# A Theoretical Analysis of Modern Assumptions and International Spillovers in New Open Economy Macroeconomics

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# Abstract

This thesis seeks to disentangle the impact of common assumptions in contemporary two-country NOEM models on macroeconomic outcomes. We introduce completeness of markets, a Taylor-type monetary policy rule, and Calvo-style price rigidities in the seminal Obstfeld & Rogoff (1995) paper (OR henceforth). Although not unique to our model, we emphasise analytical tractability by examining how these assumptions affect macroeconomic mechanics and spillover effects. First, in Chapter 1 we introduce market completeness by enforcing perfect risk-sharing across countries. Compared to OR's results, a home money supply shock increases Home output while reducing Foreign output more than in OR. We also find that a monetary shock generates counter-intuitive welfare effects. Next, in Chapter 2 we introduce a Taylor rule for monetary policy, where the central bank adjusts interest rates in response to inflation deviations from target. Under complete markets, world consumption moves one-to-one with the monetary policy shock, while the sign of the output spillover is ambiguous and depends on the relative strength of price elasticity and central bank responsiveness. Under incomplete markets, the output spillover remains as under complete markets. The consumption spillover, while ambiguous, is still negative for a reasonable set of parameter values. We also show that the exchange rate solution can be determined independently from consumption and output. Finally, in Chapter 3 we introduce Calvo-style pricing. This generates persistence in the output differential between Home and Foreign countries and slower adjustment to monetary shocks, departing from OR's instantaneous adjustment. With complete markets, the spillover effect on consumption follows world consumption, so that a contractionary policy shock yields a positive spillover sign. The spillover sign on output is again ambiguous, though positive for a set of plausible parameter values.

# Contents

# List of Figures

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# Author's declaration

I declare that this thesis is a presentation of original work and I am the sole author. This work has not previously been presented for an award at this, or any other, University. All sources are acknowledged as References.

# Introduction

The increasing sophistication of New Open Economy Macroeconomic (NOEM) models in recent decades, achieved by incorporating a broad array of features and assumptions, has made it increasingly difficult to fully grasp their fundamental macroeconomic mechanisms. This PhD thesis aims to directly tackle this trade-off, by disentangling the impact of common features and assumptions in contemporary two-country NOEM models on their basic macroeconomic mechanics. This investigation is necessary because researchers in international macroeconomics have steadily moved towards large-scale Dynamic Stochastic General Equilibrium (DSGE) models that incorporate a wide range of frictions and shocks (Smets & Wouters 2007), or even heterogeneous agents (Kaplan et al. 2018). While, the richness of these frameworks allows researchers to replicate empirical observations more closely and to answer detailed policy questions, it often comes at the expense of analytical tractability and a clear understanding of core model mechanics.

To address this trade-off, we build upon the seminal work of Obstfeld & Rogoff (1995). We do so because it offers a balance of tractability and realism that makes it a good benchmark for open-economy analysis. The model's straightforward structure provides a solid foundation for introducing additional complexities while keeping the model tractable enough for the implications to be examined analytically. Therefore, across this thesis and through a general equilibrium framework, we will investigate the signs and magnitudes of the spillover effects generated by assumptions of market completeness, Taylor rule monetary policy, and Calvo price stickiness, individually and in combination, and compare them to the seminal OR framework. The focus on international spillover effects is motivated by the increasing interconnectedness of global economies, where domestic macroeconomic policies often have unintended yet significant consequences beyond national borders. Understanding these cross-country transmission mechanisms is crucial for designing effective and coordinated policies, especially in the face of global shocks. By quantifying how key structural assumptions influence the strength and direction of spillovers, this thesis aims to contribute to the literature on open economy macroeconomics and inform both theoretical modeling and policy analysis in an interdependent world.

The publication of Obstfeld & Rogoff (1995) and Obstfeld & Rogoff (1996) jump-started a new type of two-country models that what would be later known as NOEM, mainly because they facilitated the study of how shocks and policies are transmitted across countries with a relatively solid set of microfoundations. Subsequently, a rich literature rapidly emerged, focused on extending and refining the original OR model.[1]

**Extensions to OR**

Some of this work tried to make sense of the six fundamental puzzles in the international macroeconomics literature outlined by Obstfeld & Rogoff (2001). While all six are important, much of the literature has concentrated on several key ones. The first is the Feldstein-Horioka puzzle, which highlights the surprisingly high correlation between domestic saving and investment, suggesting limited international capital mobility. The second is the Home Bias puzzle, which shows that countries engage in far less international trade and invest much less in foreign assets than standard models would predict. The third is the Consumption Correlations puzzle, which reflects the weak co-movement of consumption across countries, despite theoretical expectations of strong international risk sharing in integrated markets. The fourth is the Purchasing Power Parity puzzle, which points to the persistent deviations in price levels across countries, indicating that goods prices do not adjust as theory would suggest. The fifth is the Exchange Rate Disconnect puzzle, where exchange rates appear excessively volatile and poorly linked to macroeconomic fundamentals. Lastly, the sixth is the Home Bias in Equity puzzle, which refers to the observed preference of investors for

[1]For a full treatment of the literature from the late 1990s please see Lane (2001). For a more updated literature review, please see Corsetti et al. (2010).

domestic equities despite the potential benefits of international diversification. These puzzles collectively challenge the assumptions of standard open-economy models and underscore the need to incorporate features such as market frictions, incomplete financial integration, and nominal rigidities to better explain real-world international dynamics.

Warnock (1998) explored Home bias through preferences for traded goods, by assuming that consumers gain more utility by consuming domestic tradeables. Under this model, a domestic monetary expansion increases Home welfare more than Foreign welfare, in contrast to the OR model where gains are equally distributed. Hau (2000) similarly addressed Home bias, but through the lens of non-traded goods, demonstrating that demand expansion favours domestic goods in the presence of non-tradeables. Home bias would subsequently become a permanent feature in large part of the literature, featuring in the benchmark NOEM model developed by Corsetti et al. (2010).

Another important assumption in OR is that the Law of One Price (LOOP) must always hold. However, numerous authors have relaxed this by incorporating Pricing to Market (PTM) (Betts & Devereux 2000, Chari et al. 2002), allowing some firms to set divergent prices domestically and abroad. Under PTM, changes in the exchange rate have a smaller effect on consumption, creating the possibility of short-run exchange rate overshooting that does not appear in the original OR model. When prices at Home and Abroad are sticky, a shift in the nominal exchange rate changes the real exchange rate and drives Home and Foreign consumption in different directions. At the same time, the correlation between Home and Foreign output rises because higher domestic demand leads to greater import demand at a fixed relative price of imports. This pattern of consumption and output comovement is more in line with observed international business cycles.

Furthermore, under full PTM and unlike in OR, the current account remains balanced. PTM allows exchange rate depreciation which can improve a country's terms of trade by raising export prices in domestic currency without affecting domestic-currency import prices. Conversely, an unexpected expansion in the Home country's money supply can worsen the Foreign country's terms of trade. PTM also influences the consumption elasticity of money demand. Under PPP, countries share the same real interest rates and consumption growth, so the exchange rate's volatility merely reflects the monetary shock. But under PTM, Home and Foreign prices diverge, and the consumption elasticity of money demand becomes crucial in shaping the exchange rate response to shocks.

Authors like Chari et al. (2002) or later Peneva (2009) for closed economy have also incorporated capital, suggesting that positive monetary policy shocks impact investment, potentially altering model magnitudes and current account dynamics. The former finds that while exchange rates are consistent with empirical evidence in that are volatile, persistent and correlated, volatility is too low, while correlation between real exchange rates and the ratio of consumption across countries is too high. The latter finds that, when an economy has sectors with differential capital intensities, prices of labour intensive goods present more stickiness than the capital intensive counterparts.

Finally, a further important line of research examines the welfare implications of international policy decisions. Corsetti & Pesenti (2001) demonstrate that the policy response function's sign hinges on the relative magnitudes of intertemporal and intratemporal elasticities of substitution between Home and Foreign goods. Specifically, if the intertemporal elasticity is larger, policies become substitutes: Foreign expansion diminishes Home output, incentivising domestic central banks to also expand. In this case, achieving efficient output levels necessitates coordinated monetary policy. Tille (2001) builds on this idea by dropping the assumption of a unitary elasticity of substitution between Home and Foreign goods, showing the potential for beggar-thy-neighbour policy outcomes. Benigno (2002) also extend the Corsetti & Pesenti (2001) framework to countries of different sizes, finding that cooperative policies are superior due to non-cooperative policies' contractionary bias, as individual countries neglect the positive spillovers of their monetary expansions on partner countries.

This focus on welfare analysis has remained highly influential, with a growing body of literature examining monetary policy coordination and optimal policy. Corsetti et al. (2010) offers the latest benchmark model using OR as a starting point, albeit with significant differences that have also become dominant in the rest of the literature. Monetary policy is introduced using an interest rate rule similar to those in benchmark closed-economy models by Clarida et al. (2002) and

Benigno & Benigno (2006), and the model is underpinned by two assumptions: a set of complete markets that provide full insurance against all possible contingencies, and sticky producer prices in domestic currency in the form of staggered pricing. However, as discussed previously, the sophisticated nature of these models makes it challenging to fully understand their comparative statics. Therefore, this thesis aims to systematically analyse the individual and combined impacts of market completeness, Taylor rules, and Calvo pricing on signs and magnitudes of international spillovers, using the tractable OR framework as a benchmark. By prioritizing analytical clarity, this research seeks to provide a deeper understanding of these modern assumptions, often obscured in more complex DSGE models. This understanding is crucial for understanding the impact that this implications then have in DSGE models which are then used for policy analysis and for informing the design of effective macroeconomic policies in open economies. Furthermore, this thesis briefly explores the relevance that some of this findings may have to emerging market economies, where assumptions of completeness of markets and optimal monetary policy may have particularly relevant implications.

**Completeness of markets**

The OR model only allowed for a single riskless bond to be traded. However, a number of authors have since introduced a complete set of asset markets by virtue of analytical convenience. For example, Chari et al. (2002), using a PTM model, compared the effects of a monetary policy shock on an economy with a set of fully contingent bonds against a single non-contingent bond, and shows how incompleteness of financial markets has very little effect on the persistence of monetary shocks. Benigno & Benigno (2008) develop a framework with completeness of markets for analysing exchange rate determination in economies where policymakers follow interest rate rules.

Although our thesis mostly focus on retaining complete markets, it is important to note that relaxing the complete markets assumption allows the model to incorporate more realistic financial frictions such as limited contingency claims and borrowing constraints. In many economies, agents cannot insure perfectly against country-specific shocks which is a feature reflected in observed cross-country consumption dispersion and volatility in real exchange rates. Heathcote

& Perri (2002) show that models with incomplete markets better replicate these empirical patterns, suggesting that such environments alter both the persistence and propagation of monetary shocks. By adopting incomplete markets, the model can reveal how risk-sharing limitations modify international transmission mechanisms.

Under complete markets, countries can fully insure against idiosyncratic shocks, eliminating cross-country consumption dispersion and leading to symmetric transmission of monetary policy shocks. In such an environment, the effects on consumption, output, and welfare are largely uniform across economies. Corsetti et al. (2010) note that this benchmark setting provides a useful reference point for evaluating monetary coordination, since it abstracts from distributional consequences and highlights the pure mechanisms of international transmission. Although this assumption likely overstates real-world financial integration, its analytical clarity allows us to focus on fundamental macroeconomic linkages without the added complexity of modelling financial frictions. By removing these confounding factors, the framework sharpens the interpretation of results and enables a more precise understanding of the role of monetary policy in a fully integrated world.

There is some debate, however, on whether introducing this assumption contributes to these models' challenges in aligning with empirical data. Indeed, Obstfeld & Rogoff (2001) argues that the puzzles they observe in the literature are not caused by completeness of markets assumptions, but rather by trade frictions. More recently, Eaton, Kortum & Neiman (2016), building on their multi-country dynamic model of international trade (Eaton, Kortum, Neiman & Romalis 2016), quantitatively assess the role of trade frictions in explaining these puzzles. They find that these puzzles do disappear even while holding the completeness of markets assumption. Hagedorn et al. (2018) also compared the effects of completeness of markets in monetary policy, concluding that an expansionary monetary policy shock renders identical results for both complete and incomplete market economies in terms of aggregate variables. However, Senay & Sutherland (2019) introduce incompleteness of markets allowing for international trade in multiple assets, showing that this more realistic assumption has meaningful consequences for optimal monetary policy. Additionally, (Bakshi et al.

2018) argue that completeness of markets is a poor approximation of reality when comparing cross-country correlations on consumption, wealth, dividend returns, and asset returns. Finally, recent Heterogeneous Agent New Keynesian (HANK) models increasingly incorporate incomplete markets both in single-country (Kaplan et al. 2018) and two-country (Bayer et al. 2024) models. Nevertheless, the complete markets framework remains an analytically tractable and theoretically clean setting for isolating the pure mechanisms of monetary transmission across economies.

**Taylor rules**

While OR modelled monetary policy using a money supply rule, Taylor rules have emerged as the dominant approach since the early 2000s, becoming the field's standard, following their introduction in the seminal paper Taylor (1993). Adopting Taylor rules to model central bank behaviour, which are essentially a feedback rule for the nominal interest rate, can offer a few advantages: (1) The Reserve Bank of New Zealand was the first central bank to introduce inflation targeting at the core of their monetary policy objectives. Since then, this has become the main way in which central banks around the world conduct monetary policy. Therefore, a Taylor Rule more closely resembles central bank behaviour as it allows us to specify the reaction function of a central bank with respect to inflation deviations from target. (2) A Taylor Rule allows for greater flexibility to modify the reaction function of the central bank, readily accommodating factors such as the output gap and forward-looking parameters.

Modelling monetary policy with an interest rate feedback rule, such as the Taylor rule, captures the systematic behaviour of modern central banks responding to deviations in inflation and output. Asso et al. (2010) examine how policymakers use the Taylor rule in practice, showing it provides a framework for stabilising monetary policy deliberations while retaining room for discretion. By including such a rule, the model gains realism and analytical clarity in characterising how monetary authorities anchor expectations and respond to shocks.

Interest rate rules also facilitate the exploration of policy trade-offs and coordination across open economies. Evidence suggests that when central banks assign even modest weight to exchange rate stability, the performance of such rules improves, especially in small open economies vulnerable to currency swings Froyen & Guender (2018). This motivates the relaxation of strict money supply targeting in favour of interest rate rules, as it better captures the environment where central banks must balance inflation control with exchange rate considerations and spillover effects.

However, central bank don't always follow the behaviour implied by Taylor rule. Several papers have used the Taylor-implied interest rate to state that monetary policy was too accommodative in the run-up to the 1970s inflation spikes (Clarida et al. 2000), and again in the run-up to the Global Financial Crisis (Taylor 2007, Hofmann & Bogdanova 2012). Furthermore, recent research indicates that that even among inflation-targeting countries (approximately 45 globally in recent years, according to the IMF's AREAER), adherence to Taylor rules may be more of an exception than the norm (El-Shagi & Ma 2023). Finally, another obvious circumstance where Taylor rules may fail to represent optimal central bank behaviour is when nominal interest rates lie near the effective lower bound (Iwata & Wu 2006). However, Belke & Klose (2013) show that a Taylor rule can be augmented to still work in such an environment, by targetting real interest rates and incorporating the size of the central bank's balance sheet to the rule.

Despite this mixed evidence, the Taylor rule still remains the consensus specification in modern macroeconomic models such as the benchmark DSGE model presented in Vines & Wills (2018), or workhorse central bank models around the world; indeed the Federal Reserve's primary macroeconomic model, FRB/US, also uses a prototypical Taylor rule (Brayton & Tinsley 1997).

**Calvo pricing**

While OR introduces nominal rigidities, it does so in a relatively simple manner, where prices are sticky only for one period. However, nominal rigidities are considered to be a key component for macroeconomic models to match the observed data. Indeed, empirical micro-level studies such as Bils & Klenow (2004) and Nakamura & Steinsson (2008), as well as DSGE models (Christiano et al. 2005), find strong evidence to support it. Consequently, a significant focus in the literature has been to refine the representation of nominal rigidities, through either sticky wages or price staggering. For instance, Hau (2000) examines a scenario in which prices are flexible but nominal wages are predetermined, with both product and labour markets being monopolistic. In this model, wages being sticky leads to optimal prices remaining fixed in the short run, resulting in the same international transmission effects as domestic product price rigidities in the OR model.

Introducing staggered price setting in the form of Calvo contracts enables the model to generate realistic nominal rigidity and inflation persistence. Empirical microeconomic evidence indicates that prices do not adjust continuously but exhibit delays. For example, Nakamura & Steinsson (2008) report that in U.S. micro price data, the median duration between price changes is around four months. Incorporating Calvo-style stickiness ensures that the model reflects such delays and produces propagation dynamics that align with data.

Sticky prices fundamentally shape the transmission and duration of monetary policy shocks. When firms cannot adjust prices immediately, monetary disturbances have prolonged effects on output and inflation, which a model without such frictions fails to capture. Empirical studies such as those summarised by Mankiw & Reis (2002) highlight how sluggish price adjustment magnifies macroeconomic responses. By incorporating staggered pricing, the model can assess how central banks' actions reverberate over time and across borders, offering more policy-relevant insights into effectiveness under real-world nominal rigidities.

Conversely, numerous papers tried introducing staggered prices as opposed to one-step-ahead prices. Staggered price setting allows for a smooth price adjustment, since firms take into account previous and future price decisions when optimally setting the price. The most widely used method in the literature is Calvo (1983), which assumes that the ability for a firm to adjust prices in any given period is stochastic, where $\alpha$ gives the percentage of firms that are not able to adjust prices at time $t$, and $1-\alpha$ the percentage of firms that are able to adjust prices. Some authors also extend this logic to determination of staggered wages Erceg et al. (2000) and (Daros & Rankin 2009).

The responsiveness and persistence of prices are affected by the sensitivity of costs to output and the sensitivity of prices to those costs. Chari et al. (2002) found that, if prices are simply a markup over marginal costs and marginal costs increase with output, staggering alone will not lead to endogenous price persistence. That is, the persistence cannot be amplified by the model and firms will raise prices as soon as they can. However, if firms face convex demand schedules with increasing price elasticity of demand, they will be slower to adjust prices.

Another method used in the macroeconomic literature, albeit less common particularly in international spillover models, is that of Rotemberg (1982). This approach assumes that monopolistic firm face quadratic costs when adjusting nominal prices, which can be measured in terms of the final good. Similar to Calvo pricing, it also incorporates a parameter $\phi$ which measures the degree of nominal price rigidity. This adjustment cost, which represents the detrimental effects of price changes on the customer-firm relationship, has a positive relationship with the size of the price change and the total economic activity. In a model that is log-linearised around the steady state with zero inflation, both Calvo and Rotemberg pricing specifications lead to the same reduced-form macroeconomic dynamics (Roberts 1995), and to similar welfare implications (Nistico 2007). Consequently, it is often argued that choosing a model is just a macroeconomist's preference (Ascari et al. 2011). However, it could also be argued that that the microeconomic foundations behind the Rotemberg model are less compelling: if prices genuinely are affected by the customer-firm relationship, then that relationship should be explicitly modelled, instead of having a costs function that may or may not accurately reflect them.

Finally, this finalise this literature review we briefly addresses the emerging market literature. NOEM research specifically focused on emerging market economies is scarce. Furthermore, much of the existing spillover literature – both theoretical and empirical– emphasises replicating empirical features often associated with emerging market economies, such as foreign currency borrowing, balance of payment crisis, or spillovers from advanced economies into emerging economies (Diebold & Yılmaz 2015, Canova 2005, Ahmed et al. 2021, Akinci & Queralto 2024). Research on spillovers amongst emerging markets are even scarcer, although some authors have explored this issue both from a DSGE (Comin et al. 2014) and time-series econometrics perspectives (Huidrom et al. 2020).

**Summary of results**

In Chapter 1, we extend the well-known Obstfeld & Rogoff (1995) model by introducing the assumption of market completeness, imposing the additional constraint that Home and Foreign consumption must be equal for all periods $t \geq 1$. Following the methodology first outlined by Aoki (1981), we analyse the model by decomposing it into a differences system, a sum system, and individual variables. The differences system reveals that a money supply shock results in a one-to-one pass-through to exchange rates, rather than a less than one-to-one pass-through seen in the original OR model. Meanwhile, the sum system shows that money supply changes also generate a one-to-one pass-through to world consumption, consistent with the original OR model.

Solving for individual variables and examining the welfare effects of asymmetric monetary shocks yields several key findings. For a positive money supply shock in the Home country, we find that Home output increases and Foreign output decreases by more than in the original model. However, Home consumption increases by less, while Foreign consumption increases by more. Interestingly, Home utility can either rise or fall, depending on the specifics parameter. This result highlights the counterintuitive implications of market completeness: while perfect risk-sharing equalizes consumption across countries, it cannot hedge against shocks affecting labour markets, where mobility remains constrained. Additionally, the introduction of market completeness removes hysteresis effects present in the original model, restricting the impact of monetary shocks to the short term.

In Chapter 2 we extend the framework presented in Chapter 1 by introducing a Taylor rule for monetary policy. This Taylor rule represents a central bank that sets nominal interest rates as a function of deviations in Consumer Price Index (CPI) inflation from target. It is also symmetric across both countries. This formulation allows us to analyse the implications of a more realistic monetary policy regime, moving beyond the money supply rule of the original OR model.

We first solve the model under completeness of markets. The sign of the output spillover effect is ambiguous, dependent on parameter values: when the price elasticity of demand ($\theta$) exceeds the central bank's inflation responsiveness ($\phi_\pi$), we obtain the orthodox result of negative spillover effects from expansionary monetary policy. When $\theta < \phi_\pi$, however, we obtain unorthodox results: an expansionary monetary policy shock will have positive spillover effect in the Foreign economy. We also solve the model by relaxing the assumption of completeness of markets. We find that the international consumption spillover effect generally remains negative across most parameter values, becoming positive only when $\theta > 150$. Finally, the output spillover remains the same as under complete markets.

In Chapter 3 we introduce Calvo pricing to incorporate staggered price setting. We find that it generates several interesting results. First, increased price rigidity generates persistence in output deviation, as opposed to the instantaneous return to steady state observed under OR-style price stickiness. The spillover effects of a monetary policy shock are also impacted. Higher degrees of price stickiness are associated with a slower adjustment process and a more muted impact on the terms of trade when compared with previous chapters. Finally, the spillover effects on output remain ambiguous, although are positive and consistent with Chapter 2 and the literature when using plausible parameter values.

The remainder of this thesis is organised as follows: Chapter 1 introduces and analyse the impacts of completeness of markets; Chapter 2 introduces and analyses monetary policy through a Taylor-type interest rate rule; Chapter 3 introduces and analyse the presence of Calvo-style price-staggering; finally, in Conclusion, we conclude.

# Chapter 1

# Completeness of markets in NOEMs

## 1.1 Introduction

In the thesis introduction we discussed some of the extensions to the workhorse Obstfeld–Rogoff model. In this first chapter, we focus on the assumption of financial market completeness. This assumption simplifies the analysis by allowing households to insure perfectly against all possible states of the world. We introduce it directly into the budget constraint by adding a set of fully contingent bonds, enabling agents to share risk efficiently across countries. The clean analytical structure provided by market completeness will facilitate the incorporation of later modelling extensions, where the focus will be on more complex frictions.

Beyond its technical convenience, the assumption of complete markets serves a broader purpose in open-economy macroeconomics. It provides a benchmark environment in which international risk sharing is perfect, so any deviations from this benchmark in later chapters can be interpreted as the effects of incomplete markets or additional frictions. This framework helps us address real-world policy questions, such as how international financial integration shapes the transmission of shocks, the dynamics of exchange rates, and cross-country consumption correlations. By starting from this idealised setting, we can better isolate the role of specific frictions such as price rigidities or monetary policy rules when moving towards more realistic environments.

## 1.2 Model

OR is a two-country model with symmetric Home and Foreign agents, where there is a continuum between 0 and 1 of different varieties of a good, where the first part of that index [0, n) is physically located in the Home country, and (n, 1] are physically located in the Foreign country, and where $n$ is the size of the Home country and $1 - n$ of the Foreign country. Foreign variables are denoted with an asterisk. Agents are infinitely lived, and act as both producers and consumers of differentiated goods. The model emphasises monopolistic competition and introduces price stickiness by allowing Home-currency prices of Home goods $p(h)$ and Foreign-currency prices of Foreign goods $p^*(f)$ to be set one period in advance. Note that even if that holds, it also has to be true that the Home-currency price of Foreign goods $p(f)$ and the Foreign-currency price of Home goods $p^*(h)$ must be able to fluctuate with the exchange rate, in order for LOOP and PPP to hold. It is a model where $t = 0$ is the pre-shock period, $t = 1$ is the short-run in which a shock occurs, and $t \geq 2$ is the long-run. Due to this long-run variables will be denoted with a bar to be easily identified. Finally, only riskless real bonds are available.

In this section, we will introduce the completeness of markets by modifying the last assumption but only between periods 0 and 1. We will do this by allowing for the presence of Arrow-Debreu securities in the form of a full set of state-contingent bonds through the budget constraint. In theory, there are nine possible combinations of monetary shocks between the Home and Foreign countries, considering each can independently experience an increase, decrease, or no change in money supply. However, for tractability and to maintain alignment with the benchmark OR framework, we group these possibilities into four broad states based on the direction and symmetry of the shocks. These are defined as follows: (1) a change in the Home country's money supply while the Foreign remains constant, (2) a change in the Foreign country's money supply while the Home remains constant, (3) simultaneous changes in both countries' money supply, and (4) no change in either country. Within each state, the change may be an increase or a decrease, but for clarity and consistency, we focus specifically on the case where the Home country experiences a positive money supply shock while the Foreign remains unchanged. This allows us to isolate the international spillover effects of a unilateral policy action under different structural assumptions, without the added complexity of multiple shock directions.

To justify the assumption that both countries will always agree to maintain equal consumption levels after any shock, we assume that from an ex-ante perspective, they face identical uncertainties, the same degree risk aversion, and start with equivalent endowments. This explains our choice of introducing two possible realisations of money supply for each country.

To maintain tractability while incorporating state-contingent uncertainty, we assume that the probability of monetary shocks occurring, whether in the Home country, the Foreign country, or both, is very small, for example, less than 1 percent. This implies that the probability mass is concentrated on the state where neither country's money supply changes, and the economy remains in its pre-shock steady-state equilibrium. As a result, when firms set prices in period 0, they do so under the expectation that a monetary shock is highly unlikely. Consequently, the prices they set will be nearly identical to those they would choose in a deterministic environment without uncertainty. This simplifying assumption allows us to introduce the possibility of shocks while preserving analytical clarity in the equilibrium pricing behavior.

A distinct feature of the OR model compared to previous literature was the introduction of hysteresis on wealth distribution. The concept of hysteresis refers to the idea that temporary shocks can have permanent effects on economic outcomes, even after these shocks (or their direct mechanisms) have dissipated. In the OR model, this hysteresis is reflected in the fact that a positive money supply shock leads to a permanent increase in wealth in the Home country and a permanent decrease in wealth in the Foreign country. This process arises through the bond channel: higher interest income for the Home agent leads to a shift away from work and towards leisure, while the opposite occurs for the Foreign agent.

For brevity, we will not delve into the full micro-foundations at this point, as they are largely the same as those in OR [1]. However, it will be useful to the reader to note that the utility function takes the following form:

$$U_t^j = \sum_{s=t}^{\infty} \beta^{s-t} \left[ \log C_s^j + \chi \log \frac{M_s^j}{P_s} - \frac{\kappa}{2} y_s(j)^2 \right] \tag{1.1}$$

where $C$ is a CES real consumption index given by

$$C^j = \left[ \int_0^1 c^j(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}} \tag{1.2}$$

where $c^j(z)$ is the $j$ th Home individual's consumption of good $z$ and $\theta > 1$. We will think of $\theta$ it as the price elasticity of demand faced by each monopolist. The Home price index is given by

$$P = \left[ \int_0^1 p(z)^{1-\theta} dz \right]^{\frac{1}{1-\theta}} \tag{1.3}$$

Also, the main difference with respect to the original OR model is that we will introduce the additional condition that Home consumption should be equal to Foreign consumption for any period $t \geq 1$:

$$C_t = C_t^* \; \forall \, t \geq 1. \tag{1.4}$$

We know this will be the case because prior to any shocks, both countries are in a perfectly symmetric state. If in addition agents are risk averse and the model allows for complete risk-sharing, they will optimally choose to fully insure themselves, resulting in equal ex-post consumption. This follows from the findings from previous literature such as (Obstfeld & Rogoff 1996, Chapter 5), Chari et al. (2002) and Benigno (2004), where the authors show how complete asset markets, in setups similar to ours, lead to identical rates of consumption growth across countries, irrespective of state-dependent outcomes. This is a result of the fact that the composite consumption bundle in the Home has the same composition as the bundle in the Foreign country. Therefore, Home and

[1]The model and the non-linear condition can be found in the appendix A.1

Foreign consumption have a relative consumption of one, and thus the real exchange rate will be exogenous and equal to 1. Consequently, we can assume that consumption levels are also equal, given our assumptions of symmetry and an equal likelihood of shocks across both countries:

$$\frac{C_1(s)}{C_0} = \frac{C_1^*(s)}{C_0^*} = \frac{Y_1^w(s)}{Y_0^W}$$

where $s$ represents the state. To solve the model, we obtain the following set of equations log-linearised around the symmetric steady state[2]. First, the log-linearised price index equations:

$$p_t = np_t(h) + (1-n)[e_t + p_t^*(f)] \tag{1.5}$$

$$p_t^* = n[p_t(h) - e_t] + (1-n)[p_t^*(f)] \tag{1.6}$$

where $e$ is the log deviation of the exchange rate. The PPP condition does not need approximation:

$$e_t = p_t - p_t^* \tag{1.7}$$

The money-demand (MD) equations are:

$$m_t - p_t = c_t - \frac{\hat{r}_{t+1}}{1+\delta} - \frac{p_{t+1} - p_t}{\delta} \tag{1.8}$$

$$m_t^* - p_t^* = c_t^* - \frac{\hat{r}_{t+1}}{1+\delta} - \frac{p_{t+1}^* - p_t^*}{\delta} \tag{1.9}$$

where $r$ is the real interest rate, and $\delta$ is defined as $\delta = \bar{r} \equiv \frac{1-\beta}{\beta}$. The world demand schedules are:

$$y_t = \theta[p_t - p_t(h)] + c_t^w \tag{1.10}$$

$$y_t^* = \theta[p_t^* - p_t^*(f)] + c_t^w \tag{1.11}$$

Note that the composite commodity of the goods produced in the country is

[2] From now on, unless specified otherwise, we will use lower case variables to denote deviations from the symmetric state, following the general formula $x \equiv \frac{X - \bar{X}_0}{\bar{X}_0}$, where $\bar{X}_0$ is the symmetric steady state. Additionally, to differentiate the interest rate $r$ from its steady state, we will denote the latter as $\hat{r}$.

different from the composite of goods consumed in the country, and therefore the relative price of $y$ to $y^*$ will not be the same, which also means that the terms of trade will be endogenous. By taking the population-weighted average of the world demand schedules equations, we can get the world goods market equilibrium condition:

$$c_t^w = nc_t + (1-n)c_t^* = ny_t + (1-n)y_t^* \equiv y_t^w \tag{1.12}$$

The labour-leisure trade-off conditions, presented below, arise partly from the assumption that output generates disutility through a quadratic function:

$$(\theta+1)y_t = -\theta c_t + c_t^w \tag{1.13}$$

$$(\theta+1)y_t^* = -\theta c_t^* + c_t^w \tag{1.14}$$

Equations (1.13) and (1.14) are assumed to only hold for periods $t \geq 2$, but not for period $t = 1$. In period $t = 1$, $y_t$ and $y_t^*$ are determined solely by the demand functions and the assumptions that $p_t(h)$ and $p_t^*(f)$ were set in period $t = 0$. This captures the notion that the short-run outputs are purely demand-determined, and that supply factors only play a role from period 2 onward.

Following from our new consumption condition outlined in equation (1.4), we obtain the following log-linearised version:

$$c_t = c_t^* \; \forall \, t \geq 1 \tag{1.15}$$

Similarly, we log-linearise the consumption Euler equations for both Home and Foreign countries:

$$c_{t+1} = c_t + \frac{\delta}{1+\delta} r_{t+1} \tag{1.16}$$

$$c_{t+1}^* = c_t^* + \frac{\delta}{1+\delta} r_{t+1} \tag{1.17}$$

where $\delta$ is the rate of time preference and follows the condition $\bar{r} = \delta \equiv \frac{1-\beta}{\beta}$. Finally, subtracting (1.9) from (1.8) and applying PPP (1.7) we obtain the following Exchange Rate Equation:

$$m_t - m_t^* - e_t = c_t - c_t^* - \frac{1}{\delta}(e_{t+1} - e_t) \tag{1.18}$$

Again, it is important to note that the only difference with respect to OR in the system of equations is the introduction of equation (1.15) and the removal of the balance of payments equation.

## 1.3 Results

Since all the equations are log-linear and leveraging the symmetry properties of the model, we follow Aoki (1981) who demonstrates that, instead of solving the complete system of equations simultaneously, the model can be decomposed into two separate, tractable systems. Furthermore, in this section, we consider the Dornbusch (1987) and OR exercise of an unanticipated permanent increase in relative Home money supply at $t = 1$.

### 1.3.1 Differences system

The difference system is constructed by taking the differences between the log-linearised Home and Foreign equations, allowing the model to capture cross-country dynamics in a tractable form. While obtaining the final system involves some algebraic manipulation, the underlying method follows the standard approach used in OR.

We begin by solving for differences between Home and Foreign variables. The change in relative money supply is given by:[3]

$$\bar{m} - \bar{m}^* = m - m^* \tag{1.19}$$

[3] From now onward, and for simplicity, we will denote the long-run variables with a bar, and short-run variables without one. It is possible to eliminate the need for time subscripts altogether by exploiting the principle that in this economy, the long-run equilibrium is achieved within a single period, which corresponds to the time required for nominal prices to adjust. Consequently, variables with a bar symbol denote long-term values (period 2 and onward), while without time subscripts or a bar symbol indicate short-run values.

where $m$ is the percentage deviation of the time 1 money supply from the initial steady state, given by:

$$m \equiv \frac{M_1 - \bar{M}_0}{M_0}$$

Using that, we complete the solution of the differences system as follows:

$$e = (m - m^*) \tag{1.20}$$

$$y - y^* = \theta e \tag{1.21}$$

$$\bar{c} - \bar{c}^* = c - c^* = 0 \tag{1.22}$$

$$\bar{e} = e \tag{1.23}$$

$$\bar{y} - \bar{y}^* = 0 \tag{1.24}$$

To derive the steady state exchange rate equation, we employ the saddlepath solution logic. Since $m_t - m_t^* = m - m^* \quad \forall\, t$ we know that $m_t - m_t^*$ is constant for all $t$. Additionally, due to the assumption of completeness of markets we know that $c_t = c_t^*$. From these two statements it then follows that (1.18) becomes a first order difference equation of $e_t$. In order to solve for (1.18), and given that there is an infinite number of potential time paths which satisfy the difference equation, we must examine the stability properties of the equation. Because $e_t$ is non-predetermined, a unique (or saddlepath) solution requires the difference equation to be unstable. The only non-divergent, or bounded time path is the one that starts and remains at the steady state. Thus, (1.20) represents the steady state of equation (1.18).

The terms of trade in the long run are given by $[\bar{e} + \bar{p}^*(f) - \bar{p}(h)]$ and by $[e + p^*(f) - p(h)]$ in the short run. In short run it will depend on the exchange rate which in turn depends on the money supply, meaning the terms of trade will increase in the same amount of money supply. In the long run however, the terms of trade are 0 since both $p(h)$ and $p^*(f)$ are set to 0.

It's also clear that assuming the completeness of markets makes the results more straightforward. This is because in OR the exchange rate in the short run is determined by the following expression

$$e = m - m^* - (c - c^*) \tag{1.25}$$

where $m - m^*$ is exogenous and the $c - c^*$ endogenous. However, in our equation (1.20) we lose the endogenous variable so that the exchange rate depends purely on an exogenous variable. This has two main advantages: (1) It facilitates adding new features to the model and (2) It allows us to more clearly see the idiosyncratic effects that get introduced when adding other building blocks to the model.

This implies that in OR, and because $c - c^* > 0$, an increase in money supply will lead to its effects being dampened by consumption adjustments, therefore transmitting the shock in a proportion of less than one-to-one. In our model, however, since $c - c^* = 0$, consumption does not absorb the shock at all, and the pass-through to exchange rates becomes one-to-one. Consequently, when facing a 1% positive money supply shock, the exchange rate $e$ will increase, i.e. depreciate, by 1%.

When observing eq. (1.21), due to the fact that the differential output in the short run is $\theta e$ and since we know that $e$ depends on the differential of money, then we can assert that when $m$ increases $e$ will increase by that amount, and the differential of output will increase by the same amount multiplied by $\theta$.

Equation (1.22) indicates that the difference in the long run consumption changes are equal to the difference in the short run consumption changes. That is, the changes (relative to $t = 0$) in the Home and Foreign consumption level are permanent.

Equation (1.23) shows that the exchange rate instantaneously jumps to its long-run equilibrium following a money supply shock. From this, we can conclude that there is no exchange rate overshooting or undershooting in this model, given that we take into account the stability condition.

Lastly, due to the labour-leisure trade-off and the condition $c = c^*$, we find that the output differential in the long run is equal to zero, as shown in equation (1.24).

### 1.3.2 Sum system

The sum system is derived for a population-weighted world aggregate using the following formula. For any variable $x$, the following example applies:

$$x^w = nx + (1-n)x^*$$
$$x = x^w + (1-n)(x - x^*)$$
$$x^* = x^w - n(x - x^*)$$

which when applied to our model could be summarised into:

$$y^W = c^W = m^W \tag{1.26}$$

$$\bar{y}^W = \bar{c}^W = 0 \tag{1.27}$$

The sum system reveals that world consumption moves one-for-one with changes in global money supply, consistent with the findings of Obstfeld and Rogoff. This occurs because a monetary expansion reduces the world real interest rate, which in turn stimulates aggregate consumption across countries. The result highlights that global monetary policy has positive-sum effects , which means that an expansion in one country can raise global demand rather than simply redistributing it across borders. This insight is important in understanding international policy spillovers, particularly in a highly integrated world economy, where coordinated or uncoordinated monetary actions can have amplifying or mitigating effects on global output and welfare. It also provides theoretical support for empirical findings that monetary shocks in large economies, such as the United States, often produce global demand spillovers rather than pure substitution effects.

### 1.3.3 Individual levels

Finally, having derived solutions for differences and world aggregates, we can now solve for the individual levels of all variables. First, we begin by obtaining the individual output levels for Home and Foreign. Combining equations (1.21) and (1.26) we get equation (1.28) and (1.29).

$$y = m^W + (1-n)\theta e \tag{1.28}$$

$$y^* = m^W - (n)\theta e \tag{1.29}$$

Furthermore, because our analysis focuses on the first state (when Home money supply increases), we can further simplify the previous expressions to (1.30) and (1.31).

$$y = [n + (1-n)\theta]m \tag{1.30}$$

$$y* = n[1-\theta]m \tag{1.31}$$

Following Obstfeld & Rogoff (1996) we know that the counterparts in the OR model are

$$y_{OR} = \frac{\delta(1+\theta) + 2[n(1-\theta)+\theta]}{\delta(1+\theta)+2}m \tag{1.32}$$

$$y^*_{OR} = \frac{(n)2(1-\theta)}{\delta(1+\theta)+2}m \tag{1.33}$$

One way to compare Home output in our model (1.30) with Home output in the OR model (1.32), and to assess which is larger, is to subtract the latter from the former:

$$y - y_{OR} = \frac{\delta(1+\theta)(1-n)(\theta-1)}{\delta(1+\theta)+2}m > 0 \tag{1.34}$$

Equation (1.34) shows that this model's Home output (1.30) is larger than the Home output of OR. Therefore, a positive money supply shock in the Home country leads to a bigger increase in Home output in our model compared to OR's.

We can also observe that the output spillover effect is negative in both models, as shown by the fact that both expressions (1.31) and (1.33) are negative. This is the outcome of two competing effects: on one hand, demand for Foreign exports increases because Home production increases; on the other hand, however, demand for Foreign export decreases because the Foreign exchange rate appreciates. The latter effect dominates the former resulting in the negative spillover.

In order to compare the Foreign output in both models, we compare (1.33) and (1.31), simplify and rearrange, yielding the following expression:

$$y^*_{OR} = \frac{2}{\delta(1+\theta)+2} y^* \tag{1.35}$$

This equation implies that Foreign output in our model is more negative than in the OR framework. This difference reflects the impact of introducing market completeness. A one-percent monetary expansion in the Home economy leads to a larger increase in Home output and a larger decrease in Foreign output relative to OR. The parameter $\theta$, which determines the elasticity of substitution between Home and Foreign goods, plays a central role in shaping the magnitude of these spillover effects. A higher theta means goods are more easily substitutable across countries, which amplifies the expenditure-switching effect. As a result, demand responds more strongly to changes in relative prices, leading to greater output gains for the Home country and correspondingly larger output losses for the Foreign country. This outcome is intuitive, as the asymmetry between Home and Foreign responses is driven by exchange rate movements. In the short run, output effects are demand-determined and arise from the expenditure-switching mechanism that shifts consumption away from Foreign goods and toward Home goods.

These effects are clearly reflected in the model's output and consumption equations. For instance, the positive difference in Home output relative to the OR benchmark increases with higher values of $\theta$, particularly when $\theta$ is greater than one. In the OR framework, larger values of $\theta$ also lead to greater redistribution in consumption between Home and Foreign, as the expenditure-switching effect becomes stronger. In contrast, our model imposes an equality condition on consumption, which prevents long-run divergence between countries. As a result, changes in $\theta$ amplify output asymmetries but do not translate into persistent differences in consumption levels.

As we can follow a similar exercise to compare the individual levels of consumption. We begin by combining equations (1.22) and (1.26) to obtain the individual levels of consumption for Home and Foreign:

$$c = nm \tag{1.36}$$

$$c^* = nm \tag{1.37}$$

In Obstfeld & Rogoff (1996) the individual levels of consumption for Home and abroad in the OR model are given by

$$c = \frac{(1-n)\delta(\theta^2 - 1) + n[\theta\delta(1+\theta) + 2\theta]}{\theta\delta(1+\theta) + 2\theta} m \tag{1.38}$$

$$c^* = \frac{n\delta(\theta^2 - 1) - n[(\theta\delta(1+\theta) + 2\theta)]}{\theta\delta(1+\theta) + 2\theta} m. \tag{1.39}$$

We can simplify and rewrite equation (1.38) to

$$c = \left\{ n + \frac{(1-n)(1+\theta)\delta(\theta-1)}{\theta\delta(1+\theta) + 2\theta} \right\} m \tag{1.40}$$

and do the same for (1.39) as follows

$$c^* = \left\{ n - \frac{n(1+\theta)\delta(\theta-1)}{\theta\delta(1+\theta) + 2\theta} \right\} m \tag{1.41}$$

It is straightforward to see that consumption in equation (1.36) is smaller than consumption in equation (1.40), and that therefore our individual level of Home consumption is lower than OR's. Moreover, it is also easy to see that Foreign consumption in our model will be greater than OR's, as $c^*$ in equation (1.37) is larger than in (1.41).

An important observation that follows from these results is that in OR, a permanent money supply shock generates a permanent increase in wealth for the Home country, leading to higher Home consumption. The opposite is true for the Foreign country, where they experience a permanent decrease in wealth and a corresponding decrease in consumption. This means Home would always consume more than Foreign. In our model however, given that $c = c^*$, neither country persistently consumes more than the other. This key difference arises because in our model there is no current account surplus in $t = 1$ nor accumulation of net Foreign assets, features that are present in the OR model.

Another implication of the consumption equality condition relates to the behavior of the real exchange rate. In our model, complete markets imply perfect risk-sharing, which requires that agents in both Home and Foreign equate their marginal utility of consumption. This condition leads directly to identical consumption paths, as shown in equations (1.36) and (1.37), so that $c = c^*$. As a result, the consumption differential remains constant over time and independent of macroeconomic shocks. Since the real exchange rate is determined by relative prices between Home and Foreign goods, it can still move in response to monetary shocks. However, with no corresponding movement in relative consumption, the model implies a zero correlation between the real exchange rate and the consumption differential.

This prediction stands in contrast to the standard theoretical benchmark under complete markets, which implies a negative correlation between relative consumption and the real exchange rate. That is, when Home consumption rises relative to Foreign, the Home real exchange rate should depreciate to maintain equality in marginal utilities. In our case, however, perfect consumption risk-sharing removes variation in relative consumption altogether. This disconnect between theory and data is well documented in the literature, most notably in the Backus & Smith (1993) puzzle, where empirical studies consistently find little to no correlation, or even a positive one, between the real exchange rate and relative consumption. Our model reproduces this feature mechanically due to the imposed symmetry in consumption, which reflects the limitations of complete markets in accounting for observed cross-country consumption dynamics.

Regarding percentage deviations in consumption, a money supply shock will lead to increases in consumption in both countries. However, the increase in Home consumption will be smaller than in OR, because under OR hysteresis effects result in a permanent increase in Home wealth and a permanent decrease in Foreign wealth. In our model this hysteresis mechanism is absent due to the imposed consumption equality condition, $c = c^*$.

### 1.3.4 Welfare effects

In this analysis, we log-linearise the real components of the intertemporal utility function (1.1), namely consumption and leisure, since these directly determine welfare through their effects on household preferences. While money balances do enter the utility function in the model, they are nominal by nature and typically serve to facilitate transactions rather than contribute significantly to welfare. As such, they are excluded from the welfare-relevant log-linearisation in this section, resulting in the expression.

$$U_t^R = \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left( \log C_\tau - \frac{\kappa}{2} y_\tau^2 \right) \tag{1.42}$$

Note that like OR we assume that utility will only depend on output and consumption and will not depend on real money balances. This is because as long as the utility from money balances remains small enough, the change in $U^R$ will dominate changes in $U$.

Since we know that the steady state is reached after just one period, we can differentiate the previous equation to obtain

$$dU^R = c - \kappa \bar{y}_0^2 y + \frac{1}{\delta} \left( \bar{c} - \kappa \bar{y}_0^2 \bar{y} \right) \tag{1.43}$$

where $\bar{y}_0 = \bar{y}_0^* = (\frac{\theta-1}{\theta\kappa})^{\frac{1}{2}}$. If we substitute $\bar{y}_0$ out, the previous equation will become

$$dU^R = c - \frac{\theta-1}{\theta} y + \frac{1}{\delta} \left[ \bar{c} - \frac{\theta-1}{\theta} \bar{y} \right] \tag{1.44}$$

From here, we can now evaluate this welfare effect directly by using the reduced-form solutions for $c$, $\bar{c}$, y and $\bar{y}$ mentioned earlier. Because no consumption or output deviations from the steady state occur in the long run, $\bar{y} = 0$ and $\bar{c} = 0$, reducing the equation to the first half of the expression below:

$$dU^R = nm - \frac{\theta-1}{\theta} [n + (1-n)\theta] m \tag{1.45}$$

The same steps would apply to finding $dU^{*R}$ which will yield the equation below

$$dU^{*R} = nm - \frac{\theta - 1}{\theta}[n(1-\theta)]m \tag{1.46}$$

Finally, if we simplify equation (1.45) and (1.46), we will obtain the real utility for Home and Foreign respectively.

$$dU^{R} = [(1-\theta)(1-n) + \frac{n}{\theta}]m \tag{1.47}$$

$$dU^{*R} = [n(\theta - 1) + \frac{n}{\theta}]m \tag{1.48}$$

It is important to clarify that the welfare expressions derived above are computed conditional on the realisation of the monetary shock, and not in an ex-ante sense. This means we are evaluating the utility change once the asymmetric monetary expansion in the Home country has occurred. In contrast, an ex-ante welfare analysis would involve taking expectations over all possible states of the world, weighted by their probabilities, prior to the shock being realised. While both approaches are valid, conditional welfare is more tractable here given the model's short-term horizon and focus on transitional dynamics between periods 0 and 1. Moreover, since the model features a complete return to steady state after one period, the welfare impact is fully captured by the period-0 response, simplifying the analysis.

Perfect risk sharing also plays a critical role in shaping the welfare results. Because agents are fully insured through Arrow-Debreu markets, consumption is equated across countries despite differences in output. This implies that the welfare differences arise entirely from the disutility of production rather than consumption inequality. In the absence of risk sharing, one would expect changes in consumption to be a dominant driver of utility differences across countries. However, in this model, since $c = c^*$, only output enters differentially into utility across countries, highlighting how production effort, not expenditure, drives welfare asymmetries under complete markets.

Analysing these equations reveals a surprising result: the sign of the change in Home utility is ambiguous. Given that $\theta \in [1, \infty)$, if $\theta$ is close to 1 the change in Home's utility will be positive, but it will become negative if $\theta$ is sufficiently large.

This apparent counterintuitive result can be understood by considering that the magnitude of the increase in Home output is determined by the strength of the expenditure-switching effect following a exchange rate depreciation. If $\theta$ is very large, the increase in Home output will also be very large. Accordingly, it is precisely this increase in output that generates disutility. This is because in our model agents are both producers and consumers, and increased output generates disutility: agents cannot consume all of the additional output that they produce because they have insured consumption, which means they have to transfer a portion of this extra output to the Foreign country. Therefore, for a sufficiently large $\theta$, the net effect on Home utility can be negative. This contrasts sharply with the change in Foreign utility, which remains unambiguously positive, even through Foreign output output is always reduced by the Home monetary shock.

## 1.4 Conclusion

In this chapter, we have introduced the assumption of completeness of markets to the prominent Obstfeld & Rogoff (1995) model by adding the binding constraint that Home and Foreign consumption should be equal for all periods $t \geq 1$.

We follow Aoki (1981) and investigate our model by decomposing it into difference and sum systems, and then analysing individual levels. Using the differences system we show that, unlike in OR, a money supply shock will have a one-to-one pass-through to exchange rates. Using the sum system we show how a money supply shock generates a one-to-one pass-through following world consumption, consistent with OR. Finally, solving for individual variables and examining the welfare effects of asymmetric money supply shocks reveals several key results. We find that when the Home country faces a positive money supply shock: (1) Home output increases and Foreign decreases by more than in OR; (2) Home consumption increases by less than in OR; (3) Foreign consumption increases by more than in OR; and (4) the change in Home utility can be either positive or negative.

This last result might appear surprising. While our model features complete markets and thus perfect risk sharing, we find that the equality $dU^R = dU^{*R}$, which holds in the OR framework, no longer holds in our setting. This divergence arises despite full consumption insurance, and can be attributed to the fact that complete markets cannot insure against asymmetric shocks that affect immobile factors such as labour. However, it is important to note that this is a result based on conditional, ex-post utility. A proper assessment of the welfare benefits of risk-sharing would require comparing ex-ante expected utility across regimes, taking into account the full distribution of possible shocks.

In summary, it is clear that the assumption of completeness of markets simplifies the model's results, as the exchange rate becomes fully determined by exogenous variables, independent of specific parameter values. Another difference from OR is the absence of hysteresis. In the original OR model, money supply shocks play a crucial role in permanently altering the levels of consumption and production, while in our model the effects are restricted to the short term.

# Chapter 2

# Taylor rules in NOEMs

## 2.1 Introduction

A key characteristic of second-generation NOEM models is the use of Taylor-type monetary policy rules, replacing the money supply rules prevalent in earlier models. Under this framework, the central bank instrument is typically given by a one-period nominal interest rate. One simple and common form of the Taylor rule is given by (Walsh 2017):

$$i_t = r + \gamma\pi_t + \gamma x_t + v_t \tag{2.1}$$

where the Central Bank adjusts the nominal interest rate $i_t$, in response to deviations in inflation $\pi_t$, and the output gap $x_t$, from their respective targets. In this chapter we will build upon the model presented in Chapter 1, and compare the macroeconomic implications of introducing a similar Taylor rule. We first explore the complete markets case, which provides a clean benchmark. We then relax this assumption, allowing us to assess how limited international risk sharing affects policy transmission.

The adoption of Taylor-type rules in this chapter follows the broader shift in the NOEM literature away from money supply targeting toward interest rate–based policy frameworks. Unlike the money supply rules used in earlier models such as Obstfeld & Rogoff (1995), Taylor rules allow central banks to respond more realistically to macroeconomic conditions, particularly inflation, using a nominal interest rate as the policy instrument. This is more aligned with actual central bank practice, especially in inflation-targeting regimes where monetary aggregates play a limited role in operational decision making Woodford (2003), Clarida et al. (1999).

Moreover, compared to exchange rate regimes, Taylor rules offer greater monetary autonomy and help insulate domestic economies from external shocks, particularly under incomplete markets.

In this chapter, we consider a simplified Taylor rule that responds to inflation only. This exclusion of the output gap reflects the stylised fact that many central banks, especially in emerging markets, tend to respond more to price stability concerns than to cyclical fluctuations in output Calvo & Reinhart (2002). It also facilitates a cleaner comparison with the benchmark model in Chapter 1. However, the implications of including output gap terms, as suggested by Senay (2008), remain an important area for future work.

The motivation for employing a Taylor-type rule in this open economy context is both empirical and theoretical. In practice, central banks rarely control monetary aggregates; instead, they adjust short-term interest rates as the main policy tool, particularly under inflation-targeting regimes where money plays a limited operational role. Notably, in many emerging economies, monetary authorities also respond to exchange rate fluctuations—a reaction that can be as strong as, or stronger than, their response to inflation or output variations Mohanty & Klau (2004). The experience of emerging markets further reveals that monetary policy is often procyclical, deviating from the standard Taylor prescription due to external vulnerabilities Kaminsky et al. (2004*a*). At the same time, some emerging economies have successfully adopted transparent inflation targeting frameworks despite higher volatility and credibility constraints Mishkin & Schmidt-Hebbel (2001). This evidence strengthens the case for analysing Taylor rules in open economy settings, as they align with actual central bank practices and allow us to explore how policy design performs under varying degrees of financial openness.

The rest of the chapter proceeds as follows. Section 2.2 presents the complete markets model and its implications under a Taylor rule. Section 2.3 introduces incomplete markets and discusses the resulting differences in equilibrium dynamics. Section 2.4 provides a comparative analysis of the two cases, and followed by Section 2.5, the conclusion.

## 2.2 Model

### 2.2.1 Introducing a Taylor Rule

In our efforts to continue updating some of the components of Obstfeld & Rogoff (1995), we will now introduce a Taylor rule in its simplest form, where the policymaker will set the nominal interest rate $\hat{i}$ as a function of inflation $\pi$, subject to random monetary policy shocks $v_t$ that last for one period only. Using a Taylor rule that does not include an output gap term may be a desirable feature under certain circumstances. For example, Kaminsky et al. (2004*b*) find that monetary policy in emerging markets does not tend to include countercyclical features. However, future research may wish to test different Taylor rule specifications. Indeed, Senay (2008) finds that alternative Taylor rules can have significant impacts on the effects of monetary policy under a two-country open economy setting.

While the canonical Taylor rule typically includes both inflation and the output gap as policy targets, we adopt a simplified version that responds only to inflation. This choice reflects several empirical and theoretical considerations. First, evidence shows that inflation targeting has become the predominant monetary policy framework in many emerging economies. Petrevski (2023) surveys the evolution of inflation targeting and finds that in emerging markets, central banks often emphasize price stability more strongly than cyclical output stabilization, largely because of institutional and fiscal constraints. Similarly, International Monetary Fund (2025) case studies on inflation targeting highlight that central banks with a history of high inflation tend to concentrate on anchoring inflation expectations, which reduces the role of the output gap in practice. Omitting the output gap from the Taylor rule therefore reduces the model's complexity without distorting the transmission channels of monetary shocks we are examining. Our focus is on how shocks propagate under different asset market structures rather than on specifying an optimal policy rule. The simplified rule also allows for clearer interpretation of the inflation–output trade-off, consistent with the earlier chapter where preferences do not include habit formation or strong countercyclical features. While richer Taylor rule specifications could be explored in future work, this form remains both tractable and theoretically informative for our purposes.

As in the previous chapter, we denote variables without star as Home country, and variables with a star as Foreign country.

$$\hat{i}_{t+1} = \rho + \phi_\pi \pi_t + v_t \tag{2.2}$$

$$\hat{i^*}_{t+1} = \rho + \phi_\pi \pi_t^* + v_t^* \tag{2.3}$$

We define the nominal interest rate as $\hat{i}_{t+1} = ln(1 + i_{t+1}) - ln(1 + \delta)$, and the real interest rate as $\hat{r}_{t+1} \equiv \ln(1 + \hat{r}_{t+1}) - \ln(1 + \delta)$. This definition, which expresses interest rates as log deviations, is also consistent with Woodford (2003), Galí (2015).

Inflation, denoted as $\pi_t = p_t - p_{t-1}$, is defined as the change in the CPI, which reflects prices of paid by consumers. While the Producer Price Index (PPI) is also used in the literature, we chose CPI because adjustments in consumer prices may be different in both magnitude and timing from adjustments in producers prices, with potentially relevant consequences for our results for for policy implications. The intercept term of the Taylor Rule, $\rho$, determines the steady state inflation rate. $\phi_\pi$ determines how aggressive the central bank reacts to deviations of inflation from its target. Empirically, this parameter is likely to be country and context dependent, as different central banks are likely to behave differently over time and across different economies.

Finally, we assume that the Taylor rule is symmetric across both countries. This serves two purposes: first, it aligns with the symmetry present in the original OR model; and second, it allows us to isolate the impacts of monetary policy shocks more clearly. This is likely to be a reasonable assumption when both countries are in similar stages of economic development, although it may be less appropriate when considering interactions between developed and developing countries. Indeed, developed countries usually follow tighter inflation targets of around $2\%$, while developing countries follow more flexible and often higher inflation targets (Siklos 2008, Hammond 2012).

### 2.2.2 Closed Economy with Taylor Rule

Before introducing a Taylor rule in our two-country model, it is appropriate to first examine the closed economy case, which mirrors the behavior of the world economy in a symmetric two-country framework. For consistency, we denote variables in this section with a $w$ superscript. It is also worth noting that household preferences remain unchanged from the previous chapter, ensuring continuity in utility specification across setups.

The log-linearised Euler equation for consumption is given by

$$c_{t+1}^w = c_t^w + \hat{r}_{t+1}^w \tag{2.4}$$

Since we are using a different definition of the real interest rate deviation when compared to OR, our Euler equation will also be different. While OR used $c_t^w = -\frac{\delta}{1+\delta}\hat{r}_{t+1}^w$, we will use

$$c_t^w = -\hat{r}_{t+1}^w \tag{2.5}$$

Note that $c_{t+1}^w$ is absent from the Euler equation, even though consumption Euler equations by definition relate consumption in the current period to consumption in the future. We will now show why that is the case.

By converting the log-linearised labour-leisure equation from the previous chapter into a population-weighted sum, we can obtain the following expression relating world income to world consumption: $(1+\theta)y_t^w = (1-\theta)c_t^w$. If we combine a barred version of this expression with the steady-state version of the log-linearised goods market clearing condition, we obtain

$$\bar{y}^w = \bar{c}^w = 0 \tag{2.6}$$

which also implies that $c_{t+1}^w = 0$. In an economy with flexible prices in every period, we know that $y_t^w = \bar{y}^w$. Since we also assume that the market clearing condition holds in ever period, $c_t^w = \bar{y}^w$, it follows from (2.4) that $\hat{r}_{t+1}^w = 0$.

If we plug it into a standard Fisher's equation $\hat{r}^w_{t+1} = \hat{i}^w_{t+1} - \pi^w_{t+1}$, we obtain $0 = \hat{i}^w_{t+1} - \pi^w_{t+1}$ . If we plug in the Taylor rule, we obtain $0 = \rho + \phi_\pi \pi^w_t - \pi^w_{t+1}$, which we can rearrange as

$$\pi^w_{t+1} = \rho + \phi_\pi \pi^w_t \tag{2.7}$$

We know that in the steady state world inflation is constant, such that $\pi^w_t = \pi^w_{t+1} = \pi^w$. Imposing this condition into (2.7) and rearranging for $\pi$ yields the steady state solution:

$$\pi^w = \frac{\rho}{1 - \phi_\pi} \tag{2.8}$$

In order for the model to have a unique stationary solution, it must satisfy the Blanchard-Khan condition (Blanchard & Kahn 1980). We show below that this is only met if $\phi_\pi > 1$.

Since inflation is endogenous to the model, we want to use our model to determine it. However, we do not know what inflation is at $t = 0$ because it's not naturally pre-determined. In Figure 1 we compare the solutions of (2.7) when $\phi_\pi > 1$ and $\phi_\pi < 1$. Additionally, we also assume that $\rho = 0$ in both panels. If inflation were to be predetermined, we would need to be in the situation shown in the chart to the right, where for any $\pi_0 \neq 0$ (indeed it could also be negative but we have omitted this from the charts for simplicity) the equilibrium will converge back to 0. However, since inflation in our model is not pre-determined, we must hope to be in the situation of the chart to the left, where we can see that the bounded time path solution is unique, as if $\pi_0 \neq 0$, inflation acts in an explosive manner and therefore can not converge to 0. Therefore, $\pi_0 = 0$ is the only bounded time path.

We will now look at the case where there is price stickiness, where the shock occurs in $t$ yet prices are pre-determined in $t - 1$. Since our shock only last one period only, we will denote $t \geq 2$ as the long run. Under sticky prices, and contrary to the previous case, $y^w_t \neq \bar{y}^w$.

To show the impact of price stickiness in the results of the model we can write a modified version of the IS curve[1] show in (2.5).

[1]Note that in our chart vertical intercept of the curve is positive, while (2.5) means that $\bar{y}^w = 0$ - we do so for simplicity of illustration.

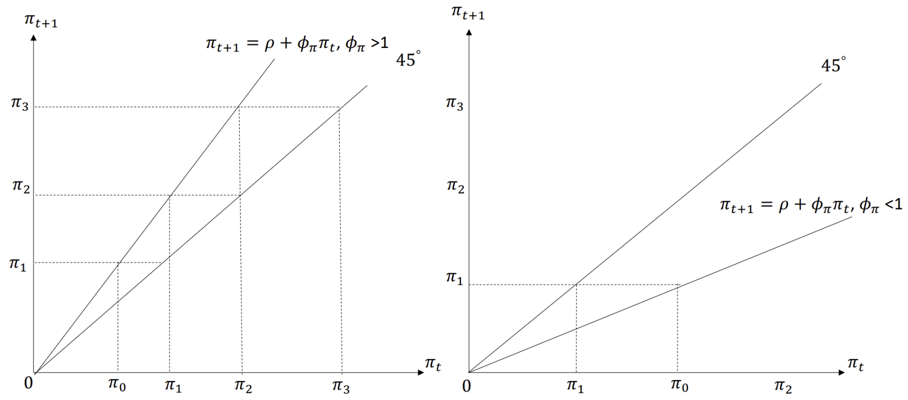


Figure 2.1: Determinacy of equilibrium

$$\bar{y}^w = y_t^w + \hat{r}_{t+1}^w \tag{2.9}$$

If we plug the Taylor rule into the Fisher's equation we get $\hat{r}_{t+1}^w = \rho + \phi_\pi \pi_t^w + v_t^w - \pi_{t+1}^w$. We can then substitute the steady state solution from $\pi^w$ into $\pi_{t+1}^w$, obtaining the equation below which we can think of as a substitute of the LM curve

$$\hat{r}_{t+1}^w = \rho + \phi_\pi \pi_t^w + v_t^w - \frac{\rho}{1 - \phi_\pi} \tag{2.10}$$

Since in our substitute for the LM curve the real interest rate does not depend on output, and since $\pi_t^w$ is predetermined at zero, the LM curve will be a horizontal line, with an intercept which depends on the monetary policy shock. The mechanics of this model are then exemplified in Figure 2.2.

In the event of a positive monetary shock where $v_t$ increases, the LM curve will shift upward from LM to LM'. Because of this, there will be a shift of the equilibrium from point A to point B, an increase of real interest rate, and a shift in output from $y_0$ to $y_1$. After the shock, the economy will return to the old long run real interest rate and output since the shock only happened in period $t$. The increase in real interest rates and decrease in output corroborates what one might intuitively expect.

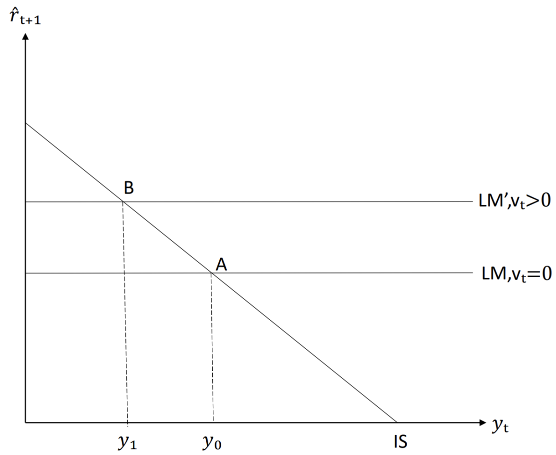


Figure 2.2: World ISLM curves

### 2.2.3 Two-country model with a Taylor Rule and complete asset markets

Having analysed the case of a closed economy, we now extend the model to a two-country setting. We do so by first setting out the long run difference system.

#### Long Run and Short Run Difference System

Recall that we denote the short run as $t = 1$ and long run as $t \geq 2$ onwards. As our shocks only last one period, variables will reach their steady state during period 2.

The consumption difference system is given by equation (2.11) below, where the differential consumption in the long run and the differential consumption of the short run are equal to zero. This is due to the assumption of completeness of markets discussed in Chapter 1, where we established that $c_t = c_t^*$ for $t \geq 1$.

Completeness of markets, implemented through the introduction of Arrow-Debreu securities in the form of a full set of state-contingent bonds, implies four possible states of the world. These states correspond to the combination of two potential values (zero or non-zero) for $v$ and $v^*$. As explained in Chapter 1 the four states are: 1) $v$ is non-zero and $v^*$ is zero; 2) $v^*$ is non-zero and $v$ is zero; 3) Both $v$ and $v^*$ are zero; 4) Both $v$ and $v^*$ are non-zero. In this framework, non-zero values of $v$ or $v^*$ are assumed to occur with a low probability. This ensures that prices set one period in advance are determined without any significant expectation of such shocks.

$$c_{t+1} - c^*_{t+1} = c_t - c^*_t = 0 \tag{2.11}$$

The long-run output difference system is given by the equation below

$$y_{t+1} - y^*_{t+1} = 0 \tag{2.12}$$

Let us define the depreciation rate as $d_t \equiv e_t - e_{t-1}$. In order to derive the long run depreciation rate by we must start by subtracting (2.3) from (2.2)

$$\hat{i}_{t+1} - \hat{i}^*_{t+1} = \phi_\pi(\pi_t - \pi^*_t) + v_t - v^*_t \tag{2.13}$$

We assume UIP is present, which is given by the following

$$e_{t+1} - e_t = \hat{i}_{t+1} - \hat{i}^*_{t+1} \tag{2.14}$$

Since PPP holds, the following will be true

$$e_{t+1} - e_t = \pi_{t+1} - \pi^*_{t+1} \tag{2.15}$$

If we plug the differential nominal interest rate into UIP we get

$$e_{t+1} - e_t = \phi_\pi(\pi_t - \pi^*_t) + v_t - v^*_t \tag{2.16}$$

If we plug the left-hand side of (2.15) lagged once into (2.16), we get

$$e_{t+1} - e_t = \phi_\pi(e_t - e_{t-1}) + v_t - v^*_t \tag{2.17}$$

Finally, plugging in $d_{t+1} \equiv e_{t+1} - e_t$ into (2.17), we get the law of motion for the depreciation rate

$$d_{t+1} = \phi_\pi d_t + v_t - v^*_t \tag{2.18}$$

Now, let us consider a positive monetary policy shock in the Home country occurring at time $t$, represented by an increase in $v_t$ in the short run only. From (2.18) we can argue that the depreciation rate in period $t+1$ will be 0. Because $d_{t+1}$ is not pre-determined, its starting value is unknown. To obtain a unique, perfect foresight time path, we need to find a unique starting value that prevents the time path from diverging. In the absence of shocks, that starting value of $d_t$ has to be 0. This condition for a unique, bounded solution is satisfied only if $\phi_\pi > 1$, as otherwise multiple non-divergent paths would exist. Furthermore, this implies that once the shock occurs and the exchange rate $e_t$ is affected, the exchange rate in the subsequent period, $e_{t+1}$, will reach its steady state value.

It is also important to note that, despite applying a shock to the Taylor Rule itself (which governs the nominal interest rate), the differential nominal interest rate does not change. We can see this in (2.14), where since we know that $d_{t+1} = 0$, the right hand side will also equal 0.

Since we have just shown that $d_{t+1} = 0$, we can use (2.18) to solve for $d_t$ as a function of $v_t - v_t^*$. And if we rearrange again in terms of depreciation rate, we will get

$$d_t = \frac{-v_t + v_t^*}{\phi_\pi} \tag{2.19}$$

Since $e_{t-1}$ is pre-determined, when $v_t$ increases, $e_t$ has to decrease

$$e_t - e_{t-1} = \frac{-v_t + v_t^*}{\phi_\pi} \tag{2.20}$$

This shows that examining today's depreciation rate in light of the current differential Taylor rule reveals a key finding: a contractionary monetary policy shock ($v_t \uparrow$) leads to a decrease in $d_t$, indicating an appreciation of the exchange rate. The extent of this impact is determined by $\phi_\pi$.

As we have described, under a Taylor Rule, an increase in $v_t$ is a contractionary monetary policy shock; whereas under a money supply rule, an increase in $m_t$ is an expansionary monetary policy shock. This also means that a temporary, contractionary monetary policy shock in the Home country will result in a permanent appreciation of the exchange rate for $t \geq 1$, implying that in our model there is no exchange rate overshooting.

This analysis also demonstrates that, under the assumption that both countries use Taylor rules and follow flexible exchange rates, we can determine the exchange rate independently of price stickiness, or the demand side of the economy or the goods market.

### World levels

In this section we introduce the world-level variables for nominal interest rates and inflation. Since by definition the world version of any variable is given by a weighted average, we can calculate the weighted average nominal interest rate by applying $n$ and $1-n$ to Home and Foreign,

$$\hat{i}^w_{t+1} = n\hat{i}_{t+1} + (1-n)\hat{i}^*_{t+1} \tag{2.21}$$

and the same for inflation

$$\pi^w_{t+1} = n\pi_{t+1} + (1-n)\pi^*_{t+1} \tag{2.22}$$

Since the world's real interest rate equals the individual countries' real interest rate, we can write it as

$$\hat{r}^w_{t+1} = \hat{r}_{t+1} = \hat{r}^*_{t+1} \equiv \hat{i}^w_{t+1} - \pi^w_{t+1} \tag{2.23}$$

In order to express the world interest rate in terms of the previously defined country-level real interest rates, it is trivially true that this also satisfies the general relationship

$$\hat{r}^w_{t+1} = (n)\hat{r}_{t+1} + (1-n)\hat{r}^*_{t+1} \tag{2.24}$$

To find the steady state solution for $\pi_{t+1}$ and $\pi^*_{t+1}$, we can make use of (2.15), re-written as $d_{t+1} = \pi_{t+1} - \pi^*_{t+1}$, and of the fact that $d_{t+1} = 0$ in every period other than the current period. Therefore, $\pi_{t+1} = \pi^*_{t+1}$ for every period other than the current one. From (2.22) this means that $\pi^w_{t+1} = \pi_{t+1}$ and $\pi^w_{t+1} = \pi^*_{t+1}$.

As shown previously, the steady state solution for world inflation is given by

$\pi^w_{t+1} = \frac{\rho}{1-\phi_\pi}$. Using this solution, we can obtain the real interest rates for Home and Foreign by substituting this expression into equations (2.25) and (2.26).

$$\hat{r}_{t+1} = \rho + \phi\pi_t + v_t - \frac{\rho}{1-\phi_\pi} \tag{2.25}$$

$$\hat{r}^*_{t+1} = \rho + \phi_\pi\pi^*_t + v^*_t - \frac{\rho}{1-\phi_\pi} \tag{2.26}$$

As the market clearing condition ($c^w = y^w$) applies, by combining (2.5), (2.9), and (2.10) we can obtain the Euler equation

$$y^w_t = c^w_t = -[\rho + \phi\pi^w_t + v^w_t - \frac{\rho}{1-\phi_\pi}] \tag{2.27}$$

From this, and as already shown in Figure 2.2, we can see that if $v^w_t$ increases, $c^w$ will decrease, and vice-versa.

$$y^w_t = c^w_t = -[\rho(1 - \frac{1}{1-\phi_\pi}) + \phi_\pi\pi^w_t + v^w_t] \tag{2.28}$$

**Levels of individual-country variables**

In this section, we derive the individual levels for output ($y$, $y^*$), and consumption ($c$, $c^*$) for the Home and Foreign countries. We obtain these individual levels by combining the new sum system with the difference system, as well as some of the equations presented in Chapter 1. As established previously, under certain assumptions, the time paths of variables in this model will consist of two values: a short run and a long run (or steady state) value, consistent with both Chapter 1 and OR. Therefore, to simplify notation we will henceforth drop the subscript $t$ when denoting short run variables (i.e., for $t = 1$), and denote long-run variables (for periods $t \geq 2$) by dropping the $t$ subscript and adding a bar above the variable.

In order to find the individual level Home output $y$ and Foreign output $y^*$ we use the same equations as in the previous chapter.

$$y = y^w + (1-n)(y - y^*) \tag{2.29}$$

$$y^* = y^w - n(y - y^*) \tag{2.30}$$

We can also use the differential output equation from our previous chapter

$$y - y^* = \theta e \tag{2.31}$$

In order to derive the individual output levels for Home and Foreign, we can insert (2.28) and (2.31) into (2.29) and (2.30), and obtain (2.32) and (2.33)

$$y = -[\rho(1 - \frac{1}{1 - \phi_\pi}) + \phi_\pi \pi^w + v^w] + (1 - n)\theta e \tag{2.32}$$

$$y^* = -[\rho(1 - \frac{1}{1 - \phi_\pi}) + \phi_\pi \pi^w + v^w] - n\theta e \tag{2.33}$$

Below we find the individual consumption levels for Home and Foreign ($c$ and $c^*$).

We can use the differential consumption equation from our previous chapter where we first introduced the completeness of markets

$$\bar{c} - \bar{c}^* = c - c^* = 0. \tag{2.34}$$

In order to find the individual level for the Home country we can use

$$c = c^w + (1 - n)(c - c^*) \tag{2.35}$$

and for Foreign country we use

$$c^* = c^w - n(c - c^*) \tag{2.36}$$

We can insert (2.28) and (2.34) into (2.35) and (2.36), to obtain (2.37) and (2.38).

$$c = c^w \tag{2.37}$$

$$c^* = c^w \tag{2.38}$$

Substituting equation (2.28) into the equation above we observe that a positive monetary shock, $v_t$, leads to a decrease in consumption for both Home and Foreign following a one-to-one relationship. This demonstrates that completeness of market eliminates the differential consumption components that would be present otherwise. This occurs because the insurance set up beforehand affects Home and Foreign consumption to the same degree, for a given contractionary shock in the Foreign country.

Just as a check, let us show that, even though Home and Foreign inflation rates ($\pi_t$ and $\pi_t^*$) are not predetermined, world inflation ($\pi_t^w$) is. Recall that the log linearised price indices for both Home and Foreign are given by:

$$p_t = np_t(h) + (1-n)(e_t + p_t^*(f)) \tag{2.39}$$

$$p_t^* = n[p_t(h) - e_t] + (1-n)[p_t^*(f)] \tag{2.40}$$

We can plug (2.39) and (2.40) into (2.22) to get

$$\pi_t = n[p_t(h) - p_{t-1}(h)] + (1-n)[e_t - e_{t-1}] + (1-n)[p_t^*(f) - p_{t-1}^*(f)] = n\pi_t(h) + (1-n)[d_t + \pi_t^*(f)] \tag{2.41}$$

and

$$\pi_t^* = (1-n)\pi_t^*(f) + n[-d_t + \pi_t(h)] \tag{2.42}$$

Substituting (2.41) and (2.42) into (2.22), we obtain

$$\pi_t^w = n\{n\pi_t(h) + (1-n)[d_t + \pi_t^*(f)]\} + (1-n)\{(1-n)\pi_t^*(f) + n[-d_t + \pi_t(h)]\} = [n^2 + (1-n)n]\pi_t(h) + [n(1-n) + (1-n)(1-n)]\pi_t^*(f) = n\pi_t(h) + (1-n)\pi_t^*(f) \tag{2.43}$$

Since $d_t$ cancels out we are left with the average rate of producer price inflation. $\pi_t(h)$ is defined as the inflation rate of the PPI for the Home country, while $\pi_t(f)$ is the inflation rate of the PPI of Foreign country.

This shows that $\pi_t^w$ is predetermined because both $\pi_t(h)$ and $\pi_t^*(f)$ are also predetermined since prices are set one period in advance for period t.

### The International Spillover Effect

To investigate the aggregate effects of a monetary policy shock on output, we rearrange (2.20) as a function of $e_t$[2], and plug it into (2.32) and (2.33), obtaining (2.44) and (2.47) respectively. Differentiating (2.44) with respect to Home and Foreign monetary policy shocks ($v$ and $v^*$), yields the Home output multiplier (2.45) and the international spillover on Home output (2.46), while differentiating equation (2.47) with respect to $v$ and $v^*$ yields the Foreign output multiplier (2.48) and the international spillover on Foreign output (2.49) respectively.

We can then see that the effects of a monetary policy shock in equations (2.46) and (2.49) are ambiguous, as the spillover effect of a monetary policy shock under a Taylor rule is not guaranteed to be negative (i.e., $dy/dv^*$ is not guaranteed to be positive), unlike in the situation under a money supply rule that we saw in the previous chapter. That is because $\frac{\theta}{\phi_\pi} - 1$ could take either sign, for values of $\theta$ and $\phi_\pi$ within their valid theoretical ranges.

$$y = -[\rho(\frac{\phi_\pi}{\phi_\pi - 1}) + \phi_\pi \pi^w + nv + (1-n)v^*] + (1-n)\theta[e_0 - \frac{v - v^*}{\phi_\pi}] \tag{2.44}$$

$$\frac{dy}{dv} = -n - \frac{(1-n)\theta}{\phi_\pi} \tag{2.45}$$

$$\frac{dy}{dv^*} = (1-n)[-1 + \frac{\theta}{\phi_\pi}] \tag{2.46}$$

$$y^* = -[\rho(\frac{\phi_\pi}{\phi_\pi - 1}) + \phi_\pi \pi^w + (n)v + (1-n)v^*] - n\theta[e_0 - \frac{v - v^*}{\phi_\pi}] \tag{2.47}$$

$$\frac{dy^*}{dv^*} = -(1-n) - \frac{n\theta}{\phi_\pi} \tag{2.48}$$

$$\frac{dy^*}{dv} = -n + \frac{n\theta}{\phi_\pi} \tag{2.49}$$

Nevertheless, to narrow down the potential sign of the spillover effect on the Foreign country, we can consider plausible values for $\phi_\pi$. Recall that $\theta$ represents

[2] Note that for this section, and since our monetary shock occurs in period 1, $e_{t-1}$ becomes $e_0$.

the price elasticity of demand faced by each monopolist, while $\phi_\pi$ reflects the aggressiveness of the central bank response to inflation deviations. Obstfeld & Rogoff (1996) show that $\theta > 1$ is required to ensure positive output levels, as otherwise marginal revenue would be negative, similar to Chapter 1. Empirical estimates for $\theta$ in the literature vary, but typically range between 5 and 10. Similarly, values of $\phi_\pi$ also vary, although 1.5, consistent with Taylor (1993), remains widely used in recent literature (Galí 2015).

If we assume these parameter values and ranges, and consequently that $\theta > \phi_\pi$, the ambiguity of the spillovers disappears. In this case, an expansionary monetary policy shock (represented by a decrease in $v^*$) in the Foreign country will result in a negative spillover on the Home country. Conversely, an expansionary monetary policy shock (represented by a decrease in $v$) in the Home country will result in a negative spillover on the Foreign country.

However, and still under a set of reasonable parameter estimations, a scenario where $\theta < \phi_\pi$ remains possible. This situation may occur in countries where significant portions of their economy are characterised by high-monopoly power industries, such as regulated utilities or essential services, and where central banks are also known to implement aggressive monetary policies. In this case, and contrary to standard findings in the literature, the sign of the spillover would be positive. Specifically, when $\frac{\theta}{\phi_\pi} < 1$, an expansionary monetary policy shock (a decrease in $v*$) in the Foreign country will result in positive spillover effect for the Home country, and an expansionary monetary policy shock (a decrease in $v$) in the Home country will result in positive spillover effect for the Foreign country.

Our results show that the effect of a monetary policy shock under a Taylor rule are transmitted to the economy through two main channels: (1) the real interest rate channel, operating through the goods market, and (2) the exchange rate channel. Equations (2.29) and (2.30) show that these two channels operate through the world output level (2.28) and the differential output (2.31), respectively. However, the international spillover effects present some ambiguity, as shown in equations (2.46) and (2.49), contrasting with the unambiguous results obtained in Chapter 1 without a Taylor rule.

This ambiguity in the sign of spillover effects can also be understood by unpacking the underlying transmission mechanisms. In our model, monetary policy affects output through both the real interest rate and the exchange rate. The exchange rate channel operates in part through an expenditure-switching effect: an appreciation of the Home currency following a positive Foreign monetary shock lowers the relative price of Foreign goods, which reduces Home demand for domestic output. The strength of this channel depends on the price elasticity of demand, captured by $\theta$, and the responsiveness of monetary policy to inflation, $\phi_\pi$. When $\theta$ exceeds $\phi_\pi$, the exchange rate channel dominates, and spillovers are negative. However, if $\phi_\pi$ is large relative to $\theta$, central banks are effectively stabilising domestic conditions before the expenditure-switching effects can fully materialise, which can flip the sign of the spillover. This highlights how the interaction between transmission channels and parameter values governs the direction of cross-border effects.

Following Blanchard & Galí (2007), we now examine whether a Taylor rule which completely suppresses demand side shocks also suppresses the effects on both output and inflation. To do so, we assume that the central banks from Home and Foreign coordinate their monetary policies and react very aggressively to inflation deviations, allowing the parameter $\phi_\pi$ to approach infinity.

In contrast to Blanchard & Galí (2007), where their output multiplier approaches zero, our multipliers (2.45) and (2.48) converge to $-n$ and $-(1-n)$, due to $\frac{(1-n)\theta}{\phi_\pi}$ and $\frac{n\theta}{\phi_\pi}$ also converging to zero. Therefore, in our model the own-country multiplier will remain negative, implying that highly aggressive central banks, even with coordinated action, not only fail to eliminate the effects of random monetary policy shocks, but generate negative output deviations.

While our analysis assumes a symmetric Taylor rule across countries, it is worth briefly considering how the results might change if monetary policy rules were asymmetric. For example, if the Home central bank responded more aggressively to inflation or output than the Foreign authority, the domestic transmission channel of monetary shocks would strengthen, amplifying the domestic effects of a shock. Conversely, asymmetric rules would also alter the spillover effects. A more responsive Home central bank could offset part of the expenditure-switching effect, weakening the negative spillover from a Foreign monetary shock. On the other hand, if Foreign policy were less responsive, its monetary shocks could induce stronger relative price movements and lead to more pronounced spillovers into the Home economy. These asymmetries would complicate the interpretation of cross-country effects, and in some parameter ranges, could even reverse their signs.

## 2.3 Extension to the case without completeness of markets

The original Obstfeld & Rogoff (1995) framework does not impose the assumption of completeness of markets, and uses money supply as the key monetary policy instrument. In an attempt to update the model we have introduced completeness of markets and an interest rate rule in the form of a Taylor rule. For this extension, however, and in order to facilitate the comparison of our monetary policy rule with OR's, we relax the assumption of completeness of markets.

Recall that we introduced the completeness of the markets assumption by assuming perfect consumption risk sharing, $C = C^*$ or this model extension, we relax this constraint, allowing monetary policy shocks to generate differential consumption effects across countries.

### 2.3.1 Global Equilibrium

Previously, the simplifying assumption of completeness of markets allowed us to bypass explicit equilibrium conditions for markets for different types of assets. However, now there is a single real risk-free bond. To calculate the global equilibrium condition of said bond, we start by assuming that global net Foreign assets are equal to zero

$$nB_{t+1} + (1-n)B^*_{t+1} = 0. \tag{2.50}$$

The previously set out condition that world output equals world expenditure will still hold as below

$$C^w_t \equiv nC_t + (1-n)C^*_t = n\frac{P_t(h)}{P_t}y_t(h) + (1-n)\frac{P^*_t(f)}{P^*_t}y^*_t(f) \equiv y^w_t. \tag{2.51}$$

### 2.3.2 Symmetric steady state

Studying the dynamics of the model in the steady state allows us to determine whether variables grow at the same rate over time or remain constant. In our model, consumption and production are constant over time, as there is no underlying source of growth. Additionally, the Euler equation implies that, given any monetary policy shock, consumption will remain equal to one another across any two period ($C_t = C_{t+1}$ or $C_t = \bar{C}$ and $C^*_t = C^*_{t+1}$ or $C^*_t = \bar{C}^*$) as long as the real interest rate is equal to the inverse of the discount factor ($1+r_{t+1} = \frac{1}{\beta}$).

With constant output and consumption, the real interest rate $r$ is determined by the consumption Euler equation (1.16) and therefore given by

$$\bar{r} = ln(\delta + 1) = \frac{1-\beta}{\beta} \tag{2.52}$$

In the steady state net foreign assets must also be constant over time, implying that current accounts must equal zero. This is achieved by setting consumption equal to the Gross National Product (GNP). When $\beta(1+r) = 1$, $\ln(1+\delta) = r$, and income is constant, the steady state consumption will equal to the steady state real income. Therefore, when GNP is expressed in the units of composite consumption goods, it will yield the equations below.

$$\bar{C} = \delta\bar{B} + \frac{\bar{P}(h)\bar{Y}}{\bar{P}} \tag{2.53}$$

$$\bar{C}^* = -\left(\frac{n}{1-n}\right)\delta\bar{B} + \frac{\bar{P}^*(h)\bar{Y}^*}{\bar{P}^*} \tag{2.54}$$

Log-linearising these two equations around the symmetric steady state, where steady state consumption will be equal to real income for both Home and Foreign, we obtain the following:

$$\bar{c} = \delta\bar{b} + \bar{p}(h) + \bar{y} - \bar{p} \tag{2.55}$$

$$\bar{c}^* = -\left(\frac{n}{1-n}\right)\delta\bar{b} + \bar{p}^*(f) + \bar{y}^* - \bar{p}^* \tag{2.56}$$

These two expressions will only apply for the long run because in the short run the steady state consumption does not need to equal the steady state real income. Additionally, note that they are missing time subscript and they have over bars as they are only valid for steady state changes.

Given that the initial symmetric steady state of Foreign asset exists ($\bar{B}_0 = 0$), we can define $\bar{b}$ as $d\bar{B}/\bar{C}_0^w$. And since that $\bar{C}_0 = \bar{C}_0^w = \bar{y}_0$, we are able to normalize the changes in Home bond holdings using initial Home consumption or output.

One aspect not previously examined is the behavior of $\bar{B}$. In an infinitely lived two-country model, the steady-state value of net foreign assets, $\bar{B}$, exhibits a 'unit root' property. This means that while $\bar{B}$ can be determined, its value depends on the full dynamics of the model rather than the steady-state equilibrium condition alone. As observed in the OR model (though not explored here due to the assumption of complete markets mitigating this effect), the 'unit root' property implies hysteresis: the steady-state value of $\bar{B}$ is shaped by the economy's historical trajectory, meaning transitory shocks to $B_t$ can have lasting effects.

In order to obtain $\bar{B}$, we must first obtain the world differential demand equation, which can be derived from subtracting the world demand schedule from representative Home and Foreign products:

$$y_t - y_t^* = \theta[e_t + p^*(f) - p_t(h)] \tag{2.57}$$

The relative price of typical Foreign and domestic products or terms of trade, given by $e_t + p^*(f) - p_t(h)$, generally shifts when international wealth distribution becomes uneven. As a result, and now even in the long run, the symmetric property of Home and Foreign producers is lost, opening the possibility to heterogeneous pricing of produced products, even when prices are still compared using the same currency. The differential labour-leisure trade off is given by:

$$y_t - y_t^* = -\frac{\theta}{1+\theta}(c_t - c_t^*) \tag{2.58}$$

The differential wealth transfer equation is obtained from subtracting (2.56) from (2.55) and making use of PPP equation from chapter 1 (1.7):

$$\bar{c} - \bar{c}^* = \left(\frac{1}{1-n}\right)\delta\bar{b} + \bar{y} - \bar{y}^* - [\bar{e} + \bar{p}^*(f) - \bar{p}(h)] \tag{2.59}$$

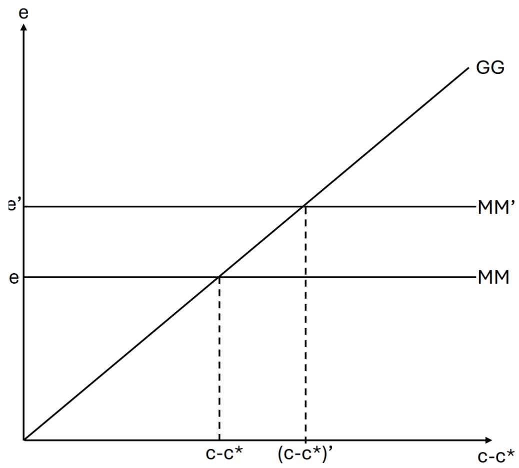


Figure 2.3: GG-MM curves

If we were to substitute the barred version of equations (2.57) and (2.58) into the equation above (2.59), we can see the effects of wealth transfer on differential consumption:

$$\bar{c} - \bar{c}^* = \left(\frac{1}{1-n}\right)\left(\frac{1+\theta}{2\theta}\right)\delta\bar{b} \tag{2.60}$$

If output were exogenous, a wealth transfer to Home of $\bar{b}$ would lead to a steady state of the per capita international consumption differential of $[\frac{1}{(1-n)}]\delta\bar{b}$. In this scenario, Home agents would increase consumption by the amount given by the per capita interest on the transfer $\delta\bar{b}$, while Foreign agents would lower it by $\frac{n\delta\bar{b}}{(1-n)}$. However, given that in our model output is endogenous, the effects of a wealth transfer on the consumption differentials will be smaller (recall that $\theta > 1$). This is because when they benefit from improved incomes due to higher interest receipts, Home agents shift out of work into leisure, while poorer Foreign agents do the opposite. Given this, the Home's steady state terms of trade is

$$\bar{p}(h) - \bar{e} - \bar{p}^*(f) = \left(\frac{1}{1-n}\right)\left(\frac{1}{2\theta}\right)\delta\bar{b} \tag{2.61}$$

Driven by the labour-leisure decision, and after receiving the transfer the steady state, terms of trade for the Home country improve. After the shock hits, and unlike the long-run case, in the short-run income will not equal expenditure. Instead, both countries could run a current account imbalance following the equation below.

$$B_{t+1} - B_t = rB_t + \frac{P_t(h)y_t}{P_t} - C_t \tag{2.62}$$

The linearised short run current account equation will be given by the equations below, which are derived by combining the log-linearised price index and the fact that $p(h)$ and $p(f)$ are set at zero one period in advance:

$$\bar{b} = y - c - (1-n)e \tag{2.63}$$

$$\left(\frac{-n}{1-n}\right)\bar{b} = \bar{b}^* = y^* - c^* + ne \tag{2.64}$$

$\bar{b}$ is present in the aforementioned equations because, given a single-period price setting, any net Foreign asset stocks that emerge at the conclusion of the first period become the new steady-state levels from the second period onward. Consequently, $b_t = \bar{b}$ for $\forall t \geq 2$, as all agents have equal discount rates and outputs remain constant. This equality forms an essential link between the short-term and long-term equilibrium equations. When prices are perfectly flexible and all shocks are lasting, there are no fluctuations, and the global economy immediately settles into the steady state defined by the current wealth distribution. Though price stickiness in this context is temporary, unforeseen monetary elements will ultimately influence the global distribution of wealth over the long run. Subtracting the two equations above will yield:

$$\bar{b} = (1-n)[(y - y^*) - (c - c^*) - e] \tag{2.65}$$

Finally, if we plug in equations (2.57), (2.60) and $\bar{c} - \bar{c}^* = c - c^*$ into the equation above, we get the GG equation.

$$e = \frac{\delta(1+\theta) + 2\theta}{\delta(\theta^2 - 1)}(c - c^*) \tag{2.66}$$

In (2.18) we have derived the solution for the exchange rate, showing that it is not affected by the presence of completeness of markets. We can also rearrange it as below to use it as our MM schedule.

$$e_t = e_{t-1} - \frac{v_t - v_t^*}{\phi_\pi} \tag{2.67}$$

On the world level, equation (2.28) remains the same. This is because the absence of the completeness of markets assumption does not affect the world level of consumption and output because, as we have shown earlier, they behave as if they were in a closed economy.

In Figure 2.3 we represent graphically the MM and GG curves we have just derived. The MM schedule, which represents the solution to the exchange rate, is a horizontal line because its solution does not depend on $c - c^*$. The GG curve illustrates the steady-state consumption differential as a function of net Foreign asset positions, linking the short- and long-term systems. It is characterised by an upward slope, indicating that domestic consumption increases in relation to Foreign consumption only if the exchange rate depreciates in the short term. This enables domestic output to grow compared to Foreign output.

We can now combine both schedules to obtain the equilibrium for $e$ and $c - c*$. In Figure 2.3, the solid MM line corresponds to the pre-shock equilibrium where the exchange rate is at level $e$. Following a negative shock to $v_t$, the new schedule is then given by the horizontal line MM'.

We can obtain the solution for $\bar{b}$ by substituting (2.31) and a rearranged version of (2.66) into (2.65)

$$\bar{b} = (1-n)\left\{\theta e - \left[e\frac{\delta(\theta^2-1)}{\delta(1+\theta)+2\theta}\right] - e\right\} \tag{2.68}$$

Factoring out $e$ will then give us

$$\bar{b} = (1-n)e\left\{\theta - 1 - \left[\frac{\delta(\theta^2-1)}{\delta(1+\theta)+2\theta}\right]\right\} \tag{2.69}$$

Finally, $e$ can then be substituted out with (2.67) and, since our monetary shock occurs in period 1, $e_{t-1}$ becomes $e_0$.

$$\bar{b} = (1-n)\left[e_0 - \frac{v - v^*}{\phi_\pi}\right]\left\{\theta - 1 - \left[\frac{\delta(\theta^2-1)}{\delta(1+\theta)+2\theta}\right]\right\} \tag{2.70}$$

As explained earlier the sign of $\bar{b}$ is important as it connects the imbalances (if any) of the short-run current account and the long-run current account. If there is a positive monetary shock in Home country ($v_t < 0$) then $\bar{b}$ will be positive because of the inequality below.

$$(1-n)(\theta-1)\left\{1-\left[\frac{\delta(1+\theta)}{\delta(1+\theta)+2\theta}\right]\right\} > 0 \tag{2.71}$$

If there is a positive monetary shock ($v_t < 0$) in the Home country, then according to equation (2.67), and since $e_{t-1}$ is pre-determined, $e_t$ will have to increase. This will in turn increase $\bar{b}$, meaning the Home country will experience a current account surplus, while the Foreign country will face a current account deficit. Additionally, and resembling OR's result, the larger the Home country is given by variable $n$, the smaller the impact of a Home monetary shock on its current account will be.

Combining the terms of trade (2.61) with the solution for $\bar{b}$ and the solution for the exchange rate $e$ we show that a expansionary (decrease of $v_t$) policy in the Home country will improve the Home's steady state terms of trade. This is because Home agents shift out of work into leisure, while poorer Foreign residents do the opposite.

$$\begin{aligned}\bar{p}(h)-\bar{e}-\bar{p}^*(f) = &\left(\frac{1}{1-n}\right)\left(\frac{1}{2\theta}\right)\delta(1-n)\left(e_0-\frac{v-v^*}{\phi_\pi}\right)\\ &\times\left\{\frac{2\theta(\theta-1)}{\delta(1+\theta)+2\theta}\right\}\end{aligned} \tag{2.72}$$

### 2.3.3 Levels of individual-country variables

The absence of completeness of market does not change the output level of the economy. Equations (2.32) and (2.33) continue to describe the output level for Home and Foreign countries. This is due to the fact that changes in the completeness of markets assumption will only affect the consumption levels of both countries, leaving the determinants of output level unchanged.

Because there is no completeness of markets, the long run and short run differential consumption equation is given by equation (2.73), which was derived from the Euler Equation from Chapter 1. Equation (2.74) is the wealth transfer differential equation derived earlier

$$\bar{c} - \bar{c}^* = c - c^* \tag{2.73}$$

$$\bar{c} - \bar{c}^* = \left(\frac{1}{1-n}\right)\left(\frac{1+\theta}{2\theta}\right)\delta\bar{b} \tag{2.74}$$

The world level in this economy is also the same as when the economy has completeness of market

$$c^w = y^w = -\left[\rho\left(1 - \frac{1}{1-\phi_\pi}\right) + \phi_\pi \pi^w + v^w\right] \tag{2.75}$$

Plugging in $c^w$ and $c - c^*$ into the formula for individual-country variable (2.35) and (2.36) will result in the consumption level for Home and Foreign countries respectively

$$c = -\left[\rho\left(1 - \frac{1}{1-\phi_\pi}\right) + \phi_\pi \pi^w + v^w\right] + \left(\frac{1+\theta}{2\theta}\right)\delta\bar{b} \tag{2.76}$$

$$c^* = -\left[\rho\left(1 - \frac{1}{1-\phi_\pi}\right) + \phi_\pi \pi^w + v^w\right] - \left(\frac{n}{1-n}\right)\left(\frac{1+\theta}{2\theta}\right)\delta\bar{b} \tag{2.77}$$

### 2.3.4 International Spillover Effect

To analyse the impact of a monetary policy shock on Home and Foreign consumption, we first expand equations (2.76) and (2.77) by substituting in the solution for $\bar{b}$, resulting in the two equations below:

$$\begin{aligned} c = &-\left[\rho\left(1 - \frac{1}{1-\phi_\pi}\right) + \phi_\pi \pi_t^w + (n)v + (1-n)v^*\right] \\ &+ \left(\frac{1+\theta}{2\theta}\right)\delta(1-n)\left(e_0 - \frac{v - v^*}{\phi_\pi}\right)\left(\frac{2\theta(\theta-1)}{\delta(1+\theta)+2\theta}\right) \end{aligned} \tag{2.78}$$

$$c^* = -\left[\rho\left(1 - \frac{1}{1-\phi_\pi}\right) + \phi_\pi \pi^w + (n)v + (1-n)v^*\right] - \frac{n}{1-n}\left(\frac{1+\theta}{2\theta}\right)\delta(1-n)\left(e_0 - \frac{v-v^*}{\phi_\pi}\right)\left(\frac{2\theta(\theta-1)}{\delta(1+\theta)+2\theta}\right) \quad (2.79)$$

Differentiating the consumption level of the Home country with respect to the monetary shock of Home (v) and Foreign(v*), and after simplifying and rearranging we obtain:

$$\frac{\partial c}{\partial v} = -n - \frac{(1+\theta)(\theta-1)\delta(1-n)}{\phi_\pi[\delta(1+\theta)+2\theta]} \quad (2.80)$$

$$\frac{\partial c}{\partial v^*} = -(1-n) + \frac{(1+\theta)(\theta-1)\delta(1-n)}{\phi_\pi[\delta(1+\theta)+2\theta]} \quad (2.81)$$

It is straightforward to see that the resulting sign of (2.80) is negative. The magnitude of the first term, given by $-n$, comes from $c^w$, while the second term comes from $c - c*$. On the other hand, equation (2.81) shows an ambiguous sign where the $c^w$ term is negative and the $c - c^*$ term is positive. To determine the sign of the equation, we can consider the effects that changes in the parameters $\phi_\pi$, $n$, $\delta$ and $\theta$ values may have on the final sign.

The elasticity of substitution between domestic and foreign goods, $\theta$, plays a central role in shaping spillover dynamics. A higher $\theta$ increases the sensitivity of households to changes in the terms of trade, which strengthens the expenditure-switching channel. This channel becomes more prominent under market incompleteness, as households reallocate consumption in response to relative price movements.

The preference for consumption smoothing is captured by $\delta$, which affects the intertemporal response to income fluctuations. When $\delta$ increases, agents place more weight on future consumption, which enhances the role of expected income paths in shaping current spending. As a result, the spillover term becomes more positive, potentially offsetting the direct negative effect of foreign monetary shocks.

The parameter $\phi_\pi$ reflects the aggressiveness of the monetary authority's response to inflation. A higher $\phi_\pi$ means a stronger policy reaction to inflation deviations, which suppresses nominal volatility and reduces exchange rate fluctuations. In the limit as $\phi_\pi$ approaches infinity, the central bank fully stabilises inflation, and the exchange rate channel becomes inactive. This dampens cross-border spillovers and brings the model closer to the complete markets benchmark.

Under complete markets, the insurance mechanism ensures that consumption adjusts mainly through shared risk rather than price movements. This setup eliminates the exchange rate-driven expenditure-switching channel and leads to unambiguous, always negative spillover effects. Unlike the incomplete markets case, changes in $\theta$, $\delta$, or $\phi_\pi$ do not reverse the sign of the spillover effect in this environment.

In equation (2.80), if the monetary authority were to practice a strict inflation-targeting policy to counteract inflation shocks in their country, the parameter $\phi_\pi$ would approach infinity. In this scenario, the second term would cancel out, leaving the overall spillover sign negative. This finding is consistent with the results obtained earlier in the chapter under the assumption of completeness of markets. We illustrate this relationship graphically in Figure 2.4, which plots the consumption multiplier against values of $\phi_\pi$ approaching infinity. For the other parameters, we use values commonly found in the literature ($\theta = 5$, $\delta = 0.02$), and assume countries of equal size ($n = 0.5$).

We can run the same exercise for (2.81). Figure 2.4 shows that as $\phi_\pi$ approaches infinity, the spillover multiplier approaches $-0.5$, mirroring the Home country multiplier. Furthermore, it also shows that, since $\phi_\pi > 1$ due to the determinacy of the equilibrium, the spillover effect can never be positive when $n = 0.5$, $\theta = 5$ and $\delta = 0.02$.

Figure 2.5 illustrates that the Home country multiplier is downward sloping and negative as a function of Home country size, $n$. While the spillover effect is an increasing linear function of $n$, that approaches 0 as the size of the country equals 1, it remains negative for our baseline parameter values ($\theta = 5$, $\delta = 0.02$, and $\phi_\pi = 1.5$). The impact of a Foreign contractionary shock (an increase in $v^*$) exhibits the opposite effect, where the larger the Home country, the less impacted Home country consumption will be. The finding that the size of the country does not alter the sign is an expected result that can already be seen through (2.80) and (2.81).

Figure 2.6 shows that the Home country multiplier is negative for all values of $\delta$, becoming increasingly negative as $\delta$ approaches infinity. The spillover multiplier, however, presents more ambiguity. For low values of $\delta$ it is negative, but as it is a positive function of $\delta$, it becomes positive when $\delta > 1$.

As $\theta$ approaches infinity, the Home country multiplier becomes increasingly negative in response to a Home contractionary monetary policy shock. The spillover effect, (2.81), is increasing with $\theta$, although it remains negative for empirically plausible values of $\theta$, only becoming positive when $\theta > 150$.

Having analysed the model under incomplete markets, we now compare with those obtained under complete markets and with the original OR model.

Equations (2.76) and (2.77) represent the consumption levels for the Home and Foreign countries in the model without completeness of markets, while (2.37) and (2.38) describe consumption levels under completeness of markets. Comparing these pairs of equations reveals that with complete markets, not only does the exchange rate effect vanish, but the weighting of individual consumption variables also diminishes relative to the consumption levels observed in the absence of completeness of markets. Furthermore, the spillover effects in (2.37) and (2.38) are unambiguous, unlike the case without market completeness.

To compare our result with OR's, we will be referring back to the equations (1.40) and (1.41) presented in Chapter 1. Comparing (2.80) and (1.40) indicates that, given a contractionary monetary policy shock, the own country multiplier will be negative. In OR, the consumption spillover effect given $m^*$ is negative. That is, a permanent contractionary Home money supply shock permanently decreases consumption in the Home country due to a permanent increase of wealth in the Foreign country. In the model we have just presented, however, the spillover effect becomes ambiguous.

Another notable finding is that there is a theoretical possibility of an unorthodox result: when $\delta > 1$ and $\theta > 150$, the consumption spillover $dc/dv^*$ becomes positive. This outcome is unorthodox because it is not feasible under market completeness or in the original OR framework. However, it is important to note that is unlikely to hold for empirically plausible parameter values.

Although this section assumes symmetric monetary responses, relaxing that

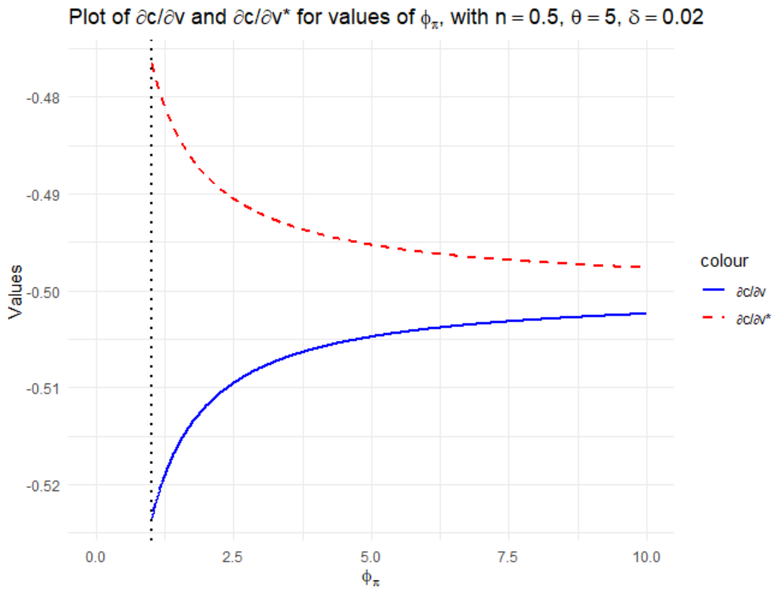


Figure 2.4: Plot of (81) and (82) for values of $\phi_\pi$

assumption may reveal further asymmetries in cross-border effects. Heterogeneous Taylor rules, particularly under incomplete markets, could amplify or dampen spillovers depending on the direction of the shock and the relative policy stance. A full treatment of this issue is beyond the scope of the current model and is left for future work.

## 2.4 Policy discussion

The analysis in Chapter 1 and Chapter 2 suggests that the degree of asset market completeness can have significant implications for the choice of a monetary policy regime. Specifically, Chapter 1 demonstrates that, under a money supply rule, incomplete asset markets dampen the responses of the exchange rate and of both Home and Foreign output to a random shock to the money supply. In contrast, Chapter 2 demonstrates that under a Taylor rule, incompleteness of asset markets exerts no comparable dampening effect on these variables, because their responses to a random shock in the Taylor rule are the same under both incomplete and complete markets.

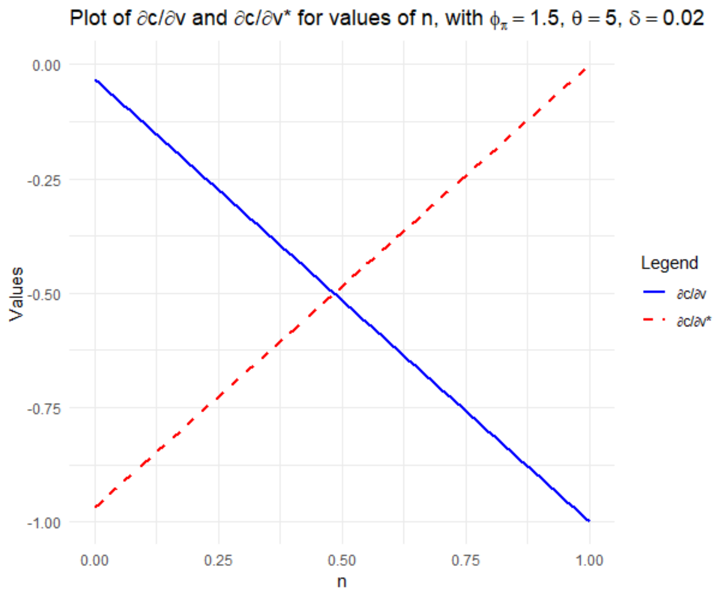


Figure 2.5: Plot of (81) and (82) for values of $n$

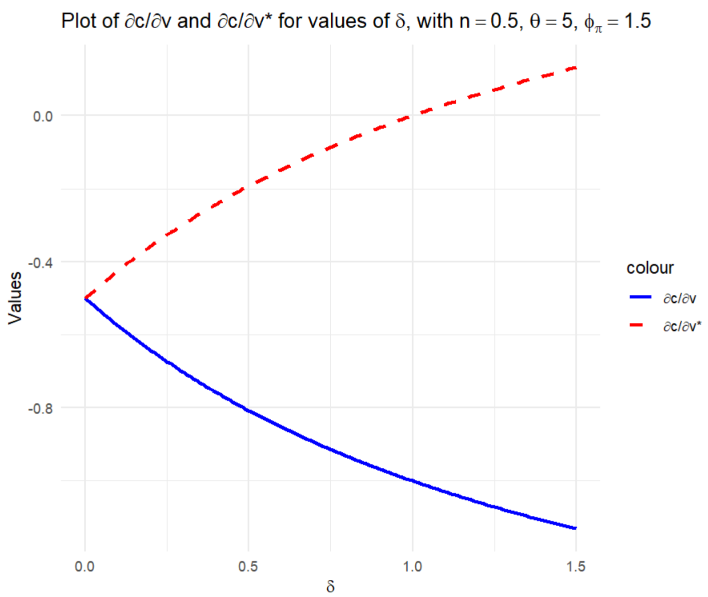


Figure 2.6: Plot of (81) and (82) for values of $\delta$

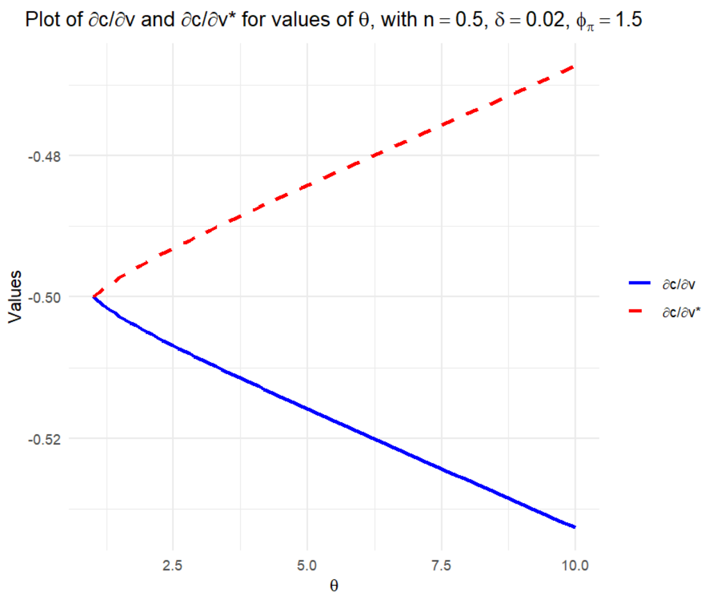


Figure 2.7: Plot of (81) and (82) for values of $\theta$

This asymmetry has potential implications for monetary policy design. Suppose that random errors in the implementation of monetary policy are unavoidable under both regimes. Further suppose, although this is a strong and specific assumption, that under complete markets, policy errors affect the exchange rate and output equally across the two regimes. In that case, in an environment with incomplete markets, the money supply rule would be preferable because market incompleteness would dampen the transmission of policy errors. This dampening effect does not occur under a Taylor rule.

Empirical studies (Prasad & Zhang 2015, Bakshi et al. 2018) have shown that developing countries tend to have lower levels of financial market completeness than advanced economies. If this pattern holds in the ASEAN region, then the logic above suggests a possible case for viewing money supply rules as a more robust option in those settings. Although this conclusion relies on several strong assumptions, it points to a specific channel through which asset market structure could affect how monetary regimes influence macroeconomic stability.

## 2.5 Conclusion

In this Chapter we have introduced a Taylor rule for monetary policy in the framework presented in Chapter 1. To facilitate comparisons across chapters and with OR, we do so in two stages: first in a model with complete markets, and second in a version with incomplete markets.

The Taylor rule we use (Taylor 1993) follows a simple form, where the central bank sets nominal interest rates as a function of deviations in CPI inflation from its target. We use log-deviation variables for the nominal and real interest rates, following standard methods in the literature Woodford (2003), Galí (2015), to keep our approach consistent with widely used macroeconomic models. We also assume symmetric Taylor rules across both countries. While doing so helps simplify the analysis, we recognise this may not hold in cases where economies have different inflation targets or institutional frameworks.

Under complete markets we impose the condition $c = c^*$, implying that the consumption spillover and consumption multiplier of a monetary policy shock will mirror the response of world consumption. World consumption, in turn, moves on a one-to-one relationship with the shock. The output spillover effect is initially ambiguous, depending on the magnitudes of $\theta$ and $\phi_\pi$. When $\theta > \phi_\pi$ we obtain an orthodox result, where an expansionary monetary policy shock delivers negative spillover effects. When $\theta < \phi_\pi$, however, we obtain unorthodox results: an expansionary monetary policy shock will generate positive spillover effects in the Foreign economy. Finally, the Home country output multiplier is negative.

With incomplete markets, where $c$ is not constrained to equal $c^*$, consumption and spillover effects become more nuanced. The consumption spillover is ambiguous, and its analysis is further complicated by the larger parameter space. Therefore, we numerically solve the output and spillover equations using plausible parameter values. We find that the international consumption spillover effect remains negative for most parameter values, becoming positive only when $\theta > 150$, and that the output level of Home country is identical that under complete markets.

# Chapter 3

# Calvo pricing in NOEMs

## 3.1 Introduction

In the original OR model, firms simultaneously set prices one period in advance. This assumption is a convenient way of allowing all price adjustments to occur within a single period, but it also implies that prices will suffer from arbitrary discrete jumps. In contrast, price staggering offers a more sophisticated approach to introducing price stickiness, allowing for smoother price adjustments. The most common method for doing so in the literature has been Calvo (1983) as described by Yun (1996). It introduces price stickiness by assuming that in each period a proportion of firms $\alpha$ are unable to adjust their prices, while the remaining proportion $1 - \alpha$ can do so optimally.

This chapter extends the model developed in Chapter 1 and Chapter 2 by incorporating Calvo-style staggered pricing. First, we outline the main components of the model. Following the approach outlined in previous chapters, we then solve the model using Aoki's method, by independently analysing the sum system and the difference system. Finally, the chapter derives the individual-level countries variables and analyse their spillover effects.

While this chapter shares with Gali & Monacelli (2005) a focus on open economy New Keynesian models with Calvo-style price stickiness, there are key differences. Their framework features a small open economy interacting with an exogenous rest of the world, whereas this chapter develops a symmetric two-country model where both economies are of equal size and jointly determine global variables. Moreover, while they primarily analyse domestic policy transmission, the two-country structure here allows for a richer analysis of cross-country spillovers and mutual feedback effects. This broader setting enables a more detailed examination of how price stickiness influences international business cycle co-movement and the propagation of monetary shocks under complete financial markets.

The adoption of Calvo pricing in this context is motivated by both empirical evidence and policy relevance. Empirically, microdata show that firms adjust prices infrequently and in a staggered fashion, rather than simultaneously, which leads to a gradual pass-through of shocks into aggregate prices. This feature is particularly important in open economies, where exchange rate movements and imported inflation transmit unevenly over time. From a policy perspective, incorporating Calvo stickiness enables the model to capture short-run trade-offs faced by central banks, such as how quickly monetary policy actions affect inflation and output, and to examine whether these effects differ across countries when shocks spill over internationally. By embedding staggered price adjustment in a two-country framework, the analysis can shed light on how nominal rigidities shape the timing, magnitude, and asymmetry of monetary policy transmission across borders.

## 3.2 The set up of the model

In previous chapters the micro-founded components of the model remained largely similar to OR, thus requiring minimal explanation. However, the introduction of Calvo pricing requires substantial changes to the micro-foundations. Therefore, in this section we provide a comprehensive description of our model setup under Calvo pricing.

### 3.2.1 Utility Function

In this model, individuals across both economies are assumed to share uniform preferences for the consumption index, real money holdings, and productive effort.

To simplify the analysis, we represent each country's population through a single, representative agent indexed by $j \in [0,1]$, reflecting symmetric preferences and constraints within the nation. The lifetime utility of a representative agent $j$ is expressed in the following general form:

$$E_t(U_t^j) = E\sum_{s=t}^{\infty} \beta^{s-t}\left[lnC_s^j + \chi ln\frac{M_s^j}{P_s} - \frac{1}{2}\kappa y_s(j)^2\right] \tag{3.1}$$

where $C_s^j$ is the real consumption index, $\frac{M_s^j}{P_s}$ is real money holdings, and $\frac{1}{2}\kappa y_s(j)^2$ is the disutility from labour. The real consumption index is in turn given by

$$C^j = \left[\int_0^1 c^j(z)^{\frac{\theta-1}{\theta}} dz\right]^{\frac{\theta}{\theta-1}}, \tag{3.2}$$

where $c^j(z)$ denotes the consumption of good $z$ by the representative agent in the Home country. The parameter $\theta$ plays a central role because it governs the degree of substitutability across varieties. A higher $\theta$ means goods are closer substitutes, which makes demand more sensitive to price differences, limiting firms' pricing power and reducing the persistence of price stickiness on the terms of trade. Conversely, a lower $\theta$ amplifies the role of relative prices in allocating demand between Home and Foreign goods, making exchange rate movements and nominal rigidities more impactful. Since $\theta > 1$ represents the price elasticity of demand faced by monopolists, this condition is essential to ensure positive output levels—otherwise marginal revenue would be negative. The consumption index generalizes the Constant Elasticity of Substitution (CES) function, extending it from a two-good case to a continuum of goods.

The price deflator for nominal money balances corresponds to the consumption based price index, derived from the consumption index. Let $p(z)$ represent the price of good $z$ in Home currency. The overall money price level is then given by:

$$P = \left[\int_0^1 p(z)^{1-\theta} dz\right]^{\frac{1}{1-\theta}} \tag{3.3}$$

The term $\frac{1}{2}\kappa y_s(j)^2$ in the utility function captures the disutility experienced by the representative Home agent as a result of increasing production levels.

This reflects the rising marginal cost of effort or resources required to produce additional output.

The utility function for a Foreign agent is analogous to that of a Home agent, reflecting the assumption if symmetric preferences across countries. The primary difference is that Home money is used exclusively by Home agents, while Foreign money is used exclusively by Foreign agents. The deflator for Foreign money balances, $M^*$, is defined by the equation below

$$P^* = \left[ \int_0^1 p^*(z)^{1-\theta} dz \right]^{\frac{1}{1-\theta}} \tag{3.4}$$

where $p^*(z)$ denotes the Foreign-currency price of good $z$.

To introduce Calvo pricing we must extend the utility function to account for its dynamic implications. Under the Calvo framework, an agent can adjust their price in any given period with a probability of $1-\alpha$, where $0 \leq \alpha < 1$. This friction is crucial because it prevents immediate full adjustment of all prices after a monetary shock. As a result, relative prices and competitiveness adjust gradually rather than instantaneously, which underpins the persistence of real effects such as output and terms of trade deviations. For a firm that last adjusted its price in period $t$, the probability that this price will remains unchanged in period $s$ (where $s \geq t$) is given by $\alpha^{s-t}$.

Following the approach outlined by Galí (2015), we introduce the notation $X_{s|t}$ to represent any variable $X_s$ that is determined by an agent, conditional on their most recent price adjustment occurring in period $t$. Similarly, $X_{s|>t}$ denotes any variable $X_s$ set by an agent, contingent on the last price change taking place before period $t$. Using these notations, the utility function can be reformulated to explicitly account for Calvo pricing as follows:

$$\begin{aligned} E_t(U_t^j) = E_t \Bigg\{ & \sum_{s=t}^{\infty} (\alpha\beta)^{s-t} \left[ \ln C_{s|t}^j + \chi \ln \frac{M_{s|t}^j}{P_s} - \frac{1}{2} \kappa\, y_{s|t}(j)^2 \right] \\ & + \sum_{s=t}^{\infty} (1-\alpha^{s-t})\, \beta^{s-t} \left[ \ln C_{s|>t}^j + \chi \ln \frac{M_{s|>t}^j}{P_s} - \frac{1}{2} \kappa\, y_{s|>t}(j)^2 \right] \Bigg\} \end{aligned} \tag{3.5}$$

This expression divides the expected lifetime utility into two components. The first summation captures states where the agent has not had any price adjustment opportunity since period $t$. The weight $(\alpha\beta)^{s-t}$ combines the discount factor $\beta^{s-t}$ and the probability $\alpha^{s-t}$ of the price remaining fixed up to period $s$. The second summation corresponds to states where the agent has already had at least one opportunity to change its price after period $t$, which occurs with probability $(1-\alpha^{s-t})$ This term does not affect current price-setting decisions, as any new price chosen in a future period would replace the price set at $t$.

When deriving the first-order conditions, only the first summation matters for price-setting, since the second is independent of the price chosen at $t$. In contrast, the consumer's optimisation problem requires considering the entire utility function because consumption and money demand decisions apply in both states. To simplify, we assume the existence of an insurance mechanism that allows the agent to maintain equal levels of consumption, $C_s$, and money holdings, $M_s$, regardless of whether they receive a price-change opportunity after period $t$. Accordingly, we impose $C_{s|t} = C_{s|>t} = C_s$ and $M_{s|t} = M_{s|>t} = M_s$ when optimizing with respect to $C_s$ and $M_s$.

Finally, note that the optimal price chosen in any later period $s > t$, when the agent receives new adjustment opportunities, constitutes a separate optimisation problem that is beyond the current scope.

### 3.2.2 Price structure

To establish the relationship between $P$ and $P^*$, we begin by assuming the absence of trade barriers, which ensures that the LOOP holds for every individual good. Let $\varepsilon$ denote the nominal exchange rate, defined as the amount of Home currency required per unit of Foreign currency. If $p(z)$ and $p^(z)$ represent the prices of good $z$ in Home and Foreign currency respectively, then the LOOP implies the following relationship:

$$p(z) = \varepsilon p^*(z) \tag{3.6}$$

Using the LOOP, we can decompose the Home CPI into components reflecting goods produced domestically (goods 0 to $n$) and abroad ($n$ to 1).

$$P = \left[ \int_0^1 p(z)^{1-\theta} \, dz \right]^{\frac{1}{1-\theta}} = \left[ \int_0^n p(z)^{1-\theta} \, dz + \int_n^1 \left[ \varepsilon p^*(z) \right]^{1-\theta} \, dz \right]^{\frac{1}{1-\theta}} . \quad (3.7)$$

Similarly, the Foreign price index $P^*$ can be written as:

$$P^* = \left[ \int_0^1 p^*(z)^{1-\theta} \, dz \right]^{\frac{1}{1-\theta}} = \left[ \int_0^n \left( \frac{p(z)}{\varepsilon} \right)^{1-\theta} dz + \int_n^1 p^*(z)^{1-\theta} \, dz \right]^{\frac{1}{1-\theta}} . \quad (3.8)$$

By comparing the two equations above, we observe that the Home and Foreign consumer price indices are linked through the principle of PPP:

$$P = \varepsilon P^* \quad (3.9)$$

While PPP holds in levels under these assumptions, the presence of Calvo pricing implies that deviations from PPP can arise in the short run because producer prices adjust sluggishly. These deviations, reflected in the terms of trade, transmit monetary shocks into persistent differences in demand allocation across countries.

In this context, PPP holds because preferences are identical across countries, and there are no deviations from the LOOP. However, it is important to note that the relative prices of individual goods are not required to remain constant. Changes in the terms of trade (the relative price of Home and Foreign tradable goods) will play a significant role in shaping these dynamics.

Its impact can be most directly seen in the PPI, which is determined by:

$$P(h) \equiv \left\{ \frac{1}{n} \int_0^n \left[ P_t(h) \right]^{1-\theta} \, dz \right\}^{\frac{1}{1-\theta}} . \quad (3.10)$$

The PPI is a history-dependent index because past price decisions continue to influence the current aggregate price level. This history dependence is a key channel through which monetary shocks generate inertia in inflation and relative prices.

Under Calvo pricing, the total weight of Home producers, represented by $n$, can be broken down based on when their last price adjustment occurred. A fraction $(1-\alpha)$ of producers updates their prices in the current period and sets them to $X_t$, while $(1-\alpha)\alpha$ adjusted one period earlier and set their prices to $X_{t-1}$. Similarly, $(1-\alpha)\alpha^2$ adjusted two periods ago and set their prices to $X_{t-2}$, continuing this pattern over time.

To calculate the price index $P_t(h)$, we need to consider the different groups of Home producers based on when they last adjusted their prices. Producers who adjusted prices in the current period are weighted by $(1-\alpha)n$. Producers who adjusted one period prior, contribute $(1-\alpha)\alpha n$, and so on. Incorporating these weights into the formula for $P_t(h)$ gives us:

$$P_t(h) = \left\{ \frac{1}{n} \Big[ (1-\alpha) n X_t^{1-\theta} + (1-\alpha)\alpha n X_{t-1}^{1-\theta} + (1-\alpha)\alpha^2 n X_{t-2}^{1-\theta} + \dots \Big] \right\}^{\frac{1}{1-\theta}}. \tag{3.11}$$

If we cancel out $n$ and re-write the equation in its general form we obtain:

$$P_t(h) = \left\{ (1-\alpha) \sum_{s=0}^{\infty} \alpha^s X_{t-s}^{1-\theta} \right\}^{\frac{1}{1-\theta}}. \tag{3.12}$$

Finally, log-linearising this equation around a zero inflation steady state results in:

$$p_t(h) = (1-\alpha) \sum_{s=0}^{\infty} \alpha^s x_{t-s} \tag{3.13}$$

where $p_t(h)$ and $x_{t-s}$ represent the log-deviation from their steady state values. This equation could also be re-written as equation (3.29) for Home and (3.30) for Foreign.

### 3.2.3 Individual Budget Constraint

To fully define the individual's problem, we now introduce the agent's budget constraint. Assuming market completeness in this model, the period budget constraint for a representative Home individual $j$, in nominal terms, is:

$$\begin{aligned} P_s B^j_{s+1|t} + M^j_{s|t} &= P_s(1+r_s)B^j_{s|t} + M^j_{s-1|t} \\ &\quad + p_s(j)\, y_{s|t}(j) - P_s C^j_{s|t} - P_s \tau_{s|t} + P_s A^j_{s|t} \end{aligned} \tag{3.14}$$

This constraint highlights the trade-offs faced by households: consumption today versus saving via bonds or money balances. In the open economy setting, these choices interact with exchange rate movements and interest differentials, shaping international capital flows and consumption smoothing across states.

In this setting, $r_s$ denotes the real interest rate on bonds, while $y_{s|t}(j)$ is the output of good $j$, produced solely by agent $j$. The domestic currency price of this good is $p_s(j)$, which, due to product differentiation, can vary across producers. However, in equilibrium, symmetric Home producers will optimally set the same price.

The term $M_{s-1}$ reflects agent $j$'s nominal money balances carried into the period $s$, and $\tau_s$ represents lump-sum taxes, which are paid in units of the composite consumption good, $C_s$. $A^j_{s|t}$ is the insurance payment for the completeness of market. This insurance applies across countries for consumption, and across producers for prices. $B_{s|t}$ is the riskless bonds. There are also 4 states, that could be realised that are linked to the monetary policy. Those states are 1) for when $v$ increases, 2) when $v^*$ increases, 3) when both increase, and 4) when neither increase.

To simplify the analysis, we assume that the Ricardian Equivalence holds. This allows us to impose, without loss of generality, that the government budget is balanced in each period.

### 3.2.4 Demand Curve Facing Each Monopolist

Using the CES consumption index introduced earlier, the demand for good $z$ by both Home and Foreign individuals can be expressed as:

$$c^j(z) = \left[\frac{p(z)}{P}\right]^{-\theta} C^j \tag{3.15}$$

$$c*^{j}(z) = \left[\frac{p*(z)}{P*}\right]^{-\theta} C*^{j} \tag{3.16}$$

By integrating the demand for good $z$ across all agents, we obtain the population-weighted average of Home and Foreign demand. Leveraging the PPP condition, which implies that $p(z)/P = p^{*}(z)/P^{*}$ for any good $z$, we find that the total world demand[1] for good $z$ adopts a constant elasticity of substitution (CES) form:

$$y^{d}(z) = \left[\frac{p(z)}{P}\right]^{\theta} C^{w} \tag{3.17}$$

$C^{w}$ here is given by

$$C^{w} \equiv \int_{0}^{n} C^{j} dj + \int_{n}^{1} C^{*} dj = nC + (1-n)C^{*} \tag{3.18}$$

In this equation, $C$ and $C^{*}$ represent the consumption of the representative agent from Home and Foreign countries, respectively. In order to simplify the notation, we drop the superscript $j$ and impose symmetry among identical agents within each country.

This specification shows that a firm's demand depends not only on its own price relative to the domestic price index, but also on the exchange rate because foreign prices are converted into Home currency. Consequently, monetary shocks that affect the exchange rate immediately alter relative demand, even before firms have a chance to reset prices.

### 3.2.5 Monetary Policy

Consistent with Chapter 2, we model the behaviour of the central bank using a simple interest rate rule, specifically a Taylor rule. Under this rule policymakers react only to changes in inflation, and is given by the equations below for Home and foreign respectively:

$$\hat{i}_{t+1} = \rho + \phi_{\pi}\hat{\pi}_{t} + v_{t} \tag{3.19}$$

[1]The reader can refer to section A.2 of the Annex for a demonstration of how the world demand in this economy is the same as in OR.

$$\hat{i}^*_{t+1} = \rho + \phi_\pi \hat{\pi}_t + v^*_t \tag{3.20}$$

### 3.2.6 First-Order Conditions for the Individual's Problem

To solve the model, we first use the demand curve to substitute $y_s(j)$ into the period budget constraint. This expression is then used to replace $C^j_s$ in the intertemporal utility function. We also maintain the assumption that an insurance scheme ensures agents have equal levels of money holdings and bonds, regardless of whether they are able to adjust their prices after period $t$. This leads to the following unconstrained maximization problem:

$$\begin{aligned} E_t(U^j_t) = \sum_{s=t}^{\infty} (\alpha\beta)^{s-t} \Big\{ \ln\Big[ (1 + r_{s|t})\, B^j_s + \frac{M^j_{s-1}}{P_s} \\ + \left(\tfrac{X_t}{P_s}\right)^{1-\theta} C^w_s - \tau_{s|t} - B^j_{s+1} - \tfrac{M^j_s}{P_s} - A^j_{s|t} \Big] \\ -\chi \ln\left(\tfrac{M^j_s}{P_{s|t}}\right) - \tfrac{k}{2} \left(\tfrac{X_t}{P_{s|t}}\right)^{-2\theta} C^w_s \Big\} \\ + \sum_{s=t}^{\infty} 1 - \alpha^{s-t} (\beta)^{s-t} \Big\{ \ln\Big[ (1 + r_{s|>t})\, B^j_s + \tfrac{M^j_{s-1}}{P_s} \\ + \left(\tfrac{X_t}{P_s}\right)^{1-\theta} C^w_s - \tau_{s|>t} - B^j_{s+1} - \tfrac{M^j_s}{P_s} - A^j_{s|>t} \Big] \\ -\chi \ln\left(\tfrac{M^j_s}{P_s}\right) - \tfrac{k}{2} \left(\tfrac{X_t}{P_s}\right)^{-2\theta} C^w_s \Big\} \end{aligned} \tag{3.21}$$

When maximising with respect to $B_{s+1}$ $\frac{M_{s|t}}{P_s}$ and $X_t$, the individual takes $C^w$ as given. The resulting first-order conditions with respect to $B^j_{s+1|t}$, $M^j_{s|t}$, and $X_t$, after rearrangement, are:

$$C_{s+1} = \beta(1 + r_{s+1}) C_s \tag{3.22}$$

$$\frac{M_{s|t}}{P_s} = \chi C_{s|t} \left( \frac{1 + i_{s+1|t}}{i_{s+1|t}} \right). \tag{3.23}$$

$$X^{\theta+1}_t = \left( \frac{\theta\kappa}{\theta - 1} \right) \frac{\sum_{s=t}^{\infty} (\alpha\beta)^{s-t} (P_s)^{2\theta} (C^w)^2}{\sum_{s=t}^{\infty} (\alpha\beta)^{s-t} \left[ \frac{(P_s)^{\theta-1}}{C_s} C^w_s \right]}. \tag{3.24}$$

Equation (3.22) represents the standard first-order Euler equation for consumption, applicable when the intertemporal elasticity of substitution is unity. Equation (3.23) corresponds to the typical first-order condition for money demand. It arises from the equilibrium condition that agents must be indifferent between two options: consuming an additional unit of goods in period $t$, or allocating the same funds to increase cash balances, thereby gaining transaction utility in period $t$ and converting the surplus cash back into consumption in the subsequent period. The final first-order condition, given by (3.24), characterises the firm's optimal price-setting decision under Calvo pricing. It shows that firms set prices by balancing current markups with the expected discounted value of future markups, conditional on not resetting prices. This forward-looking behaviour highlights the role of expectations in inflation dynamics and explains how monetary policy credibility influences current pricing decisions.

Note that these first order conditions, along with the period budget constraint, do not fully characterise the equilibrium. Equilibrium also requires the transversality condition to hold, similar to the OR model.

### 3.2.7 General equilibrium conditions

The next natural step following the microfoundations of the model is to outline the general equilibrium conditions of the model. The goods market clearing condition requires world output to equal world expenditure, expressed as:

$$C_t^w \equiv nC_t + (1-n)C_t^* = n\frac{P_t(h)}{P_t}y_t(h) + (1-n)\frac{P_t^*(f)}{P_t^*}y_t^*(f) \equiv Y_t^w. \quad (3.25)$$

This condition ensures that global resources are fully allocated, so any shift in demand between Home and Foreign goods due to relative price changes must be offset by corresponding adjustments in output or consumption.

Additionally, PPP (3.9), real CPI for both home and foreign (3.2), and the log-linearised Uncovered Interest Parity (UIP) below must all hold.

$$e_{t+1} - e_t = i_{t+1} - i_{t+1}^* \quad (3.26)$$

The UIP condition links interest rate differentials to expected exchange rate changes, making monetary policy an important driver of currency fluctuations and, through relative prices, of output dynamics in both countries.

We also define the Home and Foreign per capita real GDP as:

$$Y_t = \frac{[\frac{1}{n}\int_0^n P_t(z)Y_t(z)dz]}{P_t(h)} \tag{3.27}$$

$$Y_t^* = \frac{[\frac{1}{n}\int_n^1 P_t^*(z)Y_t^*(z)dz]}{P_t^*(f)}. \tag{3.28}$$

## 3.3 Log-linearised equations

Most log-linearized equations presented in this chapter closely resemble those presented in Chapter 1. These include core relationships such as the CPI, the PPP, the world demand schedules, the world goods market equilibrium condition, the consumption Euler equations, the money demand equations, and the completeness of markets condition.

However, some log-linearised[2] equations are new to this chapter. These include the PPI and the law of motion for the price level, which is another way of writing equation (3.13) and its foreign counterparts.

$$p_t(h) = (1-\alpha)x_t + \alpha p_{t-1}(h) \tag{3.29}$$

$$p_t^*(f) = (1-\alpha)x_t^* + \alpha p_{t-1}^*(h) \tag{3.30}$$

The labour-leisure trade-off is part of the log linearised price-setting equations. We also assume completeness of markets, implying $c = c* = c^w$. This equates individual country consumption levels with world consumption[3]:

$$x_t = (1-\alpha\beta)\sum_{s=t}^{\infty}(\alpha\beta)^{s-t}\left[p_s + \frac{1}{\theta+1}\left(c_s^w + c_s\right)\right] \tag{3.31}$$

[2]The general formula used here is $x \equiv \frac{X-\bar{X}_0}{\bar{X}_0}$ where $\bar{X}_0$ is the symmetric steady state.

[3]For the full step-by-step derivation, please see Appendix A.3

$$x_t^* = (1-\alpha\beta)\sum_{s=t}^{\infty}(\alpha\beta)^{s-t}\left[p_s^* + \frac{1}{\theta+1}\left(c_s^w + c_s^*\right)\right] \tag{3.32}$$

The forward-looking structure of $x_t$ reflects firms' optimal response to price rigidity. Because only a fraction of firms can adjust prices each period, those that can will factor in not just current costs and demand conditions, but also expectations of future conditions. This makes current pricing decisions sensitive to anticipated future inflation and output, which helps explain the sluggish adjustment of aggregate prices to shocks. In this way, the aggregate price-setting process links today's price decisions to an infinite horizon of future aggregate prices, ensuring internal consistency and rationality in pricing behavior under uncertainty.

World consumption, $c_s^w$, is an index of goods demand and is positively related to $x_t$. That is, increases in the demand for Home goods will be reflected in prices today. This occurs through the labour-leisure channel: given that in our economy our agents are both consumer and producers, higher consumption will increase the producer's wealth, which in turn will reduce their willingness to work –thus generating increases in their output prices. Lastly, $\beta$ is the discount factor and $1-\alpha$ is the probability that producers will be able to change their price in any given period.

We will now examine other log-linearised equations that are similar to those in OR. First, the log-linearised Euler equations for Home and Foreign consumption are:

$$c_{t+1} = c_t + \hat{r}_{t+1} \tag{3.33}$$

$$c_{t+1}^* = c_t^* + \hat{r}_{t+1} \tag{3.34}$$

The Euler equation describes how consumption evolves over time in response to changes in the real interest rate, $\hat{r}_{t+1}$. It implies that consumption growth is directly linked to the real interest rate. Higher interest rates encourage saving leading to faster future consumption growth; lower interest rates reduce saving incentives, slowing consumption growth. This inter-temporal trade-off is a key channel through which monetary policy influences real activity: a rise in interest rates makes future consumption relatively cheaper, incentivising households to defer spending. This leads to lower current demand, impacting output and inflation.

The log-linearised price index is given by:

$$p_t = np_t(h) + (1-n)[e_t + p_t^*(f)] \tag{3.35}$$

$$p_t^* = n[p_t(h) - e_t] + (1-n)[p_t^*(f)] \tag{3.36}$$

These equations show that a country's overall price level is a weighted average of domestic and imported goods prices. The Home price index, $p_t$, depends on the price of Home produced $(p_t(h))$ and imported goods $(p_t^*(f))$, adjusted by the exchange rate, $e_t$. Likewise, the Foreign price index, $p_t^*$, is determined by the prices of foreign-produced $(p_t^*(f))$ and imported Home goods $(p_t(h))$, also adjusted for exchange rate fluctuations. The world goods market equilibrium condition is given by:

$$c_t^w = nc_t + (1-n)c_t^* = ny_t + (1-n)y_t^* = y_t^w \tag{3.37}$$

This condition is crucial in open economy models, ensuring world consumption, $c_t^w$, and production, $y_t^w$, remain aligned. It reflects how changes in one country's output or consumption affect the global market. It assumes no net global saving or investment, where total world output is fully consumed each period. In equilibrium, global consumption equals global output, $c_t^w = y_t^w$, ensuring that all goods produced are eventually consumed. The log-linearised completeness of market assumption states that

$$c_t = c_t^*, \quad t > 1 \tag{3.38}$$

This assumption implies that individuals in different countries can fully insure against country-specific monetary shocks occurring in period 1 through international financial markets. Before any shocks, both countries are in a perfectly symmetric state. As a result, consumers can smooth consumption, leading to identical consumption levels in the Home and Foreign countries, $c_t = c_t^*$. In other words, perfect risk-sharing ensures that all agents, regardless of their country, experience the same consumption path.

In order to simplify the analysis of inflation dynamics, the price index for Home $p_0(h)$ and Foreign $p_0^*(f)$ are normalised to 0 at $t = 0$. This assumption facilitates finding the solution and ensures stationarity without affecting the key economic relationships.

$$p_0(h) = p_0^*(f) = 0 \tag{3.39}$$

The log linearised PPP equation is given by:

$$e_{t+1} - e_t = (\pi_{t+1} - \pi_t^*) \tag{3.40}$$

PPP implies that without shocks, the real exchange rate is constant, and the nominal exchange rate reflects inflation differentials between two countries. The equation above shows that changes in the exchange rate are driven by the inflation difference between Home $(\pi_{t+1})$ and Foreign country's inflation $(\pi_{t+1^*})$. Higher Home inflation leads to nominal exchange rate depreciation (i.e., the home currency falls), and vice versa. If we define the depreciation rate as $d_{t+1} = e_{t+1} - e_t$, PPP can be written as:

$$d_{t+1} = \pi_{t+1} - \pi_{t+1}^* \tag{3.41}$$

Lastly, the log-linearised world demand schedules are:

$$y_t = \theta[p_t - p_t(h)] + c_t^w \tag{3.42}$$

$$y_t^* = \theta[p_t^* - p_t^*(f)] + c_t^w \tag{3.43}$$

It is important to note that the composite of goods produced in the country differs from the composite of goods consumed within the country. As a result, the relative price of $y_t$ to $y_t^*$ differs, allowing for endogenous determination of the terms of trade.

### 3.3.1 Deriving NKPC

The equations (3.31) and (3.32) can be rewritten in terms of inflation by subtracting $p_{t-1}$ from both sides. Defining PPI inflation as $\pi_t(h) = p_t(h) - p_{t-1}(h)$

and $\pi_{t+1}(h) = p_{t+1}(h) - p_t(h)$ within the price-setting equation, and eliminating the summation component, we obtain inflation equations expressed in terms of the PPI, $\pi_t(h)$ and $\pi_t(f)$.

To introduce output as a variable in our NKPC, we use equations (3.42) and (3.43). By rearranging these equations in terms of the price index, we can substitute out $p_s$ and $p_s^*$ in (3.31) and (3.32), allowing output to be incorporated into the inflation dynamics.

$$\begin{aligned} x_t - p_{t-1}(h) = (1-\alpha\beta)\left[\frac{y_t}{\theta} + \pi_t(h) + \frac{1}{\theta+1}\left(c_t^w + c_t\right)\right] \\ + \beta\alpha\left[E_t\big(x_{t+1}\big) - p_t(h) + \pi_t(h)\right] \end{aligned} \tag{3.44}$$

$$\begin{aligned} x_t^* - p_{t-1}^*(f) = (1-\alpha\beta)\left[\frac{y_t^*}{\theta} + \pi_s^*(f) + \frac{1}{\theta+1}\big(c_t^w + c_t^*\big)\right] \\ + \beta\alpha\big[E_t\big(x_{t+1}^*\big) - p_t^*(f) + \pi_t^*(f)\big]. \end{aligned} \tag{3.45}$$

For the substitution of $x_t - p_{t-1}(h)$ and $x_t^* - p_{t-1}^*(f)$ and its $t+1$ counterparts we make use of the law of motion for Calvo pricing of Home and Foreign in equations (3.29) and (3.30). Subtracting the lagged PPI in the left hand side and right hand side of the law of motion for Calvo price setting for both countries and after some re-arranging we obtain:

$$x_t - p_{t-1}(h) = \frac{\pi_t(h)}{1-\alpha} \tag{3.46}$$

$$x_t^* - p_{t-1}^*(f) = \frac{\pi_t^*(f)}{1-\alpha} \tag{3.47}$$

We then substitute equations (3.46) and (3.47) into (3.44) and (3.45). Additionally, we impose the market completeness condition, which ensures that $c_t^w = c_t = c_t^*$. This allows us to combine the consumption variables into a single expression.

After rearranging the equations in terms of $\pi_t(h)$ and $\pi_t^*(f)$, we derive the New Keynesian Phillips Curve (NKPC) for both the Home and Foreign countries.

$$\pi_t(h) = \frac{1-\alpha}{\alpha}(1-\alpha\beta)\left[\frac{y_t}{\theta} + \frac{\theta-1}{\theta(\theta+1)}c_t^w\right] + \beta E_t \pi_{t+1}(h) \tag{3.48}$$

$$\pi_t^*(f) = \frac{1-\alpha}{\alpha}(1-\alpha\beta)\left[\frac{y_t^*}{\theta} + \frac{\theta-1}{\theta(\theta+1)}c_t^w\right] + \beta E_t \pi_{t+1}^*(f). \tag{3.49}$$

The Home and Foreign NKPC equations describe the evolution of inflation based on both domestic and global economic conditions. Inflation in each country is influenced by domestic output, which captures demand, world consumption, and expected future inflation due to forward-looking price setting. Firms adjust prices in response to current marginal costs and anticipated inflation. Since the equations are symmetric, both economies exhibit similar inflation dynamics, driven by their respective output levels and shared global consumption.

### 3.3.2 Transforming other equations in terms of rate of change

We can also express other equations from earlier sections in this chapter in terms of rates of change. After some manipulation, the log-linearised price index yields[4]:

$$p_t - p_t(h) = (1-n)\left[e_t + p_t^*(f) - p_t(h)\right] \tag{3.50}$$

$$p_t^* - p_t^*(f) = n\left[p_t(h) - e_t - p_t^*(f)\right] \tag{3.51}$$

Plugging these two equations into the log-linearised world demand equations (3.42) and (3.43) results in the equations below for Home and Foreign respectively:

$$y_t = \theta(1-n)\left[e_t + p_t^*(f) - p_t(h)\right] + c_t^w \tag{3.52}$$

$$y_t^* = \theta n\left[p_t(h) - e_t - p_t^*(f)\right] + c_t^w \tag{3.53}$$

[4] A step by step derivation can be found in Section A.5 of the Annex.

These equations show world demand in terms of the terms of trade, which in turn are given by $[e_t + p_t^*(f) - p_t(h)]$.

To transform the price index in terms of inflation, we first-difference all the variables (CPI, PPI, and exchange rate) with a one-period lag:

$$p_t - p_{t-1} = n\left[p_t(h) - p_{t-1}(h)\right] + (1-n)\left[(e_t - e_{t-1}) + p_t^*(f) - p_{t-1}^*(f)\right] \quad (3.54)$$

$$p_t^* - p_{t-1}^* = n\left[p_t(h) - p_t(h) - (e_t - e_{t-1})\right] + (1-n)\left[p_t^*(f) - p_{t-1}^*(f)\right] \quad (3.55)$$

Which can be re-written into:

$$\pi_t = n\pi_t(h) + (1-n)\left[d_t + \pi_t^*(f)\right] \quad (3.56)$$

$$\pi_t^* = n\left[\pi_t(h) - d_t\right] + (1-n)\pi_t^*(f) \quad (3.57)$$

## 3.4 Sum System

Having derived all the necessary log-linearized equations and applying the Aoki method as in previous chapters, we now solve the model by separating it into a sum system and a difference system.

The sum system represents the world economy by taking a weighted average of variables from both the Home and Foreign countries. This interpretation allows us to analyse the global response in a manner analogous to that of a closed economy model. To denote these aggregate variables, we assign them a subscript $w$.

The solution for individual variables within the sum system is based on three fundamental equations: the IS equation, the NKPC, and the Taylor Rule. The IS equation establishes that real income increases with autonomous expenditure and declines with higher real interest rates, while the NKPC links inflation to output dynamics. The Taylor rule captures the central bank's policy behaviour. The seminal work of Woodford (2003) and the influential contribution by Clarida et al. (1999) both employ a three-equation framework as a foundational analytical tool. This framework enables a detailed examination of macroeconomic shocks, identifies the structural determinants of interest rate rules, and understanding of the mechanisms underlying inflation bias.

In this paper, we adopt a two-equation framework by combining the IS equation with the Taylor Rule, following Galí (2015). This approach, along with our focus on simple one-period Taylor rule shocks, allows us to reduce the model to a second-order difference equation system, significantly simplifying its analysis.

### 3.4.1 World NKPC equation

The world NKPC is derived through the log-linearised NKPC equations, by combining equations (3.48) and (3.49) as a weighted average for the world economy:

$$\pi_t(w) = \frac{1-\alpha}{\alpha}(1-\alpha\beta)\left[\frac{y_t^w}{\theta} + \frac{\theta-1}{\theta(\theta+1)}c_t^w\right] + \beta E_t\pi_{t+1}(w). \tag{3.58}$$

We know that in every period $c^w = y^w$. And for simplicity we denote $\frac{1-\alpha}{\alpha}(1-\alpha\beta) = k$. Therefore, we can rearrange and simplify the equation to:

$$E_t\pi_{t+1}^w = \beta^{-1}\pi_t^w - \beta^{-1}k\left[\frac{2}{\theta+1}y_t^w\right]. \tag{3.59}$$

### 3.4.2 IS-TR equation

The IS equation is derived from the weighted average of the consumption Euler equation:

$$c_{t+1}^w = c_t^w + \hat{r}_{t+1}^w \tag{3.60}$$

Since in every period it is true that $\bar{y}_t^w = \bar{c}_t^w$, we can rewrite the IS curve in terms of output:

$$y_{t+1}^w = y_t^w + \hat{r}_{t+1}^w \tag{3.61}$$

Our world Taylor rule is:

$$\hat{i}_{t+1}^w = \rho + \phi_\pi\pi_t^w + v_t^w \tag{3.62}$$

Using Fisher's equation $\hat{r}_{t+1}^w = \hat{i}_{t+1}^w - \pi_{t+1}^w$ and substituting the world Taylor rule, we can rewrite it as:

$$\hat{r}^w_{t+1} = \rho + \phi_\pi \pi^w_t + v^w_t - E\pi^w_{t+1} \tag{3.63}$$

Substituting this into the IS equation yields the IS-TR equation:

$$y^w_{t+1} = y^w_t + \rho + \phi_\pi \hat{\pi}^w_t + v^w_t - E\pi^w_{t+1} \tag{3.64}$$

Replacing $E\pi^W_{t+1}$ with equation (3.59), and simplifying and rearranging, we obtain:

$$y^w_{t+1} = y^w_t \left[1 + \beta^{-1} k \left(\frac{2}{\theta + 1}\right)\right] + \rho + \pi^w_t \left[\phi_\pi - \beta^{-1}\right] + v^w_t. \tag{3.65}$$

The two system equations, (3.59) and (3.65), form a second-order dynamic system, whose stability properties are determined by the two eigenvalues characterizing the system. For a unique, bounded, and rational expectations solution to exist, the system must exhibit the necessary degree of saddlepoint stability, as defined by Blanchard & Kahn (1980).

In this context, it is important to note that the two state variables, $\pi^w$ (world inflation) and $y^w$ (world output), are inherently non-predetermined. Consequently, for the system to satisfy the determinacy condition, both eigenvalues must be unstable, meaning they must lie outside the unit circle. Verifying this condition requires algebraic exploration, which we will not delve into here.

As Galí (2015) explains, this determinacy condition imposes a restriction on the Taylor Rule parameters, particularly $\phi_\pi$, which must be sufficiently large. In our case, the requirement simplifies to $\phi_\pi > 1$. The logic behind this condition is that when inflation rises, the central bank must raise the nominal interest rate by more than the increase in inflation to raise the real rate. A higher real rate reduces demand, thereby cooling inflation. If $\phi_\pi < 1$, real interest rates would fall in response to inflation, amplifying the shock rather than stabilising it. Thus, $\phi_\pi > 1$ ensures that monetary policy stabilises both inflation and output by anchoring expectations.

Examining the world IS-TR and world NKPC equations above, and assuming that the determinacy condition holds, we observe that the system remains in its steady state as long as the variable $v_t^w$ does not change over time (i.e., there are no monetary shocks in either country). This behaviour arises because the system has no predetermined state variables, making it entirely forward-looking. This purely forward-looking nature arises because the model excludes backward-looking elements like capital accumulation, habits in consumption, or adjustment costs. Without these frictions, current variables adjust solely in response to expectations about the future. Thus, any deviation from steady state is corrected immediately once the shock vanishes, as agents have no reason to anticipate continued effects.

Consequently, once any $v^w$ shock dissipates, the unique, bounded, rational-expectations solution reverts to the steady-state solution. While other time paths may satisfy the system, they are unbounded and, therefore, divergent. Following the standard methodology for solving such models, we eliminate these divergent paths from consideration.

It might seem counter-intuitive that the system reverts immediately to its steady state. One might expect that the price stickiness introduced by Calvo-style staggered pricing would cause the world economy to take time to return to the natural level of output after a shock.

In this model, prices adjust gradually, suggesting that recovery from a disturbance might occur slowly. We find, however, that in the sum system they return to the steady state instantaneously, reflecting a lack of output persistence. This lack of persistence may seem counterintuitive given the presence of Calvo frictions. However, when aggregating across two symmetric economies, the pricing frictions cancel out in the sum. Since both countries face identical structures and shocks hit symmetrically, the sticky price adjustment that normally induces inertia does not operate at the aggregate level. As a result, inflation in the sum system adjusts fully on impact and returns to steady state immediately, despite the underlying price rigidity. Nevertheless, we will show later how this changes when examining the difference system under an open economy framework, where output persistence may actually emerge.

What drives output persistence in this model is the interplay between several structural parameters and forward-looking behavior of economic agents. The discount factor, $\beta$, reflects how much agents value future consumption relative

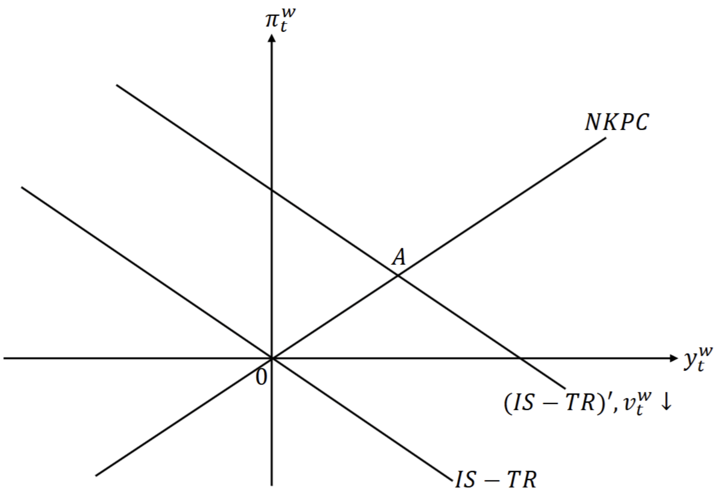


Figure 3.1: World economy system

to current consumption; a higher $\beta$ means they place more weight on future expected variables, which slows the economy's return to steady state after a shock. The parameter $\eta$, representing the elasticity of intertemporal substitution, determines how responsive consumption is to changes in real interest rates; a lower $\eta$ implies consumers are less willing to shift consumption over time, muting the immediate effect of shocks and prolonging their impact. Nominal rigidity is captured by the Calvo parameter $\lambda$, which governs price stickiness; higher values of $\lambda$ imply that prices adjust more slowly, generating more persistent deviations of output and inflation from their steady states. The labor share parameter, $\alpha$, influences how changes in wages feed into inflation dynamics, affecting the real effects of monetary shocks. Lastly, the monetary policy response parameter $\phi$ measures how aggressively the central bank reacts to inflation; stronger policy responses (higher $\phi$) tend to dampen persistence by anchoring inflation expectations more firmly. Together, these parameters shape the speed and magnitude of the economy's adjustment, determining how long monetary shocks continue to affect output and inflation over time.

Figure 3.1 illustrates the world economy system. Equation (3.66) gives the upward-sloping NKPC curve, with slope $k\frac{2}{\theta+1}$. The downward-sloping curve results from combining the IS and Taylor rule equations, as expressed in (3.67).

Equations (3.66) and (3.67) are derived by setting $\pi_{t+1}^w$ and $y_{t+1}^w$ to zero. The IS-TR curve at point 0 represents the case where no monetary shock occurs.

A reduction in $v_t^w$, from zero to a negative value, clearly shifts the IS-TR curve upwards due to the negative relationship between $\pi^W$ and $v^W$ and (3.67). Consequently, equilibrium moves from 0 to A, and output and inflation both increase. The effect on $\hat{i}_{t+1}^w$ itself may at first seem ambiguous, because the indirect effect of the increases in $\pi_t^w$ and $y_t^w$ via the Taylor rule counteracts the direct effect of the fall in $v_t$. However, if we look instead at the original IS equation(3.61) , we can see that the rise in $y_t^w$ implies that the net effect on interest rate must be a decrease.

Overall, a one-period fall in $v_t^w$ captures the effects of a temporary monetary expansion in a fairly natural way. The only unexpected results is that these effects do not persist in the world economy.

Using straightforward algebra, we can determine the values of inflation and output in period $t+1$. From equations (3.59) and (3.65), assuming no further monetary shocks, and as in Chapter 2 that $\phi_\pi > 1$, and $\rho = 0$, the steady-state values of output and inflation are $y_{t+1}^w = 0$ and $\pi_{t+1}^w = 0$. Rearranging equations (3.59) and (3.64) in terms of $\pi_t^w$, we obtain:

$$\pi_t^w = k[\frac{2}{\theta+1} y_t^w] \tag{3.66}$$

$$\pi_t^w = -\frac{1}{\phi_\pi}(y_t^w + v_t^w) \tag{3.67}$$

### 3.4.3 Finding the world variables

To determine world output, $y_t^w$, we equate equations (3.66) and (3.67). Rearranging and factorizing terms, we get the world output expression:

$$y^w = \frac{-v_t^w}{k\frac{2}{\theta+1}\phi_\pi + 1} \tag{3.68}$$

Substituting (3.68) into (3.66), we obtain:

$$\pi_t^w = k\frac{2}{\theta+1}\left[\frac{-v_t^w}{k\frac{2}{\theta+1}\phi_\pi+1}\right]. \tag{3.69}$$

## 3.5 Difference System

The difference system, using the Aoki method, is constructed by subtracting the log-linearized Foreign country variables from their corresponding Home country variables.

### 3.5.1 Exchange rate solution

In order to find the exchange rate solution we first subtract the Taylor rule for Home, (3.19) and Foreign, (3.20):

$$\hat{i}_{t+1} - \hat{i}^*_{t+1} = \phi_\pi(\pi_t - \pi_t^*) + v_t - v_t^* \tag{3.70}$$

Since the log-linearized PPP and UIP hold, we substitute (3.70) into (3.26) and replace the differential inflation of period $t$ using its lagged value from (3.40). Next, applying the definition of the depreciation rate, $d_{t+1} \equiv e_{t+1} - e_t$, along with its lagged form, we derive the law of motion for the depreciation rate:

$$d_{t+1} = \phi_\pi d_t + v_t - v_t^* \tag{3.71}$$

The depreciation rate at $t+1$ must be zero because $d_{t+1}$ is not predetermined, meaning its initial value is unknown. To ensure a unique perfect foresight path, $d_t$ must have a unique starting value that prevents divergence. In the absence of shocks, this requires $d_t = 0$, which holds if $\phi_\pi > 1$, ensuring that multiple stable paths are ruled out. As a result, while $e_t$ may rise or fall in response to a one-period monetary shock, the exchange rate eventually returns to its steady state, $e_{t+1}$.

We can now solve for $d_t$ and $e_t$ by setting $d_{t+1} = 0$. Since $e_{t-1}$ is predetermined, it can be treated as a constant. This allows us to derive (3.72), matching the results from Chapter 2.

$$e_t = e_{t-1} + \frac{-v_t + v_t^*}{\phi_\pi} \tag{3.72}$$

This equation shows that a contractionary Home monetary policy shock (increase in $v_t$) reduces $d_t$, causing exchange rate appreciation, with the magnitude depending on $\phi_\pi$. Exchange rate appreciation is permanent for period $t > 1$, thus ruling out overshooting.

Finally, we can see that when both countries follow a simple Taylor rules and flexible exchange rates, the exchange rate can be determined without considering price stickiness a la Calvo, demand dynamics, or the goods market.

### 3.5.2 Differential output

After simplifying the model, the essential equations for the difference system are reduced to the two most critical ones: the differential NKPC and the differential world demand schedule. We can obtain the differential NKPC by subtracting (3.48) and (3.49):

$$\pi_t(h) - \pi_t^*(f) = k\left[\frac{y_t - y_t^*}{\theta}\right] + \beta\left[E_t\pi_{t+1}(h) - E_t\pi_{t+1}^*(f)\right]. \tag{3.73}$$

And we can obtain the differential output by subtracting (3.52) and (3.53):

$$y_t - y_t^* = \theta[e_t - (p_t(h) - p_t^*(f))] \tag{3.74}$$

A key aspect linking CPI and PPI lies in the differential NKPC, which incorporates only PPI due to the Calvo-style staggered price-setting mechanism. In contrast, differential output originally (3.42) and (3.43) before substituting the substituting log-linearised price index depends on both CPI and PPI. To bridge this gap and capture the structure as a whole, we can substitute (3.74) into (3.73):

$$\pi_t(h) - \pi_t^*(f) = k[e_t - (p_t(h) - p_t^*(f))] + \beta[E_t\pi_{t+1}(h) - E_t\pi_{t+1}^*(f)] \tag{3.75}$$

At this point, we introduce a new variable, defined as $\eta_t \equiv p_t(h) - p_t^*(f)$. The variable $\eta_t$ represents the difference between Home and Foreign producer prices, which determines relative prices and therefore the terms of trade. This matters because, in an open economy, the terms of trade influence demand allocation between Home and Foreign goods, which in turn drives relative consumption and output. Under flexible prices, $\eta$ would jump immediately to its new equilibrium after a monetary shock, fully offsetting the effect on competitiveness. However, with Calvo-style staggered pricing, most firms keep their prices unchanged each period, so only a subset of firms respond to the shock. This slow adjustment means that $\eta$ cannot fully offset the initial exchange rate movement, leaving relative prices temporarily misaligned. As a result, Home goods remain relatively cheaper (or more expensive) for several periods, leading to persistent deviations in the terms of trade and in relative output. The second-order difference equation for $\eta$ captures this dynamic: it shows how past prices (predetermined) and future expectations jointly determine the speed of adjustment, making $\eta$ the key state variable linking staggered pricing to persistent real effects.

Using this definition and noting that $\pi_t(h) = p_t(h) - p_{t-1}(h)$ (and similarly for Foreign), we can rewrite the equation:

$$\eta_t - \eta_{t-1} = k(e_t - \eta_t) + \beta(E_t \eta_{t+1} - \eta_t) \tag{3.76}$$

Bringing all $\eta$ terms to the left-hand side, factoring, and rearranging, we get the following second-order difference equation for $\eta_t$:

$$-\beta E_t \eta_{t+1} + (1 + \beta + k)\eta_t - \eta_{t-1} = e_t k \tag{3.77}$$

In an open economy, the system is no longer fully forward-looking as in the closed economy. Therefore, the difference equation for $\eta_t$ depends both on the predetermined variable $\eta_{t-1}$ and on the expected variable $E\eta_{t+1}$. This introduces a backward-looking component alongside the forward-looking element. As a result, the economy adjusts gradually to shocks rather than instantly, causing persistent fluctuations in the terms of trade and, consequently, in relative consumption levels. This persistence arises from the hybrid nature of the difference equation, which includes both forward- and backward-looking components.

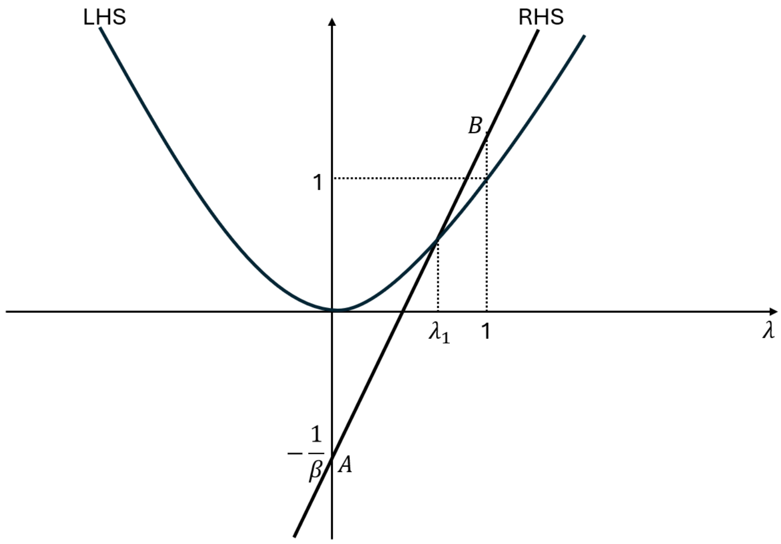


Figure 3.2: Eigenvalues of the second order difference system of equation (3.79)

To analyse this in greater detail, we derive the rational expectations solution for $\eta_t$. Given that the system includes one natural initial condition in any period $t$, specifically the predetermined value of $\eta_{t-1}$, a unique and bounded time path requires one eigenvalue of the difference equation to lie inside the unit circle, while the other must lie outside.

To demonstrate saddlepoint stability, we examine the characteristic equation for (3.77). Since we have already solved for the exchange rate (3.72), we can treat $e_t$ as constant, with $\lambda$ representing the generic eigenvalue.

$$-\beta\lambda^2 + (1+\beta+k)\lambda - 1 = 0 \tag{3.78}$$

We rearrange this to see the parameter effects on $\lambda$:

$$\lambda^2 = \frac{1+\beta+k}{\beta}\lambda - \frac{1}{\beta} \tag{3.79}$$

We now discuss the left- and right-hand side terms separately as plots of $\lambda$ functions in the diagram 3.2: The graph shows that the right hand side is a straight line through point A $(0, -\frac{1}{\beta})$ and point B $(1, \frac{1+k}{\beta})$.

Point $A$ lies below the parabola, while point $B$ is positioned above it. Consequently, the parabola and the line must intersect. It is evident that one intersection occurs within the range $\lambda \in (0, 1)$, indicating that one eigenvalue lies inside the unit circle and is therefore stable.

Additionally, although not explicitly shown in the diagram, there must be a second intersection in the range $\lambda \in (1, \infty)$, confirming that the other eigenvalue lies outside the unit circle and is unstable. This ensures that the condition for a unique, bounded, rational expectations solution (or saddle-point stability), is inherently satisfied in the difference system (3.77), without requiring any additional parameter restrictions.

Since no randomness is assumed in the exogenous variables, and sunspot solutions are excluded due to saddle-point stability, the solution for $\eta_t$ is non-stochastic. As a result, we can omit the expectation operator, $E_t$, from the difference equation.

Now that we have confirmed the stability of the solution we can algebraically find the saddle-path solution for $\eta_t$ from the equation:

$$\eta_t - \bar{\eta} = A_1 \lambda_1^t + A_2 \lambda_2^t \tag{3.80}$$

Since the saddle path solution is $A_2 = 0$, we could then re-write it

$$\eta_t = A_1 \lambda_1^t + \bar{\eta} \tag{3.81}$$

Here $\bar{\eta}$ is the steady state value of $\eta$. Also, through some simple algebra, we know that $\bar{\eta} = e$. $A_1$ is a coefficient to be determined from the initial conditions.

To solve for $A_1$, consider the scenario where the economy is initially in a steady state, such that $\bar{\eta} = e_t = \eta_t = 0$. At $t = 0$, the exchange rate, $e_t$, is raised to a strictly positive value, resulting from either a decrease in $v$ or an increase in $v^*$. This new value of $e_t$ is permanent.

In $t = 0$, we assume that the interest rate increase is announced before price adjustments. Consequently, the new price level is set with knowledge of the updated exchange rate through the interest rate, allowing $\eta_0$ to respond to the shock. However, $\eta_{-1}$ remains predetermined at zero.

Using the solution for $t = -1$ yields $A_1 = -\lambda_1 \bar{\eta}$. Therefore, the complete

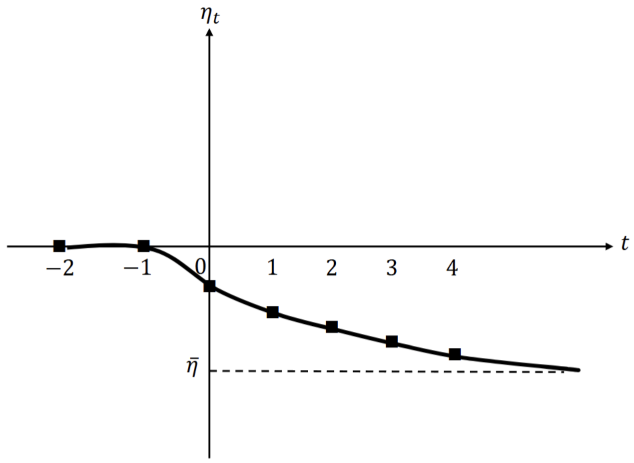


Figure 3.3: Behaviour of $\eta$ in difference system

solution for $\eta_t$ is:

$$\eta_t = (1 - \lambda^{t+1})\bar{\eta} \tag{3.82}$$

Figure 3.3 describes the behaviour of $\eta_t$ following a monetary policy shock, showing how it will slowly converge to its steady state $\bar{\eta}$ in the long run.

Given an expansionary monetary shock in the Home country, where $v_0$ decreases in period 0, $\eta_0$ will increase in period 0 due to the exchange rate depreciation. Over time, it will slowly converge to its steady state, with the convergence rate depending on the value of $\lambda_1$.

In an open economy, the model shows output persistence. This can be seen in the output differential equation (3.74) where $\eta_t$ is negatively related to $y_t - y_t^*$. When a monetary policy shock causes the exchange rate to rise, the economy does not revert to its steady state within a single period, unlike in the world economy model. Instead, $\eta_t$ gradually approaches zero over time.

A good supplement to these results can be seen in Benigno & Benigno (2008). It very closely follows OR, while also updating the model with the several modern assumptions. They find that, under a fixed exchange rate regime, a monetary policy shock causes persistence in the terms of trade, similar to our finding of output persistence under flexible exchange rates. This comparison naturally raises the question of whether exchange rate flexibility is desirable or whether a fixed regime might mitigate or amplify such persistence. A full analysis of optimal

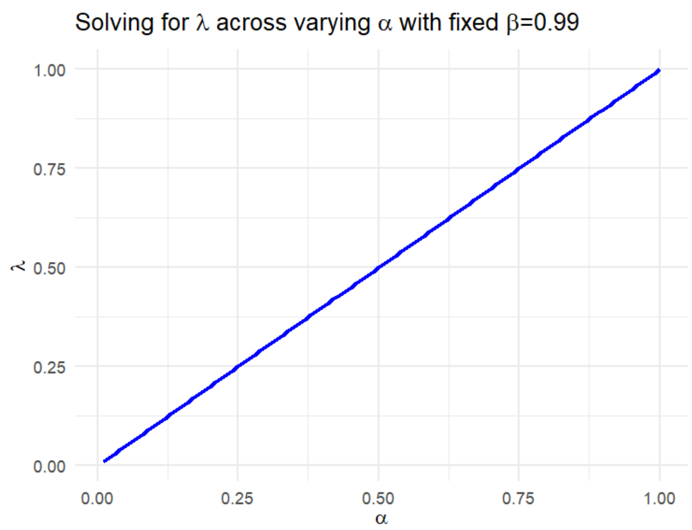


Figure 3.4: Solving for $\lambda$ across varying $\alpha$

exchange rate policy lies beyond the scope of this chapter, but it represents an important direction for further research.

Going back to our paper, the rate of this $\eta_t$ adjustment depends on the value of $\lambda_1$, which varies between 0 and 1. A smaller $\lambda_1$ leads to a faster return to the steady state, whereas a value closer to 1 results in a slower convergence.

Equation (3.79) shows that $\lambda_1$ depends on $\beta$ and $k$, which in turn depends on $\beta$ and $\alpha$. Figure 3.4 shows that $\lambda_1$ is positively related to $\alpha$, for a $\beta$ fixed at 0.99. This shows that higher price rigidity will translate into an economy that takes longer to return to its steady state equilibrium, implying that monetary policy interventions have longer-lasting effects.

## 3.6 Levels of individual-country variables

Having independently analysed the sum and difference systems, we can now combine the results from both to examine the behaviour of individual-country variables.

Considering the consumption levels in the Home and Foreign countries, their responses to a shock mirror those of world consumption and world output. This outcome arises due to the pre-established insurance arrangement between the

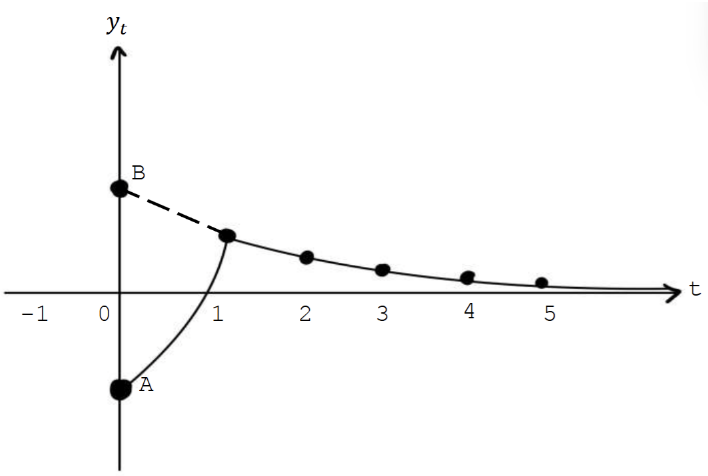


Figure 3.5: Impact of a foreign contractionary monetary policy shock on $y_t$

two countries, which ensures that any imbalances in consumption levels at the end of each period are corrected, resulting in equal consumption levels, $c = c^*$. Consequently, the consumption levels of the Home and Foreign countries are given by:

$$c_t^w = c_t^* = c_t = \frac{-v_t^w}{k\frac{2}{\theta+1}\phi_\pi + 1} \tag{3.83}$$

As discuss in Chapter 2, when $c = c^*$, the difference consumption between Home and Foreign will disappear. Moreover, this implies there is no persistence in consumption levels, with $c_t, c_t* = 0$ for all $t \geq 1$.

Now, we can calculate the output levels for Home and Foreign countries by substituting (3.68) and (3.74), and using $\eta_t \equiv p_t(h) - p_t^*(f)$ in the formulas $y_t = y_t^w + (1-n)(y_t - y_t^*)$ and $y_t^* = y_t^w - n(y_t - y_t^*)$. This yields the following two equations:

$$y_t = \frac{-v_t^w}{k\frac{2}{\theta+1}\phi_\pi + 1} + (1-n)\theta(e_t - \eta_t) \tag{3.84}$$

$$y_t^* = \frac{-v_t^w}{k\frac{2}{\theta+1}\phi_\pi + 1} - n\theta(e_t - \eta_t) \tag{3.85}$$

## 3.7 International Spillover Effects

To determine the international spillover effects, we expand equations (3.84) and (3.85). The variable $v^w$ can be expressed as the weighted average of shocks from the Home and Foreign countries, given by $v^w = nv + (1-n)v^*$. Here, $v^w$ captures the aggregate monetary shocks impacting the world economy, weighted by country size. This means that the overall output response depends not only on the magnitude of shocks in each country but also on how large each economy is, reflecting their relative influence in global trade and financial markets. Also using (3.82), we can then obtain:

$$y_t = \frac{-nv_t - (1-n)v_t^*}{k\frac{2}{\theta+1}\phi_\pi + 1} + (1-n)\theta[e_t - (1-\lambda^{t+1})\bar{\eta}] \tag{3.86}$$

$$y_t^* = \frac{-nv_t - (1-n)v_t^*}{k\frac{2}{\theta+1}\phi_\pi + 1} - n\theta[e_t - (1-\lambda^{t+1})\bar{\eta}] \tag{3.87}$$

Using the fact that in the long run $\eta$ equals the exchange rate, we find the exchange rate solution in (3.72). Focusing on the short-run effects of the shock, we note that the equation below applies specifically to period 0:

$$y_0 = \frac{-nv_0 - (1-n)v_0^*}{k\frac{2}{\theta+1}\phi_\pi + 1} + (1-n)\theta\lambda(e_{-1} - \frac{v_0 - v_0^*}{\phi_\pi}) \tag{3.88}$$

$$y_0^* = \frac{-nv - (1-n)v^*}{k\frac{2}{\theta+1}\phi_\pi + 1} - n\theta\lambda(e_{-1} - \frac{v_0 - v_0^*}{\phi_\pi}) \tag{3.89}$$

We can now differentiate (3.88) and (3.89) with respect to the Home and Foreign monetary shocks, $v$ and $v^*$, respectively. For analytical purposes, we also expand on $k$. This yields:

$$\frac{dy_0}{dv_0} = \frac{-n}{(\frac{1-\alpha}{\alpha})(1-\alpha\beta)\frac{2}{\theta+1}\phi_\pi + 1} - (1-n)\lambda\frac{\theta}{\phi_\pi} \tag{3.90}$$

$$\frac{dy_0}{dv_0^*} = (1-n)\left\{\frac{-1}{\left(\frac{1-\alpha}{\alpha}\right)(1-\alpha\beta)\frac{2}{\theta+1}\phi_\pi + 1} + \lambda\frac{\theta}{\phi_\pi}\right\} \tag{3.91}$$

Given that the two countries are symmetric, our discussion will primarily focus on the Home country's perspective.

With the differentiation now established, it becomes apparent that monetary shocks propagate through two distinct channels: the interest rate effect and the terms of trade effect. These corresponds to the first and second terms respectively on the right hand sides. The latter, representing the terms of trade, aligns with the results found in Chapter 2, but with a key difference: here, the terms of trade effect is dampened by $\lambda$ due to the price staggering mechanism introduced in this chapter.

The differential output equation is partly determined by $\lambda$[5]. The parameter $\lambda$ represents the degree of price rigidity values close to 1 imply highly sticky prices, slowing adjustment in relative prices and terms of trade. This stickiness dampens how quickly countries can respond to shocks through changing competitiveness, thus muting the spillover effects transmitted via the terms of trade channel. And since $\lambda$ is an endogenous parameter that can take values between 0 and 1, its presence reduces the terms of trade effects. Additionally, an $\eta_t$ absent from (3.86) and (3.87) yields similar results as when $\lambda = 1$, showing that partial price flexibility can only be seen when $\lambda < 1$.

A key difference from Chapter 2 is that the exchange rate channel no longer directly affects the output equation. Instead, its role is replaced by the terms of trade. This shift reflects the real-world notion that nominal exchange rate changes alone do not fully determine international competitiveness and instead, the relative price levels (terms of trade) which adjust more gradually due to price rigidities become the critical factor influencing trade balances and output. As shown in equations (3.90) and (3.91), the partial price flexibility introduced in this chapter means that the exchange rate effect is no longer pure. Instead, it manifests through the terms of trade, driven by differential output dynamics.

The interest rate channel also presents a key difference compared to Chapter 2. In this chapter, we have introduced the world NKPC, which directly influences the derivation of world output and, therefore, Home output. Consequently, even if there is no $\lambda$ term in the differential output, the multiplier here will not match

[5] Remember that $\eta_t \equiv p_t(h) - p_t^*(f)$ and that $\eta_t = (1 - \lambda^{t+1})\bar{\eta}$. Therefore, the differential output equation can be rewritten as $y_t - y_t^* = \theta[e_t - (1 - \lambda^{t+1})\bar{\eta}]$

the one in Chapter 2. The interest rate channel in this model is dampened by a denominator term that is absent in Chapter 2. The interest rate effect and the terms of trade effect critically shape the signs and dynamics of the spillover effects. We illustrate their roles in Figure 3.5, showing the evolution of $y_t$ in the short run, medium run, and long run.

In the case where the interest rate effect dominates, a positive shock to $v^*$ at period 0 will cause $y_0$ to decrease to point $A$. Over time, output will gradually return to its steady state but from above. The gradual return to steady state after a shock is driven by a combination of price stickiness and the forward-looking nature of agents. Prices adjust slowly due to staggered contracts, and agents form expectations about future inflation and output, which together create inertia in the economy's response. This occurs because an increase in $v^*$ leads to an immediate adjustment of the exchange rate to its steady-state value post-shock, while output converges more slowly. Conversely, if the terms of trade effect dominate, an increase in $v^*$ will cause $y_0$ to rise to point $B$ in period 0, followed by a gradual decline back to its steady state.

The Home country's output multiplier, in response to a contractionary monetary policy shock (where $v$ increases), is negative, indicating a decline in output. This outcome is consistent across all three chapters and in the original OR model.

Finally, we now examine the spillover effects from a Foreign country contractionary monetary shock ($v^*$ increases). The sign of the multiplier given by equation (3.91) appears ambiguous. This ambiguity, also found in Chapter 2, comes from $\phi_\pi$ and $\theta$. By solving for the spillover effect for different values of each parameter, we are able to clarify its sign[6]. The sign ambiguity arises because parameters like $\phi_\pi$ and $\theta$ govern how aggressively the central bank responds to inflation and how sticky prices are, respectively. When the central bank's reaction is very strong or prices are very flexible, the transmission mechanisms can reverse, causing non-standard spillover effects.

Figure 3.6 illustrates that, for a wide range of reasonable values for $\alpha$, $\beta$, and $n$, the spillover effect consistently exhibits a positive sign. This finding suggests that adopting parameter values commonly used in the literature yields

[6] Following the literature, the baseline parameters for our simulations are fixed at $n = 0.5$, $\alpha = 0.75$, $\beta = 0.99$, $\phi_\pi = 1.5$ and $\theta = 5$.

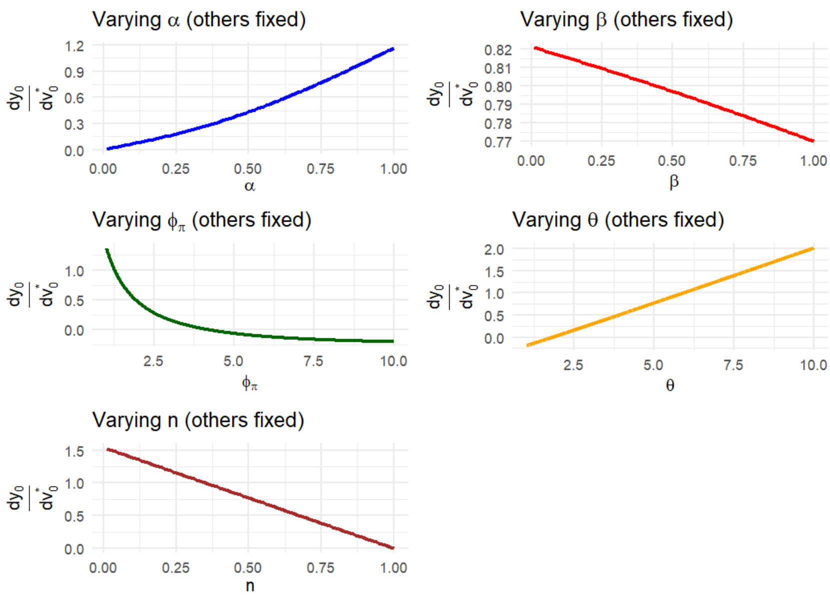


Figure 3.6: International spillover effect on Home country for contractionary monetary policy shock on Foreign country

an orthodox sign, aligning well with theoretical intuition and matching the results from Chapter 2. However, there are certain values of $\phi_\pi$ and $\theta$ that yield a non-orthodox result: values of $\theta < 1.75$ or values of $\phi_\pi > 4.75$ return negative spillover effects. Other interesting results that follow from this figure are that the less sticky the price setting (lower $\alpha$), or the larger the country ($n$ approaches 1), the magnitude of the spillover will converge to 0. Intuitively, larger countries (higher $n$) have more domestic market insulation, reducing the relative impact of foreign shocks. Similarly, more flexible prices allow quicker adjustment in relative competitiveness, which can dampen the persistence and magnitude of spillovers.

## 3.8 Conclusion

In this chapter, we introduced Calvo pricing into the framework developed in previous chapters, extending the model to account for a more flexible type of price stickiness. Also, following previous chapters, we solved the model by decomposing it into a sum system and a difference system.

We find that in the sum system, Calvo pricing does not alter the fundamental result that a monetary policy shock affects world consumption and output in a 1-to-1 fashion. However, the difference system reveals that this increased price rigidity generates persistence in output deviation, as opposed to the instantaneous return to the steady state observed under OR-style price staggering.

Cross-country spillover effects of monetary policy shocks are also affected by Calvo pricing. Higher degrees of price stickiness are associated to a slower adjustment process. Indeed, when compared to previous chapters, the impacts of monetary policy shocks on terms of trade are less pronounced.

Finally, we find that the spillover effects on output are ambiguous. However, after solving the spillover multiplier for a plausible set of values for each relevant parameter, we find that, like in Chapter 2, it is positive and in line with the literature.

# Conclusion

In this thesis we have directly addressed a gap in the NOEM literature by systematically investigating the impact of three modern assumptions –market completeness, Taylor-type monetary policy rules, and Calvo-style staggered pricing– on the model mechanics and international spillovers, all within a tractable two-country framework. Building upon the foundational work of Obstfeld & Rogoff (1995), this research deliberately prioritized analytical clarity to precisely disentangle the individual and combined effects of these assumptions, which are often obscured within modern DSGE models.

Chapter 1 shows that the introduction of completeness of markets, which we implement by imposing the additional constraint that Home and Foreign consumption must be equal for all periods $t \geq 1$, can generate counterintuitive outcomes. Specifically, perfect risk-sharing can lead to results where Home utility declines following a positive monetary policy shock. Moreover, it eliminates the persistent effects of monetary policy shocks that were present under incomplete markets, restricting their impact to the short term.

In Chapter 2 we move beyond the money supply rule of OR and incorporate a Taylor-type interest rate rule, where the central bank adjusts interest rates in response to deviations of CPI inflation from their target. Under completeness of markets, the international spillover effects are ambiguous, crucially depending on the magnitudes of price elasticity and central bank responsiveness. Notably, when $\theta < \phi_\pi$ we obtain unorthodox results where an expansionary monetary policy shock has positive spillover effects. Relaxing the assumption of completeness of markets also impacts the spillover dynamics, revealing that the sign of the international spillover effect remains negative, becoming positive only for exceptionally high values of $\theta$ (above 150). Reflecting on the policy relevance of these findings, our results suggest a potentially important implication: in emerging economies characterized by less developed financial markets, money supply rules may offer greater stability against monetary policy implementation errors compared to Taylor rules.

Chapter 3 further advances the analysis by incorporating Calvo-style staggered pricing. We shot that this introduces persistence into output deviations and slows the adjustment to monetary shocks relative to OR, even under complete markets. Crucially, the sign of the spillover effects of monetary policy shock remained ambiguous, although consistent with Chapter 2, it remained orthodox when using parameter values in line with the literature.

The model developed across these three chapters makes two key contributions to the NOEM literature. First, it has enabled a detailed and systematic analysis of the impact that key modern assumptions have in the main results of two country models –an analysis that is significantly more challenging to do in more complex models. And second, it has yielded a benchmark-like model that is fully tractable but that still incorporates all the essential features necessary for researchers to build upon and extend with further complexities[7].

We believe there are many extensions that could be built upon this thesis, and that could be highly relevant to the current economic and political climate. Given the increasing prominence of fiscal policy in the public and policy discourse in recent years, investigating the spillover effects of fiscal policy shocks represents a particularly valuable extension. Furthermore, the recent global socks, including both the COVID-19 pandemic and the energy crisis triggered by Russia's invasion of Ukraine, have also demonstrated how supply side shocks can significantly destabilise economies, something that much of the literature had ignored. Therefore, exploring the international transmission of supply-side shocks within out tractable framework also represents a highly policy-relevant direction for future research.

Finally, as previously mentioned, we identify a notable gap in applying these

[7] Gali & Monacelli (2005) is a prominent benchmark model which focuses on a continuum of small open economies with similar characteristics to ours and that also remains highly tractable. However, our model is different in that it does not assume agents to have home bias, does not account for elements that are unique to small open economies, and analyses the international spillover effects of monetary policy shocks.

models to emerging market economies, particularly for regions like ASEAN. While some literature explores the impacts of US monetary policy spillovers on emerging markets, much less attention has been given to spillover effects amongst emerging economies or in other economic regions. Our results from Chapters 1 and 2 suggest that future research could investigate the complexities of applying interest rate rules for monetary policy in economies with high levels of market incompleteness. Alternatively, another interesting extension would be to apply the framework outlined by Peneva (2009) to study spillovers between countries with differential labour-to-capital ratios, a common characteristic in emerging market economies.

# Appendix

## A.1 Obstfeld and Rogoff model and non-linear equilibrium condition

Euler equation

$$C_{t+1} = \beta(1 + r_{t+1})C_t \tag{A.1}$$

Money-in-the-utility function

$$\frac{M_t}{P_t} = \chi C_t \left( \frac{1 + i_{t+1}}{i_{t+1}} \right) \tag{A.2}$$

Labor-leisure trade-off condition

$$y_t^{\frac{\theta+1}{\theta}} = \frac{\theta - 1}{\theta\kappa}(C_t^t)^{\frac{1}{\theta}} \frac{1}{C_t} \tag{A.3}$$

Goods-market clearing condition

$$C_t^w \equiv nC_t + (1-n)C_t^* = n\frac{p_t(h)}{P_t}y_t(h) + (1-n)\frac{p_t^*(f)}{P_t^*}y_t^*(f) \equiv Y_t^w \tag{A.4}$$

Purchasing power parity condition non-linear

$$P = \varepsilon P^* \tag{A.5}$$

Exchange rate determination (nominal)

$$1 + i_{t+1} = \frac{P_{t+1}}{P_t}(1 + r_{t+1}) \tag{A.6}$$

## A.2 Comparing our world demand equation with OR's

Calvo style price setting means that, unlike in OR, different Home producers have different opportunities to change their prices. This generates heterogeneity for output levels and prices across Home producers. Consequently, while OR can simply equate Home per capita GDP to the output of a representative Home producer and Home PPI to the price of a representative consumer, under Calvo style price setting GDP and PPI must be calculated as averages across heterogenous outputs and prices. They are given by the expressions below:

$$Y_t = \frac{[\frac{1}{n}\int_0^n P_t(z)Y_t(z)dz]}{P_t(h)} \tag{A.7}$$

$$P(h) \equiv \left\{ \frac{1}{n}\int_0^n [P_t(z)]^{1-\theta}\ dz \right\}^{\frac{1}{1-\theta}}. \tag{A.8}$$

Let $n$ denote the total weight of Home producers. In period $t$, a fraction $1-\alpha$ of these producers have an opportunity to update their prices to $X_t$. Similarly, in period $t-1$ a fraction $(1-\alpha)\alpha$ updated their prices to $X_{t-1}$; in period $t-2$ a fraction $(1-\alpha)\alpha^2$ updated their prices to $X_{t-2}$, and this pattern continues into the past. Applying this to (A.8), and noting that the integral is just the sum over the various weights of the different prices, times these prices raised o the power of $1-\theta$, we obtain:

$$\begin{aligned} P_t(h) &= \left\{ \frac{1}{n}\Big[(1-\alpha)nX_t^{1-\theta} \right. \\ &\quad \left. +(1-\alpha)\alpha n X_{t-1}^{1-\theta} + (1-\alpha)\alpha^2 n X_{t-2}^{1-\theta} + \cdots \Big] \right\}^{\frac{1}{1-\theta}} \\ &= \left\{ (1-\alpha)\sum_{s=0}^{\infty} \alpha^s X_{t-s}^{1-\theta} \right\}^{\frac{1}{1-\theta}}. \end{aligned} \tag{A.9}$$

Log-linearising this around a zero inflation steady state we then get (3.13):

$$p_t(h) = (1-\alpha)\sum_{s=0}^{\infty}\alpha^s x_{t-s} \tag{A.10}$$

which can also be written as (3.29):

$$p_t(h) = (1-\alpha)x_t + \alpha p_{t-1}(h) \tag{A.11}$$

Having shown how the Calvo low of motion looks like, we will now turn to examining the world demand function for Home aggregate output. In OR, the lack of heterogeneity among Home producers implies that the demand function for an individual Home-produced good is identical to that of the aggregate per capita output of the Home economy. In contrast, under Calvo-style price setting, where producers are heterogeneous, the demand for Home real per capita GDP must be calculated by aggregating demand across different good types. Using the definition of Home per capita real GDP above, together with Calvo style price setting we obtain:

$$\begin{aligned} Y_t = \Bigg\{\frac{1}{n}\Big[&(1-\alpha)n\,X_t Y_{t|t} \\ &+ (1-\alpha)\alpha n\,X_{t-1}Y_{t|t-1} \\ &+ (1-\alpha)\alpha^2 n\,X_{t-2}Y_{t|t-2} + \cdots\Big]\Bigg\}/P_t(h) \end{aligned} \tag{A.12}$$

where $Y_{t|t-s}$ denotes the output of a producer whose most recent opportunity to adjust its prices occurred in period $t-s$.

We also know what the demand function for an individual Home good-type is $Y_t(z) = [P_t(z)/P_t]^{-\theta}C_t^w$, where $z$ is the good-type, $P_t$ is the CPI and $C_t^w$ is world composite consumption. Having already shown the relationship between $P_t(z)$, $P_t(h)$, and $X_{t-s}$, we can also write the demand function as $Y_{t|t-s} =$

$[X_{t-s}/P_t]^{-\theta}C_t^w$, which when substituting into (A.12) yields:

$$\begin{aligned} Y_t = \Big\{ & (1-\alpha)X_t^{1-\theta}P_t^{\theta}C_t^w \\ & + (1-\alpha)\alpha X_t^{1-\theta}P_t^{\theta}C_t^w + \cdots \Big\}/P_t(h) \\ = P_t^{\theta}C_t^{\theta}\Big\{ & (1-\alpha)X_t^{1-\theta} \\ & + (1-\alpha)\alpha X_{t-1}^{1-\theta} + \cdots \Big\}/P_t(h) \end{aligned} \tag{A.13}$$

Notice that the bracketed term equals $[P_t(h)]^{1-\theta}$. Therefore, the equation can be rewritten as:

$$Y_t = P_t^{\theta}C_t^w[P_t(h)]^{-\theta} = [P_t(h)/P_t]^{-\theta}C_t^w \tag{A.14}$$

This shows that the demand function for Home per capital real GDP turns out to be identical o the demand function for a single good type, except that the individual good's price $P_t(z)$ is replaced by the PPI.

## A.3 Derivation of the log-linearised price-setting equation

In order to derive the log-linearised price-setting equation, we can start by rewriting equation (3.31) as below:

$$X^{\theta+1}\sum_{s=t}^{\infty}(\alpha\beta)^{s-t}\frac{P_s^{\theta-1}}{C_s}C_s^w = \frac{\theta\kappa}{\theta-1}\sum_{s=t}^{\infty}(\alpha\beta)^{s-t}P_s^{2\theta}(C_s^w)^2 \tag{A.15}$$

If we total differentiate it with respect to the three variables it depends on $(X_t\{P_s, C_s^w, C_s\}_{s=t}^{\infty})$ we obtain:

$$\begin{aligned}
&\{(\theta+1)X_t^{\theta}\sum_{s=t}^{\infty}(\alpha\beta)^{s-t}\frac{P_s^{\theta-1}}{C_s}C_s^w\}\,dX_t \\
&\quad+\Bigg[X_t^{\theta+1}\sum_{s=t}^{\infty}(\alpha\beta)^{s-t}\{(\theta-1)\frac{P_s^{\theta-2}}{C_s}C_s^w\}\,dP_s \\
&\qquad+\sum_{s=t}^{\infty}(\alpha\beta)^{s-t}\{-\frac{P_s^{\theta-1}}{C_s^2}C_s^w\}\,dC_s \\
&\qquad+\sum_{s=t}^{\infty}(\alpha\beta)^{s-t}\{\frac{P_s^{\theta-1}}{C_s}\}\,dC_s^w\Bigg] \\
&=\frac{\theta\kappa}{\theta-1}\Bigg[\sum_{s=t}^{\infty}(\alpha\beta)^{s-t}\{2\theta P_s^{2\theta-1}(C_s^w)^2\}\,dP_s \\
&\quad+\sum_{s=t}^{\infty}(\alpha\beta)^{s-t}\{P_s^{2\theta}2(C_s^w)\}\,dC_s^w\Bigg]
\end{aligned} \tag{A.16}$$

We can express the differentials in terms of proportional differentials replacing $dX_t$ with $\frac{dX_t}{X_t}X_t$. This transformation allows us to rewrite the equation so that $\frac{dX_t}{X_t}$ represents the proportional change in $X_t$. This factor $X_t$, which appears alongside the proportional differential, is then absorbed into the surrounding terms, effectively modifying their coefficients.

$$
\begin{aligned}
&\left\{(\theta+1)X_t^{\theta+1}\sum_{s=t}^{\infty}(\alpha\beta)^{s-t}\frac{P_s^{\theta-1}}{C_s}C_s^w\right\}\frac{dX_t}{X_t} \\
&\quad + X_t^{\theta+1}\left[\sum_{s=t}^{\infty}(\alpha\beta)^{s-t}\left\{(\theta-1)\frac{P_s^{\theta-1}}{C_s}C_s^w\right\}\frac{dP_s}{P_s}\right. \\
&\qquad + \sum_{s=t}^{\infty}(\alpha\beta)^{s-t}\left\{-\frac{P_s^{\theta-1}}{C_s}C_s^w\right\}\frac{dC_s}{C_s} \\
&\qquad\quad \left. + \sum_{s=t}^{\infty}(\alpha\beta)^{s-t}\left\{\frac{P_s^{\theta-1}}{C_s}C_s^w\right\}\frac{dC_s^w}{C_s^w}\right] \\
&= \frac{\theta\kappa}{\theta-1}\left[\sum_{s=t}^{\infty}(\alpha\beta)^{s-t}\{2\theta P_s^{2\theta}(C_s^w)^2\}\frac{dP_s}{P_s}\right. \\
&\quad \left. + \sum_{s=t}^{\infty}(\alpha\beta)^{s-t}\{P_s^{2\theta}2(C_s^w)^2\}\frac{dC_s^w}{C_s^w}\right]
\end{aligned}
\tag{A.17}
$$

Next, we evaluate the coefficients on the proportional differentials in the zero-inflation steady state, where $P_s$, $C_s$, and $C_s^w$ remain constant for all values of $s$. Consequently, we drop the $s$ subscript. The same occurs for $X_t$. Since such terms no longer depend on $s$, we can factor out the constant terms (curly bracketed terms) from the intertemporal sums ($\sum_{s=t}^{\infty}(\alpha\beta)^{s-t}$), obtaining:

$$\begin{aligned}
&\left\{(\theta+1)X^{\theta}\frac{P^{\theta-1}}{C}C^{w}\sum_{s=t}^{\infty}(\alpha\beta)^{s-t}\right\}\frac{dX_t}{X_t} \\
&\quad + X^{\theta+1}\Bigg[(\theta-1)\frac{P^{\theta-1}}{C}C^{w}\sum_{s=t}^{\infty}(\alpha\beta)^{s-t}\frac{dP_s}{P_s} \\
&\qquad - \frac{P^{\theta-1}}{C}C^{w}\sum_{s=t}^{\infty}(\alpha\beta)^{s-t}\frac{dC_s}{C_s} \\
&\qquad + \frac{P^{\theta-1}}{C}C^{w}\sum_{s=t}^{\infty}(\alpha\beta)^{s-t}\frac{dC^w_s}{C^w_s}\Bigg] \\
&= \frac{\theta\kappa}{\theta-1}\Bigg[2\theta P^{2\theta}(C^{w})^{2}\sum_{s=t}^{\infty}(\alpha\beta)^{s-t}\frac{dP_s}{P_s} \\
&\quad + P^{2\theta}2(C^{w})^{2}\sum_{s=t}^{\infty}(\alpha\beta)^{s-t}\frac{dC^w_s}{C^w_s}\Bigg]
\end{aligned} \tag{A.18}$$

Notice that $X^{\theta+1}P^{\theta-1}C^{w}/C$ and $\frac{\theta k}{k-1}P^{2\theta}(C^{w})^{2}$ are common factors in all the coefficients in the LHS and RHS respectively. Therefore, if we set all variables in (A.16) to their zero-inflation steady state values, we obtain:

$$X^{\theta+1}\frac{P^{\theta-1}}{C}C^{w}\sum_{s=t}^{\infty}(\alpha\beta)^{s-t} = \frac{\theta k}{\theta-1}P^{2\theta}(C^{w})^{2}\sum_{s=t}^{\infty}(\alpha\beta)^{s-t} \tag{A.19}$$

Additionally, the summation $\sum_{s=t}^{\infty}(\alpha\beta)^{s-t}$ is essentially the sum of an infinite geometric series with a common ratio $\alpha\beta$. Given that this ratio will be between 0 and 1, we can also write it as $1/(1-\alpha\beta)$. This allows us to cancel the term from both sides in our previous equation, yielding:

$$X^{\theta+1}\frac{P^{\theta-1}}{C}C^{w} = \frac{\theta k}{\theta-1}P^{2\theta}(C^{w})^{2} \tag{A.20}$$

This means that in the totally-differentiated equation there are common factors both on the LHS and RHS which can also be cancelled. If we also use the fact that $\sum_{s=t}^{\infty}(\alpha\beta)^{s-t}$ is an infinite geometric series, we can therefore simplify (A.17) into:

$$
\begin{aligned}
(\theta+1)\frac{1}{1-\alpha\beta}\frac{dX_t}{X_t}+(\theta-1)\sum_{s=t}^{\infty}(\alpha\beta)^{s-t}\frac{dP_s}{P_s} \\
-\sum_{s=t}^{\infty}(\alpha\beta)^{s-t}\frac{dC_s}{C_s}+\sum_{s=t}^{\infty}(\alpha\beta)^{s-t}\frac{dC_s^w}{C_s^w} \\
= \quad 2\theta\sum_{s=t}^{\infty}(\alpha\beta)^{s-t}\frac{dP_s}{P_s}+2\sum_{s=t}^{\infty}(\alpha\beta)^{s-t}\frac{dC_s^w}{C_s^w}
\end{aligned}
\tag{A.21}
$$

Which after rearranging, gives:

$$
\frac{dX_t}{X_t}=\frac{1-\alpha\beta}{\theta+1}\sum_{s=t}^{\infty}(\alpha\beta)^{s-t}\left[(\theta+1)\frac{dP_s}{P_s}+\frac{dC_s}{C_s}+\frac{dC_s^w}{C_s^w}\right] \tag{A.22}
$$

The last step needed to derive equation (3.31) is to replace proportional differentials by the log-deviations of variables where $z_t \equiv lnz_t - lnz$, where $z$ is the value of $z_t$ around the zero-inflation steady state and symmetric across countries. This will then yield (3.31).

We will now proceed to check on whether our log-linearised price-setting equation is intuitively sensible, using two different approaches:

1. Comparison with the original OR model. OR never directly outlines the optimal price-setting equation. Nevertheless, we can derive it by combining the log-linearised world demand equation –which is the same as our (3.42), and their log-linearised labour-leisure trade off equation, yielding:

   $$
   p_t(h)=p_t+\frac{1}{\theta+1}(c_t^w+c_t) \tag{A.23}
   $$

   Note that, in a zero-inflation steady state, (3.31) in fact reduces to exactly the same. This makes sense because in steady state there is enough time for all prices to adjust, so firm behaviour should be the same as under flexible prices.

2. Comparison with Calvo's original closed economy model of price staggering (Calvo 1983), in discrete form. Calvo did not derive a his-price-setting

equation, and instead just assumed an equation which he considered plausible. He also did not incorporate a discount factor. A basic version of his equation could be:

$$x_t = (1-\alpha)\sum_{s=t}^{\infty}\alpha^{s-t}[p_s + \gamma y_s] \tag{A.24}$$

where $y_s$ is the log-deviation of output and $\gamma > 0$ is the sensitivity of the 'new price' to output.

If we imagine a closed economy version of our model, we would have $c_s^w = c_s = y_s$, so that our equation (3.31) would become:

$$x_t = (1-\alpha\beta\sum_{s=t}^{\infty}(\alpha\beta)^{s-t}[p_s + \frac{2}{\theta+1}y_s] \tag{A.25}$$

showing that Calvo's equation is the same as ours under the special case where $\beta = 1$ and $2/(\theta+1) = \gamma$.

## A.4 Derivation of the NKPC

Our optimal price setting equations (3.31) and (3.32) do not include output. In order to relate output to prices, we can start by rearranging (3.42) and (3.43) as follows:

$$p_t = \frac{1}{\theta}[y_t - c_t^w] + p_t(h) \tag{A.26}$$

$$p_t^* = \frac{1}{\theta}[y_t^* - c_s^t] + p_t^*(f) \tag{A.27}$$

After plugging them into (3.31) and (3.32) and simplifying further, we then obtain:

$$x_t = (1-\alpha\beta)\sum_{s=t}^{\infty}(\alpha\beta)^{s-t}\{\frac{y_t}{\theta} + p_t(h) + \frac{c_t}{\theta+1} - \frac{1}{\theta(\theta+1)}c_t^w\} \tag{A.28}$$

$$x_t^* = (1-\alpha\beta)\sum_{s=t}^{\infty}(\alpha\beta)^{s-t}\{\frac{y_t^*}{\theta} + p_t^*(f) + \frac{c_t^*}{\theta+1} - \frac{1}{\theta(\theta+1)}c_t^w\} \tag{A.29}$$

Since $c_t = c_t^* = c_t^w$:

$$x_t = (1-\alpha\beta)\sum_{s=t}^{\infty}(\alpha\beta)^{s-t}\{\frac{y_t}{\theta} + p_t(h) + \frac{\theta-1}{\theta(\theta+1)}c_t^w\} \tag{A.30}$$

$$x_t^* = (1-\alpha\beta)\sum_{s=t}^{\infty}(\alpha\beta)^{s-t}\{\frac{y_t^*}{\theta} + p_t^*(f) + \frac{\theta-1}{\theta(\theta+1)}c_t^w\} \tag{A.31}$$

We can then re-write the equations to remove the summation:

$$x_t = (1-\alpha\beta)\left[\frac{y_t}{\theta} + p_t(h) + \frac{\theta-1}{\theta(\theta+1)}c_t^w\right] + \beta\alpha(x_{t+1}) \tag{A.32}$$

$$x_t^* = (1-\alpha\beta)\left[\frac{y_t^*}{\theta} + p_t^*(f) + \frac{\theta-1}{\theta(\theta+1)}c_t^w\right] + \beta\alpha(x_{t+1}^*) \tag{A.33}$$

We can re-write these equations in terms of inflation by subtracting $p_{t-1}$ from both left and right hand sides, and using the fact that inflation deviation is given by $\pi_t(h) = p_t(h) - p_{t-1}(h)$ and $\pi_{t+1}(h) = p_{t+1}(h) - p_t(h)$:

$$\begin{aligned} x_t - p_{t-1}(h) = (1-\alpha\beta)&\left[\frac{y_t}{\theta} + \pi_t(h) + \frac{\theta-1}{\theta(\theta+1)}c_t^w\right] \\ &+ \beta\alpha\left[E_t\big(x_{t+1}\big) - p_t(h) + \pi_t(h)\right] \end{aligned} \tag{A.34}$$

$$\begin{aligned} x_t^* - p_{t-1}^*(f) = (1-\alpha\beta)&\left[\frac{y_t^*}{\theta} + \pi_s^*(f) + \frac{\theta-1}{\theta(\theta+1)}c_t^w\right] \\ &+ \beta\alpha\big[E_t\big(x_{t+1}^*\big) - p_t^*(f) + \pi_t^*(f)\big]. \end{aligned} \tag{A.35}$$

Subtracting $p_{t-1}$ from the left and right hand side of the law of motion equations for Calvo pricing ((3.29) and (3.30)) we obtain

$$x_t - p_{t-1}(h) = \frac{\pi_t(h)}{1-\alpha} \tag{A.36}$$

$$x_t^* - p_{t-1}^*(f) = \frac{\pi_t^*(f)}{1-\alpha} \tag{A.37}$$

Substituting (A.36), (A.37) and their $t+1$ counterparts into (A.34) and (A.35) we get

$$\begin{aligned}\frac{\pi_t(h)}{1-\alpha} &= (1-\alpha\beta)\left[\frac{y_t}{\theta} + \pi_t(h) + \frac{\theta-1}{\theta(\theta+1)}c_t^w\right] \\ &+ \beta\alpha\left[E_t\frac{\pi_{t+1}(h)}{1-\alpha} + \pi_t(h)\right]\end{aligned} \tag{A.38}$$

$$\begin{aligned}\frac{\pi_t^*(f)}{1-\alpha} &= (1-\alpha\beta)\left[\frac{y_t^*}{\theta} + \pi_s^*(f) + \frac{\theta-1}{\theta(\theta+1)}c_t^w\right] \\ &+ \beta\alpha\big[E_t\frac{\pi_{t+1}^*(f)}{1-\alpha} + \pi_t^*(f)\big].\end{aligned} \tag{A.39}$$

Finally, bringing $\pi_t(h)$ and $\pi_t^*(f)$ to the left hand side, and simplifying the remaining right hand side, will yield the Home (3.48) and Foreign (3.49) NKPCs.

## A.5 Derivation of log-linearised price index in terms of rate of change

In this section we show step by step the derivation of the log-linearised price index in terms of rate of change. We start with the two equations below, which represent the log-linearised price indices for Home and Foreign, and which we originally showed in Chapter 3.

$$p_t = np_t(h) + (1-n)\left[e_t + p_t^*(f)\right] \tag{A.40}$$

$$p_t^* = n\left[p_t(h) - e_t\right] + (1-n)p_t^*(f) \tag{A.41}$$

Subtracting $(1-n)p_t(h)$ and $(1-n)p_t(f)$ ensures that the weights of Home and Foreign price sum to 1, a property that will be needed normalization. This step simplifies the equations, making price deviations more transparent by isolating the effects of exchange rates and foreign prices. As a result, the log-linearized derivation becomes more intuitive and easier to interpret in terms of rate of change.

$$p_t = np_t(h) + (1-n)p_t(h) - (1-n)\left[e_t + p_t^*(f) - p_t(h)\right] \tag{A.42}$$

$$p_t^* = n\left[p_t(h) - e_t - p_t^*(f)\right] + np_t^*(f) + (1-n)p_t^*(f) \tag{A.43}$$

which after rearranging, yields the log-linearised prices indices in terms of rate of change presented earlier ((3.50) and (3.51)):

$$p_t - p_t(h) = (1-n)\left[e_t + p_t(f) - p_t^*(h)\right] \tag{A.44}$$

$$p_t^* - p_t^*(f) = n\left[p_t(h) - e_t - p_t^*(f)\right] \tag{A.45}$$

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