INTERFERENCE SUPPRESSION FOR SATELLITE NAVIGATION SIGNALS

MPHIL THESIS

BY

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Abstract

Interference suppression for satellite navigation system has become one of the hottest topic in the research area. By far, there are two main methods for reducing the effect of jamming signals: traditional adaptive array signal processing and blind signal processing techniques. Normally, we need to know where the navigation signals come from when we are using the traditional adaptive array signal processing technique. For blind techniques, the main advantage is that we do not need any information about DOA angles of the desired signals, but its performance is worse than the traditional adaptive techniques. One of the most popular methods is the power minimization method, which belongs to the class of blind methods.

In this thesis, I focus on the research of blind signal processing techniques. Based on the traditional power minimization approach, we propose an improved algorithm for suppressing strong interferences for satellite navigation based on antenna arrays. It is known that the principle component analysis (PCA) technique can extract the principle components from the original data. Thus, the idea is to replace the auxiliary antenna outputs in the traditional power minimization method by the principal components of received array signals. leading to an improved performance, as verified by our simulations.

Moreover, we also give an in-depth study of the power minimization method. Using the same approach adopted in the performance analysis of the minimum variance beam former. We analyze the performance of the power minimization method in detail in terms of its output signal to interference plus noise ratio (SINR). The results are very close to the expected value when the sample size is small.

Contents

List of Publications

1. B. Qiu, W. Liu and R. B. Wu, “Blind Interference Suppression for Satellite Navigation Signals Based on Antenna Arrays”, in Proceedings of the Conference of Signal and Information Processing (ChinaSIP), pp. 370-373, July 2013.

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Chapter 1

Introduction

1.1 Introduction

The current design of the Global Positioning System (GPS) was proposed in the late 1960s, which is based on a constellation of 24 man-made satellites orbiting around the earth at an attitude of 20000 km. Each satellite keeps on transmitting a position message with precision timing among all system components and between satellites.

By far, the satellite-based navigation system has been improved for more than 40 years, which can provide the informiation of position, velocity and timing for people in all kinds of environment conditions. A special case of the three-dimensional space-time processor (3D STAP) was proposed for radar systems to cancel ground interference has been proposed in [1, 2, 3, 4]. As a result, it has been not only used as a worldwide military algorithm, but also applied widely in civil communities. However, the performance of the GPS or a general satellite navigation system is significantly susceptible to interference from either intentional or unintentional sources, due to that the satellite signals arrive at the receiver with a very low power, typically 20-30 dB below the received thermal noise level [5]. Therefore, interference suppression has become one of the hot topics for using GPS without distortion of desired GPS signal.

Interference suppression has been studied by many researchers in this field and many methods have been proposed based on either the specific structure (such as cyclostationarity) or the direction of arrival (DOA) information of the signals or both, employing traditional adaptive beamforming algorithms or some blind signal processing techniques. [6, 7, 8, 9]. Adaptive beamforming approaches are very traditional ways to suppress interference, which employ spatial degrees to form nulls in the direction of interfering signals and give good performance over very narrow bandwidths. In order to form broadband nulls, both spatial and temporal adaptive degrees of freedom are required [10, 11, 12]. In a general environment, an algorithm is proposed which uses derivative constraints on the directions of interferers into Hung-Turner and sample matrix inversion (SMI) algorithms to broaden the width of nulls [13, 14].

Blind signal processing methods stand an important position in the area of interference suppression. Although they are not as good as the normal adaptive beam forming methods, like the linearly constraint minimum variance (LCMV) beam former, they can be applied in highly dymanic environments when the DOA information of the desired signal is not available. In [15], an interference suppression method based on space-time adaptive processing (STAP) without loss or distortion of the desired signal is proposed. A self-coherence anti-jamming scheme is introduced in [9, 16], which relies on the unique structure of the coarse/acquisition (C/A) code of the satellite signals. To reduce the computational complexity of the STAP system, reduced-rank processing methods are studied in [17]. A blind interference suppression method by applying the cyclic adaptive beam forming (CAB) algorithm with a subspace technique is proposed in [18]. Unlike the traditional beam forming methods, the power minimization method can form deep nulls in the interference directions without knowledge of the desired satellite signals when the interfering signals are much stronger than the desired signals, and it also has a rather low computational complexity due to its simple structure [19, 20]. An improved algorithm based on the power minimization method is proposed in [21]. Moreover, Domain Weighted Principle Component Analysis (DW-PCA) is an effective way to cancel interfering signals [22, 23]. Applying this technique to form a robust adaptive beam former, an improved performance can be obtained, as the principal components are exracted from the received data [24, 25].

Nowadays, the minimum variance beam former has become one of the most important interference suppression techniques. For improving the performance of this beam forming technique, the output signal-to-interference-plus-noise ratio (SINR) is considered as the parameter of interest. This ratio is related to many parameters including the input signal-to-noise ratio (SNR), the input interference-to-noise ratio (INR) or the signal-to-interference ratio (SIR). However, most of the researches so far have not considered all of the parameters. The approaches that consider the effect of SNR and angular separation are proposed in [26, 27, 28]. [11, 29] have proposed the analysis by considering the effect of finite sample size. In 1996, a complete analysis of the SINR was presented as a function of all parameters in [30]. The approach is based on the case that the signal and the noise are Gaussian and the number of samples is large compared with the array size.

In this section, I will give a brief description of the proposed methods, which constitute original contribution of this thesis.

1.2 Original Contributions

The following is a list of the original contributions of the thesis:

- **Blind interference suppression for satellite navigation signals based on antenna arrays**
  The traditional power minimization method can cancel the effect of interference and provide good performance effectively, when the power of interference is large compared to that of the desired signal. However, the degradation of the output SINR will occur with this method, when the power of the desired signal is getting closer to that of interfering signal. Built on the success of the power minimization method, a novel technique is proposed by incorporating principal component analysis (PCA) into the structure and replacing the auxiliary antenna outputs by the principal components of the received array signal. The principal components extracted by the PCA operation can be considered as a better representation of the very strong interfering signals and therefore can cancel the interfering signas present in the refenrece antenna more effectively, leading to an improved performance. According to the simulation results, the performance of the proposed method has not been degraded the high power of the desired signal, while the traditional one has suffered as the power of the desired signal is increasing. In [22, 23, 24, 25], the power of the desired signal is higher than that of interfering signals, so the effect of

the desired signal will be enhanced by applying PCA.

- **Performance analysis of the power minimization method**
  Generally, the traditional performance analysis is based on the asssumption that the sample size is large and the signals and noise are Gaussian which is widely used signal model, and presented as a function of all parameters. However, this method is very complex and difficult to use for the power minimization method, as the power minimization method does not need any information of the directions of signals. To transfer the general expression to a simple one, a novel method is presented, which can analyze the performance of the power minimization method. Meanwhile, the proposed expression can also analyze the performance of LCMV with consideration of finite sample number and the error of direction of signals. An approximation of the expected value of the output SINR for the power minimization method has been derived in this thesis, and the simulation results show that it can be considered as an effective representation of the true values.

1.3 Thesis Outline

The thesis is structured as follows:

In Chapter 2, an introduction is given to the model and characteristic of GPS satellite signal. Some common features of GPS are discussed. Then turning our eyes to the basics of array signal processing, and the signal model is introduced. Some adaptive beam forming approaches based on different parameters are introduced, including optimization using reference signal, null-steering beam forming and beamforming for maxmizing the output SINR. Subsequently, LCMV beam former, one of the approaches in minimum variance beam forming, is discussed in detail. The constraints form a response on the direction of the desired signal and suppress interfering signals effectively. Another structure is also introduced, which is called Generalize Sidelobe Canceller (GSC). It is an alternative implementation of the LCMV beam former. Moreover, compared with a general beam forming structure, the advantages of space-time adaptive processing (STAP) are discussed. The STAP technique has been widely used to reduce the effect of interfering signals because it greatly increases the freedom of degree under the same antenna conditions.

Chapter 3 focuses on blind interference suppression algorithms. In general, it is very difficult to obtain the knowledge of directions of signals normally. Therefore, the power minimization method is proposed, which does not need any prior information of directions of signals. This approach forms nulling in the directions of the jamming signal, so it can reduce or cancel the effect of interference effectively. More importantly, a useful technique for separating interfernces from the received data is presented, which is called PCA. Finally, I propose an improved algorithm for interference suppression which is based on a combination of the power minimization method and PCA.

In Chapter 4, I continue to investigate the power minimization method and give further insight of it. Firstly, the traditional performance analysis of the minimum variance beam former is discussed. The general expression is obtained, but it is too complex for us to apply it to the algorithm of power minimization directly. Thus, based on the general expression, an alternative expression is presented, which can be used for analyzing performance of the power minimization method and LCMV beam former.

Finally, conclusions and an outlook on possible future work are given in Chapter 5.

Chapter 2

Review of Interference Suppression Algorithms for Satellite Navigation

In this chapter, the background of interference suppression for satellite navigation signals will be introduced. Many approaches, which are investigated frequently these years, will be reviewed and discussed. Subsequently, I will mainly introduce the adaptive beamforming algorithms and nulling steering technology. We will provide and explain more details and informantion of some interference suppression algorithms, such as the LCMV beamformer and the power minimization approach. Besides, the GSC will be discussed, which is an alternative implementation of the LCMV beam former.

2.1 GPS Background

2.1.1 Overview of GPS

Nowadays, GPS has been widely used in every aspect of our lives, which was established in the 1960s for an optimum positioning system. The users can be provided with postion, velocity and worldwide information by the equipment of GPS receivers. This design is based on the constellation which consists of twenty four satellites arranged in 6 orbital planes with 4 satellites on each plane. GPS can provide service to every user becasue the receivers operate passively. The satellites transmit ranging codes and navigation information on two frequencies with the code division multiple access (CDMA) technique.

Table 2.1: GPS signal structure

Signal priority	Primary	Secondary
Signal designation	L1	L2
Carrier frequency (MHz)	1575.42	1227.6
Pseudorandom noise code (10^6 chips/sec)	P(Y)=10.23, C/A=1.023	P(Y)=10.23, C/A=1.023
Navigation message data rate (bps)	50	50

Two frequencies are $L1 = 1575.42$ MHz and $L2 = 1227.6$ MHz, respectively. Each satellite transimits on these two frequenices, but with different ranging codes, which will be discussed later. The navigation data provides the means for GPS receivers to ensure where the satellite is, when the sigal is transmitted, and for the receiver to determine the transmit time of the signal. The user receiver must contain a clock to apply this technique to measure the receiver's three dimensional location. Only three range measurements are required when the receiver clock is synchronized with the satellite clocks. A crystal clock is employed in GPS receivers to minimize their size.

Besides, GPS consits of two services. One is the Standard Positioning Service (SPS), which is designated for civil community, and the other one is Precise Positioning Service (PPS), which is reserved for U.S military and government agency users.

2.1.2 GPS Satellite Signal Characteristics

Modulation Format of GPS

As shown in Fig. 2.1, both L1 and L2 carrier frequencies are modulated by the binary product in the modulator using the Binary Phase Shift Keying (BPSK) scheme. The L1 frequency (1575.42 MHz) is modulated by two pseudorandom noise (PRN) codes (plus the navigation message data), the coarse/acquisition code (C/A code) and the precision code (P code), respectively. The GPS C/A code is a Gold code with a sequence length of 1023 chips [31]. Because the chipping rate of the C/A code is 1023 MHz, the period of the pseudorandom sequence is 1023 or 1 millisecond. The P code sequence length will be

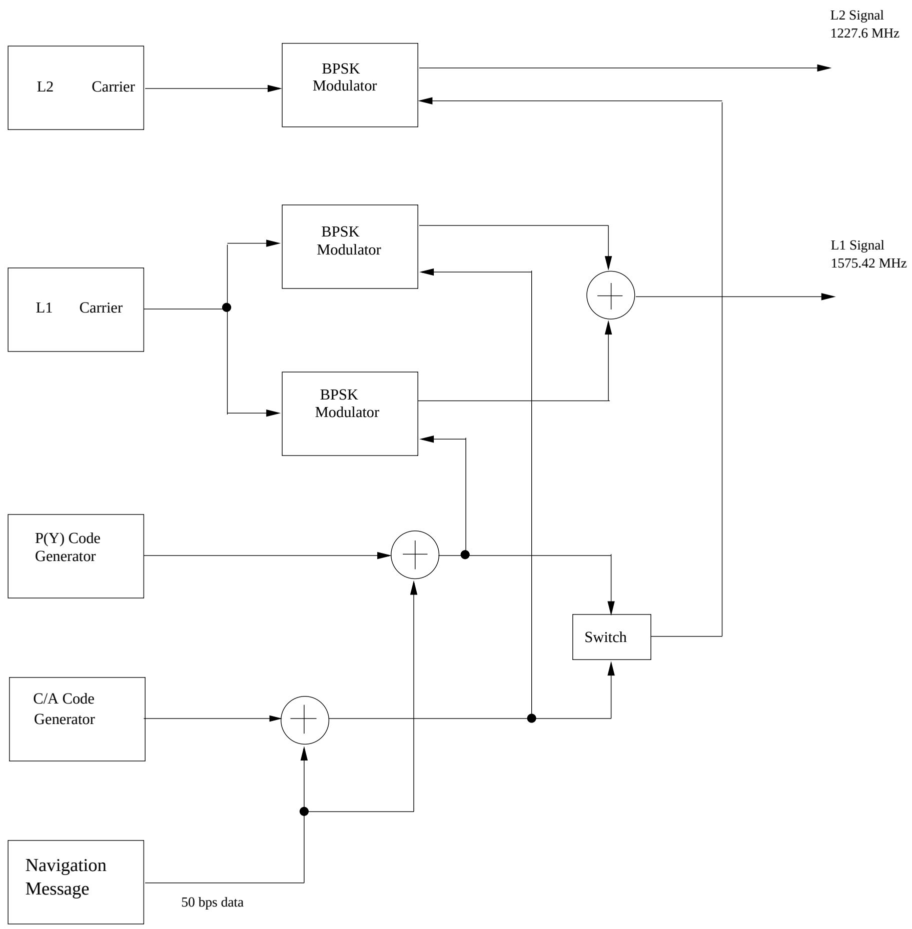

Fig. 2.1: Block diagram of the GPS satellite transmitter structure.

more than 38 weeks in length, but is divided into 37 unique sequences that are truncated at the end of each week. The L2 frequency is transmitted by only one PRN code. Besides, navigation message is 50 bps data with binary values $+1$ and -1, which is combined with PRN codes before modulating with $L1$ and $L2$ carriers. The navigation message consists of the ephemeris and the almanac data that are needed for the navigation solution.

I summarize the GPS structure on L1 and L2 in Table 2.1. For the GPS receiver, the C/A code has a chipping rate of 1.023×10^6 chips/sec and the P code has a chipping rate of 10.23×10^6 chips/sec.

Acquisition

- **Acquisition**

  The first step of the GPS signal processing scheme is acquisition. The GPS receiver must detect the presence of the signals before they can be tracked and decoded for positioning computation. The received signal must be acquired at the correct code phase and carrier frequency in the acquisition.

- **Acquisition Scheme and Search Band**

  Many acquisition methods have been improved to acquire the GPS signal more effectively [32, 33]. A method called circular correlation by Fourier transforms performs a faster acquisition over other approaches and was proposed in [31].

  The satellite movement induces a Doppler shift of up to 5 kHz from the GPS L1 frequency [34]. The line of sight velocity of the satellite causes a Doppler effect in different frequencies. In most of cases, it is sufficient to search the frequencies when the maximum error will not be more than 500 Hz [35].

- **Detection Thresholds**

  The correlation process has a number of peaks. A threshold signal detector is required to determine the presence of GPS signals [36]. The correlation peak in the minimum value should exceed the threshold for the acquisition process to declare the signal.

Tracking

The key purpose of the tracking channel is to achieve precise Doppler and code delay estimation [37]. In carrier correlation, the input digital IF signal from the front-end is correlated with the carrier signals to obtain In-phase (I) and Quadrature-phase (Q) data. The carrier signals are synthesized by the carrier Numerically Controlled Oscillator (NCO) and the discrete sine and cosine mapping functions. In general, the I and Q signals are correlated with replica code synthesized by the code generator. In the case that the code phase is not tracked properly, the correlations between the replica code phase and the incoming Space Vehicles (SV) code phase are different.

Signal Level

The minimum received signal powers for three kinds of GPS signals are summarized in Table 2.2. The main feature of GPS signals is their low signal power level. The power

Table 2.2: Minimum received GPS signal power.

Power (dBW)	L1 (C/A Code)	L1 (P Code)	L2 (P Code or C/A Code)
At 3 dB gain linearly polarized antenna	-160	-163	-166
At unity gain antenna	-3	-3	-3
At typical RHCP antenna	3.4	3.4	3.8
At unity gain RHCP antenna	-159.6	-162.6	-165.2

of the signals received at a user equipment is nearly 10^{-16} watt, or one-billionth of a billionth of the power consumed by a single 100 watt light bulb. In normal operation, the C/A signal power density at a receiver is below the noise floor and it is not clear on a spectrum analyser. For an antenna with a 3 dB gain, the minimum received signal power of a L1 C/A signal is -160 dBW [38], while that of a L1 P code signal is -163 dBw.

Moreover, L1 signal with either C/A code or P code has -3 dB for a unity gain antenna. The minimum received GPS signal power is 3.4 dB, for both either C/A code and P code, for a typical Right-Hand Circular Polarization (RHCP) antenna.

2.1.3 Effects of Interference on GPS Satellite Signal Receivers

GPS Signal Model

The complex baseband GNSS signal received by an N-element antenna array of arbitrary structure can be expressed as

$$\begin{aligned}\mathbf{x}(nT_s) = \sum_{m=0}^{M} \alpha_m d(nT_s - \tau_m) c(nT_s - \tau_m) \mathbf{a}_m e^{j\phi_m + j2\pi f_m n T_s} \\ + \sum_{j=0}^{J} v_j(nT_s)\mathbf{b}_j + \eta(nT_s)\end{aligned} \tag{2.1}$$

where M and J are the numbers of multipath and interferer components, respectively. T_s is the sample interval, τ_m is the code delay, ϕ_m is the phase shift, α_m is the gain factor and f_m is the Doppler frequency. The general GPS $\mathbf{x}(nT_s)$ stands for the transmitted navigation data bits and $c(nT_s)$ is a PRN code. We suppose that $v_i(nT_s)$ is the waveform of the jth interferer and $\eta(nT_s)$ represents the spatial temporal white zero-mean Gaussian noise and its variance σ^2. Moreover, $\mathbf{a}_m$ and $\mathbf{b}_j$ are the steering vectors of the mth multipath and the jth interferer components, respectively.

Therefore, I can put the received data **x** in a more compact form by defining the steering matrices **A** and **B**, and the vectors of GPS and interfering signals can be transformed as

$$\mathbf{x} = \mathbf{A}\mathbf{s} + \mathbf{B}\mathbf{v} + \mathbf{n} \tag{2.2}$$

where

$$\mathbf{A} = [\mathbf{a}_0, \cdots, \mathbf{a}_M]$$

$$\mathbf{B} = [\mathbf{b}_1, \cdots, \mathbf{b}_J]$$

$$\mathbf{s} = [\alpha_0 c(nT_s - \tau_0)e^{j\phi_M + j2\pi f_0 nT_s}, \cdots, \alpha_M c(nT_s - \tau_M)e^{j\phi_M + j2\pi f_M nT_s}]^T$$

$$\mathbf{v} = [v_1(nT_s), \cdots, v_j(nT_s)]^T \tag{2.3}$$

noting that the steering matrice **A** and **B** incorporate the whole spatial characteristics of the array and have full rank. Therefore, it is assumed that all samples are located in the same bit [39].

Interference Suppression at GPS Receivers

Any navigation system can be affeceted by an interferer with a high power. GPS signal is very easy to be influenced by interfering signals from unknown directions because it arrives at the receiver at a very low-power level, 20-30 dB below the receiver's thermal noise level. Therefore, the performance of GPS navigation degrades significantly in the presence of high-power interference.

Interference is normally defined as eirher wideband or narrowband, which depend on its bandwidth relative to the bandwidth of the desired GPS signal. Here the L1 C/A signal is discussed. The ultimate limit in narrowband interference is a signal consisting of a single tone, referred to as a Continuous Wave (CW). Table 2.4 and Table 2.3 summarize various types and potential sources of interferers [37].

Table 2.3: Types of interference and potential sources for wideband environment

Type	Potential Sources
Band-limited Gaussian	International matched bandwidth noise jammers
Phase/frequency modulation	Television transmitters or nearband microwave link transmitters
Matched spectrum	International matched-spectrum jammers
Pulse	Any type of burst transmitter such as radar or ultrawideband

Table 2.4: Types of interference and potential sources for narrowband environment

Type	Potential Sources
Phase/frequency modulation	Intentional chirp jammers from AM radio station
Swept continuous wave	Intentional swept CW jammers or FM stations
Continuous wave	near-band unmodulated transmitted's carriers

The main strategy of interference mitigation is to eliminate the jamming signal or reduce the power of interference. Many interference mitigation techniques have been carried out for GPS receivers , including the RF/IF filtering, the use of sufficient number of bits and AGC, augmentation of GPS by adaptive antenna array processing [40, 31, 7, 10]. Interference can be filtered out by either a GPS antenna or RF/IF filters in the front end of a commercial GPS receiver. The use of augmentation of GPS by ground is too rich to implement. Since interference in a GPS environment comes from specific directions, the adapive antenna array processing techniques are considered in most of the time [37, 38].

There are two representative adaptive array algorithms. One is known as a nulling antenna system [41, 42], which is used to cancel interfering signals, and the other one is called a beam former [43, 44, 45], which does not only cancel the interfering signals, but also leads to the components of the satellite signals at each element aligned in phase, providing useful gain in the direction of the signal of interest.

Although the adaptive nulling technique can adaptively place nulls in the directions of interfering signals, it may be inadequate for wideband operation. Thus, space-time

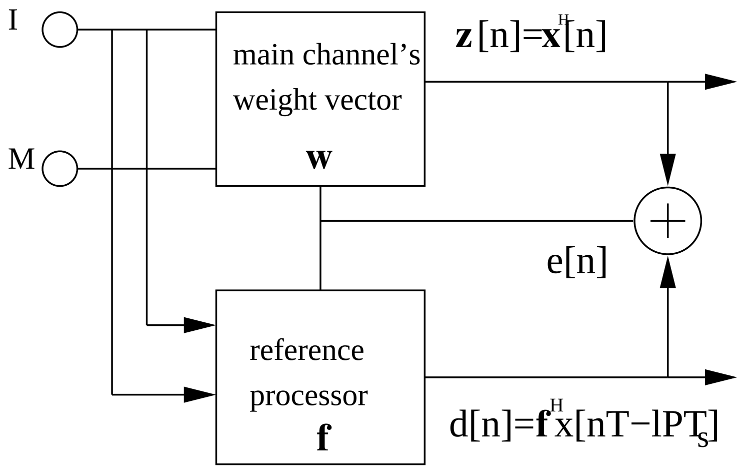

Fig. 2.2: The structure of self-coherence of GPS signal.

adaptive processing (STAP) [7] has been proposed in recent years to solve these problems. The STAP technique has been widely used to reduce the effect of interference because it greatly increases the degree of freedom under the same antenna conditions.

An interference suppression method based on STAP technique has been proposed which uses self-coherent feature of GPS signals [46]. This approach is based on the structure of GPS signal. A block diagram of the proposed algorithm is shown in Fig 2.2, which consists of a main channel and a reference channel. The samples in both channels are processed by the beam forming weight vector **w** and another processor **f**. The samples of the reference channel are lP chips ($P = 1022$, $1 \le l < 20$) delay of the main channel's data. The model of the main channel is given by

$$\mathbf{x}[n] = \mathbf{a}_0 s_0[n] + \sum_{j=1}^{J} \mathbf{a}_j s_j[n] + \mathbf{n}[n] \tag{2.4}$$

where $\mathbf{x}[n]$ is the $M \times 1$ data vector, $s_0(n)$ is the desired GPS signal and $s_j[n]$ is the jth interferer, $\mathbf{a}_0$ and $\mathbf{a}_j$ are the $M \times 1$ steering vectors of the desired GPS signal and the jth interferer, respectively, and $\mathbf{n}[n]$ is the noise vector. I assumed in these thesis that the desired signal, interference and noise are uncorrelated.

Because of the repetition of the GPS signal, the GPS signals of two channels in Fig 2.2 have the same values as long as they are within the same symbol period. However, the interferer samples have different values because they have a different periodic signal

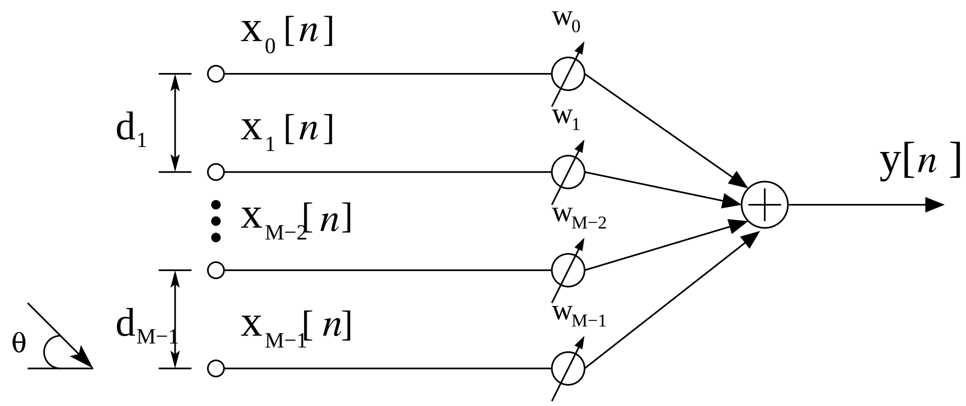

Fig. 2.3: A narrowband beam forming structure.

structure from that of the GPS signal. The samples of the reference channel are given by

$$\mathbf{x}[nT - lPT_s] = \mathbf{a}_0 s_0[n] + \sum_{j=1}^{J} \mathbf{a}_j s_j[nT - lPT_s] + \mathbf{n}[nT - lPT_s] \tag{2.5}$$

The algorithm can find the weight vectors **w** and **f** by maximizing the cross-correlation between the output of both the main channel and the reference channel.

The main advantage of the algorithm is that is makes full use of the strucure of GPS signals and does not need any prior knowledge of the transmitted signal or the direction of the satellites. Besides, this method is not susceptible to steering vector errors and therefore very robust. So the algorithm may be used more widely in GPS interference cancellation.

Although the STAP technique is effective in interference suppression, it has a high computational complexity. Some techniques for solving this problem are proposed in [6, 15, 47, 48].

Moreover, we can provide better performance by improving equipment with jammer rejection and more robust receiver signal processing with complex correlators. We can also avoid interferer environments by using navigation systems. Finally, the interfering signals can be cancelled effectively by attacking the jammer directly.

2.2 General Array Signal Model

In the beam forming research [6, 15, 47, 48], an array of sensors is employed, and the signal of interest or the desired signal is assumed to arrive from some specific directions with interfering signals from other directions. These sensors are located at different spatial positions. The main task of interference suppression is that collecting spatial samples and processing them to null out or reduce the effect of interfering signals and extract the desired signal. Thus, a specific beam pattern of the array system is achieved with the main beam pointing to the desired signals and nulls towards the interferers.

According to the bandwidth, beam forming systems can be divided into two categories: narrowband and wideband.

A narrowband beam forming structure based on a linear array with M sensors is shown in Fig. 2.3. The output $y(t)$ at time t is given by a linear combination of these spatial samples $x_m(t), m = 0, 1, 2, \ldots, M-1$, as

$$y(t) = \sum_{m=0}^{M-1} x_m(t) w_m^* \tag{2.6}$$

where w_m and * denotes the coefficients of the sensors and the complex conjugate, respectively.

This model is only useful for sinusoidal or narrowband signals, where the bandwidth of the impinging signal should be small enough to make sure the time delay between received sensor signals can be approximated by a simple phase shift. Besides, I normally assume the sensors are identical and omnidirectional [49, 50, 51].

Now I study the array's response to an impinging complex plane wave $e^{j\omega t}$ with an angular frequency ω and a DOA angle θ, where θ is measured from the broadside of the linear array. Without loss of generality, I assume the phase of the signal is zero at the first sensor. Hence the signal received by the first sensor is $x_0(t) = e^{j\omega t}$ and by the *m*th sensor is $x_m(t) = e^{j\omega(t-\tau_m)}$, $m = 1, 2, 3, \ldots, M-1$, where τ_m is the propagation delay for the signal from sensor zero to sensor m and it is a function of θ. Therefore, the beam former output is given by

$$y(t) = e^{j\omega t} \sum_{m=0}^{M-1} e^{-\omega\tau_m} w_m^* \tag{2.7}$$

with $\tau_0 = 0$. The response of the output changes to:

$$P(\omega, \theta) = \sum_{m=0}^{M-1} e^{-j\omega\tau_m} w_m^* = \mathbf{w}^H \mathbf{a}(\omega, \theta) \tag{2.8}$$

where $\{\cdot\}^H$ is the Hermitian transpose operation and the weight vector **w** holds the M coefficients of the sensors, given by:

$$\mathbf{w} = [w_0, w_1, \ldots, w_{M-1}]^T \tag{2.9}$$

and $\mathbf{a}(\omega, \theta)$ is known as the steering vector, which is given by:

$$\mathbf{a}(\omega, \theta) = [1, e^{-j\omega\tau_1}, \ldots, e^{-j\omega\tau_{M-1}}]^T \tag{2.10}$$

where $\{\cdot\}^T$ is transpose operation.

In array processing, if the inter-element spacing of the impinging signals array is too large, then the sources at different locations may have the same array steering vector, which leads to the spatial aliasing problem, because I cannot determine their locations based on the received array signals.

We assume a uniformly spaced linear array with an inter-element spacing d, and I have $\tau_m = m\tau = m(dsin\theta)/c$ and $\omega\tau_m = m(2\pi dsin\theta)/\lambda$, where λ is the corresponding wavelength and ω is the angular frequency. Then the steering vector can change to

$$\begin{aligned} \mathbf{a}(\omega, \theta) &= [1, e^{-j\omega\tau_1}, \ldots, e^{-j\omega\tau_{M-1}}]^T \\ &= [1, e^{-j(2\pi dsin\theta)/\lambda}, \ldots, e^{-j(M-1)(2\pi dsin\theta)/\lambda}]^T \end{aligned} \tag{2.11}$$

To avoid aliasing, the condition $|2\pi(\sin\theta)d/\lambda| < \pi$ has to be satisfied. Then I have $|d/\lambda \sin\theta| < 1/2$. Therefore the sensor spacing should be less than half of the signal wavelength λ.

Normally, I always set $d = \lambda/2$, so $\omega\tau_m = m\pi\sin\theta$ and the steering vector is given by:

$$\mathbf{a}(\theta_k) = [1, e^{-j\pi\sin\theta_k}, \cdots, e^{-j(M-1)\pi\sin\theta_k}]^T \tag{2.12}$$

If there are K signals impinging from different DOA angles $\theta_0, \cdots \theta_{K-1}$, then the steering vector can be written into a matrix form as

$$\mathbf{A} = \begin{bmatrix} 1 & e^{-j\pi\sin\theta_0} & \dots & e^{-j(M-1)\pi\sin\theta_0} \\ 1 & e^{-j\pi\sin\theta_1} & \dots & e^{-j(M-1)\pi\sin\theta_1} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & e^{-j\pi\sin\theta_{K-1}} & \dots & e^{-j(M-1)\pi\sin\theta_{K-1}} \end{bmatrix} \tag{2.13}$$

the matrix **A** can be briefly written as

$$\mathbf{A} = [\mathbf{a}_0(\omega,\theta_0), \mathbf{a}_1(\omega,\theta_1), \mathbf{a}_2(\omega,\theta_2), \cdots, \mathbf{a}_{K-1}(\omega,\theta_{K-1})]^T \tag{2.14}$$

then the array output **x**(t) can be computed as

$$\mathbf{x}(t) = \mathbf{A}\cdot\mathbf{s}(t) + \mathbf{n}(t) \tag{2.15}$$

where

$$\mathbf{x}(t) = [x_0(t), x_1(t), \cdots, x_{M-1}(t)]^T$$

and

$$\mathbf{s}(t) = [s_0(t), s_1(t), \cdots, s_{K-1}(t)]^T$$

are the received array signal vector and source signal vector, respectively. **n**(t) is a random noise vector.

As the signal bandwidth increases, the performance of a narrowband beam former will degrade. For wideband signals, each of them consists of infinite number of different frequency components, and the value of the weights should be different for different frequencies. Thus, the new weight vector for wideband environment should be written as

$$\mathbf{w}(\omega) = [w_0(\omega), w_1(\omega), \cdots, w_{M-1}(\omega)]^T \tag{2.16}$$

Traditionally, a method to form such a set of frequency dependent weights is to use tapped delay-lines (TDLs) and FIR/IIR filters in its dicrete form. Either TDLs or FIR/IIR filters perform a temporal filtering process to form a frequency dependent response for each of the received wideband sensor signals to cancel out the difference of phase for different frequency components. The output of such a wideband beam former can be computed as:

$$y(t) = \sum_{m=0}^{M-1}\sum_{f=0}^{F-1} x_m(t - fT_s) \times w_{m,f}^* \tag{2.17}$$

where $F-1$ is the number of delay elements associated with each of the M sensors and T_s is the delay between adjacent taps of the TDLs.

The weight vector of a wideband beam former can be expressed as

$$\mathbf{w} = [\mathbf{w}_0, \mathbf{w}_1, \cdots, \mathbf{w}_{F-1}]^T \tag{2.18}$$

where each vector w_f, $f = 0, 1, \cdots, F-1$, contains the M complex conjugate coefficients found at the ith tap position of F TDLs, and is expressed as

$$\mathbf{w} = [w_{0,f}, w_{1,f}, \cdots, w_{M-1,f}]^T \tag{2.19}$$

Note that $a_w(\theta, \omega)$ is the steering vector for wideband signals and its elements correspond to the complex exponentials $e^{j\omega(\tau_m + iT_s)}$

$$\begin{aligned}\mathbf{a}_w(\theta, \omega) = [&e^{-j\omega\tau_0}, \cdots, e^{-j\omega\tau_{M-1}}, \cdots, e^{-j\omega(\tau_0+T_s)}, \cdots, e^{-j\omega(\tau_{M-1}+T_s)} \\ &, \cdots, e^{-j\omega(\tau_0+(F-1)T_s)}, \cdots, e^{-j\omega(\tau_{M-1}+(F-1)T_s)}]^T\end{aligned} \tag{2.20}$$

Written the array output as a digital form, then the nth snapshot vector $\mathbf{x}[n]$ of the received array signals can be expressed as

$$\mathbf{x}[n] = \mathbf{A} \cdot \mathbf{s}[n] + \mathbf{n}[n] \tag{2.21}$$

where

$$\mathbf{x}[n] = [x_0[n], x_1[n], \cdots, x_{M-1}[n]]^T \in \mathbb{C}^{M\times 1}$$

and

$$\mathbf{s}[n] = [s_0[n], s_1[n], \cdots, s_{L-1}[n]]^T \in \mathbb{C}^{L\times 1}$$

For finding out a weight vector, it is very important to estimate an effective correlation matrix of received array data $\mathbf{R}_{xx}$.

2.3 Basics of Beam Former

2.3.1 Optimization Using a Reference Signal

If there is a reference signal $r[n]$ available, a narrowband beam forming structure can employ it to estimate the weights of the beam former [52], which is shown in Fig 2.4. The

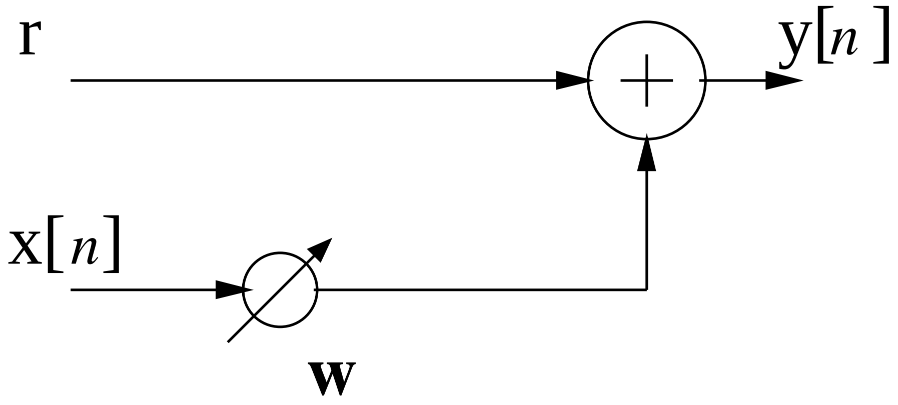

Fig. 2.4: The reference signal based (RSB) beam forming structure.

array output is substracted from an available reference signal to generate an error signal, and I have:

$$
\begin{aligned}
e[n] &= r[n] - y[n] \\
&= r[n] - \mathbf{w}^H \mathbf{x}[n]
\end{aligned} \tag{2.22}
$$

Using the minimum mean-squared error (MSE), given by

$$
\begin{aligned}
MSE &= E\{e[n]e^*[n]\} \\
&= E\{(r[n] - \mathbf{w}^H[n]\mathbf{x}[n])^H(r[n] - \mathbf{w}^H[n]\mathbf{x}[n])\} \\
&= E\{|r[n]|^2\} - E\{\mathbf{w}^H[n]\mathbf{x}[n]r^*[n]\} - \\
&\quad E\{r[n]\mathbf{x}^H[n]\mathbf{w}[n]\} + E\{\mathbf{w}^H[n]\}\mathbf{R}_{xx}E\{\mathbf{w}[n]\}.
\end{aligned} \tag{2.23}
$$

Taking the gradient of (2.23) with respect to $\mathbf{w}^*$ leads to

$$
\nabla_{\mathbf{w}^*} = -\mathbf{z}_{xr} + \mathbf{R}_{xx}\mathbf{w}[n] \tag{2.24}
$$

where

$$
\mathbf{z}_{xr} = E\{\mathbf{x}[n]r^*[n]\}
$$

let $\nabla_{\mathbf{w}^*}$ equal to zero and solving it leads to the following optimum weight vector, which is the well-known Wiener-Hoff equation,

$$
\mathbf{w}_{Mopt} = \mathbf{R}_{xx}^{-1}\mathbf{z}_{xr}. \tag{2.25}
$$

The scheme is employed to acquire a weak signal in the presence of a strong interferer by setting the reference signal to zero and initializing the weights to provide an omnidirectional pattern [53]. Besides, the MSE minimization scheme is also called the Wiener filter. In general, the Wiener filter provides higher output SNR compared to the MVDR in the presence of a weak signal source. The two processors provide almost the same performance because the input signal power is much lower than the background noise. This result is discussed in [54, 55].

The required reference signal for the Wiener filter may be produced in many different ways, depending upon the application. The approaches based on using reference signals to estimate array weights have been discussed in [43, 44, 45].

Although the reference signal based beam former may look unrealistic, it provides a chance for the standard adaptive algorithms to be employed in many adaptive beam formers, such as the least mean square (LMS) algorithm and the recursive least squares (RLS) algorithm [56, 57].

2.3.2 Null-Steering Beam Former

The nulling-steering beam forming method is used to cancel a plane wave arriving from a direction and produces a null in the response pattern in the DOA of the plane wave. DICANNE is an early scheme, which achieves this by the estimation of the signal arriving from a specific direction by steering a conventional beam in the direction of the source and substracting the output of this from each element [41, 42]. It is very effective to cancel strong interference and could be repeated for multiple interference cancellation.

A beam with unity response in the direction of the desired signal and nulls in the directions of interferers may be formed by estimation of the weights of a beam former, using some constraints. Assume that a_o is the steering vector in the direction where unity response is required and that $a_1, \cdots, a_{K-1}$ are $K-1$ steering vectors corresponding to $K-1$ directions where nulls are required. the desired weight vector can be obtained by solving the following equation:

$$\mathbf{w}^H \mathbf{a}_0 = 1 \tag{2.26}$$

$$\mathbf{w}^H \mathbf{a}_k = 0, \tag{2.27}$$

the solution is given by using matrix notation

$$\mathbf{w}^H \mathbf{A} = \mathbf{c}^T \tag{2.28}$$

where **A** is a matrix associated with all directional sources, given by

$$\mathbf{A} = [\mathbf{a}_0, \mathbf{a}_1, \cdots, \mathbf{a}_{K-1}] \tag{2.29}$$

and **c** is a vector of all zero except for the first element, given by

$$\mathbf{c} = [1, 0, \cdots, 0]^T \tag{2.30}$$

Assume that the inverse of A is valid, which requires that all steering vectors are linearly independent. Thus, the solution of the weight vector is given by

$$\mathbf{w} = \mathbf{A}^{-1}\mathbf{c} \tag{2.31}$$

Because of the structure of the vector **c**, the first row of matrix $\mathbf{A}^{-1}$ forms the weight vector. In other words, the weights have the desired properties of unity response in the desired direction and nulls in the directions of interferers.

When the number of required nulls is less than $M-1$, $i.e.$ $K < M-1$, the matrix **A** is not a square matrix. So the solution of the weight vector is given by

$$\mathbf{w} = \mathbf{A}(\mathbf{A}^H\mathbf{A})^{-1}\mathbf{c} \tag{2.32}$$

Although this beam forming method has nulls in the directions of interferers, it is not effective to minimize the uncorrelated noise at the array output. It is possible for us to minimize the mean output power subject to some constraints [58].

An application based on null-steering beam forming for detecting an amplitude modulated signal by placing nulls in the known directions of interference is described in [59], which can reduce the power of the strong interference in a mobile communications system. The application of a null-steering scheme for a transmitting array employed at a station is discussed in [60], by minimizing the interference toward other channel mobile users. Moreover, the performance analysis of a null-steering algorithm is presented in [61].

2.3.3 Beam Former for Maximizing the Output SINR

The shortcoming of null-steering beam forming is that it needs the knowledge of the directions of interferers to estimate the weight vectors instead of maximizing the output

SNR. In this section, the optimal beam forming method is introduced, which overcome the limitations.

Assume that an M-dimensional complex vector $\widehat{\mathbf{w}}$ represent the weights of a beam former, which can maximize the output SNR without any constraints. An expression for $\widehat{\mathbf{w}}$ can be given by [52, 62]

$$\mathbf{w} = \mu_0 \mathbf{R}_N^{-1} \mathbf{a}_0 \tag{2.33}$$

where

$$\mathbf{R}_N = E\{\mathbf{n}[n]\mathbf{n}^H[n]\} \tag{2.34}$$

with

$$\mathbf{n}[n] = [\mathbf{n}_0[n], \mathbf{n}_1[n], \cdots, \mathbf{n}_{M-1}[n]]^H \tag{2.35}$$

is the correlation matrix of the noise alone, and $\mathbf{n}[n]$ is the $M \times 1$ noise vector. It means that it does not contain any information about the directions of desired signals and jamming signals and μ_0 is a constant. For an array constrained to have a unit response in the direction of the desired signal, μ_0 can be given by

$$\mu_0 = \frac{1}{\mathbf{a}_0 \mathbf{R}_N^{-1} \mathbf{a}_0} \tag{2.36}$$

when $g = 1$, a distorted response will be achieved. Thus, the weight vector can be expressed as

$$\mathbf{w} = \frac{\mathbf{R}_N^{-1} \mathbf{a}_0}{\mathbf{a}_0^H \mathbf{R}_N^{-1} \mathbf{a}_0} \tag{2.37}$$

In practice, the correlation matrix of noise alone is not usually available, so the

$$\mathbf{R}_{xx} = E\{\mathbf{x}[n]\mathbf{x}^H[n]\} \tag{2.38}$$

is used to estimate the weight vector instead of $\mathbf{R}_N$. An expression for the weight is given by

$$\hat{\mathbf{w}} = \frac{\mathbf{R}_{xx}^{-1} \mathbf{a}_0}{\mathbf{a}_0^H \mathbf{R}_{xx}^{-1} \mathbf{a}_0} \tag{2.39}$$

For the optimal beam former to maximize the SNR by cancelling interferers, the number of interfering signals must be less than $(M - 2)$, because a linear array with M

elements has $(M-1)$ degrees of freedom and one is for the direction of the desired signal. However, the array beam former may not be truly able to achieve the maximization of the output SNR by reducing in every interferer because of environment effects.

According to the researches of mobile communication, the optimal beam former is often seen as the optimal combiner. Discussion on the application of the optimal combiner to improve the performance of output SINR can be found in [63, 64, 65].

2.4 Linearly Constrained Minimum Variance Beam Former

This adaptive beam forming algorithm assumes that the reference signal is not available, but the information of DOA angle of the signal of interest and their bandwidth range are known. Thus, some constraints can be applied on the array coefficients and minimize the power of the output subject to the applied constraints. The response of the beam former is constrained by some specific directions of the impinging desired signals, which are preserved subject to a phase response. Therefore, the output components will obtain a gain due to the interfering signals being minimized.

The main idea of LCMV beam forming is to ensure that any signal having a frequency ω_0 and DOA angle θ_0 passes the beam former with a specified response f, where f is a constant, given by:

$$\mathbf{w}^H \mathbf{a}_0(\theta_0) = f \tag{2.40}$$

and the value of the mean output power is given by

$$\begin{aligned} E\{\|y[n]\|^2\} &= E\{\mathbf{w}^H \mathbf{x}[n]\mathbf{x}[n]^H \mathbf{w}\} \\ &= \mathbf{w} E\{\mathbf{x}[n]\mathbf{x}[n]^H\}\mathbf{w} \\ &= \mathbf{w}^H \mathbf{R}_{xx} \mathbf{w} \end{aligned} \tag{2.41}$$

Thus, the LCMV problem can be computed as:

$$\begin{aligned} &\min_{\mathbf{w}} \mathbf{w}^H \mathbf{R}_{xx} \mathbf{w} \\ &\text{subject to } \ \mathbf{w}^H \mathbf{a}_0(\theta_0) = f \end{aligned} \tag{2.42}$$

This single constraint formulation can be generalized by applying to multiple linear constraints for more control of the beam former's response, for example, by using more information of DOA angles and frequencies.

To find the optimum weight vector **w**, the method of Lagrange can be used. Firstly, a new function F_{LCMV} is formulated to transform the constrained optimizing problem into a unconstrained one by using the Lagrange multiplier method, which is given by

$$F_{LCMV} = \mathbf{w}^H \mathbf{R}_{xx} \mathbf{w} + \lambda_1 (\mathbf{w}^H \mathbf{a}(\theta_0) - G_0) + \lambda_1^H (\mathbf{w}^H \mathbf{a}(\theta_0) - G_0)^H \tag{2.43}$$

where λ_1 is an indeterminate parameter. Secondly, differentiating the function in Equation (2.43) with w^H, I have:

$$\nabla_{w^H} F_{LCMV} = \mathbf{R}_{xx} \mathbf{w} + \lambda \mathbf{a}(\theta_0) \tag{2.44}$$

Thirdly, setting this result equal to zero, the optimal weight vector $\mathbf{w}_{opt}$ can be calculated in term of the Lagrange multiplier:

$$\mathbf{w}_{opt} = -\mathbf{R}_{xx}^{-1} \mathbf{a}(\theta_0) \lambda \tag{2.45}$$

Noticing that the parameter λ satisfies the constraint Equation (2.43), I have:

$$-\mathbf{a}(\theta_0)^H \mathbf{R}_{xx}^{-1} \mathbf{a}(\theta_0) \lambda = f \tag{2.46}$$

Finally, solving this equation for λ and substituting λ into Equation (2.45) yields:

$$\mathbf{w}_{opt} = \frac{f \mathbf{R}_{xx}^{-1} \mathbf{a}(\theta_0)}{\mathbf{a}(\theta_0)^H \mathbf{R}_{xx}^{-1} \mathbf{a}(\theta_0)} \tag{2.47}$$

which is the final solution to the LCMV problem.

As described, the processor weights are selected by minimizing the output power of the processor while maintaining a non-zero response in the look direction. The LCMV beam former is an optimum beam former in terms of maximizing the output SINR. This is also known as minimum variance distortionless response (MVDR) [66], if I constrain the array with a unit response to the desired signal, namely, $f = 1$.

For other cases, $i.e.$ a non-broadside arrival, the constraint matrix can be formulated by sampling the frequency band of interest of the signal and constrain the response of beam former to those frequency points to be the desired ones, which are usually some pure delays or zero if we want to null out those signals.

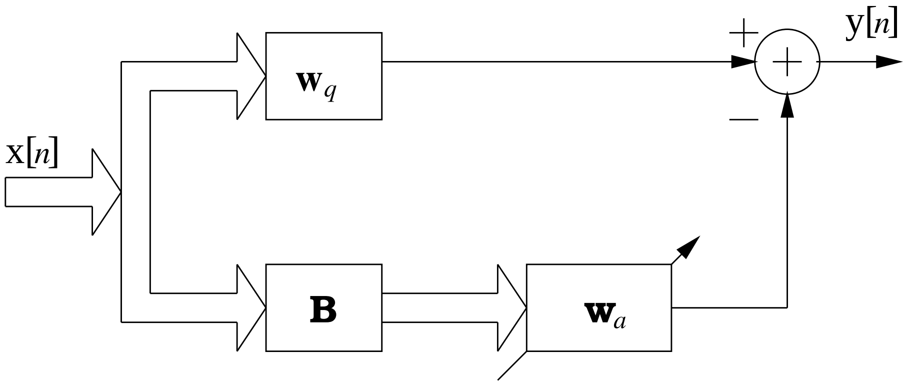

Fig. 2.5: The GSC beam forming structure.

2.5 Generalized Sidelobe Canceller

Generalized Sidelobe Canceller can be viewed as a method for transforming the constrained minimization problem like LCMV into an unconstrained one in order that the well-known standard unconstrained adaptive algorithms can be employed in this structure, which is shown in Fig. 2.5.

The LCMV beam former can be considered as having two conditions: one is the constraint:

$$\mathbf{w}^H \mathbf{a}(\theta_0) = f, \tag{2.48}$$

and the other one is to minimize the output variance:

$$\min_{\mathbf{w}} \mathbf{w}^H \mathbf{R}_{xx} \mathbf{w}. \tag{2.49}$$

Then $\mathbf{w}$ can be decomposed into two orthogonal component $\mathbf{w}_q$ and $-\mathbf{v}$, which is defined as [67, 68]

$$\mathbf{w} = \mathbf{w}_q - \mathbf{v} \tag{2.50}$$

subject to

$$\mathbf{a}^H(\theta_0)\mathbf{w}_q = f \tag{2.51}$$

lies in the range of the steering vector $\mathbf{a}(\theta_0)$, which can be rewritten by

$$\mathbf{w}_q = \mathbf{a}(\theta_0)(\mathbf{a}^H(\theta_0)\mathbf{a}(\theta_0))^{-1} f \tag{2.52}$$

while the component **v** is the null space of $\mathbf{a}(\theta_0)$, $i.e$, the space of all **v** fulfilling $\mathbf{a}^H(\theta_0)\mathbf{v} = 0$. We can use a vector $\mathbf{w}_a$ to linearly combine the basis vectors in an $M \times M$ matrix **B** to form **v**, then I have:

$$\mathbf{v} = \mathbf{B}\mathbf{w}_a \tag{2.53}$$

where **B** is the blocking matrix which can be obtained from $\mathbf{a}(\theta_0)$ using some orthogonalization methods such as the cascaded columns of difference (CCD) method, the singular value decomposition (SVD) method and QR decomposition [69].

Given the choice for $\mathbf{w}_q$ and **B**, Equation (2.50) can be changed into

$$\mathbf{w} = \mathbf{w}_q - \mathbf{B}\mathbf{w}_a \tag{2.54}$$

which satisfies the constraint equation for any choice of $\mathbf{w}_a$.

Then the LCMV problem is reduced to that of finding the weights $\mathbf{w}_a$ without any constraints any more. A modified LCMV formulation is given by:

$$\begin{aligned}\mathbf{w}_{a,opt} &= \arg\min_{\mathbf{w}_a}[\mathbf{w}_q - \mathbf{B}\mathbf{w}_a]^H \mathrm{E}\{\mathbf{x}[n]\mathbf{x}^H[n]\}[\mathbf{w}_q - \mathbf{B}\mathbf{w}_a] \\ &= \arg\min_{\mathbf{w}_a}[\mathbf{w}_q - \mathbf{B}\mathbf{w}_a]^H \mathbf{R}_{xx}[\mathbf{w}_q - \mathbf{B}\mathbf{w}_a] \end{aligned} \tag{2.55}$$

The solution to the problem in Equation (2.55) can be given by Equation (3.9). According to $\mathbf{w}_{opt} = \mathbf{w}_q - \mathbf{B}\mathbf{w}_{a,opt}$, I have:

$$\mathbf{B}\mathbf{w}_{a,opt} = \mathbf{w}_q - \frac{G_0\mathbf{R}_{xx}^{-1}\mathbf{a}(\theta_0)}{\mathbf{a}(\theta_0)^H\mathbf{R}_{xx}^{-1}\mathbf{a}(\theta_0)} \tag{2.56}$$

Multiplying both sides of Equation (2.56) by $\mathbf{B}^H\mathbf{R}_{xx}$, for $\mathbf{B}^H\mathbf{a}(\theta_0) = 0$, I have:

$$\begin{aligned}\mathbf{B}^H\mathbf{a}(\theta_0)\mathbf{B}^H\mathbf{w}_{a,opt} &= \mathbf{B}^H\mathbf{a}(\theta_0)\mathbf{w}_q - \mathbf{B}^H\mathbf{a}(\theta_0)\frac{G_0\mathbf{R}_{xx}^{-1}\mathbf{a}(\theta_0)}{\mathbf{a}(\theta_0)^H\mathbf{R}_{xx}^{-1}\mathbf{a}(\theta_0)} \\ &= \mathbf{B}^H\mathbf{a}(\theta_0)\mathbf{w}_q - 0 \end{aligned} \tag{2.57}$$

Then multiplying both sides by $(\mathbf{B}^H\mathbf{R}_{xx}\mathbf{B})^{-1}$, the solution to $\mathbf{w}_a$ can be obtained by:

$$\mathbf{w}_{a,opt} = (\mathbf{B}^H\mathbf{R}_{xx}\mathbf{B})^{-1}\mathbf{B}^H\mathbf{R}_{xx}\mathbf{w}_q \tag{2.58}$$

2.6 Other Interference Suppression Algorithms

The traditional LCMV beam former and GSC introduced are two classical algorithms for interference suppression. In addition, there are many adaptive beam forming algorithms which are not introduced in this thesis, such as the soft constrained minimum variance (SCMV) beam former and the correlation constrained minimum variance beamformer (CCMV). All of those algorithms are mainly proposed for two reasons, one is improving the performance of output SINR, the other one is the computational complexity.

A new beam forming technique to suppress interfering signals is proposed in [39]. This method consists of two stages. Firstly, using the fact that the global navigation system signal (GNSS) is well below the noise floor, the subspace method is applied to find the interference subspace. Next, after despreading the signal, the interference subspace is used as a constraint in the following optimization problem.

Another method is proposed in [70], where based on circular antennas arrays, the minimum norm (min-norm) and LCMV methods are considered for interference suppression.

In theory, I usually assume that the signals are uncorrelated with each other, the antennas are ideal and the position is correct. However, in practice, many factors invalidating these ideal assumptions exist and sometimes some of them are very serious and can not be ignored. Thus, robust adaptive beam forming algorithms are developed to deal with this problem [71, 72, 73, 74, 75, 76, 77].

Among them, many approaches have tried to overcome the mismatch error between the real DOA of signal of interest and the designed look direction of the array to obtain the better improve the robustness of the beam former. For example, I can use a calibration signal to find the quiescent vector and the blocking matrix in GSC, or I can apply some target tracking methods to estimate the true DOA angle or the signal subspace to reduce or even cancel the mismatch error. If only the desired signal is present during a period I know, it can be used to adjust the array data to the right direction.

2.7 The Computational Complexity Analysis

The computational complexities of RSB algorithm, null-steering beam former, LCMV Beam former and the GSC algorithm are compared in term of real multiplications with respect to the sensor number M , which is shown in Table 2.4.

For RSB algorithm, I need O(NM^2) real multiplications to calculate $\mathbf{R}_{xx}$ and O(NM)

Table 2.5: A summary of computational complexities.

RSB	Null-steering	LCMV	GSC
$O(NM^2) + O(NM)$	$KM^2 + 4M$	$O(NM^2) + 22M$	$8M^2 + 4M - 4$

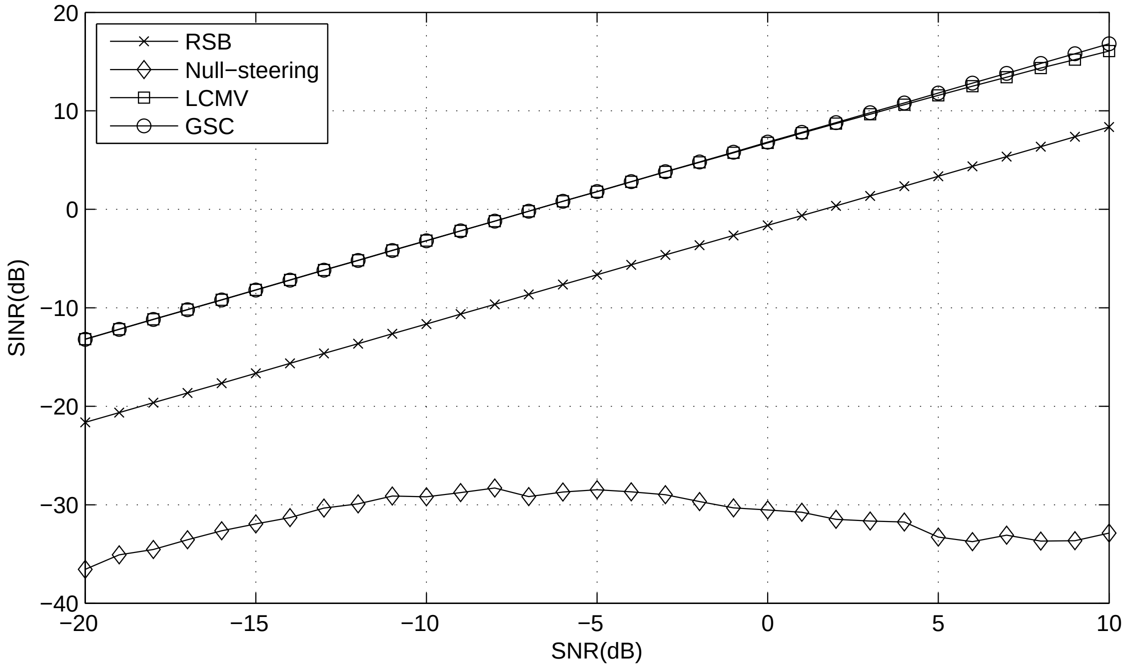

Fig. 2.6: Output SINR versus the input SNR with one interfering signal for $M = 5$.

real multiplications to obtain $\mathbf{z}_{xr}$. We also need additional $4M$ real multiplication to get the weight vector. Thus, the total computational complexity will be O$(NM^2)+$O(NM).

For null-steering beam former, the total computational complexity will be $KM^2{+}4M$.

For LCMV Beam former, the total computational complexity will be $O(NM^2){+}22M$.

For GSC Beam former, the total computational complexity will be $8M^2 + 4M - 4$.

2.8 Simulation

In this part of simulation, the reference signal beam former, null-steering beam former, LCMV beam former and GSC beam former are compared. I suppose it is based on a uniform linear array with 5 antennas and half-wavelength spacing. One arriving angle of narrowband interferer is randomly generated between $-\pi$ and π, and the input INR is

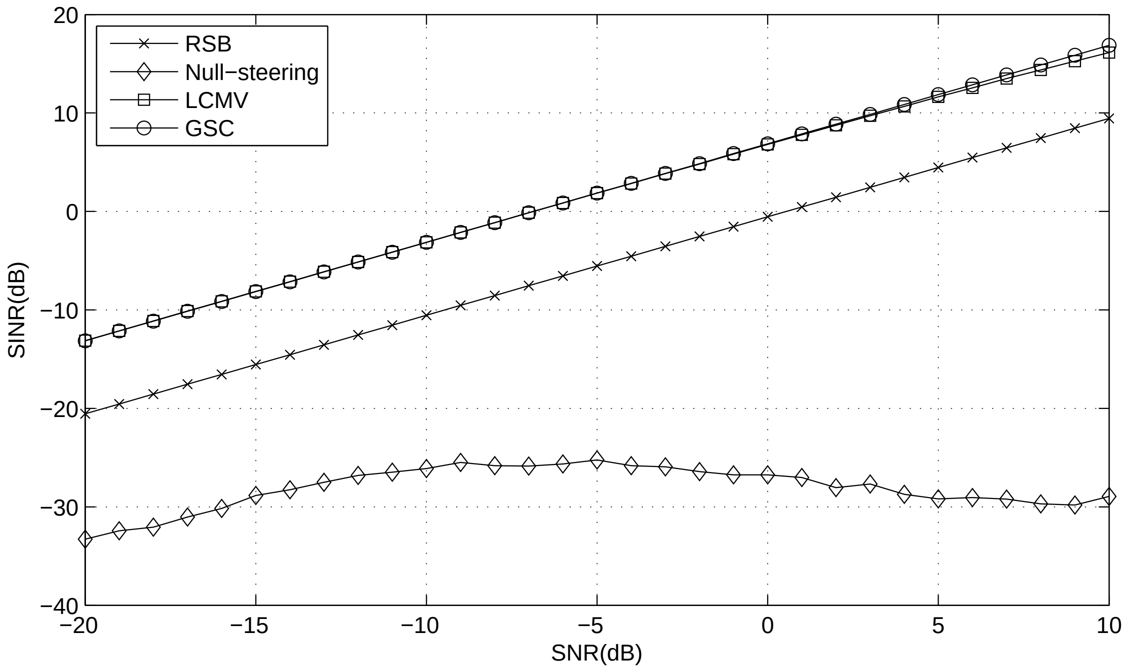

Fig. 2.7: Output SINR versus the input SNR with two interfering signals for $M = 5$.

fixed at 15 dB. The SNR is varied from -20 dB to 10 dB The input SINR is varied from 0.0014 dB to 1.1933 dB. The output SINR versus the input SNR, averaged over 1000 simulation runs, is shown in Fig. 2.6.

In this four algorithms, the null-steering algorithm provides the worst performance as it does not need any information of DOA angles and just forms null in the direction of interfering signals. The performance of LCMV algorithm and GSC algorithm are quite similar although LCMV provides better performance than GSC algorithm when SNR is getting closer to INR.

Then I add one more interfering signal to the received data. The results are very similar as the previous simulations.

2.9 Summary

In this chapter, firstly, I have reviewed the background of GPS, and introduced the structure of GPS signals. GPS signals are susceptible to all kinds of interferers from different sources, such as television and microwave. Then I introduce the signal level of GPS signal and different steps of GPS. All kinds of interference suppression methods are reviewed, such as the STAP method. In particular, I have focused on methods based on antenna array signal processing. A review about general reference signal beam former model and null-steering model are provided and two representative beam formers, the LCMV beam former and the GSC are described in detail. GSC algorithm can be looked as an improved method compared with LCMV. Other algorithms for interference suppression which are not mentioned in this thesis are also important, like SCMV and CCMV. In the simulation part, I compare with these four difference algorithms by varying the input SINR.

Chapter 3

Blind Interference Suppression Algorithms Based on Antenna Arrays

In Chapter 2, some adaptive beam forming algorithms for interference suppression are discussed, such as the LCMV beam former, which needs to know the directions of the desired signals. However, in practice, it is very difficult for us to gain the knowledge of directions of signals. Thus, the power minimization method is reviewed in this chapter, which does not need any prior information about directions of signal. This algorithm forms nulls in the the directions of interference, so it can reduce or cancel interfering signals effectively. Besides, a useful technique for separating interference from the received array data is also introduced in this chapter, called PCA. Finally and more importantly, I will propose an improved power minimization structure for interference suppression which combined with PCA. Simulation results show that the proposed method is more effective to increase the output SINR than the conventional power minimization method.

3.1 Power Minimization Method

3.1.1 Basics of the Power Minimization Method

The null steering technique has been widely used in signal navigation system, because it provides good performance for cancelling interference or reducing the effect of interference. The power minimization method is applied when the power of the desired signal is well below the noise level and the power of interference [6]. In a highly dynamic environment, it is very difficult to acquire the prior information about the signals. The main advantage of the power minimization method is that I do not exploit any priori information on the angular directions of the impinging signals [21].

As mentioned, only narrowband signals are discussed in this thesis. Consider a uniform linear array with M antennas. The array receives a narrowband desired signal s_1 from the direction θ_1 measured from the broadside of the array, and $K-1$ narrowband interferers s_k, $k=1,\cdots,K-1$ from directions θ_k, $k=2,\cdots,K-1$, respectively. We suppose that the signals are uncorrelated with each other and of zero mean, and the sensor noise is temporally and spatially white. Thus, I can rewrite the array data in a matrix format:

$$\mathbf{X}_{M\times N} = \mathbf{A}_{M\times K}\mathbf{S}_{K\times N} + \mathbf{N}_{M\times N} \tag{3.1}$$

where N is the sample number, and X, S and N are data matrices denoting the received signals, source signals and noise, respectively, which are defined as

$$\mathbf{S} = [\mathbf{s}[1], \mathbf{s}[2], \ldots, \mathbf{s}[N]]$$

$$\mathbf{X} = [\mathbf{x}[1], \mathbf{x}[2], \ldots, \mathbf{x}[N]] \tag{3.2}$$

with

$$\mathbf{x}[n] = [x_1[n], x_2[n], \ldots, x_M[n]]^T, \tag{3.3}$$

$$\mathbf{s}[n] = [s_1[n], s_2[n], \ldots, s_K[n]]^T$$

for $n = 1, \ldots, N$.

The structure of the power minimization method is shown in Fig 3.1, where the first received signal $x_1[n]$ is estimated as the reference antenna. The remaining antennas are viewed as auxiliary elements with the remaining received signals $x_m[n]$, $m=2,\cdots,M$. The weight vector is formed as

$$\mathbf{w} = [w_1, w_2, \cdots, w_M]^T \tag{3.4}$$

where $w_1 = 1$ for the power minimization method.

The main idea behind this method is to minimize the average power P_y of the output signal $y[n]$ subject to the constraint of $w_1 = 1$. Thus, the problem can be formulated as

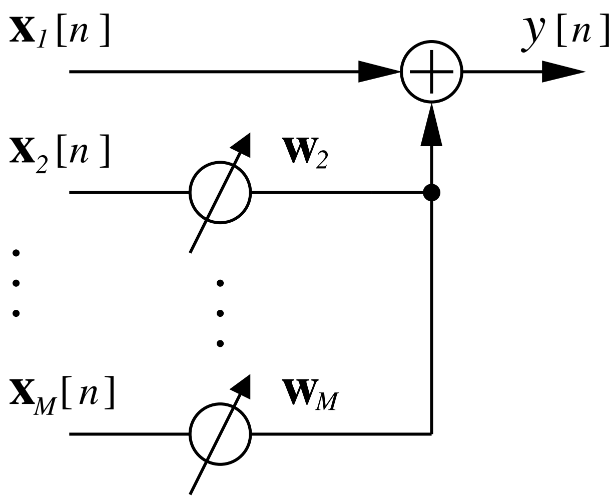

Fig. 3.1: Structure of the power minimization method.

$$\min_{\mathbf{w}} P_y \quad \text{subject to } \mathbf{w}^H\mathbf{c} = 1 \tag{3.5}$$

where

$$\begin{aligned} P_y &= E\{y^2[n]\} \\ &= \mathbf{w}^H\mathbf{R}_{xx}\mathbf{w} \end{aligned} \tag{3.6}$$

and $\mathbf{c} = [1, 0, \cdots, 0]^T$ is an $M \times 1$ vector.

As the sample number N is finite, it is impossible for us to find the true covariance matrix $\mathbf{R}_{xx}$ in practice. Thus, the approximation of $\mathbf{R}_{xx}$ is used instead, which is given by

$$\mathbf{R}_{xx} \approx \frac{1}{L}\mathbf{X}\mathbf{X}^H \tag{3.7}$$

To find an optimum solution to the power minimization problem, I can again apply the method of Lagrange multipliers. Firstly, a new function F_1 is formulated to transfer the constrained optimizing problem into an unconstrained one using the Lagrange multiplier method, which is given by

$$F_1 = \mathbf{w}^H\mathbf{R}_{xx}\mathbf{w} + \lambda(\mathbf{w}^H\mathbf{c} - 1) + \lambda^H(\mathbf{w}^H\mathbf{c} - 1)^H \tag{3.8}$$

where λ is the Lagrange multiplier.

solving this equation for λ and obtain the optimal solution:

$$\mathbf{w}_{opt} = \frac{\mathbf{R}_{xx}^{-1}\mathbf{c}}{\mathbf{c}^H\mathbf{R}_{xx}^{-1}\mathbf{c}} \tag{3.9}$$

Compared with the LCMV beam former, the power minimization method replaces the steering vector of the desired signal $\mathbf{a}(\theta_0)$ by an $M \times 1$ vector $\mathbf{c}$. This method do not need any angular information about directions of the signals, either the desired signal or interference. The power minimization method is very effective in a highly dynamic environment since it is very difficult for the GPS receivers to obtain directions of signals. Moreover, this method can form $M-1$ nulls in the directions of $M-1$ interferers.

3.1.2 Eigen-Decomposition of the Covariance Matrix

Eigen-decomposition based on the subspace technique has been carried out and applied to GPS signal in [6]. The estimation of array correlation matrix $\mathbf{R}_{xx}$ has been used in the power minimization method, which is defined as

$$\mathbf{R}_{xx} = \sum_{k=0}^{K-1} \sigma_k^2 \mathbf{a}_k \mathbf{a}_k^H + \sigma_n^2 \mathbf{I} \tag{3.10}$$

where $E\{\cdot\}$ represents the expectation operation, $\mathbf{I}$ is the identity matrix, σ_k^2 and σ_n^2 denotes the power of the kth source signal and the noise, respectively. Since signals and noise are uncorrelated, all cross-terms are cancelled out, i.e.

$$\begin{aligned} &E\{\mathbf{s}_i[n]\mathbf{s}_j^*[n]\} = 0 \ \text{ for any } \ i \neq j \\ &E\{\mathbf{s}_i[n]\mathbf{s}_j^*[n]\} = \sigma_i^2 \ \text{ for } \ i = j \\ &E\{\mathbf{s}_i[n]\mathbf{n}^*[n]\} = 0 \ \text{ for any } \ i \end{aligned} \tag{3.11}$$

Then, $\mathbf{R}_{xx}$ can be expressed by its eigenvalues and their corresponding eigenvectors. Moreover, the eigenvalues of $\mathbf{R}_{xx}$ can be divided into two sets when the received array data consists of uncorrelated directional source signals and uncorrelated white noise.

The eigenvalues that are of equal values are contained in one set. Their value is equal to the variance of the noise. The eigenvalues which are a function of the parameters of the directional sources are contained in the second set, and their number is equal to the number of these sources. The $\mathbf{R}_{xx}$ of an array of M elements immersed in K directional source signals and spatially white noise has K signal eigenvalues and M-K noise eigenvalues.

Denoting the M eigenvalues of $\mathbf{R}_{xx}$ in a descending order by λ_m, $m = 1, 2, \cdots, M$ and their associated eigenvectors by $\mathbf{U}_m$, $m = 1, 2, \cdots, M$, the matrix $\mathbf{R}_{xx}$ can be written by using eigen-composition as:

$$\mathbf{R}_{xx} = \mathbf{\Sigma}\mathbf{\Lambda}\mathbf{\Sigma}^H \tag{3.12}$$

with a diagonal matrix

$$\mathbf{\Lambda} = \begin{bmatrix} \lambda_1 & 0 & \dots & 0 \\ 0 & \lambda_2 & \dots & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & \dots & \lambda_M \end{bmatrix} \tag{3.13}$$

and eigenvector matrix

$$\mathbf{\Sigma} = [\mathbf{U}_1, \mathbf{U}_2, \cdots, \mathbf{U}_M] \tag{3.14}$$

where $\mathbf{U}_m$ is the mth eigenvector corresponding to the mth eigenvalue λ_m.

Using the fact that the eigenvectors form an orthonormal set, the following expression for $\mathbf{R}_{xx}$ is given by

$$R = \sum_{m=1}^{M} \lambda_m \mathbf{U}_m \mathbf{U}_m^H + \sigma_n^2 I \tag{3.15}$$

3.1.3 Signal Subspace Based on Power Minimization

For the general GPS case, the spatial covariance matrix can be divided into three components: the covariance matrix of the desired GPS signal, the covariance matrix of interference and the covariance matrix of noise. Equation (3.10) can be expressed as

$$\mathbf{R}_{xx} = \rho_{gps}^2 \mathbf{a}_{gps} \mathbf{a}_{gps}^H + \mathbf{A}_I \mathbf{P}_I \mathbf{A}_I^H + \sigma_n^2 \mathbf{I} \tag{3.16}$$

where $\mathbf{a}_{gps}$ is an $M \times 1$ steering vector of the GPS signal, ρ_{gps}^2 is the power of the GPS signal, $\mathbf{A}_I$ is an $M \times K$ matrix containing the steering vectors for the K interferers, which is defined as

$$\mathbf{A}_I = [\mathbf{a}_0, \mathbf{a}_1, \cdots, \mathbf{a}_{K-1}] \tag{3.17}$$

with

$$\mathbf{a}_k = [a_{1,k}, a_{2,k}, a_{3,k}, \cdots, a_{M,k}], \tag{3.18}$$

$\mathbf{P}_I$ is an $K \times K$ diagonal matrix with the powers of the interfering sources because the interference is uncorrelated with each other, which is defined as

$$\mathbf{P}_I = \begin{bmatrix} \rho_0 & 0 & \dots & 0 \\ 0 & \rho_1 & \dots & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & \dots & \rho_{K-1} \end{bmatrix}. \tag{3.19}$$

Finally, σ_n^2 is the noise power. Since the GPS signal is well below the noise floor, Equation (3.17) can be simplified into

$$\mathbf{R}_{xx} \approx \mathbf{A}_I \mathbf{P}_I \mathbf{A}_I^H + \sigma_n^2 \mathbf{I} \tag{3.20}$$

Note that $\mathbf{R}_I = \mathbf{A}_I \mathbf{P}_I \mathbf{A}_I^H$, whose rank is equal to the number of interferers. Thus, the number of interference K must be less than that of antennas M. The Equation (3.21) can be decomposed into

$$\mathbf{R}_{xx} = \sum_{m=1}^{K} \lambda_m \mathbf{U}_m \mathbf{U}_m^H + \sum_{m=K+1}^{M} \sigma_m \mathbf{U}_m \mathbf{U}_m^H \tag{3.21}$$

where λ_m and $\mathbf{U}_m$, $m = 1, \cdots, K$ are K larger eigenvalues and the associated eigenvectors, respectively, and $\mathbf{U}_m$, $m = K+1, \cdots, M$, are the eigenvectors associated with the smaller eigenvalues σ_n^2. Subsequently, $\mathbf{U}_m$, $m = 1, \cdots, K$ form the K-dimensional interference subspace and $\mathbf{U}_m$, $m = K+1, \cdots, M$ form the $N-K$-dimensional noise subspace. The inverse of $\mathbf{R}_{xx}$ can be expressed as

$$\begin{aligned}
\mathbf{R}_{xx}^{-1} &= \sum_{m=1}^{K} \frac{1}{\lambda_m} \mathbf{U}_m \mathbf{U}_m^H + \sum_{m=K+1}^{M} \frac{1}{\sigma_n^2} \mathbf{U}_m \mathbf{U}_m^H \\
&= \sum_{m=1}^{K} \frac{1}{\lambda_m} \mathbf{U}_m \mathbf{U}_m^H + \frac{1}{\sigma_m^2} [I - \sum_{m=1}^{K} \mathbf{U}_m \mathbf{U}_m^H] \\
&= \sum_{m=1}^{K} \frac{\sigma_m^2}{\sigma_n^2 \lambda_m} \mathbf{U}_m \mathbf{U}_m^H + \frac{1}{\sigma_m^2} [\mathbf{I} - \sum_{m=1}^{K} \mathbf{U}_m \mathbf{U}_m^H] \\
&= \frac{1}{\sigma_n^2} [\sum_{m=1}^{K} \frac{\sigma_n^2}{\lambda_m} \mathbf{U}_m \mathbf{U}_m^H + \mathbf{I} - \sum_{m=1}^{K} \mathbf{U}_m \mathbf{U}_m^H] \\
&= \frac{1}{\sigma_n^2} [I - \sum_{m=1}^{K} (1 - \frac{\sigma_n^2}{\lambda_m}) \mathbf{U}_m \mathbf{U}_m^H]
\end{aligned} \tag{3.22}$$

Assume that the power of interference is significantly greater than that of noise, I can consider that $1 - \frac{\sigma_n^2}{\lambda_m} \approx 1$. Equation (3.22) can be given by

$$\mathbf{R}_{xx}^{-1} = \frac{1}{\sigma_n^2}\{\mathbf{I} - \sum_{m=1}^{K} \mathbf{U}_m \mathbf{U}_m^H\} \tag{3.23}$$

where $\sum_{m=1}^{K} \mathbf{U}_m \mathbf{U}_m^H$ is the interference subspace and $\mathbf{I} - \sum_{m=1}^{K} \mathbf{U}_m \mathbf{U}_m^H$ is the orthogonal complement of the interference subspace.

According to Equation (3.23), I can easily see that when the interference is much stronger than noise, the orthogonal complement of the interference subspace can be substituted by $\mathbf{R}_{xx}^{-1}$. Thus, when the weight vector is calculated, I do not need any information of the directions of arriving interference or GPS signals.

However, to calculate the weight vector for interference suppression by the power minimization method, the following conditions must be satisfied:

Firstly, the desired signal must be well below the noise floor in order that I can ignore the effect of the desired signal.

Secondly, the power of interference must be significantly greater than that of noise in order that the inverse of $\mathbf{R}_{xx}$ can be replaced by the the orthogonal complement of the interference subspace.

Thirdly, the number of interference is not allowed to be more than that of antennas.

Although the interference can be cancelled and optimize the output SINR using the power minimization method, it is unavoidable that its performance degrades significantly when the input SNR is increases, which will be shown in the simulation section.

3.2 Principal Component Analysis

3.2.1 Introduction

PCA is the oldest and best known techniques of multivariate analysis. It was first introduced and improved by Pearson and Hotelling. Nowadays, as the advance of computing technologies, it is widely used in some statistical programming packages.

In this section, the PCA technique will be applied to the area of array signal processing. The main idea of PCA is to reduce the dimensionality of a data set in which there are a large number of signal samples.

3.2.2 Definition of Principle Components

Suppose that $\mathbf{x}[n]$ is a vector of M random variables, and the variances of the M random variables and the structure of the covariances or correlations are considered. As usual, I can not simply look at the M variables and all of the covariances and correlations. PCA allows us to find a few derived variables that keep most of information given by the original data. Firstly, I need find a linear combination $\mathbf{w}_1^H\mathbf{x}[n]$ which contains the maximum variance of $\mathbf{x}[n]$, where

$$\mathbf{w}_1 = [w_{1,1}, \cdots, w_{1,M}]^T \tag{3.24}$$

is an $M \times 1$ vector and $\{\cdot\}^H$ denotes conjugate transpose, that

$$\mathbf{w}_1^H\mathbf{x}[n] = \sum_{m=1}^{M} w_{1,m}x_m[n] \tag{3.25}$$

Secondly, find a linear combination $\mathbf{w}_2^H\mathbf{x}[n]$, which is uncorrelated with $\mathbf{w}_1^H\mathbf{x}[n]$ and having maximum variance. At the kth stage, a linear combination $\mathbf{w}_k^H\mathbf{x}[n]$ is found which has maximum variance and is uncorrelated with $\mathbf{w}_1^H\mathbf{x}[n]$, $\mathbf{w}_2^H\mathbf{x}[n]$,$\cdots$ $\mathbf{w}_{k-1}^H\mathbf{x}[n]$. In other words, the kth linear combination $\mathbf{w}_k^H\mathbf{x}[n]$ denotes the kth principal component of the original data **x**[n]. Generally, The number of principal components is less than that of the random variables, i.e. $k \leq m$. The matrix for transforming the original data into principal components can be obtained by many methods, such as singular value decomposition (SVD).

To apply principal component analysis to the study of array signal processing, I need to find the weight vectors $\mathbf{w}_1, \mathbf{w}_2, \cdots \mathbf{w}_M$. Firstly, consider the covariance matrix $\mathbf{R}_{xx}$ of data vector $\mathbf{x}[n]$, which has been defined in the previous section. This matrix contains the covariance between the ith and the jth elements of $\mathbf{x}[n]$ when $i \neq j$ and the variance of the jth element of $\mathbf{x}[n]$ when $i = j$. In practice, the covariance matrix is unknown, and I can replace it by a sample covariance matrix, which has been defined as Equation (3.7).

Then I define the output vector $\mathbf{y}[n]$ as

$$\mathbf{y}[n] = \mathbf{W}^H \mathbf{x}[n] \tag{3.26}$$

where

$$\mathbf{W} = [\mathbf{w}_1, \mathbf{w}_2, \cdots, \mathbf{w}_M] \tag{3.27}$$

and

$$\mathbf{y}[n] = [y_1[n], y_2[n], y_3[n], \cdots, y_M[n]]^T \tag{3.28}$$

As the linear functions $\mathbf{w}_1^H\mathbf{x}[n]$, $\mathbf{w}_2^H\mathbf{x}[n]$,$\cdots$ $\mathbf{w}_M^H\mathbf{x}[n]$ are uncorrelated with each other, I can consider var$(y_1[n], y_2[n]) = 0$, var$(y_1[n], y_3[n]) = 0$ and so on. We denote that E$\{\mathbf{y}_m[n]\mathbf{y}_m[n]^T\} = \delta_m^2, m = 1, 2, \cdots, M$. The correlation matrix $\mathbf{R}_{yy}$ can be defined as

$$\begin{aligned} \mathbf{R}_{yy} &= E\{\mathbf{y}[n]\mathbf{y}^H[n]\} \\ &= \begin{bmatrix} \delta_1^2 & 0 & \dots & 0 \\ 0 & \delta_2^2 & \dots & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & \dots & \delta_M^2 \end{bmatrix} \end{aligned} \tag{3.29}$$

Secondly, to derive the form of the principal components, $\mathbf{w}_1^H\mathbf{x}[n]$ is considered first. The problem can be formulated as

$$\begin{aligned} &\max_{\mathbf{w_1}} \mathbf{w}_1^H \mathbf{R}_{xx} \mathbf{w}_1 \\ &\text{subject to } \ \mathbf{w}_1^H \mathbf{w}_1 = 1 \end{aligned} \tag{3.30}$$

The constraint $\mathbf{w}_1^H\mathbf{w}_1 = 1$ means that the sum of squares of elements of $\mathbf{w}_1$ equals 1 in order that the maximum is achieved for a non-zero $\mathbf{w}_1$. To solve the problem (3.30), the approach is to use the method of Lagrange multipliers. Maximize

$$\mathbf{w}_1^H \mathbf{R}_{xx} \mathbf{w}_1 - \lambda(\mathbf{w}^H \mathbf{w} - 1) \tag{3.31}$$

where λ is a Lagrange multiplier. Differentiating with respect to $\mathbf{w}_1$, I have:

$$\mathbf{R}_{xx}\mathbf{w}_1 - \lambda \mathbf{w}_1 = 0 \tag{3.32}$$

Then I have:

$$(\mathbf{R}_{xx} - \lambda \mathbf{I}_m)\mathbf{w}_1 = 0, \tag{3.33}$$

where $\mathbf{I}_m$ is an $M \times M$ identity matrix. Therefore, λ is an eigenvalue of $\mathbf{R}_{xx}$ and $\mathbf{w}_1$ is the eigenvector. Thus, to maximize the function $\mathbf{w}_1^H \mathbf{R}_{xx} \mathbf{w}_1$, the optimal weight vector is the associated eigenvector. The quantity to be maximized is

$$\mathbf{w}_1^H \mathbf{R}_{xx} \mathbf{w}_1 = \mathbf{w}_1^H \lambda \mathbf{w}_1 = \mathbf{w}_1^H \mathbf{w}_1 \lambda = \lambda, \tag{3.34}$$

so λ is the largest eigenvalue, denoted as λ_1, corresponding to the eigenvector $\mathbf{w}_1$. Thus, Equation (3.34) can be written as

$$\mathbf{w}_1^H \mathbf{R}_{xx} \mathbf{w}_1 = \lambda_1 \tag{3.35}$$

Similarly, the kth principal component of $\mathbf{x}[n]$ is $\mathbf{w}_k^H \mathbf{x}[n]$ and $\mathbf{w}_k^H \mathbf{R}_{xx} \mathbf{w}_k = \lambda_k$, where λ_k is the kth largest eigenvalue of $\mathbf{R}_{xx}$, and $\mathbf{w}_k$ is the corresponding eigenvector.

In summary, the correlation matrix $\mathbf{R}_{yy}$ can be also defined as

$$\begin{aligned}
\mathbf{R}_{yy} &= E\{\mathbf{y}[n]\mathbf{y}[n]\} \\
&= \begin{bmatrix} \delta_1^2 & 0 & \dots & 0 \\ 0 & \delta_2^2 & \dots & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & \dots & \delta_M^2 \end{bmatrix} \\
&= \begin{bmatrix} \lambda_1 & 0 & \dots & 0 \\ 0 & \lambda_2 & \dots & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & \dots & \lambda_M \end{bmatrix}
\end{aligned} \tag{3.36}$$

As shown above, the eigenvectors $\mathbf{w}_1, \mathbf{w}_2, \cdots, \mathbf{w}_M$ correspond to the eigenvalues $\lambda_1, \lambda_2, \cdots, \lambda_M$, which are the first largest, the second largest, $\cdots$, and the smallest eigenvalues, respectively.

3.3 The Proposed Structure Based on PCA only

The theory of principal component analysis has been discussed in the last section. The eigenvectors can be used to separate the principal components from the original data $\mathbf{x}[n]$,

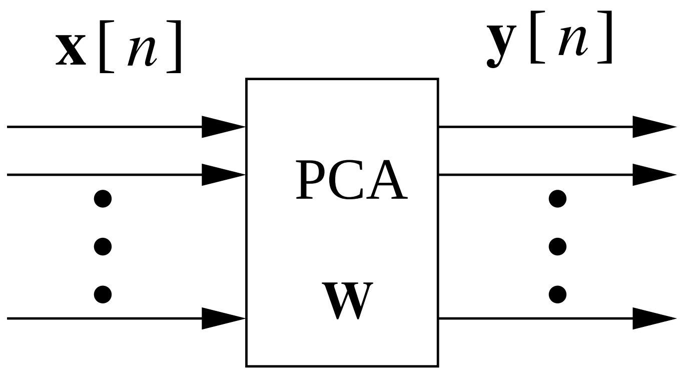

Fig. 3.2: Proposed Structure of the PCA technique.

and the output vector $\mathbf{y}[n]$ contains the principal components from large to small, i.e. $y_1[n]$ contains the largest component and $y_M[n]$ contains the smallest component. The structure for PCA is shown in Fig 3.2.

In this case, I can apply the PCA technique to the original data $\mathbf{x}[n]$ for interference suppression, as the PCA technique can separate the components of interfering signals from original data effectively. If there is one interferer in the received data, the principal component of the interference is mostly contained in the first row of output $\mathbf{y}[n]$, that is $y_1[n]$. Thus, the remaining $M-1$ outputs are only affected by noise and the desired signal. As the desired signal is very weak compared to noise, it is difficult decide which element of output contains the component of the desired signal. Thus, by taking the average of the remaining $M-1$ components, I expect that I can obtain good performance in terms of the output SINR. The weight vector is given by

$$\mathbf{w}_{pca} = \frac{1}{M-1}\sum_{m=2}^{M}\mathbf{w}_m \tag{3.37}$$

If there are K interferers contained in the original data, K outputs contain the principal components of interferers and $M-K$ elements of output $\mathbf{y}[n]$ are applied to calculate the weight vector. The weight vector for multiple interferers can be expressed as

$$\widehat{\mathbf{w}}_{pca} = \frac{1}{M-K}\sum_{m=K+1}^{M}\mathbf{w}_m \tag{3.38}$$

In practice, the PCA technique is carried out by SVD. Assume that the received data $\mathbf{X}$ is an $M \times N$ matrix and it is expressed as Equation (3.2).

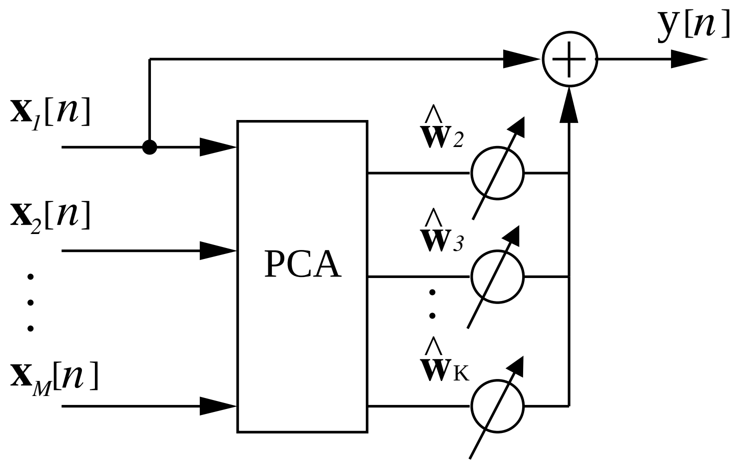

Fig. 3.3: The improved power minimization method based on PCA.

Then apply SVD to decompose **X** into the following form,

$$\mathbf{X} = \mathbf{U}\mathbf{L}\mathbf{V}^H \tag{3.39}$$

where

(i) **U** is an $M \times M$ matrix with the property $\mathbf{U}^H\mathbf{U} = \mathbf{I}$, which contains the eigenvectors of $\mathbf{X}\mathbf{X}^H$;

(ii) **L** is an $M \times N$ matrix with nonnegative values on the diagonal and zeros off the diagonal;

(iii) **V** is an $N \times N$ matrix with the property $\mathbf{V}^H\mathbf{V} = \mathbf{I}$, which contains the eigenvectors of $\mathbf{X}^H\mathbf{X}$.

3.4 Improved Power minimization Method

PCA can separate the principal components from the original data and keep most of information and characteristics. It means that good estimation of the strong signals can be provided by applying principal component analysis when the interference is much stronger than the desired navigation signals and noise. In this case, I can combine this kind of technique with the traditional power minimization method. In the power minimization method , the key is to minimize the difference between the reference antenna and the auxiliary antennas in order to cancel the interfering components at the reference antenna effectively. We here propose a new method which combines the power minimization method and the PCA technique. The new structure is shown in Fig. 3.3. We replace the auxiliary paths of the traditional power minimization method by the output of PCA which contains principal components of original data. As the principal components can be a better representation of strong interference than the original data. I hope that the interference at the reference antenna will be cancelled more effectively in this way.

Suppose that there are $K-1$ strong interfering signals contained in the original signal, and $K-1$ outputs of PCA are represent as the auxiliary elements. Each auxiliary element contains the component of a strong interferer. To formulate the new problem, I construct an $M \times K$ transformation matrix $\mathbf{T}$ as follows

$$\mathbf{T} = [\mathbf{c}, \mathbf{u}_1, \mathbf{u}_2, \cdots, \mathbf{u}_{K-1}] \tag{3.40}$$

where $\mathbf{c} = [1, 0, 0, 0, \cdots]^T$ is an $M \times 1$ vector, and $\mathbf{u}_1, \mathbf{u}_2, \cdots, \mathbf{u}_{K-1}$ are the eigenvector associated with the principal components. Then applying $\mathbf{T}$ to the original data $\mathbf{X}$, I have a new set of data $\hat{\mathbf{X}}$

$$\hat{\mathbf{X}} = \mathbf{T}^H \mathbf{X}, \tag{3.41}$$

where

$$\hat{\mathbf{X}} = [\hat{\mathbf{x}}[1], \hat{\mathbf{x}}[2], \hat{\mathbf{x}}[3], \ldots, \hat{\mathbf{x}}[N]]$$

Then applying the power minimization method to the transformed data $\hat{\mathbf{X}}$, I have

$$\min_{\hat{\mathbf{w}}} \ \hat{\mathbf{w}}^H \hat{\mathbf{R}}_{xx} \hat{\mathbf{w}} \tag{3.42}$$

$$s.t. \ \ \mathbf{c}^H \hat{\mathbf{w}} = 1$$

where

$$\hat{\mathbf{R}}_{xx} = E\{\hat{\mathbf{x}}[n]\hat{\mathbf{x}}[n]^H\} = \mathbf{T}^H E\{\mathbf{x}[n]\mathbf{x}[n]^H\}\mathbf{T}\ , \tag{3.43}$$

$\hat{\mathbf{w}} = [1, \hat{w}_2, \ldots, \hat{w}_K]^T$, and $\mathbf{c} = [1, 0, 0, \ldots]^T$ is an $K \times 1$ vector. Following the solution in the power minimization method, I can obtain the optimum solution for the new weight vector $\hat{\mathbf{w}}_{opt}$

$$\hat{\mathbf{w}}_{opt} = (\mathbf{c}_1^H (\mathbf{T}^H \mathbf{R}_{xx} \mathbf{T})^{-1} \mathbf{c}_1)^{-1} (\mathbf{T}^H \mathbf{R}_{xx} \mathbf{T})^{-1} \mathbf{c}_1 \tag{3.44}$$

Table 3.1: A summary of computational complexities.

Power Minimization	PCA	Proposed Method
$O(NM^2) + 16M$	$O(M^3) + 4(M - K)$	$O(M^3) + O(M^2) + 16K$

3.5 The Computational Complexity Analysis

The computational complexities of the traditional power minimization method, the proposed algorithm and the PCA algorithm are compared in term of real multiplications with respect to the sensor number M , which is shown in Table 3.1.

For traditional power minimization method, $O(NM^2)$ real multiplications are needed to calculate the $\mathbf{R}_{xx}^{-1}\mathbf{c}$, $8M$ real multiplications are needed to calculate the $\mathbf{c}^H\mathbf{R}_{xx}^{-1}\mathbf{c}$, $4M$ real multiplications are needed to calculate the optimal weight vector $\mathbf{w}_{opt}$ and additional $4M^2$ are needed to obtain $\mathbf{R}_{xx}^{-1}$, so it totally to $O(NM^2) + 16M$ real multiplications.

For PCA algorithm, the eigenvectors $\mathbf{w}_M$, $m = K + 1, \ldots, M$ are calculated by SVD, which need $O(M^3)$ real multiplications. The weight vector need $M - K$ real multiplications. So it totally to $O(M^3) + 4(M - K)$ real multiplications.

For the proposed algorithm, I need $O(M^3)$ real multiplications to calculate the SVD to obtain eigenvectors by SVD, $O(M^2)$ real multiplications are needed to calculate $\mathbf{T}^H\mathbf{R}_{xx}\mathbf{T}$ and $16K$ real multiplications is needed to calculate the optimal weight vector, so so it totally to $O(M^3) + 8M^2 + 16K$ real multiplications.

3.6 Simulations

In this section, compare the performances of our the improved power minimization method, the traditional power minimization method, the principal component analysis technique and the LCMV beam former by using computer simulations with different sample number, antenna number and the input signal to noise ratio (SNR). We assume that the desired signal, interference and noise are uncorrelated with each other and they are all narrowband signals. The optimal solution of the LCMV beam former can be found in Chapter 2.

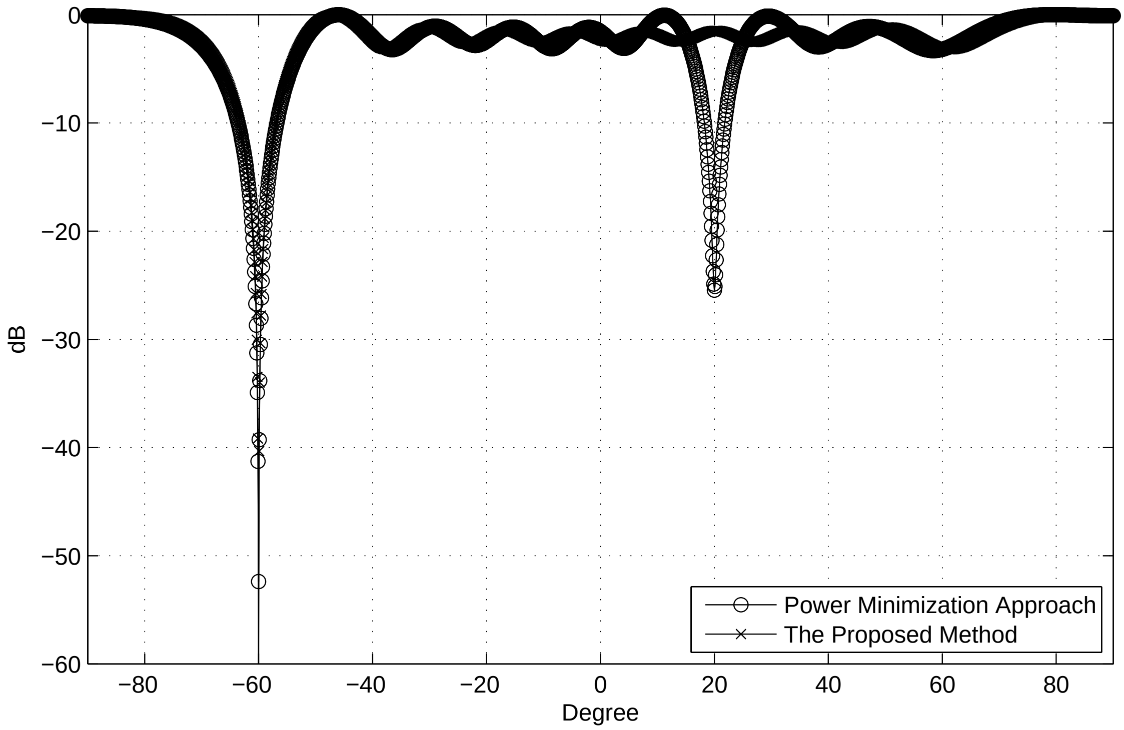

Fig. 3.4: The beam pattern of the proposed method and power minimization method.

3.6.1 Simulation I

In this set of simulations, the beam pattern of our improved power minimization method is compared with that of the traditional power minimization method. There are $M = 10$ sensors and $L = 1000$ samples for each sensor. The aim is to receive a desired signal from the broadside ($\theta = 20°$) and suppress one narrowband interfering signals arriving from DOA angle $\theta = -60°$. The input SNR is 5 dB and the input INR is 10 dB.

We can see the result in Fig. 3.4. Both the improved power minimization method and the power minimization method have a deep null in the direction of interference. However, the traditional power minimization method also forms a deep null in the direction of the desired signal. We can realize that the desired signal is seen as interference and cancelled by minimizing the difference between the reference antenna and auxiliary elements when the SNR is close to the INR. Apparently, the proposed method has overcome the limitation and achieved better performance as long as the SNR is lower than the INR.

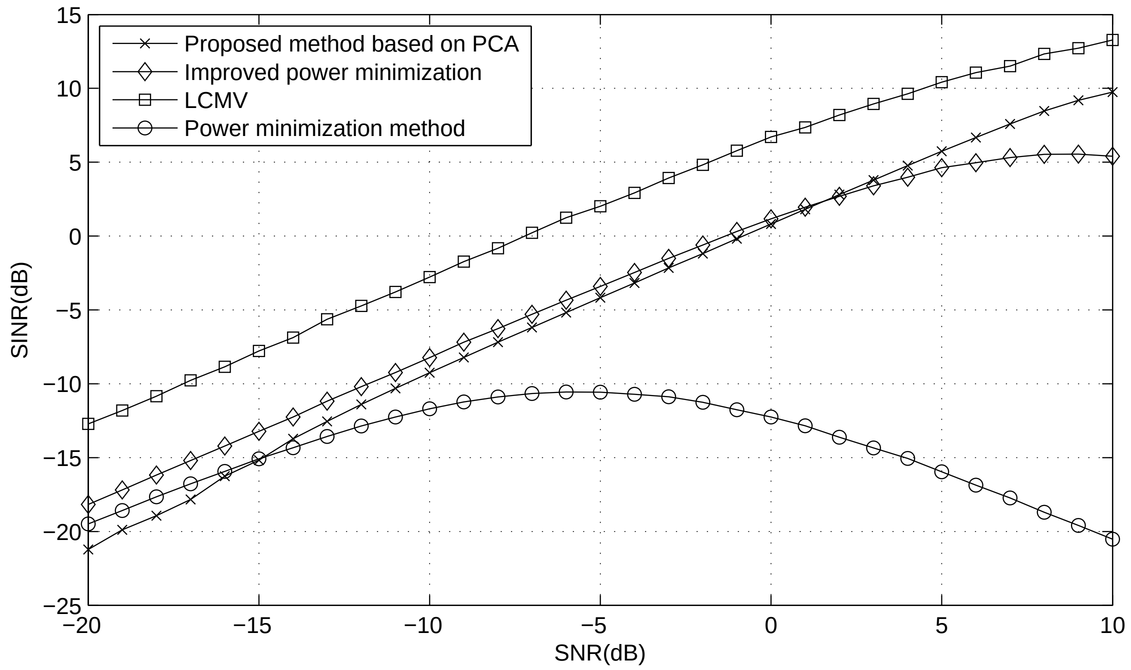

Fig. 3.5: Output SINR versus the input SNR with one interfering signal for $M = 5$.

3.6.2 Simulation II

Part A

In this part, the input SNR of the SOI varies from -20 dB to 10 dB. Firstly, I suppose it is based on a uniform linear array with 5 antennas and half-wavelength spacing. One arriving angle of narrowband interference is randomly generated between $-\pi$ and π, and the input INR is fixed at 15 dB. The input SINR is varied from 0.0014 dB to 1.1933 dB. The output SINR versus the input SNR, averaged over 1000 simulation runs, is shown in Fig. 3.5. It can be seen that the performance of the power minimization method declines when the input SNR increases, while the improved method has always achieved a better performance and the improvement becomes significant for larger SNR values. For the PCA method, when the input SNR is less than about -10 dB, it gives the worst performance, and it outperforms the power minimization method when SNR is larger than about -10 dB, but still not as good as the improved power minimization method. Moreover, the performance of the LCMV beam former is always the best in all of methods.

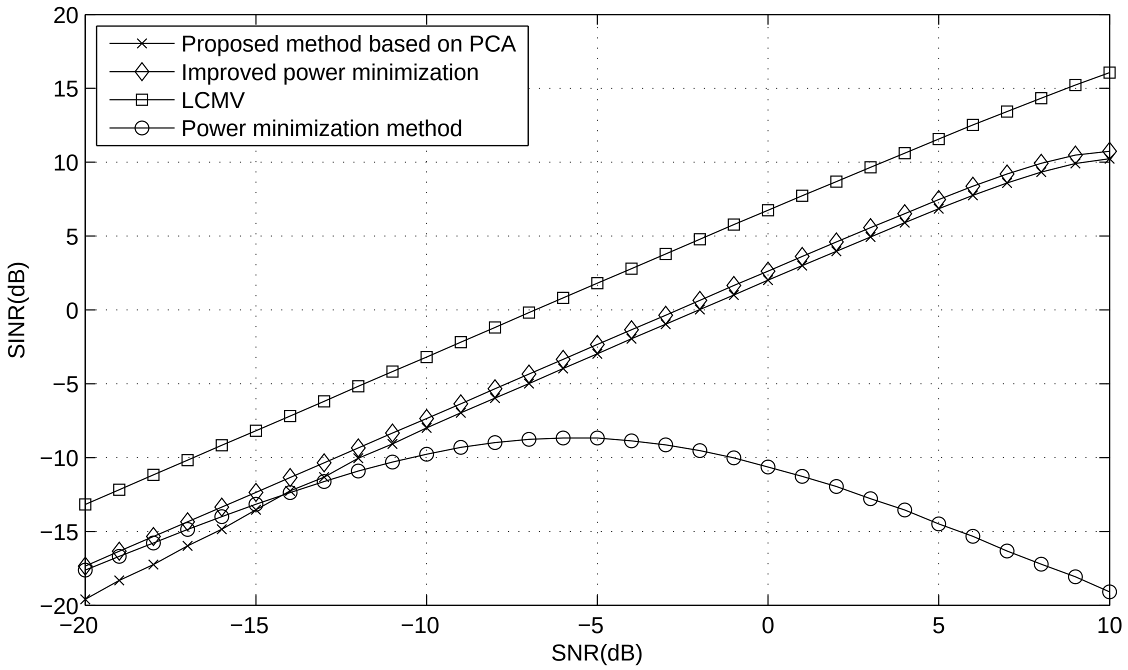

Fig. 3.6: Output SINR verus the input SNR with two interfering signals for $M = 5$.

Part B

We add one more interfering signal with random angle between $-\pi$ and π to the received data. The INR is fixed at 15 dB and 20 dB, respectively. All of settings is the same as Part A. Figure 3.6 shows the result, which is very similar to the result in Part A. The LCMV method always gives the best performance among the four solutions. However, compared with Part A, the PCA only method has reached a worse performance. The performance of four methods degrades because of the effect of two interfering signals.

3.6.3 Simulation III

In the third set of simulations, I increase the antenna number to 10 and the remaining parameters are the same as in Simulation II. Firstly, I discuss the case of one interferer. The output SINR versus input SNR is shown in Fig. 3.7, with a similar result as in Fig. 3.5. The main difference is that now the turning point is not -10 dB, but about -15dB. When SNR is 10 dB, the output SINR of the improved power minimization method method reaches almost 9 dB, while that in Simulation II is only 6 dB. Now I increase the number of interference to two. The result is shown in Fig. 3.8. A difference between Fig. 3.6 and

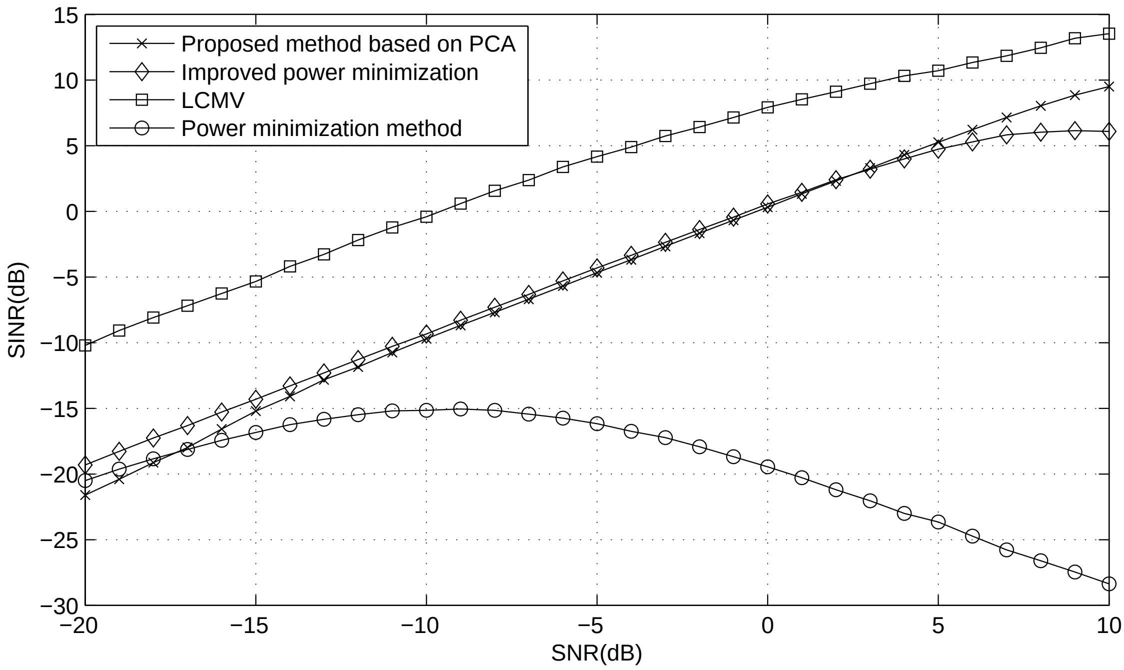

Fig. 3.7: Output SINR verus the input SNR with one interfering signal for $M = 10$.

Fig. 3.8 is that the output SINR of the traditional power minimization method begins to decrease after SNR is -10 dB with 10 sensors, while the output SINR of the traditional power minimization method begins to decrease after SNR is -5 dB with 5 sensors. When the input SNR is increasing, the output SINR of the PCA method is getting closer to that of the improved method.

3.6.4 Simulation IV

In this set of simulation, a uniform linear array with 10 antennas and half-wavelength spacing is used. I suppose that there are seven interfering signals arriving from $20°$, $30°$, $10°$, $50°$, $80°$, $-40°$, $-60°$, respectively. The INRs of these signals are fixed as 15 dB. There is also a deisired signal arriving from $0°$ and SNR is -10 dB.

Fig. 3.9 shows that when the number of interfering signals is close to the number of sensors, the performance of the proposed method is much better than that of PCA operation before the SNR is 5 dB. When the SNR is higher than 5 dB and closer to the INR. the curves of PCA operation and the improved power minimization method method decrease dramatically.

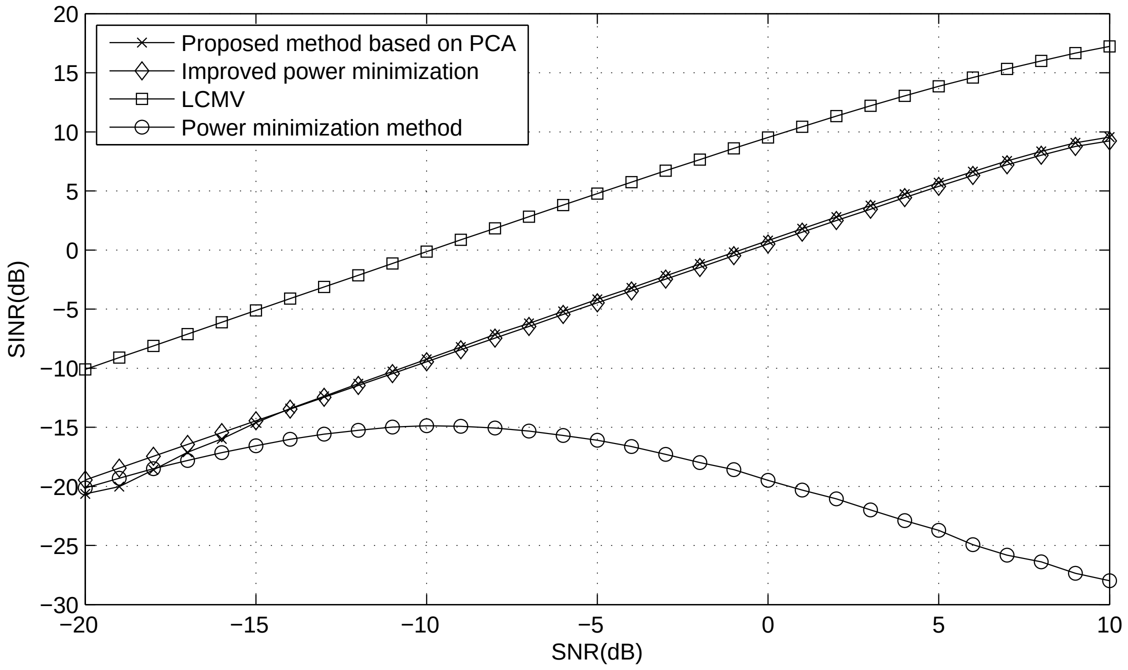

Fig. 3.8: Output SINR verus the input SNR with two interfering signals for $M = 10$.

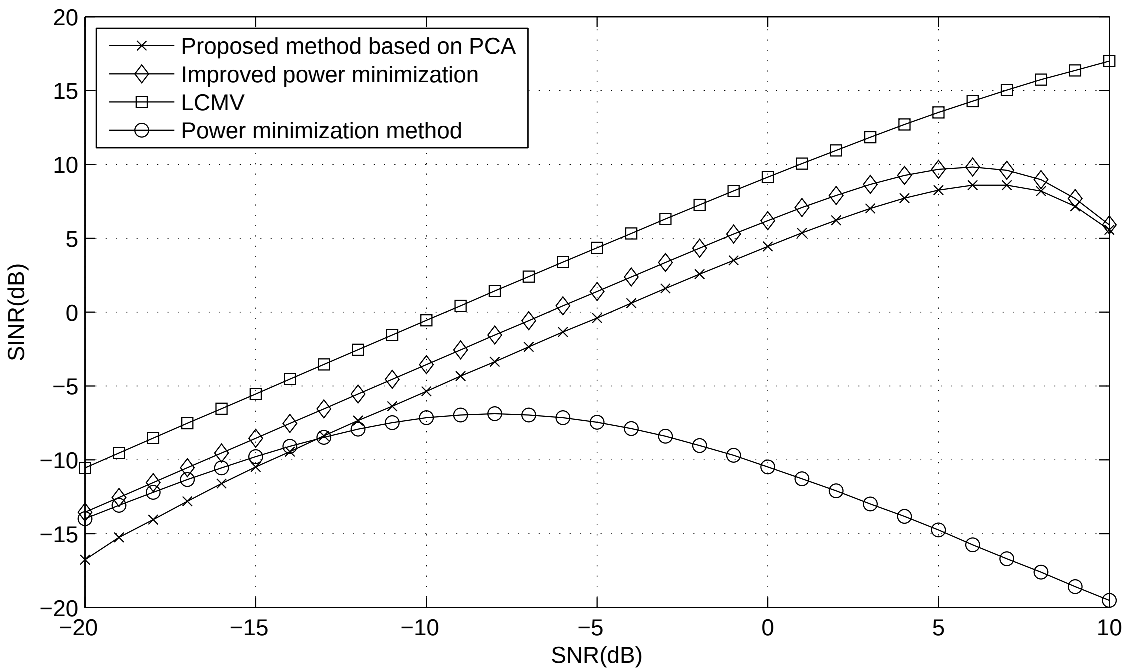

Fig. 3.9: Output SINR verus the input SNR with seven interfering signals for $M = 10$.

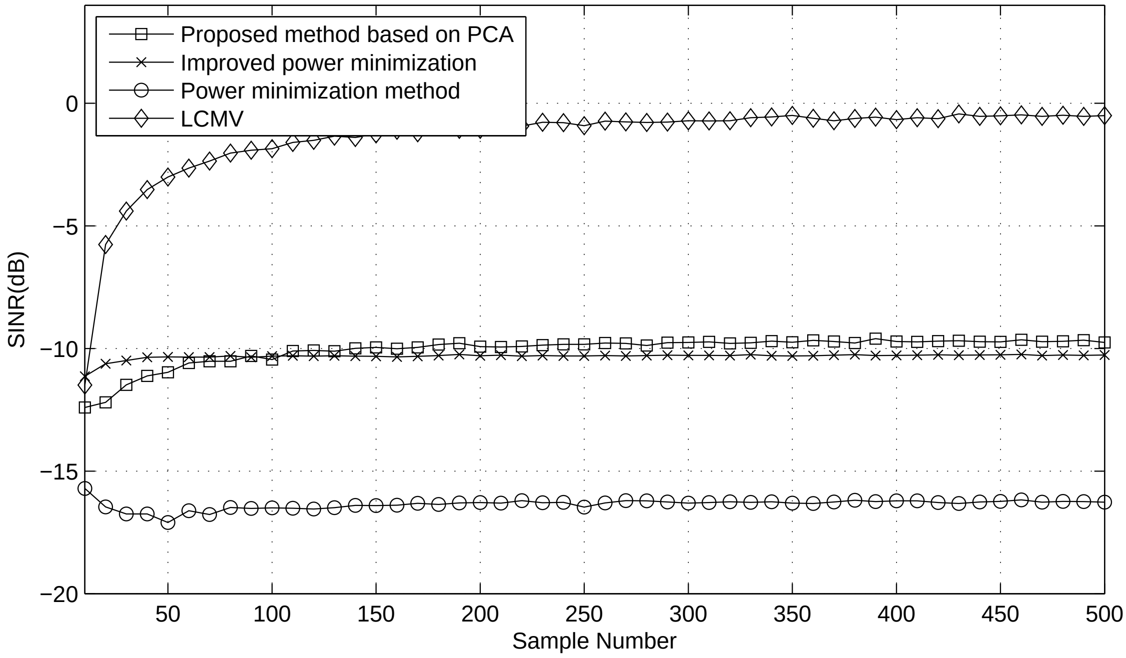

Fig. 3.10: Output SINR versus number of samples.

3.6.5 Simulation V

In this section, the sample number is varied from 10 to 500. The received data is tested every 10 samples. There are one narrowband interferer and one narrowband desired signal, and SNR and INR are fixed as -10 dB and 10 dB, respectively. A antenna number is $M = 10$. The desired signal arrives from the broadside ($\theta = 0°$) and the narrowband interfering signals arrives from $\theta = -60°$.

Figure 3.10 shows the result of simulation. When the number of samples is increasing from 10 to 50, the output SINRs rise significantly, especially for the LCMV method. The performance of PCA method is almost the same as that of the proposed method, and the difference between these two methods is extremely small and can be ignored. All of the four methods can provide better output SINRs with a larger number of samples. More importantly, the proposed method can achieve better performance than the traditional power minimization method whether the number of samples is smaller or larger. The LCMV method provides the best SINR among the four methods because it forms beam adaptively with the additional information of the direction of the desired signal.

3.7 Summary

In this chapter, I have introduced a classic blind algorithm for interference suppression, which is called power minimization. It can achieve good performance if the power of the desired signal is very small compared with that of interference and noise. Then principal component analysis is reviewed, which is a well-known statistical approach. Here I appled it to suppress interference, as it can separate the principal components of interference from the original data effectively. Combining PCA and the traditional power minimization method, a novel method is proposed by replacing the auxiliary elements of power minimization by principal components of interference, which are a better representation of the strong interference than the original signal. As shown in our simulation results, LCMV beam former provides the best performance in the four algorithms as it contains the information of DOA angles. The proposed method can be looked as an improved method compared with the traditional power minimization method and provide the better performance. When the number of the interfering signal is very small. the results of PCA operation provide better output SINR thant that of the proposed method. However, when the number of the interference is getting closer to the sensor number the proposed method has apparent advantage compared with the PCA operation.

Chapter 4

Performance Analysis of the Minimum Variance Beam Former

In this chapter, I will continue investigating the power minimization method. The beam former performance is measured by the ratio of the desired signal power to the interference-plus-noise power, referred to as the SINR. The output SINR is affected by many parameters, such as sample number, information of DOA and the correlation between the desired signal and interference. Firstly, a conventional performance analysis of the minimum variance beam former will be reviewed, one of the popular and important adaptive beam forming techniques. However, the expression of this conventional method is extremely complex and is affected by a lot of parameters. Then a simplified SINR analysis of the minimum variance beam former is considered and it is applied to the power minimization method. Finally, simulation results are provided for a comparison between our derived SINR results and the simulated one.

4.1 Problem Formulation

The signal model has been discussed in Chapter 2, which is given by

$$\begin{aligned} \mathbf{x}[n] &= \mathbf{A}\mathbf{s}[n] + \mathbf{n}[n] \\ &= \mathbf{a}(\theta_0)s_0[n] + \sum_{k=1}^{K-1} \mathbf{a}(\theta_k)s_k[n] + \mathbf{n}[n] \\ &= \mathbf{a}(\theta_0)s_0[n] + \mathbf{v}[n] \end{aligned} \tag{4.1}$$

where

$$\mathbf{v}[n] = \sum_{k=1}^{K-1} \mathbf{a}(\theta_k)s_k[n] + \mathbf{n}[n] \tag{4.2}$$

denotes the interference-plus-noise vector and $s_0[n]$ is the desired signal.

The output of the minimum variance beam former can be expressed as

$$\hat{y}[n] = \mathbf{w}^H\mathbf{x}[n], \tag{4.3}$$

I can obtain the weight vector **w** by minimizing the output power subject to a unity gain constraint. The expression has been discussed before:

$$\mathbf{w} = \frac{1}{\mathbf{a}^H(\theta_0)\mathbf{R}_{xx}^{-1}\mathbf{a}(\theta_0)}\mathbf{R}_{xx}^{-1}\mathbf{a}(\theta_0) \tag{4.4}$$

where $\mathbf{R}_{xx}$ is the covariance matrix, $\mathbf{a}(\theta_0)$ is the steering vector of the desired signal.

We set that the weight vector satisfies the unity gain constraint toward the direction of the desired signal:

$$\mathbf{w}^H\mathbf{a}(\theta_0) = 1 \tag{4.5}$$

The SINR of the beam former output can be analyzed through (4.4). For analyzing the SINR at the beam former output, firstly, another expression will be derived for the weight vector, and then apply it to calculate the SINR performance.

The covariance matrix can be expressed as

$$\mathbf{R}_{xx} = \sigma_{s_0}^2\mathbf{a}(\theta_0)\mathbf{a}^H(\theta_0) + \mathbf{a}(\theta_0)\mathbf{r}^H + \mathbf{Q} \tag{4.6}$$

where $\sigma_{s_0}^2$ is the power of the desired signal s_0, **r** denotes the correlation matrix between the desired signal and the interference-plus-noise, which is given by

$$\mathbf{r} = E\{\mathbf{s}_0^\star\mathbf{v}\} \tag{4.7}$$

and **Q** denotes the correlation matrix of interference-plus-noise

$$\mathbf{Q} = E\{\mathbf{v}\mathbf{v}^H\} \tag{4.8}$$

In practice, the real value of the correlation matrix can not be obtained because of the finite sample number. Thus, they will be replaced by the approximation

$$\mathbf{r} = \frac{1}{N}\sum_{n=1}^{N} s_0[n]^\star\mathbf{v}[n] \tag{4.9}$$

$$\mathbf{Q} = \frac{1}{N}\sum_{n=1}^{N}\mathbf{v}[n]\mathbf{v}^H[n] \tag{4.10}$$

Thus, (4.6) can be rewritten as

$$\mathbf{R}_{xx} = \mathbf{D} + \mathbf{b}\mathbf{b}^H \tag{4.11}$$

where

$$\mathbf{b} = \sigma_{s_0}\mathbf{a}(\theta_0) + \sigma_{s_0}^{-1}\mathbf{r} \tag{4.12}$$

and

$$\mathbf{D} = \mathbf{Q} - \sigma_{s_0}^{-2}\mathbf{r}\mathbf{r}^H \tag{4.13}$$

Using the matrix inversion lemma in (4.11), then I get

$$\begin{aligned}\mathbf{R}_{xx}^{-1} &= (\mathbf{D} + \mathbf{b}\mathbf{b}^H)^{-1} \\ &= \mathbf{D}^{-1} - \mathbf{D}^{-1}\mathbf{b}(\mathbf{b}^H\mathbf{D}^{-1}\mathbf{b} + 1)^{-1}\mathbf{b}^H\mathbf{D}^{-1}\end{aligned} \tag{4.14}$$

Applying the same method to (4.13)

$$\begin{aligned}\mathbf{D}^{-1} &= (\mathbf{Q} - \sigma_{s_0}^{-2}\mathbf{r}\mathbf{r}^H)^{-1} \\ &= \mathbf{Q}^{-1} - \mathbf{Q}^{-1}\mathbf{r}(\mathbf{r}^H\mathbf{Q}^{-1}\mathbf{r} - \sigma_{s_0}^2)^{-1}\mathbf{r}^H\mathbf{Q}^{-1} \\ &= \mathbf{Q}^{-1} + \frac{\mathbf{Q}^{-1}\mathbf{r}\mathbf{r}^H\mathbf{Q}^{-1}}{\sigma_{s_0}^2 - \mathbf{r}^H\mathbf{Q}^{-1}\mathbf{r}}\end{aligned} \tag{4.15}$$

Thus, (4.14) can be rewritten as

$$\begin{aligned}\mathbf{R}_{xx}^{-1} = \; & \frac{1}{\mathbf{r}^H\mathbf{Q}^{-1}\mathbf{r}}[(\mathbf{r}^H\mathbf{Q}^{-1}\mathbf{r})\mathbf{Q}^{-1} + \mathbf{Q}^{-1}\mathbf{r}\mathbf{r}^H\mathbf{Q}^{-1}]\mathbf{I} - \\ & \{\mathbf{b}\mathbf{b}^H[(\sigma_{s_0}^2 - \mathbf{r}^H\mathbf{Q}^{-1}\mathbf{r})\mathbf{Q}^{-1} + \mathbf{Q}^{-1}\mathbf{r}\mathbf{r}^H\mathbf{Q}^{-1}]\}/ \\ & \{(\sigma_{s_0}^2 - \mathbf{r}^H\mathbf{Q}^{-1}\mathbf{r}) + \mathbf{b}^H \\ & [(\sigma_{s_0}^2 - \mathbf{r}^H\mathbf{Q}^{-1}\mathbf{r})\mathbf{Q}^{-1} + \mathbf{Q}^{-1}\mathbf{r}\mathbf{r}^H\mathbf{Q}^{-1}]\mathbf{b}\}\end{aligned} \tag{4.16}$$

According to (4.16), (4.12) and (4.1), the weight vector can be expressed by

$$\mathbf{w} = \frac{1}{\mathbf{a}^H(\theta_0)\mathbf{Q}^{-1}\mathbf{a}(\theta_0)}\mathbf{Q}^{-1}\mathbf{a}(\theta_0) - [\mathbf{I} - \frac{\mathbf{Q}^{-1}\mathbf{a}(\theta_0)\mathbf{a}^H(\theta_0)}{\mathbf{a}^H(\theta_0)\mathbf{Q}^{-1}\mathbf{a}(\theta_0)}]\mathbf{Q}^{-1}\mathbf{r} \tag{4.17}$$

According to (4.17), the matrix

$$\mathbf{P} = \mathbf{I} - \frac{\mathbf{Q}^{-1}\mathbf{a}(\theta_0)\mathbf{a}^H(\theta_0)}{\mathbf{a}^H(\theta_0)\mathbf{Q}^{-1}\mathbf{a}(\theta_0)} \tag{4.18}$$

is a projection matrix, which obeys

$$\mathbf{PP} = \mathbf{P} \tag{4.19}$$

The definition of this type of projection can be found as oblique projections [78, 79].

Using the projection notation, (4.17) can be rewritten as

$$\mathbf{w} = \frac{1}{\mathbf{a}^H(\theta_0)\mathbf{Q}^{-1}\mathbf{a}(\theta_0)}\mathbf{Q}^{-1}\mathbf{a}(\theta_0) - \mathbf{PQ}^{-1}\mathbf{r} \tag{4.20}$$

Notice that the sample correlation is nonzero, even though the interference is uncorrelated with the desired signal because of the finite sample number.

This approximation of the weight vector is valid for a large number of samples and even for a moderate sample number, but it provides poor performance for very low sample numbers.

4.2 Performance Analysis for the Power Minimization Method

4.2.1 Weight Vector

The probability density function for the Sample Matrix Inversion method has been proposed in [80, 81]. We can apply the results to the case of power minimization method, and then calculate the expected value and the covariance matrix of the weight vector $\mathbf{w}_{opt}$, which are denoted as $E\{\mathbf{w}_{opt}\}$ and $Cov(\mathbf{w}_{opt})$, respectively. We can derive the closed-form approximation of the expected value of the SINR accounting for both the effect of finite sample number and interference.

The problem formulation of the power minimization method has been discussed in Chapter 3. To begin, $\mathbf{X}$ can be represented as a linear transformation of $\mathbf{Z}$, which is expressed as

$$\mathbf{X} \stackrel{\mathrm{d}}{=} \mathbf{R}_{xx}^{\frac{1}{2}}\mathbf{Z} \tag{4.21}$$

where $\mathbf{X}$ is the matrix of the received data, $\mathbf{Z}$ is an M by L matrix comprised of zero mean i.i.d. normal complex random variables with unit variance. $\mathbf{R}_{xx}^{\frac{1}{2}}$ is the positive square root matrix of $\mathbf{R}_{xx}$, i.e., $\mathbf{R}_{xx}^{\frac{1}{2}}\mathbf{R}_{xx}^{\frac{1}{2}} = \mathbf{R}_{xx}$. Note that $\stackrel{\mathrm{d}}{=}$ is the equality in distribution.

Suppose that there is such an M $\times$ M unitary matrix **U** that $\mathbf{Z} \stackrel{\mathrm{d}}{=} \mathbf{U}^H\mathbf{Z}$, is independent of **Z**, and $\mathbf{U}\mathbf{U}^H = \mathbf{I}_M$. Then I have

$$\mathbf{X} \stackrel{\mathrm{d}}{=} \mathbf{R}_{xx}^{\frac{1}{2}}\mathbf{U}^H\mathbf{Z} \tag{4.22}$$

Applying (**??**) into (4.22), I obtain

$$\bar{\mathbf{R}}_{xx}^{-1} = \mathbf{R}_{xx}^{\frac{-1}{2}}\mathbf{U}^H(\mathbf{Z}\mathbf{Z}^H)^{-1}\mathbf{U}\mathbf{R}_{xx}^{\frac{-1}{2}}L \tag{4.23}$$

This outer product matrix arises in multivariate statistical analysis. After a bit of algebra then a stochastic expression for the weight vector $\mathbf{w}_{opt}$ can be expressed as

$$\hat{\mathbf{w}}_{opt} \stackrel{\mathrm{d}}{=} \frac{\mathbf{R}_{xx}^{-\frac{1}{2}}\mathbf{U}^H(\mathbf{Z}\mathbf{Z}^H)^{-1}\mathbf{U}\mathbf{R}_{xx}^{-\frac{1}{2}}\mathbf{c}}{\mathbf{c}^H\mathbf{R}_{xx}^{-\frac{1}{2}}\mathbf{U}^H(\mathbf{Z}\mathbf{Z}^H)^{-1}\mathbf{U}\mathbf{R}_{xx}^{-\frac{1}{2}}\mathbf{c}} \tag{4.24}$$

The unitary matrix **U** can be chosen such that

$$\mathbf{U}\mathbf{R}_{xx}^{-\frac{1}{2}}\mathbf{c} = (\mathbf{c}^H\mathbf{R}_{xx}^{-1}\mathbf{c})^{\frac{1}{2}}\mathbf{p} \tag{4.25}$$

where **p** is an $M \times 1$ vector with $\mathbf{p} = [1, 0, \cdots, 0]^T$.

Note that

$$\begin{aligned}\mathbf{R}_{xx}^{-1}\mathbf{c} &= \mathbf{R}_{xx}^{\frac{-1}{2}}\mathbf{U}^H\mathbf{U}\mathbf{R}_{xx}^{\frac{-1}{2}}\mathbf{c} \\ &= (\mathbf{c}^H\mathbf{R}_{xx}^{-1}\mathbf{c})^{\frac{1}{2}}\mathbf{R}_{xx}^{\frac{-1}{2}}\mathbf{U}^H\mathbf{p}\end{aligned} \tag{4.26}$$

Applying (4.25) into (4.24), I have

$$\hat{\mathbf{w}}_{opt} \stackrel{\mathrm{d}}{=} \frac{\mathbf{R}_{xx}^{-\frac{1}{2}}\mathbf{U}^H(\mathbf{Z}\mathbf{Z}^H)^{-1}\mathbf{p}}{(\mathbf{c}^H\mathbf{R}_{xx}^{-1}\mathbf{c})^{\frac{1}{2}}\mathbf{p}^H(\mathbf{Z}\mathbf{Z}^H)^{-1}\mathbf{p}} \tag{4.27}$$

Then I can partition $\mathbf{R}_{xx}^{\frac{-1}{2}}\mathbf{U}^H$ as

$$\mathbf{R}_{xx}^{\frac{-1}{2}}\mathbf{U}^H = [\mathbf{g}, \mathbf{G}] \tag{4.28}$$

where **g** is an $M \times 1$ vector and **G** is and $M \times (M-1)$ matrix.

Substituting (4.28) into (4.26), I have

$$\mathbf{g} = (\mathbf{c}^H\mathbf{R}_{xx}^{-1}\mathbf{c})^{\frac{1}{2}}\mathbf{R}_{xx}^{-1}\mathbf{c} \tag{4.29}$$

With the relation $\mathbf{R}_{xx}^{-1} = \mathbf{g}\mathbf{g}^H + \mathbf{G}\mathbf{G}^H$, I have

$$\begin{aligned} \mathbf{G}\mathbf{G}^H &= \mathbf{R}_{xx}^{-1} - \mathbf{g}\mathbf{g}^H \\ &= \mathbf{R}_{xx}^{-1} - \frac{\mathbf{R}_{xx}^{-1}\mathbf{c}\mathbf{c}^H\mathbf{R}_{xx}^{-1}}{\mathbf{c}^H\mathbf{R}_{xx}^{-1}\mathbf{c}} \end{aligned} \tag{4.30}$$

We partition $\mathbf{Z}$ into $\mathbf{Z} = [\mathbf{z}_1, \mathbf{Z}_2]^H$, where $\mathbf{z}_1$ is an $N \times 1$ vector with a normal distribution and unity variance as $\eta(0,1)$, and $\mathbf{Z}_2$ is an $N \times (M-1)$ matrix with the same distribution as $\eta_{M-1}(0, \mathbf{I}_{M-1})$. $\mathbf{z}_1$ and $\mathbf{Z}_2$ are independent of each other. Then $\mathbf{Z}_2^H\mathbf{Z}_2$ has a Wishart distribution and $(\mathbf{Z}_2^H\mathbf{Z}_2)^{-1}$ has the inverse-Wishart distribution [82].

Inserting the Wishart distribution into (4.28), (4.27) and (4.26), I can get

$$\hat{\mathbf{w}}_{opt} \stackrel{\mathrm{d}}{=} \frac{\mathbf{R}_{xx}^{-1}\mathbf{c}}{\mathbf{c}^H\mathbf{R}_{xx}^{-1}\mathbf{c}} + \frac{\mathbf{G}\mathbf{f}}{(\mathbf{c}^H\mathbf{R}_{xx}^{-1}c)^{\frac{1}{2}}} \tag{4.31}$$

where $\mathbf{f} = -(\mathbf{Z}_2^H\mathbf{Z}_2)^{-1}\mathbf{Z}_2^H\mathbf{z}_1$

Since $\mathbf{z}_1$ is independent of $\mathbf{Z}_2$ and has zero mean, I have E$\{\mathbf{G}\mathbf{f}\}$=0. Therefore, the expected value of $\mathbf{w}_{opt}$ is

$$E\{\hat{\mathbf{w}}_{opt}\} = \mathbf{w}_{opt} = \frac{\mathbf{R}_{xx}^{-1}\mathbf{c}}{\mathbf{c}^H\mathbf{R}_{xx}^{-1}\mathbf{c}} \tag{4.32}$$

The covariance matrix of $\mathbf{w}_{opt}$ is

$$\begin{aligned} \mathrm{Cov}(\hat{\mathbf{w}}_{opt}) &= E\{(\hat{\mathbf{w}}_{opt} - E\{\hat{\mathbf{w}}_{opt}\})(\hat{\mathbf{w}}_{opt} - E\{\hat{\mathbf{w}}_{opt}\})^H\} \\ &= \frac{E\{\mathbf{G}\mathbf{f}\mathbf{f}^H\mathbf{G}^H\}}{\mathbf{c}^H\mathbf{R}_{xx}^{-1}\mathbf{c}} \\ &= \frac{\mathbf{G}E\{\mathbf{f}\mathbf{f}^H\}\mathbf{G}^H}{\mathbf{c}^H\mathbf{R}_{xx}^{-1}\mathbf{c}} \end{aligned} \tag{4.33}$$

Also with the fact that $\mathbf{z}_1$ is independent of $\mathbf{Z}_2$, the expectation $E\{\mathbf{f}\mathbf{f}^H\}$ can be calculated from the conditional expectation as

$$\begin{aligned} E\{\mathbf{f}\mathbf{f}^H\} &= E\{E\{\mathbf{f}\mathbf{f}^H|\mathbf{Z}_2\}\} \\ &= E\{(\mathbf{Z}_2^H\mathbf{Z}_2)^{-1}\mathbf{Z}_2^H E\{\mathbf{z}_1\mathbf{z}_1^H\}\mathbf{Z}_2(\mathbf{Z}_2^H\mathbf{Z}_2)^{-1}\} \\ &= E\{(\mathbf{Z}_2^H\mathbf{Z}_2)^{-1}\mathbf{Z}_2^H\mathbf{I}_N\mathbf{Z}_2(\mathbf{Z}_2^H\mathbf{Z}_2)^{-1}\} \\ &= E\{(\mathbf{Z}_2^H\mathbf{Z}_2)^{-1}\} \\ &= \frac{\mathbf{I}_{M-1}}{N-M+1} \end{aligned} \tag{4.34}$$

then I have

$$\mathrm{Cov}(\hat{\mathbf{w}}_{opt}^H) = \gamma\left(\mathbf{R}_{xx}^{-1} - \frac{\mathbf{R}_{xx}^{-1}\mathbf{c}\mathbf{c}^H\mathbf{R}_{xx}-1}{\mathbf{c}^H\mathbf{R}_{xx}^{-1}\mathbf{c}}\right) \tag{4.35}$$

where

$$\gamma = \frac{\left(\mathbf{c}^H \mathbf{R}_{xx}^{-1} \mathbf{c}\right)^{-1}}{N - M + 1}$$

Then the correlation matrix $E\left\{\hat{\mathbf{w}}_{opt}\hat{\mathbf{w}}_{opt}^H\right\}$ can be calculated as

$$\begin{aligned} E\left\{\hat{\mathbf{w}}_{opt}\hat{\mathbf{w}}_{opt}^H\right\} &= Cov(\hat{\mathbf{w}}_{opt}) + E\left\{\hat{\mathbf{w}}_{opt}\right\} E\left\{\hat{\mathbf{w}}_{opt}\right\}^H \\ &= \gamma \left[\mathbf{R}_{xx}^{-1} + (N - M + 1)\frac{\mathbf{R}_{xx}^{-1}\mathbf{c}\mathbf{c}^H \mathbf{R}_{xx}{-1}}{\mathbf{c}^H \mathbf{R}_{xx}^{-1}\mathbf{c}}\right] \end{aligned} \tag{4.36}$$

4.2.2 The Approximation of the Expected Value of the SINR

The SINR associated with $\hat{\mathbf{w}}_{opt}$ can be expressed as [83]

$$\text{SINR}(\hat{\mathbf{w}}_{opt}) = \frac{\sigma_{s_0}^2 |\hat{\mathbf{w}}_{opt}^H \mathbf{a}(\theta_0)|^2}{\hat{\mathbf{w}}_{opt}^H \mathbf{R}_{in} \hat{\mathbf{w}}_{opt}} \tag{4.37}$$

where $\mathbf{R}_{in}$ is the correlation matrix of interference-plus -noise data.

Then a close-form approximation can be calculated to the expected value of the output SINR for the power minimization based beam former, where the effect of finite sample size has been considered. The expected value of SINR can be expressed as

$$E\left\{\text{SINR}(\hat{\mathbf{w}}_{opt})\right\} = E\left\{\frac{\sigma_{s_0}^2 |\hat{\mathbf{w}}_{opt}^H \mathbf{a}(\theta_0)|^2}{\hat{\mathbf{w}}_{opt}^H \mathbf{R}_{in} \hat{\mathbf{w}}_{opt}}\right\} \tag{4.38}$$

Therefore, (4.38) can be divided into two parts: numerator and denominator, which can be viewed as two functions of random variable $\hat{\mathbf{w}}_{opt}^H$.

Assume C and D are these two parts, then the expected value $E\left\{\frac{C}{D}\right\}$ can be given by [84]

$$\begin{aligned} E\left\{\frac{C}{D}\right\} &= E\left\{\frac{E\{C\} + C - E\{C\}}{D}\right\} \\ &= E\left\{\frac{E\{C\}}{D}\right\} + E\left\{\frac{C - E\{C\}}{D}\right\} \\ &= E\left\{\frac{E\{C\}}{E\{D\}}\frac{E\{D\}}{D}\right\} + E\left\{\frac{C - E\{C\}}{D}\right\} \\ &= \frac{E\{C\}}{E\{D\}} E\left\{\frac{E\{D\}}{D}\right\} + E\left\{\frac{C - E\{C\}}{D}\right\} \end{aligned} \tag{4.39}$$

Then (4.38) can be transformed as

$$E\left\{\text{SINR}(\hat{\mathbf{w}}_{opt})\right\} = \text{SINR}_{apr}\Gamma + \Delta \tag{4.40}$$

where SINR_{apr} is an approximation of $E\left\{\text{SINR}(\hat{\mathbf{w}}_{opt})\right\}$, Γ is a multiplicative factor and Δ is an additive factor. Therefore $E\left\{\text{SINR}(\hat{\mathbf{w}}_{opt})\right\}$ is given by

$$\begin{aligned}\text{SINR}_{apr} &= \frac{\left\{\sigma_{s_0}^2|\hat{\mathbf{w}}_{opt}^H\mathbf{a}(\theta_0)|^2\right\}}{\left\{\hat{\mathbf{w}}_{opt}^H\mathbf{R}_{in}\hat{\mathbf{w}}_{opt}\right\}} \\ &= \frac{\sigma_{s_0}^2\mathbf{a}(\theta_0)^H E\left\{\mathbf{w}\mathbf{w}^H\right\}\mathbf{a}(\theta_0)}{tr\left[\mathbf{R}_{in}E\left\{\hat{\mathbf{w}}_{opt}\hat{\mathbf{w}}_{opt}^H\right\}\right]}\end{aligned} \tag{4.41}$$

where $tr\left\{\right\}$ is the trace operator. The multiplicative factor Γ and the additive factor Δ are given by

$$\begin{aligned}\Gamma &= E\left\{\frac{E\{\hat{\mathbf{w}}_{opt}^H\mathbf{R}_{in}\hat{\mathbf{w}}_{opt}\}}{\hat{\mathbf{w}}_{opt}^H\mathbf{R}_{in}\hat{\mathbf{w}}_{opt}}\right\} \\ &= E\left\{\frac{tr\left[\mathbf{R}_{in}E\left\{\mathbf{w}\mathbf{w}^H\right\}\right]}{\hat{\mathbf{w}}_{opt}^H\mathbf{R}_{in}\hat{\mathbf{w}}_{opt}}\right\}\end{aligned} \tag{4.42}$$

$$\begin{aligned}\Delta &= E\left\{\frac{\sigma_{s_0}^2|\hat{\mathbf{w}}_{opt}^H\mathbf{a}(\theta_0)|^2 - E\{\sigma_d^2|\hat{\mathbf{w}}_{opt}^H\mathbf{a}\}}{\hat{\mathbf{w}}_{opt}^H\mathbf{R}_{in}\hat{\mathbf{w}}_{opt}}\right\} \\ &= E\left\{\frac{\sigma_{s_0}^2|\hat{\mathbf{w}}_{opt}^H\mathbf{a}(\theta_0)|^2 - \sigma_{s_0}^2\mathbf{a}^H(\theta_0)E\left\{\hat{\mathbf{w}}_{opt}\hat{\mathbf{w}}_{opt}^H\right\}\mathbf{a}(\theta_0)}{\hat{\mathbf{w}}_{opt}^H\mathbf{R}_{in}\hat{\mathbf{w}}_{opt}}\right\}\end{aligned} \tag{4.43}$$

Γ is close to one and Δ is very small compared with SINR_{apr} [84]. Then SINR_{apr} can be used as the approximation of $E\left\{\text{SINR}(\hat{\mathbf{w}}_{opt})\right\}$, i.e.,

$$E\left\{\text{SINR}(\hat{\mathbf{w}}_{opt})\right\} \approx \text{SINR}_{apr} \tag{4.44}$$

By substituting (4.41) and (4.36), the approximation of SINR can be expressed as

$$\text{SINR}_{apr} = \frac{\sigma_{s_0}^2\left\{\mathbf{a}^H(\theta_0)\mathbf{R}_{xx}^{-1}\mathbf{a}(\theta_0) + (N-M)\frac{\mathbf{c}^H\mathbf{R}_{xx}^{-1}\mathbf{a}(\theta_0\mathbf{a}^H(\theta_0)\mathbf{R}_{xx}^{-1}\mathbf{c}}{\mathbf{c}^H\mathbf{R}_{xx}^{-1}\mathbf{c}}\right\}}{tr\left(\mathbf{R}_{in}\mathbf{R}^{-1}\right) + (N-M)\frac{\mathbf{c}^H\mathbf{R}^{-1}\mathbf{R}_{in}\mathbf{R}_{xx}^{-1}\mathbf{c}}{\mathbf{c}^H\mathbf{R}_{xx}^{-1}\mathbf{c}}} \tag{4.45}$$

This equation is a general result for approximation considering the effect of finite sample size. When the sample size is infinite, the expression can be simplified as

$$\text{SINR}_{apr}|_{N\to\infty} = \frac{\sigma_{s_0}^2\mathbf{c}^H\mathbf{R}_{xx}^{-1}\mathbf{a}(\theta_0)\mathbf{a}^H(\theta_0)\mathbf{R}_{xx}^{-1}\mathbf{c}}{\mathbf{c}^H\mathbf{R}_{xx}^{-1}\mathbf{R}_{in}\mathbf{R}_{xx}^{-1}\mathbf{c}} \tag{4.46}$$

Besides, the closed-form approximation of the expected value of output SINR for the LCMV beam former has been already study and the approximation can be expressed as

[84]

$$\text{SINR}_{apr|\text{LCMV}} = \frac{\sigma_{s_0}^2 \mathbf{a}^H(\theta_0)\mathbf{R}_{xx}^{-1}\mathbf{a}(\theta_0)}{\frac{\text{tr}(\mathbf{R}_{in}\mathbf{R}_{xx}^{-1})}{N-M+1} + \frac{N-M}{N-M+1}\frac{\mathbf{a}^H(\theta_0)\mathbf{R}_{xx}^{-1}\mathbf{R}_{in}\mathbf{R}_{xx}^{-1}\mathbf{a}(\theta_0)}{\mathbf{a}^H(\theta_0)\mathbf{R}_{xx}^{-1}\mathbf{a}(\theta_0)}} \tag{4.47}$$

When the sample covariance matrix is estimated in the absence of the desired signal, i.e., $\mathbf{R}_{xx} = \mathbf{R}_{in}$, $\sigma_{s_0}^2 \mathbf{a}^H(\theta_0)\mathbf{R}_{xx}^{-1}\mathbf{a}(\theta_0)$ is the optimal SINR. Therefore, the ratio of the approximation value to the expected value is $(N+1-M)/N$. It means that the SINR_{apr} is a very good approximation to the real expected value of that for the LCMV beam former [84].

4.3 Simulation and Results

A uniform linear array with different sensors is considered in this simulation section. In all examples, the DOA angle of the desired signal is fixed at $\theta_0 = 0°$. The interference-to-noise ratio is fixed at 10 dB. The performance of power minimization method is compared with that of LCMV. The approximation and the expected value of SINR for the power minimization method are given. The approximation SINR is calculated by (4.45), and the expected value of SINR is given by (4.38), which is averaged over 1000 runs, as the parameters are changing randomly. For LCMV, the approximation SINR is given by (4.47), and the expected value of SINR is expressed as $\text{SINR}(\mathbf{w}_{lcmv}) = \sigma_{s_0}^2|\mathbf{w}_{lcmv}^H\mathbf{a}(\theta_0)|^2/\mathbf{w}_{lcmv}^H\mathbf{R}_{in}\mathbf{w}_{lcmv}$. We will change the number of samples, sensors and interference for comparison with these results.

4.3.1 SINR Versus SNR

In this part, the figures are plotted by altering the input SNR of the desired signal. The input SNR is varied from -30 dB to -5 dB and the input SINR is varied from 0.0014 dB to 1.1933 dB. I consider that there is only one interference in the array data, which arrives at $-60°$. The INR is fixed at 10 dB. Firstly, the sample size used is $N = 20$ and the sensor number is $M = 5$. The results are shown in Fig. 4.1 and Fig. 4.2, respectively. According to the results, the values of SINR for LCMV are always higher than that for power minimization, as LCMV has the information of DOA angles of the desired signal,

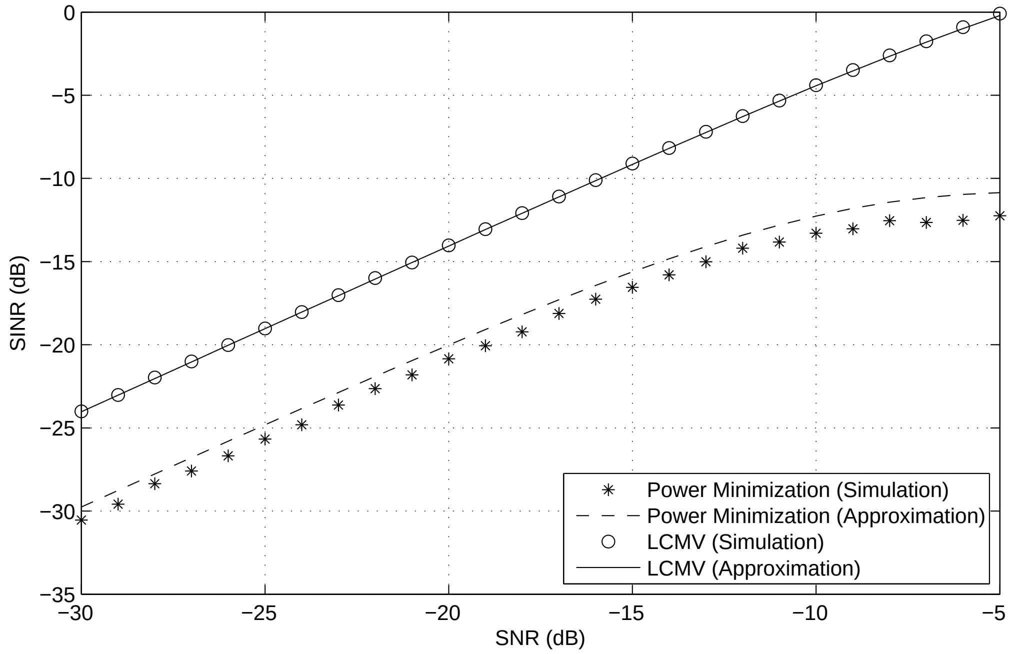

Fig. 4.1: Output SINR versus the input SNR for $N = 20, M = 5$.

while power minimization method is a blind algorithm. Although the sample size is small, the approximation SINR of LCMV is almost the same as the expected SINR value. For power minimization method, the approximation value of SINR is about 1 dB higher than the expected value SINR, especially at higher INR values.

When the sample size used is increased to $N = 100$, the main advantage is that the simulation SINR for the power minimization method becomes very close to the expected value, while the curves of LCMV still coincide with each other very well.

Similar results are shown in Fig. 4.3 and Fig. 4.4, with sensor number $M = 10$. When the sample size is $N = 20$, there is 3 dB gap between the simulated and the desired results for the power minimization method, while it reduces to much smaller than 1 dB for the N=100. For LCMV, both curves match each other very well.

4.3.2 SINR Versus Sample Size

In the second part, figures with different sample sizes are plotted. The sample size is varied from 10 to 300. The DOA angle of the desired signal is fixed at $\theta_0 = 0°$, and the input SNR is -10 dB. Firstly, I set the sensor number to be $M = 5$, and only one interferer is considered, which arrives from $-60°$. The INR is 10 dB. The results are shown in Fig.

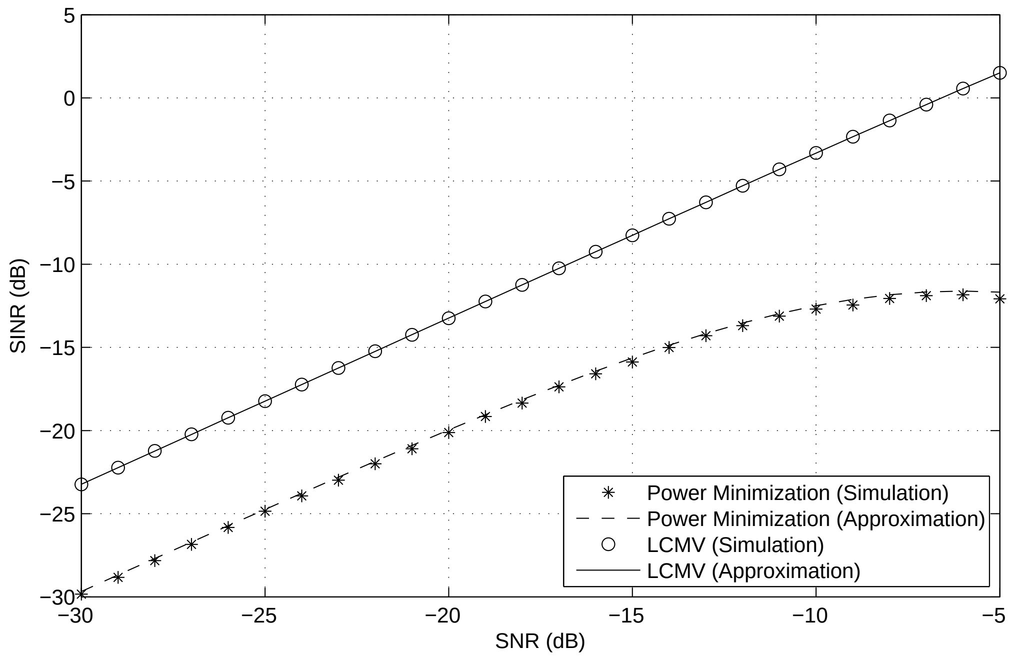

Fig. 4.2: Output SINR versus the input SNR for $N = 100, M = 5$.

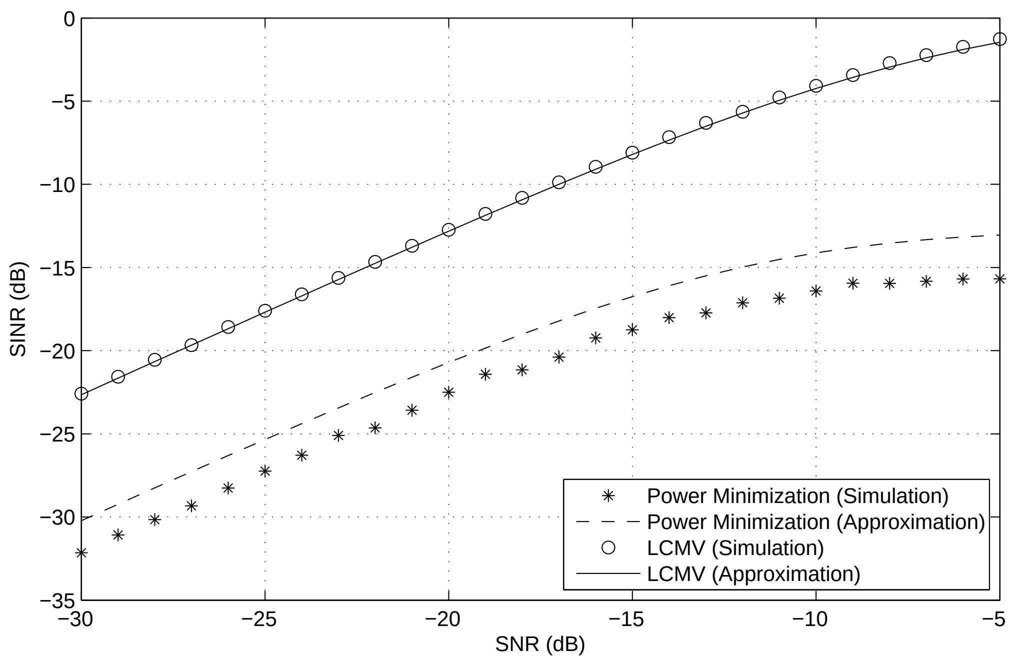

Fig. 4.3: Output SINR versus the input SNR for $N = 20, M = 10$.

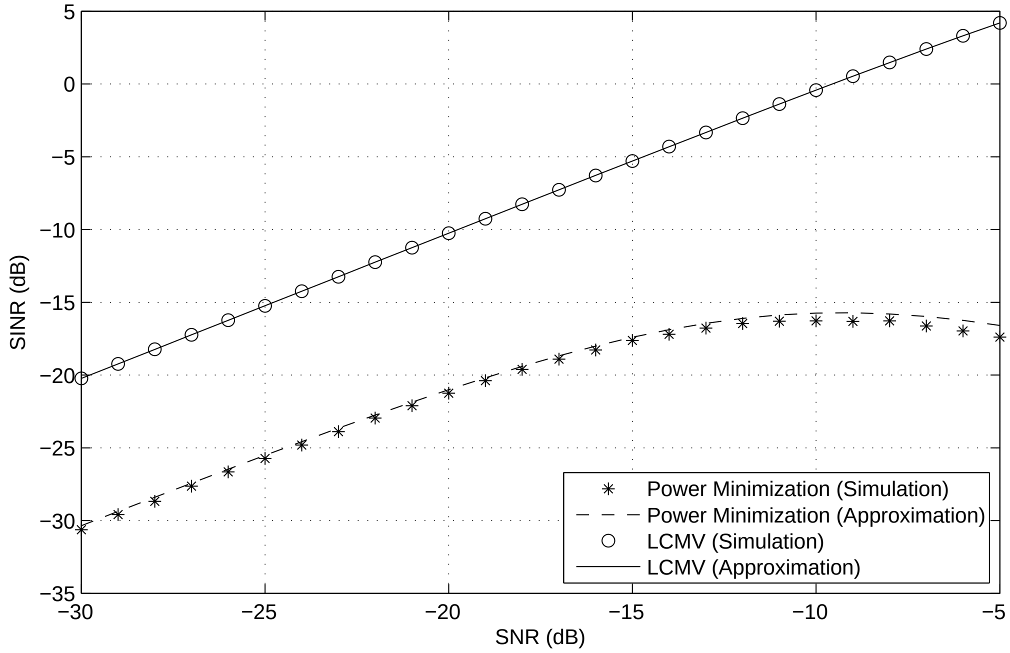

Fig. 4.4: Output SINR versus the input SNR for $N = 100, M = 10$.

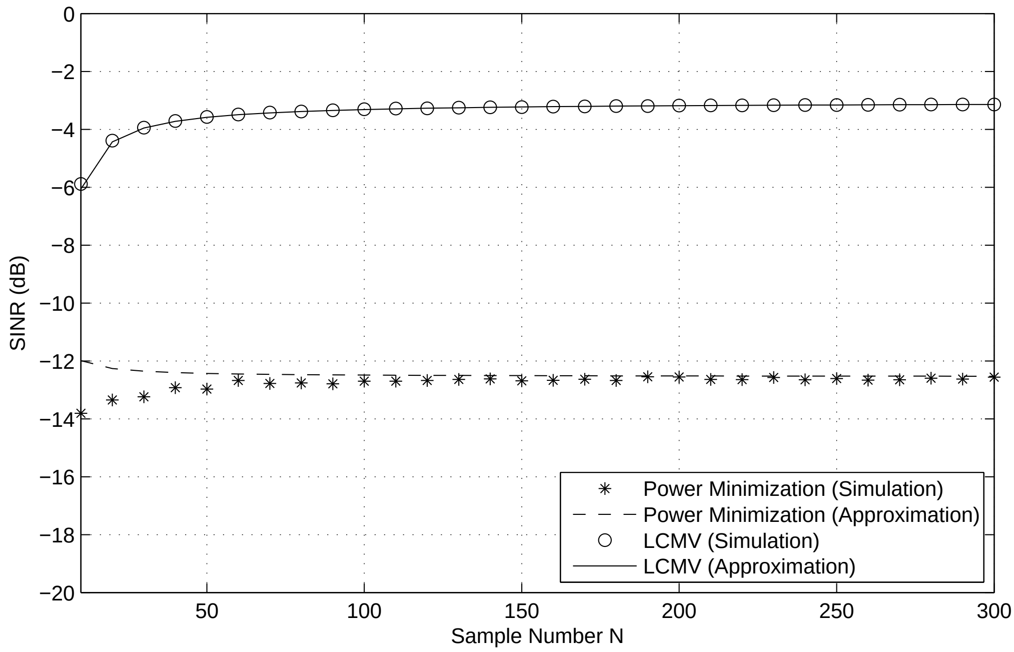

Fig. 4.5: Output SINR versus the sample size with one interference for $M = 5$.

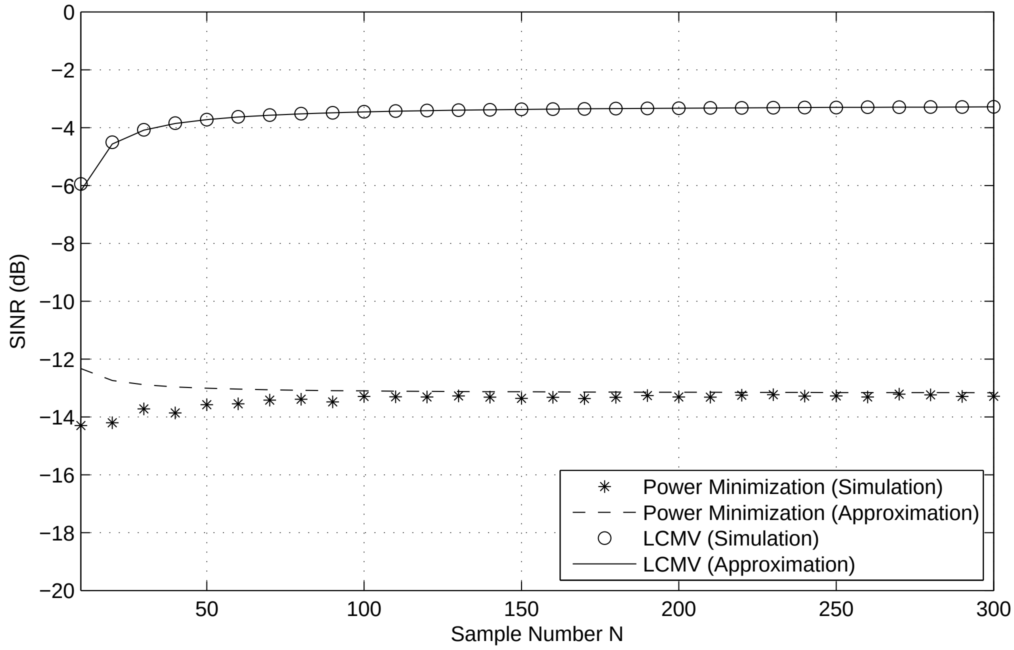

Fig. 4.6: Output SINR versus the sample size with two interfering signals for $M = 5$.

4.5. The curves for LCMV increase much quickly than those for the power minimization method. For LCMV, its SINR reaches the constant level at 40 samples, while the SINR of power minimization reaches the steady state at almost 80 samples. Then one more interference is added, arriving from $30°$. The INR is also fixed at 10 dB. The simulation curves are shown in Fig. 4.6. For power minimization, the curves converge at 120 samples and its SINR reaches the constant level at 150 samples.

4.4 Summary

In this chapter, based on some early work about the output SINR analysis of traditional beam forming schemes, I derived the statistical properties of the optimum weight vevtor of the power minimization method considering the finite sample effect. Then a closed-form approximation output SINR value is given for this method. Simulation results have compared between our derived SINR results and the simulated one and shown that the derived results of LCMV have a close match to the simulated ones, especially for larger sample sizes. Besides, the approximation of the output SINR for power minimization method does not have a close match to the simulated ones. The reason is that the factor Γ is not close to one and Δ is not small enough compared with SINR_{apr}. In the future, the further research is needed to have a better representation to the real output SINR for those ranges.

Chapter 5

Conclusions and Future Work

5.1 Conclusions

In this thesis, I have been mainly focused on the study of interference suppression for satellite navigation systems based on the antenna array signal processing techniques.

First of all, I have given a review of the GPS system, and considered the characteristics and model of GPS, and the effects of interference in navigation environments. General GPS background is discussed as well, such as signal level, the source of interfering signals and the format of GPS signal. After that, the basics of array signal model is reviewed, both narrowband and wideband array signal models are mentioned. Then basics of beam former are reviewed. There are three beam former models mentioned here: null-steering beam forming, maximizing output SINR and minimizing mean square error. Most of interference suppression algorithms are based on these three models. I have also discussed a well-known adaptive array signal processing algorithms: LCMV and its alternative implementation generalized sidelobe canceller (GSC). LCMV beam former has two conditions on the weight vector. The first one is the response on the direction of the desired signal, the second one is minimize the output variance. The GSC can be considered as a scheme for transforming the LCMV beam former. Besides, a space-time adaptive processing algorithms was also discussed, which has been widely used in GPS area in the recent years.

Secondly, I focus on the blind interference suppression algorithms, which is absence of the direction of DOA angles. It may accurate the speed of algorithms. The power minimization method is a classical interference suppression algorithm, which is well-known and effective way to reduce or cancel the interferer effects, it only works when the power of the desired signal is small compared with that of interfering signals. However, its main advantage is that it does not need the information of angles. Moreover, principal component analysis was introduced and applied to the interference suppression area, where the weight vector is calculated by averaging the eigenvectors of the received data corresponding to those minor components. PCA is a technique which can extract the principal components from the original data. The PCA technique is applied to the received data in this thesis and the results show that the output SINR has been improved. More importantly, the improved power minimization method is proposed, which combined the PCA technique and the power minimization method, Based on this improved method, the auxiliary elements of the structure of the power minimization method are replaced by the principal components of the received array signals in order to cancel interfering signals more effectively. When the input SNR is very close to the input INR, the output SINR of my improved power minimization method always stand at a higher level compared with the power minimization method, as shown in our simulations. Besides, my proposed method based on PCA only also provide similar performance as the improved power minimization method.

Lastly, an in-depth study of the power minimization method is proposed, Based on the results in [84], the approximation values can be a good representation as the simulated ones when the sample number is lower. Thus, I have developed a novel expression of the output SINR and try to apply it to the power minimization method by considering the finite sample size effect. Simulation results have shown that the approximation of the output SINR for power minimization method does not have a close match to the simulated ones. The reason is that the factor Γ is not close to one and Δ is not small enough compared with SINR_{apr}. In the future, the further research is needed to have a better representation to the real output SINR for those ranges.

5.2 Future Work

In this thesis, I focus on the interference suppression algorithms based on satellite array. I review some basics of beam former, well-know LCMV algorithm and GSC structure. After presenting a classical power minimization method , an improved way combined PCA and power minimization structure is proposed. Then I try to only use PCA technique to suppress interfering signals. Furthermore, a performance analysis of the power minimization method and LCMV is present in Chapter 4. However, there is still a lot of work to do based on the array property.

Firstly, I have assumed that the signals are uncorrelated and narrowband, the antennas are ideal and their positions are correct, and besides, there is no multipath existed in the propagation environment, etc. However, in real communication environments, the signal is not completely uncorrelated and its banwidth is increasing. The gaps between each antenna have to be considered reasonably. All kinds of factors exist to affect those assumptions. As a result, if some real factors are considered, the performance of the algorithms may degrade significantly. How to improve algorithms to mitigate those influences as much as possible is one of the major future works.

Secondly, I focus on the determined structure of array signals, where the number of sources is less or equal to the number of antennas and most algorithms introduced were designed for such problems. However, in practice, the number of sources is unknown and indefinitely. It may well exceed the antenna number. This problem is very challenging and can be improved from many aspects. Future research can focus on this area [85].

Thirdly, the method of performance analysis is used in LCMV beam former previously. I can see for very small sample sizes, the gap between our derived results and the simulated ones are relating big when I applied the same method to power minimization method. The further research is needed to have a closer approximation to the real output SINR for those ranges.

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