Essays on Fiscal Sustainability: Assessment and Adjustment

Tingli Shen

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The University of Sheffield
School of Economics

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To my parents

Declaration

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Abstract

This thesis comprises three chapters on fiscal sustainability, focusing on ageing society. Chapter 1 discusses the assessment of fiscal sustainability, while Chapter 2 and 3 explore fiscal policy measure aimed at improving it.

Chapter 1 evaluates fiscal sustainability using three separate approaches: (1)undertaking an econometric test of the PVBC based on the series of debt, deficit and other macro variables pertinent to fiscal policies as Campbell (1987) test; (2)regressing the primary surplus on debt to estimate a Bohn (2005) rule and (3) measuring the required adjustment in a fiscal tool to achieve a given debt target in the medium-term, the so-called Blanchard (1990) medium-term debt dynamics. Chapter 1 conducts the empirical analysis through the length of a unified VAR model that incorporates demographic change. The combined empirical evidence suggests that fiscal policies in most countries have been on an unsustainable path since 2008.

In Chapter 2, I use a transformed VAR model to analyse the response required by the Bohn (2005) rule to converge the Blanchard (1990) medium-term debt dynamics to zero. The results suggest that major economies should strengthen their responses to rising debt ratios and the responses lies within the range of previous significant response bands. Nonetheless, the reliability of the counterfactual results is challenged by the Lucas (1976) critique.

In Chapter 3, I use a large-scale OLG model calibrated to China and compare two steady states (2020 vs 2100). Exogenous variation in the labour tax rate yields an inverted-U "debt Laffer effect" and model-implied debt limits (fiscal space) in each steady state. Sustainability is assessed by comparing these limits with the sustainable debt ratio implied by the Bohn (2005) rule. The implied sustainable debt ratio is 72.38%, well below the debt limits in both steady states, indicating sustainability after ageing. However, ageing tightens fiscal space, so feasibility typically requires a higher labour tax rate in 2100.

Contents

List of Figures

List of Tables

Chapter 1

Fiscal Policy Sustainability in Ageing Economies: A VAR Analysis

1.1 Introduction

Since the global financial crisis, many advanced and developing countries have experienced significant increase in their debt. Those economies have experienced greater fiscal pressures through using expansionary fiscal policies to alleviate the negative effects of the crises, thereby generating high levels of public debt. At the same time, population ageing has become an important fiscal challenge (International Monetary Fund (2025); Organisation for Economic Co-operation and Development (2024)). While population growth rates in some advanced economies remain low, others have already entered a phase of negative growth. The ongoing decline in population growth rate is a clear signal of ageing, which serves as an important transmission variable for fiscal pressures across public finance, social security, and labour markets and population ageing would contribute to rising debt-to-GDP ratio, posing challenges to fiscal sustainability.

Many countries place a high priority on fiscal sustainability, with a growing consensus that population ageing represents a critical challenge to the medium- and long-term stability of public finances. Major economies emphasize fiscal sustainability, as documented in reports by State Council of China (2021), European Commission (2022), and Cabinet Office of Japan (2025). Some have even established offices whose reports serve as key references for fiscal policy design, all of which highlight the impact of demographic changes on fiscal sustainability (for example, Parliamentary Budget Officer of Canada (2024), Office for Budget Responsibility of the UK (2024), Congressional Budget Office of the US (2024)). To strengthen fiscal sustainability in medium term, many countries have taken actions: European Commission (2024) has introduced Medium-Term Plan (MTP), requiring member states to design expenditure paths that help reduce debt ratios and enhance fiscal sustainability while addressing ageing-related challenges; and Cabinet Office of Japan (2024) has set a medium-term target for primary balance, which takes population ageing into account.

In a dynamic general equilibrium model, fiscal sustainability requires the present value government budget constraint (PVBC) to hold. The empirical macro-econometric literature so far has devised three separate approaches to evaluate fiscal policy sustainability. The first approach is to undertake a econometric test of the PVBC based on the series of debt, deficit and other macro variables pertinent to fiscal policies (e.g. Barro (1984) ; Hamilton and Flavin (1986); Campbell (1987); Trehan and Walsh (1988). This approach focuses on the long-run implications of a deterministic version of the PVBC and tries to answer whether the debt-to-GDP ratio will remain finite or explode in the long term. It nevertheless has some drawbacks such as the problem of unit root tests, the weak restriction set on short-term government behaviour and the backward-looking nature. The second approach is to regress the primary surplus on debt to estimate the Bohn (2005) rule. Bohn (2005) finds that the fiscal sustainability requires the positive relationship between primary balance and the debt. The third approach is the indicator of fiscal stance which is proposed by Blanchard (1990), and Buiter et al. (1993).

This approach looks for the sustainable debt-output ratio when the government budget constraint is at steady state (see Blanchard et al. (1991). The quality of those indicators relies on the projections on which they are based and it also requires limited horizon since the forecast would be less accurate if using longer horizon.

The contribution of Chapter 1 lies in its empirical analysis of fiscal policy sustainability, integrating all three widely used approaches within a unified VAR model. This model incorporates demographic transition dynamics, without focusing on a dedicated analysis of the welfare or macroeconomic effects of population ageing. VAR model as a time series model has the well-documented advantages for forecasting and can capture the future dynamics of fiscal and macroeconomic variables in the PVBC. These three tests can take advantage of the benefit of VAR model and deliver different economic interpretation. The Campbell (1987) test directly examines if the restriction required for the PVBC to hold are met in a VAR. When the cross equation restrictions are not rejected, the PVBC holds and the debt does not need adjustment. Nevertheless, when the result shows that the policy is unsustainable, this test cannot tell what method should be used to restore the fiscal sustainability. The second test Bohn (2005) rule illustrates the systematic relationship between primary balance and debt. Though Bohn (2005) rule is only based on the historical data and implicitly answers whether the fiscal policy is undertaking an unsustainable path, Bohn (2005) rule sheds light on the degree of the response of primary balance to debt should change and describes how the primary balance is affected when the debt ratio increases 1%. The third test Blanchard (1990) medium-term debt dynamics measures the required adjustment in a fiscal tool to achieve a given target in the medium term. It compares the debt value forecast by the VAR model with the debt value set as the target in the economy. Though it only implicitly says whether the fiscal policy is on unsustainable path or not, it comments on if the debt is too much or too little through the comparison. Blanchard (1990) medium-term debt dynamics indicates if the fiscal policy is too loose or too tight.

This empirical results show that in most countries fiscal policy has undertaken an unsustainable path from 2008 onward. Though Campbell (1987) test results show that fiscal policy are all sustainable, Bohn (2005) rule results show that the governments do not react to debt and China is significantly negatively react to the debt. Blanchard (1990) medium-term debt dynamics shows that the fiscal policies in those major economies are all too loose. The main implication from the three tests is that those major economies should implement fiscal adjustment.

The chapter is structured as follows: Section 1.2 presents the literature review. Section 1.3 describes the methodology. Section 1.4 details the data used. Section 1.5 reports the empirical results. Section 1.6 conducts a robustness check. Section 1.7 provides the conclusion.

1.2 Literature Review

In a dynamic general equilibrium model, fiscal sustainability requires the PVBC to hold. This is directly based on several equations. PVBC is derived from the government budget identity:

$$(1+\rho)b_{t-1} = pb_t + b_t, \tag{1.1}$$

where b_t is the debt to GDP ratio at the end of period t; $\rho \simeq R - g - \pi$ equals the nominal interest rate, R, minus the growth rate of total real GDP, g, and the inflation rate, π; pb_t is the primary balance to GDP ratio. Solving forward the government budget identity in equation (1.1) across j periods yields:

$$b_t = E_t \sum_{i=1}^{j} \frac{1}{(1+\rho)^i}(pb_{t+i}) + \frac{1}{(1+\rho)^j} b_{t+j}, \tag{1.2}$$

where E_t is the expectation operator. When j goes to infinity and the transversality condition or the no-Ponzi condition is satisfied:

$$\lim_{j \to \infty} \frac{1}{(1+\rho)^j} b_{t+j} = 0, \tag{1.3}$$

and equation (1.2) becomes:

$$b_t = E_t \sum_{i=1}^{\infty} \frac{1}{(1+\rho)^i}(pb_{t+i}). \tag{1.4}$$

Equation (1.4) says that the present value of primary surpluses of the government until infinity period should equal to current debt liabilities and this equation is called the government present value budget constraint (PVBC). Most research for analysing fiscal sustainability are based on the PVBC (see Afonso (2008) and Economides and Philippopoulos (2023)). The other equations discussed above are also important, as they often provide the basis for empirical analysis of fiscal sustainability.

The empirical analysis on fiscal policy sustainability is essentially dominated by three approaches.

The first method is a econometric test of the PVBC based on the series of debt, deficit and other macro variables pertinent to fiscal policies (e.g. Hamilton and Flavin (1986); Trehan and Walsh (1988); Afonso et al. (2023)). This econometric approach is faithful to the theoretical definition and remains widely used today. These econometric tests examine for the existence of unit root in government debt and budget deficits series, and/or for cointegration relationship between revenues and expenditures (e.g Trehan and Walsh (1988, 1991); Wilcox (1989); Hakkio and Rush (1991); Quintos (1995)). Chalk and Hemming (2000) indicate that these tests capture sufficient conditions for solvency, and directly examine fiscal policy sustainability.

The PVBC described in equations (1.4) is analogous to the present value model, which is widely used in economics and finance (see Sargent (1979) and Hall (1978) etc.). While the present value model serves as a general framework, the PVBC represents a specific case within it. They are similar in that b_t is a linear function of the present discounted value of pb_{t+j}. For this type of constraint, Campbell (1987) developed a test to assess its empirical validity. The Campbell (1987) test can be applied to determine whether government debt is sustainable. Additionally, Galvão et al. (2011) used the present value model to examine fiscal sustainability. In their empirical implementation, they emphasize a GMM formulation that consolidates the VAR restrictions into simple orthogonality conditions, thus mitigating sensitivity to lag-length selection. However, the underlying identification remains consistent with the logic of Campbell (1987).

However, the shortcomings of this approach are quite obvious. First, the unit root tests are often hard to reject the alternative hypothesis that the variable is close to unity. When the series is actually stationary but has a root close to one, tests are hard to reject the null hypothesis that there is a unit root, and this is what the variable debt to GDP ratio is faced with. This weakness would then make the test result have little power, which would affect the conclusion of those tests based on the integrate order of fiscal variables. While the hypothesis proposed by Hamilton and Flavin (1986), Trehan and Walsh (1988), and Quintos (1995) base their tests on that the integrate order of the debt should be less or equal to one, Bohn (2007) proves that PVBC can hold as long as the debt variable is integrated of finite order. Rejecting those hypotheses nevertheless does not provide evidence that PVBC does not hold since they do not rule out the higher-order integration of those fiscal variables. Second, the PVBC sets very weak restriction on government behaviour in the short term. Under certain conditions any level of government borrowing, and even an ever-increasing debt-GDP level, can be maintained in the long run. Third,,the tests are backward-looking whereas the constraint itself is forward-looking and do not take into account changes in fiscal policy over time; typically the tests suggest that fiscal policy has been constant in the past, being either always sustainable or unsustainable and will remain the same in the future. Not only may fiscal policy have changed over the data period analysed, but there might be breaks in the data that are unrelated to the current conduct of fiscal policy or its future development. Further, when the tests reject fiscal sustainability, they do not provide any information about how the fiscal stance should adjust.

The second approach stems from the research of Bohn (2005). Bohn (2005) proved that a positive response of the primary balance to debt, when controlling the fluctuations in government expenditure and output, is sufficient to guarantee that fiscal policy satisfies the PVBC. Bohn (2005) proves a fiscal sustainability condition, while others using control variables other than just cyclical component of output and government expenditure are just mean-reverting. Since Bohn (2005) is based on Barro (1984)'s theory. The related equation is called the "fiscal reaction function" since it examines the response of primary balance to debt. Bohn (2005) rule does not require assumptions about debt management, the maturity and/or denomination structure of debt, or the residence of debt holders. Bohn (2005) rule is valid regardless of how debt and the primary balance are measured, at constant or current prices, and in levels or as shares of GDP. Taking this test also does not require explicit knowledge of fiscal policy rules or the portfolio of public debt instruments (i.e., indexed bonds, nominal bonds, foreign currency bonds, etc.). When the test result is significantly negative, it tells that how much the response should adjust. The linear, conditional response of the primary balance to debt of Bohn (2005) rule is sufficient but not necessary condition for PVBC. A non-linear and/or time varying response can also support fiscal solvency as long as the response is strictly positive above a certain threshold debt–output ratio, or almost surely in the long run. Estimating Bohn (2005) rule does not need unit root test for the variables and it sheds light on the degree of the response of primary balance to debt. Many recent studies use ARDL or ECM specifications to capture a stable long-run relationship—often framed as cointegration—between the primary balance and its determinants. (see Lima Campos and Cysne (Lima Campos and Cysne)). However, this approach is still backward-looking since it just uses historical data.

The third approach is the indicator of fiscal stance, proposed by Blanchard et al. (1991) and Buiter (1985). In the steady state, equation (1.1) would show that the steady-state debt ratio b can be taken as the annuity value of the steady state primary balance pb, In policy applications, this condition is used either as an indicator of the primary balance-output ratio needed to stabilize a given debt-output ratio (the so-called "debt stabilizing" primary balance), or as an indicator of the sustainable target debt-output ratio that a given primary balance-output ratio can support. There are also variations of this approach that use the constraint $b_t + (1 + \rho_t)b_{t-1} = pb_t$ to construct estimates of primary balance targets needed to produce desired changes in debt at shorter horizons than the steady state. This approach is widely used by the European Commission (2007, 2010), and International Monetary Fund (2025). Blanchard (1990) medium-term debt dynamics that uses this approach measures the required adjustment in a fiscal tool to achieve a given target in the medium term. It compares the debt value forecast with the debt value set as the target in the economy. The quality of the indicator ultimately relies on the projections on which they are based. The forecast could be external projection or could be from a specific model and as pointed out by Horne (1991) and Cuddington (1997), and more recently by Leeper (2010). Unlike the first two approaches, the indicators of the fiscal stance are forward-looking. It requires the forecast of debt variables or the primary balance variables. It also takes the future dynamics of macroeconomic and fiscal variables into account when using an econometric model for forecasting. Though it cannot tell whether the fiscal policy is sustainable or not, it can quantifies if the debt is too high or too low relative to a given target. In the similar way, Blanchard (1990) medium-term debt dynamics indicates if the fiscal policy is too loose or too tight.

As global population ageing continues to intensify, a growing body of research has focused on the implications of demographic change for macroeconomic outcomes and public finances. Population ageing has been shown to affect a range of macroeconomic variables, including government debt as documented by IMF (a) and Eur (2018), inflation as analysed by Bullard (2012) and Juselius and Takáts (2018), and the natural rate of interest as discussed by Carvalho et al. (2016). Against the backdrop of rising public debt levels, Ramos-Herrera and Prats (2020) study the impact of ageing on fiscal sustainability and find that population ageing poses significant challenges to maintaining sustainable public finances. These findings highlight the importance of explicitly accounting for demographic developments when assessing fiscal sustainability.

The existing literature generally agrees that population ageing affects government finances and explicit public debt dynamics primarily through two key channels, one of which is the age-related public expenditure channel. In major economies, one strand of the literature conducts forward-looking assessments of the fiscal consequences of ageing based on demographic projections and public expenditure forecasts. These studies consistently show that, as the share of the elderly population rises, social security expenditures—particularly pensions and healthcare—tend to increase as a share of GDP, thereby exerting persistent upward pressure on public debt paths under unchanged fiscal policies, as shown by Congressional Budget Office of the US (2024) and IMF (b). Distinct from these projection-based analyses, another strand of the literature adopts an empirical approach and directly examines the specific channels through which ageing affects fiscal outcomes. Empirical evidence provided by Börsch-Supan (2001), Horioka (2007), and Cribb, Emmerson, and Karjalainen (Cribb et al.) suggests that population ageing significantly increases public pension spending and alters the composition of government expenditure, with effects that are mainly reflected in medium- to long-term fiscal dynamics rather than short-term fiscal fluctuations.

Maestas et al. (2016) exploit predetermined variation in demographic structure to identify population-ageing shocks and show that ageing significantly reduces the growth rate of GDP per capita. This negative growth effect is often attributed to a slowdown in total factor productivity (TFP) growth. Focusing on Europe, Aiyar and Ebeke (2016) examine the relationship between labour-force ageing and productivity and find that a shift in the workforce age composition toward older cohorts depresses labour productivity growth, largely through weaker TFP performance. Using OECD economies, Lee and Shin (Lee and Shin) conduct a channel-decomposition analysis and show that declining trend TFP growth is the most important mechanism through which population ageing dampens aggregate economic growth.

While some studies use indicators such as the pension expenditure ratio or the old-age dependency ratio as a proxy for population ageing to examine its macroeconomic effects, Weiske (Weiske) employ population growth rate in a macroeconometric framework to characterise demographic change and analyse its dynamic interactions with macroeconomic variables, since this variable reflect changes in fertility, the pace of population expansion, and long-run labour supply growth. Moreover, under the government's intertemporal budget constraint, population growth enters the effective discounting of future fiscal variables. Accordingly, this paper uses the population growth rate as a parsimonious state variable to capture demographic dynamics, the speed of population transition, and their macro-fiscal implications.

1.3 Methodology

1.3.1 Linear Approximation

Before estimating the VAR model, I linearized the government budget identity, leading to the derivation of the PVBC. This linearization offers several advantages. First, it allows the discount factor to enter the PVBC in a linear form, simplifying the analytical framework. Second, it does not require the assumption of the steady state as required by structural models. (see Campbell and Shiller (1987) and Bergin and Sheffrin (2000)), ensuring that the approximation is not based on any specific theoretical model. As a result, the calculation can be performed regardless of the statistical properties of the data. Polito and Wickens (2005) instead use log-linear approximation when taking the dynamics of tax revenue and government expenditure into account. In this thesis the focus is the relationship between primary balance and debt. Primary balance can be negative so log linearisation is not applicable.

To explain why we do this form of linear approximation. That is, in the end, we can have the growth rate of population entering the PVBC. While some studies use indicators such as the pension expenditure ratio or the old-age dependency ratio as a proxy for population ageing to examine its macroeconomic effects, Weiske (Weiske) employ population growth rate in a macroeconometric framework to characterise demographic change and analyse its dynamic interactions with macroeconomic variables, since this variable reflect changes in fertility, the pace of population expansion, and long-run labour supply growth. Moreover, under the government's intertemporal budget constraint, population growth enters the effective discounting of future fiscal variables. Accordingly, this paper uses the population growth rate as a parsimonious state variable to capture demographic dynamics, the speed of population transition, and their macro-fiscal implications.

The nominal government budget identity is:

$$B_t = G_t - V_t + (1 + R_t) \cdot B_{t-1}, \tag{1.5}$$

where B_t is the nominal debt, G_t is the nominal government expenditure, V_t is the nominal tax revenue, R_t is the average interest rate on debt at the end of period t-1.

Divide both sides of equation (1.5) by nominal GDP, Y_t, and rearrange it:

$$\frac{G_t}{Y_t} + (1 + R_t)\frac{B_{t-1}}{Y_t} = \frac{V_t}{Y_t} + \frac{B_t}{Y_t}. \tag{1.6}$$

To take population aging into account, write nominal variables, G_t, B_t, V_t, and Y_t as the product of the population N_t, the price level P_t, and their real per capita form. Then equation (1.6) then becomes:

$$\frac{N_t P_t \mathcal{G}_t}{N_t P_t \mathcal{Y}_t} + (1 + R_t)\frac{N_{t-1} P_{t-1} \mathcal{B}_{t-1}}{N_t P_t \mathcal{Y}_t} = \frac{N_t P_t \mathcal{V}_t}{N_t P_t \mathcal{Y}_t} + \frac{N_t P_t \mathcal{B}_t}{N_t P_t \mathcal{Y}_t}, \tag{1.7}$$

where $\mathcal{G}_t$ is the real per capita government expenditure, $\mathcal{B}_t$ is the real per capita debt, $\mathcal{V}_t$ is the real per capita tax revenue, and $\mathcal{Y}_t$ is the real per capita output. The ratios in equation (1.7) are just equal to the ratio between the nominal variable and the nominal GDP. After simplify equation (1.7) yields:

$$b_t = -pb_t + (1 + \rho_t) \cdot b_{t-1}, \tag{1.8}$$

where b_t is the debt to GDP ratio, and pb_t is the primary balance-to-GDP ratio. The term ρ_t is defined as $\rho_t = \frac{(1+R_t)}{(1+n_t)(1+\pi_t)(1+\gamma_t)}$, where n_t is the population growth rate, π_t is the inflation rate, and γ_t is the real economic growth rate per capita. Hence, ρ_t can be approximated as $\rho_t \simeq R_t - n_t - \pi_t - \gamma_t$. Equation (3.25) can then be linearised around the fix point. So the steady state can be written as:

$$b = -pb + (1 + \rho)b, \tag{1.9}$$

which can be simplified as:

$$pb = b\rho. \tag{1.10}$$

To prepare the equation for linearisation, each variable is decomposed into its steady-state value and a deviation from the steady state. Specifically, write

$$\tilde{b}_t \equiv b_t - b, \quad \tilde{pb}_t \equiv pb_t - pb, \quad \tilde{\rho}_t \equiv \rho_t - \rho \tag{1.11}$$

Put this in equation (1.40), there becomes:

$$b + \tilde{b}_t = -\left(pb + \tilde{pb}_t\right) + (1 + \rho + \tilde{\rho}_t)\left(b + \tilde{b}_{\mathrm{t}-1}\right) \tag{1.12}$$

After Taylor first series expansion:

$$b + \tilde{b}_t = -pb - \tilde{pb}_t + (1 + \bar{\rho})b + (1 + \rho)\tilde{b}_{\mathrm{t}-1} + b\rho_{\mathrm{t}} + \tilde{b}_{\mathrm{t}-1}\tilde{\rho}_t \tag{1.13}$$

With steady state equation to cancel the constant parts:

$$\tilde{b}_t = -\tilde{p}\tilde{b}_t + (1 + \rho)\tilde{b}_{t-1} + b\tilde{p}_t + \tilde{b}_{t-1}\tilde{p}_t \tag{1.14}$$

With Taylor approximation, I ignore the second-order small terms:

$$\tilde{b}_t \approx (1 + \rho)\tilde{b}_{t-1} + b\tilde{\rho}_t - \tilde{p}b_t \tag{1.15}$$

Write the equation in levels:

$$
\begin{aligned}
b_t - b &\approx (1+\rho)\,(b_{t-1} - b) + b\,(\rho_t - \rho) - (pb_t - pb) \\
b_t - b &= b_{t-1} - b + \rho b_{t-1} - b\rho + b\rho_t - b\rho - pb_t + pb \qquad (1.16) \\
b_t &= -pb_t + pb + b_{t-1} + \rho b_{t-1} - b\rho + b\rho_t - b\rho
\end{aligned}
$$

The equation can be further simplified since $pb = b\rho$ and a linear approximation is then given by:

$$
\begin{aligned}
b_t &= -pb_t + (1+\rho)b_{t-1} + b\rho_t - b\rho \\
(1+\rho)b_{t-1} &= b_t + pb_t - b\rho_t + b\rho
\end{aligned}
\qquad (1.17)
$$

Let $\mathcal{I}_t$ denote the information set available at time t. Define the conditional expectation operator as

$$E_t[\cdot] \equiv E[\cdot \mid \mathcal{I}_t]\,.$$

The equation can be written as:

$$b_{t-1} = \frac{1}{1+\rho} E_t\left[b_t + pb_t - b\rho_t + b\rho\right] \qquad (1.18)$$

Put the equation one period ahead, it becomes:

$$
\begin{aligned}
b_t &= \frac{1}{1+\rho} E_t\left[b_{t+1} + pb_{t+1} - b\rho_{t+1} - b\rho\right] \\
b_t &= \frac{1}{1+\rho} E_t b_{t+1} + \frac{1}{1+\rho} E_t\left[pb_{t+1} - b\rho_{t+1} - b\rho\right]
\end{aligned}
\qquad (1.19)
$$

The above becomes

$$b_t = \frac{1}{1+\rho} E_t(pb_{t+1} - b\rho_{t+1} + pb) + \frac{1}{1+\rho} E_t(b_{t+1}).$$

Forward iterating the above for j periods yields

$$b_t = \sum_{i=1}^{j} (1+\rho)^{-i} E_t(pb_{t+i} - b\rho_{t+i} + pb) + (1+\rho)^{-j} E_t(b_{t+j}). \tag{1.20}$$

Let s_t denote effective primary balance:

$$s_t = pb_t - b\rho_t + pb. \tag{1.21}$$

Equation (1.20) can also be written as

$$b_t = \sum_{i=1}^{j} (1+\rho)^{-i} E_t(s_{t+i}) + (1+\rho)^{-j} E_t(b_{t+j}). \tag{1.22}$$

When j goes to infinity,

$$b_t = \sum_{i=1}^{\infty} (1+\rho)^{-i} E_t(s_{t+i}), \tag{1.23}$$

and $\lim_{j\to\infty}(1+\rho)^{-j}E_t(b_{t+j}) = 0$. In this way, the discount rate, ρ, in the model is constant.

1.3.2 Model

The empirical framework is based on a VAR for several reasons. First, the VAR can represent structured macroeconomic models including DSGE model through imposing restrictions. Second, time series models are known for forecasting, as it can take into account for structural breaks and not require the prior testing that identifying breaks. These advantages make VAR models convenient to generate forecasts and to automate the calculation. Third, VAR models do not impose particular view of the economy. For instance, when using DSGE model one should impose the transversality conditions to compute the solution. This means that the debt ratio never explodes in a DSGE model, or equivalently it is sustainbale by assumption. Using VAR model avoids any contentious issues that might arise form choosing a specific macroeconomic model and possibility from imposing restrictions not satisfied by the data. Last, VAR model can also take into account the dynamics of population aging. To take population aging into account, many research uses OLG model (e.g Puhakka (2005), Chalk and Hemming (2000), Rankin and Roffia (2003)). Using OLG model requires for assuming the type of pension system and the way that population aging affects fiscal sustainability. Nevertheless, Yoshino et al. (2019) conclude that the impact of population aging is generally analysed through two channels: one is the impact on the growth rate of GDP and the other is the effect of pension increase on government expenditure. Those effects would all eventually be captured by public finance variables which are related to fiscal sustainability and then would be captured by VAR model used in this chapter.

The VAR model is specified as follows:

$$\boldsymbol{z_t} = \boldsymbol{A_0} + \boldsymbol{A(L)}\boldsymbol{z_{t-1}} + \boldsymbol{u_t} \tag{1.24}$$

where $\boldsymbol{A_0}$ is a constant matrix, $\mathbf{A}(L) = \sum_{i=0}^{p-1} \mathbf{A}_i L^i$, with $\mathbf{A}_i$ indicating a $q \times q$ matrix of lag i coefficients, L is the lag operator, and $u_t \sim$ i.i.d.$[\mathbf{0}, \mathbf{\Sigma}]$. The vector $\boldsymbol{z_t}$ is specified as $\boldsymbol{z_t} = [b_t,\, n_t,\, pb_t,\, \tilde{y}_t,\, \tilde{g}_t,\, \pi_t,\, r_t,\, \gamma_t]$. b_t is the debt to GDP ratio, n_t is the growth rate of population, pb_t is the primary balance to GDP ratio, $\tilde{y}_t$ is the cyclical component of output, $\tilde{g}_t$ is the cyclical component of government expenditure, r_t is the long-term interest rate, π_t is the inflation rate, and γ_t is the real output growth rate per capita. $\boldsymbol{z_t}$ vector include all fiscal variables in the PVBC. Bohn (2005) finds that without controlling the cyclical component of government expenditure and output would make the react of primary balance ratio to debt ratio higher, thereby those two variables should be controlled.

The vector of variables $\boldsymbol{z_t}$ can be I(0) or I(1) and it is quite different from the previous literature is that there is neither discussion about the non-stationarity nor about the first difference of the variable. VAR model in this chapter is mainly used for forecasting and expectations will not be affected by the fact that data follows a random walk thus the variables used in the VAR model does not need to be first differenced. The integrate orders of variables do not affect the choice of the VAR model as well. The cointegrated VAR may be required when the integrate order is greater than zero, yet the cointegrated VAR can be written in levels VAR.

Forecast could be based on the companion form of the VAR model since this form would make the forecast easier:

$$\boldsymbol{Z_t} = \boldsymbol{\mathcal{A} Z_{t-1}} + \boldsymbol{U_t} \tag{1.25}$$

where $\boldsymbol{Z_t} = [\boldsymbol{z'_t}, \boldsymbol{z'_{t-1}}, ..., \boldsymbol{z'_{t-p+1}}, \boldsymbol{1}], \boldsymbol{U_t} = [\boldsymbol{u'_t}, \boldsymbol{0'}, ..., \boldsymbol{0'}]'$, and

$$\mathcal{A} = \begin{bmatrix} \boldsymbol{A_1} & \boldsymbol{A_2} & . & . & \boldsymbol{A_p} & \boldsymbol{A_0} \\ \boldsymbol{I} & \boldsymbol{0} & . & . & . & . \\ \boldsymbol{0} & \boldsymbol{I} & \boldsymbol{0} & . & . & . \\ . & . & . & . & . & . \\ \boldsymbol{0} & . & . & \boldsymbol{I} & \boldsymbol{0} & . \\ \boldsymbol{0} & \boldsymbol{0} & . & . & \boldsymbol{0} & \boldsymbol{1} \end{bmatrix}.$$

The size of $\boldsymbol{Z_t}$ is 8(p+1)*1, $\boldsymbol{\mathcal{A}}$ is 8(p+1)*8(p+1), and $\boldsymbol{U_t}$ is 8(p+1)*1. In this chapter, recursive VAR is adopted to document the break in fiscal policy.

1.3.3 Campbell (1987) Test

Trehan and Walsh (1991) comment that the present value model Campbell and Shiller (1987) use is formally equivalent to PVBC. The way that Campbell (1987) test the permanent income hypothesis is the same as the one that Campbell and Shiller (1987) test the present value model. Koo (2011) uses this method of Campbell and Shiller (1987) to test whether the initial debt for Korea is sustainable. This method is forward-looking since it forecasts the sum of the present value of future primary surpluses while most of the research testing the econometric test on government PVBC does not have this advantage. The test result also can tell that the current level of debt is too much if the restriction is rejected due to the implication from the test that the sum of present value of future primary balance cannot pay off the current level of debt.

The Campbell test is a test on the cross-equation restriction. This cross-equation restriction requires both sides of the PVBC be regressed on the same vector and the test result can determine whether the fiscal policy is sustainable or not. Campbell (1987) test is based on equation (1.22). According to the linear approximation of the government budget identity described in the previous section, the intertemporal government budget constraint is given by equation (1.22). Setting $b\rho \cong pb$ in equation (1.21) returns the intertemporal government budget constraint of equation (1.4). I base my estimate of Campbell (1987) test on this restricted version.

To forecast primary balance I forecast first the vector $\boldsymbol{Z_{t+j}}$ using parameters estimated from the companion form VAR model through equation (1.25)

$$\boldsymbol{E}_t[\boldsymbol{Z}_{t+j}|\boldsymbol{\Omega}_t] = \boldsymbol{\mathcal{A}}^j \boldsymbol{Z}_t, \tag{1.26}$$

where $\boldsymbol{\Omega_t}$ stands for the information included in $\boldsymbol{Z_t}$. Thereafter, the present value of the variables in $\boldsymbol{Z_t}$ from period 1 to infinity can be forecast through the below equation:

$$\sum_{j=1}^{\infty}(1+\rho)^{-j}\boldsymbol{E_t}[\boldsymbol{Z_{t+j}}|\boldsymbol{\Omega_t}] = \frac{\boldsymbol{\mathcal{A}}}{1+\rho}[\boldsymbol{I} - \frac{\boldsymbol{\mathcal{A}}}{1+\rho}]^{-1}\boldsymbol{Z_t} \tag{1.27}$$

where ρ is calibrated to the mean value of all discount rate observations. The left hand side of this expression is a $8(p+1)\times 1$ vector whose third element is $\sum_{j=1}^{\infty}(1+\rho)^{-j}E_t[pb_{t+j}|\Omega_t]$. Thus it can be written as:

$$\sum_{j=1}^{\infty}(1+\rho)^{-j}E_t[pb_{t+j}|\boldsymbol{\Omega_t}] = \begin{bmatrix}0 & 0 & 1 & 0 & . & . & . & 0\end{bmatrix}\frac{\boldsymbol{\mathcal{A}}}{1+\rho}[I - \frac{\boldsymbol{\mathcal{A}}}{1+\rho}]^{-1}\boldsymbol{Z_t}. \tag{1.28}$$

Let

$$\boldsymbol{F} = \boldsymbol{Ra} = -\begin{bmatrix}0 & 0 & 1 & 0 & . & . & . & 0\end{bmatrix}\frac{\boldsymbol{\mathcal{A}}}{1+\rho}[I - \frac{\boldsymbol{\mathcal{A}}}{1+\rho}]^{-1} \tag{1.29}$$

with $\boldsymbol{R} = \begin{bmatrix}0 & 0 & 1 & 0 & . & . & . & 0\end{bmatrix}$ and $\boldsymbol{a}$ stands for parameters in the rest part of the equation. Now consider running separate regressions of the left- and right-hand sides of equation onto vector $\boldsymbol{Z_t}$. That is, regress separately b_t onto $\boldsymbol{Z_t}$ and $\sum_{j=1}^{\infty}(1+\rho)^{-j}E_t[pb_{t+j}|\boldsymbol{\Omega_t}]$ onto $\boldsymbol{Z_t}$. Since $\boldsymbol{Z_t}$ includes b_t as its first element, the regression coefficient of the left-hand-side regression is the vector $\boldsymbol{Q} = \begin{bmatrix}1 & 0 & . & . & . & .\end{bmatrix}$. The regression coefficient of the right-hand-side regression is **F**. Therefore, the model implies the following restriction on the vector **F** which is test using the Wald test:

$$\boldsymbol{F} = \boldsymbol{Q} = \begin{bmatrix}1 & 0 & . & . & . & .\end{bmatrix} \tag{1.30}$$

The Wald tests statistics are computed as: $W = [\boldsymbol{F} - \boldsymbol{Q}]'[\boldsymbol{\partial F/\partial a \Lambda \partial F/\partial a'}]^{-1}[\boldsymbol{F} - \boldsymbol{Q}]$ where $\boldsymbol{F} - \boldsymbol{Q}$ is the difference between the coefficient vectors, $\boldsymbol{\partial F/\partial a}$ is the matrix derivative of the **F** vector with respect to the VAR parameters and $\boldsymbol{\Lambda}$ the variance-covariance of the underlying parameters. Since this is a linear model, the restriction should be $\boldsymbol{Ra} - \boldsymbol{Q} = \boldsymbol{0}$, the partial derivative should be $\boldsymbol{\partial(Ra - Q)/\partial a} = \boldsymbol{R}$. The test statistic has χ^2 distribution with 8*(p+1) degrees of freedom since there are eight independent variables plus the constant. The rejection of these restrictions implies the rejection of the PVBC.

Nevertheless, when estimating a VAR in levels, Campbell (1987) note that heteroskedasticity may affect standard inference. Hence, I employ a heteroskedasticity-robust (White) covariance estimator for the Wald statistic, given by

$$\boldsymbol{\Lambda} = \left(\boldsymbol{Z}'_{\text{stack},-1}\boldsymbol{Z}_{\text{stack},-1}\right)^{-1} \boldsymbol{Z}'_{\text{stack},-1}\,\boldsymbol{\Theta}\,\boldsymbol{Z}_{\text{stack},-1}\left(\boldsymbol{Z}'_{\text{stack},-1}\boldsymbol{Z}_{\text{stack},-1}\right)^{-1}, \tag{1.31}$$

where $\boldsymbol{Z}_{\text{stack},-1} = [\boldsymbol{Z}_0, \boldsymbol{Z}_1, \ldots, \boldsymbol{Z}_{T-1}]'$ stacks the lagged regressors $\boldsymbol{Z}_{t-1}$ across $t = 1, \ldots, T$, with T denoting the effective sample size used for estimation (i.e., the number of usable time observations after accounting for lags), and

$$\boldsymbol{\Theta} = \text{diag}(\omega_1, \ldots, \omega_T), \qquad \omega_t = \widehat{u}_t^2. \tag{1.32}$$

1.3.4 Bohn (2005) Rule

The second test is the Bohn (2005) rule which is used to examine the systematic relationship between the current primary balance ratio and the current debt ratio. Bohn (2005) concludes that the primary balance should react positively to the debt at the end of last period. The related equation is called the 'fiscal reaction function'. Mendoza and Ostry (2008) conclude that the essence of Bohn (2005) fiscal reaction function is the relationship between debt and primary balance hence the systematic relationship between primary balance and debt ratio. Controlling other fiscal variables that can affect primary balance is quite important when examining Bohn (2005) rule since omitting those fiscal variables would create bias on the final results. Bohn (2005) controls the cyclical component of output by using HP filter and the cyclical component of government expenditure by using a simple time series model when studying the fiscal sustainability in the US. Mendoza and Ostry (2008) use the cyclical components of government expenditure and output calculated by the Hodrick–Prescott filter.

I estimate Bohn (2005) rule by using a structural VAR model based on the unrestricted VAR model in (1.24). Using the Cholesky decomposing to the structural VAR, I can then recover the Bohn (2005) rule in the structural VAR model and analyse the relationship between primary balance ratio and debt ratio while controlling for other lagged fiscal variables in PVBC that would affect primary balance ratio.

When planning the fiscal policy, especially when planning the budget or planning the annual fiscal adjustment, policy makers quite often refer to the macroeconomic situation in the last year. Controlling the components of the discount rate in the last period when estimating Bohn (2005) rule is consistent with the inertia of annual fiscal response and the basic logic of annual-based policy making.

The structural VAR model based on the unrestricted VAR model in (1.24) is:

$$\boldsymbol{C z_t} = \boldsymbol{D_0} + \boldsymbol{D(L) z_{t-1}} + \boldsymbol{v_t}, \tag{1.33}$$

where $\boldsymbol{C}$ is the matrix of parameters, $\boldsymbol{D_0}$ is a constant matrix, $\mathbf{D}(L) = \sum_{i=0}^{p-1} \mathbf{D}_i L^i$, with $\mathbf{D}_i$ indicating a $q \times q$ matrix of lag i coefficients, $\boldsymbol{L}$ is the lag operator, and $\boldsymbol{v_t}$ is the residual, $\boldsymbol{v_t \sim [0, \Sigma_v]}$. Some coefficients that are components of the response of primary balance to GDP ratio to debt ratio are inside matrix $\boldsymbol{C}$ and I compute this matrix by using Cholesky decomposition.

When Pre-multiply $\boldsymbol{C^{-1}}$ to each part in the structural VAR model in eq (1.33) can derive the unrestricted VAR model in eq (1.24):

$$\begin{aligned} \boldsymbol{z_t} &= \boldsymbol{C^{-1} D_0} + \boldsymbol{C^{-1} D(L) z_{t-1}} + \boldsymbol{C^{-1} v_t} \\ &= \boldsymbol{A_0} + \boldsymbol{A(L) z_{t-1}} + \boldsymbol{u_t}. \end{aligned}$$

Doing Cholesky decomposition to $\boldsymbol{\Sigma}$, the variance-covariance matrix for residuals in the reduced-form VAR model in eq (1.24):

$$\boldsymbol{\Sigma} = \boldsymbol{P}\boldsymbol{P}', \tag{1.34}$$

where $\boldsymbol{P}$ is a triangular matrix. To recover $\boldsymbol{C}$ from $\boldsymbol{P}$, define the structural variances $\boldsymbol{\Gamma}$ as the square of the diagonal elements of $\boldsymbol{P}$. Since

$$\boldsymbol{\Sigma} = \boldsymbol{u}_t\boldsymbol{u}_t' = \boldsymbol{C}^{-1}\boldsymbol{v}_t\boldsymbol{v}_t'(\boldsymbol{C}^{-1})' = \boldsymbol{C}^{-1}\boldsymbol{\Gamma}(\boldsymbol{C}^{-1})', \tag{1.35}$$

which implies that $\boldsymbol{P} = \boldsymbol{C}^{-1}\boldsymbol{\Gamma}^{-\frac{1}{2}}$ and hence

$$\boldsymbol{C} = (\boldsymbol{P}\boldsymbol{\Gamma}^{-\frac{1}{2}})^{-1}. \tag{1.36}$$

Now the equation can be expanded as:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ c_{21} & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ c_{31} & c_{32} & 1 & 0 & 0 & 0 & 0 & 0 \\ c_{41} & c_{42} & c_{43} & 1 & 0 & 0 & 0 & 0 \\ c_{51} & c_{52} & c_{53} & c_{54} & 1 & 0 & 0 & 0 \\ c_{61} & c_{62} & c_{63} & c_{64} & c_{65} & 1 & 0 & 0 \\ c_{71} & c_{72} & c_{73} & c_{74} & c_{75} & c_{76} & 1 & 0 \\ c_{81} & c_{82} & c_{83} & c_{84} & c_{85} & c_{86} & c_{87} & 1 \end{bmatrix} \begin{bmatrix} b_t \\ n_t \\ pb_t \\ \tilde{y}_t \\ \tilde{g}_t \\ \pi_t \\ r_t \\ \gamma_t \end{bmatrix} = \begin{bmatrix} d_{10} \\ d_{20} \\ d_{30} \\ d_{40} \\ d_{50} \\ d_{60} \\ d_{70} \\ d_{80} \end{bmatrix} +$$

$$\begin{bmatrix} d_{11}(L) & d_{12}(L) & d_{13}(L) & d_{14}(L) & d_{15}(L) & d_{16}(L) & d_{17}(L) & d_{18}(L) \\ d_{21}(L) & d_{22}(L) & d_{23}(L) & d_{24}(L) & d_{25}(L) & d_{26}(L) & d_{27}(L) & d_{28}(L) \\ d_{31}(L) & d_{32}(L) & d_{33}(L) & d_{34}(L) & d_{35}(L) & d_{36}(L) & d_{37}(L) & d_{38}(L) \\ d_{41}(L) & d_{42}(L) & d_{43}(L) & d_{44}(L) & d_{45}(L) & d_{46}(L) & d_{47}(L) & d_{48}(L) \\ d_{51}(L) & d_{52}(L) & d_{53}(L) & d_{54}(L) & d_{55}(L) & d_{56}(L) & d_{57}(L) & d_{58}(L) \\ d_{61}(L) & d_{62}(L) & d_{63}(L) & d_{64}(L) & d_{65}(L) & d_{66}(L) & d_{67}(L) & d_{68}(L) \\ d_{71}(L) & d_{72}(L) & d_{73}(L) & d_{74}(L) & d_{75}(L) & d_{76}(L) & d_{77}(L) & d_{78}(L) \\ d_{81}(L) & d_{82}(L) & d_{83}(L) & d_{84}(L) & d_{85}(L) & d_{86}(L) & d_{87}(L) & d_{88}(L) \end{bmatrix} \begin{bmatrix} b_{t-1} \\ n_{t-1} \\ pb_{t-1} \\ \tilde{y}_{t-1} \\ \tilde{g}_{t-1} \\ \pi_{t-1} \\ r_{t-1} \\ \gamma_{t-1} \end{bmatrix}$$
$$+ \begin{bmatrix} v_{bt} \\ v_{nt} \\ v_{pb\,t} \\ v_{\tilde{y}t} \\ v_{\tilde{g}t} \\ v_{\pi t} \\ v_{rt} \\ v_{\gamma t} \end{bmatrix}. \tag{1.37}$$

The third equation in the equation is actually a ARDL model. The response of Bohn (2005) rule in the primary balance equation in equation (1.33) is then:

$$\begin{aligned} pb_t = d_{30} - c_{31}b_t - c_{32}n_t + \sum_{j=1}^{p} \Big(& d_{31}^{(j)} b_{t-j} + d_{32}^{(j)} n_{t-j} + d_{33}^{(j)} pb_{t-j} + d_{34}^{(j)} \tilde{y}_{t-j} \\ & + d_{35}^{(j)} \tilde{g}_{t-j} + d_{36}^{(j)} \pi_{t-j} + d_{37}^{(j)} r_{t-j} + d_{38}^{(j)} \gamma_{t-j} \Big) + v_{pb\,t}. \end{aligned} \tag{1.38}$$

When in the steady state, every variable in their steady state level and cyclical components are 0,

$$pb = d_{30} + \phi_b\, b + \phi_n\, n + \phi_\pi\, \pi + \phi_r\, r + \phi_\gamma\, \gamma, \tag{1.39}$$

where $\phi_b = \frac{\sum_{j=1}^{p} d_{31}^j - c_{31}}{1-\sum_{j=1}^{p} d_{33}^j}$. In this way, ϕ_b measures the conditional the response of primary balance ratio to debt ratio. The standard error is calculated by Delta method. If the response coefficient δ_b is negative or the test statistic shows that the result is not significant, I can conclude that the government does not proactively react to the increase in government debt. This probably could result in default and increase the probability of fiscal unsustainability.

In this model, the primary balance-to-GDP ratio equation can be written in an ARDL form, a specification widely used in the fiscal sustainability and fiscal reaction function literature (see Berti et al. (2016) and Lima Campos and Cysne (Lima Campos and Cysne)). Relative to a static regression in levels, a key advantage of the ARDL framework is that it models fiscal behaviour through a distributed-lag and autoregressive structure, so concerns about spurious regression are less acute: spuriousness typically arises when non-stationary level series are related in a purely static setting, whereas in an ARDL the long-run interpretation is constrained by the overall dynamic structure rather than by contemporaneous level correlations. Moreover, the long-run interpretation is internally disciplined: when the primary-balance process is highly persistent and exhibits weak mean reversion, the implied long-run relationship is unlikely to be stable or economically plausible, cautioning against treating the estimates as a robust long-run fiscal reaction. Finally, including lagged primary-balance terms captures the well-documented policy smoothing in the fiscal stance and helps absorb serial correlation, thereby improving the reliability of statistical inference.

There are two caveats when examining Bohn (2005) using the VAR model. First, Bohn (2005) is still backward-looking since it only uses parameter estimated from the VAR model. It cannot determine if the fiscal policy is sustainable. Second, a positive reaction in this research is only a sufficient condition for government proactive attitude. A nonlinear and/or time varying response can also support fiscal solvency as long as the response is strictly positive above a certain threshold debt-output ratio, or almost surelv in the long run. (see Mendoza and Ostry (2008)).

1.3.5 Blanchard (1990) Medium-Term Debt Dynamics

The third approach is the medium-term debt dynamics (MTDD). This fiscal indicator is proposed by Blanchard (1990), Blanchard et al. (1991), Buiter (1985) and Buiter et al. (1993). This debt dynamics is used to measure the gap between the target level of debt ratio and the forecast level of debt ratio in period t+j derived from VAR model thereby this fiscal indicator is forward-looking and determined endogenously from a model instead of being set through ad hoc assumption and based on external projections. The gap between the targeted debt ratio and the forecast level of debt ratio tells if the debt is too much or too little and it also answers how much the debt ratio should change to go back to the targeted level.

This indicator is derived by using the research method similar to that in Polito and Wickens (2012a). Polito and Wickens (2012a) choose to use VAR model to forecast the logarithmic equivalent of the primary deficit and then to forecast the targeted debt ratio and forecast the future debt ratio. Since the focus now is on the relationship between primary surplus ratio and debt ratio, I use linear approximation directly and do not separate the component of primary surplus. Essentially calculating Blanchard (1990) medium-term debt dynamics needs to estimate a VAR that includes the variables in $\boldsymbol{z_t}$; take n-periods ahead forecasts of these variables; set a debt-to-GDP target; compute the present value of the VAR forecasts and calculate the difference between the target one and the forecast one in period t+j. The proposed index seeks to determine whether fiscal policy is expected to achieve a target level of the debt ratio for a given j-period horizon.

To form the Blanchard (1990) medium-term debt dynamics, equation (1.22) should be arranged as:

$$MTDD(t,j) = (1+\rho)^{-j} \cdot b^*_{t+j} - \left[b_t - \sum_{i=1}^{j}(1+\rho)^{-i}E_t(s_{t+i})\right] \tag{1.40}$$

where b^*_t is the debt-to-GDP ratio target, b_t is the debt-to-GDP ratio in period t, and $s_t = pb_t - b\rho_t + pb$ is the effective primary surplus-to-GDP ratio. MTDD(t,j) measures the gap between two parts: $(1+\rho)^{-j}b^*_{t+j}$, the discounted value of debt-GDP ratio in period $t+j$, and $[b_t - \sum_{i=1}^{j}(1+\rho)^{-i}E_t(s_{t+i})]$, the forecast of the discounted value of debt in period $t+j$. $E_t(s_{t+i})$ is forecasted by the companion form of VAR model. Expressing s_t as the following linear function of $\boldsymbol{z_t}$ to collect coefficients describing relationship between the two:

$$s_t = \alpha + \boldsymbol{\beta' z_t} \tag{1.41}$$

and defining the selection matrix $\boldsymbol{S} = [\boldsymbol{I, 0, 0,}..., \boldsymbol{0}]$ such that $\boldsymbol{z_t} = \boldsymbol{SZ_t}$ I obtain

$$s_{t+i} = \alpha + \boldsymbol{\beta' S}(\boldsymbol{\mathcal{A}^i Z_t}) \tag{1.42}$$

5-year time horizon is chosen in this research and this choice makes the Blanchard (1990) medium-term debt dynamics, the Campbell (1987) test of infinite-term, and the Bohn (2005) of short-term form an assessment framework with multi dimension.

The result of Blanchard (1990) medium-term debt dynamics can be divided into three cases:

(i) $MTDD(t,j) = 0$ then the forecast debt ratio for period t+j is on target;

(ii) $MTDD(t, j) > 0$ then the debt ratio in period t+j is forecast to be below target;

(iii) $MTDD(t, j) < 0$ then the debt ratio in period t+j is forecast to be above target.

Only in case (iii) the forecast value of the level of discounted debt-GDP ratio insufficient to achieve its desired value, suggesting that fiscal policy is too loose. The caveat using Blanchard (1990) medium-term debt dynamics is that it does not tell whether fiscal policy is sustainable, since choosing too far horizon result in severe inaccuracy.

This is used for medium-term plan.According to the needs of major economies, the horizon would be set as longer than 5 years.

1.3.6 Discussion

In the VAR framework, these three approaches described above jointly constitute a comprehensive evaluation system in the sense that they balance faithfulness to the theoretical definition with practical policy relevance. The system can be described using the equation below:

Faithfulness to the Theoretical Definition	⟷	Practical Policy Relevance
Campbell (1987) Test	Bohn (2005) Rule	MTDD (t, j)

In this section, I use table 1.1 to summarize the difference among three approaches in the system and the advantage, especially the information that each of them provided. The Campbell (1987) test keeps faithfulness to theorectical definition It examines fiscal sustainability by regressing the present value sum of the primary balance ratio predicted by the VAR model and the debt ratio included in the VAR vector onto the VAR model vector, testing whether the two coefficient vectors are equal to each other. However, Campbell (1987) test is still backward looking, and while it can indicate that a policy is unsustainable, it does not specify whether the level of debt is excessive or insufficient, nor does it provide guidance on the measures needed to restore fiscal sustainability.

The second test, the Bohn (2005) rule, describes the systematic relationship between the primary balance and debt. Although the Bohn (2005) rule is based solely on historical data, it reveals and quantifies how the primary balance responds to the change in debt ratio. This response helps assess whether the level of debt is stable or unstable.

The third test, the Blanchard (1990) medium-term debt dynamics, measures the fiscal adjustments required to achieve a given medium-term debt target. It compares the debt values predicted by the VAR model with the target debt values set in the economy. Through this comparison, it assesses whether debt levels are too high or too low and how much should adjust.

Table 1.1: Comparison among the Three Tests

	Forward Looking	Fiscal Sustainability	Debt Level Evaluation	Degree of Fiscal Adjustment
Campbell (1987) test		✓		
Bohn (2005) Rule			✓	
Blanchard (1990) medium-term debt dynamics	✓		✓	✓

1.4 Data

This chapter analyses the fiscal sustainability for major economies: G7 countries (Canada, France, Germany, Italy, Japan, UK, and US) and China. Since the global financial crisis of 2007–2009, public debt in major economies has increased persistently, both in magnitude and growth rate, raising concerns about the accumulation of fiscal risks. Since the COVID-19 pandemic, macroeconomic uncertainty has increased substantially, further amplifying these concerns and making the sustainability of fiscal policy in major economies one of the most pressing policy questions in practice.

Including all the G7 economies and China allows the analysis to capture two representative pathways of population ageing. On the one hand, most G7 countries are already in an advanced stage of ageing, where fiscal pressures mainly arise from ongoing increases in age-related spending such as public pensions and healthcare. On the other hand, China faces rapid ageing alongside a growth transition, implying that fiscal sustainability challenges are more closely linked to the joint constraints of demographic shifts and a slowing expansion of the tax base. Examining these two groups together therefore provides a broader assessment of fiscal sustainability under demographic change and helps evaluate whether the empirical regularities remain robust across different institutional environments.

I used data series from the OECD Economic Outlook database, the IMF World Economic Outlook (WEO) database and the IMF International Financial Statistics database to create datasets on eight variables for the G7 countries and China: government debt as a share of GDP (b_t), growth rate of population (n_t), cyclical component of output ($\tilde{y}_t$), cyclical component of government spending ($\tilde{g}_t$), primary balance as a share of GDP (pb_t), inflation (π_t), long-term nominal interest rate (r_t), and real economic growth rate per capita(γ_t). These data are shown in Figures 1.1 - 1.8 with

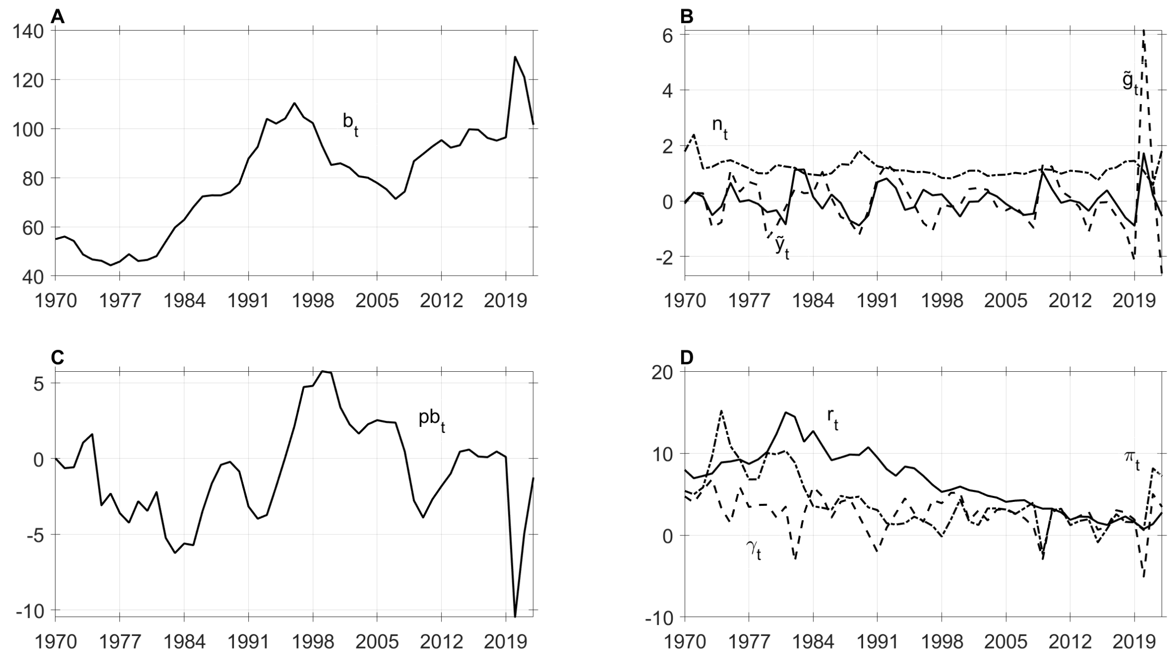

Figure 1.1: Canada: Data Plot

This figure presents the model variables for Canada (1970–2022). Panel A: debt-to-GDP ratio. Panel B: cyclical component of government expenditure (dashed line), cyclical component of output (solid line), and population growth rate (dotted line). Panel C: primary balance-to-GDP ratio. Panel D: long-term interest rate (solid line), real GDP growth rate (dashed line), and inflation rate (dotted line). All units are expressed as percentages.

Panel A presenting the b_t; Panel B displaying $\tilde{g}_t$, $\tilde{y}_t$, and n_t; Panel C illustrating pb_t; and Panel D showing r_t, γ_t, and π_t. All data range from 1970 to 2022, with the exception of China. The data for China only available from 1980.

n_t is calculated by applying N_t, the overall population level in year t which is available for all countries analysed in the OECD database to the equation $n_t = (N_t - N_{t-1})/N_{t-1}$. The 1970 n_t for most countries analysed and the 1991 n_t for Germany, and the 1980 n_t for China are calculated in each country using the average n_t over the next 2 years.

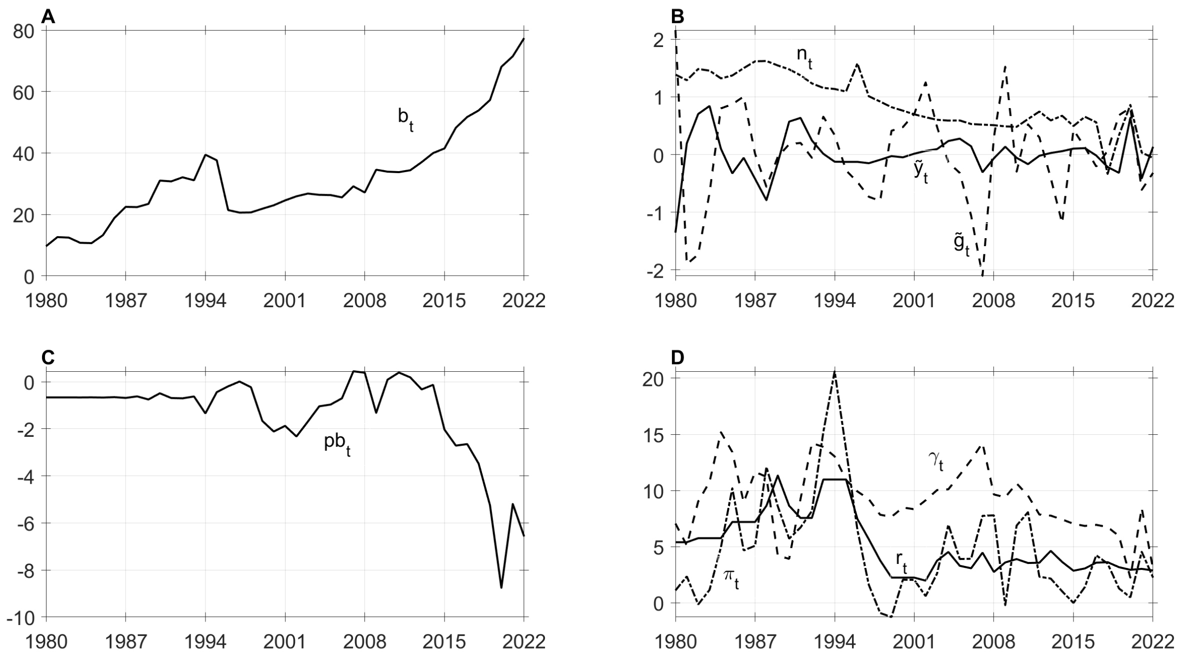

Figure 1.2: China: Data Plot

This figure presents the model variables for China (1980–2022). Panel A: debt-to-GDP ratio. Panel B: cyclical component of government expenditure (dashed line), cyclical component of output (solid line), and population growth rate (dotted line). Panel C: primary balance-to-GDP ratio. Panel D: long-term interest rate (solid line), real GDP growth rate (dashed line), and inflation rate (dotted line). All units are expressed as percentages.

In the German sample, the 1991 reunification constitutes a one-off level shift in population statistics, which generates an abnormal spike in the population-growth series around that year. In a VAR setting, such a single-period outlier can do more than distort the population-growth equation itself: because the system is estimated jointly, it may affect coefficient estimates across equations and inflate the estimated residual covariance matrix. This can, in turn, alter shock identification in a VAR (e.g., under a recursive Cholesky scheme). To mitigate this issue, I "take out" the reunification spike from the German population-growth series. The advantage of this approach is that it removes the spike from the population series ex ante, yielding a smoother population-growth series.

Operationally, I use an impulse dummy (D_t) that equals one in 1991 and zero otherwise. I first estimate the auxiliary regression $[n_t = c + \gamma D_t + u_t,]$ and then remove the estimated one-off component from the original series: $[n_t^{adj} = n_t - \hat{\gamma} D_t.]$ This adjustment affects only the reunification-related observation while leaving all other years unchanged.

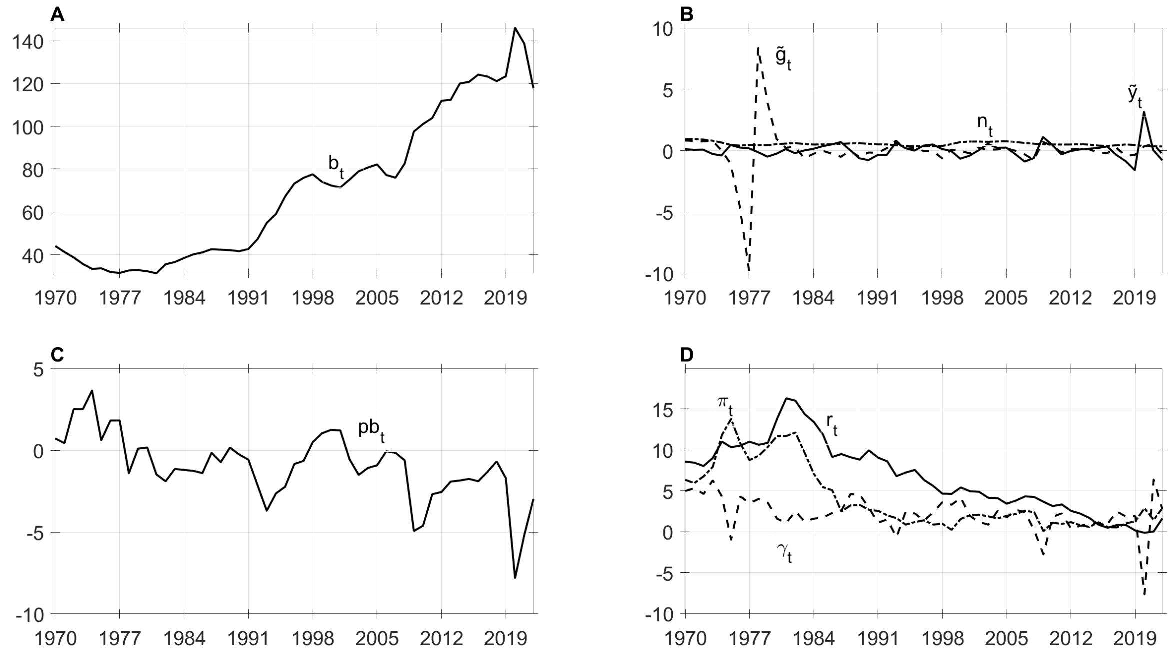

Figure 1.3: France: Data Plot

This figure presents the model variables for France (1970–2022). Panel A: debt-to-GDP ratio. Panel B: cyclical component of government expenditure (dashed line), cyclical component of output (solid line), and population growth rate (dotted line). Panel C: primary balance-to-GDP ratio. Panel D: long-term interest rate (solid line), real GDP growth rate (dashed line), and inflation rate (dotted line). All units are expressed as percentages.

$\tilde{y}_t$ is obtained by detrending the real output per capita using the Hodrick-Prescott filter with the smoothing parameter set at 6.25. The real output is calculated by dividing the nominal gross domestic product value at market price in the OECD database by the gross domestic product deflator at market price, and the real output per capita is obtained by dividing the real output by N_t.

$\tilde{g}_t$ is calculated by first dividing the gross general government interest payments by the Nominal Gross domestic product value at market price to form the interest payment of the general government to GDP ratio. The total disbursements of the general government as a percentage of GDP minus interest payment of the general government to GDP ratio equals the non-interest government expenditure to GDP ratio. g_t is derived by using a hp filter to the non-interest government expenditure to GDP ratio with smoothing parameter 6.25. The OECD database lacks data on total general government disbursements for France from 1970 to 1977. Therefore, I use total government expenditure from the IMF World Economic Outlook database as a substitute.

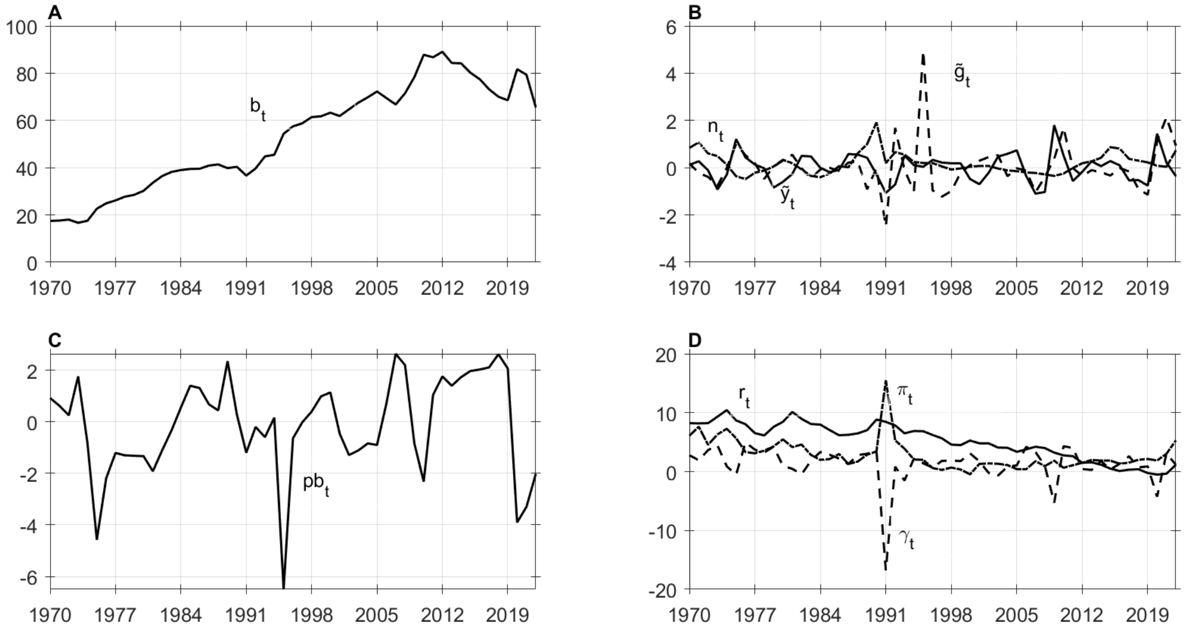

Figure 1.4: Germany: Data Plot

This figure presents the model variables for Germany (1970–2022). Panel A: debt-to-GDP ratio. Panel B: cyclical component of government expenditure (dashed line), cyclical component of output (solid line), and population growth rate (dotted line). Panel C: primary balance-to-GDP ratio. Panel D: long-term interest rate (solid line), real GDP growth rate (dashed line), and inflation rate (dotted line). All units are expressed as percentages.

The method to calculate the π_t is to use use the formula $\pi_t = (P_t^{GDP} - P_{t-1}^{GDP})/P_t^{GDP}$, where P_t^{GDP} is the implicit GDP deflator index in OECD database. π_t is then obtained as the percentage change in the GDP deflator. The 1970 π_t for most countries analysed and the 1991 π_t for Germany, and the 1980 π_t for China are calculated in each country using the average π_t over the next 2 years.

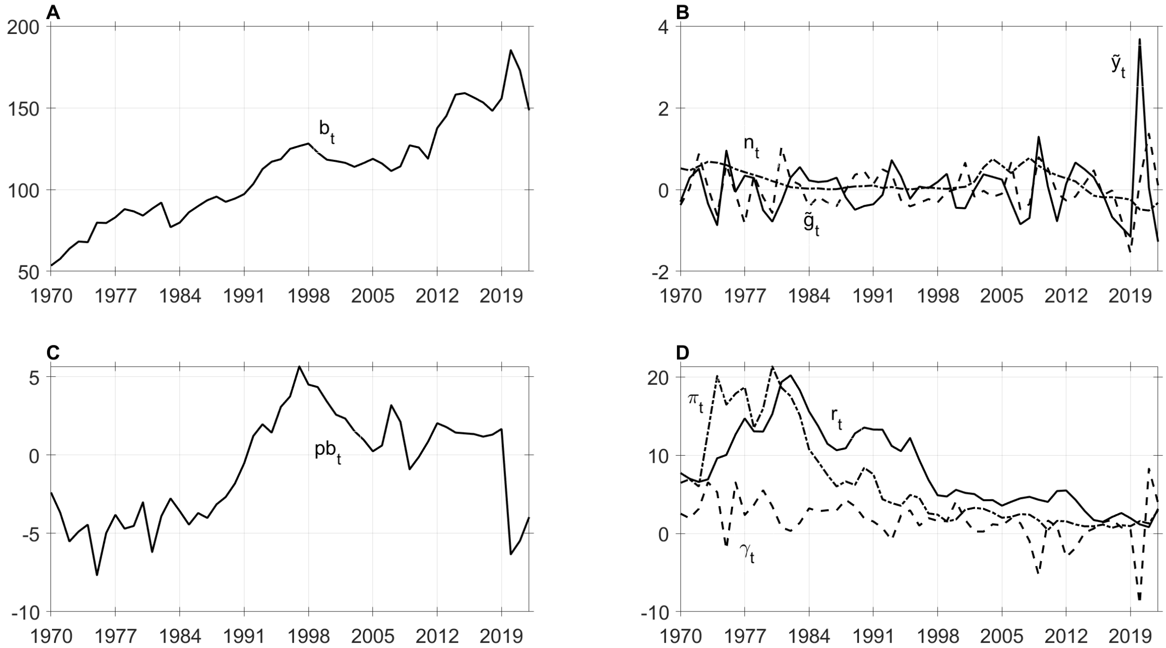

Figure 1.5: Italy: Data Plot

This figure presents the model variables for Italy (1970–2022). Panel A: debt-to-GDP ratio. Panel B: cyclical component of government expenditure (dashed line), cyclical component of output (solid line), and population growth rate (dotted line). Panel C: primary balance-to-GDP ratio. Panel D: long-term interest rate (solid line), real GDP growth rate (dashed line), and inflation rate (dotted line). All units are expressed as percentages.

r_t is taken directly from the OECD database. The long-term nominal interest rate refers to government bonds that mature in ten years. The deposit rate from the IMF International Financial Statistics are used to replace missing data for China (1980-2002).

γ_t is derived using the formula $\gamma_t = (y_t - y_{t-1})/y_t$, where y_t is the real GDP per capita. The 1970 γ_t for most countries analysed and the 1991 γ_t for Germany, and the 1980 γ_t for China are calculated in each country using the average γ_t over the next 2 years.

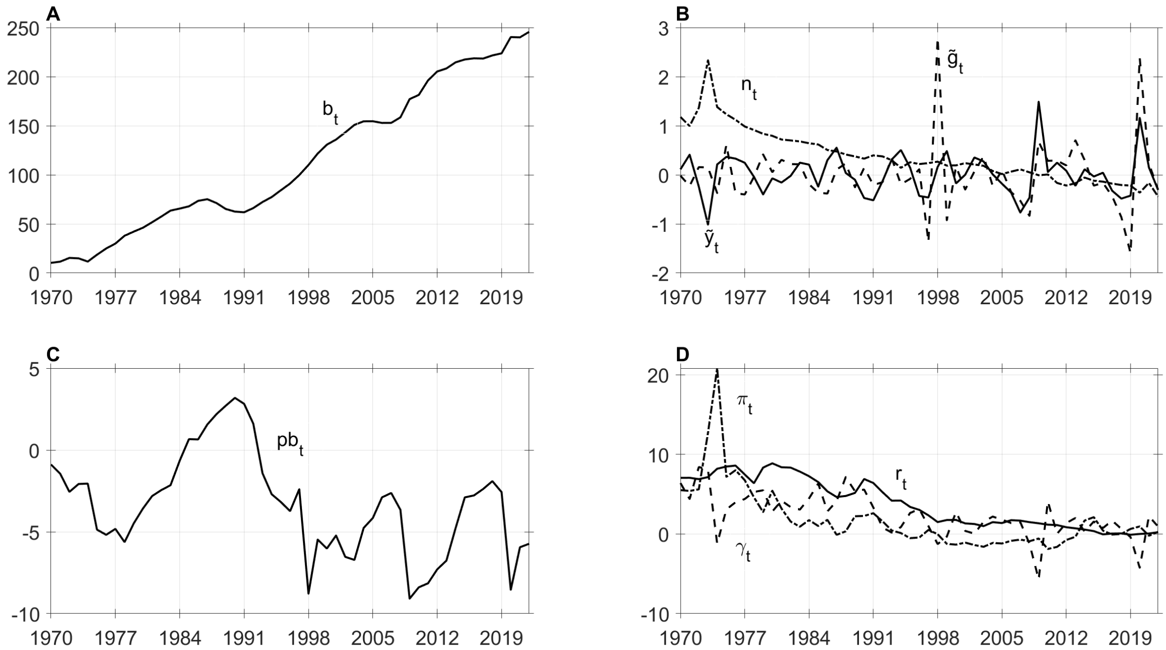

Figure 1.6: Japan: Data Plot

This figure presents the model variables for Japan (1970–2022). Panel A: debt-to-GDP ratio. Panel B: cyclical component of government expenditure (dashed line), cyclical component of output (solid line), and population growth rate (dotted line). Panel C: primary balance-to-GDP ratio. Panel D: long-term interest rate (solid line), real GDP growth rate (dashed line), and inflation rate (dotted line). All units are expressed as percentages.

b_t is measured using data on general government gross financial liabilities as a percentage of GDP directly available from the OECD Economic Outlook database. Time series for pb_t is directly taken from the OECD database using the general government primary balance as a percentage of GDP. pb_t for China in 1980-1989 are calculated in each country using the average pb_t over the next 2 years. pb_t for France in 1970-1977 are collected from the IMF World Economic Outlook database.

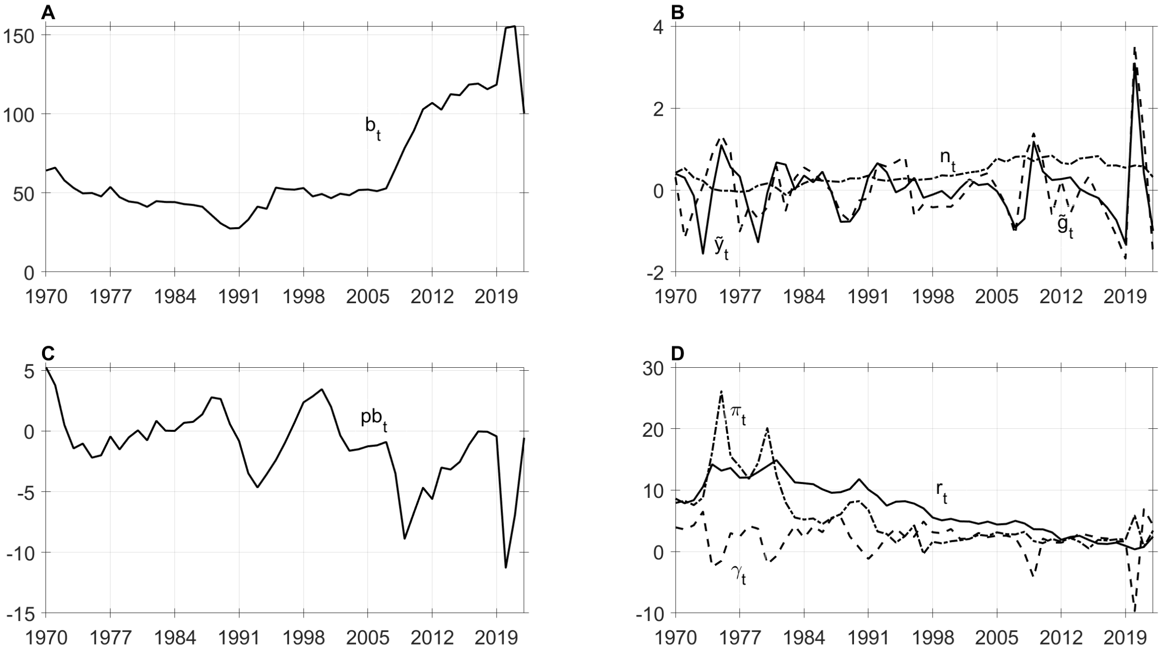

Figure 1.7: UK: Data Plot

This figure presents the model variables for the UK (1970–2022). Panel A: debt-to-GDP ratio. Panel B: cyclical component of government expenditure (dashed line), cyclical component of output (solid line), and population growth rate (dotted line). Panel C: primary balance-to-GDP ratio. Panel D: long-term interest rate (solid line), real GDP growth rate (dashed line), and inflation rate (dotted line). All units are expressed as percentages.

There are two trends in the data: one is that the debt of major economies have been increasing following 2007-2009 global financial crisis and the COVID-19 pandemic; the other is that two components of the discount rate ρ_t: long-term interest rates r_t and the growth rate of population n_t have continued to fall. Panel A for each country plots the debt to GDP ratio. Canada's debt ratio was hump-shaped between 1980 and 2007. It has been on an upward trend since 2007, soaring to 117.46% in 2020. China's data shows that the debt-to-GDP ratio remained at around 20% between 2002 and 2006, fluctuated after 2006, and began to rise after 2008. The COVID-19 pandemic has led to a further increase in the debt ratio. From 1980 to 2021, France's debt ratio rose from only 20.83% to more than 100%. Germany's debt ratio generally rose between

1992 and 2010, then gradually fell to 60% in 2019, and then jumped again. Italy's debt ratio rose between 1980 and 1994. It declined until 2007, and has been rising since then. Japan's debt ratio has generally been on an upward trend and has remained high, reaching 246% in 2021. the UK debt ratio is generally on an upward trend, reaching around 100%. US's debt ratio has a similar trend to that of UK and US's debt is at a higher level than that of UK.

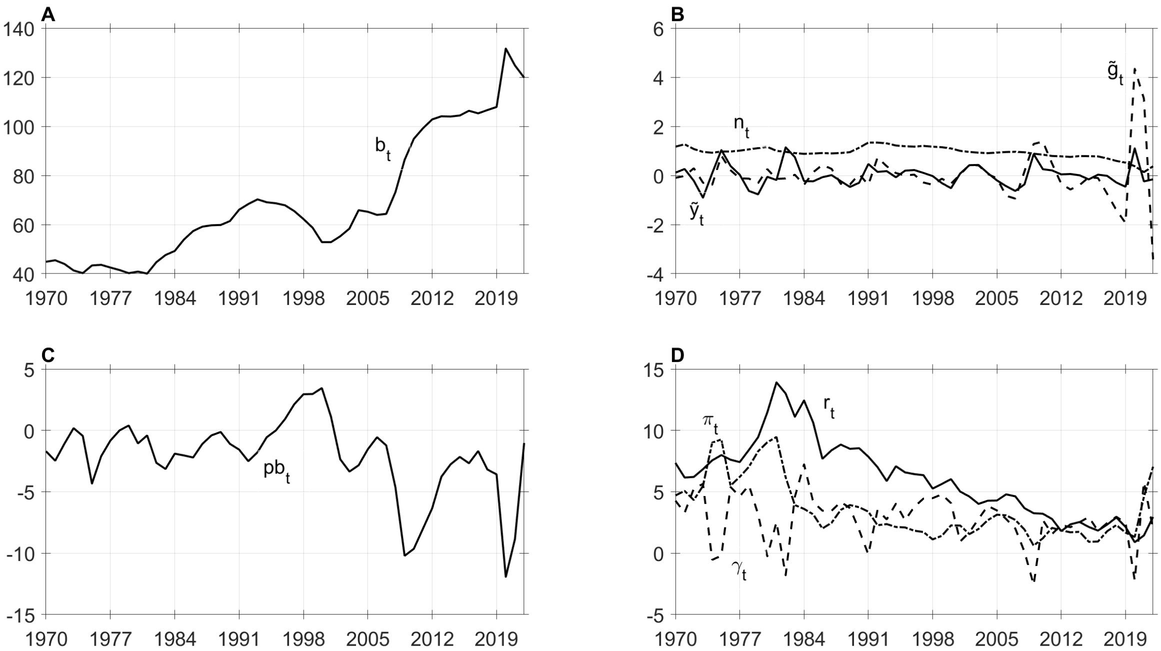

Figure 1.8: US: Data Plot

This figure presents the model variables for the US (1970–2022). Panel A: debt-to-GDP ratio. Panel B: cyclical component of government expenditure (dashed line), cyclical component of output (solid line), and population growth rate (dotted line). Panel C: primary balance-to-GDP ratio. Panel D: long-term interest rate (solid line), real GDP growth rate (dashed line), and inflation rate (dotted line). All units are expressed as percentages.

Long-term interest rates are gradually declining in major economies (see Panel D in Figure 1.1-1.8). During the COVID-19, the long-term interest rates of the euro zone countries - France and Germany - even became negative. The declining tendency of n_t (see Panel B in Figure 1.1-1.8) is clear China, Italy, and Japan. For the rest of the major economies analysed decline after the COVID-19 shock. This pulls down the discount rate even further. π_t (see Panel B in Figure 1.1-1.8) decrease and keep low during the Great Moderation. The inflation jumped up and down when hit by the shock, nevertheless it remained low after the financial crisis.

Panel C for each country shows the primary balance to GDP ratio. Canada's primary surplus remained constant in the mid-2010s, which allowed the debt ratio to fall slightly over the same period. China's primary balance was mostly negative. France's primary balance was relatively stable in the 1980s, but there were fiscal deficits. After 2001, the primary balance was negative, and the financial crisis and lockdown caused the balance to fall sharply. Germany's surge in the primary balance was accompanied by a surge in the debt ratio. Italy's primary balance ratio was mostly positive. however, the debt ratio started from a high level —— 97.98%. Japan has been in a primary deficit state except for the period 1985-1992. The government deficit levels in 2008 and 2020 were very high. the UK's primary balance to GDP ratio showed little or no trend. the surge in the US debt to GDP ratio was accompanied by high levels of government deficits.

This chapter employs a recursive (expanding-window) estimation strategy, rather than a rolling (fixed-window) approach. The estimation sample starts from a fixed initial period and is subsequently expanded by adding one additional observation at each time.

The recursive estimation framework offers several advantages. First, it provides a systematic way to assess parameter stability over time, which is particularly relevant in the context of fiscal policy, where institutional arrangements, demographic structures, and policy regimes may evolve gradually rather than through discrete structural breaks. Second, because each estimation uses only information available up to time T, the method avoids look-ahead bias and is consistent with real-time policy analysis. Third, compared to rolling-window estimation, the expanding-window approach preserves long-run information contained in earlier observations and yields more efficient estimates under parameter stability.

1.5 Empirical Result

Using annual data, the lag length of the VAR model is set to one. To ensure correct model specification, I conduct Ljung-Box test for serial correlation and examined the eigenvalues of the coefficient matrix. The model is estimated separately for each country, including the G7 economies and China, and recursively estimated starting with the first 25 observations through to 2022.

The model specification results and empirical results for the G7 countries and China are shown in Figure 1.9-1.16. Panel A shows p value of Campbell (1987) test results, W, and maximum eigenvalues of the estimated coefficient matrix, Eig. Panel B presents the Ljung-Box test statistics of the residuals in each equation of the VAR. The Ljung Box test checks whether there is serial correlation in the residuals of the model to ensure that the model is correctly specified. The null hypothesis is that there is no residual autocorrelation over the 20 lag periods. The critical value for a chi-square distribution with 20 degrees of freedom and more than 22 observations is 31.41, with a significance level of 5%. The relative critical value is slightly smaller for fewer than 22 observations. Overall, the results indicate that the null hypothesis of no serial correlation in the residuals cannot be rejected. Panel A also displays the largest absolute eigenvalue of the coefficient matrix obtained from the recursive estimation of the VAR model. This eigenvalue serves as a key indicator of the model's stability over time. If the largest eigenvalue exceeds one, it suggests the presence of a unit root, indicating potential instability in the system. By tracking how the eigenvalue evolves as additional observations are incorporated, I can assess whether the estimated VAR model remains stable throughout the sample period. Generally speaking, VAR (1) model can specify the dynamics for those major economies. Panel C shows the responses of Bohn (2005) rule : the estimated response coefficient of the debt ratio on the primary balance δ_b in equation (1.39); Panel D-H shows Blanchard (1990) medium-term debt dynamics and its components.

For MTDD(t,j), I set j as 5 and the debt target as the average of debt ratio between 1998 and 2007. In this chapter, the horizon is set to five years (j=5) to align with the medium-term policy relevance for the G7 countries. I use the average debt from 1998 to 2007 as the debt target for MTDD for the following reasons. First, the global economic environment during this period was relatively stable, marked by expansion and favourable fiscal conditions in the G7 countries and China. Second, debt level was comparatively low in this period, as both advanced and emerging economies pursued prudent fiscal policies. Third, using a multi-year average instead of a single year helps smooth short-term fluctuations and better captures the structural characteristics of debt, minimizing the influence of temporary shocks or exceptional events.

I choose to display the average forecast values of the variables in the figures. MTDD(t,j) is derived by subtracting the target level of debt-GDP ratio from the present value of the forecast debt ratio. The most crucial component of MTDD(t,j) is the debt forecast, which primarily relies on the forecast of the effective primary balance. Since MTDD(t,j) examines whether fiscal policy can achieve the target debt ratio within a given horizon j, this requires forecasting the effective primary balance from period t+1 to period t+j. To present the results intuitively, I display the average forecast values of the components of the effective primary balance and use † to signal the averages. The averages of these components are as follows:

$$x_{t,j} = \frac{1}{j}\sum_{i=1}^{j} E_t x_{t+i}, \tag{1.43}$$

where x_t stands for $PV(\rho_t)$, $PV(r_t)$, $PV(n_t)$, $PV(\gamma_t)$, $PV(\pi_t)$, $PV(s_t)$, and $PV(pb_t)$. PV stands for the present value.

1.5.1 Canada

For Canada, the Campbell (1987) test results showed that the null hypothesis could not be rejected. This means that the fiscal policy for Canada is sustainable. The coefficient of Bohn (2005) rule is positive until including the observations in 2022. This means that the response for Canada was mean-reverting until 2022. The fluctuations of MTDD(t,5) is mainly from primary balance. In the meantime, the fluctuations from π_t also contributes to the debt dynamics. The fluctuations in inflation and economic growth rate also help to reduce debt dynamics.

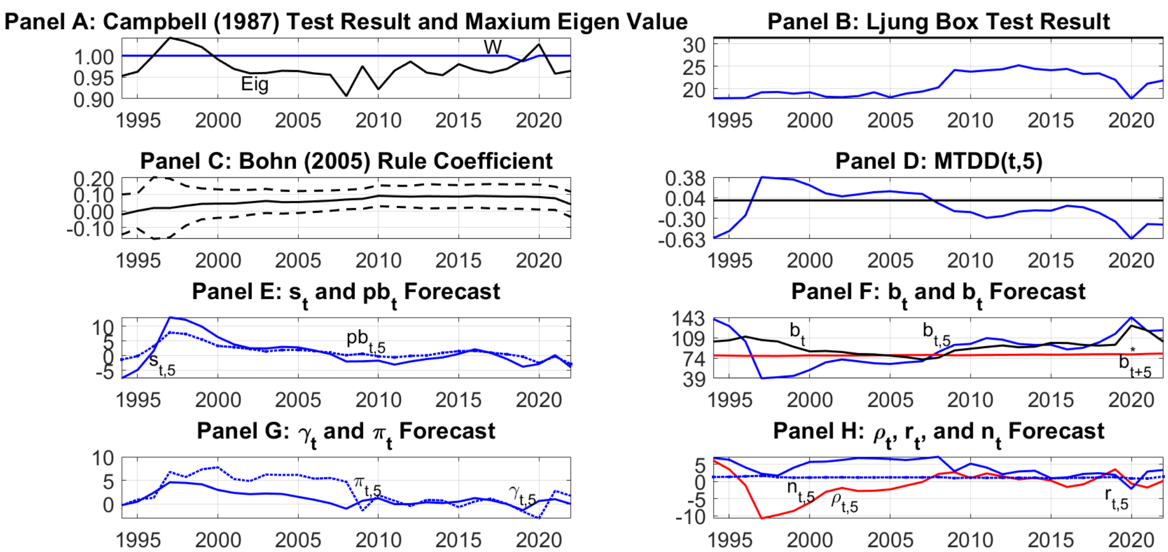

Figure 1.9: Canada (1970-2022).

Panel A presents the p-values from the recursive Campbell (1987) test result (W, blue line) and the maximum eigenvalue (Eig, black line) in the model. Panel B shows the Ljung box test results (blue line) and the corresponding critical values (black line). Panel C depicts the estimated response coefficient, δ_b in equation (1.39), based on Bohn (2005) (solid black line) along with its 95% confidence interval (dashed black lines). Panel D illustrates the MTDD(t,5) (blue line) and the zero-dynamics benchmark (black line) where the debt ratio target is the average debt ratio between 1998 and 2007. Panel E-H shows the components of MTDD(t,5) and the unit is percent (%).

1.5.2 China

For China, Campbell (1987) test results show that China's fiscal policy has always been sustainable. China's Bohn (2005) rule coefficient has always been insignificant. In 2020, the response of fiscal policy to the debt ratio became unstable and then the response coefficient became significantly negative. Blanchard (1990) medium-term debt dynamics show that the fiscal policy is too loose. The fluctuation in debt dynamic is mainly from primary balance and γ_t contributes to discount rate. Nevertheless, the inflation contribution more after 2018.

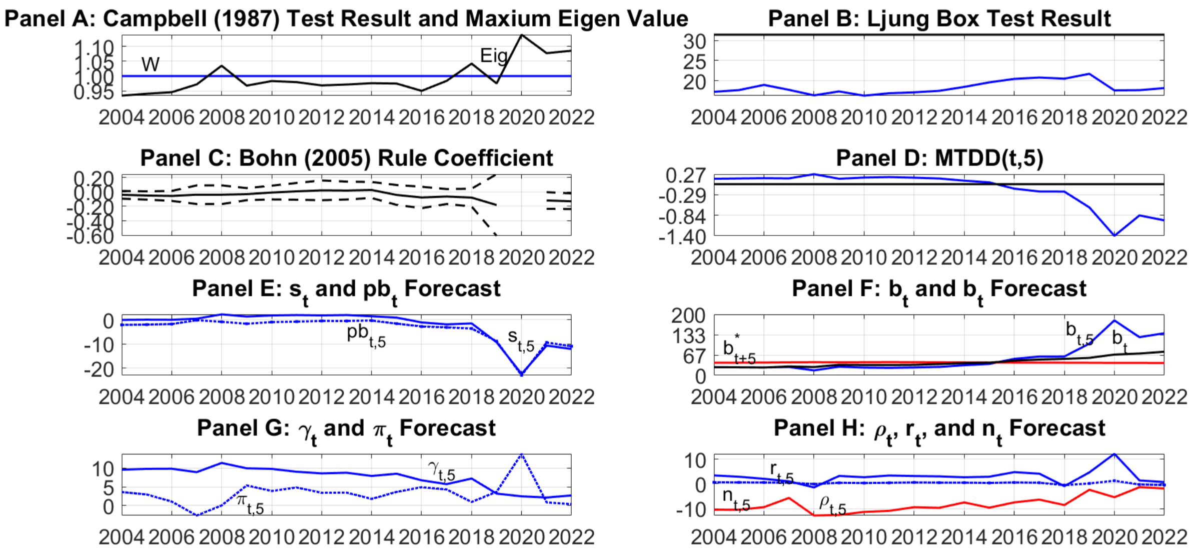

Figure 1.10: China (1980-2022).

Panel A presents the p-values from the recursive Campbell (1987) test result (W, blue line) and the maximum eigenvalue (Eig, black line) in the model. Panel B shows the Ljung box test results (blue line) and the corresponding critical values (black line). Panel C depicts the estimated response coefficient, δ_b in equation (1.39), based on Bohn (2005) (solid black line) along with its 95% confidence interval (dashed black lines). Panel D illustrates the MTDD(t,5) (blue line) and the zero-dynamics benchmark (black line) where the debt ratio target is the average debt ratio between 1998 and 2007. Panel E-H shows the components of MTDD(t,5) and the unit is percent (%).

1.5.3 France

Campbell (1987) test results show that France's fiscal policy is sustainable but it is noticeable that Campbell (1987) test result is insignificant when including data until 2021. Bohn (2005) rule results show that the response of primary balance to debt has always been insignificant; MTDD(t,5) fluctuation is mainly from primary balance and the fluctuation between 2006 and 2013 is mainly from the fluctuation of interest rate.

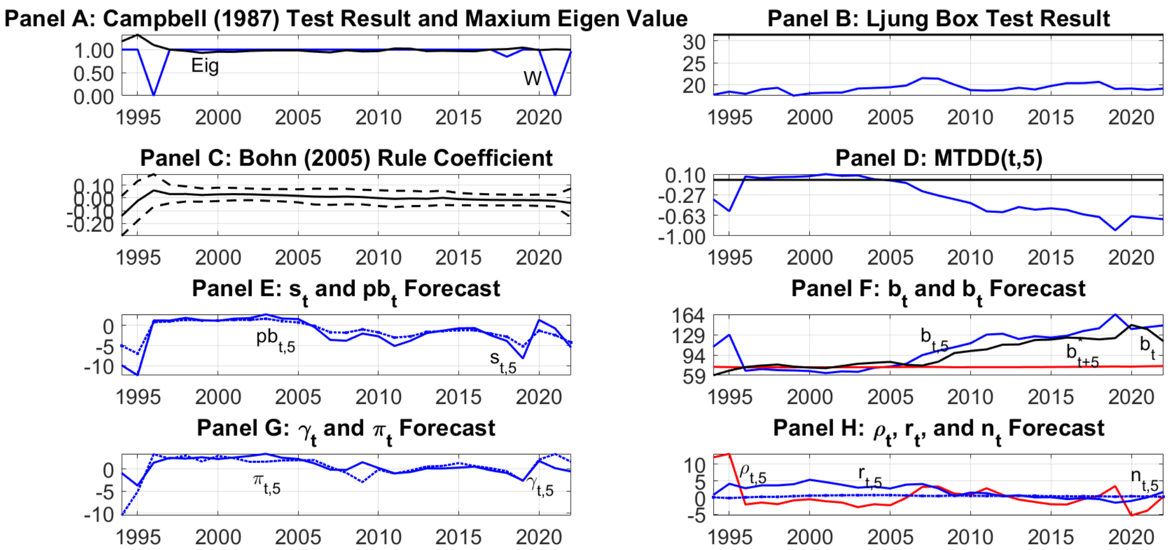

Figure 1.11: France (1970-2022).

Panel A presents the p-values from the recursive Campbell (1987) test result (W, blue line) and the maximum eigenvalue (Eig, black line) in the model. Panel B shows the Ljung box test results (blue line) and the corresponding critical values (black line). Panel C depicts the estimated response coefficient, δ_b in equation (1.39), based on Bohn (2005) (solid black line) along with its 95% confidence interval (dashed black lines). Panel D illustrates the MTDD(t,5) (blue line) and the zero-dynamics benchmark (black line) where the debt ratio target is the average debt ratio between 1998 and 2007. Panel E-H shows the components of MTDD(t,5) and the unit is percent (%).

1.5.4 Germany

Campbell (1987) test results show that German's fiscal policy is sustainable . The results of Bohn (2005) rule is insignificant. From the perspective of Blanchard (1990) medium-term debt dynamics is mainly from γ_t and is mainly from primary balance after 2011.

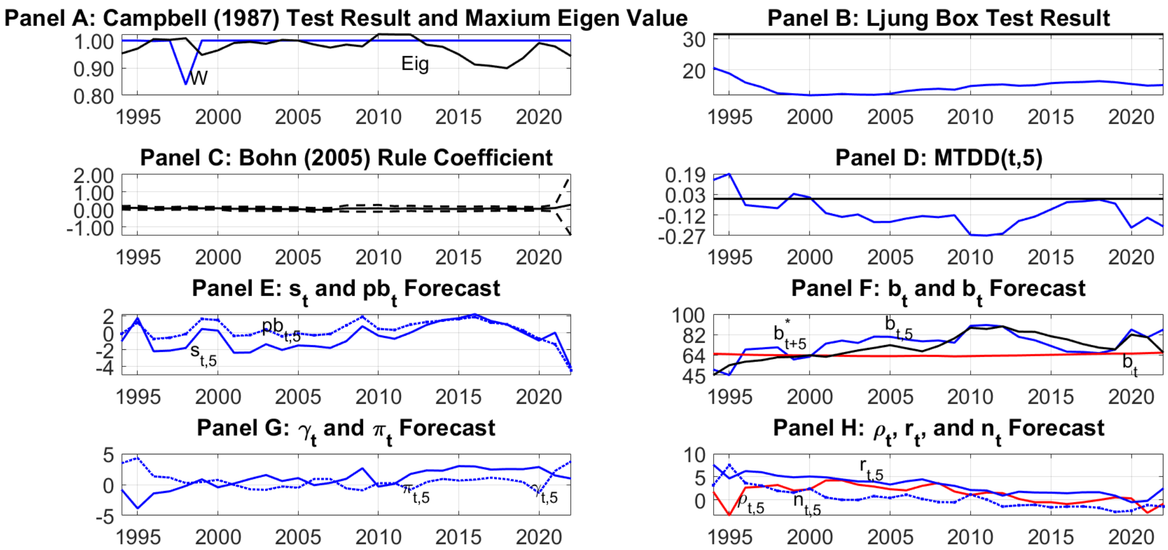

Figure 1.12: Germany (1970-2022).

Panel A presents the p-values from the recursive Campbell (1987) test result (W, blue line) and the maximum eigenvalue (Eig, black line) in the model. Panel B shows the Ljung box test results (blue line) and the corresponding critical values (black line). Panel C depicts the estimated response coefficient, δ_b in equation (1.39), based on Bohn (2005) (solid black line) along with its 95% confidence interval (dashed black lines). Panel D illustrates the MTDD(t,5) (blue line) and the zero-dynamics benchmark (black line) where the debt ratio target is the average debt ratio between 1998 and 2007. Panel E-H shows the components of MTDD(t,5) and the unit is percent (%).

1.5.5 Italy

Campbell (1987) test results show that Italy's fiscal policy is sustainable. The coefficient of Bohn (2005) rule is positive until including the observations in 2022. At the same time, The fluctuation of MTDD(t,5) is mainly from that of inflation rate.

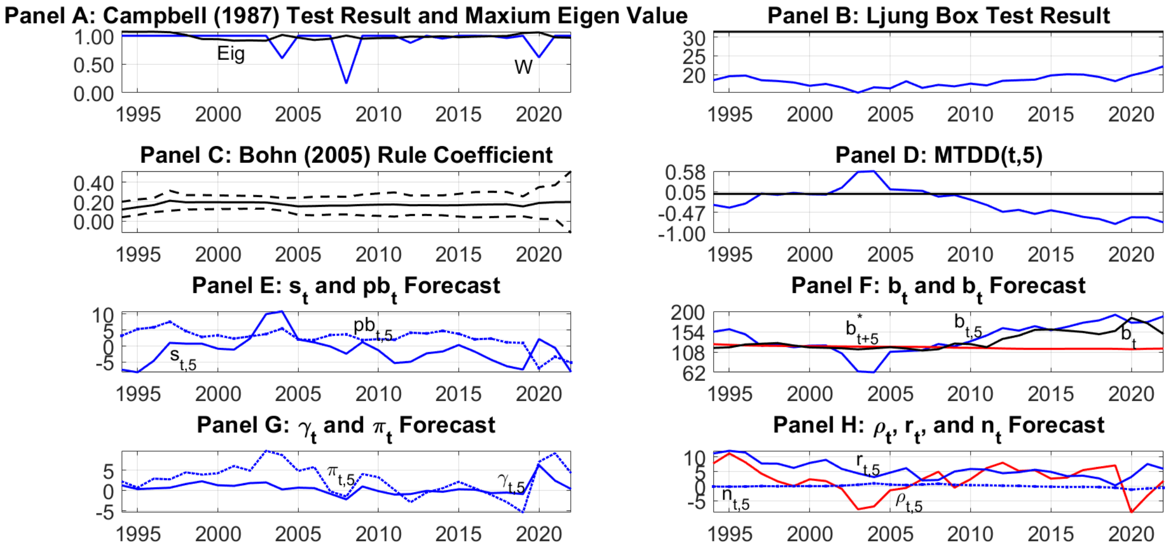

Figure 1.13: Italy (1970-2022).

Panel A presents the p-values from the recursive Campbell (1987) test result (W, blue line) and the maximum eigenvalue (Eig, black line) in the model. Panel B shows the Ljung box test results (blue line) and the corresponding critical values (black line). Panel C depicts the estimated response coefficient, δ_b in equation (1.39), based on Bohn (2005) (solid black line) along with its 95% confidence interval (dashed black lines). Panel D illustrates the MTDD(t,5) (blue line) and the zero-dynamics benchmark (black line) where the debt ratio target is the average debt ratio between 1998 and 2007. Panel E-H shows the components of MTDD(t,5) and the unit is percent (%).

1.5.6 Japan

Campbell (1987) test results for Japan proved that the PVBC of Japan can be satisfied, which means that fiscal policy is sustainable. The results of Bohn (2005) rule show that the coefficient is insignificantly positive. The fluctuations of Blanchard (1990) medium-term debt dynamics is mainly from inflation rate. After 2008, the debt dynamics have been on a downward trend. By 2021, the estimated debt dynamics are -1.2, and the gap between the expected debt ratio and the target debt ratio is quite large.

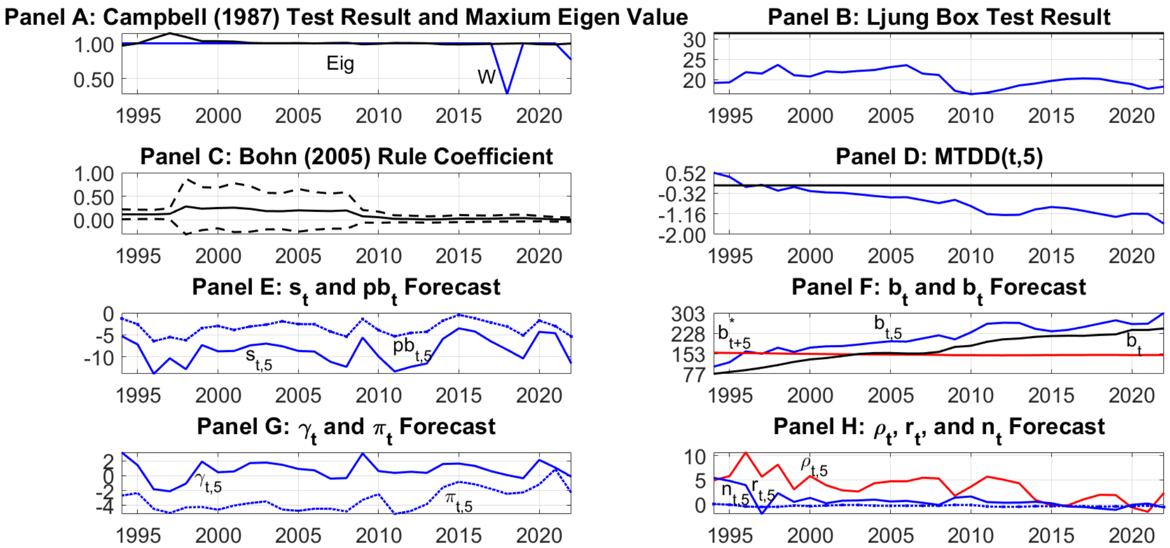

Figure 1.14: Japan (1970-2022).

Panel A presents the p-values from the recursive Campbell (1987) test result (W, blue line) and the maximum eigenvalue (Eig, black line) in the model. Panel B shows the Ljung box test results (blue line) and the corresponding critical values (black line). Panel C depicts the estimated response coefficient, δ_b in equation (1.39), based on Bohn (2005) (solid black line) along with its 95% confidence interval (dashed black lines). Panel D illustrates the MTDD(t,5) (blue line) and the zero-dynamics benchmark (black line) where the debt ratio target is the average debt ratio between 1998 and 2007. Panel E-H shows the components of MTDD(t,5) and the unit is percent (%).

1.5.7 UK

For the UK: Campbell (1987) test results show that the hypothesis F=Q cannot be rejected. The null hypothesis that the sum of all present values of the primary balance equals the debt-to-GDP ratio cannot be rejected. Although the UK's fiscal policy is sustainable, the gradual decline in the response coefficient is not significant. The fluctuations of Blanchard (1990) medium-term debt dynamics is mainly from those of inflation rate. These results suggest that UK fiscal policy should also be tightened. Overall, although the UK's fiscal policy is sustainable, it should still strive to tighten fiscal policy and increase its response to the increase in the debt ratio.

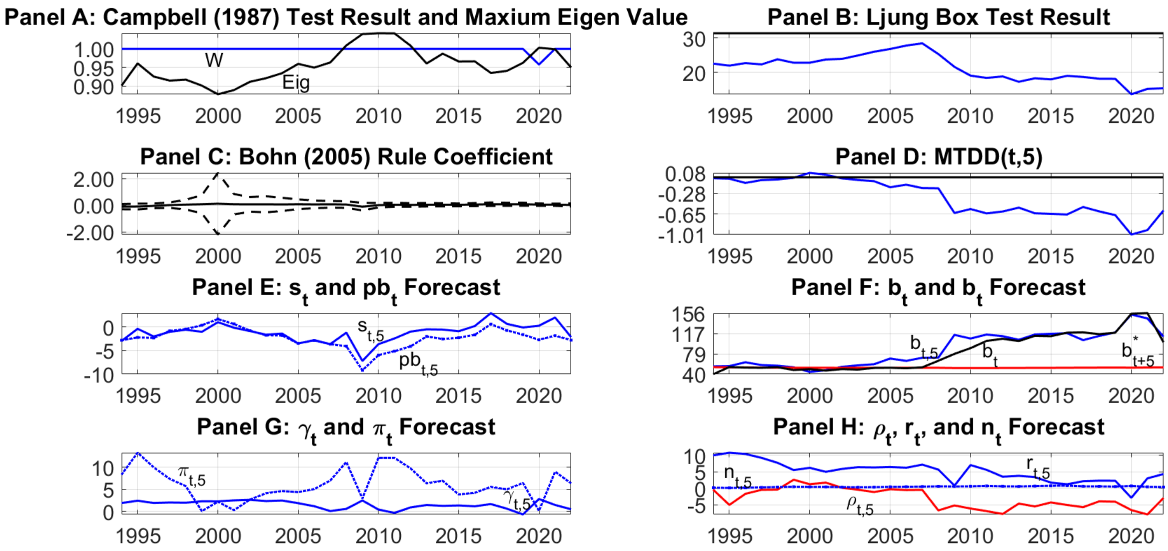

Figure 1.15: UK (1970-2022).

Panel A presents the p-values from the recursive Campbell (1987) test result (W, blue line) and the maximum eigenvalue (Eig, black line) in the model. Panel B shows the Ljung box test results (blue line) and the corresponding critical values (black line). Panel C depicts the estimated response coefficient, δ_b in equation (1.39), based on Bohn (2005) (solid black line) along with its 95% confidence interval (dashed black lines). Panel D illustrates the MTDD(t,5) (blue line) and the zero-dynamics benchmark (black line) where the debt ratio target is the average debt ratio between 1998 and 2007. Panel E-H shows the components of MTDD(t,5) and the unit is percent (%).

1.5.8 US

For the United States, Campbell (1987) test results show that the null hypothesis cannot be rejected and the United States' fiscal policy is sustainable. It can be seen from the figure that the response of US fiscal policy is insignificant. The debt dynamics is mainly from that of primary balance. Blanchard (1990) medium-term debt dynamics is -1.0357, implying that US fiscal policy should be tightened.

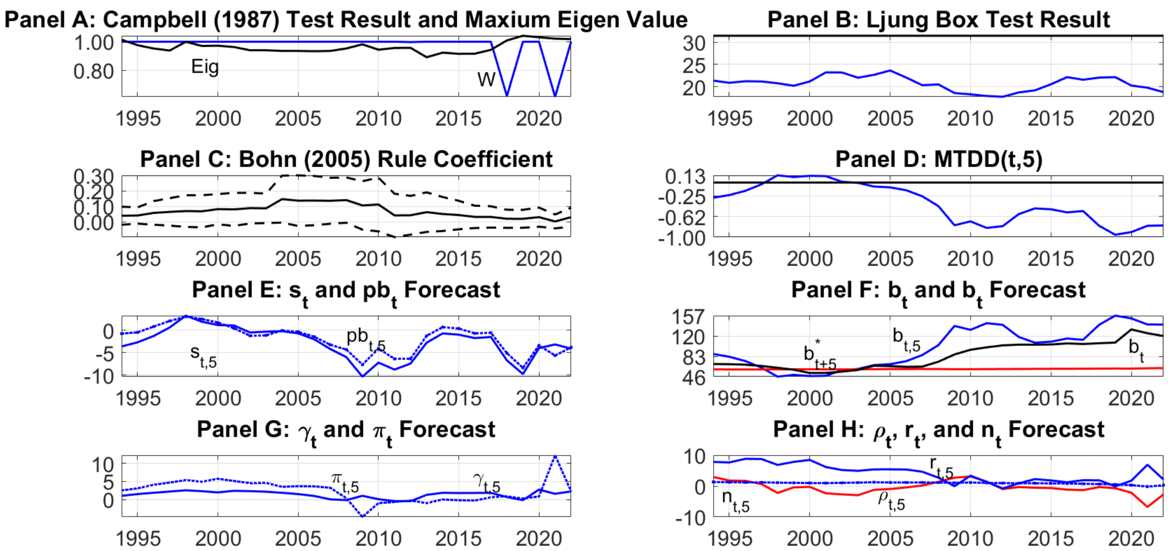

Figure 1.16: US (1970-2022).

Panel A presents the p-values from the recursive Campbell (1987) test result (W, blue line) and the maximum eigenvalue (Eig, black line) in the model. Panel B shows the Ljung box test results (blue line) and the corresponding critical values (black line). Panel C depicts the estimated response coefficient, δ_b in equation (1.39), based on Bohn (2005) (solid black line) along with its 95% confidence interval (dashed black lines). Panel D illustrates the MTDD(t,5) (blue line) and the zero-dynamics benchmark (black line) where the debt ratio target is the average debt ratio between 1998 and 2007. Panel E-H shows the components of MTDD(t,5) and the unit is percent (%).

When comparing the results across countries, it is easy to find that there are very obvious different results among Bohn (2005) rule and MTDD(t,j) and not among Campbell and Shiller (1987) test. This is consistent with the expectation that the

PVBC can be easily hold. It is also shown in the result that the response for Bohn (2005) rule for several countries, Canada, France and Germany, are clearly exaggerated, showing that the uncertainty increase. For MTDD. If it is possible, we should see the trend backward to 0 debt dynamics. Unfortunately, this is not the case. Many major economies have their MTDD bounce back a bit after the 2020 pandemic, but in 2022 is shows the tendency to decrease again.

The findings from alternative econometric tests are broadly in line with those from the Campbell (1987) test. Magazzino, Brady, and Forte (Magazzino et al.) see the PVBC hold for G7 countries.

Everaert and Jansen (2018) estimate a panel fiscal reaction function for a set of advanced (OECD) economies—hence including the G7—and show that once cross-country heterogeneity is allowed, the mean-group response of the primary balance to lagged debt is small and statistically insignificant. Afonso and Jalles (2017) use a large cross-country panel (covering both advanced economies and the G7) and argue that the estimated debt-feedback in fiscal reaction functions is highly sample- and specification-dependent, with statistical insignificance being common in several groups/subsamples. Sun (2015) shows that when the baseline specification is augmented with additional macro controls—most notably inflation and a financial-crisis dummy—the coefficient on lagged debt becomes statistically insignificant, which is very similar to my research and derives the same result as mine.

Medium-term projections from both the IMF and the OECD point to a rising public-debt trajectory under broadly unchanged policy settings. In particular, International Monetary Fund (2024) projects that China's general government gross debt continues to increase over the medium term, reaching about 111.1% of GDP by 2029. Meanwhile, Organisation for Economic Co-operation and Development (2023) shows that, in the absence of additional adjustment, ageing-related fiscal pressures substantially worsen debt dynamics across the G7, with the median G7 country experiencing a large rise in net government financial liabilities (or debt) by 2040.

The policy implication of this chapter is that for major economies analysed fiscal policies have undertaken an unsustainable path from 2008 onward. Though Campbell (1987) test results show that fiscal policy are all sustainable, the restriction is quite weak and would allow ever increasing debt, which is not suitable for government debt control. Bohn (2005) rule results indicate that, in general, governments do not respond to debt, while China exhibits a significantly negative response. Blanchard (1990) medium-term debt dynamics shows that the fiscal policies in those major economies are all too loose. The main implication from the three tests is that a fiscal policy change is required.

Cochrane (2023) nevertheless argues that the VAR model implicitly assumes that the agents in the economy does not have more information than the history variables and this would make the results unreliable.

1.6 Robustness Check

This section reports three robustness checks to assess whether the main findings are sensitive to (i) the choice of filter used to extract cyclical components, (ii) the assumed debt target used to construct MTDD, and (iii) the credibility of the VAR forecasts as evaluated by an in-sample forecasting exercise.

1.6.1 Alternative Filter: Hamilton versus HP

In the baseline specification, I use the Hodrick–Prescott (HP) filter to extract cyclical components of output and government spending. A well-known concern is that the HP filter can induce endpoint distortions and may generate a trend that reacts sluggishly to turning points, thereby potentially misclassifying short-run movements as cyclical. As an alternative, I apply the Hamilton filter, which provides a regression-based decomposition and is less prone to the endpoint problem. Appendix Figures A.1–A.8 show that the results under the Hamilton filter are qualitatively similar to those in the baseline, suggesting that the empirical findings are not driven by the filtering method.

1.6.2 Different Debt Target

In the baseline, the debt target is set to the average debt-to-GDP ratio over 1998–2007. For robustness, I instead use the full-sample average debt ratio as the target. This change does not affect the VAR-based forecast of the primary balance; accordingly, MTDD becomes mechanically more negative than in the baseline specification, reflecting the higher (or different) target level rather than a change in the predicted fiscal response.

1.6.3 In-sample Forecast Validation

To assess the forecasting performance of the VAR and to provide an internal validation of the forecast inputs used in constructing MTDD, I conduct an in-sample forecasting exercise for the debt-to-GDP ratio. Figure 1.17 plots the observed series (solid line) against the recursively constructed in-sample forecast (dashed line). Specifically, at each time t, the VAR is re-estimated using data from the beginning of the sample up to t, and a five-year-ahead forecast is produced conditioning on information dated $t-5$. The close alignment between the forecast and the realised debt dynamics supports the use of the VAR forecasts in the MTDD calculations. The results are shown in 1.17.

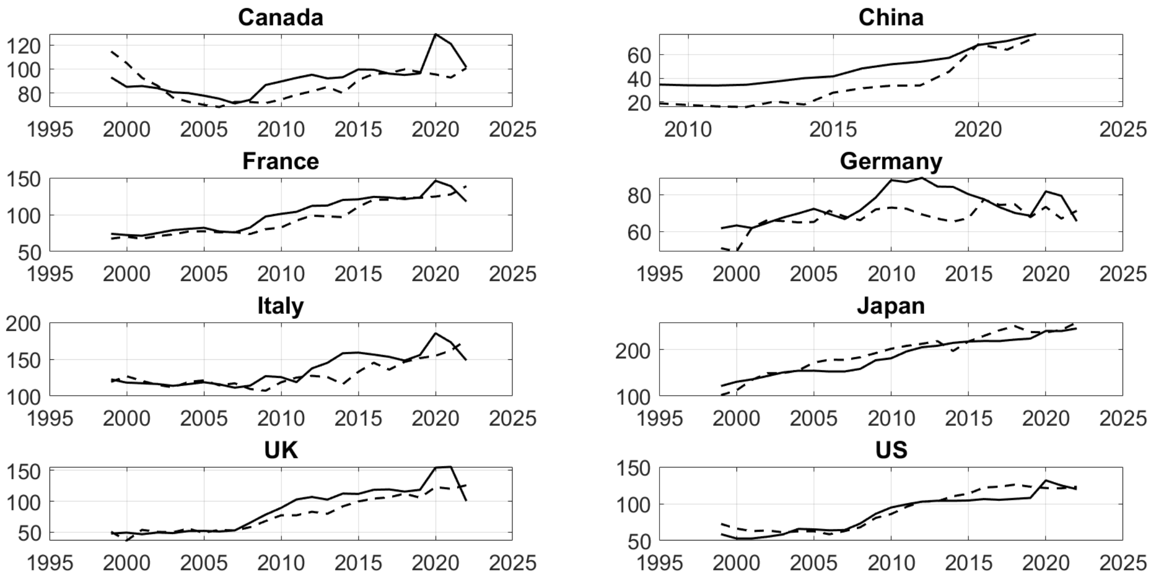

Figure 1.17: In sample forecast for debt-GDP ratio.

Notes: The solid line shows the observed debt-to-GDP ratio. The dashed line shows the five-year-ahead in-sample forecast at time t, constructed using coefficients estimated recursively with data from the beginning of the sample up to t, and conditioning on information dated $t-5$. All unit in percent (%).

1.7 Conclusion

The contribution of Chapter 1 lies in its empirical analysis of fiscal policy sustainability, integrating all three widely used approaches within a unified VAR model of the economy that accounts for demographic change. These three tests can take advantage of the benefit of VAR model and deliver different economic interpretation.

The empirical results show that in most countries fiscal policy has undertaken an unsustainable path from 2008 onward. Though Campbell (1987) test results show that fiscal policy are all sustainable, Bohn (2005) rule results show that the governments do not react to debt and China is significantly negatively react to the debt. Blanchard (1990) medium-term debt dynamics shows that the fiscal policies in those major economies are all too loose. The main implication from the three tests is that a fiscal policy change is required.

In the aftermath of the 2008 financial crisis, major economies accumulated sizable public debts, while demographic shifts further intensified long-run fiscal pressures—fueling renewed calls for fiscal consolidation. The evidence in Chapter 1 aligns with this concern. Across the G7 countries and China, the Campbell (1987) PVBC-based test does not reject fiscal sustainability at the 5% level. However, the estimated fiscal reaction functions under the Bohn (2005) rule reveal that primary balances do not respond positively to higher debt ratios and are in some cases significantly negative, implying weak policy correction as debt rises. Consistently, the Blanchard (1990) medium-term debt dynamics (MTDD), evaluated relative to the debt-to-GDP ratio target (average debt-GDP ratio between 1998-2007), indicates that these countries have on average followed a loose fiscal stance. Taken together, the results point to a material gap between the solvency conditions suggested by PVBC tests and the observed policy behaviour and medium-term debt arithmetic, underscoring the need for credible fiscal adjustment to prevent a persistent deterioration of public finances.

Chapter 2

Fiscal Adjustment and Fiscal Sustainability: Counterfactual Analysis

2.1 Introduction

As major economies accumulated large debts following the 2008 financial crisis, and demographic changes further intensified pressures on fiscal sustainability, calls for fiscal austerity has grown louder. This concern is also reflected in the empirical results of Chapter 1. In the previous chapter, I employed three approaches from the empirical macroeconomic literature to assess fiscal sustainability for the G7 countries and China. According to Campbell (1987) test — an econometric test of the present value of government budget constraint (PVBC) based on the series of debt, deficit and other macro variables pertinent to fiscal policies — the fiscal policies of the analysed economies are all found to be sustainable at the 5% significance level. However, when applying Bohn (2005) rule, which assesses the responsiveness of the primary balance-to-GDP ratio to debt-to-GDP ratio, the estimated responses are not significantly positive and, in some cases, significantly negative. This suggests that the governments of these major economies do not react to debt-to-GDP ratio. Furthermore, Blanchard (1990) medium-term debt dynamics (MTDD), which evaluates the required fiscal adjustment to return to the debt target, the average debt-to-GDP ratio prior to the financial crisis, all the countries appear to be pursuing loose fiscal policies. Overall, these results indicate that fiscal policy adjustments are necessary necessary to avoid a continued trajectory of unsustainable public finances. Many research has the consensus that the fiscal policy should be adjusted and this is also align with the research results found in Chapter 1.

Existing work on fiscal sustainability often relies on accounting-style indicators to quantify how much fiscal effort is needed to stabilise debt. For instance, the tax-gap concept in Blanchard (1990) backs out the tax rate required to make debt dynamics equal to zero, and the S1/S2 indicators from International Monetary Fund (2009) similarly compute the structural primary balance needed to meet a debt objective under given macroeconomic conditions. These approaches are useful for benchmarking, but they typically treat the primary balance as if it can be moved exogenously toward a target (or a steady state) and then infer the implied tax rate or adjustment. In practice, however, tax-rate changes are slow-moving and institutionally constrained, and it is difficult for governments to keep a positive primary balance for many consecutive years — as these indicators would often require — so gap-type indicators are less informative about how fiscal policy is actually implemented when debt rises.

A more policy-relevant question is therefore: when the debt ratio increases, how strong must the endogenous fiscal response be —– in particular, the responsiveness of the primary balance to debt —–to restore sustainability? Although fiscal reaction functions, especially the Bohn (2005) rule, have been widely estimated, they are often used mainly to test whether a reaction exists, rather than to quantify the required response that would bring debt back onto a sustainable path. Starting with Ghosh, Kim, Mendoza, Ostry, and Qureshi (Ghosh et al.), the literature began to use fiscal reaction functions not only as a diagnostic test of "whether policy reacts", but as a policy-evaluation device to assess whether the rule-implied primary balance can offset self-rolling debt(i.e., the debt-stabilising requirement). Nevertheless, they do not further discuss the magnitude of Bohn (2005) rule should be taken.

This chapter contributes to the fiscal-adjustment literature by applying a fiscal policy rule that is often estimated but rarely used as an operational adjustment device. While the rule itself does not quantify the fiscal effort required to restore sustainability, integrating it with MTDD helps bridge this gap. MTDD pinpoints the size of the required adjustment by measuring the deviation between projected and target debt paths, but on its own it offers limited guidance on how policy should be implemented. Combining MTDD with the Bohn (2005) a concrete behavioural mechanism —–namely, a primary-balance response to debt —–thereby delivering a more precise and implementable fiscal adjustment path.

The final results suggest that countries accustomed to conducting loose fiscal policies should strengthen the responsiveness of the primary balance-to-GDP ratio to the debt-to-GDP ratio. Moreover, the magnitude of the required response are mostly lying within the previous response bands. I've done bootstrap Chow test and the results show that most of these are insignificant. Nevertheless, these findings remain subject to the Lucas (1976) critique, as policy effectiveness may vary under different expectations and structural environments.

This chapter proceeds as follow. Section 2.2 is the literature review; Section 2.3 describes the methodology; Section 2.4 shows the results and Section 2.5 is the conclusion.

2.2 Literature Review

There are mainly two ways to conduct counterfactual policy analysis. One is DSGE model, and the other is VAR model (see Eichengreen and Wyplosz (1998), Creel et al. (2012), and Polito and Spencer (2016)). Stock and Watson (2001) show that VAR model can be used to analyse policy. I choose the VAR model and the advantage of this model is: First, VAR model has the advantage in fitting the data. For the policy analysis, the fitness of data is essential for the Central bank and international financial institution. They even allow the data to determine the dynamic structure among variables, paying less attention to theoretical purity. Second, VAR model tend to give better forecasts than structure model, such as DSGE model, while the forecast for future debt is the aim in this chapter. VAR model gives better forecast particularly if structural breaks are allowed for, see for example Clements and Hendry (2006) and Hendry (2006). This is especially important when the focus of this chapter is debt dynamics. Third, it can give a close approximation to any structural model. Though the resulting rule would not be first best compared with using a correct structure of the economy, the estimation can be simple and transparent.

Analysing different policy rule under old mechanism is not a rare research type. Sims and Zha (2006)). They set policy rule in Greenspan regime for the whole data and derives that during this period inflation were not affected.

The counterfactual analysis for policy is mainly concerned with its macroeconomic effects (see e.g. Gordon and Leeper (1994),Christiano et al. (1997), Leeper et al. (1996)and Cushman and Zha (1997)). In order to analyse the effects after fiscal policy changes, there are three methods of counterfactual analysis in the literature for different issues or fiscal policy studies. The first method is to change the policy shocks to 0 and then compare the simulated data with the actual data. Favero (2002) assumes that the monetary and fiscal authorities have never deviated from the rules so as to retain macroeconomic shocks when simulating the model, but set all monetary and fiscal shocks to zero. This is to compare what happens with what would happen if the monetary and fiscal authorities never deviated from their rules over the available sample. The second method is to directly change the series. This is for the surprise policy rule change. Eichengreen and Wyplosz (1998) use the VAR model to analyse the impact on the output of EU member states if the Stability and Growth Pact (SGP) was launched 30 years before the establishment of the EU. Their research analyse the impact of SGP on EU member states, while SGP mainly requires the member states to control the primary deficit-to-GDP ratio below 3%. Eichengreen and Wyplosz then directly capped the primary deficit ratio series to be no larger than 3% and then compare the output series derived from the model with the actual ones. Sims (1982) discuss the situation when increasing FFR 50 base point. The method is to use the series of monetary policy innovations to hold the FFR at a specific level for some time, taking into account that in the VAR, actions on interest rates in earlier period affect those in later period. The third is to change the policy rule (see Sims and Zha (2006)). They set the 1970-1980 the same policy rule in Greenspan regime and derives that during this period inflation were not affected. For example, Rossi and Zubairy (2011) construct the counterfactual changes in output when fiscal expenditure shocks occurred to reflect the macroeconomic effects of policies. Bachmann and Sims (2012) study the transmission mechanism of policy expenditures. They obtain the immediate response of national output to a fiscal shock without the effect of market subject confidence by closing the immediate response of market subject confidence to fiscal policy shock. They set the coefficient to close the endogenous response of one variable to the shock of another variable, and then compare whether there is a significant difference in the pulse response pattern of the variable before and after the closure. While the last method mainly studies the transmission mechanism of government spending, it is more suitable for studying fiscal reaction functions to improve fiscal stances, which is concerned with changing the response policy by changing the coefficient.

There are several strands of the fiscal-adjustment literature that rely on gap-type concepts, most notably the tax gap, which provides a useful normative benchmark for the tax rate consistent with a given debt objective. However, treating the tax rate as an instrument that can be adjusted continuously and quasi-instantaneously is a strong abstraction. In practice, tax policy is shaped by voter preferences and party competition, and is therefore characterised by institutional inertia and non-trivial political and administrative costs; as a result, tax systems typically change through infrequent and discrete reforms rather than through small annual adjustments (see the political-economy perspective in Hettich and Winer (1999)). Moreover, in the optimal taxation and public finance tradition, governments have incentives to smooth distortionary taxes over time: fiscal shocks are absorbed primarily through debt issuance and intertemporal budget reallocation, instead of frequent changes in tax rates. This tax-smoothing logic implies that closing a primary-balance gap via tax-rate adjustment is better interpreted as a long-run normative reference than as a policy lever that can be repeatedly and mechanically deployed at short- or medium-term horizons (see Barro (1979); also Bohn (Bohn) for discussion on tax smoothing and the allocation of fiscal instruments).

I choose the medium-term debt dynamics (MTDD) measure of Blanchard (1990) as the object for the counterfactual exercise because it is explicitly path-based: it summarises fiscal prospects in terms of the implied trajectory of the debt-to-GDP ratio, which is the key outcome of interest in this chapter. A central advantage of MTDD is that it depends on model-based forecasts rather than being purely backward-looking. While the inputs are disciplined by historical data through estimation, the evaluation is forward-looking in the sense that it embeds projected macroeconomic conditions into the debt accumulation mechanism and therefore delivers a coherent debt path under a given scenario. This feature makes MTDD particularly suitable for the conditional forecasting approach adopted here. Within a fixed forecasting system, changing only the fiscal reaction rule—most importantly, the Bohn (2005) rule—generates a new counterfactual forecast for the primary balance and hence a new implied debt path, holding the rest of the forecast environment constant. In this setting, the policy experiment can be assessed directly by comparing the counterfactual debt path with the benchmark trajectory, without requiring a detailed decomposition of the policy change into its separate macroeconomic transmission effects. This "rule-change within a conditional forecast" strategy is also in line with existing conditional-forecast applications in the literature (see, e.g., Lenza et al. (2010)).

As to identification, different from identification method Bernanke and Mihov (1998), I use PVAR identification method proposed by Polito and Wickens (2012b) to analyse Bohn (2005) rule that decreases MTDD to avoid invalid identification. The method avoids invalid identification issues that arise when assuming VAR disturbances of the non-policy and policy equations in the estimated VAR are uncorrelated but they are actually not. The identification of fiscal policy based on the VAR method has a common assumption that fiscal policy responds to economic changes with a lag. Fatas and Mihov (2000) distinguish discretionary fiscal policy from automatic stabilizers by assuming that government spending responds to macroeconomic conditions with a lag. However, simply replacing the equations for the policy instruments in the original VAR by the rule would alter the stochastic structure of the other equations. Polito and Wickens (2005) used the VAR model to analyse the Taylor rule to improve fiscal stance. They proposed an identification method for policy VAR (PVAR), which changes the random structure of the solution of the policy tool equation when directly using the original VAR model for counterfactual analysis and this identification method can avoid the problem mentioned above. This method is used in this chapter for couterfactual analysis.

Although the primary gap is a convenient accounting benchmark—because it back-solves the primary balance implied by the debt-accumulation identity under an assumed (r-g) differential and a given debt objective (see, e.g., Escolano (2010))—this chapter does not take the primary gap as its central object of analysis for three reasons. First, it is an accounting-based requirement: it indicates how large the primary balance would need to be to meet the stated debt objective, but it does not speak to whether such an adjustment is feasible or sustainable in practice over long horizons. Second, the primary-gap exercise typically treats fiscal adjustment as an exogenously imposed path, thereby abstracting from the endogenous policy response observed in the data (i.e., how primary balances adjust in response to debt and macroeconomic conditions) and from implementation constraints. Third, sustained episodes of very large primary surpluses are historically rare; accordingly, using the primary gap as a baseline scenario may be unduly optimistic, as emphasized by Eichengreen and Panizza (Eichengreen and Panizza). For these reasons, the analysis instead evaluates fiscal sustainability through an empirical fiscal reaction function (and the implied debt dynamics), which directly assesses whether the adjustment required by debt arithmetic is consistent with historically observed policy behaviour.

Many literature uses directly the conditional forecast to analyse the effect of unconventional policies, such as quantitative easing (see Lenza et al. (2010), Kapetanios et al. (2012) and Dahlhaus et al. (2018)). This method imposes the paths of artificial policy variables as forecast conditions and examines the resulting trajectories of other macroeconomic variables. By comparing these outcomes with those under actual policy scenarios, it allows researchers to infer policy impacts.This analysis differs from the impulse response function (IRF) approach. While IRFs estimate the causal effects of isolated shocks, conditional forecasts capture the most likely evolution of the economy under the joint influence of multiple shocks, given the imposed policy path. As such, this method does not require identification of a single structural shock, but still illustrates the whole dynamic adjustment process. It thereby offers a more realistic basis for policy evaluation.

2.3 Methodology

This section focuses on the counterfactual analysis, while the estimation of the MTDD is primarily discussed in Section 1.3.5.

For the counterfactual analysis, this chapter adopts the PVAR identification method proposed by Polito and Wickens (2012b), which requires transforming the VAR model before applying alternative fiscal policies. To derive new Bohn (2005) rules aimed at improving the MTDD, it is not appropriate to simply replace the policy instrument equations in the original VAR with the rule. Doing so would alter the stochastic structure of the remaining equations, thereby undermining the integrity of the model. This process begins with the reduced-form VAR(p) model:

$$\boldsymbol{z_t} = \boldsymbol{A_0} + \boldsymbol{A(L)z_{t-1}} + \boldsymbol{u_t}, \tag{2.1}$$

where $\boldsymbol{z'_t = (z'_{1,t}, z'_{2,t})}$, $\boldsymbol{z_{1,t}}$ is an m $\times$ 1 vector of economic variables, and $\boldsymbol{z_{2,t}}$ is a $k \times 1$ vector of policy instruments, $q = m + k$, and $\boldsymbol{u_t \sim (0, \Sigma)}$. In equation (2.1), $\boldsymbol{A_0}$ is a vector of constant terms, $\boldsymbol{L}$ is the lag operator, and $\boldsymbol{A(L)}$ is an autoregressive lag polynomial expressed as $\boldsymbol{A(L) = \Sigma_{i=1}^{p} A_i L^{i-1}}$, where $\boldsymbol{A_i}$ is a $q \times q$ matrix of lag i coefficients. The vector of non-policy variables $\boldsymbol{z_{1,t}}$ in the VAR is

$$\boldsymbol{z'_{1,t}} = \begin{bmatrix} n_t & \tilde{y}_t & \tilde{g}_t & \pi_t & r_t^l & \gamma_t & b_t \end{bmatrix}',$$

which includes the growth rate of population (n_t), the cyclical component of output($\tilde{y}_t$), the cyclical component of government expenditure ($\tilde{g}_t$), the inflation rate (π_t), the long-term interest rate (r_t^l), the real output growth rate (γ_t), and the debt to GDP ratio (b_t). These are the economic variables related to PVBC. For simplicity in explanation, I use $\boldsymbol{X_t}$ to represent all non-policy variables except b_t, the debt-to-GDP ratio. Then $\boldsymbol{z}'_{1,t}$ can be expressed as $[\boldsymbol{X_t}', b_t]$. The vector of the policy instrument $\boldsymbol{z_{2,t}}$ contains only one variable — primary balance to GDP ratio (pb_t). In the model, the variance-covariance matrix $\boldsymbol{\Sigma}$ is unrestricted, thereby allowing the VAR disturbances to be correlated.

The VAR model can be partitioned into two groups: the non-policy variables group and the policy instrument group:

$$\begin{bmatrix} \boldsymbol{z_{1,t}} \\ \boldsymbol{z_{2,t}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{A_{10}} \\ \boldsymbol{A_{20}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{A_{11}(L)} & \boldsymbol{A_{12}(L)} \\ \boldsymbol{A_{21}(L)} & \boldsymbol{A_{22}(L)} \end{bmatrix} \begin{bmatrix} \boldsymbol{z_{1,t-1}} \\ \boldsymbol{z_{2,t-1}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{u_{1,t}} \\ \boldsymbol{u_{2,t}} \end{bmatrix}. \tag{2.2}$$

In equation (2.2), $\boldsymbol{A_{11}(L)} = \sum_{i=1}^{p} \boldsymbol{A_{11,i}L^{i-1}}$, $\boldsymbol{A_{12}(L)} = \sum_{i=1}^{p} \boldsymbol{A_{12,i}L^{i-1}}$, $\boldsymbol{A_{21}(L)} = \sum_{i=1}^{p} \boldsymbol{A_{21,i}L^{i-1}}$, $\boldsymbol{A_{22}(L)} = \sum_{i=1}^{p} \boldsymbol{A_{22,i}L^{i-1}}$ and $\begin{bmatrix} \boldsymbol{u_{1,t}} \\ \boldsymbol{u_{2,t}} \end{bmatrix} \sim \boldsymbol{iid}$ $\left(\begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{0} \end{bmatrix}, \begin{bmatrix} \boldsymbol{\Sigma_{11}} & \boldsymbol{\Sigma_{12}} \\ \boldsymbol{\Sigma_{21}} & \boldsymbol{\Sigma_{22}} \end{bmatrix} \right)$ where $\boldsymbol{\Sigma_{12}} = \boldsymbol{\Sigma}'_{\boldsymbol{21}}$. There are two ways to transform the VAR for counterfactual analysis: one is to set $\boldsymbol{\Sigma_{12}} = \boldsymbol{0}$, meaning that the shocks to the non-policy variables may affect the policy instrument variables contemporaneously but no the other way around; the other one is to set $\boldsymbol{\Sigma_{21}} = \boldsymbol{0}$, meaning that the shocks to the policy instrument variables may affect the non-policy variables contemporaneously and still no the other way around.

I choose the first one due to the implementation lag fiscal policy (this is discussed by Blanchard and Perotti (2002)) and the characteristics of fiscal policy as an automatic stabilizer. As an automatic stabilizer, when there is a negative shock to the cyclical component of output, for example, this would automatically result in the decrease in tax base and then the decrease in primary balance, the policy instrument in this chapter. However, the operation lag for fiscal policies is longer than that for monetary policy. The fiscal policy should go through legislation and there are implementation lag from the announcement to implementation. Therefore, the fiscal policy does not contemporaneously react to the shocks of non-policy variables is a sensible identification restriction.

With $\boldsymbol{\Sigma_{12}} = \mathbf{0}$, If $\boldsymbol{u_{1,t}}$ is correlated with $\boldsymbol{u_{2,t}}$ then I can decompose $\boldsymbol{u_{2,t}}$ into a component $\boldsymbol{\epsilon_t}$ that is uncorrelated with $\boldsymbol{u_{1,t}}$ and a component correlated with $\boldsymbol{u_{1,t}}$. Hence those residuals can be written in this way:

$$\boldsymbol{u_{2,t}} = \boldsymbol{G u_{1,t}} + \boldsymbol{\epsilon_t},$$

or in matrix term,

$$\begin{bmatrix} \boldsymbol{u_{1,t}} \\ \boldsymbol{u_{2,t}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{I} & \mathbf{0} \\ \boldsymbol{G} & \boldsymbol{I} \end{bmatrix} \begin{bmatrix} \boldsymbol{u_{1,t}} \\ \boldsymbol{\epsilon_t} \end{bmatrix} = \boldsymbol{H} \begin{bmatrix} \boldsymbol{u_{1,t}} \\ \boldsymbol{\epsilon_t} \end{bmatrix}.$$

Since the variance-covariance matrix is:

$$\boldsymbol{\Sigma} = \begin{bmatrix} \boldsymbol{\Sigma_{11}} = \boldsymbol{E}\left[\boldsymbol{u_{1,t}u'_{1,t}}\right] & \boldsymbol{\Sigma_{12}} = \boldsymbol{E}\left[\boldsymbol{u_{1,t}u'_{2,t}}\right] \\ \boldsymbol{\Sigma_{21}} = \boldsymbol{E}\left[\boldsymbol{u_{2,t}u'_{1,t}}\right] & \boldsymbol{\Sigma_{22}} = \boldsymbol{E}\left[\boldsymbol{u_{2,t}u'_{2,t}}\right] \end{bmatrix},$$

it follows that

$$\boldsymbol{E}[\boldsymbol{u_{2,t}u'_{1,t}}] = \boldsymbol{\Sigma_{21}} = \boldsymbol{E}[\boldsymbol{G u_{1,t}u'_{1,t}}] = \boldsymbol{G\Sigma_{11}}.$$

Thus,

$$\boldsymbol{G} = \boldsymbol{\Sigma_{21}\Sigma_{11}^{-1}}. \tag{2.3}$$

To transform the model in equation (2.1), pre-multiply each term by the matrix $\boldsymbol{H^{-1}}$ to obtain:

$$\boldsymbol{H}^{-1}\boldsymbol{z}_t = \boldsymbol{H}^{-1}\boldsymbol{A}_0 + \boldsymbol{H}^{-1}\boldsymbol{A}(\boldsymbol{L})\boldsymbol{z}_{t-1} + \boldsymbol{H}^{-1}\boldsymbol{u}_t. \tag{2.4}$$

We can rewrite (2.4) in partitioned form:

$$\begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} \\ -\boldsymbol{G} & \boldsymbol{I} \end{bmatrix} \begin{bmatrix} \boldsymbol{z}_{1,t} \\ \boldsymbol{z}_{2,t} \end{bmatrix} = \begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} \\ -\boldsymbol{G} & \boldsymbol{I} \end{bmatrix} \begin{bmatrix} \boldsymbol{A}_{10} \\ \boldsymbol{A}_{20} \end{bmatrix} + \begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} \\ -\boldsymbol{G} & \boldsymbol{I} \end{bmatrix} \begin{bmatrix} \boldsymbol{A}_{11}(L) & \boldsymbol{A}_{12}(L) \\ \boldsymbol{A}_{21}(L) & \boldsymbol{A}_{22}(L) \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{z}_{1,t-1} \\ \boldsymbol{z}_{2,t-1} \end{bmatrix} + \begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} \\ -\boldsymbol{G} & \boldsymbol{I} \end{bmatrix} \begin{bmatrix} \boldsymbol{u}_{1,t} \\ \boldsymbol{u}_{2,t} \end{bmatrix}. \tag{2.5}$$

Then expand it row-by-row to obtain:

$$\boldsymbol{z}_{1,t} = \boldsymbol{A}_{10} + \boldsymbol{A}_{11}(\boldsymbol{L})\boldsymbol{z}_{1,t-1} + \boldsymbol{A}_{12}(\boldsymbol{L})\boldsymbol{z}_{2,t-1} + \boldsymbol{u}_{1,t}, \tag{2.6}$$

$$\boldsymbol{z}_{2,t} = [\boldsymbol{A}_{20} - \boldsymbol{G}\boldsymbol{A}_{10}] + \boldsymbol{G}\boldsymbol{z}_{1,t} + [\boldsymbol{A}_{21}(\boldsymbol{L}) - \boldsymbol{G}\boldsymbol{A}_{11}(\boldsymbol{L})]\, \boldsymbol{z}_{1,t-1} + [\boldsymbol{A}_{22}(\boldsymbol{L}) - \boldsymbol{G}\boldsymbol{A}_{12}(\boldsymbol{L})]\, \boldsymbol{z}_{2,t-1} + \boldsymbol{\varepsilon}_t. \tag{2.7}$$

It is important to note that $u_{1,t}$ and ϵ_t are uncorrelated so that two equations can be separated and equation (2.6) now can combine with any rules with the form of equation (2.7) to do counterfactuals.

Bohn (2005) proved that the significant positive relationship between primary balance and the initial debt, the debt at the end of period $t-1$, when controlling the fluctuations in government expenditure and output is sufficient to guarantee that the fiscal policy satisfies the PVBC. Mendoza and Ostry (2008) later conclude that in essence the fiscal reaction function is trying to determine whether the rise in public debt would result in the increase in primary balance. Therefore, Mendoza and Ostry (2008) regress primary balance-GDP ratio on debt-GDP ratio while controlling cyclical components of government expenditure and output. This simple Bohn (2005) rule focusing on the contemporaneous response of primary balance to debt is estimated using the equation (2.8):

$$pb_t = \delta_0 + \delta_1 b_t + \delta_2 \tilde{g}_t + \delta_3 \tilde{y}_t + \mu_t, \tag{2.8}$$

where $\mu_t \sim iid\left(0, \sigma_\mu^2\right)$.

It is evident to see that the second equation in the transformed VAR model (eq (2.7)) is isomorphic to Bohn (2005) rule in equation (2.8). Therefore, to see the effect of new Bohn (2005) rules on MTDD, I set new response coefficients in the Bohn (2005) rule and the new fiscal rule is represented as:

$$pb_t = \delta_0^* + \delta_1^* b_t + \delta_2^* \tilde{g}_t + \delta_3^* \tilde{y}_t, \tag{2.9}$$

where * is the new parameters objectively set. The second equation in the **VAR(p)** model then becomes:

$$\boldsymbol{z_{2t}} = \boldsymbol{A_{20}^*} + \boldsymbol{\Lambda z_{1,t}} + \boldsymbol{A_{21}^*(L) z_{1,t-1}} + \boldsymbol{A_{22}^*(L) z_{2,t-1}} + \boldsymbol{\epsilon_t}, \tag{2.10}$$

where $\boldsymbol{A_{20}^*} = \delta_0^*$, $\boldsymbol{\Lambda} = \begin{bmatrix} \mathbf{0} \\ \delta_1^* \end{bmatrix}'$. Since Bohn (2005) rule is non-stochastic and has no lagged dynamics, $\boldsymbol{A_{21}^*(L), A_{22}^*(L)}$ and $\boldsymbol{\epsilon_t^*}$ would all be zero.

The model then becomes:

$$\begin{bmatrix} \boldsymbol{I} & \mathbf{0} \\ -\boldsymbol{\Lambda} & \boldsymbol{I} \end{bmatrix} \begin{bmatrix} \boldsymbol{z_{1,t}} \\ \boldsymbol{z_{2,t}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{I} & \mathbf{0} \\ -\boldsymbol{G} & \boldsymbol{I} \end{bmatrix} \begin{bmatrix} \boldsymbol{A_{10}} \\ \boldsymbol{A_{20}^*} \end{bmatrix} + \begin{bmatrix} \boldsymbol{I} & \mathbf{0} \\ -\boldsymbol{G} & \boldsymbol{I} \end{bmatrix} \begin{bmatrix} \boldsymbol{A_{11}(L)} & \boldsymbol{A_{12}(L)} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{z_{1,t-1}} \\ \boldsymbol{z_{2,t-1}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{u_{1,t}} \\ \mathbf{0} \end{bmatrix}. \tag{2.11}$$

Let $\boldsymbol{M} = \begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} \\ -\boldsymbol{\Lambda} & \boldsymbol{I} \end{bmatrix}$ and use $\boldsymbol{M}^{-1}$ to pre-multiply both sides of the previous VAR to get the reduced form VAR under new rule:

$$\begin{bmatrix} \boldsymbol{z}_{1,t} \\ \boldsymbol{z}_{2,t} \end{bmatrix} = \begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} \\ -\boldsymbol{\Lambda} & \boldsymbol{I} \end{bmatrix}^{-1} \begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} \\ -\boldsymbol{G} & \boldsymbol{I} \end{bmatrix} \begin{bmatrix} \boldsymbol{A}_{10} \\ \boldsymbol{A}_{20}^{*} \end{bmatrix}$$
$$+ \begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} \\ -\boldsymbol{\Lambda} & \boldsymbol{I} \end{bmatrix}^{-1} \begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} \\ -\boldsymbol{G} & \boldsymbol{I} \end{bmatrix} \begin{bmatrix} \boldsymbol{A}_{11}(\boldsymbol{L}) & \boldsymbol{A}_{12}(\boldsymbol{L}) \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{z}_{1,t-1} \\ \boldsymbol{z}_{2,t-1} \end{bmatrix} + \begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} \\ -\boldsymbol{\Lambda} & \boldsymbol{I} \end{bmatrix}^{-1} \begin{bmatrix} \boldsymbol{u}_{1,t} \\ \boldsymbol{0} \end{bmatrix}. \tag{2.12}$$

Simplify equation (2.12) and there derives:

$$\begin{bmatrix} \boldsymbol{z}_{1,t} \\ \boldsymbol{z}_{2,t} \end{bmatrix} = \begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{\Lambda} - \boldsymbol{G} & \boldsymbol{I} \end{bmatrix} \begin{bmatrix} \boldsymbol{A}_{10} \\ \boldsymbol{A}_{20}^{*} \end{bmatrix} + \begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{\Lambda} - \boldsymbol{G} & \boldsymbol{I} \end{bmatrix} \begin{bmatrix} \boldsymbol{A}_{11}(\boldsymbol{L}) & \boldsymbol{A}_{12}(\boldsymbol{L}) \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{z}_{1,t-1} \\ \boldsymbol{z}_{2,t-1} \end{bmatrix}$$
$$+ \begin{bmatrix} \boldsymbol{u}_{1,t} \\ \boldsymbol{\Lambda}\boldsymbol{u}_{1,t} \end{bmatrix}. \tag{2.13}$$

A clearer form for equation (2.13) :

$$\begin{bmatrix} \boldsymbol{z}_{1,t} \\ \boldsymbol{z}_{2,t} \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}_{10}^{\#} \\ \boldsymbol{A}_{20}^{\#} \end{bmatrix} + \begin{bmatrix} \boldsymbol{A}_{11}^{\#}(\boldsymbol{L}) & \boldsymbol{A}_{12}^{\#}(\boldsymbol{L}) \\ \boldsymbol{A}_{21}^{\#}(\boldsymbol{L}) & \boldsymbol{A}_{22}^{\#}(\boldsymbol{L}) \end{bmatrix} \begin{bmatrix} \boldsymbol{z}_{1,t-1} \\ \boldsymbol{z}_{2,t-1} \end{bmatrix} + \begin{bmatrix} \boldsymbol{u}_{1,t}^{\#} \\ \boldsymbol{u}_{2,t}^{\#} \end{bmatrix}, \tag{2.14}$$

where terms marked with the superscript $^{\#}$ denote the corresponding coefficients and variables in equation (2.13). Equation (2.14) can also be expressed in its reduced form as:

$$\boldsymbol{z}_t = \boldsymbol{A}_0^{\#} + \boldsymbol{A}^{\#}(\boldsymbol{L})\boldsymbol{z}_{t-1} + \boldsymbol{u}_t^{\#}. \tag{2.15}$$

This is the reduced form VAR under the new policy rule. The companion form of the VAR model used later for forecast of debt is:

$$\boldsymbol{Z_t} = \boldsymbol{\mathcal{A}^{\#}} \boldsymbol{Z_{t-1}} + \boldsymbol{U_t^{\#}}. \tag{2.16}$$

where $\boldsymbol{Z_t'} = [\boldsymbol{z_t'}, \boldsymbol{z_{t-1}'}, ..., \boldsymbol{z_{t-p+1}'}, \boldsymbol{1}], \boldsymbol{U_t^{\#\prime}} = [\boldsymbol{u_t^{\#\prime}}, \boldsymbol{0'}, ..., \boldsymbol{0'}]$, and

$$\mathcal{A}^{\#} = \begin{bmatrix} \boldsymbol{A_1^{\#}} & \boldsymbol{A_2^{\#}} & . & . & \boldsymbol{A_p^{\#}} & \boldsymbol{A_0^{\#}} \\ \boldsymbol{I} & \boldsymbol{0} & . & . & . & . \\ \boldsymbol{0} & \boldsymbol{I} & \boldsymbol{0} & . & . & . \\ . & . & . & . & . & . \\ \boldsymbol{0} & . & \boldsymbol{0} & \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & . & . & \boldsymbol{0} & \boldsymbol{1} \end{bmatrix}.$$

$E_t s_{t+j}$ for the debt dynamics under new Bohn (2005) rule then should be forecast using equation (2.16). The forecast of the effective primary balance under changed fiscal rule $E_t s_{t+j}^{\#}$:

$$E_t s_{t+j}^{\#} = \alpha + \boldsymbol{\beta' S}(\boldsymbol{\mathcal{A}^{\#j} Z_t}). \tag{2.17}$$

For the new Bohn (2005) rule, I first assign the value of δ_1 to δ_1^* and observe the status of the MTDD. Then I either increase or decrease the response rate,δ_1^*, by 1% each time until the debt dynamics reaches zero. δ_0^* is assigned the value of δ_0, which is the constant estimated using Equation (2.8).

2.4 Results

With the annual data, the lag length of the VAR model is chosen as 1. I estimate the model separately for each country, G7 countries and China, and estimate recursively starting with the first 25 observations until 2022. The key findings are as follows: the final results suggest that countries accustomed to conducting loose fiscal policies should strengthen the responsiveness of the primary balance-to-GDP ratio to the debt-to-GDP ratio. I've done bootstrap Chow test and the results show that most of these are insignificant. Nevertheless, these findings remain subject to the Lucas (1976) critique, as policy effectiveness may vary under different expectations and structural environments. For MTDD(t,j) in this chapter, the horizon is set as 5 years, fulfilling the needs for G7 countries, j=5. I use the average debt from 1998 to 2007 as the debt target for MTDD for the following reasons.

I choose to display the average forecast values of the variables in the figures. MTDD(t,j) is derived by subtracting the target level of debt-GDP ratio from the present value of the forecast debt ratio. The most crucial component of MTDD(t,j) is the debt forecast, which primarily relies on the forecast of the effective primary balance. Since MTDD(t,j) examines whether fiscal policy can achieve the target debt ratio within a given horizon j, this requires forecasting the effective primary balance from period t+1 to period t+j. To present the results intuitively, I display the average forecast values of the components of the effective primary balance and use † to signal the averages. The averages of these components are as follows:

$$x^{\dagger}_{t,j} = \frac{1}{j}\sum_{i=1}^{j} E_t x_{t+i}, \tag{2.18}$$

where x_t stands for $PV(\rho_t)$, $PV(r_t)$, $PV(n_t)$, $PV(\gamma_t)$, $PV(\pi_t)$, $PV(s_t)$, and $PV(pb_t)$. PV stands for the present value.

I specifically illustrate $PV(s_t)$ and $PV(pb_t)$. I display $PV(s_t)$ as it is one of the main components of MTDD(t,j). The variations of s_t is mainly from the change in its components, pb_t and ρ_t. When the huge gap between $PV(pb_t)$ and $PV(pb_t^{\#})$ is large, it indicates that pb_t contributes significantly in bringing the debt ratio within the target. I also show $PV(pb_t)$ in comparison with $PV(s_t)$ to facilitate understanding of whether ρ_t plays the dominant role in aligning the debt ratio forecast with the target. If the gap between $PV(s_t^{\#})$ and $PV(pb_t^{\#})$ is larger than its counterparts in the VAR model, it suggests that ρ_t plays a greater role.

Table 2.1: Empirical Results and Counterfactual Results for MTDD(2022,5)

	MTDD (2022,5)	δ_1 in equation (2.8)	δ_1^* in equation (2.9)	PV (b_t^*)
Canada	-0.2948	0.0379**	0.07	81.47%
China	-0.9892	-0.0900***	0.01	40.42%
France	-0.6767	-0.0322***	0.04	74.98%
Germany	0.0496	0.0561***	0.03	65.56%
Italy	-0.7526	0.0569***	0.09	115.65%
Japan	-1.5772	-0.0188***	0.07	146.98%
UK	-0.5603	-0.0417***	0.12	52.84%
US	-0.7494	-0.0620***	0.05	61.66%

* Each row presents the situations and the empirical results for G7 countries and China respectively. Column 2 reports MTDD(2022,5), where a negative value implies an overly loose fiscal policy and a positive value indicates a tight one. Column 3 shows the estimated fiscal response coefficient (δ_1) based on the Bohn (2005) rule (equation (2.8)). Column 4 displays the hypothetical fiscal response coefficient (δ_1^*) that would stabilize $MTDD^{\#}(2022,5)$ around zero under the PVAR model. Column 5 reports the present value of debt ratio target. Stars denote statistical significance: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

The average values for the forecast of the components of $MTDD^{\#}(t,5)$ under the PVAR model is written as:

$$x_{t,j}^{\#\dagger} = \frac{1}{j}\sum_{i=1}^{j} E_t x_{t+i}^{\#}. \tag{2.19}$$

where $x_t^{\#}$ stands for $PV\rho_t^{\#}$, $PVr_t^{\#}$, $PVn_t^{\#}$, $PV\gamma_t^{\#}$, $PV\pi_t^{\#}$, $PV(s_t^{\#})$, and $PV(pb_t^{\#})$.

$PV(s_t^{\#})$ is the present values of $s_t^{\#}$, and $PV(pb_t^{\#})$ is the present values of $pb_t^{\#}$.

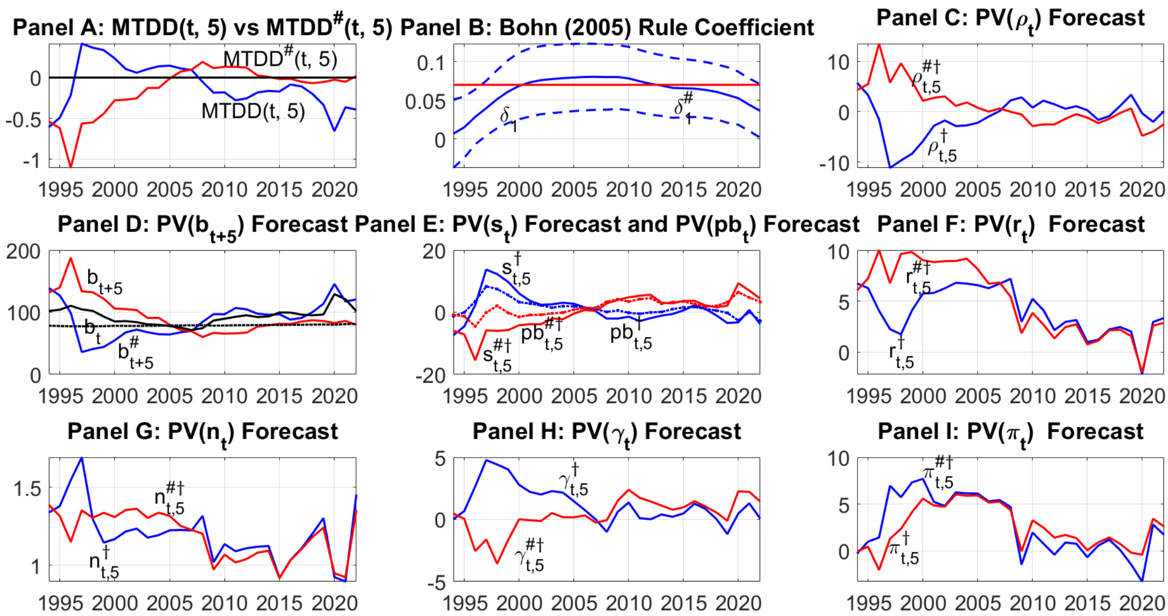

Figure 2.1: Canada MTDD(t,5), $MTDD^{\#}(t,5)$ and Their Components

Panel A shows MTDD(t,5) (blue line) and $MTDD^{\#}(t,5)$ (red line). Panel B shows the recursively estimated Bohn (2005) rule coefficients δ_1 (blue line), its 95% confidence interval (blue dashed line), and $\delta_1^{\#}$ conducted to make $MTDD^{\#}(t,5)$ closer to 0. Panel C to Panel I display the forecast of component of MTDD and $MTDD^{\#}$, with superscript † meaning the average of the forecast based on the VAR model for the variables between t+1 period and t+5 period (blue line) and superscript #† the average of the forecast based on the PVAR model for the variables between t+1 period and t+5 period (red line).

The response coefficient in the Bohn (2005) rule is adjusted by 1% for each country to examine its effect on MTDD(2022, 5) until the value converges to zero. Here, MTDD(2022, 5) represents the Blanchard (1990) medium-term debt dynamics as of 2022, projected over a 5 -period horizon. For all the country analysed, MTDD(2022, 5) values are negative, indicating that fiscal policies have been loose. Raising the MTDD value (i.e., bringing it closer to zero) requires a reduction in the debt-to-GDP ratio, which in turn necessitates a stronger response in the Bohn (2005) rule. This implies that government would need to generate higher primary balances in order to stabilize debt and reduce the debt dynamics. The results are presented in the Table 2.1. For policy change based on the VAR model, displaying the recursive estimates is also important; these are shown in Figures 2.1-2.8.

2.4.1 Canada

According to MTDD(t,5), Canada's fiscal policies have been quite loose since 2008. The response coefficients δ_1 in the Bohn (2005) rule, when estimated recursively, were not significantly positive at the 5% level in the early years. In 1997 δ_1 was 0.04 and significantly positive. The response increased to 0.08 in 2009, then exhibited a downward trend; however, overall, the response has remained significantly positive since 1996. If MTDD(2022,5) for Canada were to converge to 0, the response in Bohn (2005) rule would need to increase. Based on the counterfactual results, the required response would be 0.07 — a level Canada implemented in the past and the response was also significant. Adopting such a stronger fiscal response would lead to a higher $pb_{t,5}^{\#\dagger}$ and a lower discount rate $\rho_{t,5}^{\#\dagger}$, both of which would help reduce the debt projection. Furthermore, it is noteworthy that under new policy γ_t is forecast to be higher, especially 2018.

2.4.2 China

According to MTDD(t,5), China's fiscal policies have been quite loose since 2015. The response coefficients δ_1 in the Bohn (2005) rule showed an upward trend between 2004 and 2010; however they were not statistically significant at the 5% level. Since 2018, the response has turned significantly negative and has remained so. If the MTDD(2022,5) for China were to converge to 0, the response in the Bohn (2005) rule would need to increased. According to the results, the response would rise to 0.01 — a level previously implemented by China, though it was not significant. Adopting such a positive response would result in higher $pb^{\#\dagger}_{t,5}$ and lower discount rate $\rho^{\#\dagger}_{t,5}$. Among these, $pb^{\dagger}_{t,5}$ was the primary contributr to the decline in MTDD(t,5). Under the new fiscal policy, ρ_t and its components are forecast to be higher.

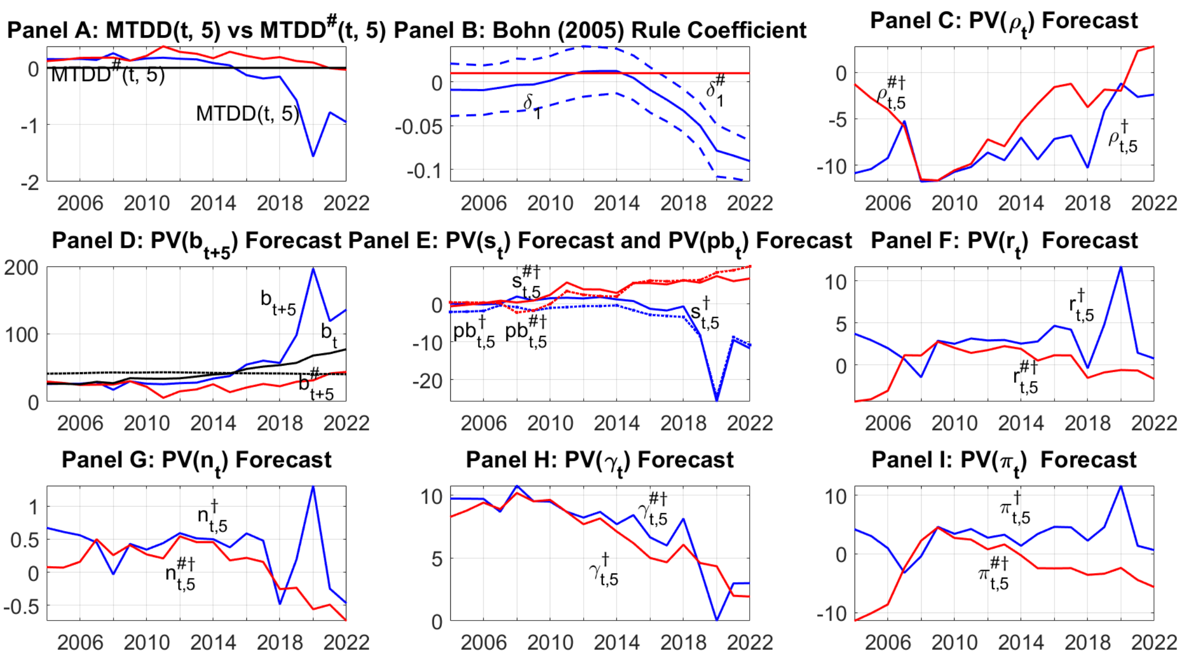

Figure 2.2: China MTDD(t,5), $MTDD^{\#}(t,5)$ and Their Components

Panel A shows MTDD(t,5) (blue line) and $MTDD^{\#}(t,5)$ (red line). Panel B shows the recursively estimated Bohn (2005) rule coefficients δ_1 (blue line), its 95% confidence interval (blue dashed line), and $\delta_1^{\#}$ conducted to make $MTDD^{\#}(t,5)$ closer to 0. Panel C to Panel I display the forecast of ρ_t, PVb_{t+j}, $PV(s_t)$, $PV(pb_t)$, r_t, n_t, γ_t, π_t, with superscript † meaning the average of the forecast based on the VAR model for the variables between t+1 period and t+5 period (blue line) and superscript #† the average of the forecast based on the PVAR model for the variables between t+1 period and t+5 period (red line).

2.4.3 France

According to MTDD(t,5), France's fiscal policies have been loose since 2006. The response coefficients δ_1 in the Bohn (2005) rule were significantly negative in the early period and became statistically insignificant after 1996. Although this later indicated an improving trend, the response remained significantly negative after 2009. If the MTDD(2022,5) for France were to converge to 0, the response in Bohn (2005) rule would need to increase. According to the results, the required response would be 0.04 — a level France has not previously implemented, but one that remains plausible. Implementing such a positive response would result in higher $pb^{\#\dagger}_{t,5}$ and lower discount rate $\rho^{\#\dagger}_{t,5}$, both of which contribute to the decrease in the debt forecast. Notably, the components of the discount rate, $\pi^{\#\dagger}_{t,5}$, $n^{\#\dagger}_{t,5}$, and $\gamma^{\#\dagger}_{t,5}$, are all higher than their counterparts under the original fiscal policy.

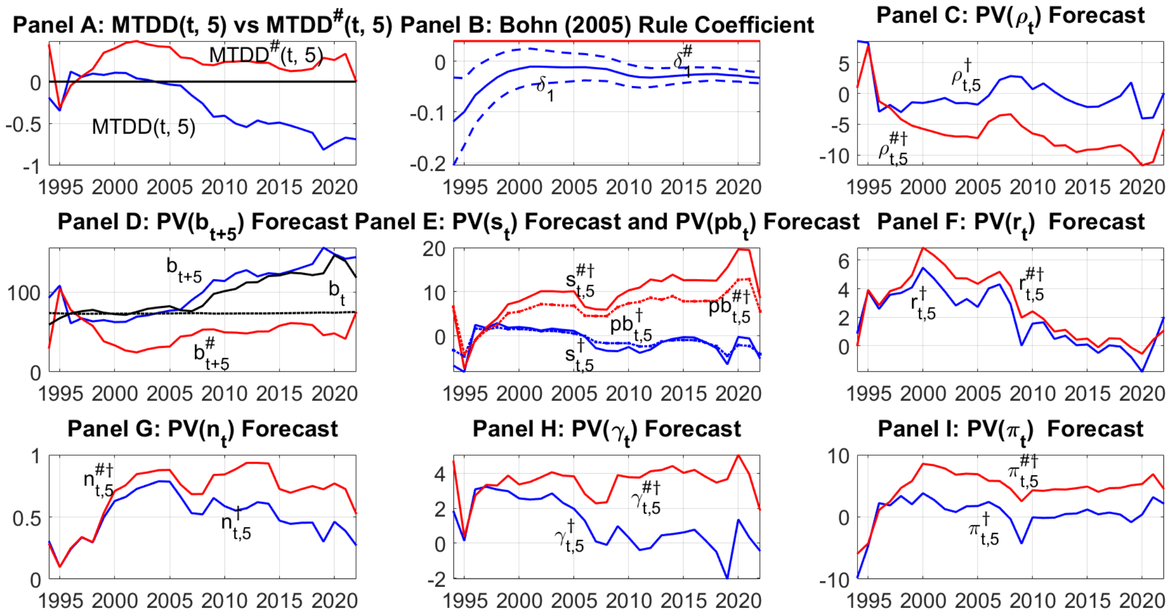

Figure 2.3: France MTDD(t,5), $MTDD^{\#}(t, 5)$ and Their Components

Panel A shows MTDD(t,5) (blue line) and $MTDD^{\#}(t, 5)$ (red line). Panel B shows the recursively estimated Bohn (2005) rule coefficients δ_1 (blue line), its 95% confidence interval (blue dashed line), and $\delta_1^{\#}$ conducted to make $MTDD^{\#}(t, 5)$ closer to 0. Panel C to Panel I display the forecast of ρ_t, PVb_{t+j}, $PV(s_t)$, $PV(pb_t)$, r_t, n_t, γ_t, π_t, with superscript † meaning the average of the forecast based on the VAR model for the variables between t+1 period and t+5 period (blue line) and superscript #† the average of the forecast based on the PVAR model for the variables between t+1 period and t+5 period (red line).

2.4.4 Germany

According to MTDD(t,5), Germany's fiscal policies have been mostly loose since 2000. The response coefficients δ_1 in the Bohn (2005) rule have remained around 0.02 and are significant at 5% level. If the MTDD(2022,5) for Germany were to converge to 0, the response in Bohn (2005) rule

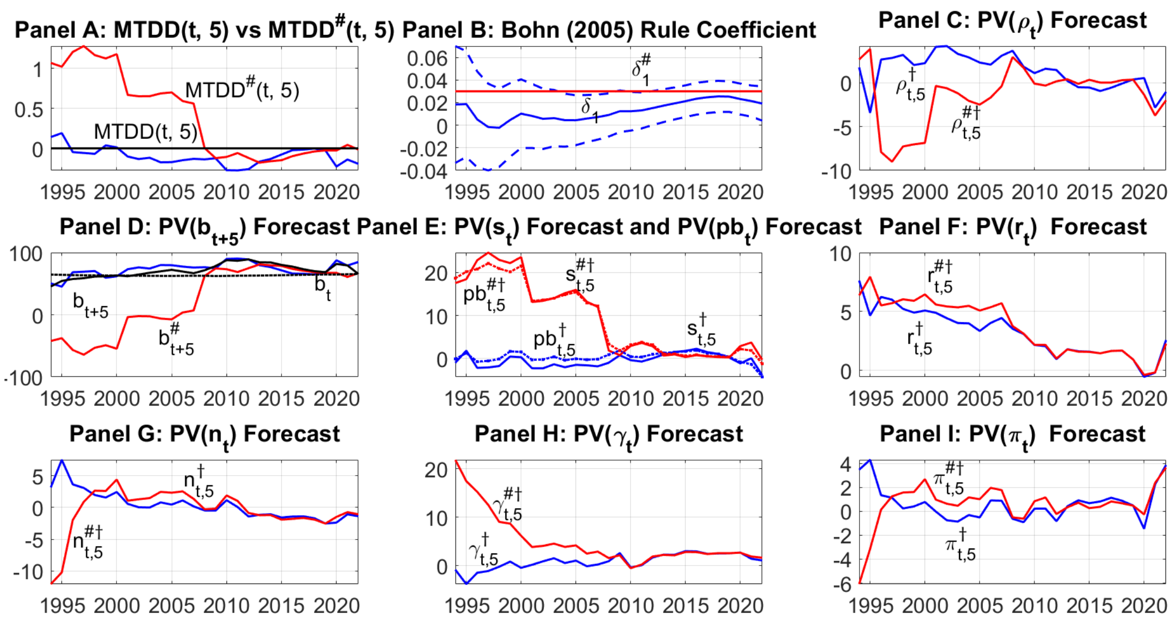

Figure 2.4: Germany MTDD(t,5), $MTDD^{\#}(t,5)$ and Their Components

Panel A shows MTDD(t,5) (blue line) and $MTDD^{\#}(t,5)$ (red line). Panel B shows the recursively estimated Bohn (2005) rule coefficients δ_1 (blue line), its 95% confidence interval (blue dashed line), and $\delta_1^{\#}$ conducted to make $MTDD^{\#}(t,5)$ closer to 0. Panel C to Panel I display the forecast of ρ_t, PVb_{t+j}, $PV(s_t)$, $PV(pb_t)$, r_t, n_t, γ_t, π_t, with superscript † meaning the average of the forecast based on the VAR model for the variables between t+1 period and t+5 period (blue line) and superscript #† the average of the forecast based on the PVAR model for the variables between t+1 period and t+5 period (red line).

would need to increase. According to the results, the required response would be 0.03. Implementing this modest increase in the response would result in higher $pb_{t,5}^{\#\dagger}$, which is the main contributor to the reduction in MTDD(t,5). The policy has little impact on $\rho_{t,5}^{\#\dagger}$ and its components, particularly after 2008.

2.4.5 Italy

According to MTDD(t,5), Italy's fiscal policies have been loose since 2008. The response coefficients δ_1 in the Bohn (2005) rule, estimated recursively, have remained significantly positive. The response rose from 0.1, showing an upward trend

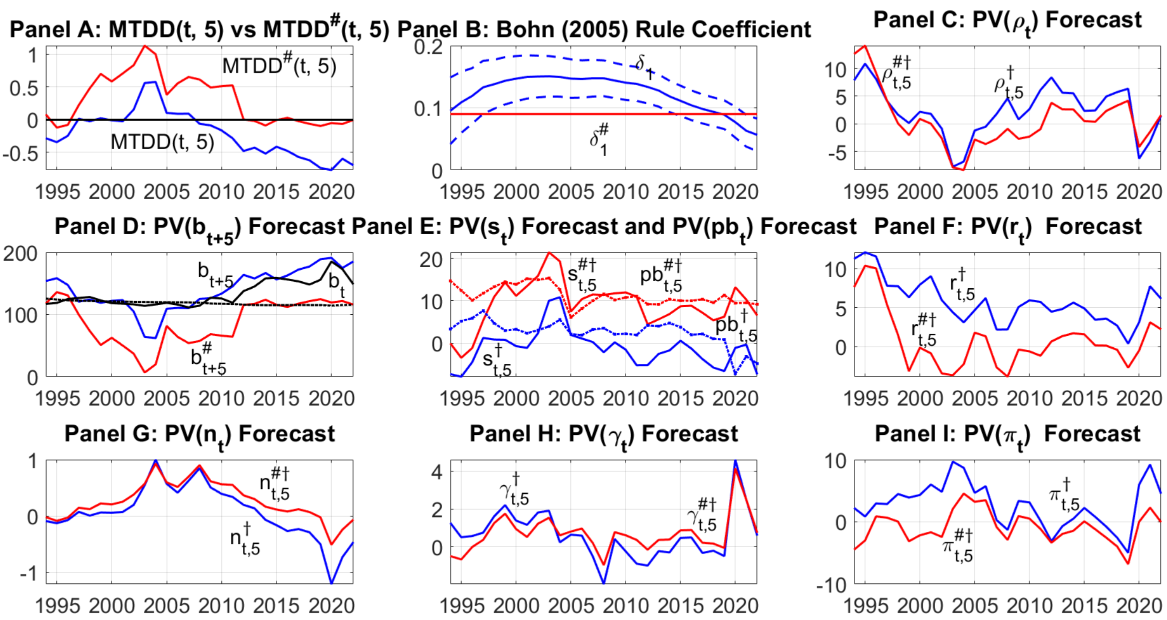

Figure 2.5: Italy MTDD(t,5), $MTDD^{\#}(t,5)$ and Their Components

Panel A shows MTDD(t,5) (blue line) and $MTDD^{\#}(t,5)$ (red line). Panel B shows the recursively estimated Bohn (2005) rule coefficients δ_1 (blue line), its 95% confidence interval (blue dashed line), and $\delta_1^{\#}$ conducted to make $MTDD^{\#}(t,5)$ closer to 0. Panel C to Panel I display the forecast of ρ_t, PVb_{t+j}, $PV(s_t)$, $PV(pb_t)$, r_t, n_t, γ_t, π_t, with superscript † meaning the average of the forecast based on the VAR model for the variables between t+1 period and t+5 period (blue line) and superscript #† the average of the forecast based on the PVAR model for the variables between t+1 period and t+5 period (red line).

between 1994 and 2007. After 2008, it declined to 0.06, though overall, the coefficients have consistently been significantly positive. If MTDD(2022,5) for Italy were to converge to 0, the response in Bohn (2005) rule would need to increase. According to the results, the required response would be 0.09 — a level Italy had implemented around 2019. Adopting this stronger response would result in higher $pb^{\#\dagger}_{t,5}$, the main contributor to the increase in MTDD(t,5). Furthermore, it is noteworthy that implementing this response would lead to lower values of $r^{\#\dagger}_{t,5}$, $\pi^{\#\dagger}_{t,5}$, but higher values of $n^{\#\dagger}_{t,5}$, especially after 2008, compared to their counterparts under the Bohn (2005) policy.

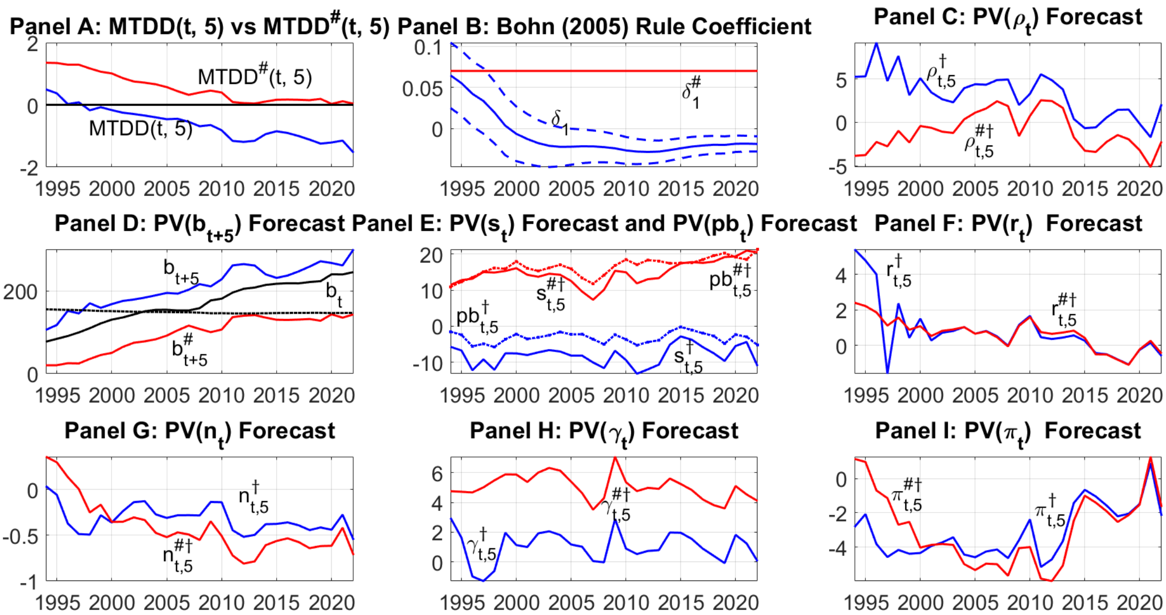

Figure 2.6: Japan MTDD(t,5), $MTDD^{\#}(t,5)$ and Their Components

Panel A shows MTDD(t,5) (blue line) and $MTDD^{\#}(t,5)$ (red line). Panel B shows the recursively estimated Bohn (2005) rule coefficients δ_1 (blue line), its 95% confidence interval (blue dashed line), and $\delta_1^{\#}$ conducted to make $MTDD^{\#}(t,5)$ closer to 0. Panel C to Panel I display the forecast of ρ_t, PVb_{t+j}, $PV(s_t)$, $PV(pb_t)$, r_t, n_t, γ_t, π_t, with superscript † meaning the average of the forecast based on the VAR model for the variables between t+1 period and t+5 period (blue line) and superscript #† the average of the forecast based on the PVAR model for the variables between t+1 period and t+5 period (red line).

2.4.6 Japan

According to MTDD(t,5), Japan's fiscal policies have been quite loose since 1997. the response coefficients δ_1 in the Bohn (2005) rule ,estimated recursively, were initially significantly positive, hovering around 0.04. However, the response became insignificant after 1996 and even turned significantly negative — around -0.02 — after 2004. If MTDD(2022,5) for Japan were to converge to 0, the response in Bohn (2005) rule would need to increase. According to the results, the required response would be 0.07 — a level Japan had implemented around 1994. Implementing this negative response would result in higher $pb^{\#\dagger}_{t,5}$ than its counterpart under the original policy. $pb^{\#\dagger}_{t,5}$ is the main contributor to the rise in the MTDD(t,5). Furthermore, it is noteworthy that implementing this response would result in higher $\gamma^{\#\dagger}_{t,5}$ but lower $n^{\#\dagger}_{t,5}$ than their counterparts under original fiscal policy.

2.4.7 UK

According to MTDD(t,5), UK's fiscal policies have been loose since 2002. The response coefficients δ_1 in the Bohn (2005) rule, estimated recursively, were not significantly positive in the early years. The response declined noticeably during the global financial crisis and the subsequent sovereign debt crisis. Although it recovered slightly after 2013, the response has generally remained significantly negative since 2011. If UK's MTDD(2022,5) had converged to 0, the response in Bohn (2005) rule would have needed to increase. According to the results, the required response would be 0.12 — a level UK had never implemented before. Implementing this positive response would result in higher $pb^{\#\dagger}_{t,5}$ and lower discount rate $\rho^{\#\dagger}_{t,5}$ compared to their counterparts under the original policy and $pb^{\dagger}_{t,5}$ was the main reason for the increase in the MTDD(t,5). Furthermore, it is noteworthy that implementing this response would result in lower $r^{\#\dagger}_{t,5}$, $\pi^{\#\dagger}_{t,5}$ but higher $\gamma^{\#\dagger}_{t,5}$, especially after 2018, compared to their counterparts under the original policy.

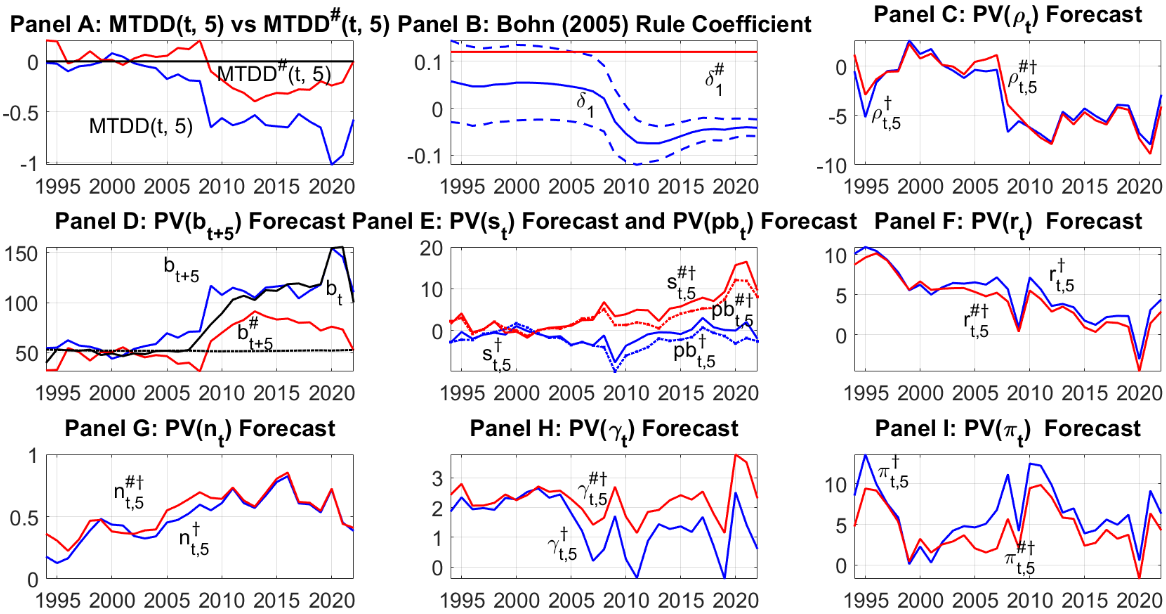

Figure 2.7: UK MTDD(t,5), $MTDD^{\#}(t,5)$ and Their Components

Panel A shows MTDD(t,5) (blue line) and $MTDD^{\#}(t,5)$ (red line). Panel B shows the recursively estimated Bohn (2005) rule coefficients δ_1 (blue line), its 95% confidence interval (blue dashed line), and $\delta_1^{\#}$ conducted to make $MTDD^{\#}(t,5)$ closer to 0. Panel C to Panel I display the forecast of ρ_t, PVb_{t+j}, $PV(s_t)$, $PV(pb_t)$, r_t, n_t, γ_t, π_t, with superscript † meaning the average of the forecast based on the VAR model for the variables between t+1 period and t+5 period (blue line) and superscript #† the average of the forecast based on the PVAR model for the variables between t+1 period and t+5 period (red line).

2.4.8 US

According to MTDD(t,5), US's fiscal policies have been loose since 2003. The response coefficient δ_1 in the Bohn (2005) rule recursively estimated was not significant in the early years but it is significantly positive between 1997 and 2003. After 2003, it became insignificant and even significantly negative after 2011. If the MTDD(2022,5) for US were to converge to 0, the response in Bohn (2005) rule would need to rise. According to the results, the required response would be 0.05 — a level US had implemented before the financial crisis. Implementing this positive response would result in higher $pb^{\#\dagger}_{t,5}$ and lower discount rate $\rho^{\#\dagger}_{t,5}$ compared to their counterparts under the original policy and $pb^{\dagger}_{t,5}$ was the main reason for the rise in the MTDD(t,5). Though ρ_t do not contribute much to this, it is noteworthy that implementing this response would result in higher $\gamma^{\#\dagger}_{t,5}$ compared to its counterparts under the original policy.

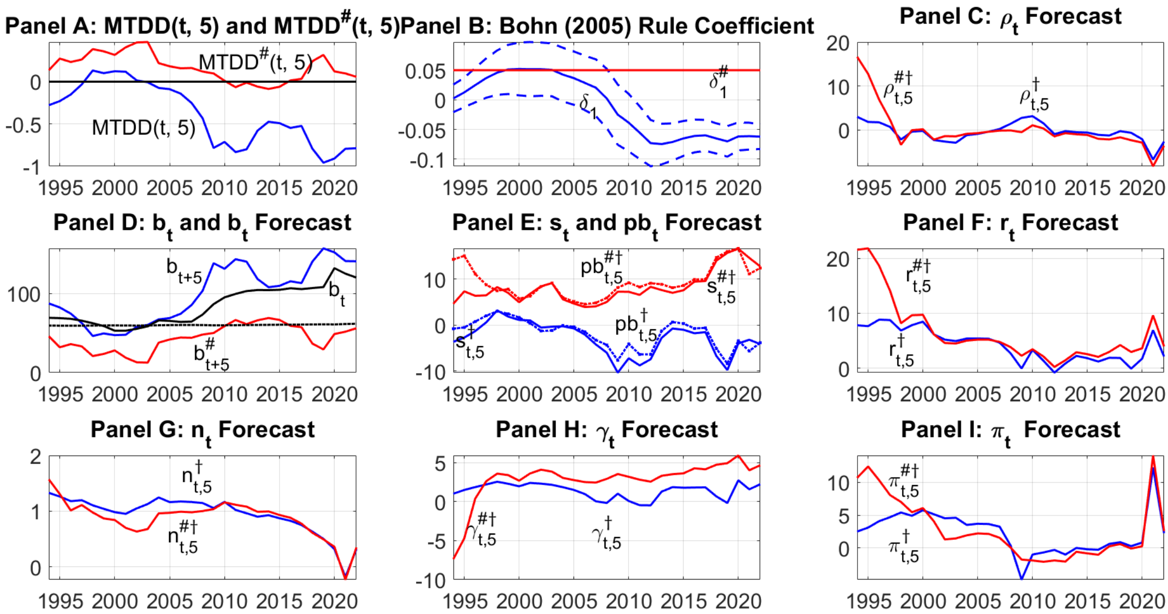

Figure 2.8: US MTDD(t,5), $MTDD^{\#}(t,5)$ and Their Components

Panel A shows MTDD(t,5) (blue line) and $MTDD^{\#}(t,5)$ (red line). Panel B shows the recursively estimated Bohn (2005) rule coefficients δ_1 (blue line), its 95% confidence interval (blue dashed line), and $\delta_1^{\#}$ conducted to make $MTDD^{\#}(t,5)$ closer to 0. Panel C to Panel I display the forecast of ρ_t, PVb_{t+j}, $PV(s_t)$, $PV(pb_t)$, r_t, n_t, γ_t, π_t, with superscript † meaning the average of the forecast based on the VAR model for the variables between t+1 period and t+5 period (blue line) and superscript #† the average of the forecast based on the PVAR model for the variables between t+1 period and t+5 period (red line).

In general, the results show that all major economies need to increase their response to increase in debt ratio. Moreover, the magnitude of the response are mostly lying within the previous response bands.

2.4.9 Stability Analysis

Using a VAR model for counterfactual analysis is subject to the Lucas (1976) critique: the effects of alternative polices may lack credibility if they fail to account fro changes in agents' expectations. Rudebusch (2005) noted, the parameters in empirical models implicitly reflect agents' expectations about policies but these expectations may not remain stable when the alternative policies were implemented. Nevertheless, the instability would manifest as parameter shifts within the model. To ensure parameters stability, I use a bootstrap Chow test to examine the structural stability of the VAR model.

To do a bootstrap Chow test in this chapter, I follow the method of Candelon and Lütkepohl (2001). I choose bootstrap Chow test because the sample is small, most just 50 observations and the normal Chow test would reject the stability very frequently. I did a sample split test since there is no significant break point and bootstrap the test for 4000 times. Most results show no significant instability, thereby reducing the significance of Lucas (1976) critique.

2.5 Conclusion

The empirical analysis in Chapter 1 shows that while the fiscal policies of G7 countries and China appear sustainable under Campbell (1987)'s present value budget constraint (PVBC) test, they do not exhibit a significant response to debt accumulation based on Bohn (2005) rule. This suggests that major economies are not proactively adjusting their fiscal stance. Additionally, Blanchard (1990) medium-term debt dynamics analysis indicates that all these countries follow relatively loose fiscal policies.

Building on this background, Chapter 2's contribution is to move the Bohn (2005) rule beyond a purely descriptive question of whether a debt response is present, and instead develop a quantifiable and comparable framework for assessing medium-term fiscal adjustment. Specifically, using the reduced-form VAR estimated in Chapter 1, I still conduct conditional forecasts in which a strengthened response of the primary balance to debt is imposed as a policy-rule counterfactual, just as Lenza et al. (2010) etc. By comparing the counterfactual outcome, $MTDD^{\#}$, with the MTDD under the realised scenario, the chapter quantifies the extent to which a stronger debt-responsive rule can improve medium-term debt dynamics. Unlike accounting-style indicators that merely infer how large a primary-balance gap would be required, this approach embeds the adjustment within the empirically estimated dynamic interactions in the data, thereby providing quantitative evidence that is closer to the way fiscal policy is implemented in practice.

I base on VAR model, the reduced-form VAR model in the last chapter, to analyse the Bohn (2005) rule to decrease Blanchard (1990) medium-term debt dynamics. Stock and Watson (2001) show that VAR model can be used to analyse policy. VAR model is good at forecasting and fitting the data. As to identification, I use PVAR identification method proposed by Polito and Wickens (2012b) to analyse Bohn (2005) rule that decreases Blanchard (1990) medium-term debt dynamics to avoid invalid identification. The results are displayed by comparing $MTDD^{\#}$ under conditional forecast and MTDD under real scenarios to show the better effect of Bohn (2005) rule on Blanchard (1990) medium-term debt dynamics comparing to the original one.

The results suggest that countries accustomed to loose fiscal policies should increase their responses and most of them only need to restore their response to debt. However, these conclusions remain subject to the Lucas (1976) critique, highlighting the need for further research on optimizing fiscal policy.

Chapter 2 uses an empirical model to characterise the response of the primary balance to debt and to quantify the medium-term fiscal adjustment required for debt stabilisation. Demographic change is therefore captured only implicitly through the data and the estimated coefficients, but a detailed discussion of ageing mechanisms and their policy implications is not the focus of that chapter. At the same time, tax-rate changes are typically slow-moving and politically constrained in the medium term, so relying on tax adjustments to deliver medium-term consolidation is often unrealistic. These considerations motivate Chapter 3 to adopt a longer-horizon perspective: over longer horizons, demographic ageing becomes a central driver of fiscal dynamics and the relevant policy trade-offs, making the feasibility and effects of tax-policy changes more meaningful to evaluate. Accordingly, Chapter 3 focuses on China and introduces the ageing process explicitly in a structural model to study its implications for fiscal sustainability and the design of tax-adjustment paths.

Chapter 3

Fiscal Sustainability: Evidence from an OLG Model

3.1 Introduction

China's debt level and its pace of increase in recent years have become a growing concern. China's general government gross debt rose from about 22.7% of GDP in 2000 to about 88.3% of GDP in 2024; in particular, over the last decade it more than doubled.[1] In Chapter 1, I estimate the Bohn (2005) fiscal reaction function and find that, for China, the primary balance response to debt is significantly negative, suggesting that fiscal policy is not mean-reverting with respect to debt. However, Bohn (2005) rule is backward looking and assumes that the fiscal policy has been constant and will remain the same. When broader horizon is taken and changing the tax rate is a sensible option, with a rational expectation model we can tell if fiscal policy is sustainable or not in China this ageing society. This long-run focus is well-motivated:

[1]Source: *General government gross debt (% of GDP)* for China from the International Monetary Fund World Economic Outlook (WEO) database (as disseminated via the Federal Reserve Bank of St. Louis FRED series "GGGDTACNA188N").

tax rates are difficult to change frequently in the short run, but they are a feasible and durable revenue-side instrument in long-run institutional and tax-system reforms.

This chapter makes two contributions. First, it explicitly recognizes that an increase of the labour tax rate can lead to either higher or lower tax revenue, depending on the initial tax rate and the responsiveness of taxpayers. This so-called Laffer effect has been extensively studied in the literature; see, for example, Davig et al. (2010), Trabandt and Uhlig (2011), and Polito and Wickens (2015). I show that, in a life-cycle setting, the Laffer effect determines an upper limit on a government's ability to raise debt through labour income taxation. Second, I compare the exogenous sustainable debt ratio, derived from Bohn (2005) rule, with the model-implied debt limits to examine whether the current fiscal policy would still be sustainable in the new steady state. The sustainable debt ratio is derived from combining the Bohn (2005) rule with the government budget constraint. I compare this ratio to the debt limits in both the 2020 and 2100 steady states. When the sustainable debt ratio falls below both limits, it suggests that the current fiscal policy is sustainable under demographic change. In addition, I calculate the effective consumption tax, effective labour tax, and effective capital tax for China using the method of Mendoza et al. (1994).

Chapter 3 adopts a dynamic general equilibrium (DGE) model, which explicitly incorporates rational expectations. And the model that simulate population ageing, the important risk that I pay attention to in this thesis, is overlapping generation (OLG) model. The model is developed along the lines of the large-scale OLG framework used by Auerbach et al. (1981), Auerbach and Kotlikoff (1987), and Heer, Polito, and Wickens (2023). This model includes overlapping generations of households, a competitive production sector and a government that finances consumption and transfers using distortionary labour, capital and consumption taxation, issues non-contingent debt, runs a pay-as-you-go pension system and conduct fiscal policy following Bohn (2005) rule. The OLG framework allows for the integration of survival probabilities and population growth rates. To also take population ageing into account, this chapter assumes that the economy is in the steady state in 2020 that is implied by the stationary population for the 2020 survival probabilities and population growth rate. Population then evolves and the economy achieves new steady state in 2100 with higher survival probabilities and lower population growth rate, implying that the population ages. This model combines the Bohn (2005) rule to examine fiscal sustainability for China.

My main conclusions are as follows: China's current fiscal policy remains sustainable, though this would need the change in fiscal environment. When the Laffer curve effect sets limits for debt, it indicates that the direct tax can no longer support the issuance of debt beyond the threshold. I refer to this threshold as the debt limit, which is shaped by the economic structure, fiscal policy design and demographic conditions. I compare the debt limit and the sustainable debt ratio derived from Bohn (2005) rule to examine fiscal sustainability for China and the result is that the debt limits in two steady states—before and after population ageing— are larger than the sustainable debt ratio, showing that the fiscal policy of China is sustainable. It is also worthy of notice that the space between the debt limits and the sustainable debt ratio decrease significantly from the steady state in 2020 to the one in 2100. Nevertheless, the labour tax rate would need to increase to meet the sustainable debt ratio implied by Bohn (2005) rule.

The structure of this chapter is as follows. Section 3.2 reviews the literature on population aging, fiscal policy sustainability, and related modeling approaches. Section 3.3 introduces the overlapping generations model used in this study, while Section 3.4 details its calibration. Section 3.5 presents the simulation results and discusses their policy implications. Finally, Section 3.6 summarizes the main findings and conclusions.

3.2 Literature Review

3.2.1 OLG Model for Demographic Structure Change

The overlapping generations (OLG) model is widely used for analysing fiscal change. (see the works of Auerbach and Kotlikoff (1987), De Nardi et al. (1999), Imrohoroğlu and Kitao (2012), Kitao (2014) and Heer et al. (2020)) due to its ability to capture intergenerational resource allocation. The OLG literature provides a standard framework for analysing long-run fiscal issues under demographic change: by modelling life-cycle households and intergenerational resource allocation, it makes the incidence of taxes, transfers and public debt explicit across age cohorts (e.g., Auerbach et al. (1981), Auerbach and Kotlikoff (1987), Heer, Polito, and Wickens (2023)). A recurring conclusion in quantitative OLG studies is that ageing systematically reshapes life-cycle profiles: as population weight shifts toward older cohorts, labour supply declines earlier and more steeply with age, while saving and consumption profiles adjust as households anticipate longer retirement and changes in after-tax returns —— together tightening the government's long-run financing problem. Building on this tradition, I model ageing through shifts in survival probabilities and population growth as in Heer et al. (2020), and embed a standard fiscal block with distortionary taxation, transfers, a pay-as-you-go pension system, and non-contingent government debt so that all experiments respect the government budget constraint. Consistent with the literature's emphasis on age profiles, my results also show that demographic change materially reallocates labour, saving, and consumption across cohorts and compresses fiscal space, making the sustainability requirement more demanding in the post-ageing steady state.

OLG model offers significant advantages. First, by explicitly distinguishing individuals by age, OLG models can naturally capture demographic dynamics such as birth, death, and ageing, allowing for a direct analysis of how changes in population structure affect the economy. Second, the OLG framework permits systematic differences across cohorts in terms of labour supply, savings, consumption, and tax burdens, enabling a more accurate assessment of the intergenerational distributional effects of fiscal policy. Moreover, when facing long-term structural shifts — such as declining birth rates or increasing life expectancy — OLG models demonstrate strong dynamic adjustment capabilities, capturing the path-dependent nature of economic transitions.

Many literature focuses on the sustainability of local government debt for China (Lu and Sun (2013); Zhang and Barnett (2014), Bai and Liu (2022)). Sun (2019) follows the method Ghosh et al. (2013) to build the fiscal space for China. Though Sun (2023) considered general government debt in his OLG model, the model combined with the growth-maximizing theory instead.[2] Nevertheless, he shows that the present fiscal policy for China is sustainable.

China is ageing rapidly and faces rising debt and social-security burdens, and as Zhai (Zhai) says that China-focused OLG studies quantify the long-run macroeconomic consequences of demographic change. Different from China, economies such as Japan and Korea are already highly visible ageing-related fiscal pressures. In recent OLG work for these countries typically models social insurance in detail and evaluates concrete reform packages within a general-equilibrium setting—for Japan, see İmrohoroğlu et al. (2019), and for Korea, see Baksa, Bonthuis, Guo, and Munkacsi (Baksa et al.). Therefore, this chapter focuses on the consequences as well.

3.2.2 Fiscal Sustainability

In DGE model, fiscal sustainability often requires the intertemporal government budget constraint (IGBC) to hold: (see for example Mendoza et al. (2014) and Heer

[2] Growth-maximizing theory assumes that the public capital can increase labour productivity and the marginal output of private capital so as to affect the total output (see Aschauer (1989, 2000)).

et al. (2020))

$$b_0 = \sum_{t=0}^{\infty} \left[\left(\prod_{i=0}^{t-1} \frac{1}{1+\rho_i} \right) pb_t \right].$$

For DGE model, government solvency is typically ensured by construction. In order to compute the solution, the transversality conditions should be imposed. As a result, explosive debt paths are ruled out ex ante, and "fiscal unsustainability" tends to manifest itself not as an observed debt explosion within the model, but as the absence of a stationary equilibrium / determinacy, or an unstable (non-convergent) solution. In this sense, the solvency condition is primarily a modelling restriction tied to stability and closure (see, e.g., Schmitt-Grohé and Uribe (2003)). This directly means that fiscal policy would always hold. This also implies a limitation: if stability fails, the model may simply reject the specification, without directly addressing how fiscal rules should be redesigned when the environment changes (e.g., under demographic transitions).

3.2.3 Laffer Curve Effect

Since in equilibrium-based DGE/OLG analyses the intertemporal government budget constraint and solvency conditions are typically imposed when computing steady states, the analysis often does not focus on generating (or "observing") explosive debt paths within the model. A common approach in this literature is therefore to characterise the government's fiscal capacity using the model-implied Laffer curve, and to evaluate fiscal space (or fiscal limits) as the gap between the maximum steady-state revenue attainable by varying a distortionary tax rate and the benchmark steady-state revenue under the calibrated tax system (see, e.g., Heer et al. (2020)). A growing body of work using infinitely-lived-agent models quantifies Laffer effects and fiscal space for the consolidated government budget (e.g., Davig et al. (2010), Trabandt and Uhlig (2011), Bi (2012), Ghosh et al. (2013), Polito and Wickens (2015)). This chapter complements that literature by employing an OLG model and focusing on the well-known fiscal sustainability condition in the Bohn (2005) rule: alongside fiscal space, it provides a policy-rule-based benchmark to assess whether the implied fiscal adjustment is consistent with the economy's fiscal capacity. Recent work has also incorporated Laffer effects in large-scale OLG models to study how fiscal constraints restrict the scope for tax progressivity and redistribution (Guner et al. (2016); Holter et al. (2019)), and to identify demographic thresholds beyond which income taxation can no longer fund general government spending (Heer et al. (2020)).

In these studies, the Laffer curve is typically computed under a closure in which one component of the government budget is treated as a residual so that the government budget constraint holds exactly in steady state. Drautzburg and Uhlig (2011) shows that alternative financing/closure assumptions (e.g., distortionary versus lump-sum instruments) can lead to materially different quantitative outcomes, especially at longer horizons. Accordingly, I keep the model structure and the set of endogenous variables unchanged across scenarios and adopt a consistent closure so that comparisons remain meaningful—an approach that is standard in policy DGE implementations (see, e.g., Albonico, Calès, Cardani, Croitorov, Ferroni, Giovannini, Hohberger, Pataracchia, Pericoli, Raciborski, Ratto, Roeger, and Vogel (Albonico et al.)).

3.3 Model

In this section, I introduce the environment of the OLG model, which includes the Bohn (2005) rule that the government should implement and the concept for the sustainable debt ratio to check the fiscal sustainability after the population ageing.

I adopt an OLG framework with exogenous long-run productivity growth to better match the trend-growing nature of modern economies. This setup provides a realistic macroeconomic environment in which wages, output and tax bases evolve over time, and allows fiscal and pension-related variables to be interpreted in standard ratio or per-effective-worker terms that are comparable to the data. Importantly, introducing trend growth improves the external validity of the ageing analysis without requiring the chapter to focus on growth-driven mechanisms.

3.3.1 Environment

This chapter uses a large-scale OLG model base on that of Heer et al. (2023) to study the impact of demographic transition. This OLG model is calibrated to closely match the overall structure of the Chinese economy, the pension system, and tax revenue. This model does not have the characteristics of cohort heterogeneity, and all members of a given cohort are equal. Under the pay-as-you-go system, contributions from the younger generation are distributed to current retirees, and the pension system does not incur debt. Under the pension system, the government collects pension contributions from the younger generation through labor taxes (denoted as tau_p) and pays pension benefits to the older generation (denoted as pen_t).

In this model, a period t represents 1 year. In every period $t \geq 0$ a new generation of households is born. The age of newborns is $s = 1$, which means the real-life age 20. All cohorts retire at the end of R=40 years old (corresponding to the actual age of 60 years old) and the maximum age is J = 70 years old (i.e., the actual age is 89 years old). The number of periods during retirement equals $J - R + 1 = 31$. Let N_t be the total population in period t, and $N_t(s)$ be the population of age s, so that the proportion of this age group in the total population is $\mu_t^s \equiv N_t(s)/N_t$. The population grows at rate $n_t = \frac{N_{t+1}}{N_t - 1}$ in each period. An s-year-old household in period t survives to the next period $(s+1)$ with a probability of $\phi_{t,s}$. According to the age settings in the model, $\phi_{t,0} = 1.0$ and $\phi_{t,J} = 0$.

Household

Each household consists of a single individual, who may be either working or retired. Households maximize the expected intertemporal lifetime utility in each period t:

$$U_t = \sum_{s=1}^{J} \beta^{s-1} \left(\prod_{j=1}^{s} \phi_{t+j-2,j-1} \right) \left(u\left(c_{t+s-1}^s, l_{t+s-1}^s\right) + v\left(g_{t+s-1}\right) \right), \tag{3.1}$$

where β is the discount factor, c_t^s is the household consumption at age s in period t, l_t^s is the labor supply of the household at age s in period t. During the working period, the labor supply of an s-year-old household is $l_t^s \geq 0$ for $s = 1, \ldots, R-1$, while during retirement, for $s = R, R+1, \ldots, J$, $l_t^s = 0$. $v(g_t)$ is the household utility from the government consumption.

The discount factor β measures the extent to which households tend to consume in early years rather than in later years; thus, a higher discount factor β indicates that households tend to consume in later years and save a higher proportion of their wages, which means a higher savings rate. $v(g_t)$ is assumed to be additive, which means that government consumption per capita g_t has no direct effect on household behaviour since it only affects behaviour indirectly through its effect on transfers and taxes. Therefore, in the utility optimization problem below, utility from the government consumption will no longer be considered.

The utility function of the household , which depends on consumption and labor, follows the specification used by Trabandt and Uhlig (2011):

$$u\left(c_{t+s-1}^s, l_{t+s-1}^s\right) = \frac{1}{1-\sigma}\left(\left(c_{t+s-1}^s\right)^{1-\sigma}\left[1-\kappa(1-\sigma)\left(l_{t+s-1}^s\right)^{1+1/v_1}\right]^{\sigma}-1\right), \qquad (3.2)$$

where ν_1 is the Frisch labor supply elasticity and σ denotes the inverse of the intertemporal substitution elasticity, and κ is a scaling parameter. The utility function takes this form so that it is consistent with long-term growth, see Trabandt and Uhlig (2011); it has a constant Frisch labor supply elasticity. The budget constraint of the household is formulated as

$$(1+\tau^c)c_t^s = x_t^s + (1+(1-\tau^K)r_t)\omega_t^s + tr_t - \omega_{t+1}^{s+1}. \qquad (3.3)$$

The Consumption of the household is taxed at rate τ^c, and the household also should pay income taxes on capital income at rate τ^K. Households in each period t receive transfer tr_t. ω_t^s represent the assets of the s-year-old household at the beginning of period t. ω_t^s consist of equity k_t^s and government bonds $b_t^s, \omega_t^s = k_t^s + b_t^s$. The model assumes that the household is born without assets and does not intend to leave bequests, which means that the household has no financial assets at the beginning and end of its lifetime, implying $\omega_t^1 = \omega_t^J = 0$. In equilibrium, the household is indifferent whether it holds assets in the form of physical capital or government bonds, since both produce the same (deterministic) after-tax returns. If this were a two-period overlapping generation model, the proportions of asset holdings would be identical at both the household and aggregate levels. However, this is not the case in a multi-period model, as discussed in this chapter. Therefore, without loss of generality, I assume that each household holds the same proportions of both assets. Net non-capital income x_t^s is expressed as

$$x_t^s = \begin{cases} (1-\tau_t^w - \tau_t^p)\, w_t \bar{y}^s A_t l_t^s & s = 1,\ldots,R-1, \\ \text{pen}_t & s = R,\ldots,J. \end{cases} \tag{3.4}$$

where $w_t \bar{y}^s A_t l_t^s$ is total pre-tax labor income, w_t is the wage rate, the age-efficiency factor $\bar{y}^s$, aggregate labor productivity A_t, and working hours l_t^s. $\bar{y}^s$ is the productivity of the s-year-old household, where the age-productivity profile $\{\bar{y}^s\}_{s=1}^{R-1}$ is a hump-shaped function. τ_t^w is personal income tax on wage income (wage-income tax rate). Households also pay contributions to the pension system at a tax rate of τ_t^p and when they retire, they receive pensions pen_t. τ_t^w and τ_t^p added together is labour tax rate $\tau_t^l = \tau_t^w + \tau_t^p$.

In each period t, the lifetime utility in equation (3.1) is maximized subject to the household budget constraint in equation (3.3) results in the equilibrium conditions for the optimal allocation of consumption, labour and assets:

$$\lambda_t^s (1+\tau^c) = (c_t^s)^{-\sigma} \left[1 - v_0(1-\sigma)\,(l_t^s)^{1+1/\nu_1}\right]^{\sigma}, \quad s = 1,\ldots,J \tag{3.5}$$

$$\lambda_t^s (1-\tau_t^w-\tau_t^p)\, \bar{y}^s A_t w_t = v_0 \sigma \left(1 + \frac{1}{v_1}\right) (c_t^s)^{1-\sigma} \left[1 - v_0(1-\sigma)\,(l_t^s)^{1+1/v_1}\right]^{\sigma-1} \cdot (l_t^s)^{1/\nu_1}, \quad s = 1,\ldots,R-1 \tag{3.6}$$

$$\lambda_t^s = \beta \phi_{t,s} \lambda_{t+1}^{s+1} \left[1 + \left(1-\tau_{t+1}^K\right) r_{t+1}\right], \quad s = 1,\ldots,J-1 \tag{3.7}$$

$$\lambda_t^s = \beta \phi_{t,s} \lambda_{t+1}^{s+1} (1 + r_{t+1}^B), \quad s = 1,\ldots,J-1, \tag{3.8}$$

where λ_t^s is the Lagrange multiplier of the household budget constraint in equation (3.3).

Firms

In this model, firms are assumed to operate in competitive goods and factor markets. They maximize profits:

$$\Pi_t = Y_t - r_t K_t - w_t A_t L_t - \delta K_t, \tag{3.9}$$

where capital depreciates at a rate of δ, and A_t grows at an exogenous rate of γ:

$$A_{t+1} = (1+\gamma)A_t. \tag{3.10}$$

Enterprises are assumed to operate under a standard Cobb–Douglas production technology, given by:

$$Y_t = K_t^{\alpha} \left(A_t L_t\right)^{1-\alpha}. \tag{3.11}$$

Since production is perfectly competitive, in equilibrium, the rewards of labour and capital are calculated according to their marginal products, that is:

$$r_t = \alpha K_t^{\alpha-1} \left(A_t L_t\right)^{1-\alpha} - \delta, \tag{3.12}$$

$$w_t = (1-\alpha) K_t^{\alpha} \left(A_t L_t\right)^{-\alpha}. \tag{3.13}$$

Government

The aggregate disbursement of the government includes public consumption G_t, transfer payments Tr_t and public debt interest $r_t^b B_t$, where r_t^b represents the interest that the government needs to pay for issuing bonds and B_t represents public debt. These government expenditures are paid by aggregate revenue: taxes T_t, new debt issued B_{t+1}, and bequests Beq_t charged to households that do not survive at the end of any given period t. Therefore, the government budget constraint in each period t is written as:

$$G_t + \mathrm{Tr}_t + \left(1 + r_t^B\right) B_t = B_{t+1} + T_t + Beq_t. \tag{3.14}$$

Following Kindermann and Krueger (2022), accidental bequests collected by the government include the after tax gross return on assets and are formulated as:

$$Beq_{t+1} = \sum_{s=1}^{J} (1 - \phi_{t,s}) N_t(s) \left(\left[1 + \left(1 - \tau_{t+1}^K\right) r_{t+1}\right] \omega_{t+1}^{s+1}\right). \tag{3.15}$$

T_t consists of taxes on consumption, interest income, and wage income:

$$T_t = \tau^c C_t + \tau_t^w w_t A_t L_t + \tau_t^K r_t \Omega_t. \tag{3.16}$$

Social Security

Social security institutions implement a balanced budget:

$$\tau_t^p w_t A_t L_t = \sum_{s=R}^{J} N_t(s) pen_t. \tag{3.17}$$

While labour tax rate τ^l is exogenously calibrated, τ^p as a part of τ^l is endogenously to satisfy the PAYG pension budget in steady state. τ^p absorbs the pension financing pressure generated by population ageing and the replacement-rate benefit rule, while τ^n adjusts residually so that the overall tax wedge faced by households does not mechanically drift with demographics.

In this model, I assume that the government budget constraint and social security constraint are both balanced. First, this assumption is meant to focus on the impact of the system on dynamic paths of economic variables, avoiding the complicated feedback from the extra policy response, such as tax rate change and transfer payment adjustment. This is to say that assuming a balanced budget constraint is to isolate the effect of the pension system itself. Ignoring other policy response is to focus on the laffer curve effect, instead of forecasting the real paths. Second, the reasonability of system design. Balancing the budget does have some realistic foundation in the running system, especially for the PAYG pension system. This pension system usually is designed to cover the expense of the retiree by the working age population payment. Even if there are occasional deficit or primary balance, they are balanced in the long term from the perspective of long-term design. Third, the requirement of steady state. From the perspective of theoretical stability, balanced budget assumption can avoid the problem of debt accumulation or the infinite growth of capital, making sure the existence or the robustness of the model.

Equilibrium Conditions

At the aggregate level, consumption, labour in terms of efficiency units, assets and transfers are determined as the sum of the corresponding individual variables, as indicated in equations (3.18)-(3.21):

$$C_t = \sum_{s=1}^{J} N_t(s) c_t^s, \tag{3.18}$$

$$L_t = \sum_{s=1}^{R-1} N_t(s) \bar{y}^s l_t^s, \tag{3.19}$$

$$\Omega_t = \sum_{s=1}^{J} N_t(s) \omega_t^s, \tag{3.20}$$

$$Tr_t = \sum_{s=1}^{J} N_t(s) tr_t^s. \tag{3.21}$$

The goods market equilibrium is then:

$$Y_t = C_t + G_t + K_{t+1} - (1-\delta)K_t. \tag{3.22}$$

whereas equilibrium in the capital market requires that aggregate assets purchased by the households are equal to the sum of aggregate capital and government bonds demanded by firms and the government, respectively:

$$\Omega_t = K_t + B_t. \tag{3.23}$$

The no-arbitrage condition implies that in equilibrium all assets pay the same after tax rate of return:

$$r_t^B = \left(1 - \tau_t^K\right) r_t. \tag{3.24}$$

3.3.2 Sustainable debt ratio

Mendoza and Ostry (2008) proposed the concept of a sustainable debt ratio. This is the mean debt–GDP ratio to which the economy converges in the long run. It is derived by using the government's debt dynamics and the Bohn (2005) rule.

The government budget identity (this is also called debt dynamics equation) is:

$$b_t = -pb_t + (1+\rho_t)\cdot b_{t-1}, \tag{3.25}$$

where pb_t is the primary balance as a proportion of GDP, b_t is the debt to GDP ratio, and b_{t-1} is the initial debt, the debt at the end of last period. This is a intra-temporal government budget constraint, which is also the non-linear difference equation in b_t.

Bohn (2005) rule is a systematic relationship between the primary balance ratio and the debt ratio:

$$pb_t = \delta_1 b_{t-1} + \delta_t + \varepsilon_t, \tag{3.26}$$

where $\delta_t = \delta_0 + \delta_2 \tilde{g}_t + \delta_3 \tilde{y}_t$, δ_0 is the constant, $\tilde{g}_t$ is the cyclical component of government expenditure, and $\tilde{y}_t$ is the cyclical component of real output and ϵ_t is a zero-mean error. When δ_1 is positive and significant, the data indicate that fiscal solvency holds.

If the δ_t and ρ_t are stationary, with $\bar{\delta}$ and $\bar{\rho}$ the intra-temporal government budget constraint in (3.25) and the Bohn (2005) rule in equation (3.26) yield the following equation for the long-run expected value for the public debt–GDP ratio:

$$E[b_t] = -(\delta_1 E[b_{t-1}] + \bar{\delta}) + (1 + \bar{\rho}) \cdot E[b_{t-1}]. \tag{3.27}$$

In the long-run, $E[b_t] = E[b_{t-1}]$:

$$E[b_t] = -(\delta_1 E[b_t] + \bar{\delta}) + (1 + \bar{\rho}) \cdot E[b_t]. \tag{3.28}$$

Equation (3.28) can be simplified as:

$$\begin{aligned} E[b_t] &= -\ (\delta_1 E[b_t] + \bar{\nu}) + (1 + \bar{\rho}) \cdot E[b_t] \\ (1 + \delta_1 - (1 + \bar{\rho}))E[b_t] &= -\ \bar{\delta} \\ (\delta_1 - \bar{\rho})E[b_t] &= -\ \bar{\delta} \\ E[b_t] &= \frac{\bar{\delta}}{\bar{\rho} - \delta_1}. \end{aligned} \tag{3.29}$$

This is a condition for computing sustainable debt ratios, defined as the mean debt–GDP ratio to which the economy converges in the long run.

3.4 Calibration

Parameter	Value Calibrated for China	Description
n_{2020}	0.43%	The steady-state population growth rate in 2020
n_{2100}	-1.19%	The steady-state population growth rate in 2100
α	0.34	Production elasticity of labour
δ	6.8%	Depreciation (annual)
L	0.35	Steady state average working hours
r^B	4.0%	After-tax return on government bond
γ	2%	Real Economic Growth Rate
σ	2.0	(Inverse of) Intertemporal elasticity of substitution
ν_1	0.50	Frisch labour elasticity
G/Y	23.65%	Government consumption / GDP
Tr/Y	2%	Government Transfer/GDP
$pen/(wA\bar{l})$	22.3%	Gross replacement rate of pension
τ^c	20%	Consumption tax rate
τ^K	37%	Capital tax rate
τ^L	13%	Labour tax rate
κ	5	Preference parameter: weight of labour
β	1.0078	Discount factor
B/Y	41%	Government debt/ GDP
τ^p	9%	Social security tax rate

Table 3.1: Calibration of the OLG model

The calibration of parameters in this study follows two main approaches: most are calibrated directly using external data sources, while the remainder are determined endogenously from the model's equilibrium conditions. For parameters calibrated with external data, the process draws not only on values commonly adopted in the related literature but also relies heavily on available economic and fiscal statistics. However, the scope of usable data is limited: the period from 2008 to 2019 is the only span that provides complete coverage of all required variables. Accordingly, the calibration relies on averages over this period to approximate the steady state.

Table 3.1 reports the parameter values calibrated from external sources.

3.4.1 Demographics

Newborns are assumed to have a real-life age of 20, corresponding to s=1, and live up to a real-life age of 89 , thus J=70. Retirement ages right now in China is 60 for male, 55 for female civil servants, and 50 for female workers. In this chapter, I follow Hsu et al. (2022) who use a similar OLG model to study the impact of population ageing on China and set the retirement age as 60, irrespective of gender. Therefore, I assume agents in the model retire at age R=41(real-life age of 60) and the retirement period is $J - R + 1 = 30$.

- Population growth rates $n_{2020} = 0.43\%$ in the steady state of 2020 is calculated as the average of the annual population growth rates, from 2000 to 2020, obtained from the UN (2023).

- I assume that the n_{2100} in the steady state of 2100 and $n_{2100} = -1.19\%$ is calculated as the average of the annual population growth rates from 2080 to 2100, obtained from UN (2023).

- In period t, the agent has a certain probability of surviving to period $t+1$. In period t, the survival probability of the 0-year-old household to survive until age 1 is 1 ($\phi_{t,0} = 1$), the survival probability of the J-year-old household to survive until next period $t+1$ is zero ($\phi_{t,J} = 0$) ; the survival probability of the s-year-old household to survive until age $s+1$, $\phi_{t,s}$, is non-zero and it is calculated using the following method. Survival probabilities $\phi_{t,s}$ are computed from annual data on life expectancy for both sexes combined (from age 20 to age 89) which are also obtained from UN (2023). The calibration employs the average survival probabilities from 1980 to 2020. Available data there include estimates of the annual life expectancy for age s ranging from 0 , i.e. a newly born, to age 100, from 1950 to 2100. Life expectancy for a s-year old agent in period t, LF_t^s, can be written as: $LF_t^s = \phi_{t,s}(5 + LF_{t+5}^{s+5})$. This means that the s-year old agent in period t can survive with probability $\phi_{t,s}$ to next group (5-year group) and enjoy the life expectancy when he/she is $(s+5)$-year old in next period, $t+5$ period. Thus the survival probability can derived with the data of life expectancy:

$$\phi_{t,s} = \frac{LF_t^s}{5 + LF_{t+1}^{s+1}}.$$

I then use the spline interpolation method to calculate the survival probability for each age and adopt the average survival probabilities for each age group between 2000 and 2020. For survival probability in the steady state in 2100, I use the average survival probability for each age group between 2080 and 2100.

3.4.2 Preferences

- Frisch labour elasticity has a core effect of labour tax rate when determining the degree of distortion to labour supply behaviour. It is an important parameter since it not only illustrates the degree of labour supply to the intertemporal change of real wage, but also the effect of tax policy and social welfare system on household labour supply. Following Song et al. (2015), who also employ an OLG model to examine pension system reforms for fiscal sustainability under population ageing and set the Frisch elasticity at 0.5, I adopt the same value in this study. This relatively low elasticity reflects the inelasticity of labour supply, implying that tax policy does not easily affect labour supply. Such a setup is consistent with the reality of China: households' labour supply response to tax changes is limited, and the distortionary effect of taxation is small due to labour market inelasticity, the inflexibility of working hours, and generally moderate marginal tax rates (see Wang et al. (2024)).

- In this model, intertemporal elasticity of substitution $1/\sigma$ is set as 0.5, implying that Chinese citizens' low intertemporal substitution preference, high risk aversion and insensitiveness to interest for saving. Zhang (2023) estimates that σ for China is 0.5. This value is empirically supported by other literature (such as Gu et al. (2013) who estimate IES as 0.3-0.5, emphasizing Chinese's the conservative saving tendency). This value is wildly used for modelling Chinese economic behaviour (see Liu et al. (2021)), thereby the choice combining the theoretical and empirical reasonability. This value is also widely adopted in policy-related macroeconomic modelling, thereby ensuring both empirical support and practical relevance.

3.4.3 Production

- The average working hour L is a very important parameter in the model. It is calculated using following formula:

$$\frac{\text{Hours Worked per Person } (15-64)}{\text{Hours to be Allocated Between Work and Leisure } * \text{ Days per calendar year 365}}, \tag{3.30}$$

  where the hours worked per person is the total annual hours worked by individuals aged between 15 and 64, the hours allocated between work and leisure are set at 14 hours as the common choice, and the number of days in a year is taken as 365, representing the total days in a calendar year. The data for total annual hours worked is sourced from the Total Economy Database from the Groningen Growth and Development Centre and the Conference Board. Based on this calculation, the normalized hours worked per person are determined to be 0.35.

In this chapter, parameters such as the capital share α and the depreciation rate δ are not directly matched to observed data, but instead are backed out from steady-state conditions.

- Specifically, the capital share parameter α is derived from the stationary versions of (3.7) and (3.12), resulting in:

$$\alpha = \frac{K}{Y}\left[\frac{1}{1-\tau^K}\left(\frac{(1+\gamma)^\sigma}{\beta}-1\right)+\delta\right]. \tag{3.31}$$

- The depreciation rate δ is obtained from the steady-state form of the capital accumulation equation:

$$\delta = \frac{\tilde{i}}{\tilde{k}} - \gamma - n - \gamma n, \tag{3.32}$$

  which corresponds to the stationary transformation of:

$$\tilde{i}t = (1+\gamma)(1+n)\tilde{k}t + 1 - (1-\delta)\tilde{k}_t.$$

  This approach ensures internal consistency among parameters and allows the model to generate well-defined steady-state outcomes under varying tax rates—providing a coherent basis for constructing Laffer curves.

- The economy growth rate γ in this chapter is calibrated as 2.0%. This setup is the same as the one set by Fernández-Villaverde et.al2023. They base on the Ramsey-Cass-Koopmans model, and set economy balanced growth rate for US as 0.02, and believe that China's economy growth rate should be the same after it finishing the catch-up. Hsu et.al (2022) also set their balanced economy growth rate the same value.

- Heer (2019) uses the method of Hanse (1993) to calibrate the age-profile productivity. However, the publicly available data does not include the average wage for different age groups. In this chapter, I use the estimates from Hsu et al. (2022). They utilize data from CHFS 2017 [3] to construct an average wage series

[3] The CHFS is a nationally representative survey in China on household finance conducted by Survey and Research Center of Southwestern University of Finance and Economics. This survey has been conducted four rounds in 2011, 2013, 2015, and 2017 respectively. The survey collects microlevel

for different age groups and then standardize this series using the average wage value of the age group 18. They derive an age-efficiency equation as follows:

$$\bar{y}^s = 1.000 + 0.06181 \times \mathrm{s} - 0.001299 \times s^2.$$

This equation gives the age-productivity profile a single-hump shape. For this analysis, I use the data starting from the age of 20.

3.4.4 Government

- After-tax return on government bond r^B is calibrated using the average long-term interest rate on government bond between 2008 and 2019 collected from IMF database.

- The government consumption to GDP ratio G/Y is the average value of the final consumption of the general government and the interest payment of the general government between 2008 and 2019. The final consumption expenditure of the general government is from the World Bank Databases and the interest payment of the general government is from the public finances in modern history of the IMF database.

- The transfer to GDP ratio, Tr/Y is 2%, calibrated by using the average value of transfer payment to GDP ratio between 2008 and 2019.

- The replacement rate is set as 22.3%, which is the average revenue(proportional to the revenue before retirement) of male from OECD (2023).

information on individual and household assets, including housing, business assets, financial assets, and other household assets. The survey also collects information on household age, income, expenditures, and socioeconomic status. Gan et al. (2014) provided an informative introduction on CHFS sampling design and survey methodology; Zhang (2016) introduced several studies based on CHFS.

I use the method of Mendoza et al. (1994) to calibrate tax rates used in this chapter: labour tax rate τ^l, capital tax rate τ^K, and consumption tax rate τ^c. Following their method, these tax rates are mechanically computed from a fixed set of OECD tax revenue categories and national accounts aggregates. However, the OECD database does not report all series required for China; I therefore supplement the missing series using the World Bank database. The data sources for all variables are reported in Table 3.3.

- The consumption tax rate is calculated using the following equation:

$$\tau^C = \frac{5110 + 5121}{C + G - GW - 5110 - 5121}, \tag{3.33}$$

  where 5110 is general taxes on goods and services, 5121 is the Excise taxes, C is private final consumption, G is government final consumption expenditure, and GW is compensation of employees paid by producers of government services.

- To calculate other effective tax rate, I need to calculate household's average tax rate on total income, τ^h:

$$\tau^h = \frac{1100}{OSPUE + PEI + W}, \tag{3.34}$$

  where 1100 is taxes on income, profits, and capital gains of individuals, OSPUE is the mixed income of the enterprises, PEI is the household's property and entrepreneurial income, W is the wages and salaries.

- The equation for labour tax rate τ^l is:

$$\tau^l = \frac{\tau^h * W + 2000 + 3000}{W + 2200}, \tag{3.35}$$

  where 2000 is the total social security contributions, 3000 is the taxes on payroll and workforce, and 2200 is the employer's social security contributions. Notice that $\tau^l = \tau^w + \tau^p$. Labour tax rate is composed by wage-income tax rate (τ^w) and social security tax rate (τ^p).

- Capital tax rate τ^k:

$$\tau^K = \frac{(\tau^h * (OSPUE + PEI) + 4400 + 4100 + 1200)}{OS}, \tag{3.36}$$

where 4400 is the taxes on financial and capital transactions, 4100 is recurrent taxes on immovable property, 1200 is the taxes on income, profits, and capital gains of corporations, and OS is the net operating surplus and mixed income of the economy.

Table 3.2 also reports other 4 parameters which are endogenously calibrated to ensure the equilibrium solution of the model. They are the closure variables in this model, ensuring that the constraints holds in the steady state. These parameters include utility parameter κ, which is set to yield average working hour $L = 0.35$. The discount factor β, which is set to yield real interest rate 4.0%. Though β is larger than unity, the effective discount factor $\frac{\beta}{(1+\gamma)^\sigma}\phi_{t,s}$ in the intertemporal Euler condition is still smaller than unity. In (3.8) The government debt ratio B/Y, set to ensure that the general government budget constraint in equation (3.14) is satisfied, and the tax rate τ_p, set to ensure that the social security budget in equation (3.17) is met.

Parameter	Value Calibrated for China	Description
κ	5	Preference parameter: weight of labour
β	1.0078	Discount factor
B/Y	41%	Government debt/ GDP
τ^p	9%	Social security tax rate

Table 3.2: Calibrated parameters from model equilibrium solution

Symbol	Original Series Name[a]	Resource
1100	Taxes on income, profits, and capital gains of individuals	OECD Revenue Statistics
1200	Taxes on income, profits, and capital gains of corporations	OECD Revenue Statistics
2000	Total social security contributions	OECD Revenue Statistics
2200	Employer's social security contributions	OECD Revenue Statistics
3000	Taxes on payroll and workforce	OECD Revenue Statistics
4100	Recurrent taxes on immovable property	OECD Revenue Statistics
4400	Taxes on financial and capital transactions	OECD Revenue Statistics
5110	General taxes on goods and services	OECD Revenue Statistics
5121	Excise taxes	OECD Revenue Statistics
C	Private final consumption	OECD National Accounts
G	general government final consumption expenditure	World Development Indicators Database
OS	Net operating surplus and mixed income of the economy	Calculation[b]
OSPUE	gross operating surplus and mixed income of the total economy [c]	OECD National Accounts
PEI	Household's property and entrepreneurial income	Calculation[d]
W	Received compensation of employees[e]	OECD National Accounts

Notes: [a] Numeric symbols are OECD database tax codes, while letter symbols follow the naming convention used in Mendoza et al. (1994). [b] Constructed series, defined as "OSPUE - consumption of fixed capital". Consumption of fixed capital is computed as (fixed capital consumption as a share of GNI) × GNI, with both inputs taken from the World Bank database; the construction follows Trabandt and Uhlig (2011). [c] Code in the database: SB2G_B3G. [d] Constructed series; computed as "property income receivable (SD4R) minus property income payable (SD4P)." Both components are taken from the OECD database; the construction follows Trabandt and Uhlig (2011). [e] Code in the database: SD1R.

Table 3.3: Tax Code for Calibration

3.5 Results

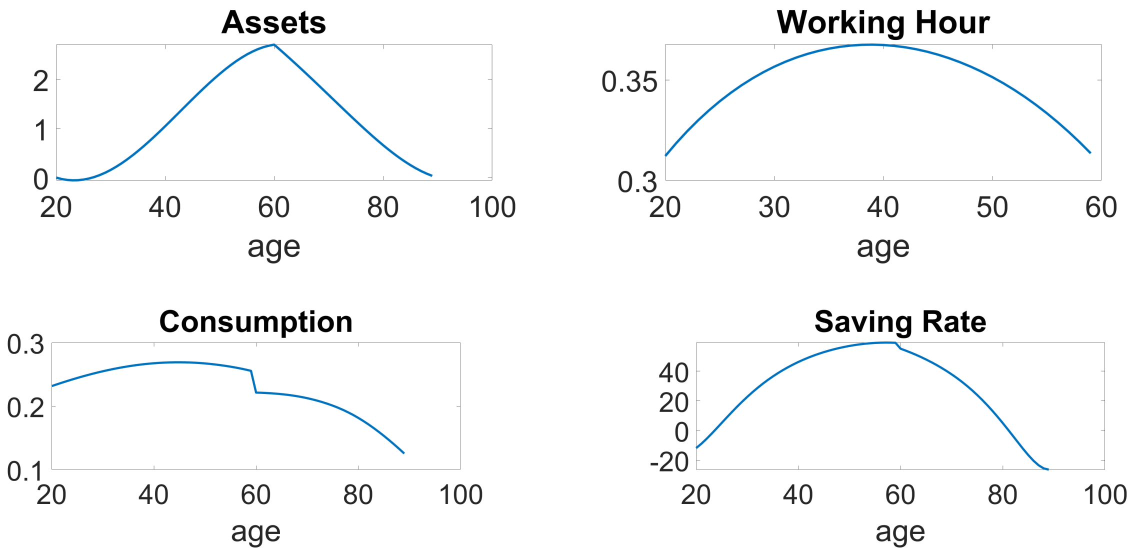

Figure 3.1: Age Profile in 2020 Equilibrium

In this chapter, I analyse the impact of labour tax rate policy on macroeconomic variables before and after population ageing. To do so, I assume that the economy is initially in a steady state in 2020. By 2100, the economy is in a new steady state driven by demographic changes, characterised by higher survival probabilities across cohorts and a lower population growth rate. In these two steady states, survival probabilities and population growth rate follow the United Nations medium-term projections, as outlined in UN (2023).

This section presents the age profiles under both the benchmark calibration and the ageing steady state. The benchmark age profiles are shown to ensure that the model replicates key empirical moments, while the ageing steady-state profiles illustrate how demographic shifts alter individual behaviour over the lifecycle. The analysis then focuses on the fiscal implications of these demographic changes, with particular attention to the Laffer curves for tax revenue and debt, which represent the core findings of this chapter.

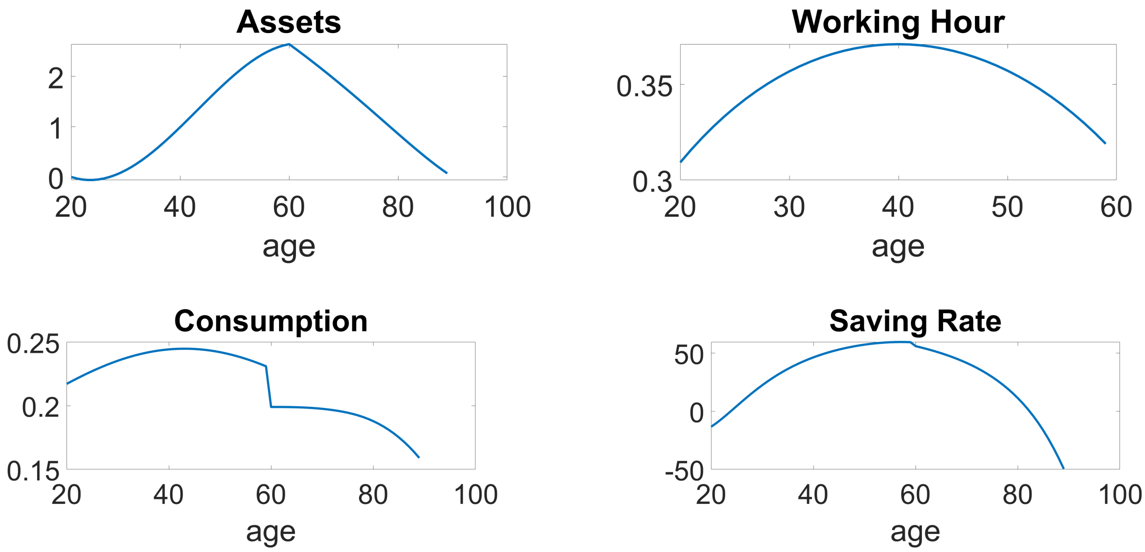

Figure 3.2: Age Profile in 2100 Equilibrium

3.5.1 Age Profiles

The age profiles in steady states 2020 and 2100 are illustrated in Figure 3.1 and 3.2. The age profiles in the 2020 steady state broadly align with empirical moments, capturing key patterns in wealth accumulation, labour supply, consumption, and savings over the lifecycle. In 2020, wealth accumulates over the working life and peaks just before retirement, at the real lifetime age of 60 — one period prior to retirement. Working hours follow a hump-shaped pattern, reaching their maximum at age 40. Throughout the working life, consumption increases but declines after retirement due to the cessation of labour income. Consumption does not exhibit smoothing, primarily due to the very low replacement rate. The saving rate is defined as the ratio of savings $y-(1+\tau^c)\,c$ to disposable income y, where income y consists of net labour and capital income plus transfer payments. During the working years, savings are positive, while in retirement, they become negative. The highest savings rate—reaching 59% of net income—is observed during the period of peak labour supply, around age 59. In

2100 steady state, the effects of population ageing are reflected in individual behaviour. Across all age groups, individuals in the new steady state work longer than those in 2020. Moreover, agents consume less. In the 2100 steady state, agents younger than 30 save at a higher rate than their counterparts in 2020, whereas agents older than 30 save at a lower rate. A higher survival probability strengthens the incentive to accumulate assets during working life, because agents expect a longer retirement period and therefore raise saving earlier in the life cycle.

3.5.2 Laffer Curve

In this chapter, I analyse the quantitative impact of unilateral changes in labour taxes on debt by constructing a Laffer curve. considered in the context of demographic change, direct taxation — particularly labour taxation — deserves special attention in the case of China. The relatively low effective labour tax rate in China suggests considerable scope for increases to support fiscal sustainability in the face of population ageing. However, any such increase must be carefully considered, as labour taxation is inherently distortionary to labour participation. Despite this drawback, labour tax is closely linked to government budget, thereby giving rise to the Laffer curve effect. When labour tax rate change, there exists an upper limit on the actual revenue under Laffer curve effect. In this model, we use debt as the residual instrument, whose level is determined residually from the government budget constraint so that the constraint holds in steady state. It also follows an inverted U-shaped trajectory, which is interpreted as the Laffer curve for debt.

In Figure 3.3, I plot the labour tax rate on the horizontal axis and solve for the intertemporal series of total tax revenue, while government purchases and transfer payments remain constant. Although the two steady states represent different demographic structures, the Laffer curves for tax revenues exhibit similar shapes. The tax revenues in 2100 is lower than the ones in 2020, indicating that for a given labour tax rate the tax revenue declines as the population ages. The initial labour tax rate is set at 13%, based on the benchmark calibration. In both steady states, the Laffer curve for tax revenue increases with the labour tax rate initially, but begins to decline beyond a certain threshold. Specifically, the Laffer curve for tax revenue peaks at a 70% labour tax rate in 2020, and at 69% in 2100.

To analyse the effect of a unilateral change in the labour tax rate on the debt-to-GDP ratio, I plot a diagram with the labour tax rate on the vertical axis and the corresponding debt ratio on the horizontal axis in 3.4. The Laffer curves for debts in two different steady states also exhibit similar shapes. This is because debt-GDP ratio is the residual instrument for the government budget constraint. This is because when we hold other variables constant, changes in the wage-income tax rate reshape labour-tax revenues, and—through the government budget constraint—translate into corresponding movements in the debt-to-GDP ratio. The debt ratios in 2100 is lower than the ones in 2020, implying that the 2020 economy can sustain a higher debt level at a lower fiscal cost. In the 2020 steady state, the debt ratio peaks at 359%, while in 2100 it peaks at 323%, suggesting that population ageing reduces fiscal space. The labour tax rate corresponding to the maximum debt ratio is 62% in 2020 and 64% in 2100. These results indicate that the debt limit in 2020 is higher than in 2100, and is achieve at a lower labour tax rate.

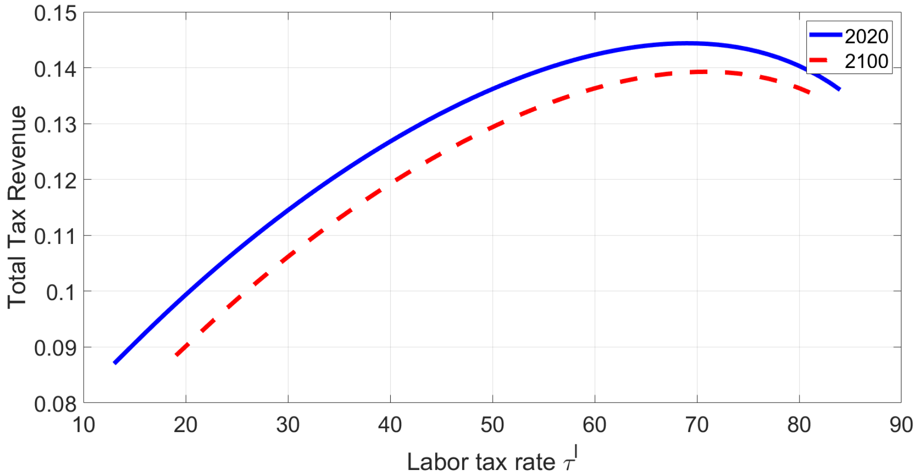

Figure 3.3: Laffer Curve on Total Tax Revenue From Labour, China, and 2100

The fiscal policy that China is implementing is estimated using the data of the sample period between 2008 and 2019. $\bar{\delta_1}$ is estimated as -0.11 and $\bar{\rho}$ is estimated as -0.0825. According to equation (3.29), the sustainable debt ratio for China is 72.38%, which is lower than the debt limit in both steady states. This shows thus that the present fiscal policy for China is sustainable. The same conclusion is also derived by Sun (2023).

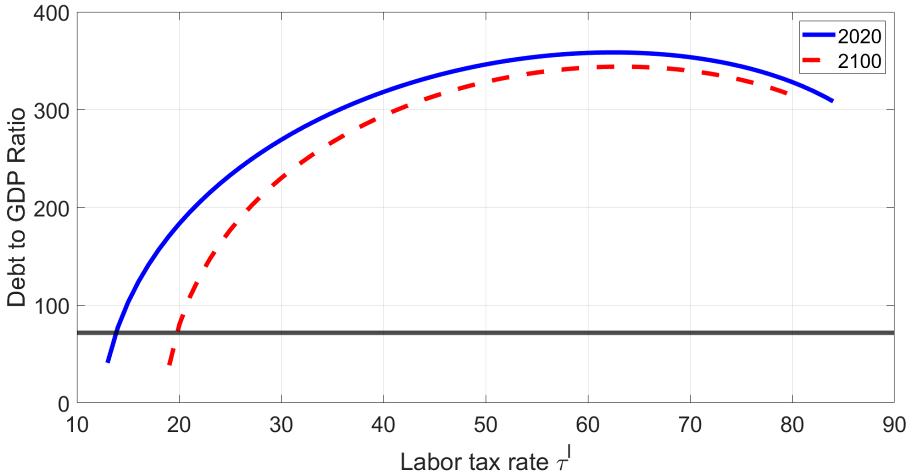

Figure 3.4: Laffer Curve for Debt Ratio, China, 2020 and 2100

Note: The Laffer curve illustrates the relationship between the debt ratio and tax rates, showing the peak response before a decline. The red dots in the diagram highlight the sustainable debt ratio and the corresponding labour tax rates in the steady states of 2020 and 2100.

3.6 Conclusion

In this chapter, I employ a large-scale OLG model to analyse fiscal sustainability in China in the context of population ageing. I consider two steady states—one in 2020 and the other in 2100—representing the economic environment before and after demographic transition. The chapter contributes by moving the discussion from an empirical diagnosis of sustainability to a structural, quantifiable assessment of whether sustainability remains feasible under ageing and what tax adjustment is required to maintain it. Specifically, the model delivers a non-linear Laffer effect for debt: when the labour tax rate is varied exogenously, the equilibrium debt position implied by the government budget constraint is not monotone, but instead rises initially and then declines, generating an inverted-U relationship between the labour tax rate and the debt-to-GDP ratio. Building on this mechanism, I compare an exogenous "sustainable debt ratio" implied by the Bohn (2005) rule with the model-implied debt limit in each steady state, thereby assessing whether China's current fiscal behaviour remains inside the feasible debt space once ageing is explicitly incorporated. I then embed the Bohn (2005) rule within the OLG framework to characterise how the required fiscal response ——i.e., the strength of the primary —balance reaction to debt needed for feasibility—changes as the demographic structure shifts from 2020 to 2100.

The results indicate that China's current fiscal policy remains sustainable. By exogenously increasing the labour tax rate, I construct Laffer curves for debt. For any given labour tax rate, the debt ratios in 2100 are lower than those in 2020, implying that the 2020 economy can sustain a higher debt level at a lower fiscal cost. The sustainable debt ratio for China is estimated as 72.38%, which is significantly below the debt limits identified in both steady states. This indicates that China's current fiscal policy remains sustainable and this conclusion is consistent with Sun (2023).

3.7 Stationary Equilibrium

The OLG model described in Section 3.3 is nonstationary because the aggregate technology A_t and the population N_t grow over time at the rates γ and n, respectively. Consequently, to derive an equilibrium solution, it is necessary to rescale individual variables by the aggregate technology and aggregate variables by both the aggregate technology and the size of the population. It is then necessary to rewrite all equilibrium conditions in terms of the rescaled variables. Individual variables are rescaled using $\tilde{x}_t^s \equiv x_t^s/A_t$, for $x_t^s = c_t^s, y_t^s, k_t^s, \omega_t^s, b_t^s,\ tr_t^s, \tilde{pen}_t \equiv pen_t/A_t$ and $\widetilde{\lambda}_t \equiv \lambda_t/A_t^{-\sigma}$. Similarly, aggregate variables are rescaled using $\widetilde{X}_t \equiv X_t/\left(A_t N_t\right)$, where $X_t = Y_t, C_t, K_t, \Omega_t, B_t, G_t, Tr_t, Beq_t, Tax_t$, and $\widetilde{L}_t \equiv L_t/N_t$. Table 3.4 and 3.5 presents the normalised counterparts of all equilibrium conditions in equations (3.3)-(3.8) and (3.12)-(3.23).

Stationary counterpart	**Eq. No**
$(1+\tau_t^c)\tilde{c}_t^s = \tilde{x}_t^s + \left[1+(1-\tau_t^K)r_t\right]\tilde{\omega}_t^s - (1+\gamma)\tilde{\omega}_{t+1}^{s+1}$	(3.3)
$\tilde{x}_t^s = \begin{cases} (1-\tau_t^w-\tau_t^p)w_t\bar{y}^s l_t^s, & s=1,\dots,R-1 \\ \tilde{\text{pen}}_t, & s=J \end{cases}$	(3.4)
$\tilde{\lambda}_t^s(1+\tau^c) = (\tilde{c}_t^s)^{-\sigma}\left[1-\kappa(1-\sigma)(l_t^s)^{1+1/\nu_1}\right]^{\sigma},$ $s=1,\dots,J$	(3.5)
$\tilde{\lambda}_t^s(1-\tau_t^w-\tau_t^p)\bar{y}^s w_t = \kappa\sigma(1+\frac{1}{\nu_1})(\tilde{c}_t^s)^{1-\sigma}$ $\times\left[1-\kappa(1-\sigma)(l_t^s)^{1+1/\nu_1}\right]^{\sigma-1}(l_t^s)^{1/\nu_1}, \quad s=1,\dots,R-1$	(3.6)
$(1+\gamma)^{\sigma}\tilde{\lambda}_t^s = \beta\phi_{t,s}\tilde{\lambda}_{t+1}^{s+1}\left[1+(1-\tau_{t+1}^K)r_{t+1}\right], \quad s = 1,\dots,J-1$	(3.7)
$(1+\gamma)^{\sigma}\tilde{\lambda}_t^s = \beta\phi_{t,s}\tilde{\lambda}_{t+1}^{s+1}(1+r_{t+1}^B), \quad s=1,\dots,J-1$	(3.8)

Table 3.4: Stationary counterparts of the model equilibrium conditions (right column)

Stationary counterpart	**Eq. No**
$r_t = \alpha \tilde{k}_t^{\alpha-1} \tilde{L}_t^{1-\alpha} - \delta$	(3.12)
$w_t = (1-\alpha)\tilde{k}_t^{\alpha} \tilde{L}_t^{-\alpha}$	(3.13)
$\tilde{g}_t + \tilde{tr}_t + (1+r_t)\tilde{b}_t = (1+\gamma)(1+n)\tilde{b}_{t+1} + \tilde{tax}_t + \tilde{beq}_t$	(3.14)
$(1+n)\tilde{beq}_{t+1} = \sum_{s=1}^{J} \mu_t^s (1-\phi_t^s) \left[1 + \left(1-\tau_{t+1}^k\right) r_{t+1}\right] \tilde{\omega}_{t+1}^{s+1}$	(3.15)
$\tilde{tax}_t = \tau_t^c \tilde{c}_t + \tau_t^w w_t \tilde{L}_t + \tau_t^k (r_t - \delta) \tilde{\omega}_t$	(3.16)
$\tau_t^p w_t \tilde{L}_t = \sum_{s=\mathrm{R}}^{J} \mu_t^s \tilde{pen}_t$	(3.17)
$\tilde{c}_t = \sum_{s=1}^{J} \mu_t^s \tilde{c}_t^s, \quad \tilde{L}_t = \sum_{s=1}^{J} \mu_t^s \bar{y}^s \tilde{l}_t^s,$ $\tilde{\omega}_t = \sum_{s=1}^{J} \mu_t^s \tilde{\omega}_t^s, \quad \tilde{tr}_t = \sum_{s=1}^{J} \mu_t^s \tilde{tr}_t^s$	(3.18)- (3.21)
$\tilde{y}_t = \tilde{c}_t + \tilde{g}_t + (1+\gamma)(1+n)\tilde{k}_{t+1} - (1-\delta)\tilde{k}_t$	(3.22)
$\tilde{\omega}_t = \tilde{k}_t + \tilde{b}_t$	(3.23)

Table 3.5: Stationary counterparts (left column) of the OLG model equilibrium conditions (right column) (Continued)

3.8 Numerical Algorithm

I solve a non-linear equations problem in 113 variables consisting of the 69 individual asset levels, $\tilde{\omega}^s = \tilde{k}^s + \tilde{b}^s, s = 1, \ldots, 70$, (with $\tilde{\omega}^1 \equiv 0$), the 39 individual labor supplies, $l^s, s = 1, \ldots, 39$, and the aggregate variables, $\tilde{k}, \tilde{L}, \tilde{\omega}, \tau^p$, and $\tilde{b}$.

The system of non-linear equations consists of the 108 first-order conditions of the household (the 69 Euler conditions and the 39 first-order conditions of the household with respect to the labor supply) as presented in (3.6) and (3.7) (after the substitution of $\tilde{\lambda}_t^s$ from (3.5) and the following 5 aggregate equilibrium conditions: government budget constraint in (3.14), social security budget constraint in (3.17), total labour equation in (3.19), total asset equation in (3.20), and capital market equilibrium in (3.23).

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Appendix A

Appendix to Chapter 1

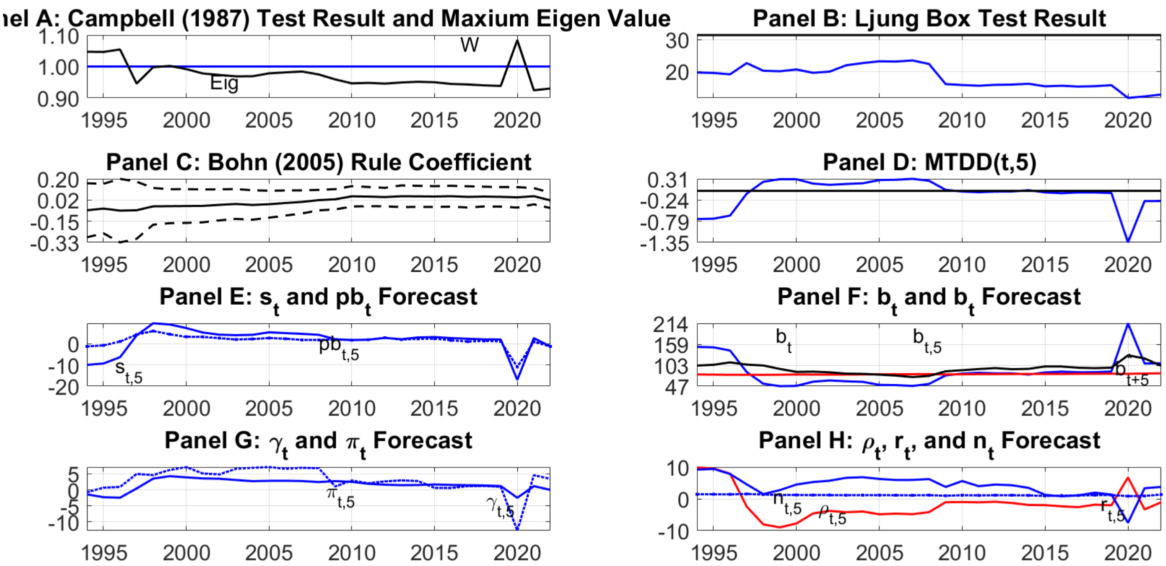

Figure A.1: Canada (1970-2022) Robustness Check.

Panel A presents the p-values from the recursive Campbell (1987) test result (W, blue line) and the maximum eigenvalue (Eig, black line) in the model. Panel B shows the Ljung box test results (blue line) and the corresponding critical values (black line). Panel C depicts the estimated response coefficient, δ_b in equation (1.39), based on Bohn (2005) (solid black line) along with its 95% confidence interval (dashed black lines). Panel D illustrates the MTDD(t,5) (blue line) and the zero-dynamics benchmark (black line) where the debt ratio target is the average debt ratio between 1998 and 2007. Panel E-H shows the components of MTDD(t,5).

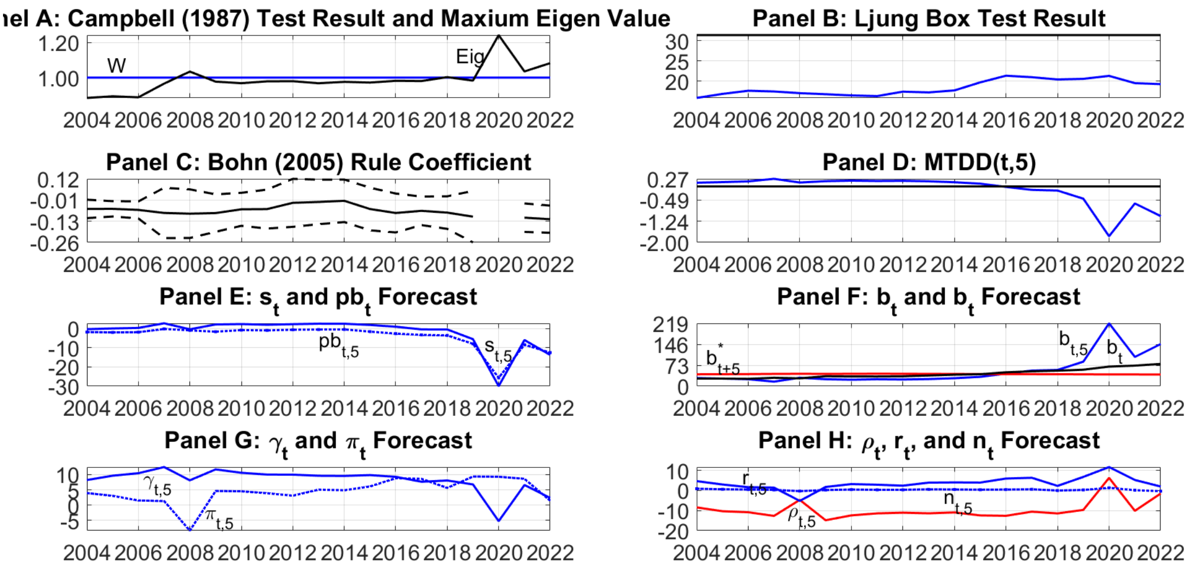

Figure A.2: China (1980-2022) Robustness Check.

Panel A presents the p-values from the recursive Campbell (1987) test result (W, blue line) and the maximum eigenvalue (Eig, black line) in the model. Panel B shows the Ljung box test results (blue line) and the corresponding critical values (black line). Panel C depicts the estimated response coefficient, δ_b in equation (1.39), based on Bohn (2005) (solid black line) along with its 95% confidence interval (dashed black lines). Panel D illustrates the MTDD(t,5) (blue line) and the zero-dynamics benchmark (black line) where the debt ratio target is the average debt ratio between 1998 and 2007. Panel E-H shows the components of MTDD(t,5).

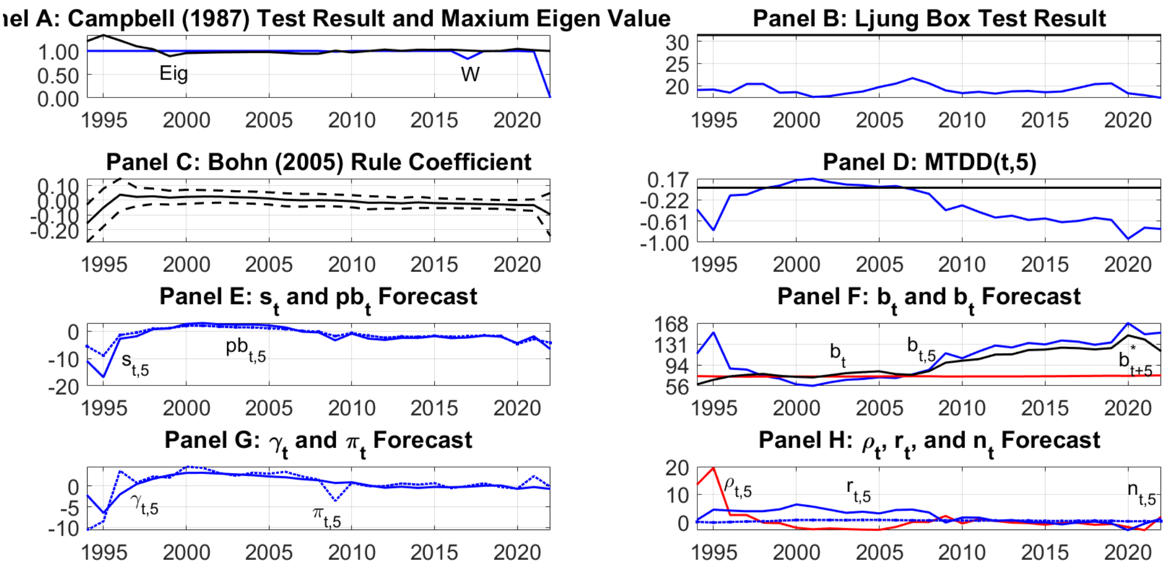

Figure A.3: France (1970-2022) Robustness Check.

Panel A presents the p-values from the recursive Campbell (1987) test result (W, blue line) and the maximum eigenvalue (Eig, black line) in the model. Panel B shows the Ljung box test results (blue line) and the corresponding critical values (black line). Panel C depicts the estimated response coefficient, δ_b in equation (1.39), based on Bohn (2005) (solid black line) along with its 95% confidence interval (dashed black lines). Panel D illustrates the MTDD(t,5) (blue line) and the zero-dynamics benchmark (black line) where the debt ratio target is the average debt ratio between 1998 and 2007. Panel E-H shows the components of MTDD(t,5).

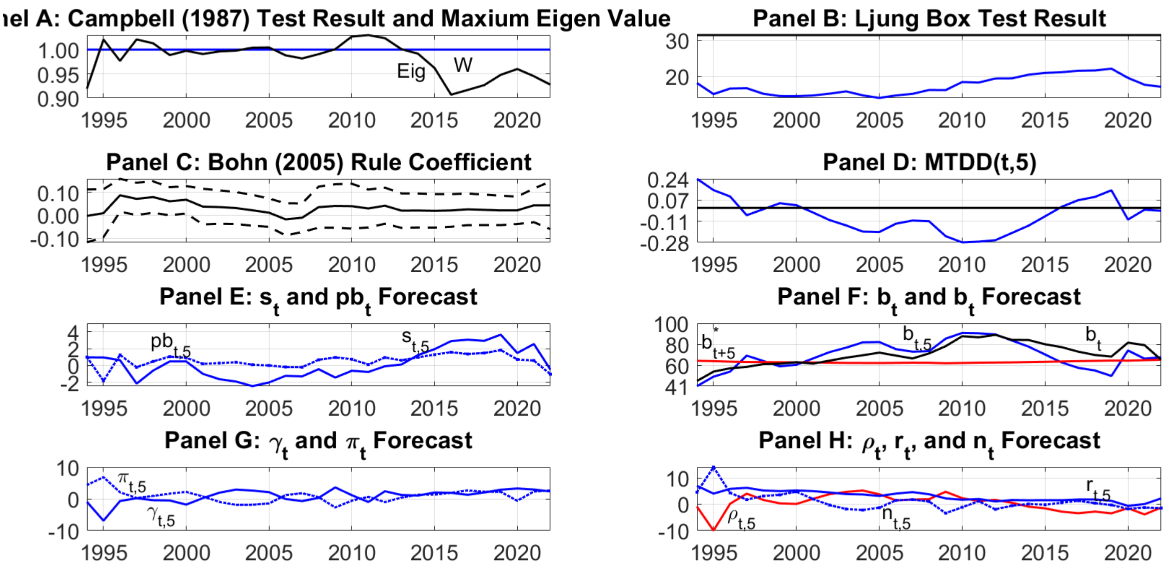

Figure A.4: Germany (1970-2022) Robustness Check.

Panel A presents the p-values from the recursive Campbell (1987) test result (W, blue line) and the maximum eigenvalue (Eig, black line) in the model. Panel B shows the Ljung box test results (blue line) and the corresponding critical values (black line). Panel C depicts the estimated response coefficient, δ_b in equation (1.39), based on Bohn (2005) (solid black line) along with its 95% confidence interval (dashed black lines). Panel D illustrates the MTDD(t,5) (blue line) and the zero-dynamics benchmark (black line) where the debt ratio target is the average debt ratio between 1998 and 2007. Panel E-H shows the components of MTDD(t,5).

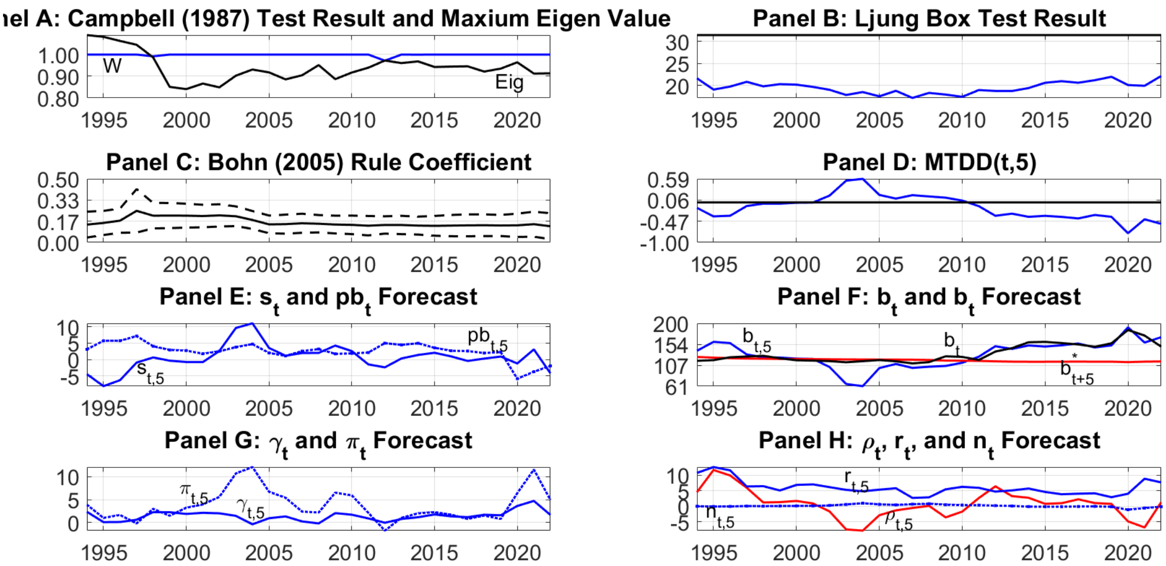

Figure A.5: Italy (1970-2022) Robustness Check.

Panel A presents the p-values from the recursive Campbell (1987) test result (W, blue line) and the maximum eigenvalue (Eig, black line) in the model. Panel B shows the Ljung box test results (blue line) and the corresponding critical values (black line). Panel C depicts the estimated response coefficient, δ_b in equation (1.39), based on Bohn (2005) (solid black line) along with its 95% confidence interval (dashed black lines). Panel D illustrates the MTDD(t,5) (blue line) and the zero-dynamics benchmark (black line) where the debt ratio target is the average debt ratio between 1998 and 2007. Panel E-H shows the components of MTDD(t,5).

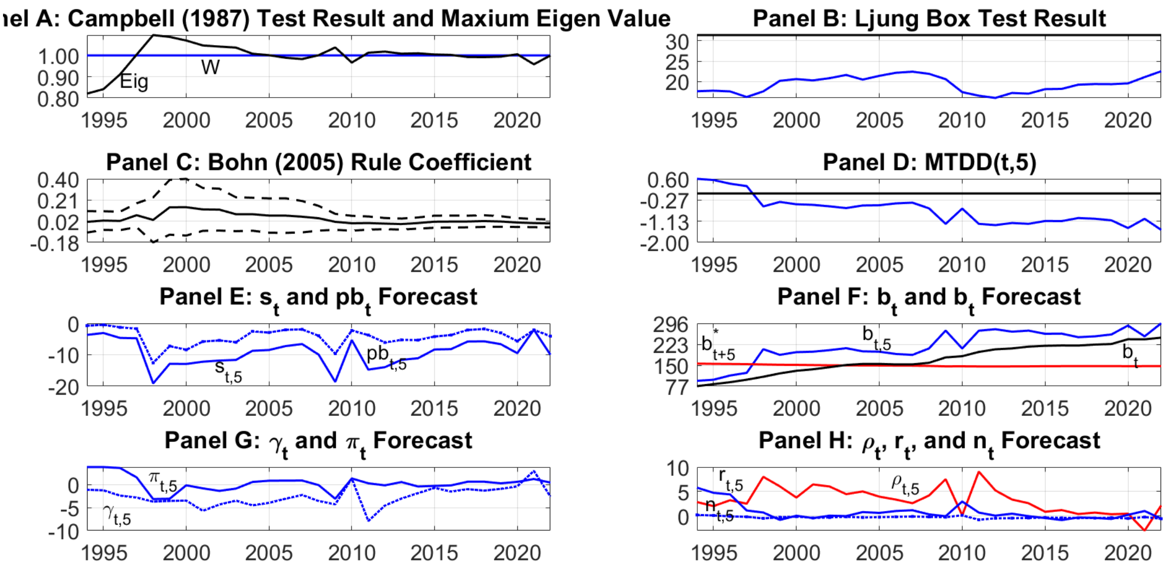

Figure A.6: Japan (1970-2022) Robustness Check.

Panel A presents the p-values from the recursive Campbell (1987) test result (W, blue line) and the maximum eigenvalue (Eig, black line) in the model. Panel B shows the Ljung box test results (blue line) and the corresponding critical values (black line). Panel C depicts the estimated response coefficient, δ_b in equation (1.39), based on Bohn (2005) (solid black line) along with its 95% confidence interval (dashed black lines). Panel D illustrates the MTDD(t,5) (blue line) and the zero-dynamics benchmark (black line) where the debt ratio target is the average debt ratio between 1998 and 2007. Panel E-H shows the components of MTDD(t,5).

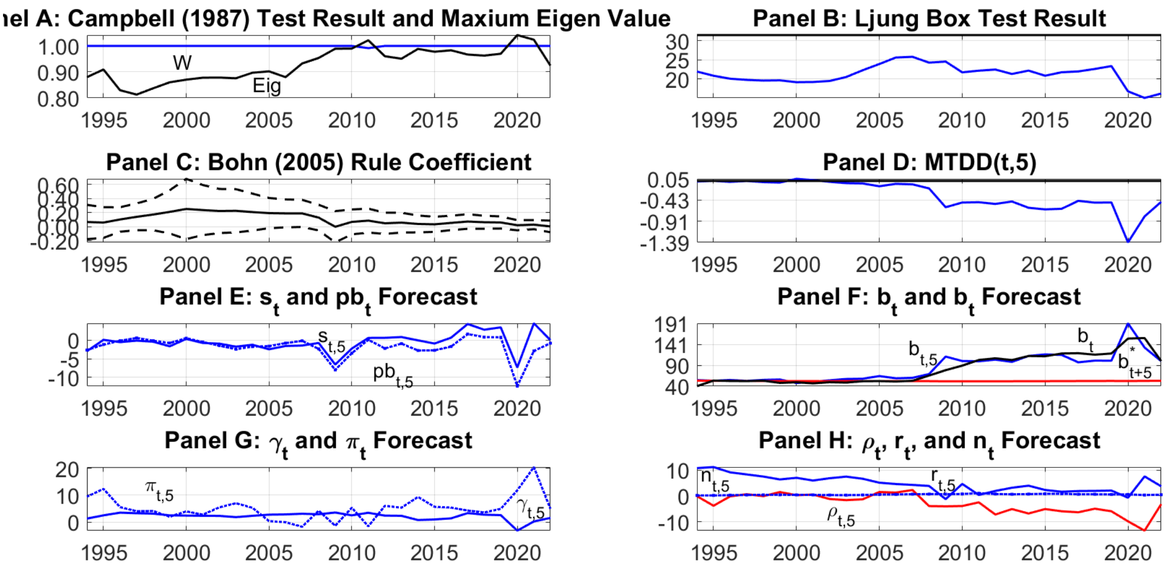

Figure A.7: UK (1970-2022) Robustness Check.

Panel A presents the p-values from the recursive Campbell (1987) test result (W, blue line) and the maximum eigenvalue (Eig, black line) in the model. Panel B shows the Ljung box test results (blue line) and the corresponding critical values (black line). Panel C depicts the estimated response coefficient, δ_b in equation (1.39), based on Bohn (2005) (solid black line) along with its 95% confidence interval (dashed black lines). Panel D illustrates the MTDD(t,5) (blue line) and the zero-dynamics benchmark (black line) where the debt ratio target is the average debt ratio between 1998 and 2007. Panel E-H shows the components of MTDD(t,5).

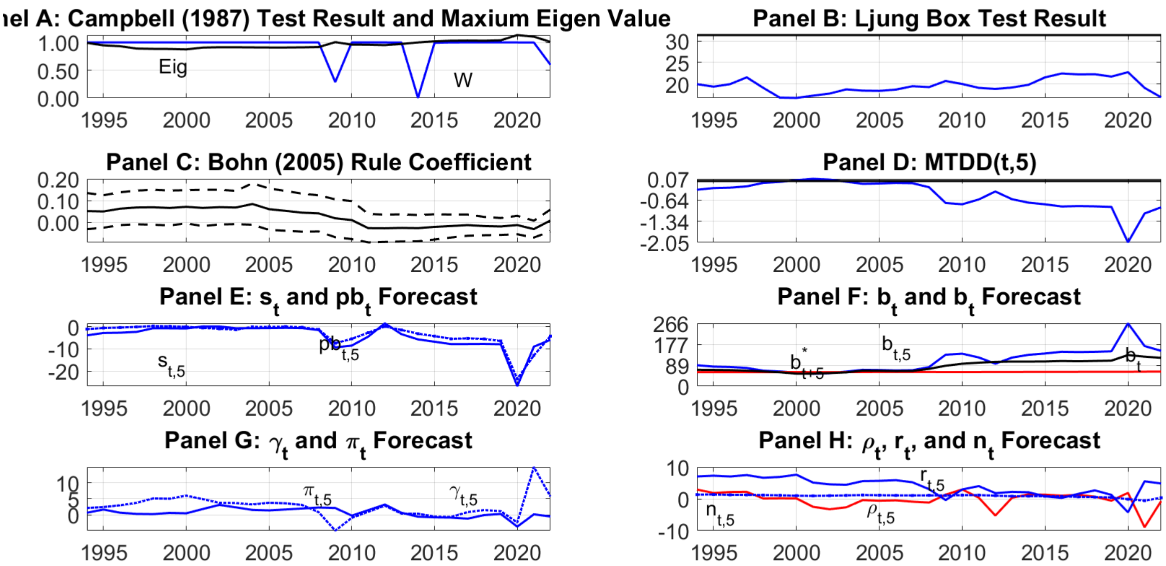

Figure A.8: US (1970-2022) Robustness Check.

Panel A presents the p-values from the recursive Campbell (1987) test result (W, blue line) and the maximum eigenvalue (Eig, black line) in the model. Panel B shows the Ljung box test results (blue line) and the corresponding critical values (black line). Panel C depicts the estimated response coefficient, δ_b in equation (1.39), based on Bohn (2005) (solid black line) along with its 95% confidence interval (dashed black lines). Panel D illustrates the MTDD(t,5) (blue line) and the zero-dynamics benchmark (black line) where the debt ratio target is the average debt ratio between 1998 and 2007. Panel E-H shows the components of MTDD(t,5).