Study of Asset Pricing: Insights from Factor Models and Dynamic Methods

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Business and Society

May 2025

Abstract:

This thesis examines various aspects of the Fama-French five-factor model (FF-5) through different methodologies, including deep learning models, nonparametric methods, and traditional FF-5 model with liquidity factor.

The 1st empirical chapter examines the Fama-French five-factor model's ability to explain stock returns in Chinese market, focusing on the size effect and liquidity effect. It also analyses the relationship between idiosyncratic volatility and expected stock returns. The study finds that liquidity effects are weaker than size, value, and investment effects, with the six-factor model outperforming the five-factor model in certain portfolios. Additionally, it finds a negative correlation between idiosyncratic volatility and stock returns.

The second study uses a nonparametric method to estimate the conditional factor model, offering an alternative to traditional linear models. By applying nonparametric approach, the study estimates the optimal bandwidth for conditional alphas and betas and compares high-frequency and low-frequency factors. The results indicate that the nonparametric method outperforms the OLS method in reducing pricing errors, making it a valuable tool for high-frequency empirical applications.

The third research explores the application of the Fama-French five-factor model in financial forecasting, utilising deep learning techniques, specifically CNN, LSTM, CNN-LSTM, and TCN-LSTM models, to predict excess returns based on high-frequency data. Among the models, CNN-LSTM achieves the best performance, effectively balancing feature extraction and modelling.

Declaration

I declare that this thesis is a presentation of original work and I am the sole author. This work has not previously been presented for an award at this, or any other, University. All sources are acknowledged as References.

Publications arising from this thesis:

Hou, Z., Manahov, V. and Stafylas D. (2025). Fama and French Five-Factor Modelling: New Evidence from a Nonparametric Method, Studies in Nonlinear Dynamics & Econometrics.

Content

Chapter 1. Introduction

The capital Asset Pricing Model is the core of modern financial market price theory and is widely used in investment decision-making and corporate financial management. Markowitz (1952) proposes the theory of portfolio selection. Markowitz takes the price change of the portfolio as a random variable, using its mean value to measure expected return, and its variance to measure risk. Based on this, Classical asset pricing-related theories, including Sharp's capital asset pricing model CAPM, the Fama-French three-factor and the further improved Fama-French five-factor model, have been excavated.

1.1 Research Background

As early as the 1950s, asset pricing theory had sprouted. Markowitz (1952) first uses mathematical concepts to clearly define and quantify risks and returns and pioneered the portfolio theory. However, how to quantitatively describe the relationship between risk and return is still elusive. Sharpe (1964) established a capital asset pricing model based on the basic principle of the securities portfolio. The model proposes the concept that the return on assets depends on the risk degree of assets and uses quantitative methods to describe the relationship between risk and return for the first time, which became the basis of financial market pricing theory. The Capital asset pricing model CAPM[1] mainly includes the premise that there is no defect in the capital market, investors are risk-averse, borrow at a risk-free interest rate, and cannot affect the stock price through trading. This theory is mainly based on Markowitz's portfolio theory, which is expanded and extended by Sharpe (1964), Lintner (1965) and Mossin (1966). The most important contribution of the CAPM model is to put forward the concept of beta value, which represents the sensitivity of asset returns to market risk and reflects the role of an asset in the overall undivided market risk. In the CAPM, the expected return of all stocks and asset portfolios is mainly related to the beta value of the stock or asset portfolio, that is, the expected excess return of any stock exceeding the risk-free return rate is only proportional to

[1] The standard assumptions underlying the Capital Asset Pricing Model (CAPM) include frictionless markets, homogeneous investor expectations, a single-period investment horizon, the ability to borrow and lend unlimited amounts at a risk-free rate, and normally distributed asset returns.

the beta value of the stock, which is a single-factor model. This simple relationship has laid the foundation for the subsequent study of a large number of multi-factor pricing models (Blume et al., 1972).

As research in finance advances, it becomes increasingly evident that a single market factor is insufficient to capture the relationship between systematic risk and the cross-sectional variation in stock returns. In reality, stock returns are influenced by multiple sources of risk beyond the market portfolio. To address the limitations of the Capital Asset Pricing Model (CAPM), Ross (1976) proposes a more general multi-factor framework—Arbitrage Pricing Theory (APT). APT argues that arbitrage is central to maintaining price equilibrium in an efficient market. Risk-free arbitrage opportunities emerge when the market deviates from equilibrium and help restore balance. While APT shares conceptual similarities with CAPM, it differs in a fundamental way. Rather than assuming that asset returns are driven solely by the market portfolio, APT models expect returns as a linear function of several unknown systematic risk factors. It relies on the principle of no arbitrage to establish an approximate linear relationship between asset returns and these risk factors. Although APT provides a broader theoretical framework, it faces practical challenges, particularly in identifying the relevant risk factors and estimating their corresponding risk premiums.

Although these alternative models offer important insights, this study focuses on the FF-3 and FF-5 models because of their foundational role in the evolution of empirical asset pricing and their extensive application in both academic research and practical investment analysis. The FF models provide a structured and interpretable framework for understanding the cross-sectional variation in stock returns while maintaining a balance between model complexity and explanatory power, which is crucial for evaluating model performance across different datasets and market environments.

The three-factor model proposed by Fama and French (1993) is representative of the development of the single-factor model to the multi-factor model. The three-factor model provides a better explanation for the excess return of the US capital market at the beginning of its introduction. However, with the progress of the times and the evolution and improvement of the multi-level capital market, more financial indicators such as the profitability and the asset growth rate for measuring the company operating ability increase the diversity of evaluating the company performance. As financial markets evolve, it also makes the investment method more diversified and more difficult to quantify simply. Therefore, the three-factor model makes it increasingly difficult to explain various anomalies in the market, and the three-factor model is also inconsistent in different capital markets. Fama and French (2015) extend the three-factor model to the five-factor model to verify its interpretation ability in the US capital market. The five-factor model successfully solves the problems that the APT model does not solve.

Compared to the traditional CAPM model and the Fama-French three-factor model (FF-3), the FF-5 model extends its ability to explain changes in asset returns by adding profitability (RMW) and investment style (CMA) factors. The RMW (Robust Minus Weak) factor captures the return difference between firms with high and low operating profitability. The CMA (Conservative Minus Aggressive) factor reflects the return spread between firms that invest conservatively and those that pursue aggressive investment strategies. This extension provides a more comprehensive view of the multiple risk factors present in the market when analysing fund returns. The study shows that the FF-5 model performs well across different fund types, particularly in explaining the risk-adjusted returns of equity and hybrid funds. Furthermore, the FF-5 model generates the lowest GRS statistic in time-series tests, indicating that its risk-adjusted return estimates are more accurate. The model also demonstrates a high level of overall fit in explaining returns, outperforming the Carhart four-factor model (FFC) and other competing models (Fama & French, 2015; Carhart, 1997). This suggests that a greater portion of the variation in the model's alpha distribution arises from sampling error rather than genuine pricing errors. Thus, the FF-5 model not only exhibits strong statistical significance in explaining fund returns but also excels across a variety of evaluation metrics.

1.1.1 The Development and Challenges of Chinese Stock Market

After more than two decades of development, the Chinese stock market has expanded rapidly, hosting over 4,000 listed companies as of 2025, which places it among the largest equity markets globally. Alongside this expansion, the market retains several characteristics typical of emerging economies, including persistent structural frictions, an evolving institutional framework, and an investor composition that differs notably from those of mature markets such as those in the United States and Europe. These distinctive features imply that asset pricing mechanisms documented in developed markets cannot be directly extended to the Chinese context without careful empirical validation.

As a core component of the Chinese multi-tier capital market system, the stock market plays a vital role in capital allocation, corporate financing, and broader economic transformation. The co-existence of rapid growth and ongoing structural imperfections makes it a particularly informative setting for studying asset pricing models under time-varying liquidity and market frictions. Consequently, understanding how such frictions shape return dynamics is essential for evaluating the applicability and performance of standard asset pricing frameworks in this market.

Liquidity is a fundamental characteristic of financial markets and plays a central role in the asset pricing process. In frictionless markets, assets can be traded quickly, in large quantities, and at low transaction costs without significantly affecting prices. In reality, however, liquidity is time-varying and subject to market frictions such as information asymmetry, transaction costs, and institutional constraints. These frictions are particularly pronounced in emerging markets, where market depth, trading continuity, and investor composition differ markedly from those in developed markets.

The Chinese stock market provides a distinctive setting for studying liquidity and asset pricing. Since its establishment, Chinese equity market has experienced rapid expansion accompanied by substantial structural transformation, regulatory intervention, and shifts in trading mechanisms. These features have resulted in pronounced fluctuations in market-wide liquidity over time, especially during periods of market stress and policy adjustment. As a consequence, liquidity risk is likely to be priced in asset returns, beyond the risk factors captured by standard asset pricing models.

Recent studies highlight the importance of incorporating liquidity-related information into asset pricing frameworks. In particular, Liu et al. (2019) demonstrate that liquidity conditions significantly affect expected returns by influencing trading activity, price discovery, and risk compensation mechanisms. Their findings suggest that liquidity not only reflects market trading capacity but also captures investors' exposure to systematic trading frictions, thereby serving as a distinct source of priced risk. This perspective provides a direct link between market microstructure characteristics and cross-sectional return variation.

Against this background, analysing liquidity within the Chinese stock market is especially relevant. Compared with developed markets, Chinese equity market exhibits higher participation by retail investors, greater sensitivity to policy changes, and more frequent liquidity shocks. These characteristics amplify the role of liquidity in shaping asset prices and raise important questions regarding the adequacy of traditional factor models that abstract from market frictions. Understanding how liquidity evolves and how it is priced at the market level therefore constitutes a necessary step before examining liquidity effects within specific market segments.

1.1.2 Limitations of Traditional Methods

With the development of financial markets and the continuous emergence of various financial instruments, it is particularly important to accurately measure the volatilities of financial series and properly describe its dynamic system structure. The typical characteristics of financial time series mainly focus on the characteristics of volatilities, such as time variability, aggregation, asymmetry, and long memory. Most of these characteristics are difficult to get through intuitive description or induction. Most studies on the Fama-French model use low-frequency data (daily, weekly, monthly). For example, Fama and French (2015) employ monthly data to construct and validate the five-factor model, while Hou et al. (2015) rely on monthly returns to examine investment-based factor structures. Similarly, Cakici et al. (2013) use monthly data to test the applicability of Fama–French factor model in emerging markets. However, market micro-noise always exists. When using relatively low-frequency data such as daily data, the monthly estimate may be noisy.[2] Through some additional averaging, such as forming a stock portfolio with a similar estimation factor beta, the noisy estimation problem caused by a small number of time series observations can be alleviated to some extent. The nonparametric test model[3] can more accurately describe the nonlinearity and heteroscedasticity

[2] Market microstructure noise refers to short-term distortions in asset prices caused by trading frictions, such as bid-ask bounce, discrete pricing, or order processing delays, which can bias volatility estimates and weaken regression results.

[3] Nonparametric models do not require prior assumptions about the functional form of the relationship between variables. They allow the data to determine the shape of the model, making them suitable for capturing complex,

in the financial market and better deal with the noise and volatilities in high-frequency data. Therefore, using high-frequency data and a nonparametric test method to regress the Fama-French model can improve the accuracy of the model. Because factor betas are actually quantities similar to covariance, they benefit from additional data collected at high frequencies. Using a relatively short time window, I can decompose the specific risk into its continuous and jumping parts. The specific risk is usually measured as the change in stock returns after removing the influence of common factors, and this framework allows the time-varying coefficients（Aït-Sahalia, et al. 2020）.

Factor models have been successful in capital asset pricing and are widely used in portfolio management. In the framework of these models, the risk premium of an asset is linearly related to the beta coefficient of the asset. The beta coefficient reflects the sensitivity of assets to various market risk factors. The slope of the linear relationship should be the same for all assets, representing the market price per unit of beta risk associated with each risk factor. However, a growing proportion of empirical research suggests that risk premiums are not static but evolve (see Keim and Stambaugh, 1986; Ferson and Harvey, 1991; Ferreira, GilBazo, and Orbe, 2011). Ang and Kristensen (2012) argue that traditional factor models are distorted when testing alpha equal to zero if the factor loadings vary over time; the previous can lead to misleading statistical inferences about the validity of factor models, especially the CAPM model and Fama-French three-factor model (FF-3). These findings challenge the stability assumptions of traditional models and suggest that risk pricing may have more dynamic properties.

The noise caused by limited time series observations can be mitigated through additional averaging methods, such as combining stocks with similar factor beta estimates (Aït-Sahalia et al., 2021). Thus, it is necessary to conduct time-varying tests on the widely used FF-5 model. The motivation for using nonparametric estimation of the factor loadings in the Fama-French model is to address the limitations of parametric methods and to provide a more accurate and robust analysis of the factor loadings. Nonparametric methods perform well in capturing the nonlinear relationship between the stock returns and the factors, and produce more accurate estimates of the factor coefficients, especially in the presence of outliers or other data

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nonlinear, and irregular patterns in financial time series.

irregularities (Ang and Kristensen, 2012; Fan et al., 2012; Connor et al., 2012).

In response to these challenges, I utilises high-frequency data in conjunction with nonparametric estimation techniques to address the challenges posed by time-varying factor loadings, nonlinearity, and estimation noise. The subsequent empirical chapters, in particular Chapters 3 and 4, demonstrate how these methodological advancements provide a more comprehensive understanding of dynamic risk pricing mechanisms.

1.1.3 Stock Prediction: Deep Learning Techniques

Forecasting financial risk premium has a central role in the empirical research in Asset Pricing. The representative asset pricing model can be traced back to the Markowitz mean-variance model proposed in 1952. The Capital Asset Pricing Model (CAPM) is proposed jointly by Sharpe (1964) and Lintner (1965). As the earliest model to describe stock return, the Capital Asset Pricing Model (CAPM) is a single-factor model that incorporates only the market factor. Fama (1970) proposes the influential joint hypotheses framework, which highlights that any test of market efficiency is inherently a joint test of both the efficiency hypothesis and the asset pricing model employed. This framework complicates the interpretation of empirical results, as a rejection may imply either market inefficiency or model misspecification. Carhart (1997) combines the Fama-French three-factor model with the momentum factor (MOM)， capturing the tendency of stocks with high past returns to continue performing well in the short term. The model improves the explanatory power for mutual fund returns and addresses the short-term persistence in performance that previous models fail to capture. Novy Marx (2013) constructed a four-factor model, which includes the market factor (MKT), value factor (HML), momentum factor (UMD), and profitability factor (PMU). He shows that profitability, measured as gross profits over assets, has strong predictive power for returns even when controlling for value. Barillas and Shanken (2018) show that profitability-based models perform comparably to the Fama-French five-factor model in explaining cross-sectional returns. Liu et al. (2021) apply profitability-augmented models in emerging markets and find significant improvements in risk-adjusted return predictions, especially during periods of market stress. Fama and French (2015) propose the Fama-French five-factor model by adding profitability and investment factors to the original three factors. To further test the efficiency of the market, it is necessary to first determine a reasonable asset pricing model as a standard. The focus of the third empirical chapter 3 is not to further improve the explanation of stock returns but to identify models that can efficiently predict future returns. Creating accurate stock return forecasts is not only crucial for financial institutions in determining potential investment opportunities but also crucial for investors to create investment portfolios that efficiently utilize excess alpha that traditional asset pricing models have not captured.

In the field of factor investment or asset pricing, there are two main types of traditional econometric methods, cross-sectional regression, and time series regression. The former often uses lagged stock characteristics to regress the future returns of individual stocks; The latter often regresses the overall returns of the investment portfolio to some macro variables. Fama and MacBeth (1973) use a two-step cross-sectional regression method to test CAPM. Compared to general cross-sectional regression, Fama and MacBeth (1973) treat the regression results of each cross-section as an independent sample, effectively avoiding the influence of random perturbation on cross-sectional correlation. This method is known as the "Fama-MacBeth" regression method and is widely used in research. Notable applications of the Fama–MacBeth procedure include Jagannathan and Wang (1996), Daniel and Titman (1997), and Lewellen, Nagel and Shanken (2010). In addition to the multi-factor model, the consumption-based pricing model (CCAPM) is also an important asset pricing model, which assumes that the risk premium of an asset is determined by the covariance of its return with the growth of aggregate consumption. This reflects the extent to which an asset co-moves with the risk of consumption faced by investors. Hansen (1982) pioneers the discussion of the properties of the Generalised Method of Moments (GMM) estimator and how to select weight matrices. They propose using the GMM method to test CCAPM. In recent years, GMM has also been increasingly used to estimate the risk exposure of stocks and the prices of specific risk factors. Robert Shiller, as the main founder of behavioral finance, proposes the noise trader model in Shiller (1984): rational investors can analyse fundamentals reasonably and make investment decisions; the presence of noise traders has caused deviations in prices and intrinsic value. And the excessive fluctuation of prices comes from personal irrational behavior leading to an overreaction to fundamental information.

Stock prediction involves estimating future stock value changes based on past and present market-related information. It is a judgment, estimation, and prediction of changes in stock market activities. Transforming investors subjective investment concepts into mathematical models and applying objective methods such as data mining and analysis to explore the patterns of stock fluctuations has become a crucial quantitative approach for effective stock prediction. In the early stages, the most extensive and mature quantitative tools for stock price prediction are time series models, such as ARIMA and GARCH, which are classic examples. These classic models are particularly effective for capturing linear trends, autocorrelation, and volatility clustering in financial time series data (Box and Jenkins, 1976; Engle, 1982; Bollerslev, 1986). However, the financial field is a complex dynamic environment, and financial time series data often exhibit high noise, non-stationarity and non-linearity. Therefore, classical econometric models based on statistical theory often fail to make accurate predictions due to their strict data assumptions.

Over the decades, computational intelligence technology in financial prediction has gradually replaced traditional statistical analysis methods. Machine learning algorithms and deep learning techniques have gained attention and development in stock price prediction problems. Currently, the most commonly used nonlinear methods for quantifying stock price prediction include traditional machine learning algorithms such as support vector machines (Steinwart, 2008) and ensemble learning methods like random forests (Breiman, 2001) and popular deep learning models (such as RNNs and LSTMs). Research has shown that neural networks offer significant advantages in stock price prediction compared to classical econometric and machine learning models, providing more accurate and reliable prediction results. Fischer and Krauss (2018) demonstrate that long short-term memory (LSTM) neural networks can effectively capture temporal dependencies in financial time series, achieving high accuracy in S&P 500 stock return predictions. Atsalakis and Valavanis (2009) provide a comprehensive survey of neural network applications in financial forecasting, highlighting their flexibility and robustness in modelling non-linear relationships. Bao et al. (2017) combine wavelet transforms and deep learning methods to model the complex and dynamic structures of stock price movements, significantly improving prediction performance.

In recent years, deep learning methods have become a popular topic in machine learning.

Convolutional neural networks (CNNs) have achieved successful applications in temporal data classification and image recognition due to their powerful feature extraction and recognition capabilities (LeCunet al., 1988). Recurrent neural networks (RNNs) have performed well in sequence modeling problems, especially their improved versions, Long Short-Term Memory (LSTM) networks and Gated Recurrent Units (GRUs), which enhance the memory range of RNNs (Werbos,1990; Hochreiter and Schmidhuber, 1997; Cho et al., 2014). These improvements allow the models to connect and utilize text information from distant points in the sequence, while gate structures prevent early information forgetting. However, these models have some practical shortcomings, such as gradient explosion or disappearance, large memory requirements, and long training times. In the context of stock price forecasting, the ability of LSTM and GRU networks to retain and process long-term sequential patterns is particularly beneficial for capturing lagged effects and cyclical trends in financial markets.

The Temporal Convolutional Network (TCN) is characterised by maintaining data causality, theoretically improving the accuracy of model predictions (Bai et al., 2018). TCN is a new prediction algorithm developed by CNN to solve time series problems, representing an improvement over CNN. Compared to the RNN architecture, TCN mainly alters the receptive field by increasing the number of network layers, changing the dilation coefficient, and adjusting filter size. This better controls the model's memory length. Furthermore, since the backpropagation path of TCN differs from the time direction of the sequence, it avoids the vanishing and exploding gradient problems of RNNs. Unlike a typical CNN, which contains fewer convolutional layers, TCN can generate a larger convolutional kernel by continuously increasing the number of convolutional layers. This allows it to capture long-term dependencies in sequences and achieve comprehensive information extraction. Such architectural strengths enable TCN to better model the complex temporal structures found in stock price data, thereby improving the robustness and precision of predictive models.

TCN integrates the architectures of CNN and RNN, enabling large-scale feature extraction and concurrent processing like CNN, thereby saving computational time. It also allows the use of data sequences of any length and directly maps to output sequences of the same length, like RNN. By extending convolutional structures and incorporating residual links, TCN expands the receptive field, resulting in a significant amount of effective historical length in model predictions and improving prediction accuracy. These characteristics make TCN particularly suitable for financial forecasting tasks, where the need to process long historical records and detect intricate patterns in stock movements is essential for producing high-quality forecasts.

1.2 Research Motivation

To better discover the role of the market micro-structure, such as asset price formation and liquidity provision, into stock markets I mainly explore the mechanisms of asset price discovery and formation. However, the actual financial market is influenced by various factors, such as information asymmetry and transaction costs, and it is not the ideal market with frictionless and sufficient liquidity. In reality, there are many financial anomalies, such as illiquidity premiums, and the issue of liquidity has been a controversial area of research in capital markets (Amihud and Mendelson, 1986; Pastor and Stambaugh, 2003; Acharya and Pedersen, 2005; Bali et al., 2014). Since 1986, when Amihud introduced the idea of a liquidity premium, integrating liquidity factors into asset pricing has opened up new directions in the traditional theoretical framework.

Since its introduction, the three-factor model (FF-3) by Fama and French (1993) has been trendy in empirical research. Fama and French (2015) extended their three-factor model and introduced the five-factor model (FF-5), which prompted a further round of empirical testing by academics. However, the Fama-French model does not consider liquidity risk when explaining stock returns. Fama and French (2015) discovered that the parameters provided by Pástor and Stambaugh (2003) to capture liquidity risk cannot explain expected return. However, Pástor Stambaugh's (PS) liquidity factor is based on their price effect metrics and does not obtain a considerable liquidity premium (Ma and Zhang., 2021). In the literature, additional liquidity factors are also observed, some of which are derived from liquidity substitution indicators other than price effect indicators. For instance, Liu's (2006) liquidity risk factor is a measure of transaction continuity based on liquidity that captures many characteristics of liquidity and generates a substantial premium.

In a closely regulated, order-driven stock market, the influence of illiquidity risk on asset pricing should be emphasized, particularly in extreme market situations such as Chinese. The

1st empirical chapter uses panel data from the Chinese stock market to determine if illiquidity risk across dimensions such as price effect, trade speed, trading volume, transaction costs, and asymmetric information can explain stock returns. Yu, et al. (2019) discover that nearly every dimension of stock illiquidity is positively correlated with excess stock returns. Moreover, smaller, less liquid equities incur greater liquidity costs, which is strong evidence of a "move to liquidity."

With the continuous development of the financial market and the continuous improvement of statistical theories, the application of statistics in the financial field is receiving more and more attention. Various methods and models, such as the Cointegration test (Engle and Granger, 1987), VAR model (Sims, 1980), non-stationary time econometric model, error correction model, provide powerful analysis for empirical cases (Phillips and Perron, 1988; Johansen, 1991). Scholars have conducted empirical analysis on financial trends and policies at the micro level, such as the operating trends of stock markets and stock index futures, through the use of GJR (Glosten et al., 1993), GARCH-M models (Engle et al., 1987), generalized moment estimation (Hansen, 1982), and cross autocorrelation models (Lo and MacKinlay, 1990). Scholars also use artificial neural network methods based on nonlinear nonparametric models, assuming that the collected data is fully utilized without limiting parameters, and then the structure and parameters of the model are determined from the data to analyse and predict financial aspects such as option pricing, exchange rates, stocks, and bankruptcy.

Regarding stock returns, among the factors that affect stock returns, statisticians can use regression models to model and analyse a particular indicator or data Y (response variable) of interest, attempting to identify several indicators or data that can explain as much as possible the overall changes in Y, namely $X_1 ... X_p$ (explanatory variable). However, in many cases, the data collected often have non-linear relationships, and the commonly used linear regression models are no longer applicable due to their limitations. A saturated nonparametric model can lead to the final model being too complex, with excessive variance in parameter estimates and model inefficiency. At this point, some nonparametric or semi-parametric models with specific structures exhibit advantages.

According to Gallagher (2015), the capital asset pricing model and the Fama-French factor model, which are conditional factor models, offer abundant research resources for capital asset pricing. However, the outcomes of these studies are generally unclear or open to interpretation. Typically, researchers build models based on the available condition information to match the empirical data. Cochrane (2005) pointed out that CAPM and other models imply an Information set about investors. However, the best thing he can hope to do is to test the meaning of variables that he can observe and include in the test. Therefore, the conditional linear factor model is not testable. While this approach allows for fitting the data, the condition information itself cannot be known in advance. In contrast, non-parametric regression does not necessitate researchers to specify the functional form of the estimated data. Consequently, they argue that factors should be determined by the data rather than predetermined variables.

The traditional approach to allowing factor loadings to depend on observable state variables in a conditional model is to estimate them in terms of instrumental state variables. However, this approach has limitations because the estimation of factor loadings is very sensitive to instrumental state variables and is not observable in advance. In general, the traditional approach assumes that factor loadings are constant, although there is evidence that they vary over time, particularly for standard capital asset pricing models and Fama-French models.

The Fama-French factor model is a widely used asset pricing model that explains portfolio returns based on risk factors: market risk, size, value, investment, and profitability. The model assumes that each stock's return is a linear combination of these factors, where the factor loadings represent the sensitivity of the stock's return to these factors. The traditional approach to estimating the factor loadings in the Fama-French model is through parametric methods such as linear regression. However, chapter three uses a non-parametric regression of the factor loadings of the Fama-French factor model, which is different from the assumption that the factor loading does not change with time in conventional research. So, the estimated value of the factor loading is closer to its true value under limited samples, and its distribution is closer to the true distribution under limited samples, providing a better basis for testing.

The traditional approach to estimating the factor loadings in the Fama-French model is through parametric methods such as linear regression. The ordinary least squares method (OLS) can only fit the model linearly and may ignore the nonlinear situation. Under the assumption that the factor loadings change with time, the significance of the factor loading is examined by the test method of individual test and joint test, which will determine whether the factor loading of the Fama-French factor model changes with time in a statistical sense. Generally, when the regression method is adopted in multi-factor stock selection, the multi-factor model will be constructed by polynomial linear regression, and the regression coefficient will be used to determine the factor weight. Although the regression method can reduce the subjectivity of factor coefficient determination to a certain extent compared with the scoring method, the linear regression assumes that the random error term supports the normal distribution with zero mean value, while the financial data often presents the distribution state of peak and the fat tail, which does not meet the assumption of the normality of the error term. At this time, it is meaningless to use the P-value and t-test to test the significance of the model or explanatory variables, thus affecting the accuracy of the model.

The research motivation for using nonparametric estimation of the factor loadings in the Fama-French model is to address the limitations of parametric methods and to provide a more accurate and robust estimation of the factor loadings. As discussed in the work of Connor, Hagmann and Linton (2012), it proposes a semiparametric framework that overcomes the rigidity of traditional estimation and enhances the flexibility of the model. Nonparametric methods have been shown to perform well in capturing the nonlinear relationship between the stock returns and the factors and to produce more accurate estimates of the factor loadings, especially in the presence of outliers or other data irregularities.

The multi-factor models established by Fama and French (1993,2015) are linear models that take historical feature data as input and stock returns as output. However, the stock market is chaotic, complex, and dynamic, so linear model assumptions may be unreasonable. It is particularly important to consider nonlinear models to achieve the mapping between features and stock returns. From previous research in the fields of finance and economics, such as MacKinlay (1995), Cooper et al. (2004), and Lo (2004), data models emphasize the establishment of correct models, but rarely consider the compatibility between models and data, which leads to the dual problem of complex models and unsatisfactory prediction results. However, machine learning does not emphasize the structure of models, only needs to test the accuracy of predictions based on the characteristics of input data, which can better adapt to the rapid changes in the financial market and the complex data structure. This has led research to pay attention to how to apply machine learning methods to the field of financial research.

Baldi (1989) found through research that neural networks have better predictive ability than traditional models in dealing with nonlinear and chaotic time series data. Grudnitski (1993) uses neural networks to predict gold futures prices and found that neural networks were superior to traditional models. The predictive ability of neural networks is superior to traditional data models. Kaastra (1995) applied neural networks to predict monthly futures trading volume, However, it is found that neural networks have problems such as overfitting, difficulty in convergence, and difficulty in model training. In 1995, Vapnik proposes a support vector machine model based on neural networks. This method maps linearly indivisible data to high-dimensional space by introducing kernel function theory, thereby achieving linear separability of data. It is found that this model performs better than neural networks in all aspects. After this, support vector machine models began to comprehensively surpass neural networks and have been widely used in various industries. In the financial field, support vector machine models gradually replaced neural networks as the main machine learning method for financial price prediction. Drucker (1997) first applied the concept of SVM in regression problems and widely applied it. In the prediction of stock and bond prices, Kim (2003) proposes a method of using SVM to study stock prices, and empirical results show that, SVM can replace traditional stock prediction methods.

Deep learning is an advanced algorithm in machine learning that represents and learns data. Its high-dimensional characteristics enhance the flexibility of prediction techniques and significantly outperform traditional nonlinear learning tools in statistics. Deep learning is characterised by speed, efficiency, and low error rates, providing cost-effective and efficient solutions for asset optimization research. For example, Fischer and Krauss (2018) use deep learning algorithms to predict economic returns through investment portfolio Sharpe ratios.

1.3 Research Gaps

Although classical asset pricing models such as CAPM and the Fama-French multifactor models have been widely studied, numerous empirical anomalies continue to challenge their explanatory power. This section discusses key return anomalies—such as the size, valuation, momentum, and liquidity effects—as well as limitations in modelling assumptions, including time variation and nonlinearity. These gaps underscore the need for more adaptive approaches, particularly those informed by machine learning.

1.3.1 “Size Effect” and “P/E Ratio Effect

Banz (1981) finds a regular negative correlation between the average return of stocks and the size of the company; that is, the stocks of smaller companies have a higher rate of return, while the stocks of larger companies have a lower rate of return. Therefore, the "size effect" is also known as the "small company effect." Fama and French (1996) uses the three-factor model to study the New York Stock Exchange, the American Stock Exchange, NASDAQ, and stocks. The empirical results show that the stock portfolios of small-size companies tend to produce higher returns than those of large-size companies. At the same time, the stock portfolio with a high book-to-market ratio can produce a higher return than the stock portfolio with a low book-to-market ratio. The research results of FF further confirm the existence of the "small-firm effect."

The "P/E ratio effect," also known as the "low price-earnings ratio effect," is found by Basu (1983), who discovers that the performance of stocks with a low P/E ratio is better than that of portfolios with a high P/E ratio. As the cost that investors are willing to pay for each unit of profit of the company, the P/E ratio represents the number of years that investors need to invest in a specific stock to recover the investment cost. On the other hand, the P/E ratio reflects the company's future growth potential. Therefore, the higher the P/E ratio, the higher the market's evaluation of the stock and the more optimistic about the future development potential of the company.

However, in the actual capital market, the empirical studies performed by many scholars reveal many phenomena that cannot be explained by the capital asset pricing model and efficient market hypothesis (Fama and French, 1992; Jegadeesh and Titman, 1993). "Size effect" and "P/E ratio effect," as anomalies of the securities market, directly challenge the dominance of the efficient market hypothesis and CAPM model in academia. According to the efficient market theory, the stock price reflects current information. If the "size effect" exists, small companies have an excess return rate that is significantly higher than that of large companies. So, investors can easily obtain excess return by building a portfolio composed of company stocks with small circulating share capital.

While studies such as Pastor and Stambaugh (2003) and Acharya and Pedersen (2005) provide theoretical and empirical support for the role of liquidity risk, few have examined how the inclusion of a liquidity factor may moderate or mediate the strength of these anomalies across varying market environments. Moreover, the interaction between firm size, valuation ratios and liquidity characteristics remains underexplored, particularly in segmented or underdeveloped equity markets. A more granular factor model incorporating liquidity measures could yield improved explanatory accuracy and offer new insights into the conditional behaviour of size and valuation effects.

1.3.2 Excessive Volatility and Asset Bubbles

Early studies document that asset prices often display excess volatility relative to fundamentals, raising questions about the adequacy of frictionless asset pricing models (Shiller, 1981; LeRoy and Porter, 1981). Subsequent research attributes part of this excess volatility to market frictions, among which liquidity plays a central role. When trading becomes costly or constrained, prices may deviate from fundamental values and adjust in a discontinuous manner (Amihud and Mendelson, 1986; Pastor and Stambaugh, 2003).

A growing body of literature further links liquidity fluctuations to asset mispricing and bubble-like behavior. Brunnermeier and Oehmke (2013) and Gromb and Vayanos (2018) show that funding constraints and liquidity shortages can amplify price movements and generate persistent deviations from fundamentals. Empirical evidence suggests that liquidity shocks are associated with heightened volatility and return dispersion, particularly during periods of market stress (Acharya and Pedersen, 2005; Amihud, 2002).

In emerging markets, these effects tend to be more pronounced due to weaker market depth, higher retail participation, and stronger regulatory intervention. Studies on the Chinese stock market document substantial time variation in market liquidity and its close association with return volatility and mispricing (Chordia et al., 2008; Liu et al., 2019). These findings suggest that liquidity is not merely a trading characteristic but a systematic risk component that affects asset prices.

Despite these advances, much of the existing literature relies on linear factor models or parametric specifications that may not fully capture nonlinear interactions between liquidity conditions and price dynamics. This limitation motivates further research on incorporating liquidity into asset pricing models using more flexible approaches, particularly in markets characterized by frequent liquidity shocks.

1.3.3 Momentum Effect and Reversal Effect

Return predictability in the form of momentum and reversal has been extensively documented in the literature (Jegadeesh and Titman, 1993; De Bondt). While early studies treat these patterns as market anomalies, subsequent research emphasizes the role of trading frictions and liquidity conditions in shaping return dynamics. In particular, momentum profits are found to be stronger during periods of high market liquidity, whereas reversals tend to occur following liquidity contractions (Chordia and Shivakumar, 2002; Sadka, 2006).

Several studies argue that liquidity provision and investor trading behavior jointly generate nonlinear and state-dependent return predictability. Pastor and Stambaugh (2003) and Korajczyk and Sadka (2008) show that assets with higher sensitivity to liquidity risk earn higher expected returns, and that this sensitivity varies over time. More recent work highlights that liquidity-driven return patterns are difficult to reconcile within linear factor models, especially when market conditions shift abruptly.

Empirical evidence from the Chinese stock market also supports the presence of liquidity-related return predictability. Studies document that momentum and reversal effects are closely linked to trading activity, investor composition, and liquidity cycles (Hsu et al., 2016; Liu et al., 2019). These findings suggest that return predictability in emerging markets is inherently nonlinear and may depend on complex interactions between liquidity and investor behavior.

However, the majority of existing studies continue to rely on parametric models that impose linear pricing relationships. This methodological constraint limits their ability to capture the full dynamics of liquidity-driven momentum and reversal effects. As a result, there is growing interest in applying nonparametric and machine learning methods to asset pricing and return prediction, which can accommodate nonlinearities and high-dimensional interactions more effectively.

1.3.4 Liquidity Indicator

Liquidity acts as a fundamental indicator of the capital market and development. Without liquidity, there will be no volatility and no stock return. Therefore, liquidity also runs through the development of the stock market and has an important impact on the return of stocks. As suggested by Harris (1990), in a liquid market, the round-trip cost of any number of securities is minimal. If the cost is too high, no one wants to trade. As a result, the market becomes illiquid, and capacity declines accordingly. They also summarize theoretical evidence for liquidity premiums and empirical work on the corporate bond listing. Vayanos and Wang (2013) conduct a literature review on three major topics: how to quantify insufficient liquidity, the relationship between insufficient liquidity and hidden market defects and other asset characteristics, and the effect of insufficient liquidity on the expected return on assets. Kumar and Misra (2015) examine existing liquidity modeling frameworks and macroeconomic and firm-specific liquidity determinants. Matt et al. (2016) provide an overview of global stock market liquidity as well as a study of liquidity measurement for international research. Le and Gregoriou (2020) review the literature on low-frequency liquidity measurement, concentrating on the technical features of the measurement and its impact on asset pricing.

Nonetheless, these studies classify liquidity measures based on specific characteristics of low-frequency measures; broader aspects of liquidity measures, such as perspectives, data frequency, computing cost, accessibility of information, and other specificities such as periodicity, predictability, and potential application, remain unexplored. In addition, the dynamic nature of financial markets requires updating expertise regularly and modifying the techniques and tools used to evaluate liquidity in the event of a problem. Without buy- or sell-side barriers, the market lacks urgency. The bid and ask quantities at the best available prices represent the depth of the market. LOB has a dynamic structure, which implies that pricing and quantities fluctuate frequently. The bid-ask spread expands when the quantity at the best bid or best ask is exhausted. If a trader files a considerable sell order, prices are anticipated to decline. The total number of blocks at each price level indicates the breadth of the market.

Moreover, there is no unified approach for selecting and validating liquidity indicators for inclusion within multifactor asset pricing frameworks. Numerous studies have introduced proxies to capture various dimensions of liquidity, including bid-ask spreads, trading volume, turnover ratios, and price impact measures (Harris, 1990; Amihud, 2002; Pastor and Stambaugh, 2003). Despite this diversity, existing empirical applications tend to employ a single liquidity proxy, often relying on low-frequency data such as the Amihud (2002) measure, without adequately assessing the sensitivity of such indicators to market microstructure conditions or sampling frequency (Goyenko et al., 2009; Amihud et al., 2015; Hasbrouck, 2009). These practices raise concerns regarding the robustness and generalisability of findings across different market environments, particularly in settings with high-frequency trading activity or fragmented liquidity. This lack of standardisation reduces the generalisability of findings and impedes cross-market comparisons. In addition, the increasing complexity of electronic trading systems, characterised by time-varying bid-ask spreads, depth fluctuations, and shifting market-making activity, demands more dynamic and granular liquidity constructs. A model that integrates flexible liquidity measures into the Fama and French framework could enhance explanatory power and align more closely with the realities of modern market microstructure (Acharya and Pedersen, 2005; Goyenko, Holden, and Trzcinka, 2009).

1.3.5 Time-Variation of Factor Loadings

One of the first studies to evaluate CAPM in a conditional sense is from Jagannathan and Wang (1996), which allows the beta coefficient and market risk premium to change over time. They qualitatively review their work by noting that several studies have empirically assessed the performance of static versions of CAPM in the twenty years before their research. Nagel and Lewellen (2006) utilise short window regression testing to evaluate the conditional CAPM. They challenge the previous literature (eg. Dybvig and Ross, 1985; Jensen, 1968; Jagannathan and Wang,1996) and discover that the conditional alpha value of a stock may be zero while the unconditional alpha value is not; the previous arises when the beta value of a stock is allowed to change over time and is related to stock premium or market volatilities.

Using short-window regression, Lewellen (2006) estimates the parameters for size, B/M, and momentum investment portfolios. A direct evaluation of conditional CAPM is an alpha estimation. They find that alpha is statistically significant and reasonably close to unconditional alpha, even though the average dependent alpha, assuming the conditional CAPM holds, is zero (Ferson and Qian, 2004). The nonparametric method (Leave-one-out cross-validation) used by Li and Yang (2011) estimates and tests conditional factor models. It also provides insights to understand better why conditional FF models cannot explain well-known asset pricing anomalies. The study shows that the cross-sectional distribution of stock returns cannot be explained by either conditional CAPM or FF models. Furthermore, the method is effective and resistant to the dimensional problem in the nonparametric literature. Nonetheless, the method is only effective with a limited sample size, even if it is entirely data-driven. It is worth mentioning that it only provides a distribution of pricing errors rather than a complete asymptotic distribution (Ang and Kristensen 2012).

To interpret the constant terms in OLS regressions as conditional alphas, Ang and Kristensen (2012) suggest using a nonparametric methodology to test long-run and conditional alpha and beta. They create a continuous-time version of the discrete-time factor model and impose constraints on the data-generating procedure. They conclude that the proposed estimators can be used asset-by-asset and expanded to different sets of portfolios; the estimators can also be used to assess conditional factor models and can be expanded to include adaptive estimators to produce estimates of conditional alphas or factor loadings without using future data. Even though they find significant changes in factor loadings, overwhelming evidence suggests that conditional CAPM and Fama and French (1993) models cannot explain value premiums or momentum effects.

While these studies highlight the potential of flexible, data-driven approaches in capturing time-varying factor behaviour, there remains a lack of empirical frameworks that are both robust and scalable across different market conditions and portfolio dimensions. Existing non-parametric methods are not yet optimised for large-scale, high-frequency, or multi-factor environments, which limits their generalisability. Furthermore, few studies attempt to integrate such techniques into established factor pricing models in a way that allows for structural interpretation of evolving risk exposures. There is a need for more computationally efficient and theoretically coherent conditional modelling approaches that can simultaneously accommodate dimensional complexity and yield interpretable insights into the dynamics of risk premia.

1.3.6 Method Selection of Machine Learning

Stock prediction using statistical models is known for its high accuracy and interpretability, making it valuable for investors in risk management and asset allocation. However, these models have notable limitations. They assume stock index data is linear over a fixed time series, whereas real stock indices exhibit nonlinearity. Additionally, statistical models often fail to account for other significant factors influencing stock prices. Machine learning methods address these shortcomings by allowing for flexible modelling of non-linear relationships and capturing complex interactions among multiple factors that influence stock prices. Unlike traditional statistical models, machine learning techniques do not rely on rigid assumptions about data structure and are capable of dynamically adapting to changes in market conditions, thereby enhancing prediction accuracy and robustness. Despite the advantages of neural networks, they may not always accurately predict financial markets. Therefore, deep networks are required to effectively analyse long-term trends and data fluctuations (Mahdavisharif, 2021). Deep learning algorithms autonomously learn and identify complex relationships within data, making them highly effective. Consequently, deep learning technology has garnered significant attention and development in recent years, with applications spanning finance, meteorology, and transportation. Luo et al. (2017) propose an event-driven stock prediction method using an L1 regularized logistic regression model. Sreelekshmy et al. (2017) utilized recurrent neural network (RNN) and long short-term memory (LSTM) models for stock price prediction.

Simple RNN models, however, are prone to gradient vanishing and exploding issues, leading to the development of various RNN variants (Saud and Shakya, 2017;Chen et al., 2020; Zhang et al., 2023; Gao and Zhao, 2023). The gated recurrent unit (GRU) model, which employs reset and update gates, addresses these gradient issues. Shen et al. (2018) propose the GRU model and its improved version for predicting stock index trading signals for the Hang

Seng Index (HSI), German Stock Market Index (DAX), and S&P 500 Index achieving favorable results. Lin et al. (2021) and Kelotra et al. (2020) employ the CEEMDAN-LSTM model, Deep Conv-LSTM model, and two LSTM variants for prediction. Researchers have also explored stacked models for financial time series prediction, such as stacked simple RNN, GRU, and LSTM models, as well as combinations of these models. Stacking increases model depth and enhances feature extraction capabilities.

Despite these advances, Existing studies typically focus on forecasting individual stocks or single indices, with limited evaluation of the generalisability of model performance across broader asset classes or varying market regimes (Fischer and Krauss, 2018; Chen et al., 2019; Zhang et al., 2017). In addition, relatively few models are designed to integrate factor information into neural architectures, resulting in a disconnect between predictive accuracy and interpretability. Further, although advanced models such as TCN-LSTM or transformer hybrids have demonstrated potential, they remain underexplored in the context of return magnitude prediction. There is a need for more unified frameworks that combine financial domain knowledge with deep learning capabilities to address both the complexity and structure of financial return dynamics.

To address these issues, this study constructs intraday factor models based on 30-minute stock returns to better capture high-frequency market dynamics. By embedding factor information into deep learning architectures, it systematically integrates financial theory with flexible non-linear modelling. Moreover, the research applies and compares multiple neural networks, including hybrid structures, to assess predictive accuracy and robustness across different market conditions, thereby improving both the generalisability and interpretability of asset pricing models.

1.4 Research Aims and Objectives

Firstly, due to the traditional standard asset pricing theory, models cannot provide a plausible explanation for many anomalous situations in actual financial markets, and market investors consider the impact of liquidity in asset pricing models caused by micro factors such as transaction costs and information asymmetries. This has become a popular research topic on the linkage between market micro theory and asset pricing models. In recent years, market investors increasingly focus on the influence of micro factors such as transaction costs and information asymmetry in asset pricing models. Consequently, the 1st empirical chapter aims to address the following questions:

1. How do the characteristics of stocks affect their excess returns?

2. How does the liquidity factor influence the marginal performance of the FF-5 factor model?

3. How does idiosyncratic volatility affect expected stock returns?

To answer these questions, the 1st empirical chapter explores the impact of liquidity factors on excess investment returns in Chinese stock market using portfolio time series analysis and cross-sectional regression analysis.

Secondly, the noise caused by limited time series observations can be mitigated through additional averaging methods, such as combining stocks with similar factor beta estimates (Aït-Sahalia et al., 2021). Thus, it is necessary to conduct time-varying tests on the widely used FF-5 model. The motivation for using nonparametric estimation of the factor loadings in the Fama-French model is to address the limitations of parametric methods and to provide a more accurate and robust analysis of the factor loadings. Nonparametric methods perform well in capturing the nonlinear relationship between the stock returns and the factors, and produce more accurate estimates of the factor coefficients, especially in the presence of outliers or other data irregularities (Ang and Kristensen, 2012; Fan et al., 2012; Connor et al., 2012). Based on the previous, the 2nd empirical chapter considers the underlying nonlinear relationships and is the first to apply the nonparametric regression approach proposed by Ang and Kristensen (2012) to the FF-5 model using a higher data frequency and constructing factors at the 30-min level. In the 2nd empirical chapter, I shed light on the following questions:

1. Does factor loading vary over time in the Fama-French five-factor model, and what is the impact of this time variation on model performance?

2. What is the effect on alpha, if time variation in factor loading is incorporated into the model?

Finally, this study aims to evaluate mainstream asset pricing models, key factors, and the application of machine learning in stock prediction. Specifically, it examines how various factors affect stock returns and how machine learning can predict price trends. The objective is to compare the performance of traditional linear regression models with various machine learning models. While numerous studies have explored the use of single neural networks for stock price prediction, few have investigated the application of hybrid models and predicted stock or portfolio return. To address this gap, this chapter focuses on the TCN-LSTM hybrid model. Initially, the model uses TCN for feature extraction. Subsequently, LSTM extracts information and long-term dependencies from input features such as company fundamentals, before finally performing regression prediction on the Fama-French five-factor model constructed by the Dow Jones index. The 3rd empirical chapter constructs a relatively simple yet reasonable TCN prediction model based on the basic principles of Time Convolutional Neural Network (TCN) and optimised model parameters. The 3rd empirical chapter first compares and analyses the performance of the Temporal Convolutional Network (TCN) model with several traditional prediction models. Second, starting from the idea of combination prediction, the TCN and LSTM networks are integrated to enhance the structure of the basic TCN model. This integration is achieved by connecting LSTM and TCN in an end-to-end manner, aiming for stock return prediction.

This thesis is driven by a unified objective: improving the precision of asset pricing and reducing systematic pricing errors. While standard factor models provide a parsimonious benchmark, their empirical performance often deteriorates when returns exhibit time variation, regime dependence, and nonlinear interactions—features that become more salient at higher frequencies and under changing market conditions. Accordingly, the thesis is structured as a coherent sequence of increasingly stringent tests and model refinements, each designed to diagnose and reduce residual mispricing rather than to present disconnected model comparisons.

The analysis begins with conventional linear factor frameworks to establish a baseline level of pricing performance and to quantify pricing errors in a disciplined, interpretable manner. This benchmark stage is essential because it provides a transparent reference point for evaluating whether subsequent extensions deliver economically meaningful reductions in mispricing. The study then introduces model augmentations motivated by economically relevant return drivers and market frictions, assessing whether the added information set yields incremental explanatory power and systematically compresses intercepts across test portfolios. In this step, switching across market settings or portfolio constructions serves an identification purpose: it examines whether pricing improvements are robust across different trading environments and whether residual errors concentrate in specific segments, thereby revealing where standard models are most prone to misspecification.

Recognizing that a substantial component of pricing error may reflect functional-form restrictions rather than omitted variables alone, the thesis further relaxes parametric assumptions by incorporating nonparametric and deep learning methods. These approaches are introduced not as alternatives to factor models but as complementary tools to capture nonlinear, state-dependent, and high-dimensional relationships that linear specifications cannot represent. By comparing pricing errors and predictive performance across model classes and data frequencies, each chapter contributes to the same overarching theme: achieving more accurate pricing by systematically reducing unexplained return variation and clarifying the conditions under which errors arise.

To sum up, the thesis presents a unified research strategy in which changes in market focus, model specification, and methodology are all explicitly motivated by a single criterion—whether they deliver more precise pricing and smaller, less systematic pricing errors.

1.5 Research Contribution

This study integrates three complementary approaches to asset pricing and stock return prediction, making several contributions to the literature surrounding the Fama-French five-factor model. First, by analysing data from the Chinese stock market, an emerging and highly policy-sensitive environment, it extends the FF-5 framework into a six-factor model through the incorporation of a liquidity factor. This extension responds to the growing concerns in the literature regarding the limited explanatory power of the FF-5 model in emerging markets, where institutional characteristics and market frictions are more prevalent (Bali et al., 2017; Hou et al., 2020).The empirical results indicate that the extended FF-6 model, through the incorporation of the LIQ factor, significantly improves the explanation of excess returns. LIQ displays robust, statistically significant, and consistent pricing power across different portfolio sorts, indicating that it captures a dimension of excess not accounted for by traditional factors. Therefore, future multi-factor modelling for the Chinese market should consider including LIQ as a core pricing factor to enhance both the theoretical relevance and empirical accuracy of asset pricing models. Notably, the size effect (SMB) remains prominent, whereas the value factor (HML) demonstrates weak explanatory capability, and the investment factor (CMA) appears largely redundant. These findings reaffirm previously identified inconsistencies in the application of the FF-5 model across different economic contexts and highlight the necessity for market-specific adaptations in empirical asset pricing research (Fama & French, 2015; Barillas & Shanken, 2018).

Second, the study addresses the limitations inherent in conventional linear models that assume static factor loadings by employing a nonparametric estimation framework to capture time-varying exposures. Traditional asset pricing models often impose the restrictive assumption of constant factor loadings, which neglects behavioural shifts and evolving macroeconomic conditions (Jagannathan & Wang, 1996; Lewellen & Nagel, 2006). Through the application of nonparametric techniques, this research uncovers substantial temporal variation in factor sensitivities, thereby challenging the validity of static modelling assumptions. This methodological refinement enhances the flexibility and realism of asset pricing models, particularly in environments characterised by high-frequency trading, volatility and rapid shifts in market sentiment (Ang & Kristensen, 2012). The results offer a dynamic perspective on systematic risk exposure that is better aligned with the realities of modern financial markets.

Third, by integrating deep learning methodologies into the prediction of excess returns, this study advances the frontier of machine learning applications in finance. Through a comprehensive evaluation of TCN-LSTM, CNN-LSTM, LSTM and CNN models using high-frequency data from the Dow Jones Industrial Average, the research demonstrates that the CNN-LSTM architecture achieves the highest predictive accuracy and stability. This suggests that hybrid models capable of jointly extracting spatial and temporal features possess substantial advantages in modelling the complex dynamics of financial time series (Fischer & Krauss, 2018;Gu et al., 2020). Although the TCN-LSTM model effectively captures local temporal dependencies, it exhibits limitations in learning persistent long-term patterns due to the nature of convolutional architectures (Lea et al., 2017). These empirical findings underscore the inadequacy of traditional linear models in capturing nonlinearities and higher-order interactions in financial datasets, reaffirming the promise of deep learning approaches in predictive financial modelling.

In addressing these issues, the study fills several important gaps in the extant literature. It responds to the limited generalisability of the FF-5 model in emerging markets, challenges the assumption of constant factor loadings and introduces machine learning innovations into the asset pricing domain. The findings provide actionable implications for practitioners, offering enhanced methodologies for return forecasting, portfolio optimisation and dynamic risk management. They also inform regulatory authorities by highlighting the necessity of adaptive and forward-looking frameworks in systemic risk evaluation (Adrian et al., 2019).

In conclusion, this research advances theoretical knowledge by demonstrating the critical importance of liquidity, time variation and nonlinear modelling in modern asset pricing. It also enriches empirical practices by providing a unified, flexible and data-driven approach that bridges the gap between traditional finance theory and contemporary machine learning techniques, thereby laying the groundwork for future investigations in dynamic and complex financial environments.

Chapter 2. Liquidity Matters: Enhancing the Fama-French Model in Chinese Stock Market[4]

This chapter examines the expansion of the Fama-French five-factor model by incorporating a liquidity factor to better explain stock returns in the Chinese stock market. Using daily data between 2013 and 2021, the study evaluates the significance of liquidity in capturing return variations alongside traditional factors. The analysis further explores the role of idiosyncratic volatility in asset pricing, assessing its incremental explanatory power and behaviour across different market conditions. The empirical findings reveal that liquidity can capture cross-sectional risk premiums beyond the explanatory scope of traditional models, and liquidity and idiosyncratic volatility contribute to improved model performance. The study highlights the importance of enhancing traditional factor models to accommodate additional dimensions of market risk, offering insights for both academic research and practical investment strategies.

[4] This chapter has been published in the Journal of Studies in Nonlinear Dynamics & Econometrics.

2.1 Introduction

The Capital Asset Pricing Model (CAPM) is central to modern financial market price theory and widely employed in investment decision-making. Markowitz (1952) proposes portfolio selection theory, treating portfolio returns as random variables, using the mean as the expected return, and variance as a risk measure. Following this, classical asset pricing theories emerged, including CAPM, the Fama-French three-factor model (FF-3), and the more recently improved Fama-French five-factor model (FF-5).

This study examines the explanatory power of the Fama-French five-factor model (FF-5) in the context of the Chinese A-share market, with a particular focus on two key extensions. First, it incorporates a liquidity factor into the FF-5 framework to investigate whether market microstructure characteristics improve cross-sectional return prediction. Second, it analyses the role of idiosyncratic volatility, derived from model residuals, as an additional source of return variation.

Since its introduction, the three-factor model established by Fama and French (1993) has been prevalent in empirical research. However, evidence suggests the FF-3 model fails to adequately explain variations in average stock returns associated with profitability and investment style. Aharoni et al. (2013) identify a significant negative correlation between capital investment and projected returns, while Novy Marx (2013) observes a positive correlation between expected profitability and stock returns.

In response, Fama and French (2015) update the FF-3 model to the FF-5 model, prompting extensive empirical tests among academics. Nonetheless, financial anomalies such as illiquidity premiums remain unresolved. Liquidity has been a controversial research area since Amihud (1986) introduced the liquidity premium concept, and integrating liquidity factors into asset pricing has offered new perspectives beyond traditional theoretical frameworks. Notably, the Fama-French models do not address liquidity risk explicitly. Fama and French (2015) suggest that the liquidity parameters provided by Pástor and Stambaugh (2003) fail to explain expected returns. However, Ma and Zhang (2021) note that the liquidity factor developed by Pástor and Stambaugh, based on price effect metrics, does not generate a significant liquidity premium. In contrast, Liu (2006) indicates that liquidity risk factors capturing transaction continuity and liquidity characteristics can produce substantial premiums.

Idiosyncratic risk, representing residual or non-systematic risk, arises from stock return fluctuations due to specific events rather than market-wide movements. As asset pricing models evolve from FF-3 to FF-5, the representation and accuracy of idiosyncratic risk characterisation improve accordingly.

Research on idiosyncratic volatility and cross-sectional returns has been extensive, often employing regression methods from asset pricing models to isolate the residual series' standard deviation as a proxy for idiosyncratic volatility. This approach captures market characteristics and identifies stock return fluctuations excluded from market risk. Tinic and West (1986) adopt CAPM to extract residual series volatility. Ang et al. (2006, 2009) expand on CAPM by incorporating market value and book-to-market ratio factors from the FF-3 model.

The main aim of this chapter is to further explore asset pricing improvements by examining whether adding a liquidity factor to the FF-5 model enhances its explanatory power for Chinese market returns compared to traditional FF-3 and FF-5 models. Roy and Shijin (2018) include human capital in the FF-5 model, proposing a balanced six-factor model. Dirkx and Peter (2020) compare the FF-3 and a six-factor model (incorporating profitability, investment, and momentum factors) in the German market, finding only a marginal increase in explanatory power from the six-factor model.

This chapter offers several contributions to the empirical asset pricing literature. First, it extends the Fama-French five-factor model by incorporating a liquidity factor, providing a more microstructurally grounded measure of liquidity. Second, it derives idiosyncratic volatility directly from the residuals of both five-factor and six-factor models, offering a model-consistent approach to capturing firm-specific risk. Third, it applies these extensions to the Chinese A-share market, where information asymmetry and evolving market efficiency present a unique environment for empirical testing. By addressing these dimensions, the study fills notable gaps in the literature regarding the role of liquidity and idiosyncratic risks in emerging markets.

Although the efficiency of China stock market is closed to semi-strong efficiency, it has not broken the frontier of weak-form efficiency. Investors can still obtain excess returns through information asymmetry, such as using inside information. Insiders use private information to trade. For instance, irrational insiders will submit instructions to complete the transaction as soon as possible according to their expected value, and rational insiders will submit large reverse price limit orders to affect the stock price, which can facilitate the development of its profitable direction so as to obtain more profits. When insiders place orders in the market, their inside information will be leaked into the market through the transaction price. At this time, external traders are able to compare the relationship between the average expected value of the market and the market transaction price. The existence of abnormal disclosures of major company information and collusion between institutions will cause abnormal stock prices and liquidity volatility. Then it may cause deviations from the final empirical test results. This is bound to have an impact on the factors that can significantly explain the excess return of the stock market. The feature of this study is that the FF model is used to connect the full text. On the basis of studying the performance of the FF-5 model in Chinese stock market, the FF-5 model is applied to the study of liquidity pricing, the Fama-French five-factor model is applied to the study of liquidity pricing, idiosyncratic volatility and so on.

Foye (2016) first evaluates whether the FF-5 model outperforms the FF-3 model in describing stock returns in emerging markets, testing data from 18 countries across three regions. Results indicate the FF-5 model consistently outperforms FF-3 only in Eastern Europe and Latin America. In China, research predominantly favours the FF-3 model. Zhang (2018) shows that although the FF-5 model applies to Chinese A-share market, its explanatory power remains weaker than that of FF-3; newly added profitability and investment factors explain stock returns, yet the significance of the *HML* factor diminishes. Thus, consensus on the relative merits of the FF-5 versus FF-3 model remains elusive, and research examining the relationship between idiosyncratic risk and stock returns under the FF-5 framework is limited.

This chapter aims to address this gap by deriving idiosyncratic volatility from the residuals of the five- and six-factor models in emerging markets. Instead of using liquidity indices proposed by Amihud (2002) or Kang and Zhang (2014), the study utilises the effective bid-ask spread, better suited to high-frequency data, to assess micro-level impacts on stock returns. Studies by Lu et al. (2007), Gao et al. (2014), and Chen et al. (2014) indicate that liquidity enhances model predictability, suggesting that liquidity pricing research on price shocks can illuminate information transmission mechanisms within the price discovery process.

Fama (1965) first introduces the Efficient Market Hypothesis (EMH), linking information to market dynamics and evaluating stock prices through market reactions to new information. Fama (1970) further proposes specific criteria for judging market efficiency, asserting that a market is efficient if all stock prices reflect all available information. In an entirely efficient stock market, investors have equal access to vast amounts of information, already incorporated into stock prices, preventing them from achieving excess returns through information alone.

Market efficiency comprises three types: weak-form, semi-strong, and strong efficiency. Weak-form efficiency indicates that current stock prices fully reflect historical trading data (price, return, and volume), rendering technical analysis ineffective for earning excess returns. Semi-strong efficiency posits that stock prices reflect all publicly available information, including historical data, financial statements, industry and macroeconomic indicators. Fundamental analysis, therefore, does not lead to excess returns. Strong efficiency assumes that prices reflect all information, both public and private, including insider information. Since the 1970s, empirical research on market efficiency mainly investigates weak-form and semi-strong efficiency.

Chinese scholars have extensively examined weak-form market efficiency since the establishment of the Chinese stock market, yielding contradictory findings. Early research by Wu (1993), using autocorrelation analysis on the Shanghai stock market, concludes the market does not achieve weak-form efficiency. Similarly, Yu (1994) rejects weak-form efficiency through random walk tests on Shanghai and Shenzhen indices. Xie et al. (2002), Li and Zheng (2018), Chen and Li (2005), Zhao and Chen (2012), and Meng, Liu and Zhou (2012) consistently find the Chinese stock market lacking weak-form efficiency through various methodologies such as GARCH and unit root tests.

Conversely, Hu and Fan (2000) argue that the Shanghai stock market achieves weak-form efficiency using correlation tests. Li and Wang (2010) find that the CSI 300 Index demonstrates weak-form efficiency in the short term but not medium or long term. Ming (2011) supports the weak-form efficiency of the Shanghai market through unit root tests. Xu (2019), Liu (2015), and Zhao (2016) also find evidence supporting weak-form efficiency, highlighting methodological differences as reasons behind the conflicting conclusions.

The Chinese stock market, transitioning from immaturity to maturity, continues to develop rapidly. Consequently, research on market efficiency remains highly debated. Two main approaches dominate: statistical tests and empirical tests based on technical analysis strategies. Statistical tests, including unit root, random walk, variance ratio, GARCH, and Hurst tests, are prevalent, while technical analysis strategies assess whether indicators can generate excess returns. However, given that stock market reforms in China only began in 2005, existing conclusions are not definitive due to the limited research period.

Developed markets, generally achieving weak-form or even semi-strong efficiency, set the benchmark for market efficiency studies. Osborne (1962) and Fama (1965) use run tests to validate the random walk hypothesis, demonstrating stock price independence over time. Lechukwn (2016) expands the EMH study to developing countries, using wavelet unit root tests on Nigerian stock market data, confirming weak-form efficiency regardless of information asymmetry or economic development levels.

Lintner (1964) develops the CAPM to examine the relationship between expected market returns and risk. While CAPM helps investors gauge returns relative to risk, practical application reveals limitations. Empirical studies, such as those by Campbell, Andrew, and Mackinlay (1997) and Reinganum (1981), indicate CAPM has inadequate explanatory power for stock returns. Similar findings in China, by Chen et al. (2001), Yang (2011), and Du (2015), also highlight the limitations of CAPM. Improvements and criticisms of CAPM have driven theoretical and empirical developments. Miller and Scholes (1972), Black, Jensen, and Scholes (1972), and Kothari, Shanken, and Sloan (1995) suggest statistical adjustments, highlighting portfolio construction over individual assets for more reliable results. Roll (1977) raises concerns about verifying CAPM due to uncertainties surrounding market portfolios, prompting alternative empirical methods, such as those by Stambaugh (1982). Ross (1976) introduces Arbitrage Pricing Theory (APT), addressing the limitations of CAPM. Roll and Ross (1980) demonstrate the broader application of APT by identifying multiple risk factors influencing stock returns, underscoring its flexibility compared to CAPM.

Building upon CAPM, Merton (1969, 1971) proposes the Inter-temporal Capital Asset Pricing Model (ICAPM), accounting for dynamic market conditions. French, Schwert, and Stambaugh (1987), and Bali and Engle (2008a) empirically validate ICAPM, affirming the relationship between expected returns and volatility. Breeden (1979) proposes the Consumption-based Capital Asset Pricing Model (CCAPM), linking consumption utility with investment decisions. CCAPM addresses the theoretical shortcomings of CAPM by providing microeconomic foundations for investment choices, enhancing explanatory power through investor consumption and risk preferences.

Fama and French (1993, 1996) develop the three-factor model (FF-3), incorporating market risk, size, and book-to-market factors, significantly improving the explanatory power of CAPM. Griffin (2002) and Cao (2005) affirm the effectiveness of FF-3 internationally, though profitability and investment factors remain unexplained. Consequently, Carhart (1997) introduces a momentum factor, developing a four-factor model. Fama and French (2015) later expand to a five-factor model (FF-5), adding profitability and investment factors, further enhancing explanatory power across international markets. Recent developments include a six-factor model proposed by Fama and French (2018), integrating momentum into FF-5. However, alternative perspectives by Hou et al. (2021) challenge this, highlighting other growth predictors. Rahul and Santhakumar (2018) and Dirkx and Peter (2020) provide mixed evidence regarding improvements from the six-factor model compared to traditional models.

Liquidity remains a critical consideration in asset pricing. Amihud and Mendelson (1986) first demonstrate liquidity premiums, suggesting compensation for lower liquidity. Acharya and Pedersen (2005) introduce liquidity-adjusted CAPM, highlighting liquidity's role in returns through dynamic and static channels. Studies by Chordia et al. (2000) and Pástor and Stambaugh (2003) support the systemic importance of liquidity in pricing. Idiosyncratic volatility, the residual risk beyond systematic factors, garners significant research attention. Ang et al. (2006, 2009) and Fu (2009) illustrate methodologies for capturing idiosyncratic volatility, with debates around its correlation with expected returns. Scholars like Miller (1977), Merton (1987), and Barberis and Huang (2001) offer theoretical explanations, suggesting various positive and negative relationships based on behavioural and informational assumptions. Empirical research continues to explore this dynamic, providing nuanced insights into the complexities of asset pricing models.

The empirical findings indicate that while the factors in the Fama-French five-factor model remain relevant, particularly the size factor, including a liquidity variable leads to measurable improvements in cross-sectional explanatory power. Furthermore, idiosyncratic volatility, when extracted from the extended model, is positively associated with expected returns.

These results suggest that firm-specific risks and trading frictions are not fully captured by traditional factors and should be incorporated into asset pricing frameworks in markets with significant heterogeneity in trading conditions and investor behaviour.

The structure of this chapter is organised as follows. Section 2 outlines the construction of the Fama-French five-factor and six-factor model, alongside the derivation of idiosyncratic volatility measures. Section 3 presents the empirical results, including factor analysis, regression analysis, and cross-sectional return tests using the Fama-MacBeth methodology. Particular emphasis is placed on evaluating the explanatory power of liquidity and idiosyncratic volatility in the context of the Chinese stock market.

2.2 Methodology

This chapter will discuss the fundamentals of the relationship between risk and expected return. It will primarily introduce the multi-factor pricing model and the most recent research on the five-factor model so that the FF five-factor model discussed in this chapter may be properly comprehended. Fama and French (2015a) analysed the valuation model of Miller and Modigliani (1961) and constructed the theoretical foundation for the five-factor asset pricing model, demonstrating that the expected return of the stock is related to the book to market ratio (B/M), profit, and investment of listed companies. Miller and Modigliani's (1961) valuation model proved that the market value of Listed Companies in period t may be calculated using the following formula:

$$m_t = \sum_{i=1}^{\infty} E(d_{t+i})/((1+r)^i. \tag{1}$$

The m_t is the market value of a listed company at time t, and r is the long-term expected return. Divide this formula by B_t (the book value) to get:

$$m_t/B_t = \left[\sum_{i=1}^{\infty} E(d_{t+i})/((1+r)^i\right]/B_t. \tag{2}$$

When other variables remain unchanged, when the stock market value (*Mt)* decreases or the book to market ratio (*Bt/Mt)* increases, the expected stock return (*r*) increases; the expected stock return is positively correlated with the expected return (Y_{t+i}) and negatively correlated with the growth of shareholders' stock.

In order to further enhance the explanatory power of these market anomalies, Fama and French (2015) added the factors of profitability and investment ability into the three-factor model to construct a more efficient and advanced Fama-French five-factor model:

$$R_{it} - R_{ft} = \alpha_i + b_i\,(R_{mt} - R_{ft}) + s_i\,SMB_t + h_i\,HML_t + r_i\,RMW_t + c_i\,CMA_t + \varepsilon_{it.} \quad (3)$$

Let R_{it} be the return of a particular stock at time t and observe the data of M stocks during the T period. F_t represents the factor matrix of the same period in the Fama-French factor model. R_{mt} is the market return rate, R_{ft} is the risk-free interest rate, SMB_t is the size factor, HML_t is the value factor, RMW_t is the profitability factor, and CMA_t is the investment factor. Considering the variation of factor loading over time, I establish a new Fama-French five-factor model.

2.2.1 Factor Construction

According to Fama and French (2015), stocks are categorized into large-cap and small-cap based on market value. At the end of June each year (t), stocks are split into two groups—small (S) and big (B)—using the median market capitalization (stock price per share multiplied by outstanding shares). Next, stocks are sorted by their book-to-market (B/M) ratio from high to low. The top 30% are classified as high B/M (V), the middle 40% as medium B/M (N), and the bottom 30% as low B/M (G). This cross-grouping results in six stock portfolios: SG, SN, SV, BG, BN, and BV. Then, the book-to-market ratio is replaced with operating profitability and investment level, respectively, following the same sorting method. This process divides all stocks into 12 portfolios: SR, SN, SW, BR, BN, BW, SC, SN, SA, BC, BN, and BA. Here, R represents robust, W indicates weak profitability, C denotes a conservative investment style, A signifies an aggressive investment approach, and N stands for neutral profitability or investment level. The market-value-weighted average return is then calculated for each portfolio in each period. Finally, four factors are constructed based on the return differences among various portfolios, enabling further calculations to derive the values of each factor variable：

$$SMB = SMB_{B/M} + SMB_{OP} + SMB_{Inv}. \quad (4)$$

$$HML = (SH+BH)/2 - (SL+BL)/2. \quad (5)$$

Then, I form six portfolios based on the intersection of two size portfolios (S and B) and three profitability (*OP*) portfolios (represented as robust (*R*), neutral (*N*), and weak (*W*), respectively), as well as three investment (*Inv)* portfolios (represented as conservative (*C*), neutral (*N*), and aggressive (*A*) respectively). *OP* refers to annual revenue minus sales costs, interest expenses, and sales, as well as general and administrative expenses, and then divided by the carrying amount of equity at the end of the previous fiscal year. Inv refers to the total asset changes from the fiscal year at the end of the year to the fiscal year at the end—the fiscal year ending in, divided by total assets. The breakpoints between *OP* and Inv are the 30th and 70th percentage points in the index. SR, BR, SW, BW, SC, BC, SA, and BA are abbreviations for different portfolios in asset pricing models. SR refers to small stocks with robust profitability, BR to big stocks with robust profitability, SW to small stocks with weak profitability, and BW to big stocks with weak profitability. SC represents small stocks that are conservative in investment (low investment levels), BC indicates big stocks that are conservative in investment, SA refers to small stocks that are aggressive in investment (high investment levels), and BA represents big stocks that are aggressive in investment. These classifications are typically used to construct factors like *RMW* (Robust Minus Weak) and *CMA* (Conservative Minus Aggressive), capturing the return differences based on profitability and investment levels across small and large stocks. The profitability and investment factors are given by:

$$RMW = (SR+BR)/2 - (SW+BW)/2. \tag{6}$$

$$CMA = (SC+BC)/2 - (SA+BA)/2. \tag{7}$$

In this section, the size of the company is measured as the product of the stock price at the end of year T-1 (where T denotes the current fiscal year) and the number of outstanding shares. The book-to-market ratio (B/M) is calculated as the ratio of total shareholders' equity at the end of year T-1 to total market capitalization. In the study of Fama and French (2015a), operating profitability is measured as: $OP = \frac{Revenue - Cost\ of\ Goods\ sold - Interest\ Expense - Selling\ Expense}{Book\ Value}$.

In this chapter, the ratio of operating profit to book value from the income statement at the end of year T-1 is used as the proxy variable for profitability (OP). At the same time, following the approach of Fama and French (2015a), the investment level (INV) of a listed company is proxied by the growth rate of total assets, calculated as the percentage change in total assets at the end of T-1 relative to the end of T-2.

2.2.2 Construct 5x5 portfolio

The samples are sorted annually by circulating market capitalization into five groups from small to large and further subdivided into five groups based on B/M, OP, and INV, also ranked from small to large, forming a 5×5 portfolio structure (75 portfolios in total). The daily value-weighted average excess return for each portfolio is then calculated based on its current market capitalization.

The resulting average excess returns of the 5×5 portfolios are used in two ways: firstly, by averaging the excess returns across to analyse the characteristics of factor-based returns. Secondly, the excess returns are used as the dependent variable in the time-series regression analysis of the Fama-French five-factor model, with R_{it} - R_{ft} on the left side of formula (3).

2.2.3 FF-6 Factor Model

This research calculates the time-weighted average of the liquidity indicator *Esp* as its daily average, and the weight is the time interval between two adjacent records. The time weighted bid-ask spread not only reflects the spread itself, but also reflects the duration of the spread. It also reflects the quotation strategy of investors, which is closely related to the degree of information asymmetry. This indicator applies all the information of intraday high-frequency data, which is taken as the benchmark indicator of this study.

Liquidity risk and its impact in some emerging stock markets may differ from that in developed markets because of their order-driven market structure, the predominance of individual investors and strict trading rules, such as the Chinese stock market. In such stock markets, information asymmetries and transaction costs are more severe. High frequency bid-ask spreads are an ideal measure of liquidity and are considered to be a direct indicator of liquidity. Following the findings of Mcinish and Wood (1992), this chapter calculates the time-weighted efficient bid-ask spreads (Esp) for each stock for each trading day using the time interval between two consecutive quotes as the respective weights:

$$Esp = 2 \times \left| P_t - \frac{S_1 + B_1}{2} \right| / (S_1 + B_1). \quad (8)$$

Where S_l is the ask price, B_l is the bid price, and P_t is the stock trading price in the time t.
In practice, the effective spread is calculated using the above formula for each intraday time interval between consecutive quote updates. The length of each interval serves as its weight. The daily liquidity measure for a stock is then computed as the weighted average across all intervals for that day.

2.2.4 Construct Liquidity Factor

This chapter will use the daily time-weighted *Esp* to measure stock liquidity. The construction of the liquidity factor in this chapter is similar to the calculation of HML and other factors above. First, divide into large-size group and small-size group according to market value, and sort stocks according to *Esp* value. Taking 30%, 40% and 30% as split points, it is divided into three groups: high liquidity, medium liquidity and low liquidity. The liquidity factor is constructed by 2x3 grouping and the return weighted by the current market value. which will record high liquidity as *HL,* medium liquidity as *NL* and low liquidity as *LL*. Next, the weighted return on the total market value of the high liquidity stock portfolio minus the weighted return on the total market value of the low liquidity stock portfolio is recorded as the liquidity factor *LIQ,* and then adding it to the model for test:

$$SMB_{LIQ} = (r_{SL} + r_{SN} + r_{SH})/3 - (r_{BL} + r_{BN} + r_{BH})/3. \quad (9)$$

$$LML = (r_{SH} - r_{SL})/2 + (r_{BH} - r_{BL})/2. \quad (10)$$

$$R_{it} - R_{ft} = \alpha_i + b_i\,(R_{mt} - R_{ft}) + s_i\,SMB_{LIQ} + h_i\,HML_t + r_i\,RMW_t + c_i\,CMA_t + l_i LIQ + \varepsilon_{it}. \quad (11)$$

2.2.5 Idiosyncratic Volatility Test

Idiosyncratic volatility cannot be observed directly, and its estimation is relative to the systematic return of stocks, so it is model dependent. There are three main decomposition methods for idiosyncratic volatility: Campbell et al. (2001) indirect separation method based on CAPM, the direct separation method based on CAPM represented by Bali et al. (2005) and the direct separation method based on Fama-French three-factor pricing model represented by Xu and Malkiel (2003). Therefore, I first obtain the residual series from the five-factor model and six-factor model.

According to Xu and Malkiel (2003), for each stock in the sample, I conduct time series regression on the market excess return in the past 12 months. I fit the beta estimate thus generated with the daily return of the current month to obtain the daily residual. Idiosyncratic volatility is the sum of the squares of these daily residuals. Then, according to the practice of Ang et al. (2006), the realised monthly idiosyncratic volatility of portfolio ***i*** in month ***t*** is defined as:

$$iv_{i,t} = \sqrt{n_{i,t}}\, std(\varepsilon_{i,t,\tau}). \qquad (12)$$

Where $n_{i,t}$ is the trading days of the current month, and $std(\varepsilon_{i,t,\tau})$ represents the standard deviation of the daily residual returns. The daily residuals are obtained by regressing the daily excess returns of stock on the daily returns of the relevant set of risk factors. Finally, I will observe residual terms for size-OP, size-B/M, size-Inv and size-LIQ in the FF-5 model and the FF-6 model respectively.

2.2.6 Test for Performance of Models

When comparing the performance of the model, the performance of the regression intercept term is mainly used as the judgment basis. The GRS test proposed by Gibbons et al. (1989) is a commonly used method to test the efficiency of the pricing model, which can test whether all intercept terms are 0 at the same time. If the pricing model can fully explain the excess return of all stock portfolios on the cross-section, the joint test of all portfolio regression intercept terms should not reject the original hypothesis that it is 0 at the same time. The calculation logic of GRS inspection is as follows:

$$(\frac{T}{N})(\frac{T-N-L}{T-L-1})[(\hat{\alpha} \textstyle\sum_n^{-1} \hat{\alpha})/(1+\bar{\mu}'\hat{\omega}^{-1}\bar{\mu})] \sim F\ (N, T-N-L) \qquad (13)$$

Among them, the $\hat{\alpha}$ is the N*1 vector of the estimated intercept term; $\sum_{\mathrm{n}}^{-1}$ is the inverse of the covariance matrix of the unbiased estimate of the residual; $\bar{\mu}$ is the L*1vector of the average return rate of the risk factor. The *GRS* test assumes that the intercept term of any *i-regression* result α_i=0, then *GRS* statistics is zero; According to the research method in this chapter, the smaller the value of *GRS* statistics is, the larger the p-value is, the better the explanatory ability of the model to explain dependent variables is, that is, the model is better.

This research tends to use *GRS* test to test the Size-B/M, size-OP and Size-INV of FF-5 factor model and FF-6 factor model respectively. Besides *GRS*, Fama and French (2015a) also selects three other indicators to measure the performance of regression intercept. This chapter will also add $A|\alpha_i|$ index, where $\hat{\alpha}$ is the absolute value of the *i* th stock portfolio regression intercept term, and $A|\alpha_i|$ is the average of the absolute values of the 25 regression intercept terms. Therefore, this chapter calculates the GRS statistics and $A|\alpha_i|$ of size-B/M, size-OP, size-Inv and size-LIQ under the FF-5 and FF-6 models respectively under the 2x3 grouping. In addition, to ensure robustness, this chapter will simultaneously calculate the *GRS* statistics and $A|\alpha_i|$ of the above four portfolios under the 2x2 and 2x2x2x2 grouping.

2.3 Empirical Results

2.3.1 Data

This chapter selects the daily returns of 150 stocks from Chinese main board stock market as the sample, with all data sourced from the Wind and CSMAR databases. The Wind database primarily provides stock trading data, while the CSMAR database offers financial data. The data frequency is based on daily stock returns.

To analyse both the Fama-French five-factor model (FF-5) and liquidity, I select a sample of 50 large-cap, 50 mid-cap, and 50 small-cap stocks. The selected stocks cover various industries, including energy, equipment, technology, real estate, internet, transportation, and industrial manufacturing. However, financial sector stocks such as banks, insurance, and securities firms are excluded due to differences in accounting standards. Additionally, Special Treatment (ST) companies are excluded, as they are often suspected of financial fraud and are subject to bankruptcy accounting, which deviates from normal accounting practices. Considering Chinese high savings rate, where most residents prefer savings as their primary investment method, the one-year fixed deposit benchmark interest rate is chosen as a proxy for the risk-free interest rate.

The sample period spans January 1, 2011, to December 30, 2022. After the bull market from 2005 to 2007, the 2008 financial crisis triggered a 72.81% decline in the Shanghai

Composite Index (SSE Composite Index) from its peak. Following two years of recovery, Chinese stock market entered a new cycle in 2011. The market experienced another bull run in 2015, with the index surging to 5,178.19 points, marking a 211.02% increase from the 2008 low. However, after 2015, the SSE declined until mid-2020, when the stock market entered a new expansion phase, driven by the growth of the technology sector. While the index rises is moderate, overall market volume and turnover increased significantly.

Market liquidity is further enhanced through key structural reforms, including the Equity Share Reform in 2005 and the establishment of ChiNext in 2009, which diversified Chinese stock market landscape. As a result, the selected sample period spans two complete market cycles (July 1, 2011-April 30, 2022), making it well-suited to reflect the characteristics of Chinese stock market.

2.3.2 Descriptive Statistics and Correlation of Factors

Table 1 presents the liquidity factor that records the highest mean daily return (0.0016), followed closely by the market factor, thereby emphasizing the potential importance of liquidity in the Chinese stock market. In contrast, the value and profitability factors exhibit negative mean returns, which suggests that the traditional value and profitability premiums are weak or even absent during the sample period. Moreover, the market factor demonstrates the highest volatility, as reflected by its standard deviation (0.0145), and also registers the most pronounced minimum return (–0.0956), indicating heightened market fluctuations over the period. Market and SMB factors display left-skewness (–0.7438 and –1.0065, respectively) and leptokurtosis (6.8611 and 6.2542), indicating the presence of extreme downside movements and fat-tailed behaviour.

Overall, the evidence highlights the necessity of incorporating liquidity risk into factor models and questions the empirical relevance of value and profitability factors in the Chinese setting. These distributional characteristics and statistical deviations also suggest the need for robust modelling approaches when investigating return patterns in emerging markets.

Table 1. Summary statistics for six-factor model: January 1, 2013, to December 30, 2021.

The table shows average daily returns (Mean), the standard deviations of daily returns (Std dev). R_m-R_f is the value-weight return on the market portfolio of all sample stocks minus the daily benchmark interest rate of the bank's one-year fixed deposit. At the end of each June, stocks are assigned to two Size groups using the median market cap as the breakpoint. Stocks are also assigned independently to two or three book-to-market equity (B/M), operating profitability (OP), investment (INV) groups and liquidity (LIQ) groups, using 150 stocks' medians of B/M, OP, Inv and LIQ and the 30th and 70th 150 stocks' percentiles.

2x3 Factors	Rm-Rf	SMB	HML	RMW	CMA	LIQ
Mean	0.0001	0.0005	-0.0009	-0.0006	0.0001	0.0016
Std dev.	0.0145	0.0128	0.0119	0.0131	0.009	0.0131
min	-0.0956	-0.0999	-0.0594	-0.0546	-0.0404	-0.0977
max	0.0806	0.0707	0.0516	0.0819	0.0546	0.0579
Skewness	-0.7438	-1.0065	0.0516	0.0819	0.0546	0.0579
Kurstios	6.8611	6.2542	0.0779	0.6443	0.3116	-0.6891
Jarque Bera (prob.)	3986 (0.00)	3478 (0.00)	260 (0.00)	1252 (0.00)	514 (0.00)	2082 (0.00)

In figure 1, the *MKT* factor maintains a stable and positive cumulative return throughout the study period, which is consistent with asset pricing theory's expectation that systematic risk should be compensated. The *SMB* factor exhibits a clearly positive excess return within the Chinese main board sample. Although micro-cap firms listed on the Growth Enterprise Market and the Science and Technology Innovation Board are excluded, the sample still contains a substantial number of small- and mid-cap firms with market capitalisation between 5 billion and 20 billion Chinese yuan. These firms tend to demonstrate greater price and valuation elasticity in policy-driven or theme-based rallies. In contrast, firms with extremely large market capitalisation, characterised by operational stability and lower volatility, tend to lower the returns of the large-size group, thereby enhancing the relative performance of the *SMB* factor.

The extreme values of daily factor returns are largely concentrated around the 2015 boom–bust episode in the Chinese equity market, reflecting the combined effects of leverage expansion, liquidity shocks, and strong policy interventions. The market factor exhibits large positive and negative extremes during this period, consistent with the rapid transition from a leverage-driven bull market to a sharp correction under tightening financing constraints. During the market crash, the size and liquidity factors display the most pronounced tail behavior, indicating that small-cap stocks and market liquidity were particularly vulnerable to forced deleveraging and trading frictions.

In contrast, the value and profitability factors show temporary reversals as capital rotates away from speculative growth stocks toward firms with relatively stronger fundamentals, while the investment factor responds more moderately, reflecting the delayed adjustment of corporate investment decisions to market stress. Overall, the clustering of extreme factor returns highlights the dominant role of systemic liquidity conditions and institutional characteristics in shaping factor dynamics, and further supports the presence of strong non-normality and nonlinear behavior in factor returns.

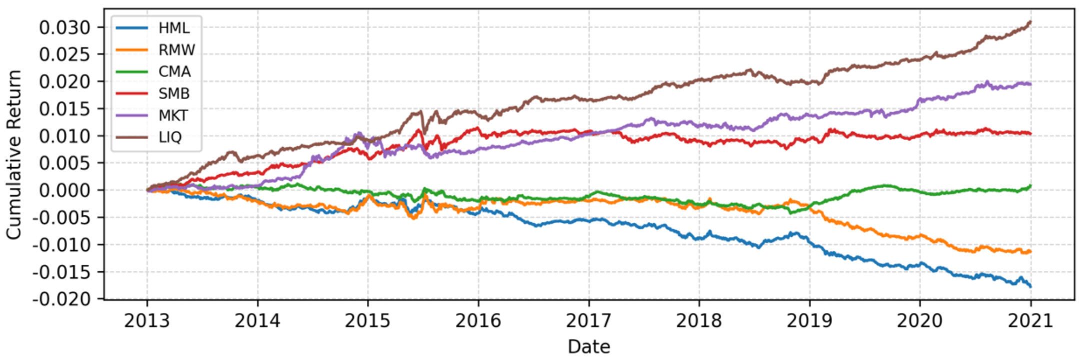

Figure 1. Accumulate Daily Return of Fama-French 5 Factors
Note: This figure presents the cumulative return of five Fama-French factors, MKT(purple line), SMB (red line), HML(blue line), RMW(orange line), CMA(green line) and LIQ(Brown line) over the period from January 2013 to December 2021.

The *RMW* factor, in contrast, shows persistently negative cumulative returns over the sample period, indicating that firms with high profitability do not consistently earn a risk premium. This phenomenon closely relates to valuation preferences in the Chinese mainboard market. In certain years, firms with low or even negative earnings attract valuation premiums due to their involvement in strategic emerging industries. Meanwhile, traditionally profitable sectors often experience regulatory constraints and valuation discounts. These distortions in the pricing mechanism reduce the explanatory power of profitability in accounting for cross-sectional returns.

The *HML* factor also appears relatively weak in the Chinese mainboard market. In developed markets, a high book-to-market ratio generally signals undervaluation or potential mean reversion. However, in the Chinese context, the book-to-market ratio is frequently affected by asset revaluations or non-operating components in financial statements, which diminishes its economic relevance. Moreover, the market increasingly favours high-growth, high-turnover sectors, exerting long-term downward pressure on the performance of the value factor.

Finally, the *CMA* factor fails to deliver consistent positive excess returns throughout the sample period. This result possibly reflects the non-market-oriented nature of capital expenditure decisions in many Chinese listed companies. Investment activities are often influenced by administrative approvals, policy incentives, or access to preferential financing, rather than being driven by market-based return expectations. Furthermore, inconsistencies in the financial reporting of investment expenditures weaken the factor's capacity to explain variations in stock returns.

Table 2 presents the correlation coefficient between the HML and RMW factors reaches as high as 0.75, indicating a strong positive relationship. This suggests that, in the Chinese main board market, value stocks—those with high book-to-market ratios—often also exhibit strong profitability. This coupling is closely related to the industrial portfolio of the Chinese market, where high B/M firms are frequently concentrated in traditional sectors such as finance, real estate, energy, and steel. Although these firms are typically valued at lower levels, they tend to maintain relatively stable or even strong earnings, leading to the observed high positive correlation between HML and RMW. Such structural overlap may result in factor redundancy when constructing multi-factor models, which in turn can reduce the model's explanatory power and stability.

Second, a significant negative correlation of -0.41 is observed between the SMB and RMW factors. This implies that within the sample of main board firms, small-cap companies generally have weaker profitability, or at least are less stable in their earnings compared to large-cap firms. This characteristic is particularly evident in the Chinese market, where small and mid-sized firms, although offering growth potential and valuation flexibility, often lack the financial robustness of large blue-chip firms and are more vulnerable during periods of economic downturn. This negative relationship reflects a typical style divergence between small-cap growth and large-cap profitability.

The correlation between *SMB* and *HML* is also negative, at -0.20, further supporting the observation that small-cap stocks in the Chinese market tend to exhibit growth-oriented rather than value-oriented characteristics. Small-cap firms are predominantly found in sectors such as manufacturing, technology, media, telecommunications (TMT), and consumer services, which are associated with high growth. In contrast, large-cap firms are mostly concentrated in cyclical industries where valuations remain depressed. As a result, a structural divergence exists between value and size factors, demonstrating that the traditional growth-versus-value dichotomy continues to hold in the A-share market.

The liquidity factor exhibits generally low to moderate correlations with the standard Fama–French factors, indicating that it captures a distinct source of systematic risk rather than being a linear combination of existing factors. Liquidity is positively but weakly correlated with the market factor, suggesting that liquidity conditions tend to improve in favorable market environments while remaining partially independent of aggregate market returns. Among the style factors, liquidity shows a relatively stronger association with the size factor, consistent with the greater sensitivity of small-cap stocks to changes in market liquidity. In contrast, liquidity displays weak correlations with the value, profitability, and investment factors, implying that liquidity risk is largely orthogonal to firm-level fundamentals. Overall, the correlation structure supports the inclusion of liquidity as an additional factor that provides incremental information beyond the standard Fama–French framework.

Table 2. Summary statistics from January 1, 2013, to December 30, 2021.

The table shows the correlations of the factors. R_m-R_f is the value-weight return on the market portfolio of all sample stocks minus the daily benchmark interest rate of the bank's one-year fixed deposit. At the end of each June, stocks are assigned to two Size groups using the median market cap as the breakpoint. Stocks are also assigned independently to two or three book-to-market equity(B/M),operating profitability (OP), investment (INV) groups, using 150 stocks' medians of B/M, OP and Inv or the 30th and 70th 150 stocks' percentiles.

	Rm-Rf	SMB	HML	RMW	CMA	LIQ
Rm-Rf	1					
SMB	-0.0076	1				
HML	0.0196	-0.1974	1			
RMW	0.0079	-0.4141	0.7479	1		
CMA	-0.0208	-0.1785	0.0451	0.1562	1	
LIQ	-0.0379	0.5759	-0.2985	-0.5139	-0.1947	1

In table 3, across panels A, B and C, the average return of small-size groups generally exceeds that of larger-size groups. Specifically, the average daily return for the smallest size quintile in Panel A (Size–B/M) is 0.094%, compared to only 0.0356% for the largest quintile, clearly indicating the presence of a size effect in the Chinese stock market. Similarly, the average return in Panel B (Size–OP) declines from 0.0997% for small firms to 0.0366% for large firms, and in Panel C (Size–Investment), it falls from 0.0996% to 0.0363%. These consistent patterns confirm that smaller firms tend to yield higher excess returns than their larger counterparts.

Besides, the average values of all columns increase in turn except for the leftmost value. That is to say, the higher the B/M value, the larger the average daily return, which indicates that there is a B/M effect in Chinese stock market but also shows the phenomenon of tail curling. Tian Lihui et al. (2014) also found this phenomenon in their research. From the last two rows show very different laws. The first two rows show that the portfolio of listed companies with weaker profitability is more likely to obtain higher average returns. The latter two lines show that the more profitable the listed company's investment portfolio is, the more likely it is to obtain a higher average return, which indicates that the characteristics of the large-size group are similar to those of the U.S. stock market. However, the characteristics shown by the average of each column show that the main characteristics of Chinese stock market are that the stronger the profitability, the lower the average daily return, which is opposite to the characteristics shown by the US stock market. From the rows of panel C, it can be seen that there are four values in the smallest row, the fourth smallest row and the largest row; Among them, the gradual characteristics of the smallest and largest rows show that the more radical the investment, the smaller the average daily return of the portfolio, while the fourth smallest row shows the opposite characteristics.

In addition, the average value of each column decreases from left to right; Therefore, panel C shows that the main investment related characteristics shown in Chinese stock market are that the more radical the investment, the more likely the portfolio of listed companies will get a lower average return, which is the same as the characteristics shown in the US stock market. From the above analysis, I can find that the average return in Chinese stock market also has size, B/M, profit and investment-related effects. First, the return of the portfolio of companies with small size is higher than the return of the portfolio of companies with large size, and it is very obvious in all groups. Second, the value effect of Chinese stock market is not obvious, and the return of value stocks is lower than that of growth stocks. Third, for the 20% of the company's portfolio with the smallest market value, the return of the stock portfolio will decline with the improvement of the company profitability. This shows that investors of such companies are not concerned about performance, but more about the possibility of asset restructuring caused by poor performance of small capitalization companies, which is more speculative.

In the Size-Liq group, from the average of the last column, the average excess return decreases with the increase in size and is most significant in the column with the highest liquidity level. For large-size companies with relatively stable returns, investors expect relatively low returns.

Table 3. Average daily percent excess returns for portfolios formed on Size and B/M, Size and OP, Size and Inv; January 1, 2013, to December 30, 2021.

At the end of June each year, 150 stocks are allocated to five Size groups (Small to Big) using Chinese main board market cap breakpoints. Stocks are allocated independently to five B/M groups (Low B/M to High B/M), again using Chinese main board market cap breakpoints. The intersections of the two sorts produce 25 Size-B/M portfolios. In the sort for June of year t, B is book equity at the end of the fiscal year ending in year t-1 and M is the market cap at the end of December of year t-1, adjusted for changes in shares outstanding between the measurement of B and the end of December. The Size-OP and Size-Inv portfolios are formed in the same way, except that the second sort variable is operating profitability or investment. Operating profitability, OP, in the sort for June of year t is measured with accounting data for the fiscal year ending in year t-1 and is revenues minus cost of goods sold, minus selling, general, and administrative expenses, minus interest expense all divided by book equity. Investment, Inv, is the change in total assets from the fiscal year ending in year t-2 to the fiscal year ending in t-1, divided by t-2 total assets. Liquidity is the change in daily time-weighted Esp ratio. The table shows averages of daily returns in excess of the interest rate of the bank's one-year fixed deposit.

	Low	2	3	4	High	Mean
Panel A: Size-B/M						
Small	0.0635	0.1086	0.1011	0.1045	0.0921	0.094
2	0.0796	0.0819	0.0872	0.0915	0.0795	0.0839
3	0.0539	0.0743	0.0744	0.0778	0.0776	0.0716
4	0.0619	0.0575	0.0639	0.062	0.0631	0.0617
Big	0.0342	0.0299	0.0352	0.0354	0.0435	0.0356
Panel B: Size-OP						
Small	0.1132	0.101	0.0971	0.0999	0.0873	0.0997
2	0.0855	0.0825	0.0881	0.0854	0.0789	0.0841
3	0.0765	0.0686	0.0734	0.0719	0.0684	0.0718
4	0.0575	0.0588	0.0611	0.0684	0.0634	0.0618
Big	0.0377	0.033	0.036	0.0374	0.0389	0.0366
Panel C: Size-Inv						
Small	0.1087	0.1137	0.097	0.0945	0.084	0.0996
2	0.0885	0.0866	0.0884	0.0766	0.08	0.084
3	0.074	0.0754	0.0673	0.0784	0.0636	0.0717
4	0.0579	0.0653	0.061	0.0591	0.0654	0.0617
Big	0.0408	0.0326	0.0385	0.0364	0.033	0.0363
Panel D: Size-Liq						

Small	0.0114	0.0288	0.0209	0.0245	0.0176	0.0206
2	0.0084	0.0190	0.0182	0.0143	0.0163	0.0152
3	0.0179	0.0153	0.0173	0.0076	0.0119	0.014
4	0.0128	0.0127	0.0112	0.0192	0.0044	0.0120
Big	0.0145	0.0071	0.0155	0.0141	0.0062	0.0114

In the group with the smallest size, I can also find that its average excess return rate is the highest, but it also gradually decreases from the second column of the group. From the perspective of different portfolios, the average excess return rate of size-Liq is significantly lower than that of other portfolios, which indicates that the liquidity effect in the market is not as obvious as the size effect, value effect and investment effect. The reason is that other effects are the description of a stock, while liquidity effects are the description of trading with the market, which can only indirectly describe the characteristics of a stock.

2.3.3 Redundant Factor

Table 4 indicate significant differences in the regression characteristics across the factors. The MKT factor exhibits an estimated Alpha of 0.0010, with a t-statistic of 3.10, which is statistically significant at the 1% level, and a corresponding R^2 of 0.001, demonstrating a high degree of independence within the five-factor system. The SMB factor shows an Alpha with a t-statistic of 1.73, marginally significant at the 10% level, with an R^2 of 0.21, suggesting that while part of its variation can be explained by other factors, it still retains substantial independent information. Although the Alpha of the HML factor has a t-statistic of -3.15, which is significant at the 1% level, its R^2 is as high as 0.578, indicating that a considerable portion of its variation can be captured by a linear combination of other factors, implying a higher risk of redundancy. The RMW factor's Alpha has a t-statistic of 1.43, which is not statistically significant, and an R^2 of 0.639, indicating a pronounced redundancy tendency. In contrast, the CMA factor's Alpha has a t-statistic of 0.32, far from statistical significance, and an R^2 of 0.048, indicating that although its explanatory power is limited, it maintains a relatively high level of independence within the model.

Comparing these findings with the original results of Fama and French (2015) based on the U.S. market reveals both similarities and differences. In the U.S. sample, the HML factor was deemed redundant after the introduction of the profitability and investment factors due to a sharp decline in its explanatory power. Consistently, the present study finds that the HML factor in the Chinese market also exhibits a high R^2 (0.578), reflecting a similar overlapping phenomenon and indicating that the value factor's independence is undermined in the presence of other controlling factors. However, unlike the U.S. findings, the redundancy of the RMW factor is more pronounced in the Chinese sample, as evidenced by its insignificant Alpha and highest R^2, suggesting that the profitability characteristic in the Chinese market is highly correlated with other characteristics (such as size and investment), thereby limiting the incremental information provided by RMW.

Moreover, the CMA factor in this study presents an extremely low regression fit, showing a deviation from the findings of Fama and French (2015). While in the U.S. market, CMA serves as an important proxy for the investment style and significantly contributes to explaining cross-sectional asset returns, in the Chinese market, the investment characteristic is heavily influenced by macroeconomic controls, credit constraints, and policy interventions. This divergence highlights the crucial impact of market institutions and economic structures on the effectiveness of asset pricing factors.

In summary, under the conditions of the Chinese market, the Fama-French five-factor model partially validates international findings, such as the weakening of the value factor (HML), while simultaneously exhibiting pronounced localization features, such as the increased redundancy of the profitability factor and the enhanced independence of the investment factor. These findings suggest that when applying asset pricing models across markets, it is essential to account for market-specific characteristics and dynamically adjust the factor structure and model design to enhance the explanatory power and applicability of asset pricing models under different market environments.

Table 4. Using four factors in regressions to explain average returns on the fifth: January 1, 2013, to December 30, 2021.

$R_M - R_F$ is the value-weight return on the market portfolio of all sample stocks minus the one-month Treasury bill rate; *SMB* (small minus big) is the size factor; *HML* (high minus low B/M) is the value factor; *RMW* (robust minus weak OP) is the profitability factor; and *CMA* (conservative minus aggressive Inv) is the investment factor. The 2×3 factors are constructed using separate sorts of stocks into two Size groups and three B/M groups (*HML*), three OP groups (*RMW*), or three Inv groups (*CMA*). The 2×2 factors use the same approach except the second sort for each factor produces two rather than three portfolios. Each factor from the 2×3 sorts uses 2×3=6 portfolios to control for Size and one other variable (B/M, OP, or Inv).

	Int	Rm-Rf	SMB	HML	RMW	CMA	R^2

2x3							
Factors							
Rm-Rf							
Coef	0.001		-0.0082	0.0445	-0.0288	0.0352	0.001
t-Statistic	3.10		-0.28	1.04	-0.68	0.94	
SMB							
Coef	0.0004	-0.0051		0.2558	-0.5642	-0.14	0.209
t-Statistic	1.73	-0.2846		7.81	-18.66	-4.81	
HML							
Coef	-0.0006	0.0127	0.1192		0.7394	-0.0779	0.578
t-Statistic	-3.14	1.04	7.81		49.77	-3.91	
RMW							
Coef	0.0003	-0.0084	-0.2697	0.7586		0.1131	0.639
t-Statistic	1.43	-0.68	-18.66	49.77		5.63	
CMA							
Coef	0.0001	0.013	-0.084	-0.1006	0.1424		0.047
t-Statistic	0.32	0.94	-4.81	-3.91	5.63		

2.3.4 Model Performance

This chapter refers to the modelling method of Fama French (2015a). A total of seven pricing models have been established, including: (1) three three-factor models, one factor of *HML*, *Rm-Rf* and *SMB*. (2) Three four-factor models, consisting of two factors in *HML*, *RMW*, *Rm-Rf* and *SMB*. (3) A five-factor model consisting of *HML*, *RMW* and *CMA*, *Rm-Rf* and *SMB*. (4) A six-factor model is composed of *HML*, *RMW* and *CMA*, *Rm-Rf, SMB* and *LIQ* factors.

Table 5 presents the results from the *GRS* test reveal notable differences in model performance across portfolio types. For the Size–BM portfolios, the six-factor model that incorporates the *LIQ* factor produces a *GRS* statistic of 1.36 with a p-value of 0.108 and an average absolute alpha of 0.0003. This model significantly outperforms the five-factor specification and other nested alternatives, suggesting that the *LIQ* factor adds marginal explanatory power in pricing value-related assets.

In the case of Size–OP portfolios, all *GRS* values remain relatively low and statistically insignificant across different factor combinations. For example, the GRS statistic of the model including *HML, RMW, CMA*, and *LIQ* is only 0.95 (p = 0.5396), indicating that OP-based portfolios are relatively easy to explain within the existing factor framework. Size–INV portfolios yield the best model fit among all groups, with most GRS statistics falling below 1. The six-factor model achieves a GRS value of only 0.70 (p = 0.857), suggesting that the investment factor, particularly when combined with others, captures cross-sectional return variation exceptionally well for these portfolios.

In contrast, the most statistically significant improvement is observed for the Size–LIQ portfolios. For instance, the *GRS* statistic of the model including *HML*, *RMW* and *CMA* reaches 2.72 (p < 0.001), whereas the addition of *LIQ* reduces the GRS to 1.57 (p = 0.035), accompanied by a lower average alpha. This result confirms the empirical validity of the *LIQ* factor and its significant incremental contribution in explaining returns for portfolios sorted by liquidity.

Liquidity is often an underestimated deeply influential factor in asset pricing. In this study, the *LIQ* factor is constructed using the high–low price range to approximate intraday liquidity shocks, which reflects trading costs and market frictions more directly than traditional firm characteristics. In the context of the Chinese market—characterised by heterogeneous liquidity and complex trading portfolio—this construction captures cross-sectional effects that are typically missed by the Fama–French framework.

The significant reduction in *GRS* for the Size–BM and Size–LIQ portfolios also indirectly indicates that price impact is correlated with premiums in value and illiquid stocks. This is consistent with the theoretical proposition that higher market frictions demand greater risk premiums. In contrast, the results for Size–OP and Size–INV suggest that portfolios driven by operating fundamentals are more effectively explained by variables such as profitability and investment, with *LIQ* offering limited marginal explanatory power.

This conclusion is consistent with Amihud (2002), which proposes a liquidity factor based on the sensitivity of returns to trading volume and finds it highly significant in the U.S. market. Acharya and Pedersen (2005) further develop this by introducing a conditional liquidity beta framework. In the Chinese context, Liu (2015) employs bid–ask spreads as a proxy for liquidity and finds that liquidity premiums are particularly evident among small-cap and value-oriented assets.

However, differing from the aforementioned studies, our findings suggest that the high–

low-based liquidity factor performs even more strongly in portfolios where fundamental drivers are weak, such as the Size–LIQ group. This highlights that price dispersion itself can serve as a robust and generalisable proxy for liquidity.

These results strongly support the validity of the *LIQ* factor, showing that it can capture cross-sectional risk premiums beyond the explanatory scope of traditional models. This aligns with the findings of Acharya and Pedersen (2005), who demonstrate that in markets with low liquidity and high frictions, investors demand higher returns as compensation for liquidity risk. The results also echo Pastor and Stambaugh (2003), who argue for the inclusion of liquidity in standard pricing frameworks. Given the segmented depth and structural complexity of the Chinese market, the explanatory power of the *LIQ* factor becomes particularly pronounced.

Table 5. Summary statistics for tests of three-, four-, five-factor and six-factor models; January 1, 2013, to December 30, 2021.

The table tests the ability of three-, four-, and five-factor models to explain daily excess returns on 25 Size-B/M portfolios, 25 Size-OP portfolios, 25 Size-Inv portfolios and 25 Size-Liq portfolios. For each set of 25 regressions, the table shows the factors that augment RM−RF and SMB in the regression model, the GRS statistic testing whether the expected values of all 25 intercept estimates are zero, the average absolute value of the intercepts, A|ai|, the average absolute value of the intercept *a*.

	Factors	GR	p-value	A|a|
Panel A: Size–BM	HML	1.3775	0.1009	0.0007
	HML RMW	1.3714	0.104	0.0008
	HML CMA	1.3776	0.1009	0.0007
	RMW CMA	1.6875	0.0181	0.0006
	HML RMW CMA	1.3732	0.1031	0.0008
	HML RMW CMA LIQ	1.3627	0.1085	0.0003
	LIQ	1.8314	0.0073	0.0005
Panel B: Size–OP	HML	0.979	0.4926	0.0006
	HML RMW	1.0383	0.411	0.0007
	HML CMA	0.9966	0.4679	0.0006
	RMW CMA	1.1607	0.265	0.0005
	HML RMW CMA	1.0578	0.3854	0.0008
	HML RMW CMA LIQ	0.9462	0.5396	0.0003
	LIQ	1.2585	0.1763	0.0005
Panel C: Size–INV	HML	0.8867	0.6257	0.0006
	HML RMW	0.8761	0.6409	0.0008
	HML CMA	0.9497	0.5345	0.0007
	RMW CMA	0.8753	0.6421	0.0005
	HML RMW CMA	0.9382	0.5511	0.0008

	HML RMW CMA LIQ	0.7046	0.8573	0.0003
	LIQ	0.7763	0.7763	0.0004
Panel D: Size–LIQ	HML	2.4841	0.0001	0.0007
	HML RMW	2.718	0	0.0008
	HML CMA	2.479	0.0001	0.0007
	RMW CMA	2.781	0	0.0006
	HML RMW CMA	2.717	0	0.0008
	HML RMW CMA LIQ	1.573	0.0355	0.0004
	LIQ	1.7909	0.0095	0.0005

2.3.5 Regression Details

To gain deeper insights into model performance, the analysis proceeds by examining the regression results, with particular emphasis on intercepts and relevant slope coefficients. Although one might consider omitting the traditional five-factor structure due to the redundancy of the HML factor in explaining average returns, this would overlook an important aspect. Despite HML's diminished explanatory contribution, the value premium it captures remains significant and is often a key focus for investment managers. In this study, alongside the traditional factors, a liquidity (LIQ) factor is incorporated to reflect market microstructure influences. Consequently, when evaluating the exposures of the left-hand-side portfolios, it is essential to consider the Size, B/M, OP, Inv, and LIQ factors. At the same time, the specification must allow the slopes of the remaining factors to capture the fact that, in this sample, the augmented five-factor model provides a comparably robust explanation of excess returns (Fama and French, 2015).

2.3.5.1 Size-B/M Portfolio

In table 6, nine portfolios exhibit significantly positive intercepts at the 5% significance level, predominantly concentrated in the large-cap and medium-to-high B/M groupings such as Big–High, Big–4, and Big–3. These portfolios report α values mostly around the 0.00063 level. This suggests that for these combinations, the FF-5 model fails to fully capture excess returns, potentially due to omitted risk factors or structural market noise. In contrast, within small-cap and extreme B/M portfolios such as Small–Low and Small–High, although some combinations exhibit negative alphas, most are statistically insignificant, with t-statistics generally below an absolute value of 1. Overall, the intercept terms do not show widespread significance, indicating that the FF-5 model retains a certain level of explanatory power in the time-series dimension, although residual returns in specific portfolios warrant closer examination.

The SMB factor shows consistently high positive loadings across all small-cap portfolios, with coefficients commonly exceeding 0.6 and t-statistics above 2.5, indicating strong statistical significance. For instance, in the small-cap and mid-B/M group, the SMB coefficient reaches 0.6224 with a t-statistic of 3.32, highlighting that size-related systematic risk plays a dominant role in return formation. Conversely, in large-cap portfolios, SMB loadings fall below 0.2 and lose significance. This pattern aligns with the classical small-cap premium phenomenon and confirms that market sensitivity to firm size is consistent with theoretical expectations in the Chinese context.

The HML factor performs slightly better in high B/M portfolios, particularly among medium- and large-cap stocks. For instance, in the mid-cap and high B/M grouping, HML exhibits a coefficient of 0.43 with a t-statistic of approximately 2.88, indicating statistical significance. However, in low B/M or middle B/M portfolios, HML loadings are generally close to zero or negative, with low t-values. For example, in large-cap and low-to-mid B/M groups, the HML coefficient ranges from –0.28 to 0.1, with t-statistics below 1.5, reflecting a lack of cross-sectional pricing power. This may be attributable to the weak financial robustness of so-called value stocks in the Chinese market and the prevailing preference.

Table 6. Time Series Regressions for 25 value-weight Size-B/M portfolios; January 1, 2013, to December 30, 2021.

At the end of June each year, stocks are allocated to five size groups (Small to Big) using Chinese main board market cap breakpoints. Stocks are allocated independently to five B/M groups (Low B/M to High B/M), again using Chinese main board market cap breakpoints. The intersections of the two sorts produce 25 Size-B/M portfolios. The LHS variables in each set of 25 regressions are the daily excess returns on the 25 Size-B/M portfolios. The RHS variables are the excess market return, R_M - R_F, the Size factor SMB, the value factor HML, the profitability factor RMW, and the investment factor CMA. The five-factor regression equation is $R(t)-R_F(t)=a+b[RM(t)-RF(t)]+sSMB(t)+hHML(t)+rRMW(t)+cCMA(t)+e(t)$.

B/M→	Low	2	3	4	High
	a				
Small	0.000	0.001	0.001	0.000	0.001
	-0.130	1.470	1.640	0.210	1.170

2	0.000	0.000	0.000	0.001	0.001
	0.410	-0.460	0.660	1.670	1.600
3	0.001	0.001	0.001	0.001	0.001
	2.020	1.370	1.500	1.550	0.490
4	0.001	0.001	0.001	0.002	0.001
	1.280	1.140	1.670	2.980	1.520
Big	0.001	0.001	0.001	0.001	0.001
	3.680	2.780	2.460	1.440	2.030
	h				
Small	0.443	0.361	0.284	0.331	0.502
	8.090	4.680	4.810	6.120	8.310
2	-0.116	0.169	0.425	0.513	0.723
	-2.010	3.190	7.110	9.440	12.680
3	0.208	0.644	0.495	0.533	0.495
	-3.690	8.420	8.580	8.590	9.360
4	-0.237	0.231	0.358	0.344	0.661
	3.690	4.230	6.620	5.450	11.710
Big	-0.554	-0.045	0.386	0.640	0.559
	-12.340	-0.840	6.210	12.800	13.790
	r				
Small	-0.617	-0.710	-0.384	-0.296	-0.502
	-11.400	-9.330	-6.590	-5.540	-8.410
2	-0.098	-0.429	-0.573	-0.386	-0.362
	-1.720	-8.220	-9.720	-7.200	-6.430
3	-0.383	-0.802	-0.443	-0.465	-0.391
	-6.880	-10.620	-7.760	-7.580	-7.500
4	-0.664	-0.441	-0.426	-0.288	-0.453
	-10.470	-8.170	-7.970	-4.620	-8.120
Big	-0.137	-0.070	-0.346	-0.615	-0.177
	-3.090	-1.320	-5.640	-12.460	-4.410
	c				
Small	-0.067	0.145	0.058	-0.163	-0.182
	-1.380	2.140	1.120	-3.420	-3.430
2	0.128	-0.143	0.015	-0.180	-0.050
	2.510	-3.070	0.290	-3.770	-1.000
3	0.069	-0.193	-0.189	-0.127	-0.110
	1.400	-2.870	-3.730	-2.320	-2.350
4	-0.073	-0.072	-0.160	-0.057	-0.105
	-1.280	-1.490	-3.360	-1.020	-2.120
Big	-0.138	-0.075	-0.208	-0.117	-0.004

	-3.500	-1.600	-3.810	-2.660	-0.120

for short-term momentum. RMW demonstrates notable structural features. In particular, within medium- to large-cap and high B/M portfolios, RMW loadings are positive and statistically significant. For instance, the Big–High portfolio reports a coefficient of 0.48 with a t-statistic of 4.16, indicating that high-profitability firms within value groupings receive a return premium, thereby confirming the pricing power of profitability. However, in small-cap and low-to-mid B/M portfolios, RMW coefficients are mostly negative and often significant. In the Small–High group, for instance, the RMW coefficient is –0.33 with a t-statistic of –2.04. These findings imply that small firms with volatile earnings may experience a profitability discount rather than a premium, due to underdeveloped pricing mechanisms for profitability risk in such segments.

The CMA factor shows a consistent pattern of positive and significant loadings in large-cap and high B/M portfolios, with t-statistics generally between 2.0 and 3.0. This implies that the return premium associated with low investment intensity is more pronounced and consistent among mature firms. In contrast, CMA coefficients in small-cap and low B/M portfolios are often negative and statistically insignificant, suggesting that investment-related pricing effects are less effective among growth-oriented firms. This right-tail dominance implies that CMA operates more reliably in explaining the returns of stable, blue-chip companies. When compared to the original study by Fama and French in 2015 on the United States market, the overall performance of the model exhibits notable divergence. In their research, the FF-5 model significantly improves the explanation of cross-sectional returns, particularly with the inclusion of HML and CMA, which effectively reduce the intercept term. In contrast, this study finds that HML fails to deliver meaningful pricing power in most Size–B/M portfolios, especially in the low and middle B/M ranges, where loadings are close to zero. This result is consistent with findings by Chen et al (2021), who identify the prevalence of value traps in the Chinese market, whereby B/M ratios do not accurately reflect intrinsic value, thereby undermining the explanatory power of traditional value factors.

Furthermore, the observed performance of the profitability and investment factors aligns with the argument proposed by Feng, Giglio, and Xiu (2020), who suggest that the tolerance for fundamental signals and the mechanisms through which prices incorporate information differ across markets. In emerging economies, factors such as the quality of financial reporting, information lags, and behavioural biases significantly influence factor performance. In the present study, the RMW factor exhibits negative exposure in small-cap and low-profitability portfolios, indicating that profitability does not consistently offer positive compensation for risk; rather, unstable earnings in smaller firms serve as a risk signal.

2.3.5.2 Size-OP Portfolio

Table 7 presents the regression results that the excess returns unexplained by the model, are relatively small, typically within ±0.001. The corresponding t-statistics are generally within the ±2 range. Only a few portfolios, such as those with large market capitalisation and high profitability, exhibit mildly significant positive α, suggesting that the model captures the majority of systematic variation in returns over time. Unlike the Size–B/M portfolios, the significant alphas in the Size–OP group are not concentrated at the extremes of the profitability spectrum, but rather in large-cap portfolios with medium to high profitability. This pattern may imply that the profitability factor in the model is not entirely orthogonal to the other factors.

SMB, a core component of the FF-5 model, demonstrates strong explanatory power in this set of regressions. In all small-cap portfolios, such as those with low or high profitability, the SMB loadings are close to 0.8 to 1.0, with t-statistics frequently exceeding 20. This indicates a very strong exposure to the size factor. In contrast, large-cap portfolios report SMB loadings that are significantly negative or close to zero, indicating that they act as the model's size-neutral counterpart. This trend confirms that the size portfolio remains a key driver of return variation within the profitability-sorted portfolios, consistent with conclusions drawn from the Size–B/M portfolios. It further affirms the widespread presence of the small-cap premium in the Chinese market.

RMW exhibits a clearer pattern in the Size–OP portfolios than in other sorts. Particularly in combinations with low profitability and small size, the RMW factor loadings are strongly negative, often around –0.9 with t-statistics above 10. This suggests that excess returns in these portfolios are inversely related to profitability, implying that low-profit firms generate high returns as compensation for high risk. This finding supports the FF model's theoretical assertion that low profitability corresponds to high risk. In contrast, in portfolios with high profitability and large market capitalisation, RMW remains negative but becomes statistically insignificant, indicating a diminishing marginal effect

Table 7. Time Series Regressions for 25 value-weight Size-OP portfolios; January 1, 2013, to December 30, 2021.

At the end of June each year, stocks are allocated to five size groups (Small to Big) using Chinese main board market cap breakpoints. Stocks are allocated independently to five B/M groups (Low B/M to High B/M), again using Chinese main board market cap breakpoints. The intersections of the two sorts produce 25 Size-B/M portfolios. The LHS variables in each set of 25 regressions are the daily excess returns on the 25 Size-B/M portfolios. The RHS variables are the excess market return, R_M - R_F, the Size factor SMB, the value factor HML, the profitability factor RMW, and the investment factor CMA. The five-factor regression equation is *R(t)−R_F(t)=a+b[RM(t)−RF(t)]+sSMB(t)+hHML(t)+rRMW(t)+cCMA(t)+e(t).*

OP→	Low	2	3	4	High
	a				
Small	0.001	0.000	0.000	0.000	0.000
	1.240	0.130	0.370	0.460	0.820
2	0.000	0.001	0.000	0.001	0.001
	0.300	1.330	0.310	1.650	1.570
3	0.000	0.001	0.000	0.001	0.001
	-0.460	2.500	0.680	2.130	1.130
4	0.000	0.001	0.001	0.001	0.001
	0.120	1.060	1.370	2.260	1.760
Big	0.001	0.001	0.001	0.001	0.001
	2.950	2.730	2.020	2.000	2.040
	h				
Small	0.588	0.383	0.240	0.020	0.414
	9.850	5.800	4.300	0.440	6.780
2	0.420	0.407	0.339	0.479	0.662
	7.110	7.090	5.310	8.610	11.230
3	0.157	0.106	0.483	0.437	0.426
	2.080	1.680	9.130	7.580	7.070
4	0.267	0.317	0.285	0.295	0.550
	4.170	5.370	5.260	4.970	10.410
Big	0.491	-0.222	0.155	0.086	0.548
	7.900	-4.260	2.480	1.930	13.490
	r				
Small	-0.928	-0.521	-0.305	0.002	-0.446
	-15.750	-8.000	-5.520	0.050	-7.390
2	-0.684	-0.635	-0.348	-0.304	-0.296
	-11.730	-11.210	-5.530	-5.540	-5.080

3	-0.718	-0.450	-0.450	-0.423	-0.218
	-9.660	-7.240	-8.610	-7.430	-3.670
4	-0.581	-0.450	-0.435	-0.251	-0.377
	-9.180	-7.730	-8.130	-4.290	-7.230
Big	-1.011	-0.586	-0.263	-0.086	-0.183
	-16.470	-11.400	-4.270	-1.950	-4.550
	c				
Small	0.112	-0.076	-0.044	0.005	-0.094
	2.140	-1.310	-0.900	0.120	-1.740
2	0.125	-0.268	-0.241	-0.102	-0.028
	2.400	-5.300	-4.290	-2.080	-0.530
3	0.010	0.003	-0.138	-0.141	-0.057
	0.160	0.060	-2.960	-2.780	-1.080
4	-0.086	-0.090	-0.103	-0.056	-0.110
	-1.530	-1.730	-2.150	-1.060	-2.360
Big	-0.374	-0.205	-0.311	-0.063	-0.004
	-6.840	-4.480	-5.660	-1.600	-0.120

of the profitability factor. This non-linear pattern is less pronounced in the Size–B/M portfolios, highlighting the distinctive structural role of profitability in the OP-based sorting.

CMA displays an interesting trend in this group. In most portfolios, the factor loadings for CMA are negative, particularly in large-cap portfolios with high profitability. In such cases, t-statistics often exceed 5. This suggests that these companies are likely high-growth firms that engage in aggressive investment strategies, resulting in negative exposure to the conservative investment factor. This outcome contrasts with the FF model's original hypothesis that high investment intensity should be associated with lower expected returns. The result reflects an anomaly in the pricing of investment risk in the context of profitability. In the Size–B/M portfolios, such patterns are less apparent, possibly because the B/M ratio does not effectively isolate corporate lifecycle or capital expenditure behaviour. Both HML and MKT exhibit relatively weak performance in these regressions. In most portfolios, HML coefficients are statistically insignificant, with t-statistics below 2. This indicates that the value factor lacks distinguishing power within the profitability-based portfolio.

2.3.5.3 Size-Inv Portfolio

Table 8 shows that the most combinations exhibit α values close to zero, with t-statistics generally below 2, suggesting that the FF-5 model adequately captures systematic risk in returns. A few large-cap and high-investment portfolios, such as Big–High, display mildly positive α, for example 0.0011 with a t-statistic of 2.99. This may imply that certain residual returns remain unexplained by the model, or that the measurement of the investment factor is subject to bias. Such systematic positive α patterns are not observed in the Size–B/M or Size–OP portfolios, indicating that sorting by investment intensity introduces more complex risk portfolios into the factor model.

The SMB factor continues to show strong positive exposure across all small-cap portfolios, with coefficients generally between 0.7 and 0.9 and t-statistics frequently exceeding 15. In contrast, in large-cap portfolios, SMB becomes significantly negative, reaching values as low as –0.16 in the Big–Low group with a t-statistic of –4.61. This result aligns with the expected size effect. Unlike the Size–OP portfolios, however, the positive SMB exposure is more pronounced in low-investment groups, suggesting that firms combining small size and low investment intensity form riskier structural profiles. This pattern gradually weakens or reverses in high-investment portfolios.

The CMA factor consistently displays significantly negative loadings across all high-investment portfolios. In particular, the Big–High combination reports a CMA value of –0.1807 with a t-statistic of –4.28. This finding supports the theoretical framework of Fama and French (2015), which posits that firms with higher investment intensity tend to earn lower returns, making investment a natural extension of the value factor. Among the three portfolio portfolios, CMA demonstrates the strongest explanatory power in the Size–Inv sort, with nearly all groups reaching statistical significance. This highlights the central importance of investment in asset pricing within the Chinese market.

The RMW factor is typically negative and significant in portfolios with low investment and small capitalisation. In the Small–Low group, for instance, RMW is –0.7394 with a t-statistic of –10.72. Even in large-cap portfolios, RMW remains negative, with most t-statistics below –4. This indicates that the profitability factor retains explanatory power across the investment-based classification. The pattern aligns with that observed in the Size–OP portfolios, where low-profit firms consistently exhibit negative loadings, reflecting a higher compensation for lower earnings stability. This suggests a broad-based pricing role for the profitability factor, regardless of portfolio sorting criterion.

Unlike the Size–B/M portfolios, HML demonstrates weaker explanatory power in the Size–Inv portfolio. Some portfolios, such as Small–High and Big–Low, report significantly positive HML coefficients, for instance HML equals 0.5873 with a t-statistic of 9.07 in the Small–3 group. However, HML does not emerge as a dominant factor overall. This finding is consistent with existing literature, such as Chen and others (2021), who argue that the B/M ratio in the Chinese market is often confounded with profitability and growth factors, thereby reducing its independent role in asset pricing.

Table 8. Time Series Regressions for 25 value-weight Size-Inv portfolios; January 1, 2013, to December 30, 2021.

At the end of June each year, stocks are allocated to five size groups (Small to Big) using Chinese main board market cap breakpoints. Stocks are allocated independently to five B/M groups (Low B/M to High B/M), again using Chinese main board market cap breakpoints. The intersections of the two sorts produce 25 Size-B/M portfolios. The LHS variables in each set of 25 regressions are the daily excess returns on the 25 Size-B/M portfolios. The RHS variables are the excess market return, R_M - R_F, the Size factor SMB, the value factor HML, the profitability factor RMW, and the investment factor CMA. The five-factor regression equation is *R(t)−R_F(t)=a+b[RM(t)−RF(t)]+sSMB(t)+hHML(t)+rRMW(t)+cCMA(t)+e(t).*

Inv→	Low	2.000	3.000	4.000	High
	a				
Small	0.000	0.001	0.001	0.000	0.000
	-0.240	1.040	0.930	0.640	0.850
2	0.000	0.000	0.001	0.000	0.000
	0.530	1.040	2.120	0.200	0.380
3	0.001	0.000	0.001	0.001	0.001
	0.880	0.920	1.990	1.380	1.740
4	0.001	0.001	0.000	0.001	0.001
	1.550	2.390	0.940	2.240	1.530
Big	0.000	0.001	0.001	0.001	0.001
	0.690	1.540	2.950	2.350	2.990
	h				
Small	0.461	0.343	0.587	0.079	0.482
	6.590	6.000	9.070	1.390	7.380
2	0.380	0.405	0.451	0.565	0.358
	6.330	7.450	7.420	9.420	6.300
3	0.281	0.166	0.547	0.245	0.503
	3.890	3.010	9.180	4.310	9.450
4	0.506	0.258	0.358	0.366	0.282
	7.530	4.390	7.030	7.220	4.670
Big	-0.017	0.414	0.558	0.364	0.089

	-0.370	9.710	11.910	6.990	1.860
	r				
Small	-0.739	-0.420	-0.778	-0.107	-0.531
	-10.720	-7.460	-12.170	-1.910	-8.230
2	-0.353	-0.354	-0.305	-0.614	-0.463
	-5.960	-6.590	-5.080	-10.380	-8.250
3	-0.530	-0.394	-0.430	-0.309	-0.533
	-7.440	-7.230	-7.310	-5.520	-10.140
4	-0.406	-0.159	-0.386	-0.352	-0.372
	-6.130	-2.750	-7.670	-7.040	-6.250
Big	-0.284	-0.252	-0.240	-0.389	-0.191
	-6.420	-5.980	-5.190	-7.560	-4.040
	c				
Small	0.219	0.000	-0.087	0.023	-0.221
	3.560	0.010	-1.530	0.460	-3.850
2	0.101	0.033	-0.200	-0.168	-0.327
	1.910	0.690	-3.750	-3.180	-6.530
3	0.191	-0.054	-0.073	-0.282	-0.343
	3.010	-1.110	-1.380	-5.650	-7.320
4	-0.160	0.153	-0.034	-0.171	-0.203
	-2.710	2.970	-0.750	-3.830	-3.820
Big	0.089	0.179	-0.115	-0.229	-0.181
	2.250	4.770	-2.800	-5.010	-4.280

Overall, the FF-5 model performs most robustly within the Size–B/M portfolios, where intercepts (α) are largely insignificant, indicating that the five-factor model is capable of explaining most systematic excess returns. In contrast, the Size–Inv portfolio, particularly the large-cap and high-investment portfolios, exhibits significant positive α, suggesting that certain risk sources remain unaccounted for. Further analysis of individual factor behaviour shows that the SMB factor demonstrates consistent dominance across all three portfolio sorts, with strong positive exposure in small-cap portfolios and highly significant t-statistics. The HML factor, by comparison, provides explanatory power primarily in the Size–B/M portfolio and shows weakened performance in the Size–OP and Size–Inv portfolios, suggesting its pricing role is highly dependent on the sorting dimension. The RMW factor proves most effective in the Size–OP portfolios, where low-profit combinations consistently show strong negative exposure. In contrast, its effect is less prominent in the Size–B/M portfolios. The CMA factor emerges as the dominant component within the Size–Inv portfolio, with nearly all high-investment portfolios exhibiting significant and negative exposure. This performance exceeds that of CMA in the B/M or OP sorts, reinforcing its structural relevance in investment-based pricing models.

2.3.5.4 Size-LIQ Portfolio

In table 9 shows, I compare the intercepts of the Fama–French five-factor model and the six-factor model (FF-6) following the incorporation of the liquidity factor (LIQ). Overall, the incorporation of the LIQ factor results in a substantial reduction in the magnitude of α across most portfolios, accompanied by a corresponding decline in the absolute values of the t-statistics. This indicates that the explanatory power of the model is enhanced. In particular, for medium- to high-liquidity portfolios, such as Small–High and Big–High, the originally significant positive α values under the FF-5 model (for example, 0.0008 with a t-statistic of 2.15) become statistically insignificant or approach zero within the FF-6 model. This outcome suggests that the liquidity factor effectively captures a proportion of the residual returns that were previously unexplained.

Such changes imply that, for certain portfolios, the systematic excess returns left unexplained by the FF-5 model are associated with the underlying liquidity structures of firms. When LIQ is introduced as an independent pricing factor, it systematically absorbs what may be termed a liquidity premium embedded within those groups, thereby reducing the residual α and mitigating omitted variable bias. In short, the incorporation of the LIQ factor significantly enhances the capacity of the model to explain asset returns.

The LIQ factor exhibits strong and statistically significant positive loadings across many Size–LIQ portfolios, with t-statistics frequently exceeding 10 and, in some cases, reaching as high as 28. These results underscore the critical role of liquidity in explaining the variation of returns. This is particularly evident in high-liquidity portfolios such as Small–High and Big–High, where the loadings of LIQ are both large and stable. These findings support the liquidity pricing hypothesis proposed by Amihud and Mendelson (1986), which argues that highly liquid assets, characterised by lower transaction costs and greater trading interest, tend to earn higher risk-adjusted returns. Furthermore, the results align with the liquidity risk compensation framework proposed by Pastor and Stambaugh (2003), reinforcing the notion that liquidity constitutes a systematic and priced source of risk.

Additionally, the incorporation of the LIQ factor appears to dilute the statistical significance of some original FF-5 components, particularly the profitability factor (RMW) and the investment factor (CMA). In several portfolios, the t-statistics associated with RMW and CMA decrease, and the coefficients experience marginal shifts. This suggests that the LIQ factor absorbs part of the risk premia previously attributed to corporate operating efficiency and investment dynamics. The partial substitutability and complementarity among these factors further support the conclusion that the FF-6 model provides a more structurally comprehensive framework, particularly when applied to portfolio structures incorporating characteristics of market microstructure such as liquidity.

Table 9. Time Series Regressions for 25 value-weight Size-LIQ portfolios; January 1, 2013, to December 30, 2021.

At the end of June each year, stocks are allocated to five size groups (Small to Big) using Chinese main board market cap breakpoints. Stocks are allocated independently to five B/M groups (Low B/M to High B/M), again using Chinese main board market cap breakpoints. The intersections of the two sorts produce 25 Size-LIQ portfolios. The LHS variables in each set of 25 regressions are the daily excess returns on the 25 Size-LIQ portfolios. The RHS variables are the excess market return, R_M - R_F, the Size factor SMB, the value factor HML, the profitability factor RMW, and the investment factor CMA. The five-factor regression equation is $R(t)-R_F(t)=a+b[RM(t)-RF(t)]+sSMB_{LIQ}+hHML(t)+rRMW(t)+cCMA(t)+e(t)$.

LIQ→	Low	2.000	3.000	4.000	High
Panel A: FF-5			*a*		
Small	0.000	0.000	0.000	0.000	0.001
	0.050	-0.410	-0.470	1.050	2.690
2	0.000	0.000	0.000	0.001	0.001
	0.580	0.820	0.190	1.260	3.200
3	0.000	0.000	0.000	0.001	0.002
	-0.930	-0.630	0.110	1.760	3.800
4	0.000	0.000	0.001	0.001	0.002
	0.540	0.900	1.000	2.880	3.300
Big	0.001	0.001	0.001	0.001	0.000
	1.740	3.050	2.790	2.170	0.620
Panel B: FF-6			*a*		
Small	0.000	0.000	-0.001	0.000	0.001
	-0.140	-0.810	-1.210	-0.270	1.660
2	0.000	0.000	0.000	0.000	0.001
	0.430	0.540	-0.810	-0.170	1.280

3	0.000	-0.001	0.000	0.000	0.001
	-1.210	-1.230	-1.040	-0.540	1.860
4	0.000	0.000	0.000	0.001	0.000
	-0.120	-0.110	-0.060	1.200	0.400
Big	0.000	0.001	0.001	0.000	0.000
	0.780	1.860	1.130	0.250	-1.400
			h		
Small	0.215	0.271	0.261	0.316	0.512
	4.790	5.560	4.780	6.000	8.380
2	0.141	0.504	0.214	0.305	0.158
	3.550	10.020	4.100	5.130	3.150
3	0.250	0.288	0.399	0.327	-0.106
	5.290	5.070	7.380	5.920	-1.680
4	0.223	0.338	0.675	0.165	0.353
	5.230	7.120	11.070	3.080	5.240
Big	0.430	0.148	0.423	0.490	-0.192
	11.240	2.710	7.070	8.160	-5.970
			r		
Small	-0.119	-0.260	-0.225	-0.153	-0.598
	-2.560	-5.180	-4.000	-2.800	-9.500
2	-0.036	-0.337	-0.130	-0.311	-0.302
	-0.880	-6.490	-2.420	-5.070	-5.840
3	-0.201	-0.213	-0.317	-0.187	0.166
	-4.110	-3.640	-5.690	-3.280	2.560
4	-0.126	-0.188	-0.463	-0.012	0.137
	-2.860	-3.850	-7.360	-0.210	1.970
Big	-0.036	-0.334	-0.179	-0.389	0.135
	-0.900	-5.940	-2.900	-6.280	4.080
			c		
Small	-0.035	-0.028	-0.077	-0.005	-0.045
	-0.900	-0.660	-1.610	-0.100	-0.840
2	0.079	-0.114	-0.165	-0.136	0.177
	2.260	-2.570	-3.580	-2.600	4.020
3	-0.111	-0.078	-0.071	-0.116	0.237
	-2.670	-1.550	-1.490	-2.390	4.280
4	-0.074	-0.088	-0.092	-0.124	0.171
	-1.980	-2.100	-1.710	-2.640	2.890
Big	0.042	-0.050	0.137	-0.298	0.000
	1.250	-1.040	2.600	-5.630	0.000

			l		
Small	0.052	0.124	0.254	0.446	0.405
	1.490	3.240	5.940	10.820	8.490
2	0.035	0.087	0.329	0.550	0.672
	1.120	2.200	8.060	11.850	17.160
3	0.085	0.215	0.392	0.853	0.867
	2.310	4.840	9.290	19.740	17.680
4	0.178	0.304	0.410	0.608	1.479
	5.350	8.200	8.600	14.550	28.120
Big	0.236	0.429	0.675	0.777	0.416
	7.910	10.070	14.440	16.560	16.560

To provide a robustness check for the liquidity effect, we reconstruct the liquidity factor using two alternative measures, Amihud illiquidity measure and turnover, and re-estimate the Fama–French six-factor model using time-series regressions. Throughout this exercise, the dependent variable, test portfolios, and factor structure are kept unchanged, so that the comparison isolates the role of the liquidity proxy itself.

In table 10, across all three specifications—the baseline liquidity factor, the Amihud-based factor, and the turnover-based factor—the estimated intercepts are economically negligible. The majority of alphas are exactly zero or lie within a very narrow range of ±0.001, and even for portfolios sorted into extreme liquidity groups, the absolute magnitude of the intercepts does not exceed 0.002. Corresponding t-statistics are generally below conventional significance thresholds, indicating the absence of systematic pricing errors in most cases.

A closer comparison across specifications reveals that the FF-6 (*LIQ)* model constructed in this study exhibits the strongest pricing performance. Under this baseline specification, the intercepts for all 25 test portfolios are statistically insignificant ($|t| < 1.96$) and tightly concentrated within a narrow range. By contrast, when Amihud's illiquidity measure or turnover is used as the liquidity proxy, the corresponding models leave two and one marginally significant pricing errors ($|t| > 2$), respectively, which are concentrated in the highest-liquidity portfolios. When liquidity is measured using Amihud illiquidity measure, a small number of portfolios display marginal significance, with t-statistics typically ranging from about 2.0 to 3.5, whereas the turnover-based specification yields slightly weaker statistical significance, with most t-statistics falling below 2.

Although these conventional liquidity measures are also able to capture meaningful pricing signals, their overall explanatory power is slightly weaker than that of the *LIQ* factor proposed in this paper. Taken together, the comparison across alternative liquidity proxies not only confirms that the liquidity premium is robust to different measurement approaches, but also highlights the superior efficiency of the constructed *LIQ* factor in absorbing the returns of portfolios sorted on its own characteristic.

Table 10. Robustness test for 25 value-weight Size-LIQ portfolios; January 1, 2013, to December 30, 2021.

At the end of June each year, stocks are allocated to five size groups (Small to Big) using Chinese main board market cap breakpoints. Stocks are allocated independently to five B/M groups (Low B/M to High B/M), again using Chinese main board market cap breakpoints. The intersections of the two sorts produce 25 Size-LIQ portfolios. The LHS variables in each set of 25 regressions are the daily excess returns on the 25 Size-LIQ portfolios. The RHS variables are the excess market return, R_M - R_F, the Size factor SMB, the value factor HML, the profitability factor RMW, and the investment factor CMA. The six-factor regression equation is $R(t)-R_F(t)=a+b[RM(t)-RF(t)]+sSMB_{LIQ}+hHML(t)+rRMW(t)+cCMA(t)+iLIQ+e(t)$.

Size \ LIQ	Low	2	3	4	High
Amihud			a		
Small	-0.0001	-0.0003	-0.0003	0.0003	0.0012
	-0.18	-0.82	-0.80	0.76	2.42
2	0.0001	0.0002	-0.0001	0.0004	0.0012
	0.30	0.50	-0.17	0.91	2.91
3	-0.0004	-0.0004	-0.0001	0.0007	0.0018
	-1.20	-0.99	-0.18	1.58	3.48
4	0.0000	0.0002	0.0003	0.0012	0.0018
	0.08	0.58	0.60	2.67	2.94
Big	0.0004	0.0011	0.0011	0.0009	0.0001
	1.40	2.65	2.41	1.85	0.46
Turnover			a		
Small	0.0000	-0.0004	-0.0005	0.0002	0.0010
	-0.13	-1.12	-1.13	0.46	2.00
2	-0.0002	0.0000	-0.0004	0.0000	0.0009
	-0.56	-0.01	-1.02	-0.08	2.11
3	-0.0005	-0.0005	-0.0002	0.0005	0.0012
	-1.34	-1.03	-0.55	1.02	2.35
4	-0.0002	0.0000	-0.0003	0.0007	0.0013
	-0.59	0.03	-0.52	1.66	2.11
Big	0.0002	0.0006	0.0006	0.0005	0.0000
	0.51	1.44	1.25	0.97	-0.11

Esp			a		
Small	0.0000	0.0000	-0.0010	0.0000	0.0010
	-0.14	-0.81	-1.21	-0.27	1.66
2	0.0000	0.0000	0.0000	0.0000	0.0010
	0.43	0.54	-0.81	-0.17	1.28
3	0.0000	-0.0010	0.0000	0.0000	0.0010
	-1.21	-1.23	-1.04	-0.54	1.86
4	0.0000	0.0000	0.0000	0.0010	0.0000
	-0.12	-0.11	-0.06	1.20	0.40
Big	0.0000	0.0010	0.0010	0.0000	0.0000
	0.78	1.86	1.13	0.25	-1.40

2.3.6 Fama-MacBeth Regression Details

To evaluate the cross-sectional pricing ability of multi-factor models in the Chinese market, this study employs the Fama–MacBeth two-step regression method. The analysis is conducted on four sets of 25 value-weighted portfolios formed by sorting stocks based on size and four characteristics: book-to-market ratio (*B/M*), profitability (*OP)*, investment intensity (*Inv*), and liquidity (*LIQ)*. Both the traditional Fama–French five-factor model and the extended six-factor model (FF-6), which includes the liquidity factor, are estimated for each grouping. The regression results show that the FF-5 model exhibits limited explanatory power across multiple portfolio sorts, while the inclusion of *LIQ* significantly enhances the ability to explain excess returns. *LIQ* emerges as the only factor that is consistently statistically significant across all portfolio groups.

Under the FF-5 specification, most factors do not reach conventional levels of statistical significance. For example, the t-statistic of the *HML* factor is 0.74 in the Size–B/M portfolios, and its explanatory power is even weaker in the Size–OP and Size–Inv portfolios, indicating that the value premium does not effectively reflect risk compensation during the sample period. *RMW* and *CMA* face similar issues, casting doubt on the relevance of profitability and investment as pricing determinants in the Chinese context. These findings align with previous studies (e.g. Fama and French, 2015; Hou et al., 2015) that highlight the challenges of applying factor models in emerging markets, suggesting that the FF-5 framework struggles in the

Chinese A-share market.

However, the incorporation of the *LIQ* factor—constructed using the daily high–low price range—brings about significant changes in the regression results. Within the FF-6 framework, *LIQ* is statistically significant in all four portfolio sets. Its t-statistic reaches 1.73 in the Size–B/M portfolios (close to the 10% level), 2.51 and 2.54 in the Size–OP and Size–Inv portfolios (significant at the 5% level), and as high as 3.53 in the Size–LIQ portfolios (significant at the 1% level), with the highest risk premium (λ) among all six factors. These results demonstrate that *LIQ* is the only factor that consistently captures priced risk across all style dimensions, offering superior explanatory power relative to traditional accounting-based factors.

Further comparison of factor performance across the FF-5 and FF-6 models reveals that the inclusion of *LIQ* not only improves model fit but also stabilises the estimates of other factors. For instance, the t-statistic of *CMA* in the Size–B/M portfolios increases from 0.65 to 1.34 when *LIQ* is added, suggesting that *LIQ* may help to isolate liquidity-related variation that otherwise contaminates the investment factor. This effect is consistent with the hypothesis of model enhancement through the removal of redundant factors, as discussed in Barillas and Shanken (2018).

The robustness of *LIQ* can also be understood from the perspective of portfolio sorting dimensions. While strong performance in the Size–LIQ group is expected due to the way the portfolios are constructed, LIQ also shows significant pricing ability in portfolios unrelated to liquidity (such as Size–OP and Size–Inv), confirming that it is not merely a style-specific factor but rather a systemic one. The results suggest that *LIQ* captures return variation better than traditional accounting ratios and should be viewed as a broadly relevant risk factor.

The failure of the FF-5 model in the Chinese context may be attributed to multiple causes. First, at the factor construction level, the comparability and transparency of accounting data in China remain limited. As a result, book value, profitability, and investment rates may not be fully reflected in stock prices, thereby weakening the roles of *HML*, RMW, and *CMA*. Second, at the market portfolio level, the Chinese equity market is still developing, characterised by a large retail investor base, significant behavioural biases, and frequent regulatory interventions, all of which distort the risk-return relationship. Additionally, market frictions such as liquidity disparities, short-selling constraints, and IPO-related restrictions further reduce the effectiveness of traditional fundamental-based models. As Hou et al. (2015) argue, profitability and investment signals may be masked by such frictions in some markets, necessitating the introduction of alternative factors.

In this context, *LIQ* is not merely a supplementary factor but a structural redefinition of what constitutes priced risk in the Chinese market. The empirical evidence presented in this study shows that liquidity may be one of the few truly priced risk factors, surpassing conventional value, profitability, and investment metrics.

In conclusion, the regression results suggest that the traditional FF-5 model exhibits limited cross-sectional pricing power in the Chinese market, particularly within value- and profitability-driven portfolio groups. The extended FF-6 model, through the incorporation of the *LIQ* factor, significantly improves the explanation of excess returns. *LIQ* displays robust, statistically significant, and consistent pricing power across different portfolio sorts, indicating that it captures a dimension of systemic risk not accounted for by traditional factors. Therefore, future multi-factor modelling for the Chinese market should consider including *LIQ* as a core pricing factor to enhance both the theoretical relevance and empirical accuracy of asset pricing models.

Table 11. Cross-sectional Regression; July 1, 2013, to April 30, 2021, 108 months.

At the end of June each year, stocks are allocated to five Size groups (Small to Big) using Chinese main board market cap breakpoints. Stocks are allocated independently to five liquidity groups (Low Esp to High Esp), again using Chinese main board market cap breakpoints. The intersections of the two sorts produce 25 Size-Liq portfolios. The LHS variables in each set of 25 regressions are the daily excess returns on the 25 Size-Liq portfolios. The RHS variables are the excess market return, R_M - R_F, the ln Size (*LMV*), the ln B/M ratio (*LBM*), the profitability *OP*, the investment Inv, and the liquidity Liq. The five-factor regression equation is R_i-R_{ft}= $a+b_1(R_{mt}-R_{ft})+b_2SIZE+b_3B/M+b_4OP+b_5INV+e_t$. The six-factor regression equation is R_{it}-R_{ft}= $a+b_1(R_{mt}-R_{ft})+b_2SIZE_{LIQ}+b_3B/M+b_4OP+b_5INV+b_6LIQ+e_t$.

FF-5

-	Size–B/M	Size–OP	Size–Inv	Size–LIQ
MKT	0.0001	0.0004	-0.0005	-0.0004
t-stat	0.12	0.83	-0.95	-0.7
HML	0.0002	0.0001	0.0001	0.0003
t-stat	0.74	0.34	0.39	0.88
RMW	-0.0002	-0.0003	-0.0001	-0.0001
t-stat	-0.61	-0.98	-0.25	-0.39
CMA	-0.0001	-0.0002	0	-0.0002
t-stat	-0.38	-0.48	-0.06	-0.62

FF-6

	Size–B/M	Size–OP	Size–Inv	Size–LIQ
MKT	-0.0002	0	-0.0006	-0.0008
t-stat	-0.4	-0.02	-1.1	-1.38
HML	0.0003	0.0001	0.0002	0.0001
t-stat	0.82	0.32	0.53	0.39
RMW	-0.0003	-0.0004	-0.0002	-0.0002
t-stat	-0.96	-1.29	-0.69	-0.71
CMA	-0.0002	-0.0002	-0.0001	-0.0003
t-stat	-0.6	-0.59	-0.19	-0.75
LIQ	0.0003	0.0001	0.0001	0.0003
t-stat	1.34	0.33	0.21	1

2.3.7 Idiosyncratic Volatility Test

Table 12 presents that the results of the next ten trading days, the five groups of data, show the higher the idiosyncratic volatility, the lower the return. And the *t*-test results show that there is a significant difference between the high return of stocks with low idiosyncratic volatility and the low return of stocks with high idiosyncratic volatility. Moreover, in the process from T+1 to T+10, except for the first group, the return of other groups shows a downward trend. It shows that the impact of idiosyncratic volatility on expected return is gradually weakening, and the greater the idiosyncratic volatility is, the greater the degree of weakening is. This result verifies the negative correlation between idiosyncratic volatility and stock return from the perspective of the Fama-French five-factor model. It further shows that taking the idiosyncratic risk cannot obtain corresponding returns and is likely to bear greater losses.

Table 12. Regression formed on IVOL.

Fama-French five-factor model is used to regress the sample data to extract idiosyncratic volatility. In the calculation of Fama-French five-factor model, the explained variable is the daily return of stocks, and the explanatory variable is MKT factor, SIZE factor, B/M factor, RMW capability factor, CMA factor. After regression analysis of each sample in turn, take the standard error of the residual of the regression model as the idiosyncratic volatility, and then rank the stocks in the same trading day in descending order, and then rank the daily return of each stock in the next 10 trading days according to the order of, Observe whether the return rate of stock is related to its idiosyncratic volatility through the order of such as and the order of return rate after such order.

FF-5	1	2	3	4	5	Difference	t-stat
T+1	0.000231	0.000652	0.000743	0.000921	0.000984	0.000753	7.6771
T+2	0.000233	0.000648	0.000741	0.000917	0.000993	0.00076	8.8569

T+3	0.000231	0.000649	0.000733	0.000919	0.000981	0.00075	7.4301
T+4	0.000245	0.000651	0.000754	0.000899	0.000948	0.000703	7.2732
T+5	0.000297	0.000646	0.000739	0.000891	0.000928	0.000631	7.0945
T+6	0.000301	0.000635	0.000717	0.000864	0.000905	0.000604	6.8943
T+7	0.000309	0.000644	0.000708	0.000871	0.000906	0.000597	6.6495
T+8	0.000316	0.000651	0.000699	0.000878	0.000902	0.000586	6.3425
T+9	0.000322	0.000638	0.000691	0.000854	0.000911	0.000589	6.5477
T+10	0.00031	0.000629	0.000701	0.000867	0.000904	0.000594	6.1963

From table 13, in the absence of any control variables, the cross-sectional relationship between expected stock returns and idiosyncratic volatility is significantly negative.. After adding the control variable, the negative correlation still exists significantly, and the coefficient has no significant change compared with -0.1490, indicating that the control variable cannot inhibit the negative correlation between idiosyncratic volatility and expected returns. When Skewness is added, the coefficient of idiosyncratic volatility changes to -0.2241. Compared with no control variable, the coefficient decreases, but it is not very obvious; After adding the return of the previous month, the idiosyncratic volatility coefficient dropped to -0.2152, and the t-value also dropped to -3.09, indicating that the impact of the return of the previous month is higher than that of Skewness. Therefore, the addition of Skewness and the return of the previous month can explain the mystery of idiosyncratic volatility to some extent, but it cannot change the negative correlation between stock expected return and idiosyncratic volatility.

This result is consistent with the research conclusions of some scholars: Deng and Zheng (2010) extracted the expected trait volatility by establishing the ARMA model. The research results show that there is a significant positive relationship between the two; Zuo, et al. (2011) estimated the expected idiosyncratic volatility using the realized idiosyncratic volatility, GARCH (EGARCH) model and ARMA model with a lag of one period, respectively. Without adding the control variable, the expected idiosyncratic volatility and the expected excess return have a significant negative correlation. When adding the control variable - turnover rate, the expected idiosyncratic volatility and the expected excess return are still negative correlations. It just becomes insignificant; Tian and Liu (2011) use the EGARCH (1,1) model to estimate the expected idiosyncratic volatility. Their research results show that the expected idiosyncratic volatility has a significant positive correlation with the stock return.

Combined with the current situation of China's stock market, the main reason for this phenomenon is the existence of short-selling restrictions. The negative correlation between idiosyncratic volatility and stock returns in the next period is due to the reversal effect of stock prices with higher returns in the current period or the performance of regression value after the overvaluation of prices due to the strong speculative atmosphere. In addition, it has been confirmed to some extent that the main reason why the results of different scholars' research on the relationship between idiosyncratic volatility and stock return are inconsistent is the choice of proxy variables of expected idiosyncratic volatility. The predicted value of idiosyncratic volatility predicted by the time series method is selects as the proxy variable of idiosyncratic volatility, which is significantly positively correlated with the expected return due to the same period of information. The essence of the study is the correlation between the lag period and the expected return, so it is a significant negative correlation. The main reason for this phenomenon is that the stocks with higher returns in the current period are more likely to have higher idiosyncratic volatility than the stocks with lower returns in the current period. This asymmetry of volatility is mainly due to the restriction of short-selling mechanisms.

Table 13. Fama-MacBeth Regression formed on IVOL.

Fama-French five-factor model is used to regress the sample data to extract idiosyncratic volatility. In the calculation of Fama-French five-factor model, the explained variable is the daily return of stocks, and the explanatory variable is MKT factor, SMB factor, HML factor, RMW capability factor, CMA factor. After regression analysis of each sample in turn, take the standard error of the residual of the regression model as the idiosyncratic volatility, and choose the skewness and the return of last month as the control variables.

	Int	Mkt	SMB	HML	RMW	CMA	IVOL	SKEW	R_{t-1}
Coefficient	0.014	0.093	0.085	0.0001	0.0089	0.0033	-0.2355		
t-stat	2.922	2.474	2.385	1.3156	2.3708	1.5713	-3.32		
Coefficient	0.013	0.011	0.008	0.0001	0.0088	0.0029	-0.2241	-0.0002	
t-stat	3.059	2.733	2.339	1.1229	2.3544	1.4004	3.21	-0.7947	
Coefficient	0.015	0.011	0.009	0.0001	0.0063	0.0023	-0.2152		-0.0262
t-stat	2.702	2.968	2.002	1.1285	1.6782	1.16	3.09		-1.0359
Coefficient	0.012	0.013	0.007	0.0001	0.0074	0.0051	0.2354	-0.0002	-0.0281
t-stat	2.898	3.26	1.73	0.9429	1.9712	1.6952	-3.31	-0.9308	-1.1114

Chapter 3. Fama-French Five-Factor Modeling: New Evidence from a Nonparametric Method

This study constructs a Fama-French five-factor model that considers the time-varying properties of the parameters and introduces a nonparametric method that estimates the factor loadings. I approach the topic from a micro perspective using high-frequency data to construct factors and models to evaluate the sensitivity of each factor on abnormal returns. The results show that the conditional alphas of portfolios are optimised, and the nonparametric model outperforms the traditional models. Our findings lead investors to consider the impact of parameter time-variation when using multi-factor stock selection models to construct asset portfolios.

3.1 Introduction

Factor models have been successful in capital asset pricing and are widely used in portfolio management. In the framework of these models, the risk premium of an asset is linearly related to the beta coefficient of the asset. The beta coefficient reflects the sensitivity of assets to various market risk factors. The slope of the linear relationship should be the same for all assets, representing the market price per unit of beta risk associated with each risk factor. However, a growing proportion of empirical research suggests that risk premiums are not static but evolve (see Keim and Stambaugh, 1986; Ferson and Harvey, 1991; Ferreira, GilBazo, and Orbe, 2011). Ang and Kristensen (2012) argue that traditional factor models are distorted when testing alpha equal to zero if the factor loadings vary over time; the previous can lead to misleading statistical inferences about the validity of factor models, especially the CAPM model and Fama-French three-factor model (FF-3). These findings challenge the stability assumptions of traditional models and suggest that risk pricing may have more dynamic properties.

This chapter uses a nonparametric estimation approach to investigate the time-varying nature of factor loadings in the Fama-French five-factor model (FF-5). Chiah et al. (2016) explore the superiority of the Fama-French five-factor model (FF-5), particularly in its ability to explain asset pricing anomalies with exceptional performance. Compared to the three-factor model, the five-factor model captures the sources of variation in market returns more comprehensively by incorporating profitability (RMW) and investment factors (CMA). The study demonstrates that the FF-5 model significantly reduces pricing errors and performs best in explaining various asset pricing anomalies, such as size, value, profitability, and asset growth anomalies.

Moreover, although Fama and French suggest that the book-to-market ratio factor might be redundant under the five-factor model in their analysis of the US market, Chiah et al. (2016) show empirically that the book-to-market ratio factor retains significant explanatory power even in the presence of profitability and investment factors. This finding has important implications for extending research and practical applications of multi-factor models and underscores the wide applicability of the FF-5 model in international markets. Additionally, the study highlights that the return estimates of the five-factor model exhibit higher statistical significance and that its explanatory power applies not only to standard asset portfolios but also remains robust when tested on market segments. This evidence suggests that the FF-5 model outperforms competing models in both theory and practice, providing a more comprehensive framework for asset pricing research. Sha and Gao (2019) argue that the FF-5 model is significantly superior in the Chinese mutual fund industry.

Compared to the traditional CAPM model and the Fama-French three-factor model (FF-3), the FF-5 model extends its ability to explain changes in asset returns by adding profitability (RMW) and investment style (CMA) factors. This extension provides a more comprehensive view of the multiple risk factors present in the market when analyzing fund returns. The study shows that the FF-5 model performs well across different fund types, particularly in explaining the risk-adjusted returns of equity and hybrid funds. Furthermore, the FF-5 model generates the lowest GRS statistic in time-series tests, indicating that its risk-adjusted return estimates are more accurate. The model also demonstrates a high level of overall fit in explaining returns, outperforming the Carhart four-factor model (FFC) and other competing models. This suggests that a greater portion of the variation in the model's alpha distribution arises from sampling error rather than genuine pricing errors. Thus, the FF-5 model not only exhibits strong statistical significance in explaining fund returns but also excels across a variety of evaluation metrics.

The noise caused by limited time series observations can be mitigated through additional averaging methods, such as combining stocks with similar factor beta estimates (Aït-Sahalia et al., 2020). Thus, it is necessary to conduct time-varying tests on the widely used FF-5 model. The motivation for using nonparametric estimation of the factor loadings in the Fama-French model is to address the limitations of parametric methods and to provide a more accurate and robust analysis of the factor loadings. Nonparametric methods perform well in capturing the nonlinear relationship between the stock returns and the factors, and produce more accurate estimates of the factor coefficients, especially in the presence of outliers or other data irregularities (Ang and Kristensen, 2012; Fan et al., 2012; Connor et al., 2012). Based on the previous, our study considers the underlying nonlinear relationships and is the first to apply the nonparametric regression approach proposed by Ang and Kristensen (2012) to the FF-5 model using a higher data frequency and constructing factors at the 30-min level. I shed light on the following questions: *(i) Does factor loading vary over time in the Fama-French five-factor model, and what is the impact of this time variation on model performance? (ii) What is the effect on alpha, if time variation in factor loading is incorporated into the model?*

The approach proposed by Ang and Kristensen (2012) considers the time-variation of factor parameters, allowing for a more flexible capture of changes in factor loadings over time. This approach applies not only to a single asset but also to portfolios of multiple assets, enabling joint tests of the stability of conditional alpha and beta. This flexibility makes the methodology suitable for more empirical studies and portfolio analysis. I compare this method with that of Li and Yang (2011), who do not provide an optimal bandwidth selection process and lack a constancy test for conditional alpha.

Our empirical findings provide several key insights into the behavior of high-frequency and low-frequency factors within the FF-5 model. Firstly, I show that the daily cumulative returns of high-frequency and low-frequency factors exhibit similar trends for most of the period under examination. This indicates consistency in the factor movements across different frequencies. However, in the latter part of the sample period, significant deviations emerge, suggesting a shift in the market dynamics that affects the behavior of these factors. This shift highlights the importance of using high-frequency data to capture such changes accurately.

Secondly, our analysis demonstrates the time-varying nature of the FF-5 model. In our joint tests, I reject the null hypothesis that long-term alpha equals zero, indicating that the estimated alpha retains statistical significance over time, even when accounting for the time-varying nature of the factors. Through statistical testing, I find evidence that the factor loadings are not static over time, which contradicts with the assumptions of traditional static models. This time-varying characteristic suggests that the sensitivity of assets to different risk factors can change, emphasising the need for models that adapt to evolving market conditions.

Finally, our model demonstrates an advantage when comparing the alpha estimates from high-frequency data to those obtained through OLS regression using lower-frequency data. Specifically, the high-frequency approach is more effective in reducing estimation errors. This reduction implies that the time-varying adjustments provided by our model capture more of the explanatory power of the market movements, leading to a more accurate representation of the actual performance of asset portfolios. Such improvements underscore the potential of high-frequency analysis to enhance the understanding of risk-adjusted returns in financial markets.

One of the first studies to evaluate CAPM in a conditional sense was from Jagannathan and Wang (1996), which allows the beta coefficient and market risk premium to change over time. They qualitatively review their work by noting that several studies have empirically assessed the performance of static versions of CAPM in the twenty years before their research. Nagel and Lewellen (2006) utilise short window regression testing to evaluate the conditional CAPM. They challenge the previous literature (e.g. Jensen, 1968; Dybvig and Ross, 1985; Jagannathan and Wang, 1996) and discover that the conditional alpha value of a stock may be zero while the unconditional alpha value is not; the previous arises when the beta value of a stock is allowed to change over time and is related to stock premium or market volatilities.

Using short-window regression, Lewellen and Nagel (2006) estimates the parameters for size, B/M, and momentum investment portfolios. A direct evaluation of conditional CAPM is an alpha estimation. They find that alpha is statistically significant and reasonably close to unconditional alpha, even though the average dependent alpha, assuming the conditional CAPM holds, is zero (Ferson and Qian, 2004). The nonparametric method (Leave-one-out cross-validation) used by Li and Yang (2011) estimates and tests conditional factor models. It also provides insights to understand better why conditional FF models cannot explain well-known asset pricing anomalies. The study shows that the cross-sectional distribution of stock returns cannot be explained by either conditional CAPM or FF models. Furthermore, the method is effective and resistant to the dimensional problem in the nonparametric literature. Nonetheless, the method is only effective with a limited sample size, even if it is entirely data-driven. It is worth mentioning that it only provides a distribution of pricing errors rather than a complete asymptotic distribution (Ang and Kristensen 2012).

To interpret the constant terms in OLS regressions as conditional alphas, Ang and Kristensen (2012) suggest using a nonparametric methodology to test long-run and conditional alpha and beta. They create a continuous-time version of the discrete-time factor model and impose constraints on the data-generating procedure. They conclude that the proposed estimators can be used asset-by-asset and expanded to different sets of portfolios; the estimators can also be used to assess conditional factor models and can be expanded to include adaptive estimators to produce estimates of conditional alphas or factor loadings without using future data. Even though they find significant changes in factor loadings, overwhelming evidence suggests that conditional CAPM and Fama and French (1993) models cannot explain value premiums or momentum effects.

An alternative approach to exploiting high-frequency data for analysing single stocks is proposed by Aït-Sahalia et al. (2020). They demonstrate that high-frequency alpha estimates are more precise and stable over time compared to daily frequency alpha estimates. They explore the possibility of high-frequency variables explaining specific stock returns by comprehensively examining all traded stocks on the New York Stock Exchange, the American Stock Exchange, and the Nasdaq Stock Exchange. They also document that surprise returns boost specificity leaps. The semi-martingales used in this study also allow for the broad time-variation of beta, allowing for continuous and idiosyncratic jumps in leverage effects, returns, and volatilities. My work differs from theirs in that our research focuses more on portfolios. I study the time-variation of the FF-5 model within portfolios formed based on different characteristics. Our approach is based on literature that estimates time-varying factor loadings using high-frequency data (e.g. Ang and Kristensen 2012). I also use the plug-in method to calculate the data bandwidth and choose a Gaussian kernel function, as choosing a symmetric kernel function can reduce estimation bias. Then, I estimate the alpha and beta for each period to obtain long-term alpha and beta. Finally, I verify the alpha constancy of the model.

The instrumental variable principal component analysis (IPCA) approach, proposed by Kelly et al. (2019), estimates hidden factors and time-varying loadings by using observable characteristics as instrumental variables. This method allows factor loadings to be partially dependent on observable asset characteristics, thereby controlling for time variation to some extent. It focuses more on selecting features and attributes feature effects to an intercept representing risk-free compensation. This approach efficiently manages many features within high-dimensional forecasting systems, enhancing the performance of factor models through dimensionality reduction. Additionally, it extends the existing Gibbons et al. (1989) test, placing a greater emphasis on parameter testing.

Significant superiority of the Fama-French five-factor model (FF-5) over the three-factor model (FF-3) in certain regions of emerging markets has been demonstrated by Foye (2018). This is particularly evident in the markets of Eastern Europe and Latin America, where the FF-5 model provides better explanations for asset pricing anomalies. However, the performance of the FF-5 model is relatively limited in the markets of Asia and does not substantially improve the explanatory power of returns. This limitation arises due to the lack of significance of profitability and investment factors in these markets, as firms are heavily influenced by differences in capital allocation efficiency and profitability. The weaker performance of the investment factor in Asian markets can be attributed to the stages of economic development, with firms adopting high-growth strategies and facing stronger policy interventions in investment behavior. Additionally, the profitability factor in Asian markets is often obscured by other factors not accounted for in the model, such as momentum effects or the influence of foreign investment. The above findings indicate that the structure of markets and the characteristics of regional economies significantly affect the performance of the FF-5 model and its explanatory capabilities.

Racicot et al. (2023) adopt the dynamic panel data approach , which uses GMM to deal with the endogeneity of liquidity factors and focuses on the impact of the liquidity premium in international markets. Their method relies on the choice of instrumental variables and although there may be weak instrumental variables, this approach is still able to capture time series characteristics. Meanwhile, they explore the FF-5 model by introducing a lagged dependent variable, which is equivalent to considering the time variation of the coefficients in another dimension, and they show that the model better captures the cyclicality of returns.

The core of this chapter is to explore the influence of the dynamic nature of factor loadings on asset pricing models, offering a crucial perspective for understanding model performance across different time periods and market environments. This aligns with the findings of Foye (2018), which indicate that the effects of profitability and investment factors can vary over time or in response to specific market conditions. Consequently, it is theoretically viable to employ non-parametric techniques to further explore the dynamics of factor loadings in the FF-5 model across various market regions and time dimensions, thereby enhancing the applicability and explanatory power of factor models.

The Economic Policy Uncertainty (EPU) factor, introduced by Arkol and Azimli (2024), extends the Fama-French five-factor model and examines its interaction with traditional factors in emerging markets. The dynamic nature of the EPU factor itself implies that its impact varies at different points in time. Thus, in applying this factor, they indirectly examine how it affects stock returns over time, which aligns with the idea of focusing on time-varying factors. Although they do not directly focus on the time-varying nature of the factor loadings using a nonparametric approach, their study does address time-varying nature to some extent. This time sensitivity to the impact of EPU makes the study somewhat different from traditional static factor models.

The main contribution of this study is the development of a robust methodology to reduce low-frequency estimation noise due to limited time series observations. Existing studies typically rely on parametric methods, which may impose rigid assumptions on the relationship between stock returns and factors, which may lead to bias and reduced robustness in the estimation, especially in the presence of outliers or non-linearities. By utilising non-parametric techniques, this study directly addresses these limitations and provides a more flexible framework for estimating time-varying factor loadings.

The structure of this chapter is organised as follows. Section 2 presents our methodology. Section 3 describes the data and presents our empirical results for the conditional Fama-French five-factor model.

3.2 Methodology

This section introduces a nonparametric approach to estimating and testing a conditional model. We first define a conditional Fama-French five-factor model. Then, we discuss how to choose the appropriate bandwidth. Then, we propose tests to evaluate its performance in explaining changes in asset returns. Finally, we conduct a constancy test to demonstrate whether the model is time-varying.

3.2.1 Discrete Parametric Model

Let R_{it} be the return of a particular stock at time t and observe the data of M stocks during the T period. F_t represents the factor matrix of the same period in the Fama-French factor model. Rmt is the market return rate, R_{ft} is the risk-free interest rate, SMB_t is the size factor, HML_t is the value factor, RMW_t is the profitability factor, and CMA_t is the investment factor. Considering the variation of factor loading over time, I establish a new Fama-French five-factor model.

The model is abbreviated as:

$$Y_{it}=\alpha_{it}+F_t\beta_{it}+\varepsilon_{it}. \tag{14}$$

$$B_{it}=(r_i, s_i, h_i, r_i, c_i). \tag{15}$$

According to the most common kernel estimation method in nonparametric estimation, this article selects the Gaussian kernel as the kernel function, aiming to estimate the time points of each stock using all samples α、β coefficient. Moreover, compared to other nonparametric estimation methods, as long as the kernel function is symmetric, kernel function selection and bandwidth do not play a decisive role in the estimation.

The objective function obtained by combining the idea of minimum variance is:

$$min \sum_{t=1}^{T}(Y_{it}-F_t\beta_{it}-\alpha_{it})^2 K(\frac{t-t_0}{Th}). \tag{16}$$

The derivation process is as follows:

$$-2\sum_{t=1}^{T}[Y_{it}-X_t(\alpha_{it},\beta_{it})']K(\frac{t-t_0}{Th})(-X_t)=0. \tag{17}$$

$$\sum_{t=1}^{T}Y_{it}X_t K(\frac{t-t_0}{Th})-\sum_{t=1}^{T}X_t'X_t K(\frac{t-t_0}{Th})(\alpha_{it},\beta_{it})'=0. \tag{18}$$

The estimated parameter obtained is:

$$[\alpha_{it},\beta_{it}]'=[\sum_{t=1}^{T}X_t'X_t K(\frac{t-t_0}{Th})]^{-1}[\sum_{t=1}^{T}Y_{it}X_t K(\frac{t-t_0}{Th})]. \tag{19}$$

Where, $K(\frac{t-t_0}{Th})$ is the kernel function, and h is the selected window width. Different window widths should be chosen because different stocks have different α、β. The change and curvature will differ, and stocks may have different heteroscedasticity. Determining the unique bandwidth of individual stocks helps to adjust the utility of the estimator. But to avoid the tedious selection process, I assume all stocks' window widths are equal and their convergence speed is similar.

3.2.2 Continuous Parameter Model

Ang and Kristensen (2012) established a continuous parameter version of the discrete model to demonstrate the consistency of the estimated parameters. Assume that the abnormal return rate of *M* stocks conforms to the following stochastic differential equation:

$$ds(t)=\alpha(t)dt+\beta(t)^{'}dF(t)+\sum^{1/2}(t)dB(t). \quad (20)$$

F(t) is a vector of *J* elements, and *B(t)* is an M-dimensional Brownian motion. This equation corresponds to the ANOVA model for realized volatility, which is proposed by Bollerslev et al. (2006) and Mykland, Zhang (2006). This framework is designed to decompose and estimate volatility dynamics by characterizing the dynamic process of asset returns through such a stochastic differential equation. Assuming that *n* time points within the time interval of [0, *T*] can be observed for *s(t)* and *F(t).* Consider the estimated *α(t)* and *β(t)* as the instantaneous drift of *s(t)* and *F(t)* coordinated volatilities, respectively. To simplify the calculation, adjacent observation points are assumed to be equidistant. The continuous model is discretised to facilitate analysis. The discrete model is:

$$\Delta s(t_i) = \alpha(t_i)\Delta t_i + \beta(t_i)\Delta F(t_i) + \Sigma^{1/2}(t_i)z(t_i)\sqrt{\Delta t_i}. \quad (21)$$

Where, z_t is subject to independent distribution and normal distribution with zero mean and I_M covariance.

$$\Delta s_i = s(t_i) - s(t_{i-1}), \quad \Delta F(t_i) = F(t_i) - F(t_{i-1}). \quad (22)$$

To ensure that *α* if the estimator is asymptotically effective, additional conditions must be added. When *Δ* approaches zero, these conditions are related to controlling the discretisation deviation to converge to zero at a sufficient speed. Bandi, Phillips (2003) and Kristensen (2010) have already described these conditions.

3.2.3 Choice of Kernel Function

Like other conventional nonparametric estimations, choosing the kernel function and bandwidth is also necessary. Due to the theoretical focus on intra-sample estimation and testing, selecting a symmetric kernel function can reduce the bias of the estimator:

$$K(z) = \frac{1}{\sqrt{2\pi}} exp(-\frac{z^2}{2}). \tag{23}$$

Referring to other research methods, this study employs a Gaussian kernel function. Ang and Kristensen (2012) demonstrate that if a single observation is weighted in terms of its temporal proximity to the time point *t*, the shape K of the kernel determines the weighting method for different observations. Tail bias is an unavoidable problem in kernel estimation methods, and many methods exist to solve this problem. This paper adopts the simplest and most commonly used method, which assumes that the estimated period belongs to a subset of the observation interval.

For the estimators to be consistent, the bandwidth selection must approach zero as the sample size increases. The window width determines the distribution of samples used to estimate alpha and beta over the period *[0, T]*. A large window width means that many observations near time *t* are weighted, which reduces the weight of closer and more accurate observations. This results in a more significant beta error but less volatility in the estimation. A small window width means only observations very close to time *t* are used to estimate parameter beta, resulting in more minor errors but significant volatilities in beta. The bandwidth determines the error and volatilities of the estimated value. Therefore, the window width should vary depending on the sample. Specifically, as the sample size expands, the window width should approach zero appropriately to eliminate the bias and variance of limited samples.

Due to the different convergence rates of conditional and long-term estimators, this article chooses bandwidth for both. If the same bandwidth is used for estimation, the analysis of long-term parameters will be too smooth. Hence, the window width for long-term parameter estimation is smaller than the conditional estimation.

3.2.4 Bandwidth of Conditional Estimator

To condition the selection of the estimator bandwidths, this paper adopts the global plug-in method to simulate the optimal bandwidths. The criterion for selecting the bandwidths is to minimise the overall mean square error in *[0, T]*, so the window chosen widths are global. In some cases, a local window width that suits the regional characteristics of the data may lead to more accurate estimation results. The global bandwidth can be found by splitting the sample into several sub-samples and then applying the international plug-in method to find the global bandwidths suitable for the sub-samples. In a symmetric kernel function, there is:

$$\int_{-\infty}^{\infty} K(u)du = 1,\ \int_{-\infty}^{\infty} uk(u)du = 0,\ \mu_2(K) = \int_{-\infty}^{\infty} u^2K(u)du < \infty\,. \tag{24}$$

The corrected set of formula (24) defines the three essential properties that any valid symmetric kernel function, $K(\cdot)$, must satisfy for the nonparametric estimation performed in this study. First, the normalisation condition $\int K(u)du = 1$ensures the kernel acts as a weighting scheme that sums to unity, guaranteeing that the local estimate is a proper weighted average of nearby data points. Second, the symmetry condition $\int uk(u)du = 0$ is fundamental for controlling bias; it ensures data points before and after the target time point t_0 are weighted equally, which is critical for obtaining an unbiased path of time-varying betas. Finally, the finite second moment condition,$\int u^2K(u)du < \infty$ quantifies the kernel spread. The specific value of u^2K for the chosen Gaussian kernel is not arbitrarily 1 but is a precise constant that directly enters the calculus of the optimal bandwidth h, via the plug-in method discussed subsequently.

Since the kernel function selected in this research is a Gaussian function, and there are specific mean and variance from the normal distribution, the window width of the conditional estimator is chosen by a simple embedding method. This is a method proposed by Silverman (1986), which states that if the kernel density function comes from an average population and has a specific mean and variance, it can be embedded using the thumb method:

$$\theta'' = \frac{3}{8\sqrt{\pi}}\sigma^5. \tag{25}$$

Where θ'' represents the result of integrating the second derivative of the standard normal kernel function. This value is used as a parameter in optimizing the bandwidth selection for kernel density estimation. When selecting the bandwidth, θ'' can serve as an intermediate variable or reference value in calculations to ensure that the smoothing level in the estimation process is appropriate. Here, y represents the distribution density function of the sum density function, and the unknown variance can be estimated by:

$$[\frac{1}{n-1}\sum_{t=1}^{T}(y_i - \hat{y})^2]. \tag{26}$$

Another more robust estimation method is $\hat{\sigma} = \frac{Q}{1.34}$. Q is the interquartile range, thus obtaining the expression of bandwidth embedded in the thumb method:

$$\hat{h} = (4\sigma^5/3n)^{1/5}. \tag{27}$$

Equation (25) provides a theoretical description of the relationship between the kernel density function and variance. Equation (27) is a practical formula derived based on this theoretical foundation, used to calculate the optimal bandwidth $\hat{h}$. It is derived by balancing the bias and variance of the estimation and incorporates the influence of the sample size n. The σ^5 term in Equation (27) is related to the factor θ'' in Equation (25). The bandwidth expression for long-term parameter estimation is:

$$\hat{h}_{LR} = \hat{h} \times n^{-2/15}. \tag{28}$$

3.2.5 Tests for constancy of estimators

Assuming it is proven that the estimated values of *α(t)* and *β(t)* are consistent across a large sample. In this case, a statistical test can be constructed to determine whether the estimated results are significant. In the test, the portfolio-by-portfolio test method can be used to obtain the time-varying factor loading, or the technique of joint test of all stocks can be used to test the time-varying factor loading of asset portfolio stocks.

Two null hypothetical expressions:

$$H_k(\alpha): \alpha_k(t)=\alpha_k \in R,[0,T]. \tag{29}$$

$$H_k(\beta): \beta_k(t)= \beta_k \in R,[0,T]. \tag{30}$$

R represents the set of real numbers. When factor loadings are correlated with factors, the actual conditional pricing error and the correlation between factors and factor loadings jointly affect the magnitude of unconditional pricing errors (Ang and Kristensen, 2012).

$$\hat{\alpha}_{LR,K} = \lim_{n\to\infty} \frac{1}{n} \sum_{t=1}^{n} \alpha_k(t). \tag{31}$$

$$\hat{\beta}_{LR,K} = \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \beta_k(t). \tag{32}$$

Under the null hypothesis, the test statistic satisfies:

$$\frac{w_k(\alpha)-m(\alpha)}{v(\alpha)} \sim N(0,1). \tag{33}$$

$$\frac{w_k(\beta)-m(\beta)}{v(\beta)} \sim N(0,1). \quad (34)$$

Where $m(\alpha)$ and v^2 denote the asymptotic mean and variance of the test statistic, respectively, given by:

$$m(\alpha) = \frac{k_2}{Th} \quad v^2(\alpha) = \frac{2(\int K(y)K(z+y)dy)^2 dz}{T^3 h}. \quad (35)$$

$$m(\beta) = \frac{k_2 J\Delta}{Th}, \quad v^2(\beta) = \frac{2(\int K(y)K(z+y)dy)^2 dz}{n^2 h}. \quad (36)$$

If I want to adopt a joint test for all stocks, I have to consider the interrelationships between stocks, and their test statistics may vary. The long-term estimates of alpha obtained using Eq. (19) are substituted into Eq. (37) and (38). The following equations outline the weighted approach used in the model.

$$\bar{W}(\alpha) = \frac{1}{n}\sum_{k=1}^{T}[\hat{\alpha}(t) - \hat{\alpha}_{LR}]' \sum_{k=1}^{-1}(t)[\hat{\alpha}(t) - \hat{\alpha}_{LR}]. \quad (37)$$

Where: $$m(\alpha) = \frac{k_2 M}{Th}, \quad v^2(\alpha) = \frac{2M(\int K(y)K(z+y)dy)^2 dz}{T^3 h}. \quad (38)$$

M is the number of portfolios. The asymptotic distribution of this test statistic is consistent with the statistical distribution constructed when tested separately.

$$\frac{w_k(\alpha_{LR})-m(\alpha_{LR})}{v(\alpha_{LR})} \sim N(0,1). \quad (39)$$

$$m(\alpha_{LR}) = \frac{k_2 M}{Th}, \quad v^2(\alpha_{LR}) = \frac{2M(\int K(y)K(z+y)dy)^2 dz}{T^3 h}. \quad (40)$$

k_2 is a constant (0.2821) derived from the kernel function used, J represents the dimensionality. $k_2 J\Delta$ indicate a scaled version of k_2 those accounts for the time step Δ and a dimension. The $w_k(a)$ and $w_k(\beta)$ refer to test statistics for estimating and testing conditional alphas and betas obtained from eq. (19).

3.2.6 Construction of Intraday Factors

The decision to construct intraday factors stems from the need to capture the behavior of factors during trading hours, which cannot be observed with traditional daily frequency models. High-frequency data reveals more subtle changes in market behavior, offering a more detailed view of the dynamics of factors (Hui et al., 2022; Holý, 2022). By constructing intraday factors, we study the immediate reactions of factors to market shocks. This granular analysis aids in understanding the influence of short-term market behavior on longer-term trends and provides valuable information for investors in a high-frequency trading environment. While the fundamental components of the factors may not change significantly over short time intervals, constructing intraday factors allows us to examine the sensitivity of these factors to instantaneous market fluctuations, leading to a more comprehensive understanding of the market price discovery process.

The data which obtained from the Center for Research in Security Prices (CRSP) and Refintiv databases. The CRSP provides fundamental stock data, and the Reinitiv provides 30-minute stock return data. We use 30-min data; although the sample period is only three years, the length of the time series provides a sufficient number of observations. Specifically, a 30-min time interval can better capture the dynamic changes in the market in the short term, especially during periods of high market volatility. Compared to daily or monthly data, high-frequency data helps us analyse the immediate response of market to unexpected events and reveal changes in the microstructure. We use Nasdaq 100 stocks for the period from January 2020 to December 2022. The sample data in this study exclude companies that delist during the sample period or that go public after the sample period begins. Additionally, we filter out trading data before 9:30 am and after 4:00 pm.

Despite the relatively short sample period, the use of 30-minute frequency data significantly increased the total sample size. As a result, each constructed return series includes a total of 9,827 observations, which provide sufficient statistical power for the analyses. For the FF-5 model, processing high-frequency data over a long period increases computational costs and leads to data redundancy. Due to the rapid changes in market dynamics, high-frequency data can still capture significant changes in factor performance and rapid market responses in the short term, which is impossible with the use of daily or monthly frequency data.

Compared to broader markets such as the New York Stock Exchange which includes more traditional industries, the characteristics of Nasdaq differ from others. In Nasdaq, there are more growth stocks with higher liquidity, which may lead to a different performance of the HML factor compared to other markets that are more value oriented. However, the distinction made by the HML factor between growth and value stocks can still provide valuable pricing signals in the market. Several studies (e.g. Fama and French, 1998; Asness et al., 2019) demonstrate that even in markets with strong growth potential, value premiums may continue to play a role. The FF-5 model is initially developed based on extensive market data in the US stock market, and its factor structure has considerable universality and can be applied to different types of markets. The concentration of the Nasdaq 100 in the technology sector means that the index may be more sensitive to sudden changes in economic policy and technological advancements. This sensitivity aligns with the focus of this paper on examining the time-varying nature of the model.

Similarly, Fama and French (2015) introduced the profitability and investment factors. Our study employs the 2x3 factor construction method, which excludes stocks in the middle 40% of OP and Inv, thus focusing more on the extremes of these variables. Compared to the traditional 2x2 sorting, the performance of the factors obtained from the two sorting methods remains comparable. Firstly, divide all stocks into two groups based on the median market value: small market value (*S*) and significant market value (*B*). Divide all stocks into three groups: high (*H*), medium (*N*), and low (*L*) based on the 30% and 70% percentiles of the book-to-market ratio. Secondly, by sorting the two indicators of market value and book-to-market ratio, all stocks can be divided into six combinations: *SH, SN, SL, BH, BN*, and *BL*; the value and size factors are given by:

$$SMB = SMB_{B/M} + SMB_{OP} + SMB_{Inv}. \tag{41}$$

$$HML = (SH+BH)/2 - (SL+BL)/2. \tag{42}$$

Then, we form six portfolios based on the intersection of two size portfolios (*S* and *B*) and three profitability (*OP*) portfolios (represented as robust), neutral (*N*), and weak (*W*), respectively), as well as three investment (*Inv)* portfolios (represented as conservative (*C*), neutral (*N*), and aggressive (*A*) respectively). *OP* refers to annual revenue minus sales costs, interest expenses, and sales, as well as general and administrative expenses, and then divided by the carrying amount of equity at the end of the previous fiscal year. Inv refers to the total asset changes from the fiscal year at the end of the year to the fiscal year at the end—the fiscal year ending in, divided by total assets. The breakpoints between *OP* and Inv are the 30th and 70th percentage points in the Nasdaq 100 index. SR, BR, SW, BW, SC, BC, SA, and BA are abbreviations for different portfolios in asset pricing models. SR refers to small stocks with robust profitability, BR to big stocks with robust profitability, SW to small stocks with weak profitability, and BW to big stocks with weak profitability. SC represents small stocks that are conservative in investment (low investment levels), BC indicates big stocks that are conservative in investment, SA refers to small stocks that are aggressive in investment (high investment levels), and BA represents big stocks that are aggressive in investment. These classifications are typically used to construct factors like RMW (Robust Minus Weak) and CMA (Conservative Minus Aggressive), capturing the return differences based on profitability and investment levels across small and large stocks. The profitability and investment factors are given by:

$$RMW = (SR\text{–}BR)/2 - (SW+BW)/2. \quad (43)$$

$$CMA = (SC\text{–}BC)/2 - (SA+BA)/2. \quad (44)$$

3.3 Empirical Results

3.3.1 Data

The decision to construct intraday factors stems from the need to capture the behavior of factors during trading hours, which cannot be observed with traditional daily frequency models. High-frequency data reveals more subtle changes in market behavior, offering a more detailed view of the dynamics of factors (Hui et al., 2022; Holý, 2022). By constructing intraday factors, I study the immediate reactions of factors to market shocks. This granular analysis aids in understanding the influence of short-term market behavior on longer-term trends and provides valuable information for investors in a high-frequency trading environment. While the fundamental components of the factors may not change significantly over short time intervals, constructing intraday factors allows us to examine the sensitivity of these factors to instantaneous market fluctuations, leading to a more comprehensive understanding of the market price discovery process.

The data is obtained from the Center for Research in Security Prices (CRSP) and Refintiv databases. The CRSP provides fundamental stock data, and the Reinitiv provides 30-minute stock return data. I use 30-min data; although the sample period is only three years, the length of the time series provides a sufficient number of observations. Specifically, a 30-min time interval can better capture the dynamic changes in the market in the short term, especially during periods of high market volatility. Compared to daily or monthly data, high-frequency data helps us analyse the immediate response of market to unexpected events and reveal changes in the microstructure. I use Nasdaq 100 stocks for the period from January 2020 to December 2022. The sample data in this study exclude companies that delist during the sample period or that go public after the sample period begins. Additionally, I filter out trading data before 9:30 am and after 4:00 pm.

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Compared to broader markets such as the New York Stock Exchange which includes more traditional industries, the characteristics of Nasdaq differ from others. In Nasdaq, there are more growth stocks with higher liquidity, which may lead to a different performance of the HML factor compared to other markets that are more value oriented. However, the distinction made by the HML factor between growth and value stocks can still provide valuable pricing signals in the market. Several studies (e.g. Fama and French, 1998; Asness et al., 2019) demonstrate that even in markets with strong growth potential, value premiums may continue to play a role. The FF-5 model is initially developed based on extensive market data in the US stock market, and its factor structure has considerable universality and can be applied to different types of markets. The concentration of the Nasdaq 100 in the technology sector means that the index may be more sensitive to sudden changes in economic policy and technological advancements. This sensitivity aligns with the focus of this chapter on examining the time-varying nature of the model.

3.3.2 Summary Statistics for factor Returns

Table 14. Descriptive Statistics.

This table reports summary statistics of Fama and French (2015) factors. At the end of each June, stocks are allocated to two size groups using the median market cap as the breakpoint. Stocks are also allocated independently to three book-to-market equity (*B/M*), operating profitability (*OP*), and investment (*Inv*) groups, using stocks' medians of size, B/M, op and inv or the 30th and 70th stocks' percentiles. $R_M - R_F$ is the value-weight return on the market portfolio of all sample stocks minus the benchmark interest rate of the bank's one-year fixed deposit. Data is at a 30-minute frequency, and the span is from Jan 2020 to Dec 2022. The sample consists of 9,827 observations.

Panel A	mean	Std	min	max
Mkt	0.0001	0.0067	-0.05	0.05
SMB	0.0003	0.0012	-0.01	0.07
HML	-0.0029	0.0022	-0.04	0.06
RMW	-0.0033	0.0024	-0.03	0.04
CMA	-0.0002	0.003	-0.04	0.06

Panel B	Kurtosis	Skewness	JB-Statistics	P-Value
Mkt	0.0683	8.2890	11447	0
SMB	0.0756	8.5347	12537	0
HML	-0.3414	9.3226	16539	0
RMW	0.1937	8.1967	11105	0
CMA	-0.1594	7.8865	9806	0

Panel C: Correlation

	MKT	SMB	HML	RMW	CMA
MKT_RF	1				
SMB	0.269	1			
HML	0.002	0.389	1		
RMW	-0.289	0.117	-0.081	1	
CMA	-0.381	-0.549	0.144	0.056	1

In Table 14, Panel A presents the summary statistics of the factors. The average return of the *Mkt* factor is the highest, reaching 0.016%. Except for the *Mkt* factor, the average return rate of the *RMW* factor is 0.003%, while the average return of the *HML* factor and *CMA* factor over the sample time are negative. The standard deviation of factors at the 30-minute level is significantly lower than that of FF-5 factor studies using daily or monthly data. This is because high-frequency data can capture small volatility in stock prices more frequently, which may lead to more average and stable returns in each period. On the contrary, low-frequency data contains more considerable price volatilities, increasing the return variance. Panel B reports the normality test results. All factor returns exhibit pronounced departures from normality, with fat-tailed and asymmetric distributions. Accordingly, the Jarque–Bera test strongly rejects normality at conventional significance levels, supporting the use of distributionally robust or nonparametric methods in subsequent analysis. I construct a return series (LHS variable) using data from all stocks, and then I obtain residuals from the FF-5 model regression. A P-value of zero indicates that the data does not follow a normal distribution. This may be due to substantial noise within high-frequency data, which is consistent with our expectations. It also suggests that traditional linear regression models are not suitable for this case.

Figure 2 compares the daily cumulative returns of *HML*, *SMB*, *RMW*, *CMA*, and *Mkt* factors based on high-frequency replication with the factor returns constructed using daily data. The high-frequency and low-frequency cumulative returns of the *Mkt* factor are almost identical, but there is a moderate difference between *RMW* and *CMA*. At the same time, there is a significant difference between *SMB* and *HML*. Regarding *HML*, the most crucial difference seems to have occurred after September 2021 due to the possibility of substantial instantaneous return volatility in high-frequency data, affecting the cumulative return at later time points. Figure 1 also compares the daily aggregated cumulative returns of the high-frequency factors. Blue lines represent high-frequency factors, and red lines represent low-frequency factors. The sampling period is January 2020–December 2022.

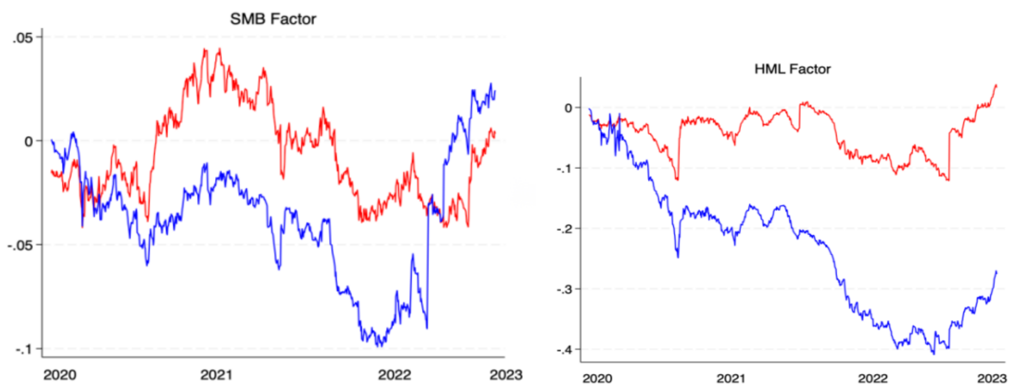

Figure 2. Comparison of the comparison between high-frequency factor returns and low-frequency factor returns

Note: This figure compares the daily aggregated cumulative returns of the high-frequency factors, in blue solid lines, with those of the low-frequency factors, in red dashed lines. The sampling period is January 2020–December 2022.

Although both the high-frequency and low-frequency MKT factors are designed to capture market-wide risk, they are constructed from fundamentally different information sets and aggregation schemes. The high-frequency factor reflects continuous intraday price discovery, incorporating real-time information flow as well as market microstructure effects prevalent during trading hours. In contrast, the low-frequency factor, constructed from daily closing prices, represents a discrete aggregation. Due to this structural difference, particularly the inclusion of overnight jumps and the non-linear compounding of intraday volatility, simply aggregating high-frequency returns does not mechanically replicate the low-frequency factor series. Nevertheless, as visually corroborated in Figure 2, the two series exhibit strongly aligned trends in their cumulative returns. This confirms that the high-frequency factor successfully captures the same underlying market risk premium, while providing a finer-grained, dynamic representation of its evolution within the trading day.

3.3.3 Regression Details

In Table 15, I present long-run alphas and factor loadings for high-frequency data through nonparametric regression. Each column of factor loadings is sorted from top to bottom based on size (small to Big), and every five rows are sorted based on the *B/M*. The smallest *B/M* ratio is marked as 1, and the most significant ratio is marked as 5. Also, Table 14 shows that the long-term market factor loadings exhibit the most minor volatility between growth and value stocks, mainly between 0.00016 and 0.00022. Meanwhile, extreme growth portfolios generate the highest intercept term in any size quintile, indicating more unexplained abnormal returns in such portfolios. No significant size and value effects are found in any portfolio. High *B/M* and small-cap stocks do not provide higher abnormal returns. I find that the intercept term of high-value investment portfolios is the smallest in any cap, which means the model has explained the value effect. In the Size-B/M Portfolio, the intercept terms are significantly smaller than those constructed using low-frequency data, indicating that I have captured more unexplained abnormal returns. The only slight negative intercept appeared in the small-cap high-value portfolio, ranging from -0.004 to -0.018. Still, there is no clear evidence to suggest that the model was ineffective in this investment portfolio. In addition, the intercept term of the microcap extreme growth portfolio is the highest, as the loadings of the *SMB* and *Mkt* factors do not provide sufficient explanatory power. The pattern of extreme growth intercept negative for small-cap stocks and positive for large-ca– stocks - disappeared from our model.

Surprisingly, the *HML* factor load is negative in extreme value portfolios, with fluctuations ranging from -0.263 to -0.211, indicating that the market prefers growth stocks. The *SMB* factor loadings (-0.465, -0.631) and *RMW* factor loadings (-0.050, -0.114) also exhibit similar extreme-cap performance, which occurs in Size-OP and Size-Inv portfolios. This represents that investors prefer stocks with weak profits and aggressive investments, and I believe this result is consistent with the fact that technology stocks mostly dominate the Nasdaq market.

Fama and French's (2015) study mentioned that FF-5 solves the problem of strong negative impact in the investment portfolios of the three smallest cap quintiles and the highest investment quintiles in FF-3 and switching to an FF-5 model will shift these intercepts to zero. The negative slope of investment and profit factors reduces the estimated values of five factors in expected returns. So, it is intuitive that the 24 positive intercept coefficients in Size-B/M can also be attributed to the loadings of *RMW* and *CMA*. In addition, in the second largest portfolios of *B/M*, *SMB*, *HML*, and *RMW* all provided the highest abnormal returns except for the most increased size quintile combination. All three-factor loadings show a downward trend in the extreme-size portfolios. This is also reasonable because investors favour large technology companies more due to their market position and better financial condition.

Table 15. Long-run Fama and French (2015) alphas and factor loadings of size-B/M portfolios.

This table reports long-run alphas and factor loadings estimates from a conditional Fama and French (2015) model applied to 25 *size-B/M* portfolios. At the end of each June, stocks are allocated to two size groups using the median market cap as the breakpoint. Stocks are allocated independently to three book-to-market equity(*B/M*), operating profitability (*OP*), and investment (*Inv*) groups, using medians of size, *B/M*, *OP*, and *Inv* and the 30th and 70th stocks' percentiles. The LHS variables in each set of 25 regressions are the excess returns on the 25 *Size-B/M* portfolios. The RHS variables are the excess market return, $R_M - R_F$, the Size factor, *SMB*, the value factor, *HML*, the profitability factor, *RMW*, and the investment factor, *CMA*, constructed using independent 2×3 sorts on Size. $R_M - R_F$ is the value-weight return on the market portfolio of all sample stocks minus the benchmark interest rate of the bank's one-year fixed deposit. The long-run estimates, with standard errors in parentheses, are computed following Eq. (29) and Eq. (30) and estimates of conditional alphas and betas. The whole data sample is from Jan 2020 to Dec 2022.

B/M	Low	2	3	4	High	Low	2	3	4	High
	Alpha					*Std (Alpha)*				
Small	0.07	0.01	0.03	0.02	0.02	0.00	0.00	0.00	0.00	0.00
2	0.02	0.00	-0.01	-0.02	-0.02	0.00	0.00	0.00	0.00	0.00
3	0.04	0.01	-0.01	0.02	0.01	0.00	0.00	0.00	0.00	0.00
4	0.01	-0.01	-0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Big	0.00	0.00	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00
	SMB					*Std (SMB)*				
Small	-0.47	0.19	0.38	0.61	-0.77	3.23	1.90	2.28	2.63	2.08
2	0.46	0.26	0.59	0.83	-0.48	3.06	1.80	2.15	2.40	1.96
3	0.26	0.28	0.54	0.78	-0.40	2.91	1.81	2.08	2.12	1.80
4	0.50	0.25	0.48	0.97	-0.23	2.42	1.63	1.83	1.93	1.78
Big	-0.60	-0.47	-0.54	-0.63	-0.57	1.28	1.22	1.29	1.28	1.23
	HML					*Std (HML)*				
Small	-0.85	-0.51	0.21	0.48	0.51	1.21	0.61	0.75	0.93	0.80
2	-0.48	-0.50	0.19	0.43	0.47	1.15	0.60	0.73	0.85	0.73
3	-0.44	-0.49	0.21	0.41	0.45	1.15	0.59	0.69	0.75	0.68
4	-0.38	-0.39	0.04	0.13	0.34	0.93	0.54	0.68	0.76	0.74
Big	-0.26	-0.35	-0.25	-0.21	-0.21	0.44	0.43	0.45	0.44	0.45
	RMW					*Std (RMW)*				
Small	0.18	-0.11	-0.05	0.18	-0.02	0.81	0.47	0.75	0.69	0.55

2	0.08	-0.16	-0.09	0.18	-0.04	0.72	0.45	0.69	0.64	0.51
3	0.19	-0.14	-0.04	0.23	0.02	0.70	0.44	0.67	0.57	0.48
4	0.16	-0.03	0.03	0.19	0.04	0.59	0.39	0.54	0.50	0.46
Big	-0.11	-0.10	-0.10	-0.08	-0.05	0.31	0.31	0.32	0.31	0.33
	CMA					*Std (CMA)*				
Small	0.15	-0.13	0.12	0.01	-0.10	0.92	0.50	0.60	0.74	0.62
2	0.19	-0.12	0.18	0.11	-0.05	0.82	0.47	0.57	0.68	0.57
3	0.14	-0.11	0.17	0.09	-0.05	0.82	0.47	0.54	0.61	0.53
4	0.14	-0.07	0.12	0.10	-0.07	0.64	0.43	0.47	0.55	0.51
Big	0.02	-0.01	0.04	0.02	0.01	0.35	0.34	0.35	0.35	0.34
	MKT					*Std (MKT)*				
Small	0.53	0.97	0.90	0.72	0.83	0.35	0.22	0.28	0.29	0.29
2	0.62	0.93	0.87	0.70	0.82	0.33	0.22	0.26	0.28	0.27
3	0.64	0.93	0.88	0.75	0.85	0.33	0.23	0.25	0.25	0.25
4	0.75	0.96	0.94	0.85	0.86	0.27	0.22	0.21	0.24	0.23
Big	0.85	0.86	0.85	0.84	0.85	0.16	0.16	0.17	0.16	0.16

In Table 16, each column of factor loadings is sorted from top to bottom based on size (small to Big), and every five rows are sorted based on the operating profitability. The smallest B/M ratio is marked as 1, and the most significant ratio is marked as 5. The tests on the 25 Size-OP portfolios show that the intercept of a moderately profitable investment portfolio is primarily negative, which does not explain abnormal returns well. In these portfolios, the factor loadings of *RMW* have a strong negative impact in the same cap, and all the *RMW* factor loadings remained negative, which contradicts the positive average return of the factors. The reason may be that the 5x5 construction method did not effectively strip out companies with better operating profitability. None of the factors can provide abnormal returns in the largest quintile, the same as in the Size-B/M portfolio. Notably, the small-cap size quintile generates three negative intercepts, and Fama (2015) proposed that the Size-Op portfolio does not isolate small stocks. Compared with Size-B/M, *CMA* factor loadings decreased without significant changes in other factor loadings. This indicates that companies with sure profitability in small caps but significant investments generate negative abnormal returns that the model cannot explain.

Table 16. Long-run Fama and French (2015) alphas and factor loadings of size-OP portfolios.

This table reports long-run alphas and factor loadings estimates from a conditional Fama and French (2015) model applied to 25 *size-OP* portfolios. At the end of each June, stocks are allocated to two Size groups using the median market cap as the breakpoint. Stocks are allocated independently to three book-to-market equity(*B/M*), operating profitability (*OP*), and investment (*Inv*) groups, using medians of size, *B/M*, *OP*, and *Inv* and the 30th and 70th stocks' percentiles. The LHS variables in each set of 25 regressions are the excess returns on the 25 *Size-B/M* portfolios. The RHS variables are the excess market return, $R_M - R_F$, the Size factor, *SMB*, the value factor, *HML*, the profitability factor, *RMW*, and the investment factor, *CMA*, constructed using independent 2×3 sorts on Size. $R_M - R_F$ is the value-weight return on the market portfolio of all sample stocks minus the benchmark interest rate of the bank's one-year fixed deposit. The long-run estimates, with standard errors in parentheses, are computed following Eq. (29) and Eq. (30) and estimates of conditional alphas and betas. The whole data sample is from Jan 2020 to Dec 2022.

OP	Low	2	3	4	High	Low	2	3	4	High
	Alpha					*Std (Alpha)*				
Small	0.07	0.01	0.00	0.03	0.03	0.00	0.00	0.00	0.00	0.00
2	0.01	-0.02	-0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00
3	0.04	-0.01	0.01	0.02	0.02	0.00	0.00	0.00	0.00	0.00
4	0.00	-0.03	-0.02	0.00	0.01	0.00	0.00	0.00	0.00	0.00
Big	0.00	0.00	0.00	0.00	-0.01	0.00	0.00	0.00	0.00	0.00
	SMB					*Std (SMB)*				
Small	1.45	1.13	-0.64	0.62	-0.23	3.38	3.01	2.16	2.25	2.60
2	1.57	1.24	-0.43	0.62	-0.08	2.63	1.99	2.07	1.95	2.40
3	1.54	1.16	-0.82	0.76	-0.11	2.54	1.47	1.93	1.88	2.14
4	1.19	0.70	-0.14	0.75	0.06	1.74	1.84	1.71	1.26	1.81
Big	-0.62	-0.48	-0.60	-0.64	-0.53	1.23	1.30	1.26	1.17	1.50
	HML					*Std (HML)*				
Small	0.17	-0.62	0.10	0.22	-0.43	1.23	1.23	0.86	0.78	0.95
2	0.17	-0.52	0.11	0.18	-0.40	1.05	0.79	0.74	0.71	0.84
3	0.13	-0.46	0.15	0.22	-0.39	1.02	0.65	0.69	0.69	0.84
4	0.03	-0.19	0.02	0.00	-0.45	0.70	0.69	0.64	0.44	0.65
Big	-0.26	-0.31	-0.23	-0.23	-0.23	0.44	0.44	0.44	0.42	0.56
	RMW					*Std (RMW)*				
Small	0.10	-0.28	-0.40	-0.01	0.36	0.98	0.77	0.50	0.59	0.71
2	0.15	-0.28	-0.38	0.01	0.34	0.69	0.46	0.53	0.51	0.64
3	0.15	-0.19	-0.30	0.03	0.35	0.67	0.40	0.50	0.49	0.52
4	0.23	-0.08	-0.18	0.06	0.31	0.43	0.48	0.44	0.32	0.32
Big	-0.10	-0.11	-0.10	-0.10	-0.04	0.31	0.32	0.32	0.30	0.38
	CMA					*Std (CMA)*				
Small	-0.11	-0.33	-0.24	-0.02	0.22	0.96	0.68	0.57	0.65	0.75
2	0.10	-0.22	-0.16	0.07	0.24	0.57	0.52	0.60	0.54	0.70
3	0.10	-0.22	-0.07	0.04	0.23	0.61	0.42	0.58	0.52	0.58
4	0.05	-0.10	-0.12	0.04	0.19	0.47	0.52	0.48	0.34	0.35
Big	0.02	-0.02	0.02	0.03	0.04	0.35	0.35	0.34	0.32	0.42
	MKT					*Std (MKT)*				
Small	0.80	0.84	0.94	0.87	0.76	0.40	0.31	0.23	0.26	0.30
2	0.74	0.81	0.92	0.86	0.76	0.26	0.21	0.26	0.23	0.27
3	0.77	0.85	0.92	0.86	0.77	0.27	0.19	0.24	0.22	0.25
4	0.91	0.89	0.95	0.92	0.78	0.20	0.22	0.20	0.16	0.16
Big	0.85	0.84	0.85	0.84	0.88	0.16	0.16	0.16	0.16	0.18

In Table 17, each column of factor loadings is sorted from top to bottom based on size (small to Big), and every five rows are sorted based on the investment. The most minor investment ability is marked as 1, and the largest is marked as 5. In size-inv, the intercept term decreases with increasing size, indicating that 'he model's explanatory power is rising, but there are no negative values. The *HML* factor loadings change from positive to negative as the size increases. This is because companies that invest aggressively are mostly growth stocks, while conservative ones are mostly value stocks. Meanwhile, this demonstrates that in size-inv, size and B/M characteristics are, to some extent, replaced by investment. All 25 portfolios show harmful exposure to *CMA*, just like companies that invest heavily despite low profitability, but the *CMA* slope is sufficient to explain their negative average returns.

Table 17. Long-run Fama and French (2015) alphas and factor loadings of size-Inv portfolios.

This table reports long-run alphas and factor loadings estimates from a conditional Fama and French (2015) model applied to 25 *size-Inv* portfolios. At the end of each June, stocks are allocated to two Size groups using the median market cap as the breakpoint. Stocks are allocated independently to three book-to-market equity(*B/M*), operating profitability (*OP*), and investment (*Inv*) groups, using medians of size, *B/M*, *OP*, and *Inv* and the 30th and 70th stocks' percentiles. The LHS variables in each set of 25 regressions are the excess returns on the 25 *Size-B/M* portfolios. The RHS variables are the excess market return, R_M-R_F, the Size factor, *SMB*, the value factor, *HML*, the profitability factor, *RMW*, and the investment factor, *CMA*, constructed using independent 2×3 sorts on Size. R_M-R_F is the value-weight return on the market portfolio of all sample stocks minus the benchmark interest rate of the bank's one-year fixed deposit. The long-run estimates, with standard errors in parentheses, are computed following Eq. (29) and Eq. (30) and estimates of conditional alphas and betas. The whole data sample is from Jan 2020 to Dec 2022.

Inv	Low	2	3	4	High	Low	2	3	4	High
	Alpha					*Std (Alpha)*				
Small	0.05	0.06	0.01	0.02	0.01	0.001	0.001	0.00	0.00	0.00
2	-0.01	0.02	-0.02	-0.01	0.02	0.00	0.00	0.00	0.00	0.00
3	0.04	0.02	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.00
4	0.01	0.01	0.01	-0.02	0.00	0.00	0.00	0.00	0.00	0.00
Big	-0.01	0.00	-0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00
	SMB					*Std (SMB)*				
Small	0.37	-0.18	0.07	0.56	0.39	2.88	2.79	2.14	2.22	2.80
2	0.55	0.00	0.39	0.66	0.55	2.41	2.68	2.09	2.06	2.50
3	0.04	0.26	0.26	0.61	0.62	2.51	1.90	1.58	2.03	2.38
4	0.48	0.12	0.39	0.61	0.44	2.11	2.11	1.79	1.77	2.13
Big	-0.65	-0.65	-0.55	-0.53	-0.44	1.29	1.28	1.17	1.26	1.24
	HML					*Std (HML)*				
Small	0.06	-0.18	0.16	-0.16	-0.53	1.06	1.05	0.89	0.81	1.15
2	0.05	-0.16	0.12	-0.11	-0.40	0.94	1.02	0.82	0.76	1.00
3	-0.15	0.12	0.12	-0.07	-0.41	0.95	0.76	0.63	0.73	0.96
4	-0.14	-0.31	0.05	-0.22	-0.23	0.84	0.76	0.78	0.64	0.83
Big	-0.23	-0.22	-0.22	-0.25	-0.33	0.46	0.45	0.42	0.43	0.44
	RMW					*Std (RMW)*				
Small	0.21	0.19	0.24	-0.03	-0.47	0.77	0.70	0.55	0.60	0.61
2	0.13	0.15	0.19	-0.05	-0.40	0.63	0.68	0.54	0.53	0.56
3	0.18	0.24	0.24	-0.01	-0.37	0.63	0.50	0.42	0.51	0.51
4	0.21	0.21	0.22	0.03	-0.18	0.54	0.53	0.44	0.46	0.50
Big	-0.10	-0.10	-0.04	-0.11	-0.10	0.32	0.31	0.30	0.31	0.32
	CMA					*Std (CMA)*				
Small	-0.07	0.12	0.09	-0.02	-0.42	0.84	0.75	0.62	0.62	0.67
2	0.02	0.17	0.17	0.04	-0.30	0.69	0.73	0.58	0.57	0.61

3	0.17	0.14	0.14	0.03	-0.29	0.68	0.54	0.45	0.57	0.58
4	0.01	0.13	0.11	0.01	-0.19	0.60	0.57	0.51	0.47	0.55
Big	0.02	0.02	0.04	0.03	-0.02	0.35	0.35	0.32	0.34	0.35
	MKT					*Std (MKT)*				
Small	0.69	0.59	0.84	0.95	0.99	0.36	0.31	0.27	0.26	0.29
2	0.66	0.59	0.83	0.91	0.95	0.31	0.30	0.26	0.24	0.26
3	0.63	0.84	0.84	0.92	0.96	0.28	0.24	0.20	0.24	0.25
4	0.82	0.73	0.86	0.97	0.98	0.30	0.28	0.22	0.21	0.24
Big	0.84	0.84	0.87	0.85	0.86	0.17	0.17	0.15	0.16	0.16

3.3.4 Constancy Tests of the Conditional Alphas

I assume that the parameter is equal to zero, and I examine whether the parameter is significantly higher or lower than this value; hence, this study uses a two-tailed test. Table 18 shows the constancy test for the portfolios. Surprisingly, all the portfolios reject the null hypothesis that alpha is constant. In all portfolios, the test statistic exceeds the critical value of 1.96 at the 5% level, and the time-variance of the alpha increases with size in all cases, showing that our model performs better on capturing the time-varying alpha for stocks above the medium cap. Among the 25 size-B/M portfolios, the extreme values (7.852, 8.315, 8.266, and 9.055) are found in the growth portfolios, excluding the largest size group, which suggests that the time-variation of growth stocks is more significant. In each quintile of the 25 size-OP portfolios, stocks with medium profitability have stronger alpha time-variation of 7.079, 7.796, 9.138, 8.313, and 10.403, respectively. In each quintile of the 25 size-Inv portfolios, the stocks with neutral and aggressive investments are better at capturing the time-varying nature of alpha. Contrary to Ang and Kristensen (2012), our study rejects the hypothesis that long-run alpha is constant. I attribute this, first, to the fact that the time-variation of alpha in the FF-3 model is now tapped by the *RMW* and *CMA* factors in the FF-5 model. Second, this represents that high-frequency factors can better capture time-varying alpha.

Table 18. Tests of conditional Fama and French (2015) alphas of Size-B/M, Size-OP, and Size-Inv portfolios.

This table reports test statistics in Eq. (40) of tests of constancy of conditional alphas loadings from a dependent Fama-French five-factor model. The long-run estimates, with standard errors in parentheses, are computed following Eq. (29) and Eq. (30) and estimates of conditional alphas and betas. At the end of each June, stocks are allocated to two Size groups using the median market cap as the breakpoint. Stocks are allocated independently to three book-to-market equity(*B/M*), operating profitability (*OP*), and investment (*Inv*) groups, using medians of size, *B/M*, *OP*, and *Inv* and the 30th and 70th stocks' percentiles. The LHS variables in each set of 25 regressions are the excess returns on the 25 *Size-B/M* portfolios. The RHS variables are the excess market return, R_M-R_F, the size factor, *SMB*, the value factor, *HML*, the profitability factor, *RMW*, and the investment factor, *CMA*, constructed using independent 2×3 sorts on Size. R_M-R_F is the value-weight return on the market portfolio of all sample stocks minus the benchmark interest rate of the bank's one-year fixed deposit. If the test statistic is greater than 1.96, I reject the hypothesis that alpha is constant at the 5% level. The test statistics are all absolute values. The whole data sample is from Jan 2020 to December 2022.

Panel A：Size-B/M	Low	2	3	4	High
Small	3.784	7.852	6.175	4.940	6.155
2	4.259	8.315	6.452	5.393	6.674
3	4.434	8.266	6.813	6.033	7.162
4	5.576	9.055	8.020	6.805	7.469
Big	10.160	9.747	9.988	10.246	9.690
Panel B：Size-OP	Low	2	3	4	High
Small	3.954	5.953	7.079	5.909	5.497
2	5.166	8.279	7.796	6.029	5.944
3	5.670	6.701	9.138	6.826	6.170
4	6.647	7.634	8.313	6.757	7.086
Big	10.076	9.905	10.403	10.364	10.635
Panel C：Size-Inv	Low	2	3	4	High
Small	4.508	6.636	5.821	6.636	6.436
2	5.276	4.659	5.994	7.310	6.556
3	5.066	5.776	6.546	6.005	7.156
4	6.545	6.211	7.275	8.688	7.317
Big	10.007	10.138	10.654	10.460	10.015

Table 19 shows *W* test (Ang and Kristensen, 2012) statistics of tests of constancy of conditional alphas from a conditional Fama and French (1993) model. Unlike Table 15, I test the time-varying nature of all observations.

Table 19. Tests of conditional Fama and French (1993) alphas of Size-B/M.

This table reports long-run alphas and factor loadings estimates from a conditional Fama and French (1993) model applied to 25 *size-B/M* portfolios. At the end of June each year, stocks are allocated to five *Size* groups (Small to Big) using market cap breakpoints. Stocks are also allocated independently to five *B/M* groups (Low *B/M* to High *B/M*). The intersections of the two sorts produce 25 *Size-B/M* portfolios. The LHS variables in each set of 25 regressions are the excess returns on the 25 *Size-B/M* portfolios. The RHS variables are the excess market return, R_M-R_F, the *Size* factor, *SMB*, the value factor, *HML*. R_M-R_F is the value-weight return on the market portfolio of all sample stocks minus the benchmark interest rate of the bank's one-year fixed deposit. The long-run estimates, with standard errors in parentheses, are computed following Eq. (20) and Eq. (21) and estimates of conditional alphas and betas. The whole data sample is from Jan 2020 to Dec 2022.

Panel A：FF-3	Low	2	3	4	High
Small	0.005**	0.004**	0.005**	0.003**	0.003**
2	0.004**	0.003**	0.002**	0.002**	0.005**
3	0.004**	0.002**	0.003**	0.002**	0.008**
4	0.004**	0.003**	0.003**	0.002**	0.007**
Big	0.005**	0.004**	0.004**	0.005**	0.01**

The results show that the original hypothesis that alpha is constant is rejected at the 5% level for all observations. Compared to the results of Fama and French (2015), the difference in our results is smaller, but I similarly find a trend of higher returns between high *B/M* (value stocks) and low *B/M* (growth stocks), especially for smaller firms. The extreme value (0.0008) occurs for high-value medium-sized companies. Compared to the results of the FF-5 model, the alpha value of FF-3 is smaller, which suggests that there are fewer unpriced risk factors, an approach that is more applicable to the FF-3 model.

In Table 20, I report long-run estimates of alphas for each portfolio and the 10–1 momentum strategy. The research results show that for the ten decile momentum portfolios, I can reject the null hypothesis that all long-run alphas are equal to zero, as the p-value for rejection is zero. The long-run alphas range from −0.985% with a standard error of 0.0003 for the first loser decile to 0.922% with a standard error of 0.0001 to the tenth loser decile. If the positive intercept of the winner portfolio and the negative intercept of the loser portfolio are statistically significant, it may indicate the presence of systematic mispricing in the market. Therefore, I conclude that the conditional four-factor model fails to price the momentum portfolios.

Table 20. Tests of conditional Carhart four-factor model (1997) alphas of momentum portfolio

This table reports estimates of long-run alphas and factor loadings from a conditional Carhart four-factor model (1997) applied to decile momentum portfolios and the 10–1 momentum strategy. The RHS variables are the excess market return, R_M-R_F, the *Size* factor, *SMB*, the value factor, *HML,* the momentum factor, *MOM*. R_M-R_F is the value-weight return on the market portfolio of all sample stocks minus the benchmark interest rate of the bank's one-year fixed deposit. The long-run estimates, with standard errors in parentheses, are computed following Eq. (26). The whole data sample is from Jan 2020 to December 2022.

Decile	Alpha	Std	p-value
Loser	-0.98	0.000336	0.00
2	-0.47	0.000077	0.00

3	-0.29	0.000061	0.00
4	-0.16	0.000052	0.00
5	-0.05	0.000048	0.00
6	0.05	0.000048	0.00
7	0.17	0.000052	0.00
8	0.29	0.000060	0.00
9	0.47	0.000076	0.00
Winner	0.92	0.000129	0.00

3.3.5 Comparison of OLS and Nonparametric Alphas

Figure 3 shows the differences of alpha in size-B/M, size-op, and size-inv based on OLS and nonparametric. I can see that among the three groups, nonparametric alphas are closer to 0. Only in size-inv, nonparametric alphas have some outliers less than 0. I use the exact data for OLS regression, and the results show that almost all intercept terms were insignificant . Compared to the results obtained by Gallagher (2015) using the plug-in method, our nonparametric alpha has very few outliers and is more stable, indicating that our model is better at processing high-frequency data. Secondly, compared with traditional FF-5 factor studies (Fama and French, 2015, 2017), the values obtained by both methods are very close to 0, which is beneficial from the factors I constructed using high-frequency data. The high-frequency stock market data contains more detailed market trading information, allowing investors to conduct a more detailed analysis of the market. This is one of the significant advantages of financial high-frequency data compared with low-frequency data. In addition, high-frequency data contains more micro noise, and the microstructure noise offset by low-frequency data will be highlighted in high-frequency data (Bandi and Russell 2006, 2008).

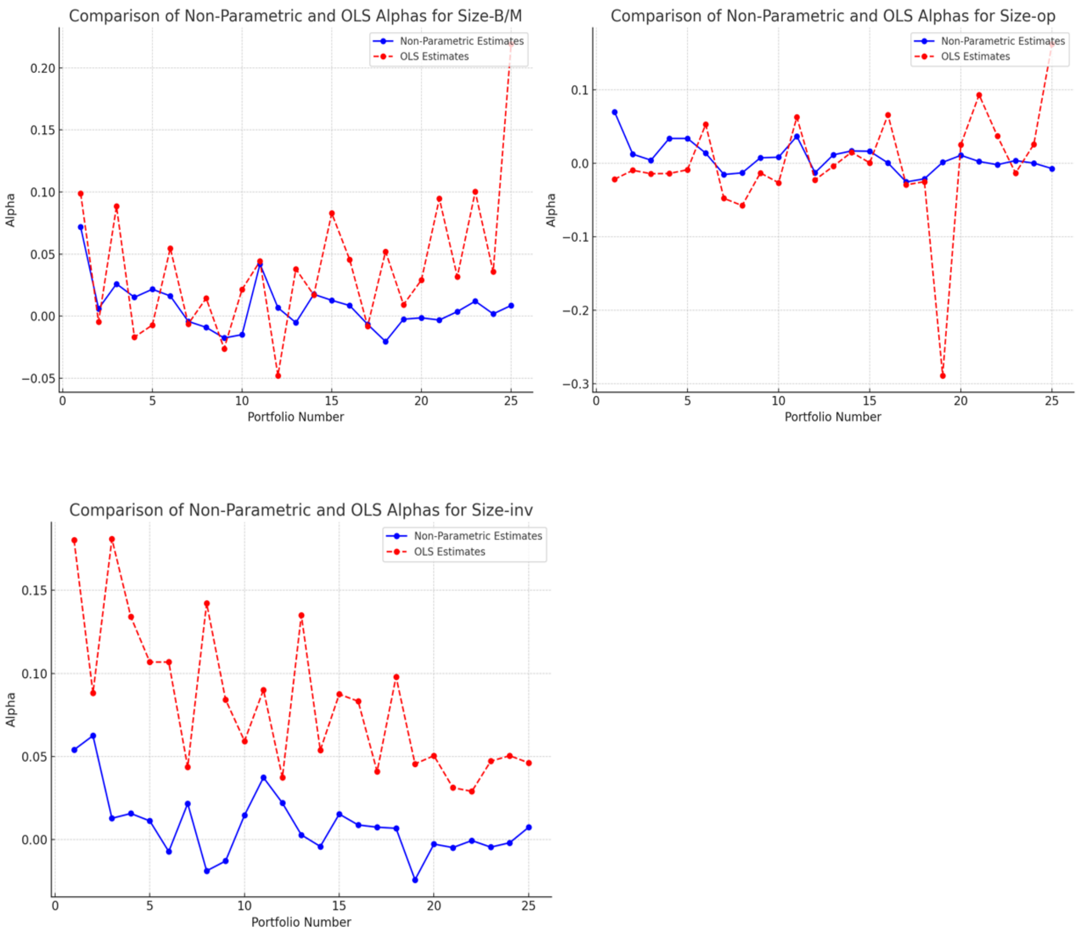

Figure 3: Comparison of Long-Run and OLS Alphas in the Fama-French Five-Factor Model.

Note: Long-run alphas versus ordinary least-squares alphas in the Fama and French (2015) model for the Size-B/M, Size-OP and Size-INV, and each sort contains 25 portfolios. I plot the long-run alphas using blue line to represent nonparametric alphas and red line to represent OLS alphas. The sampling period is January 2020–December 2022.

Chapter 4. Stock Return Prediction with TCN-LSTM Network

Forecasting stock return is essential in asset pricing research. Traditional asset pricing models, such as the Fama-French five-factor model, struggle with financial market complexity. This study applies CNN, LSTM, CNN-LSTM, and TCN-LSTM to predict stock returns using 30-minute frequency data of Dow Jones Industrial Average stocks (2021-2024). First, I use TCN-LSTM to predict the left-hand returns of the FF-3 and FF-5 models respectively. The results show that TCN-LSTM is more suitable for predicting the returns of FF-5. Then, I use four deep learning methods to predict the left-hand returns of the FF-5 model and compare the performance of different methods. A comparison between TCN-LSTM and other traditional deep learning methods shows that CNN-LSTM achieves the best overall performance, effectively balancing feature extraction and time-series modelling. LSTM tends to overfit the training data, leading to reduced generalisation on unseen data, while CNN demonstrates better generalisation despite higher training errors. TCN-LSTM, although capable of capturing temporal dependencies, exhibits higher overall prediction errors, suggesting limitations in modelling long-term financial trends.

4.1 Introduction

Before neural networks become widely popular, linear regression methods such as GARCH and ARIMA are predominantly used for modelling and predicting stock prices, achieving relatively good forecasting results (Box and Jenkins, 1976; Bollerslev, 1986). Traditional linear models have numerous advantages, including simplicity, ease of use, and strong interpretability regarding stock prices. However, in reality, stock prices are influenced by multiple factors, including unexpected events, indicating that stock markets possess both linear and nonlinear characteristics. Consequently, using traditional linear models alone to analyse stock prices and markets remains inadequate.

Neural networks, being inherently nonlinear models, have complex stacked structures that enable them to recognise the nonlinear characteristics of stock markets compared to linear models, thus providing more accurate modelling and prediction. As the depth of neural networks increases, their ability to combine and extract features from input data continually improves, giving them an advantage in analysing noisy, nonlinear financial time series.

Among current studies employing nonlinear methods for stock price prediction, traditional machine learning technique such as Naive Bayes, Random Forests, and Support Vector Machines (SVM) are most frequently applied (Huang et al., 2005; Patel et al., 2015; Ballings et al., 2015; Nti et al., 2020). These machine learning methods are widely used not only in financial time series analysis but also broadly in other time series modelling contexts. As a branch of machine learning, deep learning-based neural network models primarily focus on image, audio, and text classification tasks, initially with only Backpropagation (BP) neural networks widely used for financial time series forecasting. Among common deep learning models, RNN[5] and LSTM[6] are more suitable for modelling time series compared to TCN and CNN, thus substantial research focuses on RNN and its variant LSTM. Research combining convolutional networks with LSTM architectures is becoming increasingly mainstream.

A large number of experimental results indicate that financial markets are predictable to a certain extent (Bustos, 2020; Thakkar, 2021). The traditional prediction methods are mainly based on statistical models. In 1970, Box and Jenkins propose the autoregressive integrated moving average (ARIMA) model, which is widely used in financial time series forecasting. In 1976, statisticians Box and Jenkins provide a detailed explanation of the autoregressive moving average (ARMA) model and elaborate on the modeling principles and steps. In 1982, Engle proposes the autoregressive conditional heteroskedasticity (ARCH) model, which details the theory and methods of the ARCH model. This method makes outstanding contributions to predicting volatility in financial markets.

In 1986, Taylor proposes the Stochastic Volatility (SV) model, which is close to the model used in financial theory to represent financial price behavior. In the same year, Bollerslev propose the generalized autoregressive conditional heteroskedasticity (GARCH) model, which is obtained by improving the ARCH model. The GARCH model can provide dynamic predictions, with a narrower range during static periods and a wider range during fluctuating periods. Nelson (1991) study improves the GARCH model and names it the Exponential Generalized Autoregressive Conditional Heteroscedasticity (EGARCH) model, which fully explains the leverage effect in the stock market. With the development of computing and the increasing availability of data features, factor models start to be applied in the field of financial forecasting. Subsequently, more and more statistical models are widely applied in the field of financial time series prediction. Khan et al. (2022) find that the ARIMA model has the potential for short-term prediction.

Traditional stock prediction research mainly uses technical analysis (analysing market

[5] Recurrent neural networks (RNNs) are a class of neural networks designed to process sequential data by maintaining a dynamic memory of past information through hidden states.

[6] Long short-term memory (LSTM) networks are an extension of RNNs that address the vanishing gradient problem by introducing gated structures to preserve long-term dependencies.

price movements through historical charts), fundamental analysis (analysing macroeconomic conditions, industry conditions, company operating conditions, etc. that determine the intrinsic value of stocks and affect stock prices), and statistical models to predict stock price trends. Technical analysis focuses on forecasting stock price movements through the study of historical price and volume patterns (Murphy, 1999). Fundamental analysis evaluates macroeconomic indicators, industry trends, and firm-specific financials to estimate intrinsic value (Graham and Dodd, 2009). Statistical models such as ARIMA and GARCH are widely used to capture time-series patterns in financial data (Tsay, 2005). However, in the face of the increasingly growing stock market and the massive amount of data it exists, relying solely on traditional technical analysis cannot meet the demand of the market.

With the continuous development of financial theory and the advent of the big data era, machine learning models gradually become the mainstream analysis method for stock price prediction. Machine learning, first proposed by Arthur Samuel (1959), who uses a certain mapping to convert input data into output data, allowing for the exploration of historical stock price patterns, regression prediction of stock price time series, and price trend judgment. Currently, commonly used machine learning models in stock price prediction include Support Vector Machines (SVM), Support Vector Regression (SVR), Random Forests, Decision Trees, Naive Bayes, BP neural networks in Artificial Neural Networks (ANN), and Hidden Markov Models (HMM).

Vapnik (1999) proposes the SVM method, which is a new method developed on the basis of statistical learning theory and solves complex problems that general linear models cannot handle. Strong generalization ability, high fitting accuracy, good predictive performance, suitable for limited sample problems, effectively promoting the development of machine learning theory. Fan (2001) is the first to apply this method to investment decision-making. They conduct in-depth analysis of stock price trends in the US financial market and established an effective investment decision-making model using support vector machines. Muller et al. (1997) find that SVM can be applied to high-dimensional, complex data problems. And it can effectively solve the overfitting situation in machine learning methods, which is currently used in many aspects. Wang (2010) uses methods for handling imbalanced samples at both the data and algorithm layers, using cost-sensitive algorithms on the dataset, and selecting SVM with soft intervals on the model. The results show that the classification of this integrated model exceeds expectations, improving the recognition accuracy of minority class samples and also significantly enhancing the predictive performance of the model.

Nayak et al. (2015) propose a hybrid framework of SVM and the k-nearest neighbor algorithm for predicting the Indian stock market. Compared with the FLIT2NS and CEFLANN models, their results show that the hybrid model has better adaptability to high-dimensional data, effectively balances classifier complexity and error, and has good predictive ability. Yuxiang et al. (2022) combine the Autoregressive Integrated Moving Average (ARIMA) model with SVM to predict crude oil prices, demonstrating that the SVM model can effectively predict nonlinear and complex data such as crude oil prices through self-updating and combining with other models.

Breiman (2001) first proposes using the random forest algorithm for stock selection, and the results showed that the random forest algorithm, as an ensemble learning algorithm, has good functions in stock selection, opening up a new path for subsequent scholars to study. Booth (2013) first applied the performance weighting of random forest to predict stock price returns, and the results showed that the random forest model outperformed all other models in terms of prediction accuracy and profitability.

Michel (2017) establishes random forest models and Adapost models for comparison, and the research results showed that the random forest model performed the best in predicting stock prices and was most suitable for stock selection. Krauss (2016) constructed stock selection models based on decision trees, random forests, and neural networks using S&P 500 index components from 1992 to 2015. The results showed that the random forest model performed the best among all models.

Gu et al. (2020) conduct a comprehensive study on empirical asset pricing using 60 years of US stock data, comparing traditional linear models with various machine learning approaches. They find that regression trees and neural networks offer superior out-of-sample performance, with shallow models outperforming deep models under low signal-to-noise conditions. Price-trend indicators such as momentum and yield reversal emerge as the most predictive, followed by liquidity and volatility factors. Complementing this, Wang et al. (2020) propose a hybrid DWT-ARIMA-GSXGB model that effectively decomposes and forecasts stock index prices. Other studies apply hidden Markov models (HMMs) for stock prediction, with enhanced versions integrating trading data and news to address data sparsity and improve accuracy (Zhang et al., 2018).

Recent studies demonstrate the growing application of machine learning and deep learning models in stock prediction. Ji (2021) improves the generalisation ability of backpropagation networks using particle swarm optimisation. Bonne et al. (2021) enhance the interpretability of factor models by modelling nonlinear residual returns separately. Deep learning models such as CNNs, RNNs, LSTMs, and GRUs have shown strong learning capacity and feature extraction performance compared to shallow models like decision trees and SVMs (LeCun, Bengio, and Hinton, 2015; Schmidhuber, 2015). To improve temporal data modelling, LSTM and Temporal Convolutional Networks (TCNs) have been widely adopted.

Empirical research supports the superiority of deep architectures. Jiang et al. (2020) use CNNs to extract K-line chart features and enhance Sharpe ratios. Yan et al. (2021) and Gao et al. (2020) find LSTM and attention-based networks outperform linear and tree-based models. Wu et al. (2018) combine AdaBoost with LSTM to achieve superior directional and horizontal prediction accuracy. Zhao et al. (2021) propose an LSTM-CNN-CBAM model incorporating attention mechanisms to boost prediction on the Chinese stock market. Similarly, Kim et al. (2019) fuse image and time-series features using an LSTM-CNN framework to optimise trading strategies.

Several hybrid models further enhance accuracy and robustness. Patra et al. (2022) combine LSTM and GRU, while Kumar et al. (2020) introduce RNN-LSTM frameworks with swarm intelligence algorithms. Li et al. (2019) use LSTM within a GAN-based anomaly detection model (MAD-GAN) to capture temporal dependencies. For long-horizon learning, TCN architectures—such as ED-TCN and dilated TCN—employ convolutional filters and skip connections to improve sequence modelling (Lea et al., 2016; Gopali et al., 2021). He and Zhao (2019) report the results of using TCN for time series anomaly detection in their study, in which they fitted prediction errors using multivariate Gaussian distributions and uses these errors to calculate anomaly scores. However, they did not compare the performance of TCN with some other deep learning-based anomaly detection methods, which raises questions about whether TCN is superior to other technologies.

The structure of this chapter is organized as follows. Section 4.2 outlines the methodology, detailing the architectures of TCN, LSTM, and the proposed TCN-LSTM model, along with performance evaluation metrics. Section 4.3 presents the empirical results and discusses the predictive performance of different deep learning models.

4.2 Methodology

4.2.1 The Architecture of TCN

The Temporal Convolutional Network (TCN), also known as causal convolutional networks, is characterized by maintaining the causality of data, which theoretically can greatly enhance the accuracy of model predictions. TCN is a novel predictive algorithm developed from CNN to address time series problems and is an improvement upon the traditional CNN architecture. Compared to RNN architectures, TCN primarily modifies the receptive field by increasing the number of network layers, adjusting the dilation factor, and varying the filter sizes, which allows for better memory management and control over the model memory length. Additionally, because the back-propagation path of TCN differs from the temporal direction of sequences, it can avoid the issues of vanishing and exploding gradients commonly found in RNNs. In contrast to typical CNN architectures, which generally have fewer convolutional layers, TCN can continually increase the number of convolutional layers, resulting in larger convolutional kernels that capture long-term dependencies in the sequence, thus enabling more comprehensive information extraction.

The construction of the TCN model is shown in Fig.4. Causal convolution is first proposed in Wave Net (2016), which can predict the output at time $t+1$ using input data before time t. Its characteristic is that by zero padding the beginning of the input sequence, the network output length can be kept equal to the input length, ensuring that future information does not affect the current output.

The receptive field of basic causal convolutions increases linearly with the number of network layers. Thus, stacking more convolutional layers can expand the model receptive field. However, increasing the number of convolutional layers can lead to certain issues: excessive network depth, too many parameters, gradient vanishing, complex training, and poor fitting performance. Dilated convolutions effectively address these problems.

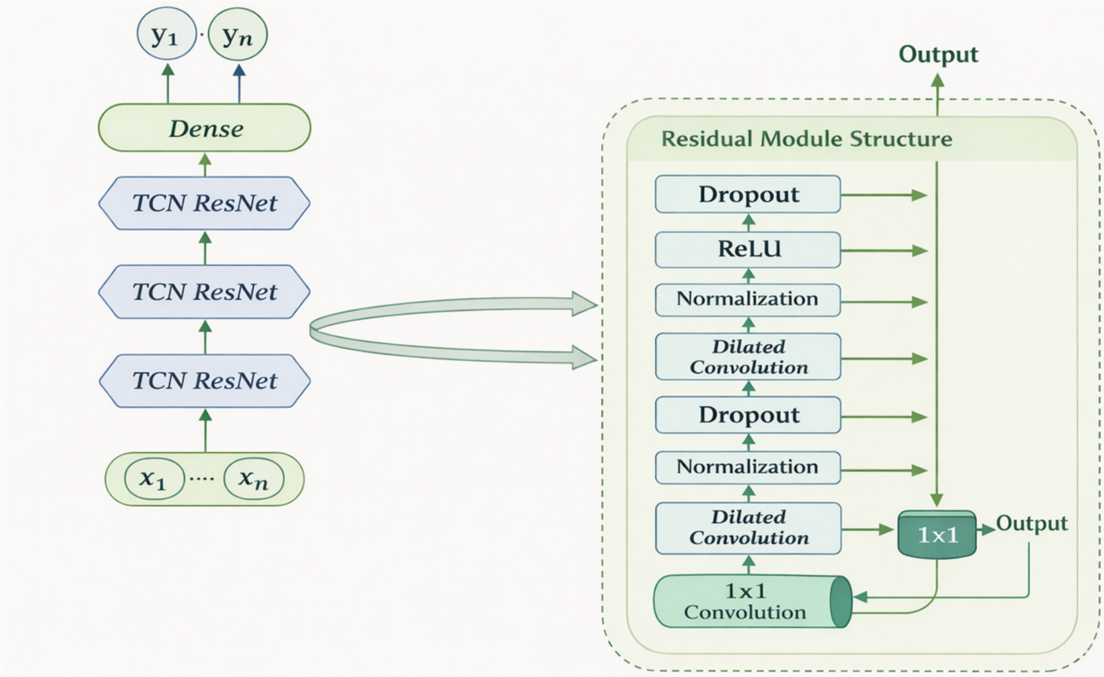

Figure 4. TCN model structure.

The defining feature of dilated convolutions is that they allow for an increase in the receptive field by adding layers and enlarging the size of the convolutional kernels. As you progress to higher layers, the convolutional kernels become larger, resulting in a broader range of output information. This ensures that the filters can capture a wider range of input features, efficiently acquiring information over long time spans. After introducing dilated convolutions, the receptive field can be expressed as:

$$RF_L = 1 + (k-1) \times \sum_{i=0}^{L-1} d_i. \tag{45}$$

Among them, k is the kernel size and d_i the expansion factor of the i-th layer causal convolution, usually denoted as $d_i = b^i$, which b^i is the dilation base. This design ensures that the receptive field expands exponentially with depth, enabling the model to integrate information over very long-time spans even with small kernels.

The TCN network is composed of multiple residual connections modules. Each residual block includes four parts: Dilated Causal Conv, Normalized Weight Norm, ReLU, and Dropout.

The Weight Norm layer is mainly used to speed up model training and improve model accuracy; The ReLU activation function is added after two convolutional layers, which can effectively avoid phenomena such as gradient vanishing in the model; Randomly inactivate neurons through Dropout after each convolutional layer to prevent overfitting of the model; Adding convolution can adjust the width of the residual tensor and solve the problem of dimensional differences that may occur between input and output. The process of features extraction by TCN has three steps.

At first, this study constructs a relatively simple yet reasonable TCN prediction model based on the basic principles of Time Convolutional Neural Network (TCN) and optimised model parameters to extract features from excess return sequence data. By comparing and analysing the TCN model with other common traditional prediction models, the advantages of the TCN model are highlighted.

Secondly, considering issues such as irrelevant features and feature dimension explosion that can affect the computational efficiency and predictive performance of the model, it is necessary to appropriately screen and reduce the dimensionality of the considered feature variables. By removing the influence of unimportant features on the model results, the prediction accuracy and efficiency of the model can be further improved.

Thirdly, starting from the idea of combination prediction, the TCN and LSTM networks are integrated to enhance the structure of the basic TCN model. This integration is achieved by connecting LSTM and TCN in an end-to-end manner.

4.2.2 The Architecture of LSTM

Deep learning is a method that, based on the input of original data, gradually extracts high-level features, implicit information, etc. from the data by using multiple hierarchical approaches, so as to build a model for prediction tasks. For example, in graphic and image processing, details such as edges can be identified at a lower level, while other information, such as letters and numbers, can be extracted at a higher level. The "depth" refers to the number of levels for data transformation and processing.

Most of the current deep learning models are based on Artificial Neural Networks (ANN).

An Artificial Neural Network is composed of interconnected units called neurons. These units are activated according to the input data and mimic the human brain during the information processing, interacting with other feature processing units. The sigmoid function is used as the transfer function at each hidden layer. The entire network uses the gradient descent method with momentum to adjust the weights of each layer to optimize the parameters and construct an appropriate prediction model.

Traditional Neural Networks (NN) have defects in information utilization, that is, they cannot effectively use previous information to predict future tasks. Recurrent Neural Network (RNN) is a deep learning model that can use internal states to store information and deal with time series. However, the RNN itself does not have the ability to learn long-term dependencies, cannot effectively use information that is too far from the current time point, and the Recurrent Neural Network is often affected by short-term memory. For long sequences, especially in time series prediction modeling and text analysis, the RNN will encounter the problem of gradient disappearance during the backpropagation process. If the gradient value shrinks to a very small value, the RNN will be unable to learn the sequence, so it has short-term memory. In order to deal with the gradient disappearance problem of the Recurrent Neural Network, the Long Short-Term Memory (LSTM) neural network model, which uses storage units and gate structures, has been proposed.

LSTM is a network model proposed by Schmidhuber et al. (1997). LSTM is a network model designed to solve the longstanding problems of gradient explosion and gradient disappearance in RNN. LSTM networks can effectively avoid the vanishing gradient problem that often plagues traditional RNNs. In scenarios with long time sequences, RNNs experience multiple instances of gradient accumulation. If the gradients are often negative, this can lead to the subsequent gradients vanishing; conversely, if the gradients remain large, it can result in gradient explosion.

LSTM addresses this by incorporating a memory cell and gating mechanisms, where some multiplicative operations are replaced with additive ones. This allows gradients to flow more stably through the network, leading to more reasonable gradient changes over time. However, the trade-off is that LSTMs have a more complex architecture, increasing computational demands and making parallel processing more challenging.

Wang and Chen (2024) state that the core of LSTM is the memory unit, which enables the network to retain information between multiple time steps. As Fig.5 shown, it consists of three specialized gating structures: input gate, forget gate, and output gate, which collectively manage the information flow of the unit. These gates allow LSTM to add, discard, or pass information, ensuring that only relevant data affects predictions. By utilizing these capabilities, LSTM provides a powerful tool for stock price prediction, which can effectively analyze financial time series data and identify meaningful patterns before major market fluctuations.

The LSTM calculation process is as follows: The output from the previous time step and the input at the current time step are fed into the forget gate. After computation, the forget gate produces an output, expressed by the following equation:

$$\mathcal{F}_t = \sigma(W_f \cdot [h_{t-1}, x_t] + b_f). \tag{46}$$

Where the value range of $\mathcal{F}_t$ is (0,1), W_f is the weight of the forget gate, and b_f is the bias of the forget gate, x_t is the input value of the current time and h_{t-1} is the output value of the last moment.

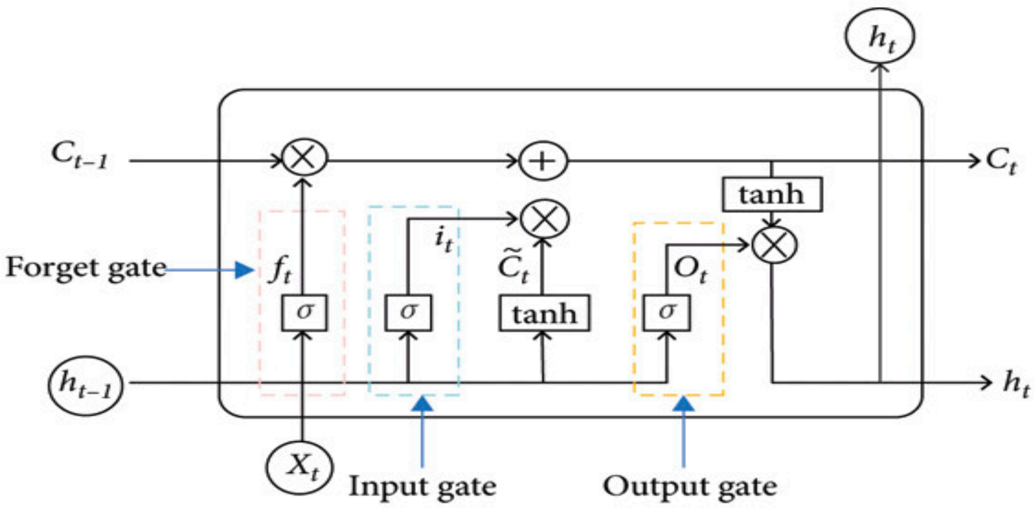

Figure. 5. LSTM Model Structure.

The output from the previous time step and the input at the current time step are fed into the input gate. After computation, the output of the input gate and the candidate cell state are obtained, as shown in the following equations:

$$i_t = \sigma(W_i \cdot [h_{t-1}, x_t] + b_i) . \tag{47}$$

$$\tilde{C}_t = tanh(W_c \cdot [h_{t-1}, x_t] + b_c). \tag{48}$$

Where the value range of i_t is (0,1), W_i is the weight of the input gate, b_i is the bias of the input gate, W_c is the weight of the candidate input gate, and b_c is the bias of the candidate input gate. Update the current cell state as follows:

$$C_t = f_t * C_{t-1} + i_t * \tilde{C}_t . \tag{49}$$

The output h_{t-1} and input x_t are received as input values of the output gate at time t, and the output o_t of the output gate is obtained as follows:

$$o_t = \sigma(W_o \cdot |h_{t-1}, x_t| + b_o) . \tag{50}$$

Where the value range of o_t is (0,1), W_o is the weight of the output gate, and b_o is the bias of the output gate. The output value of LSTM is obtained by calculating the output of the output gate and the state of the cell, as shown in the following formula

$$h_t = o_t * tanh(C_t). \tag{51}$$

Overall, the advantage of the LSTM model lies in its three gate structures, based on activation functions, which regulate the flow of information in and out. Through this filtering process, it selects more relevant historical data, enabling the model to effectively learn sequential dependencies in prediction tasks, thereby improving its forecasting performance.

4.2.3 The Architecture of TCN-LSTM

As Fig.6 shown, the TCN model extracts past data through one-dimensional causal convolution to ensure temporal order. Residual connections accelerate convergence speed, while dilated convolutions enable the extraction of temporal features. The ReLU function is used for mapping, and sequence features are derived after computation. The TCN layer captures more comprehensive sequence features through dilated and causal convolution operations, enabling it to extract extended information dependencies.

The output from the TCN layer serves as the input to LSTM network layer, which further extracts features while preserving those obtained from the TCN, and then combines them with the features captured by the LSTM. This approach helps prevent feature degradation. The fused features are then used as the input for the fully connected layer, with the Dropout mechanism applied to mitigate overfitting. As a nonlinear model, LSTM model can be used as a complex nonlinear unit to construct a wider range of deep neural networks, with nonlinear fitting ability, thus effectively extracting data features (Huo et al, 2022).

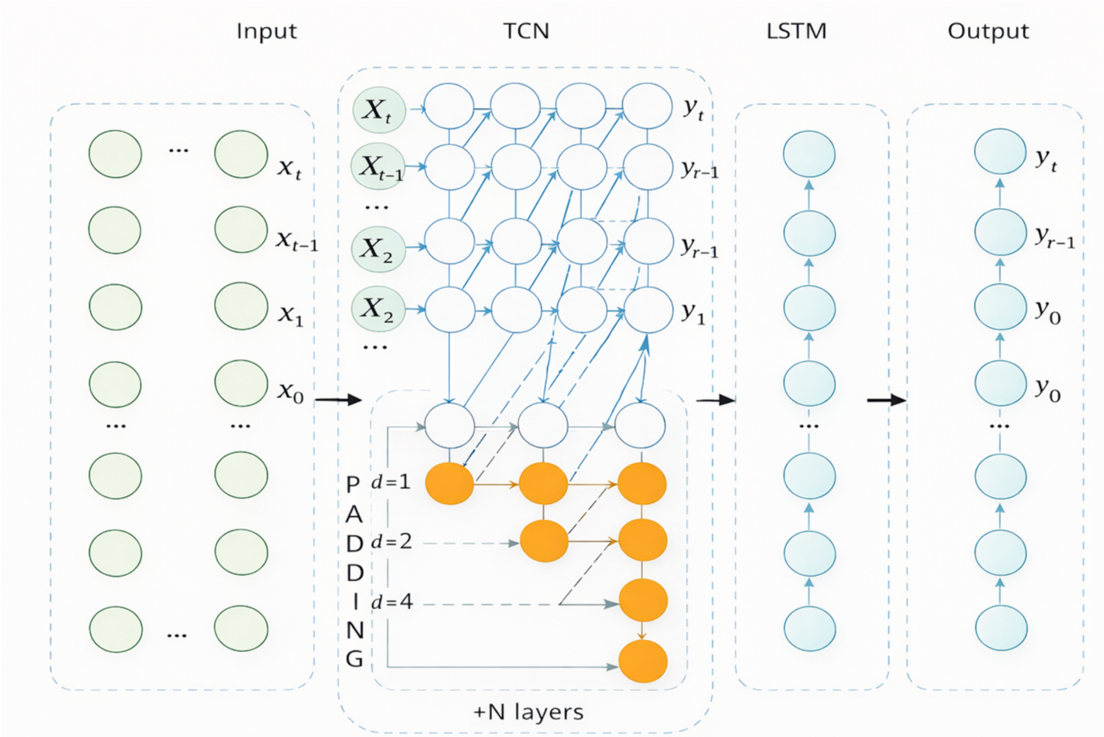

Figure. 6. Connection of TCN-LSTM

Before training the model, some basic parameters need to be set, including window_size, batch_size, and epoch. The temporal convolutional neural network falls under the category of deep learning, and it typically uses the gradient descent algorithm for model optimization. The parameter batch_size refers to the size of each batch of data, that is, the number of samples trained in one iteration. The parameter update is completed by calculating the average loss function value of these samples. Generally, the frequency of parameter updates should be inversely proportional to the size of the batch_size.

However, to prevent high randomness in the batch, which can make the model difficult to converge, batch_size should not be set too small. On the other hand, the larger the batch_size, the more accurate the gradient descent direction, and the faster the training speed. However, if batch_size is too large, it can reduce randomness, leading to a single direction in gradient descent, resulting in a local optimum instead of a global optimum. To determine an appropriate batch_size, considering prior experience, experimental data, and experimental environment constraints, batch_size is set to 32.

The parameter epoch refers to a complete training cycle of the entire training dataset. A higher number of epochs does not necessarily lead to better results; too few epochs can lead to insufficient training and failure to converge, while too many epochs can result in overfitting, reducing the model's generalisation ability. Additionally, different window_size values will produce different stock price prediction results. Window_size represents the time step, which means predicting the stock price at time step window_size + 1 based on the data of the past window_size time steps. Considering the original data time span, window_size is set to 10.

4.2.4 Performance Evaluation

To measure the prediction accuracy of each model after training, this chapter selects the following commonly used model evaluation metrics in deep learning. The calculation methods are as follows:

Root Mean Square Error:

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}. \tag{52}$$

Root Mean Squared Error (RMSE) is used to assess the square root of the average of squared differences between predicted and actual values, a method widely recommended in forecasting accuracy research (Chai and Draxler, 2014).

Mean Absolute Error:

$$MAPE = \frac{1}{n}\sum_{i=1}^{n}|y_i - \hat{y}_i| \times 100. \tag{53}$$

Mean Absolute Percentage Error (MAPE) is employed to express the prediction error as a percentage of the actual values, following the guidelines for forecast evaluation outlined by Hyndman and Koehler (2006).

Residual Predictive Deviation:

$$RPD = \frac{\sigma_y}{\sigma_e}. \tag{54}$$

Where σ_y denotes the standard deviation of the observations and σ_e denotes the standard deviation of the prediction error. The larger the RPD value, the better the predictive ability of the model. Residual Predictive Deviation (RPD) is introduced to assess the reliability of predictive models, with a value greater than 2 indicating good performance, according to Williams and Sobering (1993).

Mean Squared Error:

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2. \quad (55)$$

The y_i is actual value and the $\hat{y}_i$ is observed value. MSE measures the deviation between the prediction result and the real value by calculating the square of the error, which can effectively reflect the fitting ability of the model. It is representing the average of the squared errors, is also considered due to its effectiveness in penalising large deviations, further emphasised by Chai and Draxler (2014).The smaller the MSE value, the lower the prediction error of the model, and the better the fitting effect.

4.3 Results and Discussion

In this section, I present the empirical results of the Fama-French five-factor model using high-frequency data. The analysis begins with tests of the statistical properties and correlations among the factors, followed by an evaluation of the predictive performance of four deep learning models—CNN, LSTM, CNN-LSTM, and TCN-LSTM—through a comparison of forecasting errors across training and testing datasets.. This chapter focuses on predicting future stock returns rather than price levels. Specifically, the prediction target is the excess return of the test portfolio at time t+1, while the input information set consists of variables observed up to time t. This setup ensures that the prediction exercise is strictly out-of-sample and avoids any look-ahead bias.

4.3.1 Data Description

This chapter selects the excess return of an extreme portfolio constructed through independent double sorting on size and book-to-market (B/M) ratio—specifically, the portfolio consisting of the smallest size and the lowest B/M ratio among the constituents of the Dow Jones Industrial Average—as the core dependent variable. The portfolio is constructed following the procedure: at each time point, within the constituent stocks, independent quantile sorts are performed separately on market capitalisation and B/M ratio, and the intersection of the smallest size group and the lowest B/M group forms the target portfolio, with constituent stocks weighted by market capitalisation.

This design aims to establish a prediction task with a transparent information environment and controlled confounding factors, thereby enabling a more reliable and interpretable evaluation of the forecasting capabilities of deep learning models (Angrist and Pischke, 2009). The portfolio carries clear economic significance: it represents exposure to the "growth" style within the large-cap stock market, characterised by both high growth expectations and high uncertainty. Its short-term return fluctuations reflect both the market's pricing of future growth potential and the reassessment of growth risks, thus providing a well-defined observational window for examining the high-frequency processing mechanisms of complex information (Barberis, Shleifer, and Vishny, 1998). Furthermore, this portfolio maintains consistent construction logic and interpretative foundations within both the Fama-French three-factor and five-factor model frameworks. This allows to systematically compare the incremental contribution of different factor model specifications to the predictive performance of deep learning models.

Table 21 presents the core parameters of the following four methods, and the selection of these parameters directly influences the learning ability, generalisation capability, and computational efficiency of the model. Firstly, the training set ratio is set to 0.7, ensuring the model has sufficient data for learning while retaining enough test data to assess the generalisation ability of the model. The 16 channels and 3×1 convolutional kernel, combined with dilated convolution, allow the model to effectively extract local features of time series data while preventing information loss due to a limited receptive field. The Dropout rate is set to 0.05, and L2 regularisation weight decay is set to 0.003, both working together to mitigate overfitting and enhance the stability of the model on the test set. The model employs only one residual block, ensuring the TCN structure remains shallow, preventing gradient vanishing issues while maintaining computational efficiency.

During optimisation, the initial learning rate is set to 0.01, with Adam used for gradient updates, enabling fast convergence. After 850 epochs, the learning rate is reduced to 1% of its original value, further refining the model and preventing oscillations around the optimal solution. Additionally, the batch size is set to 32, balancing computational resource efficiency and training stability, ensuring the smooth operation of the model. These parameter choices carefully address short-term feature extraction, long-term dependency modelling, computational resource utilisation, and overfitting prevention, allowing TCN-LSTM to perform effectively in financial time series forecasting while maintaining the stability and generalisation ability of the model.

The selection of these parameters is consistent with widely adopted practices in deep learning for time series modelling. The 70% training ratio is commonly used in financial forecasting tasks to balance model learning and validation (Zhang et al., 2017). A kernel size of 3 effectively captures short-range dependencies with reduced computational cost (Bai et al., 2018). Dropout and L2 regularisation are standard strategies to mitigate overfitting, especially in financial applications where data are often noisy and overparameterisation risks exist (Srivastava et al., 2014). Batch sizes of 32 and adaptive learning rates with decay are also frequently applied in sequence-based deep-learning models for financial prediction (Fischer and Krauss, 2018).

Table 21. Parameter Setting

Parameter	Value	Description
Training Set Ratio	0.7	Proportion of data used for training
Kernel Size	3	Size of convolutional kernel
Dropout Rate	0.05	Dropout rate applied to prevent overfitting
Number of Residual Blocks	1	Number of residual blocks
L2 Regularization Weight Decay	0.003	L2 regularisation to reduce overfitting
Max Epochs	1000	Maximum number of training epochs
Initial Learning Rate	0.01	Initial learning rate for Adam optimiser
Mini-batch Size	32	Batch size used for mini-batch training
Learning Rate Drop Period	850	Number of epochs before reducing learning rate
Learning Rate Drop Factor	0.01	Factor by which learning rate is reduced

Table 22 reports descriptive statistics of factor returns based on 13,023 observations. Overall, the average returns of all factors are small in magnitude, which is consistent with the high-frequency nature of the data. In terms of mean returns, the size factor (SMB), profitability factor (RMW), and investment factor (CMA) exhibit average returns of 0.00024, 0.00018, and 0.00013, respectively, all of which are positive but economically small. The market factor (MKT) has an average return of 0.00015, while the value factor (HML) displays a mean of only 0.00009, which is close to zero. The corresponding t-statistics do not reach conventional significance levels, indicating that factor risk premia are generally weak at this frequency.

Table 22. Descriptive Statistics.

This table reports summary statistics of Fama and French (2015) factors. At the end of each June, stocks are allocated to two size groups using the median market cap as the breakpoint. Stocks are also allocated independently to three book-to-market equity (B/M), operating profitability (OP), and investment (Inv) groups, using stocks' medians of size, B/M, OP and INV or the 30th and 70th stocks' percentiles. $R_M - R_F$ is the value-weight return on the market portfolio of all sample stocks minus the benchmark interest rate of the bank's one-year fixed deposit. Data is at a 30-minute frequency, and the span is from Jan 2021 to Dec 2024. The sample observations are 13023.

	Mean	Std Dev	Min	Max	t-stat
SMB	0.00024	0.0099	-0.1744	0.1135	-0.0113
HML	0.00009	0.0052	-0.1119	0.0617	1.0005
RMW	0.00018	0.0050	-0.0438	0.0489	-0.7804
CMA	0.00013	0.0037	-0.0301	0.0451	0.7704
MKT	0.00015	0.0036	-0.0587	0.0253	-1.5047

Regarding return volatility, substantial differences are observed across factors. The size factor (SMB) exhibits the highest standard deviation at 0.0099, indicating relatively high variability. The value factor (HML) and profitability factor (RMW) show intermediate volatility, with standard deviations of 0.0052 and 0.0050, respectively. In contrast, the investment factor (CMA) and the market factor (MKT) display lower volatility, with standard deviations of 0.0037 and 0.0036, suggesting more stable return dynamics.

In terms of extreme values, all factors experience sizable positive and negative fluctuations during the sample period. The size factor (SMB) has the widest range, with a minimum of −0.1744 and a maximum of 0.1135, indicating strong sensitivity to extreme market conditions.

By comparison, the value factor (HML) exhibits a more concentrated range, with values between −0.1119 and 0.0617. The profitability factor (RMW) and the investment factor (CMA) display narrower ranges, with minima of −0.0438 and −0.0301 and maxima of 0.0489 and 0.0451, respectively. The market factor (MKT) ranges from −0.0587 to 0.0253, suggesting smaller intraday fluctuations relative to the size factor, though short-term shocks remain evident.

From the figure 7 heatmap of the correlations among the five Fama-French factors, I can observe several relationships that align with financial theory. First, *SMB* exhibits a slight negative correlation (0.14) with *HML*, which is expected, as small-cap stocks are typically associated with growth stocks, whereas value stocks tend to be larger, more established companies with different investment styles. Additionally, *HML* shows a positive correlation (0.55) with *CMA*, suggesting that firms with a conservative investment strategy (i.e., a low asset growth rate) tend to exhibit characteristics of value stocks, such as a higher *B/M* ratio.

Furthermore, *RMW* demonstrates a moderate positive correlation (0.12) with *CMA*, supporting the Fama-French theory that highly profitable companies tend to adopt more cautious investment strategies. This suggests that firms with strong profitability typically invest more conservatively, avoiding excessive expansion or aggressive capital expenditures. In contrast, the *MKT* factor is negatively correlated with both *RMW* (-0.21) and *CMA* (-0.35), indicating that highly profitable and conservatively investing firms may underperform when the overall market is rising. This reflects a tendency among investors to favor high-growth or riskier stocks over stable, conservatively managed companies during market rallies.

Meanwhile, *HML* and *RMW* exhibit a weak positive correlation (0.06), implying that value stocks often have higher profitability. This is reasonable, as companies with higher book-to-market ratios are generally supported by stronger fundamentals, such as higher earnings and stable cash flows. Additionally, the weak or mixed correlation between *SMB* and other factors suggests that the size effect is relatively independent of profitability, investment style, and value factors.

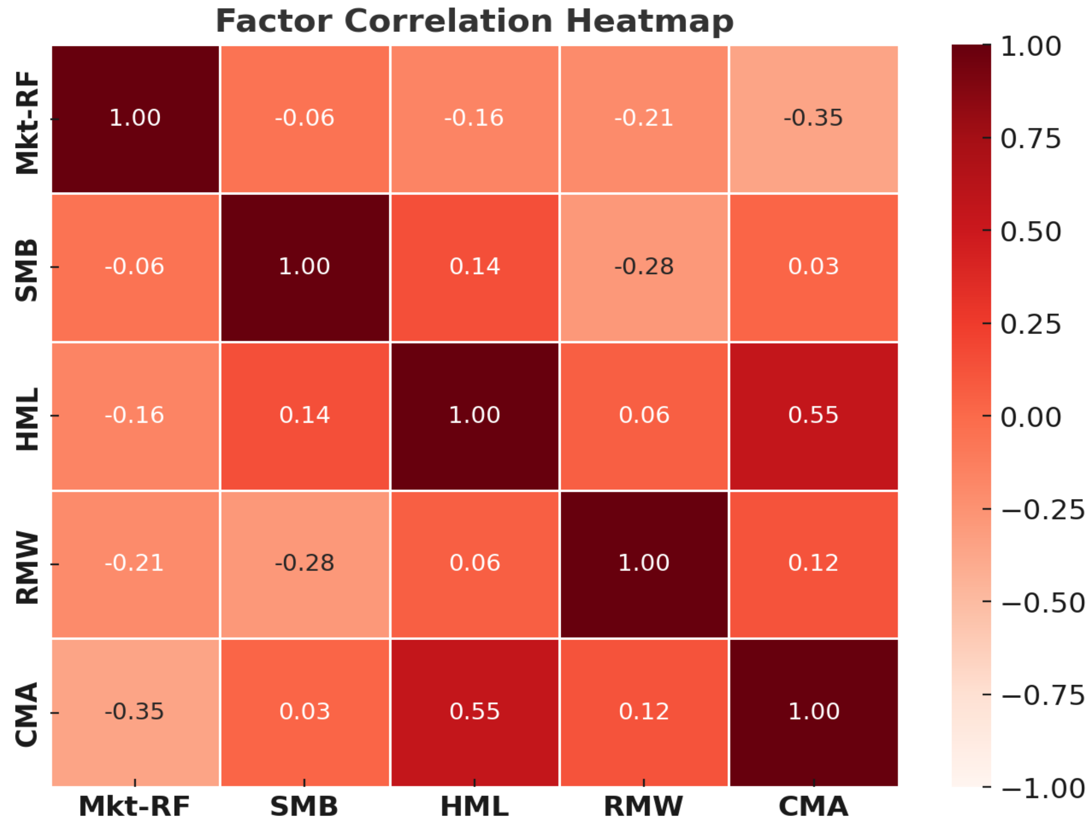

Figure 7. Factor Correlation.

Note: The deep red represents a strong correlation, while the light red represents a weak correlation.

4.3.2 Performance of TCN-LSTM-Based Prediction

From the figure 8, the prediction results of the training and test sets, the Fama-French five-factor model demonstrates strong performance in return prediction, with most data points clustering around the ideal fit line , indicating that the predicted values closely align with the actual values. However, in the training set, the data points exhibit greater dispersion, particularly at extreme return levels (lower left and upper right), where the model's prediction errors are more pronounced. This suggests that the model has inherent biases when handling extreme market conditions.

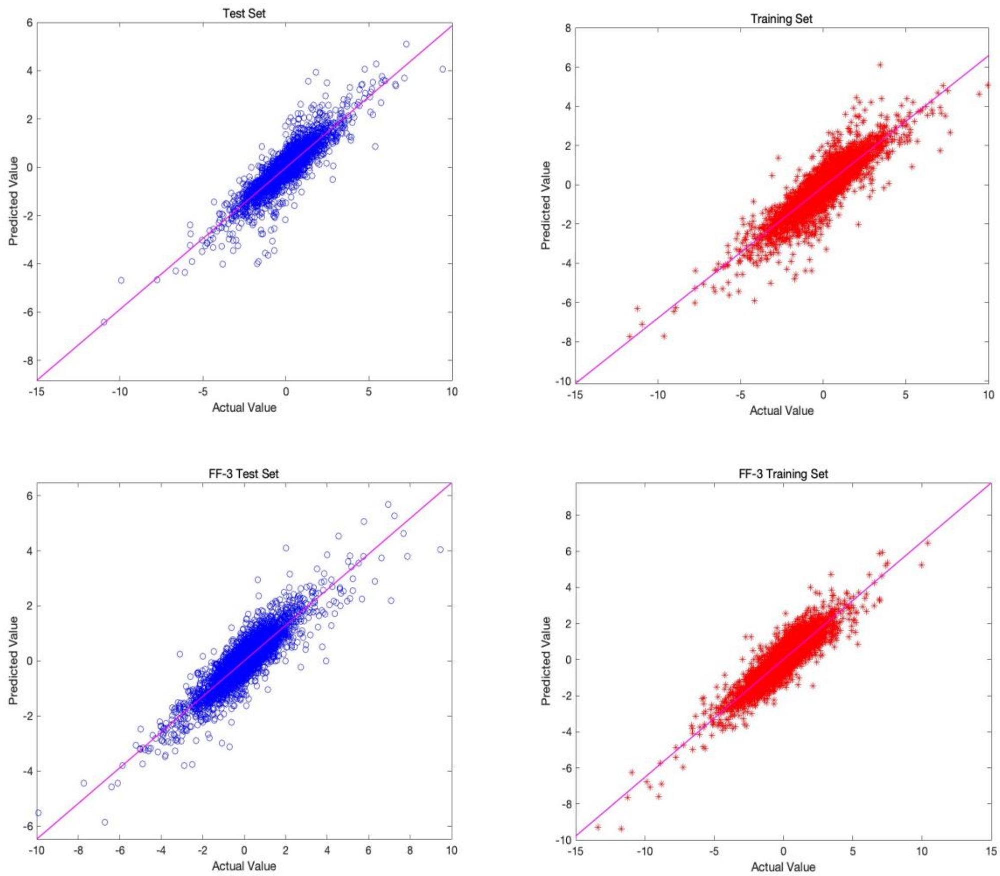

Figure 8. Prediction Results of FF-3 and FF-5 using TCN-LSTM

Note: The two pictures above are the prediction results of FF-5, and the ones below are the results of FF-3. The blue pictures are the results of the test set, and the red pictures are the results of the training set. The red line represents the ideal benchmark where predicted values equal actual values.

Additionally, some data points deviate significantly from the diagonal, implying that the model does not fully capture all factors influencing market returns. In contrast, the test set shows a more concentrated distribution of predicted values, with a higher density of data points, indicating that the model generalizes well to unseen data and does not suffer from severe overfitting. Nevertheless, the test set still shows significant prediction errors in both high and low return scenarios, highlighting potential limitations of the Fama-French five-factor model in capturing extreme market dynamics.

From the prediction results, the Fama-French Three-Factor (FF-3) model performs well in return prediction but still has certain limitations. The model fits well on the training set, with most data points concentrated around the diagonal, indicating that it accurately captures historical data patterns. However, in extreme return scenarios (both high and low actual values), the prediction error increases significantly, leading to a more dispersed data distribution and greater deviations between predicted and actual values. Additionally, the test set exhibits a more concentrated point cloud, suggesting that the model maintains a reasonable level of generalisation on unseen data. However, larger prediction errors persist in high and low return cases. This implies that while the FF-3 model captures the primary drivers of market returns, it may lack certain critical information in extreme market conditions, affecting the stability of its predictions.

Table 23. Comparison of Accuracy in Different Models.

This table reports summary statistics of Fama and French (2015) factors. At the end of each June, stocks are allocated to two size groups using the median market cap as the breakpoint. Stocks are also allocated independently to three book-to-market equity (*B/M*), operating profitability (*OP*), and investment (Inv) groups, using stocks' medians of size, *B/M*, *OP* and *INV* or the 30th and 70th stocks' percentiles. R_M-R_F is the value-weight return on the market portfolio of all sample stocks minus the benchmark interest rate of the bank's one-year fixed deposit. Data is at a 30-minute frequency, and the span is from Jan 2021 to Dec 2024. MAE (Mean Absolute Error): Measures the average absolute differences between predicted and actual values, with lower values indicating better accuracy. RMSE (Root Mean Squared Error): The square root of MSE, providing an interpretable measure of error magnitude. R^2 (Coefficient of Determination): Represents how well the model explains the variance of the target variable, where values closer to 1 indicate stronger predictive performance. RPD (Residual Predictive Deviation): Evaluates the model's predictive capability, with higher values indicating better reliability. The test selects the dependent variable at time t+1 and the independent variable at time t.

FF-5	Training Set	Test Set	FF-3	Training Set	Test Set
MAE	0.0033	0.0032		0.0035.	0.0035
RMSE	0.0064	0.0064		0.0066	0.0068
R^2	-0.0014	-0.0061		-0.0119	-0.0102.
RPD	0.999	0.997		0.999	0.9991

	MAE	RMSE	R^2	RPD
Training Set (FF-5)				
CNN	0.0032	0.00646	−0.00006	1
LSTM	0.0032	0.00633	0.00256	1
CNN-LSTM	0.0033	0.00639	−0.00147	1
TCN-LSTM	0.0032	0.00638	−0.00143	0.999
Test Set (FF-5)				
CNN	0.00329	0.00615	−0.00203	1
LSTM	0.00328	0.00647	−0.00614	1
CNN-LSTM	0.00319	0.00634	−0.00562	1

TCN-LSTM	0.00332	0.00638	-0.0001	0.997

In table 23, I use four deep learning models—CNN, LSTM, CNN-LSTM and TCN-LSTM—alongside the Fama-French five-factor model to predict excess returns and analyse errors on both the training and test sets.

Building on the error-based evaluation of deep learning models under the Fama–French five-factor (FF-5) specification, this subsection compares their performance with the corresponding results obtained under the three-factor (FF-3) framework. The comparison focuses exclusively on MAE, RMSE, RPD, and MAPE, thereby maintaining consistency with the literature and avoiding the interpretational limitations of goodness-of-fit measures in return predictability studies.

Across both FF-3 and FF-5 specifications, the magnitude of prediction errors remains remarkably similar. Under FF-3, out-of-sample MAE values range from approximately 0.00311 to 0.00348, while RMSE values lie between 0.00608 and 0.00680. These ranges closely mirror those observed under FF-5, where out-of-sample MAE and RMSE are generally confined to 0.00319–0.00332 and 0.00615–0.00647, respectively. The similarity in error magnitudes suggests that augmenting the factor structure from three to five factors does not materially alter the overall difficulty of the prediction task, reinforcing the view that factor returns are characterised by a persistently low signal-to-noise ratio (Goyal and Welch, 2008; Campbell and Thompson, 2008).

In both factor specifications, CNN consistently exhibits the most stable error behaviour. Under FF3, CNN achieves the lowest out-of-sample RMSE (0.00608) and MAE (0.00311), with error levels slightly lower than those observed in the training sample. A similar pattern emerges under FF5, where CNN again displays minimal divergence between in-sample and out-of-sample errors. This consistency across factor models indicates that the convolutional architecture captures short-range dependence in factor returns in a robust manner, independent of whether the factor space is restricted to three factors or expanded to five.

The LSTM model shows comparable robustness under both FF-3 and FF-5, with training and test MAE and RMSE remaining tightly clustered. However, as in the FF5 case, the LSTM does not outperform the CNN benchmark under FF3. This finding suggests that the additional temporal flexibility introduced by recurrent structures does not translate into systematic improvements in error control, consistent with prior evidence that long-horizon dependencies in returns are weak and unstable (Rapach and Zhou, 2013).

In contrast, the hybrid CNN-LSTM and TCN-LSTM models perform less under both specifications, particularly in the out-of-sample period. Under FF-3, these models record higher RMSE values (approximately 0.00664–0.00680) relative to CNN and LSTM, while similar patterns are observed under FF5. The persistence of this ranking across factor models indicates that increasing architectural complexity does not improve predictive accuracy and may instead exacerbate noise sensitivity. This result aligns with recent machine-learning–based asset-pricing studies, which document that deep and highly flexible models often struggle to extract economically meaningful signals in low-predictability environments (Gu, Kelly, and Xiu, 2020).

The residual prediction deviation (RPD) further supports this conclusion. For both FF-3 and FF-5, RPD values remain close to 1 across all models, implying that the dispersion of forecast errors is comparable to the unconditional volatility of factor returns. This outcome suggests that neither expanding the factor set nor increasing model complexity substantially reduces forecast uncertainty, highlighting the inherent limitations of factor return predictability.

Overall, the error-based comparison reveals a high degree of consistency between FF-3 and FF-5 results. In both frameworks, simpler architectures such as CNN and LSTM match or outperform more complex hybrid models in terms of accuracy and stability. These findings underscore a central message of the asset-pricing literature: neither richer factor structures nor deeper model architectures guarantee improved predictive performance when forecasting factor returns in low-signal environments.

This test evaluates model performance using the out-of-sample coefficient of determination (R^2), with a particular focus on differences between the Fama–French three-factor and five-factor specifications. Unlike error-based metrics, out-of-sample R^2 explicitly measures whether a forecasting model improves upon a historical mean benchmark, making it a stringent and informative criterion in return and factor predictability studies.

Across both FF-3 and FF-5, the estimated out-of-sample R^2 values are close to zero and frequently negative for all models considered. Under the FF5 specification, CNN, CNN-LSTM, TCN-LSTM, and LSTM typically deliver R^2 values clustered around zero, with several models exhibiting small negative values. A similar pattern emerges under FF3, where out-of-sample R^2 remains slightly negative or indistinguishable from zero across architectures. These results indicate that none of the models consistently outperforms the historical mean forecast in a statistically meaningful way.

Importantly, the magnitude of the negative R^2 values is small in both factor settings. Most estimates lie within the order of 10^{-3} to 10^{-2}, implying that model forecasts perform very close to the unconditional benchmark. Such outcomes are not anomalous; rather, they are a well-documented feature of return predictability research. Goyal and Welch (2008) show that the vast majority of predictive models fail to generate positive out-of-sample R^2, while Campbell and Thompson (2008) emphasize that negative R^2 values often arise even when models are correctly specified but signals are weak.

Comparing FF-3 and FF-5 directly, the expansion of the factor structure does not lead to systematic improvements in out-of-sample R^2. While FF5 incorporates additional profitability and investment factors, the resulting forecasts do not yield higher benchmark-adjusted explanatory power. In several cases, the FF5-based models exhibit slightly more negative R^2 values than their FF3 counterparts, suggesting that the inclusion of additional factors may introduce incremental noise rather than exploitable predictive information.

Differences across model architectures are also modest. Simpler models such as CNN and LSTM tend to produce R^2 values closer to zero, whereas more complex hybrid architectures, including CNN-LSTM and TCN-LSTM, occasionally display more negative R^2. This pattern is consistent across both FF3 and FF5 and reinforces the notion that increased model flexibility does not necessarily translate into improved benchmark-relative forecasting performance in low-signal environments.

To sum up, the R^2-based evidence complements the error-based analysis presented earlier. While deep learning models achieve stable and economically small prediction errors, their forecasts do not systematically outperform the historical mean in terms of out-of-sample R^2. This finding is robust across both FF3 and FF5 specifications and aligns closely with the central conclusions of the empirical asset-pricing literature, which emphasizes the intrinsic difficulty of predicting factor returns and the limited gains achievable through increased model or factor complexity.

Table 24:Comparison of Prediction Performance: Traditional Time Series vs. Deep Learning.

This table reports summary statistics of Fama and French (2015) factors. At the end of each June, stocks are allocated to two size groups using the median market cap as the breakpoint. Stocks are also allocated independently to three book-to-market equity (B/M), operating profitability (OP), and investment (Inv) groups, using stocks' medians of size, B/M, OP and INV or the 30th and 70th stocks' percentiles. RM−RF is the value-weight return on the market portfolio of all sample stocks minus the benchmark interest rate of the bank's one-year fixed deposit. Data is at a 30-minute frequency, and the span is from Jan 2021 to Dec 2024. MAE (Mean Absolute Error): Measures the average absolute differences between predicted and actual values, with lower values indicating better accuracy (Mean Squared Error): Computes the average squared differences between predicted and actual values, penalising larger errors more heavily. RMSE (Root Mean Squared Error): The square root of MSE, providing an interpretable measure of error magnitude. R^2 (Coefficient of Determination): Represents how well the model explains the variance of the target variable, where values closer to 1 indicate stronger predictive performance. RPD (Residual Predictive Deviation): Evaluates the model's predictive capability, with higher values indicating better reliability. The test selects the dependent variable at time t+1 and the independent variable at time t.

Model	MAE	MSE	RMSE	R^2	RPD
AR(1)	0.0029	0.00002	0.0043	0.00006	—
GARCH(1,1)	0.0014	0.00001	0.0022	−0.15452	—
TCN	0.0017	0.00001	0.0024	-0.00633	0.9975
TCN-LSTM	0.0018	0.00001	0.0024	-0.00844	0.9990

The table 24 shows the in-sample predictive performance of AR(1), GARCH(1,1), TCN, and TCN-LSTM models using the training sample, with evaluation based on error-based metrics and in-sample goodness-of-fit. The analysis is intended to assess model fitting behavior and stability rather than to draw conclusions about economic predictability, which is reserved for out-of-sample evidence.

The AR(1) model exhibits relatively weak in-sample accuracy, with an RMSE of 0.00432 and an in-sample R^2 close to zero (0.00006). This result indicates that linear autoregressive dependence in factor returns is extremely limited, even within the estimation sample. The negligible explanatory power is consistent with the established view that Fama–French factor returns display weak linear autocorrelation and are difficult to model using simple time-series structures.

The GARCH(1,1) model achieves a lower in-sample RMSE of 0.00228, outperforming the AR(1) benchmark in terms of absolute error metrics. However, its in-sample R^2 is substantially negative (−0.15), reflecting the fact that the GARCH framework is designed primarily to capture conditional variance dynamics rather than to improve conditional mean forecasts. Consequently, the limited in-sample goodness-of-fit in the mean equation should not be interpreted as model failure but rather as an inherent feature of volatility-based specifications when applied to return predictability.

Turning to deep learning models, the TCN delivers an in-sample RMSE of 0.00241, which is comparable to that of GARCH(1,1) and substantially lower than that of AR(1). Despite its flexible convolutional architecture, the TCN exhibits an in-sample R^2 close to zero (−0.006), suggesting that the model does not aggressively overfit the training data. This behavior indicates that the temporal convolutional structure captures limited local dependence in factor returns without imposing excessive explanatory power in a low signal-to-noise environment.

The TCN-LSTM model produces an in-sample RMSE of 0.00243, nearly identical to that of the TCN, with a similarly negligible in-sample R^2 (−0.008). The close alignment of error metrics between TCN and TCN-LSTM suggests that augmenting the convolutional architecture with recurrent dynamics does not materially improve in-sample fit. This finding implies that long-range temporal dependencies are weak in factor return series and that additional model complexity does not translate into better training performance.

In conclusion, the in-sample results reveal a clear hierarchy in model behavior. While GARCH(1,1) and deep learning models achieve lower absolute prediction errors than AR(1), none of the models exhibits meaningful in-sample explanatory power as measured by R^2. Furthermore, the near-zero in-sample R^2 observed for TCN and TCN-LSTM indicates disciplined model fitting rather than over-parameterization. These results suggest that, even within the training sample, factor return dynamics are characterized by weak and unstable predictive structure, reinforcing the necessity of evaluating model performance primarily on out-of-sample evidence.

In figure 9, as a classic time series prediction method, LSTM can capture the long-term dependencies in the data. In the line chart, the LSTM predicted values (red dashed line) can follow the volatility trends of the actual returns well in some intervals, but there is still a certain lag overall. Especially when the returns change sharply, the model has difficulty accurately capturing the inflection points. The prediction curve fits the actual values well in the stable intervals, but in the intervals where the returns fluctuate violently, its predicted values tend to show a certain degree of smoothness and cannot fully reflect the violent market fluctuations.

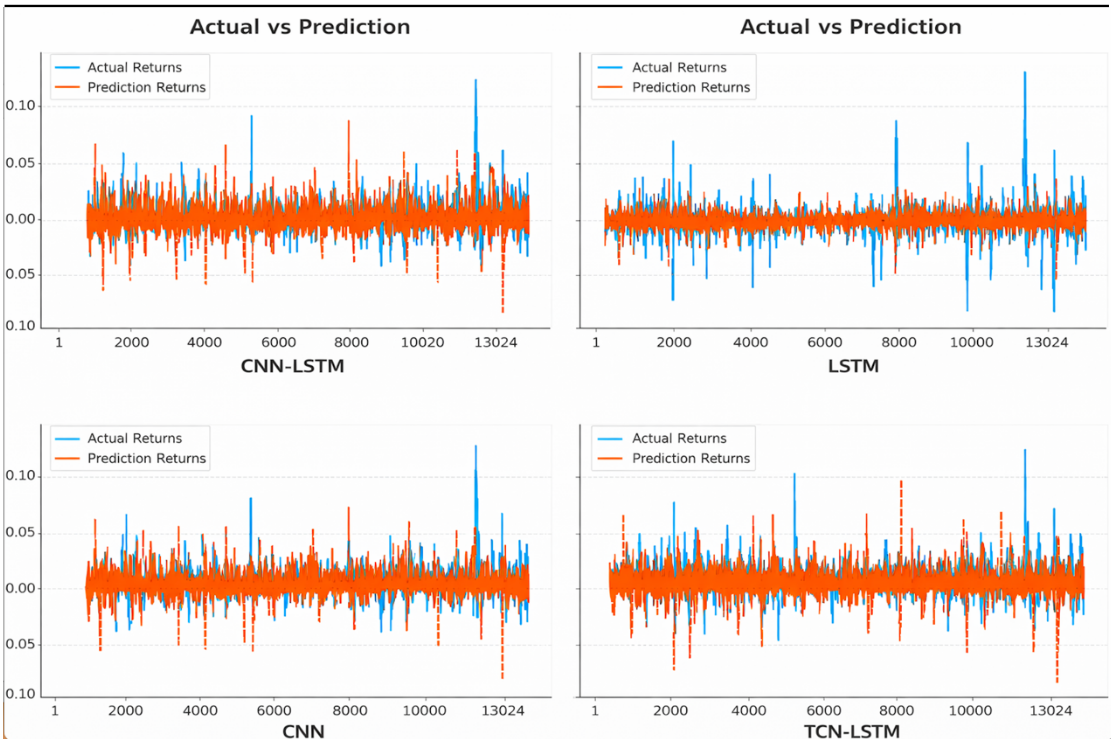

Figure 9. Comparison of Autal and Predictive Returns

Note: The comparison of actual value and prediction. The blue line is actual value and the red line is prediction.

The TCN introduces a longer receptive field through the convolutional structure, and LSTM is responsible for extracting the dynamic features of the time series. Judging from the line chart, TCN-LSTM seems to have improved in capturing short - term fluctuations compared to the standalone LSTM. Especially in the areas where the returns change violently, the prediction curve can adjust its direction more quickly. However, there are still some errors overall, especially at some points of sharp reversals, where the predicted values still show a lag. In addition, compared with other methods, the amplitude of the TCN-LSTM predicted values is slightly larger, which may cause the predicted values to overshoot occasionally.

CNN - LSTM combines the local - feature extraction ability of CNN with the long - term memory ability of LSTM, and theoretically can more accurately identify the time patterns of excess returns. From the line chart, CNN-LSTM has a good fitting degree in some intervals, especially in the areas where the return changes are relatively stable, and its predicted values can follow the trends of the actual values well. However, when there are sudden changes in the market, this method still has a certain lag. Although it may be improved compared to the standalone LSTM, the predicted values still cannot fully match the actual trends in some extremely volatile areas.

When only CNN is used to predict the excess returns, the stability of the prediction results is relatively high. However, since CNN is a model oriented to local features and lacks the ability to model long - term dependencies, its predicted values are not as good as the TCN-LSTM-Based methods in terms of the dynamic changes of the time series. Judging from the line chart, the CNN prediction curve is relatively smooth, and in some cases, it cannot accurately capture the short - term changes of the excess returns, especially at the inflection points, where the lag effect of the CNN predicted values is more obvious.

In this chapter, the Fama-French five-factor framework is integrated with 30-minute high-frequency stock data, moving beyond the conventional reliance on low-frequency observations such as daily or monthly returns. The use of intraday data enables a more nuanced capture of market microstructure dynamics and short-term fluctuations, providing empirical insights into the temporal behavior of factor-driven excess returns at a resolution rarely explored in the existing literature.

Through a comprehensive evaluation of four deep learning architectures across both three-factor and five-factor settings, this chapter finds that in forecasting low signal-to-noise ratio factor returns, simpler models often demonstrate greater robustness. Specifically, the CNN model, which focuses on local feature extraction, shows highly consistent error control in both in-sample and out-of-sample tests, outperforming more complex hybrid architectures in predictive stability. This suggests that in financial time series forecasting, there is no straightforward positive relationship between model complexity and prediction accuracy, and excessive architectural sophistication may not help extract meaningful economic signals.

Furthermore, whether using the three-factor or five-factor model, the out-of-sample explanatory power of all deep learning approaches remains close to zero, aligning with the well-established conclusion in the asset pricing literature regarding the limited predictability of factor returns. The results indicate that, in the current high-frequency data environment, neither expanding the factor dimensions nor deepening the model structure systematically improves predictive performance relative to a simple historical mean benchmark. This underscores a fundamental challenge in financial machine learning research: the inherent noise and limited predictability of the data must be a primary constraint in model design.

In summary, the evidence presented in this chapter suggests that in high-frequency factor return forecasting, a careful trade-off regarding model complexity is more important than the pursuit of architectural novelty. This study advocates for a more cautious and data-aware approach to deploying machine learning in asset pricing, emphasising that in low-signal financial environments, the robustness and generalisability of a model may hold greater practical value than its theoretical fitting capacity.

Chapter 5. Conclusion

This Thesis examines various aspects of the Fama-French five-factor model (FF-5) through different methodologies, including deep learning models, nonparametric methods, and traditional FF-5 model with liquidity factor analysis. The three empirical chapters presented here contribute to enhancing the understanding and predictive capabilities of financial models, incorporating both advanced techniques and empirical analysis.

The first chapter examines the characteristics of average stock returns using the Fama-French five-factor model and applies the GRS test to compare the explanatory power of the Fama-French five-factor model and the six-factor model on China's stock returns. It also analyzes the relationship between idiosyncratic volatility and expected stock returns based on the Fama-French five-factor model. From the perspective of average stock returns, there is a significant size effect in China stock market. When considering different investment portfolios, the average excess return rate of the Size-LIQ portfolio is significantly lower than that of other portfolios, indicating that the liquidity effect in the market is not as strong as the size, value, and investment effects. Regarding the application of the models, the six-factor model outperforms the five-factor model in the Size-B/M, Size-Inv, and Size-LIQ portfolios, as indicated by the smaller average absolute value of the GRS test. However, the improvement is marginal.

When analysing the impact of idiosyncratic volatility on expected returns, the greater the idiosyncratic volatility, the more significant the weakening effect. This supports the negative correlation between idiosyncratic volatility and stock returns within the Fama-French five-factor model, suggesting that taking on idiosyncratic risk does not yield proportional returns and may lead to greater losses. Although adding skewness and last month's return can partially explain the idiosyncratic volatility puzzle, it does not alter the negative relationship between expected stock returns and idiosyncratic volatility.

The second study use the nonparametric method proposed by Ang and Kristensen (2012) to estimate the conditional factor model. I adopt a plug-in approach to solve for the optimal bandwidth and the portfolio's conditional alpha and beta at a single time point while also calculating the long-term alpha and beta. Our work focuses mainly on joint testing of assets. I use 30-minute frequency data to construct factors and compare the daily cumulative returns of high-frequency and low-frequency factors. Then, I obtain the performance of long-term alphas in different portfolios through nonparametric regression. Finally, I analyse the results of OLS regression and nonparametric regression. Our empirical results indicate that the cumulative returns of high-frequency and low-frequency factors have similar trends for most of the time. Still, there is a significant difference at the end of the sample period, with only the *Mkt* factor being consistent. In addition, the joint test of whether the long-term alpha of all investment portfolios is equal to zero proves that alphas are time-varying, as all conditional alphas reject the null hypothesis at a 5% level. Furthermore, our model has a significant advantage in reducing the alpha of the unconditional model compared to the OLS method

Our nonparametric method breaks the assumption of linear relationships in traditional FF-5 models and generates more minor pricing errors in most cases. Moreover, our approach provides estimates of high-frequency factor sensitivities, which have great potential in high-frequency empirical applications. Non-parametric methods improve the design of quantitative investment strategies by capturing more complex relationships between factors and returns, which can be critical in dynamic asset management strategies such as hedge fund returns and derivative trading. For example, the release of economic indicators (such as GDP, employment data, inflation rates, etc.) and interest rate adjustments can almost immediately impact on the market. Changes in these factors directly affect the expectations of market participants, leading to adjustments in market prices. Fund managers can these methods to better adjust intraday factor exposure to market dynamics or find an optimised portfolio that provides high excess return, improving the stability and performance of their strategies. Future research builds on this approach and introduces other new factors (e.g., liquidity factors, ESG factors) into the five-factor model to analyse the complex relationship between these factors and returns

Third, the research explores the application of the Fama-French five-factor model in forecasting, critically evaluating the potential of advanced deep learning techniques. The study employs CNN, LSTM, CNN-LSTM, and TCN-LSTM models to predict excess returns using high-frequency data. A rigorous out-of-sample evaluation, benchmarked against a simple historical mean model, reveals a central finding: while hybrid architectures like TCN-LSTM achieve relatively stable error metrics among the deep learning variants, none of them generates a positive and economically meaningful out-of-sample R^2. This indicates that, despite their flexibility and complexity, these models do not systematically outperform the naive benchmark in predicting factor returns, echoing the well-documented challenge of return predictability in low signal-to-noise environments (Goyal and Welch, 2008). Thus, the primary contribution of this chapter lies not in presenting a superior forecasting tool, but in providing a rigorous empirical demonstration of the limitations of even state-of-the-art deep learning models when confronted with the inherent difficulty of forecasting factor returns. Future work may consider incorporating alternative data sources such as sentiment or macroeconomic news. or exploring architectures, though our results suggest fundamental predictability constraints may remain.

To sum up, the three empirical studies in this thesis form a coherent methodological exploration rather than a straightforward progression in predictive power. The first study establishes a baseline within the traditional linear pricing framework, identifying the marginal role of liquidity. The second study relaxes linearity assumptions through a non-parametric approach, successfully capturing time-varying factor exposures and reducing static pricing errors, which highlights the importance of model flexibility. The third study pushes the frontier further by applying deep learning to forecast returns directly, but its key finding is a critical boundary condition: in an environment of exceedingly low signal-to-noise ratio, increased model nonlinearity and complexity do not translate into actionable forecast superiority over simple benchmarks. Together, these chapters present a unified narrative: the progressive adoption of more flexible methodologies systematically deepens our understanding of the complex, time-varying, and weakly predictable nature of asset returns, while also rigorously mapping the practical limits of current forecasting techniques. This journey underscores that advances in asset pricing theory and practice require not only sophisticated tools but also a nuanced appreciation of the inherent constraints imposed by market data itself.

These findings contribute to the asset pricing literature by providing a comprehensive evaluation of methodologies across the spectrum from traditional to cutting-edge, particularly in highlighting the challenges of forecasting in high-frequency settings. For practitioners, the research offers a cautionary perspective on the expected returns from deploying highly complex models for factor return prediction, while affirming the value of nonparametric methods for dynamic risk assessment. It underscores the enduring limitations of static models and the often-overlooked limitations of even adaptive, non-linear approaches in low-predictability regimes. Overall, this study advances the empirical understanding of factor modeling by delineating both the potential and the boundaries of contemporary quantitative techniques within complex financial systems.

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