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Multilayer Economic Networks with Strategic Cooperation

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Doctor of Philosophy

Declaration

All work presented within this thesis is the author's own work except where specific reference has been made to the work of others.

Research material produced during the course of this PhD has been peer-reviewed or is currently undergoing peer-review for publication. Chapters which contain this material, with the permission of the co-authors, are indicated in Section 1.4.1.

Toby Willis
Monday 30th December, 2024

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Terms and Abbreviations

BPR Bureau of Public Roads. 49

NTU Non-Transferrable Utility. 36, 68

PoA Price of Anarchy. 25, 59, 65, 75

PoS Price of Stability. 25, 59, 65, 75, 76

SITC Standard International Trade Classification. 130–132, 141

SPE Subgame Perfect Equilibrium. 26

WE Wardrop Equilibrium. 48

Abstract

Understanding the implications of unpredictable and disruptive events, such as the 6-day blockage of the Suez Canal or the imposition of sanctions, is important for guiding the most appropriate response. This is particularly true when the potential for cooperative behaviour between smaller groups exists, exploiting the situation for their benefit.

The research presented in this thesis first seeks to analyse the impact of coalitional elements on basic games of network formation. This serves as a proxy for infrastructure creation and maintenance, as well as the formation of voluntary membership groups for mutual benefit. Conditions under which the players find group membership beneficial are found, as well as analysis provided on the impact of these groups on the public good. The research develops a multilayer model of the interaction between multimarket Cournot trade and congested transport and, once analysed, provides time horizon specific strategy optimisation. The developed coalitional elements and Cournot-Transport model are subsequently combined in a model to represent the imposition of sanctions, and the process by which the sanction target may try and circumvent these sanctions with the assistance of uninvolved third-parties. The model is compared to real world historical data in order to demonstrate its applicability, and that of the analysis conducted upon it. Finally, analysis is conducted on this model to capture its implications at different time horizons and under different network structures.

The original contributions in this thesis include the development of a novel approach to the formation and expansion of non-transferable utility coalitions with membership limitations. This allows for development, from the literature, in the understanding of the stability of coalitions of this form and their impact on both social dilemma and network formation games. It also provides new results on optimal responses to disrupted transportation networks in a dynamic network game. The thesis uses these results to characterise the rate of approach to a new equilibrium after disruption, as well as discussing the impact of coalitions on games that include this aspect. A model of the impact of sanctions is subsequently developed. This gives further results on the best responses to sanctions by players in this game, providing results which

help to explain the effectiveness or otherwise of sanctions compared to expectations in the real world.

Chapter 1

Introduction

This chapter provides an introduction to the work which appears in this thesis and the contextual background for the problems that it aims to address. It also outlines the structure of the rest of the thesis, as well as how the chapters are connected to the aims and objectives.

1.1 Background

The problem of unpredictable economic implications

The prediction of the impacts of both systemic change and political interventions on economic systems is a complex challenge. The disruption caused by the blockage of the Suez Canal [2] prevented its use for transport for 6 days. The Suez Canal carries 12% of maritime traffic, which itself makes up more than 90% of global trade [2]. In 2019, more than 1 billion tons of cargo were transported through the canal [3]. Even a temporary reduction in capacity results in the need for alternate routes leading to impacts on prices in all markets. Disruption is not exclusive to transport considerations however. Political concerns resulting in the imposition of sanctions also impact the economic system, reducing trade between the sanction sender and the sanction target. This consequently has implications both on the price of goods in the two principal markets, as well as in independent markets [4]. Having the availability of modelling tools, capable of predicting the implications of disruption both analytically and through simulation, allows for more appropriate responses and preparation. The use of game theory as a method to represent the competitive dynamics between economic entities is a common choice that is frequently seen [5, 6]. This use of game theory allows the prediction of both the new equilibrium after the disruption, and also the amount of time until the new equilibrium is reached.

Network Transportation and Congestion

In order to appropriately consider the impact of rare, unpredictable and typically extreme so-called black swan events [7] on transport, the transport itself must be modelled. The use of a graph-theoretical network to represent the junctions and paths of the transport network is a natural choice that is used widely in the literature [8]. Congestion dynamics depend heavily on the basis scenario. Different congestion functions, each giving a functional relationship between flow and cost, are appropriate in different cases. These can include an exponential curve, such as that used in road traffic modelling [9], or a sigmoid as used in gas transport [10]. They can even be used in trade dynamics to represent the decrease in sale price of goods to a market.

Multilayer Networks

Multilayer networks exist within the field of complex network theory and provide a tool for analysis of systems with different channels of connectivity [11]. A layer of a multilayer network represents a mode of connectivity, whilst a node may have different kinds of interaction, resulting in different neighbours on each layer. The characteristics of interaction are basis dependent. Some examples of what different layers can represent include different forms of social interaction, different types of transportation, the playing of pairwise games between connected players or a combination of these interactions.

Cooperation and Coalition in Games

Deliberate cooperation isn't something considered within the most basic implementations of game theory [12], but the development of cooperation is natural as it provides opportunity for players to improve their outcomes. A common method for the implementation of cooperation is through a coalition, wherein players elect to act as a single entity, optimising their actions with respect to the entirety of the coalition rather than just to themselves [13]. Depending on the context, this can be universally advantageous, improving the welfare of all players and affected non-players, or it can represent negative characteristics such as syndication or insider trading. An understanding of the incentives for trade allows the implementation of control techniques to encourage or discourage coalition formation as appropriate.

Oligopoly Games on a Network

Oligopolies are characterised by a few firms competing in the market, each of which has non-trivial market power. They are differentiated from monopolies, in which a single firm has total market control, and perfect competition in which a large number of firms compete with no firm having significant market power. In economics literature, oligopoly games can appear in both micro-economic and macro-economic settings. The oligopolies in these games represent large companies [5] and nations [14], respectively, and both forms of oligopoly are addressed within this thesis. A network basis restricts the decisions of the oligopolies in parallel with real life limitations and opportunities. This can be a restriction in opportunities to trade imposed by economic sanctions or transportation considerations [15], or opportunities formed through price reduction through cooperation [16].

Research gap in Multilayer Games

As will be described in more detail in Chapter 2, despite extensive work in the areas of network transport analysis and oligopoly games, the intersection of these areas is not well studied, despite the clear basis cases that would benefit from it. This results in a lack of analytical tools and modelling techniques for the study of the rate of approach to equilibrium and the effect of black swan events on trade.

1.2 Research Aim and Objectives

The aim of this research is to analyse optimal strategy in a multilayer economic system over different time horizons and when including the opportunity for cooperation with other players.

This aim will be supported by the following objectives:

1. Develop a model of network formation and cooperative strategy, building from the literature, and analysing the emergence and stability of coalitions (**Chapter 3**).

2. Develop a multilayer model linking together the dissimilar dynamics of optimal transport on a congested network and oligopolic trade following a Cournot competition structure (**Chapter 4**).

3. Extend the former two models by uniting elements analysed in each, into a multilayer model of sanction imposition and trade optimisation (**Chapter 5**).

4. Conduct a comparison of this final model with real world data and provide analytical results on the impact of coalition formation on utility (**Chapter 5**).

The first and second objectives can be classed as model development. The third is model application and the fourth is model evaluation.

1.3 Thesis Outline and Structure

A brief summary of the main content and results of each chapter is summarised below.

Chapter 1 provides the contextual background to this thesis. It discusses the real-world basis for the problems addressed. It outlines the aims, objectives and research impact of the research. It briefly discusses several important elements of the thesis and key pieces of literature that the thesis builds upon.

In Chapter 2, a literature review is presented to assess the current state of knowledge in network games. The review covers the characteristics of oligopoly games and previous models using this setting. It is found that there is no multilayer approach to oligopoly, transportation and coalition-formation currently in the literature. The importance of this combination to transportation dynamics creates an opportunity for further development. The chapter describes the use of oligopoly models to characterise trade, and the development of cooperation on those models. It further describes the nature of network modification games, a relevant area of game theory literature to this thesis. The chapter then describes key areas of cooperation and competition, including sanctions literature. Chapter 2 then discusses the relevant areas of network transport analysis and non-technical bases for research in this area. Finally, the literature review is summarised to frame the needed research required to develop original models uniting the elements of oligopoly, transportation and coalition-formation to advance the literature.

The general structure of the oligopoly game is introduced in Chapter 3. Models where this oligopoly structure is separately linked with functional transportation elements and with non-transferable utility are developed. Results from these models are used to demonstrate characteristics of coalition formation. These characteristics continue to be seen throughout later chapters where the oligopoly structure appears.

Chapter 4 focusses on transportation elements, discussing available transportation algorithms. It then introduces a model of a multilayer system where a congestion based transportation network comprises one of the layers. Results are given relating to equilibria of the system, the rate of approach to equilibrium when the networked graphs are changed and the optimal strategies under different patience characteristics, as well as the impact of coalitions on the game.

Chapter 5 establishes an explicit basis of sanctions applied to a target. The structures introduced in Chapters 3 and 4 are then developed-upon to create a multilayer model of this basis, before results about the effectiveness of sanctions, time related considerations and the importance of network structure are given. Chapter 5 also presents real-world findings relating to the models developed throughout the thesis.

Chapter 6 provides an overall discussion of the research findings and recommendations for future work.

1.4 Summary of Contributions to Knowledge

This thesis makes a contribution to the field of game theory and complex systems by addressing a research gap in the analytical approach to oligopoly games on networks. The novel contributions to knowledge of this thesis correspond directly to the research aims and objectives and are as listed below:

1. A set of techniques for the analysis of non-transferable utility games are developed, along with an analysis of the weaknesses of non-transferable utility games.

2. A method for the analysis of the rate of re-emergence of stability for a system disrupted by network characteristic changes is developed.

3. Techniques for the analysis of socially optimal return to equilibrium are developed and an analysis of the cost of efficient deployment is conducted.

4. A model, mirroring real world sanction dynamics, is developed giving predictive capabilities on the development of trade disruption under sanction imposition or transport network disruption by other means, accidental or planned.

5. An numerical analysis of the model is conducted, comparing the results given by the model to real-world observations.

1.4.1 Research Output

The following articles were developed during my PhD research. There is significant overlap between work in this thesis and these published works.

- **Toby Willis**, Giuliano Punzo. Multi-Layer Cournot-Congestion Model In: IFAC-PapersOnLine Volume 55, Issue 40, 2022, Pages 61-66.

  Material prepared for this article is featured in Chapters 4

- **Toby Willis**, Giuliano Punzo. Oligopoly Game Stabilisation Through Multilayer Congestion Dynamics In: arXiv:2406.19079.

  Material prepared for this article is featured in Chapters 4,5

Chapter 2

Review of Literature on Multilayer Economic Systems and Cooperative Game Theory

This review aims to both introduce the important areas of literature that inform Chapters 3 to 5, and to contextualise the research in this thesis by outlining the state of the art. The fundamentals of the two macro areas this work has been developed in, Game Theory and Network Theory will also be illustrated, in order to provide context with regard to why specific implementations were chosen from within them, in preference to alternative options.

2.1 Graph Theory

Graph theory is one of two key pillars of this thesis, both as a tool for defining the connections between players in a game, but also for modelling geographical connections where appropriate. Historically, Leonhard Euler is widely regarded as having published the first paper in graph theory, with his 1736 publication considering the seven bridges of Konigsberg, now a famous problem [17].

Figure 2.1 shows a graph representation of the focus of Euler's paper,

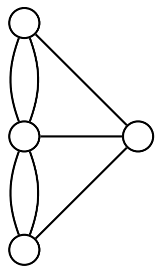

Figure 2.1: *A Graph representation of the famous bridges of Konigsberg problem.*

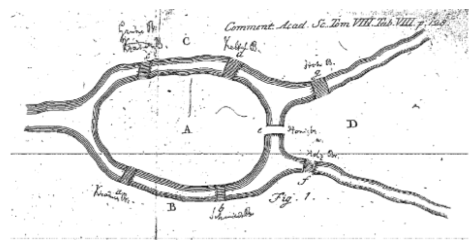

Figure 2.2: *Euler's Map of Kongisberg. Reproduced from [1].*

the bridges of Konigsberg, constructed to model the map shown in figure 2.2. The circles are nodes, representing the landmasses of Konigsberg, and the lines are edges, representing the bridges connecting them. Euler was able to prove the non-existence of a path which utilises all bridges exactly once [17]. Graph theory has developed substantially since the 1700s, including the extension of graphs to include multiple layers, where each layer represents a different form of interaction and in the use of weights in the edges, as a means of defining the importance of the connection between the nodes [11].

When considering interactions between pairs of players where some pairs cannot interact directly, such as where they have not interacted before, or when considering the transport of goods, graph theory is an important and natural tool. Graph theory is a branch of mathematics used to model pairwise relations between objects [17, 18]. Network theory is a subset of graph theory which considers networks, where a network is a graph where the edges have attributes. This allows additional information to be encoded, increasing the complexity of the system being evaluated.

Formally, a graph G is defined as a set of nodes V and edges E. The nodes within V are indexed uniquely as $v_i \in V$. Depending on the context, edges are either given indices, with an edge $e_i \in E$, or defined by the pair of nodes they connect. In the second context, the edge which connects $v_i \in V$ and $v_j \in V$ is defined as e_{ij} [11]. An example graph with seven nodes and nine edges can be seen in figure 2.3.

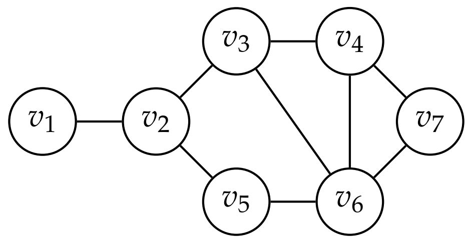

Figure 2.3: *Example of a seven node graph with nine edges.*

When considering complex systems modelled by networks, the attributes of an edge can be specific and dependent on the context. One example of this is networks where nodes are modelled as players and edges as pairwise games played between them [19]. Here the edge's attributes may represent the type of game being played, or the current state of the game. However, in a transport network, where edges are potential routes and nodes are hubs connecting them [20], the edge attributes may instead represent length, capacity or the current flow value upon that edge.

Multilayer networks also exist and can be used to examine complex systems where multiple types of interaction occur simultaneously. Some criticisms of multilayer networks with high layer multiplicity have been offered such as by [21]. The authors gives results on the reducibility of multilayer networks through the encoding of additional information into the edges in a lesser number of layers, or, in highly multilayer cases drawn through data-gathering, through the merging of layers. It is found that this can result in a significant reduction in the number of layers by as much as 75%, but the advantages of multilayer networks are not contested.

Alternative Definition of a Network

One linguistic overlap in the literature is in the definition of a network, with the second being described as follows. Here, a network is not a structure of nodes and edges, but instead relates to the idea of interconnected goods with positive or negative externalities [22]. Classical examples of network externalities include the negative externality for each of nuts and bolts, which are useless individually, but useful together. A second example is of the adoption of a particular technology standard, which has positive externality as the more it is adopted, the greater the benefit for doing so. This definition of a network is not used in this thesis. Noteworthy papers, including [22], give results on Cournot competition using this definition of a network, with Cournot competition formally introduced in section 2.2.2. This is followed by offshoots such as [23] which uses the same definition.

2.1.1 Networks in Economics

The use of networks in economic theory occurs widely, with noteworthy papers such as [24] which implements a bipartite network which connects countries to the products they export. They argue that this network approach allows them to obtain more accurate assessments of the potential for economic development. The authors' model provides support for approaches to development, focussing on developing economic complexity through investing in new products and capabilities. They assert that this would create incentives for the further development of new capabilities and products, providing analytical support for the theories proposed by Hirschman [25].

Another example of the use of networks can be seen in [26], which examines the spread of information about economic opportunities using a network. The authors obtain an improvement in their understanding through the use of network dynamics when modelling and controlling for information spread, as opposed to just examining geographical adjacency in economic decision making. This highlights the importance of networks in understanding economic choices. Many different approaches are taken in the implementation of network economics. One of these is used by Bramoullé and Kranton [27], who analyse 'public good' games played on a network, giving the example of planting a garden which all can benefit from. Another is used by Jackson [28], who discusses the impact of networked markets on seller opportunities. Here, sellers require network adjacency in order to be able to sell in a market.

2.2 Game Theory

Game theory provides the second key pillar of this thesis. It provides a tool for evaluating the interaction of multiple agents (players) attempting to achieve their objectives, for which they receive utility, in an environment where the actions of each player affect the others. Historically, early works in game theory include work in 1944 by John von Neumann and collaborators [29]. This was then developed by many, most notably by John Nash [30]. The utility received can represent anything, but a common choice given game theory's role in economic literature is money (though not always with a linear relationship [31]). A normal form game is one where players make simultaneous decisions, as opposed to extensive form, where decisions are made sequentially [12].

2.2.1 Important Games

One of the most well known types of game are single round 2-player games with symmetric utility functions. One example of this is the 'stag hunt', a game which models two hunters deciding, without prior consultation, what form of game to seek [12].

		Player 2	
		Stag	Rabbit
Player 1	Stag	$(3,3)$	$(0,2)$
	Rabbit	$(2,0)$	$(1,1)$

Figure 2.4: *The utility matrix for the Stag Hunt game*

Within the stag hunt, players obtain resources (food) according to the matrix shown in figure 2.4, with player 1 receiving the left value in each case, and player two receiving the right value. This means when player 1 hunts a rabbit and player 2 hunts a stag, player 1 receives a payoff of 2 and player 2 receives a payoff of 0. In this game, the players can catch a stag, but only if they cooperate, while hunting for rabbits provides a more consistent but less beneficial choice.

The Prisoner's Dilemma is another classical example. It poses a hypothetical case of two criminals, caught together. The existing evidence against each is enough for a small prison sentence, but if they give further evidence against the other, they receive a reduction in their own sentence while the other player is convicted for longer. Figure 2.5 shows the utility matrix game, with value's drawn from Poundstone's [32] implementation. The key characteristic of this game is that players receive greater personal utility for testifying than for staying silent regardless of the opponent's choice, but prefer the other player to remain silent regardless of their own.

		Player 2	
		Stay Silent	Testify
Player 1	Stay Silent	$(-1,-1)$	$(-3,0)$
	Testify	$(0,-3)$	$(-2,-2)$

Figure 2.5: *The utility matrix for the Prisoner's Dilemma game*

Through these two games, we introduce the concepts of Nash Equilibria, Pareto Optimality and Welfare optimisation. A Nash Equilibrium is an outcome such that no player receives greater utility through changing their strategy unilaterally. (Testify, Testify), (Stag, Stag) and (Rabbit, Rabbit) are all Nash Equilibria. Pareto Optimality represents an outcome where no player's utility can be improved without reducing the utility of another player. The Pareto Frontier is the set of all Pareto optimal outcomes. In the stag hunt, (Stag,Stag) is the only Pareto optimal outcome. In the prisoner's dilemma, all strategies except (testify,testify) are Pareto optimal, but the only move which represents Pareto improvement is from (testify,testify) to (stay silent,stay silent). The welfare of a system refers to the total utility received by all players. The welfare-maximising outcomes are therefore (Stay Silent, Stay Silent) and (Stag, Stag). A differentiation between pure strategies and mixed strategies is also made. A pure strategy represents always choosing a specific course of action, while a mixed strategy randomly selects from a number of strategies with selected probabilities.

We also utilise the existing concepts of the Price of Anarchy (PoA) and Price of Stability (PoS). The PoA refers to the potential lost utility from the players' selfish behaviour, comparing the worst Nash Equilibrium outcome to the best possible outcome. The PoS gives a slightly different value from comparing the best Nash Equilibrium to the best outcome.

A normal form game (also known as a strategic form game) is a mathematical representation of a game in which players make decisions simultaneously and independently, each aiming to maximise their own payoff. It has three elements. The set of players, the pure strategy space for each player and utility (or payoff) functions that give each player von Neumann-Morgenstern utility for each possible strategy combination by all players [12]. The utility is the value received by the players, in figure 2.4, this is the value defined by the matrix.

A repeated game is a type of extensive form game where a normal form game is played repeatedly by the same group of players. Formal analysis has been conducted of sample extensive form games such as the prisoner's dilemma such as in [33], characterising the comparative quality of equilibrium points as well as referring to the concept of impatience which the authors refer to as a discount rate.

Fudenberg [34] gives further results on the existence of Nash Equilibria in repeated games, discussing the folk theorem, so called because it was widely known in the 1950s before it was published. The theorem suggests that, as the patience (as represented by the patience variable β) of the players increases to a limiting value of one, almost any average utility can be found from rational play in a Subgame Perfect Equilibrium (SPE). This means that, where players attempt only to maximise *their* utility, and where threats to punish undesirable behaviour are not credible unless the punishment is independently rational for the threatening player, there exists a function from β to most average utility values for the game. The folk theorem therefore emphasises the importance of an appropriately selected β characteristic, as its manipulation can allow results to be chosen according to an author's preference.

2.2.2 Modelling Economic Systems

Game theory has a rich literature of framing economic and social interactions in order to provide an analytical and predictive tool for the understanding of real world dynamics. Methods have been developed to create games with varied dynamics.

The diverse nature of player types involved in the complex systems of trade results in multiple analytical tools being appropriate. This is dependent on the specific dynamics of the game each of these players experiences. When considering economic competition in an oligopolic market however, the key types of competition are Cournot competition, Bertrand competition and Stackelberg competition, each named after the game theorists who introduced them, Antoine Cournot, Joseph Bertrand and Freiherr von Stackelberg respectively.

Cournot competition involves a homogeneous product, no collusion between firms, each firm having market power, a fixed number of firms, rational and strategic behaviour by each firm and in particular, strategic choices being with respect to quantities rather than prices [35]. Cournot competition is seen where production quantities must be committed-to before reaping a profit based on the production of all players. Cournot competition occurs widely, for goods such as oil [14, 36] and grain [36, 37]. A player's strategy in a Cournot competition is how much of a good to produce.

Bertrand competition involves a homogeneous product, a fixed number of firms, sufficient capacity to supply the entire market, symmetric marginal cost for each player, simultaneous action and in particular, competition with respect to prices rather than quantities [38]. Examples of Bertrand competition can be found in selecting between different commodities such as gas, oil and coal for energy production, in the competition between consumer goods such as between Coke and Pepsi [39] and in the selection of a petrol station to fill up a car (assuming perfect information). A player's strategy in Bertrand competition is the price at which they should they sell a good.

Stackelberg competition occurs when players choose their actions sequentially, rather than simultaneously, over a single round [40]. Each player knows the order of action and they choose to act in order to optimise their utility, with the follower being able to select an optimal response to the chosen set of strategies of all other players. The leader must therefore play a strategy that optimises their outcome after all other players have attempted to exploit the situation. Examples occur where a market leader selects a price and specification, such as of a new smartphone, and their opponents respond by selecting their own price and specification, already knowing the strategy of the market leader.

These games are sometimes combined, such as by Brander and Spencer [41], who provide an analysis into a Cournot duopoly with Governmental influence. Here, the governments acting as Stackelberg first-movers. The governments act first, attempting to maximise their utility under all possible strategies of the firms, followed by the firms making a joint best response. The authors find that action by a government changes the game, such as through export subsidies. This can result in benefits to firms, with the subsidised firm receiving an increase in domestic welfare despite the terms of trade moving against the implementing government. While the strategies at play are those of macroeconomic strategy for the governments (such as placing subsidies or tariffs) and production quantity for the firms, the governments find themselves in a prisoner's dilemma, preferring to cooperate to reduce costs, but being incentivised to defect to increase their trade profit. The positive relationship between increased competitiveness in other markets and profit is replicated in Chapter 4, where increased competitiveness in a market through reduced transporation costs results in a utility increase in that market. Whilst in [41] this is achieved through governmental subsidies, and here it is achieved through increased market access, there is a clear parallel between the results.

Analysing the outcomes of these games can be carried out using four methods. The first method attempts to simulate real world conditions by introducing volunteer players to the game and the competition format, then having them play for a real world payment based on their score. An example of this was conducted in [42], where an analysis into the outcomes in Bertrand competition with asymmetric costs was conducted. This research confirmed the analytic predictions, that the expected outcome is the Nash Equilibrium outcome, where the most efficient player sets a sale price that is just below the marginal cost of the next most efficient player, capturing the market and generating a profit.

The second method conducts pure mathematical analysis on the games, calculating expected outcomes based only on game theoretic first principles. This can be seen in [43], where the outcomes of Bertrand competition with asymmetric costs is analysed, finding results on the myopic stable set, a concept introduced in [44] which parallels the Nash Equilibrium for social environments. This method is desirable due to its mathematical rigour, but can lose connection with real-world applications due to the theoretical approach.

The third method relies on a computational approach. This allows an approximation of the analytical solution in cases where it can not be easily derived. It is also cheaper than experiments with volunteer players which requires incentivising rational play with real world payments such as in [45].

The fourth and final method involves comparing the results to those derived from real-world data. This allows for a greater understanding of the relevance of the model to real-world dynamics. It gives evidence of whether or not the model appropriately represents the true characteristics of the basis case, or whether analysis would be inaccurate due to insufficient, inappropriate or excessive complexity.

A Comparison of Cournot and Bertrand Competition

A comparison of Cournot and Bertrand methods for the analysis of goods is widespread within the literature, due to the clear importance of the accuracy of models to the problem. Players sign contracts to engage in either Cournot or Bertrand competition, committing to compete in one form and preferring the one which is more profitable. Singh and Vives [46] conduct an analysis of a system with quadratic production costs (such that producing high quantities becomes extremely expensive). They build on a model of a differentiated duopoly proposed by Dixit [47] where goods can be either substitutable or complementary. They find that firms prefer to choose contracts which allow them to engage in Cournot competition in the case of substitutable products as this maximises their profit.

Choi and Lim [48] conduct further analysis into the preferences of firms between engaging in Cournot competition or Bertrand competition, and also into optimal response to each of these cases. They find that firms prefer to engage in Cournot competition when goods are complements, this is where buying one firms' good increases the appeal of the other firms' good. They also find that, whatever the competition type, optimal trade policy is to subsidize exports under Cournot competition and to tax exports under Bertrand competition. Later [49], the same authors explore firms' preferred competition types. They find a prisoner's dilemma scenario where, though firms would prefer Cournot competition, their strategies are dominated by Bertrand competition strategies, leading to moves away from the Pareto frontier. They further find that market liberalisation can move the optimal competition type from Bertrand to Cournot, and result in a net improvement in utility received by the players. Their results are supported by other authors, such as Eaton and Grossman [50], who confirm their findings that subsidies are optimal when considering Cournot competition, while taxes are optimal for Bertrand competition. They further confirm the idea that free trade is optimal.

Other authors such as Basak and Wang [51] find the opposite, giving results that while the social welfare, a measure of the total utility received by all players, is the same for either Bertrand or Cournot competition, that Bertrand competition is superior for firms where the goods have substitutability, parametrised herein by $\gamma \in [0, 1]$. This means that Bertrand contracts are superior where the goods are at any degree of substitutability except either totally isolated ($\gamma = 0$) or perfect substitutes ($\gamma = 1$).

More recent results [52] argue that mixed Cournot-Bertrand competition is the outcome of strategic trade policy, discussing the ideas of a Stackelberg competition with firms i and j, where a Cournot-Bertrand game involves firm i engaging in Cournot competition and firm j in Bertrand competition, and Bertrand-Cournot the opposite. They find that Cournot and Cournot-Bertrand competition are equivalent in terms of welfare, that a subsidy is optimal in either case, while in Bertrand, or Bertrand-Cournot competition, a tax is optimal.

2.2.3 Cournot Competition

Literature focussing on networked Cournot markets has been around for many years [53, 54, 55], though much of the application focus has been on networked electricity markets alongside more general literature on the topic of networked markets in oligopoly games [56, 57]. This extends to work which also includes substitutability of goods from firms offering multiple goods [58, 59, 60]. Bimpikis *et al.* [61, 62] implement a multimarket oligopoly analytically, developing a model from the basis which appears in [63]. The authors examine a bipartite network of sellers and markets. They give results about the formation of equilibria and the process by which players assess the expansion into additional markets. They also identify areas for future research, specifically in focussing the analytical basis on real world problems. They identify capacity constraints as one area with potential for future research. This is an area that this thesis explores in more detail in Chapter 5.

Other works have similarities to those outlined by Bimpikis *et al.*, such as Alsabah *et al.* [64] who provide an analysis of a linked oligopoly-transport system, finding counterintuitive results, such as the reduction in transport costs can result in a decrease in profits for all firms if the firm was capacity constrained.

In transport theory, the Wardrop equilibrium [65] refers to a state under which no driver would benefit from marginal change to their route selection. Similarities between the Nash equilibrium with Cournot competition and the Wardrop equilibria, are also found [66], finding that multimarket Cournot oligopoly games have analogous properties to the transportation properties in a selfish routing game [67]. This is due to the similarities between the congestion functions from transport and the increasing costs and reducing profits associated with producing more and selling more to a single market in a Cournot oligopoly game. This link suggests that joint analysis is relevant and provides opportunities for a later reduction in the complexity of the game.

More complicated functions for production and congestion in the area of Cournot competition over bipartite networks are also utilised. Here, [68] uses quadratic production costs and gives results about the possibility of convex optimisation techniques as a method for finding equilibria. Abolhassani *et al.* [68] uses natural gas transportation as an example application of their work [69, 70].

Harker *et al.* [71] examines a networked Cournot model, and additionally includes some routing elements, with the construction of networks with intermediate nodes requiring transshipment. Further work on the differentiability of solutions also occurs in [72, 73]. Differentiability here is a characteristic which indicates that that is no non-continuous behaviour from the associated functions.

Roux *et al.* [74] looks into historical data on domestic and export markets for goods within a Cournot oligopoly game. They find that their modelling does not sufficiently explain their findings that the data shows a bias towards purchasing domestic products. This occurs even when considering trade costs. Typical trade costs are in the order of 5-10% for high income countries, while assuming that transport costs are the only factor, they demonstrate that trade costs would need to be 91%. One possible explanation they explore is of collusive factors increasing the bias towards sales in the domestic market, and they hypothesise that intervention would be welfare-improving if this is the case.

The relevance of optimal transport methodology to Cournot problems is well documented analytically. Blanchet *et al.* [75] gives results on the ability to obtain Cournot-Nash equilibria through the minimisation of costs related to optimal transport.

Networked oligopoly games are not limited to Cournot, and there has been basis-specific work done on Bertrand competition [68] and Stackelberg competition [76]. Motalleb *et al.* [76] extend a model of energy distribution in networked markets by including a secondary consideration of trading stored energy in multiple markets.

Competition with information uncertainty adds an additional level of complexity to games, and when these occur on a network, this can be represented by an additional communication layer through which players improve their understanding about other player's strategies [77].

From precedents discussed in this section [61, 62, 71], the choice was made to use a networked Cournot model to represent the trade elements which appear in Chapters 4 and 5. Cournot strategy choice most closely parallels the decisions being made by real-world decision makers and, while additional complexity [41] could increase the model's accuracy, the clarity of the pure Cournot model is desirable.

2.2.4 Network Games

Games of trade, supply and demand are limited in their ability to capture long term dynamics, a key example of which is infrastructure creation and optimisation, such as in traffic allocation for server farms [78]. A consistent assumption made throughout this thesis and supported in the literature [20, 78, 79] is of the applicability of networks as a proxy for infrastructure. This is a largely basis-agnostic model which can be applied to multiple transport scales, such as rail connections or pipeline availability.

There has been research into the nature of equilibria in a network game where the graph's nodes are players. Galeotti *et al.* [19] provide a contribution to the literature in the area of network games, focussing on a form of games where players appear as nodes on a network, with the edges between that player and other players acting to modify their utility. Each player's utility function is based on their own strategies and of those of each of their adjacent players. This results in feedback from nodes beyond their direct adjacency. The particular contribution is in the area of incomplete information, with players only having local knowledge of the network. They then extend this to a game where players have a deeper knowledge of the network, examining the impact of increasing the radius of knowledge, first from their own degree to that of themselves and each of their neighbours.

Fabrikant *et al.* [80] introduce a network creation game where players appear as nodes on the graph, but such that the players' strategies are dependent on which edges they choose to place and maintain. The design intention of this game is to mimic the creation of internet-like networks, though the focus of the paper is strictly analytical. The authors give results about bounds on edge creation cost which yield trivial results, as well as suggesting a number of weaknesses, such as the regular existence of a 'central' node without consideration of congestion in this node. This form of game provides a basis for the work which appears in Chapter 3. A game including the effects congestion is then outlined in [81] which gives contributions in area, addressing pure strategy Nash Equilibria in congestion games. The paper includes contributions in the area of the computational difficulty of the problem of identifying these pure-strategy Nash Equilibria. Anshelevich [82] further examines network formation games, introducing elements of fair allocation following ideas outlined in [13], contextualising them alongside the price of stability. Corbo [83] explicitly extends the work in [80], both through the extension of Fabrikant's network creation game as well as through an analysis of the price of anarchy in this game. They refer to the form of the game introduced in [80] as a unilateral connection game, and introduce the bilateral connection game where both players are responsible for links between them. They find that in the bilateral connection game, the average price of anarchy is better than in the unilateral connection game, but that the worst-case scenario is worse in the bilateral connection game than in the unilateral game. Albers *et al.* [84] disproves a key hypothesis outlined in [80] which hypothesised that there existed a value A for which all link-creation costs $\alpha > A$ resulted in a tree graph. Albers *et al.* also provides an improvement on the upper bound on the price of anarchy found in [80]. Albers *et al* finally briefly introduce a more general form of cost sharing in edge creation than that which appears in [83], improving on previous results on both the price of anarchy, as well as demonstrating the failure of the tree conjecture posed by [80] in this case as well.

Shahrivar and Sundaram [85] develops further on the idea of a network formation game, extending to a multilayer format. Like Fabrikant *et al.* [80], they focus on a utility metric based on the distance between nodes, but introduce the idea of connections between some pairs of nodes being more important than others, as characterised by one layer of the multiplex network, with edges defined by E_1. The aim is then to form a network in the second layer with edges in E_2 which minimise the distance between nodes which are connected in E_1. They give results on the relationship between the reward function and the cost of placing an edge. These result in specific network structures being formed as a best response in the second layer. The authors also give results about the computational cost of finding these best responses, finding that the problem is NP-hard, and thus computational approaches are not appropriate in high complexity networks.

Jackson and Wolinsky [86] provide key contributions in a game where players in a social and economic network game form and break edges due to self-interest. They characterise networks that are stable under rational player action, and those which are efficient, and find that there does not always exist a stable network which is efficient.

2.3 Games as Dynamic Programming

In general within game theory, players are considered to make a myopic 'greedy' response, improving their utility as much as possible considering only the immediate outcome. When extending the time horizon over which players optimise from myopic to more longer term, an understanding of the utility the player will receive in the future is required. In order to achieve this, a method known as dynamic programming is used [87].

Dynamic programming breaks down a larger problem into a series of smaller sub-problems. This allows a value equation to be generated for each of the smaller sub problems. Dynamic programming was developed by Richard Bellman and the Bellman equation [88] is an important concept within it, giving the value of a course of action. This value can then be used to represent the optimised path through this sub-problem when being considered as part of a larger problem, reducing the complexity. This has an application in both network routing, a major focus of Chapter 4, and non-myopic strategy selection within game theory. By evaluating the expected utility of a strategy selection at a later stage, the total expected utility of a strategy at the current stage can be evaluated. This can include consideration of an impatience function, indicating how much value is placed on immediate outcomes compared to long term ones, with β as the variable associated with impatience.

A difference equation is a form of discrete differential equation [89]. Difference equations are appropriate for calculating total utility received in infinite games with reducing costs, such as in the Bellman equation. Where there is a predictable convergence to consensus (or a impatience characteristic with a uniform reduction relative to the previous stage), difference equations allow a solution to be calculated. This is used in Chapter 4 to find the utility generated over an extended time horizon.

2.4 Coalitional Games

Coalitional game theory is an area of game theory which focusses on analysing the impact of cooperation on the outcomes of a game. In a coalitional game, players can join a coalition wherein all players act to improve the utility of the coalition as a whole. This takes different forms depending on the characteristic of the game [90].

The Shapley Value [91] is a measure developed by Lloyd Shapley which characterises the importance of each member of a coalition. This is based on the additional value that each member adds by joining each possible subset of the coalition that does not include it. This is widely used and taught as a fundamental value within cooperative literature [12].

Transferrable Utility

Where the game involves transferrable utility [13], such as where utility represents money, the players act to maximise the utility of the entirety of the coalition, then share this increased utility between the players.

Non-Transferrable Utility

Where the game involves non-transferrable utility [92], such as where utility represents prison time, the players act to move towards a new position where all players receive a greater or equal utility than they had previously received, in a form of Pareto improvement.

Within modern literature, Aumann [92] provides a defence of the Non-Transferrable Utility (NTU) value introduced by Shapley against criticism levelled by Roth and Shafer [93]. Roth and Shafer introduce a number of examples which result in NTU values which they consider unintuitive and accordingly a sign that the NTU should be replaced. Aumann [92] argues these non-intuitive examples are less concrete than stated as the alternative solutions are weaker than Roth and Shafer suggest.

Coalitional Importance

Results on coalitions are visible through the glove game [94]. This is a coalitional game where coalitions receive utility only for complete pairs of gloves formed. Players with a rarer glove type have greater value to the coalition than others, resulting in them receiving a greater share of the coalition's utility. A coalition receives 1 unit of utility collectively for each completed pair of gloves and 0 for any unused gloves. Belau [94] compares the Shapley value to component restricted Shapley value [95] and the Owens value [96], giving efficient formulae for the calculation of the component restricted Shapley value and the Owens value in the glove game.

The importance of control elements in maintaining cooperation between parties who have an incentive to defect is important and can have impacts on increasing the welfare of all players. One example of this can be seen in a context where the control elements are given by a central body taxing a population of agents engaging in prisoner dilemma style interactions [97]. The authors of [97] identify the required magnitude of control to establish an initial beneficial equilibrium. They also identify that there is a reduction in the ongoing cost of control elements after some time has passed.

The incentives towards cooperation are not limited to cases where control elements are implemented. Gama *et al.* [98] analyses the impact of the entry of new firms into an oligopoly game and similarly, the impact of mergers which serve as the inverse of the entry of new firms. We are particularly interested in mergers because they are analogous to the formation of coalitions. They, in particular, discuss the 80% rule [99] which identifies that mergers are unprofitable unless there is a high degree of collusion already occurring (greater than the eponymous 80%). The authors also investigate welfare in the system, finding that mergers improve welfare. Their estimation of welfare involves evaluating the total profit made by the collection of firms, so this increase in welfare corresponds to a decrease in competition and an increase in profits. This means that companies are incentivised to merge, but doing so will have a negative impact on the outcomes for agents purchasing their goods.

The idea of coalitional behaviour and the relative power of coalitions within Cournot competition is explored in more detail by Zhang *et al.* [100]. This discusses coalitions and the competitiveness in an oligopoly. Their key contributions are on the equilibrium among competiting coalitions, the social welfare of various outcomes, as well as connecting their model to urban parking allocation and electricity markets. They also characterise the coalition size which results in a reduction in the efficiency losses for the firms involved.

Ciardiello [101, 102] also looks at cooperation within supply chains, but in contrast to the horizontal mergers looked at by Gama *et al.* in [98], they are specifically interested in vertical mergers. That is, they look at the impact of cooperation by players engaged in dissimilar activity. They find that the Shapley allocation (that is, the allocation of utility between participating members of the coalition according to their Shapley values) is a measure which satisfies many of the desirable properties for their model in environmental supply chains.

The development of cooperation when groups must contain members with different roles has been examined by Beal *et al.* [103] who refer to this difference in modality as diversity coalition games. The authors develop values for this game, such as the diversity-Shapley value, building on the existing Shapley value. They give examples of application primarily related to political voting structures, such as the requirements for constitutional reform in France, but their work can also be used to describe coalition formation for the transshipment of sanctioned goods, with the two player types being the sanctioned player and the potential intermediaries. The first provides a market for the goods, while the second provides access to the sanction sender's goods.

2.4.1 Real World Cooperative Dynamics

Direct analysis of a number of specific real world cases as games, looking in particular at the characteristics of cooperation, have been conducted. These case studies inform the choice of model and are the basis of the research presented in Chapters 4 and 5.

Small network models with multiple player characteristics are common, such as by Zheng *et al.* [104, 105] who provide results on a model with vertical cooperation wherein agents do not have symmetric strategies. Their model includes port authorities, which receive utility based on maximising local welfare, and port operators, competing to maximise profit. They find that sharing information can help to maximise welfare, but that the competitive port operators require incentivisation to engage in cooperation, providing support for the social welfare benefit of intervention.

Studies on the pressure for change in the number of firms involved in an oligopoly is also relevant, with Zhang and Wang [106] giving contributions towards understanding the impact on social welfare of the introduction of new firms which are more or less efficient than the existing average. They find that when efficient firms join, industry profit is raised. However they find that network externalities play a significant factor, reducing consumer welfare where inefficient firms enter a market where increased good sales increase demand for the product.

The effectiveness of cooperation in the reduction of congestion costs has also been studied [107]. This paper introduces a model where firms can choose to cooperate to reduce transport costs, but still compete at the final destination on sale price. Their key contributions are that firms are not incentivised to engage in cooperation as, though the welfare of those consuming the goods is improved through the reduction in costs, the firms achieve the same profits. They find however, that where the price is regulated (or where the firms also cooperate on price to the detriment of consumers) this cooperation becomes incentivised. Jacob *et al.* [107]'s contributions clearly indicate the effectiveness of transport cooperation, but also give the warning that central control is required to manage this effectively as a problem in welfare maximisation.

There are also contributions to the literature with a less analytical approach, describing real world cooperative dynamics with price competition, such as [14]. This examines the degree of cooperation which occurs in OPEC, an organisation which arranges cooperation in price setting between oil-producing countries to maximise their profit. This paper finds that the real world level of cooperation does not result in the exact outcome that would be expected if OPEC attempted to exploit its coalition to the maximum extent, and further, that the interior allocations of sales within the coalition are not exactly those that would be expected within a 'perfect' cartel, with smaller members receiving more favourable conditions. They contribute a model which allows the studying of the degree of cartelisation, by which they mean the extent to which a coalition truly engages in perfect collusion, in line with the concept of a coalition. The existence of intermediate states of cooperation between complete competition and complete cooperation is intuitive, but this intermediate state reflects a lesser degree of advantage from coalition caused by a lack of trust between players. This is identified as being due to the imperfect enforcement mechanisms in the coalition.

Results on supply chains also inform the transshipment modelling in Chapter 5, such as those by Nagurney [108, 109], which gives results on supply chains, introducing a model where players compete in production, storage and distribution. The authors' results are theoretical and are open to a future baselining against the constants of a particular industry.

2.4.2 The representation of Coalition on a Network

The formal representation of coalitions on a network will be shown to be important in Chapter 5 as methods for the transhipment of goods in order to bypass sanctions.

When representing a coalition, one approach is through a partition of nodes where joint membership of a single partition represents joint membership of a coalition. However, in some scenarios, it is appropriate for cooperation to exist in multiple formats, with players a member of multiple coalitions simultaneously. One such example is when multiple methods of bypassing sanctions are used. Here, the use of hypergraphs can be appropriate [110]. Hypergraphs are a generalisation of a graph where edges are not pairwise, but can be connected to any number of vertices.

Formally, a hypergraph $\mathcal{H}(\mathcal{N}, \mathcal{L})$ is a mathematical object that consists of a set of N nodes $\mathcal{N} = \{n_1 = 1, \dots, n_N = N\}$ and a set of L hyperlinks $\mathcal{L} = \{l_1, \dots, l_L\}$ [110, 111]. Each hyperlink is a subset of $\mathcal{N}$ representing the nodes connected by that hyperedge. This offers a way to model the interactions of players engaging in different coalitions simultaneously. Alvarez-Rodriguez *et al.* [110] use this approach to evaluate the development of cooperation on a network.

A different approach is taken by Goyal and Joshi [16] who examine pairwise edges on a network. Here an edge represents an agreement between a pair of players to cooperate to reduce production costs. They find that, counter-intuitively, stable networks are often assymetric, giving as examples, star networks, interlinked star networks and dominant group networks, as seen in figure 2.6. There are clear links between their findings and the more theoretical work [80] which appears in section 2.2.4.

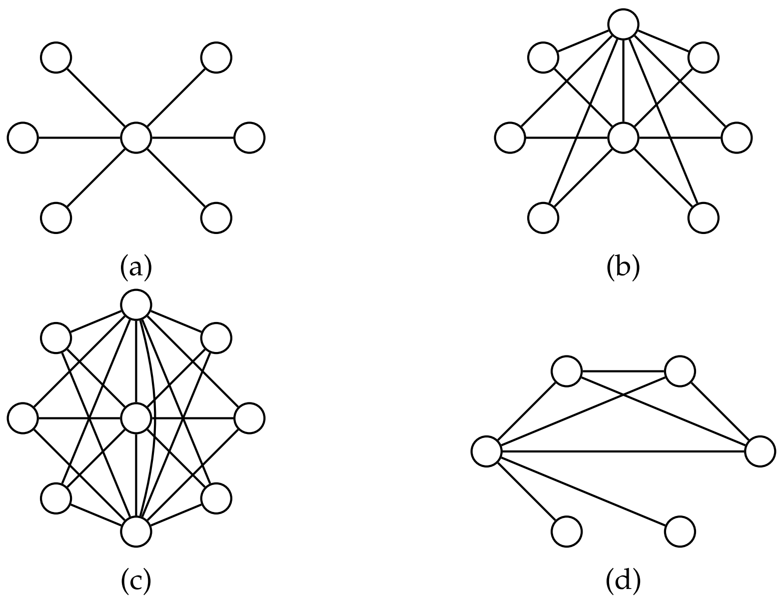

Figure 2.6: *Assymetric networks that are introduced by Goyal and Joshi as stable networks in an edge creation game. (a) A star network with a single node as its centre. (b) Interlinked star networks with two centre nodes. (c) Interlinked star networks with three centre nodes. (d) Four interlinked asymmetrical-sized star networks.*

The network creation game is developed on slightly by Sarkhel *et al.* [112] who further characterise network stability under different edge construction costs, finding bounds on the values of edge creation cost which result in complete and empty networks. This pairwise coalition formation structure was chosen for use in coalitions in Chapter 5.

2.4.3 Network Consensus

Network consensus formation is relevant as a model for the spread of information across a network. The spread of opinion across the network has a parallel in the spread of multiple limited understandings of the nature of the overall network. An example of this is of bilayer graph.

The upper layer has a state, such as the current amount of traffic on the system. The lower layer contains a communication network, with understanding of the upper layer diffusing through the lower layer following consensus formation dynamics.

Young [113] gives contributions in the area of consensus on various different graph topographies, including the complete graph, cycle graphs and path graphs, finding that higher degrees of interconnectivity (such as in the complete graph) increase the rate of convergence to consensus. This can be extended to a multilayer network [114], finding that the symmetries in the multilayer network results in partitions of nodes in each layer such that the members of each cluster within the coalition achieve consensus.

Wang *et al.* [115] also give results on the evolution of cooperation in a multilayer environment, but are focussed more on the time taken to reach consensus. They find that where there are similarities in degree between layers consensus is arrived at more swiftly.

Other authors look at consensus formation in more dynamic networks [116], such as that of multi-agent systems. They find that networks with lower network diameter, for example the small-world network, lead to faster convergence to consensus.

Network consensus approaches are suggested as an providing additional complexity and insight, in particular to the model which appears in Chapter 4.

2.5 Sanctions

Sanctions are a form of economic action typically employed within a diplomatic strategy of dis-incentivisation or punishment for actions that a state considers undesireable. They act to reduce the amount of a trade good or goods that are available to the target country. They are discussed here due to their place in Chapter 5 where the model developed through Chapters 3 and 4 is extended to analyse the effectiveness of sanctions.

Historically, sanctions have been widely imposed, and work has been done into classifying these by Hufbauer *et al.* [117], by characterising the sanction senders S, targets T, duration of the sanctions and the goal the sender was attempting to achieve. The dataset presented by the authors gives historical examples of sanctions both achieving their goal and failing, with examples such as US sanctions on the German Empire from 1914-1918 resulting in a military victory, or sanctions by the UN on Italy from 1935-1936 aimed at incentivising a withdrawal of Italian troops from Abyssinia [117].

Sanctions have been studied in more specific depth as well, including an analysis of the impact of sanctions on the international trading behaviour of firms. Crozet [118] gives statistical analysis with regard to sanctions, characterising the impact of sanctions through an analysis of the number of firms which are trading between the two countries at the time. There is an associated weakness in that the quantity of business being done by these firms is not included, but significant results are obtained regardless. The authors find clear evidence that the imposition or removal of sanctions has an immediate effect on the number of firms trading.

Further research finds that the effectiveness of sanctions is dependent on the level of commitment and conflict expectations [119]. They also find that the effect of sanctions is greater where they are multilateral (multiple senders) than where they are unilateral.

The Global Sanctions Database [120] [121] is a more recent project than Hufbauer *et al.*'s, providing a history of sanctions that is more amenable to statistical analysis. They categorise the types of good which were subject to sanction, the objective type and whether the sanctions were successful. They find that the number of sanctions has increased and at the point of publication, the number of active sanctions was at its peak [122]. They provide an explicit evaluation of the effectiveness of sanctions on Russia in 2014. They also provide an analysis of the heterogeneous effects of unilateral and multilateral sanctions on trade, which has analytic overlaps with work in [123]. A related result is found by Peksen and Jeong [124] who note that when sanctions are imposed on a wealthy regime, one with a less democratic government or with a high trade dependence on the sender, sanction reciprocity is more likely. Russia is found to follow these characteristics closely, lining-up with their real world imposition of counter-sanctions in 2014.

Other authors have found that sanctions are less effective than had previously been believed, with just 38% of cases successfully influencing target behaviour [125]. This contributes to a case study presented in Chapter 5, which looks to find reasons why sanctions by the West on Russia in 2022 were much less effective than expected. The authors propose that this is a result of the amalgamation of aid suspension statistics with direct sanctions. They find that, when examining aid suspensions and sanctions, sanctions are even less effective, with as much as 44% of aid suspensions proving effective in influencing target behaviour in comparison to just 26% of purely economic sanctions.

Employing sanctions in the most effective matter is rational for the party imposing the sanctions. Various approaches have been made for generating a robust model of sanctions effectiveness through network theory.

Eaton and Engers [126] give fundamental results in sanction effectiveness, finding that the sanctions become more uncompromising as the sender's costs decrease or as the sender's patience increases. In particular, they find that sanctions are most likely to be effective when the target's benefits from trade are high and the sender's are small, increasing the damage done to the target by the sanctions while minimising the costs to the sender.

Other papers [123, 127] focus instead on the network structure which results in sanctions having the most significant effect. This is relevant for goods where there are limited transport opportunities such as natural gas.

Finally, it is relevant to highlight case studies into sanction impact on a single nation, such as the US economic sanctions against Iran [128]. The author finds that US sanctions against Iran imposed in 2010 were less effective than initially anticipated, citing as one factor the negative perception of some international parties, who then provided opportunities to bypass the sanctions. A second case study exists in [129], discussing the effectiveness of the sanctions on Russia in response to the invasion of Ukraine in 2022 in a foreign policy research context. They find that the sanctions have been effective due to their effect as an international deterrence and as a symbol, despite their failure in achieving the stated goal of ending the invasion. However, they do not provide a specific analysis onto the relative impact economically.

2.5.1 Third Parties' Impact on Sanction Effectiveness

The literature clearly indicates that the impact of sanctions is affected not just by the relationship and characteristics of trade between the sanction sender and the sanction target, but also by independent third parties. This is important to the work presented in Chapter 5 in analysing the effectiveness of sanctions with transshipment, which can be modelled in this way. A number of methods for implementing third party interference on sanctions have been considered in order to identify the ones that most accurately model the system.

Early significant works in this field come from Basu [130], whose focus is not on sanctions specifically, but on the impact on influence and power of the extension from the 'dyad' (a pairwise model) to the 'triad' (where a third party also influences the outcome), and more generally, multiple relations. Basu proposes models which are the basis for many later publications and argues that the results demonstrate that abandoning dyadic assumptions can give improved explanations for observed behaviour in games which otherwise are only explainable by abandoning other more fundamental assumptions such as the rationality of the players.

One of the most important characteristics of sanctions is the relative proportions of trade that go to the sanction sender and to third parties. Peksen and Peterson [131] give results on the likelihood of the imposition of sanctions, finding that when the potential sanction target becomes more dependent on the sender, the likelihood of sanctioning increases, but only under the condition that the ability to deflect trade to third parties is low. Their results emphasise the importance of considering third parties.

The degree to which states are prepared to suffer negative consequences for the imposition of sanctions is related to the effectiveness of sanctions, particularly in cases where the sanctions are imposed by a coalition of senders. The senders remaining committed to the sanctions, and the imposition of more strict sanctions initially, results in high sanction effectiveness. This is supported by studies into the importance issue salience (the importance to the sanction sender of the reason that the sanctions are imposed) which are found to have an impact on the effectiveness of sanctions. Ang and Peksen [132] find that sanctions are most likely to be effective when issue salience is high in the sender state and low in the target state.

Cranmer *et al.* [133] argue that it is appropriate to address sanctions as a network phenomena. The authors also give results on the rate at which the size of a sanctioning group grows, finding that states tend to join a small group in sanctioning a target, but the likelihood diminishes as the group grows. Accordingly, they find that small coalitions collectively imposing sanctions are more likely than a sole sender, while large groups are relatively unlikely.

Chupilkin *et al.* [134] give evidence of the existence of transshipment, which they refer to as intermediated trade, through the case-study of economic implications of the Russian invasion of Ukraine. Transshipment is a possible explanation for sanction ineffectiveness in that case, finding that members of the Eurasian Economic Union had seen significant increases in trade values. This disproportionately affected goods that had been sanctioned, allowing limited sanction circumvention.

Network Sanctions

A relevant analysis into the effect of network structure on sanction effectiveness has been conducted by Joshi *et al.* [123, 127], giving contributions particularly in the area of the size of an effective sanctioning coalition. They introduce two models, a short-term model where edges can only be removed from the trade network, and a long-term model where edges can also be added, allowing the target to compensate for their loss of access to the trade network caused by edge removal by the sanctioning coalition. The edge removal methods present in this model are used to provide the basis for the trilayer sanctions model in Chapter 5, influencing the trade layer.

2.5.2 Sanction Busting

Sanction busting refers to the reduction in the effectiveness of sanctions due to action by another party. Within a microeconomic[1] context, this can refer to a firm ignoring national laws prohibiting trading with sanctioned nations and choosing to trade to the sanction target anyway. Within macroeconomics however, this refers to action by a third party to deliberately reduce the effectiveness of the sanctions [5, 6], for economic benefit, to ingratiate itself with the sanction target or to infuriate the sender.

Studies have been conducted into the process of detecting sanction busting. Early *et al.* [135] find that sanction busting does occur, and also offer a comparison of the 'realist' and 'liberal' explanations for sanction busting. The realist explanation for sanction busting focusses on geopolitical benefits for the state from which these occur, such as the state political and security purposes, while the liberal explanation is related to profit-seeking behaviour. They find evidence that the realist explanation is less likely than the liberal explanation. This informs the model which appears in Chapter 5. This model uses market profit as the primary form of utility, tacitly subscribing to the liberal explanation of the reasons for sanction busting.

Studies have also been conducted into real-world cases of sanction busting. Gutmann *et al.* [136] examine from the realist perspective. They find no evidence of sanction busting by Russia, but ambiguous results for sanction busting by China. This helps to confirm the presence of this problem, which they analyse taking into account the effect of sanctions on trade volumes, using work by Felbermayr *et al.* [137]. The latter supports the assertion that sanction busting is occurring, and supports the idea that China is involved in reducing the effectiveness of sanctions

[1]Microeconomics is a branch of economic theory which focusses on the decision making of individuals and firms with profit-seeking incentives, whereas macroeconomics is a parallel branch focussing on the functioning of an economy as a whole, and is more appropriate for analysing state-level decision making [5, 6].

on Russia through transshipment. The impact of sanction busting behaviour and the opportunities of the sanction senders to prevent it are further analysed in Chapter 5.

2.6 Congested Network Transport

Transport theory provides important contributions to this thesis, and provides the dynamics that govern transport layers in the networks which appear in Chapters 4 and 5. Most relevant to this thesis is the study of the optimal transport problem, the idea of minimising time and costs of transport. The optimal transport problem was first popularised by Gaspard Monge [138]. A common motivating problem is that of moving goods from a set of mines to a set of factories and forming a transport plan which minimises the cost. In the original statement of the problem, where each mine provides the exact resources that can be processed by one factory, this is an assignment problem.

Further literature gives results about the nature of equilibria in a stable transport system. The key introduction was that of the Wardrop Equilibrium (WE) [65]. When a system is in Wardrop Equilibrium, no marginal unit of traffic (such as a driver or shipment) will receive a reduction in costs by switching to any other route.

Later work in transportation literature [139] has extended the problem initially outlined by Monge (known as the Monge-Kantorovich problem) by introducing congestion dynamics and finding WE.

Roughgarden and Tardos [140] give further results when a continuum of rational players is introduced. Here players optimise for personal travel time rather than being centrally assigned. They find bounds on the inefficiency of selfish routing. These results are then developed further by Roughgarden [67] in an algorithmic game theory context, improving on the bounds for selfish routing as well as introducing bounds on inefficiency in resource allocation games and Shapley network design games.

An important part of the appropriate modelling of transport environments is through the use of relevant congestion functions. These congestion functions are application dependent, but within highways literature, the Bureau of Public Roads (BPR) has put forward an equation for modelling congestion. This equation has adjustable variables specific to the road being analysed [9, 141], with other characteristics of congestion outlined by [142].

The BPR function is used by Zhang *et al.* [143] who introduce additional elements to improve their model's accuracy, including the current degradation coefficient of capacity and the density distribution function of road section saturation. This results in improvements over more simple models.

2.6.1 Analytical Considerations

In order to simplify analytical research, it is common to choose an abstraction. This reduces processing time, as well as maximises generality when considering congestion over many applications, rather than for a specific case. A choice that is commonly made is that of polynomial costs in optimal routing [144, 145]. A further simplification that is sometimes made is of an arbitrarily large player count, resulting in strategy profiles in continuous space rather than in discrete space.

Support for the multilayer approach to transport networks found in Chapters 4 and 5 is widespread, with the extension of transport beyond single layer analysis having become increasingly common both due to its utility in analysing multiple-modes of transport simultaneously, alongside the interaction between them. A standard assumption is that each layer of a transport graph represents a different mode of transport, giving results on the vulnerabilities and possible improvements in the multilayer structure from the analysis of multiple European cities [146]. Other studies focus on a single city, such as the work of Ibrahim *et al.* [147] who analyse of two-layer network of transport in the city of Bordeaux, with the layers representing public transport by bus and tram. The authors propose a model and highlight its ability to give predictions on the impact of service disruptions or network improvements.

Other analytical approaches have also been used in the analysis of multilayer network travel [148]. The authors analyse performance indicators for travel in multilayer networks, before outlining some open problems in multilayer transport analysis, including that of analysing networks with three or more layers and the importance of inter-layer interaction on the outcomes.

2.7 Key Thematic Areas of the Literature

Table 2.1 contains a summary of the principal literature areas in this thesis alongside a number of the key authors whose work is important to this thesis, either through fundamental concepts or through modern work upon which this thesis builds.

Literature Area	**Key Authors**
Game Theory	John Nash; Augustin Cournot; Joseph Bertrand; Heinrich von Stackelberg; Kostas Bimpikis; Adrien Blanchet; Alex Fabrikant; Susanne Albers
Network Theory	Leonhard Euler; Stefano Boccaletti; Andrea Galeotti; Riccardo Galotti; Claude Berge
Coalitional Games	Lloyd Shapley; Robert Aumann; Alvin Roth; Wayne Shafer; Guilliermo Owen; Sanjeev Goyal
Sanction Theory	Kaushik Basu; Sumit Joshi; Jonathon Eaton; Gary Hufbauer; Dursun Peksen
Transport Theory	Gaspard Monge; Leonid Kantorovich; John Wardrop; Tim Roughgarden; Bureau of Public Roads

Table 2.1: *Summary of key literature areas.*

Chapter 3 addresses game theory and coalitional games directly, touching upon network theory. Chapter 4 addresses game theory, network theory and transport theory, and briefly addresses coalitional games. Chapter 5 combines elements from both of these and introduces a real-world sanction basis, addressing game theory, network theory, coalitional games, sanction theory and transport theory.

2.8 Summary

From this comprehensive review of the literature, it can be seen that multilayer network modelling is a powerful tool for modelling complex systems, having been applied separately to many areas of study. This includes economic games, as discussed in section 2.2, cooperative games, from section 2.4, sanction games in section 2.5 and transportation in section 2.6. Section 2.2 also discussed the various forms of game theoretic competition and argues why, for the application and level of complexity appropriate for the model in Chapters 4 and 5 a networked Cournot competition is most appropriate.

After reviewing the relevant literature, the following is needed to achieve the aims of this thesis:

- An analysis of network formation games as an approach to coalition formation, developing the understanding of the idea of coalitional stability in this environment, as well as its impact on the outcome of associated games. Existing approaches to coalition formation discussed in section 2.4 do not capture non-transferable utility cases where the costs for coalition maintenance are paid by a single member.

- The creation and analysis of a bi-layer Cournot competition and optimal transport model using techniques to analyse the characteristics of equilibrium, including the rate of approach to a new equilibrium after disruption to the transportation network. As described in section 2.2, existing approaches to networked Cournot modelling, while analytically rigorous, do not capture either the rate of approach to equilibrium or the profit incentives to the players, particularly when paired with the additional complexity of a transportation layer.

- Improved modelling is needed in the understanding of the effectiveness of sanctions under the liberal explanation of sanction ineffectiveness as described in section 2.5. Sanction literature has extensive statistical analysis and strong underlying models, but these are not appropriately combined in order to evaluate the likelihood of sanctions being effective when applied.

To meet these needs, the following are carried out. Firstly in Chapter 3, an analysis of a combined coalition formation game and network formation game is conducted under non-transferrable utility conditions, considering the impact of these games upon one another and the subset of possible coalition structures which are stable in the game. In Chapter 4, a combined model of Cournot competition and optimal transport is created and analysed, building off of existing work by Bimpikis *et al* [61]. The problem is analysed from a dynamic programming perspective, examining the incentives held by players of a variety of impatience characteristics. Chapter 5 develops the model created in Chapter 4 is further developed in order to make it appropriate for the analysis of the effectiveness of sanctions, looking to find in particular, the proportional effectiveness relative to the expected level when no malicious transshipment occurs. Finally, Chapter 6 draws together the conclusions, comparing the results to the literature, before suggesting further work.

Chapter 3

Coalition Dynamics

This chapter includes both fundamental notation and concepts that will be useful throughout the remainder of the thesis. A novel method for coalition formation in a dynamic game, requiring Pareto improvement in non-transferable utility games, is then introduced. This is applied to a prisoner's dilemma game and a stag hunt game in order to demonstrate the impact of this approach to coalition formation. A network creation game is subsequently introduced from the literature [80] with the aim of developing the model through the introduction of coalition formation. This is carried out first, by examining existing myopic and novel dynamic programming strategy results on the game without coalition formation, then developing the game to include coalitional elements. A numerical example is given, as well as results on equilibrium existence, stability, and characteristics. The models in this chapter rely on the assumptions listed below. Key supporting literature is summarised in Table 3.1.

Key Assumptions Informing Chapter 3 Models

- Players act rationally, seeking to maximise their utility.
- In dynamic games, the strategy updates are conducted with uniform delay.
- A coalition's strategy is chosen as if it were a single player maximising the collective utility of coalition members.
- Coalition membership cost is linear for each player based on the size of their coalition.
- Players will not allow additional members to join the coalition if it reduces their personal utility, even if it improves the coalition's utility as a whole.

Authors & Year	**Key Contribution**	**Relevance to Thesis**
Aumann and Shapley (1994)	Repeated Games and Coalitional Game Theory	The basis for the repeated game structure, prisoner's dilemma and introduction of coalitional structures which this Chapter builds upon
Fabrikant *et al* (2004)	Network Formation Game	The basis for the network formation game in this Chapter
Pacheco *et al* (2008)	N-player Stag Hunt	The basis for the coalitional stag hunt game in this Chapter

Table 3.1: *Summary of key literature relevant to Chapter 3.*

3.1 Preliminaries

Two structures must be formally introduced, which remain vital throughout this work. That of the normal form game within game theory, and that of a network within network theory. The structure of a coalitional game and how it extends both normal form and dynamic games is then defined.

As discussed in section 2.2, a game is defined to be a structure in which each player receives a payoff which is a function of the strategic choices made by all players. Consequently, it is naturally suited for modelling competitive structures. The following definitions are drawn from the work of Fudenberg [12].

3.1.1 Normal Form Game

A normal form game (also known as a strategic form game) has three elements.

- The set of players $i \in \mathcal{I}$, which we take to be the finite set {1,2,...,I}.
- The pure strategy space S_i for each player i, from which follows the set of all strategies S, with $S = S_1 \times S_2 \times \cdots \times S_I$.
- Payoff functions u_i for each profile $s = (s_1, \ldots, s_I) \in S$.

The payoff functions give each player i's von Neumann-Morgenstern utility (or payoff) $u_i(s)$. We will frequently refer to all players other than some given player i as 'player i's opponents' and denote them '$-i$'. S is the set of all strategies, with $S = S_1 \times S_2 \times \cdots \times S_I$ [12].

The welfare [149] of a system is defined to be the total payoff received by all players, given by

$$W(s) = \sum_{i=1}^{I} u_i(s). \tag{3.1}$$

Welfare is a measure of the overall value of a solution. In games focused on infrastructure building and maintenance, this can represent the benefits provided by the geographical access granted by the infrastructure, to all players, considering the cost of maintaining the said infrastructure.

A collection of the key variables used in the game in this chapter is provided in Table 3.2.

Symbol	Definition
$\mathcal{I}$	The Set of all Players
$\mathcal{S}$	Set of all Strategy Profiles
$\mathcal{C}$	Subset of Players $\mathcal{I}$
$u_i(s)$	Utility received by player $i \in \mathcal{I}$ from strategy profile $s \in \mathcal{S}$
$W(s)$	Utility received by all players from strategy profile $s \in \mathcal{S}$
$\zeta(i, \mathcal{C})$	Net transfer of utility from members of $\mathcal{C}$ to $i \in \mathcal{C}$
$\|X\|$	The cardinality of set X
$V(\cdot)$	The Bellman equation
$F(\cdot, \cdot)$	The result of the players' strategy update rule

Table 3.2: *Notation for Game Definition (some terms to be defined later in the chapter).*

3.1.2 Network and Graph Preliminaries

Although later chapters use multilayer network structures, a single layer is sufficient for the model introduced in this chapter. Nevertheless, for consistency, this formal definition follows the conventions that appear in Boccaletti *et al.*'s [11] definition of a multilayer network.

A network [11] is defined to be a graph

$$G = (V, E)$$

where V is a set of nodes

$$v_i \in V \text{ with } i \in \{1, 2, \ldots, N\}$$

where N is the number of nodes and E is a set of edges

$$e_{ij} = (v_i, v_j) \in E.$$

The function giving flow on edges is introduced here for completeness, but they are not used in Chapter 3. These are important concepts in Chapter 4 where they are particularly relevant for the congested transport model.

The amount of flow on an edge e_{ij} is given by $f(e_{ij})$. A collection of network variables is provided in Table 3.3.

Symbol	Definition
G	The Network
V	Set of Nodes
E	Set of Edges
$f(e_{ij})$	The flow on edge $e_{ij} \in E$

Table 3.3: *Notation for Network Definition*

3.1.3 Coalition Behaviour

A coalition between players occurs when the set of players in the coalition leave aside their selfish utility aspirations and instead collectively work to maximise their total utility. That is, for a set of players $\{c_1, c_2, \ldots, c_C\} \in \mathcal{C} \subseteq \mathcal{I}$ in a coalition $\mathcal{C}$, instead of each player choosing a strategy s_i in order to maximise their own utility equation $u_i(s_i, s_{-i})$, the set of all players in the coalition collectively choose a response to maximise their total payoff, given by

$$\max_{s \in S_{\mathcal{C}}} \sum_{i \in \mathcal{C}} u_i(s). \tag{3.2}$$

This coalition strategy is denoted as $s_{\mathcal{C}}$, from the set of all possible coalition strategies $s_{\mathcal{C}} \in S_{\mathcal{C}}$ (or $S_{\mathcal{C}_k}$ where multiple coalitions exist, requiring the coalitions to be indexed).

Coalitional game theory is divided into two distinct types, depending on whether the utility in the game is transferrable or non-transferrable in nature.

Where utility is transferable (such as in many cases where the utility represents money [5]), the players can exchange it to incentivise participation in the coalition. Within transferable utility, the share of the coalition's total utility as given in equation (3.2) remains the same, while players receive

$$u_i(s) + \zeta(i, \mathcal{C}). \tag{3.3}$$

Here $\zeta(i, \mathcal{C})$ is a function that modifies the utility of player i in coalition C according to the utility distribution function of the coalition. This implies that

$$\sum_{i=1}^{C} \zeta(i, \mathcal{C}) = 0.$$

Where utility is not transferrable (such as when utility represents less tangible values such as knowledge [150], enjoyment [151] or national GDP [152]), players are unable to exchange utility between them to incentivise coalition membership. This results in coalitions remaining stable only when every member benefits directly from their membership. That is, for player i, a coalition $\mathcal{C}$ and all other players $\mathcal{I} \setminus \{\{i\} \cup \mathcal{C}\}$, player i chooses between s_i and $s_{\mathcal{C}\cup\{i\}}$, following the core tenant of game theory to maximise their personal utility. Accordingly, they evaluate

$$u_i(s_i, s_{-i}) - u_i(s_{\{i\}\cup\mathcal{C}}, s_{\{i\}\cup-\mathcal{C}}) = r \tag{3.4}$$

preferring to join the coalition if the result r is negative, and to not join the coalition if r is positive.

3.1.4 Notable Strategy Profiles

A strategy profile $s = \{s_1, s_2, \ldots, s_I\} \in \mathcal{S}$ can have notable characteristics. A subset of these, relevant to the thesis, are listed below.

Social Optimal

A socially optimal strategy profile is one with the greatest total utility, as measured through the welfare function. In this model this is the outcome which results in the largest total profit across all companies [12]. A strategy profile s^O is socially optimal if:

$$W(s^O) \geq W(s) \text{ for all } s \text{ in } S.$$

Nash Equilibrium

A strategy profile is a Nash equilibrium if no player can improve their utility through a unilateral change in strategy [12]. As such, a strategy profile s^* is a Nash equilibrium if for every player i, such that $s^* = (s_i^*, s_{-i}^*)$,

$$u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*) \text{ for all } s_i \text{ in } S_i.$$

Pareto Optimal

A strategy profile is a Pareto optimum [153] if there are no other strategy profiles where all player receive increased or the same utility. As such, a strategy profile $s' = (s'_i, s'_{-i})$ is Pareto optimal if there exists no strategy profile $s = (s_i, s_{-i})$, such that both

$$u_i(s) \geq u_i(s') \text{ for all } i \text{ in } S_i$$

and

$$\text{there exists } j \text{ such that } u_j(s) > u_j(s').$$

3.1.5 Outcome Evaluation

The price of anarchy and the price of stability can be used to evaluate the system by giving a value that represents a comparison between Nash equilibrium states and the possible optimum state.

Price of Anarchy

The PoA [154] for a game is the result of a function giving a value representing the worst possible damage caused by selfish behaviour. The PoA is defined as:

$$\text{PoA} = \frac{\max_{s \in S} W(s)}{\min_{s^* \in S^*} W(s^*)} \tag{3.5}$$

where S^* is the set of all Nash equilibria.

Price of Stability

The PoS [82] is a similar but less well known concept. It results in a value representing the lowest possible damage caused by selfish behaviour. This is often relevant in cases of guided equilibrium formation. The players in the game remain selfish, but the initial conditions are tweaked in order to improve the outcome from the anarchical case. The PoS is defined as:

$$\text{PoS} = \frac{\max_{s \in S} W(s)}{\max_{s^* \in S^*} W(s^*)}. \tag{3.6}$$

3.1.6 The Bellman Equation

$$V(x_0) = \max_{\{S_i^t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta_t F(S^t, S_i^t) \tag{3.7}$$

is the Bellman equation [88] with $F(S^t, S_i^t)$ the utility given by the players' strategy update rule, S^t is the strategy profile at time t and β_t is impatience, representing the priority given to utility received sooner rather than later. Where $\beta_t = 0$ for $t > 0$, the player behaves myopically, only valuing utility received immediately. Where $\beta_t = 1$ for all t, the player values all future time steps equally. Usually, $\beta \in [0, 1]$. The strategy update rule assumes that at each time t, the active player chooses the best response over all times, $s_i^{*,t}$ from the set S_i^t.

These definitions and measures are used to develop and discuss models in this chapter, as well as in Chapters 4 and 5.

3.2 Coalitional Games

Coalition Implemention

This thesis introduces a specific form of coalitional game as a structure:

$$\mathbf{T} = \langle \mathcal{I}, S, \mathbf{C}, \mathbf{u} \rangle \tag{3.8}$$

where

$$\mathcal{I} = \{1, 2, \ldots, I\}$$

is a set of players,

$$S = \{S_1, S_2, \ldots, S_I\}$$

is an **I**-tuple of pure strategy sets, one for each player,

$$\mathbf{C} = \{\mathcal{C}_1, \mathcal{C}_2, \ldots, \mathcal{C}_k\}$$

is a B_I-tuple (with B_i the i$^{\text{th}}$ Bell's number) [155] of set-theoretic partitions of I. This is demonstrated in figure 3.1. Finally,

$$\mathbf{u} = \{u_1, u_2, \ldots, u_I\}$$

is an I-tuple of utility payoff functions.

The current coalition structure is represented by the structure $[x_1, x_2, x_3, \ldots, x_I]$ where x_i is an integer which gives the index of the coalition the player is a member of. Player 1 is always a member of coalition 1 and as such $x_i = 1$. A player in a coalition with no other players is considered to be in a coalition with cardinality 1. Players cannot be a member of more than one coalition as they cannot simultaneously optimise with respect to multiple coalition's utilities. Figure 3.1 shows the full set of coalition structures in a 3 player game, as well as the change that can be made by a single play in a step. Moves available to player I_1 are shown in purple, by I_2 in green, and by I_3 in red. Figures 3.2 and 3.3 show the available changes in the coalition structure in a four-player game when the decision is made by players I_3 and I_4.

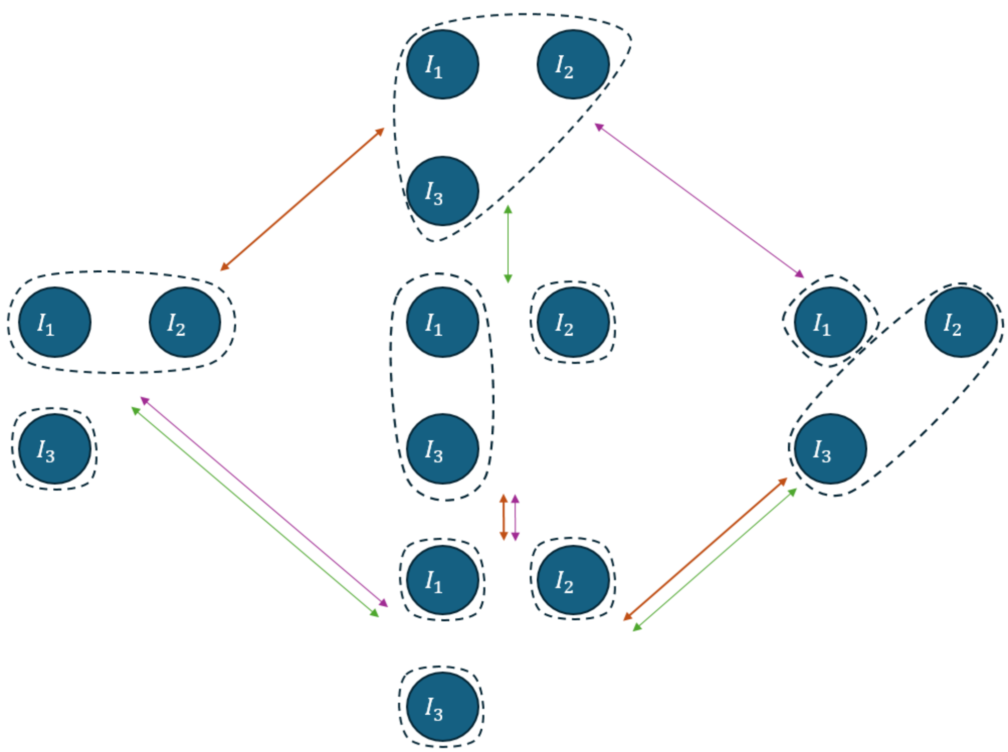

Figure 3.1: *A set of all 3-player coalitions and the possible transformations that can be made in a single step by each player.*

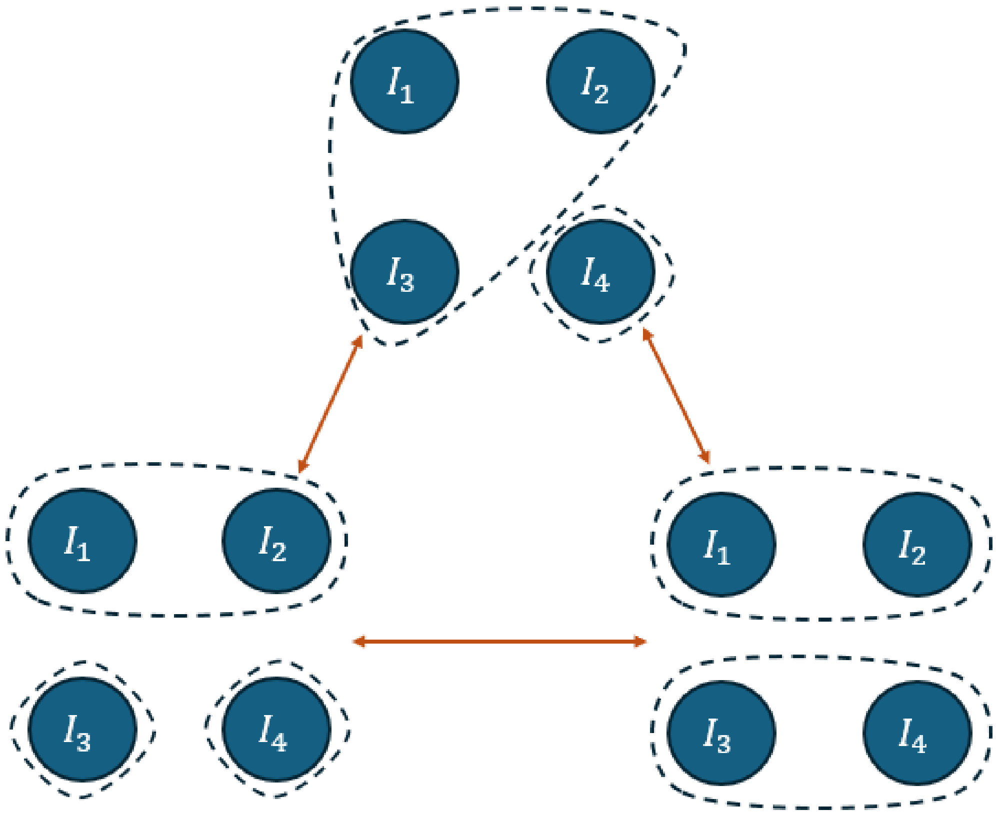

Figure 3.2: *A sample of coalition transformations with 4 players where player I_3 updates. From the starting point top centre, $[1, 1, 1, 2]$, I_3 can choose to leave the coalition, resulting in coalition structure $[1, 1, 2, 3]$ in the bottom left, join a coalition with I_4, with coalition structure $[1, 1, 2, 2]$ in the bottom right or remain in their original coalition.*

Coalition Strategy

Where all coalitions contain just a single player, the underlying game is played in the same way as it would be in a normal form game. As such, where coalition formation provides no benefit to players, the results from the normal form game without coalition elements remain. Coalition formation therefore provides players with an extra tool to improve their utility.

The concepts introduced in section 3.1 allow for some immediate novel

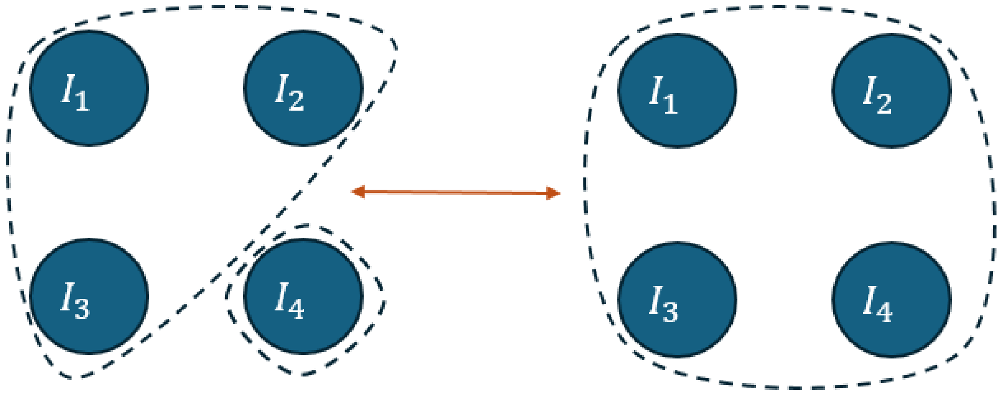

Figure 3.3: *A sample of coalition transformations with 4 players where player I_4 updates. From the starting point $[1,1,1,2]$, I_4 can choose to join the coalition, resulting in coalition structure $[1,1,1,1]$ or remain independent.*

insight into coalitional games. Coalitional games exist to provide players in non-transferable utility games with an enforced structure within which to cooperate while retaining purely selfish motivations. In this section, a form of coalition formation is introduced where the updates are sequential, local, and myopic. That is, the current state of the coalitions within the game is updated with each player, in turn, considering if there is an improvement available through a myopic change in strategy.

3.2.1 Case Study: Coalitional N-player Social Dilemmas

To demonstrate the impact of the opportunity to form a coalition, an N-player prisoner's dilemma is examined, with a specified utility function u_i [32, 33, 156]. The standard prisoner's dilemma is introduced in section 2.2.1. Each player's strategy profile is given by $s_i = t^{N-1}$, where $t = \{a, b\}$, the choice to either co-operate with or implicate each other player. A strategy profile of a^{N-1} would represent the player choosing not to implicate (betray) any other players, while a profile of b^{N-1} would represent the player implicating all other players. Each player seeks to maximise their personal utility u_i, given by

$$u_i(s) = 2d \cdot N + A_i - d \cdot B_i, \tag{3.9}$$

where A_i is the number of other players whom player i has implicated, B_i is the number of other players who have implicated player i and $d > 1$

is a constant which weights the benefits of cooperating and betraying other players.

In the normal-form game, the Nash equilibrium involves every player implicating every other player. The resulting Nash equilibrium is not Pareto optimal, and accordingly there exist alternate outcomes which all players would prefer. However, due to the strong dominance of the cooperate strategy by betray, there is no opportunity for cooperation to dynamically develop. The introduction of the opportunity to form coalitions provides an opportunity for players to improve their personal payoff, which also results in a net increase to the welfare. This represents a reduction in total prison time served by all players. Within a coalition, coalition members choose strategies that benefit the coalition as a whole while continuing to behave rationally towards players outside the coalition. As such, as a member of a coalition, each member would not implicate any other member, and would implicate every player who is not a part of the same coalition.

As introduced in section 3.1.3 the players consider whether coalition membership is beneficial to them based on whether the resultant strategy from the coalition's members is superior to the one they would obtain alone or in their current coalition. A player will join a coalition if they receive increased utility from doing so and leave a coalition if they receive reduced utility. The coalition similarly requires all players who are already members to myopically improve their utility through the addition of the new member. Otherwise, they will object to the new player joining and the new player would not be permitted to join the coalition.

It is possible for multiple coalitions to exist, but there do not exist any stable equilibria with multiple coalitions. This is because, when choosing coalition membership, players are incentivised to select the largest coalition as its membership gains the greatest increase in payoff. The result of this game is that all players join the grand coalition, emphasising why the separation of players is vital to the most common statement of the prisoner's dilemma.

Results

This modified prisoner's dilemma thus achieves homogeneous outcomes for all players because of their symmetry. With N players, where no coalitions are available, each player plays strategy b^{N-1} and receives a payoff from equation 3.9 of

$$2dN + N - 1 - d \cdot (N - 1) = dN + N + d - 1.$$

With coalitions introduced however, the only stable strategy profile has all players as members of the grand coalition, playing strategy a^{N-1} and receiving utility from equation 3.9 of $2dN$.

For this game without coalition formation, this results in a PoA of $\frac{2dN}{dN+N+d-1}$. For $N = 2$, this has a value of $\frac{4d}{d+3}$ and as $\lim_{N\to\infty} \mathrm{PoA} = \frac{2 \cdot d}{d+1}$.

Following the values given by Poundstone [32], $d = 3$, and accordingly as $\lim_{N\to\infty} \mathrm{PoA} = \frac{8}{5}$.

The PoS has the same result, $\lim_{N\to\infty} \mathrm{PoS} = \frac{2 \cdot d}{d+1} = \frac{8}{5}$.

However, by introducing coalition formation, we have the PoA and PoS for the game with coalition formation, of PoA = PoS = 1.

Results for a range of parametrisations can be seen in the heatmap in figure 3.4.

3.2.2 Case Study: Coalitional N-player Common Interest Complexities

One of the unique characteristics of the dynamic coalition formation requiring Pareto improvement is illustrated through the stag hunt. This game is introduced in section 2.2.1 [12]. Players hunt by either cooperating for a bigger reward or by prioritising safety. The extension to a multiplayer game $(n > 2)$ follows the work of Pacheco *et al.* [157]. In this n player game, there is a threshold $m \leq n$ for the number of cooperators required to receive rewards from cooperating, while cooperating

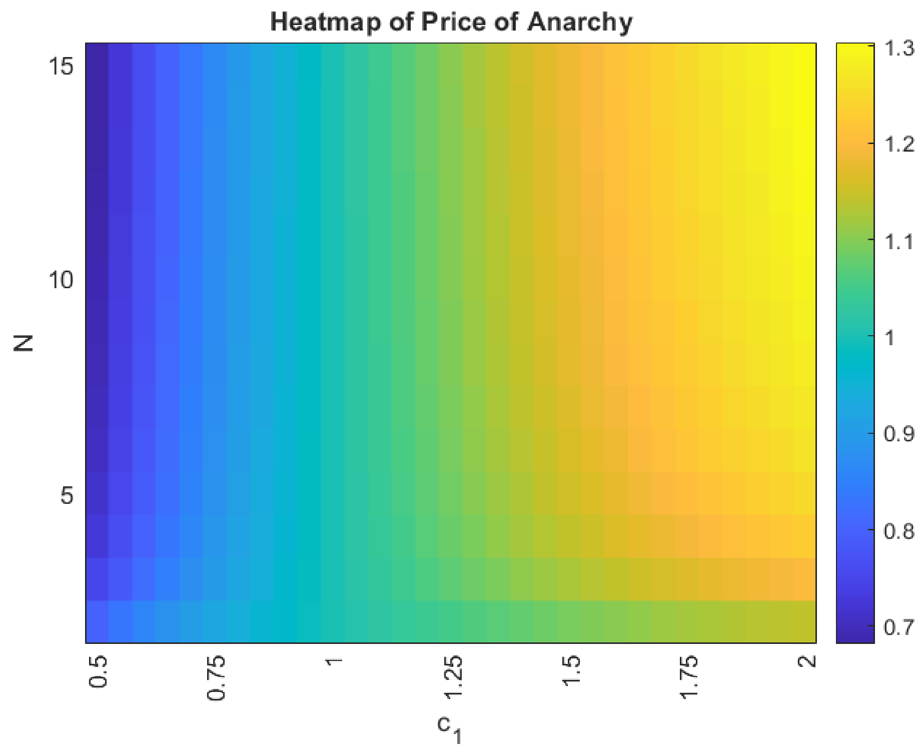

Figure 3.4: *A heatmap of the Price of Anarchy as player count and punishment for defecting are varied.*

always has a cost. An example utility table for the five-player game with threshold 3 is given in table 3.4.

In addition, players have the opportunity to join coalitions, incurring a marginal cost based on the size of the coalition they are a member of. For $F > 1$, given a randomised starting position, one of two equilibria will emerge, either $q = n$ or $q = 0$. These represent collective action to hunt stags and universal independent action to hunt hares respectively. Figure 3.5 shows this in an n-player case with $n = 10$, success threshold $m = 5$ and stag hunting reward $R > \frac{n}{m} + 1$, played as a dynamic game in which a single randomly chosen player updates their strategy at each time step. Where the starting q is much below the threshold m, or somewhat above the threshold, players will change strategy toward hunting hares where the hunt will not succeed even with their assistance, or towards hunting stags if the hunt will succeed with them. The marginal

q	$R_S(q)$	R_H
5	$F-1$	$-$
4	$4F/5-1$	1
3	$3F/5-1$	1
2	-1	1
1	-1	1
0	$-$	1

Table 3.4: *Payoffs for the 5-Player Stag Hunt Game. q is the number of stag hunters, $R_S(q)$ is the utility received by stag hunters and R_H is the utility received by hare hunters. F is the reward for a successful stag hunt where $q = n$.*

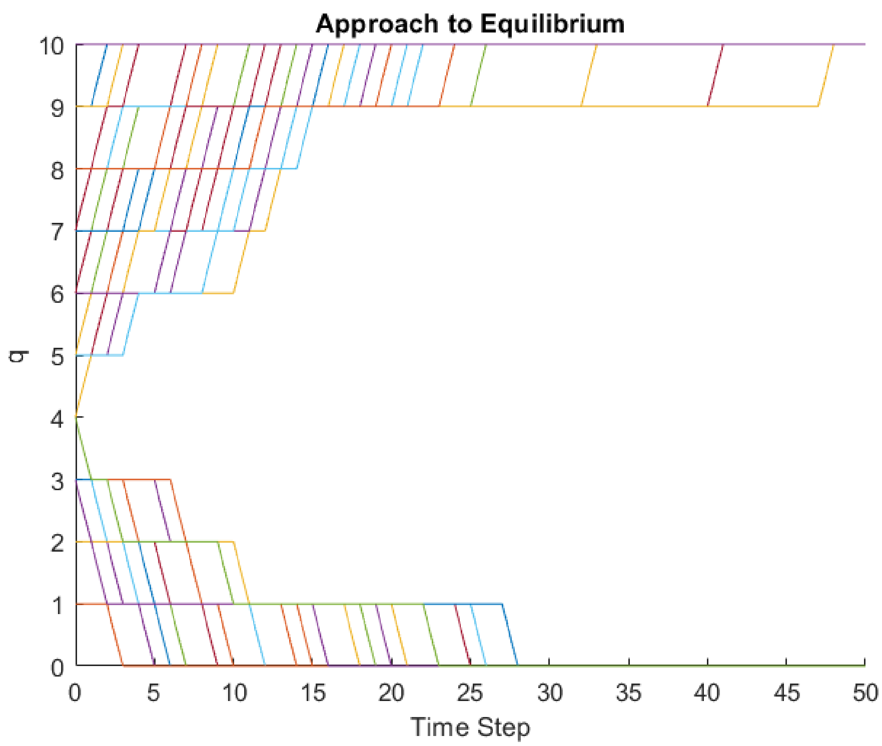

Figure 3.5: *A Graph showing the random approach to equilibrium from different starting positions in a 10-player stag hunt with success threshold $m = 5$.*

case occurs where $q = 4$, just below the threshold m. If the first player to act is a stag hunter, they change strategy to hare hunting as they cannot make the hunt profitable. Conversely, a hare hunter can make the stag hunt profitable and changes strategy towards hunting stags.

Unlike in the prisoner's dilemma game of the previous section, coalition formation is not consistently successful in incentivising a change in strategy. Coalitions with even a marginal cost for membership collapse quickly due to the stability of the equilibria. With q_{-C} the number of players not within coalition C who are hunting stags, coalition dynamics change the resultant equilibrium only where

$$|C| \geq M - q_{-C}. \tag{3.10}$$

Otherwise, existing coalitions quickly dissolve because membership requires marginal cost for no benefit.

3.2.3 Case Study: Network Formation Game

This thesis introduces a new game in which players seek to maximise their utility through network formation with the opportunity to form coalitions. Three possible basis cases are of the creation and maintenance of infrastructure links between adjacent towns, of the maintainence of internet network architecture, and of the maintenance of collaborations between academics. The game is a single layer network formation game, following the characteristics outlined in [80], who considered the problem without coalition formation. Players pay a cost for maintaining each undirected edge they have formed between them and other players. They receive positive utility from minimising the distance between them and other players in the game. The players are allowed to use the edges formed by other players when evaluating this distance. The coalitional elements of this game make it a non-transferable utility NTU game, and the players are able to form coalitions in order to collectively optimise their edge placement to maximise the coalition's utility.

Strategy Profile

Each player's strategy is given by

$$s_i \times c_i \in S_i \times C_j \text{ s.t } d_c^i(c_0, c_i) \leq 1 \tag{3.11}$$

where c_i is the new coalition partition that i would like to join, choosing from the set of possible coalitions $\mathbf{C}$ which are adjacent to c_0, the current partition. Here d_c is the 'partition distance'. The partition distance is a discrete distance metric where $d_c^i(c_j, c_k) = 1$ if and only if there is a single move (i.e. joining or leaving or changing coalition) by player i which changes the partition state from c_j to c_k. $d_c^i(c_j, c_k) \in \{0, 1, \infty\}$, as given the current structure c_j, all coalition structures can be characterised to be either $c_j = c_k$, c_k is accessible to player i from c_j in a single move or c_k is not accessible in any number of moves.

Utility Equations

Each node represents a player with the utility of player i given by:

$$u_i = -\alpha|s_i| - \sum_{j=1}^{n} d_G(i, j). \tag{3.12}$$

Here $\alpha \in \mathbb{R}$ is the cost of edge creation, $|s_i|$ the number of edges formed by player i and $d_G(i, j)$ is the topological distance through the network between nodes i and j.

In [80] Fabrikant *et al.* outline a number of basic results on this game. First, that games in which $\alpha < 1$ or $\alpha > 2$ are not generally interesting due to dominant network structures. Where $\alpha < 1$, both the social optimal and the Nash equilibrium correspond to a totally connected graph as seen in figure 3.7. Where an edge does not exist, players are incentivised to place one. Where $\alpha > 2$, a star graph as seen in figure 3.6 is the socially optimal solution, with a high degree of stability, and is the end result for most initial conditions.

The authors find that other graphs are possible for $\alpha > 2$, such as the Petersen graph, which remains stable for $\alpha \leq 4$), with a transient equilibrium at $\alpha = 4$. Figure 3.8 shows this graph. We demonstrate these

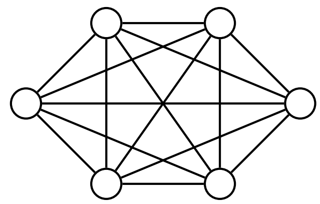

Figure 3.6: *A totally connected network resulting from low edge formation cost $\alpha < 1$.*

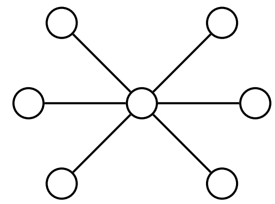

Figure 3.7: *A star network resulting from high edge formation cost $\alpha > 2$.*

results by considering a player with 1 edge connected to them from other players, selecting player I_1 at node 1 and the edge between them and node 3. Where $\alpha = 4$, there are two joint optimal responses. The first is to return to the Petersen graph, placing edges to nodes 6 and 4. The second is to place an edge to node 0, increasing the total distance as given by $\sum_{j=0}^{n-1} d_G(i,j)$ by 4, but reducing the number of edges placed by 1, decreasing costs by 4. From this new position, a series of best responses would result in the graph returning to a star graph. These results are developed in this thesis through consideration of a dynamic programming approach rather than the myopic one considered by Fabrikant *et al*, and through the introduction of coalition formation.

Optimal Response Over An Extended Time-Horizon

The introduction of less myopic strategy choices is evaluated through the Bellman function, as given in equation 3.7. This considers that the current player is optimising not only for the immediately following turn, but for all future turns, with decreasing interest given by the patience β. When evaluating whether they should diverge from the Petersen graph where $\alpha = 4$, the player balances the immediate decrease in utility resulting from the disruption against the increase they receive as either an edge or central node in the star network which will result. For $\beta = 1$, the infinite timescale means that the stability of the star network will be valued over the utility received from the Peterson graph, while for $\beta < 0.5$, the temporary disruption is always too high, and the player chooses not to move away from the Petersen graph, resulting in it remaining stable. For $\beta \in (0.5, 1)$, it could be worth changing, but it is highly dependent on turn order and somewhat dependent on what choices the players make from a number of equally advantageous ones. Where players choose to change strategy, the resultant graph will be the star network due to the high edge creation cost.

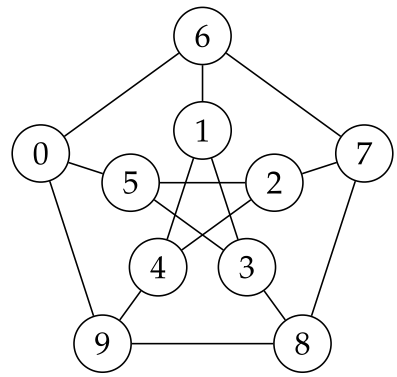

Figure 3.8: *The Petersen Graph*

3.3 Coalition Formation and Dissolution

Coalition structures are not something which always benefits a player however, as with standard games, the principal analysis involves examining what changes each player would make to a game from a given starting position. The start position is therefore the strategy profile $s_1 \times s_2 \cdots \times s_N$ of all players in the game, and a coalition structure $C_k \in \mathbf{C}$. A player i examines all possible coalition structures after a unilateral move. Player i may choose to remain in their coalition, but player i may also leave their coalition and form a new coalition in which they are the only member or leave their coalition and join another existing coalition.

3.3.1 Utility and Motivation

Although this model abstracts physical geography in favour of a topological distance metric, it has applications in the provision of national infrastructure. One example is in the case of a series of regional intercity transport links provided by towns. The formation of coalitions representing cooperation to maximise the quality of their service within a rigid infrastructure is also appropriate. This is a case of non-transferable utility in which the players seek to maximise the linear utility by minimising the costs given by equation 3.13. This is similar to equation 3.12 with an additional term considering the cost of coalition membership.

$$u_i = -\alpha|s_i| - \sum_{j=1}^{n} d_G(i,j) - \gamma|C_i|. \tag{3.13}$$

Here, $\gamma \in \mathbb{R}$ the cost of coalition membership, and $|C_i|$ the size of the coalition i is in. In equation 3.13, the relationship is linear, but this is not required, with sublinear and superlinear costs each being appropriate for some basis cases. The players first seek to optimise the distance between their citizens and every other town on the network, so as to provide the best access to other towns. This corresponds to the term $\sum_{j=1}^{n} d_G(i,j)$ in equation 3.13. Second, to minimise the number of transport routes the town is paying for to other towns, which are also available for use in reverse. This is given by $\alpha|s_i|$. Finally, the minimisation of administrative costs, as represented by the addition to the formulation originally appearing in [80] for the purpose of network structure: $\gamma|C_i|$. This increases as the coalition grows, due to the increased difficulty in coordination as the number of players increases.

3.3.2 Worked Example: Network Formation

Considering an initial condition as shown in Figure 3.9. Arrows represent edges being paid for by the player whose node they begin at.

Selecting $\alpha = 1.5$: and $\gamma = 0.2$:

$$C_j = \{(1), (2, 3, 4)\}$$

gives

$$\mathbf{u} = \{(-5.2), (-6.6), (-6.1), (-6.1)\}$$

choosing player 2, the possible network structures after a single step is

$$C_{k_1} = \{(1, 2), (3, 4)\}$$

$$C_{k_2} = \{(1), (2), (3, 4)\}$$

$$C_{k_3} = \{(1), (2, 3, 4)\}$$

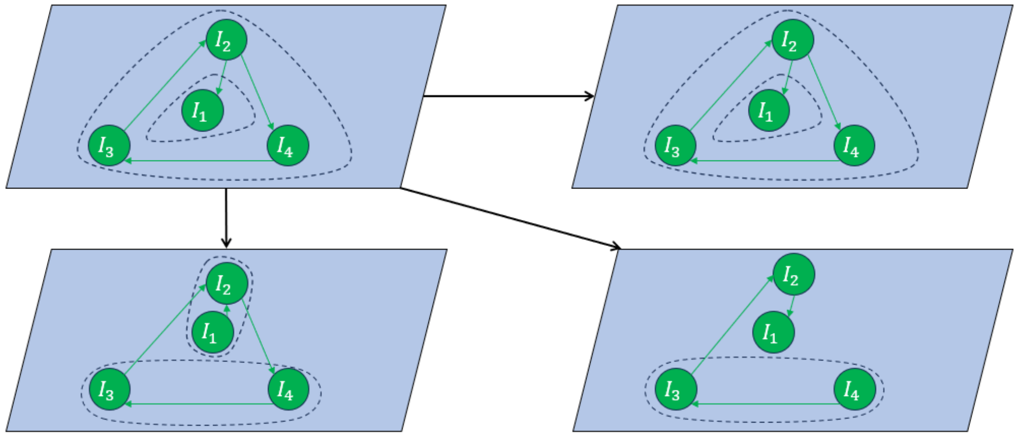

Figure 3.9: *An initial condition and possible future states under action by player* 2.

these then result in utility payouts (working anticlockwise)

$$\mathbf{u}|C_{k_1} = \{(-6.9), (-4.9), (-5.9), (-5.9)\}$$

$$\mathbf{u}|C_{k_2} = \{(-5.2), (-6.7), (-6.9), (-4.9)\}$$
$$\mathbf{u}|C_{k_3} = \{(-5.2), (-6.6), (-6.1), (-6.1)\}$$

While the second of these changes results in a higher utility for player 2, u_1 is reduced and so player 2 would not be accepted into this coalition. As such, there is no unilateral change in coalition structure that improves the utility for player 2 so the current model is stable to change by player 2 at this step.

3.3.3 Analytical Results

This thesis presents several novel findings regarding the behaviour of the system, which are strongly influenced by the chosen value of α. The analysis focuses on a general form where the edge creation cost α lies in the range $\alpha \in (1, 2)$, parameterised as $\alpha = 2 - \delta$ for $\delta \in (0, 1)$.

Proposition 3.3.1. For a coalition M with members $m_1, m_2, \ldots, m_k$, for all $m_i, m_j \Rightarrow d_G(m_i, m_j) = 1$.

This proposition implies that between every pair of members of a coalition, there will be an edge, leading to the coalition becoming fully connected.

Proof. Considering the metric of coalition utility given in equation 3.12, here referred to as u_M, and assuming towards a contradiction that there exists a graph structure such that there exists $d_G(m_i, m_j) > 1$ which maximises u_M for changes in $S_{m_1} \times S_{m_2} \times \cdots \times S_{m_k}$. This implies $d_G(m_i, m_j) \geq 2$. Adding an edge between m_i and m_j has cost $\alpha = 2 - \delta$, and the edge reduces $d_G(m_i, m_j)$ to 1. This reduces u_M by at least 2. Placing an edge between m_i and m_j thus increases u_M by a minimum of δ. This implies that there is a change to the graph structure that increases u_M with respect to $S_{m_1} \times S_{m_2} \times \cdots \times S_{m_k}$. This is a contradiction, as it was assumed that the graph structure maximised u_M, but there exists an improvement that can be made. Consequently, the existence of a graph structure such that there exists $d_G(m_i, m_j) > 1$ can be rejected. □

Proposition 3.3.2. When $|V| > 2, \delta < \gamma\frac{2(k-1)}{k-2}$, if a coalition C has cardinality $|C| = 2n$ for $n \in \mathbb{N} \Rightarrow$ there exists $c_i \in C$ s.t. there exists $s^q_{c_i} \in s'_{c_i}|s_{-c_i}$ s.t. $u_i(s_{c_i}) < u_i(s^q_{c_i})$

This proposition implies that coalitions are only stable when they have an odd number of members or with an even number of members while $\delta > \gamma\frac{2(k-1)}{k-2}$.

Proof. Considering $m_1, m_2, \ldots, m_k \in M, |M| = k, k = 2n - 1$ for $n \in \mathbb{N}$ and assuming w.l.o.g $u_{m_1} \leq u_{m_2} \leq \cdots \leq u_{m_k}$ and a unilateral change of strategy by m_1 s.t. $|C_{m_1}| = 1$. The initial cost to m_1 considering only coalition members while m_1 is a member of the coalition is:

$$k - 1 + \alpha\frac{k}{2} + \gamma k$$

after m_1 has left the coalition the best strategy is to connect to the coalition at the hub node, giving it a cost of

$$2k - 3 + \alpha + \gamma.$$

Therefore the change in cost is:

$$-k + 2 + \alpha\frac{k-2}{2} + \gamma(k-1)$$

$$= -\delta\frac{k-2}{2} + \gamma(k-1).$$

As such, leaving the coalition provides a positive change in utility for m_1 for $\delta < \gamma\frac{2(k-1)}{k-2}$. □

Corollary 3.3.1. When $\delta < \gamma\frac{2(k-1)}{k-2}$, total coalition is only stable for networks with $|\mathbf{I}| = 2n-1, n \in \mathbb{N}$.

This corollary implies that the total coalition is only possible when the graph has an odd number of members, while $\delta < \gamma\frac{2(k-1)}{k-2}$.

Proof. This is a direct consequence of Proposition 3.3.2. As even sized coalitions are unstable, if a graph has an even number of elements, the total coalition is not stable while $\delta < \gamma\frac{2(k-1)}{k-2}$. □

The implications of these results can be seen in figure 3.10 for coalition formation cost $\gamma = 0.3$.

3.3.4 Impact of Coalition Formation

The analytics give results about the prices of anarchy and stability. The PoA and the PoS as defined in equations 3.5 and 3.6 are found in a star network. In both cases, the total utility in the system is given by:

$$2(k-1)^2$$

with $k = |\mathbf{I}|$ while the social optimal has utility

$$k(k-1).$$

Accordingly the PoA and PoS without coalitional elements are both given by

$$\frac{k}{2(k-1)}.$$

In the case where coalition is made available to the players, the value of the PoA is

$$\frac{k(k-1)}{k(k-1)} = 1$$

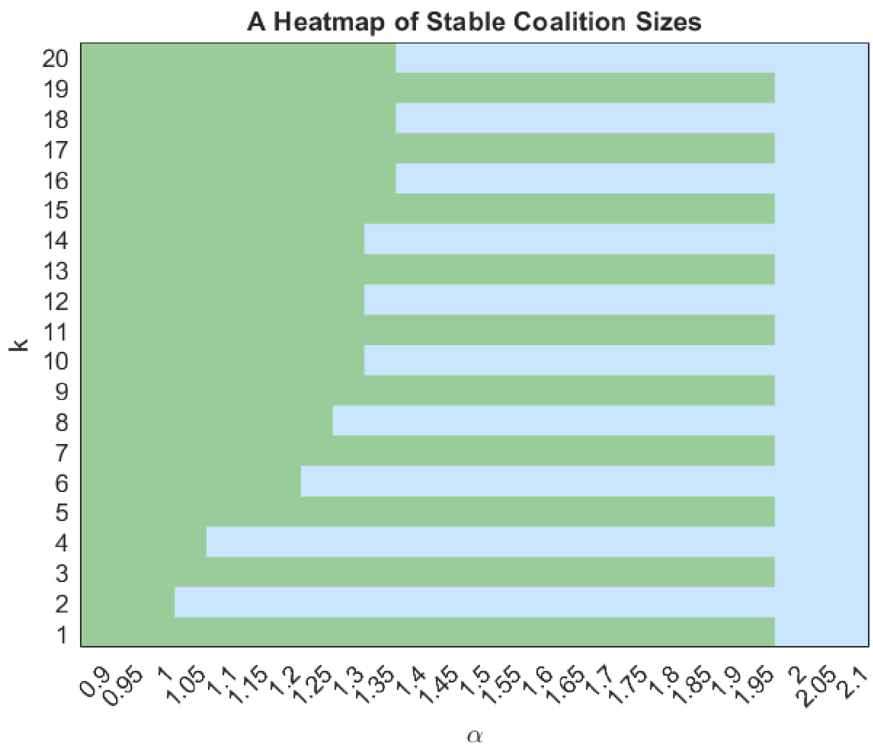

Figure 3.10: *A heatmap of stable coalition sizes. Coalition size is given by k and cost of edge formation by α. Stable coalitions can exist for values of α and k corresponding to a green cell.*

while the value of the PoS is

$$\frac{k(k-1)}{2((\gamma+1)k-1)}.$$

Accordingly, the addition of coalition elements dramatically improves the Nash equilibrium solutions in both anarchic and stable cases towards a near socially optimal solution. Due to the improved social welfare of the stable solution however, guided equilibrium development using either a selected initial condition or control elements (incentives) such as a reward for moving towards the preferred state would improve the outcome.

3.4 Discussion

Coalition formation as a tool was introduced and explored in order to analyse its impact on existing games in the literature [32, 33, 80, 156]. An original formulation for non-transferable utility games, requiring unilateral consent from the existing membership of the coalition, was introduced. This is distinct from the existing literature in the area of non-transferable utility games, which has focussed on the fair allocation of utility between the members of a coalition [13]. When combined with a network formation game [80], this unilateral acceptance requirement results in only specific coalition sizes being myopically stable, giving original results developing the understanding of the network formation game and this form of non-transferable utility coalition.

The application of coalitional formation to some real world problems is natural, due to its potential to improve the utility of all players through cooperation. The case studies of the prisoner's dilemma and stag hunt demonstrate both its ability to change the equilibrium of the system, improving the social welfare, and the limitations brought from this dynamic method.

The result of non-myopic strategy optimisation was also explored on a network formation game, showing that it was able to change the total set of stable equilibria in such a way that the social welfare of the system was improved.

This chapter provided a novel approach to coalition formation for non-transferable utility games requiring Pareto improvement to increase in size. It gave original results on the implications of that approach when applied to well known games and to the network formation game, identifying both stability and the impact on the prices of anarchy and stability. A summary of these contributions can be seen in Table 3.5.

3.4.1 Development from this model

The benefits to the players have been explored building on the classical model of the prisoner's dilemma and the more complicated network formation problem. However, while these are appropriate choices for

Area of Literature	Key Contribution
Coalitional Game Theory	The introduction of a novel form of coalition formation technique which requires Pareto Improvement for all members
Coalitional Game Theory	Analysis on repeated classical games such as the Prisoner's Dilemma and the Stag Hunt demonstrating the impact of the above approach to coalition formation, particularly on the rate of emergence of a stable equilibrium
Coalitional Game Theory	Results are given on the change to the price of anarchy at equilibrium in classical and network formation games

Table 3.5: *Summary of key contributions within Chapter 3 categorised by area of literature.*

undertaking a basic analysis of the impact of coalition formation on game dynamics, they are limited in their application.

In order to extend the use of coalition formation dynamics more widely, developing a model closely aligned with a real-world basis is required. An examination of coalitional dynamics accordingly reappears in Chapter 5 where they are united with the multilayer model that is developed in Chapter 4. The time-horizon based analysis through the Bellman equation also appears in Chapter 4 where the response to immediate disruption, which will have ongoing impacts over an extended time-frame, is a central theme.

Chapter 4

Cournot and Transport Elements

In this chapter, a novel multilayer network model is introduced, combining oligopoly trade with traffic congestion elements. Initially, additional elements required for this model are introduced, such as a multilayer network and Cournot competition. The multilayer network expands on the single layer model that appeared in Chapter 3. The constituent elements of the model are then analysed separately. Numerical simulations are performed on a case study of a linked bilayer model. The bilayer is then analysed, giving original results about the nature of the equilibria for the system. This work builds on that of Chapter 3, retaining an interest in coalition formation but applying it to a significantly more complicated underlying model.

The intention is that this model has relevance to understanding real-world trade dynamics, specifically of goods with non-trivial transport costs and a high degree of substitutability. An example is the transport of food products, such as grain. The model in this chapter relies on the assumptions listed below. Key supporting literature is summarised in Table 4.1.

Key Assumptions Informing the Model appearing in Chapter 4

- Players have the opportunity to act sequentially.
- The relationship between supply and price is strictly decreasing and twice differentiable.
- The congestion function is strictly increasing and twice differentiable.
- The goods being sold are homogeneous.
- Each player has market power following the format of an oligopoly game.
- Players are rational and do not engage in collusion unless otherways stated.

Author(s) & Year	**Key Contribution**	**Relevance to Thesis**
Cournot (1929)	Cournot Competition	The primary form of quantity-based competition that exists within each of the markets
Bimpikis *et al.* (2019)	Cournot Competition in Networked Markets	An extension of the above competition type to multiple networked markets and results on the equilibrium
Blanchet and Carlier (2016)	Cournot competition with non-congested transport	Introduces non-congested transportation elements to a Cournot problem, which this chapter improves upon

Table 4.1: *Summary of key literature relevant to Chapter 4.*

4.1 Preliminaries

The players represent firms selling goods and attempting to maximise their total profit, which depends on the costs of transporting goods and the profit from selling to the various markets. The profit made by selling to markets is captured through the dynamics in the upper layer. Each player and each market is associated with a node in the upper layer. The impact of congestion on transport cost is captured through the dynamics in the lower layer. Figure 4.1 shows an example multilayer network, with nodes x_1^α and x_2^α in layer G_α having embedded players in green and markets y_1^α and y_2^α in red. Transport transshipment hubs are shown in blue on layer G_β as nodes v_i^β.

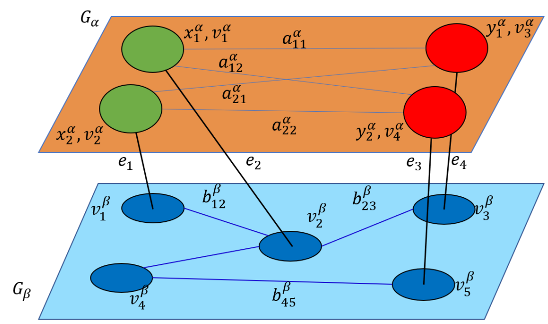

Figure 4.1: *An example bilayer network with 2 players, 2 markets and a 5 node transport layer*

4.1.1 Formal Model Network Definition

A bilayer network is a form of multilayer network appropriate where there are two forms of interaction happening in parallel with influence on each other. The following definitions use conventions established by Boccaletti *et al.* [11]. A multilayer network is a pair represented as:

$$\mathcal{M} = (\mathcal{G}, \mathcal{E}). \tag{4.1}$$

Here, $\mathcal{G}$ is the set of networks with nodes and intra-layer edges and $\mathcal{E}$ is the set of inter-layer edges. When considering a bilayer network, these can be defined more explicitly. A bilayer network is a pair represented as:

$$\mathcal{M} = (\mathcal{G}, \mathcal{E}_{\alpha\beta}). \tag{4.2}$$

Here, the indices α and β are used to reference the layers, and the family of undirected weighted graphs denoted as $\mathcal{G} = (G_\alpha, G_\beta)$ is referred to as the "layers" of the network $\mathcal{M}$. In the context of Figure 4.1, the graph G_α is also colloquially called the "Upper Layer," while G_β is referred to as the "Lower Layer".

Layer α is the graph given by $G_\alpha = (V_\alpha, E_\alpha)$ where $V_\alpha = \{v_i^\alpha, i = 1, 2 \ldots N_\alpha\}$ with N_α the number of nodes in layer α. For our problem, $N_\alpha = N + M$ where N is the number of players (sellers) and M is the number of markets. The upper layer's nodes can be partitioned into two sets: X, consisting of N players (sellers), and Y, consisting of M markets.

An edge $a_{i,j} = (v_i^\alpha, v_j^\alpha) \in E_\alpha$ where E_α is the set of intra-layer edges. Layer G_α is a bipartite graph as there is no exchange of traffic between groups of sellers or between groups of markets.

As layer α is bipartite, the following relations are true.

$$X \cap Y = \emptyset$$

and

$$V_\alpha = X \cup Y.$$

This implies that all nodes are either sellers or markets, and that no nodes are both.

An ordering exists on the elements of V_α with $v_1^\alpha = x_1, \ldots, v_N^\alpha = x_N, v_{N+1}^\alpha = y_1 \ldots, v_{N_\alpha}^\alpha = y_M$.

Layer β represents a transport network. Here nodes represent junctions and locations of players and markets, and edges represent routes between these junctions. Formally, in layer β we have the (non-bipartite) graph $G_\beta = \left(V_\beta, E_\beta\right)$, where nodes $V_\beta = \{v_i^\beta, i = 1, 2 \ldots N_\beta\}$ and edges $b_{i,j} = (v_i^\beta, v_j^\beta) \in E_\beta$ with N_β the number of nodes in layer β.

It is stated that E_α and E_β are the sets of intra-layer edges. $\mathcal{E}_{\alpha\beta}$ is defined to be the set of inter-layer edges between layers α and β.

For a pair of nodes v_i^β and v_j^β, there exists a set of k paths $P_{i,j}$, with $p_{i,j}^k$ the k^{th} path between them. Each path is a unique set of edges $b_{q,r} \in E_\beta$ that connects v_i^β and v_j^β. Edges can be used in multiple paths and each path is minimal, such that there does not exist $b_{q,r} \in p_{i,j}^k$ such that $p_{i,j}^k \backslash b_{q,r}$ still connects v_i^β and v_j^β. The set of all paths in layer G_β is defined to be P_β. The number of paths between i and j is given by $N_{P_{i,j}}$.

The flows on edges in E_α, E_β and of paths in P_β are defined as

$$f(\cdot) : E_\alpha \cup E_\beta \cup P_\beta \to \mathbb{R}_+ \tag{4.3}$$

and represent the amount of goods being sold by a player to a market and the amount of goods being transported along an edge or path for layer α and β. $f_i(\cdot)$ gives the flow from strategies of player i on an edge or path.

Wardrop Equilibrium

The Wardrop Equilibrium is a result which gives the stable equilibrium in a non-cooperative transport model. When a system is in Wardrop Equilibrium, no marginal unit of traffic (such as a driver or shipment) will receive a reduction in costs by switching to any other route [65]. This means that for u_i giving the transport costs of player i where $p^{k_1}_{j_1,j_2} \neq 0$ and $p^{k_2}_{j_3,j_4} \neq 0$,

$$\frac{\partial u_i}{\partial p^{k_1}_{j_1,j_2}} = \frac{\partial u_i}{\partial p^{k_2}_{j_3,j_4}} \text{ for all } j_1, j_2, j_3, j_4. \tag{4.4}$$

This implies that for any given player i, each path has the same marginal cost, meaning there is no benefit to changing their strategy.

4.1.2 Game Definition

Cournot competitions have been briefly introduced in section 2.2.2. They are a form of economic competition in which players decide the quantity of goods they will make available; the total quantity produced by all players results in a price. Networked Cournot competition [61] extends this by including multiple markets. It is appropriate to the dynamics of the players in the upper layer with respect to each market as the following conditions are satisfied:

- the game has multiple players;
- there is no collusion between players;
- each player has market power, that is, each player strategy changes the price in the market they participate in;

- the number of players and markets are fixed and the goods sold by all players are homogeneous;

- players choose the quantity of goods to sell rather than their price, which is a consequence of the total amount of good sold;

- players engage in rational behaviour.

In addition to the standard Cournot conditions, the condition of fixed total production is included. This modifies the original form of the Cournot game from quantity selection in a single market to the optimal distribution of quantity among several markets. Each market in isolation acts as a Cournot competition, but to increase the quantity sold to that market, a player must decrease their contribution to other markets.

An important characteristic is that of the market's price, which can be found using the so-called reserve prices of a spectrum of individuals in the market, yielding a demand curve. A reserve price (or the indifference price) is the maximum price the market is willing to pay for a good [5, 6]. In each market it is assumed that there will be a range of reserve prices held by the spectrum of buyers. This leads to a functional relationship between the supply of goods and the price at which they will all be sold.

Associated with each market y_j is a reserve price function A_{i,y_j} of the form

$$A_{i,y_j}(\cdot) : \mathbb{R}_+ \to \mathbb{R}$$

which maps from a positive supply to a profit (which could be negative). As supply increases price will monotonically decrease. Players seek to maximise their profit by splitting their sales between the available markets in order to receive maximal utility. The relationship between supply and price per unit can be seen in figure 4.2. The demand curve is often explained as giving the quantity demanded, dependent on the price the good is being sold for. Greater quantity is demanded when there are lower prices, as a greater number of potential consumers have the product available for less than the amount they value the good, their 'reserve price'. The same relationship is true in reverse however, and companies will choose a price that maximises their profit based on the supply they make available to the market. Excess profit refers to the additional value gained by buyers in the market who are able to obtain the goods for beneath their valuation.

Figure 4.2: *A sample demand curve with profit in green and excess profit in orange.*

4.1.3 Utility Equations

Each player x_i strives to optimize the difference between their income and the costs incurred in selling their goods. This multilayer game, as in [158], includes the following key aspects:

- The costs incurred related to the utilization of transport links. These costs increase as the transport links become more congested due to higher usage.

- The income derived from selling goods to markets. Players earn profit based on the supply and demand dynamics within those markets.

The utility function for each player is defined as follows:

$$u(s_i, s_{-i}) : S_i \times S_{-i} \to \mathbb{R}_+. \tag{4.5}$$

Here, s_i represents the current strategy of player x_i, S_i is the set of all possible strategies for player x_i, s_{-i} is the current strategy of all other players, and S_{-i} is the set of all possible strategies for these players.

This is a dynamic game with players moving asynchronously. Players sequentially update their strategies to optimize their responses to all other player's strategies. Players consider not just the immediate future, but seek to maximize their utility over multiple rounds, taking into account the potential future strategies of other players. The update order starts with player 1 (located at x_1), followed by player 2, and so on, looping back to player 1 after player N has updated their strategy.

Each player's strategy is a combination of actions from both the upper (player/market) layer and a lower (transport) layer, and includes:

- Choosing an allocation of markets to sell goods in the upper layer. This involves a mixed strategy in the form of $[f(a_{i,1}), f(a_{i,2}), \ldots, f(a_{i,M}))]$ for $a_{i,j} \in E_\alpha$.
- Choosing an allocation of transport paths to transport the goods in the lower layer. This involves a mixed strategy in the form $[f(p^1_{i,1}), \ldots, f(p^{(N_{P_{i,1}})}_{i,1}), \ldots, f(p^1_{i,m}), \ldots, f(p^{(N_{P_{i,m}})}_{i,N_c})]$.

This results in a strategy represented as:

$$s_i = \left([a_{i,1}, a_{i,2}, \ldots, a_{i,M}],\ [p^1_{i,1}, \ldots, p^{(N_{P_{i,1}})}_{i,1}, \ldots, p^1_{i,m}, \ldots, p^{(N_{P_{i,m}})}_{i,N_c}] \right). \quad (4.6)$$

The utility, income minus costs, for a player x_i is calculated as:

$$u_i(s_i, s_{-i}) = A_i(s_i, s_{-i}) - B_i(s_i, s_{-i}), \quad (4.7)$$

$$A_i(s_i, s_{-i}) = \sum_{j=1}^{M} A_{i,m_j}(s_i, s_{-i}). \quad (4.8)$$

Here, $A_i(s_i, s_{-i})$ represents the total profit made in the upper layer by player i, and $A_{i,m_j}(s_i, s_{-i})$ denotes the profit made by player i in market j. Similarly, $B_i(s_i, s_{-i})$ accounts for the congestion cost paid by player i. The choice of congestion functions is not restricted beyond requiring a monotone increase, continuity and convexity.

4.1.4 A Minimal Complexity Example

Consider a group of players aiming to profit from various markets. They receive updates on the current market state sequentially and aim to maximize their utility. The cost from market saturation c_i for each player i is determined as follows in a N-player M-market game:

$$c_i = \sum_{k=1}^{M} f_i(e_k) \left(\sum_{j=1}^{N} f_j(e_k) \right). \tag{4.9}$$

In equation 4.9, $f_j(e_k)$ represents the amount of traffic that player j assigns to route k, N is the total number of players and M is the total number of markets. The choice of an inverse linear demand function, as expressed in equation 4.9, is a commonly used assumption in the literature [46, 77]. In the 2-player, 2-market game, there is only one degree of freedom, with $f_i(e_1) + f_i(e_2) = 1$. This gives $f_i(e_2) = 1 - f_i(e_1)$. Substituting this in gives an equation:

$$c_i = f_i(e_1) \left(\sum_{j=1}^{2} f_j(e_i) \right) + (1 - f_i(e_1)) \left(2 - \sum_{j=1}^{2} f_j(e_i) \right) \tag{4.10}$$

Remark 1. This model is time independent and is symmetric to all players. As such, all players will use the same method to determine optimal strategy on their turn when they consider the same problem.

Players update their responses sequentially beginning with player 1, continuing to player N and returning to player 1 afterwards. Beginning with player 1, the situation before player 1 changes strategy is considered to be the set of strategies at time $t = 0$. Accordingly at the time step t, the player i has just updated where $t = i (\text{mod } N)$.

Without loss of generality we can consider the player who is about to respond to be player 1 at $t = 0$. Given a starting situation $[-, s_2, s_3, \ldots, s_N]$ player 1 wants to choose the strategy s_1 which minimises their costs according to their patience β or up to a particular time horizon t_h. One round refers to the time taken for all players to update their strategy once, and is equivalent to $t_h = N$, with player 1 being interested in maximising their utility (minimising their costs) at time steps $1, 2, \ldots, N$ and beyond.

4.2 Equilibrium Dynamics

We shall first consider the characteristics of the layers when examined individually (such that there is no feedback from the other layer) to then go on to understand the behaviour of the model as a whole.

4.2.1 Upper Layer Dynamics - Approach to Cournot Equilibrium

To analyse the dynamics within the upper layer, let us begin by establishing an assumption that will later be relaxed: the allocation of flows across all paths in the lower layer connecting the same origin and destination remains fixed. In other words, for all k and l, each within the range of 1 to $N_{p_{ij}}$, and for all i and j within the range of 1 to N and 1 to M respectively, it holds that $f(p_{ij}^k) = f(p_{ij}^l)$. Furthermore, this allocation remains constant and unaffected by the quantity of goods a player sells in a specific market.

This situation means that, while the transportation costs have an impact on a player's utility, the dynamics in the lower layer are unresponsive to changes in the upper layer. In essence, a player can adjust the volume of goods they sell in each market, but they lack the ability to alter the distribution of flow among paths with the same origin and destination. As a result, the game unfolds exclusively in the upper layer.

If we assume that the game commences from a non-equilibrium state and the players progressively respond optimally to the strategies of other players; we can analyse the utilities over multiple rounds.

Where the utility function is twice differentiable, concave, and strictly decreasing, and the costs of production are twice differentiable, convex and increasing, the Cournot Game has a unique equilibrium [61, Theorem 1]. The assumption is made that there is no cost for production in the upper layer. As such, this cost function is given by the 0-function which satisfies the above condition [61]. Consider the two player, two markets case with a profit function $A_{i,m_j}(s_i, s_{-i})$, generally defined as

$$A_{i,m_j}(s_i, s_{-i}) = f(a_{i,j}) \left(Q - \sum_{k=1}^{N} f(a_{k,j}) \right). \tag{4.11}$$

Here Q is a fixed constant giving the profit per unit where there is no supply. As the players aim to maximize their utility without the ability to adjust their production, it is important to note that the magnitude of Q does not impact the analytical outcomes, but is present to represent that profit is positive. The utility that an individual player receives in a market adheres to the curve labeled 'profit in market 1,' which is depicted in Figure 4.3 below.

The profit function above satisfies the conditions of monotonic increase, continuity and twice differentiability outlined by [61]. This implies that the game possesses a unique equilibrium [61, Theorem 1].

To identify this equilibrium in the two-player, two-market game, we can examine the sum of $A_{i,m_1}(f(a_{1,1}), s_{-i})$ and $A_{i,m_2}(1 - f(a_{1,1}), s_{-i})$ for $f(a_{1,1}) \in [0, 1]$ with $Q = 1$. Here, m_1 and m_2 are the markets, and $f(a_{1,1})$ represents the proportion of player 1's production assigned to market 1. Importantly, the sum of $A_{i,m_1}(f(a_{1,1}), s_{-i})$ and $A_{i,m_2}(1 - f(a_{1,1}), s_{-i})$ demonstrates concavity. The equilibrium is located at the maximum point, which occurs when $f(a_{1,1}) = 0.5$, as illustrated in Figure 4.3.

In this case where there are 2 players, the optimal strategy can be found analytically, as shown in the next section, for different time-horizons.

4.2.2 Time-Horizon Preferences

Following the principals of dynamic programming, we are interested in defining strategies for the players which maximise their long term utility according to different preferences. One possible example of this is the myopic response, where players select a strategy in order to maximise the utility received in the most immediate future without regard to long-term performance. Another possible example is of equal weight-

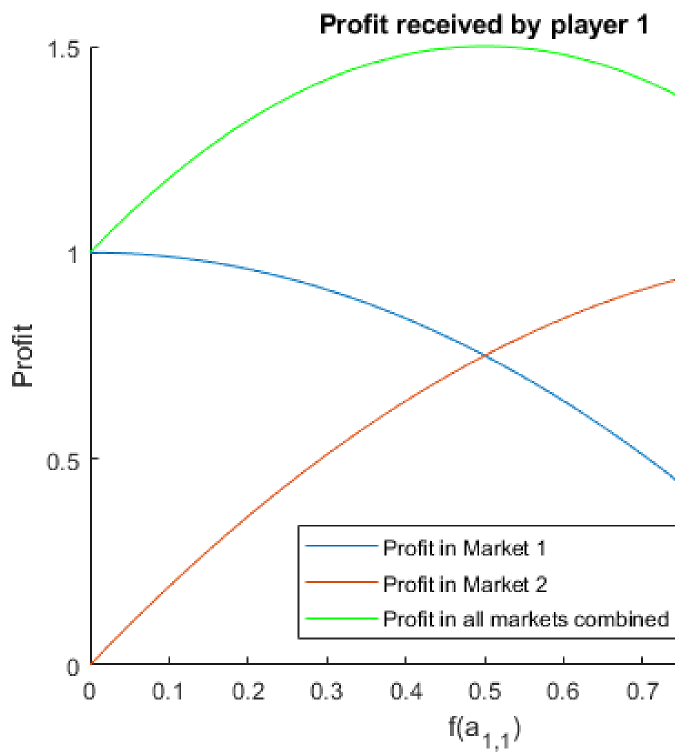

Figure 4.3: *The upper layer utility functions for player i using equation 4.12 before normalisation.*

ing for all possible future time steps. Here a different strategy at the current timestep may be appropriate. One real-world justification for a time-horizon with consistent weighting up to t_h would be of maximising results before the end of the financial year, with timesteps representing months. Another is of general impatience, leading to greater value placed on early outcomes than on longer term ones. This type of strategy is usually characterised by an impatience value $\beta \in [0, 1]$ which modifies the relative weight of each timestep when compared to the previous.

Consider t is the time step, x_t is the strategy played by player x at t and a_t is the strategy played by the opponent at t. Equation (4.12) is drawn from $A_i(s_i, s_{-i})$ and the specific formulation given in Equation (4.11)

$$u_i(s_1, t) = x_t(x_t + a_t) - x_t(-x_t - a_t)$$

$$u_i(s_1,t) = 2x_t(x_t + a_t). \quad (4.12)$$

We shall now show that the best response is time-horizon (t_h) dependent and give proofs of it for time-horizons $1,2$ and the infinite case.

The myopic best response is defined to be the strategy a player will choose if they consider only their utility for a time-horizon $t_h = 1$.

The strategy choices of the players are normalised, such that all strategies are in $-1 \leq s_i \leq +1$. For example, a strategy $[s_1, s_2] = [-0.5, 1]$ represents player 1 selling $\frac{3}{4}$ of their goods to market 1 and $\frac{1}{4}$ to market 2 and player 2 selling all of their goods to market 2. The equilibrium position is represented by 0 for each player. In this case both players contribute equally to each market. The game is assumed to have a history, and accordingly at time $t = 0$, all players have a strategy $[s_1, s_2, \ldots]$. At time $t = 1$, player 1 responds to the position of all other players. As they may choose any response, their previous strategy does not matter. The position of all other players at time $t = 0$ is denoted as a_0, and player 1's response is xa_0, a constant multiple of the other players' positions. At time $t = 2$, player 2 updates their strategy, choosing their best response. In the two player game, this results in strategies $[xa_0, x^2a_0]$.

Myopic Best Response

The myopic best response considers only time-horizon of one. The utility equation for i can then be calculated, and due to its concavity, the unique maximum will be the best response. The best response represents recognising a player is selling too much to one particular market, and switching to a more balanced portfolio.

The set of strategies played by the players at $t = 1$ (the only time step being evaluated) is $[s_1, a_0]$, where s_1 is a strategy in response to a_0. Substituting into Equation (4.12) this results in utility for player 1 of

$$u_1(s_1,t) = -2s_1^2 - 2a_0s_1.$$

This has a maximum where $\frac{\partial u_1(s_1,t)}{\partial x} = 0$.

$$\frac{\partial u_1(s_1,t)}{\partial x} = -4s_1 - 2a_0 = 0$$

and so $s_1 = \frac{-a_0}{2}$, with $x = \frac{-1}{2}$.

As a result, in the 2-player Multimarket Cournot Game with fixed transportation costs and a utility defined in Equation (4.12), the myopic best response to an opponent's strategy $s_{-i} = a_0$ is $\frac{-a_0}{2}$, where $[s_1, a_0]$ is the set of strategies played by the players at $t = 0$.

Time Horizon $t_h = 2$ Best Response

It can be shown analytically that in the 2-player multimarket Cournot Game with fixed transportation costs and a utility defined in Equation (4.12), the optimal response for a time-horizon of 2 moves (one full round), to an opponent's strategy $s_{-i} = a_0$ is $\frac{-a_0}{3}$, which gives $x = \frac{-1}{3}$, where $[xa_0, a_0]$ is the set of strategies played by the players at $t = 0$.

As in the myopic case, the optimum can be found by examining the utility over time steps 1 and 2. Due to the concavity of the utility function, the maximum will be the best response.

Due to the symmetry of player's position upon their turn, as laid out in remark 1, if a player uses strategy xa_0 at $t = 1$, resulting in $[xa_0, a_0]$ then their opponent will respond with x^2a_0 at time $t = 2$, resulting in $[xa_0, x^2a_0]$. The utility over the two timesteps is therefore given by

$$u_1(s_1, 1) + u_1(s_1, 2) = 2a_0^2x^2 + 2a_0^2x + 2a_0^2x^3 + 2a_0^2x^2$$

$$u_1(s_1, 1) + u_1(s_1, 2) = 2a_0^2(x^3 + 2x^2 + x)$$

This has fixed points where $\frac{\partial(u_1(s_1,1)+u_1(s_1,2))}{\partial x} = 0$.

$$\frac{\partial(u_1(s_1, 1) + u_1(s_1, 2))}{\partial x} = 2a_0^2(3x^2 + 4x + 1) = 0$$

has roots at $x = -1$ and $x = \frac{-1}{3}$. The root at $x = -1$ corresponds to both players repeatedly playing a market strategy equivalent to one another of the form $[-a_0, a_0]$. A strategy of $-a_0$ is a local minimum and players can improve their utility by moving away from it in either direction. For $x < -1$, the utility does increase. However this represents each player exponentially increasing the amount they sell to the market not dominated by their opponent, and recouping their losses when their opponent does the same. Due to the finite nature of markets, this is not a long term strategy and can be disregarded.

The root at $x = \frac{-1}{3}$ is the a local maximum and accordingly represents the best strategy for the time-horizon of 2.

Long Time Horizon Best Response

In the 2-player Multimarket Cournot Game with fixed transportation costs and a utility defined in Equation (4.12), the optimal response x considering for a time-horizon of $t_h \geq 1$ moves are such that $\frac{-a_0}{2} \leq \frac{-a_0}{3}$, where $[x, a_0]$ is the set of strategies played by the players at $t = 1$ in response to a starting position $[y, a_0]$ at $t = 0$. Player 1's position y does not have an impact on strategy selection, and therefore disappears.

As above, the optimum can be found by examining the utility over time steps 1 to t_h. Due to the concavity of the utility function, the maximum will be the best response. Figure 4.4 shows this for $1 \leq t_h \leq 100$, showing convergence to $\frac{-1}{1+\sqrt{2}}$. In fact, theorem 1, proved below, confirms this analytically for the limit case $t_h \to \infty$.
Theorem 1. In the 2-player Multimarket Cournot Game with fixed transportation costs and a utility defined in Equation (4.12), the optimal response for an infinite time-horizon is $\frac{-a_0}{1+\sqrt{2}}$, where $[x, a_0]$ is the set of strategies played by the players at $t = 0$.

The optimum can be found by examining the utility over all time steps due to the concavity of the utility function; the maximum will be the best response. $\frac{-1}{1+\sqrt{2}} = 1 - \sqrt{2}$, but the first is preferred due to its similarity with the optimal response for $t_h = 1$, $\frac{-1}{2}$ and $t_h = 2$, $\frac{-1}{3}$
Proof. For large t_h, the structure of the successive terms of the utility function giving the total value over all $t \leq t_h$ remains the same and is given by u_1. The fixed point therefore occurs at $\frac{\partial u_1}{\partial x} = 0$.

$$\frac{\partial u}{\partial x} = \frac{\partial(u_1(s_1,0) + u_1(s_1,1) + u_1(s_1,2) + \cdots + u_1(s_1,t_h))}{\partial x} = 0$$

$$\frac{\partial u}{\partial x} = (t_h + 1) \cdot x^{t_h} + \left(\sum_{i=1}^{t_h - 1} 2(i+1)x^i \right) + 1 = 0. \tag{4.13}$$

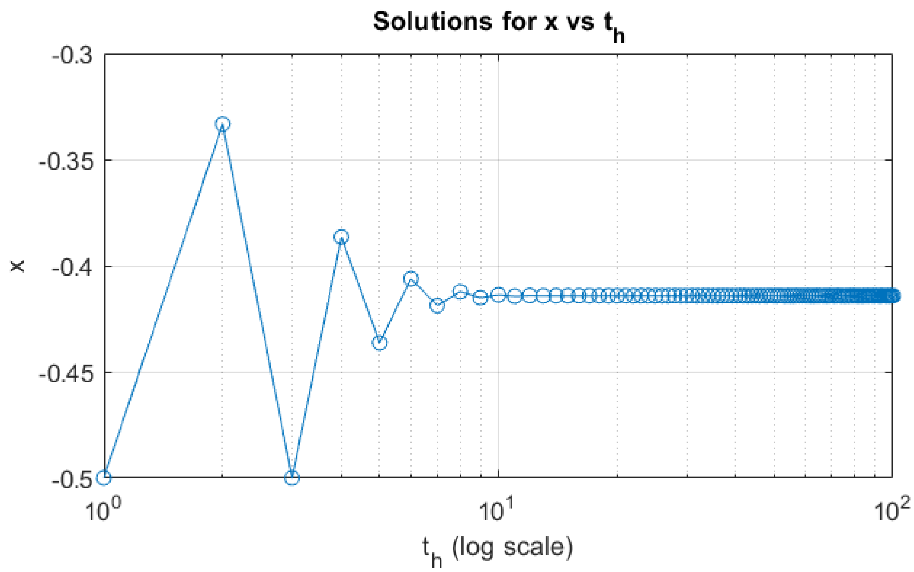

Figure 4.4: *The set of x corresponding to the optimal response xa_0 for $1 \leq t_h \leq 100$, converging to $\frac{-1}{1+\sqrt{2}}$. Analytic convergence is proved in theorem 1.*

This is the sum of successive terms of 4.12 for t_h time steps, taking into account that, when a player updates their strategy, all future time steps are affected by that update. We can use the identity

$$\frac{1}{(1-x)^2} = \sum_{n=1}^{\infty} \left(nx^{(n-1)} \right), \tag{4.14}$$

as the only values of x which make sense are $-1 < x < 1$. By substituting Equation (4.14) into Equation (4.13), we get

$$\frac{\partial u_1}{\partial x} = 2 \sum_{n=1}^{\infty} \left(nx^{(n-1)} \right) - 2 + 1 = 0$$

$$\frac{\partial u_1}{\partial x} = \frac{2}{(1-x)^2} - 2 + 1 = 0.$$

A fixed point where $\frac{\partial u_1}{\partial x} = 0$ occurs at $x = \frac{-1}{1 \pm \sqrt{2}}$. Of these, we need only consider the negative case, $x = \frac{-1}{1+\sqrt{2}}$ as the alternate solution results in players receiving negative utility relative to the equilibrium at all time steps. Accordingly, a response $x \cdot a_0$ where $x = \frac{-1}{1+\sqrt{2}}$ is optimal for an infinite time-horizon.

$\square$

Convergence occurs quickly, with the difference between x and $\frac{-1}{1+\sqrt{2}}$ less than 0.001 by $t_h = 20$.

Best response for patience factor β

Proposition 4.2.1. In the 2-player Multimarket Cournot Game with fixed transportation costs and a utility defined in Equation (4.12), the optimal response where each timestep is given weight proportional to β^t for $\beta \in (0,1)$ is given by

$$x = \frac{-(\beta + 1 - \sqrt{\beta^2 - \beta + 1})}{3\beta}.$$

where x is the constant representing the magnitude of the response, such that if S is given by $[-, a_0]$, the response is given by $[xa_0, a_0]$.
Proof. Utility is given by the current player's response x to the current state of the system. The utility given by the response from the next two time steps is given by

$$u_1(s_1, 1) = x(1-x)a_0(x-x^2)a_0^2$$
$$u_1(s_1, 2) = \beta x(x^2 - x)a_0^2$$

The best response is given at the maximum of $\omega = u_1(s_1, 1) + u_1(s_1, 2)$. This is found at the maximum of ω, which occurs where $\frac{d\omega}{dx} = 3\beta x^2 - 2(\beta+1)x + 1 = 0$, which has two roots. The positive root represents a solution which requires progressively more extreme responses to their market, outside the allowable range. As above, given the finite nature of trade goods, this root can be discarded. Accordingly, the maximum is given at the negative root, at $x = \frac{-(\beta+1-\sqrt{\beta^2-\beta+1})}{3\beta}$.

Collated solutions

Optimal response for each β value and for where this a threshold are given in table 4.2.

Case	β	Optimal Response
Myopic	$\beta = 0$	$\frac{-a_0}{2}$
Infinite	$\beta = 1$	$\frac{-a_0}{1+\sqrt{2}}$
One Round	N/A	$\frac{-a_0}{3}$
Threshold	N/A	Figure 4.4
Fixed Impatience	$\beta \in (0, 1)$	$\frac{-a_0(\beta+1-\sqrt{\beta^2-\beta+1})}{3\beta}$

Table 4.2: *Optimal Responses for Different Impatience Characteristics*

4.2.3 Robustness of Competition during the Approach to Equilibrium

We are also interested in the incentives to the players between cooperation and competition in the immediate aftermath of a disruption. We examine a marginal case, finding that a marginal change in strategy towards competition provides a guaranteed increase in utility. This proves that that a cooperative response is never optimal for the lead player, and accordingly, the other players when they become the lead player.

We now introduce and define the equilibrium mimicking strategy.
Definition 1. An equilibrium-mimicking strategy is one that results in all markets receiving the same supply that they would if the game was in stable equilibrium.

Player strategies are normalised with respect to an equilibrium mimicking strategy, wherein players consistently play $[0, 0]$. Figure 4.5 shows the normalised strategies where each player adopts a myopic best-response strategy to their opponent's strategy. From a starting position, players respond in a predictable manner, which leads to an approach to the equilibrium in the centre of the graph. The bounds on convergence are highlighted in red, and they become applicable after a single response. These represent the response to any given strategy by the opponents. Red crosses indicate some initial starting points, from which both convergence paths are illustrated, contingent on which player takes the initial action.
Theorem 2. In the two-player, two-market game, let $A_i(\cdot, \cdot)$ be the utility received by the first player in the upper layer as defined in Equation

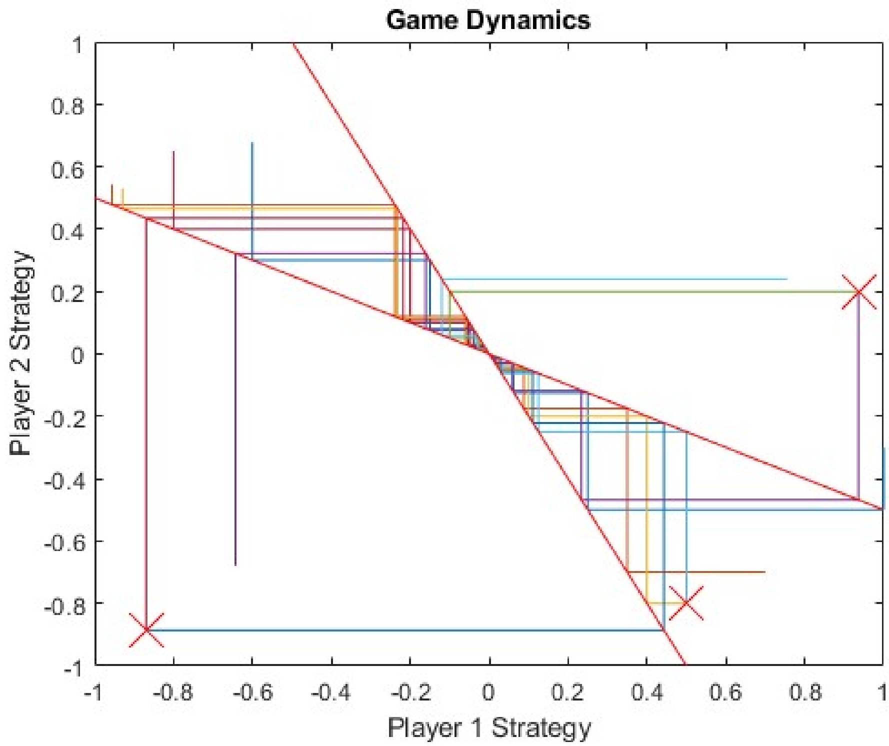

Figure 4.5: *The trajectories of strategy for discrete time steps from a variety of starting points.*

(4.8) and let $g_i \geq 0$ be the difference in utility gained after the first move as a myopic best response, compared to the utility the first player would receive if they played an equilibrium mimicking strategy. Then the utility of the first player through successive myopic best responses, compared to the final unique equilibrium strategy is $\frac{8g_i}{15} > 0$.

This implies that rational players will have no incentive to play an equilibrium strategy if all other players have not already done so.
Proof. Each player responds to opponent strategy $s_{-i} = a_0$ with strategy $\frac{-1}{2}a_0$, as derived above, resulting in successive strategy profiles $\left[\frac{-a_0}{2}, a_0\right], \left[\frac{-a_0}{2}, \frac{a_0}{4}\right], \ldots$. With g_i the additional utility gained by the myopic responder relative to the $[0,0]$, the utility gained by the same player at each subsequent change in strategy by either player is given by the sequence z_i, with $z_{i+2} = \frac{1}{16}z_i$, with terms $z_1, z_2 = g_i, -\frac{g_i}{2}$, resulting in

$g_i, -\frac{g_i}{2}, \frac{g_i}{16}, -\frac{g_i}{32}, \ldots.$

This is the sum of two infinite geometric sequences, which can be expressed as

$$\sum_{n=1}^{\infty} z_n = \frac{z_1}{1 - (\frac{z_2}{z_1})}.$$

Substituting in z_1 and z_2 from above, we find that this sums to

$$\frac{8g_i}{15} > 0.$$

□

The myopic best response thus proves to be a superior strategy to an equilibrium-mimicking strategy, which can accordingly be rejected, finding that players compete to equilibrium, rather than jumping directly to equilibrium-mimicking strategies.

Theorem 3. In the two-player game, let the utility received by the first player in the upper layer be as defined as in Equation (4.12), and let $g_i \geq 0$ be the difference in utility gained after the first move for an infinite time-horizon optimal strategy (as defined in Theorem 1), compared to the utility the first player would receive at equilibrium. Then the utility of the first player, through successive infinite time-horizon optimal moves, compared to the final unique equilibrium strategy is

$$\frac{g_i(1+\sqrt{2})}{4} > 0.$$

Proof. The optimal response for $\beta = 1$ is $\frac{-1}{1+\sqrt{2}}a_0$, leading to a succession of strategies $[\frac{-1}{1+\sqrt{2}}a_0, a_0]$, $[\frac{-1}{1+\sqrt{2}}a_0, (\frac{-1}{1+\sqrt{2}})^2 a_0]$. This results in a sequence z_i of utilities received by the first player (normalised with respect to the equilibrium utility), expressible as two infinite geometric series alternating in z_i, with $z_{i+2} = (1-\sqrt{2})z_i$, with terms $z_1, z_2 = g_i, g_i(1-\sqrt{2})$. Again, using the formula on these two infinite geometric sequence

$$\sum_{n=1}^{\infty} z_n = \frac{z_1}{1 - (\frac{z_2}{z_1})},$$

it is found that the sequence sums to $\frac{g_i(1+\sqrt{2})}{4}$.

$\square$

It can be seen that, as expected, $\frac{1+\sqrt{2}}{4} > \frac{8}{15}$, demonstrating that playing a strategy which optimises for an infinite time horizon results in a higher utility for the player than the myopic strategy when considering a large number of time steps.

Characteristics under Player Collusion

An assumption made in the classical Cournot problem is the absence of collusion between players, and each chooses their strategy independently. A group of players engaging in collusion is described as a 'coalition'. Members of a coalition improve their utility by rejecting strategies that are not Pareto-optimal for the subset of players in the coalition, in favour of acting to mutually increase their utilities.

We shall distinguish between 'equilibrium play' and 'advantage play'. 'Equilibrium play' is the specific strategy which players immediately move to the strategy they would play in the final equilibrium. This fixes their utility preventing them from making both losses and profits relative to this fixed amount. In comparison, 'advantage play' is any other strategy in which players attempt to profit from the current out of equilibrium state.

Proposition 4.2.2. For any player count N, where the starting conditions are not at equilibrium, there exists a Pareto optimum strategy where equilibrium play is dominated by advantage play.

Proposition 4.2.2 implies that where players work together, equilibrium play is never optimal. To prove that, we refer to the definition of Pareto optimality given in section 3.1.4

Proof. Assume towards a contradiction that there is no Pareto-optimal strategy wherein any players benefit from engaging in advantage play. Accordingly, all players will play an equilibrium strategy, receiving utility u. Consider player i. The system is not currently at equilibrium and accordingly $\frac{\partial u_i}{\partial x_i} \neq 0$. As such, either $x_i = +\epsilon$ or $x_i = -\epsilon$ results in a utility $u^* > u$. This results in the first player engaging in advantage play, and accordingly the system is not in equilibrium for the strategy selection of the next player to act and they are faced with the same decision. Each player would therefore follow this strategy, resulting in all players receiving utility greater than u. This would mean that every player received positive utility relative to the equilibrium case. As such, equilibrium play can be rejected in favour of advantage play, with early acting players cooperating to receive maximal benefit before responses by later players.

4.2.4 Lower Layer Dynamics - Approach to Transport Equilibrium

Concentrating on the lower layer, consider the upper layer transport dynamics to be fixed such that there is a fixed set of flows to be allocated between a series of origin-destination pairs through the lower layer.

We define Total Transport to be the amount of goods a player has available to allocate between all markets they have access to.

Consider a transport problem as a congestion game such that, in its simplest form, the lower layer has two nodes v_1 and v_2, and two symmetric paths, e_1 and e_2, each from v_1 to v_2. There are two routes from the origin to the destination, and N players all located in node v_1, attempting to minimise their transport costs to the markets, which are all located at node v_2. This bilayer network is visible in figure 4.6. A fixed total transport quantity T_i is associated with each player, where T_i is a fixed constant such that

$$\sum_{j=1}^{2} f_i(e_j) = T_i$$

for all i in $1, \ldots, N$, where N is the number of players. As such, the seller's output is bounded, but the market inputs and the capacity of the transport network are unbounded. The strategy of players in the upper layer is fixed and so there is no feedback loop from the outcome of the transport problem to the upper layer. First consider the case in which the cost to player x_i for using edge e_j is given by

$$c_i = \sum_{j=1}^{M} f_i(e_j) \sum_{k=1}^{N} f_k(e_j) \tag{4.15}$$

where $f_i(e_j)$ is the flow of player x_i on edge e_j. This is the same quadratic cost structure as is introduced in section 2 of [61]. Quadratic costs are chosen following a precedent established in the literature [46, 77].

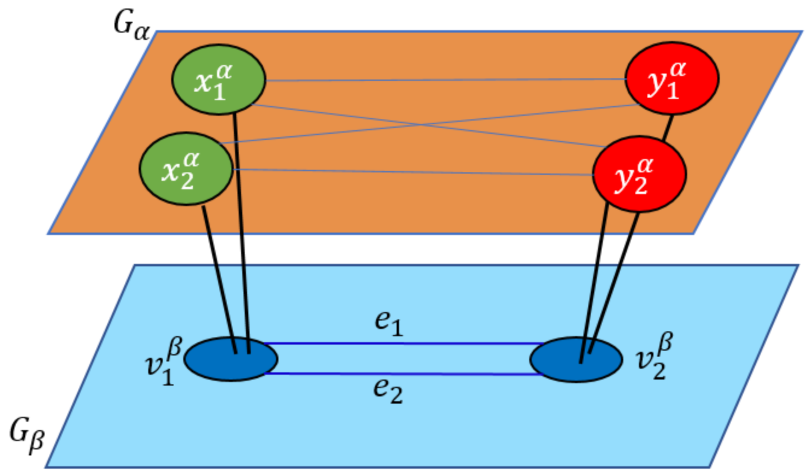

Figure 4.6: *A Network with two paths through the lower layer.*

The set of strategies in s_i has one degree of freedom as

$$f_i(e_2) = T_i - f_i(e_1).$$

This means that the strategy space s_i available to player i has only one degree of freedom. As such it can be represented by a single value. We set this value to be the amount of transport sent along route e_1. For generality, $s_i \in [-1, 1]$. This is normalised such that $s_i = -1$ represents a pure strategy in which $f_i(e_1) = 0$, $s_i = 1$ corresponding to the pure strategy in which $f_1(e_1) = T_1$ with all mixed strategies corresponding to $s_i \in (-1, 1)$. As a result, the equilibrium position is represented by $s_i = 0$. We can now state proposition 4.2.3. This states that, when transportation costs are monotonically increasing as flow increases and the amount of transport between player i and market j in the upper layer is a fixed constant, Wardrop equilibria can be found in the lower layer.

Proposition 4.2.3. For a congestion function $B(f(\cdot))$ which increases monotonically with flow $f(\cdot)$, and $\mathrm{f}(\mathrm{e}^{\alpha}_{i,j}) \in \mathbb{R}_+$ for $\mathrm{e}_j \in E_\beta$, Wardrop Equilibria exist in the lower layer.

Proof. Consider the rate of change of cost with respect to changes made in a player's transport strategy, given by

$$\frac{\partial c_i(s_i, s_{-i})}{\partial p^k_{i,j}}. \tag{4.16}$$

Here, $p_{i,j}^k$ is the k^{th} path between nodes i and j. When considering a change in the paths selected, a player can examine the marginal change in congestion cost from doing so. Increasing the amount the player uses a path will always result in an increase in costs. Due to the fixed upper layer however, $f(a_{i,j})$ is constant and

$$\sum_{k=1}^{N_{P_{i,j}}} p_{i,j}^k = f(a_{i,j}) \tag{4.17}$$

is fixed. Here, $N_{P_{i,j}}$ is the number of paths between i and j. As such any additional traffic along one path will result in a proportional reduction in traffic along a parallel path or paths. For a constant q_{ij} specific to each player-market pair, the path allocation is therefore in equilibrium when

$$\frac{\partial c_i(s_i, s_{-i})}{\partial p_{ij}^k} = q_{ij},\ q_{ij} \in \mathbb{R}_+, p_{ij}^k \neq 0 \text{ for all } i, j, k, \tag{4.18}$$

as no player gains a benefit from changing which paths they allocate their transport on due to the monotonic nature of congestion function $B(f(\cdot))$. This results in all paths available to a player having the same cost, fulfilling the conditions of a Wardrop equilibrium.

□

The inter-layer connections provide a feedback loop that, so far, here and in the literature has not been considered. It is important to note that, for Theorem 4.2.3, paths must exist in the transport layer G_β between $L(x_i^\alpha)$ and $L(y_j^\alpha)$ for which $f(a_{i,j}) > 0$. In the complete model however, when a path between $L(x_i^\alpha)$ and $L(y_j^\alpha)$ does not exist, the transport layer would give infinite congestion cost for trade between x_i and y_j, resulting in $f(a_{ij}) = 0$. As such, any trade between this player-market pair in the upper layer would stop. The players would then always have an incentive to change to another strategy in the upper layer creating a beneficial feedback loop wherein players only sell to markets they can reach.

4.2.5 Transport costs over multiple paths

Transport costs relate to edges and for each player, the cost relates to the share of transport over a number of paths. As the players allocate transport to different paths, the combined transport cost loses differentiability. In the case of proposition 4.2.4 each player wants to use different paths for which the individual transport cost is continuous and twice differentiable, the result over all paths is continuous and differentiable, but not twice differentiable. This allows us to state the following theorems.

Proposition 4.2.4. Transportation costs are continuous and differentiable, but not twice differentiable.

Proof. Consider a player attempting to minimise transportation costs from $L(x_i)$ to $L(y_j)$ across 3 three routes. The flows on the routes are given by r_1, r_2, and r_3 with the cost on each route given by $r_1(r_1 + l_1)$, $r_2(r_2 + l_2)$, and $r_3(r_3 + l_3)$ and without loss of generality $l_1 \leq l_2 \leq l_3$, with l_k a constant for all k. The total transportation cost is then given by

$$t : \mathbb{R}^3 \to \mathbb{R}.$$

The total flow which x_i routes between $L(x_i)$ and $L(y_j)$ is f and as such $r_1 + r_2 + r_3 = f$. The minimum of $t(r_1, r_2, r_3)$ is given by $T(f)$.

$$T : \mathbb{R} \to \mathbb{R}.$$

The marginal rate of change of t with respect to each edge are given by:

$$\frac{\partial t}{\partial r_1} = 2r_1 + l_1, \quad \frac{\partial t}{\partial r_2} = 2r_2 + l_2, \quad \frac{\partial t}{\partial r_3} = 2r_3 + l_3.$$

As such, there are changes of behaviour of $T(f)$ at $f = \frac{l_2 - l_1}{2}$ and at $f = \frac{2l_3 - l_1 - l_2}{2}$.

$$T(f) = \begin{cases} t(f, 0, 0), & \text{for } f < \frac{l_2 - l_1}{2}; \\ t\left(\frac{f}{2} + \frac{l_2 - l_1}{4}, \frac{f}{2} - \frac{l_2 - l_1}{4}, 0\right), & \text{for } \frac{l_2 - l_1}{2} < f \\ & < \frac{2l_3 - l_1 - l_2}{2}; \\ t\left(\frac{f}{3} + \frac{-2l_1 + l_2 + l_3}{6}, \frac{f}{3} + \frac{l_1 - 2l_2 + l_3}{6}, \frac{f}{3} - \frac{2l_3 - l_1 - l_2}{6}\right), & \text{for } f > \frac{2l_3 - l_1 - l_2}{2}. \end{cases} \tag{4.19}$$

This is continuous as

$$\lim_{f \to \frac{l_2 - l_1}{2}^-} T(f) = \frac{l_2^2 - l_1^2}{4} = \lim_{f \to \frac{l_2 - l_1}{2}^+} T(f)$$

and

$$\lim_{f \to \frac{2l_3 - l_1 - l_2}{2}^-} T(f) = \frac{-l_1^2 - l_2^2 + 2l_3^2}{4} = \lim_{f \to \frac{2l_3 - l_1 - l_2}{2}^+} T(f).$$

The first derivative is given by

$$\frac{dT}{df} = \begin{cases} 2f + l_1, & \text{for } f < \frac{l_2 - l_1}{2}; \\ f + \frac{l_1 + l_2}{2}, & \text{for } \frac{l_2 - l_1}{2} < f < \frac{2l_3 - l_1 - l_2}{2}; \\ \frac{2f}{3} + \frac{l_1 + l_2 + l_3}{3}, & \text{for } f > \frac{2l_3 - l_1 - l_2}{2}. \end{cases} \tag{4.20}$$

T is differentiable as

$$\lim_{f \to \frac{l_2 - l_1}{2}^-} \frac{dT}{df} = l_2 = \lim_{f \to \frac{l_2 - l_1}{2}^+} \frac{dT}{df}$$

and

$$\lim_{f \to \frac{2l_3 - l_1 - l_2}{2}^-} \frac{dT}{df} = l_3 = \lim_{f \to \frac{2l_3 - l_1 - l_2}{2}^+} \frac{dT}{df}.$$

The second derivative is given by

$$\frac{d^2T}{df^2} = \begin{cases} 2, & \text{for } f < \frac{l_2 - l_1}{2}; \\ 1, & \text{for } \frac{l_2 - l_1}{2} < f < \frac{2l_3 - l_1 - l_2}{2}; \\ \frac{2}{3}, & \text{for } f > \frac{2l_3 - l_1 - l_2}{2}. \end{cases} \tag{4.21}$$

Therefore, $\frac{d^2T}{df^2}$ is not continuous at $\frac{l_2 - l_1}{2}$ and $\frac{2l_3 - l_1 - l_2}{2}$, and accordingly T is not twice differentiable.

$\square$

It has been shown that T is not twice differentiable at the transition between assigning additional flow to 1 path and assigning it to 2 paths and also that T is not twice differentiable at the transition between assigning additional flow to 2 paths and assigning it to 3 paths. By similar reasoning, it can be seen that T is not twice-differentiable at the change in behaviour between any k and $k+1$ paths being used. Between k paths and $k+1$ paths, T will hold the following behaviour,

$$T(f) = \begin{cases} t\left(\frac{f}{k} + \frac{-(k-1)l_1+l_2+\cdots+l_k}{2k}, \frac{f}{k} + \frac{l_1-(k-1)l_2+\cdots+l_k}{2k}, \ldots, 0\right), \\ \qquad \text{for } \frac{2l_k-l_1-l_2-\cdots-l_{k-1}}{2} < f < \frac{2l_{k+1}-l_1-l_2-\cdots-l_k}{2}; \\ t\left(\frac{f}{k+1} + \frac{-(k)l_1+l_2+\cdots+l_{k+1}}{2k+2}, \frac{f}{k+1} + \frac{l_1-(k)l_2+\cdots+l_{k+1}}{2k+2}, \ldots\right), \\ \qquad \text{for } f > \frac{2l_{k+1}-l_1-l_2-\cdots-l_k}{2}. \end{cases} \tag{4.22}$$

While this and the first differential are continuous here, the second differential is not and accordingly there is not twice differentiability.

4.3 Numerical Simulations

In this section the results from two numerical simulations conducted on specific case studies are presented, demonstrating the convergence to equilibrium of the players' strategies, utilities and marginal utilities.

The model employed was developed using custom MATLAB functions, due to the lack of existing tools in the space of dynamic games. These functions were designed to handle utility calculations, player strategy selection, and dynamic game responses. The model takes as inputs an adjacency matrix representing layer β and the initial strategies of all players at time $t=0$. Each turn in the dynamic game operates with a computational complexity of $\mathcal{O}\left(\sum_{j=1}^{M} N_{P_{ij}}\right)$ for active player i, reflecting the interaction between the network structure and player dynamics.

The utility equation for player i is given by:

$$u_i = A_i + B_i, \tag{4.23}$$

where

$$A_i = -\sum_{j=1}^{M} f(e^{\alpha}_{ij}) \left(\sum_{k=1}^{N} f(e^{\alpha}_{kj})\right),$$

and

$$B_i = -\sum_{j=1}^{p} f_i(e_j^\beta)\left(\sum_{q=1}^{N} f_i(e_j^\beta)^2\right),$$

with M the number of markets, N the number of players, p the number of edges in the lower layer, $f(\cdot)$ a metric how much an edge is used and $f_i(\cdot)$ a metric of the amount player i uses an edge.

That is, the costs to player i are given by the amount they sell to each market multiplied by the total amount every player sells to that market and the amount player i uses each edge in the lower layer multiplied by the amount all players use that edge in the lower layer.

4.3.1 3 Player 3 Market Case

Consider the multilayer network given in figure 4.7. There are 3 players and 3 markets. Players 1 and 2 are geographically located in node 1 of the lower layer, and player 3 is located in node 6 of the lower layer. Market 1 is located in node 7 of the lower layer, and markets 2 and 3 are located in node 6. Player 1 chooses from nine strategies available. The nine strategies are drawn from the existence of 3 markets for goods, and 3 routes to each of the markets, characterised as high (travelling through nodes v_2^β and v_3^β, middle (travelling through v_4^β) and low (travelling through v_5^β). Player 2's position is exactly symmetric, with the same 9 strategies available to them. Player 3 however is located such that there is only one appropriate route from them to each of the three markets, and so has 3 strategies available to them.

The player's strategies over time are given in figure 4.9. At time $t = 0$, the players are assigned random strategies. At each timestep, one of the three players updates their strategies, beginning with player 1 at $t = 1$. On a players turn, they examine the marginal change in utility received by increasing their use of each of the strategies available to them. They then increase their usage of the most profitable strategy available and decrease their usage of the least profitable strategy. This is done until they can no longer increase their utility by changing their chosen strategy, and can be seen in figure 4.8. It can be seen that on each player i's turn, the marginal utility received from each strategy

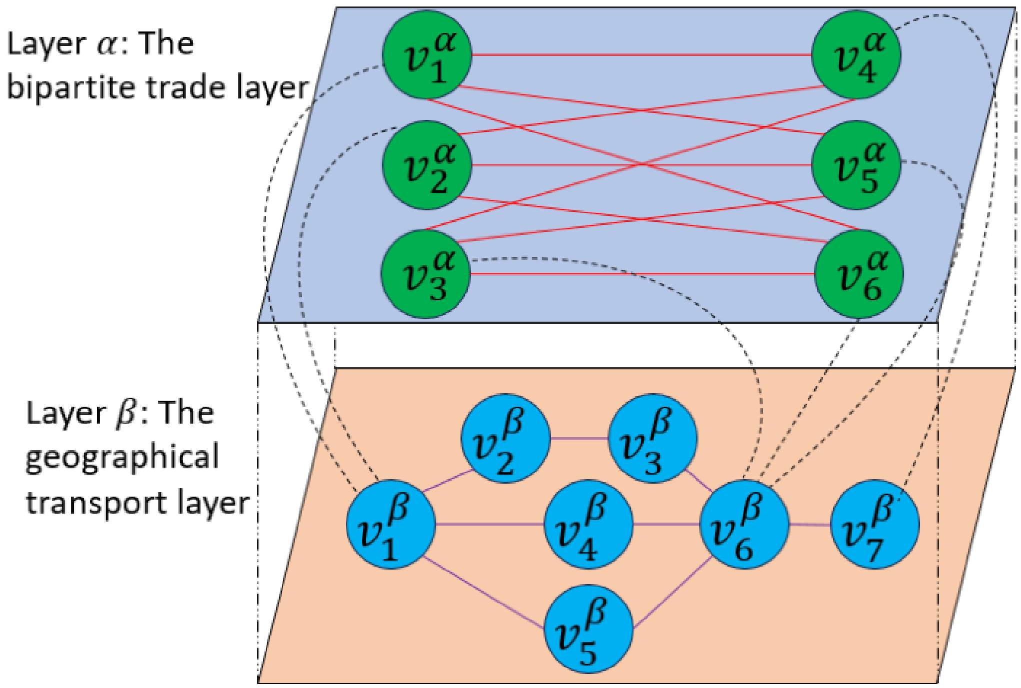

Figure 4.7: *The multilayer network for the case study.*

becomes the same, before changes in the opponent's strategies change these marginals without an opportunity for player i to respond, representing i no longer playing their optimal strategy. It can also be seen that the marginal utilities received converge, such that each strategy by each player generates the same increase. By $t = 15$, a stable equilibrium has emerged. At this equilibrium, players 1 and 2 engage in symmetric behaviour, as expected given their identical positioning on the graph. Considering player 3, it can be seen that they sell the same amount to markets 2 and 3, and a lesser amount to market 1, due to the increased trade costs of accessing it.

A more specific breakdown of player 1's strategies appears in figure 4.10. They prefer to use the middle and lower routes due to their reduced lengths, and use them the same amount due to their symmetry. They also prefer to use markets 2 and 3 due to their relative proximity.

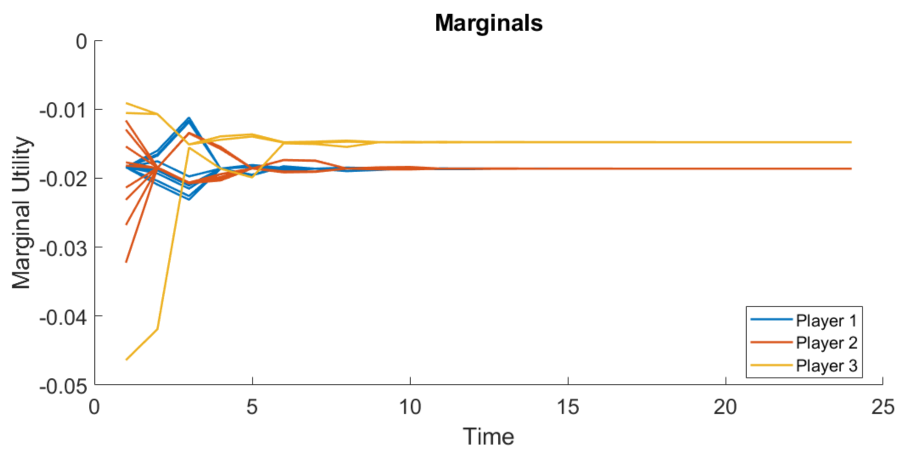

Figure 4.8: *Time against marginal utility for the 21 strategies.*

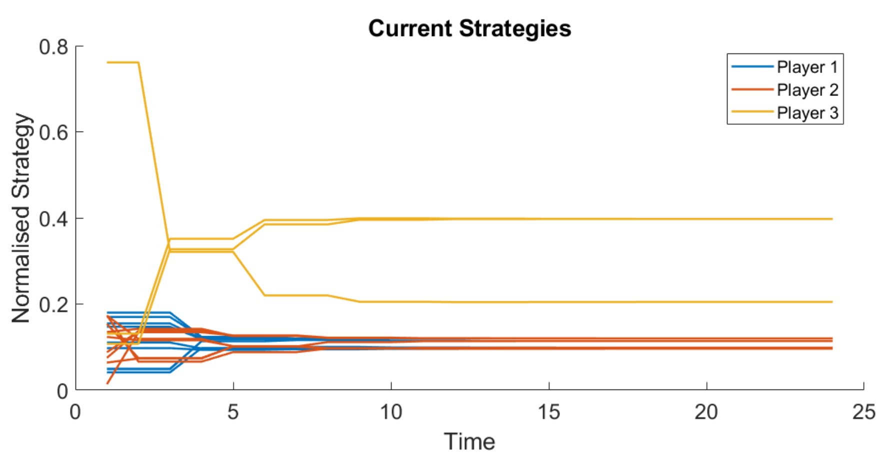

Figure 4.9: *Time against strategy for each of the 21 strategies.*

The utility received by each player at each time step is visible in figure 4.11. Player 3's superior position results in them paying less in transport costs, while player 1 and 2 receive symmetric utility.

It should be noted that market 3's location further away from the player results in fewer goods being transported there by the players, as although there is a higher price there, the increased transport cost balances this out. This can be used to calculate the price in that market.

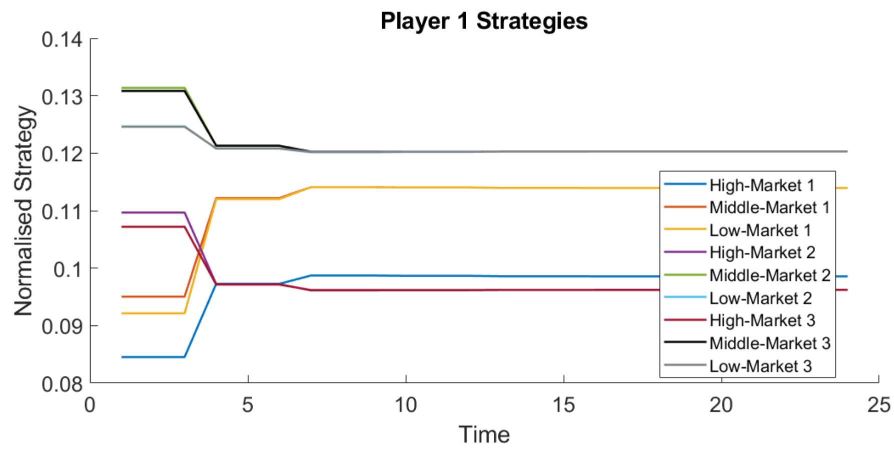

Figure 4.10: *Time against Strategy for each of the 9 strategies for Player* 1.

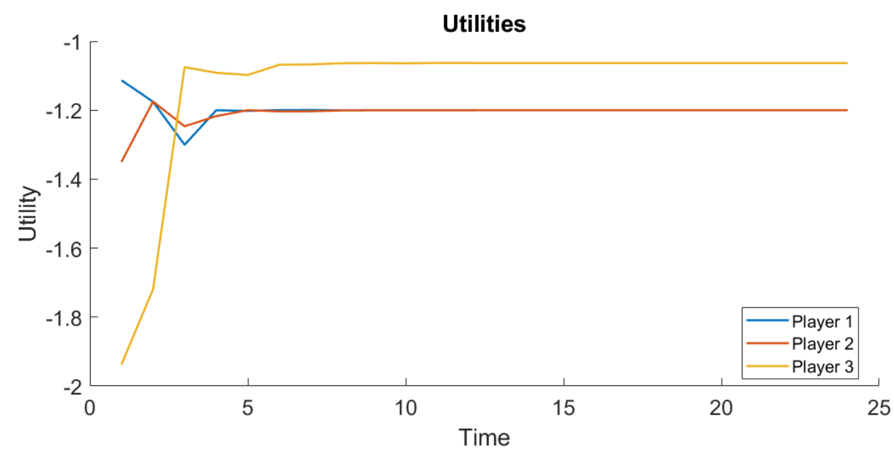

Figure 4.11: *Time Against Utility for three players.*

4.3.2 4 Player 3 Market Case

A second scenario is considered, where there exists a single player and market located distantly from the others, with a network structure as shown in figure 4.12.

The marginal utilities seen in figure 4.13 diverge as successive players act, before returning to equality on each player's turn.

These marginal utility optimisations result in strategies seen in fig-

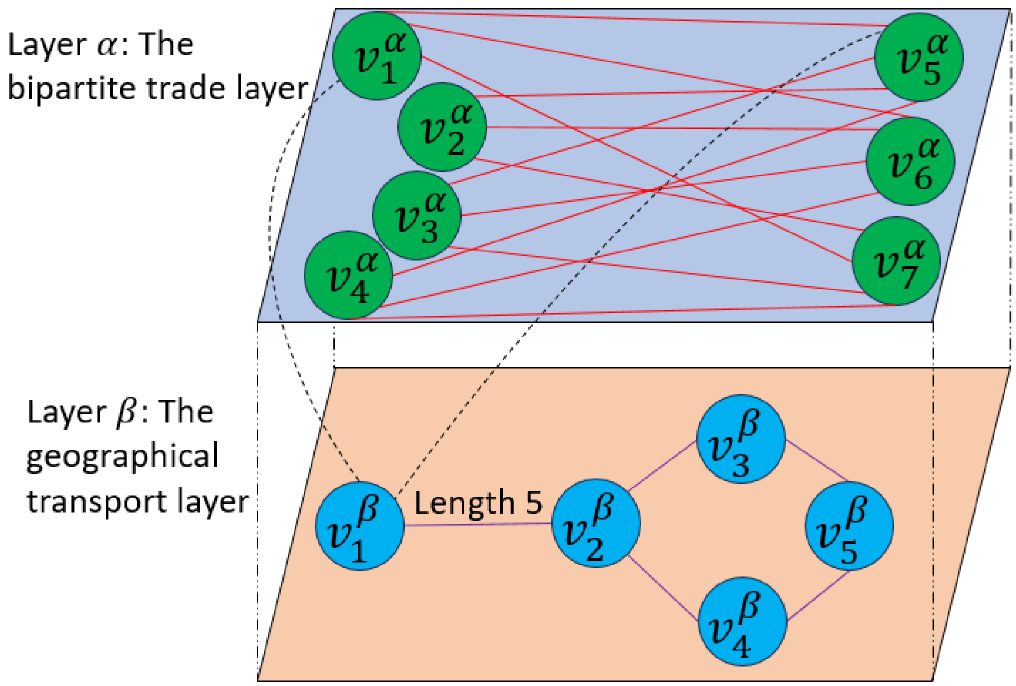

Figure 4.12: *The Multilayer network for the second case study. Players 2,3 and 4 are located in nodes v_2^β, v_4^β and v_5^β respectively. Markets 2 and 3 are located in nodes v_3^β and v_5^β. The edge between v_1^β and v_2^β is length 5, resulting in greater costs for using it.*

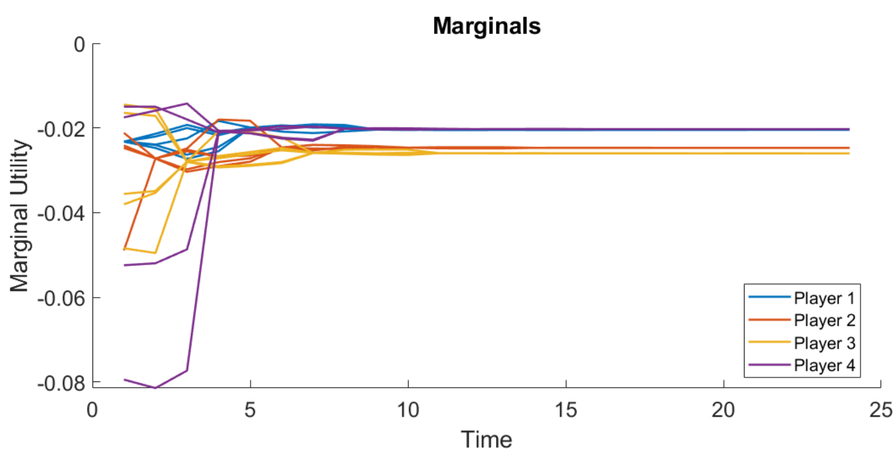

Figure 4.13: *Time against marginal utility for the 19 strategies.*

ure 4.14, which reach equilibrium by $t = 15$. Of note are the two very well-used strategies from Player 1 to Market 1 and Player 4 to Market 3, which in both cases require 0 transport cost B_i. This results in players receiving the utilities seen in figure 4.15, with player 1 benefitting from their relatively unchallenged access to market 1 and player 4 benefitting from the 0 transportation costs to market 3.

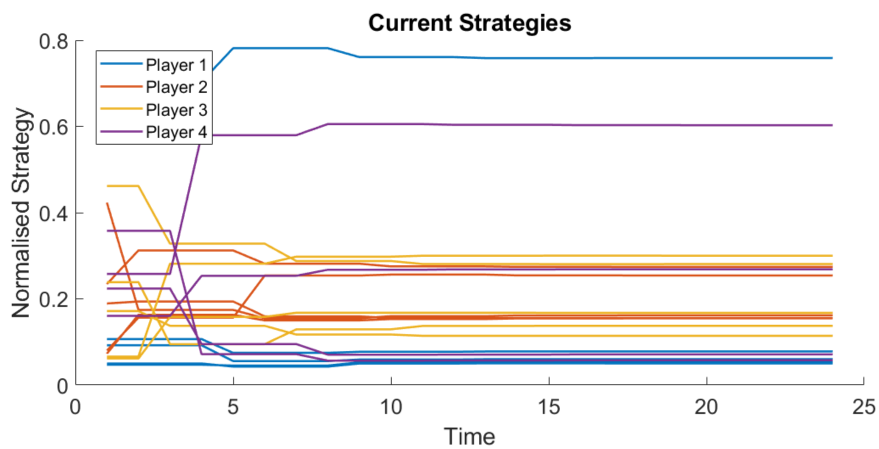

Figure 4.14: *Time against strategy for each of the 19 strategies.*

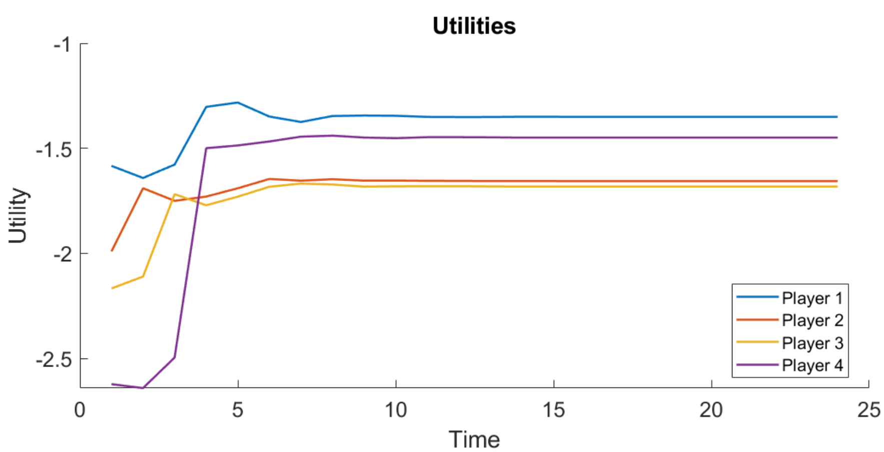

Figure 4.15: *Time Against Utility for four players.*

4.4 Bilayer Solutions

General properties of players' strategies under equilibrium are also presented in order to characterise equilibrium when considering the bilayer problem with the game played simultaneously on both layers.

When examining the interconnected layers, it is important to note that the proof of the existence of unique equilibria, as presented in [61] and utilized in section 4.2.1, is not applicable. This is due to the fact that, while the profit functions maintain their properties of being twice differentiable, convex, and strictly decreasing, the cost functions do not exhibit these characteristics; instead, they lack second differentiability, are concave, and exhibit an increasing trend as demonstrated in section 4.2.5. However, we can establish the necessary and sufficient condition for equilibrium in the multilayer game through the following theorem.

Theorem 4. The Cournot-congestion game achieves equilibrium if and only if, for each player,

$$\frac{\partial u(s_i, s_{-i})}{\partial a_{ij}} - \frac{\partial u(s_i, s_{-i})}{\partial p_{ij}^k} = r(i) \text{ for all } p_{ij}^k \neq 0 \tag{4.24}$$

for $i \in 1, 2, \ldots, N$, $j \in 1, 2, \ldots, M$, $k \in 1, 2, \ldots, N_{P_{ij}}$, and $r(i) \in \mathbb{R}_+$, with $r(i)$ representing a fixed constant for all j and k.

This implies that the system is in equilibrium if and only if each player possesses the same marginal utility for all (non-zero) strategies available to them. This is a unique and stable equilibrium.

Proof. The change in utility for a market and its associated route is given by

$$\frac{\partial u(s_i, s_{-i})}{\partial a_{ij}} - \frac{\partial u(s_i, s_{-i})}{\partial p_{ij}^k} = r(i) \ \forall p_{ij}^k \neq 0 \tag{4.25}$$

Assume towards a contradiction that there exists a system in equilibrium where there exists two market-route pairs for player i such that

$$\frac{\partial u(s_i, s_{-i})}{\partial a_{ij_1}} - \frac{\partial u(s_i, s_{-i})}{\partial p_{ij_1}^{k_1}} > \frac{\partial u(s_i, s_{-i})}{\partial a_{ij_2}} - \frac{\partial u(s_i, s_{-i})}{\partial p_{ij_2}^{k_2}} > 0,$$

and the system is in equilibrium. Here, it is not true that $j_1 = j_2$ and $k_1 = k_2$ simultaneously. By making a marginal from the ${k_2}^{\text{th}}$ path from player i to market j_2 to the ${k_1}^{\text{th}}$ path from player i to market j_1, u_i will increase. This is a contradiction, and accordingly where the system is in equilibrium, equation 4.25 is true.

Conversely, assuming towards an equilibrium that the system is not in equilibrium, with

$$\frac{\partial u(s_i, s_{-i})}{\partial a_{ij_1}} - \frac{\partial u(s_i, s_{-i})}{\partial p_{ij_1}^{k_1}} = \frac{\partial u(s_i, s_{-i})}{\partial a_{ij_2}} - \frac{\partial u(s_i, s_{-i})}{\partial p_{ij_2}^{k_2}} > 0 \text{ for all } i.$$

There must exist an alternate allocation of flow which improves the utility for some player. This player would increase the amount they use a specific route, which decreases the marginal utility given by that market-route pair while increasing the marginal utility given by another. However, due to this monotonicity of change in marginal utility for both market and transport strategies, this change would result in a decrease in utility for any non-marginal change. This is a contradiction, and accordingly, where equation 4.25 is true, the system is in equilibrium.

□

Corollary 4.4.1. The strategy profile S^E which meets the conditions outlined in equation 4.25 is a unique and stable equilibrium.

This means that, given a bilayer graph, there will exist only a single stable equilibrium for trade and transport distribution, allowing the eventual equilibrium of the network to be found assuming no further disruption.

Proof. The stability property is a result of the utility functions for both the upper and lower layer having Lyapunov properties. This results in a dampening effect on deviation from the equilibrium.

□

Theorem 4 and corollary 4.4.1 combined give the existence and characteristics of a unique stable equilibrium. This allows a prediction of the supply and, therefore, the price of goods in each market. It also allows a prediction of the resulting transport equilibrium, as well as the impact that modifying the network will have on future prices and transportation usage.

4.5 Conclusions

This chapter detailed the characteristics of an original bilayer model, combining a multimarket Cournot game with network congested transport dynamics. When examining the behaviour of players in the upper layer in isolation from the lower layer, it was observed that a unique stable equilibrium emerges. Similarly, an investigation of the lower layer revealed the presence of a unique stable equilibrium.

However, when these layers were coupled together, proofs of the existence of a unique stable equilibrium from the literature no longer held, as the assignment of traffic over multiple results results in a utility function which was shown to not be twice differentiable. However, original work was conducted proving the existence of equilibrium within this coupled framework and the dynamics of these equilibria were explored using a link-route representation of the lower layer, as well as by being demonstrated through two numerically solved case studies.

A summary of the key contributions in this chapter can be seen in Table 4.3.

4.5.1 Development from this model

One notable weakness of this model lies in the choices of assumption made with respect to market price and transportation characteristics. These choices follow the typical practice used within the literature, but were not closely linked to a specific application. This limits the utility of the model's results without further baselining. The model contains some consideration of coalitional elements, but did not consider the impact on the rate of change in equilibrium of multiple coalitions acting against one another. The threat of unilateral punishment as a response from all future players would be effective in reducing defection, albeit cooperation would be unstable. These issues are addressed in Chapter 5 which develops this model through both baselining and a more explicit cooperation dynamic.

Area of Literature	**Key Contribution**
Network Game Theory	A novel multilayer network model is introduced, combining multimarket Cournot oligopoly trade with transport congestion elements
Network Game Theory	Analysis is conducted on this model giving results on individual layer interaction before extending to general solution characteristics
Strategic Optimisation	Optimal responses are examined under both algorithmic and discrete patience characteristics providing solutions over all timeframes
Network Game Theory	Non-differentiability of the model is noted, invalidating existing proofs of the existence of unique equilibria
Network Game Theory	The existence of stability is then confirmed through the model's Lyapunov properties wherein divergence from equilibrium creates a harmonic increase in marginal utility from return to equilibrium

Table 4.3: *Summary of key contributions within Chapter 4 categorised by area of literature.*

Chapter 5

Sanction Model

In this chapter, the novel multilayer network model which appears in Chapter 4 is extended, adding additional considerations specific to a basis case where governments are players. An explicit coalition formation and utility sharing structure, implemented through a third networked layer, are also considered. The elements of sanctions literature that are relevant to diplomatic considerations are introduced, and then the layers of a tri-layer model are presented. The implications of this model are then discussed. Real-world data is also presented in order to ground the work alongside the problem it seeks to address.

The multilayer network is intended as an instrument to understand the effectiveness of real-world sanction imposition. In particular, it attempts to analyse the impact of transhipment on the effectiveness of sanctions, where transhipment is used by a third party in an attempt to assist a sanction target in avoiding diplomatic repercussions. The work in this chapter refers to two explanations for sanction busting described by Early *et al.* [135]. The liberal explanation hypothesises that market incentives are the reason for the ineffectiveness of sanctions, while the realist explanation hypothesises that geopolitical interest is responsible. The model which appears in this chapter aligns more closely with the liberal explanation, looking to examine the market incentives to the participants. In a case where the rewards for transhipment are not sufficient to incentivise assisting the target, such as where additional sanctions are placed on those who do so, this would be evidence for the realist explanation.

The initial work which became this chapter was inspired by reports that sanctions on Russia had only been one-third as effective as had been anticipated [159, 160]. Initial estimates in 2022 predicted there would be a 6% [161], 11% [159] or 12%[160] reduction in GDP in 2022, with the real figure 2.1% [162], against a predicted 3.1% increase. Though the work remains entirely general, when forced to make assumptions, ones which follow this basis were made. In addition, the model is compared to real-world data about the imposition of sanctions on Russia and the opportunities for transhipment. The model in this chapter relies on the assumptions listed below. Key supporting literature is summarised in Table 5.1.

Key Assumptions Informing the Model appearing in Chapter 5

- Players have the opportunity to act sequentially.
- The relationship between supply and price is strictly decreasing and twice differentiable.
- The congestion function is strictly increasing and twice differentiable.
- The goods being sold by each player are homogeneous and are co-located with a market.
- Each player has market power following the format of an oligopoly game.
- Players are rational and do not engage in collusion unless otherwise stated.
- Coalitions between players represent an optimisation of trade strategy, utilising both physical and trade changes.
- Sanctions are imposed by a single player on a single player unless otherwise stated.
- There is no opportunity for the transportation network to be modified from the point of defection.

Author(s) & Year	**Key Contribution**	**Relevance to Thesis**
Basu *et al* (1986)	Fundamentals of n-player negotiation	Informs work on sanctions in this Chapter
Peksen *et al* (2007,2016,2022)	Real world sanction effectiveness	Provide a basis for the analysis of sanction effectiveness and inform sanction reciprocity within the models
Shapley (1951)	The introduction of the Shapley Value	A technique used for the estimation of member's value within the coalition
Owen (1977)	The introduction of the Owens Value	A rejected alternate technique used for the estimation of member's value within the coalition
Joshi and Mahmud (2016)	An implementation of sanctions on a network	Used to provide a basis for the model of sanctions on a network which appears in the chapter
EUSTAT (2022)	Data on European imports and exports in the 2000s-2020s	Utilised in the case study in section 5.3

Table 5.1: *Summary of key literature relevant to Chapter 5.*

5.1 Preliminaries and Notation

5.1.1 Sanctions Notation

Sanctions literature [123, 127] defines three groups in relation to sanctions. These are the party or parties that impose the sanction, i.e., the sanction senders S, the party or parties being sanctioned, i.e., the targets T, and independent third parties I. These are collectively members of the set of players $\mathcal{N}$. To avoid a conflict of notation, in this chapter, the sanction senders are P, the sanction targets are Q and the third parties are X. Table 5.4 contains a statement of all entities in this chapter.

Symbol	Definition
$\mathcal{I}$	The Set of all Players
$\mathcal{S}$	The Set of all Strategy Profiles
$\mathcal{C}$	The Subset of Players $\mathcal{I}$ in a Coalition
$u_i(s)$	Utility received by player $i \in \mathcal{I}$ from strategy profile $s \in \mathcal{S}$
$\zeta(i, \mathcal{C})$	Net transfer of utility from members of $\mathcal{C}$ to $i \in \mathcal{C}$
P	The Set of Players who impose Sanctions
Q	The Set of Players who are Sanctioned
X	The Set of all remaining Players
η	The Profit made from trading in layers G_α and G_β
μ_i	The Transfer of Utility between Coalition Members $c \in C$
ζ_i	The Shapley value of coalition members $i \in \mathcal{I}$
θ_i	The proportion of the coalition's value ζ generated by $i \in \mathcal{I}$.
R_i	The Reward received by Players $q \in Q$
$t = 0$	The Time at which Sanctions are imposed
t^*	The Time at which Coalitions are formed

Table 5.2: *Notation for Sanction Definition*

The sanction target Q is assumed to receive sanctions for engaging in behaviour that P wishes to discourage, referred to as defecting (from behavioural norms). Accordingly, the sanctions act as punishment, while Q receives a reward R which incentivises their initial defection. X receives a share of the utility generated by helping bypass sanctions Q in exchange for their assistance.

The time at which sanctions are imposed is also assumed by convention to be $t = 0$. Consequently, for $t < 0$ there are no sanctions and for $t > 0$ sanctions are in place. In this work, we introduce a second significant instance, t^*, at which there is a change in network characteristics. Here, t^* represents the time at which transhipment is initiated in order to bypass sanctions. This can be at $t^* = t = 0$, but a delay, such as for the negotiation of the distribution of utility between sanctioned players, can result in $t^* > 0$.

5.1.2 Network Definition

The network that appears in this chapter is a trilayer network $\mathcal{M} = \{\mathcal{G}, \mathcal{E}_{\alpha,\beta,\gamma}\}$. The three layers, referred to hereafter as the upper layer, middle layer, and lower layer represent transportation, political connection, and coalition, respectively. An example of this can be seen in figure 5.1.

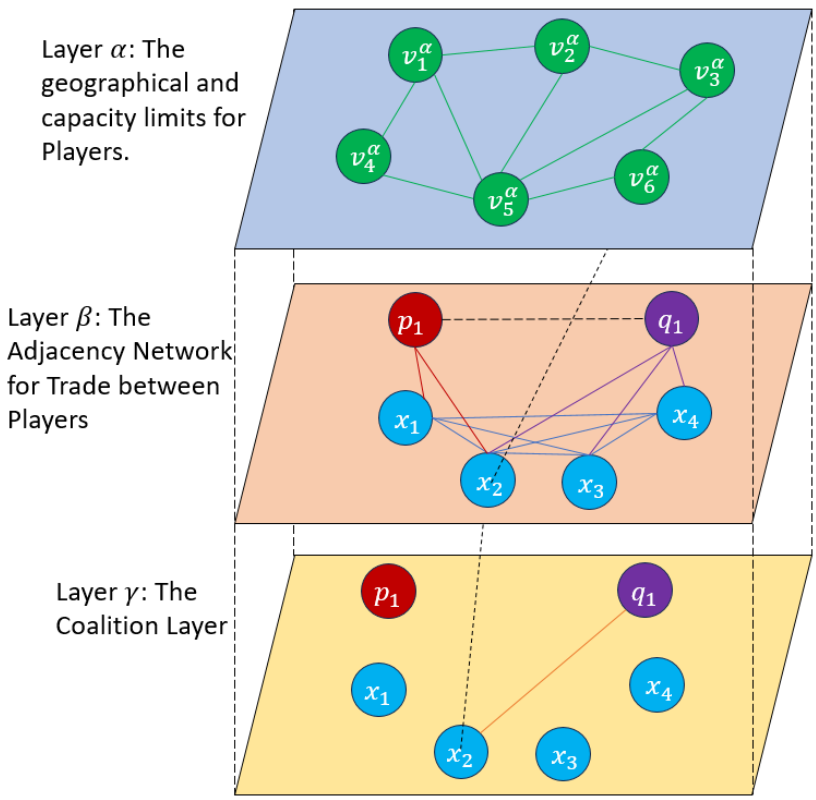

Figure 5.1: *An example of the trilayer network, with each layer labelled according to its utilisation in this work.*

Layer α

Layer α is the transportation layer and represents the transportation environment within which the trade occurs. Following an application of the transportation of energy goods, an example of this is of oil pipelines. Figure 5.2 shows a map of major European pipelines. More detailed models which include minor pipelines form a graph that traverses 38 countries with 24,000 nodes and 250,000 edges. Figure 5.3 shows an example structure for layer α.

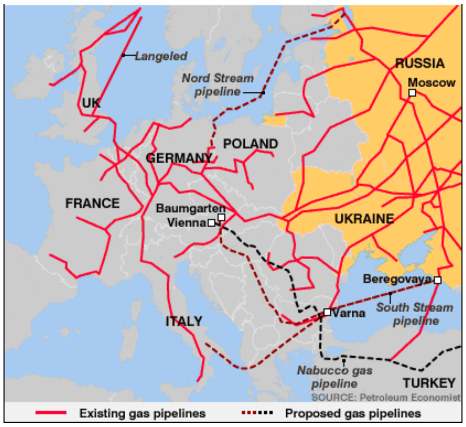

Figure 5.2: *A map of Major Oil pipelines in Europe. Reproduced from [163].*

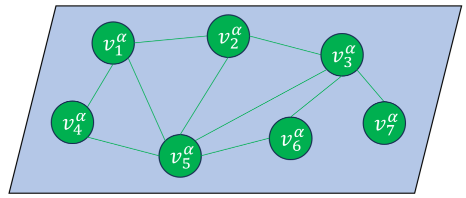

Figure 5.3: *An example layer α, with edges given by E^α. Loops, connecting each node to itself are not shown for clarity.*

Layer α uses an adjacency matrix to represent the capacity for transportation between the nodes. An example matrix corresponding to the transportation graph which appears in figure 5.1 is E_α below. Adjacency matrix E_α which indicates whether there exists adjacency (connection) between nodes, and the capacity of the edge connecting them. As an undirected network the capacity of each edge is given by the value in the adjacency matrix $a_{ij} = a_{ji}$.

$$E_\alpha = \begin{bmatrix} \infty & a_{12} & 0 & a_{14} & a_{15} & 0 \\ a_{21} & \infty & a_{23} & 0 & a_{25} & 0 \\ 0 & a_{32} & \infty & 0 & a_{35} & 1 \\ a_{41} & 0 & 0 & \infty & a_{45} & 0 \\ a_{51} & a_{52} & a_{53} & a_{54} & \infty & a_{56} \\ 0 & 0 & a_{63} & 0 & a_{65} & \infty \end{bmatrix}.$$

The unconnected nodes have an edge with 0 weight between them, which represents that 0 goods can be transported. Each node has an loop with infinite weight to itself, representing that there exists no limit on internal transport between a player and a market located at the same node. The upper layer $G_\alpha = (V_\alpha, E_\alpha)$ represents the geographical and capacity limits of transportation between players.

Layer β

The middle layer, $G_\beta = (V_\beta, E_\beta)$, represents the trade values between the markets associated with the players. Each player is embedded in a node. Each node contains both an embedded player, exporting goods to markets, and an embedded market, receiving goods from players. Each player $i \in \mathcal{I}$ plays a dynamic networked Cournot game of assigning their trade between the markets in the game to maximise their trade profit. There is an associated adjacency matrix E_β, where each element of the matrix represents the amount of trade flowing from the player embedded in the market to the other nodes.

$$E_\beta = \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} & 0 & 0 \\ b_{21} & b_{22} & 0 & b_{24} & b_{25} & b_{26} \\ b_{31} & 0 & b_{33} & b_{34} & b_{35} & b_{36} \\ b_{41} & b_{42} & b_{43} & b_{44} & b_{45} & b_{46} \\ 0 & b_{52} & b_{53} & b_{54} & b_{55} & b_{56} \\ 0 & b_{62} & b_{63} & b_{64} & b_{65} & b_{66} \end{bmatrix}$$

Figure 5.4 shows an example of layer β, with the edge between P and Q dashed to indicate that it will be removed at $t = 0$ when sanctions are imposed. Unlike in E_α, elements of E_β can be asymmetrical, such that $b_{12} \neq b_{21}$, representing unequal trade between players. When sanctions are imposed, $b_{ij} = 0$ for all b_{ij} such that $i \in P$ and $j \in Q$. The general form is

$$E_\beta = \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1I} \\ b_{21} & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ b_{I1} & \dots & \dots & b_{II} \end{bmatrix}$$

with the first $|P|$ rows and columns representing the members of P, the next $|Q|$ representing the members of Q and the remainder representing members of X.

Layer γ

The lower layer, $G_\gamma = (V_\gamma, E_\gamma)$, represents the existence of coalitions between players to bypass trade restrictions in layer β. In figure 5.5, players Q, x_1 and x_2 are in a coalition, which generates additional utility

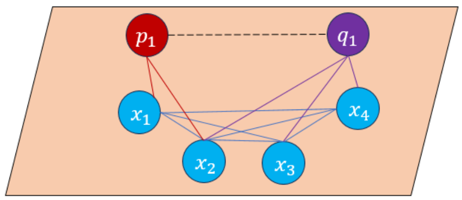

Figure 5.4: *An example layer β, with some pairs being unable to trade with one another.*

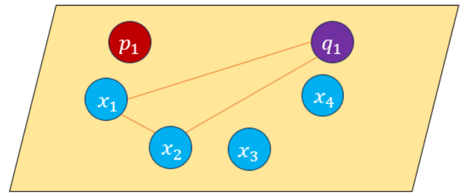

Figure 5.5: *An example layer γ, with a three member coalition existing between player q_1, x_1 and x_2. Edges exist between all pairs of coalition members.*

through cooperation to subvert the sanctions imposed by P. This trade value is then shared between them.

The Shapley value [13] is calculated by finding the amount of additional utility generated by the coalition. The Shapley value is chosen over the Owens value [96] due to the Shapley values consideration of all possible coalition structures, rather than a specific one as in the case of the Owens value. Explicitly, the Shapley value of a player within a coalition is given by:

$$\zeta_i(\omega) = \frac{1}{n} \sum_{C \subseteq N \setminus \{i\}} \left(\omega(C \cup \{i\}) - \omega(C) \right),$$

where $\omega : C \to \mathbb{R}, C \subset \mathcal{I}$ is the value function.

Coalition formation in layer γ has dynamics that are approximately analogous to the glove game [94].

The glove game, introduced in section 2.4 demonstrates relative value to a coalition by posing a game where the aim is to form complete pairs of gloves. In an environment where two players i_1 and i_2 have left gloves and a third i_3 has a right glove, i_3 has greater value to the coalition than the others. The Shapley value is found by assessing the utility received by the coalition collectively as each member is added in all possible orderings. Of the six possible orderings, there is only one where each of i_1 and i_2 add value. Accordingly, $\theta_1 = \theta_2 = \frac{1}{6}$ and $\theta_3 = \frac{2}{3}$.

The overlap between the glove game and the trilayer model presented in this chapter lies in the two characteristics of the players. The sanction targets $q \in Q$ have additional consumers willing to spend money to receive an increased utility, while independents $x \in X$ have spare capacity in the transportation network and access to the exports and of the sanction senders $p \in P$. Where these are matched together, an additional value can be generated for the coalition. Consequently, in a market with an excess of opportunity for trade pairing, targets $q \in Q$ will have greater Shapley values. Conversely, where players with significant market power are sanctioned, and the transport system does not have significant excess capacity, third parties $x \in X$ will have greater Shapley values as they are not in competition with one another.

5.2 Dynamic Game Definition

5.2.1 Utility Equations

The utilty received by player $i \in \mathcal{I}$ in the game is given by

$$U_i(t) = \eta_i + \mu_i + R_i + \lambda_i. \tag{5.1}$$

Here η_i is the profit received by player i from trading goods in layer β, given by

$$\eta_i = \sum_{k \in K} \left(b_{ik} \cdot \left(H - \sum_{m=1}^{n} b_{mk} \right) \right),$$

with n the number of players, b_{ik} is the amount of goods sold by player i to the market associated with node v_k^β. Constant H is a fixed constant giving the profit per unit where there is no supply. The transferrable utility from membership of a coalition is given by μ_i, with

$$\mu_i = \zeta_i - \sum_{c \in C} \left(U_i^*(t^* << t) - U_i^*(0 << t < t^*) \right). \tag{5.2}$$

where U_i^* is the utility equation without μ_i and ζ_i is the Shapley value to the coalition of player i. Time $(t^* << t)$ refers to times after coalitions have formed, with $<<$ giving that the competition resulting from the change has resolved to a new equilibrium. Time $(0 << t < t^*)$ refers to times after the defection has occurred and new equilibrium been found, but before coalitions have formed. Reward R_i is given to the sanction targets Q for defecting, given by

$$R_i = \begin{cases} r_i \in \mathbb{R}, & i \in Q \\ 0, & i \notin Q, \end{cases}$$

where r_i is a basis dependent constant. The reward given to the sanction sender S for successful sanctions is given by

$$\lambda_i = \begin{cases} \frac{U_Q(t>>t^*)}{|P|}, & i \in P \\ 0, & i \notin P, \end{cases}$$

representing a direct linear correlation between the damage done to Q and the benefit received by P. As such, $p \in P$ are prepared to accept a reduction in their own utility represented by ν_p if it corresponds to a proportionate or greater reduction in the utility of their targets.

5.2.2 Strategic Response

In order to simplify discussion of the trade and transport dynamics in this chapter, a myopic strategic response is introduced. This follows the dynamics which appear when considering the upper layer only in Chapter 4.

The myopic strategic response given by a player can be seen in figure 5.6 in an example n-player, 2-market game. The collective strategies played by all other players appears on the y-axis as s_{-j} and player j chooses a best response on the x-axis in response, with $s_j = 0$ representing the equilibrium position, $s_j = -10$ representing all goods assigned to market 1 and $s_j = 10$ representing all goods assigned to market 2. Section 4.2.2 gives the optimal response in this case as a response of $\frac{-1}{2}$ of the effective position of the other player. The blue curve provides the utility received by player j for a response to $s_{-j} = -9$, and has a maximum at $s_j = \frac{9}{2}$.

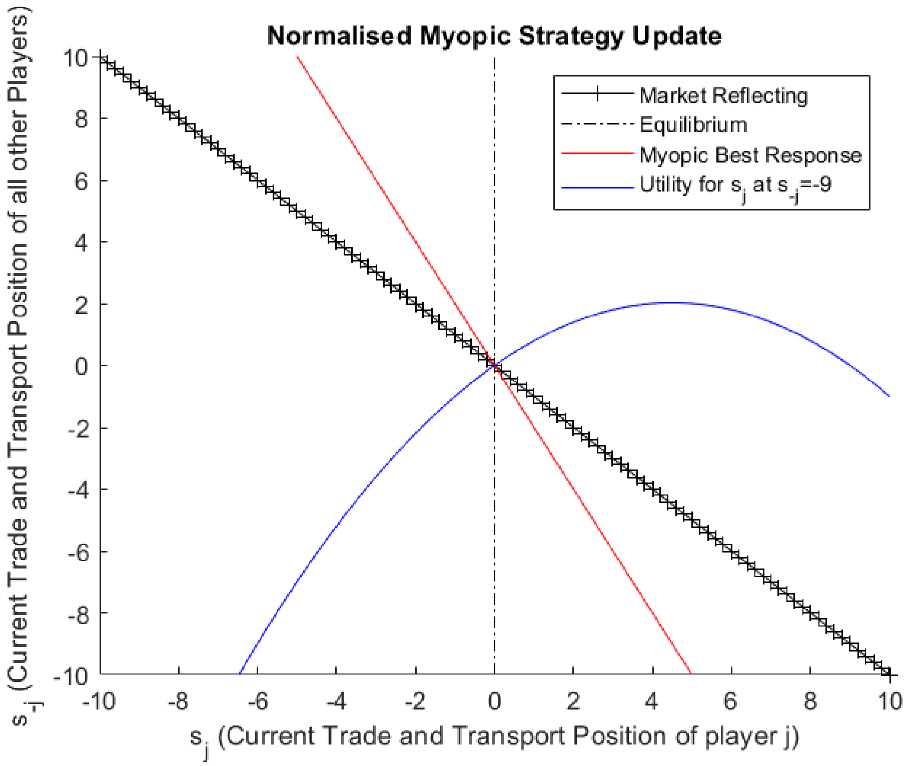

Figure 5.6: *A graph demonstrating the incentives to a player to a range of responses.*

5.2.3 Player Strategies

The strategic response is applied by each of the players in turn, with sanctions being imposed at time $t = 0$ and coalition formation finishing at $t = t^*$. The major stages of the game can be seen in figure 5.7.

Not previously discussed are the incentives on P and Q. Q is only incentivised to defect, engaging in the behaviour that P considers sanction-worthy, if their individual rewards R are greater than their loss of utility from $P's$ sanctions. If they choose to defect, P is incentivised to impose sanctions only if the negative impact on Q is more significant than the negative impact on P. The findings of Mertens *et al.* [125], that aid suspensions are more effective than imposed sanctions, are relevant here, as they represent a case where P receives a benefit for sanctioning Q, increasing their commitment to the sanctions. After both players

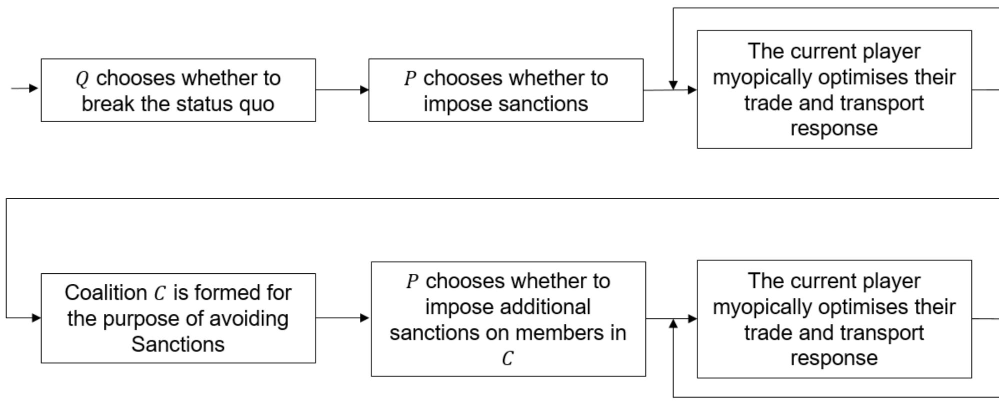

Figure 5.7: *A flowchart outlining the strategic steps in the game.*

have made these decisions, the players respond to the resulting dynamic game, making a myopic strategic response as outlined in section 5.2.2. A coalition may then form. Members of Q will join, as they are already sanctioned, but each $x \in X$ chooses whether to do so. After this, P chooses whether to extend sanctions to each member of C, preventing transhipment from being effective, but with additional cost to P without additional benefit λ_i from additional punishment inflicted on Q.

Sequential Game Form

Figure 5.8 shows the normalised utilities received for each of the decisions throughout. The relative value of $R = \sum_q r_q$ and $a_1 - a_9$ are important for evaluating the expected outcomes from the game. We can a priori know from game structure that $R > 0$, $R > a_2$, $R > a_5$, $R > a_8$, $a_1 > a_4$, $a_1 > a_7$ and $a_3 > 0, a_9 > a_3$. These results are intuitive from the game structure, except for $a_3 > 0$, which is a result of the game dynamics. As the only players without limited trade opportunities, $x \in X$ have more latitude to take advantage of the trade environment, resulting in additional utility.

We analyse each subgame beginning from the final decision made in order to find if the overall tree is reducible. Table 5.3 breaks down the meaning of each of the values appearing in figure 5.8

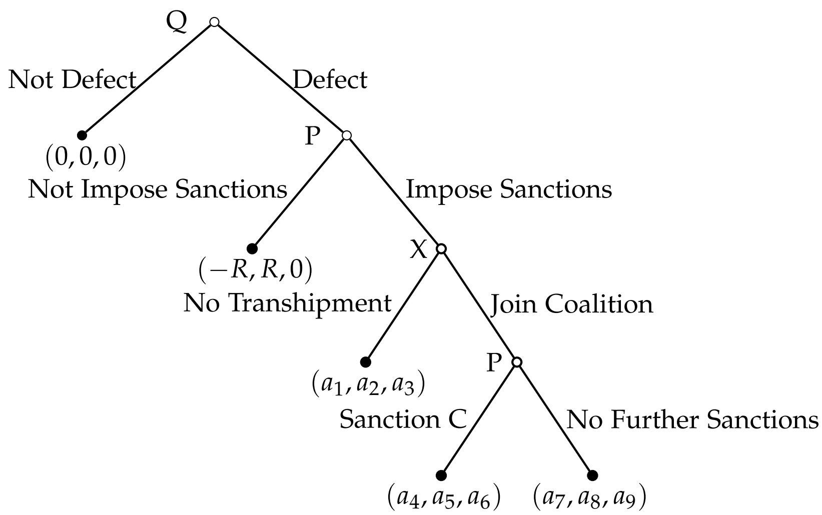

Figure 5.8: *A tree diagram showing the decisions and utilities made by P, Q and X before and after t^*.*

The Imposition of Sanctions on Transhipment Facilitators

The players $p \in P$ receive utility a_4 or a_7, receiving additional trade costs where they sanction C due to the reduced number of available markets for their goods, but receive additional utility relative to a_7 from the increased effectiveness of sanctions on Q.

The Incentives of Third Parties to Engage in Transhipment

If the players in X find P's threats of further sanctions to be not credible, they are incentivised to join the coalition as $a_9 > a_3$ as they receive the additional value for enabling transhipment without any loss from sanctions. If P does sanction C however, P compares a_3 and a_6 and make their decision based on whether their expected utility is raised or lowered.

The Imposition of Sanctions on Defectors

The result of this is then used to decide whether P should impose sanctions. They compare $-R$, received from allowing Q to defect without punishment to a_1, a_4 or a_7. A value $a_P \in \{a_1, a_4, a_7\}$ represents the outcome for P of this multistage subgame.

Value	Meaning
R	The Reward received by Q for defecting
a_1	The utility received by P where Sanctions are imposed and no coalition forms to undermine them
a_2	The utility received by Q where Sanctions are imposed and no coalition forms to undermine them
a_3	The utility received by X where Sanctions are imposed and no coalition forms to undermine them
a_4	The utility received by P where Sanctions are imposed and a coalition forms to undermine them but no further sanctions are imposed
a_5	The utility received by Q where Sanctions are imposed and a coalition forms to undermine them but no further sanctions are imposed
a_6	The utility received by X where Sanctions are imposed and a coalition forms to undermine them but no further sanctions are imposed
a_7	The utility received by P where Sanctions are imposed and a coalition forms to undermine them and further sanctions are imposed on coalition members
a_8	The utility received by Q where Sanctions are imposed and a coalition forms to undermine them and further sanctions are imposed on coalition members
a_9	The utility received by X where Sanctions are imposed and a coalition forms to undermine them and further sanctions are imposed on coalition members

Table 5.3: *Key Values in Figure 5.8.*

The Initial Choice to Defect

Finally, the initial decision can be evaluated. Potential defectors Q decide both whether the threat of P's sanctions is credible, and if it is, whether X is sufficiently incentivised to defect, considering the threat of additional sanctions from P. If the threat of sanctions is not credible, with P receiving less utility from imposing sanctions than damage is done to Q, due to $a_P \in \{a_1, a_4, a_7\}$ such that $a_P < -R$ then Q will defect. A value $a_Q \in \{a_2, a_5, a_8\}$ represents the outcome of this multistage subgame for Q. If the threat of sanctions is credible, but the impact of transhipment provides an opportunity, with the expected utility $a_Q \in \{a_2, a_5, a_8\}$ such that $a_Q > 0$, the threat of sanctions is not effective, and the sanction targets Q are incentivised to defect.

5.3 Case Study: Sanctions on Russia in 2022

The Standard International Trade Classification (SITC) classifies goods for the purpose of international trade statistics. Table 5.4 gives the groups as used by the EUROSTAT European database [164]. Results presented in [165] indicate that sanctions imposed on Russia affected transport and military equipment (SITC 7) more heavily than other areas.

SITC	Goods
$0+1$	Food, Drinks and Tobacco
$2+4$	Raw Materials
3	Mineral Fuels, Lubricants and Related Materials
5	Chemicals and Related Products
$6+8$	Other Manufactured Goods
7	Machinery and Transport Equipment

Table 5.4: *SITC Classifications [164]*

As was outlined at the beginning of the chapter, evidence of reduced effectiveness of sanctions on Russia, relative to the expected level, was one of the motivating problems leading to this research[1]. Noteworthy graphs, such as the ones which appear in figure 5.9 show a significant change in volume of SITC goods exported to a number of countries. For example SITC 7 exports to Kyrgyzstan in 2023 were approximately 1000% compared to the 2022 level. Other studies of the same events likewise find evidence that transhipment had occurred [134]. A significant exclusion from consideration was Belarus, which also faced sanctions due to their role in supporting the invasion.

It is important however to contextualise these quantities , the graphs in figure 5.10, as well as figures A.1 to A.5 in appendix A, show this

[1]This case study was investigated with the assistance of Oliver Schellenberg, who was responsible for generating an initial set of graphs from the data based on my specifications. We then discussed and refined the visualisations.

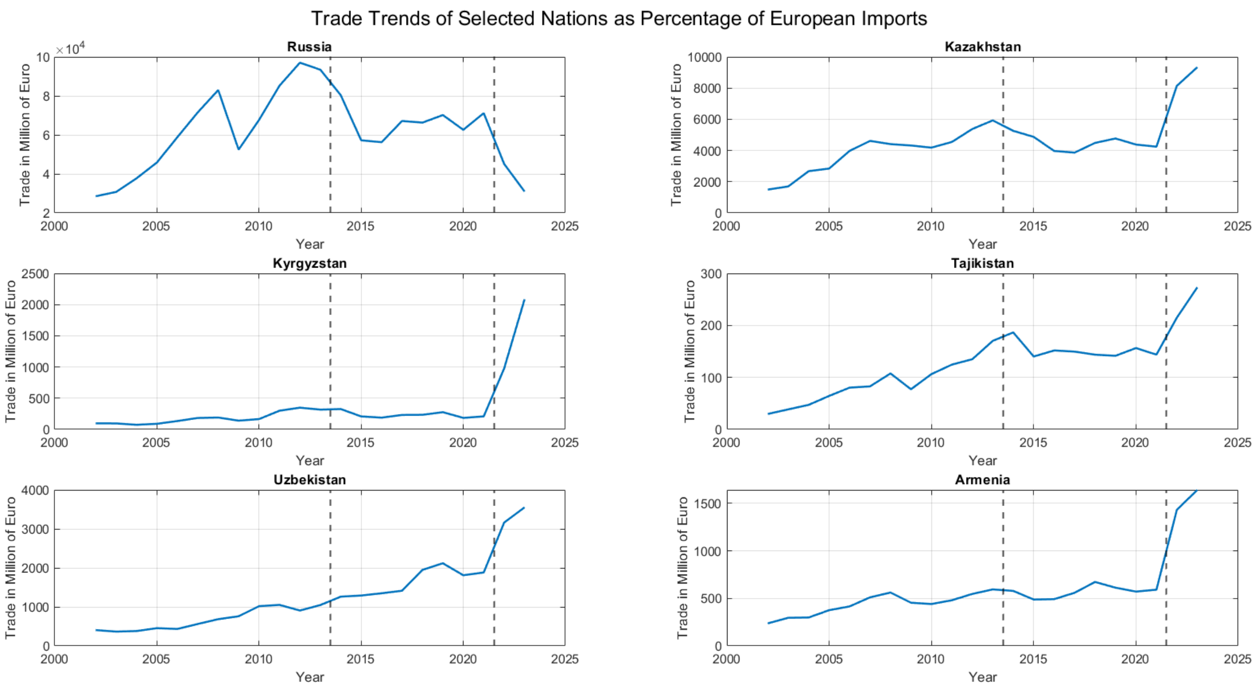

Figure 5.9: *Country specific graphs showing relative changes in European exports of SITC 7, machinery and transport equipment.*

data relative to many other countries that could be viewed as potential transhipment targets due to their proximity or political ties to Russia. It can be seen that the sanctioning of Russia apparently influences the change in the overall percentage of EU exports to the listed countries in 2014-2023. That is, there is insufficient evidence to state that the reduced effectiveness of sanctions against Russia can be explained through coalition formation for transhipment. This is not to say that no transhipment is occurring; figure 5.9 gives strong evidence supported by the literature[134]; but rather that the effects are insufficient to explain the two-thirds reduction in effectiveness [166, 161, 162].

With reference to figure 5.8, these findings lead to some understanding about the anticipated utility by the parties involved. Comparing changes in export quantities from figure 5.9 around the annexation of Crimea by Russia in 2014, it can be seen that transhipment was not con-

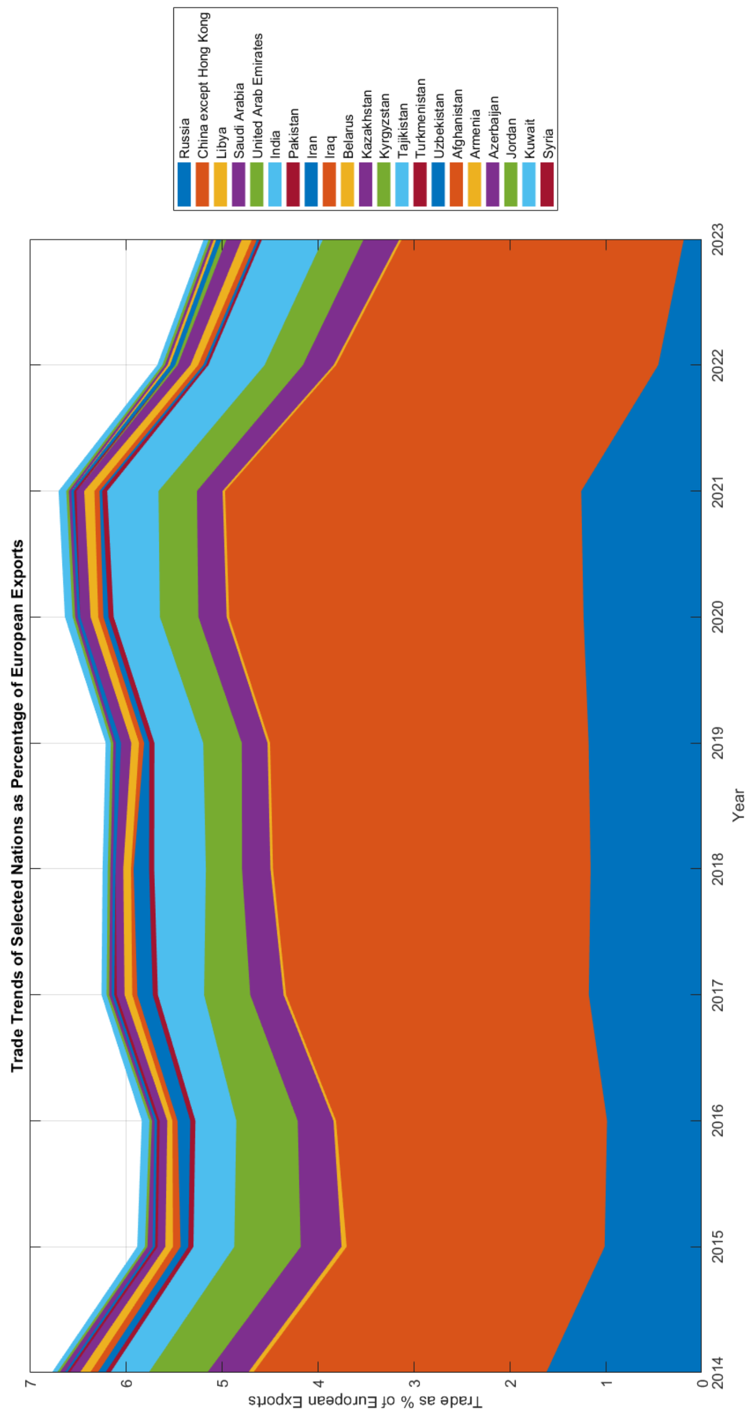

Figure 5.10: *A graph showing relative changes in European exports of SITC 7, machinery and transport equipment.*

sidered to result in an increase in utility for third parties. This represents the threat of being included in the sanctions by the west as being credible, and to be avoided. By 2022 however, data suggests that this threat was no longer as credible, with third parties being willing to engage in transhipment.

Again, with reference to figure 5.8, it is important to note that decisions are made with under assumptions of perceived utility. For example, it is generally accepted that Russia's defection through the invasion of Ukraine was anticipated to be over more quickly [167], resulting in a greater reward R for Russia. This does allow for the modification of the macro-game from single event, to a repeated game. This incentivises P (western nations) to impose sanctions even if not immediately profitable, through the expectation of a reduction of future defections. This makes the lack of credibility of potential western sanctions on third parties engaging in transhipment even more significant.

5.4 Numerical Simulations

In order to verify results, numerical simulations were conducted on a 6-player game, using the model which appeared in section 4.3 as a basis, with additional elements as appropriate for the capacity constraints and coalitional opportunities. In each model, the period of instability resulting from the change in network structure can be seen after changes in network structure at $t = 0$, $t = 100$, settling to a stable equilibrium by $t = 50$ and $t = 150$. This stable equilibrium is also a unique equilibrium, as it possesses the same properties as the dynamic game in Chapter 4. As a result, depending on the process of network state change, it may be appropriate to for a period of instability, with utility benefits to players as demonstrated in Theorem 2, or alternatively use the model purely as a method of finding these equilibria which are otherwise computationally intensive to determine.

5.4.1 The Impact of countersanctions

A result found in the literature by Peksen and Jeong [124], who note that when sanctions are imposed on a wealthy regime, is that counter sanctions are more likely. The model captures this as demonstrated by the case study below.

A 6-player game, within which all players had equal market power (as represented by the amount of goods they had available), was created. There is one sanction sender p_1, one sanction target q_1 and four third parties x_1 to x_4. The model has three key time windows. These are the periods before initial sanction imposition $t < 0$, after initial sanction imposition $0 < t < 100$ and after counter-sanction imposition $t > 100$. Figure 5.11 shows the results, including a period of instability after the change in network characteristics, followed by a stable equilibrium. The initial imposition of sanctions by p_1 results in a significant decrease in utility for q_1 [168] and a slight increase for p_1 and each member of X. This is due to a reduction of opportunities for q_1 and an associated opportunity for the other players to exploit their limitations. The imposition of countersanctions then results both in a decrease in utility for p_1 and an increase for q_1.

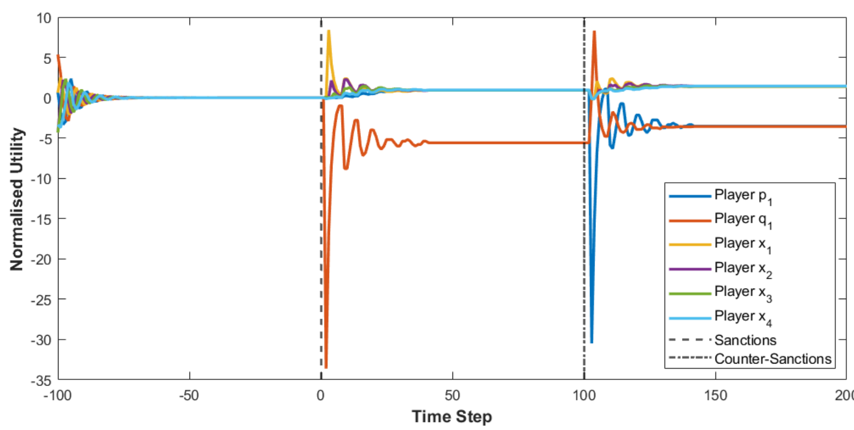

Figure 5.11: *A plot showing the relative utilities of players as sanctions and counter sanctions are imposed. Sanctions are imposed at $t = 0$ and countersanctions at $t = 100$.*

As a result, q_1 is incentivised to impose countersanctions, and will do so, both to punish p_1 for having imposed the sanctions, and to increase their own utility through the market. Figure 5.11 does not include the reward received by q_1 for their defection, but the impact is clear. For $r \geq 4$ where countersanctions are imposed, or $r \geq 7$ where they are not, q_1 will choose to defect. Similarly, the gratification utility λ received by p_1 is greater than the costs for sanctions only, and results in a net-neutral position for sanctions and countersanctions, meaning p_1 is likely to impose sanctions.

5.4.2 The Impact of Additional Members of P

It is an intuitive result that the more people sanction a particular target, the more effective sanctions become. This can be seen in figure 5.12 where at $t = 0$ sanctions and countersanctions are imposed, and at $t = 100$, x_3 and x_4 become p_2 and p_3, also sanctioning q_1. This results in a further reduction in q_1's utility, confirming the expectation.

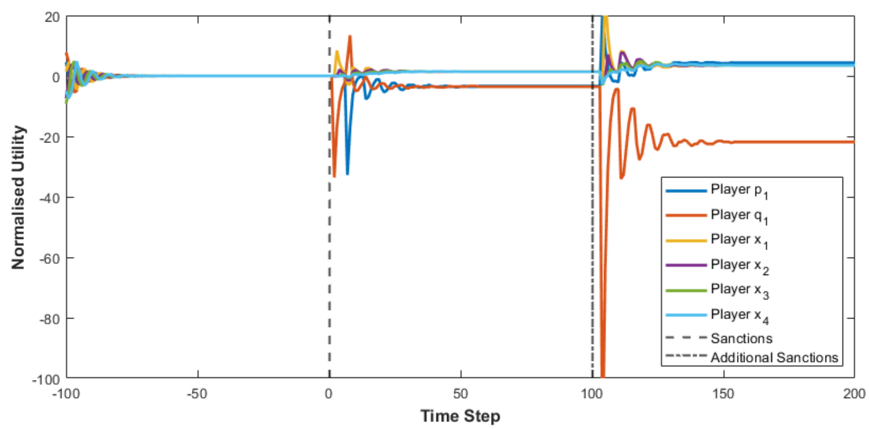

Figure 5.12: *A plot showing the relative utilities of players as sanctions and further sanctions are imposed. Sanctions and countersanctions are imposed at $t = 0$ and further sanctions at $t = 100$.*

The impact of the cost of having imposed sanctions (and received countersanctions) at $t = 0$ on p_1 is also reduced, as q_1's requirements to trade in the markets remaining to them becomes increasingly inflexible, allowing exploitation by the only slightly constrained p_1 who receives greater utility than they would in equilibrium.

5.4.3 The Impact of Capacity Constraints

One aspect of the model not considered in section 5.2 is the impact of capacity constraints on the sanction target. A 6-player model was created to model this, where sanctions and countersanctions are imposed by p_1

and q_1 at $t = 0$, and additional capacity constraints are placed upon q_1 at $t = 100$. Figure 5.13 shows the utilities receives by players over time.

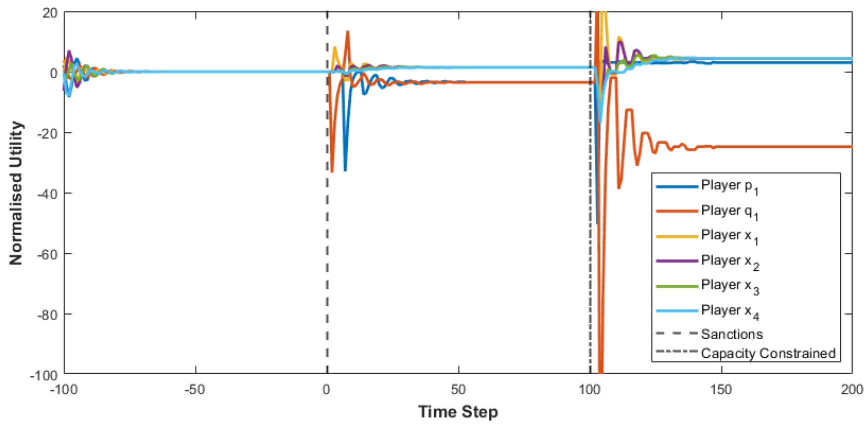

Figure 5.13: *A plot showing the relative utilities of players as sanctions, countersanctions and capacity constraints are imposed. Sanctions and countersanctions are imposed at $t = 0$ and capacity constraints at $t = 100$.*

It is worthy of note that the result is very similar to that where additional players join the sanctioning coalition, again caused by the predictability of q_1's actions, with q_1 preferring to assign additional goods to markets they only have limited access to, reducing their utility.

5.5 Results

A number of insights can be gained using this model. First, a number of propositions which follow from results found in Chapter 4.

5.5.1 Analytical Results

As a result of corollary 4.4.1:
Proposition 5.5.1. If the price in each market is strictly decreasing, concave and twice differentiable, there is a unique Nash Equilibrium for $\mathcal{M}$ where $t < t^*$.

This result finds that there will always be a unique equilibrium formed before coalition formation for the purpose of transhipment occurs at

$t < t^*$. This can be extended to the game after coalition formation as follows:

Proposition 5.5.2. If the price in each market is strictly decreasing, concave and twice differentiable and the transportation costs are strictly increasing, convex and twice differentiable, there is a unique Nash Equilibrium for the trilayer game represented by $\mathcal{M}$ where $t \geq t^*$.

This is a stronger result, which implies that coalition formation does not disrupt the existence of stable equilibrium.

Proof. For $t \geq t^*$, if the coalitions only continue $i \in Q$, there is no modification to the network from $t < t^*$ and the result remains. In the other case, where there exists a coalition C with members $c \in C$ such that $c \in X$, the players act as a single entity, collectively allocating their trade to markets in order to maximise their collective utility. This does not modify the paired market price and cost characteristics, resulting in an allocation that, while over several markets, can be considered as a single player in the style of the original problem, resulting in the existence of a unique stable equilibrium.

□

Proposition 5.5.3. A player $x \in X$ will choose to join a coalition for the purpose of sanction bypassing if and only if the reward a_4 for P through sanctioning C and preventing transhipment is greater than the reward a_7 for ignoring transhipment, such that $a_4 > a_7$.

Where a_4 is the utility received by P when they sanction the transhipment coalition C and a_7 is the utility received by P when the impose no further sanctions.

This implies that independent third parties are incentivised to cooperate with sanction targets if and only if P's threat of further sanctions is credible.

Proof. The difference between utilities a_4 and a_7 represent whether or not P receives greater utility through effectively sanctioning Q and C, or by receiving the benefits of trading with $C \backslash Q$. If they receive greater utility from continuing to trade with $C \backslash Q$, then the threat of additional sanctions is not credible and there is no economic incentive for X not to assist Q through transhipment.

□

By analysing explicit network structures, statements about the relationships between $a_1, a_2, \ldots, a_9$, which result in sanctions being effective, or ineffective can be found. These allow more specific statements to be made about the conditions under which each of the five major outcomes seen in figure 5.8 will occur.

5.5.2 Lower Bound: Ring Network

As an illustrative example, we introduce a ring network, where, in order for transhipment to occur, every intermediary must join the coalition. Figure 5.14 shows the middle layer G_β. If Q chooses to defect, at $t = 0$, sanctions are imposed, turning this ring network into a linear network, with p_1 and q_1 at the ends.

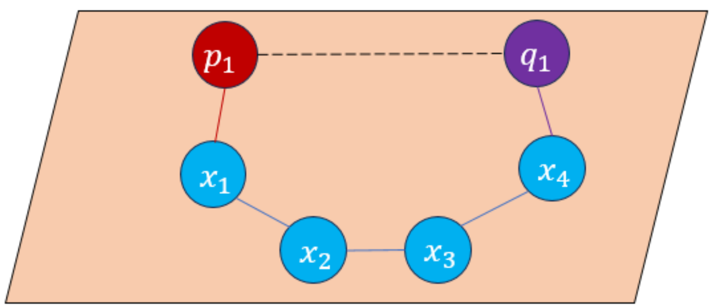

Figure 5.14: *The middle layer G_β of a network, with highly restricted trade opportunities.*

Proposition 5.5.4. In a linear network, q will choose to defect if and only if rewards, as idenitified in figure 5.8, satisfy the following inequality: $\frac{a_8+a_9-a_2-a_3}{d(p,q)} + a_2 > 0$.

Here, $d(p,q)$ is the distance between p and q through the network. This represents a comparison between the actual utility received by q after transhipment has increased their utility and that they would receive if they do not defect.

Proof. In a linear network, each of the members of the coalition is required in order to achieve successful transhipment between p and q. As such, when making an ordering of the coalition members, in each case, the last to be added generates all the value available through the coalition. This results in each member being equally valuable, and an even split of the additional value generated. The distance between p and q is equal to the number of members of the coalition, and accordingly $\frac{a_8+a_9-a_2-a_3}{d(p,q)}$ is an appropriate measure of the additional value generated by the coalition.

□

This is the minimum coalitional value that can be held by $q \in Q$, and accordingly is the structure which most benefits p assuming that sanctions will not be effective in dissuading q.

5.5.3 Upper Bound: Complete Network with High Transport Capacity

A complete network is one in which every member is connected to every other member. As a second illustrative example, we introduce a complete network, where, in order for transhipment to occur, any of a number of intermediaries would be sufficient. Figure 5.15 shows the middle layer G_β in such a case.

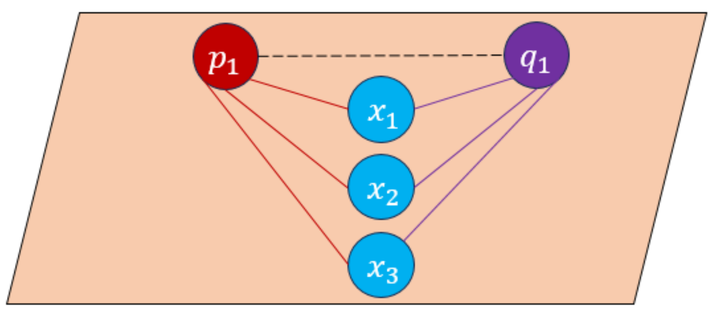

Figure 5.15: *The middle layer G_β of a network, with no restriction on trade opportunities for p and q, connections between x_1, x_2 and x_3 not shown.*

Proposition 5.5.5. In a complete network, the sanction targets Q will choose to defect if and only if rewards, as given in figure 5.8, satisfy the following inequality: $\frac{a_8+a_9-a_2-a_3}{|C-1|!} + a_2 > 0$.

Here, $|C|$ gives the cardinality of the coalition C. As in proposition 5.5.4, this is an assessment of whether q receives greater utility by defecting.

Proof. In this structure, q is the member which adds value in all possible orderings except that in which it appears first, where the next member will add value. This gives a proportion of the coalition's value equal to $\frac{1}{|C-2|!}$ to each $x \in X$ in the coalition, and the remainder (and majority for $|C| > 2$) of $\frac{1}{|C-1|!}$ to q.

$\square$

This highly connected network is the market structure which most benefits q for coalition formation. Their final utility is very close to R, having managed to bypass almost all the impacts of sanctions through their relative importance to the intermediaries.

Anticipatory Capacity Development

The increased power that q has within this high transport capacity case study informs a scenario where defecting to receive R is more advantageous. This is due to the reduced effectiveness of sanctions imposed in response. Accordingly, it is in the interest of a potential defector to prepare the environment by investing in additional transport capacity. In response, players in P (or who would be in P after defection) may take the preparation of substantial excess transport capacity as a sign of increased risk of future defection, and may choose to take steps in response. An example of a step that could be taken in response is action to reduce R, such as providing military support to a country that may be invaded.

5.6 Conclusions

The magnitude of the benefit of future defection can be predicted through an examination of the degree to which the layer G_α and G_β structures mirror those of each case study. In this context, examining the action a potential sanction target is taking prior to an anticipated defection can provide a warning of intention. Attempting to prepare significant additional capacity beyond that which is required for current optimal market distribution could be a warning sign.

The case study examining the real-world case of Russia gives an indication that while transhipment to bypass sanctions may have been occurring, the quantities were relatively small compared to the reduction in trade caused by sanctions, even when examining sanctioned SITC groups. Drawing on the literature Gutmann *et al.* [136] found that there did not exist significant evidence of sanction busting behaviour where Russia acted as third party, but that the results were more ambiguous for China. Alternate case studies are difficult for two reasons. Firstly, the sanctioned economy may have a much smaller GDP. This is the case for all recently imposed sanctions excluding counter-sanctions such as those placed by Russia on the US in 2022. The second reason is that sanctions may be less strict, such as those placed on China by the United States in 2020 for human rights reasons.

A summary of the key contributions in this chapter can be seen in Table 5.5.

5.6.1 Policy Implications

This work has direct relevance to policymakers seeking to understand real-world sanctions. It reinforces a number of expected and previously known characteristics of sanction effectiveness, that is, that a larger sanctioning coalition leads to greater disincentivisation for defection, that countersanctions are rational and should be expected, and that third parties willing to cooperate provide opportunities for a sanction target to evade sanctions. The implications of this for policymakers are that Western nations have lost the perception of credibility with respect to sanctions imposition, leading to transhipment. Rebuilding this credibility alongside watching for anticipatory capacity construction provides the opportunity to more effectively disincentivise undesireable behaviour by potential defectors from the international status quo.

Area of Literature	**Key Contribution**
Network Game Theory	The multilayer model is further developed through the addition of networked coalition formation within the network to better model the impacts of transhipment
Economics	A case study of Western sanctions on Russia was conducted, identifying a change in the anticipated threat of sanctions between 2014 and 2022
Optimisation and Network Game Theory	Numerical Modelling was conducted on the sanction network, confirming key results from the literature and giving further predictions on the impact of capacity constraints on sanction effectiveness
Coalitional Game Theory	Network structures are analysed with respect to the opportunity for sanctioning bounds on the Shapley value for all network cases.
Policy	Recommendations for how policy-makers should approach sanctions in order to increase their effectiveness, discourage their requirement and anticipate their necessity

Table 5.5: *Summary of key contributions within Chapter 5 categorised by area of literature.*

Chapter 6

Conclusions and Future Work

6.1 Conclusions

Within the broad context of economic analysis, it is important to take advantage of the additional opportunities provided by the intersection of game theory and multilayer network theory. Recognising the disruption to the status quo caused by black swan events, particularly in an international environment that is increasingly unstable and which is affected by the increasing impact of climate change, results in a competitive environment wherein players attempt to adjust to new circumstances. The aim of the research reported in this thesis is to understand the optimal response to these events by rational players when they consider different time horizons, as well as to understand the incentives for these players to cooperate.

The contributions of this thesis can be divided into three parts. The first part focussed on developing an original approach examining the principals of coalition formation with strict entry requirements. The second part developed a combined trade and transport model and considered situations in which players optimise over different time horizons. The third and final part extended this to the basis of sanction imposition and quantified the extent to which market forces and sanction-busting impacted the effectiveness of sanctions. Four objectives were set for the thesis in section 1.2, these are now considered to show how this thesis aids the research community in achieving important contributions.

6.1.1 Coalitions with Selfish Entry Criteria

There exist a number of implementations of both network formation games and coalition formation structures in the literature which can result in different end results. As such, the first steps introduced both the network structures and game-theoretic metrics that are important in evaluating the state of the game.

The thesis then goes on to introduce the implementation of coalitions with selfish entry requirements, with unanimous consent of all existing members being required before new members may join the coalition. This parallels voting structures where all players have a veto, such as the European Union. The selected coalition methodology is applied to implementations of the prisoner's dilemma and stag hunt social dilemma games, highlighting its applicability and illustrating basic results. A network formation game is then introduced which underpins the coalition model.

Results follow, examining the incentives on players in a numerical model before giving results analytical results about coalition structure, size and stability. The impact of these coalitions on the game-theoretic utility through the price of anarchy and price of stability is also found, allowing the control cost of establishing the optimal stable network to be found.

6.1.2 Linked Trade and Transport Modelling

The elements required for the novel model are introduced, developing on a networked markets model from the literature, through the addition of a second layer on which a selfish routing game is played.

This model is then analysed, examining equilibria and optimal response for different time horizons over each of the two layers individually. General formulae for the optimal response under different time preferences were found, as well as differences between cost structures in single layer implementations and with the addition of a transport layer. Results are also given on the incentives for cooperation that exist in this model, finding that players benefit more from acting selfishly than they would from cooperating.

Numerical results examining the approach to equilibrium in a sample game are presented, demonstrating the parallels between selfish routing and multi-modal market-path selection, with marginal utility being equal for each player in this model. These results are also confirmed analytically, deriving the characteristics of a bilayer solution, as well as of the uniqueness of the solution.

6.1.3 Sanction Modelling

A further extension of the model is then introduced, including requisite elements from sanction literature. This model is comprised of three layers representing: manufacturing and market nations; transport infrastructure, and; national coalitions.

The performance of a dynamic game being played out over the model structure is examined in order to evaluate the incentives for sanctions, establishing a tree of major changes to the characteristics of the network game through defection, sanctions and coalition formation. Results on utility outcomes for players through the dynamic game which result in these major choices being advantageous (or not) are given.

Analytical results are confirmed through numerical simulation, evaluating the relative changes in utility observed, confirming the model's applicability through the confirmation of the inctentives to impose countersanctions, the impact of increased sanction coalition size and of transport capacity constraints.

Finally analytical results are derived which confirm the continued existence of unique stable equilibria both where no coalitions have formed and where coalitions have developed as well as give bounds on the coalitional value a sanction target has, thereby informing their negotiating power.

6.2 Recommendations for Future Research

While working on this thesis, various potential new areas of research in the field of network games with coalition dynamics were identified. There is also space for further development of economic models through bringing them into closer alignment with real-world observations. This section outlines these research recommendations for further work that could be pursued to extend this thesis.

Future Work on Coalitions with Entry Criteria

The existing coalition formation and dissolution methodology operate under the assumption that coalition structures are rigid, undertaking collective strategy change through the impetus of a single player within the coalition. However, this precludes the opportunity for coalitions to merge with other coalitions or completely disappear in a single step.

A coalitional merge under these conditions would require a Pareto improvement for all members of both coalitions. This is a difficult requirement, but given the benefits of collective action, one that is possible, particularly where the merging process allows the consideration of more than two coalitions simultaneously. Within a basis of regional cooperation in transport infrastructure, this would represent higher levels of authority combining or dissolving regions.

A coalitional collapse would be more closely inspired by the close-to-immediate collapse of large political institutions (such as the Soviet Union) when confidence is broken. This coalitional collapse alters the network structure dramatically and the process for the formation of new coalitional structures that form in the aftermath would develop the existing work.

Finally, the application of coalitions with strict entry criteria to games other than the network formation game, generalising the results, provides the potential for further results.

Future Work on the Derivation of Optimal Strategy for Large Player Count

The work in Chapter 4 gives analytical results on optimal strategic responses in the 2-player game, which is particularly tractable given the single degree of freedom. It also gives general results for all player counts on the myopic response. However, the dimensionality of the strategy space increases linearly with player count, resulting in high complexity very quickly.

Two potential approaches to address this are the creation of bounds on optimality of strategies in the n-player case and the use of evolutionary models to approximate optimal strategy through progressive increases in fitness.

Future Work on the Baselining of the Trade-Transport Model

The work in Chapter 4 uses simple congestion functions chosen for their analytical tractability, their precedence in the literature and their computational efficiency. However, real-world transport and trade follow more complicated patterns. Accordingly, in order to use this model's characteristics in a predictive manner, an explicit baselining of the model to an application would provide additional value. In particular, the use of existing studies into the relationship between supply and price may significantly alter the incentives on players, having knock-on effects on player profit and the most desireable transportation routes.

This baselining would also involve the construction of a specific trade network; the assumption that bipartite connections are all-to-all is not appropriate. This would require an algorithmic approach, taking advantage of the numerical simulation techniques developed for the models in Chapter 5. It would also be appropriate to use graph reduction techniques to reduce the complexity of underlying networks.

The model also makes the assumption that the goods produced by the players have perfect substitutability. An examination of the impact of a reduction in substitutability on market demand should be conducted, anticipating that this would alter the equilibrium characteristics.

Future Work on the Impact of Incomplete Information on Player Strategy

It is assumed throughout this thesis that players have perfect knowledge in their decision making. Introducing information uncertainty, in particular with respect to the transportation network and market prices that influence the player's payoffs, allows for greater applicability to real world scenarios where transportation time leads to uncertainty in outcome.

A proposed implementation would retain the bilayer network structure which appears in Chapter 4, introducing a third layer. This layer has points of contact with the transport layer and the markets, with information from those locations diffusing through the network, competing against a status quo created by the existing information. Consensus dynamics would continuously bring the information network's average knowledge towards the true values of the network, but there would be a delay. The players would also be embedded in this layer, using their limited understanding to make strategy choices. Given the model's focus on the adjustment to a new equilibrium after disruption, the modelling of players acting with imperfect information could allow for additional understanding of players' actions, as well as being an interesting problem analytically.

Future Work on a Case Study for Sanctions Modelling

The sanctions modelling which appears in Chapter 5 examines a case in which the trade capacity of the sanctioned nation is many times the capacity of the independent third parties who choose to act as intermediaries for the purposes of transhipment. The examination of additional case studies where the relative power of the involved players are different, such as EU sanctions on Egypt in 2013 on human rights grounds [120]. This would give additional context to the applicability of the techniques, and potentially suggest refinements to the model.

6.3 Closing Remarks

This thesis has developed methods for assessing the formation and dissolution of coalitions and models for assessing the economic impacts of disruption caused by both network changes and geopolitical choices. These methods were compared to data from the EUSTAT database, and the insight these tools were capable of generating was discussed.

This thesis first developed a novel approach to coalition formation, applying the techniques to a prisoner's dilemma and network formation model. The characteristics for change in coalition structure were discussed before giving their implications on each of the models, demonstrating the set of stable coalitions as punishment magnitude and edge creation cost vary. Then, a focussed model of trade and transport was developed and analysed, finding optimal player strategies over any time-horizon. Single layer equilibria were discussed before analysing the complete model, both analytically and numerically. The model was then further developed, including more explicit coalition formation elements alongside a specific application to sanction-busting. The model was then used to derive characteristics of the game's equilibria that lead to specific major decisions by players. Finally, a case study was examined, observing real-world changes in behaviour that inform the model's characteristics.

Future work has been identified which could advance the knowledge contributed in this thesis. There is the potential for further development of coalitional tools through additional strategy choices and greater complexity in parallel games. There is also the potential to advance knowledge through the analysis of case studies, creating more specific models that could build on this thesis and help provide stakeholders with the confidence that secret cooperation will not prevent important interventions from being important.

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Appendix A

European Sanction Data

Appendix A contains stacked area graphs for imports and exports for SITC groups 3,6+8 and 7, using data from the european database [164]. The graph for SITC 7 exports appears in Chapter 5. SITC 7 was the most affected by European sanctions on Russia in 2022, with groups 3 and 6+8 having the most significant secondary impact. Imports are also included, despite their relative smaller quantity in order to investigate the impact of countersanctions, as discussed in Peksen and Jeong [124].

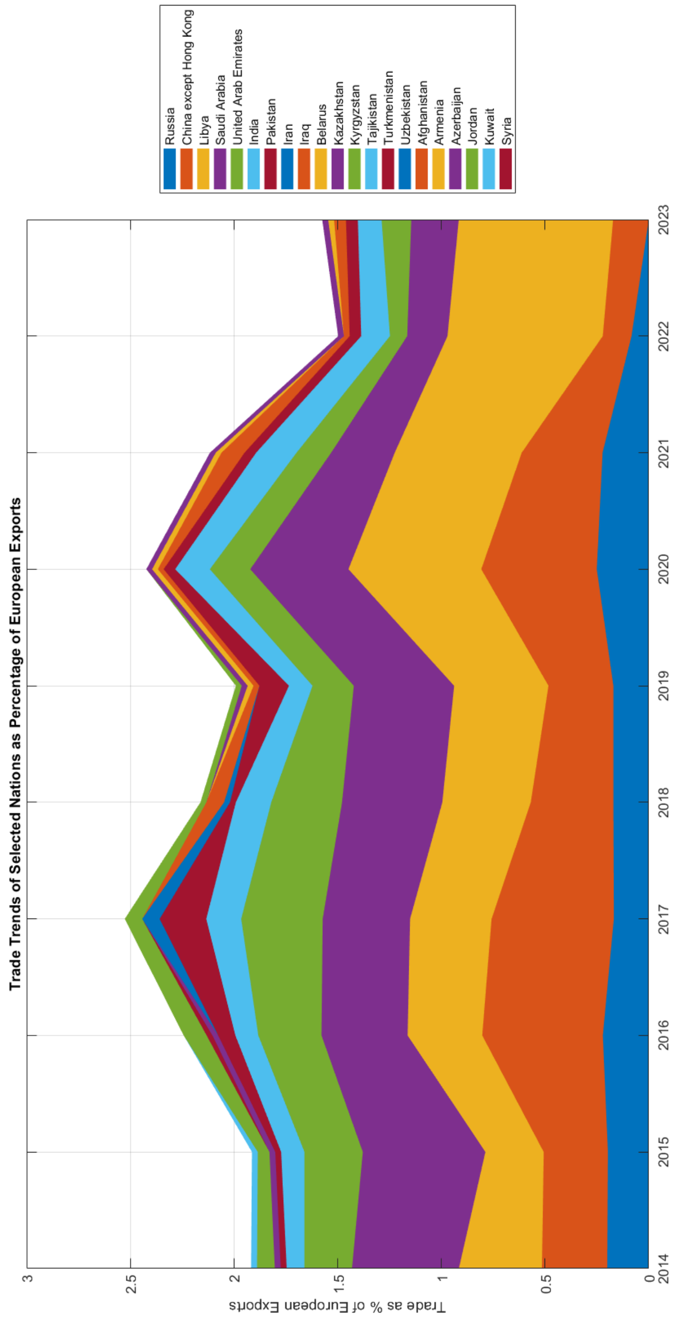

Figure A.1: *European exports of SITC 3 goods*

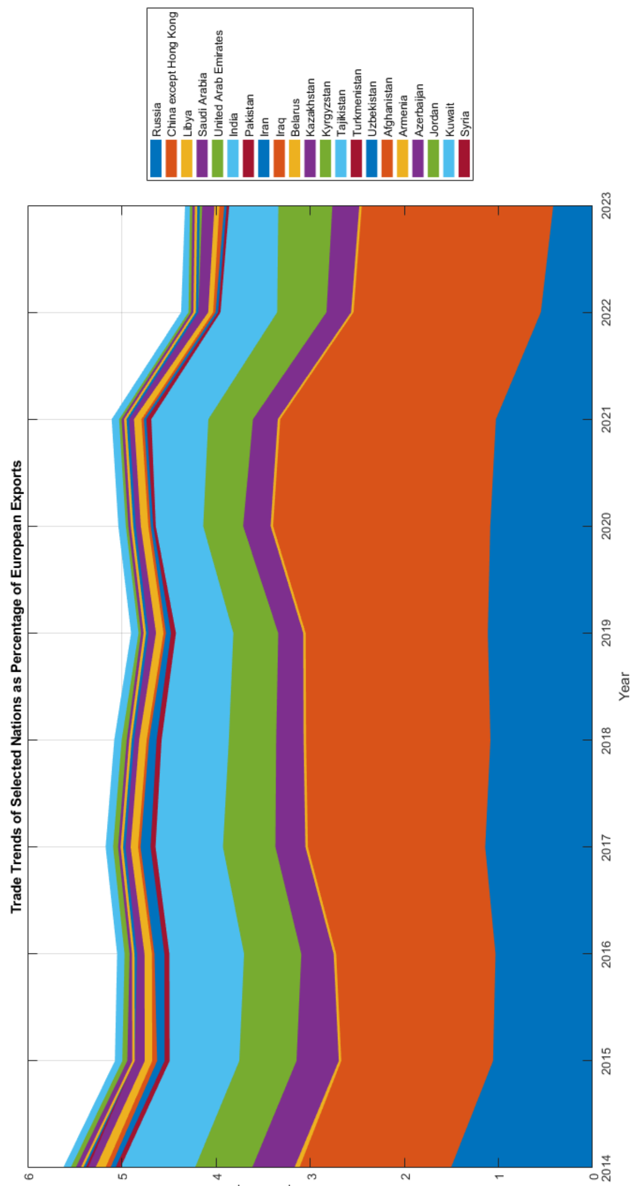

Figure A.2: *European exports of SITC 6+8 goods*

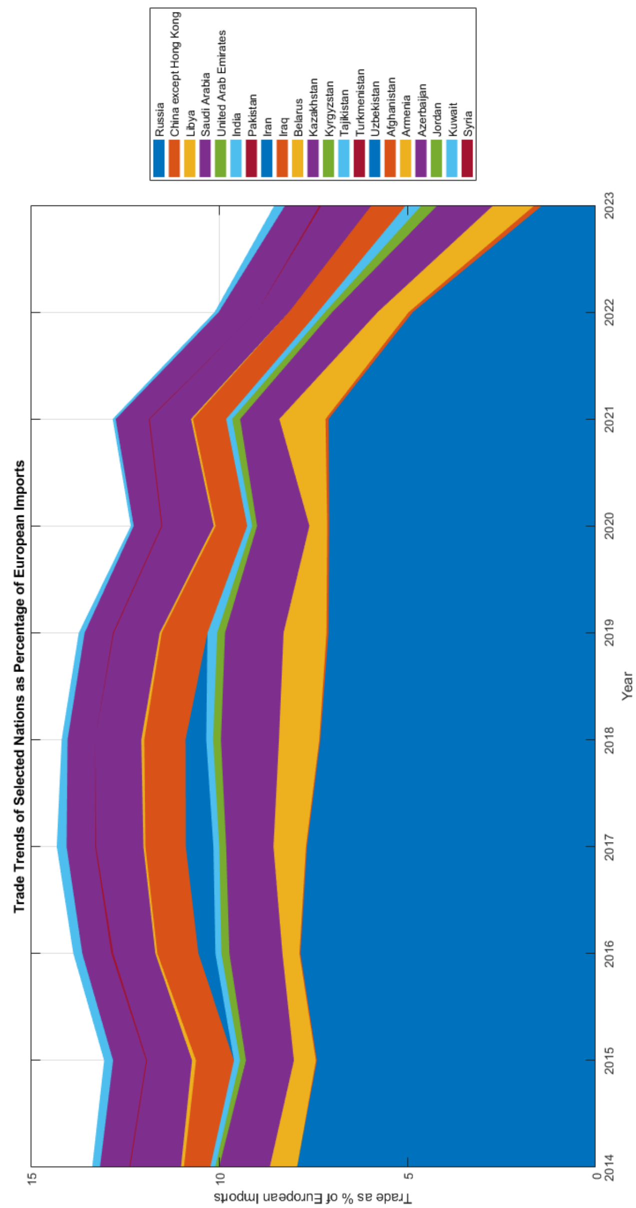

Figure A.3: *European imports of SITC 3 goods*

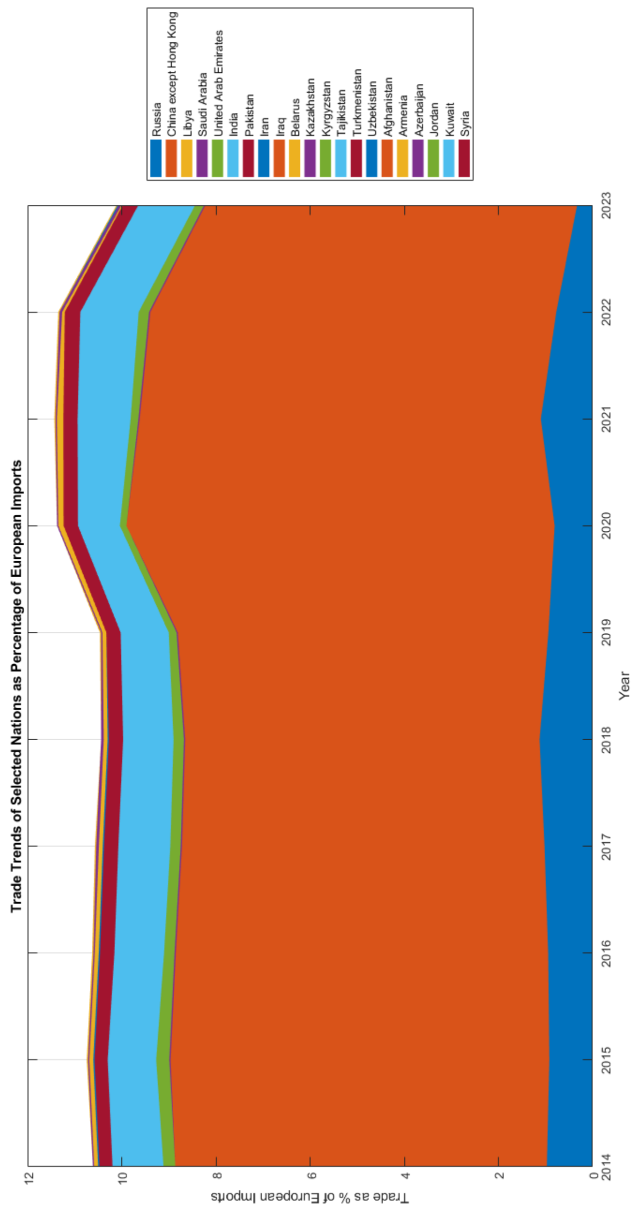

Figure A.4: *European imports of SITC 6+8 goods*

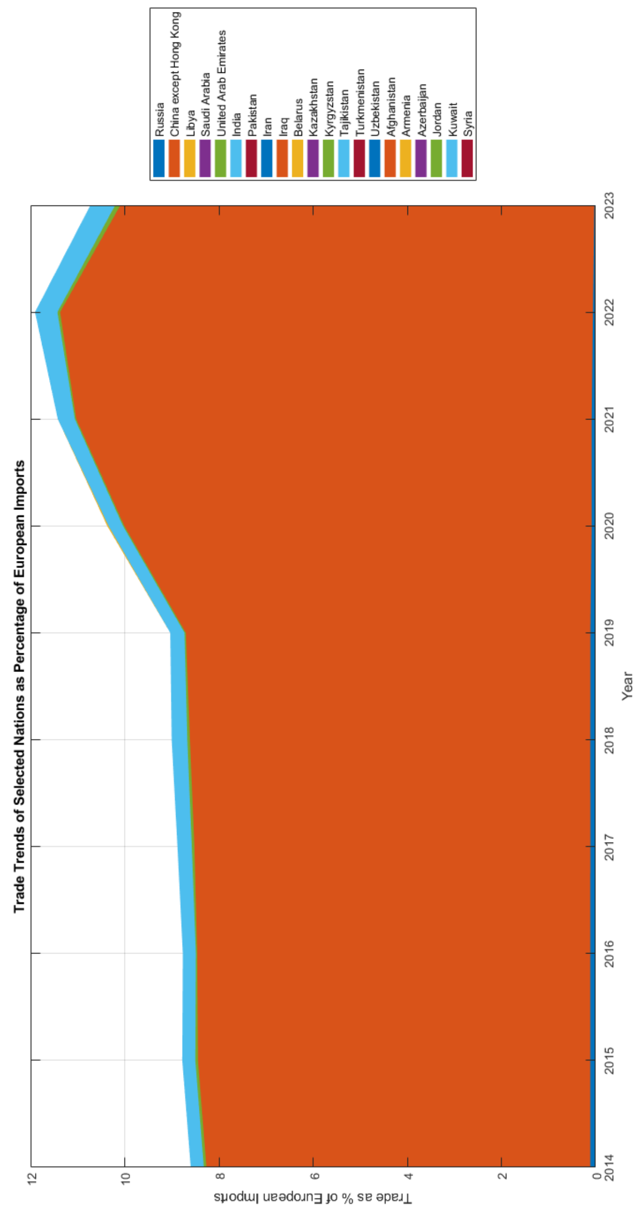

Figure A.5: *European imports of SITC 7 goods*