions and issues faced within both fields.

First, within the linear spatial literature, it is obvious that this model has become an integral part of the analysis of horizontal differentiation. However, it is also quite apparent that the specification of the model plays an integral role in determining what equilibria are possible and the strategic decisions of the firms. Yet, in spite of all of this, so much work has been done in mapping the effects of all the changes to even the most basic model specifications that these issues are no longer such a problem. That is, the basic DGT model has become an important tool within the Industrial Economist's toolbox.

Second, the addition of cost asymmetries is not without its issues either. However, the majority of work that has been carried out focuses on a few key issues. Within the linear spatial work it is well understood when pure strategy and mixed strategy equilibria exist, but the impact of endogenising the firms' cost reductions has been mostly overlooked. Instead the focus has been on spillover effects and the impact of these on the firms' location decisions. Within the non-linear models, asymmetric costs are not so well established. Rather, these have been more concerned with examining the impact of competition boosting policy on social welfare. Nonetheless, these have interesting and real world applications that should not be ignored.

Finally, within the VC literature, we observe that a huge gap appears to have been left: how do *firms* respond to venture capital *ex ante* and *ex post*. Whilst much has been done from the point of view of the venture capitalist, this is only half of the story. Consequently, much has been left to examine. Whilst the review was empirically based, these theories are, in essence, testing theoretical hypotheses and commonly held beliefs about VC. Therefore, the issues observed here do give an indication of the dearth of certain theoretical ideas within the literature.

Chapter 2

Spatial Competition and the Innovative Process

2.1 Introduction

Since Hotelling's (1929) seminal work formalised the modelling of linear spatial markets much has been done to examine and adapt his work to examine horizontal differentiation within duopoly markets. Whilst the addition of cost asymmetries has offered interesting results, one crucial question has often been overlooked: *how* does the *innovative process* (the ability to spot and implement new and innovative production methods) determine the composition of the final product market. To this end, we propose a model of endogenous cost selection in which firms are heterogeneous with respect to both their *ex ante* abilities to implement a cost reducing technology and their investment timings. In doing so, it is possible to examine the importance of both investment timing and innovative ability in determining the composition of the final product market or, more interestingly, whether it is possible for an *ex ante* inefficient firm to become an *ex post* market leader. Thus, we extend the linear spatial competition literature in two ways. First, firms are *a priori* heterogeneous in respect of both their ability to implement cost reductions and the timing of their investments. Second, the *ex post* cost differential is endogenous. The results offer two interesting additions to the existing literature: (*i*) if a firm possesses both an investment timing and efficiency advantage it always becomes the *ex post* efficient firm; and (*ii*) if a firm possesses only an investment timing advantage, it can become the *ex post* efficient firm if and only if the *a priori* ability gap is not too large.

Hotelling (1929) developed a simple model for examining firms' product choices in a spatial setting, finding that firms would have a preference to produce homogeneous goods. The result suggested that firms would be driven to agglomerate as, by moving towards their rival in the location space, firms could steal a rival firm's market share. D'Aspremont *et al.* (1979) undermined this result arguing that no pure strategy solution existed in Hotelling's original model specification finding, instead, that equilibrium only exists if transport costs are quadratic.[1] The use of quadratic transport costs, proposed by DGT, ensures a stable equilibrium for all price-location pairs but, in contrast to Hotelling's (1929) conclusion, suggested that firms would find it optimal to produce maximally differentiated products. Furthermore, were firms able to locate outside of the city's boundaries, it would be optimal for them do so (Tabuchi and Thisse, 1995; Lambertini, 1997).[2]

One criticism to be levelled at this body of work is that the firms are assumed to face symmetric production costs. The specific consideration of exogenous cost differentials has been examined within a linear spatial framework by Schulz and Stahl (1985), Ziss (1993) and Matsumura and Matsushima (2009). Ziss (1993) introduced heterogeneous production costs into a DGT framework, finding that maximum product differentiation is the only pure strategy Nash equilibrium solution, and that this (pure strategy) equilibrium outcome exists if and only if the cost differential is not too large. Matsumura and Matsushima (2009) extend this analysis to consider cases in which this small cost differential assumption is violated. They find, where cost differentials are large, the firms face contrasting incentives regarding location with the stronger (weaker) firm having an incentive to minimise (maximise) product differentiation. Thus, a mixed strategy Nash equilibrium is ensured even where the heterogeneity between firms undermines a pure strategy solution.

It is for this reason that the Hotelling (1929) model "is [assumed] incon-

[1] D'Aspremont et al. (1979) will henceforth be denoted DGT.

[2] In general, the linear spatial competition literature highlights how sensitive these models are to model specification. These include, but are not limited to, the number of dimensions in the product space (Irmen and Thisse (1998)), the distribution of consumers (Anderson *et al* (1997)) and the specification of transport costs (Anderson (1988)).

venient for investigating endogenous production costs" as a full examination of location equilibrium conditions is difficult to obtain (Matsumura and Matsushima, 2009: p216). Nonetheless, from a theoretical standpoint, the existence of mixed strategy solutions may explain differences in market structure across seemingly similar markets especially where firms face asymmetric costs. Bester *et al* (1996), using a DGT model, find an infinity of mixed strategy equilibria and argue that, without some coordination mechanism, firms face a strictly positive probability that they would locate at the same point. Thus, where firms face asymmetric production costs, simple coordination failure can be the difference between a high cost firm remaining in the market or being driven out of business. Whilst this result is intuitive, Martin (2001) argues that firms *do not* randomly select a location or product mix but, rather, make such decisions in secret. This generates asymmetric information and inherits the same propensity for coordination failure.

Some efforts have been made to endogenise production costs within a linear spatial framework, but these have had a significantly different focus. Most similar to this work, Mai and Peng (1999) and Piga and Poyago-Theotoky (2005) examine the impact of geographic spillover effects on the firms' locations. Using a three stage, location-R&D-price model with *symmetric firms* they observe that the firms are drawn closer to each to benefit from a rival's R&D spillovers. Thus, depending upon the magnitude of the spillover rate both maximal and minimal differentiation are possible.[3] Matsumura and Matsushima (2010b) examine the role of patenting and licensing noting that pure strategy Nash equilibria are ensured for any level of cost differential as long a firm can license its innovation.[4] Finally, Matsumura and Matsushima (2004) and Kumar and Saha (2008) examine the difference between public and private firms' cost reductions. The main finding in "mixed duopolies" is that the level of public ownership is crucial and that when ownership is small the results degenerate to those of the DGT model. However, when public ownership is sufficiently large, a more socially efficient equilibrium obtains.

Also pertinent to this discussion are models of competition between firms

[3] Whilst both models examine *different* innovations - cost-reducing and quality-improving respectively - Sun (2013) finds no difference between the specifications.

[4] Matsumura and Matsuhima (2010a) find similar results but where cost heterogeneity is exognenous.

facing heterogeneous costs within a circular city/Salop (1979) framework. Aghion and Schankerman (2004) and Syverson (2004) adopt a Bayesian set-up, with firms knowing only their own (stochastic) production costs and assume prices are set prior to learning their rivals' locations and costs. Analogous to the linear spatial model results, the (pure strategy) equilibrium breaks down for very large cost differentials as the (low cost) firms' incentive to limit price becomes acute. Interestingly, Alderighi and Piga (2009, 2010), who examine the maximum permissible cost disparity within these models, observe that an Eaton and Lipsey (1978) style "no mill-price undercutting rule" must be satisfied, similar to that derived by Ziss (1993), or "a highly efficient firm's reach could potentially extend beyond its nearest neighbour's position" (Alderighi and Piga, 2009: p.3). Furthermore, extending this analysis to allow firms to endogenously select their locations suggests "the distance between two direct competitors is strictly increasing in the average productivity: all else equal, more productive firms are more isolated" (Vogel, 2008: p.450).[5] Therefore, whilst issues arising from using a Hotelling model to examine endogenous cost selection do exist, these problems are comparable to other areas of the spatial competition literature.

Consequently, to the author's knowledge, no attempt has been made to examine *how* the innovative process (the ability to spot and implement innovation/research opportunities) determines the composition of the final product market. This paper, rather than simply examining the *effect* of cost differentials, analyses *the cause* of cost differentials *and* their effects.[6] That is, we examine the extent to which a firm's ability to innovate - in terms of both its ability to implement a new production technology and innovation timing

[5] To the best of the author's knowledge some papers in this area have examined endogenous cost selection. However, their focus is generally on the entry effects of R&D. See Scalera and Zazzaro (2005) or Ebina and Shimizu (2012).

[6] It is important to note that this paper only considers process innovation and, consequently, horizontal differentiation. Hotelling (1929) style models often focus on horizontal differentiation, but can be extended to include quality improving investments and vertical differentiation. In fact, using a similar model to that presented here, Sun (2013: p.138) finds that both process and product innovation - cost reduction and quality improving investment respectively - "share identical strategic properties" and that within a linear spatial model "[it is possible] to derive the equilibrium strategies of a quality improving R&D model from the ones of a cost-reducing R&D model, and vice versa". Consequently, the results derived here would likely transfer to a situation in which the products are vertically differentiated, or product innovation were used instead. Therefore, such an omission is irrelevant.

- affects the composition of the final product market. To do this, we propose a three-stage model in which the firms sequentially invest in R&D before simultaneously competing in locations and then prices. Furthermore, the firms are assumed to be heterogeneous in their abilities to reduce their costs. Our results suggest two findings unique to the literature. First, if a firm possesses both an investment timing and efficiency advantage it always becomes the *ex post* dominant firm (proposition 4). Second, where a firm possesses only an investment timing advantage, it can become the *ex post* market leader firm if and only if the *a priori* efficiency gap is not too large (summary 9). These results suggest that the innovation process plays a crucial part in determining which firm will become the *ex post* market leader. Additionally, whilst it is the *ex post* low cost firm that becomes the market leader, as noted in the literature, it is possible that this firm could be either the *ex ante* efficient or inefficient firm.

Another body of literature to which this paper relates, and another crucial aspect of the innovative process, examines patent races. Whilst not explicitly examined here, this body of literature gives some indication as to *which* firm has the most incentive to innovate. The early literature, focusing on deterministic R&D and complete information regarding the firms' innovation strategies, sheds some light as to whether *ex ante* efficient firms can be "leapfrogged" by *ex ante* inefficient firms or if they are destined to maintain their position.[7] Gilbert and Newbury (1982), examining the strategic actions of an incumbent monopolist, concluded that, as monopoly profits are always at least as large of those in any other market structure, there always exists an incentive to defend this position by preemptively investing and patenting.[8] However, it appears that these results are not robust to the existence of asymmetric firms. Rather, when one firm is granted a competitive advantage in innovation, the results

[7] A more detailed literature review of this can be found in Pollock (2008). However, it suggests that since the initial work regarding patent races, the examination of patents has become more focused on: *i*) patent design; *ii*) cumulative innovation; *iii*) licensing; *iv*) imitation; and *v*) the openess of innovation (such as the rise of open source software).

We ignore a discussion of cumulative innovation here as innovation, within this model, consists of only a single stage.

[8] Leininger (1991) notes that Gilbert and Newbury's (1982) basic model is, in essence, a simple auction for a patent. Therefore, as firms must compete to "win" the auction, rents are dissipated compared to the monopoly position. Moreover, where the firms become increasingly symmetric, rent dissipation is exacerbated.

become more extreme. Fudenberg *et al* (1983), Harris and Vickers (1985a; 1985b) and Aoki (1991) all find that granting a firm any advantage in the innovation stage - be it a larger reward from obtaining a patent, being better at R&D, or simply being further along the development path - it will dominate the competition and *always* wins the patent race.[9] Though, this may be because the "leader" in a patent race invests more heavily than the follower (Harris and Vickers, 1987). Quite simply, then, the early industrial organisation literature highlights that a firm with some innate advantage in a patent race will, inherently, become the winner.

The most recent literature regarding patent races has focused on uncertain returns to innovation by adopting a real options approach, the right - though not obligation - to invest in a new technology (Dixit and Pindyk, 1994). Quite remarkably, similar results obtain in the real options literature, directly comparable to those obtained in the earlier work. Or, as Hsu and Lambrecht (2007: p.21) eloquently state, "one [should] expect qualitative nature of previous results from the industrial organisation literature on the timing of technological innovation is likely to remain unaltered by uncertainty and asymmetric information".

When the profitability of investment is uncertain, and when firms face no competition, the real options approach suggests it is better to wait for better information regarding the profitability of investment before committing (Huisman and Kort ,1999). In contrast, when a firm faces competition, the benefits of waiting are undermined as there is the potential to be preempted which drives firms to invest earlier than they would have done otherwise; a decision made to preempt their rivals and win the patent race. Furthermore, this result becomes more pronounced as the number of firms participating in the race increases, as each firm faces a *stronger* threat of preemption which drives them to invest earlier still (Bouis *et al*, 2009; Grenadier, 2002).[10] However, the result most pertinent here are those derived between asymmetric firms. Chan and Kwok (2007) examine a real options model in which the firms face asym-

[9] These results are robust for both single- and multi-stage innovation. Furthermore, the issue of rent dissipation, as observed by Leininger (1991), still obtains.

[10] Grenadier (2002) notes that, at the limit, firms are induced to invest where their expected returns from investment are zero, or all expected rents are completely dissipated. In contrast, Bouis *et al* (2009) finding that almost all expected profits are dissipated when facing only a single competitor.

metric costs of innovation, though cost information is common knowledge, and conclude that it is the most *innovative and efficient* firm that should win the patent race. However, this result can be criticised because, if the firms compete with complete information, why don't they simply cooperate to maximise joint profits?[11] However, Lambrecht and Perraudin (2003), using a model in which the firms face uncertainty regarding the costs of their rivals, the firms still attempt to strategically preempt their rivals, though the extent to which this occurs depends upon the initial distribution of costs. Nonetheless, it is, once again, the most *innovative and efficient* that should win the race.

It is interesting that the existing literature - using two very different approaches - finds identical results: where firms compete in deterministic R&D, it is the firm with some competitive advantage in innovation that will *always* win the race.[12] In essence, these results suggest that the firms that are better at innovating - due to, for example, receiving higher payoffs from innovation, facing lower R&D costs, or being further along the development path - are the most likely to be the "winners" and, consequently, have the biggest incentive to innovate. The results presented here, however, both agree and contrast with this assertion. On the one hand, if *one* firm possesses an advantage in innovation then the results derived in the patent race literature do, indeed, align with the results presented here. That is, the firm with the advantage unambiguously becomes the dominant firm and would, therefore, have the biggest incentive to innovate. On the other hand, and in contrast to the existing literature, this paper suggests that *both* firms may possess some advantage in the innovative process, such as one firm having a timing advantage whilst another is better at conducting R&D. In this instance, the identity of the "winning" firm is determined by the balance of these advantages, and the firm with the most incentive to innovate is the firm with the most potent advantage.

Another related body of literature relates to endogenous entry timing into the final product market, or the firms choose when to select their locations. Within a linear spatial setting, it appears, at least within a single period

[11] This criticism of complete information is made by both Weeds (2002) and Lambrecht and Perraudin (2003).

[12] Indeed, the only difference between the equilibria between complete and incomplete information cases is that, in the incomplete information case, the firms optimally adjust their preemptive investment behaviour to take into account the risk associated with uncertain payoffs.

framework with unbounded location spaces, that the timing of entry has a significant impact on the composition of the final product market.[13] Indeed, as such a decision offers a considerable advantage to the first firm to choose its location, which firm would like to move first? Within a two-period Cournot duopoly game, van Damme and Hurkens (1999) observe that a high cost firm finds committing to move first riskier than a low cost firm and so it must be the low cost firm that emerges as the endogenous Stackelberg leader. This assertion backed by Branco (2008) who, within a similar framework, also finds that the low-cost firm becomes the Stackelberg leader. However, both papers disagree as to whether a Cournot equilibrium would obtain if the firms face symmetric costs.[14]

Whilst our analysis does not afford firms asymmetric location timing, let alone endogenous ones, our results again agree and contrast with those of endogenous entry games. Whilst the results presented here agree that the *ex post* low cost firm would emerge as the market leader, they also suggest that this firm could be either the *ex ante* efficient or the inefficient firm. If, as here, the firms are *a priori* heterogeneous with respect to their effectiveness at reducing costs and the timing of their investments then a firm's unit cost is no longer a measure of its efficiency but, instead, a direct consequence of strategic interactions *given* the firms' relative efficiencies (investment timing and cost reduction effectiveness). Therefore, the van Damme and Hurkens (1999) and Branco (2008) results ignore the possibility that this firm could be either the *ex ante* efficient or inefficient firm. That is, an *a priori* inefficient firm may, in fact, possess an incentive to commit to becoming the Stackelberg leader if the *ex ante* efficiency gap is not too large.

The rest of the paper proceeds as follows: section 2.2 describes the model's specification; section 2.3 analyses the model including the equilibrium conditions for prices, locations and cost reduction; and section 2.4 concludes.

[13] Lambertini (2002) finds that, under such circumstances, the first mover in the location stage locates in the centre of the market whilst the follower must locate outside of the market.

[14] It is of note that similar models specifying Bertrand competition obtain a reversed result in which the efficient firm enters first and the inefficient firm second (see van Damme and Hurkens (2004) and Tasnádi (2003)). Interestingly, they note that only when the cost differential between the firms is sufficiently large will the *ex ante* efficient firm become the *ex post* dominant firm.

2.2 Model

Consider a three-stage duopoly model. In the first stage, the firms compete in cost reduction sequentially before making symmetric location and price choices in the second and third periods respectively.

It should be noted that the sequence of moves presented here - innovation *before* location - examines only half the story. Within the context of the innovative process, what the sequence of stages should be is not *a priori* obvious. In fact, it is possible to make a reasonable case for *either*: *i*) firms innovating before choosing their locations; or *ii*) locations being chosen before firms implement their investment strategies. Either way, the specification chosen must suit the particular industry that is chosen. The sequence presented here, with cost reducing investments being made before choosing a location, works well for industries in which a piece of capital can produce a wide variety of a single good. As an example, one could think of the production of chocolate; once the factory is built it is equally capable of producing any variety of chocolate, from milk to dark chocolate. Therefore, *before* committing to the production of a specific variety of chocolate, the firm can invest in developing a low cost production method. However, if the R&D required is more specific, for example the capital required differs between varieties, then the aforementioned approach is not appropriate. In such circumstances, a firm must choose what variety of good it wishes to produce *before* investing in the appropriate capital. Thus, it would be erroneous to assume that our specification generalises to *all* industries, a claim we do not make, but suggest it is better suited to industries in which the productive technology is fairly homogeneous, even if the final products are not.[15]

As in d'Aspremont *et al*'s (1979) model, two firms, A and B, supply a physically homogeneous good from different locations on the real axis. The location of firm N is denoted by $n \in \mathbb{R}$ and, consequently, the location of each

[15] Whilst the order of moves used here is not, by any means, unreasonable, a more practical reason leads us to use this particular sequence of events. Once the sequence of investment and location decisions is reversed - with firms locating before innovating - the model becomes intractable. Consequently, whilst it would be nice to compare the two scenarios directly, it appears that this is not possible. In fact, this issue of complexity/intractability is noted by Piga and Poyago-Theotoky (2005).

firm is measured from zero. Without loss of generality, we assume $a < b$.[16] Firms maximise profits at each stage.

Consumers are uniformly distributed across a linear city of unit length and are assumed to have a density of one. For simplicity the city is defined by $[0, 1] \in \mathbb{R}$ which ensures that any $a, b \notin [0, 1]$ simply implies that a firm is locating outside of the city's boundaries. It is also assumed that consumers face unit demands and consume either zero or one units of production. Consequently, a consumer, located at $x \in [0, 1]$, would only purchase a good from firm N, located at n, if and only if

$$U_x = s - p_N - t(x - n)^2 \geq 0$$

where s is a fixed utility of consuming a good, p_N is the price charged by firm N, t is a measure of consumer heterogeneity and $t(x - n)^2$ is a quadratic transport cost incurred by a consumer having to move a distance of $|x - n|$ to consume good N. In order to ensure that total demand is equal to one, or that each consumer purchases one good, s is assumed to be large enough that $U_x \geq 0$ for all $x \in [0, 1]$ for at least one of the firms' products and consumers only purchase the good that maximises their utility.

In the first stage the firms sequentially invest in cost reduction to reduce their *ex ante* production cost. At the beginning of the first stage, we assume that both firms face a symmetric production cost, c, but these initial production costs can be reduced by $\varphi_N(I_N)$ at a cost of $C_N(I_N)$; where I_N is the investment level of firm N. Furthermore, it is assumed that the cost reduction schedule and investment cost functions are linear and quadratic in investment respectively.[17] More formally, the cost reduction and investment cost schedules are given by:

[16] It is trivial to derive the results for the case $a > b$.

[17] Whilst these functional form assumptions are not ideal, they are necessary for two reasons. First, to make the mathematics more tractible. Attempts were made to generalise the model using a convex investment cost schedule but these rendered the model intractable. Second, to keep the model specification in line with other cost selection models within the linear sptial competition literature. This linear cost reduction schedule is assumed across a broad range of the literature: georgraphic spillover effects (see Mai and Peng (1999) and Piga and Poyago-Theotoky (2005)); licensing (see Matsumura and Matsushima (2004 and 2010a)) and; public vs. private firms (see Matsumura and Matsushima(2010b)).

$$\begin{aligned}\varphi_N(I_N) &= m_N I_N \\ C_N(I_N) &= \frac{1}{2} I_N^2\end{aligned}$$

where $m_N > 0$ represents the ability of firm N to implement the cost reducing technology or the (constant) marginal cost reduction per unit of investment. The final unit cost of firm N, used in the price and location stages, is given by

$$c_N = c - \varphi_N(I_N)$$

In specifying the first stage in this manner it is possible to allow firms to differ with respect to both their investment timing and efficiency. In the case of investment timing this is obvious, as firms move sequentially and this can be thought of as the firms differing in their abilities to spot new investment opportunities. However, the cost reduction schedule allows for firm heterogeneity with respect to their investment efficiencies if $m_A \neq m_B$. Without loss of generality, the remainder of this paper assumes $m_A > m_B \geq 0$ or, more simply, that firm A is more effective at implementing cost reduction dollar-for-dollar than firm B. With both of these assumptions in place it is possible to examine the relative importance of investment efficiency and timing advantages.

One final, and crucial assumption, is that the results presented here are derived such that both firms remain active in the final product market, at *all* stages of the game. Whilst the focus of this paper is on the impact the innovative process has on the *composition* of the final product market, we limit the analysis to the most interesting cases where both firms remain active. The consequence of removing this assumption has already been documented by both Ziss (1993) and Matsumura and Matsushima (2009). Simply, where one firm can generate a sufficiently large *ex post* cost advantage they begin to act strategically to deter entry and the model breaks down, with only mixed strategy Nash equilibrium solutions remaining (Ziss, 1993). Therefore, by investing heavily in the cost reduction stage and generating large *ex post* cost differential, the Stackelberg leader in the investment stage can create a game of "cat and mouse", with the *ex post* efficient firm attempting to prevent entry by locating next to the inefficient firm and the inefficient firm trying to "run" away. That is, the Stackelberg leader uses its position to become predatory and attempts to prevent profitable entry by the follower, undermining *all* pure strategy location equilibria.[18] In fact, these conflicting agglomeration strategies lead to maximal and minimal differentiation appearing with equal probability, or entry deterrence being successful with probability 1/2. However, as this result is already well documented, we omit it here and focus on the cases in which both firms remain active in the final product market, or neither firm's cost advantage is sufficiently large that it becomes incentivised to try and deter the entry of its rival.

The game is solved by backward induction and at each stage firms maximise their profits. Only pure strategy Nash equilibria are examined in this paper. The game proceeds as follows. In the first stage, firms select their investment levels, $I_N \in [0, \infty)$, sequentially; both cases where firms A and B move first are examined. In the second stage, each firm selects its location, $n \in \mathbb{R}\ \forall\ N \in \{A, B\}$, simultaneously. Finally, in the third stage, firms select their prices, $p_N \in [c_N, \infty)$, simultaneously.

2.3 Analysis

2.3.1 Price Stage

Recall that consumers *i*) maximise utility; and *ii*) can only purchase one unit of production. In this instance, a consumer, located at x, is indifferent between purchasing either good A or good B if and only if

$$s - p_A - t(x-a)^2 = s - p_B - t(b-x)^2 \tag{2.1}$$

As $a < b$ by assumption, the total demand for good A is given by all consumers located to the left of x and demand for good B is the residual demand, $1 - x$.

[18] One should expect that this result would also occur here because our model is identical to that presented by Matsumura and Matsushima (2009), but with the *ex post* cost differential determined endogenously.

Solving equation (2.1) yields the demand functions:

$$
\begin{aligned}
D_A &= x = \frac{p_B - p_A}{2t(b-a)} + \frac{(a+b)}{2} \\
D_B &= 1 - x = \frac{p_A - p_B}{2t(b-a)} + \frac{(2-a-b)}{2}
\end{aligned}
$$

Taking these demand functions as given, firm N's profits are given by the expression:[19]

$$\pi_N = (p_N - c + \varphi_N)D_N - C_N$$

Firms simultaneously select prices, $p_N \in [c_N, \infty)$, to maximise profits, taking location and investment choices as given. The first order conditions are:

$$
\begin{aligned}
\frac{\partial \pi_A}{\partial p_A} &= \frac{p_B - 2p_A + c - \varphi_A}{2t(b-a)} + \frac{(a+b)}{2} = 0 \\
\frac{\partial \pi_B}{\partial p_B} &= \frac{p_A - 2p_B + c - \varphi_B}{2t(b-a)} + \frac{(2-a-b)}{2} = 0
\end{aligned}
$$

The second order conditions are given by $\frac{\partial^2 \pi_N}{\partial p_N^2} = -\frac{1}{b-a} < 0 \; \forall \; N$ and are, therefore, met for all price pairs, (p_A, p_B). Solving these equations simultaneously yields the equilibrium prices:

$$p_A = \frac{3c - 2\varphi_A - \varphi_B + t(b-a)(2+a+b)}{3} \tag{2.2}$$

$$p_B = \frac{3c - \varphi_A - 2\varphi_B + t(b-a)(4-a-b)}{3} \tag{2.3}$$

From these conditions, it is possible to observe that a subgame perfect Nash equilibrium in prices always exists, regardless of the locations and the *ex post* cost disparity between the two firms. Furthermore, the signs of the components within the equilibrium price functions are as one would expect, with prices increasing in the *a priori* unit cost and decreasing in the cost reduction efforts of both firms. However, assuming $\varphi_A = \varphi_B$, it is notable that the equilibrium prices of both firms are affected more by changes to their own

[19] For notational purposes, as I_N is known at this stage we abbreviate $\varphi_N(I_N)$ and $C_N(I_N)$ to φ_N and C_N respectively, $N \in \{A, B\}$.

ex post unit cost than to changes to a rival's *ex post* unit cost.

2.3.2 Location Stage

With the equilibrium prices given by equations (2.2) and (2.3), the relevant profit functions for both firms are:

$$\pi_A = \frac{[\varphi_A - \varphi_B + t(b-a)(2+a+b)]^2}{18t(b-a)} - C_A$$

$$\pi_B = \frac{[\varphi_B - \varphi_A + t(b-a)(4-a-b)]^2}{18t(b-a)} - C_B$$

Each firm, N, simultaneously selects its location, $n \in \mathbb{R}$, in order to maximise profits taking investment decisions and equilibrium prices as given. The first order conditions are given by

$$\begin{aligned}\frac{\partial \pi_A}{\partial a} &= 0 = -\frac{2}{9}\frac{t(1+a)[t(b-a)(2+a+b)+\varphi_A-\varphi_B]}{t(b-a)} \\ &\quad + \frac{1}{18}\frac{[t(b-a)(2+a+b)+\varphi_A-\varphi_B]^2}{t(b-a)^2}\end{aligned}$$

$$\begin{aligned}\frac{\partial \pi_B}{\partial b} &= 0 = \frac{2}{9}\frac{t(2-b)[t(b-a)(4-a-b)-\varphi_A+\varphi_B]}{t(b-a)} \\ &\quad + \frac{1}{18}\frac{[t(b-a)(4-a-b)-\varphi_A+\varphi_B]^2}{t(b-a)^2}\end{aligned}$$

Solving these equations simultaneously for a and b yields:

$$a = \frac{(\varphi_A - \varphi_B)}{3t} - \frac{1}{4}$$

$$b = \frac{(\varphi_A - \varphi_B)}{3t} + \frac{5}{4}$$

The relevant second order conditions are given by:

$$\frac{\partial^2 \pi_A}{\partial a^2} = \frac{1}{486}\frac{(4(\varphi_A-\varphi_B)+9t)(4(\varphi_A-\varphi_B)-27t)}{t}$$

$$\frac{\partial^2 \pi_B}{\partial b^2} = \frac{1}{486}\frac{(4(\varphi_A-\varphi_B)-9t)(4(\varphi_A-\varphi_B)+27t)}{t}$$

As these equations must be strictly negative, it is trivial to show $\varphi_A - \varphi_B \in (-\frac{9}{4}t, \frac{9}{4}t)$ must hold for both second order conditions to be met, or that the *ex post* cost disparity between the two firms is not too large relative to the transport cost.

This additional condition ensures that a pure strategy Nash equilibrium in locations exists but, unlike the pricing game, only where the *ex post* production costs are not too different. This is because, in allowing for cost heterogeneity, a highly efficient firm may have an incentive to drive its inefficient rival from the market. A similar argument is made by Alderighi and Piga (2008) and Matsumura and Matsushima (2009). Recalling the demand functions and substituting in the equilibrium price and location choices yields:

$$D_A = \frac{1}{18}\frac{(4(\varphi_A - \varphi_B) + 9t)}{t}$$

$$D_B = -\frac{1}{18}\frac{(4(\varphi_A - \varphi_B) - 9t)}{t}$$

Therefore, if $\varphi_A - \varphi_B \geq \frac{9}{4}t$ then the *ex post* high cost firm, firm B, would be driven out of the market whilst the low cost firm, firm A, would serve all demand.[20] It is for this reason that large *ex post* cost differentials undermine the existence of a pure strategy Nash equilibrium in the location stage given that, for all $\varphi_A - \varphi_B \geq \frac{9}{4}t$, firm B's demand and profits are driven to zero. Thus, there no longer exists a unique optimal location for firm B as, no matter where it locates, it would surely be driven from the market. Given that firm B no longer has a unique optimal location, a pure strategy Nash equilibrium cannot exist. Consequently, we restrict our focus to cases in which $|\varphi_A - \varphi_B| < \frac{9}{4}t$, or a pure strategy Nash equilibrium exists in the location stage.

With this restriction placed on the *ex post* cost differential, it is possible to say something of the location choice of each firm. First, where $\varphi_A = \varphi_B$, both firms locate symmetrically outside of the market (at $a = -\frac{1}{4}$ and $b = \frac{5}{4}$ respectively). As the quadratic transport costs make price competition increasingly fierce, firms prefer to locate beyond the boundaries of the city to mitigate the price competition effects on their profits (Lambertini, 1997). Second, the structure of the firms locations implies that the distance between the two

[20] This is because $D_A \geq 1$ and $D_B \leq 0$.

firms is fixed at $\frac{3}{2}$, regardless of the size of the cost disparity. Finally, where $\varphi_A \neq \varphi_B$, it is the firm with lower unit costs that locates closer to the centre of the market, with the inefficient firm moving further from the city to shield itself from aggressive price competition. Additionally, as $|\varphi_A - \varphi_B| \rightarrow \frac{9}{4}t$ the efficient firm's location converges to $\frac{1}{2}$, the centre of the market. Therefore, it is only an efficient firm that is able to locate within the city's boundaries at the expense of its inefficient rival who, fearful of limit pricing, is driven away from the centre of the market.

2.3.3 Investment Stage

In the first stage of the model, the firms select investment levels, $I_N \in [0, \infty)$ $\forall\ N$, *sequentially*. This sequential investment assumption obviously affords one firm a strategic timing advantage, but can also be thought of as another way in which the firms' relative investment abilities differ. For example, an investment timing advantage in this case may reflect a difference in the firms' abilities to spot new investment opportunities; be a consequence of a disparity in the skills of the firms respective R&D departments or; more simply, be luck.

Recalling the equilibrium locations, prices and $m_A > m_B \geq 0$, the first stage profit functions are given by:

$$\pi_A = \frac{1}{108}\frac{(4(m_A I_A - m_B I_B) + 9t)^2}{t} - \frac{1}{2}I_A^2$$
$$\pi_B = \frac{1}{108}\frac{(4(m_A I_A - m_B I_B) - 9t)^2}{t} - \frac{1}{2}I_B^2$$

In the following sections, the equilibrium investment levels are determined where (i) the *ex ante* efficient firm, firm A, moves first; and (ii) the *ex ante* inefficient firm, firm B, moves first. These are examined in turn.

Efficient Firm Moves First

In this case firm A has a Stackelberg leadership advantage in cost reduction and its investment decision is made taking firm B's optimal response as given. Assuming $R_B(I_A)$, the solution to $\frac{\partial \pi_B}{\partial I_B} = 0$, is the best response function of firm B to any the investment choice of firm A, the equilibrium investment decision of firm A is given by:

$$\arg\max_{I_A} \pi_A(I_A, R_B(I_A))$$

Solving this equation yields firm A's equilibrium investment level:

$$I_A = \frac{18tm_A(16m_B^2 - 27t)}{216tm_A^2 - (8m_B^2 - 27t)^2} \tag{2.4}$$

Substituting I_A into $R_B(I_A) = \frac{\partial \pi_B}{\partial I_B} = 0$ obtains:

$$I_B = \frac{18tm_B(16m_A^2 + 8m_B^2 - 27t)}{216tm_A^2 - (8m_B^2 - 27t)^2} \tag{2.5}$$

In order to obtain a reasonable equilibrium in the cost reduction stage two conditions must be satisfied: (i) equilibrium investment levels must be non-negative; and (ii) the second order conditions must be satisfied. However, in order for the game to yield a subgame perfect Nash equilibrium it is also necessary for the pure strategy equilibria in the cost reduction stage to be consistent with those in the price and location stages. This occurs if $\varphi_A(I_A) - \varphi_B(I_B) = m_A I_A - m_B I_B \in (-\frac{9}{4}t, \frac{9}{4}t)$.

Proposition 1 *Equilibrium investment levels are non-negative, the second order conditions are satisfied and equilibrium investment in the first stage is consistent with a subgame perfect Nash equilibrium at all stages if and only if*

$$m_A^2 \in (m_B^2, \frac{27}{16}t - \frac{m_B^2}{2}) \tag{2.6}$$
$$m_B^2 \in [0, \min\{m_A^2, \frac{27}{8}t - 2m_A^2\}) \tag{2.7}$$

Proof. *In Appendix 6.1* ■

Equations (2.6) and (2.7) ensure that a reasonable equilibrium is not only ensured in the investment stage, but also that equilibrium investment decisions do not undermine the existence of a pure strategy Nash equilibrium for the entire game. However, they also imply that, for an equilibrium to exist, the *a* *priori* heterogeneity between the two firms cannot be too large.[21] In fact, given the simplicity of (2.6) and (2.7) is not too difficult to formalise the maximum level of heterogeneity permissible in the model. This implies:

Corollary 2 *Given conditions (2.6) and (2.7), and taking m_B^2 as given, the maximum a priori heterogeneity between the firms' cost reducing efficiency is given by*

$$\max\{m_A^2\} - m_B^2 = \frac{27}{16}t - \frac{3}{2}m_B^2 = \Upsilon$$

If this holds, then:
(i) $m_A^2 \in (0, \frac{27}{16}t)$ *and* $m_B^2 \in [0, \frac{27}{24}t)$*; and*
(ii) As $m_B^2 \to \frac{27}{24}t$, $\Upsilon \to 0$

All of the above results can be derived very simply from equations (2.6) and (2.7) assuming $m_A^2 > m_B^2 \geq 0$ and so a formal proof is omitted. However, the importance of these results comes from their implications for the model with regards to ensuring a pure strategy Nash equilibrium. Given firm A possesses both an investment timing and efficiency advantage at the beginning of the game, corollary 2 simply states that the efficiency advantage cannot be too great or firm A's equilibrium actions would undermine the stability of a (pure strategy) equilibrium solution. Quite simply, then, firm A's effectiveness at cost reduction must be capped relative to that of firm B. If this were not the case, and the efficiency gap is greater than or equal to Υ, then either: (i) the equilibrium investment levels could be negative; (ii) the second order conditions are violated; (iii) the location stage has no pure strategy Nash equilibrium; or (iv) some combination of $(i) - (iii)$ occurs.

Assuming that conditions (2.6) and (2.7) are met, it is then possible to make a number of observations regarding the equilibrium investment levels of the firms.

Proposition 3 *For all* m_A^2 *and* m_B^2 *as defined in (2.6) and (2.7):*
(i) I_A *and* I_B *are strictly positive;*
(ii) $\frac{\partial I_A}{\partial m_A} > 0$, $\frac{\partial I_B}{\partial m_A} < 0 \ \forall \ m_B^2 \in [0, \frac{27}{24}t)$*;*

[21] The conditions derived in proposition 1 are derived under the assumption that both firms remain active in the final product market.

(iii) $I_B \to 0$ *as the a priori efficiency gap tends to* Υ*; and*

(iv) $I_A > I_B$

Proof. *In Appendix 6.2* ■

Proposition 3 makes four observations regarding the equilibrium investment levels of the firms if conditions (2.6) and (2.7) are met. The first, that the equilibrium investment levels of both are strictly positive, rules out the possibility that one, or both, firms remain passive in the cost reduction stage. Moreover, it goes further than the assumption made to derive proposition 1, that equilibrium investment levels simply be non-negative, and rules out any case in which one firm would simply "give up" and exit the market.

The second implies that the equilibrium investment decision of firm A (B) increases (decreases) as firm A becomes more efficient relative to firm B. This occurs because, as cost reduction in this model is a strategic substitute, firm A is able to use its investment timing advantage to temper its rival's cost reducing investment.[22] Therefore, as firm A becomes a stronger competitor, relative to firm B, it is better able to take advantage of its timing and efficiency advantages by investing more heavily in cost reduction and forcing firm B's equilibrium investment to contract.

The third states that, as the efficiency gap converges to Υ, firm A becomes sufficiently aggressive that firm B's optimal investment decision is to invest nothing. However, from corollary 2, the maximum level of heterogeneity is decreasing in m_B^2 and, as m_B^2 increases, the maximum *a priori* efficiency gap, Υ, converges to zero. Therefore, as the weaker firm becomes relatively stronger (m_B^2 increases), the size of the efficiency gap required to drive I_B to zero becomes smaller.[23] Therefore, it must be that the *ex ante* efficient firm becomes more aggressive when competing against relatively tougher rivals.

The final observation simply notes that the equilibrium investment levels of firm A are always strictly larger than those of firm B. As cost reductions in this model are treated as strategic substitutes, an efficient firm has an incentive to invest heavily in cost reduction in the first stage to limit the cost

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[22] Here:

$$R_B^{'}(I_A) = \frac{8m_A m_B}{8m_B^2 - 27t} < 0$$

[23] Observe $\frac{\partial \Upsilon}{\partial m_B^2} < 0$

reduction of its weaker rival. Therefore, as firm A also possesses an investment timing advantage, even where the firms are (almost) symmetric, it will use this advantage to cement its dominance in the final product market through aggressive and preemptive investment. Thus, it is intuitive that $I_A > I_B$.

Despite all of these observations, both (2.6) and (2.7) ensure that, whilst firm B's optimal investment strategy is restricted as its rival becomes more aggressive, it is always optimal to invest. In fact, firm B's optimal investment decision has to satisfy two contrasting incentives: (i) investing in cost reduction allows the firm to retain a small, but positive, market share; but (ii) cost reductions exacerbate fierce price competition in the final product market. Which of these incentives dominates depends on the relative strengths of the two firms but, in general, it is optimal to make a small investment to protect a small share of the market whilst ensuring the *ex post* market is not too fiercely competitive.

Knowing $I_A > I_B$ and $m_A^2 > m_B^2 \geq 0$ also implies that the *ex post* cost reduction of firm A is larger than that of firm B; as the cost reduction form is simply given by

$$\varphi_i(I_i) = m_i I_i \;\forall\; i \in \{A, B\}$$

Intuitively, then, where $I_A > I_B$ and $m_A^2 > m_B^2 \geq 0$ it must hold that $\varphi_A(I_A) > \varphi_B(I_B)$. Therefore, a firm with an investment timing and efficiency advantage is able, through an aggressive, preemptive investment strategy, to maintain these competitive advantages into the location and prices stages. For firm A, the benefits of possessing a lower unit cost and serving a larger proportion of the market are the impetus for it to act aggressively in cost reduction as it is able to cement its position in the final product market as the dominant firm.

These equilibrium investment levels yield profits for each firm given by:

$$\pi_A = -\frac{3}{4}\frac{t(16m_B^2 - 27t)^2}{216tm_A^2 - (8m_B^2 - 27t)^2} \tag{2.8}$$

$$\pi_B = -\frac{81}{4}\frac{(16m_A^2 + 8m_B^2 - 27t)^2(8m_B^2 - 27t)}{\left[216tm_A^2 - (8m_B^2 - 27t)^2\right]^2} \tag{2.9}$$

Examining these profit functions leads to one observation: the profits obtained by both firms are strictly positive. Once again, this is a direct consequence of proposition 1. Of course, if this were not the case then it would be impossible for an equilibrium to be sustained as one firm would have no unique optimal location.

All this leads to:

Proposition 4 *If m_A and m_B are defined by (2.6) and (2.7) and firm A has a Stackelberg leadership advantage in the cost reduction stage then:*

(i) $I_A > I_B > 0$;

(ii) $\varphi_A > \varphi_B$;

(iii) $\pi_A > \pi_B$; and

(iv) a pure strategy Nash equilibrium is ensured across all stages of the game.

Proof. *Follows directly from propositions 1, 3 and Appendix 6.3* ■

The third observation, that firm A obtains larger profits than firm B, should not come as a surprise given the previous propositions. Firm A invests aggressively in preemptive cost reduction to become the *ex post* low cost firm and, consequently, limits the market share that firm B can attain. Quite simply, the efficient firm marginalises the inefficient firm in the final product market. Furthermore, as firm A becomes relatively stronger, the *ex post* cost differential becomes larger such that $D_A \to 1$ and $D_B \to 0$. Obviously, as the demand for firm B becomes smaller and the cost differential increases then the profits associated with competition are pushed towards zero.

The implications of this result are clear. Where a firm possesses both an investment timing and efficiency advantage, the *ex ante* efficient firm invests aggressively in cost reduction in order to cement its place as the dominant firm in the final product market. The efficient firm's investment generates lower *ex post* unit costs, serves a greater proportion of demand and yields larger profits than its *a priori* (and, indeed, *ex post*) inefficient rival. Nevertheless, whilst the *ex ante* efficient firm invests to cement its position as the market leader, the *ex ante* inefficient firm still possesses an incentive to invest. However, the size of the investment must always remain relatively small for two reasons: first, investment is used in order to maintain the weaker firm's (niche) market position by reducing the size of the *ex post* cost asymmetry; and second, the investment is kept relatively small in order to mitigate the effects of increased price competition.

Inefficient Firm Moves First

In this case firm B has a Stackelberg leadership advantage in cost reduction and makes its investment decision taking firm A's optimal response as given. Assuming $R_A(I_B)$, the solution to $\frac{\partial \pi_A}{\partial I_A} = 0$, is the best response function of firm A to the investment choices of firm B, the equilibrium investment decision of firm B is given by:

$$\arg\max_{I_B} \pi_B(R_A(I_B), I_B)$$

Solving this equation yields firm B's equilibrium investment level:

$$I_B = \frac{18tm_B(16m_A^2 - 27t)}{216tm_B^2 - (8m_A^2 - 27t)^2} \tag{2.10}$$

Substituting I_A into $R_B(I_A) = \frac{\partial \pi_B}{\partial I_B} = 0$ obtains:

$$I_A = \frac{18tm_A(8m_A^2 + 16m_B^2 - 27t)}{216tm_B^2 - (8m_A^2 - 27t)^2} \tag{2.11}$$

As in the previous case, a reasonable (pure strategy) equilibrium exists in the cost reduction stage if two conditions are satisfied: (i) equilibrium investment levels must be non-negative; and (ii) the second order conditions must be satisfied. However, in order for the game to yield a subgame perfect Nash equilibrium it is also necessary for *all* pure strategy equilibria in the price and location stage to be consistent with the investment stage. Again, this occurs if $\varphi_A(I_A) - \varphi_B(I_B) = m_A I_A - m_B I_B \in (-\frac{9}{4}t, \frac{9}{4}t)$. This leads to:

Proposition 5 *Equilibrium investment levels are non-negative, the second order conditions are satisfied and equilibrium investment levels in the first stage are consistent with a subgame perfect Nash equilibrium at all stages if and only if*

$$m_A^2 \in (m_B^2, \min\{\frac{27}{16}t, \frac{27}{8}t - 2m_B^2\}) \tag{2.12}$$

$$m_B^2 \in [0, \min\{m_A^2, \frac{27}{16}t - m_B^2\}) \tag{2.13}$$

Proof. *See Appendix 6.4* ■

The conditions imposed in equations (2.12) and (2.13) ensure that a reasonable equilibrium exists in the cost reduction stage that does not undermine a subgame perfect Nash equilibrium.[24] However, given the form of these restrictions it no longer so simple to generalise the maximum level of heterogeneity supported within the model, but we can still infer from these conditions that the heterogeneity between the firms cannot be too large. If the efficiency gap is too large then either: (i) the equilibrium investment levels are negative; (ii) the second order conditions are violated; (iii) the location stage has no pure strategy Nash equilibrium; or (iv) some combination of $(i)-(iii)$ occurs.

Keeping this result in mind, it is possible to make some observations regarding the equilibrium investment levels of both firms. However, before this, it is necessary to define a critical value of m_B^2, denoted m_B^I, such that, for any relevant m_A^2, the equilibrium investment levels of the firms are equal if $m_B^2 = m_B^I$.[25] For all other values of m_B^2, $I_A \neq I_B$. With this in mind, we obtain:

Proposition 6 *For all m_A^2 and m_B^2 as defined in (2.12) and (2.13):*
(i) I_A and I_B are strictly positive;
(ii) $\frac{\partial I_A}{\partial m_B} < 0$, $\frac{\partial I_B}{\partial m_B} > 0\ \forall\ m_A^2 \in (0, \frac{27}{16}t)$; and
(iii) $I_A > I_B$ if, $\forall\ m_A^2 \in (0, \frac{27}{16}t)$, $m_B^2 < m_B^I \in [0, \max\{m_B^2\})$
Proof. *In Appendix 6.5* ■

Proposition 6 makes three observations about the equilibrium investment levels where the inefficient firm possesses a Stackelberg leadership advantage in the cost reduction stage and conditions (2.12) and (2.13) are met. The first, that equilibrium investment levels are strictly positive, again rules out that the possibility that one, or both, firms remain passive in the cost reduction stage. As in the previous case, this observation goes beyond the assumption that required equilibrium investment levels be non-negative, which was a key assumption made to derive proposition 5.

The second implies either firm can be rendered (almost) passive during the investment stage. That is, because the equilibrium investment levels of firm A (B) decrease (increase) as firm B becomes relatively more efficient, the initial level of heterogeneity plays an important role in determining the

[24] The conditions derived in proposition 5 are derived under the assumption that both firms remain active in the final product market.

[25] A formal definition and derivation of m_B^I can be found in Appendix 5.5.

equilibrium investment levels. As in the previous case, firm B remains passive if the initial level of heterogeneity is very large. In contrast, firm A is only rendered passive where the initial level of heterogeneity between the firms is sufficiently small. Simply, where the firms' initial parameters are more "equal", a timing advantage enables the *a priori* weaker firm to preemptively invest in cost reduction and restrict the investment level of firm A. Thus, it can become a more fierce competitor when the firms are more symmetric.

The third observation extends this analysis further. It argues that, for a given m_A^2, if $m_B^2 \leq m_B^I$ the inefficient firm is very weak relative to its rival and, consequently, its ability to preemptively invest in cost reduction is weak also. Thus, even with a first mover advantage in the cost reduction stage, the *ex ante* weaker firm is unable and unwilling to act aggressively (or at least reasonably) to restrict its rival's investment and, consequently, the equilibrium investment level of firm B is less than or equal to that of firm A. However, once $m_B^2 > m_B^I$, the inefficient firm becomes sufficiently strong, relative to its rival, to be able to take advantage of its timing advantage and cement a position in the market, beyond simply filling a niche. In this case, firm B becomes more effective at preemptively investing and, therefore, invests more. In turn, this drives down the equilibrium investment decisions of firm A and firm B is induced to invest sufficiently to drive I_A below I_B. Therefore, if the *a priori* efficiency gap is not too large then the *a priori* weaker firm will invest *more* than its *ex ante* efficient rival.

The equilibrium investment observations imply firm B can become the "investment leader", but are not sufficient to ensure that firm B will become the *ex post* low cost firm too. Rather, because $m_A^2 > m_B^2 \geq 0$, ensuring $I_B > I_A$ does not directly imply $m_B I_B > m_A I_A$. Instead, the cost reduction of firm B will only be larger if its investment levels, relative to firm A's, are sufficiently large to overcome this relative inefficiency. Comparing the firms' cost reductions yields:

$$\varphi_A - \varphi_B = \frac{18tm_A(8m_A^4 - 27t(m_A - m_B)(m_A + m_B))}{216tm_B^2 - (8m_A^2 - 27t)^2}$$

Again, it is necessary to define a critical value of m_B^2, denoted m_B^{φ}, such that, for any relevant m_A^2, the equilibrium cost reduction levels of the firms are equal if $m_B^2 = m_B^\varphi$.[26] For all other values of m_B^2, $\varphi_A \neq \varphi_B$. Consequently, one observes:

Proposition 7 *For all m_A^2 and m_B^2 as defined in (2.12) and (2.13):*
(i) $\varphi_A > \varphi_B$ if $\forall\ m_A^2 \in (0, \frac{27}{16}t)$, $m_B^2 < m_B^\varphi \in [0, \max\{m_B^2\})$
(ii) $m_B^\varphi > m_B^I$
Proof. *In Appendix 6.6* ■

Both observations in proposition 7 imply that an *ex ante* inefficient firm can become the *ex post* low cost firm if: (*i*) it is not too inefficient relative to its rival; and (*ii*) it has a Stackelberg leadership advantage in the cost reduction stage. Recall from proposition 6 that *ceteris paribus* the equilibrium investment level of firm B (A) increases (decreases) as firm B becomes relatively more efficient and, once $m_B^2 > m_B^I$, firm B's equilibrium investment is larger than that of firm A. However, as firm B is relatively inefficient, for it to become the *ex post* low cost firm it is not sufficient for firm B to simply invest more than firm A but, rather, it must invest *enough* to overcome this disadvantage. This occurs once $m_B^2 > m_B^\varphi > m_B^I$. That is, once the difference between I_A and I_B is sufficiently in firm B's favour, or the *a priori* efficiency gap sufficiently small, firm B is induced to reduce its costs to such an extent that it can overcome its initial inefficiency and become the low cost firm.

Finally, taking into account the equilibrium investment levels, the corresponding profit functions are given by:

$$\pi_A = -\frac{81}{4}\frac{t^2(8m_A^2 - 27t)(8m_A^2 + 16m_B^2 - 27)^2}{[216tm_B^2 - (8m_A^2 - 27t)^2]^2} \tag{2.14}$$

$$\pi_B = -\frac{3}{4}\frac{t(16m_A^2 - 27t)^2}{216tm_B^2 - (8m_A^2 - 27t)^2} \tag{2.15}$$

Similar to investment and cost reduction, it is possible to make some observations regarding the profit levels of the firms. Before proceeding, it is necessary to define a final critical value of m_B^2, denoted m_B^π, such that, for any relevant m_A^2, the equilibrium profit levels of the firms are equal if $m_B^2 = m_B^\pi$.[27] For all other values of m_B^2, $\pi_A \neq \pi_B$. This leads to:

[26] A formal definition and derivation of m_B^φ can be found in Appendix 5.6.
[27] A formal definition and derivation of m_B^π can be found in Appendix 5.7.

Proposition 8 *For all* m_A^2 *and* m_B^2 *as defined in (2.12) and (2.13):*
(*i*) π_A, $\pi_B > 0$
(*ii*) $\pi_A > \pi_B$ *if* $\forall\ m_A^2 \in (0, \frac{27}{16}t)$, $m_B^2 < m_B^\pi \in [0, \max\{m_B^2\})$
(*iii*) $m_B^\pi > m_B^\varphi > m_B^I$
Proof. *In Appendix 6.7* ■

The first observation of proposition 8 states that the equilibrium profit levels of both firms are strictly positive if m_A^2 and m_B^2 are defined as in (2.12) and (2.13). Of course, were this not the case then it would be impossible to sustain a (pure strategy) equilibrium as one firm would have no unique optimal location. Consequently, proposition 5 ensures this holds by the underlying assumption that a pure strategy Nash equilibrium is ensured in the location and price stages.

The second and third observations imply an *ex ante* inefficient firm with a Stackelberg advantage in the cost reduction stage can only become the *ex post* dominant firm, in all respects, if and only if $m_B^2 > m_B^\pi$. Thus, only when the *ex ante* inefficient firm is not too inefficient relative to its rival is it able to fully take advantage of its timing advantage and become the market leader. The rationale behind this result is analogous to that of proposition 7. As firm B becomes relatively more efficient, the equilibrium investment levels of firms A and B decrease and increase respectively. Therefore, as the *a priori* efficiency gap becomes smaller, the *ex ante* inefficient firm becomes better able to preemptively invest in cost reduction. Once the initial efficiency gap is small enough, $m_B^2 > m_B^\pi$, firm B is able to use its timing advantage to overcome its initial disadvantage and become the market leader in terms of costs, demand and profits.

An additional, and unstated, result can be found within proposition 8. For all m_A^2, there exists a possible range of m_B^2 such that an *a priori* inefficient firm can become the *ex post* low cost firm but earn *lower* profits in the final product market. Thus, if $m_B^2 \in (m_B^\varphi, m_B^\pi)$, firm B invests more, has lower *ex post* production costs and serves a larger proportion of the market but yields smaller profits. Over this range, the costs incurred by firm B in becoming the *ex post* low cost firm are are sufficiently large to outweigh the cost and demand benefits gained by the *ex post* efficient firm. Whilst the inefficient firm can obtain a larger market share and possesses lower unit costs, in doing so, the additional investment costs prevent it from yielding larger profits than its *ex ante* efficient rival.

Considering all of these propositions together, it is possible to state:

Summary 9 *If m_A^2 and m_B^2 are defined by (2.12) and (2.13), $m_A^2 > m_B^2$ and firm B possesses a Stackelberg leadership in the cost reduction stage, four potential equilibria may obtain for a given $m_A^2 \in (0, \frac{27}{16}t)$:*

1. $m_B^2 \in [0, m_B^I)$: $I_A > I_B$, $m_A I_A > m_B I_B$ *and* $\pi_A > \pi_B$
2. $m_B^2 \in [m_B^I, m_B^\varphi)$: $I_A \leq I_B$, $m_A I_A > m_B I_B$ *and* $\pi_A > \pi_B$
3. $m_B^2 \in [m_B^\varphi, m_B^\pi)$: $I_A < I_B$, $m_A I_A \leq m_B I_B$ *and* $\pi_A > \pi_B$
4. $m_B^2 \in [m_B^\pi, m_A^2)$: $I_A < I_B$, $m_A I_A < m_B I_B$ *and* $\pi_A \leq \pi_B$

Finally, a pure strategy Nash equilibrium is ensured across all stages of the game.

Proof. *Follows directly from propositions 5, 6, 7 and 8* ■

This result has a number of interesting implications. However, the key result is simply that, as the efficiency gap becomes smaller, an *ex ante* inefficient firm becomes a tougher competitor if it has a Stackelberg leadership advantage in the cost reduction stage. When the efficiency gap is sufficiently large the inefficient firm is always either unable or unwilling to invest aggressively in cost reduction. As the initial efficiency gap becomes smaller, the inefficient firm is better able to take advantage of its first mover advantage in the cost reduction stage and adopts an increasingly aggressive investment strategy. Therefore, by moving first it is able to compensate its inefficiency by restricting its rival's investment decision and manipulating a better *ex post* situation for itself. Consequently, the initial heterogeneity between the firms plays an important role in determining the market outcome and, in general, relatively weak firms "give up" whilst stronger firms "fight".

2.4 Conclusion

This paper examines the effects of endogenous cost selection on a firm's product and pricing decisions where the firms are heterogeneous with respect to their *ex ante* investment efficiencies and investment timing. In doing so, two results unique to the literature obtain.

First, where an *ex ante* efficient firm possesses Stackelberg leadership in cost reduction, they generate lower costs, greater demand and yield larger profits *ex post* than their *ex ante* inefficient rival. This result suggests, where a firm possesses both an investment timing and efficiency advantage, the *ex ante* efficient firm invests aggressively in cost reduction in order to cement its place as the dominant firm in the final product market. Thus, the impetus for the efficient firm to invest are the additional gains of being the dominant firm in the final product market. In contrast, the *ex ante* inefficient firm's incentive to invest is, ultimately, an attempt to balance two contrasting incentives: (i) to increase investment in order to maintain its market position by reducing the size of the *ex post* cost asymmetry; and (ii) to reduce investment so as to mitigate increased price competition in the final product market. In general, the latter incentive dominates as the *a priori* efficient firm is able to preemptively invest in cost reduction and force its rival to act "soft" in order to prevent itself undermining its profits through tough price competition.

Second, if an *ex ante* inefficient firm is the Stackelberg leader in the cost-reduction stage, there are a four potential equilibrium outcomes that depend on the relative abilities of the two firms. If the *a priori* efficiency gap is large, then the *ex ante* inefficient firm invests simply to protect some market share. This result obtains because, where the *ex ante* firm is very weak and unable to make the most of its timing advantage, investment serves only to increase price competition in the final product market and undermine its profits. Therefore, for a large efficiency gap, the *ex ante* inefficient firm "gives up". However, as the initial efficiency gap becomes smaller, the inefficient firm becomes better able to preemptively invest in cost reduction and temper the investment decision of its rival firm. Consequently, the firm begins to "fight". However, it is only when the gap between the two firms is sufficiently small that the inefficient firm is able to overcome this efficiency disadvantage, using its timing advantage to either invest more than its *a priori* efficient rival; become the *ex post* low cost firm; or, become the dominant firm in the final product market. Therefore, the initial heterogeneity between the firms plays an important role in determining the market outcome and, in general, relatively weak firms "give up" whilst stronger firms "fight".

These results are unique in the literature and suggest that the firms' abilities to innovate - in terms of timing and efficiency/ability - play a crucial role in determining the composition of the final product market. Furthermore, whilst the *ex post* low cost firm *always* becomes the market leader it is not *a priori* obvious whether this will be an *ex ante* efficient or inefficient firm. Consequently, simply examining the structure of a market in which the firms face (exogenously determined) cost asymmetries fails to comprehend the role that innovative abilities play in determining the success, or failure, of the firms.

It is notable, however, that this model only examines the innovative process within a duopoly setting, when innovation most likely occurs in markets with more than two firms. Indeed, this is a conscious decision and keeps the assumptions of this paper in line with the overwhelming majority of the existing literature. Furthermore, given that the focus in the linear spatial competition literature has been on duopolistic markets suggests that this omission is not significant.[28] In fact, the existing literature that does extend the basic model beyond two firms suggests that it is *unlikely* our results would be robust to such a change. Using quadratic transport costs and a bounded location space, Brenner (2005) finds the addition of firms undermines the principal of maximal differentiation. Simply, the "introducing a market boundary leads to asymmetry between firms" with boundary firms locating more centrally - partially agglomerating - to take advantage of their monopoly position in the corners, allowing these firms to receive higher prices and yield larger profits than more centrally located firms (Brenner, 2005: p.862).[29] Whilst we cannot directly transpose Brenner's (2005) results into our work - as this model assumes an unbounded city with *ex ante* asymmetric firms - it does highlight the fact that

[28] An interested reader may wish to examine the detailed review of the existing spatial competition literature by Biscaia and Mota (2012) which notes a similar dearth of n firm models in the field of linear spatial competition.

[29] Economides (1993) examines a similar model to Brenner (2005) but maintains Hotelling's original assumption of linear transport costs. Similar to Brenner's (2005) result, corner firms are driven to locate more centrally but, due to weaker price competition, minimal differentiation obtains. However, as in Hotelling's (1929) original specification, this undermines the existence of a location equilibrium. Andersen and Nevin (1991) analysed an identical n-firm model to Economides (1993) but examined Cournot competition in the second stage, finding that agglomeration in the centre of the market was the unique location equilibrium outcome.

equilibrium outcomes, even in the most simple linear spatial setting, can be altered by the addition of firms. Therefore, given the sensitivity of linear spatial models to the initial specification, including the number of firms, it is difficult to predict how the results presented here would change, even if one suspects they will not be robust. Consequently, this is left to future research.

Another intriguing question that comes out of this work concerns whether firms would, if possible, *choose* to innovate or locate first. Such a decision can be thought as emulating an important question regarding innovation: should a firm be *first to market* or *the best product?* Indeed, this is a scenario that has cropped up many times, for example VHS and Betamax, Blu-ray and HD DVD, and, more recently, the Playstation 4 and Xbox One. This is a question that warrants further examination and, on the face of it, appears to be a non-trivial question. On the one hand, the results presented here suggest that there exists an unambiguous advantage to innovating first; preemptively investing in cost reduction enables a firm to temper the investment decision of its rival and yields higher profits. On the other hand, Lambertini (2002) notes that within the context of linear spatial competition, with an unbounded location space, it appears that the first mover possesses a significant advantage in the location stage too. That is, the leader locates in the centre of the market whilst the follower is forced to locate outside of the market. Of course, the existence of asymmetric firms, generating asymmetric production costs, suggests that such an outcome is likely to differ but it is intuitively appealing that *some* advantage would remain were the *ex post* high cost firm were to locate first.[30] Given moving first in *either* the investment or location games leads to a competitive advantage, at the expense of the rival firm, allowing the firms to endogenously choose whether to be first to the market or the best product would be intriguing. However, it is not *a priori* obvious what asymmetric firms would choose or how this would depend upon the relative efficiencies of the firms.

However, as we have already discussed, it is not a trivial matter to simply reverse the move order and so such a model may not be possible *directly*.[31]

[30] See Ziss (1993) and Matsumura and Matsushima (2009) for a discussion of how asymmetric costs alter the location decisions with simultaneous moves. These results suggest that linear spatial models are robust to sufficiently *small* cost differentials.

[31] As a brief recap, if the sequence of investment and location decisions is reversed - with

Nonetheless, it would still be interesting to endogenise the investment timing and location timing decisions. In doing so, it would be possible to explore whether the existing results in the patent race and endogenous market entry literature hold within a linear spatial setting. That is: *i*) do firms with an innovative advantage choose to invest first?;[32] and *ii*) do *ex ante* efficient firms become *ex post* market leaders by entering the market first?[33] Whilst there is some evidence to suggest that results presented here do, at least partially, agree with the existing literature, it would be interesting to see if these hold up to more formal analysis. This is left to future research.

Finally, it has been widely noted within the literature that a key element of R&D and innovation is the existence of knowledge spillovers between the firms. The addition of spillover effects in this context would be particularly interesting as it would almost certainly alter the firms' incentives to preemptively invest in the first stage. That is, whilst the results presented here suggest that aggressive investment in the first stage serves to temper a rival's investment decision, the addition of spillovers would add an opposing effect: aggressive investment would serve to *benefit* the follower. Quite simply, the more a firm invests in R&D, the greater the spillover effect that accrues to the rival firm. Consequently, spillovers may indirectly mitigate some of benefits of investing first and reduce the potency of this advantage. Therefore, it is likely that the addition of knowledge spillovers *would* have an impact on the equilibria derived here, but a thorough analysis is left for future research.

firms locating before innovating - the model becomes intractable. Consequently, it may be difficult to give the firms a choice between investing and locating first.

[32] See Pollock (2008), Chan and Kwok (2007), and Lambrecht and Perraudin (2003)

[33] See van Damme and Hurkens (1999), and Branco (2008).

Chapter 3

Cost reductions in a Hotelling model with location based spillover effects

3.1 Introduction

In this paper we analyse whether a firm's incentive to agglomerate, when research spillovers are location based, survives the existence of asymmetric abilities that may generate *heterogeneous* costs. In essence, should we ever see asymmetric firms agglomerate, or is this simply the preserve of more symmetric firms?

The importance of R&D spillovers and their impact on innovation have been well documented since the seminal works of both d' Aspremont & Jacquemin (1988) and Kamien *et al* (1992).[1] Ever since this initial work, it has become increasingly apparent that research spillovers do not simply rain down on their beneficiaries like manna from heaven but, instead, depend upon a firm's absorptive capacity: their investment in R&D to be better able to both innovate and assimilate the ideas of others. As Kamien and Zang (2000, p.997) eloquently suggest, it is wrong to assume that "no effort is required of the recipients, not even purchase of a bucket".

More recently, there has been increasing empirical, anecdotal and theoret-

[1] Also see Suzumura (1992) and Salant and Shaffer (1998) for further examples.

ical evidence that has argued that even this assertion of absorptive capacity may not tell the whole story, especially as it appears such benefits only accrue *locally*. Consequently, a new body of spatial competition literature has evolved to examine whether firms may, quite literally, meet half way (Mai and Peng (1999), Piga and Poyago-Theotoky (2005) and Sun (2013)). The results, so far, have been clear: the greater the spillover rate, the greater the incentive for firms to cluster with partial agglomeration being the likeliest outcome. Yet, within this literature the firms are assumed to be *homogeneous*, mitigating the potential for asymmetric outcomes in the final product market. Moreover, this assumption that firms are identical, especially in terms of research and development, is not intuitively appealing. Instead, the abilities of firms to undertake R&D can vary for a significant number of reasons including, but not limited to: the research skills of their employees; the quality of the equipment and raw materials they have to work with; or simply a consequence of luck.

To address this question we employ a three-stage, innovation-location-price Hotelling (1929) model with quadratic transport costs and make two key assumptions: *i*) the closer the firms are to each other, the greater the benefit they receive from their rivals' efforts in R&D; and *ii*) firms are *a priori* heterogeneous with respect to their innovative abilities, defined as the ability to implement cost reductions and assimilate a rival's research (absorptive capacity).[2,3] It is the latter of these assumptions that is new to the literature and asks, does the (partial) agglomeration incentive survive when the firms innovative abilities are asymmetric?[4]

[2]Whilst the model assumes that the firms possess symmetric absorptive capacities, we assume that they differ in their ability to implement knowledge spillovers. Consequently, the results are the same as assuming *asymmetric* absorptive capacities.

[3]It is important to note that this paper only considers process innovation and, consequently, horizontal differentiation. Hotelling (1929) style models often focus on horizontal differentiation, but can be extended to include vertical differentiation. In fact, using a similar model to that presented here, Sun (2013) finds that there exists a duality between process and product innovation. That is, cost reduction and quality improving innovation "share identical strategic properties" and that within a linear spatial model "[it is possible] to derive the equilibrium strategies of a quality improving R&D model from the ones of a cost-reducing R&D model, and vice versa" (Sun, 2013: p.138). Consequently, the results derived here would likely transfer to a situation in which the products are vertically differentiated, or product innovation were used instead. Therefore, such an omission is irrelevant.

[4]To the author's knowledge, whilst there has been some examination of asymmetric costs in the literature, nothing has been done which implies asymmetric absorptive capacity in a spatial setting.

Empirical evidence has, for some time, suggested that agglomeration is a commonly observed phenomenon. Head *et al* (1995, p.243), examining data on 751 Japanese manufacturing plants built in the US during the 1980s, found "Japanese investors [preferred] to site their plants in areas where they [found] concentrations of previous Japanese investments in the same industry and, for auto-related firms". They conclude that, given this apparently strong incentive to agglomerate, these results could not be explained by the location of natural resources nor specialised labour alone and, instead, suggest that agglomeration benefits must play an important role. In related work, Audretsch and Feldman (1996) examine which industries are more likely to see agglomeration finding that those which place a greater relative importance on new economic knowledge often see firms locate closer to one another. Finally, Jaffe *et al* (1993) use patent citations to examine the "locality" of spillovers. The key finding, that domestic patents are most often cited domestically, is taken to determine that spillover effects are, indeed, local as has often been assumed.

However, it has taken some time for the theoretical literature to offer an understanding of *why* this occurs. Using Hotelling's (1929) framework, Mai and Peng (1999), using exogenous cost reductions, examine the incentives for homogeneous firms to agglomerate if research spillovers are inversely proportional to the distance between the firms. They find that, only when the spillover rate is sufficiently large, and convex in the distance between the firms, will firms find it optimal to even partially agglomerate, though maximal and minimal differentiation are both possible. Piga and Poyago-Theotoky (2005) examine a similar model but assume: *i*) endogenous investments; and *ii*) R&D is quality-improving. Interestingly, their results also find that the incentive to agglomerate is linked to the initial rate and convexity of spillovers, with *partial* agglomeration the most likely outcome.[5] In related work, Mota and Brandão (2004) examine cost reductions and spatial agglomeration but in which firms compete in quantities in the second stage finding that only *complete* agglomeration obtains within the model. So,why is there this discrepancy between price and quantity competition?

The reason that price competition, in general, only supports partial ag-

[5] In recent work, Sun (2013) examines the difference bewteen the cost reducing and quality improving R&D finding that within this framework they are, in fact, equivalent.

glomeration is a consequence of the transport cost specification. Within a linear spatial framework, as transport costs become increasingly convex, small gains in market share from locating closer to a rival are increasingly offset by rapidly escalating price competition (d'Aspremont *et al* (1979) and Economides (1986)). Moreover, with quadratic transport costs this effect becomes so pronounced that firms would wish to locate *outside* of the city's boundaries (Lambertini (1997) and Tabuchi and Thisse (1995)). Consequently, for even partial agglomeration to be optimal, spillovers must be sufficiently large to at least partially overcome this hurdle and this can only occur if spillovers are sufficiently convex in the distance between the firms.[6] However, no such conflict exists when firms compete in quantities and, consequently, this means any incentive to agglomerate leads to complete agglomeration.

However, whether agglomeration would ever be optimal between asymmetric firms is not *a priori* obvious. Initially noted by Schultz and Stahl (1985), the existence of asymmetric costs can *undermine* pure strategy equilibria. Formalised by Ziss (1993), using exogenous cost differentials, it appears that (maximal differentiation) location equilibria can only be supported where the cost differential between the firms are not too large. Matsumura and Matsushima (2009) then extended this result in two ways. First, for "small" cost differentials the firms would prefer to locate *outside* of the city boundaries, with the asymmetric firm locating more centrally. Second, for all "large" cost differentials, a solution in mixed strategies always existed. Intuitively, for large cost differentials the equilibrium becomes an effective game of "cat and mouse" with the *ex ante* stronger firm attempting to prevent entry by locating next to the weaker firm and the weaker firm trying to get away. It is these result that lead us to ask a *new* question: if asymmetric firms may face different investment and location incentives, should we ever expect them to agglomerate?

We find two important results: one that contradicts and one that complements the existing literature. First, with location based spillovers, the firms never find it optimal to (partially) agglomerate and maximal differentiation always obtains, and this holds even where the firms are *symmetric* and spillovers are convex in the distance between the firms. However, this result is not surprising because, once firms have selected their costs, endogenously generating

[6] See Mai and Peng (1999) and Piga and Poyago-Theotoky (2005).

an *ex post* cost differential, the ferocity of price competition remains in the location stage.[7] Therefore, when the firms are choosing their locations, the rapidly escalating price competition that occurs as the firms move towards one another, noted by d'Aspremont *et al* (1979) and Economides (1986), acts as a centrifugal force driving the firms to remain as far from each other as possible.

Second, and complementing the existing literature regarding cost differentials and spatial competition, the addition of asymmetric firms - and consequently asymmetric spillovers - leads to an increasingly asymmetric outcome. Where a firm is better able to both reduce its own costs *and* assimilate a rival's economic knowledge, it becomes more aggressive in terms of both location and investment. The location equilibrium is *more* asymmetric under location based spillovers than the generic case because of the existence of *asymmetric absorptive capacities*. As the firms locate closer to one another, the more able firm assimilates *more* knowledge than it leaks to its inefficient rival. Consequently, this enables it to become *more* predatory and push its rival further from the market. In turn, this incentivises the firm to *increase* its initial research efforts because of the greater rewards, particularly market share and profits, that it is able to obtain from doing so. The impact of location based spillovers on a less able firm are less straightforward and related to the level of consumer heterogeneity. When consumers are relatively homogeneous, and can switch between products easily, this makes a more able firm *more* of a "threat". Under such circumstances, the less able firm reduces its investment and locates further from the market in order to mitigate price competition and minimise spillover between the firms. For large transport costs, where consumers find it harder to switch between goods, the more able firm's predatory threat is less pronounced. Consequently, a less able firm may invest *more* because it considers itself better protected from the more able firm's predatory threat.

Another body of literature to which this paper relates, and another crucial aspect of the innovative process, examines patent races. Whilst not explicitly examined here, this body of literature gives some indication as to *which* firm has the most incentive to innovate. The early literature, focusing on deterministic R&D and complete information regarding the firms' innovation strategies,

[7] This is akin to Matsumura and Matsushima (2009) but with endogenous cost selection.

sheds some light as to whether *ex ante* efficient firms can be "leapfrogged" by *ex ante* inefficient firms or if they are destined to maintain their position.[8] Gilbert and Newbury (1982), examining the strategic actions of an incumbent monopolist, concluded that, as monopoly profits are always at least as large of those in any other market structure, there always exists an incentive to defend this position by preemptively investing and patenting.[9] However, it appears that these results are not robust to the existence of asymmetric firms. Rather, when one firm is granted a competitive advantage in innovation, the results become more extreme. Fudenberg *et al* (1983), Harris and Vickers (1985a; 1985b) and Aoki (1991) all find that granting a firm any advantage in the innovation stage - be it a larger reward from obtaining a patent, being better at R&D, or simply being further along the development path - it will dominate the competition and *always* wins the patent race.[10] Though, this may be because the "leader" in a patent race invests more heavily than the follower (Harris and Vickers, 1987). Quite simply, then, the early industrial organisation literature highlights that a firm with some innate advantage in a patent race will, inherently, become the winner.

The most recent literature regarding patent races has focused on uncertain returns to innovation by adopting a real options approach, the right - though not obligation - to invest in a new technology (Dixit and Pindyk, 1994). Quite remarkably, similar results obtain in the real options literature, directly comparable to those obtained in the earlier work. Or, as Hsu and Lambrecht (2007: p.21) eloquently state, "one [should] expect qualitative nature of previous results from the industrial organisation literature on the timing of technological innovation is likely to remain unaltered by uncertainty and asymmetric information".

[8] A more detailed literature review of this can be found in Pollock (2008). However, it suggests that since the initial work regarding patent races, the examination of patents has become more focused on: *i*) patent design; *ii*) cumulative innovation; *iii*) licensing; *iv*) imitation; and *v*) the openess of innovation (such as the rise of open source software).

We ignore a discussion of cumulative innovation here as innovation, within this model, consists of only a single stage.

[9] Leininger (1991) notes that Gilbert and Newbury's (1982) basic model is, in essence, a simple auction for a patent. Therefore, as firms must compete to "win" the auction, rents are dissipated compared to the monopoly position. Moreover, where the firms become increasingly symmetric, rent dissipation is exacerbated.

[10] These results are robust for both single- and multi-stage innovation. Furthermore, the issue of rent dissipation, as observed by Leininger (1991), still obtains.

When the profitability of investment is uncertain, and when firms face no competition, the real options approach suggests it is better to wait for better information regarding the profitability of investment before committing (Huisman and Kort ,1999). In contrast, when a firm faces competition, the benefits of waiting are undermined as there is the potential to be preempted which drives firms to invest earlier than they would have done otherwise; a decision made to preempt their rivals and win the patent race. Furthermore, this result becomes more pronounced as the number of firms participating in the race increases, as each firm faces a *stronger* threat of preemption which drives them to invest earlier still (Bouis *et al*, 2009; Grenadier, 2002).[11] However, the result most pertinent here are those derived between asymmetric firms. Chan and Kwok (2007) examine a real options model in which the firms face asymmetric costs of innovation, though cost information is common knowledge, and conclude that it is the most *innovative and efficient* firm that should win the patent race. However, this result can be criticised because, if the firms compete with complete information, why don't they simply cooperate to maximise joint profits?[12] However, Lambrecht and Perraudin (2003), using a model in which the firms face uncertainty regarding the costs of their rivals, the firms still attempt to strategically preempt their rivals, though the extent to which this occurs depends upon the initial distribution of costs. Nonetheless, it is, once again, the most *innovative and efficient* that should win the race.

It is interesting that the existing literature - using two very different approaches - finds identical results: where firms compete in deterministic R&D, it is the firm with some competitive advantage in innovation that will *always* win the race.[13] In essence, these results suggest that the firms that are better at innovating - due to, for example, receiving higher payoffs from innovation, facing lower R&D costs, or being further along the development path - are the

[11] Grenadier (2002) notes that, at the limit, firms are induced to invest where their expected returns from investment are zero, or all expected rents are completely dissipated. In contrast, Bouis *et al* (2009) finding that almost all expected profits are dissipated when facing only a single competitor.

[12] This criticism of complete information is made by both Weeds (2002) and Lambrecht and Perraudin (2003).

[13] Indeed, the only difference between the equilibria between complete and incomplete information cases is that, in the incomplete information case, the firms optimally adjust their preemptive investment behaviour to take into account the risk associated with uncertain payoffs.

most likely to be the "winners" and, consequently, have the biggest incentive to innovate. The results presented here are, in some way linked, by suggesting that, as in the patent race literature, the firm with the innovative advantage has the most to gain. That is, our results suggest that the firm that is better at R&D, in terms of both implementing their own research *and* assimilating knowledge spillovers, unambiguously dominate the *ex post* product market. Consequently, it is likely that, were firms able to choose *when* to invest in cost reduction, it would be the firm with the innovative advantage that would be the most likely to invest first.

The remainder of the paper is as follows. In section 3.2 we present the model. In sections 3.3 - 3.5 we solve the game using backward induction, first finding equilibrium prices, then locations and finally investment levels. Section 3.6 concludes.

3.2 Model

Consider a three-stage simultaneous-move duopoly model. In the first stage, the firms compete in cost reducing innovation before making location and price decisions in the second and third periods respectively. Firms are heterogeneous with respect to their cost reducing abilities.

This particular sequence of moves, innovation-location-price, makes sense as a way of interpreting agglomeration strategies. As observed in the empirical evidence regarding agglomeration, it is the *most* innovative firms and industries that tend to agglomerate (Head *et al*, 1993; Audretsch and Feldman, 1996). Moreover, it is the most innovative firms that are more likely to have started or committed to an innovative strategy prior to deciding a location.[14] If this is the case, then it is the impact of the firm's initial investment decision, as well as the decisions of its rivals, that prompts it to choose its location and, therefore, whether or not to agglomerate. This is especially true when one considers that

[14] As a more up-to-date example, one can consider Sony's development of the Playstation 4 console. The development cycle began with the very broad goal of "freeing developers from technological barriers" (IGN, 2013a). However, later in the development cycle Sony began to focus on developing a unique set of product characteristics, or locations. In this instance, a keen focus on improving the social aspects of gaming (IGN, 2013b). Therefore, whilst developing a new and innovative product, Sony's initial investment was kept broad before developing specific set product characteristics at a later date.

the firms also take into account: *i*) the magnitude of the knowledge spillovers, a function of both investment and the distance between the firms; and *ii*) their ability to assimilate the knowledge leaked by their rivals.[15] Therefore, as the firms' initial investment decisions has both direct and indirect implications for the agglomeration decision, this sequence of moves appears to fit the empirical observations well.

However, there is another, and more practical, reason for the adoption of this approach: changing the order of moves is non-trivial extension. Indeed, where firms are allowed to locate before making investment decisions the model: *i*) becomes intractable in its current form; or *ii*) yields meaningless/nonsensical results when using simplifying assumptions. With respect to the latter, some of the existing literature has been able to overcome the issue of complexity, but this has involved assuming that the firms locate symmetrically around the centre of the market (Piga and Poyago-Theotoky, 2005; Sun, 2013). However, where firms are *a priori* asymmetric, it is not intuitively appealing to make such an assumption.[16] Consequently, it appears that it is not possible to reverse the order of moves without making an unappealing assumption.

As in Hotelling's (1929) seminal work, two firms, A and B, supply a physically homogeneous good from different locations on the real axis. The location of firm N is denoted by $n \in \mathbb{R}$ and, consequently, the location of each firm is measured from zero. This implies that firms are free to locate anywhere on the real axis and, without loss of generality, we assume $a < b$.[17] Firms maximise profits at each stage.

Consumers are uniformly distributed across a linear city of unit length and are assumed to have a density of one. For simplicity the city is defined by $[0, 1] \in \mathbb{R}$ which ensures that any $a, b \notin [0, 1]$ simply implies that a firm is located outside of the city's boundaries. Consumers face unit demands and consume either zero or one units of production. Consequently, a consumer, located at $x \in [0, 1]$, would only purchase a good from firm N, located at n, if

[15] A recent paper by Matsumura and Matsushima (2012) use a similar sequence of stages to examine the difference between R&D in bounded and unbounded cities. However, unlike here, they assume that the firms are symmetric.

[16] See Ziss (1993) and Matsumura and Matsushima (2009).

[17] It is trivial to derive the results for the case $a > b$.

and only if

$$U_x = s - p_N - t(x-n)^2 \geq 0$$

where s is a fixed utility of consuming a good, p_N is the price charged by firm N, t is a measure of consumer heterogeneity and $t(x-n)^2$ is a quadratic transport cost incurred by a consumer having to move a distance of $|x-n|$ to consume firm N's good.[18] In order to ensure that total demand is equal to one, or that each consumer purchases a good, s is assumed to be large enough that $U_x \geq 0$ for all $x \in [0,1]$ for at least one of the firms' products and consumers only purchase the good that maximises their utility.

In the first stage the firms simultaneously invest in cost reduction to reduce their *ex ante* production cost. At the beginning of the first stage, we assume that both firms face a symmetric production cost, c, that can be reduced by $\varphi_N(I_N)$ (the specification of this function is given below) at a cost of $C_N(I_N)$ by undertaking investment I_N, $N \in \{A, B\}$. For simplicity we assume that the cost-reduction function, $\varphi_N(I_N)$, is a linear function of a firm's own investment level. Thus the final unit cost of firm N (used in the price and location stages), is given by

$$c_N = c - \varphi_N(I_N)$$

We assume that the investment cost schedule $C_N(I_N)$ is quadratic as follows:

$$C_N(I_N) = \frac{1}{2} I_N^2$$

In this paper we assume that the cost-reducing investment undertaken by a certain firm can also benefit its rival i.e. the cost-reducing investment has a spillover effect. Moreover, we assume that these spillover effects are *location based*: the closer a firm is to its rival, the more it can benefit from its rival's cost-reducing activities. The specific functional form of the cost-reduction used is given by:

[18]The assumption of quadratic transport costs is to ensure pure strategy Nash equilibria in the location and pricing stages (see d'Aspremont *et al* (1979)).

$$\text{Firm A} \quad : \quad \varphi_A(I_A) = I_A + \frac{\beta I_B}{1+b-a} \tag{3.1}$$

$$\text{Firm B} \quad : \quad \varphi_B(I_B) = \alpha I_B + \frac{\alpha\beta I_A}{1+b-a} \tag{3.2}$$

where $\alpha \in (0,1)$ represents the fact that firm B is relatively *inefficient* or has a lesser ability to implement a cost reducing technology. The second term in equations (3.1) and (3.2) illustrates the *location based* spillover effects experienced by a firm due to the cost-reducing activities undertaken by its rival. In line with both Piga and Poyago-Theotoky (2005) and Mai and Peng (1999), this spillover rate is *convex* in the distance between the firms; a necessary condition for agglomeration in the current literature. There are two crucial observations to be made about the second term in the cost reduction schedule. First, whilst the spillover rate, $\beta \in (0,\alpha)$, is symmetric between the two firms, absorptive capacities are *asymmetric* as the rate of assimilation is increasing in the ability of the firm.[19] Second, the strength of this spillover effect is inversely related to the distance between two firms, with the maximum spillover occurring where the firms agglomerate.

To exemplify the contribution of the model, and further examine the importance of location based spillovers, we also consider a benchmark case where spillover effects are *not* location based. In this generic spillover case, the cost-reduction functions take the following forms:

$$\begin{aligned} \varphi_A(I_A) &= I_A + \beta I_B \\ \varphi_B(I_B) &= \alpha I_B + \alpha\beta I_A \end{aligned}$$

These functions offer a useful comparison because they allows for similar asymmetries - both in terms of costs and absorptive capacities - to be realised as those in the location based case. Thus, they allow for some insight into how endogenising the spillovers between firms may impact upon the final product market and whether agglomeration is ever optimal when the firms are asym-

[19] At first, the assumption that $\beta \in (0,\alpha)$ seems intuitively unappealing. However, it simply states that a more able firm cannot get *more* from a rival's research than the rival itself.

metric.

Finally, we make one additional, and crucial, assumption.

Assumption 1. $t > \frac{16}{27}(1 - \frac{16}{25}\alpha\beta)^2$

This assumption is made for two reasons. First, this assumption ensures that the results presented here are derived such that both firms remain active in the final product market. In essence, it is an Eaton and Lipsey (1978) "no mill price undercutting" assumption that is common within the spatial competition literature.[20] When such an assumption is not met, no pure strategy Nash equilibria exist as *all* location equilibria are undermined (Ziss, 1993). The reason for such a result is that, were this assumption to be undermined, one firm would generate a sufficiently large *ex post* cost differential that the firms' incentives at the location stage become opposite extremes. That is, two contradictory agglomeration strategies emerge: *i*) the *ex post* efficient firm wishes to completely agglomerate to minimise the costs of entry deterrence; and *ii*) the *ex post* efficient firm "gives up", invests nothing and tries to keep maximal differentiation by "running away" (Ziss, 1993; Matsumura and Matsushima, 2009). In fact, these contradictory strategies lead to maximal and minimal differentiation appearing with equal probability, or entry deterrence being successful with probability 1/2. However, it is apparent that the existence of strategic entry deterrence completely undermines one firm's incentive to agglomerate. Consequently, in order to examine whether it is ever optimal for asymmetric firms to agglomerate, it is necessary to remove the possibility of strategic entry deterrence, or assume that both firms remain active in the final product market.[21] Second, this assumption enables a direct comparison between the equilibrium investment and location decisions derived under the generic and location based spillover regimes.

The game proceeds as follows: In the first stage, firms select their investment levels, $I_N \in [0, \infty)$ simultaneously; in the second stage, each firm selects its location, $n \in \mathbb{R} \; \forall \; N \in \{A, B\}$, simultaneously. Finally, in the third stage,

[20] For example, both Ziss (1993) and Matsumura and Matsuhima (2009) make similar assumptions when examining the impact of cost asymmetries on linear spatial competition.

[21] Given that the consequence of removing this assumption has already been well documented by both Ziss (1993) and Matsumura and Matsushima (2009), its omission is inconsequential.

firms select their prices, $p_N \in [c_N, \infty)$, simultaneously. We solve the game using backward induction.

3.3 Price Stage

A consumer, located at x, is indifferent between purchasing either good A or good B if and only if

$$s - p_A - t(x-a)^2 = s - p_B - t(b-x)^2 \tag{3.3}$$

As $a < b$ by assumption, the total demand for good A is given by all consumers located to the left of x and demand for good B is the residual demand, $1-x$. Solving equation (3.3) yields the demand functions:

$$\begin{aligned} D_A &= x = \frac{p_B - p_A}{2t(b-a)} + \frac{(a+b)}{2} \\ D_B &= 1 - x = \frac{p_A - p_B}{2t(b-a)} + \frac{(2-a-b)}{2} \end{aligned}$$

Taking these demand functions as given, firm N's profits are given by the expression:[22]

$$\pi_N = (p_N - c + \varphi_N)D_N - C_N$$

Firms simultaneously select prices, $p_N \in [c_N, \infty)$, to maximise profits, taking location and investment choices as given. Simultaneously solving the first order conditions with respect to own price for each firm yields equilibrium prices given by

$$p_A = \frac{3c - 2\varphi_A - \varphi_B + t(b-a)(2+a+b)}{3} \tag{3.4}$$

$$p_B = \frac{3c - \varphi_A - 2\varphi_B + t(b-a)(4-a-b)}{3} \tag{3.5}$$

From the second order conditions, it is possible to observe that a subgame perfect Nash equilibrium in prices always exists, regardless of the locations and

[22] For notational purposes, as I_N is known at this stage we abbreviate $\varphi_N(I_N)$ and $C_N(I_N)$ to φ_N and C_N respectively.

the *ex post* cost disparity between the two firms.[23] Furthermore, the signs of the components within p_N are all as one would expect, with prices increasing in the *a priori* unit cost and decreasing in the cost reduction efforts of both firms. The reduced form profit functions for both firms are then:[24]

$$\pi_A = \frac{[\varphi_A - \varphi_B + t(b-a)(2+a+b)]^2}{18t(b-a)} - C_A \tag{3.6}$$

$$\pi_B = \frac{[\varphi_B - \varphi_A + t(b-a)(4-a-b)]^2}{18t(b-a)} - C_B \tag{3.7}$$

where $C_N = \frac{1}{2}I_N^2$ and φ_N, $N \in \{A, B\}$, is given by equations (3.1) and (3.2).

3.4 Location Stage

In this section we examine the location decisions of both firms under generic and location based spillovers before comparing the two cases.

3.4.1 Benchmark: spillovers are independent of location

In this case, the equilibrium locations can easily be demonstrated to be

$$a_g = \frac{(1-\alpha\beta)I_A - (\alpha-\beta)I_B}{3t} - \frac{1}{4} \tag{3.8}$$

$$b_g = \frac{(1-\alpha\beta)I_A - (\alpha-\beta)I_B}{3t} + \frac{5}{4} \tag{3.9}$$

where the subscript g represents the optimum location values for the generic spillover case.

Within this framework, we are able to observe a number of comparative statics.

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[23] The relevant second order conditions are given by

$$\frac{\partial^2 \pi_N}{\partial p_N^2} = -\frac{1}{t(b-a)} < 0 \;\forall\; N \in \{A, B\}$$

Thus, profits are maximised for all location pairs where $b > a$ as we assume.

[24] For notational purposes, as I_N is known at this stage we abbreviate $\varphi_N(I_N)$ and $C_N(I_N)$ to φ_N and C_N respectively, $N \in \{A, B\}$.

Remark 10 *In the generic spillover case, firm $N \in \{A, B\}$ moves towards its rival if it increases it own investment and away if a rival does so. Moreover, as the firms become increasingly symmetric, $\alpha \rightarrow 1$, the firms' locations become increasingly symmetric. The impact of the spillover rate and transport costs are ambiguous for both firms. Formally, for firm A*[25]

$$
\begin{aligned}
\frac{\partial a_g}{\partial I_{Ag}} &= \frac{1-\alpha\beta}{3t} > 0 \\
\frac{\partial a_g}{\partial I_{Ag}} &= \frac{\beta-\alpha}{3t} < 0 \\
\frac{\partial a_g}{\partial \alpha} &= -\frac{\beta I_{Ag} + I_{Bg}}{3t} < 0 \\
\frac{\partial a_g}{\partial \beta} &= -\frac{1}{3}\frac{\alpha I_{Ag} - I_{Bg}}{t} \\
\frac{\partial a_g}{\partial t} &= -\frac{1}{3}\frac{(\alpha-\beta)I_{Bg} - (1-\alpha\beta)I_{Ag}}{t^2}
\end{aligned}
$$

In many respects, remark 10 is as one would expect. First, as a firm invests more, or its rival less, this *increases* the cost disparity between the firms and makes it *relatively* more efficient. This creates an incentive for the firm with relatively lower unit costs to locate more *aggressively* by moving towards its rival to secure a greater market share and larger profits. In turn, this induces the rival firm to "run" (move away from the more efficient firm) in order to mitigate rapidly escalating price competition and preserve some of its market share. Second, as the firms become increasingly symmetric, $\alpha \rightarrow 1$, the weaker firm becomes an increasingly able rival. At all levels of investment, increasing α *reduces* the cost disparity and leaves the efficient firm less able to "push" its rival away. In fact, once the firms are identical, a symmetric location equilibrium obtains, *à la* Lambertini (1997) and Tabuchi and Thisse (1995), such that $(a_g, b_g) = (-\frac{1}{4}, \frac{5}{4})$.[26] Finally, the ambiguity of the transport cost and spillover rate equations is driven by the fact that they affect *both* firms. Moreover, the impact that they have on both firms is dependent on the prevailing investment levels of both firms. Therefore, no general result

[25] Given the identical functional forms of a_g and b_g, the comparative statics are identical for both firms.

[26] This symmetric equilibrium result is, in part, driven by the fact that once $\alpha = 1$ the firms will also invest symmetrically.

obtains.[27]

Whilst *how* the firms move given changes to the initial parameters is important, the next remark observes an equally important result.

Remark 11 *Regardless of the investment levels, the relative abilities and the spillover rate, the distance between firms A and B is constant. More precisely,*

$$b - a = \frac{3}{2}$$

Proof. *The proof is trivial and omitted.* ■

This result states maximal differentiation *always* obtains in the generic spillover case, or the comparative statics are *identical* for both firms: if one firm moves closer to its rival, the rival moves away by the *same* amount. In fact, this result offers additional support for a result that has frequently obtained within linear spatial models with unbounded location spaces: maximal product differentiation in an unbounded linear space occurs where firms maintain a distance of 3/2 from each other (Tabuchi and Thisse, 1995; Lambertini, 1994, 1997 and 2002). Furthermore, Matsumura and Matsushima (2009) find that this result is robust to sufficiently small, exogenously determined cost asymmetries.[28] In all of the aforementioned examples, the cause of this level of differentiation is rapidly intensifying price competition exacerbated by even small moves towards a rival firm, which acts as a centrifugal force keeping the firms apart. Furthermore, the generic spillover specification offers *no* incentive to agglomerate, with spillovers being "manna from heaven" regardless of where the firms locate, and so there is no centripetal force incentivising the firms to locate closer to one another. Consequently, it is intuitively appealing that this commonly observed result would also appear here.

[27] Despite being ambiguous from the point of view of the location stage, once equilibrium investments are accounted for we observe that both are strictly negative. Formally,

$$\begin{aligned}\frac{\partial a_g}{\partial \beta} &= \frac{2}{9}\frac{(1-\alpha)(1+\alpha)\left[16(1-\alpha\beta)(\beta-\alpha)-27t\beta\right]}{t\left[8(1+\alpha^2)(1+\beta^2)-32\alpha\beta-27t\right]^2} < 0\\ \frac{\partial a_g}{\partial t} &= -\frac{6(1-\alpha)(1+\alpha)(1-\beta)(1+\beta)}{t\left[8(1+\alpha^2)(1+\beta^2)-32\alpha\beta-27t\right]} < 0\end{aligned}$$

[28] Assumption 1 ensures that the cost differentials derived here are sufficiently small that a similar result obtains.

3.4.2 When spillovers depend on locations

Even though cost reducing investments are undertaken in the first stage, the firms' locations now determine the extent of spillover benefit that firm $N \in \{A, B\}$ can reap from its rival. Using the equations (3.1) and (3.2), the equilibrium locations are given by

$$a_l = \frac{1}{75}\frac{(25 - 22\alpha\beta)I_A - (25\alpha - 22\beta)I_B}{t} - \frac{1}{4} \tag{3.10}$$

$$b_l = \frac{1}{75}\frac{(25 - 22\alpha\beta)I_A - (25\alpha - 22\beta)I_B}{t} + \frac{5}{4} \tag{3.11}$$

where the subscripts l represent the optimal location when there is location based spillover effects.

Interestingly, the comparative statics are similar to the previous case.

Remark 12 *In the location based spillover case, firm $N \in \{A, B\}$ moves towards its rival if it increases its own investment and away if a rival does so. Moreover, as the firms become increasingly symmetric, $\alpha \to 1$, the firms' locations become increasingly symmetric. The impact of the spillover rate and transport costs are ambiguous for both firms. Formally, for firm A*

$$\begin{aligned}
\frac{\partial a_l}{\partial I_{Al}} &= \frac{(25 - 22\alpha\beta)}{75t} > 0 \\
\frac{\partial a_l}{\partial I_{Bl}} &= -\frac{(25\alpha - 22\beta)}{75t} < 0 \\
\frac{\partial a_g}{\partial \alpha} &= -\frac{22\beta I_{Al} + 25 I_{Bl}}{75t} < 0 \\
\frac{\partial a_l}{\partial \beta} &= -\frac{22}{75}\frac{\alpha I_{Al} - I_{Bl}}{t} \\
\frac{\partial a_l}{\partial t} &= -\frac{1}{75}\frac{(25 - 22\alpha\beta)I_{Al} - (25\alpha - 22\beta)I_{Bl}}{t^2}
\end{aligned}$$

The intuition behind these results is identical to those of the benchmark case. Any change that makes a firm *relatively* more efficient *ex post*, which tips the balance of competition in their favour, drives that firm to locate more aggressively and its rival less so.

However, as the next remark demonstrates, agglomeration is never supported in equilibrium.

Remark 13 *In the location based spillover model maximal differentiation always obtains, or*

$$b - a = \frac{3}{2}$$

On the face of it, this result implies that regardless of the additional benefits that the firms could reap through (partial) agglomeration, this *never* happens. More surprising still is the fact that this holds true for even symmetric firms, where $\alpha = 1$. In fact, this result, it appears is robust to simple changes in the specification of the model, such as: *i*) allowing the *ex ante* weaker firms ability to complete its own research and absorb spillover effects to differ; *ii*) changing the specification of the spillover function; and *iii*) allowing spillovers, if firms were to agglomerate, to become infinite.[29]

---

[29] For completeness, we offer some results here.

First, under the original specification, allowing the *ex ante* weaker firm to differ in its ability to complete its own research and absorb spillovers implies

$$\varphi_B(I_B) = a_1 I_B + \frac{a_2 \beta I_B}{1 + b - a}$$

where $a_1 \geq a_2$. In this case, we obtain

$$\begin{aligned} a &= \frac{1}{75}\frac{I_A(25 - 22a_2\beta) - I_B(25a_1 - 22\beta)}{t} - \frac{1}{4} \\ b &= \frac{1}{75}\frac{I_A(25 - 22a_2\beta) - I_B(25a_1 - 22\beta)}{t} + \frac{5}{4} \end{aligned}$$

where the distance between the firms is constant at $\frac{3}{2}$.

Second, we adopted a more simplistic spillover specification:

$$\begin{aligned} \varphi_A(I_A) &= I_A + \beta(1 + a - b)I_B \\ \varphi_B(I_B) &= \alpha_1 I_B + \alpha_2\beta(1 + a - b)I_B \end{aligned}$$

In this case

$$\begin{aligned} a &= \frac{2I_A(2 - 5\alpha_2\beta) - 2I_B(5\beta - 2\alpha_1)}{3t} - \frac{1}{4} \\ b &= \frac{2I_A(2 - 5\alpha_2\beta) - 2I_B(5\beta - 2\alpha_1)}{3t} + \frac{5}{4} \end{aligned}$$

Again, it is trivial to check that the distance between the firms remains constant at $\frac{3}{2}$.

Finally, consider a functional form in which the spillover benefit would be *infinite* were the firms to locate next to one another,

$$\varphi_A(I_A) = I_A + \frac{\beta I_B}{b - a}$$

Under this specification, as the firms locate closer together we would expect them to benefit

Whilst this result appears to be robust to a variety of changes in specification of the spillover function, and consequently quite surprising, it should be noted that this result is not unexpected. Whilst there are clearly benefits to agglomeration, by increasing the amount of knowledge absorbed from a rival firm and subsequently lower production costs, there is a simple and intuitive explanation for the lack of agglomeration. The model presented here is very much of the style used by that of Matsumura and Matsushima (2009) except, in this case, the firms endogenously select the *ex post* cost asymmetry *before* selecting their locations and prices. This means that the ferocity of price competition, well documented in the linear spatial literature, remains in the location stage and acts as a centrifugal force.[30] Consequently, the fact that the firms choose not to agglomerate - for any specification of the spillover function - simply implies that the cost of exacerbating price competition *always* dominates the benefit of agglomeration. Indeed, this explanation would also explain why agglomeration is not optimal for symmetric firms. Therefore, it would appear that, where firms choose their costs *prior* to deciding on their locations, agglomeration is optimal, and it is this result is robust to changes

from rapidly increasing cost reductions. However, despite the tweak in functional form and different location equilibria, the constant distance equilibrium still obtains. Formally,

$$
\begin{aligned}
a &= \frac{(1-2\alpha\beta)I_A - (\alpha - 2\beta)I_B}{3t} - \frac{1}{4} \\
b &= \frac{(1-2\alpha\beta)I_A - (\alpha - 2\beta)I_B}{3t} + \frac{5}{4}
\end{aligned}
$$

This suggests that, within this specification, the fierceness of price competition is sufficiently strong to keep them apart.

Second, a more simplistic spillover specification - *à la* Piga and Poyago–Theotoky (2005) and Sun (2013) was used:

$$
\begin{aligned}
\varphi_A(I_A) &= I_A + \beta(1+a-b)I_B \\
\varphi_B(I_B) &= \alpha_1 I_B + \alpha_2\beta(1+a-b)I_A
\end{aligned}
$$

In this case

$$
\begin{aligned}
a &= \frac{2I_A(2-5\alpha_2\beta) - 2I_B(5\beta - 2\alpha_1)}{3t} - \frac{1}{4} \\
b &= \frac{2I_A(2-5\alpha_2\beta) - 2I_B(5\beta - 2\alpha_1)}{3t} + \frac{5}{4}
\end{aligned}
$$

Again, the distance between the firms remains constant at $\frac{3}{2}$.

[30] See D'Aspremont *et al* (1979), Tabuchi and Thisse (1995) and Lambertini (1994; 1997; and 2002).

in the specification of the spillover function.[31]

Despite the similarities between the two spillover regimes, the following proposition demonstrates that the firms' locations do depend upon the spillover type.

Proposition 14 *Regardless of the relative abilities of the firms and the initial spillover rate, the high ability firm locates more aggressively. Formally, where firm A is the high ability firm*

$$\begin{aligned} a_l &> a_g \\ b_l &> b_g \end{aligned}$$

Proof. *See Appendix 7.1* ■

Proposition 14 observes that firm A, the *ex ante* high ability firm, locates more aggressively when spillovers are location based. In both spillover cases, the low cost firm is able to locate more centrally by using its cost advantage and the threat of fierce price competition to drive its rival away. However, when spillovers are location based, this gives the more able firm even *more* power: by moving towards the less able firm it threatens to to obtain an even greater cost disparity by absorbing more information than can be assimilated by its inefficient rival. This creates an added incentive for the high ability firm to locate more aggressively. In contrast, the low ability firm is now *more* fearful of its rival, in particular the high ability firm's ability to act in an increasingly predatory manner. It is this fear that, in turn, induces it to locate further away from the market than it otherwise would have done to mitigate price competition and spillovers between the firms.

This aggression is also apparent in how the firms respond to changes in investment as the following proposition demonstrates.

Proposition 15 *Regardless of the firms' relative abilities and the spillover rate, the firms are more responsive to changes in investment levels when spillovers*

[31] As we have already mentioned, it is not trivial to reverse the order of moves. Consequently, it is has not been possible to check that it is, indeed, the order of moves that is driving this result. However, given the results presented here, it would appear that this is the most likely cause.

are location based. Formally,

$$\frac{\partial a_l}{\partial I_{Al}} > \frac{\partial a_g}{\partial I_{Ag}}$$
$$\frac{\partial a_l}{\partial I_{Bl}} < \frac{\partial a_g}{\partial I_{Bg}}$$

Proof. *See Appendix 7.2* ■

Comparing the investment comparative statics implies that an increase in own investment or a decrease in a rival's is met by a *greater*, *more aggressive*, location response when spillovers are location based. As noted above, when spillovers are location based, increasing one firm's investment induces it to locate closer to a rival because, by doing so, it threatens to further reduce its production costs whilst also increasing price competition. It is this added research stealing incentive, that does not exist in the generic spillover case, that incentivises the firm to locate more aggressively in response to a changes in its *own* investment levels. In contrast, this additional threat of predation makes the rival firm more fearful of this extra investment and induces them to locate further from the market than they otherwise would have done.

3.5 Investment Stage

In this section we examine the investment decisions of both firms under generic and location based spillovers. In the generic spillover case, the reduced form profit functions are given by

$$\pi_{Ag} = \frac{1}{108}\frac{[4I_A(1-\alpha\beta) - 4I_B(\alpha-\beta) + 9t]^2}{t} - \frac{1}{2}I_A^2 \qquad (3.12)$$
$$\pi_{Bg} = \frac{1}{108}\frac{[4I_A(1-\alpha\beta) - 4I_B(\alpha-\beta) - 9t]^2}{t} - \frac{1}{2}I_B^2 \qquad (3.13)$$

Solving the relevant first order conditions obtains

$$I_{Ag} = \frac{2}{3}\frac{(1-\alpha\beta)(16(\alpha-\beta)^2 - 27t)}{8(1+\alpha)^2(1+\beta)^2 - 32\alpha\beta - 27t} \qquad (3.14)$$
$$I_{Bg} = \frac{2}{3}\frac{(\alpha-\beta)(16(1-\alpha\beta)^2 - 27t)}{8(1+\alpha)^2(1+\beta)^2 - 32\alpha\beta - 27t} \qquad (3.15)$$

Similarly, in the location based spillover case, the reduced form profits are given by

$$\pi_{Al} = \frac{1}{67500}\frac{[4I_B(25\alpha - 16\beta) - 4I_A(25 - 16\alpha\beta) - 225t]^2}{t} - I_A^2 \quad (3.16)$$

$$\pi_{Bl} = \frac{1}{67500}\frac{[4I_B(25\alpha - 16\beta) - 4I_A(25 - 16\alpha\beta) + 225t]^2}{t} - I_B^2 \quad (3.17)$$

In this instance, the equilibrium investment levels are given by

$$I_{Al} = \frac{2}{75}\frac{(25 - 16\alpha\beta)[16(25\alpha - 16\beta)^2 - 16875t]}{8(256\beta^2 + 625)(1 + \alpha^2) - 12800\alpha\beta - 16875t} \quad (3.18)$$

$$I_{Bl} = \frac{2}{75}\frac{(25\alpha - 16\beta)[16(25 - 16\alpha\beta)^2 - 16875t]}{8(256\beta^2 + 625)(1 + \alpha^2) - 12800\alpha\beta - 16875t} \quad (3.19)$$

As the following proposition demonstrates, both spillover regimes yield *similar* results in general.

Proposition 16 *Regardless of the spillover regime, and for all relative abilities, spillover rates and transport costs, we observe that the ex ante high ability firm, firm A, becomes the dominant firm in all important respects. More formally, for* $x \in \{g, l\}$

1. $I_{Ax} > I_{Bx} > 0$;

2. $\varphi_{Ax}(I_{Ax}) > \varphi_{Bx}(I_{Bx})$;

3. *The ex post cost differential ensures a pure strategy Nash equilibrium in the location stage; and*

4. $\pi_{Ax} > \pi_{Bx} > 0$

Proof. *For the proof of the generic (location based) spillover case see Appendix 7.3 (7.4)* ■

Essentially, proposition 16 acts as a summary of the key results that obtain under each spillover regime. What is apparent is that the *ex ante* high ability firm becomes dominant in *all* respects: investing more, generating lower costs and obtaining strictly larger profits. Given the investments derived in equations (3.14) - (3.19) it is clear that these results are interlinked. Given its greater cost-reducing ability, firm A is induced to invest more and obtain a lower *ex post* unit cost. In turn, this creates an incentive for the firm to locate more centrally; pushing the *ex post* inefficient firm away due to the fear of fierce price competition. Consequently, the efficient firm has a greater market share and is able to use this to generate increased profits.

Whilst the *ex ante* high ability firm always becomes the dominant firm in the *ex post* product market, the following proposition demonstrates that the extent of the firm's dominance is dependent upon the initial parameters.

Proposition 17 *Regardless of the spillover regime, and for all relative abilities, spillover rates, and transport costs, the high ability firm's investment is strictly decreasing in its rival's ability and the spillover rate. Formally, for all* $x \in \{g, l\}$

$$\begin{aligned} \frac{\partial I_{Ax}}{\partial \alpha} &< 0 \\ \frac{\partial I_{Ax}}{\partial \beta} &< 0 \end{aligned}$$

In contrast, the low ability firm's investment is strictly increasing in its relative ability and increasing (decreasing) in the spillover rate if and only if the transport cost is sufficiently small (large). Formally,

$$\begin{aligned} \frac{\partial I_{Bx}}{\partial \alpha} &> 0 \\ \frac{\partial I_{Bx}}{\partial \beta} &\geq 0,\ t \leq t^{\beta x} \\ \frac{\partial I_{Bx}}{\partial \beta} &< 0,\ t > t^{\beta x} \end{aligned}$$

where $t^{\beta x}$ *is the critical value at which* $\frac{\partial I_{Bx}}{\partial \beta} = 0$.[32]

Proof. *For the proof of the generic (location based) spillover case see Appendix 7.5 (7.6)* ■

Proposition 17 observes that, regardless of the spillover regime, the firms invest in a similar way. First, as the firms become relatively more symmetric, $\alpha \to 1$, the investment level of firm A falls whilst firm B's rises. Quite simply,

[32] At present we cannot say anything about the relationship between $t^{\beta g}$ and $t^{\beta l}$.

when the firms become increasingly symmetric, the investment levels converge or, when the *ex ante* inefficient firm becomes relatively *less* inefficient, it makes it harder for the more able firm to dominate a less able firm. Consequently, the more able firm is induced to invest less, and its rival more, until the firms are symmetric. Moreover, once the firms are symmetric, and investment levels identical, they locate *symmetrically* around the centre of the market, $(a, b) = (-\frac{1}{4}, \frac{5}{4})$; the classic result of Lambertini (1997) and Tabuchi and Thisse (1995). Second, as the spillover rate increases, a more able firm's investment falls because additional investment further reduces the cost of its rival. For large transport costs, a similar result obtains for the *ex ante* less able firm. However, when the transport cost is small, it makes it increasingly easy for consumers to switch "brands". Consequently, whilst firm A reduces its investment, this gives firm B an opportunity to increase its market share; increasing its investment levels.

As the following proposition demonstrates, despite the similarities between the cases, investment levels are not symmetrical across the spillover regimes.

Proposition 18 *Irrespective of the initial parameter values, the more able firm always invests strictly more in the location based case, or* $I_{Al} > I_{Ag}$*. However, for all relative abilities, spillover rates and transport costs, there exists a critical value of* t*, given by* t^y*, that lies within the relevant range of* t*, such that for all* $t > t^y$*,* $I_{Bl} > I_{Bg}$*. For all* $t < t^y$*,* $I_{Bg} > I_{Bl}$*.*
Proof. *See Appendix 7.7* ■

For firm A, proposition 18 highlights that it invests *more* aggressively when the firms face location based spillovers. Moreover, this result *does not* disappear when the firms are symmetric.[33] This suggests that even with symmetric firms, who will invest identical amounts, firm A is induced to invest *more* despite the fact that the firms do not locate closer to one another.

The question is then, why does firm A invest more aggressively when facing location based spillovers? The answer is simple. When firms are asymmetric, $\alpha \in (0, 1)$, the *ex ante* more able firm faces an additional benefit from investing heavily in the first stage: the ability to locate closer to a rival and assimilate *more* knowledge that it gives away. This additional predatory threat makes

[33] $I_{Al} - I_{Ag}|_{\alpha=1} = \frac{6}{25}\beta$

the more able firm a more potent competitor and enables it to push its rival further from the market. In turn, it is induced to invest more. Yet, even when firms are symmetric, $\alpha = 1$, the *potential* benefits from investing aggressively remain. Therefore, even under the symmetric case, firm A invests more heavily than it would have done in the generic spillover case.[34]

In the case of firm B, it appears that it will only invest more under location based spillovers if the level of consumer heterogeneity is sufficiently large. As consumers become less willing to travel to purchase a good, it becomes harder for a more able rival firm to "steal" them using price alone. Thus, even when firm A invests heavily, the less able firm is, to some extent, protected. With the more able firm less able to use predatory behaviour, the *ex ante* weaker firm is more confident to invest and does so. However, when transport costs are low, consumers are more fickle and more likely to switch between firms. In these circumstances, firm A's predatory actions are more potent and pose a much greater threat to its less able rival. Consequently, firm B's strategy changes to one of survival. In order to mitigate fierce price competition and minimise spillovers it acts "soft", reducing its investment level and locating further from the market. Consequently, a less able firm's situation is much more volatile and dependent upon the homogeneity of consumers within a market: the *more* homogeneous the consumers, the more likely it is that it will have to "run".

3.6 Conclusion

This paper examined whether the incentives for firms to agglomerate, whilst facing location based spillovers, survive the existence of asymmetric firms. More importantly, we extend the existing literature by allowing firms to differ, simultaneously, in two ways: *i*) the firms are *a priori* heterogeneous with respect to their innovative abilities; and *ii*) they have asymmetric absorptive capacities. Given the potential disparities that may arise under this setting, we asked: is it reasonable for asymmetric firms to agglomerate.

Our results, in one crucial respect, go beyond this question. With location based spillovers and an investment-location-price specification, the firms never find it optimal to (partially) agglomerate and maximal differentiation always

[34] In many ways, this is a classic "prisoner's dilemma".

obtains. In fact, not even symmetric firms find it optimal to locate closer to one another. This result is driven by the ferocity of price competition that acts as a centrifugal force, pushing the firms away from one another. In other words, the classic d'Aspremont *et al* (1979) result of maximal differentiation holds. However, it is likely that once the order of moves is reversed, or firms locate before innovation, the incentive to agglomerate returns.

Our results also complement the existing literature regarding asymmetric firms and spatial competition. It appears that location based spillovers, combined with asymmetric firms, leads to increasingly asymmetric outcomes. Were the firms to face no spillovers, the classic results of Ziss (1993) and Matsumura and Matsushima (2009) would surely obtain. Yet, the addition of *asymmetric absorptive capacities* compounds the situation. Now, as the firms locate closer to one another, the more able firm assimilates *more* knowledge than it leaks to its inefficient rival. Consequently, this enables it to become more threatening and push its rival further from the market. In turn, this incentivises the firm to *increase* its initial research efforts because of the greater rewards, particularly market share and profits, that it is able to obtain from doing so. In contrast, the weaker firm's decisions are more reliant upon the level of consumer heterogeneity. With more homogeneous consumers, the weaker firm faces a greater threat of predatory behaviour. To mitigate this increased threat, and reduce price competition, the firm invests less and moves further from the market. However, when consumers are less "fickle", this threat is reduced and the firm is willing to invest *more*.

Whilst this paper set out to examine whether agglomeration is undermined by the existence of asymmetric firms, we cannot say that this is the case: as even symmetric firms would never optimally agglomerate, our results are inconclusive. Future work should continue to examine this important issue, especially as clustering is a commonly observed phenomenon. However, the best way to do this is unclear. Whilst some work has found a theoretical support for agglomeration within a spatial setting, these often make assumptions that are objectionable (Piga and Poyago-Theotoky, 2005; Mai and Peng, 1999). Yet, it is apparent that without such assumptions it may not be possible to determine an equilibrium outcome if firms are able to choose a location prior to investing in cost reduction (Piga and Poyago-Theotoky, 2005). Therefore, in spite of being the most appealing setting for examining agglomeration, a spatial setting may not be suitable for such research.

It should also be noted that one area of criticism for this work is the assumption that the market consists of only *two* firms when, in reality, innovative markets would consist of more firms. However, this is a conscious decision and one that has been made to keep this paper in line with the majority of the linear spatial literature. Indeed, the dearth of oligopolistic linear spatial models suggests: *i*) this is a non-trivial addition; and/or *ii*) this oversight is not significant.[35] In fact, the existing literature that does extend the basic model beyond two firms suggests that it is *unlikely* our results would be robust to such a change. Using quadratic transport costs and a bounded location space, Brenner (2005) finds the addition of firms undermines the principal of maximal differentiation. Simply, the "introducing a market boundary leads to asymmetry between firms" with boundary firms locating more centrally - partially agglomerating - to take advantage of their monopoly position in the corners, allowing these firms to receive higher prices and yield larger profits than more centrally located firms (Brenner, 2005: p.862). Similar agglomeration results have also been found by Economides (1993), using linear transport costs, and Andersen and Nevin (1991), where firms compete in quantities not prices in the final stage, though, in both of these cases, the firms are symmetric and the location space is bounded. Whilst it would appear that the addition of firms may generate agglomeration in this model, such an assertion is not possible. Given the sensitivity of linear spatial models to their initial specification, it is not a priori obvious how the addition of more *asymmetric* firms, that are able to locate outside of the city, would affect the results presented here. However, it is *unlikely* that our results would remain unchanged, and this extension is left to future research.

[35] As an example of this dearth of literature, Biscaia and Mota's (2012) extensive review of the linear spatial literature highlights only *two* oligopolistic extensions.

Chapter 4

Who Becomes the Winner? Effects of Venture Capital on Firms' Innovative Incentives - A Theoretical Investigation

4.1 Introduction

It has become established in the empirical literature that venture capital (henceforth VC) plays an important role in the promotion of innovation at industry level and the professionalisation of firms at micro-level (Da Rin *et al* (2013); Dessí and Yin (2010)). In spite of consistent empirical evidence that supports this VC-to-success link at the micro level, very little has been done to give a theoretical insight into an important and, as yet, still unanswered question: *how* does VC spur such success? This question is not simply theoretically interesting but has important implications for public policy in fostering an environment conducive to innovation. As Gompers and Lerner (1999, 2001) observe, some of the most successful high-tech innovators in the US, such as Microsoft and Apple Computers, have benefited from VC backing. Therefore, understanding the mechanisms behind *how* and *why* certain venture-backed firms are, apparently, more successful is important and has, to the authors' knowledge, been ignored within the majority of the literature. We diverge from the established literature by examining the effects from the *firms'* perspectives and aim to answer three important questions: *i*) what impact does VC have on a firm's incentives to invest in innovation?; *ii*) how do rival, non-VC-backed firms respond?; and *iii*) does the prospect of receiving VC funding in the future, and its associated benefits, spur innovation *ex ante*?

Before examining the relevant literature, we note that our review of the existing work is exclusively focused on the empirical literature. Whilst theoretical VC literature does exists, it is generally focused on optimal contract theory. An excellent review of all the theoretical literature can be found in Da Rin *et al* (2013) but, for the sake of brevity, we simply acknowledge its existence here. Yet, this dearth of theoretical literature should not be a problem because, by examining the relevant empirical literature, relevant theoretical ideas should be caught by hypothesis testing.

At the industry level, there exists a long established strong, positive relationship between VC and innovation.[1] However, at firm level, VC appears to have no link to innovation *per se* but does appear to have other, real impacts on a firm's potential for success. Nonetheless, the results offer some interesting insight into the potential benefits to firms of receiving the backing of a venture capitalist. Hellman and Puri (2000), using a selection of survey and commercially available data for 173 hand-picked Silicon Valley start-ups, observe that firms pursuing an innovator strategy are more likely to obtain VC funding and see a reduction in time needed to bring a product to market. Most intriguing, however, is their assertion that, "firms are more likely to consider VC a milestone event than obtaining financing from some other kind of financier" (Hellman and Puri, 2002, p.962). Though a reason for this is not given, all three of these findings are consistent with a venture capitalist possessing at least one of two skills: *i*) a higher ability to seek out innovative firms *ex ante*; or *ii*) offering benefits beyond those of traditional finance methods through the use value-adding services *ex post*.

Other work has found remarkably similar results. Puri and Zarutskie (2012), using US firm level data between 1981-2005, compare VC- and non-VC-backed firms to examine relative growth rates. Whilst the results suggest

[1] Given our focus on a micro level model, we do not discuss industry level results here. However, for more information see Kortum and Lerner (2000), Hirukawa and Ueda (2008), Hirukawa and Ueda (2011), Popov and Roosenboom (2009), Popov and Roosenboom (2012), Faria and Barbosa (2013) and Geronikolau and Papachistou (2012).

that VC may be irrelevant in the creation of new firms (accounting for only 0.11% of new firms within the sample), they note consistently faster growth though this does not necessarily transfer to profitability. Peneder (2010), examining the impact of VC on 132 Austrian firms, founds that such firms grew 70% quicker than equivalent non-VC-backed firms, though this growth did not extend to innovation. Chemmanur *et al* (2011), using US census data, adds that total factor productivity (TFP) is also an important signal to venture capitalists and is significantly higher both pre- and post-VC compared to non-funded firms. Da Rin and Penas (2007) find remarkably similar results using Dutch firm level data. Offering some additional insight into the growth of TFP they suggest venture capitalists push the firms they back into adopting more in-house R&D practices as well as investing in absorptive capacity.

To compare whether *ex ante* or *ex post* effects are more apparent, both Kaplan *et al* (2009) and Baum and Silverman (2004) examine the factors that are important for a firm to possess in order to receive VC backing. Kaplan *et al* (2009) examine whether venture capitalists are more likely to back "the horse" (the firm's business idea) or "the jockey" (the management team). They observe that whilst VC-backed firms do, indeed, grow much faster than those that did not receive such funding, the core business ideas also remained relatively consistent in comparison to management. Moreover, whilst management may make a firm more attractive, these are not related to post-VC performance.[2] In similar work, Baum and Silverman (2004), using data on 204 Canadian biotechnology start-ups and 407 incumbents, examine whether venture capitalists "pick" (*ex ante* selection) or "build" (*ex post* mentoring) their chosen firms. They find a combination of both effects with venture capitalists more likely to invest in firms that have demonstrated some innovation (alliance participation or patents) and, thereafter, they perform better.

In fact, these result should not be a surprise given the active role that venture capitalists have been empirically demonstrated to play within a firm. As Bottazzi *et al* (2008, p.489) astutely stated, "the VC literature identifies

[2] In a related result, Wasserman (2003) finds that manager turnover is more likely when managers have succesfully developed a product rather than when they have performed poorly. The reason for this is that, once a firm has become a success, the skills that made the initial CEO so succesful in developing a product or idea may be less important once the firm faces a different scenario.

a broad role for the investor, which goes beyond the simple provision of finance. Venture capitalists may engage in a number of value-adding activities, including monitoring, support, and control. Those activities are largely non-contractible, yet may have real consequences". Monitoring is perhaps the most obvious, and empirically tested, of all of these adding-value services. Lerner's (1995) examination of biotechnology firms finds monitoring and control, as measured by venture capitalist board representation, were increasing in the need for oversight, as measured by CEO turnover. Gompers (1995) finds a similar relationship between agency costs and the monitoring within a sample of 794 VC-backed firms. More surprisingly, it appears that venture capitalists focus more investment on early-stage projects for which information asymmetries are more pronounced.[3]

However, monitoring a firm's activity is far from a venture capitalists only value-adding service. Hellman and Puri (2002), analysing data on 170 young high-tech Silicon Valley start-ups, examine the impact of VC on the development of new firms. Similar to Chemmanur *et al* (2011), the results suggested that a venture capitalist's biggest impact was on the professionalisation of the firm. This impact is firm wide with benefits at both the top, by replacing the original founders with external CEOs, and at the bottom, by formulating HR policies and improving marketing strategies. Interestingly, this result of VC firms being more likely to replace founder CEOs with external candidates is supported by Wasserman (2003) who suggests founder CEOs' skills are often outstripped by the rapid success that VC-backing offers.[4] Hochberg (2012) also finds evidence of stronger corporate governance within VC-backed firms and this result is made stronger when accounting for endogeneity. Finally, Bottazzi *et al* (2008), using survey data collected from 124 VCs across Europe, note that the aforementioned benefits may, in fact, be related to the prior

[3] Dahiya and Ray (2011) observe a similar result to Gompers (1995). However, they add that venture capitalists may use staging as a screening tool to combat asymmetric information and abandon failing projects earlier.

Hoenen *et al* (2012), evaluating 1500 US based technology firms, find that venture capitalists use other signals, for example patents, to screen weaker firms and offer stronger firms more investment. After initial round funding the impact of such signals diminishes - no further funding benefits - adding weight to a screening argument.

[4] Despite the apparent benefits of venture capitalists replacing existing CEOs, Kaplan et al (2012) find no performance difference between internal and external candidates once skills are accounted for.

business experience of the venture capitalist. To summarise their results, the more business experience a venture capitalist has, the more active it is within the firm.

Before discussing our model, we highlight an important point made by Da Rin *et al* (2013). Whilst empirical work has done well to shed some light on how venture capitalists add-value, little has been done with regards to "forward-looking selection effects". Simply put, the empirical literature assumes that the firm's *ex ante* actions are passive and that venture capitalists are the driving force behind the VC-to-success relationship. From the viewpoint of the firms, this seems a little unfair but an idea that persists. For example, Caselli *et al*'s (2009) examination of 154 Italian IPOs (including 37 VC-backed firms) noted that VC was more likely to go to those firms that had already demonstrated some innovation and similar results have been demonstrated for the US (Hellman and Puri, 2000; Mann and Sager, 2007) and Germany (Engel and Keilbach, 2007). But why would such decisions by firms be passive? And wouldn't firms change their strategic decision knowing that the addition of VC-backing will improve their chances of success in the future?

To address this issue, we consider a stylised two-period, multi-stage game in which innovation is uncertain and firms are of different innovative abilities. In order to simplify proceedings, we turn the tables on the existing literature and assume that VC is exogenous or, like the firms of the empirical literature, passive. Nonetheless, we try not to lose any of the key features that VC possesses. Therefore, we assume VC funding is a package consisting of three things: *i*) an equity stake in the firm; *ii*) pecuniary funds; and *iii*) value-adding services such as monitoring, implementing formal HR procedures or improved marketing.[5] By examining both pre- and post-VC funding decisions, we analyse whether VC spurs innovation: *i*) directly after being granted; *ii*) indirectly by incentivising firms to increase initial research efforts to increase their chances of receiving VC funding (and its associated benefits); or *iii*) a combination of both. To our knowledge, this is the first paper of its kind to approach VC in this way.

We obtain a number of theoretical results that have not been observed

[5] To an extent, one can think of an increase in funding and/or value-adding services as a proxy for the quality of the venture capitalist (see Bottazzi *et al* (2008)). However, we do not believe the specification of our model enables us to read too much into this.

before, not even empirically. In the second stage, post-VC, we observe that, regardless of VC funding, "success breeds success". That is to say, we find that a good predictor of the likelihood of future success is past success: *ceteris paribus*, a firm that innovates early is more likely to develop a high quality product. Nonetheless, the addition of VC also has a profound impact on competition *directly after* it has been granted. In essence, VC tips the balance of competition in favour of the firm that receives it, regardless the firm's relative ability level. It does this by inducing the VC-backed firm to invest more and the rival firm less, improving the relative probability of success for the portfolio firm. Therefore, we suggest that the commonly held belief that VC spurs innovation is too simplistic, as it clearly damages the prospect of the firms it *does not* support. Instead, VC *only* spurs innovation amongst the "lucky", chosen few, but unambiguously suppresses innovation of non-VC-backed firms; an idea that has been overlooked in the empirical literature. However, the magnitude of this result is sensitive to the relative homogeneity between the firms. When firms are of relatively similar abilities, VC has a more pronounced impact on the composition of the final product market. In fact, it can single-handedly determine which firm is likely to be more innovative. In contrast, as firms become more heterogeneous, whilst still altering equilibrium investment levels, VC is unable to prevent the high ability firm from being the most likely innovator.

In the first stage, we observe two important results. First, firms may treat efforts as either strategic complements or substitutes, depending upon the relative sizes of expected *future* profits between subcases. When expected profits are relatively higher in the *symmetric* (duopoly) cases, the efforts of a rival are positively correlated with a firm's expected profits, inducing it to invest *more* when a rival does. In contrast, when *asymmetric* outcomes are more valuable, the firms "compete" in effort. Second, and most important, we find that VC does impact on the firm's effort choices *indirectly*, by altering their future expected payoffs. The equity stake of the firm impacts on initial efforts in two ways: *i*) it directly reduces initial efforts by reducing expected future profits; and *ii*) it indirectly increases (decreases) efforts if the firms treat efforts as strategic substitutes (complements). Thus, the equity stake is *negatively* correlated with effort in the first stage if the firms treat efforts as strategic complements, and *ambiguously* correlated if treated as strategic substitutes. The impact of pecuniary funding and venture capitalist expertise are also ambiguous. However, this ambiguity should *not* be misinterpreted as no effect. Rather, one should interpret our *indirect effect* results more broadly: given the specification, it is likely that *future* VC will have an impact on first period efforts, though it is not possible to say whether this impact is positive or negative.

The rest of the paper is as follows. In section 4.2 we specify the model in more detail. Section 4.3 analyses the benchmark, no-VC, case. In section 4.4, we examine the impact of venture capital on the firms' effort decisions. Section 4.5 concludes.

4.2 Model

We consider a two-period, multi-stage, asymmetric duopoly model in which the quality of innovation is uncertain. We assume that two firms, i and j, have asymmetric "innovative" abilities, $a_i > 0, a_j > 0$ such that $a_i \geq a_j$ i.e. firm i is of higher ability than firm j. The structure of the game can be detailed as follows:

First period

At the beginning of the first period, given the above abilities, firms invest in effort in order to develop a prototype product that can either be of high quality (q_h) or low (q_l), the actual value of which becomes known only at the *end* of the first period. The probability of discovering a certain quality of prototype depends on a firm's ability as well as on its effort level. We denote the (unconditional) probability that firm i develops a high-quality output in a certain period by φ_i^t, $t = 1, 2$. This probability then is a function of firm i's effort level e_i^t in period t as well as its initial ability a_i i.e. $\varphi_i^t = \varphi_i^t(a_i, e_i^t)$.[6] Thus the probability that a firm develops a high or low quality prototype (q_h

[6] This probability function however may change in the second period, depending upon whether the firm discovers a high quality prototype or not - see below for the description of the second period game.

or q_l) in the first period is given by

$$\begin{aligned} \Pr[q_h] &= \varphi_i^1(a_i, e_i^1) \\ \Pr[q_l] &= 1 - \varphi_i^1(a_i, e_i^1) \end{aligned} \tag{4.1}$$

where e_i^1 is firm i's effort level in period one. The following assumptions characterise the function $\varphi_i^t(a_i, e_i^t)$.

A1. $\partial\varphi_i^t(a_i, e_i^t)/\partial e_i^t > 0; \partial^2\varphi_i^t(a_i, e_i^t)/\partial\left(e_i^t\right)^2 < 0; \varphi_i^t(a_i, 0) = 0; \partial\varphi_i^t(a_i, e_i^t)/\partial a_i > 0.$

A2. $\partial^2\varphi_i^t(a_i, e_i^t)/\partial e_i^t \partial a_i > 0.$

A1 says that the probability function is strictly concave in effort, that a firm can never develop a high-quality product if it puts in no effort, and that, for a given level of effort, the more able the firm is, the greater its probability of success. Assumption A2, which states that a firm's marginal returns to effort are increasing in its ability, captures the idea that a more able firm is better able to target its effort along more effective research paths.

We assume that the marginal cost of effort, c, is constant in every period with $c > 0$. Firms choose their effort level, $e_i^1 \in [0, \infty)$, to maximise their expected profits. Output is then realised and the quality of the firms' prototypes are revealed to all players. There are now four possible scenarios to consider for the second period game:

Case (i). $\left(q_l^i, q_l^j\right)$: When both firms develop *low* quality prototypes.

Case (ii). $\left(q_h^i, q_l^j\right)$: When firm i develops high quality prototype while firm j develops low.

Case (iii). $\left(q_l^i, q_h^j\right)$: When firm i develops low quality prototype while firm j develops high.

Case (iv). $\left(q_h^i, q_h^j\right)$: When both firms develop *high* quality prototypes.

Second period

At the beginning of the second period, given the above realisation about the quality of the prototypes, firms compete again with respect to their effort (investment) levels to produce output that can either be high (Q_h) or low (Q_l). The realisation of the second period output Q is uncertain *ex ante.* The quality of output Q however, determines a firm's future as follows: if only one firm innovates (i.e. develops a high quality good) whilst its rival does not, then that firm becomes a monopolist (e.g. through the grant of some kind of a patent right) and earns a monopoly profit M in the future period whilst its rival earns zero; if both firms innovate (i.e. if both develop Q_h) then both earn duopoly profits of D_H whereas if neither innovates (i.e. produce the low quality product Q_l) then each makes a duopoly profit of D_L in the next period. Without any loss of generality, we assume that

$$M > 2D_H > 2D_L$$

Obviously, firms aspire to become monopolists at the end of the second period and choose effort levels $e_i^2 \in [0, \infty)$ to maximise their expected payoffs at a marginal cost of c.

Our model incorporates a 'learning by doing' effect in the following sense: if a firm has been successful in discovering q_h, then even without any VC backing, this puts the firm in a better position to produce Q_h in the second period. We capture this idea by assuming that the probability of success function is now conditional on the discovery of q_h i.e.

$$\begin{aligned} \Pr[Q_h] &= \mu_i(a_i, e_i^2) \text{ if } q_h \text{ in the first stage} \quad (4.2) \\ \Pr[Q_l] &= 1 - \mu_i(a_i, e_i^2) \text{ if } q_h \text{ in the first stage} \end{aligned}$$

with

$$\begin{aligned} \lambda\mu_i(a_i, e_i^2) &= \varphi_i^1(a_i, e_i^2) \quad (4.3) \\ \lambda &\in (0, 1) \end{aligned}$$

Equation (4.3) then simply states that, at any level of effort, $e_i^2 \in [0, \infty)$, a firm that has developed a high quality prototype has a strictly higher success probability.[7] Consequently, assumptions similar to the ones made in A1 and A2 also hold for $\mu_i(a_i, e_i^2)$ and are summarised by A3 (i.e. $\mu_i(a_i, e_i^2)$ is a strictly concave function of e, is increasing in a_i and shows increasing marginal return to investment with respect to a_i).

A3. $\partial\mu_i(a_i, e_i^2)/\partial e_i^2 > 0; \partial^2\mu_i(a_i, e_i^2)/\partial\left(e_i^2\right)^2 < 0; \mu_i(a_i, 0) = 0; \partial\mu_i(a_i, e_i^2)/\partial a_i >$ 0; and $\partial^2\mu_i(a_i, e_i^2)/\partial e_i^2 \partial a_i > 0$.

Now, in this model we consider the possibility that a firm can obtain backing from a venture capitalist. The presence of a venture capitalist then substantially changes the above scenario. First of all, whether a firm receives any assistance from a venture capitalist depends entirely upon the fact whether it has developed a high quality prototype (q_h) in period 1 or not. Moreover, a VC packages is only offered to a *single* firm: where only one firm has developed a high quality prototype, the VC offering goes to that firm; if both firms developed q_h in the first period then each faces equal probability of securing VC funding (which ultimately is assigned 'randomly' or on the basis of certain outside criteria that are not considered in our model). Finally, VC comes in a *package* consisting of:

1. An equity stake in the firm, s: The equity stake that is required by the venture capitalist as compensation for its risk.
2. Pecuniary funding, F: This denotes the finance offered to the firm.
3. Value-adding services, E: This denotes the additional benefits a venture capitalist offers to the firm beyond finance such as mentoring and expert advice.

The above assumptions keep our modelling of VC in line with those of Bottazzi *et al* (2008) in so far as they imply a venture capitalist plays a far broader role in the firm than traditional financing methods.

[7] Note that this assumption ensures that all the properties of φ_i^1 are also transferred to μ_i since λ is a scaler.

How does the acquisition of a VC package affect the winning firm's probability of success? With VC funding, a firm's probability of success in producing Q_h is *further* enhanced over and above the one given by $\mu_i(a_i, e_i^2)$. The probability of innovation is now also a function of the amount of funding received, F, and the value-adding services, E. We denote this function as follows:

$$\begin{aligned} \Pr[Q_h] &= \hat{\mu}_i(a_i, e_i^2) = \mu_i(a_i, e_i^2, E, F) \text{ if } q_h \textit{ and } \text{VC-backed} \\ \Pr[Q_l] &= 1 - \hat{\mu}_i(a_i, e_i^2) \text{ if } q_h \text{ and VC-backed} \end{aligned} \tag{4.4}$$

where, for any e_i^2 and a_i

$$\hat{\mu}_i(a_i, e_i^2) > \mu_i(a_i, e_i^2)$$

if E or F are positive. Consequently, assumptions similar to that given in A3 also apply here (and hence are not repeated). The following assumption now captures the specific benefits of receiving VC backing, namely, how mentoring and funding affect the probability of innovation[8].

A4. (i) $\partial\hat{\mu}_i(a_i, e_i^2)/\partial F > 0$; $(ii)\partial\, \hat{\mu}_i(a_i, e_i^2)/\partial E > 0$; and (iii) $\partial^2\hat{\mu}_i(a_i, e_i^2)/\partial e_i^2 \partial E > 0$.

A4 says that the impact of receiving mentoring and funding are strictly positive for the firm. Additionally, part (iii) of A4 highlights the indirect effect of mentoring via a firm's effort level: the more value-adding services that are offered by a venture capitalist, the better able a firm becomes at targeting its efforts and so the marginal returns to effort increase.

Finally, if a firm developed a low-quality prototype in the first period (i.e. q_l), then its probability of innovation remains exactly as is specified by the function φ_i^t i.e. it is given by $\varphi_i^2(a_i, e_i^2)$ in the second period.

The timing of the game can now be summarised as follows:

Stage 1: Start of first period. Firms choose effort levels, $e_i^1 \in [0, \infty)$ given their abilities a_i, a_j. Output is produced and the quality of the prototype $q_s, s \in \{h, l\}$, is revealed to all players. End of first period.

[8] We use the reduced form, $\hat{\mu}_i(a_i, e_i^2)$, throughout.

Stage 2: Start of second period. The VC package (F, E, s) is assigned to the winning player who then enjoys a probability of success given by $\hat{\mu}_i(.)$. If both have developed high quality prototypes then VC funding is offered to each of them with equal probability. If neither firm discovers q_h, neither receives VC backing. Players who do not receive VC funding have a probability of success given by $\varphi_i^2(.)$. Firms then invest in their effort levels. Output is realised at the end of period 2, and firms earn (future) payoffs according to their position in the market.

We solve the game using backward induction.

4.3 Benchmark: the no-VC case

In order to appreciate the impact of VC offering, we first consider the scenario where there is no possibility of receiving a VC package. If so, then the second period probability of innovation is given by (4.2).

4.3.1 Second stage equilibrium

First we compute the expected second stage profits corresponding to each of the cases (i)-(iv). Thus, the expected profit functions are

Case (i). $\left(q_l^i, q_l^j\right)$ – both firms develop *low* quality prototypes

$$\pi_{l,l}^i|_{NVC}^{t=2} = (1-\varphi_i^2)(1-\varphi_j^2)D_L + \varphi_i^2\varphi_j^2 D_H + \varphi_i^2(1-\varphi_j^2)M - ce_i^2 \;\forall\; i$$

Case (ii). (q_h^i, q_l^j) - firm i develops high quality prototype while firm j develops low

$$\begin{aligned}\pi_{h,l}^i|_{NVC}^{t=2} &= (1-\mu_i^2)(1-\varphi_j^2)D_L + \mu_i^2\varphi_j^2 D_H + \mu_i^2(1-\varphi_j^2)M - ce_i^2\\ \pi_{h,l}^j|_{NVC}^{t=2} &= (1-\mu_i^2)(1-\varphi_j^2)D_L + \mu_i^2\varphi_j^2 D_H + \varphi_j^2(1-\mu_i^2)M - ce_j^2\end{aligned}$$

Case (iii). (q_l^i, q_h^j) - firm i develops low quality prototype whereas firm j develops high

$$\begin{aligned}\pi^i_{l,h}|^{t=2}_{NVC} &= (1-\varphi_i^2)(1-\mu_j^2)D_L + \varphi_i^2\mu_j^2 D_H + \varphi_i^2(1-\mu_j^2)M - ce_i^2 \\ \pi^j_{l,h}|^{t=2}_{NVC} &= (1-\varphi_i^2)(1-\mu_j^2)D_L + \varphi_{ij}^2 D_H + \mu_j^2(1-\varphi_i^2)M - ce_j^2\end{aligned}$$

Case (iv). (q^i_h, q^j_h) - both firms develop *high* quality prototypes

$$\pi^i_{h,h}|^{t=2}_{NVC} = (1-\mu_i^2)(1-\mu_j^2)D_L + \mu_{ij}^2 D_H + \mu_i^2(1-\mu_j^2)M - ce_i^2 \;\forall\; i$$

In the above notation for expected profits, the first superscript denotes which firm's profits we are discussing; the first subscript, x, y, denotes the case in which firm i has developed a prototype of quality $x \in \{h, l\}$ and j of quality $y \in \{h, l\}$; the second superscript denotes the period, $t \in \{1, 2\}$; and the second subscript whether this is the benchmark case (NVC) or the VC case (VC).

In each of the above cases, firms maximise profits by choosing respective effort levels. With some manipulation of the relevant first order conditions, we obtain the following set of equations corresponding to each case:

Case (i). (q^i_l, q^j_l)

$$\frac{\partial \varphi_i^2}{\partial e_i^2} = \frac{c}{(M-D_L) - \varphi_j^2(M-D_L-D_H)} \;\forall\; i \tag{4.5}$$

Case (ii). (q^i_h, q^j_l)

$$\begin{aligned}\frac{\partial \mu_i^2}{\partial e_i^2} &= \frac{c}{(M-D_L) - \varphi_j^2(M-D_L-D_H)} \\ \frac{\partial \varphi_j^2}{\partial e_j^2} &= \frac{c}{(M-D_L) - \mu_i^2(M-D_L-D_H)}\end{aligned}$$

Case (iii). (q^i_l, q^j_h)

$$\begin{aligned}\frac{\partial \varphi_i^2}{\partial e_i^2} &= \frac{c}{(M-D_L) - \mu_j^2(M-D_L-D_H)} \\ \frac{\partial \mu_j^2}{\partial e_j^2} &= \frac{c}{(M-D_L) - \varphi_i^2(M-D_L-D_H)}\end{aligned}$$

Case (iv). (q_h^i, q_h^j)

$$\frac{\partial \mu_i^2}{\partial e_i^2} = \frac{c}{(M - D_L) - \mu_j^2(M - D_L - D_H)} \forall i$$

The solutions to the above first order conditions then yield a firm's reaction function. The following proposition shows how the optimal effort level of a certain firm changes in response to its rival's.

Proposition 19 *Second period efforts are strategic substitutes regardless of the quality of the prototypes discovered at the end of the first period.*
Proof. *See appendix 8.1* ■

According to proposition 19, second period effort levels are strategic substitutes. Any *increase* in one firm's optimal effort level leads to a *decrease* in that of its rival's. The impetus for this result is the fact that, regardless of the prototypes developed by the firms, an increase in firm i's investment has two opposing effects on firm j's expected profits. First, it unambiguously decreases the chances that firm j will become a monopolist in the final product market and, consequently, reduces their expected returns to effort. Second, it increases the expected profits of becoming a duopolist by making it more likely that the firms will act as *high quality* duopolists in the final product market. However, given assumptions A1, A3 and $M > 2D_H > 2D_L$, it is trivial to demonstrate that it is the former of these effects that dominates. Therefore, should firm i's effort level increase, firm j's expected profits are strictly lower, at all levels of e_j^2, than they would have been otherwise. It is this reduction in the expected benefits of investment that drives firm j to cut its investment level in response to an increase by firm i.

The next proposition shows that regardless of the type of prototype discovered, the optimal effort level of a firm increases in its own ability but decreases in its rival's ability. Hence,

Proposition 20 *Regardless of the type of prototype discovered*

$$\frac{de_i^2}{da_i} > 0; \; \frac{de_j^2}{da_i} < 0$$

Proof. *See Appendix 8.2* ■

The importance of this proposition is that it suggests that a firm's ability level is positively correlated with its effort; *ceteris paribus*, a more able firm invests more. The rationale behind this is a consequence of assumptions A1 and A2. As a firm's ability increases, it is induced to invest more for two reasons. First, assumption A1 states that, for a given level of effort, the more able the firm, the greater its probability of success. Consequently, at all effort levels, each unit of investment yields a *higher* expected return which, in turn, induces the firm to increase its investment level. Second, assumption A2 implies that a firm's marginal returns to effort are increasing in its ability because the firm is better able to target its effort along more effective research paths. This further increases the returns to effort, once again spurring a firm to invest more. This increased investment of a more able firm, combined with proposition 19, suggests that whilst a higher ability firm will invest more, its rival will be induced to invest less.

Proposition 20 is also interesting and implies that it is possible to determine, in every case, which firm will invest the most. In the symmetric cases, (q_l^i, q_l^j) and (q_h^i, q_h^j), this analysis is fairly trivial. Assuming that, initially, the firms are symmetric with respect to their abilities, $a_i = a_j$, we are ensured that equilibrium effort levels are symmetric too. However, where firm i is allowed to become the *high ability* firm, $a_i > a_j$, propositions 19 and 20 state that this will unambiguously increase the effort level of firm i and reduce that of firm j. Therefore, in the cases in which the firms have developed prototypes of similar qualities, the high ability firm invests more and is the more likely innovator.

A similar result holds in the case in which firm i has developed a high quality prototype and firm j low quality. In this case, after applying equation (4.3) to the relevant first order conditions, we observe

$$\frac{\partial \varphi_i^2}{\partial e_i^2} = \frac{\lambda c}{(M - D_L) - \varphi_j^2(M - D_L - D_H)} \tag{4.6}$$

$$\frac{\partial \varphi_j^2}{\partial e_j^2} = \frac{\lambda c}{\lambda(M - D_L) - \varphi_i^2(M - D_L - D_H)} \tag{4.7}$$

Assuming both firms are of the same ability and that investment levels are identical yields

$$\frac{\partial \varphi_i^2}{\partial e_i^2} < \frac{\partial \varphi_j^2}{\partial e_j^2}$$

or that a symmetric equilibrium cannot be supported. In fact, given equations (4.6) and (4.7), it is obvious, given a symmetric starting point, firm i would always prefer to invest more than firm j and *vice versa*. Therefore, even with symmetric firms we would expect to observe firm i as the more likely innovator. In fact, the addition of asymmetric abilities, $a_i > a_j$, serves only to widen the gap between the firms' investment levels (proposition 20). Therefore, it is once again the high ability firm that is the most likely "winner".

It is the final case, where firm i has developed a low quality prototype and firm j high quality, that is the most interesting. With symmetric firms, $a_i = a_j$, the result of the previous case is reversed; firm j invests strictly more than firm i. However, the addition of asymmetric abilities, $a_i > a_j$, suggests that the identity of the likeliest innovator becomes ambiguous. Where firm i's ability has increased above that of firm j, propositions 19 and 20 state the investment level of firm i (j) must unambiguously increase (decrease). Consequently, even where the more able firm has developed a lower quality prototype, it is surely possible - if it is *sufficiently able* - that it may still become the more likely firm to develop a high quality final product.

Finally, it is useful to order the effort levels and, in doing so, we observe

Proposition 21 *Regardless of the quality of a rival's prototype, a firm always invests strictly more effort when it has discovered a high quality prototype. More formally:*

$$e_i^2|_{q_h^i, q_s^j} > e_i^2|_{q_l^i, q_s^j}$$

for all $s \in \{l, h\}$.
Proof. *See Appendix 8.3* ■

This result suggests that a firm will invest more, and be more likely to innovate, if it was successful in developing a high quality prototype at the end of the first stage. In essence, one clear result obtains from proposition 21: past success is a good indicator of the likeliness of future successes. In a nutshell, once a firm has demonstrated an ability to successfully innovate, it becomes *more* likely to innovate in the future than if it had failed to innovate initially.

This ability to order the effort levels also enables us to order the firms' profit levels.

Corollary 22 *Regardless of the quality of a rival's prototype, a firm's expected profits are higher when it has developed a high quality prototype. Formally,*

$$\pi^i_{h,s}|^{t=2}_{NVC} > \pi^i_{l,s}|^{t=2}_{NVC}$$

for all $s \in \{l, h\}$
Proof. *This proof is trivial and so it is omitted.* ■

Given the result in proposition 21, corollary 22 should come as no surprise. If assumptions A1, A3 and equation (4.3) hold, it is obvious that the expected returns to effort, in terms of both expected monopoly and duopoly profits, are strictly greater when a firm has developed a high quality prototype. That is to say, when a firm has been successful in developing a high quality prototype, each additional unit of effort yields *larger* increases in expected profits. These additional returns on investment induce a firm to increase their innovative efforts which, in turn, yield higher levels of expected profits. Moreover, this intuition is *independent* of the quality of the rival's prototype. Therefore, initial success has tangible consequences: a greater level of effort and expected profits compared to the scenario in which the firm had failed to innovate initially. To reiterate a previous point, these results suggest past success is a good indicator of the likeliness of future successes.

4.3.2 First stage equilibrium

In the first period, the firms' expected profit functions are given by

$$\begin{aligned}\pi^i|^{t=1}_{NVC} = (1-\varphi^1_i)(1-\varphi^1_j)\pi^i_{l,l}|^{t=2}_{NVC} + \varphi^1_i\varphi^1_j\pi^i_{h,h}|^{t=2}_{NVC} \\ + \varphi^1_i(1-\varphi^1_j)\pi^i_{h,l}|^{t=2}_{NVC} + (1-\varphi^1_i)\varphi^1_j\pi^i_{l,h}|^{t=2}_{NVC} - ce^1_i\end{aligned}$$

$$\pi^j|_{NVC}^{t=1} = (1-\varphi_i^1)(1-\varphi_j^1)\pi_{l,l}^j|_{NVC}^{t=2} + \varphi_i^1\varphi_j^1\pi_{h,h}^j|_{NVC}^{t=2} + \varphi_i^1(1-\varphi_j^1)\pi_{h,l}^j|_{NVC}^{t=2} + (1-\varphi_i^1)\varphi_j^1\pi_{l,h}^j|_{NVC}^{t=2} - ce_j^1$$

With a little manipulation of the relevant first order conditions one obtains

$$\frac{\partial\varphi_i^1}{\partial e_i^1} = \frac{c}{(1-\varphi_j^1)(\pi_{h,l}^i|_{NVC}^{t=2} - \pi_{l,l}^i|_{NVC}^{t=2}) + \varphi_j^1[\pi_{h,h}^i|_{NVC}^{t=2} - \pi_{l,h}^i|_{NVC}^{t=2}]} \tag{4.8}$$

$$\frac{\partial\varphi_j^1}{\partial e_j^1} = \frac{c}{(1-\varphi_i^1)(\pi_{l,h}^j|_{NVC}^{t=2} - \pi_{l,l}^j|_{NVC}^{t=2}) + \varphi_i^1[\pi_{h,h}^j|_{NVC}^{t=2} - \pi_{h,l}^j|_{NVC}^{t=2}]} \tag{4.9}$$

These follow a similar functional form to those in the second stage but are now dependent on the second period's expected profits. However, as the following proposition demonstrates, each firm's first period efforts may be treated as *either* strategic substitutes *or* complements.

Proposition 23 *First period efforts can be treated as either strategic substitutes or complements. Furthermore, it is possible that one firm treats efforts as a strategic substitutes whilst the other treats them as complements.*
Proof. *See appendix 8.4* ■

It is interesting that, in contrast to second period efforts, firms may treat effort either as strategic substitutes or complements. It turns out that firms only "compete" in effort (treat effort as strategic substitutes) if and only if the expected profits of becoming the *sole* developer of a high quality prototype are sufficiently large. In this scenario, additional investment by one firm *strictly decreases* the probability that the rival firm will be able to become the sole developer of a high quality prototype. As this makes up a significant proportion of a firm's expected profits, relative to the other cases, an increase in the efforts of one firm significantly reduces the expected profits of the other. Therefore, investment by one firm reduces the incentives of its rival to invest in the first place and, consequently, the rival firm's effort level falls. In contrast, in the strategic complements case, where the expected profits of being the sole developer of a high quality prototype are smaller, there becomes a greater emphasis on the expected payoffs in the *symmetric* (duopoly) cases. When these are sufficiently large, the investment of a rival actually *increases* the expected profitability of the firm. In essence, the profits of a firm are *positively* correlated with a rival firm's investment. Therefore, when one firm increases its effort levels, this induces the other firm to do the same.

An interesting third possibility emerges here too: both firms may treat effort differently. Simply, when the firms are of asymmetric abilities, expected profits must be different too. Thus, it is possible that one firm's reaction function slopes down whilst the other firm's slopes up. In essence, the firms' effort decisions becomes a game of "cat and mouse", with one firm trying to match the other, which is trying to get away.[9] In fact, this additional result may offer some theoretical grounding for the empirical observation that some firms adopt "innovator" strategies whilst others adopt "imitator" strategies (Hellman and Puri , 2000). In our model, the "innovators" are those firm that expect to make relatively large profits if they can innovate early (the firm that treats efforts as substitutes). In contrast, "imitators" are driven to invest not because they expect to be innovators alone, but because their expected profits are positively correlated with the efforts of their rival (the firm that treats efforts as complements). Therefore, in equilibrium, both firms are trying to balance two opposing forces. In the case of the "innovator", they wish to maximise their profits without attracting too much investment by an "imitator". In contrast, an "imitator" wishes to invest as much as possible, without suppressing too much innovative effort of the "innovator".

4.4 Effects of VC on firms' innovative incentives

Now consider the possibility that a firm can receive offerings from a venture capitalist. The possibility of securing VC backing then changes the above scenario substantially. Recall that a VC package, (s, E, F), is given to *only one* firm that has developed a high quality prototype where the winning firm

[9] Mathematically this is not problematic so long as the reaction functions allow for stability and uniqueness. To that end, we must ensure that firms do not "overreact" to a change in a rival's choice. Formally (Fudenberg and Tirole, 1991),

$$\left|R_i^{'}\right|\left|R_j^{'}\right| < 1$$

now has a probability of innovation function given by equation (4.4). Further, recall that if both firms developed a high quality prototypes then each receives VC with equal probability, where the firm that is *not* successful in receiving the VC offering (despite the fact that it had developed a high-quality prototype) faces the probability $\mu_i(a_i, e_i^2)$. Finally, recall that the probability of success function for the firm that developed a *low* quality prototype remains unchanged i.e. it is given by $\varphi_i^2(a_i, e_i^2) = \lambda\mu_i(a_i, e_i^2, 0, 0)$ - see equation (4.3)

As in the No-VC case, we start our analysis with the second stage game.

4.4.1 Second stage equilibrium

In the presence of VC, the expected profits for each case are given by

Case (i). (q_l^i, q_l^j)

$$\pi_{l,l}^i|_{VC}^{t=2} = (1-\varphi_i^2)(1-\varphi_j^2)D_L + \varphi_i^2\varphi_j^2 D_H + \varphi_i^2(1-\varphi_j^2)M - ce_i^2 \;\forall\; i$$

Case (ii). (q_h^i, q_l^j)

$$\begin{aligned}
\pi_{h,l}^i|_{VC}^{t=2} &= (1-s)\left[(1-\widehat{\mu}_i^2)(1-\varphi_j^2)D_L + \widehat{\mu}_i^2\varphi_j^2 D_H + \widehat{\mu}_i^2(1-\varphi_j^2)M - ce_i^2\right] \\
\pi_{h,l}^j|_{VC}^{t=2} &= (1-\widehat{\mu}_i^2)(1-\varphi_j^2)D_L + \widehat{\mu}_i^2\varphi_j^2 D_H + \varphi_j^2(1-\widehat{\mu}_i^2)M - ce_j^2
\end{aligned}$$

Case (iii). (q_l^i, q_h^j)

$$\begin{aligned}
\pi_{l,h}^i|_{VC}^{t=2} &= (1-\varphi_i^2)(1-\widehat{\mu}_j^2)D_L + \varphi_i^2\widehat{\mu}_j^2 D_H + \varphi_i^2(1-\widehat{\mu}_j^2)M - ce_i^2 \\
\pi_{l,h}^j|_{VC}^{t=2} &= (1-s)\left[(1-\varphi_i^2)(1-\widehat{\mu}_j^2)D_L + \varphi_i^2\widehat{\mu}_j^2 D_H + \widehat{\mu}_j^2(1-\varphi_i^2)M - ce_j^2\right]
\end{aligned}$$

Case (iv). (q_h^i, q_h^j). Here,

(a) If firm i received VC

$$\begin{aligned}
\pi_{h,h}^i|_{VC_i}^{t=2} &= (1-s)\left[(1-\widehat{\mu}_i^2)(1-\mu_j^2)D_L + \widehat{\mu}_i^2\mu_j^2 D_H + \widehat{\mu}_i^2(1-\mu_j^2)M - ce_i^2\right] \\
\pi_{h,h}^j|_{VC_i}^{t=2} &= (1-\widehat{\mu}_i^2)(1-\mu_j^2)D_L + \widehat{\mu}_i^2\mu_j^2 D_H + \mu_j^2(1-\widehat{\mu}_i^2)M - ce_j^2
\end{aligned}$$

(b) If firm j received VC

$$\begin{aligned}\pi^i_{h,h}|^{t=2}_{VC_j} &= (1-s)\left[(1-\widehat{\mu}^2_i)(1-\mu^2_j)D_L + \widehat{\mu}^2_i\mu^2_j D_H + \widehat{\mu}^2_i(1-\mu^2_j)M - ce^2_i\right] \\ \pi^j_{h,h}|^{t=2}_{VC_j} &= (1-\widehat{\mu}^2_i)(1-\mu^2_j)D_L + \widehat{\mu}^2_i\mu^2_j D_H + \mu^2_j(1-\widehat{\mu}^2_i)M - ce^2_j\end{aligned}$$

Note the altered second subscript in the symmetric, high quality case, VC_x. This simply states that firm $x \in \{i, j\}$ received VC when both firms were eligible.

Each firm now maximises their second period payoffs. Then, using the first order conditions - and with a little manipulation - we find that, for each of the cases (i) - (iv), the firms' effort level decisions are given by

Case (i). (q^i_l, q^j_l)

$$\frac{\partial\varphi^2_i}{\partial e^2_i} = \frac{c}{(M-D_L)-(1-\varphi^2_j)(M-D_L-D_H)} \ \forall\ i$$

Case (ii). (q^i_h, q^j_l)

$$\begin{aligned}\frac{\partial\hat{\mu}^2_i}{\partial e^2_i} &= \frac{c}{(M-D_L)-(1-\varphi^2_j)(M-D_L-D_H)} \\ \frac{\partial\varphi^2_j}{\partial e^2_j} &= \frac{c}{(M-D_L)-(1-\hat{\mu}^2_i)(M-D_L-D_H)}\end{aligned}$$

Case (iii). (q^i_l, q^j_h)

$$\begin{aligned}\frac{\partial\varphi^2_i}{\partial e^2_i} &= \frac{c}{(M-D_L)-(1-\hat{\mu}^2_j)(M-D_L-D_H)} \\ \frac{\partial\hat{\mu}^2_j}{\partial e^2_j} &= \frac{c}{(M-D_L)-(1-\varphi^2_i)(M-D_L-D_H)}\end{aligned}$$

Case (iv). (q^i_h, q^j_h)

i) If firm i received VC

$$\frac{\partial \hat{\mu}_i^2}{\partial e_i^2} = \frac{c}{(M - D_L) - (1 - \mu_j^2)(M - D_L - D_H)}$$
$$\frac{\partial \mu_j^2}{\partial e_j^2} = \frac{c}{(M - D_L) - (1 - \hat{\mu}_i^2)(M - D_L - D_H)}$$

ii) If firm j received VC

$$\frac{\partial \mu_i^2}{\partial e_i^2} = \frac{c}{(M - D_L) - (1 - \hat{\mu}_j^2)(M - D_L - D_H)}$$
$$\frac{\partial \hat{\mu}_j^2}{\partial e_j^2} = \frac{c}{(M - D_L) - (1 - \mu_i^2)(M - D_L - D_H)}$$

Again, these functions all act as a basic reaction function for each firm. With some manipulation one obtains

Proposition 24 *Second period efforts are always strategic substitutes regardless of the quality of prototype developed.*
Proof. *The proof is identical in style to that of proposition 21 and so is omitted. Nonetheless, the result still hinges on the assumptions made in A1 - A3 and* $M > 2D_H > 2D_L$. ■

The intuition behind proposition 24 is also similar to that of proposition 21. As a brief recap, regardless of the prototypes developed by the firms, an increase in firm i's investment *decreases* firm j's expected profits at all levels of e_j^2. The reason for this is that increases to firm i's effort makes it less likely that firm j will become a monopolist, whilst strictly increasing the probability that firm j will become a high, and not low, quality duopolist. Given assumption A1, A3 and $M > 2D_H > 2D_L$, it is trivial to check that the reduction in expected monopoly profits dominates, and it is this that incentivises firm j to cut back on its investment.

First we make the observation that, similar to the No-VC case, second period effort levels are determined by a firm's relative ability.

Remark 25 *Regardless of the type of prototype discovered*

$$\frac{de_i^2}{da_i} > 0; \; \frac{de_j^2}{da_i} < 0$$

Proof. *The proof is almost identical to that of proposition 20 and so is omitted. Nonetheless, the result still hinges on the assumptions made in A1 - A3 and* $M > 2D_H > 2D_L$. ■

The intuition behind remark 25 is identical to that of proposition 20 and is driven by assumptions A1 and A2. A higher ability makes a firm more likely to develop a high quality good *and* better able to target its efforts, increasing the expected returns to effort. Consequently, the firms are induced to invest more when they are of higher ability.

However, it is no longer just ability that plays a role in the determining the future successes of the firms. Instead, the VC package plays a crucial role too. The following proposition demonstrates the impact VC has on the firms' incentives to innovate.

Proposition 26 *Assuming that firm i receives VC backing, we observe*[10]

$$\frac{de_i^2}{dE} > 0 \tag{4.10}$$

$$\frac{de_j^2}{dE} < 0 \tag{4.11}$$

$$\frac{de_i^2}{dF} > 0 \tag{4.12}$$

$$\frac{de_j^2}{dF} < 0 \tag{4.13}$$

$$\frac{de_i^2}{ds} = \frac{de_i^2}{ds} = 0 \tag{4.14}$$

Proof. *See Appendix 8.5* ■

The crucial element of proposition 26 is that VC unambiguously increases the probability of successful innovation for the firm that is chosen, by inducing them to invest *more*. In contrast, a firm that must compete against a VC-backed rival becomes less likely to develop a high quality good. Consequently, VC tips the balance of competition in favour of the firm it backs.

It is two particular elements of the VC package that generate this result. First, the addition of pecuniary funding, F, makes a firm more likely to innovate at all levels of effort. Thus, a firm with financial backing is, in a sense,

[10] Results for firm j can be derived by symmetry.

able to *buy* success, regardless of their efforts or ability, as the firm may now have access to new equipment or better quality materials. It is the addition of finance, and the greater likelihood that they innovate successfully, that makes effort more valuable and induces them to invest more. Second, it is likely that a venture capitalist would offer value-adding services, from simply mentoring firms and improving marketing strategies to overhauling corporate governance completely. No matter the level involvement, a venture capitalist's own efforts are likely to have two impacts: *i*) increases to E may simply raise the probability of success at all effort levels by allowing entrepreneurs more time to focus on innovation; or *ii*) a venture capitalist may use its expertise and market knowledge to channel the entrepreneurs efforts down more fruitful research pathways. In both cases, these strictly increase the returns to each additional unit of effort. Therefore, the existence of value adding services also create an environment that enables a firm to invest more.[11]

In fact, given proposition 26, we are able to determine whether a firm would invest more or less compared to the no-VC case, as the following corollary explains.

Corollary 27 *Compared to the benchmark case, a firm that has received VC funding invests strictly more than it would have done without VC-backing. Furthermore, a firm invests strictly less, compared to the benchmark case, when it faces a VC-backed rival.*

A formal proof of this corollary 27 is unnecessary as it follows directly from proposition 26. Moreover, the intuition behind this result follows directly from the benefits of funding and venture capitalist expertise: by increasing the returns to each additional unit of effort, venture capitalists induce VC-backed firms to invest more. In contrast, if one firm has increased its investment level, the other must invest less (proposition 24).

This corollary is interesting, but it has one important and overlooked result contained within. Whilst VC does spur innovation and increases the probability of success for the firm that receives it, there are also casualties. This result is unique within the literature, capturing a new impact of VC that has not yet

[11] Given the specification of the model it is not possible to determine which effect, E or F, is larger.

been observed. If a firm is competing against a VC backed rival it becomes *less* likely to develop a high quality final good than if no VC were present. Consequently, whilst VC does spur the future innovative efforts of an early innovator, it suppresses the efforts of firms that failed to innovate initially. Therefore, this result contrasts with the oft-empirically observed result that VC *only spurs success*. Instead, whilst venture capitalists drive success for the chosen few, it holds back the firms that must compete against their portfolio firms.

Given these results, we are able to compare the relative effort levels between each case, as we do in the following proposition.

Proposition 28 *Regardless of the quality of a rival's prototype, a firm invests strictly more if it has developed a high quality prototype. Moreover, in the cases in which a firm has developed a high quality prototype, it invests more if it receives VC-funding than if does not.*

$$e_i^2|_{q_h^i,q_s^j|VC_i} > e_i^2|_{q_h^i,q_s^j|VC_j} > e_i^2|_{q_l^i,q_s^j}$$

Proof. *See Appendix 8.6* ■

This result is analogous to that of proposition 21: regardless of the prototype developed by a rival, a firm is always more likely to develop a high quality product if it has developed a high quality prototype. Interestingly, this proposition suggests that this result holds regardless of which firm actually receives venture capital. Simply, whether or not a firm receives venture capital, it is *always* more likely to successfully innovate in the future if it has demonstrated an ability to innovate in the past. This suggests that, even with VC-backing of a firm, "success *still* breeds success" or a good indicator of future success is *still* past success. However, one should not ignore the fact that VC-backing strictly improves a firm's chances of being innovative. In essence, VC augments a firm's innovative process, suggesting that *VC-backed success* is more likely.

Corollary 29 *Regardless of the quality of a rival's prototype, a firm's expected profits are higher when it has received VC funding. More formally,*

$$\pi_{h,s}^i|_{VC_i}^{t=2} > \pi_{h,s}^i|_{VC_j}^{t=2} > \pi_{l,s}^i|_{VC}^{t=2}$$

Proof. *The proof is trivial and so it is omitted.* ■

Even though "success breeds success", the question is why? Given assumptions A1 - A4 and equation (4.4), it becomes apparent that, with no additional effort on the part of the entrepreneur, its expected profits are larger if it has successfully innovated a high quality prototype. Therefore, each additional unit of effort is more valuable and generates higher levels of marginal profit. This incentivises the firm to invest *more* and generates *larger* expected profits than if it had failed to innovate at the end of the first stage.[12]

Finally, it is important to understand which firm is most likely to develop a high quality final good.[13] Assuming that firm i is the *high* ability firm, $a_i > a_j$, it is obvious that in the (q_h^i, q_l^j) and $(q_h^i, q_h^j|VC_i)$ cases firm i is more likely to succeed. This follows directly from remark 25 and proposition 26.

However, there are now two ambiguous cases where firm j has received VC funding: (q_l^i, q_h^j) and $(q_h^i, q_h^j|VC_j)$. Intuitively, assuming that the firms are initially of equal ability, it must be that firm j invests more in the (q_l^i, q_h^j) and $(q_h^i, q_h^j|VC_j)$ cases, where it is VC-backed. However, by remark 25, an increase in firm i's ability will unambiguously increase e_i^2 and decrease e_j^2. This implies that for *any* VC package, $(s,\ E,\ F)$, as long as a_i is sufficiently large the more able firm is the more likely firm develop a high quality final product regardless of the quality of its prototype. However, as E and F increase this becomes harder and, therefore, less likely. For large values of E and F it is more probable that the likely winner is determined by who is chosen to receive VC funding. That is, the firm that receives the VC becomes, somewhat automatically, the stronger firm. Therefore, depending the entrepreneurs' relative abilities and the specification of the VC package on offer, VC funding may have either a small or large impact on the likely composition of the final product market.

Therefore, VC funding does have an impact upon the second period decisions of the firms. A firm receiving VC funding becomes unambiguously better off whilst its rival becomes less likely to develop a good quality final product. However, the extent to which this disadvantages the more able firm is depen-

[12] It is the shape of the probability functions that drives this result. As $\hat{\mu}$ and μ lie strictly above φ, firms are more likely to succeed, at any level of ability or effort, if they have demonstrated some initial innovative ability.

[13] We ignore the case in which both firms develop low quality prototypes as this is identical to the no-VC case.

dent upon the relative sizes of the VC package and the asymmetry between the firms. Consequently, VC does spur innovation but it unambiguously spurns innovation for all firms that do not receive it. Therefore, we would expect different impacts across different industries. Those industries that support a few highly innovative firms should, more often than not, observe more able firms developing the high quality products. However, where firms are more symmetric, we would expect VC to aid the development of firms that would otherwise have been unexpected to become an innovator - i.e. those that may have been initially thought to be of a lower ability.

4.4.2 First stage equilibrium

The first stage expected profits are given by

$$\pi^i|_{VC}^{t=1} = (1-\varphi_i^1)(1-\varphi_j^1)\pi_{l,l}^i|_{VC}^{t=2} + \frac{1}{2}\varphi_i^1\varphi_j^1\left[\pi_{h,h}^i|_{VC_i}^{t=2} + \pi_{h,h}^i|_{VC_j}^{t=2}\right] + \varphi_i^1(1-\varphi_j^1)\pi_{h,l}^i|_{VC}^{t=2} + (1-\varphi_i^1)\varphi_j^1\pi_{l,h}^i|_{VC}^{t=2} - ce_i^1$$

$$\pi^j|_{VC}^{t=1} = (1-\varphi_i^1)(1-\varphi_j^1)\pi_{l,l}^j|_{VC}^{t=2} + \frac{1}{2}\varphi_i^1\varphi_j^1\left[\pi_{h,h}^j|_{VC_i}^{t=2} + \pi_{h,h}^j|_{VC_j}^{t=2}\right] + \varphi_i^1(1-\varphi_j^1)\pi_{h,l}^j|_{NVC}^{t=2} + (1-\varphi_i^1)\varphi_j^1\pi_{l,h}^j|_{NVC}^{t=2} - ce_j^1$$

Where the first order conditions yield

$$\frac{\partial\varphi_i^1}{\partial e_i^1} = \frac{c}{\left\{\begin{array}{c}(1-\varphi_j^1)\left[\pi_{h,l}^i|_{VC}^{t=2} - \pi_{l,l}^i|_{VC}^{t=2}\right] \\ +\varphi_j^1\left[\frac{1}{2}\left(\pi_{h,h}^i|_{VC_i}^{t=2} + \pi_{h,h}^i|_{VC_j}^{t=2}\right) - \pi_{l,h}^i|_{VC}^{t=2}\right]\end{array}\right\}} \tag{4.15}$$

$$\frac{\partial\varphi_j^1}{\partial e_j^1} = \frac{c}{\left\{\begin{array}{c}(1-\varphi_i^1)\left[\pi_{l,h}^j|_{VC}^{t=2} - \pi_{l,l}^j|_{VC}^{t=2}\right] \\ +\varphi_i^1\left[\frac{1}{2}\left(\pi_{h,h}^j|_{VC_i}^{t=2} + \pi_{h,h}^j|_{VC_j}^{t=2}\right) - \pi_{h,l}^j|_{VC}^{t=2}\right]\end{array}\right\}} \tag{4.16}$$

Similar to the No-VC case, first period efforts are determined by expected future profits. However, as the following proposition demonstrates, the reaction functions of each firm can be *either* upward *or* downward sloping.

Proposition 30 *In the VC case, first period efforts can be treated as either strategic substitutes or complements. Additionally, one firm may treat efforts as a strategic substitutes whilst another treats efforts as complements.*
Proof. *See Appendix 8.7* ■

Interestingly, a similar mechanism to proposition 23 is the impetus behind this result. Once again, it appears that firms only "compete" in effort if and only if the expected profits of becoming the *sole* developer of a high quality prototype are sufficiently large. Even the addition of venture capital, which unambiguously increases (decreases) expected profits for the VC-backed (non-VC-backed) firm, does not alter this intuition. Simply, when the expected gains are disproportionately large in the case in which only one firm develops a high quality prototype, any increase in effort by a rival firm *significantly* reduces a firm's expected profits. Consequently, an increase in one firm's effort reduces the incentive for the other to invest, regardless of whether that firm is VC-backed or not. In contrast, when the expected profits from symmetric innovation are relatively large, the expected profits of one firm become positively correlated with the effort of its rival. Therefore, when a rival firm invests more, a firm is incentivised to invest more too.

The result here is interesting because it suggests that, whilst VC funding clearly has an impact on effort levels by influencing expected second period profits, the mechanisms by which the firms compete remain unchanged. Both firms may still treat effort as strategic complements, substitutes or a combination of the two, but they still act in a similar way to the no-VC case. This is, perhaps, one of venture capitals greatest strengths: whilst it does influence the *outcome*, it does not affect the *mechanism*.[14]

In spite of this observation, one important question remains: does the lure of VC have an impact on the first period effort levels of the firms?

Remark 31 *The impact of pecuniary funding, F, and venture capitalist effort, E, on first period effort is ambiguous, regardless of whether firms treat effort as strategic substitutes or complements.*
Proof. *See Appendix 8.8* ■

[14] It is the use of an equity stake that is the reason for this observation. This is, perhaps, why venture capitalists use equity shares and not traditional methods, so as to avoid altering the incentives of the firms (see Brander and Lewis (1986)).

The inability to sign these equations, and determine their impact, should *not* be taken as an indication that they don't affect initial effort decisions. Instead, the issue lies with our inability to gauge the *magnitude* of their numerous effects. Consequently, we suggest that funding and mentoring *should* have an impact on initial effort decisions, but whether these effects are positive or negative is impossible to determine given the current specification.

However, one element of the VC package, the equity stake, does have an interesting impact on the decisions of the firms as the following proposition demonstrates.

Proposition 32 *When firms treat effort as strategic complements, the higher the equity stake in the firm, the lower the effort level, or*

$$\frac{de_i^1}{ds} < 0 \ \forall \ i$$

However, when firms treat efforts as strategic substitutes the effect of the equity stake can be either positive or negative.
Proof. *See Appendix 8.9* ■

What proposition 32 reveals is that the venture capitalist's equity stake has both a direct and indirect impact on a firm's effort choice. The *direct effect* is unambiguously negative: as the venture capitalist's equity stake becomes larger, a firm will want to invest less in the first period. Intuitively, as a the venture capitalist's share of *future* profits become larger, there is less incentive for the firm to invest because the expected profits of innovation are reduced. In contrast, the indirect effect accounts for a firm's reaction to a fall in a rival's investment *caused* by an increase in the equity stake. Where the firms treat efforts as strategic substitutes, one firm will *increase* its research efforts in response to a *reduction* in a rival's. Therefore, as an increase in the equity stake unambiguously puts downward pressure on the investment decisions of both firms, this indirectly induces the firms to invest more; *a positive indirect effect.* Consequently, whether firms actually invest more or less is determined by the balance of these opposing forces. In contrast, where firms treat efforts as strategic complements, a fall in a rival's investment will induce a firm to invest less too; *a negative indirect effect.* The reason for this is that, in this case, a firm's expected profits are positively correlated with the effort level of its rival. Therefore, as a higher equity stake reduces the rival firm's incentives to invest, this also leads a firm to reduce their effort levels too. Thus, in the strategic complements case both the direct and indirect effects act in the *same* direction, and it must be that increasing the equity stake reduces investments.

4.5 Conclusion

This paper set out to address a notable imbalance in the VC literature, that venture capitalists were the sole force behind the VC-to-success link. Diverging from the existing literature, we examined the VC-to-success link from the *firms'* perspectives, examining pre- and post-VC funding decisions to determine whether VC-funding spurred innovation: *i*) directly after being granted; *ii*) indirectly by incentivising firms to increase initial research efforts to increase their chances of receiving VC funding (and its associated benefits); or *iii*) a combination of both. This paper finds some reasoning for VC having both direct (*ex post*) *and* indirect (*ex ante*) implications for a firm's investment decisions.

The second stage results clearly demonstrate a *direct* link between VC and innovation/success. First, it appears that "success breeds success", in so far as a predictor of future innovation appears to be past innovative success. Whilst the model presented is of a static game, this result comes from the intertemporal connection between success in the first and second stages. Quite simply, the model implies that if a firm is successful in the initial stages of research, it is more likely to reach a fruitful conclusion. Second, and most important, the addition of venture-backing to a firm tips the balance of competition in its favour. The addition of funding enables firms to, in essence, "buy success", by spending money on better equipment or materials. The addition of value-adding services is equally important for two reasons: *i*) it directly increases the likelihood of success by enabling an entrepreneur to *focus* on innovation; and *ii*) a venture capitalist's expertise may indirectly benefit a firm if it is able to direct it along more fruitful research pathways. Yet, the addition of VC *only* benefits the firm that receives it and unambiguously reduces the likelihood of innovation for *all other firms*. Therefore, the commonly held belief that VC spurs innovation may only be a partial truth. Instead, VC *may*, in general, spur innovation, but this is dependent on the balance of these two, contradictory effects. However, the extent to which VC can determine which firm is the likely innovator depends upon the level of heterogeneity: the more symmetric the firms, the more likely that VC is an important factor in determining the composition of the final product market.

We also find evidence of indirect impacts on initial effort levels, *before* VC is offered to firms. By altering future expected profits, VC *indirectly* impacts upon the firm's initial, pre-VC, effort decisions. However, given the general specification of the model, it is not possible to determine whether the impact of finance and value-adding services are positive or negative. In contrast, the venture capitalist's equity stake in the firm impacts on initial efforts in two ways: *i*) it directly reduces initial efforts by reducing expected future profits; and *ii*) it indirectly increases (decreases) efforts if the firms treat efforts as strategic substitutes (complements). Nonetheless, this "ambiguity" should not be misinterpreted as there being *no* indirect effect. Rather, it is likely that VC does, indeed, have an impact and this is left for future work.

Future work should focus on three areas. First, the model needs to be generalised to incorporate a larger number of both firms and venture capitalists, though this appears to be a rather trivial addition. Extending this model to incorporate n asymmetric firms and m venture capitalists, one should expect that a similar outcome obtains *if* the underlying assumptions remain unchanged.[15] That is, VC-backed firms should be more likely to innovate at the expense of their unselected rivals and, regardless of a rival's prototype, a firm invests more if: *i*) it has developed a high quality prototype; and/or *ii*) it receives VC funding. In essence, so long as the functional forms remain unchanged and each venture capitalist can fund a *single* entrepreneur then the general principals of propositions 26 and 28 should remain the same regardless of the number of firms or venture capitalists. That is, the results presented here are likely robust to additional firms and venture capitalists.

Second, and more interesting, the addition of a greater number of venture capitalists and firms would also allow for a broader examination of the question

[15] In reality, whether n or m is larger depends upon the situation. For example, after the development of a brand new technology it is more likely that $n < m$. However, for firms setting up in well established industries it is more likely that $n > m$.

at hand: do venture capitalists spur innovation? Within the extended, n-firm, m-venture capitalist framework, it would be interesting to examine what happens when: i) a single venture capitalist could finance more than a single firm; and ii) firms with a common backer can form strategic alliances. The recent empirical literature notes that these two factors are intertwined and offer additional benefits above and beyond those assumed in this paper; the existence of a common backer makes firms *more* likely to work together and, consequently, more innovative.[16] It would be interesting to see how these additions would alter the nature of the result presented here; would joint-VC-backed firms be increasingly likely to innovate at a greater expense of non-VC-backed firms?

Finally, it would be useful to examine the model using a specific functional form. A number of the analytical issues presented here are a consequence of not being able to determine the *magnitude* of certain effects. Therefore, specifying the (unconditional) functional forms may enable a more detailed analysis of the model at hand. Indeed, this issue would allow for a more complete examination of the first stage equilibria and see to what extent VC impacts upon a firm's initial effort levels.

[16] See Lindsey (2008), Ozmel *et al* (2013) and Lerner *et al* (2003).

Chapter 5

Conclusion

The aim of this thesis was to explore three of the numerous ways in which the relative abilities of firms can influence their innovation strategies and the composition of the final product market. Driven by the observation that, in many situations, innovation and success go hand in hand, the three essays we presented were an effort to understand *why* certain firms are more likely to innovate successfully than others. In a broad sense, we observe that abilities play an important role in determining research strategies of firms, with more able firms being the likeliest to become an *ex post* market leader. Indeed, this view has been repeatedly observed empirically and anecdotally as we highlighted in the introduction and Chapter 1. Yet, the essays presented here, as well as being able to offer some important results themselves, are only the start of a *theoretical* underpinning of the transmission mechanisms by which abilities influence an industry's innovative output. The tools for examining this important issue already exist but, to the author's knowledge, little else has been done to examine *how* ability influences the composition of the final product market.

Chapter 2 evaluated the impact of abilities on the *innovative process.* In essence, rather than simply examining the *effect* of cost differentials, this essay analysed *the cause* of cost differentials *and* their effects. That is, we examine the extent to which a firm's relative *ability* to innovate - in terms of both its ability to implement a new technology and innovation timing - affects its innovative potential. To do this, we proposed a three-stage model in which the firms sequentially invest in R&D before simultaneously competing in locations and prices. In this setting two unique results were obtained.

First, where an *ex ante* more able firm possesses a Stackelberg leadership advantage in cost reducing investment, they generate lower costs, greater demand and yield larger profits *ex post* than their *ex ante* less able rival. Simply, if a firm is a "better" innovator (in terms of both timing and implementing new technologies) it will invest aggressively in cost reduction in order to cement its place as the dominant firm in the final product market. In reality, one could think of Google as such a firm because, given its innovation strategies, it is likely better able to both spot and implement new technologies. Moreover, this result is consistent with Google's persistent pursuit of new ideas and technologies that have enabled them to remain a dominant force within all the markets in which it operates. In contrast, under such circumstances a less able rival is forced to act "soft" in order to prevent itself undermining its profits through exacerbating price competition. This may go some way to explaining why some firms, such as Google, may continue to dominate a market whilst others, such as Bing and Yahoo!, are forced to serve a niche.

Second, if an *ex ante* less able firm is the Stackelberg leader in the cost reduction stage, there are a four potential equilibrium outcomes that depend on the relative efficiencies of the two firms. To summarise, the more symmetric the firms, the better able a less efficient firm is at overcoming this the "ability gap" *if* it is able to spot new opportunities early. That is, a firm has to be sufficiently able, relative to its rival, to be able to take advantage of its timing advantage. This result helps to explain *two* phenomena. First, when a market is dominant by a relatively *very* able rival, no matter when a firm spots an opportunity one should always expect the *ex ante* dominant firm to maintain its position. Again, one may think of Google. Second, where an industry is made up of relatively symmetric firms, the most likely innovator is not *a priori* obvious. Consequently, in such circumstances we would expect the *initial* innovator, often a relatively unheralded firm, to become the most dominant. As an example, one might consider the rise of McDonalds in the 1950s from within a relatively homogeneous market.

Chapter 3 then examined whether agglomeration between asymmetric firms is ever optimal. More importantly, we extend the existing literature by allowing firms to differ, simultaneously, in two ways: *i*) the firms are *a priori* heterogeneous with respect to their innovative abilities; and *ii*) they have asymmetric absorptive capacities. Simply, if the firms face different market conditions, would they still wish to agglomerate?

Our results suggest that firms, facing location based spillovers and whether or not they are asymmetric, never find agglomeration optimal. Rather, maximal differentiation always obtains. Simply, the ferocity of price competition remains as a potent centrifugal force, keeping the firms from locating closer to one another. This result holds even when spillovers that are *convex* in distance, as used to find partial agglomeration results in other spatial models.

However, we also observe that location based spillovers, combined with asymmetric firms, leads to increasingly asymmetric outcomes. More specifically, the existence of *asymmetric absorptive capacities* drives the additional asymmetry, beyond the results obtained by Ziss (1993) and Matsumura and Matsushima (2009). As the firms locate closer to one another, a more able firm assimilates *more* knowledge than it leaks to its less able rival and it is this additional ability that enables a more able firm to become *more* predatory. By moving towards its rival, a more able firm threatens to *further* increase the *ex post* cost disparity, incentivising it to locate more centrally and push its rival further from the market. In turn, this induces the more able firm to invest more too. In contrast, the less able firm's decisions are more reliant upon the level of consumer heterogeneity. With more homogeneous consumers, the weaker firm faces a greater threat of predatory behaviour so, to mitigate this, it reduces its investment level to minimise spillovers and price competition. However, when consumers are less "fickle", this threat is reduced and the firm is willing to invest *more*.

It is this final result that is most applicable to real world situations. When a firm is better able to assimilate a rival's research, this gives it a competitive advantage. As the *effective* spillover rates differ, it is the more able firm that gains *more* than its rival. Therefore, when a firm is in a better position to "steal" a rival's economic knowledge, we would expect them to be more aggressive and dominant. Consequently, despite the fact that agglomeration never obtains, it is likely that agglomeration is *less* likely with asymmetric firms as the firms' incentives will differ and cannot be overcome by simply increasing the spillover rate to both firms.

Finally, Chapter 4 examined how venture capital and ability helps shape the landscape of the final product market. To this end, we examined the VC-to-success link from the *firms'* perspectives, examining pre- and post-VC funding decisions to determine whether VC-funding spurred innovation: *i*) directly after being granted; *ii*) indirectly by incentivising firms to increase initial research efforts to increase their chances of receiving VC funding (and its associated benefits); or *iii*) a combination of both.

The second stage results clearly demonstrated a *direct* link between venture capital and innovation/success. The second stage results clearly demonstrated a *direct* link between venture capital and innovation/success. First, it appears that "success breeds success", in so far as a predictor of future innovation appears to be past innovative success. Second, and most important, the addition of venture-backing to a firm tips the balance of competition in its favour. The addition of funding enables firms to, in essence, "buy success", by spending money on better equipment or materials. The addition of value-adding services is equally important for two reasons: *i*) it directly increases the likelihood of success by enabling an entrepreneur to *focus* on innovation; and *ii*) a venture capitalist's expertise may indirectly benefit a firm if it is able to direct it along more fruitful research pathways. Yet, the addition of venture capital *only* benefits the firm that receives it and unambiguously reduces the likelihood of innovation for *all other firms*. Therefore, the commonly held belief that venture capital spurs innovation may only be a partial truth. Instead, venture capital *may*, in general, spur innovation, but this is dependent on the balance of these two, contradictory effects. However, most pertinent to this discussion is the identity of the most likely innovator. Simply, the extent to which venture capital can determine which firm is the likely innovator depends upon the level of heterogeneity: the more symmetric the firms, the more likely that venture capital is an important factor in determining the composition of the final product market. Therefore, when the ability gap is large, the more able firm will be the most likely innovator whether or not it has demonstrated innovative potential initially.

We also find evidence of indirect impacts on initial effort levels, *before* VC is offered to firms. By altering future expected profits, VC *indirectly* impacts upon the firm's initial, pre-VC, effort decisions. However, given the general specification of the model, it is not possible to determine whether the impact of finance and value-adding services are positive or negative. In contrast, the venture capitalist's equity stake in the firm impacts on initial efforts in two ways: *i*) it directly reduces initial efforts by reducing expected future profits; and *ii*) it indirectly increases (decreases) efforts if the firms treat efforts as strategic substitutes (complements). Nonetheless, this "ambiguity" should not be misinterpreted as there being *no* indirect effect. Rather, it is likely that VC does, indeed, have an impact and this is left for future work.

To summarise our findings, it is clear that the abilities of the firms plays an important role in the strategic decision making process with regards to innovation. However, regardless of the setting, the most likely innovator is, in general, a more able firm. Indeed, the only scenarios in which a less able firm may come out on top is if: *i*) they have an advantage in another respect (perhaps timing); *ii*) the firms are relatively symmetric; and *iii*) they receive some outside help (venture capital in this case). These results are intuitive and have a number of real world applications and allow a better understanding of *why* some firms are more innovative and the underlying mechanisms that allow this to occur. Nonetheless, efforts in this field should not rest on their laurels. Much more can be done to understand the importance of relative abilities in determining the composition of markets, especially given a move towards a more innovation driven society.

Chapter 6

Appendix A: Proofs for Chapter 2

6.1 Proof of Proposition 1

Proof. The second order conditions are given by:

$$\frac{\partial^2 \pi_A}{\partial I_A^2} = \frac{216tm_A^2}{(8m_B^2 - 27t)^2} - 1 < 0 \tag{6.1}$$

$$\frac{\partial^2 \pi_B}{\partial I_B^2} = \frac{8m_B^2}{27t} - 1 < 0 \tag{6.2}$$

From equation (6.1) it is possible to observe that the denominators of equations (2.4) and (2.5) are strictly negative. Given this, it is necessary that the numerators of (2.4) and (2.5) must be non-positive for I_A and $I_B \geq 0$ to hold. Given that $m_A > m_B > 0$ and $t > 0$, this implies:

$$16m_B^2 - 27t \leq 0$$

$$16m_A^2 + 8m_B^2 - 27t \leq 0$$

Rearranging these inequalities, noting $m_A > m_B > 0$ and $t > 0$, yields:

$$m_A^2 \in (m_B^2, \frac{27}{16}t - \frac{m_B^2}{2}] \tag{6.3}$$

$$m_B^2 \in [0, \min\{m_A^2, \frac{27}{8}t - 2m_A^2\}) \tag{6.4}$$

The restrictions in (6.3) and (6.4) ensure that equilibrium investment is non-negative, but it also necessary to check that they are compatible with the second order conditions. This requires:

$$m_A^2 < \frac{(8m_B^2 - 27t)^2}{216t} \tag{6.5}$$

$$m_B^2 < \frac{27}{8}t \tag{6.6}$$

Equations (6.5) and (6.6) offer the maximum *ex ante* cost reducing efficiencies for each firm that are consistent with the second order conditions. It is most obvious that equation (6.6) is always met as it is fairly trivial to show, from (6.4), and assuming $m_A = m_B$, that $\max\{m_B^2\} = \frac{27}{24}t < \frac{27}{8}t$. Knowing that $m_B^2 \in [0, \frac{27}{24}t)$ makes it easier to demonstrate that equation (6.5) must always hold too. Taking $m_B^2 \in [0, \frac{27}{24}t)$ as given, it is possible to show

$$\max\{m_A^2\} - \frac{(8m_B^2 - 27t)^2}{216t} = -\frac{1}{432}\frac{(8m_B^2 - 27t)(16m_B^2 - 27t)}{t}$$

which is strictly negative for all m_A^2 and m_B^2 pairs as defined by (6.3) and (6.4). Therefore, the second conditions hold under these restrictions.

Finally, it is necessary to evaluate whether the restrictions in (2.6) and (2.7) yield equilibrium investment levels consistent with subgame perfect Nash equilibrium for the entire model. For this to occur we require $m_A I_A - m_B I_B \in (-\frac{9}{4}t, \frac{9}{4}t)$. The *ex post* cost differential is given by

$$m_A I_A - m_B I_B = \frac{18t[27t(m_B - m_A)(m_A + m_B) - 8m_B^4]}{216tm_A^2 - (8m_B^2 - 27t)^2}$$

which is strictly positive given the conditions in (6.3) and (6.4). Therefore, it is necessary that $m_A I_A - m_B I_B < \frac{9}{4}t$, or

$$-\frac{243}{4}\frac{t^2(16m_A^2 + 8m_B^2 - 27t)}{216tm_A^2 - (8m_B^2 - 27t)^2} < 0$$

Of course, this holds if and only if $m_A^2 < \frac{27}{16}t - \frac{m_B^2}{2}$. This implies that the restrictions derived from the second order and non-negative investment conditions are generally consistent with $m_A I_A - m_B I_B \in (-\frac{9}{4}t, \frac{9}{4}t)$. However, it is necessary to remove a single case from (6.3), where $m_A^2 = \frac{27}{16}t - \frac{m_B^2}{2}$, which yields (2.6) whilst (2.7) remains identical to (6.4). ■

6.2 Proof of Proposition 3

Proof. (*i*) It is trivial to show that the equilibrium investment levels are strictly positive as this follows directly from the proof of proposition 1.

(*ii*) Taking m_A^2 and m_B^2 defined as in (2.6) and (2.7):

$$\frac{\partial I_A}{\partial m_A} = \frac{-18t(16m_B^2 - 27t)(216tm_A^2 + (8m_B^2 - 27t)^2)}{[216tm_A^2 + (8m_B^2 - 27t)^2]^2} > 0$$

$$\frac{\partial I_B}{\partial m_A} = \frac{-288tm_A m_B(8m_B^2 - 27t)(16m_B^2 - 27t)}{[216tm_A^2 + (8m_B^2 - 27t)^2]^2} < 0$$

(*iii*) Recall firm B's investment function given in equation (2.4) and assume that the efficiency gap is given by

$$\Upsilon = \frac{27}{8}t - \frac{3}{2}m_B^2$$

If this is the case then $I_B = 0$. However, it is known that $I_B > 0$ for all other levels of the efficiency gap (proposition 1) and that, for any m_B^2, $\frac{\partial I_B}{\partial m_A} < 0$. Therefore, taking m_B^2 as given, it must be that increasing the size of the efficiency gap reduces I_B until it equals zero.

(*iv*) Where equations (2.6) and (2.7) hold:

$$I_A - I_B = \frac{-18t[(m_A - m_B)(16m_A m_B + 27t) + 8m_B^3]}{216tm_A^2 + (8m_B^2 - 27t)^2} > 0$$

This is strictly positive for all relevant m_A^2 and m_B^2 pairs which implies that $I_A > I_B$ in all relevant cases. ■

6.3 Proof of Proposition 4

Proof. (iii) Taking $m_B^2 \in [0, \frac{27}{24}t)$ as given, we note that $\pi_A - \pi_B = 0$ if and only if $m_A^2 = \underline{m_A^2}$ or $\overline{m_A^2}$ where

$$\underline{m_A^2} = \frac{1}{576}\frac{243t^2(32m_B^2 - 81t) + \Lambda}{t(8m_B^2 - 27t)}$$
$$\overline{m_A^2} = \frac{1}{576}\frac{243t^2(32m_B^2 - 81t) - \Lambda}{t(8m_B^2 - 27t)}$$

and

$$\Lambda = \sqrt{-3t(16m_B^2 - 81t)(32m_B^2 - 81t)(16m_B^2 - 27t)^3}$$

It is obvious that the domains of both $\underline{m_A^2}$ and $\overline{m_A^2}$ are given by $m_B^2 \in [0, \frac{27}{16}t] \cap [\frac{81}{32}t, \frac{27}{8}t) \cap (\frac{27}{8}t, \frac{81}{16}t]$. However, from proposition 1 and corollary 2, it is know that $m_B^2 \in [0, \frac{27}{24}t)$, which is strictly contained in this domain, and so this does not pose a problem.

As $\pi_A - \pi_B = 0$ only if $m_A^2 = \underline{m_A^2}$ or $\overline{m_A^2}$, it must be that $\pi_A - \pi_B$ has a constant for all $m_A^2 \in (\underline{m_A^2}, \overline{m_A^2})$ and so we examine if: (i) $\underline{m_A^2} < \min\{m_A^2\}$; and (ii) $\max\{m_A^2\} < \overline{m_A^2}$. Simply, that $m_A^2 \in (\underline{m_A^2}, \overline{m_A^2})$.

First, $\underline{m_A^2} = 0$ if and only if $m_B^2 \in \{0, \frac{81}{32}t\}$ and implies that $\underline{m_A^2}$ is continuous, with the same sign, over $m_B^2 \in [0, \frac{27}{24}t)$. Therefore, we can evaluate

$$\min\{m_A^2\} - \underline{m_A^2} = m_B^2 - \underline{m_A^2}$$

for all $m_B^2 \in [0, \frac{27}{24}t)$. Doing so yields

$$\min\{m_A^2\} - \underline{m_A^2} = \frac{1}{576}\frac{288tm_B^2(16m_B^2 - 81t)^2 + 19683t^3 - \Lambda}{t(8m_B^2 - 27t)} \tag{6.7}$$

Equation (6.7) is zero if and only if $m_B^2 = 0, \frac{189}{64}t \pm \frac{27}{64}t\sqrt{17}$. Therefore, it is quite trivial to check that this $\min\{m_A^2\} \geq \underline{m_A^2}$ for all $m_B^2 \in [0, \frac{27}{24}t)$. Hence, for all m_B^2 defined as in equation (2.7), m_A^2 is strictly larger than $\underline{m_A^2}$.

Second, $\overline{m_A^2} = 0$ if and only if $m_B^2 = \frac{81}{32}t$. Again, this implies that $\overline{m_A^2}$ is continuous over $m_B^2 \in [0, \frac{27}{24}t)$. This only leaves us to evaluate

$$\max\{m_A^2\} - \overline{m_A^2} = \frac{27}{16}t - \frac{m_B^2}{t} - \overline{m_A^2}$$

for all $m_B^2 \in [0, \frac{27}{24}t)$. With some manipulation

$$\max\{m_A^2\} - \overline{m_A^2} = \frac{1}{576}\frac{-9t(16m_B^2 - 27t)^2 + \Lambda}{t(8m_B^2 - 27t)} \tag{6.8}$$

Equation (6.8) is zero if and only if $m_B^2 = \frac{27}{16}t$ which lies outside the relevant range of m_B^2. In this case, it is trivial to check that this is strictly negative. Hence, for all m_B^2 defined as in equation (2.7), m_A^2 is strictly smaller than $\overline{m_A^2}$.

The implication of equations (6.7) and (6.8) being positive and negative respectively, for all relevant m_A^2 and m_B^2 pairs, is that $m_A^2 \in (\underline{m_A^2}, \overline{m_A^2})$. However, it is already known that between these critical values $\pi_A - \pi_B$ must have a constant sign. Thus, taking $m_A^2 = t$ and $m_B^2 = 0$ we obtain

$$\pi_A - \pi_B = \frac{294}{361}t > 0$$

Naturally, if $\pi_A - \pi_B$ is strictly positive for these values of m_A^2 and m_B^2, it must hold from the signs of equations (6.7) and (6.8) that this holds for all relevant values of m_A^2 and m_B^2. Thus, $\pi_A - \pi_B > 0$ holds for all m_A^2 and m_B^2 as defined in equations (2.6) and (2.7). ■

6.4 Proof of Proposition 5

Proof. The second order conditions are given by:

$$\frac{\partial^2 \pi_A}{\partial I_A^2} = \frac{8m_A^2}{27t} - 1 < 0 \tag{6.9}$$

$$\frac{\partial^2 \pi_B}{\partial I_B^2} = \frac{216tm_B^2}{(8m_A^2 - 27t)^2} - 1 < 0 \tag{6.10}$$

From equation (6.9) it is possible to observe that the denominators of equations (2.11) and (2.10) are strictly negative. Given this, it is necessary that the numerators of (2.11) and (2.10) must be non-positive for I_A and $I_B \geq 0$ to hold. Given that $m_A > m_B > 0$ and $t > 0$, this implies:

$$\begin{aligned} 16m_A^2 - 27t &\leq 0 \\ 8m_A^2 + 16m_B^2 - 27t &\leq 0 \end{aligned}$$

Rearranging these inequalities, noting $m_A > m_B > 0$ and $t > 0$, yields:

$$m_A^2 \in (m_B^2, \min\{\frac{27}{16}t, \frac{27}{8}t - 2m_B^2\}] \tag{6.11}$$

$$m_B^2 \in [0, \min\{m_A^2, \frac{27}{16}t - m_B^2\}) \tag{6.12}$$

The restrictions in (6.11) and (6.12) ensure that equilibrium investment is non-negative, but it also necessary to check that they are compatible with the second order conditions. This requires:

$$m_A^2 < \frac{27}{8}t \tag{6.13}$$

$$m_B^2 < \frac{(8m_A^2 - 27t)^2}{216t} = \widetilde{m_B} \tag{6.14}$$

Equations (6.13) and (6.14) offer the maximum *ex ante* cost reducing efficiencies for each firm that are consistent with the second order conditions. It is most obvious that equation (6.13) is always met as it is fairly trivial to show $\max\{m_A^2\} = \frac{27}{16}t$. It is then trivial to note that $\widetilde{m_B}$ is strictly positive and, consequently, that $\widetilde{m_B} > \min\{m_B^2\} = 0$. Thus, it is then a case of simply ensuring that $\max\{m_B^2\} < \widetilde{m_B}$.

First, for $m_A^2 \in (0, \frac{27}{24}t]$, $\max\{m_B^2\} = m_A^2$. Thus

$$\max\{m_B^2\} - \widetilde{m_B} = -\frac{1}{216}\frac{64m_A^4 - 81t(8m_A^2 - 9t)}{t}$$

This equals zero where $m_A^2 = \frac{81}{16}t \pm \frac{27}{16}t\sqrt{5}$ both of which are strictly larger than $\frac{27}{24}t = \max\{m_B^2\}$. Therefore, for all $m_A^2 \in (0, \frac{27}{24}t]$ and over all $m_B^2 \in (0, \frac{27}{24}t)$ it must hold that $\max\{m_B^2\} - \widetilde{m_B}$ has the same sign. It is then easy to show $\max\{m_B^2\} - \widetilde{m_B} < 0$ for all $m_A^2 \in (0, \frac{27}{24}t]$.

Second, where $m_A^2 \in [\frac{27}{24}t, \frac{27}{16}t]$, $\max\{m_B^2\} = \frac{27}{16}t - \frac{m_A^2}{2}$. This implies

$$\max\{m_B^2\} - \widetilde{m_B} = -\frac{1}{432}\frac{(8m_A^2 - 27t)(16m_A^2 - 27t)}{t}$$

This is negative for all $m_A^2 \in (0, \frac{27}{16}t)$ and equal to zero if and only if $m_A^2 = \frac{27}{16}t$. However, all possible m_B^2 are strictly less than $\max\{m_B^2\}$, which doesn't cause a problem. Therefore, this implies that the second order conditions are met for all m_A^2 and m_B^2 pairs.

Finally, it is necessary to check that $\varphi_A - \varphi_B = m_A I_A - m_B I_B \in (-\frac{9}{4}t, \frac{9}{4}t)$ also holds, or that the *ex post* cost asymmetry allows for a subgame perfect Nash equilibrium. This is true if and only if

$$m_A I_A - m_B I_B > -\frac{9}{4}t \quad (6.15)$$
$$m_A I_A - m_B I_B < \frac{9}{4}t \quad (6.16)$$

From equation (6.15) we obtain

$$\frac{243}{4}\frac{t^2(8m_A^2 + 16m_B^2 - 27t)}{216tm_B^2 - (8m_A^2 - 27t)^2} > 0$$

which implies $m_A^2 + 2m_B^2 < \frac{27}{8}t$. Equation (6.16) on the other hand requires

$$\frac{9}{4}\frac{t(8m_A^2 - 27t)(16m_A^2 - 27t)}{216tm_B^2 - (8m_A^2 - 27t)^2} < 0$$

This equation implies that $m_A^2 < \frac{27}{16}t$.

Combining this final element with (6.11) and (6.12) the non-negative investment and second order conditions it is apparent that we must remove a single case (where $m_A^2 = \frac{27}{16}t$). Doing this obtains the restriction in equations (2.12) and (2.13). ■

6.5 Proof of Proposition 6

Proof. (*i*) It is trivial to demonstrate that I_A and I_B are strictly positive as this follows directly from Proposition 5.

(*ii*) Taking $m_A^2 \in (m_B^2, \frac{27}{16}t)$ as given and m_B^2 defined as in (2.13):

$$\frac{\partial I_A}{\partial m_B} = \frac{-288tm_A m_B(8m_A^2 - 27t)(16m_A^2 - 27t)}{[216tm_B^2 + (8m_A^2 - 27t)^2]^2} < 0$$
$$\frac{\partial I_B}{\partial m_B} = \frac{-18t(16m_A^2 - 27t)(216tm_B^2 + (8m_A^2 - 27t)^2)}{[216tm_B^2 + (8m_A^2 - 27t)^2]^2} > 0$$

Consequently, it is easy to see that, taking m_A^2 as given, an increase to m_B^2

decreases (increases) the equilibrium investment levels of firm B (A).

(iii) Where equations (2.12) and (2.13) hold

$$I_A - I_B = -\frac{18t[(m_B - m_A)(16m_A m_B + 27t) + 8m_B^3]}{216tm_B^2 + (8m_A^2 - 27t)^2}$$

This function is equal to zero if and only if $m_B^2 = m_B^I$ where

$$m_B^I = \frac{1}{1024}\frac{[16m_A^2 - 27t \pm \sqrt{\Omega}]^2}{m_A^2}$$

and

$$\Omega = 729t^2 + 864tm_A^2 - 256m_A^4$$

Of course, for the function to be continuous we require that $m_A^2 \neq 0$ and $\Omega \geq 0$. The first of these follows from Proposition 5, but $\Omega \geq 0$ if and only if

$$m_A^2 \in [\frac{27}{16}t(1-\sqrt{2}), \frac{27}{16}t(1+\sqrt{2})]$$

This clearly contains all relevant m_A^2 and so is not an issue.

Note that there are two critical values of m_B^2 which we will denote as

$$\begin{aligned} m_B^{I+} &= \frac{1}{1024}\frac{[16m_A^2 - 27t + \sqrt{\Omega}]^2}{m_A^2} \\ m_B^{I-} &= \frac{1}{1024}\frac{[16m_A^2 - 27t - \sqrt{\Omega}]^2}{m_A^2} \end{aligned}$$

It is possible to observe that $m_B^{I+} > 0$. This is because $m_B^{I+} = 0$ could only occur if $m_A^2 = 0$ but, if this were the case, $m_B^{I+} = \frac{0}{0}$ which is not defined. Therefore, m_B^{I+} is continuous over $m_A^2 \in (0, \frac{27}{16}t)$ and it is trivial to demonstrate that it is always positive over this range. In contrast, $m_B^{I-} > 0$ if $|m_A^2| = \frac{27}{8}t$ and is, therefore, continuous over all $m_A^2 \in (0, \frac{27}{16}t)$ also. However, it is possible to demonstrate that m_B^{I-} is always positive over this range.

Given that both m_B^{I+} and m_B^{I-} are strictly positive over the relevant range, it is useful to observe

$$m_B^{I-} - m_B^{I+} = -\frac{1}{256}\frac{(16m_A^2 - 27t)\sqrt{\Omega}}{m_A^2} > 0$$

It is also important to note that the second order conditions are violated if[1]

$$m_B^2 > \frac{(8m_A^2 - 27t)^2}{216t} = \Gamma$$

Now

$$m_B^{I+} - \Gamma = \frac{1}{13824} \frac{(16m_A^2 - 27t)(-16m_A^2(16m_A^2 - 81t) - 729t^2 + 27t\sqrt{\Omega})}{tm_A^2}$$

which equals zero if and only if $m_A^2 = \frac{27}{16}t$. Therefore, for $m_A^2 \in (0, \frac{27}{16}t)$ the equation has the same sign. In this case, it is trivial to show that it is always negative. Likewise

$$m_B^{I-} - \Gamma = -\frac{1}{13824} \frac{(16m_A^2 - 27t)(16m_A^2(16m_A^2 - 81t) + 729t^2 + 27t\sqrt{\Omega})}{tm_A^2}$$

This function equals zero if and only if $m_A^2 = \frac{27}{16}t$ or $\frac{27}{8}t$ which implies it has the same sign for all $m_A^2 \in (0, \frac{27}{16}t)$. In this case, it is trivial to check that it must be positive.

Consequently, we can ignore m_B^{I-} (as it would violate the SOCs) and simply use $m_B^{I+} = m_B^I$. This implies that for all $m_B^2 \in (m_B^I, \Gamma)$ it must hold that $I_A - I_B$ has the same sign also. Therefore, we must examine

$$\max\{m_B^2\} - m_B^I = m_A^2 - m_B^I \; \forall \; m_A^2 \in (0, \frac{27}{24}t] \tag{6.17}$$

$$\max\{m_B^2\} - m_B^I = \frac{27}{16}t - \frac{m_A^2}{2} - m_B^I \; \forall \; m_A^2 \in (\frac{27}{24}t, \frac{27}{16}t] \tag{6.18}$$

From equation (6.17) we observe

$$m_A^2 - m_B^I = -\frac{1}{512} \frac{(16m_A^2 - 27t)\sqrt{\Omega} + 729t^2 - 512m_A^4}{m_A^2}$$

which is never equal to zero. Thus, it is continuous over all $m_A^2 \in (0, \frac{27}{24}t]$ and

[1] As this value of m_B^2 is derived from the second order conditions, it must be that all relevant m_B^2 - contained in equation (13) - meet this criteria. Therefore

$$\max\{m_B^2\} < \Gamma$$

positive over this range. Similarly, from equation (6.18) we observe

$$\frac{27}{16}t - \frac{m_A^2}{2} - m_B^I = -\frac{1}{512}\frac{(16m_A^2 - 27t)(16m_A^2 - 27t + \sqrt{\Omega})}{m_A^2}$$

which equals zero if $m_A^2 = \frac{27}{16}t$. However, as this is not a potential value for m_A^2 it must be that it is continuous over all $m_A^2 \in [\frac{27}{24}t, \frac{27}{16}t)$ and it is trivial to demonstrate that this, too, is positive.

Therefore, it must be that, for all $m_A^2 \in (0, \frac{27}{16}t)$, $m_B^I \in (0, \max\{m_B^2\})$. ■

6.6 Proof of Proposition 7

Proof. (i) The *ex post* cost differential is given by

$$m_A I_A - m_B I_B = \frac{18tm_A m_B(8m_A^4 - 27t(m_A - m_B)(m_A + m_B))}{216tm_B^2 + (8m_A^2 - 27t)^2}$$

Furthermore, this equation is equal to zero if and only if $m_B^2 = m_B^\varphi$ where

$$m_B^\varphi = -\frac{1}{27}\frac{m_A^2(8m_A^2 - 27t)}{t}$$

Naturally, this is strictly positive and implies that it must be larger than the minimum potential value of m_B^2. Therefore, for $m_A^2 \in (0, \frac{27}{24}t]$ we find

$$\max\{m_B^2\} - m_B^\varphi = m_A^2 - m_B^\varphi = \frac{8}{27}\frac{m_A^4}{t} > 0$$

which implies that $\max\{m_B^2\} > m_B^\varphi$ for all $m_A^2 \in (0, \frac{27}{24}t]$. In addition, for all $m_A^2 \in [\frac{27}{24}t, \frac{27}{16}t)$

$$\max\{m_B^2\} - m_B^\varphi = \frac{27}{16}t - \frac{m_A^2}{2} - m_B^\varphi = \frac{1}{432}\frac{(8m_A^2 - 27t)(16m_A^2 - 27t)}{t} > 0$$

which implies $\max\{m_B^2\} > m_B^\varphi$ for all $m_A^2 \in [\frac{27}{24}t, \frac{27}{16}t)$.

Together, these imply that $\max\{m_B^2\} > m_B^\varphi$ for all $m_A^2 \in (0, \frac{27}{16}t]$. Moreover, as $m_A I_A - m_B I_B = 0$ if and only if $m_B^2 = m_B^\varphi$, we can observe that the sign of the equation is the constant on either side of m_B^φ. Thus, it is trivial to check, for a given m_A^2, if $m_B^2 < m_B^\varphi$, $m_A I_A - m_B I_B > 0$. Therefore, if $m_B^2 > m_B^\varphi$ then $m_A I_A - m_B I_B < 0$.

(ii) It is easy to check the relationship between m_B^φ and m_B^I by observing

$$m_B^\varphi - m_B^I = -\frac{1}{13824}\frac{(16m_A^2 - 27t)(16m_A^2(16m_A^2 - 27t) - 729t^2 + 27t\sqrt{\Omega})}{tm_A^2}$$

This equation is zero if and only if $m_A^2 = \frac{27}{16}t$. Therefore, over all $m_A^2 \in (0, \frac{27}{16}t]$ this must have the same sign. Therefore, it is trivial to then check, for any m_A^2, that $m_B^\varphi > m_B^I$. ■

6.7 Proof of Proposition 8

Proof. (i) Taking m_A^2 and m_B^2 as defined in (2.12) and (2.13) is is trivial to demonstrate that the profits for both firms are strictly positive.

(ii) The difference in firm profits is given by

$$\pi_A - \pi_B = \frac{6t\left\{\begin{array}{c} m_A^2(81t - 32m_A^2)(8m_A^2 - 27t)^2 \\ +729t^2 m_B^2(32m_B^2 - 81t) - 864m_A^2 m_B^2(8m_B^2 - 27t) \end{array}\right\}}{[216tm_B^2 - (8m_A^2 - 27t)^2]^2}$$

Of course, it is not obvious whether this function is positive or negative. However, we can observe that $\pi_A - \pi_B = 0$ if and only if $m_B^2 = m_B^\pi$ where

$$m_B^\pi = \frac{1}{576}\frac{243t^2(32m_A^2 - 81t) \pm \sqrt{\Sigma}}{t(8m_A^2 - 27t)}$$

and where

$$\Sigma = -3t(16m_A^2 - 81t)(32m_A^2 - 81t)(16m_A^2 - 27t)^3$$

Of course, we require that $\Sigma \geq 0$. This occurs for all $m_A^2 \in [0, \frac{27}{16}t] \cap [\frac{81}{32}t, \frac{81}{16}t]$ which is contained in the feasible set of m_A^2.

However, this does imply that there are two critical values for which $\pi_A -$

$\pi_B = 0$ given by

$$\begin{aligned} m_B^{\pi+} &= \frac{1}{576}\frac{243t^2(32m_A^2 - 81t) + \sqrt{\Sigma}}{t(8m_A^2 - 27t)} \\ m_B^{\pi-} &= \frac{1}{576}\frac{243t^2(32m_A^2 - 81t) - \sqrt{\Sigma}}{t(8m_A^2 - 27t)} \end{aligned}$$

First, over the relevant range of $m_A^2 \in (0, \frac{27}{16}t)$, $m_B^{\pi+} = 0$ if and only if $m_A^2 = 0$; which is not contained in this set. Therefore, for all $m_A^2 \in (0, \frac{27}{16}t]$, over which the function is continuous, the sign will be the same. It is trivial, then, to check that this is positive. Second, over the relevant range of $m_A^2 \in (0, \frac{27}{16}t]$, $m_B^{\pi-} = 0$ if and only if $m_A^2 = 0$ also. Consequently, the sign on this is constant for all $m_A^2 \in (0, \frac{27}{16}t]$ and it is trivial to observe that this is positive.

Therefore, we know that both $m_B^{\pi+}$ and $m_B^{\pi-}$ are positive. However, it is fairly simple to observe which is larger by checking

$$m_B^{\pi+} - m_B^{\pi-} = \frac{1}{288}\frac{\sqrt{\Sigma}}{(8m_A^2 - 27t)}$$

For all m_A^2 as defined as in equation (2.12) it must be that this is negative, or $m_B^{\pi+} < m_B^{\pi-}$.

However, we must also check that they meet the necessary SOCs. This requires

$$\begin{aligned} m_B^{\pi+} - \Gamma &= \frac{1}{1728}\frac{-(16m_A^2 - 27t)(256m_A^4 - 2160tm_A^2 + 3645t^2) + 3\sqrt{\Sigma}}{t(8m_A^2 - 27t)} < 0 \\ m_B^{\pi-} - \Gamma &= \frac{1}{1728}\frac{-(16m_A^2 - 27t)(256m_A^4 - 2160tm_A^2 + 3645t^2) - 3\sqrt{\Sigma}}{t(8m_A^2 - 27t)} < 0 \end{aligned}$$

Interestingly, both of these equations only equal zero where $m_A^2 = \frac{27}{16}t$. Thus, for all $m_A^2 \in (0, \frac{27}{16}t)$ these equations have the same sign and it is not difficult to check that these must be negative. This means that both of these critical values are below the upper bound set by the SOCs. Consequently, it is possible to argue that for all $m_B^2 \in (m_B^{\pi+}, m_B^{\pi-})$ the difference in firm profits has the same sign.

Therefore, it is only left to check that $m_B^{\pi+}$ or $m_B^{\pi-}$ are contained within the feasible set of m_B^2. Obviously, as both of these are strictly positive, it must be that they are above the lower bound. However, we must check whether they are under the upper bound. First, for all $m_A^2 \in (0, \frac{27}{24}t]$ we can check

$$m_B^{\pi+} - \max\{m_B^2\} = m_B^{\pi+} - m_A^2 \tag{6.19}$$
$$= \frac{1}{576}\frac{-288tm_A^2(16m_A^2 - 81t) - 19683t^3 + \sqrt{\Sigma}}{t(8m_A^2 - 27t)}$$
$$m_B^{\pi+} - \max\{m_B^2\} = m_B^{\pi-} - m_A^2 \tag{6.20}$$
$$= \frac{1}{576}\frac{-288tm_A^2(16m_A^2 - 81t) - 19683t^3 - \sqrt{\Sigma}}{t(8m_A^2 - 27t)}$$

Equation (6.19) equals zero if and only if $m_A^2 = 0$. Therefore, for all $m_A^2 \in (0, \frac{27}{24}t]$ it must have the same sign (and is continuous). It is trivial then to show that this is negative. Equation (6.20) is zero if and only if $m_A^2 = \frac{27}{64}t(7 \pm \sqrt{17}) > \frac{27}{24}t$. Therefore, this too has the same sign over the relevant range but, in this instance, the sign is positive. Consequently, for all $m_A^2 \in (0, \frac{27}{24}t]$ only $m_B^{\pi+}$ is contained within the set of possible m_B^2.

Second, we must check, for $m_A^2 \in [\frac{27}{24}t, \frac{27}{16}t)$

$$m_B^{\pi+} - \max\{m_B^2\} = m_B^{\pi+} - (\frac{27}{16}t - \frac{m_A^2}{2}) \tag{6.21}$$
$$= \frac{1}{576}\frac{6561t^3 - 7776t^2m_A^2 + 2304tm_A^4 + \sqrt{\Sigma}}{t(8m_A^2 - 27t)}$$
$$m_B^{\pi+} - \max\{m_B^2\} = m_B^{\pi-} - (\frac{27}{16}t - \frac{m_A^2}{2}) \tag{6.22}$$
$$= \frac{1}{576}\frac{6561t^3 + 7776t^2m_A^2 - 2304tm_A^4 - \sqrt{\Sigma}}{t(8m_A^2 - 27t)}$$

Equation (6.21) is equal to zero only where $m_A^2 = \frac{27}{16}t$. Thus, for all $m_A^2 \in [\frac{27}{24}t, \frac{27}{16}t)$, this equation has the same sign and it is simply show that this is negative. The second equation, (6.22), is zero if and only if $m_A^2 = \frac{27}{16}t$. Therefore, this too has the same sign over the relevant range. However, in this case, the sign is positive. Consequently, for all $m_A^2 \in [\frac{27}{24}t, \frac{27}{16}t)$ only $m_B^{\pi+}$ is contained within the set of possible m_B^2.

Therefore, given that the sign of $\pi_A - \pi_B$ is constant for all $m_B^2 \in (m_B^{\pi+}, m_B^{\pi-})$, $m_B^{\pi+} < \max\{m_B^2\}$ and $m_B^{\pi-} > \max\{m_B^2\}$ it must be that for all $m_B^2 \in (m_B^{\pi+}, \max\{m_B^2\})$ the sign of $\pi_A - \pi_B$ is constant. We shall rename $m_B^{\pi+} = m_B^{\pi}$

as it is contained in the relevant set and it is then simple to demonstrate that, for all $m_A^2 \in (0, \frac{27}{16}t)$, if $m_B^2 > m_B^\pi$ then $\pi_A < \pi_B$.

(iii) Finally

$$m_B^\pi - m_B^\varphi = \frac{1}{1728} \frac{(16m_A^2 - 27t)(246m_A^4 - 1296tm_A^2 + 2187t^2) + 3\sqrt{\Sigma}}{t(8m_A^2 - 27t)}$$

This equals zero if and only if $m_A^2 = 0$ or $\frac{27}{16}t$ and is continuous between. This means that $m_B^\pi - m_B^\varphi$ has the same sign for all $m_A^2 \in (0, \frac{27}{16}t)$ and it is quite easy to show that this is positive.

Thus, $m_B^\pi > m_B^\varphi > m_B^I$ over the relevant range. ■

Chapter 7

Appendix B: Proofs of Chapter 3

7.1 Proof of Proposition 14

Proof. The distance between firm A across cases is given by

$$a_l - a_g = \frac{2}{75}\frac{\beta(1-\alpha)(1+\alpha)\Lambda}{\left[\begin{array}{c} t\left[8(1+\alpha^2)(1+\beta^2) - 32\alpha\beta - 27t\right] \\ \left[8(1+\alpha^2)(256\beta^2+625) - 12800\alpha\beta - 16785t\right]\end{array}\right]} \tag{7.1}$$

where

$$\begin{aligned}\Lambda = 256(25\alpha - 16\beta)(25 - 16\alpha\beta)(1 - 4\alpha\beta + \alpha^2 + \beta^2 + \alpha^2\beta^2) \\ + 432t\left[25\beta(1+\alpha^2)(16\beta^2 - 75) + 4\alpha^2(625 - 72\beta^2)\right] \\ + 1658475\beta t^2\end{aligned}$$

Given equation (7.1), it is trivial to observe that the sign of this equation depends upon Λ. The other terms are easy to sign, with both terms of the denominator negative for $\frac{27}{16}(1 - \frac{16}{25}\alpha\beta)^2 < t$. Let $\frac{27}{16}(1 - \frac{16}{25}\alpha\beta)^2 = t^*$.

Examining Λ, it is possible to observe that this function is minimised if

$$\hat{t} = \frac{8}{61425}\frac{25\beta(1+\alpha^2)(16\beta^2 - 75) + 4\alpha(625 + 72\beta^2)}{\beta}$$

and, with some manipulation, we can observe

$$t^* - \hat{t} = \frac{8}{1535625} \frac{16\beta^3(3537\alpha^2 + 625) + 100\alpha(625 - 1384\beta^2) + 625\beta(107 - 75\alpha^2)}{\beta}$$

which is positive for all $\alpha \in (0,1)$, $\beta \in (0,\alpha)$. Therefore, Λ is increasing in t over the relevant range. Thus, at $t = \hat{t}$ we find

$$\Lambda = \frac{256}{15625}\alpha\,(25 - 16\alpha\beta)\,(25 + 16\beta)\,(25 - 16\beta)\left[\begin{array}{c} 3125 - \beta^2(625 + 2081\alpha^2) \\ +125\alpha(5\alpha - 51\beta) \end{array}\right]$$

which is positive. Thus, for all relevant t, α and β. Therefore, it must be that $a_l > a_g$.

If this is true for firm A then, given the constant distance between the firms, this must hold for B by symmetry. ■

7.2 Proof of Proposition 15

Proof. First, we observe

$$\frac{\partial a_l}{\partial I_A} - \frac{\partial a_g}{\partial I_A} = \frac{1}{25}\frac{\alpha\beta}{t}$$

which is greater than zero for all $\beta \in (0,\alpha)$.

Second, with some manipulation one finds

$$\frac{\partial a_l}{\partial I_B} - \frac{\partial a_g}{\partial I_B} = -\frac{1}{25}\beta$$

which is less than zero for all $\beta \in (0,\alpha)$. ■

7.3 Proof of proposition 16 (generic spillovers)

Proof. We prove each of these in turn

1. It is simple to check that the investment levels are positive. Both the numerator and denominator are decreasing in t. Therefore, it become

trivial to check that where $t = \frac{16}{27}(1 - \frac{16}{25}\alpha\beta)^2$ both the numerator and denominator are negative. More formally

$$\begin{aligned}
16(\alpha-\beta)^2 - 27t &= \frac{16}{625}\left[\begin{array}{c} 25(1+\alpha-\beta) \\ -16\alpha\beta \end{array}\right]\left[\begin{array}{c} 16\alpha\beta \\ -25(1-\beta+\alpha) \end{array}\right] \\
16(1-\alpha\beta)^2 - 27t &= -\frac{144}{625}\alpha\beta\,(50 - 41\alpha\beta) \\
\left\{\begin{array}{c} 8(1+\alpha)^2(1+\beta)^2 \\ -32\alpha\beta - 27t \end{array}\right\} &= \frac{8}{625}\alpha\beta(113\alpha\beta - 900) - 8(1+\alpha^2+\beta^2)
\end{aligned}$$

all of which are strictly negative for $\alpha \in (0,1)$ and $\beta \in (0,\alpha)$.

The difference between the investment levels are given by

$$I_{Ag} - I_{Bg} = \frac{2}{3}\frac{(1-\alpha)(1+\beta)[16(1-\alpha\beta)(\alpha-\beta) - 27t]}{8(1+\alpha)^2(1+\beta)^2 - 32\alpha\beta - 27t}$$

Given the above analysis, we can easily sign the denominator and so our focus is simply on the sign of the numerator. Again, this is reducing in t and, for $t = \frac{16}{27}(1 - \frac{16}{25}\alpha\beta)^2$ we observe

$$16(1-\alpha\beta)(\alpha-\beta) - 27t = 512\alpha\beta(25 - 8\alpha\beta) + 16\left[(1-\alpha\beta)(\alpha-\beta) - 1\right] < 0$$

for all $\alpha \in (0,1)$ and $\beta \in (0,\alpha)$. Therefore, $I_{Ag} - I_{Bg} > 0$ or $I_{Ag} > I_{Bg}$.

2. In this case

$$\varphi_{Ag} - \varphi_{Bg} = -\frac{18t(1-\alpha)(1+\alpha)(1-\beta)(1+\beta)}{8(1+\alpha)^2(1+\beta)^2 - 32\alpha\beta - 27t}$$

Given our analysis in 1., it is trivial to observe that this must be positive for all $t > \frac{16}{27}(1 - \frac{16}{25}\alpha\beta)^2$.

3. A pure strategy Nash equilibrium exists in the location stage if

$$\varphi_{Ag} - \varphi_{Bg} \in (-\frac{9}{4}t, \frac{9}{4}t)$$

For all $t > \frac{16}{27}(1 - \frac{16}{25}\alpha\beta)^2$ we know from 3. that $\varphi_A - \varphi_B > 0$ and so we only require

$$\varphi_{Ag} - \varphi_{Bg} < \frac{9}{4}t$$

or,

$$-\frac{9}{4}\frac{t\,[16(1-\alpha\beta)^2 - 27t]}{8(1+\alpha)^2(1+\beta)^2 - 32\alpha\beta - 27t} < 0$$

which occurs if

$$t > \frac{16}{27}(1-\alpha\beta)^2$$

It is then simple to observe

$$\frac{16}{27}(1-\frac{16}{25}\alpha\beta)^2 - \frac{16}{27}(1-\alpha\beta)^2 = \frac{16}{1875}\alpha\beta(50-41\alpha\beta) > 0$$

This ensures that an equilibrium exists in all stages of the game.

4. The profit functions are given by

$$\pi_{Ag} = -\frac{1}{36}\frac{(8(1-\alpha\beta)^2 - 27t)(16(\alpha-\beta)^2 - 27t)^2}{[8(1+\alpha)^2(1+\beta)^2 - 32\alpha\beta - 27t]^2} \quad (7.2)$$

$$\pi_{Bg} = -\frac{1}{36}\frac{(8(\alpha-\beta)^2 - 27t)(16(1-\alpha\beta)^2 - 27t)^2}{[8(1+\alpha)^2(1+\beta)^2 - 32\alpha\beta - 27t]^2} \quad (7.3)$$

The key term in both equations is the first term (in brackets) of the numerator. From these we observe that when they are negative the two profit functions are positive. This occurs if

$$t > \frac{8}{27}(1-\alpha\beta)^2$$
$$t > \frac{8}{27}(\alpha-\beta)^2$$

Given that

$$\frac{16}{27}(1-\frac{16}{25}\alpha\beta)^2 > \frac{8}{27}(1-\alpha\beta)^2 > \frac{8}{27}(\alpha-\beta)^2 > 0$$

the profits must always be positive.

The difference between the firms profits is given by

$$\pi_{Ag} - \pi_{Bg} = \frac{2}{9}\frac{(1-\alpha)(1+\alpha)(1-\beta)(1+\beta)\Gamma}{[8(1+\alpha)^2(1+\beta)^2 - 32\alpha\beta - 27t]^2}$$

where

$$\Gamma = 256(1-\alpha\beta)^2(\alpha-\beta)^2 + 27t[128\alpha\beta + 81t - 32(1+\beta^2)(1+\alpha^2)]$$

It is intuitive that the sign on this latter equation determines the sign of $\pi_A - \pi_B$. It is obvious that the first term of Γ is positive and the second term is increasing in t. Thus, where $t = \frac{16}{27}(1-\alpha\beta)^2$ we find

$$\Gamma = 256(1-\alpha)(1+\alpha)(1-\beta)(1+\beta)(1-\alpha\beta)^2 > 0$$

As $\frac{16}{27}(1-\frac{16}{25}\alpha\beta)^2 > \frac{16}{27}(1-\alpha\beta)^2$ it must be that $\pi_A - \pi_B > 0$, or $\pi_A > \pi_B$.

■

7.4 Proof of Proposition 16 (location based spillovers)

Proof. We prove each of these in turn.

1. Take any $t > \frac{16}{27}(1-\frac{16}{25}\alpha\beta)^2$. If this is the case then it must be that the denominators of both I_{Al} and I_{Bl} are strictly positive. The denominator is clearly decreasing in t and, even when $t = \frac{16}{27}(1-\frac{16}{25}\alpha\beta)^2$, we observe

   $$\left[\begin{array}{c} 8(256\beta^2+625)(1+\alpha^2) \\ -12800\alpha\beta - 16875t \end{array}\right] = -88(25-16\beta)(25+16\beta)(1-\alpha)(1+\alpha) < 0$$

   Thus both I_{Al} and I_{Bl} are strictly positive.

   The difference between the investment levels are given by

   $$I_{Al} - I_{Bl} = \frac{2}{75}\frac{(1-\alpha)(25+16\beta)[16(16\alpha\beta-25)(25\alpha-16\beta) - 16875t]}{8(256\beta^2+625)(1+\alpha^2) - 12800\alpha\beta - 16875t}$$

   It turns out that the numerator is strictly negative and, consequently, the sign on $I_{Al} - I_{Bl}$ is determined by the denominator. Given the above analysis it must be that $I_{Al} - I_{Bl} > 0$ or $I_{Al} > I_{Bl}$.

2. In this case

$$\varphi_{Al}-\varphi_{Bl}=\frac{2}{25}\frac{(1-\alpha)(1+\alpha)\left\{\begin{array}{c}32\beta(16\alpha\beta-25)(25\alpha-16\beta)\\-1125t(125-32\beta^2)\end{array}\right\}}{8(256\beta^2+625)(1+\alpha^2)-12800\alpha\beta-16875t}$$

Again, the numerator must be negative given the final term in square brackets. Consequently, the sign on $\varphi_{Al}-\varphi_{Bl}$ is determined by the sign on the denominator which, again, implies that for $\varphi_{Al}-\varphi_{Bl}>0$.

3. Unlike in the previous case, we do not end with a simple function that is dependent upon the size of the cost differential. Instead, we require

$$\frac{\partial^2\pi_{Al}}{\partial a^2}\Rightarrow\frac{1}{1518750}\frac{\left(\begin{array}{c}4I_B(25\alpha-16\beta)\\-4I_A(25-16\alpha\beta)\\-225t\end{array}\right)\left[\begin{array}{c}4I_A(152\alpha\beta-125)\\+4I_B(125\alpha-152\beta)\\+3375t\end{array}\right]}{t}<0$$

$$\frac{\partial^2\pi_{Bl}}{\partial b^2}\Rightarrow\frac{1}{1518750}\frac{\left(\begin{array}{c}4I_B(25\alpha-16\beta)\\-4I_A(25-16\alpha\beta)\\+225t\end{array}\right)\left[\begin{array}{c}4I_A(152\alpha\beta-125)\\+4I_B(125\alpha-152\beta)\\-3375t\end{array}\right]}{t}<0$$

Subbing in the equilibrium investment levels makes this even more unwieldy, with a functional form given by

$$\frac{\partial^2\pi_A}{\partial a^2}\Rightarrow\frac{1}{2250}\frac{\left\{\begin{array}{c}\left[\begin{array}{c}1024\beta(1-\alpha)(1+\alpha)\\(25-16\alpha\beta)(25\alpha-16\beta)\end{array}\right]\\+234375t(81t-32-16\alpha^2)\\-38400t\beta(98\alpha^2\beta+22\beta-375\alpha)\end{array}\right\}\left(\begin{array}{c}16(25\alpha-16\beta)^2\\-16875t\end{array}\right)}{[8(256\beta^2+625)(1+\alpha^2)-12800\alpha\beta-16875t]^2}<0$$

$$\frac{\partial^2\pi_B}{\partial b^2}\Rightarrow\frac{1}{2250}\frac{\left\{\begin{array}{c}\left[\begin{array}{c}-1024\beta(1-\alpha)(1+\alpha)\\(25-16\alpha\beta)(25\alpha-16\beta)\end{array}\right]\\+234375t(81t-32\alpha^2-16)\\-38400t\beta(98\beta+22\alpha^2\beta-375\alpha)\end{array}\right\}\left(\begin{array}{c}16(25-16\alpha\beta)^2\\-16875t\end{array}\right)}{[8(256\beta^2+625)(1+\alpha^2)-12800\alpha\beta-16875t]^2}<0$$

Taking the former of these two second order equations, we can see that the sign of this equation is dependent on

$$\Lambda = \left\{ \begin{array}{c} 1024\beta(1-\alpha)(1+\alpha)(25-16\alpha\beta)(25\alpha-16\beta) \\ +234375t(81t-32-16\alpha^2) \\ -38400t\beta(98\alpha^2\beta+22\beta-375\alpha) \end{array} \right\} \tag{7.4}$$

and

$$16(25\alpha-16\beta)^2-16875t \tag{7.5}$$

where

$$\begin{aligned} \frac{\partial\Lambda}{\partial t} &= \left\{ \begin{array}{c} 38400\beta(375\alpha-22\beta)-1200\alpha^2(3136\beta^2+3125) \\ +37968750t-7500000 \end{array} \right\} \\ \frac{\partial^2\Lambda}{\partial t^2} &= 37968750 \end{aligned}$$

Thus, the equation is minimised where

$$t=\hat{t}=\frac{8}{81}(2+\alpha^2)+\frac{256}{253125}\beta(22\beta+98\alpha^2\beta-375\alpha)$$

and we can also note that

$$\hat{t}-\frac{8}{27}(1-\frac{16}{25}\alpha\beta)^2=-\frac{8}{253125}(3125-704\beta^2)(1+\alpha)(1-\alpha)<0$$

This implies that $\frac{8}{27}(1-\frac{16}{25}\alpha\beta)^2$ is strictly larger than $\hat{t}$ and that Λ is increasing over the relevant range of t. Moreover, as $\frac{8}{27}(1-\frac{16}{25}\alpha\beta)^2 < \frac{16}{27}(1-\frac{16}{25}\alpha\beta)^2$ this is also true over our relevant range. Taking any $t > \frac{16}{27}(1-\frac{16}{25}\alpha\beta)^2$, it is obvious that (7.5) is negative. Consequently, we require that (7.4) is positive. Setting $t=\frac{16}{27}(1-\frac{16}{25}\alpha\beta)^2$ we find

$$\Lambda=\frac{256}{225}(25-16\beta)(25+16\beta)(1-\alpha)(1+\alpha)(25-16\alpha\beta)(125-44\alpha\beta)>0$$

and is *increasing* in t. Therefore, it must be that for all $t > \frac{16}{27}(1-\frac{16}{25}\alpha\beta)^2$ we have $\frac{\partial^2\pi_A}{\partial a^2}<0$ which ensures an equilibrium in the location stage.

The proof for $\frac{\partial^2\pi_B}{\partial b^2}$ is remarkably similar and involves identical steps and so is omitted here.

4. The profit equations are given by

$$\pi_{Al} = \frac{1}{22500}\frac{\left\{\begin{array}{c}(16875t - 8(25-16\alpha\beta))\\ \left[16(25\alpha-16\beta)^2 - 16875t\right]^2\end{array}\right\}}{\left[\begin{array}{c}8(256\beta^2+625)(1+\alpha^2)\\ -12800\alpha\beta - 16875t\end{array}\right]^2} \tag{7.6}$$

$$\pi_{Bl} = \frac{1}{22500}\frac{\left\{\begin{array}{c}(16875t - 8(25\alpha-16\beta))\\ \left[16(25-16\alpha\beta)^2 - 16875t\right]^2\end{array}\right\}}{\left[\begin{array}{c}8(256\beta^2+625)(1+\alpha^2)\\ -12800\alpha\beta - 16875t\end{array}\right]^2} \tag{7.7}$$

The denominator is strictly positive and so the sign of both equations is dependent on the numerator. It is trivial to show that π_A is positive if

$$t > \frac{8}{16875}(25-16\alpha\beta)^2$$

and π_B is positive if

$$t > \frac{8}{16875}(25\alpha-16\beta)^2$$

which holds for all $t > \frac{8}{27}(1-\frac{16}{25}\alpha\beta)^2$. As $\frac{16}{27}(1-\frac{16}{25}\alpha\beta)^2 > \frac{8}{27}(1-\frac{16}{25}\alpha\beta)^2$, it must be that both π_{Al} and π_{Bl} must be strictly positive for all relevant values of α, β and t.

The profit differential is given by

$$\pi_{Al} - \pi_{Bl} = \frac{2}{5625}\frac{(1-\alpha)(1+\alpha)(25-16\beta)(25+16\beta)\Theta}{\left[8(256\beta^2+625)(1+\alpha^2) - 12800\alpha\beta - 16875t\right]^2}$$

where

$$\Theta = \left[\begin{array}{c}256(25-16\alpha\beta)^2(25\alpha-16\beta)^2 - 540000t\alpha^2(625+256\beta^2)\\ +34560000t\beta(25\alpha-4\beta) + 10546875t(81t-32)\end{array}\right]$$

Again, it is obvious that Θ determines the sign on this equation. In this case, we observe

$$\begin{aligned}\frac{\partial\Theta}{\partial t} &= \begin{pmatrix} 34560000\beta(25\alpha-4\beta)-540000\alpha^2(625+256\beta^2) \\ +1708593750t-337500000 \end{pmatrix} \\ \frac{\partial^2\Theta}{\partial t^2} &= 1708593750\end{aligned}$$

This indicates that Θ is minimised where

$$t=\check{t}=\frac{16}{81}(1+\alpha^2)+\frac{1024}{50625}(4\alpha^2\beta-25\alpha+4\beta)$$

Now, it is possible to observe that

$$\frac{16}{27}(1-\frac{16}{25}\alpha\beta)^2-\check{t}=\frac{16}{81}(2-\alpha^2)+\frac{512}{50625}\beta(16\alpha^2\beta-25\alpha-8\beta)$$

which is strictly larger than zero for all $\alpha\in(0,1)$ and $\beta\in[0,\alpha)$. Therefore, for all $t>\frac{16}{27}(1-\frac{16}{25}\alpha\beta)^2$ it must be that Θ is increasing and, at $\frac{16}{27}(1-\frac{16}{25}\alpha\beta)^2$, we observe

$$\Theta=256(1-\alpha)(1+\alpha)(25-16\beta)(25+16\beta)(25-16\alpha\beta)^2>0$$

Therefore, for all $t>\frac{16}{27}(1-\frac{16}{25}\alpha\beta)^2$ it must be that $\pi_A-\pi_B>0$.

■

7.5 Proof of Proposition 17 (generic spillovers)

Proof. First, the the impact of α on I_{Ag} is given by

$$\frac{\partial I_{Ag}}{\partial\alpha}=-\frac{2}{3}\frac{f(\alpha,\beta,t)}{[8(1+\alpha)^2(1+\beta)^2-32\alpha\beta-27t]^2}$$

where $f(\alpha, \beta, t)$ is given by

$$f(\alpha,\beta,t) = 729\beta t^2 + 216t\left(\begin{array}{c} 2\alpha(1+3\beta^2) \\ -\beta^3(3-\alpha^2) - \beta(1+5\alpha^2) \end{array}\right) + 128\,(\alpha-\beta)\left\{\begin{array}{c} \alpha^3\beta(1+\beta) - \alpha^2\beta^2(7-\beta^2) \\ +\alpha\beta(5+\beta^2) \\ +\beta^2(1-\beta)(1+\beta) - 2 \end{array}\right\}$$

Furthermore, we observe

$$\begin{aligned} f_t &= 216\left[2\alpha(1+3\beta^2) - \beta^3(3-\alpha^2) - \beta(1+5\alpha^2)\right] + 1458t\beta \\ f_{tt} &= 1458\beta \end{aligned}$$

Consequently, we can observe this f minimised where $t = t^f$ such that

$$t^f = \frac{4}{27}\frac{\left[2\alpha(1+3\beta^2) - \beta^3(3-\alpha^2) - \beta(1+5\alpha^2)\right]}{\beta}$$

It is then simple to check that $\frac{16}{27}(1-\frac{16}{25}\alpha\beta)^2 > t^f$. Therefore, this function is everywhere increasing and, at $t = \frac{16}{27}(1-\frac{16}{25}\alpha\beta)^2$ we find $f(\alpha,\beta,t) > 0$ for all α and β. Therefore, this must be strictly negative.

Similarly, the the impact of β on I_{Ag} is given by

$$\frac{\partial I_{Ag}}{\partial \beta} = \frac{2}{3}\frac{g(\alpha,\beta,t)}{\left[8(1+\alpha)^2(1+\beta)^2 - 32\alpha\beta - 27t\right]^2}$$

where $g(\alpha, \beta, t)$ is given by

$$g(\alpha,\beta,t) = -729\alpha t^2 + 216t\left(\begin{array}{c} \alpha(1+3\alpha^2) - \\ \alpha\beta^2(\alpha^2-5) - 2\beta(1+3\alpha^2) \end{array}\right) + 128\,(\alpha-\beta)\left\{\begin{array}{c} \alpha^3\beta(1+\beta^2) + \alpha^2(1-7\beta^2) \\ +\alpha\beta(5+\beta^2) \\ -\alpha^4(1-\beta)(1+\beta) - 2 \end{array}\right\}$$

where

$$
\begin{aligned}
g_t &= 216\left[\alpha(1+3\alpha^2) - \alpha\beta^2(\alpha^2-5) - 2\beta(1+3\alpha^2)\right] - 1458t\alpha \\
g_{tt} &= -1458\alpha
\end{aligned}
$$

Therefore, $g(\alpha, \beta, t)$ is maximised where $t = t^g$, where

$$
t^g = \frac{4}{27}\frac{\left[\alpha(1+3\alpha^2) - \alpha\beta^2(\alpha^2-5) - 2\beta(1+3\alpha^2)\right]}{\alpha}
$$

Again, it is trivial to observe that $\frac{16}{27}(1-\frac{16}{25}\alpha\beta)^2 > t^g$. Thus, this function is always decreasing and, at $t = \frac{16}{27}(1-\frac{16}{25}\alpha\beta)^2$ we find $g(\alpha, \beta, t) < 0$. Thus, this must be negative for all relevant parameter values.

For equation the impact of α on I_{Bg},we find

$$
\frac{\partial I_{Bg}}{\partial \alpha} = \frac{2}{3}\frac{u(\alpha, \beta, t)}{\left[8(1+\alpha)^2(1+\beta)^2 - 32\alpha\beta - 27t\right]^2}
$$

where $u(\alpha, \beta, t)$ is given by

$$
\begin{aligned}
u(\alpha, \beta, t) &= 729t^2 + 216t\begin{pmatrix} [\alpha^2 + 6\alpha\beta + 2\alpha\beta^2 - 3] \\ -\beta^2(1+5\alpha^2) \end{pmatrix} \\
&\quad +128\,(1-\alpha\beta)\left\{\begin{matrix} 1-\alpha^3\beta(1+\beta^2) \\ -\alpha^2(1-7\beta^2) - \alpha\beta(1+5\beta^2) \\ -\beta^2(1-2\beta^2) \end{matrix}\right\}
\end{aligned}
$$

In this instance,

$$
\begin{aligned}
u_t &= 216\left([\alpha^2 + 6\alpha\beta + 2\alpha\beta^2 - 3] - \beta^2(1+5\alpha^2)\right) + 1458t \\
u_{tt} &= 1458
\end{aligned}
$$

Thus, $u(\alpha, \beta, t)$ is minimised where $t = t^u$, where

$$
t^u = \frac{4}{27}\left([\alpha^2 + 6\alpha\beta + 2\alpha\beta^2 - 3] - \beta^2(1+5\alpha^2)\right)
$$

It is possible to observe that $\frac{16}{27}(1-\frac{16}{25}\alpha\beta)^2 > t^u$ and so for all relevant t, $u(\alpha, \beta, t)$ is strictly increasing. Therefore, where $t = \frac{16}{27}(1-\frac{16}{25}\alpha\beta)^2$ it is trivial to observe $u(\alpha,\beta,t) > 0$. Therefore, $u(\alpha,\beta,t)$ is positive for all $\alpha \in (0,1)$, $\beta \in (0,\alpha)$ and $t > \frac{16}{27}(1-\frac{16}{25}\alpha\beta)^2$.

Finally, the impact of β on I_{Bg}, given by

$$\frac{\partial I_{Bg}}{\partial \beta} = \frac{2}{3}\frac{v(\alpha,\beta,t)}{\left[8(1+\alpha)^2(1+\beta)^2 - 32\alpha\beta - 27t\right]^2}$$

which depends upon $v(\alpha,\beta,t)$. Here, $v(\alpha,\beta,t)$ is given by

$$\begin{aligned} v(\alpha,\beta,t) &= -729t^2 + 216t\left[\begin{array}{c}(3+\alpha^2)(1-2\alpha\beta)\\ -\beta^2(1-5\alpha^2)\end{array}\right] \\ &\quad -128\,(1-\alpha\beta)\left\{\begin{array}{c}1+2\alpha^4-\alpha^3\beta(5+\beta^2)\\ -\alpha^2(1-7\beta^2)-\alpha\beta(1+\beta^2)\\ -\beta^2\end{array}\right\}\end{aligned}$$

In this instance

$$\begin{aligned} v_t &= 216\left[(3+\alpha^2)(1-2\alpha\beta) - \beta^2(1-5\alpha^2)\right] - 1458t \\ v_{tt} &= -1458 \end{aligned}$$

Obviously, this is maximised at $t = t^v$, where

$$t^v = \frac{4}{27}\left[(3+\alpha^2)(1-2\alpha\beta) - \beta^2(1-5\alpha^2)\right]$$

It is not difficult to show that $t^v < \frac{16}{27}(1-\frac{16}{25}\alpha\beta)^2$ and so for all relevant value of t - given α and β - this must be decreasing. Evaluating $v(\alpha,\beta,t)$ at the minimum value of $t = \frac{16}{27}(1-\frac{16}{25}\alpha\beta)^2$ implies that the sign is dependent on the size of both α and β. More formally, we can derive a value of t, equal to $t^{\beta g}$, such that for $t < t^{\beta g}$, this value will be positive. This is

$$t^* = \frac{4}{9} - \frac{4}{27}\left[\alpha^2(2\alpha\beta - 5\beta^2 - 1) + \beta(\beta + 6\alpha)\right] + \frac{4}{27}\sqrt{\Phi}$$

where

$$\Phi = (1-\alpha)(1+\alpha)\left[\begin{array}{c}1+15\alpha^2-4\alpha\beta(3\alpha^2+5)+2\beta^2(1-\alpha)(1+\alpha)(2\alpha^2+1)\\ +4\alpha\beta^3(5\alpha^3+3)+\beta^4(1-17\alpha^2)\end{array}\right]$$

However, it is not possible to derive a value of α or β for which this ensures $t^* > \frac{16}{27}(1 - \frac{16}{25}\alpha\beta)^2$. Nonetheless, it is possible that this may occur for certain parameter levels. For example, $\alpha = 0.9$ and $\beta = 0.1$ imply

$$\begin{aligned} t^* &= 0.68 \\ \frac{16}{27}(1 - \frac{16}{25}\alpha\beta)^2 &= 0.53 \end{aligned}$$

■

7.6 Proof of Proposition 17 (location based spillovers)

Proof. This proofs for equations $\frac{\partial I_{Al}}{\partial \alpha}$, $\frac{\partial I_{Al}}{\partial \beta}$ and $\frac{\partial I_{Bl}}{\partial \alpha}$ are logically similar to those for $\frac{\partial I_{Ag}}{\partial \alpha}$, $\frac{\partial I_{Ag}}{\partial \beta}$ and $\frac{\partial I_{Bg}}{\partial \alpha}$ and so are omitted.[1]

The interesting case, as before, is the function

$$\frac{\partial I_{Bl}}{\partial \beta} = \frac{32}{75} \frac{h(\alpha, \beta, t)}{\left[8(256\beta^2 + 625)(1 + \alpha^2) - 12800\alpha\beta - 16875t\right]^2}$$

where $h(\alpha, \beta, t)$ is given by

$$\begin{aligned} h(\alpha, \beta, t) &= -284765625t^2 - 16875t \begin{bmatrix} 500(3 + \alpha^2) \\ -6400\alpha\beta(3 + \alpha^2)) \\ -2048\beta^2(1 - 5\alpha^2) \end{bmatrix} \\ &\quad -128(25 - 16\alpha\beta) \left\{ \begin{array}{c} 31250\alpha^4 - 16\alpha^3\beta(256\beta^2 + 3125) \\ +25\alpha^2(1792\beta^2 - 625) \\ -16\alpha\beta(256\beta^2 - 625) \\ +25(25 - 16\beta)(25 + 16\beta) \end{array} \right\} \end{aligned}$$

Furthermore, we observe

$$\begin{aligned} h_t &= 8437500(3 + \alpha^2) - 108000000\alpha\beta(3 + \alpha^2) - 34560000\beta^2(1 - 5\alpha^2) - 569531250t \\ h_{tt} &= -569531250 \end{aligned}$$

[1] Proofs for these are available upon request.

Consequently, the function is maximised when $t = t^h$, where

$$t^h = \frac{8437500(30+\alpha^2) - 108000000\alpha\beta(3+\alpha^2) - 34560000\beta^2(1-5\alpha^2)}{569531250}$$

It is possible to observe that $t^h < \frac{16}{27}(1 - \frac{16}{25}\alpha\beta)^2$ and so $h(\alpha, \beta, t)$ is strictly decreasing for all relevant t. However, as with the last case, we observe that the sign on $h(\alpha, \beta, t)$ is always strictly positive when $t = \frac{16}{27}(1 - \frac{16}{25}\alpha\beta)^2$, regardless of the initial parameters α and β. Moreover, there exists a value of t, noted as $t^{\beta l} \in (\frac{16}{27}(1 - \frac{16}{25}\alpha\beta)^2, \infty)$, such that $\frac{\partial I_{Bg}}{\partial \beta} = 0$. This is given by

$$t^{\beta l} = \frac{4}{27}(3+\alpha^2) - \frac{128}{16875}\beta(25\alpha\left[3+\alpha^2\right] + 8\beta\left[1-5\alpha^2\right]) + \frac{4}{16875}\sqrt{\Sigma}$$

where

$$\Sigma = (1-\alpha)(1+\alpha)\left[\begin{array}{c} 390625(1+15\alpha^2) - 1000000\alpha\beta(3\alpha^2+5) \\ +320000\beta^2(1-\alpha)(1+\alpha)(1+2\alpha^2) \\ +409600\alpha\beta^3(3+5\alpha^2) + 65536\beta^4(1-17\alpha^2) \end{array}\right]$$

It then becomes simple to check that for all α and β

$$t^{\beta l} - \frac{16}{27}(1 - \frac{16}{25}\alpha\beta)^2 > 0$$

■

7.7 Proof of Proposition 18

Proof. The difference between the investment levels in the two cases are given by

$$I_{Al} - I_{Ag} = -\frac{6}{25}\frac{\beta x}{\left\{\begin{array}{c} \left(8(1+\alpha^2)(1+\beta^2) - 32\alpha\beta - 27t\right) \\ \left[8(1+\alpha^2)(256\beta^2+625) - 12800\alpha\beta - 16785t\right] \end{array}\right\}}$$

$$I_{Bl} - I_{Bg} = -\frac{6}{25}\frac{\beta y}{\left\{\begin{array}{c} \left(8(1+\alpha^2)(1+\beta^2) - 32\alpha\beta - 27t\right) \\ \left[8(1+\alpha^2)(256\beta^2+625) - 12800\alpha\beta - 16785t\right] \end{array}\right\}}$$

where

$$
\begin{aligned}
x &= 80000\alpha(2-\alpha^2+\alpha^4) - 131200\beta(1+\alpha^2)^2 \\
&\quad +128\alpha\beta^2(2081+3281\alpha^2-400\alpha^4) - 8192\alpha\beta^3(41\alpha-4\alpha^2\beta-4\beta) \\
&\quad +216t\left[(1025\beta-625\alpha)(1+3\alpha^2)+2\alpha\beta^2(200\alpha^2-1081)\right] + 455625\alpha t^2
\end{aligned}
$$

$$
\begin{aligned}
y &= 131200\alpha\beta(1+\alpha^2)^2 + 128\beta^2(400-3281\alpha^2-2081\alpha^4) \\
&\quad -80000(1-\alpha^2+\alpha^4) + 8192\alpha^2\beta^3(41\alpha-4\alpha^2\beta-4\beta) \\
&\quad -216t\left[(1025\alpha\beta-625)\,(3+a^2)+2\beta^2(200-1081\alpha^2)\right] - 455625t^2
\end{aligned}
$$

In both of these cases the sign on the equations simply depend upon x and y. Therefore it is sufficient to evaluate these functions.

First, x is clearly continuous for all $t > \frac{16}{27}(1-\frac{16}{25}\alpha\beta)^2$, with a positive denominator. Therefore, we must simply evaluate the sign to determine the sign of x. In this instance we can observe

$$
\begin{aligned}
\frac{\partial x}{\partial t} &= -216\left[(1025\beta-625\alpha)(1+3\alpha^2)+2\alpha\beta^2(200\alpha^2-1081)\right] + 911250\alpha t \\
\frac{\partial^2 x}{\partial t^2} &= 911250\alpha
\end{aligned}
$$

Consequently, x is maximised where $t = t^*$, where

$$
t^* = -\frac{4}{16875}\frac{\left[(1025\beta-625\alpha)(1+3\alpha^2)+2\alpha\beta^2(200\alpha^2-1081)\right]}{\alpha}
$$

It is easy to check that

$$
\frac{16}{27}(1-\frac{16}{25}\alpha\beta)^2 - t^* > 0
$$

for all α and β and so this function is decreasing. Thus, evaluating x at $t = \frac{16}{27}(1-\frac{16}{25}\alpha\beta)^2$ we obtain

$$
x = -\frac{128}{625}\alpha(25-16\beta)(25+16\beta)(1-\alpha)(1+\alpha)\left[1875-\alpha^2(625-912\beta^2)-125\beta(11\alpha+5\beta)\right]
$$

which is strictly negative. Therefore, x must always be negative and $I_{Al} > I_{Ag}$ for all relevant parameters.

The case of y is slightly more complicated. It is possible to observe that $I_{Bl} = I_{Bg}$ if and only if $t = t^y$ where[2]

$$t^y = -\frac{4}{675}\beta(123\alpha + 41\alpha^3 + 16\beta) + \frac{4}{16875}\alpha^2(625 + 2162\beta^2) + \frac{4}{16875}\sqrt{\Psi}$$

and

$$\begin{aligned}\Psi &= 390625(1-\alpha)(1+\alpha)(1+15\alpha^2) - 1281250\alpha\beta(1-\alpha)(1+\alpha)(3\alpha^2+50) \\ &\quad +625\beta^2(800 + 1053\alpha^2 - 2238\alpha^4 + 1681\alpha^6) + 4100\alpha\beta^3(600 + 157\alpha^2 - 1081\alpha^4) \\ &\quad +4\beta^4(3521\alpha^2 - 200)(241\alpha^2 - 200)\end{aligned}$$

With some manipulation it is possible to demonstrate

$$t^y > \frac{16}{27}(1 - \frac{16}{25}\alpha\beta)^2$$

for all relevant α and β. Consequently, given any α and β pair, we must simply check whether y is positive or negative. Assuming $t = \frac{16}{27}(1 - \frac{16}{25}\alpha\beta)$ - which is less than t^y, we find that it is strictly negative. Therefore, for all $t < t^y$ we find $I_{Bl} < I_{Bg}$ with the reverse true for all $t > t^y$. ∎

[2] There are, in fact, two value for which this is true. However, this is the only value that lies within the relevant range of t.

Chapter 8

Appendix C: Proofs of Chapter 4

8.1 Proof of Proposition 19

Proof. The slope of the reaction functions are given by

$$R_i^{'} = -\frac{\frac{\partial^2 \pi^i}{\partial e_i^2 \partial e_j^2}}{\frac{\partial^2 \pi^i}{\partial (e_i^2)^2}}$$

Consider case (i) - $\left(q_l^i, q_l^j\right)$ first. Then observe that

$$\frac{\partial^2 \pi_{l,l}^i|_{NVC}^{t=2}}{\partial (e_i^2)^2} = \frac{\partial^2 \varphi_i^2}{\partial (e_i^2)^2}\left[(M - D_L) - \varphi_j^2 (M - D_L - D_H)\right] < 0 \quad (8.1)$$

$$\frac{\partial^2 \pi_{l,l}^i|_{NVC}^{t=2}}{\partial e_i^2 \partial e_j^2} = \frac{\partial \varphi_i^2}{\partial e_i^2}\frac{\partial \varphi_j^2}{\partial e_j^2}[D_L + D_H - M] < 0 \quad (8.2)$$

by assumption A1 (concavity) and $M > 2D_H > 2D_L$.

As both probability functions, $\varphi_i^2(a_i, e_i^2)$ and $\mu_i^2(a_i, e_i^2)$, possess similar properties (by A1 and A3) it then follows immediately that that the other cases yield the same result. ■

8.2 Proof of Proposition 20

Proof. The proof holds for all cases (i) - (iv) given the assumptions in A1 - A3. Consequently, we only prove this for the case (q_l^i, q_l^j), but similar proofs exist for all other cases. We solve this comparative static using Cramer's rule where

$$Ax = b$$

$$\begin{bmatrix} \frac{\partial^2 \pi_{l,l}^i|_{NVC}^{t=2}}{\partial (e_i^2)^2} & \frac{\partial^2 \pi_{l,l}^i|_{NVC}^{t=2}}{\partial e_i^2 \partial e_j^2} \\ \frac{\partial^2 \pi_{l,l}^j|_{NVC}^{t=2}}{\partial e_i^2 \partial e_j^2} & \frac{\partial^2 \pi_{l,l}^j|_{NVC}^{t=2}}{\partial (e_j^2)^2} \end{bmatrix} \begin{bmatrix} \frac{de_i^2}{dx} \\ \frac{de_j^2}{dx} \end{bmatrix} = \begin{bmatrix} -\frac{\partial^2 E\pi_i}{\partial e_i^2 \partial x} \\ -\frac{\partial^2 E\pi j}{\partial e_{j2} \partial x} \end{bmatrix}$$

Using this, we can obtain $\frac{de_{i2}}{da_i}$ by substituting $a_i = x$ and using

$$\frac{de_i^2}{da_i} = \frac{\left|A_{e_i^2 a_i}\right|}{|A|} = \frac{\begin{vmatrix} -\frac{\partial^2 \pi_{l,l}^i|_{NVC}^{t=2}}{\partial e_i^2 \partial a_i} & \frac{\partial^2 \pi_{l,l}^i|_{NVC}^{t=2}}{\partial e_i^2 \partial e_j^2} \\ -\frac{\partial^2 \pi_{l,l}^i|_{NVC}^{t=2}}{\partial e_{j2} \partial a_i} & \frac{\partial^2 \pi_{l,l}^j|_{NVC}^{t=2}}{\partial (e_j^2)^2} \end{vmatrix}}{\begin{vmatrix} \frac{\partial^2 \pi_{l,l}^i|_{NVC}^{t=2}}{\partial (e_i^2)^2} & \frac{\partial^2 \pi_{l,l}^i|_{NVC}^{t=2}}{\partial e_i^2 \partial e_j^2} \\ \frac{\partial^2 \pi_{l,l}^j|_{NVC}^{t=2}}{\partial e_i^2 \partial e_j^2} & \frac{\partial^2 \pi_{l,l}^j|_{NVC}^{t=2}}{\partial (e_j^2)^2} \end{vmatrix}}$$

which yields

$$|A| = \frac{\partial^2 \varphi_i^2}{\partial (e_i^2)^2} \frac{\partial^2 \varphi_j^2}{\partial (e_j^2)^2} \left\{ \begin{array}{c} [(M - D_L) - \varphi_i^2 (M - D_L - D_H)] \\ [(M - D_L) - \varphi_j^2 (M - D_L - D_H)] \end{array} \right\} - \left(\frac{\partial \varphi_i^2}{\partial e_i^2} \right)^2 \left(\frac{\partial \varphi_j^2}{\partial e_j^2} \right)^2 (M - D_H - D_L)^2$$

$$\left|A_{e_i^2 a_i}\right| = -\frac{\partial^2 \varphi_i^2}{\partial e_i^2 \partial a_i} \frac{\partial^2 \varphi_j^2}{\partial (e_j^2)^2} \left\{ \begin{array}{c} [(M - D_L) - \varphi_i^2 (M - D_L - D_H)] \\ [(M - D_L) - \varphi_j^2 (M - D_L - D_H)] \end{array} \right\} + \frac{\partial \varphi_i^2}{\partial e_i^2} \frac{\partial \varphi_i^2}{\partial a_i} \left(\frac{\partial \varphi_j^2}{\partial e_j^2} \right)^2 (M - D_H - D_L)^2$$

Signing these equations is quite simple. First, $\left|A_{e_i^2 a_i}\right|$ is strictly positive given the assumptions A1 and A2. The sign of $|A|$ is harder to interpret. However, assuming uniqueness and stability holds, or

$$\frac{1}{|R_i'|} > \left|R_j'\right| \tag{8.3}$$

we observe

$$\frac{\partial^2\varphi_i^2}{\partial(e_i^2)^2}\frac{\partial^2\varphi_j^2}{\partial(e_j^2)^2}\begin{pmatrix} [(M-D_L)-\varphi_i^2(M-D_L-D_H)] \\ [(M-D_L)-\varphi_j^2(M-D_L-D_H)] \end{pmatrix} > \left(\frac{\partial\varphi_i^2}{\partial e_i^2}\right)^2\left(\frac{\partial\varphi_j^2}{\partial e_j^2}\right)^2(M-D_H-D_L)^2$$

Consequently, $|A| > 0$ and $\frac{de_i^2}{da_i} > 0$.

The case for $\frac{de_j^2}{da_i}$ is similar and we obtain

$$\left|A_{e_j^2 a_i}\right| = -\frac{\partial^2\varphi_i^2}{\partial(e_i^2)^2}\frac{\partial\varphi_i^2}{\partial a_i}\frac{\partial\varphi_j^2}{\partial e_j^2}\begin{pmatrix} [(M-D_L)-\varphi_i^2(M-D_L-D_H)] \\ (D_H+D_L-M) \end{pmatrix} + \frac{\partial^2\varphi_i^2}{\partial e_i^2\partial a_i}\frac{\partial\varphi_i^2}{\partial e_i^2}\frac{\partial\varphi_j^2}{\partial e_j^2}[(M-D_L)-\varphi_j^2(M-D_L-D_H)](D_H+D_L-M)$$

which is strictly negative. Consequently $\frac{de_j^2}{da_i} < 0$ ■

8.3 Proof of Proposition 21

Proof. The proof for proposition 21 is similar for both firms. Therefore, we only present the proof for firm i.

First, comparing $e_i^2|_{q_h^i,q_l^j}$ and $e_i^2|_{q_l^i,q_l^j}$, and recalling $\lambda\mu_i(a_i,e_i^2) = \varphi_i(a_i,e_i^2)$, we observe

$$\begin{aligned} \frac{\partial\varphi_i^2}{\partial e_i^2}|_{q_h^i,q_l^j} &= \frac{\lambda c}{(M-D_L)-\varphi_j^2(M-D_L-D_H)} \\ \frac{\partial\varphi_i^2}{\partial e_i^2}|_{q_l^i,q_l^j} &= \frac{c}{(M-D_L)-\varphi_j^2(M-D_L-D_H)} \end{aligned}$$

Assuming across both cases firm i invests symmetrically, we observe

$$\frac{\partial \varphi_i^2}{\partial e_i^2}|_{q_h^i,q_l^j} < \frac{\partial \varphi_i^2}{\partial e_i^2}|_{q_l^i,q_l^j}$$

Given the assumptions made in A1 and A3, it must be that this implies that firm i would like to: i) invest strictly more where it has developed a high quality prototype; ii) invest strictly less where it has developed a low quality prototype; or iii) some combination of i) and ii). Either way, when firm j has developed a low quality prototype, firm i invests more when it has developed a high quality prototype.

Second, comparing $e_i^2|_{q_h^i,q_h^j}$ and $e_i^2|_{q_l^i,q_h^j}$ we observe

$$\begin{aligned}\frac{\partial \varphi_i^2}{\partial e_i^2}|_{q_h^i,q_h^j} &= \frac{\lambda^2 c}{\lambda(M-D_L)-\varphi_j^2(M-D_L-D_H)} \\ \frac{\partial \varphi_i^2}{\partial e_i^2}|_{q_l^i,q_h^j} &= \frac{\lambda c}{\lambda(M-D_L)-\varphi_j^2(M-D_L-D_H)}\end{aligned}$$

Again, a symmetric level of effort cannot be observed. More formally, we find

$$\frac{\partial \varphi_i^2}{\partial e_i^2}|_{q_h^i,q_h^j} < \frac{\partial \varphi_i^2}{\partial e_i^2}|_{q_l^i,q_h^j}$$

which implies $e_i^2|_{q_h^i,q_h^j} > e_i^2|_{q_l^i,q_h^j}$.

Therefore, regardless of the prototype developed by firm j, firm i always invests strictly more when it has developed a high quality prototype. ■

8.4 Proof of Proposition 23

Proof. The slope of firm i's reaction function depends upon

$$\begin{aligned}\frac{\partial^2 \pi^i|_{NVC}^{t=1}}{\partial (e_i^2)^2} &= \frac{\partial^2 \varphi_i^1}{\partial (e_i^1)^2}\left[\begin{array}{c}(1-\varphi_j^1)(\pi_{h,l}^i|_{NVC}^{t=2}-\pi_{l,l}^i|_{NVC}^{t=2}) \\ +\varphi_j^1[\pi_{h,h}^i|_{NVC}^{t=2}-\pi_{l,h}^i|_{NVC}^{t=2}]\end{array}\right] \\ \frac{\partial^2 \pi^i|_{NVC}^{t=1}}{\partial e_i^1 \partial e_j^1} &= \frac{\partial \varphi_i^1}{\partial e_i^1}\frac{\partial \varphi_j^1}{\partial e_j^1}\left[\begin{array}{c}\left(\pi_{h,h}^i|_{NVC}^{t=2}+\pi_{l,l}^i|_{NVC}^{t=2}\right) \\ -\left(\pi_{h,l}^i|_{NVC}^{t=2}+\pi_{l,h}^i|_{NVC}^{t=2}\right)\end{array}\right]\end{aligned}$$

Given assumptions A1, A3 and corollary 22 it must be that the former of these equations is strictly negative. In contrast, corollary 22 is not sufficient to determine the sign of the second order cross partial derivative. However, we are able to determine that the second order cross partial derivative of firm i is negative, and so efforts are treated as strategic substitutes, if and only if

$$\pi^i_{h,l}|^{t=2}_{NVC} - \pi^i_{l,l}|^{t=2}_{NVC} > \pi^i_{h,h}|^{t=2}_{NVC} - \pi^i_{l,h}|^{t=2}_{NVC} \tag{8.4}$$

Given this, it is trivial to note that in the case of firm j effort is treated as strategic substitutes if and only if

$$\pi^j_{l,h}|^{t=2}_{NVC} - \pi^j_{l,l}|^{t=2}_{NVC} > \pi^j_{h,h}|^{t=2}_{NVC} - \pi^j_{h,l}|^{t=2}_{NVC} \tag{8.5}$$

Where both of these equations are met both firms act as strategic substitutes. If neither of these are met then both firms act as strategic complements. Interestingly, it is possible that one treats effort as a strategic substitute whilst the other strategic complements. ■

8.5 Proof of Proposition 26

Proof. This proof is done for the case (q^i_h, q^i_l) but holds for all other cases due to assumptions A1, A3, A4 and $M > 2D_H > 2D_L$. Recall that this implies firm i has received VC funding as the sole developer of a high quality prototype. We solve the comparative statics using Cramer's rule, or

$$Ax = b$$

$$\begin{bmatrix} \frac{\partial^2 \pi^i_{h,l}|^{t=2}_{NVC}}{\partial (e^2_i)^2} & \frac{\partial^2 \pi^i_{h,l}|^{t=2}_{NVC}}{\partial e^2_i \partial e^2_j} \\ \frac{\partial^2 \pi^j_{h,l}|^{t=2}_{NVC}}{\partial e^2_i \partial e^2_j} & \frac{\partial^2 \pi^j_{h,l}|^{t=2}_{NVC}}{\partial (e^2_j)^2} \end{bmatrix} \begin{bmatrix} \frac{de^2_i}{dx} \\ \frac{de^2_j}{dx} \end{bmatrix} = \begin{bmatrix} -\frac{\partial^2 E\pi_i}{\partial e^2_i \partial x} \\ -\frac{\partial^2 E\pi j}{\partial e_{j2} \partial x} \end{bmatrix}$$

where x could represent either venture capitalist effort, E, or pecuniary funding, F.

First, solving the comparative statics with respect to F, we find

$$\begin{aligned}\frac{de_i^2}{dF} &= \frac{\left|A_{e_i^2F}\right|}{|A|} \\ &= \frac{\begin{vmatrix} -\frac{\partial^2\pi^i_{h,l}|^{t=2}_{VC}}{\partial e_i^2 \partial F} & \frac{\partial^2\pi^i_{h,l}|^{t=2}_{VC}}{\partial e_i^2 \partial e_j^2} \\ -\frac{\partial^2\pi^i_{h,l}|^{t=2}_{VC}}{\partial e_{j2} \partial F} & \frac{\partial^2\pi^j_{h,l}|^{t=2}_{VC}}{\partial (e_j^2)^2} \end{vmatrix}}{\begin{vmatrix} \frac{\partial^2\pi^i_{h,l}|^{t=2}_{VC}}{\partial (e_i^2)^2} & \frac{\partial^2\pi^i_{h,l}|^{t=2}_{VC}}{\partial e_i^2 \partial e_j^2} \\ \frac{\partial^2\pi^j_{h,l}|^{t=2}_{VC}}{\partial e_i^2 \partial e_j^2} & \frac{\partial^2\pi^j_{h,l}|^{t=2}_{VC}}{\partial (e_j^2)^2} \end{vmatrix}}\end{aligned}$$

where

$$\begin{aligned}|A| = \frac{\partial^2\hat{\mu}_i^2}{\partial\left(e_i^2\right)^2}\frac{\partial^2\varphi_j^2}{\partial\left(e_j^2\right)^2}\left[(1-\hat{\mu}_i^2)(M-D_L)+\hat{\mu}_i^2 D_H\right]\left[(1-\varphi_j^2)(M-D_L)+\varphi_j^2 D_H\right] \\ -\left(\frac{\partial\hat{\mu}_i^2}{\partial e_i^2}\right)^2\left(\frac{\partial\varphi_j^2}{\partial e_j^2}\right)^2\left[M-D_L-D_H\right]^2\end{aligned}$$

and

$$\left|A_{e_i^2F}\right| = \frac{\partial\hat{\mu}_i^2}{\partial e_i^2}\frac{\partial\hat{\mu}_i^2}{\partial F}\left(\frac{\partial\varphi_j^2}{\partial e_j^2}\right)^2\left[M-D_L-D_H\right]^2$$

Given assumptions A1, A3, A4 and $M > 2D_H > 2D_L$, it is trivial that $\left|A_{e_i^2F}\right| > 0$ but $|A|$ is not so trivial to sign. However, it is possible to observe that a unique and stable equilibrium exists if

$$\frac{1}{|R_i'|} > \left|R_j'\right|$$

holds. It turns out this this is the case if

$$\begin{aligned}\frac{\partial^2\hat{\mu}_i^2}{\partial\left(e_i^2\right)^2}\frac{\partial^2\varphi_j^2}{\partial\left(e_j^2\right)^2}\left\{\begin{array}{c}\left[(1-\hat{\mu}_i^2)(M-D_L)+\hat{\mu}_i^2 D_H\right] \\ \left[(1-\varphi_j^2)(M-D_L)+\varphi_j^2 D_H\right]\end{array}\right\} \\ > \left(\frac{\partial\hat{\mu}_i^2}{\partial e_i^2}\right)^2\left(\frac{\partial\varphi_j^2}{\partial e_j^2}\right)^2\left[M-D_L-D_H\right]^2\end{aligned}$$

Therefore, $|A|$ is strictly positive too and therefore $\frac{de_i^2}{dF} > 0$.

Similarly, the effects of pecuniary funding on firm j can simply be derived from

$$\left|A_{e_j^2 F}\right| = -\frac{\partial^2 \hat{\mu}_i^2}{\partial \left(e_i^2\right)^2}\frac{\partial \hat{\mu}_i^2}{\partial F}\frac{\partial \varphi_j^2}{\partial e_j^2}\left[(1-\varphi_j^2)(M-D_L)+\varphi_j^2 D_H\right]\left[D_H+D_L-M\right]$$

which is again strictly negative given assumptions A1, A3, A4 and $M > 2D_H > 2D_L$. Note that since the sign of $|A|$, positive, remains unchanged and so $\frac{de_j^2}{dF} < 0$.

Determining the impact of venture capitalist effort is derived in a similar way, with the sign on $|A|$ still unchanged. For the sake of brevity, we simply state

$$\left|A_{e_i^2 E}\right| = \frac{\partial \widehat{\mu}_{i2}}{\partial e_{i2}}\frac{\partial \widehat{\mu}_{i2}}{\partial E}\left(\frac{\partial \varphi_{j2}}{\partial e_{j2}}\right)^2 \left[D_H+D_L-M\right]^2 - \frac{\partial^2 \widehat{\mu}_{i2}}{\partial e_{i2}\partial E}\frac{\partial^2 \varphi_{j2}}{\partial e_{j2}^2}\left\{\begin{array}{c}\left[(1-\widehat{\mu}_{i2})(M-D_L)+\widehat{\mu}_{i2}D_H\right]\\ \left[(1-\varphi_{j2})(M-D_L)+\varphi_{j2}D_H\right]\end{array}\right\} > 0$$

$$\left|A_{e_j^2 E}\right| = -\frac{\partial^2 \widehat{\mu}_{i2}}{\partial e_{i2}^2}\frac{\partial \widehat{\mu}_{i2}}{\partial E}\frac{\partial \varphi_{j2}}{\partial e_{j2}}\left\{\begin{array}{c}\left[(1-\varphi_{j2})(M-D_L)+\varphi_{j2}D_H\right]\\ \left[D_H+D_L-M\right]\end{array}\right\} + \frac{\partial^2 \widehat{\mu}_{i2}}{\partial e_{i2}\partial E}\frac{\partial \widehat{\mu}_{i2}}{\partial e_{i2}}\frac{\partial \varphi_{j2}}{\partial e_{j2}}\left\{\begin{array}{c}\left[(1-\varphi_{j2})(M-D_L)+\varphi_{j2}D_H\right]\\ \left[D_H+D_L-M\right]\end{array}\right\} < 0$$

Again the signs of these equations are determined by assumptions A1, A3, A4 and $M > 2D_H > 2D_L$. Given that $\left|A_{e_i^2 E}\right|$ ($\left|A_{e_j^2 E}\right|$) is strictly positive (negative), it is easy to observe both $\frac{de_i^2}{dE} > 0$ and $\frac{de_j^2}{dE} < 0$.

As it has already been noted, assumptions A1 - A4 cover all the possible functional forms that may be present but assume that, whilst they are not identical, they all act in a similar way. Consequently, the result of this case extends to all other VC cases.

It is trivial to demonstrate that the equity stake, s, has no impact given the first order conditions are independent of s. Consequently, equity does not impact upon the optimal investment decision and has no impact on either e_i^2 or e_j^2. ■

8.6 Proof of Proposition 28

Proof. The proof of proposition 28 is complex simply because there are a large number of cases to examine. However, given the almost identical nature of the proofs, we only derive the result for the case in which $s = h$, or the rival firm has developed a high quality prototype.

First, assume that firm j has developed a high quality prototype such that the effort ordering becomes

$$e_i^2|_{q_h^i, q_h^j VC_i} > e_i^2|_{q_h^i, q_h^j |VC_j} > e_i^2|_{q_l^i, q_h^j}$$

Comparing $e_i^2|_{q_h^i, q_h^j VC_i}$ to $e_i^2|_{q_h^i, q_h^j |VC_j}$ we observe

$$\begin{aligned} \frac{\partial \hat{\mu}_i^2}{\partial e_i^2}|_{q_h^i, q_h^j, VC_i} &= \frac{c}{(M - D_L) - (1 - \mu_j^2)(M - D_L - D_H)} \\ \frac{\partial \mu_i^2}{\partial e_i^2}|_{q_h^i, q_h^j, VC_j} &= \frac{c}{(M - D_L) - (1 - \hat{\mu}_j^2)(M - D_L - D_H)} \end{aligned}$$

When $a_i = a_j$ and $E = F = 0$, it is obvious given assumptions A1 - A4 that the effort levels are equal, $e_i^2 = e_j^2 = e^*$. Furthermore, given the results derived in remark 25 and proposition 26, it is known that should a firm receive VC backing, *any* increases in venture capitalist effort or funding will strictly increase that firm's investment levels and decrease that of its rival. Therefore, starting from a purely symmetric case, $a_i = a_j$ and $E = F = 0$, when firm i receives venture capital, holding abilities constant, it must be that $e_i^2 > e^* > e_j^2$. Similarly, were firm j to receive funding, $e_j^2 > e^* > e_i^2$. Therefore, it must be that a firm invests *more* when it is VC-backed rather than its rival. The addition of asymmetric abilities does nothing to alter this result.

Second, in the other relevant case, $e_i^2|_{q_h^i, q_h^j |VC_j} > e_i^2|_{q_l^i, q_h^j}$, and after using equation (4.3) we observe

$$\begin{aligned} \frac{\partial \varphi_i^2}{\partial e_i^2}|_{q_h^i, q_h^j, VC_j} &= \frac{\lambda c}{(M - D_L) - (1 - \hat{\mu}_j^2)(M - D_L - D_H)} \\ \frac{\partial \varphi_i^2}{\partial e_i^2}|_{q_l^i, q_h^j} &= \frac{c}{(M - D_L) - (1 - \hat{\mu}_j^2)(M - D_L - D_H)} \end{aligned}$$

It is obvious that, for any given level of effort by firm j, it must be that

$$\frac{\partial \varphi_i^2}{\partial e_i^2}|_{q_h^i, q_h^j, VC_j} < \frac{\partial \varphi_i^2}{\partial e_i^2}|_{q_l^i, q_h^j}$$

Therefore, it is the case in which firm i has developed a high quality prototype, but not received VC funding, that yields a greater level of investment. ■

8.7 Proof of Proposition 30

Proof. The proof is determined by the relevant first and second order equations, given by

$$\begin{aligned}
\frac{\partial^2 \pi^i|_{NVC}^{t=1}}{\partial (e_i^2)^2} &= \frac{\partial^2 \varphi_i^1}{\partial (e_i^1)^2}\left\{(1-\varphi_j^1)\left[\pi_{h,l}^i|_{VC}^{t=2} - \pi_{l,l}^i|_{VC}^{t=2}\right] + \varphi_j^1\left[\frac{1}{2}\left(\pi_{h,h}^i|_{VC_i}^{t=2} + \pi_{h,h}^i|_{VC_j}^{t=2}\right) - \pi_{l,h}^i|_{VC}^{t=2}\right]\right\} \\
\frac{\partial^2 \pi^i|_{NVC}^{t=1}}{\partial e_i^1 \partial e_j^1} &= \frac{\partial \varphi_i^1}{\partial e_i^1}\frac{\partial \varphi_j^1}{\partial e_j^1}\left[\left(\frac{1}{2}\left(\pi_{h,h}^i|_{VC_i}^{t=2} + \pi_{h,h}^i|_{VC_j}^{t=2}\right) + \pi_{l,l}^i|_{VC}^{t=2}\right) - \left(\pi_{h,l}^i|_{VC}^{t=2} + \pi_{l,h}^i|_{VC}^{t=2}\right)\right]
\end{aligned}$$

Given the profit ordering in corollary 29, it is obvious that the former of these equations is negative. Consequently, the slope of the reaction function is determined by the second order cross partial derivative. However, corollary 29 is not sufficient to determine the sign in this case. Given assumptions A1 - A4, it is obvious that it is trivial to observe that the sign is determined by

$$\left(\frac{1}{2}\left(\pi_{h,h}^i|_{VC_i}^{t=2} + \pi_{h,h}^i|_{VC_j}^{t=2}\right) + \pi_{l,l}^i|_{VC}^{t=2}\right) - \left(\pi_{h,l}^i|_{VC}^{t=2} + \pi_{l,h}^i|_{VC}^{t=2}\right)$$

which is negative, for both firms i and j, if and only if

$$\begin{aligned}
\pi_{h,l}^i|_{VC}^{t=2} &> \frac{1}{2}\left(\pi_{h,h}^i|_{VC_i}^{t=2} + \pi_{h,h}^i|_{VC_j}^{t=2}\right) + \pi_{l,l}^i|_{VC}^{t=2} - \pi_{l,h}^i|_{VC}^{t=2} \\
\pi_{l,h}^j|_{VC}^{t=2} &> \frac{1}{2}\left(\pi_{h,h}^j|_{VC_i}^{t=2} + \pi_{h,h}^j|_{VC_j}^{t=2}\right) + \pi_{l,l}^j|_{VC}^{t=2} - \pi_{h,l}^j|_{VC}^{t=2}
\end{aligned}$$

Where both of these conditions are met, both firms treat effort as strategic substitutes. ■

8.8 Proof of Remark 31

Proof. Solving these comparative statics by Cramer's rule obtains

$$\begin{aligned}
\frac{de_i^1}{dx} &= \frac{\left|A_{e_i^1 x}\right|}{|A|} \\
&= \frac{\begin{vmatrix} -\frac{\partial^2 \pi^i|_{VC}^{t=1}}{\partial e_i^2 \partial x} & \frac{\partial^2 \pi^i|_{VC}^{t=1}}{\partial e_i^2 \partial e_j^2} \\ -\frac{\partial^2 \pi^i|_{VC}^{t=1}}{\partial e_{j2} \partial x} & \frac{\partial^2 \pi^j|_{VC}^{t=1}}{\partial (e_j^2)^2} \end{vmatrix}}{\begin{vmatrix} \frac{\partial^2 \pi^i|_{VC}^{t=1}}{\partial (e_i^2)^2} & \frac{\partial^2 \pi^i|_{VC}^{t=1}}{\partial e_i^2 \partial e_j^2} \\ \frac{\partial^2 \pi^j|_{VC}^{t=1}}{\partial e_i^2 \partial e_j^2} & \frac{\partial^2 \pi^j|_{VC}^{t=1}}{\partial (e_j^2)^2} \end{vmatrix}}
\end{aligned}$$

where $x = E,\ F$.

In both cases

$$\begin{aligned}
|A| = {} & \left(\frac{\partial^2 \varphi_i^1}{\partial (e_i^1)^2}\right)\left(\frac{\partial^2 \varphi_j^1}{\partial (e_j^1)^2}\right)\left[\begin{array}{c} \left\{\begin{array}{c} \left(1-\varphi_j^1\right)\left[\pi_{h,l}^i|_{VC}^{t=2} - \pi_{l,l}^i|_{VC}^{t=2}\right] \\ +\varphi_j^1\left[\frac{1}{2}\left(\pi_{h,h}^i|_{VC_i}^{t=2} + \pi_{h,h}^i|_{VC_j}^{t=2}\right) - \pi_{l,h}^i|_{VC}^{t=2}\right] \end{array}\right\} \\ \left\{\begin{array}{c} \left(1-\varphi_i^1\right)\left[\pi_{l,h}^j|_{VC}^{t=2} - \pi_{l,l}^j|_{VC}^{t=2}\right] \\ +\varphi_i^1\left[\frac{1}{2}\left(\pi_{h,h}^j|_{VC_i}^{t=2} + \pi_{h,h}^j|_{VC_j}^{t=2}\right) - \pi_{h,l}^j|_{VC}^{t=2}\right] \end{array}\right\} \end{array}\right] \\
& - \left(\frac{\partial \varphi_i^1}{\partial e_i^1}\right)^2 \left(\frac{\partial \varphi_j^1}{\partial e_j^1}\right)^2 \left[\begin{array}{c} \left\{\begin{array}{c} \left(\frac{1}{2}\left(\pi_{h,h}^i|_{VC_i}^{t=2} + \pi_{h,h}^i|_{VC_j}^{t=2}\right) + \pi_{l,l}^i|_{VC}^{t=2}\right) \\ -\left(\pi_{h,l}^i|_{VC}^{t=2} + \pi_{l,h}^i|_{VC}^{t=2}\right) \end{array}\right\} \\ \left\{\begin{array}{c} \left(\frac{1}{2}\left(\pi_{h,h}^j|_{VC_i}^{t=2} + \pi_{h,h}^j|_{VC_j}^{t=2}\right) + \pi_{l,l}^j|_{VC}^{t=2}\right) \\ -\left(\pi_{h,l}^j|_{VC}^{t=2} + \pi_{l,h}^j|_{VC}^{t=2}\right) \end{array}\right\} \end{array}\right]
\end{aligned}$$

Whilst this looks rather unpleasant and impossible to sign, the condition for uniqueness and stability, $\left|R_i'\right|\left|R_j'\right| < 1$, yields

$$\left(\frac{\partial^2\varphi_i^1}{\partial(e_i^1)^2}\right)\left(\frac{\partial^2\varphi_j^1}{\partial(e_j^1)^2}\right)\left[\begin{array}{c}\left\{\begin{array}{c}(1-\varphi_j^1)\left[\pi_{h,l}^i|_{VC}^{t=2}-\pi_{l,l}^i|_{VC}^{t=2}\right]\\+\varphi_j^1\left[\frac{1}{2}\left(\pi_{h,h}^i|_{VC_i}^{t=2}+\pi_{h,h}^i|_{VC_j}^{t=2}\right)-\pi_{l,h}^i|_{VC}^{t=2}\right]\end{array}\right\}\\\left\{\begin{array}{c}(1-\varphi_i^1)\left[\pi_{l,h}^j|_{VC}^{t=2}-\pi_{l,l}^j|_{VC}^{t=2}\right]\\+\varphi_i^1\left[\frac{1}{2}\left(\pi_{h,h}^j|_{VC_i}^{t=2}+\pi_{h,h}^j|_{VC_j}^{t=2}\right)-\pi_{h,l}^j|_{VC}^{t=2}\right]\end{array}\right\}\end{array}\right]$$

$$>\left(\frac{\partial\varphi_i^1}{\partial e_i^1}\right)^2\left(\frac{\partial\varphi_j^1}{\partial e_j^1}\right)^2\left[\begin{array}{c}\left\{\begin{array}{c}\left(\frac{1}{2}\left(\pi_{h,h}^i|_{VC_i}^{t=2}+\pi_{h,h}^i|_{VC_j}^{t=2}\right)+\pi_{l,l}^i|_{VC}^{t=2}\right)\\-\left(\pi_{h,l}^i|_{VC}^{t=2}+\pi_{l,h}^i|_{VC}^{t=2}\right)\end{array}\right\}\\\left\{\begin{array}{c}\left(\frac{1}{2}\left(\pi_{h,h}^j|_{VC_i}^{t=2}+\pi_{h,h}^j|_{VC_j}^{t=2}\right)+\pi_{l,l}^j|_{VC}^{t=2}\right)\\-\left(\pi_{h,l}^j|_{VC}^{t=2}+\pi_{l,h}^j|_{VC}^{t=2}\right)\end{array}\right\}\end{array}\right]$$

Consequently, the sign on both comparative statics, E and F, depend on $|A_{e_iE}|$, $\left|A_{e_jE}\right|$, $|A_{e_iF}|$, $\left|A_{e_jF}\right|$.

For the sake of brevity, we simply offer the equations here:

$$\begin{aligned}|A_{e_iE}| &= -\left(\frac{\partial\pi^j|_{VC}^{t=1}}{\partial(e_j^1)^2}\right)\left\{\frac{\partial\varphi_i^1}{\partial e_i^1}\left(\begin{array}{c}(1-\varphi_j^1)\frac{\partial\pi_{h,l}^i|_{VC}^{t=2}}{\partial E}\\+\varphi_j^1\left[\frac{1}{2}\left(\frac{\partial\pi_{h,h}^i|_{VC_i}^{t=2}}{\partial E}+\frac{\partial\pi_{h,h}^i|_{VC_j}^{t=2}}{\partial E}\right)-\frac{\partial\pi_{l,h}^i|_{VC}^{t=2}}{\partial E}\right]\end{array}\right)\right\}\\&\quad+\left(\frac{\partial\pi^i|_{VC}^{t=1}}{\partial e_i^1\partial e_j^1}\right)\left\{\frac{\partial\varphi_j^1}{\partial e_j^1}\left(\begin{array}{c}(1-\varphi_i^1)\frac{\partial\pi_{l,h}^j|_{VC}^{t=2}}{\partial E}\\+\varphi_i^1\left[\frac{1}{2}\left(\frac{\partial\pi_{h,h}^j|_{VC_i}^{t=2}}{\partial E}+\frac{\partial\pi_{h,h}^j|_{VC_j}^{t=2}}{\partial E}\right)-\frac{\partial\pi_{h,l}^j|_{VC}^{t=2}}{\partial E}\right]\end{array}\right)\right\}\end{aligned}$$

$$\begin{aligned}\left|A_{e_jE}\right| &= -\left(\frac{\partial\pi^i|_{VC}^{t=1}}{\partial(e_i^1)^2}\right)\left\{\frac{\partial\varphi_j^1}{\partial e_j^1}\left(\begin{array}{c}(1-\varphi_i^1)\frac{\partial\pi_{l,h}^j|_{VC}^{t=2}}{\partial E}\\+\varphi_i^1\left[\frac{1}{2}\left(\frac{\partial\pi_{h,h}^j|_{VC_i}^{t=2}}{\partial E}+\frac{\partial\pi_{h,h}^j|_{VC_j}^{t=2}}{\partial E}\right)-\frac{\partial\pi_{h,l}^j|_{VC}^{t=2}}{\partial E}\right]\end{array}\right)\right\}\\&\quad+\left(\frac{\partial\pi^j|_{VC}^{t=1}}{\partial e_j^1\partial e_i^1}\right)\left\{\frac{\partial\varphi_i^1}{\partial e_i^1}\left(\begin{array}{c}(1-\varphi_i^1)\frac{\partial\pi_{h,l}^i|_{VC}^{t=2}}{\partial E}\\+\varphi_i^1\left[\frac{1}{2}\left(\frac{\partial\pi_{h,h}^i|_{VC_i}^{t=2}}{\partial E}+\frac{\partial\pi_{h,h}^i|_{VC_j}^{t=2}}{\partial E}\right)-\frac{\partial\pi_{l,h}^i|_{VC}^{t=2}}{\partial E}\right]\end{array}\right)\right\}\end{aligned}$$

$$
\begin{aligned}
|A_{e_iF}| \;=\; & -\left(\frac{\partial \pi^j|^{t=1}_{VC}}{\partial (e_j^1)^2}\right)\left\{\frac{\partial \varphi_i^1}{\partial e_i^1}\left(\begin{array}{c}(1-\varphi_j^1)\frac{\partial \pi^i_{h,l}|^{t=2}_{VC}}{\partial F}\\ +\varphi_j^1\left[\frac{1}{2}\left(\frac{\partial \pi^i_{h,h}|^{t=2}_{VC_i}}{\partial F}+\frac{\partial \pi^i_{h,h}|^{t=2}_{VC_j}}{\partial F}\right)-\frac{\partial \pi^i_{l,h}|^{t=2}_{VC}}{\partial F}\right]\end{array}\right)\right\}\\
& +\left(\frac{\partial \pi^i|^{t=1}_{VC}}{\partial e_i^1 \partial e_j^1}\right)\left\{\frac{\partial \varphi_j^1}{\partial e_j^1}\left(\begin{array}{c}(1-\varphi_i^1)\frac{\partial \pi^j_{l,h}|^{t=2}_{VC}}{\partial F}\\ +\varphi_i^1\left[\frac{1}{2}\left(\frac{\partial \pi^j_{h,h}|^{t=2}_{VC_i}}{\partial F}+\frac{\partial \pi^j_{h,h}|^{t=2}_{VC_j}}{\partial F}\right)-\frac{\partial \pi^j_{h,l}|^{t=2}_{VC}}{\partial F}\right]\end{array}\right)\right\}
\end{aligned}
$$

$$
\begin{aligned}
\left|A_{e_jF}\right| \;=\; & -\left(\frac{\partial \pi^i|^{t=1}_{VC}}{\partial (e_i^1)^2}\right)\left\{\frac{\partial \varphi_j^1}{\partial e_j^1}\left(\begin{array}{c}(1-\varphi_i^1)\frac{\partial \pi^j_{l,h}|^{t=2}_{VC}}{\partial E}\\ +\varphi_i^1\left[\frac{1}{2}\left(\frac{\partial \pi^j_{h,h}|^{t=2}_{VC_i}}{\partial E}+\frac{\partial \pi^j_{h,h}|^{t=2}_{VC_j}}{\partial E}\right)-\frac{\partial \pi^j_{h,l}|^{t=2}_{VC}}{\partial E}\right]\end{array}\right)\right\}\\
& +\left(\frac{\partial \pi^j|^{t=1}_{VC}}{\partial e_j^1 \partial e_i^1}\right)\left\{\frac{\partial \varphi_i^1}{\partial e_i^1}\left(\begin{array}{c}(1-\varphi_j^1)\frac{\partial \pi^i_{h,l}|^{t=2}_{VC}}{\partial E}\\ +\varphi_j^1\left[\frac{1}{2}\left(\frac{\partial \pi^i_{h,h}|^{t=2}_{VC_i}}{\partial E}+\frac{\partial \pi^i_{h,h}|^{t=2}_{VC_j}}{\partial E}\right)-\frac{\partial \pi^i_{l,h}|^{t=2}_{VC}}{\partial E}\right]\end{array}\right)\right\}
\end{aligned}
$$

Unfortunately, given that it is not possible to determine the magnitude of the first order conditions with respect to E or F, it is not possible to sign these equations. ■

8.9 Proof of Proposition 32

Proof. Again, using Cramer's rule we observe

$$
\begin{aligned}
\frac{de_i^1}{ds} &= \frac{\left|A_{e_i^1 s}\right|}{|A|}\\
&= \frac{\begin{vmatrix} -\frac{\partial^2 \pi^i|^{t=1}_{VC}}{\partial e_i^2 \partial s} & \frac{\partial^2 \pi^i|^{t=1}_{VC}}{\partial e_i^2 \partial e_j^2} \\ -\frac{\partial^2 \pi^i|^{t=1}_{VC}}{\partial e_{j2} \partial s} & \frac{\partial^2 \pi^j|^{t=1}_{VC}}{\partial (e_j^2)^2} \end{vmatrix}}{\begin{vmatrix} \frac{\partial^2 \pi^i|^{t=1}_{VC}}{\partial (e_i^2)^2} & \frac{\partial^2 \pi^i|^{t=1}_{VC}}{\partial e_i^2 \partial e_j^2} \\ \frac{\partial^2 \pi^j|^{t=1}_{VC}}{\partial e_i^2 \partial e_j^2} & \frac{\partial^2 \pi^j|^{t=1}_{VC}}{\partial (e_j^2)^2} \end{vmatrix}}
\end{aligned}
$$

where $|A|$ is identical to that in remark 31. Thus, it is only the signs on $\left|A_{e_i^1 s}\right|$ and $\left|A_{e_j^1 s}\right|$ that are important. With come manipulation we obtain

$$
\begin{aligned}
|A_{e_i s}| &= -\left(\frac{\partial\pi^j|_{VC}^{t=1}}{\partial(e_j^1)^2}\right)\left\{\frac{\partial\varphi_i^1}{\partial e_i^1}\left((1-\varphi_j^1)\frac{\partial\pi_{h,l}^i|_{VC}^{t=2}}{\partial s}+\frac{1}{2}\varphi_j^1\frac{\partial\pi_{h,h}^i|_{VC_i}^{t=2}}{\partial s}\right)\right\} \\
&+\left(\frac{\partial\pi^i|_{VC}^{t=1}}{\partial e_i^1\partial e_j^1}\right)\left\{\frac{\partial\varphi_j^1}{\partial e_j^1}\left((1-\varphi_i^1)\frac{\partial\pi_{l,h}^j|_{VC}^{t=2}}{\partial s}+\frac{1}{2}\varphi_i^1\frac{\partial\pi_{h,h}^j|_{VC_j}^{t=2}}{\partial s}\right)\right\}
\end{aligned}
$$

$$
\begin{aligned}
\left|A_{e_j s}\right| &= -\left(\frac{\partial\pi^i|_{VC}^{t=1}}{\partial(e_i^1)^2}\right)\left\{\frac{\partial\varphi_j^1}{\partial e_j^1}\left((1-\varphi_i^1)\frac{\partial\pi_{l,h}^j|_{VC}^{t=2}}{\partial s}+\frac{1}{2}\varphi_i^1\frac{\partial\pi_{h,h}^j|_{VC_j}^{t=2}}{\partial s}\right)\right\} \\
&+\left(\frac{\partial\pi^j|_{VC}^{t=1}}{\partial e_j^1\partial e_i^1}\right)\left\{\frac{\partial\varphi_i^1}{\partial e_i^1}\left((1-\varphi_j^1)\frac{\partial\pi_{h,l}^i|_{VC}^{t=2}}{\partial s}+\frac{1}{2}\varphi_j^1\frac{\partial\pi_{h,h}^i|_{VC_i}^{t=2}}{\partial s}\right)\right\}
\end{aligned}
$$

where

$$\frac{\partial\pi_{x,y}^i|_{VC}^{t=2}}{\partial s}<0\ \forall\ x,y\in\{l,h\}$$

Therefore, when firms treat effort as strategic complements, or

$$
\begin{aligned}
\frac{\partial^2\pi^i|_{NVC}^{t=1}}{\partial(e_i^2)^2} &< 0 \\
\frac{\partial^2\pi^i|_{NVC}^{t=1}}{\partial e_i^1\partial e_j^1} &> 0
\end{aligned}
$$

we find $|A_{e_i s}|$ and $\left|A_{e_j s}\right|$ are both strictly negative. Thus,

$$\frac{de_i^1}{ds}<0$$

■

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