



Formalisation and Validation of Multiaxial Fatigue Assessment Framework for Hybrid Aluminium-to-Steel Welded Joints

Chin Tze Ng

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Supervised by:

Professor Luca Susmel
Professor Iman Hajirasouliha
Professor Buick Davison

The University of Sheffield
Faculty of Engineering
School of Mechanical, Aerospace and Civil Engineering

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Declaration of Authorship

I, Chin Tze Ng, confirm that the Thesis is my own work, except where proper acknowledgment is given. I am aware of the University's Guidance on the Use of Unfair Means (www.sheffield.ac.uk/ssid/unfair-means). This work has not been submitted previously for any degree or qualification at this or any other university.

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Abstract

The multiaxial fatigue assessment of welded joints is a critical challenge in structural engineering, particularly for hybrid aluminium-to-steel welded joints. While extensive research exists for steel and aluminium welded joints, no established methodologies currently exist for hybrid joints. This study aims to develop a validated multiaxial fatigue assessment framework for hybrid aluminium-to-steel welded joints by integrating experimental testing, stress analysis approaches, and multiaxial fatigue criteria to address existing knowledge gaps.

A comprehensive review and quantitative assessment were conducted on standard fatigue design methodologies from Eurocodes and the International Institute of Welding (IIW), as well as stress-based critical plane criteria. These criteria were evaluated using various stress analysis approaches under constant amplitude (CA) and variable amplitude (VA) loading conditions. Findings revealed that standard fatigue design methods are generally conservative, while the Modified Wöhler Curve Method (MWC) demonstrated the highest predictive accuracy under VA loading.

To characterise the fatigue behaviour of hybrid joints, experimental fatigue tests were conducted on aluminium-to-steel welded specimens manufactured using EWM ColdArc® welding technology. Failure analysis revealed that fatigue cracks consistently initiated from the aluminium side, primarily at the weld toe, weld root, or heat-affected zone (HAZ), and occasionally within the weld itself. Crack initiation was shear stress-driven, while crack propagation followed normal stress directions.

A comprehensive validation of multiaxial fatigue criteria confirmed that the IIW approach delivered the most reliable and accurate fatigue life estimations for the hybrid joints across all stress analysis methods. The MWC was effective with localised stress analysis methods but unsuitable for nominal stress in out-of-phase loading.

This study establishes a scientifically validated multiaxial fatigue assessment framework for thin hybrid aluminium-to-steel welded joints, contributing to fatigue-resistant design, structural safety, and practical engineering recommendations for real-world applications requiring weight reduction and fatigue performance.

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Nomenclature

$\Delta\sigma_x$	Normal stress range in x-direction (MPa)
$\Delta\sigma$	Normal stress range (MPa)
$\Delta\sigma_n$	Normal stress range relative to the critical plane (MPa)
$\Delta\tau_{xy}$	Shear stress range in xy-direction (MPa)
$\Delta\tau$	Shear stress range (MPa)
σ_a	Normal stress amplitude (MPa)
τ_a	Shear stress amplitude (MPa)
$\Delta\sigma_n$	Normal stress range relative to the critical plane (MPa)
$\Delta\sigma_{eq}$	Equivalent normal stress range for VA loading (MPa)
$\Delta\tau_{eq}$	Equivalent shear stress range for VA loading (MPa)
$\Delta\sigma_R$	Reference normal stress corresponding to N_R cycles to failure (MPa)
$\Delta\tau_R$	Reference shear stress corresponding to N_R cycles to failure (MPa)
$\Delta\sigma_{R,P_s=97.7\%}$	Reference normal stress at N_R cycles recalculated for a P_s of 97.7% (MPa)
$\Delta\tau_{R,P_s=97.7\%}$	Reference shear stress at N_R cycles recalculated for a P_s of 97.7% (MPa)
$\Delta\sigma_{NS}$	Effective normal stress range derived from nominal stress approach (MPa)
$\Delta\sigma_{HSS}$	Effective normal stress range derived from hot spot stress approach (MPa)
$\Delta\sigma_{ENS}$	Effective normal stress range derived from effective notch stress approach (MPa)
σ_{min}	Minimum normal stress (MPa)
σ_{max}	Maximum normal stress (MPa)
τ_{min}	Minimum shear stress (MPa)
τ_{max}	Maximum shear stress (MPa)
$\Delta\tau_{NS}$	Effective shear stress range derived from nominal stress approach (MPa)
$\Delta\tau_{HSS}$	Effective shear stress range derived from hot spot stress approach (MPa)
$\Delta\tau_{ENS}$	Effective shear stress range derived from effective notch stress approach (MPa)
σ_m	Normal mean stress (MPa)
σ_u	Ultimate tensile strength (MPa)
D_{EC3}	Allowable damage sum for EC3 criterion under CA

D_{IIW}	Allowable damage sum for IIW criterion under CA
D_{σ}	Allowable damage sum from normal stress
D_{τ}	Allowable damage sum from shear stress
D_{tot}	Total allowable damage sum from both normal and shear stress damage
D_{crit}	Critical cumulative damage threshold under VA
D_{spec}	Specified Miner sum (allowable damage sum)
D_{al}	Allowable damage sum
D_{real}	Real damage sum derived experimentally
CV	Comparison value for IIW interaction equation criterion under VA
$\Delta\sigma_{eff}$	Effective stress range (MPa)
$\Delta\sigma_{criterion}$	Effective equivalent stress range defined by the specific fatigue criterion (MPa)
$\Delta\sigma_{VM}$	Effective von Mises stress range (MPa)
$\Delta\sigma_{MP}$	Effective absolute maximum principal stress range (MPa)
$\Delta\sigma_{find}$	Effective equivalent stress range derived from Findley's criterion (MPa)
$\Delta\sigma_{CS}$	Effective equivalent stress range derived from CS criterion (MPa)
$\Delta\sigma_{KoNoS}$	Effective equivalent stress range derived from Sonsino KoNoS criterion (MPa)
$\Delta\sigma_{SWT}$	Effective equivalent stress range derived from stress-based SWT criterion (MPa)
$\Delta\tau_{MWCM}$	Effective equivalent stress range derived from MWCM criterion (MPa)
$\Delta\sigma_{R=-1}$	Fully reversed uniaxial reference stress range at N_R cycles to failure (MPa)
$\Delta\tau_{R=-1}$	Fully reversed torsional reference stress range at N_R cycles to failure (MPa)
$\Delta\tau_{R,MWCM}$	MWCM reference shear stress range determined at N_R cycles to failure (MPa)
$\tau_{MV}(t)$	Time-varying shear stress resolved along maximum variance direction (MPa)
τ_m	Mean shear stress (MPa)
T	Time interval (s)
λ	Biaxiality stress ratio
f	Fatigue damage parameter for Findley's criterion (MPa)
β	Material constant for Findley's criterion
δ	Off angle between the normal to critical plane and the weighted average direction of the first principal stress for CS criterion (rad)
ρ_w	MWCM effective critical plane stress ratio for welded joints
$\rho_{w,lim}$	Limit of the MWCM critical plane stress ratio for welded joints
R	Stress ratio
t	Weld joint plate thickness under loading (mm)

$\sigma_{0.4t}$	Stress state at 0.4t according to the hot spot stress linear extrapolation method (MPa)
$\sigma_{1.0t}$	Stress state at 1.0t according to the hot spot stress linear extrapolation method (MPa)
k	Negative inverse slope for uniaxial S-N curve
k_0	Negative inverse slope for pure torsional S-N curve
k^*	Negative inverse slope of post-knee segment for uniaxial S-N curve under CA
k_0^*	Negative inverse slope of post-knee segment for torsional S-N curve under CA
k'	Negative inverse slope of post-knee segment for uniaxial S-N curve under VA
k_0'	Negative inverse slope of post-knee segment for torsional S-N curve under VA
m_1	Slope of S-N curve above the knee point
m_2	Slope of S-N curve below the knee point
$\Delta\sigma_{i,S,d}$	Stress ranges above the knee point (MPa)
$\Delta\sigma_{j,S,d}$	Stress ranges below the knee point (MPa)
$\Delta\sigma_{L,d}$	Stress range at the knee point of SN curve (MPa)
n_i	Number of cycles at applied stress range $\Delta\sigma_i$
n_j	Number of cycles at applied stress range $\Delta\sigma_j$
λ_i	Equivalent damage factor according to EC3 for VA
N_R	Reference number of cycles to failure
N_f	Experimental number of cycles to failure
$N_{f,e}$	Estimated number of cycles to failure
$N_{f(i)}$	Experimental number of cycles to failure for test i
$N_{f,e(i)}$	Estimated number of cycles to failure for test i
N_{kp}	Number of cycles corresponding to the knee point
n	Total number of observations in each test series
$f(R)$	Enhancement factor for stress-relieved welded joints
P_{NC}	Percentage of non-conservative estimates (%)
P_c	Percentage of conservative estimates (%)
P_s	Probability of survival (%)
r_{ref}	Fictitious reference notch radius for ENS approach (mm)
$A_{ini,exp}$	Experimental crack initiation angle ($^\circ$)
$A_{pro,exp}$	Experimental crack propagation angle ($^\circ$)

$A_{ini,theo}$	Theoretical crack initiation angle calculated from Mohr's circle (°)
$A_{pro,theo}$	Theoretical crack propagation angle calculated from Mohr's circle (°)
E	Error between experimental and theoretical angles in terms of percentage (%)
M-D _v	Critical Distance for TCD PM (mm)
RMSLE	Root mean squared logarithmic error
T _{RMS}	Performance metric used to assess accuracy of each fatigue criterion; lower values indicate higher predictive accuracy
RMSLE	Root mean squared logarithmic error
AM	Area Method
ASTM	American Society for Testing and Materials
CA	Constant amplitude
CS	Carpinteri-Spagnoli
EC3	Eurocode 3
EC9	Eurocode 9
ECs	Eurocodes
EESH	Effective Equivalent Stress Hypothesis
ENS	Effective Notch Stress
ENS Avg	Modified Effective Notch Stress using Average Measured Notch Radii
FEA	Finite element analysis
FKM	Forschungskuratorium Maschinenbau
HAZ	Heat Affected Zone
HSS	Hot Spot Stress
IIW	International Institute of Welding
KoNoS	KoNoS Hypothesis Proposed by Sonsino for Semi-Ductile Materials
LM	Line Method
MP	Maximum Principal Stress Criterion
MVM	Maximum Variance Method
MWCM	Modified Wöhler Curve Method
NS	Nominal Stress
N-SIF	Notch Stress Intensity Factor
P-M	Palmgren-Miner Rule
PM	Point Method
PSB	Persistent Slip Bands
SWT	Smith, Watson and Topper Criterion
TCD	Theory of Critical Distances
VA	Variable amplitude
VM	Von Mises Stress Criterion
WR	Weld Root
WT	Weld Toe

Chapter 1

Introduction

1.1 Background

Welding is one of the most essential and widely adopted joining technologies in industries such as offshore structures, automotive, and bridge engineering, enabling the fabrication of strong, durable, and cost-effective structural assemblies. However, despite its advantages, welded joints are inherently prone to fatigue failure. The presence of residual stresses, weld imperfections, and localised stress concentrations significantly reduces fatigue strength, making welded joints more susceptible to premature failure compared to the base materials. Given that welded joints are often the weakest link in structural assemblies, ensuring their reliability and durability under service loads remains a major engineering challenge.

In recent years, lightweight structural design has gained increasing attention, particularly in the transportation and infrastructure sectors, where reducing weight without compromising structural integrity is crucial for improving efficiency and sustainability. Hybrid aluminium-to-steel welded joints offer a promising solution, combining aluminium's lightweight properties with steel's high strength to achieve enhanced mechanical performance and weight reduction. However, the significant differences in mechanical, thermal, and metallurgical properties between aluminium and steel present major challenges in the welding process. One of the primary challenges is the formation of brittle intermetallic compounds (Fe-Al phases) at the weld interface, which has historically weakened fatigue strength and restricted the practical application of these joints (Bouche, Barbier and Coulet, 1998; Shahverdi *et al.*, 2002; Goecke, 2005; Kah, Suoranta and Martikainen, 2013; Meco *et al.*, 2015; Al Zamzami *et al.*, 2018; Gullino, Matteis and D'Aiuto, 2019).

To address this challenge, low-heat-input welding techniques have been developed, with EWM coldArc® welding emerging as a promising solution. This advanced welding technology has demonstrated the potential to minimise intermetallic phase formation, thereby improving weld integrity and mechanical performance (Goecke, 2005; Kah, Suoranta and Martikainen, 2013). While coldArc® represents a major advancement in welding, the fatigue behaviour of aluminium-to-steel welded joints under complex loading conditions remains largely unexplored. Moreover, there is currently no established methodology for accurately predicting multiaxial fatigue behaviour in these joints, highlighting a critical gap in fatigue assessment methodologies.

Traditional fatigue design and assessment approaches have been predominantly developed for homogeneous steel and aluminium welded joints, often relying on standardised stress analysis methods and multiaxial fatigue criteria. However, their applicability to hybrid welded joints remains uncertain. Given the increasing demand for hybrid structures, there is a need for reliable and accurate theoretical tools capable of predicting the fatigue life of aluminium-to-steel welded joints under multiaxial loading conditions.

This study aims to address these challenges by developing a systematic and validated fatigue assessment framework for hybrid aluminium-to-steel welded joints. By integrating experimental investigations, advanced stress analysis approaches, and multiaxial fatigue criteria, this research seeks to bridge the gap between existing fatigue assessment methodologies and their applicability to hybrid welded structures. The findings will provide engineering insights and practical recommendations to enhance the design, durability, and structural performance of hybrid welded joints in real world engineering applications.

1.2 Aim and Objectives

This research aims to develop a systematic and validated multiaxial fatigue assessment framework for thin hybrid aluminium-to-steel welded joints. By addressing the existing knowledge gaps, this study seeks to develop accurate, reliable, and practical methodologies that support structural design, durability, and optimisation in engineering applications.

To achieve this aim, the following key objectives are pursued:

- Review and evaluate existing multiaxial fatigue assessment methodologies, including standard fatigue design approaches recommended by Eurocodes and the International Institute of Welding (IIW), alongside stress-based critical plane criteria to determine their applicability to steel and aluminium welded joints under constant amplitude (CA) and variable amplitude (VA) loading conditions.
- Perform a quantitative assessment of standard fatigue criteria across various stress analysis approaches to systematically assess their accuracy and limitations in predicting fatigue behaviour in steel and aluminium welded joints under CA and VA loading conditions.
- Conduct a quantitative review of stress-based critical plane criteria, integrating them with different stress analysis approaches, to establish a structured comparison of their predictive capabilities and suitability for multiaxial fatigue life estimation in steel and aluminium welded joints.
- Design and manufacture hybrid aluminium-to-steel welded joint specimens using EWM coldArc® welding technology for experimental multiaxial fatigue testing.
- Generate experimental fatigue data to characterise the fatigue behaviour of hybrid aluminium-to-steel welded joints, facilitating failure analysis and validation of multiaxial fatigue criteria for these joints.

- Investigate the failure mechanisms of hybrid aluminium-to-steel welded joints, focusing on crack initiation sites, failure modes, and crack path analysis from meso- to macroscopic scales, to improve the understanding of their fatigue behaviour.
- Perform Finite Element Analysis (FEA) for localised stress analysis, providing useful stress quantities for multiaxial fatigue criteria to enhance fatigue life estimation accuracy.
- Validate and compare the accuracy of multiaxial fatigue criteria across different stress analysis approaches by correlating estimated fatigue life with experimental fatigue data for hybrid aluminium-to-steel welded joints under uniaxial, pure torsion, in-phase, and out-of-phase CA loading conditions.
- Assess and recommend suitable multiaxial fatigue assessment methodologies based on their accuracy and applicability, providing insights for practical engineering applications and future research.

This structured approach ensures a comprehensive evaluation and validation of fatigue assessment methods, ultimately bridging the gap between existing fatigue design methodologies and their practical application to hybrid aluminium-to-steel welded joints.

1.3 Thesis Outline

This thesis is structured into eight chapters, systematically addressing the research problem, methodology, findings, and conclusions. Each chapter builds upon the previous chapter to establish a comprehensive framework for multiaxial fatigue assessment in hybrid aluminium-to-steel welded joints.

Chapter 1 introduces the research background, emphasising the significance of multiaxial fatigue assessment for hybrid aluminium-to-steel welded joints. It defines the research aim, objectives, and scope of the study while providing an overview of the

thesis structure. Additionally, this chapter declares all publications derived from this research.

Chapter 2 presents a detailed literature review on welding technology, stress analysis approaches, and multiaxial fatigue criteria. It critically evaluates existing methodologies, identifies their advantages and limitations, and highlights knowledge gaps. This chapter establishes the theoretical foundation for the study, ensuring a structured approach to the subsequent research.

Chapter 3 conducts a quantitative review of standard multiaxial fatigue criteria recommended by Eurocodes and the IIW. It systematically assesses their effectiveness across various stress analysis approaches in estimating the fatigue life of steel and aluminium welded joints under CA and VA loading conditions.

Chapter 4 extends this quantitative review to stress-based critical plane criteria, integrating them with different stress analysis methods to evaluate their applicability to multiaxial fatigue assessment. The chapter focuses on three widely used stress-based critical plane criteria, Findley's criterion, the Carpinteri-Spagnoli (CS) criterion, and the Modified Wöhler Curve Method (MWCM). Through this analysis, it examines their predictive accuracy in estimating fatigue life. Together, Chapters 3 and 4 provide essential insights into the applicability of these criteria, forming a foundation for their extension to hybrid aluminium-to-steel welded joints.

Chapter 5 details the experimental methodology for fatigue testing of hybrid aluminium-to-steel welded joints. It outlines the materials and specimen configurations, the EWM coldArc® welding process and parameters, the experimental setup for fatigue tests, and the methodology for optical microscopic measurements of weld toe and root radii. These measurements are essential for failure analysis and FE modelling.

Chapter 6 focuses on the failure analysis of hybrid aluminium-to-steel welded joints, investigating crack initiation sites, failure modes, and crack path orientation. This chapter provides critical insights into their failure mechanisms, forming the basis for the formalisation and validation of a multiaxial fatigue assessment methodology in the next chapter.

Chapter 7 establishes a robust and physically representative multiaxial fatigue assessment framework for hybrid aluminium-to-steel welded joints under multiaxial loading conditions. It begins with the fatigue characterisation of hybrid joints, including the calibration of reference S-N curves derived from experimental test data, and evaluates mean stress effects and the influence of non-proportional loading. It then synthesises the findings by validating and comparing different multiaxial fatigue criteria using experimental data and FEA. The chapter concludes by proposing a systematic fatigue assessment methodology and providing recommendations for the design of hybrid aluminium-to-steel welded joints.

Chapter 8 presents the overall conclusions of the thesis, summarising key findings from each chapter and outlining the primary contributions of this research. It also discusses limitations of the studies and provides recommendations for future research directions.

This structured approach ensures a logical progression of the research, from theoretical analysis to experimental validation and practical recommendations, bridging the gap between existing fatigue assessment methodologies and their application to thin hybrid aluminium-to-steel welded joints.

1.4 Publications

The research conducted in this thesis has led to the publication of the following peer-reviewed journal papers.:

1. Ng, C. T., Sonsino, C. M. and Susmel, L. (2024) ‘Multiaxial fatigue assessment of welded joints: A review of Eurocode 3 and International Institute of Welding criteria with different stress analysis approaches’, *Fatigue & Fracture of Engineering Materials & Structures*, 47(7), pp. 2616–2649. doi: <https://doi.org/10.1111/ffe.14319>.

This paper presents a comprehensive quantitative review of standard multiaxial fatigue criteria suggested by Eurocodes and the IIW. It systematically evaluates their accuracy across different stress analysis approaches for steel and

aluminium welded joints under both constant amplitude (CA) and variable amplitude (VA) loading conditions.

2. Ng, C. T. and Susmel, L. (2025) ‘Quantitative Review of Critical Plane Criteria and Stress Analysis Approaches for Multiaxial Fatigue of Welded Joints’, *Fatigue & Fracture of Engineering Materials & Structures*, 48(4), pp. 1393–1428. doi: <https://doi.org/10.1111/ffe.14571>.

This study extends the quantitative review to stress-based critical plane criteria, integrating them with various stress analysis methods to assess their effectiveness in multiaxial fatigue evaluation. It provides insights into the applicability of these criteria to steel and aluminium welded joints and explores their potential extension to hybrid welded joints.

3. Ng, C. T. and Susmel, L. (2024) ‘Failure analysis of hybrid Aluminium-to-Steel welded joints under Uniaxial/Multiaxial fatigue loading’, *Engineering Failure Analysis*, 163, p. 108529. doi: <https://doi.org/10.1016/j.engfailanal.2024.108529>.

This paper presents the failure analysis of hybrid aluminium-to-steel welded joints fabricated using EWM ColdArc® welding technology. It includes crack initiation analysis, crack path orientation assessment, and failure mode identification. Additionally, it details the experimental methodology and review of the coldArc® welding technology.

These publications form the foundation of this thesis, contributing to the development of a validated multiaxial fatigue assessment framework for hybrid aluminium-to-steel welded joints.

1.5 References

Bouche, K., Barbier, F. and Coulet, A. (1998) 'Intermetallic compound layer growth between solid iron and molten aluminium', *Materials Science and Engineering: A*, 249(1–2), pp. 167–175.

Goecke, S. F. (2005) 'Low energy arc joining process for materials sensitive to heat', *EWM hightec welding GmbH*, 3.

Gullino, A., Matteis, P. and D'Aiuto, F. (2019) 'Review of aluminum-to-steel welding technologies for car-body applications', *Metals*, 9(3), p. 315.

Kah, P., Suoranta, R. and Martikainen, J. (2013) 'Advanced gas metal arc welding processes', *The International Journal of Advanced Manufacturing Technology*, 67, pp. 655–674.

Meco, S. *et al.* (2015) 'Application of laser in seam welding of dissimilar steel to aluminium joints for thick structural components', *Optics and Lasers in Engineering*, 67, pp. 22–30.

Shahverdi, H. R. *et al.* (2002) 'Microstructural analysis of interfacial reaction between molten aluminium and solid iron', *Journal of materials processing technology*, 124(3), pp. 345–352.

Al Zamzami, I. *et al.* (2018) 'Static strength and design of aluminium-to-steel thin welded joints', *Welding in the World*, 62(6), pp. 1255–1272. doi: 10.1007/s40194-018-0634-2.

Chapter 2

Literature Review

2.1 Introduction

The accurate assessment of multiaxial fatigue in welded joints remains a critical challenge in structural engineering, particularly for applications involving complex loading conditions. While significant advancements have been made in understanding fatigue behaviour in steel and aluminium welded joints, the extension of these methodologies to hybrid aluminium-to-steel welded joints remains an open research area. This chapter provides a comprehensive review of existing literature on key theoretical aspects relevant to this study, focusing on welding technology, stress analysis approaches, and multiaxial fatigue criteria.

The chapter begins by examining welding techniques, with particular focus on coldArc® technology, which has demonstrated potential for improving the quality and mechanical performance of hybrid aluminium-to-steel welded joints. The discussion then progresses to stress analysis approaches, reviewing widely adopted methods such as Nominal Stress (NS), Hot Spot Stress (HSS), Effective Notch Stress (ENS), and the Theory of Critical Distances (TCD). Each approach is critically evaluated based on its strengths, limitations, and applicability to multiaxial fatigue assessment.

Building upon this foundation, the chapter explores multiaxial fatigue criteria, covering both standard recommended criteria and more advanced stress-based critical plane criteria. The review systematically assesses their suitability for steel and aluminium welded joints under constant amplitude (CA) and variable amplitude (VA) loading conditions, establishing a theoretical basis for their potential extension to hybrid welded joints.

By critically analysing the existing body of knowledge, this chapter establishes the theoretical foundation for the subsequent investigations in this study. The findings from this review will inform the methodological framework for selecting and validating appropriate multiaxial fatigue assessment methodologies for the hybrid welded joints. The chapter concludes by highlighting key research gaps and outlining how this study systematically addresses these gaps, forming a bridge between existing fatigue assessment methodologies and their applicability to hybrid welded structures.

2.2 Welding Technologies

The development of advanced welding technologies for joining dissimilar materials has been a key area of research due to the increasing demand for lightweight, sustainable and structurally efficient assemblies in industries such as automotive, offshore and bridge engineering. Among the most commonly used structural materials, aluminium and steel present an attractive combination, offering aluminium's lightweight and recyclability alongside high mechanical strength of steel. However, welding these materials together poses significant challenges due to their contrasting thermal, mechanical, and metallurgical properties, particularly the formation of brittle Fe-Al intermetallic compounds at the weld interface, which can severely degrade joint performance (Bouche, Barbier and Coulet, 1998; Shahverdi *et al.*, 2002; Goecke, 2005; Kah, Suoranta and Martikainen, 2013; Meco *et al.*, 2015; Al Zamzami *et al.*, 2018).

To address these challenges, various advanced welding technologies have been developed to capitalise on the advantages of aluminium-to-steel joining. These include laser beam welding, friction stir welding, explosion bonding, magnetic pulse welding, and coldArc® welding (Kimapong and Watanabe, 2005; Kah, Suoranta and Martikainen, 2013; Atabaki *et al.*, 2014; Cao *et al.*, 2014; Qin, Su and Wang, 2014; Meco *et al.*, 2015; Gullino, Matteis and D'Aiuto, 2019). While all these methods aim to minimise intermetallic formation and improve weld integrity, their suitability depends on processing efficiency, material adaptability, and industrial feasibility. Among them, coldArc® welding has proven to be particularly effective, offering precise heat control and stable metal transfer, which are crucial for achieving high-quality aluminium-to-steel welds (Goecke, 2005).

2.2.1 ColdArc® Welding Mechanism

ColdArc® welding, developed by EWM Hightec Welding GmbH, is a low-energy short arc welding process designed to address the challenges associated with heat input control and intermetallic formation in hybrid aluminium-to-steel welding. Conventional arc welding techniques often introduce excessive heat input, leading to porosity, microcracks, and the formation of thick Fe-Al intermetallic compound layers, which significantly degrade weld performance. ColdArc® welding overcomes these limitations by employing a dynamically regulated current system, which maintains a controlled melting process to minimise weld defects and ensure higher-quality joints.

The key mechanism behind coldArc® welding lies in its regulated arc re-ignition process, which dynamically adjusts current levels to reduce power peaks, thereby preventing excessive heat accumulation at the weld interface (Goecke, 2005; Kah, Suoranta and Martikainen, 2013). This is achieved through a digital signal processor, which extracts power from the arc within microseconds before re-ignition, ensuring a smooth and stable molten metal transfer (Goecke, 2005; Kah, Suoranta and Martikainen, 2013). As illustrated in Figure 2.1, coldArc® welding provides significantly better control over current output during the re-ignition phase compared to conventional arc welding (Goecke, 2005; Kah, Suoranta and Martikainen, 2013). This precise regulation is crucial for minimising thermal distortion and maintaining a uniform weld bead, both of which are essential for producing structurally sound hybrid joints.

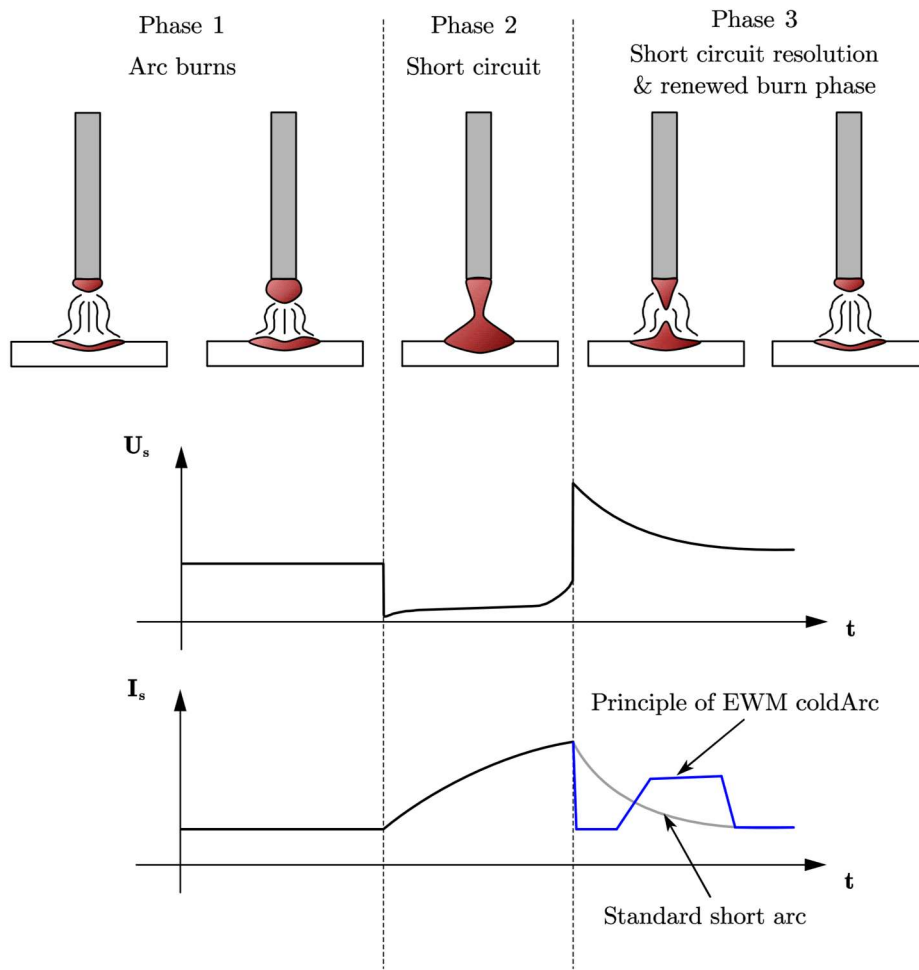


Figure 2.1: Voltage and current waveforms illustrating the arc re-ignition principle in EWM ColdArc® welding.

Following arc re-ignition, a brief melt pulse phase promotes uniform electrode melting, ensuring consistent material deposition while keeping thermal input at an optimal level (Goecke, 2005; Kah, Suoranta and Martikainen, 2013; Cao *et al.*, 2014). As shown in Figure 2.2, coldArc® welding significantly reduces power peaks, improving process stability and reducing the risk of excessive fusion and distortion (Goecke, 2005; Kah, Suoranta and Martikainen, 2013).

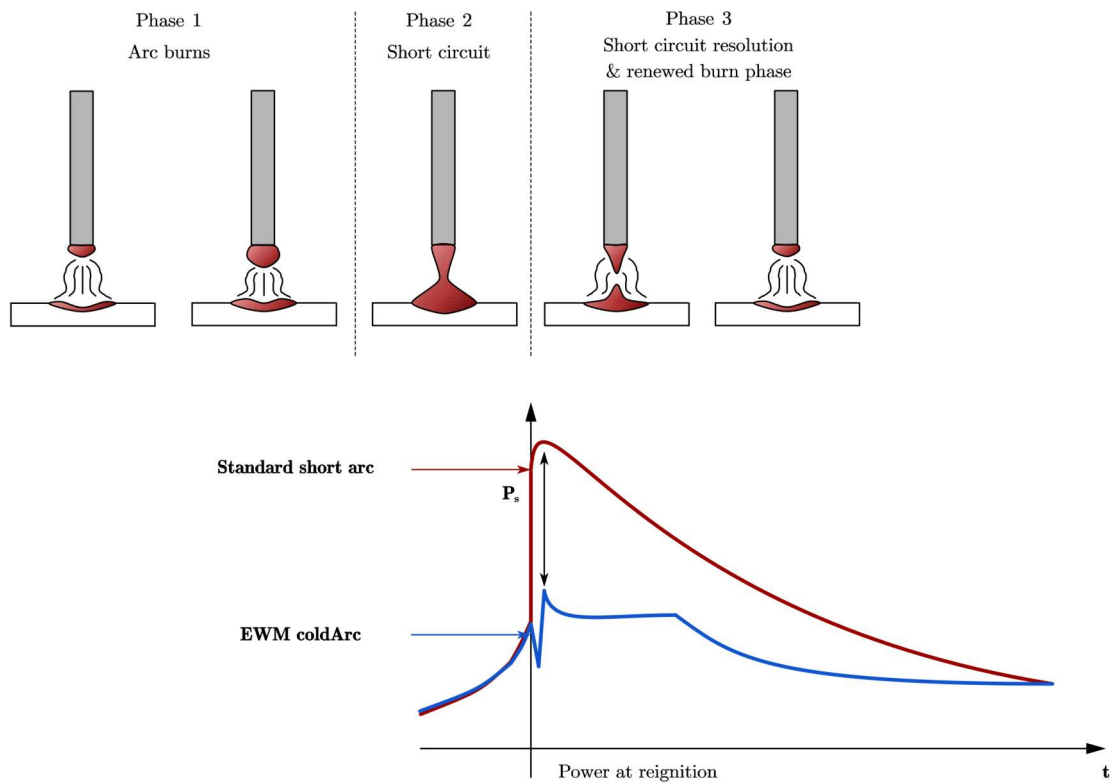


Figure 2.2: Power output reduction in EWM ColdArc® welding during the arc re-ignition phase.

The ability to finely regulate heat input allows for the welding of thin materials without burn-through, making it possible to successfully join sheets as thin as 0.3 mm using automated machines and 0.7 mm when welding manually (Goecke, 2005; Kah, Suoranta and Martikainen, 2013; Cao *et al.*, 2014; Al Zamzami *et al.*, 2018).

2.2.2 Advantages of ColdArc® Welding

The precise current regulation and optimised arc re-ignition phase of coldArc® welding offer several advantages over conventional arc welding techniques. By minimising intermetallic formation, coldArc® welding significantly improves joint ductility and fatigue resistance. Its lower heat input helps reduce excessive melting and thermal distortion, ensuring greater dimensional accuracy and structural integrity of welded components. Additionally, coldArc® welding enhances gap-bridging capabilities, accommodating minor misalignments between components without compromising weld quality (Goecke, 2005; Kah, Suoranta and Martikainen, 2013; Al Zamzami *et al.*,

2018). The process also reduces spatter and porosity, resulting in cleaner, high-quality welds with fewer post-weld defects (Goecke, 2005; Kah, Suoranta and Martikainen, 2013; Al Zamzami *et al.*, 2018).

A commonly employed strategy to further control Fe-Al intermetallic formation involves applying a zinc coating to the steel surface, effectively limiting Fe-Al phase formation in the molten material to within 10% (Richards *et al.*, 1994; Bouche, Barbier and Coulet, 1998; Goecke, 2005; Kah, Suoranta and Martikainen, 2013; Atabaki *et al.*, 2014). This approach enables partial brazing, ensuring that only the aluminium melts while the steel remains moistened, preventing the formation of brittle intermetallic phases (Goecke, 2005; Kah, Suoranta and Martikainen, 2013). ColdArc® welding has demonstrated exceptional effectiveness in maintaining this brazing-welding balance, as validated in multiple studies, establishing it as a practical and reliable method for hybrid aluminium-to-steel welding applications (Goecke, 2005; Kah, Suoranta and Martikainen, 2013; Cao *et al.*, 2014; Al Zamzami *et al.*, 2018).

Given its proven success, this zinc coating strategy was also implemented in the manufacturing of the hybrid aluminium-to-steel welded joints examined in this study, ensuring the production of strong welded joints suitable for multiaxial fatigue testing.

2.3 Stress Analysis Approaches for Welded Joints

Accurately assessing the multiaxial fatigue strength and lifetime of welded joints requires a representative stress analysis approach that effectively captures the complex stress states at critical regions. Various stress analysis methodologies have been developed and validated to facilitate this process, ranging from global stress analysis techniques to more localised approaches. These methodologies provide the necessary stress quantities for use in multiaxial fatigue criteria, ensuring reliable fatigue life estimations under different loading conditions.

Several widely accepted stress analysis approaches have been established through engineering standards and guidelines, including the Eurocodes (EC3, EC9) and recommendations from the International Institute of Welding (IIW). Additionally, more advanced approaches have demonstrated success in addressing complex fatigue loading

scenarios, such as non-proportional multiaxial loading, which are particularly critical for steel welded joints.

This study focuses on four key stress analysis approaches that are widely applied in fatigue assessment of welded joints: the NS approach, the HSS approach, the ENS approach, and the TCD approach. These methods offer varying levels of simplification and precision, each with its own strengths and applicability in evaluating fatigue performance under multiaxial loading conditions. The following sections provide a detailed review of each approach, outlining their fundamental principles, advantages, and limitations in the context of welded joint fatigue assessment.

2.3.1 NS approach

The NS approach is one of the most widely used methods for evaluating stress distributions in welded joints, relying on the nominal cross-sectional area as the reference for stress calculations, as illustrated in Figure 2.3 (Susmel and Tovo, 2004; Anon, 2005; Hobbacher, 2016; Mei *et al.*, 2021).

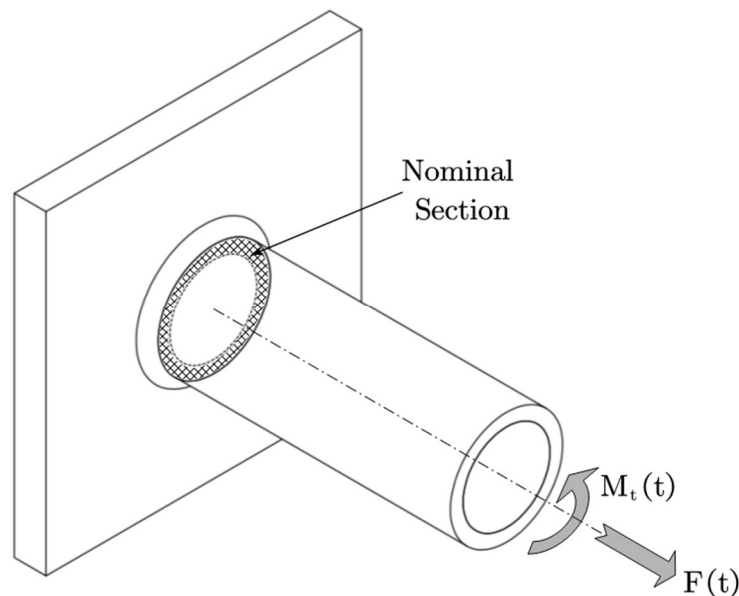


Figure 2.3: Definition of the nominal cross-sectional area for tube-to-plate welded joints based on the NS approach.

This method is particularly valued for its efficiency and practicality, making it well-suited for standard welded joint geometries where the nominal areas could be clearly

defined. Nominal stresses are typically determined using bending and torsion equations derived from simple beam theory, providing a straightforward approach to assessing fatigue strength.

A key advantage of the NS approach is its ability to account for influence of macro-scale geometric influences on stress distribution within welded joints (Niemi, 1995; Anon, 2005, 2007; Hobbacher, 2016; Mei *et al.*, 2021). As a result, it is widely adopted in fatigue design standards, including the EC3, EC9, and the IIW recommendations. These standards define reference sections for conventional welded joints, enabling practitioners to select appropriate design curves for fatigue assessment. The correct reference fatigue strength (FAT) value and negative inverse slope (k) must be identified to ensure accurate fatigue life predictions (Anon, 2005, 2007; Hobbacher, 2016). Furthermore, the classification of welded joints using the NS approach considers geometric configuration, crack initiation locations, and material type, distinguishing between aluminium and steel joints for greater accuracy in fatigue assessment (Anon, 2005, 2007; Hobbacher, 2016).

However, the NS approach has notable limitations, particularly when applied to complex welded structures. In cases where geometries are intricate, defining a nominal stress reference area is often challenging, making this method less applicable where stress concentrations play a dominant role (Niemi, 1995; Susmel and Tovo, 2004; Radaj, Sonsino and Fricke, 2006). Additionally, the NS approach does not account for localised stress effects, which are critical for accurately estimating fatigue life in complex welded joints (Niemi, 1995; Sonsino, 1995; Susmel and Tovo, 2004; Radaj, Sonsino and Fricke, 2006; Hobbacher, 2016). Due to these limitations, more advanced stress analysis methodologies are required to provide a detailed representation of critical stress states for further fatigue assessment.

2.3.2 HSS approach

The HSS approach, also known as the structural stress approach, is a widely used methodology for fatigue assessment of welded joints, particularly when defining nominal stress is impractical due to complex geometries. Unlike the NS approach, which considers stresses at the nominal cross-section, the HSS approach accounts for

stress concentration effects arising from the overall structural geometry while excluding local notch effects at the weld toe (Niemi, 1995; Anon, 2005; Radaj, Sonsino and Fricke, 2006; Hobbacher, 2016; Al Zamzami and Susmel, 2018). By incorporating both membrane stress and bending stress components, the HSS approach provides a more representative measure of the stress distribution near the weld toe (Niemi, 1995; Anon, 2005; Radaj, Sonsino and Fricke, 2006; Hobbacher, 2016).

The HSS can be determined using either finite element analysis (FEA) or strain gauge measurements taken at specific reference points (Niemi, 1995; Anon, 2005, 2007; Radaj, Sonsino and Fricke, 2006; Hobbacher, 2016; Mei *et al.*, 2021). However, since strain gauge methods require extensive experimental procedures, they are less popularly used in the literature (Archer, 1987; Razmjoo, 1997; Yousefi, Witt and Zenner, 2001; Bäckström, 2003; Costa *et al.*, 2005; Susmel, 2009c; Susmel, Sonsino and Tovo, 2011; Al Zamzami and Susmel, 2018; Alencar *et al.*, 2021). Consequently, this study focuses exclusively on the FEA-based approach for deriving hot-spot stresses.

To obtain FEA-derived HSS values, standards such as the EC3 and the IIW propose a linear stress extrapolation method using designated reference points at specific distances away from the weld toe. The selection of these reference points depends on the mesh density of the finite element model (Niemi, 1995; Anon, 2005; Radaj, Sonsino and Fricke, 2006; Hobbacher, 2009, 2016). For fine mesh models, reference points are positioned at $0.4t$ and $1.0t$ from the weld toe, where t represents the reference plate thickness. In cases where a three-point extrapolation is used, the reference points are located at $0.4t$, $0.9t$, and $1.4t$. For coarse mesh models, the reference points are taken at $0.5t$ and $1.5t$ from the weld toe (Niemi, 1995; Anon, 2005; Radaj, Sonsino and Fricke, 2006; Hobbacher, 2009, 2016).

These reference points ensure that the nearest selected point is outside the highly localised notch stress zone near the weld toe (Niemi, 1995; Anon, 2005; Hobbacher, 2016; Mei *et al.*, 2021). The HSS is then computed using linear extrapolation, following Equations 2.1 to 2.3 (Niemi, 1995; Anon, 2005; Hobbacher, 2016):

$$\sigma_{HSS} = 1.67 \sigma_{0.4t} - 0.67 \sigma_{1.0t} \quad (2.1)$$

$$\sigma_{HSS} = 2.52 \sigma_{0.4t} - 2.24 \sigma_{0.9t} + 0.72 \sigma_{1.4t} \quad (2.2)$$

$$\sigma_{HSS} = 1.50 \sigma_{0.5t} - 0.50 \sigma_{1.5t} \quad (2.3)$$

To ensure precision and reliability, fine mesh models are preferred for stress extrapolation. In this study, the two-point extrapolation method defined by Equation 2.1 was specifically adopted in the reanalysis conducted in Chapters 3 and 4 to assess standard and critical plane-based fatigue criteria.

A visual representation of the HSS derivation process using the FEA-based fine mesh approach is provided in Figure 2.4, illustrating the extrapolation of hot-spot stresses at designated reference points (Niemi, 1995; Anon, 2005; Hobbacher, 2016).

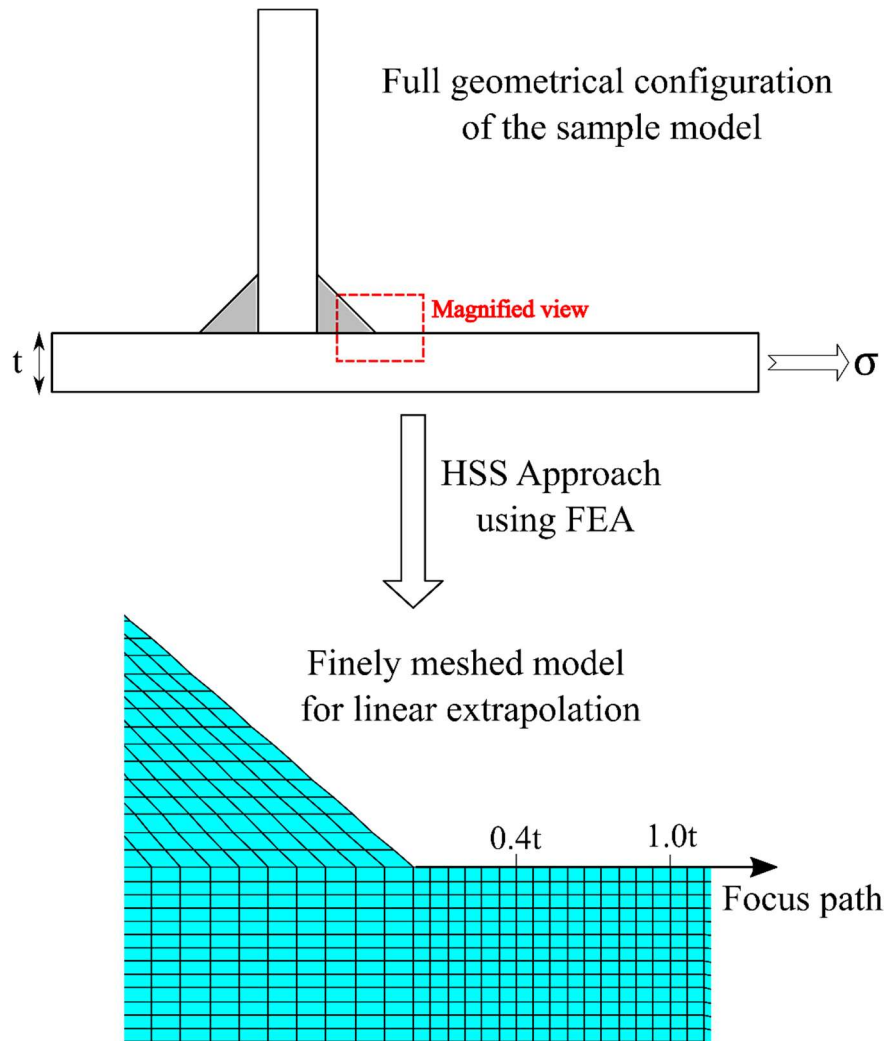


Figure 2.4: Schematic representation of the hot-spot stress extrapolation process using FEA with fine mesh models.

While the standard HSS approach is based on the maximum principal structural stress range at the crack initiation point, a modified version is necessary for multiaxial fatigue assessments, as applied in Chapter 4 (Niemi, 1995; Anon, 2005, 2007; Susmel and Tovo, 2006; Hobbacher, 2016). Originally proposed by Susmel and Tovo, this modified HSS approach separates the structural stress into normal and shear stress components, allowing for critical plane-based fatigue assessments under multiaxial loading conditions (Susmel and Tovo, 2006).

This modification is particularly relevant when fatigue failure is influenced by Mode I and Mode III stress components, while Mode II contributions remain negligible due to their non-singular nature (Lazzarin and Tovo, 1998; Tovo and Lazzarin, 1999; Susmel and Tovo, 2006; Susmel, 2009a, 2009c, 2014). The normal and shear stress components are derived using FEA, with extrapolation positions maintained at 0.4t and 1.0t reference points, following Niemi's guidelines (Niemi, 1995; Susmel and Tovo, 2006). Unlike the conventional HSS approach definition, which consolidates stress components into a single principal stress, the modified HSS approach decouples normal and shear stress states, leading to a more representative stress measure that accounts for the ductility of the material and multiaxial stress interactions (Lazzarin and Tovo, 1998; Tovo and Lazzarin, 1999; Susmel and Tovo, 2006; Susmel, 2009a, 2009c, 2014).

The ability to separately evaluate shear and normal stress contributions makes this approach particularly beneficial for multiaxial fatigue assessment, where distinguishing between shear and normal stress influences is essential (Lazzarin and Tovo, 1998; Tovo and Lazzarin, 1999; Susmel and Tovo, 2006; Susmel, 2009a, 2009c, 2014). By ensuring that stress components relative to the critical plane are correctly identified, this method provides greater accuracy in fatigue life estimations under complex multiaxial loading conditions (Lazzarin and Tovo, 1998; Tovo and Lazzarin, 1999; Susmel and Tovo, 2006; Susmel, 2009a, 2009c, 2014). The methodology of this modified HSS approach is depicted in Figure 2.5, illustrating its implementation in this study.

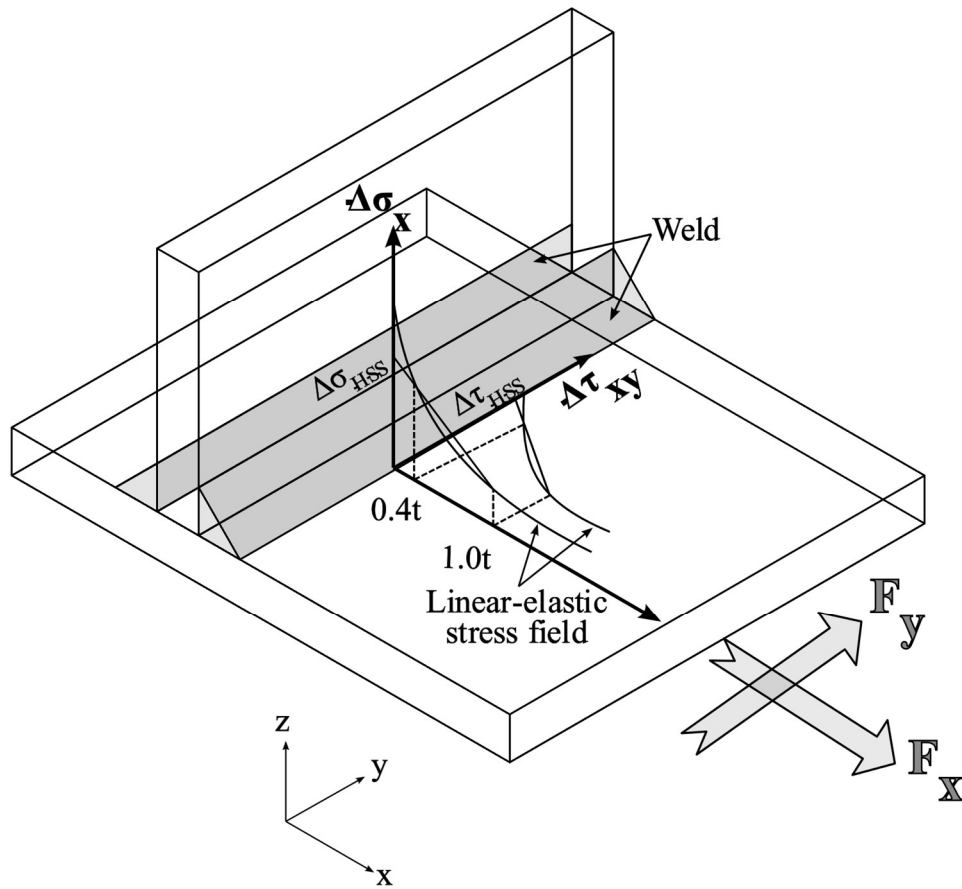


Figure 2.5: Modified HSS approach for multiaxial fatigue assessment, incorporating separate evaluation of normal and shear stress components.

Despite its advantages, the HSS approach has a fundamental limitation: it is not suitable for cases where fatigue cracks initiate at the weld root (Niemi, 1995; Radaj, 1996; Anon, 2005, 2007; Radaj, Sonsino and Fricke, 2006; Hobbacher, 2016; Mei *et al.*, 2021). The stress extrapolation methodology is inherently designed for weld toe assessments and does not accurately capture the localised stress distribution at the weld root. This limitation is the primary reason why the HSS approach was not applied in the analysis of hybrid aluminium-to-steel welded joints examined in this study. Given this constraint, alternative local stress analysis approaches are necessary, particularly for cases where weld root failure is a critical concern.

2.3.3 ENS approach

The ENS approach provides a localised and advanced methodology for evaluating welded joints, incorporating both non-linear stress concentration effects from weld toe and weld root geometries as well as the overall structural characteristics of the joint (Niemi, 1995; Radaj, 1996; Radaj, Sonsino and Fricke, 2006; Sonsino, Bruder and Baumgartner, 2010; Susmel, Sonsino and Tovo, 2011; Schijve, 2012; Fricke, 2013; Hobbacher, 2016; Pedersen, 2016; Al Zamzami and Susmel, 2018; Mei *et al.*, 2021). Unlike the NS and HSS approaches, which neglect localised notch effects, the ENS approach considers the stress gradients induced by the geometrical discontinuities of welded joints. Recognising its advantages and potential, the IIW has recommended the ENS approach as a validated fatigue assessment method, though it has yet to be formally integrated into EC3 and EC9 (Anon, 2005, 2007; Hobbacher, 2016).

According to the IIW recommendations, the effective notch stress is determined using FEA by introducing a fictitious notch at the weld toe and weld root to simulate real stress concentrations (Hobbacher, 2016). For steel and aluminium welded joints with a plate thickness (t) of 5 mm or greater, a standardised fictitious notch radius of 1.0 mm is recommended. This recommendation, originally proposed by Radaj and widely accepted within the IIW framework, has been extensively validated and is commonly applied in practical fatigue assessments of thicker welded joints (Morgenstern *et al.*, 2006; Fricke, 2007; Atzori *et al.*, 2009; Schijve, 2012; Radaj, Lazzarin and Berto, 2013; Berto, Lazzarin and Radaj, 2014; Hobbacher, 2016). The derivation of this 1.0 mm fictitious notch radius is based on Neuber's micro-support theory, which assumes an idealised zero radius notch to simulate the worst-case stress scenarios (Neuber, 1961; Sonsino, Bruder and Baumgartner, 2010; Sonsino *et al.*, 2012; Radaj, Lazzarin and Berto, 2013; Berto, Lazzarin and Radaj, 2014).

However, for thin-walled welded joints with a plate thickness below 5 mm, the IIW recommendation is not directly applicable, necessitating an alternative fictitious notch radius. To address this limitation, Sonsino proposed a refined approach, introducing a fictitious notch radius of 0.05 mm specifically for thin and flexible welded joints (Cetin Morris Sonsino, 2009; Sonsino, Bruder and Baumgartner, 2010; Sonsino *et al.*, 2012). This adjustment is derived from the relationship between stress intensity factors, crack

tip blunting, and notch stresses, ensuring a balanced compromise between FE modelling accuracy and computational feasibility in representing realistic local stress distributions (Cetin Morris Sonsino, 2009; Sonsino, Bruder and Baumgartner, 2010; Sonsino *et al.*, 2012; Al Zamzami, Davison and Susmel, 2019). Additionally, Sonsino introduced modified reference fatigue strength (FAT) values and adjusted negative inverse slope (k) parameters to better capture the fatigue behaviour of thinner welded joints, accounting for their increased flexibility and distinct fatigue responses (Sonsino, Bruder and Baumgartner, 2010; Sonsino *et al.*, 2012). The application of the ENS approach for both thin and thick-walled welded joints using FEA is illustrated in Figure 2.6.

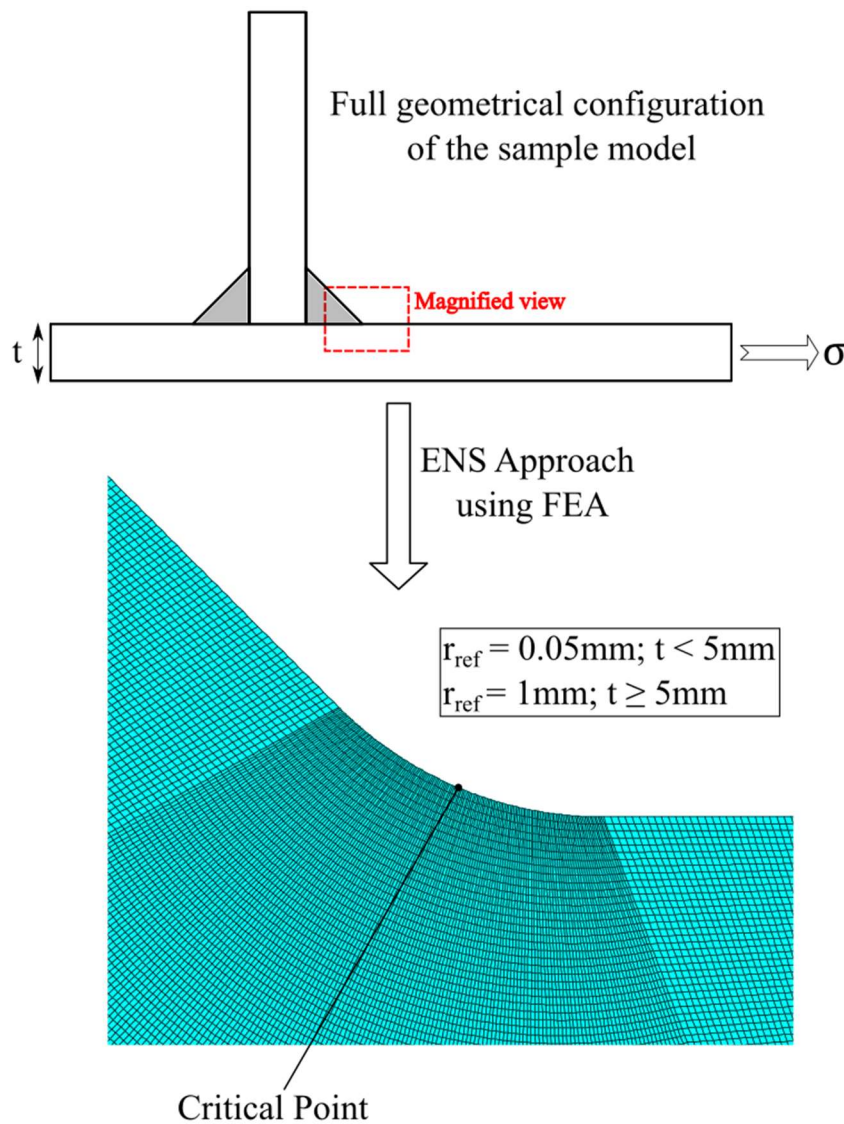


Figure 2.6: ENS approach using FEA for both thin and thick-walled welded joints.

While the ENS approach provides a more advanced and accurate evaluation of stress distributions in welded joints compared to NS and HSS approaches, it also presents practical limitations. One of the key challenges arises from the significant variability in the local weld profile along the weld seam in real-world applications. The actual geometric imperfections of welded joints can result in discrepancies between physical specimens and their numerical models, affecting the reliability of the ENS analysis approach (Hobbacher, 2016; Pedersen, 2016; Shen *et al.*, 2018; Mei *et al.*, 2021). Furthermore, the ENS approach requires explicit modelling of a fictitious notch radius and the use of fine mesh resolution to accurately capture the localised stress fields. This requirement increases computational effort and modelling complexities, making the process time-consuming, particularly when applied to complex geometries frequently encountered in engineering applications (Hobbacher, 2016; Mei *et al.*, 2021).

2.3.4 TCD approach

The TCD, introduced by David Taylor, represents a group of theories designed to account for notch effects and stress concentrators in predicting the static and fatigue strength of engineering components (Taylor, Barrett and Lucano, 2002; Taylor, 2008; Susmel and Taylor, 2012). A defining feature of the TCD approach is its reliance on a material length scale parameter, known as the critical distance, which governs failure by considering the stress distribution within a specific region surrounding the crack initiation site (Taylor, Barrett and Lucano, 2002; Luca Susmel, 2008; Taylor, 2008; Susmel and Taylor, 2012; Al Zamzami and Susmel, 2018). By incorporating this length scale, the TCD approach provides a more effective means of fatigue life estimation, particularly in cases where traditional stress analysis methods fail to capture localised stress effects.

The concept of critical distance originates from early ideas proposed by Neuber and Peterson, who proposed that fatigue damage near stress concentrators could be characterised using local stress values representative of a material's microstructural resistance to cracking (Taylor, 2008). These early ideas were later formalised through the integration of linear-elastic fracture mechanics and continuum mechanics, allowing the TCD framework to quantify damage using linear-elastic stress fields to capture

damage in the vicinity of a notch or weld. As a result, the critical distance has since been recognised as a material-dependent parameter influenced by microstructural features such as grain size, phase distribution, and boundary interfaces, as well as local constraint effects (Taylor, 2008). As such, the critical distance must be experimentally calibrated for each material system, especially when variations in alloy composition or heat treatment alter the underlying microstructural response (Luca Susmel, 2008; Taylor, 2008; Susmel and Taylor, 2012).

The TCD consists of four primary formulations: the Point Method (PM), Line Method (LM), Area Method (AM), and Volume Method (VM) (Luca Susmel, 2008; Taylor, 2008; Susmel, 2009b, 2014). These variations differ in the size and shape of the integration domain used to process local linear-elastic stress fields (Luca Susmel, 2008; Taylor, 2008; Susmel, 2009b, 2014). Among these, the Point Method (PM) is regarded as the simplest and most effective for fatigue life prediction in welded joints, particularly when combined with the Modified Wöhler Curve Method (MWCM) (Taylor, Barrett and Lucano, 2002; Susmel, 2007, 2009b, 2009a; L. Susmel, 2008). Due to its practicality, this study adopts the TCD PM to compute effective stresses in conjunction with the MWCM.

The TCD PM operates under several fundamental assumptions. Firstly, the material is assumed linear-elastic, homogeneous, and isotropic (Susmel, 2007, 2009b, 2009a; L. Susmel, 2008). Additionally, the weld toe and weld root radius are taken as zero (Susmel, 2007, 2009b, 2009a; L. Susmel, 2008). The focus path of the TCD PM follows the notch bisector, aligning with the weld toe or weld root to maintain consistency in stress evaluation (Susmel, 2007, 2009b, 2009a; L. Susmel, 2008). By employing FEA, the normal and shear stress components along the focus path are extracted at the critical distance, $M-D_v$, to determine the effective TCD PM stress state (Susmel, 2007, 2009b, 2009a; L. Susmel, 2008). A visual illustration of the focus path and critical distance $M-D_v$ is presented in Figure 2.7.

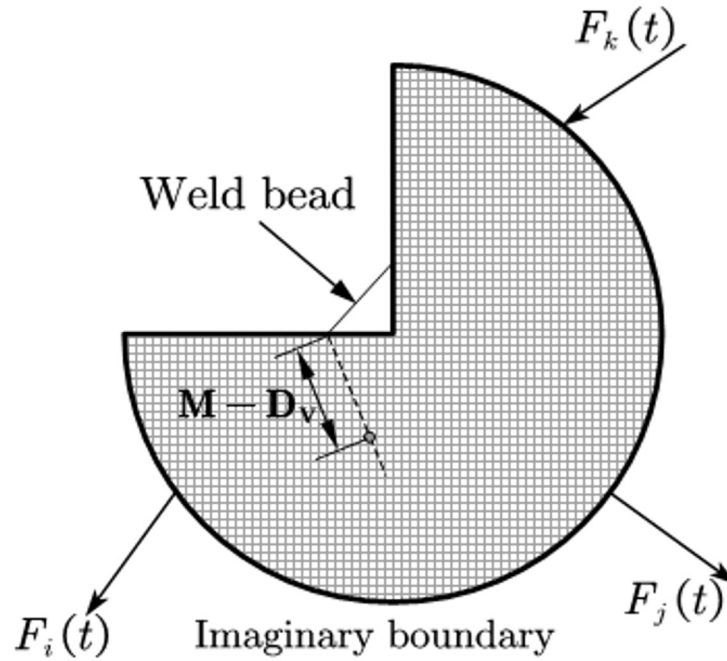


Figure 2.7: Definition of the TCD PM focus path and critical distance ($M-D_v$) for stress analysis.

One of the key advantages of the TCD approach is that it eliminates the need for fictitious notch rounding, which is a requirement in the ENS approach. By assuming a zero weld root or weld toe radius, the TCD simplifies FE modelling, making it a more efficient tool for analysing complex geometries (Susmel, 2007, 2009b, 2009a; L. Susmel, 2008). The TCD methodology is particularly well-suited for multiaxial fatigue assessments, as it enables the direct post-processing of linear-elastic stress fields at potential crack initiation sites. When integrated with the MWCM, the TCD approach effectively incorporates both the multiaxiality of the stress field and the influence of stress gradients, improving its applicability as a powerful fatigue assessment tool (Susmel, 2007, 2009b, 2009a; L. Susmel, 2008).

For steel and aluminium welded joints, the $M-D_v$ values have been previously calibrated using the Notch Stress Intensity Factor (N-SIF), EC3, EC9, and the IIW reference design curves for ground butt-welded and cruciform steel and aluminium joints under uniaxial and pure torsional fatigue loading conditions (Lazzarin and Tovo, 1998; Lazzarin and Livieri, 2001; Anon, 2005, 2007; Susmel, 2007, 2009a, 2009b; L. Susmel, 2008; Hobbacher, 2016). The established $M-D_v$ values are 0.5 mm for steel and 0.075

mm for aluminium, and these values remain constant regardless of stress complexity or joint geometry (Lazzarin and Tovo, 1998; Lazzarin and Livieri, 2001; Anon, 2005, 2007; Susmel, 2007, 2009a, 2009b; L. Susmel, 2008; Hobbacher, 2016). This consistency simplifies the application of TCD-based fatigue assessments, eliminating the need for recalibrating the critical distance power-law function with respect to cycles to failure (Luca Susmel, 2008; Susmel, 2009a; Susmel and Taylor, 2012).

This differs from other applications, such as notched steel or aluminium components, where the critical distance varies as a function of fatigue life (Luca Susmel, 2008; Susmel, 2009a; Susmel and Taylor, 2012). In such cases, shorter fatigue life is often associated with high stress gradients and the presence of microstructurally short cracks, requiring a larger critical distance to capture the broader damage process zone. At longer fatigue life, the failure mechanism is predominantly driven by stable crack propagation under more moderate stress gradients, leading to a smaller effective process zone and hence a reduced critical distance. This variation in damage mechanisms across the fatigue regime explains why the critical distance is often considered a function of fatigue life in conventional notched or plain components, typically following a decreasing power-law trend that must be recalibrated accordingly. (Luca Susmel, 2008; Susmel, 2009a; Susmel and Taylor, 2012).

The constant nature of $M-D_v$ for welded joints results from the matching negative inverse slopes (k) of calibrating S-N reference curves for ground butt-welded joints and N-SIF reference curves for cruciform joints (Lazzarin and Tovo, 1998; Lazzarin and Livieri, 2001; Anon, 2005, 2007; Susmel, 2007, 2009a, 2009b; L. Susmel, 2008; Hobbacher, 2016).

Despite its strengths, the TCD approach has notable limitations. One of the primary challenges lies in the calibration process required to determine the critical distance parameter for new materials, which demands expertise and extensive experimental validation (Taylor, Barrett and Lucano, 2002; Susmel, 2007, 2009b, 2009a; L. Susmel, 2008). Additionally, the TCD method lacks flexibility when combined with alternative fatigue criteria, as the critical distance requires recalibration if applied beyond the MWCM for fatigue assessments (Taylor, Barrett and Lucano, 2002; Susmel, 2007,

2009b, 2009a; L. Susmel, 2008). This restricts its adaptability, making it less straightforward to implement in practice when dealing with new and untested materials.

In the context of hybrid aluminium-to-steel welded joints, one of the key research objectives was to assess whether the critical distance value for aluminium welded joints could be extended to hybrid configurations, considering that aluminium is the weaker link in the joint. Previous studies on thin coldArc® aluminium-to-steel welded joints under uniaxial fatigue loading have demonstrated that this assumption holds, leading to the hypothesis that calibrated TCD PM $M-D_v$ values could be extended to multiaxial fatigue conditions (Al Zamzami and Susmel, 2018; Al Zamzami, Davison and Susmel, 2019). This hypothesis is critically evaluated in Chapter 7, where the applicability of the TCD PM approach for hybrid welded joints under multiaxial fatigue loading is investigated. If validated, this would further establish the TCD methodology as a highly effective fatigue assessment tool for practitioners, particularly with advancements in FEA that aid the stress analysis process.

2.4 Multiaxial Fatigue Criteria for Welded Joints

Fatigue induced failure is a critical concern in industrial applications where welded joints are subjected to repeated cyclic loading, particularly in complex structures such as bridges, offshore platforms, aircraft, and automobiles. The fatigue behaviour of welded joints is further complicated by multiaxial stress states, which arise due to complex external loads and geometric discontinuities at the welds. This phenomenon, known as multiaxial fatigue, presents significant challenges in accurately predicting fatigue life, as traditional uniaxial fatigue assessment methods are often inadequate in capturing the stress interactions and damage mechanisms under multiaxial loading conditions.

To address these challenges, various multiaxial fatigue criteria have been developed to reduce complex multiaxial stress states into equivalent uniaxial stress measures, allowing for fatigue life estimation through standardised approaches (Findley, 1959; Sonsino, 1995; Carpinteri, Brighenti and Spagnoli, 2000). These criteria are generally

categorised into four main groups: equivalent stress criteria, interaction equation criteria, critical plane criteria, and energy-based criteria.

The equivalent stress approach, which includes the Maximum Principal (MP) stress criterion recommended in EC9 and the Von Mises (VM) stress criterion, simplifies multiaxial fatigue assessment by reducing the stress state to a single equivalent stress value (Siljander, Kurath and Lawrence, 1992; Sonsino, 1995; Bäckström, 2003; Anon, 2007; Fricke, 2013; Pedersen, 2016). Similarly, interaction equation criteria, commonly adopted in EC3 and the IIW guidelines, define fatigue design safety limits by considering the interaction between normalised normal and shear stresses (Gough and Pollard, 1935; Anon, 2005; Hobbacher, 2016). While practical and easy to implement, these standard approaches have limitations in capturing the complexities of non-proportional loading conditions and VA loading, where stress interactions are more complex (Fricke, 2003; C M Sonsino, 2009; Pedersen, 2016).

To overcome these limitations, more advanced multiaxial fatigue criteria have been developed, particularly critical plane and energy-based approaches. Critical plane criteria are designed to identify the material plane most susceptible to fatigue failure by evaluating the normal and shear stress components acting on that plane. These criteria can be further classified based on their fundamental principles. Strain-based criteria, such as Smith-Watson-Topper (SWT), Brown-Miller, and Fatemi-Socie, are particularly effective for low-cycle fatigue applications, where plastic deformation dominates the fatigue crack initiation phase (Smith, 1970; Brown and Miller, 1973, 1979; Socie, 1987; Fatemi and Socie, 1988; Łagoda *et al.*, 2022). Stress-based criteria, including those proposed by Findley, Matake, McDiarmid, Dang Van, Carpinteri-Spagnoli (CS), Sonsino KoNoS, and the Modified Wöhler Curve Method (MWCM), are more suitable for medium- to high-cycle fatigue, where fatigue behaviour is primarily governed by elastic stress ranges (Findley, 1959; Matake, 1977; McDiarmid, 1991, 1994; Dang-Van, 1993; Carpinteri, Brighenti and Spagnoli, 2000; Sonsino and Kueppers, 2001; Kueppers and Sonsino, 2006; Carpinteri, Spagnoli and Vantadori, 2009; Susmel, 2009a). These models establish a linear relationship between stress amplitude and fatigue life, usually represented by S-N curves, making them particularly relevant for ductile materials like steel, where shear stress (Mode II) plays a dominant

role in fatigue failure. Additionally, integral-based approaches, such as the Effective Equivalent Stress Hypothesis (EESH) by Sonsino, evaluate fatigue damage accumulation by integrating stress components across multiple critical planes, providing a more comprehensive assessment method for ductile materials like welded steel joints (Sonsino, 1995; Sonsino and Kueppers, 2001).

In contrast, energy-based criteria operate on the assumption that fatigue damage is directly related to the total energy dissipated at crack initiation sites. These models estimate fatigue life by assessing strain energy density in critical regions, making them particularly effective for materials where both elastic and plastic energy contributions influence fatigue behaviour (Ellyin, 1989; Glinka, Shen and Plumtree, 1995; Macha and Sonsino, 1999; Lazzarin *et al.*, 2008; Foti and Berto, 2020; Gan, Wu and Zhong, 2021).

Given that this study focuses on the medium- to high-cycle fatigue regime of hybrid aluminium-to-steel welded joints, stress-based criteria are of primary interest. Specifically, attention will be given to standard equivalent stress and interaction equation approaches, as recommended in Eurocodes and IIW guidelines, alongside stress-based critical plane criteria, which have been widely applied in multiaxial fatigue assessments of welded joints.

The aim of this study is to establish a robust and reliable multiaxial fatigue assessment methodology for hybrid aluminium-to-steel welded joints. This requires a critical evaluation of existing criteria developed for conventional steel and aluminium welded joints to identify their strengths, limitations, and potential for extension to more complex hybrid joint configurations.

To structure this evaluation, the following two subsections are introduced. Section 2.4.1 provides a detailed review of standard multiaxial fatigue assessment criteria, including those prescribed in EC3, EC9, and the IIW recommendations. This is followed by Section 2.4.2, which examines stress-based critical plane criteria commonly applied in welded joint fatigue assessments. Although this study primarily focuses on CA loading, for completeness, both CA and VA loading scenarios are reviewed to provide broader

insights into the applicability and performance of these criteria under different loading conditions.

2.4.1 Standard and Stress Equivalent Multiaxial Fatigue Criteria

To ensure a systematic approach to fatigue design, engineering standards such as the Eurocodes (ECs) and the IIW provide well-established guidelines based on two methodologies: the safe-life design method and the damage-tolerant design method (Anon, 2005, 2007; Hobbacher, 2016). The safe-life approach focuses on designing welded structures to operate within a predefined fatigue life without requiring in-service inspections for crack detection (Anon, 2005, 2007; Hobbacher, 2016). This is achieved by selecting appropriate stress levels and standardised fatigue details while ensuring that the applied stress range remains within the endurance limit of the material (Anon, 2005, 2007; Hobbacher, 2016). However, in reality, the safe-life method is not entirely deterministic. Despite conservative design margins, there remains a small probability of premature failure due to material scatter, defects, or unforeseen service conditions. Therefore, safety-critical applications such as those in aerospace or nuclear industries require inspections to ensure the structure remains operationally safe throughout its service life. In contrast, the damage-tolerant method requires periodic inspections and maintenance, allowing for the early detection and repair of fatigue cracks before they reach a critical size (Anon, 2005, 2007; Hobbacher, 2016). This method relies on designing with fatigue-resistant materials, implementing multiple load paths, and incorporating crack arrest mechanisms to enhance structural reliability (Anon, 2005, 2007; Hobbacher, 2016). Given its practicality and alignment with established procedures for welded joints, the safe-life approach is adopted in this study as the primary framework for multiaxial fatigue assessment.

For welded joints subjected to CA loading, EC3 and the IIW propose interaction equation criteria formulated using normalised ratios of direct and shear stress components (Anon, 2005; Hobbacher, 2016). These equations provide straightforward means of assessing fatigue performance by comparing the applied stress ranges against reference fatigue strengths. A welded joint is considered safe if the accumulated fatigue damage remains below a defined threshold, which depends on type of material and

loading conditions (Anon, 2005; Hobbacher, 2016). The interaction equations for EC3 and the IIW are defined in Equations 2.4 and 2.5, respectively (Anon, 2005; Hobbacher, 2016).

$$\left(\frac{\Delta\sigma_x}{\Delta\sigma_R/SF} \right)^3 + \left(\frac{\Delta\tau_{xy}}{\Delta\tau_R/SF} \right)^5 \leq D_{EC3} \quad (2.4)$$

$$\left(\frac{\Delta\sigma_x}{\Delta\sigma_R/SF} \right)^2 + \left(\frac{\Delta\tau_{xy}}{\Delta\tau_R/SF} \right)^2 \leq D_{IIW} \quad (2.5)$$

where $\Delta\sigma_x$ and $\Delta\tau_{xy}$ represent the applied normal and shear stress ranges, respectively, while $\Delta\sigma_R$ and $\Delta\tau_R$ denote the reference fatigue strengths. The partial safety factor SF , accounts for uncertainties in fatigue assessment.

A summary of allowable damage sums under CA and VA loading for different materials and loading scenarios is presented in Table 2.1 (Anon, 2005; Hobbacher, 2016).

Table 2.1: Summary of allowable damage sums under CA and VA fatigue loading for various loading scenarios and materials.

Material	Normal & Shear Stress	Allowable Damage Sum (CA)		Allowable Damage Sum/Comparison Value (VA)		
		D_{EC3}	D_{IIW}	$D_{al,EC3}$	CV_{IIW}	$D_{al,IIW}$
Steel	Proportional	1.0	1.0	1.0	1.0	0.5
	Non-proportional	1.0	0.5	1.0	0.5	0.5
Aluminium	Proportional	1.0	1.0	1.0	1.0	0.5
	Non-proportional	1.0	1.0	1.0	1.0	0.5

It can be observed that EC3 consistently adopts a damage sum of $D_{EC3}=1$ for all loading cases, whereas the IIW differentiates between proportional and non-proportional loading, assigning $D_{IIW}=1$ for in-phase loading and reducing it to $D_{IIW}=0.5$ for out-of-phase loading conditions (Anon, 2005; Hobbacher, 2016).

For VA loading, fatigue assessment requires a more comprehensive approach that extends beyond direct stress ratios. Two main methodologies are recommended: the interaction equation criterion and the linear damage accumulation Palmgren-Miner (P-

M) rule (Anon, 2005, 2007; Hobbacher, 2016). The interaction equation criteria remain applicable for VA loading, with modifications to incorporate equivalent stress ranges, where the direct stress components, $\Delta\sigma_x$ and $\Delta\tau_{xy}$ are replaced with their equivalent counterparts, $\Delta\sigma_{eq}$ and $\Delta\tau_{eq}$ (Anon, 2005; Hobbacher, 2016). The modified interaction equations for EC3 and the IIW are expressed in Equations 2.6 and 2.7, respectively (Anon, 2005; Hobbacher, 2016).

$$\left(\frac{\Delta\sigma_{eq,EC3}}{\Delta\sigma_R/SF} \right)^3 + \left(\frac{\Delta\tau_{eq,EC3}}{\Delta\tau_R/SF} \right)^5 \leq D_{EC3} \quad (2.6)$$

$$\left(\frac{\Delta\sigma_{eq,IIW}}{\Delta\sigma_R/SF} \right)^2 + \left(\frac{\Delta\tau_{eq,IIW}}{\Delta\tau_R/SF} \right)^2 \leq CV \quad (2.7)$$

where CV representing the comparison value replaces the allowable damage sum of CA in the IIW formulation to account for VA fatigue loading complexities. The corresponding values for allowable damage sum and comparison value for VA are also summarised in Table 2.1. Unlike the IIW, EC3 retains $D_{EC3}=1$ for VA conditions (Anon, 2005; Hobbacher, 2016).

EC3 determines the equivalent stress range using a damage factor, λ_i that scales the maximum stress range from the applied load model, $\Delta\sigma_{LM}$, as given by Equation 2.8 (Anon, 2006).

$$\Delta\sigma_{eq,EC3} = \lambda_i \times \Delta\sigma_{LM} \quad (2.8)$$

The factor λ_i varies depending on the structural application, with distinct values specified for different structures such as bridges, cranes, and offshore platforms (Anon, 2005, 2006). In cases where λ_i is not defined, the P-M rule is recommended as an alternative approach (Anon, 2005).

On the other hand, the IIW directly calculates the equivalent stress range using an equivalent damage accumulation formula, incorporating the rainflow cycle counting

method to evaluate fatigue damage at different stress levels (Hobbacher, 2016). This method incorporates rainflow cycle counting to evaluate fatigue damage at various stress levels and is expressed as defined in Equation 2.9 (Hobbacher, 2016).

$$\Delta\sigma_{eq,IIW} = \sqrt{\frac{1}{D_{spec}} \cdot \frac{\sum(n_i \cdot \Delta\sigma_{i,S,d}^{m_1}) + \Delta\sigma_{L,d}^{(m_1-m_2)} \cdot \sum(n_j \cdot \Delta\sigma_{j,S,d}^{m_2})}{\sum n_i + \sum n_j}} \quad (2.9)$$

where:

D_{spec}	Specified Miner sum
m_1	Slope above the knee point of SN curve
m_2	Slope below the knee point of SN curve
$\Delta\sigma_{i,S,d}$	Applied stress ranges above the knee point
$\Delta\sigma_{j,S,d}$	Applied stress ranges below the knee point
$\Delta\sigma_{L,d}$	Applied stress range at the knee point of SN curve
n_i	Number of cycles at applied stress range $\Delta\sigma_i$
n_j	Number of cycles at applied stress range $\Delta\sigma_j$

The implementation process of EC3 and IIW interaction equation criteria using the equivalent stress range substitution method is illustrated in Figure 2.8.

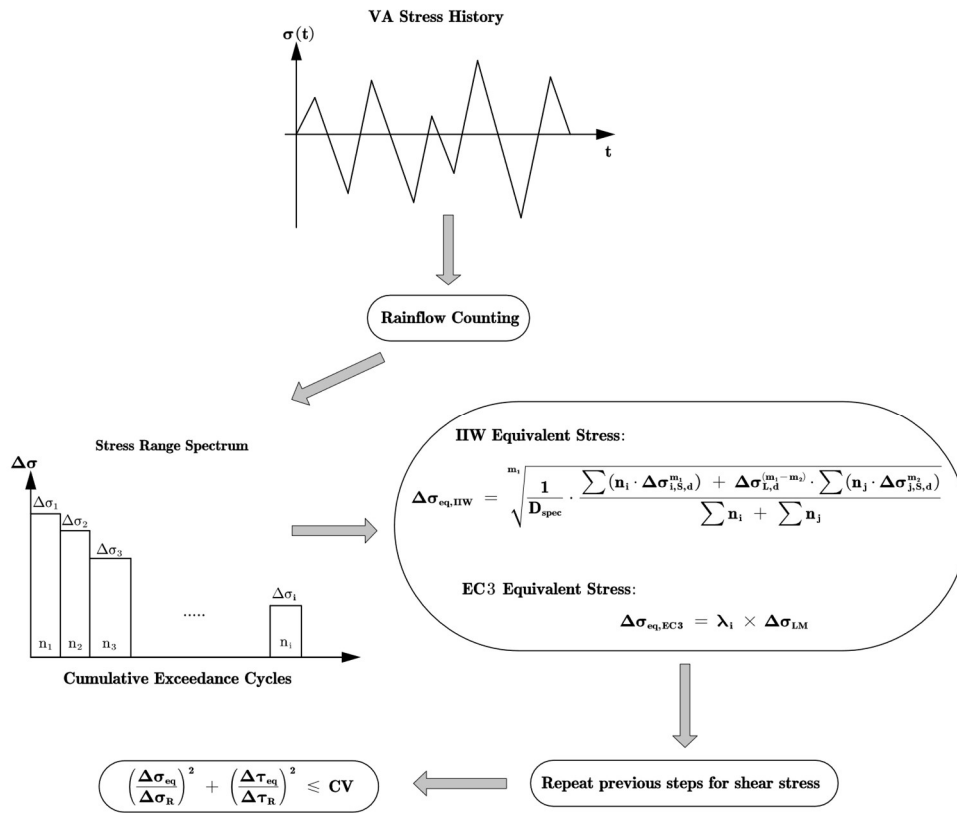


Figure 2.8: Flowchart of EC3 and IIW interaction equation with equivalent stress range substitution for VA loading.

The interaction equation methods are often visualised using interaction diagrams, which offer a graphical representation of safety margins (Anon, 2005; Hobbacher, 2016). These diagrams define the acceptable fatigue life threshold, typically recommended at two million cycles, with reference stresses recalibrated based on a probability of survival and additional safety factors margins (Anon, 2005; Hobbacher, 2016). If a data point falls within the defined boundary, the joint is considered safe for at least two million cycles. Conversely, points outside the boundary indicate failure before reaching this threshold, with greater distances signifying shorter fatigue life margins (Anon, 2005; Hobbacher, 2016).

Unlike critical plane-based fatigue criteria, the interaction equations proposed by EC3 and IIW do not explicitly provide a direct estimation of cycles to failure margins (Anon, 2005; Hobbacher, 2016). Consequently, in the quantitative review presented in Chapter 3, interaction diagrams are used as means of assessing the safety margin of different welded joint configurations. However, alternative formulations allow fatigue life

estimation using effective stress ranges (Pedersen, 2016; Mei and Dong, 2017; Mei *et al.*, 2021). To facilitate direct comparisons with other fatigue assessment methodologies, an alternative IIW formulation, as given in Equation 2.10, is adopted for the reanalysis of hybrid aluminium-to-steel welded joints (Pedersen, 2016; Mei and Dong, 2017; Mei *et al.*, 2021).

$$\Delta\sigma_{IIW} = \frac{1}{\sqrt{D_{IIW}}} \cdot \sqrt{\Delta\sigma_x^2 + \frac{\Delta\sigma_R^2}{\Delta\tau_R^2} \cdot \Delta\tau_{xy}^2} \quad (2.10)$$

For VA fatigue loading, another widely used approach is the modified P-M rule (Anon, 2005, 2007; Hobbacher, 2016), which estimates cumulative fatigue damage, D_{tot} , by summing the damage contributions from both direct and shear stresses based on their respective applied stress levels, as expressed in Equation 2.11.

$$D_{tot} = D_{\sigma} + D_{\tau} \leq D_{real} \quad (2.11)$$

Rainflow cycle counting is employed to determine the number of cycles at different applied stress levels, n_i . Once the loading spectrum is established, it is matched against the standard EC3 and IIW reference S-N curves to determine the corresponding fatigue life in cycles, N_i (Anon, 2005, 2007; Hobbacher, 2016). As a result, individual damage components, D_{σ} or D_{τ} , can be computed using Equation 2.12.

$$D_{\sigma} \text{ (or } D_{\tau}) = \sum_{i=0}^k \frac{n_i}{N_i} \quad (2.12)$$

The modified P-M rule assumes that fatigue failure occurs when the accumulated damage fraction reaches or exceeds the actual damage sum, D_{real} . Unlike the original P-M rule, which sets an allowable damage sum, D_{al} , at 1, experimental studies on steel and aluminium welded joints have shown that D_{real} values typically range between 0.01 and 10 (Eulitz, 2000; Sonsino, 2007). As a practical engineering compromise, an average value of $D_{al}=0.5$ is commonly adopted, corresponding to a 50% probability of occurrence (Eulitz, 2000; Sonsino, Maddox and Hobbacher, 2005; Sonsino, 2007). This

modification has been incorporated into the IIW recommendations to enhance the accuracy of fatigue life predictions. By applying this concept, the estimated fatigue life, $N_{f,e}$, can be determined using Equation 2.13.

$$N_{f,e} = \frac{D_{real}}{D_{tot}} \sum_{i=0}^k n_i \quad (2.13)$$

The step-by-step implementation of the P-M rule based on EC3 and IIW guidelines is illustrated in Figure 2.9.

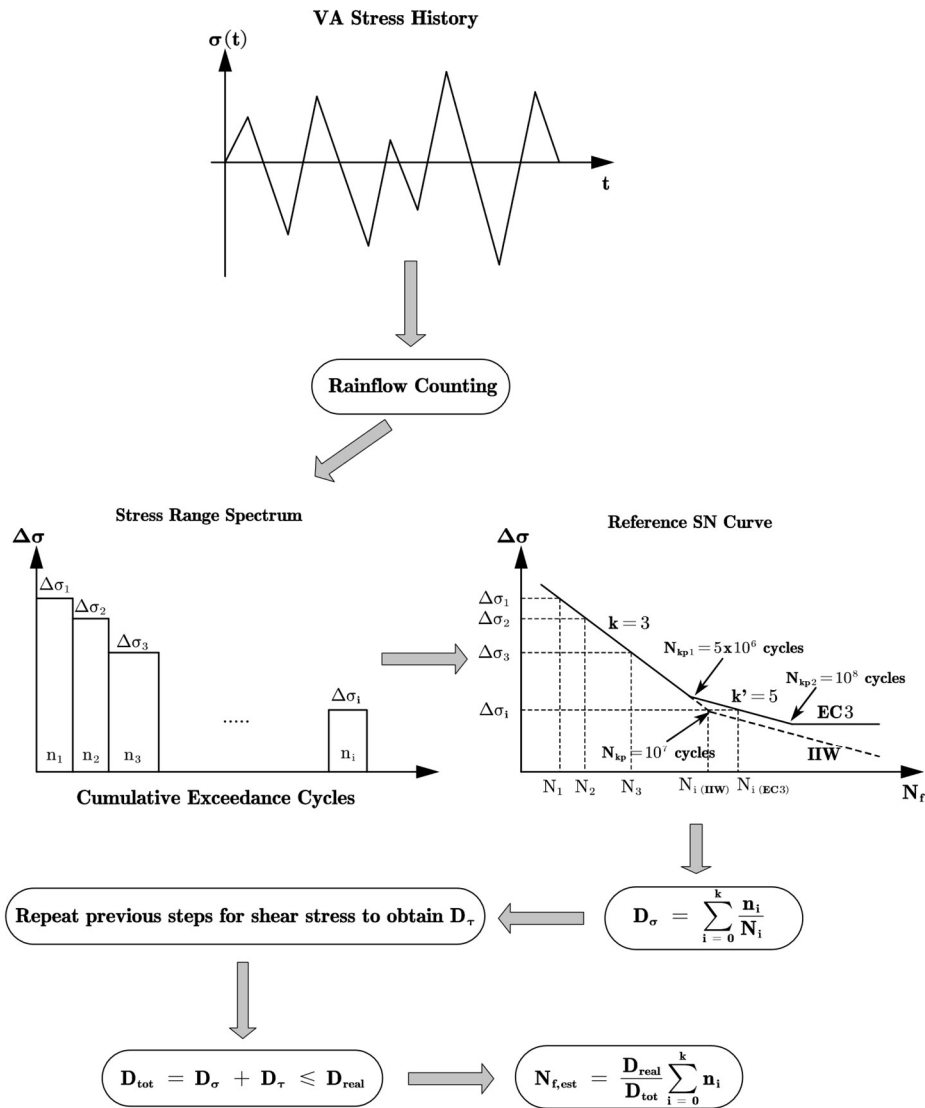


Figure 2.9: Step-by-step implementation of the P-M rule in EC3 and IIW for VA multiaxial fatigue assessment.

In contrast to EC3 and IIW, EC9 adopts the maximum principal (MP) stress criterion for evaluating both CA and VA fatigue loading conditions (Anon, 2007). The fatigue life estimation under EC9 is derived using Equation 2.14, where stress ranges are defined in terms of largest absolute maximum principal stress range (Anon, 2007; Pedersen, 2016).

$$\Delta\sigma_1 = \left(\Delta\sigma_x + \Delta\sigma_y + \sqrt{(\Delta\sigma_x - \Delta\sigma_y)^2 + 4\Delta\tau_{xy}^2} \right) / 2 \quad (2.14)$$

Similarly, the Von Mises (VM) stress criterion, which is widely used for multiaxial fatigue assessment, is formulated based on the range of stress components derived from different stress analysis approaches (Sonsino, 1995; Hobbacher, 2016; Pedersen, 2016). Equation 2.15 presents the general formulation for the VM stress criterion, which is applied in the validation exercise performed in Chapter 7 to evaluate its performance alongside other multiaxial fatigue assessment criteria.

$$\Delta\sigma_{VM} = \sqrt{\Delta\sigma_x^2 + \Delta\sigma_y^2 - \Delta\sigma_x\Delta\sigma_y + \Delta\sigma_y + 3\Delta\tau_{xy}^2} \quad (2.15)$$

Although the VM criterion is well-established for evaluating static strength in ductile materials, it is not ideally suited for fatigue analysis. It treats all stresses as positive and therefore does not account for compressive stress ranges. As a result, it can underestimate fatigue damage, particularly under multiaxial or compressive-dominant loading conditions. Its scalar formulation also neglects the role of principal stress orientation and shear-driven mechanisms, limiting its reliability in predicting fatigue crack initiation, especially under non-proportional loading.

This structured review of the theoretical aspects of standard multiaxial fatigue criteria sets the foundation for the next section, which explores stress-based critical plane approaches in greater detail.

2.4.2 Stress-Based Critical Plane Criteria

Following the discussion on standard fatigue assessment approaches, this section explores the theoretical foundation of stress-based critical plane criteria that are widely

employed for multiaxial fatigue assessments for welded joints. These criteria focus on identifying the material plane most susceptible to fatigue failure by considering both normal and shear stress components. The quantification of fatigue damage is based on key stress parameters such as amplitude, mean, and peak values, all of which play a critical role in crack initiation and propagation under cyclic loading. During each fatigue cycle, the normal stress component, $\sigma_n(t)$, varies along plane's normal direction. However, the shear stress component, $\tau_n(t)$, exhibits more complex behaviour due to continuous change in both magnitude and direction throughout the loading cycle (Carpinteri, Macha, *et al.*, 1999; Susmel, 2009a; Pedersen, 2016). Accurately determining the shear stress amplitude is not straightforward, thus necessitating the use of numerical techniques to obtain accurate and representative estimates. Several established methods, including the Longest Chord Method, Longest Projection Method, Minimum Circumscribed Circle Method, Minimum Circumscribed Ellipse Method, and the Maximum Variance Method (MVM), have been proposed for this purpose (Papadopoulos, 1998; Susmel, 2009a, 2010). Each method has its own advantages and limitations, but among them, the MVM been demonstrated to offer the highest accuracy in most situations, particularly in non-proportional and VA loading (Susmel, Tovo and Benasciutti, 2009; Susmel, 2010, 2014; Susmel and Askes, 2012; Susmel and Taylor, 2012; Faruq and Susmel, 2019). Given its robustness, MVM is adopted in this study to compute the maximum shear stress ranges in aluminium and steel welded joints especially under VA loading.

To ensure a comprehensive assessment, a selection of widely applied critical plane criteria is examined based on their applicability to both steel and aluminium welded joints. These criteria span from those suited for brittle materials to those tailored for semi-ductile and ductile materials, allowing for a broader investigation into their potential extension to hybrid aluminium-to-steel welded joints. Initially, the Findley criterion, CS criterion, and the MWCM are quantitatively reviewed in Chapter 4 to assess their predictive accuracy in estimating fatigue life for conventional steel and aluminium welded joints under both CA and VA loading conditions. Furthermore, the Sonsino KoNoS criterion and a stress-based adaptation of the SWT criterion are incorporated into the validation study for hybrid joints. The inclusion of the KoNoS criterion is motivated by its reported effectiveness in evaluating fatigue behaviour in

semi-ductile aluminium welded joints, while the stress-based SWT approach is introduced to investigate whether a criterion originally formulated for brittle materials can be effectively applied to hybrid welded joints and how its performance compares to the other selected criteria (Smith, 1970; Łagoda *et al.*, 2022; John *et al.*, 2024).

Findley's criterion identifies the critical plane by evaluating the combined influence of shear stress amplitude and maximum normal stress acting on a material plane (Findley, 1959; Bruun and Härkegård, 2015; Pedersen, 2016; Bibbo, Baumgartner and Arora, 2022). The fatigue damage parameter, f , is formulated as:

$$f = \left(\frac{\Delta\tau}{2} + \beta\sigma_{max} \right) \quad (2.16)$$

where $\Delta\tau$ represents the shear stress range, σ_{max} is the peak normal stress on the critical plane, and β is a material-dependent coefficient that quantifies the sensitivity of fatigue damage to normal stress. For ductile materials such as steel and aluminium, a typical value of $\beta=0.3$ is applied in this study based on prior research (Findley, 1959; Marquis, Bäckström and Siljander, 1997; Bruun and Härkegård, 2015; Pedersen, 2016). To facilitate fatigue life estimations and simplify fatigue assessments, Bruun and Härkegård reformulated Findley's criterion into an equivalent uniaxial stress range as defined in Equation 2.17 (Bruun and Härkegård, 2015; Pedersen, 2016).

$$\Delta\sigma_{find} = \frac{\Delta\tau + 2\beta\sigma_{max}}{1/2(\beta + \sqrt{1 + \beta^2})} \leq \Delta\sigma_R \quad (2.17)$$

This reformulated equation is employed alongside with various stress analysis approaches to simplify the assessment process. However, a quantitative review in Chapter 3 revealed that Findley's criterion tends to produce overly conservative fatigue life estimates, particularly when applied to experimental data with stress ratios greater than zero. To address this limitation, an alternative formulation is introduced for hybrid joint validation, replacing σ_{max} in Equation 2.17 with maximum normal stress range, $\Delta\sigma_{max}$. This modification results in less conservative and more realistic fatigue life estimations, particularly for cases with a stress ratio of $R=0.1$, as demonstrated in Chapter 7.

The CS criterion determines the orientation of the critical plane with the principal stress directions. It assumes that the critical plane coincides with the weighted mean direction of the principal stresses at the instant when the first principal normal stress reaches maximum during a loading cycle (Carpinteri, Brighenti, *et al.*, 1999; Carpinteri, Macha, *et al.*, 1999; Carpinteri, Brighenti and Spagnoli, 2000; Carpinteri, Spagnoli and Vantadori, 2003, 2009). The CS fatigue damage parameter is a nonlinear function incorporating both normal and shear stress ranges, adjusted by the squared ratio of fully reversed normal and shear fatigue strengths, as defined in Equation 2.18:

$$\Delta\sigma_{CS} = \sqrt{\Delta\sigma_{eq}^2 + \left(\frac{\Delta\sigma_{R=-1}^2}{\Delta\tau_{R=-1}^2}\right) \cdot \Delta\tau^2} \quad (2.18)$$

where $\Delta\sigma_{eq}$ accounts for mean stress effects using the Goodman correction defined in Equation 2.19.

$$\Delta\sigma_{eq} = \Delta\sigma + \Delta\sigma_{R=-1} \left(\frac{\sigma_m}{\sigma_u}\right) \quad (2.19)$$

Here, σ_m is the mean normal stress, and σ_u is the ultimate tensile strength of the material. Additionally, the offset angle, δ , which defines the deviation between the normal to the critical plane and the average principal stress direction is empirically determined as (Carpinteri, Brighenti, *et al.*, 1999; Carpinteri, Macha, *et al.*, 1999; Carpinteri, Brighenti and Spagnoli, 2000; Carpinteri, Spagnoli and Vantadori, 2003, 2009):

$$\delta = \begin{cases} \frac{3\pi}{8} \left[1 - \left(\frac{\Delta\tau_{R=-1}}{\Delta\sigma_{R=-1}} \right)^2 \right], & \text{if } \frac{1}{\sqrt{3}} \leq \frac{\Delta\tau_{R=-1}}{\Delta\sigma_{R=-1}} \leq 1 \\ \frac{\pi}{4}, & \text{if } \frac{\Delta\tau_{R=-1}}{\Delta\sigma_{R=-1}} < \frac{1}{\sqrt{3}} \\ 0, & \text{if } \frac{\Delta\tau_{R=-1}}{\Delta\sigma_{R=-1}} > 1 \end{cases} \quad (2.20)$$

Since standard IIW reference fatigue curves are derived for higher R-ratios to account for worst-case as-welded conditions, the fully reversed fatigue strengths, $\Delta\sigma_{R=-1}$ and $\Delta\tau_{R=-1}$ are replaced with IIW-recommended values to maintain consistency. However, when experimentally derived fatigue curves are used, the original formulation remains unchanged.

The MWCM is a shear-dominated critical plane criterion that identifies the critical plane based on maximum shear stress range while also considering the influence of the maximum normal stress range (Susmel and Lazzarin, 2002; Susmel, 2009c, 2009b, 2010, 2014; Susmel, Tovo and Benasciutti, 2009; Susmel and Askes, 2012; Al Zamzami and Susmel, 2018). To characterise the multiaxial loading condition, MWCM introduces a multiaxiality parameter, ρ_w , which is defined in Equation 2.21.

$$\rho_w = \frac{\Delta\sigma_n}{\Delta\tau} \quad (2.21)$$

where $\rho_w=1$ corresponds to uniaxial loading and $\rho_w=0$ represents pure torsional loading. This method is typically visualised using a modified Wöhler diagram, where shear stress range $\Delta\tau$ is plotted against fatigue life, N_f as shown in Figure 2.10 (Susmel and Lazzarin, 2002; Susmel, 2009c, 2010, 2014; Susmel, Tovo and Benasciutti, 2009).

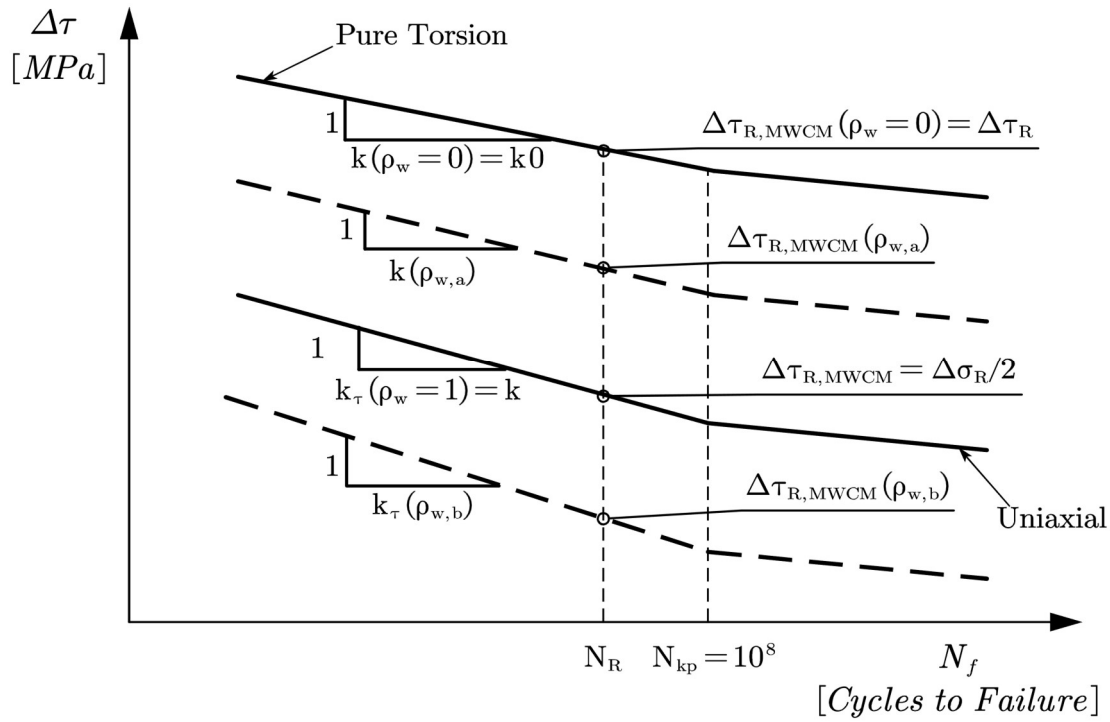


Figure 2.10: Modified Wöhler diagram illustrating the variation of ρ_w as a function of $\Delta\tau$.

From this diagram, two critical parameters are identified: the negative inverse slope, k_τ and the reference shear stress range, $\Delta\tau_{R,MWCM}$. These parameters are formulated as linear functions and calibrated using uniaxial and pure torsional reference fatigue curves as defined in Equations 2.22 and 2.23, respectively (Susmel and Lazzarin, 2002; Susmel, 2009c, 2010, 2014; Susmel, Tovo and Benasciutti, 2009).

$$k_\tau(\rho_w) = [k_\tau(\rho_w = 1) - k_\tau(\rho_w = 0)]\rho_w + k_\tau(\rho_w = 0) \quad (2.22)$$

$$\Delta\tau_{R,MWCM}(\rho_w) = \left(\frac{\Delta\sigma_R}{2} - \Delta\tau_R\right)\rho_w + \Delta\tau_R \quad (2.23)$$

To prevent excessive conservatism in fatigue life estimations at higher ρ_w values, MWCM incorporates a threshold limit to adjust its predictions, ensuring a more realistic representation of fatigue behaviour across different multiaxial loading conditions. This limit is defined as (Susmel, 2009a, 2014):

$$\rho_{w,lim} = \frac{\Delta\tau_R}{2\Delta\tau_R - \Delta\sigma_R} \quad (2.24)$$

Under CA loading, the application of multiaxial fatigue criteria is relatively straightforward, as closed fatigue cycles can be clearly defined. However, for VA loading, identifying closed fatigue cycles becomes significantly more complex, necessitating the use of cycle counting techniques (Matsuishi, 1968; Langlais, Vogel and Chase, 2003; Lee and Pan, 2005; Lee and Tjhung, 2011; Standard, 2017). A widely recognised approach for this purpose is the rainflow counting method, originally introduced by Matsuishi, based on the “pagoda roof” analogy (Matsuishi, 1968; Endo, 1974). Over time, various adaptations of the rainflow method, such as the three-point and four-point rainflow counting techniques have been developed, with the ASTM standard version being one of the most commonly adopted due to its computational efficiency (Matsuishi, 1968; Langlais, Vogel and Chase, 2003; Lee and Pan, 2005; Lee and Tjhung, 2011; Standard, 2017; Bibbo *et al.*, 2022). While this method is effective for single-component fatigue loading scenarios, such as those governed by the IIW/Gough-Pollard criterion or the EC3 interaction equation, it encounters significant limitations when dealing with multiaxial fatigue scenarios where both normal and shear stress cycles must be accounted for simultaneously.

To address these limitations, Carpinteri *et al.* proposed the CS counting method, which enables the simultaneous identification of normal and shear stress cycles (Carpinteri, Spagnoli and Vantadori, 2003). This technique first isolates closed cycles in the normal stress time history, utilising the scalar nature of normal stress to simplify cycle identification. Once the normal stress cycles are defined, the corresponding shear stress ranges are determined using methods such as the longest chord method or the minimum circumscribed circle method (Papadopoulos, 1998; Carpinteri, Spagnoli and Vantadori, 2003; Bibbo, Baumgartner and Arora, 2022). The CS counting method is particularly beneficial for multiaxial fatigue criteria that require both normal and shear stress components, including Findley’s criterion and the CS criterion (Carpinteri, Spagnoli and Vantadori, 2003; Bibbo *et al.*, 2022; Bibbo, Baumgartner and Arora, 2022).

In contrast, the MWCM, being a shear stress-based multiaxial fatigue criterion, follows a different approach to handling VA loading. Instead of explicitly defining closed fatigue cycles for both normal and shear stresses, the MVM simplifies the analysis by identifying the plane where shear stress variance is maximised (Susmel, 2010, 2014; Susmel and Taylor, 2012; Faruq and Susmel, 2019). This plane is then used to compute the critical shear stress range, reducing the need to simultaneously track both normal and shear stress cycles. The effective shear stress range based on MVM is determined as follows (Susmel, 2010, 2014; Susmel and Taylor, 2012; Faruq and Susmel, 2019):

$$\Delta\tau_{MWCM} = 2 \cdot \sqrt{2 \cdot Var[\tau_{MV}(t)]} \quad (2.25)$$

where shear stress variance, $Var[\tau_{MV}(t)]$ is computed using Equation 2.26:

$$Var[\tau_{MV}(t)] = \frac{1}{T} \int_0^T [\tau_{MV}(t) - \tau_m]^2 \cdot dt \quad (2.26)$$

and the mean shear stress, τ_m is given by Equation 2.27

$$\tau_m = \frac{1}{T} \int_0^T \tau_{MV}(t) \cdot dt \quad (2.27)$$

Since MVM reduces the shear stress to a single equivalent quantity, it allows for the application of the standard three-point rainflow counting method, as defined in ASTM standards (Susmel, 2010, 2014; Susmel and Taylor, 2012; Standard, 2017; Faruq and Susmel, 2019). This eliminates the complexities associated with defining multiaxial loading cycles and makes MVM particularly advantageous for shear-based multiaxial fatigue assessments. In this study, both the CS counting method (applied with Findley's criterion and the CS approach) and MVM (applied with MWCM) are utilised to assess VA fatigue damage in welded joints in Chapter 4.

The KoNoS criterion, developed by Sonsino, introduces a hybrid hypothesis that combines both normal and shear stress components to determine the most critical plane (Kueppers and Sonsino, 2003, 2006; Sonsino and Wiebesiek, 2007). This criterion was

specifically formulated to address the semiductile behaviour of materials under CA and VA multiaxial loading, particularly when phase shifts occur between normal and shear stresses (Kueppers and Sonsino, 2003, 2006; Sonsino and Wiebesiek, 2007). The KoNoS criterion establishes a damage parameter, S , based on the von Mises equivalent stress, which is evaluated across multiple interference planes to determine the maximum damage accumulation under both in-phase and out-of-phase loading conditions (Kueppers and Sonsino, 2003, 2006; Sonsino and Wiebesiek, 2007). The ratio of these two maximum damage values is then used to calculate an equivalent stress that captures the combined effects of normal and shear stress contributions, thereby effectively quantifying the effect of phase differences on fatigue life estimations (Kueppers and Sonsino, 2003, 2006; Sonsino and Wiebesiek, 2007). This makes KoNoS particularly suitable for aluminium welded joints subjected to complex multiaxial loading conditions. The mathematical formulation of the KoNoS equivalent stress is provided in Equations 2.28 to 2.30 (Kueppers and Sonsino, 2003, 2006; Sonsino and Wiebesiek, 2007).

$$S(\varphi, \delta) = \sqrt{\sigma_n^2(\varphi, \delta) + 3\tau_n^2(\varphi, \delta)} \quad (2.28)$$

$$\Delta\sigma_{KoNoS}(\delta) = \Delta\sigma_{eq}(\delta = 0^\circ) \cdot \left(\frac{S(\varphi^{**}, \delta)}{S(\varphi^*, \delta = 0^\circ)} \right) \quad (2.29)$$

$$\Delta\sigma_{eq}(\delta = 0^\circ) = \sqrt{\Delta\sigma_x^2 + \Delta\sigma_y^2 - \Delta\sigma_x\Delta\sigma_y + \Delta\sigma_y + 3\Delta\tau_{xy}^2} \quad (2.30)$$

Where φ represents the orientation of the critical plane and δ is the phase angle that accounts for the relative shift between normal and shear stress.

The SWT criterion, originally formulated as a strain-based critical plane approach, identifies the critical plane as the one experiencing the maximum normal strain amplitude combined with the maximum normal stress acting on that plane. The original SWT equation is expressed as (Smith, 1970; Łagoda *et al.*, 2022; John *et al.*, 2024):

$$\sigma_{max}\varepsilon_{n,a} = \frac{\sigma_f'^2}{E}(2N_f)^{2b} + \sigma_f'\varepsilon_f'(2N_f)^{b+c} \quad (2.31)$$

where σ_{max} is the peak normal stress on the critical plane, $\varepsilon_{n,a}$ is the normal strain amplitude, E is the material's elastic modulus, σ_f' and ε_f' are the fatigue strength and fatigue ductility coefficients, respectively. The exponent b and c corresponds to the material's fatigue strength and ductility exponents. This formulation is particularly effective in low-cycle fatigue, where plastic strain plays a dominant role in fatigue damage. However, for high-cycle fatigue applications, where the material exhibits predominantly linear-elastic behaviour, the original strain-based formulation becomes less suitable.

To extend the applicability of SWT criterion to high-cycle fatigue regimes, a stress-based reformulation is introduced by simplifying the equation in accordance with linear elastic behaviour, resulting in Equation 2.32 (Smith, 1970; Łagoda *et al.*, 2022; John *et al.*, 2024).

$$\sigma_{a,SWT} = \sqrt{\sigma_{max}E\varepsilon_{n,a}} \quad (2.32)$$

where $\sigma_{a,SWT}$ represents the uniaxial equivalent stress amplitude derived using the SWT criterion. This modified formulation retains the fundamental principle of SWT by capturing the interaction between normal stress and strain, while ensuring that it remains applicable within linear-elastic fatigue conditions (Smith, 1970; Łagoda *et al.*, 2022; John *et al.*, 2024).

To implement the stress-based SWT criterion, the normal strain on an arbitrary material plane, X' is estimated using Hooke's law as defined in Equation 2.33 (Smith, 1970; John *et al.*, 2024).

$$E\varepsilon_{X'}(t) = \sigma_{X'}(t) - \nu(\sigma_{Y'}(t) + \sigma_{Z'}(t)) \quad (2.33)$$

where $\sigma_{X'}$, $\sigma_{Y'}$, and $\sigma_{Z'}$, represent the stress components acting on a material plane defined by the X' , Y' , and Z' coordinate system, with X' being the orientation normal to

the plane. ν is the Poisson's ratio. The normal strain amplitude, $E\varepsilon_{x'}(t)$, is computed for each plane, with the critical plane being the one experiencing the maximum $E\varepsilon_{n,a}$ value (Smith, 1970; John *et al.*, 2024).

By adopting this stress-based reformulation, the SWT criterion is extended to high cycle fatigue regimes, where stress amplitudes, rather than strain values, provide a more representative measure of fatigue damage accumulation (Smith, 1970; Lagoda *et al.*, 2022; John *et al.*, 2024). As such, the modified SWT formulation is applied in this study to assess multiaxial fatigue in hybrid aluminium-to-steel welded joints.

Once the equivalent stress ranges for different critical plane criteria are determined, the fatigue life can be estimated using a generalised fatigue life equation based on Wöhler equation. This equation provides a unified framework for estimating the number of cycles to failure, $N_{f,e}$, across different multiaxial fatigue assessment criteria. The general expression is given by Equation 2.34.

$$N_{f,e} = N_R \cdot \left(\frac{\Delta\sigma_R}{\Delta\sigma_{\text{criterion}}} \right)^k \quad (2.34)$$

Where N_R represents the reference cycles to failure, $\Delta\sigma_R$ is the reference fatigue strength, and $\Delta\sigma_{\text{criterion}}$ is the equivalent stress range calculated according to a given multiaxial fatigue criterion and k is the negative inverse slope of the reference curve.

For each critical plane criterion, the corresponding reference fatigue strength and equivalent stress range are substituted accordingly. In the MWCM, which is based on shear stress, $\Delta\sigma_R$ is replaced by the reference shear stress range, $\Delta\tau_{R,MWCM}(\rho_w)$, while the slope parameter, k is adjusted to $k_\tau(\rho_w)$ to accommodate the effect of multiaxiality (Susmel, 2009a, 2014). This adaptation ensures that shear-based criteria are properly integrated into the unified fatigue life estimation framework.

By applying these formulations, the comparative performance of different critical plane criteria can be systematically evaluated, particularly in their ability to predict fatigue life under multiaxial fatigue loading conditions. The insights gained from these validations contribute to a more robust understanding of the applicability and

limitations of various approaches, especially in the context of hybrid aluminium-to-steel welded joints.

Despite significant research in the multiaxial fatigue criteria, several key research gaps remain unaddressed, particularly in the extension of the multiaxial fatigue assessment methodologies to hybrid welded joints. Existing fatigue life estimation models were primarily developed for homogeneous metallic materials, and their direct applicability to dissimilar aluminium-steel welded configurations remains uncertain. The following section presents a summary of key findings and identifies existing research gaps, providing a foundation for further investigations to refine multiaxial fatigue assessment methods for hybrid welded joints.

2.5 Summary and Research Gaps

This chapter has provided a comprehensive review of welding technology, stress analysis approaches, and multiaxial fatigue criteria relevant to the fatigue assessment of welded joints under multiaxial loading conditions. The discussion covered the principles, advantages, and limitations of various stress analysis approaches, including the NS, HSS, ENS, and TCD approaches, alongside an in-depth examination of multiaxial fatigue criteria, particularly those recommended by design standards and stress-based critical plane models.

While extensive research has been conducted on multiaxial fatigue in aluminium and steel welded joints, limited studies have explored hybrid aluminium-to-steel welded joints under multiaxial loading conditions. Despite the increasing adoption of lightweight and sustainable structures, there is currently no established methodology for accurately estimating the fatigue life of these hybrid joints, highlighting the need for further investigation.

To bridge this gap, several key smaller knowledge gaps must first be addressed. Existing studies lack a comprehensive and systematic comparison of multiaxial fatigue criteria across different stress analysis approaches, even for conventional steel and aluminium welded joints. Research in this area is often fragmented, with studies either

focusing on a single fatigue criterion applied to multiple stress analysis methods or evaluating multiple fatigue criteria using only one stress analysis approach (Radaj, 1996; Bäckström and Marquis, 2001; Susmel, 2009c, 2014; Papuga, 2011; Pedersen, 2016; Mei *et al.*, 2021; Bibbo, Baumgartner and Arora, 2022). This incomplete evaluation limits a holistic understanding of how different standard criteria and stress-based critical plane criteria perform across various stress analysis techniques. To address this gap, this study performs a comprehensive quantitative review under both CA and VA loading conditions, first evaluating standard fatigue criteria before progressing to advanced critical plane approaches.

To determine the most suitable multiaxial fatigue criteria for hybrid aluminium-to-steel welded joints, it is essential to first establish a fundamental understanding of their failure mechanisms. However, failure analysis of hybrid welded joints remains an underexplored area. While crack initiation and propagation have been extensively studied in homogeneous welded joints such as tubular, fillet-welded, tube-flange, and cruciform joints, these investigations primarily focus on steel or aluminium (Amstutz, Störzel and Seeger, 2001; Wang *et al.*, 2013; Caligiuri, 2015; Maggiolini *et al.*, 2016; Liu *et al.*, 2018; Shin, Chang and Muzaffer, 2021; Zhang, 2021). There is a lack of research on thin hybrid aluminium-to-steel welded joints, particularly regarding crack initiation sites, propagation paths, and failure modes under multiaxial loading. This study aims to address this knowledge gap through detailed failure analysis, contributing to a deeper understanding of fatigue behaviour in hybrid welded joints.

To systematically address these research gaps, this thesis makes the following key contributions:

- Chapter 3 presents a comprehensive quantitative review of standard multiaxial fatigue criteria, evaluating their effectiveness across different stress analysis approaches under both CA and VA loading conditions in steel and aluminium welded joints.
- Chapter 4 extends this investigation to stress-based critical plane criteria, integrating them with various stress analysis methods to establish a systematic and structured framework for multiaxial fatigue assessment.

- Chapter 6 shifts the focus to failure analysis of hybrid welded joints, examining crack initiation sites, propagation mechanisms, and failure modes to improve the understanding of fatigue failure characteristics in aluminium-to-steel joints.
- Chapter 7 synthesises these findings by validating and formalising suitable multiaxial fatigue assessment methodologies, ultimately establishing a robust predictive model for fatigue life estimation of hybrid aluminium-to-steel welded joints.

By integrating experimental investigations, stress analysis techniques, and multiaxial fatigue criteria, this research bridges the gap between existing fatigue assessment methodologies and their applicability to hybrid welded structures. The findings contribute to the development of an accurate and reliable fatigue life estimation framework, offering engineering recommendations to support the design and structural optimisation of hybrid aluminium-to-steel welded joints for real world applications.

2.6 References

Alencar, G. *et al.* (2021) ‘A finite element post-processor for fatigue assessment of welded structures based on the Master S-N curve method’, *International Journal of Fatigue*, 153. doi: 10.1016/j.ijfatigue.2021.106482.

Amstutz, H., Störzel, K. and Seeger, T. (2001) ‘Fatigue crack growth of a welded tube–flange connection under bending and torsional loading’, *Fatigue & Fracture of Engineering Materials & Structures*, 24(5), pp. 357–368.

Anon (2005) *Eurocode 3: Design of steel structures — Part 1-9: Fatigue* .

Anon (2006) ‘Eurocode 1: Actions on structures - Part 3: Actions induced by cranes and machinery’.

Anon (2007) *Eurocode 9: Design of aluminium structures — Part 1-3: Structures susceptible to fatigue* .

- Archer, R. (1987) 'Fatigue of a welded steel attachment under combined direct stress and shear stress', in *International conference of fatigue of welded constructions*, no. paper.
- Atabaki, M. M. *et al.* (2014) 'Welding of aluminum alloys to steels: an overview', *Journal for Manufacturing Science and Production*, 14(2), pp. 59–78.
- Atzori, B. *et al.* (2009) 'Fatigue design of complex welded structures', *International Journal of Fatigue*, 31(1), pp. 59–69. doi: 10.1016/j.ijfatigue.2008.02.013.
- Bäckström, M. (2003) 'Multiaxial fatigue life assessment of welds based on nominal and hot spot stresses'.
- Bäckström, M. and Marquis, G. (2001) 'A review of multiaxial fatigue of weldments: experimental results, design code and critical plane approaches', *Fatigue & fracture of engineering materials & structures*, 24(5), pp. 279–291.
- Berto, F., Lazzarin, P. and Radaj, D. (2014) 'Fictitious notch rounding concept applied to V-notches with root hole subjected to in-plane mixed mode loading', *Engineering Fracture Mechanics*, 128(C), pp. 171–188. doi: 10.1016/j.engfracmech.2014.07.014.
- Bibbo, N. D. *et al.* (2022) 'An improved rainflow counting method for multiaxial stress states using the minimum circumscribed circle method to identify shear stress ranges', *International Journal of Fatigue*, 163, p. 106997.
- Bibbo, N. D., Baumgartner, J. and Arora, V. (2022) 'Comparative study of critical plane fatigue criteria on multiaxial variable amplitude loaded welded test specimens', *International Journal of Fatigue*, 158, p. 106670.
- Bouche, K., Barbier, F. and Coulet, A. (1998) 'Intermetallic compound layer growth between solid iron and molten aluminium', *Materials Science and Engineering: A*, 249(1–2), pp. 167–175.
- Brown, M. W. and Miller, K. J. (1973) 'A theory for fatigue failure under multiaxial stress-strain conditions', *Proceedings of the Institution of Mechanical engineers*, 187(1), pp. 745–755.

Brown, M. W. and Miller, K. J. (1979) 'Initiation and growth of cracks in biaxial fatigue', *Fatigue & Fracture of Engineering Materials & Structures*, 1(2), pp. 231–246.

Bruun, Ø. A. and Härkegård, G. (2015) 'A comparative study of design code criteria for prediction of the fatigue limit under in-phase and out-of-phase tension–torsion cycles', *International Journal of Fatigue*, 73, pp. 1–16.

Caligiuri, R. D. (2015) 'Critical crack path assessments in failure investigations', *Frattura ed Integrità Strutturale*, 9(34).

Cao, R. *et al.* (2014) 'Cold metal transfer joining of aluminum AA6061-T6-to-galvanized boron steel', *Journal of Manufacturing Science and Engineering*, 136(5), p. 051015.

Carpinteri, A., Macha, E., *et al.* (1999) 'Expected principal stress directions under multiaxial random loading. Part I: theoretical aspects of the weight function method', *International Journal of Fatigue*, 21(1), pp. 83–88.

Carpinteri, A., Brighenti, R., *et al.* (1999) 'Expected principal stress directions under multiaxial random loading. Part II: numerical simulation and experimental assessment through the weight function method', *International journal of fatigue*, 21(1), pp. 89–96.

Carpinteri, A., Spagnoli, A. and Vantadori, S. (2003) 'A multiaxial fatigue criterion for random loading', *Fatigue & Fracture of Engineering Materials & Structures*, 26(6), pp. 515–522.

Carpinteri, A., Spagnoli, A. and Vantadori, S. (2009) 'Multiaxial fatigue life estimation in welded joints using the critical plane approach', *International Journal of Fatigue*, 31(1), pp. 188–196.

Carpinteri, Brighenti and Spagnoli (2000) 'A fracture plane approach in multiaxial high-cycle fatigue of metals', *Fatigue & Fracture of Engineering Materials & Structures*, 23(4), pp. 355–364.

- Costa, J. D. M. *et al.* (2005) 'Fatigue behaviour of tubular AlMgSi welded specimens subjected to bending–torsion loading', *Fatigue & Fracture of Engineering Materials & Structures*, 28(4), pp. 399–407.
- Dang-Van, K. (1993) 'Macro-micro approach in high-cycle multiaxial fatigue'.
- Ellyin, F. (1989) 'Cyclic strain energy density as a criterion for multiaxial fatigue failure', *Biaxial and multiaxial fatigue*, 3, pp. 571–583.
- Endo, T. (1974) 'Rain flow method—the proposal and the applications', *Bull. Kyushu Inst. Tech.*, 28, p. 33.
- Eulitz, K.-G. (2000) 'Damage accumulation limitations and perspectives for fatigue life assessment', in *Proceedings of Materials week 2000*, pp. 25–28.
- Faruq, N. Z. and Susmel, L. (2019) 'Proportional/nonproportional constant/variable amplitude multiaxial notch fatigue: cyclic plasticity, non-zero mean stresses, and critical distance/plane', *Fatigue & Fracture of Engineering Materials & Structures*, 42(9), pp. 1849–1873. doi: <https://doi.org/10.1111/ffe.13036>.
- Fatemi, A. and Socie, D. F. (1988) 'A critical plane approach to multiaxial fatigue damage including out-of-phase loading', *Fatigue & Fracture of Engineering materials & structures*, 11(3), pp. 149–165.
- Findley, W. N. (1959) 'A theory for the effect of mean stress on fatigue of metals under combined torsion and axial load or bending', *Journal of Engineering for Industry*, 81(4), pp. 301–305.
- Foti, P. and Berto, F. (2020) 'Fatigue assessment of high strength welded joints through the strain energy density method', *Fatigue & Fracture of Engineering Materials & Structures*, 43(11), pp. 2694–2702.
- Fricke, W. (2003) 'Fatigue analysis of welded joints: state of development', *Marine structures*, 16(3), pp. 185–200.
- Fricke, W. (2007) 'Round-robin study on stress analysis for the effective notch stress approach', *Welding in the World*, 51(3), pp. 68–79.

Fricke, W. (2013) 'IIW guideline for the assessment of weld root fatigue', *Welding in the World*, 57(6), pp. 753–791. doi: 10.1007/s40194-013-0066-y.

Gan, L., Wu, H. and Zhong, Z. (2021) 'Multiaxial fatigue life prediction based on a simplified energy-based model', *International Journal of Fatigue*, 144, p. 106036.

Glinka, G., Shen, G. and Plumtree, A. (1995) 'A multiaxial fatigue strain energy density parameter related to the critical fracture plane', *Fatigue & Fracture of Engineering Materials & Structures*, 18(1), pp. 37–46.

Goecke, S. F. (2005) 'Low energy arc joining process for materials sensitive to heat', *EWM hightec welding GmbH*, 3.

Gough, H. J. and Pollard, H. V (1935) 'The strength of metals under combined alternating stresses', *Proceedings of the institution of mechanical engineers*, 131(1), pp. 3–103.

Gullino, A., Matteis, P. and D'Aiuto, F. (2019) 'Review of aluminum-to-steel welding technologies for car-body applications', *Metals*, 9(3), p. 315.

Hobbacher, A. (2016) *Recommendations for fatigue design of welded joints and components*. Springer.

Hobbacher, A. F. (2009) 'The new IIW recommendations for fatigue assessment of welded joints and components - A comprehensive code recently updated', *International Journal of Fatigue*, 31(1), pp. 50–58. doi: 10.1016/j.ijfatigue.2008.04.002.

John, E. *et al.* (2024) 'Multiaxial fatigue of water pipe grey cast iron', *International Journal of Fatigue*, 178, p. 108002.

Kah, P., Suoranta, R. and Martikainen, J. (2013) 'Advanced gas metal arc welding processes', *The International Journal of Advanced Manufacturing Technology*, 67, pp. 655–674.

Kimapong, K. and Watanabe, T. (2005) 'Lap joint of A5083 aluminum alloy and SS400 steel by friction stir welding', *Materials transactions*, 46(4), pp. 835–841.

- Kueppers, M. and Sonsino, C. M. (2003) ‘Critical plane approach for the assessment of the fatigue behaviour of welded aluminium under multiaxial loading’, *Fatigue & Fracture of Engineering Materials & Structures*, 26(6), pp. 507–513.
- Kueppers, M. and Sonsino, C. M. (2006) ‘Assessment of the fatigue behaviour of welded aluminium joints under multiaxial spectrum loading by a critical plane approach’, *International Journal of Fatigue*, 28(5–6), pp. 540–546.
- Łagoda, T. *et al.* (2022) ‘Using the Smith-Watson-Topper parameter and its modifications to calculate the fatigue life of metals: the state-of-the-art’, *Materials*, 15(10), p. 3481.
- Langlais, T. E., Vogel, J. H. and Chase, T. R. (2003) ‘Multiaxial cycle counting for critical plane methods’, *International journal of fatigue*, 25(7), pp. 641–647.
- Lazzarin, P. *et al.* (2008) ‘Local strain energy density and fatigue strength of welded joints under uniaxial and multiaxial loading’, *Engineering fracture mechanics*, 75(7), pp. 1875–1889.
- Lazzarin, P. and Livieri, P. (2001) *Notch stress intensity factors and fatigue strength of aluminium and steel welded joints*, *International Journal of Fatigue*. Available at: www.elsevier.com/locate/ijfatigue.
- Lazzarin, P. and Tovo, R. (1998) ‘A notch intensity factor approach to the stress analysis of welds’, *Fatigue & fracture of engineering materials & structures*, 21(9), pp. 1089–1103.
- Lee, Y.-L. and Tjhung, T. (2011) ‘Rainflow cycle counting techniques’, *Metal Fatigue Analysis Handbook: Practical Problem-solving Techniques for Computer-aided Engineering*, 89.
- Lee, Y. L. and Pan, J. (2005) *Fatigue Testing and Analysis: Theory and Practice*. Elsevier.
- Liu, H. *et al.* (2018) ‘Mechanisms of fatigue crack initiation and propagation in 6005A CMT welded joint’, *Journal of Alloys and Compounds*, 741, pp. 188–196.

Macha and Sonsino (1999) 'Energy criteria of multiaxial fatigue failure', *Fatigue & fracture of engineering materials & structures*, 22(12), pp. 1053–1070.

Maggiolini, E. *et al.* (2016) 'Crack path and fracture analysis in FSW of small diameter 6082-T6 aluminium tubes under tension–torsion loading', *International Journal of Fatigue*, 92, pp. 478–487.

Marquis, G., Bäckström, M. and Siljander, A. (1997) 'Multiaxial fatigue damage parameters for welded joints: design code and critical plane approaches', in *1st North European Engineering and Science Conference (NESCO I)*. Engineering Materials Advisory Services EMAS.

Matake, T. (1977) 'An explanation on fatigue limit under combined stress', *Bulletin of JSME*, 20(141), pp. 257–263.

Matsuishi, M. (1968) 'Fatigue of Metals Subjected to Varying Stress', *Japan Society of Mechanical Engineers*.

McDiarmid, D. L. (1991) 'A general criterion for high cycle multiaxial fatigue failure', *Fatigue & Fracture of Engineering Materials & Structures*, 14(4), pp. 429–453.

McDiarmid, D. L. (1994) 'A shear stress based critical-plane criterion of multiaxial fatigue failure for design and life prediction', *Fatigue & Fracture of Engineering Materials & Structures*, 17(12), pp. 1475–1484.

Meco, S. *et al.* (2015) 'Application of laser in seam welding of dissimilar steel to aluminium joints for thick structural components', *Optics and Lasers in Engineering*, 67, pp. 22–30.

Mei, J. *et al.* (2021) 'An overview and comparative assessment of approaches to multi-axial fatigue of welded components in codes and standards', *International Journal of Fatigue*, 146, p. 106144.

Mei, J. and Dong, P. (2017) 'An equivalent stress parameter for multi-axial fatigue evaluation of welded components including non-proportional loading effects', *International Journal of Fatigue*, 101, pp. 297–311.

- Morgenstern, C. *et al.* (2006) 'Fatigue design of aluminium welded joints by the local stress concept with the fictitious notch radius of $r_f=1$ mm', *International Journal of Fatigue*, 28(8), pp. 881–890. doi: 10.1016/j.ijfatigue.2005.10.006.
- Neuber, H. (1961) 'Theory of notch stresses: principles for exact calculation of strength with reference to structural form and material', (*No Title*).
- Niemi, E. (1995) *Stress determination for fatigue analysis of welded components*. Woodhead Publishing.
- Papadopoulos, I. V (1998) 'Critical plane approaches in high-cycle fatigue: on the definition of the amplitude and mean value of the shear stress acting on the critical plane', *Fatigue & Fracture of Engineering Materials & Structures*, 21(3), pp. 269–285.
- Papuga, J. (2011) 'A survey on evaluating the fatigue limit under multiaxial loading', *International Journal of Fatigue*, 33(2), pp. 153–165.
- Pedersen, M. M. (2016) 'Multiaxial fatigue assessment of welded joints using the notch stress approach', *International Journal of Fatigue*, 83, pp. 269–279. doi: 10.1016/J.IJFATIGUE.2015.10.021.
- Qin, G., Su, Y. and Wang, S. (2014) 'Microstructures and properties of welded joint of aluminum alloy to galvanized steel by Nd: YAG laser+ MIG arc hybrid brazing-fusion welding', *transactions of nonferrous metals society of china*, 24(4), pp. 989–995.
- Radaj, D. (1996) 'Review of fatigue strength assessment of nonwelded and welded structures based on local parameters', *International Journal of Fatigue*, 18(3), pp. 153–170.
- Radaj, D., Lazzarin, P. and Berto, F. (2013) 'Generalised Neuber concept of fictitious notch rounding', *International Journal of Fatigue*, 51, pp. 105–115. doi: 10.1016/j.ijfatigue.2013.01.005.
- Radaj, D., Sonsino, C. M. and Fricke, W. (2006) *Fatigue assessment of welded joints by local approaches*. Woodhead publishing.

- Razmjoo, G. R. (1997) 'Fatigue of load-carrying fillet welded joints under multiaxial loading', in *5 th International Conference on Biaxial/Multiaxial Fatigue & Fracture*, pp. 53–70.
- Richards, R. W. *et al.* (1994) 'Metallurgy of continuous hot dip aluminizing', *International materials reviews*, 39(5), pp. 191–212.
- Schijve, J. (2012) 'Fatigue predictions of welded joints and the effective notch stress concept', *International Journal of Fatigue*, 45, pp. 31–38.
- Shahverdi, H. R. *et al.* (2002) 'Microstructural analysis of interfacial reaction between molten aluminium and solid iron', *Journal of materials processing technology*, 124(3), pp. 345–352.
- Shen, W. *et al.* (2018) 'Multiaxial fatigue analysis of complex welded joints in notch stress approach', *Engineering Fracture Mechanics*, 204, pp. 344–360. doi: 10.1016/j.engfracmech.2018.10.035.
- Shin, W., Chang, K.-H. and Muzaffer, S. (2021) 'Fatigue analysis of cruciform welded joint with weld penetration defects', *Engineering Failure Analysis*, 120, p. 105111.
- Siljander, A., Kurath, P. and Lawrence, F. V (1992) 'Nonproportional fatigue of welded structures', *Advances in fatigue lifetime predictive techniques*, pp. 319–338.
- Smith, Kn. (1970) 'A stress-strain function for the fatigue of metals', *Journal of materials*, 5, pp. 767–778.
- Socie, D. F. (1987) 'Multiaxial fatigue damage models'.
- Sonsino, C. M. (1995) 'Multiaxial fatigue of welded joints under in-phase and out-of-phase local strains and stresses', *International Journal of Fatigue*, 17(1), pp. 55–70.
- Sonsino, C. M. (2007) 'Fatigue testing under variable amplitude loading', *International Journal of Fatigue*, 29(6), pp. 1080–1089.
- Sonsino, Cetin Morris (2009) 'A consideration of allowable equivalent stresses for fatigue design of welded joints according to the notch stress concept with the reference radii $r_{ref} = 1.00$ and 0.05 mm', *Welding in the World*, 53, pp. R64–R75.

- Sonsino, C. M. (2009) 'Multiaxial fatigue assessment of welded joints—recommendations for design codes', *International Journal of Fatigue*, 31(1), pp. 173–187.
- Sonsino, C. M. *et al.* (2012) 'Notch stress concepts for the fatigue assessment of welded joints—Background and applications', *International Journal of Fatigue*, 34(1), pp. 2–16.
- Sonsino, C. M., Bruder, T. and Baumgartner, J. (2010) 'SN lines for welded thin joints—suggested slopes and FAT values for applying the notch stress concept with various reference radii', *Welding in the World*, 54(11), pp. R375–R392.
- Sonsino, C. M. and Kueppers, M. (2001) 'Multiaxial fatigue of welded joints under constant and variable amplitude loadings', *Fatigue & Fracture of Engineering Materials & Structures*, 24(5), pp. 309–327.
- Sonsino, C. M., Maddox, S. J. and Hobbacher, A. (2005) 'Fatigue life assessment of welded joints under variable amplitude loading-State of present knowledge and recommendations for fatigue design regulations'.
- Sonsino, C. M. and Wiebesiek, J. (2007) 'Assessment of multiaxial spectrum loading of welded steel and aluminium joints by modified equivalent stress and Gough-Pollard algorithms', *IIW-Document No. XIII-2158r1-07/XV-1250r1-07*.
- Standard, A. (2017) *E1049-85 (Reapproved 2017) Standard practices for cycle counting in fatigue analysis*. Tech. rep., ASTM International, West Conshohocken, PA.
- Susmel, L. (2007) 'Eurocode 3's standard curves and theory of critical distances to estimate fatigue lifetime of steel weldments', *Key Engineering Materials*, 348, pp. 21–24.
- Susmel, L. (2008) 'Modified Wöhler curve method, theory of critical distances and Eurocode 3: A novel engineering procedure to predict the lifetime of steel welded joints subjected to both uniaxial and multiaxial fatigue loading', *International Journal of Fatigue*, 30(5), pp. 888–907. doi: 10.1016/j.ijfatigue.2007.06.005.

Susmel, Luca (2008) 'The theory of critical distances: a review of its applications in fatigue', *Engineering Fracture Mechanics*, 75(7), pp. 1706–1724.

Susmel, L. (2009a) *Multiaxial notch fatigue*. Elsevier.

Susmel, L. (2009b) 'The Modified Wöhler Curve Method calibrated by using standard fatigue curves and applied in conjunction with the Theory of Critical Distances to estimate fatigue lifetime of aluminium weldments', *International Journal of Fatigue*, 31(1), pp. 197–212. doi: 10.1016/j.ijfatigue.2008.04.004.

Susmel, L. (2009c) 'Three different ways of using the Modified Wöhler Curve Method to perform the multiaxial fatigue assessment of steel and aluminium welded joints', *Engineering Failure Analysis*, 16(4), pp. 1074–1089. doi: 10.1016/j.engfailanal.2008.05.016.

Susmel, L. (2010) 'A simple and efficient numerical algorithm to determine the orientation of the critical plane in multiaxial fatigue problems', *International Journal of Fatigue*, 32(11), pp. 1875–1883. doi: 10.1016/j.ijfatigue.2010.05.004.

Susmel, L. (2014) 'Four stress analysis strategies to use the Modified Wöhler Curve Method to perform the fatigue assessment of weldments subjected to constant and variable amplitude multiaxial fatigue loading', *International Journal of Fatigue*, 67, pp. 38–54. doi: 10.1016/j.ijfatigue.2013.12.001.

Susmel, L. and Askes, H. (2012) 'Modified Wöhler Curve Method and multiaxial fatigue assessment of thin welded joints', *International Journal of Fatigue*, 43, pp. 30–42. doi: 10.1016/j.ijfatigue.2012.01.026.

Susmel, L. and Lazzarin, P. (2002) 'A bi-parametric Wöhler curve for high cycle multiaxial fatigue assessment', *Fatigue & Fracture of Engineering Materials & Structures*, 25(1), pp. 63–78.

Susmel, L., Sonsino, C. M. and Tovo, R. (2011) 'Accuracy of the modified wöhler curve method applied along with the $r_{ref} = 1$ mm concept in estimating lifetime of welded joints subjected to multiaxial fatigue loading', *International Journal of Fatigue*, 33(8), pp. 1075–1091. doi: 10.1016/j.ijfatigue.2010.09.010.

- Susmel, L. and Taylor, D. (2012) 'A critical distance/plane method to estimate finite life of notched components under variable amplitude uniaxial/multiaxial fatigue loading', *International Journal of Fatigue*, 38, pp. 7–24. doi: <https://doi.org/10.1016/j.ijfatigue.2011.11.015>.
- Susmel, L. and Tovo, R. (2004) 'On the use of nominal stresses to predict the fatigue strength of welded joints under biaxial cyclic loading', *Fatigue & Fracture of Engineering Materials & Structures*, 27(11), pp. 1005–1024.
- Susmel, L. and Tovo, R. (2006) 'Local and structural multiaxial stress states in welded joints under fatigue loading', *International Journal of Fatigue*, 28(5–6), pp. 564–575. doi: 10.1016/j.ijfatigue.2005.08.009.
- Susmel, L., Tovo, R. and Benasciutti, D. (2009) 'A novel engineering method based on the critical plane concept to estimate the lifetime of weldments subjected to variable amplitude multiaxial fatigue loading', *Fatigue and Fracture of Engineering Materials and Structures*, 32(5), pp. 441–459. doi: 10.1111/j.1460-2695.2009.01349.x.
- Taylor, D. (2008) 'The theory of critical distances', *Engineering Fracture Mechanics*, 75(7), pp. 1696–1705.
- Taylor, D., Barrett, N. and Lucano, G. (2002) 'Some new methods for predicting fatigue in welded joints', *International Journal of Fatigue*, 24(5), pp. 509–518.
- Tovo, R. and Lazzarin, P. (1999) 'Relationships between local and structural stress in the evaluation of the weld toe stress distribution', *International journal of fatigue*, 21(10), pp. 1063–1078.
- Wang, H. T. *et al.* (2013) 'An experimental investigation of local fracture resistance and crack growth paths in a dissimilar metal welded joint', *Materials & Design*, 44, pp. 179–189.
- Yousefi, F., Witt, M. and Zenner, H. (2001) 'Fatigue strength of welded joints under multiaxial loading: experiments and calculations', *Fatigue & Fracture of Engineering Materials & Structures*, 24(5), pp. 339–355.

Al Zamzami, I. *et al.* (2018) ‘Static strength and design of aluminium-to-steel thin welded joints’, *Welding in the World*, 62(6), pp. 1255–1272. doi: 10.1007/s40194-018-0634-2.

Al Zamzami, I., Davison, B. and Susmel, L. (2019) ‘Nominal and local stress quantities to design aluminium-to-steel thin welded joints against fatigue’, *International Journal of Fatigue*, 123, pp. 279–295.

Al Zamzami, I. and Susmel, L. (2018) ‘On the use of hot-spot stresses, effective notch stresses and the Point Method to estimate lifetime of inclined welds subjected to uniaxial fatigue loading’, *International Journal of Fatigue*, 117, pp. 432–449. doi: 10.1016/j.ijfatigue.2018.08.032.

Zhang, Z. (2021) ‘Quantitative characterization on fatigue fracture features of A6005 aluminum alloy welded joints’, *Engineering Failure Analysis*, 129, p. 105687.

Chapter 3

Quantitative Review of Standard Multiaxial Fatigue Assessment Methods for Steel and Aluminium Welded Joints

3.1 Introduction

This chapter presents a quantitative review of the multiaxial fatigue assessment of steel and aluminium welded joints, focusing on the guidelines provided by Eurocode 3 (EC3), Eurocode 9 (EC9), and the International Institute of Welding (IIW). While the overarching objective of this research is to establish effective multiaxial fatigue assessment methodologies for hybrid aluminium-to-steel welded joints, it is first essential to critically assess how existing multiaxial fatigue criteria and stress analysis approaches perform when applied separately to steel and aluminium joints. This foundational analysis examines the fatigue behaviour of these materials under both constant amplitude (CA) and variable amplitude (VA) fatigue loading conditions.

Ensuring the structural integrity of welded joints under multiaxial fatigue loading requires adherence to established design codes and recommendations. Eurocodes (ECs) and IIW guidelines provide essential criteria and methods for assessing the fatigue performance of welded joints. To date, there has not been an extensive and comprehensive review that evaluates these standard multiaxial fatigue criteria in conjunction with various stress analysis approaches. This chapter addresses this gap by systematically reanalysing experimental fatigue data under both CA and VA loading conditions, offering novel insights for researchers while providing structural engineers with practical guidance for designing safe and reliable welded joints.

The chapter is structured to offer a systematic review of the performance and conservatism of the EC3, EC9, and IIW criteria, evaluated through different stress analysis approaches: nominal stress (NS), hot spot stress (HSS), and effective notch stress (ENS). The Validation Methodology section outlines the joint geometries, experimental datasets, and loading conditions used in the reanalyses, along with the methodology employed to assess the accuracy and conservatism of the criteria. The chapter is organised based on stress analysis approaches, with each section evaluating the criteria's performance under both CA and VA loading conditions within a specific stress framework.

Key parameters, including uniaxial and torsional reference fatigue strengths, fatigue curve slopes, and material properties, are presented in the relevant sections to support the analysis. However, the core evaluation of the performance and accuracy of each criterion in conjunction with the stress analysis approaches is quantified by the degree of conservatism, expressed as the percentage of non-conservative estimates (P_{NC}). The P_{NC} measures the proportion of fatigue life predictions that fall outside conservative design limits. Additionally, fatigue life graphs are employed to provide a visual representation of the accuracy and reliability of the estimations under both CA and VA loading conditions.

The chapter concludes with a comparative analysis, highlighting the strengths, limitations, and applicability of each assessment approach. This in-depth evaluation establishes a crucial foundation for subsequent investigations, guiding the transition towards more advanced critical plane approaches, which will be reviewed in Chapter 4, before addressing the complexities of hybrid aluminium-to-steel welded joints later in this thesis.

3.2 Validation Methodology

This section details the validation methodology employed to quantitatively assess the multiaxial fatigue criteria recommended by EC3, EC9, and the IIW for steel and aluminium welded joints. The validation focuses on reanalysing extensive experimental

fatigue data under both CA and VA loading using three stress analysis approaches: NS, HSS, and ENS (Niemi, 1995; Tovo and Lazzarin, 1999; Anon, 2005, 2007; Radaj, Sonsino and Fricke, 2006; Sonsino, Bruder and Baumgartner, 2010; Sonsino *et al.*, 2012; Fricke, 2013; Susmel, 2014; Van Lieshout, Den Besten and Kaminski, 2016; Hobbacher, 2016; Al Zamzami and Susmel, 2018; Mei *et al.*, 2021). To provide a clear understanding of how each stress analysis approach is applied to welded joints under multiaxial fatigue, Fig. 3.1 illustrates a weld seam subjected to both tension and shear stresses. This figure highlights the key distinctions and conceptual differences between the NS, HSS, and ENS methods, offering visual context for the subsequent analysis.

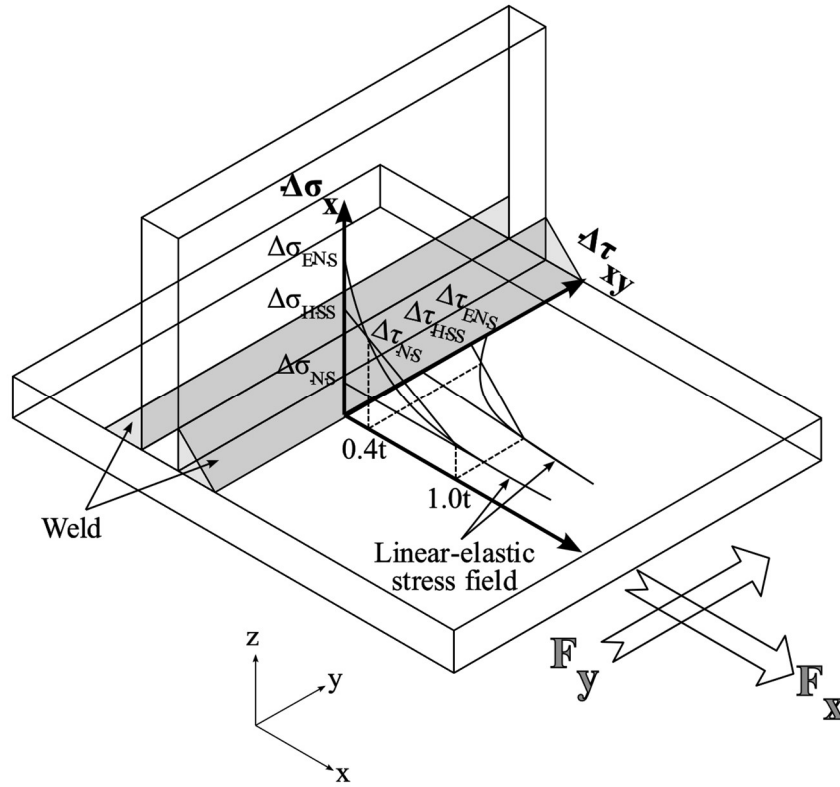


Figure 3.1: Overview of stress analysis approaches used in this study: nominal stress (NS), hot spot stress (HSS), and effective notch stress (ENS).

The theoretical foundations of the stress analysis approaches and multiaxial fatigue criteria, including detailed interaction equations and definitions of equivalent stress ranges, are comprehensively discussed in Chapter 2. This ensures that the focus of this chapter remains on the validation process, reanalysis methodology, and evaluation of

the predictive accuracy of the criteria under different loading scenarios and stress approaches.

3.2.1 Experimental Datasets and Joint Geometries

The experimental datasets compiled for this quantitative review encompass a wide range of welded joint configurations and loading conditions collected from the literature (Archer, 1987; Yung and Lawrence Jr, 1989; Siljander, Kurath and Lawrence Jr, 1989; Sonsino, 1995, 1997; Razmjoo, 1997; Amstutz, Störzel and Seeger, 2001; Sonsino and Kueppers, 2001; Yousefi, Witt and Zenner, 2001; Bäckström, 2003; Kueppers and Sonsino, 2003, 2006; Costa *et al.*, 2005; Sonsino *et al.*, 2006; Wiebesiek and Sonsino, 2010; Wiebesiek *et al.*, 2011; Pyttel, Grawenhof and Berger, 2012; Shams and Vormwald, 2017). The geometries cover diverse structural designs with varying dimensions, as illustrated in Figure 3.2. Weld leg sizes range from 2 mm to 11 mm, plate thicknesses from 2 mm to 10 mm, and joint lengths between 265 mm and 2040 mm. These dimensions represent real-world structural applications, providing a robust foundation for the validation of multiaxial fatigue assessment criteria.

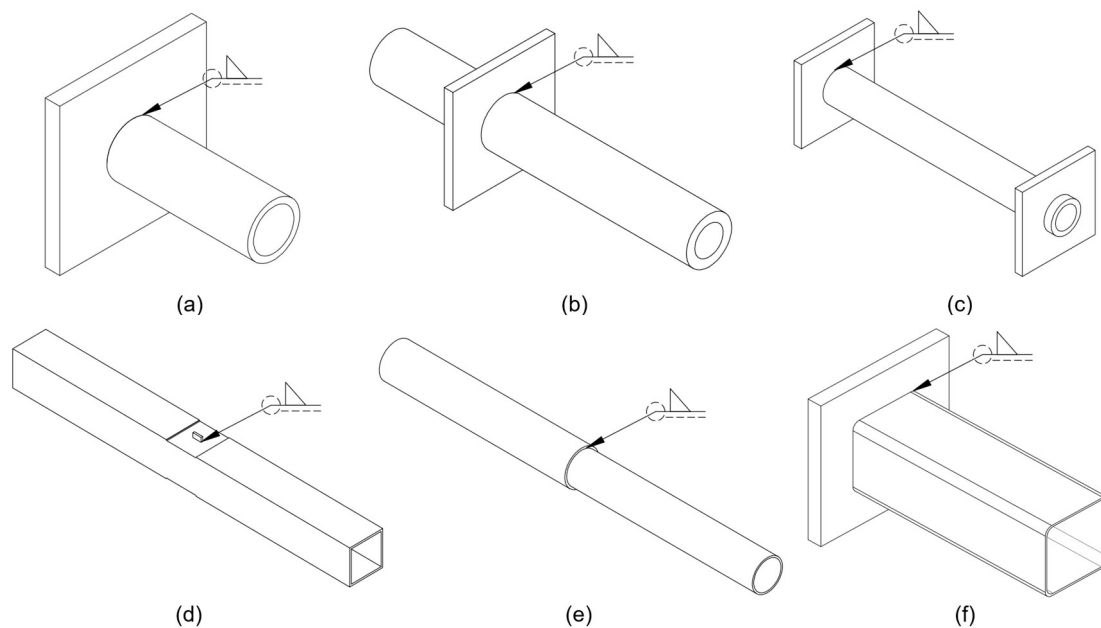


Figure 3.2: Geometries of the reanalysed steel and aluminium welded joints

The fatigue data include CA testing under multiple loading conditions, such as bending, tension, and torsion. These tests evaluate four primary loading scenarios: uniaxial, pure torsional, in-phase, and out-of-phase loading. The stress ratios ($R = \sigma_{\min}/\sigma_{\max}$ or $R = \tau_{\min}/\tau_{\max}$) examined include -1, 0, and 0.1, covering a range of cyclic loading profiles (Archer, 1987; Yung and Lawrence Jr, 1989; Siljander, Kurath and Lawrence Jr, 1989; Sonsino, 1995; Razmjoo, 1997; Amstutz, Störzel and Seeger, 2001; Yousefi, Witt and Zenner, 2001; Bäckström, 2003; Kueppers and Sonsino, 2003; Costa *et al.*, 2005; Shams and Vormwald, 2017). Typical loading paths for CA fatigue testing are depicted in Fig. 3.3.

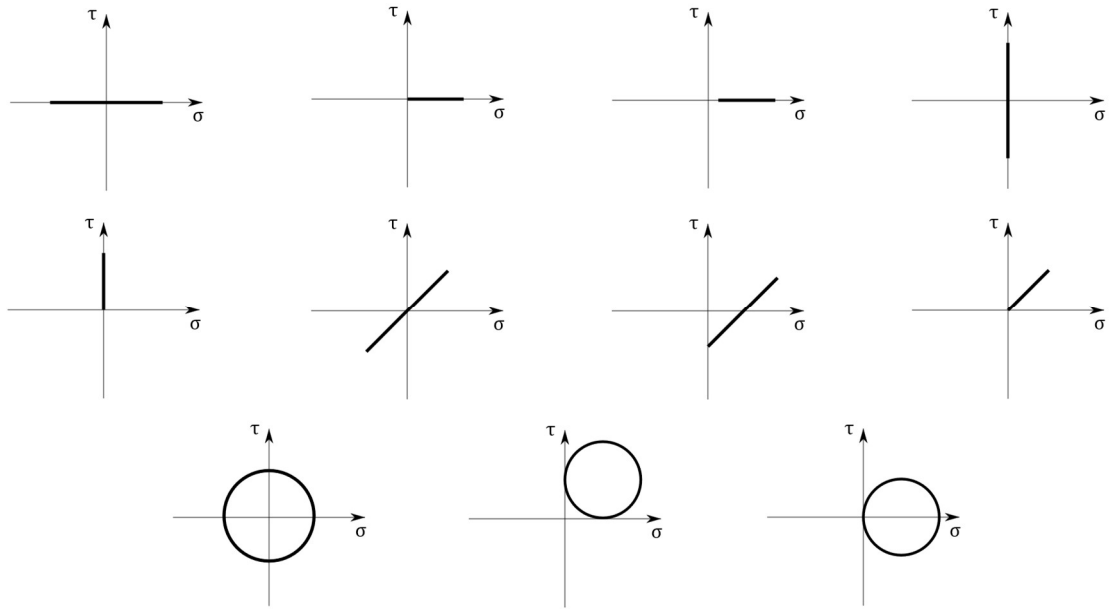


Figure 3.3: Loading paths for CA fatigue loading

For VA fatigue loading, a Gaussian load spectrum with a sequence length of 5×10^4 cycles, as presented in Fig. 3.4, was applied to all reanalysed welded joints. This spectrum was consistently adopted across the datasets in the reviewed literature (Sonsino and Kueppers, 2001; Yousefi, Witt and Zenner, 2001; Kueppers and Sonsino, 2006; Pyttel, Grawenhof and Berger, 2012).

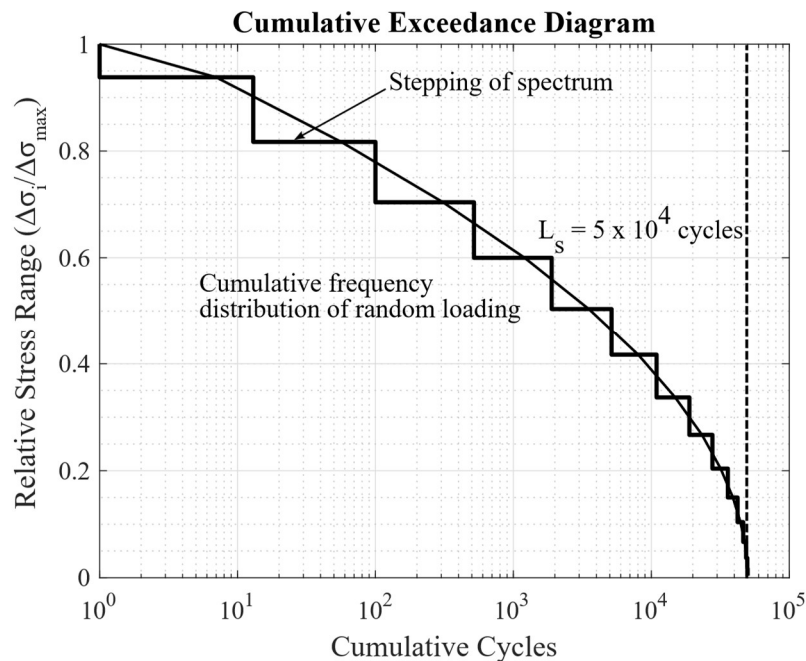


Figure 3.4: LBF Gaussian loading spectrum for VA

These comprehensive datasets enable a reliable evaluation of the predictive accuracy and conservatism of the EC3, EC9 and IIW criteria for both steel and aluminium welded joints. Furthermore, the same experimental data, welded joint geometries, and loading configurations will be employed in the next chapter for the quantitative review of critical plane fatigue assessment approaches, which will be presented in Chapter 4.

3.2.2 Mean Stress Effects and Probability of Survival Considerations

The validation process accounts for the distinct fatigue behaviours of as-welded and stress-relieved joints, both of which are influenced by residual stresses. In as-welded joints, the welding process induces high residual tensile stresses near the fatigue crack initiation zone. These stresses increase the local stress ratio, even when the global stress ratio is set to $R=-1$, thereby diminishing the effect of mean stresses (Atzori, 1983; Susmel and Tovo, 2004). As a result, mean stress effects can be treated as negligible in the fatigue assessments for as-welded joints (Atzori, 1983; Susmel and Tovo, 2004; Susmel, 2009).

In contrast, stress-relieved joints undergo heat treatment to reduce residual stresses, resulting in improved fatigue strength. To prevent overly conservative fatigue life predictions for these joints, only 60% of the compressive stress range is included in the assessment, in accordance with EC3 and IIW guidelines (Anon, 2005; Hobbacher, 2016). This adjustment is expressed through the following equation:

$$\Delta\sigma_{eff} = |\sigma_{max}| + 0.6 |\sigma_{min}| \quad (3.1)$$

Incorporating this effective stress range enhances the accuracy of stress distribution evaluations, ensuring that stress-relieved joints are assessed without excessive conservatism.

EC9 adopts a different strategy for addressing mean stress effects, introducing a mean stress enhancement factor, $f(R)$ to refine fatigue life predictions based on varying stress ratios (Anon, 2007). The enhancement factor is defined by the following conditions:

$$\begin{aligned} f(R) &= 1.3; \text{ if } R \leq -1 \\ f(R) &= -0.4 \times R + 0.9; \text{ if } -1 < R < 0.25 \\ f(R) &= 1; \text{ if } R \geq 0.25 \end{aligned} \quad (3.2)$$

For consistency and reliability, reference stresses used in fatigue assessments are based on standard reference fatigue strength, commonly referred to as FAT-values. These values correspond to stress levels at 2 million cycles to failure and are based on experimental fatigue data from uniaxial and pure torsional loading tests on welded joints. During the reanalysis, reference stresses are recalculated using a probability of survival (P_s) of 97.7%, which is equivalent to the mean minus two standard deviations (Anon, 2005, 2007; Hobbacher, 2016). Although some design codes recommend a P_s of 95% with a two-sided confidence interval, the difference between P_s of 97.7% and P_s of 95% is less than 2% (Anon, 2005, 2007; Hobbacher, 2016). This difference is considered negligible given the inherent scatter in fatigue data.

Therefore, adopting $P_s=97.7\%$ aligns with the recommendations of both ECs and the IIW (Anon, 2005, 2007; Hobbacher, 2016), ensuring a consistent and reliable assessment of reference stresses. This approach enhances the robustness of fatigue

performance evaluations, minimising the risk of non-conservative predictions and adhering to established industry standards.

3.2.3 Application of Partial Safety Factors

In this study, partial safety factors (SF) are incorporated into the fatigue strength assessment during the reanalysis, as recommended by both EC3 and IIW (Anon, 2005; Hobbacher, 2016). These factors ensure that design safety is maintained while accounting for uncertainties in loading, material properties, and failure consequences. The interaction equations used in this validation are defined in Chapter 2 (Equations 2.4 to 2.7).

The selected partial SF of 1.15 reflects the low consequence of failure for the steel and aluminium welded joints under review and aligns with the safe-life design approach. Table 3.1 summarises the recommended partial SFs for different assessment methods and levels of failure consequence. The SF is applied by reducing the allowable reference fatigue strength (FAT-values) at a P_s of 97.7%.

Table 3.1: Partial safety factors (SF) for assessment methods and failure consequence levels.

Assessment Method	Consequence of Failure	
	Low	High
Damage Tolerant	1.00	1.15
Safe Life	1.15	1.35

In contrast, EC9 recommends a partial SF of 1 for the aluminium welded joints considered in this study (Anon, 2007).

These partial safety factors play a crucial role in the validation of multiaxial fatigue criteria, which is carried out under both CA and VA loading conditions, as discussed in the following section.

3.2.4 Validation of Standard Criteria: CA and VA Loading

The validation of multiaxial fatigue criteria is conducted under both CA and VA loading conditions, following the guidelines specified by EC3, EC9, and IIW.

For CA loading, EC3 and IIW employ interaction diagrams to illustrate the relationship between normalised direct and shear stress components (Anon, 2005; Hobbacher, 2016). These diagrams serve as a visual tool for assessing the performance of the interaction equation criteria. On the other hand, EC9 applies the maximum principal stress criterion and evaluates fatigue life predictions through fatigue life graphs, enabling direct comparisons between experimental and estimated cycles to failure (Anon, 2007).

A key distinction between EC3 and IIW criteria lies in the exponents used in the interaction equations. EC3 incorporates the negative inverse slopes of reference fatigue curves, which are set to 3 for uniaxial and 5 for torsional fatigue reference curves (as shown in Equation 2.4) (Anon, 2005). This approach adds complexity, as variations in experimental fatigue curves across different sources make overall accuracy assessments challenging. Conversely, IIW interaction equation criterion, based on the Gough-Pollard criterion, consistently applies an exponent of 2 for both normalised direct and shear stresses, offering a standardised straightforward evaluation under CA loading conditions (Hobbacher, 2016).

For VA loading, the standard criteria are validated using two approaches: the interaction equations defined by EC3 and IIW, and the modified Palmgren-Miner (P-M) rule (Anon, 2005; Hobbacher, 2016). The P-M rule quantifies cumulative fatigue damage over VA loading sequences, allowing cycles to failure to be estimated for each welded joint (Anon, 2005; Hobbacher, 2016).

To evaluate the accuracy of the P-M rule, experimental cycles to failure, N_f are plotted against the estimated number of cycles to failure, $N_{f,e}$. Ideally, the estimated data points should cluster around the uniaxial and torsional $P_s=97.7\%$ line. Most points are expected to lie above the diagonal $P_s=50\%$ line (where $N_f=N_{f,e}$), indicating that the criterion yields safe fatigue life predictions. If the reference stresses were recalculated

at $P_s=50\%$, the points would mainly fall within the scatter bands of uniaxial and torsional fatigue data, providing further insights into the reliability of fatigue predictions under VA loading.

When applying the interaction equation approach for VA loading, the main difference from CA validation is the substitution of stress ranges. Instead of applying constant stress ranges, equivalent stress ranges derived from the VA loading spectra are used in the interaction equations (Anon, 2005; Hobbacher, 2016). This method is applicable only under EC3 and IIW criteria, whereas EC9 relies solely on the modified P-M rule with equivalent stress ranges defined by maximum principal stress ranges (Anon, 2007).

3.2.5 Reference S-N Curves, Knee Points, and Application of EC3 Simplified Coefficients

The validation methodology also considers the role of knee points and S-N curve slopes in fatigue life estimations. These reference fatigue curves and their corresponding knee points differ across standards, affecting both the structure of the curves and the evaluation of fatigue life.

Under EC3 guidelines, two knee points are defined at $N_{kp1}=5$ million cycles and at $N_{kp2}=10^8$ cycles, resulting in three linear segments with negative inverse slopes of 3, 5, and ∞ , respectively (Anon, 2005). In contrast, EC9 recommends slightly different negative inverse slopes of 3.4, 5.4, and ∞ (Anon, 2007). Moreover, EC9 simplifies the fatigue assessment by using a single uniaxial S-N curve, where both normal and shear stresses are transformed into equivalent maximum principal stresses (Anon, 2007).

The IIW reference fatigue curves adopt a different configuration, with a single knee point at $N_{kp}=10^7$ cycles (Hobbacher, 2016). This results in two distinct linear segments for uniaxial loading, with pre-knee slopes of $k=3$ and post-knee slopes of $k'=5$ (Hobbacher, 2016). For torsional loading, the IIW applies Haibach's modification, where the inverse slope transitions from 5 before the knee point to 9 after $N_{kp}=10^8$ cycles, following the relationship $2k-1$ (Haibach, 1989; Hobbacher, 2016). Under CA

loading, however, the IIW modifies the post-knee-point slope to a constant value of 22 for both uniaxial and torsional S-N curves (Hobbacher, 2016). These slope variations are crucial considerations in predicting fatigue life under different loading scenarios and are incorporated into the validation.

In addition to the S-N curve characteristics, EC3 introduces the simplified coefficient λ_i , which is intended to adjust equivalent stresses based on structural application and loading conditions (Anon, 2005). This coefficient varies according to the structural domain, such as crane structures, where λ_i ranges from 0.198 to 1.587 for direct stresses and from 0.379 to 1.320 for shear stresses (Anon, 2006). Since this study focuses on general scientific analysis rather than specific structural applications, the least conservative λ_i values suggested for the crane structures are utilised in the reanalysis for VA loading under EC3 interaction equation criterion. Specifically, the lowest values, 0.198 for direct stress and 0.379 for shear stress, are adopted (Anon, 2006).

This deliberate choice highlights the importance of selecting appropriate λ_i values balance accuracy and safety in fatigue predictions. It also underscores the safety margin associated with these recommended values for λ_i , particularly when applied to worst-case scenarios under the EC3 interaction criterion for VA loading. These considerations ensure that the validation process remains both reliable and representative of diverse structural applications.

3.2.6 Evaluation Metrics for Standard Multiaxial Fatigue Criteria Performance

The performance of the multiaxial fatigue assessment criteria outlined in EC3, EC9, and IIW is rigorously evaluated by analysing their predictive capabilities across various stress analysis approaches. This evaluation focuses on quantifying both the accuracy and the degree of conservatism of the criteria when applied to the fatigue life prediction of welded joints.

One of the key metrics used is the P_{NC} . This metric measures the proportion of fatigue life predictions that exceed allowable design limits, representing potentially unsafe

predictions. The P_{NC} is calculated by dividing the number of non-conservative estimates (i.e., points that fall outside the interaction curve boundary, often marked with hollow markers) by the total number of reanalysed specimens. The result is expressed as a percentage and provides a clear indication of the criteria's reliability and safety margin. A lower P_{NC} suggests that the criteria are generally conservative, thus ensuring safer design predictions.

For visual validation, fatigue life graphs are also employed, particularly for the modified P-M method under VA loading and for the EC9 maximum principal stress approach. In these graphs, points representing experimental cycles to failure are plotted against estimated cycles to failure. Since reanalyses were all based on a $P_s=97.7\%$, points that fall below the diagonal $P_s=50\%$ line indicate non-conservative estimates. The percentage of non-conservative points are quantified to provide further insights into prediction accuracy.

The upcoming sections of this chapter will build on these evaluation metrics to provide a detailed comparative analysis of the standard criteria's performance when applied with different stress analysis approaches, including NS, HSS and ENS. This quantitative assessment aims to uncover the strengths, limitations, and practical applicability of the criteria under both CA and VA loading conditions, providing valuable insights into their reliability and suitability for structural fatigue design.

3.3 Reanalyses Using the NS Approach

This section presents the reanalyses of steel and aluminium welded joints under both CA and VA loading conditions using the NS approach, in conjunction with multiaxial fatigue criteria from EC3, IIW, and EC9. The NS approach is widely recognised for its efficiency and practicality, especially for standard welded joint geometries, as it captures the macro-level stress distribution across the nominal cross-sectional area of the joint (Niemi, 1995; Radaj, 1996; Susmel and Tovo, 2004; Anon, 2005; Hobbacher, 2016; Mei *et al.*, 2021).

3.3.1 CA Loading

Table 3.2, provided at the end of this subsection for reference, summarises critical details of the reanalysed joints, including joint geometries, material properties, uniaxial and torsional reference fatigue strengths, fatigue curve slopes, and relevant literature sources. These reference values serve as a foundation for evaluating the performance of the standard criteria under CA loading conditions.

The reanalysis results for the EC3 interaction equation criterion with NS approach are illustrated in Figure 3.5, which shows data for as-welded and stress-relieved steel joints.

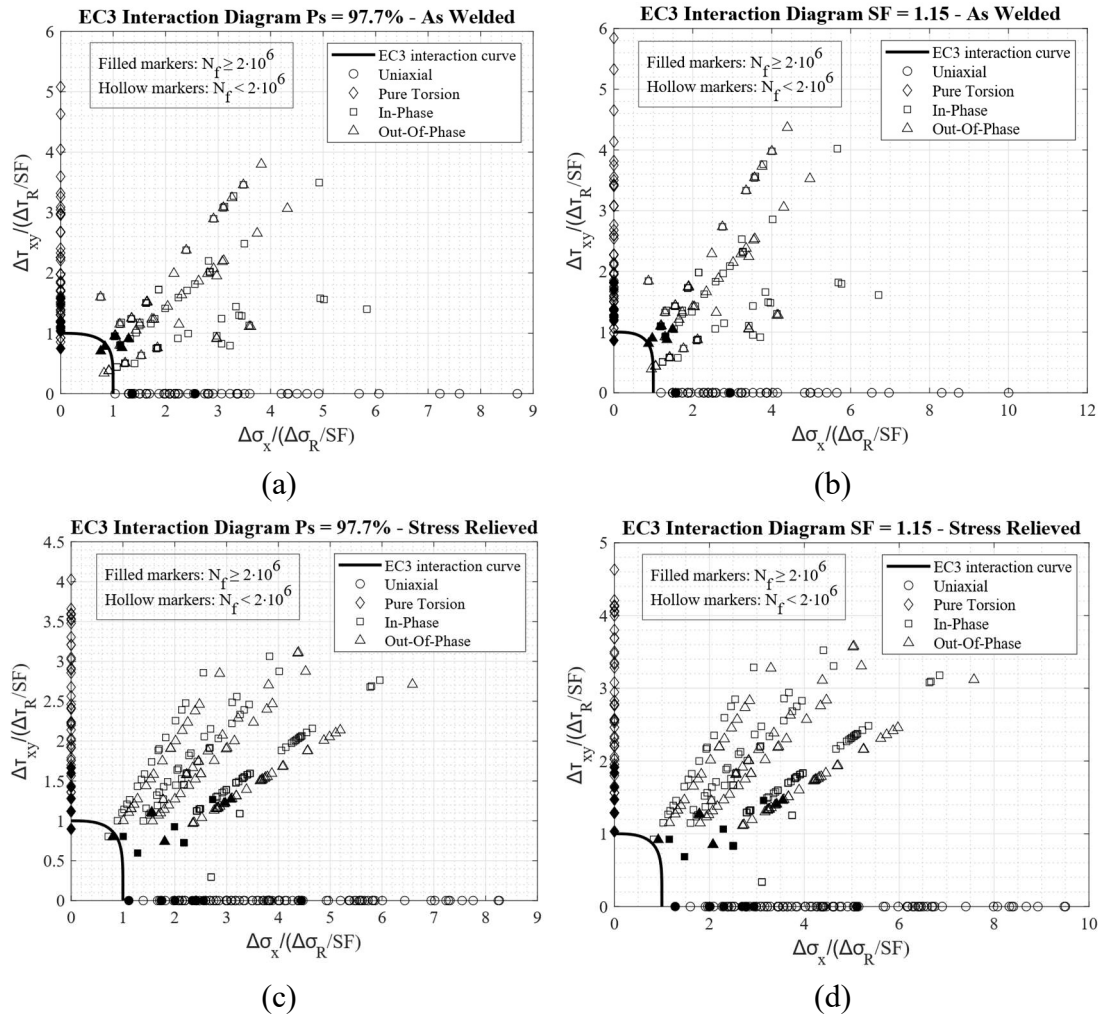
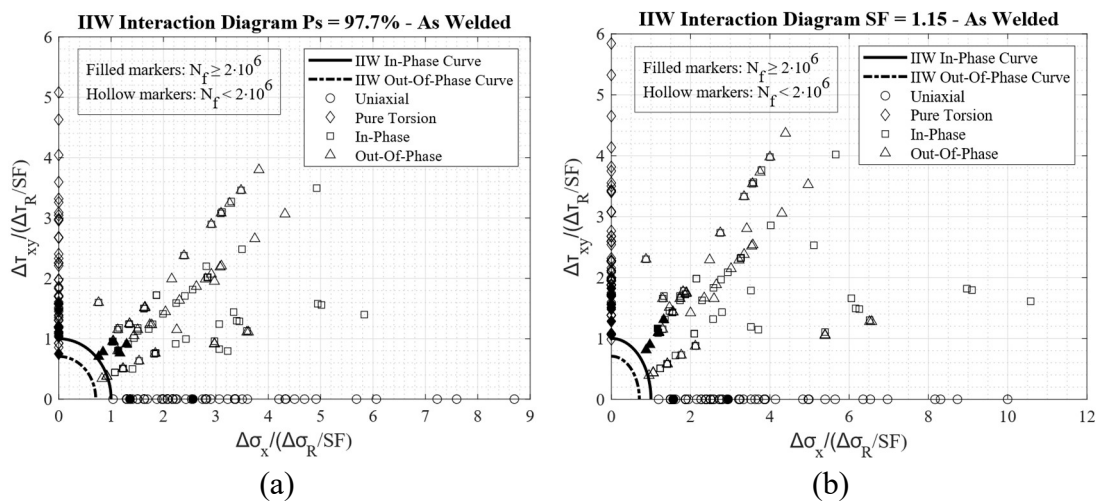


Figure 3.5: NS approach: EC3 interaction diagram for as-welded (a and b) and stress-relieved (c and d) steel welded joints under CA fatigue loading, with reference stresses ($\Delta\sigma_R$ and $\Delta\tau_R$) recalculated at $P_s=97.7\%$ and $SF=1.15$.

The recalculated reference stresses employ a P_s of 97.7% and a SF of 1.15. In the graph, black-filled markers represent experimental fatigue life exceeding 2 million cycles. Points that fall within the interaction curve or boundary indicate accurate predictions, as the criteria successfully estimate that the joints can endure more than 2 million cycles to failure. Conversely, points located outside the curve or boundary are classified as conservative estimates, meaning the criteria predicted failure before 2 million cycles, even though the experimental results showed that the specimens survived beyond this threshold.

The reanalysis reveals six instances of non-conservative estimates for as-welded steel joints, resulting in a P_{NC} of 2.54%. When an additional SF of 1.15 is applied, the number of non-conservative estimates reduces from 6 to 3 instances, yielding a P_{NC} of 0.85%. On the contrary, for stress-relieved steel joints, only one non-conservative estimate is identified when the reference stresses are recalculated at P_s of 97.7%, resulting in an overall P_{NC} of 0.28%. The incorporation of the additional SF of 1.15 further reduces the P_{NC} to 0% as shown in Figures 3.5c and 3.5d.

In the reanalysis based on the IIW criteria, Figure 3.6 shows the results for steel joints, while Figure 3.7 presents the outcomes for aluminium joints. Reference stresses for both joint types are recalculated using $P_s=97.7\%$, $SF=1.15$, and corresponding experimental data.



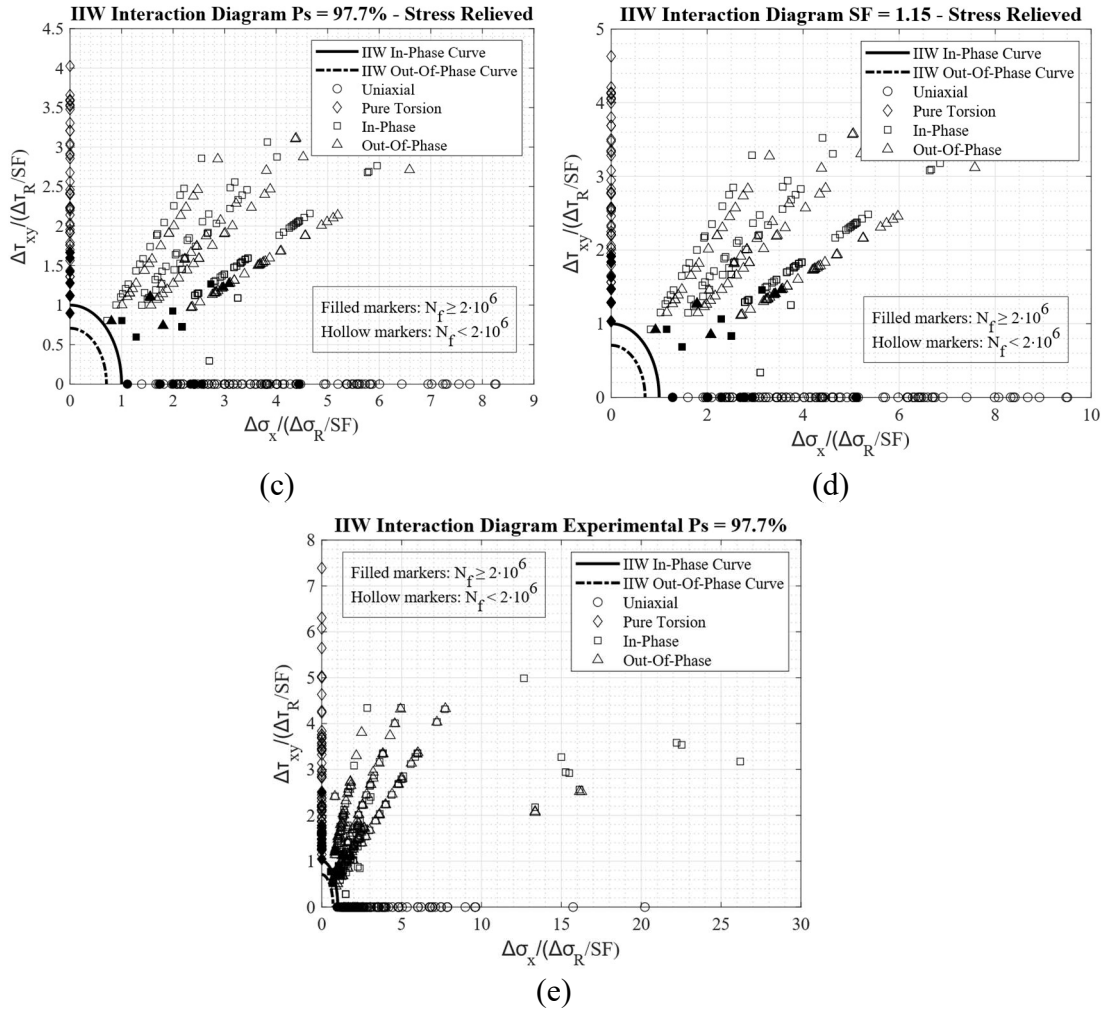


Figure 3.6: NS approach: IIW interaction diagram for as-welded (a and b) and stress-relieved (c and d) steel welded joints under CA fatigue loading, with reference stresses ($\Delta\sigma_R$ and $\Delta\tau_R$) recalculated based on $P_s=97.7\%$, $SF=1.15$, and experimental data.

A significant observation emerges: the IIW criteria exhibit fewer non-conservative estimates compared to the EC3 criteria, particularly for as-welded steel joints. This suggests that the IIW criteria provide a more conservative yet safer prediction of fatigue life under these conditions.

However, for aluminium welded joints, the reanalysis reveals a high degree of conservatism. Data points for both as-welded and stress-relieved joints are positioned significantly distant from the curve boundary that defines the design safety limit.

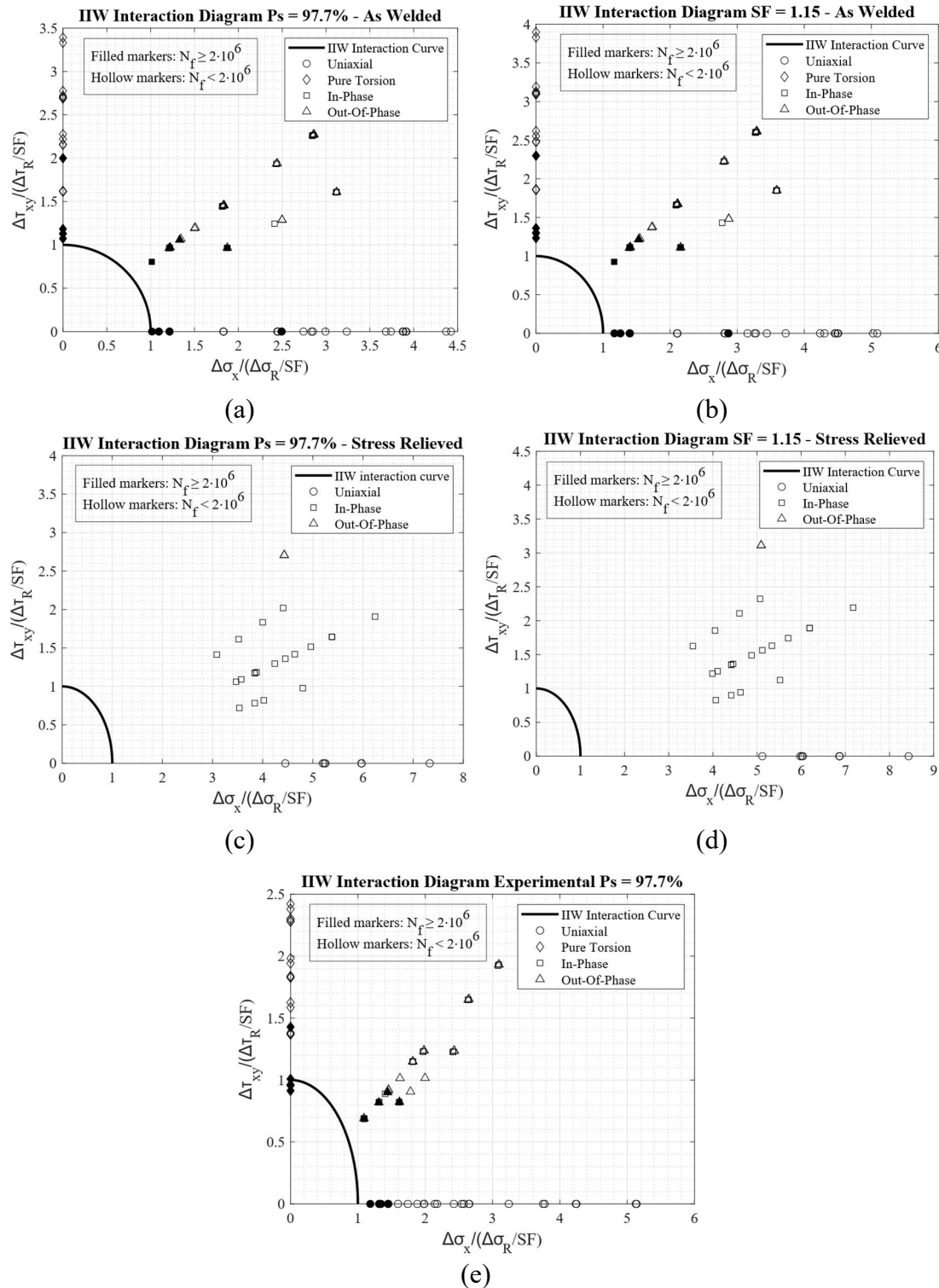


Figure 3.7: NS approach: IIW interaction diagram for as-welded (a and b) and stress-relieved (c and d) aluminium welded joints under CA fatigue loading, with reference stresses ($\Delta\sigma_R$ and $\Delta\tau_R$) recalculated based on $P_s=97.7\%$, $SF=1.15$, and experimental data.

Although this indicates overly conservative estimations, the level of conservatism is moderately reduced when reference stresses derived from experimental data are used instead of those reference S-N curves recommended by the IIW. In these cases, the data points move closer to the boundary curve, improving the balance between safety and accuracy.

The fatigue life estimations of aluminium welded joints reanalysed using the EC9 criterion are shown in Figure 3.8.

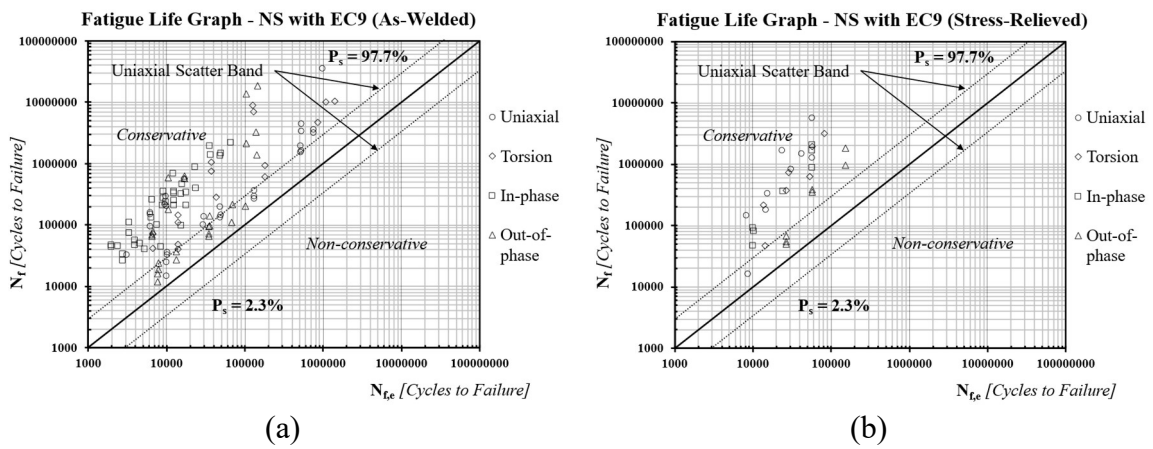


Figure 3.8: NS approach: EC9 fatigue life graph for as-welded (a) and stress-relieved (b) aluminium welded joints under CA fatigue loading, with reference stress ($\Delta\sigma_R$) recalculated based on $P_s=97.7$.

The data points show strong alignment with the uniaxial and torsional scatter bands at P_s of 97.7%. All data points fall above the $N_f=N_{f,e}$ diagonal line, indicating that the maximum principal stress criterion as applied in EC9, yields conservative predictions for aluminium welded joints. This conservatism ensures a high level of safety in fatigue life estimations under CA loading conditions.

These comparative reanalyses provide a comprehensive assessment of the predictive performance of the multiaxial fatigue criteria under CA loading, highlighting the differences in accuracy and conservatism between the EC3, IIW, and EC9 standards.

Table 3.2: Summary of reanalysed welded joints using NS approach under CA fatigue loading, including uniaxial and torsional reference fatigue strengths, geometries, materials, fatigue curve slopes, and data sources.

Material (CA)	$\Delta\sigma_{R,P_s=97.7\%}^{(1)}$ (MPa)			Uniaxial curve slope, k			$\Delta\tau_{R,P_s=97.7\%}^{(1)}$ (MPa)			Torsional curve slope, k0		Geometry	Reference
	EC3/ EC9	IIW	Exp ⁽²⁾	EC3/ EC9	IIW	Exp ⁽²⁾	EC3	IIW	Exp ⁽²⁾	EC3/ IIW	Exp ⁽²⁾		
UM StE 460 *	71	71	142.7	3	3	4.4	100	100	127.5	5	4.9	Figure 3.2a	(Sonsino, 1995)
M StE 460 *	71	71	166.5	3	3	4.6	100	100	-	5	-	Figure 3.2a	(Sonsino, 1997)
UM StE 460 *	71	71	194.7	3	3	4.4	100	100	-	5	-	Figure 3.2e	(Sonsino, 1997)
M StE 460 *	71	71	321.6	3	3	8.2	100	100	128.0	5	6.3	Figure 3.2e	(Sonsino, 1997)
StE 460 *	71	71	116.4	3	3	3.9	100	100	180.9	5	7.3	Figure 3.2a	(Yousefi, Witt and Zenner, 2001)
StE 460	71	71	122.9	3	3	5.4	100	100	80.5	5	6.1	Figure 3.2a	(Amstutz, Störzel and Seeger, 2001)
A519	71	71	96.3	3	3	5.4	80	80	94.2	5	3.7	Figure 3.2b	(Yung and Lawrence Jr, 1989)
A519-A36 *	80	80	144.4	3	3	3.8	100	100	104.2	5	5.5	Figure 3.2d	(Siljander, Kurath and Lawrence Jr, 1989)
BS4360 Gr.50E	71	71	65.6	3	3	3.0	80	80	66.3	5	4.5	Figure 3.2c	(Razmjoo, 1997)
Fe 52 steel	45	45	15.8	3	3	2.3	100	100	44.1	5	3.5	Figure 3.2f	(Bäckström, 2003)
BS4360	80	80	-	3	3	-	80	80	-	5	-	Figure 3.2d	(Archer, 1987)
S340+N, E355+N	71	71	204.2	3	3	6.1	100	100	207.8	5	6.6	Figure 3.2e	(Shams and Vormwald, 2017)
S340+N, E355+N *	71	71	44.6	3	3	2.9	100	100	71.3	5	3.6	Figure 3.2e	(Shams and Vormwald, 2017)
St 35 (t = 1 mm)	71	71	69.2	3	3	4.9	100	100	106.4	5	8.4	Figure 3.2e	(Wiebesiek and Sonsino, 2010)
St 35 (t = 2 mm)	71	71	66.4	3	3	5.3	100	100	74.5	5	6.1	Figure 3.2e	(Sonsino <i>et al.</i> , 2006)
6082-T6 *	28	32	55.0	3.4	3	6.9	-	36	50.4	-	6.2	Figure 3.2a	(Kueppers and Sonsino, 2003)
6060-T6	23	22	84.0	3.4	3	5.5	-	36	-	-	-	Figure 3.2e	(Costa <i>et al.</i> , 2005)
AW 6082	18	22	20.3	3.4	3	4.2	-	36	42.3	-	5.8	Figure 3.2e	(Wiebesiek <i>et al.</i> , 2011)
AW 5042	18	22	16.6	3.4	3	4.1	-	36	42.5	-	5.9	Figure 3.2e	(Wiebesiek <i>et al.</i> , 2011)

⁽¹⁾ Reference normal and shear stresses extrapolated at 2 million cycles to failure, with probability of survival (P_s) = 97.7%.

⁽²⁾ Exp denotes experimental data.

* Stress-relieved

UM – Unmachined; M – Machined.

3.3.2 VA Loading

In addition to the results for CA loading, Table 3.3 summarises the key data for steel and aluminium welded joints reanalysed under VA loading conditions. The table includes crucial details such as joint geometries, material properties, and uniaxial and torsional fatigue curves and slopes derived from EC3, EC9, and IIW reference standards, alongside relevant literature sources. This dataset serves as the basis for evaluating the performance of the standard multiaxial fatigue criteria under VA loading. As with the previous subsection, the table is placed at the end for reference.

The reanalysis results for stress-relieved steel joints under VA loading, using the EC3 multiaxial fatigue criteria and the NS approach, are illustrated in Figure 3.9.

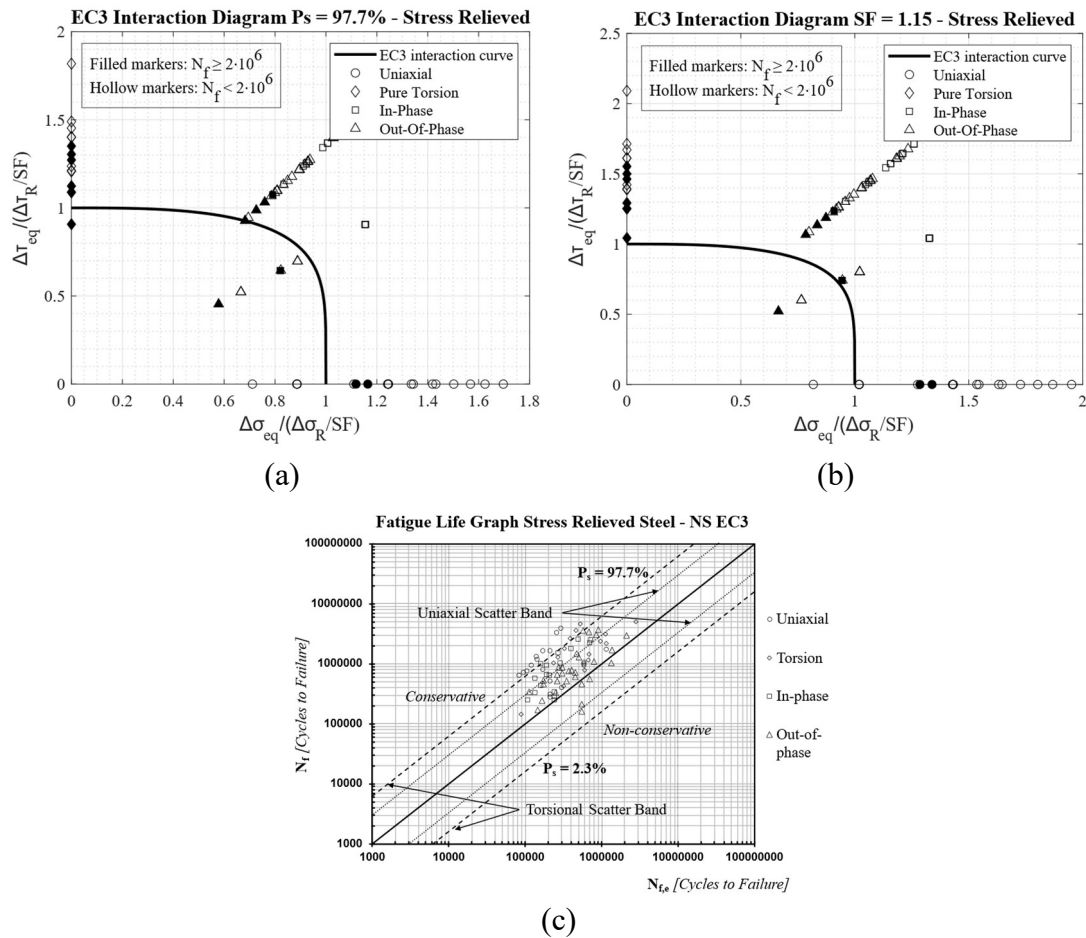
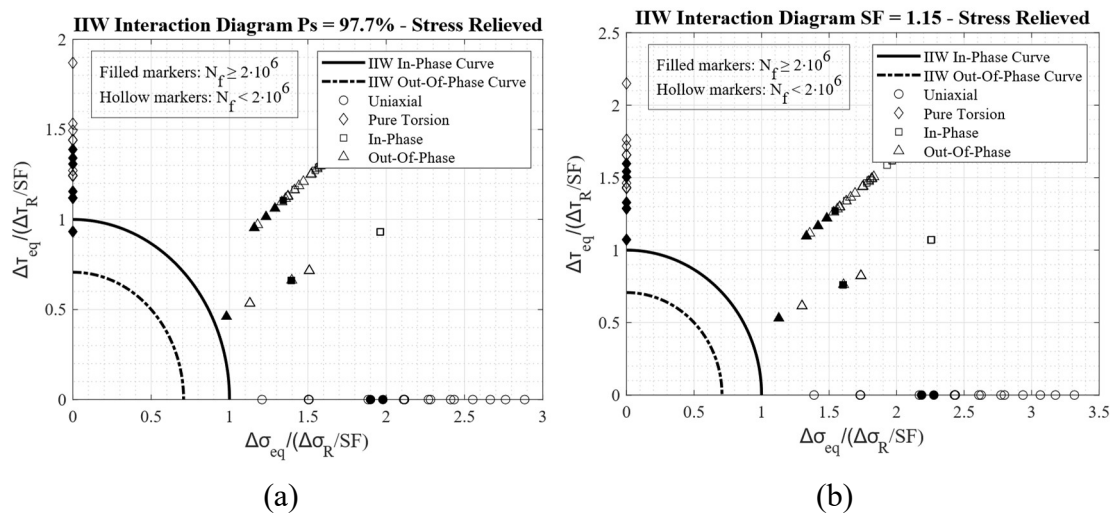
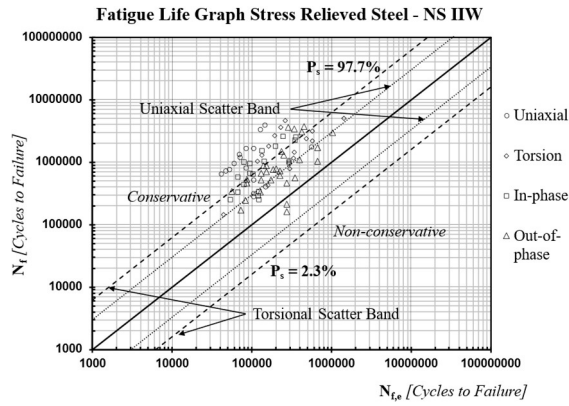


Figure 3.9: NS approach: EC3 interaction diagram (a and b) and fatigue life graph (c) for stress-relieved steel welded joints under VA loading, with reference stresses ($\Delta \sigma_R$ and $\Delta \tau_R$) recalculated based on $P_s=97.7\%$ and $SF=1.15$.

Reference stresses were recalculated with a P_s of 97.7% and an SF of 1.15, consistent with the CA analysis. When applying the EC3 interaction equation with equivalent stress range substitution as shown in Figure 3.9a, nine non-conservative estimates were identified, leading to a P_{NC} of 11.1%. However, applying an additional SF of 1.15 reduces the number of non-conservative estimates to three, lowering the P_{NC} to 3.7%, as depicted in Figure 3.9b. On the other hand, the EC3 P-M rule as shown in Figure 3.9c demonstrated that majority of fatigue life estimations fall within the uniaxial and torsional scatter bands and above the $N_f = N_{f,e}$ diagonal line. Only five fatigue life estimations fall below the diagonal line at a P_s of 50%, yielding a P_{NC} of 6.2%. Notably, these non-conservative estimates were all associated with out-of-phase loading condition. These findings indicate that, while the EC3 P-M rule generally provides conservative and accurate predictions, it exhibits slight underestimation for out-of-phase fatigue life estimations.

The reanalysis results for steel joints using the IIW interaction equation criterion with equivalent stress range substitution are presented in Figures 3.10a and 3.10b. Remarkably, no non-conservative estimates were observed, suggesting that the IIW criterion offers safe and reliable fatigue life predictions under VA loading. As shown in Figure 3.10c, the fatigue life estimations clustered tightly within the uniaxial and torsional scatter bands, with most points situated above the $N_f = N_{f,e}$ diagonal line.



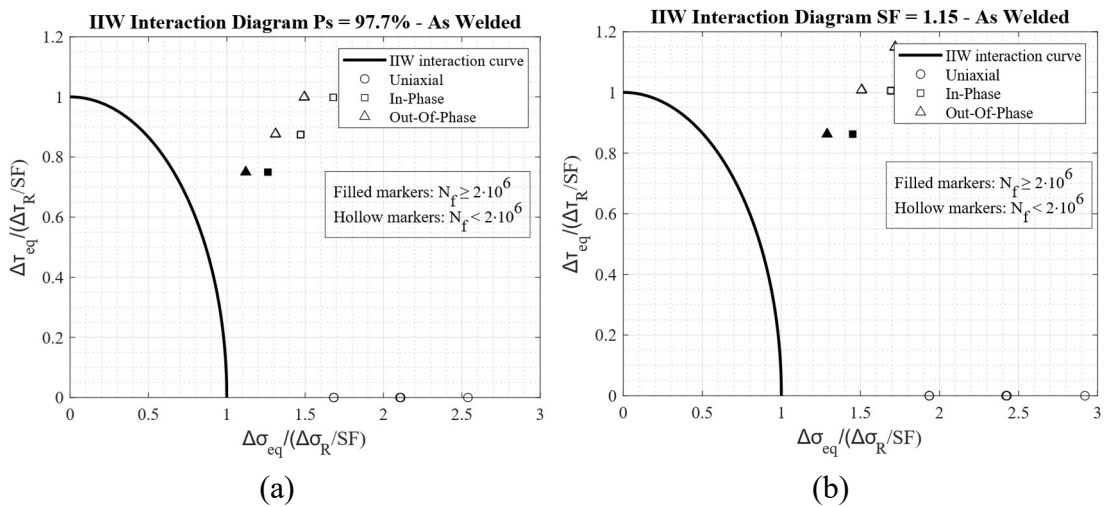


(c)

Figure 3.10: NS approach: IIW interaction diagram (a and b) and fatigue life graph (c) for stress-relieved steel welded joints under VA loading, with reference stresses ($\Delta\sigma_R$ and $\Delta\tau_R$) recalculated based on $P_s=97.7\%$ and $SF=1.15$.

Only two out-of-phase fatigue life estimations fell below the diagonal line, resulting in a P_{NC} of 2.5%, a slight improvement compared to the EC3 P-M rule. This reinforces the conservative reliability of the IIW P-M rule for steel joints under VA loading.

For aluminium welded joints, the reanalysis results using the IIW interaction equation criterion are displayed in Figures 3.11a and 3.11b. Similar to the steel case, no non-conservative estimates are identified. However, the data points for both as-welded and stress-relieved joints lie significantly distant from the curve boundary that defines the design safety limit, indicating high level of conservatism.



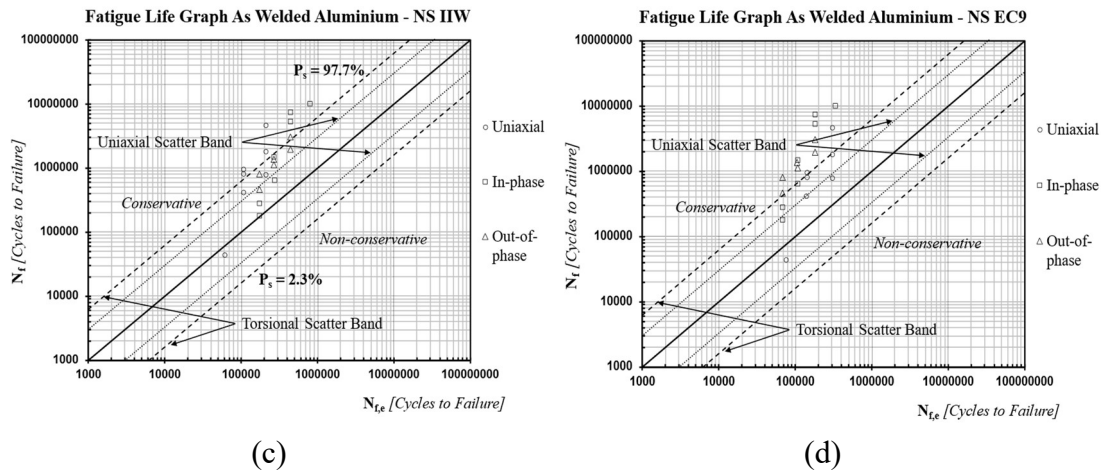


Figure 3.11: NS approach: IIW interaction diagram (a and b), IIW fatigue life graph (c), and EC9 fatigue life graph (d) for as-welded aluminium welded joints under VA fatigue loading, with reference stresses ($\Delta\sigma_R$ and $\Delta\tau_R$) recalculated based on $P_s=97.7\%$ and $SF = 1.15$.

Further assessments using the NS approach in conjunction with both the IIW and EC9 P-M rules as shown in Figures 3.11c and 3.11d reveal that fatigue life estimations for aluminium joints consistently fall above the $N_f=N_{f,e}$ diagonal line, with only one uniaxial fatigue life estimation falling below the diagonal line in both cases. This underscores the conservative nature of these estimations, particularly when compared to steel joint reanalysis results.

The VA loading reanalysis highlights critical differences in the predictive performance of the EC3, IIW, and EC9 criteria. These results underscore varying levels of accuracy and conservatism in fatigue life predictions for both steel and aluminium welded joints. By comparing these outcomes, this study enhances understanding of the strengths and limitations of standard multiaxial fatigue criteria under VA loading scenarios.

Table 3.3: Summary of reanalysed welded joints using NS approach interaction equation criterion under VA fatigue loading, including uniaxial and torsional reference fatigue strengths, geometries, materials, fatigue curve slopes, and data sources.

Material (VA)	$\Delta\sigma_{R,P_s=97.7\%}^{(1)}$ (MPa)		Uniaxial curve slope, k		$\Delta\tau_{R,P_s=97.7\%}^{(1)}$ (MPa)		Torsional curve slope, k ₀	Geometry	Reference
	EC3/EC9	IIW	EC3/EC9	IIW	EC3	IIW	EC3/IIW		
StE 460 *	71	71	3	3	100	100	5	Figure 3.2a	(Sonsino and Kueppers, 2001)
StE 460 *	71	71	3	3	100	100	5	Figure 3.2a	(Yousefi, Witt and Zenner, 2001)
42CrMo4 *	-	-	3	3	100	100	5	Figure 3.2e	(Pyttel, Grawenhof and Berger, 2012)
6082-T6	28	32	3.4	3	-	36	5	Figure 3.2e	(Kueppers and Sonsino, 2006)

⁽¹⁾Reference normal and shear stresses extrapolated at 2 million cycles to failure, with probability of survival (P_s)=97.7%.

⁽²⁾Exp denotes experimental data.

* Stress-relieved

3.4 Reanalyses Using the HSS Approach

The HSS approach, also referred to as the structural stress approach, is widely adopted for fatigue assessment of welded joints with complex geometries, particularly in cases where NS derivation is impractical. Unlike the NS approach, which considers stress at the nominal cross-section, HSS incorporates stress concentrations resulting from structural effects while excluding localised notch stresses at the weld toe (Niemi, 1995; Radaj, 1996; Anon, 2005, 2007; Hobbacher, 2016; Mei *et al.*, 2021). This distinction makes it a more representative method for evaluating critical stress distributions in intricate welded joint configurations.

To ensure methodological consistency with previous studies, this reanalysis exclusively employs the finite element analysis (FEA)-based derivation of hot spot stress, as this is the dominant approach in the reviewed literature (Niemi, 1995; Radaj, 1996; Anon, 2005, 2007; Radaj, Sonsino and Fricke, 2006; Hobbacher, 2016; Mei *et al.*, 2021). The quantitative evaluation applies the EC3, EC9, and IIW multiaxial fatigue criteria within the HSS framework, assessing their predictive accuracy for steel and aluminium welded joints under both CA and VA loading conditions.

3.4.1 CA Loading

Table 3.4, provided at the end of this subsection, summarises the key characteristics of the welded joints reanalysed under CA loading using the HSS approach, complementing the dataset previously presented for the NS-based reanalysis.

The EC3 multiaxial fatigue criterion was reanalysed using the HSS approach for both as-welded and stress-relieved steel welded joints, with the results illustrated in Figure 3.12. Reference stresses were recalculated with $P_s=97.7\%$ and an $SF=1.15$, ensuring consistency with the NS-based reanalysis. Importantly, no instances of non-conservative estimates were identified, reinforcing the robustness of the EC3 interaction equation criterion when applied using the HSS approach.

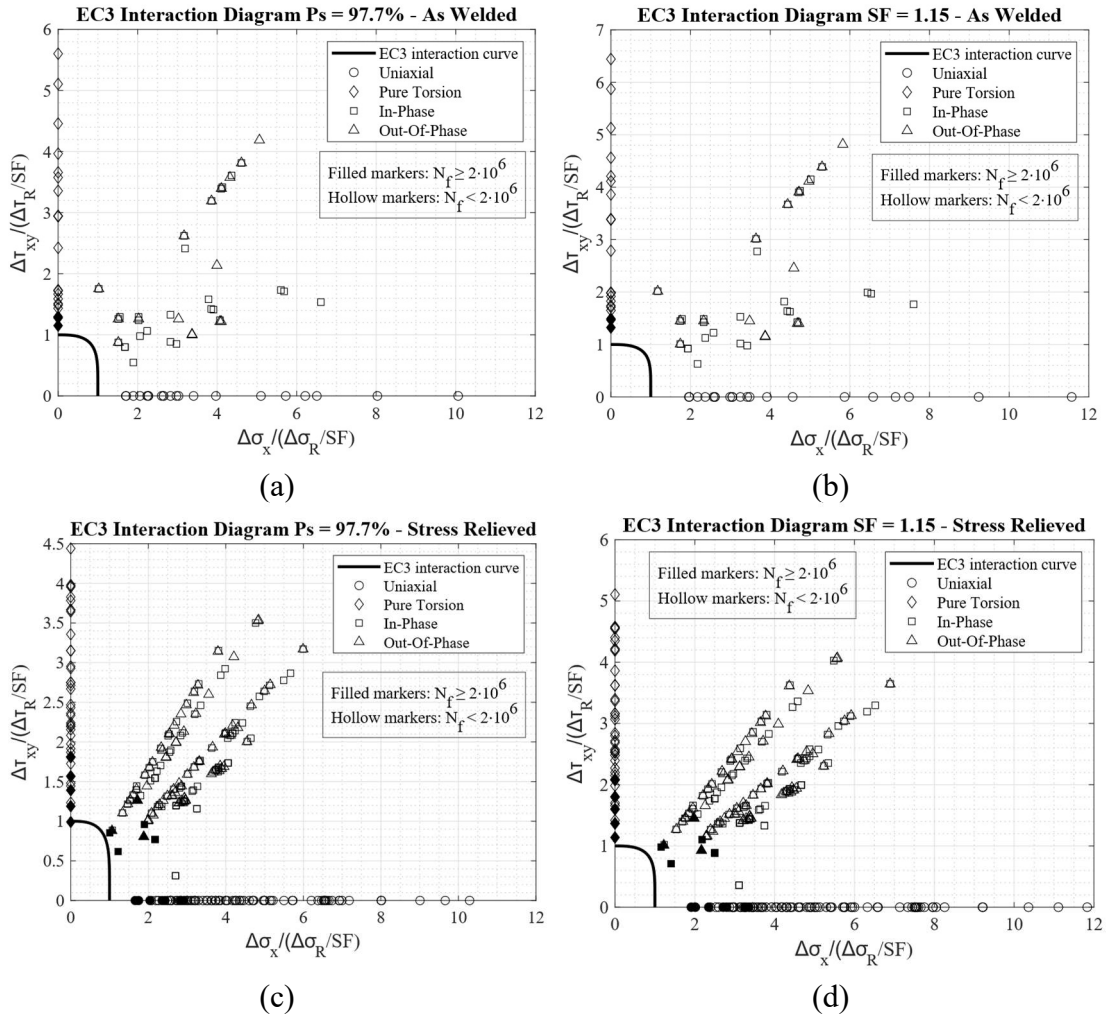


Figure 3.12: HSS approach: EC3 interaction diagram for as-welded (a and b) and stress-relieved (c and d) steel welded joints under CA fatigue loading, with reference stresses ($\Delta\sigma_R$ and $\Delta\tau_R$) recalculated based on $P_s=97.7\%$ and $SF=1.15$.

A similar reanalysis was conducted for the IIW multiaxial fatigue criterion, with results for steel and aluminium welded joints under CA loading presented in Figures 3.13 and 3.14, respectively. Once again, no non-conservative estimates were observed for both as-welded and stress-relieved steel joints, further validating the reliability of the IIW interaction equation criterion in conjunction with the HSS approach.

However, consistent with findings from the NS-based reanalysis, the HSS approach also exhibited a high degree of conservatism for aluminium welded joints, with fatigue life estimations positioned well above the design curve boundary, suggesting overly conservative estimations.

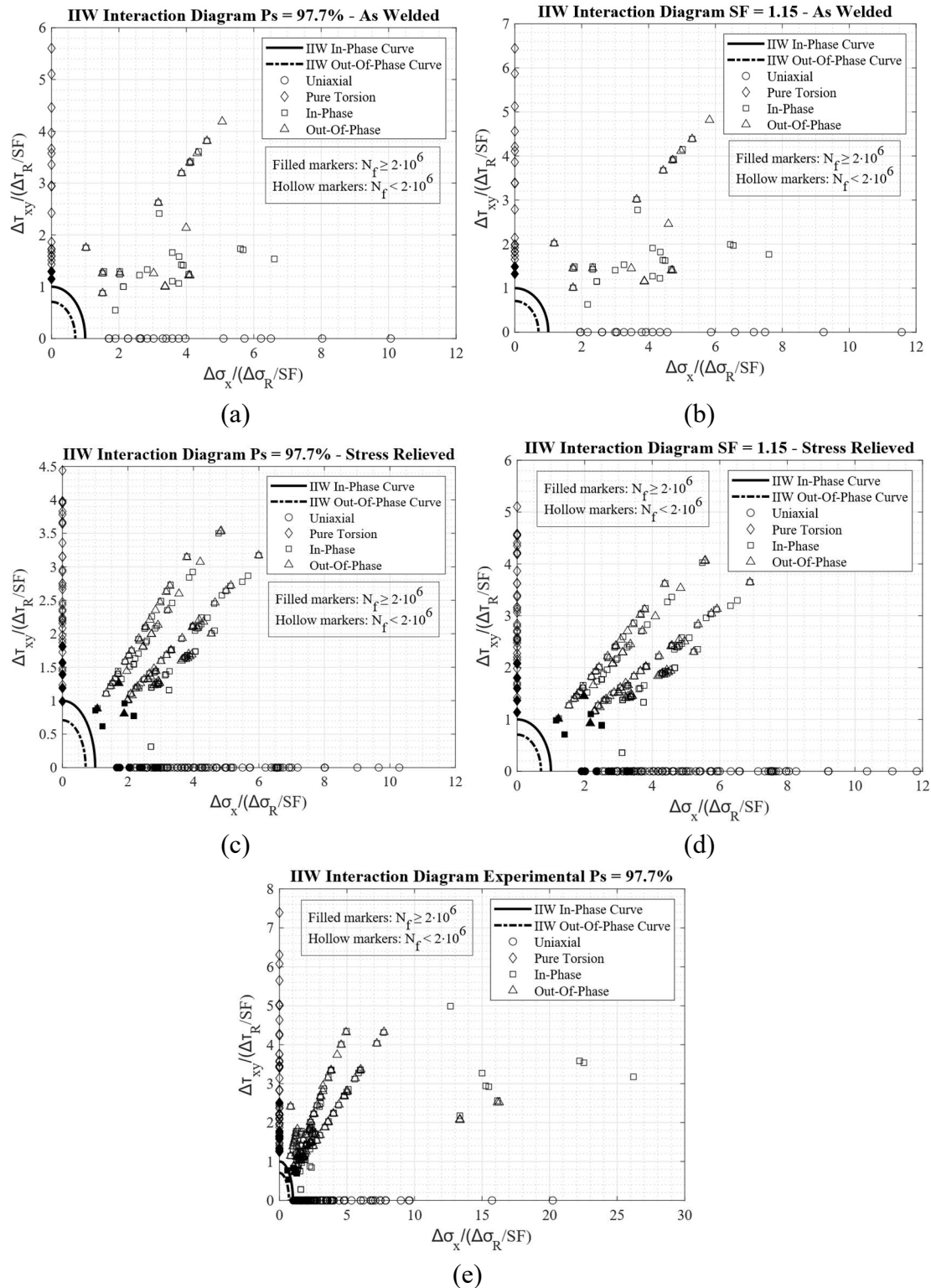


Figure 3.13: HSS approach: IIW interaction diagram for as-welded (a and b) and stress-relieved (c and d) steel welded joints under CA fatigue loading, with reference stresses ($\Delta\sigma_R$ and $\Delta\tau_R$) recalculated based on $P_s=97.7\%$, $SF=1.15$, and experimental data.

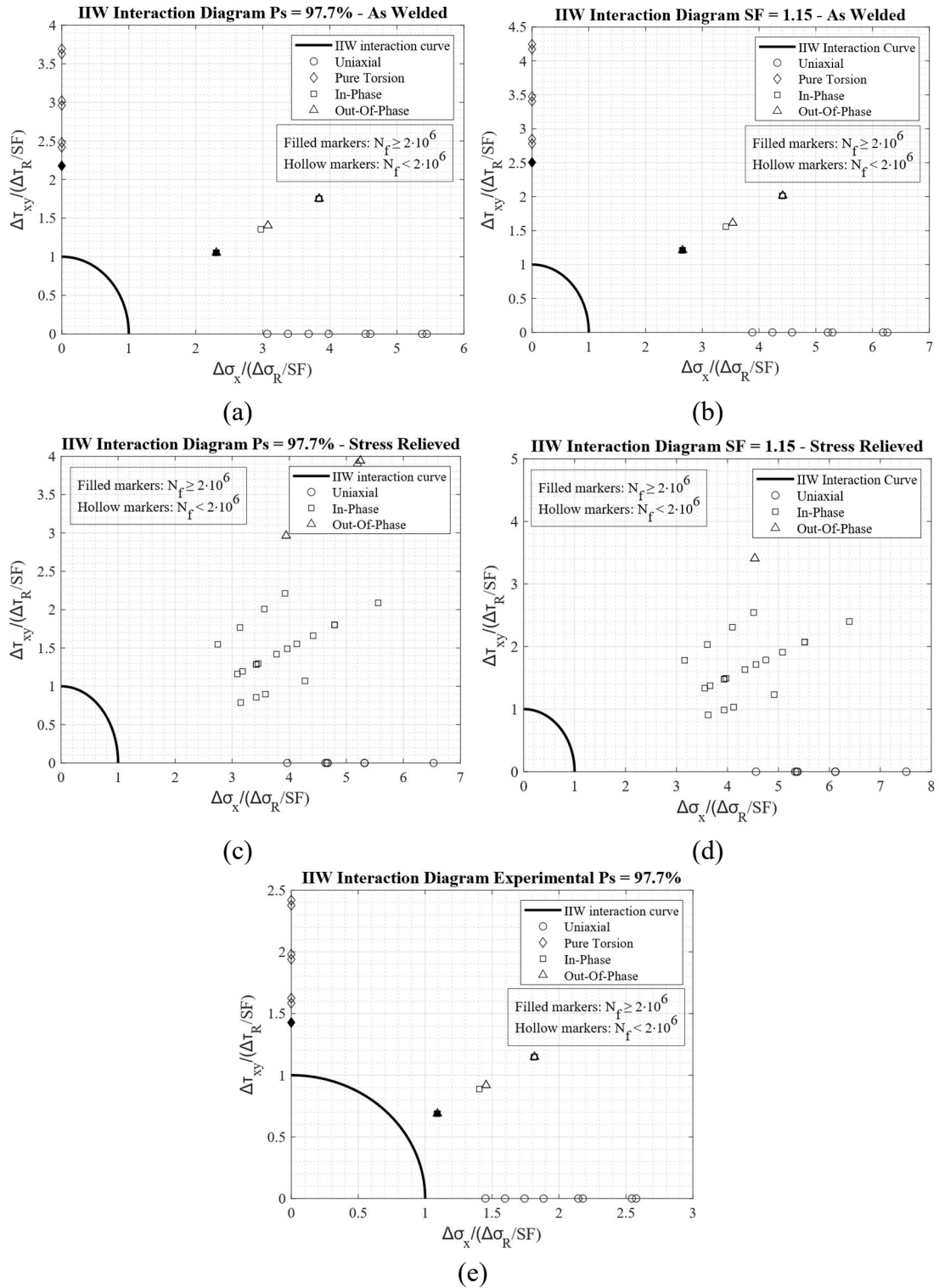


Figure 3.14: HSS approach: IIW interaction diagram for as-welded (a and b) and stress-relieved (c and d) aluminium welded joints under CA loading, with reference stresses ($\Delta\sigma_R$ and $\Delta\tau_R$) recalculated based on $P_s=97.7\%$, $SF=1.15$, and experimental data.

The fatigue life estimations for aluminium welded joints reanalysed using the EC9 criterion are presented in Figure 3.15.

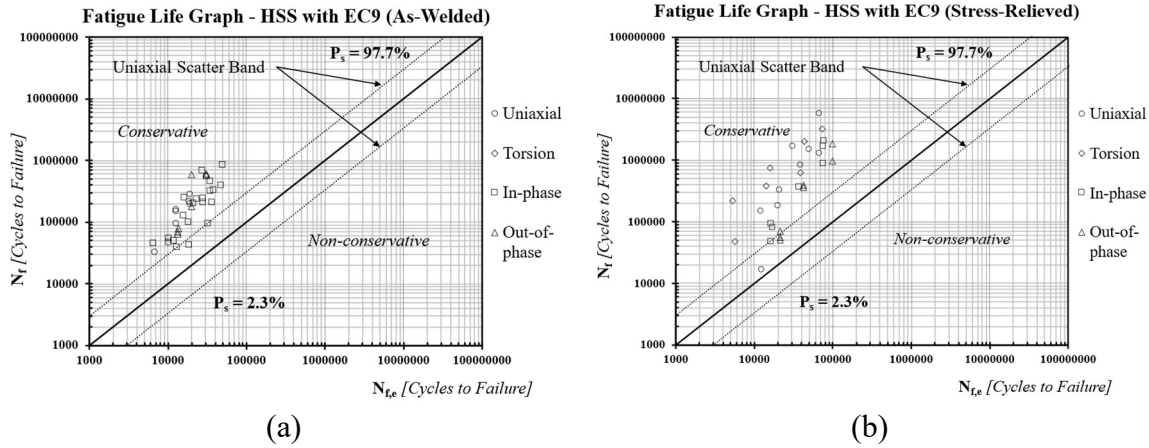


Figure 3.15: HSS approach: EC9 fatigue life graph for as-welded (a) and stress-relieved (b) aluminium welded joints under CA fatigue loading, with reference stress ($\Delta\sigma_R$) recalculated based on $P_s=97.7$.

Similar to observations from the NS-based reanalysis, all data points fall above the $N_f=N_{f,e}$ diagonal line, confirming that the maximum principal stress criterion in EC9 yields consistently conservative fatigue life estimations for aluminium welded joints. Interestingly, the fatigue life estimations shift slightly closer to the diagonal line compared to the NS-based EC9 reanalysis, suggesting a marginal reduction in conservatism. Despite this, the overall results confirm that conservatism remains substantial, ensuring that fatigue life estimations for aluminium welded joints using the EC9 criterion in conjunction with the HSS approach remain well within safe design limits under CA loading conditions.

Table 3.4: Summary of reanalysed welded joints using HSS approach under CA fatigue loading, including uniaxial and torsional reference fatigue strengths, geometries, materials, fatigue curve slopes, and data sources.

Material (CA)	$\Delta\sigma_{R,P_s=97.7\%}^{(1)}$ (MPa)			Uniaxial curve slope, k			$\Delta\tau_{R,P_s=97.7\%}^{(1)}$ (MPa)			Torsional curve slope, k ₀		Geometry	Reference
	EC3/ EC9	IIW	Exp ⁽²⁾	EC3/ EC9	IIW	Exp ⁽²⁾	EC3	IIW	Exp ⁽²⁾	EC3/ IIW	Exp ⁽²⁾		
UM StE 460 *	90	90	188.7	3	3	4.4	100	100	138.7	5	4.9	Figure 3.2a	(Sonsino, 1995)
M StE 460 *	90	90	210.0	3	3	4.6	100	100	-	5	-	Figure 3.2a	(Sonsino, 1997)
UM StE 460 *	90	90	208.4	3	3	4.4	100	100	-	5	-	Figure 3.2e	(Sonsino, 1997)
StE 460 *	90	90	163.1	3	3	3.9	100	100	206.7	5	7.3	Figure 3.2a	(Yousefi, Witt and Zenner, 2001)
A519	90	90	112.7	3	3	5.4	80	80	100.7	5	3.7	Figure 3.2b	(Yung and Lawrence Jr, 1989)
A519-A36 *	90	90	153.6	3	3	3.8	100	100	110.7	5	5.5	Figure 3.2d	(Siljander, Kurath and Lawrence Jr, 1989)
BS4360 Gr.50E	90	90	111.7	3	3	3.0	80	80	72.7	5	4.5	Figure 3.2c	(Razmjoo, 1997)
Fe 52 steel	90	90	22.7	3	3	2.3	100	100	48.4	5	3.5	Figure 3.2f	(Bäckström, 2003)
S340+N, E355+N	90	90	343.0	3	3	6.1	100	100	229.2	5	6.6	Figure 3.2e	(Shams and Vormwald, 2017)
S340+N, E355+N *	90	90	74.8	3	3	2.9	100	100	74.8	5	3.6	Figure 3.2e	(Shams and Vormwald, 2017)
6082-T6 *	36	36	76.1	3.4	3	6.9	-	36	55.0	-	6.3	Figure 3.2a	(Kueppers and Sonsino, 2003)
6060-T6	36	36	122.5	3.4	3	-	-	36	-	-	-	Figure 3.2e	(Costa <i>et al.</i> , 2005)

⁽¹⁾ Reference normal and shear stresses extrapolated at 2 million cycles to failure, with probability of survival (P_s)=97.7%.

⁽²⁾ Exp denotes experimental data.

* Stress-relieved

UM – Unmachined; M – Machined.

3.4.2 VA Loading

Table 3.5, positioned at the end of this subsection, provides a summary of the welded joints reanalysed under VA loading using the HSS approach, complementing the dataset previously presented for CA loading.

The HSS-based reanalysis of stress-relieved steel welded joints under VA loading using the EC3 multiaxial fatigue criteria is illustrated in Figure 3.16.

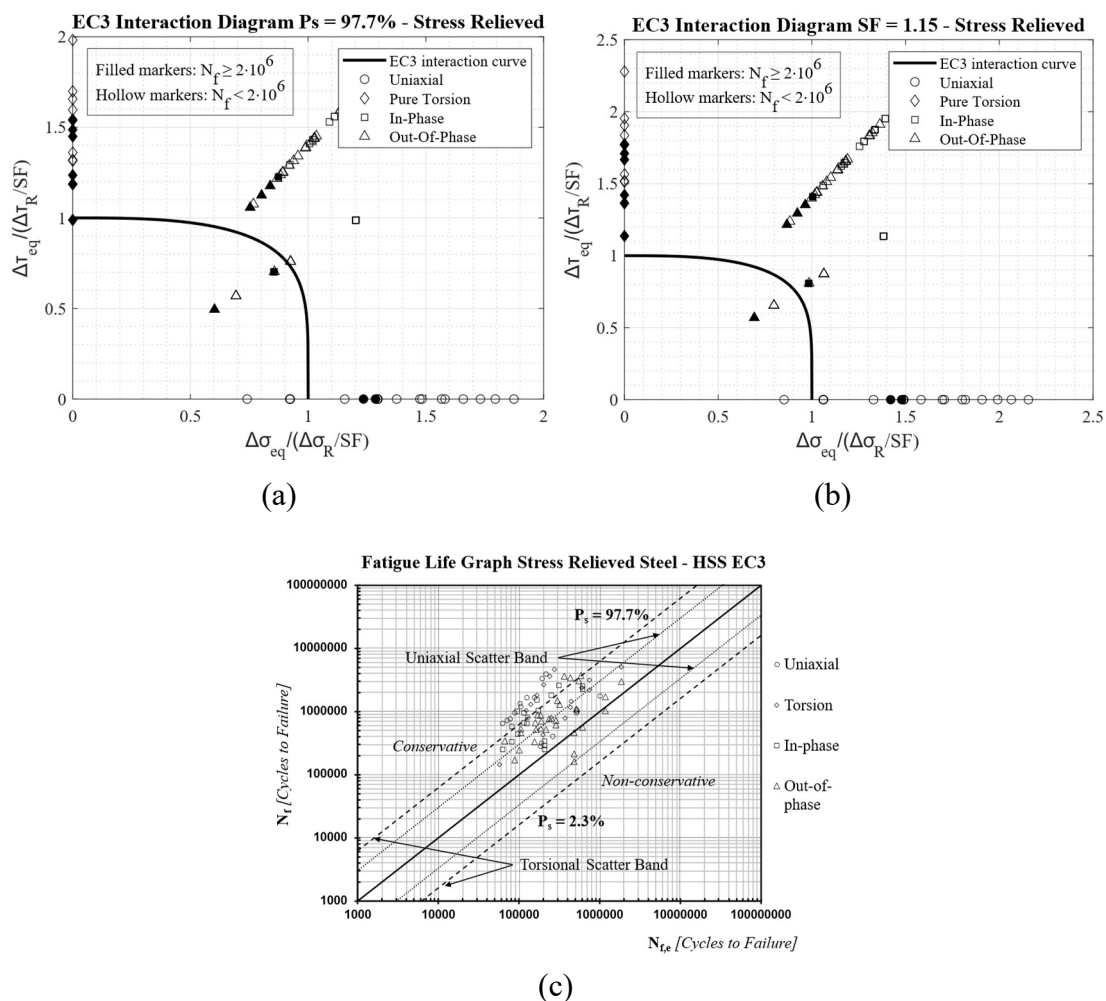


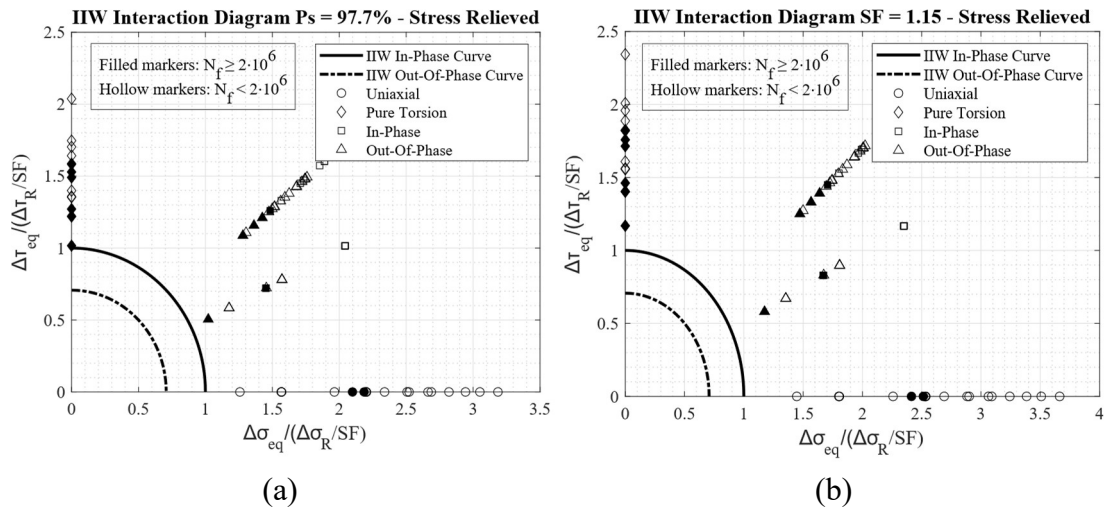
Figure 3.16: HSS approach: EC3 interaction diagram (a and b) and fatigue life graph (c) for stress-relieved steel welded joints under VA fatigue loading, with reference stresses ($\Delta\sigma_R$ and $\Delta\tau_R$) recalculated based on $P_s=97.7\%$ and $SF=1.15$.

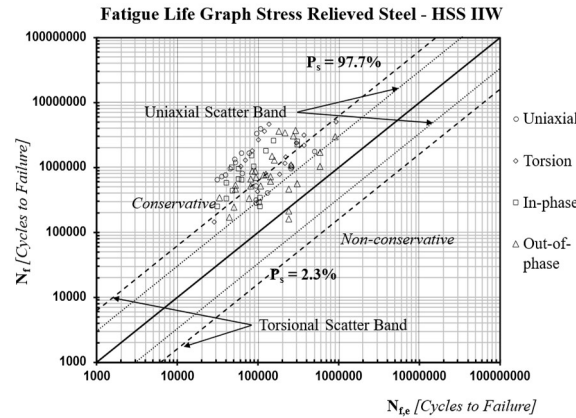
The assessment revealed six non-conservative estimates, resulting in a P_{NC} of 7.4% when applying the EC3 interaction equation criterion with equivalent stress range substitution. However, introducing an additional SF of 1.15 significantly reduced the

number of non-conservative estimates to three, thereby lowering the P_{NC} to 3.7%, demonstrating the effectiveness of the safety factor in enhancing the reliability of fatigue predictions.

Further analysis using the EC3 P-M rule in conjunction with the HSS approach, as shown in Figure 3.16c, revealed that most fatigue life estimations were clustered within the uniaxial and torsional scatter bands and positioned above the $N_f=N_{f,e}$ diagonal line, confirming conservative and generally accurate predictions. However, five fatigue life estimations fell below the diagonal line at $P_s=50\%$, corresponding to a P_{NC} of 6.2%. These findings are consistent with observations from the NS-based reanalysis, suggesting that while the EC3 P-M rule generally yields reliable predictions, its accuracy under out-of-phase loading remains slightly non-conservative.

The IIW multiaxial fatigue criterion was also reanalysed using the HSS approach, with results for steel welded joints under VA loading presented in Figure 3.17. Unlike EC3, the IIW criterion exhibited no non-conservative estimates, affirming its robustness in fatigue life estimations.



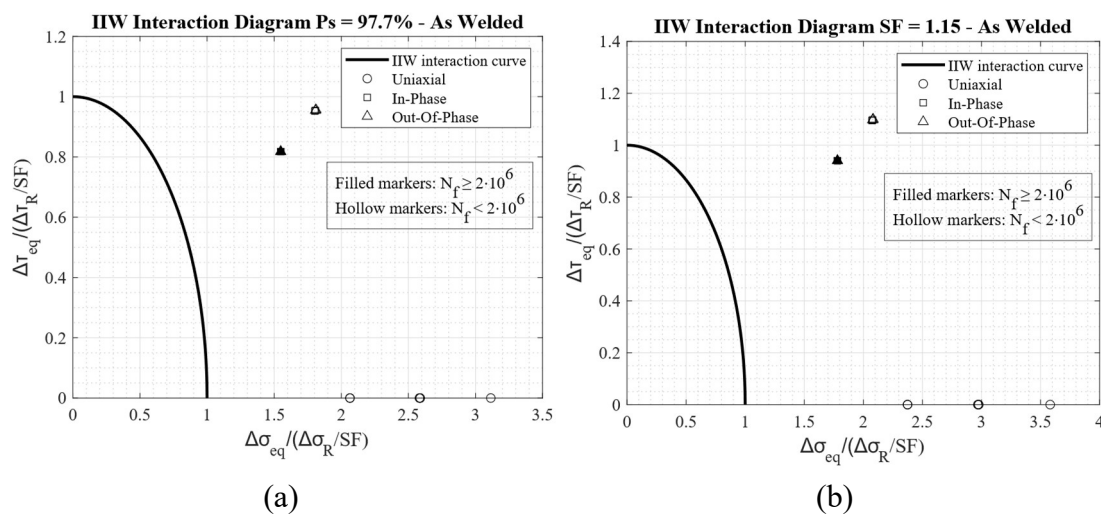


(c)

Figure 3.17: HSS approach: IIW interaction diagram (a and b) and fatigue life graph (c) for stress-relieved steel welded joints under VA fatigue loading, with reference stresses ($\Delta\sigma_R$ and $\Delta\tau_R$) recalculated based on $P_s=97.7\%$ and $SF=1.15$.

Consistent with findings from the NS-based reanalysis, the fatigue life estimations reanalysed using the IIW P-M approach remained well within the uniaxial and torsional scatter bands as shown in Figure 3.17c, with only two out-of-phase fatigue life estimations falling below the diagonal line, yielding a P_{NC} of 2.5%, which is slightly more conservative than the EC3 P-M rule in out-of-phase conditions.

For aluminium welded joints, the IIW interaction equation criterion under VA loading also exhibited zero non-conservative estimates, as depicted in Figure 3.18.



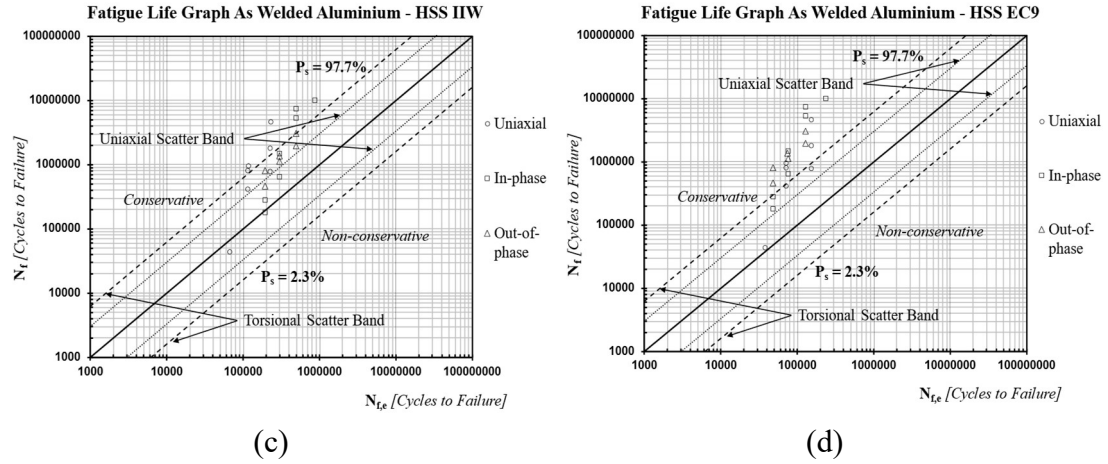


Figure 3.18: HSS approach: IIW interaction diagram (a and b), IIW fatigue life graph (c), and EC9 fatigue life graph (d) for as-welded aluminium welded joints under VA fatigue loading, with reference stresses ($\Delta\sigma_R$ and $\Delta\tau_R$) recalculated based on $P_s=97.7\%$ and $SF=1.15$.

However, similar to findings in the NS-based reanalysis, the fatigue life estimations were significantly distant from the curve boundary, indicating a high degree of conservatism. Additionally, further evaluations integrating the HSS approach with the IIW and EC9 P-M rules, as shown in Figures 3.18c and 3.18d, demonstrate that fatigue life estimations for aluminium welded joints consistently remained above the $N_f=N_{f,e}$ diagonal line. Notably, only one uniaxial fatigue life estimation fell below the diagonal line for the IIW P-M approach, whereas EC9-PM approach showed zero non-conservative estimates. These findings confirm the inherent conservatism of the IIW and EC9 criteria, particularly in aluminium welded joints.

Table 3.5: Summary of reanalysed welded joints using HSS approach interaction equation criterion under VA fatigue loading, including uniaxial and torsional reference fatigue strengths, geometries, materials, fatigue curve slopes, and data sources.

Material (VA)	$\Delta\sigma_{R,P_s=97.7\%}^{(1)}$ (MPa)		Uniaxial curve slope, k		$\Delta\tau_{R,P_s=97.7\%}^{(1)}$ (MPa)		Torsional curve slope, k0	Geometry	Reference
	EC3/EC9	IIW	EC3/EC9	IIW	EC3	IIW	EC3/IIW		
StE 460 *	90	90	3	3	100	100	5	Figure 3.2a	(Sonsino and Kueppers, 2001)
StE 460 *	90	90	3	3	100	100	5	Figure 3.2a	(Yousefi, Witt and Zenner, 2001)
42CrMo4 *	-	-	3	3	100	100	5	Figure 3.2e	(Pyttel, Grawenhof and Berger, 2012)
6082-T6	36	36	3.4	3	-	36	5	Figure 3.2e	(Kueppers and Sonsino, 2006)

⁽¹⁾Reference normal and shear stresses extrapolated at 2 million cycles to failure, with probability of survival (P_s) = 97.7%.

⁽²⁾Exp denotes experimental data.

* Stress-relieved

3.5 Reanalyses Using the ENS Approach

The ENS approach provides a localised fatigue assessment methodology that accounts for non-linear stress concentration effects at both the weld toe and root geometries. Unlike NS and HSS approaches, which primarily consider macro-level stress distributions, ENS incorporates geometric discontinuities that significantly influence fatigue failure mechanisms (Niemi, 1995; Radaj, Sonsino and Fricke, 2006; Sonsino, Bruder and Baumgartner, 2010; Susmel, Sonsino and Tovo, 2011; Schijve, 2012; Sonsino *et al.*, 2012; Fricke, 2013; Hobbacher, 2016; Pedersen, 2016; Al Zamzami and Susmel, 2018; Mei *et al.*, 2021). Although the IIW recommends the ENS method, it has yet to be formally integrated into EC3 and EC9, despite its proven effectiveness in predicting fatigue life in welded structures (Anon, 2005, 2007; Hobbacher, 2016).

As detailed in Section 2.3.3, the ENS approach introduces a fictitious notch radius is introduced at the weld toes and roots. For steel and aluminium welded joints with a plate thickness (t) of 5 mm or greater, the IIW-recommended fictitious notch radius of 1.0 mm, as proposed by Hobbacher, is applied (Niemi, 1995; Radaj, 1996; Morgenstern *et al.*, 2006; Fricke, 2007; Atzori *et al.*, 2009; Schijve, 2012; Radaj, Lazzarin and Berto, 2013; Berto, Lazzarin and Radaj, 2014; Hobbacher, 2016; Pedersen, 2016). However, for thin-walled welded joints with t less than 5 mm, the standard IIW recommendation lacks a validated guideline. To address this gap, Sonsino's alternative proposal which recommends a fictitious notch radius of 0.05 mm is applied in this reanalysis, as it has been experimentally validated for thin and flexible welded joints (Sonsino, 2009; Sonsino, Bruder and Baumgartner, 2010; Sonsino *et al.*, 2012). By incorporating the recommendations of both Hobbacher and Sonsino, this reanalysis ensures that the IIW ENS approach is applied consistently across a broad spectrum of welded joint thickness.

3.5.1 CA Loading

Table 3.6, presented at the end of this subsection, provides a comprehensive summary of the welded joints reanalysed under CA loading using the ENS approach, complementing the datasets previously presented for NS- and HSS-based reanalyses.

The table includes uniaxial and torsional reference fatigue strengths (FAT-values) recommended by IIW standards and Sonsino's recommendations, as well as key details on welded joint geometries, material properties, fatigue curve slopes, and relevant literature sources.

The ENS-based reanalysis for steel welded joints under CA loading, illustrated in Figure 3.19, was conducted with reference stresses recalculated at $P_s=97.7\%$ and an additional $SF=1.15$, maintaining consistency with the NS- and HSS-based reanalyses.

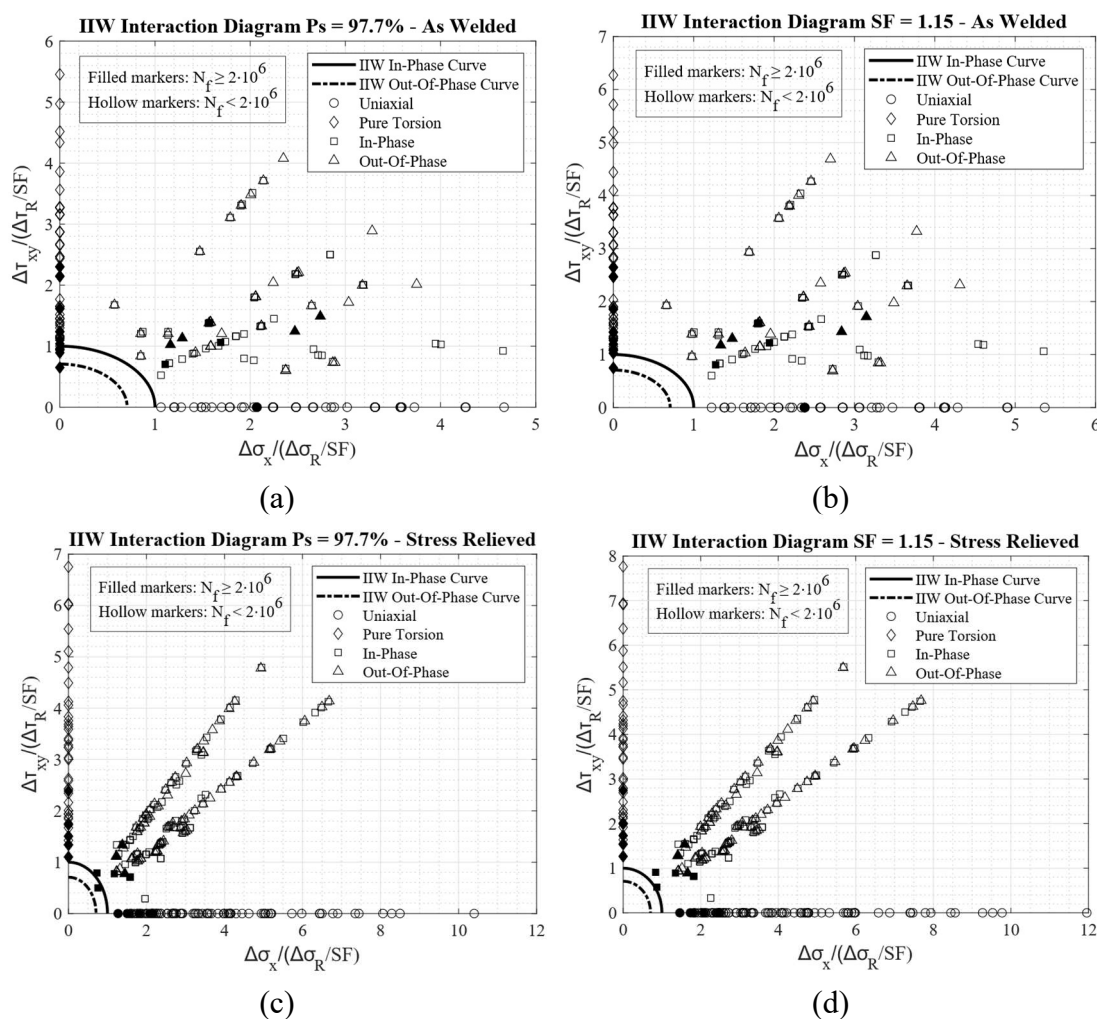


Figure 3.19: ENS approach: IIW interaction diagram for as-welded (a and b) and stress-relieved (c and d) steel welded joints under CA loading, with reference stresses ($\Delta\sigma_R$ and $\Delta\tau_R$) recalculated based on $P_s=97.7\%$ and $SF=1.15$.

The results confirm a high level of accuracy and safety, with only two non-conservative estimates at $P_s=97.7\%$. However, the application of $SF=1.15$ completely eliminated

these non-conservative instances, reducing the P_{NC} to 0%. This reinforces the robustness of the ENS approach in providing reliable fatigue life predictions for steel welded joints.

Similarly, the ENS-based reanalysis for aluminium welded joints, presented in Figure 3.20, exhibited zero non-conservative estimates, further demonstrating the inherent conservatism of the ENS approach.

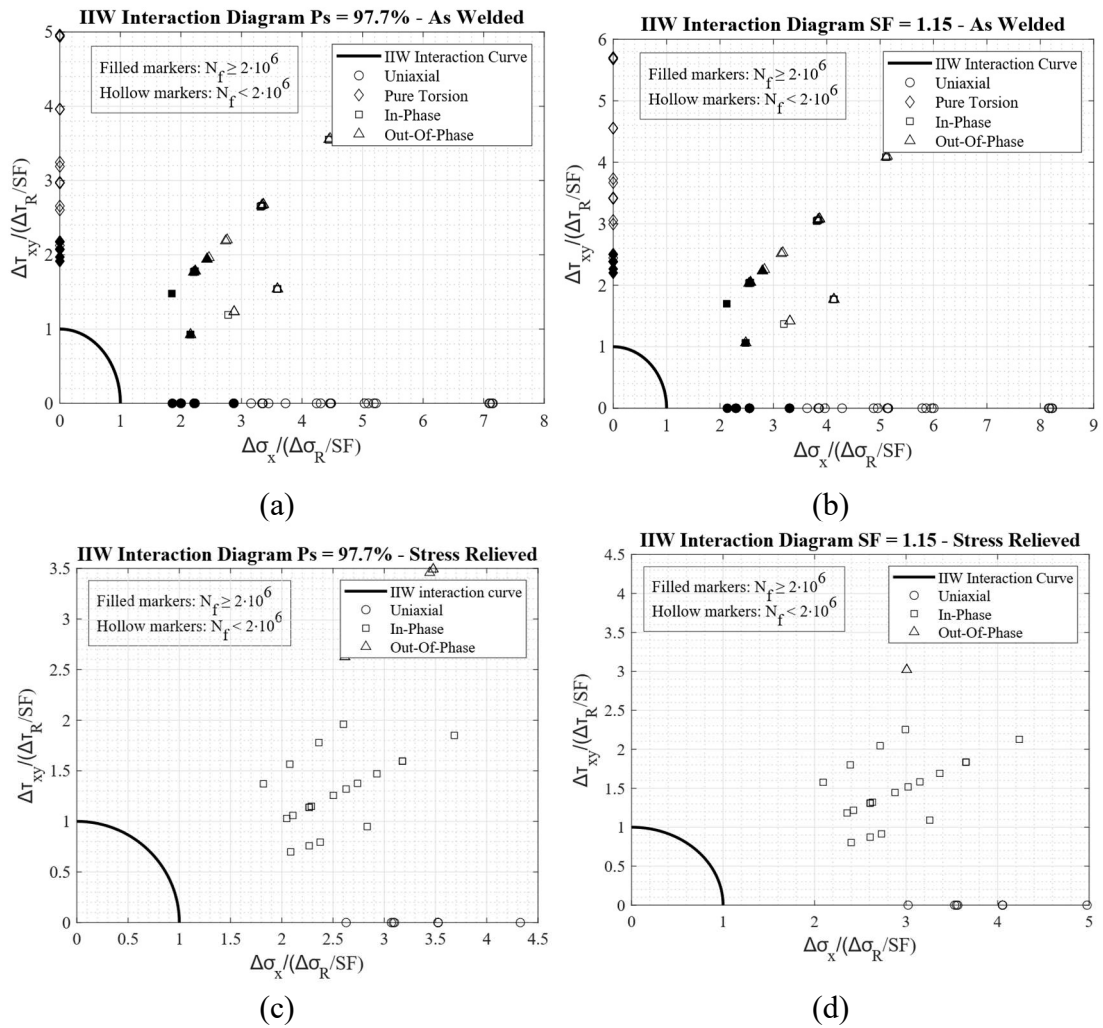


Figure 3.20: ENS approach: IIW interaction diagram for as-welded (a and b) and stress-relieved (c and d) aluminium welded joints under CA fatigue loading, with reference stresses ($\Delta\sigma_R$ and $\Delta\tau_R$) recalculated based on $P_s=97.7\%$ and $SF=1.15$.

As observed in the NS- and HSS-based approaches, all fatigue life estimations were positioned well above the design curve boundary, indicating a significant margin of safety.

Table 3.6: Summary of reanalysed welded joints using ENS approach under CA fatigue loading, including uniaxial and torsional reference fatigue strengths, geometries, materials, fatigue curve slopes, and data sources.

Material (CA)	$\Delta\sigma_{R,P_s=97.7\%}^{(1)}$ (MPa) IIW	Uniaxial curve slope, k IIW	$\Delta\tau_{R,P_s=97.7\%}^{(1)}$ (MPa) IIW	Torsional curve slope, k ₀ IIW	Geometry	Reference
UM StE 460 *	225	3	160	5	Figure 3.2a	(Sonsino, 1995)
M StE 460 *	225	3	160	5	Figure 3.2a	(Sonsino, 1997)
UM StE 460 *	225	3	160	5	Figure 3.2e	(Sonsino, 1997)
StE 460 *	225	3	160	5	Figure 3.2a	(Yousefi, Witt and Zenner, 2001)
A519	225	3	160	5	Figure 3.2b	(Yung and Lawrence Jr, 1989)
A519-A36 *	225	3	160	5	Figure 3.2d	(Siljander, Kurath and Lawrence Jr, 1989)
Fe 52 steel	225	3	160	5	Figure 3.2f	(Bäckström, 2003)
BS4360	630	3	250	5	Figure 3.2d	(Archer, 1987)
S340+N, E355+N	630	3	250	5	Figure 3.2e	(Shams and Vormwald, 2017)
S340+N, E355+N *	630	3	250	5	Figure 3.2e	(Shams and Vormwald, 2017)
St 35 (t = 1 mm)	630	3	250	5	Figure 3.2e	(Wiebesiek and Sonsino, 2010)
St 35 (t = 2 mm)	630	3	250	5	Figure 3.2e	(Sonsino <i>et al.</i> , 2006)
6082-T6 *	71	3	63	5	Figure 3.2a	(Kueppers and Sonsino, 2003)
6060-T6	180	3	90	5	Figure 3.2e	(Costa <i>et al.</i> , 2005)
AW 6082	180	3	90	5	Figure 3.2e	(Wiebesiek <i>et al.</i> , 2011)
AW 5042	180	3	90	5	Figure 3.2e	(Wiebesiek <i>et al.</i> , 2011)

⁽¹⁾Reference normal and shear stresses extrapolated at 2 million cycles to failure, with probability of survival (P_s) = 97.7%.

* Stress-relieved

UM – Unmachined; M – Machined.

3.5.2 VA Loading

Table 3.7, positioned at the end of this subsection, provides a comprehensive summary of the welded joints reanalysed under VA loading using the ENS approach, complementing the dataset presented for CA loading. In contrast to the observations obtained from CA loading, the ENS approach demonstrated high accuracy and reliability under VA fatigue conditions, with only one non-conservative estimate, corresponding to a P_{NC} of 1.23%. However, after applying a $SF=1.15$, all non-conservative estimates were eliminated, reinforcing the conservative nature of the ENS approach in fatigue life predictions for steel welded joints, as illustrated in Figure 3.21.

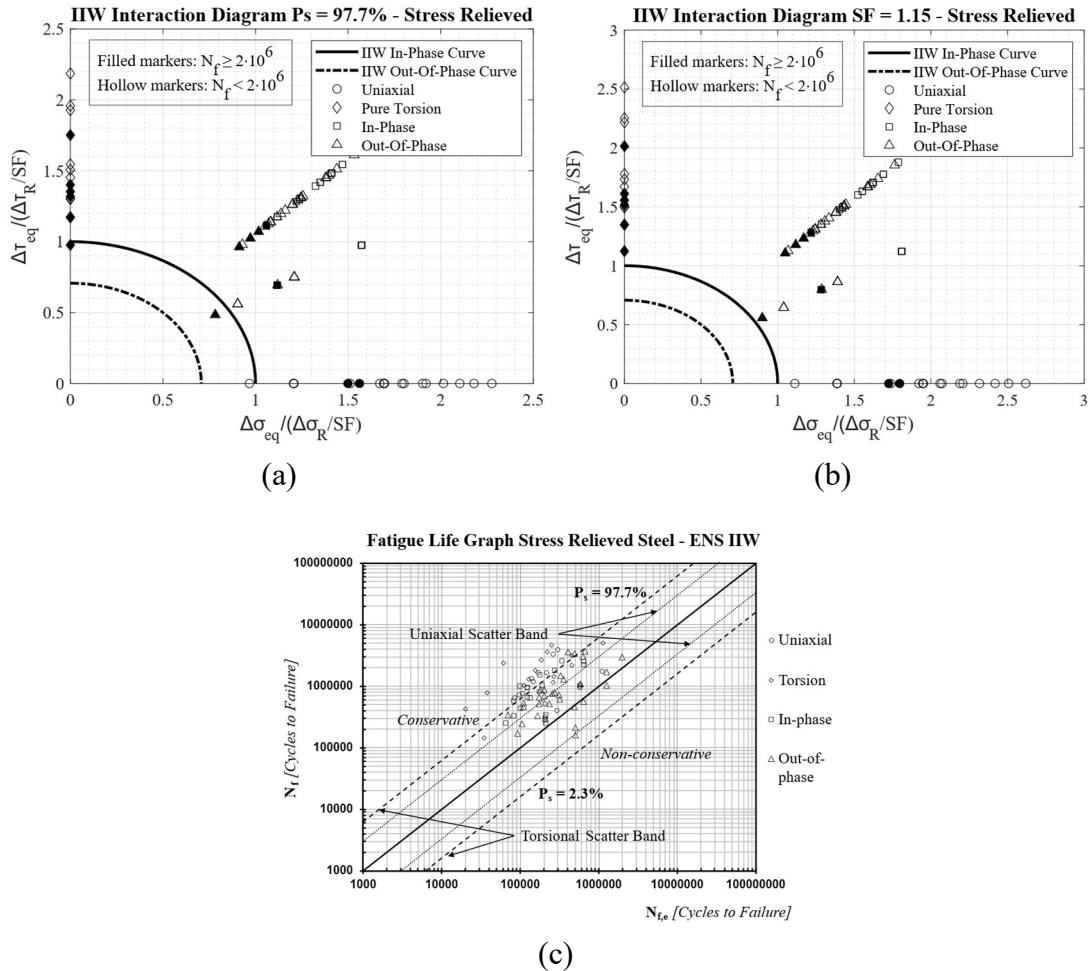


Figure 3.21: ENS approach: IIW interaction diagram (a and b) and fatigue life graph (c) for stress-relieved steel welded joints under VA fatigue loading, with reference stresses ($\Delta\sigma_R$ and $\Delta\tau_R$) recalculated based on $P_s = 97.7\%$ and $SF = 1.15$.

A similar trend was observed in the reanalysis of aluminium welded joints under VA loading, where no instances of non-conservative estimates were identified, as shown in Figure 3.22, which further validates the conservative nature of the ENS method.

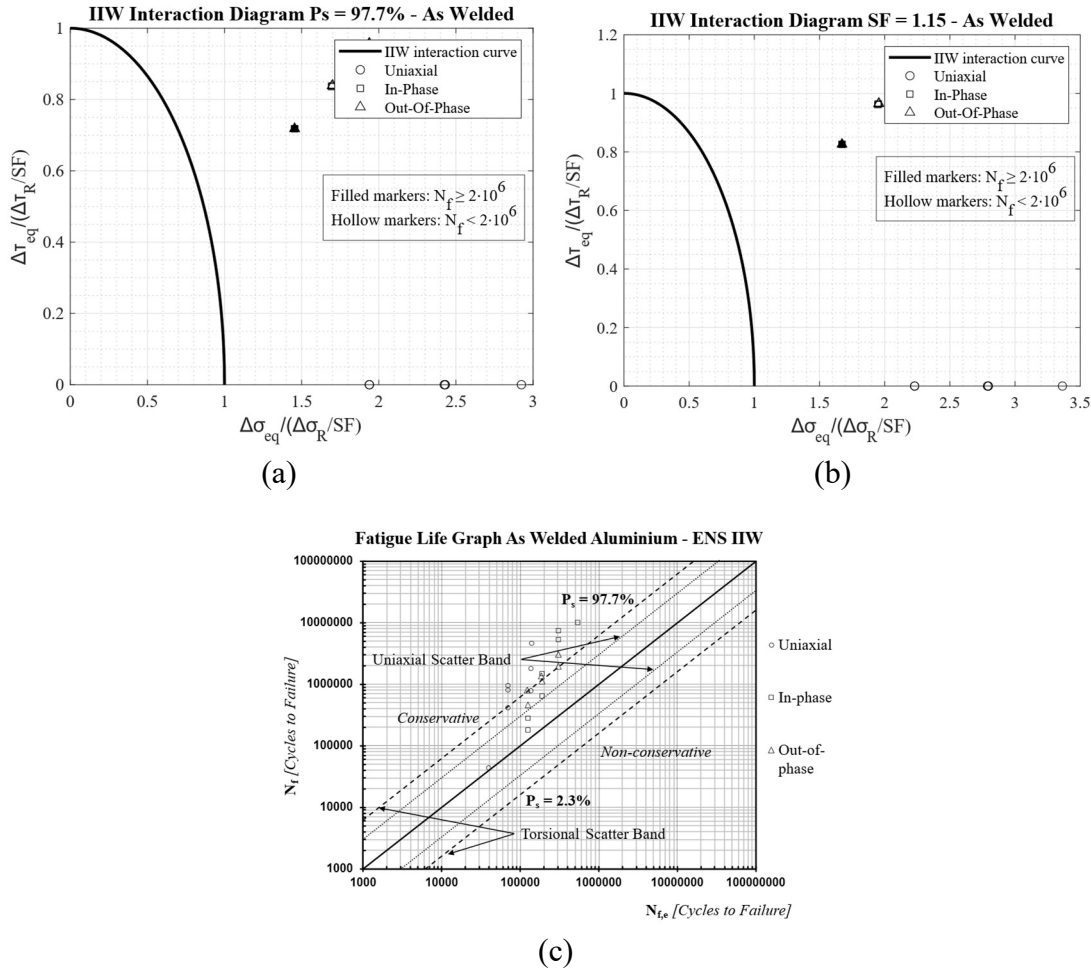


Figure 3.22: ENS approach: IIW interaction diagram (a and b) and fatigue life graph (c) for as-welded aluminium welded joints under VA fatigue loading, with reference stresses ($\Delta\sigma_R$ and $\Delta\tau_R$) recalculated based on $P_s=97.7\%$ and $SF=1.15$.

Additionally, when the IIW P-M rule was applied in combination with the ENS approach under VA loading, fatigue life estimations for both steel and aluminium welded joints were predominantly clustered within the scatter band and positioned above the $N_f = N_{fe}$ diagonal line, as illustrated in Figures 3.21c and 3.22c. However, for steel welded joints subjected to VA loading, five instances of out-of-phase loading estimates fell below the diagonal line, resulting in a P_{NC} of 6.2%. Conversely, for aluminium welded joints, all fatigue life estimations remained above the diagonal line.

Table 3.7: Summary of reanalysed welded joints using ENS approach interaction equation criterion under VA fatigue loading, including uniaxial and torsional reference fatigue strengths, geometries, materials, fatigue curve slopes, and data sources.

Material (VA)	$\Delta\sigma_{R,P_s=97.7\%}^{(1)}$ (MPa) IIW	Uniaxial curve slope, k IIW	$\Delta\tau_{R,P_s=97.7\%}^{(1)}$ (MPa) IIW	Torsional curve slope, k0 IIW	Geometry	Reference
StE 460 *	225	3	160	5	Figure 3.2a	(Sonsino and Kueppers, 2001)
StE 460 *	225	3	160	5	Figure 3.2a	(Yousefi, Witt and Zenner, 2001)
42CrMo4 *	225	3	160	5	Figure 3.2e	(Pyttel, Grawenhof and Berger, 2012)
6082-T6	71	3	63	5	Figure 3.2a	(Kueppers and Sonsino, 2006)

⁽¹⁾Reference normal and shear stresses extrapolated at 2 million cycles to failure, with probability of survival (P_s) = 97.7%.

* Stress-relieved

3.6 Key Findings and Implications

The reanalysis of welded joints using various multiaxial fatigue assessment criteria has provided valuable insights into the conservatism embedded within current standards. This assessment is based on design lines and FAT-values at $P_s=97.7\%$, incorporating an additional SF of 1.15, as recommended by EC3, EC9 and the IIW. This differs from assessments based on $P_s=50\%$, which would yield different conclusions regarding conservatism in fatigue life estimations.

A notable trend observed across all approaches is the significant conservatism in fatigue life predictions for aluminium welded joints. This is primarily due to the limited availability of experimental validation data for their reference fatigue strength, leading to cautious design recommendations (Kueppers and Sonsino, 2003; Costa *et al.*, 2005; Wiebesiek and Sonsino, 2010; Wiebesiek *et al.*, 2011). While this ensures safety, it may also result in overly conservative designs, potentially increasing material usage and costs. To refine fatigue life predictions and achieve a more balanced design approach, additional experimental studies are needed to improve the accuracy of aluminium fatigue assessments.

It is also important to recognise that the level of safety determined in this study is specific to the examined welded joints and should not be generalised across all loading conditions. However, these findings serve as a crucial reference for engineers conducting fatigue life assessments in real-world applications. In cases where direct experimental validation is impractical, such as large-scale structures like bridges and ships, mandatory inspections remain essential for ensuring structural integrity. Insights from these inspections contribute to refining fatigue life calculations, improving their accuracy and applicability to future design scenarios.

A key consideration in fatigue assessments is the difference in exponents between the EC3 and IIW interaction equations. EC3 employs exponents of 3 and 5, aligning with the inverse slopes of uniaxial and torsional S-N curves, yet it lacks formal scientific derivation (Anon, 2005; Sonsino, Wallmichrath and Kueppers, 2005). In contrast, IIW

applies an exponent of 2, derived from the Gough-Pollard relationship, with an additional 0.5 modification to account for non-proportional loading effects. While this modification improves applicability, it remains an engineering compromise aimed at accommodating non-proportional loading effects (Sonsino, Maddox and Hobbacher, 2005; Sonsino, Wallmichrath and Kueppers, 2005; Sonsino and Fricke, 2006; Sonsino and Wiebesiek, 2007; Hobbacher, 2016). This further underscores the need for experimental validation and in-service inspections to enhance confidence in fatigue life estimations.

Another significant factor influencing fatigue life estimations is the inverse slope assumptions used in standard S-N curves. EC3 and IIW recommend inverse slopes of 3 and 5 for uniaxial and torsional fatigue life predictions, respectively (Anon, 2005; Hobbacher, 2016). While appropriate for thick and rigid joints, thinner and more flexible welded joints tend to exhibit shallower experimentally derived slopes, leading to discrepancies in fatigue life predictions, particularly under VA loading conditions (Sonsino, Bruder and Baumgartner, 2010; Susmel and Askes, 2012). Shallower slopes reduce fatigue life predictions at high stress levels but increase life estimations at lower stress levels (Sonsino, Bruder and Baumgartner, 2010). To address this, adopting an inverse slope of 5 for uniaxial loading and 7 for torsional loading could improve estimations (Sonsino, Bruder and Baumgartner, 2010; Susmel and Askes, 2012).

A consistent trend in VA fatigue life estimations is the greater conservatism of the IIW P-M rule compared to EC3 and EC9 criteria. This difference in conservatism is likely attributed to the higher allowable damage sum factors in EC3 and EC9, whereas IIW applies stricter limits, leading to more cautious predictions. Despite this, both approaches remain within conservative design margins, ensuring safe and reliable fatigue life estimations.

Given the limited availability of experimental fatigue data for steel and aluminium welded joints under VA multiaxial loading, further research should focus on validating fatigue assessment criteria to enhance accuracy and cost-effectiveness. Such validation would help optimise design parameters, ensuring practical applicability in real-world welded structures while avoiding overly conservative assumptions.

In comparing the performance of different stress analysis approaches (NS, HSS, and ENS), the findings confirm that EC3, EC9, and IIW criteria consistently exhibit a commendable level of conservatism, regardless of the assessment method or loading condition (CA or VA). Among these, the HSS approach, as per IIW recommendations, proves to be the most conservative, maintaining a P_{NC} of 0% across all conditions. However, its applicability is limited when fatigue failure originates at the weld root, making the IIW-ENS approach the next most effective alternative, accommodating a broader range of welded joint configurations.

These comparative reanalyses are further supported by Tables 3.8 and 3.9, which summarise the P_{NC} across different reanalysis approaches for steel and aluminium welded joints. Table 3.8 presents the P_{NC} results for CA fatigue loading, providing a quantitative comparison of conservatism across the assessed methods.

Table 3.8: Summary of percentage of non-conservative estimates, P_{NC} values from reanalyses of as-welded and stress-relieved steel and aluminium welded joints under CA loading using EC3, EC9, and IIW criteria with NS, HSS, and ENS approaches.

Multiaxial Fatigue Criterion	Material	Stress Analysis Approaches	Percentage of Non-Conservative Estimates, P_{NC} (%)	
			$P_s = 97.7 \%$	SF = 1.15
EC3	Steel	NS	2.54	0.85
		HSS	0	0
	Steel *	NS	0.28	0
		HSS	0	0
EC9	Aluminium	NS	0	-
		HSS	0	-
	Aluminium *	NS	0	-
		HSS	0	-
IIW	Steel	NS	1.27	0.42
		HSS	0	0
		ENS	1.02	0
	Steel *	NS	0	0
		HSS	0	0
		ENS	0.92	0
	Aluminium	NS	0	0
		HSS	0	0
		ENS	0	0
	Aluminium *	NS	0	0
		HSS	0	0
		ENS	0	0

* Stress-relieved

Similarly, Table 3.9 focuses on VA fatigue loading, offering further insights into the relative conservatism of the different standard multiaxial fatigue criteria.

Table 3.9: Summary of percentage of non-conservative estimates, P_{NC} values from reanalyses of as-welded and stress-relieved steel and aluminium welded joints under VA loading using EC3, EC9, and IIW interaction equation and P-M criteria with NS, HSS, and ENS approaches.

Multiaxial Fatigue Criterion	Material	Stress Analysis Approaches	Percentage of Non-Conservative Estimates, P_{NC} (%)	
			$P_s = 97.7 \%$	SF = 1.15
EC3: Interaction Eq.	Steel *	NS	11.1	3.70
		HSS	7.41	3.70
EC3: P-M	Steel *	NS	6.17	-
		HSS	6.17	-
EC9: P-M	Aluminium	NS	5.0	-
		HSS	0	-
	Aluminium *	NS	0	-
		HSS	0	-
IIW: Interaction Eq.	Steel *	NS	0	0
		HSS	0	0
		ENS	1.23	0
	Aluminium	NS	0	0
		HSS	0	0
		ENS	0	0
IIW: P-M	Steel *	NS	2.47	-
		HSS	2.47	-
		ENS	6.17	-
	Aluminium	NS	5.0	-
		HSS	5.0	-
		ENS	0	-

* Stress-relieved

The findings emphasise the reliability of these simplified multiaxial fatigue assessment methods, particularly in comparison to more complex critical plane approaches proposed by EC3, EC9, and IIW. This study provides structural engineers with a practical and dependable framework for designing welded joints under multiaxial fatigue loading conditions, supporting safer and more efficient fatigue design methodologies.

3.7 Conclusions

This study presents a comprehensive evaluation of multiaxial fatigue assessment methods for steel and aluminium welded joints under both CA and VA fatigue loading. The reanalyses were conducted using EC3, EC9, and IIW criteria in conjunction with NS, HSS, and ENS approaches. The key findings from this quantitative assessment are summarised as follows:

- Both EC3 and IIW criteria exhibit a high level of conservatism for steel welded joints, with a P_{NC} close to 0% across all stress analysis methods, particularly under CA loading. This highlights their reliability in ensuring safe fatigue life estimations.
- The fatigue life estimations for aluminium welded joints display greater conservatism compared to steel welded joints under both CA and VA loading conditions. This can be attributed to the limited availability of experimental validation data, which results in cautious reference fatigue strengths and more conservative design recommendations (Kueppers and Sonsino, 2003; Costa *et al.*, 2005; Wiebesiek and Sonsino, 2010; Wiebesiek *et al.*, 2011).
- Among all methods assessed, the IIW-HSS approach consistently yields zero non-conservative estimates across all loading conditions, making it the most conservative and safest approach. However, its applicability is limited when fatigue failure originates at the weld root. In such cases, the IIW-ENS approach emerges as the most suitable alternative.
- Based on experimentally derived fatigue life data, the recommended inverse slopes for uniaxial and torsional S-N curves in both EC3 and IIW should be revised from 3 and 5 to 5 and 7, respectively (Sonsino, Bruder and Baumgartner, 2010). This adjustment is particularly necessary for thin and flexible welded joints, where the current slope assumptions lead to slight discrepancies in fatigue life estimations under VA loading conditions.

- The EC3 criterion adopt exponents of 3 and 5 for normal and shear stresses, respectively. However, these values lack formal scientific derivation and should undergo further verification and potential revision. In contrast, the IIW interaction equation employs an exponent of 2, which aligns with the Gough-Pollard relationship and provides more conservative and safer estimations (Sonsino, Wallmichrath and Kueppers, 2005).
- The conservatism levels observed in this study apply only to the specific reanalyses and loading configurations examined. These findings should not be generalised to all service loading conditions. Experimental validation and in-service inspections remain essential in practical applications, especially for critical welded structures such as bridges, ships, and offshore platforms.

These findings imply the importance of refining existing standard multiaxial fatigue criteria to enhance their accuracy and practical applicability. While EC3, EC9 and IIW criteria demonstrate a commendable level of conservatism, further experimental studies and refinements are required to ensure that fatigue life estimations remain both safe and efficient, without excessive conservatism that may lead to overdesign.

3.8 References

Amstutz, H., Störzel, K. and Seeger, T. (2001) ‘Fatigue crack growth of a welded tube–flange connection under bending and torsional loading’, *Fatigue & Fracture of Engineering Materials & Structures*, 24(5), pp. 357–368.

Anon (2005) *Eurocode 3: Design of steel structures — Part 1-9: Fatigue* .

Anon (2006) ‘Eurocode 1: Actions on structures - Part 3: Actions induced by cranes and machinery’.

Anon (2007) *Eurocode 9: Design of aluminium structures — Part 1-3: Structures susceptible to fatigue* .

Archer, R. (1987) 'Fatigue of a welded steel attachment under combined direct stress and shear stress', in *International conference of fatigue of welded constructions*, no. paper.

Atzori, B. (1983) 'Trattamento termici e resistenza a fatica delle strutture soldate'.

Atzori, B. *et al.* (2009) 'Fatigue design of complex welded structures', *International Journal of Fatigue*, 31(1), pp. 59–69. doi: 10.1016/j.ijfatigue.2008.02.013.

Bäckström, M. (2003) 'Multiaxial fatigue life assessment of welds based on nominal and hot spot stresses'.

Berto, F., Lazzarin, P. and Radaj, D. (2014) 'Fictitious notch rounding concept applied to V-notches with root hole subjected to in-plane mixed mode loading', *Engineering Fracture Mechanics*, 128(C), pp. 171–188. doi: 10.1016/j.engfracmech.2014.07.014.

Costa, J. D. M. *et al.* (2005) 'Fatigue behaviour of tubular AlMgSi welded specimens subjected to bending–torsion loading', *Fatigue & Fracture of Engineering Materials & Structures*, 28(4), pp. 399–407.

Fricke, W. (2007) 'Round-robin study on stress analysis for the effective notch stress approach', *Welding in the World*, 51(3), pp. 68–79.

Fricke, W. (2013) 'IIW guideline for the assessment of weld root fatigue', *Welding in the World*, 57(6), pp. 753–791. doi: 10.1007/s40194-013-0066-y.

Haibach, E. (1989) 'Verfahren und Daten zur Bauteilberechnung'. Springer.

Hobbacher, A. (2016) *Recommendations for fatigue design of welded joints and components*. Springer.

Kueppers, M. and Sonsino, C. M. (2003) 'Critical plane approach for the assessment of the fatigue behaviour of welded aluminium under multiaxial loading', *Fatigue & Fracture of Engineering Materials & Structures*, 26(6), pp. 507–513.

- Kueppers, M. and Sonsino, C. M. (2006) 'Assessment of the fatigue behaviour of welded aluminium joints under multiaxial spectrum loading by a critical plane approach', *International Journal of Fatigue*, 28(5–6), pp. 540–546.
- Van Lieshout, P. S., Den Besten, J. H. and Kaminski, M. L. (2016) 'Comparative study of multiaxial fatigue methods applied to welded joints in marine structures', *Frattura ed Integrità Strutturale*, 10(37), pp. 173–192.
- Mei, J. *et al.* (2021) 'An overview and comparative assessment of approaches to multi-axial fatigue of welded components in codes and standards', *International Journal of Fatigue*, 146, p. 106144.
- Morgenstern, C. *et al.* (2006) 'Fatigue design of aluminium welded joints by the local stress concept with the fictitious notch radius of $r_f=1$ mm', *International Journal of Fatigue*, 28(8), pp. 881–890. doi: 10.1016/j.ijfatigue.2005.10.006.
- Niemi, E. (1995) *Stress determination for fatigue analysis of welded components*. Woodhead Publishing.
- Pedersen, M. M. (2016) 'Multiaxial fatigue assessment of welded joints using the notch stress approach', *International Journal of Fatigue*, 83, pp. 269–279. doi: 10.1016/J.IJFATIGUE.2015.10.021.
- Pyttel, B., Grawenhof, P. and Berger, C. (2012) 'Application of different concepts for fatigue design of welded joints in rotating components in mechanical engineering', *International journal of fatigue*, 34(1), pp. 35–46.
- Radaj, D. (1996) 'Review of fatigue strength assessment of nonwelded and welded structures based on local parameters', *International Journal of Fatigue*, 18(3), pp. 153–170.
- Radaj, D., Lazzarin, P. and Berto, F. (2013) 'Generalised Neuber concept of fictitious notch rounding', *International Journal of Fatigue*, 51, pp. 105–115. doi: 10.1016/j.ijfatigue.2013.01.005.

Radaj, D., Sonsino, C. M. and Fricke, W. (2006) *Fatigue assessment of welded joints by local approaches*. Woodhead publishing.

Razmjoo, G. R. (1997) 'Fatigue of load-carrying fillet welded joints under multiaxial loading', in *5 th International Conference on Biaxial/Multiaxial Fatigue & Fracture*, pp. 53–70.

Schijve, J. (2012) 'Fatigue predictions of welded joints and the effective notch stress concept', *International Journal of Fatigue*, 45, pp. 31–38.

Shams, E. and Vormwald, M. (2017) 'Fatigue of weld ends under combined loading', *International Journal of Fatigue*, 100, pp. 627–638.

Siljander, A., Kurath, P. and Lawrence Jr, F. V (1989) 'Proportional and non-proportional multiaxial fatigue of tube to plate weldments', *University of Illinois at Urbana-Champaign, Urbana, Illinois, Report of the Welding Research Council*.

Sonsino, C. M. (1995) 'Multiaxial fatigue of welded joints under in-phase and out-of-phase local strains and stresses', *International Journal of Fatigue*, 17(1), pp. 55–70.

Sonsino, C. M. (1997) 'Fatigue behaviour of welded components under complex elastoplastic multiaxial deformations', *ECSC-Report No. EUR 16024 DE*.

Sonsino, C. M. *et al.* (2006) 'Fatigue strength of laser beam welded thin steel structures under multiaxial loading', *International journal of fatigue*, 28(5–6), pp. 657–662.

Sonsino, C. M. (2009) 'A consideration of allowable equivalent stresses for fatigue design of welded joints according to the notch stress concept with the reference radii $r_{ref}=1.00$ and 0.05 mm', *Welding in the World*, 53, pp. R64–R75.

Sonsino, C. M. *et al.* (2012) 'Notch stress concepts for the fatigue assessment of welded joints—Background and applications', *International Journal of Fatigue*, 34(1), pp. 2–16.

Sonsino, C. M., Bruder, T. and Baumgartner, J. (2010) 'SN lines for welded thin joints—suggested slopes and FAT values for applying the notch stress concept with various reference radii', *Welding in the World*, 54(11), pp. R375–R392.

- Sonsino, C. M. and Fricke, W. (2006) ‘Some remarks for improving the assessment of multiaxial stress states and multiaxial spectrum loading in the IIW-fatigue design recommendations’, *IIW-Document No. XIII-2158r1-07/XV-1250r1-07XIII-2128-06/XV-1222-06*, pp. 4–19.
- Sonsino, C. M. and Kueppers, M. (2001) ‘Multiaxial fatigue of welded joints under constant and variable amplitude loadings’, *Fatigue & Fracture of Engineering Materials & Structures*, 24(5), pp. 309–327.
- Sonsino, C. M., Maddox, S. J. and Hobbacher, A. (2005) ‘Fatigue life assessment of welded joints under variable amplitude loading-State of present knowledge and recommendations for fatigue design regulations’.
- Sonsino, C. M., Wallmichrath, M. and Kueppers, M. (2005) ‘Assessment of Multiaxial Fatigue Test Results on Welded Joints by Application of the IIW-Formula and Modifications’, *IIW-Document No. XIII-2046-05*.
- Sonsino, C. M. and Wiebesiek, J. (2007) ‘Assessment of multiaxial spectrum loading of welded steel and aluminium joints by modified equivalent stress and Gough-Pollard algorithms’, *IIW-Document No. XIII-2158r1-07/XV-1250r1-07*.
- Susmel, L. (2009) *Multiaxial notch fatigue*. Elsevier.
- Susmel, L. (2014) ‘Four stress analysis strategies to use the Modified Wöhler Curve Method to perform the fatigue assessment of weldments subjected to constant and variable amplitude multiaxial fatigue loading’, *International Journal of Fatigue*, 67, pp. 38–54. doi: 10.1016/j.ijfatigue.2013.12.001.
- Susmel, L. and Askes, H. (2012) ‘Modified Wöhler Curve Method and multiaxial fatigue assessment of thin welded joints’, *International Journal of Fatigue*, 43, pp. 30–42. doi: 10.1016/j.ijfatigue.2012.01.026.
- Susmel, L., Sonsino, C. M. and Tovo, R. (2011) ‘Accuracy of the modified wöhler curve method applied along with the $r_{ref} = 1$ mm concept in estimating lifetime of welded joints subjected to multiaxial fatigue loading’, *International Journal of Fatigue*, 33(8), pp. 1075–1091. doi: 10.1016/j.ijfatigue.2010.09.010.

Susmel, L. and Tovo, R. (2004) 'On the use of nominal stresses to predict the fatigue strength of welded joints under biaxial cyclic loading', *Fatigue & Fracture of Engineering Materials & Structures*, 27(11), pp. 1005–1024.

Tovo, R. and Lazzarin, P. (1999) 'Relationships between local and structural stress in the evaluation of the weld toe stress distribution', *International journal of fatigue*, 21(10), pp. 1063–1078.

Wiebesiek, J. *et al.* (2011) 'Multiaxial fatigue behaviour of laserbeam-welded thin steel and aluminium sheets under proportional and non-proportional combined loading', *International journal of fatigue*, 33(8), pp. 992–1005.

Wiebesiek, J. and Sonsino, C. M. (2010) 'New results in multiaxial fatigue of welded aluminium joints', *IIW-Document No. XIII-2314-10/XV-1349-10*, 13, pp. 2310–2314.

Yousefi, F., Witt, M. and Zenner, H. (2001) 'Fatigue strength of welded joints under multiaxial loading: experiments and calculations', *Fatigue & Fracture of Engineering Materials & Structures*, 24(5), pp. 339–355.

Yung, J.-Y. and Lawrence Jr, F. V (1989) 'Predicting the fatigue life of welds under combined bending and torsion', *Mechanical Engineering Publications, Biaxial and Multiaxial Fatigue*, pp. 53–69.

Al Zamzami, I. and Susmel, L. (2018) 'On the use of hot-spot stresses, effective notch stresses and the Point Method to estimate lifetime of inclined welds subjected to uniaxial fatigue loading', *International Journal of Fatigue*, 117, pp. 432–449. doi: 10.1016/j.ijfatigue.2018.08.032.

Chapter 4

Quantitative Review of Stress-Based Multiaxial Fatigue Criteria for Steel and Aluminium Welded Joints

4.1 Introduction

Building on the findings of Chapter 3, which quantitatively evaluated standard multiaxial fatigue criteria recommended by Eurocode 3 (EC3), Eurocode 9 (EC9), and the International Institute of Welding (IIW), this chapter extends the analysis by investigating stress-based critical plane criteria. While interaction equation approaches offer a practical framework for fatigue design, they exhibit limitations, particularly in handling non-proportional loading and variable amplitude (VA) conditions (Fricke, 2003; C M Sonsino, 2009; Pedersen, 2016). These constraints necessitate the adoption of more advanced methodologies capable of capturing the complex multiaxial stress states in welded joints.

Fatigue in welded joints remains a significant challenge due to stress concentrations at geometric discontinuities, residual stresses from welding, and material inhomogeneity between the weld metal, base metal, and heat-affected zone (HAZ). These factors, combined with complex multiaxial loading conditions, complicate fatigue life estimation, as both normal and shear stress components contribute to fatigue damage. In the high-cycle fatigue regime, stress-based critical plane criteria have been developed to provide a more physically representative assessment of fatigue damage by identifying the most critical plane based on material properties and the governing failure mode.

This study examines Findley's criterion, the Carpinteri-Spagnoli (CS) criterion, and the Modified Wöhler Curve Method (MWCM) for their applicability to high-cycle fatigue assessments. Unlike interaction equation-based approaches, these criteria explicitly account for both shear and normal stress components, enabling a more refined fatigue evaluation. The quantitative reanalysis presented in this chapter applies these criteria to the same experimental datasets examined in Chapter 3, utilising different stress analysis approaches, including the nominal stress (NS), hot-spot stress (HSS), effective notch stress (ENS), and the Theory of Critical Distances-Point Method (TCD PM). Findley's criterion and the CS criterion are reanalysed using NS, HSS, and ENS, while MWCM is assessed with NS, HSS, ENS, and TCD PM.

Despite extensive research on multiaxial fatigue, most studies take a fragmented approach by either evaluating a single criterion across multiple stress analysis methods or applying multiple fatigue criteria using only one stress analysis technique (Radaj, 1996; Bäckström and Marquis, 2001; Susmel, 2009c; Papuga, 2011; Susmel and Askes, 2012; Susmel, 2014; Pedersen, 2016; Mei *et al.*, 2021; Bibbo, Baumgartner and Arora, 2022). This narrow scope limits a holistic comparison of stress-based critical plane criteria across different methodologies, leaving gaps in understanding their relative predictive performance and applicability. This chapter addresses this gap by systematically comparing three widely used multiaxial fatigue criteria across multiple stress analysis approaches for steel and aluminium welded joints under both constant amplitude (CA) and VA loading conditions. This integrated assessment provides a more rigorous evaluation, offering deeper insights into their predictive accuracy and reliability for fatigue life estimations in welded joints.

To achieve this, the chapter first outlines the validation methodology, detailing the application of the three selected multiaxial fatigue criteria along with the evaluation metrics employed to quantify predictive accuracy. The subsequent sections present fatigue life estimations across different stress analysis approaches, highlighting key trends, strengths, and limitations. A comparative analysis follows, discussing the relative performance of the approaches, before concluding with a summary of key findings and recommendations for future research.

By systematically comparing stress-based critical plane criteria with standard multiaxial fatigue assessment methods, this chapter offers a refined understanding of their predictive capabilities, contributing to the advancement of fatigue life estimation methodologies for welded joints. These findings support the broader objective of enhancing structural reliability in engineering applications where multiaxial fatigue loading is a critical design consideration.

4.2 Validation Methodology

This section details the validation methodology employed to assess the predictive accuracy of stress-based critical plane criteria for welded joints under multiaxial fatigue loading. To ensure consistency and enable direct comparisons, the experimental datasets, joint geometries, and loading path conditions under CA and VA are identical to those in Chapter 3 (Archer, 1987; Yung and Lawrence Jr, 1989; Siljander, Kurath and Lawrence Jr, 1989; Sonsino, 1995, 1997; Razmjoo, 1997; Amstutz, Störzel and Seeger, 2001; Sonsino and Kueppers, 2001; Yousefi, Witt and Zenner, 2001; Bäckström, 2003; Kueppers and Sonsino, 2003, 2006; Costa *et al.*, 2005; Sonsino *et al.*, 2006; Wiebesiek and Sonsino, 2010; Wiebesiek *et al.*, 2011; Pyttel, Grawenhof and Berger, 2012; Shams and Vormwald, 2017). The analysis is performed using four stress analysis approaches: NS, HSS, ENS, and the TCD PM (Niemi, 1995; Anon, 2005, 2007; Sonsino *et al.*, 2012; Fricke, 2013; Susmel, 2014; Hobbacher, 2016; Al Zamzami and Susmel, 2018; Mei *et al.*, 2021).

Findley and CS criteria are reanalysed using the NS, HSS and ENS stress analysis approaches, whereas the MWCM is reanalysed alongside NS, HSS, ENS, and the TCD PM. The inclusion of the TCD PM is exclusive to the MWCM, as its critical distance, $M-D_v$ has been specifically calibrated for use with MWCM but not with other critical plane criteria under investigation (Taylor, Barrett and Lucano, 2002; Susmel, 2007; L. Susmel, 2008; Susmel, 2009a, 2009b).

4.2.1 Fatigue Life Estimations and Reference Strengths

To ensure consistency with established fatigue design frameworks, this study incorporates both IIW-recommended fatigue slopes and experimentally derived slopes, enabling a direct comparison between the standard multiaxial fatigue criteria examined in Chapter 3 and the critical plane approaches evaluated in this chapter.

Fatigue life estimations have been reanalysed using a probability of survival (P_s) of 50%, while a P_s of 97.7% is retained in tabulated datasets for standard and experimental reference strengths to ensure consistency with conventional reference strength (FAT) values. The normal and shear reference stresses are reported at P_s of 97.7% to facilitate comparison with existing design, while fatigue life estimations are calibrated using P_s of 50%, ensuring a representative distribution of predictions within the scatter bands.

The reference strengths ($\Delta\sigma_R$ and $\Delta\tau_R$) used in fatigue life estimations are obtained from both IIW-recommended values and experimentally derived values. The latter are determined through linear regression analysis of uniaxial and pure torsional CA fatigue tests, extrapolated at reference number of cycles to failure (N_R) of 2 million cycles to failure and recalculated at P_s of 50% to provide a standardised basis for comparisons.

Similar to the IIW S-N curve formulations detailed in Chapter 3, fatigue life estimations in this chapter incorporate characteristic knee points, reflecting slope transitions at defined cycle thresholds. For uniaxial CA loading, the IIW defines a knee point at $N_{kp}=10^7$ cycles, with a pre-knee slope of $k=3$, while in pure torsional loading, the knee point occurs at $N_{kp}=10^8$ cycles, with a pre-knee slope of $k_0=5$ (Hobbacher, 2016). Post-knee slopes follow the IIW-recommended values of $k^*=22$ for both loading conditions (Hobbacher, 2016). For VA loading, the Haibach modification rule ($2k - 1$) is applied, adopting $k'=5$ for uniaxial loading and $k_0'=9$ for torsional loading (Haibach, 1989; Hobbacher, 2016).

This approach ensures that fatigue life estimations remain consistent with industry standards, while simultaneously allowing for a more detailed evaluation of critical plane criteria under varying fatigue loading conditions.

4.2.2 Influence of Residual Stresses and Stress-Relieved Conditions

Given the established influence of residual stresses on fatigue behaviour, this study distinguishes between as-welded and stress-relieved joints, as outlined in Chapter 3. As-welded joints retain high residual tensile stresses, which elevate the local load ratio, thereby reducing sensitivity to mean stress effects in fatigue assessments (Atzori, 1983; Susmel and Tovo, 2004). In contrast, stress-relieved joints, subjected to heat treatment to mitigate residual stresses, exhibit improved fatigue strength (Atzori, 1983; Susmel and Tovo, 2004; Susmel, 2009a).

To account for these variations, this study applies FKM guidelines and Sonsino's rules, incorporating enhancement factors derived from established methodologies (Haibach, 1989; Cetin Morris Sonsino, 2009; Rennert *et al.*, 2013). The corresponding enhancement factors for steel and aluminium welded joints are defined by Equations 4.1 and 4.2, respectively (Haibach, 1989; Cetin Morris Sonsino, 2009; Rennert *et al.*, 2013; Susmel, 2014).

$$\begin{aligned}
 f(R) &= 1.32; \text{ if } R < -1 \\
 f(R) &= -0.22 \times R + 1.1; \text{ if } -1 \leq R \leq 0 \\
 f(R) &= -0.2 \times R + 1.1; \text{ if } 0 < R \leq 0.5 \\
 f(R) &= 1; \text{ if } R > 0.5
 \end{aligned} \tag{4.1}$$

$$\begin{aligned}
 f(R) &= 1.88; \text{ if } R < -1 \\
 f(R) &= -0.55 \times R + 1.33; \text{ if } -1 \leq R \leq 0 \\
 f(R) &= -0.66 \times R + 1.33; \text{ if } 0 < R \leq 0.5 \\
 f(R) &= 1; \text{ if } R > 0.5
 \end{aligned} \tag{4.2}$$

These guidelines are applied in evaluating multiaxial fatigue criteria across all stress analysis methods, except when using MWCM in combination with TCD PM. For TCD PM, only standard recommended constants are used, as this approach is derived from EC3 and EC9 (L. Susmel, 2008; Susmel, 2009a, 2009b; Susmel and Askes, 2012; Susmel, 2014). The FAT values for both steel and aluminium welded joints are extrapolated at 5×10^6 cycles to failure, ensuring consistency with industry standards.

While the negative inverse slopes for steel align with IIW recommendations, the pre-knee slope for aluminium under uniaxial loading is adjusted to $k=4.5$, reflecting its distinct fatigue behaviour as outlined in EC9 (Anon, 2007; Susmel, 2009a, 2014).

When validating MWCM combined with TCD PM, EC-based assumptions are employed, where the effective shear stress range is limited to 60% of the compressive stress range, as prescribed by EC3 and EC9 (Susmel, 2009a, 2014; Susmel and Askes, 2012).

4.2.3 Evaluation and Validation of Critical Plane Criteria

To quantify the accuracy of fatigue life estimations, three key statistical metrics are employed, which are the percentage of non-conservative estimates (P_{NC}), percentage of conservative estimates (P_c), and Root Mean Squared Logarithmic Error (RMSLE). The P_{NC} measure the proportion of predictions that underestimate fatigue life, while P_c quantifies overestimation. RMSLE provides an objective measure of the deviation between estimated and experimental fatigue life (Walat and Łagoda, 2014; John *et al.*, 2024).

Regarding fatigue criteria calibration, both the CS approach and MWCM are directly calibrated using uniaxial and pure torsional CA fatigue data, eliminating the need to reassess their accuracy under these specific conditions. Findley's criterion, however, is calibrated solely with uniaxial fatigue data, so its predictive accuracy in uniaxial loading is not re-evaluated. Instead, all three criteria are assessed under more complex multiaxial conditions, including in-phase, out-of-phase, and VA loading scenarios. Notably, both standard and experimentally derived scatter bands are based on uniaxial and torsional CA test data. To ensure consistency across results, only the widest scatter bands displayed in fatigue life graphs are used when determining the degree of conservatism, given the variability in experimentally derived scatter band sizes across different fatigue data series.

The accuracy of fatigue life estimations is computed using the RMSLE, given by Equation 4.3.

$$RMSLE = \sqrt{\frac{\sum_{i=1}^n \left[\left(\log \frac{N_{f(i)}}{N_{f,e(i)}} \right)^2 \right]}{n}} \quad (4.3)$$

where $N_{f(i)}$ denotes the experimentally observed number of cycles to failure for test i , $N_{f,e(i)}$ represents the estimated number of cycles to failure for the same test, and n refers to the total number of observations in each test series. Whereas the T_{RMS} metric, defined in Equation 4.4, provides further assessment of the deviation between estimated and experimental fatigue life results (Walat and Łagoda, 2014; John *et al.*, 2024).

$$T_{RMS} = 10^{RMSLE} \quad (4.4)$$

Fatigue life estimates are classified as conservative (above the widest scatter band, $P_s=97.7\%$) or non-conservative (below the scatter band, $P_s=2.3\%$), ensuring consistency across evaluations while providing a meaningful assessment of the reliability of each criterion.

4.2.4 Validation of Fatigue Life Estimations under VA Loading

Fatigue life validation under VA loading requires additional considerations compared to CA loading, where fatigue cycles are clearly defined. In VA loading, fluctuating multiaxial stress components introduce complexities that necessitate the use of advanced cycle counting techniques to ensure an accurate assessment of fatigue damage.

To improve predictive accuracy and reduce the inherent conservatism of standard damage sum assumptions, this study incorporates real damage sum values, D_{real} , as proposed by Eulitz and Sonsino (Eulitz, 2000; Sonsino, 2007). Instead of adopting the conventional $D=0.5$ assumption prescribed by EC3 and IIW, the analysis applies $D_{real}=0.27$ for aluminium welded joints and $D_{real}=0.45$ for steel welded joints when using experimentally derived reference curves (Eulitz, 2000; Sonsino, 2007). These modifications offer a more representative estimation of cumulative fatigue damage effects, resulting in improved fatigue life predictions.

A critical challenge in VA loading is the identification of closed fatigue cycles, which is relatively straightforward in unidirectional CA loading but significantly more complex in multiaxial stress conditions. Conventional rainflow cycle counting is effective for single component loading but struggles to accurately capture fatigue cycles in multiaxial scenarios (Matsuishi, 1968; Endo, 1974; Langlais, Vogel and Chase, 2003; Lee and Pan, 2005; Lee and Tjhung, 2011; Standard, 2017). To address this limitation, this reanalysis applies the CS counting method for fatigue criteria that consider both normal and shear stress components, specifically Findley's criterion and the CS criterion (Carpinteri, Spagnoli and Vantadori, 2003; Bibbo *et al.*, 2022; Bibbo, Baumgartner and Arora, 2022). As detailed in Chapter 2, the CS counting method effectively considers simultaneous normal and shear stress cycles, ensuring more reliable fatigue life estimations for welded joints subjected to VA loading when using Findley's criterion and the CS criterion (Carpinteri, Spagnoli and Vantadori, 2003).

For the MWCM, which is inherently shear stress-based, an alternative strategy is necessary. Instead of tracking both normal and shear stress cycles, Susmel's Maximum Variance Method (MVM) is applied to extract a single effective shear stress range, simplifying the stress history and enabling the use of conventional rainflow counting techniques (Susmel, 2010, 2014; Susmel and Taylor, 2012; Faruq and Susmel, 2019).

By systematically implementing the CS counting method for Findley's criterion and the CS criterion, along with MVM for MWCM, this quantitative reanalysis establishes a comprehensive validation framework for fatigue damage in welded joints under VA loading. The integration of these methodologies reinforces the reliability of stress-based critical plane criteria, providing deeper insights into their applicability for multiaxial fatigue life estimations in VA loading conditions.

4.3 Reanalyses Using the NS Approach

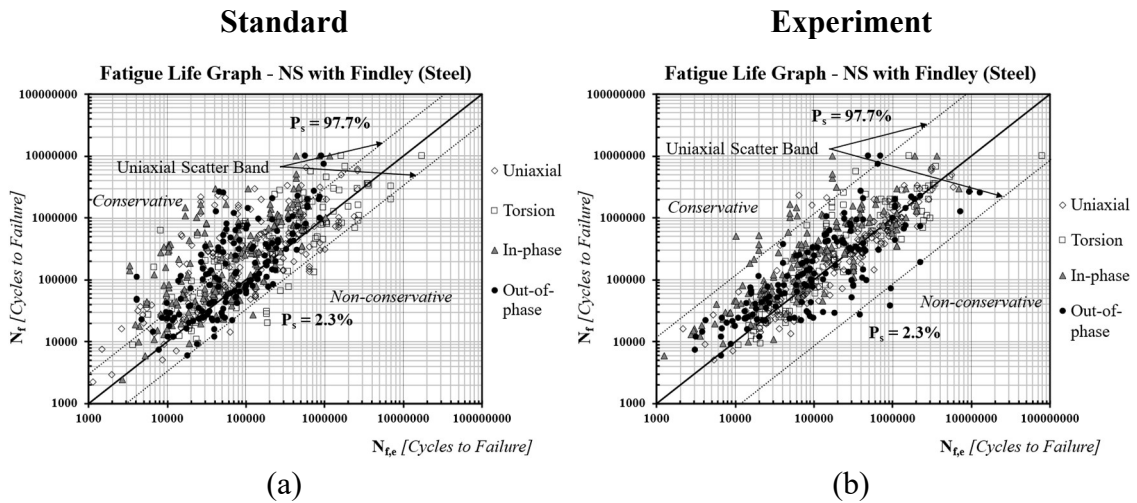
The NS approach, previously discussed in Chapter 3, serves as the foundation for reanalysing fatigue life estimations using critical plane criteria. While its simplicity makes it a practical starting point for fatigue assessment, its application in conjunction

with Findley's criterion, the CS approach, and the MWCM is further explored here to evaluate its effectiveness under CA and VA loading conditions.

4.3.1 CA Loading

Table 4.1, presented at the end of this subsection, provides a summary of the reanalysed steel and aluminium welded joints under CA loading using the NS approach, including recalculated standard and experimental reference fatigue strengths at $P_s=97.7\%$ and the corresponding k values. Fatigue life estimations for Findley's criterion, the CS approach, and the MWCM are illustrated in Figures 4.1 and 4.2 for steel and aluminium welded joints, respectively.

Findley's criterion demonstrates strong predictive capability for steel welded joints, as shown in Figures 4.1a and 4.1b, with most data points falling within the widest scatter band, ensuring conservative yet reliable estimates when using standard reference curves.



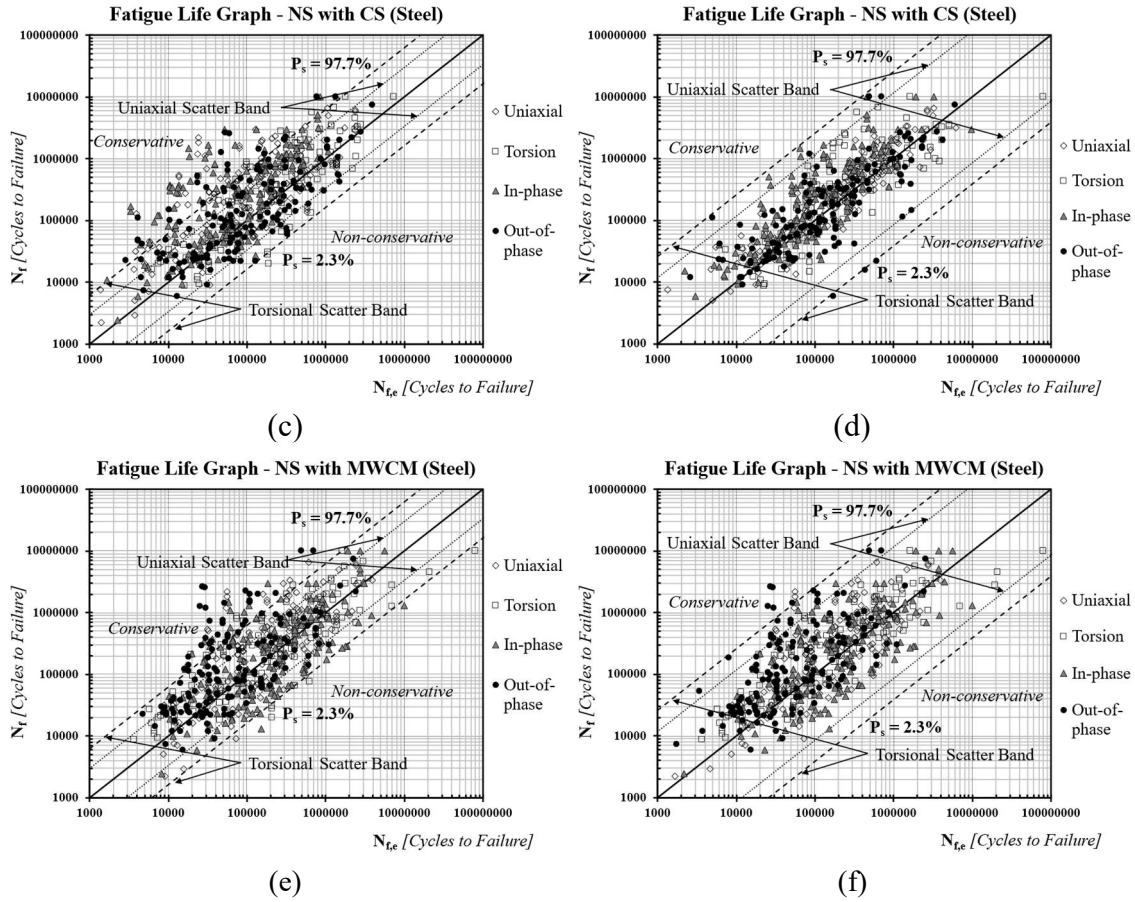


Figure 4.1: Fatigue life curves for as-welded and stress-relieved steel joints using the NS approach under CA loading: Findley's criterion (a, b), CS criterion (c, d), and MWCM (e, f), comparing standard and experimental reference strengths at $P_s = 50\%$.

When experimentally derived constants are applied, the P_c decreases from 32% to 3.5%, while the P_{NC} slightly reduces from 2.8% to 2.1%, reflecting improved predictive accuracy. Similar trends are observed for the CS approach and MWCM, where experimentally constants enhance accuracy compared to standard values. For steel joints under CA loading, CS criterion exhibits the highest accuracy, followed by Findley's criterion and the MWCM when using experimental constants. However, differences among the three criteria remain minimal, with all methods providing satisfactory predictions.

In contrast, aluminium welded joints exhibit greater conservatism, particularly in out-of-phase loading, resulting in higher P_c values. The MWCM delivers the most accurate fatigue life predictions, followed by the CS criterion, with Findley's criterion exhibiting the largest scatter.

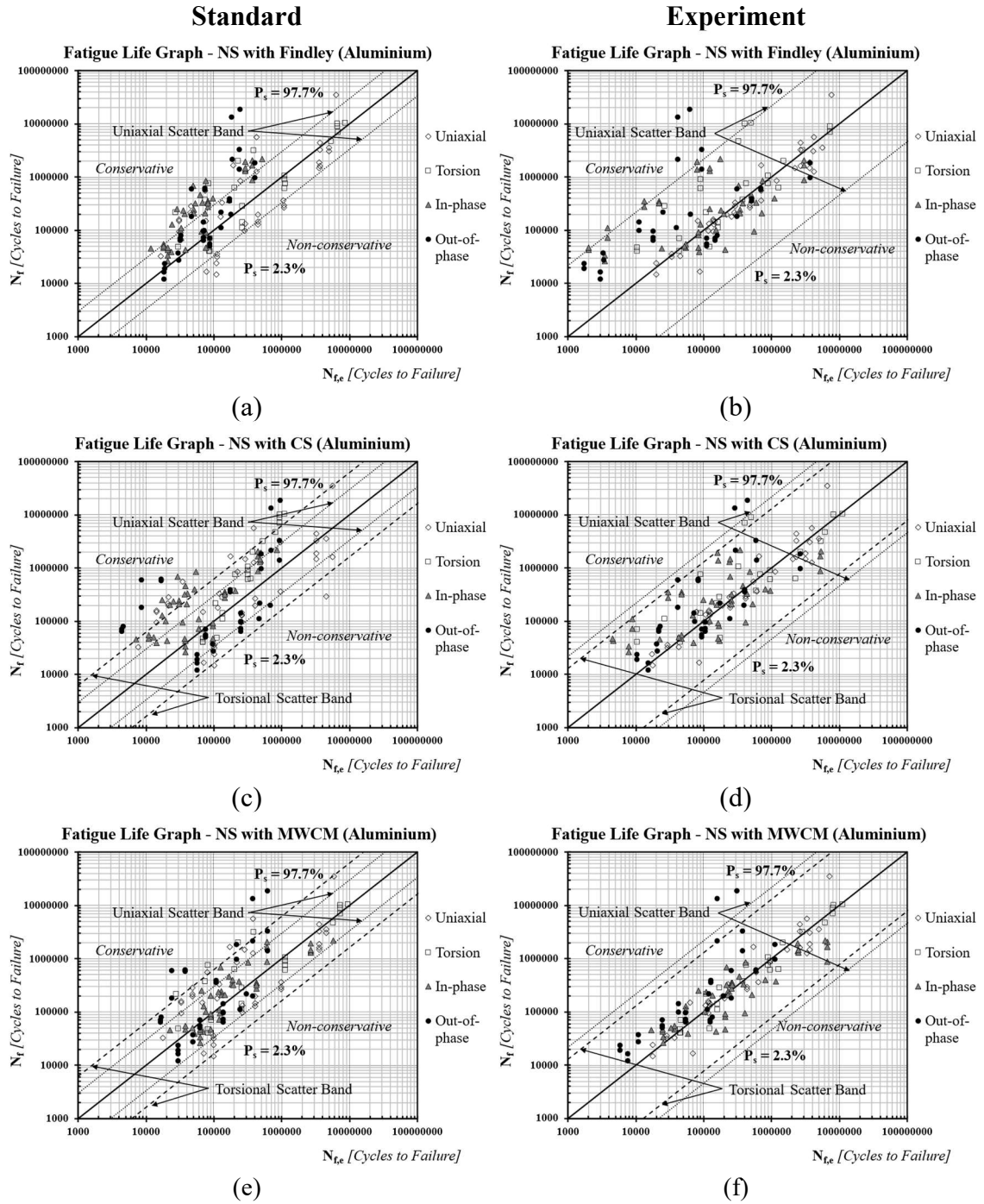


Figure 4.2: Fatigue life curves for as-welded and stress-relieved aluminium joints using the NS approach under CA loading: Findley's criterion (a, b), CS criterion (c, d), and MWCM (e, f), comparing standard and experimental reference strengths at $P_s=50\%$.

Despite this variability, all predictions remain conservative, maintaining an acceptable safety margin for practical applications.

Table 4.1: Summary of reanalysed welded joints using NS approach combined with critical plane criteria under CA loading, detailing uniaxial and torsional reference fatigue strengths, joint geometries, materials, fatigue curve slopes, and data sources.

Material (CA)	$\Delta\sigma_{R,P_s=97.7\%}^{(1)}$ (MPa)		Uniaxial curve slope, k		$\Delta\tau_{R,P_s=97.7\%}^{(1)}$ (MPa)		Torsional curve slope, k ₀		Geometry	Reference
	IIW	Exp ⁽²⁾	IIW	Exp ⁽²⁾	IIW	Exp ⁽²⁾	IIW	Exp ⁽²⁾		
UM StE 460 *	71	142.7	3	4.4	100	127.5	5	4.9	Figure 3.2a	(Sonsino, 1995)
M StE 460 *	71	166.5	3	4.6	100	-	5	-	Figure 3.2a	(Sonsino, 1997)
UM StE 460 *	71	194.7	3	4.4	100	-	5	-	Figure 3.2e	(Sonsino, 1997)
M StE 460 *	71	321.6	3	8.2	100	128.0	5	6.3	Figure 3.2e	(Sonsino, 1997)
StE 460 *	71	116.4	3	3.9	100	180.9	5	7.3	Figure 3.2a	(Yousefi, Witt and Zenner, 2001)
StE 460	71	122.9	3	5.4	100	80.5	5	6.1	Figure 3.2a	(Amstutz, Störzel and Seeger, 2001)
A519	71	96.3	3	5.4	80	94.2	5	3.7	Figure 3.2b	(Yung and Lawrence Jr, 1989)
A519-A36 *	80	144.4	3	3.8	100	104.2	5	5.5	Figure 3.2d	(Siljander, Kurath and Lawrence Jr, 1989)
BS4360 Gr.50E	71	65.6	3	3.0	80	66.3	5	4.5	Figure 3.2c	(Razmjoo, 1997)
Fe 52 steel	45	15.8	3	2.3	100	44.1	5	3.5	Figure 3.2f	(Bäckström, 2003)
BS4360	80	-	3	-	80	-	5	-	Figure 3.2d	(Archer, 1987)
S340+N, E355+N	71	204.2	3	6.1	100	207.8	5	6.6	Figure 3.2e	(Shams and Vormwald, 2017)
S340+N, E355+N *	71	44.6	3	2.9	100	71.3	5	3.6	Figure 3.2e	(Shams and Vormwald, 2017)
St 35 (t = 1 mm)	71	69.2	3	4.9	100	106.4	5	8.4	Figure 3.2e	(Wiebesiek and Sonsino, 2010)
St 35 (t = 2 mm)	71	66.4	3	5.3	100	74.5	5	6.1	Figure 3.2e	(Sonsino <i>et al.</i> , 2006)
6082-T6 *	32	55.0	3	6.9	36	50.4	5	6.2	Figure 3.2a	(Kueppers and Sonsino, 2003)
6060-T6	22	84.0	3	5.5	36	-	5	-	Figure 3.2e	(Costa <i>et al.</i> , 2005)
AW 6082	22	20.3	3	4.2	36	42.3	5	5.8	Figure 3.2e	(Wiebesiek <i>et al.</i> , 2011)
AW 5042	22	16.6	3	4.1	36	42.5	5	5.9	Figure 3.2e	(Wiebesiek <i>et al.</i> , 2011)

⁽¹⁾Reference normal and shear stresses extrapolated at 2 million cycles to failure, with probability of survival (P_s)=97.7%.

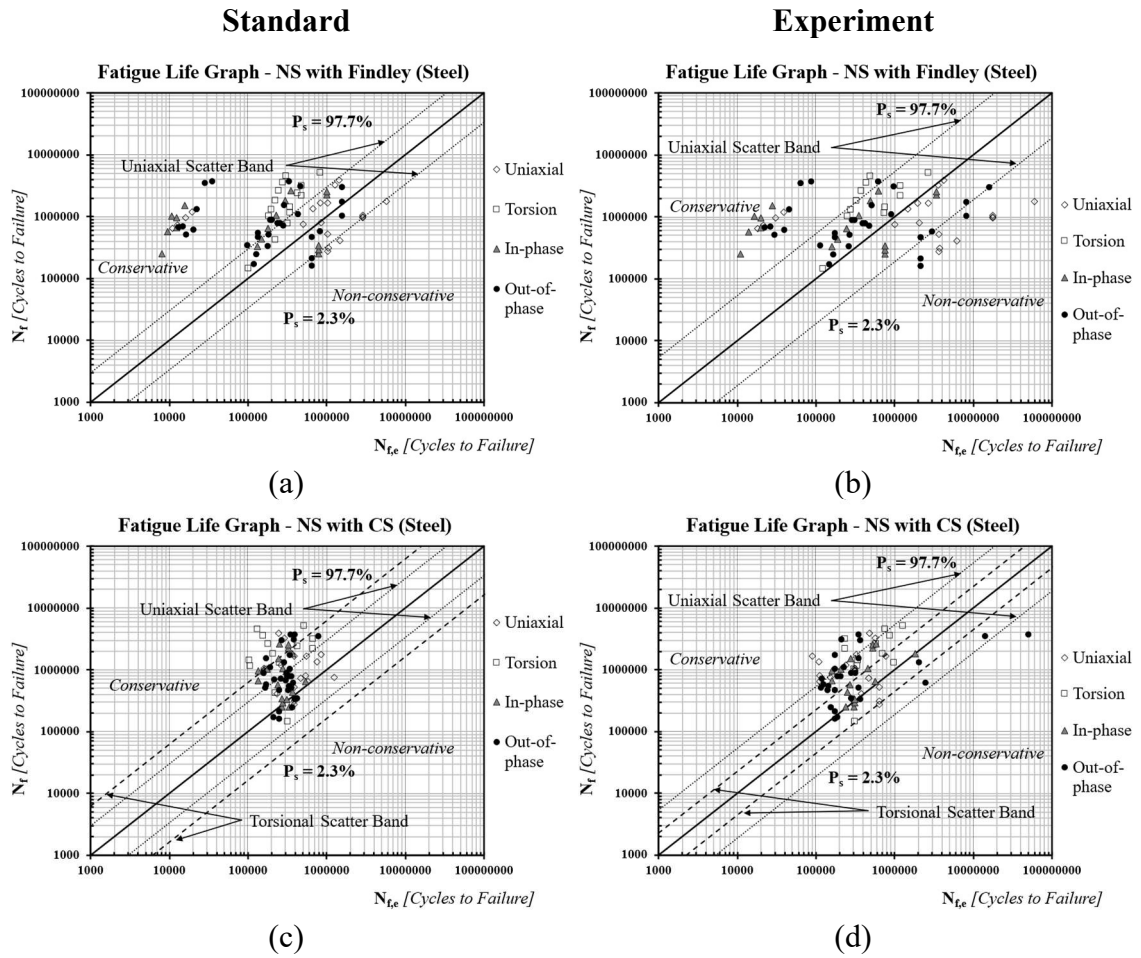
⁽²⁾Exp denotes experimental data.

* Stress-relieved

UM – Unmachined; M – Machined.

4.3.2 VA Loading

An overview of the reanalysed steel and aluminium welded joints under VA loading using the NS approach is provided in Table 4.2 at the end of this subsection. The table includes recalculated standard and experimental reference fatigue strengths at $P_s=97.7\%$, along with their corresponding k values. Figures 4.3 and 4.4 illustrate the fatigue life estimations for steel and aluminium welded joints under VA loading using this approach.



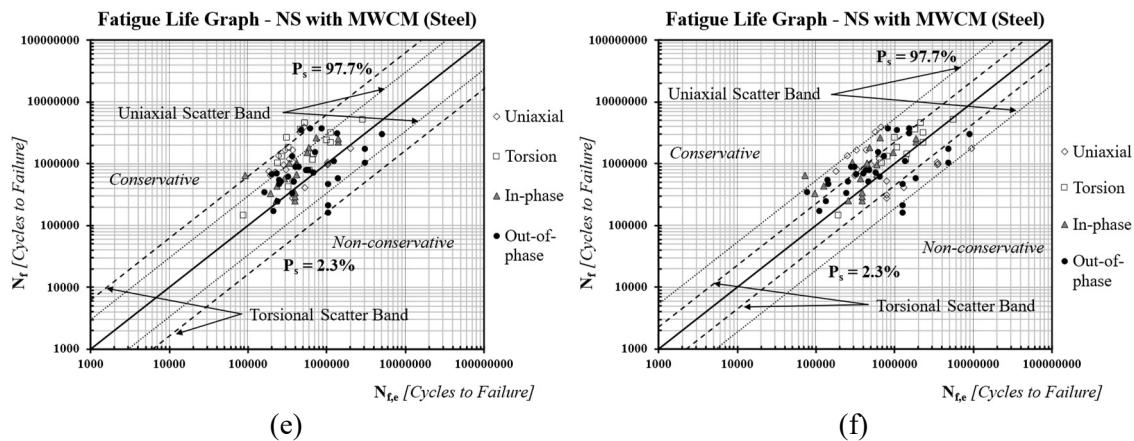


Figure 4.3: Fatigue life curves for as-welded and stress-relieved steel joints using the NS approach under VA loading: Findley's criterion (a, b), CS criterion (c, d), and MWCM (e, f), comparing standard and experimental reference strengths at $P_s=50\%$.

For steel welded joints, Findley's criterion exhibited significant scatter, particularly at loading ratios $R \geq 0$, leading to highly conservative fatigue life predictions. This conservatism stems from the inclusion of maximum normal stress in its formulation, which, combined with residual welding stresses already accounted for in the design reference curves, results in overly cautious estimations (Findley, 1959; Bruun and Härkegård, 2015).

The effect is more pronounced for stress-relieved steel joints, where the applied enhancement factors in the reanalysis were insufficient to fully mitigate conservatism. However, when experimentally derived constants were implemented, the predictions became less conservative, but an increase in P_{NC} was observed. This discrepancy suggests that Findley's criterion, originally calibrated using only uniaxial fatigue data, may not fully capture the multiaxial fatigue behaviour of welded steel joints (Bruun and Härkegård, 2015; Pedersen, 2016).

Among the three critical plane criteria, the MWCM demonstrated the highest accuracy for steel joints under VA loading, followed by the CS approach and Findley's criterion, as reflected in their respective average T_{RMS} values. As expected, VA loading introduces greater scatter in fatigue life predictions compared to CA loading; however, most estimates remain conservative and fall within the widest scatter bands, ensuring a sufficient safety margin for design purposes.

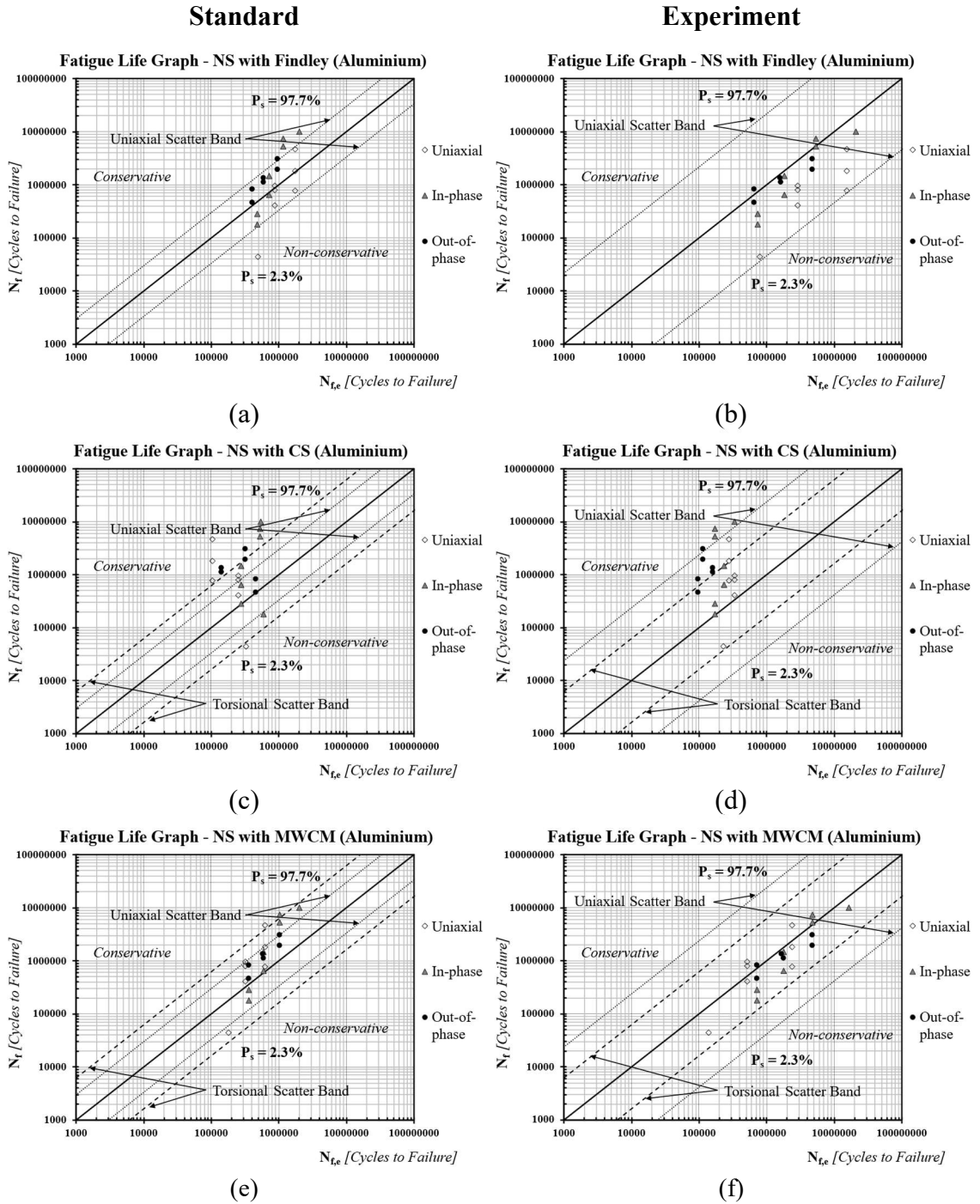


Figure 4.4: Fatigue life curves for as-welded and stress-relieved aluminium joints using the NS approach under VA loading: Findley's criterion (a, b), CS criterion (c, d), and MWCM (e, f), comparing standard and experimental reference strengths at $P_s=50\%$.

For aluminium welded joints, a different trend is observed. The CS approach exhibited greater scatter in fatigue life estimates than both Findley's criterion and the MWCM. The MWCM remained the most accurate, followed by Findley's criterion, which

produced conservative predictions but with a higher degree of variability. This suggests that while all three criteria can be applied to aluminium welded joints under VA loading, MWCM provides the most reliable and consistent fatigue life estimates.

Given the limited availability of experimental data for aluminium welded joints under VA loading, further research is necessary to comprehensively validate these fatigue criteria (Costa *et al.*, 2005; Kueppers and Sonsino, 2006; Wiebesiek and Sonsino, 2010; Wiebesiek *et al.*, 2011). Nonetheless, the application of the NS approach in combination with stress-based critical plane criteria provides safe and reliable fatigue life predictions for both steel and aluminium welded joints, regardless of loading complexity or joint condition. While experimentally derived constants yield more precise predictions, standard reference curves remain effective in maintaining safe design thresholds without excessive conservatism.

Table 4.2: Summary of reanalysed welded joints using NS approach combined with critical plane criteria under VA loading, detailing uniaxial and torsional reference fatigue strengths, joint geometries, materials, fatigue curve slopes, and data sources.

Material (VA)	$\Delta\sigma_{R,P_s=97.7\%}^{(1)}$ (MPa)		Uniaxial curve slope, k		$\Delta\tau_{R,P_s=97.7\%}^{(1)}$ (MPa)		Torsional curve slope, k0		Geometry	Reference
	IIW	Exp ⁽²⁾	IIW	Exp ⁽²⁾	IIW	Exp ⁽²⁾	IIW	Exp ⁽²⁾		
StE 460 *	71	142.7	3	4.4	100	127.5	5	4.9	Figure 3.2a	(Sonsino and Kueppers, 2001)
StE 460 *	71	116.4	3	3.9	100	180.9	5	7.3	Figure 3.2a	(Yousefi, Witt and Zenner, 2001)
42CrMo4 *	-	-	-	-	100	-	5	-	Figure 3.2e	(Pyttel, Grawenhof and Berger, 2012)
6082-T6	32	55.0	3	6.9	36	42.3	5	6.3	Figure 3.2e	(Kueppers and Sonsino, 2006)

⁽¹⁾Reference normal and shear stresses extrapolated at 2 million cycles to failure, with probability of survival (P_s)=97.7%.

⁽²⁾Exp denotes experimental data.

* Stress-relieved

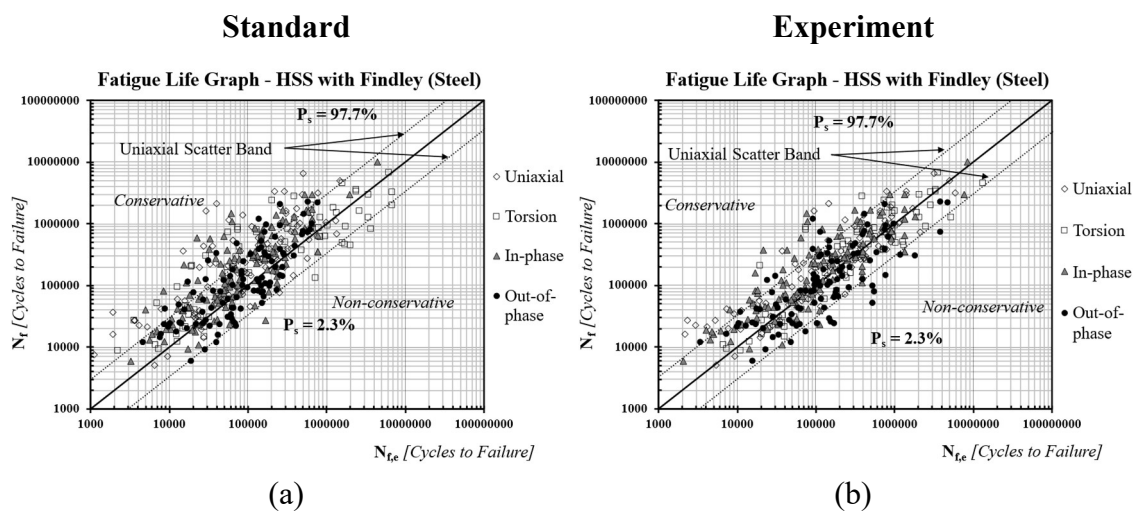
4.4 Reanalyses Using the HSS Approach

The HSS approach, previously introduced in Chapter 2, serves as an alternative to the NS approach for assessing welded joints with complex geometries. Its theoretical foundation and conventional applications have been discussed in Chapters 2 and 3 (Niemi, 1995; Radaj, 1996; Anon, 2005, 2007; Radaj, Sonsino and Fricke, 2006; Hobbacher, 2009, 2016; Alencar *et al.*, 2021; Mei *et al.*, 2021).

This section applies the HSS approach to the quantitative reanalysis of multiaxial fatigue using stress-based critical plane criteria. Fatigue life predictions are evaluated under CA and VA loading, providing a direct comparative assessment of the predictive accuracy of Findley's criterion, the CS approach, and the MWCM.

4.4.1 CA Loading

Table 4.3, provided at the end of this subsection, summarises the key characteristics of the reanalysed steel and aluminium welded joints under CA loading using the HSS approach. It includes recalculated standard and experimental reference fatigue strengths at $P_s=97.7\%$, along with their respective k values. The corresponding fatigue life predictions using Findley's criterion, the CS approach, and the MWCM are illustrated in Figures 4.5 and 4.6.



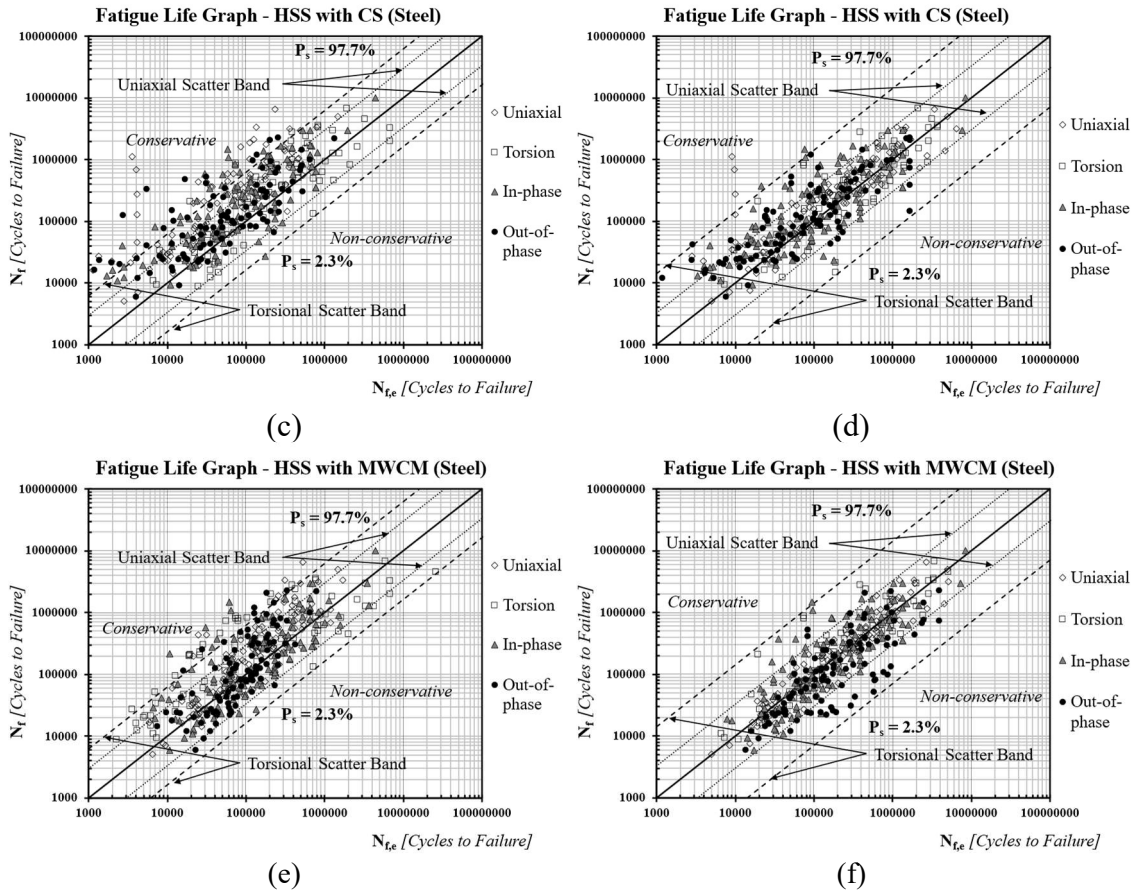


Figure 4.5: Fatigue life curves for as-welded and stress-relieved steel joints using the HSS approach under CA loading: Findley's criterion (a, b), CS criterion (c, d), and MWCM (e, f), comparing standard and experimental reference strengths at $P_s=50\%$.

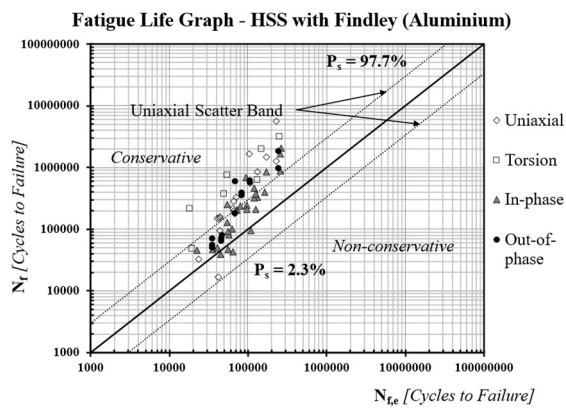
Across all fatigue criteria, the majority of fatigue life predictions fall within the widest scatter bands, with P_{NC} values consistently low, regardless of whether standard or experimentally derived constants are used. Consistent with the NS approach, implementing experimentally derived constants improves predictive accuracy, as evidenced by lower T_{RMS} values and a notable reduction in the average P_c from 23% to 10.1%.

The HSS approach demonstrates a clear improvement in predictive accuracy across all fatigue criteria compared to the NS approach, with lower T_{RMS} values observed for both steel and aluminium welded joints under CA loading. Among the fatigue criteria, the MWCM exhibits the highest accuracy, followed closely by Findley's criterion and the CS approach, particularly for steel joints. Although differences between the criteria are

minimal, all three approaches yield satisfactory results when experimentally derived constants are applied.

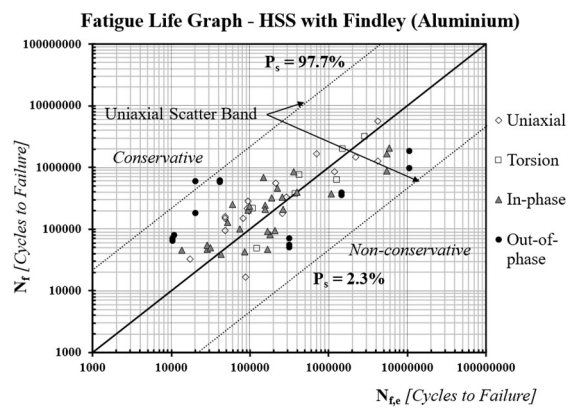
When employing standard reference curves, the MWCM again delivers the most accurate fatigue life predictions, followed by Findley's criterion and the CS approach, with P_{NC} values consistently below 10% for standard constants and remaining under 13% for experimental constants.

Standard

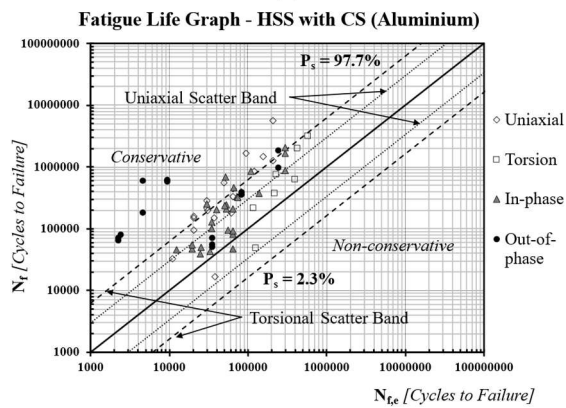


(a)

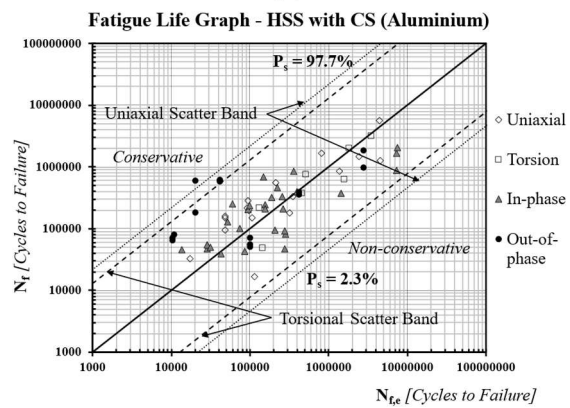
Experiment



(b)



(c)



(d)

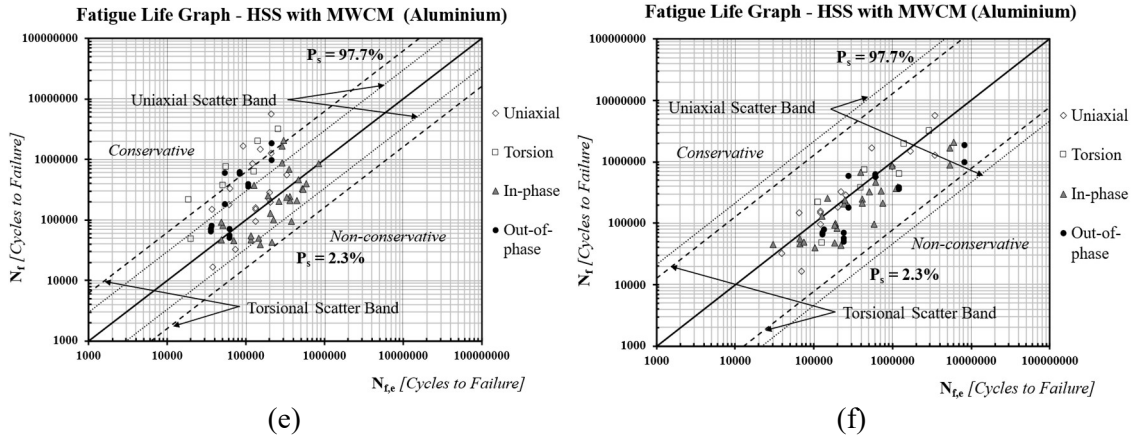


Figure 4.6: Fatigue life curves for as-welded and stress-relieved aluminium joints using the HSS approach under CA loading: Findley's criterion (a, b), CS criterion (c, d), and MWCM (e, f), comparing standard and experimental reference strengths at $P_s=50\%$.

For aluminium welded joints, similar trends to those observed with the NS approach emerge, including greater conservatism and increased scatter compared to steel joints, as reflected by higher P_c and T_{RMS} values. Nevertheless, the MWCM provides the most reliable fatigue life predictions, followed by the CS approach, while Findley's criterion exhibits the largest scatter. Despite this increased scatter, all predictions remain conservative, ensuring sufficient safety margins in fatigue design.

Table 4.3: Summary of reanalysed welded joints using HSS approach combined with critical plane criteria under CA loading, detailing uniaxial and torsional reference fatigue strengths, joint geometries, materials, fatigue curve slopes, and data sources.

Material (CA)	$\Delta\sigma_{R,P_s=97.7\%}^{(1)}$ (MPa)		Uniaxial curve slope, k		$\Delta\tau_{R,P_s=97.7\%}^{(1)}$ (MPa)		Torsional curve slope, k0		Geometry	Reference
	IIW	Exp ⁽²⁾	IIW	Exp ⁽²⁾	IIW	Exp ⁽²⁾	IIW	Exp ⁽²⁾		
UM StE 460 *	90	188.7	3	4.4	100	138.7	5	4.9	Figure 3.2a	(Sonsino, 1995)
M StE 460 *	90	210.0	3	4.6	100	-	5	-	Figure 3.2a	(Sonsino, 1997)
UM StE 460 *	90	208.4	3	4.4	100	-	5	-	Figure 3.2e	(Sonsino, 1997)
StE 460 *	90	163.1	3	3.9	100	206.7	5	7.3	Figure 3.2a	(Yousefi, Witt and Zenner, 2001)
A519	90	112.7	3	5.4	80	100.7	5	3.7	Figure 3.2b	(Yung and Lawrence Jr, 1989)
A519-A36 *	90	153.6	3	3.8	100	110.7	5	5.5	Figure 3.2d	(Siljander, Kurath and Lawrence Jr, 1989)
BS4360 Gr.50E	90	111.7	3	3.0	80	72.7	5	4.5	Figure 3.2c	(Razmjoo, 1997)
Fe 52 steel	90	22.7	3	2.3	100	48.4	5	3.5	Figure 3.2f	(Bäckström, 2003)
S340+N, E355+N	90	343.0	3	6.1	100	229.2	5	6.6	Figure 3.2e	(Shams and Vormwald, 2017)
S340+N, E355+N *	90	74.8	3	2.9	100	74.8	5	3.6	Figure 3.2e	(Shams and Vormwald, 2017)
6082-T6 *	36	76.1	3	6.9	36	55.0	-	6.3	Figure 3.2a	(Kueppers and Sonsino, 2003)
6060-T6	36	122.5	3	-	36	-	-	-	Figure 3.2e	(Costa <i>et al.</i> , 2005)

⁽¹⁾Reference normal and shear stresses extrapolated at 2 million cycles to failure, with probability of survival (P_s)=97.7%.

⁽²⁾Exp denotes experimental data.

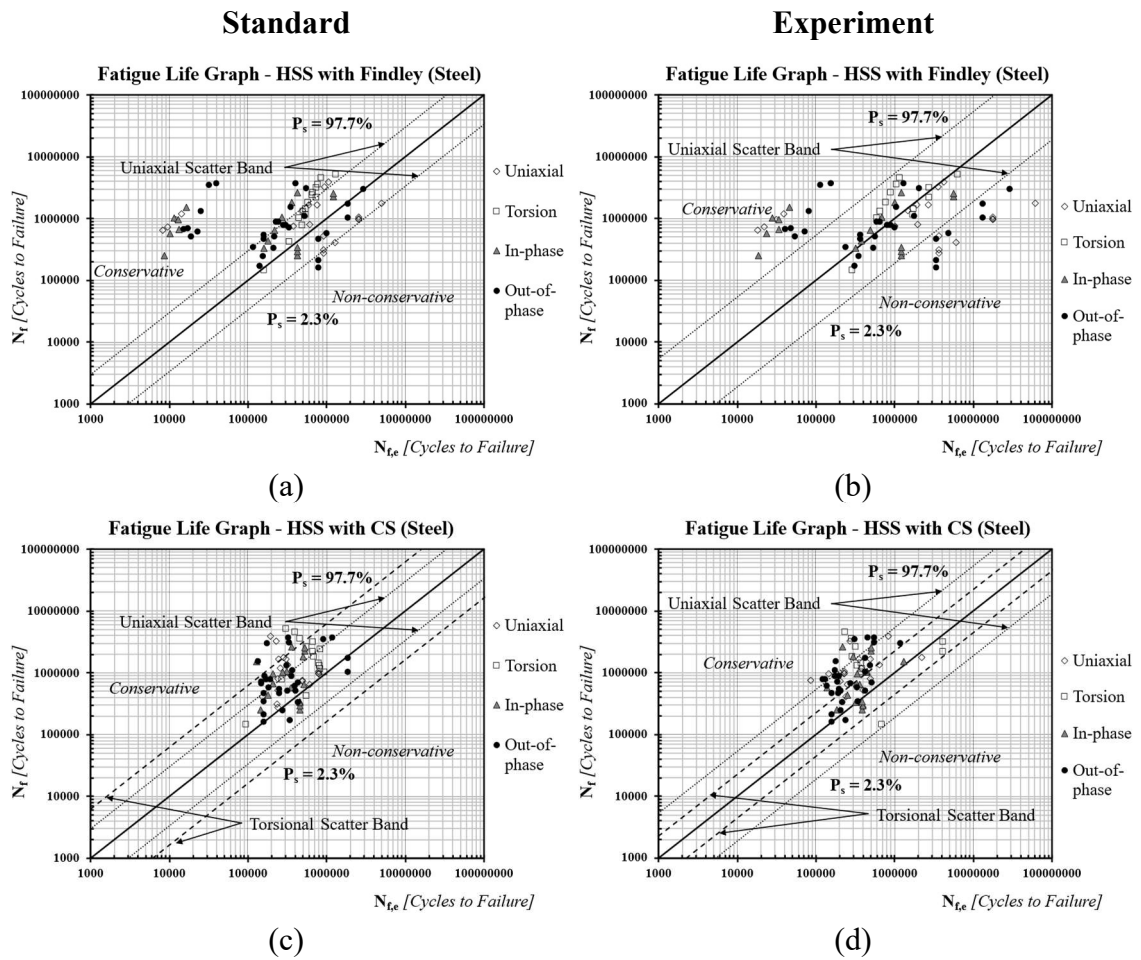
* Stress-relieved

UM – Unmachined; M – Machined.

4.4.2 VA Loading

The essential characteristics of the reanalysed steel and aluminium welded joints under VA loading are detailed in Table 4.4 found at the end of this subsection, which includes recalculated reference fatigue strengths at $P_s=97.7\%$ and their associated k values.

Under VA loading, steel welded joints exhibited greater scatter in fatigue life predictions compared to CA loading, as illustrated in Figure 4.7.



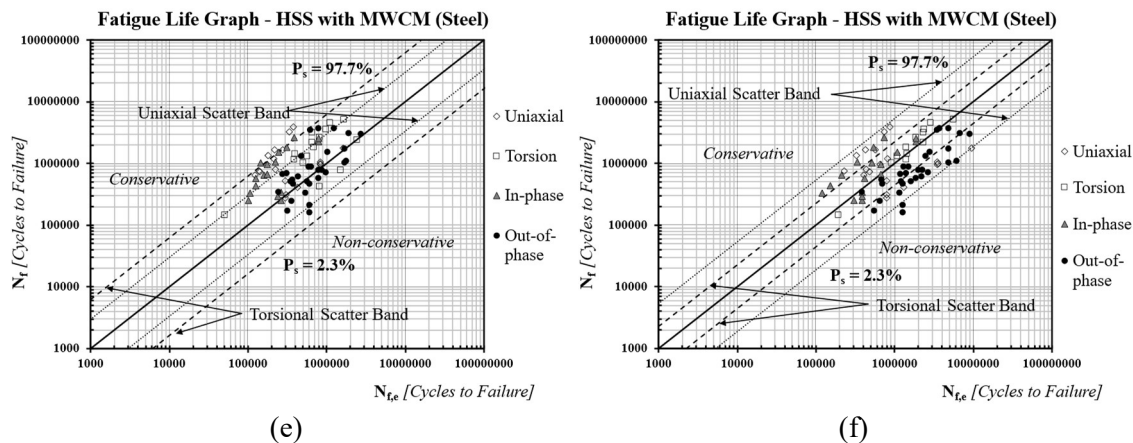


Figure 4.7: Fatigue life curves for as-welded and stress-relieved steel joints using the HSS approach under VA loading: Findley's criterion (a, b), CS criterion (c, d), and MWCM (e, f), comparing standard and experimental reference strengths at $P_s=50\%$.

Among the evaluated criteria, the MWCM provided the most accurate fatigue life estimates, followed by the CS approach, while Findley's criterion produced the largest scatter. This ranking is consistent with the results observed under CA loading, reinforcing the HSS approach's ability to improve accuracy by incorporating overall structural geometry rather than solely relying on nominal stress distributions. Despite the observed scatter, the HSS approach still yielded less variation in fatigue life predictions compared to the NS approach, indicating an overall improvement in predictive reliability.

For aluminium welded joints under VA loading, presented in Figure 4.8, the CS criterion demonstrated the highest degree of scatter, followed by Findley's criterion and the MWCM. The MWCM continued to provide the most accurate predictions, while Findley's criterion produced conservative yet more widely dispersed estimates. The increased variability in aluminium joint predictions compared to steel joints highlights the material's greater sensitivity to stress variations, particularly under multiaxial VA loading conditions.

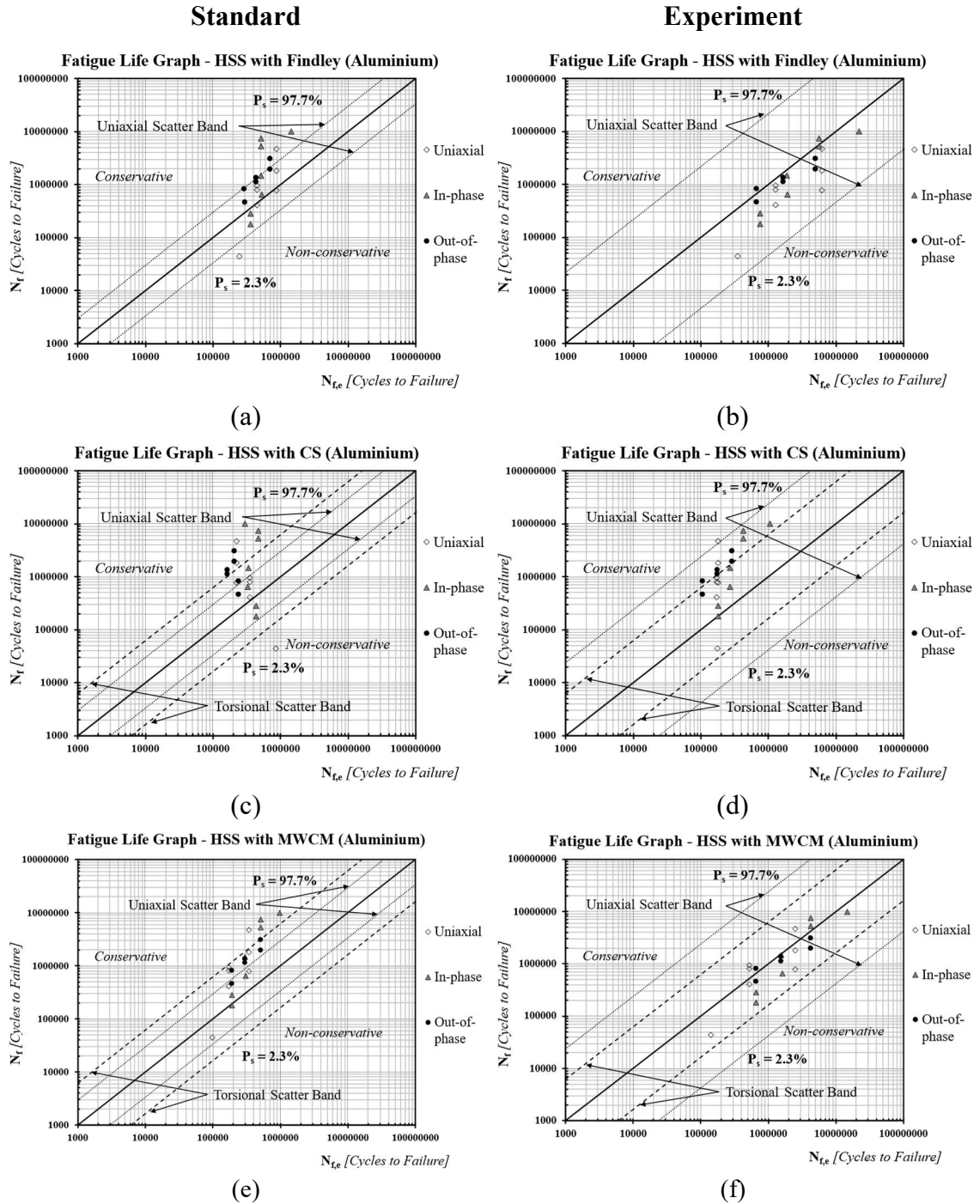


Figure 4.8: Fatigue life curves for as-welded and stress-relieved aluminium joints using the HSS approach under VA loading: Findley's criterion (a, b), CS criterion (c, d), and MWCM (e, f), comparing standard and experimental reference strengths at $P_s=50\%$.

In summary, the application of stress-based critical plane criteria using the HSS approach yields reliable fatigue life predictions for both steel and aluminium welded joints, particularly in scenarios where critical regions are located at the weld toe.

Table 4.4: Summary of reanalysed welded joints using HSS approach combined with critical plane criteria under VA loading, detailing uniaxial and torsional reference fatigue strengths, joint geometries, materials, fatigue curve slopes, and data sources.

Material (VA)	$\Delta\sigma_{R,P_s=97.7\%}^{(1)}$ (MPa)		Uniaxial curve slope, k		$\Delta\tau_{R,P_s=97.7\%}^{(1)}$ (MPa)		Torsional curve slope, k0		Geometry	Reference
	IIW	Exp ⁽²⁾	IIW	Exp ⁽²⁾	IIW	Exp ⁽²⁾	IIW	Exp ⁽²⁾		
StE 460 *	90	188.7	3	4.4	100	138.7	5	4.9	Figure 3.2a	(Sonsino and Kueppers, 2001)
StE 460 *	90	163.1	3	3.9	100	206.7	5	7.3	Figure 3.2a	(Yousefi, Witt and Zenner, 2001)
42CrMo4 *	-	-	-	-	100	-	5	-	Figure 3.2e	(Pyttel, Grawenhof and Berger, 2012)
6082-T6	36	76.1	3	6.9	36	55.0	5	6.3	Figure 3.2e	(Kueppers and Sonsino, 2006)

⁽¹⁾Reference normal and shear stresses extrapolated at 2 million cycles to failure, with probability of survival (P_s)=97.7%.

⁽²⁾Exp denotes experimental data.

* Stress-relieved

4.5 Reanalyses Using the ENS Approach

Building on the ENS approach outlined in Chapter 2 and implemented in Chapter 3 following IIW recommendations, this reanalysis evaluates multiaxial fatigue using stress-based critical plane criteria. To account for variations in welded joint thickness, both the standard 1 mm fictitious notch radius and the alternative 0.05 mm fictitious notch radius proposed by Sonsino are adopted (Radaj, 1996; Morgenstern *et al.*, 2006; Radaj, Sonsino and Fricke, 2006; Cetin Morris Sonsino, 2009; Sonsino, Bruder and Baumgartner, 2010; Susmel, Sonsino and Tovo, 2011; Schijve, 2012; Sonsino *et al.*, 2012; Radaj, Lazzarin and Berto, 2013; Hobbacher, 2016; Pedersen, 2016). This adjustment ensures a more representative fatigue assessment across different welded joint configurations, particularly for thin and flexible joints where conventional ENS parameters may not fully capture fatigue behaviour.

This section extends the ENS approach to a quantitative reanalysis of multiaxial fatigue behaviour, comparing the predictive accuracy of Findley's criterion, the CS approach, and the MWCM under both CA and VA loading conditions. By systematically evaluating these criteria, this study provides a direct comparison of their effectiveness in fatigue life estimation for welded joints, offering deeper insights into their applicability across various loading conditions.

4.5.1 CA Loading

A summary of the key parameters for reanalysed steel and aluminium welded joints under CA loading using the ENS approach is provided in Table 4.5 at the end of this subsection, including standard and experimental reference fatigue strengths recalculated at $P_s=97.7\%$ along with their corresponding k values. The fatigue life predictions obtained from Findley's criterion, the CS approach, and the MWCM are illustrated in Figures 4.9 and 4.10 for steel and aluminium welded joints, respectively.

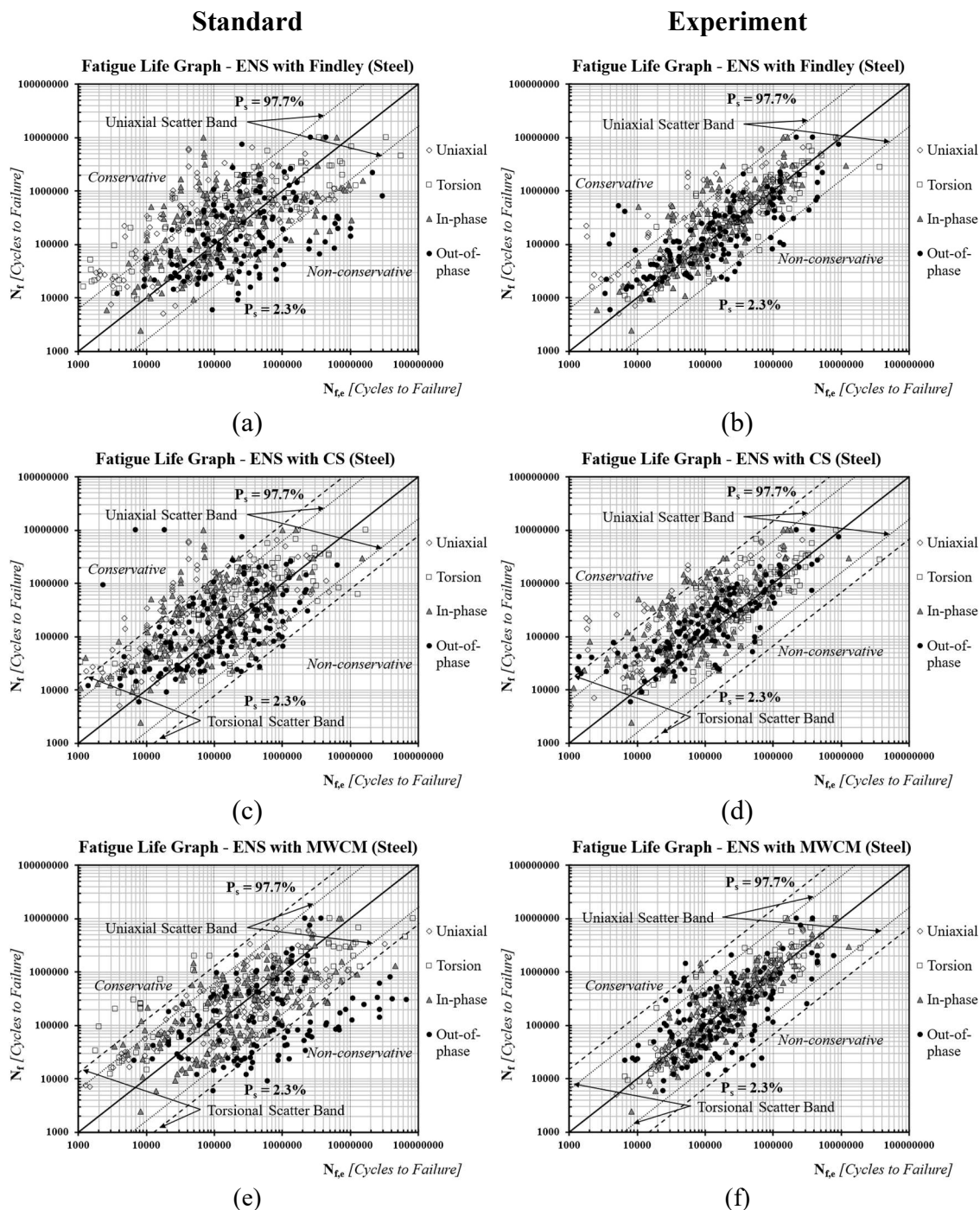


Figure 4.9: Fatigue life curves for as-welded and stress-relieved steel joints using the ENS approach under CA loading: Findley's criterion (a, b), CS criterion (c, d), and MWCM (e, f), comparing standard and experimental reference strengths at $P_s=50\%$.

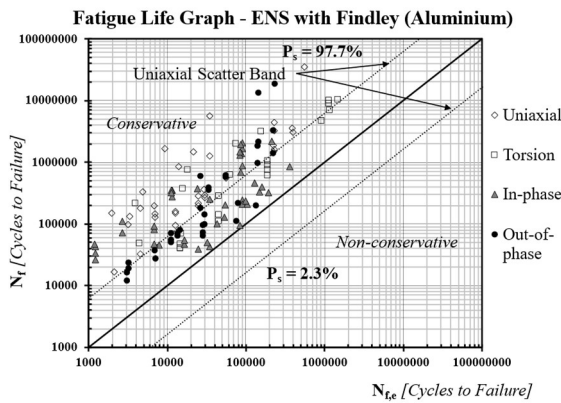
For steel welded joints under CA loading, the ENS approach exhibited greater scatter, and a higher P_{NC} compared to the NS and HSS approaches, particularly when using standard reference curves. This increased variability was primarily influenced by the

stress-relieved steel joints from Shams's study, particularly those thinner than 5 mm (Shams and Vormwald, 2017).

To address this limitation, Sonsino's approach recommends a 0.05 mm fictitious notch radius, with k of 5 for uniaxial loading and k' of 7 for pure torsion reference curves, along with reference fatigue strengths of 630 MPa and 250 MPa, respectively (Cetin Morris Sonsino, 2009; Sonsino, Bruder and Baumgartner, 2010; Sonsino *et al.*, 2012). However, experimental results revealed significant deviations from these standard values, with lower observed reference fatigue strengths of 243 MPa for uniaxial loading and 191 MPa for torsional loading. Additionally, steeper experimental slopes were noted, with $k=2.9$ for uniaxial loading and $k'=3.6$ for torsional loading. These deviations contributed to higher scatter and increased non-conservative fatigue predictions, particularly in stress-relieved steel joints. This highlights the limitations of standard ENS parameters in capturing the fatigue behaviour of thin and flexible joints.

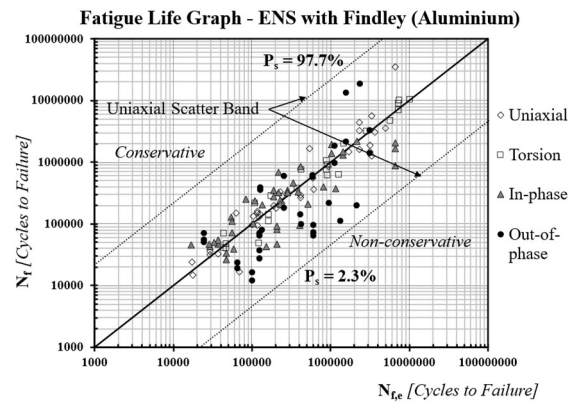
Despite these discrepancies, the use of experimentally derived fatigue curves significantly improved predictive accuracy, with most fatigue life estimations falling within acceptable scatter bands. Among the three critical plane criteria, Findley's criterion demonstrated the highest accuracy, followed by the MWCM, while the CS criterion exhibited the largest scatter. These findings emphasise the importance of experimental validation, as standard ENS parameters may not be universally applicable to thin-walled structures. Therefore, for unconventional welded joints, individual testing is recommended to ensure that reference constants are appropriate.

Standard



(a)

Experiment



(b)

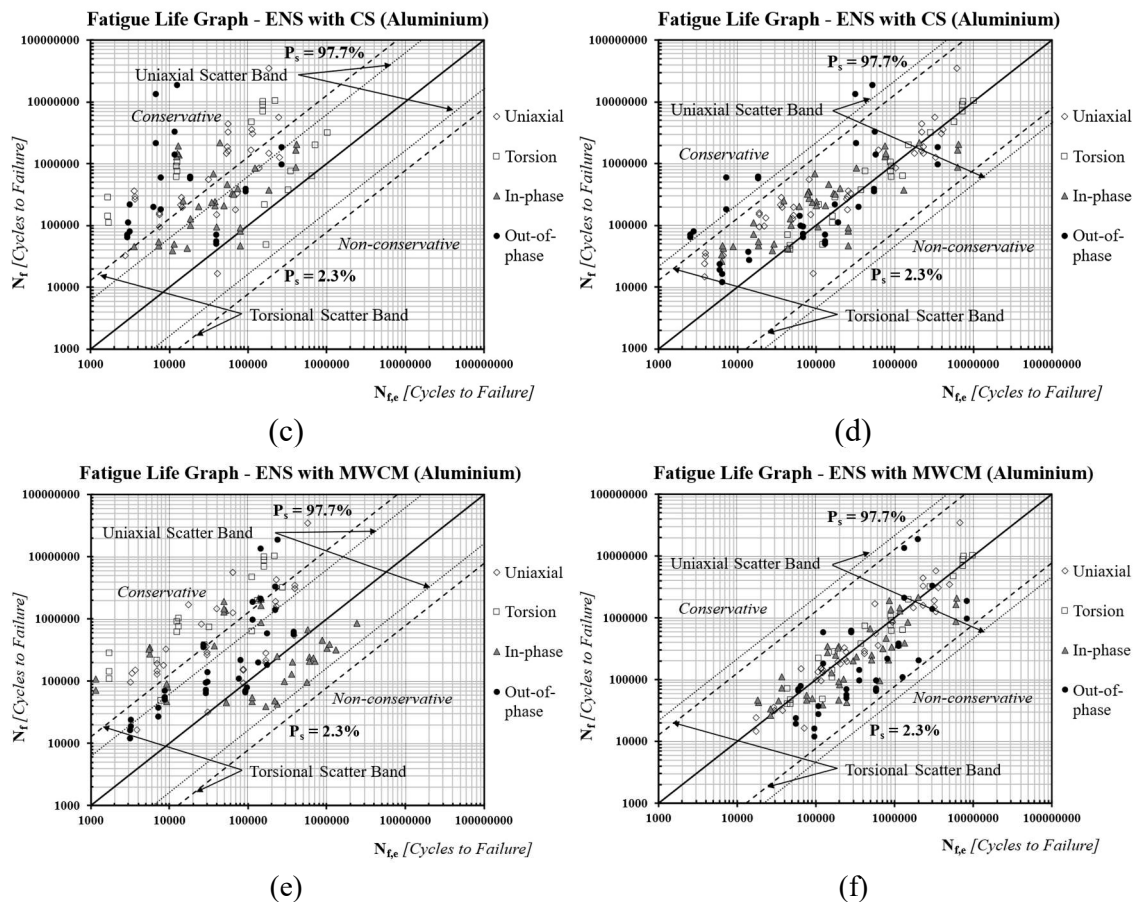


Figure 4.10: Fatigue life curves for as-welded and stress-relieved aluminium joints using the ENS approach under CA loading: Findley's criterion (a, b), CS criterion (c, d), and MWCM (e, f), comparing standard and experimental reference strengths at $P_s=50\%$.

For aluminium welded joints under CA loading, a similar trend was observed. Findley's criterion again provided the most precise fatigue life estimates, while the MWCM and CS criterion ranked second and third, respectively. Although Findley's criterion and the CS criterion exhibited slightly greater scatter, all predictions remained conservative, ensuring adequate safety margins for fatigue design.

Table 4.5: Summary of reanalysed welded joints using ENS approach combined with critical plane criteria under CA loading, detailing uniaxial and torsional reference fatigue strengths, joint geometries, materials, fatigue curve slopes, and data sources.

Material (CA)	$\Delta\sigma_{R,P_s=97.7\%}^{(1)}$ (MPa)		Uniaxial curve slope, k		$\Delta\tau_{R,P_s=97.7\%}^{(1)}$ (MPa)		Torsional curve slope, k ₀		Geometry	Reference
	IIW	Exp ⁽²⁾	IIW	Exp ⁽²⁾	IIW	Exp ⁽²⁾	IIW	Exp ⁽²⁾		
UM StE 460 *	225	362.6	3	4.4	160	221.0	5	4.9	Figure 3.2a	(Sonsino, 1995)
M StE 460 *	225	332.1	3	4.6	160	-	5	-	Figure 3.2a	(Sonsino, 1997)
UM StE 460 *	225	322.9	3	4.4	160	-	5	-	Figure 3.2e	(Sonsino, 1997)
StE 460 *	225	365.1	3	3.9	160	321.5	5	7.3	Figure 3.2a	(Yousefi, Witt and Zenner, 2001)
A519	225	192.8	3	5.4	160	145.6	5	3.7	Figure 3.2b	(Yung and Lawrence Jr, 1989)
A519-A36 *	225	295.2	3	3.8	160	183.1	5	5.5	Figure 3.2d	(Siljander, Kurath and Lawrence Jr, 1989)
Fe 52 steel	225	40.0	3	2.3	160	53.2	5	3.5	Figure 3.2f	(Bäckström, 2003)
BS4360	630	-	5	-	250	-	7	-	Figure 3.2d	(Archer, 1987)
S340+N, E355+N	630	1114.0	5	6.1	250	551.1	7	6.6	Figure 3.2e	(Shams and Vormwald, 2017)
S340+N, E355+N *	630	243.0	5	2.9	250	191.3	7	3.6	Figure 3.2e	(Shams and Vormwald, 2017)
St 35 (t = 1 mm)	630	935.0	5	4.9	250	361.7	7	8.4	Figure 3.2e	(Wiebesiek and Sonsino, 2010)
St 35 (t = 2 mm)	630	1016.1	5	5.3	250	490.5	7	6.1	Figure 3.2e	(Sonsino <i>et al.</i> , 2006)
6082-T6 *	71	140.5	3	6.9	63	84.5	5	6.2	Figure 3.2a	(Kueppers and Sonsino, 2003)
6060-T6	180	406.0	5	5.5	90	-	7	-	Figure 3.2e	(Costa <i>et al.</i> , 2005)
AW 6082	180	110.5	5	4.2	90	193.8	7	5.8	Figure 3.2e	(Wiebesiek <i>et al.</i> , 2011)
AW 5042	180	215.7	5	4.1	90	194.9	7	5.9	Figure 3.2e	(Wiebesiek <i>et al.</i> , 2011)

⁽¹⁾ Reference normal and shear stresses extrapolated at 2 million cycles to failure, with probability of survival (P_s)=97.7%.

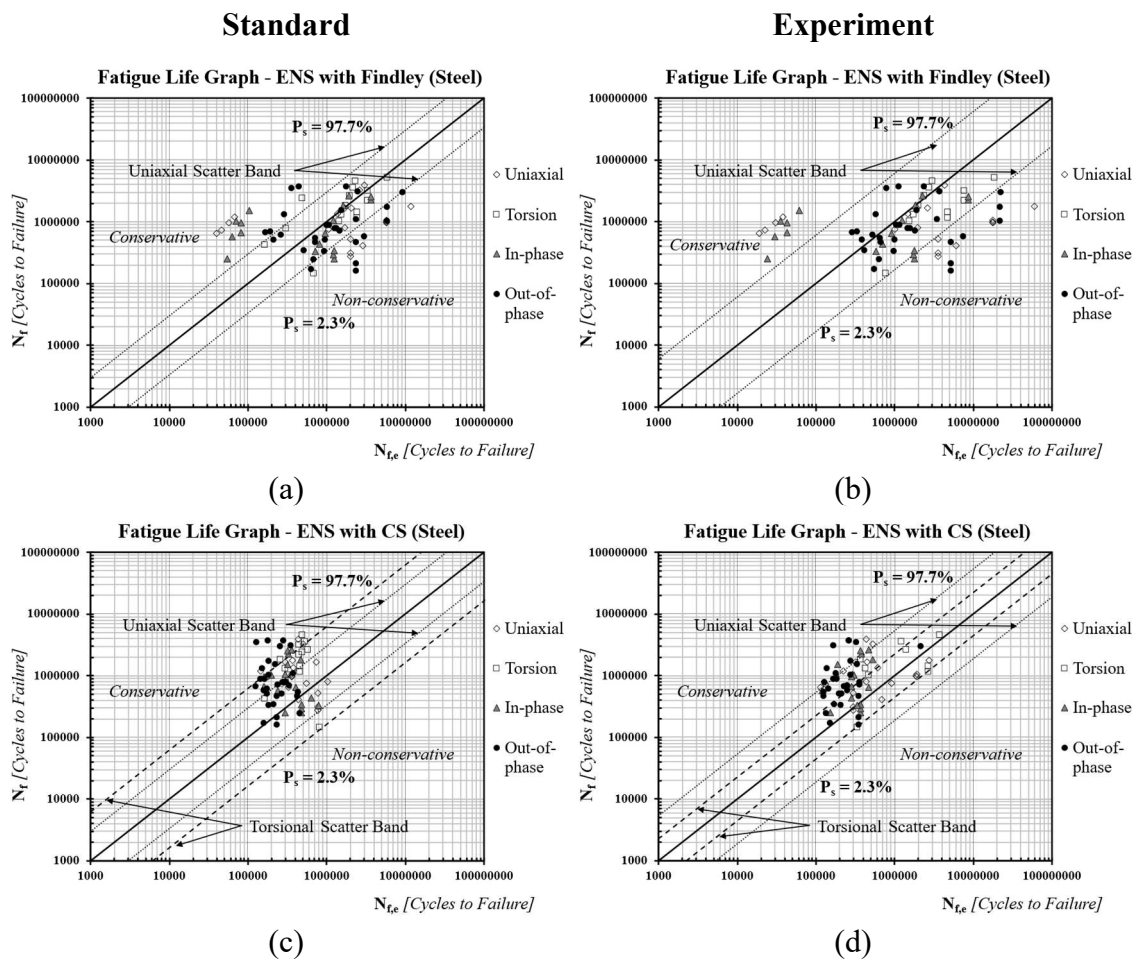
⁽²⁾ Exp denotes experimental data.

* Stress-relieved

UM – Unmachined; M – Machined.

4.5.2 VA Loading

The reanalysed steel and aluminium welded joints under VA loading using the ENS approach are summarised in Table 4.6, located at the end of this subsection, which presents the recalculated reference fatigue strengths at $P_s=97.7\%$, along with their corresponding k values. Figures 4.11 and 4.12 illustrate the fatigue life predictions obtained using Findley's criterion, the CS criterion, and the MWCM for steel and aluminium welded joints, respectively.



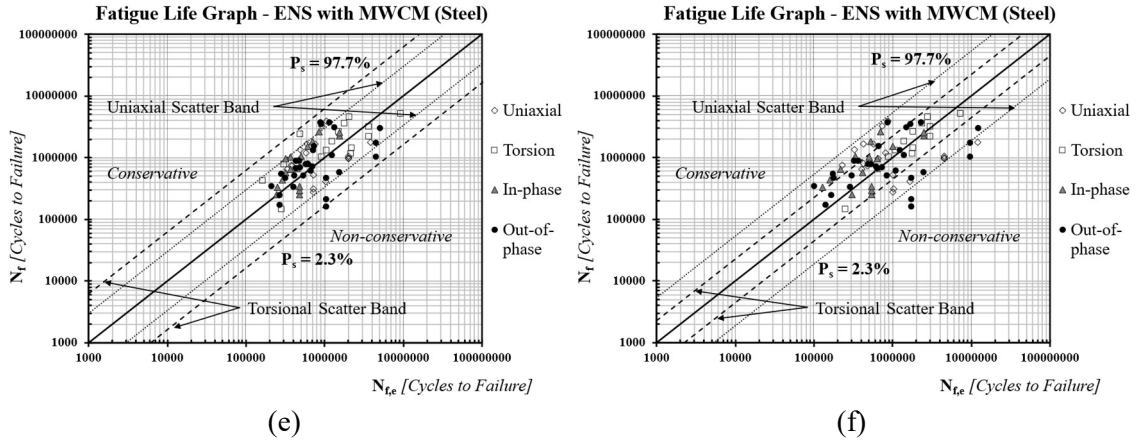
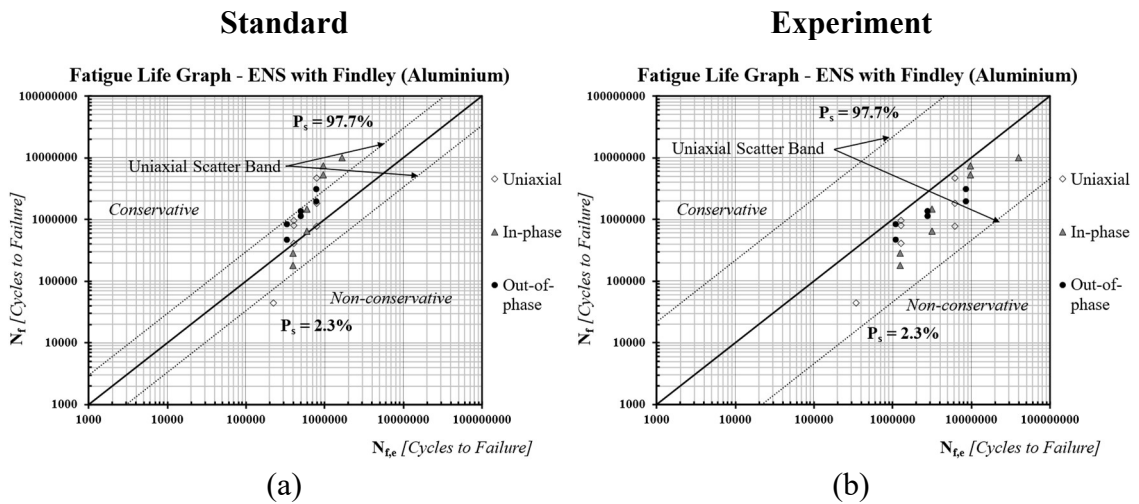


Figure 4.11: Fatigue life curves for as-welded and stress-relieved steel joints using the ENS approach under VA loading: Findley's criterion (a, b), CS criterion (c, d), and MWCM (e, f), comparing standard and experimental reference strengths at $P_s=50\%$.

For steel welded joints under VA loading, fatigue life predictions demonstrated a notable improvement when using experimentally derived reference curves, with most estimates falling within the acceptable scatter range. Among the three critical plane criteria, the MWCM exhibited the highest accuracy, followed by the CS approach, while Findley's criterion displayed greater scatter and lower predictive reliability. A similar trend was observed for aluminium welded joints, where the MWCM consistently provided the most reliable fatigue life estimates, followed by the CS approach, with Findley's criterion ranking last in terms of accuracy.



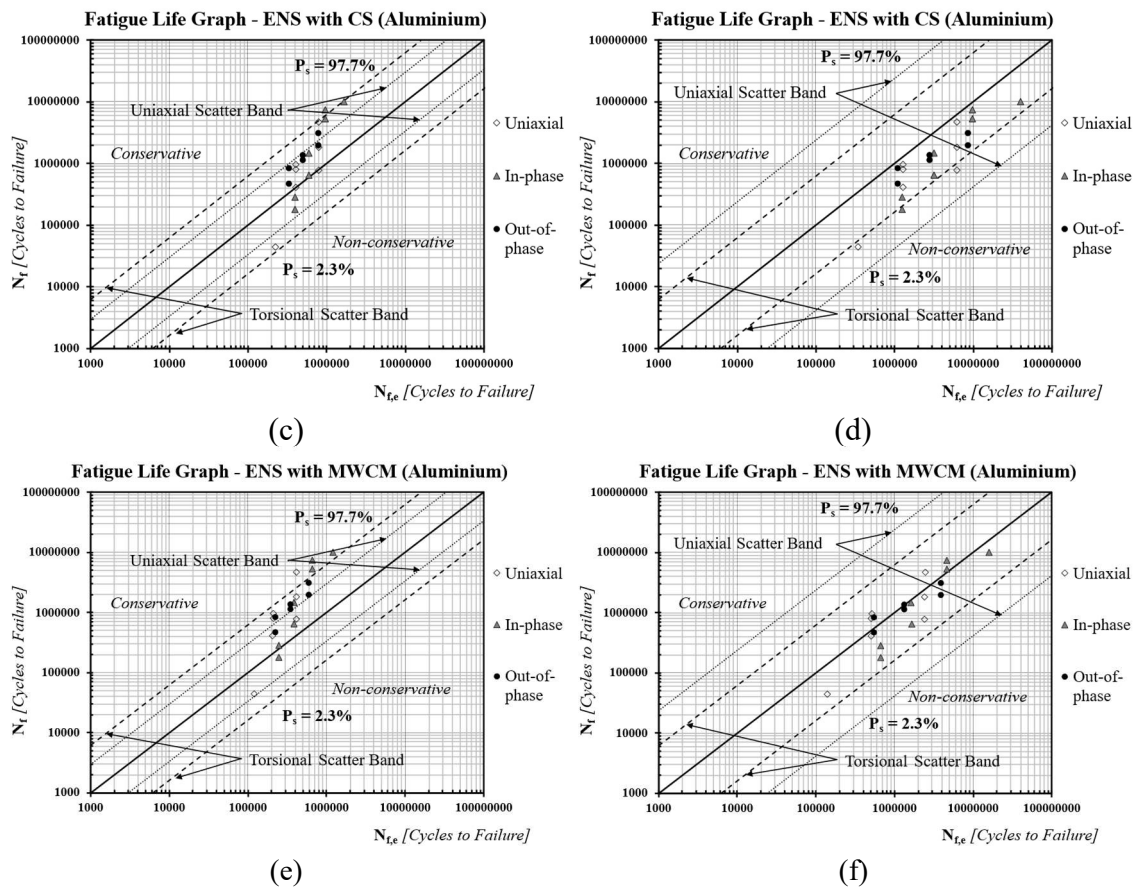


Figure 4.12: Fatigue life curves for as-welded and stress-relieved aluminium joints using the ENS approach under VA loading: Findley's criterion (a, b), CS criterion (c, d), and MWCM (e, f), comparing standard and experimental reference strengths at $P_s=50\%$.

Although the ENS approach exhibited higher scatter and increased P_{NC} when applying standard reference curves, particularly in comparison to the NS and HSS methods, the accuracy of fatigue life predictions improved significantly when experimentally derived constants were used. This suggests that despite its inherent variability, the ENS approach, in combination with multiaxial fatigue criteria, remains a valid and effective method for predicting fatigue life in welded joints subjected to VA loading.

Table 4.6: Summary of reanalysed welded joints using ENS approach combined with critical plane criteria under VA loading, detailing uniaxial and torsional reference fatigue strengths, joint geometries, materials, fatigue curve slopes, and data sources.

Material (VA)	$\Delta\sigma_{R,P_s=97.7\%}^{(1)}$ (MPa)		Uniaxial curve slope, k		$\Delta\tau_{R,P_s=97.7\%}^{(1)}$ (MPa)		Torsional curve slope, k0		Geometry	Reference
	IIW	Exp ⁽²⁾	IIW	Exp ⁽²⁾	IIW	Exp ⁽²⁾	IIW	Exp ⁽²⁾		
StE 460 *	225	362.6	3	4.4	160	221.0	5	4.9	Figure 3.2a	(Sonsino and Kueppers, 2001)
StE 460 *	225	365.1	3	3.9	160	321.5	5	7.3	Figure 3.2a	(Yousefi, Witt and Zenner, 2001)
42CrMo4 *	-	-	-	-	160	-	5	-	Figure 3.2e	(Pyttel, Grawenhof and Berger, 2012)
6082-T6	71	140.5	3	6.9	63	84.5	5	6.3	Figure 3.2e	(Kueppers and Sonsino, 2006)

⁽¹⁾Reference normal and shear stresses extrapolated at 2 million cycles to failure, with probability of survival (P_s)=97.7%.

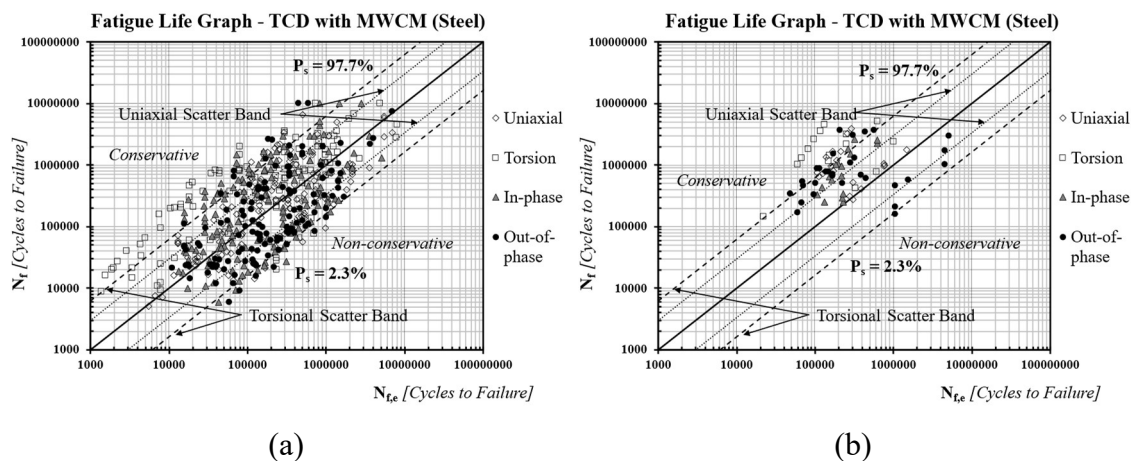
⁽²⁾Exp denotes experimental data.

* Stress-relieved

4.6 Reanalyses Using the TCD Approach

Building on the theoretical foundation of the TCD outlined in Chapter 2, this section presents a quantitative reanalysis of multiaxial fatigue using the TCD PM in conjunction with the MWCM. The TCD PM is selected due to its simplicity and proven effectiveness in welded joint fatigue assessments (Taylor, Barrett and Lucano, 2002; Susmel, 2007; L. Susmel, 2008; Susmel, 2009a, 2009b). Unlike other TCD formulations, such as the Volume, Area, and Line Methods, the PM is widely validated for welded joints and offers a more straightforward application in stress-based multiaxial fatigue life estimations (Luca Susmel, 2008; Taylor, 2008; Susmel, 2009b, 2014; Susmel and Taylor, 2012).

Tables 4.7 and 4.8, presented at the end of this subsection, provides a summary of the recalculated standard and experimental reference fatigue strengths at $P_s=97.7\%$ and their corresponding k values for steel and aluminium welded joints. Table 4.7 presents the results for CA loading, while Table 4.8 provides the values for VA loading. The reanalysed fatigue life predictions, shown in Figure 4.13, highlight the accuracy and conservatism of the TCD PM when applied with the MWCM.



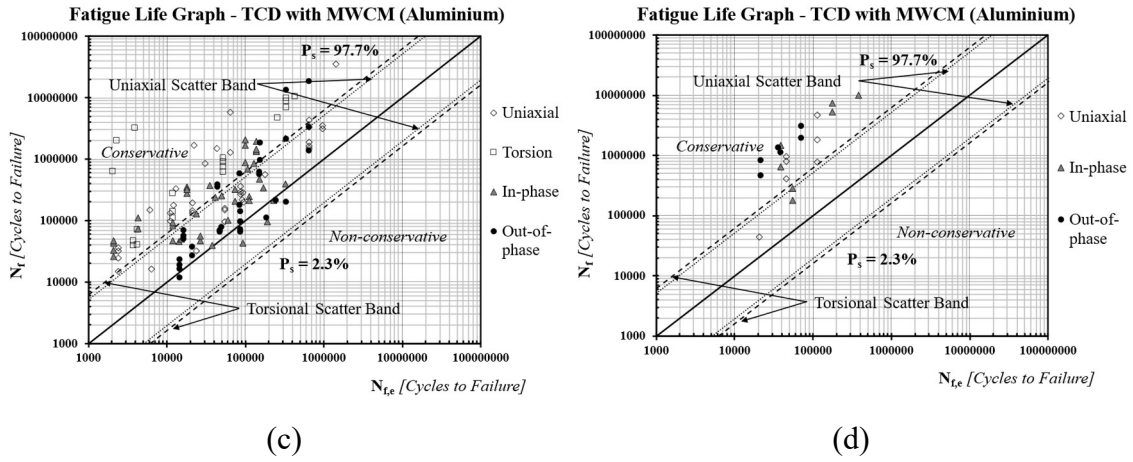


Figure 4.13: TCD PM approach: Fatigue life graphs for as-welded and stress-relieved steel (a, b) and aluminium (c, d) welded joints using the MWCM under CA (a, c) and VA (b, d) loading with standard reference strength based on $P_s=50\%$.

For steel welded joints under CA loading as shown in Figure 4.13a, fatigue life predictions obtained using TCD PM and MWCM generally align with the widest scatter band, though the method exhibits a higher degree of conservatism compared to the NS, HSS, and ENS approaches. This is particularly evident under uniaxial, torsional and in-phase fatigue loading conditions, where P_c values show a slight increase, reinforcing the safety margins of the method.

Under VA loading, the TCD PM's conservatism becomes even more pronounced, particularly in torsional fatigue tests, where P_c reaches 80%, as illustrated in Figure 4.13b. For aluminium welded joints, the TCD PM remains the most conservative approach across all loading scenarios, consistently ranking as the most conservative approach for both CA and VA loading conditions, as observed in Figures 4.13c and 4.13d.

The reanalysis confirms that the TCD PM systematically produces the most conservative fatigue life predictions, with P_{NC} values remaining at 0% across most loading scenarios. These findings align with previous studies, reinforcing the method's robustness and reliability in fatigue assessments (L. Susmel, 2008; Susmel and Taylor, 2012; Susmel, 2014). However, it is important to note that the TCD PM calibration is based on welded joints with thicknesses ranging from 6 mm to 24 mm, raising

uncertainties regarding its applicability to thinner welded joints, which thicknesses are below 5 mm (Livieri and Lazzarin, 2005).

While the TCD PM's inherent assumptions, such as zero weld root and toe radii and fixed calibration distances may contribute to its conservatism, they also provide a necessary safety margin in fatigue design (Susmel, 2007, 2009a, 2009b; L. Susmel, 2008). To enhance predictive accuracy, practitioners may consider alternative reference curves or adjustments to $M-D_v$ based on experimental validation, as suggested in relevant literature (Susmel, 2009a, 2009b).

Despite its higher estimation errors, reflected in elevated T_{RMS} values, the TCD PM remains a practical and reliable approach for multiaxial fatigue assessments when used with the MWCM. Its application ensures safe design margins for both steel and aluminium welded joints under CA and VA multiaxial fatigue loading.

Table 4.7: Summary of reanalysed welded joints using the TCD PM approach with the MWCM under CA loading, including uniaxial and torsional reference fatigue strengths, joint geometries, materials, fatigue curve slopes, and data sources.

Material (CA)	$\Delta\sigma_{R,P_s=97.7\%}^{(1)}$ (MPa) TCD	Uniaxial curve slope, k TCD	$\Delta\tau_{R,P_s=97.7\%}^{(1)}$ (MPa) TCD	Torsional curve slope, k ₀ TCD	Geometry	Reference
UM StE 460 *	86	3	67	5	Figure 3.2a	(Sonsino, 1995)
M StE 460 *	86	3	67	5	Figure 3.2a	(Sonsino, 1997)
UM StE 460 *	86	3	67	5	Figure 3.2e	(Sonsino, 1997)
StE 460 *	86	3	67	5	Figure 3.2a	(Yousefi, Witt and Zenner, 2001)
A519	86	3	67	5	Figure 3.2b	(Yung and Lawrence Jr, 1989)
A519-A36 *	86	3	67	5	Figure 3.2d	(Siljander, Kurath and Lawrence Jr, 1989)
Fe 52 steel	86	3	67	5	Figure 3.2f	(Bäckström, 2003)
BS4360	86	3	67	5	Figure 3.2d	(Archer, 1987)
S340+N, E355+N	86	3	67	5	Figure 3.2e	(Shams and Vormwald, 2017)
S340+N, E355+N *	86	3	67	5	Figure 3.2e	(Shams and Vormwald, 2017)
St 35 (t = 1 mm)	86	3	67	5	Figure 3.2e	(Wiebesiek and Sonsino, 2010)
St 35 (t = 2 mm)	86	3	67	5	Figure 3.2e	(Sonsino <i>et al.</i> , 2006)
6082-T6 *	46	4.5	28	5	Figure 3.2a	(Kueppers and Sonsino, 2003)
6060-T6	46	4.5	28	5	Figure 3.2e	(Costa <i>et al.</i> , 2005)
AW 6082	46	4.5	28	5	Figure 3.2e	(Wiebesiek <i>et al.</i> , 2011)
AW 5042	46	4.5	28	5	Figure 3.2e	(Wiebesiek <i>et al.</i> , 2011)

⁽¹⁾ Reference normal and shear stresses extrapolated at 2 million cycles to failure, with probability of survival (P_s)=97.7%.

⁽²⁾ Exp denotes experimental data.

* Stress-relieved

UM – Unmachined; M – Machined.

Table 4.8: Summary of reanalysed welded joints using the TCD PM approach with the MWCM under VA loading, including uniaxial and torsional reference fatigue strengths, joint geometries, materials, fatigue curve slopes, and data sources.

Material (VA)	$\Delta\sigma_{R,P_s=97.7\%}^{(1)}$ (MPa) IIW	Uniaxial curve slope, k IIW	$\Delta\tau_{R,P_s=97.7\%}^{(1)}$ (MPa) IIW	Torsional curve slope, k0 IIW	Geometry	Reference
StE 460 *	86	3	67	5	Figure 3.2a	(Sonsino and Kueppers, 2001)
StE 460 *	71	3	67	5	Figure 3.2a	(Yousefi, Witt and Zenner, 2001)
42CrMo4 *	-	-	67	5	Figure 3.2e	(Pyttel, Grawenhof and Berger, 2012)
6082-T6	46	4.5	28	5	Figure 3.2a	(Kueppers and Sonsino, 2006)

⁽¹⁾ Reference normal and shear stresses extrapolated at 2 million cycles to failure, with probability of survival (P_s)=97.7%.

⁽²⁾ Exp denotes experimental data.

* Stress-relieved

4.7 Key Findings and Discussion

This section presents the key insights gained from the quantitative reanalysis of multiaxial fatigue criteria applied across various stress analysis approaches. The predictive performance of NS, HSS, ENS, and the TCD approaches was assessed using P_{NC} , P_c , and T_{RMS} values, as detailed in Tables 4.9 to 4.12 at the end of this section. The results confirm that when combined with stress-based critical plane criteria, these methods provide effective fatigue life assessments for welded joints under multiaxial loading. Among the stress analysis approaches, the HSS approach consistently exhibited the best balance between accuracy and conservatism, making it a robust choice for fatigue design.

For steel welded joints under CA loading, all three critical plane criteria, Findley's criterion, the CS criterion, and the MWCM produced accurate and conservative fatigue life predictions, with minimal variability across different loading conditions. However, under VA loading, MWCM demonstrated the highest predictive accuracy, followed by the CS approach, while Findley's criterion exhibited greater scatter and higher conservatism. This increased conservatism, particularly for $R \geq 0$ loading conditions, is attributed to Findley's reliance on maximum normal stress component, which indirectly incorporates mean stress effects. Since Findley's criterion was originally formulated for unwelded components, its direct application to welded joints may require modification (Findley, 1959). A potential improvement could involve replacing the maximum normal stress with the maximum normal stress range on the critical plane, which may enhance its predictive accuracy. Further research is recommended to validate this adjustment, particularly for fatigue tests with $R \geq 0$. Additionally, the greater scatter observed in Findley's predictions is likely due to its calibration being based solely on uniaxial fatigue data, whereas both the CS approach and MWCM were calibrated using a broader dataset, incorporating both uniaxial and pure torsional fatigue data.

For aluminium welded joints, fatigue life predictions exhibited higher conservatism and greater scatter than those observed for steel joints under both CA and VA loading. This

increased variability is likely a consequence of the limited availability of experimental fatigue data for aluminium joints, highlighting the need for further experimental validation to improve predictive accuracy (Costa *et al.*, 2005; Kueppers and Sonsino, 2006; Wiebesiek and Sonsino, 2010; Wiebesiek *et al.*, 2011). This trend was also noted in Chapter 3, where standard multiaxial fatigue criteria exhibited similar conservatism for aluminium joints. Interestingly, the performance of the multiaxial fatigue criteria differed significantly between aluminium and steel joints. Unlike in steel welded joints, where Findley's criterion was the most conservative, the CS approach exhibited the highest conservatism and scatter for aluminium joints. This difference was especially pronounced when standard reference constants were used in the calibration process. The discrepancy in performance is likely due to variations in material ductility, as noted by Sonsino, which influence critical plane orientation in fatigue analysis (Sonsino and Kueppers, 2001; Kueppers and Sonsino, 2003, 2006). These findings suggest that linear combinations of shear and normal stresses may provide more accurate fatigue life estimations for aluminium welded joints than non-linear combinations (Findley, 1959; Carpinteri, Spagnoli and Vantadori, 2009; Susmel, 2014; Bruun and Härkegård, 2015; Pedersen, 2016). However, further experimental investigations on various aluminium alloys and thicknesses are required to confirm this hypothesis and refine multiaxial fatigue models for aluminium structures. Additional research should also explore the role of material strength and ductility in influencing critical plane orientation, contributing to a more comprehensive fatigue life assessment framework for aluminium welded joints under multiaxial fatigue loading.

A comparison between standard reference constants (such as those provided by IIW and ECs) and experimentally derived constants reveals that standard constants generally yield more conservative and scattered fatigue life estimates for both steel and aluminium welded joints (Anon, 2005, 2007; Hobbacher, 2016). This result is expected, as standard constants account for higher R ratios and are designed to be conservative to ensure safe design across a broad range of welded joint configurations and welding conditions (Anon, 2005, 2007; Hobbacher, 2016). However, an exception was observed in stress-relieved steel welded joints from Shams datasets under proportional and non-proportional loading, where the ENS approach, when used with standard constants,

resulted in non-conservative estimates (Shams and Vormwald, 2017). This discrepancy is likely due to significant differences in fatigue strength and k values between experimentally derived fatigue curves and standard reference curves, as previously discussed in the ENS approach reanalysis section.

Conversely, experimentally derived fatigue curves significantly improved predictive accuracy, as reflected by lower T_{RMS} values across multiple fatigue criteria and stress analysis methods. This highlights the importance of experimental calibration for non-standard welded joints, ensuring that multiaxial fatigue criteria are accurately tuned to provide reliable fatigue life estimations. While standard constants introduce greater estimation errors, they remain conservative, reinforcing their suitability for engineering design codes such as the IIW and ECs. Consequently, standard reference curves remain a dependable choice, particularly for welded joints manufactured using conventional fabrication techniques, providing a practical alternative in cases where experimental testing is not feasible or cost-effective.

Under VA loading, all three multiaxial fatigue criteria demonstrated their ability to account for complex loading conditions, consistently producing conservative fatigue life predictions. However, VA loading resulted in increased scatter and greater conservatism compared to CA loading, as expected due to the complexities associated with multiaxial fatigue under variable amplitude conditions. Despite this, fatigue life estimates remained safe and within acceptable ranges, particularly when experimentally derived constants and average D_{real} values were applied for both aluminium and steel welded joints. The effectiveness of Findley's criterion and the CS criterion under VA loading can be partially attributed to the success of the CS counting method, which enables simultaneous tracking of normal and shear stress cycles, leading to more precise fatigue life estimations (Carpinteri, Spagnoli and Vantadori, 2003; Bibbo, Baumgartner and Arora, 2022). Similarly, for MWCM, the MVM combined with conventional rainflow cycle counting proved to be a robust approach, allowing for accurate critical plane identification and effective shear stress range computation across different stress analysis methods (Susmel, 2010).

Among all evaluated methods, the TCD PM approach was the most conservative, particularly for welded joints with thicknesses between 6 mm and 24 mm, as discussed in Section 4.6. However, for thinner welded joints with thicknesses below 5 mm, the TCD PM's reliance on standard calibration methods from EC3 and EC9 may lead to reduced predictive accuracy. The inherent conservatism of the TCD PM, combined with simplifying assumptions, such as zero weld root and toe radii, results in greater estimation variability. To improve its applicability, particularly for non-standard welded configurations, future research should focus on refining calibration procedures through alternative reference curves and empirical fatigue data. Nonetheless, the TCD PM remains a viable method for multiaxial fatigue assessments, particularly in safety-critical applications.

Overall, this study highlights the effectiveness and validity of alternative stress-based critical plane criteria, including Findley's criterion, the CS criterion, and the MWCM when applied across various stress analysis methods. These approaches serve as a valuable complement to standard interaction equations, offering engineers greater flexibility in selecting appropriate fatigue assessment methodologies (Anon, 2005, 2007; Hobbacher, 2016). The findings emphasise the importance of understanding the strengths and limitations of each fatigue criterion and stress analysis approach, allowing engineers to tailor fatigue design strategies to specific welded joint configurations, ultimately enhancing structural safety and efficiency.

Table 4.9: Summary of P_{NC} , P_c , and T_{RMS} values from reanalyses of steel joints under CA loading, using critical plane criteria with various stress analysis approaches.

Multiaxial Fatigue Criterion	Condition	Stress Analysis Approaches	Standard ⁽¹⁾		Experimental ⁽²⁾		$T_{RMS,Std}^{(5)}$	$T_{RMS,Exp}^{(6)}$
			P_{NC} (%) ⁽³⁾	P_c (%) ⁽⁴⁾	P_{NC} (%) ⁽³⁾	P_c (%) ⁽⁴⁾		
Findley	Uniaxial	NS	3.2	35.7	-	-	30.5	-
		HSS	0	36.6	-	-	22.3	-
		ENS	1.3	29.9	-	-	78.2	-
	Torsional	NS	6.8	27.1	4.2	0.8	16.8	7.2
		HSS	8.0	29.3	1.3	5.3	15.2	6.0
		ENS	2.7	21.6	1.8	4.5	44.0	6.4
	In-Phase	NS	0.6	32.7	0	7.6	60.7	24.3
		HSS	0.8	11.1	3.2	15.9	14.9	10.0
		ENS	11.6	18.0	2.2	9.5	49.1	11.2
	Out-Of-Phase	NS	0.7	32.7	2.1	2.1	18.8	12.7
		HSS	4.0	15.0	13.0	9.0	8.0	9.0
		ENS	24.6	4.5	8.4	3.8	64.4	16.1
CS	Uniaxial	NS	3.2	36.9	-	-	30.8	-
		HSS	0	52.7	-	-	62.8	-
		ENS	2.1	33.1	-	-	97.6	-
	Torsional	NS	0.8	2.5	-	-	12.2	-
		HSS	0	5.3	-	-	9.6	-
		ENS	0.9	0.9	-	-	23.7	-
	In-Phase	NS	0	20.7	0	0	43.5	14.2
		HSS	0.8	11.9	0	1.6	19.5	11.7
		ENS	0.6	8.7	0	5.6	51.8	65.7
	Out-Of-Phase	NS	0	15.6	1.4	0.0	19.5	11.9
		HSS	0	20.0	0	1.0	44.9	9.8
		ENS	1.5	3.7	0	3.9	60.9	14.3
MWCM	Uniaxial	NS	8.3	23.6	-	-	13.8	-
		HSS	0	14.6	-	-	8.7	-
		ENS	5.7	6.4	-	-	19.2	-
		TCD	20.3	23.6	-	-	14.2	-
	Torsional	NS	4.2	7.6	-	-	12.0	-
		HSS	0	24.0	-	-	26.4	-
		ENS	1.0	11.7	-	-	66.0	-
		TCD	4.5	30.6	-	-	48.5	-
	In-Phase	NS	2.8	7.3	0	0	15.7	15.7
		HSS	0	4.8	0	0.8	9.6	6.6
		ENS	3.7	2.5	0	0.6	25.3	6.3
		TCD	5.7	14.5	-	-	24.9	-
	Out-Of-Phase	NS	0	20.4	0	3.4	28.9	33.1
		HSS	0	7.0	0	0	8.4	10.5
		ENS	29.8	2.5	1.5	0.8	294.8	19.3
		TCD	4.6	6.9	-	-	18.4	-

⁽¹⁾ Reanalysis using constants derived from standard reference curves.⁽²⁾ Reanalysis using constants derived from experimental reference curves.⁽³⁾ P_{NC} represents the percentage of non-conservative estimates.⁽⁴⁾ P_c represents the percentage of conservative estimates.⁽⁵⁾ $T_{RMS,Std}$ quantifies the error in fatigue life predictions when using constants from standard curves.⁽⁶⁾ $T_{RMS,Exp}$ quantifies the error in fatigue life predictions when using constants from experimental curves.

Table 4.10: Summary of P_{NC} , P_c , and T_{RMS} values from reanalyses of steel joints under VA loading, using critical plane criteria with various stress analysis approaches.

Multiaxial Fatigue Criterion	Condition	Stress Analysis Approaches	Standard ⁽¹⁾		Experimental ⁽²⁾		$T_{RMS,Std}^{(5)}$	$T_{RMS,Exp}^{(6)}$
			P_{NC} (%) ⁽³⁾	P_c (%) ⁽⁴⁾	P_{NC} (%) ⁽³⁾	P_c (%) ⁽⁴⁾		
Findley	Uniaxial	NS	21.1	21.1	42.1	21.1	111.6	259.6
		HSS	5.3	31.6	42.1	21.1	153.0	257.5
		ENS	42.1	21.1	42.1	21.1	55.9	253.6
	Torsional	NS	0	80.0	41.7	33.3	60.3	24.1
		HSS	0	46.7	0	0	12.9	5.7
		ENS	6.7	6.7	0	0	5.8	8.8
	In-Phase	NS	23.5	58.8	0	35.3	490.3	222.1
		HSS	0	52.9	0	35.3	383.6	110.4
		ENS	17.6	35.3	11.8	35.3	34.0	95.0
	Out-Of-Phase	NS	3.3	53.3	10.0	26.7	160.7	97.0
		HSS	6.7	43.3	23.3	23.3	128.5	64.6
		ENS	23.3	16.7	23.3	0	18.6	32.4
CS	Uniaxial	NS	0	36.8	36.8	68.4	20.3	34.3
		HSS	0	42.1	36.8	57.9	23.0	20.4
		ENS	0	36.8	0	42.1	18.0	15.6
	Torsional	NS	0	46.7	0	25.0	126.9	33.7
		HSS	0	20.0	0	36.4	24.0	50.3
		ENS	0	28.6	0	0	36.3	8.6
	In-Phase	NS	0	17.6	0	0	23.0	11.5
		HSS	0	11.8	0	11.8	16.4	12.5
		ENS	0	17.6	0	17.6	19.5	15.6
	Out-Of-Phase	NS	0	16.7	3.6	21.4	19.0	28.5
		HSS	0	13.3	0	26.7	19.0	19.2
		ENS	0	26.7	0	23.3	92.2	29.9
MWCM	Uniaxial	NS	0	36.8	36.8	68.4	14.3	17.2
		HSS	0	42.1	36.8	57.9	26.2	12.6
		ENS	0	36.8	0	42.1	6.7	16.4
		TCD	0	57.9	-	-	37.9	-
	Torsional	NS	0	46.7	0	25.0	19.1	3.0
		HSS	0	20.0	0	36.4	11.9	1.8
		ENS	0	28.6	0	0	5.1	2.2
		TCD	33.3	80.0	-	-	291.6	-
	In-Phase	NS	0	17.6	0	0	7.3	8.6
		HSS	0	11.8	0	11.8	22.8	5.4
		ENS	0	17.6	0	17.6	4.8	5.7
		TCD	0	11.8	-	-	22.3	-
	Out-Of-Phase	NS	0	16.7	3.6	21.4	9.1	10.4
		HSS	0	13.3	0	26.7	4.9	10.9
		ENS	0	26.7	0	23.3	6.7	12.0
		TCD	0	33.3	-	-	42.1	-

⁽¹⁾ Reanalysis using constants derived from standard reference curves.

⁽²⁾ Reanalysis using constants derived from experimental reference curves.

⁽³⁾ P_{NC} represents the percentage of non-conservative estimates.

⁽⁴⁾ P_c represents the percentage of conservative estimates.

⁽⁵⁾ $T_{RMS,Std}$ quantifies the error in fatigue life predictions when using constants from standard curves.

⁽⁶⁾ $T_{RMS,Exp}$ quantifies the error in fatigue life predictions when using constants from experimental curves.

Table 4.11: Summary of P_{NC} , P_c , and T_{RMS} values from reanalyses of aluminium joints under CA loading, using critical plane criteria with various stress analysis approaches.

Multiaxial Fatigue Criterion	Condition	Stress Analysis Approaches	Standard ⁽¹⁾		Experimental ⁽²⁾		$T_{RMS,Std}^{(5)}$	$T_{RMS,Exp}^{(6)}$
			P_{NC} (%) ⁽³⁾	P_c (%) ⁽⁴⁾	P_{NC} (%) ⁽³⁾	P_c (%) ⁽⁴⁾		
Findley	Uniaxial	NS	21.6	32.4	-	-	20.5	-
		HSS	0	64.7	-	-	45.1	-
		ENS	0	100	-	-	1505.3	-
	Torsional	NS	0	21.7	0	4.3	11.5	27.8
		HSS	0	85.7	0	0	156.5	3.7
		ENS	0	43.5	0	0	151.7	2.4
	In-Phase	NS	0	58.1	0	7.0	24.0	57.5
		HSS	0	25.0	0	0	10.9	7.5
		ENS	0	53.5	0	0	198.2	6.0
	Out-Of-Phase	NS	0	30.3	0	12.1	33.7	154.6
		HSS	0	42.9	0	7.1	19.3	126.9
		ENS	0	42.4	0	0	120.6	23.0
CS	Uniaxial	NS	13.5	40.5	-	-	37.0	-
		HSS	0	82.4	-	-	100.6	-
		ENS	0	82.1	-	-	1364.6	-
	Torsional	NS	0	17.4	-	-	18.7	-
		HSS	0	71.4	-	-	12.7	-
		ENS	0	63.2	-	-	2264.0	-
	In-Phase	NS	0	25.6	0	0	32.8	24.0
		HSS	0	14.3	0	0	24.1	10.0
		ENS	0	21.2	0	0	243.7	16.3
	Out-Of-Phase	NS	0	27.3	0	6.1	76.7	23.5
		HSS	0	35.7	0	7.1	774.7	56.4
		ENS	0	68.2	0	27.3	7620.3	98.4
MWCM	Uniaxial	NS	18.9	32.4	-	-	19.5	-
		HSS	0	41.2	-	-	33.2	-
		ENS	0	64.9	-	-	655.2	-
		TCD	18.9	48.6	-	-	157.9	-
	Torsional	NS	0	17.4	-	-	11.5	-
		HSS	0	71.4	-	-	153.6	-
		ENS	0	82.6	-	-	8461.3	-
		TCD	0	100	-	-	15829.0	-
	In-Phase	NS	0	0.0	0	0	4.6	6.3
		HSS	0	3.6	0	0	7.7	8.0
		ENS	0	27.9	0	0	403.1	7.8
		TCD	0	39.5	-	-	104.2	-
	Out-Of-Phase	NS	0	21.2	0	6.1	36.3	25.3
		HSS	0	28.6	0	0	22.8	13.6
		ENS	0	18.2	0	0	80.5	30.0
		TCD	0	18.2	-	-	24.9	-

⁽¹⁾ Reanalysis using constants derived from standard reference curves.

⁽²⁾ Reanalysis using constants derived from experimental reference curves.

⁽³⁾ P_{NC} represents the percentage of non-conservative estimates.

⁽⁴⁾ P_c represents the percentage of conservative estimates.

⁽⁵⁾ $T_{RMS,Std}$ quantifies the error in fatigue life predictions when using constants from standard curves.

⁽⁶⁾ $T_{RMS,Exp}$ quantifies the error in fatigue life predictions when using constants from experimental curves.

Table 4.12: Summary of P_{NC} , P_c , and T_{RMS} values from reanalyses of aluminium joints under VA loading, using critical plane criteria with various stress analysis approaches.

Multiaxial Fatigue Criterion	Condition	Stress Analysis Approaches	Standard ⁽¹⁾		Experimental ⁽²⁾		$T_{RMS,Std}^{(5)}$	$T_{RMS,Exp}^{(6)}$
			P_{NC} (%) ⁽³⁾	P_c (%) ⁽⁴⁾	P_{NC} (%) ⁽³⁾	P_c (%) ⁽⁴⁾		
Findley	Uniaxial	NS	14.3	0	0	0	11.6	116.4
		HSS	14.3	14.3	0	0	10.5	19.5
		ENS	14.3	14.3	0	0	11.1	19.3
	In-Phase	NS	0	42.9	0	0	15.8	6.6
		HSS	0	42.9	0	0	40.3	7.0
		ENS	0	42.9	0	0	20.3	19.0
	Out-Of-Phase	NS	0	16.7	0	0	5.7	2.9
		HSS	0	16.7	0	0	11.5	3.2
		ENS	0	16.7	0	0	8.6	9.1
CS	Uniaxial	NS	14.3	71.4	0	0	159.2	36.6
		HSS	14.3	42.9	0	14.3	82.3	84.5
		ENS	14.3	14.3	0	0.0	11.1	19.3
	In-Phase	NS	0	42.9	0	42.9	84.3	283.7
		HSS	0	42.9	0	0	111.6	67.8
		ENS	0	14.3	0	0	20.3	19.0
	Out-Of-Phase	NS	0	50.0	0	16.7	55.3	246.5
		HSS	0	66.7	0	0	84.4	92.2
		ENS	0	0	0	0	8.6	9.1
MWCM	Uniaxial	NS	14.3	14.3	0	0	14.3	5.3
		HSS	0	57.1	0	0	35.9	5.5
		ENS	0	57.1	0	0	26.3	5.4
		TCD	0	85.7	-	-	423.8	-
	In-Phase	NS	0	14.3	0	0	16.9	6.0
		HSS	0	42.9	0	0	55.8	5.3
		ENS	0	42.9	0	0	35.9	5.4
		TCD	0	57.1	-	-	920.4	-
	Out-Of-Phase	NS	0	0	0	0	5.8	3.1
		HSS	0	0	0	0	25.8	2.6
		ENS	0	0	0	0	18.3	2.3
		TCD	0	100	-	-	2920.9	-

⁽¹⁾ Reanalysis using constants derived from standard reference curves.

⁽²⁾ Reanalysis using constants derived from experimental reference curves.

⁽³⁾ P_{NC} represents the percentage of non-conservative estimates.

⁽⁴⁾ P_c represents the percentage of conservative estimates.

⁽⁵⁾ $T_{RMS,Std}$ quantifies the error in fatigue life predictions when using constants from standard curves.

⁽⁶⁾ $T_{RMS,Exp}$ quantifies the error in fatigue life predictions when using constants from experimental curves.

4.8 Conclusions

This quantitative review provides a comprehensive evaluation of stress-based critical plane criteria, Findley's criterion, the CS approach, and the MWCM, for assessing the multiaxial fatigue behaviour of aluminium and steel welded joints under both CA and VA loading conditions. The key findings are summarised as follows:

- Under CA loading, all three criteria exhibited comparable accuracy, with minimal variation in fatigue life estimations. However, under VA loading, the MWCM consistently demonstrated the highest predictive accuracy, followed by the CS approach, while Findley's criterion produced more scattered and conservative estimates due to its indirect consideration of mean stress effects via the maximum normal stress term.
- Fatigue life estimations for aluminium welded joints displayed greater conservatism and higher scatter than those for steel welded joints, with the CS approach exhibiting the most scatter. This finding highlights the need for additional experimental data to enhance the predictive accuracy of multiaxial fatigue criteria for aluminium welded structures.
- Standard calibration constants, such as those provided by IIW and ECs, generally result in conservative fatigue life estimates, offering a safe baseline for design applications where experimental data is unavailable (Anon, 2005, 2007; Hobbacher, 2016). However, incorporating experimentally derived calibration constants significantly improves predictive accuracy, particularly for non-conventional welded joint configurations.
- Among the stress analysis methods, the TCD PM combined with MWCM was identified as the most conservative approach. While this method ensures high safety margins, it may require further refinement to enhance its predictive accuracy, especially for thinner or more flexible ($t < 5$ mm) or non-standard welded joints that fall outside standard calibration assumptions.

- The critical plane criteria assessed, including Findley's criterion, the CS criterion, and the MWCM demonstrated reliable performance in predicting multiaxial fatigue life in welded joints. These criteria serve as viable alternatives to traditional interaction equation-based approaches outlined in design codes such as the IIW and ECs, particularly when experimentally calibrated constants are used.
- While the evaluated fatigue criteria have proven effective for steel welded joints, further research is needed to improve their applicability to aluminium welded structures, particularly under multiaxial CA and VA loading conditions. Compared to steel welded joints, there is a notable lack of experimental data for aluminium welded joints across different grades and thicknesses. Addressing this knowledge gap will enhance fatigue life prediction accuracy and ensure the robustness of these criteria for a wider range of aluminium welded joints.

This study confirms that stress-based critical plane criteria, when properly calibrated, provide a reliable framework for multiaxial fatigue assessment of welded joints, complementing and, in some cases, enhancing traditional standard design methodologies.

4.9 References

- Alencar, G. *et al.* (2021) 'A finite element post-processor for fatigue assessment of welded structures based on the Master S-N curve method', *International Journal of Fatigue*, 153. doi: 10.1016/j.ijfatigue.2021.106482.
- Amstutz, H., Störzel, K. and Seeger, T. (2001) 'Fatigue crack growth of a welded tube-flange connection under bending and torsional loading', *Fatigue & Fracture of Engineering Materials & Structures*, 24(5), pp. 357–368.
- Anon (2005) *Eurocode 3: Design of steel structures — Part 1-9: Fatigue* .
- Anon (2007) *Eurocode 9: Design of aluminium structures — Part 1-3: Structures susceptible to fatigue* .

- Archer, R. (1987) 'Fatigue of a welded steel attachment under combined direct stress and shear stress', in *International conference of fatigue of welded constructions, no. paper*.
- Atzori, B. (1983) 'Trattamento termici e resistenza a fatica delle strutture soldate'.
- Bäckström, M. (2003) 'Multiaxial fatigue life assessment of welds based on nominal and hot spot stresses'.
- Bäckström, M. and Marquis, G. (2001) 'A review of multiaxial fatigue of weldments: experimental results, design code and critical plane approaches', *Fatigue & fracture of engineering materials & structures*, 24(5), pp. 279–291.
- Bibbo, N. D. *et al.* (2022) 'An improved rainflow counting method for multiaxial stress states using the minimum circumscribed circle method to identify shear stress ranges', *International Journal of Fatigue*, 163, p. 106997.
- Bibbo, N. D., Baumgartner, J. and Arora, V. (2022) 'Comparative study of critical plane fatigue criteria on multiaxial variable amplitude loaded welded test specimens', *International Journal of Fatigue*, 158, p. 106670.
- Bruun, Ø. A. and Härkegård, G. (2015) 'A comparative study of design code criteria for prediction of the fatigue limit under in-phase and out-of-phase tension–torsion cycles', *International Journal of Fatigue*, 73, pp. 1–16.
- Carpinteri, A., Spagnoli, A. and Vantadori, S. (2003) 'A multiaxial fatigue criterion for random loading', *Fatigue & Fracture of Engineering Materials & Structures*, 26(6), pp. 515–522.
- Carpinteri, A., Spagnoli, A. and Vantadori, S. (2009) 'Multiaxial fatigue life estimation in welded joints using the critical plane approach', *International Journal of Fatigue*, 31(1), pp. 188–196.
- Costa, J. D. M. *et al.* (2005) 'Fatigue behaviour of tubular AlMgSi welded specimens subjected to bending–torsion loading', *Fatigue & Fracture of Engineering Materials & Structures*, 28(4), pp. 399–407.

Endo, T. (1974) 'Rain flow method—the proposal and the applications', *Bull. Kyushu Inst. Tech.*, 28, p. 33.

Eulitz, K.-G. (2000) 'Damage accumulation limitations and perspectives for fatigue life assessment', in *Proceedings of Materials week 2000*, pp. 25–28.

Faruq, N. Z. and Susmel, L. (2019) 'Proportional/nonproportional constant/variable amplitude multiaxial notch fatigue: cyclic plasticity, non-zero mean stresses, and critical distance/plane', *Fatigue & Fracture of Engineering Materials & Structures*, 42(9), pp. 1849–1873. doi: <https://doi.org/10.1111/ffe.13036>.

Findley, W. N. (1959) 'A theory for the effect of mean stress on fatigue of metals under combined torsion and axial load or bending', *Journal of Engineering for Industry*, 81(4), pp. 301–305.

Fricke, W. (2003) 'Fatigue analysis of welded joints: state of development', *Marine structures*, 16(3), pp. 185–200.

Fricke, W. (2013) 'IIW guideline for the assessment of weld root fatigue', *Welding in the World*, 57(6), pp. 753–791. doi: 10.1007/s40194-013-0066-y.

Haibach, E. (1989) 'Fatigue strength'.

Hobbacher, A. (2016) *Recommendations for fatigue design of welded joints and components*. Springer.

Hobbacher, A. F. (2009) 'The new IIW recommendations for fatigue assessment of welded joints and components—A comprehensive code recently updated', *International journal of fatigue*, 31(1), pp. 50–58.

John, E. *et al.* (2024) 'Multiaxial fatigue of water pipe grey cast iron', *International Journal of Fatigue*, 178, p. 108002.

Kueppers, M. and Sonsino, C. M. (2003) 'Critical plane approach for the assessment of the fatigue behaviour of welded aluminium under multiaxial loading', *Fatigue & Fracture of Engineering Materials & Structures*, 26(6), pp. 507–513.

- Kueppers, M. and Sonsino, C. M. (2006) 'Assessment of the fatigue behaviour of welded aluminium joints under multiaxial spectrum loading by a critical plane approach', *International Journal of Fatigue*, 28(5–6), pp. 540–546.
- Langlais, T. E., Vogel, J. H. and Chase, T. R. (2003) 'Multiaxial cycle counting for critical plane methods', *International Journal of Fatigue*, 25(7), pp. 641–647.
- Lee, Y.-L. and Tjhung, T. (2011) 'Rainflow cycle counting techniques', *Metal Fatigue Analysis Handbook: Practical Problem-solving Techniques for Computer-aided Engineering*, 89.
- Lee, Y. L. and Pan, J. (2005) *Fatigue Testing and Analysis: Theory and Practice*. Elsevier.
- Livieri, P. and Lazzarin, P. (2005) 'Fatigue strength of steel and aluminium welded joints based on generalised stress intensity factors and local strain energy values', *International Journal of Fracture*, 133(3), pp. 247–276.
- Matsuishi, M. (1968) 'Fatigue of Metals Subjected to Varying Stress', *Japan Society of Mechanical Engineers*.
- Mei, J. *et al.* (2021) 'An overview and comparative assessment of approaches to multi-axial fatigue of welded components in codes and standards', *International Journal of Fatigue*, 146, p. 106144.
- Morgenstern, C. *et al.* (2006) 'Fatigue design of aluminium welded joints by the local stress concept with the fictitious notch radius of $r_f=1$ mm', *International Journal of Fatigue*, 28(8), pp. 881–890. doi: 10.1016/j.ijfatigue.2005.10.006.
- Niemi, E. (1995) *Stress determination for fatigue analysis of welded components*. Woodhead Publishing.
- Papuga, J. (2011) 'A survey on evaluating the fatigue limit under multiaxial loading', *International Journal of Fatigue*, 33(2), pp. 153–165.

Pedersen, M. M. (2016) 'Multiaxial fatigue assessment of welded joints using the notch stress approach', *International Journal of Fatigue*, 83, pp. 269–279. doi: 10.1016/J.IJFATIGUE.2015.10.021.

Pyttel, B., Grawenhof, P. and Berger, C. (2012) 'Application of different concepts for fatigue design of welded joints in rotating components in mechanical engineering', *International journal of fatigue*, 34(1), pp. 35–46.

Radaj, D. (1996) 'Review of fatigue strength assessment of nonwelded and welded structures based on local parameters', *International Journal of Fatigue*, 18(3), pp. 153–170.

Radaj, D., Lazzarin, P. and Berto, F. (2013) 'Generalised Neuber concept of fictitious notch rounding', *International Journal of Fatigue*, 51, pp. 105–115. doi: 10.1016/j.ijfatigue.2013.01.005.

Radaj, D., Sonsino, C. M. and Fricke, W. (2006) *Fatigue assessment of welded joints by local approaches*. Woodhead publishing.

Razmjoo, G. R. (1997) 'Fatigue of load-carrying fillet welded joints under multiaxial loading', in *5 th International Conference on Biaxial/Multiaxial Fatigue & Fracture*, pp. 53–70.

Rennert, D. *et al.* (2013) 'Analytical strength assessment of components made of steel, cast iron and aluminium materials in mechanical engineering', *FKM Guideline, 6th ed, Forschungskuratorium Maschinenbau (FKM), Frankfurt/Main, Germany*.

Schijve, J. (2012) 'Fatigue predictions of welded joints and the effective notch stress concept', *International Journal of Fatigue*, 45, pp. 31–38.

Shams, E. and Vormwald, M. (2017) 'Fatigue of weld ends under combined loading', *International Journal of Fatigue*, 100, pp. 627–638.

Siljander, A., Kurath, P. and Lawrence Jr, F. V (1989) 'Proportional and non-proportional multiaxial fatigue of tube to plate weldments', *University of Illinois at Urbana-Champaign, Urbana, Illinois, Report of the Welding Research Council*.

- Sonsino, C. M. (1995) 'Multiaxial fatigue of welded joints under in-phase and out-of-phase local strains and stresses', *International Journal of Fatigue*, 17(1), pp. 55–70.
- Sonsino, C. M. (1997) 'Fatigue behaviour of welded components under complex elasto-plastic multiaxial deformations', *ECSC-Report No. EUR 16024 DE*.
- Sonsino, C. M. *et al.* (2006) 'Fatigue strength of laser beam welded thin steel structures under multiaxial loading', *International journal of fatigue*, 28(5–6), pp. 657–662.
- Sonsino, C. M. (2007) 'Fatigue testing under variable amplitude loading', *International Journal of Fatigue*, 29(6), pp. 1080–1089.
- Sonsino, Cetin Morris (2009) 'A consideration of allowable equivalent stresses for fatigue design of welded joints according to the notch stress concept with the reference radii $r_{ref} = 1.00$ and 0.05 mm', *Welding in the World*, 53, pp. R64–R75.
- Sonsino, C M (2009) 'Multiaxial fatigue assessment of welded joints—recommendations for design codes', *International Journal of Fatigue*, 31(1), pp. 173–187.
- Sonsino, C. M. *et al.* (2012) 'Notch stress concepts for the fatigue assessment of welded joints—Background and applications', *International Journal of Fatigue*, 34(1), pp. 2–16.
- Sonsino, C. M., Bruder, T. and Baumgartner, J. (2010) 'SN lines for welded thin joints—suggested slopes and FAT values for applying the notch stress concept with various reference radii', *Welding in the World*, 54(11), pp. R375–R392.
- Sonsino, C. M. and Kueppers, M. (2001) 'Multiaxial fatigue of welded joints under constant and variable amplitude loadings', *Fatigue & Fracture of Engineering Materials & Structures*, 24(5), pp. 309–327.
- Standard, A. (2017) *E1049-85 (Reapproved 2017) Standard practices for cycle counting in fatigue analysis*. Tech. rep., ASTM International, West Conshohocken, PA.

Susmel, L. (2007) 'Eurocode 3's standard curves and theory of critical distances to estimate fatigue lifetime of steel weldments', *Key Engineering Materials*, 348, pp. 21–24.

Susmel, L. (2008) 'Modified Wöhler curve method, theory of critical distances and Eurocode 3: A novel engineering procedure to predict the lifetime of steel welded joints subjected to both uniaxial and multiaxial fatigue loading', *International Journal of Fatigue*, 30(5), pp. 888–907. doi: 10.1016/j.ijfatigue.2007.06.005.

Susmel, Luca (2008) 'The theory of critical distances: a review of its applications in fatigue', *Engineering Fracture Mechanics*, 75(7), pp. 1706–1724.

Susmel, L. (2009a) *Multiaxial notch fatigue*. Elsevier.

Susmel, L. (2009b) 'The Modified Wöhler Curve Method calibrated by using standard fatigue curves and applied in conjunction with the Theory of Critical Distances to estimate fatigue lifetime of aluminium weldments', *International Journal of Fatigue*, 31(1), pp. 197–212. doi: 10.1016/j.ijfatigue.2008.04.004.

Susmel, L. (2009c) 'Three different ways of using the Modified Wöhler Curve Method to perform the multiaxial fatigue assessment of steel and aluminium welded joints', *Engineering Failure Analysis*, 16(4), pp. 1074–1089. doi: 10.1016/j.engfailanal.2008.05.016.

Susmel, L. (2010) 'A simple and efficient numerical algorithm to determine the orientation of the critical plane in multiaxial fatigue problems', *International Journal of Fatigue*, 32(11), pp. 1875–1883. doi: 10.1016/j.ijfatigue.2010.05.004.

Susmel, L. (2014) 'Four stress analysis strategies to use the Modified Wöhler Curve Method to perform the fatigue assessment of weldments subjected to constant and variable amplitude multiaxial fatigue loading', *International Journal of Fatigue*, 67, pp. 38–54. doi: 10.1016/j.ijfatigue.2013.12.001.

Susmel, L. and Askes, H. (2012) 'Modified Wöhler Curve Method and multiaxial fatigue assessment of thin welded joints', *International Journal of Fatigue*, 43, pp. 30–42. doi: 10.1016/j.ijfatigue.2012.01.026.

- Susmel, L., Sonsino, C. M. and Tovo, R. (2011) 'Accuracy of the modified wöhler curve method applied along with the $r_{ref} = 1$ mm concept in estimating lifetime of welded joints subjected to multiaxial fatigue loading', *International Journal of Fatigue*, 33(8), pp. 1075–1091. doi: 10.1016/j.ijfatigue.2010.09.010.
- Susmel, L. and Taylor, D. (2012) 'A critical distance/plane method to estimate finite life of notched components under variable amplitude uniaxial/multiaxial fatigue loading', *International Journal of Fatigue*, 38, pp. 7–24. doi: <https://doi.org/10.1016/j.ijfatigue.2011.11.015>.
- Susmel, L. and Tovo, R. (2004) 'On the use of nominal stresses to predict the fatigue strength of welded joints under biaxial cyclic loading', *Fatigue & Fracture of Engineering Materials & Structures*, 27(11), pp. 1005–1024.
- Taylor, D. (2008) 'The theory of critical distances', *Engineering Fracture Mechanics*, 75(7), pp. 1696–1705.
- Taylor, D., Barrett, N. and Lucano, G. (2002) 'Some new methods for predicting fatigue in welded joints', *International Journal of Fatigue*, 24(5), pp. 509–518.
- Walat, K. and Łagoda, T. (2014) 'Lifetime of semi-ductile materials through the critical plane approach', *International Journal of Fatigue*, 67, pp. 73–77.
- Wiebesiek, J. *et al.* (2011) 'Multiaxial fatigue behaviour of laserbeam-welded thin steel and aluminium sheets under proportional and non-proportional combined loading', *International journal of fatigue*, 33(8), pp. 992–1005.
- Wiebesiek, J. and Sonsino, C. M. (2010) 'New results in multiaxial fatigue of welded aluminium joints', *IIW-Document No. XIII-2314-10/XV-1349-10*, 13, pp. 2310–2314.
- Yousefi, F., Witt, M. and Zenner, H. (2001) 'Fatigue strength of welded joints under multiaxial loading: experiments and calculations', *Fatigue & Fracture of Engineering Materials & Structures*, 24(5), pp. 339–355.

Yung, J.-Y. and Lawrence Jr, F. V (1989) 'Predicting the fatigue life of welds under combined bending and torsion', *Mechanical Engineering Publications, Biaxial and Multiaxial Fatigue*, pp. 53–69.

Al Zamzami, I. and Susmel, L. (2018) 'On the use of hot-spot stresses, effective notch stresses and the Point Method to estimate lifetime of inclined welds subjected to uniaxial fatigue loading', *International Journal of Fatigue*, 117, pp. 432–449. doi: 10.1016/j.ijfatigue.2018.08.032.

Chapter 5

Experimental Methodology

5.1 Introduction

This chapter presents the experimental methodology developed to investigate the multiaxial fatigue behaviour of hybrid aluminium-to-steel welded joints. By addressing critical knowledge gaps, this research contributes to the development of reliable models for estimating the fatigue life of these joints under multiaxial loading conditions. Through an integrated approach, the study addresses key challenges in welding dissimilar materials and evaluating their fatigue performance, with the ultimate goal of advancing design practices and supporting broader industrial applications.

A key focus of this investigation is the application of EWM coldArc® welding technology to produce high-quality hybrid joints capable of withstanding multiaxial fatigue loading. While previous research (Al Zamzami, Davison and Susmel, 2019) have examined this welding process under uniaxial conditions, its performance under multiaxial stress states remains unexplored. This study evaluates the welding quality achieved using coldArc® technology and extends its application to the medium-to-high cycle fatigue regime under constant amplitude (CA) multiaxial loading. By addressing this unexplored domain, the research provides novel insights into hybrid welding technology and its suitability for advanced engineering applications.

Another critical aspect of this research involves analysing cracking behaviour and failure mechanisms under multiaxial fatigue conditions. Understanding fracture processes and their progression is essential for developing predictive models and reliable design strategies for hybrid joints under complex multiaxial fatigue loading scenarios. The experimental findings aim to validate and formalise design

methodologies that enable engineers to predict fatigue life accurately in practical applications.

This chapter introduces the methodologies employed, including tailored specimen design for multiaxial fatigue testing, optical microscopy for measuring weld toe and root notch geometries, and the integration of these measurements into finite element (FE) models. These methodologies collectively support the application of an alternative effective notch stress approach and the validation of multiaxial fatigue criteria, providing a comprehensive framework for fatigue assessment.

The chapter is structured to provide a clear and logical overview of the experimental methodology. It begins with a description of the materials used, the welding process employed, and the design considerations for test specimens. This is followed by a detailed explanation of the experimental setup, including the equipment and configurations used to simulate multiaxial fatigue conditions. The subsequent sections cover testing protocols, criteria for test termination, and data acquisition techniques. The chapter concludes with the methodology for microscopic measurements of weld toe and weld root notch radii, which serve to hypothesise potential critical fatigue failure locations and support the validation of the multiaxial fatigue assessment of these hybrid joints through FE modelling.

By presenting these methodologies cohesively, this chapter establishes the foundation for understanding the experimental framework and its critical role in achieving the research objectives.

5.2 Materials and Specimen Design

This section outlines the selection of materials, welding process, and geometric configuration of the specimens designed to systematically investigate the multiaxial fatigue behaviour of thin hybrid aluminium-to-steel welded joints. The design process prioritised replicating practical joint configurations while incorporating necessary adjustments to ensure compatibility with the testing equipment and grips available in the laboratory.

5.2.1 Materials

This study utilised aluminium alloy AA6082 T6 and zinc-coated S235 steel, both compliant with EN 10255 standards, as the primary materials for the hybrid welded joints. To ensure effective bonding between the dissimilar metals, AWS A5.10 ER4043 filler material was selected for its ability to provide adequate weld strength while ensuring joint reliability and durability.

A critical challenge in joining aluminium and steel lies in their limited solubility, as highlighted by the Fe-Al phase diagram (Richards *et al.*, 1994; Bouche, Barbier and Coulet, 1998; Goecke, 2005; Kah, Suoranta and Martikainen, 2013; Atabaki *et al.*, 2014). This inherent incompatibility often results in the formation of brittle intermetallic Fe-Al phases when their concentration in the molten material exceeds 10% (Richards *et al.*, 1994; Bouche, Barbier and Coulet, 1998; Goecke, 2005; Kah, Suoranta and Martikainen, 2013; Atabaki *et al.*, 2014). To overcome this challenge, the S235 steel tubular components were coated with zinc prior to welding. The zinc coating served as a protective barrier, regulating the melting process by allowing partial melting of the aluminium while limiting the melting of steel. This approach facilitated the creation of a robust joint, with the aluminium side forming a welded connection and the steel side forming a brazed bond (Goecke, 2005; Kah, Suoranta and Martikainen, 2013). By preventing the formation of brittle intermetallic phases, the zinc coating ensured the structural integrity and durability of the hybrid joint. The mechanical properties and chemical compositions of the materials are summarised in Table 5.1 and 5.2, respectively.

Table 5.1: Mechanical properties of welding materials.

Material	Ultimate Tensile Strength,	Young's Modulus,	Poisson's Ratio,
	UTS (MPa)	E (GPa)	ν (-)
AA6082-T6	275	70	0.33
S235	360	210	0.30

Table 5.2: Chemical composition of investigated materials by weight percentage (%).

Material	Chemical Composition (wt%)								
AA6082-T6	Mg	Si	Mn	Fe	Zn	Cu	Ti	Cr	Al
	0.6-1.2	0.7-1.3	0.4-1.0	0.5	0.2	0.1	0.1	0.25	Balance
S235	Mn	C	P	S					
	1.4	0.2	0.035	0.035					
ER4043	Mg	Si	Mn	Fe	Zn	Cu	Ti	Al	
	0.05	4.5-6.0	0.05	0.8	0.1	0.3	0.2	Balance	

5.2.2 Welding Process and Technology

The fabrication of tube-to-tube hybrid aluminium-to-steel joints in this study was carried out using EWM coldArc® welding technology. This advanced process was selected for its proven ability to address critical challenges associated with hybrid joints, including the control of intermetallic compound layer thickness and the minimisation of welding defects such as pores, spatter, and microcracks. Its precision and adaptability make it particularly effective for joining dissimilar materials like aluminium and steel (Goecke, 2005; Kah, Suoranta and Martikainen, 2013).

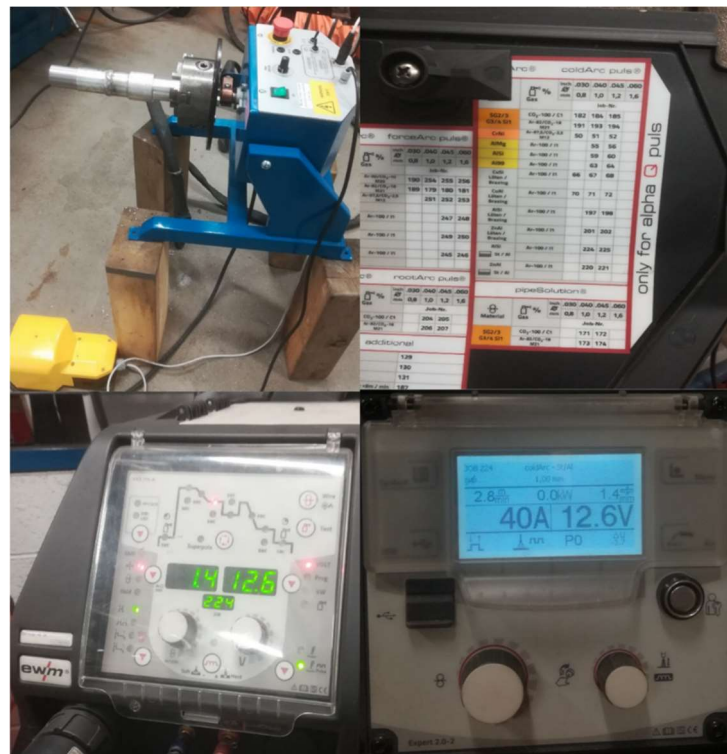
The process employs dynamic control of the current output during the arc re-ignition phase, reducing power peaks. This reduction in power minimises spatter formation, distortion, material thinning, and damage to the zinc coating while maintaining good gap-bridging capabilities (Goecke, 2005; Kah, Suoranta and Martikainen, 2013; Al Zamzami *et al.*, 2018). For a more detailed explanation of this modified short arc process, please refer to Section 2.2 of the Literature Review.

The EWM coldArc® technology includes pre-programmed welding parameters tailored for various material combinations and thicknesses. By selecting the appropriate program code and specifying the material thickness, the machine automatically configures the required parameters. If necessary, these parameters can also be manually adjusted to meet specific experimental requirements. The optimised welding parameters used in this study for fabricating tube-to-tube hybrid aluminium-to-steel welded joint specimens are summarised in Table 5.3.

Table 5.3: Parameters for EWM coldArc® welding.

Weld Settings	
Program	Pulse 224
Arc Voltage	12.6 V
Current	40 A
Wire Feed Rate	2.8 m/min
Weld Speed	3.27 mm/s

The specimens in this investigation were manually welded by an experienced technician. The welding process utilised the EWM alpha Q551 pulse welding machine, which integrates the coldArc® technology. A detailed view of the welding setup and machine interface is shown in Figure 5.1.

**Figure 5.1:** Setup and interface of the EWM alpha Q551 pulse welding machine.

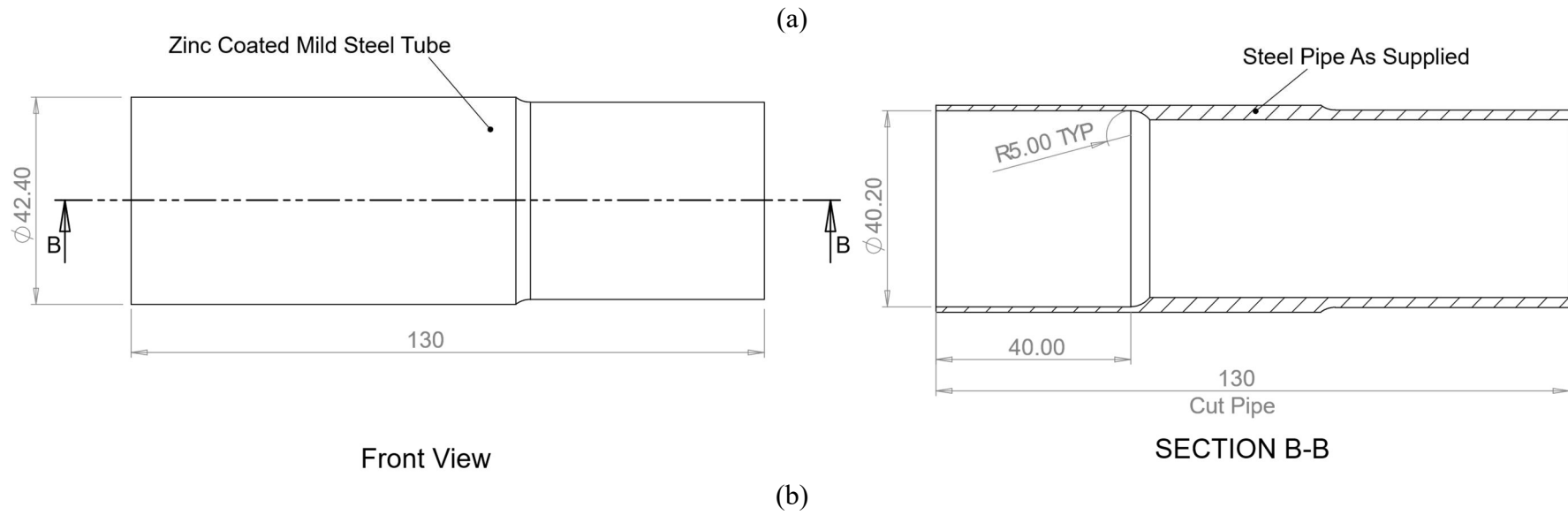
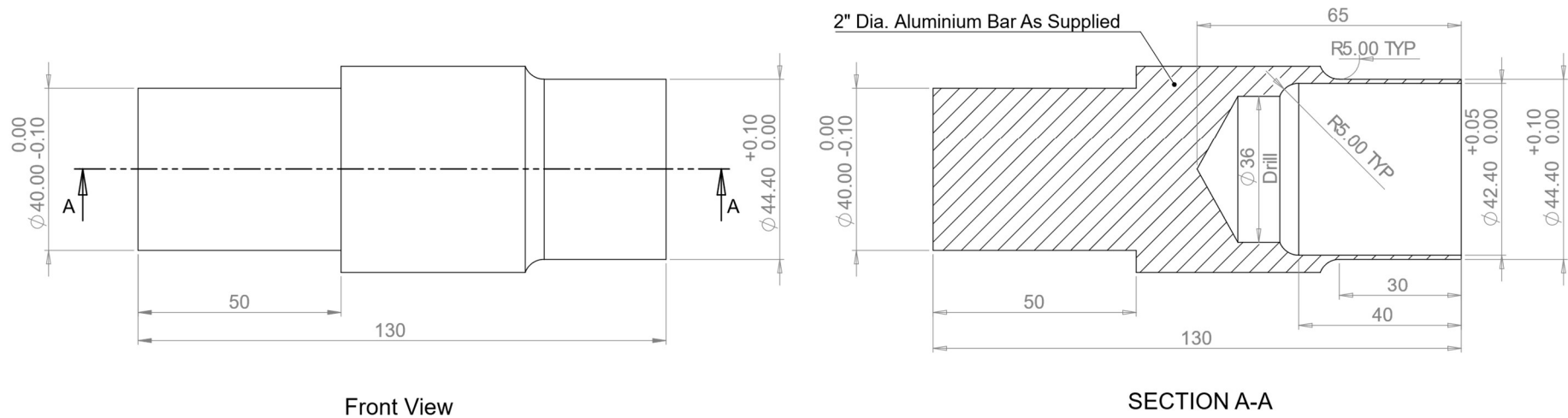
5.2.3 Specimen Geometry and Design

The design of the tube-to-tube hybrid aluminium-to-steel welded joint specimens was carefully developed to meet the specific requirements of multiaxial fatigue testing while addressing practical constraints. Key considerations included ensuring the weld region

served as the critical area for fatigue failure, achieving compatibility with testing equipment, and maintaining precise alignment of the components. The welding interface consisted of overlapping 1 mm thick hollow tubular aluminium with 1 mm thick galvanised hollow tubular steel, with an overlap distance of 15 mm to provide adequate joint strength. Each specimen had an overall length of 245 mm, with a clamping distance of 140 mm between the grips. The aluminium tube near the weld region had an outer diameter of 44.40 mm and an inner diameter of 42.40 mm.

To address challenges associated with hybrid aluminium-to-steel joints, several design modifications were introduced. Both the aluminium and steel components were precisely machined to create a thin overlap weld region and to ensure seamless fitting into the laboratory collet grips. Careful alignment during welding prevented eccentric loading and ensured uniform stress distribution. Additionally, a root radius of 5 mm was introduced in non-welded regions to prevent localised stress concentrations and to keep the weld region as the primary focus for fatigue failure. The raw aluminium material, initially larger and solid, was drilled and machined to achieve the desired hollow geometry. Similarly, the as-supplied steel tubes, originally 42 mm in diameter, were machined down to 40 mm to fit the hydraulic collet grips, which had a maximum diameter capacity of 40 mm.

The tube-to-tube configuration was selected for its suitability for combined tension and torsion fatigue testing. However, the absence of specific standards for the geometrical configuration of hybrid aluminium-to-steel welded joints in multiaxial fatigue testing, as outlined by bodies such as ASTM, IIW, or Eurocode, necessitated a tailored geometry design approach. The final geometry achieved a balance between functional testing requirements, manufacturing feasibility, and cost efficiency, ensuring reliable performance under multiaxial loading conditions. As illustrated in Figures 5.2a to 5.2c, the final specimen configuration was designed to fit the collet grips while addressing the unique challenges of hybrid welding. These design adjustments ensured accurate and reliable testing under multiaxial fatigue loading, enabling a thorough evaluation of the fatigue performance and failure mechanisms of the welded joint specimens.



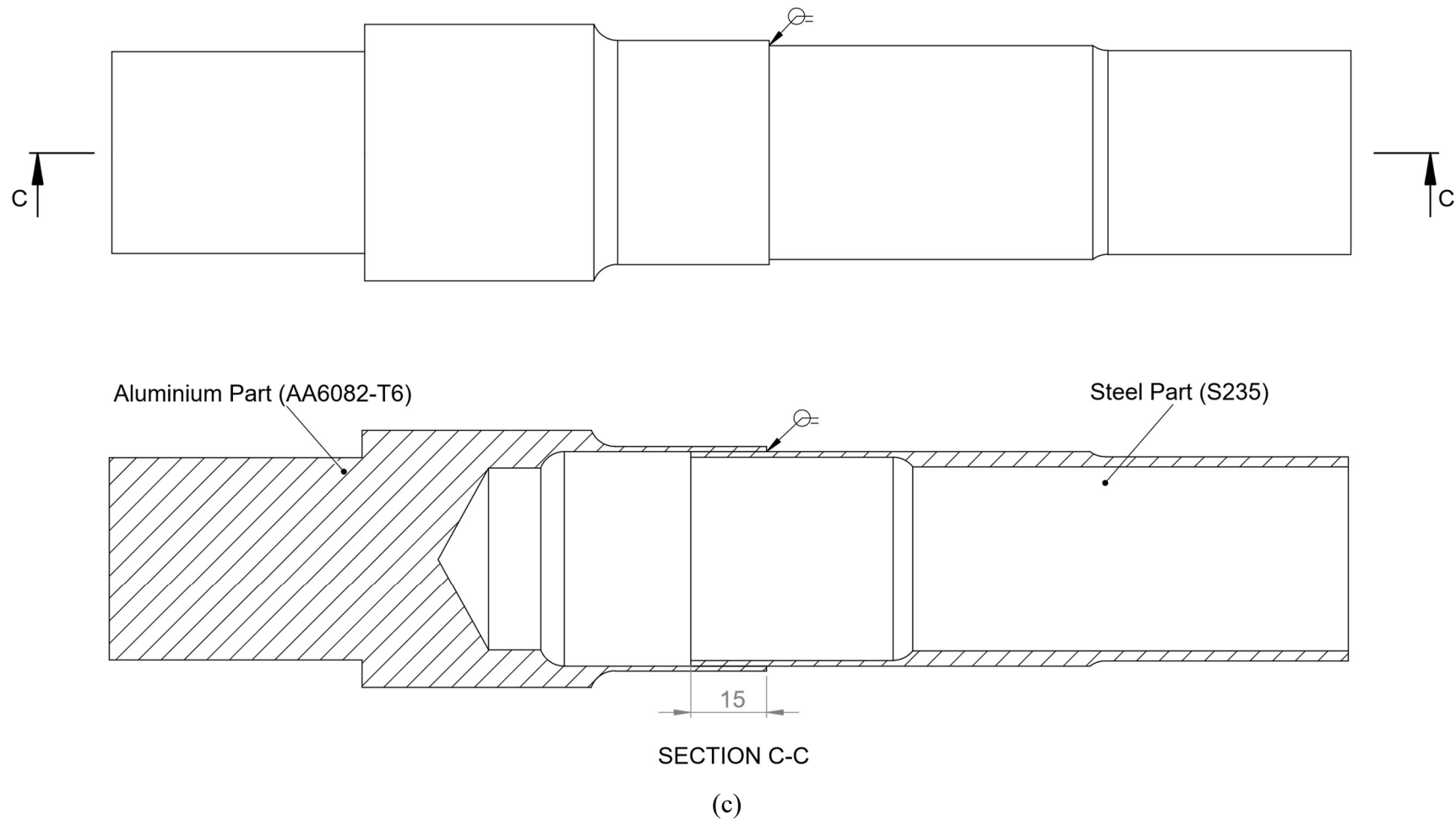


Figure 5.2: Geometrical configuration of the investigated hybrid aluminium-to-steel welded joint samples: (a) aluminium (AA6082-T6) part, (b) steel (S235) part, and (c) complete hybrid welded joint.

5.3 Experimental Setup

The multiaxial fatigue tests were performed at room temperature using a Schenck servohydraulic tension–torsion fatigue testing machine controlled by an FCS SmarTEST ONE Servo Controller, which managed fatigue loading across multiple channels. Hybrid welded joint specimens were securely clamped using MTS 646 Hydraulic Collet Grips, designed to withstand a clamping pressure of up to 45 MPa, with an axial load capacity of 250 kN and a torsional capacity of 2200 Nm. The experimental setup for these tests is illustrated in Figure 5.3.

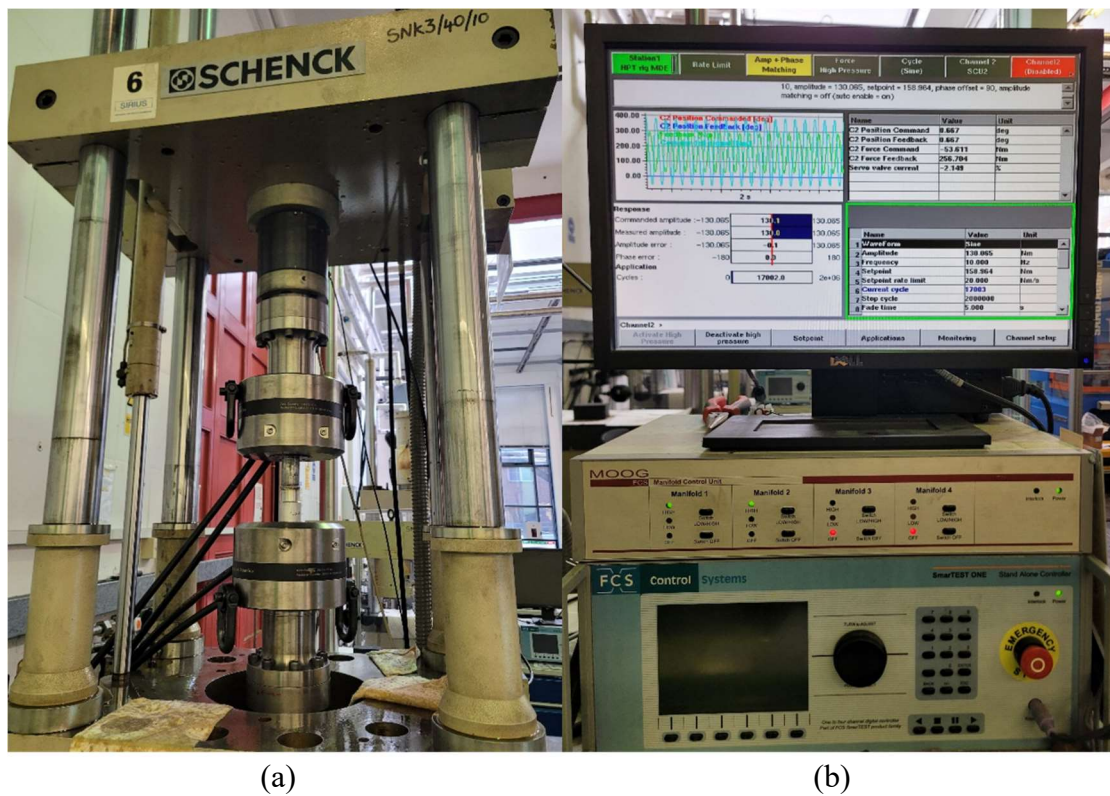


Figure 5.3: Experimental setup for fatigue testing: (a) Schenck servohydraulic tension-torsion fatigue machine, (b) FCS SmarTEST ONE Servo Controller.

The specimens were tested in their as-welded condition, without any heat treatment prior to mechanical testing. To ensure consistency and reliability, only high-quality welded joints were selected, with any specimens exhibiting visible defects such as spatter, pores, or cracks excluded from testing. Fatigue tests were designed to evaluate the effects of mean stress and combined tension-torsion loading conditions. Load ratios ($R = \sigma_{\min}/\sigma_{\max}$ or $R = \tau_{\min}/\tau_{\max}$) of -1 and 0.1 were applied to assess the mean stress effect.

Multiaxial fatigue tests were conducted under both in-phase and 90° out-of-phase loading conditions to examine the effect of rotating principal stress directions on the fatigue life. The biaxiality stress ratio, λ , defined as the ratio of axial stress amplitude, σ_a to the torsional stress amplitude, τ_a , was calculated using Equation 5.1:

$$\lambda = \frac{\sigma_a}{\tau_a} \quad (5.1)$$

During fatigue testing, λ values were fixed at 1 and $\sqrt{3}$ to investigate its impact on the critical plane and fatigue life. All multiaxial fatigue tests employed sinusoidal CA loading at a frequency of 10 Hz. The loading path diagrams, which include variations in λ for each configuration are illustrated in Figure 5.4.

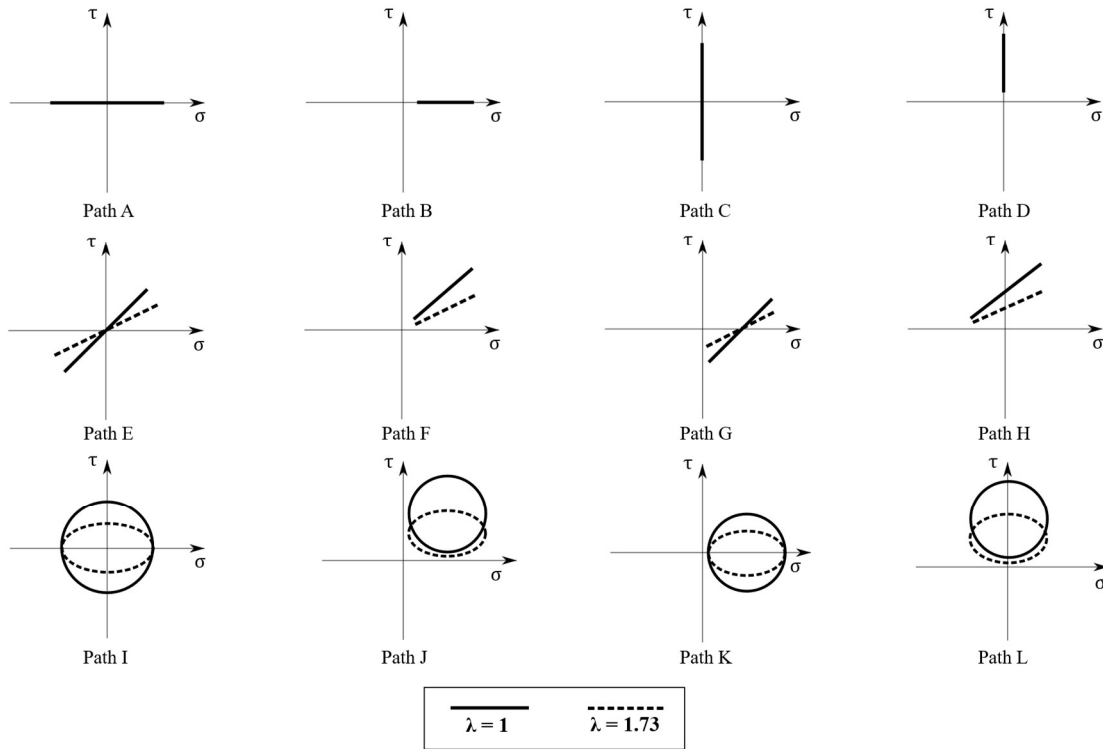


Figure 5.4: Fatigue loading path diagrams for the investigated configurations.

To ensure statistical reliability, the testing protocol adhered to ASTM E739-10 guidelines for research and development purposes (ASTM, 2015). Each test series comprised 6 to 12 tests, with a minimum of two specimens tested per stress level. For the primary series at $R=-1$, at least eight specimens were tested, with a replication level of 50%, which exceeded the recommended minimum replication percentage of 33% (ASTM, 2015).

The fatigue testing matrix, summarised in Table 5.4, provides a comprehensive overview of the test configurations, including uniaxial, pure torsion, in-phase, and out-of-phase fatigue tests. It also outlines the corresponding loading paths, λ values, and the number of hybrid welded joint specimens tested.

Table 5.4: Fatigue test matrix, including loading paths, biaxiality ratio (λ), and number of specimens.

Load Type	Load Path	Biaxiality ratio, λ	No. of Samples	Total No. of Samples
Uniaxial	A	∞	8	12
	B	∞	4	
Pure Torsion	C	0	8	12
	D	0	4	
In Phase	E	1	6	36
		1.73	6	
	F	1	4	
		1.73	4	
	G	1	4	
		1.73	4	
	H	1	4	
		1.73	4	
Out of Phase	I	1	6	36
		1.73	6	
	J	1	4	
		1.73	4	
	K	1	4	
		1.73	4	
	L	1	4	
		1.73	4	

Given the focus of this research on the medium to high cycle fatigue regime, nominal axial stress amplitudes were applied in the range of 30 to 85 MPa, with fatigue lives spanning from 10^3 to 2 million cycles to failure. Runout was defined at 2 million cycles and specimens reaching this threshold were retested at higher stress amplitudes. Fatigue tests were terminated upon observing a 10% reduction in material stiffness, which corresponded with the appearance of visible cracks measuring between 10 and 20 mm in length. This stiffness reduction indicated a loss of mechanical strength and functionality as a safe structural element, and the corresponding number of cycles to failure was recorded.

Nominal normal and shear stresses were calculated based on the surface area of the aluminium tube at the critical weld region rather than the steel tube. Aluminium, identified as the weaker link between the two materials, was expected to be the primary location of fatigue failure. This approach ensured that the derived nominal stresses accurately reflected the mechanical behaviour and physical failure mechanisms observed during multiaxial fatigue testing of these hybrid joints.

5.4 Measurements for Notch Geometry and Fatigue Assessment

Notch radii near the weld toe and weld root regions were measured by examining eight sectioned specimens, comprising four undamaged welded joints and four specimens that had failed during fatigue testing. These samples were analysed using a Nikon optical microscope with 5x and 10x magnification lenses, enabling precise measurements of notch radii and weld root opening angles. This data was critical for identifying potential critical fatigue failure locations and forming hypotheses about the failure mechanisms of these hybrid joints, further explored in Chapter 6. Additionally, the average notch radii and weld root geometries derived from these measurements were incorporated into the finite element (FE) model to accurately represent the joint geometry. This step was essential for applying the alternative effective notch stress approach and validating multiaxial fatigue criteria, as discussed in Chapter 7.

The failure analysis in this study was primarily conducted at a mesoscopic to macroscopic scale, utilising front and plan views to identify crack initiation sites and surface characteristics. This approach ensured a practical focus aligned with the study's primary aim of developing multiaxial fatigue design methodologies for aluminium-to-steel welded joints in engineering applications. While advanced techniques such as scanning electron microscopy (SEM) could provide additional microscopic insights into crack surfaces and failure mechanisms, these methods were beyond the scope of this project, as the emphasis was on practical design solutions rather than detailed microscopic investigations.

Future research could expand on this work by incorporating advanced microscopic techniques to enhance understanding of failure mechanisms at a microscopic level, as outlined in Section 8.2.

5.5 References

ASTM (2015) *ASTM E739-10: Standard Practice for Statistical Analysis of Linear Or Linearized Stress-Life (S-N) and Strain-Life (ϵ -N) Fatigue Data*. ASTM International.

Atabaki, M. M. *et al.* (2014) ‘Welding of aluminum alloys to steels: an overview’, *Journal for Manufacturing Science and Production*, 14(2), pp. 59–78.

Bouche, K., Barbier, F. and Coulet, A. (1998) ‘Intermetallic compound layer growth between solid iron and molten aluminium’, *Materials Science and Engineering: A*, 249(1–2), pp. 167–175.

Cao, R. *et al.* (2014) ‘Cold metal transfer joining of aluminum AA6061-T6-to-galvanized boron steel’, *Journal of Manufacturing Science and Engineering*, 136(5), p. 051015.

Goecke, S. F. (2005) ‘Low energy arc joining process for materials sensitive to heat’, *EWM Hightec Welding GmbH*, 3.

Kah, P., Suoranta, R. and Martikainen, J. (2013) ‘Advanced gas metal arc welding processes’, *The International Journal of Advanced Manufacturing Technology*, 67, pp. 655–674.

Richards, R. W. *et al.* (1994) ‘Metallurgy of continuous hot dip aluminizing’, *International materials reviews*, 39(5), pp. 191–212.

Al Zamzami, I. *et al.* (2018) ‘Static strength and design of aluminium-to-steel thin welded joints’, *Welding in the World*, 62(6), pp. 1255–1272. doi: 10.1007/s40194-018-0634-2.

Al Zamzami, I., Davison, B. and Susmel, L. (2019) ‘Nominal and local stress quantities to design aluminium-to-steel thin welded joints against fatigue’, *International Journal of Fatigue*, 123, pp. 279–295.

Chapter 6

Failure Analysis of Hybrid Aluminium-to-Steel Welded Joints

6.1 Introduction

Building on the experimental methodology detailed in Chapter 5, this chapter presents a comprehensive failure analysis of hybrid aluminium-to-steel welded joints, with a particular emphasis on crack initiation sites, failure locations, and crack path behaviour under multiaxial fatigue loading. These hybrid joints hold significant potential for structural applications, particularly in the automotive, offshore, and bridge engineering sectors, where reducing weight while maintaining mechanical performance is critical. By integrating lightweight and fully recyclable aluminium with high-strength steel for load-bearing components, hybrid welded joints offer a promising approach to enhancing structural efficiency and sustainability (Kah, Suoranta and Martikainen, 2013; Meco *et al.*, 2015; Gullino, Matteis and D'Aiuto, 2019). However, to fully exploit their benefits, it is essential to understand their fatigue failure mechanisms, particularly their crack path behaviour, under complex loading conditions.

Past studies have investigated welding and fatigue behaviour in dissimilar metal joints, particularly for laser arc welding and friction stir welding of hybrid aluminium-to-steel structures (Shahverdi *et al.*, 2002; Kimapong and Watanabe, 2005; Kah, Suoranta and Martikainen, 2013; Atabaki *et al.*, 2014; Cao *et al.*, 2014; Qin, Su and Wang, 2014; Meco *et al.*, 2015; Gullino, Matteis and D'Aiuto, 2019). These studies have provided valuable insights into fracture mechanisms, microstructural evolution, and failure behaviour (Liu *et al.*, 2023; Kalinenko *et al.*, 2024). Similarly, crack path analysis has been explored in various welded joint configurations, including tubular, fillet weld, tube-flange, and cruciform joints, under tensile, torsional, and bending fatigue loads

(Amstutz, Störzel and Seeger, 2001; Wang *et al.*, 2013; Caligiuri, 2015; Maggiolini *et al.*, 2016; Liu *et al.*, 2018; Shin, Chang and Muzaffer, 2021; Zhang, 2021). Further studies have also examined stress analysis approaches and fatigue life prediction models to assess the accuracy of different fatigue design methods for steel and aluminium welded joints under multiaxial fatigue conditions (Yousefi, Witt and Zenner, 2001; Macdonald and Haagenzen, 2009; Susmel, 2009b; Al Zamzami and Susmel, 2018).

Despite these advancements, coldArc® welding for thin hybrid aluminium-to-steel welded joints under multiaxial fatigue loading remains unexplored. While previous research has considered other welding methods such as laser arc and friction stir welding, coldArc®-welded thin-walled hybrid joints have not been studied under these complex loading conditions.

This chapter aims to bridge this knowledge gap by conducting a failure and crack path analysis of coldArc® welded hybrid aluminium-to-steel joints subjected to uniaxial, pure torsional, in-phase, and out-of-phase constant amplitude (CA) fatigue loading. The study begins with a microscopic examination to identify critical fatigue failure locations and assess local geometric features influencing crack initiation. This is followed by a detailed crack path analysis, mapping crack initiation trends, failure modes, and the number of cracks observed under different fatigue loading conditions. The study also evaluates crack surface morphology, including ductile failure characteristics, to provide a comprehensive understanding of the governing fatigue failure mechanisms.

Furthermore, the findings presented in this chapter serve as a foundation for Chapter 7, where they will contribute to the validation and formalisation of multiaxial fatigue design criteria and stress analysis approaches for accurately predicting the fatigue life of hybrid aluminium-to-steel welded joints. By systematically analysing failure locations, crack path trends, and theoretical-experimental correlations, this chapter provides valuable insights into the fatigue performance of these joints. The outcomes of this study will ultimately help refine fatigue assessment methodologies, guiding the design and optimisation of hybrid welded structures for enhanced durability and performance.

6.2 Identification of Critical Fatigue Crack Initiation Sites

A crucial step in understanding the fatigue behaviour of coldArc® welded hybrid aluminium-to-steel joints is determining where cracks are most likely to initiate. This involves formulating hypotheses based on joint geometry, stress concentration effects, and prior studies, followed by experimental validation through optical microscopy and crack surface analysis. Accurately identifying these failure locations provides insight into the underlying failure mechanisms under multiaxial fatigue loading, ultimately contributing to the development of more effective fatigue design methodologies.

Considering material discontinuities, geometric features, and expected stress concentrations, several potential crack initiation sites were initially hypothesised: the weld toe on both the steel and aluminium sides, the weld root regions (weld root (WR) 1 at the steel-aluminium interface and WR 2 within the aluminium region), the weld start/stop position, the weld itself, and the aluminium heat-affected zone (HAZ). These locations were selected based on established fatigue failure mechanisms in welded joints, where geometric discontinuities serve as stress concentrators and are often the most vulnerable points under cyclic loading. Figure 6.1 illustrates these hypothesised failure locations, providing a visual reference for the discussion that follows.

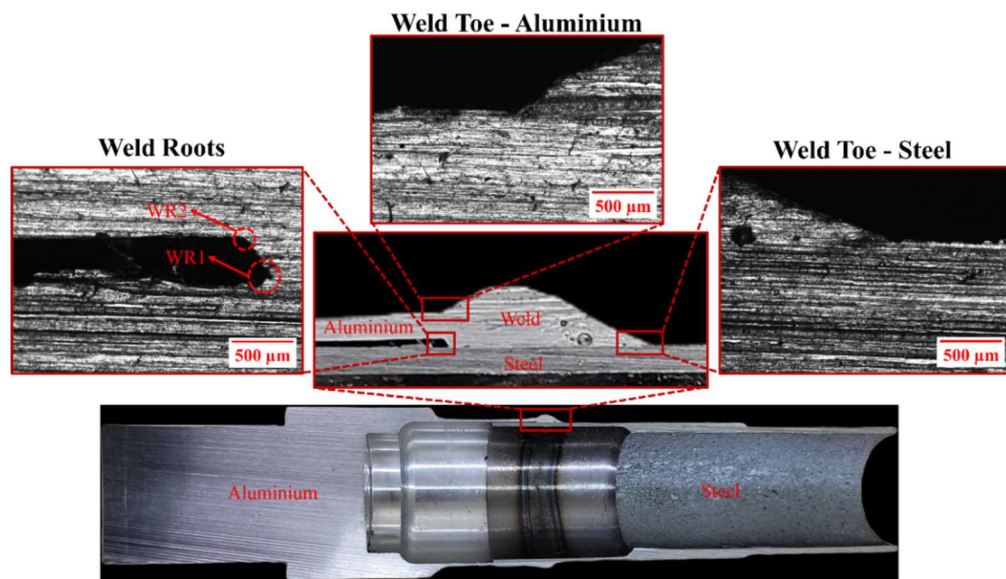


Figure 6.1: Sectioned hybrid aluminium-to-steel joint specimen illustrating potential fatigue crack initiation sites. Enlarged views highlight the weld toe regions on both aluminium and steel sides, as well as the weld root locations (WR1 and WR2).

To validate these hypotheses, optical microscopy was conducted on sectioned specimens, allowing for direct observation of potential critical fatigue crack initiation locations. The weld toe and root radii were measured using the radius function built into the microscope software, with the average values presented in Table 6.1. While the measurements are not exact and depend on how well the notch profile can be defined for circle fitting, they are sufficient to provide a reasonable approximation of the local weld geometry. Given the inherent variability of the weld profile along the joint, these average values serve as a practical representation of the typical notch geometry across different regions of the specimen.

Table 6.1: Measured average notch radii at potential fatigue crack initiation sites in aluminium-to-steel welded joints.

Potential Fatigue Crack Initiation Sites	Average Notch Radius (mm)
Weld Toe – Steel Side	1.82
Weld Toe – Aluminium Side	1.62
Weld Root (WR) 1 – Aluminium/Steel Interface	0.071
Weld Root (WR) 2 - Aluminium	0.073

Experimental findings confirmed that not all hypothesised locations exhibited crack initiation, enabling a refinement of the list of critical fatigue failure sites by eliminating locations where no fatigue failure was observed.

One of the most notable findings was that no specimens exhibited crack initiation from the steel weld toe or steel components, despite variations in weld profile geometry. Even in some cases where the weld toe notch radius on the steel side was smaller than other critical regions, which implies a higher stress concentration effects and cracks did not initiate from these locations. This observation aligns with previous research, which reported similar findings for coldArc®-welded butt, lap, and fillet joints under uniaxial and static loading conditions (Al Zamzami *et al.*, 2018; Al Zamzami, Davison and Susmel, 2019). The higher fatigue strength of steel made it less susceptible to failure, leading to the elimination of the steel weld toe as a critical failure location and shifting the focus to the aluminium side of the joint.

Further microscopic observations confirmed the presence of two weld roots, a feature resulting from the welding process, which would have remained undetected without

sectioning the specimens. As a result, the potential failure locations were refined to include the aluminium weld toe, weld root 1 at the steel-aluminium interface, weld root 2 within the aluminium region, the aluminium HAZ, and the weld itself. However, detailed crack path analysis demonstrated that weld root 1 did not serve as a primary crack initiation site, as there was no clear evidence of cracks propagating from its bisector toward the weld toe based on the analysed crack surfaces. Consequently, weld root 1 was ruled out, leaving four dominant failure locations: the aluminium weld toe/weld root 2 region, the weld itself, the weld start/stop position, and the aluminium HAZ.

Distinguishing between crack initiation at the aluminium weld toe and weld root 2 posed a key challenge due to the thin-walled geometry of the tubular specimens, which made direct observation of the weld root during fatigue testing impossible. Furthermore, post-failure crack surface analysis was complicated by fatigue-induced damage, preventing precise determination of the primary initiation site. The localised weld profile, which varies between specimens, also played a significant role in determining the exact failure mode. It is plausible that crack initiation occurred at either location depending on the specific weld geometry, with some cases suggesting the possibility of simultaneous crack initiation at both the weld toe and weld root 2, which then propagated and merged into a single failure path. Evidence supporting this was observed through surface roughness variations, where smoother regions with slower and more stable crack formations indicated crack initiation sites, while rougher areas indicating unstable crack growth corresponded to crack propagation zones, marking the transition to Stage II fracture (Pook, 1995; Reis, Li and De Freitas, 2006; Totten, 2008; Susmel, 2009a; Macdonald, 2011; Caligiuri, 2015; Liu *et al.*, 2018). Given these constraints, the aluminium weld toe and weld root 2 were classified together as a single failure mode, acknowledging their combined contribution to fatigue failure.

In contrast, fatigue cracks originating from the weld start/stop position, the weld itself, or the aluminium HAZ were more straightforward to identify. These locations exhibited distinct crack initiation patterns, directly correlated with localised weld imperfections, material discontinuities, or softening effects in the HAZ. The following section will

further investigate these finalised crack initiation failure locations, providing deeper insight into crack path development under different fatigue loading conditions.

6.3 Crack Initiation Site Analysis

Following the identification of critical fatigue crack initiation locations in Section 6.2, this section examines how these failure sites vary under different fatigue loading conditions. The primary objective is to analyse which locations dominate as crack initiation sites under uniaxial, torsional, in-phase, and out-of-phase loading and to illustrate these findings using experimental images and statistical data. The analysis was conducted at a mesoscopic to macroscopic scale, utilising optical microscopy, front, plan and section views to observe and analyse crack initiation trends of the hybrid aluminium-to-steel welded joints.

Observations from crack path analysis and failure surface examinations revealed that cracks consistently initiated from a single categorised crack initiation site, with most cases exhibiting initiation from a single localised point. However, distinguishing between weld root 2 and the aluminium weld toe remained challenging, leaving open the possibility of simultaneous crack initiation at both locations. Regardless of loading conditions or stress amplitude, crack initiation was consistently confined to a specific failure category, underscoring the dominant role of localised weld geometry and stress concentration effects in governing fatigue failure.

Although crack initiation in theory should be randomly distributed under nominally uniform stress application, experimental observations indicate that localised geometric discontinuities dictate failure locations. As established in Section 6.2, four primary crack initiation sites were identified: the aluminium weld toe/weld root region, the weld start/stop position, the weld itself, and the aluminium HAZ. The distribution of failure modes across different fatigue loading conditions is summarised in Figure 6.2, which presents a bar chart illustrating the correlation between loading conditions and crack initiation site occurrences.

This statistical analysis, based on all tested hybrid welded joints from the fatigue testing matrix (Table 5.4) in Chapter 5, confirms that local stress concentration effects and loading conditions, rather than stress amplitude levels, are the primary factors influencing crack initiation locations.

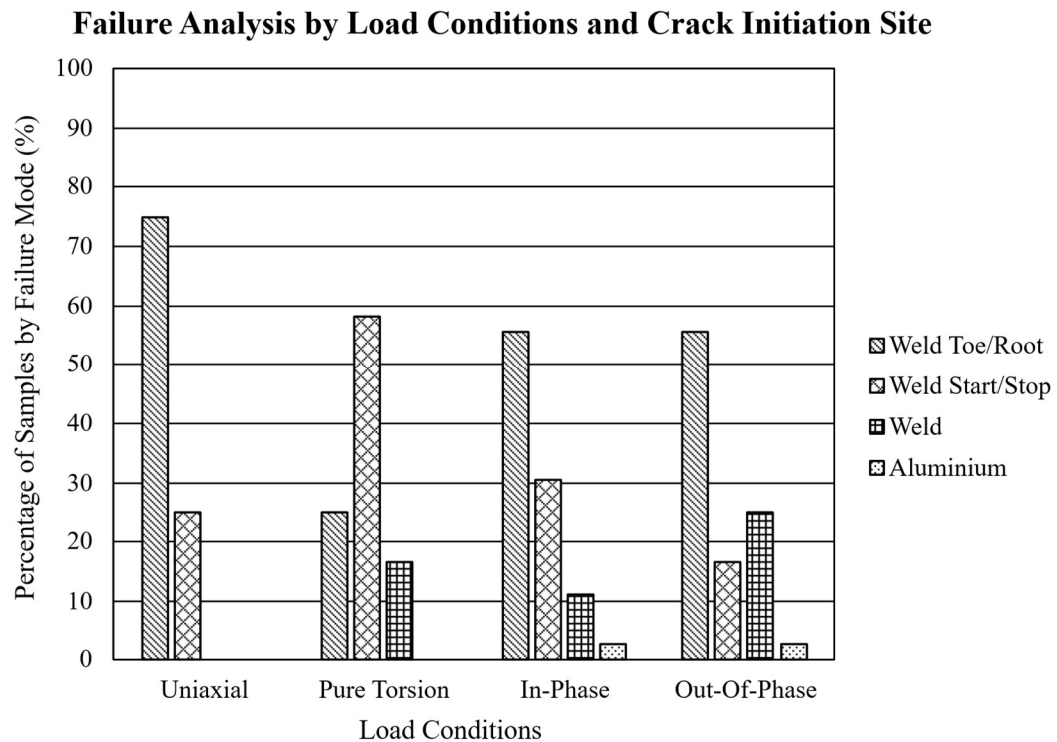
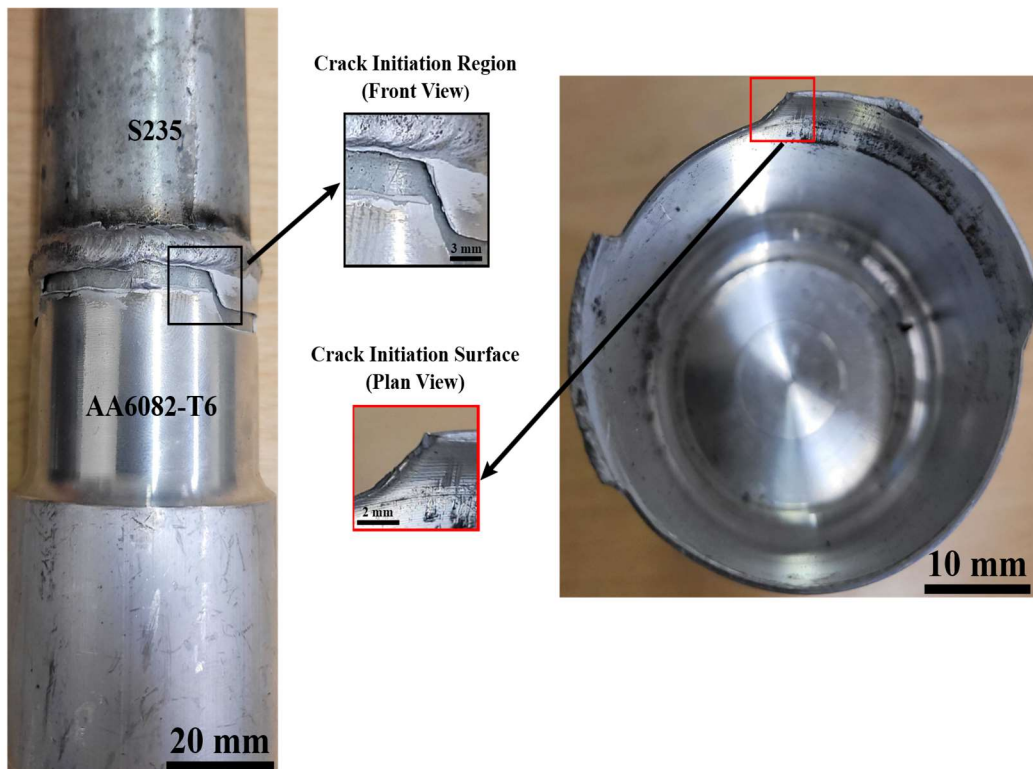
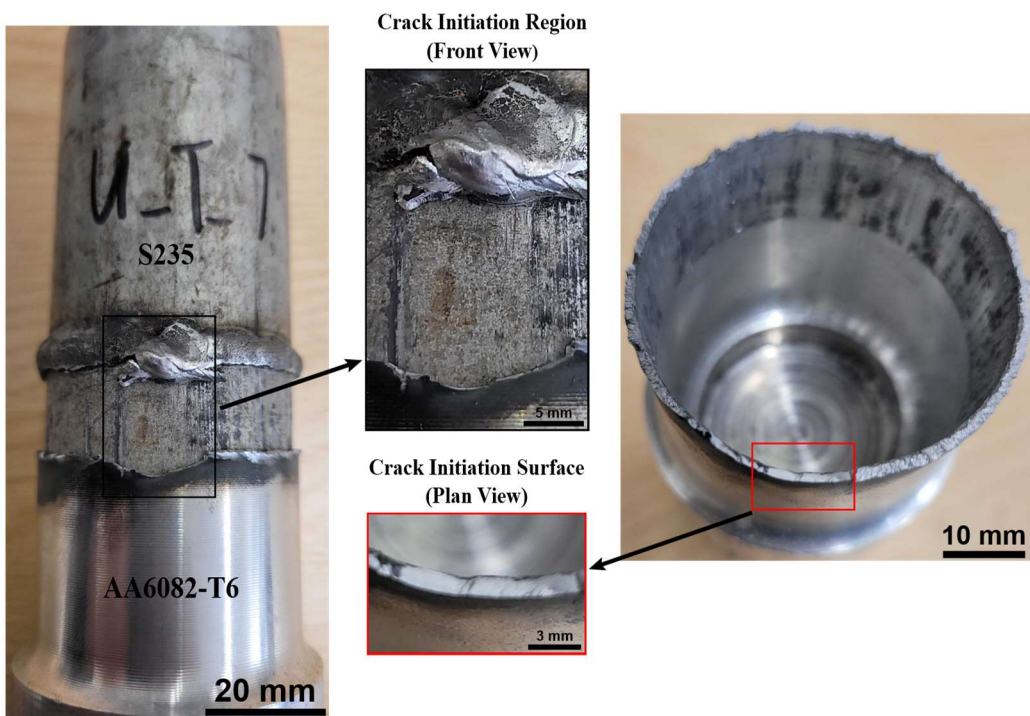


Figure 6.2: Distribution of crack initiation sites across different loading conditions.

To provide a clearer visual representation, Figures 6.3 to 6.6 present representative crack initiation site examples for each fatigue loading type. Under uniaxial fatigue loading, the weld toe/weld root region was the most frequently observed crack initiation site, accounting for 75% of the tested specimens. Figure 6.3a illustrates a crack initiating from the weld toe/root position, highlighting its susceptibility to fatigue failure. The remaining 25% of failures occurred at the weld start/stop position, as depicted in Figure 6.3b, emphasising the influence of weld discontinuities on crack formation. Notably, no crack initiation was observed at the weld itself or the aluminium HAZ under uniaxial fatigue loading.

Uniaxial (Failure Mode: Weld Toe/Root)

(a)

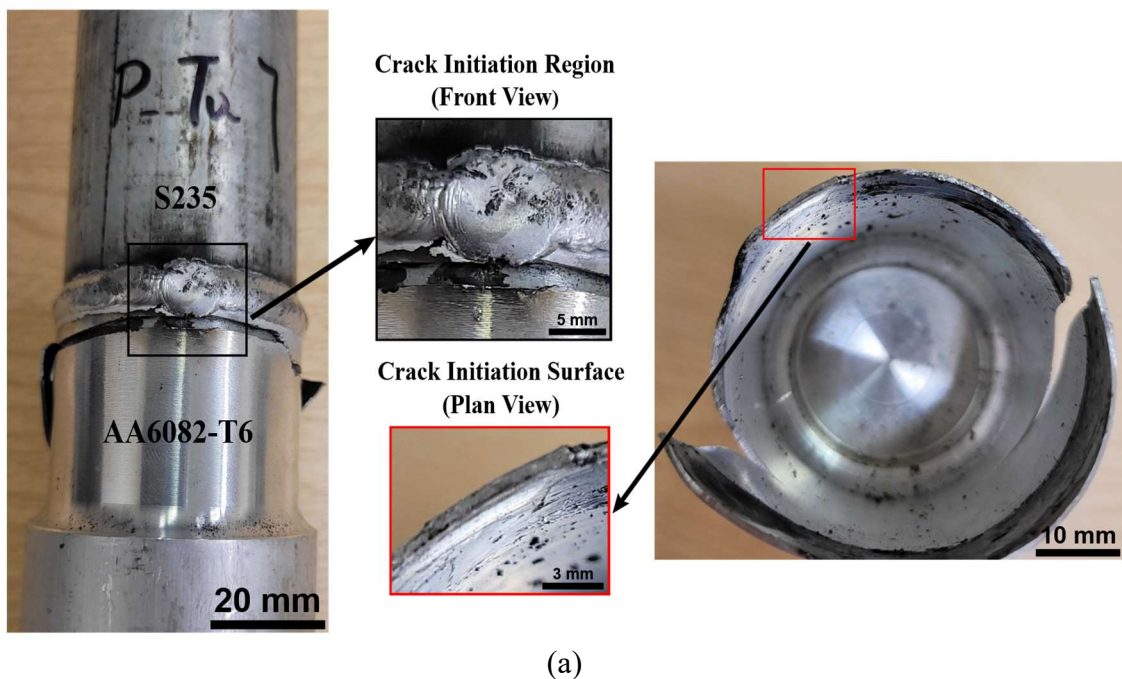
Uniaxial (Failure Mode: Weld Start/Stop)

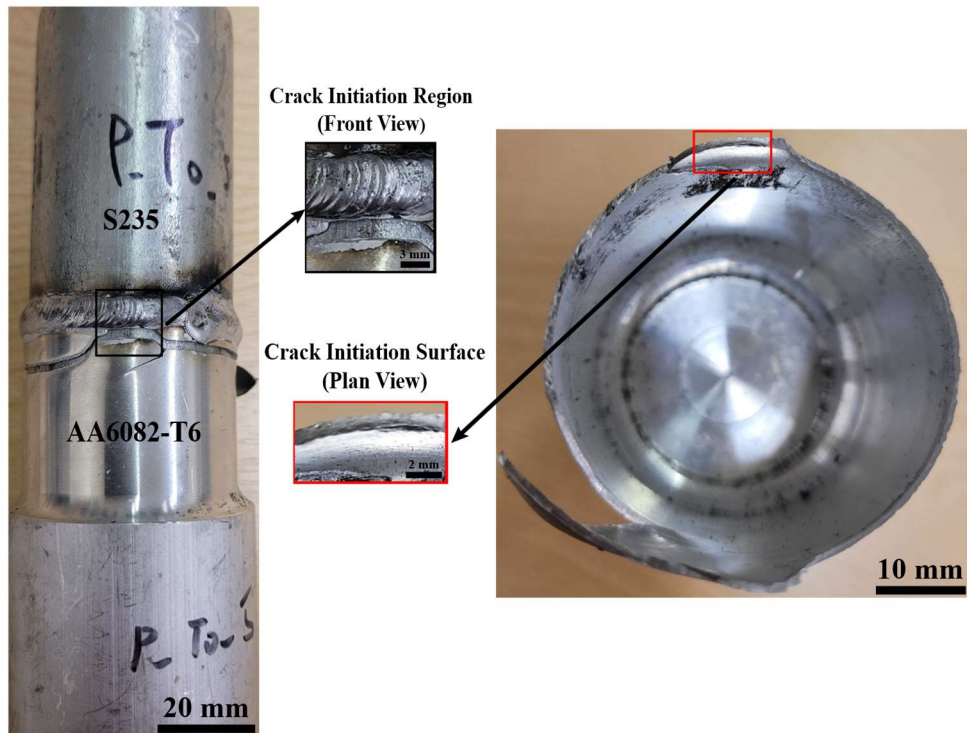
(b)

Figure 6.3: Representative crack initiation sites under uniaxial fatigue loading: (a) weld toe/root and (b) weld start/stop position.

Under pure torsional fatigue loading, crack initiation locations exhibited a distinct shift from uniaxial loading. The majority, approximately 58% of the samples experienced crack initiation at the weld start/stop position. This failure mode is illustrated in Figure 6.4a, where cracks initiated from the weld start/stop bulge region. The weld toe/root region remained a significant crack initiation site, accounting for 25% of specimens, as depicted in Figure 6.4b. Additionally, 17% of specimens exhibited failure from cracks initiating within the weld itself, as shown in Figure 6.4c.

Pure Torsion (Failure Mode: Weld Start/Stop)



Pure Torsion (Failure Mode: Weld Toe/Root)

(b)

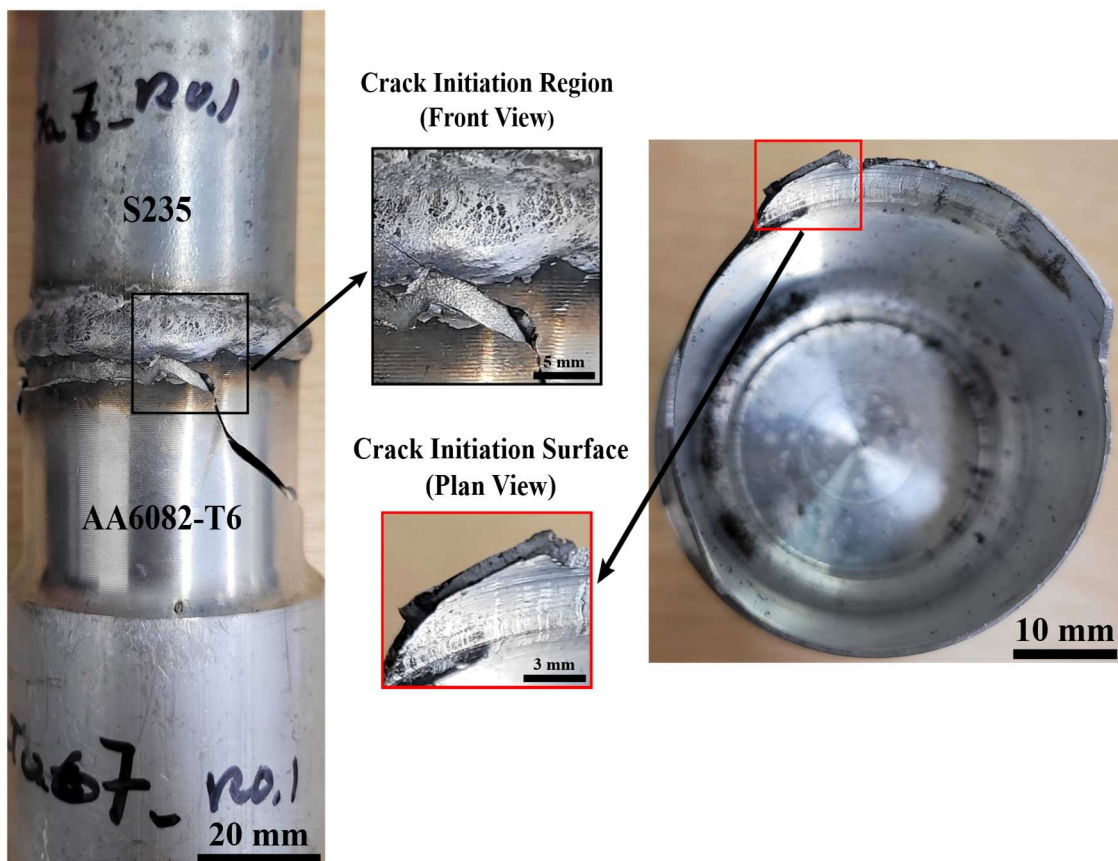
Pure Torsion (Failure Mode: Weld)

(c)

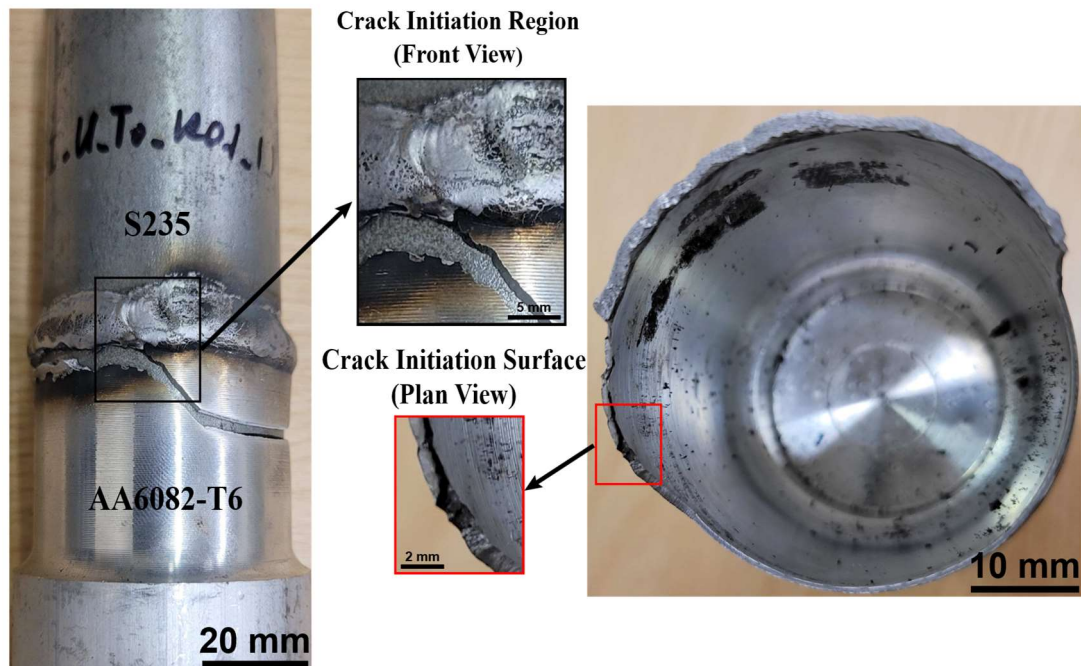
Figure 6.4: Representative crack initiation sites under pure torsional fatigue loading: (a) weld start/stop position, (b) weld toe/root and (c) weld itself.

For in-phase fatigue loading, failure trends remained similar to uniaxial loading, with the weld toe/ root region continuing to be the most dominant failure site, accounting for 56% of samples. A representative failure example is shown in Figure 6.5a. The weld start/stop position was the second most common crack initiation site with 31% of failures initiating from this location, as depicted in Figure 6.5b. A smaller percentage, 11%, showed failure originating from the weld itself as shown in Figure 6.5c. Crack initiation at the aluminium HAZ was rare, occurring in only a minimal 2% of cases as depicted in Figure 6.5d, suggesting that stress concentration at geometric discontinuities has a greater influence on crack initiation than material softening in the HAZ.

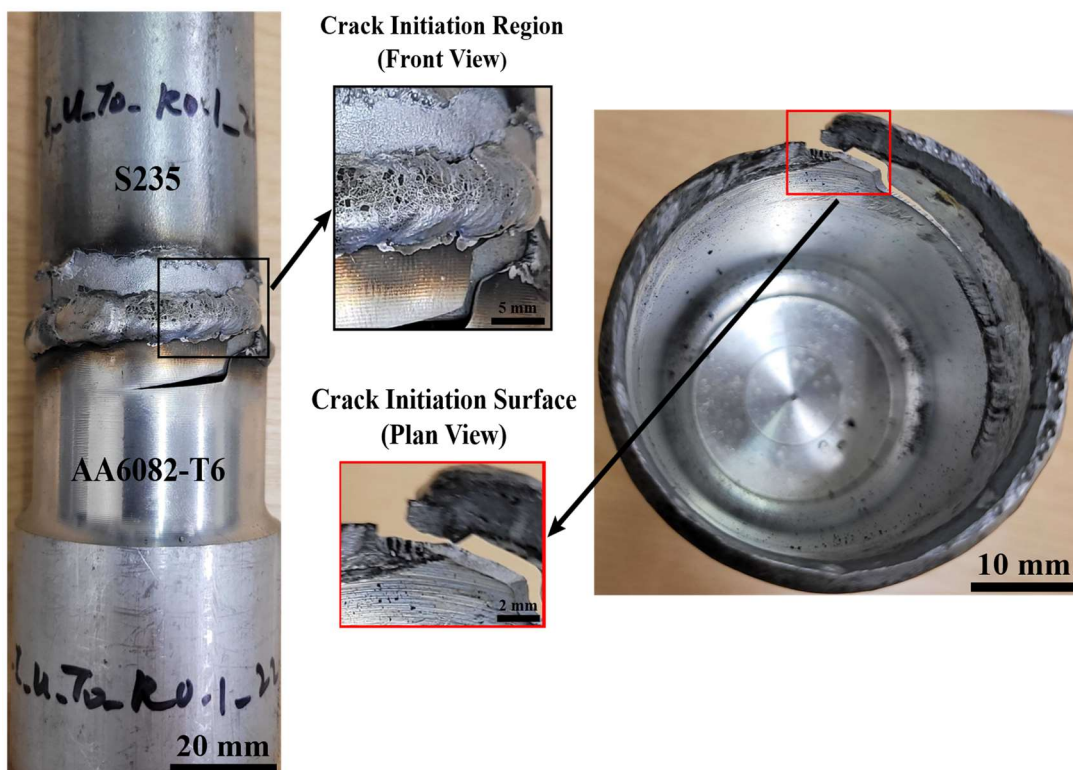
In-Phase (Failure Mode: Weld Toe/Root)



(a)

In-Phase (Failure Mode: Weld Start/Stop)

(b)

In-Phase (Failure Mode: Weld)

(c)

In-Phase (Failure Mode: Aluminium)

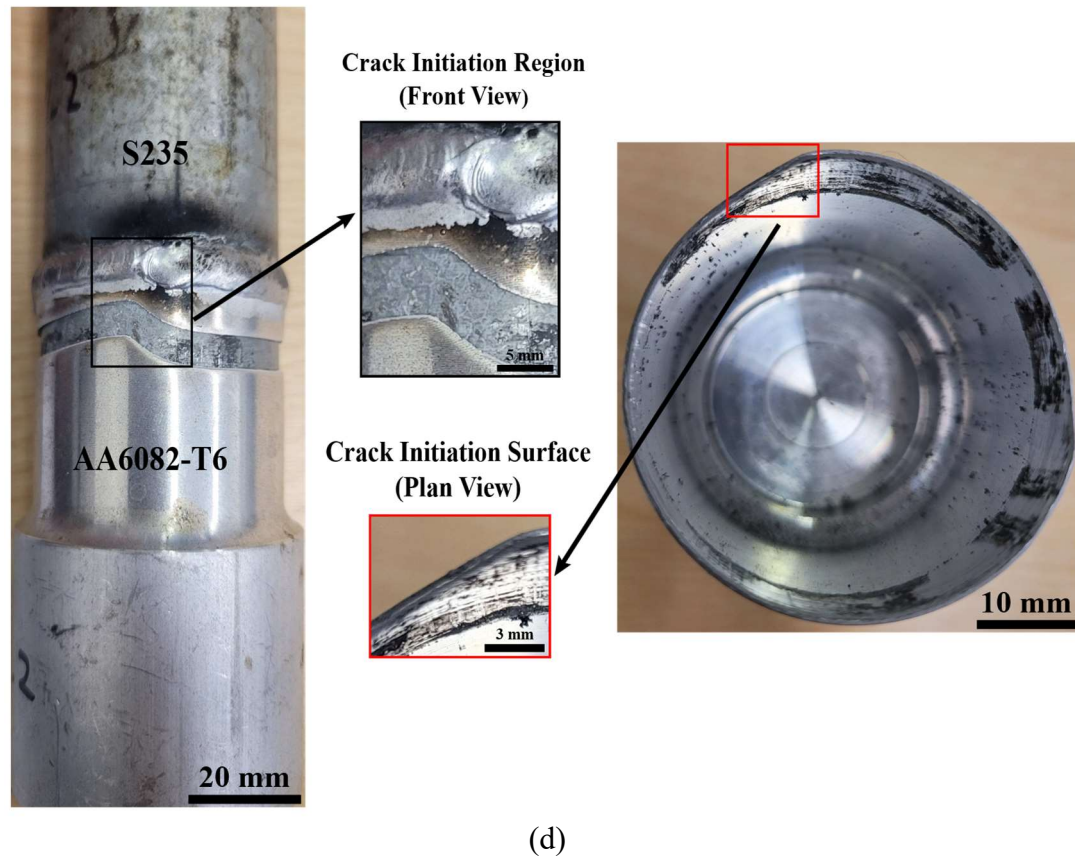
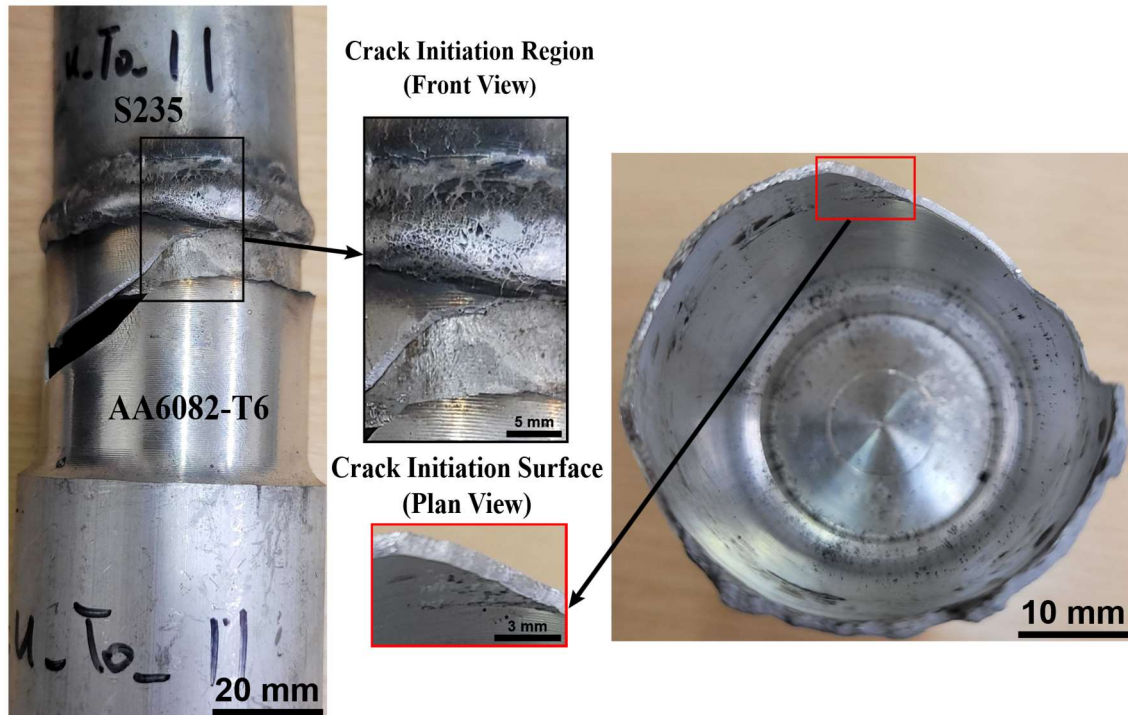
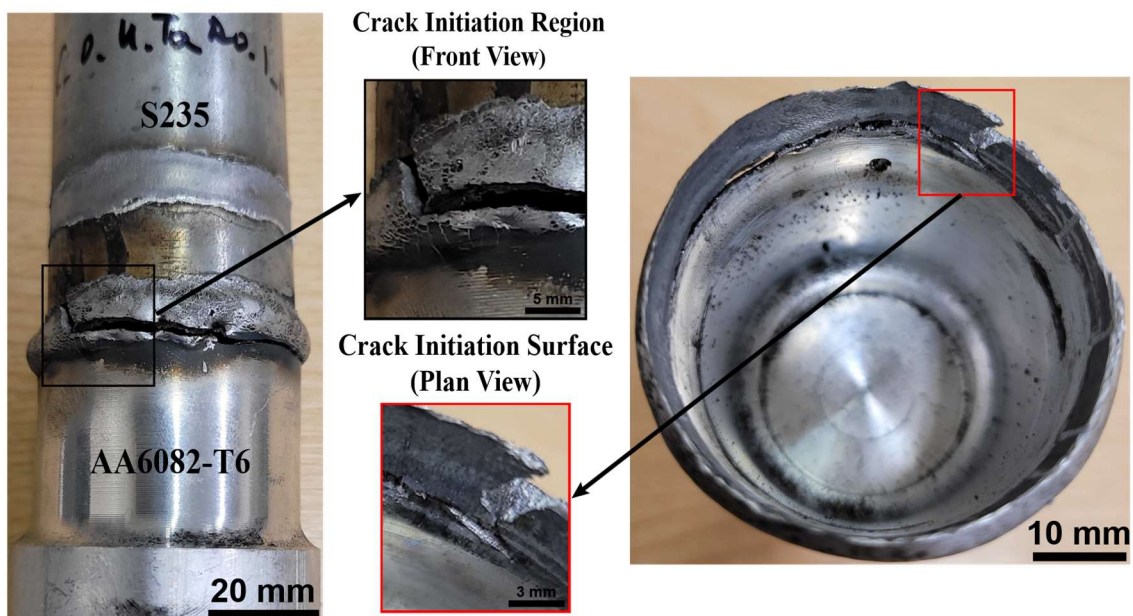


Figure 6.5: Representative crack initiation sites under in-phase fatigue loading: (a) weld toe/root, (b) weld start/stop position, (c) weld itself, and (d) aluminium HAZ.

For out-of-phase fatigue loading, a similar four-category distribution was observed. The weld toe/weld root region remained the most frequently observed failure site, representing 56% of specimens, as shown in Figure 6.6a. However, unlike in-phase loading, where the weld start/stop position was the second most common failure location, out-of-phase loading resulted in a greater proportion of failures occurring within the weld itself. The weld itself accounted for 25% of failures, as illustrated in Figure 6.6b, while the weld start/stop position contributed to 17% of failures, as shown in Figure 6.6c. Similar to previous loading conditions, crack initiation in the aluminium HAZ remained the least frequent, occurring in only 2% of specimens, as depicted in Figure 6.6d.

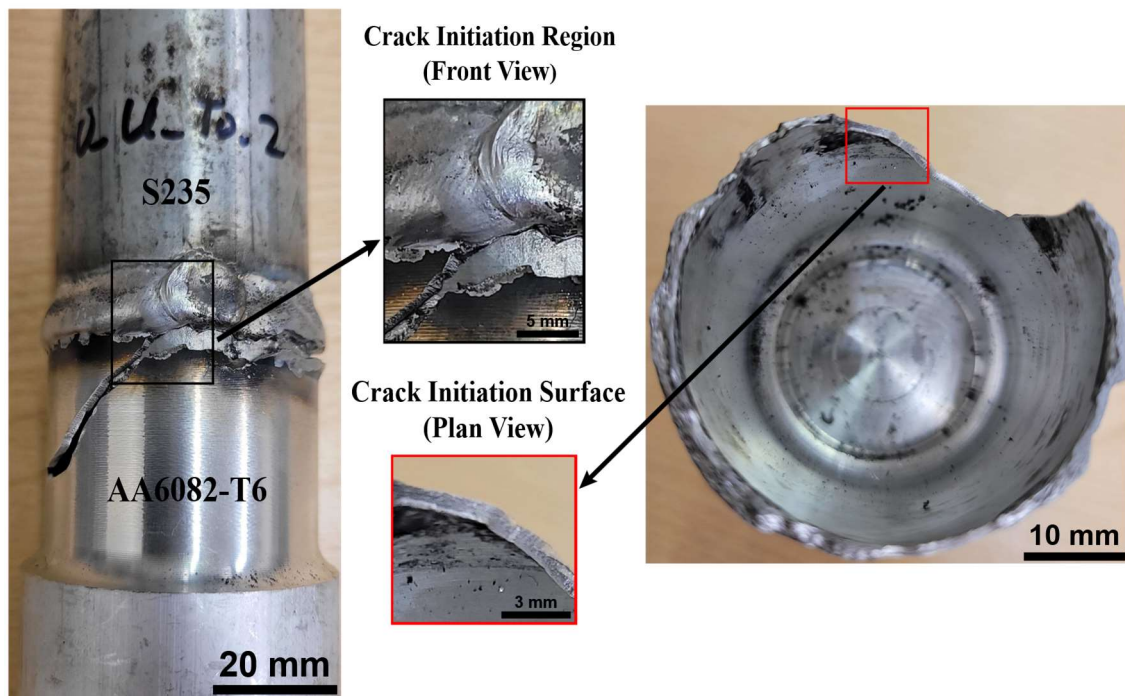
Out-of-Phase (Failure Mode: Weld Toe/Root)

(a)

Out-of-Phase (Failure Mode: Weld)

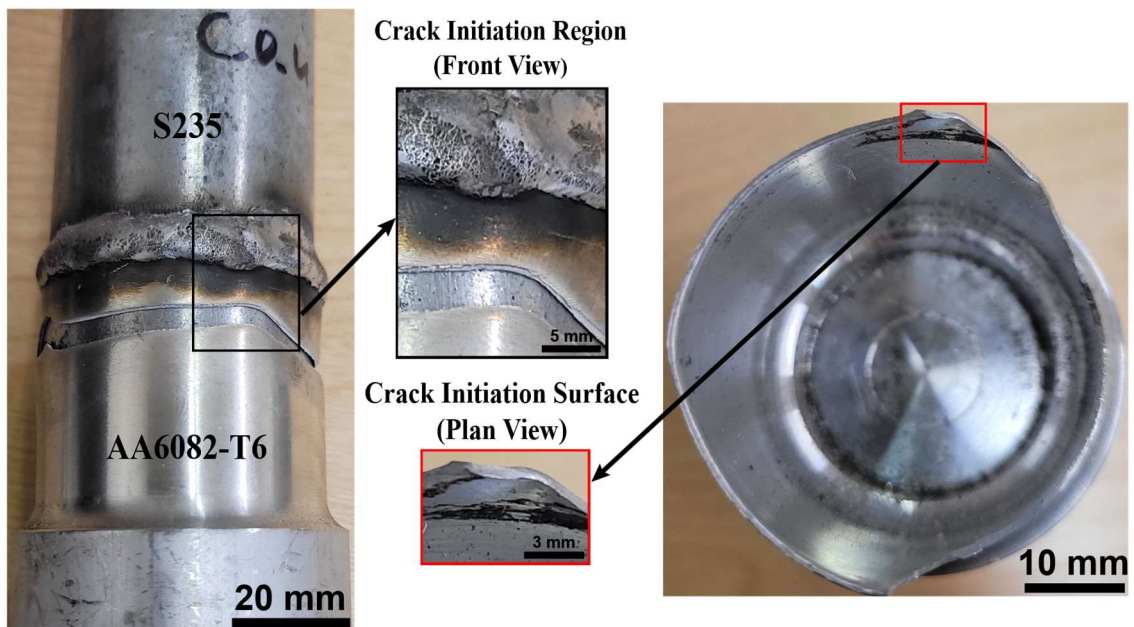
(b)

Out-of-Phase (Failure Mode: Weld Start/Stop)



(c)

Out-of-Phase (Failure Mode: Aluminium)



(d)

Figure 6.6: Representative crack initiation sites under in-phase fatigue loading: (a) weld toe/root, (b) weld itself, (c) weld start/stop position, and (d) aluminium HAZ.

Overall, these findings provide a comprehensive understanding of how crack initiation locations vary with different fatigue loading conditions in hybrid aluminium-to-steel welded joints. The analysis of failure site distributions demonstrates a clear correlation between loading condition and failure mode, confirming that stress concentration effects at specific weld locations are the dominant factors governing crack initiation. These results provide important insights into the fatigue behaviour of hybrid aluminium-to-steel welded joints and the influence of multiaxial loading conditions on failure modes. The following section will focus on crack initiation and propagation orientations, comparing experimentally measured crack angles with theoretical predictions to assess the correlation between theoretical critical plane orientation and crack path angles in the examined specimens.

6.4 Crack Initiation and Propagation Plane Orientations

Besides identifying crack initiation locations, understanding the orientation of crack initiation and propagation is essential in assessing fatigue failure mechanisms in hybrid aluminium-to-steel welded joints. Whenever possible, crack initiation and propagation angles were measured using Image Processing Software Fiji, allowing for a quantitative evaluation of failure plane orientations. In the context of this study, crack initiation is defined as the early stage of fatigue failure where a detectable crack first forms and begins to extend from the weld toe or root region, typically up to a length of approximately 1 to 2 mm. This definition is consistent with observable features on the fracture surface and is constrained by the resolution limits of optical microscopy. It is acknowledged that this operational definition may encompass part of the early propagation phase, particularly in aluminium, where microstructurally short cracks can rapidly grow to millimetre-scale lengths before becoming visible. As such, the identified initiation regions in this study are best understood as representing the transition from micro-crack formation to stable Stage II crack growth. This practical interpretation ensures consistency across specimens and reflects limitations in detecting very short or incipient cracks in fractured components.

Figure 6.7 presents a representative case of crack propagation under in-phase biaxial fatigue loading, where the crack initiated from the weld toe/root region.

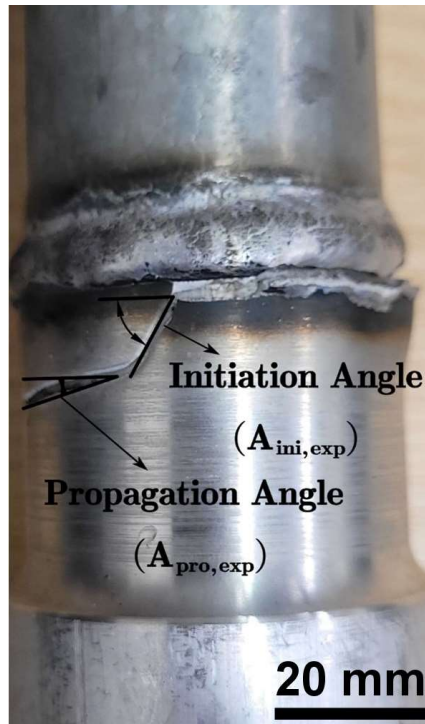


Figure 6.7: Example illustrating the determination of experimental crack initiation, $A_{ini,exp}$ and experimental crack propagation angles, $A_{pro,exp}$ under in-phase loading.

This case was selected due to its clear and well-defined crack initiation and propagation angles, unlike scenarios under pure torsional loading, where cracks exhibited zero-degree initiation angle, or in highly fractured specimens where measurement was no longer feasible due to severe post-failure damage. While the visible crack length in Figure 6.7 prior to transitioning into Mode I propagation is relatively long, which is approximately 10 mm, and exceeding the defined initiation scale of 1 to 2 mm, it was chosen for its clarity in illustrating the measurement of initiation and propagation angles. It is therefore acknowledged that this example likely includes part of the early propagation phase. Nonetheless, it remains representative of the directionality and origin of crack growth relevant to fatigue initiation in multiaxial loading scenarios.

The experimental crack initiation angle, $A_{ini,exp}$ and experimental crack propagation angle, $A_{pro,exp}$, were determined whenever the fracture surface permitted accurate identification. Notably, in uniaxial fatigue loading, the Stage I crack initiation phase was considerably shorter than in pure torsional and biaxial fatigue loading scenarios. This led to difficulties in measuring initiation angles due to the extremely short crack length at the early stage of failure. Similarly, propagation angles were challenging to

determine in specimens that experienced severe post-failure damage, where cracks rapidly progressed beyond their initiation phase. In cases where failure occurred within the weld itself, it was impossible to obtain either initiation or propagation angles, as the weld together with the aluminium component had detached entirely from the specimen.

To evaluate the relationship between the experimentally observed failure planes and theoretical predictions, the critical plane orientation was estimated using the Mohr's circle approach. The theoretical crack initiation, $A_{ini,theo}$ and crack propagation, $A_{pro,theo}$ angles were derived based on nominal stress conditions. While weld-induced localised stress states are crucial in complex joint geometries, nominal stress calculations remain widely adopted for engineering design and fatigue assessment in accordance with the International Institute of Welding (IIW) and Eurocodes (ECs) (Anon, 2005, 2007; Hobbacher, 2016). Although more advanced critical plane models are available for detailed fatigue analysis, the Mohr's circle approach was employed as an initial validation tool to establish correlations between experimental and theoretical orientations.

In medium- to high-cycle fatigue regimes, crack initiation in ductile materials is often attributed to shear stress, which promotes the formation of persistent slip bands (PSBs) and subsequent meso-crack initiation (Brown and Miller, 1979; Socie, 1987; Totten, 2008; Susmel, 2009a; Sangid, 2013). Fatigue loading induces intrusions and extrusions along slip planes, creating stress concentrations that act as primary crack initiation points (Brown and Miller, 1979; Lin, Fine and Mura, 1986; Suresh, 1998; Susmel, 2009a; Sangid, 2013). Consequently, the $A_{ini,theo}$ aligns with the maximum shear stress plane, whereas the $A_{pro,theo}$ is expected to correspond to the maximum normal stress (Mode I loading) plane (Brown and Miller, 1979; Socie, 1987; Pook, 1995; Totten, 2008; Susmel, 2009a).

To assess this hypothesis, Figure 6.8a presents a comparison between experimentally measured and theoretically predicted crack initiation angles, while Figure 6.8b illustrates the corresponding propagation angles. The degree of correlation between experimental and theoretical orientations was quantified using the percentage error (E) method, expressed as in Equations 6.1 and 6.2:

$$E (\%) = \frac{A_{ini,exp} - A_{ini,theo}}{A_{ini,theo}} \times 100 \quad (6.1)$$

$$E (\%) = \frac{A_{pro,exp} - A_{pro,theo}}{A_{pro,theo}} \times 100 \quad (6.2)$$

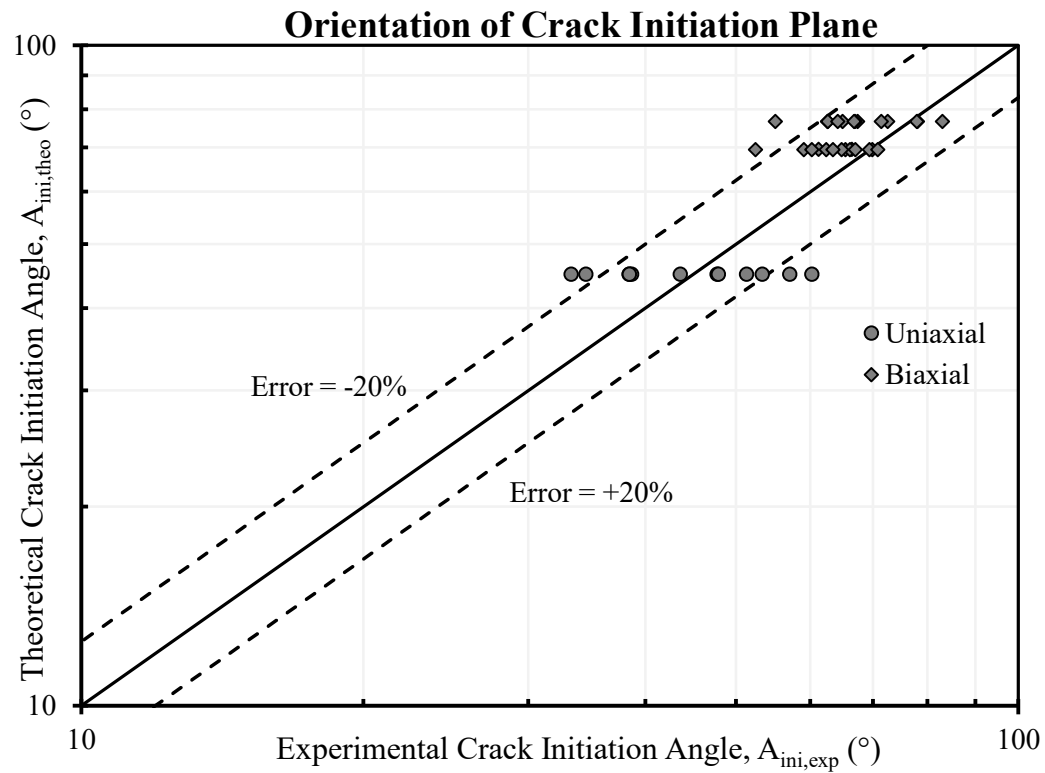
To further assess accuracy, the root mean square error (RMSE) method (Zhu *et al.*, 2023) was applied to both initiation and propagation angles, as described in Equations 6.3 and 6.4:

$$RMSE (^\circ) = \sqrt{\sum_{i=1}^n \frac{(A_{ini,exp,i} - A_{ini,theo,i})^2}{n}} \quad (6.3)$$

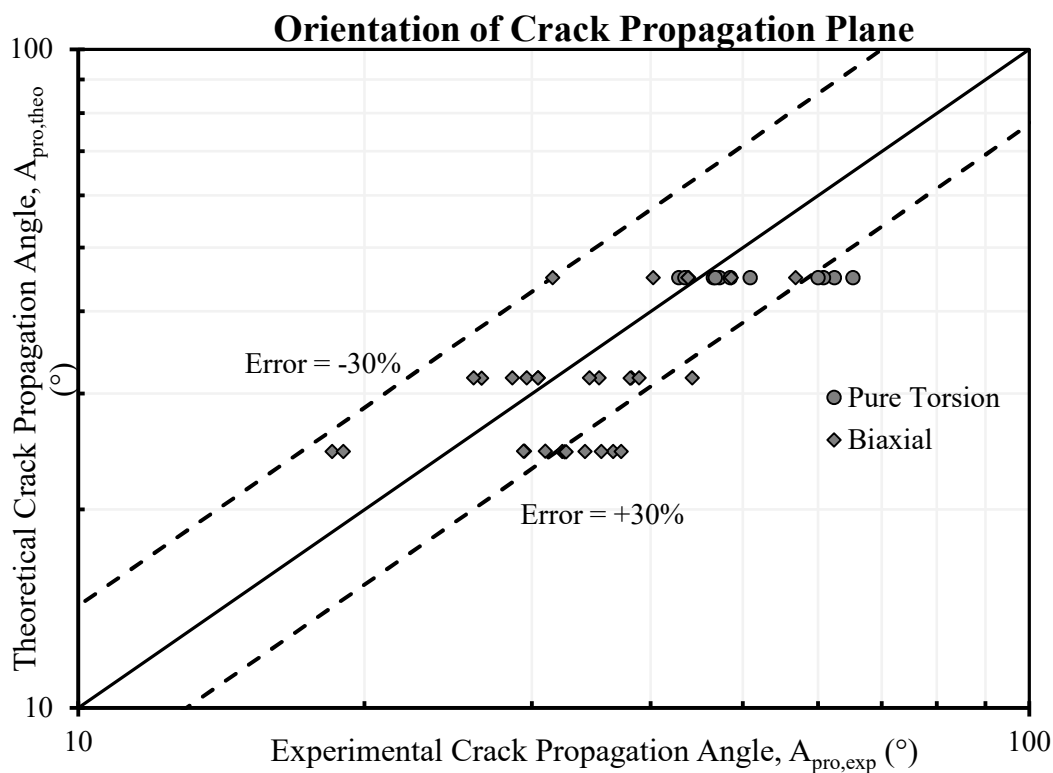
$$RMSE (^\circ) = \sqrt{\sum_{i=1}^n \frac{(A_{pro,exp,i} - A_{pro,theo,i})^2}{n}} \quad (6.4)$$

where n represents the total number of observations. The calculated RMSE values were 8.9° for crack initiation angles and 8.3° for propagation angles, indicating a reasonable level of agreement between experimental and theoretical predictions.

Under uniaxial fatigue loading, cracks typically initiated at approximately 45° , extending along the 0° axis, consistent with Mode I dominant failure. In pure torsional fatigue loading, the crack initiation phase was notably longer compared to uniaxial loading. Cracks initially formed along the 0° maximum shear stress plane, later propagating at approximately 45° , aligning with the maximum normal stress plane. Similar trends were observed under out-of-phase loading conditions.



(a)



(b)

Figure 6.8: Correlation between experimental and theoretical crack plane angles: (a) crack initiation ($A_{ini,exp}$ vs $A_{ini,theo}$) and (b) crack propagation ($A_{pro,exp}$ vs $A_{pro,theo}$).

For biaxial in-phase fatigue loading, the measured crack initiation angles were approximately 77° for biaxiality ratio (as defined in Equation 5.4), λ of 1 and 70° for λ of $\sqrt{3}$. Corresponding crack propagation angles were found to be 32° and 25° , respectively. These values were observed across all crack initiation locations, further supporting the correlation between experimental failure planes and theoretical stress orientations.

Figure 6.8a demonstrates a strong correlation between experimental and theoretical crack initiation plane orientations, with most data points falling within a $\pm 20\%$ error margin. This level of agreement is particularly significant given the inherent variability in weld geometries, microstructural defects, and potential localised stress redistribution. These findings reinforce the dominant role of shear stress in crack initiation within hybrid aluminium-to-steel welded joints.

In contrast, Figure 6.8b illustrates the correlation between experimental and theoretical crack propagation angles, with most data points lying within a $\pm 30\%$ error margin. The slightly larger deviation as compared to crack initiation angles is likely attributed to microstructural hard inclusions, complex geometric features, and local weld discontinuities, which introduce variability in the propagation path. Despite these challenges, the results confirm that crack propagation remains primarily governed by normal stress, validating the assumption that crack growth aligns with the maximum normal stress plane in Stage II fatigue failure.

These findings highlight the importance of localised stress effects and weld geometry in influencing crack orientation under multiaxial fatigue loading. The next section will present key findings and implications drawn from the crack path analysis and failure plane orientation assessments, exploring trends and their significance in the multiaxial fatigue assessment of hybrid aluminium-to-steel welded joints.

6.5 Key Findings and Discussion

The statistical analysis of crack initiation site distribution revealed a clear correlation between fatigue loading conditions and failure modes. Under uniaxial and biaxial fatigue loading, failure predominantly occurred at the weld toe/root region, highlighting

the stress concentration effects associated with weld geometry. This finding aligns with established fatigue failure mechanisms, where geometric discontinuities act as stress concentrators, promoting crack initiation under cyclic loading. In contrast, under pure torsional fatigue loading, a significant shift in crack initiation was observed, with the weld start/stop position emerging as the dominant failure site. This observation supports the hypothesis that shear-dominated loading magnifies stress concentrations at localised weld discontinuities, making these regions more susceptible to fatigue failure.

The low occurrence of crack initiation in the aluminium HAZ further underscores the dominant influence of stress concentration effects over material softening. While thermal degradation in the HAZ may contribute to localised weakening, it does not appear to play a decisive role in crack initiation when compared to stress-raising weld profile features. This suggests that weld geometry and localised discontinuities are the primary factors governing fatigue failure, rather than bulk material degradation effects.

Moreover, in-phase fatigue loading with a λ of $\sqrt{3}$ resulted in a notable increase in failures at the weld start/stop position, with 75% of specimens exhibiting crack initiation from this region. This suggests that the interaction between Mode II loading, λ of $\sqrt{3}$ and unique localised weld start/stop geometry induces multiaxial stress states that amplifies stress concentration effects, making this location more critical under such conditions (Suresh, 1998; Kueppers and Sonsino, 2003; Schijve, 2009; Susmel, 2009a).

The transition from uniaxial or in-phase biaxial loading to out-of-phase loading further influenced failure modes, particularly in cases where torsional mean stresses were present. A distinct shift was observed, where the weld itself became a more frequent crack initiation site under out-of-phase loading, a trend that was not observed under in-phase loading conditions. Crack path analysis results revealed that out of nine specimens exhibiting crack initiation at the weld itself, six were tested under torsional mean stress conditions, while two experienced both tension and torsional mean stresses. This implies that while torsional mean stresses significantly contribute to weld failure, but its effect is further intensified under 90° out-of-phase loading, which appears to promote weld detachment and peeling mechanisms.

Although failures initiating at the weld itself were less common overall, its increased occurrence under torsional mean stress loading suggests that the combined effect of torsion and multiaxial out-of-phase fatigue loading plays a key role in accelerating damage. The presence of localised weld defects, such as lack of fusion, microcracks, or porosity, further increases susceptibility to failure in these cases (Al Zamzami *et al.*, 2018; Shin, Chang and Muzaffer, 2021).

Insights from the crack orientation analysis further confirmed that crack plane orientations in hybrid aluminium-to-steel welded joints are governed by both shear and normal stress ranges. The crack initiation phase was found to be shear-dominated, whereas crack propagation was primarily influenced by normal stress. This was evident from the strong correlation between experimental crack orientations and theoretical critical plane estimations, where deviations were primarily associated with local microstructural variations and weld geometric complexities.

These insights have direct implications for multiaxial fatigue assessment methodologies. Fatigue models or criteria that incorporate both shear and normal stress ranges are more likely to yield accurate and physically representative fatigue predictions for hybrid aluminium-to-steel welded joints. Conversely, approaches that rely solely on either normal or shear stress components are likely to produce less accurate assessments, as they fail to capture the multiaxial stress interactions observed experimentally. This conclusion aligns with Sonsino's findings on multiaxial fatigue assessment of aluminium welded joints, reinforcing the idea that the combined effect of normal and shear stress is essential in developing a robust multiaxial fatigue assessment framework for hybrid aluminium-to-steel welded joints (Kueppers and Sonsino, 2003, 2006; Sonsino *et al.*, 2006).

As demonstrated in this study, fatigue failures consistently originated from the aluminium side, indicating that aluminium was the weaker link in the tested hybrid welded joints. However, this observation is specific to the materials and configuration investigated and should not be generalised. The relative strength of aluminium and steel depends on their respective alloys and heat treatments, and in some cases, aluminium alloys can exceed steels in yield strength. Nonetheless, for the coldArc®-welded specimens examined here, the findings support the applicability of fatigue assessment

models originally developed for aluminium welded joints, providing partial validation of the initial research hypothesis formulated based on prior investigations into static and uniaxial fatigue behaviour of coldArc®-welded thin aluminium-to-steel joints.

Overall, these findings are consistent with established fatigue failure mechanisms in welded structures and provide valuable insight into how hybrid aluminium-to-steel welded joints behave under multiaxial fatigue loading conditions. The results underscore the critical influence of weld geometry, loading conditions, and stress interactions on fatigue failure, forming a strong foundation for the subsequent chapter, which focuses on the formalisation and validation of a robust multiaxial fatigue assessment methodology for hybrid aluminium-to-steel welded joints.

6.6 Conclusions

This chapter presented a comprehensive failure analysis of coldArc®-welded hybrid aluminium-to-steel joints under multiaxial loading, focusing on crack initiation sites and crack orientation plane analysis. The key findings are as follows:

- Fatigue cracks consistently initiated from four primary locations: the aluminium weld toe/weld root region, the weld start/stop position, the weld itself, and, in rare cases, the aluminium HAZ. The weld toe/weld root region was the dominant initiation site, particularly under uniaxial, in-phase and out-of-phase fatigue loading.
- Under pure torsional loading, crack initiation predominantly shifted to the weld start/stop position, likely due to the combined effect of the localised bulge geometry and shear stress concentration arising from complex multiaxial stress states in this region. Out-of-phase loading further altered failure modes, leading to an increased frequency of crack initiation at the weld itself, particularly in the presence of torsional mean stresses.
- Stress concentration effects at weld discontinuities played a more critical role in fatigue failure than material softening in the aluminium HAZ, explaining the rarity of crack initiation from this region.

- Crack initiation was primarily governed by shear stresses, while crack propagation followed normal stress directions. Experimental crack angles showed strong agreement with theoretical predictions based on nominal stresses analysis and Mohr's circle, with minor deviations attributed to local weld geometry variations and microstructural effects, particularly during crack propagation.
- Findings suggest that multiaxial fatigue models should consider incorporating both shear and normal stress components to improve fatigue life predictions. The observed correlation between crack orientation and maximum shear and normal stress planes reinforces the potential benefits of physically representative fatigue criteria. Given that all failures originated from the aluminium side, existing fatigue assessment models for aluminium welded joints may be applicable to hybrid aluminium-to-steel welded joints with suitable modifications.

Overall, this chapter highlights the critical influence of localised weld geometry, loading conditions, and multiaxial stress interactions in fatigue failure mechanisms. These findings provide a strong foundation for the next chapter, which focuses on the formalisation and validation of a robust multiaxial fatigue assessment methodology for hybrid aluminium-to-steel welded joints.

6.7 References

Amstutz, H., Störzel, K. and Seeger, T. (2001) 'Fatigue crack growth of a welded tube–flange connection under bending and torsional loading', *Fatigue & Fracture of Engineering Materials & Structures*, 24(5), pp. 357–368.

Anon (2005) *Eurocode 3: Design of steel structures — Part 1-9: Fatigue* .

Anon (2007) *Eurocode 9: Design of aluminium structures — Part 1-3: Structures susceptible to fatigue* .

Atabaki, M. M. *et al.* (2014) 'Welding of aluminum alloys to steels: an overview', *Journal for Manufacturing Science and Production*, 14(2), pp. 59–78.

Brown, M. W. and Miller, K. J. (1979) 'Initiation and growth of cracks in biaxial fatigue', *Fatigue & Fracture of Engineering Materials & Structures*, 1(2), pp. 231–246.

Caligiuri, R. D. (2015) 'Critical crack path assessments in failure investigations', *Frattura ed Integrità Strutturale*, 9(34).

Cao, R. *et al.* (2014) 'Cold metal transfer joining of aluminum AA6061-T6-to-galvanized boron steel', *Journal of Manufacturing Science and Engineering*, 136(5), p. 051015.

Gullino, A., Matteis, P. and D'Aiuto, F. (2019) 'Review of aluminum-to-steel welding technologies for car-body applications', *Metals*, 9(3), p. 315.

Hobbacher, A. (2016) *Recommendations for fatigue design of welded joints and components*. Springer.

Kah, P., Suoranta, R. and Martikainen, J. (2013) 'Advanced gas metal arc welding processes', *The International Journal of Advanced Manufacturing Technology*, 67, pp. 655–674.

Kalinenko, A. *et al.* (2024) 'Dissimilar friction-stir welding of aluminum alloys 2519, 6061, and 7050 using an additively-manufactured tool', *Engineering Failure Analysis*, 156, p. 107851.

Kimapong, K. and Watanabe, T. (2005) 'Lap joint of A5083 aluminum alloy and SS400 steel by friction stir welding', *Materials transactions*, 46(4), pp. 835–841.

Kueppers, M. and Sonsino, C. M. (2003) 'Critical plane approach for the assessment of the fatigue behaviour of welded aluminium under multiaxial loading', *Fatigue & Fracture of Engineering Materials & Structures*, 26(6), pp. 507–513.

Kueppers, M. and Sonsino, C. M. (2006) 'Assessment of the fatigue behaviour of welded aluminium joints under multiaxial spectrum loading by a critical plane approach', *International Journal of Fatigue*, 28(5–6), pp. 540–546.

- Lin, M. R., Fine, M. E. and Mura, T. (1986) 'Fatigue crack initiation on slip bands: theory and experiment', *Acta Metallurgica*, 34(4), pp. 619–628.
- Liu, H. *et al.* (2018) 'Mechanisms of fatigue crack initiation and propagation in 6005A CMT welded joint', *Journal of Alloys and Compounds*, 741, pp. 188–196.
- Liu, Y. *et al.* (2023) 'Fracture mechanism of aluminum/steel laser-arc welded-brazed joints during quasi-static tensile and dynamic fatigue tests', *Engineering Failure Analysis*, 152, p. 107461.
- Macdonald, K. (2011) *Fracture and fatigue of welded joints and structures*. Elsevier.
- Macdonald, K. A. and Haagenzen, P. J. (2009) 'Fatigue of welded aluminium hollow section profiles', *Engineering Failure Analysis*, 16(1), pp. 254–261.
- Maggiolini, E. *et al.* (2016) 'Crack path and fracture analysis in FSW of small diameter 6082-T6 aluminium tubes under tension–torsion loading', *International Journal of Fatigue*, 92, pp. 478–487.
- Meco, S. *et al.* (2015) 'Application of laser in seam welding of dissimilar steel to aluminium joints for thick structural components', *Optics and Lasers in Engineering*, 67, pp. 22–30.
- Pook, L. P. (1995) 'On fatigue crack paths', *International journal of fatigue*, 17(1), pp. 5–13.
- Qin, G., Su, Y. and Wang, S. (2014) 'Microstructures and properties of welded joint of aluminum alloy to galvanized steel by Nd: YAG laser+ MIG arc hybrid brazing-fusion welding', *transactions of nonferrous metals society of china*, 24(4), pp. 989–995.
- Reis, L., Li, B. and De Freitas, M. (2006) 'Analytical and experimental studies on fatigue crack path under complex multi-axial loading', *Fatigue & Fracture of Engineering Materials & Structures*, 29(4), pp. 281–289.
- Sangid, M. D. (2013) 'The physics of fatigue crack initiation', *International journal of fatigue*, 57, pp. 58–72.
- Schijve, J. (2009) *Fatigue of structures and materials*. Springer.

Shahverdi, H. R. *et al.* (2002) 'Microstructural analysis of interfacial reaction between molten aluminium and solid iron', *Journal of materials processing technology*, 124(3), pp. 345–352.

Shin, W., Chang, K.-H. and Muzaffer, S. (2021) 'Fatigue analysis of cruciform welded joint with weld penetration defects', *Engineering Failure Analysis*, 120, p. 105111.

Socie, D. F. (1987) 'Multiaxial fatigue damage models'.

Sonsino, C. M. *et al.* (2006) 'Fatigue strength of laser beam welded thin steel structures under multiaxial loading', *International journal of fatigue*, 28(5–6), pp. 657–662.

Suresh, S. (1998) *Fatigue of materials*. Cambridge university press.

Susmel, L. (2009a) *Multiaxial notch fatigue*. Elsevier.

Susmel, L. (2009b) 'Three different ways of using the Modified Wöhler Curve Method to perform the multiaxial fatigue assessment of steel and aluminium welded joints', *Engineering Failure Analysis*, 16(4), pp. 1074–1089. doi: 10.1016/j.engfailanal.2008.05.016.

Totten, G. (2008) 'Fatigue crack propagation.', *Advanced Materials & Processes*, 166(5), pp. 39–42.

Wang, H. T. *et al.* (2013) 'An experimental investigation of local fracture resistance and crack growth paths in a dissimilar metal welded joint', *Materials & Design*, 44, pp. 179–189.

Yousefi, F., Witt, M. and Zenner, H. (2001) 'Fatigue strength of welded joints under multiaxial loading: experiments and calculations', *Fatigue & Fracture of Engineering Materials & Structures*, 24(5), pp. 339–355.

Al Zamzami, I. *et al.* (2018) 'Static strength and design of aluminium-to-steel thin welded joints', *Welding in the World*, 62(6), pp. 1255–1272. doi: 10.1007/s40194-018-0634-2.

-
- Al Zamzami, I., Davison, B. and Susmel, L. (2019) ‘Nominal and local stress quantities to design aluminium-to-steel thin welded joints against fatigue’, *International Journal of Fatigue*, 123, pp. 279–295.
- Al Zamzami, I. and Susmel, L. (2018) ‘On the use of hot-spot stresses, effective notch stresses and the Point Method to estimate lifetime of inclined welds subjected to uniaxial fatigue loading’, *International Journal of Fatigue*, 117, pp. 432–449. doi: 10.1016/j.ijfatigue.2018.08.032.
- Zhang, Z. (2021) ‘Quantitative characterization on fatigue fracture features of A6005 aluminum alloy welded joints’, *Engineering Failure Analysis*, 129, p. 105687.
- Zhu, S.-P. *et al.* (2023) ‘Machine learning-based probabilistic fatigue assessment of turbine bladed disks under multisource uncertainties’, *International Journal of Structural Integrity*, 14(6), pp. 1000–1024.

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Chapter 7

Multiaxial Fatigue Assessment of Hybrid Aluminium-to-Steel Welded Joints

7.1 Introduction

Building upon the failure analysis presented in Chapter 6, this chapter focuses on the multiaxial fatigue assessment of thin coldArc-welded hybrid aluminium-to-steel joints. Given the complex stress interactions at the weld toe and weld root, which have been identified as the primary fatigue crack initiation sites, this study aims to establish a robust and physically representative fatigue assessment methodology for these joints under constant amplitude (CA) multiaxial fatigue loading.

Despite the increasing use of hybrid welded joints, no prior research has formalised fatigue assessment methodologies for coldArc-welded thin aluminium-to-steel joints under multiaxial loading conditions. Previous studies have focused exclusively on uniaxial fatigue loading, leaving a knowledge gap regarding their behaviour under complex multiaxial stress states (Al Zamzami, Davison and Susmel, 2019). Furthermore, while multiaxial fatigue assessment models have been extensively developed for aluminium and steel welded joints individually, as quantitatively reviewed in Chapters 3 and 4, their applicability to thin hybrid aluminium-to-steel joints remains unverified. Given the distinct differences in mechanical properties between aluminium and steel (Bouche, Barbier and Coulet, 1998; Al Zamzami *et al.*, 2018), the direct application of existing fatigue models may not be suitable, necessitating a formal validation framework.

To address this gap, this chapter begins by calibrating reference fatigue curves derived from experimental uniaxial, pure torsional, and multiaxial fatigue tests. These

calibrated fatigue properties serve as the foundation for assessing the predictive capabilities of different stress-based multiaxial fatigue assessment approaches.

The Nominal Stress (NS) approach, commonly used in fatigue design codes, is first evaluated as a baseline assessment method (Anon, 2005, 2007; Hobbacher, 2016). This is followed by the Effective Notch Stress (ENS) approach, which considers localised stress concentrations at critical weld regions and enhances prediction accuracy by incorporating the geometrical stress raising effects of the weld profile (Hobbacher, 2009, 2016; Schijve, 2012; Radaj, Lazzarin and Berto, 2013; Chang, Sun and Qiu, 2020). Additionally, an alternative version of the ENS approach, ENS Avg, is introduced, incorporating experimentally measured average notch radii to further refine stress estimations. Finally, the Theory of Critical Distances – Point Method (TCD PM) is examined as an approach that eliminates the need for predefined notch radii, assuming sharp notches at weld toes and roots (Susmel, 2008, 2009b, 2009c; Al Zamzami and Susmel, 2018).

Each methodology is applied in conjunction with various multiaxial fatigue criteria, ranging from standard design models to critical plane-based approaches, enabling a comprehensive evaluation of their accuracy, reliability, and applicability. The predictive performance of these approaches is quantified using statistical metrics, facilitating a comparative analysis of their effectiveness in fatigue life estimations.

Through this systematic investigation, this study establishes the first validated multiaxial fatigue assessment framework for coldArc-welded thin hybrid aluminium-to-steel welded joints, offering critical insights into the most suitable methodologies for estimating fatigue life under complex loading conditions. The chapter concludes by identifying the most effective fatigue assessment models, providing a scientific basis for improving fatigue design guidelines for hybrid welded structures.

7.2 Validation Methodology

To establish a robust and physically representative fatigue life assessment framework for thin coldArc-welded hybrid aluminium-to-steel welded joints, a systematic

validation methodology is employed. Given the multiaxial nature of loading and the complex stress states at weld toe and root regions, this study applies three distinct stress analysis approaches, namely the NS approach, the ENS approach, and the TCD PM to reassess and validate the experimental fatigue test results. Since weld root failure is a potential fatigue-critical location, the hot-spot stress (HSS) approach is not considered, as it is only valid when failures originate from weld toe regions (Niemi, 1995; Anon, 2005, 2007; Radaj, Sonsino and Fricke, 2006; Hobbacher, 2016).

Both ENS and TCD PM require finite element analysis (FEA) to accurately determine local stress states at critical locations. For the ENS approach, Sonsino's recommended fictitious notch radius of 0.05 mm is adopted, as the welded joint thickness is below 5 mm (Sonsino, 2009; Sonsino, Bruder and Baumgartner, 2010; C M Sonsino *et al.*, 2012). Additionally, an alternative ENS Avg approach is introduced, where experimentally measured average notch radii are used to evaluate whether a measured geometric approach improves predictive accuracy over the existing fictitious notch method.

For the TCD PM approach, FEA is also employed with a critical distinction, where zero-radius notches are set at weld toe and weld root regions, and stress states are extrapolated at a critical length ($M-D_v$) of 0.075 mm from the notch bisector (Susmel, 2008, 2009b, 2009a). This follows the calibrated TCD PM methodology for aluminium welded joints, as aluminium is the weaker link in the hybrid joint, making it the assumed governing failure location (Al Zamzami and Susmel, 2018). This assumption aligns with observations from Chapter 6, where failure consistently initiated from the aluminium side of the welded joint.

Before applying these fatigue assessment methodologies to multiaxial fatigue conditions, the uniaxial and pure torsional fatigue datasets at $R=-1$ and $R=0.1$ are used for calibration. This calibration is repeated based on both nominal stress ranges and effective notch stress ranges, allowing the derivation of experimental fatigue curves to establish reference fatigue strengths and negative inverse slopes. These parameters are then used to calibrate the multiaxial fatigue criteria, which are subsequently employed to estimate the fatigue life of hybrid welded joints under in-phase and 90° out-of-phase loading conditions.

The calibration approach varies depending on the fatigue criterion. In this study, eight different multiaxial fatigue criteria are assessed. For Findley's criterion, the maximum principal stress criterion (MP) as recommended in Eurocode 9 (EC9), Von Mises (VM) criterion, stress-based Smith-Watson-Topper (SWT) criterion, and Sonsino KoNoS criterion, only uniaxial fatigue datasets are utilised (Findley, 1959; Smith, 1970; Kueppers and Sonsino, 2003; Anon, 2007; Bruun and Härkegård, 2015; Łagoda *et al.*, 2022). Conversely, for criteria such as the International Institute of Welding (IIW)/Gough-Pollard, Carpinteri-Spagnoli (CS), and the Modified Wöhler Curve Method (MWCM), both uniaxial and pure torsional reference fatigue curves are incorporated to ensure a more comprehensive calibration (Gough and Pollard, 1935; Susmel, 2014; Hobbacher, 2016).

Beyond the experimental fatigue curves, standard recommended fatigue curves from EC9, IIW, and Sonsino's recommendations are also included in the validation process. This comparative analysis provides insight into whether existing fatigue design guidelines offer safe and reliable fatigue life predictions for hybrid welded joints or whether modifications are necessary for these thin coldArc-welded hybrid structures. The results of this assessment are presented as fatigue life graphs for each multiaxial fatigue criterion, under the NS, ENS, ENS Avg, and TCD PM approaches, illustrating the accuracy of each fatigue assessment methodology.

A key aspect of this validation process is the assessment of mean stress effects. Given that as-welded hybrid joints inherently experience high local tensile residual stresses due to the welding process (Atzori, 1983; Susmel and Tovo, 2004), it is necessary to evaluate whether additional mean stresses, such as those present in $R=0.1$ loading conditions, significantly influence fatigue life estimations. If the fatigue data for $R=0.1$ remain within the scatter band of the fully reversed ($R=-1$) fatigue data, this would confirm that mean stress effects in as-welded hybrid joints are negligible, provided no stress-relief treatments are applied.

Furthermore, the fatigue behaviour under in-phase and 90° out-of-phase loading conditions is systematically examined. The fatigue life estimations from both loading configuration are compared to determine whether out-of-phase loading results in greater fatigue damage than in-phase loading, or whether no significant detrimental

effects arise due to out-of-phase conditions, as reported in Sonsino's findings for aluminium welded joints. Given the semi-ductile characteristics of aluminium, Sonsino proposed that out-of-phase loading effects may not significantly amplify fatigue damage (Kueppers and Sonsino, 2003, 2006). This study evaluates whether this trend holds for coldArc-welded hybrid joints by directly analysing the fatigue test data obtained in this study.

The effectiveness of each stress analysis approach and multiaxial fatigue criterion is quantified using statistical metrics, including the percentage of non-conservative (P_{NC}) and conservative (P_c) estimations, as well as T_{RMS} values, consistent with the validation methodology in Chapter 4. Through this systematic validation, this study identifies the most reliable and physically representative fatigue assessment approach for hybrid welded joints under multiaxial fatigue loading. The findings serve as a foundation for developing optimal fatigue design methodologies, offering critical insights into the structural performance and engineering design of hybrid welded structures.

7.3 Experimental Fatigue Behaviour Characterisation and S-N Curve Analysis for Calibration

This section presents the characterisation of the fatigue behaviour of coldArc-welded thin hybrid aluminium-to-steel welded joints based on uniaxial, pure torsional, and multiaxial fatigue tests. By systematically processing the experimental data, S-N curves are constructed to gain insights into the fatigue performance of these joints under different loading conditions. These curves serve as the foundation for calibrating multiaxial fatigue criteria in the subsequent sections. Additionally, standard reference fatigue curves from EC9, IIW, and Sonsino's recommendations are incorporated as benchmarks, allowing for a comparative assessment of whether existing fatigue design methodologies provide conservative, non-conservative, or well-aligned fatigue life estimations for these thin coldArc-welded hybrid joints.

In this study, the number of cycles to failure, N_f is defined based on the termination criterion established in Chapter 5, corresponding to a 10% reduction in global stiffness. This typically aligns with a surface crack length of approximately 10 mm. As such, the

measured fatigue life encompasses the full crack initiation phase as well as a portion of the early crack propagation phase but does not capture the complete crack growth to final fracture. This definition reflects a conservative yet practical estimate of fatigue life for welded joints in service. Consequently, the initiation phase constitutes a substantial proportion of N_f , particularly in notched aluminium components, where crack nucleation and microstructurally short crack growth are dominant in the early stages of fatigue.

The fatigue test data are processed using a log-normal linear regression model, where stress ranges are plotted against the number of cycles to failure on a log-log scale. To represent the scatter in fatigue life, scatter bands at probabilities of survival (P_s) of 50%, 2.3%, and 97.7% are included, aligning with standard recommendations in Eurocodes (ECs) and the IIW (Anon, 2005, 2007; Hobbacher, 2016). For NS approach, the nominal stresses are calculated based on the most critical nominal sectional area of the aluminium tube near the weld region, instead of the steel tube. The uniaxial and pure torsional fatigue data, obtained under fully reversed ($R=-1$) loading conditions, are plotted in Figures 7.1a and 7.1b, respectively, incorporating key parameters such as reference fatigue strength at $P_s=50\%$, the negative inverse slope (k), and the associated scatter ratios (T_σ , T_τ) (Sonsino, 2007). The T_σ and T_τ are calculated based on Equation 7.1.

$$T_\sigma = \frac{\Delta\sigma_{R,P_s=2.3\%}}{\Delta\sigma_{R,P_s=97.7\%}} \quad (7.1)$$

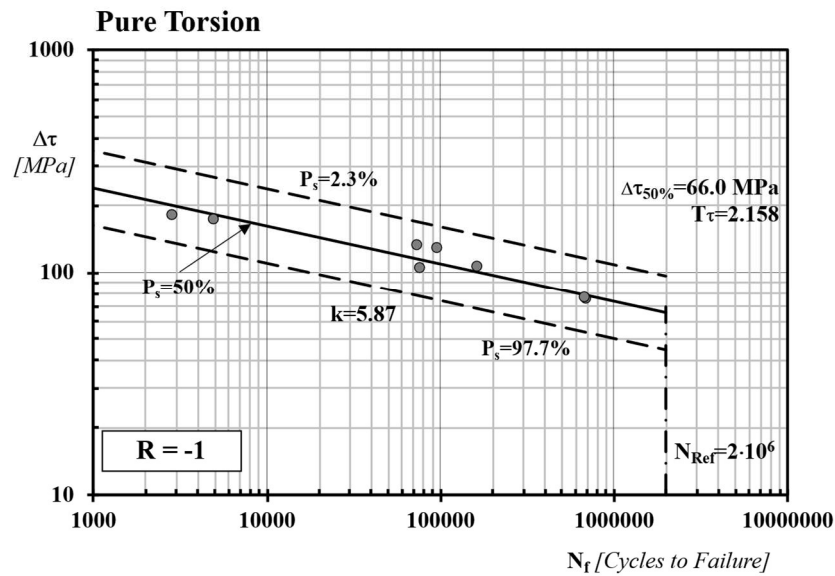
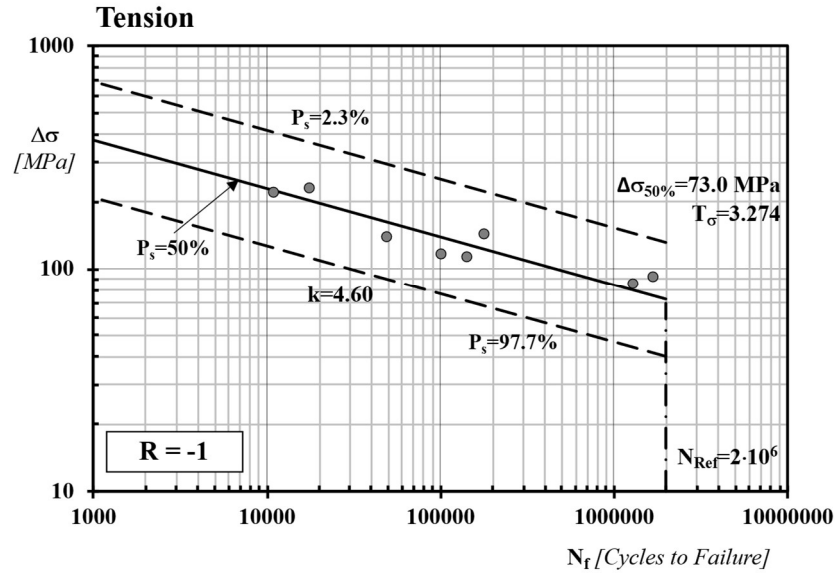
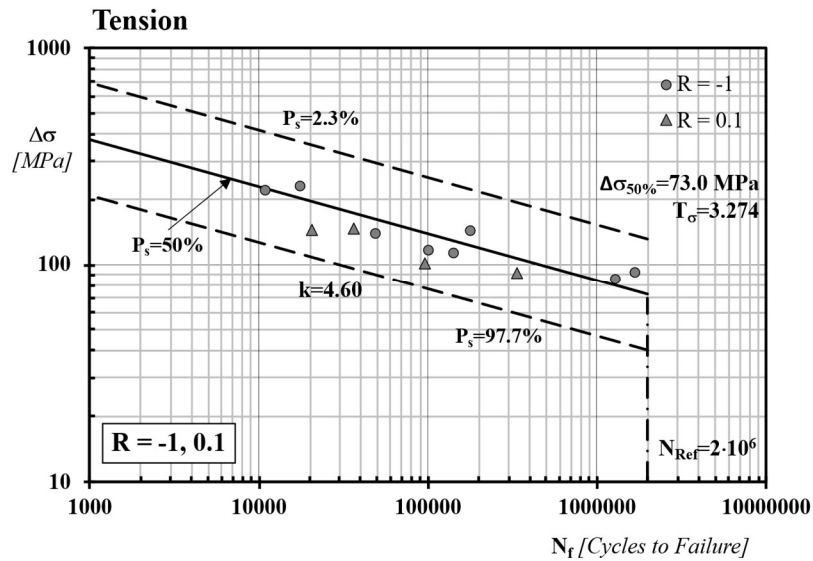


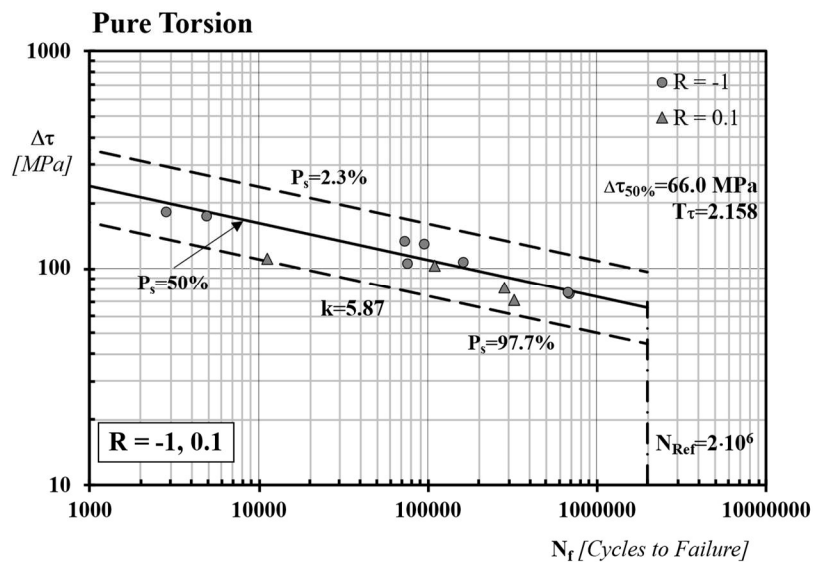
Figure 7.1: Uniaxial (a) and pure torsional (b) S-N curves for thin aluminium-to-steel welded joints derived only from fully reversed ($R=-1$) fatigue test data.

Given the mean stress effects are known to influence fatigue performance in plain metals, particularly under uniaxial loading, fatigue life estimations often require higher R ratios ($R>0$) to be considered (Findley, 1959; Susmel, 2009a; Susmel and Taylor, 2012). However, in as-welded joints, including steel and aluminium, mean stress effects are often negligible due to the presence of welding-induced residual stresses, which increase the localised stress ratio (R) at the weld toe, even when the nominal global R ratio remains negative (Atzori, 1983; Susmel and Tovo, 2004). To assess whether this

assumption applies to the investigated coldArc-welded hybrid joints, the initial S-N curves were generated using only fully reversed ($R=-1$) fatigue data. Subsequently, $R=0.1$ uniaxial and pure torsional fatigue data were plotted onto the existing $R=-1$ S-N curves and scatter bands, without altering the original linear regression analysis, as shown in Figures 7.2a and 7.2b.



(a)



(b)

Figure 7.2: Uniaxial (a) and pure torsional (b) S-N curves for thin aluminium-to-steel welded joints, incorporating $R=0.1$ fatigue data within $R=-1$ scatter bands.

If the $R=0.1$ fatigue data points remain within the $R=-1$ scatter band, it indicates that mean stress effects can be disregarded in fatigue life estimations, provided that the

maximum applied stress does not exceed the yield strength of the AA6082-T6 component. Figure 7.2 confirms that this assumption, as all $R=0.1$ data points fall within the $R=-1$ scatter band, reinforcing the assumption that mean stress effects are negligible for these as-welded hybrid joints. Consequently, S-N curves incorporating both $R=-1$ and $R=0.1$ fatigue data are generated, as illustrated in Figures 7.3a and 7.3b, aligning with standard fatigue design practices, where higher R ratio fatigue data are included to ensure a conservative and safe approach in defining fatigue design curves.

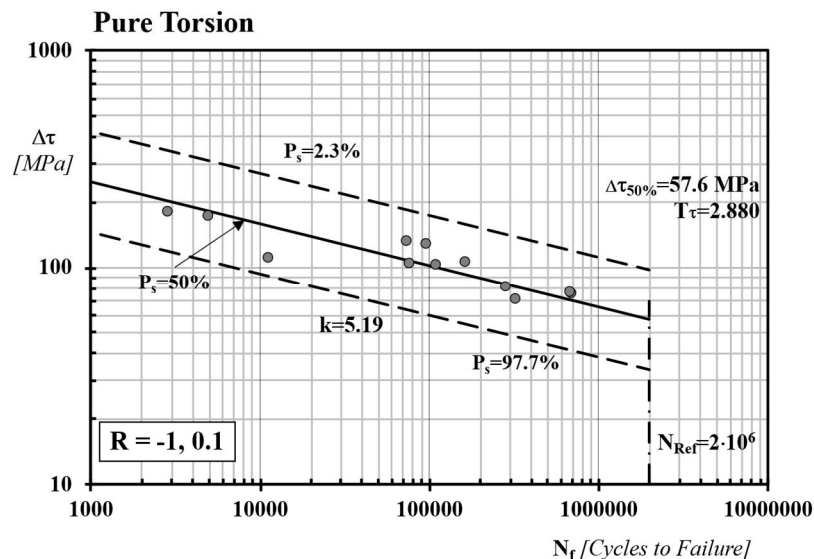
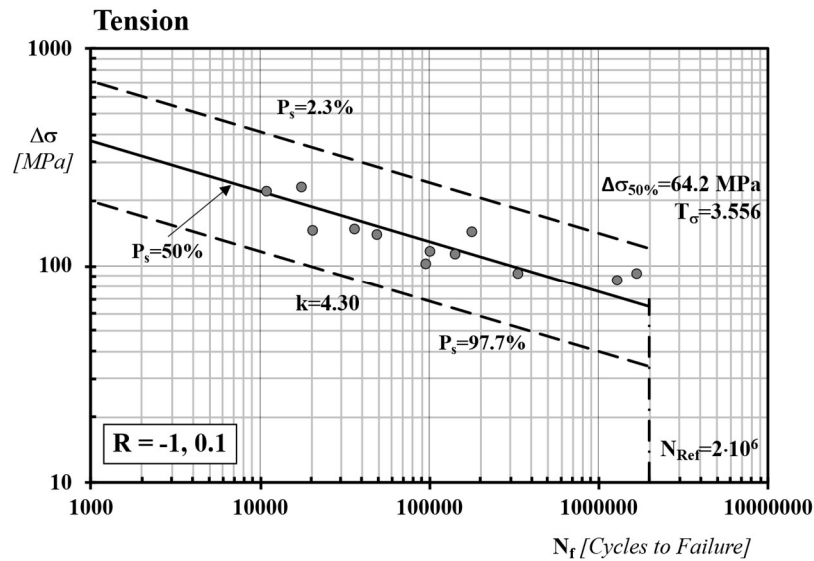


Figure 7.3: Uniaxial (a) and pure torsional (b) S-N curves for thin aluminium-to-steel welded joints, incorporating both $R=-1$ and $R=0.1$ fatigue data within the scatter bands.

Certain multiaxial fatigue criteria, such as the CS criterion, use reference fatigue strength and negative inverse slopes derived solely from $R=-1$ fatigue data, as the CS criterion inherently incorporates a Goodman-like mean stress correction term. Conversely, most other multiaxial fatigue criteria utilise the S-N curves derived from both $R=-1$ and $R=0.1$ data to capture a more comprehensive representation of fatigue behaviour.

Other than mean stress effects, another critical aspect of fatigue behaviour characterisation is the comparison of in-phase and out-of-phase loading conditions to assess whether non-proportional loading increases fatigue damage. While non-proportional loading generally reduces fatigue strength in conventional and welded steel joints, the same trend is not always observed in aluminium welded joints. Sonsino's findings for aluminium tube-to-plate welded joints indicate that out-of-phase loading does not lead to additional fatigue damage compared to in-phase loading, whereas Susmel's investigations on 6082-T6 plain specimens suggest that out-of-phase loading does have a detrimental effect on fatigue strength (Kueppers and Sonsino, 2003, 2006; Susmel and Petrone, 2003). These observations highlight that different materials exhibit varying sensitivities to out-of-phase loading, depending on the interaction between material ductility and principal stress rotation during fatigue loading (Sonsino, 1995).

To determine the influence of out-of-phase loading on the investigated hybrid aluminium-to-steel welded joints, S-N curves for both in-phase and out-of-phase conditions are compared. Figure 7.4 illustrates these S-N curves, recalculated at $P_s=50\%$ based on experimental datasets. To enhance clarity, scatter bands and individual fatigue data points are omitted to prevent excessive clutter when presenting both loading conditions within the same plot. The trends associated with biaxiality ratios (λ) of 1 and $\sqrt{3}$, as detailed in Chapter 5, are also included.

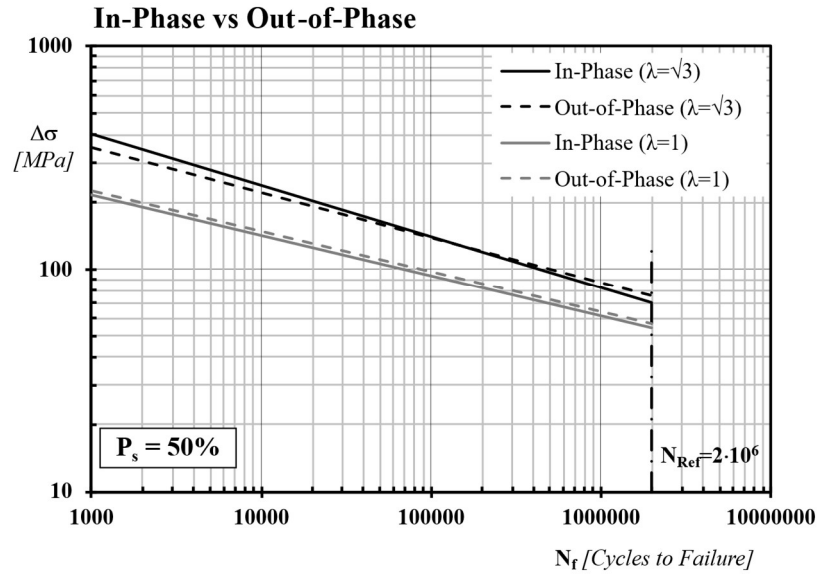


Figure 7.4: S-N curves for in-phase and out-of-phase loading conditions at $\lambda=1$ and $\lambda=\sqrt{3}$, illustrating the effect of non-proportional loading on fatigue strength.

The trends in Figure 7.4 indicate that out-of-phase loading has a negligible effect, as reflected by the similar reference fatigue strengths and negative inverse slopes of the in-phase and out-of-phase S-N curves, considering the inherent large scattering of the data as quantified by the T_σ and T_τ which is typical for welded joints due to its variability in the welding process. The observed fatigue behaviour aligns with Sonsino's findings for aluminium welded joints under multiaxial fatigue (Kueppers and Sonsino, 2003, 2006). These insights provide valuable understanding of the fatigue mechanisms governing hybrid joints under complex stress states.

To further assess the fatigue performance of these joints, the experimentally derived S-N curves are compared against standard reference fatigue curves from EC9, IIW, and Sonsino's recommendations. For the NS approach, standard reference fatigue strengths (FAT) values and negative inverse slopes (k) are obtained from EC9 and IIW, where they are selected based on weld geometry and loading configuration, irrespective of the specific aluminium grade series (Anon, 2007; Hobbacher, 2016). For the ENS approach, Sonsino's recommended fatigue strength values for welded joints thinner than 5 mm are applied to account for localised notch effects (Sonsino, Bruder and Baumgartner, 2010; Cetin Morris Sonsino *et al.*, 2012). These reference curves serve as benchmarks for validation, facilitating a systematic comparison between experimentally observed fatigue behaviour and existing fatigue design

recommendations. This assessment also helps determine whether modifications to current fatigue design methodologies are necessary to improve fatigue life estimation accuracy for hybrid joints.

The insights from S-N curve analysis and comparison with standard reference curves provide the necessary calibration data for multiaxial fatigue criteria, particularly when reanalysing fatigue performance using the NS, ENS, and ENS Avg approaches. In contrast, the TCD PM approach does not require new experimental calibration, as it is not feasible to produce ground butt-welded hybrid joints using coldArc® welding technology for direct calibration. Instead, TCD PM applies pre-established reference data, rigorously calibrated by Susmel for aluminium welded joints, including ground butt, tube-tube, and cruciform configurations (Susmel, 2009b, 2009a). The reference strength calibration and derivation of the M-D_v are assumed to be extendable to hybrid aluminium-to-steel joints, given that fatigue failure consistently initiates in the aluminium region.

By establishing these reference fatigue curves and validating them against standard design recommendations and experimental datasets, this section lays the foundation for the calibration of multiaxial fatigue criteria in the subsequent sections. The reference fatigue strengths and corresponding k values for both standard-recommended and experimentally derived S-N curves, recalculated at P_s=97.7% for 2 million cycles, are summarised in Table 7.1.

Table 7.1: Reference fatigue strengths and slopes for uniaxial and torsional fatigue curves of hybrid aluminium-to-steel joints across different stress analysis approaches.

Stress Analysis Approach	$\Delta\sigma_{R,P_s=97.7\%}^{(1)}$ (MPa)		Uniaxial curve slope, k		$\Delta\tau_{R,P_s=97.7\%}^{(1)}$ (MPa)		Torsional curve slope, k ₀	
	Std ⁽²⁾	Exp ⁽³⁾	Std ⁽²⁾	Exp ⁽³⁾	Std ⁽²⁾	Exp ⁽³⁾	Std ⁽²⁾	Exp ⁽³⁾
NS	22	34.1	3	4.3	36	33.9	5	5.2
ENS	180	180.4	5	4.3	90	55.6	7	5.2
ENS Avg	180	123.8	5	4.3	90	64.2	7	5.2
TCD	55	-	4.5	-	28	-	5	-

⁽¹⁾ Reference normal and shear stresses at 2 million cycles to failure, with P_s=97.7%.

⁽²⁾ Std refers to standard recommended values by IIW, EC9 and Sonsino's guidelines.

⁽³⁾ Exp denotes experimental data

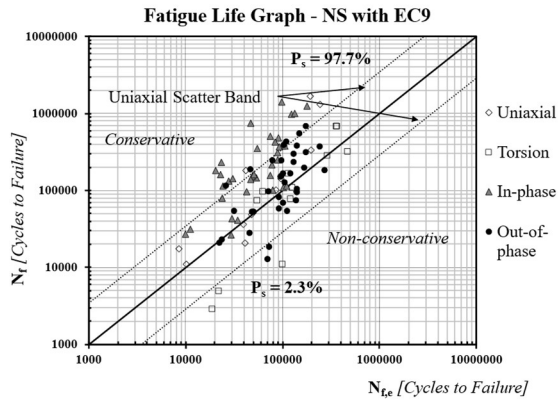
The full fatigue test datasets are provided in Appendix A. The findings ensure that fatigue life estimations in the following analyses are based on well-characterised fatigue data, leading to a systematic and robust validation of fatigue assessment methodologies for coldArc-welded thin hybrid aluminium-to-steel welded joints.

7.4 Reanalysis Using the NS Approach

The NS approach for hybrid aluminium-to-steel welded joints is based on the measured average sectional area of the aluminium part near the weld region. This choice aligns with Eurocodes (ECs) and IIW recommendations, where NS approach is widely used due to its simplicity (Anon, 2005, 2007; Hobbacher, 2016). However, in practical scenarios, identifying a clear and unambiguous nominal reference sectional area can be challenging, particularly in complex welded configurations. Despite its limitations, the NS approach serves as a preliminary fatigue assessment method, providing an initial understanding of the fatigue behaviour of the welded joints before applying more complex localised stress approaches such as the ENS and TCD approaches.

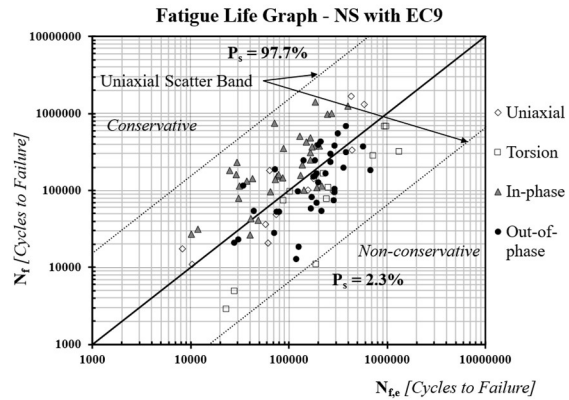
In this study, the NS approach is used in conjunction with eight multiaxial fatigue criteria: EC9/MP criterion, IIW/Gough-Pollard criterion, VM criterion, Sonsino KoNoS, MWCM, Findley's criterion, CS criterion, and the stress-based SWT criterion (Gough and Pollard, 1935; Findley, 1959; Smith, 1970; Kueppers and Sonsino, 2003; Anon, 2007; Susmel, 2014; Bruun and Härkegård, 2015; Hobbacher, 2016; Łagoda *et al.*, 2022). Each criterion is applied using both standard and experimentally derived fatigue constants, as summarised in Table 7.1. The main difference between EC9 and IIW criteria lies in the adopted reference fatigue strength and negative inverse slope values, where EC9 uses FAT 32 with $k=3.4$, whereas IIW recommends FAT 22 with $k=3$ (Anon, 2007; Hobbacher, 2016). Additionally, a modification was applied to Findley's criterion to replace the maximum normal stress term with the maximum normal stress range, following observations in Chapter 4 that the original formulation exhibited excessive conservatism for $R=0.1$ conditions. This modification aligns with the conclusion that mean stress effects can be neglected for as-welded hybrid joints. The performance of these criteria using the NS approach is illustrated in the fatigue life graphs presented in Figure 7.5.

Standard

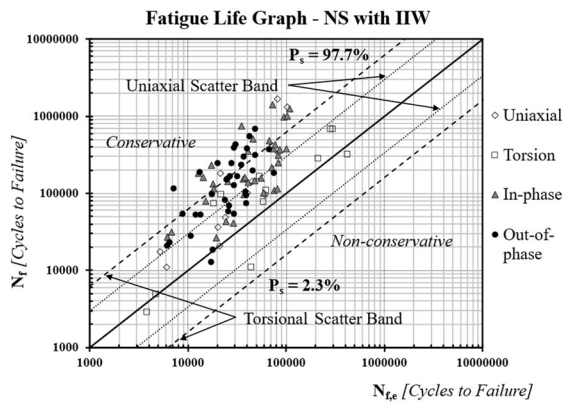


(a)

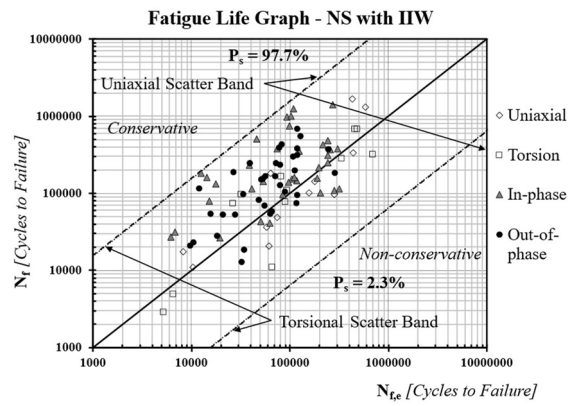
Experiment



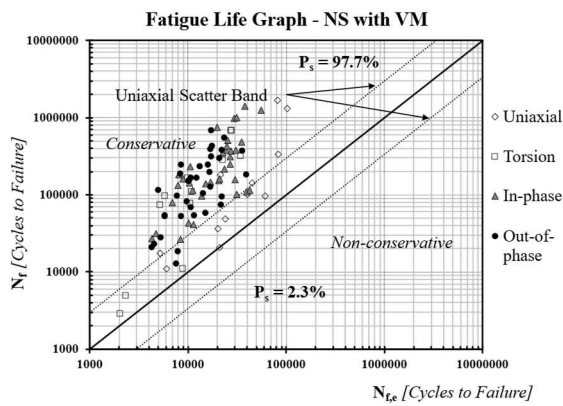
(b)



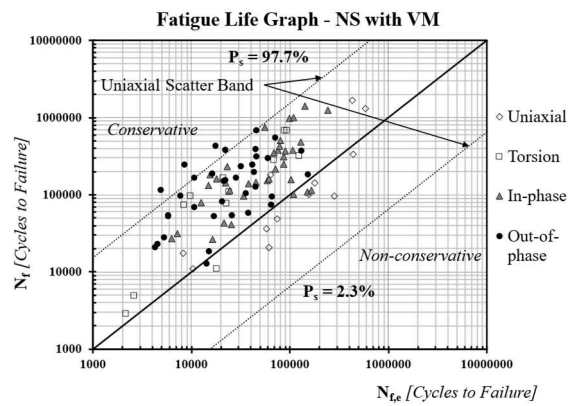
(c)



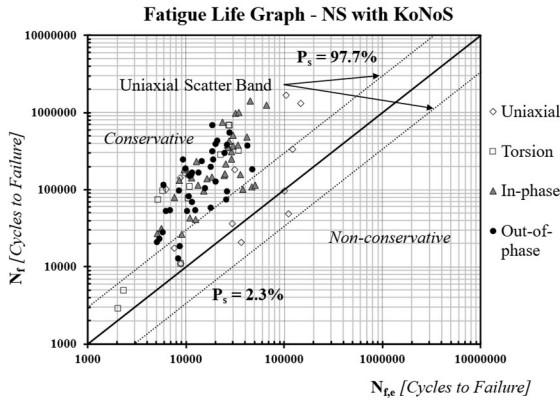
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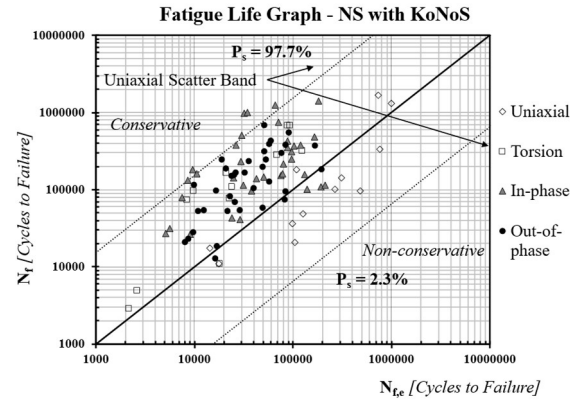
(e)



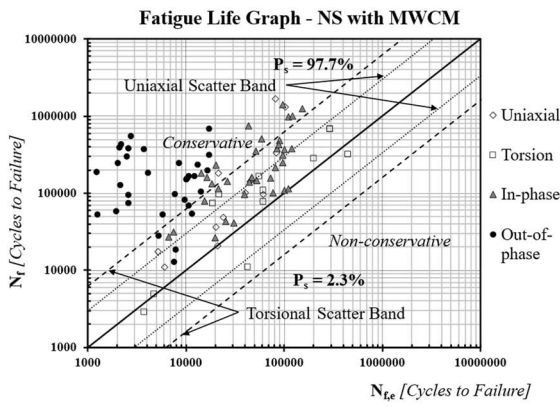
(f)



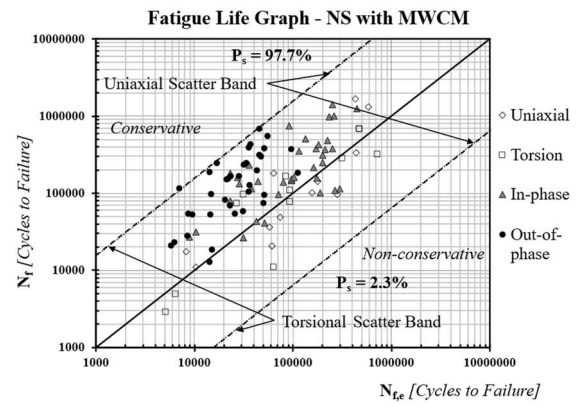
(g)



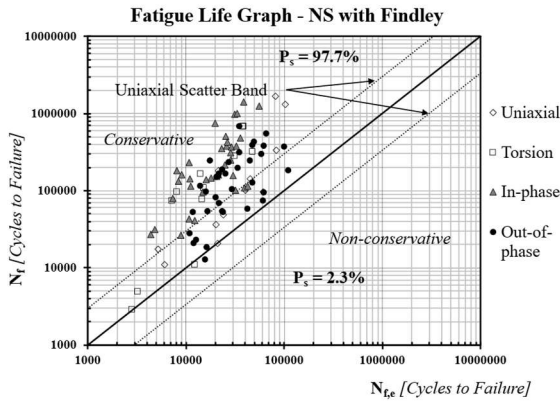
(h)



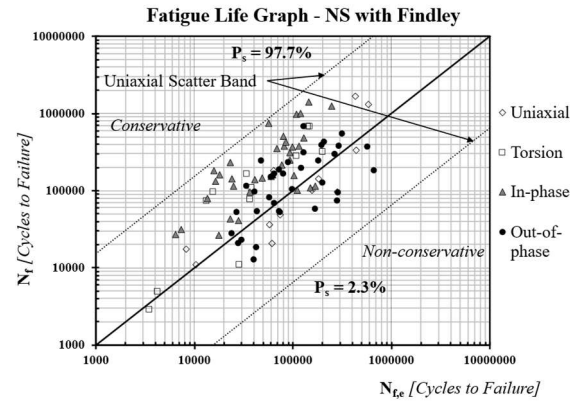
(i)



(j)



(k)



(l)

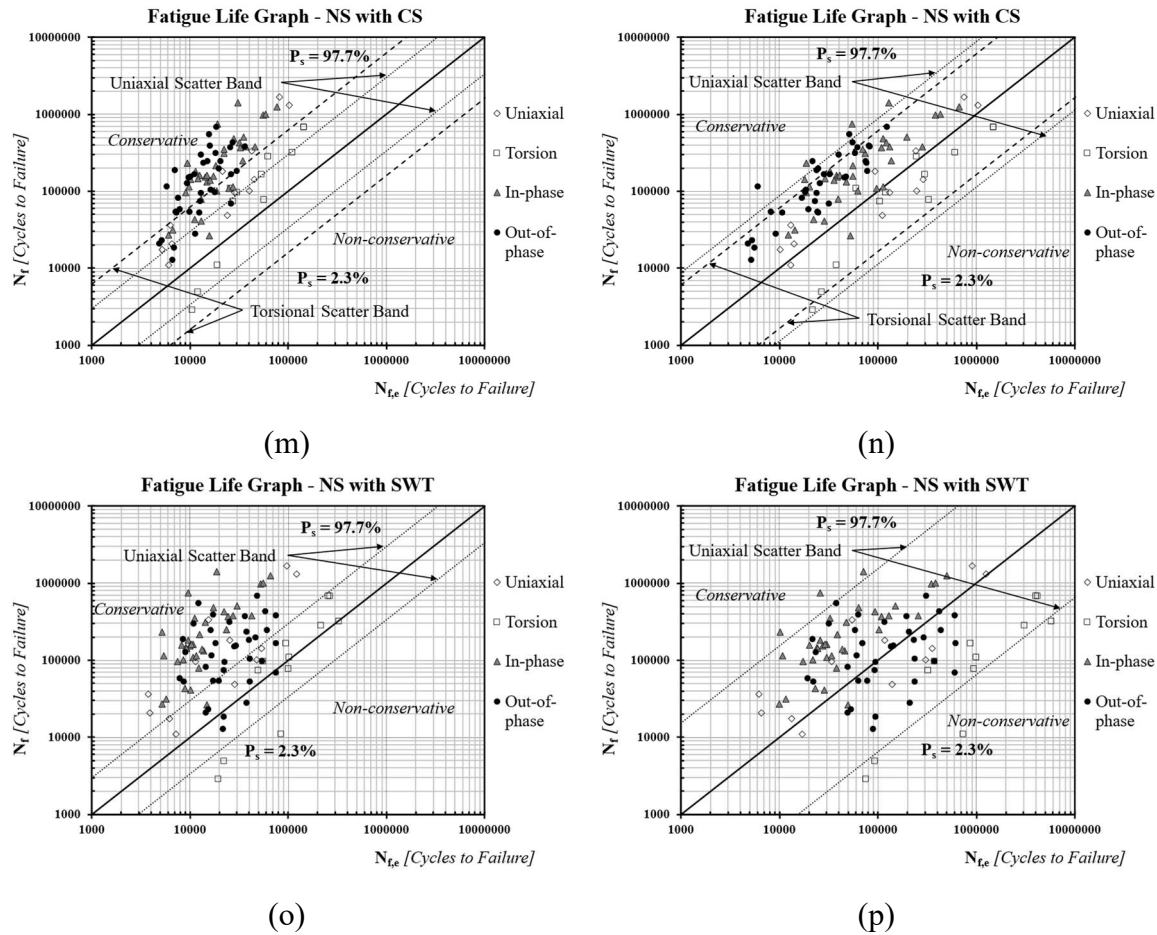


Figure 7.5: Fatigue life predictions for as-welded hybrid aluminium-to-steel joints using the NS approach with eight multiaxial fatigue criteria, comparing standard and experimental reference strengths at $P_s=50\%$.

A general observation from Figure 7.5 is that experimentally derived constants consistently improve predictive accuracy compared to standard constants, which tend to be more conservative. The conservatism of standard constants is reflected in higher P_c values. Furthermore, the scatter bands of experimentally derived uniaxial and torsional S-N curves are broader than those of the standard reference curves, reflecting the higher variability in fatigue test data.

Among the evaluated criteria, EC9 (MP) and IIW (Gough-Pollard) criteria perform notably well across different loading conditions. As shown in Figures 7.5a and 7.5b, the EC9 criterion exhibits slightly less conservative predictions for pure torsional loading but provides the most accurate estimates for uniaxial, in-phase, and out-of-phase loading. This is confirmed by its lowest average T_{RMS} values of 14.1 (standard) and 13.5 (experimental), making it the most reliable NS approach-based fatigue

criterion. The IIW (Gough-Pollard) criterion, shown in Figures 7.5c and 7.5d, is slightly more conservative than the MP criterion, particularly for out-of-phase loading when standard constants are used. Nonetheless, the IIW criterion remains highly effective, achieving the second-lowest T_{RMS} values of 37.5 (standard) and 17.3 (experimental). Notably, the IIW criterion performs exceptionally well under pure torsional loading conditions, reinforcing its applicability for multiaxial fatigue assessment.

The VM criterion, illustrated in Figure 7.5e, is observed to be consistently conservative across all loading conditions, with fatigue life estimates positioned well above the $N_f = N_{f,c}$ diagonal line. This suggests that among the three standard criteria (MP, IIW, and VM), the VM criterion is the most conservative.

Among the stress-based critical plane criteria, Sonsino KoNoS, MWCM, and Findley's modified criterion exhibit different levels of accuracy and conservatism. Sonsino KoNoS, shown in Figure 7.5h, performs well for out-of-phase loading, as its formulation inherently accounts for the observed non-damaging effect of out-of-phase loading in aluminium welded joints. However, its predictions for in-phase loading are more conservative than other criteria due to its Von Mises-based equivalent stress formulation. Despite this, all predictions remain within an acceptable accuracy range.

MWCM, when used with standard IIW constants as shown in Figure 7.5i, is found to be overly conservative for out-of-phase loading, exhibiting an excessive P_c value of 86.1%. This can be attributed to the high stress ratio, ρ_w , which influences MWCM's multiaxiality sensitivity, leading to highly conservative fatigue life estimates under out-of-phase conditions (Susmel, 2008, 2009c, 2009b, 2014). However, when experimentally derived constants are applied as illustrated in Figure 7.5j, the level of conservatism is reduced, resulting in a lower T_{RMS} of 31.8, making MWCM a viable but still conservative fatigue assessment criterion.

The modified Findley's criterion, depicted in Figure 7.5l, demonstrates accurate fatigue life estimations, particularly when using experimental constants. It achieves the third-lowest average T_{RMS} of 18.5, further validating that the maximum normal stress range is more appropriate than maximum normal stress for as-welded joints, where mean stress effects are negligible.

The CS criterion, though included in the study, does not exhibit any notable advantages or disadvantages in its performance. It provides fatigue life estimates that are generally safe and accurate, with slightly higher non-conservatism for pure torsional loading, but remains within an acceptable range.

The SWT criterion, shown in Figures 7.5o and 7.5p, produces the most scattered fatigue life estimations, yielding the highest average T_{RMS} of 128.3. In particular, pure torsional loading cases show the least conservative estimates, which aligns with expectations as SWT is inherently more suited to brittle materials (Smith, 1970; Łagoda *et al.*, 2022). This finding further reinforces that the hybrid aluminium-to-steel welded joints, having semi-ductile characteristics, are less suitable for assessment using the SWT criterion.

Overall, the reanalysis using the NS approach highlights that EC9 (MP) and IIW (Gough-Pollard) criteria are the most reliable for fatigue life estimation, with MP offering the highest accuracy across all loading conditions and IIW demonstrating strong performance, particularly under pure torsional loading. The VM criterion is consistently conservative, overpredicting fatigue life across all conditions. Among the critical plane criteria, MWCM and Sonsino KoNoS exhibit varying accuracy, with MWCM showing excessive conservatism in out-of-phase loading when used with standard constants, while Sonsino KoNoS remains accurate under out-of-phase conditions but tends to be more conservative for in-phase loading. The modified Findley's criterion proves to be an effective adaptation, achieving high accuracy when applied with experimental constants. In contrast, SWT is the least suitable, displaying excessive scatter and non-conservative estimations, particularly under pure torsional loading.

Based on the P_c , P_{NC} , and T_{RMS} metrics, the top three most suitable criteria for use with the NS approach are EC9, followed by IIW, and then Findley's criterion. While other criteria remain viable, SWT seems unsuitable due to its scattered estimations.

The insights gained from this NS approach assessment provide a baseline understanding of fatigue behaviour in these hybrid joints and serve as a reference point for subsequent ENS, ENS Avg, and TCD PM approaches, where localised stress effects will be incorporated to further refine fatigue life estimations.

7.5 Reanalysis Using the ENS Approach

The ENS approach adopted in this study follows Sonsino's recommendations for thin welded joints with plate thicknesses below 5 mm, which are not explicitly covered in IIW guidelines (Hobbacher, 2016). Unlike the conventional fictitious reference radius, r_{ref} of 1 mm used for thicker welded joints, which was originally derived from Neuber's micro-support hypothesis, a smaller reference radius of 0.05 mm is recommended for thin-walled welded structures (Sonsino, Bruder and Baumgartner, 2010; Susmel, Sonsino and Tovo, 2011; C M Sonsino *et al.*, 2012). This value is based on the relationship between the stress intensity factor, notch stresses, and crack tip blunting, providing a practical balance between finite element (FE) modelling accuracy and computational feasibility (Sonsino, Bruder and Baumgartner, 2010; Susmel, Sonsino and Tovo, 2011; C M Sonsino *et al.*, 2012). The ENS approach is applied alongside various multiaxial fatigue criteria, utilising Sonsino's recommended reference fatigue strengths, as summarised in Table 7.1. For most criteria, the reference fatigue strength values are 180 MPa for uniaxial and 90 MPa for torsional loading, based on the Peak Stress Hypothesis (PSH) (Sonsino, Bruder and Baumgartner, 2010; Susmel, Sonsino and Tovo, 2011; C M Sonsino *et al.*, 2012). However, for the VM criterion, a different reference fatigue strength of 160 MPa is adopted for both uniaxial and torsional fatigue assessments due to its distinct formulation (Sonsino, Bruder and Baumgartner, 2010; Susmel, Sonsino and Tovo, 2011; C M Sonsino *et al.*, 2012). Despite these differences in reference fatigue strengths, the negative inverse slope (k) values remain consistent across both PSH and VM approaches, with k of 5 for axial loading and k of 7 for torsional loading (Sonsino, Bruder and Baumgartner, 2010; C M Sonsino *et al.*, 2012).

7.5.1 FEA for ENS Approach

Before evaluating the fatigue life estimations using the ENS approach with various multiaxial fatigue criteria, the FE modelling process is detailed to ensure clarity and reproducibility. A linear elastic material model is assumed, with FEA performed in ANSYS APDL using the PLANE 25 elements. This 2D axisymmetric representation is chosen as it effectively models both axial and torsional loading while accurately capturing local stress distributions at critical weld regions. The model geometry is

constructed based on the measured localised features detailed in Chapter 6, and the material properties of AA6082-T6 aluminium and S235 steel are assigned as reported in Chapter 5.

To validate the suitability of a 2D approach, a comparison with a simple full 3D cylindrical model under axial and torsional loading was performed. The results confirmed that both models produced identical stress distributions, justifying the use of the more computationally efficient 2D model. Given that ENS analysis relies on precise local stress evaluations, a mesh convergence study was carried out, as illustrated in Figure 7.6.

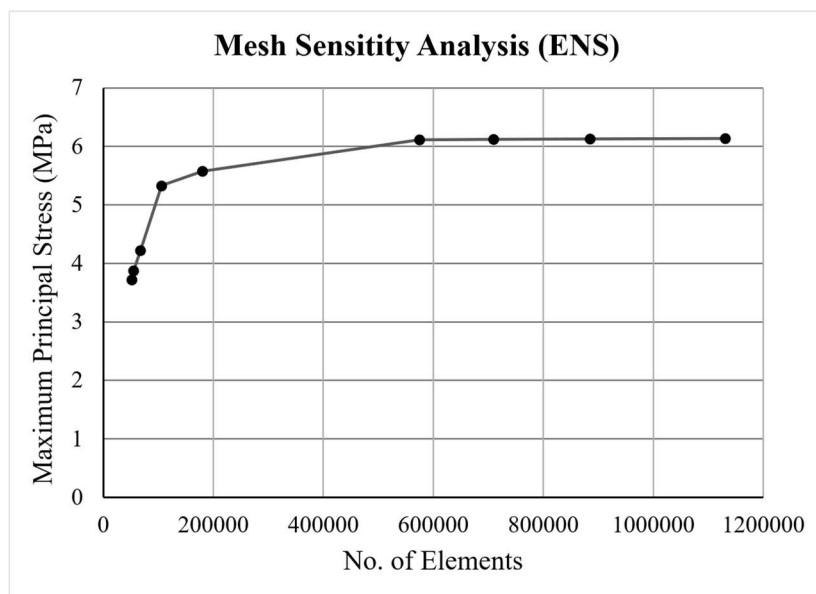


Figure 7.6: Mesh convergence analysis based on maximum principal stress under a nominal 1 MPa axial load.

As shown in Figure 7.6, maximum principal stress convergence was achieved at 574678 elements, corresponding to a refined mesh size of around 0.01 mm at the critical region. Beyond this point, the maximum principal stress remains stable, ensuring an optimal balance between computational efficiency and accuracy in local stress assessments.

The FE model and representative maximum principal stress distributions under applied nominal axial and torsional loading conditions of 1 MPa are presented in Figure 7.7. The stress distributions indicate that the critical stress locations differ between axial and torsional loading. However, due to the assumption of linear elasticity, stress

superposition is valid when evaluating combined loading conditions. All possible local stress combinations were systematically computed, and fatigue life estimations were assessed across multiple potential failure locations.

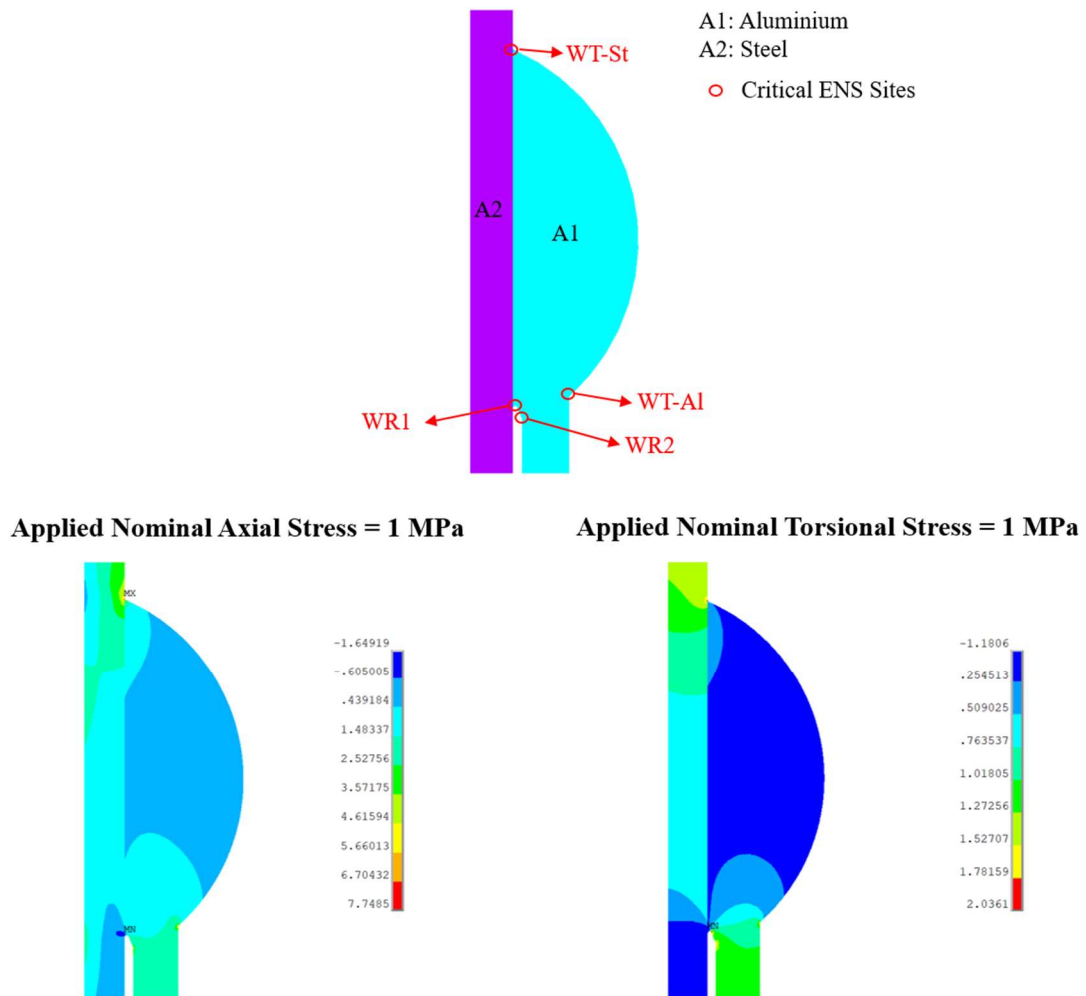


Figure 7.7: FE model and representative maximum principal stress distributions using the ENS approach under a nominal 1 MPa axial and torsional stress, illustrating localised stress variations at critical weld regions.

The analysis confirmed that the weld toe on the aluminium side (WT-Al) consistently exhibited the lowest fatigue lifetime estimates, identifying it as the most critical failure location under combined axial and torsional loading. This finding is in strong agreement with the failure analysis in Chapter 6, where experimental observations revealed predominant fatigue crack initiation at the aluminium weld toe/root region. Interestingly, while the maximum principal stress at the steel weld toe was found to be higher due to local geometric effects and material dissimilarity, the corresponding

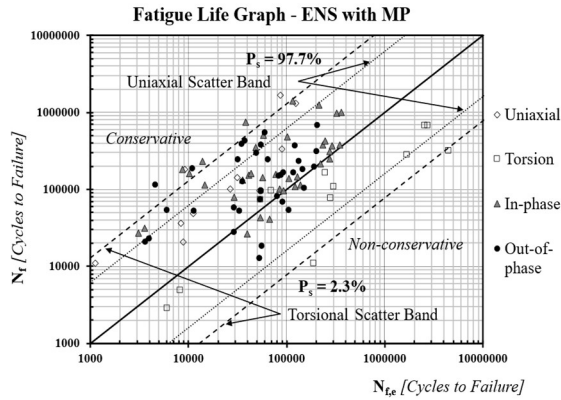
estimated fatigue lifetime was significantly longer. This outcome is attributed to the fact that the fatigue strength of steel is considerably greater than that of aluminium, making the aluminium weld toe the governing failure site.

These findings highlight the effectiveness of the ENS approach in identifying fatigue critical regions within hybrid welded joints. By integrating local stress distributions derived from FEA, the ENS methodology ensures a conservative and physically representative assessment of fatigue life, providing a strong foundation for subsequent validation using multiaxial fatigue criteria.

7.5.2 Fatigue Life Estimation Using the ENS Approach with Multiaxial Fatigue Criteria

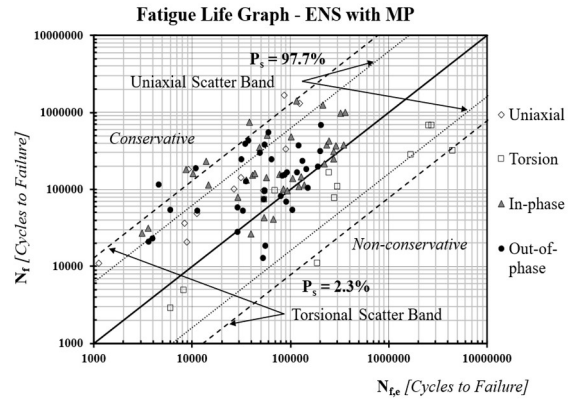
The ENS approach, similar to the NS approach, is applied alongside eight multiaxial fatigue criteria: the MP criterion, IIW/Gough-Pollard criterion, VM criterion, Sonsino KoNoS, MWCM, Findley's criterion, CS criterion, and the stress-based SWT criterion. Each criterion is evaluated using both standard and experimentally derived fatigue constants tailored to the ENS approach, as summarised in Table 7.1. A key distinction from the NS approach is that EC9 is replaced by the MP criterion, as EC9 does not provide specific recommendations for ENS-based assessments (Anon, 2007). Additionally, in contrast to the NS approach, torsional reference fatigue strengths and S-N curves are now available for the MP and Sonsino KoNoS criteria, as Sonsino specifically provided appropriate torsional reference fatigue strengths (Sonsino, Bruder and Baumgartner, 2010; C M Sonsino *et al.*, 2012). The fatigue life estimations for each criterion using the ENS approach are illustrated in Figure 7.8 and assessed based on the P_c , P_{NC} , and T_{RMS} metrics.

Standard

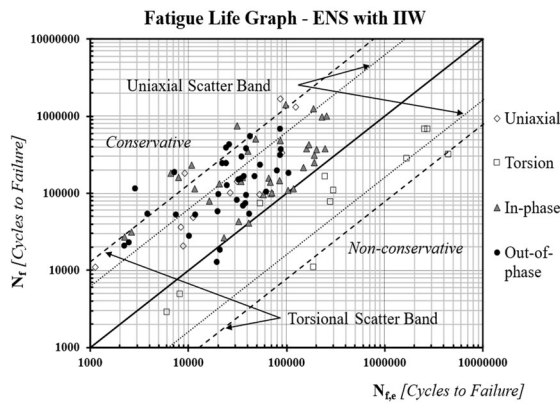


(a)

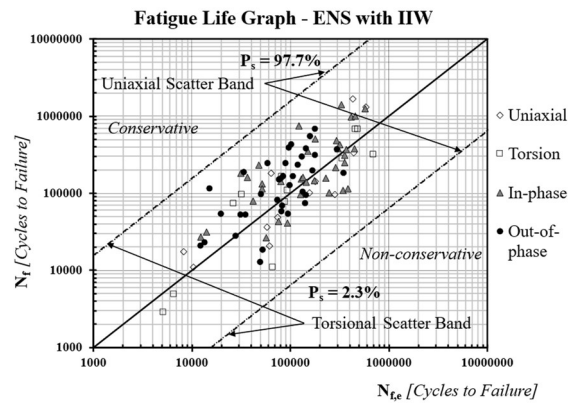
Experiment



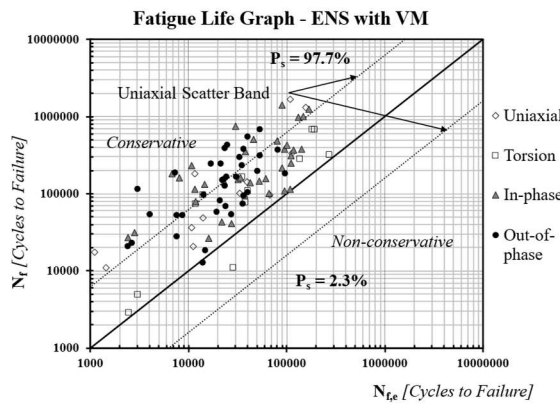
(b)



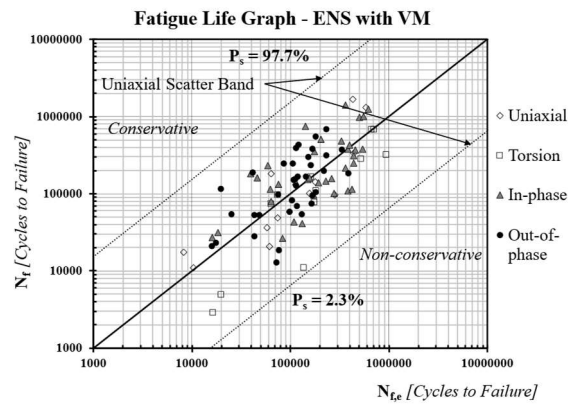
(c)



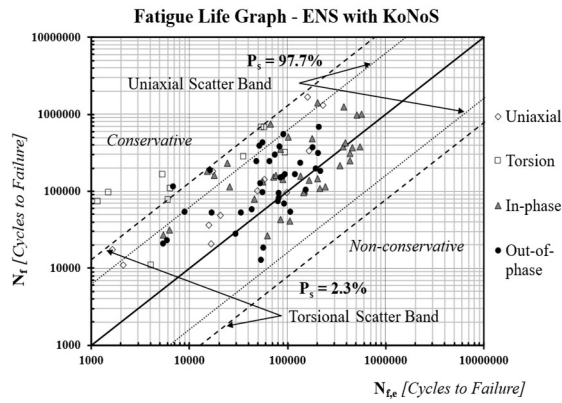
(d)



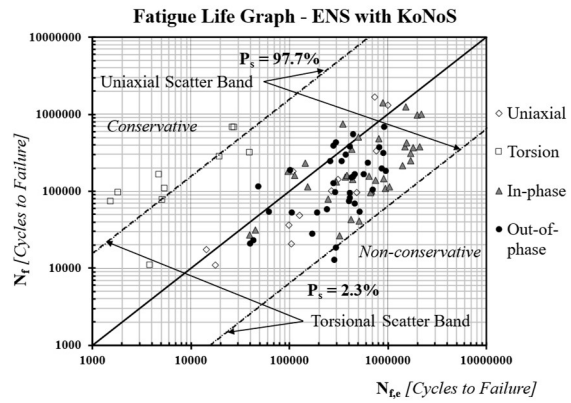
(e)



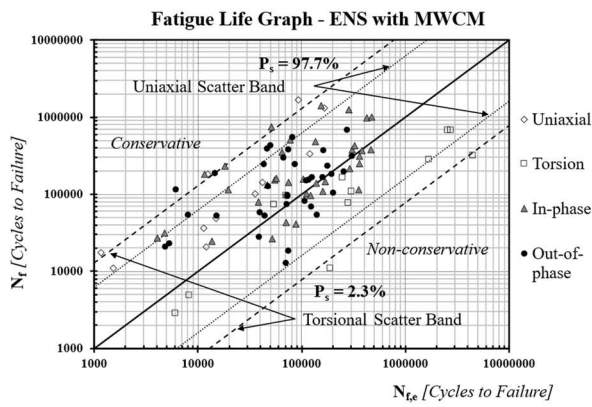
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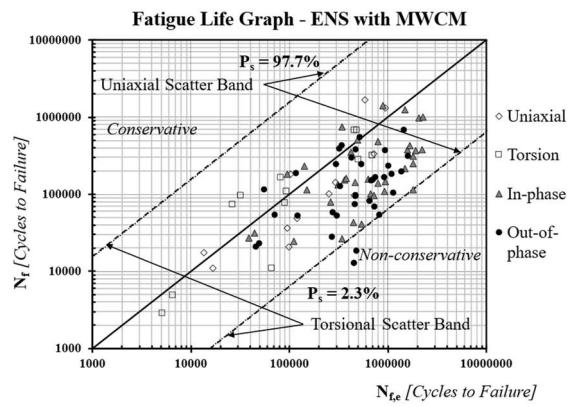
(g)



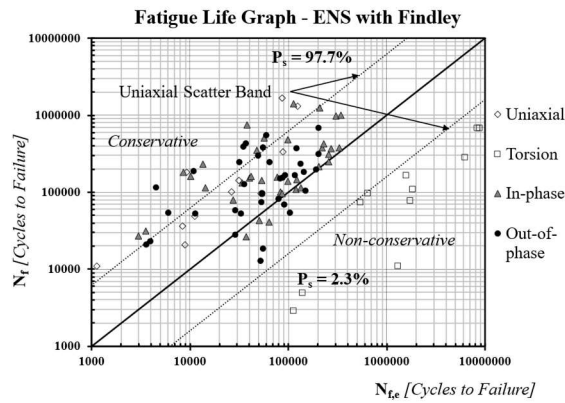
(h)



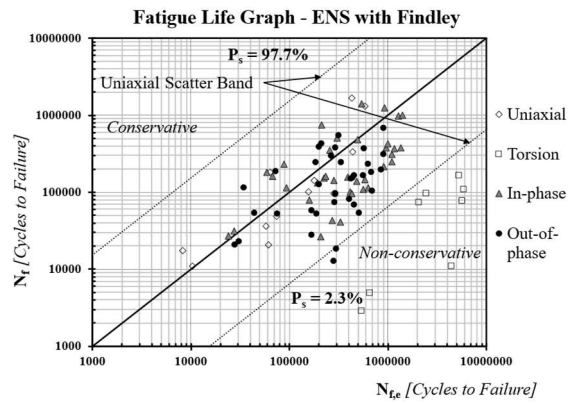
(i)



(j)



(k)



(l)

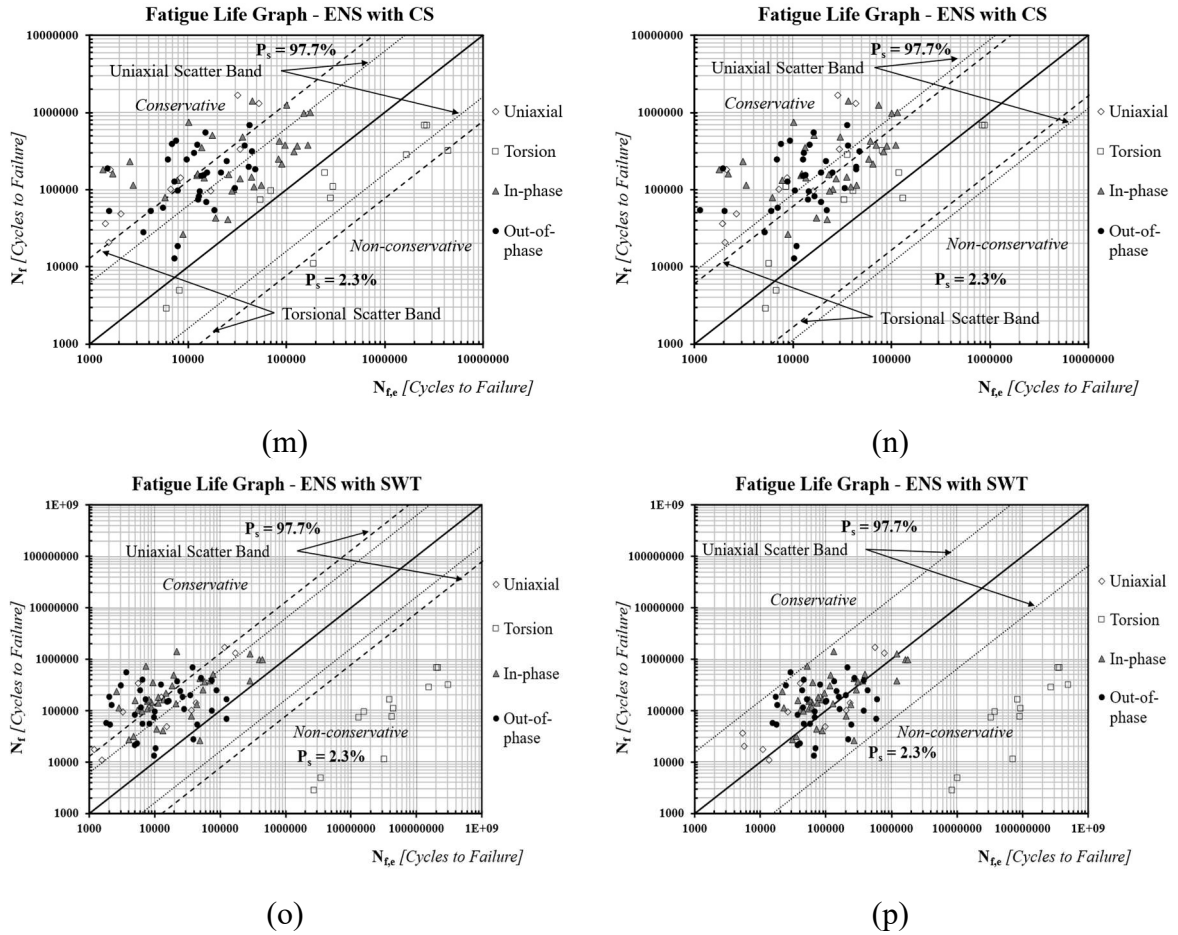


Figure 7.8: Fatigue life estimations for as-welded hybrid aluminium-to-steel joints using the ENS approach with eight multiaxial fatigue criteria, comparing standard and experimental reference strengths at $P_s=50\%$.

As observed with the NS approach, the ENS approach benefits from experimentally derived constants, which generally improve the accuracy of fatigue life estimations while reducing conservatism. However, shifting from the nominal stress-based approach to a localised stress analysis introduces notable changes in the performance of different fatigue criteria. In particular, the MP, IIW, and VM criteria, which are relatively straightforward stress-based approaches, exhibit significant improvements in predictive accuracy, especially when experimental constants are used. As with the findings from the NS approach, both the MP and IIW criteria continue to demonstrate strong performance in fatigue life estimation. However, in this case, the IIW criterion surpasses the MP criterion, achieving the lowest average T_{RMS} value of 7.4, as shown in Figures 7.8b and 7.8d. Fatigue life estimations across all loading conditions are remarkably accurate when experimental constants are used with the IIW criterion. The

MP criterion remains a strong criterion, with an average T_{RMS} of 12.8 when experimental constants are applied. However, its estimates for pure torsional loading tend to be slightly non-conservative, resulting in a higher P_{NC} compared to the IIW and VM criteria. Notably, the performance of the VM criterion improves considerably with the ENS approach, in contrast to its performance with the NS approach, further suggesting that localised stress states are more appropriate for use with the VM criterion. These findings reinforce the suitability of these relatively simple models for multiaxial fatigue assessments of hybrid welded joints, particularly when combined with localised stress approaches.

The role of localised stress states in governing fatigue performance is further evident in the behaviour of critical plane-based criteria. With the ENS approach, Sonsino KoNoS criterion shows reduced conservatism for in-phase fatigue loading but increased conservatism for pure torsional loading. Despite these shifts, the estimations remain within an acceptable accuracy range. On the other hand, the MWCM, which was previously highly conservative for out-of-phase loading under the NS approach due to its higher ρ_w values (which quantify multiaxiality at the critical plane), demonstrates a substantial improvement when applied with the ENS approach. The local stress states derived from FEA reveal that, unlike in NS approach-based calculations, the actual ρ_w values for out-of-phase loading are nearly identical to those for in-phase loading, approximately 1.1. As a result, MWCM produces more accurate fatigue life estimations, particularly for out-of-phase loading conditions. The conservatism observed with the NS approach is significantly reduced, especially when standard reference fatigue curves are used. This highlights the importance of capturing localised stress states, as the NS approach was unable to properly account for the degree of multiaxiality at the critical plane. The improvements seen in MWCM shift its ranking among the assessed criteria, making it the most accurate criterion when standard constants are used, with an average T_{RMS} of 30.9. When applied with experimental constants, the degree of conservatism under out-of-phase loading conditions is further reduced, leading to a slight tendency toward non-conservative estimates. However, MWCM remains a strong fatigue assessment criterion, with a relatively low T_{RMS} of 25.8. Its consistent performance with both standard and experimental constants makes the MWCM a highly viable option when using the ENS approach.

In contrast, the modified Findley's criterion, shown in Figures 7.8k and 7.8l, experiences a decline in accuracy under the ENS approach compared to the NS approach. This is primarily due to its calibration relying solely on uniaxial fatigue data, which leads to non-conservative estimates for pure torsional loading. However, for in-phase and out-of-phase loading conditions, Findley's criterion remains reasonably effective. The CS criterion when applied with the ENS approach, becomes excessively conservative, suggesting that its Goodman mean-stress correction, may not be well suited for hybrid welded joints. A similar trend was previously observed for aluminium welded joints reanalysed using the CS criterion with the ENS approach, as discussed in Chapter 4 during the quantitative review of critical plane criteria. Lastly, although the SWT criterion shows a slight improvement in terms of reduced scatter for uniaxial, in-phase, and out-of-phase loading, its fatigue life estimations for pure torsional loading remain highly non-conservative. This further confirms its unsuitability for hybrid welded joints, particularly under shear-dominated stress conditions.

Overall, the ranking of the best performing criteria shifts slightly when using the ENS approach compared to the NS approach. With the ENS approach, the IIW criterion emerges as the most accurate, followed by the MP criterion and the VM criterion, with the latter providing safer estimates for pure torsional loading than the MP criterion. MWCM also becomes a more viable option, particularly when both standard and experimental constants are considered, surpassing Findley's criterion as the most suitable critical plane-based approach under the ENS framework. These findings highlight the importance of incorporating localised stress states in multiaxial fatigue assessments, particularly for hybrid welded joints, where material dissimilarities and localised geometric effects significantly influence fatigue strength and lifetime.

While the ENS approach improves predictive accuracy through a localised stress assessment, it relies on a standardised fictitious reference radius of r_{ref} of 0.05 mm for all weld toes and roots. However, this standardised assumption does not account for actual variations in local notch geometry, which may influence fatigue behaviour. To explore whether a more representative evaluation can be achieved, the next section investigates the ENS Avg approach, which incorporates experimentally measured average notch radii in place of a fixed r_{ref} . While this modification aims to refine fatigue

life estimations by better capturing local stress conditions, its effectiveness remains uncertain and will be critically examined in the following analysis.

7.6 Reanalysis Using the ENS Avg Approach

The ENS Avg approach serves as an alternative refinement to the standard ENS methodology proposed by Sonsino. Unlike the conventional ENS approach, which employs a fixed r_{ref} of 0.05 mm, the ENS Avg approach incorporates the experimentally measured average notch radii obtained from sectioned specimens, as detailed in Chapter 6. The objective of this modification is to determine whether using actual notch geometries, specific to each weld toe and root region, provides a more representative local stress evaluation and enhances the accuracy of fatigue life estimations for aluminium-to-steel welded joints. The underlying hypothesis is that by replacing the standardised notch radius with the measured values, the local stress distribution can be better captured, potentially leading to improved fatigue life predictions. The key questions addressed in this section are whether this modification leads to a significant improvement in accuracy and, if so, to what extent it refines the existing fatigue life estimations.

7.6.1 FEA for ENS Avg Approach

The FEA methodology for the ENS Avg approach follows the same fundamental framework as the conventional ENS approach to maintain consistency in comparative analyses. The same material properties, element type (PLANE 25), and linear-elastic assumptions are retained. However, the key distinction lies in replacing the fixed r_{ref} of 0.05 mm with experimentally measured average notch radii at the weld toe and weld root locations. Notably, the measured weld toe radii are significantly larger, with values of 1.8 mm for the steel weld toe and 1.6 mm for the aluminium weld toe. Similarly, the smallest measured weld root radius is 0.07 mm, which, while smaller than the toe radii, remains larger than the standard r_{ref} previously assumed in the ENS approach.

To ensure numerical accuracy, a mesh convergence analysis was conducted, confirming that, as with the ENS approach, a refined mesh size of approximately 0.01 mm at the

critical region was sufficient to achieve stress convergence. The FE model and representative maximum principal stress distributions under nominal axial and torsional loading conditions of 1 MPa using the ENS Avg approach are shown in Figure 7.9.

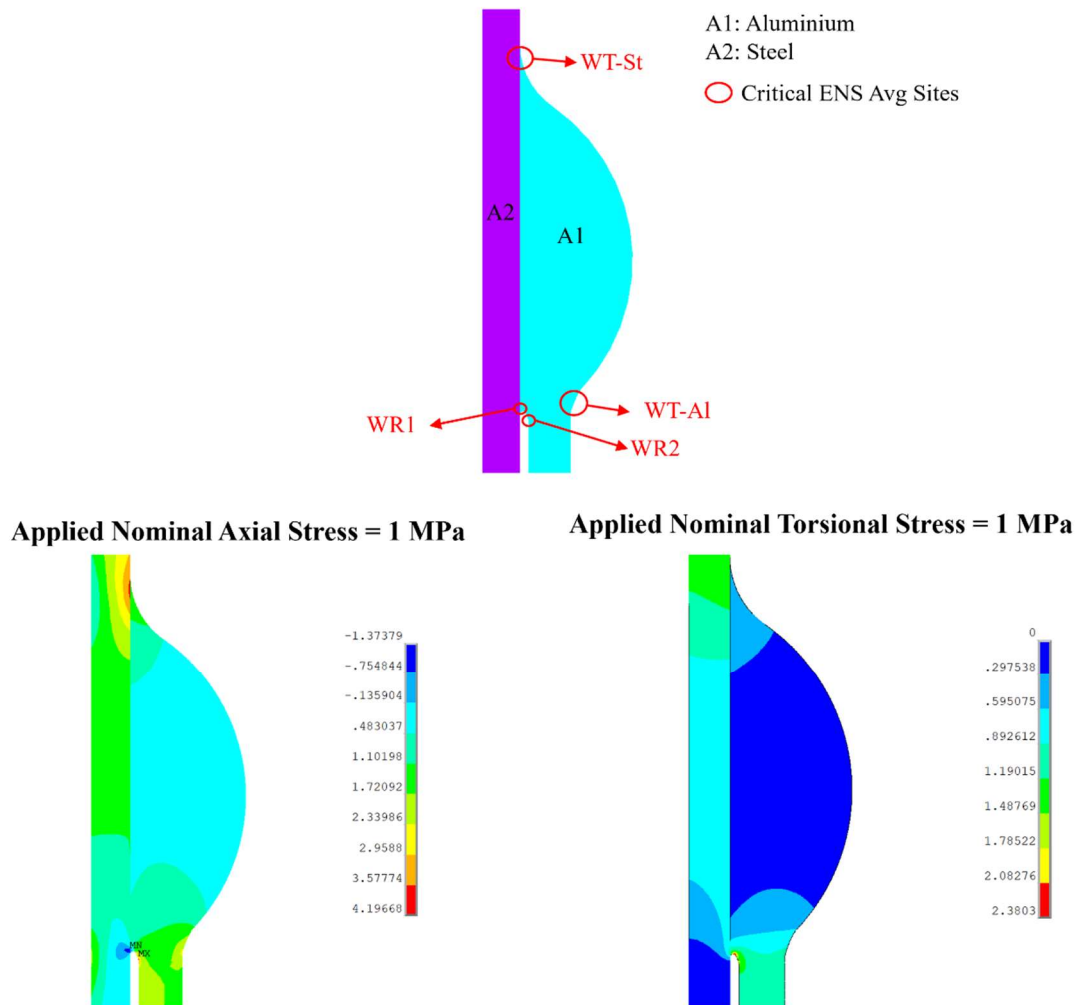


Figure 7.9: FE model and representative maximum principal stress distributions using the ENS Avg approach under a nominal 1 MPa axial and torsional stress, capturing localised stress variations at critical weld regions based on measured notch radii.

Following the same methodology as the ENS approach, the computed localised stress states at all potential critical fatigue failure regions were superimposed, and all possible stress combinations were analysed to determine the most critical fatigue failure location.

A major distinction between the ENS Avg and ENS approaches is the shift in the predicted critical failure location. While the ENS approach identified the aluminium weld toe (WT-Al) as the most vulnerable region, the ENS Avg approach instead

highlights the weld root (WR2) as the location with the lowest estimated fatigue life. This shift is attributed to the refined notch radius measurements. Since the weld root features a smaller radius than the weld toe, it experiences higher localised stress concentrations, making it more susceptible to fatigue failure. Importantly, this finding aligns with the failure analysis presented in Chapter 6, where the weld root was identified as one of the primary crack initiation sites.

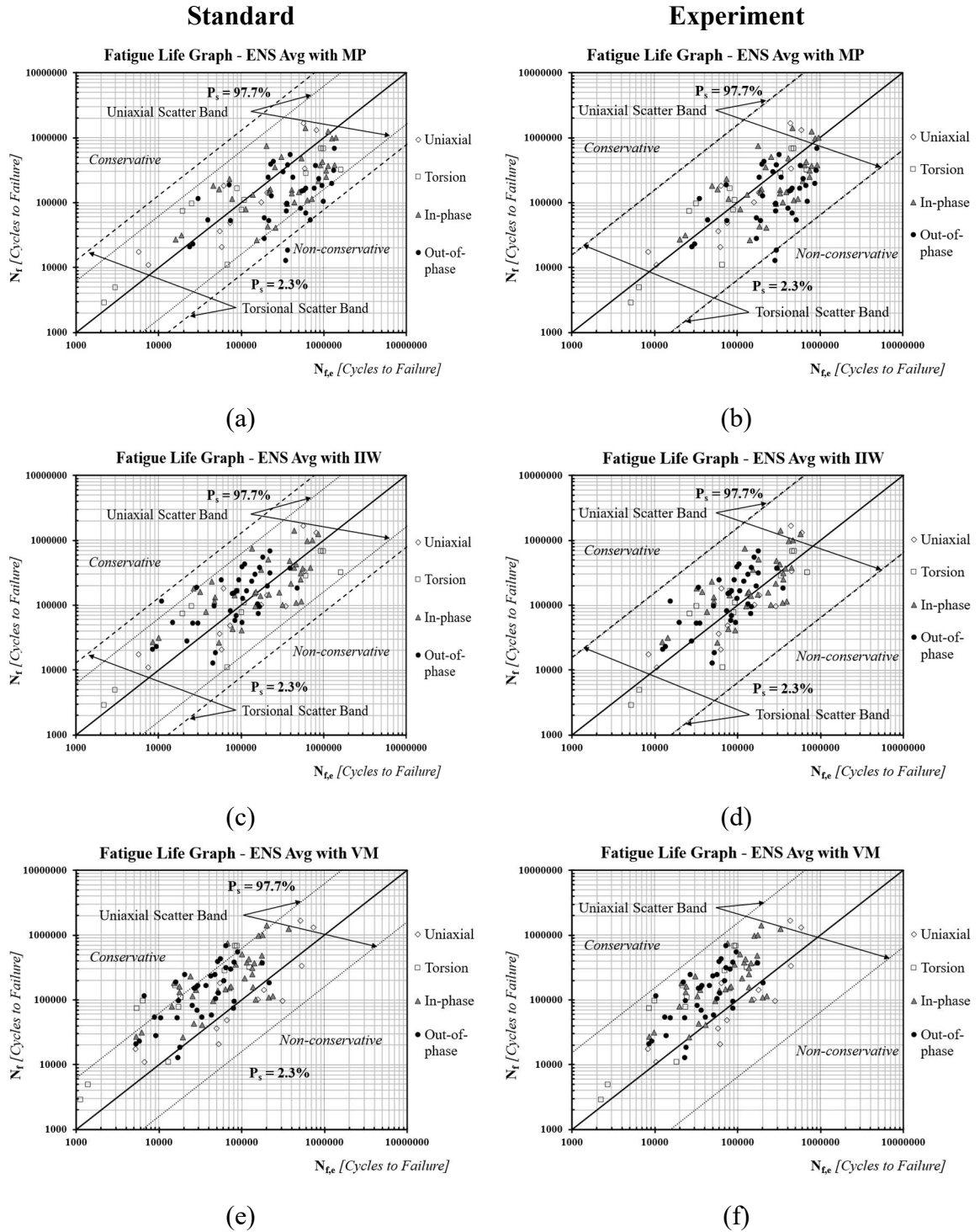
However, due to the complex hollow tube geometry, distinguishing between failures originating from the weld root or weld toe remains challenging in experimental observations. Failure analysis suggests that cracks may initiate from either location or even simultaneously at both. The FEA results from the ENS Avg approach indicate that the weld root (WR2) exhibits the highest localised stresses and the shortest estimated fatigue life. This finding is considered reasonable, as it remains consistent with the observed failure mechanisms in hybrid welded joints.

Given the identification of WR2 as the critical failure location, the ENS Avg approach is now applied in conjunction with various multiaxial fatigue criteria in the following section. The accuracy and predictive capability of this alternative methodology are rigorously evaluated to determine whether it provides a more representative fatigue life estimation than the conventional ENS approach.

7.6.2 Fatigue Life Estimation Using the ENS Avg Approach with Multiaxial Fatigue Criteria

Building upon the ENS approach, this section evaluates the alternative ENS Avg approach, which incorporates experimentally measured average notch radii instead of a standardised fictitious notch radius. The ENS Avg approach is applied alongside six multiaxial fatigue criteria: MP criterion, IIW/Gough-Pollard criterion, VM criterion, Sonsino KoNoS, MWCM, and Findley's criterion. The CS and SWT criteria, which were previously identified as unsuitable in the ENS approach due to excessive conservatism and poor accuracy, respectively, are excluded from this analysis. As before, each criterion is evaluated using both standard and experimentally derived fatigue constants, as summarised in Table 7.1. A key distinction in the ENS Avg approach is that while the standard fatigue reference strengths remain unchanged from

the ENS approach, the experimentally derived reference strengths differ. Specifically, the experimental uniaxial reference fatigue strength decreases, whereas the torsional reference fatigue strength increases slightly. The fatigue life estimations for each criterion using the ENS Avg approach are presented in Figure 7.10 and assessed based on the P_c , P_{NC} , and T_{RMS} metrics.



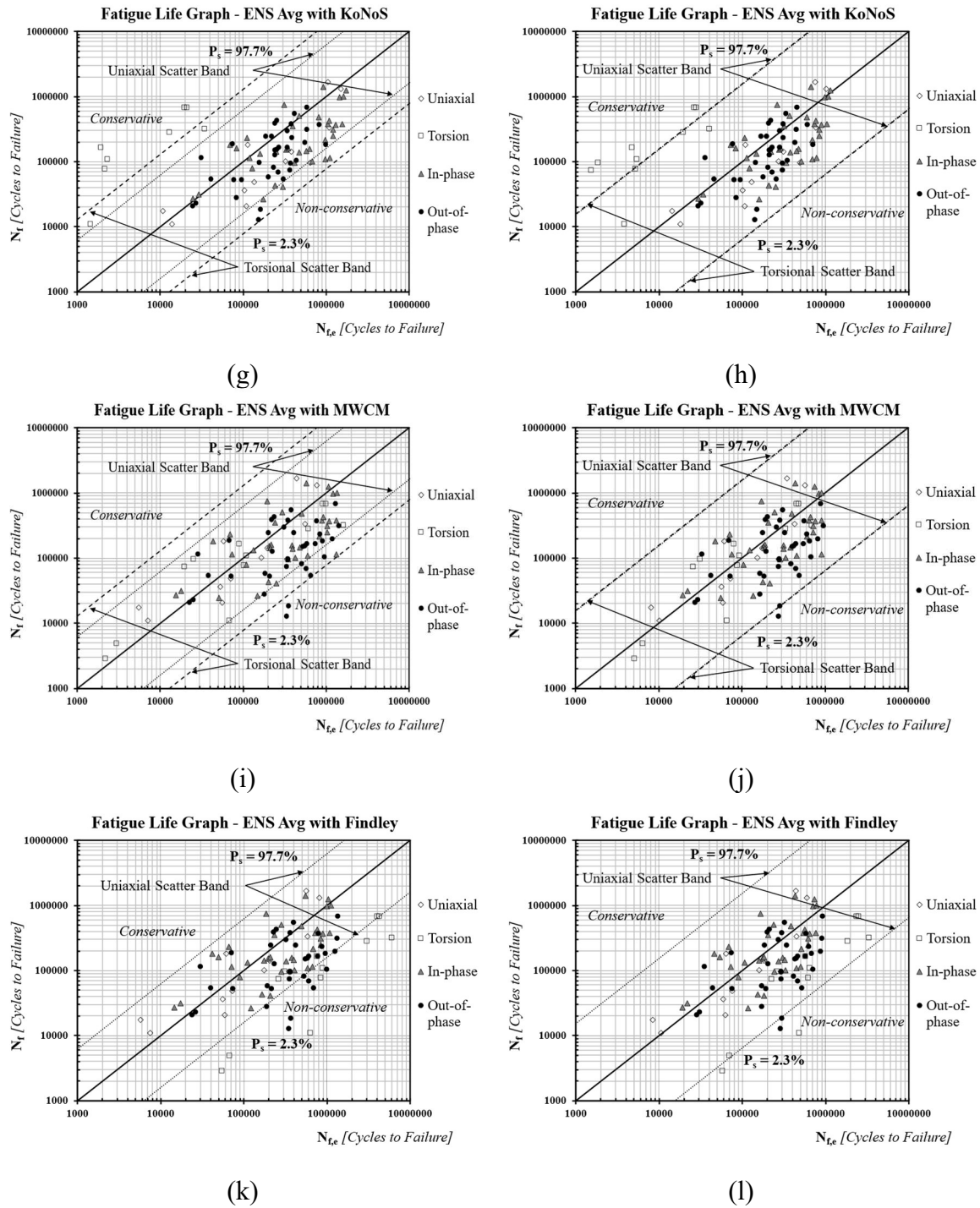


Figure 7.10: Fatigue life estimations for as-welded hybrid aluminium-to-steel joints using the ENS Avg approach with six multiaxial fatigue criteria, comparing standard and experimental reference strengths at $P_s=50\%$.

Consistent with previous findings, the ENS Avg approach benefits from the use of experimentally derived constants, which improve the accuracy of fatigue life estimations. However, unlike the ENS approach, applying standard constants in the

ENS Avg approach results in fatigue life predictions that are less conservative while still maintaining a sufficient safety margin. This is primarily due to the fact that while localised stress states at the critical region are reduced, the standard reference strengths remain unchanged from the ENS approach, leading to slightly longer estimated fatigue lifetime. Importantly, the underlying hypothesis of the ENS Avg approach, which involves replacing a standardised notch radius with experimentally measured average notch radii, appears to be valid. The improved accuracy of fatigue life estimations across all criteria, regardless of whether standard or experimental constants are used, supports the effectiveness of this modification.

The overall ranking of criteria in terms of fatigue life estimation accuracy remains largely consistent with the ENS approach. The MP criterion, shown in Figures 7.10a and 7.10b, continues to provide reliable fatigue life predictions. A key improvement observed with the ENS Avg approach is the mitigation of the slight non-conservatism previously seen for pure torsional loading in the ENS approach. However, with the ENS Avg approach, a slight shift in non-conservatism is observed for out-of-phase loading instead, with a P_{NC} of 5.6% when standard constants are used and 2.8% for experimental constants. The IIW criterion remains the most accurate among all examined criteria. Notably, the use of standard constants with the ENS Avg approach results in a substantial reduction in fatigue life scatter, as evidenced by a significant drop in the average T_{RMS} metric from 61.6 in the ENS approach to 8.0 in the ENS Avg approach. Similarly, the VM criterion maintains a comparable level of accuracy, with minor improvements in predictive performance.

For Sonsino KoNoS, the general trend of predictions remains similar to that observed with the ENS approach. The in-phase loading conservatism seen in the ENS approach continues to be reduced, but pure torsional loading remains highly conservative. In contrast, MWCM exhibits an improvement in accuracy when using the ENS Avg approach, with a reduction in T_{RMS} values for both standard and experimental constants. The Findley criterion, while benefiting from improved accuracy in the ENS Avg approach, still struggles with non-conservative fatigue life estimations for pure torsional loading conditions.

Overall, the ranking of the best-performing criteria remains unchanged when using the ENS Avg approach. The IIW criterion remains the most accurate, followed by the MP criterion and the MWCM. These findings suggest that incorporating experimentally measured average notch radii provides a more realistic representation of localised stress distributions, thereby improving overall fatigue life estimation accuracy compared to the conventional ENS approach. Nevertheless, while all criteria investigated in this section remain generally viable, Findley's criterion and Sonsino KoNoS exhibit greater variability, particularly under pure torsional loading conditions, where Findley's criterion remains non-conservative and KoNoS being overly conservative.

Despite these improvements, a critical question remains. The notch radii measurements used in the ENS Avg approach were obtained from a limited number of sectioned specimens, raising concerns about whether the averaged values adequately account for worst-case welded joint conditions, where extremely sharp weld toe or weld root notches may exist. Since weld profile variations extend across the entire circumference of the tubular hybrid joints, it is unclear whether assuming an average notch radius sufficiently captures the most severe fatigue conditions. A more conservative approach would be to assume weld toe and root radii approaching zero, representing an extreme worst-case scenario.

To address this concern, the following section explores the TCD PM, which assumes a zero-radius notch. This alternative stress analysis method is applied in conjunction with MWCM to assess whether it can reliably estimate the fatigue life of hybrid aluminium-to-steel welded joints.

7.7 Reanalysis Using the TCD Approach

The TCD approach, specifically in its simplest Point Method (PM) formulation, is adopted as an alternative localised stress analysis method in conjunction with the MWCM to reanalyse the hybrid welded joints (Taylor, Barrett and Lucano, 2002; Susmel, 2007, 2008, 2009a, 2009b). A key advantage of the TCD approach is that it eliminates the need for fictitious notch rounding, as required in the ENS approach. Instead, the TCD assumes zero weld root and toe radii, simplifying FE modelling

process (Susmel, 2007, 2008, 2009a, 2009b). This method relies on the critical distance, $M-D_v$, which is calibrated based on Notch Stress Intensity Factors (N-SIFs) and standard reference fatigue curves recommended by EC9 for uniaxial loading and IIW for pure torsional loading in aluminium welded joints (Livieri and Lazzarin, 2005; Susmel, 2009b, 2009a). Notably, the $M-D_v$ value of 0.075 mm is an inherent material property, remaining independent of geometry or stress complexity (Taylor, Barrett and Lucano, 2002; Susmel, 2009b, 2009a). In this approach, stress states are extracted at this fixed distance along a focus path positioned at the notch bisector of the weld root and weld toe.

7.7.1 FEA for TCD Approach

The FEA methodology follows the same fundamental framework as the ENS and ENS Avg approaches, assuming a linear-elastic material model with the same material properties and element type PLANE 25. However, the primary difference lies in setting the notch radii at the weld toes and roots to zero, representing the sharpest possible notch geometry. A mesh convergence analysis, shown in Figure 7.11, was performed to ensure numerical accuracy.

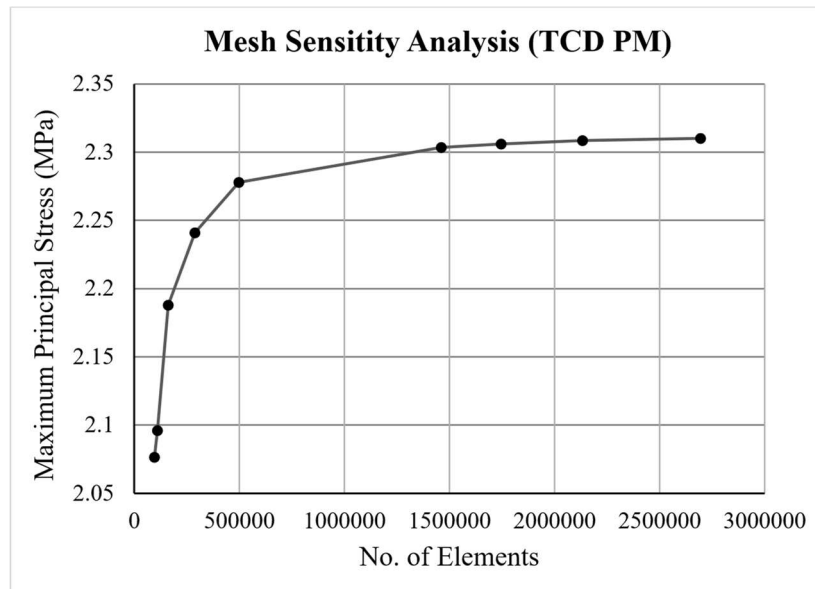


Figure 7.11: Mesh convergence analysis showing the maximum principal stress extracted at $M-D_v$ of 0.075mm along the notch bisector along the focus path under a nominal 1 MPa axial stress.

The results indicate that at 1724560 elements, corresponding to a mesh size of 0.008 mm, the maximum principal stress at 0.075 mm from the notch bisector along the focus path reaches convergence. The FE model, illustrating potential fatigue failure locations along with their associated focus paths for extracting localised stress states, is presented in Figure 7.12. Additionally, the representative maximum principal stress distributions under nominal axial and torsional loading of 1 MPa using the TCD PM approach are depicted.

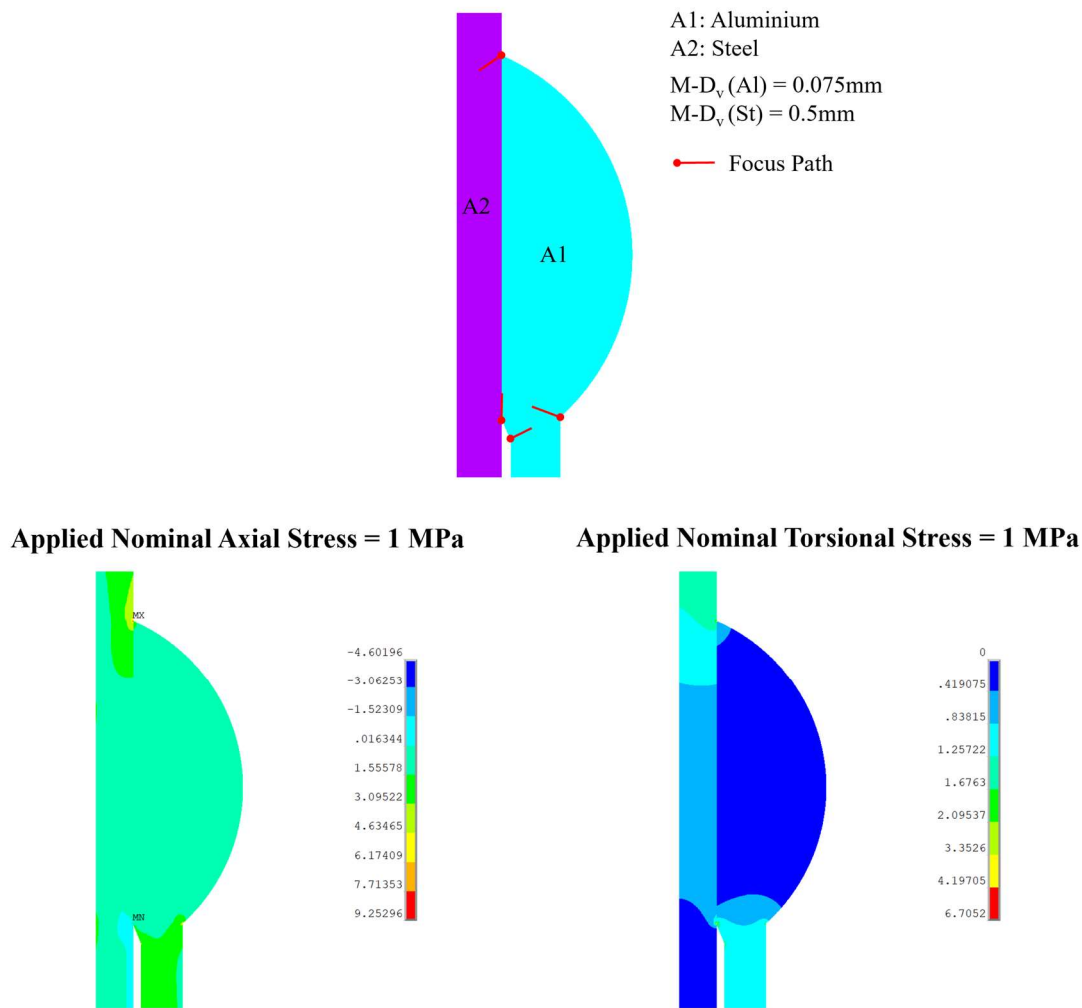


Figure 7.12: FE model with defined focus paths and representative maximum principal stress distributions using the TCD approach under a nominal 1 MPa axial and torsional stress, highlighting localised stress variations at critical weld regions.

Following the same methodology used in the ENS and ENS Avg approaches, localised stress states were extracted along the focus paths at all potential failure regions, and all possible stress combinations were analysed. The fatigue life estimations revealed that,

under the TCD PM approach, the aluminium weld toe (WT-A1) remains the most critical failure location, exhibiting the lowest estimated number of cycles to failure. This result is consistent with the prediction obtained using the ENS approach, reinforcing the reliability of the TCD method in identifying critical fatigue sites.

7.7.2 Fatigue Life Estimation Using the TCD PM Approach with MWCM

Since the TCD PM approach is calibrated specifically for use with the MWCM, no additional multiaxial fatigue criteria are examined in this section. The reference fatigue strength values are exclusively based on the standard values reported in Table 7.1. The corresponding fatigue life estimations using the TCD PM with MWCM are presented in Figure 7.13.

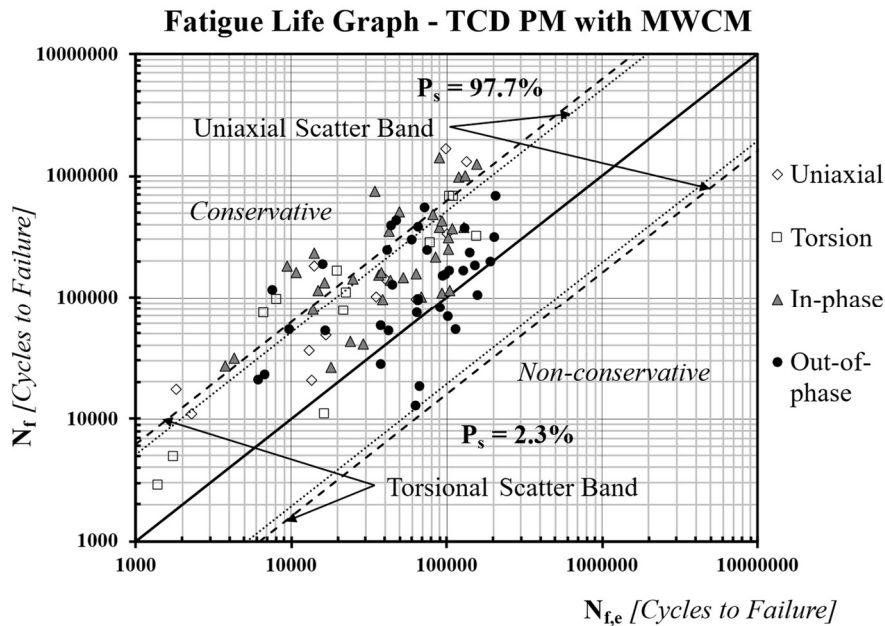


Figure 7.13: Fatigue life estimations for as-welded hybrid aluminium-to-steel joints using the TCD PM approach with the MWCM, based on standard reference strength at $P_s=50\%$.

Remarkably, the fatigue life estimations derived from this approach are safe and exhibit an adequate level of conservatism across all loading conditions, with a P_{NC} of 0% and an average T_{RMS} of 40.1. This is particularly significant considering that the M-D_v value was originally calibrated using cruciform aluminium welded joints with plate

thicknesses ranging from 6 mm to 24 mm. Despite the assumption of zero weld root and toe radii, the level of conservatism remains non-excessive, and the accuracy of estimations is satisfactory. These results suggest that the TCD PM with MWCM is a practical and robust multiaxial fatigue assessment tool, providing a straightforward and effective method for evaluating hybrid aluminium-to-steel welded joints. Given that the same TCD PM procedure has already been established for designing aluminium welded joints, its direct applicability to hybrid joints further strengthens its suitability for engineering practice.

The next section provides a comprehensive comparison of all stress analysis approaches, synthesising the critical insights gained from the NS, ENS, ENS Avg, and TCD PM approaches. This discussion will focus on formalising reliable fatigue assessment methodologies, highlighting key trends in fatigue life estimations, and evaluating the practical implications of these findings for engineering applications.

7.8 Key Findings and Discussion

This section presents a comparative evaluation of the NS, ENS, ENS Avg, and TCD PM approaches, assessing their effectiveness in estimating the fatigue life of hybrid aluminium-to-steel welded joints. By synthesising insights from previous reanalyses, this discussion identifies key trends in fatigue life estimations, evaluates the performance of different multiaxial fatigue criteria, and considers the practical implications for fatigue assessment and structural design. To support this discussion, Table 7.2 detailing the P_{NC} , P_c , and T_{RMS} values for the top three multiaxial fatigue criteria under each stress analysis approach is provided at the end of this section.

The NS approach, despite its simplicity, has demonstrated strong reliability, particularly when used with standard stress-based criteria such as EC9 (MP) and IIW (Gough-Pollard). Originally developed to provide straightforward and practical solutions, these methodologies performed exceptionally well without requiring localised stress assessments (Anon, 2007; Hobbacher, 2016; Pedersen, 2016). However, MWCM under the NS approach was overly conservative for out-of-phase loading conditions, highlighting its sensitivity to multiaxiality when used without a

refined local stress evaluation. Additionally, SWT exhibited substantial scatter and non-conservatism, confirming its unsuitability for hybrid welded joints due to its incompatibility with semi-ductile or ductile fatigue behaviour. These findings validate that existing standard design methodologies, particularly IIW and EC9, can be confidently applied to hybrid aluminium-to-steel welded joints without requiring additional modifications.

The introduction of localised stress approaches such as the ENS and ENS Avg significantly improved fatigue life estimation accuracy by incorporating stress concentration effects at critical weld locations, such as the weld toe and root. Unlike the NS approach, which assumes nominal stress distributions, ENS-based methods provide a more precise representation of local stress states, leading to better predictions across various multiaxial fatigue criteria. Critical plane-based criteria, such as MWCM, performed notably better under ENS and ENS Avg than under NS, demonstrating how a more refined stress evaluation enables a more accurate quantification of multiaxiality at fatigue-critical locations. However, Findley's criterion struggled with pure torsional loading, as its calibration is based solely on uniaxial fatigue data, making it less reliable for shear-dominated conditions. A key improvement was observed when modifying Findley's formulation by replacing maximum normal stress with the normal stress range, which enhanced its applicability to welded joints. Meanwhile, the IIW criterion consistently remained the most reliable, reaffirming its robustness and accuracy across all stress analysis approaches.

While ENS Avg provides a slight accuracy improvement over the ENS approach through the incorporation of experimentally measured average notch radii, its practical feasibility is questionable. The process of sectioning welded joints to measure average notch radii is labour-intensive and time-consuming, and given the relatively minor improvements in fatigue life estimations, the added effort may not be justified in engineering practice. Since the conventional ENS approach already produces safe and satisfactory estimates, it remains the more viable choice for most practical fatigue assessments.

A consistent observation across all approaches is that pure torsional loading conditions introduce the highest degree of inaccuracy in fatigue life estimations. This stems from

the limitations of equivalent stress-based approaches, which prioritise normal stress components over shear stresses, leading to less conservative predictions for torsion-dominated cases. Findley's criterion was particularly problematic, producing non-conservative fatigue life estimations under pure torsional loading, while Sonsino KoNoS proved excessively conservative in such cases, emphasising its sensitivity to shear-driven fatigue failures. These findings highlight the importance of selecting appropriate criteria for different loading conditions, as some methodologies inherently favour normal stress-driven failures, while others better capture shear-dominated fatigue mechanisms.

An alternative method explored in this study is the TCD PM approach, which eliminates the need for predefined notch radii by assuming zero weld root and toe radii. This significantly simplifies FE modelling, while still producing realistic and sufficiently conservative fatigue life estimations. When applied alongside MWCM, TCD PM provided safe and accurate fatigue predictions, achieving a P_{NC} of 0% and an average T_{RMS} of 40.1. Given that M-D_v was originally calibrated using aluminium cruciform welded joints with thicker plate geometries, its successful application to hybrid aluminium-to-steel welded joints suggests that TCD PM with MWCM is a robust alternative (Livieri and Lazzarin, 2005; Susmel, 2009b, 2009a). This method offers engineers a practical solution that eliminates the need for explicit notch modelling, while still ensuring safe and reliable fatigue life assessments.

A particularly significant finding of this study is that out-of-phase loading does not significantly amplify fatigue damage in hybrid aluminium-to-steel welded joints. This explains why standard approaches such as IIW and EC9 performed exceptionally well, despite not explicitly accounting for non-proportional loading effects. In contrast, critical plane-based criteria were originally developed to capture fatigue damage mechanisms associated with non-proportional loading, which is often a concern in plain steel and other ductile materials (Susmel, 2008, 2009c; Al Zamzami and Susmel, 2018). However, for hybrid welded joints, the observed insensitivity to non-proportional loading effects means that simpler stress-based approaches remain highly effective, reducing the need for computationally intensive critical plane methods. This aligns with IIW recommendations, where aluminium welded joints were assigned a fatigue damage

sum ($D_{IIW}=1$), unlike steel welded joints, which are more sensitive to non-proportional loading effects (Hobbacher, 2016).

From a practical engineering perspective, standard reference fatigue strength values, while conservative, remain within an acceptable range. This level of conservatism is anticipated, as the same reference values are used for lower graded aluminium welded joints. Given that AA6082-T6 was used in this study, the observed conservatism in fatigue life estimations is well within expectations, confirming that the existing methodology ensures a sufficient safety margin for engineering applications.

Based on these findings, the most effective methodologies for the fatigue assessment of hybrid aluminium-to-steel welded joints are as follows. The IIW criterion remains the most reliable, particularly when applied with both NS and ENS-based methods. The MP (EC9) criterion also provides a strong alternative, particularly for engineers who prefer a simplified yet accurate fatigue design method under the NS approach. MWCM is a viable critical plane-based alternative, especially when paired with ENS, ENS Avg, or TCD PM, but it should not be used with the NS approach, particularly under out-of-phase loading conditions. For engineers seeking a time-efficient method that avoids explicit notch radius modelling, TCD PM with MWCM presents a strong alternative, as it enables fatigue assessment without requiring predefined notch radii.

Finally, mean stress effects were found to be negligible for these as-welded joints, as fatigue life estimations at $R=0.1$ were not excessively conservative. This reinforces the applicability of existing fatigue assessment methodologies without the need for additional mean stress corrections. These findings provide a strong foundation for developing reliable and practical fatigue design guidelines, ensuring that current engineering methodologies can be confidently applied without major modifications.

Table 7.2: Summary of P_{NC} , P_c , and T_{RMS} values from the reanalysis of hybrid joints, using various stress analysis approaches with different multiaxial fatigue criteria.

Stress Analysis Approach	Condition	Multiaxial Fatigue Criterion	Standard ⁽¹⁾		Experimental ⁽²⁾		$T_{RMS,Std}^{(5)}$	$T_{RMS,Exp}^{(6)}$
			P_{NC} (%) ⁽³⁾	P_c (%) ⁽⁴⁾	P_{NC} (%) ⁽³⁾	P_c (%) ⁽⁴⁾		
NS	Uniaxial	EC9	3.2	25.0	-	-	9.3	-
		IIW	0	50.0	-	-	31.8	-
		Findley	0	50.0	-	-	31.8	-
	Torsional	EC9	25.0	0	8.3	0	10.3	20.6
		IIW	0	0	0	0	7.7	6.0
		Findley	0	75.0	0	0	106.9	15.9
	In-Phase	EC9	0	41.7	0	0	30.6	12.9
		IIW	0	33.3	0	0	55.9	23.0
		Findley	0	88.9	0	0	279.8	32.9
	Out-Of-Phase	EC9	5.6	16.7	0	0	6.5	7.2
		IIW	0	33.3	0	0	54.8	11.4
		Findley	0	66.7	0	0	39.5	6.6
ENS	Uniaxial	IIW	0	25.0	-	-	103.1	-
		MP	0	33.3	-	-	103.0	-
		MWCM	0	41.7	-	-	59.5	-
	Torsional	IIW	16.7	2.5	-	-	28.8	-
		MP	16.7	0	-	-	28.9	-
		MWCM	16.7	0	-	-	29.0	-
	In-Phase	IIW	0	13.9	0	0	48.1	7.2
		MP	0	11.1	2.8	0	30.0	13.7
		MWCM	0	5.6	2.8	0	17.5	29.8
	Out-Of-Phase	IIW	0	13.9	0	0	66.3	7.7
		MP	0	5.6	0	0	26.0	18.7
		MWCM	0	2.8	5.6	0	17.6	41.3
ENS Avg	Uniaxial	IIW	0	0	-	-	6.4	-
		MP	0	0	-	-	6.4	-
		MWCM	5.6	6.4	-	-	6.8	-
	Torsional	IIW	0	0	-	-	9.0	-
		MP	0	0	-	-	9.0	-
		MWCM	0	0	-	-	9.1	-
	In-Phase	IIW	0	0	0	0	8.3	7.2
		MP	0	0	0	0	11.9	8.6
		MWCM	0	0	0	0	12.2	8.9
	Out-Of-Phase	IIW	0	0	0	0	8.4	7.6
		MP	5.6	0	2.8	0	28.4	19.0
		MWCM	5.6	0	2.8	0	26.9	18.0
TCD PM	Uniaxial	MWCM	0	33.3	-	-	48.8	-
	Torsional	MWCM	0	25.0	-	-	40.7	-
	In-Phase	MWCM	0	38.9	-	-	54.0	-
	Out-Of-Phase	MWCM	0	13.9	-	-	17.0	-

⁽¹⁾Reanalysis using constants derived from standard reference curves.⁽²⁾Reanalysis using constants derived from experimental reference curves.⁽³⁾ P_{NC} represents the percentage of non-conservative estimates.⁽⁴⁾ P_c represents the percentage of conservative estimates.⁽⁵⁾ $T_{RMS,Std}$ quantifies the error in fatigue life predictions when using constants from standard curves.⁽⁶⁾ $T_{RMS,Exp}$ quantifies the error in fatigue life predictions when using constants from experimental curves.

7.9 Conclusions

This chapter systematically evaluated the multiaxial fatigue behaviour of coldArc-welded hybrid aluminium-to-steel joints using four stress analysis approaches: NS, ENS, ENS Avg, and TCD PM. These approaches were assessed in conjunction with various multiaxial fatigue criteria to determine their accuracy in fatigue life estimations. The key findings from this study lead to the following conclusions:

- The NS approach, despite its simplicity, provided safe and reliable fatigue life estimations, particularly when used with IIW (Gough-Pollard) and EC9 (MP) criteria. These standard methodologies can be confidently extended to hybrid welded joints.
- The ENS and ENS Avg approaches significantly improved fatigue life predictions by capturing notch effects at critical weld locations. MWCM performed notably better under these approaches, demonstrating that a more refined local stress evaluation allows critical plane-based criteria to capture multiaxial fatigue behaviour more effectively.
- Although ENS Avg marginally improved accuracy, the effort required to measure average notch radii makes it impractical for routine engineering applications. The conventional ENS approach already provides safe and sufficiently accurate estimations.
- The TCD PM approach with MWCM eliminated the need for predefined notch radii, simplifying finite element modelling while maintaining conservative and reliable fatigue life predictions. This method is a practical alternative for engineers seeking a computationally efficient fatigue assessment approach.
- While standard fatigue strength values were found to be conservative, they remain within an acceptable range for practical applications. The same reference values are used for lower graded aluminium welded joints series, making the observed conservatism expected and reasonable for hybrid welded joints.

- The fatigue behaviour of as-welded hybrid joints was found to be largely insensitive to mean stress effects, as fatigue life estimations for $R=0.1$ conditions were not excessively conservative. This suggests that mean stress corrections are unnecessary for these welded joints in their as-welded condition.
- Among all examined approaches, IIW remains the most reliable criterion, followed by EC9 (MP) for simplified assessments. MWCM is a viable option when used with ENS, ENS Avg, or TCD PM, but should not be applied with NS for out-of-phase loading. For engineers seeking to bypass notch modelling, TCD PM with MWCM provides a strong alternative.

These conclusions establish a validated fatigue assessment framework for hybrid aluminium-to-steel welded joints, ensuring that existing design methodologies can be applied confidently. The next chapter will further contextualise these findings and discuss their broader implications for engineering applications and future research.

7.10 References

- Anon (2005) *Eurocode 3: Design of steel structures — Part 1-9: Fatigue* .
- Anon (2007) *Eurocode 9: Design of aluminium structures — Part 1-3: Structures susceptible to fatigue* .
- Atzori, B. (1983) ‘Trattamento termici e resistenza a fatica delle strutture soldate’.
- Bouche, K., Barbier, F. and Coulet, A. (1998) ‘Intermetallic compound layer growth between solid iron and molten aluminium’, *Materials Science and Engineering: A*, 249(1–2), pp. 167–175.
- Bruun, Ø. A. and Härkegård, G. (2015) ‘A comparative study of design code criteria for prediction of the fatigue limit under in-phase and out-of-phase tension–torsion cycles’, *International Journal of Fatigue*, 73, pp. 1–16.
- Chang, Y., Sun, C. and Qiu, Y. (2020) ‘Effective notch stress method for fatigue assessment of sheet alloy material and bi-material welded joints’, *Thin-Walled Structures*, 151, p. 106745.

- Findley, W. N. (1959) 'A theory for the effect of mean stress on fatigue of metals under combined torsion and axial load or bending', *Journal of Engineering for Industry*, 81(4), pp. 301–305.
- Gough, H. J. and Pollard, H. V (1935) 'The strength of metals under combined alternating stresses', *Proceedings of the institution of mechanical engineers*, 131(1), pp. 3–103.
- Hobbacher, A. (2016) *Recommendations for fatigue design of welded joints and components*. Springer.
- Hobbacher, A. F. (2009) 'The new IIW recommendations for fatigue assessment of welded joints and components - A comprehensive code recently updated', *International Journal of Fatigue*, 31(1), pp. 50–58. doi: 10.1016/j.ijfatigue.2008.04.002.
- Kueppers, M. and Sonsino, C. M. (2003) 'Critical plane approach for the assessment of the fatigue behaviour of welded aluminium under multiaxial loading', *Fatigue & Fracture of Engineering Materials & Structures*, 26(6), pp. 507–513.
- Kueppers, M. and Sonsino, C. M. (2006) 'Assessment of the fatigue behaviour of welded aluminium joints under multiaxial spectrum loading by a critical plane approach', *International Journal of Fatigue*, 28(5–6), pp. 540–546.
- Łagoda, T. *et al.* (2022) 'Using the Smith-Watson-Topper parameter and its modifications to calculate the fatigue life of metals: the state-of-the-art', *Materials*, 15(10), p. 3481.
- Livieri, P. and Lazzarin, P. (2005) 'Fatigue strength of steel and aluminium welded joints based on generalised stress intensity factors and local strain energy values', *International Journal of Fracture*, 133(3), pp. 247–276.
- Niemi, E. (1995) *Stress determination for fatigue analysis of welded components*. Woodhead Publishing.
- Pedersen, M. M. (2016) 'Multiaxial fatigue assessment of welded joints using the notch stress approach', *International Journal of Fatigue*, 83, pp. 269–279. doi: 10.1016/J.IJFATIGUE.2015.10.021.

Radaj, D., Lazzarin, P. and Berto, F. (2013) 'Generalised Neuber concept of fictitious notch rounding', *International Journal of Fatigue*, 51, pp. 105–115. doi: 10.1016/j.ijfatigue.2013.01.005.

Radaj, D., Sonsino, C. M. and Fricke, W. (2006) *Fatigue assessment of welded joints by local approaches*. Woodhead publishing.

Schijve, J. (2012) 'Fatigue predictions of welded joints and the effective notch stress concept', *International Journal of Fatigue*, 45, pp. 31–38.

Smith, Kn. (1970) 'A stress-strain function for the fatigue of metals', *Journal of materials*, 5, pp. 767–778.

Sonsino, C. M. (1995) 'Multiaxial fatigue of welded joints under in-phase and out-of-phase local strains and stresses', *International Journal of Fatigue*, 17(1), pp. 55–70.

Sonsino, C. M. (2007) 'Course of SN-curves especially in the high-cycle fatigue regime with regard to component design and safety', *International Journal of Fatigue*, 29(12), pp. 2246–2258.

Sonsino, C. M. (2009) 'A consideration of allowable equivalent stresses for fatigue design of welded joints according to the notch stress concept with the reference radii $r_{ref}=1.00$ and 0.05 mm', *Welding in the World*, 53, pp. R64–R75.

Sonsino, C M *et al.* (2012) 'Notch stress concepts for the fatigue assessment of welded joints—Background and applications', *International Journal of Fatigue*, 34(1), pp. 2–16.

Sonsino, C. M., Bruder, T. and Baumgartner, J. (2010) 'SN lines for welded thin joints—suggested slopes and FAT values for applying the notch stress concept with various reference radii', *Welding in the World*, 54(11), pp. R375–R392.

Susmel, L. (2007) 'Eurocode 3's standard curves and theory of critical distances to estimate fatigue lifetime of steel weldments', *Key Engineering Materials*, 348, pp. 21–24.

- Susmel, L. (2008) 'Modified Wöhler curve method, theory of critical distances and Eurocode 3: A novel engineering procedure to predict the lifetime of steel welded joints subjected to both uniaxial and multiaxial fatigue loading', *International Journal of Fatigue*, 30(5), pp. 888–907. doi: 10.1016/j.ijfatigue.2007.06.005.
- Susmel, L. (2009a) *Multiaxial notch fatigue*. Elsevier.
- Susmel, L. (2009b) 'The Modified Wöhler Curve Method calibrated by using standard fatigue curves and applied in conjunction with the Theory of Critical Distances to estimate fatigue lifetime of aluminium weldments', *International Journal of Fatigue*, 31(1), pp. 197–212. doi: 10.1016/j.ijfatigue.2008.04.004.
- Susmel, L. (2009c) 'Three different ways of using the Modified Wöhler Curve Method to perform the multiaxial fatigue assessment of steel and aluminium welded joints', *Engineering Failure Analysis*, 16(4), pp. 1074–1089. doi: 10.1016/j.engfailanal.2008.05.016.
- Susmel, L. (2014) 'Four stress analysis strategies to use the Modified Wöhler Curve Method to perform the fatigue assessment of weldments subjected to constant and variable amplitude multiaxial fatigue loading', *International Journal of Fatigue*, 67, pp. 38–54. doi: 10.1016/j.ijfatigue.2013.12.001.
- Susmel, L. and Petrone, N. (2003) 'Multiaxial fatigue life estimations for 6082-T6 cylindrical specimens under in-phase and out-of-phase biaxial loadings', in *European Structural Integrity Society*. Elsevier, pp. 83–104.
- Susmel, L., Sonsino, C. M. and Tovo, R. (2011) 'Accuracy of the modified wöhler curve method applied along with the $r_{ref} = 1$ mm concept in estimating lifetime of welded joints subjected to multiaxial fatigue loading', *International Journal of Fatigue*, 33(8), pp. 1075–1091. doi: 10.1016/j.ijfatigue.2010.09.010.
- Susmel, L. and Taylor, D. (2012) 'A critical distance/plane method to estimate finite life of notched components under variable amplitude uniaxial/multiaxial fatigue loading', *International Journal of Fatigue*, 38, pp. 7–24. doi: <https://doi.org/10.1016/j.ijfatigue.2011.11.015>.

Susmel, L. and Tovo, R. (2004) ‘On the use of nominal stresses to predict the fatigue strength of welded joints under biaxial cyclic loading’, *Fatigue & Fracture of Engineering Materials & Structures*, 27(11), pp. 1005–1024.

Taylor, D., Barrett, N. and Lucano, G. (2002) ‘Some new methods for predicting fatigue in welded joints’, *International Journal of Fatigue*, 24(5), pp. 509–518.

Al Zamzami, I. *et al.* (2018) ‘Static strength and design of aluminium-to-steel thin welded joints’, *Welding in the World*, 62(6), pp. 1255–1272. doi: 10.1007/s40194-018-0634-2.

Al Zamzami, I., Davison, B. and Susmel, L. (2019) ‘Nominal and local stress quantities to design aluminium-to-steel thin welded joints against fatigue’, *International Journal of Fatigue*, 123, pp. 279–295.

Al Zamzami, I. and Susmel, L. (2018) ‘On the use of hot-spot stresses, effective notch stresses and the Point Method to estimate lifetime of inclined welds subjected to uniaxial fatigue loading’, *International Journal of Fatigue*, 117, pp. 432–449. doi: 10.1016/j.ijfatigue.2018.08.032.

Chapter 8

Conclusions and Future Work

8.1 Conclusions

8.1.1 Overview of Research Aim, Objectives and Significance

The increasing demand for sustainable and lightweight structural solutions has driven rising interest in hybrid aluminium-to-steel welded joints, particularly in automotive, bridges, and offshore structures, where fatigue resistance is a critical design consideration. Advancements in welding technologies have enabled the successful joining of dissimilar materials, presenting new opportunities for optimised material efficiency by combining the high strength of steel with the lightweight and recyclable properties of aluminium. This integration allows for more efficient material distribution, where aluminium is used for non-load-bearing components, while steel provides superior load-bearing capacity.

Despite these advancements, hybrid aluminium-to-steel welded joints remain largely unexplored in fatigue design. Existing fatigue assessment methodologies have been developed independently for aluminium and steel welded joints, yet no formalised framework exists to predict their multiaxial fatigue behaviour when combined. This knowledge gap poses a significant challenge for engineers, as conventional fatigue models may not accurately capture the complex stress interactions in hybrid joints. To fully utilise their potential, a validated multiaxial fatigue assessment framework is essential to ensure reliable fatigue life estimations and enable safe and efficient implementation in engineering applications.

This PhD research directly addresses this critical gap by developing a robust and evidence-based multiaxial fatigue assessment methodology tailored for thin coldArc®-

welded aluminium-to-steel joints. Through rigorous validation, the study provides a practical and reliable foundation for engineering design, ensuring that these hybrid joints can be assessed with confidence under multiaxial fatigue loading.

To achieve this aim, the research followed a structured and progressive approach, systematically increasing in complexity to formalise a validated multiaxial fatigue assessment framework. The key objectives were:

- To critically review and quantitatively assess existing multiaxial fatigue criteria using different stress analysis approaches for steel and aluminium welded joints under constant amplitude (CA) and variable amplitude (VA) loading.
- To experimentally investigate and characterise the fatigue behaviour of hybrid aluminium-to-steel welded joints under uniaxial, pure torsional, in-phase, and out-of-phase CA fatigue loading.
- To perform a detailed failure analysis, identifying critical fatigue initiation sites and gaining deeper insight into the governing failure mechanisms.
- To validate and establish a reliable multiaxial fatigue assessment framework by comparing different stress analysis methods and multiaxial fatigue criteria, determining the most suitable approach for accurate fatigue life predictions in hybrid welded joints.

Through this systematic investigation, the study successfully fulfilled these objectives and developed a scientifically validated multiaxial fatigue assessment methodology that bridges the existing knowledge gap. The findings contribute to both academic research and engineering practice, ensuring that thin hybrid aluminium-to-steel welded joints can be designed with confidence for real-world structural applications.

8.1.2 Key Findings and Contributions

This research establishes a scientifically validated multiaxial fatigue assessment framework for thin coldArc®-welded aluminium-to-steel joints, addressing a critical gap in fatigue design methodologies. A systematic approach was taken, progressing

from theoretical evaluation to experimental validation before formalising a reliable fatigue assessment methodology for hybrid welded joints.

Before developing a dedicated assessment framework, it was first necessary to examine how aluminium and steel welded joints behave independently under multiaxial fatigue conditions. A comprehensive review was conducted, assessing both standard design methodologies, including Eurocodes (EC3, EC9) and IIW guidelines, and stress-based critical plane criteria, namely Findley, Carpinteri-Spagnoli (CS), and the Modified Wöhler Curve Method (MWCM). These criteria were reanalysed using nominal stress (NS), hot-spot stress (HSS), effective notch stress (ENS), and the Theory of Critical Distances-Point Method (TCD PM) with MWCM under CA and VA loading. The key findings from this quantitative assessment revealed that:

- Standard fatigue design approaches generally provide safe but conservative fatigue life estimations, particularly for aluminium welded joints, which exhibited greater scatter than steel due to the limited availability of multiaxial fatigue data.
- IIW-HSS was the most conservative and safest approach for steel welded joints, whereas IIW-ENS was better suited for aluminium welded joints.
- MWCM showed the highest predictive accuracy under VA loading, while Findley's criterion was more conservative and scattered.
- TCD PM combined with MWCM offered high conservatism, but refinements may be required for thin or flexible joints ($t < 5$ mm).

To characterise the fatigue behaviour and establish a reliable dataset for validation, experimental fatigue tests were conducted on coldArc®-welded hybrid aluminium-to-steel joints. A detailed failure analysis was performed to identify crack initiation sites, failure locations, and the governing fatigue mechanisms, leading to the following key observations:

- Fatigue cracks consistently initiated from the aluminium side, confirming aluminium as the weaker link in these hybrid joints.

- The four dominant crack initiation sites were the aluminium weld toe/root region (most common failure location), weld start/stop position, the weld itself, and the aluminium heat-affected zone (HAZ).
- Crack initiation was primarily driven by shear stresses, while crack propagation followed normal stress directions, aligning with theoretical predictions.

Building on these insights, a comprehensive validation of multiaxial fatigue assessment methods was conducted, evaluating IIW (Gough-Pollard), EC9 (Maximum Principal), Von Mises, SWT, and Sonsino KoNoS, alongside Findley, CS, MWCM, and various stress analysis approaches (NS, ENS, ENS Avg, TCD PM). The key conclusions confirmed that:

- The most effective multiaxial fatigue assessment methodology for hybrid aluminium-to-steel welded joints is IIW (Gough-Pollard) with NS and ENS approaches, ensuring general conservatism and applicability. The Maximum Principal (MP) criterion also proved suitable for direct application to aluminium and hybrid welded joints.
- The IIW and MP criteria, originally developed for aluminium welded joints, were found to be applicable to hybrid joints.
- MWCM is a viable alternative when used with ENS, ENS Avg, or TCD PM, but should not be applied with NS for out-of-phase loading.
- For engineers seeking to avoid notch modelling, TCD PM with MWCM provides a strong alternative.
- Existing fatigue models for aluminium welded joints can be successfully extended to hybrid joints, ensuring accurate fatigue life estimations.

This study provides a scientifically validated multiaxial fatigue assessment framework, ensuring that hybrid aluminium-to-steel welded joints can be designed with confidence for real-world engineering applications. By systematically assessing both standard and advanced fatigue life estimation techniques, this research bridges the gap between experimental findings and fatigue design methodologies, offering:

- Improved fatigue life estimations accuracy, enabling safer and more efficient engineering design.
- A validated methodology for engineers, ensuring that hybrid aluminium-to-steel welded joints can be assessed with confidence in structural applications.
- Practical recommendations for industry, guiding the selection of appropriate fatigue assessment criteria based on loading conditions and stress analysis approaches.

These findings fill a critical gap in multiaxial fatigue design, providing both academic and practical contributions to ensure the safe and reliable implementation of hybrid aluminium-to-steel welded joints in structural applications where fatigue resistance is a primary concern.

8.2 Suggestions for Future Work

This research has established a scientifically validated multiaxial fatigue assessment framework for hybrid aluminium-to-steel welded joints, yet further investigation is required to refine fatigue life predictions, enhance applicability, and facilitate industrial implementation. Future studies should focus on extending the assessment methodologies developed in this study, addressing gaps related to VA loading, thickness effects, residual stress influence, aluminium grade variations, and alternative welding techniques to optimise hybrid joints for real-world applications.

One of the most critical areas for further research is the effect of VA loading, as real-world structures rarely experience simple CA loading. To improve the accuracy of fatigue life predictions, future studies should:

- Evaluate the fatigue response of hybrid joints under complex VA loading conditions, ensuring predictions reflect realistic service conditions.
- Determine the allowable damage sum for hybrid joints, as existing values for steel and aluminium welded joints vary significantly (0.01 to 10), and their applicability to hybrid joints remains uncertain.

Another key aspect requiring further investigation is the influence of joint thickness on multiaxial fatigue behaviour. This study primarily focused on thin-walled hybrid welded joints, but it remains unclear whether fatigue performance varies with thickness. Further research should:

- Assess whether fatigue performance and stress distribution change with increasing joint thickness, particularly for joints thicker than 5 mm.
- Determine whether existing fatigue assessment methodologies remain valid for thicker hybrid joints or require modifications to account for thickness-related stress variations.

The influence of residual stresses and stress-relieving treatments on fatigue performance also warrants further study. While this research focused on as-welded conditions, post-weld treatments could enhance fatigue life. Future work should:

- Evaluate the effect of stress-relieving treatments on fatigue life and its resistance.
- Assess whether existing enhancement factors for aluminium and steel joints remain applicable to hybrid welded joints or require adjustments.
- Investigate the residual stress distribution in as-welded hybrid joints using non-destructive techniques such as X-ray diffraction to quantify their influence on local stress states and fatigue performance.
- Integrate experimentally measured residual stresses into fatigue life models to enhance the accuracy and reliability of fatigue assessments in hybrid aluminium-to steel welded structures.

Further microscopic investigations using scanning electron microscopy (SEM) would provide valuable insights into fatigue behaviour at the microstructural level. Future studies should:

- Examine crack initiation and propagation mechanisms at the grain level, improving the understanding of local stress distribution and material interactions.

- Investigate the role of ductility and phase interactions at the aluminium-steel interface, providing deeper understanding of microstructural influences on fatigue behaviour.

The effect of aluminium grade variations on fatigue performance should also be explored, as different alloy compositions and microstructures could significantly influence fatigue resistance. Research should:

- Compare the fatigue performance of different aluminium grades to determine how mechanical properties, microstructure, and heat treatment history affect fatigue life.
- Assess whether existing fatigue assessment methodologies can be generalised across different aluminium grades or require modifications.

Additionally, further investigation is needed to determine whether the fatigue assessment methodologies developed for coldArc® welding are applicable to other hybrid welding processes. Future research should:

- Validate whether friction stir welding (FSW), laser arc hybrid welding, or other advanced welding techniques produce comparable fatigue performance and joint integrity.
- Assess whether different welding processes alter the multiaxial fatigue behaviour of hybrid joints, potentially requiring adjustments in fatigue life estimation approaches.

The industrial implementation of hybrid aluminium-to-steel welded joints remains a crucial area for further exploration. Future research should focus on:

- Applying the validated fatigue assessment methodology to full-scale structures, such as bridges, offshore platforms, and automotive components, where fatigue resistance is critical.
- Optimising hybrid joints for structural lightweighting, ensuring aluminium is used for non-load-bearing sections while steel carries primary loads, enhancing both efficiency and sustainability.

By addressing these research gaps, future work can further unlock the potential of hybrid aluminium-to-steel welded joints, leading to more durable, lightweight, and sustainable structural solutions across a wide range of industries and applications.

Appendix A: Experimental Fatigue Test Results

Code	$\Delta\sigma_{nom}$ [MPa]	$\Delta\tau_{nom}$ [MPa]	$R_\sigma^{(1)}$ [-]	$R_\tau^{(1)}$ [-]	$\delta^{(2)}$ [°]	N_f [Cycles]	FM ⁽³⁾
U_T_1	112.4	0	-1	-	0	142470	WT/WR
U_T_2	116.2	0	-1	-	0	102215	WT/WR
U_T_3	91.8	0	-1	-	0	1684898	WT/WR
U_T_4	85.4	0	-1	-	0	1305955	WS
U_T_5	143.5	0	-1	-	0	181007	WS
U_T_6	138.2	0	-1	-	0	48990	WS
U_T_7	218.3	0	-1	-	0	10892	WT/WR
U_T_8	229.9	0	-1	-	0	17562	WT/WR
U_T_9	101.2	0	0.1	-	0	95979	WT/WR
U_T_10	91.2	0	0.1	-	0	334167	WT/WR
U_T_11	144.5	0	0.1	-	0	20532	WT/WR
U_T_12	146.2	0	0.1	-	0	36170	WT/WR
P_To_1	0.0	173.7	-	-1	0	4912	W
P_To_2	0.0	181.5	-	-1	0	2851	WT/WR
P_To_3	0.0	106.8	-	-1	0	164727	WS
P_To_4	0.0	104.9	-	-1	0	76790	WT/WR
P_To_5	0.0	75.8	-	-1	0	686004	WS
P_To_6	0.0	76.6	-	-1	0	681711	WS
P_To_7	0.0	132.6	-	-1	0	73458	WS
P_To_8	0.0	127.9	-	-1	0	95737	WS
P_To_9	0.0	81.2	-	0.1	0	284460	WS
P_To_10	0.0	70.7	-	0.1	0	321082	WT/WR
P_To_11	0.0	103.8	-	0.1	0	109045	W
P_To_12	0.0	111.0	-	0.1	0	11122	WS
C_I_U_To_1	68.9	68.9	-1	-1	0	375424	WS
C_I_U_To_2	98.4	98.4	-1	-1	0	26492	WS
C_I_U_To_3	100.4	100.4	-1	-1	0	132324	WT/WR
C_I_U_To_4	64.6	64.6	-1	-1	0	976928	W
C_I_U_To_5	63.2	63.2	-1	-1	0	997868	WT/WR
C_I_U_To_6	104.6	104.6	-1	-1	0	79265	WT/WR
C_I_U_To_6_1	45.4	45.4	-1	-1	0	2000000*	-

C_I_U_To_7	92.4	92.4	0.1	-1	0	43453	WT/WR
C_I_U_To_8	68.2	68.2	0.1	-1	0	425460	WT/WR
C_I_U_To_9	88.5	88.5	0.1	-1	0	41265	W
C_I_U_To_10	66.8	66.8	0.1	-1	0	250461	WT/WR
C_I_U_To_11	69.8	69.8	0.1	0.1	0	217322	WT/WR
C_I_U_To_12	77.6	77.6	0.1	0.1	0	146225	WT/WR
C_I_U_To_13	66.8	66.8	0.1	0.1	0	309597	WT/WR
C_I_U_To_14	83.0	83.0	0.1	0.1	0	96381	WT/WR
C_I_U_To_15	66.0	66.0	-1	0.1	0	366617	WT/WR
C_I_U_To_16	80.6	80.6	-1	0.1	0	138061	WT/WR
C_I_U_To_17	63.6	63.6	-1	0.1	0	379907	WS
C_I_U_To_18	91.5	91.5	-1	0.1	0	141600	WT/WR
C_I_U_To_19	74.1	42.8	-1	-1	0	1263962	WS
C_I_U_To_20	96.1	55.5	-1	-1	0	507067	WS
C_I_U_To_21	136.1	78.6	-1	-1	0	162690	WT/WR
C_I_U_To_22	140.8	81.3	-1	-1	0	181957	WS
C_I_U_To_23	173.3	100.1	-1	-1	0	27196	WS
C_I_U_To_24	167.9	96.9	-1	-1	0	31616	WS
C_I_U_To_25	86.0	49.7	-1	-1	0	484321	WT/WR
C_I_U_To_26	103.1	59.5	0.1	-1	0	151449	WT/WR
C_I_U_To_27	83.9	48.4	0.1	-1	0	1401080	WT/WR
C_I_U_To_28	102.2	59.0	0.1	-1	0	162502	WS
C_I_U_To_29	90.7	52.4	0.1	-1	0	156933	WT/WR
C_I_U_To_30	83.1	48.0	0.1	0.1	0	108138	AL
C_I_U_To_31	81.0	46.7	0.1	0.1	0	115647	W
C_I_U_To_32	89.3	51.6	0.1	0.1	0	101125	WT/WR
C_I_U_To_33	104.5	60.3	0.1	0.1	0	739803	W
C_I_U_To_34	128.3	74.1	-1	0.1	0	234227	WS
C_I_U_To_35	99.7	57.6	-1	0.1	0	347373	WT/WR
C_I_U_To_36	126.6	73.1	-1	0.1	0	114846	WS
C_O_U_To_1	97.3	97.3	-1	-1	90	245331	WT/WR
C_O_U_To_2	114.1	114.1	-1	-1	90	27707	WT/WR
C_O_U_To_3	111.1	111.1	-1	-1	90	52981	WT/WR
C_O_U_To_4	90.6	90.6	-1	-1	90	68432	WS
C_O_U_To_5	90.4	90.4	-1	-1	90	164273	WT/WR
C_O_U_To_6	100.5	100.5	-1	-1	90	96411	WT/WR
C_O_U_To_7	101.2	101.2	0.1	-1	90	12867	WT/WR
C_O_U_To_8	82.0	82.0	0.1	-1	90	104377	AL

C_O_U_To_9	100.0	100.0	0.1	-1	90	18207	WT/WR
C_O_U_To_10	78.2	78.2	0.1	-1	90	196031	WT/WR
C_O_U_To_11	86.2	86.2	0.1	0.1	90	163647	WT/WR
C_O_U_To_12	93.0	93.0	0.1	0.1	90	81558	W
C_O_U_To_13	88.2	88.2	0.1	0.1	90	53776	WT/WR
C_O_U_To_14	77.3	77.3	0.1	0.1	90	313637	WT/WR
C_O_U_To_15	91.4	91.4	-1	0.1	90	154696	W
C_O_U_To_16	92.3	92.3	-1	0.1	90	148405	W
C_O_U_To_17	77.2	77.2	-1	0.1	90	672474	WT/WR
C_O_U_To_18	84.1	84.1	-1	0.1	90	234437	WT/WR
C_O_U_To_19	108.3	62.5	-1	-1	90	430385	WS
C_O_U_To_20	100.2	57.9	-1	-1	90	375500	WT/WR
C_O_U_To_21	165.1	95.3	-1	-1	90	114271	WT/WR
C_O_U_To_22	169.7	98.0	-1	-1	90	22815	W
C_O_U_To_23	173.0	99.9	-1	-1	90	20911	WT/WR
C_O_U_To_24	156.0	90.1	-1	-1	90	54050	WT/WR
C_O_U_To_25	100.5	58.0	-1	-1	90	93017	WS
C_O_U_To_26	85.7	49.5	0.1	-1	90	368770	WS
C_O_U_To_27	82.7	47.7	0.1	-1	90	183014	WS
C_O_U_To_28	100.8	58.2	0.1	-1	90	73891	WS
C_O_U_To_29	114.0	65.8	0.1	-1	90	57091	WT/WR
C_O_U_To_30	109.6	63.3	0.1	0.1	90	125012	W
C_O_U_To_31	98.5	56.9	0.1	0.1	90	539467	WT/WR
C_O_U_To_32	102.5	59.2	0.1	0.1	90	299370	WT/WR
C_O_U_To_33	138.7	80.1	0.1	0.1	90	185333	W
C_O_U_To_34	137.7	79.5	-1	0.1	90	52199	W
C_O_U_To_35	109.9	63.4	-1	0.1	90	390447	W
C_O_U_To_36	111.7	64.5	-1	0.1	90	246360	W

(1) R_{σ} and R_{τ} denote the stress ratio for normal and shear stress, respectively.

(2) δ represents out-of-phase loading angle.

(3) FM indicates the failure mode, where cracks initiate from: WT/WR - Weld Toe/Root, WS – Weld Start/Stop, W - Weld itself, and AL – Aluminium Heat Affected Zone (HAZ).

* Run Out