Three Essays on Economics of Two-Sided Markets

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Abstract

This thesis analyzes economic issues arising in two-sided markets.

Chapter 1 studies merger issues on two-sided markets where platforms are located in a modified city circular market. Under non-localized competition, platforms on the circle and platform in the center choose pricing strategies on the buyer side and seller side to maximize its profits. Compared with premerger stage, the equilibrium results at the post merger stage are differentiated with the total number of platforms on the circle. In a market with $N+1$ platforms ($N>2$), as N increases, any platform on the platform will increase its incentive to be merged, while the platform in the center will decrease its incentive to merge.

Chapter 2 studies the privacy concern issues on a monopoly social media platform where a user discloses heterogeneous types of personal information according to his privacy trade-off under each type of information on the user side. In equilibrium, the user behaves more cautiously in disclosing information about revealing his personal identities, while he is more naive in disclosing information about revealing his shopping preferences. When the platform collects a positive amount of information of each type, in equilibrium, it will charge perfect price discrimination on the advertiser side.

Chapter 3 studies the advertising issues on a duopoly two-sided market, where two symmetric platforms are located at two endpoints of a Hotelling line and compete for buyers and sellers, who are distributed on the line. The seller's optimal advertising choice is independent of the total size of the buyer side and is only determined by how much advertising fees the platform is willing to charge on each unit of ad. Additionally, platform's maximized profits imply that a higher investment in advertising technology may not always increase the platform's profitability.

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Declaration

I declare that this thesis is a presentation of original work and I am the sole author. This work has not previously been presented for a degree or other qualification at this University or elsewhere. All sources are acknowledged as references.

Introduction

With the development of digital technologies, two-sided markets become increasingly popular in the field of economic research. In two-sided markets, two group of agents are connected via an intermediary. For example, buyers and sellers can make transactions through E-commerce platforms; Internet users receive advertising content through social media platforms. It is interesting to note that, when different groups of agents interact on a platform, an agent on one side will generate cross-group externalities, which depends on the total size of the other side. The cross-group externalities can be a typical characteristic of two-sided markets, since it can affect both platform and agent's strategy decisions. Another characteristic is the platform's pricing pattern on each side. Platform can either charge a fixed price or charge a transaction-based price on each side of the market. For example, media platforms usually charge a fixed subscription fees to attract viewers, while credit card systems may charge a transaction-based fees on their service. Furthermore, the platform may charge asymmetric fees on each side. For example, users usually do not need to pay registration fees to search an item on a E-commerce platform, while sellers have to pay registration fees to be served on the platform. The different way of setting the pricing may also affect the degree of agents' cross-group externalities enjoyed. The last worth-mentioning characteristic of two-sided market is each agent can choose to join either a single platform or multi platforms. The agent's entry decision can significantly affect the way of platform competing for market shares under different scenarios.

This thesis is inspired by the main characteristics of two-sided markets as well as real-world issues. Although each essay considers a specific context, the research ideas of each topic are interrelated. From the perspective of market type, Chapter 1 considers both oligopoly and competitive market, Chapter 2 considers a monopoly platform, and Chapter 3 considers a duopoly market with two symmetric platforms. Specifically, both Chapter 1 and Chapter 3 involves a specific spatial city model setting. From the perspective of platform's decision strategy, in each chapter, platform chooses a specific pricing strategy to maximize its profits. Specifically, both Chapter 2 and Chapter 3 charges asymmetric pricing strategy on each side. Additionally, the model work of each chapter can be applied to explain main issues of online platforms in today's world.

The research ideas of Chapter 1 are inspired by merger cases of Google in recent years. As a giant searching platform, Google has acquired more than 100 platforms in the past twenty years. It is notable that in some cases, the merger does not increase the pricing significantly, which is not consistent with the situation in single-sided market. Also, the merged companies are either startups or developing own brand loyalties. Therefore, the incentive of Google company to merge may not be consistent either. Then it is interesting to discuss the merger effects on two-sided markets in a theoretical way. Thus, Chapter 1 studies a city circular market with a number of platforms, where one platform is in the center of the circle, and other platforms are located around the circle. This modified circular model will then involve both localised competition between platforms on the circle and non-localised competition between platform on the circle and platform in the center. Under each type of competition, platforms choose its pricing strategy to compete for two groups of agents in the market, buyers and sellers. The analysis is based on comparing a benchmark of a three-platform market and a competitive market with more than two platforms on the circle. The results highlight that the merger effect on two-sided markets differentiated with the total number of platforms in the market. In a three-platform model, the merger strategy will be profitable for any platform as if it does not generate any market shares without merger. In contrast, in a market with more than two platforms on the circle, as the total number of platforms on the circle increase, any platform on the circle will have more incentives to be merged than the platform in the center. The equilibrium results also imply how cross-group externalities significantly internalise the platform's pricing decision.

The research ideas of Chapter 2 are inspired by user's privacy concern on social media platforms. Due to the advancement of information technology, the online platform become readily to collect and process user's information. On one hand, analysing data can generate more targeted service and social media interacting experience. On the other hand, collecting too much information may arise user's concern about privacy risks, such as cybersecurity attack and data breach. However, in the real world, users sometimes still disclose too much personal information even they believe they care about protecting their privacy. It is then interesting to discuss how user's privacy concern on social media platform can affect their information disclosure strategy. Additionally, whether platform will make more efforts on holding and protecting user's collected information. Thus, Chapter 2 studies user's disclosure behaviour by considering two heterogeneous type of personal information, say type 1 and type 2. Disclosing type 1 information can reveal user's personal identities and bring interaction benefits among users while a cybersecurity risk may exist. Disclosing type 2 information can reveal user's shopping preferences and bring matching benefit with the advertiser, while the risk of data breach may exist. Then, user will have distinct privacy trade offs under each type of information. When collecting user's information, the platform has to spend a greater amount of expenditure on holding the information. Then, the platform may choose to trade a certain amount of information with the advertiser, so that advertiser can generate more potential benefits. The advertiser in this model plays a passive role that he only decides whether to receive the trading offer by observing platform's strategy decision. The results highlight that user behaves more cautious of disclosing type 1 information with the privacy concern of cyberbullying risk than disclosing type 2 information with the privacy concern of advertising nuisance. Therefore, platform can always collect positive amount of type 2 information. Additionally, the social media platform always has an incentive to trade user's information as if the holding cost of that type information is higher.

The research ideas of Chapter 3 is actually inspired by Chapter 2 when considering the matching service between users and the advertiser. Advertising revenue is an important component of representing profitability on most of online platforms. While the advancement of digital technologies promote the capacity of personalised advertising, some sellers may still choose to directly display their ads by reaching the total size of buyers on the platform. This advertising type may also become popular for the platform if it charges zero registration fees to attract buyers to enter. However, buyers may feel annoying if too much ads are sent to them. Thus, the advertising level may affect both seller's and buyer's entry choice. Therefore, Chapter 3 studies the role of advertising on platform competition. To simplify the analysis, the model considers two symmetric platforms located at two endpoints of a Hotelling line, where buyers and sellers are also distributed on the line. Each platform charges zero registration fees to attract buyers and positive fees to attract sellers. Besides that, each platform also introduces advertising service to the sellers by charging extra advertising fees. After joining the platform, the seller can decide how much ads to send out by observing platform's advertising pricing decision. The results highlight that seller's advertising choice is not determined by the total size of buyers on that platform. Additionally, buyer's nuisance cost of advertising will affect each platform's advertising pricing level charged on each unit of ad. Finally, each platform may not be profitable by increasing its investment on advertising technology.

Chapter 1

Merger Analysis in Two-sided Market

1.1 Introduction

Two-sided markets in real world usually involve shopping malls, payment card system, and Internet-based platforms (Rochet and Tirole, 2006). In two-sided markets, two groups of agents, for example sellers and buyers, usually make interactions via a platform. One crucial characteristic of two-sided markets is network effects, which show the relationship between the number of agents and the value they can enjoy. Direct network effects exist when the benefit enjoyed by a user depends on the number of users on the same side. However, two-sided markets usually lead to indirect externalities. That is, the benefits of interaction enjoyed by a user will depend on how many users platform can attract on the other side (Armstrong, 2006). Network effects can cause higher market concentration and a higher entry barrier (Argentesi et al., 2021). For online platforms, strong network effects can generate powerful data sources so that platforms can provide better services and attract more users. Sometimes collecting data from existing users may be insufficient; therefore, platforms tend to acquire additional data through merger strategies (Chen et al., 2022). Thus, the horizontal merger can achieve higher product values for users and become more competitive than other outsider platforms (Cosnita-Langlais and Rasch, 2022).

According to DePamphillis (2009), firms usually attempt to develop merger plans for several reasons, such as increasing market power and integrating core technology. The incumbent can either merge with a competitor directly or merge with a firm of providing complementary product or service. For example, in digital markets, new firms can achieve rapid growth of innovation if they acquire investment opportunities, and therefore they may become merged targets of the incumbent. As a leading company in the digital market, there are about 168 merger cases of Google between 2008 and 2018. One transaction that can be regarded as a horizontal merger was the Google/Waze merger decision taken by UK authorities. Before the merger, Google provided free mapping and turn-by-turn navigation services through the application Google Maps, while Waze also provided a turn-by-turn navigation service application on mobile devices. The merger was then cleared in 2013. This decision did not appear to have an impact on potential market competition, as Apple Maps was still active in the fight for market share. After joining the merger, Waze also achieved further growth in many countries. The evaluation of post-merger market performance revealed that the Google/Waze merger not only reduced the entry cost of new map information under Waze's crowdsourcing business model, but also improved user's efficiencies through Google Maps' high market penetration (Argentesi et al., 2021).

The results of traditional merger analysis may not directly apply to mergers in two-sided markets (Correia-da-Silva et al., 2019). Evans and Noel (2008) first considered the importance of indirect network effects on merger issues in two-sided markets by empirically evaluating traditional tools for merger analysis in one-sided markets. The results show a significant estimation bias, which implies that merger analysis in two-sided markets should consider indirect network effects and its effect on pricing structures. Chandra and Collard-Wexler (2009) provided another starting point for merger analysis in two-sided markets. By applying data before and after the mergers in the Canadian newspaper industry in the 1990s, the result concluded that mergers in two-sided markets do not cause a higher price-setting on either side, which is different from the result in traditional one-sided merger analysis. However, the result could not prove the possibility that platforms with greater market power choosing merger strategy could lower their price setting in the post-merger stage. Also, the results could not support the possibility post-merger prices could keep unchanged since the increasing market power could be offset by the cost-reduction effect. Therefore, this paper will extend a theoretical framework for merger analysis in two-sided markets to provide further implications behind existing empirical results.

In recent times, the growing market dominance of certain digital industry players has raised concerns among Antitrust Authorities (AAs) and governments, leading to a heightened sense of unease about market concentration in these sectors. Merger control faces a pressing issue in the form of the acquisition of potential competitors, which can come from rapidly growing firms in adjacent markets, sometimes even small in size. This can pose challenges to established incumbents, making it important to address this concern in order to prevent such acquisitions from occurring (Motta and Peitz, 2021). Blocking a merger can prevent long-term harm but it also prevents potential benefits from being realized; approving a merger with remedies presupposes an effective system of monitoring and enforcement, without which problems can be amplified after the merger has been approved and cannot be undone (Chen et al., 2022).

The aim of this chapter is to discuss the effect of mergers on two-sided markets by following a modified city circular market. In this model, there exists a platform located in the center of the circle, while all other platforms are located around the circle. Thus, platforms have either localized competition or non-localized competition. Agents, divided into buyers and sellers, are distributed on the circle, and each agent is single homing for exogenous reasons. The merger analysis is based on comparative statics, where the platforms choose the price levels charged on each group of agents to maximize their profits both at the premerger stage and at postmerger stage. The analysis also shows how the merger effect is determined by the total number of platforms on the circle. The results highlighted that the cross-group network externalities play a significant role in affecting post-merger equilibrium prices as well as the generated market shares. In a three-platform platform, the market is less competitive, so the exogenous merger strategy will be profitable for any platform that does not generate market shares without merger. In a market with $N+1$ platforms ($N > 2$), a more competitive market leads to a more effective merger strategy to increase the gap between the profits of the insider and the outsider. Furthermore, the insider platform on the circle will increase their incentives to be merged compared to the insider platform in the center.

The remaining sections are structured as follows. Section 1.2 reviews the related literature. Section 1.3 provides a merger analysis of a three-platform model. Section 1.4 discusses the effect of mergers in a market with $N+1$ platforms ($N \geq 0$). Section 1.5 concludes.

1.2 Literature Review

This research will be based on the theoretical framework of competition in two-sided markets. Armstrong (2006) focused on pricing strategies and the role of cross-group externalities. The study highlighted that the platform often charges zero or lower prices on the buyer side to attract buyers to join the platform and charges the seller side higher prices. Although this subsidy strategy may cause price distortion, it can effectively seize the buyers' market size and gain profits from sellers who have a stronger willingness to pay through cross-group network effects. In addition, the study also discussed both how single-homing competition, where users choose one platform to join, and multi-homing competition, where users choose multiple platforms to join, works in two-sided markets. Since the larger size of user group can generate greater network effects, a platform may therefore gain a dominant advantage in market competition. This will lead to a more concentrated market structure. Therefore, platforms must carefully set pricing strategies to ensure sufficient network effects and maximize profitability. This chapter will explore merger issues and competition in two-sided markets with buyers and sellers, which also emphasizes the importance of cross-group externalities on merging strategies. Although the model setting introduces direct network externalities, they are offset in the analysis results.

The difference in merger strategies between traditional one-sided and two-sided markets stems from the existence or nonexistence of network externalities. When an agent enters the platform, both the direct network effect and the indirect network effect are created. Cosnita-Langlais and Rascg (2022) conducted a systematic analysis of pricing strategies, competition dynamics, and policy regulations in two-sided markets. This study considered horizontal mergers in a circular single-sided market and focused on the relationship between merger cost savings and network effects between firms and showed that both of them could influence the merger's profitability. The result also highlighted that when platforms integrate more service functions through merger strategies, the intensity of cross-group network externalities may evolve from buyers attracting sellers in a one-way manner to buyers and sellers in a two-way coordination, which challenges the applicability of static pricing models. Since traditional antitrust reviews rely on static market share indicators and ignore the impact of dynamic changes in cross-group network externalities on competition in two-sided markets, antitrust policies for two-sided markets should fully consider bilateral characteristics. This chapter will discuss in depth the impact of merger strategies in two-sided markets under non-localized competition, specifically analyzing pricing decisions based on cross-group network externalities. Many studies related to the two-sided market set the direct network effect equal to zero (Rochet and Tirole, 2003). In this chapter, although both the direct externality parameters and indirect externality parameters are introduced in the model setting, the direct network effects are actually offset when identifying the number of agents on each side. Therefore, this chapter mainly focuses on analyzing how cross-group externalities influence market size and platforms' pricing strategies. In addition, I also assume that the settings of network effect parameters are the same on all platforms. Furthermore, there are two basic structures for price setting. Platforms can charge each side either on fixed basis or on per-transaction basis (Armstrong, 2006). In this paper, I assume that each platform will offer lump-sum prices on each side. Finally, the merger analysis will be based on the situation that each agent will choose only one platform, which was also introduced by Armstrong (2006).

This research also considers spatial competition. Previous studies of two-sided markets usually introduced the Hotelling model to identify the number of agents on each side. For example, Armstrong (2006) presented the Hotelling specification, with two platforms located at the endpoints of a linear city. Darguad and Reggiani (2015) also applied another spatial market, the spokes model, to analyze multi-firm differentiated product competition. The result showed that the merger could drive both insiders and outsiders to increase their prices. In this study, a city circular model was involved introduced by Salop (1979). In his paper, both agents and firms are located around the circle, and the real competition only exists between two neighboring firms, which also allows the limitation of non-localized competition. Cosnita-Langlais and Rascg (2022) also laid an important foundation for the merger analysis based on the Salop model. However, I suppose a platform located in the center of the circular city which attempts to develop a merger plan. The idea is similar to Bouckaert (2000), where a mail-order business was introduced in the city, and traveling to the business was independent of the location. I extend this non-localized idea and assume that the center platform in the city takes a higher market position compared with other platforms on the circle. For example, the center platform has fixed-cost core technology, so the competition with other platforms on the circle is irrelevant of its location. In this way, the multi-platform spatial competition will involve both localized effects and non-localized effects. The model with a center platform in a city circular market is more in line with the actual two-sided market ecology. The standard Salop model cannot deeply analyze the internal mechanism of the center platform to strengthen cross-group network externalities with the support of a large agent base. In addition, adding a center platform can further explore the competition relationship between the center platform and the platform on the circle. When the center platform participates in the merger, its pricing strategy forms a squeeze effect on the outsider platforms, which then triggers the reconstruction of market share. This process fully reflects the role of the center platform in reshaping the market competition pattern and is difficult to present in the standard model. Furthermore, the model with a central platform constructs a more accurate analytical framework for antitrust policy design. The model containing a central platform can provide policymakers with more sophisticated tools to evaluate the welfare effects of merger behavior and the role of cross-group network externalities in two-sided market merger.

Regulators are mainly concerned about the effect of mergers on social welfare issues. Traditional analysis showed that horizontal mergers could reduce total welfare due to the increasing price (Cosnita-Langlais and Rasch, 2022). Correla-da-Silva et al. (2019) concluded that the effect of mergers with multisided platforms on users is determined by comparing externality-adjusted prices and average marginal cost. Specifically, under linear demand functions, the merger can benefit users on both sides when the externalities are large. Toshimitsu (2019) studied the relationship between merger-related network externalities and the social welfare effect under the assumption that merging firms increase the compatibility level. It concluded that consumer surplus in the post-merger stage would be higher than that in pre-merger stage because of the sufficiently large degree of network compatibility between merging firms. Thus, the proposed merger could be approved by the relevant antitrust authorities. There also exists a link between merger and cost savings. When a firm participates in the merger and becomes an insider, the post-merger market share will increase, which can lead to lower production costs (Cosnita-Langlais and Rasch, 2022).

Motta and Peitz (2021) have identified the acquisition of possible rivals as a critical concern for merger control in digital sectors. Their model highlights that in the digital industry, even small startups can develop successful projects, and if the startup has the ability to proceed with their project, the merger can lead to anti-competitive outcomes, either through a "killer acquisition" or an upgrade with reduced competition. Only when the start-up is unable to pursue its project without the merger and the incumbent has an incentive to develop the project after acquiring the start-up can the merger be considered pro-competitive according to their analysis. Gautier and Lamesch (2021) also highlighted several reasons why a product can be discontinued after an acquisition. Firstly, the product may not have performed as expected, leading the acquirer to abandon the project. Secondly, the acquisition may have been motivated not by the product or brand itself but rather by the assets or innovation capabilities of the company. In this scenario, the acquired assets may be transferred to the acquirer and the target company is ultimately shut down. Finally, the product may be discontinued to safeguard the acquirer's market position. Furthermore, Chen et al. (2022) have centered their analysis on mergers that are data-driven and possess two fundamental characteristics, personalization and cross-market interaction due to complementarity.

This chapter contributes to studies of merger issues in two-sided markets. Whilst the previous literature also discusses the merger effect in a Salop's circular market, this chapter will also involve non-localized competition by setting a platform in the center of the circle. Thus, platforms among insiders are asymmetric in changing their pricing levels and market shares when compared with premerger equilibrium results. Therefore, the incentive of merging may depend on the platform's position in the market. Additionally, this chapter distinguished merger effects on two-sided markets when the total market size is either smaller or larger. This could strongly demonstrate the role of indirect network externalities on internalizing platform's pricing decisions, especially when platforms on the circle are quite competitive.

1.3 A Three-Platform Model

1.3.1 Preliminary

The spatial model introduced is based on Salop (1979) and Bouckaert (2000). This section considers a circular market with two platforms on the circle, denoted 1 and 2, and one platform in the center of the circle, denoted M (see Figure 1). The radius of the circular market equals f and the perimeter equals to 2.

Agents

Agents are divided into two groups, sellers and buyers, denoted s and b. Both sellers and buyers are uniformly distributed on the circle. First, this setting simplifies the description of agent behavior, assuming that all agents have similar behavioral tendencies and economic characteristics as a whole, to characterize the overall trend and general laws of the market. Specifically, the uniform distribution for agents allows the model to focus more on analyzing the basic interactive relationship between the platform and agents, and focuses the research on the impact of key factors such as cross-group network externalities on the two-sided market, providing a relatively concise theoretical basis for understanding the economic phenomena of the two-sided market. Second, the assumption of uniform distribution means that the status and role of agents on both sides of the market are relatively equal, enabling the model to better analyze the interaction mechanism of agents on both sides of the market. In addition, when studying the competition of different two-sided market models, the uniform distribution assumption provides a common starting point for the comparison of different markets, which helps to accurately evaluate the effect of merger strategies in different market environments.

Assume that each agent chooses to be single-homing for exogenous reasons. In reality, agents may not be limited to choosing a single platform based on their own needs and the characteristics of different platforms. This also reflects that platforms will attract users through more differentiated competition strategies in the market, making competition among platforms more complex and diversified. In contrast, assuming that each agent can only choose one platform, the model can focus on the direct effect of merger strategy on platform competition and clearly show how the post-merger platform directly affects the competitive situation through changes in market share and pricing strategy. In addition, under the assumption of single-homing, the change of merger strategy on the platform's cross-group network externalities is more significant, thereby affecting market competition. Finally, the single-homing assumption simplifies the platform decision-making process. In a two-sided market, the platform needs to set different prices for agents on both sides. Under the assumption of single homing, the platform does not need to consider the multi-homing costs of agents between different platforms, and can more directly formulate the optimal pricing strategy based on factors such as the market position of the post-merger platform.

In the model setting, the market is fully covered, so there are no agents who have not chosen a platform. On the one hand, the agent's choice is driven by his location on the circular market, which eliminates the complexity brought by some agents not choosing to join any platform. Therefore, the model can focus more on analyzing core issues such as the platform's pricing strategy, market equilibrium, and competitive behavior when all agents participate. Furthermore, the model will focus on the impact of cross-group network externalities on the two-sided market. Based on the role of cross-group network externalities, some agents joining the platform will attract other agents to join one after another. This interaction effect prompts agents in the market to continue gathering on different platforms and ultimately achieve full market coverage.

If an agent from one group joins a platform, he will enjoy interaction benefits depending on the total size of the other group. Denote α_s as the parameter of the indirect network effect of the seller, measuring the benefit of a seller of interacting with each buyer on the same platform. Denote α_b as the parameter of the indirect network effect of a buyer, measuring the benefit for a buyer to interact with each seller on the same platform. Each seller and buyer also enjoy a direct benefit by joining the platform, denoted β_s and β_b, where $\alpha, \beta > 0$. To join a platform, each seller and buyer should also pay a lump sum fee, which is charged by the platform. If a seller travels around the circle to enter platform 1 or platform 2, the transportation cost is linear with the travel distance, where the transportation parameter is denoted by t_s. If a buyer travels around the circle to enter platform 1 or platform 2, the transportation cost is also linear with the travel distance, where the transportation parameter is denoted by t_b. If an agent travels to the platform M, the transportation cost is fixed at f, which is independent of his location.

In order to conduct a more tractable analysis, the model setting is simplified by following the assumptions below.

Assumption 1. *In this circular market with* $2+1$ *platforms, there exists perfect symmetry between the two sides so that* $\alpha_s = \alpha_b = \alpha$, $\beta_s = \beta_b = \beta$ *and* $t_s = t_b = t$.

Thus, any agent on one side enjoys the same benefit by interacting with an agent on the other side, and any agent between two neighboring platforms has the same brand differentiation. The externalities of the cross-group network are the core characteristic assumptions of two-sided markets. If cross-group network externalities are strong, the platform can achieve profitability and development by attracting agents on one side to attract agents on the other side, thus supporting propositions about the pricing strategy of the platform and market equilibrium. The value of the symmetric parameter α is set to more clearly reflect how the scale of the agents significantly affects the strength of cross-group network externalities. Similarly, setting a symmetric transportation parameter makes the agents on both sides have the same properties in facing brand differentiation, thereby reducing the complexity in the model and making it easier to draw key conclusions about the platform pricing strategy and market equilibrium.

Assumption 2. *For a market with 2 platforms on the circle*

$$t^2 > 36\alpha^2 \tag{1}$$

where α^2 captures the overall indirect influence between buyers and sellers, and t^2 captures the combined effect of transportation cost on the efficiency of the total interaction. This assumption simplifies the model analysis, making brand differentiation the main driving force for agents to choose a platform, ensuring balanced competition among multiple platforms and avoiding extreme market concentration. Otherwise, the emergence of monopoly platform will make it impossible to analyze merger strategies.

Platform 1 and platform 2, locate at equal distances from each other around the circle. Thus, the distance between two platforms equals to 1, while platform M locates at the center of the circle. Therefore, the utility of an agent from each group on each platform is determined in the following way.

- An agent's utility of joining platform 1

$$u_s^1 = \alpha_s n_b^1 + \beta_s - p_s^1 - t_s x_s^1 \tag{2}$$

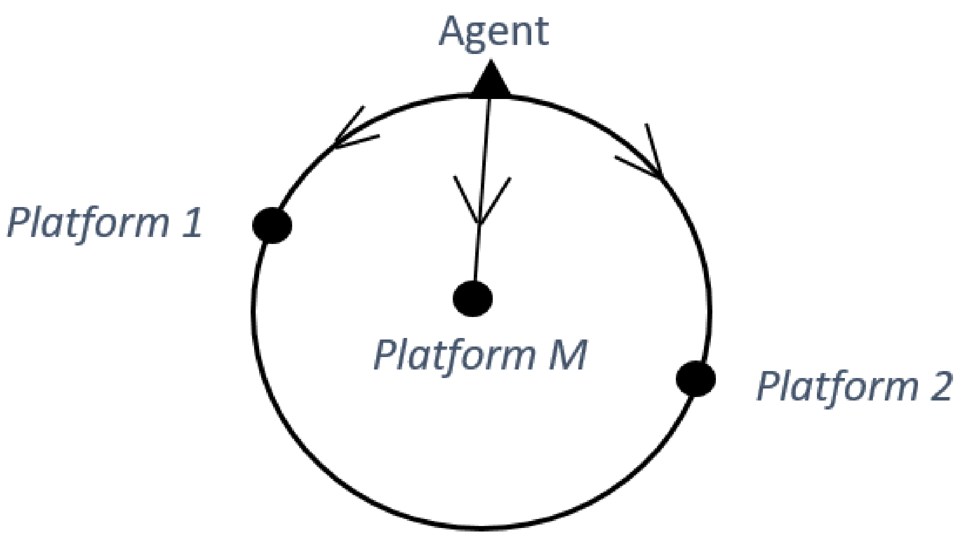

Figure 1: A Three-Platform Market

$$u_b^1 = \alpha_b n_s^1 + \beta_b - p_b^1 - t_b x_b^1 \tag{3}$$

where $\{n_s^1, n_b^1\}$ are the number of sellers and buyers attracted by platform 1; $\{p_s^1, p_b^1\}$ are the prices charged by platform 1;$\{x_s^1, x_b^1\}$ are the distance a seller and a buyer located from platform 1, where $x^1 \in [0, 1]$.

- An agent's utility of joining platform 2

$$u_s^2 = \alpha_s n_b^2 + \beta_s - p_s^2 - t_s(1 - x_s^1) \tag{4}$$

$$u_b^2 = \alpha_b n_s^2 + \beta_b - p_b^2 - t_b(1 - x_b^1) \tag{5}$$

where $\{n_s^2, n_b^2\}$ are the number of sellers and buyers attracted by platform 2; $\{p_s^2, p_b^2\}$ are the prices charged by platform 2.

- An agent's utility of joining platform M

$$u_s^M = \alpha_s n_b^M + \beta_s - p_s^M - f \tag{6}$$

$$u_b^M = \alpha_b n_s^M + \beta_b - p_b^M - f \tag{7}$$

where $\{n_s^M, n_b^M\}$ are the number of sellers and buyers attracted by platform M; $\{p_s^M, p_b^M\}$ are the prices charged by platform M.

The utility functions above show that $\{\alpha_s, \alpha_b\}$ and $\{\beta_s, \beta_b\}$ do not change with the platform joined by an agent. Therefore, the degree of both indirect network effect through interacting with an agent of the other group and direct benefit through joining a platform are the same on all platforms.

Platforms

In this model, platforms provide horizontally differentiated products and compete in prices. Suppose each platform has a same constant per-agent serving cost on each side, denoted c_s and c_b. The cost setting indicates that when the platform provides services to two groups, the marginal costs brought by each group are different, which directly affects how the platform chooses pricing strategies on each side to maximize profits. The profit function of each platform can be written as:

- Profit of platform 1

$$\pi^1 = (p_s^1 - c_s)n_s^1 + (p_b^1 - c_b)n_b^1 \tag{8}$$

- Profit of platform 2

$$\pi^2 = (p_s^2 - c_s)n_s^2 + (p_b^2 - c_b)n_b^2 \tag{9}$$

- Profit of platform M

$$\pi^M = (p_s^M - c_s)n_s^M + (p_b^M - c_b)n_b^M \tag{10}$$

1.3.2 Premerger Equilibrium

Given utility choices and brand preferences of platform 1, 2 and M, agents can either travel around the circle to platform 1 or platform 2, or travel to the center to platform M. Thus, each platform can generate its market shares and maximized their profits under the following indifferent situations.

(1) The seller is indifferent between platform 1 and platform 2.

$$u_s^1 = u_s^2 \tag{11}$$

This equation generates platform 1's market shares on the seller side $n_s^1 = 2x_s^1$, platform 2's market shares on the seller side $n_s^2 = 2(1 - x_s^1)$, and platform M's market shares on the seller side $n_s^M = 0$. That means, it is too costly for the seller to visit platform M. Thus, market shares are only captive to platform 1 and platform 2. Hence, substituting utility functions of u_s^1 and u_s^2 into the equation and obtain

$$n_s^1 = 1 + \frac{2\alpha_s(n_b^1 - 1) - (p_s^1 - p_s^2)}{t_s} \tag{12}$$

$$n_s^2 = 1 + \frac{2\alpha_s(n_b^2 - 1) - (p_s^2 - p_s^1)}{t_s} \tag{13}$$

Since two sides are symmetric, so each platform's market shares on the buyer side is

$$n_b^1 = 1 + \frac{2\alpha_b(n_s^1 - 1) - (p_b^1 - p_b^2)}{t_b} \tag{14}$$

$$n_b^2 = 1 + \frac{2\alpha_b(n_s^2 - 1) - (p_b^2 - p_b^1)}{t_b} \tag{15}$$

while $n_b^M = 0$.

It is noteworthy that platform's market shares on each side do not rely on the direct benefits from joining the platform $\{\beta_s, \beta_b\}$. Whether an agent on one side is attracted by a platform depends on the valuation of interacting with the other side through the platform. When platform M generates zero market shares, for either platform 1 or platform 2, if all prices keep fixed, an extra buyer will attract extra $2(\frac{\alpha_s}{t_s})$ number of sellers, and an extra seller will attract extra $2(\frac{\alpha_b}{t_b})$ number of buyers.

Solving equations (12) and (14) simultaneously can yield the demand functions of platform 1, which are only determined by prices of each platform.

- Platform 1's Demand Functions

$$\begin{aligned} n_s^1 &= \frac{1}{t_s t_b - 4\alpha_s\alpha_b}[t_b(t_s - p_s^1 + p_s^2) + 2\alpha_s(-2\alpha_b - p_b^1 + p_b^2)] \\ n_b^1 &= \frac{1}{t_s t_b - 4\alpha_s\alpha_b}[t_s(t_b - p_b^1 + p_b^2) + 2\alpha_b(-2\alpha_s - p_s^1 + p_s^2)] \end{aligned} \tag{16}$$

Solving equations (13) and (15) simultaneously can yield the demand functions of platform 2, which are only determined by prices of each platforms.

- Platform 2's Demand Functions

$$\begin{aligned} n_s^2 &= \frac{1}{t_s t_b - 4\alpha_s\alpha_b}[t_b(t_s - p_s^2 + p_s^1) + 2\alpha_s(-2\alpha_b - p_b^2 + p_b^1)] \\ n_b^2 &= \frac{1}{t_s t_b - 4\alpha_s\alpha_b}[t_s(t_b - p_b^2 + p_b^1) + 2\alpha_b(-2\alpha_s - p_s^2 + p_s^1)] \end{aligned} \tag{17}$$

Assumption 2 implies that both platform 1 and platform 2's demands are also positive. Under the indifferent situation between two neighboring platforms, each platform's demand function on each side is actually a proportion of total market shares divided by $t_s t_b - N^2\alpha_s\alpha_b$ (where in a three-platform model, $N = 2$). Also, each platform's demand function on one side decreases with the pricing on both sides, and increases with the rival platform's pricing on both sides. However, agents respond much more sensitively to the change of pricing on the same side than to the change of on the other side of whether the price is charged by the platform itself or its rival. This can be derived by comparing the coefficients of each price variable. For example, an additional change of p_s^1 or p_s^2 will cause a demand change of $\frac{t}{t^2-4\alpha^2}$ on n_s^2, while an additional change of p_b^1 or p_b^2 will cause a demand change of $\frac{2\alpha}{t^2-4\alpha^2}$ on n_s^2. Since $\frac{t}{t^2-4\alpha^2} > \frac{2\alpha}{t^2-4\alpha^2}$, sellers of platform 2 will respond much more sensitively to the change of pricing on the seller side. In addition, keep all other things fixed, if platform 1 lowers p_b^1, both n_s^1 and n_b^1 will increase, while the increase of n_b^1 is greater than the increase of n_s^1. At the same time, both n_s^2 and n_b^2 will decrease, while the decrease of n_b^2 is greater than n_s^2. To respond, platform 2 should also be lower p_b^2.

In the premerger stage, platform 1 chooses $\{p_s^1, p_b^1\}$ to maximize the profit function π^1 and platform 2 chooses $\{p_s^2, p_b^2\}$ to maximize the profit function π^2. In the case of a symmetric equilibrium, each platform sets the same price levels on each side and derives the following outcome.

Proposition 1. *In a circular market with "$2+1$" platforms, in the pre-merger stage, when the center platform generates zero market shares, the pricing competition between two neighboring platforms derives the symmetric equilibrium that:*

$$p_s^* = c_s + t_s - 2\alpha_b = c_s + t - 2\alpha \tag{18}$$

$$p_b^* = c_b + t_b - 2\alpha_s = c_b + t - 2\alpha \tag{19}$$

Proof. *The first order conditions of platform 1's profit maximized problem are*

$$\begin{aligned} \frac{\partial \pi^1}{\partial p_s^1} &= (p_s^1 - c_s)\frac{\partial n_s^1}{\partial p_s^1} + n_s^1 + (p_b^1 - c_b)\frac{\partial n_b^1}{\partial p_s^1} = 0 \\ \frac{\partial \pi^1}{\partial p_b^1} &= (p_b^1 - c_b)\frac{\partial n_b^1}{\partial p_b^1} + n_b^1 + (p_s^1 - c_s)\frac{\partial n_s^1}{\partial p_b^1} = 0 \end{aligned} \tag{20}$$

In the symmetric case, $p_s^* = p_s^{1*} = p_s^{2*}$ and $p_b^* = p_b^{1*} = p_b^{2*}$. The first order conditions *can be then written as*

$$\begin{aligned} t_s t_b - 4\alpha_s\alpha_b - t_b(p_s^* - c_s) - 2\alpha_b(p_b^* - c_b) = 0 \\ t_s t_b - 4\alpha_s\alpha_b - t_s(p_b^* - c_b) - 2\alpha_s(p_s^* - c_s) = 0 \end{aligned} \tag{21}$$

Solving two equations simultaneously can derive the equilibrium outcome. The second-order conditions are $\frac{\partial^2\pi^1}{\partial(p_s^1)^2} = \frac{-2t_b}{t_s t_b - 4\alpha_s\alpha_b} < 0$, $\frac{\partial^2\pi^1}{\partial(p_b^1)^2} = \frac{-2t_s}{t_s t_b - 4\alpha_s\alpha_b} < 0$, $\frac{\partial^2\pi^1}{\partial p_s^1 \partial p_b^1} = \frac{-2(\alpha_s+\alpha_b)}{t_s t_b - 4\alpha_s\alpha_b} < 0$ *and* $\frac{\partial^2\pi^1}{\partial p_b^1 \partial p_s^1} = \frac{-2(\alpha_s+\alpha_b)}{t_s t_b - 4\alpha_s\alpha_b} < 0$, *which prove the stability of equilibrium results.*

The platform maximizes profits by balancing serving cost, market power, and cross-group network effect incentives. The profit margin on each side is determined by agents' brand differentiation between two platforms adjusted downward of network effects by interacting total market shares. Transportation cost measures the differences in users preferences for platforms. Therefore, the higher the value of t, the higher the premium the platform can charge. Therefore, the difference between price and marginal cost increases with the marginal cost of transportation. In addition, since the cross-group externalities brought by the buyer side will enhance the attractiveness to the seller side, the platform returns this part of the value to the seller side in the form of price subsidies. Thus, p_s^* decreases with α_b and p_b^* decreases with α_s. Without considering the characteristics of the two-sided market, $p_s^* = c_s + t_s$ and $p_b^* = c_b + t_b$. This makes it impossible for the platform to capture bilateral interactions and turns them into unilateral decision-making, thus charging agents higher levels, resulting in a reduction in the size of agents.

Under equilibrium prices, the equilibrium demand and profits of each platform can be obtained.

$$\begin{aligned} n_s^{1*} = n_s^{2*} = 1 \\ n_b^{1*} = n_b^{2*} = 1 \end{aligned} \tag{22}$$

$$\pi^{1*} = \pi^{2*} = (t_s - 2\alpha_s) + (t_b - 2\alpha_b) = 2(t - 2\alpha) \tag{23}$$

The consumer surplus of platform 1 and platform 2 is written as

$$CS^1 = CS_s^1 + CS_b^1 = \int_0^{x_s^1} u_s^1 dx + \int_0^{x_b^1} u_b^1 dx \tag{24}$$

$$CS^2 = CS_s^2 + CS_b^2 = \int_{x_s^1}^1 u_s^1 dx + \int_{x_b^1}^1 u_b^1 dx \tag{25}$$

Under equilibrium prices, the consumer surplus of each platform can be obtained as

$$CS^{1*} = CS^{2*} = \frac{(3\alpha + \beta - t - c_s)^2 + (3\alpha + \beta - t - c_b)^2}{2t} \tag{26}$$

The numerator reflects the driving effect of net utility. 3α reflects the enhancement of cross-group network externalities when the two platforms are in competitive equilibrium, while $\beta - t - c$ reflects that the basic utility of users joining the platform is adjusted downward by transportation costs and the platform's serving costs. Therefore, consumer surplus accelerates with the expansion of cross-group network externalities and the differences between basic utility and costs. The denominator reflects the dilution effect of transportation costs on consumer surplus. The more agents care about brand differentiation, the lower the consumer surplus led by net utility.

The social welfare function of platform 1 and platform 2 are written as

$$SW^1 = CS^1 + \pi^1 \tag{27}$$

$$SW^2 = CS^2 + \pi^2 \tag{28}$$

To find the social optimal pricing level, the following proposition is derived.

Proposition 2. *In a circular market with "2+1" platforms, in the pre-merger stage, when the center platform generates zero market shares, the social welfare maximization problem derives the symmetric equilibrium that:*

$$p_s^{SW} = c_s + \frac{2\alpha\beta + t\beta - 4t\alpha^2}{t^2 - 4\alpha^2} \tag{29}$$

$$p_b^{SW} = c_b + \frac{2\alpha\beta + t\beta - 4t\alpha^2}{t^2 - 4\alpha^2} \tag{30}$$

Proof. *In the symmetric case, $p_s^1 = p_s^2 = p_s$ and $p_b^1 = p_b^2 = p_b$. Therefore, the first order conditions of social welfare maximization problem are*

$$\frac{\partial SW}{\partial p_s} = 2[-\frac{\alpha_s n_b^1 + \beta_s - p_s}{t_s} + n_s^1 - \frac{(p_s - c_s)t_b + (p_b - c_b)2\alpha_b}{t_s t_b - 4\alpha_s\alpha_b}] = 0 \tag{31}$$

$$\frac{\partial SW}{\partial p_b} = 2[-\frac{\alpha_b n_s^1 + \beta_b - p_b}{t_b} + n_b^1 - \frac{(p_s - c_s)2\alpha_s + (p_b - c_b)t_s}{t_s t_b - 4\alpha_s\alpha_b}] = 0 \tag{32}$$

Solving two equations simultaneously can derive the equilibrium outcome. The second-order conditions are $\frac{\partial^2 SW}{\partial p_s^2} = \frac{\partial^2 SW}{\partial p_b^2} = \frac{-2}{t(t-2\alpha)} < 0$ and $\frac{\partial^2 SW}{\partial p_s p_b} = \frac{\partial^2 SW}{\partial p_b p_s} = \frac{-\alpha}{t^2-4\alpha^2} < 0$, which show the stability of the equilibrium results.

Different from analyzing the profit maximization problem, welfare issues need to consider both cross-group network externalities and the basic utility (β) of agents when joining the platform. When comparing p^{SW} and p^*, it can be found that when $\beta < \frac{t(t-2\alpha)}{t+2\alpha}$, $p^{SW} < p^*$. Otherwise, excessive basic utility will increase the weight of consumer surplus greater than the platform's profit, so that the platform has to raise prices to balance profits and consumer surplus. Therefore, under the strong cross-group network externalities, social planners are more inclined to lower prices to attract more agents to expand social welfare, even when profits are reduced.If the platform's profit-maximizing pricing is significantly higher than the social welfare level, social planners need to approach the social optimum through price controls or promoting competition.

(2) The seller is indifferent between platform 1 and platform M.

$$u_s^1 = u_s^M \tag{33}$$

This equation generates platform 1′ market shares on the seller side $n_s^1 = 2x_s^1$, platform 2's market shares on the seller side $n_s^2 = 0$, and platform M's market shares on the seller side $2(1 - x_s^1)$. That is, it is too costly for the seller to visit platform 2. Thus, market shares are only captive to platform 1 and platform M. Hence, substituting utility functions of u_s^1 and u_s^M into the equation and obtain

$$n_s^1 = \frac{2f}{t_s} + \frac{4\alpha_s(n_b^1 - 1) - 2(p_s^1 - p_s^M)}{t_s} \tag{34}$$

$$n_s^M = 2 - \frac{2f}{t_s} + \frac{4\alpha_s(n_b^M - 1) - 2(p_s^M - p_s^1)}{t_s} \tag{35}$$

Symmetrically, each platform's market shares on the buyer side is

$$n_b^1 = \frac{2f}{t_b} + \frac{4\alpha_b(n_s^1 - 1) - 2(p_b^1 - p_b^M)}{t_b} \tag{36}$$

$$n_b^M = 2 - \frac{2f}{t_b} + \frac{4\alpha_b(n_s^M - 1) - 2(p_b^M - p_b^1)}{t_b} \tag{37}$$

while $n_b^2 = 0$.

When platform 2 generates zero market shares, for either platform 1 or platform M, if all prices keep fixed, an extra buyer will attract extra $4(\frac{\alpha_s}{t_s})$ number of sellers, and an extra seller will attract extra $4(\frac{\alpha_b}{t_b})$ number of buyers.

Solving equations (34) and (36) simultaneously can yield the demand functions of platform 1, which are determined by prices of both platform 1 and platform M.

- Platform 1's Demand Functions

$$
\begin{aligned}
n_s^1 &= \frac{1}{t_s t_b - 16\alpha_s\alpha_b}[2t_b(f - 2\alpha_s - p_s^1 + p_s^M) + 8\alpha_s(f - 2\alpha_b - p_b^1 + p_b^M)] \\
n_b^1 &= \frac{1}{t_s t_b - 16\alpha_s\alpha_b}[2t_s(f - 2\alpha_b - p_b^1 + p_b^M) + 8\alpha_b(f - 2\alpha_s - p_s^1 + p_s^M)]
\end{aligned}
\tag{38}
$$

Platform 1's demand functions can be rewritten as $n_s^1 = \frac{2(f-2\alpha)}{t-4\alpha} + \frac{2t}{t^2-16\alpha^2}(p_s^M - p_s^1) + \frac{8\alpha}{t^2-16\alpha^2}(p_b^M - p_b^1)$ and $n_b^1 = \frac{2(f-2\alpha)}{t-4\alpha} + \frac{2t}{t^2-16\alpha^2}(p_b^M - p_b^1) + \frac{8\alpha}{t^2-16\alpha^2}(p_s^M - p_s^1)$. Assumption 1 implies that $t - 4\alpha$ is positive. Therefore, unlike the first indifferent situation, the rational condition for realizing positive demands of platform 1 should be $f > 2\alpha$. That is, the brand preferences of the platform M is also greater than the network effects of the interaction of the total number of agents on the market. The platform uses the reasonable relationship between users brand preferences (t and f) and cross-group externalities (α) to ensure that the user scale is positive. Also, agents respond much more sensitively to the change of pricing on the same side than to the change of on the other side of whether the price is charged by the platform itself or its rival. This can be derived by comparing the coefficients of each price variable. For example, an additional change of p_s^1 or p_s^M will cause a demand change of $\frac{2t}{t^2-16\alpha^2}$ on n_s^1, while an additional change of p_b^1 or p_b^M will cause a demand change of $\frac{8\alpha}{t^2-4\alpha^2}$ on n_b^1. Since $\frac{2t}{t^2-16\alpha^2} > \frac{8\alpha}{t^2-16\alpha^2}$, sellers of platform 1 will respond much more sensitively to the change of pricing on the seller side. Finally, agents also respond more sensitively to the price change on both sides than in the first indifferent situation $\frac{2t}{t^2-16\alpha^2} > \frac{t}{t^2-4\alpha^2}$ and $\frac{8\alpha}{t^2-16\alpha^2} > \frac{2\alpha}{t^2-4\alpha^2}$. That is, when buyers or sellers choose between the center platform (platform M) and the circle platform (platform 1), the balance between their brand preferences and the strength of cross-group network externalities leads to a greater price elasticity of demand.

Solving equations (35) and (37) simultaneously can yield the demand functions of platform M, which are determined by prices of both platform 1 and platform M.

- Platform M's Demand Functions

$$\begin{aligned} n_s^M &= \frac{1}{t_s t_b - 16\alpha_s\alpha_b}[2t_b(f - 2\alpha_s - p_s^M + p_s^1) + 8\alpha_s(f - 2\alpha_b - p_b^M + p_b^1)] \\ n_b^M &= \frac{1}{t_s t_b - 16\alpha_s\alpha_b}[2t_s(f - 2\alpha_b - p_b^M + p_b^1) + 8\alpha_b(f - 2\alpha_s - p_s^M + p_s^1)] \end{aligned} \tag{39}$$

The following proposition can be obtained.

Proposition 3. *In a circular market with "$2+1$" platforms, there exists $f \leq \frac{1}{2}t$.*

Proof. *The total market shares of agents on each side is 2. That is, $n_s^1 + n_s^M \leq 2$ and $n_b^1 + n_b^M \leq 2$. Combine the equation of $n_s^1 + n_s^M$ and obtain*

$$\frac{2}{t_s t_b - 16\alpha_s\alpha_b}[2t_b(f - 2\alpha_s) + 8\alpha_s(f - 2\alpha_b)] \leq 2 \tag{40}$$

$$2t_b(f - 2\alpha_s) + 8\alpha_s(f - 2\alpha_b) \leq t_s t_b - 16\alpha_s\alpha_b \tag{41}$$

Following Assumption 1, inequality (41) can be rewritten as

$$2(t + 4\alpha)(f - 2\alpha) \leq (t + 4\alpha)(t - 4\alpha) \tag{42}$$

Therefore, $2(f - 2\alpha) \leq t - 4\alpha$ derives $f \leq \frac{1}{2}t$.

f captures any agent's preference differentiation between a platform on the circle and a platform in the center, and t captures any agent's preference differentiation between two neighboring platforms on the circle. For an agent located in the middle of platform 1 and platform 2, the transportation cost of choosing platform M is equal to or lower than the transportation cost of choosing platform 1. Therefore, when non-localized competition exists, platform M may need to rely more on cross-group network externalities to attract or retain agents. Proposition 3 ensures that the demand for platform M is positive under non-localized competition. Otherwise, users will give up choosing platform M because the strength of cross-group network interaction effect is weakened by the relatively high transportation cost.

In the premerger stage, platform 1 chooses $\{p_s^1, p_b^1\}$ to maximize the profit function π^1 and platform M chooses $\{p_s^M, p_b^M\}$ to maximize the profit function π^M. In the case of a symmetric equilibrium, each platform sets the same price levels on each side and derives the following outcome.

Proposition 4. *In a circular market with "$2+1$" platforms, at the premerger stage, the pricing competition between any platform on the circle and the platform in the center derives the symmetric equilibrium that:*

$$p_s^* = c_s + f - 2\alpha_b = c_s + f - 2\alpha \tag{43}$$

$$p_b^* = c_b + f - 2\alpha_s = c_b + f - 2\alpha \tag{44}$$

Proof. *The first order conditions of platform M's profit maximization problem are:*

$$\begin{aligned} \frac{\partial \pi^M}{\partial p_s^M} &= (p_s^M - c_s)\frac{\partial n_s^M}{\partial p_s^M} + n_s^M + (p_b^M - c_b)\frac{\partial n_b^M}{\partial p_s^M} = 0 \\ \frac{\partial \pi^M}{\partial p_b^M} &= (p_b^M - c_b)\frac{\partial n_b^M}{\partial p_b^M} + n_b^M + (p_s^M - c_s)\frac{\partial n_s^M}{\partial p_b^M} = 0 \end{aligned} \tag{45}$$

In the symmetric case, $p_s^* = p_s^{1*} = p_s^{M*}$ and $p_b^* = p_b^{1*} = p_b^{M*}$.The first order conditions can be then written as

$$\begin{aligned} 2t_b(f - 2\alpha_s) + 8\alpha_s(f - 2\alpha_b) - 2t_b(p_s^* - c_s) - 8\alpha(p_b^* - c_b) = 0 \\ 2t_s(f - 2\alpha_b) + 8\alpha_b(f - 2\alpha_s) - 2t_s(p_b^* - c_b) - 8\alpha_b(p_s^* - c_s) = 0 \end{aligned} \tag{46}$$

Solving two equations simultaneously can derive

$$
\begin{aligned}
p_s^* &= c_s + \frac{1}{t_s t_b - 16\alpha_s\alpha_b}[(t_s t_b - 16\alpha_b^2)(f - 2\alpha_s) + 4t_s(\alpha_s - \alpha_b)(f - 2\alpha_b)] \\
p_b^* &= c_b + \frac{1}{t_s t_b - 16\alpha_s\alpha_b}[(t_s t_b - 16\alpha_s^2)(f - 2\alpha_b) + 4t_b(\alpha_b - \alpha_s)(f - 2\alpha_s)]
\end{aligned}
\tag{47}
$$

For simplicity, applying Assumption 1 that $\alpha_s = \alpha_b$, thus the equilibrium outcome can written as $p_s^* = c_s + f - 2\alpha_b$ and $p_b^* = c_b + f - 2\alpha_s$. Since the platforms on the circle are horizontally differentiated, so the pricing competition between platform 2 and platform M should generate the same equilibrium outcome. The second-order conditions are $\frac{\partial^2 \pi^M}{\partial (p_s^M)^2} = \frac{\partial^2 \pi^M}{\partial (p_b^M)^2} = -\frac{4t}{t^2 - 16\alpha^2} < 0$ and $\frac{\partial^2 \pi^M}{\partial p_s^M \partial p_b^M} = \frac{\partial^2 \pi^M}{\partial p_b^M \partial p_s^M} = \frac{-16\alpha}{t^2 - 16\alpha^2} < 0$, which prove the stability of the equilibrium results.

Similarly to the first indifferent situation, without considering bilateral relationships, $p_s^* = c_s + f$ and $p_b = c_b + f$. Considering bilateral relationships, the profit margin on each side is the brand differentiation adjusted downward by network effects, while differently in the first indifferent situation, f captures the brand differentiation between the platform on the circle and the platform M. According to Proposition 3, it can also be obtained that the equilibrium price level charged on each side under non-localized competition is lower than the price level charged on each side under localized competition since the brand differentiation parameter f is lower than t. In the context of localized competition, platform 1 or platform 2 can lock in buyers and sellers through brand differentiation, so a relatively low pricing strategy will not become the main means of competition. In the context of non-localized competition, The mobility of buyers or sellers not traveling on the circle is enhanced. When some buyers or sellers choose the platform M, both platforms need to attract and retain user scale through relatively lower prices. Therefore, equilibrium prices are lower when platform M commands positive market share.

Under the equilibrium prices, the equilibrium demand and profits of each platform can be obtained.

$$\begin{aligned} n_s^{1*} = n_s^{M*} &= \frac{1}{t_s t_b - 16\alpha_s\alpha_b)}[2t_b(f-2\alpha_s) + 8\alpha_s(f-2\alpha_b)] = 1 \\ n_b^{1*} = n_b^{M*} &= \frac{1}{t_s t_b - 16\alpha_s\alpha_b)}[2t_s(f-2\alpha_b) + 8\alpha_b(f-2\alpha_s)] = 1 \end{aligned} \tag{48}$$

$$\pi^{1*} = \pi^{M*} = (f-2\alpha_s) + (f-2\alpha_b) = 2(f-2\alpha) \tag{49}$$

Again, compared with the first indifferent situation, platform 1 generates the same equilibrium market shares while charging a lower price on each side. Thus, platform's maximized profits is also lower than in the first indifferent situation.

The consumer surplus of the platform M is written as

$$CS^M = CS_s^M + CS_b^M = \int_{x_s^1}^1 u_s^M dx + \int_{x_b^1}^1 u_b^M dx \tag{50}$$

Under equilibrium prices, the consumer surplus of each platform can be obtained as

$$CS^{1*} = CS^{M*} = \frac{(3\alpha+\beta-f-c_s)^2 + (3\alpha+\beta-f-c_b)^2}{2t} \tag{51}$$

The change in agent brand differentiation parameter from t to f makes consumer surplus greater than the first indifference situation. The social welfare function of platform M is written as

$$SW^M = CS^M + \pi^M \tag{52}$$

To find the social optimal pricing level, the following proposition is derived.

Proposition 5. *In a circular market with "2+1" platforms, in the pre-merger stage, when the center platform competes with a platform on the circle, the social welfare maximization problem derives the symmetric equilibrium that:*

$$p_s^{SW} = c_s + \frac{t - 4\alpha}{2} + \frac{2\alpha(f - 2\alpha)}{t - 4\alpha} \tag{53}$$

$$p_b^{SW} = c_b + \frac{t - 4\alpha}{2} + \frac{2\alpha(f - 2\alpha)}{t - 4\alpha} \tag{54}$$

Proof. *In the symmetric case, $p_s^1 = p_s^M = p_s$ and $p_b^1 = p_b^M = p_b$. Therefore, the first order conditions of social welfare maximization problem are*

$$\frac{\partial SW}{\partial p_s} = 2[-\frac{\alpha_s n_b^1 + \beta_s - p_s}{t_s} + n_s^1 - \frac{(p_s - c_s)t_b + (p_b - c_b)2\alpha_b}{t_s t_b - 4\alpha_s \alpha_b}] = 0 \tag{55}$$

$$\frac{\partial SW}{\partial p_b} = 2[-\frac{\alpha_b n_s^1 + \beta_b - p_b}{t_b} + n_b^1 - \frac{(p_s - c_s)2\alpha_s + (p_b - c_b)t_s}{t_s t_b - 4\alpha_s \alpha_b}] = 0 \tag{56}$$

Solving two equations simultaneously can derive the equilibrium outcome. The second-order conditions are $\frac{\partial^2 SW}{\partial p_s^2} = \frac{\partial^2 SW}{\partial p_b^2} = \frac{-2}{t} < 0$, which shows the stability of the equilibrium results.

Firstly, compared to the equilibrium results of Proposition 2, p_s^{SW} and p_b^{SW} do not include β, which represents the agent's direct network externalities of joining a platform. In the first indifferent situation, the number of agents attracted by each platform is determined by localized competition, which allows β to directly affect social welfare through consumer surplus and thus affect equilibrium pricing. However, in the second indifferent situation, since the transportation cost of agents to platform M is fixed at f, the effect of β on the number of agents is limited and eventually offset in the search for equilibrium pricing solutions. Secondly, equations (53) and (54) are composed of three parts. c_s and c_b ensure that each platform can cover basic serving costs. $\frac{t-4\alpha}{2}$ reflect the balance between agent brand differentiation t and cross-group network externalities α. If this term becomes larger due to the relative dominance of t, then the platform will have to intervene directly in agent choices through pricing. The last term $\frac{2\alpha(f-2\alpha)}{t-4\alpha}$ reflects the impact of asymmetric transportation costs between platforms, suggesting that platforms maycompensate for the asymmetric brand by raising prices while guiding agent choices to optimize social welfare. Thirdly, compared with profit-maximized equilibrium results, when transportation cost is high but the cross-group network externalities are not completely offset, maximization of social welfare requires lowering prices to balance market share, while the profit-maximization platform is more inclined to maintain relatively high prices, leading to $p_s^{SW} < p_s^*$.

(3) The seller is indifferent between platform 2 and platform M.

$$u_s^2 = u_s^M \tag{57}$$

This equation generates platform 1's market shares on the seller side $n_s^1 = 0$, platform 2's market shares on the seller side $n_s^2 = 2(1 - x_s^1)$, and platform M's market shares on the seller side $n_s^M = 2x_s^1$. That means it is too costly for the seller to visit platform 1. Thus, market shares are captive to platform 2 and platform M. Hence, substituting utility functions of u_s^2 and u_s^M and obtain

$$n_s^2 = \frac{2f}{t_s} + \frac{4\alpha_s(n_b^2 - 1) - 2(p_s^2 - p_s^M)}{t_s} \tag{58}$$

$$n_s^M = 2 - \frac{2f}{t_s} + \frac{4\alpha_s(n_b^M - 1) - 2(p_s^M - p_s^2)}{t_s} \tag{59}$$

Symmetrically, each platform's market shares on the buyer side is

$$n_b^2 = \frac{2f}{t_b} + \frac{4\alpha_b(n_s^2 - 1) - 2(p_b^2 - p_b^M)}{t_b} \tag{60}$$

$$n_b^M = 2 - \frac{2f}{t_b} + \frac{4\alpha_b(n_s^M - 1) - 2(p_b^M - p_b^2)}{t_b} \tag{61}$$

while $n_b^1 = 0$.

The third indifferent situation is analyzed in the same way as the second indifferent situation. Hence, the demands of each platform is given by

- Platform 2's Demand Functions

$$\begin{aligned} n_s^2 &= \frac{1}{t_s t_b - 16\alpha_s\alpha_b}[2t_b(f - 2\alpha_s - p_s^2 + p_s^M) + 8\alpha_s(f - 2\alpha_b - p_b^2 + p_b^M)] \\ n_b^2 &= \frac{1}{t_s t_b - 16\alpha_s\alpha_b}[2t_s(f - 2\alpha_b - p_b^2 + p_b^M) + 8\alpha_b(f - 2\alpha_s - p_s^2 + p_s^M)] \end{aligned} \tag{62}$$

- Platform M's Demand Functions

$$\begin{aligned} n_s^M &= \frac{1}{t_s t_b - 16\alpha_s\alpha_b}[2t_b(f - 2\alpha_s - p_s^M + p_s^2) + 8\alpha_s(f - 2\alpha_b - p_b^M + p_b^2)] \\ n_b^M &= \frac{1}{t_s t_b - 16\alpha_s\alpha_b}[2t_s(f - 2\alpha_b - p_b^M + p_b^2) + 8\alpha_b(f - 2\alpha_s - p_s^M + p_s^2)] \end{aligned} \tag{63}$$

In the premerger stage, platform 2 chooses $\{p_s^2, p_b^2\}$ to maximize the profit function π^2 and platform M chooses $\{p_s^M, p_b^M\}$ to maximize the profit function π^M. In the case of a symmetric equilibrium, each platform set the same price levels on each side and derives the outcome which should be the same as Proposition 2 that $p_s^* = c_s + f - 2\alpha_s$ and $p_b^* = c_b + f - 2\alpha_b$. Therefore, the equilibrium demands should be $n_s^{2*} = n_s^{M*} = 1$ and $n_b^{2*} = n_b^{M*} = 1$. The maximized profits should be $\pi^{2*} = \pi^{M*} = (f - 2\alpha_s) + (f - 2\alpha_b)$.

Comparing the three indifferent situations above, the following results can be derived. (1) Platform M can only generate market shares under non-localized competition, where any platform on the circle competes with platform M by choosing pricing levels on each side simultaneously. (2) Under each situation, the equilibrium prices are determined by three components. The first component captures the platform's marginal serving cost on each side. The second component captures the brand differentiation between two competing platforms. The third component captures how network externalities on each side adjust downward each platform's pricing level charged. (3) Compared localized competition with non-localized competition, two competing platforms equally distribute the total market shares in equilibrium. However, the equilibrium pricing level charged on each side under non-localized competition is lower than equilibrium pricing level charged on each side under localized competition, since the brand differentiation of two platforms under non-localized competition is lower than localized competition.

1.3.3 Postmerger Equilibrium

Consider an exogenous merger between platform M and platform 1 (note: the analysis will not differ if there is an exogenous merger between platform M and platform 2), and assume that no new platforms enter the market. Thus, platform 1 and platform M become the insider, denoted by platform I, while platform 2 remains the outsider, denoted by platform O. The two-sided market in this model setting is more like digital platforms, such as social media platforms and e-commerce platforms, etc. Therefore, the rationale for a merger is that the merger strategy can quickly integrate the user base of the target platform and expand the user scale in the two-sided market, thereby enhancing the platform stickiness through cross-group network externalities. Another incentive would be that digital platforms can eliminate market threats and reduce competitive pressure by merging potential competitors. For example, an exogenous merger incentive of two ride-hailing platforms could be that each merger participant wants to realize a surge in the number of drivers and passengers while curbing potential competition in the ride-hailing platform sector.

Insider (Platform 1 + Platform M)

The insider can lead to joint pricing decisions $\{p_s^I, p_b^I\}$, without changing the location of any platform. Furthermore, agents from any insider platform can benefit from the total group size on the other side by both platforms. Denote the number of agents of the insider as $\{n_s^I, n_b^I\}$, where $n_s^I = n_s^1 + n_s^M$, and $n_b^I = n_b^1 + n_b^M$. Thus, the utility of an agent from each group on the insider is determined by

$$u_s^I = \alpha_s n_b^I + \beta_s - p_s^I - f \tag{64}$$

$$u_b^I = \alpha_b n_s^I + \beta_b - p_b^I - f \tag{65}$$

This merger strategy has separated platform 1 from localized competition. Although the location of platform 1 has not changed, as part of an insider, platform 1 is supposed to have stronger utility in the post-merger stage. This utility is achieved by significantly increasing the participation value of the agents on the other side due to the expansion of the agent scale on one side, rather than the higher brand preferences (t). In addition, the setting of transportation cost f in the insider's utility function is also to better compare the analysis results of "2+1" platform mergers and "$N+1$" platform mergers. When there are N ($N > 2$) platforms on the circle, platform M can simultaneously attract agents distributed between any two adjacent platforms. This enables the platform M to be more dominant than other insider participants through the agent scale advantages accumulated at the pre-merger stage. Therefore, platforms that choose the merger strategy will operate at the transportation cost of platform M (f).

The profit function of the insider can be written as:

$$\pi^I = (p_s^I - c_s)n_b^I + (p_b^I - c_b)n_b^I \tag{66}$$

Outsider (Platform 2)

The outsider makes pricing decision $\{p_s^O, p_b^O\}$, and attracts number of agents $\{n_s^O, n_b^O\}$. Suppose a seller is located at a distance $y_s^1 \in [0,1]$ from platform 1 and a buyer is located at a distance $y_b^1 \in [0,1]$ from platform 1. Thus, the utility of an agent from each group on the outsider is given by

$$u_s^O = \alpha_s n_b^O + \beta_s - p_s^O - t_s(1 - y_s^1) \tag{67}$$

$$u_b^O = \alpha_b n_s^O + \beta_b - p_b^O - t_b(1 - y_b^1) \tag{68}$$

The profit function of the outsider can be written as:

$$\pi^O = (p_s^O - c_s)n_s^O + (p_b^O - c_b)n_b^O \tag{69}$$

Similar to premerger stage, given utility choices and brand preferences of insider and outsider, either insider and outsider platform can generate its market shares and maximized their profits under the following indifferent situation.

The seller is indifferent between insider platforms and the outsider platform if and only if

$$u_s^I = u_s^O \tag{70}$$

This equation generates insider's market shares on the seller side $n_s^I = 2y_s^1$, outsider's market shares on the seller side $n_s^O = 2(1 - y_s^1)$. Hence, substituting utility function of u_s^I and u_s^O into the equation and obtain

$$n_s^I = \frac{2f}{t_s} + \frac{4\alpha_s(n_b^I - 1) - 2(p_s^I - p_s^O)}{t_s} \tag{71}$$

$$n_s^O = \frac{2f}{t_b} + \frac{4\alpha_s(n_b^O - 1) - 2(p_s^O - p_s^I)}{t_s} \tag{72}$$

Since two sides are symmetric, insider and outsider's market shares on the buyer side is

$$n_b^I = \frac{2f}{t_s} + \frac{4\alpha_b(n_s^I - 1) - 2(p_b^I - p_b^O)}{t_b} \tag{73}$$

$$n_b^O = \frac{2f}{t_b} + \frac{4\alpha_b(n_s^O - 1) - 2(p_b^O - p_b^I)}{t_b} \tag{74}$$

Compared with the indifferent seller between platform 1 and platform 2 in the premerger stage, it is noteworthy that platform M, as a participant of insider, also generates market shares on each side. Also, in the postmerger stage, for either insider or outsider, if all prices keep fixed, an extra buyer will attract $4(\frac{\alpha_s}{t_s})$ sellers, and an extra seller will attract extra $4(\frac{\alpha_b}{t_b})$ number of buyers, which are more than the premerger stage. Compared with the indifferent seller between platform 2 and platform M in the premerger stage, it is noteworthy that platform 1, as a participant of insider, also generates market shares on each side. Therefore, in the postmerger stage, each platform, either as a participant of insider or a participant of outsider, can generates market shares.

Solving equations (71) and (73) simultaneously can yield the demand functions of insider platform, which are only determined by prices of each platform.

- Insider's Demand Functions

$$\begin{aligned} n_s^I &= \frac{1}{t_s t_b - 16\alpha_s\alpha_b}[2t_b(f - 2\alpha_s - p_s^I + p_s^O) + 8\alpha_s(f - 2\alpha_b - p_b^I + p_b^O)] \\ n_b^I &= \frac{1}{t_s t_b - 16\alpha_s\alpha_b}[2t_s(f - 2\alpha_b - p_b^I + p_b^O) + 8\alpha_b(f - 2\alpha_s - p_s^I + p_s^O)] \end{aligned} \tag{75}$$

Solving equations (72) and (74) simultaneously can yield the demand function of outsider platform, which are only determined by prices of each platform.

- Outsider's Demand Functions

$$\begin{aligned} n_s^O &= \frac{1}{t_s t_b - 16\alpha_s\alpha_b}[2t_b(f - 2\alpha_s - p_s^O + p_s^I) + 8\alpha_s(f - 2\alpha_b - p_b^O + p_b^I)] \\ n_b^O &= \frac{1}{t_s t_b - 16\alpha_s\alpha_b}[2t_s(f - 2\alpha_b - p_b^O + p_b^I) + 8\alpha_b(f - 2\alpha_s - p_s^O + p_s^I)] \end{aligned} \tag{76}$$

Since $n_s^I + n_s^O = 2$ and $n_b^I + n_b^O = 2$, following Proposition 2 can obtain $f = \frac{1}{2}t$. In the postmerger stage, insiders (platform 1 and platform M) choose $\{p_s^I, p_b^I\}$

to maximize the profit function π^I. The maximization problem can be written as

$$\max_{p_s^I, p_b^I} \pi^I = (p_s^I - c_s)n_s^I + (p_b^I - c_b)n_b^I \tag{77}$$

The outsider (platform 2) chooses $\{p_s^O, p_b^O\}$ to maximize outsider's profit function π^O. The maximization problem can be written as

$$\max_{p_s^O, p_b^O} \pi^O = (p_s^O - c_s)n_s^O + (p_b^O - c_b)n_b^O \tag{78}$$

In the case of a symmetric equilibrium, each platform set the same price levels on each side and derives the following proposition.

Proposition 6. *In a circular market with "2+1" platforms, at postmerger stage, the pricing competition between insider and outsider derives the symmetric equilibrium that*

$$p_s^{I*} = p_s^{O*} = c_s + f - 2\alpha_b = c_s + f - 2\alpha \tag{79}$$

$$p_b^{I*} = p_b^{O*} = c_b + f - 2\alpha_s = c_b + f - 2\alpha \tag{80}$$

Proof. *The first order conditions of insider platform's profit maximization problem can obtain:*

$$\begin{aligned} p_s^{I*} &= \frac{1}{2}c_s + \frac{1}{2}f - \alpha_s + \frac{1}{2}p_s^{O*} \\ p_b^{I*} &= \frac{1}{2}c_b + \frac{1}{2}f - \alpha_b + \frac{1}{2}p_b^{O*} \end{aligned} \tag{81}$$

The first order conditions of outsider platform's profit maximization problem can obtain:

$$\begin{aligned} p_s^{O*} &= \frac{1}{2}c_s + \frac{1}{2}f - \alpha_s + \frac{1}{2}p_s^{I*} \\ p_b^{O*} &= \frac{1}{2}c_b + \frac{1}{2}f - \alpha_b + \frac{1}{2}p_b^{I*} \end{aligned} \tag{82}$$

Solving equations (81) and (82) can then derive the symmetric equilibrium. The second-order conditions are $\frac{\partial^2 \pi^I}{\partial (p_s^I)^2} = \frac{\partial^2 \pi^I}{\partial (p_b^I)^2} = -\frac{4t}{t^2 - 16\alpha^2} < 0$ *and* $\frac{\partial^2 \pi^I}{\partial p_s^I \partial p_b^I} = \frac{\partial^2 \pi^I}{\partial p_b^I \partial p_s^I} = \frac{-16\alpha}{t^2 - 16\alpha^2} <$

0, *which prove the stability of the equilibrium results.*

The implications of Proposition 4 are as follows. Compared with localized competition equilibrium results at the premerger stage, where the agent is indifferent between platform 1 and platform 2, the equilibrium pricing level charged at the postmerger stage becomes lower. That is, both the insider and outsider platforms decrease its pricing level on each side. Since the platform M did not gain any market share at the premerger stage, the merger strategy will pay more attention to agent retention rather than price increases. In reality, many platforms will first expand their agent base and consolidate their market position through mergers, and then achieve profitability through value-added services at a certain stage in the future, rather than adopting a strategy of directly raising prices at the beginning of the merger. However, compared with nonlocalized competition equilibrium results at the premerger stage, where the agent is indifferent between platform 1 (or platform 2) and platform M, the equilibrium price level charged at the postmerger stage remains unchanged (see Proposition 3). That is, the merger strategy has no effect on both the insider and the outsider. In a circular market with "2+1" platforms, the type of market competition caused by this merger strategy is still an indifferent situation between the center platform and any platform on the circle. Therefore, compared to the results of Proposition 4,there will be no significant changes in insider or outsider pricing strategies at the premerger stage.

Under the equilibrium prices, the equilibrium demand and profits of insider and outsider can be obtained as

$$\begin{aligned} n_s^{I*} &= n_s^{O*} = 1 \\ n_b^{I*} &= n_b^{O*} = 1 \end{aligned} \tag{83}$$

$$\pi^{I*} = \pi^{O*} = (f - 2\alpha_s) + (f - 2\alpha_b) = 2(f - 2\alpha) \tag{84}$$

The consumer surplus of insider and outsider can be obtained as

$$CS^{I*} = CS^{O*} = \frac{(3\alpha + \beta - f - c_s)^2 + (3\alpha + \beta - f - c_b)^2}{2t} \tag{85}$$

Compared with localized competition equilibrium results at premerger stage, where agent is indifferent between platform 1 and platform 2, the merger strategy has no market share effect on the outsider and platform 1 as a participant of insider. Thus, the profit of platform 1 and platform 2 at postmerger stage becomes lower by charging a lower pricing level. However, the merger strategy has a positive effect on the market share on the platform M, which also captures half of the total market share on each side. Thus, platform M makes positive profits π^{I*} as a participant of insider than zero profits at premerger stage. Furthermore, there is a significant increase in consumer surplus for all platforms in the post-merger stage. For platform M, the merger strategy has increased the number of buyers and sellers at the same time, forming a denser interactive network and breaking through the zero consumer surplus at the premerger stage. For platforms 1 and 2, the merger strategy changes the transportation costs of agents from being dominated by the t parameter to being dominated by f parameter, which directly increases their consumer surplus.

Compared with non-localized competition equilibrium results at premerger stage, where agent is indifferent between platform 1 (or platform 2) and platform M, the merger strategy has no market share effect on platform M, therefore its profit maintains unchanged that $\pi^{M*} = \pi^{I*}$. The merger strategy also has no market share effect on the platform which participated non-localized competition under each indifferent situation at premerger stage, which also realize the same profit. Therefore, the merger strategy does not change platform 1 and platform M's consumer surplus. Finally, the merger strategy has a positive market share effect on the platform which did not participate non-localized competition under each indifferent situation at premerger, which therefore realizes positive profits at post-merger stage than zero profit at premerger stage. Thus, the consumer surplus also increases at the postmerger stage.

1.3.4 Effects of Merger

Comparing the results of the premerger equilibrium and the postmerger equilibrium, the following propositions can be obtained.

Proposition 7. *In a circular market with "2+1" platforms, merger strategy will not change the profitability of either an insider platform or an outsider platform if that platform generated non-zero profits under any non-localized competitions at premerger stage.*

Proof. *According to Proposition 3 and Proposition 4, if there exists non-localized competition between platform 1 and platform M at premerger stage, then* $\pi^{1*} = \pi^{M*} = \pi^{I*} = (f - 2\alpha_s) + (f - 2\alpha_b)$*. Similarly, if there exists non-localized competition between platform 2 and platform M at premerger stage, then* $\pi^{2*} = \pi^{M*} = \pi^{I*} = \pi^{O*} = (f - 2\alpha_s) + (f - 2\alpha_b)$.

The implications of Proposition 7 are as follows. The profit margin on each side is determined by the brand differentiation and network externalities. At the postmerger stage, the insider and the outsider only have non-localized competition, so the brand differentiation term in equilibrium is given by f. If any insider platform or outsider platform also generated market shares in non-localized competition at the premerger stage, the brand differentiation term in equilibrium should also be f. In addition, the adjustment of network externalities does not change at postmerger equilibrium as well. Therefore, the combined effect on the profit margin of that platform should be the same at both stages. Thus, the profitability remains unchanged at the post-merger stage. Under non-localized competition, buyers and sellers at the post-merger stage still face the same indifferent situation as at the pre-merger stage. Therefore, the merger strategy will not lead to significant agent mobility, which will also limit the expansion of the market power of the insider and the outsider. In addition, whether at the pre-merger stage or at the post-merger stage, a city circular market with "2+1" platforms meets the typical characteristics of an oligopoly market structure, so the insider and the outsider have strong strategic interaction with each other. Both will avoid triggering competitive price cuts by maintaining the original price. Despite the center platform merging with a platform on the circle, the remaining platform can still maintain their competitiveness through brand differentiation, forcing the insider platform to be unable to increase profits by raising prices. Therefore, if the merger strategy does not significantly change the agent scale or the strategic response of competitors, the equilibrium price and the profit level will remain unchanged.

Proposition 8. *In a city circular market with "2+1" platforms, merger strategy will be profitable for either an insider platform or an outsider platform as if that platform generated zero market shares under any types of competition at premerger stage.*

Proof. *According to Proposition 1 and Proposition 6, if there exists localized competition between platform 1 and platform 2 at premerger stage, then* $\pi^{M*} = 0 < \pi^{I*} = (f - 2\alpha_s) + (f - 2\alpha_b)$. *According to Proposition 3 and 4, if there exists non-localized competition between platform 1 and platform M at premerger stage, then* $\pi^{2*} = 0 < \pi^{O*} = (f - 2\alpha_s) + (f - 2\alpha_b)$. *If there exists non-localized competition between platform 2 and platform M at the premerger stage, then* $\pi^{1*} = 0 < \pi^{I*} = (f - 2\alpha_s) + (f - 2\alpha_b)$.

The implications of Proposition 8 are as follows. Compared with premerger stage, merger strategy in this model setting plays an effective role of distributing total market shares among all platforms in this circular market. For any platform that generated zero market shares at premerger stage, either participating as an insider and enjoying joint market shares or remaining as an outsider and competing with insiders can cause that platform to make positive profits at the post-merger stage, which is equal to $(f - 2\alpha_s) + (f - 2\alpha_b)$.

Proposition 9. *In a city circular market with "2+1" platforms, merger strategy will not be profitable for either an insider platform or an outsider platform if it generates non-zero profits under localized competition at premerger stage.*

Proof. *According to Proposition 1 and Proposition 6, if there existed localized competition between platform 1 and platform 2 at premerger stage, then* $\pi^{1*} = \pi^{2*} = (t_s - 2\alpha_s) + (t_b - 2\alpha_b)$. *While at postmerger stage, both platform 1 and platform 2's profit decrease to* $\pi^{I*} = \pi^{O*} = (f - 2\alpha_s) + (f - 2\alpha_b)$.

The implications of Proposition 9 are as follows. At premerger stage, agent's brand differentiation between platform 1 and platform 2 under localized competition is given by the parameter t. While at postmerger stage, platform 1 becomes an insider and platform 2 becomes an outsider, so the competition changes to non-localized competition. Thus, the agent's brand differentiation is given by $f = \frac{1}{2}t$. This reduction of brand differentiation leads to both platform 1 and platform 2's pricing level decreasing. Since merger strategy has no effect on the market shares of platform 1 or platform 2 compared to the localized competition at premerger stage, the maximized profits of both platforms decrease by t.

Proposition 10. *In a city circular market with "2+1" platforms, merger strategy will not change social welfare if platforms have non-localized competition at premerger stage.*

Proof. *The social welfare of platform I is written as* $SW^I = CS^I + \pi^I$ *and the social welfare of platform O is written as* $SW^O = CS^O + \pi^O$. *Therefore, in the sym-*

metric case, the social welfare maximization problem derives the following first order conditions.

$$\frac{\partial SW}{\partial p_s} = 2[-\frac{\alpha_s n_b^I + \beta_s - p_s}{t_s} + n_s^1 - \frac{(p_s - c_s)t_b + (p_b - c_b)2\alpha_b}{t_s t_b - 4\alpha_s \alpha_b}] = 0 \tag{86}$$

$$\frac{\partial SW}{\partial p_b} = 2[-\frac{\alpha_b n_s^I + \beta_b - p_b}{t_b} + n_b^1 - \frac{(p_s - c_s)2\alpha_s + (p_b - c_b)t_s}{t_s t_b - 4\alpha_s \alpha_b}] = 0 \tag{87}$$

Solving two equations simultaneously can derive the equilibrium outcome.

$$p_s^{SW'} = c_s + \frac{t - 4\alpha}{2} + \frac{2\alpha(f - 2\alpha)}{t - 4\alpha} \tag{88}$$

$$p_b^{SW'} = c_b + \frac{t - 4\alpha}{2} + \frac{2\alpha(f - 2\alpha)}{t - 4\alpha} \tag{89}$$

The second-order conditions are $\frac{\partial^2 SW}{\partial p_s^2} = \frac{\partial^2 SW}{\partial p_b^2} = \frac{-2}{t} < 0$*, which shows the stability of the equilibrium results.Compared to the result of Proposition 5,* $p_s^{SW'} = p_s^{SW}$ *and* $p_b^{SW'} = p_b^{SW}$.

The implication of Proposition 10 are as follows. In a city circular market with "2+1" platforms, the merger strategy does not change the inherent stability of non-localized competition among platforms. Therefore, the pricing levels of insider and outsider after the merger maintain the original market segmentation between the center platform and any platform on the circle and avoid excessive concentration of agents. Thus, social welfare will remain stable after the merger.

In summary, in a circular market with "2+1" platforms, an exogenous merger strategy removes the localized competition, and therefore causes agent's brand differentiation to maintain at a lower level. Thus, either insider platforms or the outsider platform does not charge a higher pricing level on each side. Since there are only two platforms on the circle, the market is less competitive. Hence, the merger strategy only gets all three platforms engaged from the perspective of generating market shares, while no significant results are shown on demand expansion issues. Thus, whether platform 1 or platform M has an incentive to merge depends on whether they generated zero market shares under competition at the pre-merger stage. Platform 2, as an outsider, will be profitable at postmerger stage only if it generated zero market shares under non-localized competition at premerger stage. Therefore, from the perspective of efficiency gains, the merger strategy can help the platform that failed to make profits before participating in mergers to integrate more buyers and sellers by expanding market share. Thus, consumer surplus of that platforms increase significantly in the postmerger stage. In contrast, the merger strategy does not change the total social welfare if there exists non-localized competition in the pre-merger stage. Finally, although the merger strategy increases market concentration, monopoly pricing does not occur due to the existence of the outside platform and the restriction of $f \leq \frac{1}{2}t$, so the monopoly trend is limited. In the next section, a market with $N+1$ ($N > 2$) platforms will be considered, and the analysis will be based on the characteristics of a more competitive market.

1.4 Market with $N + 1$ Platforms ($N > 2$)

This section considers a circular market with N platforms located equally distant on the circle, where $N > 2$, and platform M in the center of the circle. The radius of the circular market equals f and the perimeter equals N. Thus, the distance between two adjacent platforms is equal to 1. The number of platforms in a circular market will directly affect the market competition results and the impact of merger strategies. In a circular market with "2+1" platforms, due to the limited number of platforms, the two-sided market is more in line with the characteristics of an oligopoly market. A circular market with "$N+1$" platforms

($N > 2$) is more competitive. In non-localized competition, the platform in the center can simultaneously attract agents located between any two platforms on the circle, so that each agent will obtain different network effects when choosing the central platform or the platform on the circle. This may further lead to each platform adopting pricing and merger strategies that are different from those in the market with "2+1" platforms. Therefore, the market with "$N+1$" platforms cannot directly replicate the analysis results of the market with "2+1" platforms. Sellers and buyers are still distributed uniformly on the circle and choose to be single-homing for exogenous reasons. Similarly to the market with "2+1" platforms, the following assumptions are provided to conduct a more tractable analysis.

Assumption 3. *In this circular market with $N+1$ platforms, there exists perfect symmetry between the two sides so that $\alpha_s = \alpha_b = \alpha$, $t_s = t_b = t$ and $\beta_s = \beta_b = \beta$.*

Thus, any agent on one side enjoys the same benefit by interacting with an agent on the other side, and any agent between two neighboring platforms has the same brand differentiation.

Assumption 4. *For a market with N platforms on the circle, $t^2 > 4(N+1)^2\alpha^2$.*

Again, this assumption simplifies the analysis of the model, making brand differentiation the main driving force for agents to choose between two neighboring platforms on the circle and avoiding extreme market concentration. Therefore, Assumption 4 also ensures the stability of multiple platforms coexisting in the two-sided market. Denote any two adjacent platforms as platform i and platform j ($i \neq j$). Suppose a seller is located at a distance x_s^i from the platform i and a buyer is located at a distance x_b^i from platform i. Therefore, an agent located between the platform i and the platform j derives the utility of each platform as:

- An agent's utility of joining platform i

$$u_s^i = \alpha_s n_b^i + \beta_s - p_s^i - t_s x_s^i \tag{90}$$

$$u_b^i = \alpha_s n_s^i + \beta_b - p_b^i - t_b x_b^i \tag{91}$$

where $\{n_s^i, n_b^i\}$ are the number of sellers and buyers attracted by platform i; $\{p_s^i, p_b^i\}$ are the prices charged by platform i.

- An agent's utility of joining platform j

$$u_s^j = \alpha_s n_b^j + \beta_s - p_s^j - t_s(1 - x_s^i) \tag{92}$$

$$u_b^i = \alpha_s n_s^j + \beta_b - p_b^j - t_b(1 - x_b^i) \tag{93}$$

where $\{n_s^j, n_b^j\}$ are the number of sellers and buyers attracted by platform j; $\{p_s^j, p_b^j\}$ are the prices charged by platform j. An agent's utility of joining platform M is given by equation (5) and (6).

In this section, platforms still provide horizontally differentiated products and compete in prices. Then the profit function of each platform can be written as:

- Profit of platform i

$$\pi^i = (p_s^i - c_s)n_s^i + (p_b^i - c_b)n_b^i \tag{94}$$

- Profit of platform j

$$\pi^j = (p_s^j - c_s)n_s^j + (p_b^j - c_b)n_b^j \tag{95}$$

Profit of platform M is given by equation (10).

1.4.1 Premerger Equilibrium

Given utility choices and brand preferences of platform i, platform j and platform M, agents can either travel around the circle or travel to the center.

Thus, each platform can generate its market shares and maximized their profits under the following indifferent situations.

(1) The seller is indifferent between platform i and platform j.

The first indifferent situation leads to localized competition between any two adjacent platforms, where the platform M generates zero market shares on each side. That is, the indifferent situation between platform i and platform j is the same as the indifferent situation between platform 1 and platform 2 in the circular market with 2+1 platforms. Therefore, the equilibrium result of the pre-merger should also be the same as shown in the circular market with "2+1" platforms that $p_s^* = c_s + t_s - 2\alpha_b$ and $p_b^* = c_b + t_b - 2\alpha_s$.

(2) The seller is indifferent between platform i and platform M.

$$u_s^i = u_s^M \tag{96}$$

This equation generates platform i's market shares on the seller side $n_s^i = 2x_s^i$ and its neighboring platform, platform j's market shares on the seller side $n_s^j = 0$. Platform M is independent of location and, therefore, can generate the rest of market shares on the seller side from every two adjacent platforms that $n_s^M = N(1 - x_s^i)$. Hence, substituting utility functions of u_s^i and u_s^M into the indifferent equation and obtain

$$n_s^i = \frac{2f}{t_s} + \frac{2N\alpha_s(\frac{N+2}{2N}n_b^i - 1) - 2(p_s^i - p_s^M)}{t_s} \tag{97}$$

$$n_s^M = N(1 - \frac{f}{t_s}) + \frac{2N\alpha_s(\frac{N+2}{2N}n_b^M - 1) - N(p_s^M - p_s^i)}{t_s} \tag{98}$$

Symmetrically, each platform's market shares on the buyer side is

$$n_b^i = \frac{2f}{t_b} + \frac{2N\alpha_b(\frac{N+2}{2N}n_s^i - 1) - 2(p_b^i - p_b^M)}{t_b} \tag{99}$$

$$n_b^M = N(1-\frac{f}{t_b}) + \frac{2N\alpha_b(\frac{N+2}{2N}n_s^M - 1) - N(p_b^M - p_b^i)}{t_b} \tag{100}$$

For either platform i or platform M, if all prices keep fixed, an extra buyer will attract extra $(N+2)\frac{\alpha_s}{t_s}$ number of sellers, and an extra seller will attract extra $(N+2)\frac{\alpha_b}{t_b}$ number of buyers. The more platforms exist on the circle, the more number of extra agents that can be captive to each platform. Furthermore, the coefficient of the price variables in equations (97) and (99) is $\frac{2}{t}$, while the coefficient of price variables in equation (98) and (100) is $\frac{N}{t}$. Since $\frac{2}{t} < \frac{N}{t}$, agents who join platform M respond much more sensitively to changes in pricing levels than agents who join platform i.

Solving equations (97) and (99) simultaneously can yield demand functions of the platform i, which are determined by the prices of the platform i and the platform M.

- Platform i's Demand Functions

$$\begin{aligned} n_s^i &= \frac{2t_b(f - N\alpha_s - p_s^i + p_s^M) + 2(N+2)\alpha_s(f - N\alpha_b - p_b^i + p_b^M)}{t_s t_b - (N+2)^2\alpha_s\alpha_b} \\ n_b^i &= \frac{2t_s(f - N\alpha_b - p_b^i + p_b^M) + 2(N+2)\alpha_b(f - N\alpha_s - p_s^i + p_s^M)}{t_s t_b - (N+2)^2\alpha_s\alpha_b} \end{aligned} \tag{101}$$

Assumption 4 implies that $t > 2(N+1)\alpha > (N+2)\alpha$, therefore $t_s t_b - (N+2)^2\alpha_s\alpha_b > 0$. Thus, platform i can generate positive demands on each side. Solving equations (98) and (100) simultaneously can yield the demand functions of platform M, which are determined by prices of both platform i and platform M.

- Platform M's Demand Functions

$$\begin{aligned} n_s^M &= \frac{Nt_b(t_s - f - 2\alpha_s - p_s^M + p_s^i) + N(N+2)\alpha_s(t_b - f - 2\alpha_b - p_b^M + p_b^i)}{t_s t_b - (N+2)^2\alpha_s\alpha_b} \\ n_b^M &= \frac{Nt_s(t_b - f - 2\alpha_b - p_b^M + p_b^i) + N(N+2)\alpha_b(t_s - f - 2\alpha_s - p_s^M + p_s^i)}{t_s t_b - (N+2)^2\alpha_s\alpha_b} \end{aligned} \tag{102}$$

Different from a 2+1 market, the demand functions of each platform under non-localized competition in a "$N+1$" market is asymmetric. When platform M can simultaneously attract buyers between any two platforms on the circle through non-localized competition, this directly affects the seller's utility as the cross-group network externalities brought by the buyer increase, so platform M will also simultaneously attract sellers between any two platforms on the circle. In contrast, this non-localized competition can only allow platform i to attract buyers and sellers located near it, so the cross-group network externalities that each buyer or seller can enjoy are far less than those of buyers and sellers on platform M. For platform i, the number of agents on each side is adjusted by $\frac{2t}{t^2-(N+2)^2\alpha^2}$ of both platform's pricing decision on the same side and adjusted by $\frac{2(N+2)\alpha}{t^2-(N+2)^2\alpha^2}$ of both platform's pricing decision on the different side. For platform M, the number of agents on each side is adjusted by $\frac{Nt}{t^2-(N+2)^2\alpha^2}$ of both platform's pricing decision on the same side and adjusted by $\frac{N(N+2)\alpha}{t^2-(N+2)^2\alpha^2}$ of both platform's pricing decision on the different side. When $N > 2$, the total number of platforms on the circle plays a more significant role on affecting platform M's demands than on affecting platform i's demands. In addition, the sensitivity of buyer demand to sellers pricing is more affected by the number of platforms than the sensitivity of buyer demand to buyers pricing is affected by the number of platforms.

In the premerger stage, the platform i chooses $\{p_s^i, p_b^i\}$ to maximize the profit function π^i and the platform M chooses $\{p_s^M, p_b^M\}$ to maximize the profit function of π^M. The maximization problem of each platform can be written as

$$\max_{p_s^i, p_b^i} \pi^i = (p_s^i - c_s)n_s^i + (p_b^i - c_b)n_b^i \tag{103}$$

$$\max_{p_s^M, p_b^M} \pi^M = (p_s^M - c_s)n_s^M + (p_b^M - c_b)n_b^M \tag{104}$$

The equilibrium results are shown in the following proposition.

Proposition 11. *In a circular market with $N+1$ platforms ($N > 2$), at premerger stage, the non-localized price competition of platform i and platform M derives the unique solution $\{p_s^{i*}, p_b^{i*}, p_s^{M*}, p_b^{M*}\}$ that:*

$$p_s^{i*} = c_s + \frac{1}{3}(f+t) - \frac{2}{3}(N+1)\alpha \tag{105}$$

$$p_b^{i*} = c_b + \frac{1}{3}(f+t) - \frac{2}{3}(N+1)\alpha \tag{106}$$

$$p_s^{M*} = c_s + \frac{1}{3}(2t-f) - \frac{1}{3}(N+4)\alpha \tag{107}$$

$$p_b^{M*} = c_b + \frac{1}{3}(2t-f) - \frac{1}{3}(N+4)\alpha \tag{108}$$

Proof. *The first order conditions of platform i's maximization problem can obtain*

$$\begin{aligned} p_s^{i*} &= \frac{1}{2}c_s + \frac{1}{2}(f-N\alpha) + \frac{1}{2}p_s^{M*} \\ p_b^{i*} &= \frac{1}{2}c_b + \frac{1}{2}(f-N\alpha) + \frac{1}{2}p_b^{M*} \end{aligned} \tag{109}$$

Solving equations simultaneously can then derive the solutions. The second-order conditions are $\frac{\partial^2\pi^i}{\partial(p_s^i)^2} = \frac{\partial^2\pi^i}{\partial(p_b^i)^2} = \frac{-2t}{t^2-(N+2)^2\alpha^2} < 0$ and $\frac{\partial^2\pi^i}{\partial p_s^i \partial p_b^i} = \frac{\partial^2\pi^i}{\partial p_b^i \partial p_s^i} = \frac{-8(N+2)\alpha}{t^2-(N+2)^2\alpha^2} < 0$, which prove the stability of the equilibrium results.

The first order conditions of platform M's maximization problem can obtain

$$\begin{aligned} p_s^{M*} &= \frac{1}{2}c_s + \frac{1}{2}(t-f-2\alpha) + \frac{1}{2}p_s^{i*} \\ p_b^{M*} &= \frac{1}{2}c_b + \frac{1}{2}(t-f-2\alpha) + \frac{1}{2}p_b^{i*} \end{aligned} \tag{110}$$

Solving equations simultaneously can then derive the solutions. The second-order conditions are $\frac{\partial^2\pi^M}{\partial(p_s^M)^2} = \frac{\partial^2\pi^M}{\partial(p_b^M)^2} = \frac{-2Nt}{t^2-(N+2)^2\alpha^2} < 0$ and $\frac{\partial^2\pi^M}{\partial p_s^M \partial p_b^M} = \frac{\partial^2\pi^M}{\partial p_b^M \partial p_s^M} = \frac{-4N(N+2)\alpha}{t^2-(N+2)^2\alpha^2} < 0$, which prove the stability of the equilibrium results.

The implications of Proposition 11 are as follows. First, the pricing strategy of each platform is also determined by three parts: serving cost, brand differentiation term, and cross-group network term. In the $N+1$ model, the brand differentiation term includes both t and f. The price of the platform i is adjusted upward by $\frac{1}{3}(f+t)$, which is $\frac{1}{3}(t-2f)$ higher than the brand differentiation term in equations (43) and (44). The price of the platform M is adjusted upward by $\frac{1}{3}(t-2f)$, which is $\frac{2}{3}(t-2f)$ higher than the brand differentiation term in equations (43) and (44). Therefore, if only the brand preference of buyers or sellers is considered, platforms in more competitive markets will set the same or even higher pricing levels as those in oligopoly markets. In addition, the price of the platform i is adjusted downward by $\frac{2}{3}(N+1)\alpha$, which is $\frac{2}{3}(N-2)\alpha$ compared to the cross-group network term in equations (43) and (44). The price of the platform M is adjusted downward by $\frac{1}{3}(N+4)\alpha$, which is $\frac{1}{3}(N-2)\alpha$ compared to the cross-group network term in equations (43) and (44). Therefore, when the number of platforms in a two-sided market increases, the cross-group network externalities are stronger, and the platforms will adopt lower pricing strategies to attract a large number of buyers and sellers to join. Taking into account the combined effects of brand differentiation and cross-group network externalities, platforms have incentives to set lower prices in more competitive markets than in oligopolistic markets.

Under equilibrium prices, the demands of each platform on both sides are symmetric. Therefore, the demands and maximized profit of platform i are given by

$$n_s^{i*} = n_b^{i*} = \frac{2[f+t-2(N+1)\alpha]}{3[t-(N+2)\alpha]} \tag{111}$$

$$\pi^{i*} = \frac{4[f+t-2(N+1)\alpha]^2}{9[t-(N+2)\alpha]} \tag{112}$$

The demands and maximized profit of platform M are given by

$$n_s^{M*} = n_b^{M*} = \frac{N[2t - f - (N+4)\alpha]}{3[t - (N+2)\alpha]} \tag{113}$$

$$\pi^{M*} = \frac{2N[2t - f - (N+4)\alpha]^2}{9[t - (N+2)\alpha]} \tag{114}$$

Comparing equations (111) and (113) can derives that under equilibrium prices, the market share attracted by platform i is smaller than the market share generated by platform M. This is also because the cross-group network externalities of platform M are stronger than those of the platform i, attracting more buyers and sellers to choose the platform M, allowing it to obtain a larger market share. Comparing equations (112) and (114) can derive that, under equilibrium prices, the platform M generates higher profits than the platform i. Although platform M has not much advantage in pricing compared to platform i, the increase in the number of platforms in the "$N+1$" model has enabled platform M to significantly expand its market size by achieving stronger cross-group network externalities under non-localized competition. Therefore, platform M makes more profit than platform i. As N increases, the profitability difference between platform i and platform M will become more significant.

The consumer surplus of the platform i is written as

$$CS^i = CS_s^i + CS_b^i = \int_0^{x_s^i} u_s^i dx + \int_0^{x_b^i} u_b^i dx \tag{115}$$

Under equilibrium prices, the consumer surplus of each platform can be obtained as

$$CS^{i*} = \frac{(\alpha n_s^{i*} + \beta - p_s^{i*})^2 + (\alpha n_b^{i*} + \beta - p_b^{i*})^2}{2t} \tag{116}$$

$$CS^{M*} = \frac{(\alpha n_s^{M*} + \beta - p_s^{M*} - f)^2 + (\alpha n_b^{M*} + \beta - p_b^{M*} - f)^2}{2t} \tag{117}$$

The consumer surplus of platform i depends on the agent scale between two adjacent platforms on the circle and pricing levels, while the consumer surplus of platform M depends more on the diffusion of cross-group network effects and fixed transportation cost coverage. When comparing the consumer surplus of the platform i and the platform M, when N is large, the externalities of the network between groups of the platform M are significantly enhanced, leading to $CS^{M*} > CS^{i*}$.

The social welfare function of platform M is written as

$$SW^i = CS^i + \pi^i \tag{118}$$

To find the social optimal pricing level, the following proposition is derived.

Proposition 12. *In a city circular market with "$N+1$" platforms, in the pre-merger stage when the center platform competes with a platform on the circle, the social welfare maximization problem derives the equilibrium that:*

$$p_s^{SWi} = c_s + \frac{t - 2(N+1)\alpha}{2} + \frac{2\alpha(f - 2\alpha)}{t - (N+2)\alpha} \tag{119}$$

$$p_b^{SWi} = c_b + \frac{t - 2(N+1)\alpha}{2} + \frac{2\alpha(f - 2\alpha)}{t - (N+2)\alpha} \tag{120}$$

$$p_s^{SWM} = c_s + \frac{t - 2\alpha}{2} + \frac{2\alpha(f - 2\alpha)}{t - (N+2)\alpha} \tag{121}$$

$$p_b^{SWM} = c_b + \frac{t - 2\alpha}{2} + \frac{2\alpha(f - 2\alpha)}{t - (N+2)\alpha} \tag{122}$$

Proof. *The total social welfare function is written as $SW = SW^M + N(SW^i)$. The first-order condition of p_s^i is*

$$\frac{\partial SW}{\partial p_s^i} = N\left(\frac{\partial CS^i}{\partial p_s^i} + \frac{\partial \pi^i}{\partial p_s^i}\right) + \frac{\partial \pi^M}{\partial p_s^i} = 0 \tag{123}$$

Similarly, we derive $\frac{\partial SW}{\partial p_b^i} = 0$, $\frac{\partial SW}{\partial p_s^M} = 0$, *and* $\frac{\partial SW}{\partial p_b^M} = 0$. *Solving equations simultaneously can derive the equilibrium outcome. The second order conditions are* $\frac{\partial SW^2}{\partial (p_s^i)^2} = \frac{\partial^2 SW^2}{\partial (p_b^i)^2} = N(-\frac{1}{t} - \frac{4t}{t^2-(N+2)^2\alpha^2}) < 0$ *and* $\frac{\partial SW^2}{\partial (p_s^M)^2} = \frac{\partial^2 SW^2}{\partial (p_b^M)^2} = -\frac{1}{t} - \frac{2Nt}{t^2-(N+2)^2\alpha^2} < 0$, *which show the stability of equilibrium results.*

The implications of Proposition 12 are as follows. Similarly to the second indifferent situation of the "2+1" model, the equilibrium prices are composed of three terms. c_s and c_b ensure that the platform can cover basic serving costs. The second term of each equilibrium equation shows the balancing of t and the cross-group network externalities. The last term of each equilibrium equation reflects f interaction with cross-group network externalities to optimize resource allocation. Compared equations (119) and (120) with (121) and (122), $p_s^{SWi} < p_s^{SWM}$ and $p_b^{SWi} < p_b^{SWM}$. Since there are N homogeneous platforms i in the market, competition dilutes its cross-group network effect and limits its regulatory capacity. However, platform M has a larger agent base, so its cross-group network effects are stronger. Therefore, the price adjustment of the platform M has a more significant impact on social welfare. Furthermore, $p^{i*} - p^{SWi} = \frac{1}{12}(f+t) + \frac{1}{12}(N+1)\alpha > 0$ and $p^{M*} - p^{SWM} = \frac{1}{12}(2t-f) - \frac{1}{12}(N+4)\alpha > 0$. Therefore, $p^{SWi} < p^{i*}$ and $p^{SWM} < p^{M*}$.Social welfare maximization embodies collective rationality, and corrects market failures through price adjustments. Therefore, reducing p^{SWi} and p^{SWM} can promote the expansion of user scale, amplify the positive externalities of network effects, and improve overall market efficiency.

1.4.2 Postmerger Equilibrium

Consider an exogenous merger between platform M and platform i and assume that no new platforms enter the market. Thus, platform M and platform i become the insider, while the rest of $(N-1)$ platforms on the circle remain as the outsiders. Similarly to a three-platform model, the insider leads to a joint pricing decision and generates total demands of both platforms. In reality, even in more competitive markets, platforms can eliminate possible market threats and consolidate market shares by merging potential competitors. The utility of an agent from each group on the insider is determined as equations (48) and (49), and the profit function of the insider is written as equation (50).

Unlike a three-platform model, the outsider in this section consists of $(N-1)$ platforms, and localized competition still exists between any two adjacent outsider platforms. Suppose a seller is located at a distance $y_s^j \in [0,1]$ from the platform j and a buyer is located at a distance $y_b^j \in [0,1]$ from platform j. The utility of an agent from each group on platform j, where platform j is a participant of outsiders ($j \neq i$, and $j = 1, 2, ...N-1$), is given by

$$u_s^j = \alpha_s n_b^j + \beta_s - p_s^j - t_s y_s^j \tag{124}$$

$$u_b^j = \alpha_b n_s^j + \beta_b - p_b^j - t_b y_b^j \tag{125}$$

where $\{p_s^j, p_b^j\}$ are the prices charged by platform j, and $\{n_s^j, n_b^j\}$ are the number of agents attracted by platform j. Either insider or outsider platforms can generate their market shares in the following indifferent situations.

Then, there are two indifferent situations to consider: (1) The seller is indifferent between platform j and platform $j+1$, where $j+1 \neq i$. This leads to localized competition between any two adjacent outsiders, and therefore zero market shares are generated by the insider. (2) The seller is indifferent between insider platforms and platform j if and only if

$$u_s^I = u_s^j \tag{126}$$

This equation generates platform j's market shares on the seller side $n_s^j = 2y_s^j$, and the insider can generate the rest of market shares on the seller side from every two adjacent outsider platforms that $n_s^I = (N-1)(1-y_s^i)$. Hence, substituting utility functions of u_s^I and u_s^j into the indifferent equation and obtain

$$n_s^j = \frac{2f}{t_s} + \frac{2(N-1)\alpha_s[\frac{N+1}{2(N-1)}n_b^j - 1] - 2(p_s^j - p_s^I)}{t_s} \tag{127}$$

$$n_s^I = (N-1)(1-\frac{f}{t_s}) + \frac{2(N-1)\alpha_s[\frac{N+1}{2(N-1)}n_b^I - 1] - (N-1)(p_s^I - p_s^j)}{t_s} \tag{128}$$

Symmetrically, each platform's market shares on the buyer side is

$$n_b^j = \frac{2f}{t_b} + \frac{2(N-1)\alpha_b[\frac{N+1}{2(N-1)}n_s^j - 1] - 2(p_b^j - p_b^I)}{t_b} \tag{129}$$

$$n_b^I = (N-1)(1-\frac{f}{t_b}) + \frac{2(N-1)\alpha_b[\frac{N+1}{2(N-1)}n_s^I - 1] - (N-1)(p_b^I - p_b^j)}{t_b} \tag{130}$$

For either insider or outsider, if all prices keep fixed, an extra buyer will attract $(N+1)\frac{\alpha_s}{t_s}$ number of sellers, and an extra seller will attract $(N+1)\frac{\alpha_b}{t_b}$ number of buyers. The capability of attracting extra agents on the insider or outsider is better than a three-platform model at the postmerger stage (where extra $4\frac{\alpha}{t}$ agents are captive) only when there are more than three platforms on the circle. Furthermore, as the total number of platforms in the competition is reduced to N, each platform's ability to attract additional agents on the other side is also less than premerger stage. From the perspective of pricing, each outsider's market shares on each side is adjusted downward by the same influence of market's pricing strategy shown as $\frac{2}{t}$, while insider's market shares on each side are adjusted downward by a less influence shown as $\frac{N-1}{t}$ than at the premerger stage. Furthermore, compared to a three-platform model, insider agents respond more sensitively to price changes only when N increases.

Solving equations (127) and (129) can derive the demand functions of each outsider platform,given by

$$\begin{aligned} n_s^j &= \frac{2t_b[f-(N-1)\alpha - p_s^j + p_s^I] + 2(N+1)\alpha_s[f-(N-1)\alpha_b - p_b^j + p_b^I]}{t_s t_b - (N+1)^2\alpha_s\alpha_b} \\ n_b^j &= \frac{2t_s[f-(N-1)\alpha - p_b^j + p_b^I] + 2(N+1)\alpha_b[f-(N-1)\alpha_s - p_s^j + p_s^I]}{t_s t_b - (N+1)^2\alpha_s\alpha_b} \end{aligned} \tag{131}$$

Assumption 4 implies that $t > (N+1)\alpha$ and therefore, each outsider can generate positive demands on each side. Solving equations (128) and (130) simultaneously can derive the demand functions of insider.

$$\begin{aligned} n_s^I &= \frac{(N-1)t_b(t_s - f - 2\alpha_s - p_s^I + p_s^j) + (N^2-1)\alpha_s(t_b - f - 2\alpha_b - p_b^I + p_b^j)}{t_s t_b - (N+1)^2\alpha_s\alpha_b} \\ n_b^I &= \frac{(N-1)t_s(t_b - f - 2\alpha_b - p_b^I + p_b^j) + (N^2-1)\alpha_b(t_s - f - 2\alpha_s - p_s^I + p_s^j)}{t_s t_b - (N+1)^2\alpha_s\alpha_b} \end{aligned} \tag{132}$$

The demand functions of insider and outsider platforms are also asymmetric. Compared with the premerger stage, for each outsider platforms, the number of agents on each side is adjusted downward by a smaller influence of the platform's pricing decision on the same side, shown as $\frac{2t}{t^2-(N+1)^2\alpha^2}$, and adjusted downward by a smaller influence of the platform's pricing decision on the different side, shown as $\frac{2(N+1)\alpha}{t^2-(N+1)^2\alpha^2}$. For the insider, the number of agents on each side is adjusted downward by a smaller influence of the platform's pricing decision on the same side, shown as $\frac{(N-1)t}{t^2-(N+1)^2\alpha^2}$, and adjusted downward by a smaller influence of the platform's pricing decision on the different side, shown as $\frac{(N^2-1)\alpha}{t^2-(N+1)^2\alpha^2}$. When N increases, the adjustment of the platform pricing decision to affect the insider's market shares is more significant than affecting the outsider's market shares. Furthermore, compared to a three-platform model, both the insider and outsider platform pricing decision to adjust their own market share becomes more significant when N increases.

In the postmerger stage, insiders (platform i and platform M) choose $\{p_s^I, p_b^I\}$ to maximize the profit function π^I. Each outsider platform (platform j) chooses $\{p_s^j, p_b^j\}$ to maximize its profit function π^j. The equilibrium results are shown in the following proposition.

Proposition 13. *In a circular market with $N+1$ platforms ($N > 2$), at postmerger stage, the pricing competition between insider and outsider derives the unique solution $\{p_s^{I*}, p_b^{I*}, p_s^{O*}, p_b^{O*}\}$ that*

$$p_s^{I*} = c_s + \frac{1}{3}(2t - f) - \frac{1}{3}(N+3)\alpha \tag{133}$$

$$p_b^{I*} = c_s + \frac{1}{3}(2t - f) - \frac{1}{3}(N+3)\alpha \tag{134}$$

$$p_s^{O*} = c_s + \frac{1}{3}(t + f) - \frac{2}{3}N\alpha \tag{135}$$

$$p_b^{O*} = c_b + \frac{1}{3}(t + f) - \frac{2}{3}N\alpha \tag{136}$$

Proof. *The first order conditions of platform j's maximization problem can obtain*

$$\begin{aligned} p_s^{j*} &= \frac{1}{2}[f - (N-1)\alpha] + \frac{1}{2}p_s^{I*} \\ p_b^{j*} &= \frac{1}{2}[f - (N-1)\alpha] + \frac{1}{2}p_b^{I*} \end{aligned} \tag{137}$$

Solving equations simultaneously can then derive the equilibrium solutions. The second-order conditions are $\frac{\partial^2 \pi^j}{\partial (p_s^j)^2} = \frac{\partial^2 \pi^j}{\partial (p_b^j)^2} = \frac{-4t}{t^2 - (N+1)^2\alpha^2} < 0$ and $\frac{\partial^2 \pi^j}{\partial p_s^j \partial p_b^j} = \frac{-16(N+1)\alpha}{t^2 - (N+1)^2\alpha^2} < 0$, which show the stability of the equilibrium results. Since each outsider platform sets a symmetric price, therefore, $p_s^{O*} = p_s^{j*}$ and $p_b^{j*} = p_b^{O*}$.

The first order conditions of insider's maximization problem can obtain

$$
\begin{aligned}
p_s^{I*} &= \frac{1}{2}c_s + \frac{1}{2}(t - f - 2\alpha) + \frac{1}{2}p_s^{j*} \\
p_b^{I*} &= \frac{1}{2}c_b + \frac{1}{2}(t - f - 2\alpha) + \frac{1}{2}p_b^{j*}
\end{aligned}
\tag{138}
$$

Solving equations simultaneously can then derive the equilibrium solutions. The second-order conditions are $\frac{\partial^2 \pi^I}{\partial (p_s^I)^2} = \frac{\partial^2 \pi^I}{\partial (p_b^I)^2} = \frac{-2(N-1)t}{t^2-(N+1)^2\alpha^2} < 0$ *and* $\frac{\partial^2 \pi^I}{\partial p_s^I \partial p_b^I} = \frac{-4(N^2-1)\alpha}{t^2-(N+1)^2\alpha^2} < 0$*, which show the stability of the equilibrium results.*

The implications of Proposition 13 are as follows. Unlike a three-platform model, when $N > 2$, the postmerger equilibrium prices of insider and outsider are asymmetric. First, the impact of merger strategies on different insider platforms is different. After platform i chooses the merger strategy, its pricing for agents on both sides increases by $\frac{1}{3}[t + (N-1)\alpha]$, while platform M only increases its pricing for agents on both sides by $\frac{1}{3}\alpha$ after choosing the merger strategy. Since platform i becomes an insider, it can also participate in attracting agents between any two adjacent platforms on the circle, resulting in an expansion of market share in the two-sided market and an increase in market power. Therefore, platform i has stronger control over the needs of agents on both sides and can obtain higher profits by raising prices. As N increases, the gap in the pricing decision of the platform i between the premerger stage and the postmerger stage becomes larger. In contrast, the price increase of the platform M after becoming an insider is not significant. Since it has accumulated a relatively large agent base before participating in the merger, the platform M may be more willing to consolidate its market share through merger strategies. By maintaining a relatively stable price, it can attract more agents to join the platform, thereby enhancing the platform's cross-group network externalities and market competitiveness. In the long run, this will help the platform achieve more sustainable development and higher profits in the market, so the platform M will not achieve a significant price increase in the short term. Secondly, the merger strategy has resulted in the outsider platform (platform j) increasing its pricing for users on both sides by only $\frac{2}{3}\alpha$. That is, the pricing of each outsider platform is adjusted downward by a less degree of network externalities. Therefore, the gap in the price decision of an outsider platform between the pre-merger stage and the post-merger is independent of the total number of platforms in the market. Although the merger strategy will increase market concentration, there is still competition in the market. Platforms that are not involved in mergers need to maintain competitive prices to attract agents. If the price increase on one side is too significant, more agents on this side will be lost to the insider platform, which will further affect the cross-group network externalities of agents on the other side. Therefore, in order to maintain the number of agents on both sides, outsider platforms will also avoid significantly raising prices. Finally, by comparing the equilibrium pricing of insider platforms and outsider platforms, it can be seen that the merger strategy further strengthens cross-group network externalities, so outsider platforms have no incentive to set higher prices than insider platforms. As N increases, the price decision gap between the insider and the outsider increases. From the first-order conditions, it can also be seen that the merger strategy does not change the way the platform M responds to the pricing strategy of its rival platform, while the merger strategy causes each platform on the circle, either insider or outsider, to respond more positively to the pricing strategy of its rival platform.

Under the equilibrium prices, the demands and maximized profit of insider platforms are given by

$$n_s^{I*} = n_b^{I*} = \frac{(N-1)[2t - f - (N+3)\alpha]}{3[t-(N+1)\alpha]} \tag{139}$$

$$\pi^{I*} = \frac{2(N-1)[2t - f - (N+3)\alpha]^2}{9[t-(N+1)\alpha]} \tag{140}$$

Under the equilibrium prices, the demands and maximized profit of each outsider platforms are given by

$$n_s^{O*} = n_b^{O*} = \frac{2[f + t - 2N\alpha]}{3[t-(N+1)\alpha]} \tag{141}$$

$$\pi^{O*} = \frac{4[f + t - 2N\alpha]^2}{9[t-(N+1)\alpha]} \tag{142}$$

Platform i has integrated the agent scale and market power of platform M through merger strategies, resulting in a significant increase in its market share. In this way, the number of agents on both sides will further increase, and more agents on one side will attract more agents on the other side to join, strengthening cross-group network externalities and enhancing the value of the platform to agents. However, for the platform M, the growth of its platform market share after participating in the merger is limited. Even if a merger occurs, there are still a large number of competing platforms in the market that adopt various strategies to maintain or expand their market share, making it difficult for the platform M to significantly seize more market share at the postmerger stage. In addition, without considering network externalities, the outsider platform's market shares on each side do not change in postmerger equilibrium. Considering network externalities, each outsider platform generates fewer market shares than the premerger equilibrium. As N increases, the gap of each outsider's market shares generated by each outsider between the pre-merger equilibrium and the post-merger equilibrium becomes larger. Since the merger strategy has expanded the market size of insider platforms and strengthened their advantages in market competition, the market share of outsider platforms has declined. The comparison of profitability will be analyzed in the next section.

The consumer surplus of the insider and the outsider is written as

$$CS^I = CS_s^I + CS_b^I = \int_{y_s^j}^{1} u_s^I dy + \int_{y_b^j}^{1} u_b^I dy \tag{143}$$

$$CS^O = CS_s^O + CS_b^O = \int_{0}^{y_s^j} u_s^O dy + \int_{0}^{y_b^j} u_b^O dy \tag{144}$$

Under equilibrium prices, the consumer surplus of each platform can be obtained as

$$CS^{I*} = \frac{(\alpha n_s^{I*} + \beta - p_s^{I*} - f)^2 + (\alpha n_b^{I*} + \beta - p_b^{I*} - f)^2}{2t} \tag{145}$$

$$CS^{O*} = \frac{(\alpha n^{O*} + \beta - p_s^{O*})^2 + (\alpha n^{O*} + \beta - p_b^{O*})^2}{2t} \tag{146}$$

Since outsiders attract fewer users at the postmerger stage and the pricing level increase, outsiders' consumer surplus decrease significantly. For platform i, agents on both sides realize greater cross-group network externality utility due to the merger strategy, which leads to an increase in consumer surplus in the post-merger stage. However, for platform M, due to the merger strategy, its price growth and agent scale growth are quite limited, which results in the change in consumer surplus of platform M in the post-merger stage being less significant than that of platform i.

The social welfare function of the insider is written as

$$SW^I = CS^I + \pi^I \tag{147}$$

The social welfare function of each outsider is written as

$$SW^j = CS^j + \pi^j \tag{148}$$

To find the social optimal pricing level, the following proposition is derived.

Proposition 14. *In a city circular market with "$N+1$" platforms, in the postmerger stage, the social welfare maximization problem derives the equilibrium that:*

$$p_s^{SWI} = c_s + \frac{(N-1)(f-2\alpha)[t+(N+1)\alpha]}{t^2-(N+1)^2\alpha^2} \tag{149}$$

$$p_b^{SWI} = c_b + \frac{(N-1)(f-2\alpha)[t+(N+1)\alpha]}{t^2-(N+1)^2\alpha^2} \tag{150}$$

$$p_s^{SWO} = c_s + \frac{2[f-(N-1)\alpha][t+(N+1)\alpha]}{t^2-(N+1)^2\alpha^2} \tag{151}$$

$$p_b^{SWO} = c_b + \frac{2[f-(N-1)\alpha][t+(N+1)\alpha]}{t^2-(N+1)^2\alpha^2} \tag{152}$$

Proof. *The total social welfare function is written as $SW = SW^I + (N-1)SW^O$. To solve the problem, derive $\frac{\partial SW}{\partial p_s^I} = 0$, $\frac{\partial SW}{\partial p_b^I} = 0$, $\frac{\partial SW}{\partial p_s^O} = 0$, $\frac{\partial SW}{\partial p_b^O} = 0$. Solving first-order equations simultaneously can derive the equilibrium outcome. The second order conditions are $\frac{\partial^2 SW}{\partial (p_s^I)2} = \frac{\partial^2 SW}{\partial (p_b^I)2} = -\frac{2t+2(N+1)\alpha}{t^2-(N+1)^2\alpha} < 0$, and $\frac{\partial^2 SW}{\partial (p_s^O)2} = \frac{\partial^2 SW}{\partial (p_b^O)2} = -\frac{4t+4(N+1)\alpha}{t^2-(N+1)^2\alpha} < 0$, which show the stability of equilibrium results.*

The implications of Proposition 14 are as follows. In the post merger, the equilibrium pricing that maximizes social welfare also takes into account both producer profits and consumer surplus. By lowering prices, the platform can attract more agents and increase the consumer surplus. Although the platform's profit may decrease, the increase in total welfare brought about by the expansion of the agent base reflects the inclination towards consumer welfare under the social welfare goal and achieves a balance of interests between market participants through price adjustment. Specifically, $N-1$ reflects the intensity of competition faced by insiders. The more intense the competition, the more significant the competition-driven pricing adjustment in the adjustment item (the second component of each equilibrium price equation). This will drive the platform to expand its agent base by reducing prices and improving social welfare. $f - 2\alpha$ (insiders) and $f - (N-1)\alpha$ (outsiders) reflect the adjustment of pricing due to cross-group network externalities, which will attract agents through competition and enhance the collaborative benefits of the bilateral network. Similarly, $t + (N+1)\alpha$ reflects the synergy between transportation cost t and cross-group network externalities. In summary, the equilibrium prices imply the balance between expansion of agent scale and profitability of platforms. Compared with equations (133)-(136), $p^{SWI} < p^{I*}$ and $p^{SWO} < p^{O*}$. Therefore, social welfare maximization will lower prices to attract more agents and increase consumer surplus. At the same time, this pricing level can ensure that the platform obtains sufficient profits (covering serving costs and retaining some surplus) to maintain market supply. Ultimately, through pricing adjustment, the total social welfare is optimized.

1.4.3 Effects of Merger

Compared the results of premerger equilibrium and postmerger equilibrium, the following propositions can be obtained.

Proposition 15. *In a circular market with $N+1$ platforms ($N > 2$), the merger strategy will be more profitable for the insider platform located on the circle than the insider platform located in the center.*

Proof. *The merger strategy increase platform i's profit by $\Delta\pi^i = \pi^{I*} - \pi^{i*}$ and increases the profit of the platform M by $\Delta\pi^M = \pi^{I*} - \pi^{M*}$.*

$$\Delta\pi^M - \Delta\pi^i = \frac{2N[2t - f - (N+4)\alpha]^2 - 4[f + t - 2(N+1)\alpha]^2}{9[t - (N+2)\alpha]} \tag{153}$$

Since $[2t - f - (N+4)\alpha] - [f + t - 2(N+1)\alpha] = t - 2f + (N-2)\alpha > 0$, the whole denominator part should be positive either. Therefore, $\Delta\pi^M > \Delta\pi^i$, gives that *the merger strategy allows platform i to achieve greater growth in profits.*

The implications of Proposition 15 are as follows. After the platform i chooses the merger strategy, its pricing level and market share increase significantly. This combined effect enables platform i to achieve higher profitability in the post-merger stage. In contrast, after the platform M chose the merger strategy, its pricing level and market share do not change significantly. This combined effect makes the profit growth of the platform M' in the post-merger stage less prominent than that of the platform i. Cross-group network externalities are the key factor that leads to this result. If the market share that a platform can achieve before participating in a merger is very limited, then the merger strategy can enable it to achieve the maximum degree of cross-group network externalities, thereby continuously improving the platform's attractiveness to agents and forming a virtuous circle. However, if the platform already has a certain degree of market power and has accumulated a relatively large market share before participating in a merger, it will be difficult to adopt more aggressive market expansion measures after participating in the merger; otherwise, agents may flow to competitors. Therefore, for the platform M, the economic significance of the merger strategy is to consolidate its advantages in market competition and obtain long-term sustainable profitability.

Proposition 16. *In a circular market with $N+1$ platforms ($N > 2$), platforms on the circle that choose to join the insider gain more profits than those that choose to remain as the outsider.*

Proof. *The merger strategy increase platform i's profit by $\Delta\pi^i = \pi^{I*} - \pi^{i*}$ and increases the profit of the platform j by $\Delta\pi^j = \pi^{O*} - \pi^{j*}$.*

$$\Delta\pi^i - \Delta\pi^j = \frac{2(N-1)[2t - f - (N+3)\alpha]^2 - 4[f + t - 2N\alpha]^2}{9[t - (N+1)\alpha]} \tag{154}$$

Since $[2t - f - (N+3)\alpha] - [f + t - 2N\alpha] = t - 2f + (N-3)\alpha > 0$, *the whole denominator part should be positive either. Therefore,* $\Delta\pi^i > \Delta\pi^j$, *gives that the merger strategy allows platform i to achieve greater growth in profits.*

The implications of Proposition 16 are as follows. On the one hand, the merger strategy enables insiders to occupy a more advantageous position in the market, resulting in more intense competition for platforms that do not participate in the merger and their market share will be squeezed. Therefore, outsiders will appropriately raise prices to maintain profitability. On the other hand, considering that agents' sensitivity to prices on one side of the platform may affect the cross-group network externalities of agents on the other side, outsiders will not significantly increase prices. In contrast, platform *i*'s pricing and market share have increased significantly after participating in the merger. Therefore, the merger strategy will lead to a higher profit level for platform *i* than for platform *j*. As N increases, the cross-group network externalities play a more significant role in the decrease of its market share on both sides, while the decrease of the price level is independent of N. As a result, any outsider platform makes less profits by charging a lower price level and generating less market shares on both sides, and the profits will be much lower than the insider's profit.

Proposition 17. *In a circular market with* $N+1$ *platforms* $(N > 2)$, *as N increases, the merger strategy will be more profitable for the insider platform located on the circle.*

Proof.

$$\pi^{I*} - \pi^{i*} = \frac{2\alpha(N+1)(N+3)(f+t-N\alpha)}{9[t-(N+1)\alpha][t-(N+2)\alpha]} \tag{155}$$

Since $t > 2(N+1)\alpha$, $\pi^{I*} - \pi^{i*} > 0$. *That is, the merger strategy leads to the platform i increasing its profit. The equation also shows that changes in N have a significant impact on incremental changes in profits. As N increases, the denominator gradually decreases while the numerator gradually increases, making the whole equation increase.*

The implications of Proposition 17 are as follows. Before participating in the merger, the market share that platform i can attract in market competition is only the agents located between platform i and its adjacent platforms. After participating in the merger, the platform i integrates the market advantages of the platform M, resulting in the expansion of its market scale and significantly enhancing cross-group network externalities. And this effect will become stronger as the number of platforms increases. In reality, when there are too many platforms in a two-sided market, market resources are relatively dispersed, and the cross-group network externalities that a single platform can achieve are relatively weak. The merger strategy can allow the platform to increase the attractiveness of the platform to agents with stronger market power. The more platforms there are, the more agents are attracted, the more frequent the cross-group network interactions are, and the higher the platform's profitability will be. Finally, if insiders attract more agents through cross-group network externalities, they may form stronger market power. When N increases, the merger strategy will significantly increase the profits of insiders, which may lead to stronger monopoly tendencies. Therefore, from the perspective of antitrust policy, the long-term dynamic impact of cross-group network externalities should be considered, while maintaining the competitiveness of outsiders and preventing potential monopolies caused by excessive concentration of insiders are also important.

The above implications clearly show that it is not possible to apply the analysis of mergers in traditional one-sided markets to two-sided markets. This is mainly because two-sided markets have cross-group network externalities, and the platform's pricing strategy needs to consider the demand elasticity and cross-group network externalities of agents on both sides, both before and after the merger. In a two-sided market with $N + 1$ platforms, without considering cross-group network externalities, effects of merger can be distorted. From the perspective of pricing strategies, the merger strategy does not change the pricing levels of platform M and platform j. For platform i, becoming an insider only changes the degree of brand differentiation of agents towards it, thereby affecting its price level and the number of platforms N will have no impact on the pricing strategy at the postmerger stage. From the perspective of agent demands, since agents only pay attention to changes in platform pricing, the merger strategy reduces the number of agents that insiders can attract. In contrast, the agent scale of outsiders has not changed. Therefore, following the analytical logic of the traditional one-sided market will give platforms a stronger incentive to stay as an outsider, which distorts the evaluation of merger strategies in two-sided markets.

Proposition 18. *In a circular market with $N+1$ platforms ($N > 2$), as N increases, the merger strategy lowers the pricing levels of all platforms that maximize social welfare than the pricing levels at the pre-merger stage.*

Proof. *Comparing the premerger equilibrium prices with postmerger equilibrium prices can derive*

$$p^{SWi} - p^{SWI} = \frac{t-2(N+1)\alpha}{2} + \frac{(f-2\alpha)}{t-(N+2)\alpha} - \frac{(N-1)(f-2\alpha)}{t-(N+1)\alpha} > 0 \quad (156)$$

$$p^{SWM} - p^{SWI} = \frac{t-\alpha}{2} + \frac{2\alpha(f-2\alpha)}{t-(N+2)\alpha} - \frac{(N-1)(f-2\alpha)}{t-(N+1)\alpha} > 0 \quad (157)$$

$$p^{SWi} - p^{SWO} = \frac{t-2(N+1)\alpha}{2} + \frac{2\alpha(f-2\alpha)}{t-(N+2)\alpha} - \frac{2[f-(N-1)\alpha]}{t-(N+1)\alpha} > 0 \quad (158)$$

Therefore, both insiders and outsiders have lower social welfare maximized pricing levels in the postmerger stage.

The implications of Proposition 18 are as follows. The merger strategy has reduced the number of platforms in the market. In the postmerger stage, the pricing strategy of the platforms has shifted from "competitive defense" to "scale expansion", maximizing agent participation by lowering prices, thereby increasing total social welfare. For insiders, the market share expands in the postmerger stage, which enhances cross-group network externalities. The benefits of agents joining the insider platform increase due to more participants, allowing the platform to attract agents at lower prices. Hence, Proposition 18 reflects the welfare optimization path of "scale before profit" in the platform economy, achieving Pareto improvement in resource allocation efficiency through price adjustment.

1.4.4 Policy Responses

Based on the analysis of the model above, policies can be designed by adjusting the cost parameters. Define the policy parameters τ^I for the insider cost adjustment and τ^O for the outsider cost adjustment. Then the new cost parameters become

$$c_s^{I'} = c_s + \tau^I \tag{159}$$

$$c_b^{I'} = c_b + \tau^I \tag{160}$$

$$c_s^{O'} = c_s + \tau^O \tag{161}$$

$$c_b^{O'} = c_b + \tau^O \tag{162}$$

IF $\tau > 0$, a restrictive policy is introduced. If $\tau < 0$, a incentive policy is introduced. Then the new pricing levels of insiders and outsiders under profit-maximization problem are given by

$$p_s^{I'} = (c_s + \tau^I) + \frac{1}{3}(2t - f) - \frac{1}{3}(N+3)\alpha \tag{163}$$

$$p_b^{I'} = (c_b + \tau^I) + \frac{1}{3}(2t - f) - \frac{1}{3}(N+3)\alpha \tag{164}$$

$$p_s^{O'} = (c_s + \tau^O) + \frac{1}{3}(t + f) - \frac{2}{3}N\alpha \tag{165}$$

$$p_b^{O'} = (c_b + \tau^O) + \frac{1}{3}(t + f) - \frac{2}{3}N\alpha \tag{166}$$

Through the analysis of the postmerger stage, it can be seen that the insider platform has significantly enhanced cross-group network externalities and increased consumer surplus through joint pricing and user integration. The outsider's consumer surplus has significantly decreased due to reduced market share and increased pricing. From the perspective of profit changes, the insider's profits are significantly higher than before merging, while the outsider's profits are lower than the premerger stage. In addition, Proposition 18 shows that the socially optimal pricing in the postmerger stage is lower than premerger stage, so the agent scale expansion of the insider is the core driving force of the merger. Therefore, although the consumer surplus and profits of the external platform have declined, the insider has significantly increased the agent scale and consumer surplus through joint pricing and the strengthening of cross-group network externalities, resulting in a positive net change in social welfare. Thus, this merger strategy improves social welfare.

Therefore, the government should adopt incentive policies for scale expansion. First, the adjustment of the cost parameter can be implemented on the insider such that $\tau^I < 0$, the essence of which is to reduce the operating costs of the insider. According to equations (163) and (164), the reduction in costs $c_s^{I'}$ and c_b^{I*} directly leads to a lower price for the insider. This makes the platforms involved in the merger more price-competitive in the two-sided market. Lower pricing will attract more agents, and then the cross-group network externalities will be further strengthened. The effect of cross-group network externalities will not only expand the agent scale of the platform, but also improve the overall utility of agents, ultimately enhancing social welfare. Secondly, the adjustment of the cost parameter can be implemented on the outsider such that $\tau^O < 0$. According to equations (165) and (166), the reduction in costs $c_s^{O'}$ and $c_b^{O'}$ directly leads to a lower price for the outsider, which can help it maintain competitive pricing. As market concentration increases after the merger, outsider platforms can support low-price strategies that allow outsiders to retain some market shares and to avoid completely losing the market. This competition maintains market vitality and ultimately has a positive impact on social welfare. Therefore, adjusting the cost through τ^I and τ^O can balance market competition and concentration. For mergers that improve social welfare, incentive policies can optimize resource allocation and implies that mergers in two-sided markets require intervention due to cross-group network externalities, ensuring market outcomes align with social welfare goals.

1.5 Conclusion

This chapter was motivated by an empirical analysis of mergers in two-sided markets. Traditional merger theory usually involves one-sided markets, so the result can have an estimated bias because the cross-group externalities are ignored. This chapter considers modified city circular model to show how the merger of platforms affects pricing setting and generating market shares on each side, and also provides theoretical explanations on the significance of indirect network effects. By comparing Bertrand game in pre-merger stage and post-merger stage, the equilibrium prices show how the price setting of each side is strongly adjusted by the indirect network effects. The effect of merger on platforms also depends on the total number of platforms on the circle. In a three-platform market, the merger strategy will be profitable for either an insider platform or an outsider platform if that platform does not attract any market shares without merger. In a market with $N + 1$ platforms ($N > 2$), the merger strategy leads to higher profits for insider platforms than for outsider platforms. However, the profitability of each insider platform is different. As N increases, the insider platform on the circle will increase their incentive to merge, while the insider platform in the center decreases their incentive to merge. In addition, the socially optimal pricing in the post-merger stage is lower than that before the merger. Although the consumer surplus and profit of the outsider platform have declined, the insiders have significantly increased the agent scale and consumer surplus through joint pricing and the strengthening of cross-group network externalities, resulting in a positive net change in social welfare. For this type of merger, incentive policies should be adopted for scale expansion. It would be interesting to further explore the effect of endogenous mergers on the circular market so that the incentive for mergers can be extended.

Chapter 2

Privacy Concern on Social Media Monopoly

2.1 Introduction

Online platforms usually generate revenue by providing advertisers with access to data about their user base (Acquisti, Taylor and Wagman, 2016). There are third party businesses, such as Acxion and Bloomberge, that exist on the market to enable the collection and trading of data. The most popular technology that data providers use is cookies. When a user visits a data providers partner website for the first time, a cookie will be sent to the browser and used to record any behaviors of that user on the website. Thus, data providers can collect detailed data on each user and identify their consumption characteristics (Bergemann and Bonatti, 2015). In addition, the number of users participating in social networks is nowadays increasing, so social media platforms themselves, such as Facebook and Twitter, are constantly enhancing their interaction effects with users by collecting and analyzing information about users, in order to explore more profitability. For example, users may share information about their preferences for a product or service on the platform (Kirpalani, and Philippon, 2020). By disclosing that information to advertisers, the platform can then deliver the most relevant ads to the consumer (Acquisti, Taylor and Wagman, 2016). While users can enjoy a personalized social experience on the platform, the collection and use of personal information may also raise public concerns of privacy issues. For example, when a person posts an image of him dining at a well-known restaurant on a social media platform, it may reveal information about the likeness of other customers. Or, one's purchasing history and personal information on shopping websites may be resold to third-party companies without one's knowledge. This could let one receive a large number of spam emails or spam phone calls for a short period of time afterwards.

The understanding of the term privacy is not uniformly defined in the relevant fields of study. Research interest in privacy issues generally focuses on the control and protection of personal information, since personal information that can be collected, analyzed and transacted usually has significant economic values (Acquisti, Taylor and Wagman, 2016). First, users can enjoy direct benefits from sharing information, for example, by saving the time cost of matching the most desired advertisements (Kirpalani and Philippon, 2020). Second, the sharing of personal information may create positive or negative externalities because those information can be nonrival. In some cases, users who share their personal information may allow the platform to infer more information about non-users (Choi, Jeon, and Kim, 2019). Users may not know how platforms will handle their personal data. Therefore, they are likely to be concerned about the various harms caused by the direct or accidental use of data (Lee, Ahn, and Bang, 2011). For example, some studies focus on the relationship between consumer data and retail price discrimination. There is no open market for personal data in which users themselves can participate currently. Therefore, how to protect privacy without diminishing the benefits of sharing information has attracted widespread research interest. The main policy adopted by the platform is to allow users to customize the privacy settings when using the platform (Acquisti, Taylor, and Wagman, 2016). For example, access to the user's location can be prohibited when not using the platform's services.

From an economic perspective, the impacts of control and protection of personal information depend on specific context (Acquisti, Taylor, and Wagman, 2016). For example, users can reject some terms in the cookies or connect their social media platform with an e-Commerce platform (Ichihashi, 2020). The user's decision to share information is actually a trade-off between the gain from sharing information and the loss or risk that the user is willing to bear by sharing information. In many cases, users claim to care about privacy while providing excessive information to the platform at the same time, which is known as privacy paradox (Norberg et al., 2007). Pallant et al. (2022) suggest five factors that influence user information sharing, which are value, risk, vulnerability, transparency, and control. In addition, their empirical results suggest that platforms should aim to provide transparency and control over privacy policies, since users highly value these two factors.

This paper aims to provide answers to questions in a novel way such as what are user's privacy concerns when a user discloses heterogeneous personal information on the social media monopoly, and what are the effects of user's different concerns on the social media platform's profitability when considering a trading market of user i's personal information? In our paper, when a certain user joins a social media platform, she will disclose two different types of personal information. The first type of information reveals the user's personal identity, such as name, email address, hobbies, etc., which are mainly used to create a personal profile. The second type of information reveals the user's other characteristics, such as shopping preferences and similar behaviors. Then the user's privacy concerns under each type of information will be different. In our paper, the user's privacy concern arises due to the user's willingness to disclose a certain amount of each type of personal information (instead of the accuracy of personal information disclosed). Then, for each type of information, users face a trade-off between privacy benefits and privacy risks. Upon collecting information from the users, the platform needs to process and protect such information, which normally requires a substantial amount of investment on the platform's part. The platform also provides a separate trading service to the advertiser by deciding on both the amounts to be traded for each type of information and the corresponding prices to be charged for each type of sale. The advertiser in our model plays a passive role: he only decides whether to receive the trading service after observing the platform's decision.

Our results highlight that the user's information disclosure strategies are quite different when they face different types of privacy concerns. Faced with the privacy concern of cybersecurity risks such as cyberbullying attacks, users behave cautiously about disclosing information that reveals their personal identities. However, faced with privacy concerns of preference information breaches, users behave less cautiously, by which they always disclose a certain amount of information on the platform.

The remaining sections are structured as follows. Section 2.2 reviews related literature. Section 2.3 provides the basic setting of the model. Section 2.4 discusses the user's privacy concern with equilibrium outcomes. Section 2.5 provides an extension analysis by introducing naive users. Section 2.5 concludes.

2.2 Literature Review

Few studies have discussed the willingness of consumers to provide marketers with different types of personal data. Phelps, Nowak, and Ferrell (2009) focused on consumers' attention to information-behavioral consistency and their perceptions of exchange relationships with marketers who collect and use personal information. Specifically, consumers' general concerns about how companies use their personal information depend on four general factors: (1) the type of personal information requested, (2) the amount of control over the information provided, (3) the potential exchange offerings, consequences, and benefits, and (4) consumer characteristics. Both consumers and marketers often look at privacy issues in terms of information control i.e. control over who has access to personal data (i.e. disclosure), how personal data are used (i.e. misappropriation and fake light), and the amount of advertising and marketing offers coming from use (i.e. hacking) of personal data. A high degree of information control means that consumers can meaningfully influence the use of information about them. Two key underlying assumptions are: (1) most consumers want more control, and (2) giving consumers more control over how information about them is used will alleviate their privacy concerns.

For the platform, protecting users' data from malicious third parties is often costly. In addition, the platform should also decide on the extent to which it should maintain privacy protection. For example, Google and Apple have already developed privacy protection tools to restrict data flowing freely to other parties (Goldfarb and Que, 2023). From a public policy perspective, the most important factors are the type of information collected and the degree of consumer control over subsequent dissemination. This research will analyze a theoretical model to examine the relationship between user privacy concerns and the degree of control provided by the platform.

Since misuse of information is a particularly prominent form of online risk, and respect for privacy is often closely associated with trust in consumer surveys, building consumer trust is a valuable way for platforms to encourage consumers to disclose personal data. Lutz (2018) also developed and tested a framework for analyzing the impact of privacy concerns on sharing that considers institutional and societal privacy threats, trust, and social and monetary motives. User trust, such as classifying a particular service provider as trustworthy, will allow users to rely on their services and enjoy their benefits without the need for complex risk calculations or extensive protective actions. A well-documented heuristic for forming trust beliefs is based on fair information practices (i.e. proactive communication of security and privacy policies, assurances, and further customer service). Similarly, according to Martin (2018), consumers find that breaches of privacy expectations, especially secondary use of information, reduce trust in websites. Companies that violate privacy expectations are penalized twice: once through privacy violations directly affecting consumers' trust; and through the (perceived) reductions of the importance of trust factors such as integrity and competence to trust. Trust therefore mitigates privacy concerns by removing the risk of misuse of information, thereby highlighting the importance of building trust in maintaining online consumer relationships. Consumers can make trust judgments regardless of the content of the privacy statement. Potential competitive advantages can be achieved by respecting privacy, increasing trust, and placing greater emphasis on a company's competence and integrity. In our research, consumers' trust in the platform will depend upon the effect of the degree of control exercised by the platform, which will then affect the user's decision on data sharing.

Information externalities, positive or negative, usually arise when users share data on social media platforms. This creates a series of interconnected effects: First, a certain user's interactions with other users can reveal his or her connections. Consequently, other people's preferences can be revealed through their interactions with the current user. Finally, all users' behaviors in other markets, such as online shopping, are also revealed through the information they may share on the platform (Goldfarb and Que, 2023). Choi, Jeon and Kim (2019) considered indirect information externalities and discussed a theoretical model of privacy in which data collection requires consumer consent, and consumers are fully aware of the consequences of such consent. However, excessive collection of personal information occurs in the monopolistic market equilibrium, which leads to an excessive loss of privacy compared to the social optimum. The main mechanisms for this outcome are information externalities and user coordination failures, some of whom decide to share their personal information, which may allow data controllers to infer more information about non-users. However, they specify that information is heterogeneous from the perspective of information externalities and lack of research on the role of policy on privacy control. Acemoglu et al. (2022) also consider a monopoly market where the platform can trade data. They find that excessive use of data by the platform will lead to negative externalities and therefore the data will be underpriced which may not affect consumer's valuation of privacy.

Acquisti, Taylor, and Wagman (2016) considered an economic analysis of privacy issues and showed that, based on different theories and evidence, privacy protection can increase or decrease welfare. In addition, the definition of privacy also depends on different situations, so the trade-offs of sharing personal data should be discussed specifically. According to previous studies, firms usually protect consumers data when consumers realize how the company can benefit from their data so that they can change their purchasing decisions. Sometimes, companies can also gain more benefits under privacy protection behaviors. As intermediaries for data sharing, such as Google and Facebook, they may provide relevant services on one side and sell ad positions on the other side. When users on one side open their personal data to the platform, they will usually be matched with more relevant advertisements, whilst for those who completely hide their data, the content of an advertisement is likely to be random. Furthermore, the study also reviewed a monopoly data-sharing platform with heterogeneous consumers on the one side and advertisers on the other. By comparing situations under complete information, they show that the platform and advertisers may not achieve a socially optimal match, when the advertisers acquire only part of the information. Bergemann and Bonatti (2015) also studied transactions between the platform and the advertisers. Their result shows that direct data sharing to advertisers can lead to a decrease of advertisement purchasing positions on the advertiser side, so the platform will limit the accuracy of shared data to make profits.

Taylor and Wagman (2014) discussed several models under the oligopoly market with firms and consumers to compare the profit maximization and social welfare issues under the privacy and non-privacy contexts. In linear city and circular city models, firms set uniform prices in a privacy context, where firms have no information about consumers. Firms set prices based entirely on consumer types in a non-privacy context in which consumer information is common knowledge. Their results showed that consumers are better off without privacy. In a vertical differentiation model, two firms are differentiated by quality, and consumers are differentiated by their willingness to pay. The result shows that consumers with a higher willingness to pay would prefer privacy and that more consumers would choose high-quality firms without privacy. In a multi-unit symmetric demand model, firms set pricing strategies with complementary features. Compared to the privacy context, consumer surplus and social welfare are both reduced under the non-privacy context, although the profits were higher. Therefore, the effect of privacy enforcement should be considered in the specific economic model.

Duan, Liu, and Feng (2022) study pricing strategies in online platforms based on privacy concerns. The online platform provides new original content, charges a subscription price on the user side, provides user information to improve the targeting level in online advertising, and charges a price for the advertising space on the advertiser side. The analysis shows that the platform's pricing decisions depend on the level of information disclosure, and when the information disclosure is at an intermediate level, the maximum surplus can be achieved. Ichihashi (2023) introduced a dynamic model of consumer privacy choices on the platform. The study highlights the platform's information collection strategies through a commitment to not collect a great amount of information in the early period. As a result, consumers are more willing to use the platform's services. Fainmesser, Galeotti and Momot (2023) theoretically measured the effect of data protection regulations. Their result shows that social efficiency will be satisfied if the authority imposes policies of combining minimum data protection levels with taxes or fines.

Our paper contributes to the literature on privacy concerns in two-sided markets in a novel way. Whilst the previous literature has discussed the role of privacy concerns on a platform's decision to monetize user's information, the research question in our paper focuses on how users' trade-off between different types of information disclosure decisions affect a platform's strategic decision about what type of information and how much information to be traded and prices to be charged in the trading market. Our model specifically considers the trade-offs between different types of information to be disclosed. By disclosing personal information about personal identities, while users will enjoy positive network effects among a vast size of users, they will also be subjected to the risk of being exposed to attacks of cyberbullying and reputation damage. Whilst the user will experience better matching of advertisers' products by disclosing personal information about shopping preferences and behaviors, they may also be subjected to the nuisance cost due to the information breach to the advertiser.

2.3 Model Setup

Consider a market with a monopoly social media platform, a mass one of Internet users and an advertiser.

2.3.1 Users

Internet user's valuation of the platform service is uniformly distributed over $[\underline{v}, \bar{v}]$, with distribution function F. Then the mass of users who join the platform is measured by $1 - F(\underline{v})$. When users join the platform, they do not pay any subscription fees, but should provide a certain amount of personal information to create an account, such as user names, email addresses and gender information etc. In addition, users can also share interaction information, such as adding their interests and hobbies on the personal profile, posting a selfie picture and clicking the "like" button on other's comments. Both types of user information are shared with the platform, which can provide users further personalized and targeted service.

Suppose user i joins the platform and discloses two types of personal information, say type 1 and type 2. By disclosing type 1 personal information, user i can enjoy network effects from interacting with other users. In a traditional two-sided market analysis, an agent's network effect enjoyed on one side usually relates to the number of agents on the other side. Our model assumes that such network externalities arise on the *same* side of the market since, depending on the mass of users, a certain user can enjoy interaction utilities with other like-minded users who have found this particular platform's characteristics more suitable to their tastes. This assumption focuses on the market scenario of social media platforms, making the model more in line with the actual market structure of the "same-sided drive". In this way, the platform can concentrate resources on improving user experience in order to collect more user information. Hence, if user i chooses x_1 amount of type 1 information to disclose, his social network utility on the user side is given by $x_1\alpha[1-F(\underline{v})]$, where $\alpha > 0$ is the network effect parameter. At the same time, user i may also be exposed of potential cybersecurity risk, such as identity theft and cyberbullying attack. Denote cybersecurity risk parameter as $s(s > 0)$, then user i's privacy cost of disclosing x_1 amount of type 1 information is given by $x_i s[1-F(\underline{v})]$. User i's privacy cost of disclosing type 1 personal information relies on both the amount of information disclosed and the mass of users on the platform. That is, disclosing more specific personal information on a relatively popular social media platform will cause higher risk of exposing cyberbullying attack. To make the following analysis more tractable, user i's utility of disclosing x_1 amount of type 1 information is then defined as a quadratic form

$$u_i(x_1) = x_1(\alpha - s)[1-F(\underline{v})] - \frac{1}{2}x_1^2 \tag{1}$$

The rationale behind the linear term is the trade-off between network utility and privacy cost, reflecting the direct gains and losses of users disclosing type 1 information. If the cybersecurity risk is higher that $\alpha < s$, user i's network effects of interacting users will be negative and therefore leads to disutility of disclosing type 1 information. The rationale behind the quadratic term is that the marginal cost of disclosing information on behalf of users is increasing. It ensures that the utility function has a unique maximum value - when the marginal benefit is equal to the marginal cost, the user maximizes the utility. Without the quadratic term, the user may disclose information without limit, which is inconsistent with the rational decision-making behavior of users in reality.

In contrast, type 2 information mainly reveals user i's shopping preferences and behaviors. Thus, by disclosing type 2 personal information, user i can enjoyed the benefits of targeted service. For example, by analyzing information disclosed by user i, platform can match user i a product of the advertiser which is more preferred by user i. If user i chooses x_2 amount of type 2 information to disclose, his utility of being matched the advertiser is given by mx_2, where m represents the matching benefit. At the same time, user i may also be exposed to potential risks. For example, if user i's information about his willingness to pay a specific good is breached to the advertiser, he may then suffer price discrimination from the advertiser. User i's privacy cost of disclosing x_2 amount of type 2 information is also defined as a quadratic function $\frac{1}{2}\phi x_2^2$, where $\phi > 0$ is the nuisance cost parameter. Therefore, the net utility of user i by disclosing x_2 amount of type 1 information is

$$u_i(x_2) = mx_2 - \frac{1}{2}\phi x_2^2 \tag{2}$$

The rationale behind the linear term is the utility that users obtain through the precise matching service.The more detailed the shopping preferences disclosed by users, the more the products recommended by the platform will meet their needs, and the higher the service value users will enjoy. The rationale behind the quadratic term is the accelerated accumulation of privacy risks, It also reflects that the marginal cost of disclosing type 2 information on behalf of users is increasing. If there is no quadratic term, users may over-disclose information (because there is no increasing cost constraint), which goes against the cautious attitude of users towards privacy risks in reality.

Thus, user i's total utility function of disclosing both types of information is written as

$$u_i = x_1[\alpha - s][1 - F(\underline{v})]] - \frac{1}{2}x_1^2 + mx_2 - \frac{1}{2}\phi x_2^2 \tag{3}$$

It is worth noting that x_1 and x_2 both represent the amount of information disclosed by the user. Since the user cannot disclose negative amounts of information, $x_1 \geq 0$ and $x_2 \geq 0$. This dimensionless setting simplifies model analysis and makes it applicable to different types of information shared by users (such as text information, image information, etc.).

2.3.2 Platform

The monopoly model excludes the impact of platform competition on user privacy decisions, allowing the model to focus more on studying how the platform's market power affects its data collection behavior. In addition, the characteristics of the social media industry (such as Facebook and Twitter) are more likely to lead to natural monopoly tendencies, so the monopoly model setting can better help explain why privacy leakage and relevant issues frequently occur on social media platforms in reality. The monopoly platform collects all amount of information $\{x_1, x_2\}$ disclosed by user i. Holding user's personal information is usually costly, because information management system requires higher level of investment. Therefore, trading user's information can help reduce platform's holding cost. The platform can then decide whether to trade a certain amount of information with the advertiser. Suppose the platform charges p_1 on each amount of type 1 information and p_2 on each amount of type 2 information. The trading cost is assumed to be zero. Besides setting the price, the platform also chooses a fraction γ_1 of x_1 amount of type 1 information and a fraction γ_2 of x_2 amount of type 2 information to trade, $\gamma_1, \gamma_2 \in [0, 1]$. Thus, platform's total revenue of trading user i's personal information is $p_1\gamma_1x_1 + p_2\gamma_2x_2$.

Suppose that c_1 is the holding cost of each unit of collected type 1 information, and c_2 is the holding cost of each unit of collected type 2 information, $c_1, c_2 > 0$. Then the holding cost of keeping non-traded information is $c_1(1-\gamma_1)x_1 + c_2(1-\gamma_2)x_2$. Thus, the total payoff function of the platform for trading user i's information is written as

$$\pi_P = [p_1\gamma_1x_1 - c_1(1-\gamma_1)x_1] + [p_2\gamma_2x_2 - c_2(1-\gamma_2)x_2] \tag{4}$$

where the first two terms represent the net payoff from trading γ_1x_1 amount of type 1 information, and the last two terms represent the net payoff from trading γ_2x_2 amount of type 2 information.

2.3.3 Advertiser

In this model, the advertiser plays a relatively passive role. Without buying user i's information, the advertiser could only build an official social presence and display basic information of all products on the platform and connect user i through platform's matching service. However, if the advertiser buys user i's information, it can generate separate target capabilities as both types of user i's information can be analyzed and identified as a number of targeting options. Thus, the advertiser can enjoy further benefits such as increased revenues from selling user i a specific product. Denote the benefit enjoyed from buying each unit of user i's type 1 information as b_1, and the benefit enjoyed from buying each unit of user i's type 2 information as b_2. Then, the total utility function of the advertiser from buying user i's information can be defined as a quadratic form

$$u_A = [(b_1 - p_1)\gamma_1 x_1 - \frac{1}{2}(\gamma_1 x_1)^2] + [(b_2 - p_2)\gamma_2 x_2 - \frac{1}{2}(\gamma_2 x_2)^2] \tag{5}$$

where the first two terms capture advertiser's net utility of buying user i's type 1 information, and the last two terms capture the net utility of buying user i's type 2 information.

2.3.4 Stage of the game

The following analysis will be based on user i's disclosure strategies and platform's trading strategies. Specifically, the timing of the game is as follows.

- At stage 1, user i joins the platform and decides how much of type 1 information to disclose, x_1, and how much of type 2 information to disclose, x_2.
- At stage 2, platform collects user i's disclosed information $\{x_1, x_2\}$ and decides on the proportion of user i's type 1 information to trade, γ_1, and in corresponding price p_1 to charge, how much proportion of user i's type 2 information to trade, γ_2, and in corresponding price p_2 to charge.
- At stage 3, advertiser observes the platform's decisions $\{p_1, \gamma_1, p_2, \gamma_2\}$ and decides whether to buy user i's personal information.

2.4 Equilibrium Analysis

The game will be solved by backward induction.

2.4.1 Advertiser's Purchasing Decision

At stage 3, the advertiser will purchase user i's information if his total utility is non-negative. That is,

$$[(b_1 - p_1)\gamma_1 x_1 - \frac{1}{2}(\gamma_1 x_1)^2] + [(b_2 - p_2)\gamma_2 x_2 - \frac{1}{2}(\gamma_2 x_2)^2] \geq 0 \tag{6}$$

The rational condition of advertiser's purchasing behavior can be obtained as follows.
(1) If $\gamma_1 x_1 > 0$ and $\gamma_2 x_2 = 0$, then the advertiser has non-negative utility from purchasing type 1 information and has zero utility of type 2 information since no information of type 2 will be provided by the platform. Thus,

$$(b_1 - p_1)\gamma_1 x_1 - \frac{1}{2}(\gamma_1 x_1)^2 \geq 0 \tag{7}$$

Hence, the advertiser will purchase user i's type 1 information if and only if

$$b_1 \geq p_1 + \frac{1}{2}\gamma x_1 \tag{8}$$

(2) If $\gamma_1 x_1 = 0$ and $\gamma_2 x_2 > 0$, then the advertiser has non-negative utility from purchasing type 2 information and has zero utility of type 1 information since no information of type 1 will be provided by the platform. Thus

$$(b_2 - p_2)\gamma_2 x_2 - \frac{1}{2}(\gamma_2 x_2)^2 > 0 \tag{9}$$

Hence, the advertiser will purchase user i's type 2 information if and only if

$$b_2 \geq p_2 + \frac{1}{2}\gamma_2 x_2 \tag{10}$$

(3) If $\gamma_1 x_1 = 0$ and $\gamma_2 x_2 = 0$, then the advertiser has zero utility since neither type 1 information nor type 2 information is provided by the platform.
(4) If $\gamma_1 x_1 > 0$ and $\gamma_2 x_2 > 0$, then the advertiser will purchase user i's information if and only if inequality (6) holds.

Thus, the advertiser's rational condition of purchasing user i's information will become the constraint of solving the game at stage 2.

2.4.2 Platform's Trading Decision

At stage 2, the platform makes profits by facing both traded-fraction designing problem and traded pricing problem of each type of user i's collected information. The optimization problem can be defined as

$$\max_{\gamma_1,\gamma_2,p_1,p_2} \pi_P = [p_1\gamma_1 x_1 - c_1(1-\gamma_1)x_1] + [p_2\gamma_2 x_2 - c_2(1-\gamma_2)x_2] \tag{11}$$

s.t.

$$[(b_1 - p_1)\gamma_1 x_1 - \frac{1}{2}(\gamma_1 x_1)^2] + [(b_2 - p_2)\gamma_2 x_2 - \frac{1}{2}(\gamma_2 x_2)^2] \geq 0 \tag{12}$$

By solving the optimization problem, the following proposition is obtained.

Proposition 1. *The monopoly platform will not make any trading decisions on either type of information if users disclose nothing about that type. If user i only discloses type 1 information, then the monopoly platform's profit is maximized by the solution* $\{\gamma_1^*, p_1^*\}$ *that* $\gamma_1^* = \frac{b_1+c_1}{x_1}$, $p_1^* = \frac{b_1-c_1}{2}$. *If user i only discloses type 2 information, then the monopoly platform's profit is maximized by the solution* $\{\gamma_2^*, p_2^*\}$ *that* $\gamma_2^* = \frac{b_2+c_2}{x_2}$, $p_2^* = \frac{b_2-c_2}{2}$. *If user i discloses both type 1 and type 2 information, then the monopoly platform's profit is maximized by the solution* $\{\gamma_1^*, p_1^*, \gamma_2^*, p_2^*\}$ *that* $\gamma_1^* = \frac{b_1+c_1}{x_1}$, $p_1^* = \frac{b_1-c_1}{2}$, $\gamma_2^* = \frac{b_2+c_2}{x_2}$, $p_2^* = \frac{b_2-c_2}{2}$.

Proof. *To solve the maximization problem, constructing the Lagrange function L* *by introducing a parameter λ.*

$$\begin{aligned} L =& [p_1\gamma_1 x_1 - c_1(1-\gamma_1)x_1] + [p_2\gamma_2 x_2 - c_2(1-\gamma_2)x_2] \\ &+\lambda[(b_1 - p_1)\gamma_1 x_1 - \frac{1}{2}(\gamma_1 x_1)^2 + (b_2 - p_2)\gamma_2 x_2 - \frac{1}{2}(\gamma_2 x_2)^2] \end{aligned} \tag{13}$$

The Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial \gamma_1} = p_1 x_1 + c_1 x_1 + \lambda[(b_1 - p_1)x_1 - \gamma_1 x_1^2] \leq 0, \gamma_1 \geq 0 \tag{14}$$

$$\frac{\partial L}{\partial \gamma_2} = p_2 x_2 + c_2 x_2 + \lambda[(b_2 - p_2)x_2 - \gamma_2 x_2^2] \leq 0, \gamma_2 \geq 0 \tag{15}$$

$$\frac{\partial L}{\partial p_1} = \gamma_1 x_1 - \lambda\gamma_1 x_1 = 0 \tag{16}$$

$$\frac{\partial L}{\partial p_2} = \gamma_2 x_2 - \lambda\gamma_2 x_2 = 0 \tag{17}$$

$$\frac{\partial L}{\partial \lambda} = (b_1 - p_1)\gamma_1 x_1 - \frac{1}{2}(\gamma_1 x_1)^2 + (b_2 - p_2)\gamma_2 x_2 - \frac{1}{2}(\gamma_2 x_2)^2 = 0 \tag{18}$$

The solutions are derived from the following different situations. (1) If $x_1 = 0$ and $x_2 > 0$, the platform will set neither γ_1 nor p_1. Then the Kuhn-Tucker conditions can be simplified as

$$\begin{aligned} &p_2 + c_2 + \lambda(b_2 - p_2 - \gamma_2 x_2) \leq 0, \gamma_2 \geq 0 \\ &\gamma_2(1-\lambda) = 0 \\ &(b_2 - p_2)\gamma_2 - \frac{1}{2}\gamma_2^2 x_2 = 0 \end{aligned} \tag{19}$$

If $\gamma_2 = 0$, then the Kuhn-Tucker conditions lead to $(p_2 + c_2) + \lambda(b_2 - p_2) \leq 0$. Since $p_2 + c_2 > 0$ and $\lambda > 0$, the inequality will make sense only if p_2 is sufficiently higher than b_2, which will not be accepted by the advertiser. Therefore, the platform will not set zero trading proportion of collected type 2 information. If $\gamma_2 > 0$, then the Kuhn-Tucker conditions can derive $\lambda = 1$, $\gamma_2^* = \frac{b_2+c_2}{x_2}$ and $p_2^* = \frac{b_2-c_2}{2}$. (2) If $x_1 > 0$ and

$x_2 = 0$, the platform will set neither γ_2 nor p_2. Then the Kuhn-Tucker conditions can be simplified as

$$\begin{aligned}
&p_1 + c_1 + \lambda(b_1 - p_1 - \gamma_1 x_1) \leq 0, \gamma_1 \geq 0 \\
&\gamma_1(1-\lambda) = 0 \\
&(b_1 - p_1)\gamma_1 - \frac{1}{2}\gamma_1^2 x_1 = 0
\end{aligned} \tag{20}$$

If $\gamma_1 = 0$, the Kuhn-Tucker conditions lead to $(p_1 + c_1) + \lambda(b_1 - p_1) \leq 0$. Since $p_1 + c_1 > 0$ and $\lambda > 0$, the inequality will make sense only if p_1 is sufficiently higher than b_1, which will not be accepted by the advertiser. Therefore, the platform will not set zero trading proportion of collected type 1 information. If $\gamma_1 > 0$, then the Kuhn-Tucker conditions can derive $\lambda = 1$, $\gamma_1^* = \frac{b_1+c_1}{x_1}$ and $p_1^* = \frac{b_1-c_1}{2}$. (3) If $x_1 > 0$ and $x_2 > 0$, then the Kuhn-Tucker conditions can be simplified as

$$\begin{aligned}
&p_1 + c_1 + \lambda(b_1 - p_1 - \gamma_1 x_1) \leq 0, \gamma_1 \geq 0 \\
&p_2 + c_2 + \lambda(b_2 - p_2 - \gamma_2 x_2) \leq 0, \gamma_2 \geq 0 \\
&\gamma_1(1-\lambda) = 0 \\
&\gamma_2(1-\lambda) = 0 \\
&(b_1 - p_1)\gamma_1 - \frac{1}{2}\gamma_1^2 x_1 + (b_2 - p_2)\gamma_2 - \frac{1}{2}\gamma_2^2 x_2 = 0
\end{aligned} \tag{21}$$

Similar to the first two situations, either $\gamma_1 = 0$ or $\gamma_2 = 0$ will not be accepted. When $\gamma_1 > 0$ and $\gamma_2 > 0$, solving the Kuhn-Tucker conditions can derive $\gamma_1^* = \frac{b_1+c_1}{x_1}$, $p_1^* = \frac{b_1-c_1}{2}$, $\gamma_2^* = \frac{b_2+c_2}{x_2}$ and $p_2^* = \frac{b_2-c_2}{2}$. (4) If $x_1 = 0$ and $x_2 = 0$, then the platform will not make any trading decisions either.

The implication of Proposition 1 is as follows. Platform's trading decision is made by observing user *i*'s disclosure choice. The platform's profit can be maximized only when at least one type of personal information is disclosed by user *i*. Note that regardless of whether the platform trades only one type of information or both, the functional forms for the optimal values of γ_i and p_i

remain the same although the actual equilibrium values will differ depending upon the equilibrium values of x_i that are decided by the users (see the analysis in the next sub-section). When the platform collects a certain amount of information of either type, the optimal traded fraction of each type of information is determined by advertiser's marginal benefit enjoyed from purchasing that type of information, platform's marginal cost of holding that type of collected information, and user i's disclosed level of that type of information. For example, for the type 1 information x_1 collected, either a higher marginal benefit enjoyed by the advertiser b_1, or a higher marginal holding cost c_1 undertaken by the platform will increase the traded volume of type 1 information $\gamma_1^* x_1$. Similarly, for type 2 information x_2 collected: either a higher value of b_2 or c_2 or both will increase the traded volume of type 2 information $\gamma_2^* x_2$. Therefore, when holding user i's personal information is costly, platform always has an incentive to increase traded proportion of user i's personal information.

The optimal pricing level charged on each type of information is determined by advertiser's marginal benefit enjoyed from purchasing each type of information and platform's marginal cost of protecting that type of information. To charge a positive price p_1, the platform's marginal holding cost of type 1 information c_1 should be less than the advertiser's marginal benefit of type 1 information b_1. To charge a positive price p_2, the platform's marginal holding cost of type 2 information c_2 should be less than the advertiser's marginal benefit of type 2 information b_2. If $\{b_1, b_2\}$ is higher, than the platform will also choose a higher pricing level $\{p_1^*, p_2^*\}$. The platform has a significant advantage in terms of user i's information that advertiser requires, so it will make perfect price discrimination under the complete information of advertiser's willingness to pay. That is, the platform charges a higher price and trade more amount of information of a type, for which the advertiser's benefit is higher. If $\{c_1, c_2\}$ is higher, then platform will choose a lower pricing level $\{p_1^*, p_2^*\}$. A higher per-unit holding cost of a specific type of information can lead to a higher traded amount of information of that type, which will significantly reduce platform's total holding cost of that type's collected information. Therefore, this may allow the platform to maintain its profitability with charging a lower price.

With the optimal solution, when $x_1 > 0$ and $x_2 > 0$, platform's maximized profit is given by

$$\pi_P^* = [\frac{(b_1 + c_1)^2}{2} - c_1 x_1] + [\frac{(b_2 + c_2)^2}{2} - c_2 x_2] \tag{22}$$

The first term captures platform's maximized profit from trading user i's type 1 information, and the second term captures platform's maximized profit from trading user i's type 2 information. Each term shows how traded information offsets the holding cost of total amount of each type of information. The rational condition for platform making non-negative profits should be

$$c_1 x_1 + c_2 x_2 \leq \frac{(b_1 + c_1)^2}{2} + \frac{(b_2 + c_2)^2}{2} \tag{23}$$

With the optimal solution, advertiser's utility $u_A^* = 0$, which also implies perfect price discrimination.

2.4.3 User's Disclosure Decision

At stage 1, user i chooses the optimal disclosure level of type 1 and type 2 information by solving the following utility maximization problem.

$$\max_{x_1, x_2} u_i = x_1(\alpha - s)1 - F(\underline{v})]] - \frac{1}{2}x_1^2 + m x_2 - \frac{1}{2}\phi x_2^2 \tag{24}$$

The optimal solution then is obtained in the following proposition.

Proposition 2. *With privacy concern of both type 1 and type 2 information, user i's utility is maximized by the solution* $\{x_1^*, x_2^*\}$ *that*

$$x_1^* = (\alpha - s)[1 - F(\underline{v})] \tag{25}$$

if and only if $\alpha \geq s$*, and*

$$x_2^* = \frac{m}{\phi} \tag{26}$$

Proof. *The first order conditions are*

$$\frac{\partial u_i}{\partial x_1} = (\alpha - s)[1 - F(\underline{v})] - x_1 \leq 0, x_1 \geq 0 \tag{27}$$

$$\frac{\partial u_i}{\partial x_2} = m - \phi x_2 \leq 0, x_2 \geq 0 \tag{28}$$

If $x_1 = 0$*, then inequality (27) leads to* $\alpha = s$*, which is possible. If* $x_1 > 0$*, then solving* $\frac{\partial u_i}{\partial x_1} = 0$ *can derive* $x_1^* = (\alpha - s)[1 - F(\underline{v})]$*, where* $s > c$*.*

If $x_2 = 0$*, then inequality (28) leads to* $m \leq 0$*. Since* $m > 0$*,* $x_2 = 0$ *is impossible. If* $x_2 > 0$*, then solving* $\frac{\partial u_i}{\partial x_2} = 0$ *can derive* $x_2^* = \frac{\phi}{m}$*.*

The implication of Proposition 2 is as follows. For $x_1^* \geq 0$ to hold, $\alpha \geq s$ must hold. User *i*'s optimal disclosure level of type 1 information is determined by the extent of marginal net externalities obtained from interacting with other users on the user side. If marginal net externalities of interacting with the user side is non-positive, that $\alpha \leq s$, then disclosing type 1 information will generate disutilities. If net externalities of interacting with the user side is sufficiently positive, then user *i* will disclose type 1 information as much as possible to realize the greatest network utilities. Furthermore, the distribution function has an important impact on users' information disclosure decisions and final results by reflecting user participation and the strength of network effects in the model, as well as determining the optimal information disclosure level and the existence conditions of the solution in mathematical derivation. When $[1 - F(\underline{v})]$ is greater, more users on the platform can interact with the user i, thereby enhancing the network effect obtained by the user i disclosing type 1 information. As long as $\alpha > s$, users are likely to disclose more type 1 information. In contrast, if $[1 - F(\underline{v})]$ is smaller, even if $\alpha > s$, the network effect obtained by users is limited, which can lead to reduced information disclosure. In addition, $[1 - F(\underline{v})]$ also affects the conditions for the existence of the optimal solution. When $x_1 > 0$, if $x_1^* \geq 0$, it must also satisfy both $\alpha \geq s$ and $[1 - F(\underline{v})] \geq 0$.

In contrast, x_2^* should be always positive that user i is always willing to disclose a certain amount of type 2 information. User i's optimal disclosure level of type 2 information is determined by $m = \phi x_2^*$. The left hand side captures the marginal benefit enjoyed from matching the advertiser, while the right hand side captures the marginal privacy cost of disclosing type 2 information. With ϕ fixed, the higher the value of matching benefits user i enjoys, the higher the disclosure level of type 2 information the agent chooses. If on the other hand, with m fixed, a higher value of ϕ implies that the user i will choose to disclose a lower level of type 2 information. User i knows the platform may disclose her type 2 information to achieve more targeted advertising, but she does not know exactly how much of type 2 information will be breached. So if user i behaves naively and has fewer privacy concerns, she will disclose type 2 information as much as possible. On the other hand, if user i is cautious and feels sensitive about the information breach, she may still disclose type 2 information but will likely disclose a relatively small level of information. Therefore, user i's privacy concerns of disclosing type 2 information rely both on the matching performance with the advertiser and her attitudes toward information breach.

With the optimal solution, user i's total utility is presented by

$$u_i^* = \frac{1}{2}(\alpha - s)^2[1 - F(\underline{v})]^2 + \frac{1}{2}\frac{m^2}{\phi} \tag{29}$$

where $\alpha \geq s$.

The maximized utility implies that even user i generates zero net utilities from disclosing type 1 information, he can also make total positive utilities by disclosing type 2 information.

With the optimal solution of $x_1^* > 0$, $x_2^* > 0$, platform's optimal traded fraction $\{\gamma_1^*, \gamma_2^*\}$ is written as

$$\gamma_1^* = \frac{b_1 + c_1}{(\alpha - s)[1 - F(\underline{v})]} \tag{30}$$

$$\gamma_2^* = \frac{\phi(b_2 + c_2)}{m} \tag{31}$$

The rational conditions of $\gamma_1^*, \gamma_2^* \in [0, 1]$ are given by

$$\alpha - s > 0 \tag{32}$$

$$b_1 + c_1 \leq (\alpha - s)[1 - F(\underline{v})] \tag{33}$$

$$b_2 + c_2 \leq \frac{m}{\phi} \tag{34}$$

Under the optimal solutions, inequality implies that user i generates positive net utilities from type 1 information. Inequality (33) implies that user i's marginal utility of disclosing type 1 information is greater than the sum of platform's marginal cost of holding type 1 information and advertiser's marginal benefit of purchasing type 1 information. Inequality (36) implies that disclosing level of type 2 information is greater than the sum of platform's marginal cost of holding type 2 information and advertiser's marginal benefit of purchasing type 2 information.

The platform's maximized profit π_P^* is written as

$$\pi_P^* = \{\frac{(b_1+c_1)^2}{2} - c_1(\alpha - s)[1 - F(\underline{v})]\} + \{\frac{(b_2+c_2)^2}{2} - \frac{c_2 m}{\phi}\} \tag{35}$$

Proposition 3. *When keeping other things fixed, a higher value of holding cost $\{c_1, c_2\}$ will lead to a lower profit of platform π_P^*.*

Proof.

$$\frac{\partial \pi_P^*}{\partial c_1} = b_1 + c_1 - (\alpha - s)[1 - F(\underline{v})] \leq 0 \tag{36}$$

If c_1 becomes higher, then the profit π_P^* will become lower. Similarly,

$$\frac{\partial \pi_P^*}{\partial c_2} = b_2 + c_2 - \frac{m}{\phi} \leq 0 \tag{37}$$

If c_2 becomes higher, then the profit π_P^* will become lower.

The implication of Proposition 3 is as follows. The platform's profitability of trading user i's collected information of each type is a trade off between platform's holding cost and its service performance on both sides. When the platform invest greater amount on holding user i's personal information, the total benefits of collecting and purchasing information may not offset the holding cost, which will negatively affect platform's profitability.

Proposition 4. *When keeping other things fixed, a higher value of ϕ leads to both platform's higher traded fraction γ_2^* and platform's higher profits π_P^*.*

Proof. $\frac{\partial \gamma_2^*}{\partial \phi} = \frac{b_2+c}{m} > 0$. *Therefore, when ϕ becomes higher, γ_2^* will also becomes higher.* $\frac{\partial \pi_P^*}{\partial \phi} = \frac{m}{\phi^2} > 0$. *Therefore, when ϕ becomes higher, π_P^* will also become higher.*

Proposition 4 can be explained as follows. The privacy cost of user i for disclosing x_2^* amount of type 2 information is $\frac{1}{2}\phi x_2^{*2} = \frac{1}{2}\frac{m^2}{\phi}$. When ϕ increases, the equilibrium value of user i's privacy costs for this type of information actually decreases adding to an increased value of net utilities (see equation (18)). Thus, user i will be willing to disclose a higher level of type 2 information. When the platform observes user i's disclosure level, it increases the traded fraction of γ_2 to maintain a higher profitability.

Proposition 5. *When keeping other things fixed, a higher value of s leads to platform's higher profits π_P^*.*

When s becomes higher (but still lower than α), user i will increase their privacy concern of cybersecurity risk, and therefore disclose less amount of type 1 information. This also leads to a lower cost of holding user i's type 1 information. Thus, platform's profit will become higher.

2.5 Extension - Naive Users

2.5.1 Model Setting

In this section, assume that there are rational and naive users in the market. The utility function of rational users continues the model setting and equilibrium analysis in Section 2.4, therefore, the utility function of rational users is given by

$$u_i^R = x_1[\alpha - s][1 - F(\underline{v})]] - \frac{1}{2}x_1^2 + mx_2 - \frac{1}{2}\phi x_2^2 \tag{38}$$

Under the constraint $x_1 \geq 0$ and $x_2 \geq> 0$, the rational user chooses x_1^R and x_2^R to maximize its utility.

In contrast, naive users are defined as users who have cognitive biases about privacy risks. Therefore, their relevant risk parameters are $s' < s$ and

$\phi' < \phi$. Therefore, the utility function of naive users is given by

$$u_i^N = x_1[\alpha - s'][1 - F(\underline{v})]] - \frac{1}{2}x_1^2 + mx_2 - \frac{1}{2}\phi' x_2^2 \tag{39}$$

Similarly, under the constraint $x_1 \geq 0$ and $x_2 \geq 0$, the naive user chooses x_1^N and x_2^N to maximize its utility.

Assume that the proportion of rational users in the market is θ, and the proportion of naive users in the market is $1 - \theta$, where $\theta \in [0, 1]$. Thus, the total amount of information collected by the platform $\{x_1, x_2\}$ will be

$$x_1 = \theta x_1^R + (1 - \theta)x_1^N \tag{40}$$

$$x_2 = \theta x_2^R + (1 - \theta)x_2^N \tag{41}$$

The platform's profit function and advertiser's utility function remain unchanged. The timing of the game will change as follows.

- At stage 1, the rational user joins the platform and decides how much of type 1 information to disclose, x_1^R, and how much of type 2 information to disclose, x_2^R. At the same time, the naive user joins the platform and decides how much of type 1 information to disclose, x_1^N, and how much of type 2 information to disclose, x_2^N.

- At stage 2, platform collects user's disclosed information $\{x_1, x_2\}$ and decides on the proportion of type 1 information to trade, γ_1, and in corresponding price p_1 to charge, how much proportion of type 2 information to trade, γ_2, and in corresponding price p_2 to charge.

- At stage 3, advertiser observes the platform's decisions $\{p_1, \gamma_1, p_2, \gamma_2\}$ and decides whether to buy user's personal information.

2.5.2 Main Results and Analysis

User's Disclosure Decision

Proposition 2 gives the rational user's disclosure decision that for type 1 information, if $\alpha \geq s$, then $x_1^R = (\alpha - s)[1 - F(\underline{v})]$. If $\alpha < s$, then $x_1^R = 0$. For type 2 information $x_2^R = \frac{m}{\phi}$. For naive users, since $s' < s$, it is easier to satisfy $\alpha > s'$ even if $\alpha < s$. Even if rational users choose not to disclose type 1 information, naive users may disclose some type 1 information due to a lower risk of cybersecurity. Therefore, the level of disclosure of type 1 information will be $x_1^N = (\alpha - s')[1 - F(\underline{v})]$ and $x_1^N > x_1^R$. The level of disclosure of type 2 information will be $x_2^N = \frac{m}{\phi'}$. Since $\phi' < \phi$, $x_2^N > x_2^R$. Therefore, naive users may disclose more type 2 information because they underestimate the privacy risks.

Platform's Trading Decision

According to Proposition 1, $\gamma^{1*} = \frac{b_1+c_1}{x_1}$ and $\gamma^{2*} = \frac{b_2+c_2}{x_2}$. Without considering the change of b and c, naive users increase x_1 and x_2, resulting in a decrease in the transaction ratio of the platform. However, when naive users disclose excessive information, the expansion of the total amount of tradable information resources held by the platform not only brings the potential for growth in trading revenue but also pushes up holding costs, so it is necessary to adjust strategies to rebalance profits. Therefore, the platform may choose to increase γ_1 and γ_2.On the one hand, this can directly drive the growth of transaction revenue because, for advertisers, more information can be used to analyze user preferences. On the other hand, when x_1 and x_2 increase due to naive users, increasing $\gamma 1$ and $\gamma 2$ can reduce the proportion of information that needs to be held, avoiding the cost out of control due to the increase in the total amount of information. Therefore, when increasing γ_1 and γ_2, if the growth rate of trading income exceeds the increase in holding costs, the platform profit will increase.

Furthermore, in Proposition 1. $p_1^* = \frac{b_1 - c_1}{2}$ and $p_2^* = \frac{b_2 - c_2}{2}$. Excessive disclosure by naive users causes a surge in the information supply of the platform. If b_1 and b_2 remain unchanged, the excessive supply can push the platform to lower their pricing level p_1 and p_2 to motivate advertiser's purchasing behaviors. Thus, the platform can increase the transaction volume by reducing prices appropriately while increasing the trading proportion and reducing the amount of untraded information held. This can not only improve the willingness of advertisers to buy, but can also control costs, thus achieving profit optimization.

Welfare Loss of Users

Naive users can suffer welfare losses due to underestimating privacy risks and over-disclosing information. Welfare loss caused by disclosing type 1 information is given by

$$\Delta W_1 = (s - s')[\theta x_1^R + (1 - \theta x_1^N][1 - F(\underline{v})] \tag{42}$$

$(s - s')$ is the difference between the actual cybersecurity risk and the risk perceived by naive users, reflecting the degree to which naive users underestimate the risk. The larger the difference, the more type 1 information disclosed by the user, resulting in more significant welfare losses for the user. The last term reflects the correlation between information disclosure risk and user interaction scale.

Welfare loss caused by disclosing type 2 information is given by

$$\Delta W_2 = (\phi - \phi')[\theta \frac{1}{2}(x_2^R)^2 + (1 - \theta)\frac{1}{2}(x_2^N)^2] \tag{43}$$

Similarly, $\phi - \phi'$ is the difference between the actual nuisance cost and the private cost perceived by naive users. The second term shows the total privacy cost combined with the user structure, where the quadratic form reflects the increasing marginal privacy cost. The greater the deviation in naive users' perception of privacy costs, the more type 2 information the naive users disclose, resulting in greater welfare losses.

Policy Suggestions

Although the platform and the advertiser may realize increased benefits due to increased information transactions, user welfare losses may dominate, leading to a decline in total social welfare. In reality, feasible solutions can be to have government intervention in the market to ensure the protection of privacy rights and fairness of information sharing. Government policies can impose mandatory risk disclosure that requires the platform to make privacy risks clear to users. This can increase users awareness of cybersecurity risks and privacy costs, reducing $s - s'$ and $\phi - \phi'$. In addition, the government can establish disclosure limits to curb excessive disclosure of information by naive users and reduce privacy risks. For example, the European Commission introduced the Digital Service Act and the Digital Market Act in 2020 to protect consumer's fundamental rights on digital platforms, which can provide users with a better cybersecurity environment.

2.6 Conclusion

This paper analyzes the role of Internet users' privacy concerns in a monopoly social-media platform, where users can interact with other users and can potentially get matched with advertising bodies. Platforms provide such matching services by collecting information from the users and trading some of the collected information with the advertisers, in most cases, without the users' knowledge. In our paper, we consider two types of privacy concerns by the user. The first type of privacy concern is related to the user's disclosure choice of certain personal information that can also create network effects on the user side. The disclosure decision is determined by interactions between benefits enjoyed and the potential for facing cyber-security risks. The equilibrium disclosure level is determined by weighing these two aspects. The second type of privacy concern is related to the user's disclosure choice of information which can generate matching benefits through getting matched with the right advertiser. We find that in equilibrium, this disclosure decision is determined by weighing matching benefits with the nuisance cost of an information breach. Specifically, the equilibrium level of information is such that the marginal matching benefit equals to marginal privacy cost. The monopoly platform collects both types of user information and decides whether to trade a certain amount of each type with the advertiser and how much to charge for each category of information.

Our results imply that the platform practices perfect price discrimination on the advertiser side, which generates zero utility for them from trading information. Differences in the equilibrium price levels charged for each type of information traded rely on the advertiser's marginal benefit of purchasing each type of information and the platform's marginal cost of holding that type of information. Comparing users' privacy concerns under two different types of personal information, we find that the user behaves more cautiously in disclosing type 1 (i.e. personal identity related) information than disclosing type 2 (e.g. shopping behaviour related) information: Whilst the user may not disclose any (additional) type 1 information if she believes that the risk of cybersecurity is high, she will always disclose type 2 information even with privacy concerns of an information breach. Our results explain how information holding costs affect the platform's profitability. Whilst high holding costs always increase the platform's incentives to trade users' personal information more, it may also reduce the platform's profitability because the total benefits of collecting and purchasing information may not be offset by the information holding cost.

In the extension work, a framework is developed to distinguish rational users and naive users, where naive users underestimate privacy risks. Due to risk perception bias, naive users disclose more type 1 and type 2 information, thereby changing platform strategies. Although their increased information supply initially reduces the platform's trading proportion, platforms may strategically increase these proportions to balance transaction revenue and holding costs. At the same time, excess information supply can prompt platforms to lower prices to encourage advertisers to buy. More importantly, naive users suffer welfare losses from underestimating risks. To address these issues, policy recommendations aim to enhance users' risk awareness, curb over-disclosure, and protect privacy. This analysis enriches the understanding of the dynamics of information disclosure in heterogeneous user markets and provides practical insights for platform regulation and user welfare protection.

It would be interesting to explore the role of user privacy concerns further in the context of other market configurations. For example, the monopoly platform framework can be extended to an oligopolistic setting. Each platform will then compete for collecting user's information on one side while competing for advertisers to trade users' information on the other side. Such extensions remain in our future research plans.

Chapter 3

Advertising and Platform Competition

3.1 Introduction

The advancement of digital technologies has dramatically changed the way an individual can buy online. Massive information is collected and analyzed by platforms to target their subscriber's shopping preferences. For example, e-commerce platforms, such as Amazon and eBay, may display targeted ads on user's browse page based on his recent product browse history. Google may place relevant sponsor links on the top of user's keyword search result pages. On YouTube, an online video sharing and social media platform, users can receive ads from either outside or inside of viewing user-generated content. By introducing advertising, both the platform and the advertiser can develop their brand awareness and promote more potential sales (Zhang et al., 2020). Advertising revenues can be an important reflection of the profitability of an online platform. Some platforms may even provide users services for free with higher degree of advertising levels (Crampes, Haritchabalet and Jullien, 2009).

From an economic perspective, advertising can affect consumption behaviors in different ways. One main view is defined as persuasive advertising, which believes that the advertising can change consumer's valuation of the product, and therefore "persuade" the consumer to buy the product. Thus, the entry barrier for new advertisers may exist because the consumer has established the brand loyalty through persuaded advertising. Another view is defined as informative advertising, which believes that advertising can tell consumers information about the product, and therefore lead to cost saving effect, as consumers do not need to make too much effort on searching the product information (Bagwell, 2007).

There are two methods of providing advertising through the platform. Firstly, the platform can directly display advertiser's ads so that users or consumers can receive them as if they enter the platform. Secondly, the platform can match a targeted group of users or consumers with a specific advertiser (De Corniere and De Nijs, 2016). In real-world issues, both two methods can be applied to improve advertiser's potential revenues. Since the first method can reach the total size of consumers on the platform, an advertiser who has larger brand competitive advantages in a mass market can easily realize actual sales. In contrast, the second method can achieve a higher target capacity; therefore, advertiser whose product is characterized by specific preferences can realize actual sales by a more personalized matching.

Although the development of advertising technology brings greater convenience to buyers when contacting the seller side on the platform, some consumers may find too much advertising annoying and try to avoid advertising (Johnson, 2013). For example, consumers may block the function of delivering ads when using the platform or choose not to subscribe any advertising emails sent by the advertiser. Thus, from a marketing perspective, sending too much advertising may not be beneficial in generating further revenues for both the platform and the advertiser. Then, it is also interesting to discuss whether user's annoying attitude toward advertising will affect seller's advertising quantity decision and platform's advertising pricing decision.

The aim of this chapter is to analyze the role of advertising in two-sided duopoly markets, where two symmetric platforms serve buyers on one side for free and sellers on the other side by charging registration fees. Each platform also charges extra advertising fees to the seller side to let them send out a certain number of ads. On the seller side, platform's registration pricing decision will affect seller's perceived utility from contacting the buyer side, while platform's advertising pricing decision will affect seller's perceived payoff of sending ads. On the buyer side, although buyers enjoy platform's service for free, they believe that they will be annoyed by receiving too much ads. Therefore, the seller's ad quantity decision will also affect the buyer's entry. The results highlight that the seller's advertising decision is independent of the total size of the buyer side. With considering the indirect network externalities effect, platform's registration pricing decision is adjusted downward by net brand differentiation between two platforms. Considering the buyer's nuisance cost of advertising, the platform's advertising pricing decision will be adjusted upward by a higher value of nuisance parameter. Finally, the results show that the platform may not always make higher profits with a higher investment in advertising technology.

The remaining sections are structured as follows. Section 3.2 reviews related literature. Section 3.3 provides the basic setting of the model. Section 3.4 analyzes the equilibrium results. Section 3.5 provides an extension analysis by introducing ad blockers and buyer heterogeneity. Section 3.6 concludes.

3.2 Literature Review

Previous literature contributes to the theoretical analysis of advertising with differentiated products. Barter(1978) presented a classical model of advertising and horizontal product differentiation, where buyers can only know about product information through advertising. This assumption emphasizes the indispensable role of advertising in market transactions. The model adopts a structural analysis framework to analyze the impact of advertising on the competition structure of the market. By taking advertising as a key variable in the buyer's choice mechanism, it explores how companies use advertising to perform horizontal product differentiation, thereby reshaping the market competition dynamics. This modeling approach has inspired further studies to further explore how non-price factors such as advertising regulate market interactions and competition outcomes. Bloch and Manccau (1999) studied the effect of persuasive advertising by using a Hotelling model either. In the model setting, firms are located at the two ends of a linear city. Consumers are evenly distributed along the line and each consumer has a preferred product variant. The results highlight that firms strategically adjust advertising intensity to alter consumers perceived product distances, thereby reshaping market shares and competition outcomes. They also compare firm profitability of two scenarios, where two products on the Hotelling line are either provided by a monopoly or a duopoly. These studies inspire the basic model setting of this chapter. Related to advertising nuisance, Johnson (2013) studied the effect of targeted advertising and showed that a higher targeted capacity will improve firm's profits while consumers may worse off since the consumer is likely to be matched a less preferred product and too much advertising will become annoying. This study highlights the importance of advertising nuisance in market dynamics, which directly inspires the specification of buyer advertising aversion costs in this chapter. This chapter quantifies buyers' negative perception of advertising as a utility loss term and systematically explores how advertising nuisance affects buyers' platform joining decisions, as well as the advertising strategies of platforms and sellers, thus expanding the research on advertising nuisance.

The analysis of advertising on two-sided markets in previous studies mainly focuses on media market, where the media platform connects audience and advertisers through its either advertising pricing strategy or advertising quantity strategy. Armstrong (2006) presented a model of informative advertising where two media platforms generates profits by charging either a lump-sump advertising fee or a per-reader advertising fee. The equilibrium results highlight how the way of charging advertising fees will affect platform's profits when readers may or may not like ads. Crampes, Haritchabalet and Jullien (2009) also analyze media platforms competing for views on one side and advertisers on the other side by charging a price level on both sides, while the equilibrium results highlight the effect of different setting on advertising technology. By introducing non-linear advertising technology, the platform's profitability relies on the change of returns in the number of viewers on the platform. The results highlight that seller's advertising decision only relies on how much advertising fees the platform will charge on each unit of ad and the value of revenue of sending an ad. Reisinger, Ressner and Sechmidtke (2009) mainly analyzed the effect of both participation externality and pecuniary externality on media platform competition, and the results highlight advertising can be an either substitute strategy or complementary strategy under platform competitions. This conclusion prompts this chapter to rethink the complex mechanism of advertising-related decisions in broader platform competition. Therefore, this chapter further explores the nonlinear impact of advertising technology investment on platform profits, and derives the relationship between the marginal cost of advertising technology and the seller's advertising revenue parameters by constructing a platform profit function. Athey and Gans (2010) analyzed the effect of targeting technology on the media market where consumers are heterogeneous. However, these studies are closely related to the context of the media market. In media platforms, interactions typically revolve around viewers consuming content with embedded ads, and advertisers target that audience, leading to specific advertising pricing and quantity strategies. This is fundamentally different from the general two-sided platforms studied in this chapter. For example, in the model setting of this chapter, sellers actively determine advertising volume after joining the platform, reflecting more proactive advertiser behavior. Therefore, the conclusions drawn from these media-centric studies cannot be applied directly to general platforms. This limitation highlights the need for the research in this chapter, which provides insight into advertising decisions beyond the narrow media market framework by analyzing a wider range of platform types.

This chapter contributes to the study of advertising issues in two-sided markets. The previous literature mainly explores the role of advertising levels in media markets. By distinguishing between sellers' voluntary advertising intentions and platforms' advertising attraction strategies, this chapter constructs a more general analytical framework, fills the research gap in general platform advertising decisions, and provides theoretical support for the analysis of advertising strategies in non-media two-sided markets. In the model setting, the advertising level specifies how much advertising sellers are willing to place on the platform after joining the platform, rather than how many advertisers the platform is willing to attract. Second, the model setting of this chapter distinguishes between the perceived utility of the connection between the subjects and the perceived utility (or disutility) of advertising. Specifically, buyers may incur additional interference costs by receiving a certain number of advertisements, while sellers may obtain additional benefits by sending a certain number of advertisements. Third, this chapter also emphasizes the asymmetry of registration pricing setting between the two parties. Fourth, this chapter also discusses whether a higher level of investment in advertising technology is always beneficial to the platform. Finally, the existing literature rarely explores the impact of ad blockers and buyer heterogeneity on advertising decisions. This chapter innovatively introduces the classification of active buyers (using ad blockers) and passive buyers (not using ad blockers) and deeply analyzes how buyer heterogeneity affects platform advertising strategies and sellers' advertising decisions.

3.3 Model Setting

The model considers a duopoly market, where two symmetric platforms, denoted platform i ($i = 1, 2$), are located at two endpoints of a Hotelling line and competing for two groups of agents, denoted buyers and sellers. Agents are uniformly distributed on the Hotelling line, and assume that each agent chooses to join a single platform for exogenous reasons.

3.3.1 Platform

On the buyer side, each platform charges zero registration fees on their service. On the seller side, platform i charges a lump-sum registration fee per seller by providing basic services, denoted p_i. Per-agent serving cost of each platform is assumed to be zero. In addition, each platform also provides an advertising service that a seller can send the buyer side a certain amount of ads by paying extra advertising fees. Suppose platform i charges advertising fees A_i on each unit of ad. In this model, each platform's cost of introducing advertising technology is defined as a linear function of how much ads are requested by a seller. Denote the marginal cost of introducing advertising technology as c. Then, the profit function of platform i is given by

$$\pi^i = (p_i + A_i a_i - c a_i) n_s^i \tag{1}$$

where a_i represents seller's amount of ads requested on platform i, and n_s^i represents the number of sellers attracted by platform i.

3.3.2 Sellers

By paying a registration fee p_i, a seller can join platform i and enjoy positive network externalities, which depend on total number of buyers on that platform. The more buyers are on the platform, the more potential sales a seller can realize, which will in turn let him generate more profits. Suppose a seller can perceive a value of v_s from contacting each buyer, then the seller's network externalities of contacting the buyer side on platform i is given by $v_s n_b^i$, where n_b^i represents total number of buyers attracted by platform i. Furthermore, the seller also chooses to send a certain amount of ads to the buyer side by paying extra advertising fees. Sellers believe that advertising can help them boost buyer's awareness of their products, which will in turn improve potential transactions and increase their sales revenues. In this model, the seller's total revenues of sending ads is defined as a non-linear function of how much ads are requested by the seller. That is, seller's marginal revenue of sending an ad decreases total amount of ads requested. Denote the revenue parameter of sending an ad as r. Then, seller's total utility function of joining platform i is given by

$$u_s^i = v_s n_b^i - p_i + ra_i - \frac{1}{2}a_i^2 - aA_i - ty_i \tag{2}$$

The utility function involves three components. The first component, $v_s n_b^i - p_i$, captures seller's net benefit tradeoff of joining platform i. $v_s n_b^i$ reflects that the more buyers there are, the greater the chance for sellers to increase sales by reaching potential customers. p_i is the direct cost for sellers to join the platform. If $v_s n_b^i > p_i$, positive net benefits will encourage sellers to join the platform. The second component, $ra_i - \frac{1}{2}a_i^2 - aA_i$ captures seller's trade-off between the benefits and costs of advertising strategies. ra_i reflect the direct benefits of advertising that revenue increases linearly with the amount of advertising, showing the promotion effect of advertising on sales. The quadratic term $-\frac{1}{2}a_i^2$ reflects the increasing marginal cost of advertising. When the amount of advertising is small, the market is more receptive to advertising and the marginal cost is lower. However, as the amount of advertising continues to increase, buyers may resist advertising and the marginal cost will rise rapidly. This setting mathematically ensures that the seller has the optimal amount of advertising to avoid over-advertising. aA_i is the direct cost of the seller participating in the advertising service. The seller needs to weigh the advertising revenue against the cost. If the platform charges too high, the seller will reduce advertising; otherwise, the seller may increase advertising. Therefore, the seller maximizes the utility of advertising by optimizing a_i, which reflects the rational decision-making logic of participants in the two-sided market in advertising strategies. The last component captures, ty_i captures seller's transportation cost to platform i on the Hotelling line, where $y_i \in [0, 1]$ denoted seller's located distance from platform i, and t denoted an agent's transportation parameter. In summary, sellers seek to maximize utility in the decision of joining the platform and placing advertisements.

3.3.3 Buyers

A buyer can join a platform by paying zero registration fees. This specifies some characteristics of shopping platforms in the real world, that buyers usually enter a shopping mall or online shopping website (e.g. Amazon.com) for free. When a buyer joins a platform, he can also enjoy positive network externalities, which depend on total number of sellers on that platform. The more number of sellers are on a platform, the more product choices a buyer can have, which will in turn bring him a better shopping experience on that platform. Suppose a buyer can perceive a value of v_b from contacting each seller, then the buyer's network externalities of contacting the seller side on platform i is given by $v_b n_s^i$. However, buyers also believe that "*there's no such a thing as a free lunch*", so they understand that they may also receive a certain amount of ads, which are independent of basic services provided by the platform. For example, buyers would have to read some ads before browsing the platform's basic web page, or before checking out. By receiving these ads, buyer may have to spend longer time on the platform by viewing the ad information and even spend more expenditures on impulse purchasing. Thus, too much ads may annoying buyers and lead to a nuisance cost. Define buyer's nuisance cost of receiving ads as a non-linear function, $\frac{1}{2}\phi a_i^2$, where ϕ represents the nuisance cost parameter. The non-linear form implies that, buyer's marginal nuisance costs of receiving ads increase with how much ads he will receive. Thus, buyer's total utility function of joining platform i is given by

$$u_b^i = v_b n_s^i - \frac{1}{2}\phi a_i^2 - t x_i \tag{3}$$

The utility function of buyers also involves three components. The first component $v_b n_s^i$ reflects the main source of benefits for buyers in a two-sided market. The more sellers there are, the more product choices buyers have and the more likely the shopping experience will improve. This is a classic manifestation of cross-group network externalities in two-sided markets. The second component is the cost to the buyer of receiving the advertisement. The quadratic form implies that the marginal aversion cost of advertising increases as the amount of advertising increases. The greater the amount of advertising, the faster the buyer's time spent browsing ads, the risk of impulse consumption, and the degree of disgust increase, which is consistent with the logic of the intensified negative experience caused by "advertising overload" in reality. The third component, tx_i captures buyer's transportation cost to platform i on the Hotelling line, where $x_i \in [0, 1]$ denoted buyer's located distance from platform i. This model assumes that buyers and sellers have symmetric brand differentiation tastes, therefore, the transportation parameter is the same on both sides that $t_s = t_b = t$. In summary, buyers gain cross-group network benefits through the scale of sellers on the platform, while bearing the aversion costs and other potential costs brought by advertising. Buyers decide whether to join the platform and the extent of their use of the platform by evaluating these factors, which reflects the rational logic of user decision making in a two-sided market.

In order to conduct more tractable analysis, the following assumption is applied throughout the analysis.

Assumption 1. *In this market, an agent's brand taste is sufficiently larger than his indirect network externality that* $t^2 > 4v_s v_b$.

In the Hotelling model, t reflects consumers' preference for brand differentiation of the platform. The larger t, the more participants care about the differences in the brand positioning of the platform. v_s and v_b represent the indirect network externalities of sellers who contact buyers and buyers contacting sellers. That is, sellers may get more transaction opportunities due to more buyers, and buyers may get more product choices because of more sellers. Therefore, this assumption shows that the intensity of brand preference of participants in the platform significantly exceeds the impact of indirect network externalities. In the competition of duopoly platforms, even if the network externalities of a platform are weak, as long as its brand differentiation is prominent enough, the platform can still attract users.

3.3.4 Stage of the Game

The following analysis will be based on platform i's pricing strategies and seller's advertising strategy. Specifically, the timing of the game is as follows.

- At the first stage, platform i sets both a per-seller registration fee p_i and a per-unit advertising fee A_i to attract sellers to join. Platform i also charges zero registration fee on buyers to attract them to join.
- At the second stage, buyers and sellers, having observed platform i's pricing decisions, decide whether to join platform i.
- At the third stage, sellers, having joined platform i, decide how much ads to send out on the buyer side, a_i.

The game will be solved by backward inductions in the next section.

3.4 Equilibrium Analysis

3.4.1 Seller's Advertising Decision

At the third stage, a seller, have joined platform i, chooses a_i to maximize his utility function. The maximization problem is written as

$$\max_{a_i} u_s^i = v_s n_b^i - p_i + r a_i - \frac{1}{2} a_i^2 - a_i A_i - t y_i \tag{4}$$

The equilibrium result is derived in the following proposition.

Proposition 1. *Seller's utility on platform i is maximized by the unique solution a_i^* that*

$$a_i^* = r - A_i \tag{5}$$

Proof. *The first order condition is given by*

$$\frac{\partial u_s^i}{\partial a_i} = r - a_i - Ai = 0 \tag{6}$$

which derives $a_i^* = r - A_i$.

The implication of Proposition 1 is as follows. In equilibrium, seller's marginal revenue of choosing a_i^*, given by $r - a_i^*$, equals to seller's marginal cost of choosing a_i^*, given by A_i. Seller's advertising decision is independent of platform i's performance of generating market shares on either buyer side or seller side, but only cares about how much advertising fees charged on each unit of ads. If platform i charges a higher price, then the seller, who joined platform i, will decrease his demands of advertising. When platform i charges A_i higher than r, then the seller will not send out any ads.

With the optimal solution a_i^*, the seller's utility function is given by

$$u_s^i(a_i^*) = v_s n_b^i - p_i + \frac{1}{2}(r - A_i)^2 - ty_i \tag{7}$$

The utility function is then determined by how much number of buyers on platform i, platform i's pricing strategies of p_i and A_i, and the total transportation cost to platform i. The next subsection will solve both buyer and seller's demand generated by platform i, n_s^{i*} and n_b^{i*}, using indifferent situations.

3.4.2 Agent's Demands on Platform i

At the second stage, having observed two platform's pricing decisions, a seller's utility function of joining platform i is given by equation (7). A seller's utility function of joining platform j ($i \neq j$) is given by

$$u_s^j = v_s n_b^j - p_j + \frac{1}{2}(r - A_j)^2 - t(1 - y_i) \tag{8}$$

Then, platform i's demands generated on the seller side is given by the following indifferent equation.

$$v_s n_b^i - p_i + \frac{1}{2}(r - A_i)^2 - ty_i = v_s n_b^j - p_j + \frac{1}{2}(r - A_j)^2 - t(1 - y_i) \tag{9}$$

Thus, the number of sellers on platform i, $n_s^i = y_i$, is obtained as

$$n_s^i = \frac{1}{2} + \frac{v_s(2n_b^i - 1) - (p_i - p_j) + \frac{1}{2}(r - A_i)^2 - \frac{1}{2}(r - A_j)^2}{2t} \tag{10}$$

while the number of sellers on platform j, $n_s^j = 1 - y_i$, is obtained as

$$n_s^j = \frac{1}{2} + \frac{v_s(2n_b^j - 1) - (p_j - p_i) + \frac{1}{2}(r - A_j)^2 - \frac{1}{2}(r - A_i)^2}{2t} \tag{11}$$

The number of sellers on each platform decreases with that platform's pricing levels and increases with its rival platform's pricing levels. Furthermore, on each platform, an extra buyer can attract extra $2(\frac{v_s}{t})$ number of sellers.

Similarly, having observed two platform's pricing decisions, a buyer's utility function of joining platform i is given by

$$u_b^i = v_b n_s^i - \frac{1}{2}\phi(r - A_i)^2 - tx_i \tag{12}$$

A buyer's utility function of joining platform j is given by

$$u_b^j = v_b n_s^j - \frac{1}{2}\phi(r - A_j)^2 - t(1 - x_i) \tag{13}$$

Then, platform i's demands generated on the buyer side is given by the following indifferent equation.

$$v_b n_s^i - \frac{1}{2}\phi(r - A_i)^2 - tx_i = v_b n_s^j - \frac{1}{2}\phi(r - A_j)^2 - t(1 - x_i) \tag{14}$$

Thus, the number of buyers on platform i, $n_b^i = x_i$, is obtained as

$$n_b^i = \frac{1}{2} + \frac{v_b(2n_s^i - 1) - \frac{1}{2}\phi(r - A_i)^2 - \frac{1}{2}\phi(r - A_j)^2}{2t} \tag{15}$$

while the number of buyers on platform j, $n_b^j = 1 - n_b^i$, is obtained as

$$n_b^j = \frac{1}{2} + \frac{v_b(2n_s^j - 1) - \frac{1}{2}\phi(r - A_j)^2 - \frac{1}{2}\phi(r - A_i)^2}{2t} \tag{16}$$

Similarly, on each platform, an extra seller can attract extra $2(\frac{v_b}{t})$ number of buyers. However, the pricing effect on determining number of buyers is asymmetric to the pricing effect on determining number of sellers. Firstly, each platform charges zero registration fee on the buyer side, so the number of buyers are not adjusted by $\{p_i, p_j\}$. Secondly, since buyers care about how much ads received by the buyer side, the number of buyers are also adjusted by $\{A_i, A_j\}$, where the adjustment coefficient is related to the nuisance cost parameter ϕ. Solving equation (10) and (15) simultaneously can yield the demand functions of platform i, which are determined by $\{p_i, p_j, A_i, A_j\}$.

$$\begin{aligned} n_s^i &= \frac{1}{2} - \frac{t(p_i - p_j) - \frac{1}{2}(t-\phi)(r-A_i)^2 + \frac{1}{2}(t-\phi)(r-A_j)^2}{2(t^2 - v_s v_b)} \\ n_b^i &= \frac{1}{2} - \frac{v_b(p_i - p_j) - \frac{1}{2}(1-\phi t)(r-A_i)^2 + \frac{1}{2}(1-\phi t)(r-A_j)^2}{2(t^2 - v_s v_b)} \end{aligned} \tag{17}$$

The expression (17) depicts the internal mechanism by which the duopoly platforms compete for the user scale of buyers and sellers through pricing and advertising strategies. The demand function on the seller side is adjusted by both the registration fee and the advertising strategy. $\frac{1}{2}$ reflects the baseline equilibrium state under the initial symmetric competition of the two oligopoly platforms. That is, if there are no other interference factors, sellers are evenly distributed between the two platforms. $\frac{t(p_i-p_j)}{2(t^2-v_s v_b)}$ reflects the competitive effect of platform registration fee p_i. If $p_i > p_j$, the cost for sellers to join platform i increases, resulting in a decrease in n_s^i. Although registration fees directly affect sellers, the number of sellers also affects the number of buyers through network externalities, and the number of buyers in turn has a reverse effect on the sellers' network external benefits. Therefore, differences in registration fees will indirectly affect sellers' final utility and platform selection through this chain reaction. The rest of the function reflects the role of advertising strategy. If $A_i > A_j$, sellers will reduce the amount of advertising and even reduce their willingness to join the platform i due to compression of advertising profit margins, resulting in a decrease in n_s^i. In addition, brand differentiation preference and buyer advertising aversion costs mediate the impact of advertising strategy through $t - \phi$. If participants care more about brand differentiation or buyers have a high degree of advertising aversion, then the impact of A_i on the number of sellers will be amplified, reflecting the interaction between advertising strategy and market characteristics. Similarly, the demand function on the buyer side shows that the registration fee p_i that the platform charges sellers will affect the number of sellers and then affect the benefits of buyers through v_b. In addition, the amount of advertising from the seller decreases as the advertising fees increase, which can cause buyers to be less disturbed by advertising. However, ϕt reflects the combined effect of buyers aversion to advertising and brand preference. Therefore, advertising strategies indirectly regulate buyers decision to join the platform by affecting their advertising aversion costs.

Similarly, the demand functions of platform j, which are determined by $\{p_i, p_j, A_i, A_j\}$, are

$$\begin{aligned} n_s^j &= \frac{1}{2} - \frac{t(p_j - p_i) - \frac{1}{2}(t-\phi)(r-A_j)^2 + \frac{1}{2}(t-\phi)(r-A_j)^2}{2(t^2 - v_s v_b)} \\ n_b^j &= \frac{1}{2} - \frac{v_b(p_j - p_i) - \frac{1}{2}(1-\phi t)(r-A_j)^2 + \frac{1}{2}(1-\phi t)(r-A_i)^2}{2(t^2 - v_s v_b)} \end{aligned} \tag{18}$$

3.4.3 Platform's Pricing Decision

At the first stage, platform i chooses $\{p_i, A_i\}$ to maximize its profit function π^i. The maximization problem is written as

$$\max_{p_i, A_i} \pi^i = [p_i + A_i(r - A_i) - c(r - A_i)]n_s^i \tag{19}$$

Platform j chooses $\{p_j, A_j\}$ to maximize its profit function π^j. The maximization problem is written as

$$\max_{p_j, A_j} \pi^j = [p_j + A_j(r - A_j) - c(r - A_j)]n_s^j \tag{20}$$

The equilibrium results are derived in the following proposition.

Proposition 2. *In this market where two symmetric platforms compete for buyers and sellers, each platform's profit is maximized by the unique solution $\{p^*, A^*\}$, where $p^* = p_i^* = p_j^*$ and $A^* = A_i^* = A_j^*$ are given by*

$$p^* = \frac{t^2 - v_s v_b}{t} - \frac{\phi t(r-c)^2}{(\phi + t)^2} \tag{21}$$

$$A^* = \frac{ct + \phi r}{\phi + t} \tag{22}$$

Proof.*The first order conditions of platform i's maximization problem are*

$$\frac{\partial \pi^i}{\partial p_i} = n_s^i + [p_i + A_i(r - A_i) - c(r - A_i)]\frac{\partial n_s^i}{\partial p_i} = 0 \tag{23}$$

$$\frac{\partial \pi^i}{\partial A_i} = (r - A_i - A_i + c)n_s^i + [p_i + A_i(r - A_i) - c(r - A_i)]\frac{\partial n_s^i}{\partial A_i} = 0 \tag{24}$$

Substituting the equation of n_s^i into first order conditions and solving them simultaneously can derive the equilibrium results. The second order conditions are $\frac{\partial^2 \pi^i}{\partial p_i^2} = -\frac{t}{t^2 - v_s v_b} < 0$, $\frac{\partial^2 \pi^i}{\partial A^{i2}} = -1 + \frac{t-\phi}{t^2 - v_s v_b}\{(r - 2A^* + c)(r - A^*) - \frac{1}{2}[p^* + (A^* - c)(r - A^*)]\} < 0$, and $\frac{\partial^2 \pi^i}{\partial A^i \partial p^i} = \frac{\partial^2 \pi^i}{\partial p^i \partial A^i} = \frac{(t-\phi)(r-A^*) - t(r-2A^*+c)}{2(t^2 - v_s v_b)}$. Therefore, $\frac{\partial^2 \pi^i}{\partial p_i^2}\frac{\partial^2 \pi^i}{\partial p^i \partial A^i} - (\frac{\partial^2 \pi^i}{\partial A^i \partial p^i})^2 > 0$, which satisfies the negative definiteness condition for profit maximization.

The implication of Proposition 2 is as follows. The optimal registration fees charged on the seller side is determined by two components. The first component $\frac{t^2-v_s v_b}{t}$ captures the net brand differentiation between two platforms with considering agent's indirect network externalities. The second component $\frac{\phi t(r-c)^2}{(\phi+t)^2}$ captures how the pricing charged on the seller is adjusted by considering buyer's advertising nuisance ϕ, seller's advertising benefit parameter r, platform's advertising technology c, and brand differentiation between two platforms, t. Although each platform's marginal serving cost is zero, if the cost of introducing advertising technology is higher, each platform will also increase its registration fees on the seller side. If seller's benefit from sending an ad is higher, then the platform will decrease its registration fees on the seller side. To analyze the effect of nuisance parameter on determining p^*, derive $\frac{\partial p^*}{\partial \phi} = -\frac{(r-c)^2(\phi-t)}{(\phi+t)^3}$. Since $t > \phi$, $\frac{\partial p^*}{\partial \phi} > 0$, which implies that a higher value of nuisance parameter will increase platform's registration fees charged on the seller side. To analyze the effect of brand differentiation parameter on determining p^*, derive that $\frac{\partial p^*}{\partial t} = \frac{t^2+v_s v_b}{t^2} - \frac{\phi(r-c)^2(\phi-t)}{(\phi+t)^3} > 0$. Therefore, a larger brand differentiation between two platforms will lead to each platform increase its registration fees on the seller side.

The optimal advertising fees charged on the seller side are adjusted upward by both c and r. That is, either a higher cost of introducing advertising technology or a higher value of seller's benefit from sending an ad will increase platform's advertising fees charged on the seller. To analyze the effect of nuisance parameter on determining A^*, derive $\frac{\partial A^*}{\partial \phi} = \frac{(r-c)t}{\phi+t}$. If $r > c$, then $\frac{\partial A^*}{\partial \phi} > 0$, which implies that a higher value of nuisance parameter will also increase platform's advertising fees charged on the seller side. If $r < c$, then $\frac{\partial A^*}{\partial \phi} < 0$, which implies that a higher value of nuisance parameter will decrease platform's advertising fees charged on the seller side. To analyze the effect of brand differentiation parameter on determining A^*, derive $\frac{\partial A^*}{\partial t} = \frac{(c-r)\phi}{(\phi+t)^2}$. If $r > c$, then $\frac{\partial A^*}{\partial t} < 0$, which implies that a larger brand differentiation between two platforms will lead to each platform decrease its advertising fees on the seller side. If $r < c$, then $\frac{\partial A^*}{\partial t} > 0$, which implies that a larger brand differentiation between two platforms will lead to each platform increase its advertising fees on the seller side. Therefore, compared with the determination of p^*, the determination of A^* relies more on the value of $r - c$.

With the optimal solution A^*, $a_i^* = r - A^*$ is then given by

$$a_i^* = \frac{(r-c)t}{\phi + t} \tag{25}$$

The rational condition of seller to sending a positive number of ads on platform i is given by the following proposition.

Proposition 3. *Sellers will send a positive number of ads on platform i if and only if $r > c$.*

Thus, $\frac{\partial A^*}{\partial \phi} > 0$ while $\frac{\partial A^*}{\partial t < 0}$. Seller's optimal advertising level is adjusted upward by r and adjusted downward by t and ϕ. To analyze the effect of brand differentiation parameter on determining a_i,I derive $\frac{\partial a_i^*}{\partial t} = \frac{\phi(r-c)}{(\phi+t)^2} > 0$, which implies that a larger brand differentiation between two platforms will lead to each platform increase its advertising fees on the seller side. In the symmetric equilibrium, each platform distribute the total market shares on each side evenly, therefore $n_s^* = n_s^{i*} = n_s^{j*} = \frac{1}{2}$ and $n_b^* = n_b^{i*} = n_b^{j*} = \frac{1}{2}$. Thus, each platform's maximized profit is given by

$$\pi^* = \pi^{i*} = \pi^{j*} = \frac{t^2 - v_s v_b}{2t} + \frac{\phi t c(r-c)}{(\phi+t)^2} \tag{26}$$

To analyze the effect of advertising technology cost on platform's profit, the following proposition is obtained.

Proposition 4. *In the symmetric equilibrium of this duopoly platform competition* *model, when analyzing the impact of the cost of advertising technology on the profit of the platform, it holds that: a higher investment in advertising technology leads to an increase in the profit of the platform if and only if the marginal cost of advertising technology c satisfies $c < \frac{r}{2}$, where r is the revenue parameter of seller advertising.*

Proof. *Derive* $\frac{\partial \pi^*}{\partial c} = \frac{\phi t(r-2c)}{(\phi+t)^2}$. *For* $\frac{\partial pi^*}{\partial c} > 0$, *the ineuality must hold. Solving this gives* $c < \frac{r}{2}$. *If* $c \geq \frac{r}{2}$, *then* $\frac{\partial pi^*}{\partial c} \leq 0$, *meaning higher c does not increase profit. Thus, the "if and only if" condition* $c < \frac{r}{2}$ *is rigorously proven.*

The implications of Proposition 4 are as follows. c reflects the marginal cost of the platform that provides advertising services. When $c < \frac{r}{2}$, if c increases, sellers' advertising revenue $r - A^*$ can still cover costs. Therefore, the incremental profits gained by the platform through advertising services exceed the costs caused by the increase in c, ultimately driving up the platform's total profits. When $c \geq \frac{r}{2}$, the marginal cost of advertising technology is too high, and sellers will reduce advertising due to cost pressure, resulting in a decrease in platform's advertising revenue and profits. Proposition 4 reveals the cost-benefit trade-off of platforms investments in advertising technology. When the marginal cost of advertising technology is at a lower level, the platform can increase its investment in advertising technology to stimulate sellers' advertising demand and enhance the network externalities of the two-sided market, thereby achieving profit growth. Therefore, the platform needs to control the cost of advertising technology within a reasonable range to avoid suppressing sellers' advertising willingness due to excessively high costs, and affecting the platform's profits.

It is also interesting to note that $\frac{\partial \pi^*}{\partial \phi} = \frac{tc(r-c)(t-\phi)}{(\phi+t)^2} > 0$. This implies that a higher value of nuisance parameter will increase platform's maximized profits, because a higher value of ϕ causes platform to enjoy a higher payoff from selling ads with charging a higher advertising fee and generating a less quantity of ads demanded. However, a higher value of nuisance parameter will decrease seller's payoff of buying ads, because it leads to a higher A^* and a^*, which makes the payoff $\frac{1}{2}(r - A_i)^2 = \frac{(2-c)^2t^2}{2(\phi+t)^2}$ decreases with ϕ.

With the optimal solution, buyer's nuisance cost is given by $\frac{1}{2}\phi a_i^* = \frac{\phi t^2(r-c)^2}{2(\phi+t)^2}$. A higher value of nuisance parameter also leads to a higher nuisance cost of buyer, since $\frac{\partial(\frac{1}{2}\phi a_i^*)}{\partial\phi} = \frac{(r-c)^2t^2(t-\phi)}{2(\phi+t)^3} > 0$.

3.5 Extension - Ad-blockers and Buyer Heterogeneity

3.5.1 Model Setting

Based on the previous analysis, now considering ad-blockers and distinguishing two types of buyers.

Passive Buyers

Passive buyers do not use ad-blockers and they are fully exposed to ads. Their utility function is given by

$$u_b^{i,p} = v_b n_s^i - \frac{1}{2}\phi a_i^2 - t x_i \tag{27}$$

where $\frac{1}{2}\phi a_i^2$ captures full nuisance cost from ads.

Active Buyers

Active buyers use ad-blockers and reduces ad exposure. Suppose ad-blockers eliminate α proportion of ads ($0 < \alpha < 1$), so their nuisance costs becomes $\frac{1}{2}\phi(1-\alpha)a_i^2$. Therefore, their utility function is given by

$$u_b^{i,a} = v_b n_s^i - \frac{1}{2}\phi(1-\alpha)a_i^2 - t x_i \tag{28}$$

Let λ denote the proportion of active buyers in the market and $1 - \lambda$ denote the proportion of passive buyers. Thus,

$$n_b^i = \lambda n_b^{i,a} + (1-\lambda) n_b^{i,p} \tag{29}$$

3.5.2 Equilibrium Analysis

Seller's Advertising Decision The maimization problem is written as

$$\max_{a_i} u_s^i = v_s[\lambda n_b^{i,a} + (1-\lambda) n_b^{i,p}] - p_i + r a_i - \frac{1}{2} a_i^2 - a_i A_i - t y_i \tag{30}$$

The equilibrium result is derived in the following proposition.

Proposition 5. *In the setting of ad blockers and heterogeneous buyers under the duopoly platform competition model, the seller's utility on platform i is maximized by the unique solution $a_i^{*'}$ that*

$$a_i^{*'} = r - A_i - \lambda \alpha A_i \tag{31}$$

Proof. *The first order condition is given by $\frac{\partial u_s^i}{\partial a_i} = r - a_i - A_i + v_s \frac{\partial n_b^i}{\partial a_i} = 0$, where $\frac{\partial n_b^i}{\partial a_i} = \lambda(1-\alpha)\frac{\partial n_b^{i,a}}{\partial a_i} + (1-\lambda)\frac{\partial n_b^{i,p}}{\partial a_i}$. Solving the first-order condition can then derive $a_i^{*'} = r - A_i + v_s[\lambda(1-\alpha)\frac{\partial n_b^{i,a}}{\partial a_i} + (1-\lambda)\frac{\partial n_b^{i,p}}{\partial a_i}]$. To simplify the model, assume that the impact of advertising on the number of buyers is the same, and only the weakening of the advertising effect by α is considered. Therefore, $a_i^{i*} = r - A_i - \lambda\alpha A_i$.*

The implications of Proposition 5 are as follows. Equation (31) reflects the rational decision making of sellers under the influence of buyer heterogeneity and ad blocking. A_i directly increase the sellers' costs, prompting them to reduce advertising. The last term $-\lambda\alpha A_i$ shows the effect of the proportion of active buyers and the proportion of ad blocking on the reduction in advertising volume. The more active buyers there are, the stronger the blocking effect, and the sellers will reduce the amount of advertising due to the decrease in the actual advertising reach. As the actual effect of advertising weakens, sellers reduce advertising investment to optimize their utilities.

With the optimal solution $a_i^{*'}$, the seller's utility function is given by

$$u_s^i(a_i^{*'}) = v_s n_b^i - p_i + \frac{1}{2}r^2 - rA_i + \frac{1}{2}A_i^2(1 + \lambda\alpha - \lambda^2\alpha^2) - ty_i \tag{32}$$

The indifferent situation between platform i and j can then derive the number of sellers on platform i that

$$n_s^i = \frac{1}{2} - \frac{t(p_i - p_j) - \frac{1}{2}(t-\phi)[r - A_i - \lambda\alpha A_i]^2 + \frac{1}{2}(t-\phi)[r - A_j - \lambda\alpha A_j]^2}{2(t^2 - v_s v_b)} \tag{33}$$

Platform's Pricing Decision Platform i chooses $\{p_i, A_i\}$ to maximize its profit function. The equilibrium results are derived in the following proposition.

Proposition 6. *In this market where two symmetric platforms compete for buyers and sellers, in the setting of ad blockers and heterogeneous buyers, each platform's profit is maximized by the unique solution $\{p^{*'}, A^{*'}\}$ that $p^{*'} = p_i^{*'} = p_j^{*'}$ and $A^{*'} = A_i^{*'} = A_j^{*'}$ are given by*

$$p^{*'} = \frac{t^2 - v_s v_b}{t} - \frac{\phi t(r - c - \lambda ac)^2}{(\phi + t)^2} \tag{34}$$

$$A^{*'} = \frac{ct + \phi(r - \lambda\alpha r)}{\phi + t} \tag{35}$$

Proof. *The first order conditions of platform i's maximization problem are*

$$\frac{\partial \pi^i}{\partial p_i} = n_s^i + [p_i + A_i(r - A_i - \lambda\alpha A_i) - c(r - A_i - \lambda\alpha A_i)]\frac{\partial n_s^i}{\partial p_i} = 0 \tag{36}$$

$$\frac{\partial \pi^i}{\partial A_i} = (r - 2A_i - \lambda\alpha A_i + \lambda\alpha c)n_s^i + [p_i + A_i(r - A_i - \lambda\alpha A_i) - c(r - A_i) - \lambda\alpha A_i]\frac{\partial n_s^i}{\partial A_i} = 0 \tag{37}$$

Substituting the equation of n_s^i into first order conditions and solving them simultaneously can derive the equilibrium results. The second order conditions are $\frac{\partial^2 \pi^i}{\partial p_i^2} = -\frac{t}{t^2 - v_s v_b} < 0$, $\frac{\partial^2 \pi^i}{\partial A^{i2}} = -1 + \frac{t-\phi}{t^2 - v_s v_b}\{(r - 2A^{*'} + c - \lambda\alpha c)(r - A^{*'} - \lambda\alpha A^{*'}) - \frac{1}{2}[p^{*'} + (A^{*'} - c)(r - A^{*'} - \lambda\alpha A^{*'})]\} < 0$, *and* $\frac{\partial^2 \pi^i}{\partial A^i \partial p^i} = \frac{\partial^2 \pi^i}{\partial p^i \partial A^i} = \frac{(t-\phi)(r - A^{*'} - \lambda\alpha A^{*'}) - t(r - 2A^{*'} + c - \lambda\alpha c)}{2(t^2 - v_s v_b)}$. *Therefore,* $\frac{\partial^2 \pi^i}{\partial p_i^2}\frac{\partial^2 \pi^i}{\partial p^i \partial A^i} - (\frac{\partial^2 \pi^i}{\partial A^i \partial p^i})^2 > 0$, *which satisfies the negative definiteness condition for profit maximization.*

The implications of Proposition 6 are as follows. Equation (34) is combined with two components. The first component $\frac{t^2 - v_s v_b}{t}$ shows the net benefits of brand differentiation. When brand preference t is significant, the platform can attract sellers even if the network externalities $v_s v_b$ are weak. Therefore, the registration fee increases with t, reflecting the platform's strategy of setting higher registration fees by leveraging its brand advantages. If $v_s v_b$ is significant, the platform should reduce registration fees to balance the attractiveness of the network externalities. The second component shows the impact of advertising-related adjustments on registration fee pricing. If ϕ is greater, the buyer's aversion cost to the advertisement will be higher. The platform will then compensate sellers for the loss caused by the restriction of advertising by reducing the registration fee. Therefore, the increase in ϕ will reduce the registration fee, reflecting the compromise of the platform with the aversion of buyers to advertising to maintain the participation of the seller. In addition, the proportion of active buyers λ and the proportion of ad blocking α constitute the ad blocking effect. As $\lambda\alpha$ increases, the actual reach rate of the seller's advertising will decrease, and therefore advertising revenue is compressed. The quadratic form strengthens the negative impact of ad blocking on registration fees, reflecting the platforms compensation mechanism for the reduced benefit of sellers due to ad blocking. In equation (35), the numerator component $ct + \phi(r - \lambda\alpha r)$ shows the cost-benefit trade-off. As c increases, the platform will increase advertising fees to cover costs, reflecting the cost-driven pricing logic. $\phi(r - \lambda\alpha r)$ implies the secondary regulation of the aversion of buyers to advertising in advertising revenue. If ϕ is greater, even though advertising revenue is high, platforms need to be cautious with pricing to balance the buyer experience and advertising revenue. The denominator component $\phi + t$ implies the pricing strategy of the platform between the attention of buyers to the differentiation of the brands and the aversion to advertising. Therefore, equilibrium results reflects the multi-dimensional decision-making of the platform in the two-sided market. The platform must use the brand differentiation advantage to set the registration fee, and it must also deal with the impact of buyers' ad aversion and blocking behavior.

Proposition 7. *In this market where two symmetric platforms compete for buyers and sellers, in the setting of ad blockers and heterogeneous buyers, platforms increase advertising fees if active buyer proportion rises, and sellers reduce ad volume if ad-blocker proportion increases.*

Proof. *Derive* $\frac{\partial A^{*'}}{\partial \lambda} = \frac{\phi\alpha(r-c)}{\phi+t}$. *When* $r > c$, $\frac{\partial A^{*'}}{\partial \lambda} > 0$, *implying that an increase in the proportion of active buyers will drive up advertising fees. Derive* $\frac{\partial a^{*'}}{\partial \alpha} = -\lambda A^{*'}$. *Since* $\lambda > 0$ *and* $A^{*'} > 0$, $\frac{\partial a^{*'}}{\partial \alpha} < 0$, *this implies that improving the effectiveness of the ad blocker will directly lead to a reduction in the amount of seller advertising.*

The implications of Proposition 7 are as follows. When the proportion of active buyers increases, more buyers use ad blockers, which reduces the actual reach of ads to active buyers. In order to maintain advertising revenue, the platform will focus advertising resources on passive buyers. Since passive buyers' reception of ads is relatively stable, the platform can obtain higher advertising revenues from passive buyers by increasing advertising fees. Furthermore, improving the effectiveness of ad blockers means that the proportion of ads actually reaching buyers decreases. In order to avoid wasting resources, sellers will rationally reduce the amount of ads to save costs. This adjustment reflects the seller's adaptive decision to the decline in advertising effectiveness. By reducing the amount of advertising to optimize their own profits, it reflects the behavioral adjustment logic of market participants under ad blocking. Therefore, ad blockers help platforms perform implicit price discrimination, and the distinction between active buyers and passive buyers allows the platform to maximize profits. In addition, although ad blockers reduce buyer annoyance, excessive use will affect interactions between buyers and sellers. A policy response to excessive advertising could be to consider capping the amount of advertising so that sellers maximize their services with the constraint that $a_i \leq \bar{a}$. This will help lower the seller's advertising cost. However, if $\bar{a} < a_i^{*'}$, the network externalities $v_s n_b^i$ can be reduced if the ads are critical to buyer awareness.

3.6 Conclusion

This chapter evaluates the effect of paid advertising on a duopoly two-sided market. To determined the platform's pricing strategies and seller's advertising strategy, equilibrium results of realizing platform's maximized profits and seller's maximized utility are provided. The result shows that seller's advertising decision does not rely on the total number of buyers on the platform which he joined, but rely on the advertising fees charged on each unit of ads, Platform's registration pricing decision is determined by the net brand differentiation between two platforms and agents' cost and benefit of advertising and platform's advertising cost. The platform's maximized profits also show that a higher investment on advertising technology may not always lead to a higher profit. However, a higher value of buyer's nuisance parameter will increase platform's maximized profits by charging a higher advertising fee and generating a less quantity of ads demanded. For the seller, a higher value of buyer's nuisance parameter will decrease seller's payoff of buying ads. Furthermore, I extend the model to include ad blockers and buyer heterogeneity, distinguishing between passive and active buyers. As ad coverage decreases, sellers adjust the amount of advertising to optimize utility based on the proportion of active buyers and the effect of ad blocking. Meanwhile, platforms make pricing decisions by weighing brand differentiation, buyer ad aversion, and the impact of ad blocking. The main results highlight how ad blockers allow platforms to practice implicit price discrimination, maximizing profits by differentiating between buyer types. However, while ad blockers can alleviate buyer annoyances, overuse can undermine buyer-seller interactions. Policy measures such as setting advertising limits can balance seller costs and maintain network externalities. In general, this extension analysis deepens the understanding of the market dynamics of ad blockers and heterogeneous buyers, providing insights for platform strategies and regulatory approaches.

For the shortcoming of this chapter, the setting of two symmetric platforms and homogeneous agents tends to derive basic equilibrium results. Therefore, it is interesting to study how different platform's advertising technology affect their advertising service quality and therefore change both buyer and seller's entry choice under asymmetric competition. Furthermore, it is also interesting to study the effect of advertising on platform competitions when agents on each side are multi-homing.

Conclusions

This thesis conducts diverse stream of theoretical economic research on the economic of privacy. Chapter 1 studies the merger effect of platform competition in a modified city circular market. The result shows that merger strategy may not increase platform's pricing level charged on each side when the market is less competitive. While in a market with more than three platforms, the merger strategy may lead to asymmetric pricing decision between insider and outsider. The outsider will not be profitable by charging a lower level of price and generating less market shares. The merger effect on each insider platform is also differentiated. Therefore, as the total number of platforms in the market increase, the platform on the circle will increase its incentive to be merged, while the platform in the center will decrease its incentive to be merged. Chapter 2 studies user's privacy concern of disclosing different types of personal information on a social media monopoly platform. The result shows that with the privacy concern, users may not disclose information about revealing his personal identities, but they will always disclose information about revealing his shopping preferences. Furthermore, the platform will always has an incentive to trade a certain amount of user's information as if it collects positive amount of information. Chapter 3 studies the effect of platform's pricing decisions on competing buyers and sellers in a duopoly market. The result shows that seller's advertising level choice does not depend on the total market shares of buyers generated by the platform. Platform's pricing decision will be affected by considering buyer's nuisance cost of receiving ads. Additionally, increasing the cost of advertising technology may not cause a higher profit of the platform. It would be interesting to further explore the economic issues in two-sided market by introducing more endogenous setting, such as an endogenous merger model setting. Also, more heterogeneous characteristics can be involved in the analysis of two-sided market. For example, platform may compete for market shares by providing different advertising qualities. Or users may choose multihoming, which can also affect its information disclosure choices.

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