The Rôle of Analogy in Aristotle's Theory of *Particular Justice*

George Michael Greenwood

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The candidate confirms that the work submitted is his own and that appropriate credit has been given where reference has been made to the work of others

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The dissertation is the result, in part, of an old interest in justice, but more directly it originated in Roger White's work on Aristotle's use of analogy. His influence on me has been enormous, greater than any other single influence, and not only on this thesis as Supervisor but on every aspect of thinking; for which I am indebted. Above all I am grateful to my wife Linda for her support through many difficulties, and also for the many ideas she suggested in our discussions of every part of the work.

Abstract

The thesis is that Aristotle's theory of justice, particularly Particular Justice, is only properly explicable in terms of proportion theory. In the innumerable assessments of the theory that have been made nods have been given in the direction of analogy, but nods only. Almost always these have been accompanied by complaints about Aristotle's persistently dragging in 'mathematical' figures, references, and explanations. What these complaints have overlooked is that the theory is *soaked* in the language of proportionality because his conception of Particular Justice is inherently (and in a sense *only*) about greater and less. Hence it is about equality; and hence about the modes of equality. I have, therefore, begun by looking at the nature of ratio and proportion; at the models of proportion known by Aristotle's time; and especially at the then new Eudoxian theory of proportions. Aristotle combines these with his own heuristic development of generic structure. His use of generic structure in the life-sciences has become better understood than hitherto, but not so much its combination with analogy. In the areas of justice and exchange the combination has hardly been appreciated at all.

From the 17th century onwards, both the structure of the theory, and the nature of the forms of justice within the structure, have been almost universally misrepresented. The result has been the belief among all commentators that the structure of the theory is unsatisfactory. The dissatisfaction stems from suppositions that Aristotle presents (in at least *some* sense) 'Corrective Justice'. These (mis)perceptions then result in claims for a third species or genus of justice, or that the given species are not firmly drawn, or that, whether there are two or three species, the issues are inadequately conceived. Against all these modern interpretations I defend Aquinas's presentation of the theory (of Particular justice, not his understanding of justice in general). The detailed analysis of the text I offer—giving due weight to the models of proportion which Aristotle uses (and declares) throughout—refutes, I believe, all the charges of inconsistency, confusion, and incompleteness, that have often been levelled against it.

Aristotle works through his model of justice step-by-step; the last part of the model, the doctrine of exchanges, is treated by the last part of the thesis. This doctrine has been maligned even

more, perhaps, than the earlier proposals of the species of justice. What, so far as I can tell, has never been grasped is that in chapter 5 Aristotle applies the Eudoxian theory of proportions in detail. Although he might not have been writing 'economics', the structure he gives is the deepest model of economic interactions that has yet been proposed. What are commonly dismissed as obvious, baffling, or unfortunate references to cobblers, variables, and beds are illustrations of the Eudoxian general theory of magnitude applied to the interactions which bind the participants into a community.

As the issues touched on are inevitably wide-ranging I have attempted to write the narrative discussing only the central themes, but I have supplied footnotes (sometimes very extensive) elaborating many of the allied issues.

Contents

Abbreviations

Aristotle:

Pr. An.	*Prior Analytics*
Post. An	*Posterior Analytics*
HA	*History of Animals*
GA	*Generation of Animals*
PA	*Parts of Animals*
NE	*Nicomachean Ethics*
MM	*Magna Moralia*
EE	*Eudemian Ethics*

Aquinas:

In Sent.	*in Quatuor Libros Sententiarum,* vols. VI, VII(i), (ii)
Opera	*Sancti Thomæ de Aquino Opera Omnia* (Leonine edition)
Expositio	*Ethicorum Aristotelis ad Nicomachum Expositio*
Sententia	*Sententia Libri Ethicorum*
Commentary	*Commentary on the Nicomachean Ethics*
Summa	*Summa Theologiæ*

Others:

Super Ethica	*Alberti Magni Opera Omnia* XIV - 1 *Super ethica*
Archive	*Archive for History of Exact Sciences*
AL	*Aristoteles Latinus*
Companion	*Companion Encyclodedia of the History and Philosophy of the Mathematical Sciences*
Dictionary	*Dictionary of Scientific Biography*
Gotthelf & Lennox	A.Gotthelf & J.G.Lennox (eds.) *Philosophical issues in Aristotle's Biology*
Grant	A.Grant *The Ethics of Aristotle,* vol. I
Jackson	H.Jackson *The Fifth Book of the Nicomachean Ethics of Aristotle*
Joachim	H.H.Joachim *The Nicomachean Ethics: A Commentary*
Lewis & Short	C.T.Lewis & C.Short *A Latin Dictionary*
Liddell & Scott	H.G.Liddell & R.Scott *Greek-English Lexicon*
Lowry	S.T.Lowry *Aristotle's Mathematical Analysis of Exchange*
Meikle	S.Meikle *Aristotle's Economic Thought*
Recherches	*Recherches de Théologie ancienne et médiévale*
Soudek	J.Soudek *Aristotle's Theory of Exchange*
Stewart	J.A.Stewart *Notes on the Nicomachean Ethics of Aristotle*, vol. I

Chapter 1

RATIO AND PROPORTION

1. 1 Ratio

The notion of ratio was used from the beginning of mathematics but we don't know of a definition until one was formulated by Eudoxus towards the middle of the fourth century BC. Preserved by Euclid that definition is given in two propositions:

5.3 Λογος ἐστι δυο μεγεθων ὁμογενων ἡ κατα πηλικοτητα ποια σχεσις

5.4 Λογον ἐχειν προς ἀλληλα μεγεθη λεγεται ἁ δυναται πολλαπλασιαζομεν ἀλληλων ὑπερεχειν

Heath's widely used translation of 5.3:

A ratio is a sort of relation with respect of size between two magnitudes of the same kind[1]

doesn't quite capture Euclid's expression. Two aspects of 5.3 need clarification, (i) ratio is not merely a sort of relation but *the* relation with respect to size. The expression "a sort of relation" is too vague[2], for whatever relations there might be, the terms must be capable of comparison.

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[1] T.Heath *The Thirteen Books of Euclid's Elements II* (1926). I.Thomas's translation is the same (*Greek Mathematics I*, Loeb edition, 1934). It is used by B.van der Waerden *Science Awakening* (1954, p.187); by I.Bulmer-Thomas (in his *Euclid*, in *Dictionary IV*, 1971, pp.414-37); by P.Dedron & J.Itard in their *Mathematics and Mathematicians I* p.89 (1973); and by D.H.Fowler 'Ratio and Proportion' in *Science and Philosophy in Classical Greece*, (ed. A.C.Bowen, 1991); and by P.Byrne *Analysis and Science in Aristotle*, (1997) p.252 n.71. E.B.Plooij (*Euclid's Conception of Ratio and his Definition of Proportional Magnitudes as Criticised by Arabian Commentators*, 1950, p.48) translates Heiberg's rendering as:

Ratio is some state of two magnitudes in connection with size.

Theon of Smyrna (2nd century AD: *Mathematical Introduction to the Study of Plato* p.73, 16) wrote:

ratio in the sense of proportion is a form of relation of two homogeneous terms one to another, as for example double, triple.

He says that ratio "may be of greater, less or equal". Nicomachus (1st century AD: *Introduction to Arithmetic II*, 21,3) gave "a ratio is a relation of two terms to one another" (Heath p.292). In addition Plooij (p.55) translates the definition given by al-Nairîzî:

Ratio is a certain relation as to measure between two magnitudes of the same species.

I.Todhunter, in his translation *Euclid's Elements* (1862), says "a mutual relation of two magnitudes". See also § 4.1, note 243.

[2] Al-Jayyânî's (11th century) *Commentary on Ratio* (ibid. p.18):

Ratio is size of a magnitude as compared with another magnitude of the same species, viz. a comparison is made between the two magnitudes for the purpose that the size may be known of one of them as compared with the other

more exactly captures Euclid's sense of the priority of ratio as *the* relation at work. (Al-Jayyânî does not follow up with the definitions 5.4 and 5.5, but moves on to the proposal "Proportion is equality of ratios".)

Comparison shows, first of all, if the magnitudes are the same or not. If they are different then one of the terms must be greater than the other. Being equal or, if not, being greater and less, is a characteristic a pair of terms has which stands apart from (and prior to) whichever 'sort of relation' or sort of interaction supervenes. Whether or not one term is a division of the other (as in the definition 5.1)[3] or a multiple of it (as in 5.2)[4], if they are magnitudes of the same kind they will be comparable. And where there is a difference between the terms ratio aims to quantify that difference exactly[5]. So whatever the difference between the two magnitudes is *is* "the relation with respect of size". As ratio expresses the comparison *rather* than such interactions as division or multiplication (or subtraction or addition) Knorr's translation of 5.3 is closer:

> 'Ratio' is of two homogeneous magnitudes the manner of relation (they have to each other) with respect to size.[6]

Heath joined a long tradition in finding a weakness about 5.3[7]; he said it

> is as vague and as little practical use as that of a straight line; it was probably inserted for completeness, and in order merely to aid the conception of a ratio.[8]

Simson thought it was not genuinely Euclid's[9], and Fowler complains that "the *Elements* does not contain a definition of ratio"[10]. But Berggren objects to Fowler that Euclid rightly thought it was exact enough[11]. What seems to be overlooked by the critics (though half-suggested by Heath's

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[3] 5.1 (Todhunter):

> A less magnitude is said to be a part of a greater magnitude, when the less measures the greater; that is, when the less is contained a certain number of times exactly in the greater.

[4] 5.2 (Todhunter):

> A greater magnitude is said to be a multiple of a less, when the greater contains the less a certain number of times exactly.

[5] See § 2.3.

[6] W.Knorr *What Euclid Meant,* (in Bowen, p.125, n.15). (He also objects to Heath's giving the indefinite "a sort of..." for ἡ ... ποια σχεσις.)

[7] He has an extended note (1926 II pp.116-19) in which he largely agrees with (and repeats nearly word for word) I.Barrow (*Lecture III, 1666)* on the supposed flaws in the definition.

[8] T.Heath *A History of Greek Mathematics* vol.I (1921) p.384, and see his extended note, 1926 II pp.116-19.

[9] R.Simson *Euclid* (1756).

[10] 'Ratio in Early Greek Mathematics' in *Bulletin of the American Mathematical Society* (1979) p.830. Bulmer-Thomas (op.cit.) and E.Carruccio (*Mathematics and Logic in History and in Contemporary Thought,* 1964 p.100) also find 5.3 unsatisfactory.

[11] J.L.Berggren 'History of Greek Mathematics: A Survey of Recent Research' in *Historia Mathematica* (1984) p.399; he cites I.Mueller (*Philosophy of Mathematics and Deductive Structure in Euclid's Elements,* 1981), but Mueller attributes the weakness to Euclid, not to the translation. Byrne (op.cit.) claims that from Aristotle's point of view 5.3 is redundant when placed alongside 5.4.

comment) is that ratio is defined through the use of two propositions, not one. The objectors to 5.3 have treated it in isolation from 5.4; this Heath translates as:

> Magnitudes are said to have a ratio one to another which are capable, when multiplied, of exceeding one another.[12]

It is the combination of these two proposals which yields the notion of ratio. There is nothing objectionable in breaking a notion down into principles better presented separately. In the present case two distinct aspects of ratio are given by the two proposals—the relation between two quantities (5.3), and more exactly what can be done with the relation (5.4)—and these points need to be grasped independently rather than run together.

(ii) The second questionable feature of 5.3 is "with respect of size". Augustus de Morgan translated 5.3 as defining ratio not as the relation between sizes but in terms of the number of times one quantity may be subtracted from the other. He read "πηλικοτητα" as indicating periodicity not quantity[13]. His reading followed Wallis and Gregory[14], and was followed in turn by Todhunter in his notes[15], and by Apostle, for example, who also treats ratio as the number of interactions between two quantities[16]. Heath firmly rejected de Morgan's interpretation, he said that the meaning of πηλικος is 'how great'[17]. (Aristotle uses the word with this sense in *NE V* 1134b11 when relating a child and father: ἑως ἀν ᾑ πηλικον). Eutocius, citing Nicomachus and Heron, used πηλικος for magnitude, as did Iamblichus[18]. Ptolemy also referred to the "size" (or "length"—Περι της πηλικοτητος των ἐν τῷ κυκλῷ εὐθειων) of chords in a circle[19]. *Liddell & Scott* also give the meaning as "how great or large". I believe Heath's comments on πηλικοτης must be correct, but that notwithstanding, Euclid's

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[12] This definition excludes the infinitesimal: for any quantities A and B, where A < B there is a ratio where for some natural number n, $n\text{A} > \text{B}$. (These became known as Archimedean magnitudes). Mueller (ibid.), followed by Knorr (in Bowen 1991), insists that def. 4 treats any inequalities between *multiples* of the terms. This would have to represented differently, as e.g., for natural numbers: m, n, p, q, $m\text{A} > n\text{B}$, $p\text{A} < q\text{B}$. (5.4 has suffered far greater misunderstanding than 5.3; this is reported briefly in § 3.7.)

[13] *Differential and Integral Calculus* (1842, p.18).

[14] J.Wallis *Treatise of Algebra, Both Historical and Practical* (1685); D.Gregory *ΕΥΚΛΕΙΔΟΥ ΤΑ ΣΩΖΟΜΕΝΑ* (1703).

[15] 1862, p.280, but not, curiously, in his translation (see note 1).

[16] H.Apostle *Aristotle's Philosophy of Mathematics* (1952) pp.60-66.

[17] 1926, II, pp.116-19.

[18] Iamblichus (*In Nicomachi*, p.8, 3-5). Eutocius (6th century AD) *Commentary on Archimedes's Measurement of a Circle;* this and Nicomachus's allusion is given in Heath II, 1926 p.117.

[19] *Syntaxis* 1.10 (Table of Sines: Introduction), *Greek Mathematics* vol.2 (I.Thomas, 1941 p.412).

presentation of ratio, combining 5.3 and 5.4, and when contrasted with, yet related to 5.1 and 5.2, if carefully translated, is precise enough to make clear the rules and principles required.

1. 2 Two approaches to mathematics

The two ways of reading 5.3 reflect two conceptions as to the nature of mathematics which appear to have flourished in antiquity (and indeed still survive). One approach was to focus on the performance of regular interactions. This conception might be thought of as (proto)algebraic in character—the quantities acted upon being thought of as secondary to the interactive processes. The other perception focused on the relations among quantities. Accounting for all the possible relations between quantities proved to be the more successful conception in resolving the main difficulty which faced the mathematics of ratios (as will be discussed below). The earlier approach centred on the periodicity; it gave rise to the mathematics of ἀνθυφαιρεσις, and is associated with the work of Theodorus and Theaetetus. This earlier conception was unwittingly assigned to Euclid for many centuries, and wittingly assigned today by Fowler, who argues that anthyphairetic ratio-theory, not proportion-theory, was pursued at the Academy; and assigned to Eudoxus by Knorr, who argues that a variant of the process was developed by him[20].

The conception of ratio that evolved in the Arab world and in Christendom during the Middle Ages was closer to de Morgan's than to Heath's. The "essence" of ratio was sought by such mathematicians as Ahmed ibn Jusuf (in the 9th century) and al-Tûsî (in the 13th) and declared to be the measure of one magnitude by another[21]. This notion, even though resulting from intensive study of the *Elements*, was non-Euclidean (and more importantly non-Eudoxian). The sequence of definitions (1-7) through which Euclid builds up the general theory of proportions in Book *V* shows

[20] Discussed at §§ 2.5, 2.6 and 2.7. Knorr *The Evolution of the Euclidean Elements* (1975), *Archimedes and Pre-Euclidean Proportion Theory* (1978a), and (1991). Fowler in several publications, in addition to 1979 and 1991: 'Anthyphairetic Ratio and Euxodan Proportion' in *Archive 24*, pp. 69-72 (1982), and *The Mathematics of Plato's Academy* (1987). I have found that A.Thorup also sees this distinction:

> The ancient [sc. anthyphairetic] concept is the assertion of equality of two *processes*, the Euclidean concept is merely an equivalence *relation* of two pairs of magnitudes.

(pp.3-4, 'A Pre-Euclidean Theory of Proportion', in *Archive 45*, 1992 pp.1-25; my italics).

[21] Plooij, p.65.

that his notion of ratio was quite different in character from the interactive principle: it covers all cases of quantity, as prior to, and quite apart from, whether or not they measure one another[22].

Two terms are *in ratio* where a multiple of either term exceeds the other; this is expressed symbolically as $a : b$ (for this symbol see below, § 1.6). To be in ratio the terms must be comparable; the principle of making a comparison determines quantities "as to the more or the less". (And of course when they are equal the terms are neither greater nor less than each other.) Such comparison has built in to it the constraint that the terms refer to objects which fall within the same genus (we would not recognise as making sense a claim that a given angle, say, is greater than a quarter of an hour, or that a scalene triangle is less than 53). So unless the terms refer to homogeneous mathematical objects there can be neither equality nor any relation of greater or less; nor could a multiplication of either meaningfully be said to exceed the other.

The possibility of a comparison's being made forms the core of Aristotle's account of exchange value in chapter 5, Book *V* of the *Ethics* (and the subject-matter of chapter 6 below). It is also the platform upon which the biological writings are based (below, chapter 3), and Aristotle also considers the issue in such other places as *Physics VII* chapter 4[23] In all these the concept of equality is intimately connected with the possibility of the application of the notions of greater and less. For there to be a comparison as to the greater or less the senses in which the words are used must also not be equivocal. Degrees of sharpness, for instance, could be meaningfully compared only where the same sense of the word is used: sharp angles can be compared, and sharp sounds, but not a sharp angle with a sharp sound (Aristotle is also especially keen to exclude figuration from careful, analytical and scientific language[24])

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[22] Definition 5.5 is given at § 2.7. 5.6 (Τα δε τον αὐτον ἐχοντα λογον μεγεθη ἀναλογον καεισθω) corresponds exactly to Aristotle's assertion at *NE V* 1131a32 "proportion is the equality of ratios". 5.7 gives the case for *ratios* (not magnitudes) being "greater or less".

[23] There is a useful discussion of this in W.van Leyden's *Aristotle on Equality and Justice* (1985, chapter 2, and an appendix).

[24] *Topics* 158b9 (Pickard-Cambridge):

> The hardest, however, of all definitions to treat in argument are those that employ terms about which, in the first place, it is uncertain whether they are used in one sense or several, and, further whether they are used literally or metaphorically by the definer.

See also § 2.5.

1. 3 Terms in ratio

Having noted that the terms need be used in the same sense, and must refer to homogeneous entities, what entities are they that might be joined in ratio? An expression was sought which would cover all candidates: lines, planes, weights, periods of time, angles, and volumes. The name 'magnitude' (μεγεθος) applies to them; what they have in common is that they are all continuous extensions. Greek mathematicians and philosophers were acutely conscious of the differences between all these and plurality[25]. Plurality *per se* is conveyed by numbers. (By 'number' is meant cardinal number, i.e., of separate extensions or discrete units of any sort the answers that would be given to the question "How many?"). Characteristically we *count* separate units but we *measure* continuous extensions. Measuring involves counting, and in both there is a continuous and a separate element: in measuring the continuous seems primary (and the separate secondary), and in counting it is the reverse. Geometry studies continuous quantities *as* such magnitudes, and arithmetic studies separated units purely as discrete quantities. The interconnectedness of the continuous and the discrete is also evident in that though the term 'magnitude' applies to continuous extensions it has the virtue that we can also make sense of it applied to separated units as well; we could refer to 'quantities of any magnitude'[26]. Despite the interconnectedness Aristotle prefers to keep the branches of mathematics apart; he regards quantity (ποσον) as the genus for the study of mathematics[27] but is reluctant to endorse it as a generic title[28]. He disallows magnitude as the

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[25] Nichomachus (op.cit. *II* 21, 5; 23.2,3) says that the basic division in mathematics into the continuous and the discrete was Pythagorean. Aristotle *Categories* 4b20 (Ross):

> quantity is either discrete or continuous;

Metaphysics Δ 1020a7 (Kirwan):

> we call a quantity what is divisible into constituents each of which has the nature of a one and a this. A certain quantity is a plurality if it is countable, a magnitude if it is measurable.

[26] Plato is sometimes translated as applying the notion of magnitude to both numbers and continuous extensions. In *Gorgias* 451B, in a passage distinguishing arithmetic from calculation (ἀριθμητικη from λογιστικη), "odd and even numbers of whatever magnitude" is given (by W.Hamilton) even though the word μεγεθος does not appear in the text. J.Klein (*Greek Mathematical Thought and the Origin of Algebra,* 1934-36, pp.18-19) argued that the distinction between ἀριθμητικη and λογιστικη was not between number theory and practical calculation but between counting and calculation. (A view widely supported, e.g. by Annas, 1976 pp.5-6—see note 29.)

[27] See also J.Lear 'Aristotle's Philosophy of Mathematics' in *Philosophical Review* 61(2) (1982) pp.161-91, and § 2.8, note 145.

[28] Perhaps his outlook was connected with the use of variables. Aristotle is the first writer we know of to make use of them; as the variable abstracts from any given 'quantity' all actual quantity, leaving a sign to stand as proxy, then if these were to form part of mathematics, the term 'quantity' could not be sufficiently universal. The modern custom of referring to

generic term because it implies divisibility, and divisibility is not a property of points or units *qua* units. Although he speaks of a universal mathematics[29] he is not satisfied with any term to represent the objects of mathematics as a whole[30] (with good reason as it turns out—see § 3.6). However, if we *may* take "quantity" as the term for the objects which mathematics of every sort deals, then quantities (as the entities capable of being greater or less[31]) will be *in ratio* when compared to other quantities of the same sort.

Fowler holds that natural numbers were differentiated not only into cardinals and ordinals, but what we would call cardinals included series which could be named 'adverbial numbers' (once, twice, three-times, ...) and 'repetition numbers' (half, third, quarter, fifth, ...); these, he says, should not be confused with ordinals[32]. It seems rather unnatural to speak of ordinals as magnitudes; *qua* ordinals, numbers would not be comparable as to the greater or less, hence they could not be in ratio (the 89th in a series, for example, is not greater or less than the 4th in virtue of being the 89th). However, any of the mathematical entities capable of being in ratio could have been used by Aristotle in models for application to non-mathematical topics, so the care which would be needed to assess the differing mathematical terms if this were a *mathematical* inquiry would add little more that is helpful to the present subject. What *is* of importance for Aristotle's use of ratio is the requirement to observe

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variables or constants as "quantities" is metaphorical, so would have been anathema to Aristotle for scientific terminology (*Topics IV* 123a33-123b, *VI* 139b12-18, 140a9-16, *VIII* 158b8-24). The need for some term to convey a notion conceived of and used strictly apart from whatever quantity it might replace, in addition to the need for some term to convey separated and continuous quantities *qua* either separated or continuous, perhaps ruled out any known word.

[29] *Categories 4b20-25, Metaphysics K* 1064b8-9, *M* 1077a9-12 (see § 2.8 and note 146), and notably at b17-22:

> Just as general propositions in mathematics are not about separate objects over and above magnitudes and numbers, but are about these, not only *as* having magnitude or being divisible (Annas).

J.Annas in her notes to chapter 3 of Books *M* and *N* of the *Metaphysics* (1976), Heath (1949, pp.223-26), and P.Byrne (1997, chapter 6), all discuss the metaphysical reasons why Aristotle is wary of labelling the objects of universal mathematics. It is an important issue to his philosophy of mathematics, and indicative of his aim in metaphysics in general, but the lack of a universal term is only marginally relevant to our present study; that relevance appears in § 3.6.

[30] *Post. An.* 74a21 (Tredennick):

> but since there was no single term to denote the common quality of numbers, lengths, time and solids, and they differ in species from one another ...

[31] *Categories* 6a26-35. Of course quantity *per se* does not vary in degree: 32 is not any the less a quantity than is 98,328 (see 6a19-25).

[32] 1979, pp.14-15.

homogeneity for any *given* ratio[33]. Ratios (to repeat) bind mathematical objects of the same kind, hence $18^\circ : 6^\circ$ and $45^3 : 80^2$ are possible ratios, but $18^\circ : 45^3$ is not.

1.4 Polarity

The Greek word for ratio was λογος; its origin appears to have been "something said". It was the term used for *account, argument, assertion, communication, computation, definition, discourse, doctrine, explanation, expression, inference, language, measure, message, narrative, proposition, rationality, reason, reckoning, speech*, and *word*[34] (De Morgan translated it as "communicating instrument".) Even as a technical mathematical expression we should not entirely neglect its other senses; i.e., it still retains some of the character of this enormous range of uses. It is not too difficult to see that the spirit with which the word is used in all this vast range of cases is that something (the subject to which it applies) is brought *into* the realm of knowledge[35]. It was contrasted with ἀλογον, in which the subject is left in a world of ignorance. 'Αλογος is used by Aristotle, as it had been by Plato and other thinkers such as Democritus, Empedocles, Heraclitus, Leucippus, Parmenides, Philolaus and Protagoras for *absurd, contrary to reason, groundless, inexplicable, irrational, speechless, unaccountable, unintelligent*[36], *unintelligible, unreasoning, unutterable, without expression* and *incommensurable*[37]. I am not here concerned with the obviously enormous differences between or among these terms, but to indicate that they broadly share a common force: they can, by and large, be mutually substituted with the senses of the sentences in which they occur intact. (Not, of course, where their differences are being in some way underlined—as when a contrast is made between *groundless*, say, and *inexplicable*—but in general terms.) The word λογος, then, indicated the presence of reason in any of its appearances, and ἀλογον the absence.

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[33] An issue treated in §§ 3.4 and 3.6.

[34] *Liddell & Scott.*

[35] Questions as to the possibility of there having been an original or a focal usage for λογος, or a use upon which other uses rely, or a cluster of overlapping meanings and uses, which might or might not be explicable analogically do not, I think, need to be explored in order to make the rather general claim that the context and associations of a novel term should be borne in mind as giving clues when attempting to grasp its significance.

[36] Philolaus (*Fragment 11*) included this among the titles of το ἀπειρον.

[37] In *Physics* 188a5, 252a25; *On the Heavens* 289a6, b34; *Meteorology* 355a21, 36, 362a14, 366a9; *NE* 1095b14, 1102a18, 34, b13, 34; *Politics* 1334b18-21; *EE* 1218a29; *Economics* 1343b13; *Rhetoric* 1370a18; *Poetics* 1460a18, 1461b14; *MM* 1198a17. See also Fowler (1987 p.194).

For the Pythagoreans, as for later mathematicians, the presence of reason was shown by an exact calculation or measurement, or else a rule whereby these could be performed. In view of the litany of polysemes above, the term λογος looks to have been a natural choice to convey the rationale—the exact, ac(countable) manner in which quantities will be linked. And indeed the use of λογος in mathematics and formal reasoning already had a long history before Plato's time. It went back to Pythagoras (about 530BC) at least. The terminology used in the learned discussions was largely Pythagorean, with one notable characteristic: it was a language of dichotomies. Rational thought was taken to be embodied in polarities; the issues surrounding such polarities as 'one/many', 'limiting/unlimited', 'odd/even', 'bounded/boundless', 'countable/uncountable', and 'rational/irrational' continually absorbed Plato and the Academy. Whatever we think of rational enquiries couched in terms of such grand-looking polarities (and we tend not to think much of them) nevertheless, distaste for the style of thought and language, or suspicion of their worth, should not lead us to reject out of hand the respectability of the thought *developed through* the dichotomies. (By which I mean good sense and clear thinking about the nature of the world evolved through the use of them.) These pairings had their source in the idea of a primary dichotomy, often called the contrariety of περας/απειρον. Philolaus's (Pythagorean) tract opens:

> Nature in the cosmos was fitted together of *peras* and *apeiron*, the order of the all as well as of all things in it.[38]

Aristotle says the Pythagoreans used ten primary περας-απειρον contraries, the table he gives is: limited/unlimited, odd/even, one/many, right/left, male/female, rest/motion, straight/curved, light/dark, good/bad, square/oblong[39]. The περας-απειρον dichotomy

> appear in the traditional table of opposites and were important opposed first principles in Pythagorean theory ... *apeiron* receives the traditional Pythagorean characterisation as that without beginning, middle, or end[40].

Throughout the *Philebus,* where Plato treated the contrariety as 'determinate/indeterminate' and as the 'differentiated/undifferentiated', the extent to which something was known was the extent

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[38] Quoted by G.de Santillana *The Origins of Scientific Thought* (1961 p.60).

[39] *Metaphysics A* 986a24. Whatever doubts there may be as to Aristotle's reliability in reporting the doctrine are irrelevant for the present purpose; even had he been completely wrong about the Pythagorean doctrines he nevertheless acquired any 'misinformation' during his twenty years at the Academy or at the latest from Aristoxenos (see note 55). Hence even if the tradition was false it is still true that it was the tradition that reached Aristotle.

[40] J.Gosling (1976, p.166) in the notes to his translation of the *Philebus*, referring to 31A9-10.

to which it was not indeterminate or undifferentiated. Knowing and limiting were treated as in important ways equivalent: to define a concept was to set its limits. To περας—what is finite (limited, determined, bounded, differentiated or known)—was understood as what is rational, as λογος. To number, itemise or list were the paradigmatic methods of coping with the unlimited[41]. Progress in knowledge was to be had through the application of το περας—notions of counting and measuring—to the unlimited (το ἀπειρον). (Indeed Aristotle brings up this "old Pythagorean imagery" of "good" being a form of το περας and "evil" a form of το ἀπειρον when setting out the nature of virtue in *NE II* 1106b29.) Correspondingly, what could not be tamed through το περας fell outside the limit of knowledge, there, where exact order failed, only the uncountable (unaccountable), immeasurable, realm of ignorance was left.

Note that the 'one/many' dichotomy was not seen by Plato, as might be supposed, as expressing this 'rational/irrational' (λογος-ἀλογον) division. The *one* did not figure, properly speaking, as indicating knowledge (with *many* then designating ignorance or the irrational) but the dichotomy gave the prerequisite for the possibility of knowledge. The same term "many" is used by Plato for two importantly different spheres: the <u>countably</u> many, which enjoy the virtues του περατος (of the rational, ordered, limited, differentiated), and the <u>uncountably</u> many (which are unlimited, unbounded, undifferentiated and irrational). On the border between the countably and the uncountably many, the bounded and unbounded (hence eventually the border between the rational and the irrational) are methods of approximation. These are invoked where exact measurement or calculation is not available. An approximation gives (from the Platonic viewpoint) a bogus exactitude where properly there is only 'the unlimited'[42]. Hence approximation has irrationality and ignorance built-in. In the early days the notion of ratio gave precision to the contrariety 'greater/less' (e.g. 3 : 2). The mathematical processes of approximation[43], by contrast, were pursued from a default

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[41] To "count" was pre-eminently to differentiate, determine or "account for". The puns "giving an account", "what counts as" ("a count") were as obvious in the Greek as they are in English.

[42] Issues of the "indefinite dyad" are explored at length by Plato, notably in the *Timaeus*, and may be associated with the 'heavenly tradition' in the *Philebus*.

[43] See O.Neugebauer *The Exact Sciences in Antiquity*, chapter 2 (1957), or J.Høyrup *Babylonian Mathematics* in *Companion* (1994) for Egyptian and Babylonian methods of approximation. The Babylonians had developed methods of approaching what was, in effect, the value of $\sqrt{2}$. The notion of proof was a definitively Greek contribution to rational thought, and a proof was associated with exact demonstration which rests on the assumption of an exhaustive application of known, delimited, principles. Hence there was an unsatisfactory air

position: only where the exacting standards of knowledge and reason—supposed to be exemplified by ratio—could not be met.

1. 5 Greater and less

The polarity which the notion of ratio embodies is that of 'greater/less', but it is not found in Aristotle's record of the Pythagorean decalogue. For Aristotle, to differ by the more or the less characterises comparisons between species within a genus (and correspondingly what cannot be accounted for in terms of degree is the mark of heterogeneity). Aristotle consciously develops his own ideas against the background of the Pythagorean and Platonic doctrines. For example at *Metaphysics A* 986b25 he points to a contrast between Pythagoras and Plato where Pythagoras saw the great and the small as 'unity' made by mixing the elements to which they applied according to some ratio: so many parts dry to so many wet; so many parts fast to so many slow; the vibration of strings in a certain ratio for the octave (2 : 1)—and 4 : 3 for the fourth, 3 : 2 the fifth[44]. Plato had objected (*Philebus* 24-25) that 'more' and 'less' are not mixtures (which would require exact quantities) but *comparisons*. They form a dyad falling within το ἄπειρον (the unlimited), the application of which is always indefinite, unlike the notion of equality, or that of any given quantity or measurement[45]. Indefinite, that is, except in one important respect: the comparison is definite in relation to the other term. In a comparison there is always one limit; one of the comparables is larger or faster or hotter, wetter, longer, louder or sharper than the other. With respect to its correlate what otherwise falls within the realm of ignorance is capable of yielding some exact knowledge. In relation to its counterpart, and only in relation to it, it is limited (determined, known and rational). Hence the greater/less relation alone delivers only the *minimally* precise account of a subject; the only precision or boundary it has (i.e., the only knowledge it yields) is with respect to its counterpart. In ratio, on the other hand, the aim was to bestow on this 'greater/less' opposition a

surrounding demonstration or proof through approximation. (Even Eudoxus's infinitesimal analysis could only *aim* for precision via approximations—this 'method of exhaustions', however, *sometimes* achieved exactitude: with it Archimedes was able to square the parabola.)

44 Recorded in Philolaus *Fragment 6* (K.Freeman *Ancilla to the Pre-Socratic Philosophers: A Complete Translation of the Fragments in Diels 'Fragmente der Vorsokratiker'*, 1948). Diogenes Laertius *Lives of Eminent Philosophers* (8, 26) also reports Alexander of Aphrodisias's account of the Pythagorean concept.

45 *Philebus* 24C-D.

bounded and exhaustively precise value. Ratio was to specify exactly the degree by which one measure differs from the other. To be able to quantify and measure exactly, and to be able to display the precise relation to another object that is equally precisely accounted for, gave ratio the status of a paradigm for rationality (conveyed by the very term λογος), but the 'greater/less' polarity was merely the ground from which this hoped-for precision sprang.

1. 6 Proportion

Ratio and proportion are often spoken of interchangeably[46]; this is harmless enough much of the time, yet for the present purpose we need to observe a cluster of important logical differences between them. Ratios take quantities as their subject-matter, whereas proportions do not take quantities directly for their subject-matter, but take these same ratios (ratio 'acts upon' quantities, but proportion 'acts upon' the action). In ratios any inequalities between quantities are formalised, but proportions make formal the equalities between the ratios. In taking mathematical objects as the subject-matter ratio may be seen as a 'first-order' process. Proportion then is to be seen as a 'second-order' process acting upon the *activity* of the first.

Proportion is shown formally as $a : b :: c : d$. The difference of the formal expression of proportion from that of ratio helps to show the logical difference between them. Ratio ($a : b$) is the relation between two quantities in which something is done to the magnitudes (a, b). The colon-like dots (":") shows the ordered linking of the objects; each dot standing for a term. (Originally pebbles would stand for terms, these would be arranged as here, or in rows or squares or triangles, with lines drawn between them. The lines indicated the relations between the terms.) A proportion is a relation between *relations*, i.e., something is done to the something that is doing something to the quantities. The doubling of the symbol ("::") indicates this recursion—the doubling of the 'colon' expresses the doubling up of the relation. What survives in the squared dots is a relic of ancient

[46] Translators of Aquinas tend not to distinguish them, see e.g., the Blackfriars editions of the *Summa.* Similarly G.Phelan (*St.Thomas and Analogy*, 1943)—see § 1.9 below. (For the mediæval evolution of the terms see § 2.2 note 103.) A.Jones refers to "Eudoxus's ratio theory" (p.48 *Greek Mathematics to* AD *300*, in *Companion* vol.1 pp.46-56), and A.Wilson (*The Infinite in the Finite*, 1995, pp.253, 255) refers to proportion as ratio whilst analysing inferences in Book V of *Elements*. *Meikle* (p.7) also calls a ratio what is in fact a proportion. (For the terminology of ratio and proportion, § 2.2.)

geometric demonstration; they indicate the universality of formulae, standing for units or quantities of any kind[47].

In this thesis the terms 'analogy' and 'proportion' are used interchangeably. The Greek word for proportion was ἀναλογια: the word λογος prefixed by ἀνα. By Plato and sometimes by Aristotle the prefix is still kept separate: ἀνα λογος[48] This early usage perhaps shows the then novelty of the term together with its logical character—that some additional process is applied to a ratio[49]. Like λογος ἀνα was a highly polysemous word[50], though it usually conveys *some* notion of 'higher than'. Analogy is some process 'higher' than ratio which *processes* ratios. Where ratio makes a comparison between two magnitudes, analogy, in the pairing of a pair of terms, compares that comparison with some other two.

In focusing on (and re-iterating) the first- and second-order difference between ratio and proportion I aim to emphasise a feature of analogy that is often overlooked even in discussions of classical proportion: the *logical* character of analogy. It is often stated that (classical) analogy is the equality of ratios, and requires four terms; although this is perfectly correct it does not bring out the logical significance of the difference between the two. Fowler, for example, pursues the notions of mathematics attributed to Theaetetus as essentially ratio-theoretic, not as proportion theory[51], even so he does not bring out the distinction between the two that I wish to emphasise: that there was a difference in character between 'first order' ratio and 'second-order' *proportion*. Fowler treats proportion as holding among four terms, but applying to four rather than to two terms, whilst central to the principle of analogy, does not, *of itself*, indicate any great logical distinction. Ratio compares

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[47] The square created by the four dots '::' had a related use as the "square of opposition", a format greatly used in antiquity and the Middle Ages, and widely thought to be used by Aristotle in chapters 3 and 5 of the Book on justice (see § 6.3 figure *3*).

[48] For example *Timaeus* 37A and *Republic* 511E. At 509D, in the same discussion as 511E, Plato has the expression ἀνα τον αὐτον λογον. *Liddell & Scott* say the jointed and the disjointed uses were "plainly equivalent".

[49] The significance of the prefix is often missed; de Santillana, for example, says "*Logos* means 'discourse', 'reason', 'argument', 'inference', and also 'proportion'." (op.cit. p.65).

[50] *Liddell & Scott* record: *on, upon, upward motion, from the bottom to the top, spreading all over, throughout, increase, improvement, arise!, up!, continuous.*

[51] 1979 p.16; 1987 p.31. Fowler adopts the same notion of ratio as de Morgan, though without *following*, or even referring to him. He avoids anachronisms such as 'continued fractions' or 'Euclidean algorithm' when speaking of the anthyphairetic processes of ratio; see § 2.5, note 119.

(and counts) quantities, but proportion compares the comparisons, rather than counting the steps between the terms.

1.7 The modes of proportion and the mean

The early Pythagoreans developed proportion theory in association with their interest in music[52] For them a proportion was generated where a point (corresponding to a place on a string) divides a line into two parts such that they will be *in some sense* equal. Indeed discovering equality where equality is not apparent links all the threads associated with proportion theory; Theon of Smyrna wrote:

> Eratosthenes says that ratio is the source of proportionality and the origin for the generation of everything which is produced in an ordered way. For all proportionalities arise from ratios and the source of all ratios is equality.[53]

The point at which the division was made was called the "mean" ("the mean between the extremes"). Three points were found which generated an equality, hence there were three "means". Archytas defined these three classical proportions:

> There are three proportions in music, the arithmetic, the geometric, and the sub-contrary or so-called harmonic. We have an arithmetic proportion when three terms are related with respect to excess[54], as follows: the first exceeds the second by as much as the second exceeds the third. ... We have a geometric proportion when the first term is to the second as the second is to the third. ... The sub-contrary proportion, which we will call harmonic, is that in which the terms are such that if the first exceeds the second by a certain part of the first, the second will exceed the third by the same part of the third.[55]

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[52] A.Szabó (*The Beginnings of Greek Mathematics* (1978), and endorsed by Berggren (1984 pp.395-96)) presents philological arguments for the origins of the terminology for proportion-theory in music and sacred ritual.

[53] Quoted by van der Waerden (1954 p.231); he goes on to quote Pappus:

> Proportionality is composed from ratio, and equality is the origin of all ratios. Geometric mediety indeed has its first origin in equality; it establishes itself and also the other medieties. It shows us, as says the divine Plato, that proportionality is the source of all harmonics and of all rational ordered existence.

Plato mentioned here is mediated through the eponymous character in the dialogue *Platonicus* by Eratosthenes.

[54] The expression "with respect to excess" prefigures Aristotle's explanation of his use of the mean in the *Ethics*.

[55] M.Cohen & I.Drabkin *A Source Book in Greek Science*, 1948, pp.6-7. (See also § 6.3 and notes 372 and 374.) In his book on justice Aristotle relies on the Archytan definitions of proportion. His familiarity with and closeness to Archytas is evident when we recall that Archytas was a friend and supporter of Plato, and that Aristoxenus, disciple and biographer of Archytas, joined Aristotle when he opened the Lyceum. Aristotle also wrote three books on Archytas's work. (See G.Allman *Greek Geometry from Thales to Euclid* (1889) p.107.) Iamblichus (op.cit., p.100, 19-25) also records a definition of proportions:—

(i) The "arithmetic" is the simplest equality, the mean was struck in the middle, mid-way between the extremes, producing two equal lengths. This creates plain every-day equality—the sort of equality which anyone will recognise as what equality means. It is at once *both* the simple quantitative equality (and as such appears to stand apart from, and prior to, notions of proportion) *and* a mode of proportion[56]. The very simplicity of this analogy has proved disastrously misleading to commentators on Aristotle's use of it in the *Ethics*. (It is the outstanding *simplicity* of this proportion that Aristotle relies on to distinguish the justice of exchanges from that of distribution, which characterises [ii], the second mean.)

(ii) This was called the "geometric"; here a line is divided at a point where the difference between the two lengths created have an equal multiple, i.e., the mean is located where it exceeds the first term by the same multiple as it is exceeded by the third. Where the arithmetic was the simplest, the geometric is the purest mode of proportion. It is often taken to be proportion *per se*. This elevation of the species 'geometric proportion' to the genus 'proportion' has also proved seriously misleading to commentators of both the *Ethics* and the *Politics*[57].

(iii) The third mean was the first sub-contrary, and because of its strong musical associations, sometimes called "harmonic" (notably by Archytas, though not by Plato or Aristotle)[58]. Ἁρμονια referred, in Pythagoras's time, to the arrangements of parts in a whole[59]. This mean is located where it exceeds the first quantity by the same part (fraction or division) of that quantity as it falls short of the third. (This mode of proportion may have a rôle in Aristotle's theory of exchange value.)

'Middle' is the origin of μεσοτης ("mean"); the mean being *in some sense* the mid-point of a set-up which is in balance. (*Liddell & Scott* give "balance" as a translation of μεσοτης.) The mean

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In antiquity, in the time of Pythagoras ... there were only three means, the arithmetic, the geometric, and a third in order, that which was once called sub-contrary, but was later re-named *harmonic* by Archytas and Hippasus.

[56] Discussed below in §§ 2.1, 5.6, 5.11, 5.12.

[57] §§ 2.1, 4.9, 4.10.

[58] See §§ 2.1, 6.6.

[59] *Liddell & Scott* give it a considerable range of meanings, including: means of joining, fastening, joint, union, adjustment, framework, covenant, agreement, settled government, order, musical scale, stringing, due arrangement, remedy; and as the Pythagorean term for three.

can be seen as the point which generates the given mode of equality. Aristotle defines the mean (*NE 2* 1106a30):

> By the mean of the thing I denote a point equally distant from either extreme.

It is, as it were, the fulcrum or centre-of-gravity of a weighted body or line. The proportion is the whole set-up (as it were) of the line balanced at the fulcrum. Although the notion of the mean has generated extensive debate[60] it is its rôle as the point of balance in a proportion that needs to be retained when considering Aristotle's theory of justice. The mean provides the balance; it is (as it were) the *position* of the fulcrum for differently weighted lines. (The distinction between mean and proportion needs to be kept in mind for the resolution of the errors attributed to Aristotle discussed in chapter 2.)

1. 8 Continuous and separated proportions

The Archytan definitions and the early Pythagorean studies treated the proportion of continuous magnitudes. For such extensions there were three terms, the two extremes and the mean (*a, m, b*). If the three terms (the mean and the extremes) are not treated as, or expanded to four, then it is better not to speak of a proportion (there will not be a relation between *ratios*). Hence Pappus's remark:

> A mean differs from a proportion in this respect that, if anything is a proportion it is also a mean, but not conversely.[61]

Euclid's definition 5.8, which stipulates the minimum of three terms, is possibly an interpolation[62], but whether it is or not, dividing the magnitude into two related extensions generates four terms where there had been three: *a : mean,* and *the mean : b* (*a : m :: m : b*). To compare the relations between the magnitudes which the application of the mean creates, the mean is given twice. Aristotle explains this point in *NE* 1131a34-b3:

> That a discontinuous proportion has four terms is plain, but so also has a continuous proportion, since it treats one term as two, and repeats it: for example [pointing to a diagram] as the line representing the first term A is to the line representing the second B, so is B to the line

[60] See for example the introduction to any edition of the *Ethics*.

[61] *Commentary on Book X of Euclid's Elements, III*, p.70, 17.

[62] *Elements V* def. 8: Ἀναλογια δε ἐν τρισιν ὁροις ἐλαχιστη ἐστιν. Heath (1926 II p.131) reports Hankel's view that it is a later, unnecessary, addition. The definition is very similar to Aristotle's remark (*NE V* 1131a31) that a proportion has a minimum of four terms; in either case Aristotle clarifies the terminology in the lines which follow.

> representing the third term C. Hence B is mentioned twice, so that if it be counted twice, there will be four analogates.[63]

Treated as a proportion the mid-term is repeated[64], and the four terms created could be separately labelled. (Letters (A, B, Γ, Δ)[65] were convenient for this purpose.) Aristotle is usually careful to use *analogia* for the 4-term proportion. Now the 4-term proportion, being 4-term, looks discontinuous, and so does not appear to have a mean, but there is a mean to all proportions in that there is a mid-point in the equality of the ratios. I.e., understood as equivalent to $aRb = cRd$, the mean is located at the sign for equality[66].

Pythagoreans assumed an equivalence between geometric and numerical values; for them the elements of all things could be accounted for, i.e., counted (arithmetically) and arranged (geometrically) in ways such that comparisons of continuous magnitudes were expressible in whole numbers. For example, for the arithmetical proportion, by 2 : 4 : 6; for the geometric by 2 : 4 : 8; and for the harmonic by 6 : 8 : 12.[67] For these examples treated as continuous *proportions* they will be: 2 : 4 :: 4 : 6 (arithmetic), 2 : 4 :: 4 : 8 (geometric), and 6 : 8 :: 8 : 12 (harmonic).

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[63] Translation adapted from the Loeb edition which doesn't translate the letter-labels, and Ross who does. MSS differ in preserving, on the one hand, letter-labels (not alphabetical numerals—see P.Keyser 'A Proposed Diagram in Aristotle EN V.3, 1131a24-b20 for Distributive Justice in Proportion', *Apeiron* 25ii, 1992, pp.135-44, and §§ 4.10 and 6.7 below), and adverbial numbers (rather than ordinals—see § 1.3 above) on the other. (K_a, CCC, and H_a have πρωτερου, δευτερου, τριτου, as do P_b and N_b, in the main. Letter-labels, or cardinals, appear in L_b, O_b, Q, and M_b. NC and B_1 have adverbials in three places and letter-labels in others). *Jackson,* p.81, *Stewart,* pp.426-27, and J.Burnet (*The Ethics of Aristotle,* 1900 pp.211f.) discuss the mathematical representation in the various surviving manuscripts. Jackson changed his mind between 1872 and 1879; earlier he felt that labels standing for cardinals was the original text, but on reflection he thought that ordinals, which had been the usage since Michael of Ephesus (11th century), was correct. Using the same language as Aristotle uses here Archimedes (and later Theon) distinguished continuous (συνεχης: 3-term) proportions from separated (διηρημενη: 4-term) ones. Continuous proportion has the consequent of one ratio as the antecedent of the next. This is not so with separated proportions. Nicomachus (II, 21, 5-6—see *Jackson,* p.81, and Heath 1926, II p.293) refers to "connected (συνημμενη) and disjointed (διεζευγμενη) proportions". Euclid often uses "proportional in order, or successively" for numbers in continuous proportion.

[64] Aristotle's use of the diagram explains his concern with the distinction between the separated and continuous quantities—see § 4.10 below.

[65] M.N.Tod 'The Alphabetic Numeral System in Attica' in *The Annual of the British School at Athens* 45, pp.126-39 (1950), and 'Letter Labels in Greek Inscriptions', ibid. vol. 49, pp.1-8. (1954). Phoenician letters were used for alphabetic numerals, rather as we use Greek or Latin. These were as the Greek alphabet but with three additional signs; the first of these, the *digamma "Ϝ"* appeared sixth, in place of Z. As the digamma was obsolete in Greece its use as a numeral is clear. (See P.Keyser, op.cit., and § 4.10.)

[66] This contrast in the uses of the mean is put to use by Aristotle when defining virtue and justice in Books *II* and *V* of the *Ethics* (§§ 4.3(vi, vii), 4.5, 4.6 below).

[67] 8 exceeds 6 by 1/3, and falls short of 12 by 1/3. This proportion is generated by the formula C = 2(AB)/A+B.

For discontinuous proportions the ratio between one term and a second, and between the second and a third, will be the same as that between two quite other terms. In the arithmetic proportion the ratio 2 : 4 is not only the same as the ratio 4 : 6, but for any integers $m : n$. Similarly with the geometric proportion the ratio of 2 : 4 is the same as 8 : 16 or 25 : 50. The harmonic proportion preserves the same part of the third term (in the fourth) as the second is of the first (e.g., if the first term is 20, and the second is, say, 3/4 of that (15), the same relation—the same portion or division—will hold for 48 (36), and for any other quantity, say, 16 (12), so that 48 : 36 :: 16 : 12)[68].

The distinction between the separate and the continuous was observed for proportion theory throughout the classical period; Euclid gives his account of these in different books (*V* and *VII*). He is content mainly to edit the differing traditions and preserve the differing sources[69]. Yet what may have contributed to the difficulty commentators have found in Aristotle's writing connecting proportions with means (discussed in § 2.1) is the difference between 3-term (continuous) and 4-term (separate) proportions. (Indeed misplacing and misunderstanding Euclid's treatment of differences between continuous and separated proportions seriously distorted the mediæval reading of *Elements V,* and prevented the understanding of the definitions 5.4 and 5.5 for hundreds of years—this is outlined in § 3.7.) Properly, as has just been said, *analogia* requires four terms; the old 3-term proportion (with "the mean between the extremes") treated *as a proportion* needs four terms. The analogical principle whereby the same ratio holds between the second pair of elements as the first is more generally expressible by four terms than with three, even where the proportion is continuous and the mid-term repeated, hence analogy came to be expressed as "the first term is to the second as the third is to the fourth".

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[68] See §§ 2.1 and 6.6.

[69] Euclid as an editor preserved the formulations of his originals and did not re-cast them without very good reason. Knorr (1975) p.303 writes:

> He sought to change as little as possible of the completed treatises which were brought together in his compilation. Such is claimed by Pappus, apparently on the authority of Apollonius.

The repetitions in the structure of the *Elements* show that Euclid presents overlapping issues from different originals, and hence from different perspectives. See also Artmann (1991, p.2).

1.9 Mediæval and modern conceptions of analogy

In classical proportion the format *a : b :: c : d* is equivalent to *aRb = cRd* (the same relation holds for both pairs of analogates—there is "an equality of ratios"). Conceptions of analogy which have emerged over the last few hundred years differ significantly from this equivalence. At bottom, modern notions of analogy suppose some form of ordered resemblance in disparate phenomena or ambiguities of expression[70]. The difference between this and the classical conception needs to be recognised in order to avoid misunderstanding as to the nature of the inferences involved[71]. Thomists have written on analogy at length, especially in connexion with metaphysics, and often relating univocal to equivocal uses of language[72]. Thomists such as Phelan hold that mathematical analogy is not *strict* analogy because it applies only to quantifiable entities:

> mathematical analogy — although the word analogy was first used to designate the proportion of one quantity to another — cannot be regarded as a strict analogy since it is valid only within the genus of dimensive quantity.[73]

He says, however, that St Thomas's treatment of mathematical analogy coincides in the essentials of the Euclidean doctrine, and cites several passages to support this. Most of these turn out to be rather vague[74], but certain of Aquinas's remarks do clearly show a grasp of the essential mathematical principles of analogy[75]. He also says that analogy relates two [not four] things, and that:—

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[70] As an example of current general philosophical conceptions of analogy (as apart from the specialist treatments in the following pages) analogy is defined by P.Angeles in the *Dictionary of Philosophy* (1981, p.8):

> Originally a mathematical term ... The Greek term came later to mean the (usually linguistic) *comparison of similarities in concepts or things.* 1 *The pointing out of similarities or resemblances between things.* 2 A form of (usually inductive) inference in which *from the assertion of similarities between two things* it is then reasoned that the things will probably also be similar in yet other respects.

(My italics.)

[71] Misunderstanding classical inference damages accounts given of Aristotle's use of it in such fields as biology and ethics (chapter 3 below).

[72] Influentially Cajetan (*De Nominum Analogica III* and *De Ente et Essentia Commentaria*) elaborated (but departed from) Aquinas's uses of analogy. He divided analogy into differing types: those of inequality, attribution, and proportionality. Those of proportionality are further separated into the 'improper' and the 'proper'. (In the improper the analogates form into one pair which have the same formal meaning, but in the other pair there is a 'metaphorical' predication. In 'proper' proportionality all the analogates are formally 'proportional'.) See D.Burrell [1973] *Analogy and Philosophical Language* (1973) pp.11-12; G.Phelan (op.cit), R.M.McInery *The Logic of Analogy* (1961) or B.Davies *The Thought of Thomas Aquinas* (1992).

[73] Ibid. p.18. Burrell (ibid. pp.10-11) also explicitly rejects classical analogy.

[74] Ibid p.52 n.36; the commentaries on *Ethics I,* ref. § 96; *Physics,* lect. 10, n.7, 13, n.9; *On the Heavens,* lect. 14 nn. 3 ff., which Phelan mentions, do not clearly show the precise mathematical basis of analogy.

[75] See § 4.1.

> The basic proposition in the doctrine of Thomistic analogy, in its strict and proper meaning, is that whatever perfection is analogically common to two or more beings is intrinsically (formally) possessed by each, not however, by any two in the same way or mode, but by each in proportion to its being.[76]

which takes the term "analogy" into fields far removed from Aristotle's use of it.

A great deal of work has also been done in cognitive science and related disciplines assuming the modern conceptions of analogy. The emphasis has not been on the equality of ratios, or on the principle that a *comparison* is to be compared, or on the condition holding among four terms, but rather on:

> statements about the number of properties in common or about the degree of similarity between particulars[77]

sometimes between "large and complex domains of information"[78]. Both in modern Thomist philosophy and in the development of cognitive models analogy is taken to be a mapping procedure *which specifically excludes classical analogy*[79]. Models of classes or domains are favoured (often referred to as the *base* and *target* domains), in which it is hoped one will map onto the other. These are models of two terms or classes, not of four. It is curious, perhaps even ironic, that the notion of a mathematical model itself was generated by the initial application of the classical notion of analogy which has been so comprehensively ditched—the rules in *Elements VI* to determine the conditions to be met for two geometric figures to be similar, when generalised, were the rules for one complex of terms to map onto another[80].

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[76] 1943 pp.12-13.

[77] J.Weitzenfeld 'Valid Reasoning by Analogy', *Philosophy of Science Journal* 5 (1984 pp.137-149). A recent text which treats analogy as essentially 'noticing similarities' is *Fluid Concepts and Creative Analogies: Computer Models of a Fundamental Mechanism of Thought,* by D.Hofstadter and the Fluid Analogies Research Group (1998).

[78] M.Keane *Where's the Beef?: the Absence of Pragmatic Factors in Pragmatic Theories of Analogy, Human Cognitive Research Laboratory Technical Report n.28* (1987).

[79] (Ibid. p.2) Keane traces the history of research on analogical inference for cognitive science:

> I shall begin by considering the work of Winston (1980, 1982, 1986) rather than with the seminal work of Evans (1968). My reason for excluding Evans lies in the fact that his concern lies with a special type of analogy (i.e., proportional analogies).

(T.Evans 'Program for the Solution of a Class of Geometric Analogy Intelligence Test Questions', in *Semantic Information Processing;* P.H.Winston 'Learning and Reasoning by Analogy', in *Communications of the Association for Computing Machinery* (1980), 'Learning New Principles from Precedents and Exercises', in *Artificial Intelligence* (1982), 'Learning by Augmenting Rules and Accumulating Censors', in *Machine Learning II* (1986).)

[80] See § 2.8.

An influential approach in the modern investigations into analogical inference has been taken by M.Hesse[81] who bases analogy on a principle of similarity, not of identity. Her notion is that analogical inference rests on a similarity of relations, not their equality; in this respect hers may be taken as representative of modern conceptions of analogical inference. She regards this position as traditional, but it is a tradition which excludes Aristotle and Kant (and stems ultimately from Aquinas's attempts to reconcile differing treatments of 'substance' in Aristotle's writings). It might be thought that it is from Kant that the application of analogical inference to empirical inquiry as a form of systematic resemblance is derived, but this is not so. What he says is

> In philosophy analogies signify something very different from what they represent in mathematics. In the latter they are formulas which express the equality of two quantitative relations, and are always *constitutive*; so that if three members of the proportion are given, the fourth is likewise given, that is, can be constructed. But in philosophy the analogy is not the equality of two *quantitative* but of two *qualitative* relations; and from three given members we can obtain *a priori* knowledge of the relation to a fourth, not the fourth member itself. The relation yields, however, a rule for seeking the fourth in experience and a mark whereby it can be detected.[82]

Kant retains the equality of the relations. Outside mathematics the *character* of the fourth term is not guaranteed but there is no weakening of the relation. He does not view analogy as a treatment of similarity but as 'perfect similarity' (as with similar triangles). I.e., although Kant's use of analogy differs very greatly from Aristotle's, like Aristotle he uses it as a structure to locate the *appropriate* equality. He goes further, he regards analogy as the form of inference which operates exactly where there is no resemblance:

> Such knowledge is knowledge *by analogy*, which means not, as the word is commonly taken, an imperfect similarity of two things, but a perfect similarity of two relations between quite dissimilar things.[83]

If we take the classical (Aristotelian) format of *a : b :: c : d,* it displayed the *same* relation between the two pairs of terms. Like Kant's Aristotle's use of analogy is perfectly applicable to "quite dissimilar things" without any requirement to discover some common property or degrees of resemblance in virtue of which disparate clusters of phenomena will be found to correspond. (This application forms the core of Aristotle's solution to the problem of exchange-value; a solution which students of the history of economics have failed to grasp exactly because of the lack of familiarity

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[81] 'On Defining Analogy', in *Proceedings of the Aristotelian Society* (1959-60 pp.79-100).

[82] *Critique of Pure Reason* A179 (trans. Kemp-Smith).

[83] *Prolegomena to Any Future Metaphysic,* s.580 (trans. P.G.Lucas). (Berkeley had issued a similar warning in *Alciphron,* fourth dialogue.)

with Aristotle's use of the Eudoxian proportion theory[84].) For both Aristotle and Kant the correspondence of the relations among wholly dissimilar clusters is quite often the virtue of analogy.

The analogical relation in its classical format is expressed as $a\mathrm{R}b = c\mathrm{R}d$. With Hesse's conception of analogical inference the equality is reduced to a mere similarity of relations. The classical formula does not hold, and must be replaced by some other formula which connects $a\mathrm{R}b$ with $c\mathrm{R}'d$. Hesse says (p.80):

> for the assertion in the case of an analogy of the relation between *a* and *b*, and a relation between *c* and *d*, was always qualified by the remark that these two relations are not *identical,* but only *similar* in some relevant respect .... thus it seems that we should make the analogy equivalent to the existence of two *similar* relations R, R', such that $a\mathrm{R}b$ and $c\mathrm{R}'d$.

For the question "How are R and R′ connected?" Hesse gives a 'distributive lattice'—which is an algebra of lattices drawn from set-theory[85]. For Hesse an analogy occurs where a sufficient number of *properties* are found to correspond between classes. The lattice is to enable an adequate match for a qualifying number of such properties[86].

In addition to the weakening of the relation from equality to one of mere similarity ("an imperfect similarity") the modern conceptions of *formal* analogy, such as those of Hesse, Weitzenfeld, Winston, Keane, and Hofstadter, imagine a connexion between two clusters, i.e., they attempt some comparison between a base and target. Mathematical models are used, but by not isolating the relations in the models from the properties, the comparison reduces to a 'first order' comparison between two clusters, i.e., to a *direct* comparison between two (groups of) things. Together, the reduction of the relation to some principle of similarity, and the similarity being between "two things", not merely departs from Kant's formulation but inverts it. He objected that analogy is not an imperfect similarity of two things but a perfect similarity of the *relations* between things. In classical and Kantian analogy the comparison must be *indirect* exactly because a direct comparison may not be available (e.g., 47° is not directly comparable with any number of square feet). The outstanding value of analogy for Aristotle and Kant is its capacity to draw into an *exact*

[84] Many writers on the history of economic thinking, as a result, grossly undervalue Aristotle's proposals. Even his greatest admirers, such as Karl Marx, could not recognise the nature of Aristotle's argument (see chapter 6).

[85] Lattice theory uses principles modelled on intersection (join) and union (meet) which allows for differing levels of sets.

[86] See also Burrell's comments on Hesse's proposals (from a very different stance from mine), 1973 pp.18-19.

account things which cannot be compared directly. This virtue is lost where the first- and second-order distinction is blurred into a 'sufficient' resemblance of properties and an inexact "equivalent to the existence of two *similar* relations". (This supposition that analogy is the pursuit of an imperfect similarity of properties has had an adverse effect even on the study of Aristotle's use of it; see note 71 and §§ 3.4-3.6.)

In origin analogy was a mathematically technical notion; this was later loosened up to be used to indicate (mere) resemblances between things. This weaker, vague sense became so widespread that, in turn, technically mathematical programmes were devised to capture it. The original rigour and sophistication of mathematical analogical inference was then replaced by cruder and vaguer methods to treat this shallower understanding. Its no use complaining that the concept of analogical inference as it has developed is "wrong", but we may insist that, in becoming once again mathematical, as it is normally conceived and practised in several fields, it is a much more feeble mathematically technical conception than it had been, or as had been used by either Aristotle or Kant.

There is an outstanding exception among current researchers into analogical modes of inference: Dedre Gentner, with her collaborators, isolates the *relational* character of analogical mapping processes; i.e. they take the relations between objects in the "base" to project onto the "target". This treats the very feature which I have said distinguishes proportions from ratios. In proportion it is the <u>relation between</u> the terms, i.e., the ratio itself, that is mapped, not the terms. Gentner's method[87] captures this central distinction. Her work has been very influential, but that has been despite her understanding of this central feature of analogical inference, not because of it.

[87] Known as the Structure-Mapping Engine (it is proposed in several articles, see under 'Gentner' in the works cited). Where Hesse borrows from set theory, Gentner borrows rules from the predicate calculus whereby the properties (one-place predicates) of the objects in the base domain are disregarded and only the relations (two or more-place predicates) between them are preserved; these are then mapped onto the target domain. The mapping will fall under a higher-order relation acting as a higher-order function taking the lower-order relations as arguments.
An earlier exception to the general approach to analogy (from a very different standpoint) is found in Fr. S.Brown's *The World of Imagery* (1927) in which he argues for the open-ended hierarchical character of analogical comparison.

Chapter **2**

IRRATIONALS

2. 1 ΚΥΡΙΩΣ ΑΝΑΛΟΓΙΑ

Interpreters have often objected to Aristotle's expression "arithmetic proportion"; Heath says it is not an expression we would use now, nor one used by Euclid nor Apollonius nor Archimedes[88] Heath overstates this; we don't know if they did or not, very little of what they actually wrote survives. Commentators have tended not to see the arithmetic proportion *as* a proportion but as mere quantitative equality, and complain that Aristotle's repeated uses of the arithmetic proportion (in the *Ethics* especially) are really demonstrations of arithmetic progression[89]. This insistence has led to difficulties for the interpretation of the theory of justice. The main difficulty has been the vagueness and seeming inconsistency in Aristotle's 'proportion' language; problems have arisen partly because Aristotle's terminology looks anomalous and partly because he used the three then contemporary senses of the term ἀναλογια, one of which became obsolete. These commentators indicate that Aristotle's usage is both mathematically incorrect and needlessly complicated. But Archytas's definitions show Aristotle's usage was the standard mathematical vocabulary of the age. His definitions (§ 1.7 above) show that Aristotle was not alone, we know from Iamblichus that

> "the most perfect proportion" consisting of four terms and called 'musical', which, according to tradition, was discovered by the Babylonians and was first introduced into Greece by Pythagoras. It was used, he says, by many Pythagoreans, e.g. (among others) Aristaeus of Croton, Timaeus of Locri, Philolaus and Archytas of Tarentum, and finally by Plato in the *Timaeus*.[90]

So the mathematicians had more than one form of proportion, *and* they called the other sorts something other than "geometrical". When at *NE V* 1131b13 Aristotle says

> this kind of proportion is termed by mathematicians geometrical proportion

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[88] *Mathematics in Aristotle* (1949) p.273.

[89] *Grant* (ref. Book *II* 1106a35) insists on "arithmetic progression" as against geometric proportion, but raises no objection to the expression as used in Book *V* chapter 4. Burnet (1900 pp.216-17) and H.Rackham (*NE* 1926 pp.91 and 274) object to the arithmetic proportion as a proportion.

[90] Heath 1921, I p.86 referring to *In Nicomachi* p.118.
The *Timaeus* reference is 36A, though Plato does not name the proportion. He says:—

he is reporting the mathematicians own usage. As Iamblichus refers to "the most perfect proportion" he must allow that there are others, even if less perfect. Nicomachus (as Iamblichus indicates) also commented on the arithmetic proportion, as did Proclus. Nicomachus used the term 'proportion' to cover more than one mode of connexion; he wrote (op.cit. *II*, 21, 2):

> Proportion, pre-eminently, is the bringing together to the same (point) of two or more ratios; or more generally of two or more relations, even though they are not subjected to the same ratio but to a different or some other (rule).

Proclus said that Eudoxus added three new proportions, and a further four were found later[91]—and were all spoken of as "proportions". Thrasyllus also said there were the three proportions[92]. Difficulty emerged because, as Iamblichus said,

> it is premissed that it was the geometric which the ancients called proportion pre-eminently, though it is now common to apply the name generally to the remaining means as well.

Again

> the second, the geometric mean has been called proportion pre-eminently because the terms contain the same ratio, being separated according to the same proportion.[93]

And of the proportions Adrastus (reported by Theon[94]) said:

> the geometric was called both proportion pre-eminently and primary ... though other means were also commonly called proportions by some writers.

What seems to me to be the anachronistic objection to Aristotle's expression "arithmetical proportion" has its source in the comments found in such authors. Falling in with them Burnet treated the geometric as pure proportion, he wrote[95]

> originally αναλογια was confined to geometric proportion (ratio) but that by Aristotle's time it was already extended to series in arithmetical progression (1106a35) and later it was used for all manner of series.

> there were two kinds of means, the one exceeding and exceeded by equal parts of its extremes, ... the other being that kind of mean which exceeds and is exceeded by an equal number. (Jowett.)

[91] Proclus (*A Commentary on the First Book of Euclid's Elements*, p.67).
The three Eudoxian additions were (i) $(a-b):(b-c) = a:c$; (ii) $(a-b):(b-c) = c:b$; (iii) $(a-b):(b-c) = b:a$. The four later discoveries were (iv) $(a-c):(a-b) = b:c$; (v) $(a-c):(a-b) = a:b$; (vi) $(a-c):(a-b) = a:c$; and (vii) $(a-c):(b-c) = b:c$. (See van der Waerden 1954 p.232.)

[92] Heath, who reports this (1926 II p.292), thinks that Thrasyllus was confused.

[93] *On the Pythagorean Life* (p.98; 14; p.100, 15; p.104, 19).

[94] Heath, ibid.

[95] 1900 p.216; Burnet refers to J.Gow (i.e., to *A Short History of Greek Mathematics*, 1884, p.93) to support his view.

But despite Theon and the conflicts in Iamblichus's account this does not accord with what is known of mathematical principles from before Aristotle's time. Of the original three proportions the third (the first sub-contrary or harmonic) could have been discovered only in the presence of the simpler modes[96]. Since all these explorations took place before Aristotle's day (evidenced by Archytas's definitions) Burnet has inverted the history of the subject; the sub-contrary proportion, in particular, was explored by early Pythagoreans in connexion with their harmonic researches. Contrary to his earlier remarks that ἀναλογια originally applied only to the geometric proportion Burnet later, in his *Early Greek Mathematics* (1930 p.106), says that the first sub-contrary was associated with the discovery of the octave, and almost certainly went back to Pythagoras himself. (Any 'original' mathematical use of non-geometric ἀναλογια would then, on his own account, have had to have been even earlier than Pythagoras.) That the geometric proportion was spoken of both as pure proportion and as one of the classical modes of it has been a confusing feature of Aristotle's uses of analogy. He frequently presents the very same mathematical objects anaglyphically as contrasting:

(i) proportional vs. quantitative equality,

with

(ii) geometric vs. arithmetic equality.

Spoken of *qua* (ii) both terms are proportional, but *qua* (i) only one term is proportional. Aristotle's vagueness in borrowing these mathematical contrasts has more to it than mere vagueness. The whole of Particular justice is conceived in terms of proportionality yet, at the same time, proportion is invoked only when it is needed (Aristotle does not, as he is routinely accused, drag in proportional and mathematical principles where they do not apply). In some instances plain equality expresses perfectly well the circumstance in which justice prevails. Even though the plain equality is *also* the arithmetic proportion, there would have seemed to Aristotle (and his audience, who were familiar with discussions couched in terms of proportionality[97]) little need to bring-up the more complex principle when the simpler contrast would work. Yet properly speaking in terms of his theory,

[96] The Pythagorean interest in the different forms of proportion, not merely ratio, is seen in Philolaus's exploration of geometric figures. The cube exhibits the harmonic proportion 12 : 8 :: 6 : 4, i.e., 12(edges) : 8(corners) :: 6(faces) : 4(sides to each face). See also §§ 6.3 and 6.4.
[97] The language of proportionality would have been very much 'in the air' at the Academy and the Lyceum.

Aristotle ought to speak of the terms *qua* their mode of proportion: the arithmetical mode in the case of plain equality, and the geometric mode where only 'proportion' is mentioned—even though to do that might seem pedantic. This running together of two ways of construing the same terms is the single most complicating factor in Aristotle's theory of justice.

There has been a second difficulty in respect of means and proportions. Concurring with Jackson, Heath (1926 *II* p.292, commenting on *Elements VII*.20) accepts that there were differences of definition of proportion among the writers of later antiquity, yet his decision (p.293) that:

> the natural conclusion to be that of Nesselmann, that originally the geometric proportion was called analogia, the others, the arithmetic, the harmonic, etc. *means;* but later usage had obliterated the distinction[98]

cannot be correct; the three modes were clearly differentiated from at least the time of Pythagoras—and Archytas, we have seen, referred to the "three proportions in music". It seems most likely that the wider notion covering the three classical forms were spoken of as proportions *alongside* the more limited geometric usage. The habit of referring to the geometric, arithmetic or harmonic *proportion* interchangeably with *mean* was quite unproblematic and unambiguous whenever the difference between a mean and a proportion was not the issue[99]. When it becomes an issue the distinction between a mean and a proportion (indicated above at § 1.7) will need to be kept in mind. Those objecting to the language of arithmetic and harmonic proportions—such as Iamblichus (intermittently), Theon of Smyrna, Nesselmann, Gow, *Grant*, Burnet, Heath[100], Rackham and *Soudek*[101]—tend to mistake what is merely the position (as it were) of the fulcrum for the equilibrium of the whole structure it supports. To say of one such set-up—the geometric—that it is "properly" a proportion but that the harmonic and arithmetic set-ups are not is to misconceive the nature of means and proportions (the mean being, as said earlier, the central point of what is in balance rather than the whole). What probably led commentators to elevate the Geometric was the success of that

[98] Heath and Jackson (p.87) both refer to G.H.F.Nesselmann (*der Algebra der Greichen,* 1842 pp.210-212), who claimed that the geometric was a (presumably *the*) proportion, and the others μεσοτητες.

[99] See § 1.7 note 56 for Iamblichus's reference to the three proportions in music.

[100] In his remarks on *Elements V*.6 (1926 *II* p.129) Heath defends from M.Simon (*Euclid und die sechs planimetrischen Bücher*, 1901 p.110), the translations "proportion", "in proportion", and "proportional" as all suitable for ἀναλογον, on the basis that the meaning in each case is perfectly in order. Simon claimed the translation should express 'standing in the relation of proportion', and asked 'What is proportional to what?'; Heath thought this too fussy, but he didn't extend an equivalently laudable flexibility in usage to the ancients.

[101] Pp.54-58.

proportion when expanded into the general theory of magnitudes. Thereafter the capacity to bear an equal multiple became a vital criterion in deciding a proportion. The difficulty for the distinction of forms of proportion from forms of equality, *and* between means and proportions, have arisen because, in both cases, it was simultaneously convenient to use the terms both narrowly and broadly. But the fact that it is no longer the custom to employ the notion of the arithmetic proportion is not reason enough to cavil at Aristotle's use of it. Neither is there reason to suppose that the original users were unable to discriminate the uses[102].

2. 2 Terminology

The note of uncertainty over the correct terminology for means, proportions and ratios that was sounding by later antiquity increased during the Middle Ages, and has unfortunately been projected forward, we have seen, to more recent times. But that uncertainty should not be projected back to the classical period. In his theory of justice Aristotle employs proportion theory throughout, and assumes a familiarity with it in his audience. The confusion which arose—even before the onset of the Dark Ages—has been in part due to terminology, but reciprocally, what generated the unclear language was the confusion in the ideas. The doctrine of proportion in Euclid's *Elements,* especially Book *V*, upon which Aristotle relies, is the very text which suffered the most subfusc misapprehension for so long. The vicissitudes of *Elements V* have a bearing on the understanding of the theory of justice in that it is only through the awareness of Aristotle's use of proportion theory that the cogency of his argument may be seen. Correspondingly, unfamiliarity with the true classical proportion theory made use of by Aristotle vitiated both the translation and the interpretation of Book *V* of the *Ethics.* The tale is also instructive in demonstrating just how elusive the central principles of analogy have been[103].

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[102] There is no lack of clarity when Aristotle speaks of ratio rather than proportion (such as at *NE II* 1106a35, *V* 1131b13, 1131b34-1132a).

[103] Of the several routes by which the *Elements* have survived:

(i) Theon of Alexandria (4th century AD) produced an edition of the *Elements* which differed somewhat from the original. All the Western and most of the Eastern editions stemmed from this version until 1809 when Peyrard discovered a more accurate manuscript descending from a pre-Theonine script: all modern editions are based on this more reliable text.

(ii) The oldest direct Greek-Latin translation to survive into the Middle Ages was Boethius's (5th century AD) through Cassiodorus's *Institutiones, Book II* (8th-9th century; see J.Murdoch, *The Transmission of Euclid,* in *Dictionary IV* p.437-59).

We might expect the Latin for ratio to have been *ratio*, but it became *proportio*; and for proportion *proportionalitates* (which can be a little confusing[104]). Roger Bacon noticed that in his Special Edition of the *Elements* Æthelhard calls *medietates* what he called *proportionalitates* in the initial translation and his earlier abridged commentary[105]. The principles and terminology of the theory of proportions were so muddied by Bacon's time (mid-13th century) that he could gain no idea of what the theory was. (The errors of interpretation which undermined the theory will be reported briefly after the theory itself has been given at §. 2.9.)

(iii) There were two distinct sources of the Arabic translations and commentaries, (a) a Theonine copy of the *Elements* was sent by the emperor Constantine V to the Caliph in AD 760. About 40 years later Haroun ar-Raschîd's vizier had it translated into Arabic by al-Hajjâj (who translated it twice). From this many Arabic translations (and also translations into Syriac, Hebrew, Persian and Armenian) were derived. Æthelhard of Bath translated the work from this Arabic tradition in Toledo in 1126 (where he also made use of the Boethian text), followed by an abridgement (*Commentum*) and later a fuller *Editio Specialis.* (M.Clagett 'The Medieval Latin Translations from the Arabic of the *Elements* of Euclid, with Special Emphasis on the Versions of Adelard of Bath' in *Isis 44*, 1953, pp.16-42). The second Arabic source (b) is of unknown but pre-Theonine provenance. This was the tradition used in the 9th century school of Ishâq ibn Hunain and Thâbit ibn Qurra. (J.Shönbeck *Euclidean and Archimedean Traditions in the Middle Ages and the Renaissance*, in *Companion I*, pp.173-84.) Gerard of Cremona translated both variants in the mid-twelfth century—the Theonine text taken from a renowned commentary by al-Nairîzî. Campanus produced his re-working of Æthelhard's *Commentum* (not the more extensive *Specialis* text) in 1259; this became the most widely used of the many versions of the *Elements* in the West during the Middle Ages.
(iv) The Byzantine emperor Manuel I sent a number of ancient works to Sicily with Aristippus as a gift to King William the Bad. (G.Sarton *Introduction to the History of Science 2i,* 1931.) Of these Aristippus translated the *Phaedo*, *Meno* and Book *IV* of Aristotle's *Meteorology,* but not the *Elements* nor Ptolemy's *Almagest.* These were translated shortly after 1160 by an anonymous ex-medical student from Salerno. His was the most exact translation of the *Elements* there has ever been. (Murdoch 'Euclides graeco-latinus: A Hitherto Unknown Medieval Latin Translation of the *Elements* Made Directly from the Greek', in *Harvard Studies in Classical Philology 71* (1966) pp.249-302). Unfortunately this fine work was little known.
(v) Cardinal Bessarion had three Greek MSS of the work in his great library, in addition to the (by then traditional) Campanus recension. In 1468 Bessarion gave to Venice his collection of nearly six hundred texts saved from the fall of Constantinople. In Venice in 1505 Zamberti used these manuscripts for the first printed translation of the *Elements* from the Greek (the text from the Arabic-Æthelhard-Campanus tradition had been printed there in 1482). Thereafter innumerable translations were printed, even in the sixteenth century; of these Tartaglia's commentary (of 1543) was of special importance in that Vincenzo Galilei knew it (see § 3.7, note 239). (It was first translated into English in 1570 by Billingsley.)
[104] Murdoch keeps to this usage (*The Medieval Language of Proportions: Elements of the Interaction with Greek Foundations and the Development of New Mathematical Techniques,* in A.C.Crombie (ed.) *Scientific Change*, 1963 pp.237-71). Translating λογος as *proportio* rather than *ratio* seems to stem from Boethius's *Arithmetica* and *De Institutio Musica*—see L.Minio-Paluello's response to Murdoch in Crombie p.309. *Ratio* came to be used for 'character' or 'essence' through a development which traces back to Empedocles. In *PA I* 642a20ff. Aristotle speaks of the essence of bone as a λογος, i.e., that it is composed of elements combining in a given ratio, rather as we think of chemical elements. This was a principle of Empedocles.
[105] Roger Bacon *Communia Mathematica* (trans. R.Steele, 1940, p.125).

2. 3 The problem of the irrationals

The earliest Greek mathematicians had supposed that every ratio of continuous magnitudes would correspond to a ratio of whole numbers[106]. A pair of lines provides the simplest example where such an equivalence was expected. Any pair of lines should be exactly measurable by some smaller length, but at some stage it was discovered that it this is not always so. Neither was the absence only from remote cases, but in the plainest, most unavoidable examples. The side of the square was found to be incommensurable with its diagonal, and the circumference of the circle with the diameter. The line divided in "extreme and mean ratio"[107]—the "golden section" as Kepler later called it—was also found to yield incommensurable lines. The construction of the regular pentagon used the golden section, as did the dodecahedron, icosahedron[108], and the inscribed pentagram[109]. Attempts to locate the mid-point of the octave also led directly to the discovery of √2.[110] The kind of problem which the discovery generated can be seen most vividly with the simple example of a rectangle of 1 x 9 units compared with one of 1 x 8. A square 3 x 3 units covers exactly the same area as the figure 1 x 9, but no square can be constructed which precisely covers the same area as 1 x 8 units. The fact appears counter-intuitive, and yet in a physically obvious way it demonstrates how the rational understanding of the world was undermined. To account for the world through reason had meant, fundamentally, that it was exhaustively and precisely ordered, both numerically and geometrically. It was not merely that Pythagoras had concocted some 'mystical' view of the world 'as

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106 Philolaus, *Fragments 4, 11,* and Aristotle in *Metaphysics A* 990a18ff., *N* 1090a20-25.

107 *Elements VI* def.3 (Heath):

> A straight line is said to have been cut in extreme and mean ratio when, as the whole line is to the greater segment, so is the greater to the less.

VI, 30 gives the procedure to cut a given (finite) straight line in mean and extreme ratio.

108 *Elements XIII,* 16 gives the rules to construct the icosahedron, and *XIII,* 17 the dodecahedron.

109 The pentagon with inscribed pentagram creates an infinite regress. The pentagram with inscribed pentagon was an emblematically Pythagorean figure. The early Pythagorean interest in the figures associated with the pentagon is confirmed by coins from Chalcis of 480 BC showing the five-pointed wheel. Other coins from Metapontum from 440 BC have been found—Metapontum was a Pythagorean stronghold until they were ejected at about that time (see Artmann 1993). R.Herz-Fischler *A Mathematical History of Division in Extreme and Mean Ratio* (1987), and *The golden number, and division in extreme and mean ratio,* in *Companion,* vol.2, pp.1576-83 (1994) examines issues of mean and extreme ratio. See also J.Høyrup *Babylonian Mathematics* (ibid. vol.1, pp.21-28). Von Fritz ('The Discovery of Incommensurability by Hippasus of Metapontum' in *Annals of Mathematics,* 1946) claims that it was the anthyphairetic process applied to the lines in extreme and mean ratio which revealed their irrationality (see Knorr 1975 pp.29-30).

110 A.Wasserstein 'Theaetetos and the History of the Theory of Numbers' in *Classical Quarterly* (1958).

number', and that this pseudo-science was threatened by the irrational. On the contrary it was the most mundane facts of experience (two straight lines) that defied a complete rational explanation. The side and diagonal of a square taken together meet the requirements for ratio in Euclid's definitions (*V* 1-5): they were related with respect to size, they were of the same kind, and they were capable when multiplied of exceeding one another. And yet the relation was indefinite, inexact and inexhaustive. Even though the definitions imposed a limit (περας) on the contrariety 'greater/less' (which fell under το ἀπειρον—§ 1.4 above) the ratio of no two numbers whatever could show the relation of the two lines. Λογος, the paradigm for the rational, was found unavoidably to contain ἀλογον (as with the methods of approximation). This is why on page 9 I said that where there was a difference between the terms ratio only *aims* to quantify that difference, rather than "ratio quantifies the difference exactly". At the heart of the most exact science lurked το ἀπειρον, the inexact, irrational, unlimited unreason. Λογος had failed.

2. 4 The discovery

Speculation about the discovery of the problem has been enormous[111] but who discovered it, and when, is of marginal relevance to our inquiry. The speculations are relevant in only one respect: the impact the discovery of the irrationals had on philosophical, not strictly mathematical, issues. The accounts of the discovery by scholars have been bound-up with their views on the nature of Greek mathematics. The more traditional view has been that the discovery had a shattering effect on the Pythagoreans in that it undermined their unified account of the world[112]. They were faced with problems which could not be tackled arithmetically; it was beyond the hope of numbers to double the

[111] The ancient references to the discovery are found in Iamblichus (circa AD 250-325), *On the Pythagorean Life;* Pappus (circa AD 320), *Commentary on Book X of Euclid's Elements;* and Proclus (circa AD 410-85, op. cit.). Speculation as to the discovery is found in Tannery (1887), Zeuthen (1910, 1915), Heath (1921), von Fritz (1945), van der Waerden (1954), Neugebauer (1957), Wasserstein (1958), Burkert (1972 ch.6), Knorr (1975 ch.2), Fowler (1979 pp.294ff.).

[112] Zeuthen, Tannery, Heath, Neugebauer, van der Waerden and Burkert felt that the discovery brought about a shift from arithmetic and (proto)algebraic thinking which had been absorbed from Egyptian, Phoenician and Babylonian sources. The Mesopotamians had an ancient tradition of what largely corresponds to algebraic mathematics (including quadratic equations of considerable sophistication). After the appearance of the irrationals it was found that arithmetic or algebraic techniques could only offer methods of approximation, not exactitude.

cube[113], trisect an angle, or square the circle. So the world could not be explained by, nor correspond to, natural numbers. Not only was the irrational found in the most inconvenient places, a proof appeared which showed that any attempt to explain certain magnitudes numerically leads to a contradiction[114]

More recent historians[115] have tended to minimise the impact of the discovery; they claim that the *mathematical* evidence (such as it is) appears unperturbed by the presence of irrationals. But even if we were to accept the views of those who say that the irrationals presented no embarrassment to Greek mathematics, numbers and spatial magnitudes could no longer enjoy a common set of rules, or fall within a single general theory. As a result, over and above any difficulties that might be faced by the philosophy of mathematics, cosmology was deeply affected. The Pythagorean aim was more than mathematical, rather it was to achieve a comprehensive account of reality[116]. Since magnitudes were found to which no number would correspond priority could no longer be assigned to arithmetic,

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[113] Apollo, as god of both music and mathematics, set this problem: to double the size of His insufficient altar at Delos. The problem reduced to the location of a second mean: $a : x :: x : y :: y : b$. Plato also speaks of the question of a second mean in the *Timaeus* 32B (Jowett):

> as the world must be solid, and solid bodies are always compacted not by one mean but by two, God placed water and air in the mean between fire and earth, and made them to have the same proportion–so far as was possible (as fire is to air so is air to water, and as air is to water so is water to earth).

[114] Aristotle refers to this proof in *Pr. An.* 41a27-29. Using the odd/even polarity, with a *reductio* argument, it is assumed that the side of a right isosceles triangle is commensurable with its diagonal. Odd numbers result in being equal to even numbers. A variant of the proof is given by Euclid in *Elements X;* this is equivalent to the demonstration that given a right isosceles triangle *(abc)* with the hypotenuse *(ab)* held to be commensurable with the sides *(ac, cb)* of mutually prime integers (h and s), then $h > 1$. $h : s \Rightarrow h^2 : s^2$; but from Pythagoras's theorem $h^2 = 2s^2$. Hence, as h equals 2 *x something*, h is even; and as they are mutual primes <u>s must be odd.</u> Now h being even, let $h = 2n$; again, from Pythagoras, $(2n)^2 = 2s^2$, so $2s^2 = 4n^2$; then $2n^2 = s^2$. As s^2 equals 2 *x something*, <u>s must be even</u>. As the assumption that the hypotenuse is commensurable with the side results in the absurdity that the side is both odd and even the assumption must be false, and the terms must be incommensurable.

[115] Such as Szabó (1978), S.Unguru ('On the Need to Rewrite the History of Greek Mathematics', in *Archive 15* pp.67-114, 1975), Knorr (1975, ch.2), Mueller (1981), and Fowler (who reviews the evidence, 1987 pp.294-308). The tenor of their approach is that mathematicians, then as now, liked to get on with the mathematics leaving foundational crises to Sunday afternoons.

[116] Reported by Aristotle in *Metaphysics A* 985b23ff (Ross):

> Contemporary with these philosophers [sc. Leucippus and Democritus], and before them, the Pythagoreans, as they are called, devoted themselves to mathematics; they were the first to advance this study, and having been brought up in it they thought its principles were the principles of all things. Since of these principles numbers are by nature the first, and in numbers they seemed to see many resemblances to the things that exist and come into being ... since, again, they saw that the attributes and the ratios of the musical scales were expressible in numbers; since then, all other things seemed in their whole nature, they supposed the elements of numbers to be the elements of all things.

nor to any 'arithmetic geometry'. The sudden appearance of the proof of the impossibility of their aim being met could not but have been of central importance: their single comprehensive account of the world had evaporated. Those thinkers whose interests went beyond purely mathematical questions[117] were inevitably concerned at the impossibility of giving an exact value to magnitudes arithmetically, and by the lack of correspondence between numerical and spatial quantities. Exact integrated and exhaustive knowledge was now felt to be impossible to attain (the pursuit of exactitude now leading to an infinite regress). Only methods of approximation were available where it had been confidently assumed that precise and complete knowledge was within reach.

I have so far treated *irrational* and *incommensurable* as equipollent, but properly speaking incommensurable terms do not share a common measure, whereas 'irrational' applies where the value of a term is not expressible as a natural number. Irrationals lack precision. It is misleading to refer to irrational 'numbers', numbers, to *be* numbers, had to be exact. The failure to locate any exact measure is what most concerned Greek thinkers; the term they used (ἄλογος) indicates the profoundly disturbing character of the issues their discovery raised (see § 1.4). Treated geometrically, if not arithmetically, all magnitudes will have some exact measure, but even where an exact measure was possible an exact *and complete* account was not. When incommensurable magnitudes are in ratio (i.e., where they are of the same kind, and a multiple of the lesser will exceed the greater) all that could be shown (as with methods of approximation) was the *minimal* information required to count as knowledge. I.e., that no more than the principle 'greater than' applies. The precise measurable comparison was missing.

Whether the discovery was early or late, shattering or a mere anomaly, the conflicting aetiologies agree on at least one thing: the irrationals were known prior to Plato's establishment of the Academy (about 387BC)[118]. A major aim of the Academy was to bring a higher standard of rigour to mathematics; three of the mathematicians associated with it—Theodorus, Theaetetus, and

[117] I.e., mathematicians such as Archytas, Eudoxus, Euclid.
[118] That the issue was an entirely familiar, standard, subject is shown by Plato's joking about irrational quantities in the *Republic VII* 534D. He says that children who are to become the rulers should not be irrational quantities, as in geometry. (But Plato treats commensurability very seriously in *Laws VII*, 819 D ff..)

Eudoxus—were concerned with the problem of the irrationals, and it is clear that these issues were at the heart of the mathematics associated with the Academy.

2. 5 ΑΝΤΑΝΕΙΡΕΣΙΣ vs. ΑΝΘΥΦΑΙΡΕΣΙΣ

The earliest method of dealing with irrational magnitudes was that of reciprocal subtraction (ἀνθυφαιρεσις)[119]. Berg translates this term as "removing of the immediate opposite"[120]; *Liddell & Scott* give a less restricted meaning: *take away again* or *in turn.* Augustus de Morgan interpreted the definitions of ratio with which we began (*Elements* 5.3 and 5.4) as (in effect) definitions of the anthyphairetic process. In his view "the relation is with respect to quantuplicity not size" (i.e., ratio tells how many times one quantity may be taken from the other). Against a foundational status for ἀνθυφαιρεσις Becker argued that the anthyphairetic definition of proportion was developed solely to extend the numerical notion of proportion to cover incommensurable magnitudes[121]. Knorr also claims that Theaetetus developed it as a necessary part of his exploration of irrationals, but with no programme to resolve the difficulties which the irrationals posed to mathematics as a whole. But whether it was foundational or not, when applied to incommensurable magnitudes the anthyphairetic technique does not escape the flaw that it is only a method of approximation; it must count or match an unending sequence. (Given any pair of natural numbers the process of reciprocal subtraction will terminate, but given a pair of continuous extensions the process might be unending; in such cases the magnitudes are incommensurable.)

Zeuthen, Dijksterhuis, and Becker[122] each independently deduced that what we call the anthyphairetic theory of proportion was the source of the definition which Aristotle refers to in *Topics:*

> It is easily proved, for instance, that the line parallel to the side and cutting the plane figure divides similarly the base and the area. But once the definition is stated, what is said becomes

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[119] Later known as continued fractions or as the Euclidean algorithm, though Fowler (e.g., 1987, p.31) objects to these terms as harmful anachronisms for discussion of classical ratio or proportion theory. The ancient processes of ratio were, he says, subtractions not divisions, and continued fractions tend to be associated with more sophisticated fractions and with real numbers rather than with the original, principally geometric, methods.

[120] C.Berg *Græsk-Dansk Ordbog*, 1864.

[121] O.Becker 'Eudoxus-Studien 1', *Quellen und studien zur Geschichte der Mathematik, Astronomie und Physik,* Berlin, 2B, 1933 pp.311-333 (summarised in Van Der Waerden 1954, pp.175-179). His reading was accepted by the later Heath, and by Plooij and Szabó.

[122] Discussed by van der Waerden (ibid.).

> immediately clear. For the areas and the bases have the same mutual exhaustion [ἀνταναιρεσις]; such is the definition of the same ratio.[123]

But commenting on this passage Alexander of Aphrodisias said:

> What is said expressed in such-like terms is not familiar, but it becomes familiar once the definition of "in proportion" is stated, that the line and the space are divided analogously by the drawn parallel. Now this is the definition of proportionals which the ancients used: these magnitudes are proportional to each other of which the reciprocal subtraction [ἀνθυφαιρεσις] is the same. But he [Aristotle] has called reciprocal subtraction "mutual exhaustion" [ἀνταναιρεσις]. But those which are proportional to each other are also said to be connected in a similar manner [ὁμοιως] to each other. For this reason he has said "such is the primary definition of 'same ratio'" instead of "this is the definition of 'proportion'".[124]

Ἀνταναιρεσις was a much commoner word than ἀνθυφαιρεσις, with a wider range of uses; Berg translated ἀνταναιρεσις as 'cancelling out'; van der Waerden (who discusses Alexander's remark)[125] says that it was drawn from *balancing against each other. Liddell & Scott* say it was used for *corresponding diminution, alternate removal, cancellation, to be struck-off, cancelling opposite sides of an account,* and *to be cancelled correspondingly.* In their record the two words look pretty interchangeable, and indeed commentators have often assimilated them[126]. But if we take Berg's more exacting translation there is a marked difference between the terms: on the one hand (with ἀνθυφαιρεσις) there is an interactive process, and on the other (with ἀνταναιρεσις) only a corresponding sequence.

The extract from *Topics* is taken from a passage in which Aristotle reviews the difficulties in capturing 'first principles' in exact definition. Here, more than anywhere, we should expect Aristotle to be precise in his choice of expression (the need for exact terminology being the point of the discussion[127]). The mathematical illustration is chosen to display the care necessary when

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[123] 158b29, as quoted by Knorr (1975 pp.157-58).

[124] *In Topica,* p.545. Translation by Walleis, slightly modified (Walleis left ἀνθυφαιρεσις, ἀνταναιρεσις and ὁμοιως in the Greek).

[125] 1954 p.176; he reviews the writings of earlier commentators such as Heiberg, Zeuthen, Heath, Dijksterhuis, Becker, Plooij (1950). The issue was later treated by Szabó (1978, and earlier), Knorr (1975 p.290 n.26), Larsen (1984), Thorup (1992), and extensively by Fowler (1979, 1982, 1987, 1991).

[126] Though not Heiberg or Heath; Heath attributes to Heiberg the proposal that in using ἀνταναιρεσις rather than ἀνθυφαιρεσις Aristotle highlights the supposed fact that the rule could not supply a general proof in the pre-Eudoxian model (it treated commensurate magnitudes only), whilst he himself takes advantage of the new general theory. Heath thinks this interpretation unlikely (and needlessly complicated) whereas (1926 II pp.120-21 n.22):

> though Eudoxus had formulated the new definition, the old one was still current in the textbooks of Aristotle's time, and was taken by him as being a good enough illustration of what he wished to bring out in the passage of the *Topics* referred to.

[127] It is by means of first principles that exact definition is achieved; How then, asks Aristotle, are these first principles themselves to be decided? Equivocation, polysemy and figuration are particularly to be guarded against for the correct rendering of the definitions of scientific

determining first principles. Aristotle's preference for ἀντανειρεσις is well chosen: unlike ἀνθυφαιρεσις mutual exhaustion figures in both reciprocal subtraction and corresponding bisection. When the measures are correspondingly bisected ἀντανειρεσις is plainly seen. But equally, when measures are reciprocally subtracted mutual exhaustion is present. In both techniques the quantities correspondingly diminish, but there is *interaction* in only one. Consequently ἀντανειρεσις, which occurs with both, is the 'first principle', and ἀνθυφαιρεσις subsidiary. This is the reason, it seems to me, Aristotle stipulates ἀντανειρεσις as the definition of *same ratio*.

Aristotle does not, as Alexander thinks he does, distinguish *same ratio* from *proportion*—in fact in the *Ethics* Aristotle gives the definition of proportion as 'same ratio', just as Euclid did later (Euclid says ' "being in the same ratio" is defined ...' when giving the explanation of proportion). Yet Aristotle, as we have seen, sees a difference between ἀντανειρεσις and ἀνθυφαιρεσις. Alexander's observation is therefore misleading; he treats the pair of terms ἀντανειρεσις/ἀνθυφαιρεσις as, in effect, mapping *same ratio/proportion*, which was far from Aristotle's thought.

Probably due to Alexander's assigning ἀντανειρεσις to *same ratio*, and ἀνθυφαιρεσις to *proportion*, commentators have taken Alexander to be identifying the one method as the other[128]. This is not quite what is said; rather what he says indicates that the two methods are *reducible* to practical equivalence. To be so reducible there had to exist the two distinct principles; there was, in Alexander's reading of Aristotle's mind, *some* difference between the terms *same ratio* and *proportion* which were, nevertheless treatable ὁμοιως. In mistakenly (in my view) projecting the two mathematical techniques onto a supposed difference between same ratio and proportion Alexander did not bring out with sufficient clarity Aristotle's considered preference for the term

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principles, and the inferences closely tied to them (or, indeed to any scientific or analytic principles).

[128] Artmann (1993 p.31 n.46) regards them as synonymous. M.Larsen (p.2 of 'On the Possibility of a Pre-Euclidean Theory of Proportions', in *Centaurus 27,* 1984 pp.1-25) explicitly treats the two methods as equivalent; and Byrne (1997, pp.112-17, 139) speaks of *antaneiresin* but in fact describes ἀνθυφαιρεσις. Fowler (1987 p.32) says

> Some [e.g., Heiberg and Heath] have argued that the two words have different meanings, but I shall treat them as synonymous, as did Alexander of Aphrodisias.

In fact Alexander did not treat them as synonymous, but just how much he grasped of Aristotle's reason in selecting ἀντανειρεσις I'm not sure. Fowler cites documents from the *Zenon* archive (as he does again in 1991, p.107) which show their indifferent use in commercial accounts for the autumn of 251BC and the summer of 243. For many such purposes the terms, of course, would have been equivalent.

ἀνταναιρεσις. By Alexander's day (2nd century AD) these earlier conceptions had long been superseded by the Euclidean theory, but Alexander's puzzlement tells us (when we look into the context of the *Topics* extract and the etymology of the word which Alexander thought Aristotle oddly selected) that there once had been two distinct pre-Euclidean conceptions of analogy (which Alexander wrongly saw as the distinction between *proportion* and *same ratio*). The two approaches reflect the two suppositions (perhaps even presuppositions) as to the nature of mathematics which I alluded to at § 1.2. The periodicity was conceived as either a *process* or a *condition*. As a process it was the active, de Morgan-like notion, associated with ἀνθυφαιρεσις and Theaetetus. Thought of as a condition it was the passive, Heath-like concept, resting on the ἀνταναιρεσις of successive bisections.

Although the *Topics* extract is frequently cited, the broader passage from which it is taken is rarely mentioned, consequently the rather casual (though sometimes insistent—see the last few notes) identification of the terms has obliterated the logical finesse which Aristotle brings. His sensitive choice of expression should be recognised, but unfortunately Fowler's dismissive:

> The mathematical proposition that Aristotle is describing, in his typically vague fashion

is all too representative. Aristotle is not directly concerned to make a mathematical proposition; his purpose is to illustrate the need to choose the *exact* mathematical term. In so doing his "typically vague fashion" is more precise than his critics' precision.

2. 6 An intermediate theory ?

The conception of proportion theory as either antaneiretic or anthyphairetic was largely replaced by the definition given in Euclid's *Elements*. *Elements V* has been attributed to Eudoxus since ancient times, but Knorr argues (1978a) that, based on deductions he makes from the work of Archimedes, Eudoxus had quite a different theory. Archimedes's antaneiretic usage, and the proximity to Eudoxus's method of exhaustions, in Knorr's view, support the existence of this alternative conception. Knorr "reconstructs" the theory from propositions 6 and 7 of Archimedes's *On Plane Equilibria, Book I*[129]. This book proposes the Lever Law[130], and these propositions

[129] Archimedes also employs an antaneiretic lemma both in his preface to *Quadrature of the Parabola* and *On Spiral Lines;* (as given by Knorr):—

specifically deal with the principle of Balance. Proposition 6 treats commensurable measures and 7 incommensurables. Unfortunately the book survives only in highly corrupted manuscripts in which these propositions are incomplete and, as they stand, invalid[131]. To make the propositions tenable, then, a good deal of reconstruction is necessary. To support his claim that 6 and 7 echo much earlier formulae devised by Eudoxus Knorr "reconstructs" the principles keeping to the Archimedean division into commensurable and incommensurable parts, but treating all classes of magnitude alike[132]. The account for incommensurable weights is more complicated than that for commensurables, and for these Knorr turns to a later scholium on Theodosius's (Ist century BC) *Sphærica, Book III*[133]. This reclaimed theory Knorr believes is the one referred to by Aristotle, not so much in *Topics* but in other passages, especially those from *Posterior Analytics*[134].

> Of unequal areas the excess by which the greater exceeds the lesser is capable when added to itself of exceeding any pre-assigned finite area.

[130] Mentioned by Pappus, and attributed to Archimedes by Heron. For the connexion with the Lever Law see § 3.7, note 241 and § 6.6.

[131] Mach said that the proof of 6 was not valid; Dijksterhuis (*Archimedes* pp.305-306) said that though 7 might be improved, it couldn't be quite saved. Heiberg called 7 obscure and imperfect, and Berggren says that the proof is just "wrong" (see Knorr 'Archimedes and the *Elements*: Proposal for a Revised Chronological Ordering of the Archimedean Corpus', *Archive 19*, 1978b pp.211-90).

[132] For the full account of this alternative theory see Knorr 1978a, Appendix III, pp.230-35. For the commensurable cases:
Commensurable weights a, b, c, d, will be proportional when if the magnitudes a, b have a common measure E, and c, d the measure F, there will be natural numbers n, m such that if $nE = a, nF = c, mE = b, mF = d$; then $a : b :: c : d$.

[133] Following a remark by Eutocius (6th century AD) referring to:

> the beginning of the tenth book of Euclid's *Elements* and the third book of Theodosius's *Sphærica*

Heiberg (*Archimedes*, p.270) traced the latter to Book III propositions 9 and 10, which treat the arcs of great circles. The *Elements* reference is to X proposition 1. This Euclidean proposition is notably antaneiretic, unlike its succeeding proposition X,2 which is anthyphairetic. X,1 says (Heath):

> Two unequal magnitudes being set out, if from the greater there be subtracted a magnitude greater than half, and from that which is left a magnitude greater than its half, and if this process is repeated continually there will be left some magnitude which will be less than the lesser magnitude set out

whereas X,2 says:

> If, when the less of two unequal magnitudes is continually subtracted in turn from the greater, that which is left never measures the one before it, the magnitudes will be incommensurable.

The later scholium on Theodosius's III proposition 9 may be interpreted:
For natural numbers m, n, given three homogeneous magnitudes a, b, c, let c be arbitrary and $a > b$. Construct a fourth proportional d commensurable with c whereby $a > d > b$.
Where necessary, bisect successively c until x remains which is much less than $a - b$; then either (i), when b and m are commensurable, some multiple of m will exceed b exactly. Or (ii), when b and m are not commensurable, some multiple of m will exceed b by less than itself. There is then some magnitude such that $n - 1(m) < b$, but $n(m) > b$. Let $n(m) = d$. As d is commensurable with c (being obtained from a division of it) a magnitude has been

Knorr does not mention what must be the most obvious interpretation of his evidence: that Eudoxus first worked-out this intermediate Special Theory in connexion with the bisection principle and his method of exhaustions (Aristotle demonstrates his familiarity with this method in *Physics VIII*[135]). Then he later arrived at the General Theory by fusing the concept of the equimultiples with the principle that magnitudes of the same kind might exceed or be exceeded by others, whether or not they are commensurable. Such a reading fits the evidence we have from antiquity: the references to Eudoxus by Archimedes and Apollonius, the reports derived from Eudemus, and the comments by the later Greek and Roman historians.

2. 7 A general anthyphairetic theory ?

In 1978 when Knorr proposed the intermediate model it was thought that the anthyphairetic approach was incapable of generating a comprehensive theory of proportions. As such it could not be what Aristotle was referring to in *Posterior Analytics.* Becker had thought that the technique could produce the rules for numbers and (rectangular) areas mentioned in *Topics*, but not treat such other continuous extensions as volumes, weights, periods of time, or angles. Then in 1984 Larsen demonstrated that the εναλλαξ theorem is provable using ανθυφαιρεσις—though only with the aid of techniques not known before the 20th century[136]. In 1992 Anders Thorup published a paper

> to show that the alternative theory of proportions may be built up on a very simple interpretation of "anthyphairesis"; without any reference to modern notational machinery.[137]

The stumbling-block to a comprehensive anthyphairetic account of proportions was always thought to be the alternation rule; it is this rule which Thorup overcomes with his proposal[138]. If he is correct

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constructed such that by however little *a* exceeds *b* there is a magnitude *d* commensurable with *c* falling between them.

[134] 74a17ff. (Tredennick):

> Again, if there were no triangle except the isosceles, the proof that it contains angles equal to the sum of the two right angles would be supposed to apply to it *qua* isosceles. Again, the law that *proportionals alternate* might be supposed to apply to numbers *qua* numbers, and similarly to lines, solids and periods of time; as indeed it used to be demonstrated of these subjects separately .... but now the law is proved universally; for the property did not belong to them *qua* lines or *qua* numbers, but *qua* possessing this special quality which they are assumed to possess universally.

For the related passage 99a9 see § 2.8.

[135] 266b2-3 (Wicksteed & Cornford):

> for by successive additions I can make the power exceed any given limit, and by corresponding subtractions can make the time fall short of any.

[136] Op.cit.

[137] 'A Pre-Euclidean Theory of Proportions', in *Archive 45*, 1 pp.1-16.

then a *general* anthyphairetic proportion theory was within the orbit of mid-fourth century mathematicians.

Thorup is particularly careful to avoid claiming that his construction reflects what actually happened at the time; he restricts himself to the claim only that it was quite possible. Yet his work, and that of Larsen, appears to reduce the call for a distinctively antaneiretic intermediate theory. Knorr's reconstruction is, in any case, somewhat tenuous[139]; the thread being found in a commentary some time later than the 1st century BC on Theodosius's use of a principle of Archimedes who, in any case, flourished a hundred years after the discovery of whichever method it was that generated a comprehensive theory. The variant theories both rely on two main features:

(i) that homogeneous magnitudes may exceed or be exceeded whether they are commensurable or not,

(ii) equimultiples.

(i) is an obvious characteristic, but it needed to kept in mind in connexion with (ii), which carries the main burden for both the suggested alternatives (in addition to the Euclidean general theory). It is equimultiples that make the alternative theories tenable. As all agree that equimultiples were used by the mid-fourth century, it *would* seem reasonable to project their use back a few years to allow for the general *anthyphairetic* theory. But doing this might be putting the cart before the horse (as Thorup himself clearly recognises): relying on the very insight of the equimultiples in the Euclidean theory which make the alternative models possible. It is much less extravagant to keep to the traditional view that Eudoxus devised the (Euclidean) general theory to resolve the difficulties to comprehensive and exact calculation posed by the irrationals. Keeping to this stand Eudoxus's

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[138] Thorup's suggestion may be represented:
For homogeneous magnitudes a, b, c, d, and natural numbers m, n, then:
(A) when, respectively, a, b, and c, d, follow different sequences of reciprocal subtraction they are said to differ in *order*. Equimultiples of a and c, and of b and d respectively, will differ in order.
(B) When the *order* of reciprocal subtraction (ἀνθυφαιρεσις) of a and c differs from that of b and d, the ἀνθυφαιρεσις of (a, b) differs from that of (c, d).
Using A and B to demonstrate that $(a : c :: b : d)$ assume $(a : c \neq b : d)$, then:
(i) from A: $n(a, b)$ and $m(c, d)$, then (na, mc) differs in *order* from (nb, md).
(ii) Then from B: $n(a : b) \neq m(c : d)$.
But as by hypothesis $(a : b) = (c : d)$, $(a : b) \neq (c : d) = \perp$ (i.e., absurd).
Hence $(a : b = c : d) \Rightarrow (a : c = b : d)$.

[139] Knorr's proposal doesn't seem to have been widely taken up: Larsen (1984), Thorup (1992), and A.Jones (*Greek Mathematics to AD 300,* in *Companion I* pp.46-56, 1994) do not mention it.

introduction of equimultiples can be seen as forming the bridge from the 'first order' thinking of ratio theory to the fully realised 'second order' conception of analogy.

2. 8 Aristotle's use of the models of proportion

The importance of these possible aetiologies for Aristotle's thinking is, if they do anything, to increase his range of options. In *Topics* 158b29 he gives the antaneiretic definition, as Knorr thinks he also does in *Physics III* 206b6ff.—and indeed the wording there is strictly antaneiretic[140]. Other passages from *Physics* could be based on any of the models for proportion, e.g., at *IV* 215b-16a17 where Aristotle makes a detailed application of proportion theory to velocity through differing media (water, air, earth). And in *VII* 249b31-250a28 where he relates the quantities of force and motion to distance and time ("A will move B over distance C in time D") he might possibly have relied on an anthyphairetic conception. Aristotle goes immediately on (250a29ff.) to apply the formula to living things: growth will be determined as "A alters B to degree C in time D". This again could have depended on any of the suggested models, as could the argument against the possibility of an infinite body (*On the Heavens I* 272a22-74a13). But the language of yet other passages in *Physics,* such as *VI* 237b31-33:

> For if we take any fraction of the motion the whole motion will be some multiple of that fraction, and the time occupied by the whole motion will be the *same* multiple of the time occupied by the fraction of the motion. Consequently since the fractions are finite both in magnitude and in number the time also will be finite; for it will be a multiple of the time occupied by the fractional motion and it will be equal to that period of time multiplied by the number of fractional motions[141]

anticipate the terminology of the *Elements.* Aristotle here speaks of the equimultiples of finite magnitudes which, although *possibly* borrowed from the very complicated alternative constructions, seem far more likely to have been based on the much simpler, incisive and comprehensive, theory we know from Euclid. And if *Physics VI* assumes the Euclidean model there is no good reason at all for Aristotle to have supposed the more awkward and elaborate preliminary theories in books *III, IV* and

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140 Knorr had earlier (1975, p.285) thought that there Aristotle drew on both the anthyphairetic and the Eudoxian theories. Wicksteed's and Cornford's (Loeb) translation, altered slightly (where they give *proportion* I have put *ratio*: the Greek is τῷ αὐτῷ λογῷ, and later τον λογον) is:

> For if one should take a definite piece away from a limited magnitude and then go on to take away the same ratio of what is left (not the same fractions of the original whole) and so on and on, one will never work through to the end of the original magnitude.

141 237b31-33 (Wicksteed & Cornford).

VII of the same work. The recommendation in *Topics* to study equivalences between the differentiae of both close and remote genera[142] could also *perhaps* have been taken from the earlier models, but it reads much more naturally as benefiting from the Eudoxian theory. The treatment of the winds in *Meteorology III* 362a34-64b22 and the more elaborate use of proportion theory for rainbows at 375b-76b22[143] depend on the principle of mathematical modelling[144], and though they also *could* have been taken from the other theories the complexities of the formulae indicate that this was unlikely. It is far more plausible to suppose that the principle of the mathematical model rested on the general theory. A further extract from *Posterior Analytics,* 85a38 (Barnes):

> for as they go on they prove as in the case of proportion, e.g. that whatever is of such a type — neither number nor solid nor plane but something apart from these — will be proportional,

and one from *Metaphysics M* 1077a9-12 (Ross):

> Again, there are certain mathematical theorems of a universal character, extending beyond these substances .... a substance which is neither number nor point nor spatial magnitude nor time[145]

also read most naturally as drawing on the general theory of proportions, being indications of something like 'real number' (which he might or might not regard as fictitious). Indeed Aristotle's remarks show that the speculation that there might be mathematical entities of such an abstract nature as to parallel what would later be thought of as real numbers arises from concepts introduced by the general theory's revelation of what a mathematical entity might be[146]. And of one of the most vivid uses of the general theory (which occurs in *Posterior Analytics*):—

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[142] 108a6-14 (Pickard-Cambridge):

> Likeness should be studied, first, in the case of things belonging to different genera, the formulae being 'A : B = C : D' (e.g. as knowledge stands to the object of knowledge, so is sensation related to the object of sensation), and 'As A is in B, so is C in D' (e.g. as sight is in the eye, so is reason in the soul, and as a calm is in the sea, so is windlessness in the air). Practice is more especially needed in regard to terms that are far apart; for in the case of the rest, we shall be more easily able to see in one glance the points of likeness.

[143] This corresponds to a lost theorem of Apollonius. Simson worked out the theorem from *ElementsVI*.3; it is given by Heath 1949 pp.181-83.

[144] The principle of mathematical modelling, in turn, follows directly from the later theory. The notion of a model is found in *Elements VI* where Euclid applies the general principles defined in Book *V* to continuous magnitudes. Book *VI* demonstrates the conditions to be met for two geometric figures to be 'similar', and in so doing establishes what it is for one figure to be a model of another. The equivalence is generated (i.e., the model is facilitated) through the ordered sequence of the proportionals. For two triangles *abc* and *def*, $a : b :: d : e :: b : c :: e : f :: c : a :: f : d$. This *can* be performed using anthyphairetic methods, but in an improbably round-about way.

[145] Also 1077b17-19, see § 1.3, note 27.

[146] Knorr had earlier (1975 p.302) thought Aristotle was referring to the Eudoxian theory. Klein (op.cit. p.162) says Aristotle is here "using the model of Eudoxus"; Heath also said (1949 pp.43-44) that it seems clear that Aristotle was already familiar with the general theory

> Why do proportionals alternate? The cause is different for lines and for numbers, and yet the same; different if the lines are considered as lines, and the same if they are considered as exhibiting a given increment. So with all proportionals[147]

it imposes quite a strain to read this as other than drawing from the proportion theory more or less as it appears in Euclid.

Aspects of the use of analogy in the biological works further show a familiarity with the general theory[148]. In his elaboration of the structure of the life sciences Aristotle pioneers the concept of one species being used as a model for another[149]. His use of the principle is taken directly from that of mathematical modelling reproduced in *Elements VI*[150], thus demonstrating his familiarity with that notion.

Together with his book on justice, with which we are primarily concerned, the reliance on the principle of models throughout the biological works and in the *Meteorology*, his remarks in *Physics VI* (and therefore also in *III, IV,* and *VII*), those on mathematics in *Metaphysics* and *Posterior Analytics,* all show a dependence on the general theory of proportions. The research and speculation by historians of mathematics—whether there was a general anthyphairetic theory, or a limited anthyphairetic theory combined with a complex antaneiretic model, or the full Euclidean theory—all demonstrate that *some* comprehensive proportion theory was known to Aristotle. The evidence we have *can* fit any of the models of proportion theory that have been suggested; the only case it will not fit is the absence of *some* general theory (by *general* is meant a formula for every class of magnitude whether commensurable or not). And we should note that the *presence* of either or both of the alternative models is no evidence for the *absence* of the Eudoxian model.

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of Eudoxus, and that he is alluding to it here. Van der Waerden makes a similar point (1954 p.178), as does J.Lear (p.166 of 'Aristotle's Philosophy of Mathematics', in *Philosophical Review* 91, pp.161-91). Lear's position is the reverse of Knorr's; where Knorr thinks the treatment of proportions in Aristotle's time was less general than in Euclid, Lear thinks it was more. Speaking of the ἐναλλαξ theorem (for this theorem, *V* prop. 16, see note 162) Lear says that

> the proof with which Aristotle was familiar probably had a slightly more algebraic character than Euclid V-16.

[147] 99a8-11 (Tredennick). See also Byrne's useful remarks (1997, p.142).

[148] Throughout the biological corpus, but notably at *PA I,* chapter 1.4-5, *II* 653b32ff., and *HA* 486a5-491b26, 588a25.

[149] He does not appear to *state* the principle of a model in the biological writings, even though that is what is assumed for analogical comparisons. *That* a corresponding function holds between genera is what permits a filling-out of the relation between the pairs of analogates.

[150] The geometric continuous magnitudes there he boldly transfers to self-moving (i.e. living) things, which will be comparable either *directly* according to the greater or less, or *indirectly* through analogy.

It has been necessary to examine these recent speculations as they might have revealed a history in which a general theory of proportions did not exist in Aristotle's time; what they have actually thrown-up is the possibility of a wider range of models for Aristotle to have drawn on. If it was the case that Eudoxus produced the intermediate Special Theory, then later the General Theory of Proportions, or if Eudoxus or other mathematicians proved the ἐναλλαξ theorem anthyphairetically, we may read Aristotle as making use of these differing formulations of proportion as they served his own researches. Yet it turns out that not only is there no good reason to doubt Aristotle's knowledge of the Euclidean theory. By positing his knowledge of it a great deal in his work across the several fields indicated above, that would otherwise be obscure, becomes explicable. We must conclude that the comprehensive theory known to Aristotle was the Eudoxian as traditionally ascribed.

2. 9 The general theory of proportions

In *Elements* Euclid gives the general theory in Book *V*. At 5.5, following the definition of ratio, he says:

> 'being in the same ratio' - a first is to a second and a third to a fourth - is predicated whenever, in regard to equimultiples of the second and fourth, according to any multiplication whatsoever, the former alike exceed the latter, or alike equal, or alike fall short, taken in the same order[151]

I.e., for any quantities a, b, c, d, there will be natural numbers m and n:

$$\begin{aligned} ma > nb &\Rightarrow mc > nd \\ ma = nb &\Rightarrow mc = nd \\ ma < nb &\Rightarrow mc < nd. \end{aligned}$$

Book *VII* gives the older formulation, 7.20 (Heath):

> Numbers are proportional where the first is the same multiple or the same part, or the same parts of the second that the third is of the fourth.

[151] Knorr's translation (in Bowen 1991, p.127), slightly altered.
(Ἐν τῷ αὐτῷ λογῷ μεγεθη λεγεται εἶναι πρωτον προς δευτερον και τριτον προς τεταρτον ὅταν τα του πρωτου και τριτου ἰσακις πολλαπλασια των του δευτερου και τεταρτου ἰσακις πολλαπλασιων καθ᾽ ὁποιονουν πολλαπλασιασμον ἑκατερον ἑκατερου ἢ ἅμα ὑπερχῃ ἢ ἅμα ἰσα ᾖ ἢ ἅμα ἐλλειπῃ ληφθεντα καταλληλα.)
De Morgan's gloss on 5.5 ('Proportion', *The Penny Cyclopaedia XIX* p.51) is:
For magnitudes, A and B of one kind, and C and D of the same or another kind, are proportional when all the multiples of A can be distributed among the multiples of B in the same intervals as the corresponding multiples of C among those of D.

It is said that Euclid retained this version of the theory in line with his policy of keeping the work of his originals without undue editing. The Book *VII* formula is given for numbers and the Book *V* formula for magnitudes (i.e. for discrete and continuous quantities separately). 7.20 reflects the older, anthyphairetic solution; it is less general than the other in that since it treats numbers there is no issue of incommensurability. The two presentations are said by all commentators, no doubt correctly, to reflect the arithmetic/geometric distinction. Even so, the formulation for numbers strikingly accords with the three classical means developed from the Pythagorean analysis of strings. (i) The geometric mode of proportion "the same multiple" (the equal *multiple*). (ii) The harmonic mode "the same part" (the equal *division*). (iii) The arithmetic mode "the same parts" (the equal *quantities*).

The force of 5.5 is that quantities of any kind can be related such that, whether or not they share a common measure, the truth conditions under which they fall can be specified exactly. By means of Eudoxus's formula, whether or not a precise direct comparison is available, in all cases a precise and exhaustive *indirect* comparison is. Any kind of quantity is equal to, or greater or less, than some other quantity *of the same kind*; this general fact of quantity *qua* quantity is used by Eudoxus to resolve the problem. The theory recursively applies the ancient polarity 'greater/less'; Eudoxus takes what it is to *be* a quantity (the very capacity to be '<', '>', and so '='), and applies this feature to quantity itself. (Quantity, to which the principle of 'greater and less' is to apply, *qua* quantity itself is the principle of being capable of greater or less—all other attributes having been abstracted.) Just as a magnitude of any kind is comparable with another of the same genus, so now *that* comparison can always be compared to another such comparison (and a *comparison* itself being precisely the application of 'greater and less'). Unlike the antaneiretic or anthyphairetic methods (which count the steps in the corresponding states or processes of interaction), by exploring all the possibilities of the dyad 'greater and less' the general theory supplies the necessary and sufficient conditions for there to be a proportion.

Before Eudoxus mathematicians had sought a common measure to achieve a proportion between pairs of ratios[152]. They looked for it through the subtraction (anthyphairetically) or the division (antaneiretically) of the quantities. Through these processes they aimed to find the measure

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[152] See Byrne's recent (and particularly lucid) account of the Eudoxian proportion theory (1997, pp.137-42).

either *qua* a quantity, or *qua* a quantified number of steps[153]. Eudoxus turned their procedures upside-down; he ignored the *apparent* need for a common measure entirely, and allowed the quantities to measure one another by co-ordinating their multiplication. He thus removed (with what I am calling a 'second-order' concept not requiring the elusive common measure) the supposition that some measure (some x) had to be found to solve the problem. This principle Aristotle applies *directly* to the problem of exchange-value.

Book *V* goes on to specify the patterns in which analogical formulation could be varied. Euclid demonstrates that from $a : b :: c : d$ it follows that the analogates a, b, c, d will be connected by alternation $a : c :: b : d$ (*V* definition 12); inversely $b : a :: d : c$ (*V* def. 13); by combination or composition (*V* def. 14); by separation or division (*V* def.15); and by conversion (*V* def. 16)[154]. Book *VI* applies the principles developed in Book *V* to continuous magnitudes, i.e., to geometric figures (it is here that the principles for mathematical modelling are given). *VI* def. 2 survived only in a form thought to be unintelligible (Heath 1926 II, p.189)[155], Simson substituted a rather loose definition[156] which is met by two formulae, neither of them valid as universal proportions[157]. Though usually cited as the source of the definition of 'reciprocal proportion' ($a : b :: c : d \Rightarrow a : b :: d : c$)[158] by commentators on the *Ethics*, it is not directly generated by Simson's reconstruction. Stewart in fact quotes a rather better formulation[159] of the definition (which applies to equiangular geometric figures not to quantities in general). *VI* propositions 14 and 15 apply the rule to parallelograms and triangles respectively. Although Book *VI* is concerned with geometric figures, it does produce a rule

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[153] In what Fowler (op.cit.) calls adverbial and repetition numbers respectively; see § 1.3.

[154] *V* def.14 $(a+b) : b :: (c+d) : d$; def.15 $(a-b) : b :: (c-d) : d$; def.16 $a : (a-b) :: c : (c-d)$.

[155] (Heath p.189):
Figures are reciprocally related when there are in each of the two figures antecedent and consequent ratios.

[156] Simson (1756):
Two magnitudes are said to be reciprocally proportional to two others when one of the first is to one of the other magnitudes as the remaining one of the last two is to the remaining one of the first.

[157] This definition yields both:
$a : b :: c : d \Rightarrow a : c : d : b$ and $a : b :: c : d \Rightarrow a : d :: c : b$.

[158] *Jackson*, for example, repeatedly in pp.87-99; *Stewart* pp.451-53, 464-65; Burnet 1900, p.223, Gauthier-Jolif, 1959 II p.376.

[159] *Stewart* p.442 (he does not mention his source):
Two sides of one figure are said to be *reciprocally proportional* to two sides of another, when one of the sides of the first is to one of sides of the second, as the remaining side of the second is to the remaining side of the first.

in association with proportions that is of universal application: *VI* proposition 16[160] is generalisable as the biconditional: $a : b :: c : d \Leftrightarrow a \times d = b \times c$. This mutual dependence of proportionality and the product of the reciprocating terms is a much more likely starting point for "reciprocity on the basis of proportion" which interests Aristotle (discussed in chapter 6 below) than the dubious 'reciprocal proportion' that is commonly cited.

There are four features of these elaborations of the formulae which need to be noted here:

(i) that the 'reciprocal proportion' ($a : b :: d : c$) is not the inverse proportion ($b : a :: d : c$)[161]; just what the reciprocal proportion is will be a factor in understanding the theory of exchange-value (see chapter 6).

(ii) Euclid does not mention the reciprocal proportion, only that under certain conditions, i.e., if the quantities belong to equiangular figures, they will have reciprocally proportional parts.

(iii) The formula drawn from *VI* prop. 16 will provide the criteria to be met for a just exchange.

(iv) Although for ratio the terms must be homogeneous there is no parallel requirement for *all* cases of analogy. Homogeneity is a requirement for just those formulae which use the principle of alternation. The adverb ἐναλλαξ was used adjectivally as with "alternate magnitudes" and "alternate angles" to indicate any rule of proportion in which the terms in one ratio may cross-over to the other (and *V*, proposition 16, the *alternando* rule, stipulates homogeneity)[162]. To alternate the terms in the ratios $a : b$ and $c : d$ so that $a : c$ and $b : d$ all the terms must be capable of being in ratio with all the others. (I.e., they must all belong to the same genus.) The rules which operate without the use of the ἐναλλαξ principle, on the other hand, may apply to analogates from other categories. *V* propositions 17 and 18, for example, hold across genera[163] (i.e., there is no requirement for c and d to belong to the same genus as a and b). This feature of homogeneity for only *some* analogies is fully developed in Euclid's *Elements,* and was of significance for Aristotle's account of exchange-value.

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160 See § 6.3 figure 7 and note 382.

161 V.Karasmanis (p.376 of 'The Mathematical Passage in "Nicomachean Ethics"1131b5-15', in *Ancient Philosophy*, 1993, pp.373-378) speaks of the 'reciprocal' proportion ($a : b :: d : c$) as the inverse; see §§ 4.10, 6.3, 6.4, and 6.6.

162 *V* prop.16 (Todhunter):
If four magnitudes of the same kind be proportionals, they shall also be proportionals when taken alternately.

163 From definitions 15 and 14 respectively (see note 154).

Chapter 3

CLASSIFICATION

3. 1 Dichotomous division

Homogeneity figures in Aristotle's use of proportion theory, albeit in a way that is not well understood. But before looking at that, or even at the connexion between analogy and generic sequence, we should bring to mind Aristotle's interest in classification. Or, as that interest might be taken to characterise a great part of the whole corpus, two aspects of classification related to analogy. Of these two aspects one is positive and the other negative, the negative being his response to dichotomous division. That method of investigation followed quite naturally from the polarities outlined in §1.4 and had had something of a stranglehold on philosophy since the time of Pythagoras. Plato began to be dissatisfied with dichotomous division; he recommended that if bisection failed to accord with the "joints in nature" then division should be made into as many parts as would accord[164]. Despite the apparent strengths of dichotomy (seen, for example, in the zeugmatic passage defining an angler in the *Sophist*[165]—the angler is defined through ten dichotomous stages starting from "skill", a high genus[166]) Plato became increasingly uncomfortable with the method (which is demonstrated throughout the *Statesman*).

What was an increasing dissatisfaction for Plato becomes a settled hostility in Aristotle. He has several objections to the method, but the first is logical[167]. To rely on dichotomous division, as had been attempted, to draw inferences or arrive at a definition (or refute an argument or ascertain matters of fact)[168] is question-begging. If it should be asked, for example:—

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[164] The aim was to order the Forms as they really occur in nature (*Philebus* 16D, *Statesman* 287C); *Phaedrus* 265E (Wright):

> by dividing at the joints, as nature directs, and not attempting to break any limb in half after the fashion of a bungling carver.

[165] The sophist himself being metaphorically an angler; the purpose of division is to 'hunt down' exactly what a subject is.

[166] Skill (τεχνη) is divided into that of making or of acquiring; acquisition then by consent or capture, capture into open or secret. Secret, i.e., hunting of inanimate or animate things; animals are then divided into terrestrial or not: the non-terrestrial into bird or fish. Fishing takes place either with nets or by a form of striking; striking by night or day. In daylight either from above or below. From below by a barbed hook at an angle into the mouth; hence "angling".

[167] *Pr. An. I* 46a31ff; *Post. An. II* 91b12ff.

[168] *Pr. An. I* 46b26.

"Is man animate or lifeless?" The definer assumes that man is an animal; he has not proved it.[169] All substances are here assumed to be animate or inanimate. Man is reasonably taken to be a substance; it does not follow that man is animate but merely either animate or inanimate. Similarly with any predication, e.g., to establish if man is footed (or wingless, blooded, rational, or mortal) instead of narrowing down a major premiss, what would be the middle term of a syllogism linking the major to the minor premiss is widened into a universal disjunction: all animals are either footed or footless, winged or wingless, or whatever the predicate. The point at issue—being footed or wingless or whatever—is always merely assumed, never derived.

Division does have a use but plays "*only* a small part"[170], that small part is to help set out the order of things to be investigated. Having decided (in the *Analytics*) that dichotomous division is logically incapable of defining a subject or determining a matter of fact, Aristotle has no difficulty in showing its ineffectiveness for the classification of animals. Continuous bisection (in effect ἀνταναιρεσις as applied to zoology) does not follow the joints in nature at all. Dividing animals, for example, into *terrestrial* and any one other class (*aquatic,* say), leaves out of account another group of equal standing (*birds*). Yet dichotomous division goes much further than mere bisection, it selects some highlighted attribute which it then bisects either through privation or into opposites. The opposition will often be no more than privation, or come very close to that principle, e.g., in oppositions such as *white/black*[171] and *whole/split*, the *black* may be the absence of light, and *split* (as with cloven-hoof) the absence of wholeness. Other opposites, such as *left/right, straight/curved, heavy/light,* or *hard/soft* may indicate something more than the absence of a property, but little can be done with them as headings for further division (few subordinate groupings will be only on the right, or only curved, or light, or soft[172]). Aristotle goes on to reject privation: the *lack* of a property does not create a positive life-form[173]. The mere absence of an attribute does not necessarily count as

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[169] *Post. An.* 91b18.
[170] See Ross's comments in his edition of *The Prior and Posterior Analytics of Aristotle*, p.398; his italics.
[171] *PA I* 643a22.
[172] *Hard/soft*, however, was to some extent a usable opposition: aquatic creatures were divisible into the *hard* and the *soft*. The 'soft' being the cephalopods, distinguished from other molluscs. The *hard* were further divided into those hard on the outside and soft within (crustaceans) and those soft outside covering a hard structure (vertebrates).
[173] *PA I* 642b23 (Ogle):

There can be no specific forms of a negation.

There are discussions in the *Categories* 11b onwards, *Topics* 143b33 and *Metaphysics Δ* on the senses in which it is appropriate to speak of privation.

privation, privation is the lack of some attribute natural to a subject area. Being feathered, for example, is a property of certain bipeds and of certain aquatic animals, so it might be said to be natural to such life forms, but the featherlessness of monkeys and jelly-fish is not a useful truth to know about them: there is nothing about featherlessness to contribute to the understanding of their essences or definitions, or classification. (Very broadly, species are not defined or arrived at through the absence of some property.)

Dichotomous division also, rather than providing a mechanism which uniquely isolates or classifies essential features, results in the same creatures falling into several classes. Balme thinks this is what Aristotle regards as the outstanding defect of the method[174]. The Academics had not presented just any differentiae to be dichotomously processed, but tried to find which of the important requirements should be primary. They hoped that some essential attribute—such as movement—would be the key property from which to derive the others. The chosen differentia was then sequentially divided to reach the individual species. Aristotle's method is to concentrate on the functions vital for an animal to *be* an animal, such as respiration, locomotion, growth, or reproduction. He crucially perceives the most important requirements *as* functions, and from among these vital functions he opposes the Academic attempts to, as it were, choose a first among equals[175]. He demonstrates how it is impossible to derive the other defining attributes from any one characteristic, however important. If locomotion, for example, is chosen as the prime function then animals will be divided along the lines which implement locomotion: into the footless and the footed; then into bipeds and the many-footed; the many-footed into quadrupeds and any others. We then find that (*GA II* 732b15-24; Platt):

> These classes admit of much cross-division. Not all bipeds are viviparous (for birds are oviparous), nor are they all oviparous (for man is viviparous), nor are all quadrupeds oviparous (for horses, cattle, and countless others are viviparous), nor are all viviparous (for lizards, crocodiles, and many others lay eggs). Nor does the presence or absence of feet make the difference between them, for not only are some footless animals viviparous, as vipers and the cartilaginous fishes, while others are oviparous, as the other fishes and serpents, but also among those which have feet, many are oviparous and many viviparous, as the quadrupeds above mentioned.

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[174] D.M.Balme, e.g., in *Aristotle's Use of Division and Differentiae*, in *Gotthelf & Lennox*, pp.69-89.

[175] For a very different reading, however, see P.Pellegrin *Logical Difference and Biological Difference: The Unity of Aristotle's Thought* (ibid., pp.313-38, and *Aristotle's Classification of Animals*, 1982 (trans. A Preus, 1986).

And so on and on throughout the animal kingdom. For such reasons as these Aristotle rejects dichotomous division; the method does not respect natural divisions nor does it yield the classes of living things to be found in the world. He concludes[176] that some more positive method of research is necessary, which would be both accurate and comprehensive.

3. 2 Hierarchy

Aristotle's more positive approach to classification required an expansion of the concept of generic order. Plato introduced this principle of classifying into γενος and εἶδος to the Academy, and it became a major theme of study (as fragments from Speusippus's *Homologies* testify)[177]. As we see with small children there are two primitive impulses towards classification: to pull things apart and to put them together. These impulses may evolve into sophisticated procedures of analysis and synthesis—or as Plato puts them into συναγωγη (collection) and διαιρεσις (division). Although he mentions συναγωγη[178] Plato gives much more attention to διαιρεσις[179], but the beauty of a fully developed generic classification is that it respects both these opposing tendencies: it aims to bring unity and diversity into a comprehensive whole. Whilst rejecting the Academic orthodoxy of defining an εἶδος through the dichotomous division of the γενος Aristotle took over the programme of devising a generic system. His approach (I think it can be said) treats collection and division more even-handedly[180]; he focuses as much on the collective principle for εἴδη and γενη as their separability. Aristotle's starting point, as we have just seen, is the observation that living things are not characterised, nor can be defined, by any one pre-eminent function, but by several functions

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[176] *Pr. An. I* 46b36 (Jenkinson):

> It is clear then that this method of investigation is not suitable for every inquiry, nor is it useful in those cases in which it is thought to be most suitable

and (*PA I* 644b19, a11, Peck):

> dichotomy is either (a) impossible or (b) futile ... so it is impossible for those who follow the method of twofold division to arrive at any of the particular animals.

[177] It is said (by Balme for example in *De Partibus Animalium I, De Generatione Animalium I* 1972 pp.101-105) that neither Plato nor Aristotle were primarily interested in classification, but rather to account for phenomena. The first requirement would then be correct definition not placement in a rigid order. Classification however is not taxonomy, but a heuristic tool; accounting for phenomena will require their placement in relation to the investigator and to other phenomena.

[178] In *Philebus* 18C, *Phaedrus* 249B, 265D, *Sophist* 226A, 267A-B but particularly 265D.

[179] In *Phaedrus, Sophist, Statesman* and *Philebus*.

[180] Aristotle does not oppose division *per se* but rather dichotomy; according to Diogenes Laertius he wrote a book *On Divisions* (see Ross, *Select Fragments*, p.105).

collected together[181]. It is essential for any living creature to move, for example, and to grow and obtain nutrition; it is through the simultaneous division of the collection of such vital functions as these that the accurate and comprehensive account of living things is made possible[182]. (Aristotle applies his much more developed conception of generic order not only to the life-sciences but also to his other fields of interest, including the theory of justice.) To make his system more effective he inspires generic order with two important principles: teleology and the Eudoxian proportion theory.

Before considering the rôles these ideas play we need to outline those features of the generic hierarchy most closely tied to analogy. A caveat must be entered here: Aristotle also uses the terms γενος and εἶδος non-hierarchically. Deciding how he is using the terms at any given point is controversial; I will join the controversy in the next section. For the hierarchical use of the term an account is given in *Metaphysics Δ* 1024a29ff. of what a γενος is. It is

(i) a continuous series of things of the same form.
(ii) The source, or prime-mover, or unifying principle of (i).
(iii) That which underlies, or is common to (i).
(iv) The first, logically speaking, of a series of descriptors which is stated as part of the essence (and where the terms which follow are the differentiae).

Already, without going any further, it is plain that the subject of γενος (like γενος itself) will generate an extensive issue. For its rôle in the theory of justice, and the connexion with analogy, however, we may limit attention to certain minimal aspects of the use of the term. When speaking of generic order we find that certain notions recur and interact, these include not only γενος, διαφορα and εἶδος, but universality and individuation, and the principle of the level they occupy within a hierarchy.

The individual object—pre-eminently from the separate items of every-day experience: *this* pen, *that* chair, *Fido*—is what primarily is meant by substance. In a perhaps weaker sense that which is represented by the subject rôle of a sentence is also treated as a substance: it is to the extent to which something is individuated and predicable that it is the substance. Whatever takes up that position is to be distinguished from its predication or attributes as nouns are from adjectives. The individuated thing might be the individual object, or a differentia of such an object, or the form it

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[181] *PA I* 644a4:
No single differentia, I repeat, either by itself or with its antecedents, can possibly express the essence of an eidos.

[182] *PA I* 645b5-28.

shares with other objects, or the class under which the forms and differentiae fall. Whatever the subject is, at whichever level, *qua* the subject it is not to be confused with its attributes.

The individual object shares (in most cases) a common form with certain other objects. The form is common, i.e., universal for *that* level of classification. It defines the essence[183] the individuals have in common and what an individual item is taken to be (it is *a* pen, *a* chair, *a* Spaniel). The form shows both what an object is and what it is not: it is a structure which individuates the items, not from other items, but from other forms[184] (the pen is a pen not a pencil, Fido is a Spaniel not a Labrador). This essential, individuating, defining form is the εἶδος.

What it is that is structured is the range of properties (or attributes or qualities) known as διαφοραι (differentiae) shared by all the individuals of the same γενος. The origin of the term διαφορα was 'moving hither and thither'[185]; and Furth sources its logical root as 'being different from' (and this remains its most telling sense)[186]. The rôle of differentiae *qua* differentiae is adjectival, unlike γενη and εἰδη which will be the subjects the differentiae qualify. (This grammatical point will be of significance for deciding the forms of justice.) The range of features that will differ from other features is arranged into the differing structures (the εἰδη)[187]. The εἰδη are structures *of* something in two senses: they are structures composed *of* their differentiae (as Spaniels are composed of blood, bone, muscle, hair ...), and they are forms *of* some general kind (Spaniels are sorts of dog, pens are sorts of writing implement, chairs of furniture). What characterises a γενος (which covers all the differentiae that combine into the εἰδη) is that it is the widest appropriate class in any given inquiry to which an object will be assigned. The key term here is 'appropriate' as, in the process of definition, the assignment must be to the nearest available (relevant) γενος. In making this point at *Topics* 143a15 Aristotle gives the example of justice. Justice should not be defined merely as *some* state which produces equality, or distributes according to the appropriate mode of

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183 See Balme pp.296-98 of *Aristotle's Biology is not Essentialist* in *Gotthelf & Lennox*, pp.291-312 or M.Furth (*Substance, Form and Psyche: An Aristotelian Metaphysics*, 1988).

184 Which echoes Plato, e.g., in *Theaetetus* 208C-D.

185 *Liddell & Scott.*

186 1988, p.101; Furth cites *Theaetetus* 208C. In the attempt to define knowledge as "a correct belief coupled with an account" Socrates mentions:

> being able to name some particular characteristic which marks off the thing in question from everything else.

187 The arrangement may be in any number of ways; in addition to the arrangements of the organs of animals into their characterising structures *Metaphysics H* 1042b15 gives e.g., blending, tying, gluing, nailing, and by position or location in time or space.

equality, but is a higher γενος of the highest γενος *virtue*. Leaving out of the definition that justice is a virtue would omit what is most important about it.

The differentiae of a γενος are, as it were, a common pool from which the forms take the properties they need[188]. It should be noted that the full range of differentiae will not usually be present in each εἰδος. One sort of bird may have a spur or crest[189] or marking absent from others of the same γενος. Εἰδη *as* εἰδη contain only the essential features which define an object; the differentiae used in the definition being those necessary for the thing to be the thing that it is[190]. *Qua* εἰδη they are concerned only with such essentials (essential differentiae being those which implement the vital functions)[191]; non-essential properties (i.e. accidents) falling within the γενος may be essential differentiae for other εἰδη of the γενος. Aristotle uses a favourite example of the triangle to illustrate the nature of a form: in all triangles the angles add up to two right angles. Hence this fact is a property of triangles *per se*. Even so, it is a *per se accident* and not of the essence of a triangle, in that we do not need to know it to know that a figure *is* a triangle[192]. (But adding up to two right angles could be an essential attribute of some other pattern of the γενος *plane figure* to which triangles belong.)

It is evident that several levels of generality of forms and kinds might be invoked for any given sequence of classification. Series of generality will reach a highest γενος, which is the widest category to which an individual item, or individuated class, can usefully be said to belong. Highest γενη would be e.g., *substance, relation, living (thing), virtue:* an individual dog Fido might be a Spaniel, which is a breed, a sub-εἰδος, separable from the other sub-εἰδη of the εἰδος *dog*, which is

[188] Γενος can also be thought of as equivalent to category or predicate (and it is mentioned in this sense in *Metaphysics Δ* 1024b13): it is the category to which the parallel forms belong, and the *most* that can be predicated of an individual object. (Arisotle's word for predicate is often 'category', e.g., in *On Interpretation* 21a9.)
[189] *HA* 486b10.
[190] *PA I* 643a27.
[191] Aristotle speaks of the "differentia of the differentia" (in *Metaphysics Z* 1038a8 and *PA I* 643bff. for example) these are usually essential differentiae which qualify an already qualified γενος. If the γενος is (say) *animal*, a subaltern γενος might be that γενος qualified by the highest level differentia which implements a vital function (e.g., locomotion). *Footed* would be such a high level differentia (since being footed enables certain modes of locomotion). The differentia of the differentia then will be an attribute which relates to feet (such as *split* or *quadruped*), not to some other organ, which would implement a different function (see §§ 5.5 and 5.6 for the distinction of εἰδος from διαφορα in the classifications of justice).
[192] *Metaphysics Δ* 1925a30, *E* 1026b12, *PA I* 643a30. A zoological parallel is found e.g., in *GA I*. See also Gotthelf *First Principles in Aristotle's* Parts of Animals, in *Gotthelf & Lennox*, pp.167-98.

individuated from the other εἰδη of the γενος *terrestrial quadruped*, which might in turn be isolated from the other γενη of the higher *mammalian* γενος, which will then be separated from the other higher γενη of the even greater γενος *animal*, which could be individuated from the other greater γενος (*plant*) of the highest γενος *living thing*. An exactly corresponding generic sequence is employed for the layout of the subject of justice in the *Ethics* (see § 4.4).

3. 3 ΓΕΝΟΣ and ΕΙΔΟΣ

Aristotle's use of the machinery of γενος and εἶδος is so extensive, especially in his books on the life-sciences, that until quite recently he was assumed to be proposing the taxonomy of living things[193]. Read as taxonomy his references to γενος and εἶδος were bewildering, with classes of animals cited without apparent consistency. For example in *History of Animals* 523b14 he speaks of two *species* of cicada, than at 535b of one *genus* of cicada, and at 556a30 he says there are two *genera* of cicada. He also refers to a γενος of ox which has a bone in the heart, where there are no species (*HA II* 506a8). In *Generation of Animals* 719a7, 11, remarking that the uterus in ovoviviparous animals such as cartilaginous fishes (such as sharks) and vipers differs from both vivipara and ovipara "because they participate in both *species*", he then (751a28 and 754a20) refers to the *genus* of oviparous fishes[194]. But through writers such as Balme and Pellegrin it has become clear that Aristotle's references are not taxonomic[195]. We should not read *his* use of γενος and εἶδος as the biological *genus* and *species* (this is why I have avoided these terms in the foregoing paragraphs). Aristotle applies the terms at any level of generality; they do not (or need not) fasten on to a taxonomic series in which Fido is a King Charles, a sub-subaltern class of Spaniel, which is a breed (an artificial selection) from the Familiaris species of the canine genus of the carnivorous family of the placentalian order of the mammalian class of the vertebrate phylum of the animal kingdom.

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[193] See Balme pp.80-81 of *Aristotle's Use of Division and Differentiae* (*Gotthelf & Lennox* pp.69-89), and J.Lennox (*Kinds, Forms of Kinds, and the More and the Less in Aristotle's Biology*, ibid. pp.339).

[194] Translators had long been embarrassed by such apparent inconsistencies; D'Arcy Thompson omits a group reference here entirely, and Platt writes "both classes".

[195] From Balme's 'ΓΕΝΟΣ and ΕΙΔΟΣ in Aristotle's Biology', *Classical Quarterly* n.s.12 pp.81-93 of 1962 onwards, and Pellegrin 1982.

Since the work of Balme and then Pellegrin it is widely accepted that when speaking of γενος and εἰδος Aristotle is using the terms indifferently as designating one group or another, just as we would say "kind of hawk" or "sort of bird" without suggesting that hawks form the wider class containing birds. Yet when the terms occur together their use is strictly hierarchical; the γενος-εἰδος relation is invoked as a significant and rigid structure to inform any given inquiry. Although they may well be used at any level of generality (so that the γενος spoken of might be *winged, feathered, bird, predator,* or *animal*) the εἰδη will always be sorts or forms within whatever serves as the γενος. The question whether there is semantic significance for the other separated uses of the terms other than as markers for groups is still controversial. Pellegrin believes (1987) that Aristotle speaks of γενος to indicate phenomena viewed as a class ready for division into sub-classes, and εἰδη as the outcomes of the division. The phenomena treated as γενη will be perceived as being distinct from other whole classes; the objects (which could be the very same phenomena) viewed as εἰδη will be considered as parallel forms within some broad category[196].

I think there is something right and something wrong about this reading. The first half of it is simply wrong; Pellegrin just ignores passages such as *HA II* 506a8 where Aristotle refers to the genus of ox that is not divisible into sub-groups, but is an ultimate species. Plainly there are no infima species and the class of ox is the outcome of some broader division. The second part of the interpretation fares better; Aristotle does seem to use γενος when separating the subject from all other groups (perhaps relying on the old *same/difference* polarity). The difference of the subject from other classes comes to the fore, with any samenesses (equivalences) pushed to the background. Conversely, with the term εἰδος Aristotle might be focusing on the equivalences among the phenomena; their differences will then be relegated. The class of ox, even though it is not a group to be further divided into species, would, on this interpretation, be viewed *qua* distinct from other classes of animal.

Since Aristotle builds his theory of justice on the generic order (to be argued later), what we need to be clear about is (i) that when used in association the two terms γενος and εἰδος form a mutually rigid structure. (ii) There is great flexibility in the scope of application of the generic order; (iii) otherwise the terms designate any class, with (iv) the proviso that perhaps γενος will

[196] See also Balme's exposition in *Aristotle's Use of Division and Differentia* (op. cit.).

emphasise separation from other whole groups, and εἶδος parallel classes of animals within some overall kind.

3. 4 Heterogeneity

Having taken a brief look at generic structure we need to examine the connexion between that structure and analogy. It is undoubtedly the great strength of proportion theory that it enables precise and exhaustive comparison to be made between elements remote from each other, and from widely separated fields. It is tempting to suppose that in the biological works at least Aristotle treats proportion as exclusively heterogeneous. And Pellegrin, who has argued convincingly for the non-taxonomic reading of Aristotle's use of the generic structure, fixes analogical inference at the level higher than the genus[197] (e.g. 1982 pp.56, 72, 88-89, and 1987 pp.329-30). He writes:

> Above the generic gap, analogy can establish relationships between *heterogeneous* beings, permitting propositions of the type: "that which is (a) in *genos* A is (b) in *genos* B".[198]

even though this is no more than just one of the many permutations of the analogical formula. Such a severe restriction inevitably reduces the power of analogical inference. Pellegrin then further distorts the picture by interpreting his reading of the formula to mean:

> what nail is for *genos* A is hoof for *genos* B.[199]

He takes this interpretation of an already unnecessarily restrictive formulation to be the paradigm for Aristotle's understanding of analogy. Yet if we look at the passages from which Pellegrin's example is taken they do not in fact treat the 2nd and 4th terms as γενη, with the 1st and 3rd as 'parts'. Even allowing for any degree of looseness in γενος-εἶδος relations, nail and hoof in Aristotle's examples do not relate to γενη, but to particular animals. However, putting that misreading to one side, treating the analogical formula as representing no more than something (a differentia or 'part') being contained in one γενος in relation to something contained in another (which Pellegrin has represented as $a : A :: b : B$), is not only *not* Aristotle's sole conception of analogical inference, it is not even his usual. In § 2.8 above we found that Aristotle uses analogy throughout the corpus, but

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[197] For my discussion of Pellegrin in the following pages I am much indebted to R.M.White's forthcoming *The Rôle of Analogy in Aristotle's Biology*.

[198] 1982 p.72, with references to the discussion of 'form', 'privation', and 'matter' in *Metaphysics Λ* 1070b17.

[199] Ibid. p.89.

commonly, i.e., in *Topics, Physics, Posterior Analytics, On the Heavens, Meteorology,* and various sections of the *Metaphysics*, the uses cannot be made to fit the structure which Pellegrin construes from the biological works (and from some passages of the *Politics*[200] and *Metaphysics*). Neither can it fit uses in the *Ethics, Rhetoric*[201] or *Poetics*[202]. As Pellegrin holds (rightly I think) that there is no hard division to be made between the biological and the logico-metaphysical uses, there can be no question of isolating Aristotle's zoological reading of analogy from the rest.

At the very least, in extending the principle of analogy to non-strictly-mathematical fields, Aristotle is aware of *some* general principle (§§ 1.8, 2.5, 2.8. above) to the effect that analogy applies (i) to 'any magnitudes whatsoever', (ii) distinguishes heterogeneous from homogeneous cases[203], and (iii) generates a range of permutations. As we saw at § 2.9 several variants of the analogical rule were recorded by Euclid; how many of them were familiar to Aristotle cannot be known, but he must have known some of them. And although the formula $a : b :: c : d$ may well be legitimately interpreted as $a : A :: b : B$, that is not in fact the way Aristotle himself typically uses it. Nor does such an interpretation benefit from the moral he draws from the mathematics of proportion theory that all phenomena, be they γενη or εἰδη of any level, or differentiae or 'parts', *just in case they are quantifiable* are candidates to be represented by the terms in a proportion.

Pellegrin's doctrine that analogy must be heterogeneous[204] also conflicts with Aristotle's explicit statements that some analogies take place within the genus. The ἐναλλαξ principle (which stipulates homogeneous proportion) does not appear to have as ready a use in zoology as it has in

[200] *I* 1252aff.; *IV* 1290b25ff.

[201] 1407a11ff. The genus covering the whole metaphor is that of gods and their paraphernalia. The sense of "cup of Ares" is achieved intuitively (there is a particular resonance in that the Dionysian cup was a wide, flatish bowl). If a = *shield*, b = *Ares*, c = *cup*, d = *Dionysus*, then: $a : b :: c : d \Rightarrow a : d :: c : b$ (i.e., *shield : Dionysus :: cup : Ares*). This is not, as we saw, a true proportion, but applicable only in the cases of figures such as equal triangles as in the reconstructed definition *VI*.2, and propositions 14 and 15 (see § 2.9). The linguistic trick works because the elements are conceived as homogeneous—that is why they can be swapped over. The analogical basis of the metaphor cannot be $a : A :: b : B$, as that format stipulates two genera. (Finding the rule whereby d is assigned to a (and c to b) is the task of *V* chapter 5.)

[202] 1457b16; Aristotle contrasts the analogical relation with the immediately preceding γενος-εἰδος relations.

[203] To be argued in the following two sections.

[204] Which has been accepted by other writers; Lennox, for example, writes (p.358, op.cit.):
Below the level of analogical likeness every *eidos* of a *genos* may serve as a *genos* for further division.

other fields, but Aristotle relies on it in *Posterior Analytics* (see §§ 2.8 and 2.9 above), the *Ethics*[205], and the *Rhetoric*[206]. Pellegrin himself cites two of the works (*History of Animals* and *Metaphysics Δ*) in which Aristotle says that analogy occurs within the genus. The extract from *History of Animals* is a little unclear, but Pellegrin agrees that the text states the relevant point that the analogical relation holds between εἴδη not γενη:

> In some animals the parts are not of the same *eidos* and do not differ by excess and defect, but by analogy.[207]

Of the *Metaphysics* passage, *Δ* 1016b31, (as quoted by Pellegrin):

> Things that are one in number are also one in *eidos*, while things that are one in *eidos* are not all one in number; things one in *eidos* are all one in *genos*, while things one in *genos* are not all one in *eidos* except by analogy; while things that are one by analogy are not all one in *genos*.

Pellegrin indeed points out the common error of supposing the analogical "one" here to be extra-generic, whereas the text reads "things one in *genos* are not all one in *eidos*, except by analogy" (i.e., that εἴδη not γενη are here analogically related). These passages all contradict his claim that analogical inference takes place only across, and not within, genera. Pellegrin's response is that there must be some "involuntary lapse", agreeing with Ross's view[208] that it is by "mere inadvertence" that Aristotle places the analogical relation where he does. Pellegrin says that

> the passage says that even a relationship at the level of the *eide* of the same *genos* can be said to be analogical. But this statement is terminologically un-Aristotelian, since the relation is between *gene*.[209]

An insistence not only flatly contradicted by Aristotle but one which leads Pellegrin into an extremely distorted interpretation of the texts. In 1982 p.89 (referring to *PA I* 655b2) he says that nails, hoof, claws, horns, and beaks are:

> considered as *eide* of the *genos* "organs of defense", even if Aristotle does not say so explicitly,

and that *qua* organs of defence:—

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[205] Book 5 ch. 5.

[206] 1407a17 (Freese):
But in all cases the metaphor from proportion should be reciprocal and applicable to either of the two things of the same genus.

[207] 486b17 as given by Pellegrin. The unclarity of the text is more evident in Ogle's wording: Once again we have to do with animals whose parts are neither identical in form nor yet identical save for the differences in the way of excess and defect: but they are the same only in the way of analogy.

[208] Pellegrin 1982 p.93; Ross *Aristotle's Metaphysics*, v.1 p.305, 1924.

[209] 1982 p.94 (repeated almost exactly in 1987 p.330).

nail and hoof are no longer analogous.[210]

This construal misses both the letter and the spirit of Aristotle's treatment of zoological comparison. Being analogous is exactly what Aristotle says they are. Beaks differ from other beaks by the more and the less, as do horns, claws, and nails from other horns, claws, and nails respectively. But claws do not differ from beaks by the more or the less. They are comparable analogically, and only insofar as they perform corresponding functions[211].

3. 5 Teleology

The 'parts' of animals are not, of course, determined as inanimate things might be (by dividing into half, third, 5/8, and so on), but are recognised *as* parts in terms of their functions. The parts of animals are in Aristotle's terminology the instruments[212] which carry out the functions that exist "for the sake of the whole"[213]. The need each whole animal has to perform the several functions vital to its being an animal (and being the specific animal that it is) gives rise to (and explains) its differing parts. The parts all exist to serve functions such as respiration, nutrition, growth, defence, reproduction or locomotion[214]. In the classification of life-forms it is hardly possible to separate the rôle of teleology from the connexion between analogy and generic order[215]: it is just because a

[210] 1987 p.329.

[211] Insisting that analogy can operate only at the level higher than the genus also has the complicating (but not insuperable) drawback of what is to count as the genus. It also runs strangely counter to Pellegrin's celebrated work in finally liberating Aristotle's classificatory machinery from a rigid taxonomy. Since genera are not fixed, what might count as extra-generic is not fixed either; this can be overcome by determining that whatever the scope of the genus, analogical comparison takes place at a higher level. Pellegrin makes just this point in 1987 pp.328-29, but this banishes analogy to the fringes of the classificatory scheme and does not gel with Aristotle's extensive reliance on it.

[212] More exactly the parts are divided (*HA I*, 486a1ff.) into the uniform tissues (such as blood, flesh, bone, hair) and the non-uniform organs (such as hand, foot, lung). Only the latter are spoken of as "instruments" (*PA* 647b22ff.—ὄργαν, the Greek for instrument). Certain parts, such as the heart and veins, have the character of either tissues or organs (*PA II* 647a31, b18).

[213] When speaking of analogy Pellegrin does not particularly distinguish the organs from the functions they implement. In 1982, p.71 he speaks of the vital functions as "characteristic properties (nutrition, sensation, movement)" yet he has just spoken of the organs which carry out these functions *also* as "characterized by a property ... having wings". Referring in the same passage to *both* the principle of locomotion and the instrument which performs it as "properties" would not be so misleading were it not for the failure to differentiate the rôle of function from the other elements present in an analogical comparison.

[214] *Politics IV* 1290b25.

[215] No grander claim for the meaning of teleology is being made. Notions of purpose and function assigned to non-conscious entities, and to Darwinian evolution, are hotly debated. The point being made here is that Aristotle accounts for the parts of animals in terms of the ends they serve.

comparison is aimed towards some end that analogy is useful. There is an especially illuminating passage at *PA* 681b12-30 (Peck):

> But there is yet another part which every animal must have ... these creatures must have some part which is analogous to the parts which in blooded animals are connected with the control of sensation. In the Cephalopods this consists of a fluid contained in a membrane ... An organ just like this, also called the *mystis*, is present in the Crustacea ... the *mystis* occupies a place which corresponds exactly with the heart in blooded creatures: which shows that it is the counterpart of it.

The organ in fact corresponds to the liver, not the heart. The error in observation *is* an error exactly in virtue of there having to be analogues of the heart and liver in creatures of other genera. That we recognise it to be a mistake confirms Aristotle's model for the analogical correspondence of equivalent functions.

Remote elements *can* be 'analogically' aligned in any old way: the beak of the Occelated turkey might be narrower than the reed bunting's by a measure *exactly* equal to the degree by which the blood of the antelope coagulates less firmly than that of sheep. But the equality of two such measures is of no interest, even to men in anoraks. What decides the futility of such a comparison is not the remoteness of the analogates but the lack of a common purpose. There must be a point to the exercise; for Aristotle the point is always in the way animals in each genus carry out the same function. Locomotion, for example, is achieved in one genus by flying, in another by swimming, and by undulating, walking or creeping in yet others[216]. In terms of analogy these several actions are regarded as the same process, and without the functional correspondence comparison of the organs which implement locomotion would be a mistake (as with the *mystis* and the heart), or at best (as with the beaks and the blood) otiose. The implementing organs may all differ in form, and not only in genus—depending where the genus is drawn (which depends in turn on the particular question being asked[217])—yet are to be compared indirectly, i.e., not directly according to the greater or less of their differentiae, but by analogy.

Pellegrin believes Aristotle uses analogy to:—

> relate one group of animals to another, and ultimately ... to ... Man.[218]

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[216] *PA I* 639b3.

[217] In *Metaphysics Δ* 1016a20 for example: wine, oil, and water, might be treated either as distinct genera or as species of one genus, liquid.

[218] Ibid. Pellegrin cites G.E.R.Lloyd (*Science, Folklore and Ideology: Studies in the Life Sciences in Ancient Greece,* section 3: 'Man as Model', 1983) as supporting a similar view.

Such a programme need not conflict with its teleological rôle. It employs the principle of the model but still requires the corresponding organs in the other animals to be understood as the corresponding parts because they implement the same functions (of movement or reproduction or whatever). But he says that Aristotle's use of analogy:

> is not limited to the morphological or functional domain[219]

yet beaks and claws are comparable analogically only insofar as they are organs of defence, but insofar as the organs also perform other, different, tasks (feeding on the one hand, and locomotion on the other) they are not comparable at all, either by analogy or by the greater or less. Since no non-functional explanation of analogy is given, and functional priority denied, it seems reasonable to suppose that Pellegrin conceives analogy to be what (virtually) all other writers of the post-mediæval world take it to be: as an ordering of resemblances. In case it should be denied that he relies on the modern conception Pellegrin scotches any such disclaimer: he says that analogy is not limited to the morphological *or* the functional *or* the psychological. Hence he relies on some principle of ordered resemblance as prior to, and independent of, function. Yet for analogy to work towards the understanding of animals the organ in one creature must perform the task which corresponds to the equivalent process in another, *not* that the organs should merely look alike[220]. So the use of analogy relies *in practice* on the priority of function—albeit in Pellegrin's case through the back door. (If function is set aside, how it is that good analogies are to be distinguished from bad is left radically unexplained.) But through the front door Pellegrin only admits function as being no more than on a par with morphology and psychology. His lack of recognition of the primary rôle of function falls in with the modern notions that analogy is the creation or explication of resemblance independently of function.

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[219] 1987 p.330.

[220] It often happens that similarities flow from the analogical correspondence; e.g., there will often be certain regularities in the position of organs (a fact which misled Aristotle about the *mystis*). The resemblances are the results, or symptoms, of the analogical relation, not the cause. That one element may resemble another is for Aristotle strictly irrelevant if they do not relate functionally. The post-mediæval conception of analogy as a resemblance relation infects most modern approaches to its use (§ 1.9 above) but it has no place in the structural and relational understanding of classical analogy.

3. 6 Resemblance

I can only suppose Pellegrin supposes, but Olshewsky declares:[221]

> The thesis of this paper is that [Aristotle] treated *analogia* as a device for the explication of resemblances.

As we saw in chapter 1, despite the warnings of Kant and Berkeley, the idea that that is what analogy is has become completely established. Even though (I repeat) the classical conception was both more powerful and more exact, there can be no complaint against the current notion *per se*: it is what it is. The error here has been to suppose that the one notion is the other. The presupposition that analogical inference seeks "an imperfect similarity of two things" (§ 1.9), which Kant exposed as just what classical analogy does *not* do, has been damaging to the attempts to explain Aristotle's use of analogy. When Olshewsky (p.6) says

> [Aristotle] seeks an analogical identity across categories in order to explain resemblances,

referring to the passage at the end of *Metaphysics* (*N* 109b317ff., Ross):

> For in each category of being an analogous term is found— as the straight line is in length, so is the plane in surface, perhaps the odd in number, and the white in colour,

his explanation is at total variance with the written passage. There is *no* resemblance between the odd (rather than the even) and a level surface. White does not resemble what is straight any more than jonquil and viridian are similar to what is bent, or puce curved. Aristotle's point is diametrically opposite: that there may be important characteristics held to correspond through the analogical formula despite the total lack of any resemblance whatsoever. The salient feature of Aristotle's use of analogy in the life-sciences is precisely its ability to locate this equivalence of function, whether any resemblance can be found or not[222].

Regarding the generic status of analogy Pellegrin's approach is the reverse of Olshewsky's; Olshewsky sees analogy for Aristotle as operable strictly within a genus:—

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221 T.M.Olshewsky, p.1 of 'Aristotle's use of *Analogia*', in *Apeiron* vol.2, pp.1-10 (1968).

222 When we say that one thing resembles another it is in virtue of this or that respect(s). When one thing does not resemble another it is always possible to contrive some respect in accordance with which a similarity can be found. Many (usually feeble) jokes are of just this sort ("Why is the moon like a bottle of Guinness? Because ...."). Where everything is allowed to be similar to everything else the principle of resemblance loses its use. In an earlier section of the *Metaphysics* (*Λ* 1070b17) Aristotle analyses phenomena in terms of *form, privation,* and *matter*, as these principles will hold across the categories in spite of the absence of shared, not because of the possibility of occult, properties.

> in order for *analogia* to reveal knowledge, the terms must be commensurate in this sense because alternation is a property of proportionality ... Thus knowledge, properly speaking, cannot be transferred from one genus to another, not even if they are as closely allied as arithmetic and geometry.[223]

Olshewsky thinks that "because alternation is a property of proportionality" it is a property of *all* proportionality. His non-sequitur leaves out of account the immense range of applications (which decided Pellegrin's interpretation) that not only happen to cross genera, but do so with the clear purpose of crossing them. Pellegrin insists that Aristotle never uses analogy within a genus; Olshewsky says he only uses analogy within the genus, but Mueller thinks Aristotle doesn't know the difference[224]. He claims that Eudoxus, and following him Aristotle, was indifferent to the issue of homogeneity. He assigns the first consciousness of this issue to Euclid (or at the earliest to some unknown mathematician who flourished in the years between). Like Olshewsky Mueller cites the passage from *Posterior Analytics*[225] to support his claim. There Aristotle says, with respect to the ἐναλλαξ theorem, that "now" the principle is proved universally which used to be demonstrated for each class of magnitude separately. Mueller holds that by abstracting the character which makes lines, solids, numbers, and time periods mathematical, viz. their quantity (or magnitude), from these several objects Aristotle "obliterates the distinction" between homogeneous and heterogeneous classes. He says (p.2)

> For [Aristotle] the theory involves abstraction from all differences between magnitudes. Thus the objects of the theory are necessarily homogeneous.

The first sentence is misleading, and in any case the second doesn't follow. The homogeneity of "the objects of the theory" flows from the nature of ratios (see §§ 1.1-1.5) not from their nature as magnitudes. Only homogeneous elements can be in ratio (half an hour does not have a ratio to a pyramid). The homogeneity of the *alternando* rule distinguishes alternable elements from mathematical entities which cannot be interchanged (i.e., homogeneous from heterogeneous quantities). Like Olshewsky Mueller believes that Aristotle takes the alternation formula to be paradigm for all cases of analogy, but over and above that misapprehension, Mueller ignores one of Aristotle's deepest metaphysical commitments: to be spoken of at all objects need a form (εἶδος). Neither matter nor differentiae can be left to float free within a genus without being enformed, i.e.,

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[223] 1968 p.9. Olshewsky cites Heath 1949 pp.43f., 223 for Aristotle's use of the alternation principle.

[224] 'Homogeneity in Eudoxus's Theory of Proportions', in *Archive 7*, pp.1-6 (1970).

[225] 74a17ff. given in § 2.6 n.134 above (related to 99a8-11 given in § 2.8).

without specification. They are, as mentioned in § 3.2, *qua* differentiae, adjectives. The alternation theory, Mueller says, "involves abstraction", which of course it does, but the abstraction should not be such as to leave the elements formless—mere quantities with all distinctions removed. Mueller's reading obliterates not only the heterogeneous/homogeneous division, but all distinctions quantities possess. He misconceives the level of the homogeneity involved, and takes Aristotle's thought that there is a sense in which the elements of any science will be homogeneous, to be the level of the homogeneity at stake. All comparisons are homogeneous in *that* sense; even specifiably heterogeneous formulations. A theorem such as the ἐναλλαξ does not obliterate the specific distinctions between the modes of quantity, any more than calling rabbits, crabs, dragonflies, and wildebeest "animals" removes *their* generic differences. In mathematics the genus (i.e., the highest genus) is quantity; yet (as mentioned in § 1.3) Aristotle resists giving that name, or any other, to all elements of the genus. He does this *precisely* because to do so raises the wrong expectations: it would flag the notion of quantities *qua* mathematical entities standing (or capable of being spoken of) apart from their several forms. There is something bizarre in Mueller accusing Aristotle of blurring these very distinctions, when it is Aristotle who points out the dangers—devoting much of the last two books of the *Metaphysics* to the issue. Aristotle does not maintain that the *alternando* rule governs all proportions; he simply selects a homogeneous class for his example. The objects in any given application of the *alternando* theory are homogeneous *as distinct from* other possible objects within the scope of mathematics. Aristotle would only be removing the specific differences between lines, numbers, solids, and periods of time, if he were to say that these several classes of quantity could *interact* in the ἐναλλαξ theorem. Which he does not[226].

[226] Aristotle calls the classes of magnitude in the passage εἴδη, which could easily have led to the thought that they make up a single genus. Mueller is perhaps assuming the γενος-εἶδος terminology as applying a rigid taxonomy to mathematics parallel to the old interpretation of the biological works. It is worthwhile here to follow Pellegrin's suggestion (see § 3.3) that where Aristotle focuses on features held in common he uses εἶδος regardless of the level of generality. (But Aristotle is perfectly aware, as the passages 74a17 and 99a9 themselves show, that solids, for example, are not greater or less than periods of time.) The discussion from which the extract 74a17 is taken examines the rôles of the principles of '*qua*' and '*per se*', and the level of universality of attributes. Particularly in such a discussion the probability is that Aristotle would have been alive to the sense and extent (the *qua* and *per se*) of the homogeneity of the terms. Over and above this specific probability, in view of his overall interest in the generic status of classes and objects, it is *prima facie* unlikely that Aristotle would neglect the homogeneous/heterogeneous distinction. (Mueller discusses Aristotle's concept of mathematical objects further in *Aristotle's Doctrine of Abstraction*, in Sorabji, 1990 pp.463-80.)

Imputations that Aristotle was unaware of, or indifferent to, the importance of homogeneity for certain analogies, or heterogeneity for others, are not well founded. He stipulates homogeneity where the mode of analogy is drawn from the ἐναλλαξ principle (as in the *Rhetoric*). In other cases analogy is used for the very reason that the objects belong to different genera. Aristotle's approach to the relation between generic status and analogy is not, as Pellegrin thinks, heterogeneous with lapses of consistency; nor is it, as Mueller thinks, indifferent to generic status (which would then, in effect, reduce the terms to homogeneity, where Olshewsky thinks they belong). These extreme interpretations all misconceive the connexion between genus and analogy. Neither heterogeneity nor homogeneity form part of the definition of it, nor are they even accidents *per se* of analogy. They each figure for certain extensions of the analogical formula, and not for others. The spellings-out of the rules for each were recorded by Euclid (given above in § 2.9), and we have no good reason to think that Aristotle did not know of these, and many to think that he did.

There is only one passage in the Aristotelian corpus I can recall which defies the explanation of the connexion between analogy and generic order I have been proposing. In *History of Animals* Aristotle says

> In those blooded and footed animals which are viviparous, the bones do not differ much: they differ only "by analogy" i.e., in hardness and softness, and in size.[227]

It is not an extract which Pellegrin mentions; nor is it one I can make sense of. The section in which it appears is concerned with comparisons of bones among the various species. Possibly Aristotle means that the classes of animals involved vary too much to allow a direct comparison of their bones. Perhaps by 'not differing very much, but only by analogy' he indicates *minor*, i.e., homogeneous analogies rather than *major*, heterogeneous ones. But even so, for the bones of fairly closely related animals (the footed, blooded, viviparous ones), to differ in hardness and size, is a matter of degree, not analogy, however minor. Good sense of the text is made by dropping the expression κατ' ἀναλογιαν (on the grounds that it crept in through a later scribe who did not understand the issues). This is exactly what D'Arcy Thompson does in his translation[228]. But these are only tentative suggestions; I am loath to claim that the text is corrupt, or there was a slip of the pen: these, time

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[227] Peck, *HA III* 516b3ff. (Ὅσα μεν οὖν των ἐναιμων και πεζων ζῴοτοκα ἐστιν οὐ πολυ διαφερει τα ὀστα ἀλλα κατ' ἀναλογιαν μονον σκληροτητι και μαλακοτητι και μεγεθει.)

[228] *The History of Animals* (1910): "in the way of relative hardness, softness, or magnitude".

and again, have been the cries of commentators whenever the text conflicts with their understanding. All I can say is I don't understand it.

There are two strands of thought to be combined to grasp the use Aristotle makes of analogy: (i) analogy discovers a common purpose; (ii) the Eudoxian model applies to all quantities.

(i) Although 'explicating resemblance' (where there may be little or no apparent similarity) forms the basis of virtually all modern conceptions of analogy, it is harmfully anachronistic to read that as the use Aristotle makes of it. For him there is an analogy only where an equivalent function can be assigned, whether there is any resemblance or not. Despite the lack of any resemblance, wholly disparate phenomena can fruitfully be regarded as in important ways equivalent provided that a common purpose is found. The job analogy is able to do is to account for phenomena exactly and comprehensively, however close or remote the elements, whether they share a common measure or not. (This facility Aristotle applies especially to the problem of exchange-value.)

(ii) The General Theory of Proportions applied (a) even if the analogates were incommensurable; whether they fell (b) into the same genus; or (c) into different genera. Elements from differing genera will usually be (relevantly[229]) incommensurable, but so might certain analogates within a genus (e.g., the side and diagonal). In the teeth of Aristotle's plain assertions to the contrary Pellegrin is misled by the great value of analogy for (c) into supposing it applies *only* to (c). What provokes analogy however is the absence of a *direct* measure, whether inside or outside the genus. It makes an *indirect exact* measure by comparing the comparisons[230]. The reason it is not called on for those commensurable quantities within a genus is simply that there it isn't necessary: the direct application of greater and less suffices.

To recap, to account for the life-sciences Aristotle fuses three distinct principles: teleology, generic structure, and proportionality. The (perhaps I should say *a*) teleological principle joins analogy to the generic order. Perceiving the biological works as non-taxonomic has lifted much of

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[229] Differentiae will often be irrelevantly commensurable, of course, e.g., the weight of a turkey's egg and a sheep's bladder can be directly compared (and see § 6.5).

[230] Notice that the same referents might be compared either directly or indirectly; there could be an analogy: *triangle : hexagon :: rectangle : octagon*, even though all the analogates are plane figures comparable as to the greater and less (see also *Meteorology* 347b12 in which hail, rain, snow, dew and frost are related both analogically and by degree).

the darkness from the zoological writings, though more light would be shed by a clear grasp of the rôle of analogy. Unlike γενος and εἰδος, which he employs casually along-side more structured uses, Aristotle employs ἀναλογια only technically, keeping very close to its mathematical provenance. He always puts to use the *mathematical* principles of proportion (both the classical forms of it and the Eudoxian general theory—these being the traditional discipline and what was then the most up-to-date mathematics), and the failure to appreciate the impact of these mathematical principles on him has long resulted in complaints of inconsistency and obscurity in his theory of justice. That theory still suffers from its dislocation from the proportion theory upon which it was modelled. I say "has long resulted" because knowledge of the relevant mathematics began to evaporate before the fall of Rome. Aristotle's treatment of justice is so intimately bound-up with the Eudoxian formulae, and so dependant on the power of the central definitions, that we should bring to mind the fortunes of those definitions.

3. 7 Disproportionate palimpsests

We have seen in §. 2.5 that by the heyday of the Roman empire there was already some lack of clarity as to the principles of proportion that had once been known. For the next millennium the understanding of *Elements V* was sadly confused. In the Levant the presentations of the definitions were quite accurate, but with one exception[231] there was scarcely any understanding of the thought lying behind them. What foxed the Levantine mathematicians was the notion of 'equimultiple'. (The mediæval difficulties with equimultiples are indicated below in connexion with Campanus's edition of the *Elements*.) Worry over the meaning of equimultiple formed the core of Ahmed ibn Yusuf's (9th century) *Letter*[232]. He sought "the cause and essence" of ratio yet he did not clearly distinguish ratio from proportion; indeed Ahmed defined proportion where he was supposed to be defining ratio ("*ut sit in primo de partibus secundi quantum in tertio de partibus quarti*"). And his troubles with equimultiples were echoed throughout the Middle Ages in both the East and the West.

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[231] The 11th century mathematician al-Jayyânî demonstrated the effective equivalence of proportionality of Books *V* and *VII*. I.e., although *VII*.20 does not provide enough to cover incommensurables, it does follow that there cannot be a proportion when (taken in order) nA > B, and nC < D. (See Plooij, his translation pp.16-46.)

[232] Translated in the 12th century by Gerard of Cremona as *Epistola de Proportione et de Proportionalitate* (Murdoch 1963 p.252).

The errors in the West were much more complicated than the those in the East. The difficulties started with the definition 5.4. This was wrongly translated, but the texts from which it was taken were quite correct. The mistranslation started with Æthelhard; Boethius's (via Cassiodorus) version of 5.4, and both the Arabic traditions (via Gerard) were all accurate[233]. Æthelhard, however, both inserted a principle of continuity properly belonging to Book *X*, and confounded proportions with ratios:

> *Quantitates que dicuntur continuam proportionalitatem habere, sunt quarum eque multiplicea aut equa sunt aut eque sibi sine interruptione addunt aut minuut.*[234]

This spurious definition infected the understanding of Book *V*. It is true that there is a principle of continuity in Euclid's definition 5.4, but only in that only finite quantities are capable of being continuous with others. (I.e., it treats Archimedean not non-finite quantities—if the magnitudes were infinitesimal or infinite they could not be directly compared through multiplication, or any other process.) The requirement that for there to be a ratio magnitudes be continuous *in this sense* (which amounts to their being of the same finite genus) was read by Æthelhard as a proposal for '*continuam proportionalitatem*'—continuous proportions as distinct from *dis*continuous ones[235]. 5.4

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[233] *Elements V*.4 in Boethius's version is:

proportionem vero ad se invicem magnitudines habere dicuntur, quæ possunt sese invicum multiplicatæ transcendere.

Gerard of Cremona translated from the (Theonine) text of al-Nairîzî stemming from al-Hajjâj:

Quantitates, inter quas dicitur esse proportio, sunt quarum possibile, cum multiplicatur, alias addere.

Gerard's (pre-Theonine) translation from Ishâq/Thâbit:

Quantitates quarum quedam ad alias proportionales esse dicuntur, sunt quarum quasdem cum multiplicantur super alias addere possibile est.

The Sicilian translation directly from the Greek was also perfectly accurate:

Proportionem ad se invicum habere quantitates dicuntur que possunt multiplicate se invicem superare.

[234] Which I translate as

Quantities which are said to have a continuous proportion are those with an equal multiple, or else are equal, or else themselves, without interruption, increase or reduce.

A further version of 5.4 commonly read in the Middle Ages is very close to this, and belongs to the tradition that Clagett and Murdoch call "the Boethius-Adelard *mélange*" (Crombie p.241):

Illa continue proportionalitates esse dicuntur, quorum uno modo multiplicia aut equalia sibi sunt aut equalitur sese continue superant et a se continue superantur.

(Paris Bibliothèque Nationale 10257.)

[235] Since 5.4 refers to ratios, with no mention of proportion, it suggests that, together with the uncertainty between *mediates* and *proportionalitates* (mentioned §§ 2.1, 2.2 above), the unfortunate custom of translating λογος as *proportio* may be implicated in the mix-up. Book *V* does contain the implication of the difference between continuous and discrete proportions in the definition 5.8: ἀναλογια δε ἐν τρισιν ὁροις ἐλαχιστη ἐστιν. The three terms being those in a continuous proportion (which Aristotle explains at *NE* 1131a34ff., see § 1.8).

was thus misconstrued as the rule for merely one class of *proportions* instead of being the (latter part of the) definition for all *ratios*.

Equally seriously, the notion (or rather *a* notion) of equimultiple was brought forward from the law for proportions (5.5) to the definition of ratio (5.4). In the process of being thus misplaced (equimultiples being a feature which distinguished proportions *from* ratios) the idea of an equimultiple was itself misunderstood in the West as in the East. As with the Levantines from Ahmed onwards Campanus took Euclid's term ἅμα—from 5.5 (ἢ ἅμα ὑπερχῇ ἢ ἅμα ἴσα ᾖ ἢ ἅμα ἐλλειπῇ)—to mean '(by) the same amount' (*similes vel additione vel diminutione vel equalitate*). This replaced Eudoxus's principle that the greater and less be greater and less *correspondingly*, or "in the same way" or "simultaneously"[236] (which was more accurately in Latin "*simul vel additione...*") not that the greater and less were of the actual amount. The error was not primarily a fault of translation; although the most widely disseminated text and commentary of the *Elements* was Campanus's revision of Æthelhard's second version (the *Commentum*), many of the copies of this edition contained the correct "*simul*(taneous)". And the other Latin translators, Boethius, Hermann of Carinthia, Gerard (from both his Arabic sources), and the Sicilian, in addition to the Arabic translators themselves, all gave the correct translation. Unlike 5.4 all the translations of 5.5 were quite accurate; the failure in both the East and the West was in understanding what it meant. What it meant was that the ratios could be manipulated to measure each other, not that there was any 'amount' to be found which would resolve difficulties[237]. (In effect the mediæval readings were variants on the old anthyphairetic approach[238]: not perceiving the character of the general theory has

[236] For ἅμα *Liddell & Scott* give: *at once, at the same time, together with, at one and the same time, as soon as,* and, without reference to time, *together, both.*
[237] See §§ 2.9 and 6.9.
[238] Not grasping the correct definitions in Book *V* did not signify a decline of interest in proportion and ratio, on the contrary, associated with the more than fifty editions and commentaries produced in the Arab world between the 9th and the 14th centuries, at least eight texts (those by al-Jauharî, Ahmed ibn Jusuf, Sanad ibn Alî, al-Mahanî, al-Hasan, al-Fârâbî, al-Jayyânî, al-Samarqandî) were separately devoted to the issues. Even conceived as an anthyphairetic work (this indeed was stated by al-Mâhânî, al-Nairîzî, al-Jayyânî, and Omar Khayyam—see Plooij pp.25, 50-51, and Omar Khayyam (11th-12th century) *Explanation of the Difficulties in Euclid's Postulates*) the definitions in Book *V* were regarded highly. This regard was expressed in the West by Roger Bacon (excluding 5.5) as (op.cit., p.125):

> prior to all, since they are appropriate to figures and numbers and all things mathematical, and through the medium of mathematics, to yet other things and sciences.

Bacon's failure to appreciate 5.5 is hardly surprising in view of the vacuity assigned to equimultiples, and the assumption that only discontinuous quantities were covered. He thought Euclid's language there was obscure, and proper to neither mathematics nor correspondingly undermined every account of Aristotle's treatment of exchange-value—see chapter 6.)

Being assumed to be (i) an anthyphairetic theory set the stage for the complex reconstructing of the definitions *V*.3-5 in a manner which grossly undervalued them. (ii) The latter part of the definition of ratio became a rule for just one class of proportions, taking (iii) the notion of continuity to be a characteristic of that class, where (iv) in reality it underscored needs to be met for all ratios; (v) vitiating the concept of the equimultiple to express no more than 'the same amount' rather than *at the same time* or *in the same way*, and (vi) projecting this debased concept onto the preceding definition, in effect (vii) replacing definition 5.5 by the arithmetic 7.20, when (viii) 5.5 had been devised explicitly to replace the anthyphairetic theory. These multiple factors rather than merely (ix) mistranslation, were responsible for the evaporation of the meaning of the theory of proportions[239]. A

philosophy (pp.86, 94). In the West the latter part of the Middle Ages saw a flourishing of work on proportion theory; it was extensively discussed by writers such as Bradwardine, Oresme and Albert of Saxony, and was explored notably in connexion with astronomy and mechanics (Oresme *De Proportionibus Proportionum;* Albert of Saxony *Tractatus Proportionum*). Archbishop Bradwardine, in his *Tractatus de Proportionibus Velocitatum in Motibus,* connected variations of force and resistance to the differentials of velocity. He developed a 'calculus' of ratio seeking a mean between terms of given ratios and a 'ratio of ratios'—although this (*proportio proportionum*) was Oresme's expression. (See Clagett *The Science of Mechanics in the Middle Ages,* 1959, chapter 7.)
(So we see that Galileo's use of proportion to give a general theory of motion followed in the scholastic tradition, his breach with that tradition being his use of the true Eudoxian definitions to explain motion as continuous, not discrete.)

[239] The loss was not recovered until the very end of the Middle Ages. In the 1460's Regiomontanus compared the Greek MSS with the Campanus editions (§ 2.2, note 103(v) above). He appreciated the genuine 5.5 definition (see Dedron & Itard 1973 I, p.199) but died (in 1476) before he could publish. Zamberti (1505) did not clearly rectify the palimpsests (S.Drake 'Galileo Gleanings XXII: Velocity and Eudoxian Proportion Theory, in *Physis 15*, 1973), which is very strange as he inveighed against Campanus as a "barbarous translator" who had filled the text "with extraordinary scarecrows, nightmares and phantasies". Heath (1926, I p.98) points out that Zamberti did not realise that Campanus had to deal with the text as he received it.

The crucial rôle of the general theory of proportions in the development of science is vividly demonstrated by its impact on Galileo. Throughout the Middle Ages, as we have seen, the central definitions of *Elements V* were misunderstood as variants of Book *VII*, in which arithmetic quantities primarily figured. Apparently using Tartaglia's commentary (note 103(v)) Galileo deployed the Eudoxian formula to re-define the physics of motion. In *De Motu* chapter 15 (1591-92) he relied on 5.5 to specify the truth conditions for relating disparate elements such as speed, weight, volume, and height; and straight and circular motion (this last in opposition to Aristotle—although Galileo follows Aristotle, e.g., from *Physics VII* chapter 5, in using proportion to connect motion and force with distance and time). Also in *Mechanics* (about 1602), in his proof of the Lever Law (which was a simplification and improvement on that of Archimedes: note that Archimedes had utilised a pre-Euclidean, anthyphairetic model of proportions—ref. § 2.6 notes 129-133), Galileo set aside the commensurable/incommensurable distinction by means of 5.5 (see Drake *Galileo at Work: His Scientific Biography,* 1978, pp.15, 27, 57, 302).

consequence of this loss was that the mediæval commentators of the *Ethics* did not have the model of proportion theory upon which Aristotle relied available to them.

Chapter 4

THE PATTERN OF THE THEORY

4. 1 Aquinas and analogy

I don't think there has ever been a satisfactory commentary on the account of justice in the *Ethics,* but the best of the many that have been made was that by St Thomas Aquinas. In the following chapters I shall promote his interpretation (except for one major issue—§ 4.7) as accurate and perceptive, and I especially support him against criticism of recent centuries. Not that St Thomas's well-known doctrine of analogy[240] touches on the commentary on justice. (Aquinas's, and subsequent Thomist discussions of analogy are concerned with quite other issues of metaphysics[241] and theology.) But it is plain from certain details in his remarks[242] that he was aware of the mathematical principles of ratio and proportionality—at least as they were known to the translators of Euclid by his time[243]. Aquinas distinguished indirect comparison *via* analogy from direct comparison within a genus, and he knew that analogical similitude, as against resemblance, operates wholly independently from the proximity or otherwise of the analogates. In *De Veritate*[244] he says

> *talis enim similitudo similiter invenitur in multum vel parum distantibus: non enim est maior similitudo proportionalitatis inter duo et unum et sex et tria quam inter duo et unum et centum et quinqaginta;*

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[240] Some modern Thomists (Davies 1992, p.70—see § 1.9 notes 73-75—following B.Montagnes (1963) *La Doctrine de l'analogie de l'être d'après S. Thomas d'Aquin*) say that Aquinas did not have a doctrine of analogy, but that Cajetan is the source of the 'doctrine'. Whether what he thought amounted to a theory or not, St Thomas treated analogy as a mode of predication distinct from, and falling between, univocity and equivocity; from which stems much of the subsequent metaphysics of analogy.

[241] Mentioned in § 1.9 and notes 73-77.

[242] Not all those referred to by Phelan (see § 1.9, note 75) but rather in *De Veritate* (see following note) and corresponding passages on the *Metaphysics V,* lect. 8 § 879 (referring to 1016bff.) and *Summa III (Supplement)* q. 92, art. 1, r. 6.

[243] The works St Thomas relied on most were those of Gerard of Cremona and the *Commentum* edition of Æthelhard, but also the translation by Hermann of Carinthia. Aquinas refers to Euclid by name 20 times, with several references to ratio and proportion. *Elements V.* 3, for example, is cited in *III Sent.* d.1. q.1. a.1. ad. 3 in Gerard's translation:

> *Proportio est* certitudo *mensurationis quantitatum unius generis.*

He was also familiar with Euclid through Cassiodorus's preservation of Boethius's *Ars Geometrica* (see *Aquino* I*ii, pp.65*-86*). Thomas's early studies took place at Naples, but there is no sign of his knowing the Sicilian translation made 80 years earlier.

[244] Q. 2, art. 11 ad. 4 (*Aquino* 22 Ii-Iii, p.80, § 238-42). Aquinas also shows his familiarity with proportionality, not just ratio, at q. 2, art. 3, ad. 4.

i.e., that there is no greater *analogical* closeness between very close and very distant terms (as he says, between 2 : 1 :: 6 : 3 and 2 : 1 :: 100 : 50). Aquinas was aware, unlike some modern Aristotelian commentators, that analogy functions independently of the heterogeneity or homogeneity of the analogates, *and* that direct comparison must be homogeneous[245].

My disagreement with St Thomas's account of universal justice is, as mentioned, best left (to § 4.7) until after the layout of the theory has been presented. That layout is, in turn, best seen after the many threads I have left leading up to Aristotle's theory have been drawn together.

4. 2 Resumé

Proportion begins with ratio, which is the manner of comparison between quantities, wherever comparison is possible. Two distinct ideas as to the nature of mathematics informed how ratio was to be conceived: (a) as primarily a relation between the objects of mathematics, or (b) primarily as an interactive process (stressing the activity rather than the objects acted upon). In either case, since heterogeneous forms of quantity cannot be greater or less than one another (50 yards is not more or less than 17 ounces, or an acute angle) the objects had to be of the same kind. Ratio formalises the *greater-less* relation; the notions of *more* and *less* form a polarity related to the ancient περας-ἀπειρον dichotomy which, in a number of guises, pervaded pre-Aristotelian thinking. It had been supposed that ratio would achieve precision for the notions of greater and less, fulfilling the aims to bring knowledge, i.e., reason and exactitude (περας), to what was otherwise inexact and boundless (ἀπειρον). Even though it is usual to speak of ratio and proportion as if they are much the same there is a vital distinction between them: ratio is a 'first-order' connexion between quantities. Proportion, on the other hand, is a connexion between the acts of connexion; a 'second-order' process. This logical difference is not observed very much in ordinary usage, nor even in technical discussions. Where ratio encodes the difference between quantities, proportion records the sameness of the relation despite the differences. It fastens-on to what is equal, whether or not the equality is apparent. Analogy is the equality of the ratios. Ἀναλογια (proportionality) had been developed as a study by the Pythagoreans, from whom stemmed the three classical modes: arithmetic, geometric,

[245] He says (ibid.) that although direct comparison between us and God is impossible, an indirect (but nevertheless literal not metaphorical) comparison is possible heterogeneously *via* analogy.

and harmonic, locating respectively the equal in terms of quantity, multiplication, and division, between pairs of ratios.

The differences between geometry and arithmetic lead to the difference between continuous and separated proportions; Aristotle, however, generalises both of these into 4-term proportions (in effect treating then as separated, since the features of 3-term proportion are expressible in four terms). They all require a 'mean'. The doctrine of the mean is approachable in many ways, but the mean can usefully be thought of as the mid-point, centre, or point of comparison, for the whole of the proportion. I have defended the overlapping references to *mean* and *proportion* among the ancients, and Aristotle's pristine use of *arithmetical* proportion. His conception of proportion is strictly in line with the mathematical principles of his time, and should not be confused with notions of analogy that have become familiar since the Middle Ages. These later conceptions work on the principle of resemblance, whereby similarities are located or explicated or created. Such ideas emerged from the classical notion by recognisable steps, but they distort our understanding when projected onto the ancient usage.

The discovery of the irrationals had a considerable impact on Greek thinkers, and we looked at the early anthyphairetic methods of tackling the problems raised. I hope to have clarified the reason for Aristotle's long-puzzling choice of the term ἀνταναιρεσις when referring to this issue. I suggested that the alternative expressions ἀνθυφαιρεσις and ἀνταναιρεσις reflect the differing conceptions of mathematics. The proposals of Knorr, Larsen, and Thorup, raise the possibility of there having been an earlier general solution than that traditionally ascribed to Eudoxus. But it turns out that the traditional view remains the most likely, and in any case, had any alternative theories existed they would have benefited Aristotle by providing him with a greater choice for the many uses he makes of analogy. Either way, the impact of proportion theory in general on Aristotle's thinking and what we know as the Eudoxian theory in particular, was immense.

Aristotle draws proportionality into his entire method of classification; that method starts out in dissatisfaction with Plato's (or the Academy's) method of classifying by dichotomous division. This he exposes as unsuited to classification, especially in biology, except as a minor aid. His own proposal is a generic structure made effective by principles of analogy and teleology. We contrasted his often casual use of the terms γενος and εἶδος (although these are sometimes, and always when used together, structured) with ἀναλογια, which he always uses technically. Where the εἶδος (or the γενος or the individual object) is the subject of an inquiry, *as* the subject it is to be distinguished from attributes (usually differentiae) predicated of it; these will, typically, be adjectives. 'Αναλογια was then a mathematical term which Aristotle begins to apply outside mathematics. In the life-sciences he uses the principle of *function* to integrate analogy with the generic order. The prime notion of function also determines how animals are divided into parts; their parts being organs to implement the (principally vital) functions. Comparison of animate parts or wholes is made, both within and across genera, according to their functions, not according to resemblance. (Similarity might be treated as either leading to, or resulting from, expectations as to an equivalence of function. I have not otherwise looked into other meanings of teleology, or its metaphysics.)

We found radically opposing claims for the place of analogy in the generic sequence. Ross and Pellegrin see only heterogeneous analogies, Olshewsky and Mueller only homogeneous. In fact both camps mistake the nature of classical analogy; analogy provides an *exact indirect* measure. Sometimes, as when the alternation principle applies, homogeneously, and sometimes heterogeneously. The outstanding value of proportion theory for Aristotle is that it enables the precise comparison of phenomena *whether or not* they are widely separated (N.B., *not* only when they are widely separated, or only when they are not).

The opposing (mis)perceptions of the generic standing of proportion in Aristotle's philosophy reflect a lack of awareness of the importance to him of the work of Eudoxus. Not appreciating this close association has vitiated accounts of the theories of justice and exchange, but has also done damage to the understanding of the biological works and his philosophy of mathematics. So important, and it seems novel, is the Eudoxian theory (in addition to certain key elements in proportion theory in general) to our understanding of Aristotle's thinking, that I have outlined some of the principle theorems, and traced the vicissitudes of the salient definitions. It will be immediately objected that proportion theory is well-known to have been known to Aristotle, and nearly always mentioned by commentators. So it is; it is mentioned, and sometimes elaborated, but not appreciated[246]. In the specific accounts of the importance of proportion for Aristotle (in *Soudek's* for example) the Eudoxian theory is not even mentioned, and in *Lowry's,* although the issue of

[246] An exception was Heath (1949), and recently Byrne (op. cit.).

proportionality is explored in detail and Eudoxus's influence on Aristotle highlighted, Eudoxus's general theory is ignored[247].

4. 3 Seven preliminaries

(i) Δικαιοσυνη *and* δικαιον:

With what I hope is now an adequate picture of the factors supporting Aristotle's reliance on analogy, and especially of the actual principles he deploys, we are equipped to examine his presentation of the theory of justice. Aristotle's conception is derived from Plato; there are some divergences, but these reflect his delving into what the application of proportion theory entails, and certain limitations he imposes as a result, rather than a conflict of approach[248]. A terminological preliminary should be mentioned. Δικαιοσυνη and (το) δικαιον are often both translated as 'justice'; the adjective "δικαιον" was also used (in the neuter nominative singular "το δικαιον") as a general noun to mean "the just ______": the just *thing,* or the just *act*. Much of the time little may hang on this grammatical detail, but at certain crucial points we find that it will be a considerable help in rectifying mistakes of interpretation (§§ 3.2 and chapter 5).

(ii) *Righteousness:*

The Platonic notion of δικαιοσυνη was perhaps in certain respects closer to the biblical principle of righteousness than to what is meant by 'justice' in modern English. (It also included the harmony of the connexions among the parts of the soul; Aristotle comments on this at the end of the book, where he treats it as metaphorical, and equivalent to the derivative sense of justice which applies in such cases as in (iii) following.) But whether δικαιοσυνη expressed a broader concept than δικαιον is not an issue that I shall explore: what we need to observe is that Aristotle distinguishes overall righteousness from the rightness of interactions with others of the πολις. The traits of character of the righteous will be pretty much the same as those of the just. What are designated as righteous are the traits judged as inward dispositions; what is said to be just are these

[247] *Lowry* traces the influences on Aristotle of proportion theory, and especially the Pythagorean sources *via* Archytas, but he surprisingly mentions only Eudoxus's method of exhaustions in relation to the reciprocal or harmonic proportion (see chapter 6).
[248] But see note 252 for Burnet's view.

same traits judged as outward relations to others[249]. He thus imposes a limitation on the Platonic conception; but it is thought that for Aristotle also δικαιοσυνη (justice) retains a broader sense than δικαιον (a just action or thing).

(iii) *Plato:*

The proportionality central to Aristotle's notion of justice is found in a number of Plato's dialogues; in *Gorgias* 508B (Hamilton) for example:

> you have not observed how great a part geometry plays in heaven and earth, and because you neglect the study of geometry you preach the doctrine of unfair shares

and *Laws VI* 757Bff. Plato says (Bury)

> For there are two kinds of equality which, though identical in name, are often almost opposite in their practical results .... the equality determined by measure, weight and number .... but the truest and best form of equality is not an easy thing for everyone to discern .... it produces all things good; for it dispenses more to the greater and less to the smaller, giving due measure to each according to nature .... it assigns in proportion what is fitting to each.

These passages might appear to prefigure only *distributive* justice, but they contain the germ of Aristotle's more elaborate formulation of justice in general (explored below). At 744C:

> so that by rule of symmetrical inequality they may receive offices and honours as equally as possible, and may have no quarrelling,

distributive justice is more exactly anticipated.

(iv) *The limitations of equality:*

Justice as a specific virtue, i.e., *particular* justice (see below), is intimately bound-up with equality; as justice is the appropriate, i.e., the right or fair equality, it can only apply when *some* form of equality is available. This association leads Aristotle to limit the scope of justice in ways we find unappealing nowadays. Only adult male citizens of the πολις are, he thinks, capable of some mode of quantifiable comparison (comparison as to the greater or less) with one another. The imperatives of equality require that no action, or state of affairs, can be just unless some mode of equality can be arrived at between those involved. Where no measure in common can be found there is no place for fairness—which can only apply to the genus of quantifiable justice (i.e., Particular justice). Fairness is a species of the genus *equality*, so where the *genus* cannot apply no species of that genus can be applicable. Strictly speaking, because the worth of foreigners, slaves, children, and

[249] This distinction is sometimes confounded with the distinction of *universal* from *particular* justice (e.g., by Burnet 1900, pp.202-3) rather than as the contrast between (universal) justice and the other virtues.

women, cannot be measured against that of citizens, comparison is impossible. Hence there can be no mode of equality, hence there can be no justice or injustice applied to them. Foreigners don't qualify because, although they may well be autonomous adult males, they are not members of the *polis.* Slaves may well be adult males within the *polis,* but they are not autonomous (such a notion contradicts the very condition of slavery), hence they are not properly 'members' of it. Children could be male, free, and of the *polis*, but are only potentially autonomous. Women will be adult and might be free and of the *polis*, but they are not even potentially male. All the excluded groups have debilities which prevents a quantifiable comparison as to the greater and less with citizens. Unlike non-citizens the worth of any citizen can be, in principle, measured against that of any other[250], so that *some* ratio will express the comparison. Non-citizens do not occupy the same scale—they are incommensurable with citizens, so no ratio can express the worth of a non-citizen with a citizen. Women, children, and slaves, will belong to some adult male citizen (women to a father, brother, husband, uncle, or cousin), and just as justice governs interactions with others of the *polis,* it does not govern one's relations to one's own belongings. (There is an extended discussion on whether it is possible to be unjust to oneself (1136a10ff.) in which he concludes that it isn't.) It is their incommensurability that bars those excluded from justice; Aristotle's restriction is (as with the structure of his whole theory) purely mathematical: there can be no mode of equality where the (worth of) the elements involved is not comparable. No multiplication will equate the value of a slave and master, or father and child, or us and God (1137a28), which is why justice is strictly inappropriate to such connexions.

But, Aristotle is clearly unsettled by the logic of his position; he concedes that in a secondary or metaphorical sense[251] justice may apply even where no comparison of worth between the parties is possible. Taking Plato's notion of justice regulating the "differing parts of one's nature" (as Aristotle, *NE* 1138b7, puts it) the elements involved are treated *as if* they are capable of quantifiable comparison[252]. The strongest claimants for such *ersatz* justice are wives[253]. They are well placed

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[250] *MM I* 1194b9, 17, 20-21. (This and any other appeals to *MM* are intended only as general support for the argument—indicating Aristotle's views as reported by a close follower.)

[251] 1194b7 (Stock):

> But the justice in these cases would only seem to share the name of particular justice without sharing the nature.

[252] Burnet (1900 p.246), however, sees Aristotle's remarks at the end of the book about the metaphorical extension of the word as "disposing" of Plato's notion of δικαιοσυνη.

because although as wives (and probably as women) their worth cannot be measured on the same scale as adult male citizens bound together through laws, *qua* partners who entered a contract according to laws and agreements susceptible to measurement, the worth of many of the issues that might arise between husband and wife will be "embodied in law". Insofar as the wife has entered a legal agreement and "who shares equally in ruling and being ruled" (1134b15) (presumably the household) comparison is possible, but insofar as she is not an independent citizen comparison is not; her rôle is then ambiguous. The ambiguity seems enough for Aristotle to treat this form of domestic justice as the strongest of the extended senses of the principle. So long as anyone can be viewed as if their worth could be measured against a citizen, or against any other party, then within that viewing they can be treated as sharing a common measure. Principles of greater and less can then apply, and so some mode of equality, and so justice. Moreover, the principle of analogy is used not only to extend justice to non-citizens, but however faint the justice for them is, it reflects the same rules as full-bodied justice *as if* they had the franchise.

(v) *Parallel Injustice:*

Justice and Injustice are presented as largely parallel genera of virtue and vice respectively, but they are not parallel in every respect; they are after all, most importantly, opposites. Whereas the other virtues hold the mean between the extremes of their opposing vices (as Nemesis is the mean between the vices of Envy and Malice), as higher-order conditions of vice and virtue, Justice is the mean between extremes of the same vice: Injustice (1133b30ff.)[254].

Injustice is also a particular vice parallel to Particular justice (see below); as with the case of justice, Particular injustice will be the specific vice connected with (in)equality and all issues of greater and less. Aristotle begins his treatment of Particular justice with a discussion of Injustice covering certain aspects of the subject which, in the equivalent account of Justice, he passes over. The remarks about Injustice are particularly useful in giving details which help to clarify the corresponding structure for Justice. They are especially helpful in establishing the scope of the subject.

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[253] *MM I* 1194b23.

[254] *Stewart* pp.471-75 has a useful extended note treating this passage; but many of the difficulties there flow from his not distinguishing sufficiently the Universal from the Particular conditions of justice that are discussed in the following pages.

(vi) *The mean for us:*

The doctrine of the mean applied to ethics requires that the right stance is to be found between excesses and defects. Virtue will find a position between (say) extravagance and meanness; but, Aristotle says, what is to count as meanness or extravagance will depend on circumstances. What is generosity, even extravagance, in a pauper would be miserly from a millionaire. Similarly, what is courageous for one person to do might veer between the reckless and the cowardly for some others, depending on the situation, and who they are. Consequently there needs to be, in addition to a mean between the extremes, a further principle of the mean relative to the individual and the specific context.

(vii) *Higher-order virtue:*

Justice is placed as the last of the virtues of character (what we would call moral virtues). The earlier ones are principally dispositions for the individual's own good; these personal virtues, such as courage, wit, or magnanimity, take up an ideal position between greater and less[255]. The relevant excellence is found as the mean between an excess and a defect (the excesses of the virtues just cited would be recklessness, buffoonery, and extravagance; the defects cowardice, boorishness, and meanness). Such virtues are of the same order (lying on the same scale) as their vices. Justice, by contrast, turns outward, and applies only to the individual's inter-relations with others. It is more complex than the previous, specific, virtues; it is a mean but not, Aristotle says, in the same way. (It is the point of equality in a 4-term proportion (§ 1.8), not the mean of a 3-term proportion[256].) It has the character of virtue itself, and acts upon all the specific virtues insofar as they relate outwards. It is placed after the others to sum-up the moral excellences; Aristotle quotes the proverb (1129b30):

In Justice is all virtue found in sum.[257]

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[255] In the section of *EE* (1220b40-21a12) which corresponds to *NE II* 1107a32 Aristotle lists fifteen virtues as means between their excesses and defects: courage is the mean between cowardice and recklessness, generosity between meanness and extravagance, and so on. In some cases there is no name for an extreme. Aristotle uses slightly different examples of virtues in different texts (*MM, NE, EE,* and *Rhetoric* 1366b1-22).

[256] It is not the outward stance of the virtue which renders it a 4-term proportion. Specific virtues might be purely self-directed or other-directed; thrift is a mean involving only the self, and *nemesis* is the mean between envy and malice (1108b1) which can hardly be applied to oneself. Perhaps also a purely inward virtue such as temperance is not a 3-term proportion holding the balance between two extremes, but rather a 4-term proportion inwardly supervising the particular ἕξεις.

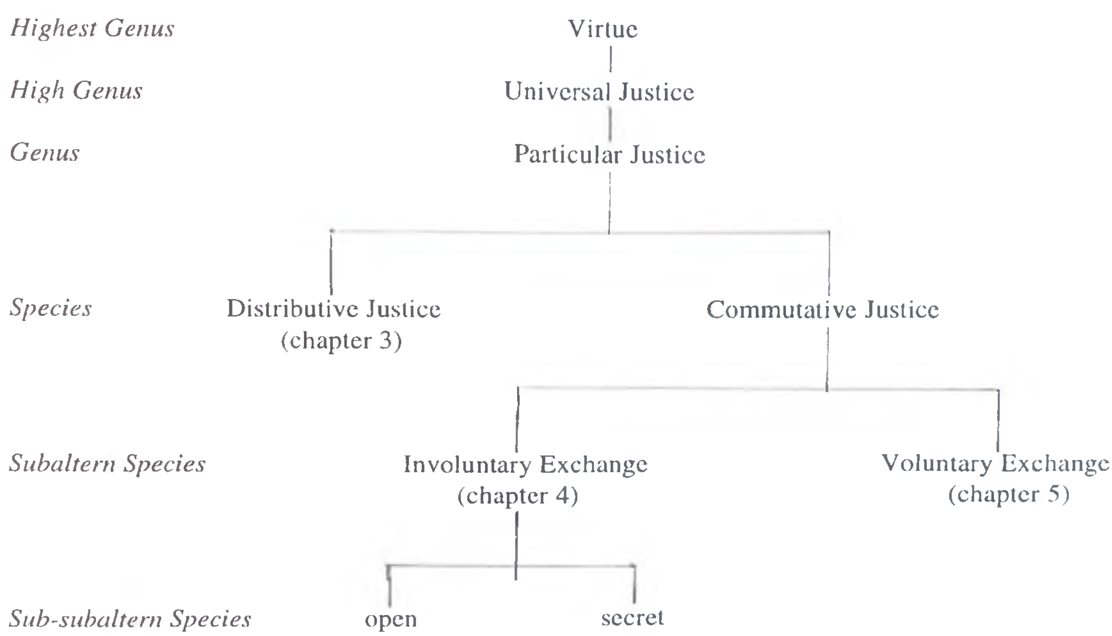
[257] ἐν δε δικαιοσυνῇ συλληβδην πασ᾽ ἀρετη ᾽νι.

This recursive, second-order, virtue acts upon (governs) what is to count as a virtue for any of the specific dispositions. Unlike the earlier virtues, such as courage or wit, it is not reflected by the continuous 3-term (what I am calling 'first-order') format, but by the 4-term (second-order) separated proportion (1131a15ff.—also *MM I* 1193b39-40). The universal character is *contrasted* with the 3-term sequences and is represented by the higher-order 4-term proportion.

4. 4 The layout of the theory

The theory of justice is set out in Book *V* of the *Ethics* (and Book *IV* of the *EE*, it forming part of the Common Books). For the last 150 years or so all commentators (that I have found) complain that the layout of the subject is a mess. What is supposed to be wrong with it differs from writer to writer, but whatever each individual scholar's view has been, they all agree that as it stands the text is confused and unsatisfactory. Against all these and related objections I maintain that Aristotle is not confused; that the many obscurities that have distressed them rest with the interpreters themselves, and that Aristotle presents a well-organised and unified theory.

The clearest way to show the structure of the theory is to set out its headings diagramatically. This is the plan Aristotle declares (1130b30ff.) and which he sticks to throughout, working faithfully through the issues step-by-step:



(figure *1*)

4. 5 The highest genus

The Highest Genus here is ἀρετη (virtue or excellence). The nature of virtue had been defined in Book *II* (1105b19ff.) as a quality; the substance qualified was ψυχη (soul)[258]. The soul is divided into specific affections παθη (feelings or emotions) on the one hand, and on the other, more general faculties. (Several specific emotions are listed at 1105b20[259].) The general faculties are then split into the naturally occurring and the acquired. The naturally occurring capacity (δυναμις) is the rather fleeting and unstable liability we have to have a specific emotion. These natural inclinations are not (or ought not to be) blamed or praised; praise and blame attach rather to what we do with them. What we do is educate, cultivate, or train, these innate but unsteady faculties into settled dispositions. Such a trained condition was known as a ἑξις (variously translated as a state, condition, habit, tendency, or disposition). The term and the notion was taken from surgery and medical practice, and then from athletic training at the gymnasium, where it meant a firm posture or trained readiness[260]. Virtue is defined as just such an acquired settled disposition (and to be immediately distinguished from other ἑξεις such as τεχνη, and mediately from παθη and δυναμεις, other qualities). The cultivated tendency to excellence produces either (i) what is worthwhile or good in itself, or (ii) what functions well. These aims will, in turn, be directed either (a) inward to benefit the individual (as we have seen these personal virtues are presented in the earlier books), or (b) outward. Justice is the characterisation of any and all of the specific virtues viewed as they relate to others[261], but it is also a specific virtue (*V* chapter 2; 1130a14ff.) largely parallel to the other specific virtues, but hybrid, in that it operates according to the higher-order 4-term proportion, not the more limited 3-term. It might have been clearer if it had had some other name, but another name would be misleading in that justice as a specific virtue still has the higher character of justice, rather

[258] In the text of the *Ethics* three divisions of the soul are given, but the principles reflect the more complex analysis in *Categories* 8b25ff.
[259] Desire, anger, fear, daring, envy, joy, friendship, hatred, longing, jealousy, pity.
[260] *Liddell & Scott.*
[261] The lack of clarity about Aristotle's use of generic structure has led to many confusions in accounting for the relations between virtue and justice. E.J.Weinrib, for example (pp.135ff. in *Aristotle's Forms of Justice,* in S.Panagiotou (ed.) *Justice, Law and Method in Plato and Aristotle* (1987, pp.133-52), writes:

> The movement of Aristotle's argument is from virtue to a form of justice congruent with virtue, then to a justice that admits but does not require virtue, and finally to a justice which completely denies virtue's relevance,

which doesn't recognise the pattern that Aristotle lays out. *All* the classes of justice fall squarely *within* the γενος *virtue.*

than the (in the appropriate sense) ordinary character of the preceding virtues. Justice in this specialised sense deals with all issues of greater and less *purely as* greater and less. It is this particular notion of justice that is our principal subject.

The connexions among the concepts ordering the relation of virtue to the soul it qualifies do not fit into the above diagram. There would be too much cross-division; even if the relevant concepts could be presented diagrammatically, the diagram would need to be so heavily cross-referenced as to subvert any hope of lucidity. (As with biology, a generic structure applies to a chosen range of issues, but does not attempt to cover the whole field.)

4. 6 A high genus

Justice is a *High Genus*; it is often said that *universal* justice[262] is a distinct class falling within the genus *justice,* with *particular* justice standing along-side as a parallel species (or sub-genus)[263]. But that is a profound misconstrual of the text—which the corresponding account of Injustice makes clear (1130a23ff., Rackham):

> Therefore there is another sort of Injustice, which is a part of Injustice in the universal sense, and there is something unjust which is a part of the unjust in general, or illegal.

Aristotle repeats (1130a34):

> Hence it is manifest that there is another sort of Injustice besides universal Injustice, the former being a part of the latter.

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[262] T.Engberg-Pedersen calls it 'comprehensive' justice (see *Aristotle's Theory of Moral Insight,* 1983, pp.54ff.).

[263] For example by T.H.Irwin who says (*Aristotle's First Principles,* 1988 pp.424ff.) that General and Special (Universal and Particular) justice are two distinct parallel virtues, not hierarchically connected with Special falling within General justice. To this misconception of the structure of the theory Irwin adds the usual modern assumption that the second species of Particular justice is the 'corrective'. From these errors Irwin constructs his own model of Aristotle's theory in terms of 'prospective' and 'retrospective' justice. General justice is prospective: it gives the rules for guidance. Special justice is mainly retrospective, although Distributive justice is also in part prospective: it guides initial distributions (Irwin does not allow the second species to be partially prospective, to guide initial exchanges; indeed he, like many others, thinks exchanges are governed by an undeclared third class (pp.625-26 nn.11 & 12). Irwin then finds fault with Aristotle for not sticking to his (Irwin's) criteria. F.Rosen ('The Political Context of Arisotle's Categories of Justice', in *Phronesis* XX, pp.228-40, 1975) also treats Universal justice along lines similar to Irwin's, and leading to the "ambiguous" (p.237) condition of certain 'corrective' issues. C.J.Rowe ('The Eudemian and Nicomachean Ethics: A Study in the Development of Aristotle's Thought', in *Proceedings of the Cambridge Philological Society,* suppl. 1-3, part 3, 3, 1971) similarly treated the forms of justice non-hierarchically; Professor Rowe, however, tells me (conversation, Spring 1997) that he is inclined to accept my objections.

These two extracts lie in a section with an uncertain history; there was uncertainty because several MSS included expressions such as "και το πλεον" in a couple of places. Jackson wrestled with the differing readings but eventually accepted Trendelenburg's endorsement of Muretus's correction of 1130b12[264] to "ἐπει δε το ἀνισον και το παρανομον....", which has been adopted by all subsequent translators. The intrusion of "more" (πλεον) confused further a passage which at first sight appears the reverse of our immediate intuitions: we usually think that there are many more things that are unfair than illegal. But what Aristotle appears to be arguing is that there is more to legality than issues relating to equality. Much of Law stipulates that we conform to some rule: murder, theft, and driving on the wrong side of the road, all transgress rules (we shouldn't only *more or less* keep to the left). Justice, universally (or generally) understood, is the whole of virtue vis-à-vis others, encompassing every sort or sense of right action. Law formalises right action into rules which supervene both on justice as a special virtue, dealing with all issues of more and less, and on all the other virtues insofar as they impinge on others. This allows justice (contrary to frequent interpretations of Aristotle's texts) to apply to non-citizens. Law will require right action in relation to others; the limitation that the absence of a common measure disallows the possibility of any mode of equality—and so the possibility of Particular justice—only disqualifies judgements with respect to issues of comparison between the parties. Many actions (promise-keeping, for example) have little to do with comparisons or equality, and fall under Universal justice. In addition, Aristotle allows the rules of Particular justice to be borrowed, so that justice-by-courtesy (§ 4.3 (iv)) is available for those who do not qualify for the strictly comparative rules.

A point needs to be made about *legal* justice. I have not itemised it in the diagram because it is not a separate class (genus or species) of justice, but is rather *universal* justice considered vis-à-vis Law (this is repeated in *MM I* 1193b1-11). On the (sublime) understanding that the laws are just, they codify the rules of virtue in general, for dealings in general. There is a key extract at 1130b23 (Thomson):

> For, broadly speaking, what we do as a result of practising virtue in general are those very actions[265] which conform to law;

[264] *Jackson*, pp.73-74; see also *Stewart*, pp.406-9.
[265] Michael of Ephesus said this refers to the various trades, businesses, and crafts, rather than to moral conduct; see *Stewart*, p.410.

which follows the above passage connecting 'the unequal' to 'the unlawful' as part to whole. Formalised justice-in-general is the 'whole' of Legal justice of which issues of greater and less are a 'part'.

4. 7 Whole and part

The one quarrel I have with Aquinas's interpretation of Aristotle's theory is his reading of General (Universal or Legal) justice. The interpretation with which I disagree did not appear in the *Commentary* (which gave a faithful account of Aristotle's text) but, in drawing-in features of the *Politics* when writing the *Summa,* Aquinas constructed an erroneous interpretation of the principles of General and Particular justice which have remained as staple readings ever since. Starting gradually at *Summa II-II* q. 58, art. 5, there is a logical shift which is completed by the end of q. 61: Q.58 a.5, *Responsio* (referring to his commentary for Book *V* of the *Ethics,* chapter I):

> we have seen justice directs a man in his relations with others,

and ad. 3:

> Accordingly general or legal justice, as directing to the common good, may be called a general virtue.

Ad.6:

> *Sed Contra: sed virtus boni civis est iustitia generalis, per quam aliquis ordinatur ad bonum commune*
> (This virtue is general justice, which orders our acts for the common good)

Responsio:

> so general or legal justice, which regards the common good as its proper object ...

By art.7 there is a movement in the scope of 'common good'. *Responsio:*

> we have seen that legal justice, which directly charges a man with the common good .... so also is a particular justice, which orders his dealings with another individual person

ad.1:

> legal justice is indeed sufficient to govern us in our dealings with others, immediately when they comprise the common good, yet mediately in the case of the good of one individual person. Hence the need of a particular justice immediately engaged with this

ad.2:

> the common good of the community and the particular good of the individual differ,

Aquinas here draws on *Politics I* 1252a7. At q.59, art.1, ad.1:

> legal justice is relative to the common good in human terms.

Then at q.61, referring to the above passages, he says: *Reponsio:*

> *iustitia particularis ordinatur ad aliquam privatum personam, quæ comparatur ad communitatem sicut pars ad totum*
> (particular justice is directed towards the private person, who may be compared to the community as a part to a whole).

He then specifies the tripartite division of the field of justice:

part → whole	(General, Universal, or Legal, justice);
whole → part	(Distributive justice);
part → part	(Commutative justice).

Then ad.4:

> *et ideo ad iustitiam legalem pertinet ordinare ea quæ sunt privatarum personarum in bonum commune*
> (Accordingly [general or] legal justice aims to conduct the dealings of private persons to the good of the community).[266]

Thus St Thomas reduces the extension of his opening (and Aristotle's original) sense in which:

(a) General justice is justice *common* to all our dealings,

to the much more limited:

(b) General justice is justice for all our *common dealings*.

Aquinas does not appear to notice the (in modern terminology) reduced scope of the universal quantifier; rules for (a) will include rules for (b), but they are far from co-extensive. The established Thomist interpretation of *universal* justice was (b), and is retained to this day by the many writers in fields of law, politics, and jurisprudence, influenced (knowingly or not) by Aquinas[267].

Placing *universal* and *particular* as mutually exclusive classes has led to many difficulties for jurisprudence; the principles appear to leave lacunae and demarcation conflicts that have proved impossible to resolve[268]. Were it not for his non-sequitur when presenting General justice St Thomas

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[266] *Summa XXXVII* 2i-2ii; translations Gilby, slightly amended.

[267] Including John Finnis, for example in chapter 8 (*Distribution, Exchange, and Restitution*) of his forthcoming work on Aquinas, and in *Natural Law and Natural Rights* (1980, pp.164-65).

[268] Ibid. pp.178-79. Finnis discusses the complexities further in 1997; if I do not misrepresent him Professor Finnis says that Aquinas was undecided as to the species of justice, and to the seriousness with which they should be held. In his early *In. Sent.* St Thomas treated the species of Particular justice as two among others, including *vindicatio, observantia,* and *innocentia* (*III, Sent.* d.33 q.3a. 4 sol. 1c and sol. 2c). I don't quite know what Aristotle would make of *observantia* (promise-keeping and obedience to law) and

need not have been fearful for the elusiveness, or the seeming overlap or conflict, of rôles in the classifications of justice. But inevitably treating General justice as merely (b)—justice for dealings with the general community, rather than (a)—justice general-to-all-dealings, whether general or specific, results in confusions[269]. For Aristotle General justice is not limited to (b), but embraces (a), pervading all moral dealings. Within this General justice every feature of right action that has to do with equality or greater and less is governed by the sub-genus of Particular justice, and more directly through its appropriate sub-divisions.

It was this illegitimate restriction from what is common to *all* interactions to (merely) our *common* interactions that set the stage for the tripartite political divisions of law. Universal justice was seen as a *part→whole* connexion relating individuals or lesser groups to the state. Particular justice was then correspondingly perceived to govern either (i) *part→part* connexions relating any individual or internal group to any other part of the community, or as (ii) a *whole→part* relation governing the individuals or groups by the central whole authority. Such ideas were developed rather more by Thomists such as Cajetan and de Soto, and by others such as Bodin[270], than by Aquinas himself, but they were more than adumbrated in the *Summa.*

There has also been a further, oblique and unexpected, consequence of the principle of a tripartite construction of the genus. Variations of the three part division of justice became so well-established by end of the Middle Ages that the thought that Aristotle divides forms of justice into three species would appear natural enough to a writer in the 17th century, when just such a new and hugely influential tripartition was proposed (discussed below).

innocentia (not harming or depriving others), but I'm inclined to think he would treat them as distinct virtues, along-side e.g., even-temper, courage, or friendliness. He certainly designates *vindicatio* as a specific virtue—νεμεσις (1108b1), *not* as a species of justice. Finnis says that eventually, via the *Commentary* (but presumably also via the *Tabula Ethicorum*) St Thomas came to the position of the *Summa* (given above). That position reflects Aristotle's account of the species of Particular justice.

[269] Finnis aims to resolve the conflicts by treating the classes of justice as no more than vague markers, which is hardly a resolution; but it is a response to what need not be a problem.

[270] J.Bodin *Six Books on the Republic* (1576).

4. 8 Particular justice

Having objected to Aquinas's treatment of General (Universal) justice, I support his interpretation of Particular justice. *Particular* falls within *universal* (General or Legal) justice (rather than being a class parallel to it), and is a genus of *legal* justice in that it treats the rules of General justice encoded in law. It deals with equality[271]; wherever issues of greater or less are at stake it is the job of Particular justice to adjust the amounts, until the fair balance is found. There is an intimate association between Particular justice and Equity (ἐπιεικες): Equity is concerned with the appropriate equality for those circumstances where rules have not been provided (1137b12ff.)[272].

The province of Particular justice is wider than that of *suum cuique tribuens*, as innumerable myths, legends, fairy stories, and tales, from the *Oresteia* to the *Merchant of Venice,* testify. Although rendering what is due is taken by many to form the whole of justice[273] it often happens that what is due cannot be rendered: the pristine value has often been destroyed, and *sunt lacrimæ rerum*. Of this almost all philosophers of justice seem oblivious, and proceed as if the *status quo ante* is, generally speaking, restorable; and is the rule rather than the exception. Aristotle's theory has at least the virtue that his model allows for new balance to be found. The factors in a judgement, either

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[271] *MM I* 1193b19 (Stock):

> The just, then, in relation to one's neighbour is, speaking generally, the equal. For the unjust is the unequal.

[272] Some rules of law have arisen through Equity; in England law for insolvency developed through it, i.e., through Mercantile law, though partly and indirectly *via* Canon from Roman law.

[273] The outstanding modern supporter of justice as no more than the rendering of what is due is probably R.Nozick. In *Anarchy, State and Utopia* (1974) he argues for a purely 'entitlement' theory and against 'end-state' theories (such as Aristotle's, though without mentioning Aristotle) which look to some desired outcome to render an action just. Nozick, however, acknowledges what are in effect Aristotelian principles for a certain difficult sort of case (p.153n), to which should be added a related but distinct sort not noticed by Nozick. Where the claim to entitlement is unresolvable in terms of entitlement (i.e., where what is to count as entitlement is the issue), for example in cases relating to dispossession as with aborigines in Australia and native Americans, or certain evils consequent upon slavery, some *other* principle has to be invoked. To such sorts of case should be added insolvency; this issue has (virtually) never been treated by philosophers (excepting to some extent Finnis 1980, pp.185-93), and Law in this field derives not from statute but Equity. In Insolvency the difficulty is not 'Who is entitled?' because all creditors, by definition, are entitled. In neither of these areas can the concept of justice as rendering what is due be adequate. Entitlement itself must rest on the very principles of Distributive justice Nozick rejects. On p.153 Nozick recognises that "the principle of distributive justice and equality" is required, but thinks it is needed for only "subsidiary" cases. Not so, cases where rendering what is due is impossible, either through insolvency or the impossibility of discovering entitlement, still require justice to be done. For a very large number of the cases with which legal systems have to deal, including the great majority of the *difficult* cases, entitlement is not the main issue.

new or old, can be assessed as to the greater and less. What it is that is to be assessable to create the right and appropriate equality is exactly the same for every kind of case: ἀξια (worth, merit, desert, or value)[274], which therefore forms the subject-matter of justice. Different kinds of problems will require proportionally different solutions; Particular justice will add or divide, multiply or subtract, the relevant factors to achieve the equal value. Justice is the *right* equality; i.e., the fair equality is in place where the *value* among the factors involved is made equal. This will require a 'second-order' principle which must take into account the equitableness of the relevant equalities.

For Aristotle the essences of Particular justice and of proportionality correspond: both can only apply when *some* principle of equality is acted upon. For such reasons as these he uses models for justice taken directly from proportion theory, and applies them consistently throughout. His remark that in political science, of which ethics is a part, we should not look for the precision we expect in mathematics (*NE I* 1094b13) should not be taken as meaning that exact principles are really only vague. If precision is much less easily attainable, all the more reason to use it as far as it will go. The 'mathematical' references should not be thought of as vague markers or regrettable idiosyncrasies, as they so often have been; they are not dispensable elaborations unsuited to the subject, but the bases from which to launch the whole project. By getting the analogical, i.e., mathematical, inferences clear the entire theory is clarified.

4. 9 Distributive justice

Aristotle often says with perfect clarity that Particular justice is divided into two sorts; this has not prevented many from claiming he really means three. About the first species there has never been any disagreement, at any rate about what it should be called, and its scope as it is first presented. About the second, and a putative third, controversy has been enormous—we come to these after considering the first.

The first species of Particular justice, to which Aristotle devotes chapter 3, is unanimously called Distributive. It is the justice which deals with all issues involving the common stock, i.e., those matters that are the business of the community *as* the community. The well-ordered community holds resources for the common good. It exacts duties and assigns benefits to its

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[274] *Liddell & Scott.*

members. The *polis* may retain wealth, places of shelter, stocks of materials, food, and medicines. Duties might be imposed to ensure the security of these common goods, and for the safety of the *polis*. Tasks and services will have to be performed, and rewards may be bestowed. People vary enormously in their contributions to the common good, and in their needs or standing; and to the extent to which they share the giving and receiving of the resources. The job of Distributive justice, among these fluctuations, is to regulate the distribution and redistribution of all things relating to the common stock. Following Plato Aristotle holds that the distribution of what arises as issues to be decided should match the ἀξίαι of the parties concerned. Sharing-out according to quantitative equality would be plainly inequitable (an active stevedore, for example, could need much more protein than a sedentary octogenarian—who might require far more medicine) and so would be the inappropriate (unfair) mode of equality to apply. Aristotle says that the right equality is found through the geometric rule. It is a 'geometric' disposal since it follows the geometric proportion; the equal *axia* is thereby preserved, and justice prevails. (The principle of geometric proportion is outlined in §§ 1.7-1.8.) On the basis that all citizens are comparable in worth, *some* ratio will quantify that comparison. Corresponding multiples will then achieve the just distribution of whatever is in question. How the worth of the parties is decided in the first place Aristotle says will vary according to circumstances[275].

It must be emphasised that Aristotle uses the geometric proportion *only* for Distributive issues, *and* that Distributive (geometric) justice does not apply to any area of exchange. In not understanding the proportionality at work—and therefore the structure of the theory—many writers have supposed that Aristotle applies geometric (distributive) rules to commercial associations. This mistake has resulted in extremely complicated distortions of the theory. There is a passage at the end of chapter 5 (1134aff.) which might seem to contradict what I argue for the authority and extension of Distributive justice. Aristotle there speaks of someone "distributing things between himself and another or between two others". First it should be noted that this passage occurs after the discussion

[275] 1131a24ff.; democracies, oligarchies, monarchies, and so on, are likely to assess value differently. It seems reasonable to extend this flexible principle to quantify the constraints of scarcity or even, perhaps, the exigencies of fashion. Treating qualitative differences quantitatively is not so artificial as it might seem; it is the standard practice throughout public life, industry, and commerce. (MP's income, e.g., is pegged to a certain grade of the Civil Service, and Judges are rated more highly than prison officers by exact quantitative degrees.) See also § 6.9.

of justice in exchanges has been completed; then that exchange or commercial interaction or agreements are not mentioned. The just individual will act as the *polis* acts; this accords with the above argument for the scope of Distributive justice. There is a feature of the Thomist tripartite account of justice that is useful in clarifying who has authority to administer Distributive justice. In the view of writers such as Cajetan, de Soto, and in this century B-H Merkelbach[276], Distributive justice guides the *whole→part* relation of the *polis* to its members[277]. It would therefore seem inappropriate for 'authorities' other than the *polis* to implement it. Although I think that identifying the *whole→part* function as uniquely Distributive is not what Aristotle proposes[278], Distributive justice, as a branch of Legal justice (see above § 4.6), is the responsibility of the *polis*, and not, it would appear, of any individual or other (and necessarily lesser) group. It seems to me that Aristotle would grant an analogical extension for an association that was sufficiently *polis*-like. Similarly, as just mentioned, where an individual distributes things (1134aff.) the geometric rule applies insofar as Particular justice is involved exactly because such distribution is *not* an exchange. To the extent that some association is a structured community (as might be certain business, charitable, sporting, or industrial ventures) it may take on, by courtesy, principles of Distributive justice. But the justice

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[276] Quoted by Finnis 1980 p.185, n.26.

[277] Finnis (1980 pp.185-88) holds that Aquinas's own position differed from those of his followers. Aquinas's view, Finnis says, is that individuals

> in charge of an item of 'common stock' will have duties of distributive justice; hence any property-holder will have such duties.

But this is only a question of what counts as the common stock. St Thomas's view of who might be lawfully entitled to make Distributive (or any other) judgements is given in *Summa II-II* q.61: a judge must act for, and be appointed by, public authority.

[278] *Whole→part* constructions are inadequate as models for Distributive justice in that they fail to show all the distributive issues. There is not only the flow from the central authority to its parts of goods to be regulated (geometrically) according to merit (of payments, honours, supplies etc.), there is also value taken from the members by the *polis* for the common good—such as taxes, jury service, or conscription. These are usually guided by 'geometric' rules. Aquinas saw such *part→whole* flows as ruled by General (Legal) not Distributive justice. But General justice is not regulated by modes of equality of any sort (as argued above in § 4.7 against the view that General justice only treats the *polis's* dealings-in-common), General justice would determine a rule that taxes or conscription are needed; it would not specify which degrees of greater and less should apply. That is the province of Distributive justice. Even claims that taxes should be flat-rate, not proportional, are distributive claims. Flat-rate taxes such as the Community Charge of the 1980's would appear to offend Aristotelian principles for 'geometrically just' reasons. Flat-rate charges on common goods, however, are not widely thought to be unjust, but they might be if imposed on necessaries. Conscription does not appear, initially, to be applied 'geometrically', but it conforms to Aristotle's principle of the relevant ἀξία, in that it is generally limited to the youngish, able-bodied, adult, (and until recently) male section of the *polis* (conscription of children or the elderly if able-bodied younger adults are available would be a *distributive* absurdity).

which governs commerce properly falls under the second species. Strictly, for an individual or group within the *polis* to distribute from their own resources would not be a matter of justice, however it was divided. In times of urgent need the *polis* will requisition the stocks needed and the resourceful individual will be its agent.

Mixing-up distributions from *the common stock of the polis* with 'distributions' in lesser κοινωνιαι such as business partnerships has been misconstrued since at least the time of Albertus (or Heliodorus, whenever that was[279]). Endorsed by Stewart, Trendelenburg[280] held that Distributive rules κατ' ἀξιαν applied to the sharing of the common stock *or* exchanges of trade[281]. Grant (p.108) said

> in all bargains the principle of geometric proportion comes in (which does not belong to corrective justice) ... With regard to this principle the text is not explicit, yet it appears to be (1) ... applicable in all cases of awards made by the state, (2) ideally to be capable of a wider application as a regulative principle of distribution of property and all the distinctions of society,

which is the reverse of Aristotle's theory. Yet it has been disastrously influential; the gross, illegitimate, elevation in (2) of κοινωνιαι *other* than the *polis* has made endless mischief for the understanding, and even for the later translation, of the text. Jackson, who accurately translates the (1131b26ff.) passage, nevertheless in his comments (p.76) says

> it is obvious that his [Aristotle's] remarks also apply to smaller κοινωνιαι such as companies of merchants or manufacturers.

Stewart goes further (pp.432-33) he aligns an officer (i.e., a high official representing the *polis*) distributing prizes with a board of directors apportioning dividends. He even raises the payment of wages and dividends as

> far the most important form of [distribution which] ... results from the operation of 'economic laws' regulating wages and profits.

Which, of course, they may well be, but they are not the Distributive principles Aristotle assigns to the *polis*; they are transactional, and governed by the justice of exchanges (and not the geometric

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[279] He is traced to 1367 but neither his real name nor the millennium in which this legendary paraphrast lived is known. R.Sorabji comments (ed., *Aristotle Transformed: the ancient commentators and their influence,* p.23 n.106) that it might have been anytime between the 3rd century AD and 14th (see § 6.2).

[280] F.A.Trendelenburg *Historische Beiträge zur Philosophie,* vol.3, p.412 (see *Stewart* p.415).

[281] As will be shown below, associations *other than* (and necessarily inferior to) the *polis* are, in the only relevant sense in the text, *private* (i.e., where the *polis* is not involved). Private interactions are governed through the second species—the justice of exchanges. *Joachim* (pp.138-39, 143-44) also reads into the text distributions of the common property of a joint-stock company or club (but he does not appeal to the *Politics* as others do).

form of proportion which these writers imagine). Trendelenburg, Grant, Jackson, and Stewart, mingle the very distinction that Aristotle is at such pains to keep apart, with the result that a subsequent translator (Ross) will actually *translate* the passage 1131b29-31 as referring to commercial activities. Ross (1925) inserts words such as "partners" and "business":

> in which the distribution is made from the common funds of a partnership it will be according to the same ratio which the funds put into the business by the partners bear to one another.

Relying on this extraordinary anachronism Soudek quite naturally confuses issues of exchange with those of distribution; he cites Burnet to support this privatisation of the state's function, but Burnet is outstanding among the leading commentators in not being drawn-in to the confusion of the geometric and arithmetic proportions—and the consequent overflowing of the Distributive principle. Burnet refers only (p.218) to Aristotle's illustration of contributions (τα εἰσεχθεντα) by the wealthy to the *polis* in war-time[282]; he doesn't mention any supposed business partnerships[283].

It was Grant who (p.113) linked the passage 1131b26ff. to *MM I* and *Politics III*, chapter 9. Stewart says of *MM I*

> It is interesting to compare in this connexion the remarkable passage MM.i.33.1193b36-94a25, in which distributive justice is described as determining the returns of labour, and regulating the exchanges which in E.N.v are discussed in the chapter on το ἀντιπεπονθος.

The connexion had been made by Jackson, and through Stewart's influence[284] especially it went on to be made by many others, including Ross, Soudek, Gauthier-Jolif, Marc-Wogau, McNeill, Keyser, Miller, Sparshott, and Judson[285], making it the orthodox account of these texts. The custom has been

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[282] They were also required to pay for the frequent festive liturgies (χορηγιαι). The Athenian legal system had an ingenious double-bluff to catch-out tax dodgers: they could be challenged to an exchange of properties at their self-assessed valuation (J.Davidson *Courtesans & Fishcakes* 1997, pp.238-46). See also § 6.7.

[283] Anachronistic distortion is a central issue separating 'primitivist' from 'modernist' readings of chapter 5—see § 6.5.

[284] His extension of the geometric, Distributive, principle is in some ways rather odd as Stewart himself says (p.423, in referring to the *Politics*) that Distributive justice is concerned with the perfection of human life, not an insurance or joint stock company.

[285] (i) *Soudek* (pp.58, 61-62), using Ross's translation, which explicitly imports commercial associations into the text, believes that the geometric formula operates in chapter 5 of *Ethics V*, as well as in the *Politics*. Soudek then says (p.63) that:

> As long as the problem is that of reducing skills to want satisfaction for the sake of establishing equality, distributive justice is at work.

He goes on to combine arithmetic and geometric proportions in a way that confuses the issues of exchange-value (chapter 6, below).

(ii) Gauthier-Jolif (1970 II, pp.360-61) believe that Aristotle surreptitiously introduces the geometric (distributive) principle to issues of commercial exchange at 1132a2ff. Weinrib (op. cit. pp.135-36, note 9) objects (rightly, in my view) that Aristotle would not rely on the geometric rule at the very point at which he distinguishes *that* rule from the arithmetic.

to add the *NE, EE,* and *MM* passages to chapter 9 of *Politics III* where, because they suppose that the Distributive function extends to trading associations, correspondingly suppose that the *Politics* chapter also extends to these lesser κοινωνιαι. Which is as gross a distortion of the *Politics* chapter(s)[286] as it is possible to make: it reverses the whole point. The thrust of the chapter is to show the *polis* as qualitatively distinct from, and far superior to, any other association that might aspire to its elevated status. To take its characteristic virtues, which have been presented to contrast with the ἐργα of lesser organisations (trading companies, or even 'night-watchmen' states), as applying to just those lower structures, stands Aristotle's whole argument on its head. There is not, and could not be, any extension of the principle which marks-out the πολις *from* other groupings *to* these inferior organisations, either in the texts, or in the spirit of Aristotle's theory. Any temptation to extend the Distributive principle analogically is forestalled not because Aristotle hasn't thought of it but because he makes detailed provision for all commercial and other associations in his treatment

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(iii) K.Marc-Wogau (*Philosophical Essays: History of Philosophy, Perception, Historical Explanation*, Library of Theoria X: *Aristotle's Theory of Corrective Justice and Reciprocity* 1967, pp. 21-40) falls foul of the vagueness in the language of proportions (examined in § 2.1). He takes all proportion to be geometric, and that the arithmetic is only a "special case" of the geometric proportion. He then treats the geometric as being re-introduced to chapter 5 to handle commercial exchanges (or rather, that Aristotle never leaves the geometric proportion at any stage).

(iv) D.McNeill (p.60 in 'Alternative Interpretations of Aristotle on Exchange and Reciprocity', in *Public Affairs Quarterly* vol.4 no.1, 1990, pp.55-68) says that for Aristotle exchanges should be on a geometrically proportional basis in some ways like the distribution of booty. It differs in that in Distributive justice there is a 'conjunction' of citizens and their shares, where in exchanges there is a 'cross-conjunction' (see chapter 6).

(v) P.Keyser ('A Proposed Diagram in Aristotle EN V. 3 1131a24-b20 for Distributive Justice in Proportion', in *Apeiron* 25ii, 1992, pp.135-44) supposes (pp.138-39 and n.11, 143) that the assignment of the shares to the parties according to their worth (1131bff.) refers to their wealth. Fortunately this doesn't damage his proposed model (see § 4.10).

(vi) Sparshott (1994, pp.169-70) jumbles the arithmetic and geometric principles so far as to remove the distinctions Aristotle is set on preserving. He complains that Aristotle:

> slides into this account of distributive justice, which is firstly said to be merely one species of justice, the other being "rectificatory". It is not clear to the reader exactly when the discussion stopped being about justice (as fairness) in general and began to be about distributions in particular.

This travesties Aristotle. Aristotle *starts out* treating (Particular) justice as fairness—the appropriate equality, either as Distributive or as Transactional (to be argued below), and not as Sparshott, following countless others, thinks 'rectificatory'.

(vii) F.D.Miller (p.71, in *Nature, Justice, and Right in Aristotle's Politics,* 1995) also thinks that Aristotle extends Distributive justice to commercial ventures at the very place where he is actually explaining the unique superiority of the *polis* over any such lesser associations.

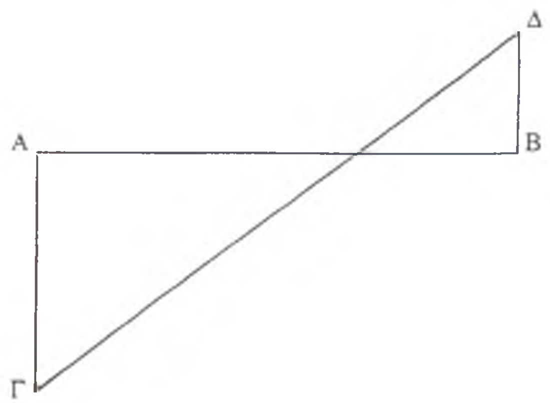
(viii) L.Judson (draft copy of 'Aristotle on Fair Exchange', in *Oxford Studies in Ancient Philosophy,* Autumn, 1997) bases his analysis on Aristotle's use of a geometric model of proportion for chapter 5.

[286] In Book II, chapter 2 Aristotle also speaks of the qualitative priority of the *polis* over any other form of association.

of exchanges. Aristotle does not mention geometric proportion in chapter 5 (see chapter 6 below), the reason he does not use the *arithmetic* proportion in the chapter, even though it governs all exchanges, is that he is by then no longer concerned to explain the justice for the act of exchange—he had just done that in chapter 4—but to assess the fair *value* of what is to be brought to an exchange. What has tricked so many into imagining him to be returning to the geometric (distributive) formula is the plain fact of his leaving the arithmetic.

4. 10 The diagram for distribution

Having laid out the order of Particular justice at the end of chapter 2 Aristotle goes on to explain justice in terms of proportions and means in chapter 3, speaking of separated and continuous proportions. In doing so he points to a diagram (1131b)[287] to explain Distributive justice. It is for this passage that the MSS preserve the alternate methods of coding (mentioned in §§ 1.3, 1.8, and notes 63, 65). What diagram is Aristotle pointing to ? The traditional answer has been the square format as mentioned in § 1.6[288] but Keyser (op. cit.) suggests a different pattern. The lines referred to are, he says: A-B, which is the line representing the link between the parties A and B, and Γ-Δ, a line between the extremes of what is to be shared between them. So far this is a fairly standard account, but the lines are not then drawn-up into a square, but cross so as to create the basis of a pair of similar triangles (such as those in *Elements VI* in which the rules for producing mathematical models are presented—see §§ 2.8, 2.9, and note 144):



(figure 2)

[287] A second diagram is said to be indicated at 1131b6 (see Rackham p.270(c), or Thomson p.146); if there are two, it is the second that is under discussion.
[288] And by Burnet, for example, 1900, p.216.

Keyser's proposal shows the pattern of the argument more vividly than the diagrams it replaces. It also seems to me to explain Aristotle's immediately preceding allusion to the difference between continuous and disjointed proportions (although Keyser (p.137) thinks that Aristotle's bringing in this distinction is "irrelevant to his discussion"). The point at which the two continuous proportions cross allows the figure to be viewed both as two distinct similar figures, *and* as two related continuities. This triangular model also leads to a more general conclusion than Keyser himself sees. In §§ 3.2 and 3.5 I mentioned Aristotle's replacing the method of dichotomous division—which privileged some given attribute of living things—with a cluster of vital attributes needed to define or classify animals. The beauty of a mathematical (in the present case a geometric) model is that it preserves clusters of attributes with no need to privilege any one. To the question 'What are the "wholes" referred to in the expression και το ὁλον προς το ὁλον ("and the whole to the whole") at 1131b7 and again at 1131b15 ?'[289] Keyser (p.139) says they are the whole lines. From this he draws certain conclusions against the traditional reading of the proportions. Karasmanis[290] also, though without the benefit of Keyser's diagram, claims that the traditional representation of the proportion is mistaken. Keyser and Karasmanis both say that all scholars support the traditional reading of the proportion[291] (which they give slightly differing formulations[292]) whereby primacy is given to the relation between the parties (A and B). They replace that primary ratio with one between a citizen and a share (A : C)[293]. I am not especially drawn to their new formulae, as it seems to me that Aristotle gives priority to the relative standing of the citizens, but from the point of view of the lesson which I think Aristotle takes from his diagram, it doesn't make any difference which inference is given priority. The mathematical model guarantees all (valid) sequences. The "whole to the whole" is not a reference limited to the whole lines, nor to one proportional sequence rather than

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[289] Rackham and Thomson ignore this phrase even though it is repeated.
[290] V.Karasmanis 'The Mathematical Passage in "Nicomachean Ethics" 1131b5-15', in *Ancient Philosophy*, 1993, pp.373-378.
[291] Cited writers are Ramsauer, Jackson, Bywater, Stewart, Burnet, Heath, Ross, Rackham, Joachim, Ostwald, Dirlmeier, Gauthier-Jolif, Hardie, Νικολευδης, von Leyden.
[292] Keyser presents the standard reading (p.138) as: $a : b = (a+c) : (b+d)$.
Karasmanis is more elaborate:
$(a : b = c : d) \Rightarrow (a : c = b : d) \Rightarrow (a+b) : (c+d) = a : b.$
[293] They prefer (Keyser): $(a+b) : (c+d) = a : c.$
(Karasmanis): $(a : b = c : d) \Rightarrow (a : c = b : d) = (a+b) : (c+d).$

another[294], but to the relation of one *whole triangle* to the other. Given that Aristotle points to the diagram, and that that diagram is likely to be of a pair of similar triangles, *any* sequence of valid inferences will follow equally "as the whole is to the whole", exactly because one figure is a model for the other, preserving a whole cluster of vital inferences. The advantage in using the geometric figures is that they display *all* the relationships of the elements represented (and not merely any one particular mathematical sequence).

Karasmanis usefully emphasises the term συζευξις as meaning *coupling, yoking,* or *assignment*, and not, as it is sometimes rendered, *addition* or *sum*[295]. The term indicates not the addition but the joining of the kind of objects (sometimes heterogeneous) that *would* go together[296]. The *polis* will not "add" jury-service or a reward of money to a citizen, but it might *assign* these to its members. The idea of assignment, rather than addition will be useful for understanding chapter 5.

In speaking of A and B standing in a ratio shown by the line connecting them being divided at some point, and of Γ and Δ being shared according to that ratio, what is being discussed is value. In distributions the value of the shares is principally for the use of the recipient (and a duty would need to be actually performed). If whatever is of value is not used it might be retained for later use, or it might be exchanged for something else that is needed or preferred. Aristotle is credited by Marx, for example, with first distinguishing use- from exchange-value[297]; it seems quite possible that it was in reflecting on the differences between Distributive and the other form of Particular justice that led him to the contrast. In Aristotle's theory the relations between the *polis* and its parts or members are *public* relations. The *polis* may (and will have to) engage in trade with some of its members, but in such associations it acts (or ought to act) merely as a party to an exchange—*as if* it were a private body. In the treatment of exchanges the *polis* acts as an arbitrator, it not being a party to the case; but in distributive issues the *polis* itself is directly involved. The Thomists (§ 4.7) construe Distributive justice as a *one-way* relation of the *polis* to its members; what they fail to capture is that

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294 Karasmanis says (p.376(i), and note 8) that "the whole to the whole" denotes the ratio of the sum of numerators to the sum of denominators, and cites *Elements V* prop.19, *VII* props. 7, 8, and 11.
295 By Welldon, Thomson, and Rackham, for example.
296 Gauthier-Jolif refer to the term designating objects with a particular affinity: male and female, night and day, sleep and waking. *Liddell & Scott* indicate *yoking* as the underlying thought, and give marriage as the prime example. Something like *horse* and *cart* might be nearer to the sense here.
297 At *Politics I* 1257a6ff.

although it is one-way, it is one-way-at-a-time. Distributive justice governs both the flow from the *polis* to its members and the flow from its constituents to the central authority (§ 4.9, note 278). Interactions and exchanges are private not public affairs and, rather obviously, their justice is a two-way process (in relation to the *polis* they are *part* ⇆ *part* relations), in which the *polis* acts as a judge between them[298].

[298] See also § 6.2.

Chapter 5

CORRECTING CORRECTIVE JUSTICE

5. 1 The second species

Where everyone agrees that the first species of Particular justice is the Distributive, characterized by the geometric proportion, there is very little agreement about the remaining sort. There is complete agreement on only two aspects of this second class of justice, (i) that it is *in some sense* 'the corrective', and (ii) that the account of it in the text is unsatisfactory. I believe both these ideas to be wrong; the second species is no more or less corrective than the first, and Aristotle's notion of the second is very well considered. There is also a feeling, not quite so unquestioned, but still very widespread, that this *diorthotic* species, even when combined with the Distributive, leaves out important areas of human affairs which Aristotle goes on to discuss under a further, though obscure and undeclared, heading. These (erroneous) notions stem from the failure to appreciate the structure of the theory. Aristotle does not propose a diorthotic *eidos*. The thought that he does has generated a bewildering range of names for the imagined species, but *prima facie* there is something fishy about a subject where there are so many attempts to capture its sense, and so little agreement even about what it should be called. It has been variously labelled Aequatrix, Collective, Corrective, Compensatory, Constitutive, Diorthotic, Directive, Emendatory, Epanorthotic, Equalising, Equating, Judicial, Parifying, Rectificatory, Regulative, Remedial, Reparative, Restitutive, Restorative, Retributive, Retrospective, and as justice in Redress. Sometimes some of these names are used indifferently but more often they have been carefully chosen in preference to the others[299]. Also some

[299] Shuchman and Webster use *remedial,* as does Ross in 1923; Ackrill, Annas, Barker, Hardie, Haren, Joachim, Judson, Lloyd, Lowry, McKeon, McNeill, Prior, Rowe, Ryan, Sparshott, and Urmson prefer *rectificatory;* as does Ross by 1925. There Ross, following Burnet and Stewart, uses *corrective* but only for the single entry επανορθωτικον, as does Moraux. Gomperz, like Burnet, generally prefers the scholastic *directive*; Kiernan gives *collective.* Ferrari and Barthélemy-Saint-Hilaire use *reparative,* Kirchmann has *constitutive.* Victorius and Eikema Hommes employ *restitutive*, *emendatory* and *compensatory.* Trojano and Donati speak of it as *judicial* as do Trendelenburg and Keyt, in addition to *corrective.* Bowie, Mercken, Petrone and Spicer prefer *retributive.* Aubenque, Bern and Engberg-Pedersen refer to "*corrective or commutative".* Others, including Appleton, Bodenhaimer, Browne, Bywater, Cairns, Castariadis, Chase, Dias, Edel, Epstein, Felden, Finley, Finnis, Fletcher, Flew, Freidmann, Friedrich, Gauthier, Grant, Grote, Hamburger, Hantz, Heath,

of these names, in addition to others such as Catallactic, Commercial, Reciprocal, Retaliatory, and Commutative, have been assigned to the supposed third class which Aristotle is said not to have distinguished clearly from the second (see § 6.1). Hardie attributes to Ross[300] the view that Aristotle treats the justice of exchanges in chapter 5 only as an afterthought. The claim had been made much earlier by Peters (in 1881), and by Ritchie and Richards in the 1890's[301] that this class amounts to a third category of justice.

The idea of a 'corrective' species succeeds a much older tradition going back to St Thomas, who read Aristotle as exhaustively dividing the issues into the Distributive and the Commutative. 'Corrective' and 'commutative' are as different as chalk and cheese, so it is remarkable that they should ever have been taken to designate the same subject. Since I maintain that Aquinas's presentation was accurate I shall need to defend it in some detail where it conflicts with all the modern(ish) accounts. (That defence should also help to make plain the actual model of the sorts of justice which Aristotle proposes.)

The term *iustitia commutativa* (commutative justice) was introduced for the second species by St Thomas in his early work *In Sententia*[302] (and perhaps originated from his taking notes from

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Hooker, Husserl, Irwin (who also calls the species *retrospective*), Jackson, Janet, Jolif, Kelson, Keyser, Lapie (in addition to *rectificatory*), Lee, Lewes, van Leyden, Lucas, MacIntyre, Marshall (together with *regulative*, alongside Dareste and Rashdall) Meikle, Miller, Monro, Myers, Newman, Oates, Ostwald, Peters (who also uses *rectificatory* and *redress*), Posnar, Preston, Pufendorf (who refers to this also as *commutative*, though in a more restricted sense than Aquinas's), Rackham, Raphael (and Spengler: *rectificatory* or *corrective*), Ravaisson, Richards, Ritchie, Rosen, Sherman, Shorey, Smith, Solomon, Soudek, Springborg, Thomson, (who also gives *emendatory*), Tricot, del Vecchio (who, influenced by Vico's *aequatrix*, also uses (translated by Guthrie) *equalizing*, *parifying*, *rectifying*, and *equating*), Vinogradoff, Waluchow, Weinrib, Welldon, Williams (B), Williams (R), Winthrop, Zeller, and Zuccante (interchangeably with *compensatory*) prefer *corrective*, which, democratically, I adopt for the time being. Only Gronovius's expression *iustitia contractoria*, written in 1720, approximates to Aquinas's notion, treating the species as transactional: to do with exchanges.

300 Hardie (op. cit., pp.194-95; Ross (1923 pp.10, 212-14) calls the 'third' species the Retaliatory.

301 F.H.Peters *The Nicomachean Ethics of Aristotle* p.148 (Peters may have got this idea from page 291 of Hildenbrand's *Geschichte und System der Rechts- und Staatsphilosophie* of 1860). *Ritchie* p.191, refers to the species as 'catallactic' or commercial justice, as does H.Richards in *Classical Review vii*, 1893, but he does not say it was an afterthought. Donati *Fondatione della Scienza del Diritto* (1929, p.22) follows Trojano *I Primordi della Riflessime Morale ed Economica in Grecia* (1897). Much more recent appearences of the doctrine occur in M.Haren *Medieval Thought* (2nd ed., 1992), Miller (p.300n.50 in Keyt & Miller 1991, pp.279-306), who thinks the place of "the reciprocal" is obscure but he nevertheless sees it as a distinct species; *Meikle* pp.156-81.

302 *In Sent.* was written in the mid-1250's; Aquinas refers to justice "*iustitia distributiva, et commutativa*" at II, ds.27, q.1 ar.3, and repeatedly thereafter: ar.3, r.4; ar.4, r.2; III, ds.18,

Albertus Magnus's lectures on the *Ethics*). St Albert had come rather close with the expression *iustitia communicativa*[303], but generally he preferred the term *iustitia directiva*[304]. Aquinas referred to Commutative justice thereafter throughout his life, and particularly in his commentary on the *Ethics*, and in the *Summa II-II*, both written around 1270-72.

The notion of 'corrective' justice as an explicit nomination has been fully accepted since at least the time of *Jackson*, but in fact it is much older; the principle that the second species is in some sense the *diorthotic* was introduced in 1653 by Felden in his objections to Grotius's remarks[305] on Aquinas's account of Aristotle's theory. It was here I think that the rot first set in; he simultaneously introduced the principles that the second species is the Corrective, and that Aristotle distinguishes three, not two, species of justice:

> sed statuit is tres species justitiae: (1) διανεμητικην distributivam, (2) διορθωτικην correctivam, (3) το αντιπεπονθος retributivam.[306]

These ideas soon began to be discussed, by Boecler for example in 1663; they were accepted by Pufendorf (1672) and have been in debate ever since[307]. In accordance with the predominant, the virtually unquestioned assumption that the second species is the diorthotic[308] (wholly apart from issues of a 'third species'), by not treating the second species as *specifically* in some sense the Corrective Aquinas is held to have misrepresented Aristotle—it has been a matter of debate whether this 'misrepresentation' was deliberate.

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q.1 ar.2; ds.33, q.1 ar.3; q.3 ar.4; IV ds.14, q.1 ar.1; q.2, ar.1; ds.46, q.1. Also it appears in the *Summa* and in the *Tabula Ethicorum* in addition to the *Commentary*.

303 Commenting on *NE* 1131a1-9 (*Alberti Magni Opera Omnia XIV-1 Super Ethica* Aschendorff 1968) he says "*et ideo divisit communicativam iustum*" (ref.391, p.330:95), and at 391, p.331:8 "*et sit circa hoc potest esse communicativa iustitia*." At 392, p.331:14-15 "*quae in communicationibus directiva*".

304 As at 389, p.329:8; 399, p.338:40, 66-67; 400, p.339:2, 23-24, 72 (ibid.).

305 Grotius (*De Jure Belli ac Pacis*, 1646) referred to the second species as the *Expletrix*.

306 J. a Felden *Annotata in H.Grotium De Juri Belli ac Pacis* Amsterdam 1653, p.10.

307 J.H.Boecler *In H.Grotii Jus Belli ac Pacis Commentatio* 1663; S.Pufendorf *De Jure Naturae et Gentium* 1672. Discussed or referred to by, among others, Thomasius, 1688; Buddeus, 1697; Voet, 1698; Glafey, 1723; H and S Cocceji, 1744; Zanotti, 1754; Michelet, 1848; Trendelenburg, 1867; Ramsauer, 1878; Peters, 1881; Richards, 1893; Ritchie, 1894; Trojano, 1897; Burnet, 1900; Masci, 1911; Ross, 1923; del Vecchio, 1952; A.R.W.Harrison, 1957; Hardie, 1968; Eikema Hommes, 1979; McNeill, 1990; Miller, 1995; Meikle, 1995; Hooker, 1995; Judson, 1997.

308 So rare is it not to speak of the second species as in *some* way the diorthotic that among commentators other than in the strictly Thomist tradition I am aware only of Ueberweg (1903) and D.S.Hutchinson (*The Cambridge Companion to Aristotle*, ed. J.Barnes, 1995) who clearly avoid it.

Was Aquinas's 'misrepresentation' just a mistake, or did he make a deliberate change? Ritchie influentially held that Aquinas mistakenly thought he was presenting Aristotle's own views:

> From all that he says in this part of the *Summa* and also from his *Commentary on the Ethics* it is quite clear that Aquinas considers that he is only following the opinion of 'the philosopher'[309]

Gauthier and Jolif, following Ritchie, hold that Aquinas was confused by the text, being misled by faulty translation:

> l'exegese thomiste repose essentiellement sur un contresens, occasione par l'ambiguite de la traduction latine qu'il utilisait.[310]

There are two ways in which St Thomas could have been confused: (i) if the text he relied on misrepresented the original, and (ii) if he himself simply misread a good translation. In *Natural Law and Natural Rights* John Finnis claims that St Thomas was not confused at all, that on the contrary he deliberately, but secretively, altered the meaning he found:

> So it was that Thomas Aquinas, purporting to interpret Aristotle faithfully, silently shifted the meaning of Aristotle's second class of particular justice, and invented a new term for it: 'commutative justice'.[311]

St Thomas looks damned either way: he stands accused as either a fool or a knave.

Against the claims that Aquinas was confused, Finnis argues that Thomas devoted far too much attention to the relevant notions in writing the *Summa* to have been unaware of the meaning of the terms he employed:

> Gauthier and Jolif .... argue that the invention [of commutative justice] rests on a misunderstanding occasioned by the ambiguous Latin translation that Aquinas used. This seems unlikely in view of the extremely elaborate treatment of commutative justice that Aquinas undertook in S.T.II-II, qq.64-78, and in view of the conceptual gaps left by Aristotle's emphasis on correction.[312]

Finnis does not mention it but Aquinas had examined the issues in detail in his *Commentary* immediately prior to writing Part II of the *Summa;* the Commentary being to some extent written as research for the *Summa II-II*[313]. In it the subdivisions of justice are thoroughly explored in lectures IV to IX; there are analytical tables distinguishing Distributive from Commutative, and there are

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309 D.G.Ritchie 'Aristotle's Subdivisions of 'Particular Justice'' *Classical Review viii* 1894, p.187.
310 1970, pp.370-71.
311 1980, p.179.
312 Ibid. p.196.
313 In the main each section of the *Commentary* corresponds to a section in the *Summa*. For example *Commentary* I, vi with *Summa* II-II qq.80-81, and VII, x with II-II q.155. See Gauthier 'La date du Commentaire de saint Thomas sur l'Éthique à Nicomaque' in *Recherches* 18, 1951, pp.66-105.

sections for the exposition of this second type of justice listed as Commutative[314]. When we consider the detailed examination of the notions involved in the *Commentary*, coming after his frequent employment of the term in his earlier work, together with its repeated use in the Table of ethical terms he had had his secretaries draw up for his use in writing the *Commentary* and *Summa II-II*[315], and add to this Finnis's point that the issue is treated extensively in the *Summa*, it becomes incredible that Aquinas might have simply misread the terms. Rather, for Thomas to have been misled it would have to be shown that the translations upon which he relied were seriously in error.

Finnis further argues that Aquinas had to work out a means of assimilating "conceptual gaps left by Aristotle's emphasis on correction" (p.196) which, of course, could not have taken place if Aquinas had not even been aware of the "emphasis on correction". Regarding this emphasis it will emerge that Finnis's remarks need to be seen from an entirely new angle.

But Finnis's own, conflicting, claim that Aquinas deliberately altered the terms to give a new meaning is just as improbable as the idea that he simply misread them. The *Summa* was not a work of Aristotelian exposition but of systematic theology, so, although it was written in close association with the *Commentary*, it would have been legitimate for Thomas to adapt Aristotle's notion to his own purpose. Thomas nevertheless does not in any way indicate that he is departing from or adapting Aristotle. The meaning employed in the *Summa* is exactly the same as (being derived from) that he had just elaborated in the *Commentary*; he presents the issues just as if they were Aristotle's ideas, as if <u>Aristotle</u> distinguishes Commutative not Corrective from Distributive justice. Whilst to have changed Aristotle's meaning in the *Summa* would have been legitimate, his not mentioning the

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[314] Lectio IV synopsis, I dividit 3; Lectio V, 2; *Expositio* pp.253, 259, 265.

[315] The *Tabula Ethicorum* was compiled by his assistants from January 1269 onwards. (The glosses in the *Tabula Ethicorum* refer directly to A (below), and obliquely to B (at 358), but also contain other references to the second species as Commutative.)

Under *IVSTITIA* (referring to 1130b30-31) they put:

Quod iusticia specialis diuiditur in distribucionem et commutacionem

and

Quod iusticia distributiua constitit in distribucionibus et honorum et pecuniarum, commutatiua uero constitit in communicacionibus,

Continuing (ref. 1131a3)

Quod iusticie commutatiue quedam est uolunitaria sicut ...

For 1132a2

Quod in iusticia commutatiua attenditur ad nocumenti differenciam solum

Under *IVSTUM*, referring to the analogical extension of justice to the domestic context (1134b8-9):

Quod iustum dominatiuum et paternum non est idem iustum distributiuo et commutatiuo, set simile.

alteration would not. That deception would have been bad enough, but to alter the sense in the *Commentary* would have been worse. Unlike the *Summa* it was a work whose entire purpose was the presentation of Aristotle's theory. St Thomas would have had to suppress the Philosopher's thought and substitute his own notion in the very work wholly devoted to the careful presentation of that thought. Quite apart from the extreme psychological improbability of such a deception, Aristotle's words were openly displayed in his text: Thomas would have had every reason to suppose that he would have been found out. Consciously to have attempted such less than angelic doctoring would have required St Thomas to have been both a knave *and* a fool.

5. 2 Aquinas's sources

As the alternatives that St Thomas either simply misread the text or consciously altered it are both untenable, the thesis that he misrepresented Aristotle depends on the scripts which could have influenced him being at fault. If they had been at fault then we can easily see how he would have been misled. In Thomas's time the Latin translation of the *Nicomachean Ethics* was still new, fragments from other works indicate that virtually the whole of the *Ethics* had been translated early in the thirteenth century, and references to it were being made by the 1230's. The translation being used at that time, which Gauthier refers to as the *Translatio Antiquior*, has been lost. In 1247 the bishop of Lincoln, Robert Grosseteste, completed his own translation (or perhaps completed his editing of the earlier translations) of the *Ethics;* it was taken directly from Greek, not, as is still often thought, from Arabic manuscripts. The original version of this *Translatio Lincolniensis*, known as the *Textus Purus*, was used by St Albert in his lectures—where Thomas would have been familiar with it[316]. From 1248 to 1252 Thomas Aquinas attended lectures on the *Nicomachean Ethics* given by Albertus Magnus in Cologne, where he took down notes[317] to assist Albertus, who wrote a commentary[318]. Albertus was gathering in whatever sources that could be obtained[319], and although

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316 See H.P.F.Mercken 'The Greek Commentators on Aristotle's Ethics', in *Aristotle Transformed* R.Sorabji (ed.) 1990, p.442. It is thought that Books II and III, known as the *Ethica Vetus* had been translated late in the 12th century. Book I, with fragments of VII and VIII, known as the *Ethica Nova*, was translated early on in the 13th century by a different scribe, and Book I of this, at least, circulated (Gauthier-Jolif 1959 II, p.77*).

317 See *Opera* 47i, pp.235*-57*.

318 *Super Ethica* (Aschendorff, 1968). Albertus wrote a further commentary in the late 1260's (see J.Dunbabin 'The Two Commentaries of Albertus Magnus on the Nicomachean Ethics', in *Recherches* 30, 1963 pp.232-50).

these many documents are likely to have coloured his reading, St Albert's presentation of the extracts we are concerned with, nevertheless, is very close to the *Purus* of Grosseteste. Hence any defects in the *Purus* might have influenced Albert who might, in turn, have influenced Aquinas.

A revision of the *Purus* was completed by 1260. This new edition, known as the *Textus Recognitus*, included Grosseteste's translation of the Greek and Byzantine commentaries on the *Ethics* with his own extensive notes in the margins[320]. It is a corrupt variant of this text which St Thomas eventually obtained. His version of the text was cobbled together from the *Recognitus* and from elements which Gauthier attributes to the *Translatio antiquior,* together with poor copies of the earlier *Purus.* Had the relevant sections in this debased copy suffered, that particularly could have influenced Thomas's understanding. The common belief that Aquinas commissioned William of Brabant (Moerbeke) to translate the works of Aristotle (or that Urban IV launched such a collaboration) has been shown by Gauthier to be mistaken[321]. Aquinas's own text, in the sections we are concerned with, differed from Grosseteste's *Purus* and *Recognitus* in word order and in punctuation, but not in the terms chosen: these are virtually identical (see below). As it is rather improbable that an independent translator (such as Moerbeke) would choose identical Latin terms, and as the quality of the text of the *Ethics* that Thomas was able to acquire was so bad, it is clear that Moerbeke was not responsible for it.

The texts then, which might possibly have influenced St Thomas, that Ritchie and Gauthier-Jolif refer to as the Latin tradition, and which they blame for Aquinas's errors, are the patched-up script directly available to him and the *Purus* edition as it was represented in the commentaries of Albert. Standing behind these were the original *Purus* and *Recognitus* editions. The upshot is that all the translations that Aquinas could have been familiar with stem from Grosseteste, either directly

[319] Such as the works of Michael of Ephesus, Eustratius, Nemesius, John of Damascus, and Hermann the German, who had completed his translation from Arabic of Averroes's *Middle Commentary on the Nicomachean Ethics* on 3 June 1240.

[320] There had been anonymous scholia on Books II to V dating from the 3rd century AD. A further commentary was written by Michael of Ephesus during the earlier part of the 11th century. At the beginning of the 13th the existing studies were compiled with the *Ethics* and published in Constantinople. It was these Byzantine works which Grosseteste translated and published with the *Recognitus* edition and added his own notes (*notulae Lincolniensis).* See also *Opera* 48(B), pp. 32,34.

[321] This is not to say that Aquinas did not associate with Moerbeke; he made use of any Moerbeke translation he could get his hands on (see Gauthier-Jolif, 1970 pp.125-31).

or indirectly. Any poor handling of the relevant passages by him could then very well have been misleading.

5. 3 The Latin tradition

In order to compare all the texts of the *Ethics* that Aquinas could possibly have read I have collected the relevant passages from the texts of the *Purus* and *Recognitus*, and also the copy of the *Purus* used by Albert. Together with St Thomas's own quotations in the *Commentary*[322] these exhaust the Latin tradition. Aristotle refers to the second type of Particular justice six times in Book V of the *Ethics*, these (labelled A-F, and with the Oxford revised translation[323]) are:

1130b34 (A) ἐν δε το ἐν τοις συναλλαγμασι διορθωτικον τουτου δε μερη δυο των γαρ συναλλαγματων τα μεν ἑκουσια ἐστι τα δ' ἀκουσια

[...,] and another kind is that which plays a rectifying part in transactions. Of this there are two divisions; of transactions some are voluntary and others involuntary

1131b25 (B) Το δε λοιπον ἐν το διορθωτικον ὃ γινεται ἐν τοις συναλλαγμασι και τοις ἑκουσιοις και τοις ἀκουσιοις

The remaining one is the rectificatory, which arises in connexion with transactions both voluntary and involuntary.

1131b33 (C) το δ' ἐν τοις συναλλαγμασι δικαιον ἐστι μεν ἰσον τι και το ἀδικον ἀνισον ἀλλ' οὐ κατα την ἀναλογιαν ἐκεινην ἀλλα κατα την ἀριθμητικην

But the justice in transactions is the sort of equality indeed, and the injustice a sort of inequality; not according to that kind of proportion, however, but according to arithmetical proportion.

1132a19 (D) ὡστε το ἐπανορθωρτικον δικαιον ἀν εἰη το μεσον ζημιας και κερδους

[...,] therefore corrective justice will be the intermediate between loss and gain.

1132b18 (E) ὡστε κερδους τινος και ζημιας μεσον το δικαιον ἐστι των παρα το ἑκουσιον το ἰσον ἐχειν και προτερον και ὑστερον

Therefore the just is the intermediate between a sort of gain and a sort of loss, viz. those which are involuntary; it consists in having an equal amount before and after the transaction.

1132b24 (F) Το δ' ἀντιπεπονθος οὐκ ἐφαρμοττει οὐτ' ἐπι το διανεμητικον δικαιον οὐτ' ἐπι το διορθωτικον

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322 There are 86 extant manuscripts of the *Commentary*, the most reliable being collated into two slightly varying presentations edited by Gauthier (*Sententia*), and by R.M.Spiazzi (*Expositio*).

323 Ross revised by J.Urmson (*The Revised Oxford Translation of the Complete Works of Aristotle* (ed.) J.Barnes, 1984).

Now reciprocity fits neither distributive nor rectificatory justice.

The Latin tradition in the translation of these passages is given in the note below[324]. The extracts show that there was no *significant*[325] variation in the mediæval translations of the six entries, as there

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[324] Listed A-F:
A (1130b34):
<u>Grosseteste *textus purus:*</u>
Una autem que in commutacionibus directiva. Huius autem partes due. Commutacionum enim hee quidem voluntarie sunt
(*AL* 1972 XXVI. 1-3(3) p.231; the following *purus* entries pp.233-36.)
<u>Grosseteste *textus recognitus:*</u>
Una autem que in commutacionibus directiva; *huiusmodi* autem partes due; commutacionum enim hee quidem voluntariae sunt,
(*AL* 1973 XXVI. 1-3(4) p.457; the following *recognitus* entries pp.459-62.)
<u>Albertus (*purus*):</u>
(...;) una autem, quae in commutationibus directiva. Huius autem partes duae; commutationum enim hae quidem voluntariae sunt,
(*Super ethica* p.329; the following extracts from Albertus pp.338-40, 342.)
<u>Aquinas *sententia:*</u>
Una autem quae in commutationibus directiva. Huius autem partes duae; commutationum enim hae quidem voluntariae sunt,
(*Sententia* p.275; the following entries pp.282, 286, 289.)
<u>Aquinas *expositio:*</u>
Una autem, quae commutationibus directiva. Huius autem partes duae. Commutationum enim hae quidem voluntariae sunt,
(*Expositio* p.254 ref. 659-60; the following entries p.260 refs. 671, 674, 677, p.263 ref. 684 and p.266 ref. 686-87.)
B (1131b25):
<u>Grosseteste *purus:*</u>
Reliqua autem una, directivum quod fit in commutacionibus et in voluntariis et in involuntariis.
<u>Grosseteste *recognitus:*</u>
Reliqua autem una directivum. Quod fit *et involuntariis et in commutacionibus et in voluntariis*;
<u>Albertus:</u>
Reliqua autem una directivum, quod fit in commutationibus, et in voluntariis et involuntariis
<u>Aquinas *sententia:*</u>
Reliqua autem una directivum quod fit in commutationibus et voluntariis et involuntariis.
<u>Aquinas *expositio:*</u>
Reliqua autem una directivum eius quod fit, et in voluntariis commutationibus et involuntariis.
C (1131b33):
<u>Grosseteste *purus:*</u>
In commutacionibus autem iustum est quidem equale quid et iniustum inequale, set non secundum proporcionalitatem illam, set secundum arismeticam.
<u>Grosseteste *recognitus:*</u>
In commutacionibus autem iustum est quidem equale quid et iniustum inequale, set non secundum
proporcionalitatem illam, set secundum arismeticam.
<u>Albertus:</u>
(...;) in commutationibus autem iustum est quidem aequale quid et iniustum inaequale, sed non secundum proportionalitatem illam, sed secundum arithmeticam.
<u>Aquinas *sententia:*</u>
In commutationibus autem iustum est quidem aequale quid et iniustum inaequale, sed non secundum proportionalitatem illam, sed secundum arismaticam.
<u>Aquinas *expositio:*</u>—

well might have been despite having all passed through Grosseteste's hands. They also show that

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In commutationibus autem iustum est quidem aequale, et iniustum inaequale; sed non secundum proportionalitatem, illam secundum arithmeticam.
D (1132a18):
Grosseteste *purus:*
Quorum erat medium, equale; quod dicimus esse iustum. Quare directivum iustum utique erit, medium dampni et lucri.
Grosseteste *recognitus:*
Quorum erat medium equale, quod dicimus esse iustum. Quare directivum iustum utique erit medium dampni et lucri.
Albertus:
(...,) quorum erat medium aequale, quod dicimus esse iustum; quare directivum iustum utique erit medium damni et lucri.
Aquinas *sententia:*
(...,) quorum erat medium aequale, quod dicimus esse iustum. Quare directivum iustum utique erit medium damni et lucri.
Aquinas *expositio:*
Quorum erat medium aequale, quod dicimus esse iustum. Quare directivum, utique erit medium damni, et lucri.
E (1132b18):
Grosseteste *purus:*
Quare lucri cuiusdem et dampni medium iustum est eorum que circa voluntarium equale habere quod et prius et posterius.
Grosseteste *recognitus:*
Quare lucri cuiusdem et dampni medium iustum est, quod preter voluntarium equale habere et prius et posterius.
Albertus:
Quare lucri cuiusdem et damni medium iustum est eorum quae circa voluntarium, aequale habere, quod et prius et posterius.
Aquinas *sententia:*
Quare lucri cuiusdam et damni medium iustum est, quod praeter voluntarium aequale habere et prius et posterius.
Aquinas *expositio:*
Quare lucri cuiusdam et damni medium iustum est eorum in praeter voluntarium aequale habere, et prius et posterius.
F (1132b24):
Grosseteste *purus:*
Contrapassum autem non congruit, neque in distributivum iustum, neque in directivum, quamvis volunt hoc dicere et Rhadamanthis iustum.
Grosseteste *recognitus:*
Contrapassum autem non congruit, neque in distributivum iustum, neque in directivum, quamvis *voluerit* hoc dicere. Et Rhadamantis iustum.
Albertus:
Contrapassum autem non congruit neque in distributivum iustum neque in directivum, quamvis volunt hoc dicere et Rhadamanthis iustum:
Aquinas *sententia:*
Contrapassum autem non congruit neque in distributivum iustum. Neque in diretivum, quamvis volunt hoc dicere; et Radamanti iustum:
Aquinas *expositio:*
Contrapassum autem non congruit, neque in distributivum iustum. Neque in directivum: quamvis voluerit hoc dicere et Rhadamantis iustum.

[325] Variations in quotation and paraphrasing show he did not have Albertus's text to hand when actually writing his own commentary (G.Wieland 'The Reception and Interpretation of Aristotle's *Ethics*' *The Cambridge History of Later Medieval Philosophy*, 1982, p.662). For Albert's influence on Thomas, and influences via Albert see Gauthier 'Appendix: St.Thomas et l'Éthique à Nicomaque', *Opera* 48, 1971.

Ritchie and Gauthier must be mistaken: there is no mistranslation. What we find is a failure to preserve Aristotle's use of an alternative expression at D—this is examined below—but it does not help their case. If Aquinas *was* misled then it could not have been through faulty translation of the text, and as we have seen neither through a simple misreading of it, and nor did he consciously alter it (which exhausts the possibilities of his having been misled).

5. 4 A simple misreading ?

As he was not misled by mistranslation did Aquinas simply misunderstand the text? Ritchie and Gauthier-Jolif discuss only the entries A and B. I argue below that Aquinas could not have misread D and F. With respect to *directivum* might Aquinas have been simply confused? Ritchie says:

> Aquinas read the *Ethics* and the *Politics* in the version of William of Moerbek. Now in this old translation the words of ...(1130b30) [A above] are rendered as follows: *una autem quae in commutationibus directiva.* In [B above] we find: *Reliqua autem una directivum* (sic) *ejus fit et in quod voluntariis commutationibus et involuntariis.* In these passages Aquinas's attention was obviously drawn by the phrase *in commutationibus* and not by the vague word *directiva* which fails to give the force of διορθωτικον. In the sentences ... from the *Summa* it will be observed that Aquinas uses *directiva* and *dirigit* of Distributive as well as of Corrective Justice: so he has clearly missed the significance of the term διορθωτικον.[326]

Gauthier-Jolif, correcting his reference to Moerbeke but otherwise in agreement with Ritchie, say:

> Le *dikaion to en tois sunallagmasi diorthotikon* d'Aristote etait en effet ainsi rendu dans la traduction de Robert Grosseteste: *una autem quae in commutationibus directiva...; reliqua autem una directivum quod fit in commutationibus et in voluntariis et in involuntariis...* Le mot important etait *directivum, diorthotikon* mais saint Thomas a souligne au contraire le mot *commutationibus* et, en face de la justice distributive, il ne connait dans son commentaire sur l'*Éthique* que la justice commutative, qui dirige les echange quant a la justice corrective proprement dite, elle n'est plus qu'un aspect secondaire de la commutative, et l'importance qu'Aristote attachait au *diorthotikon* passe inaperçue.[327]

Neither Ritchie nor Gauthier-Jolif demonstrate that there is any ambiguity due to the Latin tradition, nor what is significantly faulty in the mediæval translations. Ritchie merely says that Aquinas's attention was "obviously attracted" by the terms for transactions, not that those terms do not occur in the Greek scripts. It could well have been the case that different translations would have rendered differing senses, but as it happens they have not. The only claim that is argued for is that the important word is *directivum*, while Thomas chose rather to highlight *commutationibus*, treating the

[326] 1894, p.188.
[327] 1959 (1970), pp.370-71. (Ritchie and Gauthier-Jolif prefer slightly differing MSS).

corrective function of this type of justice as secondary. Thereby, they argue, mistaking the importance which Aristotle attaches to the corrective function of the second species; specifically to διορθωτικον. Their views indicate that Grosseteste ought to have translated the passages so as to stress the term *directivum* rather than the expression *in commutationibus*. It is not apparent how Grosseteste might have changed his translation of Aristotle's text to yield such an emphasis without violating his sense of accuracy. Gauthier-Jolif are not quite so scrupulous: notice that they create a term that is not present in the Greek:

> Le δικαιον το εν τοις συναλλαγμασι διορθωτικον d'Aristote etait en effet ainsi rendu dans la traduction de Robert Grosseteste *una autem quae in commutationibus directiva.*

But this was Grosseteste's translation of ἐν δε το ἐν τοις συναλλαγμασι διορθωτικον. They achieve the emphasis they believe to be intended by putting words in Aristotle's mouth (well, one important word). The introduction of δικαιον here illegitimately props-up what Ritchie calls "the vague word *directiva* which fails to give the force of διορτηθωτικον"[328]. Ritchie spells out his failure to grasp the central point understood by St Thomas:

> It will be observed that Aquinas uses *directiva* and *dirigit* of Distributive as well as of Corrective Justice: so he has clearly missed the significance of the term διορθωτικον.

My claim is that it is Ritchie (along with very many others) not Aquinas who has missed the significance of it.

5.5 Ambiguity of parsing

In both the Greek and its Latin translations A and B refer to both *corrective* and *exchanges*, C and E only to *exchanges*, whereas D and F use only *corrective* (apparently leaving the field wide open). The passages A and B are complicated by two quite distinct ambiguities. I discuss the second in connexion with B below. As for the first ambiguity, I believe that the mediævals have been wrongly accused of focusing on the wrong term. Aquinas discusses A in his Commentary at 928:

> secundo ibi...<una autem> Ponit secundam speciem particularis iustitiae. Et dicit, quod alia species particularis iustitiae est, quae constituit rectitudinem iustitiae in commutationibus, secundum quas transfertur ab uno in alterum; sicut prima species iustitiae attendebatur secundum quod transfertur aliquid a communi ad singolos.

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[328] Ritchie's claim is opposed by Burnet. W.F.R.Hardie (*Aristotle's Ethical Theory*, 2nd ed. 1980, p.211) says that Burnet ignores Ritchie's argument, but this is not so, he explicitly rejects it (Burnet, 1900, p.217).

C.I.Litzinger translates this as:

> Next ... at 'another species' he gives a second kind of particular justice. He says that another species establishes a measure of justice in transactions, by which a thing is transferred from one person to another - in the first species the transfer of a thing from the community to the individual was concerned.[329]

Litzinger's English rendering omits a translation of *rectitudinem*—he simply leaves out the very term which many claim definitively signifies the species. Thomas's expression was "*quae constituit rectitudinem iustitiae in commutationibus*"; the key phrase may be parsed as either:

rectitudinem iustitiae in commutationibus, or as *rectitudinem iustitiae in commutationibus.*

I.e., the expression "corrective justice in exchanges" may be understood in either of the two ways:

1 Corrective Justice in exchanges

2 corrective Justice-in-Exchanges.

The subject may either be disambiguated as Corrective Justice (applied to exchanges), or it may be Justice-in-Exchanges (qualified by the adjective 'corrective'—see §§ 3.2 and 4.2). *Iustitia commutativa,* the term employed by Aquinas, stems from *commutatio*—to alter or change—hence 'Commutative Justice'. Drawing out the options in the ambiguity of the parsing shows exactly the way that the term emerged from the two passages (A) *quae in commutationibus directiva,* ... and (B) *directivum ... in ... commutationibus.* Aquinas disambiguated the expressions as 2, whereas his accusers would disambiguate them as 1.

Regarding the first objection to St Thomas—that the translations he used were poor—no evidence has even been offered. All Ritchie and Gauthier-Jolif have actually done is merely *assert* that Aquinas read the text wrongly. Was it then the case that Aquinas simply 'saw the meaning' as 2 rather than as 1, mis-parsing the expression? Could an innocent error, i.e., one that was not a deliberate re-interpretation as Finnis claims, nevertheless not be due to mistranslation? For this to have been the case St Thomas would have had read the later two passages (D and F)—which refer to this form of justice without mentioning transactions—as if they referred exactly to transactions. Unlike Ritchie and Gauthier-Jolif St Thomas did not neglect the later passages; to have misread the text in them he would have had to read *quare directivum* and *neque in directivum* as meaning *in*

[329] *Commentary,* 1964, p.400.

commutationibus (as if the word 'direction' means transaction[330]). That Thomas did not simply misread 'direction' as 'transaction' at D and F is shown by his application of *directivum* to both Distributive and Commutative justice in the *Summa*[331]. It is absurd to suppose that St Thomas unconsciously mixed-up such totally unrelated meanings of the terms in several places and contexts *both* in his analysis of Aristotle's theory in the *Commentary* and in his application of it in the *Summa:* whether he was right or wrong he had to have deliberated. Hence we may exclude the possibility of a simple misreading.

From these considerations it appears that St Thomas could not have been confused by poor translation and did not otherwise *unconsciously* shift the meaning from 1 to 2. Following this it may seem unavoidable to conclude that, however extreme the moral, psychological and practical absurdities involved, he consciously shifted it. An explanation close to that of Finnis's would be that St Thomas noticed the ambiguity in the first two entries and then looked to C and E, which unambiguously refer to justice in exchanges with no 'corrective' reference, to confirm the meaning of A and B as 2 rather than as 1. (But he would then have had to accommodate his interpretation to the passages D and F.)

The reason for choosing Commutative rather than Directive to characterise the second type of justice, in Finnis's view, seems to be that Aquinas saw the immense potential of the notion of exchanges: that exchange takes precedence over correction (flatly contradicting Gauthier-Jolif). Regarding the depth and importance of Aquinas's insight into the notion of exchanges I agree with Finnis; nevertheless I believe that St Thomas neither consciously nor unconsciously shifted the meaning of Aristotle's text. On the contrary, he gave a careful and faithful presentation. What is

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330 At first glance it might appear that he had done exactly this when commenting on F at 967. There Aquinas had treated *neque in directivum* as referring only to commutative justice (rather than to both species), even though it is headed simply *directivum*:

> *Secundo ibi* [687] "neque in directivum". *Improbat praedictam positionem quantu ad iustiam commutativam,*

in fact, like Albertus, he was merely following the customary method of locating the text.

331 *II-II* q.61 a.1:

> *Sed contra est quod Philosophus in V* Ethic. *ponit duas partes iustitiae, et dicit quod* una est directiva in distributionibus, alia in commutationibus.

and q.61 a.3:

> *In contrarium est quod dicitur in V* Ethic. *quod una species iustitiae est directiva in distributionibus, alia in commutationibus.*

also in the *Responsio* (I am grateful to Professor Finnis for pointing this out to me):

> *nam distributiva iusticia est directiva distributionum, commuutativa vero iusticia est directiva commutationum quam attendi possunt iter duas personas.*

required is an explanation of why it is that he emphasised exchanges rather than correction in those entries (A and B) which contain both terms, followed by an account of the meaning of those passages (D and F) which do not even refer to exchanges. (C and E refer to exchanges only.) Part of that explanation will show that the modern interpretation—that Aristotle drew his distinction between distribution and correction—creates major discrepancies which the mediæval understanding wholly avoids. As we look at the passages the discrepancies emerge.

5. 6 (A) 1130b34

> (..,) and another kind is that which plays a rectifying part in transactions. Of these there are two divisions: of transactions some are voluntary and others involuntary

In the first two entries, where it is being introduced, Aristotle describes the species more fully than in the later references. We have seen that A and B could be parsed so as to define the second species according to its extension or else according to its character. In the modern view part of the character becomes the defining noun (see §§ 3.2, 4.2, 5.5); now this could very well be arrived at via the sequence 'differentia of the differentia' where the last in the series of attributes is corrective, isolating the species from any other within the genus. But in Thomas's view this character is understood as an adjective not as the sortal noun; it is a differentia which is part of the sequence of attributes shared with the other species. The extension of the distributive species we know to be whatever may be quantified and shared (wealth, duties, honours, goods) of the common stock. What then is the *specific* character of distributive justice?

As species of a common genus the attributes of the two forms of justice will differ from one another (paradigmatically) in degree—'as to the more or less', though possibly 'by excess or defect'. Any defect could not be a generically defining differentia, i.e., the species could not differ to the extent that one of them *has* a character where the other has not[332]. Nor could we even make sense of the notion that only one of them has an extension. For distributive justice not to possess a 'character' at all would, even if such an idea is coherent, eject it from the genus. I.e., if we said that, unlike the

[332] The notion of species differing 'by excess and defect' is often treated interchangeably with difference in degree, but even if we allow that these two expressions might not be wholly equivalent, the differentiae to which they apply need to be generically non-defining attributes. (A table may have drawers, but drawers are not defining attributes of tables. Aristotle gives the examples of the presence or not of a spur or crest—these are not defining differentiae of birds, as are feathers and beaks.)

second species, Distributive justice has a character that is simply "just", this could not be accommodated alongside the other species falling under the genus Particular justice. (Particular justice contains subsidiary forms of justice; for them to be species of the same genus, where one is attributed a character and an area of application, then so will the other.)

Aristotle presents the species of Particular justice as having attributes which differ as to the more or less *complex*. The more complex is the 'geometric' arrangement for just distribution (1131a10ff.) involving a minimum of four terms: the two parties in a case and the division of what is at stake into two corresponding parts. It is 'geometric' in that it is the *ratios* between the sets of terms that are to be determined, not merely the *quantities*. The more simple arrangement is the 'arithmetical' where, since the worth of the parties is irrelevant, only the value of the items at stake is involved, i.e., only the *quantity* is to be determined. As outlined in § 2.1 Aristotle is not altogether clear in his terminology with respect to proportion, and the confusions that have bedevilled interpretation have, to that extent, not really been the fault of the commentators and translators. Aristotle relies on the context to clarify his exact sense. Sometimes, as here, he speaks quite loosely of proportion to mean *geometric* proportion, to be contrasted with plain quantitative equality (κατ' ἀναλογιαν και μη κατ' ἰσοτητα, 1132b33). At other times (1106a35: Τουτο δε μεσον ἐστι κατα την ἀπιθμητικην ἀναλογιαν) he draws on the careful application of geometric and arithmetic equalities as, both of them, modes of proportion—as they are in proportion theory. This equivocation has been distinctly unhelpful (that simple, quantitative equality is *both* the plain everyday equal amount, understood by everyone to be what equality is, yet *simultaneously* a mode of proportion, is a feature of his theory of justice that could have done with a much more consistent terminology). The contrast Aristotle makes whilst presenting the difference between the distributive and transactional species is that between the complex model for geometric equality for distributions, and a simple model of quantitative equality for transactions. Properly speaking the arithmetic formula is every bit as complex as the geometric, but the complicating factors (the factors which treat the parties to a case), in effect, cancel-out by being reduced to a unity (A : B = 1). The complex arithmetically-proportional formula is replaceable with a simple arithmetic formula of quantities, and the result is a marked contrast between the geometric and the arithmetic models for justice. Even as a form of proportion arithmetic equality remains the simplest of the modes—it maps the equal quantity across a pair of ratios[333]—but here Aristotle is using arithmetic equality to contrast with the complex geometric equality for distributions. The reader is expected (somewhat unreasonably perhaps) to be aware, as his original students undoubtedly were, that Aristotle is drawing on both aspects of arithmetic equality.

Apart from the *simple/complex* distinction, as a species of a common genus the more complex form *geometric justice in distribution* corresponds in structure to the more simple *arithmetical justice in exchanges.* This correspondence is present in Aquinas's reading of the theory; it is a minimal, logical, requirement that each will have a character and an area of application (an extension). The correspondence, however, depends upon the passage A being parsed as 2. But reading it as 1—the way it is in modern discussions—the second species has markedly differing generic traits from the first; i.e. they are not species of the same genus. Where the character is taken to be corrective, and this to be peculiar to the second species, a complex asymmetry is created; one which renders the interpretation incoherent. That the modern parsing 'Corrective Justice in exchanges' does not allow anything to correspond to what could count as the character and extension of the first species becomes obvious if we take a quasi-schema:

Corrective Justice *for extension x,*

it should map

Distributive Justice *for extension y.*

It doesn't. Distribution belongs to the extension, not to the character of the species (and of course 'Distributive Justice in the distribution of y_1, y_2, ...y_n' is trivially circular, as would be a corresponding 'Corrective Justice in correction', or 'Transactional Justice in exchanges'). The alternative is to parse the second species as St Thomas did: '(corrective) Justice in Exchanges'. Some corresponding character then is required for '(...............) Justice in Distribution'. Treated as a quasi-schema:

some z (character of) Justice in Distribution

is to correspond to:—

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333 Marc-Wogau (op.cit.) comes close to the style of Aristotle's thinking in recognising proportions as the key to the text; but he misconceives the arithmetic proportion as a special case of the geometric, rather then as one of the three classical parallel modes of proportion. He constructs the forms of justice accordingly—as Distributive justice *containing* Corrective as a subaltern class, which in turn *contains* justice for exchanges as a further sub-species. He would not have been misled had Aristotle's terminology with proportion theory been clearer.

some w (character of) Justice in Exchanges.

It is agreed that w = *corrective*, so here we need a 'value' for z. 'Proportional' is no use for z because both sorts of justice are proportional. Entered in the schemas the two proportions are:

geometrically z (..................) Justice in Distribution.

arithmetically w (corrective) Justice in Exchanges

These do nothing to repair the logical mis-match in the orthodox interpretation, where without a 'value' z marks no more than a lacuna in the first species to parallel the rôle of *corrective* in the second. There is nothing in the text which looks even faintly available to occupy the hiatus other than *corrective* itself—*and nothing to prevent it.* The traits z and w are generic; the corrective character of justice is general not specific. 'Geometrically corrective Justice in Distribution' is perfectly in order, but conflicts with the modern presentations in which *corrective* is taken to distinguish the second species <u>from</u> the Distributive.

5. 7 (B) (1131b25)

> The remaining one is the rectificatory, which arises in connexion with transactions both voluntary and involuntary.

In this entry the second ambiguity emerges, but it has been entirely overlooked because the contrast that Aristotle draws has been taken to isolate the Corrective as a species (Ritchie even rehearses its terms without realising their ambiguity). This passage

> Το δε λοιπον ἐν το διορθωτικον ὁ γινεται ἐν τοις συναλλαγμασι και τοις ἑκουσιοις και τοις ἀκουσιοις

may be understood in two very different ways; Ross's translation is representative of the modern understanding:

3 The remaining one is *the rectificatory*, which arises in connexion with transactions both voluntary and involuntary,

where only one species rectifies. If we remove the (interpolated) punctuation (there was of course none to the original) we have the reading clearly understood by St Thomas:

4 The remaining one is *the-rectificatory-which-arises-in-connexion-with-transactions* both voluntary and involuntary,

where both species rectify but only one rectifies exchanges. The Latin tradition from Grosseteste may be re-translated:—

> However, there remains one giving direction, it applies to exchanges both voluntary and involuntary;

in line with which St Thomas disambiguated the passage as 4. St Thomas saw the principle as giving direction in exchanges rather than giving it in distribution. The modern interpretations, on the other hand, have no choice but to view the passage as 3. It is indisputable that an explicit distinction is being drawn by Aristotle, but by rendering that distinction as between Corrective and Distributive (or as between Corrective and *anything at all*) the passage must be read as 3.

Yet interpreting the passage as 3 flows from a non-sequitur; it is one which commits a classic error of scope with the negation operator. The mistake has been the inference that

$$\neg \text{ (A asserts } \varphi)$$

means

$$\text{A asserts } \neg \varphi.$$

I.e., because Aristotle does not say that the first species is corrective it has been taken to mean that it is not. But Aristotle nowhere says that Distributive justice is not corrective, only that the other sort *is*. This non-sequitur is the starting point for all the misinterpretations and most of the confusions that have grown up around Aristotle's theory of Particular justice since the mid seventeenth century.

5. 8 (C) (1131b31)

> But the justice in transactions is the sort of equality indeed, and the injustice a sort of inequality; not according to that kind of proportion, however, but according to arithmetical proportion.

The third entry occurs in the same passage as B but no reference to the character of justice is made at this point. That the character of this sort of justice is in some sense directive or corrective is not disputed (the distinctions between *directive* and *corrective*, or any of the other near-synonyms given in § 5.1 is examined in § 5.9 immediately below). The claim I am making is that Aristotle applies some notion of direction to *any* form of justice and so it could not be the distinction that he makes for just *this* form. For this reason the objections to Aquinas's reading of the contrast are misplaced. The corrective character of this form of Particular justice *is* secondary, contrary to Gauthier-Jolif's claim[334], exactly because it is an attribute shared with the other species. The entry C most simply and clearly expresses Aristotle's notion of the second type of justice: Justice in Exchanges—or as

[334] 1970, p.371.

Aquinas expressed it, Commutative Justice. (For the Archytan origin of the expression and the principle of justice in exchanges see § 6.3, note 372.)

We have seen that both species share a broadly directive or corrective character, and that this character differs as to the more or less complex. Corresponding to the differing degrees of complexity are the differing areas of application. In both the Involuntary and the Voluntary sub-species the simple vs. complex distinction will be a factor in separating the assessment of the value of what is to be exchanged from the things exchanged (in chapter 6).

5. 9 (D) (1132a19)

(...,) therefore corrective justice will be the intermediate between loss and gain.

The fourth entry appears in the same paragraph as B and C; here Aristotle refers to the subject in a new way; instead of διορθωτικον and συναλλαγμασι δικαιον he now uses the term ἐπανορθωτικον. Grosseteste failed to pick up the new term; he used the same Latin translation *directivum*[335] here as for the other entries. The mediæval custom was to render the Greek as literally as possible, even where this created ugly or contrived Latin expressions[336], which clearly shows that Grosseteste did not think the difference between διορθωτικον and ἐπανορθωτικον significant.

Ἐπανορθωτικον is a term with a prefix consisting of two prepositions, ἐπι to indicate that the action referred to is somehow secondary, and ἀνα to indicate (in this context) that it represents a return to, or restoration of, an earlier or superior situation. Ross, taking note that a new term has been employed, uses 'corrective' here instead of the 'rectificatory' that he gives for the other entries. Burnet suggested that διορθωτικον should be translated as 'directive', with 'corrective' reserved for ἐπανορθωτικον[337]. Diorthotic (directive) justice would then contain the epanorthotic (corrective) as a subaltern species. Διορθωτικον stems from διορθ-ωμα: to make straight, or set right; and ultimately from ὄρθος: straight. *Directivum* from *dirigo*: to set in order, or to give a particular direction, does not necessarily indicate that something having already gone wrong needs re-setting[338]. So the conflicting interpretations are whether διορθωτικον is a term only for putting-something-

335 *AL* XXVI. 1-3(5) pp. 703, 707.
336 See J.Dunbabin 'Robert Grosseteste as Translator, Transmitter, and Commentator: the *Nicomachean Ethics*' in *Traditio* 28 1972, pp.460-72.
337 1900 pp.213, 220.
338 *Liddell & Scott*; *Lewis & Short*.

back-in-place or for *both* placing and replacing. As far as I can tell the etymology of the root term ὀρθος (straight) will allow either sense: we take a straight path from the outset as well as requiring guidance to keep straight; we seek help straight away and not only when we've gone astray. If Aristotle was using the broader sense the geometric analogy will both guide initial distributions and re-direct re-distributions. The arithmetical analogy will be used in voluntary exchanges also both to guide initial transactions and to restore broken contracts. By contrast justice for involuntary exchanges *can* only give redress—the assault or fraud has first to take place. Aristotle uses the narrower term ἐπανορθωτικον δικαιον in the one case where, logically, there is no room for the wider sense.

Bearing in mind the double preposition in the prefix, the expression has built-in a reference to some prior state, some circumstance which, although it need not have been a transaction, a transaction is a likely, natural, and representative circumstance to stand in for the prior state. Some sense of a prior inter-action is then a perfectly respectable import for this expression. The expression conveys "justice as a subsequent action which restores the situation to an earlier, superior, condition". Which also strongly suggests that the wider term διορθωτικον, that he uses everywhere else, carries the wider sense—all *their* circumstances call for justice at the outset, not merely in redress[339]. For this reason it is better to use 'direction' for διορθωτικον and 'corrective' for the more restricted range of epanorthotic issues; although 'corrective' can be used in English for the broader sense, it is much more immediately understood to convey remedy rather than initial guidance.

It was a marked failing in Grosseteste not to pick up Aristotle's single use of the more restrictive term (he had available the stems *corrigo* and *emendo* which would have flagged the reactive function unique to this sub-species). His successors have often marked the new term (though Jackson, Peters, Welldon, Thomson and Irwin haven't) but they either do not notice that it has any significance (Chase, Grant, Ross, Joachim, del Vecchio) or have denied that it has any (Stewart. Rackham, Gauthier-Jolif, Harrison, Hardie)[340]. Stewart and del Vecchio, like Grosseteste himself, have missed the point of the distinct usage for this necessarily reactive sub-class of

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339 Book *IX*, especially 1164b13ff. again refers to the need for initial guidance in transactions.

340 *Stewart*, p.435; Rackham, p.275; Gauthier-Jolif 1959, p.358; A.R.W.Harrison ('Aristotle's Nicomachean Ethics, Book V, and the Law of Athens', in *Journal of Hellenic Studies* 77, 1957 pp.42-47) says the difference is "too subtle"; Hardie (op. cit., p.194).

exchanges, even though they have not treated the justice for the whole of the species as remedial. Many commentators *have* asserted that the whole of the species is remedial[341], so they find little need to see any significance in the more restrictive term for a sub-group. For these interpreters, insofar as they have noticed the difference of expression[342], the new term merely *confirms* the remedial character of the whole.

The passing-over of the special usage shows an insensitivity to the logic of Aristotle's proposals as much as to the linguistic care with which he expresses them. The justice for the *involuntary* sub-division of exchange is remedial simply because it must wait until there's been one; this limitation is a logical requirement of one part of the extension, not a differentia of the species as a whole. Aquinas, who did not have the novel term highlighting the reactive nature for this sub-species preserved for him, had to rely on the coherence of Aristotle's position, and accordingly interpreted the reference as an ellipsis.

5. 10 (E) (1132b18)

> Therefore the just is the intermediate between a sort of gain and a sort of loss, viz. those which are involuntary*; it consists in having an equal amount before and after the transaction

Here Aristotle repeats the point that he is borrowing the terms 'gain' and 'loss' from voluntary exchange; he is using them to express the relations *greater than* and *less than* in a way that vividly connects the two branches of Exchange. A number of translators have responded to the nuance in Aristotle's choice of παρα το ἑκουσιον ("other than the voluntary") rather than ἀκουσιον ("involuntary") either by using 'non-voluntary' (Jackson[343], Thomson) or by retaining the word

[341] Hardie (op.cit., p.193) writes:

> The account I have given of rectificatory justice agrees with Jackson, Ross, J.A.Smith (in the Introduction to the Everyman translation), Vinogradoff, Joachim, and Gauthier-Jolif that it is concerned with redress or the rectification of wrongs done.

He might have added *Grant*, p.112-13; Trendelenburg (op. cit., p.405); Trojano (op. cit. p.58f.); Masci *Etica* p.160 (Naples, 1911); Donati (op. cit. pp.25f.); M.Ostwald (*Nicomachean Ethics*, 1962, p.121). Since Hardie wrote A.Edel (*Aristotle and his Philosophy*, 1996 p.299), J.O.Urmson (*Aristotle's Ethics*, 1988, p.95), McNeill (op. cit., p.59), *Meikle*, Miller (1995, pp.72-73), and Judson (op. cit.) have continued to repeat this extremely widely held view.

[342] *Joachim* for example (pp.144, 146) treats the terms διορθωτικον and ἐπανορθωτικον as equivalent, without comment.

[343] *Jackson* however also inserts (p.27) the term 'corrective' "Thus το [διορθωτικον] δικαιον is a mean between a sort of profit ... in matters not voluntary", as does Williams (Welldon (1912, p.149) also imports "That which is just then in *corrective justice*" that is not in the MSS).

'voluntary' (Chase[344], Peters, and the Latin tradition: see § 5.3). In many translations 'involuntary' is substituted (Grant, Williams, Welldon, Ross* (quoted above), Rackham, Ostwald, Litzinger, Thomson, Dirlmeier) and the connexion is lost. Irwin does not translate the expression at all[345]. It is a detail, but one of those little turns of phrase which helps to reveal the broader viewpoint; the whole section from 1132a7, and especially from 1132b13, explains justice for involuntary exchanges *in terms of voluntary ones*[346]. The spirit of the section is to emphasise the correspondence between the two sub-groups. In line with this close connexion Aristotle chooses παρα το ἑκουσιον where, were it not for his purpose of explicating one branch of the subject in the terms belonging to the other, he could more easily have written ἀκουσιον. You may say that nothing hangs on such a small point; on its own nothing does, but the reversal of Aristotle's own preferred expression (and if the subjects fall under different genera or species, as many think, it is one that is pointlessly contrived), even though it is at the merely verbal level, is a distortion. As with ἐπανορθωτικον, with παρα το ἑκουσιον Aristotle uses a different mode of expression where that different mode does some work. The expression indicates that the two groups *are* parallel subaltern classes of the same species, reinforcing the "strong presumption that these two fall under the same kind of justice"[347]. Ignoring or denying Aristotle's sensitive modification of expression contributes to the overall distortion to the meaning of the text.

5. 11 (F) (1132b24)

Now reciprocity fits neither distributive nor rectificatory justice

The only place where the text unequivocally has Directive justice without referring to exchanges is at the last mention of this second form of justice. It occurs in connexion with Aristotle's opposition to the doctrine of the Pythagoreans that reciprocity (ἀντιπεπονθος) is just. His view is that reciprocity must be based on proportion. At this point Aristotle refers to justice as either Distributive or as Directive—which appears to undermine my argument. My claim is that 'directive' is an adjective,

[344] 1847, though in much later editions (Everyman, 1912) this is reversed to 'involuntary' without comment.
[345] Irwin (op. cit.) alters the Bekker sequence slightly and loses the phrase.
[346] *Joachim* (p.138) says Aristotle's reason for the explanation of justice in Involuntary matters in terms of contracts (1132a12, b13) was probably the absence of any single name for issues of (involuntary) active and passive conditions in law.
[347] Burnet (1900 pp.222-23 n.13).

not the noun, and applies to both species. It could not then be used to distinguish one species from the other. Well, suppose Aristotle needs a taxi, and there are two taxi-drivers, both wearing hats. He says "I'll take the one in the *hat*". It might appear that he had picked out the very attribute whereby they *couldn't* be distinguished. Not so, Aristotle replies that there are hats and *hats;* as he had said "the one in the *hat*" it is perfectly clear which of the two he chose. In dropping the reference to transactions in F Aristotle emphasises the contrast between a complex four-term set of relations needed to account for Distributive justice—elaborated at 1131a15-b24—and the simpler two-term relation between loss and gain which applies in transactions. By this stage Aristotle is sliding past the proportional nature of the arithmetic mode of analogy (§ 2.1 above); he habitually refers to the geometric analogy as "proportion", and the arithmetic proportion as "equality" *along-side* his more elaborate designation treating them both as proportions. Here, in the further repetition of the two species, Aristotle picks on the distributive use of the first and the simply directive character of the second. It does not mean that the first species is not directive, any more than the second is not proportional, but merely that Aristotle has a natural tendency to be elliptical. Any *simple* directive disposal of quantities is inadequate and inappropriate to distributive issues, whereas in non-distributive issues what *is* adequate and appropriate is the *simple* directive process applied to simple quantities—that simplicity is what stands out, like the hat, as the striking feature. Intent on rejecting the Pythagorean doctrine which failed to distinguish simple from complex reciprocity (see below) Aristotle glides over the whole expression, picking out its distinctively simple character from the proportionally complex character of the other species. But what is absent from Distributive justice is not rectitude or direction, but *simplicity*; the ellipse has fostered the misinterpretations of the text which place the adjectival differentia as the sortal noun (an entangling of differentia with species which violates a central tenet of Aristotle's thinking—see § 3.2).

5.12 Summary

The two forms of Particular justice which Aristotle distinguishes are the one which regulates according to a complex quasi-geometric formula for matters of distribution, and the other according to a simpler quasi-arithmetic formula for interactions of every sort which do not directly involve the *polis*. To avoid the confusions which the emphasis in his last reference to the second species has generated, and to display the pattern of his argument in the text, the full contrast which Aristotle makes between the two species of Particular justice should be read as:

1 Geometrically complex directive Justice in Distribution

2 Arithmetically simple directive Justice in Exchanges.

Looking at Aristotle's references to the second species in detail has shown that Aquinas's treatment of it was much more accurate than later commentators:

(A) he parsed so as the diorthotic term functions adjectivally.

(B) he disambiguated in a way that did not rest on the non-sequitur involving the scope of what is negated.

(C) is purely transactional, with no diorthotic qualification at that point.

(D) was not adequately translated for him so he did not see the fine logical distinction Aristotle makes in fixing the purely reactive sub-species (in chapter 4), distinguishing it from the broader pro- and re-active quality of the whole species. Yet he nevertheless respected the careful balance which Aristotle sets up in bringing together the two branches of exchange (explaining the one sub-group in terms to display the structure of the other) and treated the ellipsis as expressing 'directive Justice-in-Exchanges'.

(E) Unlike D had the nuance preserved by the Latin tradition of translation. This may have helped in seeing the two classes of exchange—voluntary and "along-side the voluntary" (*in praeter voluntarium* or *circa voluntarium*) as belonging to the same species.

(F) he treats as plainly elliptical.

The misapprehension that Aristotle had divided Particular justice into Distributive and Corrective fields led Felden and subsequent writers to the supposition that vitally important concerns fall outside the initial account. The modern orthodoxy requires Aristotle to have been hopelessly muddled both about the rôle these important issues play in his account of justice and in the structure of the whole book. In opposition to all this I believe, as did Aquinas, that Aristotle does what he sets out to do. He does <u>not</u> set-out the plan (1130b30ff.) to examine the modes of equality appropriate to each sort of case only to ignore it. He does <u>not</u> leave vacant the final sub-division of the plan to turn rather to some otherwise unmentioned species (or as some think, γενος) under which to place the later issues, all the while insisting that there are only two, the originally prescribed, species. Many scholars think Aristotle explores the later issues under this undeclared third class of justice (though not all, Burnet, del Vecchio, Irwin, and Hutchinson see no such construction). Even though there are many signs that his thoughts on justice occupied Aristotle for some considerable period (not so much from its mere appearance in the Common Books[348] and *MM*, but from related discussions in the *Politics*, and from his having written a long lost dialogue *Justice*[349]), Peters said that Aristotle's thoughts on the justice of exchanges "upon which the existence of the *polis* depends" was "an afterthought" (and many have echoed him[350]). Trendelenburg and Stewart argued for this third species to be aligned with the first (Distributive) and to have priority over the second[351]. The less radical of the interpretations for a third class see it as an additional species of justice alongside the Distributive and the Corrective[352] (Marc-Wogau is one who holds that the class for Voluntary exchanges is a sub-species of the Involuntary, see § 4.9, note 285(iii), and § 5.6, note 333). The more radical of the claims for a third class of justice stress the great importance of the issues omitted from the initial division of the genus; Finnis, for example, writes[353]:

> The real problem with Aristotle's account is its emphasis on correction, on the *remedying* of the inequality that arises when one person injures or takes from another, ...This is certainly one area of problems of justice, but even when added to the field of distributive justice it leaves untouched a wide range of problems. 'Correction' and 'restitution' are notions parasitic on some prior determination of what is to count as a crime, a tort, a binding agreement etc.

if Aristotle <u>had</u> differentiated 'Corrective' from Distributive justice then this concern at the inadequacy of his treatment would be justified. St Thomas's radical alteration of <u>such</u> a text would have represented a profound advance; he would have filled the "conceptual gaps left by Aristotle's emphasis on correction". But there weren't any. The conceptual gaps appear only with the general

[348] A point made to me by Professor Rowe.
[349] In four books according to Diogenes Laertius who cites Cicero and Suetonius, see *Select Fragments*, Ross ed., op. cit., p.vii.
[350] See § 5.1 and note 301.
[351] Trendelenburg (op.cit., p.412). *Stewart* pp.415-19; a view strongly rejected by Zeller (*Aristotle and the Earlier Peripatetics*, vol.2, 1897, pp.170-73 note 3).
[352] In addition to the writers mentioned earlier from Felden onwards who hold this very common opinion were H.A.Fechner (*Uber der Gerechtigkeitsbegriff des Aristoteles*, p.35) and E.Haecker ('Das fünfte Buch der Nicomachischen Ethik', *Zeitschrift f.d. Gymnasialwesen d. Berlin Gymnasiallehre-Vereins* 16, 1826, vol.2, p.554). L.Berns ('Aristotle and Adam Smith on Justice: Co-operation between Ancients and Moderns?', in *Review of Metaphysics* 48(i), 1994, pp.71-90) thinks that there is a third class that is "like" the earlier species, but he leaves it extremely vague whether it forms a definite species or a distinct genus. Edel's view (op. cit.), endorsed by Sparshott (op. cit.), is that the classification is all too vague to fit exact species or genera.
[353] 1980, pp178-79.

misapprehension of that emphasis by the translators and commentators[354]. Ritchie was especially influential in stressing the priority of the issues treated by the 'third class' of Particular justice; he, and those who have adopted his approach[355] hold that it is their very importance which places the issues beyond the extension of Particular justice. Contrary to Ritchie's claim, it is this importance which makes it impossible for them to fall outside Particular justice. Particular is the branch of Universal justice which Aristotle designates for issues of fairness and equality, i.e., it is the genus for *appropriate* equality. Chapter 5 is concerned with the means to ascertain appropriate equality; it deals exactly with the central theme for which Particular justice was devised. The more important the issues there, the more centrally they must occupy Particular justice. Ritchie's view, for all its celebrity, is bizarre: that Aristotle abandons his treatment of the justice which locates appropriate equality just as he approaches the issues which most require it. He leads the (already misguided) interpretations of Felden, Trendelenburg and Jackson even further astray; they only dislodged the questions of voluntary exchange from their place at the heart of the genus, Ritchie kicks them out altogether.

All these difficulties and obscurities disappear with Aquinas's reading of the text. Rather than his shifting the meaning, it is more natural to regard him as following the logic in Aristotle's classification, treating D and F as ellipses for 'simply directive Justice in Exchanges'. The misplaced emphasis on correction has given rise to the incongruity of defining the species according to differing criteria and has saddled Aristotle with the blame for an amazing range of confusions. Every one of these interpretations which reject Aquinas's presentation of the structure and character of Particular justice results in absurdities, confusions which the mediæval understanding escapes. The absurdities are fully recognised by all scholars, but they blame them on Aristotle, not their own constructions. The tenor of Finnis's remarks is to support Aquinas's deepening of Aristotle's theory. Aquinas ought

[354] The conceptual gap that *was* left had nothing to do with the emphasis on correction; it is of quite a different kind—that left by the answer Aristotle would give to the question 'Who counts?'; see § 4.3 (iv).

[355] Including Ross, Gauthier-Jolif, Hardie, Miller (1995, p.70) and *Meikle* (p.130). Harrison (op.cit pp.44-45) makes no reference to Ritchie, but effectively follows him, he seems to think that exchanges fall outside the remit of Particular justice (and thinks Aristotle's account unsatisfactory). J.Spengler ('Aristotle on Economic Imputation', in *Southern Economic Journal*, 1955 pp.371-89, reprinted in Blaug 1991 pp.55-73) claims to follow Ritchie but treats the justice of exchanges as merely a third species. Professor Finnis tells me that he does not follow Ritchie, yet I believe his approach is in line with that initiated by Ritchie.

to be defended to a considerable extent; apart from the distortion of the nature of Universal justice he gave an accurate presentation of Aristotle's theory, avoiding the non-sequitur which led to the errors of the modern interpretations of the text. The 'Latin tradition' was not guilty of mistranslating Aristotle, even though it failed to bring out an informative change of expression at one point in the original. It succeeded in preserving the two ambiguities present in the Greek where the modern tradition has disambiguated one of them wrongly, and the other it has failed to notice. More important than supporting St Thomas is to give the credit to Aristotle that all modern interpretations deny him: for having produced a carefully considered, comprehensive, and unified theory. Being clear as to the structure of the theory, and the nature of the species defined are necessary not only (rather obviously) for the understanding of the theory itself, but to have much hope of grasping what his solution to the problem of exchange-value is.

Chapter 6

THE DOCTRINE OF EXCHANGE

6. 1 The 'third species'

We found that by the end of Book *V* chapter 4 Aristotle has completed his account of the Involuntary sub-class of exchanges; he then turns, exactly in line with his stated purpose (1130b30ff.), to the remaining Voluntary sub-species. This progress, clear and rational as can be, nevertheless has been found baffling by most commentators. What has thrown them out of reckoning is the imagined Corrective species. Inevitably, were there such a species, the issues would become murky. Once set off-course by the fatal non-sequitur (see § 5.7), limiting the function of the species to the remedying of actions that have gone wrong leaves out of account the need for guidance in voluntary relations between members of the *polis*. This call for guidance and remedy is supposed by some commentators to have occurred to Aristotle only as an afterthought; but even those who do not accuse him of such gross absentmindedness think his approach to these issues is inadequate and muddled. The commonest interpretation of the theory over the last century and more (despite the opposition of Grant, Burnet, del Vecchio, Marc-Wogau, Irwin, and Aubenque) has been that these matters are dealt with through the undeclared third class of justice.

The first objection to "the afterthought" approach is that even if he had thought about such great issues "upon which the existence of the *polis* depends" (1132b34) only afterwards, and they turned out to be more important than his forethoughts, the natural action would have been to re-arrange the presentation of the theory—at the very least to make some reference to it in the layout of the plan. (Of course it would then not be known that it had been an afterthought.) There is no natural location for a third species in his plan, nor for a parallel γενος that Ritchie claims stands apart from Particular justice altogether. These omissions are indeed sins Aristotle is charged with, but his not mentioning any third class, and his not providing for it in the layout of the subject, are clues that there isn't one, not that there is one that he handles badly.

Having read the plan for Particular justice (fig. 1, § 4.4) we should expect to find, even before looking in any detail into its contents, that (i) the examination of Voluntary exchanges will follow-on immediately from the discussion of Involuntary ones. (ii) That the subject will continue to be presented in terms of proportionate equality (locating the relevant mode of equality being the function of the whole genus of Particular justice). These features are precisely what we do find in chapter 5. The mysterious fog covering the sequence of the inquiry clears once we set aside the supposed Corrective and return to St Thomas's Commutative as the second (and final) species. We then see that there is no unannounced departure from the plan, rather, Aristotle turns in the correct order in a sequence that is clear as day.

6. 2 Reciprocity

The examination of Voluntary exchanges begins with the remark that although their justice involves reciprocity, simple reciprocity does not fit either of the classes (a remark which has been taken to open the door to the third species). Distributive justice is mediated through the geometric formula, and the simpler arithmetic proportion guides the justice of exchange. Reciprocity does not appear in Distributive justice because distribution is a one-way process[356], and whatever Aristotle means exactly by reciprocity it must at least be some kind of two-way activity. For the justice of exchanges (whether Involuntary or Voluntary) what *simple* reciprocity fails to do is to distinguish objects or actions from their value. This is Aristotle's objection to the Pythagoreans, that for Involuntary exchanges the doctrine later known as *lex talionis* is unjust because the reciprocation in 'an eye for an eye, tooth for a tooth' concept might not be of the truly like-for-like. Neither in Voluntary transactions might the exchange of one item (or equal portion) for another. They would only be really like-for-like if their values were equal. Ascertaining the equal value is the business of chapter 5. It must first be understood (what is easily misunderstood) that the rules for the exchange of the goods—whether the initial exchange itself, or the rules to rectify any injustices that might follow (see § 5.9)—are not the same as the rules which guide the valuation. Among the several confusions that have beset the treatment of this chapter, *not* distinguishing the evaluation of the

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[356] Heliodorus gave another reason why reciprocity does not apply in Distributive justice; that is that in receiving dividends what is received is different in kind from the original contribution. Stewart, who upholds this claim (p.446), retails the example of a musician who is paid in money, not music. The argument is unsatisfactory, Distributive justice does not guide commercial dividends (see § 4.9), nor the payment of musicians; these are transactional issues. Heliodorus's point that a service is different in kind from its recompense is, however, a valuable one, and goes to the heart of the difficulty facing chapter 5.

action (or the object[357]) from the exchange stands as peculiarly damaging[358]. The transactional rules are arithmetically proportional, or to put it as Aristotle sometimes does (compressing the proportion), rules of simple equality rather than geometric proportion, but *that* principle is not adequate to the evaluation process which must precede the exchange. Different principles of proportion are needed to achieve that. As he does not here employ the arithmetic proportion it has been widely assumed that Aristotle reverts to the geometric (see § 4.9), but I do not think this is so. He makes no mention whatsoever of the geometric proportion in chapter 5; that fact alone should have alerted commentators to its likely absence from the chapter. In chapter 3, where Aristotle *does* obviously use it he refers to the geometric proportion constantly, and explains it in detail, using diagrams. Similarly in chapter 4 when invoking the arithmetic proportion, *that* proportion is repeatedly referred to and explained. If Aristotle uses the geometric proportion as the model for the evaluation of goods it is strange that he doesn't mention it.

Aristotle dislikes the Pythagorean concept of justice (as he represents it[359]) for its crudity; he immediately gives an example to illustrate where the Pythagorean model breaks down: at 1132b29 he speaks of a case in which an officer strikes a subordinate. The blow is an authoritative action of discipline or education, but if the subordinate returns the blow that would be an act of mutiny. The same actions (objects) therefore may carry widely differing values[360], and their exchange might not be genuinely like-for-like. The principle of just reciprocity may be seen as that of receiving *the same* (or sometimes perhaps receiving the opposite) in return; simply conceived the same action here (a blow of much the same force or physical damage) is not of the same worth (ἀξιαν), whereas the just reciprocation is that of the ἀξιαν, not of the action[361].

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[357] For Involuntary exchanges what is to be evaluated will be an action, but so it also would be for Voluntary ones where services will need to be assessed.
[358] *Meikle* (p.131) says that the issue of economic value is usually overlooked, the chapter being thought to be about fair exchange.
[359] Some (e.g., Heath, 1949 p.272, *Stewart* pp.444-45, H.Cherniss *Aristotle's Criticism of Presocratic Philosophy*, p.226 n.39) hold that Aristotle misrepresents the Pythagorean view. But *Soudek* (p.54) wisely says "we can trust Aristotle's knowledge of Pythagorean philosophy which was much more complete than ours can possibly be".
[360] A point familiar to many commentaries, see for example *Joachim* (pp.144, 146) who cites Burnet and Vinogradoff.
[361] As this is being written a Bill is going through Parliament which directly reflects the point being made. It is proposed that differing ἀξιαι are to be assigned in some cases where a particular malice towards the class to which the victim of an assault is thought by the assailant to belong. The proposal implements exactly the distinction Aristotle makes in this

Aristotle appears to choose this example as one of Involuntary exchange that is, nevertheless, on the border between the Involuntary and the Voluntary—it is placed on the border between the two chapters. He says "it makes all the difference in the world whether the action was voluntary or not"[362] (intention is still a criterion for serious crimes such as murder), and indeed assessing whether an action is voluntary or involuntary can be tricky. You might need to take into account the possibility that the blow was intended for someone else, or that the soldier was merely reckless, or that it was a joke, or the hand was controlled by a third party[363]. As with the rest of Particular justice, here justice is achieved where the appropriate equality is found. There is to be reciprocity but it must be of the like-for-*truly*-like (the whole value of the blows), not like-for-apparently-like (the physical description of the blows). The example serves to draw together the two branches of Exchange; it is discussed in the chapter on Voluntary exchanges, but is principally a matter of the Involuntary. As such it inverts and reciprocates the latter part of the previous chapter where the issues of Involuntary exchanges were explained in the language of Voluntary ones (and excused as such by Aristotle (1132a11)[364]. The two passages indicate the equivalences in the two branches of the species (again a point which ought to have guided commentators towards the generic structure I've been arguing for[365]).

6. 3 Reciprocal proportion

Reciprocity is to be based on proportion. There are a number of difficulties to be faced in sorting out what this involves, but they broadly divide into two classes. First, the surviving MSS appear disordered and repetitive, and perhaps parts of the text are missing (the diagrams certainly

chapter between the physical damage (βλαβος or *damnun*) and the harm done (ἀδικια or *iniuria*).

[362] 1132b31.

[363] There are a number of complications to the example; the officer is an agent of the *polis*, and so the issue is not private but is a public exchange. It takes place between the *polis* and one of its members (parts). Apart from this illustration Aristotle does not discuss the justice of *public exchanges*; the justice of exchange is specifically the justice for members interacting among themselves, with the *polis* as arbitrator (the notion of *criminal justice* did not then properly exist). Where the *polis* is a party to a transaction the rules of Commutative justice would probably apply; where the transaction is Involuntary the majesty of the state might demand a high valuation of the dignity, safety, and well-being of its officers, so the arithmetic rule governing the redress is likely to result in a stiff penalty.

[364] *Joachim* p.138 refers to this "avowed inaccuracy" but he doesn't notice the structural reason for it.

[365] See § 5.10 and note 347.

are). The disorder is regretted by all editors (see e.g., *Jackson* and *Stewart*[366] for extended discussion of the difficulties) and I have only one point to add to the issue of repetition (in § 6.9). Secondly, and much less reasonably, there is near universal dissatisfaction bound up with objections to Aristotle's habit of expressing the essence of Particular justice in quasi-mathematical terms. Meikle, following a long tradition, and citing Hardie and Salomon[367], dismisses the mathematics (e.g., p.143); Finley remarks[368] that this chapter isn't one of Aristotle's more transparent discussions; Bonar[369] complained it is "much tortured". Soudek thinks it "belongs to the obscurest parts of his writings", and Joachim says "it is in the end unintelligible to me"[370]. This common aversion is summed up by Smith[371]:

> The whole treatment [of Particular justice] is confused by the unhappy attempt to give a precise mathematical form to the principle of justice in the various fields distinguished.

What the critics have not appreciated is that Particular justice is exactly the area of virtue that is concerned purely with the rightness of greater and less. All the factors involved in this kind of justice relate to some magnitude being wrong and some other magnitude being right. Determining what the precise quantity is is then as close to purely mathematical principles as are straight lines or volumes. Treating the proportion language as extraneous simply fails to recognise either the character or the structure of Aristotle's programme. It is *this* failure that generated the confusion and the unhappy attempts to account for Aristotle's theory.

The form of the mathematics Aristotle uses throughout is analogical, and is exceptionally simple, yet part of the difficulty in this chapter has been in deciding just what the reciprocal analogy is. As with the earlier chapters Aristotle clearly invokes proportion, but it has not been obvious which proportion. Unlike the geometric and arithmetic forms, which are clearly recorded, the reciprocal proportion is difficult to pin down, and different candidates for it have been identified. The main contenders are as follows:—

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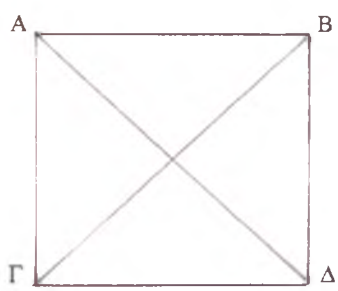
[366] Pp. 437-38 have a detailed explanation of the likely reason for the passage 1133a14-16 also appearing in chapter 4.
[367] M.Salomon *Der Begriff der Gerechtigkeit bei Aristoteles* (1979).
[368] M.I.Finley 'Aristotle and Economic Analysis', *Past and Present* 47, 1970a, pp.3-25 (p.13).
[369] J.Bonar *Philosophy and Political Economy* (1909, p.40).
[370] *Soudek*, p.45; *Joachim*, p.150.
[371] J.A.Smith *Nicomachean Ethics*, (Introduction, Everyman edition, pp.xviii-xix).

(i) *Soudek* discusses the influence of Archytas on Aristotle; he says (p.57) that Archytas introduced the term 'reciprocal proportion', and he draws attention to the rôle of proportion in the Archytan treatment of the justice of exchanges (συναλλαγματων)[372]. Soudek is in two minds (pp.56 and 59ff.) whether the proportion spoken of is the third of the classical forms, or whether it is that based on *Elements VI*. The implication is that Archytas regarded the proportion in question as being both harmonic and reciprocal[373]. (The harmonic proportion was especially associated with Archytas (see §§ 1.7, 2.1), and there can be little doubt that Aristotle obtained knowledge of it from Aristoxenus, even if he did not know of it already[374]). After rehearsing the harmonic Soudek goes on to treat the proportion used in the chapter as the entirely different figure associated with *Elements VI*.

(ii) *Lowry* (p.53) thinks that it is mysterious how the harmonic proportion came to be confused with Euclid's demonstration of reciprocal figures. He regards the proportion employed in chapter 5 as the harmonic[375]. Lowry offers an alternative to the traditional figure of exchange which Aristotle indicates at 1133a6. The usual diagram is given as a box, or a rectangle with a single diagonal, or a crossed pair of diagonals. Any of these[376] could be contained by a diagram such as this:



(figure *3*)

[372] The Archytan expression συναλλαγματα is evidently the source of Aristotle's own usage for the justice of exchanges. In the surviving fragment from the school of Archytas it has been translated as 'mutual problems' by E.L.Minar from *contractus* by G.A.Mullach (*Fragmenta Philosophorum Graecorum I,* 1875, p.562), and as 'business contracts' by Freeman (op. cit. p.80), both from the Diels Fragment 47(3). A.Dalette (*Essai sur la Politique Pythagoricienne,* 1922, pp.259ff.) argued that Archytas had applied proportion theory to the affairs of Tarentum, and that this experience stimulated Aristotle's interest in the use of proportion theory in the fields of politics, economics, and jurisprudence.

[373] Soudek cites *Stewart* p.445. Stewart refers to Alexander's record from the pseudo-Archytas, but it is not clear that this does suggest the identification of the two expressions.

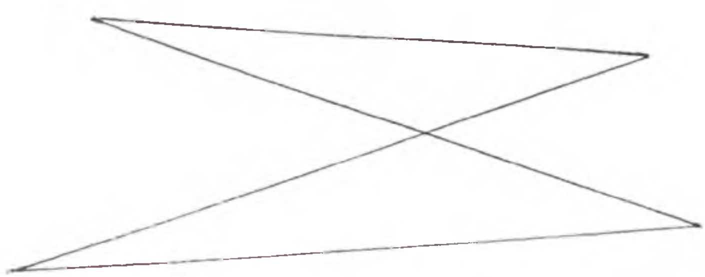
[374] See §1.7, note 55.

[375] Pp.55ff.; he cites *Select Fragments* (i.e., 25 p.96 notes 3 and 5 in Ross's edition).

[376] *Grant* and Williams presented a single intersection at the corner; *Jackson* (p.95) objects to this and gives the figure as here, as did St Thomas (see pp.291, 295, *Sententia* 47ii). *Stewart* has a blank box, Welldon a rectangle; Burnet has crossed diagonals as do Joachim, Rackham, Soudek, Ostwald, and Dirlmeier. Peters used diagonal dots; Gauthier-Jolif tentatively use the crossed diagonals (1959 II, p.376).

Lowry (pp.57-58) traces the box figure to Oresme's *Le Libre de Éthiques d'Aristote* (14th century), but it appears to have been used by Albertus in his commentaries[377]; it is certainly very ancient (being a form of the square of opposition—see § 1.6) yet is not much help in explaining the issues at stake in chapter 5. The principal notion to be illustrated is that of (diagonal) συζευξις (assignment, see § 4.10 and below), but as I do not think they are likely representations of the text, or in any way helpful in understanding Aristotle's argument, I will not explore the conflicting accounts of the relationships between the terms in these illustrations. In their place Lowry suggests a figure taken from J.Hambridge's *The Elements of Dynamic Symmetry*[378]. Like Lowry I think that Aristotle may have used some other diagram (perhaps a little less elaborate than Hambridge's), and I offer other possibilities below. What is remarkable in Lowry's account of the issues of exchange is that having considered the possibilities and explored the harmonic proportion in detail, he traces the influence of Eudoxus on Aristotle's thinking but refers only to the Eudoxian method of exhaustions in relation to the problems and completely ignores his theory of proportions.

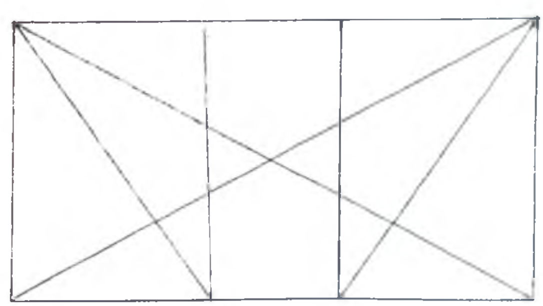
(iii) Much more common than the views of Soudek or Lowry has been the belief that the reciprocal proportion is $a : b :: d : c$ (see § 2.9), but if this is a proportion its a rum one. It is true only where c happens to equal d. It is said to be justified by (Simson's reconstruction of) *Elements VI*.2, but the justification is dubious. *VI*.2 may be illustrated:



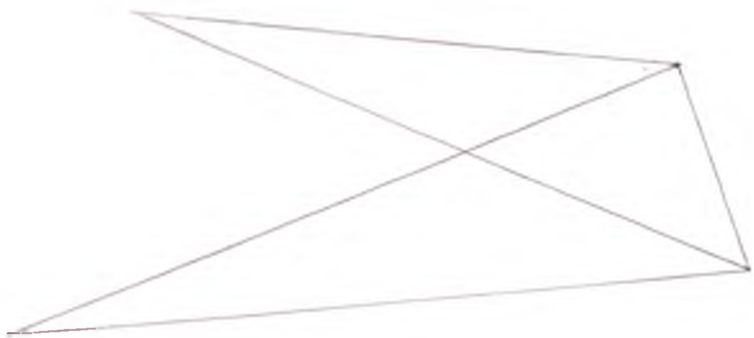
(figure 5)

[377] See *Super Ethica*, op. cit. p.243.
[378] 1948, pp.17-18.
Hambridge constructs "reciprocal figures in dynamic symmetry" in which a rectangle is divided through crossing diagonals such that pairs of proportionally smaller rectangles may be rotated, gradually reducing the overlap that has been created.

(figure 4)



Propositions 6.14 and 6.15 stem from definition *VI*.2, and 6.15 especially is taken (e.g., by *Jackson* p.89, *Stewart* p.443) to be the exemplar for the reciprocal proportion. 6.15[379] is proved with the aid of a figure such as:



(figure 6)

Euclid says that the alternating sides about the same angle in equal (i.e., equiangular) triangles are ἀντιπεπονθασιν (reciprocally proportional). The definition generates two possibilities: *a : c :: d : b*, and *a : d :: c : b*, not *a : b :: d : c;* the writers who perceive this last pattern do not explain how they derive it. There is no advantage in referring to the off-chance that *c* might happen to equal *d* as a *proportion*; together with the use of such jejune figures as *3* Aristotle is made to describe in unnecessarily cumbersome ways that *a* gets *d* in exchange for *c*. Neither the formula nor the diagram show how the exchange is just; not surprisingly the case presented by such models has been thought to be feeble.

(iv) There is a further use of the notion of reciprocity in *Elements VI* but it is not generally spoken of in connexion with the chapter[380]. This proposition (6.16, see § 2.9) is an extrapolation of 6.14, which was the implementation for parallelograms of the loose (see § 2.9) definition *VI*.2. Unlike that definition, or 6.14 and 6.15, it has universal application; i.e., although it treats purely geometric figures (rectangles) it is applicable to quantities of any class[381]. 6.16 stipulates the biconditional:—

---

[379] 6.15 (Heath):

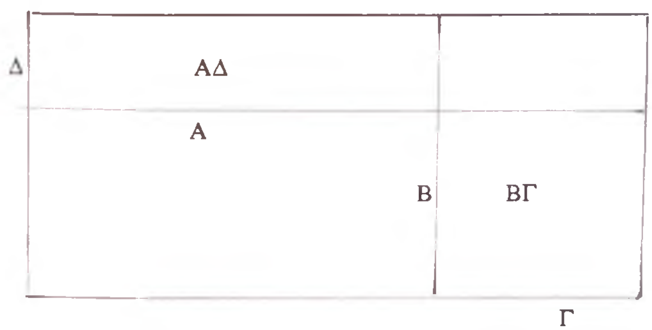
In equal triangles which have one angle equal to one angle the sides about the equal angles are reciprocally proportional; and those triangles which have one angle equal to one angle, and in which the sides about the equal angles are reciprocally proportional, are equal.

[380] Perhaps because the word 'reciprocity' is used only at a point in the working-out of the proof.

[381] As 6.16 is a special case of 6.14, and 6.14 is derived from *VI*.2, *VI*.2 and 6.14 could be regarded as being of universal application.

$$a : b :: c : d \Leftrightarrow a \cdot d = b \cdot c.$$[382]

There is a proportion if and only if the products (in the mathematical not economic sense) of the reciprocal alignments are equal. Requiring that *unless* the product of *a* and *d* equals the product of *b* and *c* there is no proportion gives precisely the rule Aristotle is demonstrating (by whichever diagram he finds most useful). The correct valuation of the products (in the economic not the mathematical sense of the word *product*) will enable the reciprocation to be just, and will be correct only when the value of $a \times d$ equals the value of $b \times c$. The format of 6.16 would have been familiar to Aristotle[383] in a simple diagram such as this:



(figure 7)

The task he faces is to apply this truth to everything capable of being quantified and exchanged. How he does this is examined below in §§ 6.7-6.9, but note that as the proposition is a variant of the ἐναλλαξ principle the terms will be homogeneous.

6. 4 Reciprocity in the text

Looking through the candidates for the reciprocal proportion that have been suggested in connexion with chapter 5 we have scarcely been able to find mathematical evidence for it. What is clear is that the reciprocal was not a proportion on a par with the two earlier ones (this is why Soudek and Lowry turn to the harmonic as the proportion in question). Aristotle undoubtedly uses the terms ἀντιπεπονθος and ἀναλογια in the chapter, but does the text really support the belief that the combination of these terms refers to this farouche proportion? There are eight occurrences of the

382 6.16 (Heath):
If four straight lines be proportional, the rectangle contained by the extremes is equal to the rectangle contained by the means; and if the rectangle contained by the extremes be equal to the rectangle contained by the means, the four straight lines will be proportional.

383 See Artmann's *Euclid's* Elements *and its Prehistory* (op. cit., p.20), but also Aristotle refers to a similar diagram in *Topics* 158b29 (see § 2.5).

word ἀντιπεπονθος (including ἀντιπεπονθεναι once) in chapter 5, these are (with Ross's translation):

(i) 1132b21, (ii) 32b22, (iii) 32b25:

> Δοκει δε τισι και το ἀντιπεπονθος εἰναι ἁπλως δικαιον ὥσπερ οἱ Πυθαγορειοι ἐφασαν ὡριζοντο γαρ ἁπλως το δικαιον το ἀντιπεπονθος (ἀλλῳ) Το δ' ἀντιπεπονθος οὐκ ἐφαρμοττει οὐτ' ἐπι το διανεμητικον δικαιον οὐτ' ἐπι το διορθωτικον
>
> Some think that *reciprocity* is without qualification just, as the Pythagoreans said; for they defined justice without qualification as reciprocity. Now 'reciprocity' fits neither distributive nor rectificatory justice

(iv) 1132b33:

> το ἀντιπεπονθος κατ' ἀναλογιαν και μη κατ' ἰσοτητα
>
> reciprocity in accordance with a proportion and not on the basis of precisely equal return

(v) 1133a10:

> εἀν οὐν πρωτον ᾗ το κατα την ἀναλογιαν ἰσον εἰτα το ἀντιπεπονθος γενηται ἐσται το λεγομενον
>
> If, then, first there is proportionate equality of goods[384] and then reciprocal action takes place, the result we mention will be effected

(vi) 1133a33:

> ἐσται δη ἀντιπεπονθος ὅταν ἰσασθῇ
>
> There will, then, be reciprocity when the terms have been equated

(vii) 1133b6:

> εἰ δ' οὑτω μη ἠν ἀντιπεπονθεναι οὐκ ἀν ἠν κοινωνια
>
> If it had not been possible for reciprocity to be thus effected, there would have been no association

(viii) 1134a24:

> Πως μεν οὐν ἐχει το ἀντιπεπονθος προς το δικαιον εἰρηται προτερον.
>
> Now we have previously stated how the reciprocal is related to the just;

The first three occur together in the passage which opens the chapter with Aristotle claiming that simple reciprocity is unsuited to either sort of Particular justice. The fourth follows-up with the adjustment that reciprocity must be based on analogy, not equality. The fifth speaks of proportional equality being necessary before reciprocation takes place (not reciprocal proportion taking place). At

[384] Interpolated by Ross, as they are by Jackson, Rackham, and Ostwald, but not by others such as Welldon, Thomson, and recently Judson, who are more accurate.

(vi) Aristotle says that producers' respective products must be rated according to a proportion that is based on the relative worth of the parties. Sandwiched between (vi) and (vii) is an account of proportion but there is no suggestion that the proportion is reciprocal. Again the passage says that unless there is a proportional valuation there can be no (just) reciprocation, i.e., no just exchange of like-for-like. The last mention of reciprocity simply refers back to the discussion relating the issue to justice. There is, then, little sign of the mathematical reciprocal proportion in the text:

(a) First of all the passages make sense as claiming that the proportion must be achieved to facilitate what is to count as fair reciprocation.

(b) There is little connexion between the words ἀντιπεπονθος and ἀναλογια; they appear together in only (iv) and (v). Of these, (iv) refers to reciprocation according to (some) proportion, and (v) to the requirement that there must *first* be proportional equality and *then* reciprocation. In the earlier chapters where modes of proportion are clearly used Aristotle labels them firmly (when speaking less directly of the character of proportion he ignores the proportional nature of the arithmetic—but not when he is introducing and defining transactional justice in terms of proportion). In chapter 5, on the other hand, there is no correspondingly clear account of the suggested third mode of proportion. No expression corresponds to "geometric proportion" or "arithmetic proportion". In fact in (iv), the one place where reciprocity and proportion are used in the same phrase (in (v) the terms occur in separate phrases linked through a conjunction), the connexion is reversed; rather than some expression such as 'proportion according to reciprocity' he says 'reciprocity according to proportion'. Stewart says (p.442, citing *Jackson* p.93):

> To ἀντιπεπονθος, literally '*that which* has suffered or received in return', is somewhat strangely used instead of το ἀντιπεπονθεναι.

I take it that Stewart does not mean merely that it is strange of Aristotle not to use the infinitive, but that the correct mathematical expression is not used. I.e., that Stewart thinks Aristotle *is* using the reciprocal proportion at (i) and (ii) but has referred to it slightly improperly. Stewart claims (p.443) that the terms ἀντιπεπονθος, ἀντιπεπονθεναι, and ἀντιπεπονθεσις are "unambiguously 'reciprocal proportion'" but that once used outside mathematics they become ambiguous. If the mathematical proportion *is* being used then the vagueness is not only in its extrapolation to non-mathematical topics but (as Stewart picks up) in the word chosen. Referring to this proportion as το ἀντιπεπονθος rather than ἀναλογια κατ᾽ ἀντιπεπονθος (or the more definite κατα το ἀντιπεπονθος) would mark an extraordinary stylistic departure from his previous custom. (This divergence alone would be reason enough to doubt that the reciprocal mode of proportion is being invoked.)

(c) Aristotle does not explain what reciprocal proportion is. He goes to great (many think excessive) pains to explain the earlier, very simple, proportions, but then when he uses a more elusive and complex one he neither names it as he does the others, nor explains how it works. Especially in view of the elaborate explanations of the geometric and arithmetic analogies whose meaning is far more obvious, his not explaining this proportion would be strangely inconsistent.

(d) What commentators (both those who see it in the text—such as Grant, Jackson, Stewart, Burnet,—and some of those who don't—such as Heath[385] and Hardie) have called reciprocal proportion ($a : b :: d : c$) is not properly a proportion (§§ 2.9, 6.3), and there is no reason to suppose that Aristotle thinks that it is.

6. 5 The problem of exchange-value

Having decided which proportions Aristotle *doesn't* use we need to discover the one he does. To appreciate his use of proportion in chapter 5 we ought first to re-state the problem Aristotle tackles, a problem which I do not think that any later thinker has faced as squarely as he does. What Meikle calls "this notoriously intractable problem" at its most stark and extreme is this:

> How is it possible to compare things with nothing in common?

Later writers—I think *all* later writers—have sought some commensurating element lurking behind the infinity of interactions, of goods and services, that are brought to an exchange. It has been simply assumed that some elusive factor (some x) must be found in virtue of which a just or at least acceptable valuation will be possible. Not only has this quest been assumed to be necessary, Aristotle is credited with having started it. Soudek quotes Kaulla admiringly[386]:

> Aristotle has brought into the world the thought .... that the comparability of values of different economic goods has as its condition a something which is common to the magnitudes to be compared.

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[385] Heath (1949 p.274) wonders if the proportion being spoken of is that "defined in Euclid VI.35" (which I suppose is 15), but decides that it isn't. Hardie, op. cit., p.199.

[386] *Soudek* p.75; R.Kaulla *Die Geschichtlichte Entwicklung der Modernen Werttheorien* (1906, p.52).

So deeply fixed is this supposition that it has prevented even his closest admirers from seeing Aristotle's actual argument. *Meikle* expresses the unquestioned supposition perfectly (p.14):

> So the problem is to discover the property they ["the most various things"] must all share, in virtue of which they are commensurable, as they must be since they are equated[387]

The quest parallels both the anthyphairetic attempts of Theodorus and Theaetetus (see §§ 1.1, 1.2, 2.5-2-7, 2.9) to locate some magnitude (some x) in virtue of which incommensurables might be resolved, and the assumptions of mathematicians throughout the Middle Ages in their attempts to represent *Elements V* (see §§ 2.2, 2.9, 3.7). The later writers have, in effect, been stuck in a pre-Eudoxian rut. In the belief that Aristotle is searching for "a something which is common" he is held to have generated (or at least to have pre-figured) different and conflicting schools of economic thought. On the one hand the various demand-led, neo-classical, and utilitarian theories, and on the other the labour theories of value.

A possibly complicating factor in supposing there to be, and then identifying, this "common substance" (as Marx called it) has been a consciousness of the danger of anachronism. Two ἕξεις have emerged; there are some ('modernists'[388]) who believe that all societies, ancient or modern, share common properties, sufficiently so at any rate to justify treating ancient societies as simpler models of modern economies. Others ('primitivists'[389]) say that modern society is different in kind from ancient communities; projecting notions drawn from modern economics onto them is then, they think, radically inappropriate[390]. Supporters of both these tendencies, nevertheless, when speaking of the issue of exchange in chapter 5, simply assume the problem to be one of determining some common element (some x). The Austrian school of marginal-utility theory, for example, "discovered" that in chapter 5 Aristotle proposes their own approach to the determination of value[391].

[387] P.14; Karl Marx, for example, was prevented from appreciating the depth of Aristotle's account of value by assuming there must be some "common substance".

[388] E.g., E.Meyer, M.Rostovtzeff (see *Meikle*, pp.3 and 158); Lowry (*The Archeology of Economic Ideas*, 1987); W.E.Thompson ('The Athenian Entrepreneur', *L'Antiquité Classique 51*, pp.53-85, 1982).

[389] E.g., Finley (*The Ancient Economy*, 1973); K.Polanyi (*Aristotle Discovers the Economy*, in Dalton 1986); *Meikle* (chapter 8) reviews the debate from a Primitivist stance.

[390] A case of the errors Primitivists fear was that discussed in § 4.9 in which commentary and then translation imposed capitalistic constructions on the text grossly distorting the nature and scope of Distributive justice.

[391] Notably J.Zmavc ('Die Werttheorie bei Aristoteles und Thomas von Aquino', *Archiv für die Geschichte der Philosophie* 12, pp.407-33, 1899) and O.Kraus ('Die Aristotelische Werttheorie in ihren Beziehungen zu den Lehren der Modernen Psychologenschule', *Zeitschrift für die gesamte Staatswissenschaft* 61, pp.573-92, 1905).

This claim was made in conscious opposition[392] to the then traditional view that the chapter gives a form of labour theory of value[393]. This older view goes back at least to Albertus Magnus who, commenting on the rather ambiguous passage 1133a14-16, says that equal amounts of labour and costs must be exchanged[394]. Aquinas adopted St Albert's reading; he determined value according to *labore et expensus*[395], but he also saw need as "the real measure of value". In doing so he was faithful to Aristotle, but it places him just as much in the other camp[396]. Over and above (or perhaps as a result of trying to reconcile) these interpretations it is also claimed that Aristotle has no theory of value at all[397], "not even a bad one" some say[398].

The 'value' about which Aristotle either did or didn't have a theory has been taken to be a quantifiable unit of (depending on your preferred school of thought) labour or need or skill or demand or labour-time or desire that, however difficult it might be to measure, will generate the right price. Accounts of the value in terms of skill (which *Soudek* interpolates continually) or labour-time or utility, merely replaces the incommensurability of the products by additional factors

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392 The χρεια-based interpretation (variously treated as *need, demand, want*) was also held by E.Barker (*The Political Thought of Plato and Aristotle*, 1906); W.Gelesnoff (*History of Economic Thought*, 1917); Van Johnson ('Aristotle's Theory of Value', *American Journal of Philology* 60, pp.445-57, 1939); *Joachim* (pp.149-51); *Soudek* (who calls it "a 'pre-marginal' utility theory" insists throughout that 'skill' is the defining factor, yet this term (τεχνη) is not used by Aristotle); J.Schumpeter (*History of Economic Analysis*, 1954). B.J.Gordon ('Aristotle and the Development of Value Theory', *Quarterly Journal of Economics* 78, pp.115-28, 1964; reprinted in Blaug, pp.113-26) credits E.Kauder ('Genesis of the Marginal Utility Theory from Aristotle to the End of the Eighteenth Century', *Economic Journal* 63, pp.638-50, 1953; reprinted in Blaug, pp.42-54) with demonstrating that Aristotle anticipated the Austrian school (see *Meikle*, p.111).

393 Apart from Marx who was inspired by what he interpreted as a labour theory of value in chapter 5 (see McNeill, op. cit. and *Meikle*) others who see it as labour-theoretic include Adam Smith (*Wealth of Nations I* chapter v; pp.31-32 of W.R.Scott edition, 1921), Ritchie, H.R.Sewall (*The Theory of Value before Adam Smith*, American Economic Association Series 3, 2(3), p.3, 1901), A.A.Trever *A History of Greek Economic Thought* (1916), *Grant, Stewart,* Burnet, Ross, and Hardie.

394 Albertus (*Opera Omnia XIV(i) Liber V*, lectio VII (404) pp.342-43) says:

> *scilicet quantum ad expensas et quantum ad laborem,* et *si non est* patiens, *idest recipiens retributionem,* passus tantum et tale, *quantum recepit, quia* communicatio non *est eorum qui sunt euisdem artis, ...*

395 R.H.Tawney traced the labour theory of value to Aquinas (*Religion and the Rise of Capitalism*, pp.38-39 in the 1947 Penguin edition).

396 A point noted by M.Beer (*Early British Economics from the XIIIth to the Middle of the XVIIIth Century*, p.166, 1938).

397 By E.Roll (*A History of Economic Thought*, revised ed., pp.26-27, 1942) for example, and *Meikle* p.190.

398 D.Winthrop ('Aristotle and Theories of Justice', *The American Political Science Review* 72, pp.1201-16, 1978), repeated by Sparshott (op. cit. p.175).

that are equally immeasurable: different skills (or needs or uses) do not share a common measure[399]. These replacements do not, then, advance the solution, they merely shift the problem sideways[400].

Schumpeter claimed that Aristotle tried to analyse "actual market mechanisms" at 1133; that he was:

> groping for some labor-cost theory of price which he was unable to state explicitly.[401]

Meikle (chapter 1, § 2, especially) elaborates this view; he believes that Aristotle makes at least two attempts[402] before admitting failure. On pp.25-26 he says:

> Aristotle is giving up as a bad job the attempt to explain commensurability ... His statement is tantamount to an admission that he does not know what is equalized in fair exchange of food, shoes, and houses ... and that he does not know what exchange value is in its technical sense of 'what *x* is'.

It has escaped all sides that Aristotle is the one thinker who makes no attempt to determine 'what *x* is'. Attempting to solve the problem of how to compare things with nothing in common by seeking this Economists Stone with which to transmute the base elements of use, need, labour, or skill, into Value is never even considered by Aristotle. He has no reason to fudge this logical impasse, indeed he is the one writer connected with economic issues who fully accepts that incommensurable means incommensurable. The problem had been solved for him by Eudoxus with the theory of proportions. As we have seen (§§ 2.8, 3.4-3.6) Aristotle makes use of the theory repeatedly throughout his work, yet the problem of the irrationals is much closer to that of exchange-value than to any of the other fields for which he borrows the solution. It has simply not occurred either to the writers on the philosophy or history of economics who discuss Aristotle, or with the commentators on the *Ethics*, that the problems of exchange-value and of the irrationals were equivalent. It seems to me, however, inconceivable that it didn't occur to Aristotle.

It will sometimes be feasible to measure one product against another, but more frequently no *relevant* comparison will be possible (a Stradivarius, a cabbage, and a paleontological report will be

399 Labour-time alone among these replacements is easily measurable, but that, in turn, rests on a prior valuation of the labourers (even piece-work operates only for those with roughly comparable skills, and within a highly restricted range of operations).

400 *Meikle* (chapter 2) argues against the χρεια-based valuations as being impossible to generate a common measure—there is no way to measure needs, wants, utilities or satisfactions. He also opposes (in chapter 9) any idea that Aristotle proposes labour-based valuations, but for metaphysical reasons connected with the incommensurability (very often) of the categories into which the objects to be exchanged will fall.

401 Op. cit. pp.60-61, note 2.

402 The attempts being first through money and then through χρεια.

commensurable in weight, but for nearly all purposes that would be as irrelevant a common measure as that connecting Occelated turkeys with the blood of antelope). Aristotle does not, as Schumpeter and Meikle claim, attempt but fail to commensurate the incommensurable; on the contrary, he accepts from the outset that incommensurables, either within or across genera, cannot be approached through any common substance. Indeed to avoid any temptation to rely on some underlying commensurating element (such as need, skill, use, or labour) for some cases, and to guarantee the completely general character of the solution for all cases, the method he adopts is the reverse of that attributed to him (and the reverse of that of later economists). Aristotle takes full advantage of Eudoxus's leap of the imagination from the 'first-order' to a 'second-order' conception of the solution. Following Eudoxus (who inverted the antaneiretic and anthyphairetic procedures attempting to locate some x to commensurate the incommensurables), Aristotle applies a precise and exhaustive indirect comparison to the factors ready to hand (not some x), and allows for the total absence of any shared property.

The principles underlying his use of the general proportion theory may best be shown by paraphrasing and adapting the description given above in § 2.9 of that theory:

> Goods or services of any kind will always be comparable with some others of the same kind. All extraneous (non-productive) attributes of the producers of the goods (such as their social standing, physique, and aesthetic tastes[403]) are to be set aside. The comparisons will form ratios between the products and their producers (this heterogeneity issue to be argued below). The quantities involved measure one another by co-ordinating their multiplication. In carrying out this project Aristotle removes the supposition that some measure (some x) has to be found to solve the problem.

Just how he solves it Aristotle shows with simple diagrams.

6.6 Aristotle's use of reciprocal figures

What the survey of the putative reciprocal proportion (§§ 6.2-6.4) showed was (i) that there has been speculation that it might have been the harmonic; (ii) that it might have been based on the same source as the 18th century reconstruction of *Elements VI* definition 2; (iii) that that definition, and (iv) its usually cited extrapolation 6.15, does not apply to quantities in general but only to similar

[403] Which is not to say that these or any other given attributes might not be productive, and so relevant to the assessment of the value of their products: the strength and agility of professional athletes are directly related to their value; a Society columnist's social standing or an art critic's sensibility could be necessary to their products.

plane figures; (v) that in any case, the definition does not justify the rogue formula $a : b :: d : c$ that is widely imposed on the text. (vi) The figures traditionally presented at 1133a6 are unhelpful, and unlikely to have been Aristotle's own (I have as yet merely asserted this, not argued it; the argument follows in the next two sections). That the box figure or crossed diagonals are ineffectual can hardly be denied; they are the target of the greater part of the abuse hurled at the text. (vii) The mathematical principle of reciprocation as it appears in 6.16, however, looks more promising as a guide to the issue faced by the chapter. (In chapter 5 the problem of finding the correct proportion is that of how to guarantee the biconditional 6.16 where the terms refer to elements that are not of the same genus.)

To support my claim that Aristotle uses a diagram different from *3* when speaking of the need for a diagonal assignment of the terms I believe it would be worthwhile to look at the kind of figures he would have been familiar with, and especially at any closely associated with notions of reciprocity and proportion. The obvious diagrams to use would then be of the proof in 6.16 (above) and those for 6.14 or 6.15 that I have been opposing as giving the 'reciprocal proportion'. Apart from its connexion with the cluster of principles which went to make up the notion of a model (and so known to Aristotle—see §§ 1.9, 2.8 note 144) 6.15 is closely related to a figure he twice uses in *Mechanics* in connexion with reciprocity. In *Mechanics* (850a39: το μηκος προς το μηκος ἀντιπεπονθεν, and at 854a25) Aristotle[404] refers to the reciprocal in speaking of the action of a lever. The diagrams there sketch the principle of the lever as used by dentists and for nutcrackers:

(figure *8*)

The figures *2*, *5*, *6*, and *8* combine two features useful to the demonstration of the case he is making, (i) the diagonal συζευξις, with (ii) the principle of reciprocity (which the traditional box figure or crossed diagonals fail to do). It seems most likely that Aristotle takes whichever figure he uses from

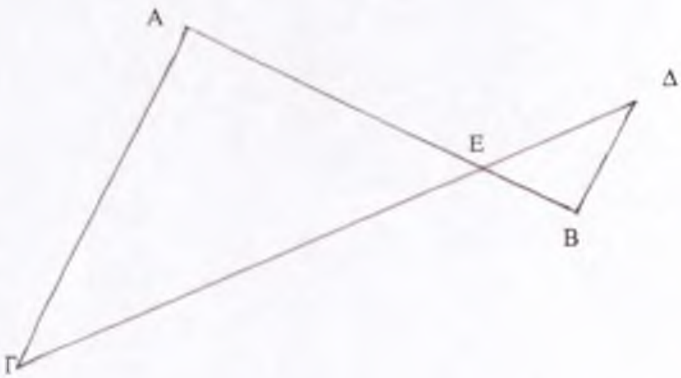
[404] *Mechanics* may have been written by an early Peripatetic from Aristotle's lectures.

the same source as Euclid, and as *Elements VI* implements rules of proportion developed in Book *V*, the source would appear to have been Eudoxus or his associates.

The question of which proportion is used in chapter 5 can now be settled. It has been left until this point because, unlike the forms of proportion in the earlier chapters, the actual *mode* of proportion used here turns out to be a comparatively minor issue. For Distributive justice the geometric proportion was a vital criterion, and Aristotle accordingly explains it at length. Similarly for the justice of Exchanges the arithmetic proportion is a defining marker, and is thoroughly explained. In both cases the use of the specific mode of analogy distinguishes it from the other. But the choice of the mode of proportion within the Voluntary sub-class of Exchanges has no such central rôle. I believe that the analogy used is the third of the three classical forms, called by others *harmonic*, but left unnamed by Aristotle. His use of that proportion is only to preserve the equal divisions across the ratios: "some portion" of one producer's output is to be valued against some portion of another's. The corresponding division is retained in the proportion *whole : part :: whole' : part'*. What is vital to the argument is not so much the specific form of the proportion but the general theory which manipulates it. Whether the ratios in $a : c$ and $b : d$ report the equivalent multiples, quantities, or divisions, what matters is *that* they are ratios, not the ratios that they happen to be.

6. 7 The first stage

Aristotle deploys the general theory through two stages; i.e., that in addition to the distinction to be made between the valuation and the exchange (mentioned earlier), the valuation itself has two stages. It should be emphasised that the 'mathematics' he uses here (and throughout his account of justice) is of the simplest possible; everything is explained through obvious geometric patterns. The first step is presented with a figure at 1133a3ff., which I suggest is a diagram such as Keyser's for Distributive justice (figure *1*, § 4.10), but tilted somewhat:—



(figure 9)

This first stage of the argument could also be demonstrated by a rectangular model similar to figure 7, or by any number of figures. (Besides, Aristotle may have used different figures at different times, just as he uses examples of different products and producers.) *But* his argument cannot be demonstrated by the box or crossed diagonals traditionally presented. The reasons for preferring a (very slightly) different figure from Keyser's chapter 3 diagram are:

(a) to contrast the two species of justice. Aristotle makes and keeps to a firm distinction between them (and we have seen the damage to the understanding of the theory when the distinction has been blurred). He may have preferred to underline the difference between the two models of Particular justice by keeping the demonstrations distinct.

(b) The diagonal alignment of the issues is more prominently displayed in a figure close to that he uses in the *Mechanics.*

(c) The notion of reciprocity is used throughout the account of the evaluation, so a figure drawn from the use of simple implements familiar to an audience (such as the nutcrackers and pliers of the *Mechanics* used to demonstrate the principle of the lever) which vividly employ (and demonstrate) the principle of reciprocity might be more helpful than an equivalent diagram which lacks the useful associations.

(d) The figure is in part the same as one needed to demonstrate the proof of 6.15, which also made use of the principle of reciprocal relations, and which also bears a marked similarity to the figure used in the *Mechanics* (as mentioned above).

These are, of course, far from compelling reasons, but they are sufficient to prompt the adjusted diagram, but what *is* necessary is that the figure used shows:—

(i) the comparison between the producers (A and B) where they may not be equal (1133a18)

(ii) the link between the producers and their own products (A - Γ and B - Δ)

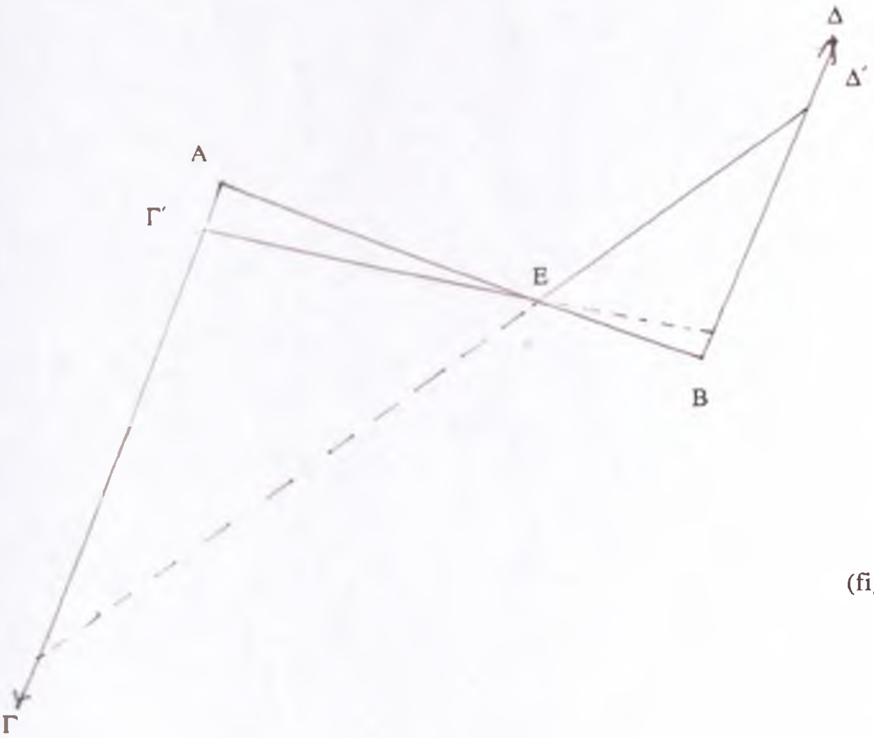
(iii) the διαμετρον συζευξις, the diagonal assignment whereby the value of part of producer A's product and the value of part of B's product are related and contrasted.

The diagonal assignment is stressed by Aristotle to contrast with the side-alignment which connects the producers to their own products (the terminology reflects the traditional contrast between side and diagonal number values). The usual box figure or crossed diagonals do not demonstrate (i) or (iii), whereas figures such as *9* or *2* show at a glance any difference in value (represented by magnitude) between A and A's product and B and B's. As with the case of Distributive justice the diagonal crosses at the mean point (E) between A and B so as to demonstrate the ratio between the parties: physician : farmer (1133a18) or builder : cobbler (1133a23).

There may be two immediate objections to the claims either that Aristotle uses some diagram such as *9* or *2*, or that by the use of such figures the argument makes sense. The first objection is 'What are to count as the 'wholes' of which the items to be exchanged are "portions"?'[405]. I believe this objection can be met in two ways. First, self-assessment formed the basis of taxation for war and liturgies at Athens[406]; it is possible that some such periodic assessment might supply the 'whole' from which a part would be for sale. More commonly a fixed-term agreement, a fee charged, an agreed contract, or military campaign, or the farming, fishing, or building season, would form the relevant 'whole'. Whatever does count as the whole product would have to be agreed as being the relevant whole by the parties to an exchange, and be based on custom and practice (and probably recognised as being the whole of what is produced by the community in general). I do not think there needs to be an *a priori* stipulation as to what the whole must be for the principle that Aristotle is appealing to (and so the figure demonstrating it) to be respectable.

Secondly, the diagram may be adjusted to show that whatever division of the producers' output is to be exchanged, they correspond. I.e., that relative to the respective producers the parts are the corresponding divisions:—

[405] This objection was made by Dr Meikle in a letter responding to an earlier draft of this chapter.
[406] See § 4.9, note 282.



(figure *10*)

Notice how by merely pointing to such diagrams the argument is seen immediately and clearly, whereas to explain the relationships would be both long winded and likely to be much less clear. Part of the reason for the difficulty universally experienced with chapter 5 has been merely because Aristotle's argument is easily explicable through the simple diagrams he uses in his lectures, but they are lost. We can just 'see' that the corresponding triangles capture our intuitions as to the differences in value.

6. 8 Producers and their products

The second objection to the proposed diagram might be that these figures could only represent producers and their products if producers and products were of the same genus. First, Aristotle clearly treats these heterogeneous elements as representable by the variables; the suggested figures 2 or 9 keep to his own usage. That still leaves the question how the ἀξιαι of producers and their products can be in ratio; Aristotle undeniably does equate these heterogeneous ἀξιαι when stipulating the diagonal συζευξις through which one producer justly acquires the other's product. For any ratio to hold between the terms—and any consequent interaction—the terms must represent homogeneous entities (as I elaborated in § 1.1 and repeatedly since, especially in connexion with the ἐναλλαξ principle which must be employed for the interactions here). I have argued against the claim that Aristotle was indifferent to, or unaware of, the issue of homogeneity in mathematics (§ 3.6), yet the generic or special differences are far more obvious between producers and artefacts than among the several species of mathematics. He must, then, be aware that a farmer is not the same kind of thing as a turnip. Yet it appears that they need to be treated as belonging to the same genus either for the diagonal συζευξις to be measurable or for the assessment of the value of the goods to be made "as farmer is to cobbler". Aristotle does not spell out how the ἀξιαι of products and of producers are connected, yet he says at 1133a24, and repeats at 33a34, that for the valuation which precedes an exchange the ἀξια of the product depends on that of the producer. The justification for equating the heterogeneous ἀξιαι of producers and products would be as follows:

The initial problem is that there will often be no direct link or comparison possible between products to be exchanged. However, there is always a direct link between a product and its own producer. Producers *qua* producers are logically bound to their products, and products to their producers, in a formal symbiosis: buildings are built by builders *qua* builders. The producer is not only indicated by the product but is defined by it: an upholsterer *qua* upholsterer does not grow cabbages, nor does a market-gardener refurbish arm-chairs. What it is to be a product, and what it is to be a producer requires their analytical and necessary connexion; even if it is *a posteriori*. Other attributes of producers should be set aside when assessing their function purely as producers. As with other objects to be measured, i.e., sensibles treated *qua* mathematical objects[407] which have their mathematical properties (i.e., continuous or discrete instantiations of greater and less) abstracted from their other attributes, producers' productive qualities need to be individuated and abstracted from *their* other characteristics. Abstracted in this way just as any other object in applied mathematics, and in virtue of the merely analytical relation between product and producer, for the purpose of assessing the exchange-value of what is produced, the producer "collapses into their product". Such equivalence and replacement of producers by their products is a perfectly standard practice of evaluation. The worth of a farmer is assessed as the worth of the farm; the value of a shopkeeper is calculated as the 'whole' made up of the value of the shop, its contents, good debts and good will, less any liabilities. It is in this spirit that Aristotle speaks of the ἀξια of the farmer,

[407] See §§ 1.3, 3.6, and *Metaphysics M;* also E.Hussey ('Aristotle and Mathematical Objects', *Apeiron* 24(4), pp.105-33, 1991) or Annas (op. cit) or Mueller (op.cit.).

builder, physician, or cobbler. And it is in this way that producers and their products, albeit heterogeneous, may be assessed homogeneously. This, it seems to me, is the general justification for Aristotle's assigning first Γ, and then Δ to A (and Δ and then Γ to B). Regarded as homogeneous in this way and for this purpose a ratio is possible between producer and product.

The most useful ratio will probably be that of *whole* → *part* which Aristotle indicates at 1133a9. *Some* amount will be part of a producer's product, quite logically, whatever is produced; a *whole : part* ratio will then correspond to the *producer-product* relation. Aristotle presents this relation both through quotidian examples (builder : house, cobbler : shoe) and variables as A : Γ (and B : Δ). Any *whole-part* comparison (A : Γ) is comparable with any other (B : Δ), such that there will be a proportion A : Γ :: B : Δ that is equivalent to a comparison *producer : product :: producer' : product'* (and to *whole : part :: whole' : part'*). Figure *10* shows such comparisons of *whole : part* ratios, but so also do figures *2* and *9*, where Γ and Δ are used by Aristotle to measure the part of each producer's output that is for sale.

The first diagram (figure 9) showed the value of the products as proportional to their producers, i.e., that the first step towards the right equality is achieved where the goods and services to be exchanged are the equivalent divisions of their respective totals. The goods up for sale are not necessarily these same divisions; furthermore in an exchange (1133a12, Thomson):

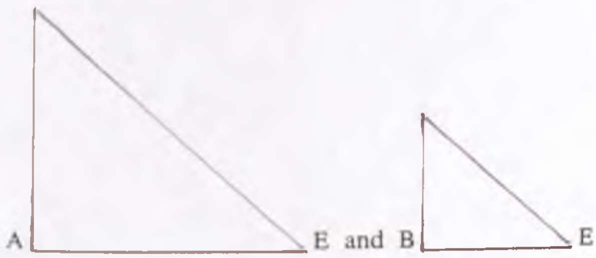
> nothing prevents the product of one of the parties from being better value than that of the other

which could mean either (i) that because of a difference in quality or standing between the producers the equal portions of their output is unequal (shown by figures *9, 2* or *10*), or (ii) that one product might be worth more because there is more of it, even though of lesser quality (shown in figure *10* where Δ'BE > Γ'AE). In either event the values need to be equalised. To bring this about Aristotle moves to the second stage of his argument.

6. 9 The second stage

In § 12 of the chapter, as it has survived, Aristotle indicates a second diagram to demonstrate this second part of the argument. The section has usually been seen as merely repeating what has gone before[408] but it seems to me that the jumble and repetition is evidence of a *development* in an argument (that would originally have been clear and distinct). The thread of argument in the unsatisfactory surviving text—with its criss-crossing accounts of money and product exchange—should be seen as having become entangled later, probably due to the loss of the original diagrams (aggravated by the lack of an algebraic notation). An explanation of any subject that relies on diagrams would almost inevitably become jumbled when the diagrams are lost.

We are told that the products are to be equalised "as builder is to farmer"; such repeated comparisons (farmer : physician, builder : cobbler) serve a purpose when there is some expected difference in value among farmers, cobblers, physicians and builders, and consequently producers and exchangers in general. If there is no difference between producers then this re-iterated comparison is pointless[409]. Their products may not be comparable but the producers clearly are; on this point Aristotle reflects our common experience. Those charged with fixing wage levels make use of such comparisons continually both within and across industries: MP's salaries are pegged to a given grade of the civil service; miners are rated against factory workers, teachers against nurses, etc. Circumstances will affect what the exact ratio will be, but some ratio (such as 8 : 5, 3 : 2, 100 : 1) is to rate the difference between physician and farmer. Where the producers are unequal A would be rated as some line A ____________________ E, to differ from some other line B ___________ E. The equivalent portions of their products Γ and Δ are then correspondingly (see *Topics* 158b29 and § 2.5) worth:



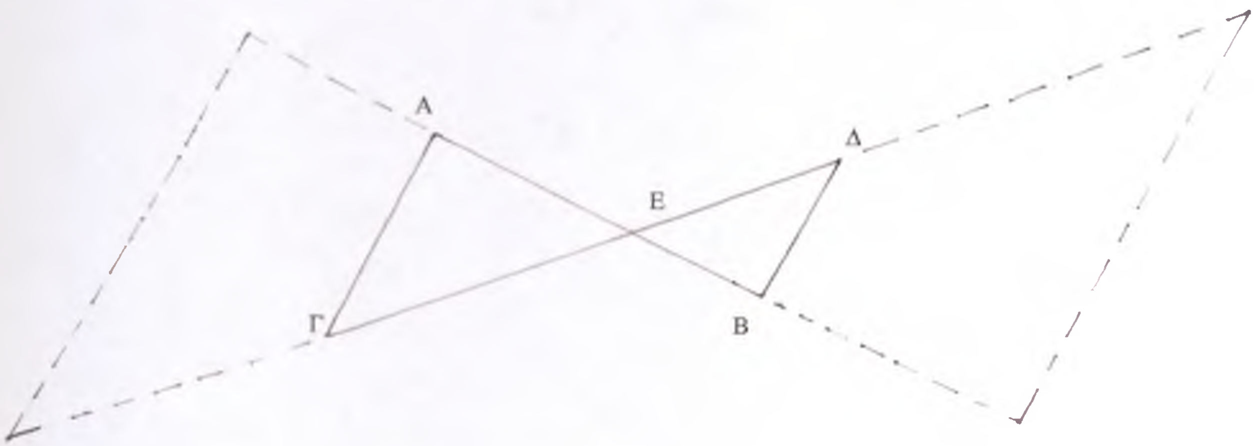
(figure *11*)

[408] See e.g., *Stewart* p.464.
[409] *Meikle* (chapter 7) claims that the parties are equal.

To bring about the desired equality some adjustment is therefore necessary to reduce the greater or increase the less, i.e., some multiple must be applied. As Aristotle is speaking of some *ratio* of farmer : cobbler to guide the value, the value will be found when

$a : c :: b : d :: a : b.$

Note that b might not be division of a (see *Elements V.*1 and *V.*2 and § 1.1 and notes 3 and 4), i.e., that provision must be made for *any* ratio (8 : 5, 4 : 1, 3 : 2, 12 : 7, or whatever). Whatever may be the ratio of A : B, the factors are to be "reduced to a proportion" (1133b) and "Δ equalised" (1133b5). This is all shown by Aristotle at 1133a34ff. with the second diagram[410] demonstrating very simply how the existing elements should be multiplied-out so that the value of the products can be equalised. Suppose the *builder : farmer* ratio (A : B) is 3 : 2 (AE : EB = 3 : 2) then



(figure *12*)

the multiples 2 and 3 will equalise the exchange-value: 2(AΓE) = 3(BΔE) (the expansion in figure *12* in fact shows these quantities doubled: 4(AΓE) = 6(BΔE)). No matter what the disparity of producers and their products, and no matter what part of their output is to be exchanged, the ratios can be multiplied-out until they are equal. For all magnitudes, there are natural numbers m, n such that m(AΓE) = n(BΔE). It is the equal divisions of the *extended* figure that can be fairly exchanged. This is a geometric demonstration of the application of the ἐναλλαξ variant of *Elements V.*5:

$m(a : c) = n(b : d).$

[410] Or by the further use of the same diagram with additional lines

A diagram such as figure *12* also shows how that unless the terms are multiplied-out in this way (1133b):

> one of the two extremes will have both the excesses,

i.e., the excess of A/B and Γ/Δ (the excess of the ἀξια of one producer over the other, and the excess of the value of one product over the other)[411]. Once the adjustment has taken place along these lines the exchange may *justly* proceed, fulfilling the requirement (*Elements* 6.16) that:

$$a \cdot d = b \cdot c \Leftrightarrow a : b :: c : d.$$

[411] The expression which follows "when they have their own" appears to have been a proverb; Aristotle uses it twice in chapter 4—at 1132a18 and 1132b17.

CONCLUSION

The actual calculations and formulae in the solution are (to repeat) very simple, but the solution applies *whether or not* the parties are equal, *whether or not* the products are homogeneous, and *whether or not* they share a common measure. It is a procedure which acts upon an existing comparison (it compares the comparisons of greater and less posited in the relation between the producer and product). These factors are grasped immediately by the use of a simple diagram (which is why Aristotle uses the diagrams—they are integral to his analysis, not mere idiosyncrasy) but with the loss of the diagrams and the lack of a convenient algebraic notation even these mathematically primitive manipulations quickly became opaque. Considering the fate of the crucial definitions of the theory of proportion (explored in chapters 2 and 3 above) which Aristotle here implements, the obscurity which descended on his use of those definitions seems to have been inevitable.

I have not looked in any detail at the rôles of χρεια or money in chapter 5, for three interconnected reasons: (i) the theme of the work is the rôle of analogy, to which they are not closely tied. (ii) They have been extensively discussed by others (and recently reviewed in *Meikle*). I broadly agree with his view that χρεια provides the framework and purpose of exchange but has no part in the actual calculation of the value. (iii) I have little to add to what has been said about these topics. There is only one point I need to make about Arisotle's treatment of χρεια and money. He says (1133b13):

> it is proper for all things to have their price set.

In economic theory there is often a difference made between economic value (that is defined one way or another, as we have seen) and price formation. This is not a distinction Aristotle makes; indeed he sees through the supposition upon which the distinction largely rests: that there is some element in virtue of which there is a value. It is not really that he side-steps the question of the economic value being something other than the price—an object needs to exist (or at least be thought to exist) to be side-stepped—and he demonstrates that there is no subjacent metaphysical common value to be accounted-for. His concern is with the moral question of how the price ought to be fixed; his answer is that it is to be arrived at through the two stages of calculation (outlined above).

I don't think he says so, but it is a reasonable inference from what he does say that in Aristotle's view if there is scarcity of important products (of food for example, or shelter, medicine, or clothing) their supply would cease to be a matter of private exchange but would become the duty of the πολις to supervise. The issue would clearly then be one of Distributive not Transactional justice. Scarcity is not then for him a factor in the moral determination of prices. And since he ignores the other great parameter of economics—fashion—only the productive factors are used to assess the value of items to be exchanged.

My arguments oppose the various modern accounts that have been given of Aristotle's theory on a number of points. Against the current orthodoxy I support St Thomas's treatment of the theory. In particular Aquinas avoided prescribing a Corrective species of any stripe. *That* solecism emerged from the mis-parsing of expressions such as *corrective justice in exchanges*, and from the non-sequitur that because Aristotle doesn't state that the first species directs (or corrects) distributions, it doesn't. There is no semi-detached third class of justice dedicated to commerce, and once the structure of the theory has been grasped there is no call for such an addition. The full understanding of Aristotle's programme, however, depends on the appreciation of his use of analogy. Aristotle uses classical analogy—not what analogy has come to mean—to relate the principle of function to that of generic order. With this fusion he develops a comprehensive as well as consistent account of the issues.

There are frequent objections to Aristotle's insistence on mathematical terminology and diagrams as lurching from the central principles of justice towards an irrelevant mathematical construal. These complaints are misdirected. Justice as a special virtue takes as its subject the occurrences of inequality. Since it is concerned purely with finding the right stance to adopt among the wrong sorts of equality, and between inequalities (principally between citizens), he borrows techniques that had been devised expressly to resolve the corresponding difficulties in mathematics. He would only be wrong to use that language if the primary conception is mistaken. But if he is right that, quite apart from its job governing the specific virtues of character, there is a further quality of justice concerned with equality (wherever and however any mode of equality is possible), he is right to use the notions of proportion to explain that quality.

Aristotle's use of proportion language is unfortunate in (only) one respect: sometimes he presents analogies fully, but more often, usually after the initial complete presentation, his references to proportions and notions associated with them are highly compressed. When focusing on some distinction to be made he will combine the geometric, the complex, and the proportional characteristics of the subject that is to be set against the notions of simplicity, of arithmetic, and of plain quantitative equality of another subject. He will often take just one term from either of these clusters to stand for the rest—and not usually the generically corresponding term. This habit has created extreme difficulty for interpreters. But that synecdoche and metonymy apart Aristotle's choice of expression is often sensitive and exact, notably where he is thought to be vague. Παρα το ἑκουσιον, ἐπανορθωτικον and ἀντανειρεσις for example, are all terms carefully chosen to meet subtle logical requirements.

The diagrams and the very simple calculations are not foibles but valuable tools of analysis and presentation. Their strengths have been grossly under-rated for at least three reasons:

(i) as just mentioned, Aristotle's whole theory is conceived as parallel to applied mathematics (the nature of which is the interaction and relation of the greater and the less, *qua* greater and less). This equivalence of subject-matter has not registered with commentators as central to the programme.

(ii) With the crucial definitions of the general theory from *Elements V* lying unrecognised the intimate association with the corresponding problems dealt with in this section of the *Ethics* was broken.

(iii) The text of the *Ethics* survived, but only without the diagrams which vividly and simply portrayed the relations between the magnitudes.

The total exclusion of the Eudoxian theory of analogy has marred the understanding of exchange-value even more than the failure to recognise Aristotle's use of analogical inference in general has his account of Particular justice. My aim has been to help restore Aristotle's original concinnity.

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