FISCAL FEDERALISM ASYMMETRY OF INFORMATION AND GRANTS-IN-AID: A THEORETICAL AND EMPIRICAL ANALYSIS

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by

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INTRODUCTION

For several years the amount spent by Government has grown both in real cost terms (that is excluding the effects of general inflation) and as a share of national output. In recent years it has been the government aim to reduce the share of government spending in national output by both trying to cut back expenditure and to rationalize the provision of services by increasing their efficiency in both the allocation and production side.

One of the most important issues in this context has been to control expenditure which was not directly decided by government itself. Between 1969 and 1974 total government expenditure rose by 33% in real terms. This growth reflected several factors: efforts to stimulate demand in the face of rising unemployment in 1973, the cost of large sector pay settlement following lengthy strikes, increases in subsidies to alleviate rapidly rising inflation, new programs in social spending and increase in industrial support.

The stimulation of domestic demand more or less coincided with and aggravated, the deterioration in the balance of payment produced by the Opec oil price rise, and the current balance of payment deficit reached £ 4 billion in 1974.

To avoid a possible collapse in the external value of pound, spending plans were began to be cut.

In 1979 a new, Conservative administration took office; it did not raise total spending to stimulate demand in the recession which began shortly after it took office and after honouring the previous administration' s commitments to paying a number of major catching up awards, public sector pay was tightly controlled and total spending was planned to remain broadly flat in real terms. The new policy resulted in a considerable slow down in public expenditure growth which was around 8% per year between 1979/84. Public expenditure cuts prove to be very difficult to be implemented for several reasons. The two main limitations of this policy are represented by pressure from client groups to increase expenditure and by the fact that about 53% of total spending cannot be directly controlled by government since it represents local government and nationalized industries expenditure.

Pressures exists to raise virtually every programme: for new and better equipment; to reduce hospital waiting time and to improve community services and so on.

The Treasury's 1984 Green Paper on long term prospects acknowledged the extent of such pressures and argued that the way to approach them was first to set up the budget constraint and then to establish priorities between competing claims. The Treasury's Green Paper has been built on a financing constraint based on targets. i.e. targets on spending were set out on the assumption that lower taxation was preferred by citizen to higher public expenditure.

The Green Paper has been criticized by several authors on the ground that it does not represent electors' feelings towards public sector expenditure; this depends, in my opinion on the value judgments and, perhaps, political views of the commentators, elements which are clearly very difficult to be assessed and rationalized. Anyway, this new policy view had and is still having important consequences in public spending administration and it is on these consequences that ,I think, it is important to concentrate. As I have already mentioned about 53% of public expenditure cannot be directly controlled by the central government.

This implies that in order to cut back the whole expenditure indirect instruments have to be used to induce lower level spending centers to follow the government's spending cuts. The effectiveness of these measures highly depends on the degree of freedom the spending agencies possess and on the information available to central government on resources, needs and effective objective of the agencies it is trying to control.

I will focus my attention on one particular and controversial aspect of these indirect controls measures which involves the relationship between central and local government. In recent years, namely from 1981 onwards, central government, in order to cut back their expenditure has continuously changed the way in which grants to local governments are allocated. While the aim of these devices was fairly well understood, the effectiveness of these systems and their implications have not been by now completely studied.

Most commentators infer from the inability of the grant system to cut back expenditure its failure, but this conclusion is, in my opinion, too simplistic. The grant system allocation has to be assessed in the more general context of the relations between central and local government.

The environment in which Government has to work is a very difficult one; on one hand, in fact, it has to give local authorities with a grant sufficient to provide a minimum level of services is provided; on the other hand it has to prevent high spending. Those two objective are incompatible, at least in the short run if local governments are better informed than the central government about their needs, preferences and resources. The grant system has to take account of these circumstances and can be used as a device to acquire relevant information from local authorities. To study this very complicated problem, the best way is, I think to start by studying the underlying model and the behaviour of the agents involved. Only after this preliminary study has been carried out it is possible to compare the optimal theoretical policy with the one observed and assess it.

The result of this exercise is the main theme of my work and the main conclusion is that Central Government's behaviour reflects the underlying problem. The grant system updating is a signal that Central Government is aware of not possessing all the relevant information to implement an optimal policy rule. The system might have failed to reach its objectives, but what it is important to assess is whether an optimal alternative solution does exist at all.

The work will be organized as follows:

Chapter one, after reviewing briefly the economic reasons for the existence of two level of government will examine some models aimed at explaining the rationale for the existence of grants from central to local governments.
I will try to explain why from a theoretical point of view, some services have to be provided locally. The main reason is, in my opinion that the local authority is better informed than the Central Government about the needs and preferences of people within each locality.
This causes an asymmetry of information problem in the Central - Local government relationship. Government has to take account of this problem in setting the grant system and this is the main reason why a first best policy cannot be implemented in this context.
The system of grant has then to take account of this important element and its effectiveness has to be judged not only in its aim at reducing expenditure but also as a device to learn the true preferences and needs of each local authority. Chapter one ends up with a very simple model that explain which would be the first best optimal strategy in a world in which all the agents share the same information.

Chapter two is devoted to a review some of the models in the asymmetry of information framework while in chapter three and four I will present the theoretical model. I will assume that local authorities are utility maximisers and I will present the grant allocation rule in both a static and a dynamic framework. At the end of chapter four I will examine some of the possible failures of the optimal system to reach its objectives due to the assumption of possible alternative behaviours.

Chapter five deals with the empirical evidence for local authorities behaviour under different assumptions. The aim of those empirical estimates is to derive a set of parameters to test the consistency of the optimal theoretical model with the actual system by which grants are allocated. Some tests will be devised for both assessing the validity of a life cycle behaviour and of some of the possible behavioural assumptions alternative to standard utility maximisation.

Chapter six will deal with the summary of all the issues by showing how the history of the changes in the grant system can be interpreted as the response of Central Government to the asymmetry of information problem it has to face.

CHAPTER ONE

1. INTRODUCTION

This chapter will discuss the economic rationale behind the provision of goods and services by local authorities. The functions of the public sector are indeed to ensure the efficient allocation and use of resources, to establish an equitable distribution of income and to maintain the economy at a reasonable high level of employment with reasonable price stability. This analysis takes for granted and accepts the assignments of tasks between central and local government as concerns the stabilization policy and the distribution of income first suggested by Musgrave (1959) and widely described and justified by Oates (1972). I shall instead concentrate on the assignment problem as concerns the efficient allocation of resources with particular reference to the provision of services. The classical analysis suggests that with respect to allocation efficiency a federal state is preferred to a unitary one.

Before examining the different models aimed at explaining and justifying the rationale for the existence of different levels of government, it is important to point out the differences in the economic and political definition of federal state and federalism. Federal state and fiscal federalism have a broader meaning in economics than in a political sense.

While in politics a federal state is characterized by a federal structure, this is not the case for economics.

In his pioneering study of federalism , Kennet C. Wheare (1959) defined federalism as:

...the method of dividing powers so that the general and regional government are each, within a sphere, co-ordinate and independent.

This definition has been widely used in political studies which concentrate on the structure of and the relations between different institutions with different sovereign powers. For economic modelling this definition is not suitable since the aim to the analysis is concentrated on different aspects. Those considerations suggest Oates (1972) to modify the definition of federal government in the following way:

"A public sector with both centralized and decentralized levels of decision-making in which choices made at each level concerning the provision of public services are determined largely by demands of those services of the residents of the respective jurisdiction.

It is clear from this definition that it makes little difference to the economists whether or not decision-making at a particular level of government is based on delegated or constitutionally guaranteed authority.
What matters, in fact, is simply that decisions regarding a particular jurisdiction reflect to a substantial extent the characteristics of the constituency of that jurisdiction. This definition implies that, in economic terms, most, if not all the systems, are federal such that the problem of

fiscal federalism concerns the economists in any country.

1.1 THE ROLE OF LOCAL GOVERNMENT IN ALLOCATING RESOURCES.

Musgrave and Oates' analyses place emphasis on the rationale for the existence of an active role of local governments in the efficient allocation of resources.

First of all, it is recognized that the local government can more effectively set up policies to cope with monopolies and externalities that limit their sphere of action within a local authority jurisdiction. While both these aspects are very important in the achievement of a first best Pareto optimal allocation of resources and have important social aspects [1], most of the literature in this area has been concentrated on the role of local government in providing public goods and services [2]. I will briefly examine some of these arguments in turn.

1.1.1 LOCAL PUBLIC GOODS.

Which are the criteria an economist would suggest in deciding the degree of decentralization in the provision of a public good? I think that one can agree with Topham's

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[1] Any environment policy and pollution control device is clearly more effective at a local rather than central government level. For reasons that will be clear later, however, those policies share with the local public goods problem the same asymmetry of information problem.

[2] the implicit assumption being that public goods are a kind of externality in the sense that they cause an externality on the consumption side.

suggestion that *public goods should be supplied in the cheapest and most efficient way* [3] where by cheapest it is meant at the smallest resource cost and by efficient it is meant that public services should be provided according to consumer's preferences.

As the intuition suggests, pure public goods should be supplied over as large a population as possible. In this context I will adopt Samuelson's (1954) definition of public goods , that is :

a pure public good can be defined as a good which is non rival in consumption and for which exclusion is impossible or too costly or it is not desirable from a social point of view.

Such goods are freely available to all who live within the jurisdiction boundaries. The more people that contribute to their costs the better; they reduce the tax bill of other contributors thereby and anyway the consumption by additional taxpayers does not detract from the consumption of these public goods by existing members of the society.

But not all public services are public goods; some are impure in the sense that they can be crowded and congested; some others spread their benefits only in a subset area of the national territory. The more people that share a given facility which is crowdable, the lower the benefit any individual derives from it. If this is the case, the

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See Topham (1983) pag. 130

existence of both club and local public goods must be recognized. In this context a local public good is defined as follows:

a local public good has the essential characteristics that it is assumed to be specific to a particular geographical location. This good spreads its benefits with spatial restriction and so the benefit is confined to one community (possibly with some spillover). Given their spatial restriction most of those goods are congestable, that is they are not available at zero cost to new residents.

Two problems arise in this context, namely finding the optimal size of people to which the congestable good should be provided and the most efficient way of providing it. In the next sections I will briefly review the literature on those aspects.

1.1.2 THEORY OF CLUBS AND OPTIMAL JURISDICTION.

The theory of clubs provides the theoretical foundations for the study of allocative efficiency for an important class of impure public goods. Club goods theory can be used in determining the need for exclusionary zoning, the efficacy of busing and the optimal size for alliances, communities and cities. The majority of economic articles examining clubs have appeared since James Buchanan seminal piece "An Economic Theory of Clubs" even though the problem had already been studied by Pigou (1946) and Knight (1924).

I will show how club theory can be used to derive the optimal conditions for the provision of local public goods. Local public goods share with club goods features like the possibility of being congested and, to some extent, the exclusiveness even though, while it is possible to exclude outsider from the benefit of a local public goods it is difficult to devise methods to exclude insiders at least from an efficient point of view. To understand this argument it must be recognized that jurisdictions hardly share with clubs the voluntary aspect. Clubs are by their nature voluntary, that is each member decides voluntarily to participate. Local jurisdictions are fixed in a way and their participation is to some extent compulsory [4]. Another important consequence to this analysis of this non perfect correspondence between clubs and jurisdictions is that local authorities as set up by Government do not correspond to the optimal club size as set up by economic theory. This causes spillover problems among jurisdictions and thus the need for Central Government intervention in order to correct them. It is possible to set up a formal model in order to show how the optimal size problem can be solved. The model I will present is quite general and it can be used for a large class of allocation problems with commodities having different characteristics as we shall see.

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[4]This argument is counterbalanced by the consideration that people, by moving, can decide which local authority to be member of such that being a member of a certain jurisdiction is a voluntary choice.
This intuition lies at the heart of Tiebout's model of "voting with the feet" i.e. revealing preferences for local public goods by choosing where to live.

An heterogeneous community formed by s individuals will be considered. Heterogeneous community in this context means that differences in preferences among individuals are allowed to exist. Only two goods are produced in the economy, namely:

x which is a private good

y which is a crowdable local public good.

We can define the quantity of a local public good enjoyed by any individual i as:

$$y_i \le y$$

where y is the quantity produced.

Define:

$$h = \sum \frac{y_i}{y} \qquad h \le s \qquad \text{since} \qquad \sum y_i \le sy$$

where s is the number of individual in the community.

h is the congestion index which can be interpreted as a crowding measure of the good we are examining. This index determines the degree to which any individual in the " club" can enjoy the commodity that is produced. High values of this index decrease the level of utility that each individual receives from good y.

Utility for the s^{th} individual can be defined as:

$$U_s = u_s(x_s , y_s, h)$$

$U'_{x_s} > 0$ $\qquad$ $U''_{x_s} < 0$

$U'_{y_s} > 0$ $\qquad$ $U''_{y_s} < 0$

$U'_h \leq 0$

Only two goods, as I have already pointed out are produced in this economy. The production function in this simplified economy is assumed to be:

$$F(x,y) = 0$$

This is an implicit production function that relates the two goods produced and assures by definition that production is efficient.

By using the definition of private goods, namely:

$$x = \sum x_i \qquad\qquad i = 1, s$$

and the public congestionable definitions:

$$y_i \leq y$$

$$h = \sum \frac{y_i}{y} \qquad\qquad i = 1, s$$

the model is closed.

By using the Pareto optimal definition of equilibrium, i.e. a point for which it is impossible to increase the utility of an individual without decreasing the utility of someone else in the economy we can solve our problem and find the optimal conditions by a standard lagrangean technique i.e. by maximising the utility of the s^{th} individual subject to the conditions previously stated and the further constraint that the utility of all other individual are not decreasing.

The formal setting will be:

$$\text{Max } U(x_s, y_s, h)$$

subject to:

$$U(x_j, y_j, h) = \bar{U}_j \quad j = 1, s-1$$

$$x = \sum_1^s x_i$$

$$y_s \leq y$$

$$h = \sum_1^s \frac{y_i}{y}$$

The Lagrangean for this problem can be written as:

$$L = U_s - \sum_1^{s-1} \alpha_j [\bar{U}_j - u_j(x_j, y_j, h)] + \beta(x - \sum_1^s x_i) +$$

$$\sum_1^s \lambda_i (y - y_i) - \mu F(x, y) - \rho (h - \sum_1^s \frac{y_i}{y})$$

The First Order Conditions for this problem can be written as:

$$\frac{\partial L}{\partial x_s} = \frac{\partial u_s}{\partial x_s} - \beta = 0$$

$$\frac{\partial L}{\partial x_i} = \alpha_i \frac{\partial u_i}{\partial x_i} - \beta = 0 \qquad i \neq s$$

$$\frac{\partial L}{\partial y_s} = \frac{\partial u_s}{\partial y_s} - \lambda_s + \frac{\rho}{y} = 0$$

$$\frac{\partial L}{\partial y_i} = \alpha_i \frac{\partial u_i}{\partial y_i} - \lambda_i + \frac{\rho}{y} = 0 \qquad i \neq s$$

$$\frac{\partial L}{\partial h} = \frac{\partial u_s}{\partial h} + \sum^{s-1} \alpha_i \frac{\partial u_i}{\partial h} - \rho = 0$$

$$i=1$$

$$\frac{\partial L}{\partial x} = \beta - \mu \frac{\partial F}{\partial x} = 0$$

$$\frac{\partial L}{\partial y} = \sum \lambda_i - \mu \frac{\partial F}{\partial y} - \rho \sum \frac{y_i}{y^2}$$

$$= \sum \lambda_i - \rho \frac{h}{y} - \mu \frac{\partial F}{\partial y}$$

$$\frac{\partial L}{\partial \lambda_i} = y - y_i$$

From these conditions and assuming $\alpha_s = 1$ we can derive that:

$$\sum \frac{\frac{\partial u_i}{\partial y_i}}{\frac{\partial u_i}{\partial x_i}} + \frac{s-h}{y} \sum \frac{\frac{\partial u_i}{\partial h}}{\frac{\partial u_i}{\partial x_i}} = \frac{\frac{\partial F}{\partial y}}{\frac{\partial F}{\partial x}} \qquad [1]$$

and:

$$\frac{\lambda_i}{\frac{\partial u_s}{\partial x_s}} = \frac{\frac{\partial u_i}{\partial h}}{\frac{\partial u_i}{\partial x_i}} + \frac{1}{y} \sum \frac{\frac{\partial u_i}{\partial h}}{\frac{\partial u_i}{\partial x_i}}$$

From the Arrow - Entoven conditions we can derive that:

$$\lambda_i (y - y_i) = 0$$

and that can be the case if:

i) $\lambda_i = 0$

ii) $y = y_i$

Let us examine the second case first. If $y_i = y$ this implies that $h = s$ and the model will give the usual Samuelsonian condition. The interpretation of those conditions is, however, a bit different. In this case each individual has not a full use of the public good since $y_i \leq y$. The congestable aspect of the public good reduces the utility that each individual can enjoy from it but it does not affect the marginal optimal conditions since at the margin there is not congestion cost.

If $\lambda_i = 0$

$$\frac{\dfrac{\partial u_i}{\partial y_i}}{\dfrac{\partial u_i}{\partial x_i}} = -\frac{1}{y}\sum \frac{\dfrac{\partial u_i}{\partial h}}{\dfrac{\partial u_i}{\partial x_i}} \qquad [2]$$

by substituting into [1] :

$$\sum \frac{\dfrac{\partial u_i}{\partial y_i}}{\dfrac{\partial u_i}{\partial x_i}} - (s-h) \frac{\dfrac{\partial u_i}{\partial y_i}}{\dfrac{\partial u_i}{\partial x_i}} = \frac{\dfrac{\partial F}{\partial y}}{\dfrac{\partial F}{\partial x}} \qquad [3]$$

by using the previous conditions we can finally write:

$$h \frac{\dfrac{\partial u_i}{\partial y_i}}{\dfrac{\partial u_i}{\partial x_i}} = \frac{\dfrac{\partial F}{\partial y}}{\dfrac{\partial F}{\partial x}} \qquad ; i = 1,2,..,s \qquad [4]$$

This model can be interpreted in terms of the utilization rate, h and in terms of the number of individuals that are allowed to use a certain facility. If it is interpreted in terms of utilization rate the model suggests that any individual will use this facility up to the point at which his marginal utility from using it is equal to the marginal cost.

If we want to interpret the model in terms of optimal size club number, we can write that :

$$s \frac{\dfrac{\partial u_i}{\partial y_i}}{\dfrac{\partial u_i}{\partial x_i}} = - \frac{\dfrac{\partial y}{\partial F}}{\dfrac{\partial F}{\partial x}} + \sum \frac{\dfrac{\partial u_i}{\partial y_i}}{\dfrac{\partial u_i}{\partial x_i}} + h \frac{\dfrac{\partial u_i}{\partial y_i}}{\dfrac{\partial u_i}{\partial x_i}}$$

The left hand side element is the marginal gain in utility, evaluated in terms of marginal utility of the private good of the marginal member derived from club inclusion and this element is expected to be nonnegative. The right hand side is represented by the marginal cost resulting from the entrant's entire utilization of the shared good in which the first term is the reallocation of the private good required to maintain the private good's marginal benefit to the entrant both before and after membership and the second element is the associated crowding cost. It is as well clear that in this model h can be also interpreted as an index of rivalness of the good. If $\frac{\partial u_i}{\partial h} = 0$, from [1] we have the usual condition that the good is a pure public good, while if h = 1 the good is a private one.

This simplified example shows that when goods can be congested or their benefits are spatially restricted it is optimal to provide them to a restricted number of individuals. This is the usual argument used to justify local provision of local public goods. While it is evident from this model that the optimal size of the "club" can be different from the nation-wide population the model does not imply that those goods and services have to be provided by decentralized rather than central governments. The club theory provides the optimal club size but does not imply that it would be more efficient to provide the goods by some peculiar institution within the club. The club theory is just the starting point for the development of the federal theory in the provision of goods and services.

As I have shown in the previous analysis the club theory is consistent with the hypothesis of different tastes among individuals in the same community. This observation offers the first justification for the federalism.

Assume that government treats individuals within the nation boundaries equally, regardless of tastes and income. If those services are either crowdable or restrict their benefits to a peculiar area there is room for a second level government whose size should correspond to the optimal facility club size. In fact, if Government provides an average level of the public crowdable good in any jurisdiction, this policy cannot be Pareto maximising, i.e. the provision of output in each jurisdiction is not Pareto efficient given that tastes for that good vary across jurisdictions.

This is very briefly the underlying theory of the decentralization theorem as stated by W. Oates:

"For a public good - the consumption of which is defined over a geographical subset of the total population, and for which the costs of providing each level of output of the good in each jurisdiction are the same for the central and the respective local government - it will always be more efficient (or at least as efficient) for local governments to provide the Pareto Efficient levels of output for their respective jurisdictions than for the central government to provide any specified and uniform level of output across all jurisdictions"

The central point of the previous analysis relies on the assumption that the Central Government will provide the same level of output in each jurisdiction. This assumption is sometimes justified on very naive grounds or on a median voter approach. I think that a more comprehensive model which takes account of all aspects I have been presenting can now be developed. I will show what is the rationale behind the average level production and that under quite reasonable assumptions the provision of public goods by local authorities is preferred from an efficient point of view to the Central Government provision.

1.1.3 STIGLER'S APPROACH.

In his seminal article in 1955 Stigler discussed the rationale for the existence of different levels of government and clearly stated the criteria to be followed in assigning tasks to the different levels of government. In Stigler's view the system of local government can be seen as a competitive market in which different firms (the local authorities) produce different goods(public goods, services and regulatory policies) for the market. The market in Stigler view is not confined within the boundary of a jurisdiction since people are allowed to move freely within the boundaries of a country. Stigler focuses his attention on the reasons why some polices are ineffective at local government level and points out three major reasons:

a) in the context of regulatory polices, when the object of such a policy can be nullified by the competition of other local authorities which do not apply the same policy. If mobility among jurisdictions is possible, in fact, people can avoid to adapt to any rule by moving to those localities which do not apply it.
b) when the source of revenue of the activity can escape financial responsibility by migration to another unit. In analogy with a perfect competitive market in which firms are price takers any local authority cannot successfully adopt a price discrimination policy in the provision of services; it does not have, in other words, the ability

to redistribute income.

The reason for this failure depends on the purely competitive organization of local services which would make impossible for a local government to obtain money from the rich to pay for the services provided to the poor, except to the extent that the rich voluntarily assumes this burden. While in Pauly model this was made possible by the assumption of an altruistic function and a fixed community of people[5], if people can freely move it is best to assign to central government the role of redistribution.

c) when the policy is incapable of efficient performance upon a local scale. This argument is to some extent similar to the problem of economies of scale in a private industry. In this respect it is efficient to provide at a national level those services which are indivisible such as, for example, a wide range of public goods; some services have to be provided at national level if their implementation requires the coordination between different authorities and that could be, for example, the case for transport.

In all other cases it would be more efficient to provide a service at local level rather than a central one. The reason for this conclusion depends on the assumption of the underlying model that Stigler had in mind, namely that individuals could freely move across jurisdictions without

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[5]In Pauly model it is also assumed that people cannot move across jurisdictions: they have to live near the poor people and this decreases their utility.

any cost or constraint and that they were perfectly informed about the different services provided by different local authorities.

Tresh interprets and formalizes Stigler's model in a very appealing way. Following Stigler's approach he suggests that the argument for federalism would appear to require the existence of a particular kind of uncertainty. The model that I will present is based on the assumption that while the local government is well informed about the preferences of the people living in its jurisdiction, central government is not. This assumption is quite reasonable in a way since it is almost impossible to think that at a national level the preferences of people in any community are perfectly known. Someone could probably argue that also local authorities cannot be perfectly informed about citizens' needs but it cannot be denied that they could be better informed than the center.

In Stigler's model jurisdictions, like firms in the private markets are in competition and this assures that the marginal benefit from services being equal to the marginal cost. This argument is analogous to Tiebout's hypothesis of "voting with the feet", that is to choose to locate in the jurisdiction offering the preferred basket of local public goods is quite evident. Even in a world with no perfect mobility and preference revelation problems I think that the assumption of local jurisdiction being perfectly informed can be interpreted as an extreme simplifying hypothesis to represent local authorities being better informed about local needs.

The model can be developed as such:

- society is faced with the problem of providing the optimal amount of a local public good. For exposition clarity suppose that this public good has to be supplied only in a certain region, that is only a subset of the population wants it.
- All the other goods are private and there is no other problem requiring government intervention for allocating reasons.
- The distribution of income is optimal and it is determined at a national level.

In a first best world of perfect certainty either the national government or a local jurisdiction comprising individuals of a m subgroup with:

$m = 1, g$ [6]

could provide the proper level of the public good which can be labelled x_g.

The optimal quantity to be provided will be determined in accordance with the standard First Order Conditions:

$$\sum_{m=1}^{g} MRS^{m}_{x_g,1} = MRT_{x_g,1}$$

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[6] i.e. the subgroup comprises g persons.

where 1 is the purely private good.

Suppose however that the local jurisdiction knows its citizens well in the sense that it can determine any individual $MRS^{m}_{x_g,1}$ with perfect certainty whereas the national government knows each of these people less well in the sense that he observes each individual's $MRS_{x_g,1}$ as a random variable:

$$\hat{MRS}^{m}_{g,1} = MRS^{m}_{g,1} + \alpha$$

where α is a random variable with E $(\alpha) = \bar{\alpha}$ or, possibly, 0

Under these conditions the social welfare will be maximised, in general by having the local jurisdiction form and decide the appropriate level of x_g rather than letting the national government determine x_g according to the F.O.C.:

$$\sum_{m=1}^{g} \hat{MRS}^{m}_{x_g,1} = MRT_{g,1}$$

If $\bar{\alpha} \neq 0$ the government rule is clearly biased, implying either over or under provision of x_g. Even if $\bar{\alpha} = 0$, however, so that $\hat{MRS}_{x_g,1}$ is an unbiased estimate of the true marginal rate of substitution a risk averse society will prefer local provision of x_g so long as the subset m = 1, g is small enough to violate the population condition of the ARROW LIND theorem. Expressed in form of indirect utility function:

$$V^m\left(\vec{g}\ ,\ I^m,\ x_g\right)\ >\ E\left[V^m\left(\vec{g}\ ,\ I^m,\bar{x}_g\right)\ \right]$$

Assuming risk aversion, person m would be willing to pay a risk premium for local rather than national provision. Proponents of federalism could have this type of uncertainty in mind when they argue that local governments best know the interests of their own citizens. The sheer geographic distance from the central government of most of the people within a given society is bound to affect adversely the transmission of information thus the need for providing services at a local basis.

2. THE ECONOMIC RATIONALE OF GRANT PROVISION.

After describing the main reasons why some services have to be provided at a local government level, I will now deal with another important problem which is related to the issue of subsidies from central to local government. The theory I have reviewed so far does not provide any insights about the need and the extent of local public expenditure finance. Had we to apply the benefit principle as put forth by Musgrave it would seem that local governments should raise the money necessary to provide services using their own resources, that is by local taxation and loans.

This argument, however, is not so straightforward and does not consider the issue in all its aspects. As I have already pointed out,the political jurisdiction [7] is seldom exactly identified with the "optimal" economic jurisdiction. This cause benefits inflows and outflows of which local authorities should take account when defining the optimal quantity of local public goods to be provided [8]. The literature suggests some reasons why grants to local authorities should be provided:

- grants designed to encourage sub central authorities to take account of external effects of their services (*spillover grants*).
- grants designed to correct fiscal imbalances between the various tiers of governments (*revenue sharing grants*).
- horizontal equalization grants.

My discussion will be merely concentrated on the economic reasons behind the provision of grants without stressing on the different ways grants can be given (either in lump-sum form or as a price subsidy i.e. in matching grant form) and the different effects they can have on expenditure [9].

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[7]that is the local authority as set up by the central government.

[8]The non perfect correspondence between political and economic jurisdiction causes an externality problem. The grant provided to correct for spillover is then given to make local authorities internalize this externality.

[9]These issues will be, in fact treated at length in chapter three and four.

2.1 GRANTS TO CORRECT FOR SPILLOVERS.

This section considers grants designed to encourage local authorities to take account of any external effect their services may generate on other jurisdictions.

Since the optimal jurisdiction in economic terms seldom corresponds to the "political jurisdiction" [10] it is possible for an authority services to spill out benefits to citizens in other areas and vice versa. The local authority, by taking account of the benefits to its own citizens does not provide the optimal quantity of local public good. In the previous section I showed that optimality requires the following condition:

$$\sum \frac{\frac{\partial u_i}{\partial y_i}}{\frac{\partial u_i}{\partial x_i}} + \frac{s-h}{y} \sum \frac{\frac{\partial u_i}{\partial h}}{\frac{\partial u_i}{\partial x_i}} = \frac{\frac{\partial F}{\partial y}}{\frac{\partial F}{\partial x}}$$

be satisfied. It is clear from this formula that the optimal amount of good y to be supplied is equal to the sum of marginal benefits from using y adjusted by the number of people using the facility and the degree of congestion.

Suppose that, although the local public good is used by s individuals the local authority which has to provide this facility is formed by $\hat{s} < s$ individuals.

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[10] The reason for this non correspondence arise from the different goals that economics and politics have and in the consideration that in real world local authorities provide a considerable amount of services with their own peculiar characteristics as concern benefit spread and congestion.

In this case the quantity of good y provided by this locality will be pushed up to the point were:

$$\sum_{i=1}^{\hat{s}} \frac{\frac{\partial u_i}{\partial y_i}}{\frac{\partial u_i}{\partial x_i}} + \frac{\hat{s} - h}{y} \sum_{i=1}^{\hat{s}} \frac{\frac{\partial u_i}{\partial h}}{\frac{\partial u_i}{\partial x_i}} = \frac{\frac{\partial F}{\partial y}}{\frac{\partial F}{\partial x}}$$

Since the local public good spreads his benefits to an optimal population s, it is clear that good y will be under provided because the locality under exam mistakenly understate the marginal benefits of that good. Such a mistake is equal to:

$$\sum_{i=1}^{\hat{s}} \frac{\frac{\partial u_i}{\partial y_i}}{\frac{\partial u_i}{\partial x_i}} + \frac{(s-\hat{s}) - h}{y} \sum_{i=1}^{\hat{s}} \frac{\frac{\partial u_i}{\partial h}}{\frac{\partial u_i}{\partial x_i}} = \frac{\frac{\partial F}{\partial y}}{\frac{\partial F}{\partial x}}$$

In order to encourage this locality to provide the optimal quantity of the public good, it is necessary to provide it with a grant that will reduce its effective marginal rate of transformation thus encouraging an increase in the production of y. The problem can in theory be solved by using a grant and in the following discussion I will use a matching grant. The analysis I will present is simplified by assuming that the only type of externality in the economy is represented by a non correspondence between optimal population size and political jurisdiction; this externality is confined to only one region and there is only one public good to be provided; no interactions between different sub governments will be considered.

If those assumptions are relaxed "*specifying an efficient* *set of subsidies could become a monstrously complicated problem, although still conceptually soluble"*[11].

This extremely simplified case is then just an example of how a matching grant could be used to bring local authorities to an efficient level of provision.

2.1.1 THE OPTIMAL MATCHING GRANT

Suppose that a grant meets a fraction g of the cost of producing y. The new optimal conditions will be given by:

$$\sum \frac{\frac{\partial u_i}{\partial y_i}}{\frac{\partial u_i}{\partial x_i}} + \frac{s - h}{y} \sum \frac{\frac{\partial u_i}{\partial h}}{\frac{\partial u_i}{\partial x_i}} = \frac{\frac{\partial F}{\partial y}}{\frac{\partial F}{\partial x}} - G$$

where $G = g \cdot \frac{\frac{\partial F}{\partial y}}{\frac{\partial F}{\partial x}}$

It is possible to show that g to make the local authority provide the optimal quantity of y will be:

$$g = \frac{\sum \frac{\frac{\partial u_i}{\partial y_i}}{\frac{\partial u_i}{\partial x_i}} + \frac{(s-\hat{s})-h}{y} \sum \frac{\frac{\partial u_i}{\partial h}}{\frac{\partial u_i}{\partial x_i}}}{\sum \frac{\frac{\partial u_i}{\partial y_i}}{\frac{\partial u_i}{\partial x_i}} + \frac{s - h}{y} \sum \frac{\frac{\partial u_i}{\partial h}}{\frac{\partial u_i}{\partial x_i}}}$$

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[11] See Oates, 1972 ,pag. 120

g can be interpreted as the fraction of the total marginal valuation of y attributable to outsiders and represent the subsidy that must be offered to local authorities to perceive correctly the benefits of the services they provide.

The previous analysis shows that determining the right level of grants is not a quite straightforward matter. In order to set up the optimal grant provision the central government should know the production function of each local public good and the indifference map of the sub central authority. While the assumption of knowing the production function a priori can be reasonable, it is clear from the discussion put forth in the previous section that the indifference map of each locality can hardly be known with precision by the central government. Indeed, as I have argued before, it is the quite reasonable assumption that the marginal rates of substitution are not well known at a central level that makes it efficient to provide local services at a local level. This at least implies that "*a lengthy trial and error process is necessary to fix the grant to the efficient level*"[12] .

The use of grants to correct for spillovers is consistent with Pigou's analysis and recommendations when dealing with externalities. Coase (1960) has suggested that externalities (of which spillovers are just an example) can be optimally solved by using a voluntary bargaining scheme between the

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[12]See King (1984) p. 136

affected parties. The gainers from external benefits are the $(s - \hat{s})$ people living in other authorities who have an incentive to bribe the producers (the local authority) to supply more, until the Pareto optimum point is reached. This suggestion is based on two very strict assumptions:

a) the bargaining process is developed at no cost;

b) participants are willing to reveal their true preferences without indulging in strategic behaviour.

While those two conditions can be probably met when few authorities are involved in the process for large sets of the population decision processes can be very expensive and a strategic behaviour in order to free ride is observed and for those reasons I think the Coasian approach could not be used to solve the spillover problem.

2.2 GRANTS TO CORRECT FOR VERTICAL IMBALANCE.

A broad purpose of grants is to compensate grantees for any mismatch there may be between their aggregate current expenditure needs on the one hand and their aggregate tax raising capacity on the other hand; such a mismatch is sometimes termed vertical fiscal imbalance. Expenditure of local authorities shows an upward trend depending on different causes;to give a flavour of these different arguments we can recall Niskanen and Machay and Weaver study and Baumol' disease. On the resources side, however, while central authorities' revenue is usually raised in forms of buoyant [13] taxes, local authorities sources of revenue, at least if the British case is considered [14] are mainly taxes which lack buoyancy.In order to finance the increased expenditure local property tax rate has to be increased and, money illusion makes the electors feel the local tax bill increases to an unsustainable level. To correct for this vertical fiscal imbalance different systems can be envisaged. I will recall here the Domestic tax relief introduced in England in the sixties and the tax base sharing. The domestic tax relief was a grant aimed at keeping domestic rates at an acceptable level; the tax base sharing allows local authorities to share with the central government the buoyant tax on income. By this system a percent of revenue raised with income tax is redistributed to local authorities, often on the basis of the so called derivation principle [15]. This system is widely used in federal governments [16] whose different history is then reflected in the way they are financed.

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[13]By buoyancy it is meant the property of a tax to increase its revenue when income goes up. The progressive tax on income is just the most common example.

In Britain up to April 1990 the main form of local taxes is represented by a property tax. With the new fiscal year the rates introduced in 1601 will be replaced by a per capita tax the so called "community charge".

[15]The derivation principle links the percent taxation redistributed to the contribution of each region in raising the total amount of taxes collected at central level.

[16]I have here implicitly used a political definition of federal government.

2.3 GRANTS TO ACHIEVE HORIZONTAL EQUITY

The final class of grants I am considering are widely used to neutralize either partially or wholly any differences that may arise in tax resources or spending needs between different areas of a nation. Following Pigou's suggestion everybody should be treated the same if they happen to be in the same situation. Discriminating on the ability to pay by charging people using the same service different sum of money is plausible, but it does not seem fair that citizens should be charged differently according to where they live. A difference in charge across authorities can arise for the following main reasons:

a) per capita resources of different localities may differ.

b) expenditure needs may differ.

In this case Central Government intervention is sought for equity reasons and it is argued that a grant has to be given to local authorities in the form of a matching grant aimed at reducing the unit cost of providing services.

The multi level system of government has thus developed a great debate among local public finance experts about which is the best form by which grants should be distributed and the problem is still unresolved. In Britain a unitary grant has usually been adopted, even though a great debate exists about the actual form of it. Unitary grants compensate simultaneously for inequalities in tax bases and inequities in the local need to spend by a system that distributes a large amount of the resources to local authorities with low tax base, high tax effort and high expenditure needs such that each local authority will be able to spend the same proportion of resources on a given service for the same level of service performance.

I will not go in further detail on the technical aspects of unitary grant since I think it should be clear from the previous discussion that a quite important gap exists between the theory justifying local governments existence and actual grant distribution. While it is recognized that local governments do exist because they are better informed about preferences and needs, the actual implementation of the system usually assumes that Central Government can observe all the relevant parameters to allocate grants. In the next three chapters I will try instead to model the optimal grant system in an asymmetry of information framework, the most appropriate to deal with the problem.

The different models I will present in chapter three and four are linked to the "principal and agent literature" with particular reference to transmission of information optimal mechanisms. This area is quite new and has received a great deal of attention in recent years. I will summarize the main issues and the most important contributions in chapter two.

CHAPTER TWO

1. INTRODUCTION

A large and interesting class of problems in economics involve delegated choices in which one individual or one organization has the responsibility for taking decisions in the interest of one or more others. A usual claim in economics is that everything works because each agent realises that he could not do better for himself than by accepting the ruling settings and maximising his objective function using those constraints.

Of course, this is only true under certain carefully specified conditions: no one agent may, for example, have a significant effect on the trading opportunities of others. Under those circumstances it is argued that not only are markets [1] efficient and the amount of information required in the economic system to work efficiently is very low - no one needs to know or care about the intentions, constraints or information pertaining to anybody else - and they insure that the information is truthful - no one else has the incentive to pretend that his preferences are different to what are in actual fact revealed to be.

On the other hand markets do not "work" in some important cases. Problems, for example, arise if one agent's action gives rise to externalities or if there are "large" non convexities in the system.

This work will be focused on the incentive problem,

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[1]The term market is here used in a very broad sense; it extends to any kind of relationship involving different economic units.

that is on situations in which individuals transact their business in such a way as to create an incentive to misrepresent important aspects of their own characteristics on their actions, The essence of the incentive problem which I shall examine here is a kind of master-servant relationship, the type of relationship that may exist between a patient and his private physician, a firm and its employee, an insurance firm and their clients or a central planning board and its satellite agencies. As it is clear from these examples, a wide range of economic problems fit this framework. As Cowell (1988) points out, "*those circumstances should not be the cue for throwing up hands in despair*". Private or government agencies can take steps to protect their interests if markets do not work and can formulate schemes to circumvent these problems.

Designing a good system of incentives is a problems that has been recognized for a long time. In principle such a system ought to encourage them for carrying out socially desirable policies. In the next sections I will review some of the common solutions offered by the literature to the asymmetry of information problem.

2. INCENTIVE MODELS.

The common feature which links together all the "incentive" models is the presence of a master - servant[2]

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[2]In this first part I shall refer to the model as master-servant relationship to stress the difference between this broader class of models and the principal and agent literature.

relationship in which the roles of the two parties have to be clearly specified. The master has a specific purpose he wants to achieve and has the power to devise rules for the servant in order to achieve his purpose. An objective function to be maximised is the usual representation of the master's problem. The master, however, has to choose between a bounded set of optimizing rules, i.e. he cannot choose any rules he likes. There are two basic reasons why the set of rules is bounded:

a) he does not have all the information that would give him a completely free hand;

b) the servant is not a slave, that is his cooperation must be obtained voluntarily.

In economic terms this means that the servant has his own objective function whose arguments can differ from the master's objective function. The servant is also allowed to have some freedom of action. Essentially he may choose whether or not to take up the deal on the term offered by the master: in most models this element is represented by a minimum reservation utility level that must be granted to the servant in any event. This participation constraint limits the set of enforceable rules that the master may choose to lay down and in some extreme cases it is itself the cause for the presence of "market failures".

When dealing with incentive problems, great care should be devoted to explain how information is spread throughout the model because the principal classification method for models dealing with incentive schemes derives either from the kind of information asymmetry (i.e. where the information asymmetry arises, what are the elements that are not known by both parties) or the time information is available to the "informed" party. This review is organized by focusing on the first of these two aspects.

According to Cowell (1986) and using the asymmetry of information as classifying aspect it is possible to distinguish between:

1) Private information models.

2) Unseen action models.

Private information models (otherwise called "adverse selection" [3] models) reflect situations in which the servant knows something that could be kept hidden from the master. Even if this private knowledge is announced, it is always possible for the servant to give a misrepresentation to his own advantage as long as the servant is relatively sure that the master will not be able to discover his cheating.

Unseen action models (otherwise called principal - agent models) reflect situations in which we can suppose that some action taken by the servant cannot be directly observed: what can be observed is the outcome which is determined in part by the unseen action and in part by random events independent of the action taken by the servant.

In this case exogenous uncertainty (not jut the

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[3]The term adverse section is here borrowed from insurance market studies and it is intended to apply to situations in which one party has got some information relevant to the optimisation process of other economic subjects but it is his selfish interest not to discover it.

misrepresentation that is endogenous to the problem) has an essential role to play since otherwise:

- the master could deduce the action from the observed outcome;
- the problem would essentially be reduced to the "private knowledge" case since the servant would know the relation between actions and outcomes but the master would not.

The master is facing a true moral hazard problem. The failure to induce the "optimal" effort level by the servant derives solely from his limited ability to monitor the servant's effort. This problem is often referred to as the "principal and agent" problem.

The standard distinction between private knowledge and unseen action model is not, I think, the most useful one because it does not focus the attention on the problem of information asymmetry which characterizes these models. Most of the differences between models and the results that can be derived depend, in fact, on the time at which the informed party acquires the information relevant to the problem; it is thus more useful to follow this second classification.

Strong and Walker (1987) classify the different situations that can arise with reference to information. The different types of models are then distinguished according to the timing and distribution of information between the principal and the agent. I have summarized their classification in table one.

TABLE ONE

	DISTRIBUTION	
	Private to the agent	Public
Pre contract	A	B
Pre effort	C	D
Pre effort - pre payoff	E	F
Post effort post payoff	G	H

This classification is fairly general since it can gather together private knowledge and unseen action models: by using this scheme the differences between the two situations will depend on the role played by the agent.

If agents are "active" in the Laffont - Maskin (1980) definition the model will be labelled a "principal and agent problem in the Cowell definition; on the other hand when the agent's role is confined to sending signals, the model will be labelled a "private information problem". This approach is quite useful since it highlights the role that timing and distribution of information plays in the results obtained using different models. Timing and distribution are key elements in the asymmetry of information literature.

Indeed the possibility of forcing the agent to reveal his private information,[4] essentially depends on the assumptions about the timing of information [5]. This is the reason why the private information literature, being confined to type A models in which the agent has pre-contractual information which has to pass onto the principal does not offer first best solutions. Even though risk aversion [6] might play an important role, it is timing that does not allow us to achieve a first best solution. This point will be made clear by reviewing the different models.

This survey will be largely confined to two party static games, that is the framework will be one in which there will be only one principal and only one agent. The model I will be using to describe the relationship between central and local government is an extension to a dynamic framework of a private knowledge model, but since in the peculiar case I will present dealing in a dynamic context does not add relevant difficulties to handling the problem, I have preferred to explain all the models in a static context where comparing the different methods of dealing with the asymmetry problem comes more easy. I shall explain why in the different models the information asymmetry arises and under which circumstances optimal incentive structures might be devised to overcome the problem. I will first

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[4] thus allowing the possibility of achieving a first-best Pareto optimal allocation.

[5] This is the reason why most of those "special cases" can only be applied in a principal and agent framework.

[6] In the Arrow - Pratt definition.

present the optimal solution in a world with perfect certainty and then I will show how the optimal conditions change according to the assumptions on the amount and timing of information.

2.1 PRINCIPAL AND AGENT THEORY.

In this first part of the review I will concentrate on models in which the agent has an active role. I will review models labelling them "principal and agent" but it should be borne in mind that some of those model could fit into the private knowledge literature depending on the classification scheme adopted.

Principal and agent theory is intended to apply to any situations with the following structure: one individual called the agent (A) must choose some action from a given set of actions {x}. The particular outcome Y which results from his choice depends also on which element from a given state of the world actually prevail at the relevant time, so that uncertainty is intrinsic to the situation. The outcome Y generates utility to a second individual, the principal, denoted by P.

A contract is to be defined under which P makes a payment to A in exchange for his effort. A's utility depends both on this payment and on the value of the action X. It is usually assumed that the principal P has a Von Newmann-Morgenster utility function U(Y-S) which is not directly dependent on the state of the world that will prevail and which is bounded and continuously differentiable to any required order. In particular $U'>0$ and $U''<0$ so that the risk loving behaviour is ruled out. In general, in order to simplify the matter risk neutrality is assumed, i.e. $U''= 0$. The agent is generally postulated to have a Newmann Morgenster utility function depending on the reward received and his effort. In general the agent's utility function can then be written as U(S,x) with $U'_S> 0$; $U''_s \leq 0$ $U'_x < 0$; $U''_x > 0$. The agent, like the principal can be either risk averse or risk neutral. The main purpose of principal-agent theory is to characterize the optimal contract under various assumptions about the information P and A possess or can acquire and thereby to explain the characteristics of such contracts which are actually observed. The main goal for the principal is thus to set up an optimal strategy among the different possibilities opened to him. Optimality of a strategy is here defined relative to the information the agent has at the time the strategy is used. This information can also be different from the initial information as it is signalled by the agent; this aspect is particularly important in a context where there is one principal and many agents or in a dynamic situation but it is worth to remember this point in this context as well in order to see how these models work.

All the individuals playing an active role in these models are assume to update their beliefs about unobservable parameters by using Bayesian rules and the sequence of past signals of other agents. Using this definition of optimal strategies we define a perfect Bayesian equilibrium concept for any mechanism. A mechanism in this context is defined as

a set of rules which specify the game to be played by agents in allocating resources and a specification of how allocation is determined given the plays of agents during the game.
Equilibrium allocations of mechanisms are allocations which depend on the actual, realized values of the parameters of the environment. Preferences orderings of agents over mechanisms therefore are naturally defined using expected utilities, based on the initial information structure. Efficient mechanisms are defined as mechanisms which cannot be improved upon by the set of all agents.

The central assumption characterizing those models is that the payment schedule can depend only upon variables which both parties can observe. In formal terms, the problem can be set up as follows:

MODEL ONE

Define:

Y = output

S = reward to the agent

x= effort of the agent

α = random variable

$y(x,\alpha)$ = production technology

Without serious loss of generality the set of states of the world (α) is given the closed unit interval [0,1]. It is assumed that $y(x,\alpha)$ is continuously differentiable to any required order with $y'_x \geq 0$; $y''_x \leq 0$ [7] and, for convenience, $y'_\alpha > 0$ so that higher values of α represent more productive states.

The principal's problem is to fix an output target y_i for the state i and a reward S_i for the agent that is optimal under the previous definition. The utility function of the agent is here defined in very general terms as:

U(S,x).

Utility is assumed , as I have already pointed out, top be increasing in S:

$$\frac{\partial U}{\partial S} > 0 \qquad \frac{\partial^2 U}{\partial S^2} < 0$$

and decreasing in x:

$$\frac{\partial U}{\partial x} < 0 \qquad \frac{\partial^2 U}{\partial x^2} < 0$$

The utility function for the principal is represented by;

$$U = E(Y,s) = \sum p_i \; U(Y_i - S_i)$$

where p_i is the probability of α_i.

In the principal and agent literature it is usually assumed that A knows his effort x and can observe Y and α.

P is assumed to know the technology process, $y(x,\alpha)$, the utility of the agent, $U(S_i,x_i)$ and can observe the outcome, Y. The differences in the different models proposed by the

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[8] y'_x refers to the first derivative of y w.r.t. x. This notation is used for the other derivative if not otherwise stated.

since $\Sigma\, p_i = 1$

literature arise on the timing and the additional information that P is assumed to have or to observe.

2.1.1 THE PRINCIPAL KNOWS x OR α.

The first class of problems I will review corresponds to type D models in table 1. It is important to start with those models since the assumptions about the timing of information in this special case will allow us to highlights what are the consequences of information asymmetry. I will start by assuming that P can observe either x or α. In this case P is always able to deduce the other ex post; an optimal solution can be found since there is no information asymmetry problem here. To make this point clear consider the solution of mode one under the hypothesis that the principal knows the effort x of the agent. In this case it is always possible for the principal to set up an optimal contract by defining a set of state contingent Y_i as to secure the agent a minimum utility level in each state .

To stress the importance of this point suppose that the effort x for the agent has been fixed to x^o and consider how the reward to the agent should be set up.

The principal's objective in this model is to maximise the expected value of his own utility subject to the constraint that the reward the agent receives produces a nonnegative utility. It is clear from this model that a first best solution can be reached in this case and no incentive problem arises. The formal proof is as follows: by assuming that P knows x, x can be fixed to an arbitrary level x^o

and the optimal solution is found solving the following maximisation problem:

$$\underset{S}{\text{Max}} \sum p_i \, U^P(Y_i - S_i)$$

s.t. $U^A(S_i \, , x^o) \geq 0$

$S_i \geq 0$

The Lagrangean for the problem can be written as:

$$\mathcal{L} = \sum p_i \, U^P\left(y(x^o,\alpha_i) - S_i\right) + \sum \mu_i \left(U^A(S_i \, , x^o) - 0 \right)$$

The First Order Conditions can be summarized as follows:

$$\text{a)} \ \frac{\partial \mathcal{L}}{\partial S_i} = - p_i \, U^{P\prime}_{s_i}\left(y(x^o,\alpha_i) - S_i\right) + \mu_i \, U^{A\prime}_{s_i}(S_i \, , x^o)$$

$$\text{b)} \ \frac{\partial \mathcal{L}}{\partial \mu_i} = U^A \, (S_i \, , x^o)$$

μ is the conventional Lagrange multiplier which is independent from α.

Condition (a) implies that:

$$\mu_i = \frac{U^{P\prime}_{s_i}\left(y(x^o,\alpha_i) - S_i\right)}{U^{A\prime}_{s_i}(S_i \, , x^o)}$$

μ is always positive then the second constraint must be satisfied as an equality which means that the agent receives his reservation utility in each state of the world. Since the effort S is fixed and independent of the state of the world that will happen.

At the optimum:

$$\frac{U^{P\prime}_{s_i}\left[\,y(x^{\circ},\alpha_i) - S_i\right]}{U^{P\prime}_{s_j}\left[\,y(x^{\circ},\alpha_j) - S_j\right]} = \frac{U^{A\prime}_{s_i}(S_i\;,\;x^{\circ})}{U^{A\prime}_{s_j}(S_j\;,\;x^{\circ})}$$

which can be rewritten, using a simpler notation as:

$$\frac{U'_p\;(\alpha_i)}{U'_p\;(\alpha_j)} = \frac{U'_a\;(\alpha_i)}{U'_a\;(\alpha_j)}$$

The implications of this condition is that both P and a's marginal rates of substitution between two states are equal. Rees (1985) investigates extensively this case and gives insights into optimal output sharing given different assumptions about the risk attitudes of the parties. I will instead immediately introduce the case in which both x and S are to be chosen optimally.

To simplify the matters I will assume that x is to be chosen before the state of the world α is known so that it will not depend upon it. In this case, a first best Pareto optimal action x for A and an associated optimal payment schedule S will be then defined for any state of the world. The contract between P and A would then specify the schedule S in exchange for A choosing x. A does have an incentive to cheat on the contract and given that he will receive S he will be able to choose some $x = x^*$. However, if P can costless observe x then the contract can contain a forcing clause to make it sufficient unattractive for A to cheat.

If P can observe x, the previous maximisation problem can be rewritten as:

$$\max_{S,x} \sum p_i \; U^P(Y_i - S_i)$$

$$\text{s.t. } U^A(S_i \;,\; x^o) \geq 0$$

The Lagrangean for the problem can be written as:

$$\mathcal{L} = \sum p_i \; U^P\Big(y(x_i,\alpha_i) - S_i\Big) + \sum \mu_i \Big(U^A(S_i \;,\; x_i) - 0 \Big)$$

and the First Order Conditions can be summarized as follows:

$$a_1) \; \frac{\partial \mathcal{L}}{\partial S_i} = - p_i \; U^{P\prime}_{s_i}\Big(y(x_i,\alpha_i) - S_i\Big) + \mu_i \; U^{A\prime}_{s_i}(S_i \;,\; x_i)$$

$$b_1) \; \frac{\partial \mathcal{L}}{\partial \mu_i} = U^A \; (S_i \;,\; x_i)$$

$$c_1) \; \frac{\partial \mathcal{L}}{\partial x_i} = p_i \; U^{P\prime}_{x_i}\Big(y(x_i,\alpha_i) - S_i\Big) \; y(x_i,\alpha_i) + \mu_i U^{A\prime}_{x_i}(S_i,x_i)$$

Again μ is positive and independent from α then the agent receives his reservation utility in each state.

Condition a_1) implies that:

$$\mu_i = \frac{U^{P\prime}_{s_i}\Big(y(x_i,\alpha_i) - S_i\Big)}{U^{A\prime}_{s_i}(S_i \;,\; x_i)} * p_i$$

then at the optimum:

$$\frac{U^{P\prime}_{s_i}\left(y(x_i,\alpha_i) - S_i\right)}{U^{P\prime}_{s_j}\left(y(x_j,\alpha_j) - S_j\right)} = \frac{U^{A\prime}_{s_i}(S_i\ ,\ x_i)}{U^{A\prime}_{s_j}(S_j\ ,\ x_j)} \qquad \text{(2a)}$$

which can be rewritten, using a simpler notation as:

$$\frac{U'_p\ (\alpha_i)}{U'_p\ (\alpha_j)} = \frac{U'_a\ (\alpha_i)}{U'_a\ (\alpha_j)}$$

Summing up c_1) for i leads to the following expression:

$$\sum \left[p_i\ U^{P\prime}_{x_i}\left(y(x_i,\alpha_i) - S_i\right)\ y'_{X_i}(x_i,\alpha_i) + \mu_i\ U^{A\prime}_{x_i}(S_i\ ,\ x_i)\right] = 0 \quad (3)$$

Since p_i is independent of the utility I can rewrite the previous expression as [9]:

$$\sum \left[\ U^{P\prime}_{x_i}\left(y(x_i,\alpha_i) - S_i\right)\ y'_{x_i}(x_i,\alpha_i) + \mu_i\ \ U^{A\prime}_{x_i}(S_i,\ x_i)\right] = 0 \quad (4)$$

which can be rewritten as:

$$\sum \left[\ U^{P\prime}_{x_i}\left(y(x)\right)\ y' + \mu_i\ U^{A\prime}_{x_i}(x)\right] = 0 \qquad \text{(4a)}$$

The optimal conditions (2a) and (4a) can be interpreted as before: the new element is represented by condition (4a) which relates to the optimal choice of x and can be

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since $\Sigma\ p_i = 1$

interpreted as follows: in any state of the world $U'[y(x)]*y'$ can be interpreted as the marginal value product of x measured in terms of P's utility. $U'(x)*\mu$ can be interpreted as the marginal cost measured in terms of P's utility. At the optimum the usual condition that the marginal cost should be equal to the product value holds and in this context it can be written as:

$$\mu_i = \frac{U'[y(x)]*y'(x)}{U'(x)}$$

μ can be interpreted as how much P has to give up in terms of his utility to yield A one extra unit of this utility (this is, in fact the marginal rate of substitution); $U'(x)$ gives the quantity of utility A would like to receive to supply the marginal bit of x. $U'[y(x)]*y'(x) + \mu*U'(x)$ is the net marginal value product of x expressed in P's utility. Because x must be chosen before the state of the world is known, the marginal value product and the marginal costs can only be equalized in expected values terms, that is on average across the states.

2.1.2 THE PRINCIPAL AND AGENT PROBLEM.

This section reviews type C and E models by showing how the different assumptions about the time information is available can change the optimal strategies.

P is assumed to know the technology used by A to produce the output in which he is interested but he knows neither the effort, x, nor the state of the world, α, which will be prevailing. In this situation a true moral hazard [10]problem arises and it is necessary for the principal to set up a strategy which forces the agent to behave in an optimal way, given the new constraint. The optimal solution will usually imply that the effort value chosen by the agent is not optimal.

The lack of observability of x and α means that P has to design a contract such that the agent does not cheat; in formal terms this is done by adding to the problem a new constraint, the so called Incentive Compatible Constraint. P must take account of the change in the environment [11] he is facing: since the value of x cannot be directly observed it will depend on the maximisation procedure of a's utility function and this procedure which cannot be controlled by the principal will affect the final equilibrium an its existence. Thus the first problem which arises when dealing with these models is the proof of the existence and uniqueness of a solution. if y(x) is not restricted to some finite interval an optimal solution to the problem may no exist [12]. One approach is thus to restrict y(x) to a finite interval and this is quite a reasonable assumption since it is possible, on theoretical grounds, to restrict the output to a minimum in the worst state of the world and to a maximum amount in the best state. Since the technology is

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[10]Moral hazard is here intended in the same sense as in insurance market literature.

[11]The definition of environment used here is borrowed from Harris and Townsend (1981).

[12]See Mirrleess (1976) for a formal proof.

given, the state of nature and the effort can have a greater influence on output but there are clearly limits to effort and its productivity (which can be reasonably represented in this context by the state of the world). Another approach developed by Mirrlees(1976) gets around this difficulty by modelling y as a random variable itself. Those assumptions allow us to simplify the matter but they usually imply that an unique optimum might be obtained only when particular conditions are met [13]. Grossman and Hart (1983) show by decomposing the principal's optimization problem into a cost versus benefits problem that an optimal solution can be found.

The existence problem has been further developed and generalized by Page (1987) to a large class of incentive schemes. One of the striking results in the optimal incentive schemes literature is that, in general, they will lead to a departure from the optimal risk sharing solution; a trade-off between the gains from sharing risk and the need to control A's choice of x which is intrinsic to the problem will in general affect the optimal solution.

2.1.3 TYPE C MODELS.

The central assumption for these models is that the agent does not know the state of the world at the time the contract is signed up bu he will know it before delivering his effort. Model two is designed to formally develop this

[13] The formal proof of this statement can be found in Grossman and Hart (1983) and Rees (1985).

approach. The first striking difference from the previous setting is related to the specification of A's utility function. In this context it is assumed that A's utility is separable in reward and effort.

MODEL 2

Define:

Y = output

S = reward to the agent

x= effort of the agent

α = random variable

$y(x,\alpha)$ = production technology

The production function for this problem corresponds to that of model one while the utility function for the agent is specified as follows:

$$U(S,x) = S - f(x)$$

which is clearly increasing in S since:

$$\frac{\partial U}{\partial S} > 0 \qquad \frac{\partial^2 U}{\partial S^2} < 0$$

and which is assumed to be decreasing in x, that is:

$$\frac{\partial U}{\partial x} < 0 \qquad \frac{\partial^2 U}{\partial x^2} < 0$$

It is important to stress the peculiar assumption that characterizes the utility function for the agent: additivity as long as separability in effort and reward is assumed throughout the analysis. The problem faced by the principal can be written as:

$$\underset{S,x}{\text{Max}} \sum p_i \; U^P(Y_i - S_i) \qquad \text{[PA]}$$

$$\text{s.t. } S_i - f(x_i,\alpha_i) \geq 0 \qquad (1)$$

$$S_i - f(x_i,\alpha_i) \geq S_j - f(x_i,\alpha_j) \geq 0 \qquad (2) \quad \text{(all i,j)}$$

Two constraints characterize the problem: the firs constraint means, as before, that the agent will receive a reward which will give him at least a nonnegative utility, that is the agent will have a nonnegative utility whatever the state of the world. The reason for this assumption is quite obvious: since the agent will know before choosing the effort the state of the world he would not be willing to to any effort which would make him worse off. The second constraint, which is also called the the incentive compatibility constraint means that the agent will always have the incentive not to cheat i.e. to reveal truthfully the state of the world which has occurred. The problem can be formalized [14] as follows:

$$\underset{S,x}{\text{Max}} \sum p_i \; U^P(Y_i - S_i) \qquad \text{[PA1]}$$

$$\text{s.t. } S_i - f(Y_i,\alpha_i) \geq 0 \qquad (1a)$$

$$S_i - f(Y_i,\alpha_i) \geq S_j - f(Y_i,\alpha_j) \geq 0 \qquad (2a) \quad \text{(all i,j)}$$

The Lagrangean for this problem can be written as:

[14] See Sappington (1981), appendix A for a formal proof.

$$\mathcal{L} = \sum p_i \; U^P(\; Y_i - S_i) + \sum_{\substack{i \\ ij}} \sum_{j} \lambda_{ij} \left[S_i - f(Y_i,\alpha_i) - S_j - f(Y_i,\alpha_j) \right] +$$

$$\sum \mu_i \left[S_i - f(Y_i,\alpha_i) - 0 \right]$$

The First Order Conditions can be summarized as follows:

$$\frac{\partial \mathcal{L}}{\partial S_i} = -\, p_i \; U^{P\prime}_{s_i}(Y_i - S_i) + \sum_{i \neq j} \lambda_{ij} - \sum_{j \neq i} \lambda_{ij} + \mu_i$$

$$\frac{\partial \mathcal{L}}{\partial Y_i} = -\, p_i \; U^{P\prime}_{Y_i}(Y_i - S_i) + \sum \lambda_{ij} \left[f'_{y_i}(Y_i,\alpha_j) - f'_{y_i}(Y_i,\alpha_i) + \right.$$

$$-\mu_i \; f'_{Y_i}(Y_i,\alpha_i)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_{ij}} = (1a)$$

$$\frac{\partial \mathcal{L}}{\partial \mu_i} = (2a)$$

Let us assume that:

$\alpha_i > \alpha_{i-1}$

and:

$$\frac{\partial f}{\partial \alpha} > 0 \quad ; \quad \frac{\partial^2 f}{\partial y_i \partial \alpha_i} < 0$$

it follows that:

a) $0 \leq Y_1^* < Y_2^* \; .... < Y_n^*$

b) if $Y_i^* = 0 \quad i < m \qquad Y_i^* > 0 \quad i > m$

c) for $i = m \qquad S_i^* - f(Y_i^*,\alpha_i) = 0$

d) for $i > m+1 \qquad S_i^* - f(Y_i^*,\alpha_i) = S_{i-1}^* - f(Y_i^*,\alpha_i)$

For any level of output Y_i^* the optimal problem can be reformulated in a rather different way. Let us assume that the principal is risk neutral such that his utility function can be written as:

$$U^p = \sum p_i \, (Y_i - S_i)$$

then his problem can be formulated as:

$$\min_S \sum p_i \, S_i$$

$$\text{s.t. } S_i - f(Y_i,\alpha_i) \geq 0 \qquad (1b)$$

$$S_i - f(Y_i,\alpha_i) \geq S_j - f(Y_i,\alpha_j) \geq 0 \qquad (2b) \quad (\text{all } i,j)$$

The Lagrangean for this problem can be written as:

$$\mathcal{L} = \sum p_i \, S_i + \sum_{\substack{i \\ ij}} \sum_{j} \lambda_{ij} \left[S_i - f(Y_i^*,\alpha_i) - S_j - f(Y_i^*,\alpha_j) \right] +$$

$$\sum \mu_i \left[S_i - f(Y_i^*,\alpha_i) - 0 \right]$$

The First Order Conditions can be summarized as follows:

$$\frac{\partial \mathcal{L}}{\partial S_i} = p_i + \sum_{i \neq j} \lambda_{ij} - \sum_{j \neq i} \lambda_{ij} + \mu_i$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_{ij}} = (1b)$$

$$\frac{\partial \mathcal{L}}{\partial \mu_i} = (2b)$$

then at the optimum:

$$p_i + \sum_{i \neq j} \lambda_{ij} = \sum_{j \neq i} \lambda_{ij} + \mu_i$$

For fixed Y^* it is possible to show that [15] in this case conditions c) and d) holds for m = 1 i.e. the agent receives his reservation utility level in state one (the worst state) and he receives the same utility level in all other states. This condition assures he does not cheat.

The optimal solution for this problem will then be one in which for any state of the world the agent will receive at least his reservation utility level and has no incentive to cheat. If there were no incentive compatible constraint the agent could have a strong incentive to cheat, but this is not sufficient to say that he would be better off. The principal would be better off without the incentive compatible constraint only in special cases since it usually end up paying more than the optimal amount.

I will now examine what kind of inefficiencies arise in this information asymmetry context. To see where inefficiencies

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[15] See Sappington (1981) appendix b for a formal proof.

arise it is necessary to define what is an efficient allocation. With reference to SAppington (1983) I will define an efficient allocation by referring to the state contingent output. Here is the definition:

The value of output which is efficient in state a is the one at which the agent's marginal disutility form generating an additional unit of output coincides with the principal's valuation of such output i.e.

$$\frac{\dfrac{\partial U^P}{\partial Y_i}}{\dfrac{\partial U^P}{\partial S_i}} = \frac{\dfrac{\partial U^A}{\partial Y_i}}{\dfrac{\partial U^A}{\partial S_i}} \qquad \text{(PEC)}$$

This is equivalent to the optimal conditions derived for model one, where, given the presence of uncertainty about the actual realization of the state of the world the equalization was on an expected basis.

In the same way it is possible to define a first best contract as the one that results in the realization of an efficient outcome whatever state of nature is ultimately realized. By recalling the optimal conditions derived before it is possible to note that conditions c) and d) imply that:

$$f(Y_{m+1}, \alpha_{m+1}) = 1 \qquad m = 1$$

$$f(Y_m, \alpha_m) < 1 \qquad m = 1$$

By the equivalence between the [PA] problem and [PA1] problem :

$$f(Y_{m+1}, \alpha_{m+1}) = 1$$

is equivalent to :

$$\frac{\dfrac{\partial U^P}{\partial Y_i}}{\dfrac{\partial U^P}{\partial S_i}} = \frac{\dfrac{\partial U^A}{\partial Y_i}}{\dfrac{\partial U^A}{\partial S_i}}$$

which is the optimal condition for a first best contract.

This proves that the contract will force the agent to produce an optimal output only in the better states of the world. In state α_i, for example, the output falls short of the optimum.

From the previous discussion it is then possible to argue that the solution derived to solve the principal and agent problem does not satisfy the requirements for a first best optimal contract since, in order to force the agent not to cheat, the optimal condition is attained only for the states in the most favourable output states.

The principal and agent model solution can be simply described by using a diagram and by making some simplifying assumptions.

Let us simplify the problem by assuming that there are only two possible states of the world, namely α_1 and α_2 with $\alpha_2 > \alpha_1$, that is α_2 is a more productive state.

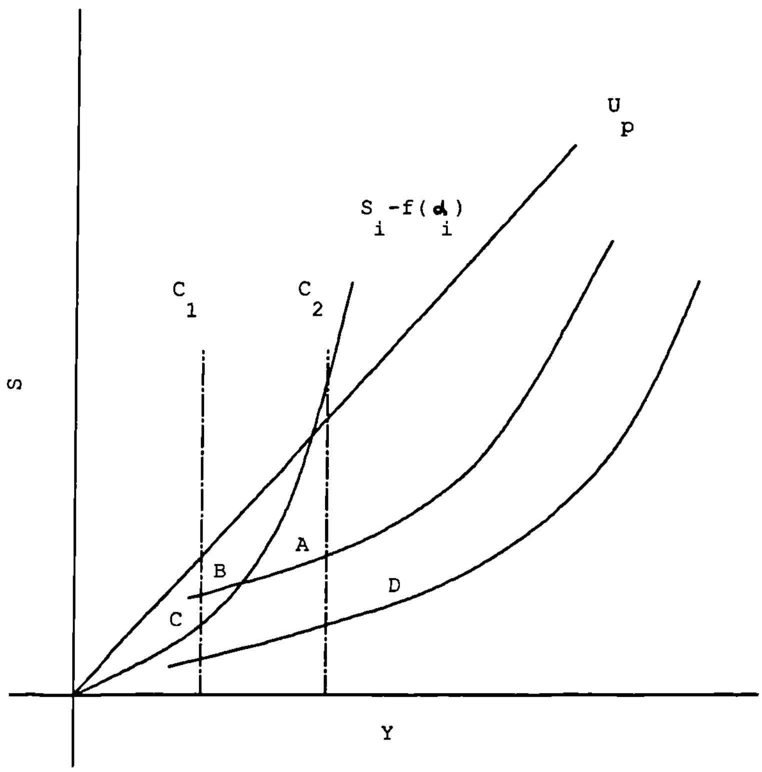

FIGURE ONE

The Principal's indifference curves can be represented by straight lines increasing SE.

The utility curves for the agent are increasing NE and depend both on the effort and on the state of the world which will occur. Allocation which satisfy condition [PEC] are shown by the vertical lines labelled C. Consider first an allocation such A for state 1 and B for state 2. This allocation does not satisfy the incentive compatibility constraint and the agent will always report that state 2 has happened. To make the agent not cheat he must be indifferent between telling the truth or lying. This can be achieved if the two rewards chosen for him lies on the same indifference curve. For example, in figure one, the combination C D is an incentive compatible allocation that can solve our problem.

It is clear that output in state 1 is not chosen optimally and that in state 2 the agent receives a reward which is greater than his reservation utility.

The same result holds if we set up the reservation utility level for the agent to any level, but the inefficiency diminishes by lowering this reservation utility level.

2.1.4 TYPE E MODELS.

Harris and Raviv (1979) observed first that, by lowering the reservation utility level inefficiency diminishes. Using this device in the contract allows to show that if A is risk neutral P can achieve a first best allocation and no incentive problem arises. The argument is developed on the basis that if A is risk neutral, a contract which specifies the reward as a function of the final output is at least as good as one which makes the reward contingent also on the effort, the state of nature and the output. If this is true, it does not matter if the principal cannot observe x or α, so long as he is able, as it has been assumed, to observe Y. However, while this statement is true, a further consideration is necessary in this context. If the contract depends only on output, irrespective of A's utility, this latter utility is not bounded; in other words the first constraint disappears.

This argument is perfectly correct, but it is worth demanding under what circumstances the reservation utility level can take any value i.e. is not bounded below. This is the way type E contracts are set out.

The only reason why the agent can accept a binding contract which can give him a disutility is that when he chooses his effort he does not know what the realization of α will be. These considerations provide the link between Harris and Raviv's model and Sappington one. As I have pointed out before "optimality" for the P-A contract depends on the reservation utility [16] set out for the agent. Sappington shows that it is possible to reach a first best allocation when there is not limited liability for the agent; that is his reservation utility can be negative. This is quite important because optimality does not just depend on the attitude to the risk of the agent but crucially on the type of contract and the relevant balance of power between the two parties as well: risk neutrality in this context is, in my opinion, a useful device to model different real - world environments with respect to timing in information and relative contractual power. This is farm more clear if we observe that when the agent is risk neutral but his reservation utility is bounded (and, for example, the assumption that a party cannot be forced to incur disutility from his effort is quite reasonable), an optimal contract cannot be set up.

[16] The term reservation utility corresponds here to "limited liability" in Sappington paper.

2.1.5 THE USE OF INFORMATION ABOUT x OR α.

Suppose we assume that x, that is the agent's effort, though not observable in its actual realization, can be observed with an error. Models in this category are halfway between C and D and E and F. The possibility for P to have information about x and α can be used to improve the incentive scheme of the contract.

In formal terms we can suppose that the principal can observe a random variable:

$$\theta \quad = \quad x + \alpha$$

with:

$E(\sigma) = 0$; $\Phi(\sigma) > 0$ on some interval $[\sigma_o , \sigma_u]$
$\Phi(\sigma) = 0$ elsewhere
Φ probability density for σ.

If we can assume that σ does not depend on α and that the σ distribution is bounded, it is possible to show that the principal can set up a first-best contract by setting out rules which forces the agent not to cheat.

This can be done by using a system of penalties which come into force if the principal observes some $\theta < x+\sigma_0$. The assumption of a bounded distribution for σ assures that no problem of hypothesis testing is incurred by the principal [17]. If the penalty is very high, A will choose everywhere x^* since the case $x > x^*$ is ruled out by the assumption

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[17] The principal has to choose a critical value for θ such that an observation $\theta < \theta^*$ would imply $x < x^*$. Hence P has to weight the type I and type II errors from the standard hypothesis testing theory.

about the behaviour of A's utility. A case of more interest is represented by a problem in which x can be inferred through the observation of another variable -z- whose distribution depends on both x and α.

In this special case, a further distinction can be made and it is related to whether z can be observed at no cost. Consider first the case in which z can be costless observed. z is assumed to depend both on the effort and the state of the world in which it will happen. In this context it is possible to define a probability distribution for z given any x. Holmstrom (1979) and Shavell (1979) have shown it is always optimal to incorporate such information in the contract because the benefit of this extra information outweighs the cost of the extra uncertainty that the presence of an extra random variable adds to the problem. The formal setting for this problem is shown in appendix one. The only difference with respect to model two is that now the realization of the output is affected by two random variables whose joint distribution determines the actual output. The optimal condition characterizing this problem can be interpreted by considering that incorporating z in the problem the principal does not get more information on the most likely value for x, rather it provides a method for the principal to design a better contract for his agent.
In other terms, the principal, by using the information about z reduces the probability of giving A an high reward for low effort in good states and vice versa.
The incorporation of z in the contract, thus, improves the

incentive properties of the contract even though it does not solve the underlying uncertainty problem.
In terms of efficient contracts as defined before the ratio between the marginal product value and its marginal cost [18] will be closer to one than in the previous situations; now that the principal can discriminate among some realizations of states he has not to offer so high a reward for output in better states. In other terms, in higher output states in which efficiency was already reached at the cost of a division of output more favourable to the agent, the principal is now able to obtain the same effort from the agent by offering a lower reward.

The second case, when a variable z can be observed by paying a fee has been extensively examined by Gale and Hellwig (1983) in an application of the incentive compatible scheme to the debt contract problem. The asymmetry of information arises because the firm can costless observe the state of the nature on which its revenue depends, which in turn affects the probability for the investor to have his money back and the interest payment on his loan. The investor is initially uninformed about the state of nature which, in this case, is represented by the actual condition of the firm but he can become informed by paying a positive fee which is again state - contingent.
The variable z of the previous model is represented here by the cost of bankruptcy. Bankruptcy can be seen as a costly

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[18] Marginal product value and marginal cost should be intended in the light of PEC condition set up before.

way of acquiring information about the firm to which the investor lent his money; before the receiver can pay the creditors of a bankrupt firm he must evaluate the firm; this corresponds to discovering the true value of the state, α. The presence of asymmetry of information here causes the level of investment to fall short of the optimal one when both parties are perfectly informed.

Credit rationing is thus the "measure" of inefficiency. Anyway, Gale and Hellwig are able to show that the presence of bankruptcy can improve the contract; the credit rationing and the interest rate would be higher in situations in which no bankruptcy costs were involved.

The conclusion is thus that the presence of variables from which information can be acquired usually improves the basic contract even if it does not solve the problem.

2.2 TYPE A MODELS.

In this section I will review models that are usually included in private information with active agent literature. Under my classification those models can be, instead, treated as principal and agent problem. The common features and assumptions characterizing these models are:

- the agent possesses some information prior to choosing an action which, if known by the principal would influence the choice of action he would like the agent to make.

The agent is then required to pass some message to the principal which depends on the "private information" he has. Since the chosen effort, outcome and payoff to the agent all depend on the message he transmits, the agent has well an incentive to misrepresent his information.

- A knows the state of the world variable, α, before taking any decision.
- He has to transmit to his principal a message which is represented by the value α.

then we are speaking of type A models in table 1.

The principal, on the basis of the α he receives will instruct the agent to take a consequent action $x(\alpha)$ in exchange for a payoff $S(\alpha)$. Given this environment the principal must take into account the incentive A May have to report a false α. Assuming that P is risk neutral and that α can be observed with :

$$\alpha_{i-1} < \alpha_i$$

the formal model relevant in this context is equivalent to model two.

In this second case, however, no risk sharing is possible between A and P. Since A perfectly knows the state of nature before the contract is set out, he will never accept contracts which do not provide for sure at least his reservation utility; in other words the first constraint in model two will by no means be eliminated: the special case examined by Shavell and Harris and Raviv will never be implemented in this context.

The equilibrium allocation shown in fig. 1 applies in this context as well. The gain to the agent from receiving in state 2 a larger reward than in a situation in which the principal were fully informed can be interpreted here as the rent he commands from the monopoly of his private information.

3. ADVERSE SELECTION MODELS.

The models concerned with adverse selection can be divided into two main classes:

a) models involving transmission mechanisms.
b) Adverse selection models with active agents.

As I have already explained in the previous section I prefer to include models with adverse selection and "active" agents in the type A principal and agent models and for this reason I will here briefly review just the transmission mechanisms models. the literature on these models is not so wide as the one on the principal and agent relationship but for what it will follow this is the relevant class of models to take into account.

3.1 MODELS REQUIRING TRANSMISSION OF INFORMATION.

These models are designed for situations in which the relationship between the two parties involves the transmission of some messages. The agent has usually the ability to misrepresent the information he has to pass on to the principal to his own advantage.

Examples in this class include allocation mechanisms, nonlinear pricing in monopoly markets and auction - design literature and, as I will explain later, in this terms it is possible to model the relationship between central and local government. Models dealing with public goods and the related problem of preference revelation are just the most known example of how optimal contracts can be devised in this context. Suppose that a planner has to implement a certain social choice rule. By social choice rule it is usually meant a "*set of feasible social states for each possible configuration of individual preferences and characteristics.*" [19]

The Pareto principle is just the most widely known example of social choice rule. If the relevant characteristics of individual agents happen to be publicly known, then the social choice rule can be implemented trivially because the choice set from which to optimize is known.

The problem of incentive compatibility in models dealing with public goods arises precisely because the characteristics are not known by the planner a priori. The planner may attempt to learn them directly by asking agents to reveal them. In general, however, if agents realize how the information they reveal is to be used, they will have an incentive to misrepresent their preferences . The planner's task is then more difficult. Those models usually assume

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[19] This definition has been borrowed by Hammond and Maskin (1986).

that individual agents send their own signals and the planner on this basis implements the social choice rule. The problem for the planner is to set out the rules of the game in such a way as to prevent each individual agent from cheating. Planning with public goods was studies by Drèze and De la Vallé Poussin and Malinvaud. In these models it is assumed that each consumer reports his marginal rate of substitution between public and private goods which is then used by the planning bureau to define the quantity of public goods to be provided and the price sharing. Agents can lie along the way but these mechanisms prove to converge to a Pareto-optimum in the long run. Another peculiar model in this class is the preference revelation rule put forth by Clarke (1971) and further developed by Tideman and Tullock (1976) in the public goods context. The Clarke principle can be stated as such:

each person is given the choice of accepting the decision that would be made without his participation or changing it to whatever he likes. In this second case he has to pay an mount of money equal to the marginal cost of all other persons of doing what he wants.

This payment is the so called "Clarke tax". Any individual has a strong incentive to reveal his true preferences because the loss in utility he suffers from not reveling them correctly is greater than the Clark tax he has to eventually pay.

Most of the models in the transmission of information literature deal with repeated games which imply a dynamic framework. Roberts (1982) has identified general necessary conditions for long term contracts to dominate short term ones but these conditions are not in the form to delineate a priori classes of models that satisfy them. In most cases, the repetition of the game can provide an efficiency gain in cases in which the contract commits either the principal or the agent to a lower payoff in some events in future periods than could be obtained from a short term contract negotiated when future periods arrive. In the case of public goods provision it is the assumption of a constant marginal rate of substitution over time that makes the agents better off by revealing their preferences.

The models in the principal-agent literature with active agents seems to be more difficult to deal with in a dynamic context, and whether the long run contract is superior to the short run one much depends on the peculiar assumptions of the model. Malcomson (1988) for example shows that efficient contracting under moral hazard alone does not require long term commitment from the principal and, provided that a short term contract can punish the agent sufficiently, it does not commit the agent either. Radner (1981, 1985), Rubinstein (1989) and Rubinstein and Yaari (1988) have instead shown that a first best solution can be obtained for a sufficiently low enough rate of discount and a sufficiently long time period.

4.CONCLUSIONS.

This brief review of the literature on incentive compatibility models shows the potential applications this theory can have. The natural extensions on theoretical grounds are concerned with dynamic models or multipart contracts. Finally it should be kept in mind that the distinction between principal and agent problem and adverse selection models is sometimes very difficult, expecially for models involving "active" agents. While this can be a problem when we want to classify models, the advantage is that the literature can provide a uniform framework to deal with problems which are apparently and substantially different.

Finally I can recall that in a number of applications the adverse selection aspect and the moral hazard problem are combined together: an important example is represented by the income tax model by Mirrlees (1987). In this model, while agents share the same preferences over consumption and leisure, they differ in their marginal labour productivity. Adverse selection is intrinsic in the problem because the tax authority (the principal) does not know the agent's marginal product. The additional problem, that is the inability of observing agents' labour-leisure choices create a moral hazard problem.

This brief review of the literature, although not exhaustive, gives some insights into the problem and offers general tools to solve problems that can be formulated in a "principal and agent" framework. The models I will present in the next sections are mainly related to the adverse selection literature since I will assume that local governments' task is to report their true preferences for public services to central government. The asymmetry of information caused by Central Government inability to observe true preferences falls for an optimal incentive scheme; however its actual implementation might be difficult, if not impossible, due to the peculiarities of the economic agents involved in the game. This problem will be discussed at length at the end of chapter four after the basic model and the optimal solution will have been presented.

APPENDIX ONE

P does not know x but has some information about the distribution of θ, a random variable depending on σ i.e.

$$\theta_i = x + \sigma_i$$

The problem can be written as:

$$\max_{S_i(Y,z),x} \sum p_i \; U^P(Y_i - S_i)$$

$$\text{s.t. } S_i(Y_i,\theta_i) - f(x_i,\alpha_i) \geq 0$$

$$S_i(Y_i,\theta_i) - f(x_i,\theta_i) \geq S_j(Y_j,\theta_j) - f(x_i,\alpha_j) \geq 0 \qquad \text{(all i,j)}$$

The only difference in the solution, if compared with the case I presented in the previous appendix is that now there is a joint distribution for Y. It is possible however to show that the payment for the agent is modified in the light of the new information acquired.

CHAPTER THREE

1. INTRODUCTION

At the end of chapter one I have argued that one of the main reasons for the existence of local authorities is their ability of being better informed than Central Government about local preferences and needs. These parameters are however essential to Central Government to devise any grant system that allows it to allocate resources efficiently [1].

This chapter is concerned with describing the optimal grant system that Central Government could use in order to reach an efficient allocation of resources and the reasons why the actual implementation of the system might be impossible. The optimal grant formula will be examined in both a static and dynamic framework and the Harris and Townsend (1983) methodological approach consisting of five steps will be followed. First the economic environment including the information structure is described, then I will define the concept of a feasible allocation mechanism for an environment. The third and fourth step consist of describing the preferences of the agents over the different mechanisms and defining the concept of an efficient mechanism. The last step describes efficient mechanisms and their allocation. Due to the peculiarities of the agents involved in my analysis I will start by explaining the reasons leading to the choice of an utility maximisation framework to describe the behaviour of collective decision bodies and its shortcomings.

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[1] Efficiency is defined in terms of minimum resources to be given up to attain an objective.

1.1 UTILITY MAXIMISATION APPROACH AND COLLECTIVE DECISIONS MAKERS: A DIGRESSION.

Decision making and choice lie at the heart of much of the discussions of human behaviour and they affect the allocative process in many ways. When dealing with individual decision making the neoclassical utility maximisation framework seems to be widely accepted. Since resources are scarce, any individual tries to do his best in order to attain the maximum level of satisfaction. Private action, at its simplest, presents little difficulty since the ultimate decision maker is assumed to be the acting individual; the problems raised when the same framework is applied to collective decision makers are, however, quite considerable. Given that individuals make decisions and organizations do not, it is worth considering whether the behaviour of collective decision makers might be well represented by a utility maximising behaviour. The widely known Arrow's impossibility theorem seems to rule out this possibility by showing analytically that there does not exist a collective choice rule or social ordering which obeys the basic axioms which any social ordering, derived from a set of individual orderings should conform to. During the fifties and sixties Arrow (1951), Black (1958), Buchanan (1954), Oliver (1955), Tullock (1958), Downs (1957) and Buchanan and Tullock (1967) have developed new approaches to collective decision making.

Downs' theory of collective decision making is focused on the behaviour of political parties. The attempt of parties to maximise voter support replaces utility maximisation in the market process. Arthur Bentley explains collective decision making in terms of the interplay of group interests while Buchanan and Tullock, by using the "theory of teams" concentrates on the behaviour of the individual in a voting process. Although those approaches are quite stimulating I will use a utility maximisation approach at least to represent local government's behaviour because, despite recent theoretical interest by economists in the subject, relatively few empirical studies have been carried out on the comparative performance characteristics of the different group decision rules. Most of the recent studies in the field are founded on utility maximisation models and they commonly uses a Cobb Douglas functional form. McMillan and Juffour(1988), however, by using data for local government expenditure on different services in Victoria (Australia) have tested alternative specifications of the underlying preferences using a translog model and rejected symmetry conditions so that data did not seem to be consistent with utility maximisation. I will take account of those factors at the end of chapter four when I will present some reasons why the optimal incentive system could fail to reach its objectives. To start with, even with the pitfalls implied by applying rational decision making processes to collective bodies, the utility maximisation approach is the most suitable to pick up the information structure of the problem I am examining. However, due to the limitations of the model when applied to collective decisions makers I will assume that local authorities behave *as if* they had a utility function which they want to maximise. This point is quite important because, for the reasons that will become clear in the proceeding of the analysis, it jeopardizes the actual implementation of the optimal grant allocation rule.

The goals pursued by Central Government and Local authorities are asymmetric since the approach I have been using assumes that Central Government's aim might be represented by a straight minimization of the size of the grant without any specific reference to the interests that Central Government have in giving the grant to local authorities in its exact size. I think this approach can be justified on the ground of efficient allocation of the resources available. Once Central Government has decided how much local services have to be provided in a specific area, it is in its best interest to reduce as much as possible the resources that have to be spent to attain this goal.

2. THE STATIC MODEL

This model assumes that the game played by both central government and local authorities is a one shot static game. Local authorities' behaviour is described by using a fairly general utility function defined over two goods, y, a composite private commodity and X which represents local public services. β is a parameter summarizing the characteristics of each local authority with respect to its preferences for expenditure.

The utility function for authority i can be written in a very general form as:

$$U(X_i, y_i, \beta_i) \qquad \frac{\partial U}{\partial X} \geq 0 \; ; \; \frac{\partial U}{\partial y} \geq 0$$

which has to be maximised subject to the budget constraint:

$$X + y = M$$

where M represents local income available. As it can be noted from the equation above the price ratio is assumed to be one. In diagrammatic terms, the budget constraint faced by a representative local authority can be depicted as follows:

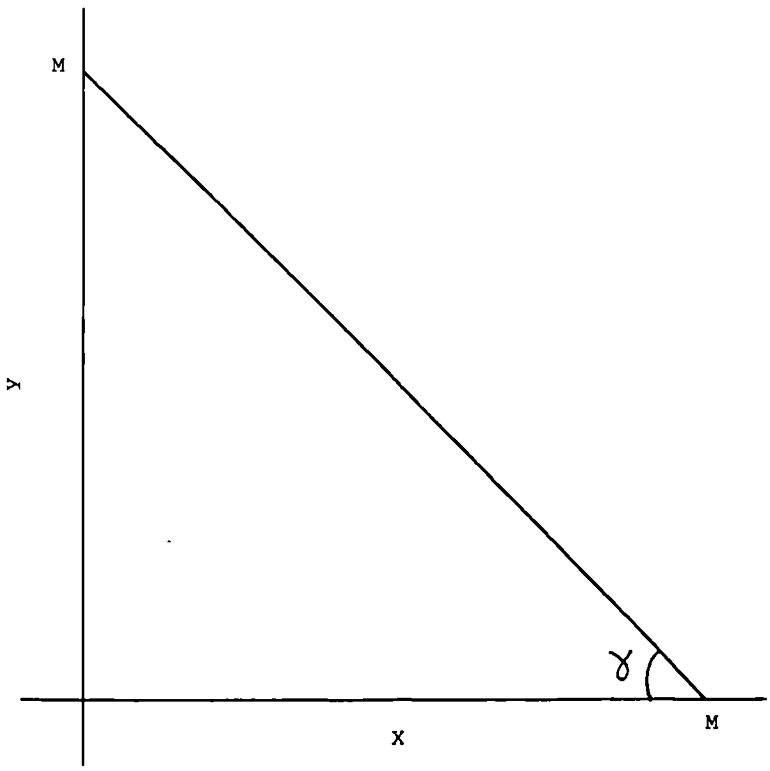

FIGURE ONE

The slope of the budget constraint, given the assumption of equal prices is $\gamma = -45^{0}$ while the price ratio is equal to $\tan(\gamma) = -1$.

Maximisation of this utility function subject to the budget constraint will lead to a demand function of the following form:

$$X_{NG} = x_{NG}(\beta, M)$$

β = preference parameter.

Central Government's objective is to make each local authority to spend at least α^{*} in local public goods. α^{*} can be thought of as the desired level of expenditure and can somehow be related in real world to the GREA [2]. α^{*} is a parameter specific to each local authority and it is known by Central Government. I will also assume that:

$$x_{NG}(\beta, M) < \alpha^{*}$$

for all the local authorities I am considering in the sample in order to rule out the possibility of negative grants that are not observed in the real world. In this environment, if Central Government wants to make each local authority spend at least α^{*}, a of subsidy has to be provided and the two principal forms can be either a lump sum of money to

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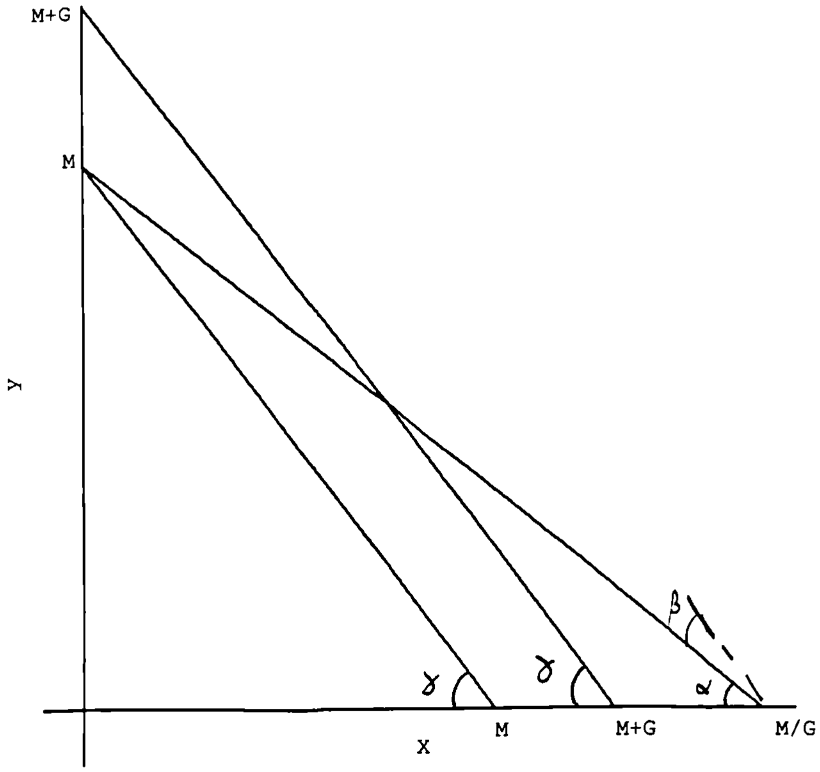
[2] GREA stands for Grant Related Expenditure Assessment.

increase local income or a per unit expenditure subsidy by which the cost of each unit of the service is partly matched by the subsidizer. Both situations are depicted in figure two.



$$\tan(\alpha) + \tan(\beta) = \tan(\gamma)$$

FIGURE TWO

As it can be noted from the above figure in the case of a lump sum grant the income available increases to M+G while in the case of a matching grant the price ratio is reduced to g with Central Government offering to pay (1−g) for each unit of expenditure. As a result of this assumption, local government's budget constraint can now be written as:

$$gX + y = M+G$$

where g represents the per unit cost faced by local authorities for providing local public expenditure after a matching grant of the form (1-g) has been introduced and G is the lump sum offered to local authorities. The new demand equation can thus be written as:

$$X_{GR} = x_{GR}(\beta, M+G, g)$$

The form and the size of the grant has clearly to be chosen in an optimal way. Optimality is here defined in terms of minimum resources necessary to make each local authority spend at least α^* on public services.

More formally Central Government behaviour can be described by:

$$\min_{g,G} \quad (gX + G)$$

$$\text{s.t. } x_{GR}(\beta, M+G, g) \geq \alpha^*$$

I will start by describing the optimal grant formula in a world with no uncertainty in order to compare the optimal contract with the one that will arise in an asymmetry of information world. The environment in which Central Government has to operate in the first case [3] can be described as follows:

i) Central Government can observe the amount of local income available in each region, M , and set the amount

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[3] i.e. when both parties are perfectly informed.

of local public services to be provided, α^*.

ii) Before the start of the period Central Government observes preferences over local public services which will be in turn used to determine the size of the grant for each authority.

iii) Open ended matching grants and lump-sum grants are the two instruments available to Central Government to reach its objective.

Once the environment is specified, the concept of an allocation mechanism for the environment has to be defined. An allocation mechanism for a given environment is simply the set of rules which specify the game to be played by the agents in allocating resources and the optimal allocation is the set of rules that both maximise the objective function and satisfies the constraint imposed by the environment. In the problem I am examining the allocation rules are grant distribution formulas that Central Government can announce at the start of the period to achieve its goal while the optimal allocation mechanism is the one that allows it to minimize the size of the grant. Given assumption ii) the model can be designed to cope with the optimal allocation rule for just one authority and it will then be replicated for the entire population by changing the relevant parameters characterizing each local authority. The separation is based on the further assumption that no ceiling exists on the amount of the grant total.[4] In terms of the exposition that follows these assumptions allow to

[4] Grant total in this context means the sum of grants distributed to all local authorities.

drop all subscripts i for local authorities.

2.1 OPTIMAL GRANT WITH BOTH PARTIES PERFECTLY INFORMED.

In a world with no uncertainty Central Government problem can be written as:

$$\min_{g,G} \; gX + G$$

$$\text{s.t.} \quad x_{GR}(M,\beta,g,G) \geq \alpha^{*}$$

Government can reach its goal by using an open ended matching grant that, by having both an income and a price effect, boosts up expenditure to a higher level than a pure lump sum grant provided the substitution effect is not zero. This can be formally shown by using the Slutsky equation for this problem. If the grant was given in a lump sum form the demand equation could be written as:

$$x_{GR}(M \;+G,\beta)$$

$$\frac{\partial x}{\partial G} = \frac{\partial x}{\partial M}$$

If the grant is given in a matching form it is equivalent to a price reduction, then we can write the Slutsky equation as:

$$\frac{\partial x}{\partial g} = - \left(\frac{\partial h(g,\bar{u})}{\partial g} - \frac{\partial x}{\partial g} * x \right)$$

where $h(g,\bar{u})$ is the Hicksian demand function.

Starting from an equilibrium point at which:

$$G(\beta,M) = (1- g) * x$$

the marginal change in the lump sum grant is approximately equal to[5]:

$$\partial G = - \partial g * x$$

then it is clear that, unless the substitution effect is zero, the matching grant has a greater effect in boosting up expenditure than a lump sum grant.

Further insight in the problem can be gained by the aid of a diagram. Consider figure three in which a local public good, X, is measured on the horizontal axis and a private composite commodity ,y , is measured on the vertical one.

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[5]The exact formula implies taking account that, the matching grant will be shared among a greater amount of expenditure, then we have to take account of this feedback we want to assess the full effect of the same marginal change in the grant distributed. I will assume here that since we deal with marginal changes we can assume that, from the point of view of grant sharing x is constant.

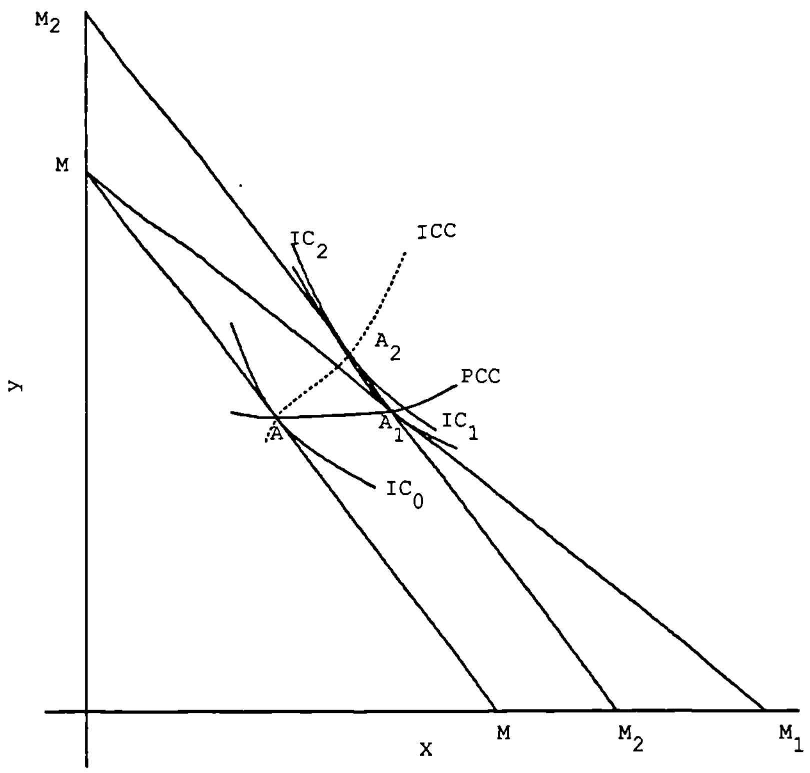

FIGURE THREE

The local authority's budget constraint is represented by the line MM The before grant optimal allocation is represented by A ,the point at which the budget constraint is tangential with the Indifference Curve (IC) I_0. Now suppose central government offers the locality a matching grant in the form of a percent contribution to the unit price. The local authority's budget constraint pivots to M M_1 The locality moves to A_1 on indifference curve I_1. Provided price elasticity of demand is not zero, demand for local public goods will increase; in this case from X_0 to X_1; the resource cost will amount to Mc, of which y_1c is financed by Central Government. If the amount y_1c had been given as a lump sum grant the local authority would have a budget line M_2M_2 and would be able to achieve a more desired position A_2 but its demand for local services would have been less than at A_1 because the matching grant is effectively a price reduction which, has I have explained before, has a substitution as well as an income effect, and this is reflected by the fact that the price consumption curve (PCC) lies to the right of the income consumption curve (ICC) after the initial position.

If the utility function for local government is a Cobb Douglas the it is possible to formally show that Central Government requires less resources by using a matching grant. The proof is given by solving the following problem:

$$\text{Max} \quad U = (1-\beta)\, \ell n\, y + \beta\, \ell n\, X \qquad\qquad 0 < \beta < 1$$

subject to:

$$X + y = M$$

which leads to the demand equation:

$$X_{NG} = \beta * M$$

When the grant distribution formula is introduced the budget constraint will be written as:

$$gX + y = M+G_L$$

and the demand equation will be equal to:

$$X_{GR} = \beta \frac{(M + G_L)}{g}$$

If all the grant is distributed in a lump sum form, then $(1-g) = 0$ and the problem faced by Central Government can be described as follows:

$$\text{MAX } (-\ G_L)$$

$$\text{s.t. } -\ (\beta * M + \beta * G_L) \leq -\ \alpha^*$$

and the optimal solution is given by:

$$G_L = \frac{\alpha^* - X}{\beta} = \frac{\alpha^*}{\beta} - M$$

If Central Government wants to offer a matching grant, G_L will be set equal to zero and the problem faced by Central Government will be:

$$\text{MAX } -\ (1-g)*\alpha^*$$

$$\text{s.t. } -\ \left(-\beta\frac{M}{g}\right) \geq \alpha^*$$

In order to allow local authorities to spend α^* the price they have to face has to be equal to:

$$g = \frac{\beta * M}{\alpha^*}$$

from which it follows that the optimal matching rate is:

$$1 - \frac{\beta * M}{\alpha^*}$$

The grant in a lump sum form will be equal to:

$$G_L = \frac{\alpha^*}{\beta} - M$$

and under matching grant will be equal to:

$$G_M = (1-g) * \alpha^* = \alpha^* - X = \alpha^* - \beta M$$

$$G_M = \beta G_L$$

since $\beta < 1$ the matching grant is less than the lump sum grant.

2.2 INCENTIVE COMPATIBLE PROBLEM.

Suppose now that government, although it knows exactly α^* and M, it cannot observe β, the preference parameter of the local authority. In terms of the description of the environment in which Central Government operates this means that assumption ii) has to be modified as follows:

iia) Central Government knows that the true β for any local authority lies in the closed interval $[\beta_1, \beta_2]$. Since no other information is available the distribution of β, $\ell(\beta)$ is assumed to be uniform [6]. Before the

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[6] It is then clear the analogy of this model with Tresh interpretation of Stigler's approach. Even all the analysis that follows will be conducted in terms of expenditure behaviour it is clearly the marginal rate of substitution between private and public goods that cannot be known with certainty.

starting of the period each local authority has to report to the Central Government its preferences over local public services which will be in turn used to determine the size of the grant. The preference parameter declared by local authorities will be labelled β_d to distinguish it from β, the true one.

Two further assumptions have to be made to give a complete description of the environment, namely:

iv) Preferences parameters are peculiar to any authority considered i.e. it is impossible to infer authority's i preferences by observing the relevant parameter for authority j [7].This assumption rules out the possibility (or the need) for coalitions between authorities then the problem can be restricted to examine the optimal contract between one local authority and the Central Government, at least in this first stage of the analysis [8].

v) β is assumed to be both independent between authorities as stated in iv) and across time [9].

The grant that each local authority receives depends negatively on β_d, i.e. the more it prefers expenditure, the less grant it needs in order to reach α^*. It is clear that it will have an incentive in reporting a β_d as low as

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[7]in other words β is not either a simple political or regional parameter.

[8]The problem could not be treated as such if the Central Government had a budget constraint on the total amount spent on grants to local authorities.

[9]This second assumption will be relaxed in dynamic model and it will actually be the crucial difference between the two setting.

possible such that it will be entitled to receive a greater amount than needed to reach α^*. The cheating clearly depends on the possibility of this behaviour to be observed and on the consequences that it might bring about. The model I am presented here is of type A [10]: the agent always knows with certainty and before the start of the period all the relevant parameters and its cheating cannot be avoided by the principal. In this framework government has to devise a system to make each local authority reveal its true preference. The uncertainty is implicit in this problem. The problem can be written in fairly broad term as such [11]:

$$\text{Min} \quad \left(1 - g(\beta_d)\right) * X_{GR}$$

$$\text{s.t. } X_{GR}(M, g(\beta_d), \beta_d) \geq \alpha^*$$

$$\text{s.t. } V\left(g(\beta), M\right) \geq V\left(g(\beta_d), M\right) \qquad \text{all } \beta$$

where V(.) is the indirect utility function which is assumed to be well behaved [12].

The second constraint in this problem is the so called Incentive Compatible constraint and its function is to

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[10] The classification of different models can be found in chapter two, table one.

[11] This approach follows closely Sappington (1984) and Harris and Townsend (1983) models. The theoretical links between the two models have been explained in the previous chapter while the application to a transmission mechanism is quite straightforward.

[12] i.e. $\frac{\partial V}{\partial X} > 0$ and $\frac{\partial V}{\partial g} \leq 0$.

assure that the optimal rule implemented will avoid cheating by the better informed party. If this constraint is satisfied, any local authority will receive an amount of money such that the level of utility it can reach by revealing its true preference is as least as high as it would be if it was cheating. It is important to note that, since in this model I am dealing with an agent that knows all the information prior to the start of the game, the inequality constraint must hold for any β [13]

According to the preferences they declare before the actual realization of expenditure local authorities will receive a matching grant whose matching rate, $(1-g(\beta_d))$, will depend on the preferences they declare. $(1-g(\beta_d))$ is negatively correlated with β_d since the higher the preference revealed, the lower is the matching rate for the same level of desired expenditure: the level of expenditure without grant is , caeteris paribus, higher and the grant is more effective. However, the indirect utility for local government is negatively correlated with g : the lower the price they have to pay for public goods the higher is the amount of private commodities they can buy.

This consideration should be sufficient to explain why in this situation the incentive compatible constraint cannot work properly. If local authorities can, as the set up of this model allows them to do, cheat they will declare the lowest β_d they can. In terms of the model described before

[13] This is one of the major differences with Sappington model in which, since the agent will know the true parameter only after the start of the period the constraint must hold on expected values terms.

this means that since V(.) is monotonically decreasing in g, the only way to have the incentive compatibility constraint satisfied is to set $g(\beta_d) = g$ thus giving the same matching grant whatever β is. This might be better illustrated by an example. Let us assume that Central Government knows a priori that the preferences for local public services in a particular locality are either β_1 or β_2 with $\beta_1 < \beta_2$ and those two event can occur with the same probability [14].The problem can be formalized as follows [15]:

$$\text{Min } 0.5\ (1-g_1)*X_{GR} + 0.5\ (1-g_2)*X_{GR}$$

$$\text{s.t.} \quad \left[x\left(\beta_1, M, g_1\right)\right] \geq \alpha^*$$

$$\text{s.t.} \quad V\left(M, g_2, \beta_2\right) \geq V\left(M, g_1, \beta_2\right)$$

$$\text{s.t.} \quad V\left(M, \beta_1, g_1\right) \geq V\left(M, \beta_1, g_2\right)$$

For the reasons I explained before, higher preferences imply a lower matching rate ,then if $\beta_1 < \beta_2$ it follows that $g_2 > g_1$ which implies that the second constraint is redundant because an authority with a comparatively low preference for public expenditure would be worse off by reporting a parameter higher than the true one.

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[14] i.e. $p(\beta_1) = p(\beta_2) = 0.5$

[15] In the following example g_i i=1,2 is used as a shorthand notation for $g(\beta_i)$ while the values with asterisk denote the optimum amounts of both public and private goods.

It is possible to illustrate the problem and its solution in the following diagram:

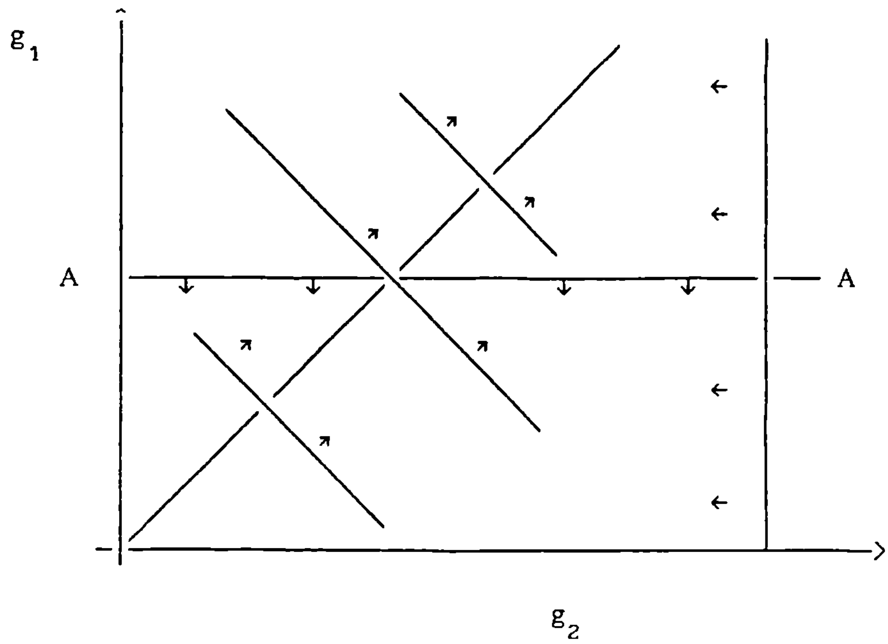

FIGURE FOUR

The diagram is drawn in the $g_1 - g_2$ space. g_1 and g_2 represent the matching rate which, since $x_{NG}(\beta,M) < \alpha^*$, have to be less than one. Constraint 1 is represented by the straight line AA while , since V(.) is decreasing in g and $g_2 > g_1$ constraint 2 is satisfied only along the 45°line from the origin. The objective function is represented by the straight lines bb that are increasing towards the origin. It follows that the optimal point is represented by c at which local authorities with preference parameter equal to β_1 spend exactly α^*[16]and any authority receives the same matching grant rate irrespective of its preferences.

If again I assume that the functional form for the utility function of each local government is a Cobb Douglas,

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[16]It is clear from the diagram that, in this case constraint 1 is binding

the problem can be formulated by the following minimization.

$$\text{Min } (1-g)\, X$$

$$\text{s.t.} \quad \frac{\beta_1 M}{g} \geq \alpha^*$$

Since it is impossible to discriminate among authorities Central Government has to set the matching grant such that local authorities with the lowest preferences will spend at least α^*.

The optimal g is equal to :

$$g_1 = \frac{\beta_1 * M}{\alpha^*}$$

An authority with preferences equal to β_2 will declare β_1 before the start of the period but it will then spend according to its his true preferences:

$$\hat{X} = \frac{\beta_2}{\beta_1} * \alpha^* \qquad > \alpha^*$$

The grant loss for government with respect to the "separating equilibrium" formula will be equal to:

$$(g_2 - g_1) \cdot \alpha^* + (1 - g_1) \cdot (\hat{X} - \alpha^*)$$

where the first expression represent how much more government is offering to the local authority for each pound it spends than it would be necessary to it to reach the optimal demand α^* and the second expression is the grant loss due to the fact that in this second case the local authority is buying a quantity of X greater than α^*.

The overall problem could be formulated in a slightly different way by assuming that Central Government is interested in seeing each authority spend at least α^* in expected value terms. I show this problem in appendix one. It is clear that in this case the size of the loss is reduced but local authorities whose preferences are lower than the expected value will not be able to reach the target level of expenditure.[17]

2.3 A TWO PRICES OPTIMAL GRANT ALLOCATION

It is then clear that in the framework I set up Central Government is unable to set a system that allows it to avoid cheating. In a static game each period local authorities can announce the lowest β_d and then spend according to their true β. In this case the best policy that it can implement is to reduce the loss as much as possible.

This goal is achieved by designing a more complicated grant distribution formula. In my analysis I will consider the possibility of setting different matching rates according to the level of expenditure [18]. In very general terms Central Government could set two different matching rates as

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[17] Because of the assumption of a uniform distribution for β, $E(\beta)$ will be in this case equal to $\dfrac{\beta_1 + \beta_2}{2}$.

[18] This system has extensively been used by Central Government in recent years as I will explain later.

follows:

(1-g) for any quantity $X < \alpha^*$

(1-h) for any unit of $X \geq \alpha^*$

The post-grant budget constraint for a representative authority under this new assumption is shown in figure five.

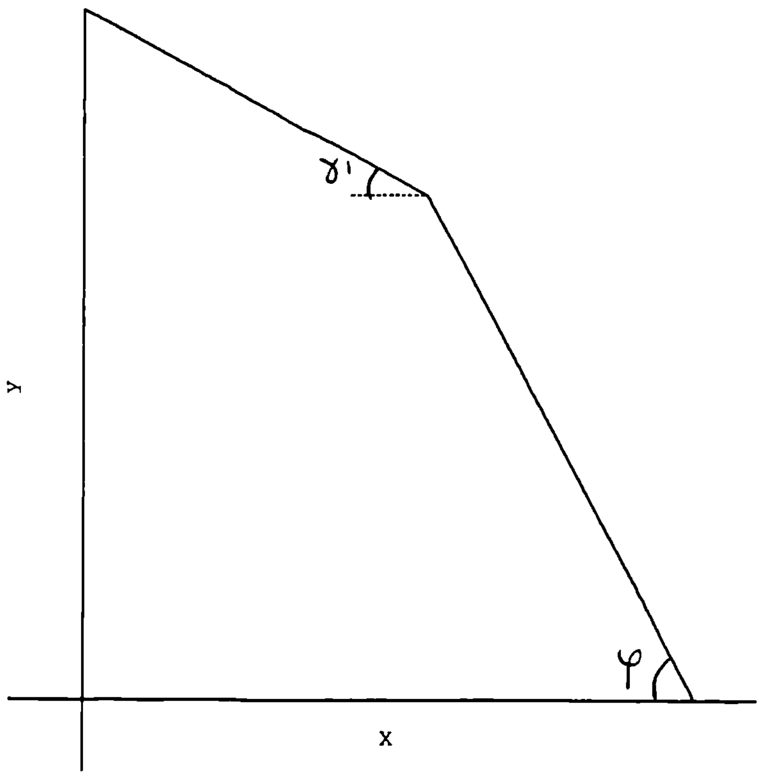

FIGURE FIVE

Central Government's problem is now to find a value for h such that any local authority, even if it is an highest spender[19] will not demand any quantity above α^*. In terms of diagram 5 this means that the tangency between the Indifference Map representing local government behaviour and

[19]Highest spenders are identified with local authorities whose true preferences are equal to β_2.

the budget constraint on the second segment must be for a level of $X^2_{GR} \leq \alpha^*$, i.e. in a non feasible region.

The new system can be implemented by using a two price system offering a subsidy for expenditure up to α^* but introducing a system of penalties in the form of grant withdrawals if local authorities over spend. If government want any local authority to spend exactly α^* the new problem can be formulated as such:

$$\text{Min } (1-g)\ \alpha^* + (1-h)\ (X_{GR} - \alpha^*)$$

$$\text{s.t. } x^1_{GR}\left(\beta_1, M, g\right) \geq \alpha^*$$

$$x^2_{GR}\left(\beta_2, M_2, h\right) \leq \alpha^*$$

x^1_{GR} = demand on the first segment.

x^2_{GR} = demand on the second segment.

$M_2 = M + (h-g)\alpha^*$

is the imputed income on the second segment [20]

The first constraint assures that local authorities with the lowest preferences will spend at least at their target level of expenditure while the new constraint is meant to stick any local authority to spend no more than α^*. This second constraint derives from the studies of utility

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[20]Following Moffit (1986) the kinked budget constraint for each local authority can be written as:
$M = gX + y$ if $X \leq \alpha^*$
$M = g\alpha^* + h(X-\alpha^*) + y$ for $X > \alpha^*$
from which it is possible to derive $M_2 = M + (h-g)\alpha^*$

maximisation subject to piecewise linear constraints. As Hall (1978) showed it is possible to derive the demand equation for a commodity whose price is non linear by maximising utility along the linearized extension of the two segments of the budget constraints. If the budget constraint is convex, the tangency of the Indifference Map with one of the extended segment in a feasible region assures uniqueness of the solution found. The problem and the technical details concerning how to derive a set of demand equations subject to a piecewise linear constraint has been studied at length by Hanoch and Honig (1978) to which the interested reader is referred. In this case Central Government has to set a price such that the quantity demanded by the highest spending authorities on the extended second segment is less than σ^{*}. This will assure that if local authorities with highest spending preferences are constrained to be at the kink, all the other ones will be spending no more than this amount.

This policy can be better illustrated by using the following diagram in which I will assume that the plotted indifference map represents the utility contours of a local authorities with preferences for expenditure $\beta > \beta_1$.

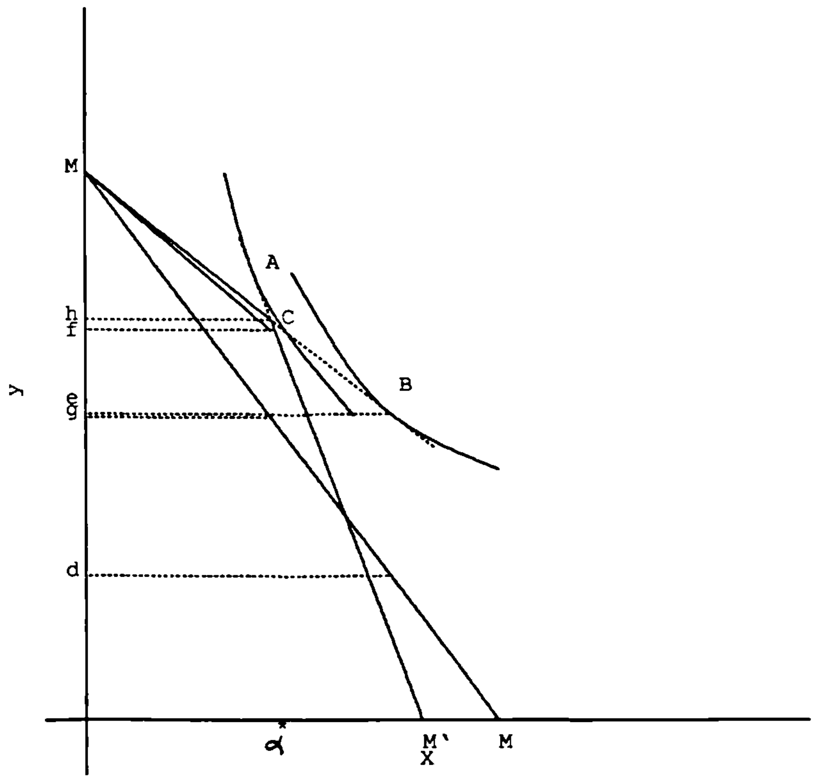

FIGURE SIX

The post-grant budget constraint for this local authority is represented by the line MM' of which the first part MC derives from the budget equation $g_1X + y = M$ and the second part, CM' corresponds to the budget equation $g_2X + y = M_2$

If Central Government was to offer a matching grant equal to $(1-g_1)$ this local authority would have chosen to spend B and local government loss would have been DG+EF. However, due to the introduction of the kink B is not a feasible allocation. On the second segment of the budget, this local authority would have spent A which is not feasible again. The authority will then be located at C, i.e. at the kink and will demand α^* for public expenditure and the loss is reduced to FH. The optimal g will be equal to g_1 and h will be :

$$h = h(\beta_2, M_2, g)$$

β_2 = highest preference

M_2 = income in the second segment.

The system is more efficient since in this case:

$$(1-h) \cdot (X^2_{GR} - \alpha^*) = 0.$$

If Central Government uses this formula it will never be able to learn the true preferences, but it will be able to reduce the loss. By observing local authorities spending at the kink, Central Government cannot infer their preferences, but this information would be of any use in a static model anyway since it could not be used to set up the system in the following period.

3. AN EXAMPLE USING A COBB DOUGLAS.

I will now present an example in which I will assume that local government behaviour can be represented by a Cobb Douglas utility function, namely:

$$\text{Min} \quad (1-g)\,\alpha^* + (1-\ h)\,(X\ -\ \alpha^*) \qquad \text{for} \quad x > \alpha^*$$

$$\text{s.t.} \quad X^1_{GR} = \frac{\beta_1 M}{g} \geq \alpha^*$$

$$X^2_{GR} = \frac{\beta_2 M_2}{h} \leq \alpha^*$$

where the first constraint assures that authorities with the lowest preferences will spend at least α^* and the second constraint means that authorities with the highest preferences will not be able to spend along the second segment of the budget constraint.

Minimization leads to:

$$g = \frac{\beta_1 * M}{\alpha^*}$$

$$h \geq \frac{g_2 - \beta_2 * g}{1 - \beta_2} \qquad g_2 = \frac{\beta_2 * M}{\alpha^*}$$ [21]

The model can again be adapted to cope with the case in which Central Government is interested in expected values as shown in appendix two.

The conclusion from the model I have just presented is that since there is no way for Central Government to know the true preferences for local authorities, the best it can do is to prevent them spending more than α^*.

It is worth noting that in this case Central Government has

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[21] The formula for h can be derived as follows:
From constraint 2 we know that:

$$h \geq \frac{\beta_2 M_2}{\alpha^*}$$

$$M_2 = M + (h-g)\alpha^*$$

then:

$$h \geq \frac{\beta_2 \left(M + (h-g)\alpha^*\right)}{\alpha^*}$$

which rearranged gives the formula in the text.

not actually any true incentive in learning β since this information will be of no use in future periods.

The dynamic game is thus clearly different and, as I shall show it allows Central Government to know the true preferences, at least if the right incentives are used.

4. THE DYNAMIC APPROACH

It is often recognized in economics that decisions taken by rational economic agents are the result of a choice among a set of opportunities taking account of the future implications of those decisions.

Optimization within a static framework seems thus to be very restrictive expecially in the light of the new techniques developed to cope with very complicated economic models. As concerns utility maximisation, for example, the conventional static framework is substituted by the discounted stream of utility which has to be maximised subject to the constraints derived from the present and future resources. Dynamic consistency in agents' behaviour seems to be now recognized and incorporated in most economic models while this aspect seems to be left behind when economists analyse government and its agents' behaviour [22]. I will suggest that local authorities behaves as if they were utility maximiser over the period they are elected. My approach is aimed to be an application to collective organizations behaviour of the

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[22] A new strand of macro economic literature is however already filling this gap by studying time consistent *models* for government intervention. For a review of this models see Stevenson et al. (1988), ch.9.

permanent income and life cycle hypothesis even if, as it will be clear later the modifications to the standard consumer's analysis are quite important. In my analysis I will use the basic assumptions of the *Modigliani Brumberg* model of lifetime consumption and the most important empirical works in this field can be considered the one carried out by Heckman and MaCurdy (1980) MaCurdy (1983) Browning, Irish and Deaton (1985). I will start by showing how a dynamic consistent set of demand equations can be derived for local authorities. The problems to solve are many and the solution seem quite restrictive and highly dependent on the assumptions used to derive it.

The model assumes that the behaviour of an individual authority can be represented by utility maximisation over two commodities, namely its own level of expenditure for local services and a composite private commodity, y. Lifetime preferences in period t might thus be written as:

$$U_t = U(X_t, y_t, \beta)$$

$$\frac{\partial U}{\partial X} > 0 \qquad \frac{\partial^2 U}{\partial X^2} < 0$$

$$\frac{\partial U}{\partial y} > 0 \qquad \frac{\partial^2 U}{\partial y^2} < 0$$

Lifetime utility in any period t may be written as the following discounted sum of a concave, twice differentiable period by period utility indices U_s:

$$W_t = \sum_s \rho^{s-t} U_s(X_s, y_s, \beta)$$

where ρ is the discount factor for utility.

Perfect foresight as well as intertemporal separability is assumed in this first part of the analysis .

Under these assumptions a two stage budgeting procedure can be used and dynamic consistent current demands can be written in terms of a single variable capturing both past decisions and future anticipations

Since the most crucial assumptions in deriving a set of dynamic consistent demand equations are those relating to the characteristics of the authority's utility function, it is important to spend some time in describing them in full detail. As I have pointed out before, intertemporal separability is assumed as well as additivity.

This aspect is very important when the perfect foresight assumption is relaxed and the individual authority is allowed to replan its demand pattern.

In standard life cycle consumption models individuals are assumed to maximise their utility over their life. The question of how long the "life" of a local authority is is nontrivial. It is worth keeping in mind that the model is designed to describe local government behaviour rather than local authority as a set of people. This implies that I have to link the concept of life cycle to the political aspect of local authorities. There are in Britain two basic systems by which local governments bodies are elected:

a) the representatives are elected simultaneously for a period of four years;

b) for each of the first three years 1/3 of the councillors are elected and no one is elected in the fourth.

I will concentrate for the sake of simplicity on examining the optimal behaviour of type a authorities. Is the life cycle of those authorities four years? How have they to be evaluated? Let us suppose that at the end of year T there are elections in authority i. The representative elected will run the authority for four years i.e. until the end of T+4.

T   T+1   T+2   T+3   T+4

However, due to the rules by which the budget is set the local government will have to set the budgets for the period T+1 - T+5.

T   T+1   T+2   T+3   T+4

T+1   T+2   T+3   T+4   T+5

On one hand it is possible to argue that the period to be taken into account is T+1 - T+5 that is the period for which the local authority is setting the budget. This approach does not to seem quite useful since the rule governing budget setting in period T+5 might be quite different and the consistency with the budgets set in the previous years crucially depends on the confidence the leading government has to be reelected.

If we consider the period T - T+4 as the right spell length, the first objection would be that local authority has not got any power in setting the budget for the first period since it has been set by the previous administrators; this argument, however, is not completely valid since, although the decision at the first stage cannot be altered, actual expenditure is still a decision variable for the new administration through the use of rate balances. The argument that the new government is always able to spend less than the budget since it could "save" by increasing the balances while it could find quite difficult to overspend is counterbalanced by the suggestion that using sophisticated creative accounting procedure it is possible to fiddle with numbers. However, it is important to point out that by assigning an arbitrary [23] ceiling to the sum of actual expenditure plus savings in the first period a quite restrictive constraint on the expenditure path through the period could potentially be introduced. Those problems,however are not very important from the theoretical point of view of the model I am going to present since, for reasons that will become clear later, local authorities, if allowed to do so transfer money forward.

Local authorities neither expect to receive nor desire to leave any inheritance. At the end of the period in which they are in charge it is quite reasonable to assume that local administrators will not leave any savings (in the form of rate balances) to the new councillors that will be elected. In order to secure reelection it is plausible to

[23] in the context of this model

think that local government will try to reduce as much as possible taxation in year T+4 since election will take place at the end of it: being able to maintain or increase expenditure without virtually any tax increase might be a determinant factor for securing reelection. The best way to achieve this result seems to be to use rate balances from previous years to finance expenditure in T+4.

4.1 THE MODEL

In this model I will not assume any proper discount rate for utility: social welfare maximisation through the political life of each government should be informed to a long run welfare perspective which suggest that the pseudo utility achieved in each period should have the same weight; however, as I pointed out before local authorities do not have a proper utility function: they behave as if they had one. This might suggest that, from a more political point of view the local government could have an interest in doing its best to maximise utility over the period T+1 - T+4 for which it is able (and has got the responsibility) to set the budget; as concerns the first period it could always blame for any perverse effect (like reduction in expenditure or increase in taxation) the flaws in the previous administration and the lack of accuracy with which they set this budget. I will no turn to present the formal setting of the new model.

In the absence of any grant, the budget constraint for the model can thus be written as:

$$X_t + \Delta BL_t(1+r) + y_t = M_t \qquad t = 1, 4$$

$BL_0 = 0$

$BL_4 = 0$

$BL_{t+1} = BL_t(1+r)$

BL = balances from previous years; this term is equivalent to wealth in a standard life cycle model.

ΔBL = the changes in balances from the previous year. This terms is equivalent to savings in a standard life cycle model.

Assuming a perfect market for borrowing and lending, the budget constraint can be rewritten as:

$$\sum R_t * X_t + \sum R_t * y_t = \sum R_t * M_t$$

$R_t = \dfrac{1}{(1+r)^t}$ i.e. the discount rate that converts money in period t to money in period 0.

To simplify the notation I will define $\sum R_t * M_t = R_v$. This term represents the discounted value of the local income through the life cycle.

At the optimum it must be true that:

$U_1(X_t, y_t, \beta) = \lambda^*$

$U_2(X_t, y_t, \beta) = \lambda^*$

where subscripts denotes partial derivatives and λ^* is the discounted Lagrange multiplier associated with the wealth constraint. According to this equation expenditure is chosen so that the marginal utility of consumption equals the marginal utility of wealth. Using the implicit function theorem it is possible to solve those equations leading to a general demand equation of the form :

$$X_{NG,t} = X\ [R_v, \lambda^*, \beta]$$
$$y_t = y[R_v, \lambda^*, \beta]$$

R_V = discounted sum of resources available to each local authority throughout its life cycle.
The functions X(.,.) and y(.,.) depend only on the functional form of U(.,.).

If a functional form is specified for U it is possible, by substituting back in the budget constraint to obtain the optimal value of λ^* and thus a complete set of equations.
Once again Central Government's objective is to minimize the grant it has to give to any local authority to make it to spend at least α^*_t in each period. To simplify the analysis I will assume that α^*_t as well as income are constant through the time period considered, they are peculiar to each local authority and known with certainty by Central Government.

To start with, I will consider a situation in which no interest rate for money available at different times is served i.e. r = 0 and, as I have pointed out before there is no time discount on utility. The implication of relaxing both assumptions will be examined later. I will also assume that:

$$X_{NG}(\beta_2, R_v, \lambda^*) < \alpha^*_t \qquad t = 1,4$$

i.e. local authorities with the highest preferences are not able to reach the expenditure target relying on their own resources. This assumption rules out the possibility of having negative grants. With the introduction of the grant the budget constraint faced by a representative local authority is modified as follows:

$$\sum R_t * X_t * g_t + \sum R_t * y_t = R_v$$

and the demand equations for both public and private consumption can in general terms be written as:

$$X_{GR,t} = X[R_v, g_t, \lambda^*, \beta]$$
$$y_t = y[R_v, g_t, \lambda^*, \beta]$$

As for the static model, the optimal allocation rule is defined as the set of rules that determines the allocation, the size and the form of the grant that each local authority will receive in the four periods.

In analogy with the results obtained in the previous section, the optimal grant setting in a system characterized by both parties fully informed is again an open ended matching grant for the reasons I explained before.

4.2 THE INCENTIVE COMPATIBLE PROBLEM

When Central Government does not know the preferences for services provided locally the asymmetry of information that characterizes the problem makes local authorities better off by cheating. By reporting a lower β than the true one, they can achieve a greater overall level of utility because they receive a grant which is greater than they one they should be entitled to had they reported their true preferences. In terms of the environment 1 described in section 2.2 assumption iv) is now replaced by:

iv)a β although independent across authorities is fixed through the life cycle of each local authority.

From assumption iv)a it follows that the optimal solution can be examined in a context characterized by Central Government and one local authority and then replicate the optimal solution to the entire population [1].

Central Government can observe local income in each period and it knows α^* but it has to rely on local government as concerns preferences for local public services. The only available information is that the true preference parameter lies in the closed interval $[\beta_1 \;\; \beta_2]$. Before the start of period one Central Government asks local authorities to report their preferences, β_d. From period one onwards it can infer β by observing the expenditure for local public

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[1] It is clear that this does not mean that all authority will receive the same incentive: what it means is that the general formula will be the same for all authorities but the actual value will depend on the parameters characterizing each authority.

services that each local authority has actually chosen.

The main difference with the static model is that in this case Central Government is able to infer from the chosen expenditure level in each period the revealed β and then local authorities'expenditure behaviour must be consistent with the declared β_d if they want to receive each year the same amount of grant. The basic differences between the two models are illustrated in the flow chart diagram in table 1.

TABLE ONE

STATIC MODEL	DYNAMIC MODEL
Period one	Period one

a) Central Government ask local authorities their β
b) on the basis of the reported β it sets up the matching grant
c) local authorities set up their expenditure

STATIC MODEL	DYNAMIC MODEL
Period 2 until period 4	Period 2 until period 4
Start from a) again.	Central Government infers β from c) in period t-1 and sets up the new matching grant accordingly.

In the next sections I will show graphically and analytically the gain a representative local authority might have in cheating in the first three years of its life cycle under two different assumptions, namely:

a) the local authority is constrained to spend all the income and grant in the same period it receives them. (i.e. its budgets has to be balanced each year);

b) it can transfer resources from one period to another one

4.2.1 CASE A) BUDGET BALANCED EVERY YEAR.

The gain from cheating for the case in which local authorities cannot transfer money across time can be quantified by comparing the indirect utility for cheating with the one it would get by declaring its preference truthfully. If our local authority declares β before the start of period one, it would be able to spend α^* and y^* in each period then its indirect utility function would be:

$$V_{(no\ cheat)} = U(\alpha^*, y^*) + U(\alpha^*, y^*) + U(\alpha^*, y^*) + U(\alpha^*, y^*)$$

as illustrated in figure seven.

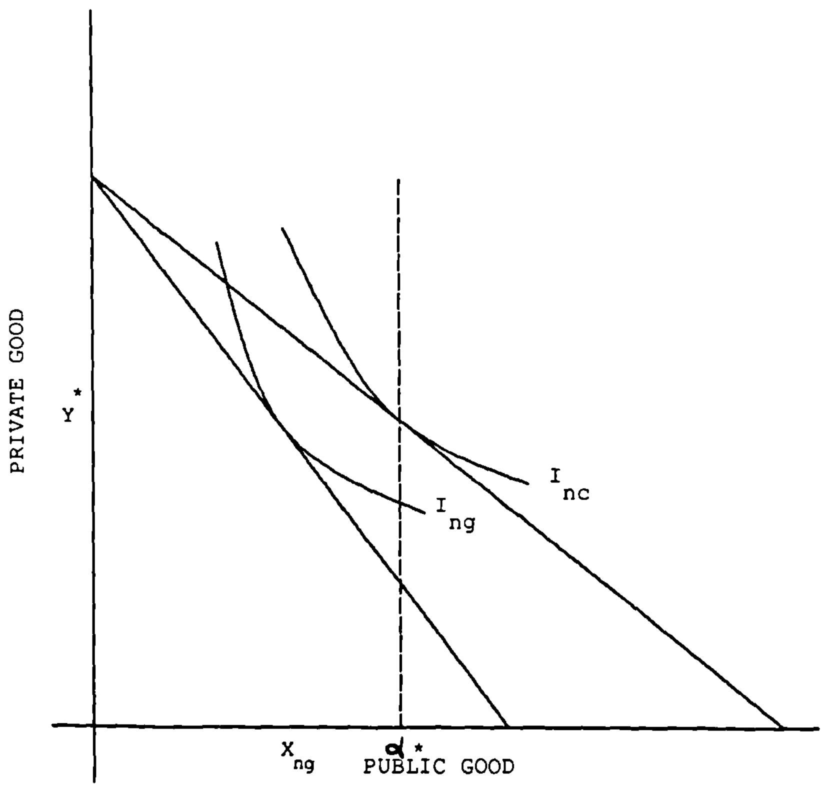


FIGURE SEVEN

The local authority in this case declares β, spends exactly α^* each year and is maximising its utility since the indifference curve is tangential to the budget constraint.

By reporting β_d before the start of period one and being consistent with its cheating in the following periods our local authority will spend α^* on local public goods but, since it is entitled to a higher grant than in the previous case, it will actually be able to spend more on private goods. The indirect utility for this case can be written as:

$$V_{(\text{cheat})} = U(\alpha^*, y^{**}) + U(\alpha^*, y^{**}) + U(\alpha^*, y^{**}) + U(X^*_{GR}, y^*)$$

$y^{**} > y^*$ which by greed implies : $U(\alpha^*, y^{**}) > U(\alpha^*, y^*)$

$X^*_{GR} > \alpha^*$ which again implies : $U(X^*_{GR}, y^*) > U(\alpha^*, y^*)$

This can be illustrated by using the following diagrams:

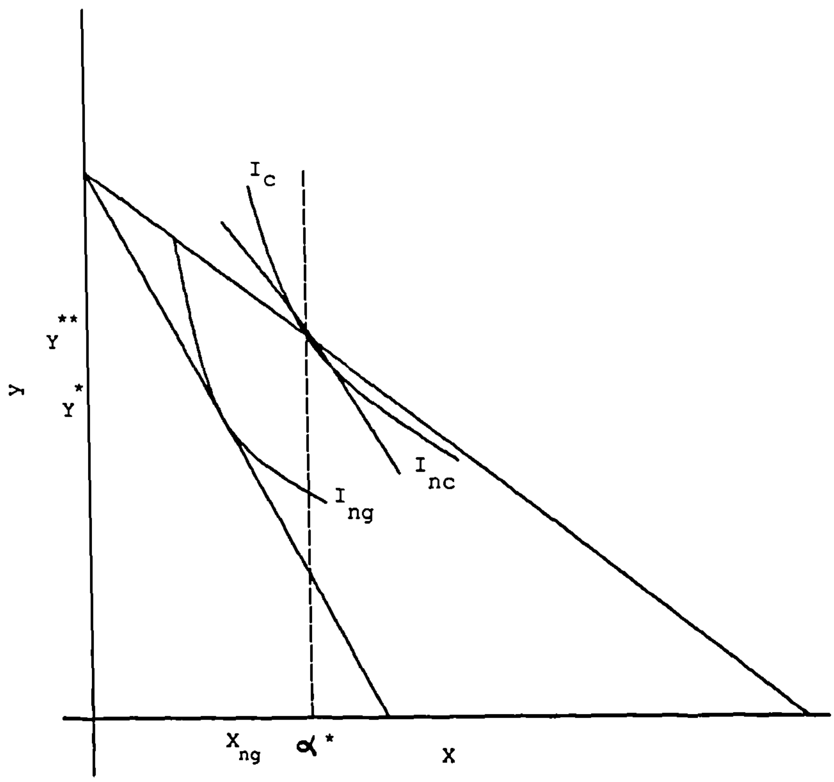


FIGURE EIGHT

In the first three periods the local authority is at an equilibrium point which is not optimum since the utility is not tangential to its budget constraint but it is on a higher indifference curve than the one labelled I_{nc} in figure seven. In period four it will reveal its true preference by choosing to locate at X^*_{GR} $y^*_{,}$ which is both an equilibrium and an optimum point since the Indifference Curve is tangential to the budget constraint. Again, by comparing IC_{ch4} with IC_{nc} the gain in utility is self evident.

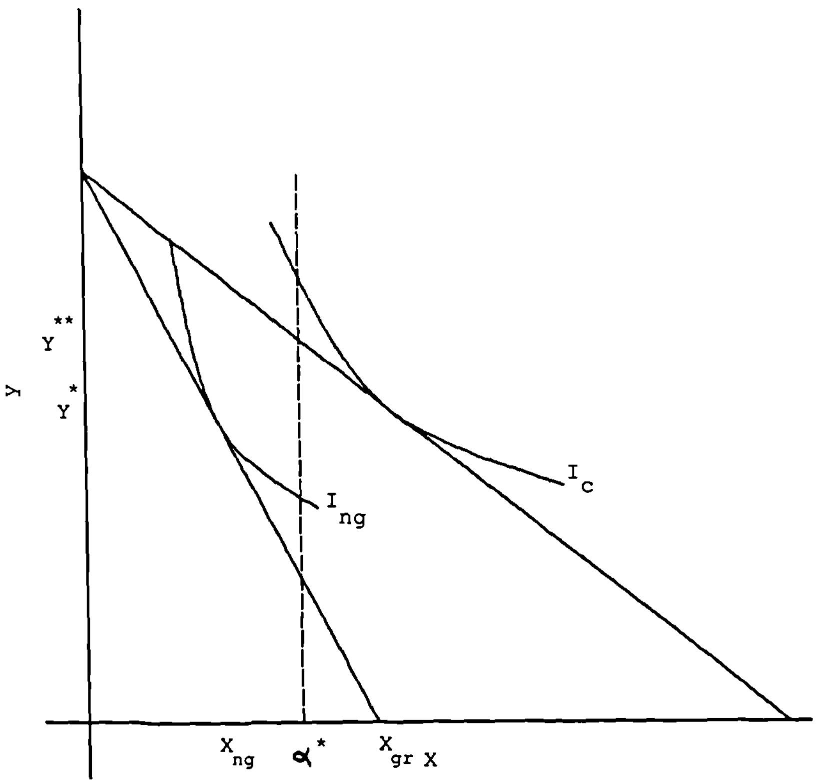


FIGURE NINE

From the following diagrams it is also possible to infer that the cheating does not depend on the assumption of no time preferences on utility: each local authority will be better off by cheating anyway because it is in each period on a higher indifference curve. Local authorities will wait until the last period to reveal their true preferences in order to maximise the benefits they receive by cheating.

Just as an illustration I will compare the indirect utility derived from revealing the true preference in the first period or in the last one. The two indirect utilities can be written as:

$$V_{(p.r.per.\ 1)} = U(X^{*}_{GR} y^{*}) + U(\alpha^{*}, y^{*}) + U(\alpha^{*}, y^{*}) + U(\alpha^{*}, y^{*})$$

$$V_{(p.r.per.4)} = U(\alpha^{*}, y^{**}) + U(\alpha^{*}, y^{**}) + U(\alpha^{*}, y^{**}) + U(X^{*}, y^{*})$$

and the gain is self evident.

4.2.2 CASE B) RESOURCES CAN BE TRANSFERRED ACROSS TIME

If local authorities can "save up" resources by reducing their consumption of private goods in the first three periods they will use part of these additional resources to finance more expenditure in the last one in which they will again reveal their preferences. This procedure is a form of "creative accounting" behaviour. I will explain this feature at length later when the basic model will be dealing with expenditure and taxation as decision variables.

It is again possible to see the gain local authorities have in cheating by comparing the indirect utility derived from cheating with the one obtained by truthfully reveal their preferences. Let consider again a representative authority. If it reports its true preference, it would be entitled to spend α^{*} and y^{*} in each period then its indirect utility function would be:

$$V_{(no\ cheat)} = U(\alpha^{\star}, y^{\star}) + U(\alpha^{\star}, y^{\star}) + U(\alpha^{\star}, y^{\star}) + U(\alpha^{\star}, y^{\star})$$

By cheating on the β it reports, our local authority will spend $\alpha^{\star}$ on local public goods but, since it is entitled to a higher grant, it will actually be able to spend more on private goods and to save resources for the last period.
The indirect utility for this case can be written as:

$$V_{(cheat)} = U(\alpha^{\star}, y) + U(\alpha^{\star}, y) + U(\alpha^{\star}, y) + U(X^{\star\star}, y)$$

$y^{\star} < y_i$ which implies: $U(\alpha^{\star}, y_i) > U(\alpha^{\star}, y^{\star})$;i=1,4

$X^{\star\star} > \alpha^{\star}$ which again implies : $U(X^{\star\star}, y_i) > U(\alpha^{\star}, y^{\star})$

This can be illustrated by using the following set of diagrams:

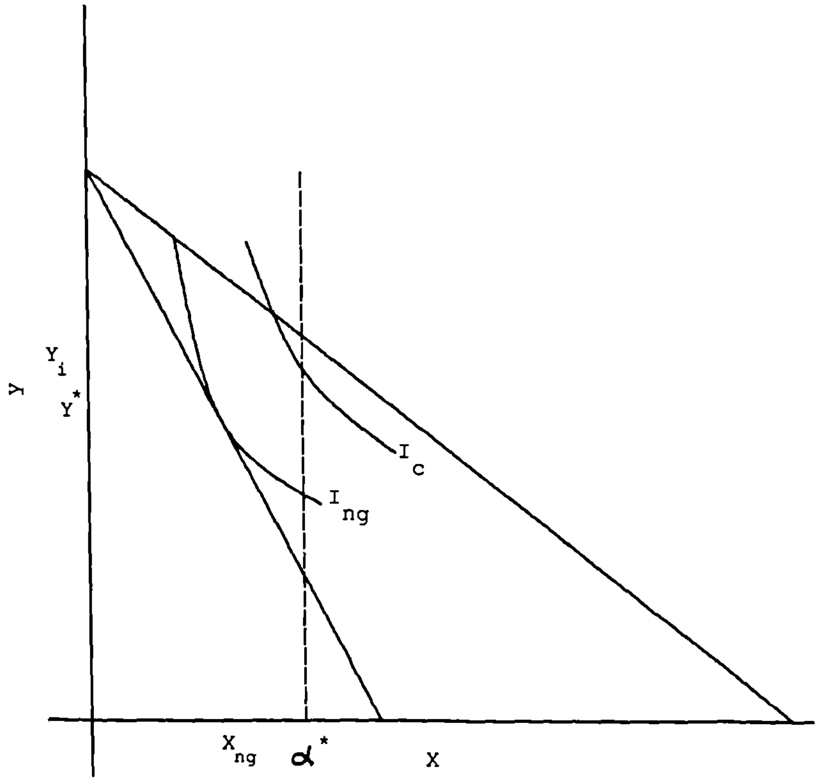
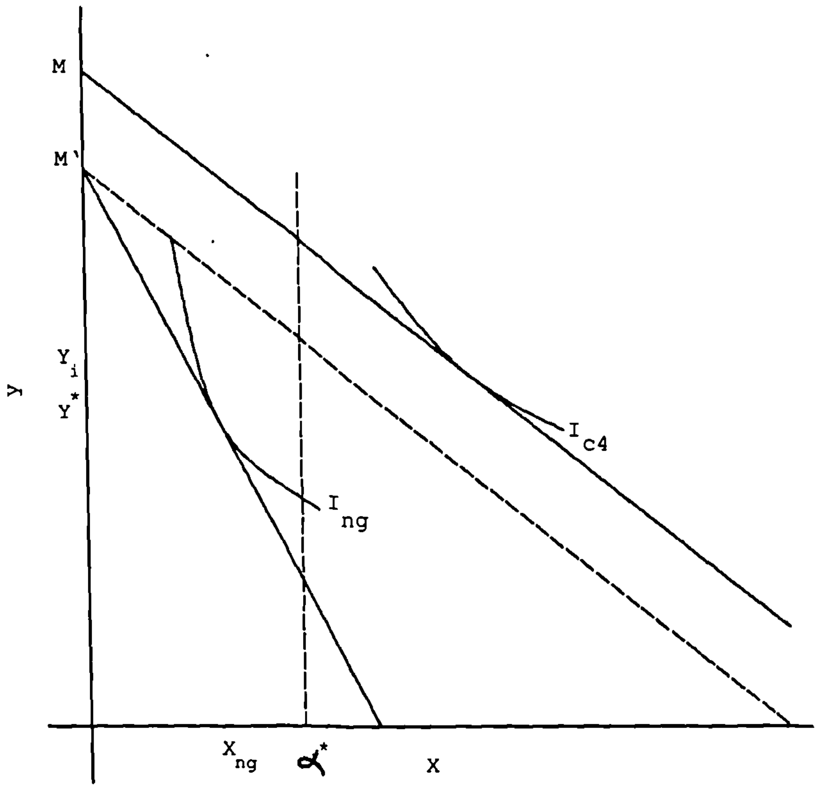


FIGURE TEN

In the first three periods, the local authority is not at an equilibrium position since the indifference curve it locates is not on the budget constraint. This policy, however allows it to save up resources for the last period as diagram eleven shows:



$$M'M = \sum_{1}^{3} (y^{**} - y_i)$$

FIGURE ELEVEN

In period four our local authority's budget constraint can be analytically written as:

$$g_4 X_4 + y_4 = M + \sum_{i=1}^{3} (y^{**} - y_i)$$

where $\Sigma(.)$ represent the savings from previous periods. As a result of its cheating then the budget constraint for our local authority in the last periods shift upwards and the price it faces is less than it should have been had it revealed its true preference in the previous periods. Since after period four local authority's decision making body will be reelected, it has an incentive to reveal its true preference since this information would only be partly used from Central Government to infer the preferences for future periods and this consideration might not affect local government decision body because in this model I do not take account of the possibility that the same body will be reelected, then utility is maximised within the period in which it is in charge of government.

In period four, then our local authority choose an optimum point and it uses the savings from previous years to shift its expenditure for public goods from α^* to X^{**}. The parallel shift in the budget constraint represents the amount of resources saved up in the first three periods, i.e. $\sum_{i=1}^{3}(y^{**} - y_i)$. As a result of this policy Central Government is worse off in this case because the matching grant has to be provided for more unit of public goods.

To avoid the cheating Central Government has to devise a system of incentives such that local authorities will be better off by revealing their true preferences. The form of incentive received will be a function of the preferences that local authorities declares to Central Government before the start of period one and I will assume that, as a result of the incentive scheme expenditure on local services is boosted up in each period by the amount:

$$\alpha_t^* \ \theta_t^* \qquad\qquad t = 1 \ , 4$$

where θ_t^* represent the optimal incentive that avoid cheating. The incentive can be introduced in many ways.I have chosen a multiplicative form because the actual grant system relates measures of needs, target expenditures and kinks by a scaling factor [25]. The problem faced by Central Government is then to minimize the grant to be given to each local authority with the double constraint of having them spending at least α^* in local public services and avoid their cheating. The problem can then be formalized as follows:

$$\underset{\theta_t g_t}{\text{Min}} \sum_{t=1}^{4} \int g_t \theta_t(\beta_d) \alpha_t^* f(\beta_d)\, d\beta_d$$

that can be rewritten as :

$$\underset{\theta_t g_t}{\text{Min}} \sum_{t=1}^{4} g_t \theta_t(\beta_d) \alpha_t^*$$

$$\begin{aligned} \text{s.t.} V_1(\theta_1(\beta), g_1(\beta), M, \beta) &+ V_2(\theta_2(\beta), g_2(\beta), M, \beta) + \\ V_3(\theta_3(\beta), g_3(\beta), M, \beta) &+ V_4(\theta_4(\beta), g_4(\beta), M, \beta) \geq \\ V_1(\theta_1(\beta_d), g_1(\beta_d), M, \beta) &+ V_2(\theta_2(\beta_d), g_2(\beta_d), M, \beta) \\ + V_3(\theta_3(\beta_d), g_3(\beta_d), M, \beta) &+ V_4(\theta_4(\beta_d), g_4(\beta_d), M, \beta) \end{aligned}$$

$$\text{s.t. } x(\beta_d, R_v, g_t) \geq \alpha^* \theta_t$$

The problem can be solved using a standard approach to constrained maximisation and the optimal set of grant allocation rules is determined by the specific form of the

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[25] In the real world, in fact, the threshold level of expenditure for a representative authority i, which should represent $\alpha^* \theta_t = GREA_i * K_i$.

utility function local governments are assumed to maximise. From constraint 2) it is possible to not that the problem can solved for just one decision variable. For example if the problem is solved with respect to θ, it is possible to observe that, since the incentive will raise the size of the grant the incentive compatible constraint will be binding then:

a) $\theta^{*}|_{\beta_1} = 1$

b) $\dfrac{\partial \theta_t}{\partial \beta_d} > 0$ at least for some t

Condition a) means that no incentive is required if the authority's true preference is the lowest value in the range while condition b) means that, in order to make an authority whose preference for expenditure is higher than β_1 reveal their true parameter the incentive itself has to be an increasing function of the preference parameter declared.
For the rest of the chapter the analysis will be conducted assuming that local authorities behaviour can be approximated by a Cobb Douglas utility function and the optimal allocation set of rules will be shown under different assumptions.

5. THE DYNAMIC OPTIMAL GRANT RULES : A COBB DOUGLAS EXAMPLE.

In analogy with the more general case presented above I will show first of all the optimal grant allocation rule for the case in which both parties are perfectly informed and I will then present the system of incentives necessary to make local authorities reveal truthfully their preferences in a world characterized by asymmetry of information.

5.1 OPTIMAL GRANT FORMULA IN PERFECT INFORMATION.

In the absence of any grant local government's demand for local services might be derived by maximising its utility function subject to the budget constraint. This can be formulated as:

$$\text{Max} \sum (1-\beta)\ \ell n\ y_t + \beta\ \ell n\ X_t \qquad\qquad t = 1,4$$

$$\text{s.t.} \sum \left[X_t + y_t\right] = \sum M_t$$

In order to simplify the analysis I will assume that M, as well as α^* is constant through time. By using a standard Lagrangean approach it is possible to derive the following set of demand equations:[26]

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[26]under the further assumption that M is constant through time.The same result would in this particular case be obtained also by assuming that local authorities cannot transfer money from one period to another one. If M is not constant through time and local authorities can transfer money from different periods the two demands equations have to be rewritten as:

$$X_t = \frac{\beta\ \Sigma M_t}{4} \quad \text{and} \quad y_t = \frac{(1-\beta)\ \Sigma M_t}{4}$$

while demands if the budget has to be balanced each period do not change.

$$X_{NG,t} = \beta M$$
$$y_t = (1-\beta) M$$

In analogy with the models I presented before, I will assume that $X_{NG,t} = \beta_2 M < \alpha^*$ then Central Government has to offer a subsidy to all local authorities. Again, the form of subsidy can either be a lump-sum grant which will increase local disposable income, M, or a matching grant whose effect is to reduce the price that local authorities have to face for expenditure. For the reasons explained in the previous sections it can be shown that a matching grant is more effective than a lump-sum grant to reach the scope of making local authorities to increase their expenditure. When the matching grant is introduced the budget constraint for local authorities can be written as:

$$\sum \left[g_t X_t + y_t\right] = \sum M$$

and the demand equations are:

$$X_t = (\beta M)/g_t$$
$$y_t = (1-\beta) M$$

In this environment Central Government problem is to find the optimal grant allocation form that:

$$\text{MAX} - \sum(1-g_t) * \alpha^*$$

$$\text{s.t.} - \left[\beta \frac{M}{g_t} \right] \leq - \alpha^*$$

The optimal solution is then:

$$g_t = \frac{\beta * M}{\alpha^*}$$

5.2 THE INCENTIVE COMPATIBLE SOLUTION.

In analogy with the previous models I will now assume that Central Government does not know the preferences for local expenditure. The environment is still represented by the assumptions stated in section four. Central Government's objective is still represented by the minimization of the sum of money it has to give to local authorities in order to make them spend at least α^* in each period. In this case one of the policies that Central Government could follow would be to set a system of matching grant in an open loop policy. Before the start of period one, since no further information is available, Central Government asks local authorities to declare their preference parameter. From period 1 onwards the level of matching grant $(1-g_t)$ can be set according to the previous level of expenditure. If local authorities behave according to a Cobb Douglas utility function, then:

$$X_t = \beta \frac{M}{g_t}$$

By observing X_t and assuming that local authorities behaviour can be represented by a Cobb Douglas Central

Government can infer:

$$\beta = \frac{X_t\ M}{g_t}$$

If local authorities report $\beta_d < \beta$, they will receive a unit matching grant of the form:

$$(1 - g_t) = \left(1 - \frac{\beta_d * M}{\alpha^*}\right)$$

in every period, but they will be constrained to spend no more than $\beta_d \frac{M}{g_t}$ in local public services in the first three periods [27]. The final remark to make is that local authorities, if they decide to cheat they must be consistent in the first three period by declaring the same β_d. As I have explained before, local authorities are supposed to maximise their utility function through time and preferences are assumed to be constant, then a change in β can only be interpreted as a signal that they are cheating. I will first examine the case in which it is not possible to transfer money from one period to the following one.

5.2.1 CASE A: BUDGET BALANCED EVERY YEAR.

Under this assumption the demand equations for the private commodity and the public one can be written as follows:

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[27]This is the result of the assumption that Central Government can infer after the first period local government's preference by observing its actual expenditure.

$$X_t = \beta_d \frac{M}{g_t} = \alpha^* \qquad t=1,3 \qquad g_t = \frac{\beta_d M}{\alpha^*}$$

$$X_4 = \beta \frac{M}{g_t} = \alpha^* \frac{\beta}{\beta_d} > \alpha^* \qquad g_4 = \frac{\beta_d M}{\alpha^*}$$

$$y_t = (1 - \beta_d) M \qquad t = 1,3$$

$$y_4 = (1 - \beta) M$$

The optimal level of cheating might be found by maximising local authorities' indirect utility function with respect to β_d.

$$\begin{aligned} \max_{\beta_d} V_a(M,g,\beta_d) = \; & (1-\beta)\, \ln M(1-\beta_d) + \beta \ln \alpha^* + \\ & (1-\beta)\, \ln M(1-\beta_d) + \beta \ln \alpha^* + \\ & (1-\beta)\, \ln M(1-\beta_d) + \beta \ln \alpha^* + \\ & (1-\beta)\, \ln M(1-\beta) + \beta \ln \alpha^* \frac{\beta}{\beta_d} \end{aligned}$$

s.t. $\beta_d \geq \beta_1$

The problem can be solved using Kuhn Tucker conditions:

$$\beta_d \geq \beta_1$$

$$\frac{\partial V}{\partial \beta_d} = -3\left(\frac{1-\beta}{1-\beta_d}\right) - \frac{\beta}{\beta_d} \leq 0$$

$$\beta_d \left(\frac{\partial V}{\partial \beta_d}\right) = 0$$

Since $0 < \beta < 1$, this derivative is always negative, then it follows that the constraint is binding and:

$$\beta_d = \beta_1$$

In order to make local authorities reveal their true preferences Central Government has to devise an incentive scheme. I will consider here an open loop policy by which central government, before of the start of period 1 offers local government the following matching grant:

$$(1 - g_t) = \left(1 - \frac{\beta_d * M}{\alpha_t^* \theta_t(\beta_d)}\right) t=1,4$$

such that local authorities are allowed to spend $X = \alpha^* \theta_t(\beta_d)$ and $y = M(1-\beta_d)$ in each period. If the form of incentives is chosen optimally, local authorities will not cheat and their indirect utility function could be written as:

$$\begin{aligned} V_b(M,g(\theta^*),\beta) = (1-\beta\ ) \ln M(1-\beta\ ) \quad &+ \quad \beta \ln \alpha^* \theta_1^* \quad + \\ (1-\beta\ ) \ln M(1-\beta\ ) \quad &+ \quad \beta \ln \alpha^* \theta_2^* \quad + \\ (1-\beta\ ) \ln M(1-\beta\ ) \quad &+ \quad \beta \ln \alpha^* \theta_3^* \quad + \\ (1-\beta\ ) \ln M(1-\beta\ ) \quad &+ \quad \beta \ln \alpha^* \theta_4^* \end{aligned}$$

where θ_t^* denotes the optimal incentive to be given to local authorities at time t in order to avoid their cheating.

Central Government's problem is to find the minimum amount of grant to give to each local authority. The total grant can be written as:

$$\sum_{i=1}^{4} \left(1 - \frac{\beta_d * M}{\alpha_t^* \theta_t(\beta_d)}\right) * \alpha_t^* \theta_t(\beta_d)$$

then, in analogy with the more general case presented in the previous section, the problem can be written as:

$$\min_{\theta_t} \sum_{t=1}^{4} \alpha^* \theta_t(\beta_d) - \beta_d M$$

s.t. $\theta_{i|\beta_1} = 1$

The second constraint in the problem just states the initial condition for the optimal incentive: the matching grant rate set up at the start of this section assures that local authorities with preferences equal to β_1 will spend α^*, then no further incentive is needed for them to reveal their preferences. If $\theta_t(\beta_d)$ is chosen optimally, it must be true that [28]:

$$\beta \sum \ln \theta_t^* \geq \beta \ln \frac{\beta}{\beta_1} + 3(1-\beta) \ln \left\{ \frac{1-\beta_1}{1-\beta} \right\}$$

This formula has a clear interpretation: the incentive to be offered to local authorities must give to them the same amount of extra utility they would have received by cheating. This is in fact the interpretation that can be given to the right hand side of the expression where the first term represents the extra utility local authorities enjoy by spending more in period four and the second term is the extra utility they enjoy by increasing the private

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[28]This result can easily be obtained by observing that if local authorities cheat they would chose β_1 in V_a. Then it is just a matter of evaluate $V_b - V_a$.

that if a local authority has preference equal β_1 θ must be equal to one, i.e. no incentive is need to make them spend at least α^* then the second constraint is redundant

Central Government's problem can thus be rewritten as

$$\underset{\theta_t(\beta_d)}{\mathrm{Min}} \quad \sum_{t=1}^{4} \left(\alpha^* \theta_t(\beta_d) - \beta_d M\right)$$

$$\text{s.t. } \beta \sum \ell n\, \theta_t^* \geq \beta \ell n \frac{\beta}{\beta_1} + 3(1-\beta) \quad \ell n \left(\frac{1-\beta_1}{1-\beta}\right)$$

This problem can be solved by using the standard Arrow - Entoven approach to maximisation. The first order conditions are:

$$\frac{\partial L}{\partial \theta_t} = -\alpha^* + \lambda / \theta_t \leq 0 \qquad\qquad t=1,4$$

$$\frac{\partial L}{\partial \lambda} = \sum \ell n\, \theta_t^*(\beta_d) \quad - \quad \ell n \frac{\beta}{\beta_1} - 3\frac{1-\beta}{\beta} \quad \ell n \left(\frac{1-\beta_1}{1-\beta}\right)$$

From the first set of constraints λ should be positive then the constraint is binding. From this set of constraints it is also clear that $\theta_1=\theta_2=\theta_3=\theta_4$ then the optimal solution, due to the equivalence between the declared β and the true one assured by the optimization of the previous problem can be written as function of β as follows:

$$\theta_t^* = \left(\frac{\beta}{\beta_1}\right)^{\frac{1}{4}} \cdot \left(\frac{1-\beta_1}{1-\beta}\right)^{\frac{3}{4}\left(\frac{1-\beta}{\beta}\right)} \qquad\qquad t = 1,4$$

The solution for this problem is rather intuitive. The constraint implies that the sum of the rate of growth of the incentives is linear and constant which implies that the functional form for the incentives should be log linear [29].
It is then clear that the best way of minimizing the incentive to be given to local authorities is to choose a common incentive for any period.

5.2.2 CASE B: MONEY CAN BE TRANSFERRED ACROSS TIME.

In this second case I will allow local authorities to transfer money from the different periods of their lifetime.
By using the same line of argument I explained for the case in which the budget had to be balanced each year a representative local authority will spend:

$$X_t = \beta_d \frac{M}{g_t} = \alpha^* \qquad t = 1,3 \qquad g_t = \frac{\beta_d M}{\alpha^*}$$

in the first three periods. However, due to the possibility of transferring money across time this local authority is no longer constrained to spend $y_t = (1 - \beta_d)\, M \qquad t = 1,3$ in private commodities.
The demand equations for both X_4 and y can be derived by solving the following problem:

$$\text{Max} \left[\sum (1-\beta)\, \ell n\, y_t\right] + \beta\, \ell n\, X_4 + 3\, \beta\, \ell n\, \alpha^*$$

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[29] β is assumed to be a given parameter determined outside the model.

$$\text{s.t.} \quad \sum \left[g_t X_t + y_t\right] = \sum M_t$$

By having declared β_d the grant they receive in the first three periods is equal to:

$$\left(1 - \frac{\beta_d * M}{\alpha^*}\right) * \alpha^* = \alpha^* - \beta_d M$$

while the price to pay in period four will be:

$$g_4 = \frac{\beta_d * M}{\alpha^*}$$

The amount spent in the first three periods for local public goods is equal to:

$$\sum g_t \alpha_t^* = 3 \beta_d M \qquad t = 1,3$$

then the budget constraint can be written as:

$$g_4 X_4 + \sum y_t = 4(M - 3\beta_d)$$

Maximising the utility function for X_4 and y_t gives:

$$X_4 = \frac{M(4-3\beta_d)\ \beta}{(4 - 3\beta)\ g_4} = \frac{(4-3\beta_d)\ \beta}{(4 - 3\beta)\ \beta_d} * \alpha^*$$

$$y_t = \frac{M(4 - 3\beta_d)}{4 - 3\beta} * (1 - \beta) \qquad t = 1, 4$$

the latter set of demand equation have been derived such that the process of maximisation leads to equalizing the marginal utility of money and then to spread resources evenly among periods on the unconstrained good. It is worth noting that the level of X_t demanded is greater than in the case in which local authorities do not cheat. This is the result of the income effect produced by the possibility of saving resources. The optimal level of cheating might be found by maximising local authorities' indirect utility function with respect to β_d.

$$\max_{\beta_d} V_a(M,g,\beta_d)=(1-\beta\ )\ \ln \frac{M(4\ -\ 3\beta_d)}{4\ -\ 3\beta} \star\ (1-\ \beta)+\ \ \beta\ \ln\ \alpha^{\star}\ \ +$$

$$(1-\beta\ )\ \ln \frac{M(4\ -\ 3\beta_d)}{4\ -\ 3\beta} \star\ (1-\ \beta)+\ \ \beta\ \ln\ \alpha^{\star}\ \ +$$

$$(1-\beta\ )\ \ln \frac{M(4\ -\ 3\beta_d)}{4\ -\ 3\beta} \star\ (1-\ \beta)+\ \ \beta\ \ln\ \alpha^{\star}\ \ +$$

$$(1-\beta\ )\ \ln \frac{M(4-3\beta_d)}{4\ -\ 3\beta} \star (1-\beta)\ +\ \beta\ \ln \frac{(4-3\beta_d)}{4\ -\ 3\beta} \star \frac{\beta}{\beta_d} \star\alpha^{\star}$$

s.t. $\beta_d \geq \beta_1$

The problem can be solved using Kuhn Tucker conditions:

$$\frac{\partial V}{\partial \beta_d} = -\ 4 \left(\frac{3(1\ \ -\ \beta\ )}{4\ -\ 3\beta_d}\right) - \frac{3\beta}{4\ -\ 3\beta_d} - \frac{\beta}{\beta_d} \leq 0$$

$$\beta_d \geq \beta_1$$

$$\beta_d \left(\frac{\partial V}{\partial \beta_d}\right) = 0$$

since the derivative is always negative, the constraint is binding then:

$$\beta_d = \beta_1$$

In order to make each local authority reveal its true preference Central Government has then to offer a form of incentive to local government. I will here consider again the policy of offering local governments a matching grant of the form:

$$(1 - g_t) = \left(1 - \frac{\beta_d * M}{\alpha^* \theta_t(\beta_d)}\right)$$

such that local authorities are allowed to spend $\alpha^* \theta_t(\beta_d)$ in each period. If the form of incentives is chosen optimally, local authorities will not cheat and their indirect utility function could, in analogy with the previous case, be written as:

$$\begin{aligned} V_b(M, g(\theta^*), \beta) = {} & (1-\beta)\, \ell n\ (1-\beta) + \beta\, \ell n\, \alpha^* \theta_1^* + \\ & (1-\beta)\, \ell n\ (1-\beta) + \beta\, \ell n\, \alpha^* \theta_2^* + \\ & (1-\beta)\, \ell n\ (1-\beta) + \beta\, \ell n\, \alpha^* \theta_3^* + \\ & (1-\beta)\, \ell n\ (1-\beta) + \beta\, \ell n\, \alpha^* \theta_4^* \end{aligned}$$

where θ_t^* denotes the optimal incentive to be given to local authorities at time t in order to avoid their cheating.

Central Government's problem can thus be written as:

$$\underset{\theta_t}{\text{Min}} \sum_{t=1}^{4} \left(\alpha^{*}\theta_t^{*} - \beta_d M\right)$$

$$\text{st. } V_b(M,g(\theta^{*}),\beta) \geq V_a(M,g,\beta_d) \qquad \text{all } \beta \in [\beta_1,\beta_2]$$

If $\theta_t(\beta_d)$ is chosen optimally, it must be true that [30] :

$$\beta\sum \ln \theta_t^{*} = \beta\ln \frac{\beta}{\beta_1} + 4(1-\beta) \ln \left(\frac{4 -3\beta_1}{4 -3\beta}\right)$$

which has the same interpretation as the analogous formula I presented in section 5.2.1.

Central Government's problem can thus be written as:

$$\underset{\theta_t(\beta_d)}{\text{Min}} \sum_{t=1}^{4} \left(\alpha^{*}\theta_t(\beta_d) - \beta_d M\right)$$

$$\text{s.t. } \beta\sum \ln \theta_t^{*} = \beta\ln \frac{\beta}{\beta_1} + 4(1-\beta) \ln \left(\frac{4 -3\beta_1}{4 -3\beta}\right)$$

This problem can be solved by using the standard Arrow - Entoven method. The first order conditions can be written

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[30]This result can easily be obtained by observing that if local authorities cheat they would chose β_1 in V_a. Then it is just a matter of evaluate $V_b - V_a$.

$$\frac{\partial L}{\partial \theta_t} = - \alpha^* + \lambda / \theta_t \leq 0 \qquad t=1,4$$

$$\frac{\partial L}{\partial \lambda} = \beta \sum \ln \theta_t^* = \beta \ln \frac{\beta}{\beta_1} + 4(1-\beta) \ln \left(\frac{4 - 3\beta_1}{4 - 3\beta} \right)$$

From the first set of constraints λ should be positive then the constraint is binding. From this set of constraint it is also clear that $\theta_1=\theta_2=\theta_3=\theta_4$ then the optimal solution, due to the equivalence between the declared β and the true one assured by the optimization of the previous problem can be written as function of β as follows:

$$\theta_t(\beta) = \left(\frac{\beta}{\beta_1} \right)^{\frac{1}{4}} \cdot \left(\frac{4 - 3\beta_1}{4 - 3\beta} \right)^{\left(\frac{1 - \beta}{\beta} \right)} \qquad t = 1,4$$

It is worth noting that in this case the incentive to give to this representative local authority to make sure it does not cheat is greater than in the case in which the budget had to be balanced every year, as we would expect.

6. A TWO PRICES SYSTEM FOR THE GRANT ALLOCATION RULE.

In the previous system the loss suffered by Central Government for its inability to know local government's preferences is equal to:

$\alpha^{*}(\theta_t - 1)$ [31]

I will now slightly modify the system in order to make Central Government learn the true preference parameter for each local authority by using a lower incentive scheme which reduces the amount Central Government has to pay to learn β.The new system foresees a set of rules by which local authorities will never be able to spend more than α^{*} [32] in each period. The implementation I will present requires the introduction of penalties in the form of grant withdrawals for expenditure in excess of the desired level. As result the system of incentive can be reduced. Each year is characterized by a double price system such that for expenditure up to α^{*} the matching rate is:

$$(1 - g_t) = \left(1 - \frac{\beta_d * M}{\alpha^{*}}\right)$$

For expenditure in excess of α^{*} the matching rate will be:

$$(1 - g_{t,2})$$

where $g_{t,2}$ is the price level assuring that any authority,

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[31] This is the loss for each period and for a representative authority. The total loss suffered by Central Government will then be:

$$\sum_{t=1}^{4} \sum_{j=1}^{n} \alpha^{*}_{tj} \; (\theta_{tj} - 1)$$

[32] The actual level of expenditure at the kink is, for the reasons I will show later, $\alpha^{*}\Omega_t$.

irrespective of its true preferences will not spend more than α^*.

I will start as usual by examining the optimal level of cheating by local authorities in this case.

I will formally present only the model based on the assumption that local authorities are allowed to transfer money from the different periods of their lifetime [33]. Let start by finding the penalty for the last period. I will assume that a representative local authority has declared β_d before the start of period one. The demand equations for the private and public goods be written as follows:

$$X_t = \beta_d \frac{M}{g_t} = \alpha^* \qquad t=1,3 \qquad g_t = \frac{\beta_d M}{\alpha^*}$$

$$X_4 = \alpha^* \qquad\qquad g_{4,1} = \frac{\beta_d M}{\alpha^*}$$

$$g_{4,2} = ?$$

$$y_t = (1 - \beta_d)\, M \qquad t = 1,4$$

In order to achieve this result Central Government has to find a set of prices to be applied on expenditure exceeding α^* such that the budget constraint of each local

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[33] Since the incentive system would be the same in both cases I have preferred to recall here only the most general. If local authorities could not transfer money across time there would be no need of increasing the severity of penalties through time.

authority will look like figure six . The grant a local authority receives for expenditure up to α^* is equal to:

$$\left(1 - \frac{\beta_d * M}{\alpha^*}\right) * \alpha^* = \alpha^* - \beta_d M$$

such that the extended intercept of the budget constraint on the second segment is:

$$M(4-3\beta_d) + \alpha^*\left(g_{4,2} - \frac{\beta_d M}{\alpha^*}\right) \quad 34$$

The demand function for the problem without constraint on how much to spend on the last period was:

$$X_4 = \frac{M(4 - 3\beta_d)}{4 - 3\beta} * \frac{\beta}{g_4}$$

then Central government has to find a $g_{4,2}$ such that:

$$\frac{M(4-3\beta_d) + \alpha^*\left(g_{4,2} - \frac{\beta_d M}{\alpha^*}\right)}{4 - 3\beta} * \frac{\beta}{g_{4,2}} \leq \alpha^*$$

$$g_{4,2} \geq M(1 - \beta_d) * \frac{\beta}{\alpha^*(1 - \beta)}$$

Since Central Government does not know β, the best it can do is fix a price so high that also the highest spenders

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[34]The derivation of this formula is analogous to the one presented for the static model, the only difference is represented by savings from previous periods.

will be stuck at the kink. In this case β has clearly to be chosen equal to β_2. A penalty system in the last period is not sufficient alone to achieve the result of having local authorities spending α^* in each period.If the two price system is introduced on expenditure only in the last period, local authorities could be better off by revealing their true preferences in the previous periods. In this case their expenditure decisions could be summarised as follows:

$$X_t = \beta_d \frac{M}{g_2} = \alpha^* \qquad t = 1,2 \qquad g_t = \frac{\beta_d M}{\alpha^*}$$

$$X_3 = ? \qquad\qquad g_3 = \frac{\beta_d M}{\alpha^*}$$

$$X_4 = \alpha^* \qquad g_{4,1} = \frac{\beta M}{\alpha^*} \qquad g_{4,2} \geq M(1 - \beta_d) * \frac{\beta}{\alpha^*(1 - \beta)}$$

X_3 can be found by maximising:

$$\text{Max} \sum (1-\beta)\, \ell n\, y_t + \beta\, \ell n\, X_3$$

$$\text{s.t.} \sum g_t X_t + y_t = \sum M$$

By having declared β_d the grant they receive in the first two periods is equal to:

$$\left(1 - \frac{\beta_d * M}{\alpha^*}\right) * \alpha^* = \alpha^* - \beta_d M$$

while the fourth period will be $(\alpha^* - \beta M)$.

The amount spent for local public in the first two periods and in period four is equal to:

$$\sum_{t=1}^{2} g_t \alpha_t^* + g_4 \alpha^* \quad = M\ (2\beta_d + \beta)$$

the budget constraint can then be rewritten as:

$$g_3 X_3 + \sum y_t = 4(M - 2\beta_d - \beta)$$

then :

$$X_3 = \frac{M(4 - 2\beta_d - \beta)}{4 - 3\beta} * \frac{\beta}{\beta_d} * \alpha^*$$

$$y_t = \frac{M(4 - 2\beta_d - \beta)}{4 - 3\beta} * (1 - \beta) \quad t = 1,\ 4$$

If local authorities were allowed to do so, Central Government will have a loss in period 3. However it is again possible to design a two price system also for this period in order to make them spend α^*. The intercept on the extended budget on the second segment is:

$$M(4 - 2\beta_d - \beta) + \alpha^* \left(g_{3,2} - \frac{\beta_d\ M}{\alpha^*} \right)$$

then Central government has to find a $g_{3,2}$ such that:

$$\frac{M(4 - 2\beta_d - \beta) + \alpha^* \left(g_{3,2} - \frac{\beta_d\ M}{\alpha^*} \right)}{4 - 3\beta} * \frac{\beta}{g_{3,2}} \leq \alpha^*$$

$$g_{3,2} \geq \frac{M(4 - 3\beta_d - \beta)}{4\ \alpha^* (1 - \beta)} * \beta$$

which can compared with the equivalent expression for $g_{4,2}$ to assess that $g_{3,2} < g_{4,2}$ i.e. the grant withdrawal is tougher in the last period.[35]

Equivalently for period one and two prices on the second segment have to be:

$$g_{2,2} \geq \frac{M(4 - 2\beta_d - 2\beta)}{4\,\alpha^*(1 - \beta)} * \beta$$

$$g_{1,2} \geq \frac{M(4 - \beta_d - 3\beta)}{4\,\alpha^*(1 - \beta)} * \beta$$

The optimal level of cheating might be found as usual by maximising local authorities' indirect utility function with respect to β_d.

$$\begin{aligned} \text{Max } V_a(M,g,\beta_d) = \; & (1-\beta)\,\ell n\, M(1-\beta_d) + \beta\,\ell n\,\alpha^* + \\ & (1-\beta)\,\ell n\, M(1-\beta_d) + \beta\,\ell n\,\alpha^* + \\ & (1-\beta)\,\ell n\, M(1-\beta_d) + \beta\,\ell n\,\alpha^* + \\ & (1-\beta)\,\ell n\, M(1-\beta_d) + \beta\,\ell n\,\alpha^* \end{aligned}$$

s.t. $\beta_d \geq \beta_1$

The problem can be solved using Kuhn Tucker conditions:

$$\frac{\partial V}{\partial \beta_d} = -4\left(\frac{1 - \beta}{1 - \beta_d}\right) \leq 0$$

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[35] As result of less savings available.

$$\beta_d \geq \beta$$

$$\beta_d \left(\frac{\partial V}{\partial \beta_d}\right) = 0$$

since this derivative is always negative, it follows that the constraint is binding then:

$$\beta_d = \beta_1$$

In order to make local authorities to reveal their true preferences Central Government has to offer a form of incentive to local government. I will here consider again the policy of offering local government a matching grant of the form:

$$(1 - g_t) = \left(1 - \frac{\beta_d * M}{\alpha_t^* \Omega_t'}\right)$$

such that local authorities are allowed to spend $\alpha_t^* \Omega_t^{\cdot}$ in each period.

If local authorities are given the optimal incentive their indirect utility function is modified as follows:

$$\begin{aligned} V_b(M.g(\Omega^*),\beta) = \; & (1-\beta\;)\; \ell n\; M(1-\beta\;) + \beta\; \ell n\; \alpha^* \Omega_1^{\cdot} + \\ & (1-\beta\;)\; \ell n\; M(1-\beta\;) + \beta\; \ell n\; \alpha^* \Omega_2^{\cdot} + \\ & (1-\beta\;)\; \ell n\; M(1-\beta\;) + \beta\; \ell n\; \alpha^* \Omega_3^{\cdot} + \\ & (1-\beta\;)\; \ell n\; M(1-\beta\;) + \beta\; \ell n\; \alpha^*\; \Omega_4^{\cdot} \end{aligned}$$

In analogy with the cases I presented before, Central Government's problem can thus be written as:

$$\underset{\Omega_t}{\text{Min}} \sum_{t=1}^{4} \left(\alpha^* \Omega_t - \beta_d M\right)$$

$$\text{s.t. } \beta \sum \ell n\, \Omega^{*}_{t} = 4(1-\beta)\ \ell n \left(\frac{1-\beta_1}{1-\beta} \right)$$

which gives as optimal solution:

$$\Omega_t(\beta) = \left(\frac{1-\beta}{1-\beta_1} \right)^{\left(\frac{\beta - 1}{\beta} \right)} \qquad t = 1,4$$

Clearly the prices on the second segment will have to be modified to take account of the incentive. The final system can be summarized as follows:

INCENTIVE SCHEME:

$$\Omega_t(\beta) = \left(\frac{1-\beta}{1-\beta_1} \right)^{\left(\frac{\beta - 1}{\beta} \right)} \qquad t = 1,4$$

PRICE ON THE FIRST SEGMENTS:

$$g_{1,i} = \frac{\beta\ M}{\alpha^{*}\ \Omega_i} \qquad i = 1,4$$

PRICES ON THE SECOND SEGMENT:

$$g_{1,2} = \frac{M(4 - \beta - 3\beta_2)}{4\ \alpha^{*}\Omega_1(1 - \beta_2)} * \beta_2 \qquad g_{2,2} = \frac{M(4 - 2\beta - 2\beta_2)}{4\ \alpha^{*}\Omega_2(1 - \beta_2)} * \beta_2$$

$$g_{3,2} = \frac{M(4 - 3\beta - \beta_2)}{4\ \alpha^{*}\Omega_3(1 - \beta_2)} * \beta_2 \qquad g_{4,2} = M(1-\beta) * \frac{\beta_2}{\alpha^{*}\Omega_4(1-\beta_2)}$$

Government loses $\alpha^{*}(\Omega_t - 1)$ in every period and for each single authority. However, since $\theta_t > \Omega_t$ there has been a reduction in it.

7. A MODEL WITH THE INTEREST RATE.

The standard model is now extended by allowing the interest rate to be different from zero. In this case, in order to derive the standard demand equations I have to discount future revenues to bring them to the same time. The basic assumptions of the model do not, however change and in particular I will assume that both Central and local government uses the same rate of interest. Local authorities still have an income in each period equal to M and Central Government wants them to spend at least α^* in each period. Perfect foresight and additive intertemporal separability separability is assumed throughout in analogy with the model presented in section 4 and 5.

7.1 OPTIMAL SETTING WITH BOTH PARTIES PERFECTLY INFORMED.

In a world with both parties informed and in the absence of any grant from Central Government the counterpart of the model presented in section 3.1 can be found by solving the following problem:

$$\text{Max} \sum (1-\beta)\, \ln y_t + \beta \ln X_t$$

$$\text{s.t. } X_1 + \frac{X_2}{(1+r)} + \frac{X_3}{(1+r)^2} + \frac{X_4}{(1+r)^3} + y_1 + \frac{y_2}{(1+r)} +$$

$$\frac{y_3}{(1+r)^2} + \frac{y_4}{(1+r)^3} = M + \frac{M}{(1+r)} + \frac{M}{(1+r)^2} +$$

$$\frac{M}{(1+r)^3}$$

In section 5.1 I define:

$$M + \frac{M}{(1 + r)} + \frac{M}{(1 + r)^2} + \frac{M}{(1 + r)^3} = R_v$$

In order to simplify the notation I will also define

$$\nu_t = (1+r)^{t-1}$$

and

$$\vartheta = (1+r)^{1-t}$$

then the solution to the previous problem can be written as follows :[36]

$$X_{NG,t} = \frac{\beta R_v}{4} * \nu_t \qquad t = 1,4$$

$$y_t = \frac{(1 - \beta) R_v}{4} * \nu_t \qquad t = 1,4$$

In analogy with the models presented in the previous sections I will assume that $X_{NG,t} < \alpha^*$ for all the local

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[36]If M is not constant through time and local authorities can transfer money across time the demand equations can be written as:

$$X_t = \frac{\beta}{4} \sum \left[\frac{M_t}{(1+r)^{(t-1)}} \right]$$

$$y_t = \frac{(1-\beta)}{4} \sum \left[\frac{M_t}{(1+r)^{(t-1)}} \right]$$

authorities which rules out the possibility of offering negative grants. In this environment Central Government problem is to find the optimal grant allocation form that:

$$\text{MAX} - \sum(1-g_t)*\alpha^{*}* \vartheta_t$$

$$\text{s. t.} - \left[\frac{\beta\ R_v}{g_t} * \nu_t\right] \leq - \alpha^{*}\vartheta_t$$

and the optimal solution is:

$$g_t = \frac{\beta * R_v}{4\alpha^{*}} * \nu_t$$

If the form of the subsidy is assumed to be either a matching grant or a lump sum the optimal policy for Central Government is again to offer a matching grant such as $(1-g_t)$

7.2 THE INCENTIVE COMPATIBLE SOLUTION.

If local authorities are better informed than Central Government about preferences, they have an incentive tocheat. I will examine consider only the case in which local authorities are allowed to transfer money among different time periods. The demand for local public goods in the first three periods can, in analogy with the model presented in section 5.2.2 be written as:

$$X_t = \beta_d \frac{R_v}{g_t} = \alpha^* \qquad t = 1,3 \qquad g_t = \frac{\beta_d R_v}{\alpha^*}$$

in the first three periods.

The demand equations for both X_4 and y can be derived by solving the following problem:

$$\text{Max} \sum (1-\beta)\, \ln y_t \quad + \quad \beta \ln X_4$$

$$\text{s.t.} \sum \vartheta_t \left(g_t X_t + y_t \right) = R_v$$

By having declared β_d the grant they receive in the first three periods is equal to:

$$\left(1 - \frac{\beta_d * R_v}{\alpha^*} \right) * \alpha^* = \alpha^* - \beta_d R_v$$

while the price in period four will be:

$$g_4 = \frac{\beta_d * R_v}{\alpha^*} \nu_t$$

The amount spent in the first three periods for local public goods is equal to:

$$\sum \vartheta_t g_t \, \alpha_t^* = 3 \, \beta_d \, R_v t = 1,3$$

and the budget constraint can be written as:

$$\sum \vartheta_t y_t \quad + \vartheta_4 X_4 = R_v \left(\frac{4 - 3\beta_d}{4} \right)$$

Maximisation of the utility function subject to the value constraint leads to the following set of demands:

$$X_4 = \frac{R_v(4 - 3\beta_d)}{4 - 3\beta} * \frac{\beta}{\beta_d} * \alpha^*$$

$$y_t = \frac{R_v(4 - 3\beta_d)}{4 - 3\beta} *(1- \beta)* \nu_t \quad t = 1, 4$$

The optimal level of cheating can be found by maximising local authorities' indirect utility function with respect to β_d.

$$\underset{(\beta_d)}{\text{Max}}\ V_a(M,g,\beta_d),=(1-\beta\ )\ \ell n\ \frac{R_v(4 - 3\beta_d)}{4 - 3\beta} *(1- \beta)+ \beta\ \ell n\ \alpha^* +$$

$$(1-\beta\ )\ \ell n\ \frac{R_v(4 - 3\beta_d)}{4 - 3\beta} *(1- \beta)*\nu_2+ \ \beta\ \ell n\ \alpha^* \ +$$

$$(1-\beta\ )\ \ell n\ \frac{R_v(4 - 3\beta_d)}{4 - 3\beta} *(1- \beta)*\nu_3+ \ \beta\ \ell n\ \alpha^* \ +$$

$$(1-\beta\ )\ \ell n\ \frac{R_v(4-3\beta_d)}{4 - 3\beta} * (1-\beta)*\nu_4+ \beta\ \ell n\ \frac{R_v(4-3\beta_d)}{4 - 3\beta} * \frac{\beta}{\beta_d} *\alpha^*$$

s.t. $\beta_d \geq \beta_1$

The problem can be solved using Kuhn Tucker conditions:

$$\frac{\partial V}{\partial \beta_d} = - 4 \left(\frac{3(1 \ - \beta\ )}{4 - 3\beta_d} \right) - \frac{3\beta}{4 - 3\beta_d} - \frac{\beta}{\beta_d} \leq 0$$

$$\beta_d \geq 0$$

$$\beta_d \left(\frac{\partial V}{\partial \beta_d}\right) = 0$$

since this derivative is always negative, it follows that the constraint is binding then:

$$\beta_d = \beta_1$$

In order to make local authorities to reveal their true preferences Central Government has to offer a form of incentive to local government. I will here consider the policy of offering local government a matching grant of the form:

$$(1 - g_t) = \left(1 - \frac{\beta_d * R_V}{\alpha_t^* \theta_t(\beta_d)}\right) \nu_t$$

such that local authorities are allowed to spend $\alpha_t^* \theta_t(\beta_d)$ in each period t. If authorities would not cheat their indirect utility function could, in analogy with the previous case, be written as:

$$\begin{aligned} V_b(M, g(\theta^*), \beta) = {} & (1-\beta)\ \ell n(1-\beta) + \beta \ell n\ \alpha^* \theta_1^* + \\ & (1-\beta)\ \ell n(1-\beta)\ \nu_2 + \beta\ \ell n\ \alpha^* \theta_2^* + \\ & (1-\beta)\ \ell n(1-\beta)\ \nu_3 + \beta\ \ell n\ \alpha^* \theta_3^* + \\ & (1-\beta)\ \ell n\ (1-\beta)\ \nu_4 + \beta\ \ell n\ \alpha^* \theta_4^* \end{aligned}$$

where θ_t^* denotes the optimal incentive to be given to local authorities at time t in order to avoid their cheating.

Central Government's problem can thus be written as:

$$\underset{\theta_t}{\text{Min}} \sum_{t=1}^{4} \left[\alpha^{*}\theta(\beta_d) - \beta_d R_v\right]$$

st. $V_b \geq V_a$

s.t. $\theta_{i|\beta_1} = 1$

The problem might be again rewritten as [37]:

$$\underset{\theta_t}{\text{Min}} \sum_{t=1}^{4} \left[\alpha^{*}\theta_t(\beta_d) - \beta_d R_v\right]$$

$$\text{s.t.}\beta\sum \ln \theta_t^{*} = \beta \ln \frac{\beta}{\beta_1} + 4(1-\beta) \ln \left(\frac{4-3\beta_1}{4-3\beta}\right)$$

then the interest rate affects the price system but makes no difference as concerns the incentive scheme.

8. THE TWO PRICES INCENTIVE SCHEME.

In analogy with the model I presented in section 6 it is possible also in this case to devise a two price system that allows Central Government to reduce the extra cost it has to pay to learn the true preferences. A possible way of dealing with the problem is again to set for each year a target expenditure represented by α^{*} and by devising a set of prices on the first segment such that local authorities

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[37]This result can easily be obtained by observing that if local authorities cheat they would chose β_1 in V_a. Then it is just a matter of evaluate $V_b - V_a$.

with the lowest preference for expenditure can just reach the target and they are at an optimal level of expenditure [38]. The price on the second segment has to be set in a way to avoid authorities with the highest preferences for expenditure to spend anyway along it. Since the method to be used is quite similar to the model without interest rate I will explain here the procedure quite briefly. By performing the same procedure it is possible to show also that in this case Central Government can do better by not allowing local authorities to spend more than α^*. However, the prices on the second segment for years two through four have to be changed in order to take account of the discount rate. For this model the optimal set of grant rules can be described by the following scheme which is analogous to the one I presented at the end of section 6. In analogy with that case I present the solution for the more general case in which local authorities are allowed to transfer money across time.

INCENTIVE SCHEME:

$$\Omega_i = \left(\frac{1 - \beta_d}{1 - \beta_1}\right)^{\left(\frac{\beta - 1}{\beta}\right)} \qquad i = 1,4$$

Prices on the first segments:

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[38] i.e. one of the Indifference Curve mapping their utility is tangential to the budget constraint.

Prices on the second segments:

$$g_{1,2} = \frac{R_v(4 - \beta \ - 3\beta_2)}{4\ \alpha^*\Omega_1(1 - \beta_2)} * \beta_2$$

$$g_{2,2} = \frac{R_v(4 - 2\beta \ - 2\beta_2)}{4\ \alpha^*\Omega_2(1 - \beta_2)} * \beta_2 * \nu_2$$

$$g_{3,2} = \frac{R_v(4 - 3\beta \ - \beta_2)}{4\ \alpha^*\Omega_3(1 - \beta_2)} * \beta_2 * \nu_3$$

$$g_{4,2} = R_v(1-\beta\ ) * \frac{\beta_2}{\alpha^*\Omega_4(1-\beta_2)} * \nu_4$$

The model I presented here can be quite easily modify to cope with inflation instead of the interest rate. It is clear that since perfect foresight was assumed in the analysis, the optimal solution will not change.

8. SOME FURTHER SUGGESTION FOR THE ANALYSIS.

The model I consider here could be extended in different ways. One possibility that is worth mentioning is the introduction of a discount rate on utility in different periods. For example, for the two price system I presented before it is quite easy to note that the incentive to be give to local authorities to make them reveal their true β should be equal to:

$$\Omega_i = \left(\sum \rho^{i-1}\right) \vartheta_i * \left(\frac{1-\beta}{1-\beta_1}\right)^{\left(\frac{\beta-1}{\beta}\right)} \qquad i = 1,4$$

I have not considered this case in detail because the introduction of a discount on time preference brings about a series of issues that I have preferred not to deal with in my analysis. On a theoretical ground it is hard to justify the existence of a discount on utility when dealing with organizations that are supposed to maximise some sort of Social Welfare Function since the common good should have the same weight in each period. On the other hand it is possible to argue that utility, expecially in later years is more important from a political point of view because the decision making body behaviour could be reelection oriented. If a weight greater than one exists, say, on utility in the last period, the system of grants has clearly to be revised. If Central Government knows the exact amount of this weight an optimal grant system could still be devised. However, in my opinion, this is not the best way to deal with the problem from a theoretical point of view. It is quite hard to believe that Central Government, although unable to know preferences for expenditure, is perfectly informed about a weight on utility which depends onlocal factors. If a time preference parameter is introduced, it should then be in the form of an additional parameter that Central Government has to learn. This clearly complicates the analysis quite a lot since in this case two variables interact and their joint distribution has to be taken into account.

CHAPTER FOUR

1. INTRODUCTION

In this chapter I will assume that the utility function for local authorities is defined over two commodities one of which is actually a bad. This new assumption is useful for estimation purposes since it is very difficult at an empirical level to find reliable data concerning local income available in each area. I will show that the basic results of the previous chapter do not change and that in particular the basic incentive scheme has to be the same. The problem presented here is more realistic because I will assume that local authorities have needs as well as preferences for local public services.

2. THE MODEL

In order to achieve a utility function consistent with utility axioms, I will use a Stone - Geary type of utility which, for a representative authority i can be written as:

$$U = (1 - \beta)\, \ell n\, (a_{1i} - T_i) + \beta\, \ell n\, (\lambda_i - a_{2i})$$

Local authorities perceive a minimum level of services to be provided and the community does not receive any form of "utility" by this provision; in my model the need element is represented by a_{2i}. Each locality has an amount a_{1i} of resources available for providing those services through taxation. a_{1i} represent the maximum amount of resources that the community can raise from taxation either as a result of a true lack of other resources available or as a result of interest groups behaviour or "political" reasons.

2.1 OPTIMAL SETTING WITH BOTH PARTIES PERFECTLY INFORMED.

In the absence of any grant from the Central Government and dropping the subscripts for ease of exposition, the budget constraint can be written as:

$$X = T$$

In analogy with the budget presented in section 2.1 of the previous chapter, the price ratio is assumed to be equal one and for a representative local authority the budget constraint can be drawn as such:

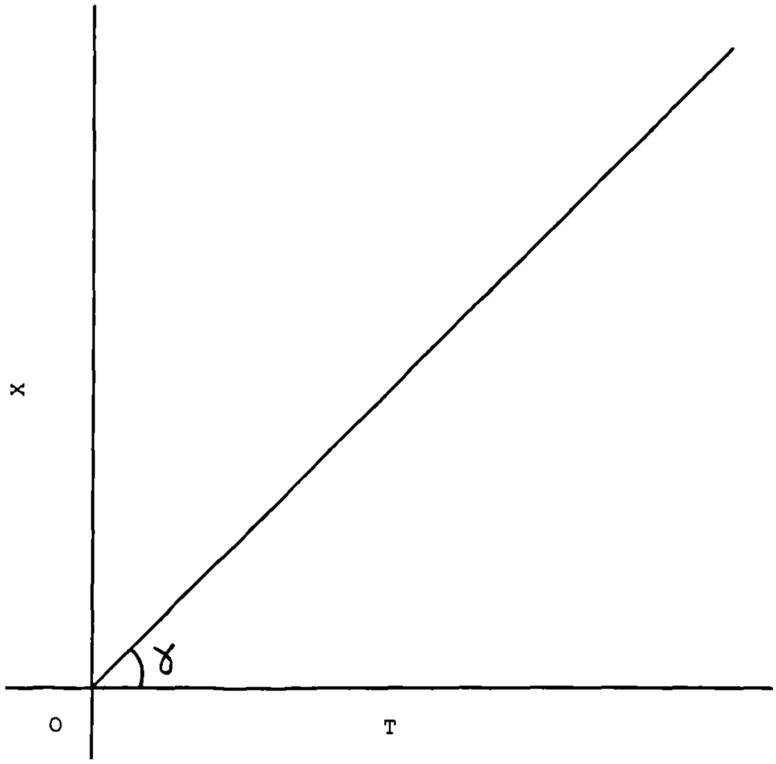

FIGURE ONE

In analogy with figure one presented in chapter three the slope of the budget constraint, γ, is equal to 45^0 and the reciprocal of the price ratio, $\tan(\gamma)$ is equal to 1.

Maximising the utility subject to the budget constraint gives the following demand equation for local public services:

$$X_{NG} = (1 - \beta)\ a_2 + \beta\ a_1$$

from which it follows that if $a_1 < a_2$ an under provision of public services would result [1].

In analogy with the two goods problem I presented in the previous chapter, I will now examine the effect of both a lump-sum grant and a matching grant on the budget constraint of a local authority representing the whole population. If a locality is provided with a pure lump sum grant equal to G its budget constraint will shift upwards by the amount G while if a matching grant is offered the budget will become steeper since the price ratio decreases. Both situations are depicted in Figure 2.

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[1] i.e. the local public services provided will be lower than the need.

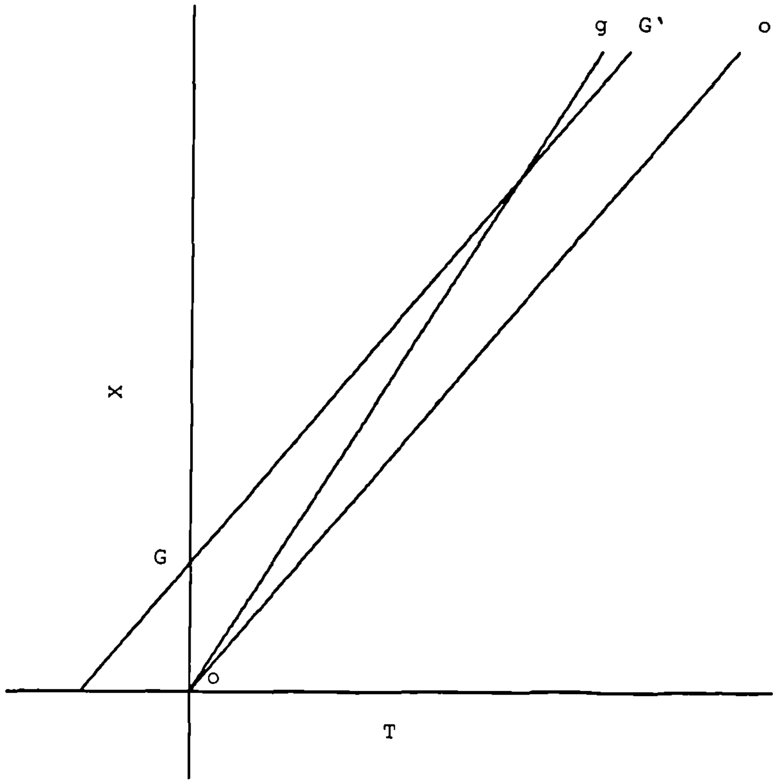

FIGURE TWO

If a lump-sum grant is introduced the original line oo shifts to GG' while if a matching grant is offered, the budget line becomes og. The budget constraint incorporating both a lump-sum and a matching grant can be written as:

$$gX - T = gG$$

Maximisation of the utility function subject to this budget constraint gives the following demand equation:

$$X_{GR} = (1 - \beta)* a_2 + \beta \left(\frac{a_1}{g} + G \right)$$

It is worth noting that if Central Government wanted each local authority to provide at least a_2, i.e. the basic need expenditure, a lump-sum grant and a matching grant would have the same effect. The total grant to be provided in this case would in fact be equal to $(a_2 - a_1)$ irrespective of the form it is granted [2]. This is the amount of resources that an authority for which $X_{NG} < a_2$ would need to provide the latter quantity. This result depends on the functional form adopted for utility: resources are first allocated to satisfy needs and then the surplus, if any, is allocated according to preferences.

I will assume, in analogy with the model presented before that Central Government wants local authorities to spend at least α^* for local services with $\alpha^* > a_2$. X_{NG} is assumed to be less than α^* for all local authorities in the sample.[3]

The environment in which Central Government has to operate can be described as follows:

i) Central Government can observe the amount of local income available in each region, $a_{1,i}$, the right amount of local public services to be provided, α^*_i and the need parameter, $a_{2,i}$.

ii) Central Government knows β and uses it to determine the size of the grant for each authority.

iii) Open ended matching grants and lump-sum grants are the two instruments available to Central Government to reach its objective.

The solution to this problem can be formally shown as follows:

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[2] See appendix four for a formal proof.

[3] This assumption again rules out the possibility of having negative grants.

$$\min_{g,G} \; G + (1-g) * \alpha^*$$

$$\text{s.t. } (1 - \beta) * a_2 + \beta \left(\frac{a_1}{g} + G \right) = \alpha^*$$

$$\text{s.t. } g \geq 0$$

The optimal solution for the problem cannot be found by using a standard Lagrangean technique because the constraint is not convex [4]. Because of the nature of the problem the optimal point will then be a corner solution and it can be shown that for this problem the best distribution formula is characterized by giving local authorities a matching grant The solution can be easily shown in diagrammatic terms [5].

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[4]This point can be easily shown by writing the first and the second derivatives of the constraint. The are as follows:

$$\frac{\partial G}{\partial g} = \frac{a_{1i}}{g^2} > 0$$

$$\frac{\partial^2 G}{\partial g^2} = -2 \frac{a_{1i}}{g^3} < 0 \quad \text{i.e. the function is concave.}$$

[5]The intuitive explanation for this result is that, as I have shown in the previous chapter, a matching grant has a greater effect than a lump sum on expenditure.

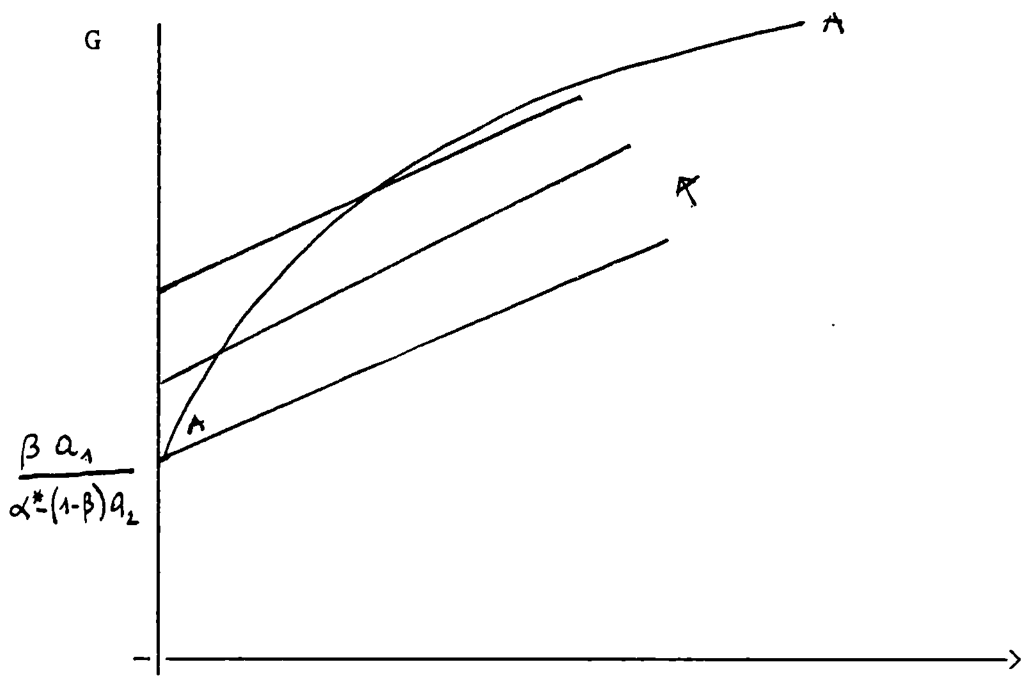

FIGURE THREE

As it is possible to note from figure three, the objective function can be depicted as a set of straight lines increasing in a north-westernly direction while the constraint is represented by the concave line AA .The optimal solution is then represented by the corner:

$$g = \frac{\beta\ a_1}{\alpha^* - (1-\beta)a_2} \qquad\qquad G = 0$$

In my analysis, however, I will make a further assumption that will prove useful when I will try to link this theoretical model to the actual grant system. The model I will present is built on the assumption that the grant system has the twofold object to bring local authorities to spend at least α^* and to provide a fully equalization as concerns their needs, irrespective of the resources available. The lump sum grant corresponds to what in the theory of inter governmental grant is known as a need equalizing grant. In order to pursue this equalization, Central Government will have to provide each local authority with a lump sum grant at least equal to a_2 which is its baseline need. The final level of expenditure will clearly still depend on preferences and resources and equalization on those elements is pursued by the matching grant. In that case the matching grant rate (1-g) will be equal to [6]:

$$(1-g) = 1 - \frac{\beta\ a_1}{\alpha^* - a_2}$$

2.2 THE INCENTIVE COMPATIBLE SOLUTION.

If, as in the case presented in chapter three, I assume that Central Government is not fully informed about preferences of local authorities, a true "adverse selection" problem arises and an incentive has to be granted to local Governments in order to make them report their preferences truthfully. The environment in which Central Government had to operate should be modified as follows:

iia) Central Government knows that the true β for any local authority lies in the closed interval $[\beta_1, \beta_2]$. Since no other information is available the distribution of β, $f(\beta)$ is assumed to be uniform [7].

Before the starting of the period each local authority has to report to the Central Government its preferences

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[6] See appendix four for a formal proof.

[7] It is then clear the analogy of this model with Tresh interpretation of Stigler's approach. Even all the analysis that follows will be conducted in terms of expenditure behaviour it is clearly the marginal rate of substitution between private and public goods that cannot be known with certainty.

over local public services which will be in turn used to determine the size of the grant. The preference parameter declared by local authorities will be labelled β_d to distinguish it from β, the true one.

iv) Preferences parameters are peculiar to any authority considered [8].This assumption rules out the possibility (or the need) for coalitions between authorities so the problem can be restricted to examine the optimal contract between one local authority and the Central Government, at least in this first stage of the analysis [9].

v) β is assumed to be both independent between authorities as stated in iii) and across time [10].

The static framework, however, does not give any insight into the problem since again there is no incentive for the Central Government to learn local authorities' preferences. The optimal solution is again represented by a two prices system such that local authorities with the lowest preferences will be able to spend α^* and to stick all the other ones at the kink[11] by using a penalty system on the

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[8] i.e. it would be impossible to infer authority's i preferences by knowing β for authority j in other words β is not either a simple political or regional parameter.

[9] The problem could not be treated as such if the Central Government had a budget constraint on the total amount spent on grants to local authorities.

[10] This second assumption will be relaxed in dynamic model and it will actually be the crucial difference between the two setting.

[11] The kink is clearly represented by α^*.

second segment.The actual formalization is not reported here since it is very similar to the two-goods model and does not give any further insights in the analysis.

3. THE DYNAMIC PROBLEM.

I will assume, in analogy with the model presented in the previous section, that local authorities' behaviour can be represented by the following utility function that they want to maximise through their lifetime. According to whether they are allowed to transfer resources from different time periods the problem faced by local authorities can be written as:

$$\text{Max } (1-\beta) \sum_{1}^{4} \ln(a_{1t} - T_t) + \beta \sum_{1}^{4} \ln(X_t - a_{2t})$$

$$\text{s.t. } g_t X_t = T_t + g_t a_2 \qquad t = 1, 4$$

$$a_2 = G$$

if their budgets have to be balanced each year and:

$$\text{Max } (1-\beta) \sum_{1}^{4} \ln(a_{1t} - T_t) + \beta \sum_{1}^{4} \ln(X_t - a_{2t})$$

$$\text{s.t. } \sum_{1}^{4} g_t X_t = \sum_{1}^{4} \left(T_t + g_t a_2\right)$$

if they are allowed to transfer resources.

In analogy with the more general model I will show first of all the optimal grant allocation rule for the case in which both parties possess all information relevant to solve the problem. To simplify the analysis I will assume that needs and resources are constant throughout the period.

3.1 THE OPTIMAL SOLUTION IN A NO-ASYMMETRY FRAMEWORK.

In the absence of any grant system but a lump-sum grant equal to a_2 the demand for local services of a representative authority might be derived by maximising its utility function subject to the budget constraint. The formal representation of the problem will then be:

$$\text{Max } (1-\beta)\sum_{1}^{4} \ln(a_1 - T_t) + \beta \sum_{1}^{4} \ln(X_t - a_2)$$

$$\text{s.t. } \sum_{1}^{4} X_t = \sum T_t + \sum_{1}^{4} a_2$$

which leads to a set of demand equations of the following form: [12]

$$X_t = a_2 + \beta\, a_1$$

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[12] The same result would in this particular case be obtained also by assuming that local authorities cannot transfer money from one period to another one. If a_1 is not constant through time and local authorities can transfer money from different periods the two demands equations have to be rewritten as:

$$X_t = \frac{\beta\, \Sigma a_{1t}}{4} + a_2$$

and

$$y_t = \frac{\beta\, \Sigma a_{1t}}{4}$$

while demands if the budget has to be balanced each period do not change.

$$T_t = \beta\ a_1$$

In this environment Central Government problem is to find the optimal grant allocation form that:

$$\text{MAX} \ - \sum (1-g_t)*\alpha^*$$

$$\text{s.t.} \ - \left[a_2 + \ \beta \frac{a_1}{g_t}\right] \le - \ \alpha^*$$

which gives:

$$g_t = \frac{\beta * a_1}{\alpha^* - a_2}$$

3.2 THE INCENTIVE COMPATIBLE SOLUTION.

If local authorities are asked to report their true β and the only way it is possible to realize their cheating is by observing actual expenditure, it is in their best interest to report a β_d lower than the true one at least in some periods of their life cycle. In analogy with the model presented in chapter three the new assumption introduced is:

v)a β although independent across authorities is fixed through the life cycle of each local authority.

such that the environment is now described by assumption i) - ii)a - iii) - iv) - v)a and the process of decision making is still described by table one in the previous chapter.

I will first examine the case in which the budget has to be balanced every year and I will then present the model in which money can be transferred across time.

3.2.1 CASE A: THE BUDGET HAS TO BE BALANCED IN EVERY PERIOD.

If local authorities are not allowed to transfer money through their life cycle the demand equation for local public goods of a representative authority can be described as follows:

$$X_t = a_2 + \beta \frac{a_1}{g_t} = a_2 + (\alpha^* - a_2) * \frac{\beta_d}{\beta_d} \quad ; \; g_t = \frac{\beta_d * a_1}{\alpha^* - a_2}$$

$$t = 1,3$$

$$X_4 = a_2 + \beta \frac{a_1}{g_4} = a_2 + (\alpha^* - a_2) * \frac{\beta}{\beta_d} \quad ; \; g_4 = \frac{\beta_d * a_1}{\alpha^* - a_2}$$

$$T_t = \beta_d a_1 \qquad t = 1,3$$

$$T_4 = \beta\, a_1$$

The optimal level of cheating can be found by maximising local authorities' indirect utility with respect to β_d.

$$\begin{aligned}\max_{(\beta_d)} \quad V_a(G,g,\beta_d) = {} & (1-\beta)\,\ln a_1(1-\beta_d) + \beta \ln (\alpha^* - a_2) + \\ & (1-\beta)\,\ln a_1(1-\beta_d) + \beta \ln (\alpha^* - a_2) + \\ & (1-\beta)\,\ln a_1(1-\beta_d) + \beta \ln (\alpha^* - a_2) + \\ & (1-\beta)\,\ln a_1(1-\beta) + \beta \ln \frac{\beta}{\beta_d} (\alpha^* - a_2)\end{aligned}$$

$$\text{s.t. } \beta_d \geq \beta_1$$

The problem can be solved using Kuhn Tucker conditions:

$$\frac{\partial V}{\partial \beta_d} = -3\left(\frac{1-\beta}{1-\beta_d}\right) - \frac{\beta}{\beta_d} \leq 0$$

$$\beta_d \geq \beta_1$$

$$\beta_d \left(\frac{\partial V}{\partial \beta_d}\right) = 0$$

since this derivative is always negative, it follows that the constraint is binding then:

$$\beta_d = \beta_1$$

In order to make local authorities reveal their true preferences Central Government has to offer an incentive to local government. I will here consider the policy of offering local government a matching grant of the form:

$$(1-g_t) = \left(1 - \frac{\beta_d * a_1}{(\alpha_t^* - a_2) * \theta_t(\beta_d)}\right)$$

such that if the incentive is chosen optimally, local authorities will not cheat and will spend $a_2 + (\alpha_t^* - a_2) * \theta_t^\bullet$ in each period. The indirect utility for not cheating under this new assumption can be written as:

$$\begin{aligned} V_b(G, g(\theta^*), \beta) = &(1-\beta)\, \ln a_1 (1-\beta) + \beta \ln (\alpha^* - a_2)\theta_1^\bullet + \\ &(1-\beta)\, \ln a_1 (1-\beta) + \beta \ln (\alpha^* - a_2)\theta_2^\bullet + \\ &(1-\beta)\, \ln a_1 (1-\beta) + \beta \ln (\alpha^* - a_2)\theta_3^\bullet + \\ &(1-\beta)\, \ln a_1 (1-\beta) + \beta \ln (\alpha^* - a_2)\theta_4^\bullet \end{aligned}$$

where $\theta_t^\bullet$ denotes the optimal incentive to be given to a representative local authority in each period to avoid its cheating. In analogy with the method presented in Chapter three, Central Government's problem can thus be written as:

$$\underset{\theta_t}{\text{Min}} \sum_{t=1}^{4} \left(\alpha^* \theta_t - \beta_d M\right)$$

st. $V_b(G, g(\theta^*), \beta) \geq V_a(G, g, \beta_d)$

s.t. $\theta^*_t(\beta_1) = 1$

At the optimum values for θ^*_t it must be true that [13]:

$$\beta \sum \ell n\, \theta^*_t = \beta \ell n \frac{\beta}{\beta_1} + 3(1-\beta) \;\; \ell n \left(\frac{1 - \beta_1}{1 - \beta}\right)$$

and the incentive compatible solution is then the same as for the two goods model.

3.2.2 CASE B: MONEY CAN BE TRANSFERRED ACROSS TIME.

In this second case I will allow local authorities to transfer money from the different periods of their lifetime by presenting a model in which local authorities are allowed to save and borrow.

In appendix five I show that the same result I present here applies for the case in which local authorities can save but

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[13] This result can easily be obtained by observing that if local authorities cheat they would chose β_1 in V_a. Then it is just a matter of evaluate $V_b - V_a$.

they are not allowed to borrow money. If the resources available are constant through time and Central Government knows the true preferences of local authorities their budget will be balanced every year, as I have previously shown. However, when Central Government does just know the range of preferences rather than the true parameter local authorities are better off if they cheat.Their expenditure decisions in the first three periods can be described as follows:

$$X_t = a_2 + \beta \frac{a_1}{g_t} = a_2 + (\alpha^* - a_2) \qquad ; \; g_t = \frac{\beta_d * a_1}{\alpha^* - a_2}$$

$$t = 1,3$$

Due to the possibility of transferring money across time this local authority is no longer constrained to raise.

$$T_t = \beta_d a_1 \qquad t = 1,3$$

The demand equations for both X_4 and T_i can be derived by solving the following problem:

$$\text{Max} \sum_{1}^{4} (1-\beta)\, \ell n\, (a_1 - T_t) + \beta\, \ell n\, (X_4 - a_2)$$

$$\text{s.t.} \; \sum \left[g_t X_t - T_t \right] = \sum g_t a_2$$

By having declared β_d the grant our local authority is entitled to receive in the first three periods is equal to:

$$a_2 + \left(1 - \frac{\beta_d * a_1}{\alpha^* - a_2}\right) * (\alpha^* - a_2)$$

then the budget constraint can be written as:

$$g_4 X_4 - \sum T_t = a_2 g_4 - 3\beta_d a_1$$

where:

$$g_4 = \frac{\beta_d * a_1}{\alpha^* - a_2}$$

Maximising the utility function subject to the budget constraint it is possible to derive the following set of demand equations:

$$X_4 = a_2 + \frac{\beta}{\beta_d} * \frac{4 - 3\beta_d}{4 - 3\beta} * (\alpha^* - a_2)$$

$$T_t = \frac{a_1}{(4-3\beta)} * \left[3\beta_d(1-\beta) + \beta\right] \qquad t = 1,4$$

which shows that the process of maximisation leads to the equalization of the marginal utility of money and then to spread resources evenly among periods on the unconstrained good. It is worth comparing the two pseudo demand equations for taxation under the two assumptions presented. If the budget has to be balanced each year, taxation in the first three periods will be equal to:

$$T_t = \beta_d \, a_1$$

while if resources can be transferred across time the amount of taxation raised will be:

$$T_t = \frac{a_1}{(4-3\beta)} * \left[3\beta_d(1-\beta) + \beta\right] \qquad t=1,4$$

The comparison of the two expressions shows that in the second case the amount of taxation raised is larger. [14] This gives further insights in creative accounting. Local authorities in the first years cheat on their preferences and raise through taxation more money than they require. This policy allow them to over spend in the last period without any further burden on the taxpayers From a political cycle point of view, the last period coincides with reelection and local Government s decision body increases its chances of reelection by using this policy. If the decision making body is reelection oriented cheating might not be quite appealing if resources cannot be transferred. In period 4, when the true preferences are finally revealed, the increase in expenditure from α^* to $\frac{\beta}{\beta_d}\alpha^*$ is partly financed by the cheating but also taxation

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[14] The proof of the last proposition is as follows:

$\beta_d\ a_1$

$$\frac{a_1}{(4-3\beta)} * \left[3\beta_d(1-\beta) + \beta\right] > \beta_d\ a_1$$

$$3\beta_d(1-\beta) + \beta > \beta_d(4-3\beta)$$

$$\beta > \beta_d$$

which is always true under the assumptions made before.

has to be increased.[15] If, as some political scientists argue [16] the electorate is more aware of taxation than expenditure, the beneficial effects of the rise in expenditure could even be outweighed by the increase in taxation, at least from the point of view of reelection.

The optimal level of cheating might be found by maximising local authorities' indirect utility function with respect to β_d.

$$\underset{(\beta_d)}{\text{Max}}\ V_a(G,g,\beta_d)=(1-\beta)\ \ell n\ \frac{a_1\ (4-3\beta_d)}{4-3\beta} *(1-\beta)+\beta\ \ell n\ (\alpha^*-a_2)\ +$$

$$(1-\beta)\ \ell n\ \frac{a_1\ (4-3\beta_d)}{4-3\beta} *\ (1-\beta)+\beta\ \ell n\ (\alpha^*-a_2)\ +$$

$$(1-\beta)\ \ell n\ \frac{a_1\ (4-3\beta_d)}{4-3\beta} *\ (1-\beta)+\ \beta\ \ell n\ (\alpha^*-a_2)\ +$$

$$(1-\beta)\ \ell n\ \frac{a_1\ (4-3\beta_d)}{4-3\beta} *\ (1-\beta)\ +\ \beta\ \ell n\ \frac{4-3\beta_d}{4-3\beta} * \frac{\beta}{\beta_d} *(\alpha^*-\ a_2\ )$$

s.t. $\beta_d \geq \beta_1$

The problem can be solved using Kuhn Tucker conditions:

$$\frac{\partial V}{\partial \beta_d} = -\ 4\left(\frac{3(1\ -\beta\ )}{4\ -\ 3\beta_d}\right) - \frac{3\beta}{4\ -\ 3\beta_d} - \frac{\beta}{\beta_d} \leq 0$$

$$\beta_d \geq \beta_1$$

$$\beta_d \left(\frac{\partial V}{\partial \beta_d}\right) = 0$$

---

[15] In this case, in fact T has to be raised from $\beta_d a_1$ to βa_1.

[16] See, for example Borooah and Van Der Ploegh (1985), ch. 9

since this derivative is always negative, it follows that the constraint is binding then:

$$\beta_d = \beta_1$$

In order to make local authorities reveal their true preferences Central government has to offer an incentive to local government. I will consider the policy of offering local government a matching grant of the form:

$$(1 - g_t) = \left(1 - \frac{\beta_d * a_1}{(\alpha_t^* - a_2) * \dot{\theta}_t} \right)$$

such that local authorities are allowed to spend $a_2 + (\alpha_t^* - a_2)$ $\dot{\theta}_t$ in each period.

If the incentive is chosen optimally the indirect utility function for not cheating can be written as:

$$\begin{aligned} V_b(G.g(\theta^*),\beta) = &(1-\beta)\ \ell n\ a_1(1-\beta) + \beta\ \ell n\ (\alpha^* - a_2) * \dot{\theta}_1 \quad + \\ &(1-\beta)\ \ell n\ a_1(1-\beta) + \beta\ \ell n\ (\alpha^* - a_2) * \dot{\theta}_2 \quad + \\ &(1-\beta)\ \ell n\ a_1(1-\beta) + \beta\ \ell n\ (\alpha^* - a_2) * \dot{\theta}_3 \quad + \\ &(1-\beta)\ \ell n\ a_1(1-\beta) + \beta\ \ell n\ (\alpha^* - a_2) * \dot{\theta}_4 \end{aligned}$$

where θ_i^* denotes the optimal incentive to be given to authorities at time t to avoid their cheating.

Central Government's problem can thus be written as:

$$\min_{\theta_t} \sum_{t=1}^{4} \left(\alpha^{*}\theta_t - \beta_d M\right)$$

st. $V_b(G,g(\theta^{*}),\beta) \geq V_a(G,g,\beta_d)$

s.t. $\theta^{*}_{t} | \beta_1$

The first constraint can be rewritten as [17]:

$$\beta \sum \ell n\, \theta^{*}_{t} = \beta \ell n \frac{\beta}{\beta_1} + 4(1-\beta)\ \ell n \left(\frac{4-3\beta_1}{4-3\beta}\right)$$

then the system of incentive should be the same as for the problem I presented in the previous chapter.

3..3 A TWO PRICE OPTIMAL GRANT DISTRIBUTION FORMULA

In the previous system the loss suffered by Central Government as a result of its inability to know the true preferences of local authorities was equal to:

$(\alpha^{*} - a_2)\ (\theta_t - 1)$

in each period and for each local authority. I will examine, in analogy with the previous section, a different system by which Central Government, although being unable to learn the true preferences parameter for each local authorities, can reduce the loss. The new system foresees a set of rules by which local authorities will never be able to spend more

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[17]This result can easily be obtained by observing that if local authorities cheat they would chose β_1 in V_a. Then it is just a matter of evaluate $V_b - V_a$.

than α^* in each period if they cheat. The actual implementation of the system requires the introduction of penalties for expenditure in excess of the desired level. This means that each year is characterized by a double matching grant as such:

$$(1 - g_{i,1}) \text{ for any } X_{GR} \leq \alpha^*$$

$$i = 1,4$$

$$(1 - g_{i,2}) \text{ for any unit of expenditure exceeding } \alpha^*$$

For expenditure up to α^* the matching rate is:

$$(1 - g_{t,1}) = \left(1 \quad \frac{\beta_d * a_1}{\alpha^* - a_2} \right)$$

For expenditure in excess of α^* the matching rate will be:

$$(1 - g_{t,2})$$

where $g_{t,2}$ is the price level securing that any authority will not spend more than α^*. I will first present the optimal level of cheating of local authorities in this case. I will formally present only the case in which local authorities are allowed to transfer money from the different periods of their lifetime [18] .Let us start by finding the penalty for the last period. The demand equations for the private commodity and the public one can be written as

[18] The incentive for this problem would not change under the assumption of local authorities not being allowed to transfer money across time. However, prices on the second segment could be set equal to $g_{1,2}$ for all the periods.

follows:

$$X_t = a_2 + \beta \frac{a_1}{g_t} = a_2 + (\alpha^* - a_2) * \frac{\beta_d}{\beta_d} \quad ; \; g_t = \frac{\beta_d * a_1}{\alpha^* - a_2}$$

$$t = 1,3$$

$$X_4 = \alpha^* \qquad\qquad g_{4,2} = \;?$$

$$T_t = \beta_d a_1 \qquad t = 1,4$$

where $g_{4,2}$ is the price on the second segment that must be chosen to secure that any local authority will not spend more than α^* irrespective of its preferences. In order to achieve this result Central Government has to find a set of prices to be applied on expenditure exceeding α^* such that the budget constraint of each local authority will look like figure four.

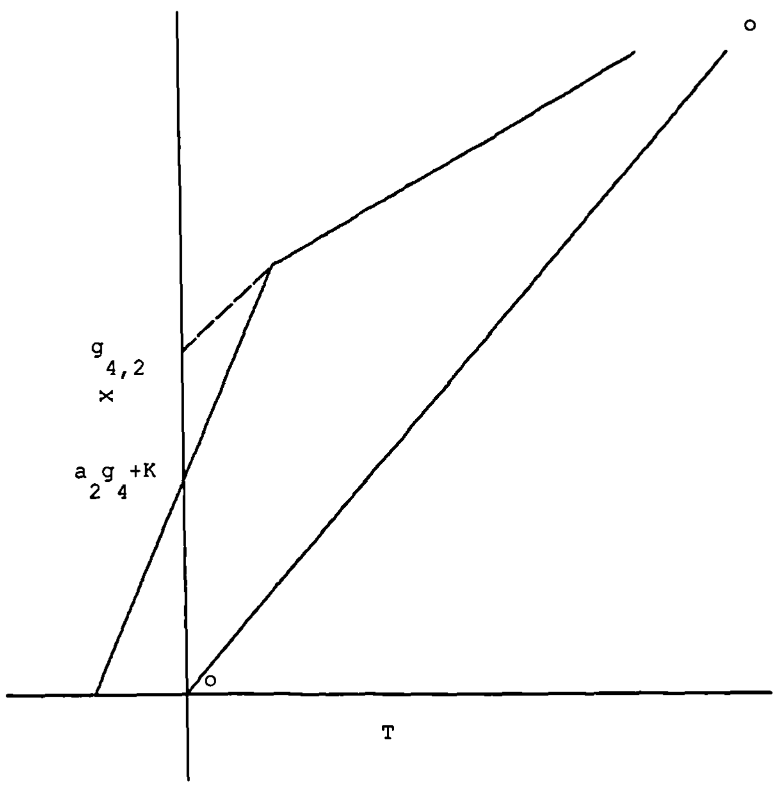

FIGURE FOUR

Appendix six presents the procedure that leads to the following equation for the price on the second segment:

$$g_{4,2} \geq \frac{\beta \left[a_1 \ (1 - \beta) + K \right]}{(\alpha^* - a_2)\ (1 - \beta)}$$

$$K = 3 \left[\frac{a_1 \quad (\beta - \beta_d)}{4 - 3\beta} \right]$$

Since Central Government does not know β, the best it can do is fix a price so high to avoid overspending. In this case β has clearly to be chosen equal to β_2. A penalty system in the last period is not sufficient alone to achieve the result of having local authorities spending α^* in each period. A further set of penalties (in the form of grant withdrawals) has to be introduced in the system because if the two price system was introduced on expenditure only in the last period, local authorities could be better off by revealing their true preferences in the previous periods. In this case their expenditure decisions could be summarized as follows [19]:

$$X_t = a_2 + \beta \frac{a_1}{g_t} = a_2 + (\alpha^* - a_2) * \frac{\beta_d}{\beta_d} \quad ; g_t = \frac{\beta_d * a_1}{\alpha^* - a_2}$$

$$t = 1,2$$

---

[19]Throughout the analysis prices on the first segment are referred to with the year in which they are in force i.e. g_t = price for expenditure on the first segment of the budget constraint in period t.

$$X_3 = x(M,\beta,BL,g_3) \qquad g_3 = \frac{\beta_d \; a_1}{\alpha^* - a_2}$$

$$X_4 = \alpha^* \; ; \quad g_4 = \frac{\beta_d a_1}{\alpha^* - a_2} \quad ; \quad g_{4,2} \geq \frac{\beta\left[a_1 \quad (1 - \beta_d) + K\right]}{(\alpha^* - a_2) \; (1 - \beta)}$$

$X_3 = x(M,\beta,BL,g_3)$ can be found by maximising:

$$\text{Max} \sum (1-\beta) \; \ell n \; (a_1 - T_t) \; + \beta \; \ell n \; (X_3 - a_2)$$

$$\text{s.t.} \sum g_t X_t + y_t = \sum g_t a_2$$

by having declared β_d the grant it receives in the first two periods is equal to.

$$a_2 + \left(1 - \frac{\beta_d * a_1}{\alpha^* - a_2}\right) * (\alpha^* - a_2)$$

while in the last period they receive a grant equal to:

$$a_2 + \left(1 - \frac{\beta \; * a_1}{\alpha^* - a_2}\right) * (\alpha^* - a_2)$$

then the budget constraint can be rewritten as:

$$g_3 X_3 - \sum T_t \quad = 4a_2 g_3 - 2\beta_d a_1 - \beta a_1$$

$$g_3 = \frac{\beta_d * a_1}{\alpha^* - a_2}$$

The maximisation procedure leads to the following demand equation.

$$X_3 = a_2 + \frac{\beta}{\beta_d} * \frac{4 - 2\beta_d - \beta}{4 - 3\beta} * (\alpha_2^* - a_2)$$

In appendix six I show the procedure that leads to the following equation for the price on the second segment:

$$g_{3,2} \geq \frac{\beta\left[a_1 \ (1 - \beta_d) + K_1\right]}{(\alpha^* - a_2)\ (1 - \beta)}$$

$$K_1 = \frac{2\ a_1 \ (\beta - \beta_d)}{4 - 3\ \beta}$$

which can compared with the equivalent expression for $g_{4,4}$ to assess that $g_{3,2} < g_{4,2}$ i.e. the grant withdrawal is tougher in the last period.[20]

Equivalently for period two and one prices on the second segment have to be:

$$g_{2,2} \geq \frac{\beta\left[a_1 \ (1 - \beta_d) + K_2\right]}{(\alpha^* - a_2)\ (1 - \beta)}$$

$$K_2 = \frac{a_1 \ (\beta - \beta_d)}{4 - 3\ \beta}$$

$$g_{1,2} \geq \frac{\beta\left[a_1 \ (1 - \beta_d) + g_{2,1}\right]}{(\alpha^* - a_2)\ (1 - \beta)}$$

[20] As a result of less savings available.

The optimal level of cheating might be found as usual by maximising local authorities' indirect utility function with respect to β_d.

$$\underset{(\beta_d)}{\text{Max}}\ V_a(G\ g,\beta_d) = (1-\beta\ )\ \ln\ a_1(1-\beta_d) + \quad \beta\ \ln\ (\alpha^*-a_2) \quad +$$
$$(1-\beta\ )\ \ln\ a_1(1-\beta_d) \quad + \quad \beta\ \ln\ (\alpha^*-a_2) \quad +$$
$$(1-\beta\ )\ \ln\ a_1(1-\beta_d) \quad + \quad \beta\ \ln\ (\alpha^*-a_2) \quad +$$
$$(1-\beta\ )\ \ln\ a_1(1-\beta_d) \quad + \quad \beta\ \ln(\ \alpha^*-\ a_2)$$

s.t. $\beta_d \geq \beta_1$

The problem can be solved using Kuhn Tucker conditions.

$$\frac{\partial\ V}{\partial\ \beta_d} = -\ 4\left(\frac{1\ -\ \beta}{1\ -\ \beta_d}\right) \leq 0$$

$$\beta_d \geq \beta_1$$

$$\beta_d\ \left(\frac{\partial\ V}{\partial\ \beta_d}\right) = 0$$

since this derivative is always negative, it follows that the constraint is binding then $\beta_d = \beta_1$ In order to make local authorities to reveal their true preferences Central Government has to offer a form of incentive to local Government. I will here consider the policy of offering local Government a matching grant of the form:

$$(1\ -g_t) = \left(1\ -\ \frac{\beta_d * a_1}{(\alpha_t^*-a_2\ ) * \Omega_t^*}\right)$$

such that local authorities are allowed to spend $a_2+(\alpha_t^*-a_2)*$

Ω^*_t in each period.

If the incentive is chosen optimally the indirect utility function for not cheating can be written as.

$$
\begin{aligned}
V_b(G,g(\Omega^*),\beta) = {} & (1-\beta)\ \ell n\ a_1(1-\beta)+\beta\ \ell n\ (\alpha^*-a_2)*\Omega^*_1 \quad + \\
& (1-\beta)\ \ell n\ a_1(1-\beta)+\beta\ \ell n\ (\alpha^*-a_2)*\Omega^*_2 \quad + \\
& (1-\beta)\ \ell n\ a_1(1-\beta)+\beta\ \ell n\ (\alpha^*-a_2)*\Omega^*_3 \quad + \\
& (1-\beta)\ \ell n\ a_1(1-\beta)+\beta\ \ell n\ (\alpha^*-a_2)*\Omega^*_4
\end{aligned}
$$

where Ω^*_i denotes the optimal incentive to be given to local authorities at time t in order to avoid their cheating.

Central Government's problem can thus be written as:

$$\underset{\Omega_t}{\mathrm{Min}} \sum_{t=1}^{4} \left(\alpha^*\Omega_t - \beta_d M\right)$$

st. $V_b(G,g(\Omega^*),\beta) \geq V_a(G,g,\beta_d)$

s.t. $\Omega^*_t|\beta_1$

The first constraint can be rewritten as [21]:

$$\beta\sum \ell n\ \Omega^*_t = 4(1-\beta)\ \ \ell n \left(\frac{1-\beta_1}{1-\beta}\right)$$

---

[21] This result can easily be obtained by observing that if local authorities cheat they would chose β_1 in V_a. Then it is just a matter of evaluate $V_b - V_a$.

then the system of incentive should be the same as for the problem I presented in the previous chapter.

4. WHY THE SYSTEM COULD FAIL TO REACH ITS GOALS.

The analysis so far has assumed that local authorities behave as if they were utility maximisers and had definite preferences for local public services. As I pointed out in the introduction to chapter three there is a lot of discussion in the literature on whether the utility maximisation framework is a useful tool to use when dealing with organizations.In this section I will present two of the possible reasons why the previous optimal grant allocation setting may fail to reach its objective. The first model is linked to the public choice literature and finds its origins from empirical evidence on local government behaviour while the second argument is more strictly linked with utility maximisation behaviour. I will present them in turn.

4.1 THE FLYPAPER MODEL.

The first failure to the system steps back from the literature on the budgetary effect of inter governmental grants. The main predictions of what I might define the conventional approach that I have implicitly adopted in the analysis so far can be summarized as follows:

a) the form as well as the total amount of the grant matters; in particular a matching grant stimulates more spending than a lump sum grant. The reason for this difference being that, while a lump sum grant push up expenditure through the income effect the matching grant creates a price as well as an income effect as I showed in section 1.1 of chapter three.

b) a lump sum grant paid to the local authority is equivalent to a series of personalised lump-sum grants paid to individuals. This second proposition is known as the equivalence theorem.[22]

The predictions of the impact of different types of grants are usually made under a set of quite restrictive assumptions that, when relaxed, make things far more complicated. The usual utility function maximisation approach to local behaviour assumes that the community is rationally trying to maximise some concept of utility that can be that of the median voter or of a representative individual in that community and it does not assume any sort of disharmony of interests between politicians and voters.

But if some disharmony exists different results can be obtained. In particular the equivalence theorem as stated before is not confirmed by any empirical testing.[23]

In an harmonious world it should not matter whether the central government cut taxes or gives revenue-sharing funds to local governments; as long as the income distributional properties are the same, either measure should increase public spending by the income elasticities of demand with the remainder going into increased private spending.

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[22] The equivalence theorem has been developed by Bradford and Oates (1971).

[23] Gramlich (1977) gives a useful summary of the different studies on the subject. See also Courant (1979).

Given the suggested failure of the existing paradigm, some authors have argued that a self interested behaviour by bureaucrats and politicians or a fiscal illusion by voters cause a lump sum grant to affect much more sub central spending than an equal value changes in the tax payment of grantee citizens to the grantor. This new development is known as flypaper theory. The main contributions to the so called flypaper theory can be found in the works by Courant (1979) and Oates (1979). The model developed here follows the one presented in Barnett (1985) and Barnett et al (1989) with some important modifications. I will assume throughout my analysis that local authorities behave as if they were social welfare maximisers over public expenditure and tax bills .Any local authority is assumed to be made of homogeneous individuals with respect to preferences and resources available such that the social welfare function is equivalent to the utility of a representative individual in that community up to a scaling factor. Since all individuals are equal, the "average" member of the community is also the median voter i.e. the decisive voter. A representative individual in local authority i is assumed to have a well behaved utility function of the form:

$$U_i = u_i(X_i, T_i, a_{1i}, a_{2i}, \beta) \tag{1}$$

$$\frac{\partial U_i}{\partial X_i} > 0 \qquad \frac{\partial^2 U_i}{\partial X_i^2} < 0 \qquad \frac{\partial U_i}{\partial T_i} < 0 \qquad \frac{\partial^2 U_i}{\partial T_i^2} < 0$$

where the parameter a_1 reflects the underlying income available in the area for local authority's service provision and a_2 can be thought of as a baseline expenditure required to provide a minimal level of services. The parameter β is determined by the preferences between local authority services and local taxation. The decision variables for the problem are X_i, the level of per capita expenditure and T_i, the tax bill.

The budget constraint derived under the two price system I presented before is drawn in figure four and can be written as:

$$X_i = G_{i1} + T_i(1-g_{i1}) + T_i g_{i1} \qquad (i=1,n) \quad \text{for } x \leq \alpha^* \qquad (2a)$$

$$X_i = G_{i2} + T_i(1-g_{i2}) + T_i g_{i2} \qquad (i=1,n) \quad \text{for } x > \alpha^* \qquad (2b)$$

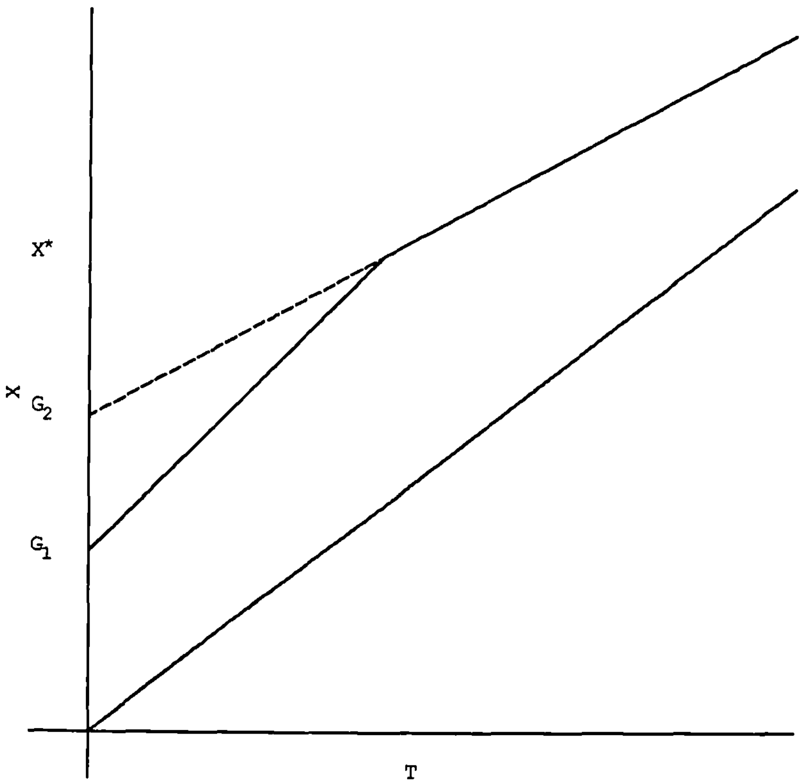

FIGURE FIVE

Where X_i is per capita expenditure, G_{ij} is the <u>implicit</u>

lump-sum grant for local authority i if it is located on segment j, T_i is the local per capita tax bill. As I noted before G_{i1} is set equal to a_2 , but for what it follows it is better to keep the two terms apart.

I assume as in the previous sections that the utility of each individual in authority i is given a specific form of the Stone Geary type then (1) can be rewritten as:

$$U_i = (1-\beta) \ln (a_{1,i} - T_i) + \beta \ln (X_i - a_{2,i}) \qquad (3)$$

In an homogeneous world without conflict of interest between voters and politicians or without misperception, maximisation of (3) subject to (2) yields a demand equation that, for segment j, can be written as:

$$X_{ij} = (1-\beta)\, a_{2,i} + \beta \left(\frac{a_{1i}}{g_{ij}} + G_{ij}\right) \qquad (4)$$

In the flypaper theory the representative voter is instead viewed as misperceiving the true cost of a unit of public services. Instead of thinking in terms of the true marginal cost of the service provision he thinks in terms of average costs. In this case the budget constraint is perceive to be:

$$X_{ik} = T_i \tilde{B}_{ik} \qquad (5)$$

$$\tilde{B}_{ik} = \frac{X_{ik}}{(X_{ik} - G_{ij})\, g_{ij}}$$

In terms of my diagram in figure four the budget line is replaced by a set of points on rays which emanate from the origin with positive slope as shown in figure six.

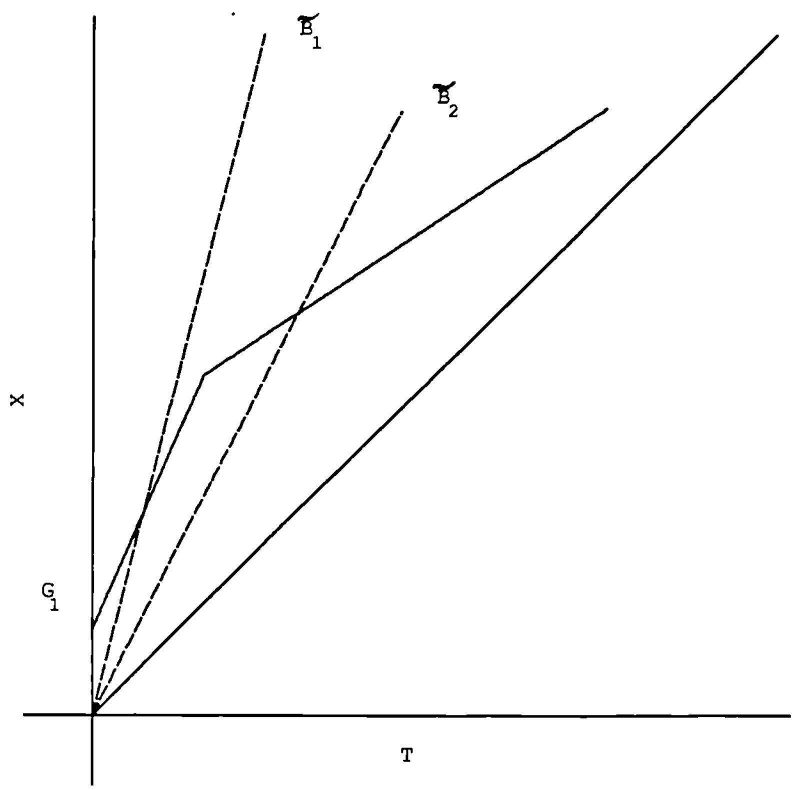

FIGURE SIX

Given a suggested expenditure level of (say) X_1 the local decision making body will concentrate on the local tax bill implication of this ,T_1, and hence an implicit post grant budget constraint of $\tilde{B}_1 = X_1/T_1$ is perceived. Similarly, for an expenditure level of X_2 the post grant budget constraint is perceived to be $\tilde{B}_2 = X_2/T_2$. More generally then the post grant budget constraint for any local authority i for a given level of expenditure, X_k is given by:

$$X_{ik} = T_{ik}\,\tilde{B}_{ij} \qquad (6)$$

where $\tilde{B}_{ik}$ is the perceived post grant tax base given expenditure level X_{ik}. Equation (6) can be rewritten to give:

$$X_{ik} = T_i(\tilde{B}_{ik} - 1) + T_{ik}B_i \qquad (7)$$

Here the first term on the right hand side of the equation represents the perceived matching grant and the second the locally raised revenue. By comparing equations (2) and (5) it can easily assessed that in the flypaper model the implicit lump sum grant disappears and revenues received in this form are perceived to be part of the implicit matching grant.[24] The reasons why bureaucrats might think in average terms can be explained in different ways: as a result of a process of decision making among a body of people with different objectives the average level of taxation is the crucial variable; on a different perspective uncertainty about the exact form of penalties ruling in different years combined with a grant setting that in its actual implementation was more complicated than the one I depicted might have induced decision makers to care about how much money they hope to receive from central government without bothering about the form and the marginal price of additional expenditure. Maximising (3) subject to (7) gives the following demand equation:

$$MsX_i = (1-\beta)\ a_{2i} + \beta\ (a_{1i}\tilde{B}_{ik}) \qquad (8)$$

---

[24] Only the points on the true budget constraint are however relevant, since by definition only at these points will the representative voter's budget constraint be balanced.

$$\hat{B}_{ik} = \frac{X_{ik}}{(X_{ik} - G_{i,j})g_{ij}} \qquad (9)$$

If I compare the two demand equations I can note that, for a given level of β, a_{2i}, a_{1i}, $G_{i,j}$, $g_{i,j}$ the demand derived under the misperceived budget constraint is greater than the one derived under the actual budget.[25] This effect is explained in figure seven; the positive implicit lump sum grant assures that the average price is always less than the marginal price; this statement in diagrammatic terms means that the misperceived budget is always steeper than the actual one[26] and that the flypaper equilibrium occurs at a point such as e_f where a tangency point on one of the perceived budget constraints coincides with a point on the true budget constraint.

---

[25]This can be readily assessed by observing that

$$a_{1,i}\hat{B} = a_{1,i}\left[\frac{G_{ij}}{T_{ik}} + 1\right]$$

This expression is equal to $\frac{a_{1,i}}{P_{ij}} + G_{ii}$ only if:

$a_{1,i} = T_{ik}$.

From (5) we know that utility function is defined only if $a_{1,i} > T_{ik}$, then the misperceived demand is greater than the standard one.

[26]The slope of the budget constraint is the inverse of the price.

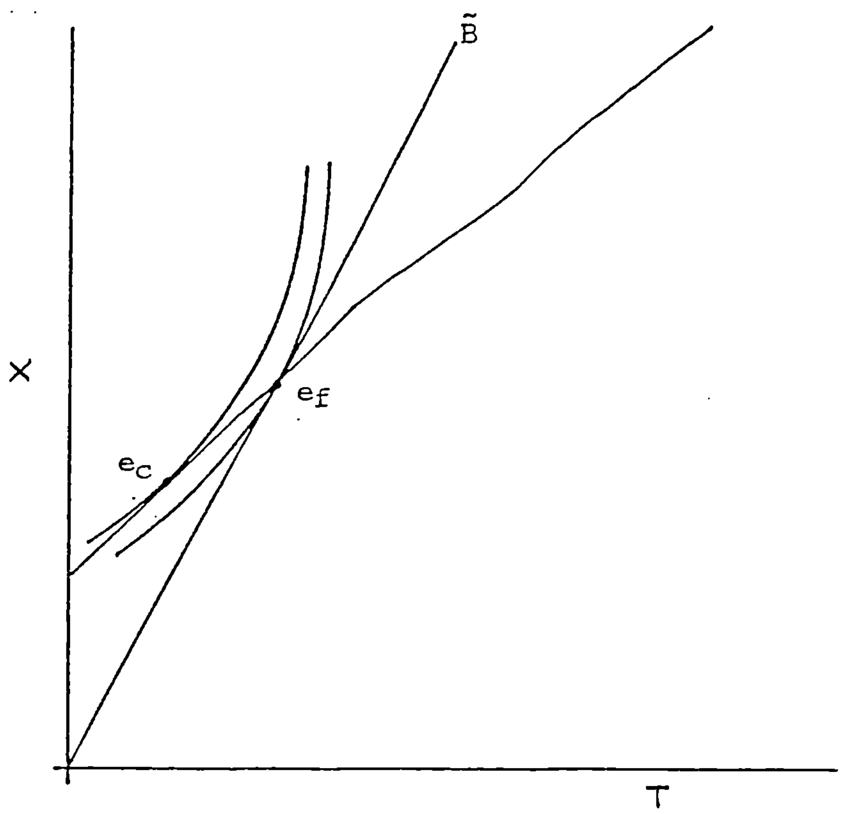

FIGURE SEVEN

Although it is not required for the discussion developed here, given the Stone-Geary preference function, the flypaper equilibrium is unique. This is the case since the price-consumption curve is downward sloping throughout and thus cuts the budget constraint, which is upward sloping throughout, once only; and the flypaper equilibrium occurs at point at which the price-consumption curve crosses the true budget constraint. Given convex preferences, had the local government correctly perceived the budget constraint, it would have been in equilibrium at a point such as e_c which involves a lower level of expenditure than does the flypaper equilibrium, e_f. From a policy point of view the existence of the flypaper effect is quite important. One of the main predictions of this theory is that the amount rather than the form of the grant matters i.e. the same amount of grant either given in lump sum , matching or block grant form should stimulate the same level of expenditure.[27] If local authorities behave according to this policy it is clear that the previous system of incentives fails to reach its objectives since it is designed to deal with marginal incentives rather than average ones. Even if local authorities revealed their true preferences, the matching grant offered to them on the first segment would be too high to achieve its objective. If Central Government knew that local authorities made their decision in average terms it would have to give either a matching grant equal to:

$$\left(1 - \frac{\beta \ a_1}{\alpha^* - \ (1-\beta)a_2}\right)$$

---

[27] The marginal effect of the lump-sum grant can however be quite different. From equations (6) and (8) I can see that the impact of a change in the implicit lump sum grant is respectively:

$$\frac{\partial X}{\partial M_{i,j}} = \beta_{nm}$$

$$\frac{\partial MsX}{\partial G_{i,j}} = \beta_m \frac{a_{1,i} P_{i,j} X_i}{(X_i - G_{i,j})^2} = \beta_m \frac{a_{1,i}}{T_i} * \frac{X_i}{(X_i - G_{i,j})}$$

If $\beta_{nm} = \beta_m$ I could conclude that the impact of a lump-sum grant is always greater if the flypaper effect exists. However, there is no reason why I should expect both values to be the same. Whether $\beta_m \dfrac{a_{1,i} B_{i,j} X_i}{(X_i - G_{i,j})^2} \gtrless \beta_{nm}$ is thus a matter of empirical investigation.

or a lump sum equal to[28]:

$$\alpha^{*} - \beta a_{1}\left(\frac{\alpha^{*}}{\alpha^{*}-(1-\beta)a_{2}}\right)$$

Under the previous system of grant the optimal total grant was equal to:

$$a_{2} + \left(1 - \frac{\beta\ a_{1}}{\alpha^{*} - a_{2}}\right)\alpha^{*} = \alpha^{*} - \beta a_{1}$$

Since $\alpha^{*} - \beta a_{1} > \alpha^{*} - \beta a_{1}\left(\frac{\alpha^{*}}{\alpha^{*}-(1-\beta)a_{2}}\right)$ the system fails to reach its objective.

The system of penalties is jeopardized as well since the average price on the second segment is clearly considerably lower than the marginal one and then it fails to restrain over spending.

4.2 DO LOCAL AUTHORITIES HAVE AN UTILITY FUNCTION AT ALL?

The second reason for the failure of the optimal grant allocation rule derives from the consideration that local authorities might not behave according to standard utility maximisation theory: although a trade off between expenditure and taxation exists, it cannot be represented by an utility function, at least in a conventional way. To

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[28] In this case, because of averaging the price the lump sum grant and the matching grant have the same effect and the equalizing grant is just a part of the total lump sum.

illustrate this point I will assume that local authorities do not have any preferences for local services in a standard definition but they can instead choose them according to the system ruling in a particular year. This clearly implies that β is not fixed through the lifetime of each local authorities and that maximisation is performed within each period. To illustrate this point I will recall that, from the previous system of grants the optimal amount of X and T can be written as:

$$X = a_2 + (\alpha^* - a_2)\,\theta$$
$$T = \beta a_1$$

Local authorities choose the best trade off according to the following utility function:

$$V_t = (1 - \beta)\,\ell n\,(a_1 - T) + \beta\,\ell n\,(X - a_2)$$

in which β is not a fixed parameter but a decision variable itself. Given the scheduled functions for X and T, β[29] will be chosen according to the following maximisation problem:

$$\text{Max } V = (1-\beta)\,\ell n\,(a_1 - \beta a_1) + \beta\,\ell n\,((\alpha^* - a_2)*\theta)$$
s.t. $\beta \leq \beta_2$

giving as optimal solution:

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[29] and X as a consequence.

$$\beta = \text{Min}\left[\beta_2 \, , \, \beta^*\right]$$

where β^* denotes the optimal value derived from the previous maximisation and it is mainly a function of a_1, the resources available for taxation and the system of incentives, Ω. The level of expenditure chosen [30], then, depend itself on the grant system and on the resources available[31]. This behaviour is clearly oriented to a maximisation of the grant received and does not respond to any life-cycle maximisation behaviour. β can vary across time according to the changes in the incentive systems offered. Such a behaviour can be justified in the context of modelling organization behaviour because the decision making process reflects the balance of different interest groups for which it is arguable that a, say, decrease in price could induce a revision in the expenditure decision process by which the pressure of the interest group claiming a high tax- high expenditure policy could prevail on the opposite low tax - low expenditure lobby.

5. CONCLUSIONS

In the last two chapters I have presented the basic theoretical model aimed at explaining and solving the asymmetry of information problem that exists between Central and Local government, Under the assumption of a static game

[30]through the choice of β

[31]Unless β that maximise the indirect utility function is greater than β_2.

no incentive compatible solution exists: the best that Central Government can do is to set targets and try to avoid local authorities to over spend. In a dynamic framework, an optimal solution can be found in the form of a set of rules by which Central Government is able to learn the true preferences for expenditure, The system requires an incentive to local authorities in the form of increasing expenditure and the rules must be set out before the start of the game through a closed loop policy which does not allow the players to revise their policies through time. The reasons why this system can fail to reach its objective can be different and I have pick up just two of them and explained how they can jeopardize the entire system. In the next chapter I will try to set up a series of test to assess whether local authorities actual behaviour reflects the assumptions made in the theoretical part by assuming that the two price system they face in real world has been set according to the optimal grant setting.

APPENDIX ONE.

The problem can be rewritten as:

$$\text{Min} \quad (1-g)\ X$$

$$\text{s.t.} \quad 0.5\left[x\left(\beta_1, M, g\right)\right] + 0.5\left[x\left(\beta_2, M, g\right)\right] \geq \alpha^*$$

Minimization of the grant leads to a price the locality has to face of the form:

$$g = f\left(E(\beta), M, \alpha^*\right)$$

Cobb Douglas example:

$$\text{Min} \ (1-g)\ X$$

$$\text{s.t.} \quad p\ \frac{\beta_1 M}{g} + (1-p)\ \frac{\beta_2 M}{g} \geq \alpha^*$$

The optimal g is equal to :

$$g = \frac{E(\beta) * M}{\alpha^*}$$

Given this g if the true value of β of the local authority is greater than the expected value of β, the local authority will buy an amount of Exp greater than α^* with a grant loss for government equal to:

$$(g_2 - g) \ * \alpha^* + (1 - g) * (X - \alpha^*)$$

where the first expression represent how much more government is offering to the local authority for each pound it spends than it would be necessary to it to reach the optimal demand α^* and the second expression is the grant loss due to the fact that in this second case the local authority is buying a quantity of X greater than α^*.

If the local authority has got a β less than its expected value, it will not reach the optimal amount α^*.
and the loss in grant that government has to face if the local authority has a β greater than β_1 is clearly reduced with respect with the case in which any authority has to spend at least α^* with certainty.

APPENDIX TWO

Cobb Douglas Example.

$$\text{Min} \quad (1-g)\,\alpha^{*} + (1-h)\,(X - \alpha^{*})$$

$$\text{s.t.} \qquad p\,\frac{\beta_1 M}{g} + (1-p)\,\frac{\beta_2 M}{g} \leq \alpha^{*}$$

$$\frac{\beta_2 M}{h} \leq \alpha^{*}$$

Minimization leads to:

$$g = \frac{E(\beta) * M}{\alpha^{*}}$$

and h :

$$h \geq \frac{g_2 - \beta_2 * g}{1 - \beta_2}$$

If Government wants any local authority to spend at least α^{*}, it has to set:

$g = g_1$ and:

$$h \geq \frac{g_2 - \beta_2 * g_1}{1 - \beta_2}$$

APPENDIX THREE

NUMERICAL EXAMPLE

INCOME = 2000

GREA = $\alpha^{*} = 450$

$\beta_1 = 0.05$

$\beta_2 = 0.2$

NO GRANT SETTING:

$X_1 = 100$

$X_2 = 400$

"SEPARATING EQUILIBRIUM" i.e. Government knows the true preferences of the local authority.

$\beta = 0.05$

Offer a matching grant $(1-g_1)$ such that:

$$\frac{\beta_1 M}{g_1} = \alpha^{*}$$

$$g_1 = \frac{2}{9}$$

Grant:

$$(1 - g_1) \cdot 450 = 450 - 400 = 50$$

$\beta = 0.2$

$$\frac{\beta_2 M}{g_2} = \alpha^{*}$$

$$g_2 = \frac{8}{9}$$

Grant:

$$(1 - g_2) \cdot 450 = 450 - 100 = 350$$

I.C.C PROBLEM.

If Government wants local authority to spend :

$E(X) = \alpha^*$

has to offer a grant equal to:

$$g = \frac{E(\beta) * M}{\alpha^*} = \frac{0.125 * 2000}{450} = \frac{5}{9}$$

Ex post:

If locality has got $\beta = 0.05$
it would spend:

$$X = .05 * 2000 * \frac{9}{5} = 180 < 450$$

If locality has got $\beta = 0.2$
it would spend:

$$X = .2 * 2000 * \frac{9}{5} = 720 >. 450$$

In this second case the grant would be:

$(1-g) * 720 = 320$

while in the no uncertainty case it was 50.

The loss of 270 pounds is made up as such:

$(g_2 - g) * 450 + (1-g) * (720 - 450)$

If Government wants the local authority to spend anyway at least 450 it has to offer:

$$g = g_1 = \frac{2}{9}$$

Ex post:

If locality has got $\beta = 0.05$
it would spend:

$$x = .05 * 2000 * \frac{9}{2} = 450 = 450$$

If locality has got $\beta = 0.2$
it would spend:

$$x = .2 * 2000 * \frac{9}{2} = 1800 > 450$$

In this second case the grant would be:

$(1-g)* 1800 = 1400$

while in the no uncertainty case it was 50.

The loss of 1350 pounds is made up as such:

$(g_2 - g_1) \cdot 450 + (1-g) \cdot (1800 - 450)$

To reduce the loss in grant Government can enforce a system of subsidies and penalties.

If it wants the local authority to spend :

$E(X) \geq \alpha^*$

the contract has to specify :

$$g = \frac{5}{9}$$

$$h = \frac{\frac{8}{9} - \frac{1}{5} * \frac{5}{9}}{1 - \frac{1}{5}} = \frac{35}{36}$$

Ex post:

If locality has got $\beta = 0.05$
it would spend:

$$X = .05 * 2000 * \frac{9}{5} = 180 < 450$$

If locality has got $\beta = 0.2$
it would spend:

$$X = .2 * 2000 * \frac{9}{5} \quad \text{for } X \leq 450 \quad ; X = 450$$

$$X = .2 * 2000 + 450 * \left(\frac{35}{36} - \frac{5}{9} \right) \cdot \frac{36}{35} = 450$$

The locality with high preference is made stuck at the kink.

In this second case the grant would be:

$(1-g)* 450 = 200$

while in the no uncertainty case it was 50.

The loss of 150 pounds is made up as such:

$(g_2 - g\ ) * 450$

If it wants the local authority to spend anyway 450 pounds the contract has to specify :

$$g = g_1 = \frac{2}{9}$$

$$h = \frac{\frac{8}{9} - \frac{1}{5} * \frac{2}{9}}{1 - \frac{1}{5}} = \frac{19}{18}$$

Ex post:

If locality has got $\beta = 0.05$
it would spend:

$$X = .05 * 2000 * \frac{9}{2} = 450 = 450$$

If locality has got $\beta = 0.2$
it would spend:

$$X = .2 * 2000 * \frac{9}{2} \quad \text{for } X \leq 450 \quad ; X = 450$$

$$X = .2 * 2000 + 450 * \left(\frac{19}{18} - \frac{2}{9} \right) * \frac{18}{19} = 450$$

The locality with high preference is made stuck at the kink.

In this second case the grant would be:

$(1 - g_1) * 450 = 350$

while in the no uncertainty case it was 50.

The loss of 300 pounds is made up as such:

$(g_2 - g_1) * 450$

The final system can be summarized as follows:

Incentive system:

$$\Omega_t = \left(\frac{1 - \beta}{1 - \beta_1} \right)^{\left(\frac{\beta - 1}{\beta} \right)} \simeq 1.9 \qquad t = 1,4$$

Prices on the first segments:

$$g_{i,1} = \frac{\beta \quad M}{\alpha^{*} \ \theta_i} \simeq \quad 0.46 \qquad\qquad i = 1,4$$

Prices on the second segments:

$$g_{1,2} = \frac{M(4 - \beta \ - 3\beta_2)}{4 \ \alpha^{*}\theta_1(1 - \beta_2)} * \beta_2$$

$$g_{2,2} = \frac{M(4 - 2\beta \ - 2\beta_2)}{4 \ \alpha^{*}\theta_2(1 - \beta_2)} * \beta_2$$

$$g_{3,2} = \frac{M(4 - 3\beta \ - \beta_2)}{4 \ \alpha^{*}\theta_3(1 - \beta_2)} * \beta_2$$

$$g_{4,2} = M(1-\beta \ ) * \frac{\beta_2}{\alpha^{*}\theta_4(1-\beta_2)}$$

The grant that they will receive will be equal to:

450*1.9 -400 = 455 in all the four periods

while in the certainty case it was equal to 50
The loss is made up as follows:

450*(1- .9) = 405

APPENDIX FOUR

The demand equation for expenditure can be written as:

$$X = (1-\beta)\, a_2 + \beta \left(\frac{a_1}{g} + G \right)$$

IF $G = 0$, in order to reach a_2 the price each local authority has to face would be equal to :

$$g = \frac{a_1}{a_2}$$

then the matching rate is equal to :

$$(1 - g) = \frac{a_2 - a_1}{a_2}$$

and the total grant is equal to:

$$a_2 - a_1$$

By setting $g = 0$, G should then be equal to:

$$a_2 - a_1$$

If $G = a_2$ and $X = \alpha^*$

$$g = \frac{\beta\, a_1}{\alpha^* - a_2}$$

APPENDIX FIVE

In the maximisation problem presented in the text I have assumed that local authorities can transfer money freely i.e. they can over spend their budget in some periods if they want to and then borrow money. It would be argued that local authorities can only use their own resources, i.e. they are not allowed to borrow.
I could have taken into account this suggestion by allowing local authorities to put forward resources but not backward. This can be done by slightly changing the constraints.
In the first period the constraint would be written as:

$$g_1\alpha_1^* + y_1 \leq M$$

$$y_1 \leq M\ (1 - \beta_d)$$

Now if I recall the previous optimal value for y_1 it is possible to show that this constraint is satisfied.

$$y_t = \frac{M(4 - 3\beta_d)}{4 - 3\beta} * (1 - \beta)$$

$$\beta_d = \beta - \Delta$$

$$\beta = \beta_d + \Delta$$

$$\Delta = \beta - \beta_d$$

$$\frac{M(4 - 3\beta_d)}{4 - 3\beta} * (1 - \beta) \leq M\ (1 - \beta_d)$$

$$\beta < 1$$

$$4 - 3\beta > 0$$

$$M(4 - 3\beta + \Delta)\ (1 - \beta) \leq M(1 - \beta + \Delta)\ (4 - 3\beta)$$

$$\Delta M\ (1 - \beta) \leq \Delta M\ (4 - 3\beta)$$

$$\Delta > 0$$

$$(1-\beta) \leq 4 - 3\beta$$

$$2\beta \leq 3$$

which is always satisfied.

APPENDIX SIX.

The budget constraint for each local authority in period four can written in an alternative way by observing that if in the first three periods:

$$T_t = \frac{a_1}{(4-3\beta)} * \left[3\beta_d(1-\beta) + \beta\right] \qquad t = 1, 3$$

while, due to their cheating, the grant system would have allowed them to be budget balanced by just raising:

$$T_t = \beta_d \ a_1 \qquad t = 1,3$$

The difference between the two expressions give the amount of resources that the local authority can transfers forward. The total amount at the end of year three will be equai to:

$$K = 3 \left[\frac{a_1 \ (\beta - \beta_d)}{4 - 3 \ \beta} \right]$$

such that the budget constraint in period four can be rewritten as:

$$g_4 X_4 - T_4 = a_2 g_4 + K$$

which clearly gives the same optimal X_4 as in the text.

If Central Government wants the representative local authority not to over spend the grant on the second segment can be found by first observing that the budget on the second segment will have to be written as:

$$g_{4,2} X_4 - T_4 = a_2 g_{4,2} + K + (\alpha^* - a_2) * (g_{4,2} - g_{4,1})$$

Maximising the utility function subject to this new budget constraint and imposing the condition $X_4 \leq \alpha^*$ gives:

$$g_{4,2} \geq \frac{\beta\left[a_1 \ (1 - \beta_d) \ + \ K\right]}{(\alpha^* - a_2) \ (1 - \beta)}$$

Analogous proof can be developed for $g_{3,2}$ and $g_{2,2}$.

In the two latter cases it the amount of savings available to be changed. As a result of less periods in which they cheat the quantity K has to be proportionally reduced.

CHAPTER FIVE

1. INTRODUCTION

In the previous chapter I have presented the optimal set of rules that in theory could enable Central Government to solve the asymmetry of information it has to face and some reasons for the failure of this system were argued. In the next two chapters I will compare the actual grant system with the optimal one and I will assess to what extent the history of the grant system in Britain reflects the learning process that Central Government has to face. This chapter presents the estimates ofestimate the parameters of the utility function for local authorities and some test for the consistency of the assumption underlying the theoretical model; the actual comparison between the two systems will be presented in chapter six.

In the first part of this chapter I will briefly recall the framework of the analysis and I will then review and compare the principal estimation techniques developed in the econometric field to cope with convex nonlinear budget constraints. In the third part, by using the model suggested by Arrufat and Zabalza I will test the hypothesis that local authorities' behaviour is consistent with a Cobb Douglas utility function and I will derive the parameter estimates for this case. In the last section I will test the existence of a flypaper effect in local government behaviour and I will try to assess whether utility maximisation, at least in a conventional definition, can approximate local government behaviour. The analysis will be restricted to the years 1986-87 - 1988-89, i.e the years in which the system of grant allocation is more similar to the setting I have derived in chapter four.

1.1 THE MODEL.

My analysis is based on the assumptions that local authorities behave as if they were social welfare maximisers and their utility can be written in very general terms as:

$$W\ (X,\ T) \qquad W_x > 0 \qquad W_T < 0 \tag{1}$$

where X is per capita expenditure and T is the (per capita) revenue each authority has to raise from taxation if it would like to spend X.[1] The utility is given the specific functional form of a Stone Geary, namely:

$$W_i = (1-\beta)\ \ln\ (a_{1i} - T_i) + \beta\ \ln\ (X_i - a_{2i}) \tag{2}$$

Defining utility over local public expenditure and taxation is a constrained choice since it is impossible to collect reliable data for local net income. The budget constraint faced by local authorities has then to be defined in terms of resources available to provide local public services. Local authorities have three major sources of finance: central government grants, the rates, and user charges. The

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[1] An alternative way of modelling utility function would have been of introducing tax rates instead of tax bills ; both methods have their own theoretical support; I argue that individuals care about tax bills and the empirical work is conducted within this framework.

bulk of central government grant is distributed as a block grant, a unitary grant intended to equalized simultaneously for differences in needs and differences in resources between authorities. By convention, expenditure is reported net of specific grants and user charges, so the budget constraint is defined only over this block grant and the local property tax, the rates. The budget constraint for each local authority can be written in a very general form as:

$$X = T + G \tag{3}$$

The system by which block grants are allocated causes the budget constraint to be piecewise linear and has been changed through years with the number of kinks varying according to the penalty system scheduled. From fiscal year 1986/7 the system of grant allocation made the budget constraint simpler by introducing only one kink at threshold [2]. The precise details of these discontinuities in the budget constraint are explained in Barnett et al. (1989b) and more detail will be given in the following chapter. For the time being it is sufficient to note that the post-grant budget constraint along any segment j can be written as:

$$g_{ij} X_i = g_{ij} \; G_{ij} + T_i \tag{6}$$

G_{ij} is the implicit lump sum grant on segment j

g_{ij} is the implicit price on segment j.

---

[2]Threshold is a parameter peculiar of the block grant and is equal to:

Both G and g derives can be derived from the Block Grant allocation rules, are peculiar to each authority and can be observed. The budget constraint for the local authorities is then equivalent to figure three in the previous chapter but it will be presented here again for the sake of completeness.

For a representative authority i the budget constraint will look like figure one.

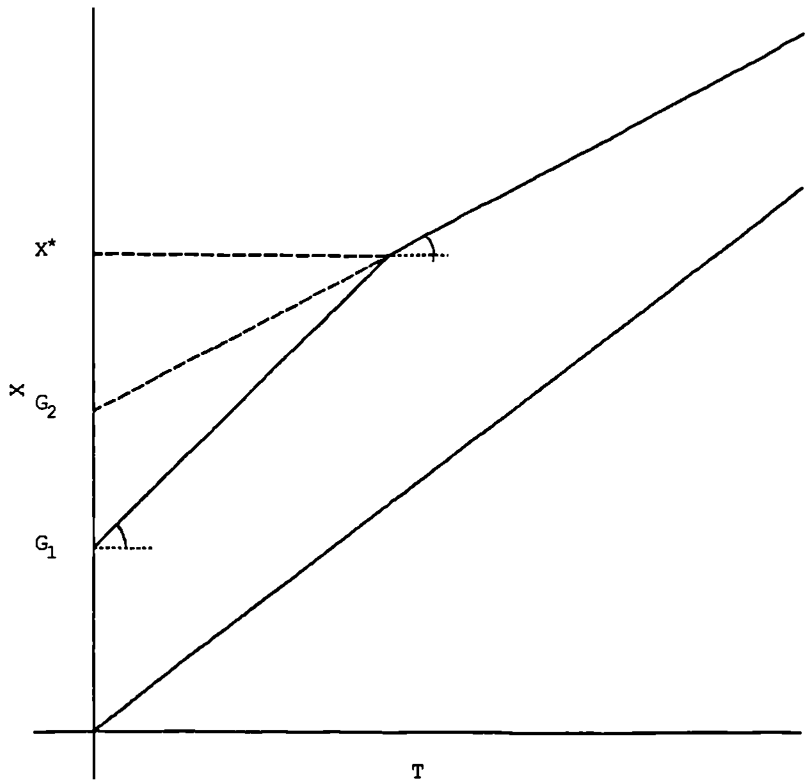

FIGURE ONE

As long as $g_{i2} > g_{i1}$ the budget constraint depicted in figure 1 is convex, as we showed in Barnett et al. (1989b)[3]

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[3]Actually the whole budget set faced by local authorities presents two regions of nonconvexity that I have not considered in this analysis.

In this study I will start by implicitly assuming that the actual grant system is set according to the optimal allocation rule and I will derive the parameters of the utility function revealed by local authorities through their expenditure decisions. The estimation of the demand equation is not straightforward because of the piecewise nature of the budget constraint; an ad hoc technique has to be devised to obtain reliable estimates.

2. NONLINEAR BUDGETS AND ECONOMETRIC ISSUES.

Traditional empirical analysis of consumer demand has usually made the assumption of linear budget sets, that is consumers are assumed to purchase any desired quantity at a constant price subject to a linear budget constraint. In recent years, expecially after the new development concerning the optimal income tax schedule a growing number of researchers have tried to assess labour supply responses to changes in the income tax rate. With a progressive tax the marginal after tax rate depends both on the tax schedule and on the total hours worked so the simple model described before can no longer be used.

The main problem when dealing with a piecewise linear budget set is that, due to the discontinuities in the price schedule the budget line consists of different segments each one associated with a different marginal price.

In principle it would be possible to use an OLS ,but in order to perform a satisfactory analysis the researcher should be able to know the particular budget line segment and thus the particular marginal price that each individual has chosen. This introduces a quite important complication since available data on expenditure may measure it with some error. If so, such data do not always correctly indicate the specific budget line that a particular authority has chosen.

In the early stage of the analysis of piecewise linear budget constraints some attempts at least squares estimations were actually performed. In a fairly well known paper Hall (1973) was the first to consider explicitly the problem. Hall showed that if the budget constraint is convex the piecewise budget can be thought of as a set of linearized segments and that tangency of the Indifference Map with one of the linearized segments assures the uniqueness of the solution. Hall applied a least squares technique by assigning observations to the segment on which they located and ignoring the kink. The method was quickly realized to generate biased and inconsistent estimates and served as an impetus to much of the subsequent research.

One of the most interesting aspects of the econometric problem involved in estimation subject to piecewise linear constraints is that the stochastic specification is of great importance and the error terms have more specific interpretations for the parameter estimates. Using a standard OLS procedure the demand equation subject to a nonlinear budget constraint would be written as:

$$X = D_1 \cdot X(g_{i1}, G_{i1}) + D_2 \cdot X(g_{2i}, G_{2i}) +$$

$$(1-D_1-D_2)\ X^* + \varepsilon \tag{5}$$

$$D_1 = 1 \quad \text{if } X^* > X(g_{i1}, G_{i1})$$
$$= 0 \quad \text{otherwise.}$$

$$D_2 = 1 \quad \text{if } X^* < X(g_{i2}, G_{i2})$$
$$= 0 \quad \text{otherwise.}$$

where $X(g_{i1}, G_{i1})$ is the demand on the first segment and:

$X(g_{i2}, G_{i2})$ is the demand on the second segment which in turn implies that: $X(g_{i1}, G_{i1}) < X(g_{i2}, G_{i2})$

As Hausman first pointed out ε, the error term, includes both optimization errors, specification errors and measurement errors and Moffit (1986) explained in great detail the reasons why OLS approach leads to biased estimates. I will recall here the most important. In the previous model the least squares approach is inconsistent because it assumes that the segment on which observations are located are nonstochastic indicators of the variables D_1 and D_2 but both those variables are not observed. Since by assumption ε has a nonzero variance, the segment on which an observation is located might not necessarily be the one on which utility maximisation occurs; it will be so only when the error term is sufficiently small as to not move the observation to a different segment. The consequence is that there is a systematic correlation between the size of the error term and the marginal price and imputed income assigned as regressors. Those problems raise some interesting technical questions about the most appropriate technique to deal with them and have received two basic treatments in econometric analysis. Some researchers have used the theory of latent variables and adapt it to cope with both participation problems and the need of modelling the error term in a different way; the most recent approach to the problem tries instead to taking account of the different sources of errors by explicitly modelling them and their behaviour in the function to be estimated.

2.1 INDEX FUNCTION APPROACH.

The statistical theory of index functions has its foundations in the literature on endogenous variables in a system of simultaneous equations and provides a conceptually simple framework for modelling corner solutions to consumer utility maximisation problems. This literature is based on the notion that discrete endogenous variables can be thought of being generated by continuous latent variables crossing thresholds. The approach has first been developed in consumer theory to cope with participation problems in which the idea of ranking observations according to the value of a specific shifter parameter has been applied to situations characterized by multiple choices. The model has been widely used in labour supply models under progressive income taxation which causes the post tax budget constraint to be piecewise linear. For any chosen utility function it is possible to determine the optimal allocation of resources for an individual authority in terms of the equivalence between the Marginal Rate of Substitution [4] and the price the authority has to face. When dealing, as in this case with convex preferences and convex constraint set, a local comparison of the marginal rate of substitution function with the price of different segments determines the location of any individual authority on the budget set.

For the specific problem at hand the marginal rate of substitution for locality i can be defined as:

$$MRS_{T_i, X_i} = \frac{d X_i}{d T_i} = - \frac{\frac{\partial W_i}{\partial T_i}}{\frac{\partial W_i}{\partial X_i}} = \frac{1 - \beta}{\beta} \cdot \frac{X_i - a_{2,i}}{a_{1,i} - T_i} \qquad (6)$$

By recalling equation (4), the budget set on segment j can be written as:

$$g_{ij} \cdot X_i - T_{ij} = g_{ij} \cdot G_{ij} \qquad (7)$$

such that an optimal tangency point is defined by the relation:

$$MRS_{T_{ij}, X_{ij}} = \frac{1}{g_{ij}} = f_{ij} \qquad (8)$$

For exposition purposes let us consider a model with only one kink as the one depicted in figure two.

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[4]The marginal rate of substitution will be referred as MRS in the following analysis.

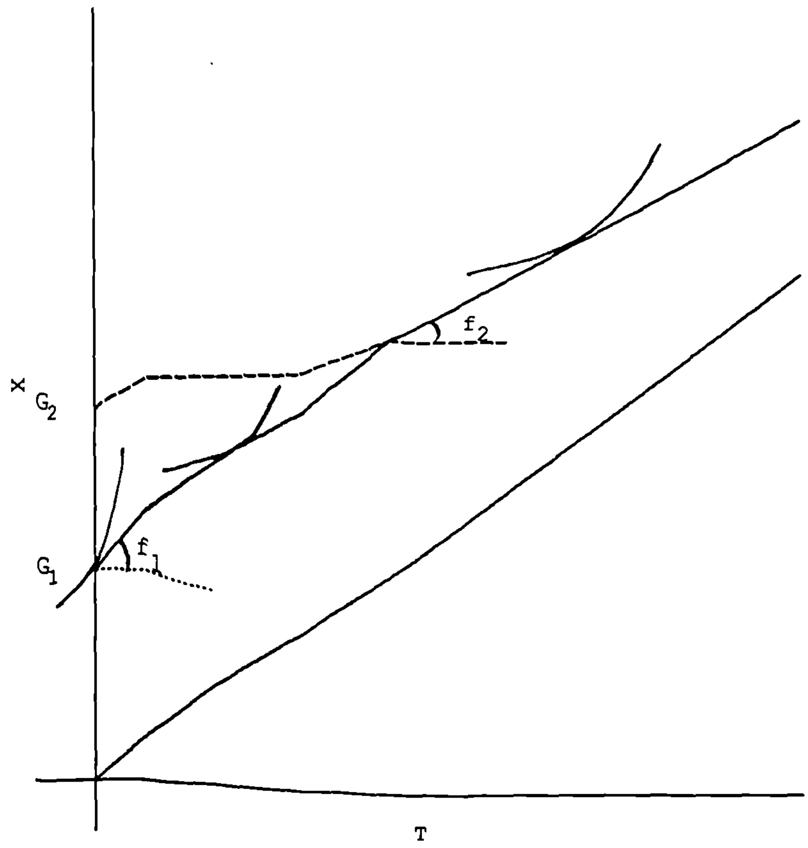

FIGURE TWO

Denoting the marginal rate of substitution along segment j by MRS_j and dropping the subscripts to simplify the notation, an authority will be at a corner solution (zero expenditure) if $MRS_1 > f_1$ it will chose to locate on the first segment if $MRS_1 < f_1$ and $MRS_2 > f_1$ it will be located at the kink if $f_1 \geq MRS_2 \geq f_2$ it will choose to locate on the second segment if: $MRS_2 < f_2$ The procedure can be easily extended to cope with more than one kink; furthermore since local authorities are not faced with a participation problem (their expenditure being greater than zero anyway) the first state will not be considered. To set up an estimation algorithm it is necessary to define a parameter on which to index the different states of the world and thus determine which state each individual authority chooses in relation to the value taken by this variable. In a recent paper [5], in order to compare the relative performances of the index function approach and the two errors model I assigned to a_{2i} the role of taste shifter, i.e. a_2 was replaced by $\alpha \sim (\bar{a}_2, \sigma^2_\alpha)$. I will use here the same approach to explain the estimation technique that can be derived using this approach. For given values of G_i, g_i, $a_{1,i}$ any authority will choose one of the possible states according to the region in which the random variable α_2 lies. These regions can easily be defined by observing that the marginal rate of substitution is monotonically decreasing in a_{2i}. The choice of a distribution for α yields a complete statistical characterization of local government behaviour For example, for the Stone Geary utility function I have used in the previous chapters to describe local authorities behaviour, α can be defined as follows:

$$\alpha = \frac{\beta}{\beta-1} * \frac{a_1 - T}{g_j} + X \tag{9}$$

Since $f(\alpha)$ has been chosen to be the normal density, an ordered probit scheme yields to consistent estimates for the parameters of the marginal rate of substitution and of the variance of α.

The procedure can be described as follows:

$$\alpha = \gamma * \frac{a_1 - T}{g_j} + \delta * X \tag{10}$$

---

[5] See Barnett et al (1989c).

$\gamma = (1-\beta)/\beta$

$\delta = 1.$

α=1 if the observed expenditure lies on the first segment

α=2 if the observed expenditure lies on the kink and so on.

The model is characterized by the assumption that the segment each locality chooses can be observed directly. Knowledge of local authorities' expenditure and the price is all information that is required to set up the model. However, it is important to note that this approach does not actually use any information about the observed expenditure data since the model is based on the region of the budget constraint in which the latter lies.

Anyway, observed expenditure ranking cannot by any means assumed to be the preferred one since the presence of different sources of errors could make the preferred position shift to a different segment [6]. The estimation procedure, by not taking account of the different sources of errors, has a poor predictive power; in particular a cluster of observations around the kink will be observed[7]. The technical reasons for this pitfall have been explained by Moffit (1986); to have an insight in the problem it is sufficient to remark that, while only one combination of parameters is compatible with maximisation over a segment a range of different values satisfying the kink.

[6] A more comprehensive explanation of the problem can be found in Moffit (1986).

[7] i.e. the distribution of the probability for the expected values will be clustered around the kinks.

2.2 TWO ERRORS APPROACH

Several new techniques have recently been developed to deal with the problem of estimating demand functions when the budget set is piecewise linear. Beginning with the study by Burtless and Hausman (1978) and later work by Hausman a very general econometric solution to the estimation problem has been shown to exist. All those approaches start with the suggestion that, due to the presence of a piecewise linear budget set, it is not efficient to assume that the random error affects the overall demand equation in the same way: the error has to be split in two parts: the so called heterogeneity error that allows observations to be located on different segments or at kinks and the random error that determines the actual allocation within a segment or kink. The two errors have then different implications for the data: whereas the measurement error tends to spread observations out evenly over the constraint, heterogeneity of preferences tends to generate clusters of observations at the kink point of a convex constraint.[8] The method has been applied to different type of problem both convex and nonconvex using different functional forms for the demand equation and different stochastic specifications for the heterogeneity error. Elsewhere I chose to implement the approach described by Moffit (1986) for estimation purposes and the comparison between the index function approach and the two errors model [9] showed that the two errors model,

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[8]See Moffit (1986) for a formal proof.

[9]See Barnett, Levaggi, Smith (1989a) for a full account of those aspects.

although requiring ad hoc algorithm to be estimated, leads to significantly more consistent estimates. For this reason I will use also in this analysis some models that explicitly take account of the different sources of errors and I will explain the technical details while I present the estimation results to which they lead.

3. EMPIRICAL ANALYSIS.

The first part of the empirical analysis is devoted to test for the hypothesis that local authorities behave as if they were maximising an utility function belonging to the Cobb Douglas class. This is the natural first step before estimating the demand equations for local authorities I presented in the theoretical part. In this analysis I will use a model similar to the one proposed by Arrufat and Zabalza (1985) which assumes the heterogeneity error [10] to enter in the model in an additive form. The model proposed by Arrufat and Zabalza uses a CES utility function of which the Cobb Douglas is just a special case.

The model I will estimate empirically will, however differ from the one proposed by Arrufat and Zabalza for two main reasons:

a) on the utility specification side, local authorities behaviour is represented by choosing the optimal amount of two commodities one of which is actually a bad.

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[10] In Barnett et al (1989a) we presented a model in which the heterogeneity error represented "needs" and entered in the demand equation in an additive form.

b) all local authorities are spending a positive amount for local public goods, so no participation problems arise.

The structural model is based on a CES utility function which, despite not generating a linear function for expenditure, turns out to be very convenient for estimation purposes. The procedure can be described as follows:

The utility of each individual in authority i is given the following specific form:

$$W_i = \left[\tau * (X_i - a_{2i})^{-\rho} + (a_{1,i} - T_i)^{-\rho} \right]^{-(1/\rho)} \qquad (11)$$

where the parameter a_{1i} reflects the underlying income available in the area for local authority service provision and a_{2i} can be thought of as a baseline expenditure required to provide a minimal level of services. At the time of budget setting[11] Central Government decision makers have to guess the baseline level of expenditure to be provided and the income available for taxation. I will assume, in analogy with the theoretical model described in chapter four that the implicit lump sum grant on the first segment is Central Government perception of the need for any authority while income available for taxation might be approximated by the maximum rate poundage increase applied by any of the authority of the same class in the previous year without incurring rate capping, multiplied by the rateable value of the local authority. The parameter τ indicates the weight on expenditure relative to taxation while ρ determines the

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[11] i.e. one year in advance with respect to actual expenditure decisions.

elasticity of substitution between expenditure and taxation, $s = 1/(1+\rho)$.

The decision variables for the problem are X_i, the level of per capita expenditure and T_i, the tax bill.Convexity of indifference curves requires that $\rho > -1$. At the maximum of (11) given the budget constraint defined by equation (5), the condition [12]:

$$\frac{X_{ij} - a_{2,i}}{a_{1,i} - T_{ij}} = \left[\frac{\tau}{g_{ij}}\right]^{s} \tag{12}$$

must be satisfied for an interior solution on each segment. The optimal behaviour can be fully described by the expenditure-taxation ratio from which it is possible to recover all the parameters needed to estimate the utility function. I will now describe the set of opportunities open to each local authority. Figure three represents the budget constraint for a representative authority which has been plotted for convenience in the $(X - a_2) - (a_1 - T)$ space.

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[12]The optimum point is characterized by the equivalence between the MRS and the price ratio. Then it follows that at the optimum:

$$\left[\frac{X_{ij} - a_{2,i}}{a_{1,i} - T_{ij}}\right]^{\frac{1}{s}} \frac{1}{\tau} = \frac{1}{P_{ij}}$$

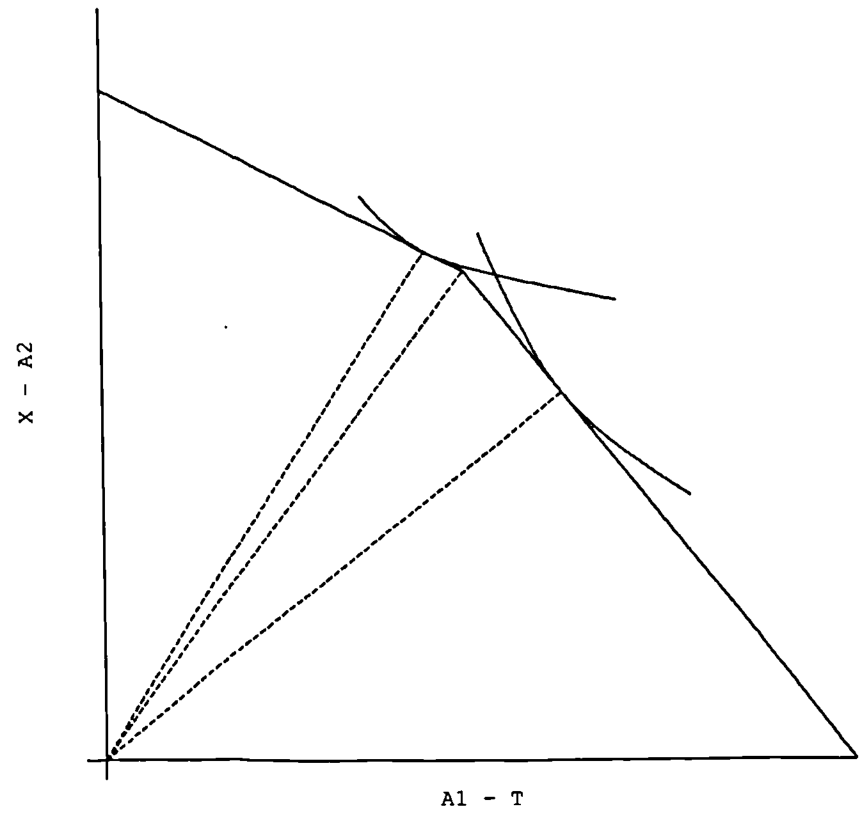

FIGURE THREE

This local authority could spend up to a_2 without any sacrifice, but beyond this point it has to give up resources according to the matching grant that Central Government offer on the different segments. I have not depicted the budget for the region in which $X < a_2$ since because of the utility function I am using local authorities are constrained to spend more than a_2 anyway. As for the empirical analysis that will follow a_2 is set to a level such that all local authorities spend more than this amount. Local authorities expenditure decisions can be summarized as follows [13]:

[13] The approach, at least at this stage is very similar to the index function model to which the reader is referred for definition of symbols and derivation of such conditions.

$MRS_1 < \frac{1}{g_1}$ $\Rightarrow$ locate on the first segment.

$MRS_2 < \frac{1}{g_2}$ $\Rightarrow$ locate on the second segment.

$\frac{1}{g_1} \geq MRS_2 \geq \frac{1}{g_2}$ $\Rightarrow$ locate at the kink.

For the CES function the Marginal rate of substitution can be written as [14]:

$$\left[\frac{X_{ij} - a_{2,i}}{a_{1,i} - T_{ij}} \right]^{\frac{1}{s}} * \frac{1}{\tau}$$

As per the index function model a variable on which to index the model is needed. The model considers two sources of stochastic variation. First I will assume that the weight on expenditure is determined by characteristics peculiar to each local authority and by a random error. I will define, dropping the i subscripts for ease of exposition:

$$\tau = \exp \left\{ \alpha - \mu D \right\} \tag{13}$$

where D is a vector of characteristics associated with local authority i and α is a random variable which is assumed to be normally distributed with zero mean and variance σ_α^2. α thus represent the heterogeneity between different authorities in the sample. The maximisation of the utility function and the definition of τ will lead to the desired expenditure-taxation ratio which is defined as:

[14]This can be obtained straightaway from equation 12.

$$\left[\frac{X - a_2}{T - a}\right]^{*}.$$

The second source of stochastic error, the random error enters in the function for the expenditure-tax ratio in the following way:

$$\left[\frac{X - a_2}{T - a_1}\right] = \left[\frac{X - a_2}{T - a_1}\right]^{*} \exp(\varepsilon) \qquad (14)$$

where ε is a random variable which is assumed normally distributed with zero mean and variance σ^2_ε I will assume, following Hausman (1981) and Arrufat and Zabalza (1986) that ε and α are independent. In order to derive the likelihood function for the problem I will take the natural logs of equation (14) and then the likelihood will be defined in terms of the expenditure-taxation ratio expressed in natural logs. This allows ε and α to enter in the estimating equation in an additive fashion. Any local authority will desire to locate along the first segment of its budget constraint, at the kink or on the second segment according to the value of the parameter α. It can be easily assessed that authorities will locate:

on the first segment if: $\alpha < \alpha_1$

on the kink if $\alpha_1 \leq \alpha \leq \alpha_2$

on the second segment if $\alpha > \alpha_2$

$$\alpha^{*}_{i} = (1+\rho)\ Q + \mu D + \ell n\ g_i \qquad i=1,2$$

$$Q = \ln\left(\frac{X^{*} - a_2}{a_1 - (X^{*}-a_2)*g_1}\right) = \text{expenditure-taxation ratio at the kink (in logs)}$$

By starting with the heterogeneity error only model the probability density for each point might be defined as:

$$\begin{aligned} Pr(E) &= Pr\left[\alpha = E - R_1;\ \alpha < \alpha_1^{*}\right] && \text{for } E < Q \\ &\quad Pr\left[\alpha = E - R_2;\ \alpha > \alpha_2^{*}\right] && \text{for } E > Q \\ &\quad Pr\left[E = Q\ ;\ \alpha_1^{*} < \alpha < \alpha_2^{*}\right] && \text{for } E = Q \end{aligned} \qquad (15)$$

where:

$$E = \ln\left(\frac{X - a_2}{a_1 - T}\right) = \text{observed expenditure-taxation ratio}$$

$$R_i = -s*\mu D - s\ln(g_i) \qquad i=1,2$$

When the second error is introduced the probability of each point is modified as follows:

$$Pr(E) = Pr\left[\alpha + \varepsilon = E - R_1;\ \alpha < \alpha_1^{*}\right] \qquad \text{for } E < Q$$

$$\Pr\left[\alpha + \varepsilon = E - R_2;\ \alpha > \alpha_2^{*}\right] \qquad \text{for} \quad E > Q$$

$$\Pr\left[\varepsilon = Q\ ;\quad \alpha_1^{*} < \alpha < \alpha_2^{*}\right] \qquad \text{for} \quad E = Q$$

(16)

The likelihood for this problem can be written as:

$$L = \int_{-\infty}^{\alpha_1} \ell\left[E - R_1^{*} \right] g(\alpha)\ d\alpha\ d\varepsilon \quad +$$

$$\int_{\alpha_1}^{\alpha_2} \ell\left[E - Q \right] \quad g(\alpha)\ d\alpha\ d\varepsilon \quad +$$

$$\int_{\alpha_2}^{\infty} \ell\left[E - R_2^{*} \right] g(\alpha)\ d\alpha\ d\varepsilon \qquad (17)$$

where :

$$R_i^{*} = s{*}\mu D - \ell n(g_i) + s{*}\alpha \qquad i=1,2$$

The derivation of the likelihood is equivalent to the one presented by Arrufat and Zabalza to which I will then refer for further technical aspects. I will record here just the final equation to be estimated:

$$L = \frac{1}{\sigma_R}\ \phi\left[\frac{E - R_1}{\sigma_R}\right]\ \Phi\left[\frac{\alpha_1 - \bar{\alpha}_1}{\bar{\sigma}_\alpha}\right] +$$

$$\frac{1}{\sigma_R}\,\phi\left[\frac{E - R_2}{\sigma_R}\right]\left\{1 - \Phi\left[\frac{\alpha_2 - \bar{\alpha}_2}{\bar{\sigma}_\alpha}\right]\right\} +$$

$$\frac{1}{\sigma_\varepsilon}\,\phi\left[\frac{E - \dot{Q}}{\sigma_\varepsilon}\right]\left\{\Phi\left[\frac{\alpha_2}{\sigma_a}\right] - \Phi\left[\frac{\alpha_1}{\sigma_a}\right]\right\} \qquad (18)$$

where:

$$\sigma_R = (s^2\sigma_\alpha^2 + \sigma_\varepsilon^2)^{1/2}$$

$$\bar{\alpha}_i = \frac{\sigma_\alpha^2\, s}{s^2\sigma_\alpha^2 + \sigma_\varepsilon^2}\left[E - R_i\right] \qquad i = 1,2$$

$$\bar{\sigma}_\alpha = \left[\frac{\sigma_\alpha^2\,\sigma_\varepsilon^2}{s^2\sigma^2 + \sigma_\varepsilon^2}\right]^{1/2}$$

$$LL = \sum \ln\,(L) \qquad (18a)$$

3.1 EMPIRICAL RESULTS FOR THE CES MODEL.

I have estimated the parameters of the utility function using equation (18a) for the fiscal years 1986-87 to 1988-89 for the Metropolitan Districts and the Shire counties. The measures of expenditure used are the budget figures announced by local authorities before the start of the fiscal year and reported by the Chartered Institute of Public Finance and Accountancy. At the time of budget setting local authorities have available the Government Rate Support Grant report which contains initial values of the parameter which determines local authorities' grant entitlements. The budget constraint arising from these data are used in this study. In the following analysis I have assumed that there is no disequilibrium at all; that is any authority is allowed to spend how much it likes to. The actual system was however fairly different since the rate capping legislation did not allow them to do so. In order to overcome this problem I have just taken the step to delete from the analysis all the rate capped authorities in the year in which they were rate capped and in the previous year[15]. The number of rate capped authorities do not exceed three in each year. In theory it would be possible to model their behaviour by setting an ad hoc model but estimation would be impossible because of lack of degrees of freedom; as far as my analysis is concerned, in most cases they should have been deleted by the sample anyway by being outliers.

The whole budget set for local authorities actually presents two different non convex regions which correspond to:

a) all expenditure financed by the block grant.

b) all expenditure financed by local taxation (i.e. due to the penalty system all the grant is withdrawn).

In this analysis I have modelled only the convex part of the budget set; this does not cause any serious problem when dealing with Metropolitan Districts and Shire Counties since all of those authorities are far away from having a zero tax rate or running out of grant. The model takes account of the possible structural differences in Metropolitan Districts - Shire counties behaviour by a dummy variable on

[15] since rate capping in year t is the result of having over spent in year t-1.

D which takes the value of 1 for Metropolitan Districts and 0 for Shire counties. The peculiar utility function and the budget constraint used do not allow to obtain consistent estimates of ρ, μ and a_{1i}[16] simultaneously. This is in part the result of the correlation between G and g but the most important cause is the role a_{1i} plays in our model. a_{1i}, being an estimate of the income available in each authority for taxation plays the same role as disposable income in a standard CES model demand equation estimates. With no constraint on how much to spend the role played by needs and preferences is clearly marginal.[17]

For this reason a_{1i} has been fixed as follows:

$$a_{1i} = K_c \cdot B_i \qquad c=1,2 \qquad (19)$$

K_1 = max rate levied by Metropolitan districts if authority i belongs to this class.

K_2 = max rate levied by Shire counties if authority i belongs to this class.

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[16] It would clearly be impossible to have a separate estimate for all a_{1i}'s. but it could be desirable to estimate a common parameter that, linked to some other variable would actually differentiate a_{1i} among observations.
In our case, for example this would account to estimate K_1 and k_2 instead of imposing their value a priori.

[17] I actually allowed a_{1i} to vary and for all year a_{1i} was set to extreme high values such that all the other parameters estimates were very unstable and with no economic meaning.

Other measures are clearly possible and I actually tried different ones but this one proved to be the most suitable to deal with this problem. The likelihood for this model[18], being based on a joint normal distribution cannot be maximised using standard statistical packages hence I wrote my own programme using the maximisation algorithm EO4UCF taken from the NAG library to get the estimates which are recorded in table 1. The asymptotical standard errors are derived using Amemyia (1986) method. In order to derive the expected values for this model it is necessary to integrate the equations for the ratio along all segments; this accounts to take the weighted mean of average expenditure ratio on the different segments and at the kinks.[19] It should be noted that, because of the peculiar estimation procedure used, no reliable statistical tests of the overall goodness of fit are available. Therefore the values for the R^2 in table 1 are only reported as a guide to the quality of the model but cannot be subjected to the statistical tests used in more traditional applications. However, if we accept Klein's (1962) suggestion that the empirical relevance of an econometric model is to be assessed on the basis of its predictive power, it is worthwhile to use the most common goodness of fit measures in order to assess how the parameters estimated for our model fit with the actual data. For this reason, I have recorded the MAE and RMSPE respectively [20]. Finally, it should be noted that because a

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[18] which corresponds to equation 19 presented before.

[19] The weight is here represented by the probability of choosing that particular ratio.

[20] MAE can be defined as:

estimates are consistent only for large sample. I have decided to present them anyway since the definition of "large" is quite unclear and because the results can be taken as a guideline into the problem anyway. It will become clear later, expecially after the presentation of the result for the Stone Geary model that cross check of parameter estimates are possible and can help in judging the consistency of the model . Close parameter estimates using quite different models cannot be used as a clear cut test but they might somehow reassure on the validity of the parameters obtained expecially when dealing with likelihood function so complicated and, possibly, not even continuous. The insight that metropolitan districts and shire counties behaviour have quite a different behaviour seems to be confirmed by this new set of estimates. Metropolitan counties' behaviour seems to be consistent with a Cobb Douglas assumption[21] and the model fits quite well with the data. For Shire counties there is a mixed evidence. In 1986-7 their behaviour could have been consistent with a Cobb Douglas utility maximisation, even though the structure of the error terms follows a completely different pattern than the one for Metropolitan districts and it is in general higher; in 1987-8 the indifference map was in general more curved than the one implied by a Cobb Douglas and again the variance of estimates was exceptionally high. The overall goodness of fit is quite good for Metropolitan districts estimates while it seems quite poor for Shire counties.

The expected value of τ for metropolitan districts in the

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[21] the value of ρ is in fact approximately equal to zero.

range of marginal rate of substitution are compatible with the kink, no reliable estimates can be obtained if there are a lot of authorities locating there; im my case the number of authorities at the kink was quite low and the estimates have been tested by using different starting values for the parameters and checking whether the algoritm was converging to the same result.

From table one it is possible to note that the results are not quite satisfactory. Even if the R^2 is not an appropriate measure of goodness of fit, its low value suggests at least that the expected values distribution follows a quite different pattern than the distribution of the observed data. The signs of the different parameters and in particular of the dummy variable suggest that the behaviour of metropolitan districts and shire counties might be so different to require a separate estimate for the two set of observations.

In order to gain further insights into the problem I have split the sample and estimate the previous model for metropolitan districts and shire counties separately. Table 2 and 3 records the results. The number of observations in the sample is now quite low and this could cause some problems when dealing with maximum likelihood whose

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$$\frac{1}{N}\sum_{i=1}^{n} \left| E_i - E(E) \right|$$

RMSPE can be defined as:

$$\left\{ \frac{1}{N} \sum_{i=1}^{n} \left(\frac{E_i - E(E_i)}{E_i} \right)^2 \right\}^{\frac{1}{2}}$$

three years is equal to 1.6 2.40 and 2.77 respectively thus implying a value for β equal to 0 .619 for 1986-87 ,0.709 for 1987-88 and 0.73 for 1988-89[22]. Those result will be used later after a demand equation consistent with the Stone Geary model will be estimated.

3.2 THE DEMAND EQUATION FOR A STONE GEARY MODEL.

In this section I will present some tests devised to outline the main similarities (or differences!) between the actual grant system and the optimal one.

From 1986-7 onwards the system by which grants to local government are allocated has been considerably simplified. In the last three years in fact the previous target system has been abolished and then the budget constraint present just one kink. This model is then very similar to the optimal set of rules I presented in chapter three and I will now test to what extent the actual system reflected the

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[22] The derivation of β can be easily obtained by observing that the marginal rate of substitution can be written as:

$$MRS_{T_i, X_i} = \frac{d X_i}{d T_i} = - \frac{\frac{\partial W_i}{\partial T_i}}{\frac{\partial W_i}{\partial X_i}} = \frac{1 - \beta}{\beta} \cdot \frac{X_i - a_{2,i}}{a_{1,i} - T_i}$$

for the Cobb Douglas and as:

$$\left[\frac{X_i - a_{2,1}}{a_{1,i} - T_i} \right]^{1-\rho} * (1/\tau)$$

for the CES utility function.

It follows then that $\beta = \tau/(\tau+1)$.

theoretical one[23]. The problem does not allow for a unique direct test but some indirect measures of consistency can be devised. For the time being I will assume that the parameter of the grant system have been chosen optimally[24] and I will derive the revealed preferences of local authorities for expenditure under the assumption of a life cycle consistent behaviour. In order to do so I would require to estimate the demand equations derived by the life cycle utility function by possibly using one of the procedures suggested in the labour supply literature. However this is not possible for different reasons:

a) most of the models in labour supply just cope with participation problems and assume a constant marginal wage on the range of the feasible hours of work;

b) the wage rate is assumed to be given, i.e. a parameter that cannot be changed by the decisions of how many hours to work. As I showed in the previous chapter local authorities can influence the price they are going to pay for expenditure in the future by deciding whether to cheat and how much; on the other hand the actual system of grant as designed in the Rate Support Grant gives a large weight to previous level of expenditure as concerns the future levels of GREAS and targets.[25]

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[23] In 1986-7 elections for local authorities which adopt the system of choosing their local representative each 4 years took place, then the three years I am considering are actually part of the same life cycle.

[24] i.e. M_{1i} reflects the basic needs, threshold represents the optimal incentive and P_{ij} are derived according to the formulas presented in the previous chapter.

[25] When the system of targets was in operation.

My approach will then be a bit different. I will derive the utility function parameters for local authorities by estimating the model separately for any of the years I am considering in my analysis and I will then devise a test to assess whether these parameters are constant through time. This procedure is perfectly valid in this case since I assume that there does not exist any time preference for expenditure and the interest rate has been set equal to zero. The analysis I am going to present is, however, tentative and incomplete since I face two other problems: the first one is that the life cycle of the authorities I am considering will end in fiscal year 1989-90 for which it is impossible to perform estimates since the system by which grants were allocated was different and some authorities in the sample use the system of electing 1/3 of their members each year.

Another important aspect to consider is that if the system of grant is set optimally, local authorities will be budget balanced each year and then it is possible to estimate their demand subject to the static budget constraint. The evidence that local authorities do actually transfer money among their life cycle is per se an evidence of some failure in the system. At the end of this chapter some possible causes for this failure will be tested.

The analysis will be performed using data on Metropolitan Districts since the assumption that they behave as if they were maximising an utility function of the Cobb Douglas class seem to be plausible [26].

The procedure can be described as follows:
The utility of each individual in authority i is given the specific form of a Stone Geary[27]:

$$W_i = (1-\beta)\, \ell n\, (a_{1i} - T_i) + \beta\, \ell n\, (X_i - a_{2i}) \qquad (20)$$

where the parameter a_{1i} reflects the underlying income available in the area for local authority service provision and a_{2i} is the baseline expenditure required to provide a minimal level of services.

As for the model I presented in the previous section, at the time of budget setting[28] Central Government decision makers have to guess the baseline level of expenditure to be provided and the income available for taxation. The implicit lump sum grant on the first segment of the budget constraint is assumed to be the basic need indicator in order to reflect the assumptions of the theoretical model, while income available for taxation might be thought of as the maximum rate poundage increase applied by any of the authority of the same class in the previous year without incurring rate capping, multiplied by the rateable value of the local authority. The parameter β is determined by the preferences between local authority services and local

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[26]This has been shown in the previous section.

[27]The Stone Geary function I am using differs from the more standard one from which the LES demand system is derived because in this case one of the commodities, namely taxation, T, is actually a bad.

[28]i.e. one year in advance with respect to actual expenditure decisions.

taxation. Maximisation of (20) subject to (5) give a demand equation of the form:

$$X_{ij} = (1-\beta)\ a_{2i} + \beta \left[\frac{a_{1i}}{g_{ij}} + G_{ij} \right] \qquad (21)$$

By assuming $a_{2i} = G_{1i}$ the demand equation on the first segment can be rewritten as:

$$(X_{1i} - a_{2i}) = \beta \frac{a_{1i}}{g_{1i}} \qquad (22a)$$

while on the second segment it will be written as:

$$(X_{2i} - a_{2i}) = \beta \left[\frac{a_{1i} + (X^{*} - a_{2i}) * (g_{2i} - g_{1i})}{g_{2i}} \right] \qquad (22b)$$

The source of error in the equation to be estimated derives from two different sources, namely an <u>heterogeneity</u> error which is represented by α and a <u>stochastic</u> error represented by ε. In the absence of any measurement error the preferred segment of the budget is determined by the heterogeneity error that in my model is represented by α. By adding the heterogeneity error and dropping the subscripts i representing different observations for ease of exposition the demand equation can be written as:

$$(X_1 - a_2) = \beta \frac{a_1}{g_1} * \exp(\alpha) \qquad (23a)$$

for the demand on the first segment and:

$$(X_2-a_2)= \beta\left[\frac{a_1 + (X^* - G_1) * (g_2 - g_1)}{g_2}\right] * \exp(\alpha) \qquad (23b)$$

for the demand on the second segment.

The second source of error is the usual random or measurement error that in my model is represented by ε. When also this second source of error is introduced the complete demand equations to be estimated can be written as:

$$(X_1 - a_2) = \beta \frac{a_1}{g_1} * \exp(\alpha) * \exp(\varepsilon) \qquad (23a)$$

$$(X_2-a_2)= \beta\left[\frac{a_1 + (X^* - G_1) * (g_2 - g_1)}{g_2}\right] * \exp(\alpha) * \exp(\varepsilon) \quad (23b)$$

By taking the logs of both sides :

$$\ln(X_1 - a_2) = \ln \beta + \ln \left[\frac{a_1}{g_1}\right] + \alpha + \varepsilon$$

$$\ln(X_2 - a_2) = \ln \beta + \ln \left[\frac{a_1 + (X^* - G_1) * (g_2 - g_1)}{g_2}\right] + \alpha + \varepsilon$$

If we consider to start with the heterogeneity error only and assuming α has a p.d.f. $f(\alpha)$ the likelihood function in this case can be written as follows:

$$L = \Pi \Pr\left[\alpha = x-g(g_1, G_1,\beta)\right] \alpha < x^*-g(g_1, G_1,\beta)$$

$$\Pi \; Pr \left[\alpha = x - g(g_2, \; G_2, \beta) \right] \; \alpha > \; x^* - g(g_2, \; G_2, \beta)$$

$$\Pi \; Pr \left[x = x^* \right] \; x^* - g(g_1, \; G_1, \beta) < \alpha^* < x - g(g_2, \; G_2, \beta) \qquad (24)$$

$$g(g_1, G_1, \beta) = \ln \beta + \ln \left[\frac{a_1}{g_1} \right]$$

$$g(g_2 \, , G_2 \, , \beta) = \ln \beta + \ln \left[\frac{a_1 + (X^* - G_1) * (g_2 - g_1)}{g_2} \right]$$

$$x = \ln (X - a_2)$$

$$x^* = \ln (X^* - a_2)$$

X^* = threshold level of expenditure.

When the second error, i.e. ε, is introduced the likelihood is modified as follows:

$$L = \pi \left[Pr(x) \right]$$

where:

$$Pr(x) = Pr\left[\alpha + \varepsilon = x - g(g_1, \; G_1, \beta)\right] \quad \alpha < \; x^* - g(g_1, \; G_1, \beta)$$

$$+ Pr\left[\alpha + \varepsilon = x - g(g_2, \; G_2, \beta)\right] \quad \alpha > \; x^* - g(g_2, \; G_2, \beta)$$

$$(25)$$

$$+ \; Pr \left[\varepsilon = x - x^* \right] \; x^* - g(g_1, G_1, \beta) \; < \alpha < \; \overset{*}{x} - g(g_2, G_2, \beta)$$

The log likelihood function can be written as:

$$LL = \sum \log \; [Pr(x)]$$

where:

$$\Pr(x) = \int_{-\infty}^{u_1} h\left[v = \left(x - g(g_1, G_1, \beta)\right), \alpha \right] d\alpha + \int_{u_2}^{\infty} h\left[v = \left(x - g(g_2, G_2, \beta)\right), \alpha \right] d\alpha + \int_{u_1}^{u_2} \frac{1}{\sigma_v} f\left(\frac{x - x^*}{\sigma_\varepsilon}\right) \frac{1}{\sigma_\alpha} f(\alpha)\, d\alpha \qquad (26)$$

$$\sigma_v^2 = \sigma_\alpha^2 + \sigma_\varepsilon^2$$

$$u_j = \left(x^* - g(g_j, G_j, \beta)\right)$$

By using the properties of the conditional normal distribution the p.d.f. for each point can be written as:

$$\Pr(x_i) = \frac{1}{\sigma_v} f(z_{1,i}) \left[F(r_{1,i}) \right] + \frac{1}{\sigma_v} f(z_{2,i}) \left[1 - F(r_{2,i})\right] + \frac{1}{\sigma_v} f(s_i) \left[F(t_{2,i}) - F(t_{1,i})\right] \qquad (27)$$

$$z_{j,i} = \left[(x_i - g(g_{j,i}, G_{j,i}, \beta) \right] / \sigma_v$$

$$s_i = [x_i - x_i^*] / \sigma_\varepsilon$$

$$t_{j,i} = \left[\left(x_i^* - g(g_{j,i}, G_{j,i}, \beta) \right) \right] / \sigma_\varepsilon$$

$$r_{j,i} = \left[t_{j,i} - \rho z_{j,i}\right] / \sqrt{1 - \rho^2}$$

$$\sigma_v = \sqrt{\sigma_\alpha^2 + \sigma_\varepsilon^2}$$

$$\rho = \frac{\sigma_\alpha}{\sigma_v}$$

3.3 EMPIRICAL ESTIMATES OF THE STONE GEARY MODEL.

I have estimated the parameter of the utility function using the technique depicted above for fiscal years 1986-7 to 1988-9 by using data for the Metropolitan districts [29]. The set of data I used is the same as for the model I presented in the previous section to which I refer for any further detail. The likelihood for this model, being again based on a joint normal distribution cannot be maximised using standard statistical packages hence I write another programme using the maximisation algorithm EO4UCF taken from the NAG library to get the estimates which are recorded in table 4. In order to derive the expected values for this model it is again necessary to integrate the demand equations along all segments which accounts to take the weighted mean of average expenditure on the different

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[29] The assumption of utility maximisation behaviour consistent with a Cobb Douglas type function is in fact consistent only for Metropolitan districts while, as I have showed in the previous section the Indifference Map for Shire counties seems to be more curved in at least two years out of three.

segments and at the kinks.[30] Again, because of the peculiar estimation procedure used, no reliable statistical tests of the overall goodness of fit are available. Therefore the values for the R^2 in table 4 are only reported as a guide to the quality of the model but cannot be subjected to the statistical tests used in more traditional applications. The results presented in table 4 show how close are the estimates for β to the one I derived from the CES estimation presented in the previous section. It is actually possible to test the hypothesis that the two β are equal by using a test first suggested by Hausman (1978). The application of Hausman test is not straightforward in this case because the variance of β for the CES model has to be evaluated. The procedure to derive this variance and the results of Hausman test are recorded in appendix one: the conclusion is that the hypothesis of the two β being equal cannot be rejected.

3.3.1 TESTING THE LIFE CYCLE HYPOTHESIS.

I have argued in the introduction to this section that, in order to be consistent with the assumptions of a life cycle utility maximisation, the estimated β should be constant through time. The results presented in table four seem to suggest that β is actually increasing through time. However, from a statistical point of view a test is needed to see if β is constant. By using a pooling cross section model I can achieve this result.

I will estimate the demand equation derived from the Stone

[30]The weight is here represented by the probability of spending that amount.

Geary utility function for the three years together and I will then test for the constancy of β. The demand equation for a representative authority i, for which the subscript has been dropped to make the notation simpler, can be written as:

$$X_{kj} = (1-\beta)\ a_{2k} + (\beta + \gamma_1 D_1 + \gamma_2 D_2) \left[\frac{a_{1k}}{g_{kj}} + G_{kj} \right] \qquad (28)$$

$j = 1,2 \qquad k = 1,3$

$D_1 = 1$ if $k = 2$

0 elsewhere

$D_2 = 1$ if $k=3$

0 elsewhere

If I again assume that $a_{2k} = G_{1k}$ I can again use the same technique I explained before to get estimates of equation (28). Constancy of β through time can be tested by imposing the restriction:

$$\gamma_1 = \gamma_2 = 0$$

and by testing its validity using the likelihood ratio test. The results for both the constrained and the unconstrained model are presented in table five.

The likelihood ratio test shows that the hypothesis that local authorities behave as if they were maximising their utility function over their life cycle is rejected [31]. The second important remark on the results of table five is

[31] the hypothesis that β is constant through time cannot be accepted at 99%.

that the values for β in the different years are .614 for 1986-7 ; .71 for 1987-8 and .732 for 1988-9, then very close to the one I obtained by performing the estimates for each year [32]. Since the estimates for β are quite close each other, it is possible to use the model in table five as reference for policy implications and for comparisons with the actual grant system.

Any definite conclusion is premature, but from the results I presented above it seems possible to infer that even though the actual grant system was designed according to the optimal rules depicted in chapter three and four, it could not have been able to fully achieve its goals because β is not constant through time. The next section deals with two possible causes of this outcome.

4 TESTING FOR ALTERNATIVE BEHAVIOURAL HYPOTHESES

In the previous chapter I have argued that one of the possible failures of the optimal grant system could arise from a misperception of the set of rules governing it and in particular I have argued that the actual budget could be misperceived. The misperception might well derive from uncertainty about the precise form of the grant or from the system being much too complicated to be fully understood. Even if they do not act as life cycle utility maximisers ,

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[32]The R^2 for the second model is however much higher, suggesting then that the predictive power of the latter is has quite improved. This result can in my opinion be related to the fact that when performing separate estimates the number of observations is not sufficient to make the structure of the two errors terms optimal for the problem.

local authorities decisions makers have to set the budget at least one year in advance when the information available is not complete[33]. This might induce bureaucrats to care just about how much money they hope to receive from central government without bothering about how many new additional kinks the actual budget will have. In addition to this problem local authorities might not maximise their utility function according to the conventional rules and the alternative behaviour could seriously jeopardize the theoretical grant system. In this section both hypothesis will be tested in turn.

4.1 A FLYPAPER MODEL OF LOCAL GOVERNMENT BEHAVIOUR

The flypaper theory assumes that local governments misperceive their true budget; instead of thinking in terms of the true marginal cost of the service provision they think in terms of average costs. If this is the case, the budget constraint will be perceived as:

$$X_{ik} = T_i \tilde{B}_{ik} \qquad (29)$$

Maximisation of (2) subject to (29) gives the following demand equation:

$$\tilde{X}_{ik} = (1-\beta)\ a_{2i} \ + \beta\ (a_{1i}\tilde{B}_{ik}) \qquad (30)$$

[33] since in turn Central Government is not in actual fact setting all the rules at the start of the new legislation.

$$\tilde{B}_{ik} = \frac{X_i}{(X_i - G_{ij})g_j} \qquad (30a)$$

From a policy point of view the existence of the flypaper effect is quite important. One of the main predictions of this theory is that the amount rather than the form of the grant matters. If local authorities behave according to this policy it is clear that the previous system of incentives fails to reach its objectives since it is designed to deal with marginal incentives rather than average ones. This is particularly important for restraining expenditure on the second segment of the budget constraint in which the average price is clearly considerably lower than the marginal one.

The estimation of (30) is more tricky than the one for the two segment budget constraint because $\tilde{B}$ depends on the level of expenditure. The model to be estimated can be written using equations (30) and (30a) together. In order to compare the results of this model with the one I presented in the previous section, I will also impose the following restriction:

$$a_{2i} = G_{1i} \qquad (31)$$

It is worth noting that equations (30) and (30a) cannot be estimated using a simultaneous equation approach since (30a) is an identity and $\tilde{B}$ cannot be substituted in (30) because this would lead to equation (20), while the main hypothesis of this interpretation of the flypaper theory is that local government bodies do not do this substitution. $\tilde{B}_{ik}$ and thus the budget constraint changes continuously according to the level of expenditure; however this is not realized until expenditure changes ; i.e. for a certain level of expenditure, the budget is perceived as being fixed[34]. This suggests that I can implement a procedure that I have labelled "*iterative Maximum Likelihood*" in which the iterations follow a type of Walrasian tatonnement process.

The log likelihood for the model can be written as:

$$LL(\beta,,\sigma^2) = -\frac{n}{2}\ln(2\pi) - n\ln(\sigma) - \frac{1}{2\sigma^2}\sum\left[X_i - \hat{X}_{ik}\right]^2 \qquad (32)$$

where X_i is the actual expenditure and $\tilde{X}_{ik}$ is derived from an iterative procedure that finds for any given level of β, G_{ij}, g_{ij} the pair of $\tilde{X}_{ik}$ and $\tilde{B}_{ik}$ compatible each other. The iterative or tatonnement procedure is implemented as follows:

1. Select an initial value for $\tilde{B}_{ik}$(in the empirical work I have taken the value of $\tilde{B}_{ik}$ which held at the previous fiscal year's actual level of expenditure as the starting value for $\tilde{B}_{ik}$, but in theory any feasible value could be selected) along with a guess for β and σ^2.
2. Use equation (30) to find the value of $\tilde{X}_{ik}$ for the assumed value of $\tilde{B}_{ik}$.

3a If at $\tilde{X}_{ik}$ the local authority's true budget is balanced stop the procedure.

3b If at $\tilde{X}_{ik}$ the local government's budget is not in balance adjust $\tilde{B}_{ik}$ and repeat step 2.

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[34]People perceive the average price as if it was the marginal price.

Continue this procedure until the level of $\tilde{B}_{ik}$ is found for which the true budget is balanced.

Step 3b requires the specification of an adjustment rule for changing $\tilde{B}_{ik}$. If at a given level of $\tilde{B}_{ik}$ the estimated expenditure exceeds the funds available the adjustment rule reduces $\tilde{B}_{ik}$. This is equivalent to increase (average) price which is the inverse of $\tilde{B}_{ik}$, if expenditure exceeds resources available. Similarly if the true budget is not all spent the adjustment rule increases $\tilde{B}_{ik}$. The adjustment rule is given by:

$$\Delta \tilde{B}_{ik} = \phi \left(\tilde{B}_{ik}^{t} - \tilde{B}_{ik}^{t-1} \right) \qquad 0 < \phi < 1$$

where the superscripts refer to the stage of the tatonnement process. The local government reaction path for this procedure is given by its price-consumption curve and the nature of the the tatonnement procedure is illustrated in Figure four.

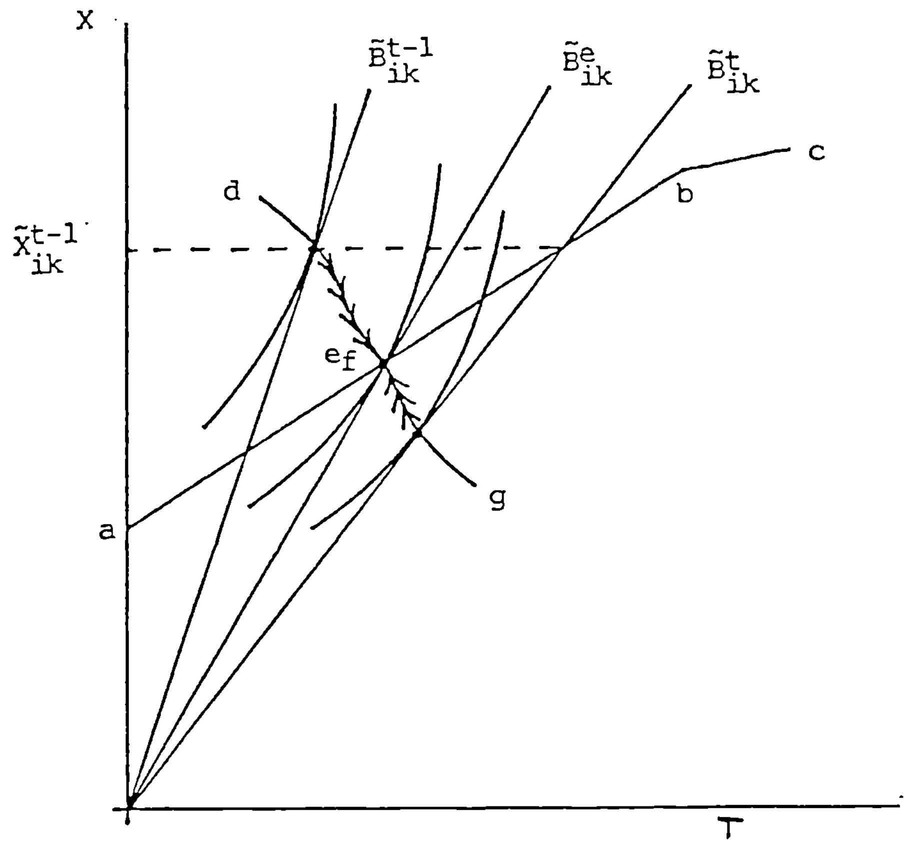

FIGURE FOUR

The starting value for $\tilde{B}_{ik}$ is given by $\tilde{B}_{ik}^{t-1}$, the budget constraint which held in the previous fiscal year, But at this level of $\tilde{B}_{ik}$ the estimated level of expenditure is $\tilde{X}_{ik}^{t-1}$ which does not lie on the true budget constraint which is given by abc. At the estimated level of expenditure the budget is exceeded and in fact the budget would be balanced for this level of expenditure only if $\tilde{B}_{ik} = \tilde{B}_{ik}^{t}$. Since $\tilde{B}_{ik}^{t} \neq \tilde{B}_{ik}^{t-1}$ the adjustment rule is implemented and $\tilde{B}_{ik}$ is reduced. The reaction path is given by de_fg and the tatonnement process will lead to a final equilibrium at point e_f[35].

4. Once step 1 to 3 is performed for all *observations*, substitute $\tilde{X}_{ik}$ and the guess for σ^2 in (32) and evaluate

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[35]The estimation procedure is then analogous to the one we used in Barnett et al (1989a).

the log-likelihood function.

5. Repeat 1-4 for a different β and σ^2. A grid search method can be used to find the parameters β and σ^2 that maximise (32).[36]

4.1.1 EMPIRICAL RESULTS.

I have estimated the parameters of the utility function by using equation (30) for the fiscal years 1986/7 - 1988/9 by using data for Metropolitan Districts. The estimates for the 'flypaper' demand equation have been obtained by using the same set of data I have been using in the previous estimates and the results are reported in table 6. The '*iterative MLE*' procedure has been implemented by writing an ad hoc program and using the NAG routine E04UCF as maximisation algorithm.

If I compare the results I can immediately note that in 1987-88 the value of β for the '*flypaper model*' demand equation is around half the one for the standard model.

This suggestion can be interpreted in terms of marginal rates of substitution which can be written as:

$$MRS_{T_i, X_i} = \frac{d X_i}{d T_i} = - \frac{\frac{\partial U_i}{\partial T_i}}{\frac{\partial U_i}{\partial X_i}} = \frac{\beta}{1 - \beta} \cdot \frac{X_i - a_{2,i}}{a_{1,i} - T_i} \quad (33)$$

Other things being equal, β determines the curvature and the steepness of the indifference map. In particular the

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[36]This procedure is analogous to the one deleloped to obtain estimates for Barnett et al (1989c)

indifference map will be flatter the greater the β. If I look at figure four again I can interpret the rationale for this finding: since the misperceived budget constraint is always steeper than the actual one, the change in β, by causing the indifference map to be steeper as well fits better with the new budget. In the last two years the gap between the two values is not so relevant, but it might be born in mind that the relationship between the curvature of the indifference curves and β is not linear.
In order to assess the empirical validity of the flypaper theory I propose two different sets of tests, namely goodness of fit and predictive power tests.

The first set of tests aims at establishing which model fits best with the data. If I look at tables 5and 6 I can easily notice that the flypaper model fits better with data since the R^2 is higher and both the RMSPE are lower. As concerns the MAE a straightforward comparison is not possible since the two models, though using the same original set of data, are estimated using a different dependent variable. In the standard model the dependent variable is $\ln (X_i - a_{2,i})$ while for the flypaper theory the dependent variable is X_i. On the basis of the R^2 and the RMSPE test it is possible to conclude that the flypaper estimates are closer to the actual data both on average and on outliers.

The second set of tests is more important from a policy point of view since it tries to assess which of the two models leads to closer estimates of the actual expenditure in year t by using only information known in period t-1.

I have carried out this set of tests by using parameters estimates from 1986-87 and 1987-8 model to forecast expenditure in 1987-88 and 1988-89. The forecasts for the flypaper model are quite straightforward: it is only necessary to use the procedure I described before by using as starting point 1986-87 estimates and iterate until we find for any authority a pair of $\hat{X}_i$ and $\tilde{B}_i$ compatible with each other. The forecasts with the model without misperception are somewhat more complicated since they depend on the value of α. One possible way to proceed would be to use the expected value of α on the overall budget but in this case all information available would not be used since the expected values of α on the different sets of the budget that are known would not be considered. For the forecasts I have used this information, i.e. the predicted expenditure is the weighted [37] expected value of expenditure on the different location with weights and expected values of the random α assumed to be constant from one year to the other. In this application I also face a different problem which is based on which model to choose for prediction. As I pointed out before, while the estimated value for β in the different years obtained by using a pooling cross section equation and a set of separate estimates are quite close together, the actual structure of the error term is not. For this reason I will present the forecast that can be obtained

[37] The weight is represented by the probability of being on that segment or at the kink.

by using both set of errors terms. Table 7 records some goodness of fit of our forecasts using both models. Those tests can be interpreted as predictive power tests in this context since they assess how close forecasts are to actual data. The flypaper theory seems to predict better both in average terms and as concerns outliers [38]. The results presented in table 7 can be also used as a straightforward test for the existence of a life cycle utility maximisation behaviour in the context of the flypaper model. By using the value of β for 1986-7 only about 40% of the variation in expenditure in 1987-8 can be explained while the unconstrained model was explaining more than 85%. This result supports the view that β is not constant through time; the forecasts and estimates for 1988-9 are closer together but the difference is still significant from a statistical point of view.

4.2 ARE LOCAL AUTHORITIES UTILITY MAXIMISERS?

At the end of chapter four I have argued that local authorities objectives, by being the result of a process of decision involving a body of individuals could not in actual fact be utility maximiser, at least in a standard way of interpreting utility maximisation. In particular I argued that β could not be constant through time and fixed outside the model, but it could itself a choice variable. If this is the case, I showed that preferences for expenditure depends

[38] The RMSPE which gives more weights to big deviations from the observed value is greater for the forecasts obtained by using the flypaper model.

both on the price system in force in each year and on resources available. This hypothesis can be tested by rewriting equations (23a) and (23b) as follows:

$$(X_1 - a_2) = \beta \frac{a_1}{g_1} \ast\exp(\delta) \ast\exp(\varepsilon) \tag{33a}$$

$$(X_2 - a_2) = \beta \left[\frac{a_1 + (X^{\ast} - G_1) \ast (g_2 - g_1)}{g_2}\right] \ast\exp(\delta) \ast\exp(\varepsilon) \tag{33b}$$

where $\delta = \mu W + \alpha$

W = resources indicator.

By taking the logs of both sides :

$$\ell n(X_1 - a_2) = \ell n\ \beta + \ell n \left[\frac{a_1}{g_1}\right] + \mu W + \alpha + \varepsilon$$

$$\ell n(X_2 - a_2) = \ell n\ \beta + \ell n \left[\frac{a_1 + (X^{\ast} - G_1) \ast (g_2 - g_1)}{g_2}\right] + \alpha + \mu W + \varepsilon$$

In this model μW determines the value of β that each local authority will choose. If local authorities were behaving according to a standard utility maximising model their preferences should be independent of resources and prices[39] then μ should not be significantly different from zero.

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[39]The actual quantity spent for local public services will clearly depend on resources and prices as a result of the type of utility chosen and of the budget constraint but this should not affect the value of β which is assumed to be given, i.e. a fixed parameter which depends on local characteristics.

I will test this hypothesis by estimating equations (33a) and (33b) using again a two error model. I will not rewrite here the log likelihood for this problem since it is very similar to equation (27) .

4.2.1 ESTIMATING THE PREVIOUS MODEL

I have estimated the parameters of the utility function by using equations (33a) and (33b) for the fiscal years 1986-87 to 1988-89 by using data for Metropolitan Districts. I have used as a proxy for resources available the previous year level of taxation. The model assumes that resources available for taxation are actually determined by the rateable value of each locality multiplied by the maximum rate observed in each year; however this measure cannot be used for W because of a multicollinearity problem. Resources available, in fact enter the model through a_{1i}. The previous level of taxation and resources available have proved to be highly correlated, so I use this variable as a proxy in my model. Table 8 reports the result of this model along with the usual goodness of fit indicators.

By using results in table 4 and 8 it is possible to test the hypothesis that μ in the different years is equal to zero and the results I have summarized in table 8 show that this hypothesis has to be rejected.

The hypothesis that preferences for local goods depend on the grant system itself cannot then be rejected. The average sample value for W in the three years of the analysis was

.228 .266 and .3004 respectively, thus implying an average value for preferences equal to .63 .68 .72 on average. The model then shows the upward trend of preferences and in this case it is possible to note that the resources available play a smooth role in the preferences pattern. If I assume that the estimated β is the true preference parameter, its variation is between .504 and .897 while the average revealed preference is in the range between .63 and .72 which is again very similar to the estimates I derived from the previous models. From a policy point of view this cause another failure to the optimal system of grant allocation outlined in the previous chapter and can possibly mean that an optimal set of rules cannot be designed at all.

5. CONCLUSIONS

This chapter has been mainly devoted to presenting the parameter estimates for the utility functions of local authorities revealed through their expenditure behaviour.

The exercise I presented implicitly assumes that the price system set up by Central Government has been set according to the rules presented in chapter four. The results show that only Metropolitan districts behaviour is consistent with the assumption of an underlying Cobb Douglas type utility function. The assumption of a life cycle maximisation seems to be rejected since the revealed preferences for expenditure are not constant through time and present an upward trend. This result, in terms of the shape of the indifference map, implies that indifference curves are flatter. Since the implicit price for expenditure on both segments of the budget constraint increased through the period of the analysis, this result can be interpreted in terms of adjusting preferences to the system ruling in a particular year and to resources available [40].

Finally I showed that local authorities could also misperceive their budget constraint by taking their expenditure decisions in terms of average price to be paid rather than marginal price.

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[40]The latter proposition has been shown using results for the model recorded in table eight.

TABLE ONE

Summary of results for years 1986/7 1988/9 . Dummy on D

£ per head

YEAR	ρ	σ_α	σ_ε	μ	dum D	R^2
1986-7	-.140	.208	.209	-.55	.093	.380
	(.01)			(.02)	(.009)	
1987-8	-.620	.280	.490	-.83	.23	.180
	(.03)			(.04)	(.01)	
1988-9	-.360	.270	.275	-.97	.16	.340
	(.02)			(.05)	(.02)	

GOODNESS OF FIT INDICATORS

	1986-7	1987-8	1988-9
MAE	8.49	0.53	3.27
RMSPE	19.40	8.20	31.76

The numbers in parenthesis are asymptotical standard errors.

TABLE TWO

Summary of results for years 1986/7 1988/9

METROPOLITAN DISTRICTS

YEAR	ρ	σ_α	σ_ε	μ	R^2
1986-7	-.0025	.28	.29	-.486	.458
	(.001)			(.045)	
1987-8	.077	.32	.31	-.879	.736
	(.002)			(.102)	
1988-9	-.004	.33	.32	-1.022	.761
	(.001)			(.168)	

GOODNESS OF FIT INDICATORS

	1986-7	1987-8	1988-9
MAE	3.5	0.23	1.27
RMSPE	4.2	1.37	14.23

TABLE THREE

Summary of results for years 1986/7 1988/9.

SHIRE COUNTIES

£ per head

YEAR	ρ	σ_α	σ_ε	C	R^2
1986-7	.006 (.03)	.149	.237	-.58 (.2)	.232
1987-8	-.370 (.002)	.312	.680	-1.05 (.18)	.100
1988-9	-.370 (.003)	.344	.304	-1.089 (.15)	.123

GOODNESS OF FIT INDICATORS

	1986-7	1987-8	1988-9
MAE	1.03	2.78	4.32
RMSPE	18.09	42.10	61.94

TABLE FOUR

Summary of results for years 1986/7 1988/9

METROPOLITAN DISTRICTS

YEAR	β	σ_{α}	σ_{ε}	R^2
1986-7	.6157 (.041)	0.128	0.132	.613
1987-8	.7124 (.062)	0.150	0.130	.310
1988-9	.733 (.073)	0.156	0.154	.207

GOODNESS OF FIT INDICATORS

	1986-7	1987-8	1988-9
MAE	0.80	1.00	1.27
RMSPE	0.32	3.48	14.23
LL	-24.30	-27.67	-24.09

TABLE FIVE

SUMMARY OF RESULTS

UNCONSTRAINED MODEL

β	σ_α	σ_ε	γ_1	γ_2	R^2
.614	.129	.138	.096	.118	.645
(.031)			(.02)	(.09)	

CONSTRAINED MODEL

β	σ_α	σ_ε	γ_1	γ_2	R^2
.680	.138	.142	-	-	.47
(.042)					

GOODNESS OF FIT INDICATORS

	UNCONSTRAINED	CONSTRAINED
MAE	.92	.94
RMSPE	3.06	4.04
LL	−58.322	−66.21

LIKELIHOOD RATIO TEST[41]: 15.78

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[41] This value corresponds already to twice the logarithm of the likelihood ratio.

TABLE SIX

RESULTS USING THE FLYPAPER MODEL

YEAR	β	σ_{α}	R^2
1986-7	.392 (.032)	.0146	.764
1987-8	.504 (.041)	.0123	.765
1988-9	.521 (.052)	.0122	.827

	MAE	RMSPE
1986-7	.0174	.0198
1987-8	.0153	.0091
1988-9	.0148	.0083

TABLE SEVEN

FORECAST FOR 1987/8 USING 1986/7 PARAMETERS.

	R^2	MAE	RMSPE
STANDARD MODEL *	.099	1.016	3.60
STANDARD MODEL **	.109	1.007	3.50
FLYPAPER MODEL	.344	.026	.38

FORECAST FOR 1988-9 USING 1987-8 PARAMETERS.

	R^2	MAE	RMSPE
STANDARD MODEL *	.049	1.028	3.835
STANDARD MODEL **	.044	1.030	3.864
FLYPAPER MODEL	.819	.016	.034

TABLE EIGHT

Summary of results for years 1986/7 1988/9

METROPOLITAN DISTRICTS

YEAR	β	σ_α	σ_ε	μ	R^2
1986-7	.504	0.102	0.103	1.025	.640
	(.021)			(.108)	
1987-8	.897	0.109	0.110	-1.045	.455
	(.031)			(.109)	
1988-9	.812	0.134	0.125	- .386	.268
	(.029)			(.02)	

GOODNESS OF FIT INDICATORS

	1986-7	1987-8	1988-9
MAE	1.23	.98	1.02
RMSPE	2.11	3.07	3.76
LL	-18.58	-21.57	-19.07
LR	11.56	11.46	10.04 [42]

[42]This value corresponds to twice the difference of the log likelihood.

APPENDIX ONE

Derivation of the standard error for β in the CES model.

As I have shown in the text, β can be derived from γ as follows:

$$\beta = \frac{e^{\gamma}}{1 + e^{\gamma}} = 1 - \frac{1}{1 + e^{\gamma}} \tag{1}$$

$$(\hat{\beta} - \beta)^2 = \left[1 - \frac{1}{1 + e^{\hat{\gamma}}} - 1 + \frac{1}{1 + e^{\gamma}} \right]^2$$

if we take the Taylor expansion of this series around $\hat{\beta}$, it is possible to write that:

$$(\hat{\beta} - \beta)^2 = \left[1 - \frac{1}{1 + e^{\hat{\gamma}}} \quad 1 - \frac{1}{1 + e^{\gamma}} \right]^2_{\gamma=\hat{\gamma}} +$$

$$- 2 \left[\frac{1}{1 + e^{\gamma}} - \frac{1}{1 + e^{\hat{\gamma}}} \right]_{\gamma=\hat{\gamma}} \frac{e^{\gamma}}{(1 + e^{\gamma})^2} (\hat{\gamma} - \gamma) +$$

$$+ \frac{e^{2\gamma}}{(1 + e^{\gamma})^4} (\hat{\gamma} - \gamma)^2 + 2 \left[\frac{1}{1 + e^{\gamma}} - \frac{1}{1 + e^{\hat{\gamma}}} \right]_{\gamma=\hat{\gamma}} \left[\frac{e^{\gamma}}{(1 + e^{\gamma})^2} - \frac{2e^{\gamma}}{(1 + e^{\gamma})^3} \right] (\hat{\gamma} - \gamma)^2$$

Using the asymptotic properties of the variance distribution it is possible to write that:

$$\text{plim} \sqrt{n} \; (\hat{\beta} - \beta)^2 = \frac{e^{2\gamma}}{(1 + e^{\gamma})^4} (\hat{\gamma} - \gamma)^2$$

$$\mathrm{AVAR}(\hat{\beta}) = \frac{e^{2\gamma}}{(1 + e^{\gamma})^4} \mathrm{AVAR}(\hat{\gamma}) \tag{2}$$

From (1) and using the results of table two it is possible to derive the following values of β

	1986/7	1987/8	1988/9
β	.619	.706	.735

From (2) it is possible to derive the asymptotical standard errors for β as follows:

	1986/7	1987/8	1988/9
STD (β)	.025	.050	.078

It is now possible compare the values of β with the one in table 4 by using Hausman (1978) test.

The values for the test are as follows:

	1986/7	1987/8	1988/9
TEST	−1.75	.902	−.20

then in the three years the hypothesis of the two β's being equal cannot be rejected.

CHAPTER SIX

1. INTRODUCTION

The history of intergovernmental grants in Britain has been characterized, expecially in the most recent years, by dramatic changes in the grant distribution formula that some commentators [1] have interpreted as a failure of the system to reach its objectives. In this section I will present a different interpretation by explaining how, in my opinion the different grant distribution formulas applied in the last decades reflect the perception of the information asymmetry problem that characterizes the relationship between Central and local government. The history of grants might, in my view divided into three different phases:

1) Central Government assumed to know all the parameters necessary to give the right amount of grant to local authorities.
2) Central Government realizes the problems caused by the asymmetry of information characterizing the game and tries to react accordingly.
3) Due to the peculiar characteristics of the agents involved in the play no optimal grant formula exists and the grant is almost arbitrary. Central Government's objectives determine the size of the grant and its amount

The dramatic changes in the grant system might then be interpreted as an evidence of Central Government's increasing awareness of the asymmetry problem that it has to face. In this light also the new drastic change in local government finance brought about by the introduction of the

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See, for example, Gibson and Travers (1986) poll tax might be seen as a response to a problem that, as I have argued in the previous chapter might not have a solution.

2. THE GRANT SYSTEM UNDER THE ASSUMPTION OF PERFECT INFORMATION

In 1967 a new grant system was introduced. In this phase Central Government was confident of being able to know, or at least to observe, all the relevant parameters to distribute the grant in an optimal way. The key feature of the system was equalization of both needs and resources between different authorities.

Equalizing needs through the grant system involved setting a national standard for all local Government services and giving each authority the ability to provide that service for each member for its population. Equalizing resources meant that each authority should have to make the same tax effort to raise revenue regardless of its taxable capacity. In order to assess the need for each area a unique method was used. A number of needs indicators were set up with the aim of taking into account variations in expenditure levels arising from:

a) the variable distribution of population and economic activity;

b) the variable concentration of different people or different types of economic activity (client groups);

c) differences in physical and social environment;

d) differences in scope and quality of services.

Until fiscal year 1973-74 the need indicators were defined a priori; from 1974-75 onwards a new method using past levels and patterns of expenditure to indicate need was adopted. The needs indicators were derived as a weighted average of past expenditures by categories of local authorities with the same characteristics by using a multiple regression analysis.
In each year since 1976-77 the best regression equation, based upon a set of explanatory variable able to provide the closest relationship with past levels and pattern of expenditure was searched and the need was then defined [2].
The value judgment underlying the distribution of the need elements was that areas should be fully compensated for differences in their needs elements then a lump sum form was the most suitable instrument to use [3]. As I have noted in chapter four this system is efficient [4] for expenditure below the need element since in this case a lump-sum grant and a matching grant have the same effect in terms of expenditure increase.

The aim of the resources element was to compensate local authorities for their deficiency in the tax base if it fell below the national standard. This element sought to

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[2] For a most comprehensive explanation of the grant system see Bennett (1985).

[3] The use of a lump-sum as the best instruments formally derived in chapter four is a well known and recognized principle in the literature. On this respect see, for example King (1986).

[4] here efficiency is defined with respect to the minimum effort (or cost) that has to be incurred to reach a predetermined objective.

overcome differences in local authorities' ability to raise revenue and to ensure that authorities were not penalized by their small tax base. A grant was paid to each local authority which fell below the standard rateable value per head as set by the Central Government and the grant was paid in a matching grant form. It must be remarked that the system was not fully equalizing since no grant withdrawals were foreseen for authorities whose rateable value exceeded the national standard.

If we recall the optimal solution for the problem with no asymmetry of information, namely:

$$\text{Min} \quad G + (1-g) * \alpha^*$$

$$\text{s.t.} \ (1 - \beta) * a_2 + \beta \left(\frac{a_1}{g} + G \right) = \alpha^*$$

and the optimal grant set given by:

$$G = a_2$$

$$(1-g) = \left(1 - \frac{\beta \, a_1}{\alpha^* - a_2} \right)$$

then similarities with the system described before are quite clear. The need equalization grant, in fact, has the same form as the theoretical one; as concerns the resources equalization part it seems that the system actually implemented is a simplified version of the optimal matching element. The implicit assumption underlying the system was that local authorities of a given class had the same preferences for local services and it was on this assumption that the standard rateable value was set out. The most important characteristic of this system, at least in the context of my discussion is that Central Government is assumed to know needs [5], resources [6] and preferences [7] and on this basis is thought to be able to bring local authorities to the level of expenditure α_i^* that was seen as the optimal level of provision of local services. Technical problems related to the estimation of the regression equation, and the special treatment of some areas [8] were blamed for the failure of this formula. The economic explanation of this failure is, however, quite different. The past levels of expenditure are not a good needs indicators since they incorporate both need and preferences then the system was actually boosting up expenditure. The inability of Central Government to distinguish between needs and preferences was the actual cause of the failure and the Government soon realizes the problem and tried to respond accordingly.

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[5] i.e. a_2

[6] i.e. a_1

[7] i.e. β.

[8] See Bennett (1982) for a full account of the different criticisms to the system.

3. THE GRANT SYSTEM AFTER THE REALIZATION OF THE ASYMMETRY OF INFORMATION PROBLEM.

The method of allocating the Rate Support Grant employed up to 1980 aroused an increasing body of criticism and it became increasingly seen by Central Government as a method undermining its ability to control public expenditure. As a result the Conservative Government that took office in 1979 changed the formula by which grant were distributed. This phase of the history of grants is characterized by a period of dramatic changes in the distribution formula that I will review only briefly.

Central government grant to local authorities was made up of three constituent parts: specific grants, supplementary grants and the rate support grants. Specific grants were made to aid local authorities in their implementation of specific projects or programmes like urban aid, which the government is keen to promote. Supplementary grants are made to assist local authorities with particular obligations often imposed on them by government, like the up keep of roads and national parks. Rate Support Grant [9] was the general grant made to subsidise spending by local authorities which would otherwise have to be financed out of the rates.

The new system for distributing the grant was sought to be a unitary grant known as block grant. Although called a unitary grant, it was concerned with the same two essential issues to which the separate resources and needs element of

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[9] which is also referred as RSG.

the old RSG were directed: resources and needs equalization. As a unitary grant, however, it was designed to achieve simultaneous need and resources equalization. This system of grant distribution is quite similar to the optimal set I described in chapter four, so I think it is interesting to compare the two systems.

I will first describe the system as has been set up by the Government and I will try later to examine the differences and similarities with the optimal theoretical grant allocation rule I derived in chapter four.

The reform of the grant system, even if has been proposed as a method to simplify the system and to give greater transparency to the way in which grants were allocated is in my opinion the clear sign that Central Government became aware of its inability to know all the parameters necessary to set a fully equalizing system at the minimum cost As stated by the Secretary of State for the Environment in fact the system had, among other scopes 'to exclude as far as possible differences in expenditure which are the result of local preferences'[10].

The block grant contained two essential features. First there is the GREA [11] for the authority. Although GREA is defined as " the cost of providing for a common standard of service in authorities with common functions" it is rather, in my opinion, Government's estimate of the optimal

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[10]Ministerial guidelines to Grants Working Group, 1980.

[11]GREA stands for Grant Related Expenditure Assessment.

level of expenditure that each local authority should reach; it is not the exact replica of the previous need element. Since it is difficult to distinguish between need and preferences GREA is rather an estimate of the optimal level of expenditure that each authority should reach. Central Government's intentions are quite clear if we observe that expenditure up to GREA is not fully financed by the Rate Support Grant as was the need element in the previous system [12]. The calculation of GREA begins by taking the national figure for relevant expenditure. This total is then distributed between authorities by first identifying measurable factors which influence the costs to authorities of providing services. These factors approximate the client groups and define the number of units of service need. The second step is the assessment of the relative importance of different factors for each service which become expressed in a set of weights.

These weights approximate to the unit cost of providing each service need. Like the previous need element five main factors in GREA are identified:

1) the population of an area;
2) the physical features of an area;
3) social and environmental characteristics of an area which may constitute particular problems;
4) differences in service costs between areas;

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[12] In the years in which the target regime was in operation for some authorities the target level was lower than GREA which would be a contradiction with the assumption of GREA=need but would instead be justified if GREA was an estimate of the optimal level of expenditure to be reached.

5) special requirements for service provision in different areas.

Within each of these categories a pound per unit of service need is specified which is then multiplied by the number of units of that service need in each relevant authority.

The second central feature of the new RSG is the definition of the Grant Related Poundage (GRP) . The standard rate poundage is determined by Central Government. In setting this poundage, as with the previous national standard rateable value per head for the resources element, central government chooses a level at which it is expected most authorities will still receive benefits from expenditure. However, unlike the previous resource element, the level chosen is not in terms of the maximum tax-base equalization attainable by dividing a given grant sum, but it is instead in terms of the maximum expenditure differences from the standard that can be supported on differing tax bases; i.e. need, preferences and resources are combined together by the use of the GRP. The first stage of the definition of GRP is the national level. This is then divided between different levels and types of authorities in terms of their different responsibilities for services.

The further features to be mentioned in the new block grant are the threshold and tapering multipliers to be employed. These tapering multiplier terms are important

since they are used to penalise those local authorities which are 'overspending' in the sense that their expenditure exceeds the GREA assessed by central government as typical of authorities with similar objective characteristics.
For those local authorities for which this 'excess' expenditure is beyond a specific threshold the level of grant is progressively reduced so that the "marginal excess expenditure" is borne increasingly by local ratepayers.

In order to compare RSG in its most simple formulation[13] with the optimal allocation in an asymmetry of information framework we have first of all to note that the term block grant is a misnomer from the point of view of the economic theory of grant: the grant is in fact a combination of a block (lump sum) grant plus a matching grant. As I noted before, the key features of the block grant are the grant related expenditure assessment (GREA) and the grant related poundage (GRP). The equalization through the block grant was envisaged as follows: if each local authority of a given class levies a specified tax rate (GRP) the block grant allows it to spend at its GREA that is, for expenditure at GREA the amount of block grant received by a local authority is the difference between its GREA and the amount of tax it would raise by applying the specified GRP to its tax base.

Figure one illustrates how the budget constraint for a

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[13] i.e. I will not consider for the time being the problems introduced by the target regime in operation from fiscal year 1983/4 through 1985/6

representative local authority is altered by the introduction of the grant from Central Government. The budget is first illustrated in terms of expenditure per head and tax rate. In the absence of any support from Central Government the budget constraint for this local authority would be written as:

$$X = rB \qquad (1)$$

where r is the tax rate and B is the tax base and in the diagram is represented by the straight line ob.

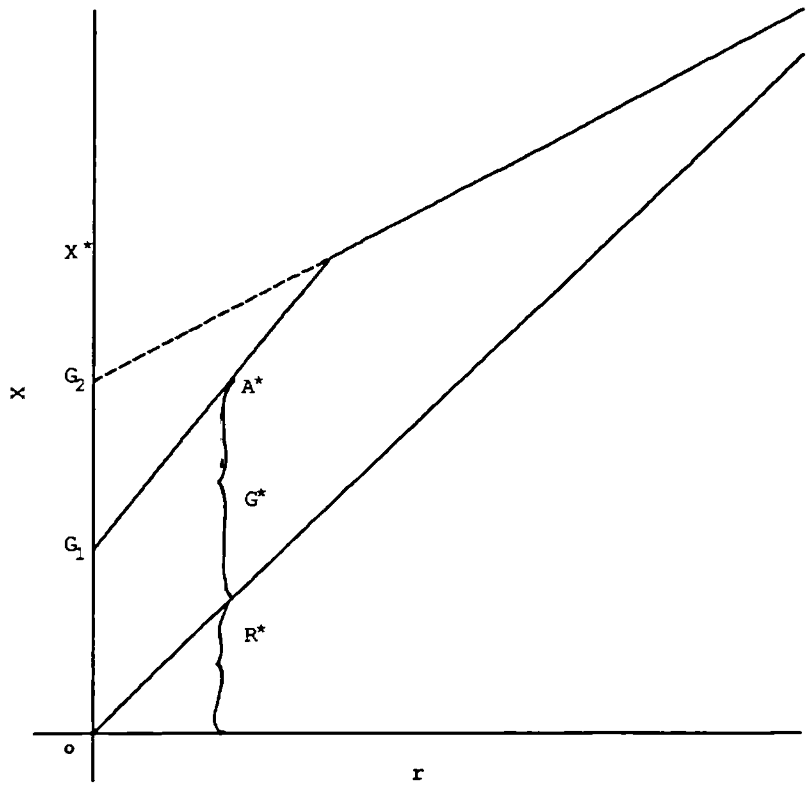

FIGURE ONE

If our authority would spend at its GREA (point A in figure one) the specified tax rate would be given by GRP (r^* in the diagram). Thus, by levying at tax rate r^*, the local authority would raise the amount R^* per capita from local taxation and qualify for amount G^* per capita in the form of block grant :

$$R^* + G^* = A.$$

As I have mentioned before GREA can be approximated by the level α_i^* of the theoretical model presented before.and it is now clear why it is not a pure need element.By spending at their GREAs all local authorities of the same class will have to raise the same tax rate. This does not clearly implies that local authorities will receive the same level of utility from this expenditure since the resource element a_1 is different unless the GREA is chosen optimally to achieve this goal.[14]

For levels of expenditure other than A, the grant related poundage schedule (GRPS) becomes relevant. This specifies the local tax rate that must be levied if a local authority is to be able to spend a given amount; again, any difference between this level of expenditure and locally raised revenue is made up by block grant. But it is useful to view any level of expenditure as a deviation from GREA. Then what the GRPS does is to ensure that any given departure of expenditure away from GREA has the same implication for the

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[14]In a world with perfect information agents the utility for local government could be defined over expenditure and tax rate as follows:

$$U_t = (1 - \beta)\, \ell n(a_1 - T) + \beta\, \ell n(X - a_2)$$

and X and the matching grant could be chosen such that:

$$U_t = (1 - \beta)\, \ell n(a_1 - T) + \beta\, \ell n(X - a_2) = U^*$$

required deviation of the local tax rate away from r^*, irrespective of the local authority's own tax base. That is, relative to their GREA's, all local authorities are able to act as if they have the same tax base. In this way different taxable resources are taken into account and the block grant system can be viewed as a needs (GREA) related district power equalizing grant. The tax base implicit in the GRPS is not a constant, but is instead dependent on a local authority's expenditure level. In the simplest formulation, once expenditure reached a given level above GREA, known as threshold, the implicit tax base was reduced. Thus a local authority's post-grant budget constraint is illustrated by line M_1de in Figure one, and it can now readily be seen how block grant comprises a lump sum grant plus a matching grant. Let the threshold level of expenditure be denoted by X_1; the local authority's post grant budget constraint is given by:

$$X = G_1 + r * B_1 \qquad : X < X_1 \qquad (2)$$

where G_1 is the implicit lump sum grant and B_1 is the tax base implicit in the section of the budget constraint G_1d. According to this system, the block grant is given by:

$$BG = G_1 + r\,(B_1 - B) \qquad : X < X_1$$

where the implicit matching grant is given by $r(B_1 - B)$ and the matching rate is $\gtrless 0$ as $B_1 \gtrless 0$.

Similarly, for expenditure in excess of X_1 we have:

$$X = G_2 + r * B_2 \qquad : X \geq X_1 \qquad (3)$$

$$BG = G_2 + r * (B_2 - B) \qquad : X \geq X_1$$

where B_2 is the implicit tax base in the section of the budget constraint de.

It is clear that if $B > B_2$ there will be an expenditure level at which block grant is exhausted. However, the institutional arrangements in England do no allow block grant payments to be negative and thus another kink is introduced at the expenditure level at which the grant is exhausted.

The system that government introduced in 1981/2, expecially in its virgin formulation which has been applied only in the last three years could have been the right response to the asymmetry of information that characterizes the relationship between the two levels of government.

The implicit lump sum grant should have had the function of giving local authorities the amount of resources necessary to provide for the baseline level of expenditure while the ranges of values between GREA and threshold with the associated GRPS could have represented the set of choices open to local government to reveal their true preferences.

The two marginal tax rate implicit in the two sections of the budget constraint can be easily converted into the system of marginal prices derived from the theoretical model I presented before by observing that since $X = T + G$ and $T =$

rB, where T is the tax payment, using equations (2) and (3):

$$X = T + G_1 + T\left(\frac{B_1}{B} - 1\right) \qquad X \leq X_1 \qquad (2a)$$

$$X = T + G_2 + T\left(\frac{B_2}{B} - 1\right) \qquad X \geq X_1 \qquad (3a)$$

The budget constraint can be depicted as in figure two.

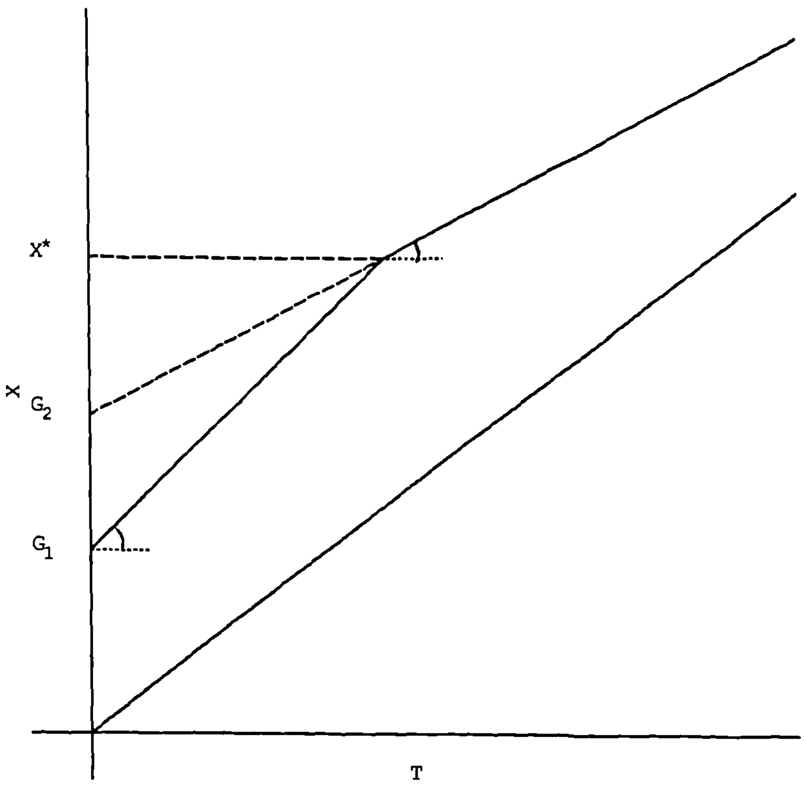

FIGURE TWO

Adjusting the level of GRPs, threshold and GREA from one year to the other one should have in theory given the right incentive to local authorities to tell the truth in (almost) the same way as α^* and all the price and incentive schedules assure the realization of an optimal grant system in the theoretical model. However, as I argued at the end of the theoretical section the system could have failed to reach its objectives if local authorities do not have a true preference parameter at all. In this case the most likely result is that local authorities instead of maximising their utility over their life cycle choose their expenditure pattern each year with references to resources available and the system of prices in force that particular year. Past experience of overspending have suggested Central Government to avoid this risk as much as possible; as a result the block grant foresees a target regime.

The targets regime operated throughout the fiscal years 1981-2 to 1985-6 and is appended to the underlying block grant system. The precise nature of the targets regime varies between different years of its operation but the essence of the system is that an expenditure target is specified for each local authority which is based predominantly on the previous year's expenditure, and is only indirectly related to expenditure need as represented by GREA . Penalties in the form of withdrawals of block grant are imposed if expenditure exceeds the target level of expenditure. The size of the penalty withdrawal of grant varies between the five years of the system's operation, but is always a monotonically increasing function of the percentage by which expenditure exceeds the target. And in general this introduces a number of additional kinks into the post - grant budget constraint. The impact of this target regime on the local authority budget constraint illustrated in Figure two is shown in figure three where the assumption is that the target level of expenditure is given by X_T (as represented by point c).

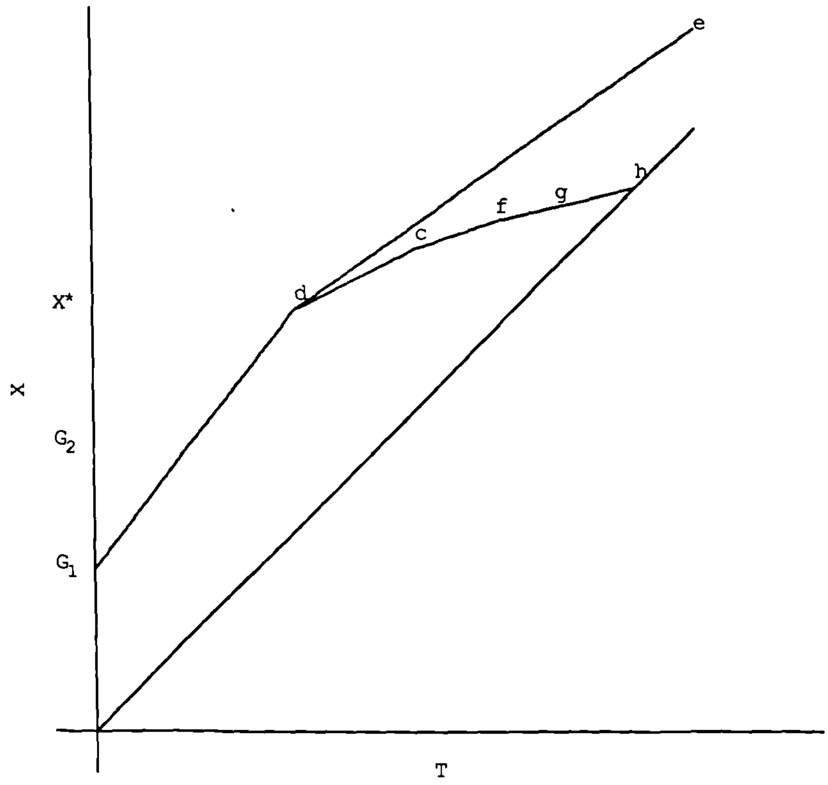

FIGURE THREE

The post-grant budget constraint in the absence of the target regime is G_1de, and following the imposition of the target regime it is given by G_1dcfg. The target system represent an attempt by the government to encourage restraint in local authority spending and its imposition means that the equalizing aspect of the underlying block grant system is to a large extent lost. Two local authorities with identical GREAs can face very different post-grant budget constraints depending on the previous level of expenditure. The history of the grant and penalty system is a well known matter and I will not go onto further detail. In the next section I will try instead to see whether similarities exist between the actual grant system and the optimal allocation rule developed in chapter four.

3.1 THEORY AND REALITY: SOME TESTS.

From fiscal year 1986-87 onwards the target regime has been abolished, the post-grant budget constraint presents just a kink, so the grant in operation is very similar to the theoretical price system described in the previous chapters[15]. The last three years also correspond to a new life cycle for local authorities since in May 1986 elections for local authorities that choose to change all representatives simultaneously took place. It is then possible to devise a series of tests aimed at assessing to which extent the grant system in operation through fiscal years 1986-87 to 1988-89 reflected the optimal allocation rule and was consistent with the preferences for expenditure revealed by local authorities. One of the most important defects of the models in the asymmetry of information literature is the lack of hypothesis to test how the models fit the actual data. The model I presented has the same defect, but in this case it is at least possible to devise some indirect tests. The first series of tests which I have labelled "*internal consistency tests*" are aimed to assess whether the actual two price system reflected the optimal system outlined in chapter four.

3.1.1 INTERNAL CONSISTENCY TESTS.

At the end of chapter four I showed how, for the case

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[15]The actual grant system in operation was an approximation of the optimal system since rate capping was in operation, then the system was per se admitting the possibility of overspending on the second segment.

in which $\rho=0$ and $r=0$[16] the optimal grant formula could be written such that the price on the first segment of the budget line was defined as:

$$g_{1i} = \frac{\beta\ a_1}{(\alpha^{*} - a_2)\theta} \qquad (5)$$

The prices on the second segment were defined as:

$$g_{4,2} \geq \frac{\beta\left[a_1\ (1 - \beta_d) + K\right]}{(\alpha^{*} - a_2)\ (1 - \beta)} \qquad (6)$$

$$K = \frac{3\ a_1\ (\beta - \beta_d)}{4 - 3\ \beta}$$

$$g_{3,2} \geq \frac{\beta\left[a_1\ (1 - \beta_d) + K_1\right]}{(\alpha^{*} - a_2)\ (1 - \beta)} \qquad (7)$$

$$K_1 = \frac{2\ a_1\ (\beta - \beta_d)}{4 - 3\ \beta}$$

$$g_{2,2} \geq \frac{\beta\left[a_1\ (1 - \beta_d) + K_2\right]}{(\alpha^{*} - a_2)\ (1 - \beta)} \qquad (8)$$

$$K_2 = \frac{a_1\ (\beta - \beta_d)}{4 - 3\ \beta}$$

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[16] i.e. for a model with no discount rate on utility and without interest rate on sums available at different time periods.

$$g_{1,2} \geq \frac{\beta\left[a_1 \quad (1 - \beta_d) \qquad\right]}{(\alpha^{*} - a_2 \;) \; (1 - \beta \;)} \tag{9}$$

while the incentive was defined as:

$$\theta = \left(\frac{1-\beta}{1-\beta_1}\right)^{\left(1 - \frac{1}{\beta}\right)} \tag{10}$$

From equation (5), by substituting in the formula all the relevant parameters, namely:

a_2 i.e. the need element is represented by M_1, the implicit lump-sum grant on the first segment. This assumption reflects the consideration that a full equalization of pure need is optimally pursued by a lump-sum grant.

α^{*}_{i} i.e. the optimal level of expenditure is represented by GREA. As I noted before, GREA, although is defined as a need element, incorporates preferences as well.

$\alpha^{*}_{i}\theta_{i}$ is the threshold which in the actual grant system represents the kink of the budget constraint for each local authority.

a_1 is, in analogy with the econometric model I presented in the previous chapter the maximum rate across Metropolitan Districts (excluding rate capped authorities from the sample) multiplied by the rateable value that year.

It is then possible to derive a distribution for β which I labelled β_a and that represents the implicit β for prices on the first segment. Two tests of consistency of the actual grant system with the optimal one can now be devised.

The first test is aimed to check the consistency of θ with the other parameters of the system. At the end of chapter four I showed that the incentive should be set according to equation (10) while the observed value for θ is equal to :

$$\theta = \frac{\text{THRESH}}{\text{GREA}} \tag{11}$$

For each year it is then possible to compare the distributions for the theoretical values of θ derived using equations (10) with the actual ones obtained by equation (11) and several tests can be designed to assess whether the two distributions are significantly different [17]. The values for θ in equation (10) are derived by assuming that θ is consistent with the distribution of β on the first segment of the budget line, that is the β_a distribution is used to obtain the θ_a distribution. The derivation of the second θ distribution using equation (11) is instead straightforward. Table 1 summarizes the principal statistical measures for each distribution in the three years under analysis. From the diagrams in appendix one it is possible to see that the two distributions follow different patterns and the log likelihood test for the hypothesis:

$$\mu_{(a)} = \mu_{(b)} \quad ; \quad \sigma_{(a)} = \sigma_{(b)}$$

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[17] from a statistical point of view.

tested by imposing the restrictions on both (a) and (b) distribution should be used to give a formal proof.

This test assumes that both distributions are normally distributed; so a test is needed to verify this hypothesis.

For a large sample, the Bearque and Jera test could be used In this case, due to the small size of the sample the test did not perform very well [18] and gave such a poor result I prefer not to record it at all. However, from the diagrams it seems possible to assess that the distributions are not normal so a nonparametric test might be used to compare them. The second test reported in table 2 is the Mann-Whitney test based on the ranking of the different values for θ and it shows that the two distributions do not have the same ranking.

Another test can now be designed. If I assume that β_1 is the lowest value of the β distribution derived using equation (5) and θ = threshold/GREA it is then possible to derive the distribution for the β implicit in the incentive scheme by substituting the relevant parameters in equation (10). I can now compare this new distribution which I will label β_b with the one I have derived using equation (5) and that was labelled β_a. The summary statistics for both distributions are recorded in table 3. These results show that the mean value of β implicit in the incentive scheme is higher than the corresponding one in the price system and

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[18] As I should have expected since it is valid only asymptotically.

that its variance is smaller. This conclusion has, however, to be carefully interpreted since the β_b distribution has been derived by implicitly assuming that the range of β [19] used for the incentive was the one derived using the price system and the validity of this hypothesis cannot be tested: the tests that I will present can assess whether the two distributions are consistent each other [20] but they give no insights for alternative behavioural hypotheses.

The tentative conclusion that can be derived is that Central Government might have designed the incentive scheme in a rather more general way by giving each authority a more homogeneous incentive while the emphasis of the grant system was on the price scheme, that is on a closer control of expenditure by using a more personalized price for each local authority. Again it is possible to compare the two distributions using the same tests I used before. I have summarized the results in table 4. As for the θ distribution it is possible to see that the two distributions are different as the tests reported at the bottom of table four show.

The second series of tests concerns the consistency of the value of the price on the second segments with the other parameters of the system. This can be done by substituting the relevant parameters in equations (6) to (9) and using the β_a and β_d for the parameter β. As I pointed out in

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[19] or at least the lowest value for β.

[20] i.e. as they should be if the actual grant system was designed according to the optimal grant allocation rule.

chapter four Central Government sets the two prices system before the start of the life cycle of local authorities, at a stage at which it does know the range of values for β but has no idea of what the true one will be. It is then optimal in this case to use the two extreme value of such distribution for the price system. This assures, in fact that all local authorities, irrespective of their preferences will not overspend.

As it can be noted from the last set of diagrams in appendix one the price derived using the β_d distribution is perfectly consistent with the actual price on the second segment, while the one derived using the β_a distribution usually leads to higher theoretical prices. This is mainly the results of the already observed comparatively higher variance of the β_a distribution. The new set of tests seem to suggest that the price and incentive system are consistent, at least on average. The main difference between the two seems to lie in the price system on the first segment for which the mean of β is assumed to be lower than for setting the other parameters .

3.2.2 THE GRANT SYSTEM AND LOCAL GOVERNMENT'S RESPONSES.

Let us now turn to the comparison between β implicit in the grant distribution formula and the ones I have derived from the estimations presented in the previous chapter.

In 1986-7 the average value of β implicit in the price system was equal to .7 while in 1987-8 it was .66 and in 1988-9 it was .67. The range of β in the different years is between .62 and .75. If we compare those values with the one derived from the estimation presented in the previous chapter, i.e. the β revealed by local authorities through their expenditure decisions, it is possible to note that the preferences for local public services implicit in the grant system are lower than those estimated. In both cases, however it seems that the hypothesis of a life cycle behaviour has to be rejected. The actual grant system proves to be quite different from the optimal grant allocation rule I presented in chapter four. This conclusion, however has to be interpreted in the light of the problems that Central Government has to solve to implement the grant formula that optimally allocates resources. Again, the actual grant system might be considered a good approximation of the formula derived in chapter four and anyway both systems seems to be inspired by the same principle.

Another interesting feature of the system can be observed by recalling the pattern of β derived from table 7 in chapter five . There seems to be an inverse correlation between the values of the estimated β and of the one implicit in the grant system, but what is important to note at this stage is that local authorities seem to adjust their preferences according to the system ruling in any particular year as well as to the resources available[21].

The number of years taken into account is clearly too limited to derive a clear cut conclusion but the hypothesis

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[21]This was the conclusion from the data examined in the last model in the previous chapter, namely in table 8.

that local governments adjust their preferences to the system in force each year cannot be at least rejected.

This consideration reinforces the argument that it might then actually be impossible to devise an optimal grant system at all. The reason for this pessimistic conclusion is based on the evidence that the preferences for local services are adjusted according to the price system in force each year. This behaviour clearly make impossible to devise any optimal system at all: whatever the rules are and the parameters are chosen, the system is bound to fail to reach its objectives.

4. THE NEW GRANT SYSTEM: ADMISSION OF A FAILURE OR AN EFFICIENT WAY TO ALLOCATE RESOURCES?

The Local Government Finance Act, which received its Royal Assent in July 1988, introduces three main changes to the system of local government finance in England and Wales. First, the domestic property tax (known as 'the domestic rates') is to be abolished, and is to be replaced by a community charge. For most individuals the community charge is to be a flat rate tax, or poll tax, although there is a sliding scale of tax eligibility for low income individuals, but everybody is to pay at least 20% of the community charge relevant to their local community. As with the property tax, each local government will in general be free to set the level of its community charge although the Act does give the Secretary of State the power to limit both the level and rate of increase of community charges. The Secretary of State has had this power with respect to the local property tax since 1984, signaling a relatively recent change in the administration of the local tax system, especially when one considers that the local property tax was introduced in 1601.

The second provision of the Act concerns the administration of the local non-domestic property tax. Whilst this tax is to be retained it is to all intents and purposes to become a central government tax. The rate of taxation is to be the same in all localities and is to be set by the central government. The yield of the tax is to go initially to central government with the proceeds being paid to local government as part of the grants-in-aid to local governments. It is important to note, however, that the grant is not to be paid out on a derivation principle, but is instead effectively to be paid out on a per capita basis. Thus there is to be no link between the amount of money raised through non-domestic property taxation in an area and the amount received back from central government.

The third part of the package of reforms concerns the nature of the main central government grant-in-aid to local governments which is to become a fixed grant in the sense that the amount received by a locality is to be independent of its expenditure. This grant, known as the Block Grant, seeks to compensate for differences between localities in expenditure needs and taxable resources.

As a result of this policy, the budget constraint for local authorities will be written as:

$$X_i = G_{ic} + t_i \Pi_i$$

where G_{ic} is the lump-sum grant distributed from Central Government which incorporates both the equalizing grant and the revenue from non domestic taxation. The equalizing grant is set in a way such that each local authority's taxpayer bill would be equal if local authorities spent at their GREAS. The non domestic taxation is the share, according to adult population, of the total tax raised through the uniform business taxation across the country.

A central objective behind the reforms is the achievement of accountability in the system of local government finance. To be accountable the government argues that two criteria should be met within a system of local government finance. First, all the eligible to vote in local elections should be liable to pay local taxes, and second, the full marginal cost of local expenditure should be met out of local taxes. The background to the government's concentration on the notion of accountability is its belief that throughout the early and mid 1980s many local governments failed to restrain their expenditure because the existing system of local government finance did not encompass accountability. The system of rate rebates and the increase of business taxation are supposed to be the most important causes of the failure of full accountability.

In recent papers[22] we showed which might be the possible

See Barnett et al., 1989 (a,b,c)

effects of the introduction of the poll tax on the pattern of local government expenditure according to the model used to represent local government's behaviour; in this context I want to explain the rationale behind the introduction of the poll tax from the point of view of the optimal grant set. The asymmetry of information which rationalizes the existence of local government is also the main cause for the failure of any efficient system for allocating resources.

If this is the case the likely response of Central Government to this problem depends on its objectives. If Central government policy is oriented towards expenditure cuts and certainty in the amount spent the best way to achieve this objective is to give any local authority a lump sum grant and give to the local decision makers full power as concerns the amount spent and the local tax to be raised. The asymmetry of information problem that, at least from efficiency, might require the existence of local government is also the cause of a never ending contrast between Central Government and the other level of decision making and it is itself the cause of other inefficient allocation of resources. Central Government has then to trade off between local provision and inefficient allocation of resources because 'its agents are cheating' and a central provision of local public goods which cannot be optimal because of a lack of information!

In synthesis then there does not seem a clear cut response to the problem: as I suggested at the beginning of this section it is Central Government's objectives that determine the balance between the trade off. What I would like to point out is that the system of grants that has been in force in the recent years might have failed to reach its objective but it is hard to think that other systems rather than this could be perform better, at least if we assume that the parties involved behave optimally and that their objectives can be represented within a neoclassical framework.

The alternative way could be to derive from alternative theories of government behaviour an optimal distribution formula. This different approach could probably be developed in a game theory framework but I think that it might be still quite difficult to formalize local government's objectives. While in abstract it might be easy to define local objectives in terms of maximising the probability of being reelected or to be relatively better off than other local authorities, when it comes to formalize the models it is even more difficult and arbitrary to restrict the agents behaviour within an equation.

TABLE 1

1986-87

	Mean	σ	Min	Max	n
θ_a	1.0562	.0417	0.993	1.144	34
θ_b	1.0987	.0079	1.0747	1.109	34

1987-88

	Mean	σ	Min	Max	n
θ_a	1.0977	.0248	1.0389	1.1678	33
θ_b	1.0979	.0070	1.0841	1.1081	33

1988-89

	Mean	σ	Min	Max	n
θ_a	1.0276	.0226	0.9730	1.0998	33
θ_b	1.0973	.0072	1.0830	1.1078	33

TABLE 2

Year	t test for $\mu_a = \mu_b$
1986-7	5.84
1987-8	17.40
1988-9	16.89

Year	MANN WHITNEY TEST	S.L.
1986-7	594	99%
1987-8	1617	99%
1988-9	769	99%

TABLE 3

1986-87	Mean	σ	Min	Max	n
β_a	0.704	.0373	0.655	0.805	34
β_b	0.740	.0081	0.716	0.752	34
1987-88					
β_a	0.666	.0253	0.620	0.751	33
β_b	0.692	.0062	0.679	0.701	33
1988-89					
β_a	0.674	.0177	0.638	0.734	33
β_b	0.732	.0073	0.719	0.743	33

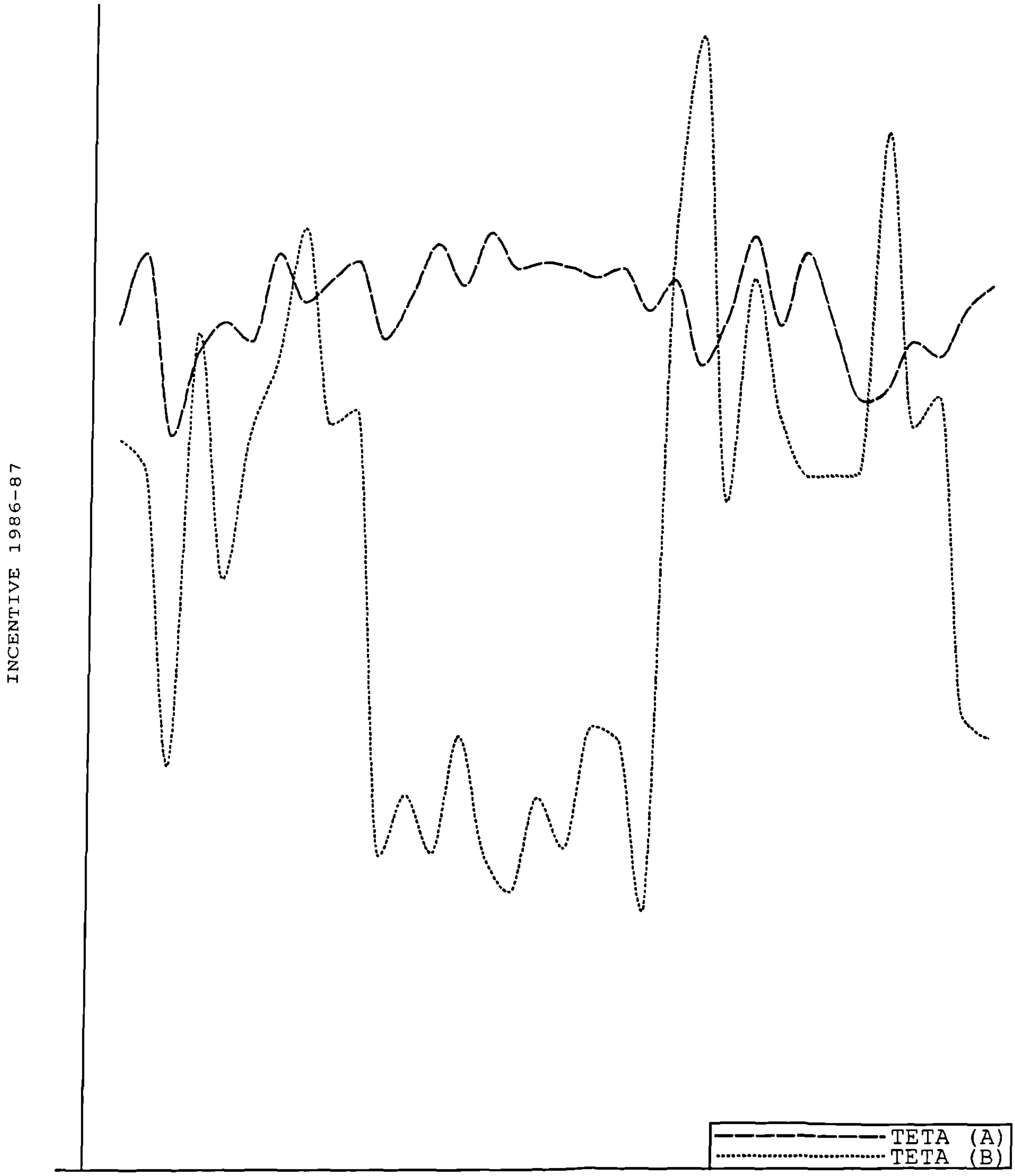
TABLE FOUR

Year	t test for $\mu_a = \mu_b$
1986-7	5.58
1987-8	5.37
1988-9	0.03

Year	MANN WHITNEY TEST	S.L.
1986-7	636	99%
1987-8	769	99%
1988-9	92	13%

APPENDIX ONE

INCENTIVE SCHEME DISTRIBUTION OF θ_a and θ_b for 1986-7

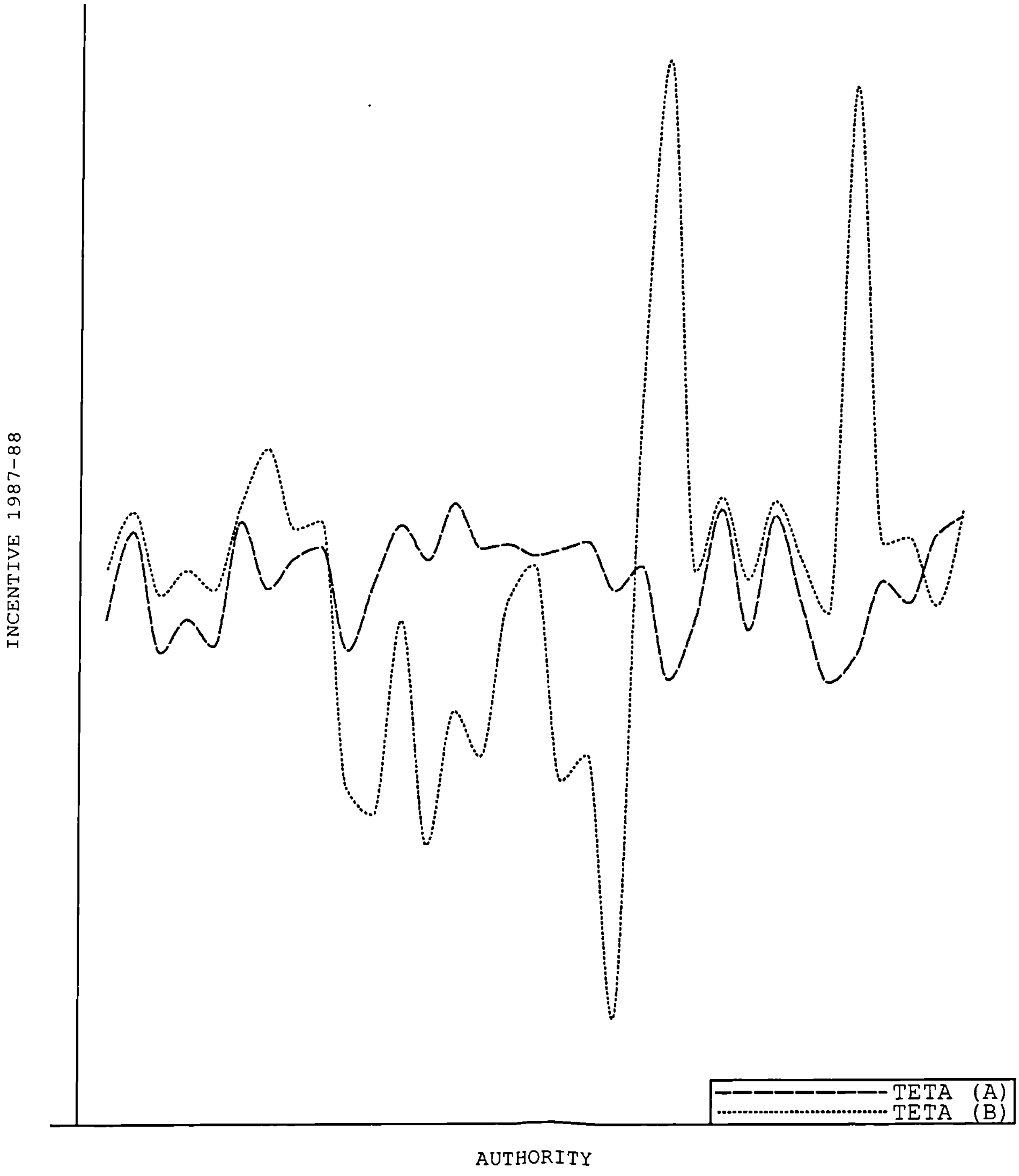


INCENTIVE SCHEME

```
HISTOGRAM - TETA87A
  PCT.    N
 0.735   25  I
 0.706   24  I
 0.676   23  I
 0.647   22  I
 0.618   21  I
 0.588   20  I
 0.559   19  I
 0.529   18  I
 0.500   17  I
 0.471   16  I
 0.441   15  I
 0.412   14  I
 0.382   13  I                              XXXXXXXXXX
 0.353   12  I                              XXXXXXXXXX
 0.324   11  I                              XXXXXXXXXX
 0.294   10  I                    XXXXXXXXXXXXXXXXXXXX
 0.265    9  I                    XXXXXXXXXXXXXXXXXXXX
 0.235    8  I                    XXXXXXXXXXXXXXXXXXXX
 0.206    7  I                    XXXXXXXXXXXXXXXXXXXX
 0.176    6  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 0.147    5  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 0.118    4  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 0.088    3  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 0.059    2  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 0.029    1  IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
             I---------I---------I---------I---------I---------I---------I
             1.08      1.08      1.09      1.10      1.11      1.11      1.12
```

```
HISTOGRAM - TETA87B
  PCT.    N
 0.735   25  I
 0.706   24  I
 0.676   23  I
 0.647   22  I
 0.618   21  I
 0.588   20  I
 0.559   19  I
 0.529   18  I
 0.500   17  I
 0.471   16  I
 0.441   15  I                              XXXXXXXXXX
 0.412   14  I                              XXXXXXXXXX
 0.382   13  I                              XXXXXXXXXX
 0.353   12  I                              XXXXXXXXXX
 0.324   11  I                              XXXXXXXXXX
 0.294   10  I                              XXXXXXXXXX
 0.265    9  I          XXXXXXXXXX          XXXXXXXXXX
 0.235    8  I          XXXXXXXXXX          XXXXXXXXXX
 0.206    7  I          XXXXXXXXXX          XXXXXXXXXX
 0.176    6  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 0.147    5  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 0.118    4  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 0.088    3  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 0.059    2  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 0.029    1  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
             I---------I---------I---------I---------I---------I---------I
           0.931     0.973     1.01      1.06      1.10      1.14      1.18
```

INCENTIVE SCHEME DISTRIBUTION OF θ_a and θ_b for 1987-8

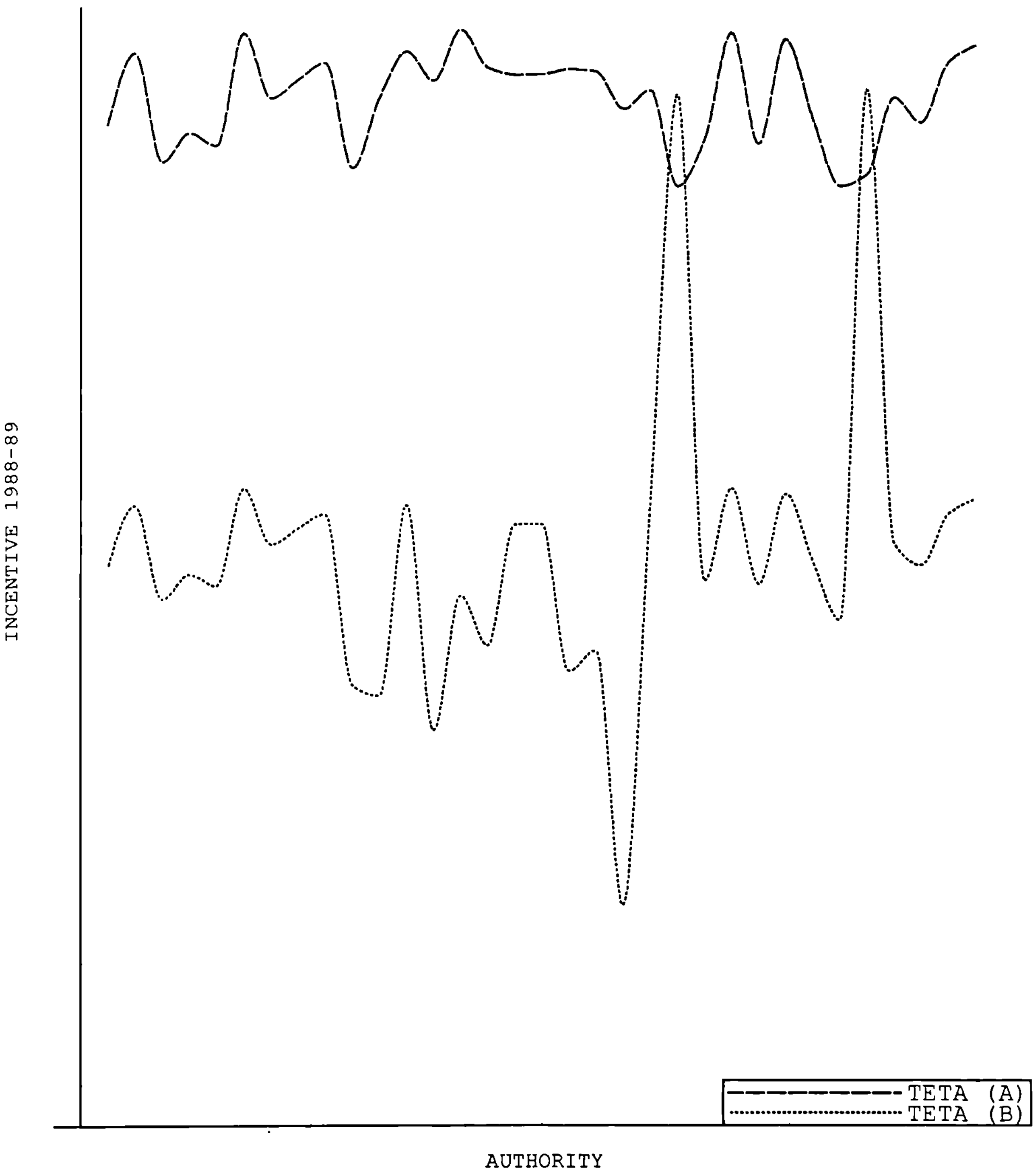


INCENTIVE SCHEME

```
HISTOGRAM - TETA88A
  PCT.    N
0.394   13  I
0.364   12  I                              XXXXXXXXXX
0.364   12  I                              XXXXXXXXXX
0.333   11  I                              XXXXXXXXXX
0.333   11  I                              XXXXXXXXXX
0.303   10  I                              XXXXXXXXXX
0.303   10  I                              XXXXXXXXXX
0.273    9  I                    XXXXXXXXXXXXXXXXXXXX
0.273    9  I                    XXXXXXXXXXXXXXXXXXXX
0.242    8  I                    XXXXXXXXXXXXXXXXXXXX
0.242    8  I                    XXXXXXXXXXXXXXXXXXXX
0.212    7  I                    XXXXXXXXXXXXXXXXXXXX
0.212    7  I                    XXXXXXXXXXXXXXXXXXXX
0.182    6  I                    XXXXXXXXXXXXXXXXXXXX
0.182    6  I                    XXXXXXXXXXXXXXXXXXXX
0.152    5  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0.152    5  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0.121    4  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0.121    4  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0.091    3  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0.091    3  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0.061    2  IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0.061    2  IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0.030    1  IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0.030    1  IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
            I---------I---------I---------I---------I---------I---------I
            1.08      1.08      1.09      1.10      1.10      1.11      1.12
```

```
HISTOGRAM - TETA88B
  PCT.    N
0.394   13  I
0.364   12  I                    XXXXXXXXXXXXXXXXXXXX
0.364   12  I                    XXXXXXXXXXXXXXXXXXXX
0.333   11  I                    XXXXXXXXXXXXXXXXXXXX
0.333   11  I                    XXXXXXXXXXXXXXXXXXXX
0.303   10  I                    XXXXXXXXXXXXXXXXXXXX
0.303   10  I                    XXXXXXXXXXXXXXXXXXXX
0.273    9  I                    XXXXXXXXXXXXXXXXXXXX
0.273    9  I                    XXXXXXXXXXXXXXXXXXXX
0.242    8  I                    XXXXXXXXXXXXXXXXXXXX
0.242    8  I                    XXXXXXXXXXXXXXXXXXXX
0.212    7  I                    XXXXXXXXXXXXXXXXXXXX
0.212    7  I                    XXXXXXXXXXXXXXXXXXXX
0.182    6  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0.182    6  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0.152    5  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0.152    5  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0.121    4  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0.121    4  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0.091    3  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0.091    3  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0.061    2  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX          XXXXXXXXXX
0.061    2  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX          XXXXXXXXXX
0.030    1  IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX          XXXXXXXXXX
0.030    1  IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX          XXXXXXXXXX
            I---------I---------I---------I---------I---------I---------I
            1.02      1.05      1.07      1.10      1.12      1.15      1.17
```

INCENTIVE SCHEME DISTRIBUTION OF θ_a and θ_b for 1988-9

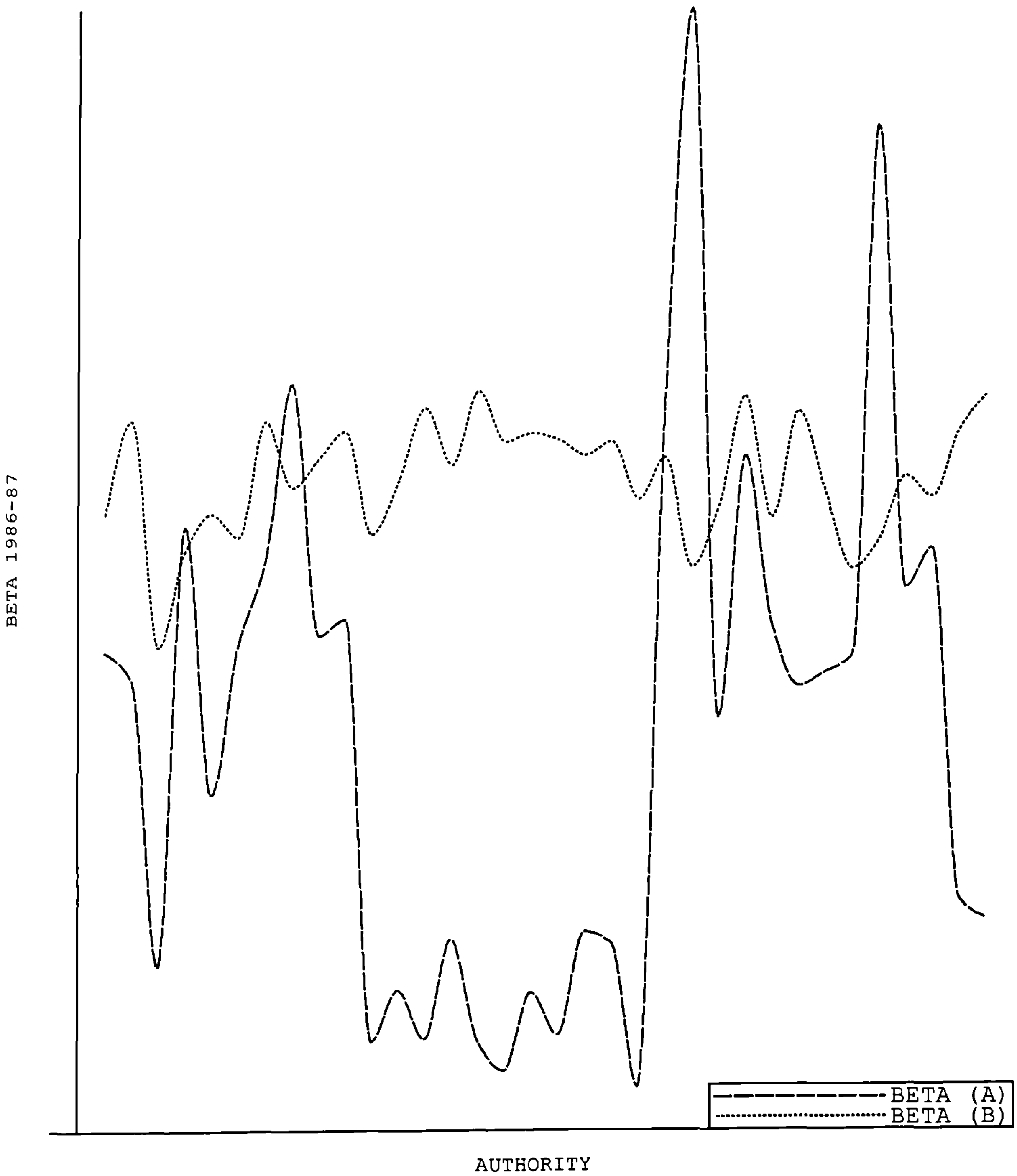


INCENTIVE SCHEME

```
HISTOGRAM - TETA89A
  PCT.    N
  0.394   13  I
  0.364   12  I                                XXXXXXXXXX
  0.364   12  I                                XXXXXXXXXX
  0.333   11  I                                XXXXXXXXXX
  0.333   11  I                                XXXXXXXXXX
  0.303   10  I                                XXXXXXXXXX
  0.303   10  I                                XXXXXXXXXX
  0.273    9  I                                XXXXXXXXXX
  0.273    9  I                                XXXXXXXXXX
  0.242    8  I                      XXXXXXXXXXXXXXXXXXXX
  0.242    8  I                      XXXXXXXXXXXXXXXXXXXX
  0.212    7  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  0.212    7  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  0.182    6  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  0.182    6  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  0.152    5  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  0.152    5  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  0.121    4  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  0.121    4  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  0.091    3  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  0.091    3  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  0.061    2  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  0.061    2  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  0.030    1  IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  0.030    1  IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
              I---------I---------I---------I---------I---------I---------I
              1.08      1.08      1.09      1.10      1.10      1.11      1.12

HISTOGRAM - TETA89B
  PCT.    N
  0.758   25  I
  0.727   24  I
  0.697   23  I
  0.667   22  I
  0.636   21  I
  0.606   20  I
  0.576   19  I
  0.545   18  I
  0.515   17  I
  0.485   16  I
  0.455   15  I                    XXXXXXXXXX
  0.424   14  I                    XXXXXXXXXX
  0.394   13  I                    XXXXXXXXXX
  0.364   12  I                    XXXXXXXXXXXXXXXXXXXX
  0.333   11  I                    XXXXXXXXXXXXXXXXXXXX
  0.303   10  I                    XXXXXXXXXXXXXXXXXXXX
  0.273    9  I                    XXXXXXXXXXXXXXXXXXXX
  0.242    8  I                    XXXXXXXXXXXXXXXXXXXX
  0.212    7  I                    XXXXXXXXXXXXXXXXXXXX
  0.182    6  I                    XXXXXXXXXXXXXXXXXXXX
  0.152    5  I                    XXXXXXXXXXXXXXXXXXXX
  0.121    4  I                    XXXXXXXXXXXXXXXXXXXX
  0.091    3  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  0.061    2  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX          XXXXXXXXXX
  0.030    1  IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX          XXXXXXXXXX
              I---------I---------I---------I---------I---------I---------I
             0.960     0.982      1.01      1.03      1.05      1.07      1.10
```

INCENTIVE SCHEME DISTRIBUTION OF β_a and β_b for 1986-7

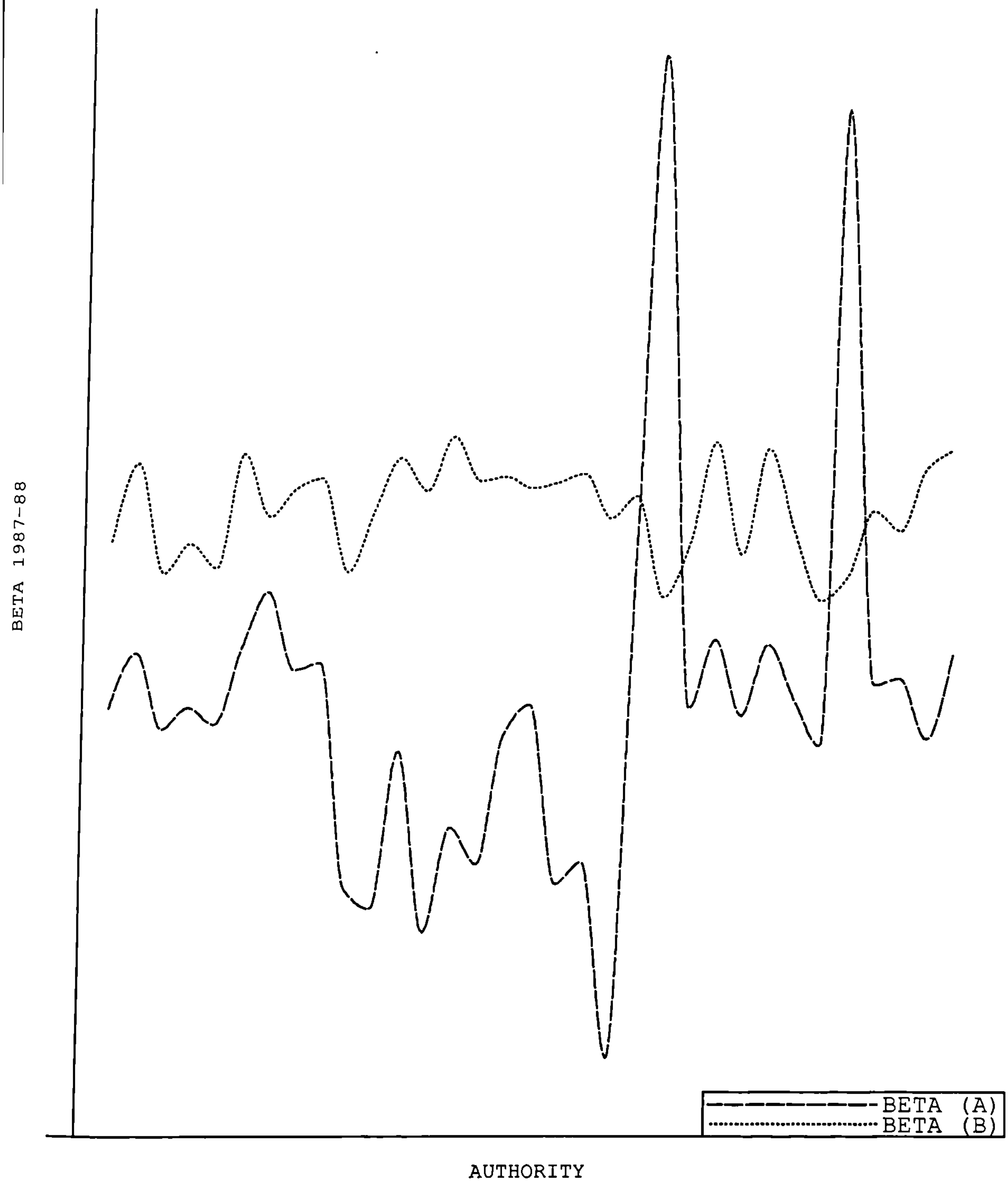


INCENTIVE SCHEME

```
HISTOGRAM - BETA87A
  PCT.    N
 0.735   25  I
 0.706   24  I
 0.676   23  I
 0.647   22  I
 0.618   21  I
 0.588   20  I
 0.559   19  I
 0.529   18  I
 0.500   17  I
 0.471   16  I
 0.441   15  I
 0.412   14  I                              XXXXXXXXXX
 0.382   13  I                              XXXXXXXXXX
 0.353   12  I                              XXXXXXXXXX
 0.324   11  I                              XXXXXXXXXX
 0.294   10  I                              XXXXXXXXXX
 0.265    9  I                              XXXXXXXXXX
 0.235    8  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 0.206    7  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 0.176    6  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 0.147    5  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 0.118    4  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 0.088    3  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 0.059    2  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 0.029    1  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
             I---------I---------I---------I---------I---------I---------I
           0.593     0.630     0.668     0.705     0.742     0.780     0.817

HISTOGRAM - BETA87B
  PCT.    N
 0.382   13  I
 0.353   12  I                              XXXXXXXXXX
 0.353   12  I                              XXXXXXXXXX
 0.324   11  I                              XXXXXXXXXX
 0.324   11  I                              XXXXXXXXXX
 0.294   10  I                    XXXXXXXXXXXXXXXXXXXX
 0.294   10  I                    XXXXXXXXXXXXXXXXXXXX
 0.265    9  I                    XXXXXXXXXXXXXXXXXXXX
 0.265    9  I                    XXXXXXXXXXXXXXXXXXXX
 0.235    8  I                    XXXXXXXXXXXXXXXXXXXX
 0.235    8  I                    XXXXXXXXXXXXXXXXXXXX
 0.206    7  I                    XXXXXXXXXXXXXXXXXXXX
 0.206    7  I                    XXXXXXXXXXXXXXXXXXXX
 0.176    6  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 0.176    6  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 0.147    5  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 0.147    5  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 0.118    4  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 0.118    4  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 0.088    3  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 0.088    3  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 0.059    2  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 0.059    2  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 0.029    1  IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 0.029    1  IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
             I---------I---------I---------I---------I---------I---------I
           0.716     0.724     0.732     0.740     0.748     0.757     0.765
```

INCENTIVE SCHEME DISTRIBUTION OF β_a and β_b for 1987-8

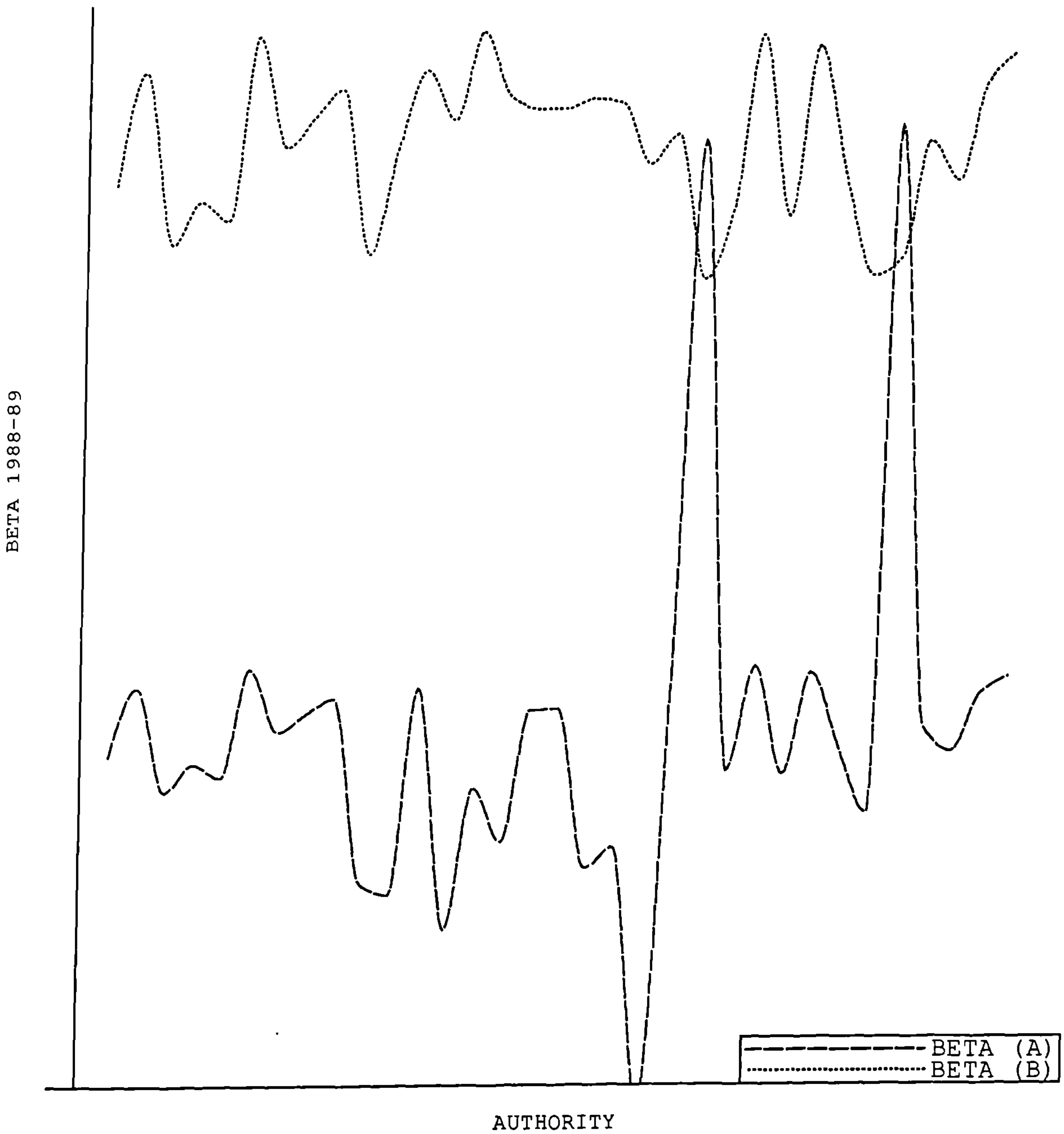


INCENTIVE SCHEME

```
HISTOGRAM - BETA88A
  PCT.     N
 0.758    25  I
 0.727    24  I
 0.697    23  I
 0.667    22  I
 0.636    21  I
 0.606    20  I
 0.576    19  I
 0.545    18  I
 0.515    17  I
 0.485    16  I                    XXXXXXXXXX
 0.455    15  I                    XXXXXXXXXX
 0.424    14  I                    XXXXXXXXXX
 0.394    13  I                    XXXXXXXXXX
 0.364    12  I                    XXXXXXXXXX
 0.333    11  I                    XXXXXXXXXX
 0.303    10  I                    XXXXXXXXXXXXXXXXXXXX
 0.273     9  I                    XXXXXXXXXXXXXXXXXXXX
 0.242     8  I                    XXXXXXXXXXXXXXXXXXXX
 0.212     7  I                    XXXXXXXXXXXXXXXXXXXX
 0.182     6  I                    XXXXXXXXXXXXXXXXXXXX
 0.152     5  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 0.121     4  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 0.091     3  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 0.061     2  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX          XXXXXXXXXX
 0.030     1  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX          XXXXXXXXXX
              I---------I---------I---------I---------I---------I---------I
            0.591     0.616     0.641     0.667     0.692     0.717     0.743

HISTOGRAM - BETA88B
  PCT.     N
 0.394    13  I
 0.364    12  I                              XXXXXXXXXX
 0.364    12  I                              XXXXXXXXXX
 0.333    11  I                              XXXXXXXXXX
 0.333    11  I                              XXXXXXXXXX
 0.303    10  I                              XXXXXXXXXX
 0.303    10  I                              XXXXXXXXXX
 0.273     9  I                    XXXXXXXXXXXXXXXXXXXX
 0.273     9  I                    XXXXXXXXXXXXXXXXXXXX
 0.242     8  I                    XXXXXXXXXXXXXXXXXXXX
 0.242     8  I                    XXXXXXXXXXXXXXXXXXXX
 0.212     7  I                    XXXXXXXXXXXXXXXXXXXX
 0.212     7  I                    XXXXXXXXXXXXXXXXXXXX
 0.182     6  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 0.182     6  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 0.152     5  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 0.152     5  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 0.121     4  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 0.121     4  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 0.091     3  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 0.091     3  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 0.061     2  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 0.061     2  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 0.030     1  IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 0.030     1  IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
              I---------I---------I---------I---------I---------I---------I
            0.673     0.680     0.686     0.692     0.698     0.705     0.711
```

INCENTIVE SCHEME DISTRIBUTION OF β_a and β_b for 1988-9

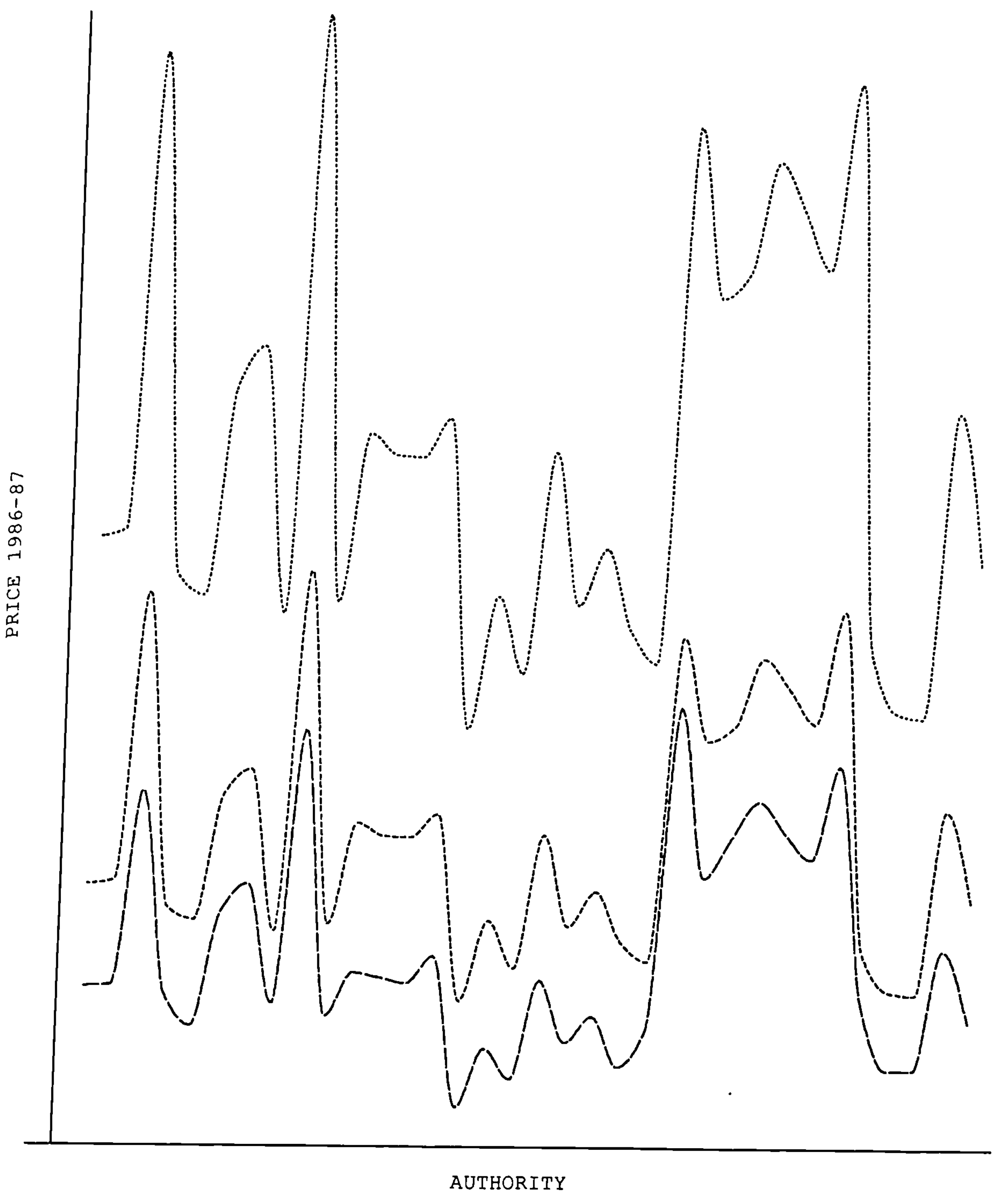


INCENTIVE SCHEME

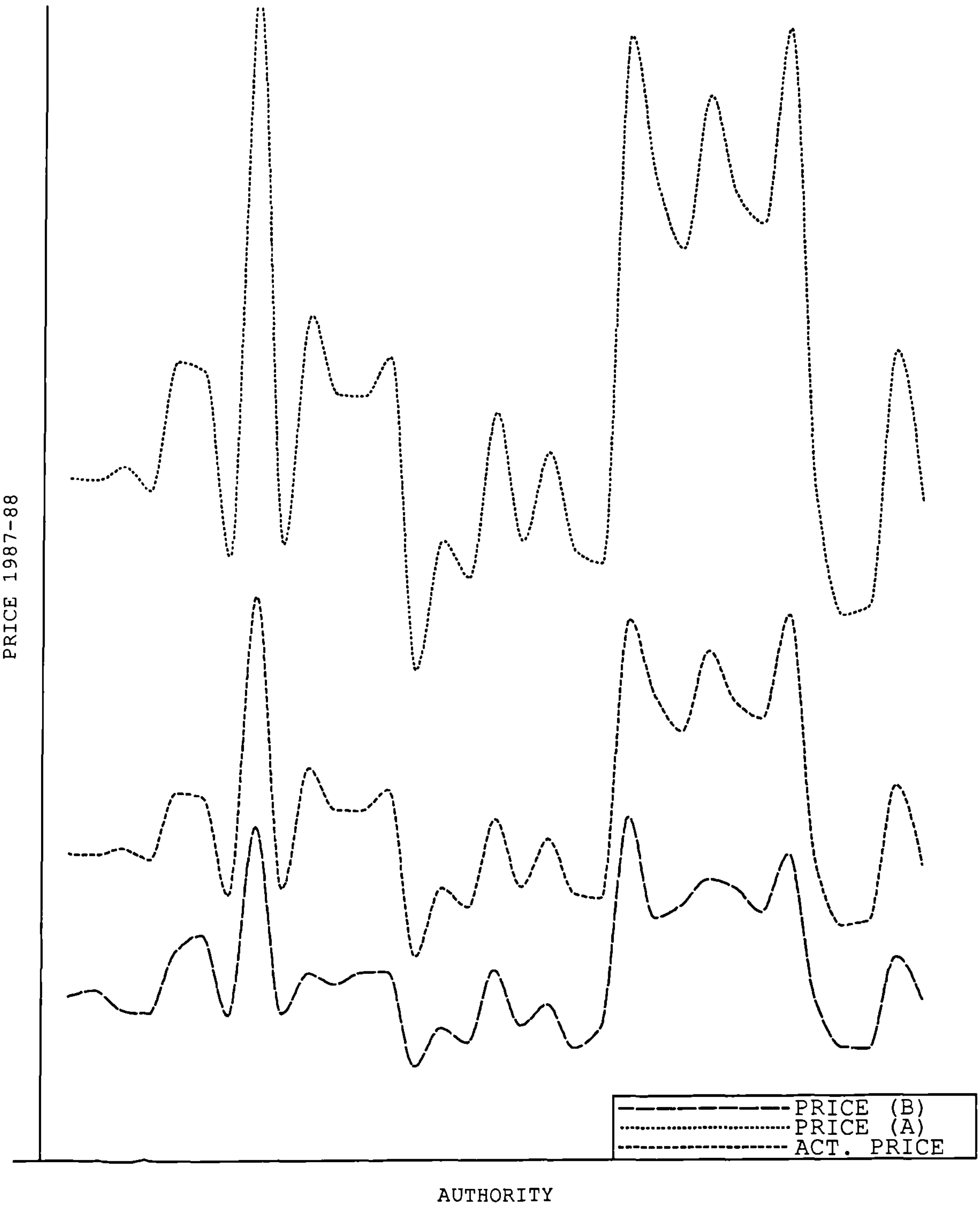
```
HISTOGRAM - BETA89A
  PCT.     N
 0.758    25  I
 0.727    24  I
 0.697    23  I
 0.667    22  I
 0.636    21  I
 0.606    20  I
 0.576    19  I
 0.545    18  I
 0.515    17  I                    XXXXXXXXXX
 0.485    16  I                    XXXXXXXXXX
 0.455    15  I                    XXXXXXXXXX
 0.424    14  I                    XXXXXXXXXX
 0.394    13  I                    XXXXXXXXXX
 0.364    12  I                    XXXXXXXXXX
 0.333    11  I                    XXXXXXXXXXXXXXXXXXXX
 0.303    10  I                    XXXXXXXXXXXXXXXXXXXX
 0.273     9  I                    XXXXXXXXXXXXXXXXXXXX
 0.242     8  I                    XXXXXXXXXXXXXXXXXXXX
 0.212     7  I                    XXXXXXXXXXXXXXXXXXXX
 0.182     6  I                    XXXXXXXXXXXXXXXXXXXX
 0.152     5  I                    XXXXXXXXXXXXXXXXXXXX
 0.121     4  I                    XXXXXXXXXXXXXXXXXXXX
 0.091     3  I                    XXXXXXXXXXXXXXXXXXXX
 0.061     2  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX          XXXXXXXXXX
 0.030     1  IXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX          XXXXXXXXXX
              I---------I---------I---------I---------I---------I---------I
            0.622     0.639     0.657     0.675     0.692     0.710     0.728

HISTOGRAM - BETA89B
  PCT.     N
 0.394    13  I
 0.364    12  I                              XXXXXXXXXX
 0.364    12  I                              XXXXXXXXXX
 0.333    11  I                              XXXXXXXXXX
 0.333    11  I                              XXXXXXXXXX
 0.303    10  I                              XXXXXXXXXX
 0.303    10  I                              XXXXXXXXXX
 0.273     9  I                              XXXXXXXXXX
 0.273     9  I                              XXXXXXXXXX
 0.242     8  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 0.242     8  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 0.212     7  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 0.212     7  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 0.182     6  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 0.182     6  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 0.152     5  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 0.152     5  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 0.121     4  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 0.121     4  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 0.091     3  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 0.091     3  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 0.061     2  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 0.061     2  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 0.030     1  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 0.030     1  I          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
              I---------I---------I---------I---------I---------I---------I
            0.711     0.718     0.725     0.733     0.740     0.747     0.755
```

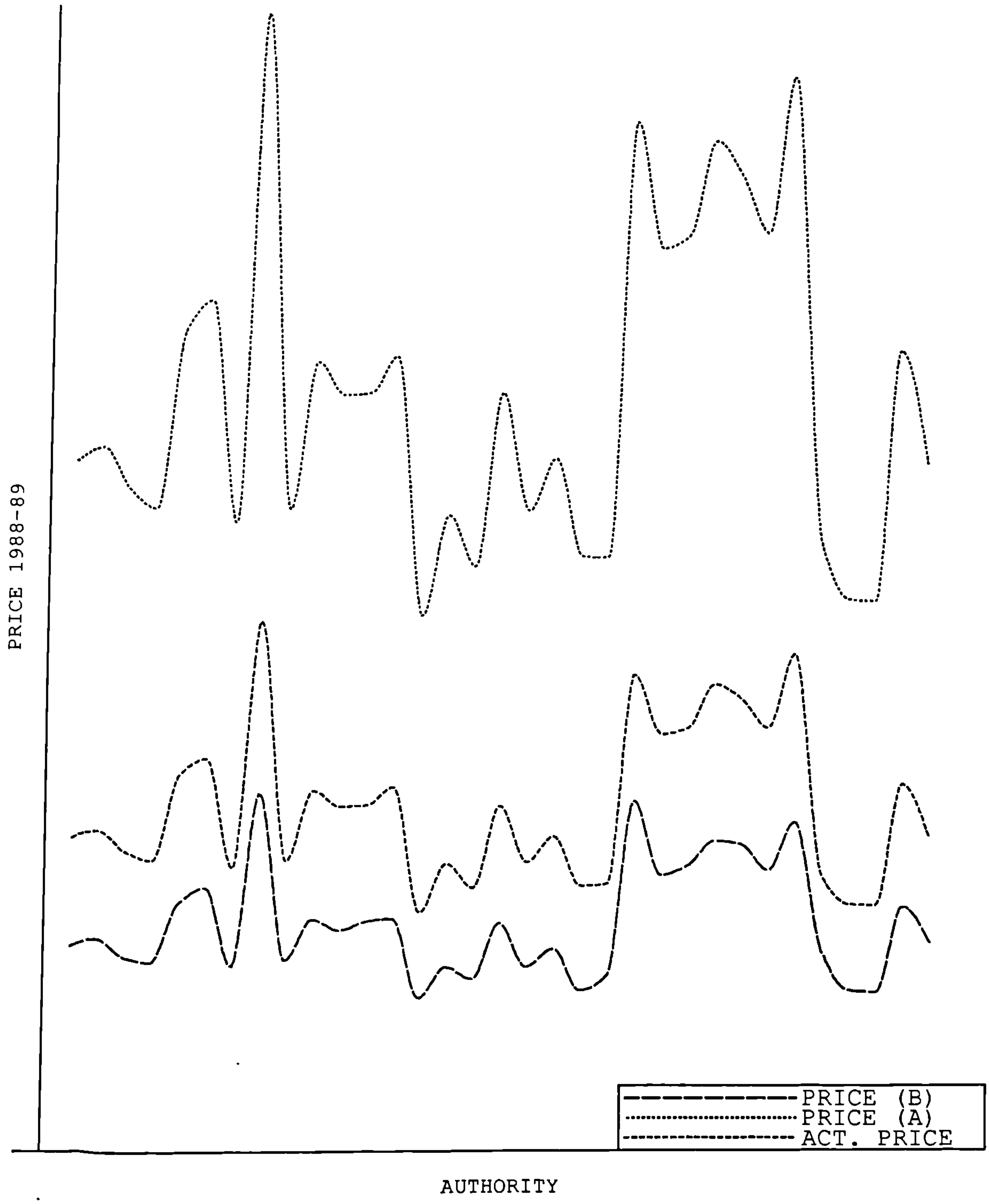
INCENTIVE SCHEME   PRICE ON THE SECOND SEGMENT 1986-7



INCENTIVE SCHEME PRICE ON THE SECOND SEGMENT 1987-8



INCENTIVE SCHEME   PRICE ON THE SECOND SEGMENT 1988-9



CONCLUSIONS

CONCLUSION

The history of intergovernmental grants in Britain has been characterized, expecially in the most recent years, by dramatic changes in the grant distribution formula that some commentators [1] have interpreted as a failure of the system to reach its objectives. In my work I present a different interpretation by explaining how, in my opinion the different grant distribution formulae applied in the last decades reflect the perception of the information asymmetry problem that characterizes the relationship between Central and local government. I focus my attention on the peculiar relationship between central and local government.

The environment in which Government has to work is a very difficult one; on one hand, in fact, it has to provide the local authorities grants to ensure a minimum level of services is provided; on the other hand it has to prevent high spending. Those two objective are incompatible, at least in the short run if local governments are better informed than the central government about their needs, preferences and resources. The grant system has to take account of these circumstances and can be used as a device to acquire relevant information from local authorities. To study this very complicated problem, I study the underlying model and the behaviour of the agents involved.

In chapter one, after reviewing briefly the economic reasons for the existence of two level of government I examine some models aimed at explaining the rationale for the existence of grants from central to local governments.

---

[1] See, for example, Gibson and Travers (1986)

I try to explain why from a theoretical point of view, some services have to be provided locally. The main reason is, in my opinion that the local authority is better informed than the Central Government about the needs and preferences of people within each locality.

This cause an asymmetry of information problem in the Central - Local government relationship. Government has to take account of this problem in setting the grant system and this is the main reason why a first best policy cannot be implemented in this context.

The system of grant has then to take account of this important element and its effectiveness has to be judged not only in its aim of reducing expenditure but also as a device to learn the true preferences and needs of each local authority. Chapter one ends up with a very simple model that explains what would be the first best optimal strategy in a world in which all the agents share the same information.

In Chapter two I review some of the models in the asymmetry of information framework while in chapter three and four I present the theoretical model to be used to optimally allocate grants to local authorities. I assume that local authorities are utility maximisers and I present the optimal grant allocation rule in both a static and a dynamic framework. At the end of chapter four I examine some of the possible failures of the optimal system to reach its objectives due to the assumption of possible alternative behaviour.

In chapter five I present the empirical evidence for local authorities behaviour under different assumptions. The aim of those empirical estimates is to derive a set of parameters to assess the consistency of the optimal theoretical model with the actual system by which grants are allocated. Some tests will be devised for both assessing the validity of a life cycle behaviour and of some of the possible behavioural assumptions alternative to standard utility maximisation.

In chapter six I deal with the summary of all the issues by showing how the history of the changes in the grant system can be interpreted as the response of Central Government to the asymmetry of information problem it has to face.

The history of grants might, in my view divided into three different phases:

1) Central Government is assumed to know all the parameters necessary to give the right amount of grant to local authorities.

2) Central Government realizes the problems caused by the asymmetry of information characterizing the game and tries to react accordingly.

3) Due to the peculiar characteristics of the agents involved in the play no optimal grant formula exists and the grant is almost arbitrary. Central Government's objectives determine the size of the grant and its amount

The dramatic changes in the grant system might then be interpreted as an evidence of Central Government's increasing awareness of the asymmetry problem that it has to face. In this light also the new drastic change in local government finance brought about by the introduction of the poll tax might be seen as a response to a problem that, as I have argued in the previous chapter might not have a solution. The Local Government Finance Act, which received its Royal Assent in July 1988, introduces three main changes to the system of local government finance in England and Wales. First, the domestic property tax (known as 'the domestic rates') is to be abolished, and is to be replaced by a community charge. For most individuals the community charge is to be a flat rate tax, or poll tax, although there is a sliding scale of tax eligibility for low income individuals, but everybody is to pay at least 20% of the community charge relevant to their local community.

A central objective behind the reforms is the achievement of accountability in the system of local government finance. To be accountable the government argues that two criteria should be met within a system of local government finance. First, all of those eligible to vote in local elections should be liable to pay local taxes, and second, the full marginal cost of local expenditure should be met out of local taxes. The background to the government's concentration on the notion of accountability is its belief that throughout the early and mid 1980s many local governments failed to restrain their expenditure because the existing system of local government finance did not encompass accountability. The system of rates rebates and the increase of business taxation are supposed to be the most important causes of the failure of full accountability.

In some recent papers[2] we showed what might be the possible

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[2] See Barnett et al., 1989 (d,e,f,g)

effects of the introduction of the poll tax on the pattern of local government expenditure according to the model used to represent local government's behaviour; in this context I want to explain which is the rationale behind the introduction of the poll tax from the point of view of the optimal grant set. The asymmetry of information which rationalizes the existence of local government is also the main cause for the failure of any efficient system for allocating resources.

If this is the case the likely response of Central Government to this problem depends on its objectives.

If Central government policy is oriented towards avoiding local authorities to play strategic games the straightforward way to achieve this objective is to give any local authority a lump sum grant and leave to the local decision makers full power as concerns the amount spent and the local tax to be raised.

Central Government has to trade off between local provision and inefficient allocation of resources because 'its agents are cheating' and a central provision of local public goods which cannot be optimal because of a lack of information! The problem does not appear to have a simple solution and some of the causes for the failure of the incentive system in this case have been envisaged.

In synthesis then there does not seem a clear cut response to the problem: as I suggested at the beginning of this section it is Central Government's objectives that determine the balance between the trade off. What I would like to point out is that the system of grants that has been in force in the recent years might have failed to reach its objective but it is hard to think that other systems rather than this could be perform better, at least if we assume that the parties involved behaves optimally and that their objectives can be represented within a neoclassical framework.

An alternative way could be to derive from alternative theories of government behaviour an optimal distribution formula. This different approach could probably be developed in a game theory framework but I think that it might be still quite difficult to formalize local government's objectives. While on abstract it might be easy to define local objective in terms of maximising their probability of being reelected or to be relatively better off than other local authorities, when it comes to formalize the models it is even more difficult and arbitrary to restrict the agents behaviour within an equation.

Finally it should be noted that Central Government's objectives in designing the grant sytem could be rather different from a mere resources maximisation. The allocation of the grant total could pursue political discrimination objectives in favour of the local authorities ruled out by the same political party as Central Government. This aspect is particularly important in the British experience since after the elections in 1986-7 most of the Metropolitan Districts and a large proportion of the Shire counties became labour dominated. Those issues are quite important and open a complete new field of analyisis.

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